Essays on Collateral and Central Counterparties

by

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Abstract

The collateral systems commonly employed by many derivatives central counterparties (CCPs), such as the Standard Portfolio Analysis of Risk (SPAN) or the Value-at-Risk (VaR) approach, fail to consider the loss dependence of their clearing members. As a consequence, CCPs are often left exposed to simultaneous extreme losses that could undermine their stability and that of the entire financial system. In this context, this thesis proposes two new collateral methodologies that address this problem. Chapter 2 uses copulas to develop a methodology that accounts for the tail dependence of market participants. This method allows individual margins to increase when clearing firms are more likely to suffer simultaneous extreme losses; thus, reducing the probability and shortfall associated with joint margin exceedances. Chapter 3 proposes a collateral methodology, called CoMargin, which generalizes the VaR approach to a multivariate setting. This method targets and stabilizes the conditional probability of financial distress across clearing members, can be generalized to any number of market participants and can be backtested using formal statistical tests. The empirical sections of Chapter 3 use proprietary data from the Canadian Derivatives Clearing Corporation (CDCC), which include daily observations of the actual trading positions of all of its members from 2003 to 2011. This dataset is the first one of its kind in the economics and finance literature and opens the door to the development of new models that do not have to rely on the strong assumptions made in the past about the trading behaviour of market participants.

Keywords: Collateral; Central Counterparties (CCPs); Derivatives Markets; Extreme Dependence; Tail Risk

A mis padres, que siempre me han brindado su apoyo incondicional y el abrigo de sus consejos, y a mis hermanas que con cariño me motivan a dar lo mejor de mí.

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Chapter 1.

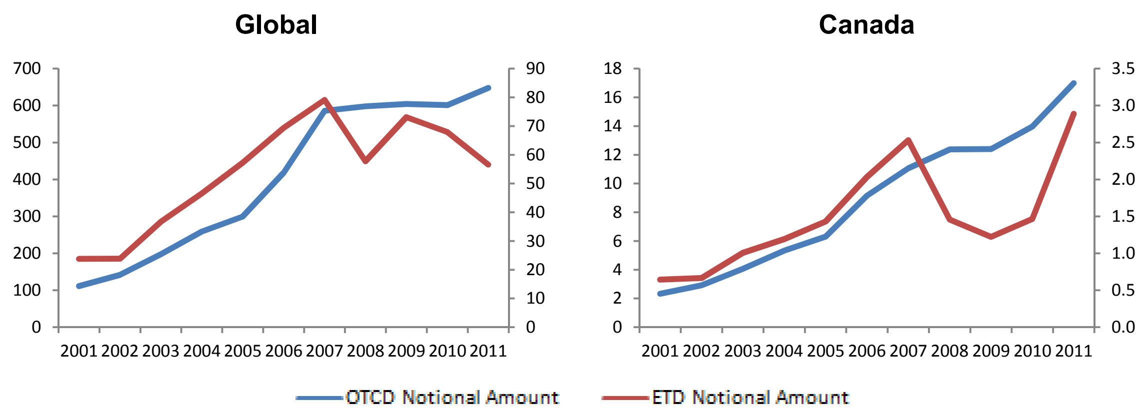
Introduction[1]

As a consequence of the recent economic crisis, one of the most important financial reforms of our lifetime is currently underway. Over-the-counter (OTC) derivatives transactions, which represent the largest financial market by size and credit exposures, are now required to be collateralized and cleared through central counterparties (CCPs). This new environment calls for regulators, academics and practitioners to assess and potentially reconsider the risk management models of CCPs, particularly those employed to set collateral requirements. Yet, despite the increasing systemic importance of these models, academic work in this area is scarce. This thesis bridges some of the gap in the literature by analyzing collateral practices in CCPs and by proposing new methodologies to enhance their stability.

Derivatives securities trade in a global market. Standardized contracts typically trade on exchanges, where market participants are allowed to deal directly with CCPs. More specialized and complex contracts, designed to meet idiosyncratic needs, are commonly traded bilaterally on the OTC market. While both markets have experienced tremendous growth over the last years, the OTC market has led the way with higher notional amounts (see Figure 1.1), current credit exposures (see Figure 1.2) and potential future exposures (see Figure 1.3 and Box 2.1).

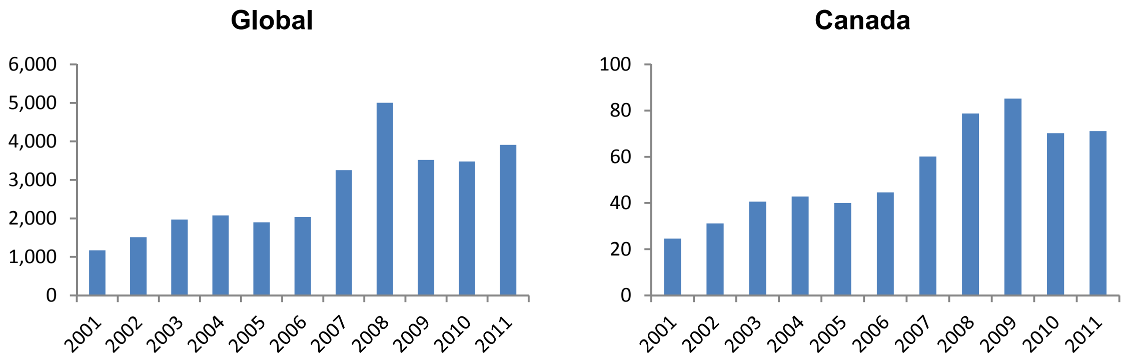
[1] Sections of this chapter are based on Cruz Lopez (2013b) and Cruz Lopez, Mendes and Vikstedt (2013). The views presented here are those of the authors and do not necessarily reflect those of the Bank of Canada.

Figure 1.1. Notional values of the OTC and exchange traded derivatives markets



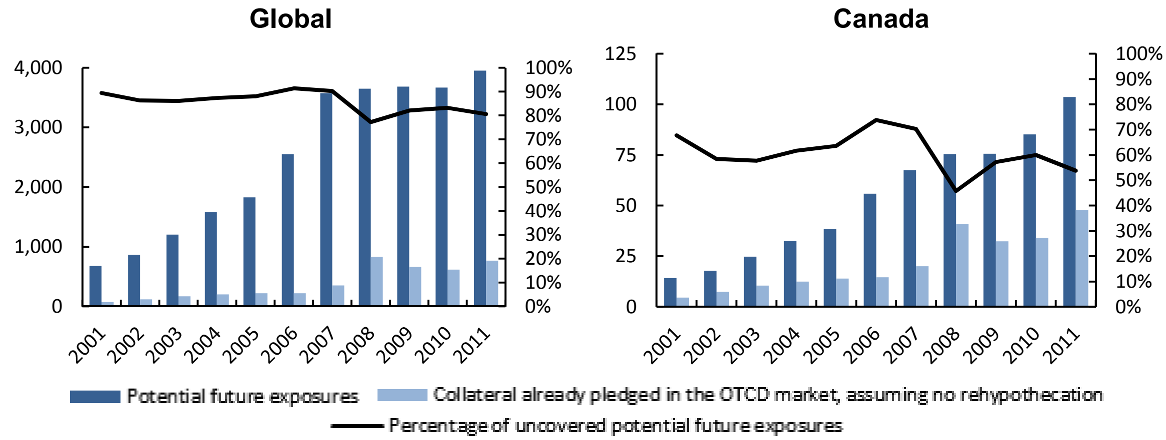
Note: Dollar values are in USD trillions. OTC amounts (blue line) are represented on the left axis. Exchange traded amounts (red line) are represented on the right axis. Global values were obtained from the Bank for International Settlements (2012). Canadian figures correspond to the six largest banks (RBC, TD, BMO, CIBC, BNS and NBC). Canadian values from 2007 to 2011 were obtained from the Office of the Superintendent of Financial Institutions (OSFI, 2012). Values from 2001 to 2006 were obtained assuming that the share of these six banks represents on average 2.10% and 2.7% of the global OTC and exchange traded market, respectively. This assumption is consistent with the average share of the Canadian market since 2007 and the values reported by CMIC (2011).

Figure 1.2. Current credit exposures in the OTC derivatives market.



Note: Dollar values are in USD billions. Global current credit exposures were obtained from the BIS Statistical Release of OTC Derivatives Statistics at end-December 2011 (BIS, 2012). Canadian values correspond to the six largest banks (RBC, TD, BMO, CIBC, BNS and NBC). From 2007 to 2011 they were obtained from the Office of the Superintendent of Financial Institutions (OSFI, 2012). From 2001 to 2006 they were obtained by assuming that the share of these six banks represents on average 2.10% of the global OTCD market. This figure corresponds to the average Canadian share of the global market in notional amounts from 2007 to 2011.

Figure 1.3. Potential future exposures and collateral pledged in the OTC derivatives markets



Note: Dollar values are in USD billions. The solid bars, measured in the left scale, represent the potential future exposures (dark blue) and the collateral pledged in the OTC market (light blue). The black line, measured in the right scale, shows the percentage of potential future exposures that would be left uncovered if all the collateral already pledged in the market was used as initial margin. The amount of pledged collateral corresponds to half of the collateral in circulation reported by ISDA (2012) after correcting for re-hypothecation. Half of this number is used to avoid double-counting, as ISDA sums collateral received and collateral posted. Following Singh (2011), it is assumed that from 2001 to 2007 and from 2008 to 2011, 67% and 58% of collateral in circulation is the product of re-hypothecation, respectively. Potential future exposures are estimated by taking the average ratio of initial margin plus default fund contribution to notional in the exchange traded market (0.61%) and applying it to the notional value of the entire OTCD market. For Canada, this approach generates values that are consistent with those reported by the Office of the Superintendent of Financial Institutions (OSFI, 2012) for the six largest Canadian banks (RBC, TD, BMO, CIBC, BNS and NBC).

Box 2.1. Credit Exposures

Current exposures are defined as the sum of positive market values (or replacement costs) in a portfolio after bilateral netting. They measure the immediate dollar amount that an investor would lose in her OTC derivatives holdings if her counterparties suddenly defaulted. In the context of central clearing, current exposures are determined after marking-to-market all positions. The clearing house then exchanges variation margin across its members to settle all outstanding claims. Therefore, current exposures provide a direct estimate of the amount of collateral needed to settle the OTC derivatives contracts currently outstanding in the market. Current exposures are also known as gross credit exposures, gross positive replacement cost, current exposure after netting or net current exposure, where the word net denotes that exposures are measured after bilateral netting (BIS, 2012).	Potential credit exposures correspond to the maximum dollar amount that an investor could lose in his OTC derivatives holdings during a pre-specified period of time (usually one day), assuming a set of scenarios or distributions. In the context of central clearing, potential credit exposures are determined through scenario analyses and an equivalent amount of initial margin is collected by CCPs to protect them against these potential losses. Therefore, potential credit exposures provide an estimate of the amount of initial margin that would be needed if outstanding OTC derivatives positions were initiated in (or migrated to) a CCP.

During the financial crisis, however, the markets for common OTC transactions, such as credit default swaps, collapsed. The opacity of these markets coupled with their loose collateral policies exacerbated liquidity constraints that lead to a rapid spread of defaults throughout the financial system. Figure 1.3, for example, shows the potential credit exposures in the OTC market that should have been covered with initial margins and compares these values to the amount of collateral actually pledged after correcting for rehypothecation. The black line shows the proportion of potential future exposures that would had been left uncollateralized if all pledged collateral had been liquidated. The figure clearly shows that OTC transactions have traditionally relied on little or no collateral. Centrally cleared transactions, on the other hand, are by construction collateralized in terms of both their current and future potential exposures. This practice allowed most CCPs to remain solvent and fully operational during the crisis.

Therefore, in an effort to reduce systemic risk, on September 2009, the G20 member countries committed to reform derivatives markets by mandating that most OTC contracts be cleared through CCPs and that those remaining bilaterally traded be subject to higher collateral and capital requirements. However, while these reforms are in principle well intended, it is obvious that their success crucially depends on the collateral practices used by CCPs.

The primary role of a CCP is to concentrate and manage credit risk in one location. In derivatives exchanges, clearing houses perform this role along with the clearing function (i.e. confirming, matching, and settling all trades). Each clearing house transacts with a limited number of market participants, known as clearing members, who are allowed to clear their own trades (i.e., conduct proprietary trading), those of their customers, and those of non-clearing firms. Through the process of novation, the clearing house becomes the counterparty and guarantor to every contract in the exchange; thus, reducing the counterparty risk faced by the clearing members. At the same time, the CCP manages its own risk exposures primarily through the use of margining systems.[2]

Margining systems require clearing members to post collateral in a margin account (see Box 2.2). This collateral is used to guarantee the trading obligations of clearing members over a period of time, usually one day, and to protect the CCP against the losses and potential default of its counterparties. Nevertheless, clearing members sometimes experience losses that exceed their posted collateral, leaving them with a negative balance in their margin accounts. If these clearing members delay their payments or default, the CCP has to cover their shortfall with its own funds in order to compensate the counterparties with wining positions. Usually, financing the shortfall of a single clearing member over a limited period of time does not impose a hefty burden on the CCP. However, financing several negative margin balances simultaneously, can significantly affect market liquidity, particularly during volatile periods, and it can eventually erode the resources of the CCP to the point of causing its failure.

[2] Other common credit risk management tools used by CCPs include requiring members to hold minimum capital levels, contribute to a default fund, enter into private insurance arrangements and segregate between client and firm margin accounts (see Jones and Pérignon, 2013).

Box 2.2. What is collateral?

Collateral has traditionally been used by financial market participants to protect against credit exposures, especially for secured lending, repurchase agreements (repos) and derivatives transactions. Depending on the nature and risk of the transaction being covered, collateral can take many forms, ranging from cash to equities or even gold.

Recent literature and financial regulation focuses on two overlapping definitions of collateral. Both define a set of assets suitable for use as a guarantee in a wide range of transactions. The first definition is based on market practice and includes financial assets that have a low risk of default. These assets are known as high-quality assets (HQA). The second definition is based on financial regulation and encompasses high-quality liquid assets (HQLA), the subset of HQA that is deemed sufficiently liquid to meet the requirements of the Basel III Liquidity Coverage Ratio. Common assets in this category include AAA, AA and OECD government securities, covered bonds, U.S. agency debt, local provincial/municipal bonds, and some corporate bonds.

Common securities used as collateral in derivatives markets include cash, U.S. treasuries, government and corporate bonds, and in some cases equities. With the exception of cash, most of these assets are subject to haircuts; that is, price adjustments set by collateral recipients and used to account for variations in the credit quality, volatility and liquidity of pledged assets.

Nevertheless, despite the fact that simultaneous losses are the major threat to the stability of CCPs, the collateral systems commonly employed by many of these institutions, such as the Standard Portfolio Analysis of Risk (SPAN) or the Value-at-Risk (VaR) approach, only focus on individual risk. As a consequence, these systems fail to consider the loss dependence of clearing members and often leave the CCP exposed to simultaneous extreme losses that could undermine its stability and that of the entire financial system.

In this context, this thesis proposes two new collateral frameworks that address this problem. Chapter 2, titled “Clearing House, Margin Requirements, and Systemic Risk”, is based on Cruz Lopez, Harris and Pérignon (2011), a publication in the journal *Review of Futures Markets* that introduces a methodology called Tail-Dependent margin. Our approach aims at reducing the probability and shortfall associated with joint margin exceedances by increasing the collateral requirements of clearing members when they are more likely to experience simultaneous extreme losses. Our methodology relies on the use of copulas to assess the P&L tail dependence coefficient of clearing firms and uses this parameter to adjust the corresponding VaR margin estimates.

Chapter 3, titled “CoMargin: A System to Enhance Financial Stability”, is based on Cruz Lopez, Harris, Hurlin and Pérignon (2013), a recently peer-reviewed Bank of Canada working paper that continues my research on optimal collateral methodologies. In this chapter my co-authors and I introduce another collateral system, called CoMargin, which generalizes the VaR approach to a multivariate setting. Like the Tail-Dependent system of Chapter 2, CoMargin depends on both the risk of a given market participant and its interdependence with other participants. However, instead of trying to minimize the occurrence of joint margin exceedances and their shortfalls, CoMargin aims at stabilizing the conditional probability of financial distress across clearing members. This mechanism allows the CCP to have a predictable and stable amount of collateral to cover its exposures even after one or more of its clearing members have suffered an extreme loss. The chapter shows theoretically and empirically that CoMargin outperforms existing margining systems, particularly when trading similarities across clearing members or comovement among underlying assets increase, as was the case during the recent crisis.

While similar in principle, the Tail-Dependent and CoMargin approaches offer features that are suitable to different environments. Tail-Dependent margins are more appropriate when the objective is to minimize the probability and shortfall associated with simultaneous margin exceedances, as would be the case when a CCP is small or a regulator or government agency is the lender of last resort. In addition, this approach could be easier to implement when standardization across contracts, market participants and CCPs is important for regulatory and risk management purposes but the financial state of clearing members is too costly to assess or monitor. More specifically, unlike the CoMargin approach, the Tail-Dependent system has the advantage that the CCP or regulator does not need to set an arbitrary threshold to define an extreme loss. Instead, extreme losses in this setting are endogenously defined as the limit to minus infinity in the P&L distribution. Although, specifying an arbitrary threshold in CoMargin is no different than, for example, setting a discretionary coverage level under the VaR approach, this could lead to heterogeneity in the implementation of the model across contracts and CCPs.

On the other hand, CoMargin is suitable for enhancing the stability of conditional coverage probabilities and the resilience of the CCP, which is important when a lender of last resort is impractical. This situation can occur, for example, when global CCPs fall under multiple-jurisdictions or are so systemically important that their failure could lead to a global cascade of defaults that cannot be easily contained. In addition, the CoMargin system has the unique advantage of offering a backtesting methodology based on formal statistical tests. Therefore, if the benefits of monitoring outweigh the costs, this system allows CCPs and regulators to have a clear and objective assessment of their collateral policies.

This thesis provides at least three significant contributions to the literature. First, it offers the first empirical assessment of the performance of different margining systems that is based on actual, and not assumed, portfolio positions. Second, it adds to the literature of financial risk management by proposing two new collateral methodologies that can be easily implemented and expanded to more general settings that require the management of counterparty risk (e.g., capital requirements, credit risk monitoring, etc.). Finally, the dataset used to conduct the empirical analyses constitutes by itself a significant contribution. A major constraint in empirical finance in general and risk management in particular is the lack of data that can be used for publishable research. In the best of cases, this data is highly confidential and in most cases, it does not exist or is just recently being collected by regulators. Through my work at the Bank of Canada and mutual cooperation with the Canadian Derivatives Clearing Corporation (CDCC), the clearing house of the TMX Montreal Exchange, I was able to collect, over a period of a year and a half, the proprietary data used in the empirical analysis of Chapter 3. This dataset, the first one of its kind in the economics and finance literature, includes the daily P&L, margin requirements, default fund contributions and trading positions on the most actively traded Canadian derivatives contracts for all CDCC members between January 2, 2003 and March 31, 2011. These data open the door to the development of new models that do not have to rely on the strong assumptions made in the past about the trading behaviour of market participants. More importantly, it allows us to depart from the traditional approach in empirical finance of implying price dynamics from other price dynamics, by letting us observe the quantities of the assets held and traded by market participants.

Chapter 2.

Clearing House, Margin Requirements, and Systemic Risk[3]

2.1. Introduction

Economic turmoil in recent years has heightened the need for well-functioning clearing facilities in derivatives markets, particularly when large market participants are in financial distress and eventually default (Acworth 2009; Pirrong 2009; Duffie and Zhu 2011). In a derivatives exchange, the clearing house is responsible for the clearing function, which consists of confirming, matching, and settling all trades. The clearing house operates with a limited number of clearing firms or futures commission merchants, which are private firms that have the right to clear trades for themselves (i.e., proprietary trading), for their own customers, and for the customers of non-clearing firms.[4]

In order to mitigate default risk, the clearing house requires clearing members to post margin (i.e., collateral). At the end of each day, the clearing house marks-to-market all outstanding trading positions and adjusts margins accordingly. A problematic situation arises, however, when the daily loss of a clearing firm exceeds its posted collateral. In this case, the firm may decide to default on its obligations and the clearing house may have to draw on its default fund to compensate the winning counterparties.[5] Eventually,

[3] This chapter is based on Cruz Lopez, Harris and Pérignon (2011).

[4] While derivatives clearing systems have been developed to deal with exchange-traded futures and options, regulations have been adopted by G20 countries to require over-the-counter derivatives to go through similar clearing processes (Acharya et al., 2009; US Congress' OTC Derivatives Market Act of 2009; US Department of Treasury, 2009; Duffie, Li, and Lubke, 2010). In response, the Chicago Mercantile Exchange, Intercontinental Exchange, and EUREX have recently created clearing facilities for Credit Default Swaps (CDS).

[5] Although exogenous events unrelated to losses in exchange-traded derivatives might also result in default, we do not specifically address these situations.

the clearing house may default as well, after its default fund has been exhausted. This scenario, as unlikely as it may appear, is plausible, especially if several large clearing firms exceed their margin simultaneously and ultimately default. It is also economically significant, because the failure of a clearing house would cause a major systemic shock that could spread default risk throughout the financial system.

Current practice in derivatives exchanges is to set the margin level of a derivative contract in such a way that it leads to a target probability of a loss in excess of collected collateral (Figlewski 1984; Booth et al. 1997; Cotter 2001). Similarly, for a portfolio of derivatives, the margin requirement is derived from a distribution of simulated losses associated to the current portfolio positions (e.g. the Chicago Mercantile Exchange's SPAN system). We depart from this traditional view and account for tail dependence in the losses of clearing firms when setting collateral requirements. More specifically, we allow the margin requirements of a particular firm to depend not only on its own trading positions, but also, potentially, on other clearing firms' positions. The basic intuition behind this concept is that the collateral requirement of a given clearing firm should increase when it is more likely to experience a loss that exceeds its posted margin at the same time as other clearing firms.

Simultaneous margin exceeding losses and defaults are more likely to occur when the trading positions across clearing firms are similar or when they have similar risk exposures due to increases in the comovement of underlying assets. Conceptually, the main source of similarities in trading positions across large clearing firms is that they share a common (and superior) information set (Jones and Pérignon, 2013). This informational advantage leads to similar directional trades. Furthermore, much of the proprietary trading activity on derivatives exchanges consists of arbitraging futures and over-the-counter (OTC) markets or cash markets (e.g. cash-futures arbitrage of the S&P 500 index, eurodollar-interest rate swap arbitrage, etc.). As a result, if large clearing firms exploit similar arbitrage opportunities, they will have similar trading positions. Empirical evidence of correlated trading among large financial institutions is found in many settings, including futures markets. Using data for all Chicago Mercantile Exchange's (CME) clearing firms, for instance, Jones and Pérignon (2013) show that extreme losses by systemically-important clearing firms tend to cluster. This finding suggests that the derivative positions of the largest trading firms can be at times very similar.

Our approach for computing margins can be summarized as follows. We start with the trading positions of each clearing firm at the end of a given day. We then consider a series of scenarios in which both the level and the volatility of all underlying assets are shocked by an arbitrary amount – in the spirit of *stress testing*. For each scenario, we mark-to-model the clearing firm's portfolio and compute the associated hypothetical profit-and-loss (hereafter P&L). These hypothetical amounts are used to estimate the Value-at-Risk (VaR) margin requirement of each clearing firm, which is defined as the α% quantile of the simulated P&Ls. In addition, the hypothetical P&Ls are used to compute the coefficient of lower tail dependence for each pair of firms. This coefficient is defined as the probability of two clearing members having simultaneous extreme losses. Our approach sets the collateral requirement of each clearing firm as a function of its highest coefficient of tail dependence with respect to every other firm.

Through the use of simulations, we show that accounting for interdependencies among clearing members decreases the likelihood of simultaneous margin exceeding losses and the shortfall associated with these events. Both of these features greatly reduce the systemic risk concerns associated with clearing houses. In addition, our methodology displays several attractive features. First, it is perfectly compatible with existing risk management techniques in place in derivatives exchanges, such as the SPAN system (CME, 2009). Second, our methodology can be applied at a daily or even higher frequency. This is important as an increasing number of derivatives exchanges mark-to-market positions twice a day (e.g. EUREX). Third, our approach differs from the "concentration risk" collateral method, which is most typically applied at the individual firm level. For instance, the Chicago Mercantile Exchange's clearing house monitors concentrations by focusing on the proportion of open interest on a given contract that is controlled by a single clearing firm, and it assigns additional margin to reflect the incremental exposure due to concentration.

In terms of methodology, this chapter is at the confluence of two streams of literature. First, we rely on modeling techniques for extreme dependence as in Longin and Solnik (2001), Ang and Chen (2002), Poon, Rockinger, and Tawn (2004), Patton

(2008), and Christoffersen et al. (2012). While previous papers focus on stock or hedge fund returns, we show that tail dependence can also be very useful to jointly model clearing members' P&L on a derivatives exchange. Second, our analysis builds on the recent literature on systemic risk. Adrian and Brunnermeier (2011) introduce the *CoVaR* measure that is the VaR of the financial system conditional on the distress of a given financial institution. Then they estimate the ΔCoVaR(firm i) = CoVaR(system|firm i) - VaR(system), which captures the marginal contribution of a particular institution to the overall systemic risk. Related studies by Acharya et al. (2010) and Brownlees and Engle (2012) focus on the *Marginal Expected Shortfall* of a given bank, defined as the expected loss of a particular firm conditional on the overall banking sector being in distress. Similar to these papers, we measure, and attempt to internalize, the potentially negative externalities of having interconnected market participants. Although in the same spirit, we use a new and different methodology which focuses on margin requirements and the risk that correlated positions pose to the clearing house.

