FUND OF HEDGE FUNDS ALLOCATION STRATEGIES WITH NON-NORMAL RETURN DISTRIBUTIONS

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PROJECT SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE IN FINANCE

In the Master of Science in Finance Program
of the
Faculty
of
Business Administration

SIMON FRASER UNIVERSITY

Fall 2015

Approval

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Degree: **Master of Science in Finance**

Title of Project: **Fund of hedge funds allocation strategies with non-normal return distributions**

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Abstract

In this paper the authors develop allocation methodologies for creating a portfolio of hedge funds. Investments in hedge funds are often used in a broader portfolio to gain exposure to the diversification and low correlation benefits of hedge fund returns. The methodologies presented allocate across multiple hedge fund strategies in a risk-controlled manner that take into account hedge fund return distributions that are asymmetric and non-normal. These alternative methodologies provide a systematic way to account for non-normality in hedge fund return distributions that the traditional mean-variance optimization framework is unable to account for. Additionally, unstable correlations among hedge fund strategies as well as serially correlated returns in some hedge fund strategies pose a problem to the allocation process. As such, in this paper we use and suggest methods for how to overcome these problems.

Keywords: Hedge Funds; Optimizations; Allocations; Fund of Hedge Funds; Investments; Portfolio Construction;

Acknowledgements

We would like to thank Professor Peter Klein for his guidance with our research. We would also like to acknowledge and thank our parents for their support of our undergraduate and graduate academic endeavours.

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1: Introduction

In the past number of years, hedge funds have increasingly been seen as an attractive investment vehicle for investors and funds looking for diversification and the low correlation benefits that hedge funds provide. One option for investors looking to allocate to hedge funds is for the investor to decide on allocation and invest directly into the funds by conducting due diligence on the managers and funds themselves. However, the more attractive option is to invest into a fund of funds where the fund of funds manager conducts the due diligence on the managers of each strategy and the fund of funds manager allocates across various hedge fund strategies. As allocating to hedge funds grows in popularity especially with institutional investors, the issue of allocation becomes ever important within a fund of funds structure relative to the issue of specific manager due diligence and selection.

The Hedge Fund Research Institute (2015) in their quarterly industry report estimated worldwide hedge fund capital deployed to all strategies to be at $2.87 trillion in 2015. Growth in the industry has given rise to distinct strategies, from the more conservative Equity Market Neutral strategy to the more aggressive Global Macro strategy. It is these strategies that investors in multiple hedge funds or managers of fund of hedge funds allocate to in a top down approach in a combination of qualitative assessment of managers and quantitative allocation based on risk and return appetite. It is precisely the exercise of quantitative allocation that our study proposes different solutions to by providing a methodology to create customized optimal portfolios of hedge funds with targeted risk and return profiles. The methodologies presented are applicable for an investor allocating to different hedge funds in their own portfolio of hedge funds or for a fund of funds manager allocating to hedge funds within the structure of a fund of funds.

Although funds of hedge funds have been around for some time, there is no consensus for the optimal systematic methodology for allocating to different strategies (Davies et al, 2009). The main issues related to why this is the case is have to do with the statistical properties of hedge fund returns and how these properties make it difficult to incorporate hedge funds into a traditional allocation optimization. As well, issues related to the data integrity of the benchmark's data plays a part in the difficulty of constructing optimal hedge fund allocations. DeSouza and Gokan (2004) summarize the main issues as the following:

- Hedge fund strategy index data may not be reliable and contains issues of survivorship bias, fund self-reporting of data, and the assumption that it possible to definitively discriminate between strategies.
- Many hedge fund strategies have significantly non-normal return distributions and can have large skewness and kurtosis.
- The stability of correlations between strategies are often non-constant and vary over time. Constant correlations between groups is an important assumption within optimal allocation frameworks.
- Many hedge fund strategies contain illiquid instruments and suffer from overly "smooth" returns and serial correlation between returns.

Expanding upon these issues, Brooks and Kat (2002) summarize the main issues that investors face when evaluating hedge fund performance and return data. Not only is there the issue of non-normal returns and autocorrelation or "smoothness" in returns, but different hedge fund strategies also exhibit higher correlation to each other than what would have been thought given the high differentiation of the strategies. This suggests many hedge fund strategies are exposed to many of the same risk factors. As well, Brooks and Kat (2002) found that due to the smoothness of returns, the Sharpe Ratios and diversification benefits of hedge funds in a portfolio may be overstated and this needs to be taken into account during the optimization process.

DeSouza and Gokcan (2004) set out to solve some of these problems in the hedge fund of funds allocation process firstly grouping similar strategies together so as to maximize correlation between strategies within the groups and to minimize correlation between the distinct groups. They have named these groups "Rational Strategy Groups", or RSG's. This process was done to minimize the problems of the instability of correlations between strategies as the correlations and covariances are a crucial input in the optimization process. As well, to address the problem of smoothness of returns they applied an "unsmoothing" technique common in evaluating real estate returns which also often exhibit smoothness and autocorrelation in returns. After these steps were taken, instead of optimizing allocation between the groups of strategies in a mean variance framework, they optimized allocation between the strategy groups by minimizing CVaR, which is a measure of "fat tail" risk common to returns with non-normal distributions, and maximizing return.

The goal of our research here is to expand upon the work and methodology of DeSouza and Gokcan (2004) firstly by replicating their research on the most recent data and on different indices. We chose to do this since Brooks and Kat (2002) bring up the issue that different indices which supposedly track the same hedge fund strategies in reality often perform quite differently. Due to this, we used a different set of indices on some of the same strategies to confirm their results are consistent across different hedge fund indices. In addition, we implemented a new method of optimizing allocation between these RSG's by minimizing the semi-variance (or downside variance) and maximizing return. The purpose of this exercise is to investigate whether returns below the average return in hedge fund returns should be a concern in the allocation process in a similar way to how optimizing around "fat tail" risk in previous research yields a different set of optimal portfolios.

2: Literature Review

In order to understand how investors and managers are using hedge funds in a broader allocation within a portfolio as well as in a fund of funds structure, we first examined the research done that analysed hedge fund performance and industry trends as a whole. In their work, Ammann and Moerth (2008) examined the performance of fund of hedge funds and whether they can offer significant outperformance. Their findings include the fact that there is a lower survivorship bias for larger fund of hedge funds compared to smaller fund of funds. They also found that fund of funds' performance is in line with their extra fees on top of the individual strategy's management fees. This is significant and indicates that a fund of funds structure can add value due to a manager's ability to select managers and allocate appropriately.

