PROBABILISTIC ASSET VALUATION APPLIED TO NATURAL RESOURCE PROJECTS

by

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Abstract

This paper develops three probabilistic asset valuation models for mining projects. Firstly, an overview of available asset valuation techniques is presented. The probabilistic asset valuation technique is described in greater detail in advance of developing the probabilistic financial models. The probabilistic models incorporate a stochastic behaviour model for the price of copper and a Chilean peso exchange rate correlated to the copper price. The stochastic model parameters are defined based on the deterministic sensitivity analysis and on academic research. A sensitivity analysis tests the influence of one parameter for which guidance from the deterministic valuation process is not available. Three potential copper projects are evaluated deterministically and probabilistically and outputs from each approach are compared. Applications of the probabilistic approach are discussed along with implications on the decision making process. Finally, a high level implementation strategy is presented aimed at overcoming barriers for adopting probabilistic asset valuation.

Executive Summary

Risk and reward typically move in tandem; higher risks demand higher reward. The nature of natural resource projects is that the stakes of this trade-off are high. These projects require large amounts of capital to develop. However, the profitability of these projects will be dictated by future operating conditions. Commodity prices clearly have the greatest impact on profitability. The most commonly used asset valuation methodology, deterministic discounted cash flows, incorporates static commodity price assumptions. Risk due to commodity price fluctuations is evaluated through sensitivity analysis. A limitation of this approach is that it considers the impact of variables in isolation by varying them in fixed intervals. Probabilistic asset valuation offers a different approach to evaluating risk. Defining variables such as commodity prices stochastically in the financial model incorporates the random element inherent in their long-term behaviour. In combination with Monte Carlo simulation, this methodology examines a large number of commodity price profiles over the asset life and captures the effects on financial metrics. Furthermore, correlations among variables can be defined in order to better model real life economic conditions. The result of the simulation is a probability distribution of selected financial metrics. This probability distribution quantifies risks associated with the project. For example, the probability that a project generates a positive net present value (NPV) is available to decision makers. In contrast, a deterministic sensitivity analysis is only able to show the financial performance under a limited range of variable assumptions and rank the relative impact among variables. This paper demonstrates the value added to project evaluation through the application of probabilistic asset valuation. The NPVs of three projects are shown to be potentially overstated based on deterministic financial models. Although lower, NPVs derived from probabilistic models were still attractive. As a result, probabilistic asset valuation would yield greater value added when applied to marginal projects, or under less favourable economic conditions, than those considered in this report. This paper concludes that probabilistic asset valuation has good potential to complement the deterministic valuation technique by improving the current methodology for sensitivity analysis. The improved sensitivity analysis provides decision makers with a tool for risk management of individual projects or among alternatives when considering the dilemma of risk and financial reward.

Dedication

I would like to dedicate this report to Robin Fowler. Her influence originally set in motion the events leading to the completion of this report and without her support, the journey would not have been completed. Kocham Cię.

Acknowledgements

I would like to extend a special thank you to Scott Powell and Dale Andres for their guidance in completing this report. Scott's extensive knowledge on the subject matter significantly assisted the learning process and added to the outcomes of the report. Dale's industry experience helped to shape the original direction of the project. In addition, thank you to members of Teck's executives who participated in the survey undertaken for this report. Their input helped generate a strategy to incorporate the concepts evaluated in this report. Ongoing support and assistance from Jason Sangha and Grant Piwek is very much appreciated.

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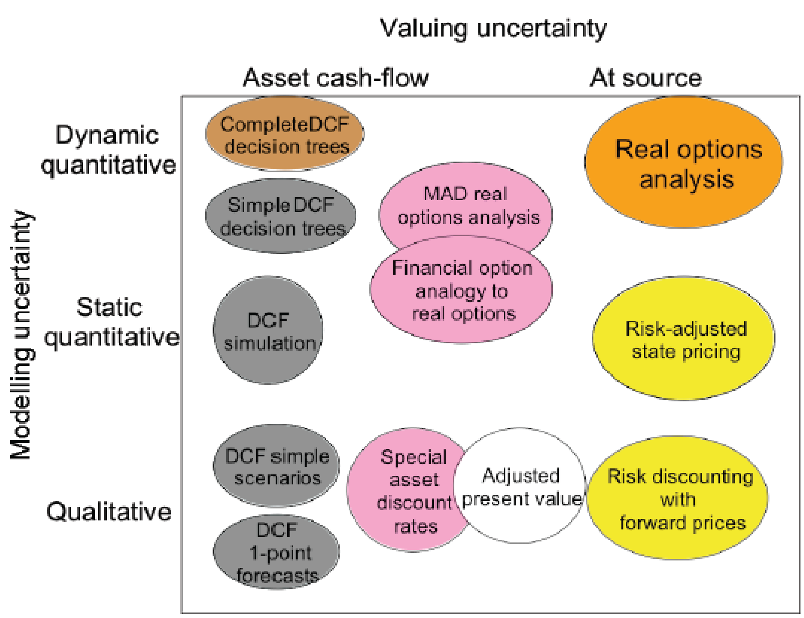
1.0 Introduction

At Teck, large scale mining projects are evaluated on the merits of safety, environmental and social sustainability, and profitability, among others. Gauging the profitability of a project is an integral part of the evaluation process. A wide range of variables can influence profitability. Unknown variables are quantified with assumptions based on research, internal expertise, and historical trends and compiled into financial models to provide an estimate of profitability. Current financial modelling utilizes a deterministic methodology. With this approach, a static set of assumptions produces a fixed output. This output for financial models includes indicators such as NPV, rates of return, and payback period. Furthermore, sensitivity analysis is performed to evaluate the effects of changes to the initial assumptions on a model's outputs. However, the current methodology fails to capture the uncertainty inherent in predicting variables outside the company's control and the uncertainty from basing assumptions on limited information. An example of such a variable might include metal prices which cannot be forecast with certainty. Therefore, as an alternative to the deterministic model, this paper will examine a probabilistic approach to financial modelling for project evaluation. The probabilistic approach incorporates the uncertainty of input assumptions, and returns a range of outputs. The use of a probabilistic financial model will allow Teck to more accurately quantify and manage the inherent risk associated with its development projects. Projects with lower risk profiles and a higher likelihood of profitability can be identified and advanced to the next stage of development. Conversely, projects less likely to be profitable can be studied further to reduce, if possible, the uncertainty of underlying risks and assumptions.

2.0 Asset Valuation Techniques

Investment decisions depend heavily on the ability of management to value assets correctly. The scale of the investment dictates the rigor of analysis applied to potential investments. Long life and capital intensive projects in the mining industry require decision makers to utilize as many tools as possible to judge projects' value. Laughton summarizes a taxonomy of valuation methods known as the Banff Taxonomy[1]. The Banff Taxonomy groups valuation methods on a multidimensional spectrum of modelling and valuing uncertainty.

Figure 1. Banff Taxonomy



Of the valuation methods found in the preceding table, discounted cash flows (DCF), decision trees, and real options account for the majority of methods employed for asset valuation, and therefore warrant a closer review.

[1] Laughton, 2007, p. 2

The DCF technique is the original valuation methodology and has been in practice for over 50 years. In these models, forecasted future cash flows are discounted using a discount rate reflecting the time value of money and a premium for the uncertainty of future cash flows. 1-point DCF forecasts include single long term variable assumptions and produce a single output. In conjunction with the 1-point analysis, firms often use DCF simple scenarios and simulation as a form of sensitivity analysis. Simple scenarios include testing different variable assumptions individually to determine the effects on the model's output. On the other hand, DCF simulation utilizes Monte Carlo simulation to run many different iterations of variable assumptions based on a stochastic distribution. The results of the simulation produce confidence levels of the expected model outputs. Decision tree analysis involves mapping all possible events and corresponding responses available to management, in sequential order. The result is a branched roadmap with many alternative paths forward. Each decision point represents a junction point in the path with corresponding expected values and probabilities of each alternative. To ascertain the value of an asset, management considers the likelihood of occurrence for each path and selects the highest value path to match their risk profile. As a result, this technique captures the implications of future decisions. Finally real options analysis takes the view that management is active, rather than passive, and can modify project decisions in the future. The DCF methodology assumes that management are passive and as a result the discount factor is used to account for possible uncertainty of future cash flows. On the other hand, real options analysis assumes that future cash flows are less uncertain because management is actively managing these cash flows and accounting for risk in their decisions. As a result, the theory is that these cash flows should be discounted at the risk free rate to reflect the lower level of uncertainty.

The ubiquitous DCF technique has been employed as the dominant methodology of asset valuation for many years and appears to be maintaining its dominance as the industry standard. In fact, Figure 2 shows a 1999 survey of CFOs[2] and confirms the popularity of DCF analysis. The traditional techniques of internal rate of return (IRR), NPV, hurdle rate, payback, and sensitivity analysis depend on the application of DCF.

[2] Graham & Harvey, 2001, p. 6

Figure 2. Percent of CFOs who always or almost always use a given technique

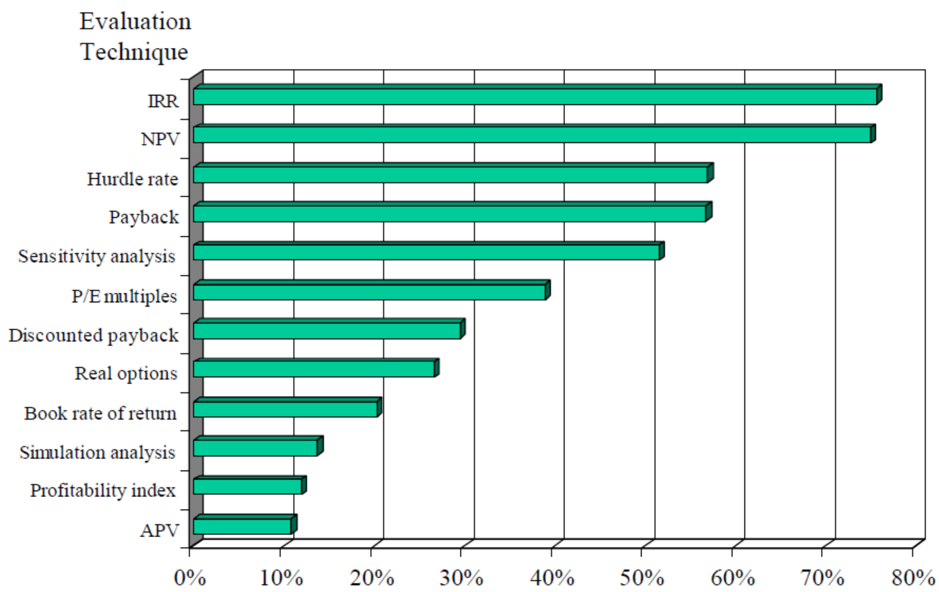


Figure 2 also highlights that evaluation techniques are not used in isolation. In most instances, techniques such as NPV and sensitivity analysis are performed in tandem during asset valuation. Other evaluation techniques are used by less than half of the surveyed CFOs. One reason might be the unfamiliarity with more advanced techniques. In addition, the lack of consensus among the academic community on fundamental aspects of these techniques, such as stochastic behaviour, adds to the reluctance to accept other valuation methods beyond DCF.

Based on the usage statistics, it will take a significant effort to gain wide spread acceptance of valuation techniques other than the traditional DCF. It should be made clear that the use of the remaining valuation methods does not preclude management from using DCF. Instead, DCF analysis can be supplemented with the more advanced methods to gain greater insight regarding uncertainty. Slow rates of adoption are to be expected. The status quo has been in practice for so many years that it has become engrained in many organizations. Moving beyond industry standards will require a cultural shift for many managers. Incremental steps will be required to achieve this cultural shift. The Banff Taxonomy and usage statistics show probabilistic analysis as the next level of analysis beyond the comfortable DCF. Ho and Pike addressed the adoption issues of probabilistic risk analysis (PRA). They surveyed firms in an attempt to answer the following question: Does the adoption of PRA lead to a shift in a firm's capital investment? Literature on the answer to this question appears to be divided. Hull[3] and Hertz[4] argue that PRA encourages investment. Since PRA can be used as a supplemental tool to gauge uncertainty, it adds to the overall data set used for decision making. Ho and Pike summarize this viewpoint as "PRA provides additional insights which may reduce descriptive and managerial uncertainty, providing managers with incentives to increase investment"[5]. On the other hand, Neuhauser and Viscione point out that some managers favour experience and judgment over quantitative methods[6]. These managers feel that an overreliance on quantitative models could overshadow the art of discerning a good investment from a poor one. As a result, these managers may not support projects justified on the basis of a probabilistic analysis. In the end, Ho and Pike's empirical research concluded that the use of PRA techniques did not have a negative impact on capital expenditures[7]. Subjectivity appears to be another hurdle for PRA adoption. Bier mentions that practitioners of PRA can steer the analysis in different directions based on differing goals and subjective judgment[8]. Furthermore, she adds that the implications of subjectivity in a probabilistic model's inputs are not fully appreciated[9]. Bier concludes that further research on the application of PRA would benefit adoption efforts of this methodology[10]. Ultimately, managers might require subject matter experts, internal or external, for guidance in the use of more sophisticated valuation techniques. This guidance might prove beneficial in the early stages of transitioning away from purely DCF valuation mindset, until a critical mass of industry use is achieved.

The use of a stochastic process in financial models affords the user a wide range of options. The absence of an industry standard in the use of stochastic models leads to variations in application and illustrates the subjectivity concerns discussed earlier. Ideally, inputs are modelled to represent the expected future behaviour. However, in most instances this is next to impossible. For some inputs, historical data can provide the evidence necessary to select an appropriate probability distribution. However, other variables, such as commodity prices, are less amenable to predicting future behaviour on the basis of the past. For these variables, many predictive behaviour models have been developed by the academic community. Two prominent behaviour

[3] Hull, 1980
[4] Hertz, 1964, p. 95-106
[5] Ho & Pike, 1992, p. 390
[6] Neuhauser & Viscione, 1973, p. 21
[7] Ho & Pike, 1992, p. 399
[8] Bier, 1999, p. 705
[9] Ibid.
[10] Bier, 1999, p. 706

models for commodity prices include the Random Walk and Mean Reverting. One example of a Random Walk model for commodity prices is based on geometric Brownian motion and takes the following form:

$$\Delta S = \alpha * S * \Delta t + \sigma * S * dz$$

Where: S = commodity price at time t

Δt = length of time between forecasting periods

$\Delta S =$ change in commodity price between forecasting periods

α = short-term price growth rate

σ = short-term price volatility

dz = standard Weiner increment = $\varepsilon * \sqrt{dt}$

ε = standard normal random variable with a mean of 0 and standard variation of 1

The Random Walk model assumes a trend for the variable being modelled, captured by the first part of the equation. If the short-term growth rate is zero, this model is referred to as a pure Random Walk model. Otherwise, the model is referred to as Random Walk with drift. The second part of the equation represents the Random Walk aspect. In this part, shocks are applied to the change in value of the variable based on a standard normal random variable. This model could be applied where there is consensus among management that the commodity in question will exhibit a constant trending behaviour. Dixit and Pindyck point out that the past behaviour of commodities resembles Random Walk characteristics when evaluating data for the previous 30 or 40 years. However, when the time horizon is expanded beyond 100 years, the Random Walk hypothesis can be rejected in favour of a Mean Reverting process[11]. One model is based on the Ornstein-Uhlenbeck process[12] which defines a Mean Reverting stochastic process with applications in modelling interest rates, exchange rates, and commodity prices. This Mean Reverting model takes the following form:

[11] Dixit & Pindyck, 1994, p. 77-78

[12] Ornstein & Uhlenbeck, 1930, p. 823

$$\Delta S = \gamma * (Sm - S) * S * \Delta t + \sigma * S * dz$$

Where: S = commodity price at time t

Δt = length of time between forecasting periods

$\Delta S =$ change in commodity price between forecasting periods

Sm = long-term equilibrium commodity price

γ = reversion rate

σ = short-term price volatility

dz = standard Weiner increment = $\varepsilon * \sqrt{dt}$

ε = standard normal random variable with a mean of 0 and standard variation of 1

The second part of this model includes the same random shock aspects as the previous Random Walk model. The difference is in the first part of the equation, where the variable being modelled is always pulled towards an equilibrium value. Application of the Mean Reverting model is appropriate where management has a long term view of a commodity but wishes to include the unpredictable nature of future commodity prices. In a study of 300 commodities, Andersson supports the mean-reverting nature of commodities. He asserts that high prices attract new entrants thereby increasing supply and reverting prices towards the marginal cost of production[13]. Further support of the Mean Reverting process in modelling future commodity price behaviour is provided by Bernard, et al.[14] and Schwartz[15].

