ROBUSTNESS OF ASSET LOCATION DECISION CONSIDERATIONS

by

Hanzhao Zhang
Bachelor of Science, Nanjing University 2008

and

Kaisheng Sun
Bachelor of Business Administration, Shandong University of Science and Technology 2010

PROJECT SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF SCIENCE IN FINANCE

In the Master of Science in Finance Program
of the
Faculty
of
Business Administration

SIMON FRASER UNIVERSITY

Fall 2014

Approval

Name: **Kaisheng Sun & Hanzhao Zhang**

Degree: **Master of Science in Finance**

Title of Project: **Robustness of Asset Location Decision Considerations**

Supervisory Committee:

_____

Dr. Peter Klein
Senior Supervisor
Professor

_____

Dr. Derek Yee
Second Reader
Adjunct Professor

Date Approved: _____

Abstract

Asset location, which means locating different types of assets to accounts with different tax treatment, is one of the investment decisions investors should consider. Conventional wisdom believes that it is preferable to hold bonds in taxable accounts and stocks in Tax-Deferred Accounts, but recent studies reveal that it is not true (Reichenstein, Hora, and Jennings, 2012). Our study further investigates the robustness of results in aforementioned paper.

Researchers believe that in most cases, it is preferable to hold bonds in TDAs and to stocks in taxable accounts. However, sensitivity analysis shows that return and risk profile of assets, risk tolerance of investors, and tax rate of different assets can reverse the preference, within reasonable ranges. The discussion also uses numerical examples to illustrate optimal weights under different assumptions, and historical data is used to prove the plausibility of numerical examples.

Keywords: asset location; taxable accounts; tax-deferred accounts; sensitive analysis.

Table of Contents

1 Introduction

When making investments, investors normally have several different types of accounts to choose to hold the money and assets. Take North America as example, financial institutions in both Canada and the US offer accounts covering three types, Taxable Account, Tax-Deferred Account (TDA), and Tax- Exempt Account (TEA). Taxable Account is the most common one, where tax is charged by the government at a percentage of the profit in a certain period and not applied with money being withdrawn from the account. TDA is a benefit the government gives to investors to defer their earnings in taxable income calculations. Tax applies when investors transfer money out of TDAs based on a percentage of the total transactions amount regardless the earning history in the account. Normally the withdrawal is considered as part of total income in the year it happens. Examples of TDA are 401(k), Individual Retirement Account (IRA) in the US and Registered Retirement Savings Plan (RRSP) in Canada. TEA is the type of account free of tax no matter how much the profit and the withdrawal. There is usually a limit on each TEA depend on the circumstance. Roth and 529 plans in the US and Tax-Free Saving Account (TFSA) in Canada are all TEAs.

In the case that investing two types of assets, stocks and bonds, in taxable accounts and TDAs, it is commonly believed that stocks should be held in TDAs and bonds should be held in taxable accounts, because deferring higher stock returns in TDAs could increase profitability in later years. However, scholars argue that conventional wisdom is actually not correct (Reichenstein, et al, 2012; Reichenstein & Meyer, 2013). They use specific cases to demonstrate that in order to maximise profits, the prevailing suggestion is to hold stocks in taxable accounts and bonds in TDAs. A few sensitivity analyses, including low returns and high risk tolerance, are also discussed as complement.

This paper is a further investigation of asset location decisions, continuing from those two papers. The discussion uses numerical examples in the first paper as a base

case, examines the robustness of previous results, revises some statements, and introduces new findings of asset location decision considerations.

Next part of this study reviews those two papers, summarises their main ideas and provides the current stage of the research. In the third part, five independent variables are tested under one-factor-at-a-time method. Results are discussed in subsections of Sensitivity Analysis part and presented in the Conclusion part. In addition, historical data is tested to verify the existence of our numerical examples.

2 Literature Review

2.1 After-Tax Asset Allocation

Asset location has been studied by a number of authors. Reichenstein (2006) focuses on the importance and necessity of using after-tax value in asset allocation.

There are several reasons why after-tax value should be used. The first reason is that in order to buy goods and service, only after-tax fund is used. The second reason is that if tax is neglected, tax-deferred accounts are actually the same as tax-exempt accounts.

In addition, models are used to test the impact of choosing different saving vehicles on investor's ownership of principal, return and risk. The models include tax-exempt accounts, tax-deferred accounts, and taxable accounts. In addition, bonds investment and different types of stocks investment are discussed under the taxable accounts. Types of stocks investment are classified according to the trading frequency and include investments performed by day trader, active investor, passive investor, and exempt investor. Multi-periods method is used and end value of assets in different accounts is estimated.

Then several conclusions are reached. The first conclusion is that for TDA accounts, it can be viewed as the investor owns $(1-t_r)$ of principal, receives all returns and bears all risks. Here t_r is the tax rate in the withdrawal year. The second conclusion is that for taxable accounts, all types of investors except the exempt investors own all principal but owns only a portion of return and risk. The exempt investors own all principal, all return, and all risk because they either donate the stock to charity or pass the stock to beneficiary at death. The portion of return and risk owned by investors vary and depends on the trading strategy. Passive investors own more than $(1-t_c)$ of return and risk; active investors own $(1-t_c)$ of return and risk; day traders own $(1-t)$ of return and risk. Here t_c refers to long-term capital gain, t_r refers to ordinary income tax after

retirement, and t refers to ordinary income tax $(t_c < t_r < t)$. The third conclusion is that the same asset can be a different asset if hold in different accounts. So, they have different return and risk characteristics when mean-variance optimization is used.

Another conclusion is that investors should put bonds in TDAs and stocks in taxable accounts. This conclusion comes from the fact that there is a spread between ordinary income tax and effective tax rate paid on stocks in the taxable account. And this asset location is especially true for investors who has high income and manage stocks passively in taxable account.

2.2 Asset Location: A Generic Framework for Maximizing After-Tax Wealth

Different approaches are used to deal with asset location problem, and one approach is to solve it case by case. Daryanani and Cordaro (2005) argue that there is no general rule for asset location, because asset location decisions are depend on client's financial profile, prevailing tax laws and tax characteristics for different asset classes.

A "Difference Approach" is used to deal with multiple assets and multiple accounts. The idea is that for a specific asset, the returns of putting it in different account is compared to determine which account is more superior. If the difference is large, then there is a high priority to put the asset in the larger return accounts. If the difference is small, then the priority of locating this asset to different accounts is low.

To evaluate the performance of different asset locations, assumptions are made that the equity exposure and asset allocation is given, and they use after-tax end wealth as the key metrics of performance. After that different cases are studied to find out the optimal asset location.

Also, asset classes are classified according to pre-tax returns and tax efficiency, and then conclude that for high pre-tax return and low tax efficiency assets, it is preferable to be hold in TDAs, because the deferring returns could bring more benefits. For the high pre-tax return and high tax efficiency assets, it is preferable to be hold in taxable accounts, because the benefits from lower tax rate in taxable accounts exceeds

benefits from deferring returns in TDAs. For the mid tax efficiency asset, the optimal asset location decisions is highly dependent on the client's financial profiles.

2.3 Two Key Concepts for Wealth Management and Beyond

Frameworks such as single period mean variance optimization (MVO) and utility function are also used to deal with asset location problem. Based on these frameworks, Reichenstein, Hora, and Jennings (2012) find that tax has significant impact on asset location decisions.

Previous studies tend to ignore tax when doing the optimization, but tax matters because that government will eventually take money from investor's accounts and therefore decrease investor's utility, and how government takes the money also affects investor's utility. Many countries set different tax rules for different accounts, such as taxable accounts and tax-deferred accounts (TDAs). Accordingly, differences among accounts bring the asset location issue, that is, what the optimal way is to distribute investment to different accounts.

The paper also reveals errors in traditional approach by using an analogy in the relationship between the government and investor and stated that government is a limited partner to investor in TDAs, and the government is a sharer of return and risk in taxable accounts.

Conventional wisdom believes that it is preferable to hold stocks in TDAs and bonds in taxable accounts, but numerical examples in the paper show opposite results, that it is preferable to hold stocks in taxable accounts and bonds in TDAs.

These numerical examples are constructed with several assumptions. First, the optimization is limited by only types of two accounts, taxable accounts and TDAs, and only two types of assets, equity and bond. A simple tax structure is assumed that tax rates only change because of different assets. Equity income tax rate applies to stock investment, and interest income tax rate applies to bond investment. Due to different relationship between government and investor, there are four assets with different return and risk characteristics: stocks in taxable accounts, stocks in TDAs, bonds in taxable accounts, and bonds in TDAs. Also, correlation between stock in taxable accounts and

that in TDAs is one, and it is the same for bonds. In addition, returns and volatilities of stocks or bonds in taxable accounts are in proportion to after tax percentage of each asset.

Then the conclusion from the numerical examples is used to further indicate implications for choice of savings vehicles, Roth Conversions, withdrawal strategies and estate planning.

2.4 The Asset Location Decision Revisited

To make conclusions in previous paper more comprehensive, Reichenstein and Meyer (2013) revisit conclusions of asset location decisions made by Reichenstein, Hora, and Jennings (2012).They reiterate the relationship between the government and investors is best viewed as a partnership, or a sharer in risk and return. In TDAs, the government only claims certain percentage of principal ownership but this does not change return and risk characteristics of remaining investment. In taxable account, on the other hand, principal is still the same, but return and risk decrease proportionally according to tax rate.

In addition, it is argued that traditional approach of making asset location is wrong. First, some of the researches are only based on limit number of specific inputs, while result may reverse under other inputs. Second, the traditional approach is theoretically not precise. It break down the investment decision into a two-step procedure, that is, determine asset allocation first, then asset location. Actually, it assumes that asset location does not change the risk profile, while numerical examples proved it to be incorrect. In addition, asset allocation will change over time because of wealth accumulation. As a result, the most appropriate way is using mean-variance optimization.

