Technology-specific capacity and the environment

by

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Abstract

Chapter 1 studies the role of investment-related emissions for the efficient distribution of investment among dirty and clean technologies. Dirty technology is not used depending on technology parameters, though clean technology may be relatively more expensive on all scales, and the societal effect of the first pollution unit may be small. In plausible cases there is a unique stationary point. Disregarding emissions from investment in dirty technology biases the stationary cost of polluting downward if dirty technology is used and the time discount factor is not too small. An inverse relationship between the cost of polluting and the marginal rate of intertemporal substitution of consumption on an optimal path is established.

Chapter 2 examines the retirement of pre-existing capital and irreversible investment in dirty and clean technologies in Pareto optimum and competitive equilibrium. Dirty capacity is optimally underutilized in equilibrium if government policy internalizes the pollution externality after such policy is sufficiently long delayed. Dirty technology capital, for example, fossil-fuel using engines and plants, should be underutilized if pollution, such as atmospheric carbon dioxide, is below its long-term level. Underutilization of the pre-installed dirty technology capital diminishes it optimally because it is not needed in the long-term or smooths it through postponing its use until investment becomes worthwhile in dirty technology. Clean technology capital, for example, solar panels or wind turbines, are efficiently underutilized to save emissions from investment or because creating new units is more costly than forwarding existing units.

Chapter 3 considers production using a dirty and reliable technology, for example, coal-using electricity generation, versus production using a clean and unreliable technology, for example, solar energy conversion into electricity, in a dynamic economy. Consumption can be equalized across states because investment absorbs the fluctuation in clean technology productivity in days in which consumption is maximized. Clean output subsidies such as feed-in premiums for grid-distributed electricity can implement a Pareto optimum. For example, the subsidy rebates a uniform energy tax or a uniform tax on investment goods. In a further example the subsidy is funded by price surcharges that are differentiated between households.

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Introduction

In this thesis heterogeneous capital is considered from a welfare point of view and in competitive equilibrium. The use of dirty technology creates emissions. The use of clean technology does not create emissions. At each date society can build assets in different technologies that have a fixed emission intensity of output by using these assets in future periods. Building new capital units in clean technology may increase pollution of the same kind that production of a general factor using dirty technology creates. Climate change is the leading example. Primary steel, cement, and mineral processiong generate carbon dioxide and methane emissions for the construction of both physical capital that converts fossil fuels coal, petroleum, or natural gas into useful energy, and physical capital that harnesses so-called renewable energy—solar, wind, geothermal, biomass, hydro, tidal and wave energy. The extraction of fossil fuels creates carbon dioxide and methane emissions. Fossil-fuel use in producing 'useful energy' generates carbon dioxide emissions. This energy can be used for producing consumption or investment goods. The use encompasses services generated from space or water cooling and heating, lighting, and motion. Renewable energy conversion does not create such emissions. Carbon dioxide and methane (and some other) emissions alter the climate with adverse effects by current scientific knowledge. These effects are modeled through direct impact of the pollution stock on the period-utility function. The following relates the assumption of irreversibility of technology-specific capital to the central contributions of the thesis, explains their relevance to controlling the climate, and contrasts inefficient investment under externalities and cyclical inefficient investment.

(i) Outlook. Introducing emissions of investment in clean technology, as Chapter 1 shows, enlarges the set of qualitatively different stationary points. These emissions do not affect the nature of the controlled dynamical system in optimum which generates unique paths of the environmental stock, human-made capital, and their marginal contributions to welfare, the so-called shadow prices. Exclusive use of clean technology, joint use of dirty and clean technology, versus exclusive use of dirty technology, in the long-term depend on technology parameters including their productivity, emission intensities, and individual maximum scale in terms of output. The emissions in building clean technology assets are responsible for a time-invariant allocation in which only clean technology is used. This insight gains some weight in the climate problem, because there is capital in energy use besides capital in energy production. The emissions in creating energy-use capital, that is, buildings including equipment and furniture, roads, and vehicle shells, can motivate such a point in the future if the construction of renewable energy technology capital becomes clean. These conclusions hold because consumption goods that spend utility, consistent with the real world, use energy that is produced by dirty or clean technologies, and utilize capital that uses such energy. The emissions in producing dirty technology assets add to the emissions of using these assets without qualitative effect for the use of dirty versus clean technologies. However, the emissions in building dirty technology capital affect the long-term cost of polluting if dirty technology is used in the long-term. Chapter 1 analyses the distribution of investment in dirty versus clean technologies when capital in each technology is fully utilized. Chapters 2 and 3 posit variable utilization of capital that leads to the efficient temporary pausing or permanent abandoning of the production using specific capital because of concerns about the environment.

Chapter 2 examines underutilizing pre-existing capital to preserve the environment building on Chapter 1. Emissions of clean technology investment can rationalize the immediate retirement of dirty technology capital. Only clean technology produces energy for consumption and investment in the long-term on such a path, on paths with initially partially utilized and successively idle dirty technology capital, and on paths in which dirty technology capital is used up in finite time and is not rebuilt. All these paths require that large renewable energy capital can be built at sufficiently low cost. This cost does not need to be lower than the cost of building fossil-fuel engines and plants. Empirical examinations have to determine if technological improvement in recent years has been sufficient for this. The former two paths apply now in this situation and in the near future if technological improvements lead to this situation within the next 50 years which is the usable time of a newly built coal power plant. Pre-existing fossil-fuel engines and power plants for cooling and heating, light, stationary drive, or mobility (dirty energy production capital) are stranded—in the former two paths—because the endogenous cost of polluting in growing the economy with energy production using solar panels, wind turbines, geothermal heat pumps, and hydroelectric dams for stationary energy use and transportation, and energy conversion from sugarcane or algae for mobility (clean energy production capital), is greater than the cost of reducing pollution by not using the dirty energy production capital.

Decommissioning productive dirty technology capital can be also optimal if the harm of affecting the environment is sufficiently large. The latter, for example, can motivate a phase-out of nuclear power with underutilization or a ban of using pre-existing genetically modified seeds, pesticides, or derivatives of fossil fuels in contact with food. In these cases the basic model may benefit from an extension to uncertain effects of pollution. The model applies to the underlying environmental concerns of practiced public policy on local air pollutants, lead pipes, acid rain, ozone-depleting substances, or ocean fish that aims at retiring specific automobiles, water pipes, refrigerants, or fishing vessels, respectively, under a strong environmental feedback. In these cases seemingly in the allocation that the policy targets, that might be an optimum, investment is banned by governments in some technologies whose (1) capital in place is used up or (2) capital is underutilized and unutilized capital becomes unproductive.

Determining an optimal climate policy taking into account the emissions of investment and locational technology scale for given productivity requires empirical modeling in further research. Postponing the use of dirty technology capital early in the planning horizon optimally smooths both the sequences of pollution and dirty technology capital if there is efficient use of such pre-installed capital. A simulation shows that this occurs for initial pollution levels in the optimization that are smaller than its long-term level given a strictly convex utility function in pollution on a competitive equilibrium path that has started without government policy and with a pollution level that does not marginally affect utility. This result is in contrast to a model in the literature with chosen underutilization of fishing vessels that fosters the regeneration of fish stock, where capital is optimally underutilized only if the biomass is smaller than its long-term level. This seems to be the only model in the literature with optimized utilization of a variable capital stock and a replenishable environmental stock. In Chapter 2 and in this fishery example there is a trade-off between consuming output and an environmental impact of production. Here production enhances pollution which adversely affects utility. There production reduces the fish stock and the fish stock positively affects productivity. Here lack of government policy leads to a large dirty technology capital stock that is efficiently underutilized because in any period the absorption of pollution in the environment is lower than pollution which thus can be persistent. There the economy escapes an open-access regime with full utilization toward the efficient long-term allocation because the open-access regime is located for steady biomass levels derived from an inverted U-shaped regeneration function of biomass. The long-term steady state of the competitive equilibrium without taxes or subsidies is in the subregion of underutilized dirty technology capital in the state plane of pollution and dirty technology capital. The difference with the level comparison of the environmental stock arises because here the marginal utility of consumption is an opportunity cost of investment and there the cost of investing is exogenous. This is important for climate policy as the current atmospheric carbon dioxide content may be below its efficient long-term level in a model in which it has deterministic effects on society.

In Chapter 3 the dispatch of dirty and clean production capacity responds to the fluctuation of clean technology inputs, in particular of renewable energy for electricity production. This contingency is considered in a structure of uncertainty and embedded in an extension to the economy of Chapter 2. As in reality, uncertainty about solar or wind energy technology productivity resolves after its investment. The result of Chapter 1 about exclusive use of clean renewable energy technology is modified in a given period that consists of multiple days with fluctuating renewable energy supply. Fossil fuels are efficiently deployed to back up clean solar or wind energy conversion when the solar radiation is sufficiently weak or the wind speed is sufficiently low in some day within a period, to smooth consumption. Chapter 3 shows that certain technology-specific government policy internalizes the pollution externality. One example is a uniform tax on energy and a clean energy subsidy that rebates the tax amount. A further example is a discriminatory surcharge between households on the uniform price of dirty and clean technology output that funds a clean output subsidy.

(ii) Efficiency. In Chapter 2 the underutilization of capital is optimal because the investment is inefficient prior to optimization. This leads to overcapitalization in dirty technology relative to an optimum. In regard to climate change this can be explained by the lack of knowledge about the dirtiness of production at the time of investment. As time goes on and the dirty character of production is known, one may argue that the political process fails to develop policies that internalize the external effect. Similarly, research may show that some reason is majourly responsible for the cyclical inefficient investment in low-productivity projects. As bubbles recur one might argue that the political process is inapt to provide a legal framework in which private agents make efficient decisions. In contrast, the overinvestment regarding the environment does not fundamentally self-correct while each cyclical bubble succeeds a correction of prices on markets. The overinvestment regarding the environment may be corrected incompletely in competitive equilibrium without taxes or subsidies when pollution affects output.

Deterministic effects of emissions on society facilitate discussion of results prone to technological explicitness. There is uncertainty about the effects of human activities on climate change, and the latter's effects on society. Understanding the implications of this uncertainty for optimal and market equilibrium allocations given heterogeneous capital is the subject of future research.

1

Capacity choice in dirty technology and clean technology

This chapter examines the efficient use of dirty and clean technologies when their investment creates emissions of the same kind that the use of dirty technology generates. Clean technology use does not create emissions. Full utilization of capital is assumed to focus on the role of investment-related emissions for the efficient long-term distribution of investment among dirty and clean technologies. Then dirty technology capital is used up in finite time if there is no dirty technology capital in the long-term. In Chapter 2 the utilization of capital is optimized so that pre-installed dirty technology capital can be optimally idle.

A novelty is the classification of efficient dirty and clean technology use given a dirty character of clean technology. Climate change is an example, because building capital in renewable energy technologies creates carbon dioxide and methane emissions. Optimal greenhouse gas emissions plans should take into account the optimal deployment of technologies that are responsible for the emissions. This chapter concludes that clean technology may be exclusively used in optimum though its investment is polluting—replacing emissions of dirty technology use. Clean technology may be used exclusively in the long-term, even if it is less productive than dirty technology on all scales of investment. Renewable energy technologies are less productive than fossil-fuel technologies on large scale. A necessary condition for the exclusive long-term use of clean technology is that the stationary technology-specific cost of pollution reduction is weakly smaller for dirty technology. This may well be true for renewable energy versus fossil-fuel technologies on small scale of aggregate investment. Future work may determine if or under what pace of technical progress the scale of high-productivity renewable energy technologies is large enough to support such a long-term optimum. Three known reasons why clean technology should be used exclusively in the long-term are (1) pollution harms society strongly, (2) abandoning the use of an input in dirty technology is optimal, because its extraction is too expensive or its stock is depleted, for example in Tahvonen (1997) depending on the extraction cost function, and (3) clean renewable energy technology takes over dirty fossil-fuel energy technology in terms of productivity, for example, through a learning effect in Hartley et al. (2010) when pollution is not controlled. In a study of directed technical change, Acemoglu et al. (2012) assume imperfect substitution of the output of clean and dirty technology types, so that optimal technological progress involves a switch in research effort toward clean technology but prevents its optimal exclusive use. In contrast to Acemoglu et al. (2012), emissions in renewable energy technology investment suggest that a carbon tax or another instrument to internalize a carbon emissions externality should be applied permanently.

The second point of interest of emissions in investment is its effect on the long-term cost of polluting if dirty technology is used in the long-term. The cost of polluting is the marginal rate of substitution of pollution reduction and consumption increase. This cost equals the relative price of pollution reduction corresponding to the dirty technology if dirty technology is used. Society is willing to pay more consumption units to preserve the environment if it has less polluting technology. Accounting a greater portion of emissions in investment lowers the complete emission intensity of dirty technology if the time discount factor is not too small. Then disregarding emissions from investment in dirty technology biases the stationary cost of polluting downward.

I use heterogeneous capital to study effects of the technology-specific rate of emissions in investment. Optimal minimum pollution can be ruled out by assuming small societal impacts by small pollution, there is no nonreproducible factor or cumulative cost for using dirty technology, and no technological progress, so that only the emissions remain as an incentive for exclusive stationary use of clean technology. The emissions from using dirty technology are proportional to valuable output. Finite resources can be consumed or invested. Dirty and clean technologies produce perfectly substitutable output. Therefore, the model differs from those used in the literature with technology choice and the environment even if technology-specific investment does not control emissions. In Acemoglu et al. (2012) finite resources can be distributed to dirty and clean production in each period. They stress an imperfect substitution of dirty and clean inputs that suits an energy-using good and a non-energy using good. In Fischer, Withagen & Toman (2004) investible resources are finite and there is no trade-off between consumption of capital services and investment. But optimal investment is unconstrained so it could be unbounded. These papers assume complementary services and emissions.[1] Keeler, Spence & Zeckhauser (1971), Brock (1977), Tahvonen & Kuuluvainen (1993), Stokey (1998), and Brock & Taylor (2010) assume that output is produced using the substitutable factors capital and emissions. This substitutability can be interpreted as technology choice in controlling the emission intensity of gross output (Stokey 1998, Copeland & Taylor 2004).

Luptačik & Schubert (1982), van der Ploeg & Withagen (1991), and Ayong Le Kama (2001) assume one technology with a specific emission intensity of output, to focus on the respective purpose of their papers. I extend their modeling approach to multiple technologies and emissions of investment. Aggregate investment in clean technologies and the cost of polluting in the following period relate weakly positively if emission intensities of investment in dirty and clean technologies are equal. This follows from the scale-dependent relative cost of dirty and clean technologies. In a study of nonrenewable resource depletion, Tahvonen & Salo (2001) formulate a scale-dependent relative advantage of a nonrenewable resource technology and an alternative renewable resource technology. Pollution is not controlled, whereas this paper has an environmental motive.

The next section characterizes stationary points and studies the dynamic behaviour of optimal plans. The section shows how the accounting of emissions in producing a factor versus proportionally to investing the factor biases the stationary cost of polluting. Section 1.2 examines optimal clean technology investment when there are multiple dirty technologies versus one dirty technology, and characterizes investment in multiple dirty technologies. Section 1.3 views delayed effects of emissions on society, and Section 1.4 concludes with a discussion of results.

[1] Tsur & Zemel (2009) make the size of the economy responsible for technology adoption in producing a factor that is used with a reproducible factor in a substitutable manner. The economy switches from a flow-cost technology to a capital-based technology when this becomes affordable starting at low amounts of the reproducible factor.

1.1 The economy

Consider a discrete-time economy with heterogeneous reproducible assets. A planner chooses a policy that internalizes feedbacks from the environment on society. For example, carbon dioxide in the atmosphere reduces the utility of consuming following floods or droughts, or diminishes health through climatic effects.

Preferences.—There is a unit mass of infinitely-lived households with identical preferences regarding consumption $c \in \mathbb{R}_+$ and pollution $Z \in \mathbb{R}$ represented by

$$J = \sum_{t=0}^{\infty} \beta^t U(c(t), Z(t))$$

where $\beta \in (0, 1)$ is the discount factor and time is t. The period-utility function $U(c, Z) \in \mathbb{R}$ is twice-differentiable for positive consumption, increasing in consumption $c \in \mathbb{R}_+$, $\partial U/\partial c > 0$, and decreasing in pollution, $\partial U/\partial Z < 0$, for $c > 0$. The utility function is strictly concave in consumption and concave in pollution, $\partial^2 U/\partial c^2 < 0$ and $\partial^2 U/\partial Z^2 \leq 0$ for $c > 0$. Both pollution reduction and consumption are noninferior goods.[2] The marginal utility of consumption $\partial U/\partial c$ approaches a large positive value M as consumption tends to zero, $\lim_{c\to 0} \partial U/\partial c = M \leq \infty$, for all Z. Then at least one household consumes a positive amount in any period $t \in \{0, 1, 2, \ldots\}$ in a Pareto optimum.

Technology.—Each technology indexed $j \in \{B, C\}$ uses capital K_j to produce a factor that is input in producing consumption and investment goods. The input amount x_j produces additions to the capital stock of technology j. The resource constraint of the factor is

$$c(t)/B + x_B(t) + x_C(t) \leq K_B(t) + K_C(t) \tag{1.1}$$

all $t \geq 0$. A change in the productivity $B > 0$ in the consumption sector has real effects.[3]

[2] Noninferiority, $\min[(\partial^2 U/\partial c^2)(\partial U/\partial Z)/(\partial U/\partial c), (\partial^2 U/\partial Z^2)(\partial U/\partial c)/(\partial U/\partial Z)] \geq \partial^2 U/\partial c \partial Z$ such that at least one inequality is strict, is a reasonable assumption. If there were markets for consumption and environmental quality then a household with greater income acquired weakly more consumption and reduction of pollution, and more of one of them. Keeler et al. (1971) assume noninferiority.

[3] A greater B makes both pollution reduction and consumption more affordable yet polluting relatively more expensive in terms of welfare. Thus it increases long-term consumption while its effect on long-term pollution is ambiguous. One may rewrite the inputs in the investment sector and capital amounts in terms of the consumption good and rescale both the productivity in the investment sector and the

The capital stock equals production capacity. The law of motion of capacity is

$$K_j(t+1) = Q_j x_j(t) \tag{1.2}$$

where $Q_j > 0$. Thus capital is useful once. Multiple use-periods of capital would yield the same timing of investment. Importantly, capital of some technology is scrapped rather than used when investment in this technology is not worthwhile for a long time. The perpetual inventory method, which the law of motion (1.2) is consistent with at full depreciation of capital, would imply that a technology is used forever when there is capital in this technology at some date at less than full depreciation. I assume the following.

Assumption 1.1 $Q_B > \beta^{-1}$.

Then growth of consumption and output is feasible and optimal absent environmental cost. I do not substitute the new capital units into the resource constraint to analyse the stability of fixed points locally using a dynamical system with shadow prices of capital. Capacity of technology $j \in \{B, C\}$ is bounded,

$$\bar{K}_j \geq K_j(t+1), \tag{1.3}$$

all $t \geq 0$ because of limited recyclable material to create capital or finite space to put capital, whichever yields the lower bound. This constraint is plausible for energy-producing capital and different for each technology $j \in \{B, C\}$ for simplicity. At least one capacity level is positive among the given $K_j(0) \in [0, \bar{K}_j]$ for $j \in \{B, C\}$.

Environment.—Production of one unit of the good using dirty technology B, for example, fossil-fuel based production of useful energy for consumption or investment, creates $d_B > 0$ units of emissions. The use of clean technology C, for example, solar or wind energy conversion, does not create emissions, $d_C = 0$. Investment-related emissions are proportional to the input in investment at rate ρ_j. Aggregate emissions are

$$E = \sum_j (d_j K_j + \rho_j x_j)$$

where $\rho_j \geq 0$ to visit the case of emission-free investment, $\rho_j = 0$. Emissions specific to

emission intensity of processes, which are described below, without affecting results.

investment occur in the production of copper, aluminum, and primary steel using coking coal in smelters and foundries, cement production using limestone, and land-use change in mining. Some emissions from agriculture and deforestation may be specifically attributed to investment. I posit the law of motion of pollution

$$Z(t+1) = Z(t) + E(t) - A(Z(t)) \tag{1.4}$$

all $t \geq 0$ to analyse a stable environmental state. The absorption $A(Z) < Z$ of pollution is a twice differentiable nondecreasing concave function, $\partial^2 A/\partial Z^2 \leq 0 \leq \partial A/\partial Z$. I make one assumption about the parameters.

Assumption 1.2 $(\rho_C - \rho_B)Q_C < d_B Q_B$.

Then there is stationary clean technology investment if its productivity is relatively greater, $Q_B < Q_C$, and it is more emission-intensive, $\rho_B < \rho_C$. The relationship holds trivially if $\rho_B \geq \rho_C$, and is plausible else, if there is a technical upper bound on Q_C.

Let the utility function U satisfy essential independence of the distribution of consumption among households and the Pareto optimal level of pollution. Then any redistribution of consumption yields the same Pareto optimal path of pollution. Bergstrom & Cornes (1983) define essential independence of the distribution of a private good and the level of a public good or bad.[4] For simplicity I focus on an allocation with equal consumption of all households. A Pareto optimal policy of consumption and input in investment $(c, x) \in \mathbb{R}^3_+$ maximizes welfare J subject to the resource constraint (1.1), the laws of motion (1.2) and (1.4) and capacity constraints (1.3) for $j \in \{B, C\}$ all $t \geq 0$. Lagranges' function

$$\begin{aligned}\mathscr{L} = \sum_{t=0}^{\infty} \beta^t \Big\{ U(c(t), Z(t)) + \epsilon(t) \Big[Z(t+1) - \sum_j (d_j K_j(t) + \rho_j x_j(t)) \\ - Z(t) + A(Z(t)) \Big] + \sum_j (q_j(t)[Q_j x_j(t) - K_j(t+1)] + \lambda_{K_j}(t+1) K_j(t+1) \\ + \beta w_j(t+1)[\bar{K}_j - K_j(t+1)] + \lambda_{x_j}(t) x_j(t)) + \lambda(t) \Big[\sum_j (K_j(t) - x_j(t)) - c(t)/B \Big] \Big\}\end{aligned}$$

contains the multipliers ϵ of the transition law of pollution and q_j of the law of motion of

[4]Kreps (1990, 161) shows that a set of nonnegative welfare weights of households exists such that each Pareto optimal allocation maximizes the weighted sum of utilities of all households.

capital in technology j, λ of the resource constraint, and w_j of the capacity constraint in technology j. These multipliers can be interpreted as the shadow prices of the respective constraints in an optimum, for example, $\epsilon(t)$ of the constraint with leading pollution $Z(t+1)$, and $q_j(t)$ of the constraint with leading capital $K_j(t+1)$. One may call these multipliers shadow prices of the respective states, and refer to λ and w_j as the shadow price of contemporaneous output and of the contemporaneous capacity bound, respectively. Maximizing $\mathscr{L}$ yields the following first-order necessary conditions. They are written as inequalities when any of the nonnegative multipliers λ_{K_j} or λ_{x_j} is positive and the respective capacity K_j or input amount x_j is zero. The first-order necessary condition of pollution in period $t \in \{1, 2, \ldots\}$,

$$\epsilon(t) = \beta(-\partial U/\partial Z(t+1)) + \beta(1 - \partial A/\partial Z(t+1))\epsilon(t+1), \quad t \geq 0, \tag{1.5}$$

is a difference equation of the shadow price ϵ. This standard condition tells that $\epsilon(t)$ is a weighted sum of future marginal disutility of pollution, the weights being the marginal contributions of current emissions to future pollution.[5] The shadow price of pollution enters the unit value $\{\lambda - d_B\epsilon\}$ of dirty technology capacity as a cost. The discounted marginal benefit from additional capital in period $(t+1)$ balances the shadow cost of holding capital,

$$\beta\{\lambda(t+1) - d_j\epsilon(t+1)\} - \beta w_j(t+1) \leq q_j(t), \quad = \text{ if } K_j(t+1) > 0, \tag{1.6}$$

all $j \in \{B, C\}$ and $t \geq 0$. The shadow rental value of space, $w_j(t)$, is zero if $K_j(t) < \bar{K}_j$. The marginal utility of consumption $\partial U/\partial c$ equals the shadow price of output divided by the productivity in the consumption sector, λ/B, since $\partial U/\partial c$ is large as consumption tends to zero. The marginal benefit of investing x_j units at most equals its marginal cost,

$$Q_j q_j(t) \leq \lambda(t) + \rho_j\epsilon(t), \quad = \text{ if } x_j(t) > 0, \quad j \in \{\text{B}, \text{C}\} \tag{1.7}$$

all $t \geq 0$. The marginal cost comprises the cost of reduced current output and envi-

[5]The law of motion (1.4) of pollution specifies $Z(t) = \Phi(E(0), E(1), \ldots, E(t-1), t)$ all $t \geq 1$. Let $\Phi(\cdot, t)$ be differentiable with respect to emissions amounts. The weighted sum $\epsilon(t) = \sum_{j=1}^{\infty} \beta^j(-\partial U/\partial Z(t+j))(\partial\Phi(\cdot, t+j)/\partial E(t))$ is the forward solution of (1.5) at $\partial\Phi(\cdot, t+1)/\partial E(t) = 1$ and $\partial\Phi(\cdot, t+j)/\partial E(t) = \prod_{s=1}^{j-1}(1 - \partial A/\partial Z(t+s))$ for $j \geq 2$.

ronmental cost by the investment sector. The next proposition states an existence and uniqueness result regarding an optimal plan, that is defined as an allocation of policy variables and the states pollution and capital.

Proposition 1.1 *There is a unique optimal plan.*

Thus, the planner's objective is well-defined. A proof of Proposition 1.1 is in the appendix. The following examines efficient investment.

1.1.1 Investment in dirty versus clean technology

Technology switching.—The optimal technology choice depends on the value of the (marginal) cost of polluting, or benefit of pollution reduction, defined as

$$\theta = \epsilon/(\partial U/\partial c)$$

which hinges on its numerator, the shadow price ϵ of pollution in terms of utils, and its denominator, the marginal utility of consumption. Precisely, θ is the relative willingness to pay for pollution reduction versus consumption increase. Furthermore define the marginal rate of substitution of consumption in periods t and $(t+1)$, $R(t+1) = (\partial U/\partial c)(t)/\beta(\partial U/\partial c)(t+1)$. Then the conditions (1.6) and (1.7) imply that

$$\frac{Q_j\{B - d_j\theta(t+1)\}}{B + \rho_j\theta(t)} \left\{ \begin{array}{c} > \\ = \\ < \end{array} \right\} R(t+1) \implies K_j(t+1) \left\{ \begin{array}{l} = \bar{K}_j \\ \in [0, \bar{K}_j] \\ = 0 \end{array} \right. , \tag{1.8}$$

all $j \in \{B, C\}$. There is investment $Q_j x_j(t) > 0$ only if the marginal rate of return on investment, the left side in (1.8), is weakly greater than the shadow return $R(t+1)$. The costs of polluting $\theta(t)$ and $\theta(t+1)$ suppress the rate of return on investment below the marginal product Q_j. Investing in a given technology costs more if building new capital units or using these capital units creates more emissions. Thus, the rate of return $Q_j\{B - d_j\theta(t+1)\}/\{B + \rho_j\theta(t)\}$ on investment decreases in the emission intensities d_j and ρ_j. The condition (1.8) confirms the intuition that if the return on investment in some technology exceeds the return on investment in another technology, then there is investment in the latter technology only if the former technology exhausts its capacity

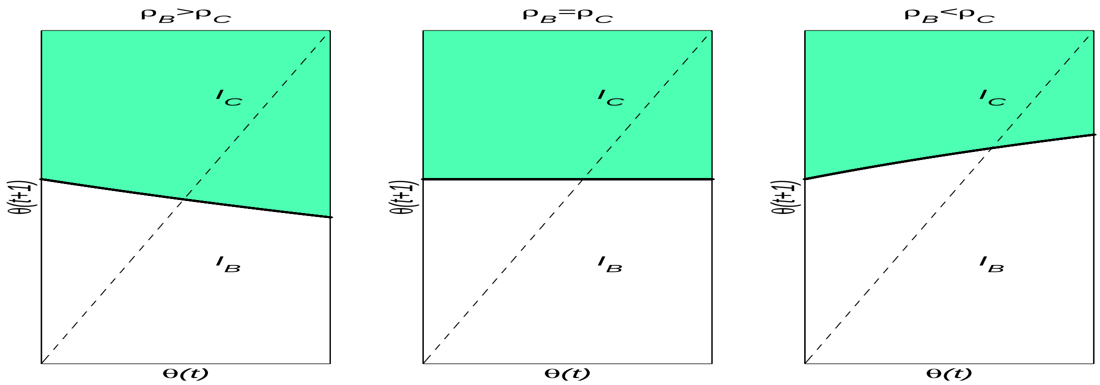


Figure 1.1: *Investment if* $Q_B > Q_C$.

bound. The following analyses dirty versus clean technology investment using the critical curve that equates their rates of return. This helps finding an optimal path knowing the long-term cost of polluting, which is examined below. One particular sequence of θ is optimal. I distinguish three cases with different qualities.

(a) *Case* $Q_B > Q_C$. A relatively small productivity of clean technology makes investing in clean technology particularly dependent on the relative emission intensity in producing capital. Figure 1.1 shows the regions of investment in the dirty and clean technology in $(\theta(t), \theta(t+1))$ space. In each of the panels there is a different relationship of ρ_B and ρ_C. In the shaded region I_C investing in clean technology has priority, $x_C \in (0, \bar{K}_C/Q_C)$ and $x_B = 0$, or $x_C = \bar{K}_C/Q_C$ and $x_B \geq 0$. The region I_B below the critical curve then designates allocations with favoured dirty technology investment, $x_B \in (0, \bar{K}_B/Q_B)$ and $x_C = 0$, or $x_B = \bar{K}_B/Q_B$ and $x_C \geq 0$. Clean technology is relatively more attractive for a large current cost of polluting $\theta(t)$ if building dirty technology capital is relatively more emission-intensive, $\rho_B > \rho_C$. The incentives to invest in dirty versus clean technology do not depend on the concurrent θ if the input use for these investment is equally polluting, $\rho_B = \rho_C$, since then the input use in either technology has the same environmental effect. These incentives still depend on the cost of polluting in the period in which the investment good is used. Clean technology must overcompensate its relatively greater emission intensity in creating new capital, $\rho_B < \rho_C$, through a greater cost of polluting $\theta(t+1)$ in the period following investment given $\theta(t)$, because $\theta(t+1)$ negatively affects the net benefit of using the dirty technology.

(b) $Q_B \leq Q_C$ *and* $\rho_B < \rho_C$. A weakly greater productivity of clean technology than the

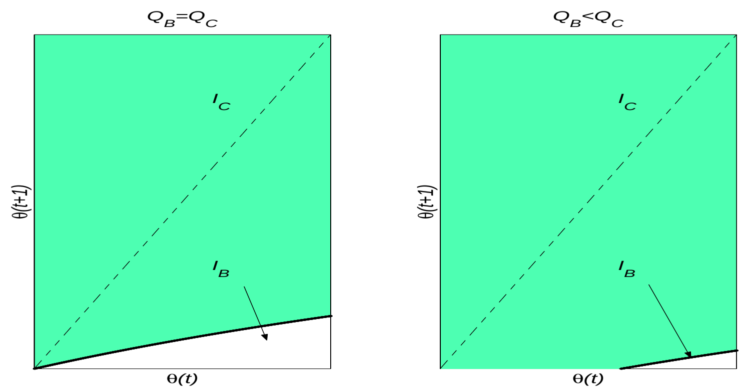


Figure 1.2: *Investment if* $\rho_B < \rho_C$.

productivity of the dirty technology, $Q_B \leq Q_C$, induces clean technology investment in a stationary allocation in which consumption, technology-specific investment, and pollution are constant (stationary point). This can be seen in Figure 1.2. The intercept of the critical curve in $\theta(t)$-$\theta(t+1)$ is nonpositive. Assumption 1.2 guarantees that the slope of this curve is less than one, so that the dashed 45-degree line that includes any stationary cost of polluting lies in the space of investment in clean technology.[6] As will become clear below, dirty technology investment then requires a current high cost of polluting and decreasing shadow cost of pollution ϵ, or small scale of clean technology. The first situation requires strictly concave utility in pollution and therefore seemingly arises only if pollution Z is greater and aggregate capacity $K_B + K_C$ is smaller than their long-term levels, respectively. This situation is unlikely the outcome of an economy subject to no taxes and subsidies in which pollution is accumulated by emissions proportional to capital.[7]

(c) $Q_B \leq Q_C$ *and* $\rho_C \leq \rho_B$. A relatively large productivity and small environmental cost of clean technology make clean technology investment optimal at any date $t \geq 0$. The critical curve has a nonpositive intercept and is nonincreasing in $\theta(t)$. Thus investment in technology C is worthwhile in the entire space $\mathbb{R}^2_+$ of nonnegative costs of polluting. Dirty technology investment may be efficient—as in case (b)—if the scale $\bar{K}_C$ of clean

[6] The general condition required for the slope is $(\rho_C - \rho_B)(d_B - d_C)Q_C Q_B < (d_B Q_B - d_C Q_C)^2$. Assume that $d_B Q_B > d_C Q_C \geq 0$. Then $(d\theta(t+1)/d\theta(t))|_{\theta(t)=0} < 1$ if and only if the symmetric condition holds.

[7] Capital destruction can lead to low capital and high pollution. This means that dirty technology investment efficiently recovers an economy after the unanticipated capital destruction before a switch to clean technology.

technology is small relative to desired output.

Investment switches between technologies if their scale $\bar{K}_j$ is large and the pair of the costs of polluting crosses the critical curve. I examine this switching after showing the uniqueness of certain types of stationary points and their stability.

Stationary points.—A stationary point is a set of values of policy variables and pollution which are constant in a dynamic plan.[8] The following two lemmata provide results that are helpful in proving the uniqueness of certain stationary points.

Lemma 1.1 *The cost of polluting is bounded above, $\theta \leq \theta_j = B(\beta Q_j - 1)/(\rho_j + \beta d_j Q_j)$, at a stationary point with investment, $x_j > 0$, in technology $j \in \{B, C\}$—dirty technology B or emission-intensive clean technology C, that is, $\rho_C > 0$.*

Proof. Either capital is interior, $K_j \in (0, \bar{K}_j)$, or at the upper bound, $K_j = \bar{K}_j$. Then condition (1.8) at $R(t+1) = \beta^{-1}$ implies that $\theta = \theta_j$ or $\theta \leq \theta_j$, respectively. *Q.E.D.*

The relative price of pollution reduction versus consumption increase is θ_j, with price of pollution reduction in the numerator. The marginal rate of substitution of consumption and pollution, θ, equals θ_j if the use of technology j is scaled to optimize pollution and weakly undermines θ_j if society uses technology j to its full extent $\bar{K}_j$. The stationary level θ depends on tastes only through the discount factor when investment is interior, $K_j \in (0, \bar{K}_j)$, for some technology j, because the technologies are linear.[9]

Lemma 1.2 *There is constant consumption or an inverse continuous relationship between consumption and pollution on a curve $\phi(c, Z) = \beta[(-\partial U/\partial Z)/(\partial U/\partial c)]/(1-\beta(1-\partial A/\partial Z))$ equal to the stationary cost of polluting θ.*

This result follows from the noninferiority of pollution reduction and consumption. A proof is in the appendix. Consumption is constant for pollution levels at which both

[8]Idling capital may depreciate so that capital does not need to be constant if the utilization of capital can be chosen. At any stationary point capital is fully utilized if investment occurs, since investment has occured in the preceding period and ongoing underutilization is wasteful. Investment in clean technology is zero if and only if clean technology capacity is zero, because there is no cost of using clean technology capital.

[9]The stationary level of θ may depend on preferences of consumption and pollution if the factor was substitutable for another factor, for example, labour in producing consumption goods and investment goods. In models with production using substitutable factors capital and emissions preferences are needed to find the levels of capital and emissions, which are necessary to compute the stationary cost of polluting.

the utility and the absorption is a linear function of pollution. The stationary cost of polluting is positive. The shadow return equals β^{-1} in a stationary allocation. Thus there is no stationary investment in the clean technology, if its marginal product is smaller than the inverse of the discount factor, $Q_C \leq \beta^{-1}$ and $\rho_C > 0$, or $Q_C < \beta^{-1}$ and $\rho_C = 0$. The next two propositions consider this.

The following lemma helps proving the next proposition and the type of an optimal stationary point when there is a continuum of stationary points.

Lemma 1.3 *Consumption increases more relative to pollution when expanding clean capacity than when increasing dirty capacity,* $(Q_C - 1)/\rho_C > (Q_B - 1)/(\rho_B + d_B Q_B)$*, if the cost of pollution reduction is weakly greater for the clean technology with emissions of investment,* $\theta_B \leq \theta_C$ *and* $\rho_C > 0$*.*

Proof. The result is immediate from $(Q_C - 1)/(Q_B - 1) > (\beta Q_C - 1)/(\beta Q_B - 1) \geq \rho_C/(\rho_B + \beta d_B Q_B) > \rho_C/(\rho_B + d_B Q_B)$ using the definition of θ_j and discounting, $\beta < 1$, if $Q_B > Q_C$. Assumption 1.2 implies that $(\rho_B - \rho_C + d_B Q_B)(Q_C - 1) > 0 \geq \rho_C(Q_B - Q_C)$ since $0 \leq \rho_C$ and $1 < Q_B$, if $Q_B \leq Q_C$ and $\rho_B < \rho_C$. Rearranging the outer relations implies the result. The same applies to $Q_B \leq Q_C$ and $\rho_B \geq \rho_C$, alternatively then $(Q_C - 1)/(Q_B - 1) \geq 1 > \rho_C/(\rho_B + d_B Q_B)$ for $d_B > 0$. *Q.E.D.*

The previous lemma does not require Assumption 1.2 if the emission intensities in investment are equal, $\rho_B = \rho_C$. The next proposition reports different stationary points.

Proposition 1.2 *Let investment in the clean technology create emissions,* $\rho_C > 0$*. There is a unique stationary point with exclusive clean technology use (CU),* $x_B = 0 < x_C$*, or joint dirty and clean technology use (JU),* $x_B > 0$ *and* $x_C > 0$*, if the cost of pollution reduction in terms of consumption decrease is smaller for the dirty technology,* $\theta_B < \theta_C$*. There is a continuum of stationary points, that includes a unique optimal point, which is either of type CU or JU, if* $\theta_B = \theta_C$ *and* $\bar{K}_B + \bar{K}_C$ *is sufficiently large. There is a unique stationary point JU, or a unique stationary point with exclusive dirty technology use (BU),* $x_B > 0 = x_C$*, if* $\theta_C < \theta_B$*.*

Proof. Lemma 1.2 implies a horizontal or downward-sloping curve ϕ in the pollution-consumption space. Consumption may be constant for some or all pollution levels. (I) If $\theta_B < \theta_C$ then let $x_B = 0$ and vary $x_C \in (0, \bar{K}_C/Q_C]$. The laws of motion at stationary levels, $Z(t) = Z(t+1)$ and $K_j(t) = K_j(t+1)$ all $j \in \{B, C\}$, read $c/B = \sum_j (Q_j - 1)x_j$ and $A(Z) = \sum_j (d_j Q_j + \rho_j)x_j$. These equations are differentiable with respect to consumption c, input amounts x_B and x_C in investment, and pollution Z. The resulting curve $\chi(c, Z)$ from these laws slopes upward and is depicted in Figure 1.3. Clean capacity is below the maximum amount $\bar{K}_C$ if the intersection with the curve $\theta_C = \phi(c, Z)$ is Southwest to the point on χ for $K_C = \bar{K}_C$. Clean capacity is constrained if this intersection is Northeast. There is no investment in dirty technology, $x_B = 0$, if the point is on the thick section in Figure 1.3. Depending on the location of the point on χ relative to $\theta_B = \phi$ for small $\bar{K}_C$ there is investment in dirty technology, $x_B > 0$. Dirty capacity is constrained so that $\theta \leq \theta_B$ if $\bar{K}_B$ is small. (II) If $\theta_B = \theta_C$ then there is a continuum of stationary points provided that there is an intersection of χ on $\{x_B = 0, x_C \in (0, \bar{x}_C]\} \cup \{x_B \in (0, \bar{x}_B], x_C = \bar{x}_C\}$ and $\theta_C = \phi$, that is, given sufficiently large $\bar{x}_j = \bar{K}_j / Q_j$ some $j \in \{B, C\}$. The optimal stationary point is unique, because by Lemma 1.3 investing in clean technology yields relatively greater consumption for given pollution increase—(i) $x_C \leq \bar{x}_C$ and $x_B = 0$, or (ii) $x_C = \bar{x}_C$ and $x_B > 0$ depending on where ϕ intersects χ with hypothetical large $\bar{K}_C$. Figure 1.3 shows these two points for different bounds $\bar{K}_C$ as the two leftmost dots. (III) The case $\theta_C < \theta_B$ is reversed to (I). The slope on χ may be relatively smaller or equal for the dirty technology, which cannot occur for the clean technology in (I) given the assumption $(\rho_C - \rho_B)Q_C < d_B Q_B$. There is relatively more consumption through expanding dirty capacity if $Q_C < \beta Q = ((d_B Q_B + \rho_B - \rho_C)/(\rho_B + d_B Q_B)) + (\rho_C/(\rho_B + d_B Q_B))\beta Q_B$. In this case and for equal consumption increase for given pollution among the dirty and clean technologies $Q_C \in [\beta Q, Q)$. Clean technology productivity is $Q_C \in [\beta Q, Q)$ if there is relatively less consumption from investing more in dirty technology despite $\theta_C < \theta_B$. *Q.E.D.*

The unique stationary point has either exclusive use of clean technology, simultaneous deployment of dirty technology and clean technology, or exclusive use of dirty technology, in contrast to dichotomy between the latter two (called Golden Age and Murky Age) in Keeler et al. (1971) in terms of emission prevention in producing output. Multiple stationary points exist only if the perpetual relative price of pollution reduction through dirty technology and clean technology is equalized, $\theta_B = \theta_C$.