Reddy, Brady and Patel (2007) noted that there has been an increasing number of institutional investors allocating to hedge funds to gain exposure to potentially higher risk adjusted returns compared to traditional strategies. In their research they found that a fund of funds structure can offer benefits that a single fund that contains many strategies cannot offer. One of these benefits is that fund of funds managers are able to conduct proper due diligence on managers, whereas in investing into a single multi strategy fund the due diligence must be done by the individual investor and exposes them to higher idiosyncratic risk in management selection. The authors also mention that the potential impact of manager selection is potentially greater than the allocation process, and the fund of funds managers that are able to allocate to the best managers more than make up for their additional layer of fees compared to the performance of single multi-strategy funds with slightly lower fees. As well, the difference in fees between a fund of funds structure and a multi-strategy hedge fund is much less than most investors think especially when the better returns of a fund of funds are taken into account. Dai and Shawky (2012) provide similar insights in their research. Their findings show that fund of funds provide investors with diversification across hedge fund strategies, professional oversight of fund managers, and necessary due diligence into fund managers. They then compare the performance of fund of funds that allocate to managers within a single strategy with fund of funds that allocate to multiple strategies. They found that fund of funds that allocated to diversified strategies performed better than those fund of funds that allocated to managers of the same strategy. This points to the significance and importance of the allocation process across strategies and shows that value can be added by using a systematic allocation methodology in addition to conducting qualitative research into selecting specific fund managers.

Previous research done into specific ways in which to do optimal allocation have yielded various methodologies. The common thread between them is the uniform finding of hedge fund returns being non-normally distributed and having auto-correlated returns in some strategies, and then subsequently how to account for these problems in the allocation methodology. Saunders, Seco, Vogt, and Zagst (2013) proposed a methodology that relies upon a regime switching model, which in essence allows the parameters in the optimization model be stochastic, or time varying. They describe market conditions, or properties of the returns of different hedge fund strategies, as being a Markov chain which is essentially a memoryless state changing random process. They note that "regime switching models allow for periods in which different market environments prevail by time varying stochastic model parameters." They modelled a class of Markov switching models for the different parameters which they used to capture volatility clustering into their optimization. Their use of Markov switching models essentially separates current market conditions into two states: normal or stressed. These market states have different risk and return characteristics and the Markov model has the ability to indicate the probability of an impending crisis. In the different market situations, different expected return and covariance matrices are used. The result is that the covariance matrix and expected returns in the optimization could be time varying, and they showed how to do this in a case example by applying the well-known Black-Litterman style mean-variance optimization. They showed that using time varying parameters in the optimization would have outperformed a strategy with constant parameters. However, their strategy has some drawbacks, as the portfolio of hedge funds was rebalanced every two weeks, and this is not a feasible liquidity requirement for most hedge funds. As well, although the time varying parameters capture tumultuous markets, they don't explicitly take into account higher moments of the return distribution in the optimization process.

Another study done by Popova, Morton, Popova and Yau (2007) set out to optimize a hedge fund allocation within a broader portfolio with other asset classes while accounting for higher moments of the distributions of hedge fund returns. They found when they allocated to hedge funds within the broad portfolio using traditional asset allocation the optimal portfolios overly allocated to hedge funds due to their higher Sharpe Ratios. To solve this problem, they accounted for higher moments in the return distributions as well as semi-variance in the optimization process. Specifically, they used methods from stochastic programming to minimize risk as measured by the second moment (standard deviation) and minimize kurtosis while maximizing return and skewness. This exercise is important since they found many of the return distributions of hedge funds had significant negative skew and positive kurtosis. The result of their work showed that within a broader portfolio, allocating to hedge funds is still advantageous in a benchmark based portfolio even when accounting for higher moments of return distributions.

Anson, Ho and Silberstein (2007) take a similar approach to Popova et al, except they apply multi-goal optimization to constructing a fund of hedge funds rather than in an entire portfolio with multiple asset classes. Additionally, they give insight into how a portfolio actually uses this allocation process in a practical way when allocating to a hedge fund of funds as two of the authors are portfolio managers at the California Public Employee Retirement System (CalPERS). They break down the hedge fund portfolio creation process into the following steps:

1. Strategy return distribution profile assessment
2. Hedge fund universe screening
3. Portfolio optimization
4. Monthly performance monitoring and risk reporting

Each of these steps are critical to creating a robust hedge fund of funds within the entire portfolio. The first step is common to many of the studies reviewed and is useful later in the optimization process. The step of screening the hedge fund universe includes a mix of quantitative and qualitative due diligence on potential investable hedge funds. Their portfolio optimization procedure is what is most applicable to our study. Their optimization procedure revolves around a multi-goal polynomial goal programming optimization, which is similar to the approach taken by Popova et al, except this is done for a portfolio of hedge funds rather than an entire portfolio. The inputs in their optimization include expected return, variances-covariance matrix, skewness-coskewness matrix and kurtosis-cokurtosis matrix. The polynomial goal programming then maximizes expected return and skewness and minimizes volatility and kurtosis, although this process can result in conflicting objectives where there is not a single optimal solution in each instance. In their results, they compare these constructed portfolios to a mean-variance optimal portfolio and found that their optimal portfolio had higher cumulative performance, less negative skewness, and less volatility than the mean-variance optimal portfolio, however their portfolio did not have a large decrease in outlying tail events when compared to the mean-variance optimal portfolio.

Another method of creating an optimal hedge fund of funds while accounting for the problems as well as higher moments of the return distributions of hedge fund strategies was done by DeSouza and Gokcan (2004). Their work is the inspiration behind our method, which will be described later. They take a different approach from the previously described methods of Anson et al and Popova et al as instead of explicitly optimizing by including the higher moments in the minimization and maximization multi-goal optimization, they optimize by simply maximizing expected return and minimizing conditional value-at-risk (CVaR). The process is similar to mean-variance optimization between hedge fund strategies, except that their risk measure is CVaR which is a measure of "fat tail" risk. CVaR calculated as the average loss of the worst 5% of losses for a particular return series. This helps to account for skewness and kurtosis since CVaR measures tail risk and so is a more appropriate measure of risk for distributions that are negatively skewed. As mentioned earlier, the optimization is done on groupings of different hedge fund strategies (rational strategy groups) where the constituents of the group are decided by an optimal clustering analysis which groups strategies that have high correlation together and minimizes inter-group correlations. They also accounted for the smoothness of many strategies' returns by applying a standard "unsmoothing" technique that adjusts the volatility of returns from the original, understated volatility of the original return series.

