BEAMFORMING FOR MULTIUSER MIMO SYSTEMS

by

Milad Amir Toutounchian

M.Sc., Iran University of Science and Technology, 2005

B.Sc.,Sharif University of Technology, 2002

A THESIS SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF

Doctor of Philosophy

in the
School of Engineering Science
Faculty of Applied Sciences

SIMON FRASER UNIVERSITY
Fall 2014

APPROVAL

Name: Milad Amir Toutounchian

Degree: Doctor of Philosophy

Title of Thesis: BEAMFORMING FOR MULTIUSER MIMO SYSTEMS

Examining Committee: Dr. Ivan Bajic
Chair

Dr. Rodney Vaughan, Professor, Simon Fraser University
Senior Supervisor

Dr. Paul Ho, Professor, Simon Fraser University
Supervisor

Dr. Jie Liang, Associate Professor, Simon Fraser University
Supervisor

Dr. Daniel C. Lee, Professor, Simon Fraser University
Internal Examiner

Dr. Lutz Lampe, Professor, University of British Columbia
External Examiner

Date Defended/Approved: September 5th, 2014

Abstract

Wireless communications systems use a multiple user scheme such as time- or frequency-division, but these do not allow truly simultaneous use of the spectrum. By deploying multiple antennas and beamforming, it is possible, in principle, for users to share the spectrum simultaneously, and this scenario is called the multiuser MIMO interference channel. This thesis presents new beamfoming design methods for this channel, derived from the convergence criteria for multi-objective optimization. Beamforming is proven to be possible for any combination of communications objective functions such as mean-square error, signal-to-interference plus noise ratio, and leakage interference. Relationships are found between the number of users and number of antennas, for different objective functions. The existence of a Nash equilibrium is guaranteed and the important networking properties of quality of service and fairness among users are accounted for. A new optimization algorithm, which is an extension of alternating optimization, is formulated for the design process. Its advantage over existing approaches is its significantly lower computational complexity. Several optimized, multi-user OFDM systems are formulated and demonstrated by simulation using statistical channel models in a multipath environment. The feedback overhead required for deploying the beamforming is quantified, showing the trade-off among complexity, minimum number of antennas required, error performance, capacity, feedback rate, and the ability to extract multi-path diversity for multiple users. When one of the users has priority access to the spectrum, the channel takes on a form of cognitive radio. This scenario is formulated as an optimization which requires solution via an evolutionary algorithm, and convergence is shown to be faster when more antennas are deployed. Finally, an architecture is presented that enables a secondary (i.e., low priority) user, whose terminals cannot directly "see" each other, to communicate in the presence of multiple primary users. The cost is the need for all the primary users to be modified to collaborate with the secondary user, and for several MIMO relays to be installed. The secondary capacity is maximized under constraints of transmission power and interference to the primary receivers, and relay selection. This concept showcases several communications techniques including eigen-beamforming, channel selection and capacity optimization.

To my beloved parents, Sousan and Iraj
for their unconditional affection, support, encourage and guidance
and to my sister and brother, Pouneh and Amirhossein
for their kindness and friendship
and to my wife, Ouldooz
for her vibes and support

"There is no greater wealth than wisdom, no greater poverty than ignorance, no greater heritage than culture, and no greater friend and helpmate than consultation."

— *Nahjul Balagha*, IMAM ALI (AS), *610-673 A.D.*

Acknowledgments

Thanks to God whose blessings and merciful have made me who I am today. The PhD is a very long journey with many ups and downs, and without collaboration with supportive people it is never being accomplished. I do remember when I wanted to start the program my senior supervisor, Professor Rodney G. Vaughan, told me that it is a big commitment. Now, I have realized it is a very true statement.

It is not easy to thank my senior supervisor at the School of Engineering Science, SFU, Dr. Rodney G. Vaughan just by some words. But what can I say, without any doubt, I could complete the program with a high level of my personal satisfaction only because of Dr. Vaughan. He has a unique academic and personal character. Dr. Vaughan is a great mentor that seeks his students' interests first and flourish them patiently. He is a very good friend for his students as well. Dr. Vaughan taught me, by his manner, that the knowledge and personality should come together. I have been extremely lucky to have a senior supervisor who cared so much about my work and my life. With his persistent encouragement, I could explore, study and accomplish research in the last couple of years. Finally, I want to acknowledge Dr. Vaughan deeply for his great support during my PhD career technically and financially.

I want to express my gratitude to Professor Daniel C. Lee, my internal examiner, for his smart questions and for his clever comments. Dr. Lee is a great faculty member with strong mathematical background. He also teaches communication's topics with solid proofs. The main contribution of my thesis, which is optimization, is aligned with Dr. Lee's interests and he helped me a lot in this regard.

I would like to acknowledge Professor Paul Ho, my supervisor. His constructive comments really improved the flow of the thesis in its final version. I managed to get the highest grade for the course Advanced Digital Communications offered by him just because he was always encouraging the motivated students. He taught me how to think analytically.

I would like to thank Professor Jie Liang, my supervisor. I have always wondered how he can get the gist of any presentation or article so quickly and manage to emphesize/critisize the most fundamental aspect of the work. During my PhD program he was the graduate chair of the department. All of the graduate students experienced one of the best administered and highly disciplined period with him.

I wish to express my appreciation to my external examiner from UBC, Professor Lutz Lampe. I have had a course with him in September 2010. I have found him a very organized faculty with diverse and deep knowledge in the wireless/wired communication systems. I really appreciate his time for revising my thesis and providing insightful feedbacks to me.

It is my honor to thank Professor Ivan V. Bajic who accepted to hold the role of the session chair for my thesis defense. For two consecutive semesters, he gave me the chance to experience the role of course developer for engineering analysis.

Finally, I am so grateful to Professor Kamal Gupta who accelerate my admission process and Professor Sami Muhaidat who was my supervisor and supported me for the first year of my program.

Last, but by no means least, I want to warmly thanks my great friends in Sierra Wireless Communications Laboratory, Mehdi Seyfi for his great help and consultation whenever I encountered to a hard problem, Saeed Ashrafinia for his vibes, Maral Dehghani for her kindness and her editorial skills, Ali Zarei, Alireza Banani, Sara Bavarian, Mahsa Najibi, Moein Shayegania, Maryam Razmhosseini, Piraj Fozoonmayeh, Omar Altrad, Ying Chen, Muhammad Naeem and Abhijit Bhattacharya. I experienced a very pleasant environment in this lab. As a visiting graduate researcher at UCLA in fall 2012, I want to deeply appreciate, Professor Ali H. Sayed for his hospitality and informative meetings, Zaid Towfic, Xiaochuan Zhao, Jianshu Chen, Harishan and Shang Kee Ting.

Besides the academic supporters and educational mentors, I owe my parents who have always been role models for me. Both of them are highly educated people who never tire of being beneficial to the society. They enlighten my professional path undoubtedly. They strongly encouraged me to expand my skills and experience academic and social life in western countries. I declare my immense gratitude to my lovely and caring wife, brother and sister as well. I dedicate this thesis to my gorgeous family.

List of Symbols

$\mathbf{a}$	A boldface lowercase letter denotes a vector.
$\mathbf{A}$	A boldface uppercase letter denotes a matrix.
$\mathrm{span}(\mathbf{A})$	Subspace generated by the columns of $\mathbf{A}$.
$\mathcal{N}(\mathbf{A})$	Null space of matrix $\mathbf{A}$.
$\dim(\mathcal{N}(\mathbf{A}))$	Dimension of the null space of matrix $\mathbf{A}$.
$\mathrm{tr}(\mathbf{A})$	The trace, sum of diagonal elements, of $\mathbf{A}$.
$\det(\mathbf{A})$	The determinant of $\mathbf{A}$.
$\mathbb{E}(X)$	The expected value of random variable X.
$(.)^T$	Transpose operation.
$(.)^{\mathcal{H}}$	Hermitian transpose operation.
$(.)^{-1}$	Inverse operation.
$\mathrm{conj}(a)$	Complex conjugate of a.
$\mathrm{Re}(a)$	Real part of a.
$\angle a$	Angle of a.
x_i^*	A solution for decision variable x_i.
$[\cdot]_{k,l}$	$(k,l)^{th}$ entry of a matrix.
$[\cdot]_k$	k^{th} entry of a vector.
$\mathbf{A}(:,i)$	Matlab notation for the ith column of matrix $\mathbf{A}$.
$\mathbf{I}_M$	Identity matrix of size M.
$\mathbf{0}_{M\times M}$	All-zero matrix of size $M \times M$.
$\boldsymbol{w}_{\max}(\mathbf{A})$	Unit-norm eigenvectors of $\mathbf{A}$ corresponds to the maximum eigenvalue of $\mathbf{A}$.
$\lambda_{\max}(\mathbf{A})$	Largest eigenvalue of $\mathbf{A}$.
$\mathbf{F}$	$N \times N$ FFT matrix with $\mathbf{F}(l,k) = 1/\sqrt{N}\exp(-\jmath 2\pi lk/N)$.
$\|\cdot\|_{\mathbf{F}}$	Frobenius norm.
$\|\cdot\|$	Euclidean norm of a vector.
$C(n,r)$	Number of all combinations of r objects, selected from n objects.
$\lvert\mathcal{S}\rvert$	The number of elements of set $\mathcal{S}$.
$\left(\!\binom{\mathcal{S}}{n}\!\right)$	Set of all n - multisets on $\mathcal{S}$.
$O(.)$	Notation to express an algorithm runtime complexity.

List of Acronyms

AO	Alternating optimization
BER	Bit error rate
BF	Beamforming
BS	Base station
CIR	Channel impulse response
CP	Cyclic prefix
CR	Cognitive radio
CSI	Channel state information
EA	Evolutionary Algorithm
EAO	Extended alternating optimization
ESA	Exhaustive Search Algorithm
FFT	Fast Fourier transform
FIR	Finite impulse response
IA	Interference alignment
IBI	Inter-block interference
IC	Interference channel
IFFT	Inverse fast Fourier transform
ISI	Inter-symbol interference
KKT	Karush-Kuhn-Tucker
LAN	Local area networks
LCP	Linear constellation precoding
LCP-OFDM	linear constellation precoded OFDM
LI	Leakage interference
LP	Linear programming
LS	Least square
LTEA	Long Term Evolution-Advanced
MAC	Multiple access channel
MIMO	Multiple-input-multiple-output
MISO	Multiple-input-single-output
ML	Maximum likelihood
MLD	Maximum likelihood detector
MMSE	Minimum mean square error
MRC	Maximal ratio combining
Rx	Receiver
MOO	Multi-objective optimization

MS	Mobile station
MSE	Mean-square-error
MU	Multi-user
OFDM	Orthogonal frequency division multiplexing
QAM	Quadrature amplitude modulation
QP	Quadratic programming
QCQP	Quadratically constrained quadratic programming
QPSK	Quadrature phase shift keying
RS	Relay station
SDP	Semi-definite programming
SDR	Semi-definite relaxation (decoder)
SER	Symbol error rate
SIMO	Single-input-multiple-output
SINR	Signal-to-interference noise ratio
SLNR	Signal-to-leakage plus noise ratio
SISO	Single-input single-output
SNR	Signal-to-noise ratio
SOCP	Second-order cone programming
S.t.	Subject to
STBC	Space-time block codes
SVD	Singular value decomposition
TDMA	Time-division multiple access
Tx	Transmitter
WPAN	Wireless personal area network
ZF	Zero-forcing

Contents

List of Tables

List of Figures

Chapter 1

Introduction

1.1 Motivation

Beamforming is a signal processing technology that is used to direct the reception or transmission (the signal energy) of a array in a chosen angular direction. Classically, beamforming works by setting the antenna element weights so that the beam is concentrated on a signal coming from one particular direction while striving to ignore interference from other directions. In multipath, where the same signal is incident from several different directions, the number of directions normally exceeds the degrees of freedom[1] available from the array. But as long as the degrees of freedom exceed the number of different signals, then beamforming is still possible, although in this case the directional beams themselves are hard to interpret. Clearly, beamforming is only possible when there are multiple antennas, and these can be at either the transmitter or the receiver, or both. In this introductory chapter, we first summarize the literature for existing beamforming methods. A particular class of beamforming has been proposed specifically for OFDM systems in the last couple of years. This class will be briefly discussed in this chapter. Two specific classes of beamforming are adaptive (classical LMS, etc.) and robust beamforming (minimum variance), and these techniques have practical disadvantages which are reviewed. Prior to interference alignment papers [7, 8], there have been a vast number of publications on beamforming designs based on mathematical optimization techniques, covering uplink or downlink in SIMO/MISO/MIMO systems [9, 10, 11, 12, 13]. MIMO in uplink or downlink cases is considered as many-to-one or one-to-many systems, respectively. However, in this thesis, the subject of interest is K decentralized users which send their data to a designated receiver while experiencing $K-1$ interference terms from other users. This system is referred to as the *multiuser MIMO interference channel.* A K-user MIMO interference channel is a many-to-many system, and beamforming designs are more challenging compared to many-to-one and one-to-many.

[1] Mathematically, degrees of freedom is the number of 'free' components

Unfortunately, the valuable results for many-to-one or one-to-many cannot be directly applied for the more complicated many-to-many case.

The formulation for the K user MIMO interference channel ends up with a hard (i.e. difficult) multi-objective multi-variable optimization problem. There is no classic mathematical approach for this kind of hard optimization problem. However, we can tackle this problem by introducing an iterative algorithm for decoupled sub-problems. The convergence of our proposed method is guaranteed if three assumptions hold. We show that the solution (convergent point) is a Nash Equilibrium (NE) for those sub-problems (games). Our contribution, in this regard, can be considered as an extended approach for the alternating optimization (AO) technique which was developed for multi-variable single objective optimization problems.

The organization of the rest of this chapter is as follows: first the general concept of beamforming is presented for the narrow-band case with a single weight per antenna, the wide-band case with a tapped delay line of weights for each antenna element. The beamforming in an OFDM system is presented including the time-domain (narrow-band) and the frequency-domain (wide-band) configurations. Beamforming is often discussed in the context of uplink or downlink, and these scenarios are also clarified as being different to the research of this thesis. The advent of *interference alignment* for beamforming in multiuser interference channel was a major advance in beamforming, and is a starting point for this thesis. Finally, a review of the main classes of optimization is given. These classes are prerequisite for understanding the following chapters.

1.2 Beamforming

A digital beamformer samples the propagating signal at the input of each antenna element, weights them based on a certain performance criterion and then combines them at the output of the beamformer. An illustration of beamforming with a desired user and three interference is depicted in fig. (1.1).

There are two types of beamformers, one for narrow-band signals and one for wide-band signals. For the ideal wireless channel, $h(t) = \delta(t)$ where $\delta(t)$ is Dirac function, the received signal at the mth antenna element is just the phase-shifted version of the signal received at the reference antenna. Consider an array of M antennas at the receiver. Let $\mathbf{a}(\theta)$ denote the response of the array to a plane wave arriving from direction θ. The $\mathbf{a}(\theta)$ for a uniformly-spaced linear antenna (ULA, see fig. (1.2)) array is:

$$\mathbf{a}(\theta) = [1 \;\; e^{-\jmath\frac{2\pi}{\lambda}d sin\theta} \cdots e^{-\jmath\frac{2\pi}{\lambda}(M-1)d sin\theta}]^T \tag{1.1}$$

where λ is the carrier wavelength and so it is clear that $\mathbf{a}$ is frequency dependent. With this ideal channel assumption, the received vector $\mathbf{x}$ is:

$$\mathbf{x}(t) = \mathbf{a}(\theta)x(t) + \mathbf{n}(t) \tag{1.2}$$

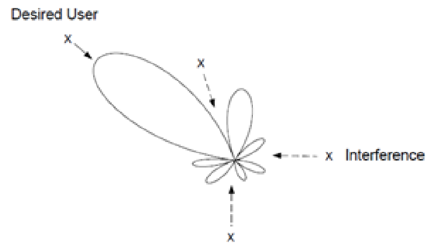

Figure 1.1: Simple illustration of beamforming.

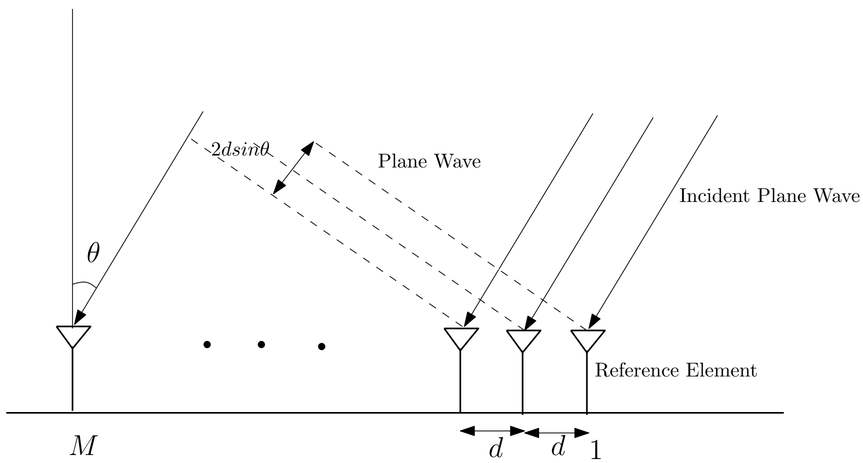

Figure 1.2: Uniform linear array of antenna.

For simplicity, we drop the time index t, so the received signal y after beamformer is (see fig. (1.3)):

$$y = \mathbf{w}^{\mathcal{H}}\mathbf{a}(\theta)x + \mathbf{w}^{\mathcal{H}}\mathbf{n} = \mathbf{w}^{\mathcal{H}}\mathbf{x} + \mathbf{w}^{\mathcal{H}}\mathbf{n} \tag{1.3}$$

The minimum variance beamformer (MVB) is chosen such that:

$$\begin{aligned} \min_{\mathbf{w}} \quad & \mathbf{w}^{\mathcal{H}}\mathbf{R_x}\mathbf{w} \\ \text{s.t.} \quad & \mathbf{w}^{\mathcal{H}}\mathbf{a}(\theta) = 1. \end{aligned} \tag{1.4}$$

where $\mathbf{R_x} = \frac{1}{N}\sum_{i=k-N+1}^{k}\mathbf{x}(i)\mathbf{x}^{\mathcal{H}}(i) \in \mathbb{C}^{M\times M}$. There is a classical closed-form solution for MVB e.g. [14]:

$$\mathbf{w} = \frac{(\mathbf{R_x} + \mu\mathbf{I})^{-1}\mathbf{a}(\theta)}{\mathbf{a}^{\mathcal{H}}(\theta)(\mathbf{R_x} + \mu\mathbf{I})^{-1}\mathbf{a}(\theta)} \tag{1.5}$$

where $0 \leq \mu \leq 1$. Because θ of the desired and interference signals are unknown, the procedure for obtaining the optimal $\mathbf{w}$ would be: 1) sweep θ from $-\pi$ to π; 2) for each θ from previous step calculate $\mathbf{a}(\theta)$ then obtain $\mathbf{w}$ from (1.5). The peaks, corresponding to $\frac{\sigma_x^2}{\mathbf{w}^{\mathcal{H}}\mathbf{R_x}\mathbf{w}}$, reveal the direction of arrival (DoA) for desired and interference signals. The MVB is defined as a robust design because it has a closed-form. The MVB beamforming design assumes that the signal at the $(m-1)$th antenna is the delayed $(\frac{2\pi}{\lambda}dsin\theta)$ version of the signal at the mth antenna. This assumption does not hold if include channel in the system.

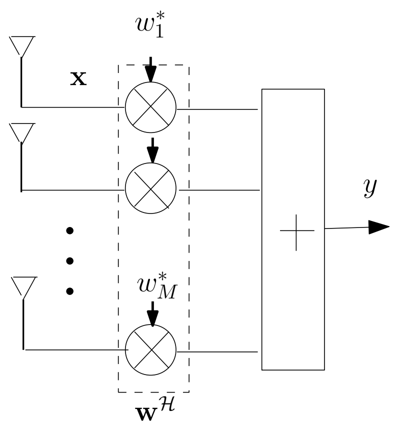

Figure 1.3: Narrow-band beamformer.

When interference signal is wide-band, then the weights appropriate for one frequency will not be appropriate for a different frequency, since the array pattern through $\mathbf{a}$ changes. This issue can be addressed by using tapped-delay line at each antenna element.

For systems such as OFDM, the adaptive algorithms update the beamformer weights, $w_1^*, ..., w_M^*$ according to the known pilots. Fig. (1.4) is a block diagram for time-domain (pre-FFT) beamformer scheme at the receiver of an OFDM system that decodes the desired signal in an interference environment.

After the CP removal, the received signal of each antenna is multiplied by its corresponding beamformer weight. These signals are added to construct the time-domain signal y. Let p denote the subcarrier of the P point FFT transformation. The signal y is then converted to the frequency domain by an FFT operation. This weighting process is formulated as:

$$y = \mathbf{w}^{\mathcal{H}}\mathbf{x} + \mathbf{w}^{\mathcal{H}}\mathbf{n} \tag{1.6}$$

Define $\mathbf{y} \triangleq [y(1); y(2); ...; y(P)]$, after applying the FFT:

$$\mathbf{z} = \mathbf{F}\mathbf{y} \tag{1.7}$$

where $[\mathbf{F}]_{p,q} = (1/\sqrt{P})e^{-\jmath(p-1)(q-1)/P}$. By comparing the pilot portion of $\mathbf{z}$, denoted as $\mathbf{z}_{\text{pilot}}$, with the desired known pilot, $\mathbf{d}_{\text{pilot}}$, the error terms in the frequency domain, $\mathbf{e}_{\text{pilot}}$, and the time-domain

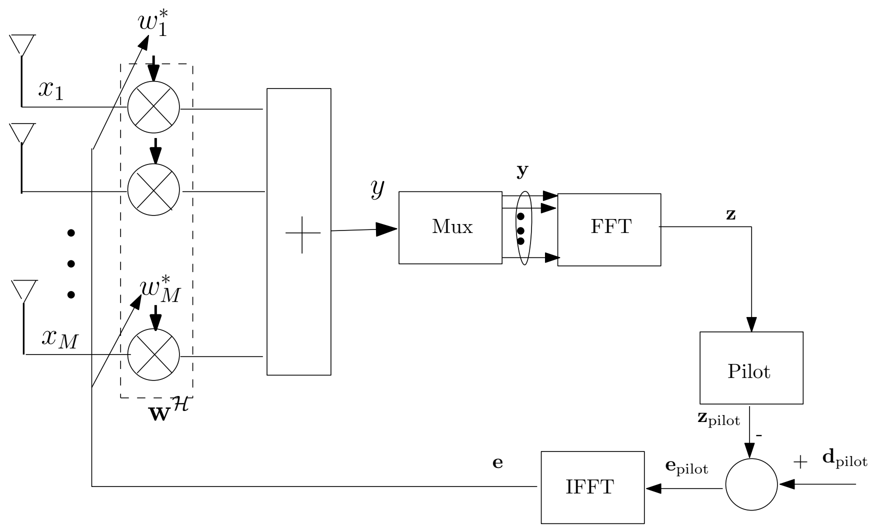

Figure 1.4: Time-domain beamformer for OFDM system (preFFT).

error, $\mathbf{e} = \mathbf{F}^{\mathcal{H}}\mathbf{e}_{\text{pilot}}$, are obtained. By using adaptive LMS, the beamformer weights are updated as:

$$\mathbf{w}(p) = \mathbf{w}(p-1) + 2\mu\mathbf{X}\mathbf{e}^* \tag{1.8}$$

where $\mathbf{X} \triangleq [\mathbf{x}(1)...\mathbf{x}(P)]$.

The post-FFT beamforming design has been also proposed in [15, 16]. The performances for both pre-FFT and post-FFT beamformer designs, by adaptive algorithms, when the powers of interferences are comparable to the power of desired user, have been evaluated in [15]. It is shown in [15] that the post-FFT gives better BER than pre-FFT and the combined pre-post-FFT beamforming provides the best performance for two equal power interferences with one desired source (SIR $= -3dB$). This is because there are more degrees of freedom for the post-FFT beamformer.

1.3 Beamforming in Uplink/Downlink Multiuser MIMO

Downlink beamforming gained more attention because of its potential of enhancing the capacity without the need of costly signal processing at the mobile station [17]. It has been shown that multiuser beamforming for uplink and downlink are closely linked and are actually dual problems. The term duality here is not used in a mathematical sense, but rather to emphasize that both problems can be solved by a unified approach. By solving the dual uplink problem, a solution for the downlink is obtained, and vice versa [17]. From a network operator's perspective, it is desirable to support a target QoS for individual users with optimal spectral efficiency. The QoS of a given link mainly depends on the signal-to-interference-plus-noise ratio (SINR). It can be modeled as a function of SINR, say $f(\text{SINR})$, where the function f takes into account various system aspects, like coding, modulation, pulse shaping, and so forth. One proposed problem and solution is to minimize the total transmission power while fulfilling $\text{SINR}_i \geq \gamma_i$ for $i = 1, ..., K$. In recent years, it has been discovered that there exist *dual* properties between the uplink and downlink channels. Duality between uplink and downlink channels has proved useful for the development of optimum transceiver strategies. This was extended and formulated in [18, 19]. Later, a connection between the uplink and downlink with Lagrangian duality was observed in [20]. For any general optimization problem, a dual function associated with the primal problem is defined. It has been proved in mathematics that for any dual feasible vector, the dual function serves as a lower bound on the optimal primal objective function [21]. Independently, an information-theoretical duality was shown for capacity regions of MIMO Gaussian broadcast channels [22].

The downlink of a multiuser MIMO system comprises a K decentralized users, each having N_r received antennas, and one base station with N_t transmit antennas. Mathematically, the base-band model of the received signal for the ith users is expressed by:

$$\mathbf{y}_i = \mathbf{H}_i \sum_{k=1}^{K} \mathbf{V}_k \mathbf{x}_k + \mathbf{n}_i \qquad i = 1, ..., K \tag{1.9}$$

where $\mathbf{H}_i \in \mathbb{C}^{N_r \times N_t}$ is the modeled flat-fading channel gains between the transmitter and ith receiver. Here, the $\mathbf{x}_i \in \mathbb{C}^{d_i \times 1}$ is the data which is intended to be received by ith users. The $\mathbf{V}_i \in \mathbb{C}^{N_t \times d_i}$ is the precoder (beamformer if $d_i = 1$ so we have $\mathbf{v}_i$) with transmit power constraint:

$$\text{tr}(\mathbf{V}_i^{\mathcal{H}} \mathbf{V}_i) \leq p_i \qquad i = 1, ..., K \tag{1.10}$$

and $\mathbf{n}_i$ is a $d_i \times 1$ vector of i.i.d. complex Gaussian random variables with zero mean, and the variance of σ_i^2.

Define $\mathbf{x} \triangleq [\mathbf{x}_1^T \cdots \mathbf{x}_K^T]^T$, $\mathbf{y} \triangleq [\mathbf{y}_1^T \cdots \mathbf{y}_K^T]^T$, $\mathbf{V} \triangleq [\mathbf{V}_1 \cdots \mathbf{V}_K]$ and $\mathbf{H} \triangleq [\mathbf{H}_1^T \cdots \mathbf{H}_K^T]^T$. The multiuser MIMO downlink signal model can be represented as:

$$\mathbf{y} = \mathbf{H}\mathbf{V}\mathbf{x} + \mathbf{n} \tag{1.11}$$

From (1.11), possible and straightforward solutions for $\mathbf{V}$ and $\mathbf{U}$ (the receive beamformers matrix, see figure (1.5)) are right and left eigen vectors of $\mathbf{H}$, respectively. However, in this chapter, we will show that eigen decomposition of the concatenated channel $\mathbf{H}$ is not a trivial solution for the transmit or receive precoder (or beamformer) for K user MIMO interference channel.

The transmit beamforming design with zero-forcing method for downlink multiuser MIMO is proposed in [23] and is summarized as:

$$\mathbf{V}_i = \mathcal{N}([\mathbf{H}_1^T \cdots \mathbf{H}_{i-1}^T \ \ \mathbf{H}_{i+1}^T \cdots \mathbf{H}_K^T]^T) \tag{1.12}$$

By this nullspace allocation, $\mathbf{H}_i \mathbf{V}_k = \mathbf{0}$ for $i \neq k$. The ZF nullspace criteria imposes $N_t \geq N_r K$ [23].

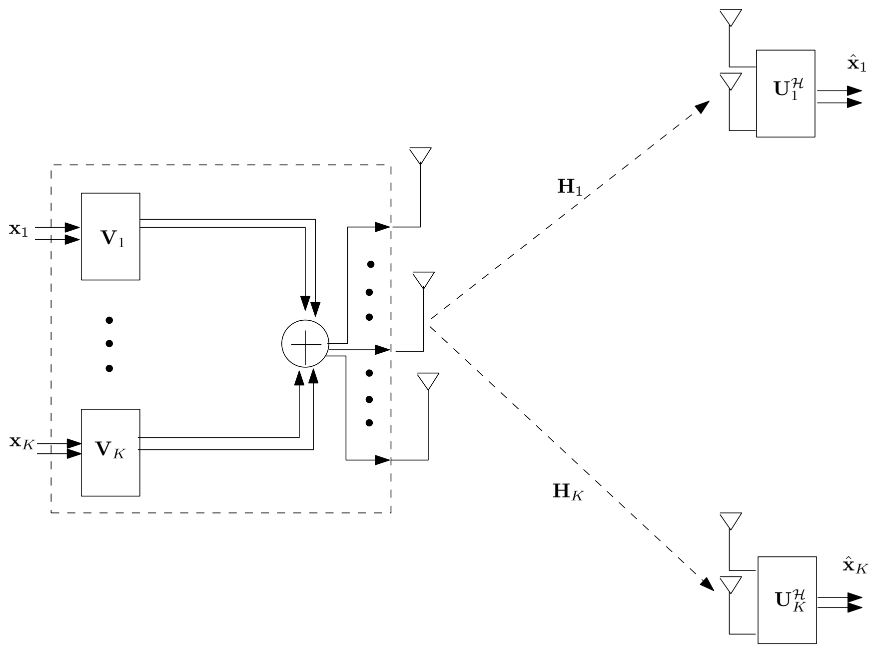

Figure 1.5: Downlink beamformer for multiuser MIMO.

1.4 Beamforming in Multiuser MIMO Interference Channels

The main focus of the thesis is beamforming in a multiuser MIMO system where K decentralized transmitters send information (data) to K decentralized receivers while using the same radio resources. A user is defined as a pair of one transmitter and one receiver (point-to-point transmission). Therefore, there are K users which are equipped with MIMO transmit and MIMO receive antennas.

For the interference channel concept, Maddah-Ali et.al. [7] and Cadambe et.al. [8] showed that for the fully connected K user wireless interference channel where the channel coefficients are time-varying and are drawn from a continuous distribution, the sum capacity is characterized as $C(\rho) = K/2\log(\rho) + o(\log(\rho))$ where ρ is signal-to-noise ratio (SNR). The degrees of freedom (DoF) in a communication system, in information theory, is defined as $\text{DoF} = \lim_{\rho\to\infty} C(\rho)/\log(\rho)$. Thus, the K user time-varying interference channel almost surely has $K/2$ degrees of freedom. Achievability of $K/2$ DoF is based on the idea of interference alignment (IA). The idea of interference alignment is that users coordinate their transmissions, using linear precoding, such that the interference signal lies in a reduced dimensional subspace at each receiver [24]. The conventional IA obtains precoders (beamformer) from considering the interference signal space. However, it is desirable to make the desired signal space roughly orthogonal the interference signal space [25]. To emphasize that the K-user MIMO interference channel is more complicated that downlink/uplink multiuser MIMO, assume that each user is sending d_i streams of information to its designated receiver in an interference channel:

$$\mathbf{y}_i = \mathbf{H}_{ii}\mathbf{V}_i\mathbf{x}_i + \sum_{j\neq i}^{K}\mathbf{H}_{ij}\mathbf{V}_j\mathbf{x}_j + \mathbf{n}_i \qquad i = 1, ..., K \tag{1.13}$$

In matrix representation:

$$\mathbf{y} = \mathbf{H}\mathbf{V}\mathbf{x} + \mathbf{n} \tag{1.14}$$

where

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{11} & \cdots & \mathbf{H}_{1K} \\ \mathbf{H}_{21} & \cdots & \mathbf{H}_{2K} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{K1} & \cdots & \mathbf{H}_{KK} \end{bmatrix}, \mathbf{V} = \begin{bmatrix} \mathbf{V}_1 & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{V}_K \end{bmatrix}.$$

Unlike uplink/downlink beamforming, $\mathbf{V}$ cannot be a left singular decomposition (SVD) of $\mathbf{H}$ in the interference channel scenario.

It is shown that IA is feasible [8, 26] if there exists $\mathbf{U}_i$ for $i = 1, ..., K$ such that:

$$\begin{cases} \text{rank}(\mathbf{U}_i^{\mathcal{H}}\mathbf{H}_{ii}\mathbf{V}_i) = d_i \\ \mathbf{U}_i^{\mathcal{H}}\mathbf{H}_{ij}\mathbf{V}_j = \mathbf{0} \quad \forall j \neq i \end{cases} \tag{1.15}$$

The IA is not the only option for designing the transmit precoders in multiuser interference channels. Various precoding (or beamforming if $d_i = 1$) designs have been proposed in the literature for transmit or receive or joint transmit and receive beamformer design in this scenario. The MIMO interference channels beamforming designs by cooperative algorithms, known as Alternating Optimization (AO) in mathematics, was discussed in [27]. Precoder designs based on IA is the subject of the recent paper [25]. The authors of [25] also derived the gradient formulation for weighted sum rate maximization which inherently is an iterative algorithm. Maximization of minimum SINR for the K-users was introduced in [3]. The authors proved that for the MISO or SIMO cases this optimization problem can be solved by a polynomial time algorithm. However, for the general MIMO case this problem is NP-hard. The joint transmit and received beamforming design cases with MSE as the cost function are treated in [27] and [1]. Reference [27] has formulated the MSE minimization as an iterative second-order-cone programming (SOCP). Reference [1] solved MSE problem by KKT. Convex optimization for precoder design in MIMO interference networks with sum rate maximization as the objective function has been recently published in [28].

1.5 Optimization

Optimization as a powerful technique that can increase a system's efficiency by considering inherent bounds and constraints. For designing any system efficiently, optimization is needed. Optimization

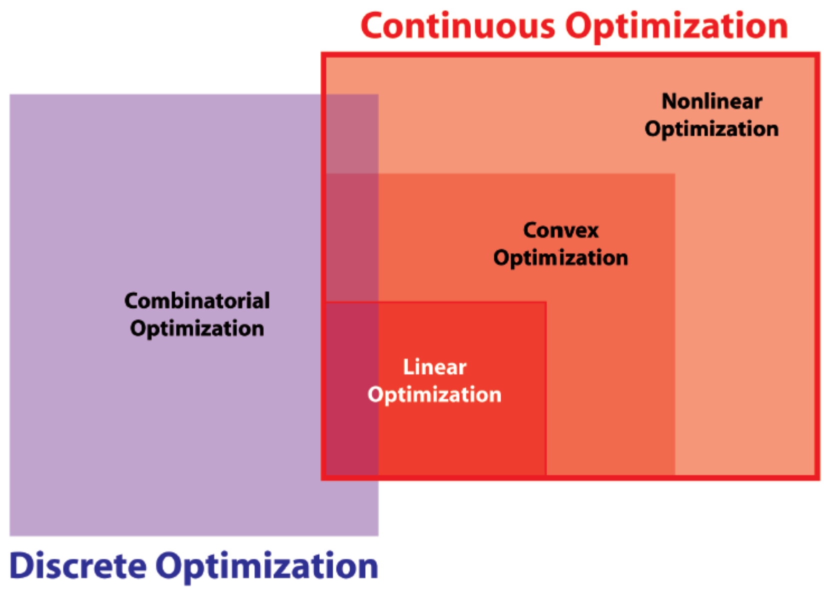

Figure 1.6: Optimization Classes.

is a broad area and consists of some classes shown in fig 1.6.

The mathematical bases behind continuous optimization are calculus, linear algebra, geometry, topology and probability. However, the principle behind discrete optimization is mainly graph theory. The field of optimization is presently at a turning point, a time at which an important change takes place which extends optimization's applications, due to:

1-Recent methodological developments:

- Convex optimization and algebraic geometry
- Non-convex optimization
- Robust optimization
- Stochastic optimization

2-Algorithmic developments:

- Polynomial time interior point methods
- New gradient-type methods for very large scale optimization problems

3-Powerful software:

- CPLEX

- Gurobi
- AMPL
- CVX
- MATLAB

1.5.1 Convex Functions and Convex Optimization Problems

A continuous function $f : \mathbb{R}^n \to \mathbb{R}$ is said to be convex if the domain of f, $\mathrm{dom}f$ (here is $\mathbb{R}^n$), is a convex set and for any $\mathbf{x}_1, \mathbf{x}_2 \in \mathrm{dom}f$ and $0 \le \theta \le 1$,

$$f(\theta\mathbf{x}_1 + (1-\theta)\mathbf{x}_2) \le \theta f(\mathbf{x}_1) + (1-\theta)f(\mathbf{x}_2) \tag{1.16}$$

Second-order condition for convexity,

$$\nabla^2_{\mathbf{xx}} f(\mathbf{x}) \succeq 0 \qquad \forall \mathbf{x} \in \mathrm{dom}f \tag{1.17}$$

Convex optimization problem in standard form is represented as:

$$\begin{aligned} \min_{\mathbf{x}} \quad & f(\mathbf{x}) \\ \text{s.t.} \quad & g_i(\mathbf{x}) \le 0 \qquad i = 1, ..., m \\ & \mathbf{Ax} = \mathbf{b}. \end{aligned} \tag{1.18}$$

where f and g_i, $i = 1, ..., m$ are convex functions and the equality constraint is affine. Convex optimization problems have three properties:

- Any locally optimal point of a convex problem is globally optimal
- Reformulating a problem in convex form (if possible) is an art, there is no systematic approach

Most problems are not convex when formulated.

Linear programming (LP) is the first and fundamental class of convex optimization. Some new standard convex problem classes are:

Quadratic Programming (QP):

$$\begin{aligned} \min_{\mathbf{x}} \quad & \frac{1}{2}\mathbf{x}^T\mathbf{Px} + \mathbf{q}^T\mathbf{x} + r \\ \text{s.t.} \quad & \mathbf{Gx} \le \mathbf{h} \\ & \mathbf{Ax} = \mathbf{b}. \end{aligned} \tag{1.19}$$

where $\mathbf{P} \succeq 0$.

Quadratically Constrained QP (QCQP):

$$\begin{aligned}
\min_{\mathbf{x}} \quad & \frac{1}{2}\mathbf{x}^T\mathbf{P}_0\mathbf{x} + \mathbf{q}_0^T\mathbf{x} + r_0 \\
\text{s.t.} \quad & \frac{1}{2}\mathbf{x}^T\mathbf{P}_i\mathbf{x} + \mathbf{q}_i^T\mathbf{x} + r_i \leq 0 \qquad i = 1, ..., m \\
& \mathbf{A}\mathbf{x} = \mathbf{b}.
\end{aligned} \tag{1.20}$$

where $\mathbf{P}_i \succeq 0$ for $i = 0, ..., m$.

Second-Order Cone Programming (SOCP):

$$\begin{aligned}
\min_{\mathbf{x}} \quad & \mathbf{q}^T\mathbf{x} \\
\text{s.t.} \quad & \|\mathbf{A}_i\mathbf{x} + \mathbf{b}_i\| \leq \mathbf{c}_i^T\mathbf{x} + d_i \qquad i = 1, ..., m \\
& \mathbf{F}\mathbf{x} = \mathbf{g}.
\end{aligned} \tag{1.21}$$

The SOCP has linear objective and second-order cone constraints. If $\mathbf{A}_i$ is a row vector, SOCP reduces to an LP. For $\mathbf{c}_i = 0$, it reduces to a QCQP. It is more general than QCQP and LP.

Semidefinite Programming (SDP):

$$\begin{aligned}
\min_{\mathbf{X}} \quad & \mathrm{tr}(\mathbf{C}\mathbf{X}) \\
\text{s.t.} \quad & \mathrm{tr}(\mathbf{A}_i\mathbf{X}) = b_i \qquad i = 1, ..., m \\
& \mathbf{X} \succeq 0.
\end{aligned} \tag{1.22}$$

In general, there is no analytical formula for the solution of convex optimization problems, but there are very effective methods for solving them. Interior-point methods work very well in practice, and in some cases can be proved to solve the problem to a specified accuracy with a number of operations that does not exceed a polynomial of the problem dimensions. It cannot yet be claimed that solving general convex optimization problems is a mature subject, such as solving least-squares or linear programming problems. Research on interior-point methods for general nonlinear convex optimization is still a very active research area, and no consensus has emerged yet as to what the best method or methods are. But it is reasonable to expect that solving general convex optimization problems will become an standard procedure within a few years.

1.5.2 General Nonlinear Constrained Optimization Problem

In the previous subsection, a convex optimization problem was introduced as a special class of general nonlinear constrained optimization problem (NCOP). However, many optimization problems in communication are non-convex. This general class can be mathematically represented as:

$$\begin{aligned} \min_{\mathbf{x}\in\mathbb{R}^n} \quad & f(\mathbf{x}) \\ \text{s.t.} \quad & g_i(\mathbf{x}) = 0 \quad i \in \mathcal{E} \\ & g_i(\mathbf{x}) \leq 0 \quad i \in \mathcal{I}. \end{aligned} \tag{1.23}$$

where $\mathcal{E}$ denotes the indices of the equality constraints, and $\mathcal{I}$ denotes the indices of the inequality constraints. The *feasible region* Ω of NCOP is the set,

$$\Omega = \{\mathbf{x}|g_i(\mathbf{x}) = 0 \text{ for } i \in \mathcal{E}, g_i(\mathbf{x}) \leq 0 \text{ for } i \in \mathcal{I}\} \tag{1.24}$$

If $\mathbf{x}$ is feasible for NCOP, $\Omega \neq \emptyset$ and $\mathbf{x} \in \Omega$, the indices of the active inequality constraints, $\mathcal{I}(\mathbf{x})$, is denoted as:

$$\mathcal{I}(\mathbf{x}) = \{i \in \mathcal{I}|g_i(\mathbf{x}) = 0\} \tag{1.25}$$

Suppose the f and g_i for $i \in \mathcal{E} \cup \mathcal{I}$ are all differentiable. The Karush-Kuhn-Tucker (KKT) necessary conditions state that if $\mathbf{x}^*$ is a local minimum of NCOP (note the asterisk does not denote complex conjugate here) then there is a Lagrange multiplier vector $\boldsymbol{\lambda}^*$ such that:

$$\begin{cases} \nabla f(\mathbf{x}^*) + \sum_{i\in\mathcal{E}\cup\mathcal{I}} \lambda_i^* \nabla g_i(\mathbf{x}^*) = \mathbf{0} \\ g_i(\mathbf{x}^*) = 0 & i \in \mathcal{E} \\ g_i(\mathbf{x}^*) \leq 0 & i \in \mathcal{I} \\ \lambda_i^* \geq 0 & i \in \mathcal{I} \\ \lambda_i^* g_i(\mathbf{x}^*) = 0 & i \in \mathcal{I} \end{cases} \tag{1.26}$$

For general NCOP, in the absence of convexity, a KKT point can be a global minimum, a local minimum or a saddle point. In order to develop sufficient conditions for a KKT point to be a local minimum, the Hessian matrix of Lagrangian function, $\mathcal{L}(\mathbf{x}, \boldsymbol{\lambda}) \triangleq f(\mathbf{x}) + \sum_{i\in\mathcal{E}\cup\mathcal{I}} \lambda_i g_i(\mathbf{x})$, is needed. By definition, for a feasible point $\mathbf{x} \in \Omega$, the set of linearized feasible direction cone is defined:

$$\mathcal{F}(\mathbf{x}) = \{\mathbf{d} \in \mathbb{R}^n|\mathbf{d}^T \nabla g_i(\mathbf{x}) = 0 \ \ \forall i \in \mathcal{E}, \mathbf{d}^T \nabla g_i(\mathbf{x}) \leq 0 \ \ \forall i \in \mathcal{I}(\mathbf{x})\} \tag{1.27}$$

The KKT sufficient conditions state that if $(\mathbf{x}^*, \boldsymbol{\lambda}^*)$ satisfy the following conditions:

$$\begin{cases} \nabla f(\mathbf{x}^*) + \sum_{i\in\mathcal{E}\cup\mathcal{I}} \lambda_i^* \nabla g_i(\mathbf{x}^*) = \mathbf{0} \\ g_i(\mathbf{x}^*) = 0 & i \in \mathcal{E} \\ g_i(\mathbf{x}^*) \leq 0 & i \in \mathcal{I} \\ \lambda_i^* \geq 0 & i \in \mathcal{I} \\ \lambda_i^* g_i(\mathbf{x}^*) = 0 & i \in \mathcal{I} \\ \mathbf{d}^T \nabla^2_{\mathbf{x},\mathbf{x}} \mathcal{L}(\mathbf{x}^*, \boldsymbol{\lambda}^*)\mathbf{d} > 0 \text{ for } \mathbf{d} \in \mathcal{F}(\mathbf{x}^*), \mathbf{d}^T \nabla f(\mathbf{x}^*) = 0 \end{cases} \tag{1.28}$$

then $\mathbf{x}^*$ is a strict local minimum of NCOP.

1.5.3 Review of The Alternating Optimization Algorithm

This section provides the necessary basics of AO, and draws heavily from [29]. AO is an iterative procedure for minimizing a general nonlinearly constrained optimization problem (NCOP) jointly over all the variables. It works by alternating minimizations over non-overlapping subsets of the variables. The set of points $\Omega_{\mathcal{J}}$ that satisfy all of the constraints of a NCOP is the feasible set for NCOP. The general form of NCOP is written:

$$\min_{\mathbf{x}\in\Omega_{\mathcal{J}}} \mathcal{J}(\mathbf{x}) = \mathcal{J}(\mathbf{x}_1, \cdots, \mathbf{x}_K) \tag{1.29}$$

where $\mathbf{x} = [\mathbf{x}_1, ..., \mathbf{x}_K]^T$. The simple idea underlying AO is to replace the sometimes difficult joint optimization of $\mathcal{J}$ over all K variables with a sequence of easier optimizations involving subsets of the variables $\mathbf{x}_1$, $\mathbf{x}_2$, ..., $\mathbf{x}_K$. Specifically, AO defines an iteration sequence $\{(\mathbf{x}_1^{(n)}, ..., \mathbf{x}_K^{(n)}) : n = 0, 1, ...\}$ that begins at $(\mathbf{x}_1^{(0)}, ..., \mathbf{x}_K^{(0)})$, and is generated by a sequence of restricted minimizations of the form:

$$\min_{\mathbf{x}_i\in\Omega_i} \mathcal{J}(\mathbf{x}_1^{(n+1)}, ...\mathbf{x}_{i-1}^{(n+1)}, \mathbf{x}_i, \mathbf{x}_{i+1}^{(n)}, ..., \mathbf{x}_K^{(n)}) \tag{1.30}$$

where $\Omega_1 \times \cdots \times \Omega_K = \Omega_{\mathcal{J}}$. Now it is assumed that the optimization problem (1.30) has a global minimizer w.r.t. $\mathbf{x}_i$. The optimal solution of (1.30), denoted by $\mathbf{x}_i^{(n+1)}$, is stated as:

$$\mathbf{x}_i^{(n+1)} = l_i(\mathbf{x}_1^{(n+1)}, ..., \mathbf{x}_{i-1}^{(n+1)}, \mathbf{x}_{i+1}^{(n)}, ..., \mathbf{x}_K^{(n)}) \quad i = 1, ..., K \tag{1.31}$$

where l_i is a non-linear function which is a solution of (1.31).

By optimality of each sub-problem, it is easy to show that:

$$\mathcal{J}(\mathbf{x}_1^{(n+1)}, \cdots, \mathbf{x}_K^{(n+1)}) \leq \mathcal{J}(\mathbf{x}_1^{(n)}, \cdots, \mathbf{x}_K^{(n)}) \tag{1.32}$$

The $\mathbf{x}^{(N)} = [\mathbf{x}_1^{(N)}, ..., \mathbf{x}_K^{(N)}]^T$, as $N \to \infty$, is a stationary point of $\mathcal{J}$ [29]. By definition, $\mathbf{x}^*$ is a stationary point of the constraint optimization problem (1.29) if:

$$(\mathbf{x} - \mathbf{x}^*)^T \nabla_{\mathbf{x}} \mathcal{J}(\mathbf{x}^*) \geq 0 \quad \forall \mathbf{x} \in \Omega_{\mathcal{J}} \tag{1.33}$$

Moreover, $\mathbf{x}^{(n)}$ converges q-linearly to $\mathbf{x}^{(N)}$ [29].

There is no guarantee for optimality by AO. Also, the relationship between $\mathbf{x}^*$, a converging point by AO, and the KKT point of the constrained optimization problem (1.29) is the subject of recent articles. AO is a powerful technique for many single-objective applications [27].