[13] Andersson, 2007, p. 781
[14] Bernard, Khalaf, Kichian, & Mcmahon, 2008, p. 289
[15] Schwartz, 1997, p. 926

3.0 Deterministic Asset Valuation

A deterministic financial model incorporates a fixed set of assumptions to produce fixed outputs. These models incorporate the DCF methodology. The current practice of asset valuation at Teck is based on deterministic models to produce metrics such as NPV, IRR, and payback period in addition to performing 1-point sensitivity analysis. In this section, three projects are presented along with deterministic DCF valuation metrics. The deterministic financial models of all three projects are Microsoft Excel based and project revenues, costs, free cash flows, and discounted cash flows. To protect corporate confidentiality, the projects are named A, B, and C. The results of this analysis will serve as a base case for comparison of a valuation process using a probabilistic financial model.

Project A is a potential copper and gold open pit mine located in Chile. Engineering studies have identified a 19 year mine life along with a production schedule. The mine will produce one concentrate, containing copper and gold, which will be sold to smelters. The deterministic financial model will project annual cash inflows based on fixed variable inputs and subtract annual outflows such as operating costs, initial and sustaining capital, and taxes. The net annual cash flows will be discounted from the year they occur and totalled to determine the NPV. In addition, the IRR and payback period will be presented. IRR is calculated as the rate of return required to achieve an NPV of zero. The payback period is the number of years required to recover initial capital costs, based on undiscounted cash flows. Project A is analyzed in first quarter 2011 US dollars, with no allowance for inflation, on the basis of 100% equity financing. A summary of the financial model is included in Appendix A.

Table 1. Project A assumptions

Project A	
Copper Price ($/lb)[1]	3
Gold Price ($/oz)[1]	1,200
Average Operating Cost ($/lb Cu)[2]	1.14
Initial Capital (US$M)[3]	900
Discount Rate (%)	8
Exchange Rate (CLP:USD)	550

Note 1. Short term prices, tapering to $2.50 for copper and $850 gold after two and four years of commercial production, respectively
Note 2. Net of by-product credits and includes realization costs
Note 3. Spread over two years

Economics for the project are modelled on an after tax basis. Chilean taxes include:

- Mining Tax – 5% of revenue
- Federal Income Tax – 17% of taxable base

Using the preceding assumptions, Project A yields the following financial results:

Table 2. Financial performance of Project A

Project A	
Valuation Methodology	Deterministic
Undiscounted Cashflows (US$M)	2,255
NPV @ 8% (US$M)	706
IRR (%)	20.7
Payback (Years)	3.9

The deterministic DCF analysis shows promising results for the project. The impressive rate of return is achieved due to the moderate capital requirements to bring the project into production. One shortfall of this analysis is that important variables are considered static. For example, with the exception of elevated short term prices during the first few years, copper and gold prices are kept constant through the project's life. In addition, other variables such as the exchange rate are modelled similarly. In reality, the prevailing values of these variables are not likely to be constant from period to period.

The initial DCF analysis fails to capture the risk associated with Project A. The favourable metrics are highly dependent on the accuracy of the input assumptions. Some of these assumptions are partially controllable by the firm whereas others are not. The company has partial control over its operating costs but little control of commodity prices. Variables such as operating costs are estimated from first principles during engineering evaluation. Commodity prices are assumed with much less accuracy. To capture the uncertain nature of these assumptions, a sensitivity analysis is carried out to gauge the risk associated with the model's inputs. One of several possible risks for this project arises from the difference between variable assumptions and their actual values at the time of execution. Actual values of these variables at the time of execution could have a significant impact on the profitability of the project. As a result, sensitivity analysis is normally carried out to test the model for different variable assumptions and the corresponding outputs. The sensitivity analysis is carried out by changing the assumptions of one variable while holding the other variables constant. This methodology isolates the impact of one variable on the model outputs. A sensitivity analysis for Project A will test the following model assumptions:

- Initial Capital
- Operating Costs
- Commodity Prices (Copper and Gold)
- Exchange Rate

The model will be tested for changes in the variables using 10% increments in the range of +30/-30%. The following table summarizes the variable assumptions to be tested:

Table 3. Range of input variables for sensitivity analysis

Variable	**-30%**	**-20%**	**-10%**	**Base Case**	**10%**	**20%**	**30%**
Initial Capital (US$M)	630	720	810	900	990	1,080	1,170
Operating Costs (US$/lb Cu)[1]	0.78	0.90	1.02	1.14	1.26	1.38	1.5
Cu Price (US$/lb)	1.75	2.00	2.25	2.5	2.75	3.00	3.25
Au Price (US$/oz)	595	680	765	850	935	1020	1105
Exchange Rate (CLP:USD)	385	440	495	550	605	660	715

Note 1 Costs are after fixed by-product credits therefore change by increments of less than 10%

Summary results of the sensitivity analysis are shown in the following figures. Full results of the analysis can be found in Appendix B.

Figure 3. Project A NPV sensitivity analysis

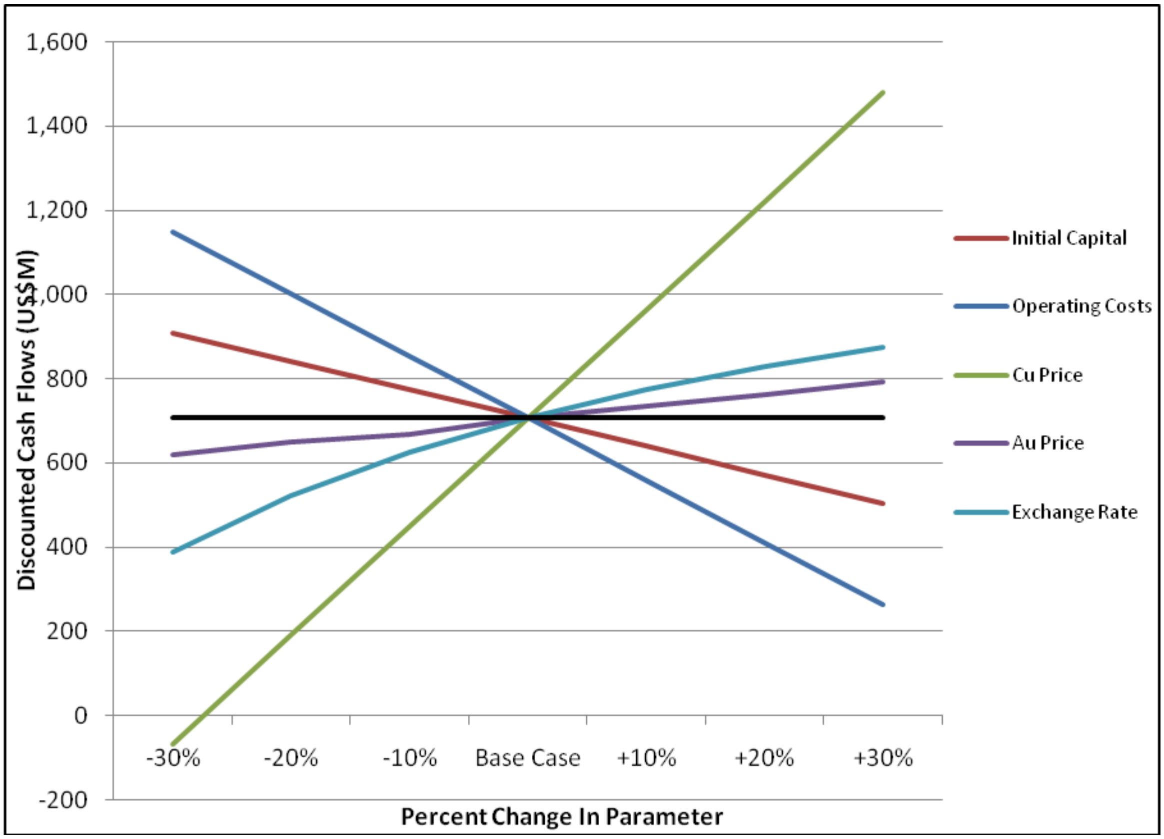


Table 4. Range of possible model outputs from sensitivity analysis for Project A

	Cash Flow (US$M)	NPV (US$M)	IRR (%)	Payback (Years)
Initial Capital (US$M)	2,042 - 2,467	504 - 908	15.3 - 30	2.8 - 4.8
Operating Costs (US$/lb Cu)	1,241 - 3,268	263 - 1,149	13.3 - 27.2	3.1 - 5
Cu Price (US$/lb)	553 - 3,956	-69 - 1,479	6.5 - 32.5	2.6 - 7.9
Au Price (US$/oz)	2,060 - 2,449	620 - 792	19.4 - 22	3.7 - 4.1
Exchange Rate (CLP:USD)	1,531 - 2,644	389 - 876	15.6 - 23.3	3.5 - 4.6

The traditional 1-point sensitivity analysis generates a range of values that valuation metrics can take depending on the input assumptions. As well, it highlights the degree of sensitivity of the model to one variable relative to others. This analysis shows that the selected valuation metrics for Project A are most sensitive to the price of copper followed by operating costs. NPV of the project ranges between -$69M for the worst case copper price scenario and $1,479M for the best case copper price scenario.

Project B is also a potential copper and molybdenum open pit mine located in Chile. This project is larger and has a greater degree of complexity than Project A and is at an earlier stage of development. It will take approximately four years for Project B to reach commercial production. The capital requirements are an order of magnitude larger compared with Project A. Over the course of a 20 year mine life, this mine is expected to produce two concentrates: copper and molybdenum. Small quantities of silver are present in the copper concentrate. The silver quantities are large enough to be payable by smelters. Both concentrates are expected to be marketed on the international market. The deterministic valuation basis and methodology are the same as described earlier for Project A. A summary of the financial model is included in Appendix C. The following table summarizes project assumptions.

Table 5. Project B assumptions

Project B	
Copper Price ($/lb)[1]	3
Molybdenum Price ($/lb)	12.5
Silver Price ($/oz)	10
Average Operating Cost ($/lb Cu)[2]	0.87
Initial Capital (US$M)[3]	3,137
Discount Rate (%)	8
Exchange Rate (CLP:USD)	550

Note 1. Short term price, tapering to $2.50/lb for the start of commercial production
Note 2. Net of by-product credits and includes realization costs
Note 3. Spread over four years

The deterministic valuation of Project B yields strong financial metrics as shown in Table 6. Processing of higher grade material early in the production schedule generates elevated cash flows during the first six years of commercial production; initial capital is recovered after five years of production. This project's value on an NPV basis appears to support a decision to move the project into the next stage of development.

Table 6. Financial performance of Project B

Project B	
Valuation Methodology	Deterministic
Undiscounted Cashflows (US$M)	7,113
NPV @ 8% (US$M)	1,249
IRR (%)	13.1
Payback (Years)	5.2

Figure 4 shows a sensitivity analysis of the NPV. The price of copper clearly has the biggest impact on the value of the project. In fact, the NPV is negative at the lower range of tested copper prices. Capital and operating costs have similar impacts on the NPV and are the next most influential variables. The project is least sensitive to the price of molybdenum and the Chilean peso exchange rate. The influence of the Chilean peso exchange rate on the profitability of the project is through the conversion of the domestic portion of operating costs to a US dollar basis. Table 7 summarizes the range of possible values of the financial metrics resulting from a 1-point sensitivity analysis. The full analysis can be found in Appendix D.

Figure 4. Project B NPV sensitivity analysis

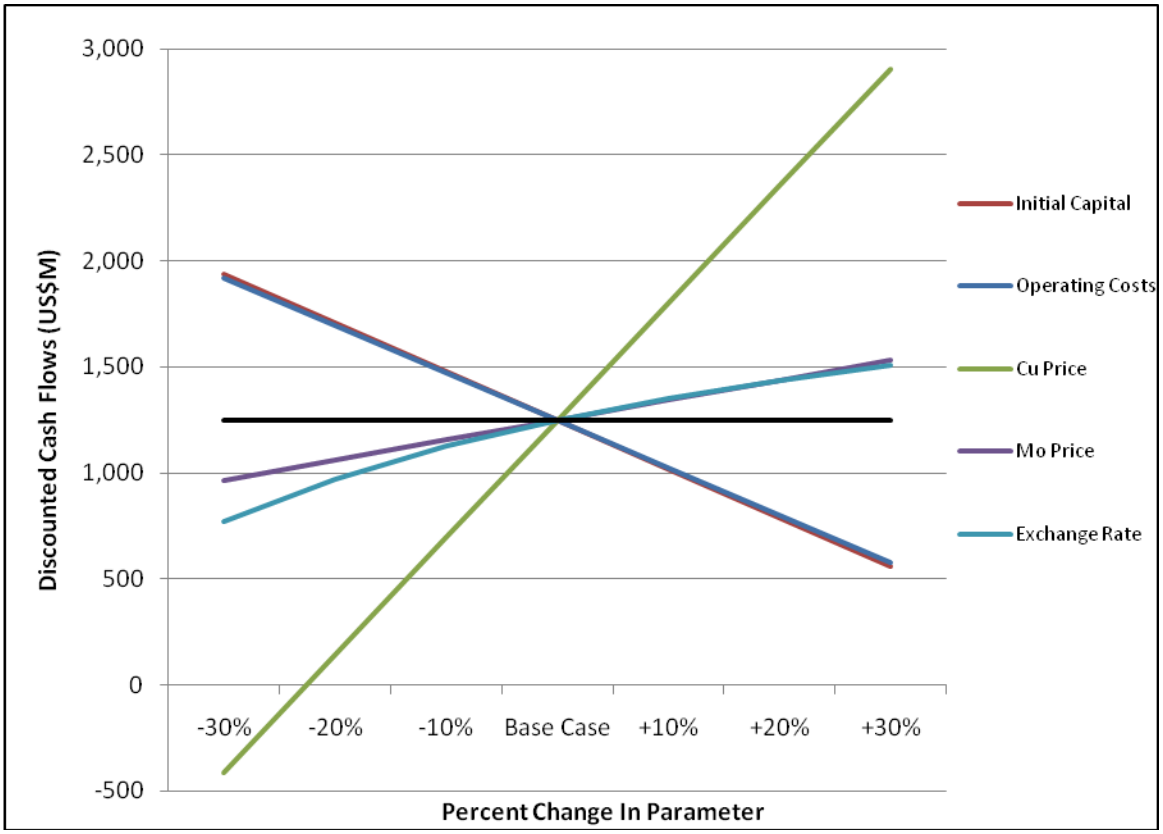


Table 7. Range of possible model outputs from sensitivity analysis for Project B

	Cash Flow (US$M)	NPV (US$M)	IRR (%)	Payback (Years)
Initial Capital (US$M)	6,367 - 7,855	560 - 1,936	3.9 - 6.5	3.9 - 6.5
Operating Costs (US$/lb Cu)	5,346 - 8,875	577 - 1,918	10.5 - 15.5	4.6 - 6
Cu Price (US$/lb)	2,723 - 11,485	-417 - 2,902	6 - 18.8	3.8 - 10.9
Mo Price (US$/lb)	6,324 - 7,898	964 - 1,533	12.1 - 14.2	4.9 - 5.6
Exchange Rate (CLP:USD)	5,852 - 7,791	770 - 1,506	11.3 - 14.1	5 - 5.7

Finally, Project C is the third potential project used to demonstrate the difference between asset valuation techniques. Similar to Project B, Project C produces copper and molybdenum concentrates with payable silver quantities. This project requires the greatest development capital over a five year construction period. Upon reaching commercial production, the project is expected to operate for 32 years. The deterministic valuation basis and methodology are the same as described earlier for Projects A and B. A summary of the financial model is included in Appendix E. The following table summarizes project assumptions.