Only clean technology is used if the cost of pollution reduction in terms of consumption decrease is smaller for the dirty technology, $\theta_B < \theta_C$, and the scale of the clean technology

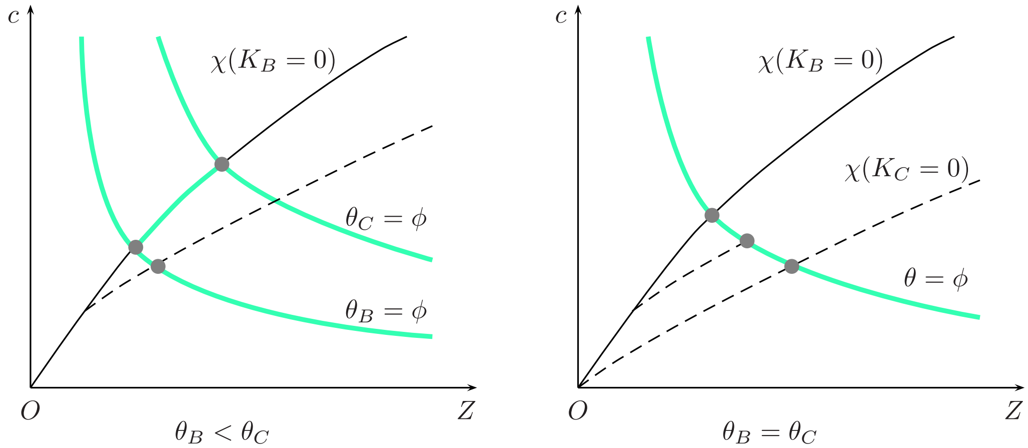


Figure 1.3: *Stationary points for* $\theta_B \leq \theta_C$.

is sufficiently large. This requires sufficiently productive clean technology, $Q_C > \beta^{-1}$. Only dirty technology is used when the pollution reduction cost of the dirty technology is relatively high, $\theta_C < \theta_B$, unless the scale of the dirty technology is too small. Table 1.1 shows the stationary points dependent on the emission intensity of clean technology holding all other parameters constant. Here $Q_C > \beta^{-1}$ is assumed in three cases of different scale of clean technology, $\bar{K}_C$. Define ϕ_C as the level ϕ in Lemma 1.2 evaluated at $c = B(1 - 1/Q_C)\bar{K}_C$ and $Z = A^{-1}((\rho_C/Q_C)\bar{K}_C)$. Let ρ^* be the level ρ_C at which $\phi_C = \theta_C$ and define ρ^{**} as such level that solves $\theta_B = \theta_C$.[10] A type CU stationary point with interior clean capacity requires a large scale of clean technology, that is, $\theta_B < \phi_C$, since only then $\theta_B < \theta_C < \phi_C$, or equivalently, $\rho^* < \rho_C < \rho^{**}$, is feasible. In the cases denoted by asterisk clean capacity is constrained, $K_C = \bar{K}_C$, because the cost of polluting θ is high relative to the clean technology scale $\bar{K}_C$ so that $\rho_C \leq \min[\rho^*, \rho^{**}]$.

An exclusive use of clean technology arises because its investment carries an environmental cost. Such an allocation can be optimal if dirty technology has a relative advantage to clean technology at all scales of clean technology, $Q_B > Q_C$, and for utility functions with small marginal utility of small pollution.[11] This result can be extended to include multiple clean technologies when there are multiple technologies. Capacity in both the

[10] There is a unique level ρ^* because ϕ_C weakly increases in ρ_C and θ_C decreases in ρ_C such that $\theta_C \to 0$ as $\rho_C \to \infty$ and $\theta_C \to \infty$ as $\rho_C \to 0$.

[11] Stationary clean technology investment follows from Lemma 1.1 if $Q_B \leq Q_C$. The Case $\rho_B < \rho_C$ yields $\theta_B < \theta_C$, because $(\rho_C - \rho_B)Q_C < d_B Q_B$ implies that $\rho_C < \rho_B + \beta(\rho_C - \rho_B)Q_C < \rho_B + \beta d_B Q_B$. The Case $\rho_C \leq \rho_B$ directly shows that $\theta_B < \theta_C$.

Large scale of clean technology, $\theta_B < \phi_C$	
$0 < \rho_C < \rho^*$	CU, $\theta \in (\theta_B, \theta_C)$ (*)
$\rho_C = \rho^*$	CU, $\theta = \theta_C$ (*)
$\rho^* < \rho_C < \rho^{**}$	CU, $\theta = \theta_C$
$\rho_C = \rho^{**}$	optimum stationary point JU, $\theta = \theta_B = \theta_C$
$\rho^{**} < \rho_C$	BU, $\theta = \theta_B$, if $\bar{K}_B$ is large, otherwise JU, $\theta \leq \theta_B$
Medium scale of clean technology, $\theta_B = \phi_C$	
$0 < \rho_C < \rho^*$	CU, $\theta = \theta_B$ (*)
$\rho_C = \rho^*$	optimum stationary point JU, $\theta = \theta_B = \theta_C$ (*)
$\rho^* < \rho_C$	BU, $\theta = \theta_B$, if $\bar{K}_B$ is large, otherwise JU, $\theta \leq \theta_B$
Small scale of clean technology, $\theta_B > \phi_C$	
$0 < \rho_C < \rho^{**}$	JU, $\theta = \theta_B$ (*)
$\rho_C = \rho^{**}$	optimum stationary point JU, $\theta = \theta_B = \theta_C$ (*)
$\rho^{**} < \rho_C$	BU, if $\bar{K}_B$ is large, otherwise JU, $\theta \leq \theta_B$

Note: CU=exclusive clean technology use, JU=joint use of dirty and clean technology, BU=exclusive dirty technology use.

Table 1.1: *Stationary points.*

dirty and clean technology is constrained in the unique stationary point if the capacity bounds are too small. Dirty technology may only have a small scale if it is endogenous. The scale of dirty energy technology may be endogenous through nonrenewable fossil fuel supply, which is the subject of further research. Dwindling fossil fuel supply and limited technical progress in converting its energy may lead to exclusive long-term use of renewable energy technologies for a wide array of energy services.

A stationary point is generally unique because goods are noninferior and the utility function is concave. These assumptions imply uniqueness if substitutable capital and emissions produce net output, for any returns to scale and proven in notes available upon request. The literature has not made this clear. Contrary to a supposition of Keeler et al. (1971) multiple stationary points do not exist in their model. The reason is that the noninferiority ensures that $(-\partial U/\partial Z)/(\partial U/\partial c)$ equal to a constant relates pollution and consumption negatively or holds one of them constant. Tahvonen & Kuuluvainen (1993) prove uniqueness under the more restrictive assumptions (1) nonincreasing marginal utility of consumption in pollution, $\partial^2 U/(\partial c \partial Z) \leq 0$, compared to noninferiority, and (2) strict concavity of utility in pollution compared to concavity.

Multiple stationary points arise from production technology—at locally constant returns to scale of dirty and clean technology. The wealth effect of the environmental asset on utility is special in assuming noninferior goods. Multiplicity in Tahvonen & Kuuluvainen (1993) using two assets pollution and a production factor can occur because of the unspecified wealth effect for the same reason as in a model with one asset in Kurz (1968). A decreasing absorption motivates multiple stationary points in Tahvonen & Withagen (1996) and Tahvonen & Salo (1996).

Without emissions from investment in clean technology, $\rho_C = 0$, dirty rechnology use is the only source of pollution inflow. There must be dirty technology capital in a stationary point unless the scale of clean technology is sufficiently large and the marginal utility of pollution at minimum feasible pollution Z_{min} is sufficiently large. The exclusive investment in clean technology in Proposition 1.2 does not rest on the latter condition. The following proposition shows the stationary level of clean capacity and when it is the only provider of output.

Proposition 1.3 *Let investment in the clean technology be emission-free, $\rho_C = 0$. There is a unique stationary point with $K_C = \bar{K}_C$ ($K_C = 0$) if the clean technology productivity Q_C is greater (smaller) than β^{-1}. There is a continuum of stationary points, and clean capacity is at its upper bound, $K_C = \bar{K}_C$, at the unique optimal point if $Q_C = \beta^{-1}$. Clean technology is exclusively used, $x_C > 0 = x_B$, in the unique stationary point if and only if $Q_C > \beta^{-1}$ and the cost of polluting ϕ evaluated at $c = (1 - 1/Q_C)\bar{K}_C$ and $Z = Z_{min}$ is weakly greater than the cost of pollution reduction of the dirty technology, θ_B, and in the unique optimal stationary point if and only if $Q_C = \beta^{-1}$ and $\phi \geq \theta_B$.*

Proof. (i) The result follows for $Q_C \neq \beta^{-1}$ since $R(t+1) = \beta^{-1}$ in a stationary point. Expanding clean capacity K_C on $[0, \bar{K}_C)$ for $Q_C = \beta^{-1}$ weakly increases consumption and strictly reduces pollution through substitution for K_B on the given curve $\chi(c, Z) = \theta_B$. Then $K_C = \bar{K}_C$ is the preferred clean capacity level. (ii) "if." Dirty technology use would raise θ above ϕ evaluated at $m_B = 0$ and thereby contradict that investment in dirty technology is worthwhile. "only if." The relation of the marginal product Q_C of clean technology and the time discount factor β follows from (i). The cost of polluting is ϕ evaluated at $c = (1-1/Q_C)\bar{K}_C$ and $Z = Z_{min}$ if there is exclusive use of clean technology. At $\phi = \theta < \theta_B$ investing in dirty technology is efficient. *Q.E.D.*

A constant level of investment in clean technology is not sustainable because investing in clean technology is too costly relative to time preference if clean technology productivity is smaller than the inverse of the discount factor, $Q_C < \beta^{-1}$. This holds with or without the emissions in building new capital, $\rho_C \geq 0$. The upper bound on clean capacity guarantees that there is a stationary point when $Q_C > \beta^{-1}$ and investment does not create emissions, $\rho_C = 0$. This bound implies finite clean capacity at all points of a continuum of stationary points if $Q_C = \beta^{-1}$. As with dirty investment multiple stationary points exist only under a special parameter constellation. The most plausible case when $\rho_C = 0$ is when the creation of dirty technology capital lacks emissions too, $\rho_j = 0$ all $j \in \{B, C\}$, though the results in Proposition 1.3 do not depend on ρ_B. Next I examine if a Pareto optimal plan converges to a unique stationary point as time tends to infinity.

Convergence of an optimal plan.—The following proposition shows that an optimal plan may converge to a unique stationary point with an interior capital stock starting at a nearby state (Z, K_B, K_C) if $\theta_B \neq \theta_C$.

Proposition 1.4 *A stationary point with interior capacity $K_j \in (0, \bar{K}_j)$ of technology j and boundary value $K_{j'} \in \{0, \bar{K}_{j'}\}$ of technology $j' \neq j$, can be locally a saddle point.*

A proof in the appendix shows that the decisive matrix of the linearized dynamical system of necessary optimality conditions and laws of motion around such a stationary point has reciprocal characteristic values. The saddle point property (of existence of a stable and unstable manifold) does not follow from linearization even if the discount factor β is not too small. In examples I found exactly one pair of complex characteristic values, which by the property that its modulus must be greater than one, precludes convergence of the linearized system. Then nevertheless the optimal policy converged in numerical simulations using grid search. A saddle point is reached asymptotically in infinite time.

Unique stationary capital levels of both the dirty and clean technology equal to their maximum levels or one of them at its maximum level and the other equal to zero should be reached in finite time. In such a case the cost of polluting and the shadow return need to reach critical levels rather than be at their long-term levels to sustain the appropriate investment.[12] Any state of pollution, dirty capacity, and clean capacity, that belongs

[12]The saddle point property, if it can be proven, or a finite-time approach path of capital stocks, if it can be constructed, implies that there is access to the stationary point. Then the sufficiency of the necessary optimality condition shows that a path on the stable manifold of the stationary point or with finite-time approach of capacities, respectively, is optimal. Claim. A convergent plan that satisfies the necessary optimality conditions (1.5)-(1.7) is optimal if utility U or absorption A is strictly concave at

to a nonoptimal stationary point and is given at the initial date is a trap. None of the stationary points in a continuum can be a saddle point. The policy of staying at any of them satisfies the necessary optimality conditions including the transversality conditions, so that if these are sufficient (see the previous footnote for a condition that ensures sufficiency) then by the uniqueness of an optimal plan convergence to the optimum stationary point is not optimal. The following presents optimal plans that converge to a unique stationary point.

Optimal plans.—Time paths of the cost of polluting and the shadow return can be characterized depending on the relation of the technology-specific cost of pollution reduction θ_B and θ_C. The most obvious candidates are paths with one switch in investment between technologies and paths with investment in a sole technology at all dates. To get a clear idea about the roles of productivity and emission intensities let the scale $\bar{K}_j$ be large for $j \in \{B, C\}$. I focus on polluting clean technology, $\rho_C > 0$, with productivity $Q_C > \beta^{-1}$ so that clean technology may be exclusively used at $\theta_C \in (0, \infty)$.

Proposition 1.5 *Let $\theta_B \neq \theta_C \in (0, \infty)$. On optimal paths with a single switch of investment from dirty technology B (C) to clean technology C (B) the cost of polluting increases (decreases) and the shadow return decreases (increases) from the date of switching to the long-term if the dirty technology is relatively more productive, $Q_B > Q_C$. There is only investment in clean technology, $x_B(t) = 0 < x_C(t)$ all $t \geq 0$ close to the stationary point if the clean technology is relatively weakly more productive, $Q_B \leq Q_C$.*

Proof. (i) $Q_B > Q_C$. The rates of return on investment in the dirty technology B and the clean technology C in (1.8) equal β^{-1} for different levels θ_B and θ_C. These rates are equal at the same level $\theta(t) = \theta(t+1) = \theta^*$ either less than or greater than θ_B and θ_C since each stationary rate of return decreases in θ. The level θ^* cannot be greater than θ_C if $\theta_B < \theta_C$ because then the critical curve for technology switching in $\theta(t)$-$\theta(t+1)$ space would imply dirty technology investment at $\theta^* = \theta_C$, which lies in the space of clean technology investment given $\theta_B < \theta_C$. Then $\theta^* < \theta_B < \theta_C$ and analogously $\theta_C < \theta_B < \theta^*$.

any Z. Proof. Convergence satisfies the transversality conditions $\lim_{t\to\infty} \beta^t \epsilon(t)$ and $\lim_{t\to\infty} \beta^t [q_j(t) - \beta w_j(t+1)] K_j(t) = 0$ all $j \in \{B, C\}$. Then a version of Arrows' Theorem, Proposition II.6.8 of Arrow & Kurz (1970*b*), implies that a plan that satisfies the necessary conditions (1.5)-(1.7) and the transversality conditions is optimal if utility U or absorption A is strictly concave at any Z. *Q.E.D.*

Interior investment, $x_j(t) \in (0, \bar{K}_j/Q_j)$, satisfies the difference equation

$$\frac{1}{\theta(t+1)} = \left(\frac{1}{\theta(t)} + \frac{\rho_j}{B}\right)\frac{1}{\varepsilon(t+1)Q_j} + \frac{d_j}{B}$$

for $j \in \{B, C\}$ where $\varepsilon(t+1) = \beta\epsilon(t+1)/\epsilon(t)$ using (1.8) and the identity $\theta(t+1) = R(t+1)\varepsilon(t+1)\theta(t)$. Investment is optimal in the technology with greatest $\theta(t+1)$ given $\theta(t)$ on these curves. There is an intersection of the curves for B and C at long-term value $\varepsilon(t+1) = \beta$ and positive $\theta(t+1)$ if $\rho_B \geq \rho_C$ and $Q_B > Q_C > \beta^{-1}$ by Assumption 1.2. Figure 1.4 draws these long-term curves. At the intersection of these curves the slope $d\theta(t+1)/d\theta(t) = (\theta(t+1)/\theta(t))^2/\varepsilon(t+1)Q_j$ is greater for the C-curve than for the B-curve since $Q_B > Q_C$. In the left panel $\theta_B < \theta_C$ and thus the intersection of these curves is to the left of θ_B. In the right panel $\theta_B > \theta_C$ and thus the intersection is to the right of θ_B. Figure 1.4 illustrates these two cases for $\rho_B = \rho_C$. The relation of ρ_B and ρ_C does not affect the conclusions The long-term C-curve lies below the long-term B-curve if there is no intersection for $\varepsilon(t+1) = \beta$ at $\theta(t+1) > 0$. This can occur only if $\rho_B < \rho_C$ by Assumption 1.2. Then the curves intersect for some $\epsilon(t) < \epsilon(t+1)$. The given curves yield the indicated flow of θ. For $\theta_B < \theta_C$ in the left panel $R(t+1)$ is greater than β^{-1} at the intersection of the curves of B and C. The cost of polluting $\theta(t)$ and the shadow return $R(t+1)$ relate negatively on the C-curve. Thus $R(t+1)$ is greater at the intersection of the B-curve and the C-curve than for $\theta(t) = \theta(t+1)$ on the indifference curve. The relation $\theta^* < \theta_C$ implies that $R(t+1)$ that equates the rates of return on investment is greater than β^{-1}. Analogously, for $\theta_B > \theta_C$ in the right panel $R(t+1)$ is smaller than β^{-1} at the intersection of the B-curve and the C-curve. The shadow return tends to β^{-1} in the long-term. (ii) $Q_B \leq Q_C$. There is no intersection of the long-term difference equations of the cost of polluting for B and C for positive $\theta(t+1)$ by Assumption 1.2. The latter is above the former for all $\theta(t) > 0$. *Q.E.D.*

The left (right) panel in Figure 1.4 designates low (high) environmental cost $\rho_B = \rho_C$ of investment. A small environmental cost of building capital advantages the clean technology. The sequence of difference equations yields the same direction in the path of the cost of polluting as the drawn curves when ϵ changes little over time. Thus there may be exclusive investment in dirty technology B or clean technology C, or a single switch in investment between these technologies. As indicated earlier, the cost of polluting and the shadow price of pollution must decrease initially, $\theta(0) > \theta(1)$ and $\epsilon(0) > \epsilon(1)$, for optimal

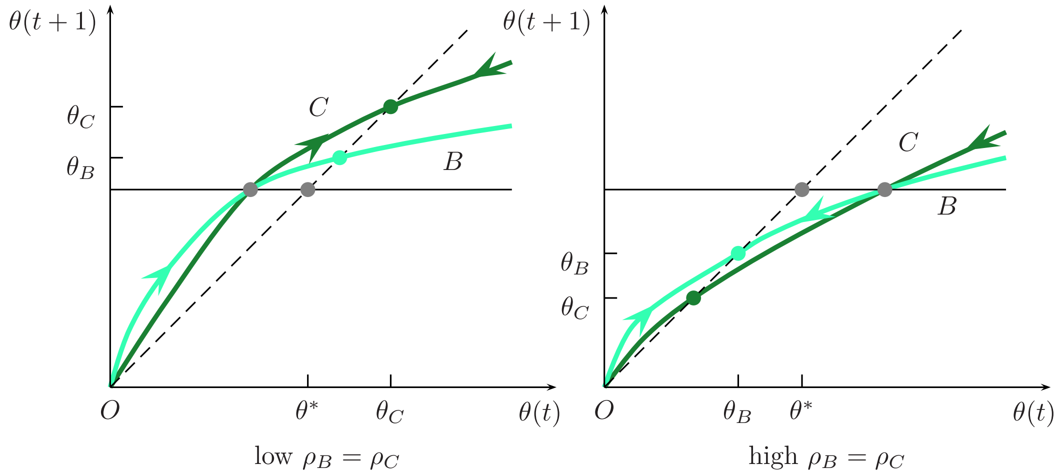


Figure 1.4: *Technology switching.*

dirty technology investment at the initial date if $Q_B \leq Q_C$ at large scale $\bar{K}_C$. The B-curve coincides with the C-curve at the unique $\varepsilon(1)$ of indifference between investing in the dirty and clean technology when $Q_B = Q_C$ that corresponds to the left panel in Figure 1.2. The B-curve is steeper than the C-curve at their intersection when $Q_B < Q_C$ in a graph that corresponds to the right panel in Figure 1.2 so that the C-curve lies above the B-curve to the left of the intersection. Dampened oscillations of θ for particular changes in ϵ cannot be excluded in general.[13] Therefore, a reversal of investment in clean and dirty technologies cannot be excluded. But a monotone motion of the cost of polluting implies a monotone path of the shadow return in the opposite direction.

Proposition 1.6 *The shadow return decreases, $R(t) > R(t+1)$, on $\{t'+1, t'+2, \ldots, t''\}$ if the cost of polluting increases, $\theta(t) < \theta(t+1)$, on $\{t', t'+1, \ldots, t''\}$ in plans with interior investment in the dirty technology, $x_B(t) \in (0, \bar{K}_B/Q_B)$, on $\{t', t'+1, \ldots, t''-1\}$. The shadow return increases, $R(t) < R(t+1)$, if the cost of polluting decreases, $\theta(t) > \theta(t+1)$, on the given time intervals. The shadow return $R(t+1)$ and the cost of polluting $\theta(t)$ are negatively related on $\{t', t'+1, \ldots, t''\}$ with interior investment in clean technology, $x_C(t) \in (0, \bar{K}_C/Q_C)$.*

Proof. The necessary optimality condition $Q_B(B - d_B\theta(t+1))/(B + \rho_B\theta(t)) = R(t+1)$ and the level $R(t)$ can be compared if $x_B(t) > 0$ and (1.3) is nonbinding for technology

[13] The drawn curves are valid at all dates, because ϵ is constant, if the marginal utility of pollution is constant and absorption is a linear function of pollution.

B. If $\theta(t-1) < \theta(t) < \theta(t+1)$ then $R(t) > R(t+1)$. If $\theta(t-1) > \theta(t) > \theta(t+1)$ then $R(t) < R(t+1)$. The necessary optimality condition $Q_C/(B+\rho_C\theta(t)) = R(t+1)$ delivers the result if $x_C(t) > 0$ and (1.3) is nonbinding for technology C. *Q.E.D.*

The shadow return $R(t+1)$ depends positively on consumption at $(t+1)$ and negatively on consumption at t. Consumption growth is lower to accommodate pollution reduction at the margin if the willingness to pay for pollution reduction, θ, in terms of the consumption good is greater. Thus the cost of polluting reflects the willingness to shift consumption over time in order to preserve the environment, which is costly in terms of consumption. This argument holds pollution at optimal values if pollution affects the marginal utility of consumption.

The shadow return tells the consumption growth rate if the effect of consumption dominates the effect of pollution on the marginal utility of consumption. Consumption should positively relate to output which positively relates to capital. I conclude that a single switch from dirty to clean (clean to dirty) technology occurs if the initial aggregate capital is sufficiently smaller (greater) than the long-term aggregate capital and $\theta_B \neq \theta_C$.

Emissions paths.—Emissions relate positively to capital. Thus the emissions path is indicated by Figure 1.4. Increasing (decreasing) cost of polluting is consistent with increasing (decreasing) emissions.

The next section turns to the role of emissions from investment in generating differences to the cost of polluting.

1.1.2 Comparative analysis of emissions from using and investing in dirty technology

The emission intensity of output of used dirty technology may be inferred by observing the ratio of emissions $d_B K_B(t) + (\rho_B/Q_B)K_B(t+1)$ and output $K_B(t)$ on a time path. What difference does it make to attribute some emissions to investments for the long-term cost of polluting?

Proposition 1.7 *The cost of polluting θ at a stationary point with investment in dirty technology, $x_B > 0$, increases (decreases) in the emission intensity of dirty technology investment, ρ_B, if the observed gross rate of change $K_B(t+1)/K_B(t)$ of dirty output is greater (smaller) than the inverse of the discount factor, β^{-1}.*

Proof. Let $g_B = K_B(t+1)/K_B(t)$. Then $d(d_B) + (g_B/Q_B)d\rho_B = 0$ and differentiation of $(\rho_B/\beta + Q_B d_B)\theta = B(Q_B - \beta^{-1})$ imply that $d\theta/d\rho_B = \theta^2 B(g_B - \beta^{-1})/(Q_B - \beta^{-1})$. The result follows from there since the denominator is positive. *Q.E.D.*

The greater the emission intensity the smaller is the cost of polluting (equal to the marginal benefit of emission reduction) because under more polluting technology consumption is smaller.[14] Disregarding the emissions from investment, which occur currently, puts too much weight on d_B, which is discounted at the rate of time preference in the cost of polluting. This explains why the discount factor plays a role. The greater the ratio $[K_B(t+1)/K_B(t)]/Q_B$ of growth rate to productivity the greater is the derived d_B for given ρ_B. Assuming $K_B(t+1)/K_B(t) > \beta^{-1}$ this shifting increases $\rho_B + \beta d_B Q_B$ and thus decreases θ, and is more pronounced the greater the level ρ_B. Conversely, inclusion of emissions from investment then increases θ, by more the greater the level ρ_B. If the discounted growth rate of capacity is too low, $\beta K_B(t+1)/K_B(t) < 1$, then the effects are reversed. The observed growth rate of dirty output depends on the preferences. With additively separable utility $U = c^{1-\psi}/(1-\psi) - \Psi(Z)$ and constant index of relative risk aversion ψ regarding consumption the growth rate of consumption is $(\beta Q_B)^{1/\psi}$ which exceeds β^{-1} for $Q_B > (1/\beta)^{1+\psi}$ and may be similar to the growth rate of output. Then for sufficiently large Q_B relative to the inverse of the discount factor the inclusion of emissions raises the long-term cost of polluting.

1.2 Multiple dirty and clean technology types

This section explains why a unique technology is chosen in the theory of substitutable capital and emissions yet there is investment in multiple clean technologies in the theory here in evaluating climate change, examines the incentives to invest in clean technology under one assumed dirty technology versus multiple dirty technologies, and discusses time paths with unconstrained dirty capacity choice. First I show that there is a unique stationary point generally, and how the local stability analysis for two technologies applies to more than two technologies.

[14] The feasibility frontier χ in pollution-consumption space tilts down and the preference curve ϕ shifts in so that at the new intersection consumption is smaller than before and pollution is smaller than at the intersection of the old ϕ and the new χ.

Proposition 1.8 *There is a unique stationary point, if the cost of pollution reduction in terms of consumption units θ_j differs over all dirty technologies $j \in \mathcal{J}$, and all clean technologies $j \in \mathcal{J}$ with emissions of investment, $\rho_j > 0$.*

Proof. The distinct relative cost θ_j and Lemma 1.1 imply that capital can be interior, $K_j \in (0, \bar{K}_j)$ only for one technology $j \in \mathcal{J}$. There is a unique intersection of the curves $\chi(c, Z)$ formed by $c/B = \sum_j (1 - 1/Q_j)K_j$ and $A(Z) = \sum_j (d_j + \rho_j/Q_j)K_j$, and $\phi(c, Z)$ given (1.5), at $K_j = \bar{K}_j$ all j with $\theta_j < \theta$ varying one other technology's investment. Else consumption and emissions are unique if $K_j = \bar{K}_j$ all j in which investment occurs, $K_j > 0$. There are no two clean technologies C' and C'' with $\rho_{C'} = \rho_{C''} = 0$ and $Q_{C'} = Q_{C''} = \beta^{-1}$. *Q.E.D.*

Proposition 1.4 applies consequently to a continuum or a discrete set of dirty or clean technologies holding a mass or sum of input in investment constant. The sums of constant capital amounts must be replaced by integrals for a continuum of technologies.[15]

Clean technology frontier versus dirty technology frontier.—In the theory without commitment Stokey (1998) interprets the unique intratemporal emission control as a unique technology chosen at the date of production. The choice set can be thought of as a dirty technology frontier, where there is a trade-off between the productivity of capital and labour and the emission intensity of output. Nordhaus (2009) views emission control in this theory as a mix of clean and dirty technologies and energy efficiency choices that can be adjusted in each period.[16] With commitment there is no unique technology choice if the marginal product Q_C of clean technologies ranges from Q_B to below one on small individual scales $\bar{K}_C$, consistent with data on the dollar cost of avoiding emissions that Nordhaus (2009) uses to calibrate the mapping between productivity and emission intensity in his model DICE. Given this wide range, and roughly equal emission intensity ρ_j of the input in investment for all technologies $j \in \{B, C1, C2, \ldots\}$ the condition (1.8) implies that investment in some types of clean technology is worthwhile if there was investment in a dirty technology.

One dirty technology versus multiple dirty technologies.—I consider clean technology investment when the capacity choice of multiple dirty technologies is unconstrained. The

[15] In the canonical system $Z(t+1) = (d+\rho)K(t) + \sum_j \rho(K_j - c(t)/B) + \sum_j (\rho_j - \rho)K_j/Q_j + Z(t) - A(Z(t))$ extends (A-1) in the appendix, and $K(t+1) = Q(K(t) + \sum_j (1 - 1/Q_j)K_j - c(t)/B)$ is the resource constraint for multiple technologies, holding constant each K_j.

[16] The factor efficiency choice seems a viable interpretation.

incentives to invest in a given clean technology may become greater or smaller from introducing multiple dirty technologies. As will be shown below the effect depends on the clean technology productivity relative to the discount factor and the cost of polluting relative to its stationary level.

Let fix the ratio $4\alpha = Q_B/d_B$ of the productivity in creating dirty technology capital and the emission intensity in using this capital for some productivity levels $Q_B \in [Q'', Q']$ subject to large scale $\bar{K}_B$ all B. The optimal interior choice $Q = 2B\alpha/\theta$ on a continuum of technologies inversely relates to the cost of polluting so that $Q = 2\beta^{-1}$ in a stationary point if $\rho_B = 0$ all B. I disregard the emissions from investment here to focus on technology choice per se. Suppose that this technology was the only dirty technology and $2\beta^{-1} \in (Q'', Q')$. This enables comparison of clean technology investment for a narrow choice set with one dirty technology and a wider choice set with more and less productive dirty technologies. The critical level for technology switching is $\theta^* = \theta(2-\beta Q_C)$ given the stationary level $\theta = B\alpha\beta$ if there is one dirty technology. Indifference levels of the cost of polluting between investing in dirty technologies versus a given clean technology are derived as follows. The level $\gamma^* = B\alpha/Q_C$ of the cost of polluting equates the rates of return on investing in any of the dirty technologies with productivity $Q_B \in [Q'', Q']$ as an unconstrained choice and in the clean technology with productivity Q_C below its maximum scale. A clean technology does not have an environmental cost here, $\rho_C = 0$ all C. The optimal dirty technology choice for $\theta(t+1) \leq \theta' = 2B\alpha/Q'$ and $\theta(t+1) \geq \theta'' = 2B\alpha/Q''$ is investing in Q' and Q'' at t, respectively. If $\gamma^* < \theta'$ then clean technology investment occurs for $\theta(t+1) > \theta'$. If $\gamma^* > \theta''$ then clean technology investment occurs for $\theta(t+1) > \theta''$. Else it occurs for $\theta(t+1) > \gamma^*$.

(i) $Q_C < \beta^{-1}$. There can be investment in a clean technology, though it would not be optimal if only one dirty technology was available, at θ greater than its stationary level. Then the switching level if there is only one dirty technology is greater than θ''. In other cases the incentives to invest in the given clean technology are smaller in an extended choice set for $\theta \in (\theta^*, \min[\gamma^*, \theta''])$ than under one dirty technology, because a dirty technology with smaller emission intensity than of this technology is feasible.

(ii) $Q_C = \beta^{-1}$. The switching level equals the stationary level when there is one dirty technology. Thus the incentives to invest in clean technology are weaker for $\theta \in (\theta^*, \min[\gamma^*, \theta''])$ because investing in a dirty technology with productivity smaller than

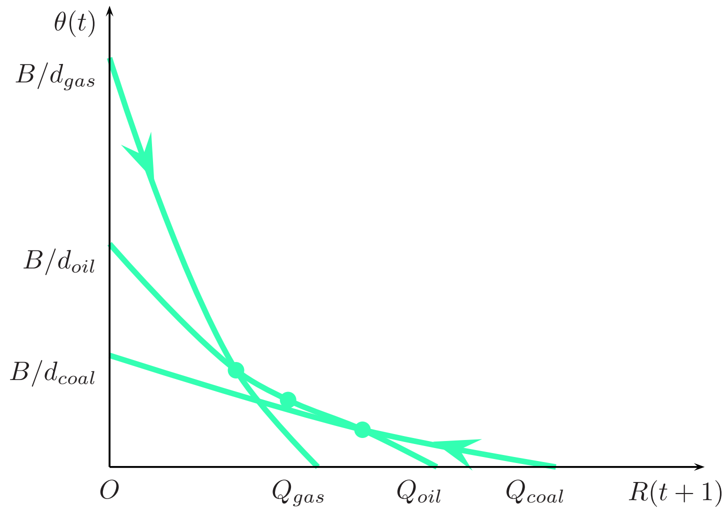


Figure 1.5: *Shadow return and cost of polluting given investment in coal, oil, and gas technologies.*

the stationary level is optimal in this range.

(iii) $Q_C > \beta^{-1}$. The hurdle to invest in a given clean technology is greater for low θ relative to its stationary level when a dirty technology with a large emission intensity is feasible, because its investment is optimal for low $\theta \in (\theta^*, \max[\gamma^*, \theta'])$.

Unconstrained dirty investment options.—Investment switches between technologies over time when the pair $(\theta(t), \theta(t+1))$ leaves the region of investment in some technology if there is a continuum or discrete array of more than two technologies characterized by (Q_j, d_j, ρ_j) each on a large scale. The rationale is the same as with two technologies.

Among coal, petroleum, and natural gas energy conversions in an economy with the relative importance of their prime uses for the single factor in the model and emission intensity in the same order (trivially) a more dirty technology is more productive so that d_B and Q_B relate positively all B.[17] Then intersections of the curves $Q_B/R(t+1) - 1 = (\rho_B + \varepsilon(t+1)d_B Q_B)\theta(t)/B$ in $R(t+1)$-$\theta(t)$ space are indifference points regarding investment at date t. This equation results from combining (1.8) and $\theta(t+1) = R(t+1)\varepsilon(t+1)\theta(t)$. Figure 1.5 plots it for three technologies. Investment in a technology

[17]The prime uses are stationary motor drive and light using electricity for coal, mobile energy for petroleum, and space and water heating for natural gas. There is scientific research in reversing this relationship for fossil fuels, for example, producing clean coal.

with maximum $R(t+1)$ given $\theta(t)$ is optimal. Let ρ_B be equal all B and suppose that the long-term allocation is based on oil. Then it is optimal to move from a coal-based economy (high d_B and Q_B) to an oil-based economy (medium d_B and Q_B) when θ increases toward its long-term level—the initial energy production capacity is small relative to its long-term capacity. In contrast, investment switches from natural gas technologies (low d_B and Q_B) to oil technologies, when θ decreases toward its long-term level—the initial energy production capacity is greater than its long-term capacity. I have used the term energy-carrier based economy. Multiple energy services may be served by different energy carriers in a given period.

1.3 Delay in societal impacts of emissions

Some scientists argue that carbon emissions from 2010 until about 2100 do not affect major measures of climate change in 2100. This suggests some inertia in the accumulation of pollutants. This section shows that under preferences that admit an additively separable utility function delay preserves the monotone or oscillatory type of convergence of an optimal plan to a unique stationary point. Winkler (2011) compares monotone approach paths with such separable objective and dampened oscillations with interaction effects in the objective in a one-state delay problem. There the approach is monotonic in the undelayed version. My first result applies.[18] Moreover I show that then a solution is readily available from the solution of a problem without such inertia. Let

$$Z(t+1) = E(t-\tau) + Z(t) - A(Z(t)) \tag{1.9}$$

be pollution at the beginning of period $(t+1)$, where the emissions amount is defined as before. The planner knows the pollution $Z(-\tau)$ for $\tau \geq 0$, and the sequence $\{E(-\tau), E(-\tau+1), \ldots, E(-1)\}$ of emissions if $\tau \geq 1$. The greater τ the greater is the inertia in the response of pollution to emissions.[19]

[18] Winkler (2011) views the time structure of differential equations that implies the preserved type of convergence albeit concludes only preserved monotonicity in a model without oscillations in the separable form in the undelayed version.

[19] Here pollution measures an environmental state that has contemporaneous adverse impacts on society. Claim. Delay between emissions and impacts yields the same results as any additive delays between emissions and accumulation of pollutants, and the latter and impacts on society, given absorption of

Optimal choice of pollution in periods $\tau+1, \tau+2, \ldots$ satisfies the difference equation

$$\epsilon^*(t) = \beta(-\partial U/\partial Z(t+1)) + \beta(1 - \partial A/\partial Z(t+1))\epsilon^*(t+1), \quad t \geq \tau, \tag{1.10}$$

of the current value shadow price ϵ^* of pollution. Define the discounted shadow price $\epsilon(t) = \beta^\tau \epsilon^*(t+\tau)$. Then the conditions (1.6) and (1.7) are necessary for an optimum. Predetermined pollution has no effect on the cost of polluting $\theta(t) = \epsilon(t)/(\partial U/\partial c)(t)$ at any date except through the effect on $\partial U/\partial c(t)$ for $t \in \{0, 1, \ldots, \tau\}$. This identifies the additively separable specification $U(c, Z) = U_0(c) - \Psi(Z)$ whose optimal plan subject to arbitrary delay solves the problem of welfare maximization for a similarly defined utility function and a one-period lagged effect of emissions on pollution, and vice versa.

Proposition 1.9 *The optimal policy for utility function $U(c, Z) = U_0(c) - \beta^\tau \Psi(Z^*)$ given pollution $Z^*(t) = Z(t+\tau)$ in (1.9) all $t \geq 0$ with one-period lagged effect of emissions on society and the optimal policy for utility $U(c, Z) = U_0(c) - \Psi(Z)$ subject to (1.9) all $t \geq \tau \geq 1$ with delayed effect of emissions on society coincide.*

Proof. All necessary optimality conditions are equivalent. In particular, substituting $Z^*(t) = Z(t+\tau)$ into

$$\epsilon(t) = \beta(\beta^\tau \partial\Psi/\partial Z^*(t+1)) + \beta(1 - \partial A/\partial Z^*(t+1)\epsilon(t+1)$$

for $t \geq 0$ yields (1.10). These necessary conditions are sufficient for a maximum of J because there is a unique optimal plan by Proposition 1.1. *Q.E.D.*

Thus for additively separable utility function the stability properties are the same and given there is a unique stationary point locally consumption, investment, pollution, and capital are on saddle paths toward this steady state.

The following representation of the shadow price $\epsilon^*(t)$ helps interpretating the cost of polluting $\theta(t) = \beta^\tau \epsilon^*(t+\tau)/(\partial U/\partial c)(t)$. The forward solution to (1.10), provided that

pollutants depends on contemporaneous pollutants. Proof. Let pollution $Z(t)$ in period t depend on the stock of a pollutant P in period $(t-j)$, and $P(t+1) = E(t+j-\tau) + P(t) - A(P(t))$, $\tau \geq j \geq 0$. Substitution of $Z(t+1) = P(t-j+1)$ and $Z(t) = P(t-j)$ yields (1.9). *Q.E.D.*

the transversality condition with respect to pollution holds,

$$\epsilon^*(t) = \left(1 - \frac{\partial A}{\partial Z(t)}\right)^{-1} \sum_{j=1}^{\infty} \left[\beta^j \prod_{s=0}^{j-1} \left(1 - \frac{\partial A}{\partial Z(t+s)}\right)\right] \left(-\frac{\partial U}{\partial Z(t+j)}\right),$$

reveals that the marginal willingness to pay for pollution reduction in terms of utils is positive, $\epsilon^*(t') > 0$, if pollution has a nonzero effect on welfare at some period $t \geq t' + 1$. Thus, the cost of polluting is a weighted sum of future marginal disutility of pollution starting at the first date when current emissions affect pollution divided by the current marginal utility of consumption.