The outline of this chapter is the following. In Section 2.2, we show how to estimate tail dependence among clearing firm losses. In Section 2.3, we formally describe our methodology to set collateral as a function of tail dependence. We assess the performance of our method using simulations in Section 2.4. Section 2.5 summarizes and concludes this chapter.

2.2. Tail Dependence

In derivatives markets, margins serve as performance bonds to guard against default. In our work, the *performance bond* $B_{i,t}$ represents the margin requirement imposed by the clearing house on clearing firm i at the end of day t, for $i = 1, \ldots, N$. This performance bond depends on the outstanding trading positions of the clearing firm. The *variation margin* $V_{i,t}$ represents the aggregate mark-to-market profit or loss of clearing firm i on day t. The *relative variation margin* $R_{i,t}$ is defined as:

$$R_{i,t} = \frac{V_{i,t}}{B_{i,t-1}} \tag{2.1}$$

Clearing firm i experiences a margin exceeding loss, or an exceedance, at time t if $R_{i,t} < -1$, or equivalently if $B_{i,t-1} + V_{i,t} < 0$, since in this case the trading loss exceeds posted collateral. In such a situation, the clearing firm may decide to default, which would generate a shortfall in the system that needs to be covered by the clearing house.

By definition, tail dependence measures the probability of two random variables having simultaneous extreme events in the same direction. We define the coefficients of upper and lower tail dependence to quantify the comovement in revenues across clearing firms in extreme market conditions. In our context, the tail dependence structure captures the degree of diversification across clearing firms and the likelihood of having simultaneous margin exceeding losses across several clearing firms. The coefficient of *upper tail dependence* of the relative variation margins of clearing firms i and j at time t is defined as:

$$\tau_{i,j}^{U} = \lim_{u \to 1} Pr\left[R_i \geq F_i^{-1}(u) | R_j \geq F_j^{-1}(u)\right] = \lim_{u \to 1} Pr\left[R_j \geq F_j^{-1}(u) | R_i \geq F_i^{-1}(u)\right] \qquad (2.2)$$

where $F_i(R_i)$ denotes the marginal cumulative distribution function of R_i for $i = 1, \dots, N$, and $u \in (0,1)$ represents the marginal cumulative distribution level. Likewise, the coefficient of *lower tail dependence* of the relative variation margins of clearing firms i and j at time t is defined as:

$$\tau_{i,j}^{L} = \lim_{u \to 0} Pr\left[R_i \leq F_i^{-1}(u) | R_j \leq F_j^{-1}(u)\right] = \lim_{u \to 0} Pr\left[R_j \leq F_j^{-1}(u) | R_i \leq F_i^{-1}(u)\right] \qquad (2.3)$$

Because we are primarily concerned with shortfalls in the clearing system, in the following sections we focus on the lower tail, and simplify the notation as follows: $\tau_{i,j} = \tau_{i,j}^{L}$.

We model trading revenue dependence across clearing firms by using a bivariate copula (Patton 2009). A copula is a function that links together marginal probability distribution functions, say $F_i(R_i)$ and $F_j(R_j)$, to form a multivariate probability distribution function, in this case $F(R_i, R_j)$. According to Sklar's Theorem (Sklar, 1959), if the marginal distributions are continuous, there exists a unique copula function C such that:

$$F(R_i, R_j) = C(F_i(R_i), F_j(R_j)) \tag{2.4}$$

Several features of copulas are useful in our context. First, marginal distributions do not need to be similar to each other. Second, the choice of the copula is not constrained by the choice of the marginal distributions. Third, copulas can be used with N marginal distributions. Fourth, the use of copula functions enables us to model the tails of the marginal distributions and tail dependence separately. This last point is very important in our case because in a multivariate setting, the likelihood of an extreme event can increase either because of fatter tails in the marginal distributions or because of fatter tails in the joint distribution function.

A natural candidate that allows us to incorporate tail dependence is the Student t-copula. Let t_v be the univariate Student t probability distribution function with v degrees of freedom. Then, for continuous marginal distributions, $F_i(R_i)$, the bivariate Student t-copula, $T_{\rho,v}$, is defined as:

$$T_{\rho,v}\left(F_i(R_i), F_j(R_j)\right) = t_{\rho,v}(R_i, R_j) \tag{2.5}$$

where $t_{\rho,v}$ is the bivariate distribution corresponding to t_v and $\rho \in [-1,1]$ is the correlation coefficient between R_i and R_j.

A Student t-copula corresponds to the dependence structure implied by a multivariate Student t distribution. It is fully defined by the correlation of the implicit variables, ρ, and the degrees of freedom, v. The degrees of freedom define the probability mass assigned to the extreme co-movements of the relative variation margins (both positive and negative). In addition, this copula assigns a higher probability to joint extreme events, relative to the Gaussian copula, the lower the degrees of freedom, because a Student *t* copula with $v_t \rightarrow \infty$ corresponds to a Gaussian copula.

Student t-copulas allow us to readily obtain an estimate of the coefficient of lower tail dependence based on the correlation coefficient and the degrees of freedom (Cherubini, Luciano, and Vecchiato 2004 and Schmidt, 2006):

$$\tau_{i,j} = 2t_{\upsilon+1}\left(-\sqrt{\upsilon+1}\sqrt{\frac{1-\rho}{1+\rho}}\right) \tag{2.6}$$

As can be seen from this equation, two parameters, the correlation coefficient and the degrees of freedom, fully describe the dependence structure of trading revenues. Intuitively, larger correlations and lower degrees of freedom lead to higher tail dependence.

We implement a two-stage semi-parametric approach to estimate the pairwise copulas across all clearing firms. The first stage consists of estimating the empirical marginal distribution of the trading revenues of each clearing firm. The second stage consists of estimating the t-copula parameters, ρ and υ, for every pair of clearing members through maximum likelihood (Genest, Ghoudi, and Rivest 1995).

2.3. Margin Requirements

In this section, we propose a new way of setting margin requirements for clearing firms. Our approach accounts for both tail risk and tail dependence structure across clearing firms. We consider a derivatives exchange with N clearing firms and D derivatives contracts (futures and options) written on A underlying assets. Let $w_{i,t}$ be the number of contracts in the derivatives portfolio of clearing firm i at the end of day t:

$$w_{i,t} = \begin{bmatrix} w_{i,1,t} \\ \vdots \\ w_{i,D,t} \end{bmatrix} \tag{2.7}$$

We consider two ways of computing the margin requirement of a clearing firm, which we present in turn below.

2.3.1. *VaR Margin*

The VaR margin requirement is applied on a firm by firm basis, without regard to correlations across firms. As in the SPAN system utilized by the CME, we consider a series of *S* scenarios based on potential one-day ahead changes in the value (ΔX) and volatility ($\Delta\sigma_X$) of the underlying assets, as well as in the time to expiration of the derivatives products. For each of the *S* scenarios, we revaluate the portfolio (i.e., we "mark-to-model" its positions) and compute the associated hypothetical P&L or variation margin of the portfolio. Thus, for each clearing firm and date t, we obtain a simulated sample of one-period-ahead variation margins, denoted $\{v_{i,t+1}^s\}_{s=1}^S$, that can be used to estimate the VaR margin requirement as follows:

Definition 2.1: The VaR margin of firm i, B_i, corresponds to the α% quantile of its P&L distribution:

$$\Pr(V_{i,t+1} \leq -B_{i,t}) = \alpha \tag{2.8}$$

Thus, VaR margin accounts for the potential margin exceeding losses of a particular clearing firm, but it ignores its interdependence with other clearing members. In this case, the total collateral collected by the clearing house at time t from all clearing firms is:

$$B_t = \sum_{i=1}^{N} B_{i,t} \tag{2.9}$$

2.3.2. *Tail-Dependent Margin*

The Tail-Dependent margin requirement is based not only on the magnitude of simulated losses (as in the VaR margin requirement), but also on the dependence structure across clearing firms' potential losses. Our objective is to increase the collateral requirement of each individual firm by an amount proportional to its degree of dependence with other firms. Therefore, the increase in collateral is proportional to the incremental risk faced by the clearing house from potentially correlated losses among its members.

Consider the portfolios of derivatives contracts of two clearing firms at the end of a given day, $w_{i,t}$ and $w_{j,t}$. For each clearing firm, we compute the variation margins generated by the S scenarios described in the previous section, $\{v_{i,t+1}^{s}\}_{s=1}^{S}$ and $\{v_{j,t+1}^{s}\}_{s=1}^{S}$, respectively, and compute $B_{i,t}$ and $B_{j,t}$ as in equation (2.8). The tail dependence between the clearing firms' simulated relative variation margins is given by:

$$\tilde{\tau}_{i,j,t} = \lim_{u \to 0} Pr\left[\tilde{R}_{i,t+1} \leq F_{i,t+1}^{-1}(u) | \tilde{R}_{j,t+1} \leq F_{j,t+1}^{-1}(u)\right] \tag{2.10}$$

where $\tilde{R}_{i,t+1} = \tilde{V}_{i,t+1}/B_{i,t}$. With N clearing firms, we end up with $N(N-1)/2$ tail dependence coefficients, which can be presented in a lower diagonal matrix:

$$\begin{bmatrix} & & & \\ \tilde{\tau}_{2,1} & & & \\ \tilde{\tau}_{3,1} & \tilde{\tau}_{3,2} & & \\ \vdots & & \ddots & \\ \tilde{\tau}_{N,1} & \tilde{\tau}_{N,2} & \dots & \tilde{\tau}_{N,N-1} \end{bmatrix}$$

For each clearing firm, we conservatively set its collateral requirement as a function of the highest coefficient of tail dependence with respect to all other clearing firms:

$$\tilde{\tau}_{i,t} = max\{\tilde{\tau}_{i,j,t}\}_{j=1, j \neq i}^{N} \tag{2.11}$$

We use this parameter to estimate the Tail-Dependent margin requirement as follows: [6]

Definition 2.2: The Tail-Dependent margin of firm i, $B_{i,t}^*$, corresponds to:

$$B_{i,t}^* = B_{i,t} \times e^{max\{\gamma(\tilde{\tau}_{i,t} - \underline{\tau});0\}} \tag{2.12}$$

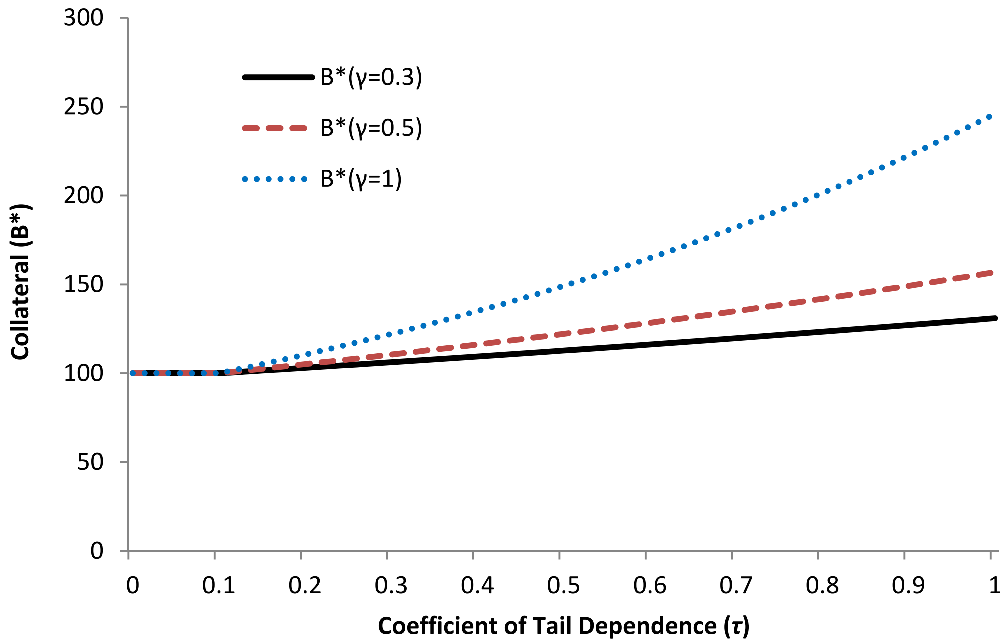
where γ is the tail dependence aversion coefficient and $\underline{\tau}$ is a threshold tail dependence coefficient below which the collateral is not affected, i.e., $B_{i,t}^* = B_{i,t}$ if $\tilde{\tau}_{i,t} \leq \underline{\tau}$. Thus, the total collateral collected by the clearing house becomes:

$$B_t^* = \sum_{i=1}^{N} B_{i,t}^* \tag{2.13}$$

Notice that in the degenerative case where $\gamma = 0$ or if $\tilde{\tau}_{i,t} \leq \underline{\tau}$, we obtain the VaR margin requirement B. Thus, the VaR collateral requirement is a special case of the Tail-Dependent margin requirement. In other words, our approach can be seen as a generalized VaR system. An implication of this result is that $B_t^* \geq B_t$. As an illustration, we plot in Figure 2.1, the level of Tail-Dependent margin for different coefficients of tail dependence aversion ($\gamma = 0.3,0.5,1$), $\underline{\tau} = 0.10$, $B = 100$, and for a tail parameter ranging between 0 and 1. Notice that no additional collateral is required for low coefficients of tail dependence. The required collateral increases with higher tail dependence aversion, a choice variable for the clearing house, and with higher tail dependence, a parameter that can be estimated from simulated trading revenues.

[6] As a nested case, the VaR margin requirement implies zero correlations.

Figure 2.1. Tail-Dependent collateral



Note: This figure presents the level of Tail-Dependent margin B^* as a function of the coefficient of tail dependence τ. The VaR margin requirement B is assumed to be equal to 100 and the threshold tail dependence coefficient $\underline{\tau}$ (below which collateral is not affected) to 0.10. The tail dependence aversion coefficient γ of the clearing house varies between 0.3 and 1.

2.4. Controlled Experiment

To demonstrate the difference between the VaR and Tail-Dependent margin requirements, we consider a derivatives exchange with N clearing firms and two call options written on different underlying assets. Four clearing firms are assumed to be systemically important $(n = 4)$ due to their size, so we focus on their margin requirements. Panel A of Table 2.1 displays the trading positions of these systemically important members in three different states: (1) low tail dependence, (2) moderate tail dependence, and (3) high tail dependence. The first state is obtained by selecting orthogonal trading positions across the systemically important firms. For the remaining states, the level of tail dependence is gradually increased by allowing the second firm to hold a position that progressively resembles that of the first. Notice, however, that the positions of the first, third, and fourth clearing firms remain constant across states. In addition, non-systemically important clearing firms are assumed to clear the market in every state. Thus, each option contract is always in zero-net supply.

In order to simulate the variation margins for each clearing firm, we define S scenarios that combine potential one-day changes in the value of the underlying assets, ΔX_1 and ΔX_2, with changes in volatility, $\Delta\sigma_{X_1}$ and $\Delta\sigma_{X_2}$. For each scenario, we mark-to-model the positions using the Black-Scholes model and generate a hypothetical change in the value of the portfolio held by each clearing firm. We then compute the coefficients of tail dependence between the simulated relative variation margins as described in equation (2.10). Panel B of Table 2.1 shows the estimated coefficients of tail dependence and Table 2.2 shows the parameter values used for this controlled experiment.

Panel C of Table 2.1 compares three ways of computing collateral. The first two are the VaR (B) and the Tail-Dependent margin requirement (B^*) discussed earlier.[7] The third collateral system aims at being budget-neutral and provides a better benchmark against which to compare the allocations of the Tail-Dependent margining system because it collects the same amount of aggregate collateral. This *Budget-Neutral* margin requirement is defined as:

$$B_{i,t}^0 = B_{i,t} + \frac{B_t^* - B_t}{n} \quad \text{for } \; i = 1, \ldots, n \tag{2.14}$$

where the budget-neutral condition is:

$$\sum_{i=1}^{n} B_{i,t}^0 = \sum_{i=1}^{n} B_{i,t}^* \tag{2.15}$$

[7] See equation (2.8) for the definition of VaR margin (B) and equation (2.12) for the definition of the Tail-Dependent margin requirement (B^*).

and from equation (2.12) it follows that $B_{i,t}^{0} = B_{i,t} = B_{i,t}^{*}$ when $\tilde{\tau}_{i,t} \le \underline{\tau}$ or $\gamma = 0$.

The results presented in Panel C of Table 2.1 show that the three margining systems are equivalent in the low-dependence state and that they diverge as the level of tail dependence increases to 0.247 in the moderate-dependence state, and to 0.908 in the high-dependence state. The equivalence across margining systems in the low-dependence state arises because the tail dependence coefficients are virtually zero, thus, $\tilde{\tau}_{i,t} \le \underline{\tau}$ and $B_{i,t}^{*} = B_{i,t}$ for all clearing firms. In other words, when default risk is well-diversified among clearing firms, the Tail-Dependent margin system converges to the VaR margin system. On the other hand, in the moderate and high-dependence states, the Tail-Dependent margin requirement for clearing firms 1 and 2 increases due to their progressively homogeneous trading positions. This homogeneity is captured by the higher coefficient of tail dependence that is incorporated into B^{*}.

Notice, however, that the VaR and Tail-Dependent margin requirements of firms 3 and 4 remain unchanged across states as their positions stay orthogonal relative to those of the other members. This is not true for the Budget-Neutral case. The Budget-Neutral margin requirement increases for all members in the moderate and high-dependence states. This situation arises because the additional collateral that would be collected under the Tail-Dependent margining system is now collected across all systemically important clearing firms. As a consequence, the Budget-Neutral margin requirements of firms 3 and 4 increase in the moderate and high dependence state due to the increased tail dependence between firms 1 and 2.

Table 2.1. Controlled experiment

	Low Tail Dependence				Moderate Tail Dependence				High Tail Dependence			
	1	2	3	4	1	2	3	4	1	2	3	4
Panel A: Trading Positions												
d=1	100	30	-50	-100	100	125	-50	-100	100	95	-50	-100
d=2	100	-170	150	-100	100	75	150	-100	100	105	150	-100
Panel B: Tail Dependence Coefficients												
$\tilde{\tau}_{2,j}$	.000	.	.	.	.247	.	.	.	.908	.	.	.
$\tilde{\tau}_{3,j}$	.000	.000	.	.	.000	.000	.	.	.000	.000	.	.
$\tilde{\tau}_{4,j}$	.000	.000	.000	.	.000	.000	.000	.	.000	.000	.000	.
$\tilde{\tau}_i$	.000	.000	.000	.000	.247	.247	.000	.000	.908	.908	.000	.000
Panel C: Margins												
B_i	3,849	6,228	4,310	5,319	3,849	3,918	4,310	5,319	3,849	3,851	4,310	5,319
B_i^*	3,849	6,228	4,310	5,319	4,022	4,094	4,310	5,319	4,905	4,908	4,310	5,319
B_i^0	3,849	6,228	4,310	5,319	3,936	4,005	4,397	5,406	4,377	4,380	4,839	5,847
p_i	.050	.050	.050	.050	.050	.050	.050	.050	.050	.050	.050	.050
p_i^*	.050	.050	.050	.050	.041	.038	.050	.050	.007	.007	.050	.050
p_i^0	.050	.050	.050	.050	.045	.044	.046	.046	.022	.022	.026	.033

Note: Panel A presents the trading positions of the four systemically important clearing firms in two option contracts (*d* = 1,2) when tail dependence is low, moderate, and high. Panel B displays the tail dependence coefficients among pairs of clearing firms ($\tilde{\tau}_{i,j}$) and the highest coefficient of tail dependence across all pairs ($\tilde{\tau}_i$). Panel C show the VaR (B_i),Tail-Dependent (B_i^*), and Budget-Neutral margin (B_i^0). When computing Tail-Dependent margins, we use a tail dependence aversion coefficient γ of 0.3 and a threshold tail dependence coefficient $\underline{\tau}$ of 0.1. Finally, the p_i variables denote the probability of a clearing firm experiencing a margin exceeding loss: $p_i = Pr[V_{i,t+1} \leq -B_{i,t}]$, $p_i^* = Pr[V_{i,t+1} \leq -B_{i,t}^*]$, and $p_i^0 = Pr[V_{i,t+1} \leq -B_{i,t}^0]$.

Table 2.2. Parameters used for the controlled experiment

Parameter	Value
A. Market and Clearing Members	
Number of derivatives securities (*D*)	2
Number of underlying assets (*A*)	2
Number of systemically important clearing members (*n*)	4
B. Underlying Assets	
Value of underlying asset 1 at $t = 0$	\$100
Value of underlying asset 2 at $t = 0$	\$100
C. Derivatives Securities	
Strike price of option contract 1	\$100
Strike price of option contract 2	\$100
Time to maturity of option contract 1	1 year
Time to maturity of option contract 2	1 year
D. Margining Systems	
Variation range in the value of the underlying assets	$\pm 50\%$
Variation range in the volatility of the underlying assets' returns	$\pm 50\%$
Number of scenarios for the value of the underlying asset and its volatility (S)	10,000
Quantile for the VaR margin system (α)	5%
Tail dependence aversion coefficient (γ)	0.3
Threshold tail dependence coefficient ($\underline{\tau}$)	0.1

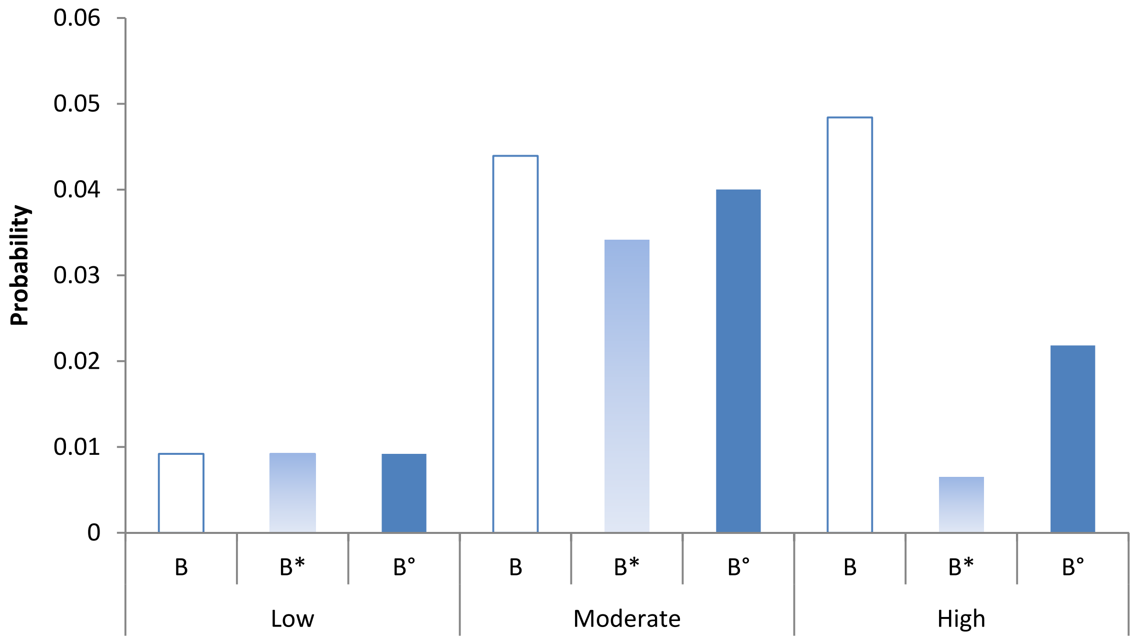
In order to assess the effectiveness of each margining system, we now turn our attention to their relative performance. In order to do this, we simulate changes in the value of the call options by randomly selecting one of the S scenarios. For each margining system, we compute the probability of margin exceedances (i.e., the probability that $B_{i,t-1} + V_{i,t} < 0$) across clearing firms, the probability of joint margin exceedances, and the magnitude of the average margin shortfall given joint margin exceedances. The bottom part of Panel C in Table 2.1 shows the probability of margin exceedance across clearing firms. Since the quantile for the VaR margining system, α, was set to 5% in the simulation (see Table 2.2), the VaR margin system has an exceedance probability of 5% in all scenarios by construction. Similarly, in the low-dependence state, when $B_{i,t} = B^*_{i,t} = B^0_{i,t}$ for all clearing firms, the probability of a margin exceedance is 5% across margining systems. In the moderate and high-dependence states, however, the probability of exceedance is lower for firms 1 and 2 under the Tail-Dependent margining system and lower for all firms under the Budget-Neutral system because more collateral is required relative to the VaR margin case.

At a first glance, this result would suggest that the Budget-Neutral system performs better than the alternatives because it reduces the unconditional probability of margin exceedances across clearing firms. However, Figure 2.2 shows that the Tail-Dependent margining system actually provides a better allocation of margin requirements. More specifically, the figure shows that the probability of joint margin exceedances is lower under the Tail-Dependent margining system, particularly when tail dependence is high.