1.5.4 Pareto Optimal and Nash Equilibrium

Definition: the feasible point $(\mathbf{x}^*, \mathbf{y}^*) \in \Omega_{f_1} \times \Omega_{f_2}$ constitutes a Nash Equilibrium of the two games $\mathcal{G}_1$ and $\mathcal{G}_2$ iff:

$$f_1(\mathbf{x}, \mathbf{y}^*) \geq f_1(\mathbf{x}^*, \mathbf{y}^*) \quad \forall \mathbf{x} \in \Omega_{f_1} \tag{1.34}$$

$$f_2\left(\mathbf{x}^*, \mathbf{y}\right) \geq f_2\left(\mathbf{x}^*, \mathbf{y}^*\right) \quad \forall \mathbf{y} \in \Omega_{f_2}. \tag{1.35}$$

where

$$\begin{aligned} \mathcal{G}_1 :& \\ \min_{\mathbf{x}} \quad & f_1(\mathbf{x}, \mathbf{y}) \\ \text{s.t.} \quad & \mathbf{x} \in \Omega_{f_1}. \end{aligned} \tag{1.36}$$

$$\begin{aligned} \mathcal{G}_2 :& \\ \min_{\mathbf{y}} \quad & f_2(\mathbf{x}, \mathbf{y}) \\ \text{s.t.} \quad & \mathbf{y} \in \Omega_{f_2}. \end{aligned} \tag{1.37}$$

Definition: the feasible point $(\mathbf{x}^*, \mathbf{y}^*) \in \Omega_{f_1} \times \Omega_{f_2}$ is the Pareto optimum of problem $\mathcal{P}$ (see below) if there does not exist another feasible point $(\tilde{\mathbf{x}}, \tilde{\mathbf{y}}) \in \Omega_{f_1} \times \Omega_{f_2}$ such that:

$$f_1\left(\mathbf{x}^*, \mathbf{y}^*\right) \geq f_1\left(\tilde{\mathbf{x}}, \tilde{\mathbf{y}}\right) \tag{1.38}$$

$$f_2\left(\mathbf{x}^*, \mathbf{y}^*\right) \geq f_2\left(\tilde{\mathbf{x}}, \tilde{\mathbf{y}}\right). \tag{1.39}$$

with at least one inequality being strict. In other words:
if $f_1\left(\mathbf{x}^*, \mathbf{y}^*\right) > f_1\left(\tilde{\mathbf{x}}, \tilde{\mathbf{y}}\right) \quad \Rightarrow \quad f_2\left(\mathbf{x}^*, \mathbf{y}^*\right) < f_2\left(\tilde{\mathbf{x}}, \tilde{\mathbf{y}}\right)$.
The problem $\mathcal{P}$ is defined as:

$$\begin{aligned} \mathcal{P} :& \\ \min_{\mathbf{x}, \mathbf{y}} \quad & [f_1(\mathbf{x}, \mathbf{y}), f_2(\mathbf{x}, \mathbf{y})] \\ \text{s.t.} \quad & (\mathbf{x}, \mathbf{y}) \in \Omega_{f_1} \times \Omega_{f_2}. \end{aligned} \tag{1.40}$$

Except for the trivial cases, Pareto optimum and Nash equilibrium do not necessarily coincide. Plenty of examples are available to confirm this.

1.6 Contributions and Organization

Objectives of this research are to propose, extend and analyze beamforming design for different multiuser MIMO communication systems. The main focus is to formulate beamforming design as an optimization problem and to propose an algorithm for finding the solution. For multiuser MIMO-IC systems, we will show that our algorithm guarantees a Quality of Service for all users. This guarantee is seldom satisfied from existing algorithms. Providing both fast and fair solutions for a difficult optimization problem is the center of attention here. The relationship between the number of antennas and the number of users, based on the selected objective functions, is derived as the part of convergence criteria for our algorithm. Issues such as feedback rate and maximum tolerable velocity for users are discussed. Beamforming designs for cognitive radios are the subjects of interest in this thesis as well. We introduce simplification steps to be able to tackle a difficult capacity maximization problem. The following summarizes the main contributions in this thesis.

1.6.1 Multi-Objective Optimization for Multiuser MIMO Interference Channels

The beamformer design for multi-user MIMO interference channels seeks solutions for high capacity and low error rate. This calls for multi-objective, multi-variable optimization. A class of single-objective, multi-variable optimization is known to be solvable by an alternating optimization (AO) technique and its convergence criteria are also known, but solution of the multi-objective case is more demanding. In this research, an efficient approximate solution for a class of multi-objective, multi-variable optimization problems is presented. It comprises a converging iterative process for obtaining the fixed point of a nonlinear continuous mapping function. In general, optimality is not guaranteed, but the solution has the useful properties in the MIMO interference channel of inherent fairness and quality of service (QoS) for all users. Simulations illustrate that the method leads to the users having both high capacities and low error rates, which are normally competing metrics in interference channels.

In this work, we propose the general framework for a class of multi-objective optimization which the extended alternating optimization (EAO) can be applied. We show that the converging point is a Nash Equilibrium (NE) for the games (sub-problems). The relationship between NE and stationary point of original problem or relationship between NE and KKT point for AO and obviously for EAO are hard open problems.

The main advantage of our proposed EAO method is its low computational complexity compared to existing methods like MSE. Generally the computational complexity of EAO for various combination of cost functions is $O(NKM^3)$ where N recalled as the number of iterations, K is number of users with M antennas at both transmitter and receiver sides. However, the computational complexity of MSE in multiuser MIMO interference channel, which is computed for the first time here, is $O(INM^2K^6)$ where I is interior-point-method (IPM) iteration.

1.6.2 Beamforming for Multiuser MIMO-OFDM Interference Channels with Multipath Diversity

We present three beamforming designs for multiuser MIMO-OFDM where the transmit and receive beamformers are obtained iteratively with closed-form steps. In the first case, the transmit (Tx) beamformers are set and then the receive (Rx) beamformers are calculated. It works by projecting the Tx beamformers into a null space of appropriate channels. This eliminates one interference term for each user. Then, the Rx-beamformer for each user maximizes its instantaneous signal-to-noise ratio (SNR) while satisfying an orthogonality condition to eliminate the remaining interferences. The second case is jointly optimizing of the Tx and Rx beamformers from constrained SNR maximization. It uses the results from the first case. The third case is also for joint optimization of Tx-Rx beamformers but combines constrained SNR and signal-to-interference plus noise ratio (SINR) maximization. The minimum number of antennas required is derived as part of the formulation. All cases can include a linear constellation precoder for extracting multipath diversity. Finally, the feedback rates are derived and compared to existing beamforming methods. Using the standardized statistical channel model for IEEE 802.11n, the simulations demonstrate faster beamforming, improved error performance and the ability to extract multipath diversity which is not possible in the least-square (LS) approach.

we show that EAO can be applied also for K sub-problems (K games). The Tx-BF and Rx-BF design for joint constrained SNR maximization in this work is transformed to K Tx only optimization problems. The LS beamforming designs for MIMO interference channels is proposed here for the first time and LS shown to be solvable by evolutionary algorithms. Therefore, our three proposed approaches are preferable to LS. The feedback rate, which can be considered as CSI overhead in IC, is introduced and compared among proposed and existing methods in this work. Simulations are performed for the more realistic standardized statistical channel model for indoor environment, IEEE 802.11n. Finally, we evaluate the computational complexity, execution time and the performance for sum-rate maximization by EAO over K games with the existing gradient method.

1.6.3 Beamforming for MIMO Cognitive Radio with Single Primary and Multiple Secondary Users

A cognitive radio system often comprises a primary user collaborating with multiple secondary users. Beamforming for such a system is presented which strives to create an interference-free environment for the primary user. The objective is to maximize the signal-to-interference plus noise ratio (SINR) for the primary user through the transmit beamformers of all users and the receive beamformer of the primary user. Finding the maximum SINR corresponds to constrained maximization of the largest eigenvalue of a Hermitian positive semidefinite matrix. This problem is not a convex optimization; however, the upper bound is known and the solution set exists, so evolutionary algorithms can be used. In the system presented here, the secondary users do not have beamformers at their multi-antenna receivers but instead use quasi-maximum-likelihood detection based on semidefinite relaxation (SDR). The bit error rate of the secondary users turns out to be comparable to the known technique of prioritized sum signal power over sum interference plus noise ratio, but our primary user has significantly better performance. The main advantages of the approach are as follows. Firstly, the calculation of the beamformers - undertaken at the primary receiver - only needs knowledge of the channels to the primary receiver. This decreases the system's overhead used for feedback. Secondly, our primary user link outperforms that from beamforming using alternating maximization which also needs full channel information and full beamforming information to be fedback.

We show that the prioritized MIMO interference channel potentially has application for a cognitive network. However, this scenario needs to be solved by evolutionary algorithms such as Genetic algorithm (GA). The more antennas at the primary and secondary networks the less GA generations required for a fixed tolerance. Our proposed method is practicable if there is one primary and a few secondary users. An advantage for this system is that only partial channel information is required for beamforming design.

1.6.4 Beamforming and Relay Selection in MIMO Cognitive Radio Networks

We consider the problem of joint relay/antenna selection and beamforming in a multiple-input-multiple-output (MIMO) amplify-and-forward (AF) relay assisted cognitive radio network. In particular, we assume a "Secondary User" with a MIMO cooperative setup, where the source uses beamforming and selects relays to assist the source-destination communication. Simultaneously, communication is performed in an underlay cognitive radio environment where the primary user(s) may tolerate only a certain amount of interference from the MIMO cooperative secondary user network. Assuming perfect channel state information (CSI) at all nodes, we propose a suboptimal relay selection scheme and find the corresponding transmit and receive beamforming for each selected relay, by taking the interference constraints into account. We show that prior to capacity maximization of secondary user, orthogonalization is needed which can be realized by two methods: semi-orthogonal singular vector selection beamforming (DS-SVSB) and projected semi-orthogonal singular vector selection beamforming (PS-SVSB) schemes. The second scheme requires one more antenna at the secondary source. These two schemes firstly define the relay selection phase and secondly transform the capacity optimization problems to a simpler formulation.

1.7 Scholarly Publications

The contributions of my thesis have resulted in four journals and one conference paper. Two other papers (one journal as the second author and one conference paper) from my time as PhD student, are published, but these are not discussed in this thesis. The related published and under-review articles for this dissertation are as follows.

1.7.1 Journal Papers

1. M. A. Toutounchian, R. Vaughan, "Beamforming with Multipath Diversity in a Multiuser MIMO-OFDM Interference Channel" , accepted for IEEE Trans. Wireless Communications

2. M. A. Toutounchian, R. Vaughan, "Multi-Objective Optimization for Multiuser MIMO Interference Channels" , submitted for IEEE Trans. Communications

3. M. A. Toutounchian, R. Vaughan, ”Beamforming for MIMO Cognitive Radio with Single Primary and Multiple Secondary Users” , submitted to IEEE Communications Letters

4. M. Seyfi, M. A. Toutounchian and R. Vaughan, ”Beamforming and Relay Selection in MIMO CognitiveRadio Networks”, is going to be submitted to IEEE Trans. Wireless Communications

1.7.2 Conference Papers

1. M. A. Toutounchian, R. Vaughan, ”SINR-based Transceiver Design in the K-user MIMO Interference Channel using Multi-Objective Optimization”, Proc. Vehicular Technology Conference (VTC Fall), 2013 IEEE 78th, Sept. 2013

Chapter 2

Multi-Objective Optimization for Multiuser MIMO Interference Channels

2.1 Introduction

Multi-user MIMO has the potential of being a breakthrough technique for improving spectral efficiency (shared use of radio resources) in wireless communications. As the demand for wireless services continues to soar, and with the spectrum being a finite, shared resource, MIMO has become a key technology for future communications systems. But multi-user MIMO systems that can perform to their potential are still far from being commercially viable. The stumbling block is the need to adapt, by optimization, the antenna weights for all the terminals. A description for multi-user MIMO is the $K-$user interference channel (IC), which refers to K users, each comprising a pair (the transmitter and receiver) of multi-antenna terminals that share the spectrum simultaneously and in the same space with all the other users. The arrays at each terminal jointly strive to suppress the interference between the different users, and at the same time maximize some measure of the quality of the users' links.

The beamformer design for multi-user MIMO-IC seeks high capacity and low error rate. This calls for multi-objective, multi-variable optimization. A class of single-objective, multi-variable optimization is known to be solvable by an alternating optimization (AO) technique and its convergence criteria are also known, but solution of the multi-objective case is more demanding. In this chapter, an efficient approximate solution for a class of multi-objective, multi-variable optimization problems is presented. It comprises a converging iterative process for obtaining the fixed point of a nonlinear continuous mapping function. Our approach is an extension to AO (EAO). In general, optimality is not guaranteed either by EAO or AO, but the EAO solution has the useful properties in the MIMO-IC of inherent fairness and quality of service (QoS) for all users. We show that the beamforming design by EAO has the lowest computational complexity compared to the existing designs which are solved by AO or gradient descent methods. Simulations discussed in this chapter illustrate that the EAO can provide all users with high capacities and low error rates, which are normally competing metrics in multiuser MIMO-IC.

The motivation for proposing the EAO is its simplicity along with its guaranteed QoS for all users and fairness among users. The QoS is not directly included in the formulation but is satisfied because of the lower-bound property of the objectives at the converging point. We will show numerically that EAO is capable of providing better performance than maximization of minimum SINR_i for $i = 1, ..., K$ which is a difficult optimization problem.

The limitations and assumptions in this chapter and throughout the thesis are summarized as follows. For the beamforming designs, we assume perfect channel state information (CSI). Additionally, one receiver node is considered as a central processing unit which gathers all of the users's CSI and computes all of the receiver beamformers and all of the transmit beamformers. These computed quantities should be fed back to other nodes. These assumptions are common for other previous works on beamforming algorithms and optimization. However, in the thesis, the feedback rates for our proposed methods and also existing methods are derived and compared. For example, in the next chapter, we propose a minimum feedback design which can be considered as a distributed beamforming design fashion. But it has poor performance compared to the centralized designs. Generally, beamformer design with constrained feedback is beyond the scope of the thesis. However, some recent works have been published to address beamforming design with limited feedback. For example, in [30], transmitter channel state information (CSIT) subject to quantization error (Grassmannian quantization) and delays of feedback channels is analyzed. But in [30], perfect channel estimation at the receivers, an error-free feedback link between each receiver and transmitter, and perfect time synchronization at each time slot, are assumed. In [31], the authors proposed Grassmannian differential feedback to reduce feedback overhead by exploiting the channel's temporal correlation. The authors also evaluated their approach both analytically and numerically as a function of channel length, mobility, and the number of feedback bits [31]. Finally, there are two CSI acquisition protocols/paradigms considered in the interference alignment literature: 1- Interference alignment via reciprocity; 2-Interference alignment with feedback. Figure 3 in [24] sketches the details of these two protocols. The complexity analysis in this chapter is performed to address only the computational burden of various optimization designs in centralized fashion. The practicality of multiuser MIMO-IC has been studied in a few articles [5, 24, 32]. Section 2.4.3, relates the maximum allowed user mobility to algorithm execution times.

2.1.1 Background on Digital Communications and Optimization

The quality or performance of a practicable link is typically expressed as some form of throughput related to the number of correctly detected bits per sec per radio frequency bandwidth. This is difficult to optimize directly because high spectral efficiency in varying channels requires communications techniques such as adaptive modulation, forward error correction coding, a complex protocol for channel sounding and exchanging the channel sounding data, and other aspects of multiple access management; and these techniques and their interactions are complicated. Therefore, indirect performance metrics that are more manageable, such as some form of signal-to-noise ratio, information-theoretic capacity, or uncoded error rate, are optimized. But it is seldom obvious which indirect metrics are the best to address. Therefore, several metrics have been presented in the literature as objective functions and these are treated here as examples for the multi-objective optimization. The current MIMO terminology for these objective functions is as follows.

2.1.2 Background on Terminology of Channels and Optimization

The spectral efficiency of optimal combining in receive or transmit diversity, and in MIMO, stems from arranging signal cancelation using beamforming with the antenna weights. The residue of the cancelation is the *self-interference*, which comprises the transmitted signals from the MIMO transmitting terminals that are inadvertently received by the receivers owing to imperfect cancelation. So interference from spectral users other than those from the MIMO system under consideration, is not part of the self-interference.

In a multi-user MIMO system, the self interference in one direction - it is convenient for now to call this the downlink (see below) - when summed across the terminal pair of all the users, has become referred to as *leakage*, or *leakage interference* (LI). However, in the uplink direction, this self interference is simply referred to as *interference*. Consequently the *sum SINR* is the ratio of the wanted signal at a single receiving terminal to the noise plus the total interference, which is caused by all users to all other users, in the uplink.

In the downlink, this quantity is referred to as the signal-to-leakage interference plus noise ratio, or *sum SLNR*. In fact the mathematical description of these signal and interference terms refer to the gains of the channels which include the effects of antenna embedded element patterns and the elements' weights. For example, the LI is the sum of these gains for the self interference in the downlink. In a reciprocal link, the sum leakage is the same as the sum interference. Sometimes, in the evolving terminology, the uplink and downlink self interference are both called leakage, and this explains the term LI-LI optimization, considered below.

If also the noise power is the same at each end of the reciprocal link (this is seldom the case in practice but it is assumed in this chapter for simplicity) then the sum SLNR is the same as the sum SINR. Along these lines, the sum of the gains for the wanted signal is referred to as signal power

(SP).

Finally, the mean square error (MSE) is also used, which refers to the difference between the sum (with the summation across the uplink and downlink directions) of all the channel gains (wanted signals and self interferences) and the gains for the wanted signal. It is emphasized that there is no external interference, i.e., from outside of the K users, considered here. (The suppression of such interference is the goal of many multiple antenna systems.) Alternatively stated, any external interference must be lumped with the noise.

In discussing the $K-$user interference channel, a clarification of the use of *uplink* and *downlink* is needed. These terms typically refer to cellular systems where the uplink is from the mobile to the basestation. If one end of the K links are all at a common basestation, then there is the possibility of coordinating (mathematically and physically) the antenna weights across all the users' antennas at the basestation. But in the K-user channel, all of users' terminals are physically separated, and the coordination required for setting the weights is more complicated. This is not just a matter of needing to interchange information between all the terminals. It stems from the mathematics of the channel decomposition which is required for optimizing the weights for the K-user channel. Finding all the antenna weights, or sets of beamformers as they are referred to hereon, is complicated mathematically. Implementation of the protocol is not considered here - the interest is just in the derivation of optimized beamformers.

In mobile communications (and other areas of communications signal processing), the solution of an optimization problem must be found quickly. The calculation time for the solution bites more as the number of variables grows. If the problem can be transformed to a convex optimization problem then solutions are at hand, but non-convex problems comprise a much larger class, and this is the subject of interest here. Alternating optimization (AO) is suitable for problems that are single-objective, multi-variable and non-convex. It is discussed in [29], where it is shown that convergence is guaranteed if the single-objective function has a unique global minimizer for each variable while the other variables are fixed. Recently, the fast-Lipschitz optimization [33] method was introduced for both convex and non-convex multi-objective functions. If some qualifying properties are satisfied, then the existence and uniqueness of the solution for the multi-objective problem is guaranteed [33]. Unfortunately, these qualifying properties do not hold for some problems, including the motivating problem of multi-user MIMO communications.

2.1.3 Contributions

In this chapter, we address a class of optimization problems where neither AO nor fast-Lipschitz methods are directly applicable. This class is therefore more general than fast-Lipschitz, although it has analogous conditions to the qualifying properties for convergence. It extends AO to multiple objectives. We show that for joint design of the transmit-receive (Tx-Rx) beamformers for sum rate maximization, the AO cannot provide a solution for Tx beamformers. (Also, the fast-Lipschitz's qualifying properties do not hold for the sum-rate problem, although this is not shown here.) In order to undertake the joint design for optimizing the sum rate, another objective function is included, such as MSE, LI, SP or SLIR. Thus, two objective functions must be solved simultaneously, e.g., LI-SINR refers to minimizing the sum LI and maximizing the sum SINR. In our approach, the Tx beamformers are taken as known at each iteration, so the maximization of the sum rate and the maximization of the sum SINR are equivalent (see Appendix A for clarifications on this point).

Our main contribution is that, by assuming a unique global minimizer for each sub-problem (one objective function and related constraints) with respect to the decoupled variables, a solution for the hard multi-objective problem is approximated by seeking a solution for a system of nonlinear equations. If this nonlinear function is contractive, or nonexpansive and satisfying certain assumptions [34, 35, 36], or a continuous mapping from a closed ball of a Euclidean space to itself [37], then existence of at least one fixed point is ensured. For each example presented below, the solution of the associated two-objective problems (viz., MSE-SINR, LI-SINR, LI-SP-SINR, SLIR-SINR) is reduced to finding a fixed point of a nonlinear function, and an iterative algorithm is also provided for finding the fixed point. However, the relation of the fixed point with the KKT point of an equally weighted sum of objective functions, including all constraints, is an open problem.

Weighting the objective functions differently does not change the outcome because in an alternating optimization each objective is optimized separately. An alternative for solving an equally weighted sum of objective functions is to use evolutionary algorithms such as a genetic algorithm (GA). But some minimum quality of service (QoS) for all of the users is generally not guaranteed using a GA, whereas the proposed fixed point method acts to maintain some minimum QoS for all users.

Previous work on joint Tx-Rx beamformer design for the K-user MIMO interference channel is summarized as follows. Different objective functions have been defined [27, 25, 38, 3] for this class of communication system. MSE-based transceiver designs are discussed in [1], LI minimization was introduced in [4], and iterative weighted sum rate maximization was formulated in [38]. By applying the AO method [39], the MSE minimization problem in [40] was formulated as an iterative second-order-cone programming (SOCP), which is a convex problem. However, even for a modest number of antennas at each terminal, M, and number of users, K, the MSE minimization by SOCP is too slow for practical deployment [2]. Therefore, a complexity comparison between our approach and the existing approaches is included in this chapter. Finding the optimal solution of max-min SINR is a strongly NP-hard problem for general K and M [3]. It has been demonstrated [2] that the fixed point solution for the LI-SP-SINR problem has lower computational complexity and outperforms max-min SINR. While LI minimization was introduced and solved by AO in [4], the optimality of the solution was not established. The formulation in this chapter includes a proof for this optimality.

In summary, the contributions of this chapter are: 1) an algorithm for a class of multi-objective optimization problems; 2) showing that the optimal solution for LI-LI is provided by AO; 3) showing that the algorithm has lower computational complexity than existing methods but has similar (mostly better) performance while fortuitously guaranteeing a QoS and providing fairness; 4) illustrating that Interference Alignment (IA) is appropriate for initialization.

The rest of the chapter is organized as follows. In Section 2.2, the $K-$user MIMO interference channel and an optimization example (sum SINR) is introduced. In Section 2.3, the basic definitions needed for optimization by the fixed point method are provided leading to a theorem which is applied in Section 2.4. Section 2.5 summarizes convergence of the algorithm and the improvement (speed or complexity, and the communications performance of the solution) over existing methods.

The mathematical notation is as follows: column vectors and matrices are denoted by boldface lower and upper case letters, respectively. Superscripts ^{T} and $^{\mathcal{H}}$ give the transpose and complex conjugate transpose, respectively. The $\boldsymbol{w}_{\max}(\mathbf{A})$ and $\boldsymbol{w}_{\min}(\mathbf{A})$ are the unit-norm eigenvectors of matrix $\mathbf{A}$ that correspond to the maximum and minimum eigenvalue of $\mathbf{A}$, respectively. $\lambda_{\max}(\mathbf{A})$ and $\lambda_{\min}(\mathbf{A})$ indicate the largest and smallest eigenvalues of $\mathbf{A}$, respectively. The range space of $\mathbf{A}$, written as span($\mathbf{A}$), is the subspace generated by the columns of $\mathbf{A}$. The null space of matrix $\mathbf{A}$ is denoted by $\mathcal{N}(\mathbf{A})$. The dimension of the null space of matrix $\mathbf{A}$, known as the nullity of $\mathbf{A}$, is denoted by $\dim(\mathcal{N}(\mathbf{A}))$. $\mathbf{I}_M$ is the $M \times M$ identity matrix, $\mathrm{tr}(\mathbf{A})$ denotes the trace of $\mathbf{A}$, and all the norms are L_2. Specific principal symbols are as follows. J, g, h, d (including subscripted variants) are real scalars; $\mathbf{u}_i$, $\mathbf{v}_i$ (i indexes the number of users) are $M \times 1$ complex vectors, $\mathcal{U}$ and $\mathcal{V}$ are sets of K complex vectors of size $M \times 1$; p_i, q_i, and f_i are functions that return a complex $M \times 1$ vector; f is a function that returns a $KM \times 1$ complex vector, and its argument $\mathbf{x}$ is also a $KM \times 1$ complex vector.

2.2 K-user MIMO Interference Channel

A K-user MIMO interference channel where each of the $2K$ terminals has M antennas is shown in Figure 2.1. The ith ($i = 1, ..., K$) user's transmission comprises a data stream (i.e., parallel data streams between users are not considered here) through a flat, Rayleigh, MIMO channel, $\mathbf{H}_{ii} \in \mathbb{C}^{M \times M}$, while experiencing similar, independently faded interference from the other $K-1$ users. Perfect channel knowledge is assumed at all terminals and perfect timing is also assumed in the usual manner to allow a linear model for the links. The transmit beamformer $\mathbf{v}_i \in \mathbb{C}^{M \times 1}$ is applied before the transmission of symbol s_i where $\mathbb{E}\{|s_i|^2\} = \sigma_s^2$. The data from across users are assumed to be independent of each other. The transmit power constraint is expressed as $\|\mathbf{v}_i\| = 1$. With the usual assumptions of synchronization for the symbols and the sampling, the received signal vector at the ith receiver can be expressed as a sum of the signal, interference and noise:

$$\mathbf{y}_i = \mathbf{H}_{ii}\mathbf{v}_i s_i + \sum_{j \neq i}^{K} \mathbf{H}_{ij}\mathbf{v}_j s_j + \mathbf{n}_i \tag{2.1}$$

where $\mathbf{n}_i$ denotes i.i.d complex Gaussian noise vector at receiver i with zero mean and $\mathbb{E}\{\mathbf{n}_i\mathbf{n}_i^{\mathcal{H}}\} = \sigma_n^2\mathbf{I}_M$. The receive beamformer $\mathbf{u}_i \in \mathbb{C}^{M\times 1}$ is applied to $\mathbf{y}_i \in \mathbb{C}^{M\times 1}$.

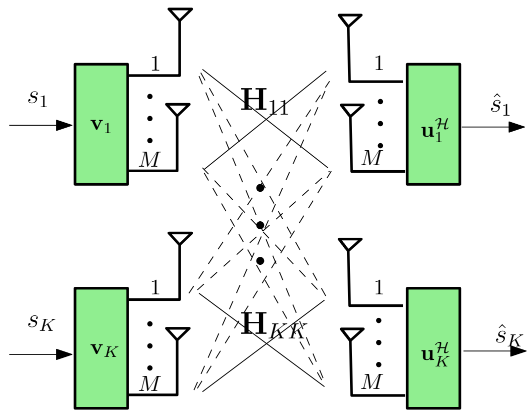

Figure 2.1: Beamforming in a K-user MIMO interference channels.

2.2.1 Motivating Example: Sum Rate Maximization

The sum rate maximization of the K-user MIMO interference channel is important because it has direct impact on capacity. The joint transmitter and receiver beamformer design for sum rate maximization takes the form

$$\begin{aligned}
\max_{\{\mathbf{u}_i\},\{\mathbf{v}_i\}} \quad & \mathcal{J}_1 \triangleq \sum_{i=1}^{K} \log_2(1+\mathrm{SINR}_i) \\
\text{s.t.} \quad & \mathbf{u}_i^{\mathcal{H}}\mathbf{u}_i = 1 \qquad i = 1, ..., K \\
& \mathbf{v}_i^{\mathcal{H}}\mathbf{v}_i = 1 \qquad i = 1, ..., K.
\end{aligned} \tag{2.2}$$

where the signal-to-interference plus noise ratio for the ith user, SINR_i, is the ratio of quadratic forms

$$\mathrm{SINR}_i = \frac{\mathbf{u}_i^{\mathcal{H}}\mathbf{H}_{ii}\mathbf{v}_i\mathbf{v}_i^{\mathcal{H}}\mathbf{H}_{ii}^{\mathcal{H}}\mathbf{u}_i}{\mathbf{u}_i^{\mathcal{H}}(\sum_{j\neq i}^{K}\mathbf{H}_{ij}\mathbf{v}_j\mathbf{v}_j^{\mathcal{H}}\mathbf{H}_{ij}^{\mathcal{H}} + \frac{\sigma_n^2}{\sigma_s^2}\mathbf{I}_M)\mathbf{u}_i}. \tag{2.3}$$

The optimal solution of (2.2) is an open problem but AO can provide a suboptimal solution for it. To solve this problem by AO, first assume that all the transmit beamformers are known, i.e., the variables $\{\mathbf{v}_i\}_1^K$ are fixed. Then, the receive beamformers $\mathbf{u}_i$ that maximize $\mathcal{J}_1$, are obtained in closed-form (see Appendix A):

$$\mathbf{u}_i = \boldsymbol{w}_{\max}\left(\left(\sum_{j\neq i}^{K}\mathbf{H}_{ij}\mathbf{v}_j\mathbf{v}_j^{\mathcal{H}}\mathbf{H}_{ij}^{\mathcal{H}} + \frac{\sigma_n^2}{\sigma_s^2}\mathbf{I}_M\right)^{-1}\mathbf{H}_{ii}\mathbf{v}_i\mathbf{v}_i^{\mathcal{H}}\mathbf{H}_{ii}^{\mathcal{H}}\right)$$

$$= \frac{\left(\sum_{j\neq i}^{K} \mathbf{H}_{ij}\mathbf{v}_j\mathbf{v}_j^{\mathcal{H}}\mathbf{H}_{ij}^{\mathcal{H}} + \frac{\sigma_n^2}{\sigma_s^2}\mathbf{I}_M\right)^{-1}\mathbf{H}_{ii}\mathbf{v}_i}{\left\|\left(\sum_{j\neq i}^{K} \mathbf{H}_{ij}\mathbf{v}_j\mathbf{v}_j^{\mathcal{H}}\mathbf{H}_{ij}^{\mathcal{H}} + \frac{\sigma_n^2}{\sigma_s^2}\mathbf{I}_M\right)^{-1}\mathbf{H}_{ii}\mathbf{v}_i\right\|} \tag{2.4}$$

Now the assumption is removed that the transmit beamformers are known, and to find them, the $\mathbf{u}_i$ given by (2.4) is substituted into $\mathcal{J}_1$. After some manipulations, this can be expressed as needing to solve

$$\max_{\{\mathbf{v}_i\}} \sum_{i=1}^{K} \log_2 \mathcal{S}$$

$$\text{s.t.} \quad \|\mathbf{v}_i\| = 1. \tag{2.5}$$

where $\mathcal{S} \triangleq$

$$\left(1 + \operatorname{tr}\left(\left(\sum_{j\neq i}^{K} \mathbf{H}_{ij}\mathbf{v}_j\mathbf{v}_j^{\mathcal{H}}\mathbf{H}_{ij}^{\mathcal{H}} + \frac{\sigma_n^2}{\sigma_s^2}\mathbf{I}_M\right)^{-1}\mathbf{H}_{ii}\mathbf{v}_i\mathbf{v}_i^{\mathcal{H}}\mathbf{H}_{ii}^{\mathcal{H}}\right)\right)$$

In general, it is not possible to solve (2.5) in closed-form. This example shows that by using AO, the receive beamformer, $\mathbf{u}_i$, can be obtained in closed-form but not the transmit beamformer $\mathbf{v}_i$.

This chapter shows that solving the following two problems simultaneously is possible and leads to closed-form solutions for both $\mathbf{v}_i$ and $\mathbf{u}_i$:

$$\begin{cases} \max\limits_{\{\mathbf{u}_i\}} & \sum\limits_{i=1}^{K} \log_2\left(1 + \text{SINR}_i\right) \\ \text{s.t.} & \mathbf{u}_i^{\mathcal{H}}\mathbf{u}_i = 1 \qquad i = 1, ..., K \\ \max\limits_{\{\mathbf{v}_i\}} & \sum\limits_{i=1}^{K} \log_2\left(1 + \text{SLNR}_i\right) \\ \text{s.t.} & \mathbf{v}_i^{\mathcal{H}}\mathbf{v}_i = 1 \qquad i = 1, ..., K. \end{cases} \tag{2.6}$$

where the signal-to-leakage plus noise ratio (SLNR) is defined below in (2.8). The objective functions in (2.6) include more information than $\mathcal{J}_1$. Optimizing the first problem in (2.6) over the receive beamformers $\mathbf{u}_i$ with the $\mathbf{v}_i$ fixed is equivalent to (see Appendix A):

$$\begin{aligned} \max_{\{\mathbf{u}_i\}} \quad & h \triangleq \sum_{i=1}^{K} \text{SINR}_i \\ \text{s.t.} \quad & \mathbf{u}_i^{\mathcal{H}}\mathbf{u}_i = 1 \qquad i = 1, ..., K; \end{aligned} \tag{2.7}$$

and the unique global solution for the receive beamformers is given by (2.4). Similarly, optimizing the second problem in (2.6) over the transmit beamformers $\mathbf{v}_i$ with the $\mathbf{u}_i$ fixed is equivalent to maximizing the following sum signal-to-leakage plus noise ratio (SLNR) [41]:

$$\begin{aligned} \max_{\{\mathbf{v}_i\}} \quad & g \triangleq \sum_{i=1}^{K} \frac{\mathbf{v}_i^{\mathcal{H}} \mathbf{H}_{ii}^{\mathcal{H}} \mathbf{u}_i \mathbf{u}_i^{\mathcal{H}} \mathbf{H}_{ii} \mathbf{v}_i}{\mathbf{v}_i^{\mathcal{H}} (\sum_{j \neq i} \mathbf{H}_{ji}^{\mathcal{H}} \mathbf{u}_j \mathbf{u}_j^{\mathcal{H}} \mathbf{H}_{ji} + \frac{\sigma_n^2}{\sigma_s^2} \mathbf{I}_M) \mathbf{v}_i} \\ \text{s.t.} \quad & \mathbf{v}_i^{\mathcal{H}} \mathbf{v}_i = 1 \qquad i = 1, ..., K; \end{aligned} \tag{2.8}$$

whose unique solution is

$$\mathbf{v}_i = \boldsymbol{w}_{\max} \left(\left(\sum_{j \neq i} \mathbf{H}_{ji}^{\mathcal{H}} \mathbf{u}_j \mathbf{u}_j^{\mathcal{H}} \mathbf{H}_{ji} + \frac{\sigma_n^2}{\sigma_s^2} \mathbf{I}_M \right)^{-1} \mathbf{H}_{ii}^{\mathcal{H}} \mathbf{u}_i \mathbf{u}_i^{\mathcal{H}} \mathbf{H}_{ii} \right). \tag{2.9}$$

Subsequently, vectors $\mathbf{u}_i^*$ and $\mathbf{v}_i^*$ can be sought that satisfy expressions (2.4) and (2.9) *simultaneously*. It is shown below how these vectors can be characterized as the fixed point of a continuous mapping.

2.3 Optimization by Fixed Point Method

Motivated by example (2.2) and the associated sub-problems (2.7) and (2.8), this section describes a general multi-objective, multi-variable optimization problem whose solution can be characterized as a fixed point. The existence of a fixed point of a vector field has been proven for three cases: a contractive mapping; a nonexpansive mapping which satisfies certain assumptions [34, 35, 36]; or a closed ball self-mapping, i.e., Brouwer's fixed point theorem [37]. Brouwer proved that a continuous function from a closed Euclidean space to itself has a fixed point.

Definition of fixed point: Consider a continuous function (a vector field) $T: \mathcal{D} \to \mathcal{D}$, $\mathcal{D} \subset \mathbb{R}^n$; and $\mathbf{x}^* \in \mathcal{D}$, with $\mathbf{x}^* = T(\mathbf{x}^*)$. Then $\mathbf{x}^*$ is a fixed point of T in $\mathcal{D}$ [42]. Below, f is used as a special example of T.

2.3.1 Optimization Framework and Assumptions

The optimization problems of interest comprise a multi-variable objective function J that is lower-bounded and can be expressed as the sum of two objective functions, g and h. The general problem is denoted by $\mathcal{P}$ and is stated in (2.10), with a collection of equality and inequality constraints where $\mathbf{u}_i \in \mathbb{C}^{M \times 1}$ and $\mathbf{v}_i \in \mathbb{C}^{M \times 1}$ for $i = 1, ..., K$.

Problem $\mathcal{P}$ is tackled by examining the solutions of two sub-problems to approximate the solution of $\mathcal{P}$. The first is

$$\boxed{\begin{aligned} \mathcal{H}: \min_{\mathbf{u}_1, \ldots, \mathbf{u}_K} \quad & h(\mathbf{u}_1, ..., \mathbf{u}_K, \mathbf{v}_1, ..., \mathbf{v}_K) \\ \text{s.t.} \quad & h_i^{\text{in}}(\mathbf{u}_i) \leq 0, \quad h_i^{\text{eq}}(\mathbf{u}_i) = 0, \quad i = 1, ..., K; \end{aligned}} \tag{2.11}$$

which assumes the $\{\mathbf{v}_i\}_{i=1}^K$ are fixed and minimizes h over the $\mathbf{u}_i$. Assuming $\mathcal{H}$ has a unique global minimum (this assumption is established for all of our example problems), and expressing the

$\mathcal{P}:$

$$\begin{aligned} \min_{\mathbf{u}_1,...,\mathbf{u}_K,\mathbf{v}_1,...,\mathbf{v}_K} \quad & J(\mathbf{u}_1,...,\mathbf{u}_K,\mathbf{v}_1,...,\mathbf{v}_K) \triangleq g(\mathbf{v}_1,...,\mathbf{v}_K,\mathbf{u}_1,...,\mathbf{u}_K) + h(\mathbf{u}_1,...,\mathbf{u}_K,\mathbf{v}_1,...,\mathbf{v}_K) \\ \text{s.t.} \quad & h_i^{\text{in}}(\mathbf{u}_i) \leq 0, \quad h_i^{\text{eq}}(\mathbf{u}_i) = 0, \quad i = 1,...,K \\ & g_i^{\text{in}}(\mathbf{v}_i) \leq 0, \quad g_i^{\text{eq}}(\mathbf{v}_i) = 0, \quad i = 1,...,K. \end{aligned} \tag{2.10}$$

minimizers as

$$\begin{cases} \mathbf{u}_1 = p_1(\mathbf{v}_1,...,\mathbf{v}_K) \\ \quad\vdots \\ \mathbf{u}_K = p_K(\mathbf{v}_1,...,\mathbf{v}_K) \end{cases} \tag{2.12}$$

where the $\{p_i\}_{i=1}^K$ are functions of only $\{\mathbf{v}_i\}_{i=1}^K$, then

$$h(\mathbf{u}_1,...,\mathbf{u}_K,\mathbf{v}_1,...,\mathbf{v}_K) \geq h(\mathbf{u}_1,...,\mathbf{u}_K,\mathbf{v}_1,...,\mathbf{v}_K)|_{\mathbf{u}_i=p_i(\mathbf{v}_1,...,\mathbf{v}_K)} \tag{2.13}$$

for any $\{\mathbf{u}_i\}_{i=1}^K \in \Omega_h$, where Ω_h is the feasible set of problem $\mathcal{H}$. Denote h_L as the minimum value of h for the given $\{\mathbf{v}_i\}_i^K$, and the corresponding $\{\mathbf{u}_i\}_i^K$ are in (2.12). To emphasize the dependencies,

$$\begin{aligned} & h_L(\mathbf{v}_1,...,\mathbf{v}_K) \triangleq \\ & h(\mathbf{u}_1,...,\mathbf{u}_K,\mathbf{v}_1,...,\mathbf{v}_K)|_{\mathbf{u}_i=p_i(\mathbf{v}_1,...,\mathbf{v}_K)} = \\ & h\left(p_1(\mathbf{v}_1,...,\mathbf{v}_K),...,p_K(\mathbf{v}_1,...,\mathbf{v}_K),\mathbf{v}_1,...,\mathbf{v}_K\right); \end{aligned} \tag{2.14}$$

then from (2.13):

$$\forall \{\mathbf{u}_i\}_{i=1}^K \in \Omega_h, \;\; h(\mathbf{u}_1,...,\mathbf{u}_K,\mathbf{v}_1,...,\mathbf{v}_K) \geq h_L(\mathbf{v}_1,...,\mathbf{v}_K). \tag{2.15}$$

The second sub-problem, $\mathcal{G}$, is the counterpart of $\mathcal{H}$, viz.,

$$\boxed{\begin{aligned} \mathcal{G}: \min_{\mathbf{v}_1,...,\mathbf{v}_K} \quad & g(\mathbf{v}_1,...,\mathbf{v}_K,\mathbf{u}_1,...,\mathbf{u}_K) \\ \text{s.t.} \quad & g_i^{\text{in}}(\mathbf{v}_i) \leq 0, \quad g_i^{\text{eq}}(\mathbf{v}_i) = 0, \quad i = 1,...,K. \end{aligned}} \tag{2.16}$$

Similar to problem $\mathcal{H}$ above, assuming $\mathcal{G}$ has a unique global minimizer for given $\{\mathbf{u}_i\}_{i=1}^K$ and expressing the solution of $\mathcal{G}$ in terms of functions $\{q_i\}_{i=1}^K$ of only $\{\mathbf{u}_i\}_{i=1}^K$, i.e.,

$$\begin{cases} \mathbf{v}_1 = q_1(\mathbf{u}_1,...,\mathbf{u}_K) \\ \quad\vdots \\ \mathbf{v}_K = q_K(\mathbf{u}_1,...,\mathbf{u}_K), \end{cases} \tag{2.17}$$

then

$$g(\mathbf{v}_1, ..., \mathbf{v}_K, \mathbf{u}_1, ..., \mathbf{u}_K) \geq g(\mathbf{v}_1, ..., \mathbf{v}_K, \mathbf{u}_1, ..., \mathbf{u}_K)|_{\mathbf{v}_i = q_i(\mathbf{u}_1, ..., \mathbf{u}_K)} \tag{2.18}$$

for any $\{\mathbf{v}_i\}_{i=1}^K \in \Omega_g$, where Ω_g is the feasible set of problem $\mathcal{G}$. Denote g_L as the minimum value of g for the given $\{\mathbf{u}_i\}_i^K$ with the corresponding $\{\mathbf{v}_i\}_{i=1}^K$ from (2.17), then

$$\begin{aligned} & g_L(\mathbf{u}_1, ..., \mathbf{u}_K) \triangleq \\ & g(\mathbf{v}_1, ..., \mathbf{v}_K, \mathbf{u}_1, ..., \mathbf{u}_K)|_{\mathbf{v}_i = q_i(\mathbf{u}_1, ..., \mathbf{u}_K)} = \\ & g(q_1(\mathbf{u}_1, ..., \mathbf{u}_K), ..., q_K(\mathbf{u}_1, ..., \mathbf{u}_K), \mathbf{u}_1, ..., \mathbf{u}_K). \end{aligned} \tag{2.19}$$

Our technique strives to solve sub-problems (2.11) and (2.16) simultaneously. Combining the associated functions in (2.12) and (2.17) calls for finding $\{\mathbf{v}_i\}_{i=1}^K$ such that

$$\begin{cases} \mathbf{v}_1 = f_1(\mathbf{v}_1, ..., \mathbf{v}_K) \\ \qquad \vdots \\ \mathbf{v}_K = f_K(\mathbf{v}_1, ..., \mathbf{v}_K) \end{cases} \tag{2.20}$$

which has composite functions $f_i(\mathbf{x}) \triangleq q_i(p_1(\mathbf{x}), ..., p_K(\mathbf{x}))$ where $\mathbf{x} \triangleq \text{col}\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_K\}$ and col concatenates vectors. With vector field $f(\mathbf{x}) \triangleq \text{col}\{f_1(\mathbf{x}), f_2(\mathbf{x}), \ldots, f_K(\mathbf{x})\}$, solving (2.20) amounts to determining a fixed point of f, i.e., a solution of

$$\mathbf{x} = f(\mathbf{x}). \tag{2.21}$$

If f is a contractive or a nonexpansive continuous mapping, which meets certain assumptions (see below), then an iterative procedure can determine fixed points. The procedure does not require the gradient vector of the original cost function J in (2.10), which can be challenging to compute.

2.3.2 Iterative Optimization Technique

The vector field f in (2.21) is a contractive mapping if

$$d(f(\mathbf{x}_1), f(\mathbf{x}_2)) \leq C d(\mathbf{x}_1, \mathbf{x}_2) \tag{2.22}$$

for some $0 \leq C < 1$ (C is the Lipschitz constant) and where $d(\mathbf{x}_1, \mathbf{x}_2)$ denotes the distance measure between the elements $\mathbf{x}_1$ and $\mathbf{x}_2$ in a Hilbert space [43]. A nonexpansive mapping means that for any $\mathbf{x}_1, \mathbf{x}_2$ in the domain of f,

$$d(f(\mathbf{x}_1), f(\mathbf{x}_2)) \leq d(\mathbf{x}_1, \mathbf{x}_2). \tag{2.23}$$

Any fixed point of f is here denoted

$$\text{Fix}(f) \triangleq \{\mathbf{x}^* \mid \mathbf{x}^* = f(\mathbf{x}^*)\}. \tag{2.24}$$

If the domain and image of the mapping f are Z and W, then from theorems in [34, 35, 36], the existence of fixed points for nonexpansive mappings f is ensured ($\text{Fix}(f) \neq \emptyset$) if W is a closed convex subset.

For brevity, the variables (antenna weights) are denoted by

$$\mathcal{U} \triangleq \{\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_K\}, \tag{2.25}$$

$$\mathcal{V} \triangleq \{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_K\}. \tag{2.26}$$

Theorem: If problems $\mathcal{H}$ and $\mathcal{G}$ of (2.11) and (2.16) have unique global minimizers and if the vector field f in (2.21) has a fixed point, then at $(\mathcal{U}^*, \mathcal{V}^*) \in \Omega_h \times \Omega_g$:

$$J\left(\mathcal{U}^*, \mathcal{V}^*\right) = h_L\left(\mathcal{V}^*\right) + g_L\left(\mathcal{U}^*\right). \tag{2.27}$$

Proof: See Appendix B.

Comments:

• In another notation [44], $(\mathcal{U}^*, \mathcal{V}^*)$ is the Nash equilibrium for game $(\mathcal{G}, \mathcal{H})$.

• The significance of the theorem is that it identifies the conditions for which $(\mathcal{U}^*, \mathcal{V}^*)$ can be approximated using an iterative scheme. For contractive, and for nonexpansive with the assumptions, an iterative algorithm is guaranteed to converge towards a fixed point. However, for the closed ball self-mapping (Brouwer's theorem) case, the convergence is not guaranteed for any specific iterative algorithm, although the existence of a fixed point is guaranteed.

• Relating the point $(\mathcal{U}^*, \mathcal{V}^*)$ to the KKT solution or stationary solution[1] of the original problem $\mathcal{P}$ is important. This relationship is an open problem and would be a powerful result but it appears that more assumptions are required in order to establish it. The ramification is that the relevance of $(\mathcal{U}^*, \mathcal{V}^*)$ to the solution of $\mathcal{P}$ is not clear for the general case.

Contractive case: If f in (2.21) is contractive, then Picard iteration [36], $\mathbf{x}^{(n)} = f(\mathbf{x}^{(n-1)})$, converges to a unique solution, $\mathbf{x}^*$, for any initialization $\mathbf{x}^{(0)}$. In other words, for contractive f, $|\text{Fix}(f)| = 1$ which means that $\mathcal{V}^*$ is unique because the modulus gives the number of fixed points. Then the iteration $\left(\mathbf{u}_i^{(n)} = p_i\left(\mathcal{V}^{(n)}\right), \mathbf{v}_i^{(n+1)} = q_i\left(\mathcal{U}^{(n)}\right)\right)$ converges towards $\mathcal{V}^*$, the unique fixed point of f.

Nonexpansive case: If f in (2.21) is nonexpansive and follows the assumptions laid out in [34, 35, 36], then there exists at least one $\mathbf{x}^* = \mathcal{V}^*$ for which $|\text{Fix}(f)| \geq 1$, and it can be obtained iteratively. Three algorithms [46] for finding a fixed point of a nonexpansive vector field f are:

I) Again using Picard iteration:

$$\mathbf{x}^{(n)} = f(\mathbf{x}^{(n-1)}); \tag{2.28}$$

[1] If for all $(\mathcal{U}, \mathcal{V}) \in \Omega_h \times \Omega_g$ we have $[(\mathcal{U}, \mathcal{V}) - (\mathcal{U}^*, \mathcal{V}^*)]^T \nabla_{\mathcal{U},\mathcal{V}} J\left(\mathcal{U}^*, \mathcal{V}^*\right) \geq 0$, then $(\mathcal{U}^*, \mathcal{V}^*)$ is a stationary point of problem $\mathcal{P}$ [45].

Table 2.1: Extended Alternating Optimization (EAO) Algorithm for Approximating the Solution of Multi-Objective Optimization

***Problem*:**

$$\begin{aligned} \mathcal{P}: \min_{\mathbf{u}_1,...,\mathbf{u}_K,\mathbf{v}_1,...,\mathbf{v}_K} \quad & J(\mathbf{u}_1,...,\mathbf{u}_K,\mathbf{v}_1,...,\mathbf{v}_K) = g(\mathbf{v}_1,...,\mathbf{v}_K,\mathbf{u}_1,...,\mathbf{u}_K) + h(\mathbf{u}_1,...,\mathbf{u}_K,\mathbf{v}_1,...,\mathbf{v}_K) \\ \text{s.t.} \quad & h_i^{\text{in}}(\mathbf{u}_i) \leq 0, h_i^{\text{eq}}(\mathbf{u}_i) = 0, \quad i = 1,...,K \\ & g_i^{\text{in}}(\mathbf{v}_i) \leq 0, g_i^{\text{eq}}(\mathbf{v}_i) = 0, \quad i = 1,...,K. \end{aligned}$$

***Assumptions*:**

1-The objective function J is lower bounded.
2-The subproblems $\mathcal{H}$ and $\mathcal{G}$, defined by (2.11) and (2.16), have unique global minimizers w.r.t. $\mathbf{u}_i$ and $\mathbf{v}_i$, respectively.
3-The vector field f, defined by (2.21), is a contractive or nonexpansive or closed ball self mapping.