In conclusion, based on mean-variance optimization approach, it is preferable to hold stocks in taxable accounts and bonds in TDAs, which is the same as his first paper. To support this conclusion, more independent variables are tested in this paper, including risk aversion levels, returns of stocks and bonds, and risk free rates. Test results show that only in some rare cases, including low interest rate and investors with extremely high risk aversion, it is better to hold bonds in taxable accounts and stocks in TDA. However,

under usual conditions, prevailing advice for investor is to hold stocks in taxable accounts and bonds in TDAs.

3 Sensitivity Analysis

The analysis in this part is based on previous studies. Reichenstein, Horan and Jennings (2012) conclude that government plays different roles in investor's taxable accounts and TDAs. And, in order to maximize utility, it is preferable for investors to hold stocks in taxable accounts and bonds in TDAs. However, these conclusions are researched from one specific scenario, though all assumptions are reasonable. As a result, Reichenstein and Meyer (2013) studies more scenarios and the concludes that only in some extreme scenarios, including scenarios that returns of stocks and bond decreased to a specified level or investor's risk aversion is very high, optimal asset location may reverse.

Nonetheless, factors other than discussed in the second paper may also influence the asset location decisions. So, this section studies tax affect in a more comprehensive way, and considers independent variables such as risk tolerance, return, volatility, correlation and tax rate. All the tests are based on the base case in section 3.1, and under one-factor-at-a-time method.

3.1 The Base Case

The analysis in this section is to replicate the base cases from previous paper "Two Key Concepts for Wealth Management and Beyond" (Reichenstein, et al, 2012). The assumptions are the same: equity market annual return is 7% with a 15% volatility, and bond market annual returns is 3% annually with a 6% volatility. The correlation between the two markets is 0.2. The tax rate is 28% for ordinary interest income, and 20% for equity income including both dividends and capital gain.

The utility function for all investors is $Utility = ER - \sigma^2 / RT$, where ER and RT stand for expected return and risk tolerance, respectively, and σ is standard deviation. The optimisation goal is maximising the utility function.

Tax is neglected in base case 1. As a result, principal in TDAs and return-risk profile in taxable accounts are not affected, and it makes no different to invest in TDAs or taxable accounts. In that case, only asset allocation is considered because utility will not change as long as the asset allocation is the same. However, in the base case 2, tax is taken into account. As a result, principal in TDAs is reduced by 28% and to the same level of taxable accounts. In the meanwhile, according to the tax rates applied to stocks and bonds, return and volatility of stocks are reduced by 20%, and returns and volatility of bonds in taxable accounts are reduced by 28%.

For base case 1, the inputs used in the optimization is stocks return (7%), stocks volatility (15%), bonds return (3%), bonds volatility (6%), correlation between stocks return and bonds return (0.2), and risk tolerance of investor (0.4725). In the meanwhile, we set constraints that 1 million is invested in TDAs and .72 million is invested in taxable accounts. Also, there is no short position in any asset and in any accounts. Then fmincon, a MATLAB built-in function, is used to perform the optimization. This optimization function gives optimal weights of stocks and bonds, and then the optimization result is combined with budget constraints in each account, and finally one possible solution is given in Table 3.1.1.

For base case 2, the return and volatility of stocks and bonds in taxable accounts are adjusted, and the following equation shows how they are adjusted:

$$r_S^{AT} = r_S^{BT} * (1 - t_E)$$
$$\sigma_S^{AT} = \sigma_S^{BT} * (1 - t_E)$$
$$r_B^{AT} = r_B^{BT} * (1 - t_I)$$
$$\sigma_B^{AT} = \sigma_B^{BT} * (1 - t_I)$$

(S-Stock, B-Bond, E-Equity, I-Interest, AT-After Tax, BT-Before Tax)

So, the inputs used in the optimization is stocks return in taxable accounts (5.6%), stocks volatility in taxable accounts (12%), bonds return in TDA (2.16%), bonds volatility in taxable accounts (4.32%), correlation between stocks return and bonds return (0.2), and risk tolerance of investor (0.4725). In the meanwhile, in order to reflect the partnership between the government and investors, we set different constraints that .72 million is invested in TDAs and .72 million is invested in taxable accounts. Similar to

base case 1, there is no short position in any asset and in any accounts. Then fmincon, a MATLAB built-in function, is used to perform the optimization. This optimization function gives optimal weights of stocks and bonds in TDAs and taxable accounts, and then the optimization result is combined with budget constraints in each account, and finally the unique solution is given in Table 3.1.1.

We generate the same results as the previous mentioned paper. Table 3.1.1 shows optimal allocation and location result when taxes are ignored. Stocks and bonds are equally held in dollar value in total, which is a reasonable and easy starting point for future comparison. Although TDAs and taxable accounts are the same due to tax neglect, we continue to use the same results as predecessor, that is, stocks and bonds are equally weighted in both accounts.

Table 3.1.1 Base Case 1

Base Case - Ignore Taxes				
	Market Value	Optimal Weights	Pre−tax Expected Return	Pre−tax Standard Deviation
Stocks in TDAs	$500,000	29.1%	7.00%	15.00%
Bonds in TDAs	$500,000	29.1%	3.00%	6.00%
Stocks in taxable account	$360,000	20.9%	7.00%	15.00%
Bonds in taxable account	$360,000	20.9%	3.00%	6.00%
Total	$1,720,000			

Table 3.1.2 shows optimal allocation and location result when taxes are considered. To be consistent with previous study (Reichenstein, et al, 2012) and make the results comparable, present value calculation of tax for TDAs is neglected. Although stocks tend to have higher tax liability in the future, this liability is more volatile. Therefore, its discount rate is higher. These two effects exactly offset each other. As a result, neglecting present value of tax does not affect the analysis of TDAs (Reichenstein, et al, 2012).

From the optimisation results, we can see that 100% taxable accounts limit is used to hold stock, which represents 50% of the total investment. All bonds and remaining stocks are located in TDAs, occupying 40.8% and 9.2% of the total investment.

Conclusion can be drawn that investors should hold stocks in taxable accounts and bonds in TDAs to maximise their utilities under this scenario.

Table 3.1.2 Base Case 2

Base Case - Consider Taxes				
	Market Value	Optimal Weights	Pre−tax Expected Return	Pre−tax Standard Deviation
Stocks in TDAs	$132,480	9.2%	7.00%	15.00%
Bonds in TDAs	$587,520	40.8%	3.00%	6.00%
Stocks in taxable account	$720,000	50.0%	5.60%	12.00%
Bonds in taxable account	$0	0.0%	2.16%	4.32%
Total	$1,440,000			

Overall, results in both Table 3.1.1 and Table 3.1.2 are exactly the same as shown in William's paper in 2012. Starting from the scenario in Table 3.1.2, the following parts in this section continues the analysis to find out how different independent variables can influence investors' assets location decisions.

3.2 Risk Tolerance (RT)

As shown in Table 3.1.2, Investors, whose RT is 0.4725, would hold stocks in taxable accounts and bonds in TDAs. This part of the paper examines whether different RTs can lead to a different decision.

Table 3.2 provides asset allocation and location results for 5 different RTs, under the condition that all other assumptions in Table 3.1.2 remain unchanged. When RT increases from 0.4725, for example to 0.6, total weights of stocks investment increases from 59.2% to 70.53%, while location outcome stays the same, with all taxable accounts amount occupied by stocks. When RT decreases, the weights of bonds are increasingly invested in total. The location outcome still does not change until RT reaches 0.3549. From this point, the share of taxable accounts begins to move from stocks to bonds. The change of outcome happens when RT move to 0.1811, where bonds in TDA slightly more than those in taxable account. When RT went down to 0.1738, all weight of taxable account is occupied by bonds.

Table 3.2 Examples of different RT

Risk Tolerance	0.6000	0.4725	0.3549	0.1811	0.1738
Stocks in TDAs	20.53%	9.20%	0.00%	13.19%	22.32%
Bonds in TDAs	29.47%	40.80%	50.00%	36.81%	27.68%
Stocks in taxable account	50.00%	50.00%	49.99%	12.34%	0.00%
Bonds in taxable account	0.00%	0.00%	0.01%	37.66%	50.00%

As a conclusion, when RT is smaller than 0.1811 (Risk Aversion is above 5.5218) the optimal asset location reverses from the base case, as bond's weight in taxable account exceeds which in TDA.

Reichenstein and Meyer (2013) find out that result could be reversed when Risk Aversion exceeds 20. And it is further argued that 20 is not a common number for Risk Aversion, so that the reverse case could only happen in rare case. Indeed, risk aversion of 20 might be ignored in the really world, since Risk Aversion is normally between 0.2 and 10 (Nestor & Hernandez, 2014). However, our paper shows that the reverse result can be found when Risk Aversion is as low as 5.5218, which is in the normal range. Therefore, in the scenario that investing bonds in taxable accounts and stocks in TDAs do exist in reality. RT is an independent variable should be considered when making asset location decisions.

The results can be also explained by the idea of partnership between investors and the government. Since the tax rate for interest income is higher than that for equity income, more risk can be avoided when investors hold bonds in taxable account than holding stocks. Therefore, when people have low RT, bond is the right choice to put in taxable to get the most risk shared by the government, and vice versa when RT is high.