Notice that the probability of joint margin exceedances increases monotonically with tail dependence under the VaR margin system. On the other hand, for the Tail-Dependent system, this probability first increases in the moderate-dependence state and then decreases in the high-dependence state. This result arises due to the value of the tail dependence aversion coefficient, $\gamma =$ 0.3, and the value of the threshold tail dependence coefficient, $\underline{\tau}$ = 0.1 (see Table 2.2), which translates into a slight increase in the required margin for firms 1 and 2 (an additional $173 and $176, respectively) in the moderate tail dependence state, and a significantly larger increase (an additional $1,056 and $1,057 respectively) in the high tail dependence state. Similar results can be observed for the Budget-Neutral system for the same reasons. A monotonic decrease of the probability of joint margin exceedances could be obtained for the Tail-Dependent collateral system if a higher value of γ or a $\underline{\tau}$ of 0 is selected.

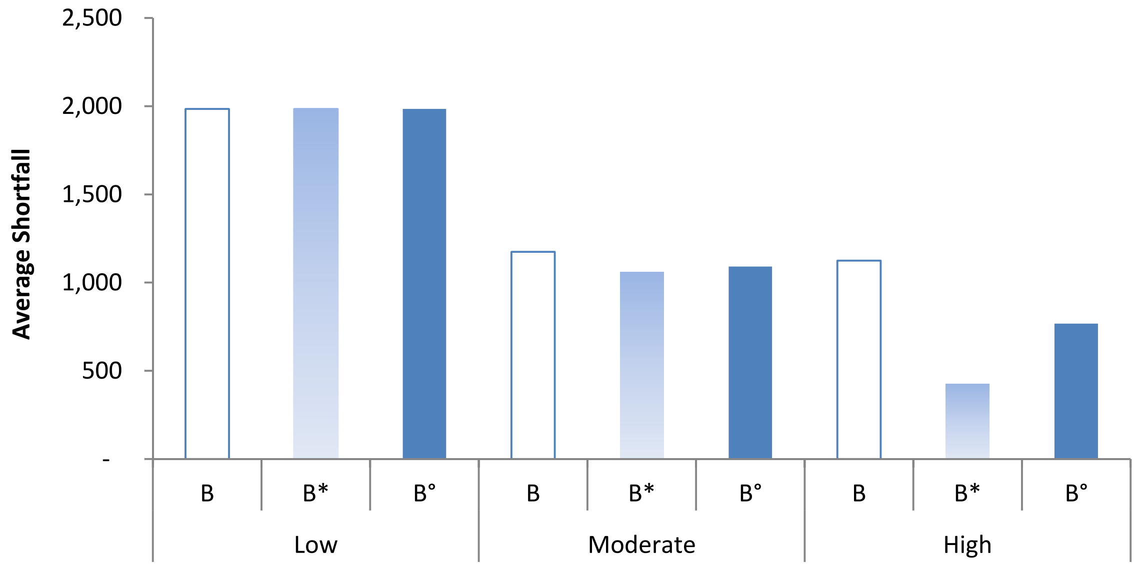
Finally, Figure 2.3 shows that the average shortfall ($B_{i,t-1} + V_{i,t}$) given a margin exceedance, is lower under the Tail-Dependent margining system in the moderate and high-dependence states. Therefore, we can conclude that the Tail-Dependent margining system is superior to the alternatives because it provides a better allocation of margin requirements. This allocation depends on the composition and homogeneity of the trading positions of the clearing members and it provides better protection against joint negative outcomes.

Figure 2.2. Probability of joint margin exceedances



Note: The figure presents, for each margining system, the likelihood of clearing firms jointly exceeding their posted margin requirements, i.e., $B_{i,t-1} + V_{i,t} < 0$, given different levels of tail dependence between them (low, moderate, and high). The three margin requirements correspond to those under the VaR (B), Tail-Dependent (B^{*}), and Budget-Neutral (B^{0}) margining systems. The results are based on 1,000,000 simulations of the actual changes in the underlying asset prices.

Figure 2.3. Average shortfall given joint margin exceedances



Note: The figure presents, for each margining system, the average shortfall ($B_{i,t-1} + V_{i,t}$) given joint margin exceedances and different levels of tail dependence between clearing firms (low, moderate, and high). The three margin requirements correspond to those under the VaR (B), Tail-Dependent (B^{*}), and Budget-Neutral (B^{0}) margining systems. The results are based on 1,000,000 simulations of the actual changes in the underlying asset prices.

2.5. Conclusion

In this chapter, we present a novel approach to compute margins for a portfolio of derivatives securities. The innovative feature of our method is that it accounts not only for the riskiness of the trading positions of an individual market participant, but also for their interdependence with those of *other* participants. Our method is a simulation-based technique that accounts for extreme tail dependence across potential trading losses.

We show that accounting for interconnections across clearing firms in a derivatives exchange lowers the probability of clearing members experiencing losses that exceed their posted collateral simultaneously. In addition, accounting for interconnections reduces the magnitude of the shortfall associated with these events. Both of these features significantly decrease the systemic risk concerns associated with clearing houses.

While our simulation analysis focuses on margins for option positions, our method can be applied to any listed derivative contract such as futures, swaps, or exchange-traded credit derivatives. Furthermore, it is important to notice that our approach is not limited to derivatives exchanges and can also be used to set collateral in any financial network. For instance, our method could be used to set collateral requirements for OTC positions.

Chapter 3.

CoMargin: A System to Enhance Financial Stability[8]

3.1. Introduction

How much collateral should a market participant post against its derivatives positions? In this chapter, we argue that margin requirements should increase with both the variability and the interdependence of the profits and losses (P&Ls) of market participants. We show that commonly used margining systems, such as the Standard Portfolio Analysis of Risk (SPAN) or the Value-at-Risk (VaR) approach, often fail to properly allocate collateral requirements because they disregard interdependencies across the portfolios of different market members. Our objective is to address this issue by proposing a new margining system, called CoMargin, which explicitly internalizes P&L interdependence. Our methodology is a model-free, scenario based approach that can be generalized to any number of market participants and, unlike the SPAN system, it can be formally backtested by using an extension of existing statistical techniques.

We focus on clearing houses in derivatives markets because these institutions concentrate a significant amount of credit risk in the financial system (Pirrong, 2009). However, our margining and backtesting methodology is general enough that it can be applied to any context where counterparty risk needs to be managed. Examples include, but are not limited to, collateral and capital requirements for repo transactions, over-the-counter (OTC) securities dealers, banks, insurance companies and newly-proposed swap execution facilities (SEFs).

[8] This chapter is based on Cruz Lopez, Harris, Hurlin and Pérignon (2013). The views presented here are those of the authors and do not necessarily reflect those of the Bank of Canada or the Canadian Derivatives Clearing Corporation.

In a derivatives exchange, the clearing house is in charge of confirming, matching, and settling all trades. Clearing houses operate with a small number of firms, called clearing members (CMs), who are allowed to submit their own trades for clearing through their firm accounts (i.e., conduct proprietary trading), as well as those of their customers or other non-clearing members through their client accounts.

The process of novation allows the clearing house to become the only counterparty to every contract. For this reason, clearing houses are sometimes known as central counterparties (CCPs). Throughout this process, the clearing house remains market-risk neutral because, by construction, the number of long positions is equal to the number of short positions outstanding for all contracts. However, the clearing house accumulates a significant amount of credit risk, which is primarily managed through the use of margining systems.[9]

A clearing house margining system requires members to post cash or financial assets as collateral in a margin account. These funds are used to protect the clearing house against the losses and potential default of its members over a period of time, usually one day. However, CMs sometimes experience losses that exceed their posted collateral, which leaves them with a negative balance in their margin accounts. We refer to these losses as *margin exceeding losses* or just *exceedances* for short. CMs that experience exceedances may delay payment or in some cases default on their obligations; thus, creating a shortfall in the market. The CCP has to cover this shortfall with its own funds and compensate the clearing members that profited from their trading positions. Usually, financing the shortfall of a single CM over a limited time does not impose a hefty financial burden on the clearing house. However, when two or more large CMs experience simultaneous margin exceeding losses, the consequences tend to be more severe. If these CMs delay their payments temporarily, the resulting shortfall tends to be short lived, but it can significantly affect market liquidity, particularly during volatile periods. On the other hand, if these CMs default, the shortfall tends to be long-lived or

[9] Other common credit risk management tools used by CCPs include requiring members to hold minimum capital levels, contribute to a default fund, enter into private insurance arrangements and segregate between client and firm margin accounts (see Jones and Pérignon, 2013).

even permanent which can erode the resources of the clearing house to the point of financial distress or even failure.

While clearing house failures are rare events, the cases of Paris in 1973, Kuala Lumpur in 1983 and Hong Kong in 1987 demonstrate that these extreme scenarios are not only possible, but also very economically significant (Knott and Mills, 2002).[10] In addition, the systemic importance of CCPs has increased in recent years due to their consolidation through economic integration and mergers and acquisitions, and due to the strong pressure from governments and market participants to facilitate or mandate the central clearing of OTC derivatives (Acharya et al., 2009; US Congress' OTC Derivatives Market Act of 2009; US Department of Treasury, 2009; Duffie, Li, and Lubke, 2010; Duffie and Zhu, 2011).[11] Therefore, it is increasingly necessary to devise appropriate risk management systems that enhance the stability and resiliency of clearing facilities.

Current margining systems employed by derivatives exchanges set collateral requirements based on a coverage level or a target probability of an exceedance event for an individual contract or portfolio of contracts (Figlewski 1984; Booth et al. 1997; Cotter 2001).[12] However, by focusing only on individual contracts or member portfolios, these systems ignore the fact that sometimes CMs face homogenous risk exposures that make their losses highly interdependent. This situation exposes the clearing house to simultaneous exceedance events that could undermine its stability.

The level of P&L dependence across clearing members increases with trade crowdedness and underlying asset comovement (Cruz Lopez, 2013a). Trade crowdedness refers to the similarity of trading positions across CMs. When positions across portfolios are very similar, they tend to have equivalent exposures and returns,

[10] Default of CMs is of course much more frequent. Recent examples in the Chicago Mercantile Exchange (CME) include Refco in 2005, Lehman in 2008, and MF Global in 2011 (see Jones and Pérignon, 2013).

[11] The CME, Intercontinental Exchange (ICE), EUREX, Euronext Liffe, and LCH-Clearnet have each recently created clearing facilities for Credit Default Swaps.

[12] For example, Kupiec (1994 and 1995) shows the empirical performance of the SPAN system for selected portfolios of S&P 500 futures and futures-options contracts and finds that, over the period from 1988 to 1992 the historical margin coverage exceeds 99% for most portfolios included in the sample.

regardless of how underlying assets behave. Underlying asset comovement refers to asset returns moving in unison. When underlying assets experience high levels of comovement, CMs tend to face similar risk exposures regardless of the composition of their portfolios because securities in all portfolios tend to move in the same direction.[13]

Both dimensions of P&L dependence are related. However, trade crowdedness is directly influenced by the individual trading behaviour of clearing members, while asset comovement is determined by aggregate market behaviour. Similar trading positions, or crowded trades, tend to arise among large clearing members because they share a common (and superior) information set. This informational advantage leads them to pursue similar directional trades, arbitrage opportunities and hedging strategies.[14] On the other hand, underlying assets tend to move in the same direction during economic slowdowns or during periods of high volatility, both of which are rarely the result of the actions undertaken by an individual market participant.[15]

In this chapter we depart from the traditional view of setting margin requirements based on individual member positions and propose a methodology that accounts for their interdependence. We estimate the margin requirement of each CM conditional on one or several other members being in financial distress. A CM is said to be in financial distress if its losses exceed its P&L VaR. By adopting this approach, we obtain individual margin requirements that increase with P&L dependence, stabilize the probability of exceedance events given financial distress, and reduce the risk that the clearing house exhausts its funds due to large or sudden shortfalls.

Our method builds on the CoVaR concept introduced by Adrian and Brunnermeier (2011) to identify systemically important financial institutions. CoVaR is defined as the VaR of the financial system (i.e., the banking sector) conditional on a

[13] The importance of asset comovement has been identified in previous studies. For example, in an early attempt to analyze the default risk of a clearing house, Gemmill (1994) highlights the dramatic diversification benefit from combining contracts on uncorrelated or weakly correlated assets.

[14] Much of the proprietary trading activity in derivatives exchanges consists of arbitraging futures and OTC or cash markets (e.g. cash-futures arbitrage of the S&P 500 index, eurodollar-interest rate swap arbitrage, etc.).

[15] Extreme dependence and contagion across assets is discussed in Longin and Solnik (2001), Bae, Karolyi and Stulz (2003), Longstaff (2004), Poon, Rockinger and Tawn (2004), Boyson, Stahel and Stulz (2010), and Harris and Stahel (2011), among others.

given institution being in financial distress (i.e., exceeding its VaR). The core of their analysis, denominated ΔCoVaR, measures the marginal contribution of a particular institution to the overall risk in the system. ΔCoVaR is calculated as the difference between the VaR of the financial system conditional on a given institution being in distress and the VaR of the financial system in the median state of the institution. Similarly, by inverting the conditioning relationship, one can assess the exposure of a given institution to the state of the financial system.

There are, however, some key differences between the CoMargin and CoVaR methodologies that are worth noticing. First, the objective of CoMargin is not to measure systemic risk. Instead, it is used to estimate margin requirements that account for the interdependence of market participants. Thus, we are not concerned with the state (i.e., VaR) of the financial system, but with the coverage that the CCP derives from collecting collateral. Second, CoMargin does not define financial distress in terms of the VaR of bank stock returns, but in terms of the VaR of the potential (one-day ahead) P&Ls of clearing members. Furthermore, unlike CoVaR, which can be estimated by conditioning on the financial distress of all members in the banking system, CoMargin can only be estimated by conditioning on a subset of market participants, because by construction the aggregate P&L across all CMs in a derivatives exchange is zero. Finally, the estimation of CoMargin is semi-parametric and much simpler than that of CoVaR, which requires a quantile regression approach.

The CoMargin estimation process starts by taking the trading positions of all clearing members at the end of the trading day as given. A series of one-day-ahead scenarios based on projected changes in the price and volatility of the underlying assets is used to assess changes in the value of the portfolio of each member. For each scenario, we mark-to-model the portfolio of each CM and obtain its hypothetical one-day-ahead P&L. Based on these hypothetical P&L calculations we compute margin requirements that target the probability of margin exceedances conditional on the financial distress of other members.

Our results show that the CoMargin system enhances the stability of the CCP by maintaining the probability of margin exceedances conditional on the financial distress of other members constant and by reducing the occurrence of simultaneous margin exceeding losses. In addition, our method increases financial resiliency because it actively adjusts the allocation of collateral as a function of market conditions that influence P&L dependence. As a result, the average magnitude of the shortfall given simultaneous exceedances is minimized relative to other collateral systems. Both of these conditions greatly reduce the systemic risk concerns associated with CCPs.

The remainder of the chapter is organized as follows. In Section 3.2, we describe how margin requirements are currently estimated under the SPAN and VaR margining systems. In Section 3.3 we define a list of properties needed to achieve a sound margining system. We present the theoretical foundations of the CoMargin system in Section 3.4 and examine its empirical effectiveness in Section 3.5. Finally, Section 3.6 concludes.

3.2. Traditional Margining Systems

Consider a derivatives exchange with N clearing members and D derivatives securities (futures, options, credit default swaps, etc.) written on A underlying assets. Let $w_{i,t}$ be the number of contracts in the derivatives portfolio of clearing member i, for $i = 1, \dots, N$, at the end of day t:

$$w_{i,t} = \begin{bmatrix} w_{1,i,t} \\ \vdots \\ w_{D,i,t} \end{bmatrix} \tag{3.1}$$

Margins are collected every day from each clearing member to guarantee the performance of their obligations and to guard the clearing house against default. Let $B_{i,t}$ be the performance bond or margin collected by the clearing house from clearing member i at the end of day t. This performance bond is a function of the outstanding trading positions of that member, $w_{i,t}$.

The variation margin, $V_{i,t}$, represents the aggregate portfolio P&L of clearing member i on day t. In this chapter, we are interested in cases when trading losses exceed margin requirements; that is, when $V_{i,t} \leq -B_{i,t-1}$. In these cases, we say that firm i has experienced a margin exceeding loss or an exceedance. Identifying firms in this state is important because they have an incentive to default on their positions or to delay payment on their obligations, which generates a shortfall in the market that needs to be covered by the CCP. Given the limited funds available to the CCP, simultaneous exceedance events can threaten its stability and survival.

3.2.1. SPAN Margin

The CME introduced the Standard Portfolio Analysis of Risk margining methodology in 1988. It has since become the most widely used margining system in derivatives exchanges around the world. Every day following the market close, clearing houses such as the Canadian Derivatives Clearing Corporation (CDCC), the Chicago Mercantile Exchange (CME), Eurex, LCH.Clearnet, Nymex and the Options Clearing Corporation (OCC), among others, use the SPAN system to estimate the margin requirements of their members.

SPAN is a scenario-based methodology that is used to assess potential changes in the value of the derivatives held by each clearing member. However, SPAN does not take a portfolio-wide approach. Instead, it divides each portfolio into contract families, defined as groups of contracts that share the same underlying asset, and estimates a charge for each family independently. Thus, for a portfolio with $d \in D$ derivatives written on $a \in A$ underlying assets, the SPAN system computes a contract family charges.

To compute a contract family charge in a portfolio, the SPAN system simulates one-day-ahead changes in the value of each contract by using sixteen scenarios that vary the price and the volatility of the underlying asset, as well as the time to expiration of the contract (see Table 3.1). The range of the potential price changes of the underlying asset usually covers 99% of its daily price movements over a historical calibration window. A similar approach is adopted for the volatility. The extreme price changes are used to assess potential changes in deep out of the money options. The scenario analysis yields a risk array for each contract that contains sixteen one-day-ahead potential value changes (i.e., each maturity and each strike price has its own array).[16] The scenario with the worst potential loss for the entire contract family is identified and that loss becomes the first part of the contract family charge.

The second part of the contract family charge consists of a discretionary adjustment that is needed because contracts with different expiration months are assumed to be equivalent in the scenario analysis. In other words, long and short positions written on the same underlying asset but with different expiration months offset each other. Therefore, risk managers are required to add an *intra-commodity spread charge* to the worst case scenario loss to account for time-spread trading. The resulting value is the contract family charge.

The collateral requirement for an entire portfolio is computed by aggregating the charges across all of its contract families. However, once again, risk managers are required to use discretionary aggregation rules to account for commodity-spread trading (i.e., simultaneous long and short positions in contracts with the same expiration months but written on different but correlated underlying assets). These adjustments are known as *inter-commodity spread charges.*

It is important to note that both intra- and inter-commodity spread charges involve the discretion of risk managers. Thus, these adjustments are rarely consistent across commodities, market conditions or clearing houses. This situation coupled with the fact that the SPAN system targets underlying price and volatility ranges, instead of the probability of portfolio-wide margin exceeding losses, make the exceedance coverage of the SPAN system inconsistent across time and markets.

[16] The projected price changes of non-linear contracts, such as options, are obtained by using numerical valuation methods or option pricing models.

Table 3.1. Scenarios used in the SPAN system

Scenario	Underlying Asset Price	Volatility Change	Time to Expiration
1	0	+ volatility range	$-1/252$
2	0	- volatility range	$-1/252$
3	$+1/3$ x price range	+ volatility range	$-1/252$
4	$+1/3$ x price range	- volatility range	$-1/252$
5	$-1/3$ x price range	+ volatility range	$-1/252$
6	$-1/3$ x price range	- volatility range	$-1/252$
7	$+2/3$ x price range	+ volatility range	$-1/252$
8	$+2/3$ x price range	- volatility range	$-1/252$
9	$-2/3$ x price range	+ volatility range	$-1/252$
10	$-2/3$ x price range	- volatility range	$-1/252$
11	$+3/3$ x price range	+ volatility range	$-1/252$
12	$+3/3$ x price range	- volatility range	$-1/252$
13	$-3/3$ x price range	+ volatility range	$-1/252$
14	$-3/3$ x price range	- volatility range	$-1/252$
15	Positive extreme change	0	$-1/252$
16	Negative extreme change	0	$-1/252$

Note: The table shows the sixteen scenarios used to determine the contract family charge in the SPAN system. Price and volatility ranges usually cover 99% of the data points over a rolling historical estimation window. Positive and negative extreme changes are designed to assess the effect of deep out of the money options.

3.2.2. *VaR Margin*

VaR is defined as a lower quantile of a P&L distribution. It is the standard measure used to assess the aggregate risk exposure of banks (Berkowitz and O'Brien, 2002; Berkowitz, Christoffersen and Pelletier, 2011), as well as their regulatory capital requirements (Jorion, 2007). VaR can also be used to set margins on a derivatives exchange. In this case, the margin requirement corresponds to a given quantile of a clearing member's one-day-ahead P&L distribution.

Definition 3.1: The VaR margin of firm i, B_i, corresponds to the α% quantile of its P&L distribution:

$$\Pr\left(V_{i,t+1} \leq -B_{i,t}\right) = \alpha \tag{3.2}$$

Like the SPAN system, the VaR margin method is applied on a firm-by-firm basis using a scenario analysis. However, the scenarios are applied to the entire portfolio (Cruz Lopez, Harris and Pérignon, 2011). More specifically, we consider S scenarios derived from simulated one-day-ahead changes in the value of the price and the volatility of the underlying assets and use them to evaluate each clearing member's entire portfolio. The hypothetical P&L or variation margin of each CM is computed by *marking-to-model* its positions in each scenario. Thus, for each CM and date t, we obtain a simulated sample of $V_{i,t+1}$ denoted $\left\{v_{i,t+1}^{s}\right\}_{s=1}^{S}$ that can be used to estimate the VaR margin requirement as follows:

$$\hat{B}_{i,t} = percentile\left(\left\{v_{i,t+1}^{s}\right\}_{s=1}^{S}, 100\alpha\right) \tag{3.3}$$

Compared to market risk VaR (Jorion, 2007), the estimation of VaR margin is much simpler. When estimating market risk VaR, there is only one observation available for each asset on date t. Therefore, the quantile of a return distribution of a given asset at time t cannot be estimated without making some strong distributional assumptions. For example, the historical simulation approach broadly used by financial institutions for market risk VaR estimations assumes that the asset returns are independently and identically distributed over time. Under these assumptions, the unconditional VaR is stationary and can be estimated from the historical path of past returns. The estimation of more refined conditional measures also requires some specific assumptions regarding quantile dynamics. For instance, the CAViaR approach proposed by Engle and Manganelli (2004), assumes an autoregressive process for the quantiles.

In the context of VaR margin, however, the situation is quite different and much simpler because we have S simulated observations of the P&L distribution of each clearing member at time t. This is an ideal situation from an econometric standpoint because the quantile of the P&L distribution can be directly implied without making any assumptions regarding its behavior over time. Thus, $\hat{B}_{i,t}$, which represents the empirical quantile based on the S simulated observations (equation 3.3), is a consistent estimate of the P&L VaR when S tends to infinity.

3.3. Characteristics of a Sound Margining System

Remarkably, there is very little guidance in the literature regarding the properties that a sound collateral system should satisfy. Nevertheless, this is a fundamental issue that needs to be addressed in order to assess the relative merits of different margining methodologies. In this section we attend to this issue by proposing five main properties that any well designed margining system must satisfy. These properties and the rationale behind them are explained below.

i. Margins must increase with P&L variability

Let $\sigma_{i,t}$ be a measure of the variability of the P&L of clearing member i at time t:

$$\text{If } \sigma_{i,t}^1 \geq \sigma_{i,t}^2 \text{ then } B_{i,t}\left(w_{i,t}, \sigma_{i,t}^1\right) \geq B_{i,t}\left(w_{i,t}, \sigma_{i,t}^2\right) \tag{3.4}$$

As Table 3.2 shows, this basic property is at the heart of existing margining methods. Intuitively, it means that since riskier trading portfolios (as measured by their variability) tend to have larger potential losses, more collateral must be collected to guarantee their performance. Or in simple words, riskier clearing members should post higher margins. The SPAN and VaR methods comply with this property because both the worst-case loss of the SPAN system and the quantile that determines VaR margin tend to increase with the variability of P&Ls.[17]

ii. Margins must increase with P&L dependence

Let $\delta_{i,t}$ be a measure of dependence between the losses of market participant i and those of other market participants at time t. P&L dependence can originate from similarities in trading positions, correlated asset prices, or both:

$$\text{If } \delta_{i,t}^{1} \geq \delta_{i,t}^{2} \quad \text{then} \quad B_{i,t}\left(w_{i,t}, \delta_{i,t}^{1}\right) \geq B_{i,t}\left(w_{i,t}, \delta_{i,t}^{2}\right) \tag{3.5}$$

The intuition behind this property is that a sound margining system should prevent (or minimize) the occurrence of simultaneous margin exceeding losses across market participants. As shown in Section 3.2, both the SPAN and VaR margin methods set margins on a firm-by-firm basis and hence completely disregard P&L dependence across clearing members.

iii. Margins should not be excessively procyclical

When margins are procyclical, market downturns and excess volatility can lead to higher initial margins and more frequent margin calls. This situation can adversely affect

[17] Artzner et al. (1999) define a coherent risk measure using four axioms: *monotonicity* (if the returns of portfolio 1 (P1) are always lower than those of portfolio 2 (P2), then P1 is riskier than P2), *translation invariance* (adding $K in cash to P1 reduces its risk by the same amount), *homogeneity* (increasing the size of P1 by a factor S increases its risk by the same factor), and *subadditivity* (risk measures need to account for diversification). Conceptually, with non-subadditive margins, it may be optimal for participants to breakdown their trading portfolio into smaller sub-portfolios in order to reduce their total margin requirements. However, in practice, clearing houses prevent financial institutions from having multiple clearing members. Furthermore, netting rules allow clearing members to post considerably less margin than what they would be required to post if they had dislocated portfolios.

funding conditions and market liquidity, and can force traders to close out their positions simultaneously; thus, intensifying market declines. Brunnermeier and Pedersen (2009) explain and model this sequence of reinforcing events which they refer to as a "margin spiral". Current margin requirements are prone to trigger these spirals because they are only a function of expected price and volatility changes. In addition, discretionary parameters, such as the intra- and inter-commodity charges used in the SPAN system, are usually modified after significant or persistent market shocks; thus, causing more variability in the margining process, which can be destabilizing for the market.

iv. Margins must be robust to outliers

Erratic margin swings due to outliers should be prevented as much as possible as they may lead to severe operational problems, such as sudden margin calls. Since SPAN margins are based on the maximum simulated loss and not a quantile, they are much more sensitive to outliers than VaR margins.

v. Margins must be testable ex-post

The only way to systematically measure the effectiveness of a margining system is by backtesting it. Backtesting aims at identifying misspecified models that lead to either excessive or insufficient coverage for the CCP relative to a target. Therefore, if a margining system cannot be backtested using formal statistical methods, we cannot identify its potential shortcomings and fine tune it to meet its objectives.