***Solution*:**

$$\text{Assign: } \mathbf{v}_1^{(0)},...,\mathbf{v}_K^{(0)}; \epsilon > 0;$$

$$\mathbf{u}_i^{(n)} = p_i\left(\mathbf{v}_1^{(n)},...,\mathbf{v}_K^{(n)}\right)$$

$$\mathbf{v}_i^{(n+1)} = q_i\left(\mathbf{u}_1^{(n)},...,\mathbf{u}_K^{(n)}\right)$$

$$\text{If } J\left(\mathcal{U}^{(n)},\mathcal{V}^{(n+1)}\right) - \left(h_L\left(\mathcal{V}^{(n+1)}\right) + g_L\left(\mathcal{U}^{(n)}\right)\right) \leq \epsilon \text{ then quit, else set } n = n+1 \text{ and repeat}$$

II) Mann iteration:

$$\mathbf{x}^{(n)} = (1 - \alpha^{(n-1)})\mathbf{x}^{(n-1)} + \alpha^{(n-1)} f(\mathbf{x}^{(n-1)}); \tag{2.29}$$

where the parameter $\alpha^{(n-1)}$ lies in [0,1);

III) Halpern iteration:

$$\mathbf{x}^{(n)} = \alpha^{(n-1)}\mathbf{w} + (1 - \alpha^{(n-1)}) f(\mathbf{x}^{(n-1)}); \tag{2.30}$$

where $\mathbf{w}$ is an arbitrary point in the domain of f.

Converging towards the fixed point is guaranteed using Mann or Halpern iterations for the case of nonexpansive mapping (with the certain assumptions) in Euclidean, Hadamard, Hilbert or CAT(κ) spaces [47, 48, 46, 42]. In communications problems, the Euclidian (vectors) and Hilbert (matrices) spaces are relevant.

Table 2.1 lists the extended alternating optimization (EAO) algorithm for approximating the solution of (2.10) from the above theorem by using Picard iteration. Let $n = N$ denote the iteration count where $J\left(\mathcal{U}^{(N)}, \mathcal{V}^{(N+1)}\right) - \left(h_L\left(\mathcal{V}^{(N+1)}\right) + g_L\left(\mathcal{U}^{(N)}\right)\right) \leq \epsilon$, then the stopping point $\left(\mathcal{U}^{(N)}, \mathcal{V}^{(N+1)}\right)$ is used as the approximation for $(\mathcal{U}^*, \mathcal{V}^*)$. The difference between the objective function and its lower bound, $J\left(\mathcal{U}^{(n)}, \mathcal{V}^{(n+1)}\right) - \left(h_L\left(\mathcal{V}^{(n+1)}\right) + g_L\left(\mathcal{U}^{(n)}\right)\right)$, is called the merit function.

2.4 Applications to MIMO Interference Channels

In this section, four examples are formulated for the joint Tx-Rx beamforming design. We show that the corresponding function f for these is a closed ball self mapping, i.e., these examples follow Brouwer's theorem. It is emphasized here that for each of these examples, the Tx beamformers are treated as known for sum rate maximization, so it is equivalent to the sum SINR maximization (Appendix A).

2.4.1 LI-SINR, SLIR-SINR, LI-SP-SINR and MSE-SINR

For these four problems, there is a common objective function (denoted h) which is the sum SINR. The second objective functions are discussed below. Different combinations of objective functions result in a different required number of antennas, shown below. It is not straightforward to know which formulation will provide the best performance in a communications context, as discussed in the Introduction. However, for the same number of antennas, LI-SP-SINR has a better sum rate and BER (as calculated in Section 2.5) than LI-SINR because the former includes more information about the signal power in its formulation.

LI-SINR

The sum SINR maximization and LI minimization problem, denoted $\mathcal{P}_1$, is

$$\begin{aligned}\mathcal{P}_1: \min_{\mathbf{u}_1,...,\mathbf{u}_K,\mathbf{v}_1,...,\mathbf{v}_K} \quad & J_1(\mathbf{u}_1,...,\mathbf{u}_K,\mathbf{v}_1,...,\mathbf{v}_K) = g_1 + h \\ \text{s.t.} \quad & \mathbf{u}_i^{\mathcal{H}}\mathbf{u}_i = 1 \qquad i = 1,...,K \\ & \mathbf{v}_i^{\mathcal{H}}\mathbf{v}_i = 1 \qquad i = 1,...,K;\end{aligned} \tag{2.31}$$

where h, the first objective function, is

$$\begin{aligned}&h(\mathbf{u}_1,...,\mathbf{u}_K,\mathbf{v}_1,...,\mathbf{v}_K) = \\ &-\sum_{i=1}^{K}\frac{\mathbf{u}_i^{\mathcal{H}}\mathbf{H}_{ii}\mathbf{v}_i\mathbf{v}_i^{\mathcal{H}}\mathbf{H}_{ii}^{\mathcal{H}}\mathbf{u}_i}{\mathbf{u}_i^{\mathcal{H}}(\sum_{j\neq i}^{K}\mathbf{H}_{ij}\mathbf{v}_j\mathbf{v}_j^{\mathcal{H}}\mathbf{H}_{ij}^{\mathcal{H}} + \frac{\sigma_n^2}{\sigma_s^2}\mathbf{I}_M)\mathbf{u}_i}\end{aligned} \tag{2.32}$$

and LI, the second objective function, denoted g_1, is

$$g_1(\mathbf{v}_1,...,\mathbf{v}_K,\mathbf{u}_1,...,\mathbf{u}_K) = \sum_{i=1}^{K}\mathbf{v}_i^{\mathcal{H}}\left(\sum_{j\neq i}^{K}\mathbf{H}_{ji}^{\mathcal{H}}\mathbf{u}_j\mathbf{u}_j^{\mathcal{H}}\mathbf{H}_{ji}\right)\mathbf{v}_i. \tag{2.33}$$

The unique global minimizer of problem $\mathcal{H}$ (cf. (2.12)) is

$$\mathbf{u}_i = \boldsymbol{w}_{\max}\left(\left(\sum_{j\neq i}^{K}\mathbf{H}_{ij}\mathbf{v}_j\mathbf{v}_j^{\mathcal{H}}\mathbf{H}_{ij}^{\mathcal{H}} + \frac{\sigma_n^2}{\sigma_s^2}\mathbf{I}_M\right)^{-1}\mathbf{H}_{ii}\mathbf{v}_i\mathbf{v}_i^{\mathcal{H}}\mathbf{H}_{ii}^{\mathcal{H}}\right) \tag{2.34}$$

and the global minimizer of problem $\mathcal{G}$ (cf. (2.17)) is

$$\mathbf{v}_i = \boldsymbol{w}_{\min}\left(\sum_{j\neq i}^{K}\mathbf{H}_{ji}^{\mathcal{H}}\mathbf{u}_j\mathbf{u}_j^{\mathcal{H}}\mathbf{H}_{ji}\right). \tag{2.35}$$

The uniqueness of this is important, and happens if the smallest eigenvalue of the nonnegative matrix between parenthesis in (2.35) has multiplicity one. This holds when $M = K$ as shown below.
Lemma 1: For $M = K$, the LI (in equation (2.33)) becomes zero and (2.35) is the unique global minimizer.

Proof. Let $\mathbf{G} \triangleq \sum_{j\neq i}^{K}\mathbf{H}_{ji}^{\mathcal{H}}\mathbf{u}_j\mathbf{u}_j^{\mathcal{H}}\mathbf{H}_{ji}$. With $\mathbf{G} \in \mathbb{C}^{M\times M}$ comprising a sum of $K-1$ i.i.d. $M\times M$ random matrices with rank 1 (because each matrix is of the form $\mathbf{a}_m\mathbf{a}_m^{\mathcal{H}}$), then $\text{rank}(\mathbf{G}) = K-1$. The $\text{rank}(\mathbf{G})+\text{nullity}(\mathbf{G}) = M$ (from rank-nullity theorem). So $\text{nullity}(\mathbf{G}) = 1$ for $M = K$. Because $\mathbf{G} \succeq \mathbf{0}$ (positive semi-definite) and $\text{nullity}(\mathbf{G}) = 1$, then for $M = K$, $\mathbf{G}$ has one zero eigenvalue and $K-1$ non-zero eigenvalues. (The rank of any square matrix is equal to the number of its non-zero eigenvalues). Therefore, if $M = K$, equation (2.35) is unique for problem $\mathcal{G}$ and equation (2.35) becomes $\mathbf{v}_i = \mathcal{N}(\mathbf{G})$. □

The f_i function corresponding to the LI-SINR problem, $\mathcal{P}_1$, is given in (2.38). Let $\mathbf{x} = \text{col}\{\mathbf{v}_1, \mathbf{v}_2, \ldots, \mathbf{v}_K\}$. From (2.38), for any $\mathbf{x}$, we have $\|f_i(\mathbf{x})\| = 1$. Therefore, for $f = \text{col}\{f_1, ..., f_K\}$, $\|f(\mathbf{x})\| = \sqrt{K}$. Because this holds for any $\mathbf{x}$, we can say that for $\|\mathbf{x}\| \leq \sqrt{K} + \delta$, then $\|f(\mathbf{x})\| \leq \sqrt{K} + \delta$, $\delta \geq 0$. This is the definition of a continuous mapping from a closed ball of a Euclidean space to itself. So the vector field f, for the LI-SINR problem, must have a fixed point (Brouwer's theorem). The lower-bounds of h and g_1 for this problem are

$$h_L(\mathcal{V}) = -\sum_{i=1}^{K} \lambda_{\max}\left(Q(\mathcal{V})\right), \tag{2.36}$$

where

$$Q(\mathcal{V}) \triangleq \left(\sum_{j\neq i}^{K} \mathbf{H}_{ij}\mathbf{v}_j\mathbf{v}_j^{\mathcal{H}}\mathbf{H}_{ij}^{\mathcal{H}} + \frac{\sigma_n^2}{\sigma_s^2}\mathbf{I}_M\right)^{-1} \mathbf{H}_{ii}\mathbf{v}_i\mathbf{v}_i^{\mathcal{H}}\mathbf{H}_{ii}^{\mathcal{H}};$$

and

$$g_{1L}(\mathcal{U}) = \sum_{i=1}^{K} \lambda_{\min}\left(\sum_{j\neq i}^{K} \mathbf{H}_{ji}^{\mathcal{H}}\mathbf{u}_j\mathbf{u}_j^{\mathcal{H}}\mathbf{H}_{ji}\right), \tag{2.37}$$

respectively. The solution to the LI-SINR problem via the fixed point of f using the EAO algorithm is summarized in Table 2.2.

Table 2.2: Iterative algorithm EAO1 for LI-SINR problem

Algorithm : Extended Alternating Optimization (EAO1)

1: Set $n = 0$; $\mathbf{v}_i^{(0)}$ for $i = 1, ..., K$

2: $\mathbf{u}_i^{(n)} = \boldsymbol{w}_{\max}\left(\left(\sum_{j\neq i}^{K} \mathbf{H}_{ij}\mathbf{v}_j^{(n)}\mathbf{v}_j^{(n)\mathcal{H}}\mathbf{H}_{ij}^{\mathcal{H}} + \frac{\sigma_n^2}{\sigma_s^2}\mathbf{I}_M\right)^{-1} \mathbf{H}_{ii}\mathbf{v}_i^{(n)}\mathbf{v}_i^{(n)\mathcal{H}}\mathbf{H}_{ii}^{\mathcal{H}}\right)$

3: $\mathbf{v}_i^{(n+1)} = \boldsymbol{w}_{\min}\left(\sum_{j\neq i} \mathbf{H}_{ji}^{\mathcal{H}}\mathbf{u}_j^{(n)}\mathbf{u}_j^{(n)\mathcal{H}}\mathbf{H}_{ji}\right)$

4: check stopping criteria and return $\mathbf{v}_i^{(N+1)}, \mathbf{u}_i^{(N)}$ if satisfied

5: else $n = n + 1$ and repeat

If $\mathbf{v}_i^{(N+1)}$ and $\mathbf{u}_i^{(N)}$ from EAO1 (Table 2.2), make the merit function less than ϵ, then the $(\mathbf{u}_i^{(N)}, \mathbf{v}_i^{(N+1)})$ point is taken as a suboptimal solution for the joint LI-SINR problem. (Note that the suboptimality is not from $\epsilon \neq 0$. Even if the point is reached where $\epsilon = 0$, then this is still likely to be a suboptimal solution since it is probably a local rather than global minimum.)

$$f_i(\mathbf{v}_1, ..., \mathbf{v}_K) =$$

$$\boldsymbol{w}_{\min}\left(\sum_{j\neq i}\mathbf{H}_{ji}^{\mathcal{H}}\boldsymbol{w}_{\max}\Big(\Big(\sum_{j\neq i}\mathbf{H}_{ji}\mathbf{v}_i\mathbf{v}_i^{\mathcal{H}}\mathbf{H}_{ji}^{\mathcal{H}}+\frac{\sigma_n^2}{\sigma_s^2}\mathbf{I}_M\Big)^{-1}\mathbf{H}_{jj}\mathbf{v}_j\mathbf{v}_j^{\mathcal{H}}\mathbf{H}_{jj}^{\mathcal{H}}\Big)\boldsymbol{w}_{\max}^{\mathcal{H}}\Big(\Big(\sum_{j\neq i}\mathbf{H}_{ji}\mathbf{v}_i\mathbf{v}_i^{\mathcal{H}}\mathbf{H}_{ji}^{\mathcal{H}}+\frac{\sigma_n^2}{\sigma_s^2}\mathbf{I}_M\Big)^{-1}\mathbf{H}_{jj}\mathbf{v}_j\mathbf{v}_j^{\mathcal{H}}\mathbf{H}_{jj}^{\mathcal{H}}\Big)\mathbf{H}_{ji}\right) \tag{2.38}$$

SLIR-SINR

The SLIR is defined as:

$$g_2(\mathbf{v}_1, ..., \mathbf{v}_K, \mathbf{u}_1, ..., \mathbf{u}_K) = -\sum_{i=1}^{K}\frac{\mathbf{v}_i^{\mathcal{H}}\mathbf{H}_{ii}^{\mathcal{H}}\mathbf{u}_i\mathbf{u}_i^{\mathcal{H}}\mathbf{H}_{ii}\mathbf{v}_i}{\mathbf{v}_i^{\mathcal{H}}(\sum_{j\neq i}\mathbf{H}_{ji}^{\mathcal{H}}\mathbf{u}_j\mathbf{u}_j^{\mathcal{H}}\mathbf{H}_{ji})\mathbf{v}_i}. \tag{2.39}$$

The corresponding problem $\mathcal{P}$ for the SLIR-SINR problem is

$$\begin{aligned}
\min_{\mathbf{u}_1,...,\mathbf{u}_K,\mathbf{v}_1,...,\mathbf{v}_K} \quad & J_2(\mathbf{u}_1, ..., \mathbf{u}_K, \mathbf{v}_1, ..., \mathbf{v}_K) = g_2 + h \\
\text{s.t.} \quad & \mathbf{u}_i^{\mathcal{H}}\mathbf{u}_i = 1 \qquad i = 1, ..., K \\
& \mathbf{v}_i^{\mathcal{H}}\mathbf{v}_i = 1 \qquad i = 1, ..., K
\end{aligned} \tag{2.40}$$

and the lower bound for g_2 is

$$g_{2L}(\mathcal{U}) = -\sum_{k=1}^{K}\lambda_{\max}\left(P(\mathcal{U})\right) \tag{2.41}$$

where

$$P(\mathcal{U}) \triangleq \left(\sum_{j\neq i}\mathbf{H}_{ji}^{\mathcal{H}}\mathbf{u}_j\mathbf{u}_j^{\mathcal{H}}\mathbf{H}_{ji}\right)^{-1}\mathbf{H}_{ii}^{\mathcal{H}}\mathbf{u}_i\mathbf{u}_i^{\mathcal{H}}\mathbf{H}_{ii}. \tag{2.42}$$

Similar to the LI-SINR problem above, the corresponding vector field for the SLIR-SINR problem is a continuous function from a closed ball of a Euclidean space to itself and so its f has a fixed point. Finding the fixed point of the SLIR-SINR problem may be obtained by using the EAO2 algorithm listed in Table 2.3. As noted above, the convergence is not guaranteed for any specific iterative algorithm, although the existence of a fixed point is guaranteed.
Again, if the $(\mathbf{u}_i^{(N)}, \mathbf{v}_i^{(N+1)})$ point, using the lower-bounds given in (2.36) and (2.41), makes the merit function less than ϵ, then this point is taken as a suboptimal solution for J_2.

In EAO2, $\sum_{j\neq i}^{K}\mathbf{H}_{ji}^{\mathcal{H}}\mathbf{u}_j^{(n)}\mathbf{u}_j^{(n)\mathcal{H}}\mathbf{H}_{ji} \in \mathbb{C}^{M\times M}$ must be invertible, and therefore it is necessary to have $M \leq K - 1$.

We also note that the solution of the SLIR-SINR problem (2.40) as given by EAO2 is different from the solution of the sum signal power (SP) over sum interference plus noise ratio problem solved by AO and studied in [5]. The sum signal power over the sum interference plus noise ratio problem

Table 2.3: Iterative algorithm EAO2 for SLIR-SINR problem

Algorithm : EAO2

1: Set $n = 0$; $\mathbf{v}_i^{(0)}$ for $i = 1, ..., K$
2: $\mathbf{u}_i^{(n)} =$
$\boldsymbol{w}_{\max}\left(\left(\sum_{j\neq i}^{K}\mathbf{H}_{ij}\mathbf{v}_j^{(n)}\mathbf{v}_j^{(n)\mathcal{H}}\mathbf{H}_{ij}^{\mathcal{H}} + \frac{\sigma_n^2}{\sigma_s^2}\mathbf{I}_M\right)^{-1}\mathbf{H}_{ii}\mathbf{v}_i^{(n)}\mathbf{v}_i^{(n)\mathcal{H}}\mathbf{H}_{ii}^{\mathcal{H}}\right)$
3: $\mathbf{v}_i^{(n+1)} =$
$\boldsymbol{w}_{\max}\left(\left(\sum_{j\neq i}\mathbf{H}_{ji}^{\mathcal{H}}\mathbf{u}_j^{(n)}\mathbf{u}_j^{(n)\mathcal{H}}\mathbf{H}_{ji}\right)^{-1}\mathbf{H}_{ii}^{\mathcal{H}}\mathbf{u}_i^{(n)}\mathbf{u}_i^{(n)\mathcal{H}}\mathbf{H}_{ii}\right)$
4: check stopping criteria and return $\mathbf{v}_i^{(N+1)}, \mathbf{u}_i^{(N)}$ if satisfied
5: else $n = n + 1$ and repeat

is

$$\begin{aligned}
\min_{\mathbf{u}_1,...,\mathbf{v}_K} \quad & \mathcal{J}_2 = -\frac{\sum_{i=1}^{K}\mathbf{u}_i^{\mathcal{H}}\mathbf{H}_{ii}\mathbf{v}_i\mathbf{v}_i^{\mathcal{H}}\mathbf{H}_{ii}^{\mathcal{H}}\mathbf{u}_i}{\sum_{i=1}^{K}\mathbf{u}_i^{\mathcal{H}}(\sum_{j\neq i}^{K}\mathbf{H}_{ij}\mathbf{v}_j\mathbf{v}_j^{\mathcal{H}}\mathbf{H}_{ij}^{\mathcal{H}} + \frac{\sigma_n^2}{\sigma_s^2}\mathbf{I}_M)\mathbf{u}_i} \\
\text{s.t.} \quad & \mathbf{u}_i^{\mathcal{H}}\mathbf{u}_i = 1 \qquad i = 1, ..., K \\
& \mathbf{v}_i^{\mathcal{H}}\mathbf{v}_i = 1 \qquad i = 1, ..., K.
\end{aligned} \tag{2.43}$$

By applying the AO method for this single-objective problem, then $\mathbf{u}_1$, for example, is the solution of:

$$\begin{aligned}
\max_{\mathbf{u}_1} \quad & \frac{\mathbf{u}_1^{\mathcal{H}}\mathbf{A}_1\mathbf{u}_1 + r_1}{\mathbf{u}_1^{\mathcal{H}}\mathbf{B}_1\mathbf{u}_1 + r_2} \\
\text{s.t.} \quad & \mathbf{u}_1^{\mathcal{H}}\mathbf{u}_1 = 1
\end{aligned} \tag{2.44}$$

where

$$\mathbf{A}_1 \triangleq \mathbf{H}_{11}\mathbf{v}_1\mathbf{v}_1^{\mathcal{H}}\mathbf{H}_{11}^{\mathcal{H}}$$

$$\mathbf{B}_1 \triangleq \sum_{j\neq 1}^{K}\mathbf{H}_{1j}\mathbf{v}_j\mathbf{v}_j^{\mathcal{H}}\mathbf{H}_{1j}^{\mathcal{H}} + \frac{\sigma_n^2}{\sigma_s^2}\mathbf{I}_M$$

$$r_1 \triangleq \sum_{i=2}^{K}\mathbf{u}_i^{\mathcal{H}}\mathbf{H}_{ii}\mathbf{v}_i\mathbf{v}_i^{\mathcal{H}}\mathbf{H}_{ii}^{\mathcal{H}}\mathbf{u}_i$$

$$r_2 \triangleq \sum_{i=2}^{K}\mathbf{u}_i^{\mathcal{H}}(\sum_{j\neq i}^{K}\mathbf{H}_{ij}\mathbf{v}_j\mathbf{v}_j^{\mathcal{H}}\mathbf{H}_{ij}^{\mathcal{H}} + \frac{\sigma_n^2}{\sigma_s^2}\mathbf{I}_M)\mathbf{u}_i;$$

and the optimal solution of (2.44) for $\mathbf{u}_1$ is

$$\mathbf{u}_1 = \boldsymbol{w}_{\max}((\mathbf{B}_1 + r_2\mathbf{I}_M)^{-1}(\mathbf{A}_1 + r_1\mathbf{I}_M)). \tag{2.45}$$

This is different from the solution in step 2 of EAO2.

$$\min_{\mathbf{u}_1,...,\mathbf{u}_K,\mathbf{v}_1,...,\mathbf{v}_K} \quad J_3 = \underbrace{\sum_{i=1}^{K} \mathbf{v}_i^{\mathcal{H}} (\sum_{j\neq i} \mathbf{H}_{ji}^{\mathcal{H}} \mathbf{u}_j \mathbf{u}_j^{\mathcal{H}} \mathbf{H}_{ji}) \mathbf{v}_i - \sum_{i=1}^{K} \mathbf{v}_i^{\mathcal{H}} \mathbf{H}_{ii}^{\mathcal{H}} \mathbf{u}_i \mathbf{u}_i^{\mathcal{H}} \mathbf{H}_{ii} \mathbf{v}_i}_{g_3(\mathbf{v}_1,...,\mathbf{v}_K,\mathbf{u}_1,...,\mathbf{u}_K)} - \sum_{i=1}^{K} \frac{\mathbf{u}_i^{\mathcal{H}} \mathbf{H}_{ii} \mathbf{v}_i \mathbf{v}_i^{\mathcal{H}} \mathbf{H}_{ii}^{\mathcal{H}} \mathbf{u}_i}{\mathbf{u}_i^{\mathcal{H}} (\sum_{j\neq i}^{K} \mathbf{H}_{ij} \mathbf{v}_j \mathbf{v}_j^{\mathcal{H}} \mathbf{H}_{ij}^{\mathcal{H}} + \frac{\sigma_n^2}{\sigma_s^2} \mathbf{I}_M) \mathbf{u}_i}$$

$$\text{s.t.} \quad \mathbf{u}_i^{\mathcal{H}} \mathbf{u}_i = 1 \quad i = 1, ..., K$$
$$\mathbf{v}_i^{\mathcal{H}} \mathbf{v}_i = 1 \quad i = 1, ..., K. \tag{2.46}$$

LI-SP-SINR

The g or h functions can comprise more than one objective function. A use of this is when the LI is to be minimized while the sum SP and also the sum SINR are to be maximized. This LI-SP-SINR optimization problem is given in equation (2.46) in which the g_3 is defined as the first two summation terms in (2.46), and the lower-bound is

$$g_{3L}(\mathcal{U}) = \sum_{i=1}^{K} \lambda_{\min} \left(R(\mathcal{U}) \right) \tag{2.47}$$

where

$$R(\mathcal{U}) \triangleq \sum_{j\neq i} \mathbf{H}_{ji}^{\mathcal{H}} \mathbf{u}_j \mathbf{u}_j^{\mathcal{H}} \mathbf{H}_{ji} - \mathbf{H}_{ii}^{\mathcal{H}} \mathbf{u}_i \mathbf{u}_i^{\mathcal{H}} \mathbf{H}_{ii}. \tag{2.48}$$

It is required that the smallest eigenvalue of $R(\mathcal{U})$ is unique.
Lemma 2: For $M > K$, the $R(\mathcal{U})$ has only one (multiplicity one) negative eigenvalue.

Proof. From a corollary of Weyl's theorem (4.3.9 in [49]), for a Hermitian matrix $\mathbf{A} \in \mathbb{C}^{M\times M}$ and vector $\mathbf{z} \in \mathbb{C}^{M\times 1}$:

$$\lambda_1(\mathbf{A} - \mathbf{z}\mathbf{z}^{\mathcal{H}}) \leq \lambda_1(\mathbf{A})$$

$$\lambda_{m-1}(\mathbf{A}) \leq \lambda_m(\mathbf{A} - \mathbf{z}\mathbf{z}^{\mathcal{H}}) \leq \lambda_m(\mathbf{A}) \quad m = 2, ..., M$$

where $\lambda_1(\mathbf{A}) = \lambda_{\min}(\mathbf{A})$ and $\lambda_M(\mathbf{A}) = \lambda_{\max}(\mathbf{A})$. Define $R(\mathcal{U})$ in (2.48) as $R(\mathcal{U}) = \mathbf{A} - \mathbf{z}\mathbf{z}^{\mathcal{H}}$ where $\mathbf{A} \triangleq \sum_{j\neq i} \mathbf{H}_{ji}^{\mathcal{H}} \mathbf{u}_j \mathbf{u}_j^{\mathcal{H}} \mathbf{H}_{ji}$ is Hermitian and $\mathbf{z} \triangleq \mathbf{H}_{ii}^{\mathcal{H}} \mathbf{u}_i$. We have $\text{rank}(\mathbf{A}) = K - 1$ (cf., Lemma 1), $\mathbf{A} \in \mathbb{C}^{M\times M}$ and $\mathbf{A} \succeq \mathbf{0}$. So $\mathbf{A}$ has $M - K + 1$ zero eigenvalues. Hence:

$$\lambda_{\min}(\mathbf{A} - \mathbf{z}\mathbf{z}^{\mathcal{H}}) \leq \lambda_{\min}(\mathbf{A}) = 0$$

$$0 \leq \lambda_2(\mathbf{A} - \mathbf{z}\mathbf{z}^{\mathcal{H}}) \leq \lambda_2(\mathbf{A}) = 0$$

$$\vdots$$

$$\lambda_{M-K}(\mathbf{A}) = 0 \leq \lambda_{M-K+1}(\mathbf{A} - \mathbf{z}\mathbf{z}^{\mathcal{H}}) \leq \lambda_{M-K+1}(\mathbf{A}) = 0.$$

Because $\mathbf{A} - \mathbf{z}\mathbf{z}^{\mathcal{H}}$ has rank K and is $M \times M$, there are $M - K$ zero eigenvalues for $\mathbf{A} - \mathbf{z}\mathbf{z}^{\mathcal{H}}$. Therefore, $\lambda_{\min}(\mathbf{A} - \mathbf{z}\mathbf{z}^{\mathcal{H}}) < 0$ (strict inequality) while $\lambda_2(\mathbf{A} - \mathbf{z}\mathbf{z}^{\mathcal{H}}) = 0$. Hence, for $M > K$, $R(\mathcal{U})$ has only one negative eigenvalue. □

Now we have the unique global minimizer w.r.t $\mathbf{u}_i$ for any M, and the unique global minimizer w.r.t $\mathbf{v}_i$ if $M > K$. Following a similar discussion of LI-SINR, f for LI-SP-SINR has a fixed point. Therefore, the assumptions for the theorem hold and we arrive as algorithm EAO3 in Table 2.4.

Table 2.4: Iterative algorithm EAO3 for LI-SP-SINR problem

Algorithm : EAO3

1: Set $n = 0$; $\mathbf{v}_i^{(0)}$ for $i = 1, ..., K$

2: $\mathbf{u}_i^{(n)} =$
$\boldsymbol{w}_{\max}\left(\left(\sum_{j\neq i}^{K}\mathbf{H}_{ij}\mathbf{v}_j^{(n)}\mathbf{v}_j^{(n)\mathcal{H}}\mathbf{H}_{ij}^{\mathcal{H}} + \frac{\sigma_n^2}{\sigma_s^2}\mathbf{I}_M\right)^{-1}\mathbf{H}_{ii}\mathbf{v}_i^{(n)}\mathbf{v}_i^{(n)\mathcal{H}}\mathbf{H}_{ii}^{\mathcal{H}}\right)$

3: $\mathbf{v}_i^{(n+1)} =$
$\boldsymbol{w}_{\min}\left(\sum_{j\neq i}\mathbf{H}_{ji}^{\mathcal{H}}\mathbf{u}_j^{(n)}\mathbf{u}_j^{(n)\mathcal{H}}\mathbf{H}_{ji} - \mathbf{H}_{ii}^{\mathcal{H}}\mathbf{u}_i^{(n)}\mathbf{u}_i^{(n)\mathcal{H}}\mathbf{H}_{ii}\right)$

4: check stopping criteria and return $\mathbf{v}_i^{(N+1)}, \mathbf{u}_i^{(N)}$ if satisfied

5: else $n = n + 1$ and repeat

MSE-SINR

In the above applications, the constraints for $\mathbf{v}_i$ and $\mathbf{u}_i$ are all equalities, viz, the norm of each user's transmit and receive beamformers are unity. But the optimization here works for inequalities as well, as long as the assumptions of the theorem hold. This is illustrated by the MSE-SINR problem. Again, h is the sum SINR maximization, but now sum MSE minimization from [1], is also used:

$$g_4(\mathbf{v}_1, ..., \mathbf{v}_K, \mathbf{u}_1, ..., \mathbf{u}_K) = \sum_{i=1}^{K}\left(\mathbf{v}_i^{\mathcal{H}}\left(\sum_{j=1}^{K}\mathbf{H}_{ji}^{\mathcal{H}}\mathbf{u}_j\mathbf{u}_j^{\mathcal{H}}\mathbf{H}_{ji}\right)\mathbf{v}_i - \mathbf{v}_i^{\mathcal{H}}\mathbf{H}_{ii}^{\mathcal{H}}\mathbf{u}_i - \mathbf{u}_i^{\mathcal{H}}\mathbf{H}_{ii}\mathbf{v}_i\right) \tag{2.49}$$

so the MSE-SINR problem, featuring an inequality constraint, can be written

$$\begin{aligned} \min_{\mathbf{u}_i, \mathbf{v}_i \in \mathbb{C}^{M\times 1}} \quad & J_4(\mathbf{u}_1, ..., \mathbf{u}_K, \mathbf{v}_1, ..., \mathbf{v}_K) = g_4 + h \\ \text{s.t.} \quad & \mathbf{u}_i^{\mathcal{H}}\mathbf{u}_i = 1 \qquad i = 1, ..., K \\ & \mathbf{v}_i^{\mathcal{H}}\mathbf{v}_i \leq 1 \qquad i = 1, ..., K. \end{aligned} \tag{2.50}$$

The solution of problem (2.50) with decision variables $\mathbf{u}_i$ and $\mathbf{v}_i$, $i = 1, ..., K$ gives the unique global minimizer for subproblems $\mathcal{H}$ and $\mathcal{G}$ of J_4. The vector field f corresponding to the MSE-SINR (not included for brevity) satisfies Brouwer's fixed point theorem. The corresponding EAO4 algorithm appears in Table 2.5. In this table, the non-negative number λ_i^*, is chosen such that $\mathbf{v}_i^{(n+1)\mathcal{H}}\mathbf{v}_i^{(n+1)} \leq 1$.

Table 2.5: Iterative algorithm EAO4 for MSE-SINR problem

Algorithm : EAO4

1: Set $n = 0$; $\mathbf{v}_i^{(0)}$ for $i = 1, ..., K$

2: $\mathbf{u}_i^{(n)} =$
$\boldsymbol{w}_{\max}\left(\left(\sum_{j\neq i}^{K}\mathbf{H}_{ij}\mathbf{v}_j^{(n)}\mathbf{v}_j^{(n)\mathcal{H}}\mathbf{H}_{ij}^{\mathcal{H}} + \frac{\sigma_n^2}{\sigma_s^2}\mathbf{I}_M\right)^{-1}\mathbf{H}_{ii}\mathbf{v}_i^{(n)}\mathbf{v}_i^{(n)\mathcal{H}}\mathbf{H}_{ii}^{\mathcal{H}}\right)$

3: $\mathbf{v}_i^{(n+1)} = \left(\sum_{j=1}^{K}\mathbf{H}_{ji}^{\mathcal{H}}\mathbf{u}_j^{(n)}\mathbf{u}_j^{(n)\mathcal{H}}\mathbf{H}_{ji} + \lambda_i^*\mathbf{I}\right)^{-1}\mathbf{H}_{ii}^{\mathcal{H}}\mathbf{u}_i^{(n)}$

4: check stopping criteria and return $\mathbf{v}_i^{(N+1)}, \mathbf{u}_i^{(N)}$ if satisfied

5: else $n = n + 1$ and repeat

2.4.2 Optimal Solution for LI-LI Problem Given by Fixed Point

A special case is when g and h are the same. This is the case for the LI-LI problem, which is treated in [4] but which does not address whether the solution by AO is suboptimal or optimal. Here, we establish the optimality of the fixed point for this single-objective problem.

Proposition: For the LI-LI problem with $M = K$, the fixed point is the optimal solution (global minimum).

Proof. The LI-LI problem is formulated as

$$\begin{aligned} \min_{\mathbf{u}_1,...,\mathbf{u}_K,\mathbf{v}_1,...,\mathbf{v}_K} \quad & J_5 = \underbrace{\sum_{i=1}^{K}\mathbf{v}_i^{\mathcal{H}}\left(\sum_{j\neq i}\mathbf{H}_{ji}^{\mathcal{H}}\mathbf{u}_j\mathbf{u}_j^{\mathcal{H}}\mathbf{H}_{ji}\right)\mathbf{v}_i}_{g_5} + \\ & \underbrace{\sum_{i=1}^{K}\mathbf{u}_i^{\mathcal{H}}\left(\sum_{j\neq i}\mathbf{H}_{ij}\mathbf{v}_j\mathbf{v}_j^{\mathcal{H}}\mathbf{H}_{ij}^{\mathcal{H}}\right)\mathbf{u}_i}_{h_5 = g_5} \qquad (2.51) \\ \text{s.t.} \quad & \mathbf{u}_i^{\mathcal{H}}\mathbf{u}_i = 1 \qquad i = 1, ..., K \\ & \mathbf{v}_i^{\mathcal{H}}\mathbf{v}_i = 1 \qquad i = 1, ..., K. \end{aligned}$$

From (B.7) and (B.8), at the point $(\mathbf{u}_i^*, \mathbf{v}_i^*)$:

$$\begin{aligned}&\sum_{i=1}^{K} \mathbf{u}_i^{*\mathcal{H}} \left(\sum_{j \neq i} \mathbf{H}_{ij} \mathbf{v}_j^* \mathbf{v}_j^{*\mathcal{H}} \mathbf{H}_{ij}^{\mathcal{H}} \right) \mathbf{u}_i^* = \\ &\sum_{i=1}^{K} \lambda_{\min} \left(\sum_{j \neq i} \mathbf{H}_{ij} \mathbf{v}_j^* \mathbf{v}_j^{*\mathcal{H}} \mathbf{H}_{ij}^{\mathcal{H}} \right),\end{aligned} \tag{2.52}$$

$$\begin{aligned}&\sum_{i=1}^{K} \mathbf{v}_i^{*\mathcal{H}} \left(\sum_{j \neq i} \mathbf{H}_{ji}^{\mathcal{H}} \mathbf{u}_j^* \mathbf{u}_j^{*\mathcal{H}} \mathbf{H}_{ji} \right) \mathbf{v}_i^* = \\ &\sum_{i=1}^{K} \lambda_{\min} \left(\sum_{j \neq i} \mathbf{H}_{ji}^{\mathcal{H}} \mathbf{u}_j^* \mathbf{u}_j^{*\mathcal{H}} \mathbf{H}_{ji} \right).\end{aligned} \tag{2.53}$$

Since for $M = K$, the smallest eigenvalue of the matrices $\sum_{j \neq i}^{K} \mathbf{H}_{ij} \mathbf{v}_j^* \mathbf{v}_j^{*\mathcal{H}} \mathbf{H}_{ij}^{\mathcal{H}}$ and $\sum_{j \neq i}^{K} \mathbf{H}_{ji}^{\mathcal{H}} \mathbf{u}_j^* \mathbf{u}_j^{*\mathcal{H}} \mathbf{H}_{ji}$ are zero, adding both sides of (2.52) and (2.53) gives

$$J_5 \left(\mathcal{U}^*, \mathcal{V}^* \right) = 0; \tag{2.54}$$

and from $J_5 \geq 0$,

$$J_5 \left(\mathcal{U}, \mathcal{V} \right) \geq J_5 \left(\mathcal{U}^*, \mathcal{V}^* \right). \tag{2.55}$$

Therefore, $(\mathcal{U}^*, \mathcal{V}^*)$, approximated by $\left(\mathcal{U}^{(N)}, \mathcal{V}^{(N+1)} \right)$, is a global minimum for objective function J_5. □

2.4.3 Complexity Analysis

Background: MSE Minimization

Let $\mathbf{r}_i \triangleq \mathbf{u}_i^{\mathcal{H}}$, then $\hat{s}_i = \mathbf{r}_i \mathbf{y}_i$. The total mean square error is

$$\text{MSE} = \sum_{i=1}^{K} \text{MSE}_i \tag{2.56}$$

where $\text{MSE}_i = \mathbb{E}\{\|\hat{s}_i - s_i\|^2\}$. The MSE minimization is

$$\begin{aligned}\min_{\mathbf{r}_i, \mathbf{v}_i} \quad & \sum_{i=1}^{K} \text{MSE}_i \\ \text{s.t.} \quad & \text{tr}(\mathbf{v}_i \mathbf{v}_i^{\mathcal{H}}) \leq 1.\end{aligned} \tag{2.57}$$

Following the procedure in [1] and using an auxiliary variable t, the MSE minimization by AO is summarized:

1. Choose $N > 1$, $\mathbf{v}_i^{(0)}$ arbitrarily for $i = 1, ..., K$

2. for $n = 1 : N$

$$\mathbf{r}_i^{(n+1)} = \mathbf{v}_i^{(n)\mathcal{H}} \mathbf{H}_{ii}^{\mathcal{H}} (\sum_{j=1}^{K} \mathbf{H}_{ij} \mathbf{v}_j^{(n)} \mathbf{v}_j^{(n)\mathcal{H}} \mathbf{H}_{ij}^{\mathcal{H}} + \sigma_n^2 \mathbf{I}_M)^{-1}$$

$$\min_{\mathbf{v}_i^{(n)}} \quad t$$

$$\text{s.t.} \begin{cases} \left\| \begin{matrix} \delta \\ [\mathbf{I}_K \otimes (\mathbf{R}^{(n+1)}\mathbf{H})]\text{vec}(\mathbf{V}^{(n)}) - \text{vec}(\mathbf{I}_K) \end{matrix} \right\| \leq t. \\ \|\text{vec}(\mathbf{v}_i^{(n)})\| \leq 1. \end{cases} \tag{2.58}$$

end

where

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{11} & \cdots & \mathbf{H}_{1K} \\ \mathbf{H}_{21} & \cdots & \mathbf{H}_{2K} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{K1} & \cdots & \mathbf{H}_{KK} \end{bmatrix}, \mathbf{V}^{(n)} = \begin{bmatrix} \mathbf{v}_1^{(n)} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{v}_K^{(n)} \end{bmatrix},$$

$$\mathbf{R}^{(n+1)} = \begin{bmatrix} \mathbf{r}_1^{(n+1)} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{r}_K^{(n+1)} \end{bmatrix}$$

$\delta = \sigma_n \sqrt{\text{tr}(\mathbf{R}^{(n+1)}\mathbf{R}^{(n+1)\mathcal{H}})}$.

The complexity of MSE minimization can be computed from complexity of a SOCP. Consider the following SOCP

$$\begin{aligned} \min_{\mathbf{x}} \quad & \mathbf{f}^T \mathbf{x} \\ \text{s.t.} \quad & \|\mathbf{A}_p \mathbf{x} + \mathbf{b}_p\| \leq \mathbf{c}_p^T \mathbf{x} + d_p, \quad p = 1, ..., P. \end{aligned} \tag{2.59}$$

where $\mathbf{x} \in \mathbb{R}^m$, $\mathbf{f} \in \mathbb{R}^m$, $\mathbf{c}_p \in \mathbb{R}^m$, $\mathbf{A}_p \in \mathbb{R}^{(m_i-1)\times m}$, $\mathbf{b}_p \in \mathbb{R}^{m_i-1}$ and $d_i \in \mathbb{R}$. The complexity of (2.59), per interior-point-method (IPM[3]) iteration, is $O(m^2 \sum_{i=1}^{P} m_i)$ [50]. Thus, the complexity of MSE with SOCP is $O(INM^2K^6)$, where I is IPM iteration.

Max-Min SINR

The SINR_i of the ith user is

$$\text{SINR}_i = \frac{\mathbf{u}_i^{\mathcal{H}} \mathbf{H}_{ii} \mathbf{v}_i \mathbf{v}_i^{\mathcal{H}} \mathbf{H}_{ii}^{\mathcal{H}} \mathbf{u}_i}{\mathbf{u}_i^{\mathcal{H}} (\sum_{j\neq i}^{K} \mathbf{H}_{ij} \mathbf{v}_j \mathbf{v}_j^{\mathcal{H}} \mathbf{H}_{ij}^{\mathcal{H}} + \frac{\sigma_n^2}{\sigma_{s_i}^2} \mathbf{I}_M) \mathbf{u}_i} \tag{2.60}$$

and the maximization of minimum SINR_i of all K-users is:

$$\max_{\mathbf{u}_i, \mathbf{v}_i} \quad \min_{i=1,...,K} \text{SINR}_i$$

[3]Interior point methods (also referred to as barrier methods) are a certain class of algorithms to solve linear and nonlinear convex optimization problems.

$$\text{s.t.} \begin{cases} \mathbf{v}_i^{\mathcal{H}}\mathbf{v}_i = 1 \\ \mathbf{u}_i^{\mathcal{H}}\mathbf{u}_i = 1. \end{cases} \tag{2.61}$$

Setting $\mathbf{A}_i \triangleq \mathbf{H}_{ii}\mathbf{v}_i\mathbf{v}_i^{\mathcal{H}}\mathbf{H}_{ii}^{\mathcal{H}}$ and $\mathbf{B}_i \triangleq (\sum_{j\neq i}^{K}\mathbf{H}_{ij}\mathbf{v}_j\mathbf{v}_j^{\mathcal{H}}\mathbf{H}_{ij}^{\mathcal{H}} + \frac{\sigma_n^2}{\sigma_{s_i}^2}\mathbf{I}_M)$, then for $i = 1, ..., K$,

$$\frac{\mathbf{u}_i^{\mathcal{H}}\mathbf{A}_i\mathbf{u}_i}{\mathbf{u}_i^{\mathcal{H}}\mathbf{B}_i\mathbf{u}_i} \geq t, \tag{2.62}$$

and so the above optimization problem is equivalent to:

$$\max_{\mathbf{u}_i,\mathbf{v}_i} \quad t$$

$$\text{s.t.} \begin{cases} \mathbf{u}_i^{\mathcal{H}}\mathbf{A}_i\mathbf{u}_i/\mathbf{u}_i^{\mathcal{H}}\mathbf{B}_i\mathbf{u}_i \geq t \\ \mathbf{u}_i^{\mathcal{H}}\mathbf{u}_i = 1. \end{cases} \tag{2.63}$$

It is known that $\lambda_{\max}(\mathbf{B}_i^{-1}\mathbf{A}_i) \geq \mathbf{u}_i^{\mathcal{H}}\mathbf{A}_i\mathbf{u}_i/\mathbf{u}_i^{\mathcal{H}}\mathbf{B}_i\mathbf{u}_i$ and the equality is when $\mathbf{u}_i = V_{\max}(\mathbf{B}_i^{-1}\mathbf{A}_i)$, where $V_{\max}(.)$ denotes the normalized eigenvectors corresponding to $\lambda_{\max}(.)$. Moreover, because $\text{rank}(\mathbf{A}_i) = 1$, then $\lambda_{\max}(\mathbf{B}_i^{-1}\mathbf{A}_i) = \text{tr}(\mathbf{B}_i^{-1}\mathbf{A}_i)$, therefore optimization (2.63) further reduces to:

$$\max_{\mathbf{v}_i} \quad \min_{i=1,\ldots,K} \text{tr}(\mathbf{B}_i^{-1}\mathbf{A}_i)$$

$$\text{s.t.} \ \|\mathbf{v}_i\| = 1 \tag{2.64}$$

with $\mathbf{u}_i = \gamma\mathbf{B}_i^{-1}\mathbf{H}_{ii}\mathbf{v}_i$, where $\gamma = 1/\|\mathbf{B}_i^{-1}\mathbf{H}_{ii}\mathbf{v}_i\|$.

By defining $\mathbf{v}_i \triangleq \mathbf{z}_i/\|\mathbf{z}_i\|$, the optimization problem (C.2) becomes unconstrained. However, because the new objective function is a highly nonlinear function of $\mathbf{z}_i$, classical derivative-based, optimization is not suitable. Therefore, GA is an appropriate candidate for solving the max-min SINR. For a multi-stream case, max-min SINR becomes even more complicated (See Appendix C).

Computational Complexity and Execution Time of Different Beamforming Methods

An alternative method to the EAO is evolutionary algorithms such as the genetic algorithm (GA). In this subsection, the complexity for the EAO suboptimal solution of: LI-SINR, SLIR-SINR, and LI-SP-SINR, is compared to the complexity of the GA for max-min SINR and SOCP for MSE minimization. These first three examples have the sum SINR as one of their objective functions (h), and so the $M \times M$ matrix inversion, with computational complexity of $O(M^3)$, is common. For N iterations of the EAO, the computational complexity is $O(NKM^3)$. For GA, the examples can be transformed to unconstrained problems so only the complexity of the sum of the objective functions (known as the fitness function) is important. For the GA with G generations, population P, and $O(KM^3)$ for the complexity of the fitness function evaluation, the computational complexity for the three problem examples is $O(GPKM^3)$. The complexities of GA and EAO are therefore compatible if $N \approx GP$. However, in the simulations below, the EAO works well for N being small.

The execution time has also been calculated for the aforementioned algorithms. Below, this is linked to the user's maximum velocity allowed by MSE, max-min SINR, joint SINR and LI, and finally LI versus K.

To give a feel for the computation requirement, these algorithms were implemented and compared using MATLAB®: version 7.6 (R2008a), under operating system: Microsoft® Windows® XP Version 5.1, using a PC with a 2.66 GHz Intel®Core™2 Quad processor with 4 gigabytes of RAM. The SOCP for the MSE minimization used CVX version 1.21 with SeDuMi solver. Table 2.6 shows

Table 2.6: Execution Time of Different Algorithms When $G = N = 16$, $P = 20$ for GA, $M = K + 1$

	MSE	**Max-Min SINR (GA)**	**Joint LI-SP-SINR**	**LI**
$K = 4$	3.53(s)	0.14(s)	0.0083(s)	0.0065(s)
$K = 8$	5.85(s)	0.43(s)	0.0420(s)	0.0363(s)

that the joint LI-SP-SINR is much faster than MSE minimization. It is slightly slower than LI, with the same computational complexity.

As a more practical illustration, consider two networks, with $K = 4$ and $K = 8$ users, in the same space and operating with the same frequency, say a WiFi band at 5 GHz. Table 2.7 summarizes the time-scale for a change of path amplitude and phase. The main assumption for these algorithms is that the channels of all users are unchanged during the computation of the Tx and Rx beamformers. In fact, such an assumption needs to be rewritten that the channel should be unchanging from the start of the channel sounding, through the interchange of channel sate information to a central site, through the calculation of the beamformers, through the interchange of beamformer data to all the terminals, to the deployment of the beamformers. It is emphasized here that the channel sounding and data interchange is not under consideration in this chapter. With this in mind, Table 2.7 allows the statement

$$t_e \leq \min(d/v, 1/D) \tag{2.65}$$

where t_e denotes the execution time of Tx-Rx beamforming algorithm.
For $K = 4$, $t_e = 0.0083$ (s) for the joint LI-SP-SINR with EAO algorithm. Therefore, at $f_c = 5$ GHz:

$$v \leq 26 \quad \text{km/h} \tag{2.66}$$

For $K = 8$, with $f_c = 5$ GHz and the joint LI-SP-SINR with EAO algorithm:

$$v \leq 5.14 \quad \text{km/h} \tag{2.67}$$

For MSE minimization with SOCP, the maximum allowable user's velocity is reduced to 0.0612 km/h and 0.0369 km/h for $K = 4$ and $K = 8$, respectively. We can state that the EAO algorithms

Table 2.7: System and Channel Parameters

System Parameters	**Symbol**
Carrier frequency	f_c
Distance between transmitter and receiver	d
User's velocity	v
Doppler shift for a path	$D = f_c v/c$
Time-scale for change of path amplitude	d/v
Time-scale for change of path phase	$1/D$

for various combination objectives will not be a bottleneck in the signal processing in this type of wireless network. In networks with a lower carrier frequency than $f_c = 5$ GHz, the maximum allowable user's velocity would be increased.