1.4 Conclusion

This chapter draws conclusions about the motions of investment in dirty versus clean technologies. Clean technology is used if the cost of polluting is sufficiently high. The relative emission intensity in investment of dirty and clean technology is less decisive for dirty versus clean technology investment for greater clean technology productivity. Clean technology is exclusively used only if the cost of reducing pollution relative to increasing consumption is weakly smaller for dirty technology than for clean technology. A sustained controlled pollution inflow from building capital enables a long-term steady state with exclusively producing clean technology. This can prevail if clean technology is less productive than dirty technology at all scales of aggregate investment.

There is investment in only one technology at a given date when all technologies can produce large output. The reason is the linear technology in the investment sector and in the production of the factor. Then on paths with roughly constant discounted marginal effects of pollution on society there is investment in one technology at all dates or switches occur in investment such that investment in any technology is optimal on a closed time interval or from some date onward infinitely. Multiple use-periods of capital will retain this switching in investment. Decreasing returns to scale in using capital may lead to simultaneous investment of multiple technologies that I relegate to further research.

A rationale for the exclusive conversion of renewable energy such as wind and solar energy into useful energy for consumption and investment is the carbon dioxide and methane emissions that occur in the manufacturing of wind turbines and solar panels, when these emissions may alter the climate. The production of capital that uses energy such as buildings, roads, and other structures currently creates these emissions too, for example, in steel and cement production, and mineral processing, and the use of wood with the effect of deforestation. The exclusive use of renewable energy technologies thus can follow if their investment does not create emissions.

A carbon tax that internalizes a carbon externality may be applied permanently if the carbon emissions in building capital of alternative non-carbon emitting technologies cannot be avoided. This likely holds when clean technology productivity increases over time. In contrast, Acemoglu, Aghion, Bursztyn & Hemous (2012) derive that decreasing use of dirty technology lowers the atmospheric carbon stock such that marginal effects of pollution on utility become zero because they disregard the emissions of investment. A topic for further research may be directing resources toward technical change in improving the emission intensity versus the productivity of such technologies.

The chapter refines a set of assumptions (noninferior goods and concave utility in pollution) that implies the uniqueness of stationary points when there is a wealth effect. The noninferiority of pollution reduction and consumption specifies the wealth effect. Multiple stationary points reside in a continuum and exist only if the relative cost of pollution reduction is equal among two technologies (and necessarily occurs if there are only two technologies that satisfy this condition). Expanding capacity of clean technology maximizes consumption and minimizes pollution so that the optimal stationary point in a continuum is unique and exhibits clean capacity if the emission intensity of investment is equal in the technologies. In an optimal stationary point either dirty and clean technologies are simultaneously used or dirty technology is exclusively used if emissions only occur in using dirty technology given that small pollution has small marginal effects on utility. The economy possesses a reciprocal root property that allows saddle-path stability and is known in the literature.

Attribution of emissions to investment versus use of dirty technology generally alters the stationary cost of polluting when dirty technology is used in the long-term. Disregarding emissions from investment in dirty technology biases the stationary cost of polluting downward if the discount factor is not too small. The reason is that the willingness to pay for pollution reduction relative to consumption increase is greater when production is less emission-intensive. This holds if the discount factor is not too small and with accounting of emissions in investmen rather than without it given an observational equivalence condition when dirty capacity grows.

Under preferences that admit an additively separable utility function in consumption and pollution delay in the impact of emissions on society preserves the monotone or oscillatory type of convergence of an optimal plan to a unique stationary point. Moreover, then a solution is available from a problem with adjusted utility function by discounting and without delay in the impact of emissions on pollution.

1.5 Appendix: Uniqueness of optimal plan, noninferiority, saddle point

Proof of Proposition 1.1. (i) Existence. A plan satisfies feasibility conditions and yields finite welfare J. The policy $c(t) = B(1 - 1/Q_B)\min(K_B(0) + K_C(0), \bar{K}_B)$, $x_B(t) = (1/Q_B)\min(K_B(0) + K_C(0), \bar{K}_B)$, $x_C(t) = 0$, is an example. Thus J is bounded from below, $J > -\infty$, for some feasible plan(s). What remains to be shown is that there is a plan that cannot be improved upon. Welfare J is bounded from above because the state space is closed and bounded and given the admissable compact set of consumption and investment and discounting. Moreover the utility function U is continuous in consumption c and pollution Z. (ii) Uniqueness. Suppose that two policies maximize J. This is to be contradicted. The vectorized average capital $K^\varepsilon = \varepsilon K^1 + (1 - \varepsilon)K^2$, $0 < \varepsilon < 1$, produces weakly more output than the weighted output $\varepsilon G(K^1, L) + (1 - \varepsilon)G(K^2, L)$ because G is concave. Then $\varepsilon G(K^1, L) + (1 - \varepsilon)G(K^2, L) \leq G(K^\varepsilon, L)$. The law of motion $K_j(t + 1) = Q_j x_j(t)$ implies that the policy (c, x) yields the same level of capital in technology $j \in \{B, C\}$ as the average capital $\varepsilon K_j^1 + (1 - \varepsilon)K_j^2$ at $t \in \{1, 2, \ldots\}$. Thus $c + B(x_B + x_C) \leq G(K^\varepsilon, L) \leq G(K, L)$ shows that the policy $(c, x) = (\varepsilon c^1 + (1 - \varepsilon)c^2, \varepsilon x^1 + (1 - \varepsilon)x^2)$ is feasible. The emissions under the policy (c, x) are equal to the ε-weighted average emissions resulting from the proposed policies, since the size of dirty technology capital is its average size, $K_B(t) = K_B^\varepsilon(t)$, all $t \geq \tau \geq 0$. The law of motion of pollution and the emissions sequence imply that pollution $Z(t)$ is weakly smaller than the average pollution $Z^\varepsilon = \varepsilon Z^1 + (1 - \varepsilon)Z^2$ in periods $t \geq t' + 1$ if the absorption $A(Z)$ is strictly concave in at least one period $t' \geq 1$. Then $J(c, Z^\varepsilon) < J(c, Z)$. Else $J(c, Z^\varepsilon) = J(c, Z)$. The utility function U is strictly concave in consumption and concave in pollution. Therefore, $\varepsilon J(c^1, Z^1) + (1-\varepsilon)J(c^2, Z^2) < J(c, Z^\varepsilon) \leq J(c, Z)$. This contradicts that the two proposed policies maximize welfare. *Q.E.D.*

The proof extends the method of using concave production functions and utility function in consumption, as outlined in Becker & Boyd III (1997), to a stock that is an argument in the welfare function such as the environment. The proof applies in the case of a lagged effect of emissions on society, $\tau \geq 1$. Furthermore, there is a unique optimum if U is concave in consumption and U or A is strictly concave in pollution at some $t \geq 1$. The constant returns to scale in producing dirty technology capital are needed in this proof, because this capital enhances pollution of which more is worse, or equivalently, reduces environmental quality of which more is better. An average policy would produce more than the average capital and thus more emissions than the average emissions under decreasing returns to scale in dirty technology investment. In economies without such an environmental asset, production of capital goods at nonincreasing returns to scale is sufficient for the uniqueness of an optimal plan.[20] Arrow's sufficiency theorem is applicable so that the same results hold at nonincreasing returns to scale in the production of investment goods of dirty technology.

Proof of Lemma 1.2. In any stationary point the conditions (1.5) and (1.8) imply that

$$\theta[\beta^{-1} - (1 - \partial A/\partial Z)] = \beta^\tau(-\partial U/\partial Z)/(\partial U/\partial c)$$

at given value of θ. In the undelayed version $\tau = 1$. (i) $\partial^2 U/\partial c \partial Z \neq 0$, $\partial^2 U/\partial Z^2 \neq 0$, or $\partial^2 A/\partial Z^2 < 0$ for a closed interval of Z. The ratio $r = (-\partial U/\partial Z)/(\partial U/\partial c)$ implicitly defines a function $c = \varphi(Z, \theta)$ with slope

$$\partial\varphi/\partial Z = -r \left(\frac{\partial^2 U}{\partial Z^2} \left[\frac{\partial U/\partial c}{\partial U/\partial Z} \right] - \frac{\partial^2 U}{\partial c \partial Z} \right) \Big/ \left(\frac{\partial^2 U}{\partial c^2} \left[\frac{\partial U/\partial Z}{\partial U/\partial c} \right] - \frac{\partial^2 U}{\partial c \partial Z} \right),$$

that is negative given noninferiority, $\partial U/\partial c > 0$, and $\partial U/\partial Z < 0$. The terms in parentheses are positive since both pollution reduction and consumption are noninferior goods and utility is strictly concave in consumption. The concave shape of absorption preserves this sign or yields it if marginal absorption $\partial A/\partial Z$ depends on pollu-

[20]One may show uniqueness of a plan with exclusive investment in clean technology using the proof of Proposition 1.1 and verifying this exclusive investment subject to decreasing returns to scale in clean technology investment.

tion locally, $\partial^2 A/\partial Z^2 \leq 0$. Marginal utility of pollution, marginal utility of consumption, and absorption are continuous since these are differentiable by assumption. (ii) $\partial^2 U/\partial c \partial Z = \partial^2 U/\partial Z^2 = \partial^2 A/\partial Z^2 = 0$ for a closed interval of Z. Consumption is constant. *Q.E.D.*

Proof of Proposition 1.4. There is a canonical system of the four equations (i) (1.5), (ii) $\beta Q q(t+1) - \beta(d+\rho)\epsilon(t+1) = q(t)$ from (1.6) and (1.7), (iii) (1.4), and (iv) $K(t+1) = Q(K(t) + (1-1/Q_j)K_j - c(t)/B)$ in $X = [Z\, K\, {-\epsilon}\, q]'$ holding constant K_j of one technology. Consumption c is an implicit function of Z, ϵ, and q from $Qq = B\partial U/\partial c + \rho\epsilon$. A first-order Taylor series expansion around a stationary point with interior capital $K \in (0, \bar{K})$ yields the linearized system $A_1 X(t+1) - A_0 X(t) = \omega$, where $A_1 = [\beta P\, \beta S;\, I\, 0]$ and $A_0 = [0\, I;\, S'\, R]$. The matrices P and

$$R = \frac{(-1)}{B^2 \partial^2 U/\partial c^2} \begin{bmatrix} \rho^2 & \rho Q \\ \rho Q & Q^2 \end{bmatrix}, \quad S = \begin{bmatrix} (1 - \partial A/\partial Z + \rho r) & rQ \\ (d+\rho) & Q \end{bmatrix},$$

are evaluated at the stationary point given $r = [(\partial^2 U/\partial c \partial Z)/(B\partial^2 U/\partial c^2)]$. The matrix P has $[\partial^2 U/\partial Z^2 + (\partial^2 A/\partial Z^2)\epsilon - B(\partial^2 U/\partial c \partial Z) r]$ in the upper left position and zeros elsewhere. S is nonsingular, in general, and in particular if $\partial^2 U/\partial c \partial Z = 0$ or $d = 0$. The matrix

$$A = {A_1}^{-1} A_0 = \begin{bmatrix} 0 & I \\ (\beta S)^{-1} & -S^{-1}P \end{bmatrix} A_0 = \begin{bmatrix} S' & R \\ (-S)^{-1}PS' & (\beta S)^{-1} - S^{-1}PR \end{bmatrix}$$

determines the stability of the linear map. Define $J = [0\, (-S')^{-1};\, S^{-1}\, 0]$. Then

$$\begin{aligned} \beta A J A' &= \begin{bmatrix} S' & R \\ (-S)^{-1}PS' & (\beta S)^{-1} - S^{-1}PR \end{bmatrix} \begin{bmatrix} 0 & \beta(-S')^{-1} \\ \beta S^{-1} & 0 \end{bmatrix} A' \\ &= \begin{bmatrix} \beta R S^{-1} & -\beta I \\ (S^{-1} - \beta S' P R) S^{-1} & \beta S^{-1} P \end{bmatrix} A' = \beta A J \begin{bmatrix} S & SP(-S')^{-1} \\ R & (\beta S')^{-1} - RP(S')^{-1} \end{bmatrix} = J \end{aligned}$$

follows exploiting the symmetry of P and R. By definition $\beta^{1/2} A$ is symplectic because of this result and the skew-symmetry of J. Symplectic matrices have a reciproal polynomial.

For any nonzero characteristic value ψ then

$$\begin{aligned}0 &= \det(A-\psi I) = \det(-\beta\psi A)\det((\beta A)^{-1} - (1/\beta\psi)I)\\ &= \det(JA'J^{-1} - (1/\beta\psi)JJ^{-1}) = \det(J)\det(A' - (1/\beta\psi)I)\det(J^{-1})\\ &= \det(A' - (1/\beta\psi)I)\end{aligned}$$

using $\det(-\beta\psi A) \neq 0$, $\det(J)\det(J^{-1}) = 1$, and $(\beta^{1/2}A)' = J^{-1}(\beta^{1/2}A)^{-1}J$. A matrix A and its transpose A' have the same characteristic values. Therefore, if $\psi \neq 0$ is a characteristic value then $1/\beta\psi$ is too. In fact, the determinant of A, which is the product of all characteristic values, is positive at β^{-2}, so that all characteristic values are nonzero. Boyd III (1989) shows that the characteristic values ψ and $1/\beta\psi$ have the same multiplicity. *Q.E.D.*

The reciprocity of characteristic values is obtained using a skew-symmetric matrix J different from $J^* = [0\,-I;\, I\,0]$. The continuous-time analog of the property that leads to this result is that the Jacobian A of the modified Hamiltonian system with time discount rate ρ satisfies $J^{-1}AJ = -A' + \rho I$ for some skew-symmetric matrix J. For a canonical system in continuous time J^* is useful. For example, van der Ploeg & Withagen (1991) use this in an economy without emissions of investment.

The emissions from investment conform to saddle-path stability. Substitution of the resource constraint into the law of motion of pollution yields

$$Z(t+1) = (d+\rho)K(t) + \rho(K_j - c(t)/B) + (\rho_j - \rho)K_j/Q_j + Z(t) - A(Z(t)) \qquad \text{(A-1)}$$

so that the next period's capital stock $K(t+1)$ is purged. The leading capital stock of the other technology j is constant. Therefore, limit cycles, which can appear when consumption is proportional to emissions (Ryder & Heal 1973), are not expected. The resource constraint $c(t)/B + K(t+1)/Q = K(t)$ and the law of motion $Z(t+1) = \rho c(t) + Z(t) - A(Z(t))$ imply a difference equation that contains $Z(t+1)$ and $K(t+1)$ as substitutes. Heal (1982) interprets Ryder & Heal (1973) in terms of the environment.

2

The timing of capital retirement in pollution control

Underutilizing pre-existing dirty technology capital prevents current emissions. With this policy the capital use can be postponed and thus emissions of investing can be avoided when delayed capital use replaces investment. Underutilization of pre-installed clean technology capital can indirectly save emissions if building it creates emissions. The leading example is climate change. Fossil-fuel technologies produce energy and carbon dioxide emissions proportionally using long-lasting capital, and their current capacity has been built irreversibly and without regard to a negative externality. This capacity may be greater than the optimum long-term energy yield of fossil fuels. Both building fossil-fuel assets and clean renewable energy technology capital creates carbon dioxide and methane emissions. I use a dynamic model with heterogeneous capital to show how allocations in optimum and in a decentralized economy without government policy differ, simulate their trajectories of pollution and capital, and characterize government policy that implements an optimum.

There are four major findings. (i) Dirty technology capital is optimally underutilized when pollution is smaller or greater than its long-term level to rapidly approach long-term levels, because consumption and investment use the same resource. (ii) Only capital that is installed at the initial date of optimization is underutilized in the deterministic setting, possibly over multiple periods. All or a portion of pre-existing dirty technology capital is idle forever, it is underutilized until it is used up or investment becomes worthwhile, or such capital is fully utilized. Investment in the dirty technology is followed by fully utilized dirty technology capital. (iii) The Pigouvian emissions tax, which is proportional to the cost of polluting, is lower in the early periods in which dirty technology use is postponed in the optimum than the tax that implements full utilization at each date by assumption in the constrained optimum, because underutilization mitigates societal effects of pollution. (iv) Clean technology capital can be optimally underutilized if the scale of other, more productive, clean technology types is large because of the environmental impact of creating new capital or because capital that is expensive to construct is pre-installed.

Emissions predictions and optimal emissions plans for climate control have focused on the investment in dirty versus clean technologies, change in energy efficiency through new equipment and retrofits, and removal of carbon from the atmosphere. Studies in the Energy Modeling Forum (EMF) 22 assume fully utilized capital stocks in testing the hypotheses of staying below given carbon levels formulated as a constraint (Clark et al., 2009). Some of these models predict that currently installed and long-lasting capital leads to atmospheric greenhouse gases in excess of 450 ppmv CO_2 equivalent units in 2100. Recent attempts to determine the social cost of carbon using continuous feedback of carbon on output, for example, Nordhaus (2009) and Golosev et al. (2011), or Barrage (2012) with output and utility feedback and distortionary fiscal policy, presume fully utilized capital. In their framework capital can be substituted for emissions (see more detailed comments below) so that a commitment to technology stock at the date of investment is lacking. In reality capital utilization can be varied beside directing investment toward technologies with fixed emission intensity of output.[1] Second these studies do not differentiate between the emissions in producing energy that is useful for consuming or investing and emissions solely for investment.

I build a model with heterogeneous capital to show the optimal timing of abandoning the use of dirty technology or postponing the use and investing in dirty and clean technologies when there are these two sources of emissions. The use of dirty technology creates emissions while the use of clean technology does not create emissions. The

[1]Utilization balances time-varying demand and supply in peak-load theories of electricity that Crew, Fernando & Kleindorfer (1995) review, and that models in the EMF may account for. The government of Mexico City has recognized that limiting the time of use of polluting technology below full capacity may save emissions. However the policy of precluding about a fifth of automobiles each weekday from driving (Hoy No Circula since 1989) may be ineffective because of double car-ownership (Davis 2008).

optimum consumption approaches a steady level in the long-term in contrast to Acemoglu et al. (2012) because I assume constant technology to focus on the retirement of capital in different technology states. The model in the present paper (given multiple clean technology types) can explain current clean technology investment by its relatively greater productivity on small scale. In a study of directed technical change, Acemoglu et al. (2012) assume nondepreciating heterogeneous capital types that produce imperfect substitutes, and cannot explain current clean production because of its relatively lower cost as the high-cost technology produces the low portion of output.[2] In a study of nonrenwable resource depletion, Tahvonen & Salo (2001) assume scale-dependent relative advantage of a nonrenewable resource technology and an alternative renewable resource technology, analogously to the dirty and clean technology types here. Pollution is not controlled, whereas this chapter has an environmental motive.

The opportunity cost of using output to invest is a consumption benefit and resources are finite in a given period as one would presume in the world economy, in contrast to literature with variable utilization and an environmental motive. Van Long (2006) allows underutilizing an endowment of time-invariant size to control pollution. The model of the sole owner fishery of Clark, Clarke & Munro (1979) and Boyce (1995) with chosen utilization of fishing vessels does not suit climate control, because it lacks a trade-off between consumption and investment. Second, the depreciation of capacity (or the productivity of capital) and utilization are linked. The finite usable time of capital with storage of unused capital contrasts the perpetual inventory method used in the fishery models, to suit the energy-climate context. A finite usable time of capital allows a sensible comparison to the constrained optimum with full utilization, because some technology may become obsolete in the long-term. Given infinite lifespan optimal and constrained optimal capital stocks would differ in the long-term if initially there was capital in a technology that is optimally unused in the long-term. Halting capital use increases the lifespan of capital. In Puu's (1977) resource extraction and in some business cycle research greater utilization raises depreciation. In contrast to my model, there fully utilized capital leaves some capital in infinite time under no investment.[3] In particular, electric power plants, heating machines,

[2] They can explain the major share of clean energy in the late 19th century, and contradict the increased share of dirty energy that is observed until the late 20th century. According to their model the research should have efficiently focused on clean technology without government incentives.

[3] In the real business cycle literature homogeneous intensity of capital use, for example Greenwood, Hercowitz & Huffman (1988), or heterogeneous intensity, for example Cooley, Hansen & Prescott (1995),

and automobile engines, that produce energy and greenhouse gas emissions are long-lived and can produce currently, or pause and produce later, and are scrapped in finite time. The production at any given productivity is bounded from above, to be consistent with renewable energy production. For example, given solar modules are more productive in California than in Washington State. The number of times in which capital is useful is exogenous as in Buhl et al. (1982). The basic model has an underpinning from vintage capital with invariable emission intensity. There capital is useful once so that a period is long. An extension exhibits vintage-dependent emission intensity and multiple times of use. Since capital is useful for an exogenous finite number of periods, no second factor is necessary for the efficient scrapping of capital in contrast to vintage capital models of Johansen (1959) or Solow et al. (1966).

Stokey (1998) argues that several technologies can be lumped into a single variable that expresses the overall emission intensity of output. This approach prevents conclusions about technology-specific capacity utilization. In the present approach instead emissions are controlled intratemporally through utilization and intertemporally through technology-specific investment which allows distinct capital to idle, or to not exist, when consumption is positive. Capital services and emissions are complementary in any period. Keeler et al. (1971), Brock (1977), Tahvonen & Kuuluvainen (1993), and Stokey (1998) assume that capital and emissions produce output at a positive substitution elasticity in the context of a pollution stock. This substitution can be interpreted as choosing the emission intensity of gross product below or at an exogenous upper bound—in Nordhaus' Dynamic Integrated Climate Economy (DICE), and by Hassler et al. (2011), given calibration of emission prevention to a mix of dirty and clean technologies and energy efficiency (Nordhaus 2009). An alternative view is controlling the emission intensity of gross output through expending resources on abatement (for example, carbon sequestration).[4] Interpreting emission control in this paradigm as utilization of capital seems troublesome. This implies underutilization at all levels of the cost of polluting given a strictly increasing emission intensity in the utilization rate, contradicting previously and

responds to changing aggregate or plant-level productivity, respectively. Lower depreciation of capital for lower utilization or the clearing of an economy-wide labour market motivate the response.

[4]An intratemporal trade-off between this emission intensity and technology choice or real emission prevention flow-expenditure, for example in model I of Keeler et al. (1971), can be rewritten in terms of net production with substitutable factors capital and emissions.

presently planned efficient plant scales in reality.[5]

Luptačik & Schubert (1982) and van der Ploeg & Withagen (1991) fix the emission intensity of output permanently. Variable utilization may improve their plans. Acemoglu et al. (2012) posit emissions and specific output in fixed proportion and distribute resources among production technologies in each period. While there is commitment to productivity, underutilization would not be optimal because of the intratemporal adjustment. Studies in the EMF 22 use heterogeneous capital to exploit data on mitigation options. Given a planner maximizes welfare subject to constraints on environmental quality in these models underutilization through postponement of using or early retirement of some technology stock will bring about a solution when the problem has no solution at full utilization.

The model includes emissions from investment, for example, in the production of steel and concrete, and mining of minerals, for fossil-fuel and renewable energy technologies. This realistic assumption can make a portion or all of the pre-installed dirty technology capital obsolete. The use of pre-installed clean technology capital that is expensive to create may be postponed because more productive types of clean technology at a large scale can sustain high consumption. This incentive for underutilization is stronger, if investment creates emissions, since then investing bears an environmental cost. (Dirty technology capital may then not be used.) This may be relevant in the future if governments continue to push expensive clean technologies by subsidies.

The next section examines the Pareto optimal retirement of capital. Section 2.2 characterizes competitive equilibrium allocations in a decentralized economy and shows how an optimum can be implemented. Section 2.3 provides results of numerical simulations. Section 2.4 presents production using bounds on physical capital, fuel that serves as an input in the dirty technology, energy-use capital and variable energy efficiency, and time-variant emission intensity of output and depreciation. Section 2.5 concludes with a discussion of results. The appendix sections contain technical results.

[5]Joskow (2010) finds that lower utilization of existing coal power plants weakly raises their emission intensity of electricity output in response to greater supply of intermittent renewable-based electricity. Underutilization of capital in DICE only at sufficiently high cost of polluting can be optimal upon introduction of complementarity. Kolstad (1996) constrains future control by current control such that a planner temporarily cannot adjust the emission intensity of gross output upward, yet does not enable underutilization. In a plan with decreasing emission intensity over time Kolstad (1996) requires uncertainty to make this constraint ever binding.

2.1 The economy

Consider a discrete-time economy with heterogeneous capital.[6] Feasibility conditions are introduced with each one dirty and clean technology. The use of dirty technology only is polluting. The investment is polluting in both these technologies. A dirty technology or clean technology may be a composite of any fossil-fuel using technologies or any renewable energy technologies, respectively, that produce energy that can be consumed or invested.

2.1.1 One dirty technology and one clean technology

Preferences.—A unit mass of infinitely-lived households populates the economy. They have an identical preference ordering represented by a period-utility function $U(c, Z) : \mathbb{R}_+ \times \mathbb{R} \to \mathbb{R}$ of consumption c and pollution Z that is twice-differentiable for positive consumption, increasing in consumption, $\partial U/\partial c > 0$ for $c > 0$, and decreasing in pollution, $\partial U/\partial Z < 0$ for $c > 0$. Marginal utility of consumption $\partial U/\partial c$ approaches a large positive value M as consumption tends to zero, $\lim_{c\to 0} \partial U/\partial c = M \leq \infty$, for all Z. Then consumption c_ℓ of at least one household ℓ is positive in any period $t \in \{0, 1, 2, \ldots\}$ in a Pareto optimum. Households have equal endowments of financial capital in the decentralized economy in Section 2.2 so that uniform transfers of government revenue to households implement uniform consumption. The utility function is strictly concave in consumption and, for small pollution levels or all pollution levels, concave in pollution. Thus $\partial^2 U/\partial c^2 < 0$ for $c > 0$ and $\partial^2 U/\partial Z^2 \leq 0$ for $c > 0$ and $Z \leq Z^*$. A household discounts utility in the welfare function

$$J = \sum_{t=0}^{\infty} \beta^t U(c(t), Z(t))$$

by a factor $\beta \in (0, 1)$.

Technology.—In each period society allocates perfectly substitutable output m_B from dirty technology B and m_C from clean technology C between the use for consumption $c = \int_0^1 c_\ell d\ell$, and as an input x_j in investment of technology $j \in \{B, C\} = \mathcal{J}$. Using

[6] An exogenous finite usable time of capital in continuous time seems to demand vintages whose productivity depreciates. Burmeister & Dobell (1970, 377) argue that exogenous full depreciation of capital in an instant requires an infinite outflow which is infeasible when (dis)investible resources are finite.

dirty technology creates emissions in proportion to output. Clean technology use does not create emissions. The emission intensities are defined below. One unit of the good yields $B > 0$ units of service that households consume. Thus,

$$c/B + x_B + x_C \leq m_B + m_C \tag{2.1}$$

is the resource constraint all $t \geq 0$. The productivity in the consumption sector is not normalized to one to point to a difference between the cost of pollution in terms of the consumption good and the willingness to pay for pollution reduction in terms of output. Production using technology $j \in \mathcal{J}$ yields $m_j = \mathrm{u}_j K_j$ units of output given chosen utilization rate $\mathrm{u}_j \in [0, 1]$ of capital $K_j > 0$. This measure of capital is production capacity since there is no substitutable factor in production. Limited recyclability of the earth's material or finite geographical space lead to the capacity constraint

$$\bar{K}_j \geq K_j(t+1) \tag{2.2}$$

all $t \geq 0$. This constraint is different for each technology $j \in \mathcal{J}$ for simplicity. At least one capital stock is positive among the given $K_j(0) \in [0, \bar{K}_j]$ for $j \in \mathcal{J}$. The number $\gamma_j \in (0, 1]$ is one minus the depreciation rate of unused capital and thus represents its portion that is useful in the next period. Investment, $x_j(t) > 0$, is not necessary to generate capital $K_j(t+1) > 0$. A positive γ_j is plausible for energy-producing capital. Incomplete depreciation refers to mothballing boilers, motors, or turbines and can be incentivized through government policy in the decentralized economy in Section 2.2. New capital of technology $j \in \mathcal{J}$ is built at the constant marginal product $Q_j > 0$. Then

$$K_j(t+1) = \gamma_j(1 - \mathrm{u}_j(t))K_j(t) + Q_j x_j(t) \tag{2.3}$$

is capital at the beginning of period $(t+1)$. Investment is irreversible, because the input in investment is nonnegative, $x_j \geq 0$. The constraint (2.2) may be binding for clean technology at an optimum and in equilibrium. Observed current clean technology investment on small scale $\bar{K}_C$ is explained in the decentralized economy absent government policy that corrects the pollution externality by $Q_C \geq Q_B$. I make the following assumption.

Assumption 2.1 $Q_B > \beta^{-1}$.

Then sustained growth of consumption and output is feasible in the absence of environmental cost. Fully utilized capital $K_j(t) > 0$, that is, if $\mathrm{u}_j(t) = 1$, completely depreciates within one period. An interpretation is that a period is long. Grubb (1997) and Nakicenovic & Grübler (2000) point to long lifespans of currently existing capital in energy production as a reason of inertia that could impede stabilizing the atmospheric content of carbon dioxide soon at a non-hazardous rate.[7] Clean capacity is underutilized only if dirty capacity is idle or absent when postponing consumption by underutilizing clean capacity yields a smaller rate of change in consumption than by using dirty capacity to invest in dirty technology. This appears if unused capital of dirty technology depreciates at a relatively weakly smaller rate, $\gamma_C \leq \gamma_B$. I assume this, which helps a clear characterization of an optimal allocation.

Assumption 2.2 $0 < \gamma_C \leq \gamma_B$.

This assumption—that the unused clean technology capital depreciates at a weakly greater rate—is plausible if dirty technology capital can be literally wrapped and clean technology capital is exposed to natural hazards, for example, in fossil-fuel using and renewable energy technologies, respectively.

Environment.—Production of one unit of output using technology j generates d_j emission units where $d_B > 0 = d_C$.[8] The emissions specific to building capital occur at rate $\rho_j \geq 0$ per unit of the quantity x_j of the input in investment. Emissions of investing in dirty technology affect both the critical cost of polluting at which full utilization and investment in dirty technology becomes optimal, and that may be reached at some date if dirty technology capital is used in the long-term, and the long-term

[7]Given this inertia in calendar time vintages that are useful once or more than once regarding model periods yields the same results on the timing of utilization and identity of underutilized units, that is, pre-installed units. The scrapping after one period of use implies that there are at most two vintages given the timing of investment and utilization that is optimal. This is formalized in Section 2.4.1. If capital vintages are useful multiple times, then underutilization distributes output over vintages and extends their lifespan early in the planning horizon. This yields the same conclusions regarding the timing of utilization and investment as one-period use if technology does not change and depreciation between ages is constant. Section 2.4.4 assumes multi-period use to examine the roles of technology improvement in regard to the emission intensity, and depreciation of capital over the lifespan, for underutilizing old versus young vintages.

[8]The optimal utilization rate of aggregate dirty technology capital is weakly greater given any pollution and capital amounts compared to the assumed no response if some dirty technology capital exhibited a greater emission intensity of output for lower utilization. The response may be relevant for current baseload coal electricity, if its underutilized plants are not wholly mothballed.

cost of polluting. The environmental cost of investing in clean technology can lead to idle dirty technology capital and to underutilized clean technology capital.[9] Emissions $E(t) = \sum_{j \in \mathcal{J}} (d_j m_j(t) + \rho_j x_j(t))$ accumulate to the stock of pollutants

$$Z(t+1) = Z(t) + E(t) - A(Z(t)) \tag{2.4}$$

at the beginning of period $(t+1)$, for example, carbon dioxide in the atmosphere, which is equal to pollution.[10] This helps characterizing long-term allocations. The absorptive capacity of the environment may vary with pollution. The absorption A is a twice differentiable nondecreasing function, which is concave, $\partial^2 A/\partial Z^2 \leq 0 \leq \partial A/\partial Z$, and less than or equal to pollution, $A \leq Z$. A plan is defined as an allocation of controls and state variables pollution and capital. Strictly concave absorption at given Z guarantees a unique optimal plan when utility is not strictly concave in pollution for such a Z. The pollution level $Z(0)$ is given.

Necessary conditions for Pareto optimal investment and utilization.—A Pareto optimum with equal consumption of all households may not be implementable in a decentralized economy with nonnegative transfers from the government to households if government revenue is small and households have unequal endowments. Other Pareto optima may then be implementable. However, the distribution of consumption does not affect the Pareto optimal allocation of pollution, if these are essentially independent, which I assume for the utility function U.[11] Then uniform consumption is without loss of generality for pollution. There are $\tau(t)$ positive capacity levels at date t which helps to define the control space. Only $\tau(t) \geq 1$ enables consumption at date t.

The planner chooses a policy of consumption, input in investment, and capital utilization $(c, x, \mathrm{u}) \in \mathbb{R}^3_+ \times [0,1]^{\tau(t)}$ on $\{0, 1, \ldots\}$ to maximize welfare J subject to the resource constraint (2.1), the upper bound on capital (2.2), and the laws of motion (2.3)-(2.4)

[9] The quantity x_j may be the sum of direct input in investment goods and inputs in material production, which generates emissions specific to investment.

[10] In general there is some mapping of past emissions onto current pollution.

[11] Following Bergstrom & Cornes (1983) a utility function satisfies essential independence of the public good or bad and the distribution of a private good if for all interior Pareto optimal allocations every other distribution of interior private goods amounts is Pareto optimal holding constant the public good or bad and the aggregate private goods amount. For example, the utility function $U(c_\ell, Z) = U'(Z)U_0(c_\ell) + U''(Z)$ where U_0 is a power function, satisfies essential independence, because there is a unique Pareto optimal $(\int_0^1 c_\ell d\ell, Z)$, and it is interior given the assumed feasibility set.

in the form $K_j(t+1) - K_j(t) = r_{K_j}(t)$ for $j \in \mathcal{J}$ and $Z(t+1) - Z(t) = r_Z(t)$ all $t \geq 0$. Let ϵ and q_j for $j \in \{B, C\}$ be the Lagrangean multipliers of the laws of motion of pollution and capital. The inclusion of the endogenous terms $r_Z(t)$ and $r_{K_j}(t)$ all $j \in \mathcal{J}$ is helpful to prove the uniqueness of an optimal plan. A vector $G \geq 0$ describes the admissable set of investment and utilization and the resource constraint and contains the nonnegativity constraints of capital. Then maximization of Lagranges' function $\mathscr{L} = \sum_{t=0}^{\infty} \beta^t [U(c(t), Z(t)) - \epsilon(t) r_Z(t) + \sum_j q_j(t) r_{K_j}(t) - \epsilon(t)(Z(t) - Z(t+1)) + \sum_j q_j(t)(K_j(t) - K_j(t+1)) + \sum_j \beta w_j(t+1)(\bar{K}_j - K_j(t+1)) + w(t)G(t)]$, where $(w_B\, w_C)$ and w are nonnegative vectors, gives rise to the following necessary optimality conditions.

The present values of the Lagrangean multipliers $\epsilon(t)$ and $q_j(t)$ are the marginal valuations $\partial J/\partial Z(t+1)$ and $\partial J/\partial K_j(t+1)$ of the state variables at an optimum, and thus $\epsilon(t)$ and $q_j(t)$ are called their (current value) shadow prices.[12] Optimal interior values of pollution $Z(t+1)$ satisfy the difference equation

$$\epsilon(t) = \beta(-\partial U/\partial Z(t+1)) + \beta(1 - \partial A/\partial Z(t+1))\epsilon(t+1), \quad t \geq 0, \tag{2.5}$$

of the shadow price ϵ of pollution. The marginal benefit from additional capital is the marginal net benefit of utilized capital plus the marginal value of nonutilized capital. The sum of these benefits discounted from $(t+1)$ at most equals the marginal cost of building capacity in period t,

$$\begin{aligned} &\beta \mathrm{u}_j(t+1)\{B(\partial U/\partial c)(t+1) - d_j \epsilon(t+1)\} \\ &\quad + \beta\gamma_j(1 - \mathrm{u}_j(t+1))q_j(t+1) - \beta w_j(t+1) \leq q_j(t), \quad = \text{ if } K_j(t+1) > 0, \end{aligned} \tag{2.6}$$

for $j \in \mathcal{J}$ all $t \geq 0$. Greater environmental cost measured by the shadow price of pollution ϵ reduces the benefit of using dirty technology capital but has no effect on the benefit of using clean technology capital since $d_B = b > 0 = d_C$. The shadow rental value of space, $w_j(t)$, is zero if $\bar{K}_j > K_j(t)$. The marginal benefit of using an output unit for investment is $Q_j q_j$ in terms of utils. This benefit cannot exceed the marginal cost of investing output,

$$Q_j q_j(t) \leq B(\partial U/\partial c)(t) + \rho_j \epsilon(t), \quad = \text{ if } x_j(t) > 0, \tag{2.7}$$

[12] The present value of the costate multiplier at t is the shadow price of the law of motion that has the state variable at its leading value at $(t+1)$ which can be taken as a parameter locally in optimum.

all $j \in \mathcal{J}$, because the input in investment is finite. The latter marginal cost comprises the cost of reduced current output and environmental cost by the investment sector. Partial capacity utilization in period $(t+1) \geq 0$ balances the values of idle and utilized units of capital on the left side in (2.6). Else all capital in a given technology $j \in \mathcal{J}$ is either idle or fully utilized. Therefore

$$
\begin{aligned}
&\mathrm{u}_j(t+1) \left\{ \begin{array}{c} =1 \\ \in (0,1) \\ =0 \end{array} \right\} \\
&\Longrightarrow \quad \beta\{B(\partial U/\partial c)(t+1) - d_j \epsilon(t+1)\} \left\{ \begin{array}{c} = \\ = \\ \leq \end{array} \right\} q_j(t) \left\{ \begin{array}{c} \geq \\ = \\ = \end{array} \right\} \beta\gamma_j q_j(t+1)
\end{aligned}
\tag{2.8}
$$

given $K_j(t+1) \in (0, \bar{K}_j)$ for $t \geq 0$. The outer relations at weak inequalities are relevant regarding utilization in period $t = 0$.[13] Before analysing these conditions an existence and uniqueness result on an optimal plan is stated—of which a proof appears in the appendix.

Proposition 2.1 *There is a unique optimal plan if utility U or absorption A is strictly concave in pollution at any pollution Z.*

Hence the welfare function is well-defined. There is a unique optimal plan because there is a unique optimal policy and the laws of motion map the current policy and states uniquely into future states.

The (marginal) cost of polluting, or equivalently, the (marginal) benefit of pollution reduction, is defined as

$$\theta = \epsilon/(\partial U/\partial c)$$

and thus hinges on the motion of its numerator, the shadow price ϵ of pollution measured in utils, and that of its denominator, marginal utility of consumption. This cost may be referred to as a social cost because ϵ measures the effect of pollution on all households

[13] The outer relations in the condition (2.8) follow from differentiation of the Lagrangean function with respect to the utilization rate. The relations to the lagged shadow price of capital result presuming that $K_j(t+1) > 0$ and examining (2.6). For $K_j(t+1) = \bar{K}_j$ the discounted land rent $\beta w_j(t+1)$ must be subtracted on the left and on the right sides.

who are identical and have a unit mass for simplicity.[14]

Critical level of cost of polluting.—The next lemma reports bounds of the cost of polluting for utilization and underutilization, which help to characterize and solve for an optimum. There is a cost of pollution reduction $\theta_j = (Q_j - \gamma_j)/(\gamma_j \rho_j + d_j Q_j)$ of any technology $j \in \{B, C\}$ that allows saving emissions by storing capital as an alternative to using output for its investment, $d_j > 0$ or $\rho_j > 0$.