VaR margins can be easily backtested because they are defined by the quantile of the P&L distribution. The intuition behind the backtesting procedure is that the actual trading losses of a given clearing member should only exceed its VaR margin α% of the time. Well known VaR validation tests can be found in Jorion (2007) and a more refined approach can be found in Hurlin and Pérignon (2012).

On the other hand, backtesting SPAN margin requirements is extremely challenging because they are based on the minimum of a simulated P&L distribution, which is very hard to identify. Validation tests, in this case, cannot be performed without making strong distributional assumptions. More important, however, is the fact that the SPAN system cannot be backtested in terms of a coverage probability of margin exceeding losses, as it targets the ranges of the underlying asset prices and volatilities instead of the coverage of portfolio-wide P&Ls. Nevertheless, since the objective of collecting margins is to guarantee the overall performance of clearing member obligations, the probability of margin exceedances is the relevant measure of the effectiveness of a margining system.

In summary, the SPAN system only complies with the first key property, whereas the VaR margin system complies with properties one, four and five. Table 3.2 summarizes these findings. It is interesting to notice, however, that existing margining techniques are unable to account for P&L dependence across market participants and to produce margin requirements that are not highly procyclical.

Table 3.2. Desirable properties of margin requirements

Properties	SPAN Margin	VaR Margin	CoMargin
Reflects P&L variability	**Yes**	**Yes**	**Yes**
Reflects P&L dependence across participants	No	No	**Yes**
Exhibits lower procyclicality	No	No	**Yes**
Is robust to outliers	No	**Yes**	**Yes**
Can be backtested	No	**Yes**	**Yes**

3.4. CoMargin

3.4.1. Concept

The VaR and SPAN collateral systems only focus on firm specific risk; that is, the unconditional probability of an individual clearing member experiencing a margin exceeding loss. By adopting either method, the clearing house guards itself from unique or independent exceedances, but it leaves itself exposed to simultaneous exceedance events. These events, however, tend to be much more economically significant because they place a more substantial burden on the resources of the clearing house.

Consider the VaR margin of firms i and j. The probability of simultaneous exceedances is given by:

$$\begin{aligned} &\Pr\left[\left(V_{i,t+1} \leq -B_{i,t}\right) \cap \left(V_{j,t+1} \leq -B_{j,t}\right)\right] \\ &= \Pr\left(V_{i,t+1} \leq -B_{i,t} | V_{j,t+1} \leq -B_{j,t}\right) \times \Pr\left(V_{j,t+1} \leq -B_{j,t}\right) \end{aligned} \tag{3.6}$$

Equation 3.6 shows that simultaneous exceedance events tend to happen more frequently not only when firm specific risk increases (i.e., when $\Pr\left(V_{j,t+1} \leq -B_{j,t}\right)$ increases), but also when P&L dependence increases (i.e., when $\Pr\left(V_{i,t+1} \leq -B_{i,t} | V_{j,t+1} \leq -B_{j,t}\right)$ increases). In the first case, firms are more likely to experience losses that exceed their collateral levels in all states of the world. In the second case, firms are more likely to experience these losses at the same time as other firms, either because they hold similar positions (i.e., trade crowdedness is high) or because underlying assets have a tendency to move together (i.e., underlying asset comovement is high). However, VaR and SPAN margins completely disregard P&L dependence and its potential effect on the stability of the CCP. In the case of the VaR system, risk managers only target unconditional exceedance probabilities by setting a coverage level, $1-\alpha$, for each clearing member individually. In the case of the SPAN system, risk managers do not have direct control over the unconditional exceedance probabilities, so the clearing house is potentially left even more vulnerable to simultaneous exceedance events.

Now, consider a fully orthogonal market; that is, a market that has firms with orthogonal trading positions and orthogonal underlying asset returns. In this case, firms have orthogonal risk exposures and their exceedance probabilities are independent. Under the VaR system this means

$$\Pr\left(V_{i,t+1} \leq -B_{i,t} | V_{j,t+1} \leq -B_{j,t}\right) = \alpha \tag{3.7}$$

and

$$\Pr\left[\left(V_{i,t+1} \leq -B_{i,t}\right) \cap \left(V_{j,t+1} \leq -B_{j,t}\right)\right] = \alpha^2 \tag{3.8}$$

Equation 3.8 shows that given a common coverage probability and at least three clearing firms, a fully orthogonal market stabilizes and minimizes the probability of simultaneous exceedance events across clearing members. In addition, a fully orthogonal market provides the best possible level of market stability, regardless of the collateral system being adopted by the clearing house, because once the risk manager selects α, the probabilities of simultaneous events are also fixed (i.e., $\alpha^2$for two events, $\alpha^3$for three events and so on). Therefore, a fully orthogonal market can be seen as a conceptual construct that provides a common benchmark for all margining systems.

With this in mind and in the spirit of the CoVaR measure of Adrian and Brunnermeier (2011), we propose a new collateral system, called CoMargin, which enhances financial stability by taking into account the P&L dependence of clearing members. Our starting point is the framework used to estimate VaR margin requirements, which was described in the previous section. Once we establish the S scenarios for each underlying asset, we jointly evaluate the portfolios of firms i and j and compute their associated hypothetical P&Ls or variation margins, $V_{i,t+1}$ and $V_{j,t+1}$, respectively, such that for each date t, we obtain a panel of simulated P&Ls, denoted $\left\{v_{i,t+1}^s, v_{j,t+1}^s\right\}_{s=1}^S$.

The CoMargin of firm i, denoted $B_t^{i|j}$, conditional on the realisation of an event affecting firm j is:

$$\Pr\left(V_{i,t+1} \leq -B_t^{i|j} | \mathbf{C}(V_{j,t+1})\right) = \alpha \tag{3.9}$$

The conditioning event that we consider is the financial distress of firm j, which we define as a loss in its portfolio in excess of its α% VaR, or equivalently, a loss in excess of its VaR margin; i.e., $\mathbf{C}(V_{j,t+1}) = \{V_{j,t+1} \leq -B_{j,t}\}$.

Definition 3.2: The CoMargin of firm i conditional on the financial distress of firm j, $B^{i|j}$, corresponds to the α% conditional quantile of their joint P&L distribution:

$$\Pr\left(V_{i,t+1} \leq -B_t^{i|j} | V_{j,t+1} \leq -B_{j,t}\right) = \alpha \tag{3.10}$$

Through Bayes theorem we know that:

$$\Pr\left(V_{i,t+1} \leq -B_t^{i|j} | V_{j,t+1} \leq -B_{j,t}\right) = \frac{\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|j}\right) \cap \left(V_{j,t+1} \leq -B_{j,t}\right)\right]}{\Pr\left(V_{j,t+1} \leq -B_{j,t}\right)} \tag{3.11}$$

where the numerator represents the joint probability of i exceeding its CoMargin requirement and j experiencing financial distress. From equations 3.2 and 3.10, we can see that the CoMargin of firm i is defined as the margin level $B_t^{i|j}$ such that:

$$\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|j}\right) \cap \left(V_{j,t+1} \leq -B_{j,t}\right)\right] = \alpha^2 \tag{3.12}$$

Notice that the CoMargin system starts by defining the financial distress level of a CM as its VaR margin. This part accounts for firm specific risk. P&L dependence is then incorporated by directly targeting the conditional exceedance probability of firm i, such that it behaves as if the market was fully orthogonal when firm j is in financial distress. Thus, when the market is indeed fully orthogonal, the CoMargin and VaR margin requirements are equivalent and produce the same results. When the market is not fully orthogonal, any differences between the collateral requirements of these two systems can be attributed to P&L dependence. More specifically, $B_t^{i|j}$ can be interpreted as the margin level that guarantees with probability α that firm i remains solvent at an optimal level when firm j experiences financial distress. The optimal level of solvency corresponds to that seen in a fully orthogonal market, where given the financial distress of firm j, firm i always has enough funds in its margin account to cover its potential losses $1-\alpha\%$ of the time. Thus, by providing coverage levels, similar to those prevalent in an orthogonal market, the CoMargin system greatly enhances financial stability.

3.4.2. Illustration

Properties

We consider a simple case with two firms that have normally-distributed P&Ls. For simplicity, we consider an unconditional distribution, with respect to past information, and consequently neglect the time index t. Let

$$(V_1, V_2)' \sim N(0, \Sigma)$$

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$$

In this setting, the CoMargins of both members, denoted ($B^{1|2}$, $B^{2|1}$), are defined by:

$$\Pr\left(V_i \leq B^{i|j} | V_j \leq -B_j\right) = \alpha \tag{3.13}$$

for $i = 1, 2$ and where $B_i = -\sigma_i \Phi^{-1}(\alpha)$ denotes the unconditional VaR of firm i and $\Phi(\cdot)$ the cdf of the standard normal distribution. The conditional distribution of V_i given $V_j < c$, $\forall c \in \mathbb{R}$ is a skewed distribution (Horrace, 2005) and is denoted by $g(\cdot)$. The CoMargin for the firm i is the solution to:

$$\int_{-\infty}^{-B^{i|j}} g(u; \sigma_i, \sigma_j, \rho) du = \alpha \tag{3.14}$$

$$g(u; \sigma_i, \sigma_j, \rho) = \frac{1}{\alpha \sigma_i} \times \phi(\frac{u}{\sigma_i}) \times \Phi(\frac{-B_j/\sigma_j - \rho u/\sigma_i}{\sqrt{1-\rho^2}}) \tag{3.15}$$

where $\phi(\cdot)$ denotes the pdf of the standard normal distribution (Arnold et al., 1993). Using the expression of CoMargin in equation 3.14, we can illustrate some of its properties:

i. The CoMargin of firm i increases with the variability of its P&L:

$$\frac{\partial B^{i|j}}{\partial \sigma_i} > 0 \tag{3.16}$$

See Appendix A1 for the proof.

ii. When there is no P&L dependence between firms i and j, CoMargin and VaR margin converge. In this example, P&L dependence is fully characterized by the correlation coefficient, ρ; thus,

$$B^{i|j} = B_i \text{ when } \rho = 0 \tag{3.17}$$

Notice, however, that this result is not specific to the normal distribution case. When there is no dependence (linear or otherwise) between the P&L of the two firms, CoMargin converges to VaR margin. See Appendix A2 for the proof.

iii. The CoMargin of firm i increases with the dependence between its P&L and that of other firms. In this example, the only other member is firm j, so

$$\frac{\partial B^{i|j}}{\partial \rho} > 0 \tag{3.18}$$

See Appendix A3 for the proof.

iv. When firms i and j have perfect P&L dependence, their CoMargin converges to α^2% VaR margin, $B_i(\alpha^2)$,

$$\lim_{\rho \to 1} B^{i|j} = B_i(\alpha^2) \tag{3.19}$$

This property shows that CoMargin is not explosive when P&L dependence is high. See Appendix A4 for the proof.

v. The CoMargin of firm i does not depend on the variability of the P&L of firm j:

$$\frac{\partial\, B^{i|j}}{\partial \sigma_j} = 0 \tag{3.20}$$

See Appendix A5 for the proof.

Theoretical Performance

In order to illustrate the performance of the CoMargin system, we now consider the case of four CMs, where two of them, members 1 and 2, have correlated P&Ls, such that:

$$V \sim N(0, \Sigma)$$

where

$$V = (V_1, V_2, V_3, V_4)' \text{ and } \Sigma = \begin{pmatrix} 1 & \boldsymbol{\rho} & 0 & 0 \\ \boldsymbol{\rho} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

We allow the correlation between the P&Ls of firms 1 and 2, ρ, to increase from 0 to 0.8. As was explained before, the rising correlation between the P&Ls of these firms can reflect an increase in the similarity of their trading positions or an increase in the comovement of the underlying assets. The left panel of Table 3.3 shows the margin requirements for each CM under both the VaR and CoMargin systems for the levels of correlation being considered. To estimate the CoMargin of a given CM, we define the conditioning event as at least one of the other three firms being in financial distress.

The table shows that VaR margin remains constant for all firms regardless of their correlation level because this method does not take into account P&L dependence. Consistent with the explanation in the previous section, CoMargin and VaR margin are equal for all firms when $\rho = 0$; that is, when there is no P&L dependence. However, notice that CoMargin is greater than VaR margin when $\rho > 0$; that is, when there is P&L dependence. In addition, CoMargin increases as ρ increases; that is, as P&L dependence increases.

It is important to notice that CoMargin could be more effective than VaR margin either because it provides a better allocation of collateral or simply because it collects additional funds. Thus, we address this issue by reporting what we call a Budget-Neutral VaR (BNVaR) margin. This margining system is designed to neutralize the second effect by collecting as much aggregate collateral as the CoMargin system, but it does so evenly across all clearing members.[18] Thus, the BNVaR margin of firm i at time t, $B_{i,t}^{0}$, is defined as:

$$B_{i,t}^{0} = B_{i,t} + \frac{\sum_{i=1}^{N} B_t^{i|j} - \sum_{i=1}^{N} B_{i,t}}{N} \tag{3.21}$$

Panels A, B and C of Figure 3.1 show the theoretical performance of the VaR, BNVaR and CoMargin systems at the clearing member level. The horizontal line in these charts highlights the values prevalent when $\rho = 0$; that is, when the market is orthogonal. Panel A plots the margin levels presented in Table 3.3 discussed above. Panel B shows the probability of a given CM exceeding its margin conditional on at least another CM being in financial distress. When $\rho = 0$, all three margining systems provide the same level of coverage. However, as ρ increases, the VaR and BNVaR margins provide less coverage when at least one clearing member is in financial distress. On the other hand, CoMargin keeps the coverage level constant. Panel C shows the probability of a CM exceeding its margin conditional on at least another CM having an exceedance. In this case, CoMargin keeps the conditional probabilities of the uncorrelated CMs stable and, unlike VaR and BNVaR, it reduces the conditional probabilities of the correlated CMs; that is, those that are more likely to experience simultaneous exceedances.

Table 3.4 reports the theoretical performance of the different margining systems at the CCP level. The table reports the unconditional probability of having a minimum number of exceedances and the probability of having additional exceedances given that one has occurred. In addition, it reports the expected shortfalls associated with these events. Panels C, D and E of Figure 3.1 extend these results to up to four exceedance events. Our findings show once again that when $\rho = 0$, all three margining systems provide the same coverage to the CCP, but as ρ increases, CoMargin provides the best overall coverage.

[18] An alternative budget-neutral margin scheme would be to redistribute the additional collateral collected from firms 1 and 2 to firms 3 and 4; that is, to collect the additional collateral from the firms that have uncorrelated P&Ls. A previous version of this paper conducted that experiment and the relative effectiveness of CoMargin is even higher than that reported here. The results are available upon request from the authors.

The unconditional probabilities in Table 3.4 and Panel D of Figure 3.1 suggest that BNVaR margin provides the best unconditional coverage as correlation increases. Nevertheless, this result is expected. In our example all four firms are identical in all respects except for their correlation level. Since BNVaR collects more aggregate funds than VaR margin and it does so evenly across CMs, it is equivalent to a VaR margin with a higher coverage level (i.e., lower α). This higher coverage level embedded in BNVaR reduces the unconditional probability of individual margin exceedances. However, as Panel D and E of Figure 3.1 show, this does not improve the unconditional and conditional probability of experiencing additional (i.e. simultaneous) exceedance events, particularly as P&L dependence increases. In simple words, collecting more VaR margin indiscriminately across CMs does not optimize the coverage to the CCP.

Finally, Panel F of Figure 3.1 shows the shortfall that the CCP is expected to cover given a minimum number of margin exceedances. Notice that both CoMargin and BNVaR margin provide similar results that outperform VaR for $\rho > 0$. CoMargin, however, has a slightly lower shortfall when simultaneous exceedances occur. In addition, recall from Panels D and E that the probability of simultaneous exceedances is lower under the CoMargin system. Therefore, the ex-ante expected shortfall for simultaneous exceedance events under the CoMargin system is less than that under the BNVaR margin system.

The right panel of Table 3.3 and Table 3.4 and Figure 3.2 repeat the previous exercise but for P&Ls that are jointly Student t distributed with degrees of freedom υ, $V \sim t_{\upsilon}(0, \Sigma)$. The variance-covariance structure, Σ, is the same as that considered under the normal distribution assumption, but in this case, we set $\rho = 0.4$ and let the degrees of freedom decrease from 30 to 5.[19] Thus, the resulting P&L distributions have progressively fatter tails.

As explained in Cruz Lopez, Harris and Pérignon (2011), changing the distributional assumption of the previous exercise from a normal to a Student t

[19] We conducted a similar experiment using $\rho = 0$ which leads to the same conclusions. The results are available upon request by contacting the authors.

multivariate distribution, allows us to create a situation where all CMs have some level of tail dependence in their P&Ls. This is consistent with empirical evidence.

The fact that a Student t multivariate distribution allows for tail dependence becomes apparent when one considers that in the bivariate case the (upper and lower) tail dependence coefficient of firms i and j, denoted $\tau_{i,j}$, is defined as

$$\tau_{i,j} = 2t_{\nu+1}\left(-\sqrt{\frac{(\nu+1)(1-\rho)}{1+\rho}}\right) \tag{3.22}$$

provided that $\rho > -1$ (Cherubini, Luciano, and Vecchiato 2004 and Schmidt, 2006).

The results of this exercise are consistent with those presented for the Gaussian assumption, but they highlight an important finding: CoMargin is able to capture P&L dependence structures that go beyond correlation. Recall that the P&L dependence structure is fully characterized by correlation only if P&Ls are normally distributed. However, asset prices and P&Ls, particularly those of non-linear portfolios, rarely follow normal distributions. Thus, at least in theory, CoMargin is more robust than other methods for a wide range of P&L distributions.

Table 3.3. Theoretical margin collected under VaR and CoMargin systems

	Jointly Normally Distributed P&Ls				Jointly Student t Distributed P&Ls			
	CM1	**CM2**	**CM3**	**CM4**	**CM1**	**CM2**	**CM3**	**CM4**
	ρ = 0				**v = 30**			
VaR	1.645	1.645	1.645	1.645	1.697	1.697	1.697	1.697
CoMargin	1.645	1.645	1.645	1.645	2.136	2.136	1.791	1.791
BNVaR	1.645	1.645	1.645	1.645	1.964	1.964	1.964	1.964
	ρ = 0.4				**v = 10**			
VaR	1.645	1.645	1.645	1.645	1.812	1.812	1.812	1.812
CoMargin	1.981	1.981	1.645	1.645	2.505	2.505	2.138	2.138
BNVaR	1.813	1.813	1.813	1.813	2.322	2.322	2.322	2.322
	ρ = 0.8				**v = 5**			
VaR	1.645	1.645	1.645	1.645	2.015	2.015	2.015	2.015
CoMargin	2.374	2.374	1.645	1.645	3.248	3.249	2.840	2.840
BNVaR	2.009	2.009	2.009	2.009	3.044	3.044	3.044	3.044

Note: This table presents the VaR (equation 3.2), CoMargin (equation 3.10), and Budget-neutral VaR (BNVaR, equation 3.21) margin requirements, assuming four clearing members whose P&Ls are jointly normally or Student t distributed. The left panel presents the case where P&Ls are jointly normally distributed, such that $V \sim N(0, \Sigma)$, $V = (V_1, V_2, V_3, V_4)'$and $\Sigma = \begin{pmatrix} 1 & \boldsymbol{\rho} & 0 & 0 \\ \boldsymbol{\rho} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ and reports the results for different levels of the correlation parameter,ρ, that range from 0 to 0.8. The right panel show the case when P&Ls are Student t distributed with degrees of freedom υ, $V \sim t_{\upsilon}(0, \Sigma)$. The variance-covariance structure, Σ, is the same as that considered under the normal distribution assumption, but in this case, we set ρ=0.4 and let the degrees of freedom decrease from 30 to 5.

Table 3.4. Theoretical performance of VaR and CoMargin systems

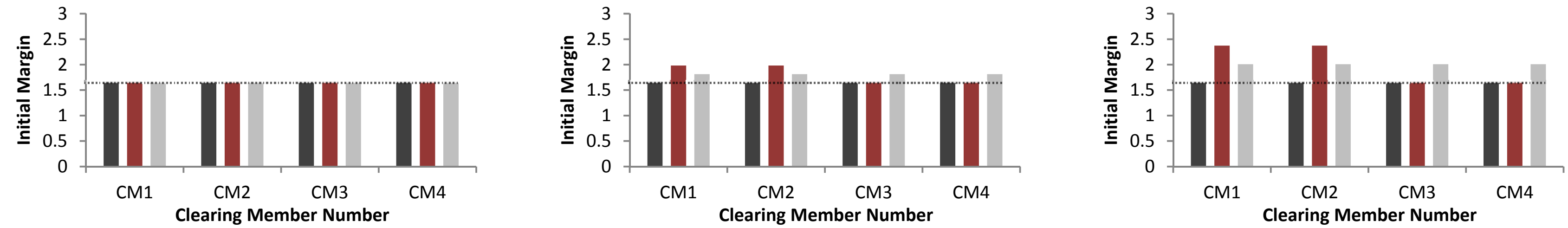
	Jointly Normally Distributed P&Ls				Jointly Student t Distributed P&Ls			
	Unconditional		Conditional on One Exceedance		Unconditional		Conditional on One Exceedance	
	Prob. of Exceedances	Expected Shortfall	Prob. of Exceedances	Expected Shortfall	Prob. of Exceedances	Expected Shortfall	Prob. of Exceedances	Expected Shortfall
	ρ = 0				**v = 30**			
VaR	0.185	0.084	0.076	0.451	0.177	0.094	0.123	0.531
CoMargin	0.185	0.084	0.076	0.451	0.115	0.056	0.074	0.485
BNVaR	0.185	0.084	0.076	0.451	0.108	0.052	0.088	0.485
	ρ = 0.4				**v = 10**			
VaR	0.179	0.084	0.109	0.466	0.171	0.119	0.151	0.695
CoMargin	0.138	0.060	0.069	0.433	0.081	0.053	0.090	0.650
BNVaR	0.129	0.055	0.083	0.430	0.077	0.051	0.104	0.658
	ρ = 0.8				**v = 5**			
VaR	0.165	0.084	0.193	0.505	0.164	0.175	0.191	1.068
CoMargin	0.110	0.048	0.062	0.432	0.051	0.060	0.129	1.171
BNVaR	0.077	0.033	0.144	0.428	0.049	0.059	0.141	1.193

Note: This table presents the theoretical performance of the VaR (equation 3.2), CoMargin (equation 3.10), and Budget-neutral VaR (BNVaR, equation 3.21) systems, assuming four clearing members whose P&Ls are jointly normally or Student t distributed. The left panel presents the case where P&Ls are jointly normally distributed, such that $V \sim N(0, \Sigma)$, $V = (V_1, V_2, V_3, V_4)'$ and $\Sigma = \begin{pmatrix} 1 & \boldsymbol{\rho} & 0 & 0 \\ \boldsymbol{\rho} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ and reports the results for different levels of the correlation parameter, ρ, that range from 0 to 0.8. The right panel show the case when P&Ls are Student t distributed with degrees of freedom υ, $V \sim t_{\upsilon}(0, \Sigma)$. The variance-covariance structure, Σ, is the same as that considered under the normal distribution assumption, but in this case, we set ρ=0.4 and let the degrees of freedom decrease from 30 to 5.