3: Data, Methodology, and Results

For this study and the analysis we have retrieved data from the Lipper TASS Hedge Fund database, where a large variety of hedge fund data is available. The Lipper TASS Hedge Fund database has been cited extensively in academic literature. The nature of our analysis is concerned with how one would allocate a fund of hedge funds; therefore we require the use of hedge fund strategy indices.

The database provides access to the Dow Jones Credit Suisse (DJCS) Hedge Fund Indices. DJCS provides a variety of hedge fund indices. DJCS tracks 9000 funds in which they select from to construct their indices. Indices are created on an asset-weighted basis. Funds included in the indices are required to have 1) minimum of $50M in AUM and 2) minimum one year track record with audited financial statements, or, if less than one year, audited financial statements and more than $500M in AUM. Performance is quoted net of fees and the indices do not include funds of funds.

A fund may be removed from the indices if 1) they do no report performance for two consecutive months or 2) fund fails to comply with necessary provisions for financial information. Survivorship bias is an inherent issue in this type of indexing. However we believe this issue is adequately addressed; as such, funds that are in the process of closing will remain in their respective index during the process of liquidation to capture all the potential negative performance before the fund is no longer operating.

For the analysis we used 8 indices that represent different, independent strategies. These strategies being: convertible arbitrage, distressed debt, risk-merger arbitrage, event driven, fixed income arbitrage, equity market-neutral, equity long-short, and global macro. These strategies closely resemble the strategies used by DeSouza (2004); the only exception is our study leaves out statistical arbitrage strategies because the data was not readily available.

We looked at monthly returns for the period Jan 1994 to May 2014 inclusive. DeSouza's (2004) study removes a half of a year in 1998 when Long Term Capital Management collapsed. However our data set includes this time period, as well as the 2008 financial crises, as we believe including these events provides a much more robust picture of the return distributions, and the actual return an investor may expect to receive while being invested in the market during times of duress. Table 1 summarizes the statistics of these strategies.

Global macro, distressed debt, event driven, and equity long-short strategies are characterized by much higher compounded annual returns than the four other strategies. The standard deviations of equity market-neutral, equity long-short, and global macro are higher than the other strategies. Three of the strategies stand out on a risk-return basis; they are characterized by significantly higher Sharpe Ratios than the other strategies. Namely, distressed debt, risk-merger arbitrage, and event driven strategies. Seven of the eight strategies have some degree of negative skewness; the outlier being global macro strategies with a slight positive skewness of 0.07. The most significant amount of negative skewness is apparent in equity market-neutral strategies with a measure of -12.18.

Distressed debt has two of the highest correlation coefficients, with event driven and equity long-short strategies of 0.94 and 0.67, respectively. The next highest correlation between two independent strategies was found to exist between fixed income arbitrage, and convertible arbitrage with a coefficient of 0.77. Most of the other strategies are relatively uncorrelated with two thirds of the coefficients of our triangular matrix (Table 3) less than 0.5. This finding is consistent with our general expectation of hedge fund strategies. Table 4 outlines the 12-month moving average rolling correlations of the various strategies. We found that the correlation structures are very unstable. The average spread, on a rolling 12 month basis among strategies, was found to be 1.286. This is strong evidence suggesting the correlations are unstable and must be accounted for when building an optimal portfolio. Table 5 also illustrates that the changes in correlation spreads is extremely high, as measured by the standard deviation.

3.1: Serial-Correlation and Unsmoothing

In previous sections and many studies it has been seen that many hedge fund strategies, including some that are used in our study, suffer from autocorrelation and smoothness in the asset values and return series. Using the common Ljung-Box Q-test for serial correlation, we found that six of the eight strategies in our study had significant autocorrelation at the 95% level, with only equity long-short and equity market-neutral not having significant autocorrelation. Some strategies display a greater coefficient of autocorrelation than others, with convertible arbitrage and fixed income arbitrage having serial correlation coefficients of 0.55 and 0.52 respectively. The strategies that displayed the most serial correlation are the strategies which commonly have the most illiquidity and asset pricing problems, as well as the fact that these strategies' potential losses are not necessarily realized as they occur economically. Although in the long run the average return of these strategies won't be affected by this serial correlation, in the meantime the volatility of the unrealized gains or losses will have a downward force on volatility, which causes an overstatement in common performance measures such as the Sharpe ratio.

In order to correct for overly smooth returns, a common method is to create a new return series with the same return characteristics but with increased volatility. Using methods for unsmoothing a series of returns is common in real estate literature. This is due to the returns in real estate having high serial correlation and smoothed returns as a result of the infrequency of valuation of real estate assets and common biases of smoothing in appraisals. The method's goal is essentially to create a new return series with corrected volatility such that the new series does not have any serial correlation. This new series is more likely to represent the true underlying characteristics of the return distribution for the asset class or strategy.

Our method to create unsmooth return series follows the approach done by DeSouza and Gokcan (2004) as well as done by Brooks and Kat (2004) and Georgiev et al (2003). We first tested each of the return series for the existence of serial correlation using the common Ljung-Box Q-test. Once the existence of serial correlation is confirmed, we determined at what lag is serial correlation is significant using sample partial autocorrelation regressions. Once we've determined the lag at which serial correlation is significant for a particular return series at a lag k, we used the following equation to determine the coefficient of correlation:

$$R_t = \alpha_0 + \alpha_1 R_{t-k} \tag{1}$$

Then the unsmoothed return series is calculated using the parameters from the previous auto regression equation as follows in the next equation:

$$R_t^{un} = \frac{(R_t - \alpha_1 R_{t-k})}{(1 - \alpha_1)} \tag{2}$$

This done such that the series of R_t^{un} contains no serial correlation. Using the corrected data series, we recalculated the summary statistics of the strategies used in our study in Table 2. The unsmoothing of the data had an effect in the following ways: the compounded return of the new return series were not significantly changed, however the standard deviation of each of the strategies increased compared to the original smooth return series. As a result of this, Sharpe Ratios across all of the strategies decreased as well. The most drastic changes in the Sharpe Ratio occurred in the strategies of convertible arbitrage, where the Sharpe Ratio went from 1.12 to 0.59, and in distressed debt, where it went from 1.68 to 1.05. However, we did not find a consistent change across the strategies in the higher moments of the distribution such as skewness and kurtosis compared to the original returns.