3.3 Return

The influence of stock and bond returns to asset location outcomes can be indicated by the results generated by different return values from the base case. Table 3.3.1 shows three examples including the base case. When both returns increase, stock return is 8% and bond return is 4% for example in the table, the asset location result does

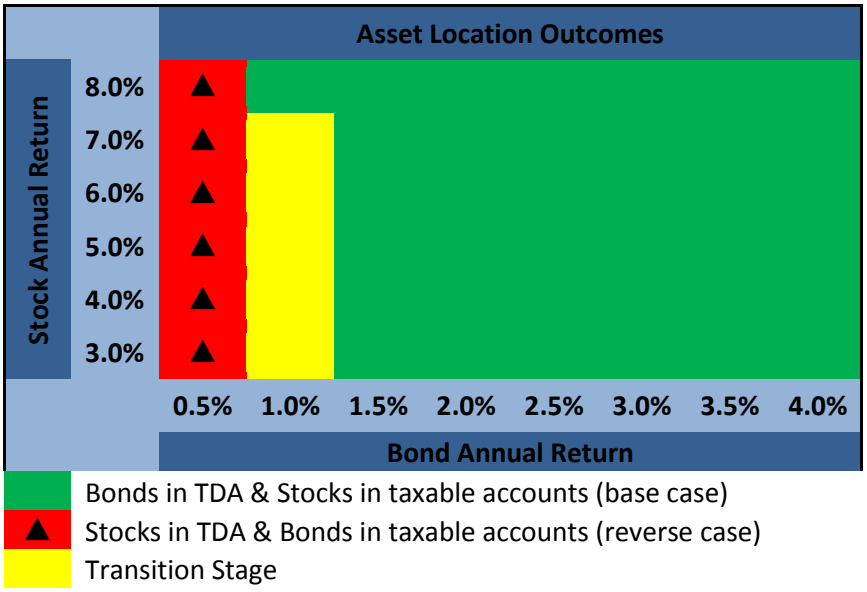
not change from the base case, where taxable account limit is used for holding stocks. However, opposite outcome appears when stock return decreases to 3% and bond return decreases to 0.5%, where the investor hold bonds in taxable accounts instead of stocks. Reichenstein and Meyer (2013) reach the same conclusion, where the data used for returns of stocks and bonds are 4% and 1%, respectively.

Table 3.3.1 Examples of different return combination

Stock Return	**3.0%**	**7.0%**	**8.0%**
Bond Return	**0.5%**	**3.0%**	**3.5%**
Stocks in TDAs	33.13%	9.20%	14.45%
Bonds in TDAs	16.87%	40.80%	35.55%
Stocks in taxable account	0.00%	50.00%	50.00%
Bonds in taxable account	50.00%	0.00%	0.00%

Next, Table 3.3.2 gives more detailed information about how changes of each return alone and both together lead to different results. The matrix provides the asset location decisions under each returns combination. For example, all three examples in Table 3.3.1 can be found in the matrix cells of their corresponding stock and bond returns. Each decision is made by comparing the value in taxable account and TDA for each type of asset. If the difference is significant, say 5%, the asset is preferred to be held in the account of larger value. If the difference is not big enough, less than 5%, then the scenario lies in the transition stage between two different outcomes. In the table, green part stands for the situation where bonds are in TDA and stocks are in taxable accounts; Red part means the reverse case where stocks are in TDA and bonds are in taxable accounts; any situations in between, not exceeding the significant level, are represented by the yellow part. And the raw data can be found in Appendix.

Table 3.3.2 Return Combination Matrix



The result of Table 3.3.2 tells that reverse cases exist when bond return is extremely low regardless the level of stock return, which means bond return plays a more significant role to asset location result. Stock return also can influence the outcome. For example, when bond has 1% annual return, stock can change the result from the base case to the transition stage when decreasing from 8% to 7%. In conclusion, both returns can influence asset location decisions and bonds are more significant than stocks. The lower the return levels are, the bigger chance the reverse case can appears.

As we can see from Appendix, lower returns of stocks and bonds are more likely to generate reverse results. One possible explanation is that when returns become lower, government shares lower returns but constant risks.

For example, we consider two asset locations in Table 3.3.3: in asset location 1, we put 50% investment in bonds in TDAs and 50% in stocks in taxable accounts. In asset location 2, we put 50% investment in stocks in TDAs and 50% in bonds in taxable accounts.

Table 3.3.3 Asset Location Starting Point

	Asset Location 1	Asset Location 2
Stocks in TDAs	0%	50%
Bonds in TDAs	50%	0%
Stocks in taxable account	50%	0%
Bonds in taxable account	0%	50%

We also make two scenarios, in scenario 1 we have the same assumptions as the base case, that is, assume 28% interest income tax, 20% equity tax, 7% equity returns, 3% bond returns, 0.4725 risk tolerance, 0.2 correlation between equity returns and bond returns, and utility function is $Utility = ER - \sigma^2 / RT$. In scenario 2, we decrease stocks return to 4% and bonds return to 0.5%, and keep other assumptions constant.

The base case we use here is that we assume there is no tax, and then asset location 1 and asset location 2 make no difference. In the following analysis, we break down the utility to return part (ER) and risk part (σ^2 / RT), as indicated in the following table.

The return and risk shared by government is the difference between base case and asset location 1 or 2.

As indicated in Table 3.3.4, for the first scenario, when changing asset location 1 to asset location 2, the return that government shares decreases from 0.7% to 0.42%. In the meanwhile, the risk that government shares decreases from 0.441% to 0.137%.

As indicated in Table 3.3.5, for the second scenario, when changing asset location 1 to asset location 2, the return that government shares decreases from 0.4% to 0.07%. In the meanwhile, the risk that government shares decreases from 0.441% to 0.137%.

In both scenarios, the change of risks shared by government is the same but the change of returns shared by government become smaller in scenario 2. As a result, when moving from asset location 1 to asset location 2, return part has smaller impact on utility function. As indicated by the utility function result asset location 1 is preferred in scenario 1 and asset location 2 is preferred in scenario 2. It also turns out that when using mean-variance optimization, we also get reverse result in scenario 2, and we represent the optimal weights in Table 3.3.6 for convenience.

Table 3.3.4 Utility Decomposition 1

	Tax = 0	Asset location 1	Asset location 2
return part (Scenario 1)	5.000%	4.300%	4.580%
risk part (Scenario 1)	1.485%	1.044%	1.348%
Utility (Scenario 1)	3.515%	3.256%	3.232%

Table 3.3.5 Utility Decomposition 2

	Tax = 0	Asset location 1	Asset location 2
return part (Scenario 2)	2.250%	1.850%	2.180%
risk part (Scenario 2)	1.485%	1.044%	1.348%
Utility (Scenario 2)	0.765%	0.806%	0.832%

Table 3.3.6 Reversed Example in Scenario 2

Stock Return = 4.0%		**Bond Return = 0.5%**		
	Market Value	Optimal Weights	Pre−tax Expected Return	Pre−tax Standard Deviation
Stocks in TDAs	$628,272	43.63%	4.00%	15.00%
Bonds in TDAs	$91,728	6.37%	0.50%	6.00%
Stocks in taxable account	$0	0.00%	3.20%	12.00%
Bonds in taxable account	$720,000	50.00%	0.36%	4.32%
Total	$1,440,000			

3.4 Volatility

The analysis method and process for the volatility is the same as previous part for the return. Standard Deviation (SD) is used to measure volatility. Table 3.4.1 shows assets allocation and location outcomes for three SD combinations, the base case, the decreased volatilities case and the increased volatilities case.

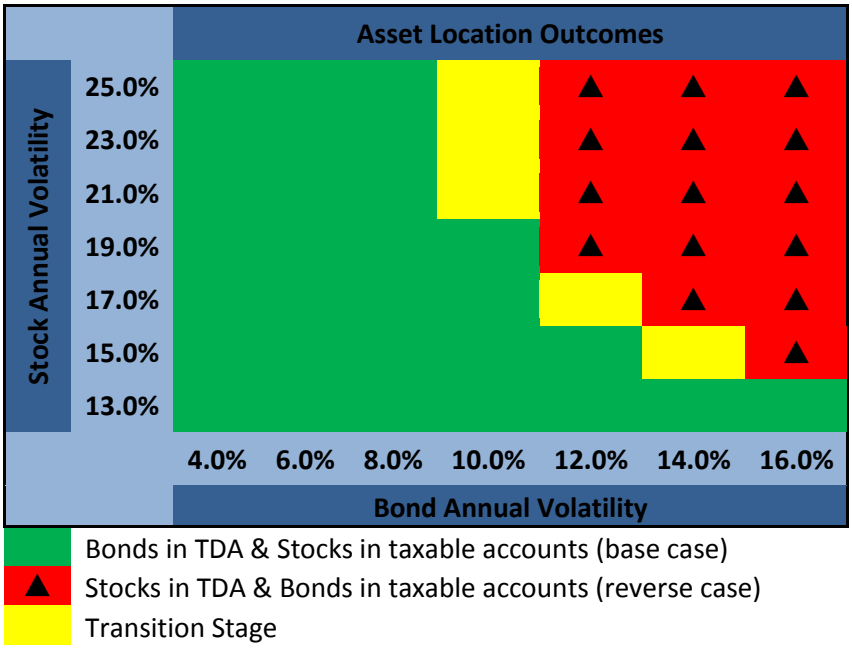
Table 3.4.1 Examples of different volatility combinations

Stock SD	**13.0%**	**15.0%**	**23.0%**
Bond SD	**4.0%**	**6.0%**	**14.0%**
Stocks in TDAs	20.62%	9.20%	34.83%
Bonds in TDAs	29.38%	40.80%	15.17%
Stocks in taxable account	50.00%	50.00%	0.00%
Bonds in taxable account	0.00%	0.00%	50.00%

As the result indicates, the drop of volatilities does not lead to a different decision for investors, but the reverse situation happens when stock and bond volatilities grow to 23% and 14% respectively.

Similar to Table 3.3.2, Table 3.4.2 gives a closer look at the influence of each and both volatilities, using the same criteria. And the raw data can be found in Appendix. The significant level of difference is still 5% here.

Table 3.4.2 Volatility Combination Matrix



In the table, reversed results occurred in the area when both volatilities are both high, such as all the outcomes in the area where stock SD is above 19% and bond SD is above 12%. Also, single high volatilities can also lead to a change in result, which can be proved by two examples in the matrix. First one is that when stock SD is 19%, bond SD moves from 10% to 12%; second one is that when bond SD is 12%, stock SD moves from 15% to 19%. In conclusion, both SDs can influence asset location decisions. The higher the volatility levels are, the bigger chance the reverse case can appears.

3.5 Correlation

Different correlations between stock market and bond market are used in making investment decisions in different scenarios. Table 3.5 provides the asset location results covering correlation range from -1 to 1.