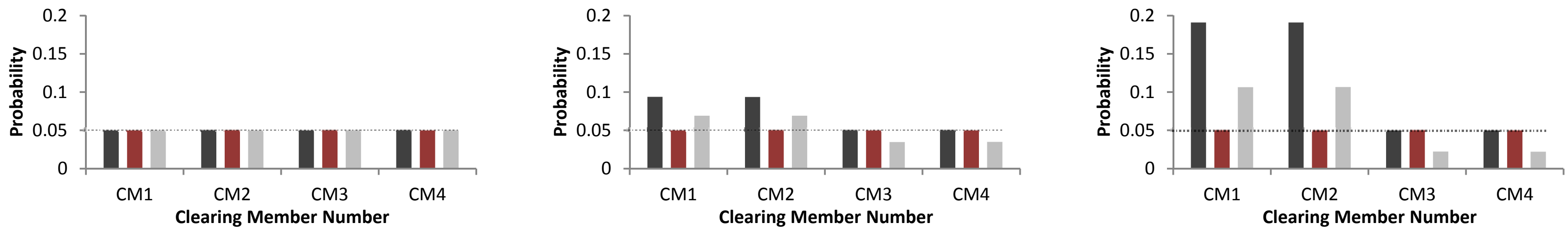
Figure 3.1. Theoretical performance of VaR and CoMargin systems assuming jointly normally distributed P&Ls

ρ = 0	ρ = 0.4	ρ = 0.8

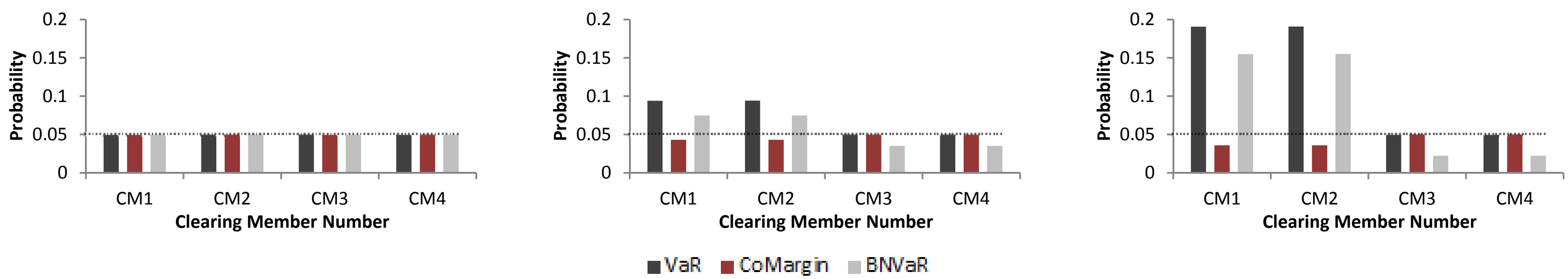
Panel A: Initial margin collected from each clearing member (dollars)



Panel B: Probability of a given CM exceeding its margin conditional on at least another CM being in financial distress

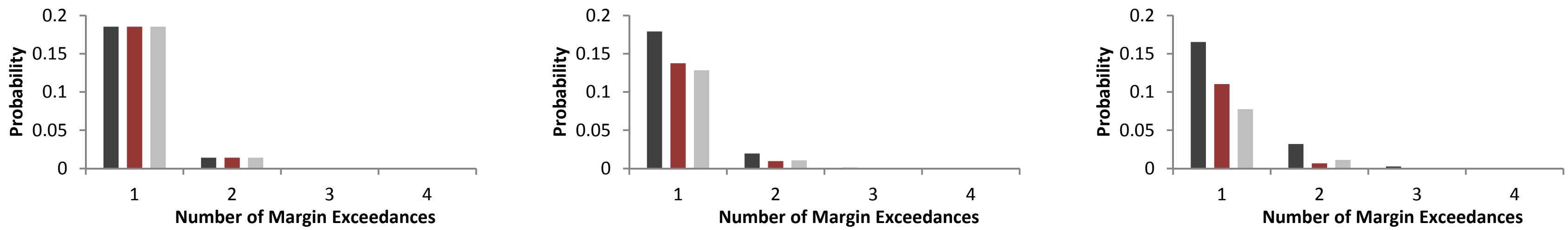


Panel C: Probability of a CM exceeding its margin conditional on at least another CM having a margin exceedance

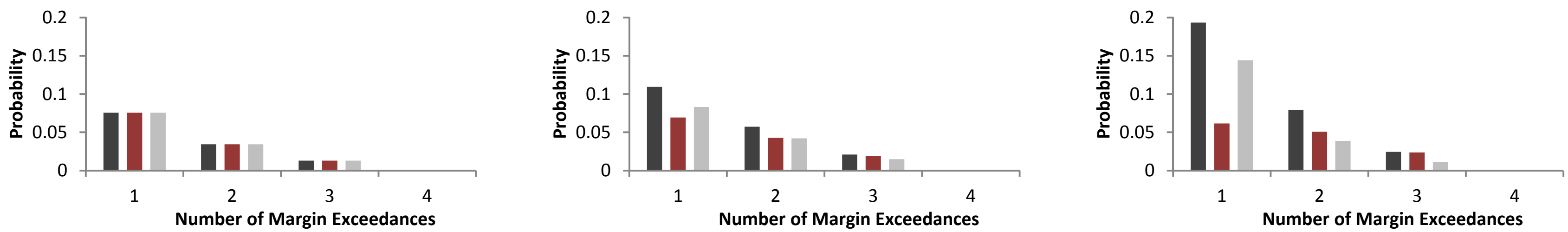


ρ = 0	ρ = 0.4	ρ = 0.8

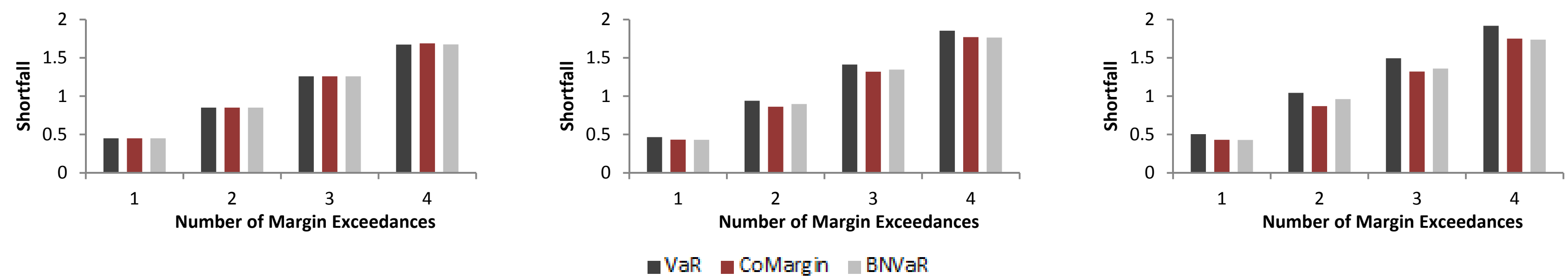
Panel D: Probability of a minimum number of margin exceedances



Panel E: Probability of additional margin exceedances given that a number of exceedances have occurred



Panel F: Conditional expected shortfall for the CCP given a minimum number of margin exceedances

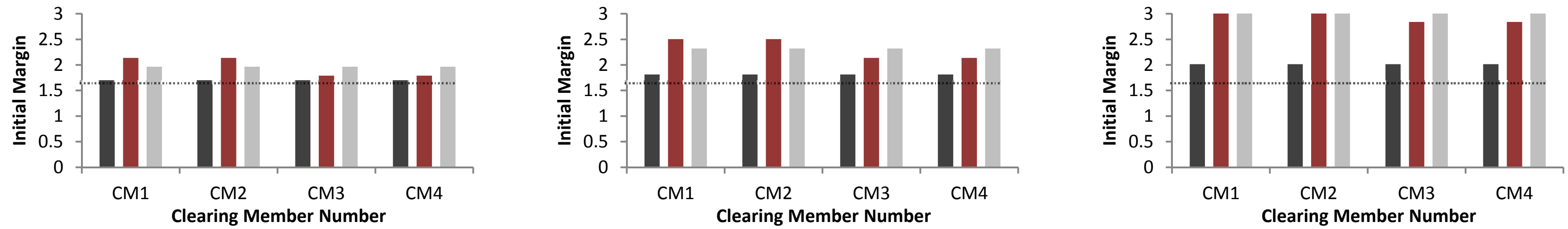


Note: This figure presents the theoretical performance of the VaR (equation 3.2), CoMargin (equation 3.10), and Budget-neutral VaR (BNVaR, equation 3.21) systems. We consider four firms with jointly normally distributed P&Ls, such that $V \sim N(0, \Sigma)$, $V = (V_1, V_2, V_3, V_4)'$and $\Sigma = \begin{pmatrix} 1 & \boldsymbol{\rho} & 0 & 0 \\ \boldsymbol{\rho} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$. We report our results for different levels of the correlation parameter,ρ, that range from 0 to 0.8.

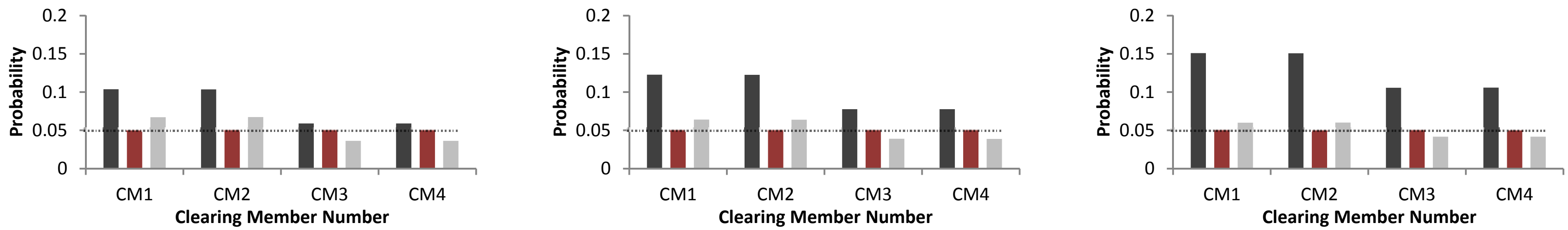
Figure 3.2. Theoretical performance of VaR and CoMargin systems assuming jointly Student t distributed P&Ls

v = 30	v = 10	v = 5

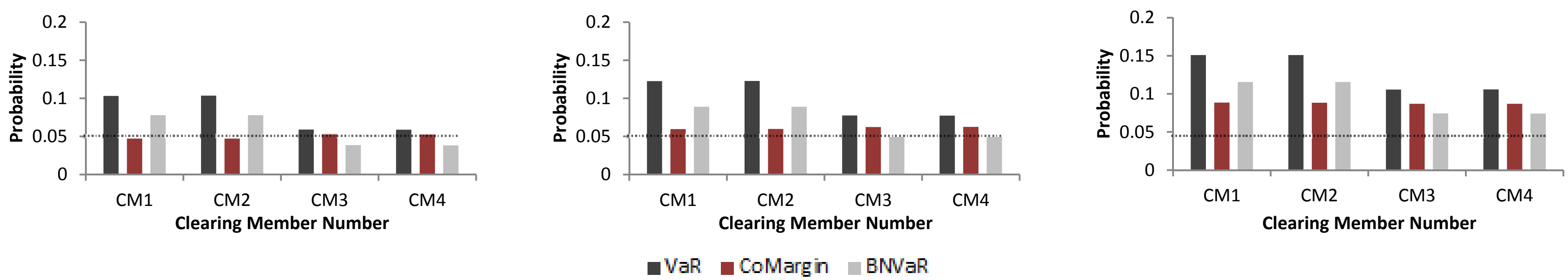
Panel A: Initial margin collected from each clearing member (dollars)



Panel B: Probability of a given CM exceeding its margin conditional on at least another CM being in financial distress

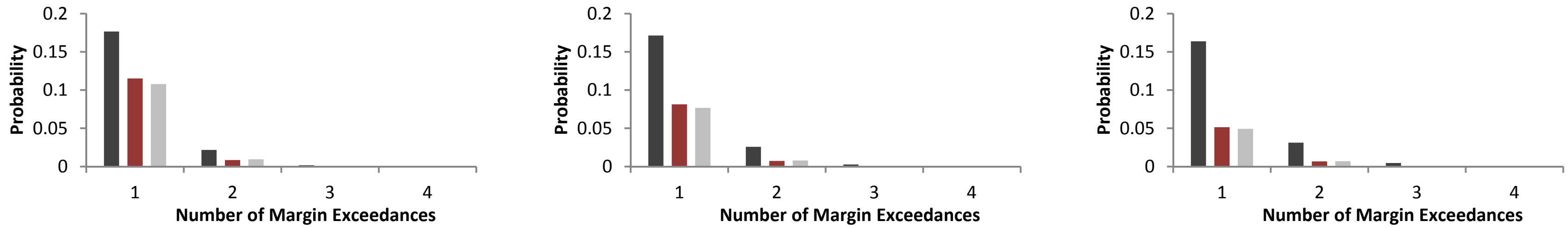


Panel C: Probability of a CM exceeding its margin conditional on at least another CM having a margin exceedance

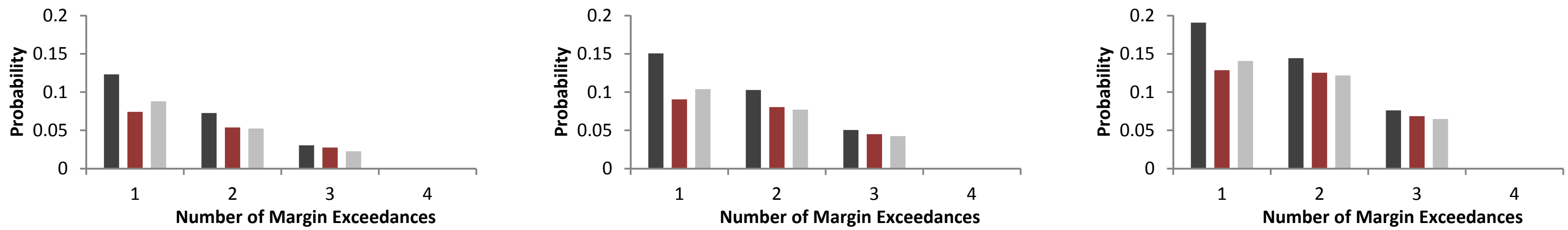


v = 30	v = 10	v = 5

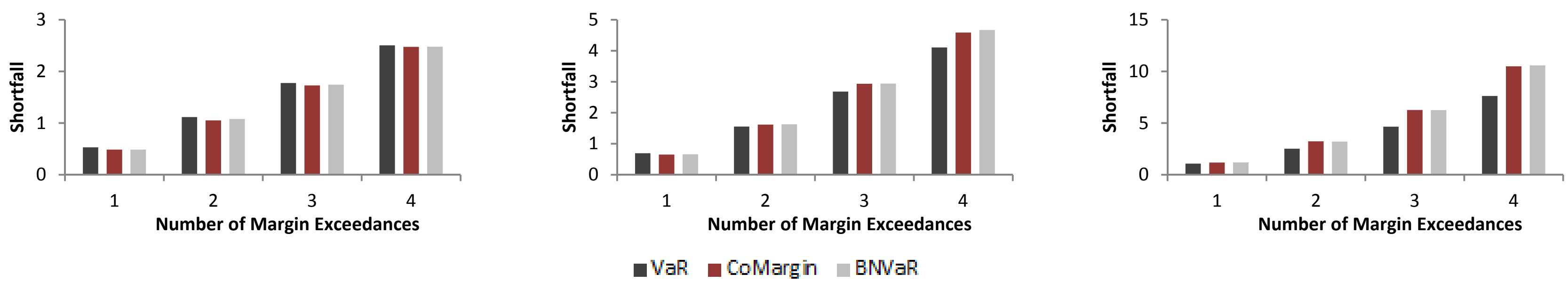
Panel D: Probability of a minimum number of margin exceedances



Panel E: Probability of additional margin exceedances given that a number of exceedances have occurred



Panel F: Conditional expected shortfall for the CCP given a minimum number of margin exceedances



Note: This figure presents the theoretical performance of the VaR (equation 3.2), CoMargin (equation 3.10), and Budget-neutral VaR (BNVaR, equation 3.21) systems. We consider four firms with joint Student t distributed P&Ls with degrees of freedom υ, such that $V \sim t_{\upsilon}(0, \Sigma)$, $V = (V_1, V_2, V_3, V_4)'$, $\Sigma = \begin{pmatrix} 1 & \boldsymbol{\rho} & 0 & 0 \\ \boldsymbol{\rho} & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ and $\rho = 0.4$. We report our results for different levels of the degrees of freedom parameter than range from 30 to 5.

3.4.3. Scenario Generation

One common feature of all margin methods is that they are scenario based. As a consequence, generating meaningful scenarios is a crucial stage when setting margin requirements. The scenario generating process used for CoMargin incorporates different dimensions of P&L dependence to simulate potential changes in the price and volatility of the underlying assets

Since the scenarios used for computing VaR margin are the starting point for the estimation of CoMargin, let us start by explaining first the VaR margin scenario generating process. Unlike the SPAN margining system, VaR margin uses a portfolio-wide approach. This allows us to take into account the asset comovement within the portfolio of each clearing member without the need for ad-hoc adjustments (i.e., inter- and intra-commodity spreads). In order to assess the potential P&L of the entire portfolio of each CM, we jointly simulate one-day-ahead changes in the underlying asset prices from a semi-parametric copula.

A copula is a function that links marginal probability distribution functions, say $F_1(r_1)$, $F_2(r_2)$,..., $F_A(r_A)$, to form a multivariate probability distribution function, $F(r_1, r_2, \dots, r_A)$, where r_a is the standardized return of underlying asset a and A is the number of assets underlying the derivatives cleared by the CCP. According to Sklar's Theorem (Sklar, 1959), if the marginal distributions are continuous, there exists a unique copula function C, such that

$$F(r_1, r_2, \dots, r_A) = C(F_1(r_1), F_2(r_2), \dots, F_A(r_A)) \tag{3.23}$$

As stated in Cruz Lopez, Harris and Perignon (2011), using copulas to model the multivariate structure of underlying asset returns is useful in this context. First, marginal distributions do not need to be similar to each other to be linked together with a copula structure. Second, the choice of the copula or multivariate structure is not constrained by the choice of the marginal distributions. Third, copulas can be used with A marginal distributions to cover all of the underlying assets cleared by the CCP (see Oh and Patton, 2012). Finally, the use of copulas allows us to model the tails of the marginal distributions and the tail dependence across underlying assets separately, which is particularly important in our case, as the likelihood of an extreme underlying asset return might increase either because of fatter tails in the marginal distributions or because of fatter tails in the multivariate distribution function.

We use Student t copulas in our modeling because, unlike their Gaussian counterparts, they resemble more closely some of the stylized features of asset returns, such as fat tails in the marginal distributions and multivariate tail dependence (Cruz Lopez, 2008). Let R be a symmetric, positive definite matrix with $diag(R) = I_A$, where I_A is the identity matrix of dimension A. Let $t_{R,\upsilon}$ be the standardized multivariate Sudent t distribution with correlation matrix R and υ degrees of freedom. Then, the multivariate Student t copula, $T_{R,\upsilon}$ is defined as:

$$T_{R,\upsilon}(F_1(r_1), F_2(r_2), \dots, F_A(r_A)) = t_{R,\upsilon}(r_1, r_2, \dots, r_A) \tag{3.24}$$

A Student t copula corresponds to the dependence structure implied by a multivariate Student t distribution. It is fully characterized by the variance-covariance matrix of the standardized returns and the degrees of freedom, υ. The degrees of freedom define the probability mass assigned to simultaneous extreme returns (both positive and negative); the lower the degrees of freedom, the higher the probability of experiencing simultaneous extreme returns relative to the Gaussian copula. However, as $\upsilon \to \infty$ the Student t copula converges to its Gaussian counterpart. In addition, notice that the Student t copula allows us to readily obtain an estimate of the coefficient of tail dependence across pairs of underlying asset returns as shown in equation 3.22.

We implement a two-stage semi-parametric approach to estimate a A-dimensional copula for the underlying asset returns. The first stage consists of estimating the empirical marginal distributions of the returns of each underlying asset. The second stage consists of estimating the t-copula parameters, R and υ, through maximum likelihood. This approach is commonly known as the canonical maximum likelihood estimation (CMLE) method (Genest, Ghoudi, and Rivest 1995). Once the copula parameters are estimated, we use the implied multivariate structure to simulate potential changes in the price of the underlying assets.

If the CCP is clearing instruments that depend on volatility, such as options contracts, the exercise can be repeated for different variations of R; that is, for different variance-covariance structures. For simplicity, in this chapter we suggest that these variations be set according to a pre-defined range relative to values predicted through the Dynamic Conditional Correlation method proposed by Engle (2002).

We use a fixed-length estimation window that is rolled daily to simulate new scenarios every day. Thus, once the S potential changes in the price of the underlying assets have been simulated, we mark-to-model all of the derivatives in the portfolio of each CM to obtain the simulated sample path $\{v_{i,t+1}^s\}_{s=1}^S$ that is required to estimate VaR margin as described in Section 3.3.

As described in Section 3.4, we use the same scenarios for estimating CoMargin, however, in this case we mark-to-model the portfolios of all clearing members simultaneously for each scenario to obtain $\{v_{1,t+1}^s, v_{2,t+1}^s, \dots, v_{n,t+1}^s\}_{s=1}^S$. This allows us to capture P&L dependence across clearing members.

3.4.4. Estimation

In the case of two clearing members, given the simulated path $\left\{v_{i,t+1}^{s}, v_{j,t+1}^{s}\right\}_{s=1}^{S}$, conditional on $B_t^{i|j}$, a simple estimate of the joint probability $\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|j}\right) \cap \left(V_{j,t+1} \leq -B_{j,t}\right)\right]$, denoted $P_t^{i,j}$, is given by:

$$\hat{P}_t^{i,j} = \frac{1}{S}\sum_{s=1}^{S} \mathbf{I}\left(v_{i,t+1}^{s} \leq -B_t^{i|j}\right) \times \mathbf{I}\left(v_{j,t+1}^{s} \leq -B_{j,t}\right) \tag{3.25}$$

where $v_{i,t+1}^{s}$ and $v_{j,t+1}^{s}$ correspond to the s^{th} simulated P&L of firms i and j, respectively. Given this result, we can now estimate $B_t^{i|j}$. For each time t and for each firm i, we look for the value $B_t^{i|j}$, such that the distance $\hat{P}_t^{i,j} - \alpha^2$ is minimized:

$$\hat{B}_t^{i|j} = \arg\min_{\left\{B_t^{i|j}\right\}} \left(\hat{P}_t^{i,j} - \alpha^2\right)^2 \tag{3.26}$$

Thus, for each firm i, we end up with a time series of CoMargin requirements $\left\{\hat{B}_t^{i|j}\right\}_{t=1}^{T}$ for which confidence bounds can be bootstrapped.

3.4.5. Backtesting

Just like with VaR margin, CoMargin allows us to test the null hypothesis of an individual member exceeding its margin requirement. More importantly, however, is the fact that we can also test the probability of exceedances conditional on the financial distress of other firms, as defined by the CoMargin of firm i, $B_t^{i|j}$. The null hypothesis in this case becomes:

$$H_0\colon \Pr\left(V_{i,t+1} \leq -B_t^{i|j} | V_{j,t+1} \leq -B_{j,t}\right) = \alpha \tag{3.27}$$

Since the null implies that $E\left[\mathbf{I}\left(V_{i,t+1} \leq -B_t^{i|j}\right) \times \mathbf{I}\left(V_{j,t+1} \leq -B_{j,t}\right)\right] = \alpha$, then a simple likelihood-ratio (*LR*) test can also be used (Christoffersen, 2009). To assess the conditional probability exceedances, we use the historical paths of the P&Ls for both members i and j; i.e., $\{v_{i,t+1}\}_{t=1}^{T}$ and $\{v_{j,t+1}\}_{t=1}^{T}$. The corresponding LR test statistic, denoted $LR_{i|j}$ takes the same form as LR_i:

$$LR_{i|j} = -2\ln[(1-\alpha)^{T-N_{i|j}}\alpha^{N_{i|j}}] + 2\ln\left[\left(1-\frac{N_{i|j}}{T}\right)^{T-N_{i|j}} \frac{N_{i|j}}{T}^{N_{i|j}}\right] \tag{3.28}$$

except that in this case $N_{i|j}$ denotes the total number of joint past violations observed for both members i and j; that is, $N_{i|j} = \sum_{t=1}^{T} \mathbf{I}\left(v_{i,t+1} \leq -B_t^{i|j}\right) \times \mathbf{I}\left(v_{j,t+1} \leq -B_{j,t}\right)$.