2.4.4 Ensured Minimum QoS by EAO

Having a minimum QoS for users means that the SINR_i is not zero for $i = 1, ..., K$. The optimal solution for the single-objective $\mathcal{J}_1$, and multi-objective functions J_1, J_2, J_3 and J_4 (if these can be found) does not guarantee a minimum QoS for the users, which means it is probable that SINR_i becomes zero for some i. This is simply because QoS is not directly addressed in the formulation. However, optimization by the fixed point method inherently addresses QoS for all the users. The mechanism is that, for all the objective functions and for given Tx beamformers, the sum SINR maximization (which is a common objective function among J_1, J_2, J_3 and J_4) is equivalent to individual users' SINR (SINR_i) maximization for $i = 1, ..., K$. Mathematically,

$$\begin{aligned} \max_{\mathbf{u}_1,\dots,\mathbf{u}_K} \quad & \sum_{i=1}^{K} \text{SINR}_i = \sum_{i=1}^{K} \frac{\mathbf{u}_i^{\mathcal{H}} \mathbf{A}_i \mathbf{u}_i}{\mathbf{u}_i^{\mathcal{H}} \mathbf{B}_i \mathbf{u}_i} \\ \text{s.t.} \quad & \mathbf{u}_i^{\mathcal{H}} \mathbf{u}_i = 1 \qquad i = 1, ..., K; \end{aligned} \tag{2.68}$$

is equivalent to

$$\begin{aligned} \max_{\mathbf{u}_i} \quad & \text{SINR}_i = \frac{\mathbf{u}_i^{\mathcal{H}} \mathbf{A}_i \mathbf{u}_i}{\mathbf{u}_i^{\mathcal{H}} \mathbf{B}_i \mathbf{u}_i} \\ \text{s.t.} \quad & \mathbf{u}_i^{\mathcal{H}} \mathbf{u}_i = 1 \qquad i = 1, ..., K; \end{aligned} \tag{2.69}$$

The optimal $\mathbf{u}_i$ for these two equivalent problems is $\mathbf{u}_i = \boldsymbol{w}_{\max}\left(\mathbf{B}_i^{-1}\mathbf{A}_i\right)$ and this solution provides $\text{SINR}_i = \lambda_{\max}\left(\mathbf{B}_i^{-1}\mathbf{A}_i\right)$, which cannot be zero. Therefore, both AO and EAO guarantee a minimum QoS for all users. We remark that solutions for $\mathcal{J}_1$, J_1, J_2, J_3 and J_4 obtained from any method other than AO or EAO (e.g., GA) do not account for a minimum QoS for all users because this property is not included in the constraints, whereas EAO/AO implicitly satisfies a minimum QoS.

Example: Consider the random channel for $K = M = 2$:

$$\mathbf{H}_{11} = \begin{bmatrix} -0.2703 - 1.1724\jmath & -1.3440 + 0.0751\jmath \\ -1.1198 - 0.6295\jmath & -1.3784 + 0.3929\jmath \end{bmatrix}$$

$$\mathbf{H}_{12} = \begin{bmatrix} -0.6212 + 0.0975\jmath & -0.2044 + 0.9125\jmath \\ -0.6791 + 1.3020\jmath & 0.0149 + 0.6245\jmath \end{bmatrix}$$

$$\mathbf{H}_{21} = \begin{bmatrix} -1.3818 + 1.1991\jmath & -0.3970 + 0.5646\jmath \\ -0.4471 + 0.7555\jmath & 1.2079 - 0.7714\jmath \end{bmatrix}$$

$$\mathbf{H}_{22} = \begin{bmatrix} -0.4010 + 0.4817\jmath & -0.9965 + 0.3224\jmath \\ 0.5476 - 0.3818\jmath & 0.4282 - 0.3676\jmath \end{bmatrix}$$

with an ensemble average SNR of 10dB, and LI-SINR.

By using the GA algorithm, the leakage interference of the first user on the second user (recall that this is one component of the LI) is 1.6660, and the leakage interference of the second user on the first user is 0.0013; the SINR of the second user is 0.0445, and the SINR of the first user is 65.0371. Therefore, the J_1 at this solution from the GA is

$$J_1^{\text{GA}} = \underbrace{1.6660}_{\text{LI of } s_1 \text{ on } s_2} + 0.0013 - \underbrace{0.0445}_{\text{SINR of } s_2} - 65.0371 = -63.4143.$$

For this (unity) level of transmit power, it is seen that the second user has a poor SINR (for detecting uncoded data) of about -13dB (the non-zero leakage from the first user contributes to this SINR being poor). This is a typical outcome in the sense that one user has good connectivity and the other has none.

From EAO, the J_1 comprising the LI and SINR is

$$J_1^{\text{EAO}} = \underbrace{0}_{\text{LI of } s_1 \text{ on } s_2} + 0 - \underbrace{5.8777}_{\text{SINR of } s_2} - 53.3140 = -59.1917$$

and now both users have zero LI, and both SINRs are acceptable with the smaller one being well above the 6dB required for a manageable error rate.

The GA gives a better objective function result than the EAO, but the EAO ensures a better outcome for this communication problem. The ratio of the weaker user's SINRs for the two optimizations is $\text{SINR}_2^{\text{EAO}}/\text{SINR}_2^{\text{GA}} \simeq 21\text{dB}$, demonstrating that the EAO is better for proving a minimum QoS for both users. Numerical results below confirm this behavior in a statistical sense. The communications behavior indicates, that despite the mathematical convenience of these objective functions, they are not very well suited to the communications problem. As explained in the Introduction, it is not always obvious which cost functions, when they are indirect, are the best.

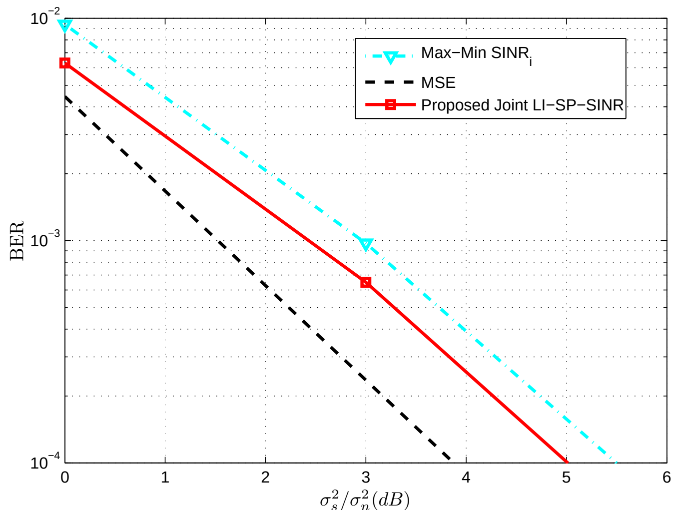

Figure 2.2: The BER performance of the proposed EAO for LI-SP-SINR, $K = 3$, $M = 4$, $N = 16$, compared with MSE by SOCP in [1] and maximization of minimum SINR in [2, 3].

2.4.5 Fairness

For the K-user channel, fairness among all users may be important. For the above examples, fairness is assured with EAO for each channel realization. But this is not the case for AO applied to the problem of sum SP over sum interference plus noise in [5]. The difference can be explained from the initial weights. In EAO, the users' initial weights are all treated the same, whereas in AO applied to sum SP over sum interference plus noise, the users are treated differently. For example, there are K sets of initial weights, say $\{\mathbf{v}_1^{(0)}, ..., \mathbf{v}_K^{(0)}, \mathbf{u}_2^{(0)}, ..., \mathbf{u}_K^{(0)}\}$. Here, $\mathbf{u}_1^{(0)}$ is missing because it is not needed yet. But $\mathbf{u}_1^{(1)}$ is then computed as a function of these initial weights, and so the first user tends to be favored. For AO applied to the sum SP over sum interference plus noise, fairness can be satisfied by rotating the initialization choice for each user, e.g., $\{\mathbf{v}_1^{(0)}, ..., \mathbf{v}_K^{(0)}, \mathbf{u}_2^{(0)}, ..., \mathbf{u}_K^{(0)}\}$ at first; then $\{\mathbf{v}_1^{(0)}, ..., \mathbf{v}_K^{(0)}, \mathbf{u}_1^{(0)}, \mathbf{u}_3^{(0)} ..., \mathbf{u}_K^{(0)}\}$, and finally $\{\mathbf{v}_1^{(0)}, ..., \mathbf{v}_K^{(0)}, \mathbf{u}_1^{(0)}, ..., \mathbf{u}_{K-1}^{(0)}\}$ at the Kth channel realization for the first, second, and Kth user, respectively. But this does not change the fact that for each realization, a single user may be favored.

2.5 Numerical Results

In this section, the communications performance for the EAO, as summarized in Tables 2.2 to 2.5, is benchmarked. Communications performance can be expressed as a bit-error rate (BER) or a sum rate, and these are usually opposing metrics. Expressions for these depend on the details of the communications techniques used. For a practical configuration for high efficiency, there is likely to be adaptive modulation and coding, and forward error correction, with a complex supporting

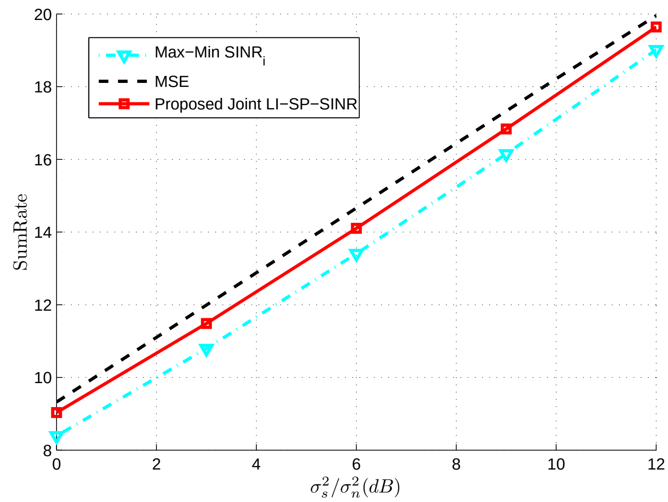

Figure 2.3: The sum rate performance of EAO for LI-SP-SINR, with $K = 3$, $M = 4$, $N = 16$, compared with MSE by SOCP in [1] and maximization of minimum SINR [2, 3].

protocol. Direct optimization of a capacity or an error performance for this situation is seldom feasible, even for a single user. Nevertheless, optimizing manageable metrics such as an information-theoretic capacity and some uncoded error rate performance can result in a configuration that also has good practicable performance [51]. To allow comparison with existing results, the communications performance is evaluated using just QPSK modulation, independent of the SNR (consequently the channel efficiency will degrade quickly when the SNR moves away from its narrow range of a few dB where it is closest to the Shannon limit), and no coding. Furthermore, all the users are assumed to have equal power, i.e., $\sigma_{s_i}^2 = \sigma_s^2$, and the channels $\mathbf{H}_{ij}$ are i.i.d with zero-mean and unit-variance. It is emphasized that the practicable communications performance is not being optimized directly, and so the results may not carry across to practicable configurations that feature adaptive modulation and coding to cater for the variable SNR of the Rayleigh channel, and have imperfect channel estimates and so on.

The BER and sum rate performance as a function of average SNR (σ_n^2/σ_s^2), with EAO, are plotted for LI-LI, LI-SINR, SLNR-SINR in Fig. 2.5 and Fig. 2.6. Also the EAO result for LI-SP-SINR is plotted in Fig. 2.2 and Fig. 2.3. For each point, 10^5 realizations were used. The EAO results are in solid lines and the existing methods in dashed lines.

The ergodic sum rate (ie., a capacity) is computed from

$$\mathrm{SR} = \mathbb{E}\Big\{\sum_{i=1}^{K} \log_2(1 + \mathrm{SINR}_i)\Big\}. \tag{2.70}$$

The number of antennas is $M = K$, $M \leq K$, $M \leq (K-1)$ and $M > K$, respectively. The lower bound on M is imposed by the assumptions of the fixed point optimization[2].

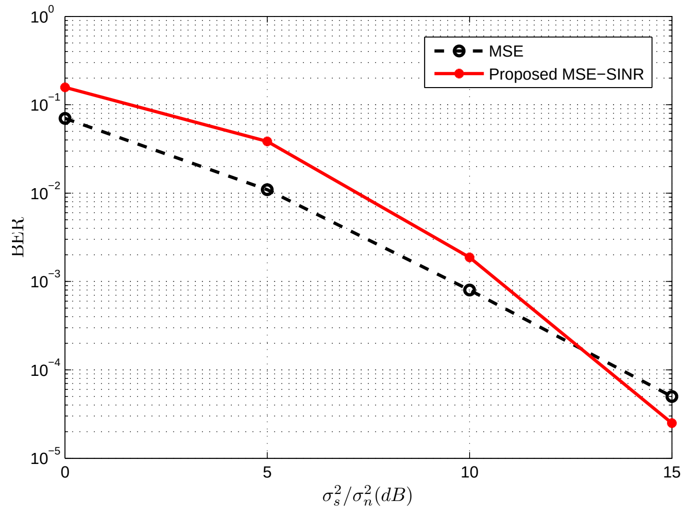

Figure 2.4: The BER performance comparison of MSE in [1] and the MSE-SINR by EAO for $K = 2$.

The benchmarks are LI-LI [4], max-min SINR [2, 3], sum signal power over sum interference plus noise ratio [5], and min sum MSE [1] by SOCP.

Figs 2.2 and 2.3 compare the BERs and sum rates, respectively, for the EAO solution of our LI-SP-SINR with the known solutions for MSE-SOCP [1] and max-min SINR [2, 3], for $K = 3$ and $M = 4$. The LI-SP-SINR outperforms the max-min SINR for both communications metrics. Against MSE with SCOP, LI-SP-SINR requires about one dB more SNR. In terms of complexity, the LI-SP-SINR is about 50 times faster than max-min SINR and 400 times faster than MSE-SOCP[3].

It is shown in [53] that beamforming design using a BER minimization is very difficult, so MSE minimization is used here instead. Fig. 2.4, for $K = M = 2$, illustrates that using the MSE-SINR objective function provides a lower BER than using MSE minimization for higher SNRs. For a larger number of users (not shown) and for lower SNRs, the MSE minimization is better. MSE-SINR and MSE have the same computational complexity and have the same execution time.

The performance of different multi-objective optimizations and their impact on BER and sum rate are illustrated in Figs 2.5 and 2.6, and compared with the optimal (see section 2.4.3) LI-LI solution obtained by AO [4] and sum signal power over sum interference plus noise ratio in [5]. In these figures, $K = M = 4$ and $N = 128$. Although the sum rate performance of the sum signal power over sum interference plus noise ratio is essentially close to that of SLNR-SINR, it turns out to have worse BER behavior. The figures demonstrate that in a multiuser MIMO interference channel, the error

[2]The lower bound on M is derived by the feasibility of interference alignment for the K-user MIMO interference channel, $2M \geq (K + 1)$ [52].

[3]These algorithms were implemented and compared using MATLAB®: version 7.6 (R2008a), under operating system: Microsoft® Windows® XP Version 5.1, using a PC with a 2.66 GHz Intel®Core™2 Quad processor with 4 gigabytes of RAM. The CVX version 1.21 with SeDuMi solver was used for MSE-SOCP.

performance and the sum rate performance both need to be investigated, with good performance in one not translating to good performance in the other (these performances naturally oppose each other), but both can be achieved when optimized together using the appropriate cost function.

The number of iterations, N, depends mainly on the initialization. For all simulations, $\mathbf{v}_i^{(0)} = \boldsymbol{w}_{\max}(\mathbf{H}_{ii})$. If the Tx-beamformer initializations come from interference alignment (IA), then N is reduced. For say $K = 3$, it is desired from IA that [54, 8]

$$\mathrm{span}(\mathbf{H}_{12}\mathbf{v}_2^{(0)\mathrm{IA}}) = \mathrm{span}(\mathbf{H}_{13}\mathbf{v}_3^{(0)\mathrm{IA}}),$$

$$\mathrm{span}(\mathbf{H}_{21}\mathbf{v}_1^{(0)\mathrm{IA}}) = \mathrm{span}(\mathbf{H}_{23}\mathbf{v}_3^{(0)\mathrm{IA}}), \tag{2.71}$$

$$\mathrm{span}(\mathbf{H}_{31}\mathbf{v}_1^{(0)\mathrm{IA}}) = \mathrm{span}(\mathbf{H}_{32}\mathbf{v}_2^{(0)\mathrm{IA}}),$$

but the set (2.71) does not take the desired signal space into account [25]. The maximum chordal distance criterion makes the desired signal space roughly orthogonal to the interference signal space. Denote $d^{\mathrm{cd}}(\mathbf{w}_1, \mathbf{w}_2) \triangleq \sqrt{1 - \tilde{\mathbf{w}}_1^{\mathcal{H}}\tilde{\mathbf{w}}_2}$, where $\tilde{\mathbf{w}}_1$ and $\tilde{\mathbf{w}}_2$ are generator vectors of $\mathbf{w}_1$ and $\mathbf{w}_2$, which can be found via QR decomposition, respectively. Note that, $d^{\mathrm{cd}}(\mathbf{w}_1, \mathbf{w}_2)$ is maximum if $\mathbf{w}_1 \perp \mathbf{w}_2$ and is minimum if $\mathbf{w}_1 \parallel \mathbf{w}_2$. Combining the IA and chordal distance criteria results in

$$t \triangleq \sum_{j \neq i} d^{\mathrm{cd}}(\mathbf{H}_{ii}\mathbf{v}_i^{(0)\mathrm{IA}}, \mathbf{H}_{ij}\mathbf{v}_j^{(0)\mathrm{IA}})$$

and

$$\mathbf{v}_i^{(0)} =$$

$$\max\left\{ \underset{\mathbf{v}_1^{(0)\mathrm{IA}} \sqsubset \mathrm{eig}(\mathbf{E})}{\arg\max} \{t\},\ \underset{\mathbf{v}_2^{(0)\mathrm{IA}} \sqsubset \mathrm{eig}(\mathbf{G})}{\arg\max} \{t\},\ \underset{\mathbf{v}_3^{(0)\mathrm{IA}} \sqsubset \mathrm{eig}(\mathbf{F})}{\arg\max} \{t\} \right\} \tag{2.72}$$

where $i \in \{1, 2, 3\}$ if the ith term in the bracket has the maximum value,

$$\begin{aligned}
\mathbf{E} &\triangleq \mathbf{H}_{31}^{-1}\mathbf{H}_{32}\mathbf{H}_{12}^{-1}\mathbf{H}_{13}\mathbf{H}_{23}^{-1}\mathbf{H}_{21},\\
\mathbf{F} &\triangleq \mathbf{H}_{23}^{-1}\mathbf{H}_{21}\mathbf{H}_{31}^{-1}\mathbf{H}_{32}\mathbf{H}_{12}^{-1}\mathbf{H}_{13},\\
\mathbf{G} &\triangleq \mathbf{H}_{32}^{-1}\mathbf{H}_{31}\mathbf{H}_{21}^{-1}\mathbf{H}_{23}\mathbf{H}_{13}^{-1}\mathbf{H}_{12},
\end{aligned} \tag{2.73}$$

and $\mathbf{X} \sqsubset \mathbf{Y}$ means that the set of column vectors of $\mathbf{X}$ is a subset of the set of column vectors of $\mathbf{Y}$, and eig($\mathbf{X}$) is a matrix whose columns are the eigenvectors of $\mathbf{X}$. For example, if $i = 1$ then $\mathbf{v}_1^{(0)} = \mathbf{e}_m$, where $\mathbf{e}_m$ is the mth normalized eigenvector of matrix $\mathbf{E}$, and

$$\begin{aligned}
\mathbf{v}_2^{(0)} &= \mathbf{H}_{32}^{-1}\mathbf{H}_{31}\mathbf{v}_1^{(0)} / \|\mathbf{H}_{32}^{-1}\mathbf{H}_{31}\mathbf{v}_1^{(0)}\|\\
\mathbf{v}_3^{(0)} &= \mathbf{H}_{23}^{-1}\mathbf{H}_{21}\mathbf{v}_1^{(0)} / \|\mathbf{H}_{23}^{-1}\mathbf{H}_{21}\mathbf{v}_1^{(0)}\|
\end{aligned} \tag{2.74}$$

Numerical experiments (not shown) demonstrate that for $K = 3$, $N = 4$ with IA-based initialization has the same sum rate performance as $N = 16$ with right singular initialization.

2.6 Conclusion

A new method is presented for finding the beamformers for multi-user MIMO-IC. The beamformer design requires the solution of multi-objective problems, and the choice of the objective functions for the best digital communications performance is not obvious. Different objective functions, namely SLNR-SINR, SLIR-SINR, LI-SINR, LI-SP-SINR, and MSE-SINR, are formulated and their solutions in small-scale systems are compared by simulation for Rayleigh channels. The method is suitable for a general class of multi-objective, multi-variable problems, under equality and inequality constraints. The mechanism is to decompose a difficult problem, for example the SLNR-SINR objectives, into two sub-problems which have guaranteed convergence. The solutions for SLNR-SINR demonstrate that a high sum rate and a low error rate can be achieved at the same time. The simulations also verify that IA provides appropriate initialization for the transmit beamformers in the sense that the number of iterations required to find all the beamformers is kept low. Including a quality of service guarantee for each of the users is not a formal part of the optimization problem since it would complicate the formulation. However, for each of the objective function examples, the presented method inherently addresses the individual users' quality of service. Other solution techniques, e.g., evolutionary methods, which are much slower, do not have this property. For the special case of LI-LI objectives, we proved that the EAO converges to optimal solution only for $M = K$. The EAO can be readily applied to the multi-stream case because a fixed-point is also defined in Hilbert space.

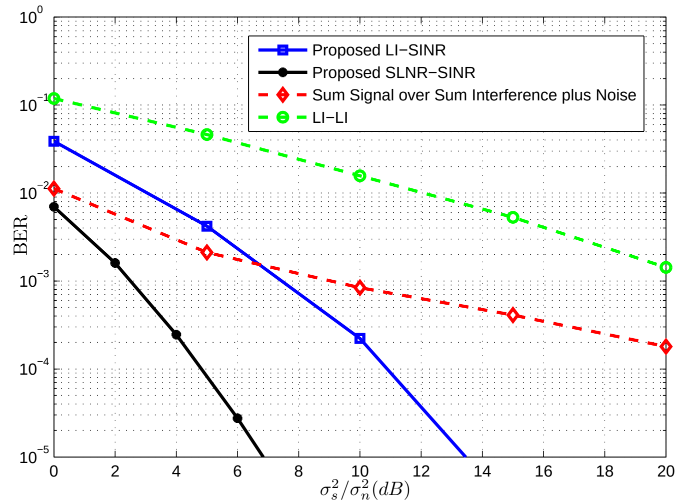

Figure 2.5: The BER performance of EAO for LI-SINR and SLNR-SINR, $K = 4$, compared with LI-LI [4] and sum signal power over sum interference plus noise ratio [5].

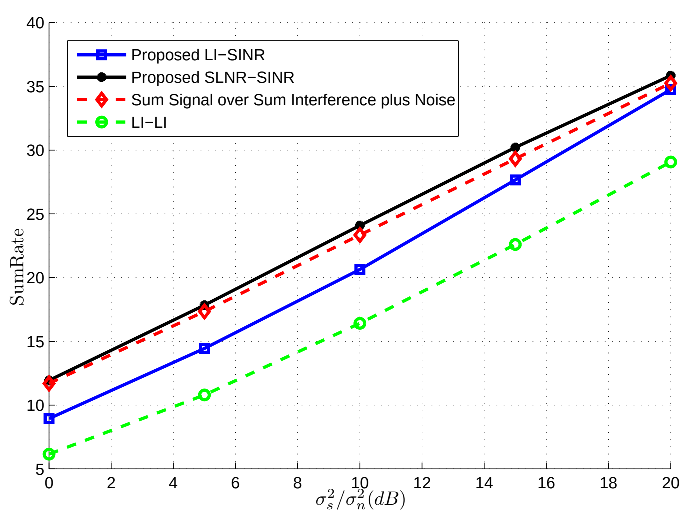

Figure 2.6: The sum rate performance of EAO for LI-SINR and SLNR-SINR, $K = 4$, compared with LI-LI [4] and sum signal power over sum interference plus noise ratio [5].

Chapter 3

Beamforming for Multiuser MIMO-OFDM Interference Channels with Multipath Diversity

3.1 Introduction

Information theory papers, such as [7, 8], show that multi-user MIMO interference channels can potentially increase the capacity of a wireless system. Beamforming enables the sum-rate of the system to approach the information theoretic capacity derived in [7, 8]. In chapter 2, we proposed a novel framework, called EAO, which has guaranteed convergence for multi-objective beamforming design. For simplicity, in chapter 2, the channel is a flat fading channel. In this chapter, the multi-path frequency-selective channel is considered. More specifically, this chapter presents three beamforming designs for multiuser MIMO-OFDM system, where the transmit and receive beamformers are obtained iteratively with closed-form steps. In the first case, the transmit (Tx) beamformers are set and then the receive (Rx) beamformers are calculated. It works by projecting the Tx beamformers into a null space of appropriate channels. This eliminates one interference term for each user. Then the Rx-beamformer for each user maximizes its instantaneous signal-to-noise ratio (SNR) while satisfying an orthogonality condition to eliminate the remaining interferences. The second case is jointly optimizing the Tx and Rx beamformers from constrained SNR maximization. It uses the results from the first case. The third case is also for joint optimization of Tx-Rx beamformers but combines constrained SNR and signal-to-interference plus noise ratio (SINR) maximization. The minimum number of antennas required is derived as part of the formulation. All cases can include a linear constellation precoder (LCP) for extracting multipath diversity. In order to further improve the digital communications error performance (without compromising the sum rate performance), the system formulation can include an LCP before the transmit beamformer and a sphere decoder (SD) following the receiver beamformer. This is to allow multipath diversity gain from the OFDM system [55]. This precoder is a fixed matrix that does not need instantaneous channel knowledge, although it does need knowledge of some channel characteristics for optimal deployment. It is noted that multipath diversity gain for OFDM systems can also be obtained by using a multi-tap receive FIR filter [11]. Finally, in this chapter, the required feedback rates are derived and compared to existing beamforming methods. Using the standardized statistical channel model for IEEE 802.11n, the simulations in this chapter demonstrate fast beamforming, with good error performance and the ability to extract multipath diversity. We show here that least-square (LS) approach for multi-user MIMO-IC does not have these desirable features. We emphasize that our proposed methods are computationally simpler than [1, 25, 56] but our design imposes a certain number of antennas at the transmitters or the receivers.

In summary, the differences between chapter 2 and this current chapter can be summarized as follows. In chapter 2, we formulated the general framework for a class of multi-objective optimization problems and proposed an algorithm for finding a solution. The application was joint Tx-Rx beamforming designs in MIMO interference channels which were transformed to two sub-problems (two games) by EAO. However, in this chapter, we show that EAO also can be applied to K sub-problems (K games). For all of the cost function examples presented in chapter 2, the two sub-problems are a function of the Tx and the Rx beamformer. However, the Tx-BF and Rx-BF design for joint constrained SNR maximization in this chapter is transformed to K Tx-only optimization problems. Also, in this chapter, LS beamforming designs for MIMO interference channels has been proposed for the first time. The feedback rate comparison between our proposed and existing methods is tabled. Simulations are performed for the more realistic standardized statistical channel model of IEEE 802.11n. We compare the computational complexity (or execution time) and the performance for sum-rate maximization by EAO over K games and gradient method in [25].

3.2 System Model, Problems Formulations and Their Solutions

The communications situation as it relates to the model are summarized as follows. There are K pairs of multi-antenna terminals which are striving to share simultaneously the spectrum in time and space. The channel is modeled as a tapped delay line (L+1 taps), and each tap is complex Gaussian (Rayleigh) and independent, and all the taps are independent between OFDM symbols (quasi-stationary). These assumptions, and more detailed ones given below, are significant simplifications with respect to the physical scenario, but are ubiquitous in digital communications research.

The time-spacing of the taps, the average energy distribution of these taps, and the maximum duration of the tapped delay line, define the modeled channel behavior. In the problem formulation (and simulations) the channel is defined statistically from this time domain model, and the frequency domain channels follow by transformation. (This is the usual approach with OFDM). This means that there is no mismatch between the instantaneous time domain and frequency domain descriptions in the channel model. Only parametric quantities are used for the channel description. No specific values are assigned for the tap duration (in seconds), total effective duration, or the shape of the power delay profile; and the corresponding total bandwidth (in Hertz), and the subcarrier bandwidth. These channel parameters can be set arbitrarily, but values according to a current IEEE Standard are used for the simulations section (Section 3.7). These standard channels feature an exponential power delay profile which is more realistic than the time domain description used in the original coder architecture [55] where an idealized uniform power delay profile was used. Such an idealized energy distribution creates overly optimistic time domain (multipath) diversity gains compared to that available in real-world channels. Further, we use Kronecker-modelled correlations for the antennas, Doppler shifts, and so on, *cf.*, [57]. The system can be time-duplexed (but this would need to be synchronized for all users), allowing a single set of antennas for receive and transmit, although the general formulation does not require this and so it uses a different number of transmit and receive antennas. Perfect channel knowledge is assumed at all the users and perfect timing is also assumed in the usual manner to allow a linear model for the link. In practice, this assumption is a challenging one in the sense that the necessary continual sounding of the channels and interchanging of information bites into the capacity which is the very quantity usually being sought by MIMO systems, and yet this aspect of its usage is not included in the formulation. This sets up a difficult interpretation for any capacity optimization. As noted above, here we plumb optimizing for analogue channel quantities, and calculate associated digital performance after the optimization. The details of the model are as follows.

Let the K users all have N_t transmit antennas and N_r receive antennas, and all users utilize the P subchannels. The formulation in this section assumes $K \geq 3$; the case for $K = 2$ is special and discussed separately below. Referring to figure 3.1, the transmit beamformers for the ith user at the pth subcarrier are written $\mathbf{v}_i(p) \in \mathbb{C}^{N_t \times 1}$, and similarly, the receive beamformers are $\mathbf{u}_i(p) \in \mathbb{C}^{N_r \times 1}$ for $i \in \{1, ..., K\}$ and $p \in \{0, ..., P-1\}$. For help fix ideas, in Figure 3.1, consider $\mathbf{\Phi} = \mathbf{I}_{P \times P}$, so the input symbol stream of user i, s_i, is multiplexed directly (unaffected by $\mathbf{\Phi}$ in this case) to the subcarriers to obtain the symbols $s_i(p)$. The output of the transmit beamformer is $\mathbf{s}_i(p) = \mathbf{v}_i(p)s_i(p)$, where $\|\mathbf{v}_i(p)\|^2 = 1$. The users' data symbols are assumed to be mutually independent.

The frequency selective channel from the μth transmit antenna of ith transmitting user to the νth receive antenna of rth receiving user is denoted by the delay-time function $h^{r,i}_{\nu\mu}(l)$ where $\nu \in \{1, ..., N_r\}$, $\mu \in \{1, ..., N_t\}$ and $r \in \{1, ..., K\}$, and $l \leq L + 1$ indexes the delay-time bin. The channel is considered unchanged for one OFDM symbol and independent between OFDM symbols. Assuming perfect OFDM symbol timing synchronization, then after removal of the cyclic prefix with length

$L_{\text{CP}} \geq L$ and after the FFT, the received signal vector for the ith user can be written:

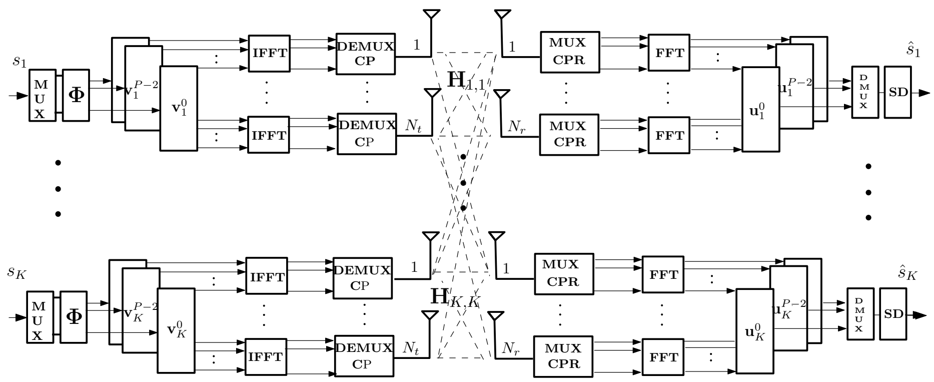

Figure 3.1: Null space and orthogonal basis multiuser beamforming with precoder.

$$\mathbf{y}_i(p) = \mathbf{H}_{i,i}(p)\mathbf{v}_i(p)s_i(p) + \sum_{i' \neq i}^{K} \mathbf{H}_{i,i'}(p)\mathbf{v}_{i'}(p)s_{i'}(p) + \mathbf{n}_i(p) \tag{3.1}$$

In (3.1), the channel at each subcarrier is $\mathbf{H}_{r,i}(p) \in \mathbb{C}^{N_r \times N_t}$. The (ν, μ) entry of it is defined as $[\mathbf{H}_{r,i}(p)]_{\nu,\mu} \triangleq H^{r,i}_{\nu\mu}(p)$, where

$$H^{r,i}_{\nu\mu}(p) := \sum_{l=0}^{L} h^{r,i}_{\nu\mu}(l) \exp(-\jmath 2\pi lp/P)$$

Applying the receiver beamformer to all the subcarriers of all the users, and simplifying the notation by dropping index p:

$$\mathbf{u}_i^{\mathcal{H}}\mathbf{y}_i = \mathbf{u}_i^{\mathcal{H}}\mathbf{H}_{i,i}\mathbf{v}_i s_i + \sum_{i' \neq i}^{K} \mathbf{u}_i^{\mathcal{H}}\mathbf{H}_{i,i'}\mathbf{v}_{i'}s_{i'} + \mathbf{u}_i^{\mathcal{H}}\mathbf{n}_i \tag{3.2}$$

Designing $\mathbf{v}_i$ and $\mathbf{u}_i$, in order to have good detection performance for all users in MIMO interference channels is a subject of research interest. In chapter 2, we showed that the following optimization:

$$\begin{aligned} \max_{\mathbf{u}_i, \mathbf{v}_i} \quad & \sum_{i=1}^{K} \log_2(1 + \text{SINR}_i) \\ \text{s.t.} \quad & \mathbf{u}_i^{\mathcal{H}}\mathbf{u}_i = 1 \qquad i = 1, ..., K \\ & \mathbf{v}_i^{\mathcal{H}}\mathbf{v}_i = 1 \qquad i = 1, ..., K. \end{aligned} \tag{3.3}$$

does not have closed-form solutions for $\mathbf{v}_i$ and $\mathbf{u}_i$, whereas we show in this chapter that the optimization problem:

$$\begin{aligned} \underset{\mathbf{u}_i}{\text{maximize}} \quad & \text{SNR}_i \\ \text{s.t.} \quad & \text{I}_j = 0 \quad j = 1, \ldots, K-2; \end{aligned} \tag{3.4}$$

where I_j is interference from jth user after applying receive beamformer, has a closed-form solution for $\mathbf{u}_i$. The one interference term elimination (before applying the receive beamformer) and the condition $\|\mathbf{v}_i\| = 1$ are both satisfied prior to the constrained SNR maximization (section 3.3). We also demonstrate that the joint design of $\mathbf{v}_i$ and $\mathbf{u}_i$ has closed-form solution for constrained SNR maximization (section 3.4). Therefore, solving constrained SNR maximization is much easier than solving (3.3).

3.3 Optimal Rx-BFs for constrained SNR maximization when the Tx-BFs are known

In this section, the Tx-BFs are found from the null space of an appropriate set of channels, and then the optimal Rx-BFs are sought. For $K \in \{2(n+1) : n \in \mathbb{N}\}$, where $\mathbb{N}$ denotes positive integers, the beamformer $\mathbf{v}_i$ is obtained by

$$\mathbf{v}_i = \mathcal{N}(\mathbf{H}_{K+1-i,i}) \tag{3.5}$$

where

$$\mathcal{N}(\mathbf{A}) \triangleq \{\mathbf{x} | \mathbf{A}\mathbf{x} = \mathbf{0}, ||\mathbf{x}|| = 1\} \tag{3.6}$$

is an orthonormal basis for the null space of $\mathbf{A}$. For $K \in \{2n+1 : n \in \mathbb{N}\}$, the $\mathbf{v}_i$ can be found by

$$\mathbf{v}_i = \begin{cases} \mathcal{N}(\mathbf{H}_{K+1-i,i}) & \text{if } K+1-i < i \\ \mathcal{N}(\mathbf{H}_{K,i}) & \text{if } K+1-i = i \\ \mathcal{N}(\mathbf{H}_{K-i,i}) & \text{if } K+1-i > i \end{cases} \tag{3.7}$$

Note that from equations (3.5) or (3.7), $\mathbf{H}_{1,K}\mathbf{v}_K = 0$. The next step is to determine $\mathbf{u}_i$ such that it maximizes the signal-to-noise ratio (SNR) of the ith user (i.e. after the Rx-BF) while suppressing the $K-2$ remaining interference terms. This optimization problem is denoted $\mathcal{P}$ for the first receiver, as an example, and the rest of the receivers beamformer design followed by the same methodology. For simplicity, $\mathbb{E}\{|s_i|^2\} = \sigma_s^2$ and $\mathbb{E}\{\mathbf{n}_i\mathbf{n}_i^{\mathcal{H}}\} = \sigma_n^2\mathbf{I}$. The problem is expressed

$$\begin{aligned} \mathcal{P} : \underset{\mathbf{u}_1 \in \mathbb{C}^{N_r} \setminus \mathbf{0}}{\text{maximize}} & \frac{\mathbf{u}_1^{\mathcal{H}}\mathbf{H}_{1,1}\mathbf{v}_1\mathbf{v}_1^{\mathcal{H}}\mathbf{H}_{1,1}^{\mathcal{H}}\mathbf{u}_1}{\mathbf{u}_1^{\mathcal{H}}\mathbf{u}_1} \\ \text{s.t.} & \begin{cases} \mathbf{u}_1^{\mathcal{H}}\mathbf{H}_{1,2}\mathbf{v}_2 = 0 \\ \mathbf{u}_1^{\mathcal{H}}\mathbf{H}_{1,3}\mathbf{v}_3 = 0 \\ \quad \vdots \\ \mathbf{u}_1^{\mathcal{H}}\mathbf{H}_{1,K-1}\mathbf{v}_{K-1} = 0. \end{cases} \end{aligned} \tag{3.8}$$

so $\mathcal{P}$ is a constrained SNR maximization formulation where maximization over a quasiconvex object function with affine constraints is sought [58]. The maximizing of the ratio of quadratic forms is a known problem with an eigen solution. But here the difference is that the $\mathcal{P}$ has constraints.

This different problem leads to a different solution. These constraints forces the interference, for first user as an example, to be eliminated. To solve $\mathcal{P}$, its Lagrangian function is needed:

$$\mathcal{L}(\mathbf{x}, \lambda) = -\frac{\mathbf{x}^{\mathcal{H}}\mathbf{Q}\mathbf{x}}{\mathbf{x}^{\mathcal{H}}\mathbf{x}} - \sum_{i=1}^{K-2} \lambda_i \mathbf{x}^{\mathcal{H}}\mathbf{q}_i; \tag{3.9}$$

where $\mathbf{x} = \mathbf{u}_1$, $\mathbf{Q} = \mathbf{H}_{1,1}\mathbf{v}_1\mathbf{v}_1^{\mathcal{H}}\mathbf{H}_{1,1}^{\mathcal{H}}$ and $\mathbf{q}_i = \mathbf{H}_{1,i+1}\mathbf{v}_{i+1}$. The linear independent constraint qualification (LICQ) holds at $\mathbf{x}^*$ if $\mathbf{x}^*$ is local solution for problem $\mathcal{P}$ (See Appendix D). Therefore, the Karush-Kuhn-Tucker (KKT) conditions for $\mathcal{P}$ are:

$$\begin{cases} \nabla_{\mathbf{x}}\mathcal{L}(\mathbf{x}^{\text{opt}}, \lambda^{\text{opt}}) = 0, \\ \mathbf{q}_i^{\mathcal{H}}\mathbf{x}^{\text{opt}} = 0. \end{cases} \tag{3.10}$$

Besides a set of solutions for the global maximum, $\mathcal{P}$ also has sets of solutions for local maxima. From (3.10), it can be shown that $\mathbf{x}^{\text{locm}}$ is a local maximum of $\mathcal{P}$ if

$$\begin{cases} \mathbf{Q}\mathbf{x}^{\text{locm}} = 0, \\ \mathbf{q}_i^{\mathcal{H}}\mathbf{x}^{\text{locm}} = 0. \end{cases} \tag{3.11}$$

The solution $\mathbf{x}^{\text{locm}}$ could be zero but the interest is in solutions where $\mathbf{x}^{\text{locm}}$ is non-zero, i.e., $\mathbf{x}^{\text{locm}} \neq \mathbf{0}$, denoted by $\mathbf{x}^{\text{locm}} \in \mathbb{C}^{N_r} \backslash \mathbf{0}$. It is recalled that $\mathbf{Q} \in \mathbb{C}^{N_r \times N_r}$ and $\mathbf{x}^{\text{locm}} \in \mathbb{C}^{N_r \times 1}$. The number of unknown parameters and number of equations in (3.11) for $\mathbf{x}^{\text{locm}} \neq \mathbf{0}$ determines the minimum number of required antennas for problem $\mathcal{P}$.

Lemma 1. *For constrained SNR maximization, the minimum number of receiver and transmit antennas are* $\min(N_r) = K$ *and* $\min(N_t) = K + 1$ *respectively.*

Proof. Recall that $\mathcal{P}$ has sets of solutions for the local maxima and a set for the global maximum, and that the interest is in the case $\mathbf{x}^{\text{locm}} \neq \mathbf{0}$ for $\mathbf{x}^{\text{locm}}$ from (3.11). There are a total of N_r unknown parameters but $K-1$ distinct equations. The number of distinct equations are $K-1$, because $\text{rank}(\mathbf{Q}) = 1$, so $\mathbf{Q}\mathbf{x}^{\text{locm}} = 0$ is counted one equation, and $K-2$ from $\mathbf{q}_i^{\mathcal{H}}\mathbf{x}^{\text{locm}} = 0$ for the linear system of equations (3.11). It is also desired that $\mathbf{x}^{\text{locm}} \in \mathbb{C}^{N_r} \backslash \mathbf{0}$. If in (3.11) the number of unknowns is greater than the number of equations ($N_r > K-1$), then $\mathbf{x}^{\text{locm}} \in \mathbb{C}^{N_r} \backslash \mathbf{0}$. Hence,

$$\min(N_r) = K. \tag{3.12}$$

On the other hand, the $\mathbf{v}_i$'s are the null space of matrices with dimension $N_r \times N_t$. Therefore $\mathbf{v}_i \in \mathbb{C}^{N_t} \backslash \mathbf{0}$, if $N_t > N_r$ or

$$\min(N_t) = K + 1. \tag{3.13}$$

□

Finding the global optimum Rx-BF from KKT of problem $\mathcal{P}$, for general case $N_r \geq K$, is a hard problem (See Appendix E). Using the Rx-BF's unitary assumption ($\mathbf{x}^{\mathcal{H}}\mathbf{x} = 1$), the problem $\mathcal{P}$ is transformed to a new optimization problem $\mathcal{P}'$. We show that the new problem $\mathcal{P}'$, has closed-form global solution if $N_r = K$. The closed-form global optimum of this new problem is presented below.

Lemma 2. *For $N_r = K$, the degree-of-freedom (DoF) of the new problem $\mathcal{P}'$ is 2.*

Proof. Problem $\mathcal{P}'$ can be written as:

$$\begin{aligned} \max_{\mathbf{x}} \quad & \mathbf{x}^{\mathcal{H}}\mathbf{Q}\mathbf{x} \\ \text{s.t.} \quad & \mathbf{B}\mathbf{x} = \mathbf{0} \\ & \mathbf{x}^{\mathcal{H}}\mathbf{x} = 1. \end{aligned} \tag{3.14}$$

where $\mathbf{B} \triangleq [\mathbf{q}_1^{\mathcal{H}}; ...; \mathbf{q}_{K-2}^{\mathcal{H}}] \in \mathbb{C}^{(K-2)\times N_r}$. If $N_r = K$ then $\mathcal{N}(\mathbf{B}) \in \mathbb{C}^{K\times 2}$, $\mathbf{x}^{\text{opt}} = \alpha[\mathcal{N}(\mathbf{B})]_1 + \beta[\mathcal{N}(\mathbf{B})]_2$. Therefore, for $N_r = K$, only two complex numbers α and β should be found, corresponding to two degrees of freedom (see below). □

Lemma 3. *For $N_r = K$, the closed-form global maximum of the problem $\mathcal{P}'$ is:*

$$\mathbf{x}^{\text{opt}} = \sqrt{1 - \frac{\mu_2}{\mu_1 + \mu_2}}[\mathcal{N}(\mathbf{B})]_1 + \sqrt{\frac{\mu_2}{\mu_1 + \mu_2}}e^{-\jmath\varphi}[\mathcal{N}(\mathbf{B})]_2 \tag{3.15}$$

where $\varphi \triangleq \angle[\mathcal{N}(\mathbf{B})]_1^{\mathcal{H}}\mathbf{Q}[\mathcal{N}(\mathbf{B})]_2$, $\mu_1 \triangleq [\mathcal{N}(\mathbf{B})]_1^{\mathcal{H}}\mathbf{Q}[\mathcal{N}(\mathbf{B})]_1$, $\mu_2 \triangleq [\mathcal{N}(\mathbf{B})]_2^{\mathcal{H}}\mathbf{Q}[\mathcal{N}(\mathbf{B})]_2$.

Proof. Based on Lemma 2, the optimization (3.14) is reformulated as:

$$\begin{aligned} \max_{\alpha,\beta} \quad & |\alpha|^2\mu_1 + |\beta|^2\mu_2 + 2\text{Re}\{\text{conj}(\alpha)\beta\sqrt{\mu_1\mu_2}e^{\jmath\varphi}\} \\ \text{s.t.} \quad & |\alpha|^2 + |\beta|^2 = 1. \end{aligned} \tag{3.16}$$

Without loss of generality assume $\beta = \sqrt{y}e^{-\jmath\varphi}$, $\alpha = \sqrt{1-y}$ which means $\text{Re}\{\beta\} = \sqrt{y}$, $\text{Re}\{\alpha\} = \sqrt{1-y}$; and it is understood (assumed) that y is real and less than unity. Hence, problem (3.16) reduces to:

$$\max_{y} \quad \mu_1(1-y) + \mu_2 y + 2\sqrt{\mu_1\mu_2}\sqrt{y(1-y)} \tag{3.17}$$

which is a square, so by taking the square root, it is equivalent to

$$\max_{y} \quad f(y) = \sqrt{1-y}\sqrt{\mu_1} + \sqrt{\mu_2}\sqrt{y}. \tag{3.18}$$

From $df/dy = 0$, $y^{\text{opt}} = \mu_2/(\mu_1 + \mu_2)$ is derived. Because $\mu_1 > 0$ and $\mu_2 > 0$, there results $0 < y^{\text{opt}} < 1$, which satisfies the above assumptions about y. Therefore, $\alpha^{\text{opt}} = \sqrt{1 - y^{\text{opt}}}$ and $\beta^{\text{opt}} = \sqrt{y^{\text{opt}}}e^{-\jmath\varphi}$, hence $\mathbf{x}^{\text{opt}} = \alpha^{\text{opt}}[\mathcal{N}(\mathbf{B})]_1 + \beta^{\text{opt}}[\mathcal{N}(\mathbf{B})]_2$ □

Therefore, the optimum Rx-BF is in closed-form for the constrained SNR maximization problem where the Tx-BFs are the null space of the appropriate channels.

A receiver design for the two-user case, $K = 2$, is now considered. Recalling that for $K = 2$, the optimization problem (3.8) has no constraint, so

$$\mathbf{v}_1 = \mathcal{N}(\mathbf{H}_{2,1})$$

$$\mathbf{v}_2 = \mathcal{N}(\mathbf{H}_{1,2}) \tag{3.19}$$

$$\mathbf{u}_1 = \boldsymbol{w}_{\max}(\mathbf{H}_{1,1}\mathbf{v}_1\mathbf{v}_1^{\mathcal{H}}\mathbf{H}_{1,1}^{\mathcal{H}}) = \mathbf{H}_{1,1}\mathbf{v}_1/\|\mathbf{H}_{1,1}\mathbf{v}_1\|$$

$$\mathbf{u}_2 = \boldsymbol{w}_{\max}(\mathbf{H}_{2,2}\mathbf{v}_2\mathbf{v}_2^{\mathcal{H}}\mathbf{H}_{2,2}^{\mathcal{H}}) = \mathbf{H}_{2,2}\mathbf{v}_2/\|\mathbf{H}_{2,2}\mathbf{v}_2\| \tag{3.20}$$

Equation (3.20) is the solution to the maximum of the ratio of quadratic forms $\mathbf{x}^{\mathcal{H}}\mathbf{Q}\mathbf{x}/\mathbf{x}^{\mathcal{H}}\mathbf{P}\mathbf{x}$ with respect to $\mathbf{x} \in \mathbb{C}^n \backslash \mathbf{0}$, where $\mathbf{P}$ and $\mathbf{Q}$ are positive semidefinite e.g., [59].