Table 8. Project C assumptions

Project C	
Copper Price ($/lb)[1]	3
Molybdenum Price ($/lb)	12.5
Silver Price ($/oz)	10
Average Operating Cost ($/lb Cu)[2]	1.26
Initial Capital (US$M)[3]	3,799
Discount Rate (%)	8
Exchange Rate (CLP:USD)	550

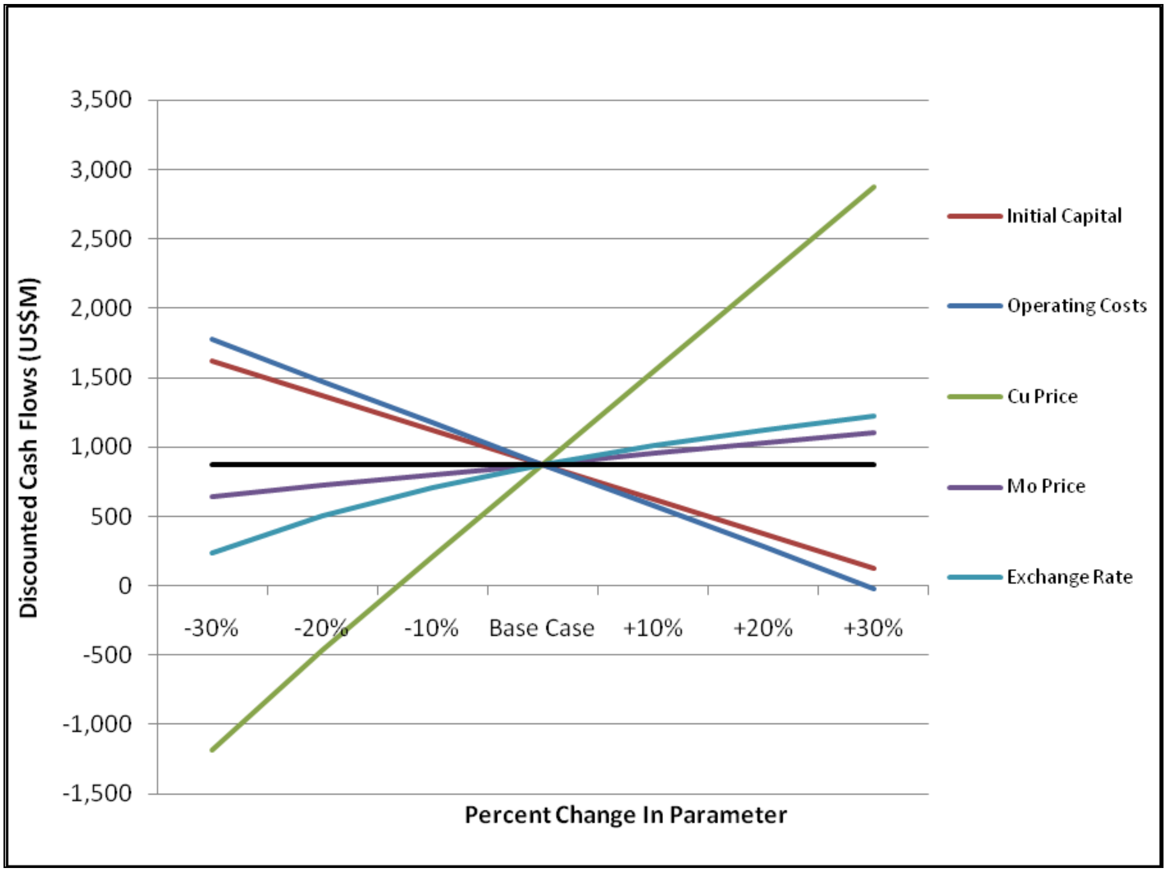
Note 1. Short term price, tapering to $2.50/lb for the start of commercial production
Note 2. Net of by-product credits and includes realization costs
Note 3. Spread over five years

Project C has the longest mine life of three projects considered. Along with the highest copper grades, this long life project generates the highest cash flow of all three projects. However, the NPV of Project C does not exceed that of Project B due to higher capital costs and the greater effect of discounting in the later stages of operation. In addition, the rate of return and payback period suffer as a result of the capital costs. Nonetheless, Project C appears to be capable of generating healthy financial metrics warranting investment in the project. Table 9 summarizes the deterministic valuation of Project C.

Table 9. Financial performance of Project C

Project C	
Valuation Methodology	Deterministic
Undiscounted Cashflows (US$M)	9,282
NPV @ 8% (US$M)	876
IRR (%)	11.0
Payback (Years)	6.8

Figure 5. Project C NPV sensitivity analysis



As expected, the sensitivity analysis of Project C highlights the sensitivity of the project to copper prices. The NPV dips to a low of almost -$1.2B under the lowest copper price protocol. Operating costs also have the ability to drive the NPV below break-even. The impact of the other variables is shown in the following table. The full 1-point sensitivity analysis can be found in Appendix F.

Table 10. Range of possible model outputs from sensitivity analysis for Project C

	Cash Flow (US$M)	NPV (US$M)	IRR (%)	Payback (Years)
Initial Capital (US$M)	8,381 - 10,181	130 - 1,620	4.9 - 8.3	4.9 - 8.3
Operating Costs (US$/lb Cu)	5,544 - 13,017	-23 - 1,773	7.9 - 13.6	5.6 - 8.3
Cu Price (US$/lb)	1,461 - 17,055	-1,183 - 2,873	2.6 - 16.7	4.5 - 17.4
Mo Price (US$/lb)	8,370 - 10,194	644 - 1,108	10.3 - 11.7	6.5 - 7.2
Exchange Rate (CLP:USD)	6,614 - 10,719	235 - 1,221	8.9 - 12.1	6.3 - 7.9

Although all three projects are showing promising results based on the deterministic financial analysis, which of them should rank highest in the development schedule? Table 11 summarizes the financial metrics of all three projects. From one perspective, Project A requires significantly less capital expenditure and offers superior rates of return. On the other hand, Project B offers the best present value but requires greater capital along with an extended construction period. Finally, Project C generates a mid-range NPV but requires the highest level of capital expenditures and generates the lowest rate of return. Decision makers subscribing to NPV as the most important valuation metric would prioritize project B, followed by Project C, leaving Project A as the least attractive.

Table 11 Deterministic Project Comparison

Project	A	B	C
Valuation Methodology	Deterministic	Deterministic	Deterministic
Initial Capital (US$M)	900	3,137	3,799
Undiscounted Cashflows (US$M)	2,255	7,113	9,282
NPV @ 8% (US$M)	706	1,249	876
IRR (%)	20.7	13.1	11.0
Payback (Years)	3.9	5.2	6.8

The deterministic valuation method considers only one possible set of assumptions for the projects. The traditional 1-point sensitivity analysis attempts to capture some of the risks associated with each project with respect to the assumed project variables. A significant weakness of the 1-point sensitivity analysis is that it considers the changes of one variable while maintaining the others constant. In reality, the interaction between the variables is more dynamic. Some economic variables will move in correlation with others while some will move independently. Furthermore, only a small number of iterations are possible with the traditional sensitivity analysis. Without running a large number of iterations, the likelihood of any outcome cannot be determined. As a result, the risks associated with both projects have not been fully captured. To truly model the financial performance of a project, an aspect of randomness should be introduced to represent the unknown nature of our assumptions. In addition, variables in the financial model should be allowed to fluctuate simultaneously in order to fully capture the range of possible outcomes. Decision makers concerned about the possible worst case scenarios might be inclined to ask how likely those scenarios are to occur. To answer this question we must move beyond the deterministic financial model and 1-point sensitivity analysis and introduce a probabilistic financial model.

4.0 Probabilistic Asset Valuation

A variable is modelled stochastically if a random process predicts its future behaviour, at least in part[16]. A probabilistic approach to asset valuation attempts to incorporate the influence of random behaviour in some or all of the variables. Using behaviour models, like those introduced earlier, and Monte Carlo simulation, a financial model can be tested for many possible scenarios. Running many iterations allows the user to develop a probability distribution of the model's outputs. This probability distribution provides decision makers with the additional information not afforded to them by the simple 1-point sensitivity analysis described in the previous section. Furthermore, behaviour of dependant variables can be correlated with other variables to develop a more realistic situational analysis. For example, probabilistic modelling was utilized for the prominent Oyu Tolgoil project in Mongolia[17]. Stochastic metal price forecasts were developed to capture cash flow uncertainty of this $4.6 billion dollar project. The use of probabilistic models takes the user towards the dynamic modelling of uncertainty as described in the Banff Taxonomy. This migration is evident in Figure 1 by an upward movement along the uncertainty axis of taxonomy. This methodology has been applied to risk analysis with applications beyond financial modelling. PRA is a general methodology used as a support tool to help quantify the risks inherent with uncertain processes[18].

To transform the deterministic models developed in the previous section into probabilistic models, select variables will be modelled stochastically. For simplicity and demonstrative purposes, one variable will be modelled stochastically and the behaviour of another will be predicted through its correlation with the stochastic variable. The traditional sensitivity analysis identified the price of copper and operating costs as having significant influence on the financial model's outputs. As a result, the price of copper will be modelled stochastically. Operating costs will fluctuate due to the exchange rate correlation with the price of copper. This probabilistic financial model will be better suited to understanding the financial performance of all three projects.

[16] Dixit & Pindyck, 1994, p. 60
[17] Oyu Tolgoil Technical Report, 2010, p. 44-47
[18] Bedford & Cooke, 2001, p. 3

The following figures show iterations of stochastic modelling of copper prices over a fifty-year period. The models were built in Microsoft Excel and incorporate the behaviour models outlined in Section 2. The deterministic, high protocol, and low protocol copper assumptions are included for comparison. The high and low protocol prices represent +30% and -30% changes from the base case, respectively.

Figure 6. One iteration of stochastic models applied to the price of copper

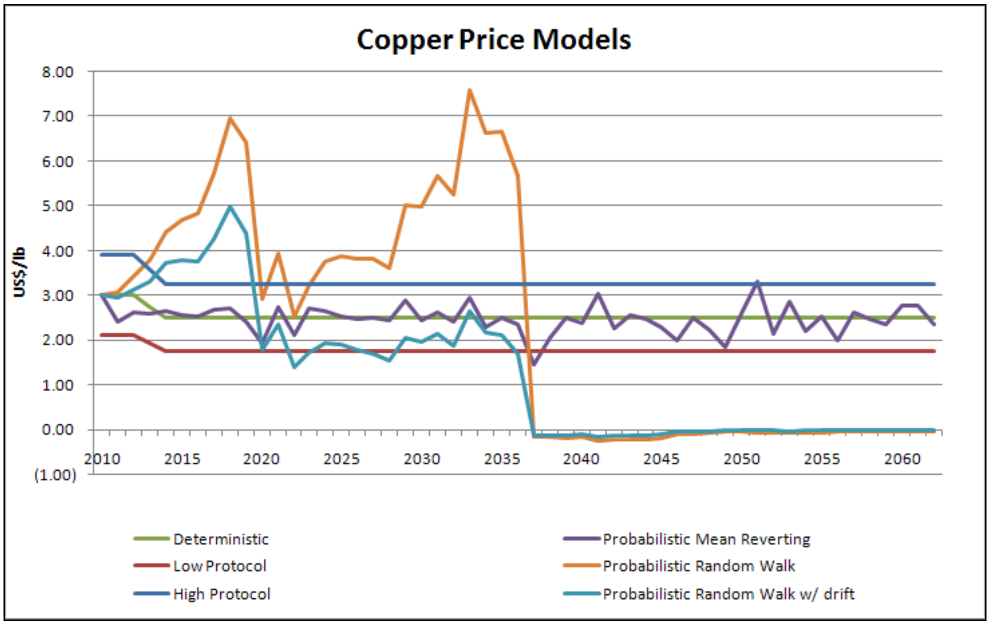
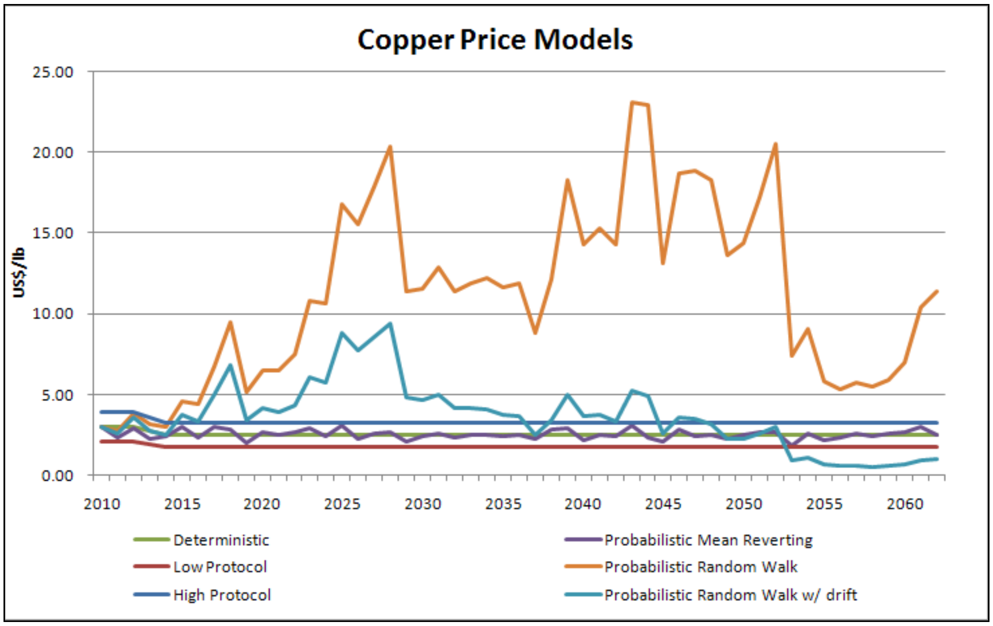


Figure 7. A second iteration of stochastic models applied to the price of copper



The deterministic, high protocol, and low protocol data points capture the static cases of copper prices evaluated during the traditional deterministic and 1-point sensitivity analysis. In contrast, the stochastic models highlight the ability to capture the real life cyclical nature of commodities. However, the preceding figures illustrate that both Random Walk models are susceptible to generating unreasonable predictions. In Figure 6, the Random Walk models predict negative copper prices. On the other hand, these same models generate very high copper prices in Figure 7. Although the presented iterations are only a small sample from a large number of iterations in a Monte Carlo simulation, they are nonetheless unrealistic. Such volatile predictions only add to the scepticism of probabilistic modelling. Based on Figures 6 and 7, more realistic behaviour is generated using the Mean Reverting model because it avoids the extreme values generated by the Random Walk models. In general, the predicted prices always trend towards the long term mean copper price with random price spikes. This behaviour captures the possible inter-period volatility without the large swings of the Random Walk models. The behaviour of the stochastic models is controlled by the parameters used in the model. As discussed in section two, the reversion rate (α), short-term price volatility (σ), and short-term price growth (α) must be defined. The stochastic models in Figures 6 and 7, used values of 0.4, 0.3, and -0.45 for the reversion rate, short-term price volatility, and short-term price growth rate, respectively. The price volatility is taken directly from the deterministic analysis of +30%/-30%. A copper price reversion rate of 0.4

is suggested by Samis and Davis[19]. In addition, without a consensus of a short-term growth rate, Samis and Davis[20] suggest setting the growth rate as a factor of the price volatility as such:

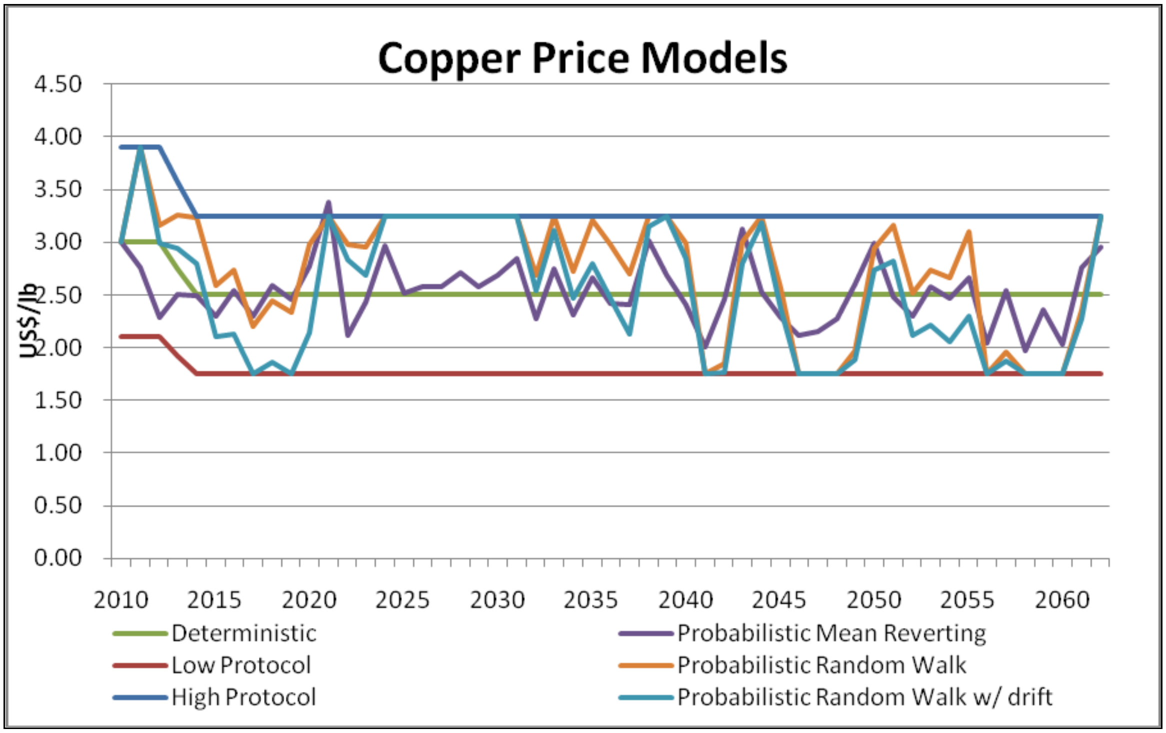
$$\alpha = -0.5 * \sigma ^\wedge 2$$

The short-term growth rate parameter is required for the Random Walk model with drift. This model will generally predict lower prices than the Random Walk model, as a result of the negative term, in the absence of organizational consensus on a growth rate. As a result, the Random Walk model with drift may not be suitable. Furthermore, the basic Random Walk model is dependant only on the volatility and the standard normal random variable. To eliminate extreme behaviour predictions such as negative price, boundary conditions could be applied. For example, the Random Walk model could be constrained by the upper and lower price protocols from the deterministic 1-point sensitivity analysis. The result of this constraint is that the predicted copper price never moves outside of the upper and lower protocols but includes more extreme shocks in the copper price behaviour relative to the Mean Reverting model.

[19]Samis & Davis, 2007, D2M.10

[20]Samis & Davis, 2007, D2M.8

Figure 8. One iteration with constrained Random Walk models



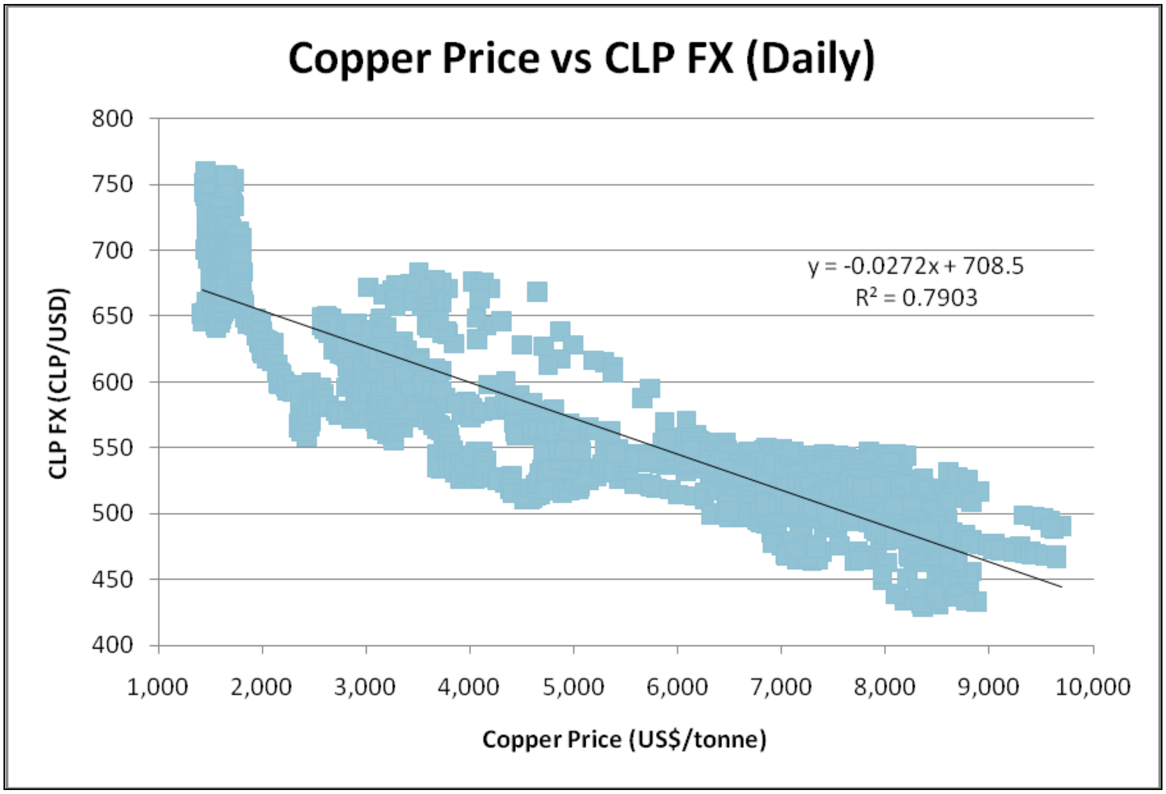
Both constrained Random Walk models take away from the original intent of the models to capture randomness and a prevailing trend. Furthermore, these constrained models provide similar copper price trends to the Mean-Reverting model evident in Figure 8. The Mean Reverting model seems more suitable because the deterministic price can be taken as the long-term price. This connection could prove a powerful force for adoption of the probabilistic methodology. For example, management might be more likely to accept the probabilistic approach if the stochastic models incorporate existing price behaviour assumptions, at least in part. As a result, the Mean-Reverting model will be used to simulate copper price behaviour in the probabilistic financial models for the three projects examined earlier.

The deterministic sensitivity analysis identified operating costs as a direct value driver for both projects. Modelling operating costs stochastically requires collecting actual data from similar existing mines to develop probability distributions. The accuracy of this exercise would not be high, due to specific operating conditions of each mine. Furthermore, internationally traded consumables such as diesel fuel are an integral component of the operating costs. As a result, developing correlations with other economic indicators would require an in-depth economic analysis beyond the scope of this paper. However, during operation, data could be collected to develop a probability distribution for operating costs based on historical performance. As a result, modelling operating costs stochastically would be more practical once a project is in operation.

Even though we will not model operating costs in our probabilistic model directly, we can incorporate their influence indirectly. Operating cost cash outflows occur in both domestic and foreign currencies. Disbursements for labour and electricity take place in domestic Chilean pesos. The remainder of the operating costs are for internationally traded goods and services and are typically paid in US dollars. The division between costs realized in domestic and foreign currencies is approximately even. As a result, the fluctuations in the Chilean peso exchange rates have a direct influence on total operating costs, on a US dollar basis. Therefore, exchange rates influence net cash flows for Project A. A government's monetary policy controls the behaviour of its currency's exchange rate. If the monetary policy were such that it is directing the course of exchange rate, then predicting its behaviour would require alignment with the government's intentions. On the other hand, if exchange rates are free floating, then their behaviour should correlate well with the economic drivers fuelling the economy. The Chilean economy is very much reliant on its natural resources, with copper extraction playing a major role. As a result, if the currency were free floating, we would expect a strong correlation between copper prices and the Chilean exchange rates. The following figure shows the relationship between copper prices and the Chilean exchange rates based on daily quotes since 2002[21].

[21] LME copper spot prices and Chilean peso exchange rates from 01/01/02 to 01/14/11

Figure 9. Copper Price and Chilean Peso exchange rate data since 2002



The data shows a good fit with a linear trend line having a coefficient of determination of 0.7903. Careful consideration of the period used in developing an economic relationship between two variables is required. Since the relationship will be used to model the future, the historical economic conditions of the dataset should resemble the expected future economic conditions. Otherwise, accuracy of the predicative model would be questionable. The period between 2002 and the beginning of 2011 is assumed to be a reasonable estimation of the future economic conditions because it captures a full economic cycle. Economic conditions were on the rebound in 2002 following the terrorist attacks in the United States before beginning to deteriorate in 2008 in advance of the most recent recession. A gradual recovery began to take shape in the second half of 2009, continuing through 2010.

Now that we have defined the stochastic model for the price of copper and correlated the behaviour of the Chilean exchange rate, we can develop probabilistic financial models for Projects A, B, and C. The Crystal Ball software was used to carry out Monte Carlo simulation with 100,000 iterations. This number of iterations far exceeds the sample size required to achieve statistically valid outputs – 95% confidence level with a 5% confidence interval. The Crystal Ball software was selected due to existing user knowledge within Teck, ease of use (Microsoft Excel add-in), and superior presentation of results. Results of the Monte Carlo simulations will be shown as histograms of NPV, IRR, and payback period. The histograms show the probability (primary vertical axis) and frequency (secondary vertical axis) of values occurring in a given interval (horizontal axis). The pink area of each histogram represents intervals which are below the deterministic value. Conversely, the blue area indicates the intervals which meet or exceed the deterministic value. In addition, a certainty of meeting or exceeding the deterministic value is shown. Percentiles and mean values of the results are represented by blue vertical lines; where P90, P50, and P10 are defined as values which are exceeded by 90%, 50%, and 10% of the simulation outputs, respectively. Outputs of the simulations for Project A are shown in the following figures.