3.4.6. *Extension to n Conditioning Firms*

Consider now that the conditioning event depends on two firms denoted j and k. In this case, the CoMargin of firm i, denoted by $B_t^{i|j,k}$, is defined as follows:

$$\Pr\left(V_{i,t+1} \leq -B_t^{i|j,k} | \mathbf{C}\left(V_{j,t+1}, V_{k,t+1}\right)\right) = \alpha \tag{3.29}$$

$$\frac{\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|j,k}\right) \cap \mathbf{C}\left(V_{j,t+1}, V_{k,t+1}\right)\right]}{\Pr\left[\mathbf{C}\left(V_{j,t+1}, V_{k,t+1}\right)\right]} = \alpha \tag{3.30}$$

The conditioning event that we consider is either firm j or firm k, or both, being in financial distress; i.e., $\mathbf{C}\left(V_{j,t+1}, V_{k,t+1}\right) = V_{j,t+1} \leq -B_{j,t} \ or \ V_{k,t+1} \leq -B_{k,t}$. In this case, the probability of the conditioning event is equal to 2α only if the financial distress events of firms j and k are mutually exclusive. In the general case, we have:

$$\begin{aligned}\Pr\left[\mathbf{C}\left(V_{j,t+1},V_{k,t+1}\right)\right] &= \Pr\left[\left(V_{j,t+1}\leq -B_{j,t}\right)\text{ or }\left(V_{k,t+1}\leq -B_{k,t}\right)\right]\\ &= \Pr\left(V_{j,t+1}\leq -B_{j,t}\right)+\Pr\left(V_{k,t+1}\leq -B_{k,t}\right)\\ &\quad -\Pr\left[\left(V_{j,t+1}\leq -B_{j,t}\right)\cap\left(V_{k,t+1}\leq -B_{k,t}\right)\right]\\ &= 2\alpha-\Pr\left[\left(V_{j,t+1}\leq -B_{j,t}\right)\cap\left(V_{k,t+1}\leq -B_{k,t}\right)\right]\end{aligned} \tag{3.31}$$

Hence, CoMargin $B_t^{i|j,k}$ satisfies the following condition:

$$\frac{\Pr\left[\left(V_{i,t+1}\leq -B_t^{i|j,k}\right)\cap\mathbf{C}\left(V_{j,t+1},V_{k,t+1}\right)\right]}{2\alpha-\Pr\left[\left(V_{j,t+1}\leq -B_{j,t}\right)\cap\left(V_{k,t+1}\leq -B_{k,t}\right)\right]}=\alpha \tag{3.32}$$

Given this result, we proceed to estimate CoMargin $B_t^{i|j,k}$. First, notice that the probability $\Pr\left[\left(V_{j,t+1}\leq -B_{j,t}\right)\cap\left(V_{k,t+1}\leq -B_{k,t}\right)\right]$, denoted $P_t^{j,k}$, does not depend on the CoMargin level $B_t^{i|j,k}$; thus, it can simply be estimated by:

$$\hat{P}_t^{j,k}=\frac{1}{S}\sum_{s=1}^{S}\mathbf{I}\left(v_{j,t+1}^{s}\leq -B_{j,t}\right)\times\mathbf{I}\left(v_{k,t+1}^{s}\leq -B_{k,t}\right) \tag{3.33}$$

Second, conditional on $B_t^{i|j,k}$, the joint probability in the numerator of equation 3.32, denoted $P_t^{i,j,k}$, becomes:

$$\begin{aligned}P_t^{i,j,k} &= \Pr\left[\left(V_{i,t+1}\leq -B_t^{i|j,k}\right)\cap\mathbf{C}\left(V_{j,t+1},V_{k,t+1}\right)\right]\\ &= \Pr\left[\left(V_{i,t+1}\leq -B_t^{i|j,k}\right)\cap\left[\left(V_{j,t+1}\leq -B_{j,t}\right)\text{ or }\left(V_{k,t+1}\leq -B_{k,t}\right)\right]\right]\\ &= \Pr\left[\left(V_{i,t+1}\leq -B_t^{i|j,k}\right)\cap\left(V_{j,t+1}\leq -B_{j,t}\right)\right.\end{aligned} \tag{3.34}$$

$$\text{or } \left(V_{i,t+1} \leq -B_t^{i|j,k}\right) \cap \left(V_{k,t+1} \leq -B_{k,t}\right)\Big]$$

$$= \Pr\left[\left(V_{i,t+1} \leq -B_t^{i|j,k}\right) \cap \left(V_{j,t+1} \leq -B_{j,t}\right)\right]$$

$$+\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|j,k}\right) \cap \left(V_{k,t+1} \leq -B_{k,t}\right)\right]$$

$$-\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|j,k}\right) \cap \left(V_{j,t+1} \leq -B_{j,t}\right) \cap \left(V_{k,t+1} \leq -B_{k,t}\right)\right]$$

Thus, a simple estimator of this probability is given by:

$$\hat{P}_t^{i,j,k} = \frac{1}{S}\sum_{s=1}^{S} \mathbf{I}\left(v_{i,t+1}^s \leq -B_t^{i|j,k}\right) \times \mathbf{I}\left(v_{j,t+1}^s \leq -B_{j,t}\right)$$

$$+\frac{1}{S}\sum_{s=1}^{S} \mathbf{I}\left(v_{i,t+1}^s \leq -B_t^{i|j,k}\right) \times \mathbf{I}\left(v_{k,t+1}^s \leq -B_{k,t}\right) \qquad (3.35)$$

$$-\frac{1}{S}\sum_{s=1}^{S} \mathbf{I}\left(v_{i,t+1}^s \leq -B_t^{i|j,k}\right) \times \mathbf{I}\left(v_{j,t+1}^s \leq -B_{j,t}\right) \times \mathbf{I}\left(v_{k,t+1}^s \leq -B_{k,t}\right)$$

and the CoMargin $B_t^{i|j,k}$ can be estimated by:

$$\hat{B}_t^{i|j,k} = \arg\min_{\left\{B_t^{i|j,k}\right\}} \left(\frac{\hat{P}_t^{i,j,k}}{2\alpha - \hat{P}_t^{j,k}} - \alpha\right)^2 \qquad (3.36)$$

Following a similar argument, CoMargin can be generalized to n conditioning firms, with $n < N - 1$. In this case, the conditioning event is that at least one of the n clearing members is in financial distress (see Appendix B for details).

3.5. Empirical Analysis

3.5.1. Data

In this section we compare the empirical performance of the SPAN, VaR and CoMargin systems by using proprietary data from the Canadian Derivatives Clearing Corporation (CDCC). The CDCC is the clearing house of the TMX Montreal Exchange, the largest derivatives Exchange in Canada. The dataset includes the daily open interest (i.e., the daily trading positions at market close) on the three-month Canadian Bankers' Acceptance Futures (BAX), the ten-year Government of Canada Bond Futures (CGB), and the S&P/TSX 60 Index Standard Futures (SXF) for the forty-eight clearing members active in the CDCC between January 2, 2003 and March 31, 2011. To the best of our knowledge no other study has ever used actual clearing member positions to analyse the performance of competing margining systems. Nevertheless, due to the proprietary nature of the data, we are only able to report aggregate results. Table 3.5 presents a short description of the data.

In a derivatives exchange, on any given day, there are many delivery dates available on each underlying asset. Over the sample period there were 45 different delivery dates available for BAX contracts and 34 delivery dates available for CGB and SXF contracts. Thus, the sample includes a total of 113 futures contracts. Table 3.6 summarizes the specifications of these contracts. The contracts in our sample do not constitute the full set of derivatives cleared by the CDCC. However, they represent a significant portion of its clearing activity. The documentation provided by CDCC states that the BAX, CGB and SXF are among the most actively traded derivatives in Canada. Furthermore, BAX and CGB are the most actively traded cleared interest rate contracts in the country (Campbell and Chung 2003 and TMX Montreal Exchange 2013a, 2013b and 2013c).

Table 3.5. Description of the data used in the empirical analysis

Item	Number	Comments
Clearing members	48	There is entry and exit in the sample, so the number of clearing members varies over time.
Trading Days	2066	The sample period is from January 2, 2003 to March 31, 2011.
Underlying Assets	3	The three underlying assets are: 1. Yield on the three-month Canadian bankers' acceptance. 2. Yield on the ten-year Government of Canada Bond Futures 3. Level of the S&P/TSX 60 Index
Three-Month Canadian Bankers' Acceptance Futures Contracts (BAX)	45	Delivery dates range from January 2003 to December 2013.
Ten-Year Government of Canada Bond Futures Contracts (CGB)	34	Delivery dates range from March 2003 to June 2011.
S&P/TSX 60 Index Standard Futures Contracts (SXF)	34	Delivery dates range from March 2003 to June 2011.
Total futures contracts	113	These represent all the futures contracts (i.e., all delivery dates) written on the three underlying assets during the sample period.
Active firm accounts	21	We report results only for this type of account.
Active client accounts	23	
Active omnibus accounts	16	

Note: The table presents an overview of the dataset used in the empirical analysis, which was obtained from the Canadian Derivatives Clearing Corporation. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset.

Table 3.6. Specifications of the contracts included in the empirical analysis

	S&P/TSX 60 Index Standard Futures (SXF)	Three-Month Canadian Bankers' Acceptance Futures (BAX)	Ten-Year Government of Canada Bond Futures (CGB)
Underlying Interest	The S&P/TSX 60 Index C$200 times the S&P/TSX 60 index futures value	C$1,000,000 nominal value of Canadian bankers' acceptance with a three-month maturity.	C$100,000 nominal value of Government of Canada Bond with 6% notional coupon.
Expiration Months	March, June, September and December.	March, June, September and December plus two nearest non-quarterly months (serials).	March, June, September and December.
Price Quotation	Quoted in index points, expressed to two decimals.	Index : 100 minus the annualized yield of a three-month Canadian bankers' acceptance.	Par is on the basis of 100 points where 1 point equals C$1,000.
Price Fluctuation	0.10 index points for outright positions. 0.01 index points for calendar spreads	0.005 = C$12.50 per contract for the nearest three listed contract months, including serials. 0.01 = C$25.00 per contract for all other contract months.	0.01 = C$10
Price Limits	A trading halt will be invoked in conjunction with the triggering of "circuit breaker" in the underlying stocks.	None	None
Settlement	Cash settlement	Cash settlement	Physical delivery of eligible Government of Canada Bonds.
Trading Hours (EST)	Early session*: 6:00 a.m. to 9:15 a.m. Regular session: 9:30 a.m. to 4:15 p.m. * A trading range of -5% to +5% (based on previous day's settlement price) has been established only for this session.	Early session: 6:00 a.m. to 7:45 a.m. Regular session: 8:00 a.m. to 3:00 p.m. Extended session*: 3:09 p.m. to 4:00 p.m. * There is no extended session on the last trading day of the expiring contract month.	Early session: 6:00 a.m. to 8:05 a.m. Regular session: 8:20 a.m. to 3:00 p.m. Extended session*: 3:06 p.m. to 4:00 p.m. * There is no extended session on the last trading day of the expiring contract month.

Source: TMX Montreal Exchange (http://www.m-x.ca).

Table 3.7 shows the summary statistics for the contracts in the sample. Panel A shows the aggregate statistics for all 113 contracts and Panels B, C and D, report the summary statistics by underlying asset. On a typical day, there were approximately 20 active contracts, 12 of them were BAX, 4 of them were CGB and the remaining 4 were SXF. On average, contracts remained active for 363 trading days. However, there is a significant dispersion across underlying assets. BAX contracts remained active for 551 days, whereas CGB and SXF contracts remained active for 239 and 237 days, respectively. BAX contracts were also the most actively traded, with an average daily gross open interest of 275,000. The corresponding value for CGB and SXF contracts was less than half of that for BAX at 131,000 and 111,000, respectively.

CDCC members have access to three accounts to submit trades for clearing: a firm, a client and an omnibus account. The firm account is used by clearing members to submit their own trades (i.e., conduct proprietary trading). The client account is used to submit trades on behalf of clearing members' clients. The omnibus account is used for all other clearing activities and is the least active account across all clearing members.

Our analysis includes 21 firm, 23 client and 16 omnibus accounts that were active on at least one day of the sample period. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. Due to disclosure restrictions, we are unable to report the owners of each active account by type. However, Table 3.8 provides the full list of clearing members that had at least one active client, firm or omnibus account during the sample period. Notice that this list includes more clearing members than those currently affiliated with the CDCC because some of them entered and exited the market during this period.

Since our objective in this section is to present an assessment of our methodology by using actual trading positions, instead of by assuming them, we report results only for the firm or proprietary-trading accounts. We consider this appropriate because these accounts best represent the actual trading decisions of the clearing members. Nevertheless, our results are consistent across all three accounts.

Table 3.7. Summary statistics of the contracts included in the empirical analysis

Variable	Average	Median	St.Dev.	Min	Max
Panel A: All Contracts					
Active contracts per day	19.81	20.00	0.9279	13.00	21.00
Trading days per contract	362.25	253.00	221.72	6.00	756.00
Panel B: BAX Contracts					
Active contracts per day	11.99	12.00	0.14	8.00	13.00
Trading days per contract	550.58	699.00	249.25	6.00	756.00
Open interest long	137.81	131.32	49.21	48.35	310.97
Open interest short	-137.81	-131.32	49.21	-310.97	-48.35
Open interest gross	275.61	262.65	98.42	96.71	621.94
Panel C: CGB Contracts					
Active contracts per day	3.93	4.00	0.49	1.00	5.00
Trading days per contract	238.91	253.00	42.73	55.00	255.00
Open interest long	65.26	60.81	27.36	16.15	176.97
Open interest short	-65.26	-60.81	27.36	-176.97	-16.15
Open interest gross	130.52	121.62	54.72	32.30	353.94

Variable	Average	Median	St.Dev.	Min	Max
		Panel D: SXF Contracts			
Active contracts per day	3.89	4.00	0.43	1.00	4.00
Trading days per contract	236.32	250.00	42.58	52.00	255.00
Open interest long	55.28	55.09	14.01	24.40	98.49
Open interest short	-55.28	-55.09	14.01	-98.49	-24.40
Open interest gross	110.56	110.17	28.02	48.79	196.99

Note: The table shows the summary statistics of the 113 futures contracts included in the empirical analysis. These contracts are divided according to their underlying assets as follows: Three-month Canadian Bankers' Acceptance (BAX), Ten-year Government of Canada Bond (CGB) and S&P/TSX 60 Index Standard (SXF). Over the sample period (January 2, 2003 to March 31, 2011), there were 45 different delivery dates available for BAX contracts and 34 delivery dates available for CGB and SXF contracts. Open interest values are reported in thousands.

Table 3.8. Clearing members included in the empirical analysis

Number	Name	Number	Name
1	Newedge Canada Inc.	25	Morgan Stanley Canada LTD.
2	RBC Dominion Securities Inc.	26	CFG Financial Group Inc.
3	Union Securities LTD.	27	MF Global Canada Co.
4	T.D. Securities Inc.	28	Haywood Securities Inc.
5	BMO Nesbitt Burns LTD.	29	Goldman Sachs Canada Inc.
6	Macquarie Private Wealth Inc.	30	Timber Hill Canada Co.
7	UBS Securities Canada Inc.	31	Credit Suisse Securities
8	Desjardins Securities Inc.	32	CIBC World Markets Inc.
9	Macquarie Capital Markets Inc.	33	NBCN Clearing Services Inc.
10	Name not reported	34	HSBC Securities (Canada) Inc.
11	Merrill Lynch Canada Inc.	35	Mackie Research Capital Corporation
12	Odlum Brown LTD.	36	Benson-Quinn GMS Inc.
13	Penson Financial Services Inc.	37	Scotia Capital Inc.
14	Dundee securities corporation	38	E*trade Canada Securities Corporation
15	Daex Commodities Inc.	39	Raymond Kames LTD.
16	Canaccord Capital Corporation	40	Lévesque Beaubien Geoffrion Inc.
17	Friedberg Mercantile Group LTD.	41	TD Waterhouse Canada Inc.
18	W.D. Latimer Co. LTD.	42	Citigroup Global Markets Canada Inc.
19	Canadian Imperial Bank of Commerce (CIBC)	43	National Bank of Canada
20	Jones, Gable & Co. LTD.	44	J.P. Morgan Securities Canada Inc.
21	Name not reported	45	Merrill Lynch Canada Inc.
22	Timber Hill Canada Company	46	Name not reported
23	Laurentian Bank Securities Inc.	47	Fidelity Clearing Canada ULC
24	Deutsche Bank Securities LTD.	48	Maple Securities Canada LTD.

Note: The table provides the full list of clearing members that had at least one active client, firm or omnibus account during the sample period (January 2, 2003 to March 31, 2011). An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. Notice that this list includes more clearing members than those currently affiliated with the Canadian Derivatives Clearing Corporation (CDCC) because some of them entered and exited the market during the sample period.

The daily settlement prices for the underlying assets and the futures contracts in the sample were obtained from Bloomberg and are plotted in Figure 3.3. Panel A shows the time series of underlying asset prices, Panel B shows the underlying asset returns and Panel C shows the settlement futures prices for all delivery dates. Lines in different colours represent different delivery dates. It is evident from Panel B that the volatility of the underlying assets increased dramatically after the onset of the financial crisis in mid-2007. In addition, Panel C shows an increase in the spread of futures prices during the same period, particularly for BAX contracts.

Figure 3.4 plots the daily stacked P&L values implied from the positions in active firm accounts. For each date t, $n^{act} \in N$ observations are plotted, which correspond to the P&L of the n^{act} clearing members that had an active account on that day. Notice how the volatility of the P&Ls increased dramatically at the beginning of the financial crisis. This is consistent with the trends described for the underlying and futures prices in Figure 3.3. Therefore, we consider two sub-periods in our analysis. The first one is the pre-crisis period, from the January 2, 2003 to July 31, 2007, and the second one is the crisis period, from August 1, 2007 to March 31, 2011.

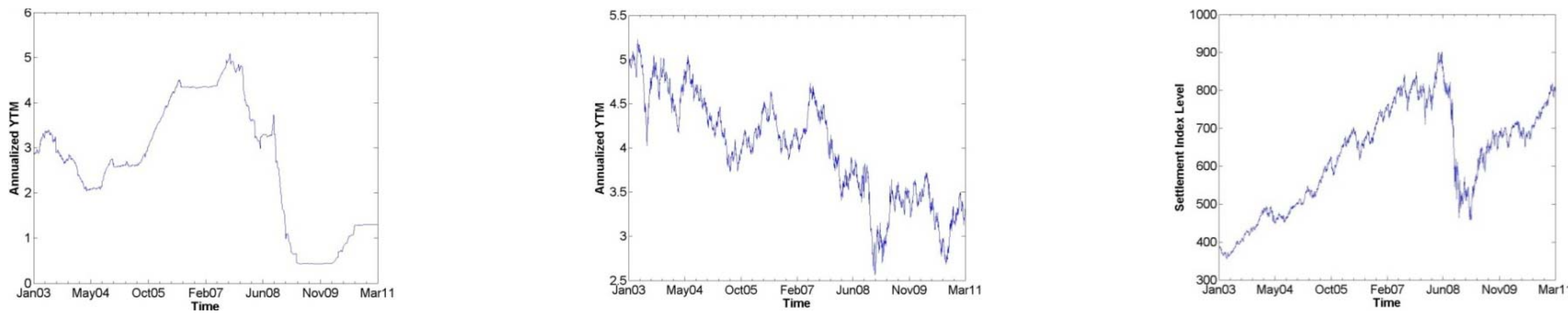
Table 3.9 presents the summary statistics for the firm accounts in the sample. Panel A reports the values for the full sample period and Panels B and C present the values for the pre-crisis and crisis periods, respectively. On a typical day, there were approximately 12 clearing members with active firm accounts. This number remained relatively stable during the pre-crisis and crisis periods. The average account was active for 56% of the days in the sample (1,146 out of 2066 days). The corresponding proportion is 75% (858 out of 1,148 days) for the pre-crisis period and 56% (516 out of 918 days) for the crisis period. The relatively shorter activity during the crisis period was partially influenced by the fact that some clearing members exited the market. The P&L numbers reported in the table focus exclusively on active accounts. The typical active account reported an implied daily loss of $60,000 on the futures contracts listed in the sample. During the pre-crisis period, these accounts reported daily losses of $164,000. However, during the crisis period, the average account reported a daily profit of $65,000. These profits were mostly derived from short positions in BAX contracts. Over the entire sample period, the typical account made an implied loss of $38,000. The corresponding numbers are a loss of $119,000 and a profit of $39,000 for the pre-crisis and crisis periods, respectively.[1]

[1] It is important to notice that the P&L values reported in this paper are those implied by the positions held by the clearing members in their firm accounts on the contracts included in the sample. The actual accounts of these clearing members, however, included positions in other contracts cleared by the CDCC that are not included in our sample. In addition, our P&L values do not include trading revenues from other sources, such as non-cleared OTC transactions.

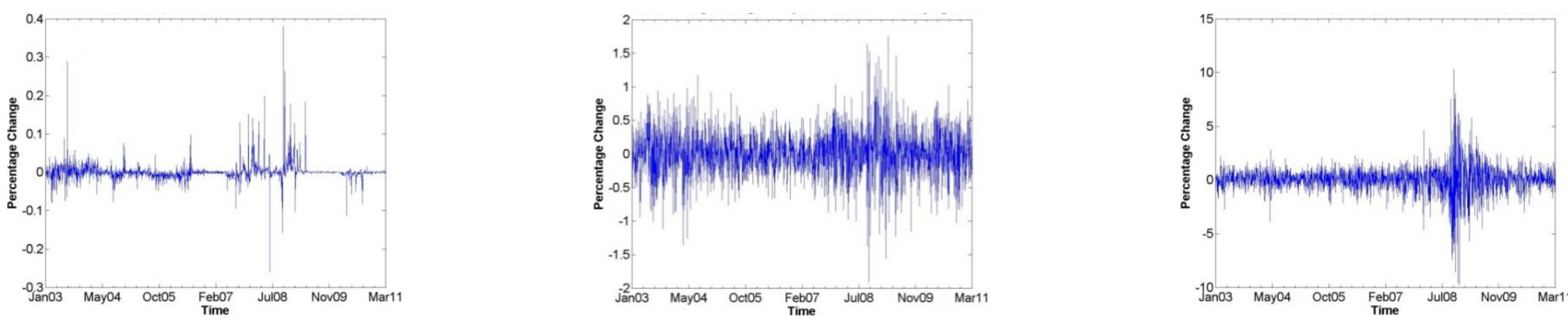
Figure 3.3. Underlying assets and futures contracts used in the empirical analysis

Three-month Canadian Bankers' Acceptance (BAX)	Ten-year Government of Canada Bond (CGB)	S&P/TSX 60 Index Standard (SXF)

Panel A: Underlying Asset Prices

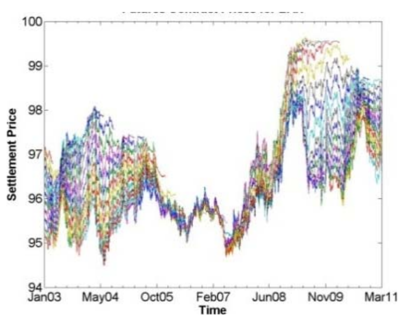


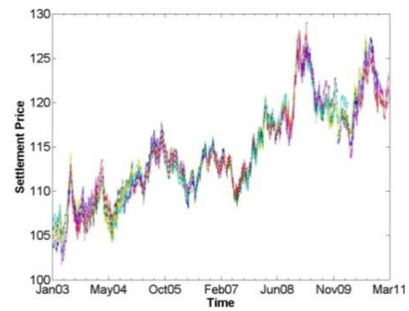
Panel B: Underlying Asset Returns

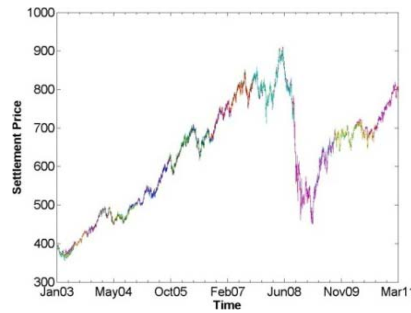


Three-month Canadian Bankers' Acceptance (BAX)	Ten-year Government of Canada Bond (CGB)	S&P/TSX 60 Index Standard (SXF)

Panel C: Futures Prices (All Maturities)

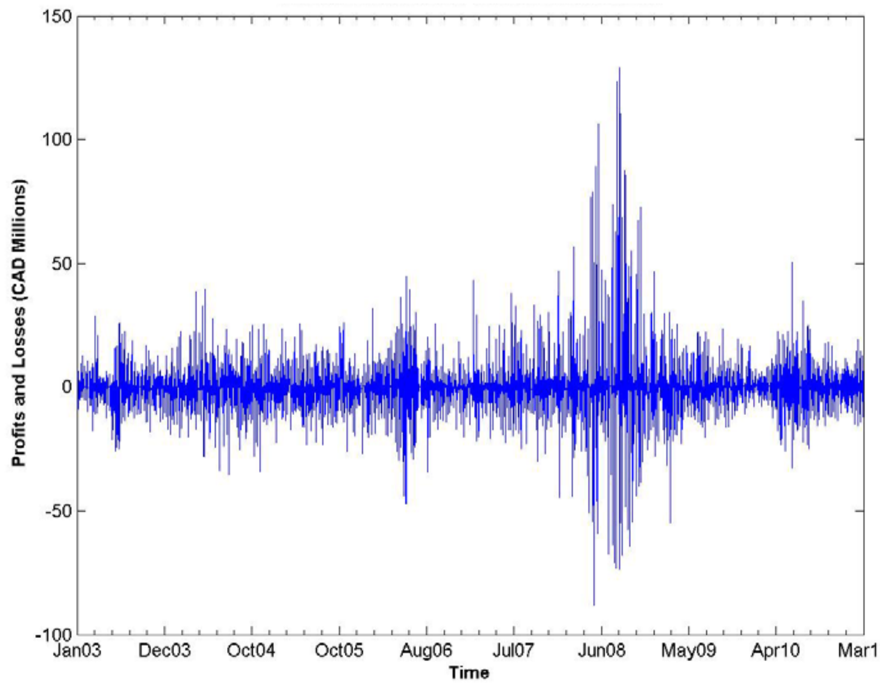






Note: Panel A presents the daily annualized settlement yield for the Three-month Canadian Bankers’ Acceptance, the annualized yield on the Ten-year Government of Canada Bond and the settlement level of the S&P/TSX 60 Index. Panel B shows the daily returns (i.e., percentage changes) of the variables presented in Panel A. Panel C presents the daily settlement futures prices for the futures contracts written on the Three-month Canadian Bankers' Acceptance (BAX), Ten-year Government of Canada Bond (CGB) and S&P/TSX 60 Index Standard (SXF), traded in the Montreal Exchange. Lines in different colours represent different delivery dates. The sample period is from January 2, 2003 to March 31, 2011. Source: Bloomberg.