Lemma 2.1 *The cost of polluting is bounded above, $\theta(t) \leq B/d_B$, if dirty technology capital $K_B(t) > 0$ is utilized, $\mathrm{u}_B(t) > 0$. Capital $K_j(t) > 0$ is underutilized, $\mathrm{u}_j(t) < 1$, only if $\theta_B \leq \theta(t)$ for dirty technology B, and $\theta_C \leq \theta(t)$ for clean technology C with $\rho_C > 0$. The cost of polluting is bounded above for dirty technology, $\theta(t) \leq \theta_B$, and for clean technology, $\theta(t) \leq \theta_C$, if $\rho_C > 0$, given investment occurs, $x_j(t) > 0$, and capital $K_j(t) > 0$ is utilized, $\mathrm{u}_j(t) > 0$.*

Proof. Define λ as the marginal product of consumption goods, B, times the marginal utility of consumption, $\partial U/\partial c$. The first result follows from the condition $\lambda - b\epsilon \geq \gamma_B q_B \geq 0$ noting that $\epsilon/\lambda = \theta/B$. The second result is immediate from $Q_j(\lambda - d_j\epsilon) \leq Q_j\gamma_j q_j \leq \gamma_j(\lambda + \rho_j\epsilon)$ given underutilization, $\mathrm{u}_j < 1$. For the upper bound of θ given investment, $x_j > 0$, and utilization, $\mathrm{u}_j > 0$, the condition (2.7) holds at equality, and the inequality in (2.8) is reversed. *Q.E.D.*

The upper bound for utilizing dirty technology capital results because this capital has a nonnegative shadow price, unlike in the constrained optimum in which utilization cannot be chosen. Clean capacity use is consistent with arbitrarily high θ because it does not affect the environment. There is a lower bound θ_j for underutilization of capacity $K_j(t) > 0$ for $j \in \{B, C\}$ because society forgives current capital services to save rather than invests in new units only if pollution reduction costs large consumption amounts. The level θ_j decreases in both the storage return γ_j and the emission intensity ρ_j of investment inputs because unused capital is worth more for smaller depreciation rate $(1-$

[14] Recall that $\gamma_B > 0$ and $\gamma_C > 0$. Dirty capacity $K_B(t) > 0$ is underutilized only in the initial period $t = 0$ because of concerns about the environment that the planner internalizes if $\gamma_B = 0$. Investment in period t is necessary for $K_B(t+1) > 0$ if unused dirty capacity becomes unproductive. Then $Q_B q_B(t) = B(\partial U/\partial c)(t) + \rho_B \epsilon(t) > 0$ from (2.7). The condition (2.8) contradicts $q_B(t) > 0$ if $K_B(t+1) > 0$ is underutilized, $\mathrm{u}_B(t+1) < 1$. Investment that precedes underutilization would be wasteful. Clean technology capital $K_C(t) > 0$ is fully utilized all time if $\gamma_C = 0$. The marginal net benefit of utilizing clean capacity, $B\partial U/\partial c$, is strictly positive all $t \geq 0$.

γ_j), and investing bears a greater environmental cost for greater ρ_j. The range between the critical technology-specific cost of pollution reduction at which both investment and underutilization are optimal and the upper bound for utilization narrows, $\theta_j \to B/d_B$, as unused capacity becomes less productive, $\gamma_j \to 0$. Second, θ_j is greater than the stationary level of the cost of polluting if there is investment in technology j, since the discounted retained fraction of unused capital, $\beta\gamma_j$, is less than one. Perpetual investment and underutilization would be wasteful. Therefore a capacity is fully utilized if there is investment in the associated technology in a plan that converges to a stationary point. The bounds of the cost of polluting can be found through the policy (c, x, u) if capital is used at decreasing returns to scale. To simplify the exposition I chose the convex production technology. Some dynamic programming results are helpful to characterize an optimum. The next Lemma presents one result.

Lemma 2.2 *There is a continuously differentiable value function* $v(Z, K_B, K_C) = \max J$ *given the initial state* (Z, K_B, K_C).

Proof. Standard arguments in Stokey & Lucas (1989) or Acemoglu (2009) can be used because the planner controls pollution. In particular, the welfare function J, which is to be maximized, is bounded from above, $J \leq U(\bar{c}, Z)/(1-\beta) < \infty$, for all feasible plans since output and pollution are bounded from above and below, respectively. Maximum consumption is $\bar{c} = B(\bar{K}_B + \bar{K}_C)$. Minimum pollution is $Z = \phi(\ldots\phi(\phi(Z(0))))$ given $\phi(Z) = Z - A(Z)$. *Q.E.D.*

Welfare J can be defined recursively for any initial state. Lemma 2.2 can be used to show that the Lagrangean multipliers of the states are the slopes of the value function, $\epsilon(t) = -\beta\partial v/\partial Z(t+1)$ and $q_j(t) = \beta\partial v/\partial K_j(t+1)$. In the following roman numeral I denotes paths with exclusive long-term use of clean technology, and those with II have long-term use of dirty technology.

Timing of investment and utilization in dirty technology.—Investment in the dirty technology may never occur (in I-1 to I-3). Its capital may be always underutilized, being idle at all dates but the initial date. This policy (I-2) in the next proposition is optimal only if the cost of polluting exceeds B/d_B at dates $t \geq 1$ by the arguments above, and clean technology has a sufficiently large productivity Q_C, a sufficiently small positive emission intensity ρ_C, and a sufficiently large scale $\bar{K}_C$ as seen as follows.[15] Chapter 1 has shown

[15]Large marginal disutility of pollution at minimum feasible pollution can lead to exclusive use of clean

that clean technology is exclusively used in the long-term at $\theta \leq B(\beta Q_C - 1)/\rho_C$.[16] This implies large Q_C or small $\rho_C > 0$. A large scale $\bar{K}_C$ makes the exclusive use of clean technology feasible at $\theta > B/d_B$.

Proposition 2.2 *Dirty technology capital $K_B(t) > 0$ is partially utilized at $t = 0$, $\mathrm{u}_B(0) \in (0,1)$, and idle, $\mathrm{u}_B(t) = 0$, at dates $t \in \{1, 2, \ldots\}$, and there is no investment in dirty technology, $x_B(t) = 0$ all $t \geq 0$, if dirty capacity $K_B(0)$ is large and clean capacity $K_C(0)$ is small, and clean technology is exclusively used in the long-term at $\theta > B/d_B$.*

Proof. There is a curve $\tau(Z, K_C)$ such that $\theta = B/d_B$ when holding $x_B = \mathrm{u}_B = 0$. This follows from $Q_C q_C(t) = (B/\theta(t))\epsilon(t) + \rho_C \epsilon(t)$ if $x_C(t) > 0$ that utilizes (2.7) for $j = C$ and the definition of θ. Then the ratio $q_C(t)/\epsilon(t)$ of shadow prices implies a continuous θ-isoquant in the space of Z and K_C because these shadow prices measure the marginal valuation of those state variables. Let $K_B(0) + K_C(0) > K > K_C(0)$ for $(Z(0), K) \in \tau$. If $\theta(0) < B/d_B$ then dirty technology capital was used up in finite time, yet the output that this generates exceeds the output at which such low $\theta(0)$ is optimal. If $\theta(0) > B/d_B$ then dirty technology capital was idle at all dates, yet initial clean capacity insufficient to afford the jump on the optimal path. *Q.E.D.*

The requirement $1/d_B < (\beta Q_C - 1)/\rho_C$, sufficiently large clean technology scale $\bar{K}_C$, and the relation of initial capacity levels that induces $B/d_B = \theta(0)$, are sufficient for an immediately retired portion of dirty capacity. The initial dirty capacity helps growing the economy with clean technology investment. Dirty capacity is idle at all dates $t \geq 1$ because the cost of pollution reduction by not using dirty capacity is smaller than the cost of polluting. Dirty capacity is never used (I-1) if the economy has the sufficient clean capacity initially for the same reason. A rationale to idle a capital stock in the literature is production cost. Some portion of a resource is efficiently unused after the marginal cost of using it has increased to the constant marginal cost of an alternative technology in Herfindahl & Kneese (1974, Chapter 4.5) and Heal (1976) or the marginal cost of a

technology. Ruling out this possibility for the climate $\rho_C > 0$ remains as a reason for exclusive use of clean technology.

[16]The stationary level θ is less than or equal to the right side of the cost of pollution reduction θ_j when replacing γ_j by β^{-1} given investment in technology j, equal at $K_j \in (0, \bar{K}_j)$. There is a unique stationary point if the stationary level differs between the technologies, and one imposes noninferior pollution reduction and consumption through the utility function, see Chapter 1. The optimal allocation converges to a unique stationary point, if such exists.

technology with diminishing returns in Tahvonen & Salo (2001). Too large pre-installed clean capacity induces underutilization of clean capacity as seen below given the same parameter values.

The following proposition shows the timing of investment and utilization in dirty technology on some paths with $\theta(t) > \theta(t+1)$ given underutilized dirty capacity at $(t+1)$. In the first case (I-3) initial dirty capacity is used up without investment because θ converges to a level above the cost of pollution reduction using reproduced capital in dirty technology, $\theta^* = B(\beta Q_B - 1)/(\rho_B + \beta d_B Q_B)$, and below the cost of pollution reduction by not using its installed capital, B/d_B, so that investing in dirty technology is inefficient but using dirty capacity is worthwhile. In this case the clean technology marginal product is large, $Q_C > \beta^{-1}$, and the clean technology scale $\bar{K}_C$ is large, which sustains such θ at exclusive clean output. In the second case (II-1) there is investment in dirty technology in the long-term because θ converges to a level less than or equal to θ^*, which is smaller than the cost of pollution reduction θ_B. In this case the stationary cost of pollution reduction is relatively smaller for the clean technology, $B(\beta Q_C - 1)/\rho_C < \theta^*$, or the clean technology scale $\bar{K}_C$ is small. In the third case of the next proposition dirty technology capital is fully utilized at all dates. This can be on a path leading to exclusive clean technology investment under the parameters that yield Proposition 2.2, because the initial aggregate capacity $(K_B(0) + K_C(0))$ is smaller than the level required for the path in this proposition (I-4), or on a path with long-term use of the dirty technology under the parameters of the second case (II-4). In the latter case the initial aggregate capacity is not sufficiently greater than the long-term aggregate capacity and the stationary clean technology cost of pollution reduction or the clean technology scale is small.

Proposition 2.3 *Dirty technology capital* $K_B(t) > 0$ *is either partially utilized,* $\mathrm{u}_B(t) \in (0,1)$*, in an early time interval* $\{0, 1, \ldots, t'\}$ *and ceases to exist,* $\mathrm{u}_B(t'+1) = 1$ *and* $K_B(t) = 0$ *all* $t > t'+1$*, or is fully utilized later,* $\mathrm{u}_B(t) = 1$ *all* $t > t'$*, or this capacity is fully utilized,* $\mathrm{u}_B(t) = 1$*, all* $t \geq 0$*, in an optimal plan in which (i) dirty technology capital* $K_B(t) > 0$ *is not idle,* $\mathrm{u}_B(t) > 0$*, and either is zero after it is positive,* $K_B(t) > 0$ *all* $0 \leq t \leq t''$ *and* $K_B(t) = 0$ *all* $t > t''$*, or always positive,* $K_B(t) > 0$ *all* $t \geq 0$*, and (ii) the cost of polluting decreases,* $\theta(t) > \theta(t+1)$*, if dirty capacity* $K_B(t+1) > 0$ *is underutilized,* $\mathrm{u}_B(t+1) < 1$*. There is no investment in dirty technology,* $x_B(t) = 0$*, all* $0 \leq t < t'$*.*

Proof. Suppose that $K_B(t) > 0$ is fully utilized, $\mathrm{u}_B(t) = 1$. Then either $x_B(t) = 0$ or

$x_B(t) > 0$. Dirty capacity $K_B(t+1)$ is zero by assumption all $t \geq 0$ if $x_B(t) = 0$. Lemma 2.1 implies that $\theta(t) \leq \theta_B \leq \theta(t+1)$ if investment in the dirty technology, $x_B(t) > 0$, and utilized dirty capacity, $\mathrm{u}_B(t) > 0$, precede underutilized dirty capacity, $\mathrm{u}_B(t+1) < 1$, which contradicts $\theta(t) > \theta(t+1)$. After capital is fully utilized either capital ceases to exist or is fully utilized. Thus partial utilization occurs early. Following investment in dirty technology, $x_B(t') > 0$, and utilized dirty technology capital $K_B(t') > 0$, dirty technology capital $K_B(t'+1) > 0$ is fully utilized, or dirty technology capital does not exist in all following periods. Then $x_B(t-1) = 0$ if $x_B(t) = 0$ all $t < t'$. *Q.E.D.*

Investment and partial utilization occur only once simultaneously, given investment occurs in the long-term. There may be no such date after underutilization (II-2)—which depends on the productivity of clean technology as will be shown below. There may be earlier dates with underutilization and without investment, and later dates with full utilization and investment. Dirty technology capital may be underutilized initially, be used up in finite time and become zero, and be built up later, in allocations without the joint investment and underutilization (II-2) and yet other allocations with it (II-3)—which depends on the effect of the pollution level on utility. The type of path depends on where the initial state is relative to the long-term state. I discuss these incentives after stating conditions that replace a statement on the endogenous θ in Proposition 2.3 in the following.

(i) Depreciation and clean technology productivity. Let the marginal rate of substitution of consumption in periods t and $(t+1)$ be $R(t+1) = (\partial U/\partial c)(t)/\beta(\partial U/\partial c)(t+1)$. In an optimum the foregone benefit of consuming dirty technology capital services, $\mathrm{u}_B(t) < 1$ and $K_B(t) > 0$, and the benefit of consuming them one period later, $\mathrm{u}_B(t+1) > 0$, are necessarily balanced by

$$\beta\gamma_B(1 - (b/B)\theta(t+1))\partial U/\partial c(t+1) = (1 - (b/B)\theta(t))\partial U/\partial c(t), \tag{2.9}$$

given $K_B(t+1) \in (0, \bar{K}_B)$.[17] Then $R(t+1)$ exceeds γ_B if and only if $\theta(t) > \theta(t+1)$. The "only if" statement given underutilization at $(t+1)$ cannot be inferred if dirty technology capital $K_B(t) > 0$ is fully utilized, or there is no dirty technology capital, $K_B(t) = 0$. In Proposition 2.3 there is utilized capital at some date, because this can be used to preclude

[17] Equalities in (2.6) and (2.8) imply that $\beta\gamma_j\{\lambda(t+1) - d_j\epsilon(t+1) - w_j(t+1)\} = \gamma_j q_j(t) = \lambda(t) - d_j\epsilon(t)$. If (2.2) is slack then $w_j(t+1) = 0$. The condition (2.9) follows by rearranging for $j = B$.

underutilization immediately following investment at this date given the assumed decrease in the cost of polluting. The latter presumption can be replaced by (i) a condition on depreciation if investing in dirty technology does not create emissions, $\rho_B = 0$, or (ii) sufficient productivity and scale of clean technology, to attain the same results.

Condition 2.1 *Pollution depreciates at smaller rate than unused dirty capacity, $\partial A/\partial Z < 1 - \gamma_B$ all Z.*

The Intergovernmental Panel on Climate Change (2007) reports that atmospheric carbon and other so-called greenhouse gases are persistent, which suggests that Condition 2.1 holds. This condition is formulated for all Z and thus is an exogenous object. The simulation in Section 2.3 satisfies it.

Condition 2.2 *$Q_C > \gamma_B(1 + (\rho_C/B)\theta_B)$ if $\rho_C \geq \rho_B$ and $Q_C > Q_B(1 - (b/B)\theta_B)$ if $\rho_C \leq \rho_B$. $\bar{K}_C$ is large.*

Under these parameters investment in clean technology is preferred to investing in dirty technology and underutilizing dirty technology capital in the next period. The simulations in Section 2.3 do not meet the large scale in Condition 2.2. Empirical work is needed to uncover the relations in the first part and the necessary scale for different pollution and dirty capacity levels. The first two conditions amount to greater productivity of clean technology investment than the preserved portion of unused dirty capacity, $Q_C > \gamma_B$, if investment in both the dirty and the clean technology is emission-free, $\rho_B = \rho_C = 0$. This case may be relevant only in the future, for example, if all steel is produced from scrap steel using the electric arc furnace rather than some steel is produced with coking coal that creates carbon dioxide emissions.[18] Condition 2.1 and $\rho_B = 0$, or Condition 2.2, rule out investment and succeeding underutilization for decreased or increased θ. The following two lemmata summarize results that lead to this conclusion. The first lemma uses results from the appendix. The second lemma exploits that the discounted marginal net benefit equals the marginal cost of investing $x_B(t) > 0$ in dirty technology,

$$Q_B(1 - (b/B)\theta(t+1))/R(t+1) = (1 + (\rho_B/B)\theta(t)), \tag{2.10}$$

[18]Interestingly, cement that uses limestone, a source of CO_2 emissions in investment, can be replaced increasingly by flyash of coal power plants or slag from coking coal of primary steel mills, in producing concrete.

if its capital $K_B(t+1) \in (0, \bar{K}_B)$ is utilized, $\mathrm{u}_B(t+1) > 0$.

Lemma 2.3 *Dirty technology capital* $K_B(t+1) \in (0, \bar{K}_B)$ *is fully utilized,* $\mathrm{u}_B(t+1) = 1$*, if there is emission-free investment in dirty technology,* $x_B(t) > 0$ *and* $\rho_B = 0$*, dirty technology capital* $K_B(t) > 0$ *is utilized,* $\mathrm{u}_B(t) > 0$*, and Condition 2.1 holds.*

Proof. Evaluating the result of Lemma 2.9 in the appendix when $x_B(t) > 0$, $\rho_B = 0$, and $\mathrm{u}_B(t+1) < 1$, shows that $R(t+1) \le \gamma_B$. This contradicts Lemma 2.8 in the appendix when $\mathrm{u}_B(t) > 0$ and $K_B(t+1) < \bar{K}_B$, which yields $R(t+1) > \gamma_B$. *Q.E.D.*

Lemma 2.4 *Dirty technology capital* $K_B(t+1) > 0$ *that is utilized,* $\mathrm{u}_B(t+1) > 0$*, is fully utilized,* $\mathrm{u}_B(t+1) = 1$*, if there is investment in dirty technology,* $x_B(t) > 0$*, and Condition 2.2 holds.*

Proof. (i) $\theta(t) \ge \theta(t+1)$. For $x_B(t) > 0$ and $\mathrm{u}_B(t+1) < 1$ Lemma 2.9 in the appendix implies that $R(t+1) \le \gamma_B$. Clean technology investment satisfies $BQ_C \le R(t+1)(B + \rho_C\theta(t)) \le \gamma_B(B + \rho_C\theta_B)$ at $\theta(t) \le \theta_B$ and $K_C(t+1) < \bar{K}_C$. Either

$$(1) \quad \rho_C\gamma_B/(\rho_B\gamma_B + Q_B b) < (Q_C - \gamma_B)/(Q_B - \gamma_B) \quad \Longleftrightarrow \quad BQ_C > \gamma_B(B + \rho_C\theta_B) \quad \text{or}$$
$$(2) \quad (\rho_B + b)\gamma_B/(\rho_B\gamma_B + Q_B b) < Q_C/Q_B \quad \Longleftrightarrow \quad BQ_C > Q_B(B - b\theta_B)$$

for (1) $\rho_C \ge \rho_B$ or (2) $\rho_C \le \rho_B$, respectively, imply that $BQ_C > \gamma_B(B + \rho_C\theta_B)$, a contradiction to the weak inequality in the reverse direction. (ii) $\theta(t) < \theta(t+1)$. Equation (2.10) holds for $x_B(t) > 0$ and $\mathrm{u}_B(t+1) > 0$. Thus $Q_B(B - b\theta_B) \ge Q_C(B + \rho_B\theta(t))/(B + \rho_C\theta(t))$. In case (1) $\rho_C[\theta_B - \theta(t)] \ge \rho_B[\theta_B - \theta(t)]$ yields $(B + \rho_B\theta(t))/(B + \rho_C\theta(t)) \ge (B + \rho_B\theta_B)/(B + \rho_C\theta_B)$ so that $BQ_C \le \gamma_B(B + \rho_C\theta_B)$. In case (2) the contradiction follows directly. Therefore capital $K_B(t+1) > 0$ is fully utilized. *Q.E.D.*

One may be cautious that clean technology is not sufficiently productive at large scale when $\rho_B > 0$. The cost of polluting may turn to θ_B from lower values after investment and full utilization in the dirty technology, because high-productivity clean technology has a small scale. Then the state moves from the region R1 $= \{(Z, K_B, K_C) \,|\, x_B > 0, \mathrm{u}_B = 1\}$ to the region R2 $= \{(Z, K_B, K_C) \,|\, x_B > 0, \mathrm{u}_B \in (0,1)\}$, if this exists, or to the region R3 $= \{(Z, K_B, K_C) \,|\, x_B = 0, \mathrm{u}_B \in (0,1)\}$, if R2 does not exist. I have found dampened oscillations in the cost of polluting in a LQ problem without such a return. In the model of a fishery in Clark et al. (1979) underutilization succeeds full utilization because capital does not vanish after it is fully utilized and investment lacks. This incentive does not exist here.

(ii) Clean technology productivity and disutility of pollution. Lemma 2.4 does not require dirty capacity $K_B(t) > 0$. Thus under Condition 2.2 the early underutilization in Proposition 2.3 extends to allocations (II-2 and II-3) with reinvestment in the dirty technology at a date at which $\theta(t) \leq \theta_B$ when there is temporarily no dirty technology capital, $K_B(t) = 0$.

Proposition 2.4 *Dirty technology capital $K_B(t) > 0$ is partially utilized, $\mathrm{u}_B(t) \in (0,1)$, only in an early time interval $\{0, 1, \ldots, t'-1\}$ and fully utilized later, $\mathrm{u}_B(t) = 1$ all $t > t'-1$, in an optimal plan in which (i) dirty technology capital $K_B(t) > 0$ is not idle, $\mathrm{u}_B(t) > 0$, and is zero after and before it is positive, $K_B(t) = 0$ all $t' < t \leq t''$, and $K_B(t) > 0$ all $0 \leq t \leq t'$ and $t > t''$, (ii) $\theta(t'') \leq \theta_B$, and (iii) Condition 2.2 holds. There is no investment in dirty technology, $x_B(t) = 0$, all $0 \leq t < t''$ on the path with underutilization.*

A proof is straightforward and thus omitted. The following discusses the allocations of Propositions 2.3 and 2.4 and how either one depends on the productivity of clean technology and the disutility of pollution. Allocations (II-1) that pass through or start in R2 are examples of Proposition 2.3. The economy may lack dirty technology capital following a period with fully utilized dirty technology capital, possibly after underutilization (in II-2), before the economy enters R1, as examples of paths in Proposition 2.4, if there is no such R2. To examine when there is a region R2, rewriting equation (2.5) as

$$R(t+1)\theta(t) = [(-\partial U/\partial Z)/(\partial U/\partial c)](t+1) + a(t+1)\theta(t+1) \tag{2.11}$$

is useful where $a(t+1) = 1 - \partial A/\partial Z(t+1)$. For any state in period t at which investing and underutilizing is optimal in the dirty technology, the equation

$$\frac{Q_B}{R(t+1)}\left[1 + \left(\frac{-\partial U/\partial Z}{\partial U/\partial c}\right)(t+1)\frac{d_B}{a(t+1)}\right] = 1 + \left(\rho_B + \frac{d_B Q_B}{a(t+1)}\right)\theta_B$$

from (2.9) and (2.11) at $\theta(t) = \theta_B$ gives the shadow return $R(t+1)$. This shadow return depends only on the successor state $(Z, K_B, K_C)(t+1)$. R2 is empty if the marginal return $Q_C/(1+(\rho_C/B)\theta(t))$ of investing in clean technology at $\theta(t)=\theta_B$ exceeds the level of $R(t+1)$ just identified when omitting clean technology from the model, though the optimal clean technology stock $K_C(t+1)$ is less than $\bar{K}_C$ (II-2). Sufficiently large productivity and scale of clean technology prohibit R2 so that there is no dirty capacity temporarily. For parameters that yield R2 dirty capacity may still vanish temporarily (II-3). R2 is characterized as follows, provided that it exists.

Proposition 2.5 *A region R2 in the state space with the policy of investment in dirty technology and partially utilized dirty technology capital $K_B(t)>0$, that is, $x_B(t)>0$ and $\mathrm{u}_B(t)\in(0,1)$, satisfies $(d_B-B/\theta_B)\partial v/\partial Z(t+1)=\gamma_B\partial v/\partial K_B(t+1)$. R2 extends to $\bar{K}_B$ as γ_B is small.*

Proof. The conditions $\epsilon=\theta_B\partial U/\partial c$ and $B\partial U/\partial c-d_B\epsilon=\gamma_B q_B$ hold in R2. The shadow prices of states are the differentials of the value function with respect to the state, $\epsilon(t)=-\beta\partial v/\partial Z(t+1)$ and $q_B(t)=\beta\partial v/\partial K_B(t+1)$. See, for example, Sargent (1987). Then $(d_B-B/\theta_B)\partial v/\partial Z(Z',K_B',K_C')=\gamma_B\partial v/\partial K_B(Z',K_B',K_C')$ describes a hyperplane in the state space that is reached from any state in R2, letting prime denote next period values. The utilization rate is $\mathrm{u}_B(t)\in[\mathrm{u}^*,1]$ for some $\mathrm{u}^*\in(0,1)$ at which the utilization rate $\mathrm{u}_B(t)$ is minimized in R2. Since $K_C'=Q_C x_C$ there is a system of four equations—the relationship of c and Z from $\epsilon=\theta_B(\partial U/\partial c)(c,Z)$ at ϵ that satisfies (2.5), which depends on the policy c' given (Z',K_B',K_C') and on Z' in general, the resource constraint, and the laws of motion—in the five unkowns consumption c, input x_B in investment and the utilization rate u_B in the dirty technology, pollution Z, and dirty capacity K_B, given clean capacity K_C. The input $x_C\in(0,\bar{K}_C/Q_C)$ has to be determined through equating $Q_C/(1+(\rho_C/B)\theta_B)$ to the shadow return $R(t+1)$ or $x_C=\{0,\bar{K}_C/Q_C\}$. In special cases the choice of x_C may be obvious. The system determines a state manifold, which is the boundary of R1 and R2, for $\mathrm{u}_B=1$, and yields a state manifold, which forms the boundary of R2 and R3, for $x_B=0$. As γ_B shrinks the dirty capacity K_B becomes arbitrarily large when $x_B=0$. *Q.E.D.*

Clean technology capital may be installed exclusively when dirty technology capital is used up following an interval of fully utilized dirty technology capital, because of high disutility from pollution at large pollution levels (II-3). This incentive prevails too in the constrained optimum in which capital is assumed to be fully utilized.[19] Other allocations

[19]The incentive does not prevail if the sequence of ϵ is roughly constant, for example, if the marginal

Exclusive long-term use of clean technology	
I-1	$(0,0),(0,0),\ldots$
I-2	$(0,\mathrm{u}),(0,0),(0,0),\ldots$
I-3	$(0,\mathrm{u}),\ldots,(0,\mathrm{u}),(0,1),(0,-),(0,-),\ldots$
I-4	$(x,1),\ldots,(x,1),(0,1),(0,-),(0,-),\ldots$
Long-term use of dirty technology	
II-1	$(0,\mathrm{u}),\ldots,(0,\mathrm{u}),(x,\mathrm{u}),(x,1),(x,1),\ldots$
II-2	$(0,\mathrm{u}),\ldots,(0,\mathrm{u}),(0,1),(0,-),\ldots,(0,-),(x,-),(x,1),(x,1),\ldots$
II-3	early interval as in II-1 until some $(x,1)$ succeeded by $(0,1),(0,-),\ldots,(0,-),(x,-),(x,1),(x,1),\ldots$
II-4	$(x,1),(x,1),\ldots$

Note: Here $K_B(0)>0$, $K_B(t)=0$ for $t\geq 1$ is indicated by "$-$", and x and u stand for interior values, $x_B>0$ and $\mathrm{u}_B\in(0,1)$, respectively.

Table 2.1: *Sequences of investment and utilization* (x_B,u_B) *in dirty technology on* $\{t,t+1,\ldots\}$.

may pass through R2 and remain in R1 for the same parameter values. Low levels of pollution may be on such paths. Table 2.1 summarizes possible sequences of dirty technology investment and utilization, which themselves or of which terminal subsequences may characterize an optimal plan.

Timing of investment and utilization in clean technology.—The analogue to the balancing condition (2.9) for dirty technology is

$$\gamma_C = R(t+1) \tag{2.12}$$

for clean technology, if $\mathrm{u}_C(t)<1$ and $\mathrm{u}_C(t+1)>0$, because using clean technology capital $K_C(t)>0$ or $K_C(t+1)\in(0,\bar{K}_C)$ is not environmentally costly. I argue with Assumption 2.2 that clean technology capital may only be underutilized if dirty technology capital is idle or is not installed. The second result in the following lemma is useful for this. The first result shows that the environmental cost is necessary for underutilizing clean capacity that can be created at a productivity that is greater than γ_C.

utility of pollution $\partial U/\partial Z$ and the marginal absorption $\partial A/\partial Z$ are nearly constant. In optimum with underutilized dirty technology capital in (II-3) $x_B(t)=0$ all $t<t'-1$ and $t^*\leq t<t''$, $x_B(t)>0$ all $t'-1\leq t<t^*$ and $t\geq t''$, $\mathrm{u}_B(t)\in(0,1)$ all $t<t'$ and $\mathrm{u}_B(t)=1$ all $t\geq t'$ when $K_B(t)>0$. Dirty technology capital is zero from t^*+1 to t''.

Lemma 2.5 *Clean technology capital $K_C(t) > 0$ is fully utilized,* $\mathrm{u}_C(t) = 1$, *if (i) the return on emission-free investment in clean technology exceeds the return from storing its unused capital, $Q_C > \gamma_C$ and $\rho_C = 0$, or (ii) the marginal rate of intertemporal substitution of consumption $R(t+1)$ is greater than γ_C.*

Proof. (i) The condition (2.8) reads $\lambda(t) - d_j\epsilon(t) \leq \gamma_j q_j(t)$ if $\mathrm{u}_j(t) < 1$. Combination with the necessary condition (2.7) of investment yields $Q_j(\lambda(t) - d_j\epsilon(t)) \leq \gamma_j Q_j q_j(t) \leq \gamma_j(\lambda(t) + \rho_j\epsilon(t))$ where $\gamma_j > 0$. The first result follows from there by contradiction for $d_C = \rho_C = 0$. (ii) Since idling and storing capital forever cannot be optimal some t exists such that $\mathrm{u}_C(t) > 0$. Utilization of capital $K_C(t) > 0$ at period t, that is, $\mathrm{u}_j(t) > 0$, requires that $\beta\{\lambda(t) - d_j\epsilon(t) - w_j(t)\} = q_j(t-1)$. Analogous reasoning to (i) for underutilization in the preceding period, $\mathrm{u}_j(t-1) < 1$, implies that $\lambda(t-1) - d_j\epsilon(t-1) \leq \gamma_j q_j(t)$. Therefore $\beta\gamma_j(\lambda(t) - d_j\epsilon(t)) \geq \lambda(t-1) - d_j\epsilon(t-1)$ yields the result. *Q.E.D.*

Using output to create new capital units without emissions, $\rho_C = 0$, is superior to forwarding unused capital if Q_C exceeds the storage return γ_C. Consuming the capital services at t is preferred to such storage if the shadow return $R(t+1)$ is greater than the return γ_C from storing the capital. The proof of Lemma 2.5 hints that clean technology capital may be mothballed if the creation of new capital units stresses the environment, $\rho_C > 0$, or its intrinsic return on investment, Q_C, is low so that the discounted return $Q_C/(1 + (\rho_C/B)\theta(t))$ from investing does not exceed the rate of return γ_C from storage, and in addition the shadow return $R(t+1)$ is not greater than γ_C.[20] The shadow return is greater than γ_B if Condition 2.1 holds and dirty technology capital is utilized. This leads to the following proposition.

Proposition 2.6 *Clean technology capital $K_C(t) > 0$ is underutilized,* $\mathrm{u}_C(t) < 1$, *only if dirty technology capital $K_B(t) > 0$ is idle,* $\mathrm{u}_B(t) = 0$, *or there is no dirty technology capital, $K_B(t) = 0$, if Condition 2.1 holds.*

Proof. The shadow return $R(t+1)$ exceeds γ_B if dirty capacity $K_B(t) > 0$ is utilized, $\mathrm{u}_B(t) > 0$, and Condition 2.1 holds. Clean capacity $K_C(t) > 0$ is fully utilized, $\mathrm{u}_C(t) = 1$, because $\gamma_C \leq \gamma_B < R(t+1)$ using Lemma 2.5. A contrapositive yields the result. *Q.E.D.*

[20]Reducing dirty output below dirty capacity to store capital is not preferred to spending output to invest in dirty technology if the marginal utility of pollution was zero in all future periods, since the return rate γ_B to storage is less than the marginal product Q_B of investment in technology B.

Long-term use of clean technology	
III-1	$(0, \mathrm{u}), \ldots, (0, \mathrm{u}), (x, \mathrm{u}), (x, 1), (x, 1), \ldots$
III-2	$(0, 1), (0, -), (0, -), \ldots, (0, -), (x, -), (x, 1), (x, 1), \ldots$
III-3	$(x, 1), (x, 1), \ldots$
Exclusive long-term use of dirty technology	
IV-1	$(0, 1), (0, -), \ldots, (x, -), (x, 1), \ldots, (x, 1), (0, 1), (0, -), (0, -), \ldots$
IV-2	$(0, 1), (0, -), (0, -), \ldots$

Note: Here $K_C(0) > 0$, $K_C(t) = 0$ for $t \geq 1$ is indicated by "$-$", and x and u stand for interior values, $x_C > 0$ and $\mathrm{u}_C \in (0, 1)$, respectively.

Table 2.2: *Sequences of investment and utilization* (x_C, u_C) *in clean technology on* $\{t, t+1, \ldots\}$.

The shadow return may exceed γ_B if Condition 2.1 does not hold, and dirty technology capital is utilized, leading to the same result as in Proposition 2.6.

Clean technology capital may be mothballed if dirty technology use is abandoned at $t = 0$. The only case in Table 2.2 that agrees to this is (I-1). The low shadow return from the Hotelling rule (2.12) means that then initial clean capacity is large relative to its long-term level. Table 2.2 summarizes the tiandng of investment and utilization in a clean technology. (III-1) matches (I-1). (III-2) agrees to (I-4). (III-3) matches (I-1 to I-3) for large scale $\bar{K}_C$ and (II-1 to II-4) for small scale $\bar{K}_C$. (IV-1) is consistent with (II-2) and (II-3). (IV-2) matches (II-1 and II-4). I refer to the discussion of dirty technology investment and utilization for the sets of the parameters beside $\bar{K}_C$ in each case.

Exclusive clean technology use in the long-term requires that $Q_C > \beta^{-1} > \gamma_C$. Thus the environmental cost in the construction of solar panels or wind turbines induces under-utilization in the clean technology that is used in the long-term. In Fischer et al. (2004) clean technology capital is built more than one period before it is used when pollution is below its long-term level because of diminishing returns to scale in its investment and a user flow cost. Renewable energy technologies do not seem to have significant operation and maintenance cost. Dirty technology does not use capital in their model. Diminishing returns of renewable energy may result from manufacturing and locating capital in geographic sites of different productivity, which can be summarized in the notion of multiple clean technology types.

2.1.2 Multiple clean technology types

The section discusses the utilization of multiple clean technology types, implications of underutilized dirty technology capital for the types of invested clean technologies, and a sufficient condition on the productivity of multiple clean technologies for the timing of the utilization of dirty technology capital.

Exclusive long-term use of clean technology.—Decreasing returns in building aggregate capital induce postponing the use of low marginal product capital. For example, each renewable energy technology is widely applicable in the world, yet at different real cost across locations of finite size. A similar effect is waiting until the marginal product of using installed capital has increased in Arrow & Kurz (1970*a*) because capital depreciates. A difference to this phenomenon in the one-sector growth model here is that investment occurs—in sites or engineering systems with high marginal product.[21] This incentive exists if investing has no environmental cost and is strengthened by the environmental cost.

The shadow return lies between the marginal returns on investment in technologies C' and C such that $Q_{C'}/(1+(\rho_{C'}/B)\theta(t)) \geq R(t+1) = \gamma_C > Q_C/(1+(\rho_C/B)\theta(t))$ if there is clean technology capital in low-productivity sites or of expensive make to begin with, $K_C(0) > 0$. Then it is optimal to not invest, and to store capital, $x_C(0) = \mathrm{u}_C(0) = 0$, in low-productivity sites or technologies and simultaneously invest and fully utilize existing capital in high-productivity sites or technologies, $x_{C'}(0) > 0$ and $\mathrm{u}_{C'}(t) = 1$. In such a plan dirty capacity is idle initially and possibly infinitely. Highly productive clean technology, that is, $Q_C > \beta^{-1}$ at sufficiently large scale $\bar{K}_C$, is necessary to sustain consumption that keeps $\theta \geq B/b$ all time, which prevents the utilization of dirty technology capital $K_B(0)$.[22]

[21] The incentive to save much is met by investment in high-productivity types and postponing the use of low-productivity types. The usefulness of capital only in one period rules out that investment lacks because past investments have been irreversible. Thus the environmental motive is solely responsible for zero investment in any technology.

[22] A growing literature considers making cleaner technology more productive, potentially reversing the relationship between minimum Q_C and maximum Q_B. Chakravorty, Roumasset & Tse (1997), in a partial equilibrium model in the spirit of Nordhaus (1973), have been overly optimistic in assuming functional forms about deterministic and exclusive progress in clean technology that predicted counterfactual reduction in greenhouse gas emissions. Edenhofer et al. (2005) posit learning-by-doing for clean technology only. Hartley et al. (2010) devise a learning effect to make a renewable resource technology more productive over time. Van Zon and Lontzek (2006) find that a carbon tax diverts research effort from clean renewables to a polluting technology through directed technical change, if there is no research funding policy that favours clean technology. Acemoglu et al. (2012) document the optimal paths of

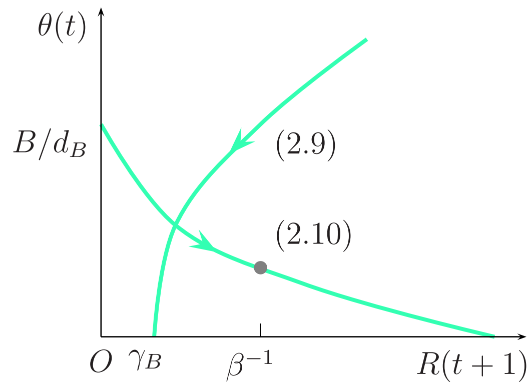


Figure 2.1: *Shadow return and cost of polluting with underutilization in case (II-1).*

There is capacity $K_C(0)$ such that $Q_B > Q_C$ at the initial date zero of optimization if governments have pushed expensive clean technologies prior to this date. It would be fully utilized in a laissez-faire equilibrium with $Q_B = R(t+1) > \gamma_C$.

Long-term use of dirty technology.—The shadow return R follows a V-shaped sequence on a path beginning with partially utilized dirty technology capital and with long-term use of this technology, for example, when the environmental disutility $(-\partial U/\partial Z)$ and marginal absorption $\partial A/\partial Z$ are constant or if Condition 2.1 holds. In this specification the shadow cost of polluting is constant. Under this condition $\epsilon(t+1)/\epsilon(t) < (\beta\gamma_B)^{-1}$, which is sufficient for the trough of the shadow return. Figure 2.1 plots the equations (2.9) and (2.10) using the identity $R(t+1)(\beta\epsilon(t+1)/\epsilon(t))\theta(t) = \theta(t+1)$ subject to $\epsilon(t) = \epsilon(t+1)$.[23] The shadow return varies positively with the marginal return $Q_C/(1+(\rho_C/B)\theta(t))$ of the marginal clean technology in which investment occurs since

$$Q_C/R(t+1) \geq (1+(\rho_C/B)\theta(t)) \quad \text{if} \quad K_C(t+1) > 0, \tag{2.13}$$

at equality for $K_C(t+1) \in (0, \bar{K}_C)$. The time path of $R(t+1)$ and $\theta(t)$ in an optimal plan (II-1) follows the indicated flow if capacity of clean technology types is bounded by sufficiently low $\bar{K}_C$. Capacity in sufficiently productive clean technologies expands to their bounds in any given period. Then investment in increasingly costly clean technologies (low Q_C, high ρ_C) is efficient when dirty technology capital is underutilized, followed by

an energy tax rate and a non-energy good research subsidy in a calibrated model of directed technical change with an exogenous rate of progress.