Recently, quasi-maximum-likelihood detection based on semidefinite relaxation (SDR) has been demonstrated to show near-maximum likelihood (i.e., SD) performance but with polynomial complexity [60]. An SDR detector is applied here for decoding the data by means of semidefinite relaxation codes for the discrete integer least squares problem [61].

Multipath diversity can be also added to this system. The first equation in (3.2), with the subcarrier index reintroduced, is

$$\mathbf{u}_1^{\mathcal{H}}(p)\mathbf{y}_1(p) = \mathbf{u}_1^{\mathcal{H}}(p)\mathbf{H}_{1,1}(p)\mathbf{v}_1(p)s_1(p)$$

$$+\mathbf{u}_1^{\mathcal{H}}(p)\mathbf{n}_1(p) \tag{3.21}$$

whose scalar nature allows the LCP matrix $\mathbf{\Phi}$ to be applied before the Tx-BF $\mathbf{v}_1(p)$. The $s_1(p)$'s are stacked for all P subcarriers and then an OFDM frame can be decoded by SD. Hence, instead of decoding $s_1(p)$ subcarrier-wise, using the LCP matrix makes it possible to decode the data frame-wise while getting multipath diversity (up to $L + 1 \neq 1$ for $\mathbf{H}_{1,1}$).

As discussed in [55], the fixed matrix $\mathbf{\Phi}$ extracts maximum multipath diversity if it is designed properly. The optimal LCP matrix design for $\mathbf{\Phi}$ is summarized as follows [55]:

- The unitary rotation matrix $\mathbf{\Theta} \in \mathbb{C}^{J\times J}$ has a Vandermonde structure

$$\mathbf{\Theta} = \frac{1}{\beta}\begin{bmatrix} 1 & \alpha_1 & \ldots & \alpha_1^{J-1} \\ 1 & \alpha_2 & \ldots & \alpha_2^{J-1} \\ \vdots & \vdots & \ldots & \vdots \\ 1 & \alpha_J & \ldots & \alpha_J^{J-1} \end{bmatrix} \tag{3.22}$$

where $\{\alpha_j\}_{j=1}^{J}$ are the roots of $x^J = \sqrt{-1}$. The parameter β is to set $\mathrm{tr}(\mathbf{\Theta}\mathbf{\Theta}^{\mathcal{H}}) = J$. (The α and β are different to those used in the proof of the above Lemma.)

- Set $P = MJ$, where $J \in \{2^n, n \in \mathbb{N}\}$, and $\mathcal{I} \triangleq \{0, 1, ..., P-1\}$ to index the P subcarriers. Subcarrier grouping can be represented by partitioning $\mathcal{I}$ into M nonintersecting subsets $\mathcal{I}_m \triangleq \{p_{m,1}, ..., p_{m,K}\}$. The mth group of subcarriers selector matrix $\mathbf{\Psi}_m \triangleq \mathbf{I}_P(\mathcal{I}_m, :)$, where $\mathbf{I}_P(\mathcal{I}_m, :)$ is a $J \times P$ permutation matrix comprising the $\{p_{m,j}+1\}_{j=1}^{J}$ rows of $\mathbf{I}_P$. The optimal subcarrier grouping would be $\mathcal{I}_{mopt} = \{m-1, M+m-1, ..., (J-1)M+m-1\}$.
- The LCP matrix is $\mathbf{\Phi} = \sum_{m=1}^{M} \mathbf{\Psi}_m^T \mathbf{\Theta} \mathbf{\Psi}_m$.

The complexity of SD for multiuser MIMO-OFDM interference system, which can extract multipath diversity through the LCP matrix, is the same as SISO-OFDM system because $h_1(p) \triangleq \mathbf{u}_1^{\mathcal{H}}(p)\mathbf{H}_{1,1}(p)\mathbf{v}_1(p)$ is a scalar channel.

3.4 Joint Rx-BF and Tx-BF for constrained SNR maximization

In the previous section, the optimal closed-form Rx-BFs were obtained by (3.15) while the Tx-BFs are the null space of channels as expressed by equations (3.5) or (3.7) according to even or odd number of users, respectively. In this section, joint Tx-BF and Rx-BF are designed for constrained SNR maximization problem by using the extended alternating optimization (EAO) algorithm for a multi-objective optimization.

Consider the following optimization problem:

$$\min_{\mathbf{x} \in \Omega_{\mathcal{J}}} \mathcal{J}(\mathbf{x}) = \mathcal{J}_1(\mathbf{x}_1, ..., \mathbf{x}_K) + ... + \mathcal{J}_K(\mathbf{x}_1, ..., \mathbf{x}_K) \tag{3.23}$$

where $\mathbf{x} = [\mathbf{x}_1, ..., \mathbf{x}_K]^T$ and $\Omega_{\mathcal{J}}$ is the feasible set. Generally, solving such a nonlinear constrained optimization problem is difficult. However, if firstly for each objective function, i.e., $\mathcal{J}_i$, $i = 1, ..., K$, there is a unique global minimizer with respect to $\mathbf{x}_i$ for fixed $\mathbf{x}_1, ..., \mathbf{x}_{i-1}, \mathbf{x}_{i+1}, ...\mathbf{x}_K$, then EAO approximates the hard problem's solution by simultaneous solving of the following K problems:

$$\min_{\mathbf{x}_i \in \Omega_i} \mathcal{J}_i(\mathbf{x}_1, ...\mathbf{x}_{i-1}, \mathbf{x}_i, \mathbf{x}_{i+1}, ..., \mathbf{x}_K) \tag{3.24}$$

where $\Omega_1 \times \cdots \times \Omega_K = \Omega_{\mathcal{J}}$.

Now it can be assumed that the optimal solution of (3.24) can be represented as:

$$\mathbf{x}_i = l_i(\mathbf{x}_1, ..., \mathbf{x}_{i-1}, \mathbf{x}_{i+1}, ..., \mathbf{x}_K) \quad i = 1, ..., K \tag{3.25}$$

where l_i is a nonlinear function and it has the following property:

$$\begin{aligned} \forall \mathbf{x}_i \in \Omega_i, \quad & \mathcal{J}_i(\mathbf{x}_1, ...\mathbf{x}_{i-1}, \mathbf{x}_i, \mathbf{x}_{i+1}, ..., \mathbf{x}_K) \geq \\ \mathcal{J}_i(\mathbf{x}_1, ...\mathbf{x}_{i-1}, \mathbf{x}_i, \mathbf{x}_{i+1}, ..., \mathbf{x}_K)&|_{\mathbf{x}_i = l_i(\mathbf{x}_1, ..., \mathbf{x}_{i-1}, \mathbf{x}_{i+1}, ..., \mathbf{x}_K)} \end{aligned} \tag{3.26}$$

Secondly, if for some a, $\|l_i\| \leq a$ for $||\text{col}\{\mathbf{x}_1, ...\mathbf{x}_{i-1}, \mathbf{x}_{i+1}, ..., \mathbf{x}_K\}|| \leq a$ (where col operator concatenates vectors), then there is a Nash equilibrium (NE) for K sub-porblems (games), see Appendix F. Finally, NE for these K games can be approximated iteratively by:

$$\mathbf{x}_i^{(n+1)} = l_i(\mathbf{x}_1^{(n+1)}, ..., \mathbf{x}_{i-1}^{(n+1)}, \mathbf{x}_{i+1}^{(n)}, ..., \mathbf{x}_K^{(n)}) \tag{3.27}$$

In the rest of the chapter, N is the fixed number of iterations after which the $\{x_i^{(N)}\}_{i=1}^K$ is the approximation for $\{x_i^*\}_{i=1}^K$.

We show that the EAO algorithm can be deployed for joint $\mathbf{v}_1$ and $\mathbf{u}_1$ design for constraint SNR problem. Assume that the $\mathbf{v}_2, ..., \mathbf{v}_K$ are assigned arbitrarily at the first iteration, the joint constrained SNR problem for the first user is formulated as:

$$\mathcal{G}:$$

$$\begin{aligned} &\underset{\mathbf{v}_1 \in \mathbb{C}^{N_t}, \mathbf{u}_1 \in \mathbb{C}^{N_r}}{\text{maximize}} \quad \mathbf{u}_1^{\mathcal{H}} \mathbf{H}_{1,1} \mathbf{v}_1 \mathbf{v}_1^{\mathcal{H}} \mathbf{H}_{1,1}^{\mathcal{H}} \mathbf{u}_1 \\ &\text{s.t.} \begin{cases} \mathbf{u}_1^{\mathcal{H}} \mathbf{H}_{1,2} \mathbf{v}_2 = 0 \\ \mathbf{u}_1^{\mathcal{H}} \mathbf{H}_{1,3} \mathbf{v}_3 = 0 \\ \qquad \vdots \\ \mathbf{u}_1^{\mathcal{H}} \mathbf{H}_{1,K} \mathbf{v}_K = 0 \\ \mathbf{u}_1^{\mathcal{H}} \mathbf{u}_1 = 1 \\ \mathbf{v}_1^{\mathcal{H}} \mathbf{v}_1 = 1. \end{cases} \end{aligned} \tag{3.28}$$

In problem $\mathcal{G}$, there are $N_r - (K-1)$ DoF. Therefore, following the similar discussion as above, $\min(N_t) = \min(N_r) = K+1$. If $\mathbf{v}_2, ..., \mathbf{v}_K$ are assumed to be known (or simply fixed), then from the optimal solution of equation (3.18) using Lemma 3, the process of finding the optimal Tx-BF of the first user, can be summarized as:

$$\begin{aligned} &\min_{\mathbf{v}_1} \quad \mathcal{J}_1 = -\mathbf{v}_1^{\mathcal{H}} (\mathbf{G}_1 + \mathbf{G}_2) \mathbf{v}_1 \\ &\text{s.t.} \quad \|\mathbf{v}_1\| = 1. \end{aligned} \tag{3.29}$$

where

$$\begin{aligned} \mathbf{C}_1 &\triangleq [(\mathbf{H}_{1,2}\mathbf{v}_2)^{\mathcal{H}}; \cdots; (\mathbf{H}_{1,K}\mathbf{v}_K)^{\mathcal{H}}] \\ \mathbf{G}_1 &\triangleq \mathbf{H}_{1,1}^{\mathcal{H}} [\mathcal{N}(\mathbf{C}_1)]_1 [\mathcal{N}(\mathbf{C}_1)]_1^{\mathcal{H}} \mathbf{H}_{1,1} \\ \mathbf{G}_2 &\triangleq \mathbf{H}_{1,1}^{\mathcal{H}} [\mathcal{N}(\mathbf{C}_1)]_2 [\mathcal{N}(\mathbf{C}_1)]_2^{\mathcal{H}} \mathbf{H}_{1,1} \end{aligned}$$

and the unique global solution of (3.29) is:

$$\mathbf{v}_1 = \boldsymbol{w}_{\max}\big(\mathbf{G}_1 + \mathbf{G}_2\big) = l_1(\mathbf{v}_2, ..., \mathbf{v}_K). \tag{3.30}$$

Generally, $\mathbf{v}_i = l_i(\mathbf{v}_1, ..., \mathbf{v}_{i-1}, \mathbf{v}_{i+1}, \mathbf{v}_K)$ where here the l_i function is maximum normalized eigenvector function. Because $\|l_i\| \leq 1$ then EAO can be deployed (Appendix F).

Algorithm 1 in Table 3.1 obtains the optimal Tx-BF for the first user at the first iteration, $\mathbf{v}_1^{(1)}$, based on some initialization $\mathbf{v}_k^{(0)}$, $k = 2, ..., K$. The $\mathbf{v}_1^{(1)}$ and $\mathbf{v}_k^{(0)}$, $k = 3, ..., K$, are then used for obtaining the $\mathbf{v}_2^{(1)}$, and this procedure is deployed for all user over N iterations. Finally, the Rx-BFs are derived by equation (3.15) for all users upon $\mathbf{v}_k^{(N)}$, $k \in \mathcal{K}$.

Table 3.1: Extended alternating optimization (EAO) for joint Tx-BF and Rx-BF in constraint SNR maximization problem with K users

Algorithm 1

Choose $N > 1$, $\mathbf{v}_k^{(0)}$ arbitrarily for $k \in \mathcal{K}$ ▷ Assume arbitrary initial values for Tx-BFs.
for $n = 0 : N-1$ **do**
  for $k = 1 : K$ **do**
    $\mathbf{C}_k = [\underbrace{(\mathbf{H}_{k,r}\mathbf{v}_r^{(n+1)})^{\mathcal{H}}; \cdots;}_{k-1} \underbrace{\cdots; (\mathbf{H}_{k,s}\mathbf{v}_s^{(n)})^{\mathcal{H}}}_{K-k}]$, $r, s \in \mathcal{K} \setminus \{k\}, r \neq s$ ▷ $\mathbf{C}_k$ is a matrix that $\mathbf{C}_k\mathbf{u}_k = \mathbf{0}$.
    $\mathbf{G}_1 = \mathbf{H}_{k,k}^{\mathcal{H}}[\mathcal{N}(\mathbf{C}_k)]_1[\mathcal{N}(\mathbf{C}_k)]_1^{\mathcal{H}}\mathbf{H}_{k,k}$
    $\mathbf{G}_2 = \mathbf{H}_{k,k}^{\mathcal{H}}[\mathcal{N}(\mathbf{C}_k)]_2[\mathcal{N}(\mathbf{C}_k)]_2^{\mathcal{H}}\mathbf{H}_{k,k}$
    $\mathbf{v}_k^{(n+1)} = \boldsymbol{w}_{\max}\big(\mathbf{G}_1 + \mathbf{G}_2\big)$ ▷ Obtain the Tx-BF for the kth user at $(n+1)$th iteration.
  end for
end for
for $k = 1 : K$ **do**
  $\mathbf{C}_k = [(\mathbf{H}_{k,r}\mathbf{v}_r^{(N)})^{\mathcal{H}}; \cdots]$, $r \in \mathcal{K} \setminus \{k\}$, compute $\mathbf{G}_1$ and $\mathbf{G}_2$ for $\mathbf{C}_k$.
  $\mu_1 = \mathbf{v}_k^{(N)\mathcal{H}}\mathbf{G}_1\mathbf{v}_k^{(N)}$, $\mu_2 = \mathbf{v}_k^{(N)\mathcal{H}}\mathbf{G}_2\mathbf{v}_k^{(N)}$
  $\varphi = \angle[\mathcal{N}(\mathbf{C}_k)]_1^{\mathcal{H}}\mathbf{H}_{k,k}\mathbf{v}_k^{(N)}\mathbf{v}_k^{(N)\mathcal{H}}\mathbf{H}_{k,k}^{\mathcal{H}}[\mathcal{N}(\mathbf{C}_k)]_2$
  $\mathbf{u}_k = \sqrt{\frac{\mu_1}{\mu_1+\mu_2}}[\mathcal{N}(\mathbf{C}_k)]_1 + \sqrt{\frac{\mu_2}{\mu_1+\mu_2}}e^{-\jmath\varphi}[\mathcal{N}(\mathbf{C}_k)]_2$ ▷ Obtain the Rx-BF for the kth user upon obtained Tx-BFs.
end for

3.5 Tx-BF and Rx-BF design for joint constrained SNR maximization and SINR maximization

In the previous section, each Rx-BF nulls its interference and then this solution is inserted to the constrained SNR objective function which yields the Tx beamformer. In this section, multi-objective optimization by the fixed point method is applied. Instead of $\mathcal{G}$, which is optimization w.r.t. $\mathbf{v}_1$ and

$\mathbf{u}_1$, define the problem $\mathcal{G}_1$ as:

$$\mathcal{G}_1 : \underset{\mathbf{v}_1 \in \mathbb{C}^{N_t}}{\text{maximize}} \;\; \mathbf{v}_1^{\mathcal{H}} \mathbf{H}_{1,1}^{\mathcal{H}} \mathbf{u}_1 \mathbf{u}_1^{\mathcal{H}} \mathbf{H}_{1,1} \mathbf{v}_1$$
$$\text{s.t.} \begin{cases} \mathbf{v}_1^{\mathcal{H}} \mathbf{H}_{2,1}^{\mathcal{H}} \mathbf{u}_2 = 0 \\ \mathbf{v}_1^{\mathcal{H}} \mathbf{H}_{3,1}^{\mathcal{H}} \mathbf{u}_3 = 0 \\ \qquad \vdots \\ \mathbf{v}_1^{\mathcal{H}} \mathbf{H}_{K,1}^{\mathcal{H}} \mathbf{u}_K = 0 \\ \mathbf{v}_1^{\mathcal{H}} \mathbf{v}_1 \leq 1. \end{cases} \tag{3.31}$$

This problem is maximization w.r.t. $\mathbf{v}_1$ only. (In an alternative notation, we are seeking a Nash equilibrium point for two games.)

Denote the global unique minimizer of $\mathcal{G}_1$ by

$$\mathbf{v}_1 = \gamma[\mathcal{N}(\mathbf{D})]_1 + \delta[\mathcal{N}(\mathbf{D})]_2 \tag{3.32}$$

where $\mathbf{D} \triangleq [(\mathbf{H}_{2,1}^{\mathcal{H}} \mathbf{u}_2)^{\mathcal{H}}; \cdots ; (\mathbf{H}_{K,1}^{\mathcal{H}} \mathbf{u}_K)^{\mathcal{H}}]$. With $|\gamma| = \omega_1$ and $|\delta| = \omega_2$, the ω_1 and ω_2 are the solution of:

$$\begin{aligned} &\underset{\omega_1, \omega_2}{\text{maximize}} \quad && \omega_1^2 \nu_1 + \omega_2^2 \nu_2 + 2\omega_1 \omega_2 \nu_3 \\ &\text{s.t.} && \omega_1^2 + \omega_2^2 \leq 1. \end{aligned} \tag{3.33}$$

where $\nu_1 \triangleq [\mathcal{N}(\mathbf{D})]_1^{\mathcal{H}} \mathbf{H}_{1,1}^{\mathcal{H}} \mathbf{u}_1 \mathbf{u}_1^{\mathcal{H}} \mathbf{H}_{1,1} [\mathcal{N}(\mathbf{D})]_1$, $\nu_2 \triangleq [\mathcal{N}(\mathbf{D})]_2^{\mathcal{H}} \mathbf{H}_{1,1}^{\mathcal{H}} \mathbf{u}_1 \mathbf{u}_1^{\mathcal{H}} \mathbf{H}_{1,1} [\mathcal{N}(\mathbf{D})]_2$, $\nu_3 \triangleq \big|[\mathcal{N}(\mathbf{D})]_1^{\mathcal{H}} \mathbf{H}_{1,1}^{\mathcal{H}} \mathbf{u}_1 \mathbf{u}_1^{\mathcal{H}} \mathbf{H}_{1,1} [\mathcal{N}$ are real. By KKT, the global unique solution of (3.33) is:

$$\begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix} = \boldsymbol{w}_{\max} \left(\begin{bmatrix} \nu_1 & \nu_3 \\ \nu_3 & \nu_2 \end{bmatrix} \right) \tag{3.34}$$

Without loss of generality,

$$\mathbf{v}_1 = \omega_1 [\mathcal{N}(\mathbf{D})]_1 + \omega_2 e^{-\jmath\theta} [\mathcal{N}(\mathbf{D})]_2 \tag{3.35}$$

where $\theta = \angle [\mathcal{N}(\mathbf{D})]_1^{\mathcal{H}} \mathbf{H}_{1,1}^{\mathcal{H}} \mathbf{u}_1 \mathbf{u}_1^{\mathcal{H}} \mathbf{H}_{1,1} [\mathcal{N}(\mathbf{D})]_2$.

Here, instead of obtaining $\mathbf{u}_1$ from problem $\mathcal{G}_1$ while $\mathbf{v}_1$ is fixed (cf. (3.35)), which is the approach of section 3.4, it is possible to obtain $\mathbf{u}_1$ from each users' SINR maximization (problem $\mathcal{G}_2$) by knowing $\mathbf{v}_1$ from (3.35):

$$\mathcal{G}_2 :$$
$$\begin{aligned} &\underset{\mathbf{u}_1 \in \mathbb{C}^{N_r}}{\text{maximize}} \quad && \frac{\mathbf{u}_1^{\mathcal{H}} \mathbf{H}_{1,1} \mathbf{v}_1 \mathbf{v}_1^{\mathcal{H}} \mathbf{H}_{1,1}^{\mathcal{H}} \mathbf{u}_1}{\mathbf{u}_1^{\mathcal{H}} (\sum_{j \neq 1}^{K} \mathbf{H}_{i,j} \mathbf{v}_j \mathbf{v}_j^{\mathcal{H}} \mathbf{H}_{i,j}^{\mathcal{H}} + \frac{\sigma_n^2}{\sigma_s^2} \mathbf{I}) \mathbf{u}_1} \\ &\text{s.t.} && \mathbf{u}_1^{\mathcal{H}} \mathbf{u}_1 = 1. \end{aligned} \tag{3.36}$$

Problem $\mathcal{G}_2$ has unique global solution w.r.t. $\mathbf{u}_1$,

$$\mathbf{u}_1 =$$
$$\boldsymbol{w}_{\max} \left((\sum_{j \neq 1}^{K} \mathbf{H}_{i,j} \mathbf{v}_j \mathbf{v}_j^{\mathcal{H}} \mathbf{H}_{i,j}^{\mathcal{H}} + \frac{\sigma_n^2}{\sigma_s^2} \mathbf{I})^{-1} \mathbf{H}_{1,1} \mathbf{v}_1 \mathbf{v}_1^{\mathcal{H}} \mathbf{H}_{1,1}^{\mathcal{H}} \right) \tag{3.37}$$

Theorem (Proof in [62]): With the unique global solution for both $\mathcal{G}_1$ and $\mathcal{G}_2$ applied for all K users, the following iterative algorithm is almost surely convergent to the fixed point of its corresponding nonexpansive vector field f:

$$\mathbf{v}_i^{(n)} = p_i(\mathbf{u}_1^{(n)}, ..., \mathbf{u}_K^{(n)})$$

$$\mathbf{u}_i^{(n+1)} = q_i(\mathbf{v}_1^{(n)}, ..., \mathbf{v}_K^{(n)})$$

(In the proof, the notation is used:

$$\mathbf{x} \triangleq [\mathbf{u}_1^T, ..., \mathbf{u}_K^T]^T$$
$$f_i \triangleq q_i(p_1(\mathbf{u}_1, ..., \mathbf{u}_K), ..., p_K(\mathbf{u}_1, ..., \mathbf{u}_K))$$

where the vector field f defined by $f \triangleq [f_1, ..., f_K]^T$ is nonexpansive [62].)

Algorithm 2 for Tx-BF and Rx-BF design for joint constrained SNR maximization and SINR maximization is summarized in Table 3.2.

Table 3.2: Tx-BF and Rx-BF design for joint constrained SNR-SINR by EAO

Algorithm 2

Choose $N > 1$, $\mathbf{u}_1^{(0)}, ..., \mathbf{u}_K^{(0)}$

for $n = 0 : N$ **do**

$$\mathbf{D}_i^{(n)} = [(\mathbf{H}_{j,i}^{\mathcal{H}}\mathbf{u}_j^{(n)})^{\mathcal{H}}; \cdots ; (\mathbf{H}_{K,i}^{\mathcal{H}}\mathbf{u}_K^{(n)})^{\mathcal{H}}] \qquad j \neq i$$

$$\nu_1 = [\mathcal{N}(\mathbf{D}_i^{(n)})]_1^{\mathcal{H}}\mathbf{H}_{i,i}^{\mathcal{H}}\mathbf{u}_i^{(n)}\mathbf{u}_i^{(n)\mathcal{H}}\mathbf{H}_{i,i}[\mathcal{N}(\mathbf{D}_i^{(n)})]_1$$

$$\nu_2 = [\mathcal{N}(\mathbf{D}_i^{(n)})]_2^{\mathcal{H}}\mathbf{H}_{i,i}^{\mathcal{H}}\mathbf{u}_i^{(n)}\mathbf{u}_i^{(n)\mathcal{H}}\mathbf{H}_{i,i}[\mathcal{N}(\mathbf{D}_i^{(n)})]_2$$

$$\nu_3 = \left|[\mathcal{N}(\mathbf{D}_i^{(n)})]_1^{\mathcal{H}}\mathbf{H}_{i,i}^{\mathcal{H}}\mathbf{u}_i^{(n)}\mathbf{u}_i^{(n)\mathcal{H}}\mathbf{H}_{i,i}[\mathcal{N}(\mathbf{D}_i^{(n)})]_2\right|$$

$$\theta = \angle[\mathcal{N}(\mathbf{D}_i^{(n)})]_1^{\mathcal{H}}\mathbf{H}_{i,i}^{\mathcal{H}}\mathbf{u}_i^{(n)}\mathbf{u}_i^{(n)\mathcal{H}}\mathbf{H}_{i,i}[\mathcal{N}(\mathbf{D}_i^{(n)})]_2$$

$$\begin{bmatrix}\omega_1\\ \omega_2\end{bmatrix} = \boldsymbol{w}_{\max}\left(\begin{bmatrix}\nu_1 & \nu_3\\ \nu_3 & \nu_2\end{bmatrix}\right)$$

$$\mathbf{v}_i^{(n)} = \omega_1[\mathcal{N}(\mathbf{D}_i^{(n)})]_1 + \omega_2 e^{-\jmath\theta}[\mathcal{N}(\mathbf{D}_i^{(n)})]_2$$

$$\mathbf{u}_i^{(n+1)} = \boldsymbol{w}_{\max}\left((\sum \mathbf{H}_{i,j}\mathbf{v}_j^{(n)}\mathbf{v}_j^{(n)\mathcal{H}}\mathbf{H}_{i,j}^{\mathcal{H}} + \frac{\sigma_n^2}{\sigma_s^2}\mathbf{I})^{-1}\mathbf{H}_{i,i}\mathbf{v}_i^{(n)}\mathbf{v}_i^{(n)\mathcal{H}}\mathbf{H}_{i,i}^{\mathcal{H}}\right)$$

end for

return $\mathbf{v}_i^{(N)}, \mathbf{u}_i^{(N+1)}$

Remark: The proposed methods here are not least square (LS) beamforming. The Tx-BF design with LS can be obtained only by evolutionary algorithms.

Proof. From (3.1), the received vector $\mathbf{y}_i$ can be also written as:

$$\mathbf{y}_i = [\mathbf{H}_{i,1}\mathbf{v}_1 \cdots \mathbf{H}_{i,K}\mathbf{v}_K] \begin{bmatrix} s_1 \\ \vdots \\ s_K \end{bmatrix} + \mathbf{n}_i \tag{3.38}$$

Let $\mathbf{H}_i \triangleq [\mathbf{H}_{i,1}\mathbf{v}_1 \cdots \mathbf{H}_{i,K}\mathbf{v}_K]$, then by applying LS decoder:

$$\begin{bmatrix} \hat{s}_1 \\ \vdots \\ \hat{s}_K \end{bmatrix} = (\mathbf{H}_i^{\mathcal{H}}\mathbf{H}_i)^{-1}\mathbf{H}_i^{\mathcal{H}}\mathbf{y}_i = \begin{bmatrix} s_1 \\ \vdots \\ s_K \end{bmatrix} + \underbrace{(\mathbf{H}_i^{\mathcal{H}}\mathbf{H}_i)^{-1}\mathbf{H}_i^{\mathcal{H}}\mathbf{n}_i}_{\mathbf{e}_i} \tag{3.39}$$

In interference channel scenario, just s_i is to be decoded from $\mathbf{y}_i$. Denote the ith row of $\mathbf{T}_i \triangleq (\mathbf{H}_i^{\mathcal{H}}\mathbf{H}_i)^{-1}\mathbf{H}_i^{\mathcal{H}}$ by $\mathbf{t}_i = \mathbf{T}_i(i,:)$. Therefore, it is desired to minimize $\mathbb{E}(\mathbf{t}_i\mathbf{n}_i\mathbf{n}_i^{\mathcal{H}}\mathbf{t}_i^{\mathcal{H}})$ which is the power of ith row of $\mathbf{e}_i$ for the s_ith symbol. As there are K users, the Tx-BF design with LS decoder is formulated as:

$$\min_{\mathbf{v}_i} \max_{i=1,\ldots,K} \mathbf{t}_i\mathbf{t}_i^{\mathcal{H}}$$

$$\text{s.t.} \|\mathbf{v}_\mathrm{i}\| \leq 1 \tag{3.40}$$

For $N_r = N_t = K$:

$$\mathbf{T}_i = \mathbf{H}_i^{-1} \tag{3.41}$$

By some matrix manipulations, the Tx-BF design with LS decoder reduces to:

$$\min_{\mathbf{v}_i} \max_{i=1,\ldots,K} \frac{1}{|\det(\mathbf{H}_i)|^2} \sum_{j=1}^{K} |\det(\mathbf{H}_{i\{ji\}})|^2$$

$$\text{s.t.} \|\mathbf{v}_\mathrm{i}\| \leq 1 \tag{3.42}$$

where $\mathbf{A}_{\{ji\}}$, known as cofactor of $\mathbf{A}$, denotes the submatrix of $\mathbf{A}$ obtained by deleting row j and column i of $\mathbf{A}$.

Solving the optimization problem (3.42) is only possible by using evolutionary algorithms. But all of the beamformer designs presented here are closed-forms. This is important for implementation. □

3.6 Feedback Rate of Proposed Beamforming Method in Comparison With Other Beamforming Schemes For Interference Channels

From the discussion in previous sections in this chapter, an issue in multi-user beamforming is the amount of information to be exchanged among receivers and transmitters, which bites into the payload capacity. (The channel sounding is an associated issue, and that is not addressed here.) In this section, the feedback of the presented beamforming is compared with existing interference channel beamforming schemes. The analysis is for flat channels, and is extended to OFDM via scaling by P. It is emphasized that the feedback rate, complexity and performance are competing factors in K-user interference channels. In the previous section, the complexity was demonstrated to be lower than existing systems.

From problem $\mathcal{P}'$, (3.5) and (3.7), it is evident that K Tx-BFs (i.e., the data describing the Tx-BF) should be fed back from the receiver nodes to the transmitter nodes (shown as arcs a in figure 3.2), and $K-1$ Tx-BFs should be fed back from each receiver node to the other receiver nodes (arcs b in figure 3.2). Figure 3.2 depicts all the feedback required for $K = 3$ in our beamforming method. Generally, $K^3 + K^2$ complex numbers should be fed back ($N_t = K + 1, N_r = K$). If a complex number is quantized by $2q$ bits information, then $2(K^3 + K^2)q$ feedback bits are needed.

As an example of existing approaches, the MSE-based transceiver design in [1] requires $K(K-1)$ channels (i.e. the data describing the sounded channel state, arcs a'), K Tx-BFs (arcs b'), and $K-1$ Rx-BFs (arcs c'), to be fed back. Figure 3.3 illustrates all the feedback required for beamforming design in [1]. This is exactly the same for maximization of the sum signal power across the network divided by the sum interference power formulated in [56] and solved by alternating maximization technique. For the iterative weighted sum rate maximization discussed in section V of [25], $K(K-1)$ channels, K Tx-BFs from receiver nodes to transmitter nodes, and K Tx-BFs among receiver nodes are required to be sent. Table III summarized feedback bits for all the cases cases. Our two beamforming methods require lower feedback bits compared with these other methods if $K+1$ antenna is considered for all systems at each terminal. This is because the exchange of average SNR is not required. Moreover multipath diversity for OFDM transmission is possible by deploying the fixed precoder matrix. The other schemes do not have this capability because their Rx-BFs are also the decoder.

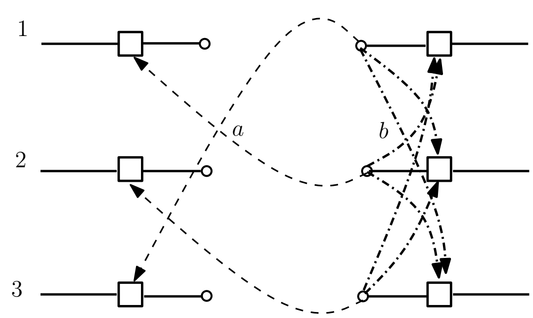

Figure 3.2: Feedback graphs for the proposed constraint SNR maximization with known Tx from nullspace when $K = 3$. The dashed arcs a represent the Tx-BF which are feedback from receiver nodes to transmitter nodes and dashed-dotted arcs b represent Tx-BF which are feedback among receiver nodes.

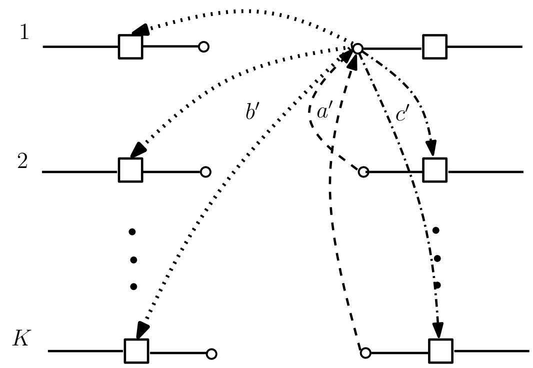

Figure 3.3: Feedback graphs for all joint Tx-Rx beamforming designs. The dashed arcs a' represent the all channels which are feedback from $K-1$ receiver nodes to one receiver node and dotted arcs b' represent Tx-BF which are feedback from one receiver node to transmitter nodes and dashed-dotted arcs c' show the Rx-BF from one receiver node to $K-1$ receiver nodes.

Table 3.3: Feedback Rate for Different Beamforming Schemes in Interference Channels

Beamforming Method	N_t	N_r	**Feedback bits**
MSE minimization [1] Iterative sum-rate maximization [25] Sum power over sum interference maximization[56]	K	K	$2(K^4-K^3+2K^2-K)q+(K-1)q$
Proposed constraint SNR maximization with known Tx from nullspace	$K+1$	K	$2(K^3+K^2)q$
Proposed joint Tx-BF and Rx-BF for constrained SNR maximization	$K+1$	$K+1$	$2(K^4+K^3+K^2-1)q$
Proposed Tx-Rx design for joint constrained SNR-SINR maximization	$K+1$	$K+1$	$2(K^4+K^3+K^2-1)q+(K-1)q$

3.7 Simulation

In this section numerical experiments are described for validating the analysis. The simulation parameters are summarized in Table 3.4. For simplicity, all the users use QPSK in the evaluation of BER performance. As discussed in the Introduction, digital communications performance is a tricky aspect of link optimization and using a single modulation cannot create high capacity (efficiency) over a range of average SNRs. (Rayleigh channels, for example, have a very large range of average SNRs.) Similarly, there is no channel coding. Strictly, the digital communications behavior should be optimized, but this is not yet possible in general as discussed. Nevertheless, optimizing with the analogue objective functions, and then applying a fixed communications configuration allows a fair performance comparison between the differently optimized beamformers.

The IEEE 802.11n standard characterizes MIMO antennas for Wireless Local Area Networks (WLAN). The IEEE 802.11n channel models [57] are designed for indoor WLAN for bandwidths of up to 100 MHz, at frequencies of 2 and 5 GHz. The channel models comprise a set of 6 profiles, labeled A to F (one tap for model A, and 9 to 18 taps for models B-F), which cover the scenarios of flat fading, residential, residential/small office, typical office, large office, and large space (indoors and

Table 3.4: Simulation Setup Parameters

Symbol	**Name**	**Value**
P	Number of subcariers	16
K	Number of users	$3, 4$
N_r	Number of receiver antenna per user	$K, K+1$
N_t	Number of transmitter antenna per user	$K+1$
$L+1$	Channel length, IEEE 802.11n model B	9
L_{CP}	Cyclic prefix	10
J	Rotation matrix rank	8
N	Iterations for EAO	16

outdoors). The maximum multipath diversity for LCP-OFDM is achieved with maximum likelihood decoding and with uniform power delay profile (PDP). To determine the benefit of LCP in the multiuser MIMO-OFDM interference channel, the IEEE 802.11n channel model B is used with the following settings: 3Hz maximum Doppler shift for all paths with Bell Doppler spectrum; 15ns rms delay spread; $\lambda/2$ element spacing at the transmit and receive antennas; and for both clusters 1 and 2: average path gains; angular spread (AS) at the receiver and at the transmitter; mean angles of departure (AoD); and mean angles of arrival (AoA) are all according to [57] standard. This channel description is still far from truly realistic (in particular the antenna aspects) but it is nevertheless a standard allowing the results to be repeatable.

The zero-forcing (ZF) Tx-BF, joint Tx-Rx BF design for leakage interference (LI) minimization and gradient based sum-rate maximization [25] are considered as benchmarks to compare with our proposed EAO method. The Tx-BF design for ZF is:

$$\mathbf{V}_i^{\text{ZF}} = \mathcal{N}\left([\mathbf{H}_{1,i}^T \cdots \mathbf{H}_{i-1,i}^T \;\; \mathbf{H}_{i+1,i}^T \cdots \mathbf{H}_{K,i}^T]^T\right) \tag{3.43}$$

Any column of $\mathbf{V}_i^{\text{ZF}}$ is a ZF solution, however to increase ZF performance (ZF with selection):

$$\mathbf{v}_i^{\text{ZF}} = \underset{c=1,\ldots,\text{size}(\mathbf{V}_\text{i}^{\text{ZF}},2)}{\text{argmax}} (\mathbf{V}_i^{\text{ZF}}(:,c))^{\mathcal{H}}\mathbf{H}_{i,i}^{\mathcal{H}}\mathbf{H}_{i,i}\mathbf{V}_i^{\text{ZF}}(:,c) \tag{3.44}$$

By this nullspace allocation, $\mathbf{H}_{i,k}\mathbf{v}_k = \mathbf{0}$ for $i \neq k$. The ZF nullspace criteria imposes that $N_t \geq N_r K$ [23]. It can be shown that the EAO over K games can be applied to sum-rate maximization as well. For brevity, we only present the final result here (See Appendix I):

$$\mathbf{v}_i^{\text{SR}} = \boldsymbol{w}_{\max}\left(\mathbf{H}_{i,i}^{\mathcal{H}}(\sum_{j\neq i}^{K}\mathbf{H}_{i,j}\mathbf{v}_j^{\text{SR}}(\mathbf{v}_j^{\text{SR}})^{\mathcal{H}}\mathbf{H}_{i,j}^{\mathcal{H}} + \frac{\sigma_n^2}{\sigma_s^2}\mathbf{I})^{-1}\mathbf{H}_{i,i}\right) \tag{3.45}$$

This problem has been also solved by gradient based method in [25].

The bit error performance of the optimized beamformers are depicted in figure 3.4 versus average SNR (σ_s^2/σ_n^2). The legend refers to: 1-Rx-BF design for constrained instantaneous SNR maximization while the Tx-BFs are fixed and Tx-BFs null one interference term; 2-joint Tx-BF and Rx-BF design for constrained instantaneous SNR maximization; 3-joint Tx-BF and Rx-BF design for constrained instantaneous SNR and SINR maximization. Each of these are depicted with and without the LCP matrix $\mathbf{\Phi}$. The $\mathbf{\Phi} = \mathbf{I}_{P\times P}$ means that there is no change invoked from the LCP matrix action at the transmitter side, and in terms of performance, symbol-wise and frame-wise detection at the receiver have the same result. In summary, the figure shows the BER for $K = 3$ users over IEEE 802.11n channel model B. The joint multi-objective Tx-BF and Rx-BF design (approach 3) has the best performance, then the joint single-objective Tx-BF and Rx-BF design (approach 2), and finally the individual design (approach 1). This simulation also demonstrates that for this channel - MIMO-WLAN with 9 paths, using the LCP matrix, still improves the system performance. Finally it is recalled that the complexity of SD decoding at the receiver increases with the constellation size but not with the number of antennas.

The key benefit of the presented beamforming approaches is their computational simplicity. This is quantified from comparing execution times with finding the Tx-BF for LS criteria with an evolutionary algorithm (GA) at a given average SNR. Averaging over 100 realizations, approach 3 is five times faster than LS with GA for $K = 4$ users. Figure 3.5 illustrates that it also has better sum-rate performance and also lower BER (not shown here) performance. The sum rate in bits/s/Hz is computed by $\sum_{i=1}^{K} \log_2(1 + \mathrm{SINR}_i)$. Here, the channel is the IEEE 802.11n channel model A, which is a flat-fading MIMO channel with average "path gain" of pdb = 0 dB, angular spreads AS = 40° at the transmitter and receiver, mean angles of departure AoD = 45° and mean angles of arrival AoA = 45°.

The BER performance and computational complexity of proposed methods have been compared with existing known methods, like ZF, LI and sum-rate maximization. From figure 3.6, approach 2 has better performance than ZF with selection and also much better than LI. The channel for this simulation is complex Gaussian with zero mean and unit variance denoted by $G(0, 1)$. Also, the sum rate maximization by EAO has been compared to sum rate maximization by gradient descent method in [25] (figure 3.7). From this figure, the performance loss is only 0.8dB. The computational complexity of this gradient method is $O(NK^2M^3)$ where $M = \max(N_r, N_t)$. However, the computational complexity of all closed-form methods presented here is $O(NKM^3)$. The required CPU times (not shown) indicate that our proposed method is five times faster for $M = 4$ and $K = 3$ (figure 3.7).

Robustness to imperfect channel information can be readily gauged in the usual way by modeling the channel with $\hat{\mathbf{H}}_{i,j} = \rho\mathbf{H}_{i,j} + \sqrt{(1-\rho^2)}\mathbf{w}$, where $\mathbf{w}$ is a zero mean, unit variance complex Gaussian random matrix. The beamformers are obtained from $\hat{\mathbf{H}}_{i,j}$ while the actual channel is $\mathbf{H}_{i,j}$. As a tie-point, we get for the first individual design (approach 1), an error performance of less than 10^{-3} at SNR = 20dB for $\rho \geq 0.995$ and $\mathbf{H}_{i,j}$ complex Gaussian with zero mean and unit variance.

3.8 Summary and Conclusion

Three new beamforming algorithms are presented for a multiuser MIMO-OFDM interference channel which can also develop multipath diversity using the known technique of applying an LCP matrix. With a unit norm for the transmit and receive beamformers, the algorithms comprise iterative procedures with closed-form steps, allowing a fast solution. Because no derivative or Lagrangian multiplier is needed, the computational complexity is less than existing beamforming methods. It is shown that the third algorithm - joint constrained SNR and SINR maximization - outperforms the least-square beamforming (LS) design, *cf.*, equation (3.40), with a much lower computational time. For quasi-realistic channels (exponential power delay profile, Kronecker antenna correlations, for the IEEE 802.11n channel model), the second algorithm may be better than third algorithm also it has lower feedback rate. The first algorithm is the simplest in the sense of having the lowest complexity and feedback rate but not performance. A lower feedback rate than existing beamforming methods is a feature of the first two algorithms, when the same antenna configuration is considered. It is known that the LCP matrix improves the error performance in strongly idealized channels (uniform power delay profile), and here our simulations demonstrate that for the more realistic IEEE 802.11n channel models, the addition of an LCP matrix (prior to the Tx-beamformer) still improves the error performance. The simplicity of the presented algorithms comes at the price of one more antenna element at each terminal, compared to existing methods. The results of this chapter can also be viewed as some quantification of the trade-offs of between algorithmic simplicity, a minimum number of antennas, feedback rate, and the capability of extracting multipath diversity, in beamforming for the MIMO-OFDM interference channel.

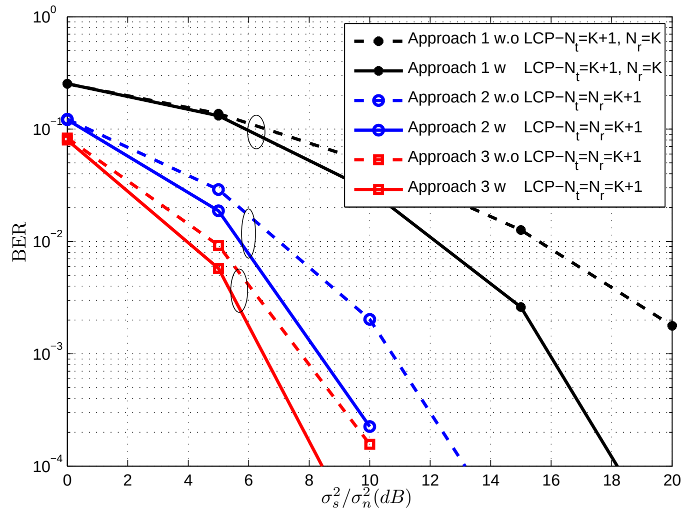

Figure 3.4: The BER performance of approaches 1,2,3 for $K = 3$ with (w) or without (w.o) deploying LCP precoder and sphere decoder with IEEE 802.11n channel model B.

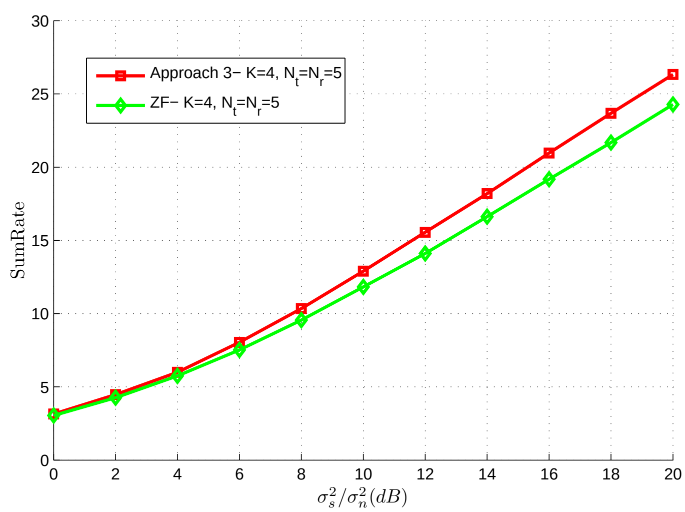

Figure 3.5: The sum rate performance of the approach 3 and LS transmit beamformer design for $K = 4$ users with IEEE 802.11n channel model A.

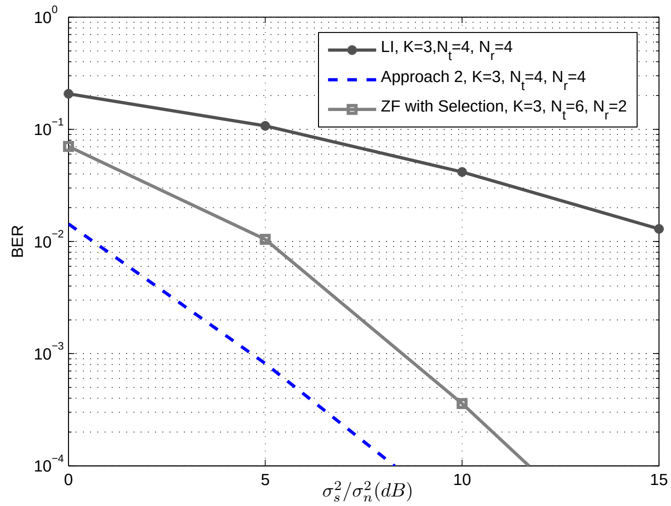

Figure 3.6: The BER performance of ZF with selection, LI and proposed approach 2 for $K = 3$ and $G(0, 1)$ channels.

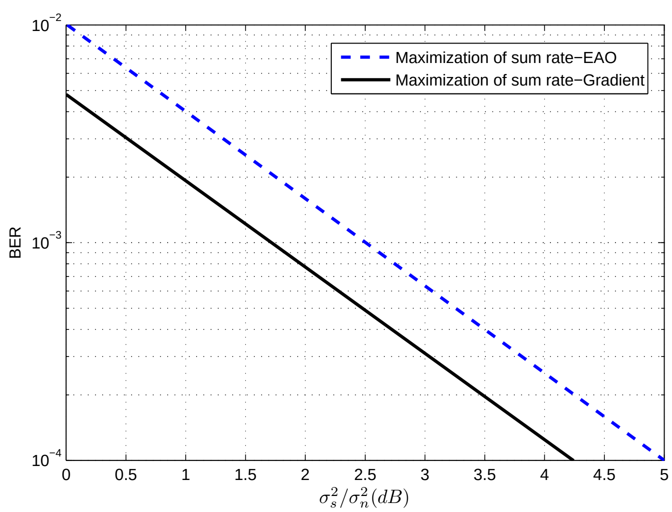

Figure 3.7: The BER performance comparison of gradient method and proposed EAO, with sum rate objective function, for $K = 3$ and $G(0, 1)$ channels.

Chapter 4

Beamforming for MIMO Cognitive Radio with Single Primary and Multiple Secondary Users

4.1 Introduction

In the previous chapters, the beamforming designs for a K user MIMO-IC system with various objective functions were discussed in detail for flat and frequency-selective channels. In chapter 2 and 3 all of the users are treated equally. However, it is possible to consider priority for one user over the others. In this chapter, we deal with a prioritized K-user MIMO-IC channel system. A cognitive radio system with one primary and $K-1$ secondary users can be considered as one application. Unlike the majority of the existing cognitive works, which control and maintain the power of secondary transmission powers below a tolerable threshold, here beamformers strive to eliminate the interference induced on a primary user from secondary users. The main advantage of our work is having lower feedback rates compared to other known methods, because only partial channel knowledge is required. Its disadvantage is that the formulated problem is non-convex.