Due to the severity of the understatement of risk in the original data series, constructing optimal portfolios out of this data would create significant allocation biases towards strategies that have unrealistically low volatility and therefore artificially higher Sharpe ratios. To demonstrate this, in Figure 2 we have shown an efficient frontier of portfolios that were constructed from the original smooth return data and another efficient frontier constructed from the unsmoothed return data. These portfolios are allocated across all 8 of the strategies included in our study and the purpose of this exercise is to show the effect of using corrected and unsmoothed data in the optimization process when compared to using the data as it is given. As seen in Figure 2, the unsmoothed frontier is below and to the right of the efficient frontier made using the original data. This shows that the original frontier underestimates the risk and return trade-off in an optimal portfolio, and the gap between these portfolios was referred to by DeSouza and Gokcan as the “smoothness gap”. In this way, the different risk for the same return in the original frontier gives overly optimistic risk vs return characteristics.

3.2: Strategy Groups

The unstable correlation structure, as previously identified, need be addressed before any attempt to optimize a portfolio efficient frontier could be considered acceptable. DeSouza (2004) utilized a statistical technique called cluster analysis to create what the authors referred to as rational strategy groups, hereon referred to as strategy groups. We offer a similar method in which we have created groupings of individual hedge fund strategies to create pseudo asset classes in which our optimizations will use to create portfolios.

More specifically, we used hierarchical cluster analysis in Matlab to create our strategy groups. The desired outcome of this process is to combine groups of individual strategies that have statistical similarities that would allow those strategies to be combined to resemble an "asset class". The hierarchical clustering works to create groups of hedge fund strategies that will have a high correlation within the groups, but also maximize the correlation between the groups, or asset classes. The end result leaves the strategy groups with a low correlation among them, so that the optimization can benefit from diversification and manage the risks of the portfolio more optimally.

We applied this statistical procedure to our unsmoothed data and equally weighted the constituents of each hedge fund strategy index within its group. This procedure mirrors the process followed by DeSouza (2004), however we have found the group constituents to be somewhat varied from DeSouza's findings. Our analysis grouped the following strategies:

1. Distressed debt, merger-risk arbitrage, event driven, and equity long-short.
2. Convertible arbitrage, and fixed income arbitrage.
3. Equity market-neutral.
4. Global macro.

Table 9 illustrates the fit of our groupings relative to the other groups. From the table we can see that the r-square between individual strategies within the same group is high, while individual strategies r-square with the next closest group is small. This suggests that our cluster analysis has achieved our goal. Intuitively these numbers tell us that the components of each group are quite similar, while each group is distinct.

3.3: Statistics of Cluster Groups

To confirm that our groupings have resulted in lowered correlations between groups and that they have also stabilized the correlation structure we look to Tables 10, 11, and 12. The correlation matrix shows that the highest correlation coefficient is now 0.527 and our lowest correlation is now -0.007. This is a large decrease over individual strategies where the highest correlation was 0.94 and the lowest was 0.09.

Because two of the cluster groups were grouped as independent strategies following our cluster analysis, they will have the same statistical properties as shown previously. Moreover, looking at Group 1 and Group 2 we can see that our grouping of those strategies has had some effect on the shape of the return distributions. We can note from Table 8 that the grouping of strategies has slightly reduced kurtosis and skewness. Groups 1 and 2 have a skewness of -1.75 and -2.45 respectively. The skew value among the constituent strategies is in the range of -0.03 to -2.26 for Group 1 and -0.96 to 2.73 for Group 2.

For the purpose of our portfolio optimization it is important that we address the impact of cluster groups in regards to the dynamic correlation structure inherent among hedge fund strategies. By clustering strategies we have successfully decreased the average correlation from 0.454 with individual strategies to 0.273 with cluster groups. This supports evidence for the purpose of optimizing a portfolio of hedge fund strategies as groups because the benefits from diversification are greater with lower correlations among strategies.

More importantly, the impact of grouping strategies should have resulted in lowered correlation spreads. This would be evidence of more stable correlations, and thus provided more accurate results when optimized. Our results show that the moving averages of the 12-month rolling correlation spreads have indeed decreased. When looking at the average of all the independent strategies' 12 month rolling correlation spreads we observe a value of 0.98. The same measure using our cluster group framework provides an average of 0.92. This change indicates that the grouping of strategies has had a beneficial effect on the correlation structure that makes the strategy more suitable for optimization over independent strategies. This is consistent with the results of DeSouza's (2004) paper.

4: Portfolio optimization

Now that we have addressed two major issues that are prevalent in hedge fund data, we are ready to begin our process of optimizing portfolios on the efficient frontier. In our previous analysis we have seen that hedge fund strategies have a meaningful amount kurtosis and skewness. We have also shown that correlation structures among independent hedge fund strategies are unstable. These problems make traditional mean-variance optimization techniques problematic. Although our data manipulation techniques have decreased some of the kurtosis and skewness, the distributions remain non-normal.

Firstly, to be able to contrast alternative methods, we start with the mean-variance approach to optimizing our cluster groups. Secondly we look at two alternatives to mean-variance, both in hope that we are able to better capture downside risk that is apparent in the skewness and kurtosis. The alternative methods being: mean-conditional value-at-risk, and mean-semi variance. Our efficient frontiers are illustrated in Figure 1.

Tables 5, 6, and 7 summarize the weights of our proposed portfolios. We have selected proposed portfolio weights from each methodologies respective efficient frontier. The portfolios were also selected to have similar returns, to make the following statistical properties more comparable in terms of risk and return.

The weights in the conservative mean-variance portfolio are weighted heavily towards group 3. As the expected return increases, the weight in group 3 shifts towards groups 1 and 4, while group 2 remains relatively unchanged. This shift is better understood by simply referring to Table 8, where we can see that the groups 1 and 4 have higher expected returns and therefore heavier weights are needed to achieve the increased expected return of the aggressive portfolio over the conservative portfolio.

Looking at the conservative mean-CVaR portfolio we see that the weight is much more evenly distributed across the four groups than the conservative mean-variance portfolio. This contrast can be explained by the skewness and kurtosis of the four groups (Table 8). It is shown that groups 2 and 3 having the highest skewness and kurtosis, measures that would be ignored in the mean-variance optimizer, but moreover the focus of the mean-CVaR optimizer. We see a comparable trend in the weights of the mean-semi variance portfolios as well.