Table 3.5 Examples of different correlation

Correlation	-0.9000	-0.6000	-0.3000	0.0000	0.2000	0.3000	0.6000	0.9000
Stocks in TDAs	7.23%	7.56%	8.00%	8.62%	9.20%	9.57%	11.18%	12.00%
Bonds in TDAs	42.77%	42.44%	42.00%	41.38%	40.80%	40.43%	38.82%	38.00%
Stocks in taxable account	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%
Bonds in taxable account	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%

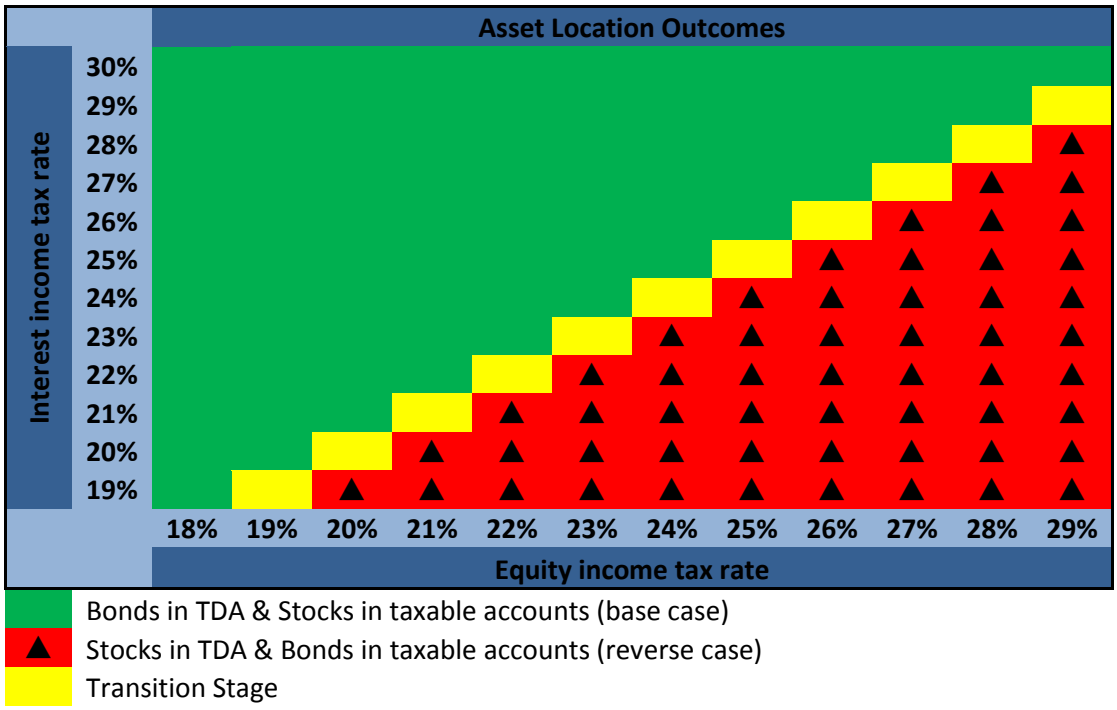
As indicated, although correlation does make investors change their total investment in stocks and bonds, stocks are still preferred in taxable account and bonds are preferred in TDAs. Therefore, conclusion can be drawn that correlation between stock market and bond market can only influence asset allocation but cannot change investors' asset location decisions.

3.6 Tax rate

Tax rates are obviously not the same in different countries or under different regulations. This part answers the question that whether tax rates difference can decide the optimal asset location results.

Table 3.6 shows the asset location results of 144 scenarios.

Table 3.6 Tax Combination Matrix



Two different outcomes are separated by a criteria, which is the relationship between two types of taxes. When interest income tax is higher than equity income tax, the result is the same as the base case, holding stocks in taxable accounts and bonds in TDAs. While when equity income tax has larger rate, bonds are preferred in taxable accounts and stocks are selected in TDAs. In addition, when two tax rates are equal, it is almost identical investing in either type account.

From the government partnership point of view, the answer is not surprising. In the case that equity income tax rate is larger, more return and risk can be share by government when bonds are hold in taxable accounts. This is an opposite situation from the base case, and as a result, the outcome is reversed.

As a conclusion, tax rates can influence asset location decisions by the relationship between interest income tax and equity income tax, but not by the overall level of tax rates. At the first glance, it is not a common case that the latter is larger. However, there still is a possibility that investors could face this situation. For example, the tax-exempt bond in US allows investors pay zero tax on bonds investment and thus is smaller than equity income tax. Also, if the investment is multinational, where more than

one tax rules apply, it is quite possible that interest income tax rate of one country is larger than equity tax rate of another country. Therefore, although the decision normally would remain when tax rates change, cross-country investors do need to check the relationship of the two type taxes.

4 Real Cases from Historical Data

The purpose of this section is to provide historical data support to previous analysis in this paper and to prove that reversed results to the base case do exist in the real world.

Different periods of historical data are used as proxies for what expected numbers might have been at various points in time, which are applied to further optimization analysis. The time period of data used in the test is the recent 20 years, from 1994 to 2014. Canadian market is selected and monthly returns of TSX Index and DEX Universe Bond Index are used to represent the performance of stocks and bonds, respectively. Appendix D shows the detailed historical data of them.

A reversed example can be found at the very beginning of the table. If the data of first 15 lines, January 1994 to March 1995, is regarded as a period for estimating future performance, then the annualised expected return of stock and bond are calculated as 3.09% and 0.89%, and the annualised expected volatility of them are calculated as 13.34% and 8.18%, as indicated in Table 4.1. If tax rate is still 28% for interest income and 20% for equity income, the after-tax numbers in taxable accounts are 2.47% and 0.64% for stock and bond return, and 10.67% and 5.89% for stock and bond SD.

Table 4.1 Real Reversed Case Example

For period from Jan. 1994 to Mar. 1995				
	Market Value	Optimal Weights	Pre−tax Expected Return	Pre−tax Standard Deviation
Stocks in TDAs	$644,510	44.76%	3.09%	13.34%
Bonds in TDAs	$75,513	5.24%	0.89%	8.18%
Stocks in taxable account	$0	0.00%	2.47%	10.67%
Bonds in taxable account	$719,977	50.00%	0.64%	5.89%
Total	$1,440,000			

The asset location result shows the optimal weights for each asset in different account. Bonds should be invested about 10% more than stocks and most of them should

be located in taxable account, occupying all account limit. The rest of bonds together with all stocks should be invested in TDAs. Obviously, this is a reversed outcome to the base case.

If we link this example to the discussion in the previous part of the paper, it is actually can be found in Table 3.3.2, the return combination matrix. Since volatility of both stock and bond, 13.34% and 8.18%, are close to those in the base case, 15% and 6%, we can locate the return combination of the example in the table. The crossing point of 3.09% stock return and 0.89% bond return is located in the bottom left corner area in the table, where there is red triangle, the reversed case section.

Next, let us stay on Table 3.3.2 and use it as an example to test the probability of the reversed case happens during the past 30 years.

Different lengths of historical data periods are used in different situations to estimate the future market performance. In the following test, 6 months to 3 years are assumed as reasonable lengths used for forecasting. Testing population used to compare conventional wisdom with recent researches comes from the 20-year data set mentioned previously. Mean-variance optimization does not apply to every subset of the data set, because every subset represents historical return and risk profile of stocks or bonds, but the expected return and risk profile of stocks and bonds are not necessarily the same. In addition, examples from Table 3.3.2 is discrete, which means there is no subset will have exactly the same return and risk profile as them. As a result, we set ranges for returns and risks to find out all cases covered by the table as the testing sample, including all outcomes. Then we use narrower return range to count the number of reversed case. If any subset falls in this range, we consider it as one reverse result in the sample.

Table 4.2 shows 7 reversed case found by the above approach, and Table 4.3 shows the corresponding expected numbers estimated by using those 7 scenarios. The total number of cases in testing sample, all cases in Table 3.3.2, is 42. Therefore, the probability that reverse result occurs is 16.67%. This is also the chance that conventional wisdom is correct.

Table 4.2 Real Results from Data Set

Reverse Results from Data Set							
	1	2	3	4	5	6	7
Period Beginning	Jan-94	Feb-94	Apr-94	May-94	Feb-99	Nov-01	Dec-09
Period Ending	Mar-95	Jul-95	Sep-94	Oct-94	Sep-99	Jun-02	May-10
Return of Stock	3.09%	4.01%	4.36%	4.38%	7.78%	8.58%	9.49%
Return of Bond	0.89%	1.79%	2.35%	2.21%	-0.93%	1.15%	1.93%
Volatility of Stock	13.34%	11.97%	13.99%	13.99%	13.93%	14.93%	14.52%
Volatility of Bond	8.18%	7.83%	6.58%	6.59%	4.20%	3.81%	4.17%

Table 4.3 Approximate Theoretical Reverse Results

Approximate Theoretical Reverse Results							
	1	2	3	4	5	6	7
Return of Stock	3.00%	5.00%	5.00%	5.00%	8.00%	8.00%	8.00%
Return of Bond	0.50%	1.00%	1.00%	1.00%	0.50%	0.50%	0.50%
Volatility of Stock	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%	15.00%
Volatility of Bond	6.00%	6.00%	6.00%	6.00%	6.00%	6.00%	6.00%

A same procedure performed to Table 3.4.2 found 93 cases in the table but none of them is the counter case. Overall, 7 out of 135 examples in real data is reversed, which means the conventional wisdom has an overall possibility of 5.19% is correct in the past 20 years.

5 Conclusion

This study has reviewed previous studies (Reichenstein, et al, 2012; Reichenstein & Meyer, 2013) about how investors choose optimal weights in different types of accounts to locate their investment portfolios, tested and revised the results of the discussion, and achieved several new findings about factors influencing asset location decisions.

This paper agrees with most ideas in these two papers. Tax can be viewed as a partnership between investors and the government, where the government shares both returns and risks by applying tax regulations on investors. This leads to investment differences among accounts under different tax rules, for example taxable accounts and TDAs. Therefore, asset location strategy becomes an important consideration when people making investment decisions, especially for private investors. Under certain assumptions, which were designed for an average investor in the real world, stocks are preferred held in taxable account and bonds are preferred held in TDAs. However, this outcome is not always holds when certain criteria changes. Reichenstein & Meyer (2013) have conducted a start to study the influence of some of those criteria, such as RT and return, and made conclusions for each factors. The following points summarises the main ideas of this paper about how asset location decisions can be influenced by investor's RT, and stock's and bond's returns, volatilities, correlation, and tax rates.