A cognitive radio system with a primary user with multiple secondary users is considered in this chapter. Beamforming for such a system is presented which seeks to create an interference-free environment for the primary user. The objective is to maximize the signal-to-interference plus noise ratio (SINR) for the primary user through the transmit beamformers of all users and the receive beamformer of the primary user. This is called the maximum achievable SINR here. Finding the maximum achievable SINR corresponds to constrained maximization of the largest eigenvalue of a Hermitian positive semidefinite matrix. This problem is not a convex optimization; however, the upper bound corresponding to the interference-free case, is known so evolutionary algorithms can be used. For the maximum achievable SINR, the secondary users do not have beamformers at their multi-antenna receivers but instead use quasi-maximum-likelihood detection based on semidefinite relaxation (SDR). The prioritized weighted mean-square-error (PWMSE) is also presented in this chapter. The bit error rate of the primary user in maximum achievable SINR is better than that of PWMSE. Another advantage of the maximum achievable SINR approach is its lower feedback rate compared to PWMSE because it requires partial channel knowledge.

4.2 Background and our contribution

There have been significant design developments in communications over the MIMO interference channel, and also over cognitive radio networks, for creating better spectrum utilization. In the multiuser MIMO interference channel, the goal is to transmit independent data streams between the transmitting terminal and the receiver terminal of K users. The transmitters and receivers share the same frequency band and time slot [63], i.e., the spectrum is shared simultaneously and this is enabled by collaborative beamforming of multiple antennas at the transmitters and receivers. In such MIMO interference channels, no priority is set for the various users, i.e., there is no primary user. However, in a cognitive radio network, the frequency spectrum is often assigned to a primary user - typically the owner of the spectrum [64] - and secondary users scavenge radio resource without significantly affecting the primary user. These techniques are different but both are trying to achieve better spectral efficiency. Both techniques require collaboration between primary and secondary users, and in particular, the burden is on the primary user to enable and manage the secondary users' access to the spectral resource. In this sense, "cognitive radio" here simply means the same as "collaborative networks", etc.

The contribution of this chapter addresses the combination of these techniques, i.e., the design of a priority-based MIMO interference channel system, which has not been previously treated. A priority-based MIMO interference channel can be viewed as a MIMO cognitive radio network. An application example would be in cellular-type networks where the primary user is the uplink from mobiles to a base station within licensed spectrum, and the secondary users are scavenged down-link transmissions from base stations (BSs) of other networks [65]. With current cellular systems, it would be easier to share the spectrum non-simultaneoulsy, for example by using different time slots or frequency channels, but it is emphasized that the characteristic of interest here is that the spectrum sharing is simultaneous.

Simultaneous spectrum sharing can only be realized through beamforming. The design of such beamformers requires some form of optimization. Because the primary user has priority, its maximum SINR can be the cost function along with some form of simultaneous quality of service (QoS) provision for the secondary users. This QoS provision in needed to ensure that the secondary users get a chance to scavenge the spectral resource. For this approach, power allocation management of the secondary users (which is the focus - and the basis of the cost functions - of most optimized cognitive radio network designs) is not required. Instead of power allocation for secondary users, here joint beamforming is deployed at the primary user's receiver and all users' transmitters.

In previous works on simultaneous spectrum-sharing cognitive radio, beamformers minimize the transmit power of secondary users to suppress interference to the primary user to an acceptable level, while maintaining a threshhold SINR for the secondary users e.g., [66]. For the general MIMO cognitive radio case, this optimization problem has been simplified by using the alternating minimization algorithm, and then solved via second-order cone programming [66]. A special case was introduced in [64] where a secondary user seeks to share the spectrum with multiple primary users. In [67], transmit beamforming for a single MIMO secondary user and a single MIMO primary user was discussed. Similar cost functions and constraints have been presented in [68] for MISO scenarios.

An issue for transceiver beamforming design in cognitive radio networks is the required amount of channel knowledge of the links. Intuitively, full channel knowledge (all links) will provide better results compared to having just partial channel knowledge. A design was developed in [66] for robustness against uncertainty in the primary link knowledge. The cases when the secondary transmitter has: complete; partial; or no knowledge of the channels which connect to the primary receivers have been treated in [64]. In our approach, each receiver (secondary or primary) need only know its own channels, so this is a form of the partial channel knowledge case. The feedback rate for our beamforming design is comparable to the other partial channel knowledge case [64] but still provides the best possible performance for a primary user.

In this chapter, we show that the maximum SINR for a primary user leads to a non-convex problem. Specifically, the problem is a constrained maximization of the largest eigenvalue of a Hermitian positive semidefinite matrix. Because this optimization problem has a known upper bound, then with negligible degradation from the interference-free case, all the transmit beamformers and the receiver beamformer of the primary user can be calculated by an evolutionary algorithm.

It is also known that efficient implementation of quasi-maximum likelihood multiuser detection based on semidefinite relaxation (SDR) has near maximum likelihood (ML) performance [69, 60]. We therefore apply SDR detection for the secondary users [61] with maximum SINR design. Also, the prioritized weighted mean-square-error (PWMSE) is addressed here. The performance of maximum SINR for the primary user is compared with PWMSE, and the feedback rates are also formulated and compared.

The notation is conventional, as follows: column vectors and matrices are denoted by boldface lower and upper case letters, respectively. Superscripts ^{T} and $^\mathcal{H}$ stand for transpose and complex conjugate transpose, respectively. The largest eigenvalue of $\mathbf{A}$ denoted by $\lambda_{\max}(\mathbf{A})$. The $\mathbf{A} \succeq 0$ indicates that $\mathbf{A}$ is positive semidefinite. $\mathbf{v}_{\max}(\mathbf{A})$ is the eigenvector of $\mathbf{A}$ that corresponds to the $\lambda_{\max}(\mathbf{A})$. The rank of a $\mathbf{A}$ shown by $\text{rank}(\mathbf{A})$ is the number of linearly independent rows or columns of $\mathbf{A}$. $\mathrm{tr}(\mathbf{A})$ denotes the trace of $\mathbf{A}$. The Euclidean norm of a vector $\mathbf{a}$ is denoted as $\|\mathbf{a}\|$.

4.3 System Model and Problem Formulation and Its Solution

4.3.1 Maximum SINR for Primary User

The model is a MIMO cognitive radio system where both a primary user and secondary users have N_t transmit antennas and N_r receive antennas. The respective transmit weight vectors are denoted by $\mathbf{v}_i \in \mathbb{C}^{N_t \times 1}$. The primary user receive beamformer is denoted by $\mathbf{u}_1$. The SDR decoder is used for secondary users.

The data symbol of user i is denoted by s_i which is assumed to be a random variable with zero mean and variance $\mathbb{E}\{|s_i|^2\} = P_i$. The output of the transmit beamformer is $\mathbf{v}_i s_i$, where $\|\mathbf{v}_i\| = 1$, so that the beamformer does not scale the transmit power.

The baseband model of flat channel from the μth transmit antenna of the jth transmitter to the νth receive antenna of the ith user is denoted by $h^{ij}_{\nu\mu}$, where $\nu \in \{1, ..., N_r\}$, $\mu \in \{1, ..., N_t\}$ and $j, i \in \{1, ..., K\}$. Define $\mathbf{H}_{ij} \in \mathbb{C}^{N_r \times N_t}$ with the (ν, μ) entry $[\mathbf{H}_{ij}]_{\nu,\mu} = h^{ij}_{\nu\mu}$. The beamformers $\mathbf{v}_i$ and $\mathbf{u}_1$ are directed at maximizing the SINR of the primary user. That is, we seek the receiver beamformer $\mathbf{u}_1 \in \mathbb{C}^{N_r \times 1}$, and transmit beamformers $\mathbf{v}_i$ such that they maximize SINR on the primary link.

Finding the maximum SINR_1 can be represented as the following optimization problem:

$$\begin{aligned} \underset{\mathbf{u}_1, \{\mathbf{v}_i\}_1^K}{\text{maximize}} \quad & \frac{\mathbf{u}_1^{\mathcal{H}} \mathbf{H}_{11} \mathbf{v}_1 \mathbf{v}_1^{\mathcal{H}} \mathbf{H}_{11}^{\mathcal{H}} \mathbf{u}_1}{\mathbf{u}_1^{\mathcal{H}} \left(\sum_{j=2}^{K} \mathbf{H}_{1j} \mathbf{v}_j \mathbf{v}_j^{\mathcal{H}} \mathbf{H}_{1j}^{\mathcal{H}} + \sigma_n^2 / P \mathbf{I}_{N_r} \right) \mathbf{u}_1} \\ \text{subject to} \quad & \|\mathbf{u}_1\| = 1, \|\mathbf{v}_i\| = 1. \end{aligned} \tag{4.1}$$

The optimum solution of (4.1) is

$$\mathbf{u}_1^{\mathrm{opt}} = \mathbf{A}^{-1} \mathbf{H}_{11} \mathbf{v}_1^{\mathrm{opt}} / \|\mathbf{A}^{-1} \mathbf{H}_{11} \mathbf{v}_1^{\mathrm{opt}}\| \tag{4.2}$$

$$\mathbf{v}_1^{\mathrm{opt}} = \mathbf{v}_{\max}(\mathbf{H}_{11}^{\mathcal{H}} \mathbf{A}^{-1} \mathbf{H}_{11}) \tag{4.3}$$

For maximum SINR_1, the $\{\mathbf{v}_i\}_2^K$ is obtained from

$$\begin{aligned} \underset{\{\mathbf{v}_i\}_2^K}{\text{maximize}} \quad & \lambda_{\max}(\mathbf{H}_{11}^{\mathcal{H}} \mathbf{A}^{-1} \mathbf{H}_{11}) \\ \text{subject to} \quad & \|\{\mathbf{v}_i\}_2^K\| = 1. \end{aligned} \tag{4.4}$$

where $\mathbf{A} \triangleq \left(\sum_{j=2}^{K} \mathbf{H}_{1j} \mathbf{v}_j \mathbf{v}_j^{\mathcal{H}} \mathbf{H}_{1j}^{\mathcal{H}} + \sigma_n^2 / P \mathbf{I}_{N_r} \right)$.

Therefore, the new optimization problem (4.4) should be solved.

The rest of this section explains the maximization of largest eigenvalue of a Hermitian positive semidefinite matrix. Optimization problem (4.4) is non-convex. Even if it is modeled as a unconstrained problem, the partial derivative of $\lambda_{\max}(\mathbf{H}_{11}^{\mathcal{H}}\mathbf{A}^{-1}\mathbf{H}_{11})$ with respect to $\{\mathbf{v}_i\}_2^K$ is difficult. However, the $\lambda_{\max}(\mathbf{H}_{11}^{\mathcal{H}}(\sigma_n^2/P\mathbf{I}_{N_r})^{-1}\mathbf{H}_{11})$ is an upper bound for SINR_1. This upper bound is obtained by putting $\{\mathbf{v}_i\}_2^K = \mathbf{0}$ in matrix $\mathbf{A}$. Therefore, we attempt to minimize the term $\lambda_{\max}(\mathbf{H}_{11}^{\mathcal{H}}(\sigma_n^2/P\mathbf{I}_{N_r})^{-1}\mathbf{H}_{11})$-$\lambda_{\max}(\mathbf{H}_{11}^{\mathcal{H}}\mathbf{A}^{-1}\mathbf{H}_{11})$. Define the normalized beamformers, viz., $\mathbf{v}_{i+1} \triangleq \frac{\mathbf{x}_i}{\|\mathbf{x}_i\|}$ for $i = 1, ..., K-1$. The maximum SINR problem is formulated in (4.5).

$$\underset{\mathbf{x}_1,\ldots,\mathbf{x}_{K-1}}{\text{minimize}} \quad \lambda_{\max}\left(\mathbf{H}_{11}^{\mathcal{H}}(\sigma_n^2/P\mathbf{I}_{N_r})^{-1}\mathbf{H}_{11}\right) - \lambda_{\max}\left(\mathbf{H}_{11}^{\mathcal{H}}\Big(\sum_{i=1}^{K-1}\mathbf{H}_{1i+1}\frac{\mathbf{x}_i\mathbf{x}_i^{\mathcal{H}}}{\|\mathbf{x}_i\|^2}\mathbf{H}_{1i+1}^{\mathcal{H}} + \sigma_n^2/P\mathbf{I}_{N_r}\Big)^{-1}\mathbf{H}_{11}\right) \tag{4.5}$$

The unconstrained optimization problem (4.5) is efficiently solved by the Genetic Algorithm (GA). Interestingly, the GA is capable of finding the solution faster, on average, for $N_t = N_r > N_u$ than for $N_t = N_r = N_u$, and the reasoning for this is in Appendix H. We also show numerically that a smaller number of generations for the GA is required for a system with $N_u = 3, N_t = N_r = 4$ compared to a system with $N_u = N_t = N_r = 3$. However, a trade-off between the number of iterations and computational burden at each iteration is inevitable.

Figure 4.1 represents the number of required generations for GA to obtain a solution for the optimization problem (4.5) over 300 independent trials for these two cases. The graphs indicate that a higher number of antennas relative to the number of total users allows a smaller number of generations for the GA, and the required number of generations is more predictable for the first case in the sense that there is monotonic decreasing behavior in its histogram (upper graph, figure (4.1)).

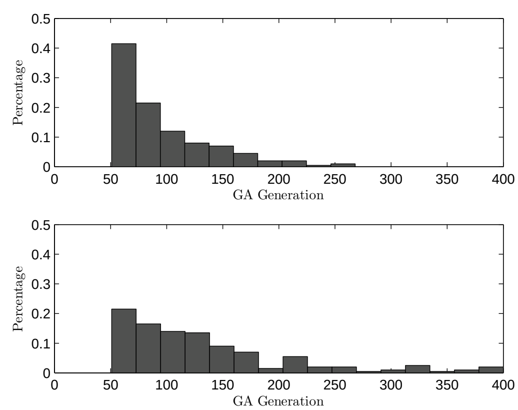

Figure 4.1: Average number of Generations for GA with $N_u = 3, N_t = N_r = 4$ (upper), $N_u = N_t = N_r = 3$ (lower) to solve optimization problem (4.5).

4.3.2 Prioritized Weighted Mean Square Error (PWMSE)

In the previous subsection SDR decodes the data for secondary users. Here, we have Tx and Rx beamformers for all users. Let $\mathbf{r}_i \triangleq \mathbf{u}_i^{\mathcal{H}}$, then $\hat{s}_i = \mathbf{r}_i \mathbf{y}_i$. The WMSE is defined:

$$\text{WMSE} = \sum_{i=1}^{K} w_i \text{MSE}_i \tag{4.6}$$

where $\text{MSE}_i = \mathbb{E}\{\|\hat{s}_i - s_i\|^2\}$. The WMSE problem is:

$$\begin{aligned} \min_{\mathbf{r}_i, \mathbf{v}_i} \quad & \sum_{i=1}^{K} w_i \text{MSE}_i \\ \text{s.t.} \quad & \text{tr}(\mathbf{v}_i \mathbf{v}_i^{\mathcal{H}}) \leq 1. \end{aligned} \tag{4.7}$$

Following the procedure in [1] and using an auxiliary variable t, the WMSE minimization by AO is summarized:

1. Choose $N > 1$, $\mathbf{v}_i^{(1)}$ arbitrarily for $i = 1, ..., K$

2. for $n = 1 : N$

$$\mathbf{r}_i^{(n+1)} = \mathbf{v}_i^{(n)\mathcal{H}} \mathbf{H}_{ii}^{\mathcal{H}} (\sum_{j=1}^{K} \mathbf{H}_{ij} \mathbf{v}_j^{(n)} \mathbf{v}_j^{(n)\mathcal{H}} \mathbf{H}_{ij}^{\mathcal{H}} + \sigma_n^2 \mathbf{I}_M)^{-1}$$

$$\begin{aligned} \min_{\mathbf{v}_i^{(n)}} \quad & t \\ \text{s.t.} \quad & \left\| \begin{matrix} \delta \\ \mathbf{D}\left([\mathbf{I}_K \otimes (\mathbf{R}^{(n+1)}\mathbf{H})]\text{vec}(\mathbf{V}^{(n)}) - \text{vec}(\mathbf{I}_K)\right) \end{matrix} \right\| \leq t, \\ & \|\mathbf{v}_i^{(n)}\| \leq 1. \end{aligned} \tag{4.8}$$

end

where

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_{11} & \cdots & \mathbf{H}_{1K} \\ \mathbf{H}_{21} & \cdots & \mathbf{H}_{2K} \\ \vdots & \ddots & \vdots \\ \mathbf{H}_{K1} & \cdots & \mathbf{H}_{KK} \end{bmatrix}, \mathbf{V}^{(n)} = \begin{bmatrix} \mathbf{v}_1^{(n)} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{v}_K^{(n)} \end{bmatrix},$$

$$\mathbf{R}^{(n+1)} = \begin{bmatrix} \mathbf{r}_1^{(n+1)} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \mathbf{r}_K^{(n+1)} \end{bmatrix}$$

$\delta = \sigma_n \sqrt{\text{tr}(\mathbf{W}\mathbf{R}^{(n+1)}\mathbf{R}^{(n+1)\mathcal{H}})}$, $\mathbf{W} = \text{diag}(w_1, \cdots, w_K)$ and $\mathbf{D} = \mathbf{I}_K \otimes \mathbf{W}^{1/2}$.

The WMSE becomes PWMSE for the first user (i.e. primary user) if $w_2 = \cdots = w_K = 1$ and $w_1 \gg 1$.

4.4 Feedback Rates and Complexity of Proposed Designs

The approach of maximizing SINR requires a lower feedback rate than through using PWMSE approach. For the two systems presented here, the data that must be exchanged between nodes is as follows. For the maximum SINR approach, the K Tx-BF must be sent from the primary receiver node to all the transmitter nodes, and $K(K-1)$ TX-BF should be fedback from the primary receiver to all the secondary receiver nodes. Each beamformer is a complex vector with K elements ($N_r = N_t = K$), and if each complex number is quantized to $2q$ bits, then a total of $2K^3q$ feedback bits is needed. For the PWMSE approach, $K(K-1)$ channels, K TX-BF and $K-1$ RX-BF should be fedback; therefore a total of $2(K^4 - K^3 + 2K^2 - K)q + (K-1)q$ feedback bits is needed. Assume $N_r = N_t = M$. The Complexity of maximum SINR is $O(GPKM^3)$ where P is GA population size and complexity of PWMSE is $O(INM^2K^6)$ where I is the number of interior-point-method (IPM) iterations.

In summary, the maximum SINR has lower feedback rate and is simpler than PWMSE.

4.5 Simulation

For this section, we evaluate communications performance using QPSK modulation, $P_i = P = 1$ and $K = N_t = N_r$. The channels are the i.i.d. Rayleigh flat fading. The simulations are statistical only, in that the beamforming is taken to mean the signal processing operation of vector multiplication. This paper does not address the spatial aspects of the beamforming, i.e., no spatial beamforming (for example, using spaced antennas) is undertaken in the simulations. In terms of the propagation environment, the instantaneous propagation scenarios are assumed to be appropriately ideal at all terminals, and the terminal's antennas are assumed to be appropriately configured with respect to their propagations scenarios, so that the beamforming will work properly.

For $K = 3$, the bit error rate (BER) is depicted in figure 4.3 from using *MaxAchieveSINR* and *PWMSE*. The initial values of the transmitter beamformer $\mathbf{v}_i^{(0)} = \sqrt{P_i}\mathbf{v}_{\max}(\mathbf{H}_{ii}^{\mathcal{H}}\mathbf{H}_{ii})$ for *PWMSE*. The number of iterations, N, for *PWMSE* is 16. It can be seen that BER for primary user with *MaxAchieveSINR* is lower than *PWMSE*. However, the BER for secodary users with *PWMSE* outperforms the *MaxAchieveSINR*.

From numerical results, we conclude that using *MaxAchieveSINR* allows a primary user to work like an interference-free environment, i.e. $\mathbf{v}_i = \mathbf{0}$ for $i = 2, ..., K$ and when $\mathbf{v}_1$ and $\mathbf{u}_1$ are designed optimally.

4.6 Conclusions

Two priority-based beamforming designs for the MIMO interference channel, referred to here as *MaxAchieveSINR* and *PWMSE*, are presented. They have applications to cognitive radio networks. The *MaxAchieveSINR* beamforming formulation leads to a non-convex problem, but because the upper bound is known the solutions can be obtained by an evolutionary algorithm. We showed that the EA converges faster with having more number because there are more candidate solutions for EA. Simulations indicate that there is negligible performance degradation compared to the interference-free case. An alternative approach is a prioritized weighted mean square error (*PWMSE*). But the *MaxAchieveSINR* method is shown to outperform *PWMSE*. Moreover, the *MaxAchieveSINR* requires a lower feedback rate than *PWMSE*.

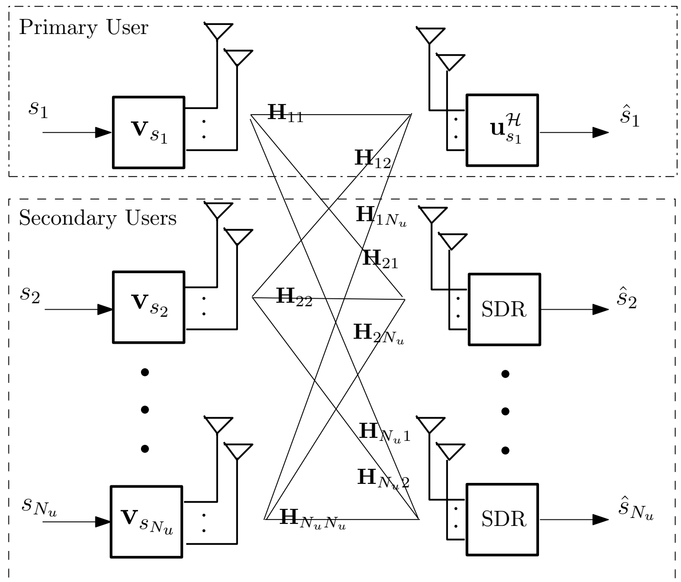

Figure 4.2: Priority-based MIMO interference channel when maximum SINR is intended for a single primary user.

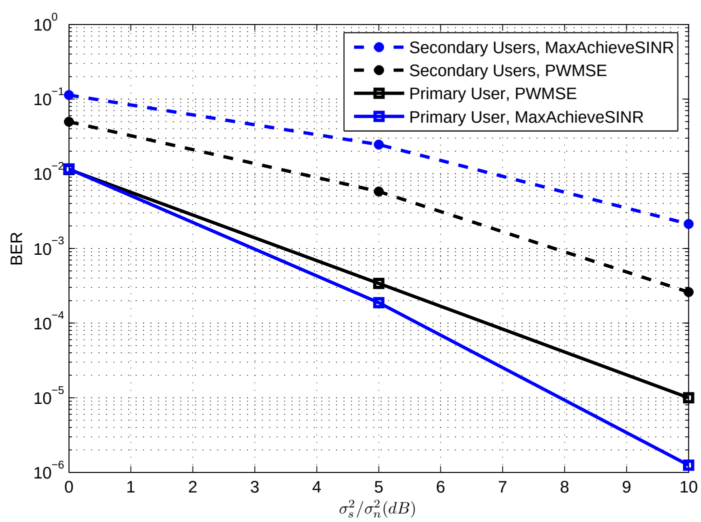

Figure 4.3: The BER comparison of two methods *MaxAchieveSINR* and *PWMSE* for $K = 3$.

Chapter 5

Beamforming and Relay Selection in MIMO Cognitive Radio Networks

5.1 Introduction

As noted in Chapter 4, cognitive radio is a technique to improve the spectral efficiency of wireless communication systems in a dynamic and opportunistic fashion. It is motivated by the observation that many networks appear to be inefficient users of the radio spectrum because for some proportional of time, the spectrum is relatively quiet. The basic idea is that instead of having another user join the incumbent system, that user can instead become an opportunistic spectrum user and scavenge some spectral resource during a time when the spectrum appears to be relatively quiet, or even unused. However, it is challenging to realize significantly better spectral usage as a feasible economic proposition using uncoordinated scavenging, or even coordinated scavenging. This is because the opportunistic user must ensure that it does not degrade the primary users' access or capacity, and this requires that its transmissions have to be coordinated with and by the primary user. In a general case, such action must: either degrade the primary users' service (capacity); or require major modifications to the primary users' hardware and software; or both. The question arises not just about the economic feasibility of the concept, but also whether the spectral usage has been improved or degraded. A basic problem here is that in order to make the spectral efficiency better, more spectral resource is required to set up and maintain the working system. In short, on one hand we are usually optimizing capacity, but on the other hand, most of the assumptions make unconstrained use of the capacity which is not part of the optimization. This type of problem was already touched on in the introductions in Chapter 2 and Chapter 3. But in particular for cognitive radio research, the question seems to linger: if the primary user is set up as a multi-user system, why introduce a scavenging spectral user that requires cooperation with all the primary users? The answer may be found for specific, specialized situations, but for general situations, this question does not seem to be well-addressed in the cognitive radio literature, and it is not taken further here, except in the discussion of the assumptions below.

This chapter uses current research ideas in cognitive radio architectures to showcase the beamforming techniques developed in the earlier chapters, along with other communications techniques, by formulating the cognitive radio management as an optimization problem. In order to make even basic progress, major assumptions are required. But this type of approach is the current state-of-the-art in cognitive radio research. Below, the system and its assumptions are made explicit so that the context of the ensuing mathematical treatment is easier to follow.

In a cognitive network, *primary users* have a license to use some part of the radio spectrum, while some *secondary users* (or cognitive users) are dynamically identifying and exploiting the spectral resources not used by primary users. Note that the term "users" is different to that of the earlier chapters. Cognitive radio paradigms have been classified in three types: underlay, overlay, and interweave [70]. The underlay type is considered in this chapter. Underlay is claimed in [70] to have a high spectral efficiency and be more practical, although such claims do not appear to be verifiable in general. Here, secondary users strive to access the same radio spectrum allocated to primary users, provided that the interference from secondary users is less than some specified limit. This limit is often expressed as an "interference temperature", a term that hints (but is seldom investigated to verify) that the interference is assumed to be noise-like for signal detection considerations. Clearly, in this kind of system, the interference control would play an essential role, and in general this would need cooperation between the primary and secondary user.

Cooperation for a secondary user involves at least three terminals. Therefore, a three-terminal network is a fundamental configuration in user-cooperation. It is obvious that deployment of cooperating multiple relays can increase the diversity order of any system, albeit at the hardware and power expense of deploying the relays and introducing a capacity-consuming protocol to run them. Multi-relay cooperation would require a time division multiplexing access (TDMA) transmission scheme, which acts to decrease the spectral efficiency of the system because extra capacity resource (extra time slots) is required by a protocol in order to deploy the relay combination algorithm. In the system described here, the relay diversity combination scheme is selection. Relay selection still requires regular usage of capacity for channel sounding and information interchange, in order to regularly deploy the selection action. The channels are taken as block-independent, so the selection must be implemented for each block (see below). Once the relay is selected, then for the duration of that selection there is a classical source-relay-destination situation, and two time-slots are used in the usual way [71] to relay a symbol or packet from the source to the destination via the relay.

The source, relay and destination all have multiple antennas for beamforming, and each is capable of multiple, independent data streams (eigen-MIMO). The goal is to maximize the capacity of the single secondary user, while its transmission power from the source and relay transmit nodes, and its interference to all the primary receivers, are all constrained. The beamformer design approach proposed here is to simplify the capacity optimization problem, and to deploy the eigen-MIMO.

There are several assumptions in the system model. Some of these are purely mathematical and are treated in the mathematics description below. But the system assumptions required to progress to a tractable problem are major as noted above. In any event, it is important to note that the presented relay selection algorithm and capacity maximization formulation are valid only with these assumptions. The system model is taken from [6], and is it is modified here by adding the multiple, beamforming relays.

The basic system is depicted in Figure 5.1. There are multiple, primary users. A single, narrow bandwidth spectrum allocation allows all the beamformers to comprise element weights that are single complex numbers. In a cellular-like structure, the primary users (shown in the figure) could be mobile terminals, each talking to a single basestation (also in the figure), in the cell. The interaction of the basestation and primary users with other basestations, and vice versa, is not considered in this model. (In practice this could be undertaken using wired connection between the basestations and a different frequency for the mobile users.) In each cell, the primary users are working in multi-user MIMO broadcast scheme for basically half of the time, the basestation is in transmit (downlink), and the mobiles are receiving, and the other half of the time, vice versa.

Only during downlink phase of primary network, the secondary user undertakes its relay transfer. At all other times (the uplink) the entire secondary user, with all its relays, is dormant. Only simplex secondary user transmission is considered in the formulation below. Duplex secondary transmission would require further protocol support and consume more capacity overhead for the extra channel sounding and related information interchange, but otherwise could use the same approach as for the simplex case. The protocol is not part of the beamforming design, and it is not addressed here.

The assumptions for the system are as follows:

- During the secondary users' transmissions' (source to relay, and relay to source), the resulting instantaneous (i.e., for each channel realization) interference at the primary receivers is assumed to be below a predefined tolerance such that the primary network still works well if this constraint is not violated. (It is not addressed here as to how this is arranged but clearly the system must consume extra capacity to achieve it. It also requires the primary users to collaborate with the secondary user, and this consumes further primary capacity.)

- The channels of the secondary links are assumed to be all known at all of the primary users' receivers, and all this channel information is in turn assumed to be supplied back to the secondary user source and relay. (Again, this means that extra capacity resource must be

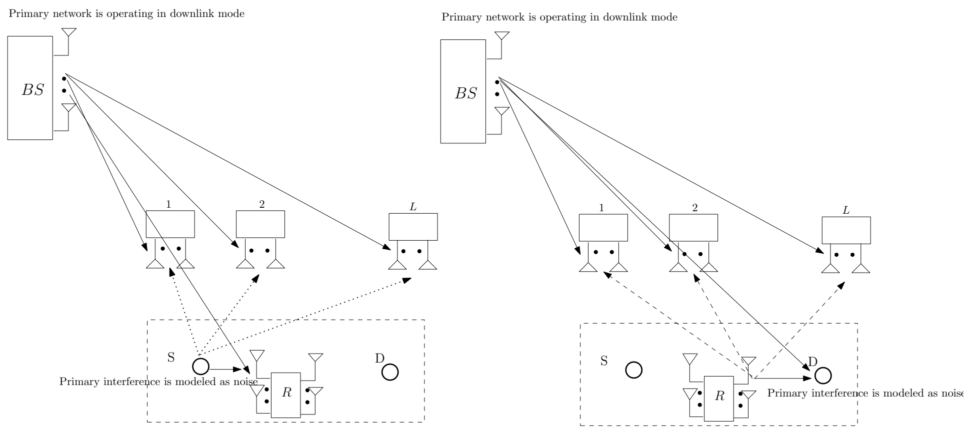

Figure 5.1: The primary network is a multi-user MIMO broadcast scheme. Within the primary downlink phase, the relaying uses two time slots to move information firstly from the source to the relay, (left) and then from the relay to the destination (right). Following [6], all channel gains are assumed to be independent flat Rayleigh, and there is an assumption that the channel gains from the secondary transmissions (both source and all relays) are 10dB (an arbitrary choice) lower than the channel gains of the primary users. The interference from the primary to the secondary is treated as noise.

consumed in order to sound all of the MIMO channels perfectly, and then interchange the channel information perfectly.)

- The channel gains from the secondary source and the relay transmissions are assumed to be 10dB less-averaged over the Rayleigh fading across the multiple antennas-compared to the primary users' channels. (This dB difference is an arbitrary choice, adopted from the original configuration in [6], where it was stated without elaboration.)

- The interference from the primary users' transmissions (i.e. from the base station) to all antennas at the relay and the destination are considered to be modelled as additive noise. This means that no beamforming resources are required at the relay and destination receivers, to selectively suppress the primary users' signals.

- The gains between all the antennas at the secondary source and those at the destination are negligible, i.e., the secondary user's terminals are assumed to not be able to "see" each other. This is not a condition for the systems' operation because the destination is not trying to listen to the relay when the source is transmitting. This is more the motivating situation for creating the relay solution of this chapter.

- All channels are flat Rayleigh, i.e. no shadow fading is included.

A way to view the situation of the penultimate bullet point is as follows. An existing communication system has the licensed spectrum and uses the space with multiple primary users. A secondary user would like to link in the same space, using the same spectrum, at the same time that the primary users are in operation. But for some reason, the secondary user cannot be considered as an extra primary user. The transmit and received terminals of the secondary user have no direct communication, i.e. they cannot "see" each other. What is needed in this communications scenario to allow the secondary user to operate? The answer is that relays are required, and moreover that cooperation with the primary users is also required. This remainder of this chapter lays out this solution. But before proceeding, let us consider the cost of the system that is being considered. Compared with the secondary user simply joining the system as an extra primary user, the cost of this solution looks to be high, including the following factors:

- The original primary users' hardware and its protocol must be replaced with a primary system that can support a highly collaborative protocol with the single secondary user.
- The primary user must also accept a penalty in its capacity because it must now allocate capacity to allow the channels to be sounded perfectly, and allow for perfect interchange of all the channel information. This penalty is essentially doubled if duplex operation of the secondary user is desired.
- Multiple relays must be added to the system.
- The single secondary user must know the number of the primary users in the cell, and the primary users must all know that there is a secondary user present.

With this interpretation of a cognitive radio system understood, it is now used as a vehicle to showcase the relay and antenna selection algorithm and optimization tools developed in this thesis. As a final note before focusing on the cognitive radio model, it is added that the presented optimization approach would also be applicable for a general multiple relay problem. Here, one terminal talks to the other through multiple relays, and the twist is that the interference from both the source and all the relays to a set of targets, is constrained.

5.2 system model

In this chapter, we consider a CR network with L primary receivers PR_l, for $l = 1, \ldots, L$, where each primary receiver is equipped with N_{p_l} antennas. We also assume a secondary cooperative network where an N_s antenna source node (S), communicates with a destination terminal (D), equipped with $N_d \geq N_s$ antennas. In this setup, $K \geq N_s$ relays R_k, $k = 1, \ldots K$, each equipped with N_{r_k} antennas are ready to assist the communication between source and destination (Fig. 5.2). We assume channel reciprocity for all the links. Furthermore, we assume the primary and the secondary networks share the same bandwidth for transmission, i.e., the primary and secondary transmission is affected by mutual co-channel interference. For the first part of this chapter, we also assume that all the source channels to the relays are known at the source terminal and that the relays to destination channels are known at the relay nodes. The source to primary receivers channels should be known at the source node. The participant relays to primary receivers channels should also be known at the corresponding relay nodes. Under these assumptions, we are able to select relays and design spatial spectrum and power allocation schemes in order to simultaneously maximize the transmission rate and minimize the imposed interference on the primary receivers. Practically, source and relays to primary receiver channels can be obtained by e.g. periodically sensing the transmitted signal from the primary receiver at the source and relay nodes respectively, if the time-division-duplexing (TDD) signaling is utilized by the primary transmission[6]. In the case where perfect CSI can not be obtained, the results in this chapter introduce upper-bounds for the secondary user network throughput.

Throughout this work we focus on the secondary network transmission and we consider half-duplex AF communication between source, relays and destination, in two non-overlapping time slots. In the first time slot the source terminal transmits data vector $\mathbf{x}_s \in \mathbb{C}^{N_s}$ which is formed by multiplexing data stream vector, $\mathbf{s} \in \mathbb{C}^{N_s}$, via N_s transmit beamforming vectors $\mathbf{w}_{s_n} \in \mathbb{C}^{N_s \times 1}$ for $n = 1 \ldots, N_s$ such that

$$\begin{aligned} \mathbf{x}_s &= \sum_{n=1}^{N_s} \mathbf{w}_{s_n} s_n \\ &= \mathbf{W}_s \mathbf{s}. \end{aligned}$$

where s_n is the n^{th} data stream and

$$\mathbf{W}_s = \left[\begin{array}{cccc} \mathbf{w}_{s_1} & \mathbf{w}_{s_2} & \ldots & \mathbf{w}_{s_{N_s}} \end{array} \right].$$

Assuming $K \geq N_s$ the source terminal sends the multiplexed data vector $\mathbf{x}_s$ to at most N_s relays such that a relay may be assigned more than one stream, one stream, or no streams at all. Define the set of selected relays by $\mathcal{R} = \{r_1, r_2, \ldots r_{N_s}\}$, where two or more members of the set may refer to the same relay. Then in the first time slot, the received signal by the k^{th} relay in $\mathcal{R}$ is given by

$$\mathbf{y}_{r_k} = \mathbf{H}_{sr_k} \mathbf{x}_s + \mathbf{n}_k \tag{5.1}$$

$$= \mathbf{H}_{sr_k} \mathbf{w}_{s_k} s_k + \underbrace{\mathbf{H}_{sr_k} \sum_{n=1, n \neq k}^{N_s} \mathbf{w}_{s_n} s_n}_{\text{Interference between secondary data streams}} + \mathbf{n}_k \tag{5.2}$$

where $\mathbf{H}_{sr_k} \in \mathbb{C}^{N_{r_k} \times N_s}$ is the $S \rightarrow r_k$ channel with independent and identically distributed (i.i.d) zero mean unit variance complex Gaussian elements. $\mathbf{n}_k \sim \mathcal{CN}(\underline{0}, \mathbf{I}_{N_{r_k}}) \in \mathbb{C}^{N_{r_k} \times 1}$ is the thermal noise at the k^{th} selected relay. The relay $r_k \in \mathcal{R}$ amplifies the signal $\mathbf{y}_{r_k}$ received from the source using the weight matrix $\mathbf{W}_{r_k} \in \mathbb{C}^{N_{r_k} \times N_{r_k}}$ and forwards the amplified signal towards the destination

$\mathcal{D}$ in the second time slot. The received signal via all the relays in $\mathcal{R}$ is then given by

$$\mathbf{y}_d = \sum_{n=1}^{N_s} \mathbf{H}_{r_k d} \mathbf{W}_{r_k} \mathbf{y}_{r_k} + \mathbf{n}_d \tag{5.3}$$

where $\mathbf{H}_{r_k d} \in \mathbb{C}^{N_d \times N_{r_k}}$ is the $r_k \to D$ channel with i.i.d zero mean unit variance complex Gaussian elements. $\mathbf{n}_d \sim \mathcal{CN}(\underline{0}, \mathbf{I}_{N_d}) \in \mathbb{C}^{N_d \times 1}$ is the thermal noise at the destination. Finally at the destination terminal, by incorporating the receive beamforming matrix $\mathbf{W}_d \in \mathbb{C}^{N_s \times N_d}$ the soft decision vector $\hat{\mathbf{s}}$, is obtained as

$$\hat{\mathbf{s}} = \mathbf{W}_d \mathbf{y}_d. \tag{5.4}$$

All the channels are assumed to be quasi-static block fading where each channel is drawn randomly at the start of each transmission clock pulse and remains constant for the whole transmission cycle. Channels from block to block are also assumed to be independently varying.

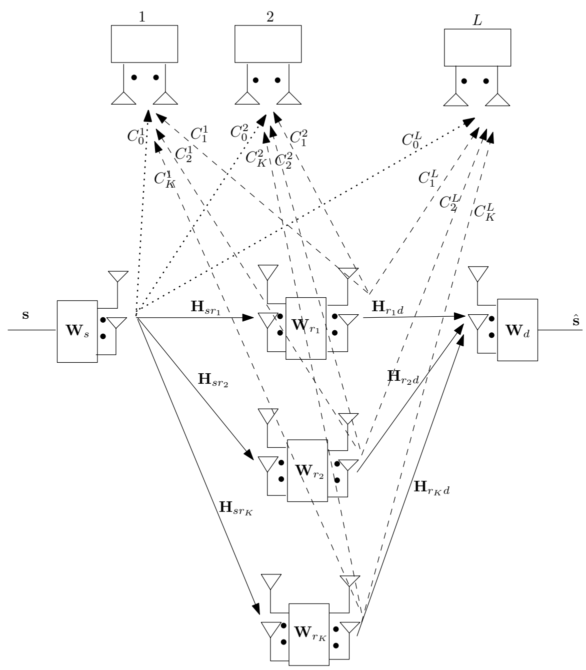

Figure 5.2: Beamforming and relay selection where cognitive radio network shares the same spectrum with L primary users.

5.2.1 Power Constraints and the CR Interference

In this section, we study the transmitted power limitations in the secondary network and tolerable interference thresholds by the primary receivers.

In the first time slot, the source terminal transmits the multiplexed vector $\mathbf{x}_s$ such that

$$\mathrm{tr}\{\mathrm{E}\{\mathbf{x}_s\mathbf{x}_s^H\}\} = \mathrm{tr}\{\mathbf{W}_s\mathbf{\Sigma_s}\mathbf{W}_s^H\} \leq P_s$$

where $\mathbf{\Sigma_s} = \mathrm{diag}(\sigma_1, \ldots, \sigma_{N_s})$ is the spatial spectrum of $\mathbf{s}$. On the other hand in the second time slot there is a power limitation of P_r such that

$$\sum_{k=1}^{N_s} \mathrm{tr}\{\mathrm{E}\{\mathbf{W}_{r_k}\mathbf{y}_{r_k}\mathbf{y}_{r_k}^H\mathbf{W}_{r_k}^H\}\} \leq P_r \tag{5.5}$$

The expectation is over the signalling set, not the channels and associated antennas weights. Assuming a power budget of P_{r_k} for each relay we may write

$$\mathrm{tr}\{\mathrm{E}\{\mathbf{W}_{r_k}\mathbf{y}_{r_k}\mathbf{y}_{r_k}^H\mathbf{W}_{r_k}^H\}\} \leq P_{r_k} \tag{5.6}$$

where $\sum_k P_{r_k} = P_r$. It can be shown that (5.6) is a special case of (5.5). However, by choosing proper quantities for P_{r_k}, (5.5) is also satisfied. Thus, in the rest of this chapter we use the individual power constraint as given in (5.6).

The secondary network is allowed to impose a limited amount of interference on the primary receiver(s). Dividing the secondary transmission into two time slots, during the first time slot the source transmission power should satisfy

$$\mathrm{tr}\{\mathbf{C}_0^l\mathbf{W}_s\mathbf{\Sigma_s}\mathbf{W}_s^H\mathbf{C}_0^{l^H}\} \leq \gamma_l$$

where $\mathbf{C}_0^l = \left[\ \mathbf{c}_{0,1}^l, \ldots, \mathbf{c}_{0,N_{p_l}}^l\ \right]^H \sim \mathcal{CN}(\underline{0}, \mathbf{I}_{N_{p_l}}) \in \mathbb{C}^{N_{p_l}\times N_s}$ is the channel from the source to the l^{th} primary receiver, and $\mathbf{c}_{0,k}^{l\ H}$ is the channel from the source to the k-th antenna of the l-th primary receiver. γ_l is the maximum tolerable amount of interference at the l^{th} primary receiver port. We do not elaborate as to what is "tolerable" here. But it is taken to mean that the resulting decrease in SINR at the primary receivers does not noticeably change their error performance. In the second time slot, data transmission of each relay imposes interference on the primary receivers. Therefore for the second time slot of secondary transmission we have

$$\sum_{k=1}^{N_s} \mathrm{tr}\{\mathrm{E}\{\mathbf{C}_k^l\mathbf{W}_{r_k}\mathbf{y}_{r_k}\mathbf{y}_{r_k}^H\mathbf{W}_{r_k}^H\mathbf{C}_k^{l\ H}\}\} \leq \gamma_l$$

where $\mathbf{C}_k^l \sim \mathcal{CN}(\underline{0}, \mathbf{I}_{N_{p_l}}) \in \mathbb{C}^{N_{p_l}\times N_{r_k}}$ is the channel from the k^{th} relay in $\mathcal{R}$ to the l^{th} primary receiver.

5.2.2 Capacity and problem formulation

Relying on equations (5.1) to (5.3), by defining

$$\begin{aligned}
\mathbf{H}_{SR} &\triangleq \left[\begin{array}{cccc} \mathbf{H}_{sr_1}^T & \mathbf{H}_{sr_2}^T & \ldots & \mathbf{H}_{sr_{N_s}}^T \end{array}\right]^T \\
\mathbf{H}_{RD} &\triangleq \left[\begin{array}{cccc} \mathbf{H}_{r_1 d} & \mathbf{H}_{r_2 d} & \ldots & \mathbf{H}_{r_{N_s} d} \end{array}\right] \\
\mathbf{n}_R &\triangleq \left[\begin{array}{cccc} \mathbf{n}_1^T & \mathbf{n}_2^T & \ldots & \mathbf{n}_{N_s}^T \end{array}\right]^T \\
\mathbf{W}_R &\triangleq \operatorname{diag}\left(\mathbf{W}_{r_1}, \mathbf{W}_{r_2}, \ldots, \mathbf{W}_{r_{N_s}}\right)
\end{aligned} \tag{5.7}$$

the input-output relation in the secondary network can be written as

$$\mathbf{y}_d = \underbrace{\mathbf{H}_{RD}\mathbf{W}_R\mathbf{H}_{SR}\mathbf{W}_s}_{\mathbf{H}}\mathbf{s} + \underbrace{\mathbf{H}_{RD}\mathbf{W}_R\mathbf{n}_R + \mathbf{n}_d}_{\mathbf{z}_d}. \tag{5.8}$$

Define $\mathbf{R}_n \triangleq \left(\mathbf{H}_{RD}\mathbf{W}_R\mathbf{W}_R^H\mathbf{H}_{RD}^H + \mathbf{I}_{N_d}\right)$, then the information-theoretic capacity computed from $\mathcal{I}(\mathbf{s}, \mathbf{y}_d)$, where $\mathcal{I}(a, b)$ is the mutual information between a and b, is:

$$\begin{aligned}
\mathcal{C} &= \frac{1}{2}\log_2 \det\left(\mathbf{I} + \mathbf{H}\boldsymbol{\Sigma}_\mathbf{s}\mathbf{H}^H\mathbf{R}_n^{-1}\right) \\
&= \frac{1}{2}\log_2 \det\left(\mathbf{I} + {\boldsymbol{\Sigma}_\mathbf{s}}^{\frac{1}{2}}\mathbf{H}^H\mathbf{R}_n^{-1}\mathbf{H}{\boldsymbol{\Sigma}_\mathbf{s}}^{\frac{1}{2}}\right).
\end{aligned} \tag{5.9}$$

The objective is to maximize the capacity subject to power and interference constraints, i.e., we define problem (**P.1**) as:

$$\underset{\boldsymbol{\Sigma}_\mathbf{s}, \mathbf{W}_{r_k}}{\text{maximize}} \ \log_2 \det\left(\mathbf{I} + {\boldsymbol{\Sigma}_\mathbf{s}}^{\frac{1}{2}}\mathbf{H}^H\mathbf{R}_n^{-1}\mathbf{H}{\boldsymbol{\Sigma}_\mathbf{s}}^{\frac{1}{2}}\right) \tag{5.10}$$

subject to:

$$\operatorname{tr}\{\mathbf{W}_s\boldsymbol{\Sigma}_\mathbf{s}\mathbf{W}_s^H\} \leq P_s, \tag{5.11}$$

$$\sum_{k=1}^{N_s}\operatorname{tr}\{\mathrm{E}\{\mathbf{W}_{r_k}\mathbf{y}_{r_k}\mathbf{y}_{r_k}^H\mathbf{W}_{r_k}^H\}\} \leq P_r, \tag{5.12}$$

for $\quad l = 1, \ldots, L:$

$$\operatorname{tr}\{\mathbf{C}_0^l\mathbf{W}_s\boldsymbol{\Sigma}_\mathbf{s}\mathbf{W}_s^H{\mathbf{C}_0^l}^H\} \leq \gamma_l, \tag{5.13}$$

$$\sum_{k=1}^{N_s}\operatorname{tr}\{\mathrm{E}\{\mathbf{C}_k^l\mathbf{W}_{r_k}\mathbf{y}_{r_k}\mathbf{y}_{r_k}^H\mathbf{W}_{r_k}^H{\mathbf{C}_k^l}^H\}\} \leq \gamma_l, \tag{5.14}$$

$$\mathbf{W}_s\boldsymbol{\Sigma}_\mathbf{s}\mathbf{W}_s^H \geq \underline{0}. \tag{5.15}$$

The above optimization problem strives to maximize the secondary user's capacity subject to the power and interference constraints in equations (5.11) to (5.15). The last constraint is due to the positive-definiteness of the source transmit spatial spectrum, $\boldsymbol{\Sigma}_\mathbf{s} \succeq \mathbf{0}$. The decision variables for optimization problem (**P.1**) are the $\boldsymbol{\Sigma}_\mathbf{s}$ and $\mathbf{W}_{r_k}$ matrices. It is emphasized that the matrix $\mathbf{W}_s$ in this optimization is not the decision variable. However the design ensures, as will be shown below, such that the inter data stream interference term in (5.2) becomes zero. Solving **P.1** for general $\mathbf{W}_{r_k}$ is not straightforward, but if it is assumed to be rank one then this problem can be transfomed to a simpler optimization problem. Finally, the received precoder at secondary destination, $\mathbf{W}_d$, is designed such that the secondary user's capacity (objcetive function of **P.1**) remains the same, before and after applying $\mathbf{W}_d$.

The formulation in **P.1**, for its objective (capacity) and its constraints, are derived for an assumed known single channel realization. The capacity for the secondary user is meaningful if **P.1** is evaluated for several channel realizations. If $\mathcal{C}_i^*$ is the objective function evaluation for ith channel realization, the average capacity for secondary users over I channel realizations is $(1/I)\sum_{i=1}^{I}\mathcal{C}_i^*$.