Figure 10. Project A NPV histogram

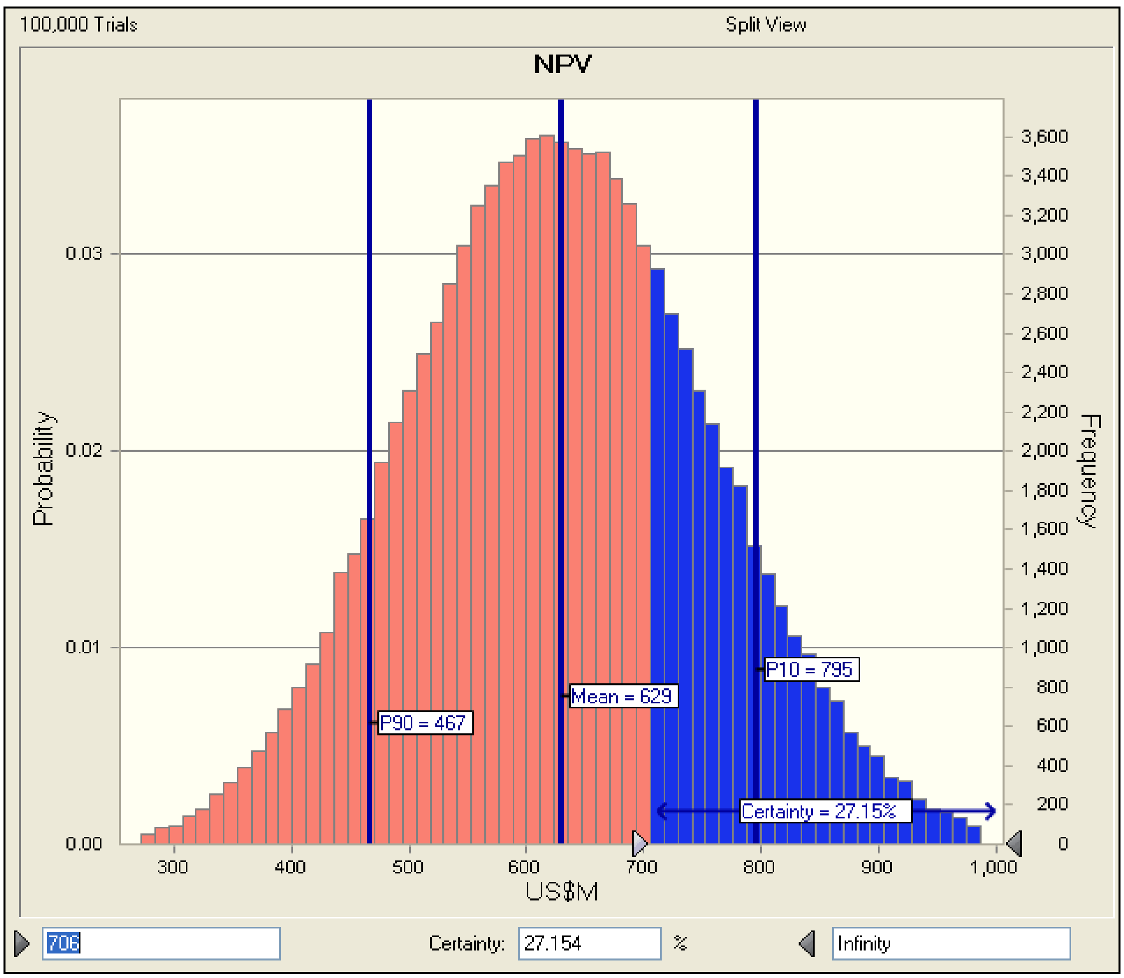


Figure 11. Project A IRR histogram

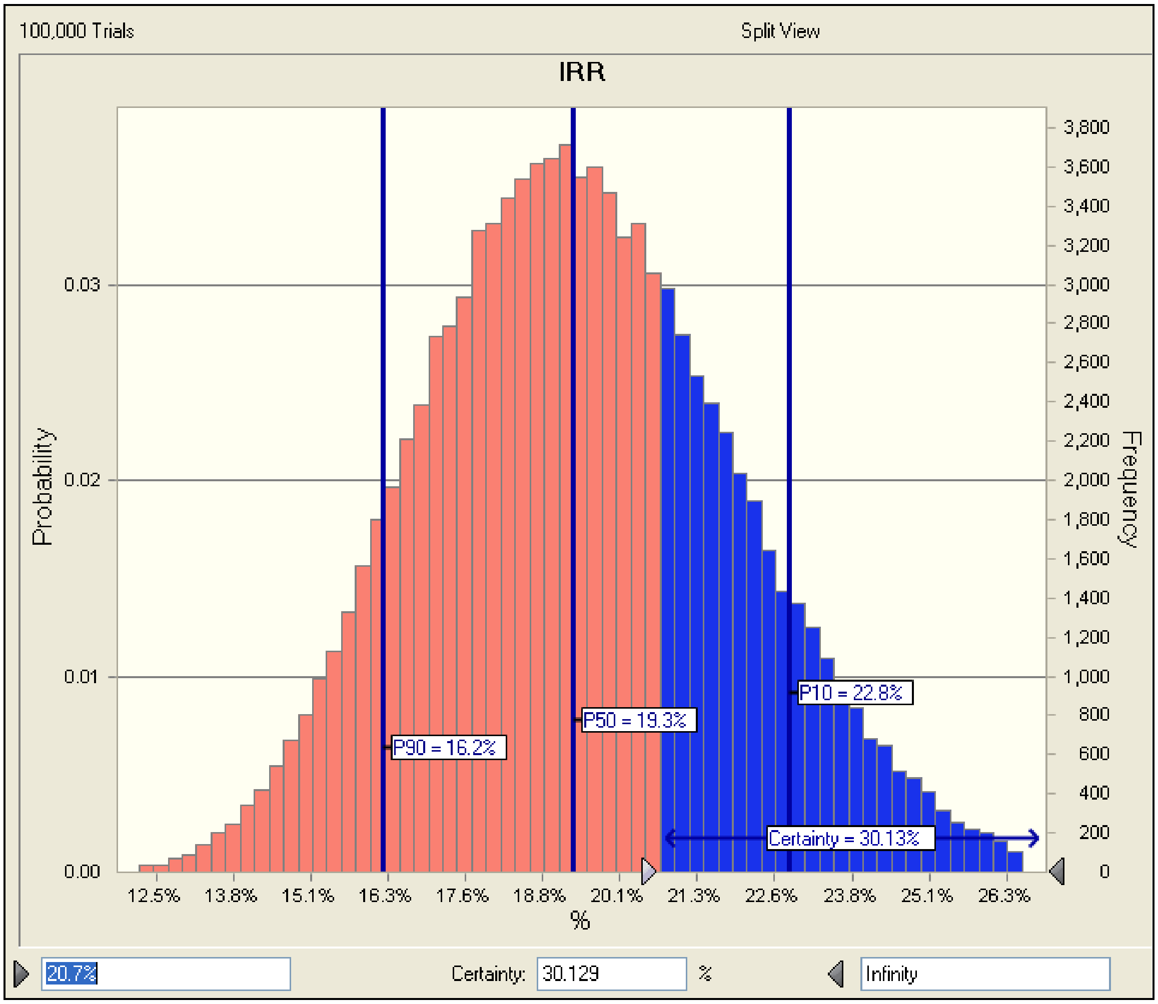
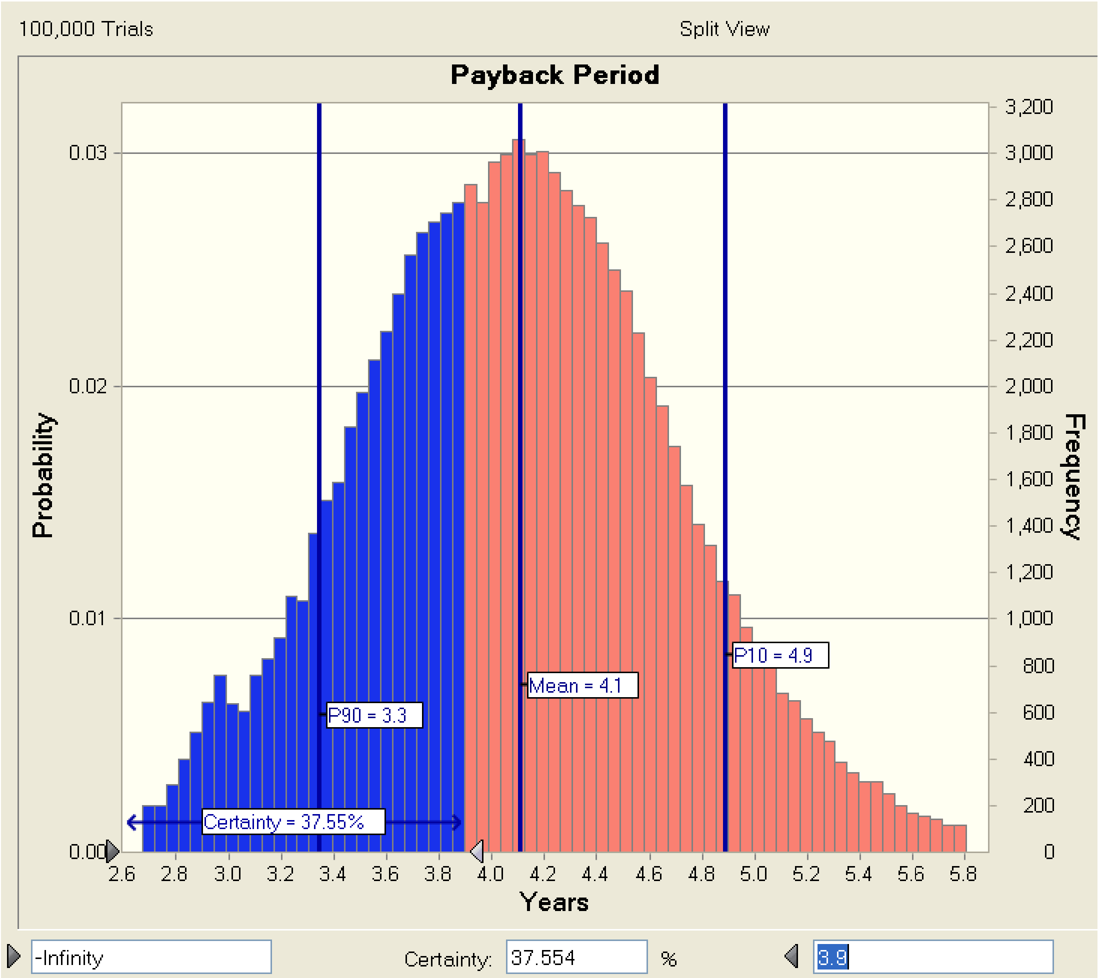


Figure 12. Project A payback period histogram



The following table compares results of the deterministic and probabilistic approaches for Project A.

Table 12. Project A deterministic and probabilistic comparison

Project A		
Valuation Methodology	Deterministic	Probabilistic
Undiscounted Cashflows (US$M) - Base Case/P50 & Mean	2,255	2,091 & 2,095
NPV @ 8% (US$M) - Base Case/ P50 & Mean	706	627 & 629
NPV Range - Low & High Price Protocols/P90 & P10	-69 & 1,479	467 & 795
IRR (%) - Base Case/P50	20.7	19.3
Payback (Years) - Base Case/P50 & Mean	3.9	4.1 & 4.1
Probability NPV>= $706M	?	27%

The probabilistic model has evaluated a large number of copper price scenarios along with the correlated Chilean Peso exchange rate. As a result, the combined affect can be captured on the project's profitability. The deterministic approach identified an NPV of $706M. The probabilistic analysis returned lower values for undiscounted cash flows, NPV, IRR, and payback period. The NPV range shows a significant difference between the two methodologies. The probabilistic model results in a much tighter range than the deterministic model. This difference can be explained by the application of a correlated variable. The NPV range in the deterministic model considered only the low and high copper price protocols. On the other hand, the probabilistic model included the correlated exchange rate behaviour for each simulated copper price. This has a dampening effect on the NPV during years with lower copper prices. On a US dollar basis, lower copper prices correspond with reduced operating costs, and vice-versa, as a result of the fluctuating Chilean peso exchange rate. These types of cause and effect relationships more closely model real life economic conditions.

The Monte Carlo simulation of the probabilistic financial model for Project B incorporates the same parameters as the simulation of Project A (number of iterations, Mean Reverting copper price model, Chilean peso and copper price correlation, rate of reversion, and copper price volatility). Histograms of the simulation for NPV, IRR, and payback period are shown in the following figures.

Figure 13. Project B NPV histogram

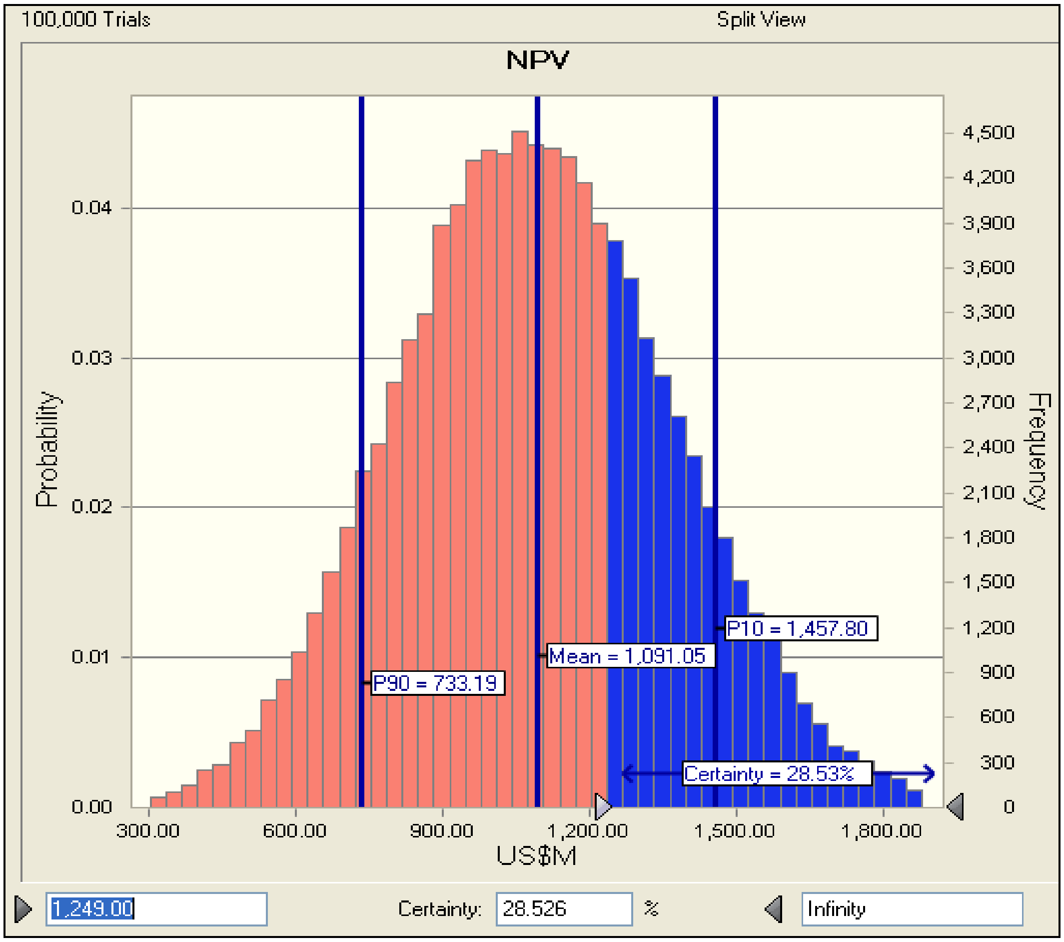


Figure 14. Project B IRR histogram

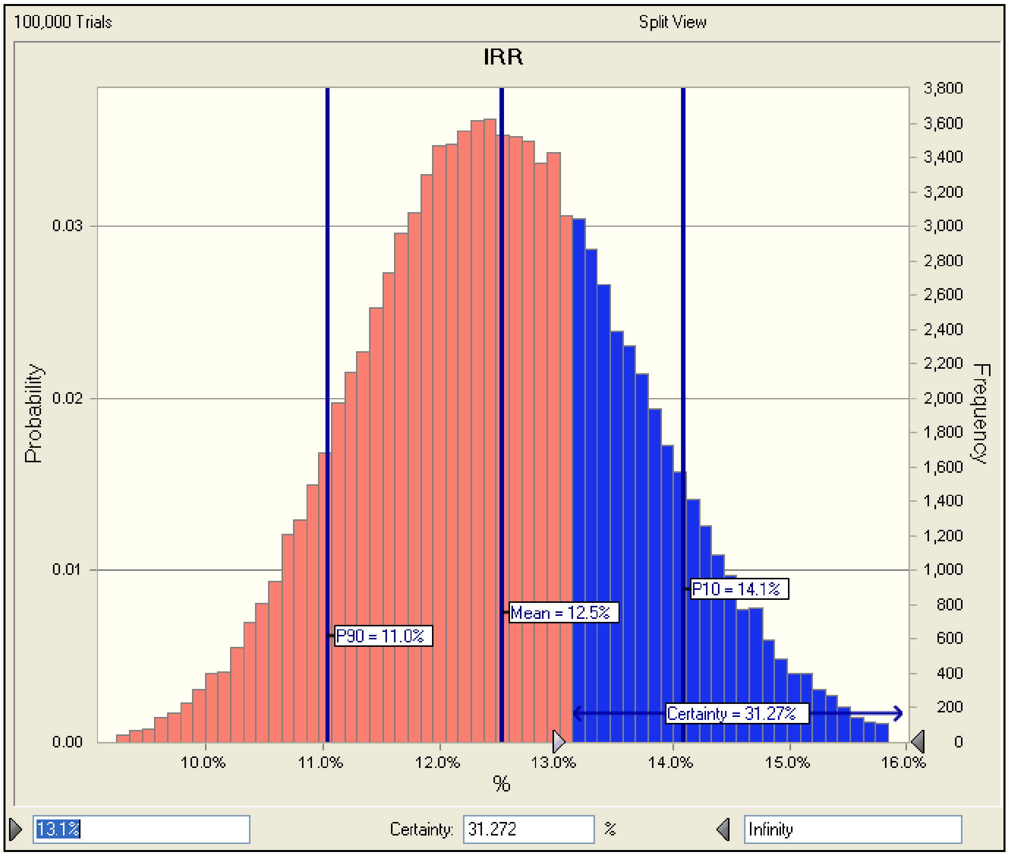
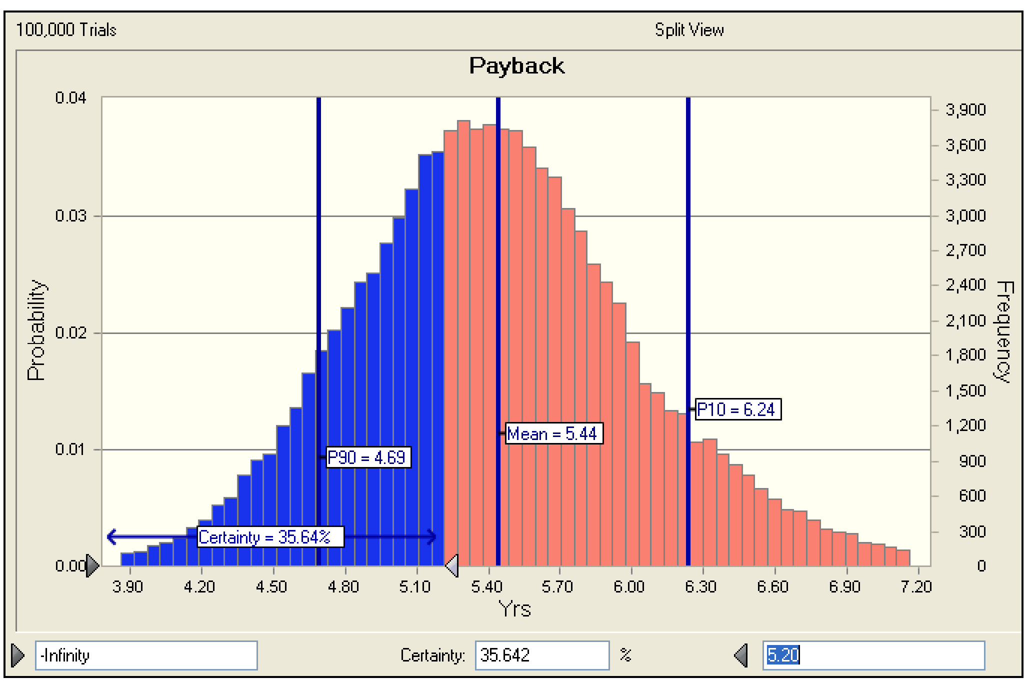


Figure 15. Project B payback period histogram



The following table compares the results of the deterministic and probabilistic approaches for Project B.