Figure 3.4. Profits and losses for active firm accounts



Note: The figure shows the daily stacked P&L implied from the positions of the 23 active firm accounts included in the sample; that is, accounts with an open interest (i.e., long or short position) in at least one underlying asset at the end of the trading day. For each date t, $n^{act} \in N$ observations are plotted, which correspond to the P&L of the n^{act} clearing members with an active account. The sample period is from January 2, 2003 to March 31, 2011 and there are $N = 48$ clearing members in the sample.

Table 3.9. Summary statistics of the firm accounts included in the empirical analysis

Variable	Average	Median	St.Dev.	Min	Max
Panel A: Full Sample period					
Active accounts per day	11.64	12.00	1.09	8.00	15.00
Active days for an account	1145.19	1420.00	911.72	3.00	2066.00
Daily P&L across CMs	-60.92	-97.80	2659.44	-15014.20	17502.52
P&L over time	-37.83	0.43	160.57	-455.52	237.50
Panel B: Pre-Crisis period					
Active accounts per day	11.96	12.00	0.95	9.00	15.00
Active days for an account	858.00	1148.00	431.12	3.00	1148.00
Daily P&L across CMs	-163.50	-156.15	2027.37	-6813.50	10381.36
P&L over time	-119.12	-0.78	225.46	-671.62	39.80
Panel C: Crisis period					
Active accounts per day	11.24	11.00	1.13	8.00	15.00
Active days for an account	516.05	684.00	418.29	3.00	918.00
Daily P&L across CMs	65.18	-57.43	3280.36	-15014.20	17502.52
P&L over time	39.83	-0.60	135.28	-110.76	484.42

Note: The table presents the summary statistics of the 23 active firm accounts used in the empirical analysis. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. The sample period is from January 2, 2003 to March 31, 2011 and there are $N = 48$ clearing members in the sample. The pre-crisis period is from January 2, 2003 to July 31, 2007 and the crisis period is from August 1, 2007 to March 31, 2011. P&L values are reported in thousands of dollars and are estimated only for active firm accounts.

3.5.2. *Empirical Performance*

Using the daily open interest in each firm account, we compute the initial margin that should be collected from each clearing member under the SPAN, VaR and CoMargin systems. The underlying price range for the SPAN approach is set at 99% and $\alpha = 2\%$ for the VaR and CoMargin systems. We use a rolling estimation window of 500 trading days in all cases. As mentioned in Section 3.2, by construction, the SPAN system is estimated using the sixteen scenarios in Table 3.1. The extreme scenarios (scenarios 15 and 16) are ignored as we are only dealing with futures (i.e., linear) contracts. For the VaR and CoMargin systems we consider $S = 100{,}000$ scenarios that are obtained using the methodology described in Section 3.4. Consistent with our earlier discussion and theoretical illustration, we set the financial distress threshold for CoMargin at the VaR margin level of the conditioning firms. The conditioning firms are the two clearing members with the highest one-day-ahead Expected Shortfall (ES) given financial distress.

For consistency across time periods, we ignore the ad-hoc inter- and intra-commodity spreads used in the SPAN system and impose a minimum margin of $10,000 on all active accounts under all systems. This amount allows us to avoid cases when clearing members are not required to post any collateral because they have matched long and short positions. These cases are likely to result in small exceedances as P&Ls in different contracts do not always offset each other. Thus, imposing a minimum margin amount prevents an upward bias in the number of SPAN exceedances. This amount, however, does not influence the rest of our results as it represents a constant that accounts for less than 0.2% of the average individual daily margin required under all systems.[21]

Table 3.10 reports the summary statistics for the daily margin collected over the full sample period under different margining methods. The table also reports budget-neutral margins for the SPAN (BNSPAN) and VaR (BNVaR) systems using the same approach as that described in equation 3.21. The average aggregate daily margin

[21] We computed our results under different minimum collateral amounts ranging from $0 to $100,000. The results are consistent in all cases. For a minimum collateral of $0, however, the SPAN system yields a high number of small exceedances.

collected across all clearing members is $112, $101 and $161 million for the SPAN, VaR and CoMargin systems, respectively. The typical clearing member posted $9.76, $8.8 3 and $14.14 million of SPAN, VaR and CoMargin collateral when it entered the market. However, on a typical day, clearing members posted $5.85, $5.34 and $8.50 million respectively. The discrepancy between the cross-sectional and time series averages is derived from the fact that the number of active clearing members changed daily due to entry and exit.

Table 3.11 and Table 3.12 report the summary statistics for the daily margin collected over the pre-crisis and crisis periods, respectively. As it would be expected given the increased volatility during the financial crisis, both aggregate and individual collateral levels are higher during the crisis period. However, the ranking of margin collections is consistent throughout the full sample period and the two sub-periods being considered. VaR margin consistently collects the least and CoMargin consistently collects the most collateral. Similarly, VaR margin consistently shows the least dispersion and CoMargin consistently shows the most dispersion of collected margin as measured by the standard deviation. This situation arises because CoMargin takes into account the variation of more factors than the other margining methods (i.e., the factors causing P&L dependence).

Panel A of Figure 3.5 shows the daily stacked initial margin requirements under the SPAN, VaR and CoMargin systems. The stacking process is the same as that used in the previous section for the P&L values of Figure 3.4. Notice that all three approaches produce margin requirements that are highly correlated. Table 3.13 shows the average cross-sectional correlation for the full sample period and the two sub-periods and displays the standard deviations in brackets. The high correlation and low dispersion between the SPAN and VaR systems coupled with the average collection values shown in Table 3.10, Table 3.11 and Table 3.12, indicate that at the individual CM level, SPAN margins behave much like VaR margins but at a higher coverage (i.e., lower α) probability. However, notice that CoMargin is the least correlated of the three systems and shows the widest dispersion. This dispersion is more pronounced during the crisis-period, when P&L dependence is higher. As mentioned in the previous section, this can be explained by the fact that CoMargin converges to VaR margin as P&L dependence decreases and diverges as P&L dependence increases.

Table 3.10. Daily margin requirements under the SPAN, VaR and CoMargin systems over the full sample period

	Mean	Median	St.Dev.	Min	Max
			Aggregate Market (CCP level)		
SPAN	112.04	105.71	38.49	49.83	328.77
VaR	101.40	95.80	36.03	42.90	301.97
CoMargin	161.31	156.20	64.93	56.89	475.27
BNSPAN	161.31	156.20	64.93	56.88	475.27
BNVaR	161.31	156.20	64.93	56.88	475.27
			Cross-sectional (CM level)		
SPAN	9.76	9.16	3.54	3.83	29.86
VaR	8.83	8.26	3.28	3.30	27.45
CoMargin	14.14	13.41	6.08	4.38	43.01
BNSPAN	14.14	13.41	6.08	4.38	43.01
BNVaR	14.14	13.41	6.08	4.38	43.01
			Time Series (CM level)		
SPAN	5.85	1.48	7.63	0.01	22.65
VaR	5.34	1.44	7.03	0.01	20.38
CoMargin	8.50	1.87	11.69	0.01	35.00
BNSPAN	10.16	5.77	7.67	3.06	27.02
BNVaR	10.55	6.95	7.06	3.81	25.69

Note: The table presents the summary statistics of the daily margin requirements under the SPAN, VaR (equation 3.2) and CoMargin (equation 3.10) systems for the 23 active firm accounts during the full sample period, from January 2, 2003 to March 31, 2011. The budget-neutral versions of the SPAN (BNSPAN) and VaR (BNVaR) systems are also presented and were computed using equation 3.21. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. Margin amounts are reported in millions of dollars.

Table 3.11. Daily margin requirements under the SPAN, VaR and CoMargin systems during the pre-crisis period

	Mean	Median	St.Dev.	Min	Max
			Aggregate Market (CCP level)		
SPAN	100.27	100.80	24.19	49.83	154.91
VaR	91.20	92.31	21.57	42.90	140.43
CoMargin	140.14	139.08	45.63	56.89	262.98
BNSPAN	140.14	139.07	45.63	56.88	262.97
BNVaR	140.14	139.07	45.63	56.88	262.97
			Cross-sectional (CM level)		
SPAN	8.49	8.32	2.38	3.83	14.98
VaR	7.72	7.47	2.15	3.30	13.79
CoMargin	11.94	11.62	4.45	4.38	25.98
BNSPAN	11.94	11.62	4.45	4.38	25.98
BNVaR	11.94	11.62	4.45	4.38	25.98
			Time Series (CM level)		
SPAN	6.30	1.76	7.87	0.01	22.05
VaR	5.73	1.56	7.27	0.01	19.62
CoMargin	8.80	2.02	11.40	0.01	32.46
BNSPAN	9.73	5.58	7.94	2.14	25.50
BNVaR	9.86	6.01	7.40	2.67	23.83

Note: The table presents the summary statistics of the daily margin requirements under the SPAN, VaR (equation 3.2) and CoMargin (equation 3.10) systems for the 23 active firm accounts during the pre-crisis period, from January 2, 2003 to July 31, 2007. The budget-neutral versions of the SPAN (BNSPAN) and VaR (BNVaR) systems are also presented and were computed using equation 3.21. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. Margin amounts are reported in millions of dollars.

Table 3.12. Daily margin requirements under the SPAN, VaR and CoMargin systems during the crisis period

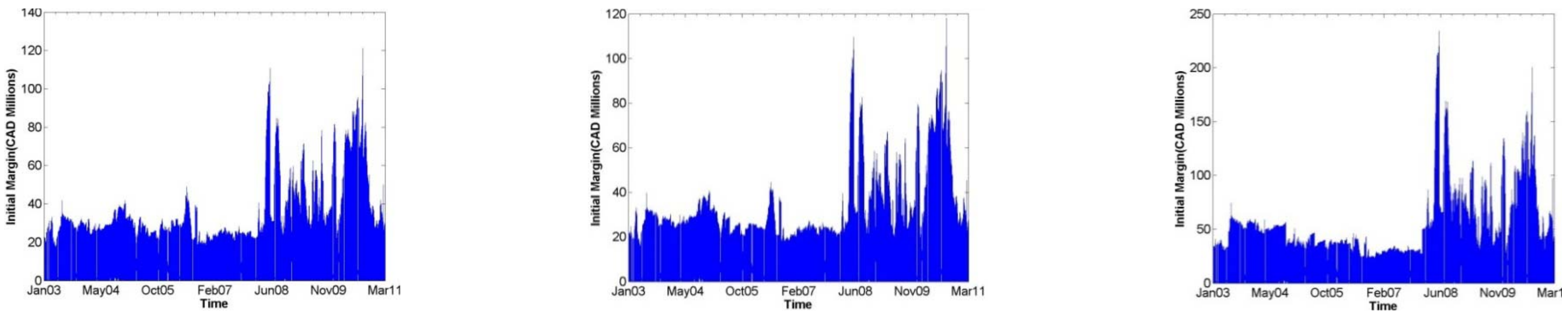
	Mean	Median	St.Dev.	Min	Max
			Aggregate Market (CCP level)		
SPAN	126.76	115.88	47.05	62.72	328.77
VaR	114.17	102.57	45.25	50.76	301.97
CoMargin	187.78	174.05	75.01	72.88	475.27
BNSPAN	187.78	174.05	75.01	72.88	475.27
BNVaR	187.78	174.05	75.01	72.88	475.27
			Cross-sectional (CM level)		
SPAN	11.36	10.85	4.07	4.88	29.86
VaR	10.21	9.51	3.87	4.46	27.45
CoMargin	16.89	16.21	6.69	5.64	43.01
BNSPAN	16.89	16.21	6.69	5.64	43.01
BNVaR	16.89	16.21	6.69	5.64	43.01
			Time Series (CM level)		
SPAN	7.21	2.24	10.44	0.01	37.10
VaR	6.58	2.09	9.67	0.01	34.05
CoMargin	10.95	3.11	16.72	0.01	60.15
BNSPAN	12.29	7.61	10.86	3.30	42.64
BNVaR	12.72	8.55	10.10	4.20	40.73

Note: The table presents the summary statistics of the daily margin requirements under the SPAN, VaR (equation 3.2) and CoMargin (equation 3.10) systems for the 23 active firm accounts during the crisis period, from August 1, 2007 to March 31, 2011. The budget-neutral versions of the SPAN (BNSPAN) and VaR (BNVaR) systems are also presented and were computed using equation 3.21. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. Margin amounts are reported in millions of dollars.

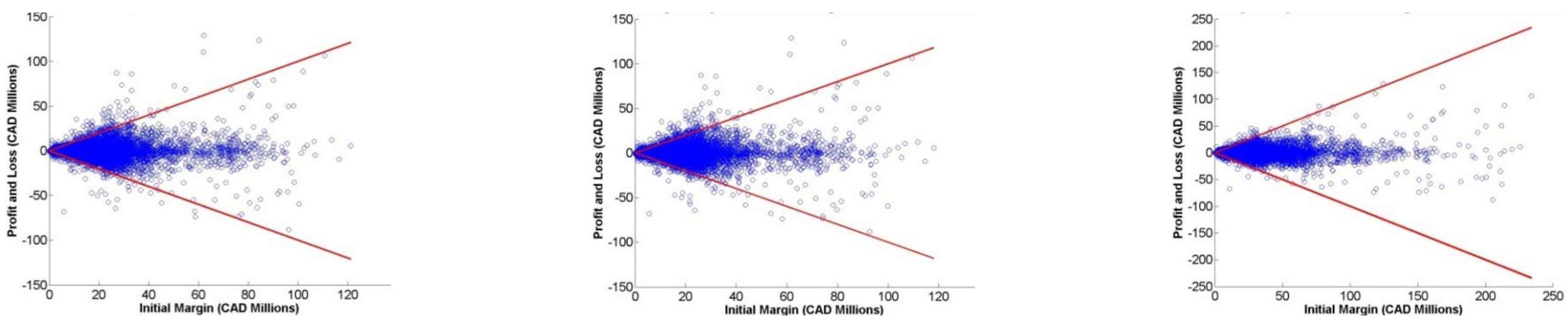
Figure 3.5. SPAN, VaR and CoMargin collateral requirements over the full sample period

SPAN	**VaR**	**CoMargin**

Panel A: Initial Margin

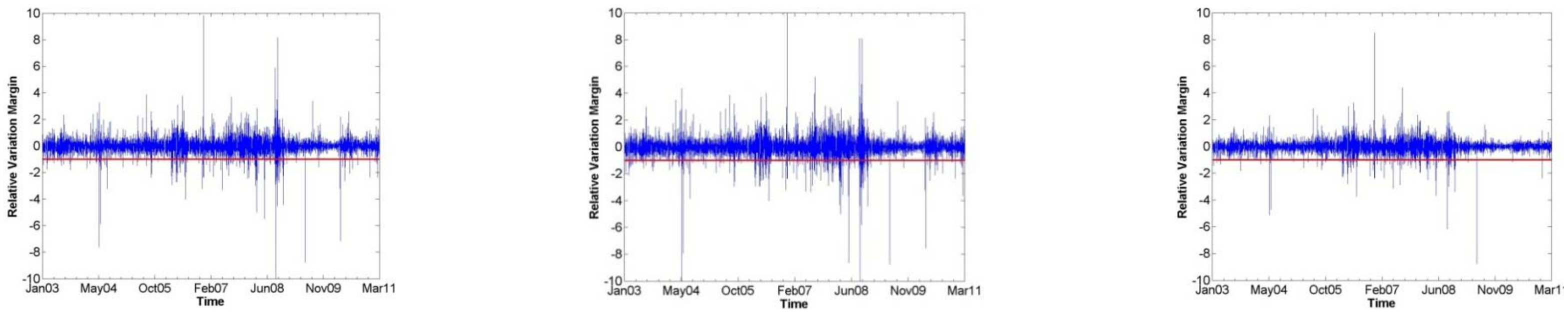


Panel B: Daily P&L and Initial Margins



SPAN **VaR** **CoMargin**

Panel C: Relative Variation Margin



Note: The figure shows the implied margin requirements from the positions in the 23 active firm accounts of the $N = 48$ clearing members in the sample under the SPAN, VaR and CoMargin systems. Panel A shows the daily stacked initial margin requirements. Panel B plots the daily implied P&L against its initial margin requirement. Panel C shows the daily stacked values of the relative variation margin, which is defined as the ratio of P&L to posted initial margin. The stacking method used in panels A and C is as follows: for each date t, $n^{act} \in N$ observations are plotted, which correspond to the observations of the n^{act} clearing members with an active account. The sample period is from January 2, 2003 to March 31, 2011.

Table 3.13. Average cross-sectional correlation between the SPAN, VaR and CoMargin systems

Variable	SPAN	VaR	CoMargin
Panel A: Full sample period			
SPAN	1.00 (0.00)		
VaR	0.99 (0.01)	1.00 (0.00)	
CoMargin	0.90 (0.28)	0.90 (0.29)	1.00 (0.00)
Panel B: Pre-Crisis period			
SPAN	1.00 (0.00)		
VaR	0.98 (0.03)	1.00 (0.00)	
CoMargin	0.94 (0.05)	0.94 (0.05)	1.00 (0.00)
Panel C: Crisis period			
SPAN	1.00 (0.00)		
VaR	0.99 (0.01)	1.00 (0.00)	
CoMargin	0.90 (0.29)	0.90 (0.29)	1.00 (0.00)

Note: The table shows the average cross-sectional correlation between the SPAN, VaR and CoMargin requirements of the 23 active firm accounts in the sample. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. The sample period is from January 2, 2003 to March 31, 2011. The pre-crisis period is from January 2, 2003 to July 31, 2007 and the crisis period is from August 1, 2007 to March 31, 2011. Standard deviations are reported in brackets.

Panel B of Figure 3.5 plots the daily P&L of each active CM against its initial margin requirement. The 45 and -45 degree lines are indicated in red. Observations falling below the -45 degree line denote margin exceeding losses. Notice that of the three margining systems, CoMargin shows the least number of margin exceedances. In addition, unlike the other systems, CoMargin tends to concentrate exceedances in low initial margin, low P&L points. These points represent clearing members with the smallest or least active portfolios; that is, those that are the least likely to pose a systemic threat to the CCP.

Panel C of Figure 3.5 shows the daily stacked values of the relative variation margin, which is defined as the ratio of P&L to posted initial margin ($V_{i,t}/B_{i,t-1}$). Once again, the stacking process is the same as that used in Figure 3.4. Observations with a relative variation margin below -1, the level depicted with a red line, represent margin exceedances. Notice that the CoMargin system exhibits the lowest number of simultaneous (i.e. clustered) margin exceedances.

Table 3.14 summarizes the performance of different margining systems over the full sample period. Table 3.15 and Table 3.16 show the corresponding values for the pre-crisis and crisis periods, respectively. The left panel of the tables measures unconditional performance in terms of the probability of experiencing at least one exceedance, the average number of exceedances and the expected shortfall if at least one exceedance occurs. The right panel of the tables reports the same measures but conditional on at least one member exceeding its margin.

Consistent with the theoretical results presented in the previous section, our empirical results show that the CoMargin system outperforms the SPAN and VaR systems in all dimensions, whether these are estimated unconditionally or conditionally. At the CCP level, notice how CoMargin consistently has the lowest probability of exceedances and the lowest average number of exceedances across the three systems. A lowest number of simultaneous exceedances also allows CoMargin to have the lowest expected shortfall.

Table 3.14. Performance of the SPAN, VaR and CoMargin systems over the full sample period

	Unconditional			Conditional on at least one exceedance		
	Prob. of Exceedances	**Avg. Exceedances**	**Avg. Shortfall (CAD Millions)**	**Prob. of Exceedances**	**Avg. Exceedances**	**Avg. Shortfall (CAD Millions)**
Aggregate Market (CCP level)						
SPAN	0.09	0.15	0.35	0.36	1.63	3.78
VaR	0.14	0.25	0.44	0.42	1.80	3.20
CoMargin	0.07	0.10	0.13	0.28	1.44	1.85
BNSPAN	0.02	0.02	0.16	0.38	1.47	10.15
BNVaR	0.02	0.03	0.16	0.36	1.47	9.45
Cross-sectional (CM level)						
SPAN	0.01	-	0.03	0.14	-	0.34
VaR	0.02	-	0.04	0.15	-	0.29
CoMargin	0.01	-	0.01	0.12	-	0.16
BNSPAN	0.00	-	0.01	0.12	-	0.90
BNVaR	0.00	-	0.01	0.12	-	0.84
Time Series (CM level)						
SPAN	0.01	-	0.02	0.15	-	0.25
VaR	0.02	-	0.02	0.15	-	0.21
CoMargin	0.01	-	0.01	0.12	-	0.11
BNSPAN	0.00	-	0.01	0.11	-	0.74
BNVaR	0.00	-	0.01	0.11	-	0.69

Note: The table compares the empirical performance of the SPAN, VaR (equation 3.2) and CoMargin (equation 3.10) systems computed for the 23 active firm accounts over the sample period, from January 2, 2003 to March 31, 2011. The left panel reports unconditional amounts and the right panel reports the same amounts conditional on at least one margin exceedance. An exceedance is defined as a loss that exceeds the margin posted by a clearing member at the end of the previous trading day. The budget-neutral versions of the SPAN (BNSPAN) and VaR (BNVaR) systems are also presented and were computed using equation 3.21. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. Average shortfall values are reported in millions of dollars and correspond to the losses expected if margin exceedances occur.

Table 3.15. Performance of the SPAN, VaR and CoMargin systems during the pre-crisis period

	Unconditional			Conditional on at least one exceedance		
	Prob. of Exceedances	Avg. Exceedances	Avg. Shortfall (CAD Millions)	Prob. of Exceedances	Avg. Exceedances	Avg. Shortfall (CAD Millions)
			Aggregate Market (CCP level)			
SPAN	0.09	0.12	0.08	0.31	1.35	0.90
VaR	0.12	0.19	0.12	0.40	1.60	1.03
CoMargin	0.07	0.09	0.04	0.25	1.27	0.60
BNSPAN	0.01	0.01	0.03	0.23	1.23	2.22
BNVaR	0.01	0.01	0.03	0.31	1.31	2.36
			Cross-sectional (CM level)			
SPAN	0.01	-	0.01	0.11	-	0.08
VaR	0.02	-	0.01	0.13	-	0.09
CoMargin	0.01	-	0.00	0.10	-	0.05
BNSPAN	0.00	-	0.00	0.10	-	0.17
BNVaR	0.00	-	0.00	0.10	-	0.18
			Time Series (CM level)			
SPAN	0.01	-	0.00	0.12	-	0.06
VaR	0.02	-	0.01	0.14	-	0.07
CoMargin	0.01	-	0.00	0.11	-	0.04
BNSPAN	0.00	-	0.00	0.09	-	0.17
BNVaR	0.00	-	0.00	0.10	-	0.18

Note: The table compares the empirical performance of the SPAN, VaR (equation 3.2) and CoMargin (equation 3.10) systems computed for the 23 active firm accounts over the pre-crisis period, from January 2, 2003 to July 31, 2007. The left panel reports unconditional amounts and the right panel reports the same amounts conditional on at least one margin exceedance. An exceedance is defined as a loss that exceeds the margin posted by a clearing member at the end of the previous trading day. The budget-neutral versions of the SPAN (BNSPAN) and VaR (BNVaR) systems are also presented and were computed using equation 3.21. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. Average shortfall values are reported in millions of dollars and correspond to the losses expected if margin exceedances occur.