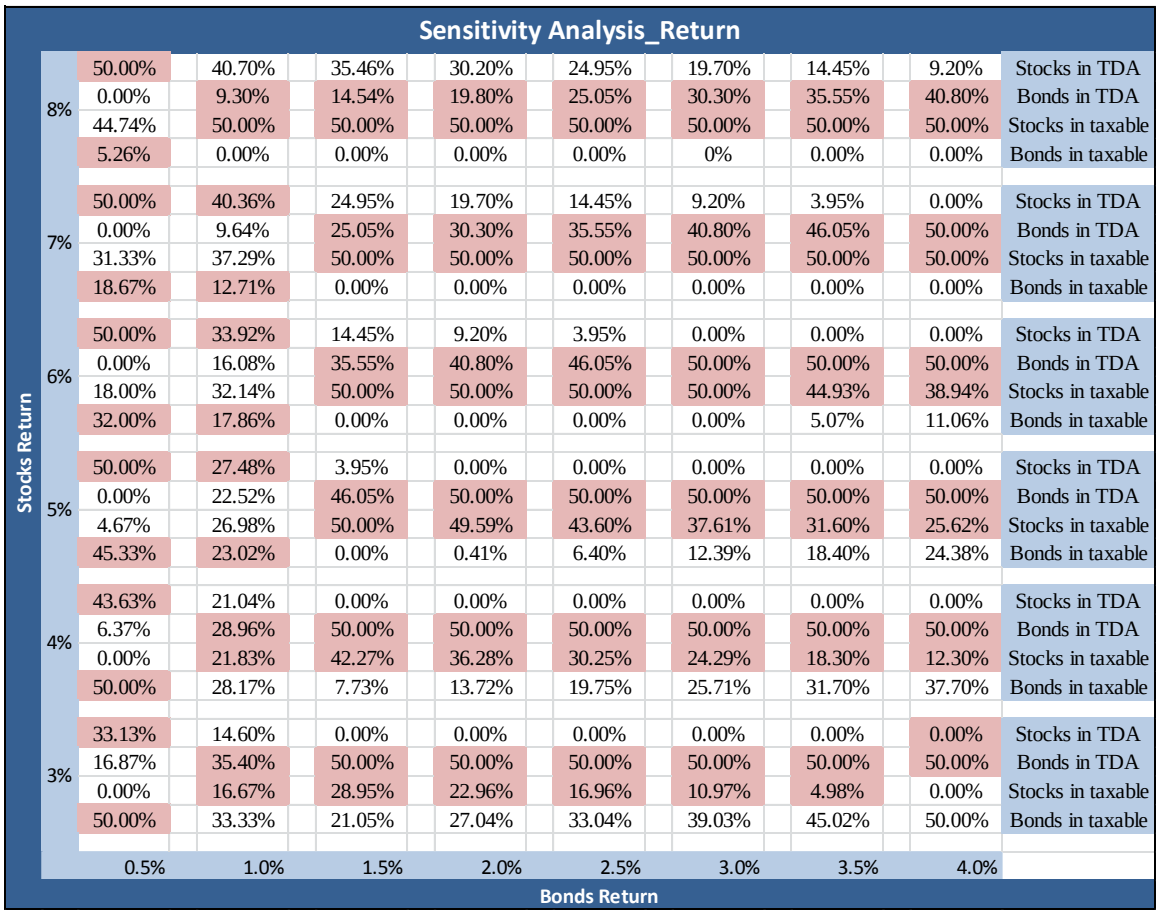
- Investors with different RTs have different asset location strategy to make the maximised utility for themselves. The lower the RTs is, or the higher the risk aversion is, the bigger chance that reverse result from base assumptions happens, where bonds in taxable account and stocks in TDA. Moreover, both outcomes have considerable possibility to be seen in the real world.
- Returns of both stock and bond have the power to decide the investment decision outcomes. In the range of this paper's study, the result is more

sensitive to bond return than stock return. Overall, lower returns could lead to an opposite result to the base case and example exits in historical performance of the real market.

- Contrary to returns, stock and bond volatilities must be high enough in order to change the original locations. Although extreme high numbers of either single SD can make the change, the reversed results are more likely to happen when both volatilities are relative high.
- Correlation between equity market and bond market is the only factor this paper found that does not have the ability to determine asset location decisions. The result stays the same no matter what the correlation is.
- Tax rates can influence location outcomes by the relationship between interest income tax and equity income tax instead of the level of the rates. Investors should compare the applying tax rules of two type of assets to make the corresponding decisions. Although it is not a normal case when income tax rate of interest is higher than that of equity under one regulation, investors should still consider about taxes when special rules apply or their investment covers two or more countries, where different countries apply different tax rules on their earnings.