Table 13. Project B deterministic and probabilistic comparison

Project B		
Valuation Methodology	Deterministic	Probabilistic
Undiscounted Cashflows (US$M) - Base Case/P50 & Mean	7,113	6,687 & 6,700
NPV @ 8% (US$M) - Base Case/ P50 & Mean	1,249	1,085 & 1,091
NPV Range - Low & High Price Protocols/P90 & P10	-417 & 2,902	733 & 1,458
IRR (%) - Base Case/P50	13.1	12.5
Payback (Years) - Base Case/P50 & Mean	5.2	5.4 & 5.4
Probability NPV>= $1,249M	?	28.5

The probabilistic valuation methodology is showing less favourable project economics than those obtained from a deterministic approach. The biggest difference is in the estimation of the NPV. The P50 and mean NPV values are approximately $160M lower using the probabilistic financial model. Furthermore, the range of possible NPVs is tighter than what was obtained in the deterministic 1-point sensitivity analysis. As described earlier, the correlated nature of the variables in the probabilistic model captures the inverse relationship of the copper price and Chilean peso exchange rate and their impact on the project economics. Finally, we see that the probabilistic model shows only 28% likelihood that Project B will deliver an NPV equal to or greater than the NPV identified in the deterministic approach.

Finally, Monte Carlo simulations of a probabilistic financial model for Project C were carried out. The simulations were performed under the same conditions as the previous two models. The following histograms show the results of the simulations.

Figure 16. Project C NPV histogram

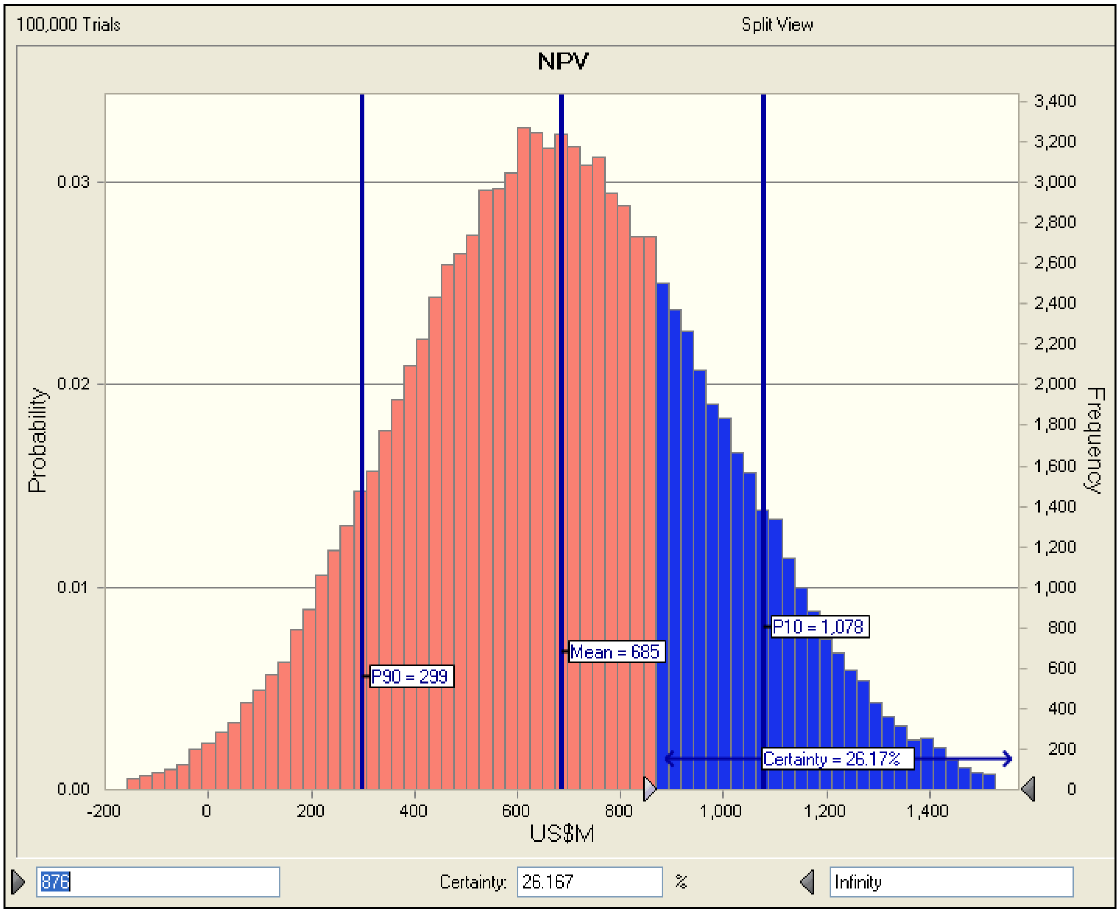


Figure 17. Project C IRR histogram

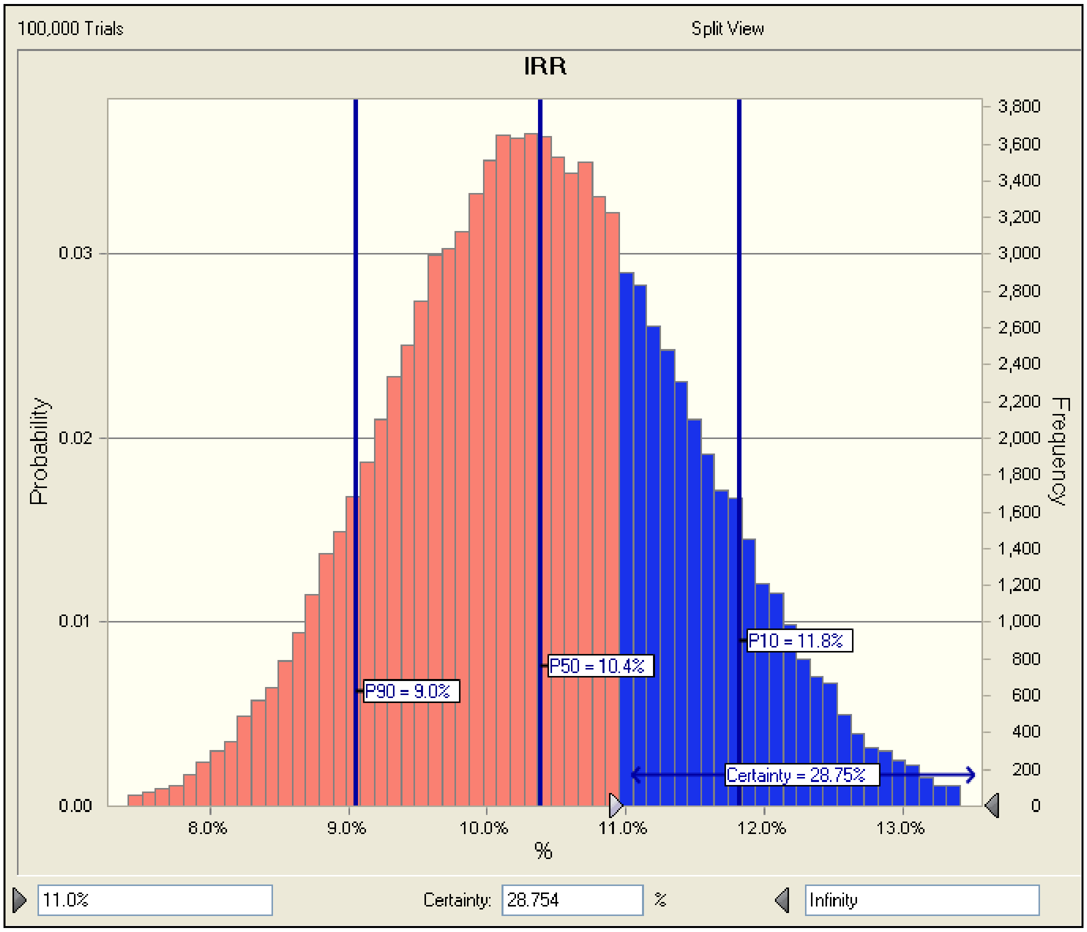
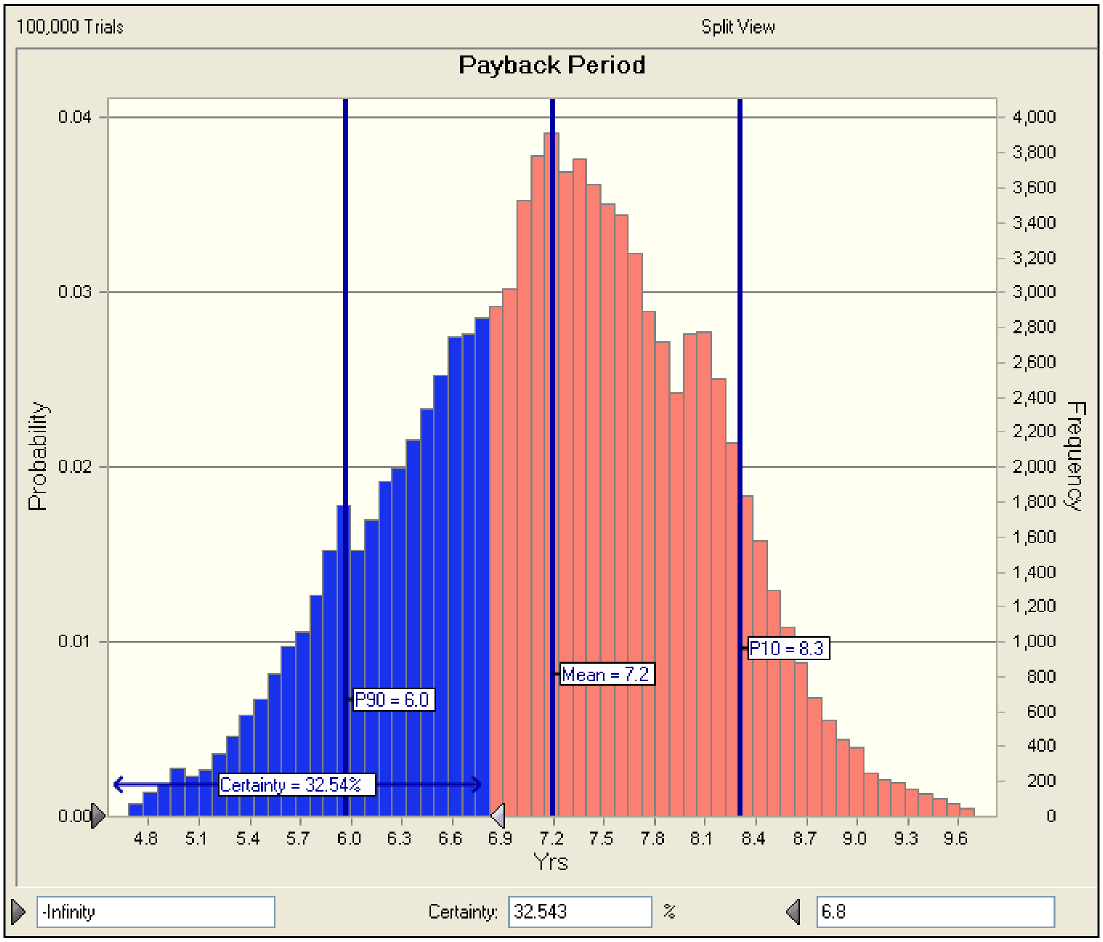


Figure 18. Project C payback period histogram



The following table compares the results of the deterministic and probabilistic approaches for Project C.

Table 14. Project C deterministic and probabilistic comparison

Project C		
Valuation Methodology	Deterministic	Probabilistic
Undiscounted Cashflows (US$M) - Base Case/P50 & Mean	9,282	8,554 & 8,565
NPV @ 8% (US$M) - Base Case/ P50 & Mean	876	679 & 685
NPV Range - Low & High Price Protocols/P90 & P10	-1,183 & 2,873	299 & 1,078
IRR (%) - Base Case/P50	11.0	10.4
Payback (Years) - Base Case/P50 & Mean	6.8	7.2 & 7.2
Probability NPV>= $876M	?	26

Results consistent with the previous projects are seen from the probabilistic model applied to Project C. Most model outputs underestimate the values derived by the deterministic model. An exception is the NPV range, which is again much tighter in the probabilistic case.

We have not explored the impact of parameters, such as reversion and volatility, used in the probabilistic model. The volatility parameter is taken directly from the original deterministic analysis. The deterministic sensitivity analysis tested copper prices of +30% and -30% from the assumed base case prices. As a result, a volatility value of 0.3 can be considered to be a good input into the probabilistic model. However, current deterministic practices do not give any indication about the reversion parameter that might be appropriate. The reversion parameter describes how quickly a variable returns to the equilibrium level and is inversely related to the time it takes deviations to return to the equilibrium level. Therefore smaller values indicate longer reversion times and vice versa. To test the impact of the reversion parameter on model outputs, a range of values was tested. The following figure summarizes the model outputs using different assumptions for the reversion parameter on Projects A, B, and C.

Table 15. Probabilistic model sensitivity to reversion parameter

Reversion Parameter	0.2	0.3	0.4	0.5	0.6
Project A					
Cashflows (US$M) - Mean	2,090	2,090	2,095	2,100	2,095
NPV @ 8% (US$M) - Mean	636	631	629	629	630
NPV Range - P90 & P10	399 & 878	437 & 831	467 & 795	491 & 769	510 & 750
IRR (%) - P50	19.6	19.4	19.3	19.3	19.3
Payback (Years) - Mean	4.1	4.1	4.1	4.1	4.1
Probability NPV >= $706M	35%	31%	27%	24%	21%
Project B					
Cashflows (US$M) - Mean	6,598	6,647	6,700	6,735	6,769
NPV @ 8% (US$M) - Mean	1,056	1,072	1,091	1,105	1,119
NPV Range - P90 & P10	548 & 1,585	651 & 1,508	733 & 1,419	796 & 1,419	856 & 1,388
IRR (%) - P50	12.4	12.5	12.6	12.6	12.6
Payback (Years) - Mean	5.55	5.5	5.41	5.41	5.38
Probability NPV >= $1,249M	31%	30%	27%	27%	27%
Project C					
Cashflows (US$M) - Mean	8,374	8,483	8,565	8,628	8,686
NPV @ 8% (US$M) - Mean	638	664	685	702	717
NPV Range - P90 & P10	92 & 1,203	207 & 1,135	299 & 1,078	371 & 1,037	435 & 1,004
IRR (%) - P50	10.2	10.3	10.4	10.5	10.5
Payback (Years) - Mean	7.3	7.2	7.2	7.2	7.1
Probability NPV >= $876M	29%	28%	26%	25%	24%

The above analysis suggests that the probabilistic model outputs remain in a tight range across all values of the reversion parameter. Although the mean NPV does not change significantly for Project A, there is more variability in NPV for Projects B and C. The sensitivity testing showed larger reversion values (shorter reversion times) to be associated with narrower NPV ranges and lower likelihoods of achieving or exceeding the deterministic NPV estimate. As expected, the larger reversion values (shorter reversion times) return prices to equilibrium levels quicker and generate less widespread price profiles, forming a narrower range of outputs. As a result, the likelihood of the deterministic outputs falling within the narrower range is lower. Aside from the range of NPV outputs, the data suggest that the rate of reversion has a little impact on the economics of Projects A, B, and C.

In this section, three probabilistic models were developed and the results contrasted with those from deterministic models. These models incorporated a Mean Reverting stochastic model to simulate a wide range of possible copper price scenarios. This model predicts less extreme copper price values than the Random Walk models and incorporates assumptions from the deterministic approach. The static copper price assumption from the deterministic model was used as the long term price to which copper prices revert in the probabilistic model. In addition, the volatility parameter required for the probabilistic model was based on the range of copper prices tested with the deterministic 1-point sensitivity analysis. The use of established parameters in the probabilistic model provides a connection with familiar processes which could assist in adoption of the probabilistic methodology. Furthermore, behaviour of the Chilean Peso exchange rate was correlated with the price of copper. As a result, the ability to capture the impacts of realistic behaviour due to inter related variables was demonstrated. In general, comparison of the deterministic and probabilistic methodologies shows that the valuation metrics are less favourable for the example projects when probabilistic models were used. The influence of variable copper price profiles and correlated exchange rate behaviour resulted in lower average values compared with the absolute deterministic outputs. The simulation approach of probabilistic financial modelling has introduced a new metric to the valuation toolkit: probability. The likelihood of achieving certain metrics, or ranges, can now be utilized in evaluating and comparing projects. The next section explores the application of information generated from the probabilistic models.