In the following sections, a relay and antennas selection algorithm will be proposed. Transforming the hard problem **P.1** to a simple optimization problem is also described. Below is a big picture description of design procedures.
1-Optimization problem **P.1**:

- Objective: Secondary user capacity maximization
- Constraints: Keep interference to all of the primary users at tolerable level, and also maintain the transmitted power at the source and relays in the budget
- Decision variable: $\mathbf{W}_{r_k}$ and $\mathbf{\Sigma_s}$
- Decision variable simplification: $\mathbf{W}_{r_k}$ can be a general matrix but to make **P.1** simple, it is assumed to be a rank one matrix. The $\mathbf{\Sigma_s}$ is considered as a diagonal matrix which means the streams are independent data

2-Relay and antenna selection:

- Relay and antennas selection are deployed to avoid ill-conditioning on the **P.1** constraints and losing information while providing the best capacity performance. The $\mathbf{W}_s$ and $\mathbf{W}_d$ beamformers are used to determine the best relay or the best set of relays with appropriate antennas.
- $\mathbf{W}_s$ is designed for simplified **P.1**, i.e. $\text{rank}(\mathbf{W}_{r_k}) = 1$. Assume $\mathbf{W}_{r_k}$ has this form, $\mathbf{W}_{r_k} = \omega_k \mathbf{a}_k \mathbf{b}_k^H$. The $\mathbf{W}_s$ and $\mathbf{b}_k$ eliminate the inter-data-stream interference at relay nodes. The $\mathbf{W}_s$ adjusts the relay transmission power while considering ill-conditioning. There are multiple solutions for $\mathbf{W}_s$
- $\mathbf{W}_d$ is designed to meet sufficient statistics for $\mathbf{s}$ and $\mathbf{y}_d$, i.e. $\mathcal{I}(\mathbf{s}, \mathbf{y}_d) = \mathcal{I}(\mathbf{s}, \mathbf{W}_d\mathbf{y}_d)$ while $\text{rank}(\mathbf{W}_{r_k}) = 1$. The $\mathbf{W}_d$ and $\mathbf{a}_k$ cancel inter-data-stream interference at destination node. There are multiple solutions for $\mathbf{W}_d$

- From those $\mathbf{W}_s$ and $\mathbf{W}_d$ candidates the one that provides the best capacity performance determines the relay and antennas selection
- The relay and antennas selection algorithm may select one relay or several relays with subsets of their antennas

5.3 Selection Beamforming One Single antenna Primary Receiver

5.3.1 Direct semi-orthogonal selection beamforming (DS-SVSB)

In this section, we discuss the proposed selection-beamforming algorithm based on the following facts. In [72], the authors show that for a single relay MIMO cooperative scenario, the optimal solution for the problem of finding single transmit and receive beamforming vectors e.g., $\mathbf{w}_s$ and $\mathbf{w}_d$ and the corresponding precoder matrix $\mathbf{W}_r$, at the relay, is matching and that the optimal $\mathbf{W}_r$ is a rank one matrix, i.e., $\mathbf{W}_r = \omega \mathbf{a}\mathbf{b}^H$, where $\mathbf{a}, \mathbf{b} \in \mathbb{C}^{N_{r_k} \times 1}$ are normalized vectors that have to be found by matching (see figure 5.3). We propose a suboptimal solution assuming that $\mathbf{W}_{r_k}$ is a rank one matrix and therefore, $\mathbf{W}_{r_k} = \omega_k \mathbf{a}_k \mathbf{b}_k^H$. The rest of the idea is that the interference from non-assigned streams to a particular relay is nulled out. In other words, assuming that $r_k \in \mathcal{R}$, equation (5.2) is deployed where we design beamforming vectors that to mitigate the interference term $\mathbf{b}_k^H \mathbf{H}_{sr_k} \sum_{n=1, n\neq k}^{N_s} \mathbf{w}_{s_n} s_n$. To target this goal, we should have

$$\mathbf{h}_{sr_k}^H \mathbf{w}_{s_n} = \delta_{nk} \tag{5.16}$$

where δ_{nk} is the Kronecker Delta function and

$$\mathbf{h}_{sr_k} \triangleq \mathbf{H}_{sr_k}^H \mathbf{b}_k \in \mathbb{C}^{N_s \times 1}. \tag{5.17}$$

Let $\{r_1, \ldots, r_{N_s}\} \in \mathcal{R}$, and

$$\mathbf{H}_{sr}^e \triangleq \left[\mathbf{h}_{sr_1}, \ldots, \mathbf{h}_{sr_{N_s}}\right]^H \in \mathbb{C}^{N_s \times N_s}, \tag{5.18}$$

one trivial choice of $\mathbf{W}_s$ to satisfy (5.16) is the pseudo inverse of $\mathbf{H}_{sr}^e$,

$$\begin{aligned} \mathbf{W}_s &= \mathbf{H}_{sr}^{e\,\dagger} \\ &= \mathbf{H}_{sr}^{e\,H}\left(\mathbf{H}_{sr}^e \mathbf{H}_{sr}^{e\,H}\right)^{-1}. \end{aligned} \tag{5.19}$$

It must be noted that $\mathbf{b}_n^H \mathbf{H}_{sr_k} \mathbf{W}_s = \delta_{nk} \mathbf{e}_k^T$ where $\mathbf{e}_k$ is the standard unit vector. This method is called zero-forcing beamforming which is extensively discussed in [73, 74, 75]. The received signal at the k^{th} relay after passing through the relay beamformer $\mathbf{b}_k$ is given by

$$\begin{aligned} x_{r_k} &= \mathbf{b}_k^H \mathbf{y}_{r_k} \\ &= s_k + n'_{r_k} \end{aligned} \tag{5.20}$$

where $n'_{r_k} \sim \mathcal{CN}(0,1) = \mathbf{b}_k^H \mathbf{n}_{r_k}$. The first hop of our secondary MIMO cooperative system can

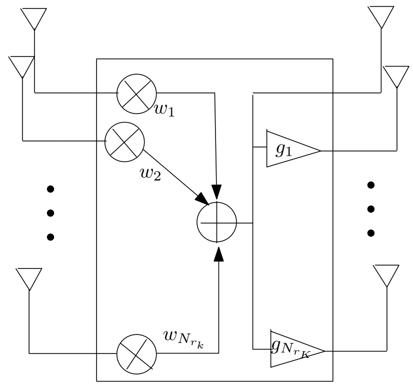

Figure 5.3: The implementation of rank one beamformer at relays

be modeled as a MIMO broadcast channel (MIMO-BC) with N_s users where at the same time the second hop can be modeled as a MIMO multiple access channel (MAC)[75].

For the second hop we desire to design the receive beamforming vectors such that multipath inter-symbol-interference (ISI) term $\mathbf{w}_{d_n}^T \sum_{k=1,k\neq n}^{N_s} \omega_k \mathbf{H}_{r_k d} \mathbf{a}_k x_{r_k}$ is eliminated at the destination terminal. To fulfil this aim, we should have

$$\mathbf{w}_{d_n}^T \mathbf{h}_{r_k d} = \delta_{nk} \tag{5.21}$$

where

$$\mathbf{h}_{r_k d} \triangleq \mathbf{H}_{r_k d} \mathbf{a}_k \in \mathbb{C}^{N_d \times 1}. \tag{5.22}$$

Defining

$$\mathbf{H}_{rd}^e \triangleq \left[\mathbf{h}_{r_1 d}, \ldots, \mathbf{h}_{r_{N_s} d}\right] \in \mathbb{C}^{N_d \times N_s}, \tag{5.23}$$

A simple choice of $\mathbf{W}_d$ to meet (5.21) is the psuedo inverse of $\mathbf{H}_{rd}^e$,

$$\begin{aligned} \mathbf{W}_d &= {\mathbf{H}_{rd}^e}^\dagger \\ &= {\mathbf{H}_{rd}^e}^H \left(\mathbf{H}_{rd}^e {\mathbf{H}_{rd}^e}^H\right)^{-1}. \end{aligned} \tag{5.24}$$

A duality as reported by [73], is obvious between the MIMO-BC hop and the MIMO-MAC hop. It is vital to notice that $\mathbf{W}_d \mathbf{H}_{r_k d} \mathbf{a}_k = \mathbf{e}_k \in \mathbb{C}^{N_s \times 1}$ and therefore,

$$\mathbf{W}_d \mathbf{y}_d = \mathbf{\Omega}\left(\mathbf{s} + \mathbf{n}'_r\right) + \mathbf{n}'_d \tag{5.25}$$

where $\mathbf{\Omega} = \text{diag}\left(\omega_1, \ldots, \omega_{N_s}\right)$, $\mathbf{n}'_r \sim \mathcal{CN}(\underline{0}, \mathbf{I}) = [n'_{r_1}, \ldots, n'_{r_{N_s}}]^T$ and $\mathbf{n}'_d \sim \mathcal{CN}(\underline{0}, \mathbf{W}_d \mathbf{W}_d^H) = \mathbf{W}_d \mathbf{n}_d$. From (5.25) we can easily find $\mathcal{I}(\mathbf{W}_d \mathbf{y}_d, \mathbf{s})$ which is the capacity of the system as

$$\begin{aligned} \mathcal{C}' &= \frac{1}{2} \log_2 \det \left(\mathbf{I} + \mathbf{\Omega} \mathbf{\Sigma_s} \mathbf{\Omega}^H \left(\mathbf{\Omega}\mathbf{\Omega}^H + \mathbf{W}_d \mathbf{W}_d^H\right)^{-1}\right) \\ &\overset{(a)}{\leq} \frac{1}{2} \sum_{k=1}^{N_s} \log_2 \left(1 + \frac{\omega_k^2 \sigma_k}{\omega_k^2 + |w_{d_{k,k}}|^2}\right). \end{aligned} \tag{5.26}$$

Table 5.1: Table I: Direct semi-orthogonal selection beamforming (DS-SVSB) Algorithm

- Set of all modes for backward channel: $\mathcal{M}_b = \{(b_1, \ldots, b_{N_s}) | b_i = \{1, .., \min(N_{r_k}, N_s)\}\}$
- Set of all modes for forward channel: $\mathcal{M}_f = \{(f_1, \ldots, f_{N_s}) | f_i = \{1, .., \min(N_{r_k}, N_d)\}\}$
- Set of all N_s relays: $\mathcal{R} = \{(r_1, \ldots, r_{N_s}) | (r_1, \ldots, r_{N_s}) \in \left(\!\binom{\mathcal{S}}{N_s}\!\right), \mathcal{S} = \{1, .., K\}\}$ $\quad |\mathcal{R}| = C(K + N_s - 1, N_s) = (K + N_s - 1)!/(K - 1)!N_s!$
- Select a subset of forward modes and subset of relays to have $\mathbf{W}_d \mathbf{W}_d^{\mathcal{H}}$ diagonal

$$\mathcal{F}_1 = \left\{ \begin{bmatrix} (r_1, \ldots, r_{N_s}) \\ (f_1, \ldots, f_{N_s}) \end{bmatrix} \Big| (r_1, \ldots, r_{N_s}) \in \mathcal{R}, (f_1, \ldots, f_{N_s}) \in \mathcal{M}_f, \|[\mathbf{u}_{r_1 d}(f_1) \ldots \mathbf{u}_{r_{N_s} d}(f_{N_s})]^H [\mathbf{u}_{r_1 d}(f_1) \ldots \mathbf{u}_{r_{N_s} d}(f_{N_s})] - \mathbf{I}\|_F < \alpha \right\}$$

$$\mathcal{F}_{1r} = \left\{ (r_1, \ldots, r_{N_s}) \Big| \begin{bmatrix} (r_1, \ldots, r_{N_s}) \\ (f_1, \ldots, f_{N_s}) \end{bmatrix} \in \mathcal{F}_1 \right\}$$

$$\mathcal{F}_{1m} = \left\{ (f_1, \ldots, f_{N_s}) \Big| \begin{bmatrix} (r_1, \ldots, r_{N_s}) \\ (f_1, \ldots, f_{N_s}) \end{bmatrix} \in \mathcal{F}_1 \right\}$$

- Select a subset of backward modes and a new subset of relays from $\mathcal{F}_{1r}$ to have $\mathbf{W}_s \mathbf{W}_s^{\mathcal{H}}$ roughly diagonal

$$\mathcal{B}_1 = \left\{ \begin{bmatrix} (r_1, \ldots, r_{N_s}) \\ (b_1, \ldots, b_{N_s}) \end{bmatrix} \Big| (r_1, \ldots, r_{N_s}) \in \mathcal{F}_{1r}, (b_1, \ldots, b_{N_s}) \in \mathcal{M}_b, \|[\mathbf{v}_{sr_1}(b_1) \ldots \mathbf{v}_{sr_{N_s}}(b_{N_s})]^H [\mathbf{v}_{sr_1}(b_1) \ldots \mathbf{v}_{sr_{N_s}}(b_{N_s})] - \mathbf{I}\|_F < \beta \right\}$$

- Select a new subset of backward modes and subset of relays from $\mathcal{B}_1$ to have backward eignevalues above a threshold

$$\mathcal{B}_2 = \left\{ \begin{bmatrix} (r_1, \ldots, r_{N_s}) \\ (b_1, \ldots, b_{N_s}) \end{bmatrix} \Big| \begin{bmatrix} (r_1, \ldots, r_{N_s}) \\ (b_1, \ldots, b_{N_s}) \end{bmatrix} \in \mathcal{B}_1, [\lambda_{sr_1}(b_1) \ldots \lambda_{sr_{N_s}}(b_{N_s})] \geq \underline{\lambda}_{\text{th}} \right\}$$

$$\mathcal{B}_{2r} = \left\{ (r_1, \ldots, r_{N_s}) \Big| \begin{bmatrix} (r_1, \ldots, r_{N_s}) \\ (b_1, \ldots, b_{N_s}) \end{bmatrix} \in \mathcal{B}_2 \right\}$$

- Select a new subset of forward modes and subset of relays from $\mathcal{B}_2$ to have forward eignevalues above a threshold

$$\mathcal{F}_2 = \left\{ \begin{bmatrix} (r_1, \ldots, r_{N_s}) \\ (f_1, \ldots, f_{N_s}) \end{bmatrix} \Big| (r_1, \ldots, r_{N_s}) \in \mathcal{B}_{2r}, (f_1, \ldots, f_{N_s}) \in \mathcal{F}_{1m}, [\lambda_{r_1 d}(f_1) \ldots \lambda_{r_{N_s} d}(f_{N_s})] \geq \underline{\lambda}_{\text{th}} \right\}$$

- The candidate relays and candidate forward and backward mode is defined as

$$\mathcal{R}^s = \left\{ (r_1, \ldots, r_{N_s}) \Big| \begin{bmatrix} (r_1, \ldots, r_{N_s}) \\ (f_1, \ldots, f_{N_s}) \end{bmatrix} \in \mathcal{F}_2 \right\}$$

$$\mathcal{M}_f^s = \left\{ (f_1, \ldots, f_{N_s}) \Big| \begin{bmatrix} (r_1, \ldots, r_{N_s}) \\ (f_1, \ldots, f_{N_s}) \end{bmatrix} \in \mathcal{F}_2 \right\}$$

$$\mathcal{M}_b^s = \left\{ (b_1, \ldots, b_{N_s}) \Big| (r_1, \ldots, r_{N_s}) \in \mathcal{R}^s, \begin{bmatrix} (r_1, \ldots, r_{N_s}) \\ (b_1, \ldots, b_{N_s}) \end{bmatrix} \in \mathcal{B}_2 \right\}$$

- Choose an element of the sets $\mathcal{R}^s$, $\mathcal{M}_f^s$ and $\mathcal{M}_b^s$ to have maximum capacity

for $i = 1 : |\mathcal{R}^s|$

$(r_1, \ldots, r_{N_s}) = \{\mathcal{R}^s\}_i$

$(f_1, \ldots, f_{N_s}) = \{\mathcal{M}_f^s\}_i$

$(b_1, \ldots, b_{N_s}) = \{\mathcal{M}_b^s\}_i$

$$\mathbf{H}_{rd}^{e,i} = \left[\sqrt{\lambda_{r_1 d}(f_1)} \mathbf{u}_{r_1 d}(f_1) \cdots \sqrt{\lambda_{r_{N_s} d}(f_{N_s})} \mathbf{u}_{r_{N_s} d}(f_{N_s}) \right]^H$$

$$\mathbf{H}_{sr}^{e,i} = \left[\sqrt{\lambda_{sr_1}(b_1)} \mathbf{v}_{sr_1}(b_1) \cdots \sqrt{\lambda_{sr_{N_s}}(b_{N_s})} \mathbf{v}_{sr_{N_s}}(b_{N_s}) \right]$$

end

$i^* = \text{argmax}_i \mathcal{C}$

where (a) is by Hadamard inequality which states that the determinant of any positive definite matrix $\mathbf{K}$ is less than or equal to the product of its diagonal elements, i.e.,

$$|\mathbf{K}| \leq \prod_i k_{i,i}$$

with equality iff $\mathbf{K}$ is diagonal. In (5.26) all the terms are diagonal except $\mathbf{W}_d \mathbf{W}_d^H$.

Lemma: $\mathcal{C}'$ and $\mathcal{C}$ are equal.

Proof. From $\mathbf{b}_k^H \mathbf{H}_{sr_k} \mathbf{W}_s = \mathbf{e}_k^T$ and $\mathbf{W}_d \mathbf{H}_{r_k d} \mathbf{a}_k = \mathbf{e}_k$, then $\mathbf{y}_d = \mathbf{H}\mathbf{s} + \mathbf{H}\mathbf{n}_r' + \mathbf{n}_d$. In another representation, $\mathcal{C} = \frac{1}{2} \log_2 \det \left(\mathbf{I} + \mathbf{H}^H (\mathbf{H}\mathbf{H}^H + \mathbf{I})^{-1} \mathbf{H} \boldsymbol{\Sigma}_\mathbf{s} \right)$ and $\mathcal{C}' = \frac{1}{2} \log_2 \det \left(\mathbf{I} + \boldsymbol{\Omega} (\boldsymbol{\Omega}^2 + \mathbf{W}_d \mathbf{W}_d^H)^{-1} \boldsymbol{\Omega} \boldsymbol{\Sigma}_\mathbf{s} \right)$. Insert $\mathbf{W}_d = \boldsymbol{\Omega} (\mathbf{H}^H \mathbf{H})^{-1} \mathbf{H}^H$ (because $\mathbf{W}_d \mathbf{H} = \boldsymbol{\Omega}$) in to $\mathcal{C}'$, then after some matrix manipulations $\mathcal{C}' = \frac{1}{2} \log_2 \det \left(\mathbf{I} + \left((\mathbf{H}^H \mathbf{H})^{-1} + \mathbf{I} \right)^{-1} \boldsymbol{\Sigma}_\mathbf{s} \right)$. By applying the Woodbury matrix inversion, $\left((\mathbf{H}^H \mathbf{H})^{-1} + \mathbf{I} \right)^{-1} = \mathbf{H}^H (\mathbf{H}\mathbf{H}^H + \mathbf{I})^{-1} \mathbf{H}$. Therefore, $\mathcal{C}' = \mathcal{C}$. □

Hence, the upper bound in the capacity ($\mathcal{C}$ or $\mathcal{C}'$) is achieved iff $\mathbf{W}_d \mathbf{W}_d^H$ is diagonal, i.e., the columns of $\mathbf{W}_d$ and subsequently $\mathbf{H}^e_{rd}$ are orthogonal. On the other hand we have

$$\mathrm{tr}\{\mathbf{W}_s\mathbf{\Sigma_s}\mathbf{W}_s^H\} = \sum_k \gamma_{e_k}^{-1}\sigma_k \leq P_s \tag{5.27}$$

where $\gamma_{e_k} = 1/\|\mathbf{w}_{s_k}\|^2 = 1/\left[\left(\mathbf{H}^e_{sr}\mathbf{H}^{e\ H}_{sr}\right)^{-1}\right]_{k,k}$ is called the effective $\mathcal{S} \to r_k$ channel gain. In the typical water-filling problem, the power is allocated to the eigen channels according to the strength of their respective effective channel. When $\mathbf{H}^e_{sr}$ is poorly conditioned, the γ_{e_k} is greatly reduced and thus the transmitter does not allocate power to its corresponding eigen channel. This is undesirable since a channel has been selected but has not been used by the transmitter. To overcome this problem, considering a large number of relays, the secondary transmitter can almost surely choose a group of N_s relays as orthogonal as possible to each other. In this way, inverting the channel $\mathbf{H}^e_{sr}$ becomes merely a rotation operation, and there is no loss of channel gains[73]. We select N_s relays from K candidates such that, $\mathbf{H}^e_{sr}$ and $\mathbf{H}^e_{rd}$ have respective orthogonal columns. Subsequently, to get $\mathbf{H}^e_{sr}$ and $\mathbf{H}^e_{rd}$ be orthogonal, it is sufficient to find $\mathbf{a}_k$s and $\mathbf{b}_k$s such that $\mathbf{a}_k\mathbf{a}_i^H = \delta_{k,i}\mathbf{I}_{N_{r_k}}$ and $\mathbf{b}_k\mathbf{b}_i^H = \delta_{k,i}\mathbf{I}_{N_{r_k}}$. Let the singular value decomposition (SVD) of $\mathbf{H}_{sR_k}$ and $\mathbf{H}_{R_kd}$ be $\mathbf{H}_{sR_k} = \mathbf{U}_{sR_k}\Lambda^{1/2}_{sR_k}\mathbf{V}^H_{sR_k}$ and $\mathbf{H}_{R_kd} = \mathbf{U}_{R_kd}\Lambda^{1/2}_{R_kd}\mathbf{V}^H_{R_kd}$. Although $\mathbf{a}_k$s and $\mathbf{b}_k$s can be any normal vectors. To give more insight to the problem, one easy choice is to set $\mathbf{b}_k = \mathbf{u}_{sr_k}(b_k)$ and $\mathbf{a}_k = \mathbf{v}_{r_kd}(f_k)$ where $\mathbf{u}_{sr_k}(b_k)$ and $\mathbf{v}_{r_kd}(f_k)$ are the b_k-th right eigenvector and f_k-th left eigenvector of $\mathbf{H}_{sr_k}$ and $\mathbf{H}_{r_kd}$, respectively. Then

$$\begin{aligned}\mathbf{H}^e_{sr} &= \left[\sqrt{\lambda_{sr_1}(b_1)}\mathbf{v}_{sr_1}(b_1), \ \ldots, \ \sqrt{\lambda_{sr_{N_s}}(b_{N_s})}\mathbf{v}_{sr_{N_s}}(b_{N_s})\right]^H \\ \mathbf{H}^e_{rd} &= \left[\sqrt{\lambda_{r_1d}(f_1)}\mathbf{u}_{r_1d}(f_1), \ \ldots, \ \sqrt{\lambda_{r_{N_s}d}(a_{N_s})}\mathbf{u}_{r_{N_s}d}(a_{N_s})\right]\end{aligned} \tag{5.28}$$

and $\mathbf{W}_s$ and $\mathbf{W}_d$ can be found from (5.19) and (5.24), respectively. Now our aim is to propose an algorithm such that

$$\mathbf{v}(i)\mathbf{v}(j)^H \simeq \delta_{i,j}\mathbf{I} \quad \text{and} \quad \mathbf{u}(i)\mathbf{u}(j)^H \simeq \delta_{i,j}\mathbf{I}.$$

To start an algorithm for relay and antenna selection, we define a set of modes containing indices which indicate the index of the selected relay, index of the backward eigen mode and the index of the forward eigen mode. The algorithm is given in Table. I. It can be verified easily that the sets $\{\mathbf{q}_1(b_1), \ldots, \mathbf{q}_{N_s}(b_{N_s})\}$ and $\{\mathbf{p}_1(f_1), \ldots, \mathbf{p}_{N_s}(f_{N_s})\}$ are individually orthogonal via the Gram-Schmidt orthogonalization procedure. On the other hand by choosing a proper α from step IV it can be deduced that only those backward and forward eigen modes that are respectively semi-orthogonal to $\mathbf{q}_n(b_n)$ and $\mathbf{p}_n(f_n)$ are chosen in the next search domain. In this way it can be seen that $\mathbf{v}_{R_n}(b_n) \simeq \mathbf{q}_n(b_n)$ and $\mathbf{u}_{R_n}(f_n) \simeq \mathbf{p}_n(f_n)$. In this way the sets $\{\mathbf{a}_1, \ldots, \mathbf{a}_{N_s}\}$ and $\{\mathbf{b}_1, \ldots, \mathbf{b}_{N_s}\}$ are respectively semi-orthogonal sets.

Let $\mathbf{c}_0^H \in \mathbb{C}^{1\times N_s}$ be the channel from the secondary source to the primary receiver and $\mathbf{c}_k^H \in \mathbb{C}^{1\times N_{r_k}}$ be the channel from the r_k to the primary receiver, then the interference imposed on the primary receivers from the secondary source and relays are given by

$$\mathbf{c}_0^H \mathbf{W}_s \mathbf{\Sigma}_s \mathbf{W}_s^H \mathbf{c}_0 \leq \gamma_1 \tag{5.29}$$

$$\sum_{k=1}^{N_s} \mathrm{E}\{\mathbf{c}_k^H \mathbf{W}_{r_k} \mathbf{y}_{r_k} \mathbf{y}_{r_k}^H \mathbf{W}_{r_k}^H \mathbf{c}_k\} \leq \gamma_1. \tag{5.30}$$

Furthermore, let $\mathbf{z} = \begin{bmatrix} z_1^H, \ldots, z_{N_s}^H \end{bmatrix}^H \triangleq \mathbf{W}_s^H \mathbf{c}_0 \in \mathbb{C}^{N_s\times 1}$ and $e_k \triangleq \mathbf{v}_{r_k d}(f_k)^H \mathbf{c}_k$, and now by choosing the obtained suboptimal values for $\mathbf{W}_s$, $\mathbf{W}_{r_k}$ and $\mathbf{W}_d$, and using (5.26) in (5.9), **P.1** is converted to problem **P.2**:

$$\underset{\sigma_k,\omega_k}{\text{maximize}} \sum_{k=1}^{N_s} \log_2 \left(1 + \frac{\omega_k^2 \sigma_k}{\omega_k^2 + |w_{d_{k,k}}|^2}\right)$$

subject to:

$$\sum_{k=1}^{N_s} \|\mathbf{w}_{s_k}\|^2 \sigma_k \leq P_s, \tag{5.31}$$

$$\sum_{k=1}^{N_s} \omega_k^2 (1+\sigma_k) \leq P_r, \tag{5.32}$$

$$\sum_{k=1}^{N_s} |z_k|^2 \sigma_k \leq \gamma_l, \quad l = 1, ..., L \tag{5.33}$$

$$\sum_{k=1}^{N_s} |e_k|^2 \omega_k^2 (1+\sigma_k) \leq \gamma_l, \quad l = 1, ..., L \tag{5.34}$$

$$\sigma_k \geq 0. \quad \forall\, k \tag{5.35}$$

It is evident that this optimization problem is non-convex. To solve it, alternating optimization (AO) can be deployed. Define $x_k \triangleq \omega_k^2$ and $y_k \triangleq \sigma_k$. Assume x_k for $k = 1, ..., N_s$ are all known.

Therefore, the first problem of (**P.2**) is simplified as:

$$\begin{aligned}\mathcal{P}_1 : \max_{\{y_k\}} \quad & \sum_{k=1}^{N_s} \log_2(1+s_k y_k) \\ \text{s.t.} \quad & \sum_{k=1}^{N_s} f_k y_k \leq P_s \\ & \sum_{k=1}^{N_s} g_k(1+y_k) \leq P_r \\ & \sum_{k=1}^{N_s} h_k y_k \leq \gamma_1 \\ & \sum_{k=1}^{N_s} l_k(1+y_k) \leq \gamma_1 \\ & y_k \geq 0\end{aligned} \tag{5.36}$$

where $s_k \triangleq x_k/(x_k+a_k)$, $a_k \triangleq |w_{d_{k,k}}|^2$, $g_k \triangleq x_k$, $f_k \triangleq \|\mathbf{w}_{s_k}\|^2$, $h_k \triangleq |z_k|^2$ and $l_k \triangleq |e_k|^2 x_k$.

Secondly, assume y_k for $k = 1, ..., N_s$ are all known. Then, the second problem of (**P.2**) is written as:

$$\begin{aligned}\mathcal{P}_2 : \max_{\{x_k\}} \quad & \sum_{k=1}^{N_s} \log_2\left(1+\frac{x_k y_k}{x_k+a_k}\right) \\ \text{s.t.} \quad & \sum_{k=1}^{N_s} d_k x_k \leq P_r \\ & \sum_{k=1}^{N_s} c_k x_k \leq \gamma_1 \\ & x_k \geq 0\end{aligned} \tag{5.37}$$

where $d_k \triangleq 1+y_k$, $c_k \triangleq |e_k|^2(1+y_k)$.

The subproblem $\mathcal{P}_1$ is a convex optimization problem because its feasible set, $\Omega_{\mathcal{P}_1}$, is convex (affine) and $\nabla^2_{\mathbf{y}} J(\mathbf{x},\mathbf{y}) \succeq \mathbf{0}$, where $J(\mathbf{x},\mathbf{y}) \triangleq -\sum_{k=1}^{N_s}\log_2(1+s_k y_k)$. Also, the subproblem $\mathcal{P}_2$ has global minimizer because its feasible set, $\Omega_{\mathcal{P}_2}$, is convex (affine), because $a_k \geq 0, y_k \geq 0$, so $\nabla^2_{\mathbf{x}} J(\mathbf{x},\mathbf{y})|_{\mathbf{x}\in\Omega_{\mathcal{P}_2}} \succeq \mathbf{0}$.

Having the global minimizers for both subproblems $\mathcal{P}_1$ and $\mathcal{P}_2$ ensures that the AO is convergent. As the AO is an iterative procedure, an initial value for x_k or y_k is needed. The initial value for y_k, $y_k^{(0)}$, can be obtained from some constraints of $\mathcal{P}_1$. One possibility for $y_k^{(0)}$, $k = 1, ..., N_s$, is $[y_1^{(0)}; ...; y_{N_s}^{(0)}] = \text{argmin}\|\mathbf{Q}\mathbf{y} - [P_s; \gamma_1]\|_2$ subject to $\mathbf{y} \geq \mathbf{0}$, where $\mathbf{Q}(:,k) = [f_k; h_k]$. This is basically linear least squares with nonnegativity constraints. The initial value for x_k, $x_k^{(0)}$, can be obtained from the power budget for each relay. From (5.6), $\text{tr}(\mathbf{W}_{r_k}\mathbf{H}_{sr_k}\mathbf{W}_s\mathbf{\Sigma}_\mathbf{s}\mathbf{W}_s^H\mathbf{H}_{sr_k}^H\mathbf{W}_{r_k}^H) + \text{tr}(\mathbf{W}_{r_k}\mathbf{W}_{r_k}^H) \leq P_{r_k}$. Substitute $\mathbf{W}_{r_k} = \omega_k \mathbf{a}_k \mathbf{b}_k^H$ on the left hand side of this inequality. We get

$x_k(\mathrm{tr}(\mathbf{H}_{sr_k}^H \mathbf{b}_k \mathbf{b}_k^H \mathbf{H}_{sr_k} \mathbf{W}_s \boldsymbol{\Sigma}_\mathbf{s} \mathbf{W}_s^H) + 1) \leq P_{r_k}$. It is easy to show that the left hand side is less than $x_k(\lambda_{sr_k}(b_k)P_s + 1)$. If $x_k(\lambda_{sr_k}(b_k)P_s + 1) \leq P_{r_k}$, then the budget power at the selected relays are satisfied. Hence, one possibility for $x_k^{(0)}$ is $x_k^{(0)} = \alpha(\lambda_{sr_k}(b_k)P_s + 1)^{-1} P_{r_k}$ where $0 \leq \alpha \leq 1$ is chosen to prohibit the infeasibility of $\Omega_{\mathcal{P}_1}$. This derivation is consistent with the results in [72].

As a summary, the non-convex optimization problem (**P.2**) can be solved by two algorithms which use AO. In Algorithm 1, $x_k^{(0)}$ is chosen, the optimal solution for y_k from $\mathcal{P}_1$ is obtained and is attributed to $y_k^{(n)}$, where $n = 1$ for the first iteration. Then $y_k^{(n)}$ is deployed for $\mathcal{P}_2$ to get the optimal solution for x_k. Attribute this solution as $x_k^{(n)}$. Repeat this procedure for some number of iterations, n. In Algorithm 2, first $y_k^{(0)}$ is chosen, then the optimal solution for x_k from $\mathcal{P}_2$ is obtained and is attributed to $x_k^{(n)}$, where $n = 1$ for the first iteration. Then $x_k^{(n)}$ is deployed for $\mathcal{P}_1$ to get the optimal solution for y_k. Attribute this solution as $y_k^{(n)}$. Repeat this procedure for some number of iterations, n.

Assume for $n \geq N$, $|J(\mathbf{x}^{(n+1)}, \mathbf{y}^{(n+1)}) - J(\mathbf{x}^{(n)}, \mathbf{y}^{(n)})| \leq \epsilon$ for arbitrarily small ϵ. Numerical evaluations show that if we choose the best result from the Algorithm 1 and Algorithm 2 then: $J(\mathbf{x}^*, \mathbf{y}^*) \leq J(\mathbf{x}^{**}, \mathbf{y}^{**})$, where $J(\mathbf{x}^*, \mathbf{y}^*)$ is cost function value from Algorithm 1 or 2 for $n = N$ and $J(\mathbf{x}^{**}, \mathbf{y}^{**})$ is the cost function evaluated at the solution from the Artificial Bee Colony (ABC) algorithm for constrained optimization by Karaboga [76]. Therefore, the simulations reveal that the AO provides better solution than ABC for problem (**P.2**).

5.3.2 Projected semi-orthogonal selection beamforming (PS-SVSB)

The objective of this scheme is to cancel the interference imposed on the primary receiver from the secondary network. For this, $\mathbf{H}_{sr_k}$ are first decomposed into components in the null space of $\mathbf{c}_0$ and range space of $\mathbf{c}_0^H$, i.e,

$$\mathbf{H}_{sr_k} = \mathbf{H}_{sr_k}^{\perp} + \mathbf{H}_{sr_k} \frac{\mathbf{c}_o \mathbf{c}_0^H}{\|\mathbf{c}_0\|^2} \tag{5.38}$$

where $\mathbf{H}_{sr_k}^{\perp}$ is the component of $\mathbf{H}_{sr_k}$ in the null space of $\mathbf{c}_0$. Similarly, for $\mathbf{H}_{r_k d}$ we have

$$\mathbf{H}_{r_k d} = \mathbf{H}_{r_k d}^{\perp} + \mathbf{H}_{r_k d} \frac{\mathbf{c}_k \mathbf{c}_k^H}{\|\mathbf{c}_k\|^2}, \quad k = 1, \ldots, N_s \tag{5.39}$$

where $\mathbf{H}_{r_k d}^{\perp}$ is the component of $\mathbf{H}_{r_k d}$ in the null space of $\mathbf{c}_k$.

To null out the interference in the primary receiver antenna, we should choose $\mathbf{W}_s$, $\mathbf{W}_{r_k}$ and $\mathbf{W}_d$ such that the channel components in the range space of $\mathbf{c}_0$ and $\mathbf{c}_k$ are nulled out. By multiplying both sides of equations (5.38) and (5.39) by $\mathbf{c}_0$ and $\mathbf{c}_k$ respectively, we have $\mathbf{H}_{sr_k}^{\perp}\mathbf{c}_0 = \underline{0}$ and $\mathbf{H}_{r_k d}^{\perp}\mathbf{c}_k = \underline{0}$. Consequently by considering SVD of the perpendicular channels as $\mathbf{H}_{sr_k}^{\perp} = \mathbf{U}_{sr_k}^{\perp} \boldsymbol{\Lambda}_{sr_k}^{\perp 1/2} \mathbf{V}_{sr_k}^{\perp H}$ and

$\mathbf{H}^{\perp}_{r_kd} = \mathbf{U}^{\perp}_{r_kd}\mathbf{\Lambda}^{\perp 1/2}_{r_kd}\mathbf{V}^{\perp H}_{r_kd}$ it is simple to show that

$$\begin{aligned}\mathbf{u}^{\perp H}_{sr_k}(b_k)\mathbf{H}^{\perp}_{sr_k}\mathbf{c}_0 &= \mathbf{u}^{\perp H}_{sr_k}(b_k)\mathbf{U}^{\perp}_{sr_k}\mathbf{\Lambda}^{\perp 1/2}_{sr_k}\mathbf{V}^{\perp H}_{sr_k} \\ &= \sqrt{\lambda_k(b_k)}\mathbf{v}^{\perp H}_{sr_k}(b_k)\mathbf{c}_0 \\ &= 0, \end{aligned} \tag{5.40}$$

similarly

$$\begin{aligned}\mathbf{c}_k^H\mathbf{H}^{\perp H}_{r_kd}\mathbf{u}^{\perp}_{r_kd}(f_k) &= \mathbf{c}_k^H\mathbf{V}^{\perp}_{r_kd}\mathbf{\Lambda}^{\perp 1/2}_{r_kd}\mathbf{U}^{\perp H}_{r_kd}\mathbf{u}^{\perp}_{r_kd}(f_k) \\ &= \sqrt{\lambda_k(f_k)}\mathbf{c}_k^H\mathbf{v}^{\perp}_{r_kd}(f_k) \\ &= 0 \end{aligned} \tag{5.41}$$

where $\mathbf{u}_\alpha(\theta)$ is the θ-th left eigen vector of $\mathbf{H}_\alpha$, $\mathbf{v}_\alpha(\theta)$ is the θ-th right eigen vector of $\mathbf{H}_\alpha$, and $\lambda_\alpha(\theta)$ is the θ-th eigen mode corresponding to $\mathbf{H}_\alpha$. Now, by letting $\mathbf{b}_k = \mathbf{u}^{\perp}_{sr_k}(b_k)$ and $\mathbf{a}_k = \mathbf{v}^{\perp}_{r_kd}(f_k)$ and defining $\mathbf{h}^{\perp}_{sr_k} \triangleq \mathbf{H}^{\perp H}_{sr_k}\mathbf{b}_k \in \mathbb{C}^{N_s\times 1}$ and $\mathbf{h}^{\perp}_{r_kd} \triangleq \mathbf{H}^{\perp}_{r_kd}\mathbf{a}_k \in \mathbb{C}^{N_d\times 1}$, we have

$$\begin{aligned}\mathbf{H}^{\perp,e}_{sr} &\triangleq \left[\sqrt{\lambda^{\perp}_{sr_1}(b_1)}\mathbf{v}^{\perp}_{sr_1}(b_1), \quad \ldots \quad, \sqrt{\lambda^{\perp}_{sr_{N_s}}(b_{N_s})}\mathbf{v}^{\perp}_{sr_{N_s}}(b_{N_s})\right]^H \\ \mathbf{H}^{\perp,e}_{rd} &\triangleq \left[\sqrt{\lambda^{\perp}_{r_1d}(f_1)}\mathbf{u}^{\perp}_{r_1d}(f_1), \quad \ldots \quad, \sqrt{\lambda^{\perp}_{r_{N_s}d}(f_{N_s})}\mathbf{u}^{\perp}_{r_{N_s}d}(f_{N_s})\right]\end{aligned}$$

then by letting $\mathbf{W}_s = \mathbf{H}^{\perp,e\dagger}_{sr}$, from (5.40) we conclude that

$$\mathbf{c}_0^H\mathbf{W}_s = \underline{0}. \tag{5.42}$$

Additionally, since $\mathbf{W}_{r_k} = \omega_k\mathbf{a}_k\mathbf{b}_k^H = \omega_k\mathbf{v}^{\perp}_{r_kd}(f_k)\mathbf{u}^{\perp H}_{sr_k}(b_k)$, from (5.41) we conclude that

$$\mathbf{c}_k^H\mathbf{W}_{r_k} = \underline{0}. \tag{5.43}$$

Moreover, from (5.1), (5.38) and (5.42) we have

$$\begin{aligned}\mathbf{y}_{r_k} &= \left(\mathbf{H}^{\perp}_{sr_k} + \mathbf{H}_{sr_k}\frac{\mathbf{c}_o\mathbf{c}_0^H}{\|\mathbf{c}_0\|^2}\right)\mathbf{W}_s\mathbf{s} + \mathbf{n}_k \\ &= \mathbf{H}^{\perp}_{sr_k}\mathbf{W}_s\mathbf{s} + \mathbf{n}_k. \end{aligned} \tag{5.44}$$

The received signal at the k-th relay after passing through the relay beamformer $\mathbf{b}_k$ is given by

$$\begin{aligned}x_{r_k} &= \mathbf{b}_k^H\mathbf{y}_{r_k} \\ &= \mathbf{u}^{\perp H}_{sr_k}(b_k)\mathbf{H}^{\perp}_{sr_k}\mathbf{W}_s + n''_k \\ &= \sqrt{\lambda_k(b_k)}\mathbf{v}^{\perp H}_{sr_k}(b_k)\mathbf{W}_s + n''_k \\ &= s_k + n''_k \end{aligned} \tag{5.45}$$

where $n''_{r_k} \sim \mathcal{CN}(0,1) = \mathbf{b}_k^H \mathbf{n}_{r_k}$. Afterwards, by setting $\mathbf{a}_k = \mathbf{v}^{\perp}_{r_k d}(f_k)$ and utilizing (5.39), (5.41) and SVD of $\mathbf{H}^{\perp}_{r_k d}$ we have

$$\begin{aligned} \mathbf{y}_d &= \sum_{k=1}^{N_s} \omega_k \mathbf{H}_{r_k d} \mathbf{a}_k x_{r_k} \\ &= \sum_{k=1}^{N_s} \omega_k \left(\mathbf{H}^{\perp}_{r_k d} + \mathbf{H}_{r_k d} \frac{\mathbf{c}_k \mathbf{c}_k^H}{\|\mathbf{c}_k\|^2} \right) \mathbf{v}_{r_k d}(f_k)^{\perp} x_{r_k} \\ &= \sum_{k=1}^{N_s} \omega_k \sqrt{\lambda_{r_k d}(f_k)} \mathbf{u}^{\perp}_{r_k d}(f_k) x_{r_k} \end{aligned} \tag{5.46}$$

Finally, we set $\mathbf{W}_d = \mathbf{H}^{\perp, e^{\dagger}}_{r_k d}$, thus the soft decisions on the output streams are

$$\mathbf{W}_d \mathbf{y}_d = \mathbf{\Omega} \left(\mathbf{s} + \mathbf{n}''_{r_k} \right) + \mathbf{n}'_d \tag{5.47}$$

where $\mathbf{n}''_{r_k} \sim \mathcal{CN}(\underline{0}, \mathbf{I}) = [n''_{r_1}, \ldots, n''_{r_{N_s}}]^T$ and $\mathbf{n}'_d \sim \mathcal{CN}(\underline{0}, \mathbf{W}_d \mathbf{W}_d^H) = \mathbf{W}_d \mathbf{n}_d$. On the other hand by (5.42) and (5.43) the interference imposed on the primary receiver in (5.29) and (5.30) is zero, therefore **P.1** is simplified to **P.3**:

$$\underset{\sigma_k, \omega_k}{\text{maximize}} \quad \sum_{k=1}^{N_s} \log_2 \left(1 + \frac{\omega_k^2 \sigma_k}{\omega_k^2 + |w_{d_{k,k}}|^2} \right)$$

subject to:

$$\sum_{k}^{N_s} \|\mathbf{w}_{s_k}\|^2 \sigma_k \leq P_s, \tag{5.48}$$

$$\sum_{k=1}^{N_s} \omega_k^2 (1 + \sigma_k) \leq P_r, \tag{5.49}$$

$$\sigma_k \geq 0. \quad \forall\, k. \tag{5.50}$$

With the same arguments for the orthogonality of $\mathbf{W}_s$ and $\mathbf{W}_d$ in Sec. 5.3.1, the relay and antenna selection method for PS-SVSB scheme is similar to Table I if $\mathbf{u}_{sr_k}(b_k)$ and $\mathbf{v}_{r_k d}(f_k)$ are replaced by $\mathbf{u}^{\perp}_{sr_k}(b_k)$ and $\mathbf{v}^{\perp}_{r_k d}(f_k)$, respectively.

5.4 Selection Beamforming Multiple Primary Receivers/Antennas

In this section we present algorithms and optimization rules when the number of antennas for primary receivers exceeds one. We consider a general case of multiple antenna primary receivers.

5.4.1 Direct semi-orthogonal selection beamforming (DS-SVSB)

The selection-beamforming scenario for DS-SVSB scheme in this case remains the same as in the previous case where the primary network contains only one single antenna receiver. In particular, the matrices $\mathbf{W}_S$, $\mathbf{W}_{r_k}$ and $\mathbf{W}_d$ are still the ones obtained in Sec. 5.3.1. In this case, defining $\mathbf{Z}^l = \left[\mathbf{z}_1^l, \ldots, \mathbf{z}_{N_s}^l\right] \triangleq \mathbf{C}_0^l \mathbf{W}_s \in \mathbb{C}^{N_{p_l} \times N_s}$ and $\mathbf{e}_k^l \triangleq \mathbf{C}_k^l \mathbf{v}_{r_k d}(f_k) \in \mathbb{C}^{N_{p_l} \times 1}$, the constraints (5.13) and (5.14) in $\mathbf{P.1}$ become $\sum_{k=1}^{N_s} \|\mathbf{z}_k^l\|^2 \sigma_k \leq \gamma_l$ and $\sum_{k=1}^{N_s} \omega_k \|\mathbf{e}_k^l\|^2 (1+\sigma_k) \leq \gamma_l$. Thus $\mathbf{P.1}$ in this case turns out to be a multilevel water-filling problem.

5.4.2 Projected semi-orthogonal selection beamforming (PS-SVSB)

In this section we are looking for choices of $\mathbf{W}_s$, $\mathbf{W}_{r_k}$ and $\mathbf{W}_d$ such that the interference in (5.13) and (5.14) are nulled out. To do so, let $\mathbf{C}_0 \triangleq \left[{\mathbf{C}_0^1}^H, \ldots, {\mathbf{C}_0^L}^H\right]^H = \mathbf{U}_{C_0} \mathbf{\Lambda}_{C_0}^{1/2} \mathbf{V}_{C_0}^H$, and $\mathbf{C}_k \triangleq \left[{\mathbf{C}_k^1}^H, \ldots, {\mathbf{C}_k^L}^H\right]^H = \mathbf{U}_{C_k} \mathbf{\Lambda}_{C_k}^{1/2} \mathbf{V}_{C_k}^H$, then by projecting the $\mathbf{H}_{sr_k}$ and $\mathbf{H}_{r_k d}$ channels to the null space of $\mathbf{C}_0^H$ and $\mathbf{C}_k^H$ we have

$$\mathbf{H}_{sr_k} = \mathbf{H}_{sr_k}^{\perp} + \mathbf{H}_{sr_k} \mathbf{V}_{C_0} \mathbf{V}_{C_0}^H \tag{5.51}$$

where $\mathbf{H}_{sr_k}^{\perp}$ is the component of $\mathbf{H}_{sr_k}$ in the null space of $\mathbf{C}_0^H$. Similarly, for $\mathbf{H}_{r_k d}$ we have

$$\mathbf{H}_{r_k d} = \mathbf{H}_{r_k d}^{\perp} + \mathbf{H}_{r_k d} \mathbf{V}_{C_k} \mathbf{V}_{C_k}^H, \quad k = 1, \ldots, N_s \tag{5.52}$$

where $\mathbf{H}_{r_k d}^{\perp}$ is the component of $\mathbf{H}_{r_k d}$ in the null space of $\mathbf{C}_k^H$.

Obviously, by multiplying both sides of equations (5.51) and (5.52) by $\mathbf{C}_0^H$ and $\mathbf{C}_k^H$ respectively from right hand side, we have $\mathbf{H}_{sr_k}^{\perp} \mathbf{C}_0^H = \underline{0}$ and $\mathbf{H}_{r_k d}^{\perp} \mathbf{C}_k^H = \underline{0}$. Consequently by considering SVD of the perpendicular channels as $\mathbf{H}_{sr_k}^{\perp} = \mathbf{U}_{sr_k}^{\perp} {\mathbf{\Lambda}_{sr_k}^{\perp}}^{1/2} {\mathbf{V}_{sr_k}^{\perp}}^H$ and $\mathbf{H}_{r_k d}^{\perp} = \mathbf{U}_{r_k d}^{\perp} {\mathbf{\Lambda}_{r_k d}^{\perp}}^{1/2} {\mathbf{V}_{r_k d}^{\perp}}^H$ it is simple to show that

$$\begin{aligned}
{\mathbf{u}_{sr_k}^{\perp}}^H(b_k) \mathbf{H}_{sr_k}^{\perp} \mathbf{C}_0^H &= {\mathbf{u}_{sr_k}^{\perp}}^H(b_k) \mathbf{U}_{sr_k}^{\perp} {\mathbf{\Lambda}_{sr_k}^{\perp}}^{1/2} {\mathbf{V}_{sr_k}^{\perp}}^H \\
&= \sqrt{\lambda_k(b_k)} {\mathbf{v}_{sr_k}^{\perp}}^H(b_k) \mathbf{C}_0^H \\
&= \sqrt{\lambda_k(b_k)} {\mathbf{v}_{sr_k}^{\perp}}^H(b_k) {\mathbf{C}_0^l}^H, l = 1 \ldots, L \\
&= 0.
\end{aligned} \tag{5.53}$$

Similarly

$$\begin{aligned}
\mathbf{C}_k^H {\mathbf{H}_{r_k d}^{\perp}}^H \mathbf{u}_{r_k d}^{\perp}(f_k) &= \mathbf{C}_k^H \mathbf{V}_{r_k d}^{\perp} {\mathbf{\Lambda}_{r_k d}^{\perp}}^{1/2} {\mathbf{U}_{r_k d}^{\perp}}^H \mathbf{u}_{r_k d}^{\perp}(f_k) \\
&= \sqrt{\lambda_k(f_k)} \mathbf{C}_k^H \mathbf{v}_{r_k d}^{\perp}(f_k) \\
&= \sqrt{\lambda_k(f_k)} \mathbf{C}_k^l \mathbf{v}_{r_k d}^{\perp}(f_k)\ , l = 1, \ldots, L \\
&= 0.
\end{aligned} \tag{5.54}$$

From (5.53) and (5.54) and regarding the new definitions of $\mathbf{H}^{\perp}_{sr_k}$ and $\mathbf{H}^{\perp}_{r_k d}$, it can be concluded that the choices of $\mathbf{W}_s$, $\mathbf{W}_{r_k}$ and $\mathbf{W}_d$ are the same as in Sec. 5.3.2. For the chosen $\mathbf{W}_s$ and $\mathbf{W}_{r_k}$ we have $\mathbf{C}^l_0\mathbf{W}_s = \underline{0}$ and $\mathbf{C}^l_k\mathbf{W}_{r_k} = \underline{0}$. The optimization problem **P.1** in this case is the same as **P.3**.