[23]In other specifications the change in the shadow price of pollution affects the curves.

backing investment out of expensive clean technologies, when dirty technology capital is fully utilized. The time path of $R(t+1)$ and $\theta(t)$ would follow the downward-sloping curve if dirty capacity was assumed to be fully utilized. Thus the possibility of underutilization of dirty technology capital lowers the incentives to invest in expensive clean technologies if the marginal Q_C for large aggregate scale in clean techology types is low, which seems currently plausible in the climate problem. The intersection of the curves in Figure 2.1 marks the turning point of $R(t+1)$ when the economy is in R2 at date t. The shadow return R is bounded from below at a level greater than this turning level if the clean technologies' minimum Q_C and its associated scale $\bar{K}_C$ are sufficiently large. Then dirty technology capital is temporarily absent after the economy was in R3 (II-2).

Suppose that ρ_C is equal all $C \in \{j \,|\, d_j = 0\}$ and let Q'' be the minimum Q_C. Proposition 2.3 follows given $Q'' > \gamma_B(1 + (\rho_C/B)\theta_B)$ if $\rho_C \geq \rho_B$ and $Q'' > Q_B(1 - (d_B/B)\theta_B)$ if $\rho_C \leq \rho_B$ all $B \in \{j \,|\, d_j > 0\}$, and given large aggregate capacity of clean technologies.

The next section turns to decision-making private agents and a government. Before I collect the optimal policies in a given period.

Consumption.—Postponing the use of capital or investing each maximizes the shadow return. In view of (2.9) and (2.10) some dirty technology capital is forwarded rather than used to produce new capital when this offers the greater shadow return, and vice versa. At a state in R2 reached in period t both the conditions (2.9) and (2.10) hold, which solve for $\theta(t) = \theta_B$.[24] Investment in clean technology may offer an even greater return. Dirty technology capital is idle, if this greater return prevails at large scale of clean technology and clean technology types are exclusively used in the long-term at sufficiently large θ. As a fourth option, postponing the use of clean technology capital is optimal if this maximizes the shadow return. Investing in high-productivity clean technology types and mothballing low-productivity clean technology types can be optimal. The shadow return $R(t+1)$ depends positively on $c(t+1)$ and negatively on $c(t)$. Thus any policy maximizes consumption growth, given pollution at t and $(t+1)$ at optimal values.

[24]The shadow price of capital is negative at states in R3 with large dirty technology capital if capital is constrained to be fully utilized. The constraint $K_B(t+1) \leq Q_B x_B(t)$ is binding in R1 and $q_B(t) \geq 0$. The constraint $K_B(t+1) \geq Q_B x_B(t)$ would be binding at states in R3 and $q_B(t) \leq 0$.

2.2 Decentralized decision-making

In each period there is a government that announces a tax and transfer policy for the current and all future periods. Emissions are taxed and the proceeds returned lump-sum to households. Actual policy may be delayed such that the announcement comprises zero tax and transfer rates before an exogenous date t^*.[25] Firms own productive capital, and households own claims to profits of firms. In an equilibrium in which firms make zero economic profit using asset-market decentralization all agents may trade a single asset. However, firms earn a differential (Ricardian) rent if the aggregate production is constrained by (2.2) because, for simplicity, there is no resource ownership that could absorb the rent.[26] Firms may use capital in dirty and clean technologies at different proportions. Thus there are financial assets specific to firms.[27] With reference to energy production the general factor is *net energy* or *useful energy* as, for example, defined in Erdmann & Zweifel (2008) and Bhattacharyya (2011), respectively.

Households.—All households have an equal endowment $\alpha_i(0)$ of tradable equity of firm i for simplicity. This assumption and equal preferences imply that each household is representative of all households. The price of the claim to firm i's profits is q_i, and the dividend it pays is d_i. One unit of the consumption good costs $\hat{p}$ units of account. The representative household receives a nonnegative transfer tr from the government. The household chooses consumption $c(t)$ and number $\alpha_i(t+1)$ of claims all $t \geq \tau$ to maximize

$$J_\tau = \sum_{t=\tau}^{\infty} \beta^{t-\tau} U(c(t), Z(t))$$

subject to the sequence of budget constraints

$$\hat{p}(t)c(t) + \sum_i q_i(t)\alpha_i(t+1) \leq \sum_i (q_i(t) + d_i(t))\alpha_i(t) + tr(t)$$

on $\{\tau, \tau+1, \ldots\}$ taking all prices, dividends, the transfer, and pollution as given. The

[25] Acemoglu et al. (2012) assume an exogenous delay and do not formalize the announcement.

[26] There is a similar rent when fossil fuel deposits are simultaneously extracted at different marginal costs which forms the basis for Harstad's (2012) proposal to acquire coal deposits to implement an optimum under spillovers.

[27] An alternative would be distinct dirty and clean technology producers and two types of assets, private equity of dirty technology users, and public equity of clean technology producers, without affecting results.

analysis of delayed government policy requires a belief of private agents. Let $\hat{\tau}$ be the emissions tax rate in the set of policy instruments $\pi = (\hat{\tau}, tr)$. For simplicity households and firms do not anticipate any change in the government's policy function $\{0\} \cup \mathbb{N} \rightarrow \{\pi(t), \pi(t+1)\ldots\}$ before the date at which this policy function changes. Given a change occurs once at $t^* \in [0, \infty)$ the representative household (and any firm) either makes one plan, if $t^* = 0$, or reoptimizes at date $t^* > 0$, so that $\tau \in \{0, t^*\}$.[28]

Firms.—The number $\hat{\alpha}_i$ of equity of firm i is variable throughout the planning horizon. Firms can pay out and issue shares. Otherwise firms could not pause producing and own capital between any two periods $t < t' - 1$ and t' in an equilibrium with nonnegative profit of all firms in every period. This would be an unnecessarily strong assumption. All firms have access to the same technologies $\mathcal{J}$ to produce energy $m_j \in [0, K_j]$, technologies to convert x_j energy units into $Q_j x_j$ investment goods, and one technology that uses x energy units to produce consumption goods amount Bx. Firms may utilize assets differently in equilibrium. Firm i may be a representative firm that is active in all sectors without change in notation. Then in an equilibrium all firms utilize capital at the same rate. Firm i's available capital in technology j follows the law of motion

$$K_{ij}(t+1) = \gamma_j(1 - \mathrm{u}_{ij}(t))K_{ij}(t) + I_{ij}(t) \tag{2.14}$$

given chosen utilization rate $\mathrm{u}_{ij}(t) \in [0, 1]$. Firm i demands I_{ij} new capital units, x_i energy units, and sells $\hat{I}_{ij} = Q_j x_{ij}$ new capital units using the proportion $\eta_{ij} = x_{ij}/x_i$ of its factor demand. It sells the energy amount $m_{ij} = \mathrm{u}_{ij} K_{ij}$, and consumption goods amount $c_i = B(1 - \sum_j \eta_{ij})x_i$. The industry capacity constraint is

$$\bar{K}_j \geq \sum_i K_{ij}(t+1) \tag{2.15}$$

all $t \geq 0$ for technology $j \in \mathcal{J}$. Equipment of technology j trades at unit price p_j. The price per unit of energy is p. Any firm is obliged to pay $\hat{\tau}$ units of account per emission unit that the firms' processes have created. Profit from using and investing in

[28]Firms may still build capital that is efficiently underutilized if private agents rationally expect a change in the course of government policy toward implementation of a Pareto optimum. This belief would require an equilibrium that connects necessary equilibrium conditions of different regimes through shadow prices of these regimes, because the allocation prior to t^* generally differs over t^* for the same initial conditions.

the dirty and clean technologies is $\Pi_{ij} = (p - d_j\hat{\tau})m_{ij} - p_j I_{ij}$ in period $t \geq 0$. Production of the consumption good, and of investment goods, using energy creates profit $\hat{\Pi}_i = (\hat{p}B - p)(1 - \sum_j \eta_{ij})x_i + \sum_j (p_j Q_j - (p + \rho_j \hat{\tau}))x_{ij}$. Firm i's profit net of equity trade

$$\Pi_i(t) = \sum_{j \in \mathcal{J}} \Pi_{ij}(t) + \hat{\Pi}_i(t) + q_i(t)[\hat{\alpha}_i(t+1) - \hat{\alpha}_i(t)] \tag{2.16}$$

in period $t \geq 0$ sums profits from the sectors for the factor energy, consumption goods, and investment goods, and the trade surplus. Each firm i chooses input demands and output supplies, and number $\hat{\alpha}_i(t+1)$ of equity, on $\{\tau, \tau+1, \ldots\}$ to maximize the present discounted value of ex-dividend profits

$$v_{i\tau} = \sum_{t=\tau}^{\infty} \frac{1}{\prod_{v=\tau}^{t} \hat{R}(v)} \{\Pi_i(t) - \hat{\alpha}_i(t) d_i(t)\}$$

subject to (2.14) and (2.15) all $t \geq \tau$ taking prices, government policy rates, and the endogenous nominal interest rate sequence $\{\hat{R}(1), \hat{R}(2), \ldots\}$ as given, where $\hat{R}(0)$ is some given positive number, and $\tau \in \{0, t^*\}$.[29] Any household or firm may offer an asset that promises $\hat{R}$ units of account return, and there is no trade of this asset in equilibrium.

Government.—A government in period $t \geq 0$ sets tax rates and lump-sum transfers $\{\hat{\tau}(t'), tr(t')\}$ for all dates $t' \geq t$ before private agents make decisions about demands and supplies at date t. The taxes and subsidies appear in a government's budget constraint

$$tr \leq \hat{\tau} \sum_i \sum_{j \in \mathcal{J}} (d_j m_{ij} + \rho_j x_{ij}) \tag{2.17}$$

all $t \geq 0$. The exogenous nature of government policy is helpful in motivating delayed policy. The tax on polluting activities in all periods that implements a Pareto optimal allocation is time-consistent. Thus if the government (or successive governments) would maximize welfare J using taxes on polluting activities and lump-sum transfers and no other instruments then equilibrium government policy was not delayed.

[29] Total dividends is $\Pi_i(t) = \hat{\alpha}_i(t) d_i(t)$ equal to total profit all $t \geq 0$ given outstanding equity $\hat{\alpha}_i(t)$. For example, a firm using clean technology j makes zero total profit from buying equipment at cost $p_j(t) I_{ij}(t) = q_i(t)\hat{\alpha}_i(t+1)$ in period t, and pays out dividend $d_i(t+1)\hat{\alpha}_i(t+1)$ and 'buyback' $q_i(t+1)\hat{\alpha}_i(t+1)$ in period $(t+1)$, in sum equal to revenue $p(t+1) I_{ij}(t)$. The equity of this firm in period $(t+2)$ may be zero, if the firm does not reinvest. Firms may maximize the discounted value of dividends taking as given $\hat{\alpha}_i(t+1)$ all $t \geq 0$ without change in results.

Equilibrium.—Markets sequentially open and close over time so that households and firms can revise decisions when government policy begins. Demand equals supply on the goods markets in a given period, if

$$\sum_i x_i = \sum_i \sum_{j \in \mathcal{J}} m_{ij}, \quad \sum_i I_{ij} = \sum_i \hat{I}_{ij}, \quad j \in \mathcal{J}, \quad c = \sum_i c_i,$$

for the general factor, investment goods, and the consumption good, respectively. An equilibrium is a system of prices $(p, \hat{p}, p_B, p_C, \hat{R})$ and $\{q_i\}_{\forall i}$, quantities of demands and supplies, and equity α_i and $\hat{\alpha}_i$, and government policy π on $\{0, 1, \ldots\}$ such that (i) the representative household and all firms solve their problems taking prices and government policy variables as given, (ii) the government satisfies its budget constraint all $t \geq 0$, (iii) the law of motion of pollution is $Z(t+1) = Z(t) + \sum_i \sum_{j \in \mathcal{J}} (d_j m_{ij} + \rho_j x_{ij}) - A(Z(t))$ all $t \geq 0$, and (iv) demand equals supply on the goods markets and securities holdings $\alpha_i(t)$ of households equals issued equity $\hat{\alpha}_i(t)$ all i, all $t \geq 0$. The following conditions are useful to characterize an equilibrium without taxes or subsidies and to determine government policy that implements a Pareto optimum.

A necessary condition for the profit-maximizing choice of capacity $K_{ij}(t+1)$ of technology $j \in \mathcal{J}$ is

$$\begin{aligned}(1/\hat{R}(t+1))(\mathrm{u}_{ij}(t+1)\{p(t+1) - d_j \hat{\tau}(t+1)\} \\ + \gamma_j(1 - \mathrm{u}_{ij}(t+1))\upsilon_{ij}(t+1) - \hat{w}_j(t+1)) \leq \upsilon_{ij}(t)\end{aligned} \tag{2.18}$$

at shadow prices υ_{ij} of (2.14) and $\hat{w}_j$ of (2.15). The condition holds at equality if the respective capacity is positive. The shadow price $\hat{w}_j(t)$ is zero if capacity $K_j(t)$ is less than the bound $\bar{K}_j$. Then utilization of the capacities at $(t+1)$ of firm i satisfies

$$\begin{aligned}\mathrm{u}_{ij}(t+1)\left\{\begin{array}{l} = 1 \\ \in (0,1) \\ = 0 \end{array}\right\} \quad \Longrightarrow \quad & \frac{1}{\hat{R}(t+1)}\{p(t+1) - d_j \hat{\tau}(t+1)\} \\ & \left\{\begin{array}{c} = \\ = \\ \leq \end{array}\right\} \upsilon_{ij}(t) \left\{\begin{array}{c} \geq \\ = \\ = \end{array}\right\} \frac{1}{\hat{R}(t+1)} \gamma_j \upsilon_{ij}(t+1)\end{aligned} \tag{2.19}$$

given $K_{ij}(t+1) \in (0, \bar{K}_j)$ for $t \geq 0$. The outer relations at weak inequalities are relevant regarding utilization at period $t = 0$. The price-taking assumption should only hold if individual capacity is smaller than maximum capacity. Equilibrium investment $I_{ij}(t) \geq 0$ in technology $j \in \mathcal{J}$ satisfies $\upsilon_{ij}(t) \leq p_j(t)$ at equality if $I_{ij}(t) > 0$ for some firm i. The shadow prices υ_{ij} are identical for all firms i that use the technology $j \in \mathcal{J}$. Choices of the consumption goods supply and the energy input demand in producing the consumption good imply that $\hat{p}(t)B = p(t)$ since consumption goods are produced in equilibrium. The necessary equilibrium condition $Q_j p_j(t) \leq p(t) + \rho_j \hat{\tau}(t)$ holds at equality if at least one firm i produces a positive quantity $Q_j x_{ij}$ of the investment good of technology $j \in \mathcal{J}$.

The next section contrasts the allocation in a competitive equilibrium without taxes or subsidies (laissez-faire equilibrium) to a Pareto optimal outcome.

2.2.1 Laissez-faire equilibrium

The policy announcements are $tr(t) = \hat{\tau}(t) = 0$ and $t \geq 0$. Firm i fully utilizes capacity $K_{ij}(t) > 0$ all $j \in \mathcal{J}$ since marginal profit per unit of output, $p(t)$, is greater than $\gamma_j \upsilon_{ij}(t)$ all $t \geq 0$.

Proposition 2.7 *Capacity $K_{ij}(t) > 0$ is fully utilized,* $\mathrm{u}_{ij}(t) = 1$, *in each firm i and all technologies $j \in \mathcal{J}$ in a laissez-faire equilibrium.*

Proof. The result follows from $p \geq Q_j p_j \geq Q_j \upsilon_{ij} > \gamma_j \upsilon_{ij}$ if $Q_j > \gamma_j$. This is true for the dirty technology. Suppose that clean technology capital is underutilized at the initial date. First $p(1)/\hat{R}(1) \leq \upsilon_{i\mathrm{C}}(0)$. Any available capital cannot be unused forever since the market price of output is positive. Then ${\gamma_C}^{t-1} \prod_{v=1}^{t} p(v)/\hat{R}(v)p(v-1) = \upsilon_{i\mathrm{C}}(0)/p(0)$ given date t of utilization for a clean technology with $Q_C \leq \gamma_C$. Underutilization at date zero cannot occur if each real interest rate $p(v)/\hat{R}(v)p(v-1)$ exceeds one. In the long-term, the real rate of interest approaches β^{-1} because both consumption and pollution converge. Consumption may temporarily decrease if the marginal rate of substitution of consumption depends on pollution. But the product of interest rate factors approaches a value greater than one. Thus clean technology capital is fully utilized all time. *Q.E.D.*

Aggregate dirty capacity $\sum_i K_{iB}$ reaches its upper bound $\bar{K}_B$ in finite time. Without emissions pricing investment occurs in the most productive technology until its capacity bound is reached. Then investment starts in the second most productive technology until exhaustion, and so on.[30] Investment in a clean technology with marginal product Q_C slightly smaller than β^{-1} or $Q_C \in [\beta^{-1}, Q_B)$ may occur while the dirty technology exhausts its capacity constraint. Investment at such productivity level depends on how pollution affects the marginal utility of consumption.[31]

Pollution is greater in some period t' than in period $(t'-1)$ if emissions $E(t) = Z(t+1) - Z(t) + A(Z(t))$ have increased long enough. The concave regeneration capacity A implies that if emissions are sufficient to generate an increase in pollution, and emissions do not decrease, then pollution increases, which Lemma 2.10 in the appendix summarizes. This lemma is useful to characterize pollution in the long-term if emissions increase.

Lemma 2.6 *Pollution approaches the level Z that solves $A(Z) = (b + \rho_B/Q_B)\bar{K}_B + (\rho_C/Q_C)K_C$ if $\partial^2 U/\partial c \partial Z \geq 0$, or $\partial^2 U/\partial c \partial Z < 0$ and the effect of consumption dominates the effect of pollution on the marginal utility of consumption, in a laissez-faire equilibrium. In particular, $\lim_{t\to\infty} \sum_i K_{i\mathrm{C}}(t) = K_C$ equals $\bar{K}_C$ if $Q_C > \beta^{-1}$, or $Q_C = \beta^{-1}$ and $\partial^2 U/\partial c \partial Z \geq 0$, and zero else.*

Proof. (i) $Q_C \geq Q_B$. Investment in dirty technology occurs in any period t only if clean capacity is at its upper bound in period $(t+1)$. On an interval with investment in dirty technology below maximum amount $\bar{K}_B/Q_B$ the real rate of return equals Q_B. Then dirty capacity increases because consumption growth is only sustainable with growth in dirty output. Growth leads to an increasing emissions sequence, $E(0) < E(1) < \ldots < E(t'')$, and continues until the capacity constraint (2.2) binds for the dirty technology. Afterwards emission is constant, $E(t'') = E(t''+s)$, $s \geq 1$. Induction implies that pollution increases if $\partial U^2/\partial c \partial Z \geq 0$. If pollution raises the marginal utility of consumption, $\partial^2 U/\partial c \partial Z > 0$, then greater uncontrolled increase in pollution accelerates growth of consumption and dirty technology capital. If pollution lowers the marginal utility of consumption, $\partial^2 U/\partial c \partial Z < 0$, then seemingly greater uncontrolled increase in pollution may eventually halt growth before K_B exhausts the carrying capacity $\bar{K}_B$ of the economy, respectively. The domination rules this out. (ii) $Q_C \in [\beta^{-1}, Q_B)$. Suppose that

[30] Given full utilization the condition (2.18) becomes $(1/\hat{R}(t+1))p(t+1)Q_j \geq p(t)$ if $\sum_i K_{ij}(t+1) = \bar{K}_j$, and equality if aggregate capacity is interior, $\sum_i K_{ij}(t+1) \in (0, \bar{K}_j)$.

[31] The value of the real interest rate at which consumption stays constant is greater than (equals, is smaller than) β^{-1} given pollution increases, if pollution lowers (does not affect, raises) the marginal utility of consumption. This suggests that if $\partial U^2/\partial c \partial Z > 0$ and Q_C is slightly smaller than β^{-1} then investment in clean technology occurs one period before the exhaustion date of dirty capacity, and consumption decreases later to its long-term level when the real interest rate becomes greater than Q_C.

$\lim_{t\to\infty} K_C(t) < \bar{K}_C$. Then as t tends to infinity $(1/\hat{R}(t+1))p(t+1)Q_C > p(t)$ since $(\partial U/\partial c)(t)/\beta(\partial U/\partial c)(t+1) = 1/\beta = \hat{R}(t+1)p(t)/p(t+1)$. This contradicts investment below maximum level. In the case $Q_C = \beta^{-1}$ at constant consumption and increasing pollution $\partial^2 U/\partial c\partial Z \geq 0$ is required to make investment in clean technology worthwhile. Otherwise, $\partial^2 U/\partial c\partial Z < 0$, the real interest rate exceeds the reciprocal of the discount factor and thereby $(1/\hat{R}(t+1))p(t+1)Q_C < p(t)$ implies that $K_C = 0$. (iii) $Q_C < \beta^{-1}$. The previous arguments lead to the result. *Q.E.D.*

Government policy is infinitely delayed, $t^* \to \infty$, in a laissez-faire equilibrium. This equilibrium is not optimal because producers not do internalize the effect of pollution on society in their decisions.

Let the limit value of pollution in a laissez-faire equilibrium be $\bar{Z}$. Underutilization of dirty technology capital can prevent a catastrophe, $U \to -\infty$ for $Z \to \hat{Z} < \bar{Z}$, when full utilization would lead to it. The following proposition summarizes this.

Proposition 2.8 *A catastrophe occurs in finite time, at date* $(t+1)$ *if* $Z(t) < \hat{Z} < d_B K_B(t) + Z(t) - A(Z(t))$, *in a laissez-faire equilibrium and can be prevented by underutilizing dirty capacity* $K_B(t) > 0$, *that is,* $\mathrm{u}_B(t) < 1$, *and investing a small amount* $x_B(t) > 0$ *or* $x_C(t) > 0$.

Proof. There is dirty technology capital $K_B(t) > 0$ which is fully utilized, $\mathrm{u}_B(t) = 1$, in the laissez-faire equilibrium. The condition $\hat{Z} < d_B K_B(t) + Z(t) - A(Z(t))$ holds at some t since $Z(t)$ approaches $\bar{Z}$ by Lemma 2.6 and this level is greater than $\hat{Z}$. The catastrophe has not occured yet. Thus $\hat{Z} > Z(t) > Z(t) - A(Z(t))$. The assumption $Z - A(Z) \leq Z$ implies that a small utilization rate $\mathrm{u}_B(t) \geq 0$ and investing a small amount $x_B(t) > 0$ or $x_C(t) > 0$ creates emissions such that $\hat{Z} > Z(t+1) = Z(t) + E(t) - A(Z(t))$. *Q.E.D.*

2.2.2 Implementation of Pareto optimum

A Pigouvian tax internalizes the pollution externality when underutilization is optimal.

Proposition 2.9 *An emissions tax equal to the product of the unit price of the consumption good and the cost of polluting,* $\hat{\tau} = \hat{p} \times \theta$, *and transfer tr for all households that satisfy (2.17) at equality, all* $t \geq 0$, *evaluated at a Pareto optimal allocation, implement this optimum.*

Proof. Let the marginal utility of income be ψ. The households' choice of consumption yields $\partial U/\partial c = \psi \hat{p}$. Its asset holdings $\hat{\alpha}_i(t+1) > 0$ satisfy $\beta(\psi(t+1)/\hat{p}(t+1))\{(q_i(t+1) + d_i(t+1))/q_i(t)\} = \psi(t)/\hat{p}(t)$. Choice of equity issue by firms implies that the term in braces is the nominal rate of return $\hat{R}(t+1)$. The necessary optimality conditions (2.18) for firms' profit maximization recover the necessary conditions (2.6) for a Pareto optimum, upon substitution of $\upsilon_{ij}/\hat{p} = q_j/(\partial U/\partial c)$, $\hat{w}_j/\hat{p} = w_j/(\partial U/\partial c)$, $p/\hat{p} = \lambda/(\partial U/\partial c) = B$, and $\hat{\tau}/\hat{p} = \epsilon/(\partial U/\partial c) = \theta$. The corresponding necessary conditions (2.19) for utilization in an equilibrium and (2.8) for utilization in a Pareto optimum coincide. The necessary equilibrium conditions $\upsilon_{ij}(t) \leq p_j(t)$ regarding investment goods purchases and $Q_j p_j(t) \leq p(t) + \rho_j \hat{\tau}(t)$ with respect to production of investment goods become the necessary social optimality condition (2.7) for investment. *Q.E.D.*

A firm and thus its owners do not receive a compensation for foregone revenue when some of its pre-installed dirty technology capital at date t^* is idle forever (I-1, I-2). Firms issue new equity to postpone the repayment of equity issued before the date t^* (buyback of equity consolidated with equity supply) when initial dirty capacity is underutilized initially and used in the long-term (I-3, II-1 to II-3). The same policy function as in Proposition 2.9 implements a constrained optimum with full utilization of capital of any type—with θ in this allocation.

Normalizing any one price in any period is possible.[32] However, the price of consumption goods may be calibrated or estimated in empirical work. Given constant price $\hat{p}$ greater income induces to trade greater consumption for additional pollution in form of the tax rate $\hat{\tau}$, if the marginal utility of consumption decreases in consumption and consumption relates positively to income. This explains why Golosov et al. (2011) observe that the Pigouvian carbon tax relates positively to income using special preferences and negative effects of pollution on output.

Government policy rates equal to zero at $0 \leq t \leq t^* - 1$ do not implement a Pareto optimum if equilibrium production creates emissions at a date t prior to $(t^* - 1)$, and pollution affects utility in any of the succeeding periods, $\partial U/\partial Z(t) < 0$ for some $1 \leq t < t^*$, independent on the optimal utilization rates.

[32]Consumption is measured in dollars so that the price $\hat{p}$ is set to one in both the approach with carbon constraint and the approach with continuous effect of carbon on production, for example, in Nordhaus (2009), on utility, for example, in Acemoglu et al. (2012), or on both production and utility, for example, in Barrage (2012) in the literature. In a summary of studies Aldy et al. (2010) refer to the imputed emissions price in models with carbon constraint as least-cost price.

2.3 Numerical examples

Some questions remain. Does delayed government policy lead to underutilized capital? Where are the regions of underutilized capacities in the state space? Is dirty technology capital underutilized when pollution is smaller than its long-term level and the economy starts with minimum pollution? What is the timing of emissions if dirty capacity is underutilized? How do the cost of polluting and the level of clean technology investment compare between the constrained optimum with fully utilized capacity and the optimum? I find answers in simulations and use two specifications for computational convenience. The focus of this section is on incentives. An empirical examination of the climate problem is relegated to future work. The appendix describes the algorithms that delivered the results. Throughout absorption is proportional to the quantitity of pollutants, $A(Z) = \varphi Z$, and the portion of preserved capacity of unused dirty technology capital is $\gamma_B = 0.72$. Pollution is persistent, as only fraction $\varphi = 0.1$ of current pollution is absorbed.

2.3.1 Strictly concave utility in pollution and one clean technology

Clean capacity is constant at $\bar{K}_C = Q_C \times 1000$ all time because investment in clean technology has a greater marginal product than investment in dirty technology, $Q_C = 1.1 \times Q_B > Q_B = (1.02)^{20}$, and this is affordable, $K_C(0) = \bar{K}_C$. There is one clean technology, or equivalently other clean technologies have a low rate of return on investment that does not make investment worthwhile, and have zero stock at the initial date. This leads to a problem with two state variables. The utility function is

$$U(c, Z) = [((1 + \xi Z) \exp(-\xi Z)c)^{1-\psi}]/(1 - \psi)$$

with constant index $\psi = 2$ of relative risk aversion and parameter $\xi = 1/442.5$. Marginal utility of pollution is finite for all pollution levels so there is no catastrophe level of pollution.[33] The values of other parameters are $\beta = 0.9675$, $B = 1$, $b = 1/30$, and $\rho_B = \rho_C = 0$. This example abstracts from emissions in the investment sector.

The economy subject to no government policy experiences increases in pollution and

[33]The utility function U is strictly concave in $Z \in (0, \xi^{-1})$ and yields the simple expression $(-\partial U/\partial Z)/(\partial U/\partial c) = (\xi c)(\xi Z)/(1 + \xi Z)$. Acemoglu et al. (2012) use a similar function whose differential with respect to pollution becomes $-\infty$ for some finite pollution level.

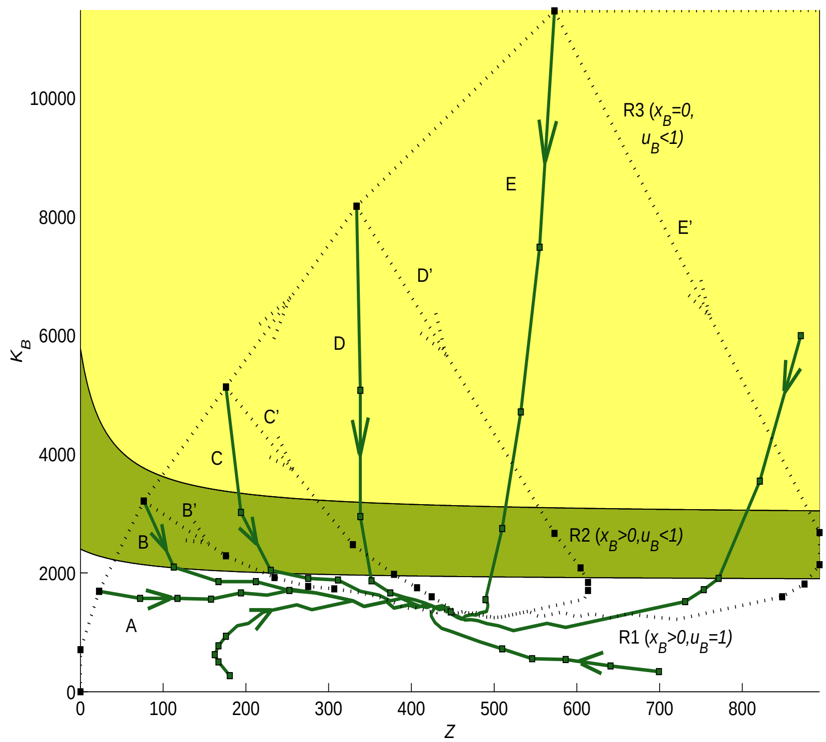


Figure 2.2: *Trajectories of optimal (solid) and constrained optimal or laissez-faire (dotted) pollution* Z *and dirty capacity* K_B *(squares on curves show values in successive periods).*

dirty capacity along the dotted upward-sloping trajectory in Figure 2.2 for the initial state $(Z, K_B, K_C) = (0, 0, \bar{K}_C)$. The squares on curves starting on this laissez-faire curve for different delay dates t^* depict optimized values of state variables in six successive periods. A date t^* corresponds to some initial date that is indexed zero in the planner problem subject to initial values of pollution and capital stocks equal to the date t^* values. Such states on solid curves arise in the global optimum with chosen utilization. These states on dotted curves are solutions to the planner problem that sets the utilization rate of capital to one, $\mathrm{u}_B(t) = \mathrm{u}_C(t) = 1$ all $t \geq t^*$ in the constrained optimum. States in further periods lie on the respective trajectory. All optimized trajectories converge to the same steady state $(448, 0.82\bar{K}_C, \bar{K}_C)$ as time goes to infinity. The constrained optimal trajectories starting at points in the designated region R1 in which dirty capacity is optimally fully utilized, such as path A, are globally optimal because these trajectories remain in this region. Underutilization of dirty capacity is efficient to smooth dirty capacity early on.

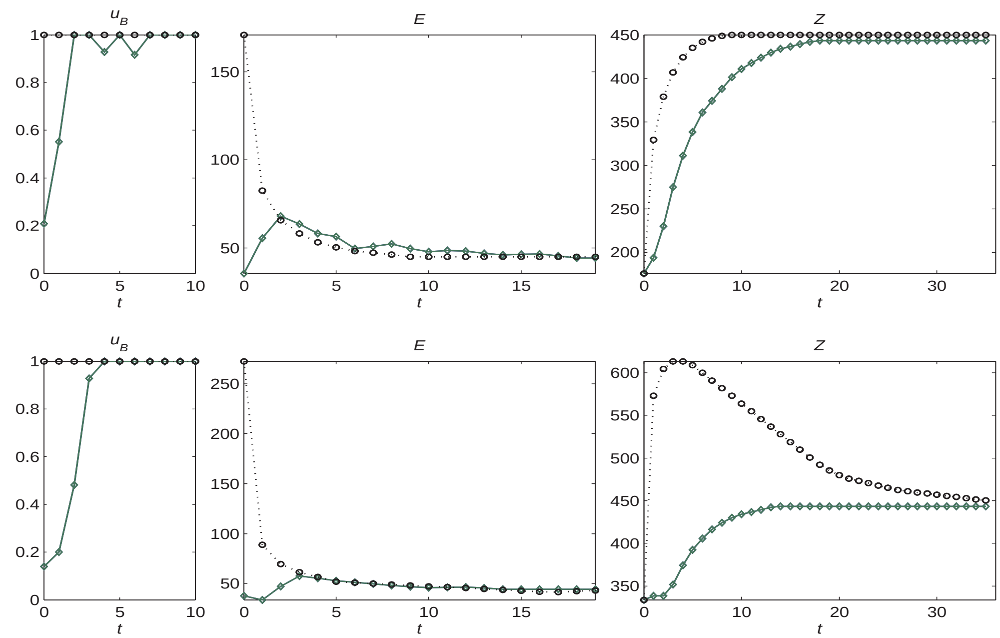


Figure 2.3: *Time paths of optimal (solid) and constrained optimal (dotted) utilization rate* u_B *of dirty technology, emissions* $b\mathrm{u}_B K_B$ *and pollution* Z.

This underutilization occurs when the pollution stock is below or above its long-term level. Capital of the dirty technology in the laissez-faire equilibrium becomes so high that optimization at date $t^* > 0$ calls for its underutilization. The simulation shows that given large upper bound $\bar{K}_B$ there may be a minimum delay date t' such that for all $t^* \geq t'$ underutilization is optimal for any initial state at date zero.[34]

The cost of polluting is $\theta_B = 15.46$ in all states $(Z, K_B, \bar{K}_C)$ in region R2 of joint investment and underutilized capital. The appendix shows how this parametric value was useful in finding the region R2 and the region R3 without investment and with underutilized capital in the dirty technology. For sufficiently large scale $\bar{K}_B$ of the dirty technology a region R3 exists.

The optimal plans B to E with initial states in R2 and R3 in Figure 2.2 obtain lower emissions in early periods and lower pollution in all periods relative to the constrained optimal plans B' to E' with same initial states. The underutilization of dirty capacity avoids current emissions that occur given assumed full utilization of capital. Figure

[34]This may not be true in a model in which the laissez-faire economy converges to a state with $K_B < \bar{K}_B$.

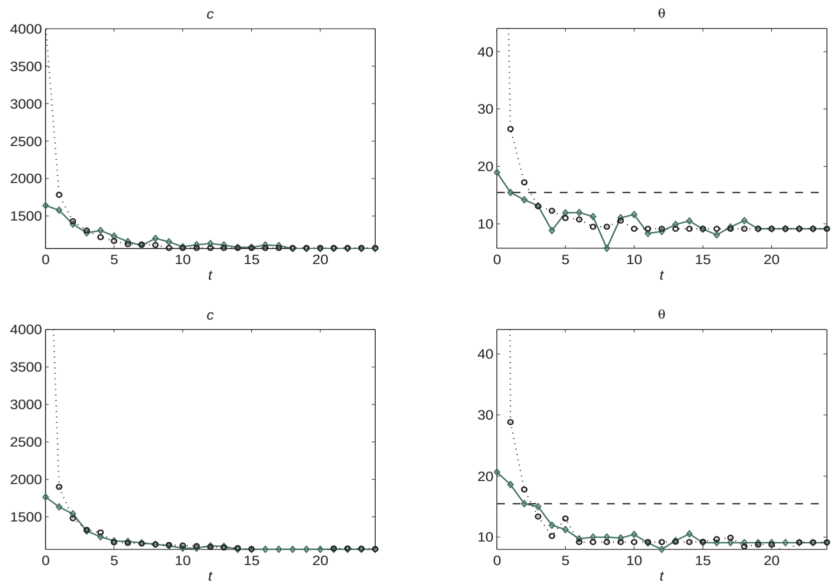


Figure 2.4: *Time paths of consumption c and cost of polluting θ in optimal (solid) and constrained optimal (dotted) plans.*

2.3 shows the time paths of utilization rate, emissions, and pollution for the two initial states $(Z, K_B) = (176, 5130)$ in the plans C and C' in the upper graphs, and $(Z, K_B) = (331, 8180)$ in the plans D and D' in the lower graphs, rounded. The utilization rate in the initial period can be read off from the time series plots of emissions by dividing the optimal emissions amount by the constrained optimal emissions amount. These rates are roughly 0.21 and 0.14 for the paths in the upper panel and lower panel, respectively. The utilization rate increases over time until it reaches one. The utilization rate u_B in the simulation of C is not one after it has been one. The utilization rate in the simulation of D is not one in the fourth period. However from the states I know that this is not optimal. The method used approximates the solution. Comparison of 0.21 and 0.14 suggests that the utilization rate is expected to be smaller in the first period of an optimal allocation the longer the delay beyond the smallest delay level at which dirty capacity is underutilized.

Consumption decreases initially in all optimal and constrained optimal plans that start at states in the regions R2 and R3. Plans with $\mathrm{u}_B(t) = \mathrm{u}_C(t) = 1$ all $t \geq t^*$ by assumption are constrained optimal. This is not surprising because these plans involve a decrease in output, and the savings rate moves monotonously, so that consumption and output comove. The consumption in Figure 2.4 corresponds to the plans C, C', D, and D' for two initial states with greater capital of dirty technology than its long-term level. Apparently underutilization lowers the rate of decrease in consumption relative to a constrained optimum. Figure 2.4 presents time paths of θ, that the summary after the next example interprets. The value of $\theta(0)$ in constrained optimum, 142.6 and 492.6, respectively, is so large that it is outside the picture at the given scale.

2.3.2 Constant marginal utility of pollution and multiple clean technologies

This example shows how the incentives to invest in dirty versus clean technologies are influenced by the possibility of underutilization. This issue was left out in the first specification for computational simplicity. The utility function

$$U = c^{1-\psi}/(1-\psi) - dZ$$

has the constant marginal disutility of pollution d to obtain a two-state problem. There is a continuum of clean technologies C on $[0, \bar{C}]$ with aggregate capital $\int_C K_C dC$. The marginal product function

$$Q(x) = Q'' + (Q' - Q'')(1 + vx)\exp(-vx), \quad 0 \le Q'' < Q',$$

decreases in the aggregate input x in clean technology investment. Along this frontier there are decreasing returns to scale in using aggregate clean technology capital in locations of varying productivity. These locations can be occupied with capital. In general, the new aggregate clean technology capital here is a function of x on geographic sites with distributed marginal product.[35] The equilibrium and efficient motion of aggregate capital in clean technologies is $\int_C K_C(t+1)dC = \int_0^{\int_C x_C(t)dC} Q(z)dz$ if all capital units of clean technologies are fully utilized.[36] In addition, then there is a value function in the

[35] Physical capital is chosen in Section 2.4.1.

[36] One may define the mass $S \subseteq \mathcal{J}'$ of a continuum $\mathcal{J}'$ of technologies with capital in the current period, $S = \{j \,|\, K_j > 0\}$, whose utilization rate is optimally either zero or one, with proper subsets $S \setminus S^- = \{j \,|\, \mathrm{u}_j = 1\}$ and $S^- = \{j \,|\, \mathrm{u}_j = 0\}$. The law of motion of this set is $S(t+1) = S^-(t) \cup S^+(t)$ when investment occurs in technologies $S^+ \subseteq \mathcal{J}' \setminus S^-$. The law of motion of aggregate capital in these technologies is $\int_j K_j(t+1)dj = [\int_{j\in S(t)} \gamma_j(1-\mathrm{u}_j(t))dj / \int_{j\in S(t)} dj] \int_j K_j(t)dj + \int_{j\in S^+(t)} Q_j dj$. Let $\mathcal{J}' = \{j \,|\, d_j = 0\}$ and $j = C$. Upon change of variable the new capital stock is $\int_{C\in S^+} Q_C dC = \int_0^{x^+(t)} Q^+(x)dx$.