Mean-variance is an acceptable method to optimizing a portfolio if the return distributions of the assets are normally distributed, or if the investor has a quadratic utility function (Cremers et al, 2005). This poses challenges for hedge funds, as we have seen that there is a meaningful amount of skewness and kurtosis. We can see that the portfolios are well diversified but tend to shift weights into asset Groups 1 and 2 with increased expected return, as those assets deliver higher returns. Within the mean-variance context, the portfolios allocations are more diversified among all strategy groups when compared to the other methodologies. The mean-variance portfolios underestimate the risk from skewness and kurtosis. These portfolios have superior Sharpe Ratios to the other methodologies because MV objective is to minimize the variance, and in doing so the portfolios overlook the risks associated in the 3^{rd} and 4^{th} moments.

Mean-CVaR portfolio optimization is a preferred method to optimizing a portfolio when skewness and kurtosis appears in the return distributions; such is the case for hedge funds. Assuming a 95% confidence interval the optimization provides an efficient frontier and portfolio weights for the cluster groups. We have observed distinctly different portfolios than those of the mean-variance approach. At the conservative portfolio level, we have a diverse set of weights that most closely resembles those of MV. As the expected return is increased the optimizer tends to dump the weights into group 4, which has the highest expected return, but also the lowest kurtosis and skewness. The portfolios have a very low allocation to group 2 and this weight very quickly goes to zero. On the basis of Sharpe Ratios, the Mean-CVaR portfolios are dominated by the MV portfolios. This is not surprising, as MV accounts for only variance-covariance matrix as a risk factor.

Mean-Semi variance provides the most extreme contrast from our traditional mean-variance portfolios. Our proposed allocation weights are less diverse than both of our previous methodologies. We find that, again, the optimizer tends to plunge into group 4, the asset with the highest expected return. Across all three risk tolerance levels, we find that the portfolios have a higher standard deviation and therefore even lower Sharpe Ratios.

We have mapped each methodologies respective efficient frontier onto the mean-standard deviation space to compare the three curves. From this graph we can visualize the differences that are evident between the three methods with their respective interpretation of risk. As we saw numerically in the Sharpe Ratios of the various portfolios, we are now able to visualize. It is clear that MV portfolios dominate M-CVaR, which subsequently dominates MSV. The MV frontier is most significantly above the other two frontiers. This result would lead a naïve investor to quickly come to the conclusion that the MV portfolios are superior, as indicated by the Sharpe Ratios. However this is not necessarily the case.

To compare each optimization technique's ability to manage the risk associated with the 3^{rd} and 4^{th} moments, we have calculated the skewness and kurtosis of each portfolio. Looking at Tables 5, 6, and 7 we can see that the kurtosis is highest and skewness is most negative across all portfolios for the MV optimization compared to the other optimizations. Although these portfolios have much higher Sharpe Ratios, it is clear that they greatly underestimate the risks associated with the 3rd and 4th moments. MSV is only dominate to M-CVaR at the conservative return level, as the expected return increases the M-CVaR portfolios dominate MSV portfolios across the measures of kurtosis and skewness. An interesting finding is that the kurtosis is decreasing for each optimization methodology as we move from the conservative to aggressive portfolios. The lowest kurtosis among aggressive portfolios is when we use a mean-CVaR optimizer. The lowered kurtosis among the "riskier" portfolios, in terms of standard deviation, can be explained by the findings of Brulhart and Klein (2005). The higher standard deviation among the aggressive portfolios, which is the denominator of statistical calculation for the 4th moment, has a downward effect on the kurtosis of the portfolios. For mean-variance and mean-semi variance we can see that the higher the standard deviation is, the lower the kurtosis. The mean-CVaR portfolios are an exception to this observation. The aggressive mean-CvaR portfolio has a lower standard deviation than the aggressive mean-semi variance portfolios but also a lower kurtosis.

5: Conclusion

We have seen that kurtosis and skewness clearly exists in the return data for hedge funds. Hedge fund strategies also contain a meaningful amount of serial-correlation in their return series. In addition, we have also illustrated that the correlations between hedge fund strategy returns are very unstable and are dynamic over time. These statistical properties pose a problem for the process of creating optimal portfolios in a mean-variance framework, and the resulting portfolios, with non-normal return distributions, greatly underestimate risks that can occur in extreme events.

This study utilized a variety of numerical procedures to address the statistical properties that exist in hedge fund data. By grouping independent strategies we have been able address the correlation dynamics among hedge fund strategies. This technique also had a positive impact on the kurtosis and skewness. Hedge funds, like other assets, have returns that are serially correlated. By unsmoothing the returns series we were able to remove this serial correlation. These statistical properties need be addressed to be able to allocate a fund of hedges funds.

The use of hedge funds in larger portfolios by institutional investors, and high-net worth individuals remains to be an attractive opportunity for diversification among more traditional assets. The attractiveness of fund of hedge funds creates the need for accurate, sophisticated allocation methodologies. We have presented a methodology for allocating a portfolio of hedge funds that can be used within a larger portfolio of more traditional assets. In doing so, we have illustrated the need to address the statistical properties that are prevalent in hedge fund returns.

This study has presented three different methodologies for quantitatively allocating a fund of hedge funds. We have shown that using standard deviation alone, as a measure of risk, is not an encompassing factor when the previously mentioned statistical properties exist in the data. Therefore, the use of a traditional mean-variance optimization does not adequately diversify risk. Comparatively, the results of the mean-variance optimization are greatly contrasted by the two other methods of optimization, namely, mean-conditional value-at-risk and mean-semi variance. We have shown that the two latter methods of optimizing portfolios create more robust portfolios in terms of risk. This is evident in the kurtosis and skewness of said portfolios.

6: Tables

Table 1: Summary Statistics
Provides the summary statistics of each independent hedge fund strategy index over the period of time covered in this study.