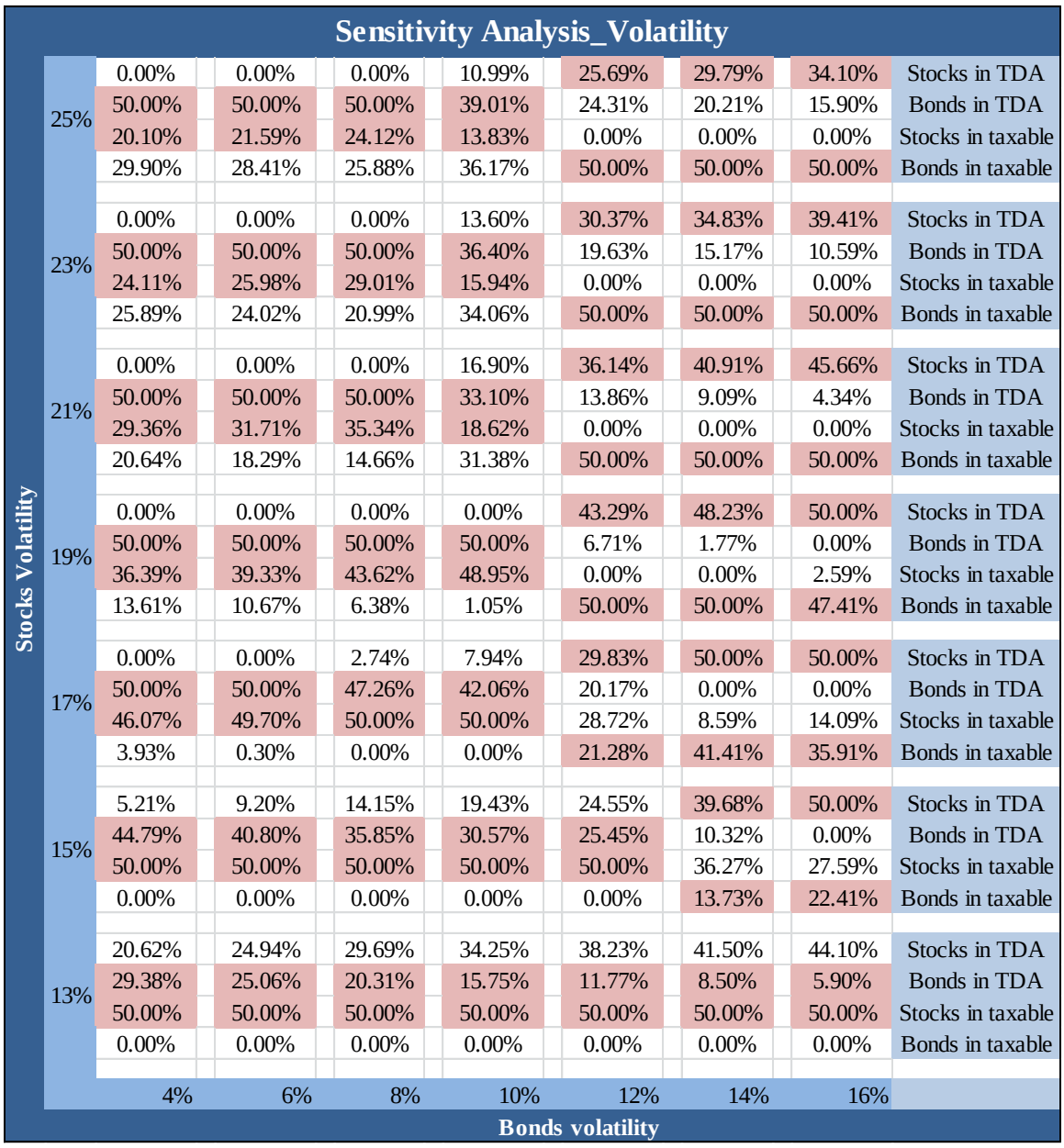
Appendices

Appendix A

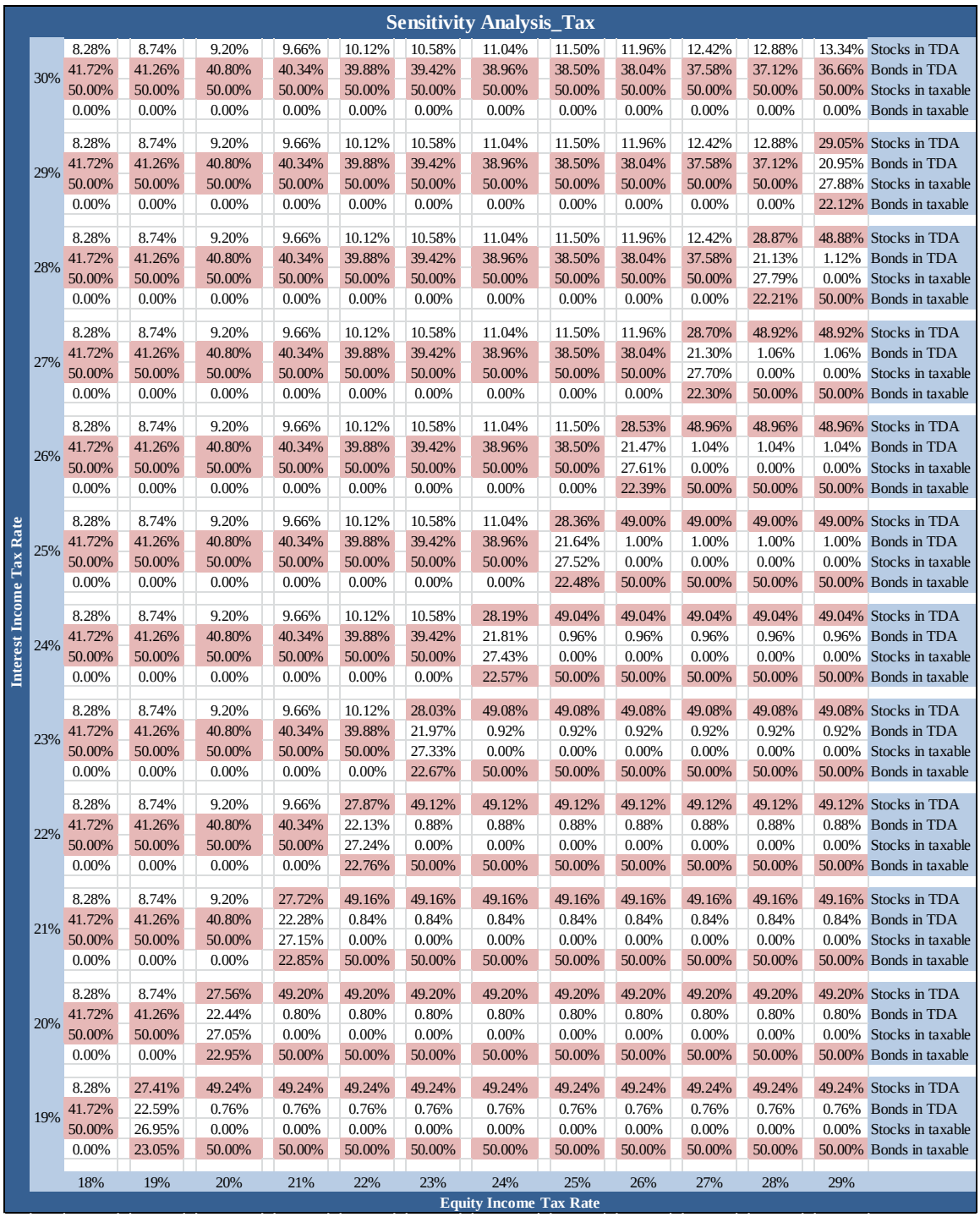


Sensitivity Analysis_Return									
Stocks Return		0.5%	1.0%	1.5%	2.0%	2.5%	3.0%	3.5%	4.0%	
		50.00%	40.70%	35.46%	30.20%	24.95%	19.70%	14.45%	9.20%	Stocks in TDA
	8%	0.00%	9.30%	14.54%	19.80%	25.05%	30.30%	35.55%	40.80%	Bonds in TDA
		44.74%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	Stocks in taxable
		5.26%	0.00%	0.00%	0.00%	0.00%	0%	0.00%	0.00%	Bonds in taxable
		50.00%	40.36%	24.95%	19.70%	14.45%	9.20%	3.95%	0.00%	Stocks in TDA
	7%	0.00%	9.64%	25.05%	30.30%	35.55%	40.80%	46.05%	50.00%	Bonds in TDA
		31.33%	37.29%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	Stocks in taxable
		18.67%	12.71%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	Bonds in taxable
		50.00%	33.92%	14.45%	9.20%	3.95%	0.00%	0.00%	0.00%	Stocks in TDA
	6%	0.00%	16.08%	35.55%	40.80%	46.05%	50.00%	50.00%	50.00%	Bonds in TDA
		18.00%	32.14%	50.00%	50.00%	50.00%	50.00%	44.93%	38.94%	Stocks in taxable
		32.00%	17.86%	0.00%	0.00%	0.00%	0.00%	5.07%	11.06%	Bonds in taxable
		50.00%	27.48%	3.95%	0.00%	0.00%	0.00%	0.00%	0.00%	Stocks in TDA
	5%	0.00%	22.52%	46.05%	50.00%	50.00%	50.00%	50.00%	50.00%	Bonds in TDA
		4.67%	26.98%	50.00%	49.59%	43.60%	37.61%	31.60%	25.62%	Stocks in taxable
		45.33%	23.02%	0.00%	0.41%	6.40%	12.39%	18.40%	24.38%	Bonds in taxable
		43.63%	21.04%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	Stocks in TDA
	4%	6.37%	28.96%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	Bonds in TDA
		0.00%	21.83%	42.27%	36.28%	30.25%	24.29%	18.30%	12.30%	Stocks in taxable
		50.00%	28.17%	7.73%	13.72%	19.75%	25.71%	31.70%	37.70%	Bonds in taxable
		33.13%	14.60%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	Stocks in TDA
	3%	16.87%	35.40%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	Bonds in TDA
		0.00%	16.67%	28.95%	22.96%	16.96%	10.97%	4.98%	0.00%	Stocks in taxable
		50.00%	33.33%	21.05%	27.04%	33.04%	39.03%	45.02%	50.00%	Bonds in taxable
		0.5%	1.0%	1.5%	2.0%	2.5%	3.0%	3.5%	4.0%	
					Bonds Return					

Appendix B


Sensitivity Analysis_Volatility
Stocks Volatility
Bonds volatility
25%
23%
21%
19%
17%
15%
13%
4%
6%
8%
10%
12%
14%
16%
0.00% 0.00% 0.00% 10.99% 25.69% 29.79% 34.10% Stocks in TDA
50.00% 50.00% 50.00% 39.01% 24.31% 20.21% 15.90% Bonds in TDA
20.10% 21.59% 24.12% 13.83% 0.00% 0.00% 0.00% Stocks in taxable
29.90% 28.41% 25.88% 36.17% 50.00% 50.00% 50.00% Bonds in taxable
0.00% 0.00% 0.00% 13.60% 30.37% 34.83% 39.41% Stocks in TDA
50.00% 50.00% 50.00% 36.40% 19.63% 15.17% 10.59% Bonds in TDA
24.11% 25.98% 29.01% 15.94% 0.00% 0.00% 0.00% Stocks in taxable
25.89% 24.02% 20.99% 34.06% 50.00% 50.00% 50.00% Bonds in taxable
0.00% 0.00% 0.00% 16.90% 36.14% 40.91% 45.66% Stocks in TDA
50.00% 50.00% 50.00% 33.10% 13.86% 9.09% 4.34% Bonds in TDA
29.36% 31.71% 35.34% 18.62% 0.00% 0.00% 0.00% Stocks in taxable
20.64% 18.29% 14.66% 31.38% 50.00% 50.00% 50.00% Bonds in taxable
0.00% 0.00% 0.00% 0.00% 43.29% 48.23% 50.00% Stocks in TDA
50.00% 50.00% 50.00% 50.00% 6.71% 1.77% 0.00% Bonds in TDA
36.39% 39.33% 43.62% 48.95% 0.00% 0.00% 2.59% Stocks in taxable
13.61% 10.67% 6.38% 1.05% 50.00% 50.00% 47.41% Bonds in taxable
0.00% 0.00% 2.74% 7.94% 29.83% 50.00% 50.00% Stocks in TDA
50.00% 50.00% 47.26% 42.06% 20.17% 0.00% 0.00% Bonds in TDA
46.07% 49.70% 50.00% 50.00% 28.72% 8.59% 14.09% Stocks in taxable
3.93% 0.30% 0.00% 0.00% 21.28% 41.41% 35.91% Bonds in taxable
5.21% 9.20% 14.15% 19.43% 24.55% 39.68% 50.00% Stocks in TDA
44.79% 40.80% 35.85% 30.57% 25.45% 10.32% 0.00% Bonds in TDA
50.00% 50.00% 50.00% 50.00% 50.00% 36.27% 27.59% Stocks in taxable
0.00% 0.00% 0.00% 0.00% 0.00% 13.73% 22.41% Bonds in taxable
20.62% 24.94% 29.69% 34.25% 38.23% 41.50% 44.10% Stocks in TDA
29.38% 25.06% 20.31% 15.75% 11.77% 8.50% 5.90% Bonds in TDA
50.00% 50.00% 50.00% 50.00% 50.00% 50.00% 50.00% Stocks in taxable
0.00% 0.00% 0.00% 0.00% 0.00% 0.00% 0.00% Bonds in taxable