5.5 Simulation Results

For the simulations, all channels, including $\mathbf{C}^l_0$, $\mathbf{C}^l_k$, $\mathbf{H}_{sr_k}$ and $\mathbf{H}_{r_k d}$, are known and are generated from independent complex Gaussian distributions. The channel from the secondary source to the primary receiver(s), $\mathbf{C}^l_0$, and channels from the secondary relays to the primary receiver(s), $\mathbf{C}^l_k$, are $\mathcal{CN}(0, 0.1)$. The channels for the secondary users are $\mathcal{CN}(0, 1)$. To have a meaningful relay network, the channels from the source to destination is considered to so weak denoted by $\mathbf{H}_{sd} \sim \mathcal{CN}(0, \epsilon)$. The secondary user's transmit-power budget P_s is varied from 1 to 100, which is equivalent to 0 dB to 20 dB average SNR denoted by σ^2_s/σ^2_n. Let $P_r = P_s$ and $P_{r_k} = P_r/N_s$. Without loss of generally, assume $l = 1$ and a single antenna primary user. Figure 5.4 illustrates the capacity performance of DS-SVSB and PS-SVSB methods for a different number of relays with respect to maximum allowable interference power, γ_l, for primary network.

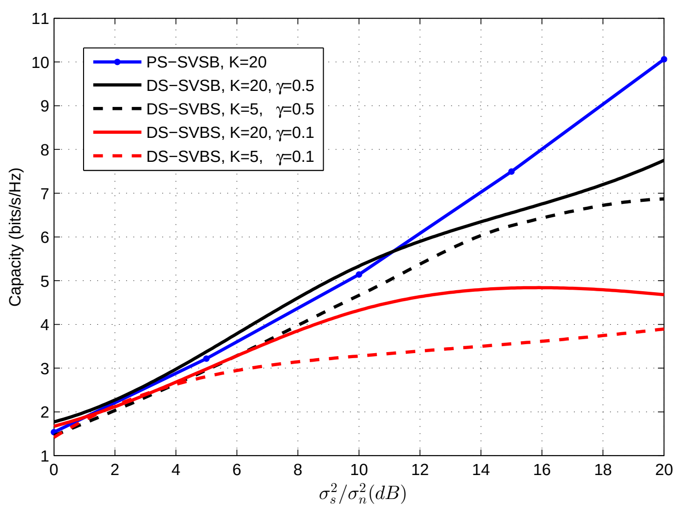

Figure 5.4: The capacity performance of DS-SVSB ($N_s = 3$) and PS-SVSB ($N_s = 4$) for $K = 5$ and $K = 20$ with respect to γ_l when $N_{r_k} = 4$, $N_d = 6$.

This figure shows that PS-SVSB provides better capacity at the price of one more antenna element at the secondary source compared to DS-SVSB. The behaviors of DS-SVSB seem predictable. By increasing the number of relays, the capacity is better for the same tolerable interference power constraint for primary users. For the same number of relays, increasing γ_l improves the capacity.

In figure 5.4, the decreasing capacity of DS-SVBS against average SNR-shown by solid red line-is because only a few channel realizations are generated for this simulation. Averaging over more channel realizations would produce a more accurate capacity curve which reaches to a limit asymptotically.

5.6 Conclusions

In this chapter, we proposed beamforming and relay selection to be used together in a cognitive and cooperative network. The relay selection and beamforming design together maximize the capacity of a single secondary user. We show that this capacity maximization problem is a difficult problem. By assuming all beamformers are rank-one at the relays and that the multiple data streams at the secondary source are independent, then this problem is simplified and can be solved by AO. However, the capacity saturates when the average SNR is increased because the interference should be maintained below a threshold at the primary receivers. By deploying one more antenna element at the secondary source and using the orthogonal projection technique, then the capacity optimization problem is even more simplified and also the capacity performance is an increasing function of average SNR. The derived formulations presented in this chapter require some major assumptions which are discussed at the introduction section in detail.

Chapter 6

Conclusion

6.1 Conclusions

In this thesis, beamforming designs have been investigated for a multiuser network with manageable interference. Having reliable communication links for all users can be guaranteed by the proposed designs. This network known as interference channels increases the total capacity compared to conventional communication systems.

The main contribution, chapter 2, is proposing a novel method of solving a class of multiple objective optimization problems. The method is basically an extension to the alternating optimization (AO) which is a well-known approach for solving non-convex single objective optimization problems. The proposed iterative algorithm, called EAO, is convergent if each objective has a unique global minimum with respect to some decision variables and additionally if the corresponding vector field, resulting from the combination of all decision variables, is a contractive or non-expansive mapping. Mathematically, we show that EAO converges to a Nash equilibrium (NE). By deploying EAO, beamformers are obtained for both transmitters and receivers with closed-form steps for all users in interference channels. In summary:

- For all combinations of two objective functions (known communication metrics such as SINR, MSE, LI), EAO is convergent;
- The communication performance of EAO is better than most existing methods;
- A Quality of Service (QoS) is guaranteed by EAO for all users in interference channels;
- The EAO has the lowest computational complexity compared to other known methods;
- For EAO applied to a specific combination of objective functions, the required assumption of having a unique global optimum has an interpretation that yields the relation between the number of antennas and the number of users.

From the communication point of view, simulations reveal the bit-error rate performance and capacity (or sum-rate) performance are not necessarily related in the multiuser MIMO interference channel. For example, a good sum-rate performance realized by maximizing total signal power over total interference power plus noise ratio, has very poor error performance.

In the context of game theory, we show that EAO can be applied also for K games (K is the number of users). Again, the main advantage of using EAO for K games compared to other existing methods is its low computational complexity. Specific new results are:

- The complexity of EAO for the K-user MIMO interference channels is $O(NKM^3)$ where N is the number of iterations and M is the number of antennas at each terminal
- The complexity of MSE by SOCP is $O(INM^2K^6)$ where I is the number of interior-point-method (IPM) iterations
- The complexity of max-min SINR and LS by the Genetic Algorithm (GA) is $O(GPKM^3)$ where G is the numbers of generations and P is the population size
- The complexity of the Gradient method for sum rate maximization is $O(NK^2M^3)$

In chapter 3, various beamforming designs and their required capacity overhead are addressed. Also, the maximum allowable speed of users for the various algorithms is discussed in terms of the algorithms' execution-time. Here, the feedback rate for the proposed and existing methods is tabled for the first time. Simulations are performed for the more realistic standardized indoor statistical channel model, IEEE 802.11n. The least-square (LS) beamforming design for MIMO interference channels is presented for the first time. We quantified the trade-offs including: low computational complexity, minimum required number of antennas, feedback overhead and the ability to extract multi-path diversity for beamforming designs in MIMO IC.

In chapter 4, the prioritized MIMO interference channel is a new view of cognitive radio systems, introduced in this dissertation. The formulated problem for this case is solvable by evolutionary algorithms. We quantitatively show that using more antennas requires *fewer* generations within an evolutionary algorithm to converge.

Finally in chapter 5, we cover three recent physical layer wireless communication scenarios including: cognitive radio; cooperative radio with relay selection; and beamforming design in secondary users as a general theoretical framework for future systems in mobile communications. The relay selection and beamforming design are performed to maximize capacity of a single secondary user, while the transmit power at both the source and the relays, and the interference induced to the primary user(s) from source and from relays, are constrained. By deploying one more antenna element at the secondary source, we show that orthogonal projection is feasible, so that the capacity optimization problem is simplified. Also, the secondary capacity performance of orthogonal projection is always an increasing function of average SNR, but, in the case of minimum antenna deployment at the secondary source, the secondary capacity performance saturates.

6.2 Future work

Increasing the capacity of wireless networks is always an important subject for research and for telecommunication companies. The capacity improvement can be realized by truly simultaneous use of spectrum. The multiuser MIMO interference channel is one step towards this goal. Extending beamformer design in interference channels to K simultaneous downlink or uplink systems, instead of a K simultaneous *point-to-point* in interference channel, may be possible. The beamforming designs under imperfect or partial channel state information are also important and have recently opened new horizons for research. Beamforming for full-duplex communication systems for multipoint-to-point, point-to-multipoint interference systems may be possible. This subject is already a research area for point-to-point interference systems.

The future work stemming from this thesis can be categorized in to three major areas including:
1-*Continuing to develop the theories of special classes of non-convex optimization problems by AO and EAO*. More specifically, tackling the open problem of multi-objective solution by EAO to establish the relationship of NE to the stationary point or the KKT point of the original optimization problem. Additionally, identifying the circumstances and assumptions for which the AO provides the global optimum for the original single-objective problem is still an open problem.
2-*Implementation issues, improvement and limitation of non-linear optimization toolkits for real-world communication problems.* For example, OPT++, an object-oriented toolkit for nonlinear optimization, is recently deployed in radio access network sharing in cellular networks to solve the resource allocation problem. Improvement of this toolkit, utilizing other packages and comparison among them, for wireless communication applications, would be both practical and useful area of research.
3-*Applying optimization for both micro and macro levels of telecommunication engineering and telecommunication market problems.*

The field of optimization is presently at a turning point due to: recent methodological developments and new theories; algorithmic developments; powerful software. The use of optimization in telecommunication is not confined to only beamforming design or power allocation or radio resource management. Many optimization applications in telecommunication are taking off. From articles such as [77, 78, 79], the applications are extremely diverse, as follows:

- Network reliability
- Dimensioning of a mobile phone network
- Routing telephone calls

- Construction of a cabled network
- Scheduling of telecommunications via satellite
- Location of LTE eNodeB
- Planning of internet based information service
- Optimizing cloud resources for delivering IPTV services through virtualization
- Planning of capacity expansion of mobile network
- Optimization in spectrum resource management

Following these, new applications of optimization in telecommunication marketing can be summarized as [80, 81]:

- Traffic modeling and cost optimization for transmitting traffic messages over a hybrid broadcast and cellular network
- Price differentiation for communication networks
- A dynamic model of prices and margins in cable TV industry
- Network operators and virtual providers: service pricing
- Media revenue management with audience uncertainty
- Incentives and pricing in communications networks
- Dynamic contract trading and portfolio optimization in spectrum markets
- Optimizing cost and quality of international calls routing

This thesis can be categorized as a branch of study for mobile telecommunications designs beyond the current 4G, i.e. the so-called 5th generation (5G). The implementation of standards for 5G are likely be around the year 2020 [82]. Appendix J summarizes the future challenges for 5G telecommunication networks.

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Appendix A

Sum Rate and Sum SINR Maximization

When the Tx beamfomers are known for all users, the sum rate maximization is equivalent to sum SINR maximization. Mathematically,

$$\begin{aligned} \max_{\mathbf{u}_1,\ldots,\mathbf{u}_K} \quad & \sum_{i=1}^{K} \log_2\left(1+\frac{\mathbf{u}_i^{\mathcal{H}}\mathbf{Q}_i\mathbf{u}_i}{\mathbf{u}_i^{\mathcal{H}}\mathbf{P}_i\mathbf{u}_i}\right) \\ \text{s.t.} \quad & \mathbf{u}_i^{\mathcal{H}}\mathbf{u}_i = 1 \qquad i = 1, ..., K. \end{aligned} \tag{A.1}$$

where $\mathbf{Q}_i \succeq \mathbf{0}$ and $\mathbf{P}_i \succeq \mathbf{0}$, is equivalent to:

$$\begin{aligned} \max_{\mathbf{u}_1,\ldots,\mathbf{u}_K} \quad & \sum_{i=1}^{K} \frac{\mathbf{u}_i^{\mathcal{H}}\mathbf{Q}_i\mathbf{u}_i}{\mathbf{u}_i^{\mathcal{H}}\mathbf{P}_i\mathbf{u}_i} \\ \text{s.t.} \quad & \mathbf{u}_i^{\mathcal{H}}\mathbf{u}_i = 1 \qquad i = 1, ..., K. \end{aligned} \tag{A.2}$$

This equivalence follows from the KKT condition for (A.1):

$$\frac{\mathbf{P}_i\mathbf{u}_i\left(\mathbf{u}_i^{\mathcal{H}}\mathbf{Q}_i\mathbf{u}_i\right) - \mathbf{Q}_i\mathbf{u}_i\left(\mathbf{u}_i^{\mathcal{H}}\mathbf{P}_i\mathbf{u}_i\right)}{\left(\mathbf{u}_i^{\mathcal{H}}\mathbf{P}_i\mathbf{u}_i\right)\left(\mathbf{u}_i^{\mathcal{H}}(\mathbf{P}_i+\mathbf{Q}_i)\mathbf{u}_i\right)} = \lambda_i\mathbf{u}_i$$

$$\mathbf{u}_i^{\mathcal{H}}\mathbf{u}_i = 1 \tag{A.3}$$

in which multiplying both sides of the first relation of (A.3) by $\mathbf{u}_i^{\mathcal{H}}$, indicates that $\lambda_i = 0$ for $i = 1, ..., K$. The same result follows from the KKT for (A.2). Thus, the unique global solution for both (A.1) and (A.2) is

$$\mathbf{u}_i = \boldsymbol{w}_{\max}\left(\mathbf{P}_i^{-1}\mathbf{Q}_i\right) \tag{A.4}$$

Appendix B

Proof of Theorem

Since f is contractive or nonexpansive, there exists $\mathbf{x}^* = f(\mathbf{x}^*)$; i.e., a set of transmit beamformers, $\mathbf{x}^* = \mathcal{V}^*$, such that

$$\mathbf{v}_i^* = q_i\left(p_1\left(\mathcal{V}^*\right), ..., p_K\left(\mathcal{V}^*\right)\right). \tag{B.1}$$

The Lipschitz function f (see (2.20) - (2.21)) is given by functions p_i (see (2.12), and, e.g., (2.34), etc.) and q_i. Let $\mathbf{u}_i^* \triangleq p_i\left(\mathcal{V}^*\right)$ and the dependencies between the transmit and receive beamformers are emphasized by the pair of equations corresponding to (2.12) and (2.17):

$$\begin{cases} \mathbf{u}_i^* = p_i\left(\mathcal{V}^*\right), \\ \mathbf{v}_i^* = q_i\left(\mathcal{U}^*\right). \end{cases} \tag{B.2}$$

Now consider an arbitrary point $\tilde{\mathcal{V}}$, with $\tilde{\mathbf{u}}_i = p_i\left(\tilde{\mathcal{V}}\right)$; and an arbitrary point $\bar{\mathcal{U}}$, with $\bar{\mathbf{v}}_i = q_i\left(\bar{\mathcal{U}}\right)$. Equations (2.13) and (2.18) give
$\forall \mathcal{V} \in \Omega_g, \forall \mathcal{U} \in \Omega_h$;

$$h\left(\mathcal{U}, \tilde{\mathcal{V}}\right) \geq h\left(\tilde{\mathcal{U}}, \tilde{\mathcal{V}}\right), \tag{B.3}$$

$$g\left(\mathcal{V}, \bar{\mathcal{U}}\right) \geq g\left(\bar{\mathcal{V}}, \bar{\mathcal{U}}\right). \tag{B.4}$$

By setting $\tilde{\mathcal{V}} = \mathcal{V}^*$ in (B.3), it follows that $\tilde{\mathcal{U}} = \mathcal{U}^*$; and setting $\bar{\mathcal{U}} = \mathcal{U}^*$ in (B.4), it follows that $\bar{\mathcal{V}} = \mathcal{V}^*$. Therefore, from (B.2), $\tilde{\mathcal{V}} = \bar{\mathcal{V}} = \mathcal{V}^*$ and $\tilde{\mathcal{U}} = \bar{\mathcal{U}} = \mathcal{U}^*$. Hence (B.3) and (B.4) can be written as

$$h\left(\mathcal{U}, \mathcal{V}^*\right) \geq h\left(\mathcal{U}^*, \mathcal{V}^*\right), \tag{B.5}$$

$$g\left(\mathcal{V}, \mathcal{U}^*\right) \geq g\left(\mathcal{V}^*, \mathcal{U}^*\right). \tag{B.6}$$

Moreover, from (2.14),

$$h_L(\mathcal{V}^*) = h\left(p_1(\mathcal{V}^*), ..., p_K(\mathcal{V}^*), \mathcal{V}^*\right),$$

$$= h(\mathbf{u}_1^*, ..., \mathbf{u}_K^*, \mathbf{v}_1^*, ..., \mathbf{v}_K^*) = h(\mathcal{U}^*, \mathcal{V}^*), \tag{B.7}$$

and

$$g_L(\mathcal{U}^*) = g(\mathcal{V}^*, \mathcal{U}^*); \tag{B.8}$$

and adding both sides of (B.7) and (B.8) gives (2.27), i.e.,

$$h_L(\mathcal{V}^*) + g_L(\mathcal{U}^*) = J(\mathcal{U}^*, \mathcal{V}^*). \tag{B.9}$$

Appendix C

Max-Min SINR for multi-stream case

Assume that user k sends d streams over the MIMO interference channel. The transmit beamformer is $\mathbf{V}_i \in \mathbb{C}^{M\times d}$ and the receive beamformer is $\mathbf{U}_i \in \mathbb{C}^{M\times d}$. Analogous to treatment above for the single-stream case, the solution to

$$\begin{aligned} &\max_{\mathbf{X}} \frac{\mathrm{tr}(\mathbf{X}^{\mathcal{H}}\mathbf{Q}\mathbf{X})}{\mathrm{tr}(\mathbf{X}^{\mathcal{H}}\mathbf{P}\mathbf{X})} \\ &\text{s.t. } \mathbf{X}^{\mathcal{H}}\mathbf{X} = \mathbf{I}_d. \end{aligned} \tag{C.1}$$

is the closed-form $\mathbf{X}^{\mathrm{opt}} = \mathbf{U}^{1:d}_{\max}(\mathbf{P}^{-1}\mathbf{Q})$ where $\mathbf{U}^{1:d}_{\max}(\mathbf{P}^{-1}\mathbf{Q})$ is the set of left singular vectors corresponding to the d strongest singular values of matrix $\mathbf{P}^{-1}\mathbf{Q}$. Therefore, the max-min SINR problem is formulated as:

$$\max_{\mathbf{V}_i} \quad \min_{i=1,\ldots,K} \frac{\mathrm{tr}\Big(\mathbf{U}^{1:d}_{\max}(\mathbf{B}_i^{-1}\mathbf{A}_i)^{\mathcal{H}}\mathbf{A}_i\mathbf{U}^{1:d}_{\max}(\mathbf{B}_i^{-1}\mathbf{A}_i)\Big)}{\mathrm{tr}\Big(\mathbf{U}^{1:d}_{\max}(\mathbf{B}_i^{-1}\mathbf{A}_i)^{\mathcal{H}}\mathbf{B}_i\mathbf{U}^{1:d}_{\max}(\mathbf{B}_i^{-1}\mathbf{A}_i)\Big)}$$

$$\text{s.t. } \mathrm{tr}(\mathbf{V}_i^{\mathcal{H}}\mathbf{V}_i) \leq d \tag{C.2}$$

where $\mathbf{A}_i \triangleq \mathbf{H}_{ii}\mathbf{V}_i\mathbf{V}_i^{\mathcal{H}}\mathbf{H}_{ii}^{\mathcal{H}}$ and $\mathbf{B}_i \triangleq (\sum_{j\neq i}^{K}\mathbf{H}_{ij}\mathbf{V}_j\mathbf{V}_j^{\mathcal{H}}\mathbf{H}_{ij}^{\mathcal{H}} + \frac{\sigma_n^2}{\sigma_{s_i}^2}\mathbf{I}_M)$. From (C.1), $\mathbf{U}_i = \mathbf{U}^{1:d}_{\max}(\mathbf{B}_i^{-1}\mathbf{A}_i)$.

Appendix D

LICQ holds for Problem $\mathcal{P}$

It is shown that if $\mathbf{x}^*$ is a local solution to non-convex problem (3.8) then LICQ holds at $\mathbf{x}^*$. Consider the following optimization problem:

$$\begin{aligned} \min_{\mathbf{x}} \quad & f(\mathbf{x}) \\ \text{s.t.} \quad & C_i(\mathbf{x}) = 0 \qquad i \in \mathcal{E} \\ & C_i(\mathbf{x}) \geq 0 \qquad i \in \mathcal{I}. \end{aligned} \tag{D.1}$$

The active set $\mathcal{A}(\mathbf{x})$ at any feasible $\mathbf{x}$ is defined as: $\mathcal{A}(\mathbf{x}) \triangleq \mathcal{E} \cup \{i \in \mathcal{I} | C_i(\mathbf{x}) = 0\}$. By definition, LICQ holds at $\mathbf{x}^*$ if the set of active constraint gradients $\{\nabla C_i(\mathbf{x}^*) | i \in \mathcal{A}(\mathbf{x}^*)\}$ is linearly independent. Problem (3.8) only has equality constraints, and $\nabla C_i(\mathbf{x}) = \mathbf{q}_i$. For the first user (say), the $\mathbf{H}_{1,i+1}$ random matrices are assumed independent and the $\mathbf{v}_{i+1}$ are non-zero random vectors so $\mathbf{q}_i = \mathbf{H}_{1,i+1}\mathbf{v}_{i+1}$. Consequently the $\nabla C_i(\mathbf{x})$ are linearly independent. Therefore, LICQ holds for problem (3.8).

Appendix E

Global optimum of problem $\mathcal{P}$ is a hard problem

The KKT for problem $\mathcal{P}$ is:

$$\begin{cases} 2\mathbf{x}(\mathbf{x}^{\mathcal{H}}\mathbf{Q}\mathbf{x}) - 2\mathbf{Q}\mathbf{x}(\mathbf{x}^{\mathcal{H}}\mathbf{x}) = (\mathbf{x}^{\mathcal{H}}\mathbf{x})^2 \sum_{i=1}^{K-2} \lambda_i \mathbf{q}_i \\ \mathbf{q}_i^{\mathcal{H}}\mathbf{x} = 0 \end{cases} \tag{E.1}$$

Define $\mathbf{B} \triangleq [\mathbf{q}_1^{\mathcal{H}}; ...; \mathbf{q}_{K-2}^{\mathcal{H}}] \in \mathbb{C}^{(K-2)\times N_r}$. Let $\mathbf{a}_j$ be an orthonormal basis for the null space of B where $j = 1, ..., N_r - K + 2$. Therefore, the solution for KKT of problem $\mathcal{P}$ is $\mathbf{x} = \sum_{j=1}^{N_r-K+2} \alpha_j \mathbf{a}_j$ where $\alpha_j \neq 0$ are obtained from:

$$\begin{aligned} &2\left(\sum_{j=1}^{N_r-K+2} \alpha_j \mathbf{a}_j\right)\left(\sum_{j=1,s>j}^{N_r-K+2} |\alpha_j|^2 \mathbf{a}_j^{\mathcal{H}}\mathbf{Q}\mathbf{a}_j + 2\mathrm{Re}\{\alpha_j^* \alpha_s \mathbf{a}_j^{\mathcal{H}}\mathbf{Q}\mathbf{a}_s\}\right) \\ &- 2\left(\sum_{j=1}^{N_r-K+2} \alpha_j \mathbf{Q}\mathbf{a}_j\right)\left(\sum_{j=1}^{N_r-K+2} |\alpha_j|^2\right) \\ &= \left(\sum_{j=1}^{N_r-K+2} |\alpha_j|^2\right)^2 \sum_{i=1}^{K-2} \lambda_i \mathbf{q}_i \end{aligned} \tag{E.2}$$

Moreover, if a solution is found, it is not known if it is global or local optimum for problem $\mathcal{P}$, and this must be checked. In short, this is a difficult situation.
By assuming $\mathbf{u}_1^{\mathcal{H}}\mathbf{u}_1 = 1$, problem $\mathcal{P}$ is transformed to a new optimization problem:

$$\begin{aligned} \max_{\alpha_1,...,\alpha_{N_r-K+2}} \quad & \sum_{j=1,s>j}^{N_r-K+2} |\alpha_j|^2 \mathbf{a}_j^{\mathcal{H}}\mathbf{Q}\mathbf{a}_j + 2\mathrm{Re}\{\alpha_j^* \alpha_s \mathbf{a}_j^{\mathcal{H}}\mathbf{Q}\mathbf{a}_s\} \\ \text{s.t.} \quad & \sum_{j=1}^{N_r-K+2} |\alpha_j|^2 = 1 \end{aligned} \tag{E.3}$$

and this problem is simpler; for the case $N_r = K$ (Section III), there are closed forms for $\alpha_1 = \alpha$ and $\alpha_2 = \beta$. Otherwise, (E.3) should be solved, which is hard.

Appendix F

Extended Alternating Optimization (EAO) for K Games

The EAO is a general form of AO for multi-objective optimization problem. Consider the sum of objective functions $\mathcal{J}_i(\mathbf{x})$:

$$\min_{\mathbf{x}\in\Omega_{\mathcal{J}}} \mathcal{J}(\mathbf{x}) = \mathcal{J}_1(\mathbf{x}_1, \cdots, \mathbf{x}_K) + \cdots + \mathcal{J}_K(\mathbf{x}_1, \cdots, \mathbf{x}_K) \tag{F.1}$$

The idea of EAO is to replace this difficult joint optimization of $\mathcal{J}$ over the sub-problems:

$$\min_{\mathbf{x}_i\in\Omega_i} \mathcal{J}_i(\mathbf{x}_1, ...\mathbf{x}_{i-1}, \mathbf{x}_i, \mathbf{x}_{i+1}, ..., \mathbf{x}_K) \tag{F.2}$$

where $\mathbf{x}_1, ...\mathbf{x}_{i-1}, \mathbf{x}_{i+1}, ..., \mathbf{x}_K$ are assumed to be known (or fixed). Let the optimization problem (F.2) has a unique global minimizer w.r.t $\mathbf{x}_i$ and be expressed by:

$$\mathbf{x}_i = l_i(\mathbf{x}_1, ..., \mathbf{x}_{i-1}, \mathbf{x}_{i+1}, ..., \mathbf{x}_K) \quad i = 1, ..., K \tag{F.3}$$

For $\mathbf{x}_1, ..., \mathbf{x}_{K-1}$, define:

$$\mathbf{x}_1 = l_1(\mathbf{x}_2, ...\mathbf{x}_{K-1}, \underbrace{l_K(\mathbf{x}_1, ..., \mathbf{x}_{K-1})}_{\mathbf{x}_K}) \triangleq f_1(\mathbf{x}_1, ..., \mathbf{x}_{K-1})$$

$$\mathbf{x}_2 = l_2(\mathbf{x}_1, \mathbf{x}_3, ..., \underbrace{l_K(\mathbf{x}_1, \mathbf{x}_2, ..., \mathbf{x}_{K-1})}_{\mathbf{x}_K}) \triangleq f_2(\mathbf{x}_1, ..., \mathbf{x}_{K-1})$$

$$\vdots$$

$$\mathbf{x}_{K-1} = l_{K-1}(\mathbf{x}_1, ..., \mathbf{x}_{K-2}, \underbrace{l_K(\mathbf{x}_1, ..., \mathbf{x}_{K-1})}_{\mathbf{x}_K})$$

$$\triangleq f_{K-1}(\mathbf{x}_1, ..., \mathbf{x}_{K-1}) \tag{F.4}$$

Denote $\mathbf{x} \triangleq [\mathbf{x}_1, ..., \mathbf{x}_{K-1}]^T$ and $f \triangleq [f_1, ..., f_{K-1}]^T$ so that the set of nonlinear equations (F.4) is:

$$\mathbf{x} = f(\mathbf{x}). \tag{F.5}$$

If for some a, $||l_i|| \leq a$ for $||\text{col}\{\mathbf{x}_1, ...\mathbf{x}_{i-1}, \mathbf{x}_{i+1}, ..., \mathbf{x}_K\}|| \leq a$ (where col operator concatenates vectors), then from Brouwer's fixed point theorem [37] $\exists \{\mathbf{x}_i^*\}_{i=1}^K$ such that:

$$\mathcal{J}_1(\mathbf{x}_1^*, \mathbf{x}_2^*, ..., \mathbf{x}_K^*) \leq \mathcal{J}_1(\mathbf{x}_1, \mathbf{x}_2^*, ..., \mathbf{x}_K^*)$$

$$\mathcal{J}_2(\mathbf{x}_1^*, \mathbf{x}_2^*, ..., \mathbf{x}_K^*) \leq \mathcal{J}_2(\mathbf{x}_1^*, \mathbf{x}_2, ..., \mathbf{x}_K^*)$$

$$\vdots$$

$$\mathcal{J}_K(\mathbf{x}_1^*, ..., \mathbf{x}_{K-1}^*, \mathbf{x}_K^*) \leq \mathcal{J}_K(\mathbf{x}_1^*, ..., \mathbf{x}_{K-1}^*, \mathbf{x}_K). \tag{F.6}$$

In another notation, $\mathbf{x}^*$ the fixed point of f, obtained by EAO, is Nash Equilibrium (NE) for K games. One approximation for obtaining $\mathbf{x}^*$ is Gauss-Seidel iteration:

$$\mathbf{x}_i^{(n+1)} = l_i(\mathbf{x}_1^{(n+1)}, ..., \mathbf{x}_{i-1}^{(n+1)}, \mathbf{x}_{i+1}^{(n)}, ..., \mathbf{x}_K^{(n)}) \tag{F.7}$$

Relating the point $\mathbf{x}^*$, the NE of K-games, to the KKT solution or stationary solution of the original problem (F.1) is an open problem.

Appendix G

Optimization of a Constrained Fractional Function Over Three Variables

It is desired to find complex vectors $\mathbf{x}$, $\mathbf{y}$, $\mathbf{z}$ for the following optimization problem:

$$\begin{aligned} \underset{\mathbf{x},\mathbf{y},\mathbf{z}}{\text{maximize}} \quad & \frac{\mathbf{x}^{\mathcal{H}}\mathbf{H}\mathbf{y}\mathbf{y}^{\mathcal{H}}\mathbf{H}^{\mathcal{H}}\mathbf{x}}{\mathbf{x}^{\mathcal{H}}\mathbf{A}\mathbf{x}} \\ \text{subject to} \quad & \|\mathbf{y}\|^2 = P_y, \|\mathbf{z}\|^2 = P_z. \end{aligned} \tag{G.1}$$

where $\mathbf{A} \succeq 0$ and $\mathbf{A} = f(\mathbf{z})$, and $\mathbf{H}$ is a complex matrix.

Preliminaries:

$P1$-The maximum of $\mathbf{x}^{\mathcal{H}}\mathbf{Q}\mathbf{x}/\mathbf{x}^{\mathcal{H}}\mathbf{P}\mathbf{x}$ with respect to $\mathbf{x} \in \mathbf{C}^n \backslash \mathbf{0}$, where $\mathbf{Q} \succeq 0$ and $\mathbf{P} \succeq 0$ is $\mathbf{x}^{\text{opt}} = \mathbf{v}_{\max}(\mathbf{P}^{-1}\mathbf{Q})$, see [59] for proof.

$P2$-For any $\mathbf{q} \in \mathbf{C}^n$, $\text{rank}(\mathbf{q}\mathbf{q}^{\mathcal{H}}) = 1$.

$P3$-$\text{rank}(\mathbf{AB}) \leq \min(\text{rank}(\mathbf{A}), \text{rank}(\mathbf{B}))$ for any matrix $\mathbf{A}$ and $\mathbf{B}$.

The optimum solution of (G.1) for $\mathbf{x}$ based on $P1$, is $\mathbf{x}^{\text{opt}} = \mathbf{v}_{\max}(\mathbf{A}^{-1}\mathbf{H}\mathbf{y}^{\text{opt}}(\mathbf{y}^{\text{opt}})^{\mathcal{H}}\mathbf{H}^{\mathcal{H}})$. Defining vector $\mathbf{q} = \mathbf{H}\mathbf{y}^{\text{opt}}$, then $\text{rank}(\mathbf{A}^{-1}\mathbf{q}\mathbf{q}^{\mathcal{H}}) = 1$ (c.f., $P2$, $P3$), so the matrix $\mathbf{A}^{-1}\mathbf{q}\mathbf{q}^{\mathcal{H}}$ has one eigenvalue which is equal to $\mathbf{q}^{\mathcal{H}}\mathbf{A}^{-1}\mathbf{q}$, therefore $\mathbf{x}^{\text{opt}} = \gamma\mathbf{A}^{-1}\mathbf{q} = \gamma\mathbf{A}^{-1}\mathbf{H}\mathbf{y}^{\text{opt}}$ where γ is an arbitrary real constant. By substituting $\mathbf{x}^{\text{opt}}$ in to (G.1), the optimum solution of (G.1) for $\mathbf{y}$ would be $\mathbf{y}^{\text{opt}} = \sqrt{P_y}\mathbf{v}_{\max}(\mathbf{H}^{\mathcal{H}}\mathbf{A}^{-1}\mathbf{H})$. By substituting $\mathbf{x}^{\text{opt}}$ and $\mathbf{y}^{\text{opt}}$ to (G.1), the optimization problem of (G.1) is reduced to following optimization problem:

$$\begin{aligned} \underset{\mathbf{z}}{\text{maximize}} \quad & \lambda_{\max}(\mathbf{H}^{\mathcal{H}}f(\mathbf{z})^{-1}\mathbf{H}) \\ \text{subject to} \quad & \|\mathbf{z}\|^2 = P_z. \end{aligned} \tag{G.2}$$

Appendix H

Analysis of GA Generations Number

On average, GA needs more generations for $N_t = N_r = N_u$ compared with $N_t = N_r > N_u$ which seems an unintuitive result.

Preliminaries: Define Ω_1 and Ω_2 which are two sets with the following properties:

$$\Omega_1 = \left\{ \begin{bmatrix} x_{11} \\ x_{12} \\ x_{22} \end{bmatrix} \middle| x_{ij} = [\mathbf{X}]_{i,j}, \mathbf{X} \in \mathbf{S}^2_{++}, \alpha - \delta \leq \lambda_{\max}(\mathbf{X}) \leq \alpha \right\} \tag{H.1}$$

$$\Omega_2 = \left\{ \begin{bmatrix} x_{11} \\ x_{12} \\ x_{22} \end{bmatrix} \middle| x_{ij} = [\mathbf{X}]_{i,j}, \mathbf{X} \in \mathbf{S}^3_{++}, \alpha - \delta \leq \lambda_{\max}(\mathbf{X}) \leq \alpha \right\} \tag{H.2}$$

where α is an arbitrary number, δ is a small number and the notation $\mathbf{S}^n_{++}$ denotes the set of symmetric positive definite $n \times n$ matrices.

Figures H.1 and H.2 show the Ω_1 and Ω_2 regions for $\alpha = 1.7$ when $\delta \to 0$. Figure H.3 represents a cross section of these two regions, where $x_{22} = 0.5$ for both Ω_1 and Ω_2. With the same α and δ, it can be concluded that:

$$|\Omega_1| < |\Omega_2| \tag{H.4}$$

$$\Omega_1' = \left\{ \begin{bmatrix} x_{11} \\ x_{12} \\ x_{22} \end{bmatrix} \middle| x_{ij} = [\mathbf{X}]_{i,j}, \mathbf{X} = \mathbf{H}_{11}^{\mathcal{H}} \mathbf{A}^{-1} \mathbf{H}_{11}, \alpha = \lambda_{\max}(\mathbf{H}_{11}^{\mathcal{H}} (\beta \mathbf{I}_{N_r})^{-1} \mathbf{H}_{11}), \alpha - \epsilon \leq \lambda_{\max}(\mathbf{X}) \leq \alpha \right\} \tag{H.3}$$

where $|\mathcal{A}|$ represents the cardinal number of the set $\mathcal{A}$.

The subregion of Ω_1 denoted by Ω_1' can be defined as (H.3), where $\mathbf{A} \triangleq \left(\mathbf{H}_{12}\mathbf{v}_{s_2}\mathbf{v}_{s_2}^{\mathcal{H}}\mathbf{H}_{12}^{\mathcal{H}} + \beta\mathbf{I}_{N_r}\right)$, $\|\mathbf{v}_p^{s_2}\|^2 = 1$.

The subregion of Ω_2 is similarly denoted Ω_2'. Figure H.4 illustrates Ω_1' and Ω_2' for $N_t = N_r = 2$ and $N_t = N_r = 3$, with $\alpha = 1.7, \epsilon = 0.1$. Again,

$$|\Omega_1'| < |\Omega_2'| \tag{H.5}$$

With the same analogy, if the solution set of problem (4.5) for $\mathbf{x}_i \in \mathbb{C}^{N_t \times 1}$ where $N_t = N_r = N_u$ is $\mathcal{A}_1$, and for $N_t = N_r > N_u$ is $\mathcal{A}_2$, then:

$$|\mathcal{A}_1| < |\mathcal{A}_2| \tag{H.6}$$

For GA, which is an evolutionary algorithm, the solution set from using more candidates means a smaller number of generations for GA for the same termination tolerance.

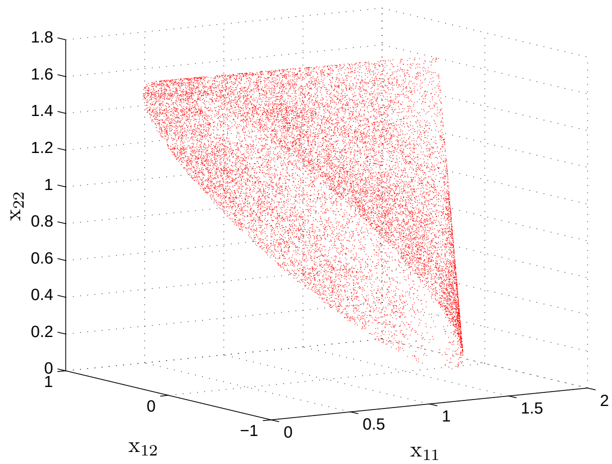

Figure H.1: The region of Ω_1 for $\alpha = 1.7$ and $\delta \to 0$

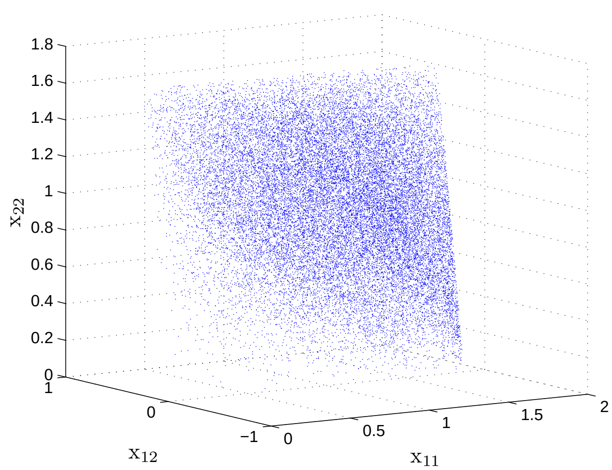

Figure H.2: The region of Ω_2 for $\alpha = 1.7$ and $\delta \to 0$

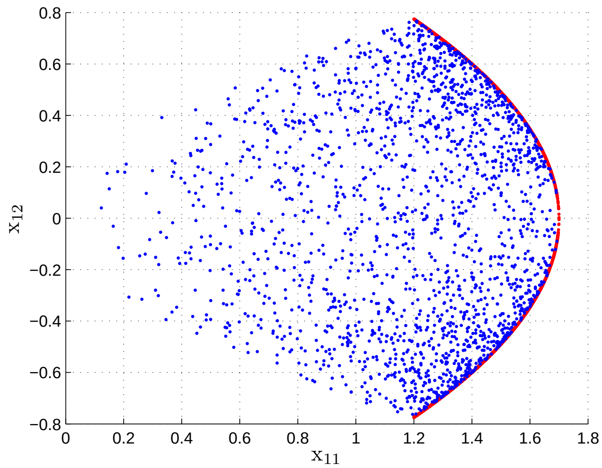

Figure H.3: A cross section of regions Ω_1 (red) and Ω_2 (blue) with $\alpha = 1.7$ and $\delta \to 0$

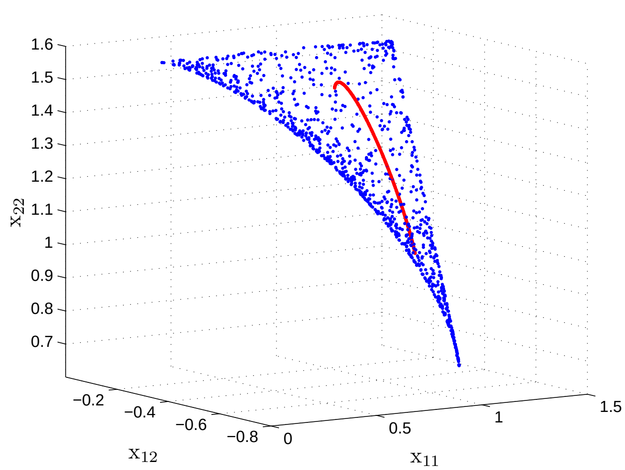

Figure H.4: The region of Ω_1' (red) and Ω_2' (blue) with $\alpha = 1.7, \epsilon = 0.1$

Appendix I

Sum Rate Maximization By EAO

Consider the sum rate problem:

$$\begin{aligned} \max_{\mathbf{v}_i,\ldots,\mathbf{v}_K} \quad & \sum_{i=1}^{K} \log_2 \left| \mathbf{I} + \mathbf{H}_{i,i}\mathbf{v}_i\mathbf{v}_i^{\mathcal{H}}\mathbf{H}_{i,i}^{\mathcal{H}} (\sum_{j\neq i}^{K} \mathbf{H}_{i,j}\mathbf{v}_j\mathbf{v}_j^{\mathcal{H}}\mathbf{H}_{i,j}^{\mathcal{H}} + \sigma_n^2\mathbf{I})^{-1} \right| \\ \text{s.t.} \quad & \mathbf{v}_i^{\mathcal{H}}\mathbf{v}_i \leq 1 \end{aligned} \tag{I.1}$$

This problem can be solved by EAO for K games as well. Define $\mathbf{A} \triangleq (\sum_{j\neq i}^{K} \mathbf{H}_{i,j}\mathbf{v}_j\mathbf{v}_j^{\mathcal{H}}\mathbf{H}_{i,j}^{\mathcal{H}} + \sigma_n^2\mathbf{I})^{-1}$. If we assume that all $\mathbf{v}_j$, $j \neq i$ are known, then the original sum rate problem is reduced to:

$$\begin{aligned} \max_{\mathbf{v}_i} \quad & \log_2 \left| \mathbf{I} + \mathbf{H}_{i,i}\mathbf{v}_i\mathbf{v}_i^{\mathcal{H}}\mathbf{H}_{i,i}^{\mathcal{H}}\mathbf{A} \right| \\ \text{s.t.} \quad & \mathbf{v}_i^{\mathcal{H}}\mathbf{v}_i \leq 1 \end{aligned} \tag{I.2}$$

For equalities $|\mathbf{I} + \mathbf{CD}| = |\mathbf{I} + \mathbf{DC}|$ and $|\mathbf{I} + \mathbf{cd}^T| = 1 + \mathbf{c}^T\mathbf{d}$, we have:

$$\begin{aligned} \max_{\mathbf{v}_i} \quad & \log_2(1 + \mathbf{v}_i^{\mathcal{H}}\mathbf{H}_{i,i}^{\mathcal{H}}\mathbf{A}\mathbf{H}_{i,i}\mathbf{v}_i) \\ \text{s.t.} \quad & \mathbf{v}_i^{\mathcal{H}}\mathbf{v}_i \leq 1 \end{aligned} \tag{I.3}$$

The solution for this simple problem is the closed form $\mathbf{v}_i = \boldsymbol{w}_{\max}(\mathbf{H}_{i,i}^{\mathcal{H}}\mathbf{A}\mathbf{H}_{i,i})$.

For multi-stream case, the sum rate maximization by EAO for K games, transformed to water-filling problem. Define $\mathbf{B} \triangleq \mathbf{H}_{i,i}^{\mathcal{H}}(\sum_{j\neq i}^{K} \mathbf{H}_{i,j}\mathbf{V}_j\mathbf{V}_j^{\mathcal{H}}\mathbf{H}_{i,j}^{\mathcal{H}} + \sigma_n^2\mathbf{I})^{-1}\mathbf{H}_{i,i}$. Similar to above, the original sum rate problem, by EAO, is reduced to:

$$\begin{aligned} \max_{\mathbf{V}_i} \quad & \log_2(1 + \mathbf{V}_i^{\mathcal{H}}\mathbf{B}\mathbf{V}_i) \\ \text{s.t.} \quad & \text{tr}(\mathbf{V}_i^{\mathcal{H}}\mathbf{V}_i) \leq P_i \end{aligned} \tag{I.4}$$

Let the SVD decomposition of $\mathbf{B}$ denoted by $\mathbf{B} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^{\mathcal{H}}$ and $\mathbf{V}_i = \mathbf{U}\begin{bmatrix} \sqrt{x_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sqrt{x_d} \\ \mathbf{0}_{M-d\times 1} & \cdots & \mathbf{0}_{M-d\times 1} \end{bmatrix}$ where $\mathbf{\Lambda}$ is a diagonal $M \times M$ matrix with λ_i on its entries. Assume $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_M$, then the problem, after some matrix manipulations, becomes:

$$\begin{aligned} \max_{x_i} \quad & \sum_{i=1}^{d} \log_2(1 + x_i\lambda_i) \\ \text{s.t.} \quad & \sum_{i=1}^{d} x_i \leq P_i \\ & x_i \geq 0 \end{aligned} \tag{I.5}$$

Its solution is obtained from water-filling:

$$\sum_{i=1}^{d} \max(0, s - \frac{1}{\lambda_i}) = P_i \tag{I.6}$$

the optimal solution ${x_i}^*$ is:

$${x_i}^* = \begin{cases} s - \frac{1}{\lambda_i} & s > \frac{1}{\lambda_i} \\ 0 & s \leq \frac{1}{\lambda_i} \end{cases} \tag{I.7}$$

Appendix J

Challenges for 5G

The challenges for 5G are: avalanche of traffic volume; massive growth in connected devices; large diversity of use cases and requirements. The components of 5G can be summarized as:

- Massive MIMO (more than 100 antennas per terminal)
- Device-to-device (D2D) communications, also known as machine-machine (M2M), internet-of-things (IofT)
- Ultra dense networks
- Higher frequencies (millimeter-wave)
- Moving networks

Also, the 5G requirements are [82]:

- Supporting 0.1-1Gbps per user, i.e. 100 Mbit/s for high mobility users and 1 Gbit/s for low mobility users
- Less energy consumption
- Latency reduction
- More coverage
- More devices per area
- D2D capability

Key concepts arising from scientific papers discussing 5G and beyond 4G wireless communications, are [83, 82, 84, 85]:

- Massive Dense Networks also known as Massive Distributed MIMO provides flexible small cells. With massive MIMO, multiple messages for several terminals can be transmitted on the same time-frequency resource, by maximizing beamforming gain while minimizing interference.

- Advanced interference and mobility management, achieved with the cooperation of different transmission points with overlapped coverage, and encompassing the option of a flexible usage of resources for uplink and downlink transmission in each cell, the option of direct device-to-device transmission and advanced interference cancellation techniques.

- Efficient support of machine-type devices to enable the D2D with potentially higher numbers of connected devices, as well as novel applications such as mission critical control or traffic safety, requiring reduced latency and enhanced reliability.

- The usage of millimeter wave frequencies (e.g. up to 90 GHz) for wireless backhaul and/or access links

- Pervasive networks providing Internet of things, wireless sensor networks and ubiquitous computing. The user can simultaneously be connected to several wireless access technologies and seamlessly move between them. These access technologies can be 2.5G, 3G, 4G, or 5G mobile networks, Wi-Fi, WPAN, or any other future access technology. In 5G, the concept may be further developed into multiple concurrent data transfer paths.

- Multi-hop networks: A major issue in beyond 4G systems is to make the high bit rates available in a larger portion of the cell, especially to users in an exposed position in between several base stations. In current research, this issue is addressed by cellular repeaters and macro-diversity techniques, also known as group cooperative relay, where also users could be potential cooperative nodes.

- Design for flexible spectrum usage for cognitive radio. It would allow different radio technologies to share the same spectrum efficiently by adaptively finding unused spectrum and adapting the transmission scheme to the requirements of the technologies currently sharing the spectrum. This dynamic radio resource management can in principle be achieved in a distributed fashion, and relies on software-defined radio. See the IEEE 802.22 standard for Wireless Regional Area Networks.

- Dynamic Adhoc Wireless Networks (DAWN), essentially identical to Mobile ad hoc network (MANET), Wireless mesh network (WMN) or wireless grids, combined with smart antennas, cooperative diversity and flexible modulation.

- Vandermonde-subspace frequency division multiplexing (VFDM): a modulation scheme to allow the co-existence of macro-cells and cognitive radio small-cells in a two-tiered LTE/4G network.

- IPv6, where a visiting care-of mobile IP address is assigned according to location and connected network.
- Wearable devices with AI capabilities, such as smart watches and optical head-mounted displays for augmented reality
- One unified global standard.
- Real wireless world with no more limitation with access and zone issues.
- User centric (or cell phone developer initiated) network concept instead of operator-initiated (as in 1G) or system developer initiated (as in 2G, 3G and 4G) standards
- World wide wireless web (WWWW), i.e. comprehensive wireless-based web applications that include full multimedia capability beyond 4G speeds.