5.0 Application

The probabilistic models have taken risk analysis beyond the capabilities of the deterministic models. More simplistically, a deterministic model, along with the 1-point sensitivity analysis, only affords the ability to test a limited number of variable combinations. The probabilistic approach is most ideally suited either to marginal projects where the sensitivity to input variables is high, or to projects with a high degree of complexity. In both cases, evaluating a large number of iterations assists in better understanding the project risks.

The main application of probabilistic modelling is to ascertain the uncertainty of cash flows generated from a project. The methodology can be applied to a single project to gauge risk, or to a series of projects in order to prioritize alternatives. For a single project, the decision process would involve evaluating the likelihood that the project generates a positive NPV. In the case of deciding between multiple projects, the exercise is one of prioritizing. Balancing financial performance with the risk profiles of alternatives is required to assign priorities. Projects could appear attractive when evaluated deterministically, but reveal significant risk when evaluated probabilistically. In the case of the three projects presented in this paper, the application of a probabilistic model showed that the financial metrics could be overstated using the deterministic models. The probabilities of achieving or exceeding the deterministic NPVs were approximately 27% for all three projects; the remainder of the Monte Carlo iterations returned lower NPV values. Although the probabilistic evaluation showed less favourable metrics for all projects, evaluated individually, each appears to support a decision to advance the projects. Recall that the deterministic analysis ranked the projects in the following order: B, C, and A. Reconsidering the order of the projects based on the probabilistic analysis reveals a possible ranking change. Project B continues to outperform the others on the NPV metric, arguably the most important metric. However, the difference between average NPVs of Projects A and C is reduced to \$55M, still in favour of Project C. The decision to prefer Project C over Project A is now more difficult considering the superior rate of return of Project A. As a result, the case could be made that the order of attractiveness should be Project B, A, and C.

The probabilistic evaluation technique also can be used as a strategic tool. Given the widespread use of deterministic evaluation, a probabilistic approach generates information possibly not captured by others. As a result, pricing decisions could be aided with this approach to asset valuation. In our examples earlier, the low likelihoods of achieving the deterministic NPVs should be taken into consideration when developing proposals or reviewing bids for the purchase or sale of assets. Significant discounts could be justified when the likelihood of realizing a stated value is low. On the other hand, high likelihoods could demand prices closer to the estimated value of the asset.

Marginal projects could benefit from a probabilistic evaluation approach. Projects showing a significant downside from a 1-point sensitivity analysis face the potential of being held back on the basis of their risk. A more realistic scenario, where variable behaviour is correlated, could lessen the negative impact of a worst case scenario. Although not necessarily a marginal project, Project B highlights this potential benefit. The project was showing NPV ranges of $-417M to $2,902M deterministically and $733M to $1,458M probabilistically. No longer is the NPV of the project negative on the extreme low end. In general, the tighter NPV range is a result of the inverse relationship on the NPV of copper prices and Chilean peso exchange rates. Although the upside of the project is limited, more importantly, the downside is limited.

The application of probabilistic evaluation has shown that risks inherent in financial models can be measured quantitatively. Monte Carlo simulations allow the user to determine the probability with which any given metric is expected to be achieved. In the case of the most widely used metric, NPV, decision makers are able to consider the chances that a project will generate a positive NPV. In order to utilize probabilistic asset valuation, acceptance of the methodology must first take place within the organization. The challenges of this acceptance are explored in the following section.

6.0 Implementation

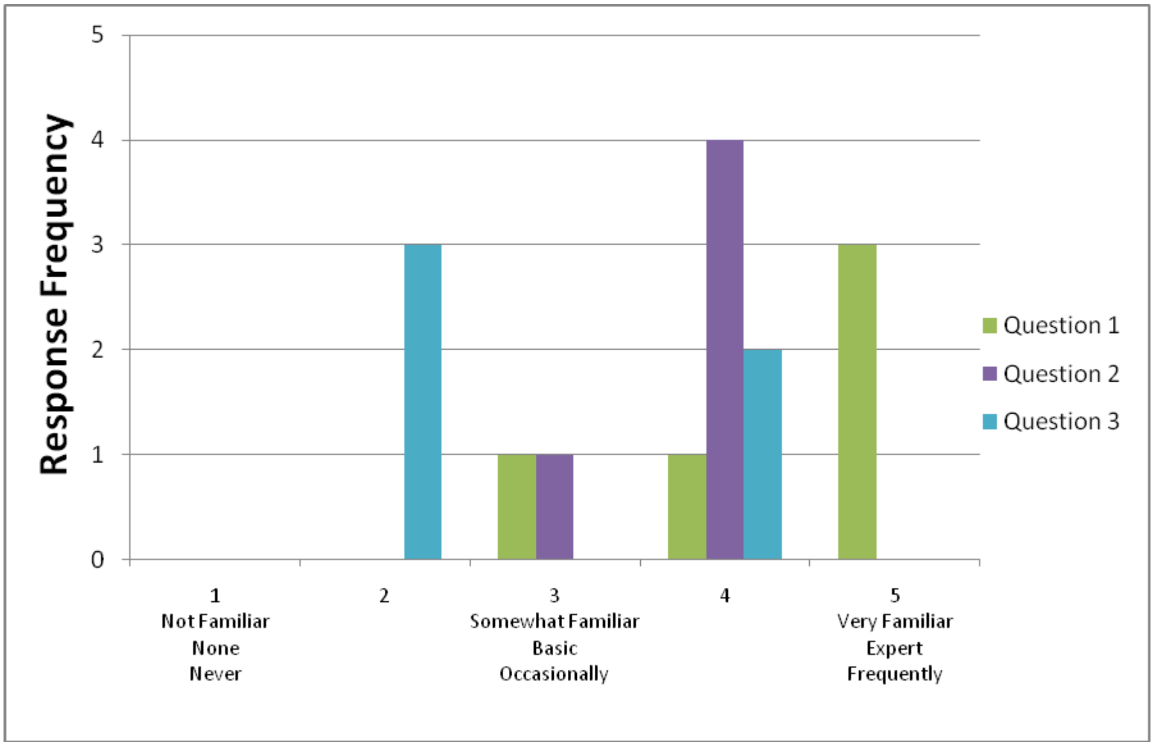
Implementation of probabilistic asset valuation is not meant to be a substitute for deterministic asset valuation, rather it is meant to complement the existing deterministic asset valuation methodology. Nonetheless, showing the potential benefits of this tool does not guarantee adoption. Existing processes with which company objectives are met are likely to be deeply rooted within the organization. As a result, if interest exists within the organization to adopt a new tool, an implementation strategy is required. Two barriers to implementation of probabilistic asset valuation were identified in section 2 of this paper and include subjectivity and understanding of the probabilistic process. In this section, attitudes towards probabilistic asset valuation at Teck are explored and an implementation strategy is derived for the adoption of this valuation methodology.

To assist in the development of an implementation strategy for the use of probabilistic asset valuation at Teck, members of Teck's executive management team were surveyed. Opinions of builders and users of financial models were solicited. Drawing a large number of firm conclusions from the survey is not possible due to the limited sample size (5 responses out of a possible 7). However, the survey can be used to assess high level attitudes toward probabilistic asset valuation. The first three survey questions focused on gauging the executive management team's familiarity with probabilistic valuation and included the following questions:

1. How would you describe your familiarity with the range of asset valuation techniques?
2. How would you describe your knowledge about stochastic processes?
3. Have you ever developed or analyzed the results from a probabilistic asset valuation model?

Responses to the survey are summarized in the following figure.

Figure 19. Teck responses to probabilistic asset valuation survey



Responses to the survey indicate that the knowledge and understanding of probabilistic techniques is present among those who participated in the survey. However, responses to the third question show that there is less experience in actually using this technique. This reflects the generally lower use of valuation techniques, other than deterministic DCF. The remaining two questions polled respondents on their interest in adopting a probabilistic asset valuation technique and the barriers to successful adoption. Interest in adopting this new technique appears to be high. There was a consensus among the respondents in favour of probabilistic asset valuation. Aspects of the responses in regards to barriers to adoption reinforce the attitudes found in academic research. Firstly, respondents identified that there must be understanding of the probabilistic process and the model outputs. The second barrier in the responses included consensus and quality of the probabilistic model inputs. Academic research identified the potential subjectivity of the process as a barrier to acceptance. Lastly, the Teck respondents felt that a change from current processes would result in resistance to adoption. The full responses to the survey can be found in Appendix G.

Kotter's model of change[22] provides eight elements necessary for the successful implementation of change. Several elements of this model are already in place throughout Teck. For example, probabilistic asset valuation has already been used at Teck's Highland Valley Copper and Antamina joint venture mine. At these sites, the use of this tool has been limited to the evaluation of potential mine expansions and other projects instead of applying the tool for a full asset valuation. However, Teck's executive management has received the use of this tool favourably. This has allowed management to witness the potential benefits of this valuation technique. The continued use at Teck's operations will help to disseminate the benefits of this methodology and create short-term wins for the effort of corporate level adoption. On the other hand, several changes are required to the status quo. As the architects of financial models, the corporate finance group would be tasked with developing the probabilistic models. Their acceptance of the additional work is subject to communicating the value added of the probabilistic models by end users. Project sponsors and decision makers within Teck's Business Units would benefit from a better understanding of risk involved with development projects. Therefore, these individuals have a significant role to play in highlighting the benefits of adding probabilistic evaluation into the overall project evaluation process. Following the initial adoption of probabilistic asset valuation, several elements will be required to sustain the momentum. Standard model inputs will need to be defined by the committee that is currently tasked with establishing project evaluation criteria on an annual basis. Subject matter experts, internal or external, can provide recommendations on elements such as the most appropriate type of behaviour models and the applicable model parameters. Finally, training for the developers and users of the probabilistic financial models will be required. Ongoing connection with the academic community would be beneficial in order to take advantage of research in the field of probabilistic analysis. Interaction with user group seminars of probabilistic evaluation would ensure that Teck keeps up with the industry best practices.

[22] Kotter, & Cohen, 2002

7.0 Conclusion

This paper has evaluated the use of an alternative methodology for evaluating assets to deterministic DCF. The probabilistic asset valuation technique goes beyond deterministic DCF in quantifying risk. A deterministic financial model approximates risk qualitatively through sensitivity analysis. In contrast, the probabilistic approach is able to quantify risk with a Monte Carlo simulation. As a result, decision makers are able to consider the likelihoods of achieving certain metrics. This additional information is useful in balancing profitability and risks associated with natural resource projects. Comparison of the two approaches indicates that the financial metrics derived by deterministic models could be overstated. The data from the three projects evaluated suggests that the NPV could be overstated by as much as 22%. The over estimation of the deterministic models is associated with the unrealistic input assumptions which are held constant throughout a project's life. However, the probabilistic model captures the randomness associated with economic variables as well as the correlations between them. The probabilistic models in this report demonstrated the use of a stochastic model for one variable along with the correlated behaviour of another variable. As a result, the inter-related behaviour of these two variables was captured in the valuation of the three projects. In practice, the greater number of variables that are defined stochastically, or are correlated with other variables, will more realistically approximate economic conditions compared with a static set of assumptions. The three projects considered in this report failed to clearly demonstrate how probabilistic asset valuation could re-prioritize alternatives due to balancing financial attractiveness and risk management. However, the results showed that the difference in NPV between Project A and C was smaller than initially identified by the deterministic DCF. This lower NPV differential could lead some decision makers to change their preference towards the projects when other metrics are considered. When applied to marginal projects, probabilistic asset valuation could show a clearer distinction for ranking projects according to financial performance and risk management. Adoption of this methodology will require a cultural shift within some organizations. Familiarity with the status quo and potential subjectivity of the methodology will provide barriers to adoption. As a result, educating builders and users of financial models about the benefits of probabilistic asset valuation is seen as a key element in gaining acceptance. Furthermore, subject matter experts could relieve concerns about subjectivity. Ultimately, probabilistic asset valuation is seen as a supplement to traditional valuation methodologies in the form of an improved sensitivity analysis, rather than as a replacement. The additional insight generated with probabilistic financial models will provide better insight into the risks of projects under consideration. Since natural resource projects tend to be complex, with large capital requirements, a better understanding of risk will allow firms to employ their capital on safer investments with greater confidence.

Appendices

Appendix A

This appendix contains a summary of the deterministic financial model for Project A. Cash flows are derived from production statistics and project assumptions. The main project assumptions are included in Section 3 of this report. These cash flows are used as a basis for comparison with the outputs of the probabilistic financial model derived in Section 4 of this report.

Year	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021
Revenues (US$M)	0	0	461	556	486	438	456	401	401	390	278
Operating Costs (US$M)	0	0	200	198	185	174	172	179	187	182	179
EBITDA (US$M)	0	0	261	359	301	265	284	222	214	208	99
Resource Taxes (Chilean Mining Taxes) (US$M)	0	0	8	12	10	8	9	6	5	5	(1)
Cash Income Taxes (US$M)	0	0	25	40	31	25	28	18	16	15	0
Capital Expenditures (US$M)	200	700	77	7	2	8	3	2	7	6	15
Free Cash Flow (before Δ WC) (US$M)	**(200)**	**(700)**	**152**	**299**	**258**	**224**	**244**	**197**	**185**	**182**	**85**
Discounted Free Cash Flow (After Taxes) (US$M)	(185)	(600)	121	220	176	141	143	106	92	84	36

Year	2022	2023	2024	2025	2026	2027	2028	2029	2030	2031	2032
Revenues (US$M)	329	259	335	338	373	337	347	330	306	327	189
Operating Costs (US$M)	171	163	159	159	157	163	165	157	160	161	107
EBITDA (US$M)	158	97	176	178	215	173	181	173	146	167	81
Resource Taxes (Chilean Mining Taxes) (US$M)	8	5	8	9	10	8	9	8	7	8	4
Cash Income Taxes (US$M)	22	15	27	28	33	27	28	27	23	26	12
Capital Expenditures (US$M)	7	2	15	2	17	2	2	5	1	14	1
Free Cash Flow (before Δ WC) (US$M)	**121**	**75**	**126**	**140**	**155**	**137**	**143**	**133**	**115**	**119**	**65**
Discounted Free Cash Flow (After Taxes) (US$M)	48	28	43	44	45	37	36	31	25	24	12

Appendix B

This Appendix contains results of the 1-point sensitivity analysis performed on the deterministic financial model for Project A. This analysis shows model outputs when individual inputs are varied, and others held constant, by -30% to +30% in 10% increments.