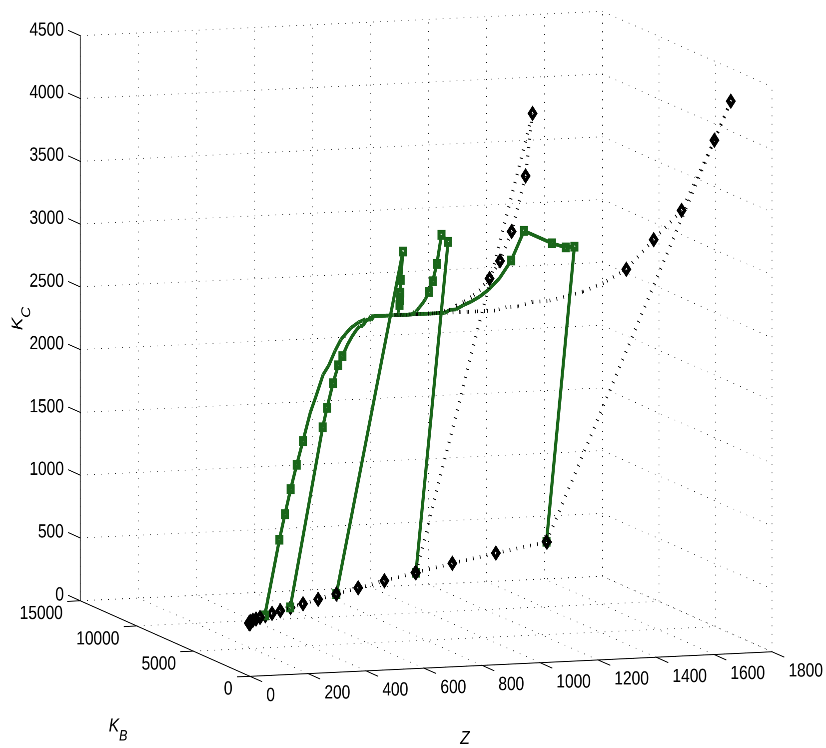


Figure 2.5: *Trajectories of optimal (solid) and constrained optimal or laissez-faire (dotted) pollution Z, dirty capacity K_B, and clean capacity $\int_C K_C dC$ (squares on curves show values in successive periods).*

states dirty technology capital and aggregate clean technology capital, $(K_B, \int_C K_C dC)$, since both utility U and absorption A are linear in pollution. I assume scale-dependent relative advantage of dirty and clean technologies, $Q' = 1.1 \times Q_B = 20 \times Q''$, and emissions from investment, $\rho_B = \rho_C = (\beta Q_B)^{-1/\psi} Q_B (1/20)(1/30)$ all C. Further parameters are $d = 7.4 \times 10^{-7}$, v that solves $\int_C K_C(0) dC = 1000/2.4$, $B = 1.2$, and $b = (19/20)(1/30)$. The horizontal dashed line runs at θ_B.

The trajectories in Figure 2.5 emanate from states in the laissez-faire equilibrium that is initialized at $Z = K_B = 0$, $\int_C x_C(0) dC = y$, and $\int_C K_C(0) dC = \int_0^y Q(x) dx$ such that $Q(y) = Q_B$. In the laissez-faire equilibrium aggregate clean capacity is $\int_C K_C(0) dC$ if dirty technology investment is below $\bar{K}_B = 14442$ in the preceding period. The laissez-faire economy does not generate sufficient output for optimal underutilization of dirty technology capital when the pollution stock is below its long-term level. In the first example, underutilization occurs at such states.

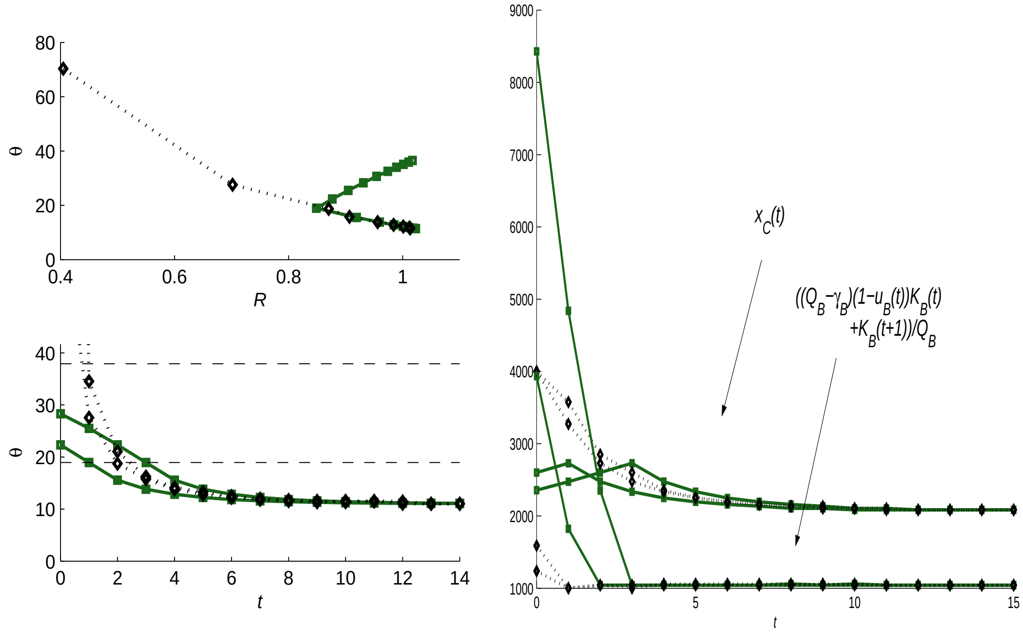


Figure 2.6: *Shadow return and cost of polluting, and time paths of dirty and clean technology savings.*

The policy of underutilization diminishes the incentives to invest in expensive clean technologies. The right panel in Figure 2.6 shows the time paths of savings. Unused dirty technology capital $(1-\mathrm{u}_B)K_B$ contributes to the savings $((1-\gamma_B/Q_B)(1-\mathrm{u}_B(t))K_B(t)+K_B(t+1)/Q_B)$ out of capacity. Clean technology investment $\int_C x_C dC$ equals its savings because clean technology capital is fully utilized following Proposition 2.6. In the constrained optimum the incentives to invest are aligned in both dirty and clean technologies. In optimum with underutilized dirty technology capital the incentives to invest in clean technologies are relatively lower early than in the constrained optimum because underutilization helps mitigating pollution effects on society. Clean technology investment peaks in the period in which both investment and underutilization in the dirty technology are optimal, when the state is in R2, below the initial level in the constrained optimum at the same initial state. This explains the V-shaped path of the shadow return. The shadow return and the cost of polluting are positively related on intervals with underutilized dirty technology capital, and negatively related on intervals with full utilization, which the curve in the upper left panel of Figure 2.6 depicts. The dotted trajectory shows a constrained optimal path. The initial values of pollution and dirty capacity are $(876, 7790)$

and $(1500, 12254)$ in this Figure. The former values initialize the constrained optimal path in the upper left panel. The cost of polluting is plotted over time in the lower left panel. The dashed lines are at the levels (B/d_B) and θ_B.

2.3.3 Summary

The simulation yields the following five insights. (i) *Delay of government policy.* Emissions are an externality. The longer emissions are unpriced the greater dirty technology capacity is built relative to the efficient long-term dirty technology output. This means that delayed government policy leads to efficient underutilization of dirty capacity. (ii) *Location of regions of underutilization of dirty capacity in the state space.* Intuitively, the society can afford underutilization well when capital is large given pollution and desires it when pollution is large given capital. In fact in the first example pollution Z and dirty capacity K_B relate negatively on the boundary of R1 and R2. In the example with constant marginal utility of pollution a constant K_B given the optimal choice K_C forms the boundary of R1 and R2. Then affordability is the dominant force behind underutilization. The slope of this boundary in the state plane (Z, K_B) is minus one (not shown) if utility is quadratic in both consumption and pollution and absorption is linear in pollution. In these examples thus underutilization of capital can be optimal when current pollution is smaller (environmental quality is greater) than its long-term stabilization level. The Propositions 2.3 and 2.4 do not tell for what initial levels of pollution dirty capacity use is postponed. (iii) *Timing of emissions.* The optimal and constrained optimal emissions on optimal paths that start at the same state in R2 or R3 differ substantially early. Variable utilization allows to start with low emissions followed by greater emissions. Fixed full utilization necessitates emissions decreases early on in the constrained optimum to approach lower capital values. (iv) *Cost of polluting.* The cost of polluting can differ substantially between an optimal plan with chosen utilization and a constrained optimal plan with assumed fully utilized capital subject to the same initial condition. The substantive difference lies in the early periods with optimal underutilization in the Figures 2.4 and 2.6. Underutilization mitigates societal effects of pollution and thus achieves a lower cost. (v) *Clean technology investment.* Expensive clean technologies are not needed when dirty technology capital utilization can be varied.

2.4 Extensions

Extensions to physical capital and a fuel technology check the robustness of results. The third extension warrants that a high portion of wealth may be affected by underutilized dirty technology assets. The fourth aspect examines if distributional effects between asset owners that have differently aged dirty technology capital are expected from underutilization, if these are efficiently fully utilized at some date.

2.4.1 Physical capital

In this section the productivity of stored unused capital vintages decreases to postulate bounds on capital rather than capacity. The boundedness of capital is more restrictive for investment than bounded capacity in Section 2.1.1. However there is an equivalence in optimum, which is to be shown. Capital $a_j(t,v) > 0$ of vintage v is utilized at chosen rate $\mathrm{u}_j(t,v) \in [0,1]$ for $j \in \mathcal{J}$. Output

$$m_j(t) = \sum_{v=-1}^{t-1} \chi_j(t-v-1)\mathrm{u}_j(t,v)a_j(t,v)$$

in period t sums production over vintages v, given productivity $\chi_j(t-v-1)$. Productivity can be interpreted as the available time of capital or net of maintenance expenditures. A widely held view among professionals is that machines, automobile engines or power plants, produce at rates independent on age yet incur increased downtime or expenditures for maintenance when becoming older. This should apply to used and unused capital. Productivity in the period after creation of capital is $\chi_j(0) > 0$. Site-specific factors relative to norm conditions and the average availability in a period may prescribe $\chi_C(0)$. Capital at the beginning of period $(t+1)$ is

$$a_j(t+1,v) = \begin{cases} \sigma_j(1-\mathrm{u}_j(t,v))a_j(t,v) \\ \varepsilon_j x_j(t) \end{cases} \quad \text{if} \quad \begin{cases} t > v \\ t = v \end{cases} \tag{2.20}$$

where $\varepsilon_j x_j(t)$ is new capital that arrives in period t and is productive with a lag of one period, and $\sigma_j \in \{0,1\}$ as explained below. For simplicity, $\chi_j(t-v-1) = {\gamma_j}^{t-v-1}\chi_j(0)$. Then productivity of stored capital units depreciates at rate $\gamma_j \in (0,1]$, or productivity of age greater than or equal to one is zero, $\gamma_j = 0$. Capital is automatically scrapped, so that σ_j equals zero if and only if unused capital became unproductive, $\gamma_j = 0$. Then unproductive capital does not block investment, as in the reduced form in Section 2.1.1. Finite recyclable supply of minerals for producing capital or finite amount of land and water for installing capital, whichever is the tight constraint, give rise to the exogenous upper bound $\bar{a}_j$ on capital of technology j. Then

$$\bar{a}_j \geq \sum_{v=-1}^{t} a_j(t+1, v) \tag{2.21}$$

all $t \geq 0$. A discussion of the relationship between welfare-maximizing choices here and in Section 2.1.1 follows after stating the planner problem.

There are $\tau(t)$ vintages with positive capital at t. A planner chooses a policy $(c, x, \mathrm{u}) \in \mathbb{R}^3_+ \times [0,1]^{\tau(t)}$ on $\{0, 1, \ldots\}$ to maximize Lagrange's function

$$\begin{aligned}
\mathscr{L} = \sum_{t=0}^{\infty} \beta^t \Bigg\{ & U(c(t), Z(t)) + \epsilon(t) \Bigg[Z(t+1) - \sum_j \Bigg(d_j \sum_{v=-1}^{t-1} \chi_j(t-v-1) \mathrm{u}_j(t,v) a_j(t,v) \\
& + \rho_j x_j(t) \Bigg) - \phi(Z(t)) \Bigg] + \sum_j \Bigg(\sum_{v=-1}^{t-1} \varphi_j(t,v)[\sigma_j(1-\mathrm{u}_j(t,v)) a_j(t,v) - a_j(t+1,v)] \\
& + \varphi_j(t,t)[\varepsilon_j x_j(t) - a_j(t+1,t)] + \beta w_j(t+1) \Bigg[\bar{a}_j - \sum_{v=-1}^{t} a_j(t+1,v) \Bigg] \Bigg) + w(t) G(t) \Bigg\}
\end{aligned}$$

given $\phi(Z) = Z - A(Z)$, and $G \geq 0$ that collects the resource constraint (2.1), nonnegativity constraints of investment inputs, feasibility constraints of utilization rates in technologies with positive capital, and nonnegativity constraints of capital. At least one initial stock is assumed positive among the given $a_B(0,-1) \geq 0$ and $a_C(0,-1) \geq 0$.

The constraint (2.2) allows greater investment $y_j(t) = \varepsilon_j x_j(t)$ than the constraint (2.21) allows if vintage capital units created more than two periods ago are around and quality depreciates ($0 < \gamma_j < 1$). Capacity equals

$$K_j(t) = \sum_{v=-1}^{t-1} \chi_j(t-v-1) a_j(t,v)$$

for technology $j \in \mathcal{J}$. The bound $\bar{a}_j$ on capital translates into the upper bound $\bar{K}_j = \chi_j(0)\bar{a}_j$ of production capacity measured in output units. The definition of capacity and the aggregate utilization rate

$$\mathrm{u}_j(t) = \left[\sum_{v=-1}^{t-1} {\gamma_j}^{t-v-1} \mathrm{u}_j(t,v) a_j(t,v) \Big/ \sum_{v=-1}^{t-1} {\gamma_j}^{t-v-1} a_j(t,v) \right] \in [0,1]$$

specific to technology j lead to the transition law (2.3) of capacity. The marginal product of investing a unit of the final good is

$$Q_j = \varepsilon_j \chi_j(0)$$

for technology $j \in \mathcal{J}$ in Section 2.1.1. The following proposition concludes from solutions here to solutions of the planner problem in this section.

Proposition 2.10 *The solutions to the planner problem here and in Section 2.1.1 coincide if the productivity of unused capital does not depreciate, $\gamma_j = 1$ all $j \in \mathcal{J}$. An optimal policy such that (i) capital $a_j(t,v) > 0$ of vintage $v < t-1$ is fully utilized, $\mathrm{u}_j(t,v) = 1$, or (ii) the constraint (2.21) in the period $(t+1)$ is not binding, if investment occurs one period before, $x_j(t) > 0$, $j \in \mathcal{J}$, solves the planner's problem in Section 2.1.1, for $\gamma_j \in (0,1)$.*

Proof. The motions (2.20) of vintage capital and (2.3) of capacity are equivalent because

$$\begin{aligned} K_j(t+1) &= \sum_{v=-1}^{t} \chi_j(t-v) a_j(t+1,v) \\ &= \gamma_j \sum_{t=-v}^{t-1} \chi_j(t-v-1)[1-\mathrm{u}_j(t,v)] a_j(t,v) + \chi_j(0) a_j(t+1,t) \\ &= \gamma_j \left\{ K_j(t) - \sum_{t=-v}^{t-1} \chi_j(t-v-1) \mathrm{u}_j(t,v) a_j(t,v) \right\} + Q_j x_j(t) \end{aligned}$$

given the capacity $K_j(t)$ and utilization rate $\mathrm{u}_j(t)$ as defined above. In view of the constraints (2.21) and (2.2) then the conditions in the Proposition deliver the result. The constraints are identical if $\gamma_j = 1$ or (i) holds. *Q.E.D.*

Suppose that the productivity of unused capital incompletely depreciates, $0 < \gamma_j < 1$. Then (ii) does not hold only if the economy both builds new capital and stores existing capital in some technology at the same date. Either the return from storing capital is greater than, equal to, or smaller than the return from investing. In the first case no investment occurs, and in the latter case investing the output from utilized capital is better than not utilizing capital. Hence the only case in question is indifference. The first-order necessary optimality conditions do not depend on the investment level and thus do not help to rule out investing up to the capacity bound and storing other capital.[37] But it is conceivable that either all vintages, $v < t-1$, are fully utilized latest at t, or if not then capital is not at its upper bound at $(t+1)$, in an optimal plan. This yields a further insight. The same allocations would be optimal if the planner had the option to scrap unutilized capital units. The planner would not do so if investment does not exhaust space when capital is underutilized.

2.4.2 Intermediate good in dirty production

This section examines the optimal utilization of capital in converting refined fossil fuel into energy that is useful for consumption and investment and capital in producing fossil fuels. Optimal policies map one-to-one to optimal policies in the basic model with underutilized dirty technology capital if the depreciation rate of unused capital in fuel-based energy production and in producing fuel are equal and the emission intensities in investing in these technologies are equal.

Technology and environment.—Production of one unit of good using technology B, for example, conversion of energy from fossil fuel, requires α_B units of an intermediate good that technology R produces. Fuel input cannot exceed fuel output,

$$\alpha_B m_B \leq m_{\mathrm{R}} \tag{2.22}$$

[37] One can rule out that old units are underutilized and young units are utilized, because the space requirement of output is weakly smaller for younger units. Claim. Underutilization of capital $a_j(t, v') > 0$ of old vintages, that is, $\mathrm{u}_j(t, v') < 1$, of technology $j \in \mathcal{J}$ is Pareto optimal only if capital $a_j(t, v) > 0$ of young vintages is idle, $\mathrm{u}_j(t, v) = 0$ and $v' < v < t$. Proof. If $\mathrm{u}_j(t, v) > 0$ and $\mathrm{u}_j(t, v') < 1$, $v' < v < t$, then lowering $\mathrm{u}_j(t, v)$ and raising $\mathrm{u}_j(t, v')$ until $(1 - \mathrm{u}_j(t, v'))\mathrm{u}_j(t, v) = 0$, achieves the same consumption and emissions and (i) keeps aggregate capital of technology j in period $(t+1)$ at same level if $\gamma_j = 1$, or (ii) lowers capital and thereby relaxes the constraint (2.21) at $(t+1)$ if $0 < \gamma_j < 1$. *Q.E.D.* Thus young units are not utilized if old units are not utilized. But young and old units may only coexist when all units are fully utilized.

all $t \geq 0$. The commitment of resources for fuel production takes the form of investment. The capital cost of fossil fuel production originates in the extraction of raw fuels, their transportation, and refinement. Accordingly,

$$c/B + x_B + x_C + x_R \leq m_B + m_C \tag{2.23}$$

is the resource constraint of energy, and $m_j = \mathrm{u}_j K_j$ equals output at chosen utilization rate $\mathrm{u}_j \in [0, 1]$ of technology $j \in \mathcal{J} = \{B, R, C\}$.

Production of one unit of fuel creates d_R emission units (notably from flaring of natural gas at extraction sites of petroleum, by ventilation of underground coal mines, at transport of raw fuel, and in refining petroleum). One unit of output of technology B produces d_B emission units (in combustion engines and power plants). Necessary conditions for a Pareto optimum follow after stating the planner's problem.

The planner chooses a policy $(c, x, \mathrm{u}) \in \mathbb{R}^4_+ \times [0, 1]^{\tau(t)}$ on $\{0, 1, \ldots\}$ to maximize J subject to the resource constraints (2.22) and (2.23), the upper bound on capacity (2.2), and the laws of motion (2.3) and (2.4) for $j \in \mathcal{J}$ all $t \geq 0$. The triple of initial capacity satisfies $0 \leq \alpha_B K_B(0) = K_R(0) < \bar{K}_R$ and $0 \leq K_C(0) < \bar{K}_C$, and contains at least one positive level.

The following conditions hold in an optimal plan. Let λ^* be the multiplier on the constraint (2.22). The discounted marginal benefit of using capital and storing unused capital at most equals the marginal cost of holding capital at the end of period t,

$$\begin{aligned} &\beta \mathrm{u}_B(t+1)\{\lambda(t+1) - \alpha_B \lambda^*(t+1) - d_B \epsilon(t+1)\} \\ &\quad + \beta\gamma_B(1 - \mathrm{u}_B(t+1))q_B(t+1) - \beta w_B(t+1) \leq q_B(t), \quad = \text{ if } K_B(t+1) > 0, \end{aligned} \tag{2.24}$$

and

$$\begin{aligned} &\beta \mathrm{u}_R(t+1)\{\lambda^*(t+1) - d_R \epsilon(t+1)\} \\ &\quad + \beta\gamma_R(1 - \mathrm{u}_R(t+1))q_R(t+1) - \beta w_R(t+1) \leq q_R(t), \quad = \text{ if } K_R(t+1) > 0, \end{aligned} \tag{2.25}$$

all $t \geq 0$. The use of fuel is costly for technology B and beneficial for technology R. This explains the sign of λ^* in the net benefits. These conditions demand to balance the values of idle and utilized capacity units in the same way as in Section 2.1.1. Therefore

$$\mathrm{u}_B(t+1)\left\{\begin{array}{l} =1 \\ \in(0,1) \\ =0 \end{array}\right\}$$
$$\Longrightarrow \quad \beta\{\lambda(t+1)-\alpha_B\lambda^*(t+1)-d_B\epsilon(t+1)\}\left\{\begin{array}{c} = \\ = \\ \leq \end{array}\right\} q_B(t)\left\{\begin{array}{c} \geq \\ = \\ = \end{array}\right\}\beta\gamma_B q_B(t+1) \tag{2.26}$$

given $K_B(t+1)\in(0,\bar{K}_B)$, and

$$\mathrm{u}_R(t+1)\left\{\begin{array}{l} =1 \\ \in(0,1) \\ =0 \end{array}\right\}$$
$$\Longrightarrow \quad \beta\{\lambda^*(t+1)-d_R\epsilon(t+1)\}\left\{\begin{array}{c} = \\ = \\ \leq \end{array}\right\} q_R(t)\left\{\begin{array}{c} \geq \\ = \\ = \end{array}\right\}\beta\gamma_R q_R(t+1) \tag{2.27}$$

given $K_R(t+1)\in(0,\bar{K}_R)$, for $t\geq 0$. Only the outer relations at weak inequalities are relevant regarding utilization in period $t=0$. The following discusses the incentives to utilize installed dirty and fuel technology capital and to invest in these technologies.

Proposition 2.11 *The statements in Propositions 2.2 and 2.3 on the timing of investment and utilization in the dirty technology hold for both the dirty technology B and the fuel technology R if these technologies have equal emission intensities of investment, $\rho_B=\rho_R$, and depreciation rates of unused capital, $\gamma_B=\gamma_R$, and the assumptions of Proposition 2.3 hold for both these technologies.*

Proof. The fuel constraint (2.23) is binding all $t\geq 0$ so that $\mathrm{u}_R(t)=\mathrm{u}_B(t)$, if $K_R(t)=\alpha_B K_B(t)$ all $t\geq 0$, which is to be confirmed. The statements in Proposition 2.2 hold since $K_R(0)=\alpha_B K_B(0)$. Consider Proposition 2.3. (i) Let $\mathrm{u}_j(t)=1$ some $j\in\{B,R\}$. Capital of technology j stays zero from $(t+1)$ onward by assumption if $x_j(t)=0$. Then $x_{j'}(t)=0$, $j'\neq j$, since investment in the other technology $j'\in\{B,R\}$ would be wasteful.

In the alternative, $x_j(t) > 0$ implies that $x_{j'}(t) > 0$, so that investment occurs in both technologies. (ii) In case $x_B(t) > 0$ and $x_R(t) > 0$ the utilization of capital implies that $\theta(t) \leq \theta^*$ defining $\theta^* = (\upsilon Q_B - [\upsilon\gamma_B + (1-\upsilon)\gamma_R])/([\upsilon\gamma_B\rho_B + (1-\upsilon)\gamma_R\rho_R] + [\upsilon d_B Q_B + (1-\upsilon)d_R Q_R])$ for some $\upsilon \in (0,1]$. Suppose that $\mathrm{u}_j(t+1) < 1$ all $j \in \{B,R\}$. Then $\theta^* \leq \theta(t+1)$. Thus the necessary optimality conditions contradict that $\theta(t) > \theta(t+1)$. Therefore $\mathrm{u}_B(t+1) = 1$ or $\mathrm{u}_R(t+1) = 1$. Suppose that $\mathrm{u}_j(t+1) = 1$ some $j \in \{B,R\}$. The reverse inequality to that in Lemma 2.9 results if $x_j(t+1) > 0$ and $\mathrm{u}_j(t+1) > 0$, as $\beta\gamma_j(\lambda(t+1) + \rho_j\epsilon(t+1)) = \beta\gamma_j Q_j q_j(t+1) \leq Q_j q_j(t) \leq \lambda(t) + \rho_j\epsilon(t)$. Thus $\mathrm{u}_j(t+1) = 1$ all $j \in \{B,R\}$ by contradiction given $\rho_B = \rho_R$ and $\gamma_B = \gamma_R$ if one of these inequalities is strict. This choice is optimal if both these inequalities hold at equality. This shows that following utilization and investment capital of both technologies B and R is fully utilized or ceases to exist simultaneously. Then underutilization occurs early and capital is proportional as claimed, because there is at most one period with underutilization and investment in both these technologies and $K_R(0) = \alpha_B K_B(0)$. *Q.E.D.*

The timing of investment and utilization is as in Section 2.1.1 if $\rho_B = \rho_R$ and $\gamma_B = \gamma_R$ because of symmetric incentives. The marginal products determine the ratio of inestment inputs.

Proposition 2.12 *Investment is proportional,* $x_R(t) = ((1-\upsilon)/\upsilon)x_B(t)$ *given constant* $\upsilon = Q_R/(Q_R + \alpha_B Q_B) \in (0,1]$*, and the utilization rates are equal,* $\mathrm{u}_R(t) = \mathrm{u}_B(t)$*, in the dirty and fuel technology all* $t \geq 0$*, if the assumptions of Proposition 2.11 hold.*

Proof. The proportionality of capital, $K_R(t) = \alpha_B K_B(t)$, and equal utilization rates, $\mathrm{u}_R(t) = \mathrm{u}_B(t)$, all $t \geq 0$, in proving Proposition 2.11 deliver the result. *Q.E.D.*

Aligning the investment expenditures and utilization rates of the dirty and the fuel technology is optimal if they have same environmental cost of investment inputs and depreciation rates of unused capital. This is one case in which redefining the dirty technology productivity to υQ_B, and writing the emission intensities $b = d_B + \alpha_B d_R$ of dirty technology output and $\rho = \upsilon\rho_B + (1-\upsilon)\rho_R$ of dirty technology investment input $x_B + x_R = x_B/\upsilon$ allow solving for an optimum without explicit use of the fuel technology.

Investing in the technology with smaller depreciation rate may be relatively delayed early in an optimal plan with underutilized dirty and fuel technology capital if the depreciation rates of unused capital are different for the dirty and the fuel technology. This follows from the law of motion (2.3) of capital. The timing of underutilization and investment in each of these technologies may still be the timing regarding the dirty technology in the model without a fuel technology. However, the necessary optimality conditions do not seem to rule out underutilization after full utilization in the technology with smaller depreciation rate of unused capital or smaller emission intensity of the input in building new capital.[38]

In practice γ_B and γ_R, and ρ_B and ρ_R, likely differ. Empirical studies have to determine yet how big the discrepancy is and the direction. The distinction between capital in fuel extraction, transport, and refinement, and of capital in energy conversion, in data will produce large portions of each of these. Fuel use in automobiles and power plants accounts for about half of the expenditure on the mobility service from vehicles and electricity generation, respectively.

2.4.3 Energy-use capital and energy efficiency

The basic model lacks explicit capital that uses energy. Each capital K_j can be written as a composite of technology-specific energy-production capital (for example, engines, turbines, and photovoltaic cells) and energy-use capital (for example, building shells, equipment, roads, and vehicle shells) leading to the same results. For empirical work it is desirable to account for the fact that the depreciation rates of major portions of energy-conversion capital and energy-use capital differ. In terms of dollar value the latter comprises mainly buildings. A building has typically a longer lifespan than solar modules mounted on its roof. Simultaneous scrapping of energy-production capital and energy-use capital is limited to bundles such as automobile engine and shell. This section accounts for different depreciation rates such that energy-use capital is perpetually inventoried and may use energy produced from dirty and clean technologies. I combine a continuum of capital types for energy use inspired by Atkeson & Kehoe (1999) with energy production using capital and an environmental motive to examine utilization of energy-use capital. Atkeson & Kehoe (1999) posit an exogenous resource cost of energy without environmental consideration to determine the response of energy efficiency to fuel price movements.

[38]On a path with some dirty output in every period full utilization of dirty or fuel technology capital $K_j(t) > 0$, $\mathrm{u}_j(t) = 1$, coincides with investment, $x_j(t) > 0$. Then arguments used in proving Proposition 2.11 show that there is no investment, $x_j(t+1) = 0$, if capital $K_j(t+1) > 0$ is underutilized, $\mathrm{u}_j(t+1) < 1$.

Capital is putty-clay in terms of its energy intensity $\bar{v} \in V = (0, v^*)$ for some supremum $v^* \in (0, \infty)$. The resource constraint

$$\int_{\bar{v}} e(\bar{v}, t) d\bar{v} \leq \sum_{j \in \mathcal{J}} m_j \tag{2.28}$$

of energy replaces (2.1). Energy use $e(\bar{v}, t) \in [0, K(\bar{v}, t)/\bar{v}]$ all $\bar{v} \in V$ may be below the energy requirement of fully utilized capital. This requirement equals capital $K(\bar{v}, t)$ divided by the energy intensity $\bar{v}$ that fixes at the date of investment. Capital of each type $\bar{v}$ follows the difference equation

$$K(\bar{v}, t+1) = ((1-\delta)\mathrm{u}(\bar{v}, t) + \gamma(1 - \mathrm{u}(\bar{v}, t)))K(\bar{v}, t) + \varepsilon(\bar{v})x(\bar{v}, t) \tag{2.29}$$

where $\delta \in [1-\gamma, 1]$ and $\varepsilon(\bar{v}) > 0$, and $\mathrm{u}(\bar{v}, t) \in [0, 1]$ is the utilization rate of $K(\bar{v}, t)$. Capital depreciates at rate $\delta = 1 - \gamma \geq 0$ regardless of its utilization, or utilized units depreciate at greater rate than nonutilized units, $1 - \delta < \gamma$. Capital services are defined as

$$z(t) = \int_{\bar{v}} \min[K(\bar{v}, t)/\bar{v}, e(\bar{v}, t)] f(\bar{v}) d\bar{v}$$

in period t given increasing and strictly concave function f. Aggregate demand for services at most equals its supply,

$$c(t)/B + \sum_{j \in \mathcal{J}} x_j(t) + \int_{\bar{v}} x(\bar{v}, t) d\bar{v} \leq \int_{\bar{v}} e(\bar{v}, t) f(\bar{v}) d\bar{v} \tag{2.30}$$

given $e(\bar{v}, t) = \mathrm{u}(\bar{v}, t)K(\bar{v}, t)/\bar{v}$ all $t \geq 0$. Aggregate emissions

$$E(t) = \sum_{j \in \mathcal{J}} (d_j m_j(t) + \rho_j x_j(t)) + \int_{\bar{v}} \rho(\bar{v}) x(\bar{v}, t) d\bar{v} \tag{2.31}$$

contain the emissions from producing energy-use capital all $t \geq 0$. The pollution stock $Z(0)$ is given. Fully utilized energy-use capital exhausts maximum energy supply in the initial period, $\int_{\bar{v}} [K(\bar{v}, 0)/\bar{v}] d\bar{v} = K_B(0) + K_C(0) > 0$, because capital units are endowed in the initial period. A Pareto optimal policy (c, x, u) and $(x(\bar{v}, t), \mathrm{u}(\bar{v}, t)) : V \to \mathbb{R}_+ \times [0, 1]$ on $\{0, 1, \ldots\}$ maximizes welfare J subject to (2.2)-(2.4) and (2.28)-(2.31). The following result about the utilization of energy-use capital in the initial period is immediate.

Proposition 2.13 *Some capital* $K(\bar{v},0) > 0$ *that uses energy is underutilized,* $\mathrm{u}(\bar{v},0) < 1$ *for at least one* $\bar{v}$ *with* $K(\bar{v},0) > 0$*, if capacity* $K_j(0) > 0$ *in energy production is underutilized,* $\mathrm{u}_j(0) < 1$*, for some* $j \in \mathcal{J}$*.*

Proof. The initial condition implies that $\int_{\bar{v}} e(\bar{v},0)d\bar{v} = \sum_j \mathrm{u}_j(0)K_j(0)$ is smaller than $\int_{\bar{v}}[K(\bar{v},0)/\bar{v}]d\bar{v}$ at $\mathrm{u}_j(0) < 1$ some $j \in \mathcal{J}$. Then $e(\bar{v},0) < K(\bar{v},0)/\bar{v}$ for at least one $\bar{v}$. *Q.E.D.*

From the viewpoint of stochastic energy prices Atkeson & Kehoe (1999) find that energy prices in the US 1960-1994 have not varied so much to induce optimal underutilization. This analysis has excluded emissions from energy production. Given long delay in optimizing pollution some units may be efficiently underutilized. It will be the least-energy efficient buildings and ports and heaviest automobiles in a class of same use value that should be permanently or temporarily retired. The retirement is thus not limited to 1/6 of dollar wealth in fuel production and energy conversion but extends to 5/6 of such assets that use energy.[39]

2.4.4 Improvement in emission intensity and time-variant return to storage

In practice plants and engines of different age that possess a vintage-dependent emission intensity of output coexist. An autonomous improvement in the emissions per kilometre driven should favour underutilization of old vehicles. Capital is useful more than once albeit it may depreciate over time. The productivity of unused capital diminishes more rapidly for older plants and engines, because more screws get loose in machines that have been used more often. The same relationship should hold for temporarily retired capital. This should instead give incentives to underutilize young vintage capital. This section studies the roles of vintage-dependent emission intensity and use-dependent depreciation for the age of underutilized plants, because renewable energy technologies may be insufficiently productive at large scale or investment in renewable energy technologies may be carbon-free in the future, both which prevents their exclusive long-term use in energy production in the realm here. I show the opposite effects as outlined above. Therefore

[39]Edenhofer et al. (2005) assume 49.2 trillion USD capital stock that produces output, and 5 and 6 trillion USD fossil-fuel extraction and conversion stock, respectively, that produce intermediate goods.

they may roughly cancel out considering both. Then the underutilization has likely mild distributional effects among owners of old versus young capital given fossil-fuel using capital is only temporarily retired. For example, old cars and young cars alike should be driven less than previously planned.

The following is a representation with capacity. Writing out productivity and physical capital gives the same results. The vintage capacity $K(t,s,v)$ originates from capital created in period v that has been utilized in $s \leq S-1$ prior periods. Output is

$$m(t) = \sum_{v=-S}^{t-1} \sum_{s=0}^{\min[S-1,t-v-1]} \mathrm{u}(t,s,v)K(t,s,v)$$

all $t \geq 0$. Capital is useful in $S \geq 2$ periods. For notational convenience investment inputs, utilization rates, capacity levels, and emission intensities of multiple technologies are appropriately stacked in vectors. The constraint of the upper bound on vintage- and use-dependent capacity,

$$\bar{K} \geq \sum_{v=-S}^{t} \sum_{s=0}^{\min[S-1,t-v]} K(t+1,s,v) \tag{2.32}$$

extends (2.2). Let $\gamma(t-v,s,v) \in [0,1]$ be the retained fraction of productivity of unutilized capital constructed in period v conditional on age $(t-v)$ and use s. The first line in

$$K(t+1,0,v) = \begin{cases} \gamma(t-v,0,v)(1-\mathrm{u}(t,0,v))K(t,0,v) \\ Qx(t) \end{cases} \quad \text{if} \quad v \begin{cases} \in \{-S,-S+1,\dots,t-1\} \\ = t \end{cases} \tag{2.33}$$

accounts for capacity from unused capital constructed before t. Addition of t-vintages enhances aggregate $(t+1)$-capacity by $Qx(t)$. Capital at $(t+1)$ that has been used at least one period and built before $(t-s)$ consists of pausing capital and used capital in period t. The pausing capital carried over from period t is used $s \in \{1,\dots,\min[S-1,t-v-1]\}$ times prior to t. The capital used in period t is used $(s-1)$ times prior to t. Used capital retains fraction $\pi(t-v,s,v)$ of productivity. This yields capacity

$$\begin{aligned} K(t+1,s,v) &= \gamma(t-v,s,v)(1-\mathrm{u}(t,s,v))K(t,s,v) \\ &\quad + \pi(t-v,s-1,v)\mathrm{u}(t,s-1,v)K(t,s-1,v), \\ &\qquad\qquad v \in \{-S,-S+1,\dots,t-s-1\} \end{aligned} \tag{2.34}$$

all $t \geq 0$. For simplicity the history of utilization does not matter for the productivity of a given vintage capital.[40] The production capacity of capital used in all periods from $(v+1)$ to t is

$$K(t+1,s,v) = \pi(t-v,s-1,v)\mathrm{u}(t,s-1,v)K(t,s-1,v), \quad v = t-s, \tag{2.35}$$

one period later all $t \geq 0$. The equations (2.33)-(2.35) replace the inventory constraint (2.3) of Section 2.1.1 for all technologies $j = 1,\dots,M$. The law of motion of pollution is

$$Z(t+1) = Z(t) + \sum_{v=-S}^{t-1} \sum_{s=0}^{\min[S-1,t-v-1]} E(t,s,v) + \rho x(t) - A(Z(t) \tag{2.36}$$

given emissions $E(t,s,v) = d(t-v,s,v)\mathrm{u}(t,s,v)K(t,s,v)$ of using capital all $t \geq 0$. There are $\tau(t)$ positive capacity levels at date t. An optimum maximizes J with respect to $(c,x,\mathrm{u}) \in \mathbb{R}_+^{1+M} \times [0,1]^{\tau(t)}$ subject to $c(t)/B + x(t) \leq m(t)$ and (2.32)-(2.36) all $t \geq 0$ given initial pollution $Z(0)$ and capacity levels, that is, maximizes Lagrange's function

$$\begin{aligned} \mathscr{L} = \sum_{t=0}^{\infty} \beta^t \Bigg\{ & U(c(t),Z(t)) + \epsilon(t)\Bigg[Z(t+1) - \Bigg(\sum_{v=-S}^{t-1} \sum_{s=0}^{\min[S-1,t-v-1]} E(t,s,v) + \rho x(t)\Bigg) \\ & - \phi(Z(t))\Bigg] + \sum_{v=-S}^{t-s-1} \sum_{s=0}^{\min[S-1,t-v]} q(t,s,v)[\bar{r}(t,s,v) - K(t+1,s,v)] + \lambda(t)[m(t) \\ & - x(t) - c(t)/B] + \beta w(t+1)\Bigg(\bar{K} - \sum_{v=-S}^{t-1} \sum_{s=0}^{\min[S-1,t-v]} K(t+1,s,v)\Bigg)\Bigg] + \lambda(t)\bar{G}(t)\Bigg\} \end{aligned}$$

where $\bar{G}$ collects the inequality constraints of input amounts $x(t)$, utilization rates $\mathrm{u}(t,s,v)$,

[40] The accounting equations of capacity hold for capital with γ and π replaced by unity. Capacity $K(t,s,v) = \chi(t-v-1,s,v)a(t,s,v)$ depends on productivity $\chi(t-v-1,s,v)$ and physical capital $a(t,s,v)$ such that $\chi(t-v,s,v) = \gamma(t-v,s,v)\chi(t-v-1,s,v)$ and $\chi(t-v,s,v) = \pi(t-v,s-1,v)\chi(t-v-1,s-1,v)$.

and capital $K(t+1,s,v)$, that form the admissable set. The term $\bar{r}(t,s,v)$ is the right side of (2.33) for $s=0$, (2.34) for $1 \leq s \leq \min[S-1,t-v]$ and $-S \leq v \leq t-s-1$, and (2.35) for $1 \leq s \leq \min[S-1,t-v]$ and $v=t-s$. The endowment of capacity at date zero consists of nonnegative stocks $\{K(0,S-1,-S),\ldots,K(0,0,-1)\}$. At least one of the elements of these vectors of length M is greater than zero. Consider a particular dirty technology with dropped index.