Appendix C



Interest Income Tax Rate	18%	19%	20%	21%	22%	23%	24%	25%	26%	27%	28%	29%	
30%	8.28%	8.74%	9.20%	9.66%	10.12%	10.58%	11.04%	11.50%	11.96%	12.42%	12.88%	13.34%	Stocks in TDA
	41.72%	41.26%	40.80%	40.34%	39.88%	39.42%	38.96%	38.50%	38.04%	37.58%	37.12%	36.66%	Bonds in TDA
	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	Stocks in taxable
	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	Bonds in taxable
29%	8.28%	8.74%	9.20%	9.66%	10.12%	10.58%	11.04%	11.50%	11.96%	12.42%	12.88%	29.05%	Stocks in TDA
	41.72%	41.26%	40.80%	40.34%	39.88%	39.42%	38.96%	38.50%	38.04%	37.58%	37.12%	20.95%	Bonds in TDA
	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	27.88%	Stocks in taxable
	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	22.12%	Bonds in taxable
28%	8.28%	8.74%	9.20%	9.66%	10.12%	10.58%	11.04%	11.50%	11.96%	12.42%	28.87%	48.88%	Stocks in TDA
	41.72%	41.26%	40.80%	40.34%	39.88%	39.42%	38.96%	38.50%	38.04%	37.58%	21.13%	1.12%	Bonds in TDA
	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	27.79%	0.00%	Stocks in taxable
	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	22.21%	50.00%	Bonds in taxable
27%	8.28%	8.74%	9.20%	9.66%	10.12%	10.58%	11.04%	11.50%	11.96%	28.70%	48.92%	48.92%	Stocks in TDA
	41.72%	41.26%	40.80%	40.34%	39.88%	39.42%	38.96%	38.50%	38.04%	21.30%	1.06%	1.06%	Bonds in TDA
	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	27.70%	0.00%	0.00%	Stocks in taxable
	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	22.30%	50.00%	50.00%	Bonds in taxable
26%	8.28%	8.74%	9.20%	9.66%	10.12%	10.58%	11.04%	11.50%	28.53%	48.96%	48.96%	48.96%	Stocks in TDA
	41.72%	41.26%	40.80%	40.34%	39.88%	39.42%	38.96%	38.50%	21.47%	1.04%	1.04%	1.04%	Bonds in TDA
	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	27.61%	0.00%	0.00%	0.00%	Stocks in taxable
	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	22.39%	50.00%	50.00%	50.00%	Bonds in taxable
25%	8.28%	8.74%	9.20%	9.66%	10.12%	10.58%	11.04%	28.36%	49.00%	49.00%	49.00%	49.00%	Stocks in TDA
	41.72%	41.26%	40.80%	40.34%	39.88%	39.42%	38.96%	21.64%	1.00%	1.00%	1.00%	1.00%	Bonds in TDA
	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	27.52%	0.00%	0.00%	0.00%	0.00%	Stocks in taxable
	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	22.48%	50.00%	50.00%	50.00%	50.00%	Bonds in taxable
24%	8.28%	8.74%	9.20%	9.66%	10.12%	10.58%	28.19%	49.04%	49.04%	49.04%	49.04%	49.04%	Stocks in TDA
	41.72%	41.26%	40.80%	40.34%	39.88%	39.42%	21.81%	0.96%	0.96%	0.96%	0.96%	0.96%	Bonds in TDA
	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	27.43%	0.00%	0.00%	0.00%	0.00%	0.00%	Stocks in taxable
	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	22.57%	50.00%	50.00%	50.00%	50.00%	50.00%	Bonds in taxable
23%	8.28%	8.74%	9.20%	9.66%	10.12%	28.03%	49.08%	49.08%	49.08%	49.08%	49.08%	49.08%	Stocks in TDA
	41.72%	41.26%	40.80%	40.34%	39.88%	21.97%	0.92%	0.92%	0.92%	0.92%	0.92%	0.92%	Bonds in TDA
	50.00%	50.00%	50.00%	50.00%	50.00%	27.33%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	Stocks in taxable
	0.00%	0.00%	0.00%	0.00%	0.00%	22.67%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	Bonds in taxable
22%	8.28%	8.74%	9.20%	9.66%	27.87%	49.12%	49.12%	49.12%	49.12%	49.12%	49.12%	49.12%	Stocks in TDA
	41.72%	41.26%	40.80%	40.34%	22.13%	0.88%	0.88%	0.88%	0.88%	0.88%	0.88%	0.88%	Bonds in TDA
	50.00%	50.00%	50.00%	50.00%	27.24%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	Stocks in taxable
	0.00%	0.00%	0.00%	0.00%	22.76%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	Bonds in taxable
21%	8.28%	8.74%	9.20%	27.72%	49.16%	49.16%	49.16%	49.16%	49.16%	49.16%	49.16%	49.16%	Stocks in TDA
	41.72%	41.26%	40.80%	22.28%	0.84%	0.84%	0.84%	0.84%	0.84%	0.84%	0.84%	0.84%	Bonds in TDA
	50.00%	50.00%	50.00%	27.15%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	Stocks in taxable
	0.00%	0.00%	0.00%	22.85%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	Bonds in taxable
20%	8.28%	8.74%	27.56%	49.20%	49.20%	49.20%	49.20%	49.20%	49.20%	49.20%	49.20%	49.20%	Stocks in TDA
	41.72%	41.26%	22.44%	0.80%	0.80%	0.80%	0.80%	0.80%	0.80%	0.80%	0.80%	0.80%	Bonds in TDA
	50.00%	50.00%	27.05%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	Stocks in taxable
	0.00%	0.00%	22.95%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	Bonds in taxable
19%	8.28%	27.41%	49.24%	49.24%	49.24%	49.24%	49.24%	49.24%	49.24%	49.24%	49.24%	49.24%	Stocks in TDA
	41.72%	22.59%	0.76%	0.76%	0.76%	0.76%	0.76%	0.76%	0.76%	0.76%	0.76%	0.76%	Bonds in TDA
	50.00%	26.95%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	0.00%	Stocks in taxable
	0.00%	23.05%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	50.00%	Bonds in taxable

Equity Income Tax Rate

Appendix D

Month	TSX Index Monthly Return	DEX Index Monthly Return	Month	TSX Index Monthly Return	DEX Index Monthly Return	Month	TSX Index Monthly Return	DEX Index Monthly Return
Jan-94	5.476%	2.263%	Jan-01	4.408%	0.740%	Jan-08	-4.720%	0.609%
Feb-94	-2.702%	-2.784%	Feb-01	-13.265%	0.895%	Feb-08	3.450%	1.346%
Mar-94	-1.814%	-4.960%	Mar-01	-5.620%	0.118%	Mar-08	-1.430%	0.970%
Apr-94	-1.360%	-0.290%	Apr-01	4.501%	-0.921%	Apr-08	4.600%	-0.517%
May-94	1.602%	-1.673%	May-01	2.831%	0.277%	May-08	5.790%	-0.163%
Jun-94	-6.662%	-1.959%	Jun-01	-4.993%	-0.005%	Jun-08	-1.410%	-0.045%
Jul-94	3.907%	1.578%	Jul-01	-0.542%	1.851%	Jul-08	-5.860%	0.908%
Aug-94	4.258%	2.956%	Aug-01	-3.656%	2.125%	Aug-08	1.540%	0.693%
Sep-94	0.393%	0.552%	Sep-01	-7.376%	0.701%	Sep-08	-14.450%	-1.988%
Oct-94	-1.353%	-0.362%	Oct-01	0.783%	3.370%	Oct-08	-16.670%	-0.797%
Nov-94	-4.422%	0.238%	Nov-01	7.956%	-1.006%	Nov-08	-4.740%	2.426%
Dec-94	3.279%	0.316%	Dec-01	3.757%	-0.260%	Dec-08	-2.640%	2.884%
Jan-95	-4.574%	0.382%	Jan-02	-0.436%	0.200%	Jan-09	-2.960%	-0.958%
Feb-95	2.845%	4.691%	Feb-02	-0.037%	0.620%	Feb-09	-6.310%	0.693%
Mar-95	4.941%	0.162%	Mar-02	3.015%	-1.855%	Mar-09	7.790%	1.792%
Apr-95	-0.706%	1.967%	Apr-02	-2.337%	1.526%	Apr-09	7.260%	0.028%
May-95	4.176%	2.803%	May-02	0.044%	0.947%	May-09	11.460%	-0.127%
Jun-95	2.080%	0.530%	Jun-02	-6.455%	0.589%	Jun-09	0.340%	1.354%
Jul-95	2.022%	-1.489%	Jul-02	-7.468%	1.495%	Jul-09	4.220%	0.668%
Aug-95	-1.946%	2.741%	Aug-02	0.222%	1.397%	Aug-09	0.950%	1.133%
Sep-95	0.570%	1.730%	Sep-02	-6.294%	1.277%	Sep-09	5.140%	0.888%
Oct-95	-1.470%	1.664%	Oct-02	1.212%	-0.229%	Oct-09	-4.040%	-0.054%
Nov-95	4.746%	2.737%	Nov-02	5.277%	0.424%	Nov-09	5.150%	1.342%
Dec-95	1.428%	1.155%	Dec-02	0.914%	2.064%	Dec-09	2.930%	-1.431%
Jan-96	5.487%	1.117%	Jan-03	-0.541%	-0.809%	Jan-10	-5.350%	1.845%
Feb-96	-0.525%	-1.642%	Feb-03	-0.021%	0.689%	Feb-10	4.970%	0.160%
Mar-96	1.032%	0.052%	Mar-03	-2.975%	-0.542%	Mar-10	3.810%	-0.731%
Apr-96	3.619%	0.052%	Apr-03	3.906%	1.619%	Apr-10	1.670%	-0.067%
May-96	2.126%	1.005%	May-03	4.320%	3.092%	May-10	-3.480%	1.180%
Jun-96	-3.568%	1.043%	Jun-03	2.055%	0.309%	Jun-10	-3.710%	1.802%
Jul-96	-2.194%	1.123%	Jul-03	4.010%	-1.363%	Jul-10	3.959%	0.456%
Aug-96	4.502%	1.499%	Aug-03	3.625%	0.516%	Aug-10	1.897%	2.021%
Sep-96	3.124%	2.318%	Sep-03	-1.000%	2.180%	Sep-10	4.088%	0.648%
Oct-96	5.918%	4.302%	Oct-03	4.840%	-1.069%	Oct-10	2.711%	0.227%
Nov-96	7.636%	2.567%	Nov-03	1.250%	0.411%	Nov-10	2.370%	-1.095%
Dec-96	-1.272%	-1.663%	Dec-03	4.830%	1.565%	Dec-10	4.091%	0.153%
Jan-97	3.168%	-0.067%	Jan-04	3.750%	1.328%	Jan-11	0.986%	-0.432%
Feb-97	0.932%	1.195%	Feb-04	3.240%	1.301%	Feb-11	4.437%	0.230%
Mar-97	-4.769%	-1.484%	Mar-04	-2.110%	0.482%	Mar-11	0.125%	-0.064%
Apr-97	2.226%	1.211%	Apr-04	-3.890%	0.686%	Apr-11	-1.022%	0.854%
May-97	6.966%	1.488%	May-04	2.250%	-2.604%	May-11	-0.867%	1.542%
Jun-97	1.098%	1.095%	Jun-04	1.730%	-0.087%	Jun-11	-3.330%	0.062%