Table 3.16. **Performance of the SPAN, VaR and CoMargin systems during the crisis period**

	Unconditional			Conditional on at least one exceedance		
	Prob. of Exceedances	**Avg. Exceedances**	**Avg. Shortfall (CAD Millions)**	**Prob. of Exceedances**	**Avg. Exceedances**	**Avg. Shortfall (CAD Millions)**
			Aggregate Market (CCP level)			
SPAN	0.10	0.19	0.69	0.42	1.93	6.91
VaR	0.16	0.31	0.83	0.45	1.99	5.32
CoMargin	0.07	0.12	0.24	0.32	1.63	3.33
BNSPAN	0.02	0.03	0.32	0.47	1.63	15.58
BNVaR	0.03	0.04	0.34	0.39	1.57	13.46
			Cross-sectional (CM level)			
SPAN	0.02	-	0.06	0.17	-	0.63
VaR	0.03	-	0.08	0.18	-	0.48
CoMargin	0.01	-	0.02	0.14	-	0.30
BNSPAN	0.00	-	0.03	0.14	-	1.40
BNVaR	0.00	-	0.03	0.14	-	1.21
			Time Series (CM level)			
SPAN	0.02	-	0.04	0.18	-	0.51
VaR	0.02	-	0.05	0.16	-	0.36
CoMargin	0.01	-	0.01	0.13	-	0.24
BNSPAN	0.00	-	0.02	0.12	-	1.15
BNVaR	0.00	-	0.02	0.12	-	0.99

Note: The table compares the empirical performance of the SPAN, VaR (equation 3.2) and CoMargin (equation 3.10) systems computed for the 23 active firm accounts over the crisis period, from August 1, 2007 to March 31, 2011. The left panel reports unconditional amounts and the right panel reports the same amounts conditional on at least one margin exceedance. An exceedance is defined as a loss that exceeds the margin posted by a clearing member at the end of the previous trading day. The budget-neutral versions of the SPAN (BNSPAN) and VaR (BNVaR) systems are also presented and were computed using equation 3.21. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. Average shortfall values are reported in millions of dollars and correspond to the losses expected if margin exceedances occur.

In addition, the relative performance of CoMargin increases when we condition on at least one exceedance event. This finding shows that CoMargin does a better job at protecting the CCP from simultaneous exceedances, even after a clearing member has surpassed its margin. Furthermore, Table 3.15 and Table 3.16 show that the relative performance of CoMargin also improves during the crisis period, when P&L dependence is more persistent. This indicates that CoMargin tends to provide more protection to the CCP when it is needed most; that is, when simultaneous exceedance events are more likely to occur.

The cross-sectional panels of Table 3.14, Table 3.15 and Table 3.16 show the probability that a typical CM surpasses its margin on any given day, and the expected shortfall associated with this event. Once again, the conditional probabilities and expected shortfalls are lower for CoMargin than for SPAN and VaR margins. This implies that under the SPAN and VaR systems, the typical CM is more likely to experience a margin exceeding loss when another CM has exceeded its margin, than under the CoMargin system. In addition, notice how both the conditional and unconditional probabilities increase for the SPAN and VaR systems during the crisis period relative to the pre-crisis period. These results imply that when P&L dependence increases, SPAN and VaR margin requirements are more likely to be exceeded by the typical CM than CoMargin requirements.

The time series panels in tables Table 3.14, Table 3.15 and Table 3.16 show the average values over the sample period for each CM. By definition, the unconditional probability of exceedances corresponds to one minus the coverage level. For VaR margin, this probability corresponds to $\alpha = 2\%$, which was used for its estimation. Similarly, CoMargin shows a constant unconditional exceedance probability of 1% during the full sample and both sup-periods. However, notice that for the SPAN system this probability increases (i.e., its unconditional coverage decreases) during the crisis period. This situation arises because the SPAN system targets price and volatility ranges instead portfolio-wide P&L quantiles, as it is the case in the VaR and CoMargin systems.

Table 3.14, Table 3.15 and Table 3.16 also show the performance of the budget-neutral SPAN and VaR systems (BNSPAN and BNVaR, respectively). As explained in the previous section, these artificial constructs allow us to test whether CoMargin performs better that its counterparts due to its allocation of collateral or due to the fact that it collects more funds. At a first glance, the results show that at the CCP and CM level, budget-neutral margins tend to perform better than CoMargin in terms of unconditional exceedance probabilities. However, as explained in the previous section, this is consistent with our theoretical results. By construction, budget-neutral methods tend to collect more aggregate collateral than the SPAN and VaR margining systems and spread the additional requirements evenly across clearing members. This allocation of collateral increases the coverage level of the SPAN and VaR systems across all CMs and reduces their unconditional exceedance probabilities.

For the pre-crisis period, Table 3.15 shows that BNSPAN slightly outperforms CoMargin in terms of conditional exceedance probabilities. However, there is an explanation for this finding that is also consistent with our theoretical results. During periods of low P&L dependence, such as the pre-crisis period, unconditional exceedance probabilities tend to be more important than conditional probabilities in determining the likelihood of simultaneous distress events (see equation 3.6). By collecting more collateral across all CMs, BNSPAN further reduces the unconditional exceedance probabilities of SPAN margins, to the point that it reduces the likelihood (but not the expected shortfall) of simultaneous distress events. This effect is further reinforced by the fact that during low P&L dependence periods, the CoMargin of many CMs converges to VaR margin (see equation 3.17), which Table 3.15 also shows has a higher unconditional exceedance probability than SPAN.

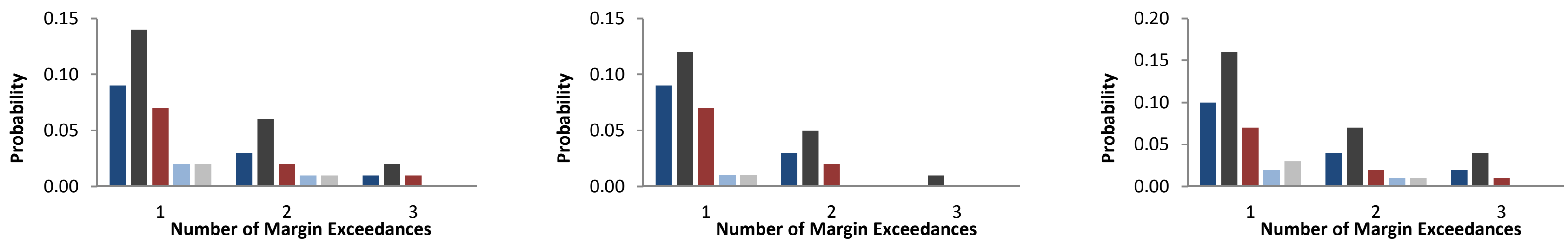
Nevertheless, despite the fact that budget-neutral measures sometimes have lower exceedance probabilities, in all cases, the results show that CoMargin yields the lowest expected shortfalls. This finding is once again consistent with our theoretical results. Relative to the CoMargin system, budget-neutral methods effectively transfer margin requirements from firms with high P&L dependence to those with low P&L dependence; thus, leaving the CCP exposed to simultaneous exceedance events. These simultaneous events account for the higher (conditional and unconditional) expected shortfalls under the budget-neutral systems. Since expected shortfalls ultimately determine the impact on the funds available to the CCP, our findings show that CoMargin allocations enhance the resilience of clearing houses by minimizing the likelihood and economic impact of adverse events.

Figure 3.6 extends our empirical findings by conditioning on up to three margin exceedance events. The results on the charts confirm that CoMargin tends to perform better than the SPAN and VaR systems even after accounting for the additional amount of collateral required (i.e., after using budget-neutral measures).

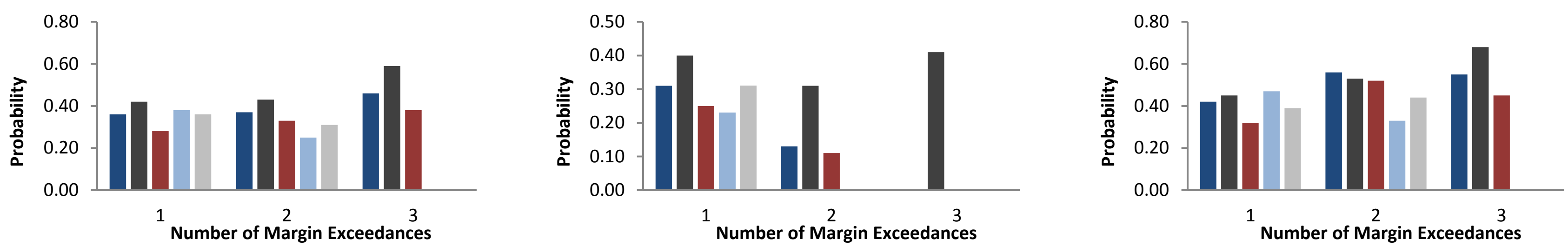
Figure 3.6. Empirical performance of the SPAN, VaR and CoMargin systems

Full Sample | **Pre-Crisis** | **Crisis**

Panel A: Probability of a minimum number of margin exceedances

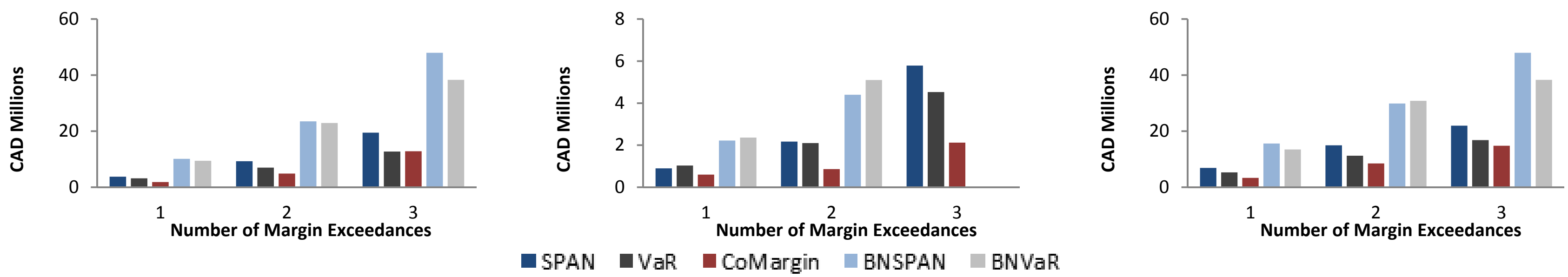


Panel B: Probability of additional margin exceedances given that a number of exceedances has occurred



Full Sample **Pre-Crisis** **Crisis**

Panel C: Conditional expected shortfall for the CCP given a minimum number of margin exceedances



Note: The figure compares the empirical performance of the SPAN, VaR (equation 3.2) and CoMargin (equation 3.10) systems computed for the 23 active firm accounts in the sample. The budget-neutral versions of the SPAN (BNSPAN) and VaR (BNVaR) systems are also presented and were computed using equation 3.21. An account is considered to be active on a given day if it has an open interest (i.e., long or short position at the end of the trading day) in at least one underlying asset. The sample period is from January 2, 2003 to March 31, 2011. The pre-crisis period is from January 2, 2003 to July 31, 2007 and the crisis period is from August 1, 2007 to March 31, 2011.

3.6. Conclusion

In this chapter, we present a new methodology, called CoMargin, to estimate margin requirements in derivatives CCPs. Our approach is innovative because it explicitly takes into account both the individual risk and the interdependence of the P&Ls of market participants. As a result, CoMargin produces collateral allocations that enhance the stability and resilience of the CCPs, which in turn reduce their systemic risk.

We show theoretically and empirically that CoMargin outperforms the widely popular SPAN and VaR margining approaches. Our method performs particularly well relative to these alternatives when the level of P&L dependence across market participants increases, as was the case during the recent financial crisis. Therefore, CoMargin provides more protection to the CCP when it needs it most.

At a technical level, we show how credit risk can be assessed using a scenario-based approach that takes into account the co-movement of underlying assets and similarities across portfolios. We also contribute to the literature by developing a backtesting methodology that relies on formal statistical tests and that can be generalized to any number of market participants. Formal backtesting techniques are particularly important in the context of structural market changes, such as, those that occurred during the financial crisis as a result of policy changes (e.g., the current G20 mandate to centrally clear OTC derivatives).The use of systems that cannot be backtested, such as SPAN, impose a challenge to both risk managers and regulators, who need to assess the effectiveness and consistency of their risk management policies.

Finally, at a more general level, the chapter illustrates the importance of accounting for simultaneous extreme events, or interdependencies, when managing credit risk. Our approach can be seen as a stepping stone that can be generalized and used in different situations, such as estimating collateral requirements for repo and other over-the-counter (OTC) transactions, assessing capital requirements for banks and insurance companies, or monitoring the accumulation of credit risk across market participants for regulatory purposes.

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Appendix A.

Proofs for CoMargin Properties

Proof A1: Let $H(B^{i|j}, \sigma_i)$ be a function such that:

$$H(B^{i|j}, \sigma_i) = \int_{-\infty}^{-B^{i|j}} g(u, \sigma_i) du - \alpha = 0 \tag{A1}$$

Note that we simplified the notation of the pdf $g(u; \sigma_i)$ compared to equation 3.15. Then, the CoMargin can be defined as an implicit function $B^{i|j} = h(\sigma_i)$. By the Implicit Functions Theorem, we have:

$$\frac{\partial B^{i|j}}{\partial \sigma_i} = -\frac{H_{\sigma_i}(B^{i|j}, \sigma_i)}{H_B(B^{i|j}, \sigma_i)} \tag{A2}$$

The derivative $H_B(B^{i|j}, \sigma_i)$ can be expressed as follows:

$$H_B\left(B^{i|j}, \sigma_i\right) = -g\left(-B^{i|j}; \sigma_i\right) < 0 \tag{A3}$$

and is negative since $g(u; \sigma_i)$ is a pdf. Thus, the sign of $\frac{\partial B^{i|j}}{\partial \sigma_i}$ is given by the sign of $H_{\sigma_i}(B^{i|j}, \sigma_i)$:

$$H_{\sigma_i}\left(B^{i|j}, \sigma_i\right) = \frac{\partial}{\partial \sigma_i}\left(\int_{-\infty}^{-B^{i|j}} g(u; \sigma_i)\, du - \alpha\right) = \int_{-\infty}^{-B^{i|j}} \frac{\partial g(u; \sigma_i)}{\partial \sigma_i} du \tag{A4}$$

For simplicity, let us consider the case where $\rho = 0$:

$$\frac{\partial g(u; \sigma_i)}{\partial \sigma_i} = \frac{\partial}{\partial \sigma_i}\left(\frac{1}{\sigma_i}\phi\left(\frac{u}{\sigma_i}\right)\right) = -\frac{1}{\sigma_i^2}\phi\left(\frac{u}{\sigma_i}\right) - \frac{u}{\sigma_i^3}\phi'\left(\frac{u}{\sigma_i}\right) \tag{A5}$$

Since $\phi'(x) = -x\, \phi(x)$, we have:

$$\frac{\partial g(u; \sigma_i)}{\partial \sigma_i} = -\frac{1}{\sigma_i^2}\phi\left(\frac{u}{\sigma_i}\right)\left(1 - \left(\frac{u}{\sigma_i}\right)^2\right) \tag{A6}$$

For any value of u such that $u < -\sigma_i$, we have $\partial g(u; \rho)/\partial \sigma_i > 0$. This condition is satisfied when $u \in \left[-\infty, -B^{i|j}\right]$ since $-B^{i|j} = \sigma_i \Phi^{-1}(\alpha) = -\sigma_i \Phi^{-1}(1-\alpha)$ and $\Phi^{-1}(1 - \alpha) > 1$ for most of the considered coverage rates (e.g. 1%, 5%). Consequently, the integral AX8 is also positive and $H_{\sigma_i}\left(B^{i|j}, \sigma_i\right) > 0$. Then we conclude that:

$$\frac{\partial B^{i|j}}{\partial \sigma_i} = -\frac{H_{\sigma_i}(B^{i|j},\sigma_i)}{H_B(B^{i|j},\sigma_i)} > 0 \tag{A7}$$

A similar result can be obtained when we relax the assumption.

Proof A2: If $= 0$, the last term in equation 3.15 becomes $\Phi\left(-B_j/\sigma_j\right) = \Phi(\Phi^{-1}(\alpha)) = \alpha$ since $B_i = -\sigma_i \Phi^{-1}(\alpha)$. Consequently, the CoMargin of firm i is the solution of the following integral:

$$\int_{-\infty}^{-B^{i|j}} \frac{1}{\sigma_i} \times \phi(\frac{u}{\sigma_i})\, du = \alpha \tag{A8}$$

By properties of the normal distribution, we have $B^{i|j} = -\sigma_i \Phi^{-1}(\alpha) = B_i$.

Proof A3: Let $F(B^{i|j},\rho)$ be a function such that:

$$F(B^{i|j},\rho) = \int_{-\infty}^{-B^{i|j}} g(u;\rho)\,\mathrm{du} - \alpha = 0 \tag{A9}$$

Note that we simplified the notation of the pdf $g(u;\rho)$ compared to equation 3.15. Then, the CoMargin can be defined as an implicit function $B^{i|j} = f(\rho)$. By the Implicit Functions Theorem, we have:

$$\frac{\partial B^{i|j}}{\partial \rho} = -\frac{F_\rho(B^{i|j},\rho)}{F_B(B^{i|j},\rho)} \tag{A10}$$

where $F_\rho(.)$ and $F_B(.)$ denote respectively the first derivative of the F function with respect to ρ and B. For any function $H(x)$ defined as:

$$H(x) = \int_{-\infty}^{-b(x)} h(t)\, dt \tag{A11}$$

we have $H'(x) = h(b(x)) \times \partial b(x)/\partial x$. Consequently, the derivative $F_B(B^{i|j},\rho)$ can be expressed as follows:

$$F_B\left(B^{i|j},\rho\right) = -g\left(-B^{i|j};\rho\right) < 0 \tag{A12}$$

and is negative since $g(u;\rho)$ is a pdf. Thus, the sign of $\partial B^{i|j}/\partial \rho$ is given by the sign of $F_\rho(B^{i|j},\rho)$:

$$F_\rho\left(B^{i|j},\rho\right)=\frac{\partial}{\partial\rho}\left(\int_{-\infty}^{-B^{i|j}} g(u;\rho)\,du-\alpha\right)=\int_{-\infty}^{-B^{i|j}}\frac{\partial g(u;\rho)}{\partial\rho}du \tag{A13}$$

Given the expression of the pdf $g(u;\rho)$ we have:

$$\begin{aligned}\frac{\partial g(u;\rho)}{\partial\rho}&=\overbrace{-\frac{1}{\alpha\sigma_i}\times\phi\left(\frac{u}{\sigma_i}\right)\times\ \phi\left(\frac{-B_j/\sigma_j-\rho u/\sigma_i}{\sqrt{1-\rho^2}}\right)}^{A}\\&\quad\times\left(\frac{-u/\sigma_i\sqrt{1-\rho^2}-(B_j/\sigma_j+\rho u/\sigma_i)\rho(1-\rho^2)^{-1/2}}{1-\rho^2}\right)\\&=A\times\left(\frac{1}{1-\rho^2}\right)^{3/2}\times\left(\frac{u}{\sigma_i}+\frac{\rho B_j}{\sigma_j}\right)\end{aligned} \tag{A14}$$

This function is positive for any value of u such that $u\leq\ \rho B_i=-\rho\sigma_i\Phi^{-1}(\alpha)$ with $-\rho\sigma_i\Phi^{-1}(\alpha)>0$. Since $B^{i|j}\geq 0$ by definition, this condition is satisfied for the interval $\left[-\infty,-B^{i|j}\right]$ and $F_\rho\left(B^{i|j},\rho\right)>0$. Then we conclude that:

$$\frac{\partial B^{i|j}}{\partial\rho}=\ -\frac{F_\rho(B^{i|j},\rho)}{F_B(B^{i|j},\rho)}>0 \tag{A15}$$

Proof A4: For $\rho=1$, the pdf $g\left(u;\sigma_i,\sigma_j,\rho\right)$ in equation 3.15 is degenerated. However, when ρ tends to one, we have:

$$\lim_{\rho\to1}\Phi\left(\frac{-\mathrm{B_i}/\sigma_j-\rho u}{\sqrt{1-\rho^2}}\right)=1 \tag{A16}$$

as long as $u<\frac{-\mathrm{B_i}}{\sigma_j}=\Phi^{-1}(\alpha)$. If we assume that the standardized CoMargin for i is larger than the standardized VaR margin for i, i.e., $-B^{i|j}/\sigma_i\leq B_j/\sigma_j$, then we have:

$$\lim_{\rho\to1}g(u)=\frac{1}{\alpha\sigma_i}\times\phi\left(\frac{u}{\sigma_i}\right) \tag{A17}$$

And consequently the CoMargin corresponds to the VaR margin defined for a coverage rate α^2 since:

$$\lim_{\rho\to1}\int_{-\infty}^{-B^{i|j}}\frac{1}{\sigma_i}\times\phi\left(\frac{x}{\sigma_i}\right)dx=\alpha^2 \tag{A18}$$

$$\lim_{\rho\to1}B^{i|j}=-\ \sigma_i\Phi^{-1}(\alpha^2) \tag{A19}$$

We can check that condition $-B^{i|j}/\sigma_i \leq B_j/\sigma_j$ is satisfied since $\Phi^{-1}(\alpha^2) \leq \Phi^{-1}(\alpha)$.

Proof A5: Since $B_j = -\sigma_i\Phi^{-1}(\alpha)$, the pdf $g(.)$ in equation 3.15 can be rewritten as:

$$g(u; \sigma_i, \sigma_j, \rho) = \frac{1}{\alpha\sigma_i} \times \phi(\frac{u}{\sigma_i}) \times \Phi\left[\frac{\Phi^{-1}(\alpha) - \rho u/\sigma_i}{\sqrt{1-\rho^2}}\right] \tag{A20}$$

As $g(.)$ does not depend on σ_j, $\partial B^{i|j}/\partial\sigma_j = 0$.

Appendix B.

CoMargin with *n* Firms

With n conditioning firms, $n < N-1$, the conditioning event of the CoMargin is that at least one of the n clearing members is in financial distress. Thus, the definition of CoMargin becomes:

$$\frac{\Pr\left[\left(V_{i,t+1} \leq -B_t^{i|n}\right) \cap \mathbf{C}\left(V_{1,t+1}, \ldots, V_{n,t+1}\right)\right]}{\Pr\left[\mathbf{C}\left(V_{1,t+1}, \ldots, V_{n,t+1}\right)\right]} = \alpha \tag{B1}$$

where the probability to observe the conditioning event is:

$$\Pr\left[\mathbf{C}\left(V_{1,t+1}, \ldots, V_{n,t+1}\right)\right] = \Pr\left[\left(V_{1,t+1} \leq -B_{1,t}\right) \, or \ldots or \, \left(V_{n,t+1} \leq -B_{n,t}\right)\right] \tag{B2}$$

Using Poincaré's formula for the probability of the union of events, we can see that:

$$\begin{aligned}
\Pr\left[\mathbf{C}\left(V_{1,t+1}, \ldots, V_{n,t+1}\right)\right] &= \sum_{\boldsymbol{j=1}}^{\boldsymbol{n}} \Pr\left[\left(V_{j,t+1} \leq -B_{j,t}\right)\right] \\
&- \underbrace{\sum_{1 \leq j_1 < j_2 \leq n}^{\boldsymbol{n}} \Pr\left[\left(V_{j_1,t+1} \leq -B_{j_1,t}\right) \cap \left(V_{j_2,t+1} \leq -B_{j_2,t}\right)\right]}_{\boldsymbol{2\ events}} \\
&+ \underbrace{\sum_{1 \leq j_1 < j_2 < j_3 \leq n}^{\boldsymbol{n}} \Pr\left[\left(V_{j_1,t+1} \leq -B_{j_1,t}\right) \cap \left(V_{j_2,t+1} \leq -B_{j_2,t}\right) \cap \left(V_{j_3,t+1} \leq -B_{j_3,t}\right)\right]}_{\boldsymbol{3\ events}} \\
&\ldots \underbrace{+(-1)^{\boldsymbol{n-1}} \Pr\left[\left(V_{1,t+1} \leq -B_{1,t}\right) \cap \ldots \cap \left(V_{n,t+1} \leq -B_{n,t}\right)\right]}_{\boldsymbol{n\ events}}
\end{aligned} \tag{B3}$$

Thus, the probability of the conditioning event can be rewritten as follows:

$$\Pr\left[\mathbf{C}\left(V_{1,t+1},..,V_{n,t+1}\right)\right] = n\alpha - P_t^n \tag{B4}$$

where P_t^n denotes the sum of the probabilities of all common events (for two events, three events, etc.). An estimator of this value, $\hat{P}_t^n$, can be obtained from the simulated path $\left\{V_{1,t+1}^s,..,V_{n,t+1}^s\right\}_{s=1}^S$. When the financial distress events of the conditioning firms are mutually exclusive, however, the probability of the conditioning events simplifies to $n\alpha$. Therefore, an estimator of the CoMargin of firm i conditional on n clearing members, $B_t^{i|n}$, is the solution of the program:

$$\hat{B}_t^{i|n} = \arg\min_{\left\{B_t^{i|n}\right\}} \left(\frac{\hat{P}_t^{i,n}}{n\alpha - \hat{P}_t^n} - \alpha\right)^2 \tag{B5}$$

where $\hat{P}_t^{i,n}$ denotes the estimator of $\Pr\left[\left(V_{i,t+1} \le -B_t^{i|n}\right) \cap \mathbf{C}\left(V_{1,t+1},..,V_{n,t+1}\right)\right]$, which is obtained by generalizing equation 3.34 conditioning on $B_t^{i|n}$.