Improvement in the emission intensity of machine output in the vintage, $d(t-v,s,v) > d(t-v-1,s,v+1)$, is observed in reality. Machines of a given vintage v may become dirtier when they deteriorate, $d(t-v,s-1,v) < d(t-v,s,v)$ if $\gamma(t-v,s,v) < 1$, because the use intervals become shorter within a period and fuel is wasted, for example, during unscheduled ramp-down intervals and following ramp-up intervals in electricity production. Intuitively, a decrease of the emission intensity $d(t-v,s,v)$ of output in the vintage v or an increase of $d(t-v,s,v)$ in the number s of periods of past use yield an incentive to underutilize old plants, which have been used more often than young plants, to maximize output per unit of emission. Weakly increasing depreciation in the age $(t-v)$ or the number s of periods used, $\gamma(t-v,s,v) \geq \gamma(t+1-v,s,v)$ or $\gamma(t-v,s-1,v) \geq \gamma(t-v,s,v)$, seems plausible because scheduled maintenance time may increase but not decrease in age or use. Increasing depreciation in the age $(t-v)$ or in the number s of periods used intuitively favours young plants for underutilization, in order to postpone the use of capital with relatively low depreciation. Convexity or concavity of the dependencies of d or γ on vintage, use, or age does not matter for these arguments. The next proposition confirms the above intuition. For simplicity let there be vintage-dependent emission intensity $d(v)$ and use-dependent depreciation of capital, $\gamma(s)$, and durability $S=2$. Used capital depreciates at weakly greater rate than unused capital, $\pi(s) \leq \gamma(s)$.

Proposition 2.14 *There is a range of the cost of polluting $\theta(t)$ such that (i) vintage capacities $K(t,0,t-1) > 0$ and $K(t+1,1,t-1) > 0$, are fully utilized,* $\mathrm{u}(t,0,t-1) = \mathrm{u}(t+1,1,t-1) = 1$, *and (ii) underutilization of vintage capacity $K(t,1,t-2) > 0$, that is,* $\mathrm{u}(t,1,t-2) < 1$, *can be optimal, and there is no range of $\theta(t)$ for the 'reverse' statement* $\mathrm{u}(t+j,j,t-1) < 1$ *for $j \in \{0,1\}$ can be optimal when* $\mathrm{u}(t,1,t-2) = 1$ *is optimal, if the emission intensity of output improves over vintages, $d(v-1) > d(v)$, and unused capacity depreciates at constant rate, $\gamma(0) = \gamma(1)$. The situation is reversed if* *the emission intensity stays constant, $d(v-1) = d(v)$, and unused capacity depreciates at greater rate when used more often, $\gamma(0) > \gamma(1)$.*

Proof. There is a critical level of $\theta(t)$ for underutilization of each vintage $v \in \{t-1, \ldots, t-S\}$ in all periods in which it would be fully utilized without environmental effect and for utilization of younger vintages for all but the youngest vintage. Comparison of these critical values yields the results. Nonnegative input in investment at t satisfies

$$Qq(t, 0, t) \leq \lambda(t) + \rho\epsilon(t) \tag{2.37}$$

and builds capital with emission intensity $d(t)$. A unit of capacity with a lower emission intensity is worth more, $q(t, 0, t-1) \leq q(t, 0, t)$ if $d(t-1) \geq d(t)$. Underutilization of the youngest capital in pristine condition at t, that is, $\mathrm{u}(t, 0, t-1) < 1$, requires that $\lambda(t) - d(t-1)\epsilon(t) + \pi(0)q(t, 1, t-1) \leq \gamma(0)q(t, 0, t-1)$ all $t \geq 0$. Combination of these three results leads to $\theta(t) \geq \theta^*(0, t-1)$ given the critical level

$$\theta^*(s, v) = B\left(Q - \gamma(s) + Q\pi(0)q(v+1, 1, v)/\lambda(t)\right)/(\rho\gamma(s) + d(v)Q)$$

all $t \geq 0$. There is capacity $K(t+1, 0, t-1) > 0$ and $K(t+1, 1, t-1) > 0$ if $K(t, 0, t-1) >$ is partially utilized, $\mathrm{u}(t, 0, t-1) \in (0, 1)$. Utilization of capacity $K(t+1, 0, t-1)$ of yet unused $(t-1)$-vintage capital, $\mathrm{u}(t+1, 0, t-1) > 0$, requires that $\beta\{\lambda(t+1) - d(t-1)\epsilon(t+1) + \pi(0)q(t+1, 1, t-1)\} = q(t, 0, t-1)$. The utilization of capacity $K(t+1, 1, t-1)$ of the same vintage regarding the portion of capital used at t, that is, $\mathrm{u}(t+1, 1, t-1) > 0$, satisfies $\beta\{\lambda(t+1) - d(t-1)\epsilon(t+1)\} = q(t, 1, t-1)$. Then

$$q(t, 1, t-1) + \beta\pi(0)q(t+1, 1, t-1) = q(t, 0, t-1)$$

all $t \geq 0$. The valuation principle implies that $q(t, 1, t-2) \leq q(t, 1, t-1)$ if $d(t-2) \geq d(t-1)$. Underutilization of capacity $K(t, 1, t-2) > 0$, that is, $\mathrm{u}(t, 1, t-2) < 1$, requires that $\lambda(t) - d(t-2)\epsilon(t) \leq \gamma(1)q(t, 1, t-2)$ since $\pi(1) = 0$. Combination of (2.37), the comparison $q(t, 0, t-1) \leq q(t, 0, t)$ of shadow prices of different vintages, the comparison $q(t, 1, t-2) \leq q(t, 1, t-1)$ of shadow prices of different vintages, and the necessary condition for $\mathrm{u}(t, 1, t-2) < 1$, yield $\theta(t) \geq \theta^*(1, t-2)$ with $\beta\gamma(1)q(t+1, 1, t-1)$ in place of $q(t, 1, t-1)$ all $t \geq 0$. The necessary condition $q(t, 1, t-1) = \beta\gamma(1)q(t+1, 1, t-1)$ for u$(t+1, 1, t-1) < 1$ implies that $\theta(t) \geq \theta^*(1, t-2)$. For $\theta(t) \in [\theta^*(1, t-2), \theta^*(0, t-1))$ underutilization of vintage $(t-1)$ at t and $(t+1)$ is not optimal while the necessary conditions for optimal utilization of these vintages at $(t+1)$ and underutilization of vintage $(t-2)$ at t hold if $d(v-1) > d(v)$ and $\gamma(0) = \gamma(1)$. For $\theta(t) \in [\theta^*(0, t-1), \theta^*(1, t-2))$ the necessary conditions for optimal underutilization of vintage $(t-1)$ at t and $(t+1)$ hold and utilization of those vintages at $(t+1)$ and underutilization of vintage $(t-2)$ at t is not optimal if $d(v-1) = d(v)$ and $\gamma(0) > \gamma(1)$. *Q.E.D.*

The improvement in the emission intensity of output favours underutilizing old vintages to use relatively unpolluting technology—in the first case in the proposition. The increasing depreciation of unused capacity in the number of periods used yields incentives to underutilize young vintages to forward capital with relatively large productivity—in the second case in the proposition. At the initial date use and vintage are negatively correlated, and age and use are positively correlated, if capital was fully utilized before optimization of pollution. The effects of use or vintage on the emission intensity of output and of age or use on the retained fraction of productivity of unused capacity thus have counterveiling effects on the age of underutilized plants.

2.5 Conclusion

This chapter examined the optimal and competitive equilibrium retirement of capital in controlling an environmental stock. Underutilization that postpones the capital use widens the policy space compared to previous literature in managing an environmental stock that causes an externality, for example accumulated carbon dioxide that affects the climate. Pre-installed dirty technology capital, such as fossil-fuel using plants and engines, may be efficiently underutilized. (i) All or some of it is idle forever because capital in clean technology, for example, renewable energy technologies, should be growing to increase the cost of polluting through investment-related emissions, or (ii) the dirty technology capital is underutilized early until investment in dirty technology becomes worthwhile and both dirty and clean technology are used in the long-term. The emissions from investing in clean renewable energy technologies, for example, in steel and cement production, rationalize the former path in controlling climate change. This path requires that large renewable energy capacities can be constructed at sufficiently low cost. Technological improvement in recent years may have made this feasible or technological progress in the near future may make it feasible. Otherwise, when clean technology is insufficiently productive at large scale, the latter path seems relevant from today's perspective. On this path at some date the dirty technology capital stock is run down sufficiently so that its investment becomes worthwhile given the technology is used in the long-term. Dirty technology capital is idle on a path when clean technology capital decreases toward its long-term level. This path arises given the same parameters as in (i) and sufficiently large initial clean technology capacity. Some low-productivity clean technology capital, such as solar panels and wind turbines in low-harvest areas, are optimally underutilized if there is much capital in more productive clean technology types, such as high-yield renewable energy harvests. The reasons are the environmental impact of replacing pre-installed low-productivity capital and the high cost to construct it relative to its depreciation when stored unused. The underutilization of clean technology capital is temporary and occurs only if all pre-existing dirty technology capital is idle forever given the plausible assumption that unused solar panels and wind turbines depreciate relatively faster than mothballed coal power plants. This points to the future if governments continue to push expensive renewable energy technologies and there is sufficient capacity of least-cost renewable energy technologies. All capital is fully utilized in an equilibrium subject to no taxes or subsidies. Delayed government policy leads to underutilized dirty technology capital in an optimal policy because too much of it is built when emissions pricing is absent. A Pigouvian tax implements an optimum.

Underutilization of fossil-fuel using capital prevents a climate catastrophe when full utilization would lead to it. This occurs when atmospheric carbon, pollution in the terms here, is greater than its efficient long-term level. Dirty capacity is optimally underutilized when pollution is below or above this level. The atmospheric carbon stock may be below its optimized long-term level. However room for additional emissions may be limited by past emissions that accumulate with a lag. I refer to an extension in Chapter 1.

Optimal utilization heavily depends on the current capital stock, rather than on the current environmental stock as found in a fishery model in Clark et al. (1979) and Boyce (1995). Here storage of unused capital smooths the capital sequence toward its long-term level since the benefit of consuming is the opportunity cost of investing output or storing capacity. The fishing vessels are underutilized only if the fish stock is smaller than its optimum long-term level because the marginal cost of investment is exogenous. At sufficiently large installed dirty technology capital no investment in dirty technology is warranted. Investment in dirty technology is lacking for small and large levels of environmental quality unlike in the partial equilibrium fishery where investment is efficient only if the fish stock is greater than or equal to its long-term level, again because here the marginal cost of investment depends on the intertemporal marginal rate of substitution of consumption and there the marginal cost of investment is constant or depends only on the level of investment. The present study and their model differ in the regeneration function of the environment. An analogue to an equilibrium with uncompensated adverse impacts of pollution is the open-access situation in a fishery. A difference lies in the motion from such a situation to an optimum. Here underutilization becomes necessary to attain a social optimum after sufficiently long absence of emissions pricing (delayed government policy) in the decentralized economy. In contrast, the regeneration function of biomass usually leads to a rest point in the open-access regime that is efficiently escapable with full utilization. Optimal underutilization of dirty technology capital is restricted to a closed early time interval of the planning horizon if this technology is used in the long-term whereas underutilization can be optimal after full utilization in Clark et al. (1979). This difference emerges because here storage and utilization are linked so that capital tomorrow given full utilization today requires investment today while there capital depreciates less than fully regardless of utilization, and the utilization critically depends on the environmental stock, so the use of capital given no investment can deplete the fish stock and underutilization can follow full utilization.

The growth effects of delayed government policy on dirty output before a regime change to an optimal plan may lend controversy to the debate on climate change. In simulations I find that (1) emissions first increase from a low level when dirty technology capital is underutilized and then decrease when dirty technology capital is fully utilized in the optimum whereas emissions decrease from a high level in the constrained optimum with assumed full utilization starting at the same pollution and capital stock amounts, (2) the Pigouvian tax, which is proportional to the cost of polluting, is lower in these early periods in the optimum than in the constrained optimum, because underutilization mitigates societal effects of pollution, and (3) expensive clean technologies are not needed when the utilization of dirty technology capital can be chosen compared to when it is fully utilized by assumption, because of the mitigation.

The model has two limitations. The linear production functions that convert a common factor into new capital units and that use this capital to produce the factor absent effects of pollution on output lend the closed form solution to a critical level of the cost of polluting at which both investment and underutilization in the dirty technology are optimal and its critical level at which dirty technology capital should not be used. These parametric levels are helpful in proving results about the timing of utilization of dirty technology capital and finding different regions of underutilized and utilized (dirty or clean) technology capital in the state space. As a result of the linearity, dirty technology capital is efficiently underutilized when government policy is sufficiently long delayed given dirty technology capacity can become sufficiently large. One question for further research is under what conditions does a laissez-faire economy converge in the long-term to a state that is characterized by efficient underutilization when there are decreasing returns to scale in using the factor in producing consumption goods and investment goods. In addition this analysis may be useful to determine if the environmental regeneration function alone is responsible for the difference to the fishery example regarding the optimal utilization in a laissez-faire steady state. In the model pollution affects utility. The level of the cost of polluting for optimal joint investment and underutilization in the dirty technology is smaller for greater pollution if pollution affects output. This effect retains the importance of the dirty technology capital stock for its underutilization when pollution is below its optimal long-term level, when the pollution effect on output is limited from above for small pollution. Implications of this effect for optimal utilization in a laissez-faire state in the long-term may be researched in further work.

There is an equivalence in the model between all machines being utilized at some fraction and this fraction of all machines producing and the remainder of machines being idle. The latter makes recycling possible. Unused capital may be saved for later use rather than being decommissioned assuming that pausing is less costly than recycling, for example from dirty technology to clean technology. Further research may find the margins of optimal pausing and recycling. Idle dirty technology capital could be recycled.

The analysis assumed equally endowed households. The postponement of use of dirty technology capital has likely only mild distributional effects in reality, because its improvement in the emission intensity over vintages and its greater depreciation of unused units that are older because they have been used more often have opposite effects on the optimal age of underutilized plants and engines. The idling of pre-installed capital has relative wealth effects between households in reality depending on the endowments of idle and utilized units if policy that implements the underutilization is not accompanied by compensating asset owners for foregone revenue. This might be important, because energy-use capital, such as buildings, equipment, and roads, is underutilized when energy production capital is underutilized in the initial period in a model with energy-use capital that is putty-clay in terms of its energy efficiency. Thus a considerable portion of aggregate wealth may be affected. Further research should examine if some types of energy-use capital are optimally idle forever if some fossil-fuel using capital is optimally idle forever.

The efficient stranding of assets suggests a political economy dimension to the climate problem worth studying. In some environmental problems capital can be modified at low cost to reduce negative environmental effects of capital use—through catalyst in vehicles whose use pollutes air locally, scrubbers in coal power plants that emit sulfur dioxide, and equipment in plants that produce refrigerants responsible for ozone depletion in the atmosphere, or replaced at low cost—lead-containing water pipes. In a fishery capital that is efficiently underutilized may have little value if it was fully utilized—old fishing boats. In contrast, fossil-fuel using plants and engines likely cannot be converted into zero-emissions capital at low cost (but there is a call for research on determining the cost), alternative renewable energy technology is more expensive at large scale, and recently constructed buildings, coal power plants and vehicles have a high market use value. While ownership and use of capital span globally, the stranding of installed capital is efficient with or without effects of carbon on households in different regions that have own governments. Because of these characteristics an international agreement among the main emitters on emissions plans may be slower for climate change than for the transboundary problem of acid rain and global problem of ozone-depleting substances. Compensation of foregone revenue to implement an optimum is an avenue for future research to explain the lack of government policy that excludes it and improve upon it. Second government policy that implements underutilization of fossil-fuel capital may motivate the observed lobbying interest in carbon storage and air capturing. The efficiency of investment in such technologies to prolong the use of pre-installed capital or perpetuate investment in fossil-fuel technology in the realm presented here is a topic for further research.

There are two cases of stranded assets though the environment can fully revert to non-hazardous levels. In the first case, capital of dirty technology is underutilized at all dates and does not receive additions through investment. The same rationale may hold given uncertain clean technology improvements as a topic for future research. In the second case capital depreciates regardless of its utilization and is efficiently underutilized in the initial period. In contrast, some climate-policy discussants argue in favour of retiring (expected future) rents from extracting fossil fuels to prepare for uncompensated Pigouvian climate policy because more carbon may be valued in assets today than should be used in the future since the effects of carbon may become irreversible beyond some carbon level (Carbon Tracker & Grantham Research Institute on Climate Change and the Environment 2013), or to make efficient climate policy with compensation under spillovers (Böhm 1993, Hoel 1994, Harstad 2012).

Empirical work is needed to find if underutilization of capital in energy production or energy use is currently optimal, or when it will be optimal under given courses of government policy. Such work may use several extensions of the present chapter.

2.6 Appendix A: Properties of Pareto optimal allocation

Multiple technologies. The following provides a composition of the vector G of inequality constraints and derives the transversality conditions which are helpful in proving the uniqueness of an optimal plan. Let u be the matrix with utilization rates of technologies with positive capital on the main diagonal and zeros elsewhere, denote e as the column vector of ones, and let I be the identity matrix. Then $G = [\sum_j (\mathrm{u}_j K_j - x_j) - c/B; x; \mathrm{ue}; (\mathrm{I}-\mathrm{u})\mathrm{e}; K]$ using vectors K of capital and x of input in investment. A planner chooses a sequence of control variables to maximize $J = \lim_{T\to\infty} J(T)$ given values of inherited pollution and capital stocks at date zero, where

$$J(T) = \sum_{t=0}^{T-1} \beta^t \{\mathscr{H}(t) - \epsilon(t)[Z(t) - Z(t+1)] + \sum_j q_j(t)[K_j(t) - K_j(t+1)] + \beta w_j(t+1)[\bar{K}_j - K_j(t+1)] + w(t)G(t)\} + \beta^T [v(T) + w(T)G(T)]$$

contains the current value Hamiltonian function $\mathscr{H}(t) = U(c(t), Z(t)) - \epsilon(t) r_Z(t) + \sum_j q_j(t) r_{K_j}(t)$, all $0 \le t \le T-1$. Here $v(T)$ is some function of $(Z(T), K(T))$. Then an optimal plan satisfies the transversality conditions $\lim_{t\to\infty}\beta^t\epsilon(t) = 0$ and

$$\lim_{t\to\infty}\beta^t[q_j(t)+\beta w_j(t+1)] \geq 0, \quad \lim_{t\to\infty}\beta^t[q_j(t)+\beta w_j(t+1)]K_j(t+1) = 0 \tag{A-3}$$

based on the following arguments. 1. *Pollution.* Define λ_Z and λ_{K_j} as the multipliers on the nonnegativity constraints $Z \geq 0$ (if one imposes it, to show that it does not matter) and $K_j \geq 0$, respectively. The differential of the limit of the present value function $\beta^T v(T)$ as T tends to infinity with respect to economic state variables $Z(T)$ and $K_j(T)$ is zero if $\beta < 1$. Then $\lambda_Z \geq 0$, $\epsilon \geq 0$, and the differential

$$\frac{\partial}{\partial Z(T)}\lim_{T\to\infty} J(T) = \lim_{T\to\infty}\beta^{T-1}\{\beta\lambda_Z(T)+\epsilon(T-1)\} = 0,$$

imply that the present value shadow price of pollution is zero in the long-term. 2. *Available capital.* The nonnegativity of λ_{K_j}, the differential

$$\frac{\partial}{\partial K_j(T)}\lim_{T\to\infty} J(T) = \lim_{T\to\infty}\beta^{T-1}\{\beta\lambda_{K_j}(T) - q_j(T-1) - \beta w_j(T)\} = 0,$$

and the nonnegativity constraints on capital, imply (A-3). The sum of the discounted shadow price of capital and discounted land rental rate is nonnegative.

In the Section 2.4.2 the constraint $\mathrm{u}_R K_R - \alpha_B \mathrm{u}_B K_B \geq 0$ is added to G all $t \geq 0$.

The following lemma establishes the sufficiency of necessary optimality conditions, and conditions for uniqueness of an optimal plan. This lemma was written earlier for a version with delay τ of emissions on pollution. (i)-(iv) refers to the necessary optimality conditions (2.6)-(2.8).

Lemma 2.7 *A plan that satisfies the necessary conditions (i)-(iv) and the transversality conditions (A-3) maximizes J. A unique plan maximizes J if U or A is strictly concave in Z at any Z.*

Proof. Any feasible control $v(t) = (c(t), x(t), \mathrm{u}(t))$ uniquely maps feasible $(Z(t+\tau), K(t))$ into feasible $(Z(t+\tau+1), K(t+1))$. Thus an admissable sequence $\{v(t)\}_{t=0}^{\infty}$ determines a unique sequence of state variables, in each period on $\mathbb{R}\times[0,\bar{K}_{\mathrm{B}}]\times[0,\bar{K}_{\mathrm{C}}]$, given initial values $(Z(\tau), K(0))$. The choice $\tilde{v}(0)$ maximizes

$$\mathscr{L}(0) = U(c(0), Z(0)) - \epsilon(0) r_Z(Z(0), K(0), v(0)) \\ + q(0) r_K(K(0), v(0)) + w(0) G(K(0), v(0))$$

at given costate variable values $\epsilon(0)$ and $q(0)$ and Lagrange multpliers $w(0)$ if $\tilde{v}(0)$ satisfies (i) of the maximum principle for $t = 0$. Thus

$$U(c(0), Z(0)) - U(\tilde{c}(0), Z(0)) \leq \epsilon(0)[Z(1) - \tilde{Z}(1)] - q(0)[K(1) - \tilde{K}(1)]$$

follows from definition of r_Z, r_K, and $\epsilon(0) = \beta^\tau \epsilon^*(\tau)$. Let the state variables $Z_\tau(t) = (Z(t), Z(t+1), \ldots, Z(t+\tau))$ and $K_\tau(t) = (K(t-\tau), K(t-\tau+1), \ldots, K(t))$ for $t \geq \tau + 1$ arise from a particular choice $(v(0), v(1), \ldots)$. The function

$$\mathscr{L}_\tau(t) = \sum_{s=t}^{t+\tau} \beta^{s-t} \big\{ U(c(s), Z(s)) - \epsilon^*(s) r_Z(Z(s), K(s-\tau), v(s-\tau)) \\ + \beta^{-\tau} q(s-\tau) r_K(K(s-\tau), v(s-\tau)) + \beta^{-\tau} w(s-\tau) G(K(s-\tau), v(s-\tau)) \big\}$$

is jointly concave with respect to all state variables that are its arguments in $(\tilde{Z}_\tau(t), \tilde{K}_\tau(t))$ if one of these state variable values is interior because U and A are concave in Z. Thus interior pollution all $t \geq \tau + 1$ implies concavity. Furthermore strict concavity follows if U or A is strictly concave in Z. The function $\mathscr{L}_\tau(t)$ is differentiable with respect to state variables since U and A are differentiable with respect to pollution. Then

$$\mathscr{L}_\tau(Z_\tau(t), K_\tau(t), \ldots) \leq \mathscr{L}_\tau(\tilde{Z}_\tau(t), \tilde{K}_\tau(t), \ldots) \\ + \sum_{s=t}^{t+\tau} \frac{\partial \mathscr{L}}{\partial Z(s)} (\tilde{Z}_\tau(t), \tilde{K}_\tau(t), \ldots)[Z(s) - \tilde{Z}(s)] \\ + \frac{\partial \mathscr{L}}{\partial K(s-\tau)} (\tilde{Z}_\tau(t), \tilde{K}_\tau(t), \ldots)[K(s-\tau) - \tilde{K}(s-\tau)]$$

given $(\epsilon^*(t), \epsilon^*(t+1), \ldots, \epsilon^*(t+\tau), q(t-\tau), q(t-\tau+1), \ldots, q(t))$, holds at strict inequality if U or A is strictly concave in Z. The classic proof of Arrow's Theorem in a (continuous time) setting without delay defines $\mathscr{L}_0(t)$ as a Lagrange function that contemporaneous control variables maximize, and invokes an envelope condition to show that the partial differential of $\mathscr{L}_0(t)$ with respect to a contemporaneous state variable involves a necessary adjoint equation from the maximum principle. The necessary condition with respect to $v(t)$ does not seem to be a prerequisite for construction of some function that is concave in state variables and whose differential with respect to any such state variable involves a necessary condition at given values of control variables. Here consumption $(c(t), c(t+1), \ldots, c(t+\tau))$ does not maximize $\mathscr{L}_\tau(t)$ if $\tau \geq 1$, so that the necessary condition of maximization of some function with respect to $v(t)$ is not used, except at $t = 0$. The differentials $\partial \mathscr{L}_\tau(\tilde{Z}_\tau(t), \tilde{K}_\tau(t), \ldots)/\partial Z(s) = \epsilon^*(s) - \beta^{-1}\epsilon^*(s-1)$ and $\partial \mathscr{L}_\tau(\tilde{Z}_\tau(t), \tilde{K}_\tau(t), \ldots)/\partial K(s-\tau) = \beta^{-\tau}[\beta^{-1}q(s-\tau-1) - q(s-\tau)]$ for $t \leq s \leq t+\tau$ evaluated at $(\tilde{Z}_\tau(t), \tilde{K}_\tau(t))$ represent conditions (ii) of the maximum principle. (If U is additively separable in functions of each c and Z then a function $\mathscr{L}_\tau(t)$ of $(Z(t+\tau), K(t))$ is definable that $v(t)$ maximizes). Rearranging and simplifying terms yields

$$\begin{aligned}
&\sum_{s=t}^{t+\tau} \beta^{s-t}[U(c(s), Z(s)) - U(\tilde{c}(s), \tilde{Z}(s))] \\
&\leq \epsilon(t)[Z(t+\tau+1) - \tilde{Z}(t+\tau+1)] - \beta^{-1}\epsilon^*(t-1)[Z(t) - \tilde{Z}(t)] \\
&\qquad - q(t)[K(t+1) - \tilde{K}(t+1)] + \beta^{-\tau-1}q(t-\tau-1)[K(t-\tau) - \tilde{K}(t-\tau)]
\end{aligned}$$

for $t = \tau+1, 2\tau+2, 3\tau+3, \ldots$. Then summation of these discounted terms over t implies that

$$\begin{aligned}
&\sum_{t=\tau+1}^{\infty} \beta^t[U(\tilde{c}(t), \tilde{Z}(t)) - U(c(t), Z(t))] \\
&\geq \lim_{t\to\infty} \beta^t\{-\epsilon(t)[Z(t+\tau+1) - \tilde{Z}(t+\tau+1)] + q(t)[K(t+1) - \tilde{K}(t+1)]\} \\
&\qquad + \beta^\tau \epsilon^*(\tau)[Z(\tau+1) - \tilde{Z}(\tau+1)] - q(0)[K(1) - \tilde{K}(1)]
\end{aligned}$$

where $\epsilon(t) = \beta^t\epsilon^*(t+\tau)$. The limit of the product of the respective discounted shadow price $\epsilon(t)$ and pollution $\tilde{Z}(t+\tau+1)$ vanishes if the transversality condition with respect to pollution holds given policy indexed by tilde. The limit term $\lim_{t\to\infty}[-\beta^t q(t)\tilde{K}(t+1)]$ can be replaced by $\lim_{t\to\infty} \beta^{t+1}w(t+1)\tilde{K}(t+1) \geq 0$ if the transversality conditions (A-3) hold for the policy $\tilde{v}(t)$, $t = 0, 1, \ldots$. Then addition to the difference of utility at $t = 0$

yields

$$U(\tilde{c}(0), \tilde{Z}(0)) - U(c(0), Z(0)) + \sum_{t=\tau+1}^{\infty} \beta^t [U(\tilde{c}(t), \tilde{Z}(t)) - U(c(t), Z(t))]$$
$$\geq \lim_{t\to\infty} \beta^t \{-\epsilon(t) Z(t+\tau+1) + q(t)K(t+1) + \beta w(t+1)\tilde{K}(t+1)\}$$

at strict inequality if U or A is strictly concave in Z. The results follow if the limit of the discounted value of pollution is zero, because then the latter line is nonnegative. The limit is nonnegative if Z is bounded from above since the limit of the shadow price of pollution is zero. Now Z is bounded from above on any feasible plan because the capacities to produce output are bounded from above. *Q.E.D.*

Proof of Proposition 2.1. (i) Existence. A plan satisfies feasibility conditions and yields finite welfare J. The policy $c(t) = B(1 - 1/Q_B) \min(K_B(0) + K_C(0), \bar{K}_B)$, $x_B(t) = (1/Q_B) \min(K_B(0) + K_C(0), \bar{K}_B)$, $x_C(t) = 0$, $\mathrm{u}_B(t) = 1$ for $t \geq 0$, and $\mathrm{u}_C(0) = 1$ if $K_C(0) > 0$, is feasible, and yields finite J. Thus not all policies yield welfare $-\infty$. There is at least one plan that cannot be improved upon because the compact state space, feasible choices of control variables (c, x, u) in closed and bounded sets, and discounting imply that J is bounded from above, the utility function U is continuous in c and Z. (ii) Uniqueness. By Lemma 2.7 a plan that satisfies the necessary conditions (i)-(iv) and the transversality conditions maximizes J, and uniquely so if U or A is strictly concave in Z at any Z. This follows because emissions have immediate impacts on pollution. For any state $(Z, K_B, K_C)(t)$ a policy at t maps one-to-one into the state (Z, K_B, K_C) at $(t+1)$. Thus, the optimal state trajectory is unique. *Q.E.D.*

The convexity of the correspondence of current states that describes the feasible set of successor states, and the strict concavity of utility in consumption, are not sufficient for a unique optimal plan with an interval of underutilization, when proposing two policies and examining a convex combination. The reason is that technology j's output in period $(t+1)$ is proportional to $\mathrm{u}_j(t+1)(1 - \mathrm{u}_j(t))$. Chow (1997) uses a perturbation of J evaluated at a given policy that satisfies necessary optimality conditions to argue that the optimal plan is unique given convex transition laws and concave return function both in controls and states. This method does not apply at corner solutions, for example, with idle capital, fully utilized capital, or no investment.

Depreciation and clean technology productivity. Condition 2.1 or 2.2 can be used to prove full utilization of dirty technology capital after investment in the dirty technology. The next lemma shows the result $R(t+1) > \gamma_B$ that is useful given emission-free investment.

Lemma 2.8 *The shadow return exceeds the retained portion of unused dirty capacity, $R(t+1) > \gamma_B$, if Condition 2.1 holds, dirty capacity $K_B(t) > 0$ is utilized,* $\mathrm{u}_B(t) > 0$*, and dirty capacity $K_B(t+1)$ is smaller than its upper bound $\bar{K}_B$.*

Proof. Pollution $Z(t+1)$ is free. Thus (2.5) and Condition 2.1 imply that $\epsilon(t) > \beta\gamma_B\epsilon(t+1)$. Then $\mathrm{u}_B(t) > 0$ in (2.8) shows that $\beta\gamma_B(\lambda(t+1) - b\epsilon(t+1)) - \beta w_B(t+1) \leq \gamma_B q_B(t) \leq \lambda(t) - b\epsilon(t)$ given $w_B(t+1) = 0$. Thus $(B/b)(R(t+1) - \gamma_B) \geq (\epsilon(t)/\beta\epsilon(t+1) - \gamma_B)\theta(t+1)$ implies the result. *Q.E.D.*

This lemma implies an upper bound on the growth of the cost of polluting, that the path in Proposition 2.2 with increasing cost of polluting satisfies. The inverse of the growth rate of the discounted willingness to pay to reduce pollution, $(\beta\epsilon(t+1)/\epsilon(t))^{-1}$, is bounded from below by a greater amount γ_B, provided that pollution is persistent relative to capital, if postponing the use of capital is more attractive to mitigate this cost, γ_B is greater.

Lemma 2.9 *The relation $\gamma_j(B + \rho_j\theta(t+1)) \geq R(t+1)(B + \rho_j\theta(t))$ holds if investment, $x_j(t) > 0$, preceded underutilization of capital $K_j(t+1) > 0$,* $\mathrm{u}_j(t+1) < 1$.

Proof. Investment in period t implies that $K_j(t+1) > 0$. Underutilizing $K_j(t+1) > 0$ requires that $\beta\gamma_j q_j(t+1) = q_j(t)$ from (2.8). Then investment satisfies $\beta\gamma_j(\lambda(t+1) + \rho_j\epsilon(t+1)) \geq \beta\gamma_j Q_j q_j(t+1) = Q_j q_j(t) = \lambda(t) + \rho_j\epsilon(t)$ by (2.7). *Q.E.D.*

Then Condition 2.1 can be used in an economy without emissions from investment in the dirty technology to derive full utilization after investment in the dirty technology.

The following interprets Condition 2.2 and examines its plausibility within the model. This assumption means that large investment in clean technology is feasible at sufficiently large productivity so that the return on investment in clean technology, $Q_C/(1 + (\rho_C/B)\theta(t))$ does not exceed the marginal rate of substitution of consumption $R(t+1)$. The first inequality in Condition 2.2 tells that this rate of return on investing in clean technology at t is greater than the rate of return from storing dirty technology capital, γ_B, when $\theta(t) \leq \theta_B$. This relation is needed if dirty technology is relatively less emission-intensive in investment. The second inequality in Condition 2.2 implies that investing in clean technology is preferable to investing in dirty technology at a date $(t-1)$, $Q_C/(1+(\rho_C/B)\theta(t-1)) > Q_B(1-(b/B)\theta(t))/(1+(\rho_B/B)\theta(t-1))$, if $\theta(t) \geq \theta_B$ and dirty technology is relatively more emission-intensive in investment. These conditions coincide if $\rho_C = \rho_B$ by definition of θ_B. If $Q_C < Q_B$ then the first inequality $Q_C > \gamma_B(1+(\rho_C/B)\theta_B)$ requires that $\gamma_B(\rho_C - \rho_B) < Q_B d_B$, which is consistent with $\rho_C - \rho_B < \beta d_B Q_B$. This plausible relation means that clean technology is less polluting than dirty technology. The second inequality $Q_C > Q_B(1-(d_B/B)\theta_B)$ is consistent with $\rho_B \geq 0$ and $Q_B > \gamma_B$.

Lemma 2.10 *Pollution increases on* $\{t', t'+1, \ldots, t''\}$ *if* $Z(t'-1) < Z(t')$, *and (i) emissions strictly increase,* $E(t-1) < E(t)$, *and* $A(Z) = Z$ *or (ii) emissions weakly increase,* $E(t-1) \leq E(t)$, *and* $A(Z) < Z$.

Proof. (i) $A(Z) = Z$. The result is obvious. (ii) $A(Z) < Z$. Then $(Z - A(Z))$ strictly increases in Z. Thus $Z(t') - A(Z(t')) > Z(t'-1) - A(Z(t'-1))$. Put this to use in $Z(t') + [E(t'-1) - A(Z(t'))] > E(t'-1) + Z(t'-1) - A(Z(t'-1)) = Z(t')$ so that $A(Z(t')) < E(t'-1) \leq E(t')$ yields that $Z(t') < Z(t'+1)$. Induction implies the result. *Q.E.D.*

2.7 Appendix B: Algorithms

(i) Strictly concave utility in pollution and one clean technology. Values for θ_B range from about 10 to 30 for $\gamma_B \in [0,1]$. The number 15.46 results from the assumed $\gamma_B = 0.72$. States with the cost of polluting $\theta(t) \leq \theta_B$ in constrained optimum are those in the region R1 of investment and full utilization in the dirty technology in Figure 2.2. The stationary point to which the trajectories converge is in this region because the cost of polluting at a stationary point with investment in the dirty technology is less than θ_B. A corollary to the first result in Lemma 2.1 is that dirty technology capital $K_B > 0$ is fully utilized if $\theta(t) < \theta_B$.

Now consider states with greater dirty capacity than in R1. A contraposition to the second result in Lemma 2.1 is that either investment occurs and capital is idle, or no investment occurs, if $\theta(t) > \theta_B$.[41] Investment and idle capital, $x_B > 0$ and $\mathrm{u}_B = 0$, for K_B close to its long-term level would yield a large change in the savings rate between states that are close to each other—states for which full utilization is optimal and states debated here, since clean technology capital is small in the example. This cannot be optimal. One can disregard no investment and full utilization, $x_B = 0$ and $\mathrm{u}_B = 1$, because lower positive available investment levels were not chosen by the program in solving the constrained problem. Then for $\theta(0) > \theta_B$ at the solution to the constrained problem capital is underutilized, $\mathrm{u}_B(0) \in [0, 1)$, in the Pareto optimum. Thus the curve for θ_B forms the boundary of the regions R1 of full utilization and investment (that includes this curve) and R2 of partial utilization and investment.

Let S be a set of triples of pollution, dirty capacity, and clean capacity numbered $i = 1, 2, \ldots, M$, and denote by S_i the feasible subset of states immediately following state $(Z(i), K_B(i), K_C(i)) \in S$. The set of S_i all i is a state correspondence. This set is found by first computing emission levels from transitions $Z = E(i) + (1-\varphi)Z(i)$ between levels of pollution in a given set, building the corresponding set of dirty capacity levels at full utilization, $E(i) = b\mathrm{u}_B K_B(i)$ at $\mathrm{u}_B = 1$, and by blocking all transitions subject to the resource constraint $c/B + x_B + x_C = \mathrm{u}_B K_B(i) + \mathrm{u}_C K_C(i)$, and the transition law $K_j = \gamma_j(1 - \mathrm{u}_j)K_j(i) + Q_j x_j$ for $j \in \mathcal{J}$, but those with $\mathrm{u}_B \in [0, 1]$, $\mathrm{u}_C \in [0, 1]$, $x_B \geq 0$, $x_C \geq 0$, and $c > 0$. Let v_0 be some initial function of states. The algorithm for finding an optimum is iterating on the problem

$$\begin{aligned} &v_{s+1}(Z(i), K_B(i), K_C(i)) \\ &\quad = \max_{c, x_B, x_C, \mathrm{u}_B, \mathrm{u}_C} \{U(c, Z(i)) + \beta v_s(Z, K_B, K_C)\} \end{aligned}$$

for $(Z, K_B, K_C) \in S_i$ all i increasing $s \in \mathbb{N}$ until the norm $\{\sum_{i=1}^{M}(v_{s+1}(i) - v_s(i))^2\}^{1/2}$ is smaller than the tolerance level 10^{-7}. The constrained optimum results by setting $\mathrm{u}_B = \mathrm{u}_C = 1$ to attain the state correspondence.

The optimum level $\theta(t)$ at a date before joint underutilization and investment results from backward calculation using (2.9), while the optimum level $\theta(t)$ afterwards comes from forward solving (2.10). The constrained optimal $\theta(t)$ for $t \geq 1$ is the series that solves (2.10) forward. The backward calculation using (2.11) yields very similar values.

[41] Closed orbits of pollution and capital with such idleness may satisfy some necessary optimality conditions but violate the transversality condition of the shadow price of pollution.

In the constrained optimum the initial level is unknown, so that comes from backward solving (2.11) using a terminal date after plans have converged on the grid. The boundary curves are obtained using the results in proving Proposition 2.5 by fitting state values to the function $K_B = r_1 + (r_2 - Z)/(r_3 + Z)$ with the Gauss-Newton method. This function yields a high coefficient of determination 0.90 (R1-R2) and 0.72 (R2-R3).

1. Compute $\theta(0)$ for all states and minimize the distance to θ_B for given Z to find states with θ close to θ_B from constrained optimization.
2. Estimate the boundary of R1 and R2 using these states.
3. Find their successors. Set $x_B = 0$ to compute the predecessors and use them to estimate the boundary of R2 and R3.

Zero investment, $x_B = 0$, is not feasible on the grid in the unconstrained optimization. Inclusion of this value would increase the grid space enormously because only one control governs the transitions of two states Z and K_B when $x_B = 0$ and $\mathrm{u}_B \in (0, 1)$, so it is better avoided. Instead the solution with $x_B = 0$ is approximated by small positive x_B. The same problem with the grid space would arise if capital is fully utilized and there are emissions of investment. This problem can be circumvented by the second specification.

(ii) Constant marginal utility of pollution and multiple clean technologies. Both utility U and absorption A are linear in pollution. The environmental shadow cost $\epsilon(t) = \beta d/(1 - \beta(1 - \varphi))$ all $t \geq 0$ uniquely solves the unstable difference equation (2.5) and satisfies both $\epsilon(t) \geq 0$ and the transversality condition $\lim_{t\to\infty} \beta^{t-1}\tilde{\epsilon}(t) = 0$ given $\epsilon = \beta\tilde{\epsilon}$. Let $K = (K_B, \int_C K_C dC)$ collect the states dirty technology capital and aggregate clean technology capital. The value function has the form $v(Z, K) = \tilde{v}(K) - \tilde{\epsilon}Z$ and satisfies the Bellman equation

$$\tilde{v}(K) = \max_{c,x,\mathrm{u}} \{c^{1-\psi}/(1-\psi) - \epsilon E + \beta\tilde{v}(K')\}$$

subject to utility function $c^{1-\psi}/(1-\psi) - \epsilon E$ and the same laws of motion of the states where prime denotes next period. In this reduced problem the current utility depends on consumption c and emissions E.

The laissez-faire trajectories solve the necessary equilibrium conditions. I deploy the following method given the sum of the equation $c/B + x_B + \int_C x_C dC = m_B + \int_C m_C dC$

from $t = 0$ to $t = t_B - 1$ weighted by $\sigma_t = (1/Q_B)^t$.