Jul-97	6.901%	3.257%	Jul-04	-0.920%	0.893%	Jul-11	-2.500%	2.052%
Aug-97	-3.750%	-0.662%	Aug-04	-0.810%	1.611%	Aug-11	-1.210%	1.170%
Sep-97	6.677%	1.720%	Sep-04	3.670%	0.298%	Sep-11	-8.660%	1.819%
Oct-97	-2.738%	1.691%	Oct-04	2.440%	1.143%	Oct-11	5.610%	-0.438%
Nov-97	-4.687%	-0.318%	Nov-04	1.940%	0.739%	Nov-11	-0.210%	0.844%
Dec-97	3.076%	0.196%	Dec-04	2.640%	1.201%	Dec-11	-1.700%	1.685%
Jan-98	0.094%	1.623%	Jan-05	-0.400%	0.956%	Jan-12	4.370%	0.498%
Feb-98	5.978%	0.055%	Feb-05	5.170%	-0.059%	Feb-12	1.670%	-0.392%
Mar-98	6.763%	1.127%	Mar-05	-0.380%	0.192%	Mar-12	-1.600%	-0.324%
Apr-98	1.480%	0.470%	Apr-05	-2.380%	1.198%	Apr-12	-0.600%	0.128%
May-98	-0.871%	0.709%	May-05	2.690%	1.761%	May-12	-6.100%	2.111%
Jun-98	-2.738%	0.405%	Jun-05	3.330%	1.447%	Jun-12	1.130%	0.011%
Jul-98	-5.843%	-0.104%	Jul-05	5.310%	-0.351%	Jul-12	0.830%	0.659%
Aug-98	-20.106%	-1.114%	Aug-05	2.500%	1.284%	Aug-12	2.630%	-0.102%
Sep-98	1.745%	3.807%	Sep-05	3.410%	-0.805%	Sep-12	3.300%	0.678%
Oct-98	10.679%	-0.083%	Oct-05	-5.650%	-0.969%	Oct-12	1.070%	-0.191%
Nov-98	2.313%	1.138%	Nov-05	4.420%	0.916%	Nov-12	-1.282%	0.618%
Dec-98	2.476%	0.859%	Dec-05	4.410%	0.751%	Dec-12	1.949%	-0.134%
Jan-99	3.813%	0.502%	Jan-06	6.060%	-0.627%	Jan-13	2.252%	-0.738%
Feb-99	-6.093%	-1.843%	Feb-06	-2.020%	0.542%	Feb-13	1.256%	1.003%
Mar-99	4.749%	2.143%	Mar-06	3.900%	-0.345%	Mar-13	-0.193%	0.435%
Apr-99	6.392%	0.081%	Apr-06	0.890%	-0.937%	Apr-13	-2.071%	1.144%
May-99	-2.346%	-0.919%	May-06	-3.560%	0.371%	May-13	1.770%	-1.461%
Jun-99	2.650%	-0.095%	Jun-06	-0.820%	-0.577%	Jun-13	-3.760%	-2.029%
Jul-99	1.103%	-0.977%	Jul-06	2.030%	2.177%	Jul-13	3.191%	0.193%
Aug-99	-1.448%	0.311%	Aug-06	2.260%	1.672%	Aug-13	1.548%	-0.602%
Sep-99	0.004%	0.679%	Sep-06	-2.320%	1.053%	Sep-13	1.398%	0.525%
Oct-99	4.369%	-1.197%	Oct-06	5.090%	0.423%	Oct-13	4.724%	1.057%
Nov-99	3.790%	0.311%	Nov-06	3.520%	1.062%	Nov-13	0.453%	-0.236%
Dec-99	12.047%	-0.080%	Dec-06	1.500%	-0.775%	Dec-13	1.989%	-0.428%
Jan-00	0.847%	-0.525%	Jan-07	1.150%	-0.120%	Jan-14	0.816%	2.604%
Feb-00	7.726%	2.693%	Feb-07	0.250%	1.283%	Feb-14	3.926%	0.342%
Mar-00	3.824%	1.036%	Mar-07	1.170%	-0.245%	Mar-14	1.228%	-0.187%
Apr-00	-1.175%	-0.682%	Apr-07	2.070%	0.027%	Apr-14	2.424%	0.507%
May-00	-0.928%	0.977%	May-07	4.990%	-1.435%			
Jun-00	10.371%	1.443%	Jun-07	-0.820%	-0.270%			
Jul-00	2.105%	0.286%	Jul-07	-0.130%	0.111%			
Aug-00	8.184%	1.509%	Aug-07	-1.290%	0.858%			
Sep-00	-7.619%	0.254%	Sep-07	3.460%	0.724%			
Oct-00	-7.080%	-0.051%	Oct-07	3.910%	0.641%			
Nov-00	-8.396%	2.015%	Nov-07	-6.220%	1.460%			
Dec-00	1.453%	0.893%	Dec-07	1.340%	0.628%			

Appendix E

```
%%
format compact;
close all;
clear all;
clc;

% Use both fmincon and portalloc, for portalloc, find out the Rf. i.e
% Sharpe ratio.

% Inputs: Expected return, Standard deviation, correlation, and other
% constraints.
% All these inputs are given in MATLAB Functions, from UtilityFunction1
to
% UtilityFunction8.


%% Case 1: Ignore taxes
% Set up the constraints
Aeq = [1 1];
beq = 1;
x0 = [.5; .5];
lb = [0; 0];
ub = [1; 1];

% Use fmincon to optimize
options1 = optimset('TolFun',1e-8,'TolCon',1e-8);
[x1,fval1] =
fmincon(@UtilityFunction1,x0,[],[],Aeq,beq,lb,ub,[],options1);


%% Case 2: Include taxes_This is base case
% Expected return and cov
% Set up the constraints
Aeq = [1 1 0 0; 0 0 1 1];
beq = [.5; .5]; % Asset Allocation: Hold 50% stocks and 50% bonds
x0 = [.25; .25; .25; .25]; % Initial testing
lb = [0; 0; 0; 0];
ub = [1; 1; 1; 1]; % No shorting

% Use fmincon to optimize
options2 = optimset('TolFun',1e-8,'TolCon',1e-8);
[x2,fval2] =
fmincon(@UtilityFunction2,x0,[],[],Aeq,beq,lb,ub,[],options2);
```

Appendix F

```
function f = UtilityFunction1(x)
% This is the first function used in Main Program
% The utility function is u = ER -sig^2/RT, where ER is the expected
% return, sig is the standard deviation of portfolio, RT is the risk
% tolerance

RStocks = .07;
RBonds = .03;
StdStocks = .15;
StdBonds = .06;
Corr = .2;
RiskTol =.4725;

f = -RStocks*x(1)-
RBonds*x(2)+1/RiskTol*(StdStocks*StdStocks*x(1)*x(1)...
    +StdBonds*StdBonds*x(2)*x(2)+2*Corr*x(1)*x(2)*StdStocks*StdBonds);
```

Appendix G

```
function f = UtilityFunction2(x)
% This is the second function used in Main Program
% The utility function is u = ER -sig^2/RT, where ER is the expected
% return, sig is the standard deviation of portfolio, RT is the risk
% tolerance

RStocks = .07;   % Change stocks return to .03, .04, .05, .06, .07
and .08,
                 % then rerun the function to get new optimal weights.
RBonds = .03;    % Change bonds return
to .005, .01, .015, .02, .025, .03, .035 and .04,
                 % then rerun the function to get new optimal weights.
StdStocks = .15; % Change stocks volatility
to .13, .15, .17, .19, .21, .23 and .25,
                 % then rerun the function to get new optimal weights.
StdBonds = .06;  % Change stocks volatility
to .04, .06, .08, .1, .12, .14 and .16,
                 % then rerun the function to get new optimal weights.
Tax_Income = .28;% Change stocks return
to .19, .20, .21, .22, .23, .24, .25, .26, .27, .29, and .30
                 % then rerun the function to get new optimal weights.
Tax_Capital = .2;% Change stocks return
to .18, .19, .21, .22, .23, .24, .25, .26, .27, .28, and .29
                 % then rerun the function to get new optimal weights.
Corr = .2;       % Change stocks return to -.9, -.6, -.3, 0, .3, .6,
and .9,
                 % then rerun the function to get new optimal weights.
RiskTol = .4725; % Change RT to .6, .3549, .1811, and .1738,
                 % then rerun the function to get new optimal weights.

RStocks_tax = RStocks*(1 - Tax_Capital);
RBonds_tax = RBonds*(1 - Tax_Income);
StdStocks_tax = StdStocks*(1 - Tax_Capital);
StdBonds_tax = StdBonds*(1 - Tax_Income);

f = -RStocks*x(1)-RBonds*x(2)-RStocks_tax*x(3)-RBonds_tax*x(4)...

+1/RiskTol*(StdStocks*StdStocks*x(1)*x(1)+StdBonds*StdBonds*x(2)*x(2)...

+StdStocks_tax*StdStocks_tax*x(3)*x(3)+StdBonds_tax*StdBonds_tax*x(4)*x
(4)...

+2*Corr*x(1)*x(2)*StdStocks*StdBonds+2*x(1)*x(3)*StdStocks*StdStocks_ta
x...

+2*Corr*x(1)*x(4)*StdStocks*StdBonds_tax+2*Corr*x(2)*x(3)*StdBonds*StdS
tocks_tax...

+2*x(2)*x(4)*StdBonds*StdBonds_tax+2*Corr*x(3)*x(4)*StdStocks_tax*StdBo
nds_tax);
```

Reference List

Daryanani, G., & Cordaro, C. (2005). Asset Location: A Generic Framework for Maximizing After-Tax Wealth. Journal of Financial Planning, 18(1), 44-54.

Gandelman, N., & Hernandez-Murillo, R. (n.d.). Risk Aversion at the Country Level. Retrieved from http://research.stlouisfed.org/wp/more/2014-005/.

Reichenstein, W. (2006). After-Tax Asset Allocation. Financial Analysts Journal, 62(4), 14-19.

Reichenstein, W., Horan, S., & Jennings, W. (2012). Two Key Concepts for Wealth Management and Beyond. Financial Analysts Journal, 14-22.

Reichenstein, W., & Meyer, W. (2013). The Asset Location Decision Revisited. Journal of Financial Planning, 26(11), 48-56.