	Convertible Arbitrage	Distressed Debt	Risk-Merger Arbitrage	Event Driven	Fixed Income Arbitrage	Equity Market Neutral	Equity Long Short	Global Macro
Compounded Return	7.36	10.64	6.37	9.60	5.49	4.86	9.42	11.10
Standard Deviation	6.60	6.35	4.02	6.11	5.44	9.84	9.55	9.27
Sharpe Ratio	1.12	1.68	1.58	1.57	1.01	0.49	0.99	1.20
Maximum Monthly Return	5.81	4.15	3.81	4.22	4.33	3.66	13.01	10.60
Minimum Monthly Return	-12.59	-12.45	-6.15	-11.77	-14.04	-40.45	-11.44	-11.55
Min/Max Spread	18.40	16.61	9.96	15.99	18.37	44.11	24.44	22.15
Maximum Drawdown	-3.17	-4.00	-2.62	-3.79	-4.24	-12.06	-1.88	-2.09
Skewness	-2.73	-2.22	-0.96	-2.26	-4.61	-12.18	-0.03	0.07
Kurtosis	20.28	14.52	7.73	14.06	36.63	175.47	6.47	7.30
JB Stat	3380.78	1566.83	267.47	1467.71	12512.60	312235.39	123.65	190.85

Table 2: Summary Statistics Unsmoothed
Provides the summary statistics of each independent hedge fund strategy index over the period of time covered in this study after the unsmoothing algorithm has been applied to the time series to remove the serial correlation in the data.

	Convertible Arbitrage	Distressed Debt	Risk-Merger Arbitrage	Event Driven	Fixed Income Arbitrage	Equity Market Neutral	Equity Long Short	Global Macro
Compounded Return	7.32	10.23	6.24	9.27	5.45	5.39	9.46	11.15
Standard Deviation	12.32	9.65	5.31	8.90	9.75	10.56	11.63	11.80
Sharpe Ratio	0.59	1.06	1.18	1.04	0.55	0.51	0.81	0.94
Maximum Monthly Return	14.04	6.94	5.42	6.86	11.47	4.05	14.14	13.93
Minimum Monthly Return	-26.65	-20.87	-8.30	-18.44	-22.05	-43.36	-14.34	-16.23
Min/Max Spread	40.69	27.81	13.72	25.30	33.52	47.41	28.48	30.17
Maximum Drawdown	-2.90	-4.01	-2.53	-3.69	-2.92	-11.71	-2.01	-2.16
Skewness	-1.97	-2.23	-0.76	-2.17	-2.83	-12.13	-0.01	-0.13
Kurtosis	18.63	17.35	7.66	15.61	24.61	174.48	5.88	7.97
JB Stat	2674.02	2325.01	247.07	1829.58	5136.07	308696.68	85.55	255.16

Table 3: Correlation Matrix

Provides the correlation matrix among independent hedge fund strategy indices

	Convertible Arbitrage	Distressed Debt	Event Driven	Equity Long Short	Equity Market Neutral	Fixed Income Arbitrage	Global Macro	Risk-Merger Arbitrage
Convertible Arbitrage	1.00	0.59	0.64	0.45	0.22	0.77	0.33	0.47
Distressed Debt		1.00	0.94	0.67	0.36	0.50	0.34	0.58
Event Driven			1.00	0.75	0.32	0.52	0.39	0.67
Equity Long Short				1.00	0.23	0.37	0.45	0.60
Equity Market Neutral					1.00	0.32	0.09	0.20
Fixed Income Arbitrage						1.00	0.40	0.30
Global Macro							1.00	0.23
Risk/merger Arbitrage								1.00

Table 4: 12-Month Moving Average Rolling Correlation Spreads

We take the 12-month rolling correlations between the individual hedge fund strategy indices and calculate the moving averages which are subsequently used to calculate the difference between the minimum and maximum values which provides the numerical values in the table

	Convertible Arbitrage	Distressed Debt	Event Driven	Equity Long Short	Equity Market Neutral	Fixed Income Arbitrage	Global Macro	Risk-Merger Arbitrage
Convertible Arbitrage	0	1.146	0.989	1.150	1.125	1.297	1.380	0.840
Distressed Debt		0	0.172	0.673	1.223	0.842	0.883	0.827
Event Driven			0	0.535	1.238	0.846	0.911	0.753
Equity Long Short				0	1.150	0.981	1.186	0.764
Equity Market Neutral					0	1.062	1.017	1.282
Fixed Income Arbitrage						0	0.955	1.078
Global Macro							0	1.039
Risk/merger Arbitrage								0

Table 5: Correlation Volatility
This table calculates the volatility of the 12-month rolling correlations.

	Convertible Arbitrage	Distressed Debt	Event Driven	Equity Long Short	Equity Market Neutral	Fixed Income Arbitrage	Global Macro	Risk-Merger Arbitrage
Convertible Arbitrage	0	0.329	0.292	0.358	0.323	0.394	0.343	0.277
Distressed Debt		0	0.058	0.203	0.325	0.259	0.265	0.266
Event Driven			0	0.150	0.330	0.222	0.277	0.238
Equity Long Short				0	0.350	0.272	0.305	0.233
Equity Market Neutral					0	0.344	0.330	0.386
Fixed Income Arbitrage						0	0.282	0.310
Global Macro							0	0.304
Risk/merger Arbitrage								0

Table 5: Mean-Variance Proposed Portfolios
This tables provide the recommended portfolio weights for Mean-Variance optimization methodology. In addition to weights these tables provide a portion of the descriptive statistics of the proposed portfolios.

	Weights								
	Group 1	Group 2	Group 3	Group 4	E[R]	Std	Sharpe	Kurtosis	Skewness
Conservative	0.2%	11.6%	83.4%	4.8%	8.3%	4.0%	2.08	185.5	-12.6
Moderate	12.9%	12.3%	55.8%	19.1%	9.5%	4.4%	2.16	101.5	-8.1
Aggressive	25.5%	12.9%	28.2%	33.3%	10.7%	5.5%	1.96	14.3	-1.9

Table 6: Mean-Conditional Value-at-Risk Proposed Portfolios
This tables provide the recommended portfolio weights for Mean-Conditional Value-at-Risk optimization methodology. In addition to weights these tables provide a portion of the descriptive statistics of the proposed portfolios.

	Weights								
	Group 1	Group 2	Group 3	Group 4	E[R]	Std	Sharpe	Kurtosis	Skewness
Conservative	33.7%	2.7%	39.1%	24.5%	8.4%	5.1%	1.65	26.1	-3
Moderate	35%	0%	16%	49%	9.6%	6.8%	1.42	7.6	-0.7
Aggressive	21%	0%	0%	79%	10.8%	9.0%	1.20	7.5	-0.7

Table 7: Mean-Semi Variance Proposed Portfolios
This tables provide the recommended portfolio weights for Mean-Semi Variance optimization methodology. In addition to weights these tables provide a portion of the descriptive statistics of the proposed portfolios.