	-30%	-20%	-10%	Base Case	10%	20%	30%
Initial Capital (US$M)	630	720	810	900	990	1,080	1,170
Cash Flow (US$M)	2,467	2,397	2,326	2,255	2,184	2,113	2,042
NPV (US$M)	908	840	773	706	639	571	504
IRR (%)	30.0	26.2	23.2	20.7	18.6	16.8	15.3
Payback (Years)	2.8	3.1	3.5	3.9	4.2	4.5	4.8

	-30%	-20%	-10%	Base Case	10%	20%	30%
Operating Costs (US$/lb Cu)	0.78	0.90	1.02	1.14	1.26	1.38	1.5
Cash Flow (US$M)	3,268	2,930	2,592	2,255	1,917	1,579	1,241
NPV (US$M)	1,149	1,001	854	706	558	410	263
IRR (%)	27.2	25.1	23.0	20.7	18.4	15.9	13.3
Payback (Years)	3.1	3.3	3.5	3.9	4.2	4.6	5.0

	-30%	-20%	-10%	Base Case	10%	20%	30%
Cu Price (US$/lb)	1.75	2.00	2.25	2.5	2.75	3.00	3.25
Cash Flow (US$M)	553	1,120	1,687	2,255	2,822	3,389	3,956
NPV (US$M)	-69	190	448	706	964	1,221	1,479
IRR (%)	6.5	11.8	16.4	20.7	24.8	28.7	32.5
Payback (Years)	7.9	5.7	4.5	3.9	3.3	2.9	2.6

	-30%	-20%	-10%	Base Case	10%	20%	30%
Au Price (US$/oz)	595	680	765	850	935	1,020	1,105
Cash Flow (US$M)	2,060	2,125	2,190	2,255	2,319	2,384	2,449
NPV (US$M)	620	649	667	706	734	763	792
IRR (%)	19.4	19.8	20.3	20.7	21.2	21.6	22.0
Payback (Years)	4.1	4.0	3.9	3.9	3.8	3.7	3.7

	-30%	-20%	-10%	Base Case	10%	20%	30%
Exchange Rate (CLP:USD)	385	440	495	550	605	660	715
Cash Flow (US$M)	1,531	1,832	2,067	2,255	2,408	2,536	2,644
NPV (US$M)	389	521	624	706	773	829	876
IRR (%)	15.6	17.8	19.4	20.7	21.7	22.6	23.3
Payback (Years)	4.6	4.3	4.0	3.9	3.7	3.6	3.5

Appendix C

This appendix contains a summary of the deterministic financial model for Project B. Cash flows are derived from production statistics and project assumptions. The main project assumptions are included in Section 3 of this report. These cash flows are used as a basis for comparison with the outputs of the probabilistic financial model derived in Section 4 of this report.

Year	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022
Revenues (US$M)	0	0	0	0	948	1,062	1,193	1,165	1,131	1,227	997	827
Operating Costs (US$M)	0	0	0	29	304	382	390	392	396	397	394	388
EBITDA (US$M)	0	0	0	(29)	644	680	803	773	734	830	602	438
Resource Taxes (Chilean Mining Taxes) (US$M)	0	0	0	0	14	16	22	20	18	23	11	2
Cash Income Taxes (US$M)	0	0	0	0	42	52	71	65	58	73	35	8
Capital Expenditures (US$M)	429	715	1,106	888	53	48	45	58	36	41	36	41
Free Cash Flow (before Δ WC) (US$M)	(429)	(715)	(1,106)	(917)	535	564	666	630	622	694	520	386
Discounted Free Cash Flow (After Taxes) (US$M)	(413)	(637)	(912)	(701)	378	369	404	354	324	334	232	159

Year	2,023	2,024	2,025	2,026	2,027	2,028	2,029	2,030	2,031	2,032	2,033	2,034
Revenues (US$M)	835	828	817	838	851	887	911	988	1,066	1,227	1,493	1,070
Operating Costs (US$M)	385	385	380	377	377	373	370	368	367	373	389	231
EBITDA (US$M)	449	443	437	461	474	514	540	620	699	854	1,103	839
Resource Taxes (Chilean Mining Taxes) (US$M)	3	20	20	21	22	24	25	29	33	41	53	36
Cash Income Taxes (US$M)	9	65	64	67	70	76	81	94	107	132	173	116
Capital Expenditures (US$M)	53	35	35	69	35	37	10	3	38	38	38	0
Free Cash Flow (before Δ WC) (US$M)	385	323	318	305	347	377	424	493	521	642	839	688
Discounted Free Cash Flow (After Taxes) (US$M)	147	114	104	92	97	98	102	110	107	123	148	113

Appendix D

This Appendix contains results of the 1-point sensitivity analysis performed on the deterministic financial model for Project B. This analysis shows model outputs when individual inputs are varied, and others held constant, by -30% to +30% in 10% increments.

	-30%	-20%	-10%	Base Case	10%	20%	30%
Initial Capital (US$M)	2,196	2,510	2,823	3,137	3,451	3,765	4,078
Cash Flow (US$M)	7,855	7,608	7,361	7,113	6,865	6,617	6,367
NPV (US$M)	1,936	1,707	1,478	1,249	1,020	791	560
IRR (%)	18.3	16.3	14.6	13.1	11.9	10.8	9.9
Payback (Years)	3.9	4.3	4.8	5.2	5.6	6	6.5
	-30%	**-20%**	**-10%**	**Base Case**	**10%**	**20%**	**30%**
Operating Costs (US$/lb Cu)	0.58	0.67	0.77	0.87	0.96	1.06	1.15
Cash Flow (US$M)	8,875	8,288	7,701	7,113	6,525	5,936	5,346
NPV (US$M)	1,918	1,695	1,472	1,249	1,026	802	577
IRR (%)	15.5	14.7	14.0	13.1	12.3	11.4	10.5
Payback (Years)	4.6	4.8	5.0	5.2	5.5	5.7	6.0
	-30%	**-20%**	**-10%**	**Base Case**	**10%**	**20%**	**30%**
Cu Price (US$/lb)	1.75	2.00	2.25	2.50	2.75	3.00	3.25
Cash Flow (US$M)	2,723	4,190	5,654	7,113	8,571	10,028	11,485
NPV (US$M)	-417	142	697	1,249	1,800	2,351	2,902
IRR (%)	6.0	8.6	11.0	13.1	15.1	17.0	18.8
Payback (Years)	10.9	7.2	5.9	5.2	4.7	4.2	3.8
	-30%	**-20%**	**-10%**	**Base Case**	**10%**	**20%**	**30%**
Mo Price (US$/lb)	8.8	10.0	11.3	12.5	13.8	15.0	16.3
Cash Flow (US$M)	6,324	6,590	6,852	7,113	7,375	7,636	7,898
NPV (US$M)	964	1,060	1,155	1,249	1,344	1,438	1,533
IRR (%)	12.1	12.4	12.8	13.1	13.5	13.8	14.2
Payback (Years)	5.6	5.4	5.3	5.2	5.1	5.0	4.9
	-30%	**-20%**	**-10%**	**Base Case**	**10%**	**20%**	**30%**
Exchange Rate (CLP:USD)	385	440	495	550	605	660	715
Cash Flow (US$M)	5,852	6,379	6,787	7,113	7,380	7,603	7,791
NPV (US$M)	770	970	1,125	1,249	1,351	1,435	1,506
IRR (%)	11.3	12.1	12.7	13.1	13.5	13.8	14.1
Payback (Years)	5.7	5.5	5.3	5.2	5.1	5.0	5.0

Appendix E

This appendix contains a summary of the deterministic financial model for Project C. Cash flows are derived from production statistics and project assumptions. The main project assumptions are included in Section 3 of this report. These cash flows are used as a basis for comparison with the outputs of the probabilistic financial model derived in Section 4 of this report.

Year	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020	2021	2022	2023	2024	2025	2026	2027	2028	2029
Revenues (US$M)	0	0	0	0	0	781	1,287	1,196	1,167	1,172	1,077	1,075	1,200	1,351	1,193	1,040	1,026	990	927
Operating Costs (US$M)	0	0	0	0	0	300	489	489	496	498	497	495	495	504	506	512	521	524	519
EBITDA (US$M)	0	0	0	0	0	481	798	706	671	674	580	579	705	846	687	527	506	466	409
Resource Taxes (Chilean Mining Taxes) (US$M)	0	0	0	0	0	3	18	14	12	11	6	6	12	19	32	24	23	21	18
Cash Income Taxes (US$M)	0	0	0	0	0	10	59	44	37	37	21	20	39	61	103	78	75	69	59
Capital Expenditures (US$M)	16	478	1,282	1,032	721	267	76	38	53	56	34	61	45	33	39	49	30	25	51
Free Cash Flow (before Δ WC) (US$M)	(16)	(478)	(1,282)	(1,032)	(721)	201	645	610	568	570	519	493	608	733	513	376	378	351	279
Discounted Free Cash Flow (After Taxes) (US$M)	(15)	(410)	(1,017)	(759)	(491)	127	376	330	284	264	223	196	224	249	162	110	102	88	65

Year	2030	2031	2032	2033	2034	2035	2036	2037	2038	2039	2040	2041	2042	2043	2044	2045	2046	2047
Revenues (US$M)	941	975	1,053	1,130	1,173	1,256	1,186	1,127	1,067	854	745	764	798	848	902	1,003	968	362
Operating Costs (US$M)	520	523	525	529	532	534	535	532	524	513	511	512	505	492	491	489	482	192
EBITDA (US$M)	420	452	527	601	641	722	650	595	543	342	234	252	293	356	410	515	486	169
Resource Taxes (Chilean Mining Taxes) (US$M)	19	21	25	28	30	34	31	28	25	15	10	10	13	16	19	24	23	7
Cash Income Taxes (US$M)	61	67	80	91	97	110	99	89	81	49	31	34	41	52	61	78	74	24
Capital Expenditures (US$M)	25	25	25	66	61	31	53	32	36	34	42	24	25	25	25	15	3	100
Free Cash Flow (before Δ WC) (US$M)	314	339	398	416	453	546	469	445	401	244	151	184	214	263	306	397	386	38
Discounted Free Cash Flow (After Taxes) (US$M)	67	67	73	71	71	80	63	56	47	26	15	17	18	21	22	27	24	2

Appendix F

This Appendix contains results of the 1-point sensitivity analysis performed on the deterministic financial model for Project C. This analysis shows model outputs when individual inputs are varied, and others held constant, by -30% to +30% in 10% increments.

	-30%	-20%	-10%	Base Case	10%	20%	30%
Initial Capital (US$M)	2,660	3,040	3,419	3,799	4,179	4,559	4,939
Cash Flow (US$M)	10,181	9,881	9,582	9,282	8,983	8,682	8,381
NPV (US$M)	1,620	1,372	1,124	876	628	379	130
IRR (%)	15.3	13.6	12.2	11.0	10.0	9.1	8.4
Payback (Years)	4.9	5.5	6.2	6.8	7.4	7.9	8.3

	-30%	-20%	-10%	Base Case	10%	20%	30%
Operating Costs (US$/lb Cu)	0.90	1.02	1.14	1.26	1.38	1.50	1.62
Cash Flow (US$M)	13,017	11,772	10,527	9,282	8,037	6,792	5,544
NPV (US$M)	1,773	1,474	1,175	876	577	278	-23
IRR (%)	13.6	12.8	11.9	11.0	10.1	9	7.9
Payback (Years)	5.6	6	6.4	6.8	7.3	7.8	8.3

	-30%	-20%	-10%	Base Case	10%	20%	30%
Cu Price (US$/lb)	1.75	2.00	2.25	2.50	2.75	3.00	3.25
Cash Flow (US$M)	1,461	4,088	6,690	9,282	11,873	14,464	17,055
NPV (US$M)	-1,183	-465	210	876	1,542	2,208	2,873
IRR (%)	2.6	6.1	8.8	11.0	13.1	14.9	16.7
Payback (Years)	17.4	9.7	8.0	6.8	5.8	5.0	4.5

	-30%	-20%	-10%	Base Case	10%	20%	30%
Mo Price (US$/lb)	8.8	10.0	11.3	12.5	13.8	15.0	16.3
Cash Flow (US$M)	8,370	8,674	8,978	9,282	9,586	9,890	10,194
NPV (US$M)	644	722	799	876	953	1,031	1,108
IRR (%)	10.3	10.5	10.8	11.0	11.3	11.5	11.7
Payback (Years)	7.2	7.1	7	6.8	6.7	6.6	6.5

	-30%	-20%	-10%	Base Case	10%	20%	30%
Exchange Rate (CLP:USD)	385	440	495	550	605	660	715
Cash Flow (US$M)	6,614	7,726	8,591	9,282	9,848	10,320	10,719
NPV (US$M)	235	503	710	876	1,012	1,125	1,221
IRR (%)	8.9	9.8	10.5	11.0	11.5	11.8	12.1
Payback (Years)	7.9	7.4	7.1	6.8	6.6	6.4	6.3

Appendix G

This Appendix contains results of a survey taken by Teck's senior executives. The intent of this survey was to gather information about the level of understanding of probabilistic asset valuation and barriers to adoption that may exist within Teck. This information was used help develop an implementation plan for probabilistic asset valuation at Teck.

1. **How would you describe your familiarity with the range of asset valuation techniques?**

Not Familiar		Somewhat Familiar		Very Familiar	Total
		1	1	3	5

   - Familiar with "traditional" asset valuation techniques as well as those regularly practiced in the Mining industry. Less familiar with the range of detailed valuation techniques commonly used in financial markets related to derivative, trading (call, put, short, options), and/or complex debt-, equity- and/or asset linked products.
   - Have been presented with evaluations from a wide range of processes, and have been the "principal client" of quite a few.

2. **How would you describe your knowledge about stochastic processes?**

None		Basic		Expert	Total
		1	4		5

- Reasonable to good understanding of the statistics behind and mechanics of stochastic modelling processes although somewhat out of practice, i.e. software, set-up routines, and discrete applications.
- Good working knowledge and use in the past, a bit rusty now.

3. **Have you ever developed or analyzed the results from a probabilistic asset valuation model?**

Never		Occasionally		Frequently	Total
	3		2		5

- Yes. However, rarely practiced within Teck's Business Development function. Used Crystal Ball, @Risk and Precision Tree at various times. I think I actually purchased licenses to each a few years back.
- Have participated in (in terms of agreeing probability distributions and ranges) and analysed results from processes such as those based on Crystal Ball.

4. **Would you consider developing a probabilistic asset valuation model to be a useful exercise? Why or why not?**

- Yes. It allows for multiple input scenarios, with ranges of possible outcomes, to be incorporated into the assessment of an asset valuation exercise which otherwise cannot be done with traditional valuation approaches. The exercise itself, combined with a pragmatic approach to understanding how the market values assets and opportunities, is useful.
- Yes. For example, in dealing with 3 looming copper projects, each with different risk characteristics, this may be a good method to "level the playing field" in assessment of project risks
- Yes. We can be more sophisticated in our evaluations by combining scenario analyses and probabilistic evaluations to get a better handle on the range of values and the risks.
- Yes. Even just to get a better feel for the range in probable outcomes. Defining and justifying assumptions can be a challenge which could make implementation/acceptance more difficult.
- Yes as it provides the range of probabilities of outcome and will give us perspective as to when and how frequently a project will or will not make money and help us assess the risk when making investment decisions. Single cases with some sensitivity does not really speak to probability of an outcome and therefore risk/reward are not properly assessed.

5. What are the barriers to implementing probabilistic asset valuation modelling at Teck?

- 1. Awareness and understanding of the processes involved in producing a reliable model and understandable output. 2. Alignment on assumptions for inputs, i.e. prices, costs, ramp up, head grade variability, process variability, and process up time. 3. Quality of data (and analysis of said data) to inform the assumptions.
- Only management decision to proceed with it, and possibly the interpretations of the results
- The biggest barrier at this stage is acceptance of the techniques in providing more robust project valuations, and how to appropriately use the outputs. I believe we have the skills and understanding at the technical levels (both engineering and financial) to apply the techniques. But developing the inputs will invoke further work by the technical specialists.
- There is an industry standard way of looking at things (which is true within Teck too). Change is not always welcome or implemented easily.
- Overall understanding of the benefits of the approach, time and training, and the fear of the need to apply probabilities which creates uncertainty in many peoples' minds when they are looking for absolutes.

6. Any other thoughts or comments?

- I wouldn't get too worked up about implementation rather I would focus on preparing three well constructed and though out models for investments: 1. Relincho, 2. An "infrastructure or mobile equipment" investment, i.e. Elk Valley or HVC, and 3. Exploration or Advanced Exploration project including an evaluation of comparable opportunities/projects in the marketplace; suggest San Nicolas. Use the output from these models to you have the conversation with key stakeholders, i.e. projects, engineering, bus dev, and finance, about how to use stochastic modelling approach to valuation, and then steps to implement more regularly.

- These techniques are being more widely adopted, and as a minimum Teck needs to keep up. Over analysis is quite possible, so an early step is to developing a broad understanding of the purpose and value of using these techniques.

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