	Weights								
	Group 1	Group 2	Group 3	Group 4	E[R]	Std	Sharpe	Kurtosis	Skewness
Conservative	57.8%	2.6%	19.8%	19.8%	8.3%	6.9%	1.21	11.3	-1.6
Moderate	56.5%	0.0%	2.6%	40.9%	9.6%	7.9%	1.21	9.2	-1
Aggressive	7.1%	0.0%	0.0%	92.9%	10.8%	11.2%	0.96	9.2	-1

Table 8: Summary Statistics of Strategy Groups
Provides the summary statistics of each group of hedge fund strategies following the cluster analysis; these groupings provide the statistical information needed to perform portfolio optimizations.

	Group 1	Group 2	Group 3	Group 4
Compounded Return	8.78	5.99	4.72	10.92
Standard Deviation	7.90	10.09	10.56	11.80
Sharpe Ratio	1.11	0.59	0.45	0.93
Maximum Monthly Return	5.58	10.70	4.05	13.93
Minimum Monthly Return	-15.49	-20.10	-43.36	-16.23
Min/Max Spread	21.07	30.80	47.41	30.17
Maximum Drawdown	-3.78	-2.88	-11.71	-2.16
Skewness	-1.75	-2.45	-12.13	-0.13
Kurtosis	12.99	20.52	174.48	7.97
JB Stat	1152.22	3406.75	308696.68	255.16

Table 9: Hierarchical Cluster Analysis
This table provides results of the Hierarchical Cluster Analysis by describing the goodness of fit.

	Strategy	r-squared with: Own Cluster Group	r-squared with: Next Closest Group	Ratio
Group 1	Distressed Debt	0.815	0.348	0.283
	Merger-Risk Arbitrage	0.538	0.180	0.563
	Event Driven	0.898	0.398	0.169
	Equity Long/Short	0.821	0.197	0.223
Group 2	Convertible Arbitrage	0.928	0.359	0.113
	Fixed Income Arbitrage	0.837	0.234	0.213
Group 3	Equity Market Neutral	1.000	0.103	0.000
Group 4	Global Macro	1.000	0.179	0.000

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Table 10: Strategy Group Correlation Matrix
Provides the correlations between the strategy groups.

	G1	G2	G3	G4
G1	1	0.527	0.256	0.423
G2		1	-0.007	0.352
G3			1	0.086
G4				1

Table 11: 12-Month Moving Average Rolling Correlation Spread.
Provides the Spread between the 12-month moving average rolling correlations.

	G1	G2	G3	G4
G1	0	0.720	1.114	0.931
G2		0	0.834	0.963
G3			0	0.983
G4				0

Table 12: Volatility of 12-Month Rolling Correlations
Provides the volatility of the rolling correlations

	G1	G2	G3	G4
G1	0	0.239	0.353	0.279
G2		0	0.313	0.293
G3			0	0.346
G4				0

7: Figures

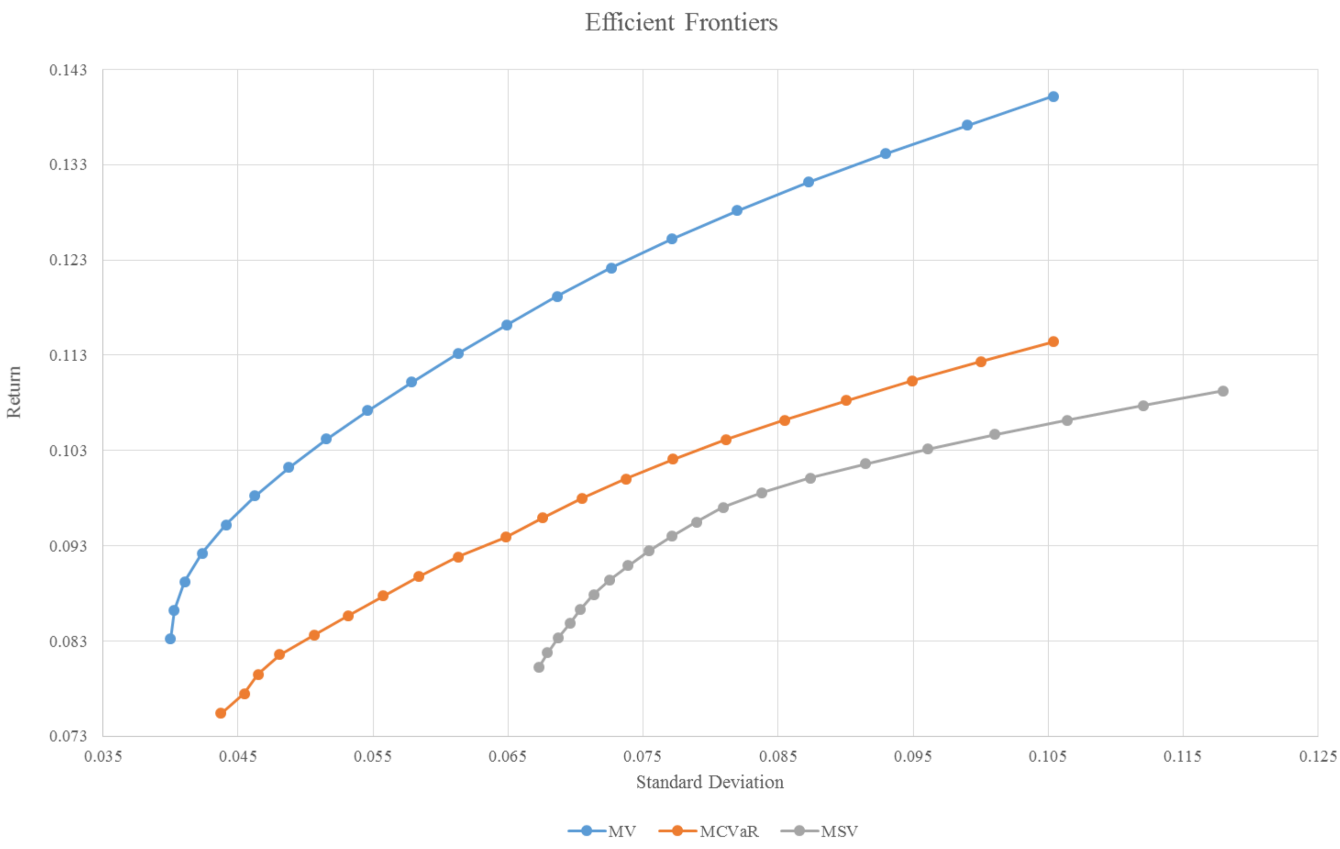

Figure 1: This graph maps the efficient frontiers for each of three optimization methodologies employed in this study. The frontiers are mapped in traditional mean-standard deviation space and provide a visualization of the risk/return trade off among each of the three optimization methodologies.

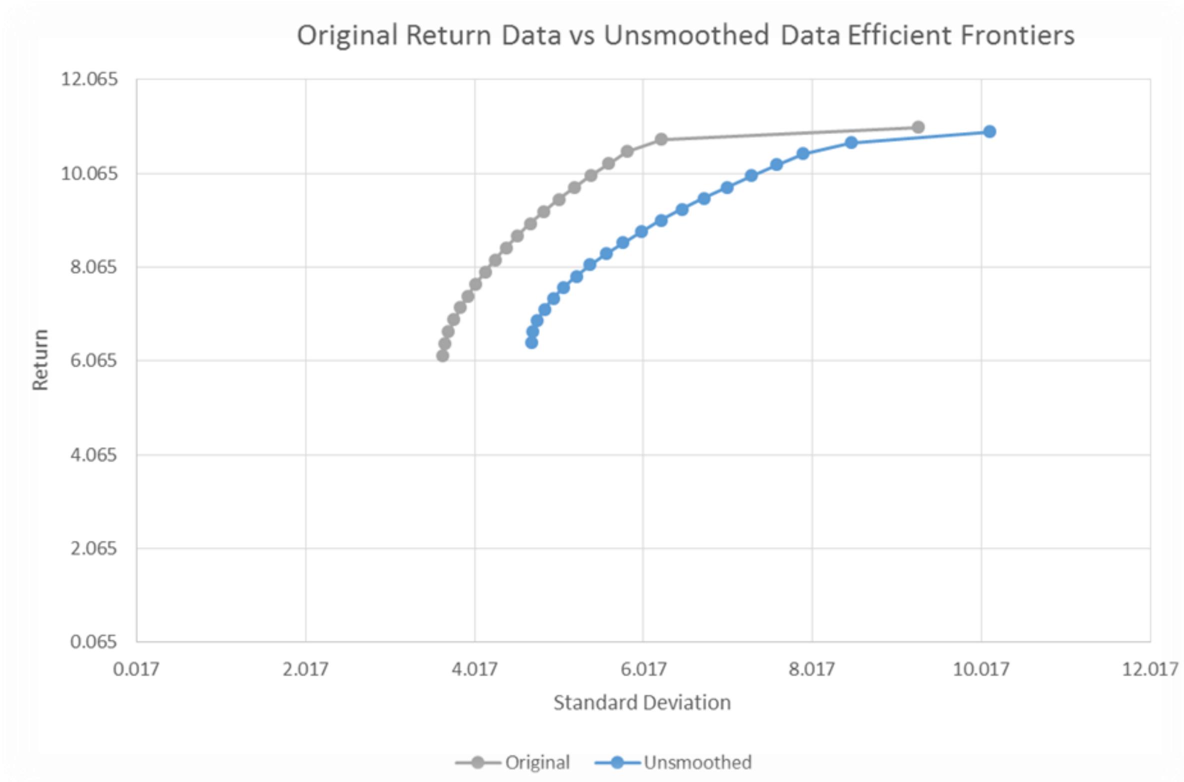

Figure 2: This graph shows the mean-variance efficient frontiers created using the eight strategies used in our studies. This shows the underestimation of risk by optimizing using the original "smooth" data. To correct for underestimation in variance and standard deviation, the return series needs to be unsmoothed to create the other "unsmoothed" frontier.

8: References

Ammann, M., & Moerth, P. (2008, July). Performance of funds of hedge funds. *The Journal of Wealth Management*, 46-63.

Anson, M., Ho, H. & Silberstein, K. (2007). Building a hedge fund portfolio with kurtosis and skewness. *The Journal of Alternative Investments. 10(1).* 25-34. Retrieved November 2015 from Business Source Complete.

Brooks, C., & Kat, H. M. (2002). The statistical properties of hedge fund index returns and their implications for investors. *Journal of Alternative Investments, 5*(2), 26.

Brulhart, T., & Klein, P. (2005). Faulty hypotheses and hedge funds. *The Canadian Investment Review,18(2).*6-13. Retrieved November 2015 from ProQuest Database.

Cremers, J.H., Kritzman, M., & Page, S. (2005). Optimal hedge fund allocations. *The Journal of Portfolio Management. 35(4).* 70-81. Retrieved November 2015 from Business Source Complete.

Dai, N., & Shawky, H. (2012). Diversification strategies and the performances of funds of hedge funds. *The Journal of Alternative Investments. 15(2)*. 75-85. Retrieved November 2015 from Business Source Complete.

Davies, R. J., Kat, H. M., & Lu, S. (2009). Fund of hedge funds portfolio selection. *Journal of Derivatives and Hedge Funds,15*(2), 91-115

DeSouza, C., & Gokcan, S. (2004, April). Allocation methodologies and customizing hedge fund multi-strategy products. *The Journal of Alternative Investments*, 7-21.

Hedge Fund Research Institute. (2015, October 20). HFR Global Hedge Fund Industry Report: Third Quarter 2015. In *Hedge Fund Research*. Retrieved from https://www.hedgefundresearch.com/pdf/pr_20151020.pdf

Popova, I., Morton, D.P., Popova, E., & Yao, J. (2007). Optimizing benchmark-based portfolios with hedge funds. *The Journal of Alternative Investments. 10(1).* 35-55. Retrieved November 2015 from Business Source Complete.

Reddy, G., Brady, P., & Patel, K. (2007, December). Are funds of funds simply multi-strategy managers with extra fees? *The Journal of Alternative Investments*, 49-61.

Saunders, D., Seco, L., Vogt, C., & Zagst, R. (2013, March). A fund of hedge funds under regime switching. *Journal of Alternative Investments, 15*(4), 8-23.

Sheikh, A.Z., & Qio, H. (2010). Non-normality of market returns: A framework for asset allocation decision making. *The Journal of Alternative Investments. 12(3).* 8-35. Retrieved November 2015 from Business Source Complete.