1. Choose the least t_B such that $c(t_B) = (\beta Q_B)^{t_B/\psi} c(0)$, using $c(0)$ from

$$\sum_{t=0}^{t_B-1} \sigma_t c(t) + \sigma_{t_B} \bar{K}_B = K_B(0) + \sum_{t=0}^{t_B-1} \sigma_t [K_C(0) - x_C(0)] + \sigma_{t_B-1} i(t_B - 1)$$

   at $i(t_B - 1) = \int_C (x_C(t_B - 1) - x_C(0))dC = 0$, is greater than $c(t_B)$ from the policy on $\{t_B, t_B + 1, \ldots\}$ given $i(t_B - 1) = 0$ that grid search yields. Then increase t_B to the greatest number that satisfies $K_B(t_B - 1) \leq \bar{K}_B$. The resulting consumption amount exceeds $c(t_B)$ in the future-looking policy.

2. Find $\int_C x_C(t_B - 1)dC$ and the future-looking policy including some grid points $\int_C x_C dC \in [x_\ell, x_h]$. Iterate on x_ℓ and x_h until $c(t_B) = (\beta Q_B)^{(t_B-1)/\psi}(\beta R(t_B))^{1/\psi}$ and $c(t_B)$ from the policy on $\{t_B, t_B+1, \ldots\}$ have sufficiently converged. The shadow return $R(t_B) = Q(\int_C x_C(t_B - 1)dC)/(1 + (\rho_C/B)\theta(t_B))$ is evaluated at the updated policy $\int_C x_C(t_B - 1)dC$.

The constrained optimal resource policies result from a grid search as described in the previous example to find $\tilde{v}$. The grid contains dirty capacity and aggregate clean capacity levels. Optimal policies that involve underutilization are computed using first-order optimality conditions. The value of $\theta_B = \epsilon/B(\partial U/\partial c)$ yields consumption c^* in R2. Then consumption, input in investment of clean technologies, and the cost of polluting can be solved backwards, for paths starting in R2 or R3, using the condition $\beta\gamma_B(B\partial U/\partial c(t + 1) - b\epsilon) = B\partial U/\partial c(t) - b\epsilon$ that is equivalent to (2.9). They can be solved forward in R1 on these paths using (2.10). The boundary of R1 and R2 approximates grid points with consumption policy c^* in the constrained optimization. Here states with $\theta(0)$ close to θ_B are those with squared distance up to 0.1. The boundary of R2 and R3 is found using the method in proving Proposition 2.5 given the successors $K_B(t + 1)$ and $\int_C K_C(t + 1)dC$ to the grid points with consumption policy c^*. The utilization rates on paths that start in R3 follow by examining the time span τ to the date of R2 and successively raising τ until $x_B(\tau) > 0$ knowing the backward solution of consumption c and input $\int_C x_C dC$ in clean technology investment, and the state $\int_C K_C dC$.

3

Capacity planning with supply uncertainty of clean technology

This chapter examines implications of daily average fluctuation of clean production for the Pareto optimal distribution of consumption and investment and analyses government policies that may or may not implement a Pareto optimum. The productivity of wind and solar energy conversion into electricity varies over the course of a day, on average daily within a season, and across seasons of the year. This production does not create carbon dioxide emissions or fuel waste. Electricity production using fossil fuel or nuclear material can be stable yet polluting. Using fossil fuels creates carbon dioxide emissions. The use of nuclear fission technologies leaves radioactive spent fuel.

There are five major findings. (i) Consumption can be equalized across days because investment absorbs the fluctuation in clean technology productivity in days in which consumption is maximized. Dirty technology backs up production in days when the productivity of clean technology is low which leads to low consumption, yet the underutilization of dirty technology capital in days when the clean technology's productivity is high requires optimally dissipating profit that implies maximum consumption and thus may not smooth consumption across all days with different wind strengths in a long period in which capital is built. (ii) I show the need of an excise tax or a contingent ad valorem tax rather than a noncontingent ad valorem tax to implement contingent underutilization through indirect taxes. These tax rates can be expressed in terms of the Pigouvian emissions tax. (iii) A clean output subsidy can implement a Pareto optimum. This subsidy may rebate a uniform output tax or equipment tax, or be funded by a discriminatory surcharge between households. In the former two systems a tax on capital purchases accounts for the societal effects of emissions in building capital. Differentiating the surcharge between households moves clean technology unit revenue closer to the price of its output than under a uniform surcharge and does not affect the relative price of consumption goods and investment goods. The system with output tax is preferred among these variants of a tax-rebate system if dirty technology capital is unequally utilized across states of clean technology productivity in optimum, because it induces the efficient utilization of capital. The system with taxed investment to internalize both the marginal effects of emissions in using and building capital does not induce underutilized capital. The system with surcharge and emissions tax that internalizes the externality in the investment sector does not induce underutilized capital. (iv) Clean technology users may not know the state, for example, cannot access wind forecasts, and clean output buyers that use equipment may direct their demand to contingent prices. This information asymmetry does not substantiate government intervention. Competitive distributors can stream contingent payments into a stable price. (v) A clean technology output subsidy fully-funded with uniform surcharges leads to overinvestment in both dirty and clean technology relative to an optimum when the optimal marginal real rate of return on investment and a weakly smaller portion of clean technology output relative to its optimal level are implemented. Clean technology earnings are too large relative to the price of output, so that there is an overinvestment in clean technologies. Then dirty technology output is too large given the relation of relative output of dirty and clean technologies. Exempting investment goods producers from the surcharge as practiced in Germany cannot implement the optimal real rate of return on investment unless emissions in the investment sector are priced.

The desirable scale of clean energy depends on the relative cost of installing dirty and clean technology capital and their available production capacities (Heal 2009). In particular, electricity can provide heating and cooling, light, and mechanical energy. Energy from fluctuating renewables supply can be stored only expensively for later transformation into electrical energy. I do not consider storage (for example, in pumped water, battery, or hydrogen) because it is currently expensive over many days independent on the location. Therefore, the availability of clean energy harvests affects optimal policy.

I use an extension of Chapter 2's model to multiple subperiods to analyse technology-specific investment and dispatch of capital use. The sequence of states is uncertain. A given period is segmented into subperiods of which each exhibits a particular productivity of clean technology capital. Users of dirty or clean technology output learn the clean technology productivity before the capital utilization can be chosen.

Relation to literature. The paper relates to previous studies of the capacity utilization of electricity generators, indirect taxation to internalize an externality, and fully-funded subsidies to clean technology. The peak-load pricing literature reviewed by Crew et al. (1995) uses models with efficient underutilization of capital motivated by time-varying demand. This variation may be within each day. Capital is underutilized when consumption is small. In Ambec & Crampes (2012) and the present paper varying success of one technology, for example, across days, induces underutilization of capital of another technology. Then capital is underutilized when consumption is large. Ambec & Crampes (2012) view a technology with constant flow cost and fixed productivity and a technology without flow cost and with uncertain productivity.[1] There output is only consumed. Here the use of output for consumption and investment implies that investment using dirty technology output caps consumption in states in which otherwise dirty technology capital was idle to smooth consumption across all states. Capital is underutilized in Ambec & Crampes (2012) because the flow cost (price of an input fuel) does not change in response to a change in the price of the output. The latter change is induced by a change in marginal utility of consumption when the productivity of installed clean technology capital changes. In the present paper the general equilibrium relaxes the price rigidity so that the environmental motive is the unique reason for underutilization.[2] The necessary equilibrium conditions provide a similar mechanism for underutilization to that under constant input price. Marginal utility of consumption induces a price change while the environmental cost is constant in a given period.

Garcia, Alzate & Barrera (2012) analyse capacity planning with uncertain positively correlated supply in locations. A subsidy to clean technology output such as a feed-in tar-

[1] The flow cost of the reliable technology can be interpreted as the sum of fuel cost and environmental cost. The competetive equilibrium allocation subject to no taxes and subsidies is not Pareto optimal if there is an environmental cost. An earlier version of this paper extended Ambec & Crampes (2012) to general convex environmental cost and analysed the planner problem.

[2] Capital is fully utilized if the environmental effect is not priced and dirty production uses complementary fuel, as in Ambec & Crampes (2012). I consider a fuel technology in an extension to make this point.

iff for grid-distributed electricity does not induce efficient investment in low-quality sites because of rationing of the sites and variable capital per site.[3] I find that a feed-in tariff leads to investment in low-quality sites which is not efficient given production capacity in high-quality sites is limited, which is plausible. In addition there is greater investment in dirty technology than in an optimum when trivially there is overinvestment in clean technology and clean technology does not provide too much output relative to dirty technology compared to an optimum. Garcia et al. (2012) conclude that a renewable portfolio standard induces underinvestment in the constant-available technology disregarding the funding through government policy, whereas I find overinvestment because I show that renewable portfolio standards with levies comprise a form of feed-in premiums with equivalent effects as feed-in tariffs.

In the literature on tax-subsidy schemes to indirectly internalize an externality, a general input or a specific output is taxed, which differs from how we shall see it done here. A general output is taxed and a subsidy accrues to specific clean output generating an offset because of the perfect substitutability of dirty and clean output. The dirty and clean outputs can be viewed as specific inputs which are perfect substitutes in producing the general output. In the model with a dirty and a clean good in Fullerton & Wolverton (1999, 2000) the provision of the general factor is taxed and the clean good purchase is subsidized. Curiously, Fullerton & Wolverton (2000) call this tax an output tax. In the model with production externality in Fullerton & Wolverton (1999) there is a sales tax of the good and a subsidy of the general (so-called clean) input that is an imperfect substitute for waste in production.[4] Walls & Palmer (2002) use a tax-subsidy system to implement an optimum. Eskeland (1994) proposes combining a dirty goods tax and a standard in controlling pollution. The tax literature lacks capital utilization which I introduce to tax-rebate systems. This literature is informally motivated by costly monitoring of emissions when invoives for goods exist. The motivation here is an interest in efficient tax-rebate systems with multiple technologies whose use requires an investment.

[3] A feed-in tariff is a price that a producer receives, for example, from a government, per unit of output that is fed to a physical network, for example, an electricity grid.

[4] In their first model Fullerton & Wolverton (1999) assume equal productivities and thus miss that the clean good may be taxed if clean technology is more productive than dirty technology provided that the dirty and the clean good are imperfect substitutes in the utility function. The general results of Wijkander (1985) apply using substitutability and complementarity defined in terms of cross price elasticities of demand, which depend on the relative productivity.

Böhringer, Hoffmann & Rutherford (2007) document the effectiveness of feed-in tariffs and renewable energy certificates in promoting renewable energy investments in the European Union (EU). The present paper sheds light on the efficiency of these output subsidies in a dynamic general equilibrium. In contrast to previous literature, a subsidy can implement an optimum: coupled with a general output tax, or through discrimination of the surcharge used to fund the subsidy. Canton & Johannesson Lindén (2010) acknowledge that a fully-funded feed-in tariff (FIT) distorts the price of renewable electricity, yet do not derive outcomes. I characterize the distorted investment in a fully-funded system. Canton & Johannesson Lindén (2010) assert that premia and tariffs affect wholesale market liquidity differently. Though this appears by definition I find that premia and tariffs lead to the same outcomes, because they attain the same balancing rule of unit earnings of the dirty and clean technology users. While I do not analyse tradeable emission permits, the renewable portfolio standard motivates trade in renewable production credits. In line with Böhringer & Rosendahl (2010), these certificates and a uniform FIT financed by consumer tax are equally effective in targeting a renewable energy output share.

Wind and solar power fluctuation. MacKay (2008) shows that wind turbines' output in Ireland varies up to a factor of seven from one day to the next, which is to say greatly. Availability of wind energy and solar energy is roughly certain in the aggregate over many days. This motivates the following structure of uncertainty. A given period $t \in \{0, 1, 2, \ldots\}$ has a constant number of subperiods equal to the positive integer W. A state measures wind speed or solar radiation and maps one-to-one to the set of productivity of technologies. Dirty technology productivity depends on the state for notational convenience. Let $\mathrm{s} = \{s_1, s_2, \ldots, s_W\}$ be a sequence of states $s \in S$. The union of all sequences s is $\hat{\mathrm{s}} = \cup_i \mathrm{s}_i$. The order of states is uncertain yet there is certainty regarding the frequency of states. Different sequences of the list $\mathcal{S}$ of all states with their multiplicity can occur. Any state s appears at the same number in all s.

The next section characterizes planning of capacity and its utilization in dirty and clean technologies subject to tax policy, presents government policy that implements a Pareto optimum, and discusses the potential of feed-in premiums and feed-in tariffs to attain an optimum. Section 3.2 extends the basic model to extraction costs, before Section 3.3 concludes with a discussion of results and usefulness of several government policies.

3.1 The economy

Firms own productive capital, and households own claims to the profits of firms. There are financial assets specific to firms, because firms may use capital in dirty and clean technologies at different proportions, so that a differential rent of clean production can create unequal profit across firms. Clean technology requires land which is not owned and thus creates a rent. This land is a complementary production factor to capital that is implicit through an upper bound on capital. Such land is not divisible by a large number of firms. In particular, this is relevant when there are multiple clean technology types, which I assume in discussing fully-funded subsidies to clean output. Consumption goods that are produced in one subperiod cannot be stored. To begin with each one dirty technology and one clean technology produce a good that is input in consumption and investment. Definition of an equilibrium follows the description of the agents' objectives and constraints.

Households.—A unit mass of infinitely-lived households populates the economy. Let $\mathrm{c}(t) = \{c(s_1,t), c(s_2,t), \ldots, c(s_W,t)\}_{\forall \mathrm{s}}$ be the list of consumption of one household in subperiods $1, 2, \ldots, W$ for all sequences s of states in period t. The welfare function

$$J = \sum_{t=0}^{\infty} \beta^t \mathrm{E}\left[\sum_{w=1}^{W} U(c(s_w,t)) - \Psi(Z(t))\right]$$

represents preferences of each household over $\mathrm{c}(t)$ and pollution $Z(t)$ for all periods t. The period-utility function is an expected utility given beliefs over $\hat{\mathrm{s}}$ that the mathematical expectation operator E expresses. There is no discounting across subperiods within a given period. The discount factor regarding the periods is $\beta \in (0,1)$. The function U is twice-differentiable, strictly increasing, and strictly concave for positive consumption $c \in \mathbb{R}_+$, $\partial U/\partial c > 0 > \partial^2 U/\partial c^2$ for $c > 0$. The function Ψ is twice-differentiable, strictly increasing, and convex in pollution $Z \in \mathbb{R}_+$, thus $\partial U/\partial Z > 0$ and $\partial^2 U/\partial Z^2 \geq 0$. Marginal utility of consumption $\partial U/\partial c$ approaches a large positive value M as consumption tends to zero, $\lim_{c\to 0} \partial U/\partial c = M \leq \infty$. Then consumption c of at least one household is positive in a Pareto optimum in all periods.

All households have an equal endowment $\alpha_i(0)$ of equity of each firm i for simplicity. The ex-dividend price of firm i's tradeable claim is $q_i(t)$ in period t. One claim pays $d_i(t)$

units of account in period t. The contingent price of the consumption good is $\hat{p}(s_w, t)$. The household chooses consumption $\{c(s_w, t)\}_{w=1,2,\ldots,W}$ and asset holdings $\alpha_i(t+1)$ all $t \geq 0$ to maximize welfare J subject to the sequence of budget constraints

$$\sum_{w=1}^{W} \hat{p}(s_w, t)c(s_w, t) + \sum_i q_i(t)\alpha_i(t+1) \leq \sum_i (q_i(t) + d_i(t))\alpha_i(t) + tr(t) \tag{3.1}$$

all s on $\{0, 1, \ldots\}$, taking all prices, dividends, the government transfer tr, and pollution as given. A household first learns the state. Then contracts about delivery of consumption goods in each subperiod can be made contingent on the state. I assume markets in each subperiod, and later use a contract to discuss an information asymmetry. Distributors receive contingent payments and make contingent deliveries in such a contract.

Energy sector.—All firms have access to the same technologies $\mathcal{J} = \{B, C\}$ to produce the factor energy, technologies to create investment goods for these technologies, and one technology to produce consumption goods. First I describe the profits from using the technologies to produce energy. Firm i demands $y_{ij}(t)$ new equipment units at unit cost $p_j(t)$ in period t. Production capacity of energy using technology $j \in \mathcal{J}$ is the productivity $\chi_j(s_w)$ times capital amount $a_{ij}(t) \geq 0$ in subperiod w of period t. Energy output $m_{ij}(s_w, t) \in [0, \chi_j(s_w)a_{ij}(t)]$ is lower than this capacity if the utilization rate $\mathrm{u}_{ij}(s_w, t) = m_{ij}(s_w, t)/\chi_j(s_w)a_{ij}(t) \in [0, 1]$ is less than one.[5] Firm i's physical capital in technology $j \in \mathcal{J}$ follows the law of motion

$$a_{ij}(t+1) = \gamma_j(1 - \max_{s_w} \mathrm{u}_{ij}(s_w, t))a_{ij}(t) + y_{ij}(t) \tag{3.2}$$

that depends on the maximum utilization rate $\max_w \mathrm{u}_{ij}(s_w, t) \in [0, 1]$ given that capital is storable, $\gamma_j > 0$, which is assumed in the following. The capacity $\chi_j(s_w)a_{ij}(t)$ was denoted capital $K_{ij}(t)$ in Chapter 2 where χ_j is constant. A firm can shift capital across its own plants to accommodate any interior firm-wide utilization rate for simplicity.[6] Some

[5] The United States Department of Energy (2010) uses a capacity factor in calculations of the levelized cost of electricity. The technology-specific capacity factor here is $\chi_j(s_w)/\triangle$ if the length of each subperiod is $\triangle$ and technology-specific capacity equals $\triangle a_{ij}(t)$. Such a factor is founded by the convention of measuring capital under some technology-specific norm conditions of (fossil fuel or renewable energy) input.

[6] The planner problem would be representable at the firm level rather than at the aggregate level though all plants are built under constant returns to scale if capital in a firm was storable only if it was idle, $a_{ij}(t+1) = \mathrm{I}_{ij}(t)\gamma_j a_{ij}(t) + (1 - \mathrm{I}_{ij}(t))y_{ij}(t)$ where $\mathrm{I}_{ij} \in \{0, 1\}$ equals zero if and only if $\mathrm{u}_{ij}(t) > 0$.

production capacity is forwarded if capital $a_{ij}(t) > 0$ is underutilized in all subperiods, $u_{ij}(s_w, t) < 1$ all s_w. In an equilibrium dirty technology capital may be efficiently underutilized if its current level is larger than its long-term level, see Chapter 2. This is expressed as a maximum utilization rate of dirty technology capital less than unity. There is an additional incentive to underutilize dirty technology capital—to utilize it below the maximum utilization rate, from the fluctuation of clean technology productivity, which the present chapter addresses. In an equilibrium with constant investment over time only the fluctuation of clean technology output yields incentives to underutilize dirty technology capital. I focus on allocations in which dirty capacity is used in at least some state in any period or or in the long-term. This rules out that some preinstalled dirty technology capital is permanently idle and there is no investment in dirty technology such as in Chapter 2. Dirty technology investment can be motivated as an insurance by sufficiently low productivity of clean technology in some state. The industry capital constraint is

$$\infty > \bar{a}_j \geq \sum_i a_{ij}(t+1) \tag{3.3}$$

all $t \geq 0$ for technology $j \in \mathcal{J}$. The bound yields simultaneous use of multiple clean technologies when otherwise investment in one clean technology would be preferred. Dirty capacity utilization can be characterized assuming one clean technology. Analysis of fully-funded subsidies to clean technology is conveniently done with multiple clean technology types. The economy starts with positive capital, $\sum_j \sum_i a_{ij}(0) > 0$.

After accounting for taxes and subsidies the sale of an energy unit produced with technology j earns $\pi_j(s_w, t)$. The composition of this unit net revenue depends on thegovernment policy regime. Profit from using and investing in the dirty and clean technologies is

$$\Pi_{ij}(t) = \sum_{w=1}^{W} \pi_j(s_w, t) m_{ij}(s_w, t) - p_j(t) y_{ij}(t), \quad j \in \mathcal{J}, \tag{3.4}$$

for any realized sequence $\{s_1, s_2, \ldots, s_W\}$ of states in period $t \geq 0$.

Investment and consumption sectors.—Firm i demands $x_i(s_w, t) \geq 0$ energy units, chooses the proportion $\eta_{ij}(s_w, t) \in [0, 1]$ of its use in the production of capital of technology j, and produces consumption goods and investment goods under constant returns to scale. The firm sells the consumption goods amount $c_i(s_w, t) = B(1 - \sum_j \eta_{ij}(s_w, t)) x_i(s_w, t)$

in subperiod w when the state is s and sells the investment goods amount $\hat{y}_{ij}(t) = \varepsilon_j \sum_{w=1}^{W} \eta_{ij}(s_w, t) x_i(s_w, t)$ in period t. Then the consumption sector and the investment sector yield profit

$$\hat{\Pi}_i(t) = \sum_{w=1}^{W} (\hat{p}(s_w, t) c_i(s_w, t) - p(s_w, t) x_i(s_w, t)) + \sum_j \varphi_j(t) \hat{y}_{ij}(t) \tag{3.5}$$

for firm i in period $t \geq 0$ given the unit price of energy $p(s_w, t)$ and the net revenue for technology-specific capital units $\varphi_j(t)$. There is no adjustment cost for varying the input amounts in producing investment goods across subperiods. Such cost seems to make optimal allocations dependent on the sequence of productivity, that is, the history of states.

Problem of firms.—Firm i's expected profit gross of equity trade

$$\Pi_i(t) = \mathrm{E}\left[\sum_{j \in \mathcal{J}} \Pi_{ij}(t) + \hat{\Pi}_i(t)\right] \tag{3.6}$$

in any period $t \geq 0$ sums profits from the sectors for energy, consumption, and investment goods. Decision-making by firms is subject to the same belief that households have about the distribution of states. The number of equity that firm i has issued is $\hat{\alpha}_i$. Firm i chooses input demands and output supplies, and issued equity $\hat{\alpha}_i(t+1)$, all $t \in \{0, 1, \ldots\}$, to maximize the present discounted value of expected ex-dividend profits

$$v_i = \sum_{t=0}^{\infty} \frac{1}{\prod_{v=0}^{t} \hat{R}(v)} \{\Pi_i(t) + q_i(t)\hat{\alpha}_i(t+1) - (q_i(t) + d_i(t))\hat{\alpha}_i(t)\}$$

subject to (3.2) and (3.3) all $t \geq 0$ taking prices, dividends, governmnent policy rates, and the endogenous nominal interest rate sequence $\{\hat{R}(1), \hat{R}(2), \ldots\}$ as given, where $\hat{R}(0)$ is some given positive number.

Emissions and pollution.—The emissions in the use of technology B are proportional to output at rate $d_B > 0$. The use of technology C does not create emissions, $d_{\mathrm{C}} = 0$. The production of capital may create emissions, which are proportional to the input use $x_{ij}(s_w, t) = \eta_{ij}(s_w, t) x_i(s_w, t)$ in investment at rate ρ_j. This saves introducing another sector that produces material or uses capital.[7] Let Z be pollution and $A(Z) \leq Z$ measure absorption of pollution by the natural environment. Then the pollution stock Z evolves according to

$$Z(t+1) = Z(t) + E(t) - A(Z(t)) \tag{3.7}$$

all $t \geq 0$ given $Z(0)$. Aggregate emissions $E(s_w, t) = \sum_i \sum_j [d_j m_{ij}(s_w, t) + \rho_j x_{ij}(s_w, t)]$ occur in state s in subperiod w. Following the definition of emissions $E(t) = \sum_{w=1}^{W} E(s_w, t)$ government policy can be described that implies certain net revenue streams. The role of government policy is to internalize the production externality.

Government.—There are three regimes of government policy. (i) Producers of energy using dirty technology and producers of investment goods pay the dollar tax $\hat{\tau}$ per unit of emissions. Then the net revenue of energy producers and investment goods producers is $\pi_j(s_w, t) = p(s_w, t) - d_j \hat{\tau}(t)$ and $\varphi_j(t) = p_j(t) - (\rho_j/\varepsilon_j)\hat{\tau}(t)$, respectively. An emissions tax at the source is based on the principles of Pigou (1920). In the other two regimes I examine the role of subsidies to clean technology output. (ii) Energy producers retain $(p(s_w, t) - \tau(t))$ per unit of energy sold to users. Setting a dollar goods tax τ may be compared to setting an ad valorem tax rate ν given by $(1 + \nu(s_w, t))(p(s_w, t) - \tau(t)) = p(s_w, t)$. Energy producers collect the unit dollar subsidy $\tau_j^*(t)$. Then $\pi_j(s_w, t) = p(s_w, t)/(1 + \nu(s_w, t)) + \tau_j^*(t)$. In contrast to Fullerton & Wolverton (1999) output that is produced using clean and dirty inputs is taxed. There a good that is input in producing the dirty and the clean good is taxed.[8] Investment goods producers bill $p_j(t) = (1 + \hat{\nu}_j(t))\varphi_j(t)$ to energy producers. The role of the dollar tax $\hat{\tau}_j$ or ad valorem tax rate $\hat{\nu}_j = \hat{\tau}_j/\varphi_j$ is to internalize the externality from emissions in the investment sector. (iii) In the system with a tax on investment goods and output subsidy the unit net revenue in energy production is $\pi_j(s_w, t) = p(s_w, t) + \tau_j^*(t)$ and the unit cost of investment is $p_j(t) = (1 + \hat{\nu}_j(t))\varphi_j(t)$. The output subsidy τ_j^* provides a rebate to users of clean technology for tax they have paid when buying newly produced capital. The role of the ad valorem tax rate $\hat{\nu}_j$ for

[7] Such material or capital may motivate a material goods tax or a tax on investment goods that are used in the investment sector, respectively, to implement an optimum.

[8] Eskeland & Devarajan (1996) implement a social optimum through a dirty goods tax and no subsidies. Extending their argument to multiple goods buyers pay a differentiated tax rate ν_j per portion of the retained revenue of sellers. To have differentiated prices for perfect substitutes I would need to make the assumption that buyers can distinguish perfectly substitutable goods by their production method. This assumption is not necessary to implement an optimum through a tax collected from buyers if there is one dirty good, because an equal tax rate for dirty and clean goods implements an optimum, and needed if there are multiple dirty goods.

investment goods is to internalize the externality of both emissions from using capital and building capital.

The government sets tax rates and lump-sum transfers on $\{0, 1, \ldots\}$ before private agents make decisions about demands and supplies. Policy satisfies the budget constraint

$$(i) \quad tr(t) \leq \sum_w \hat{\tau}(t) E(s_w, t), \quad (ii) \quad tr(t) \leq \sum_j \hat{\tau}_j(t) \sum_i \hat{y}_{ij}(t)$$
$$+ \sum_j (\tau(t) - \tau_j^*(t)) \sum_w \sum_i m_{ij}(s_w, t) \text{ or}$$
$$(iii) \quad tr(t) + \sum_j \tau_j^*(t) \sum_w \sum_i m_{ij}(s_w, t) \leq \sum_j \hat{\tau}_j(t) \sum_i \hat{y}_{ij}(t)$$

in the corresponding case all $t \geq 0$. The dollar tax and subsidy rates vary only with time t. The assumption of aggregate certainty implies that these government policy variables, when implementing a Pareto optimum, are deterministic rather than a complete contingent plan of policy rates.

Equilibrium.—Allocations are feasible if aggregate demand for each good at most equals aggregate supply of each good, and securities holdings $\alpha_i(t)$ of households equals the number $\hat{\alpha}_i(t)$ of outstanding shares of firm i, all $t \geq 0$. The feasibility conditions for goods quantities are

$$\sum_i x_i(s_w, t) \leq \sum_i \sum_{j \in \mathcal{J}} m_{ij}(s_w, t) \quad \forall s_w, \tag{3.8}$$

for energy, and

$$\sum_i y_{ij}(t) \leq \sum_i \hat{y}_{ij}(t), \quad j \in \mathcal{J}, \quad c(s_w, t) \leq \sum_i c_i(s_w, t) \quad \forall s_w, \tag{3.9}$$

for the investment goods and the consumption good. An equilibrium is a system of prices $(p(s_w, t), \hat{p}(s_w, t), p_{\mathrm{B}}(t), p_{\mathrm{C}}(t), \{q_i(t)\}_{\forall i}, \hat{R})$ and quantities of goods demands and supplies, and financial assets α_i and $\hat{\alpha}_i$, on $\{1, 2, \ldots, N\} \times \{0, 1, \ldots\}$ such that (i) the representative household and all firms solve their problems taking prices, dividends, interest rate, government policy variables, and pollution as given, (ii) the government makes policy as described above and satisfies the budget constraint at equality all ≥ 0, (iii) the law of motion of pollution is (3.7), and (iv) demand equals supply on the goods markets and $\alpha_i(t) = \hat{\alpha}_i(t)$ all i, all $t \geq 0$. The remainder of this section is devoted to characterizing the distribution of consumption, investment, and capital utilization in an equilibrium, and finding government policy that implements a Pareto optimum.

3.1.1 Equilibrium allocation

The subperiod in which a state occurs is irrelevant for decisions of households and firms. Hence the index of the subperiod can be omitted when writing prices and quantities as functions of the state. The second insight is the following. An equilibrium allocation can be characterized in terms of utilized aggregate capital if all firms simultaneously maximize the utilization rate of capital in at least one state. Then the firms do not necessarily fully utilize capital in any subperiod. Let the set of states at t in which firm i's utilization is maximized be $S_{ij}(t)$ and the set of their occurrences be $\mathcal{S}_{ij}(t)$. Firms may choose a different maximum rate so that aggregate output is optimal given individual capital is indeterminate. The necessary condition for profit-maximizing choice of capital $a_{ij}(t+1)$ of technology $j \in \mathcal{J}$ is

$$(1/\hat{R}(t+1))\Big[\sum_{s\in\mathcal{S}} \chi_j(s)\mathrm{u}_{ij}(s,t+1)\pi_j(s,t+1) + \gamma_j(1-\max_s \mathrm{u}_{ij}(s,t+1))\upsilon_{ij}(t+1) - \hat{w}_j(t+1)\Big] \leq \upsilon_{ij}(t) \quad (3.10)$$

at shadow prices $\upsilon_{ij}(t)$ of (3.2) and $\hat{w}_j(t+1)$ of (3.3). The marginal net benefit from utilizing capital and forwarding unutilized capital at most equals the marginal cost of holding capital. Using this the following lemma assures that firms maximize individual output of any given technology in at least one common state. This may occur in multiple states, for example, if capital is fully utilized in all states.

Lemma 3.1 *Suppose that there is output,* $\mathrm{u}_{ij}(s,t) > 0$ *some* i *with* $a_{ij}(t) > 0$ *and some* s*, in technology* $j \in \mathcal{J}$*. Each firm* i *maximizes the utilization rate of capital* $a_{ij}(t) > 0$ *in at least one common state, that is,* $\cap_i S_{ij}(t) \neq \emptyset$ *where* $S_{ij}(t) = \{s^* \in S : \mathrm{u}_{ij}(s^*,t) \geq \mathrm{u}_{ij}(s,t), \forall s \in S\}$.

Proof. The condition (3.10) holds at equality if the respective capital is positive. The utilization of firm i's capital in state s satisfies

$$
\begin{aligned}
\mathrm{u}_{ij}(s,t) \left\{ \begin{array}{c} = 1 \\ \in (0,1) \\ = 0 \end{array} \right\} \quad \Longrightarrow \quad \chi_j(s)\pi_j(s,t) \left\{ \begin{array}{c} \geq \\ \geq \\ \gtreqless \end{array} \right\} 0, \quad s \in S_{ij}(t); \\
\chi_j(s)\pi_j(s,t) \left\{ \begin{array}{c} \text{not defined} \\ = \\ \leq \end{array} \right\} 0, \quad s \in S \setminus S_{ij}(t),
\end{aligned}
\tag{3.11}
$$

at date $t \geq 0$. In addition,

$$
\max_s \mathrm{u}_{ij}(s,t) \left\{ \begin{array}{c} = 1 \\ \in (0,1) \\ = 0 \end{array} \right\} \quad \Longrightarrow \quad \sum_{s \in \mathcal{S}_{ij}(t)} \chi_j(s)\pi_j(s,t) \left\{ \begin{array}{c} \geq \\ = \\ \leq \end{array} \right\} \gamma_j \upsilon_{ij}(t), \tag{3.12}
$$

holds in an equilibrium. Hence in at least one state $s \in S_{ij}(t)$ the net benefit $w_j(s,t) \equiv \chi_j(s)\pi_j(s,t)$ is strictly positive if some i's maximum utilization rate at t is positive. Suppose that $\max_s \mathrm{u}_{kj}(s,t) > 0$ some k. Then $w_j(s,t) > 0$ in at least some state $s \in S_{kj}(t)$ implies that $A_{kj} = \{w_j(s,t) \leq 0 \ \forall s \in S_{kj}(t)\}$ does not hold. However A_{kj} is necessary if any firm i with positive capital in technology j does not maximize its utilization rate in any of the states in set $S_{kj}(t)$. *Q.E.D.*

The proof used $\upsilon_{ij}(t) > 0$. Thus I have ignored tax policy which makes utilization unprofitable all time, implicitly by abstracting from such allocations in optimum. The shadow price $\upsilon_{ij}(t)$ of capital in technology j is identical for all producers i in equilibrium.[9]

The following lemma helps to find the states in which investment occurs. The input choice $x_{ij}(s,t)$ in investment of technology j satisfies

$$
\varepsilon_j \varphi_j(t) \leq p(s,t), \tag{3.13}
$$

[9] Equilibrium investment $y_{ij}(t) \geq 0$ in technology $j \in \mathcal{J}$ satisfies $\upsilon_{ij}(t) \leq p_j(t)$ at equality if $y_{ij}(t) > 0$ for some firm i. Thus the shadow prices $\upsilon_{ij}(t)$ are identical for all firms i that use the technology $j \in \mathcal{J}$, if investment occurs at t. The choice of capital in period $t \geq 1$, and the result that periods of underutilization precede periods with full utilization, imply that the shadow prices $\upsilon_{ij}(t)$ are idential if no investment in technology j occurs at t.

at equality if $x_{ij}(s,t) > 0$, all $j \in \mathcal{J}$. Thus the unit price $p(s,t)$ of energy must be minimized in a state when investment occurs. Let $\psi(t)$ be the marginal utility of income, the Lagrangean multiplier of the budget constraint (3.1). The household equates marginal utility of consumption and the marginal utility cost of spending income on an additional unit of consumption,

$$\partial U/\partial c(s,t) = \hat{p}(s,t)\psi(t), \tag{3.14}$$

all $t \geq 0$. The firms' choice of producing the consumption good implies that $B\hat{p}(s,t) = p(s,t)$ because consumption is positive in each state. This leads to the following lemma that the proof of the next proposition uses.

Lemma 3.2 *Investment occurs in any technology, $x_{ij}(s,t) > 0$ some i and $j \in \mathcal{J}$, only in a state s in which consumption is maximized, $c(s,t) \geq c(t,s')$, $s \neq s'$.*

Proof. Combining the latter two results implies that $B\partial U/\partial c(s,t) = p(s,t)\psi(t)$ all $s \in S$. Then the equilibrium condition (3.13) delivers the result since marginal utility of consumption $\partial U/\partial c(s,t)$ strictly decreases in consumption. *Q.E.D.*

The household weighs utility from consuming in subperiods of the same period equally. This yields a state-invariant relationship between the marginal utility of consumption and the price of the consumption good. The opportunity cost of investment is minimized only if consumption is maximized given adjustment costs are lacking. Thus investment occurs only in states in which consumption is at its maximum level. I conjecture that investment requires sufficiently large consumption if adjusting production inputs in investment across subperiods is costly.

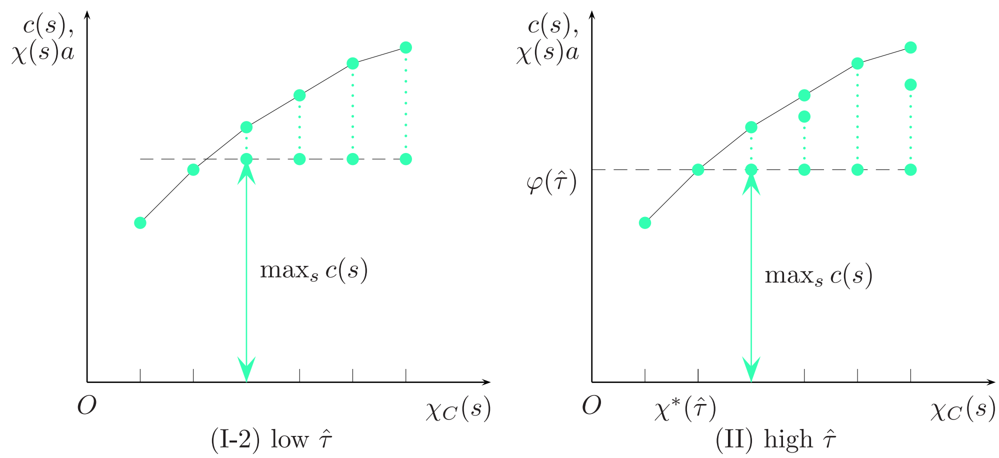
The previous lemmata on utilization and on investment, and the next proposition on the utilization of capital, help to determine when consumption fluctuates. The maximum utilization rate $\max_s \mathrm{u}_{ij}(s,t)$ is less than one in some firm i only if the distributions of consumption and investment over states are not constant over time t. Clearly, permanent underutilization would mean excess investment which is not profit-maximizing if consumption and investment did not vary across periods. The next proposition focuses on the long-term as it is valid in an equilibrium with constant distribution of consumption and investment, and possibly on other trajectories. Full utilization of capital $\sum_i a_{iC}$ in some state rules out underutilization of clean technology capital in any state because it was emission-intensive or currently installed with an average productivity less than

$\gamma_C \leq \gamma_B$, see Chapter 2. Let aggregate capital in technology j be $a_j = \sum_i a_{ij}$ and consider regime (i) in which $\pi_j(s,t) = p(s,t) - d_j\hat{\tau}(t)$.

Proposition 3.1 *Suppose that each firm i fully utilizes capital $a_{ij}(t) > 0$ in some state, $\max_s \mathrm{u}_{ij}(s,t) = 1$ all i and $j \in \{B, C\}$. Capital $a_j(t) > 0$ is fully utilized in all states, $\mathrm{u}_{ij}(s,t) = 1$ all i and $s \in S$, $j \in \{B, C\}$, if the emissions tax rate is zero, $\hat{\tau}(t) = 0$. Either (I) capital $a_j(t) > 0$ is fully utilized in each state s all $j \in \{B, C\}$, or (II) this holds for clean technology $j = C$, and dirty technology capital $a_B(t) > 0$ is fully utilized, $\mathrm{u}_{iB}(s,t) = 1$, all i for $\chi_C(s) \leq \chi^*$, and is underutilized, $\mathrm{u}_{iB}(s,t) < 1$, some i only if $\chi_C(s) > \chi^*$, for some $\chi^* > 0$, if the emssions tax rate is positive, $\hat{\tau}(t) > 0$.*

Proof. Condition (3.11) implies that in absence of emissions pricing all firms fully utilize capital, and under emissions pricing they fully utilize clean technology capital in all states. Given full utilization in some state, the set $S \backslash S_C(t)$ is empty since production using clean technology does not create emissions, $d_C = 0$. A utilization rate may be defined as unity, $\mathrm{u}_{ij}(s,t) = 1$, if capital is unproductive, $\chi_j(s) = 0$. By Lemma 3.1 all firms fully utilize capital in at least one common state. Then either (I) they do this in all states regarding dirty technology so that its aggregate capital is fully utilized in all states and the first result follows, or (II) in some states some firms underutilize dirty technology capital so that its aggregate capital is underutilized. What remains to be shown is the nature of the states with underutilization in (II). The individual utilization rate satisfies (3.11) and (3.12) given $w_j(s,t) = \chi_j(s)\{B\partial U/\partial c(s,t)/\psi(t) - d_j\hat{\tau}(t)\}$ all $j \in \{B, C\}$ for $t \geq 0$ using the firms' optimality condition $B\hat{p} = p$ and the households' optimality condition (3.14). The net benefit of dirty technology use is positive, $w_B(s,t) > 0$, for at least one $s \in S_{iB}(t)$ if dirty technology capital $a_{iB}(t) > 0$ is fully utilized in some state—in all states in $S_{iB}(t)$. Thus $w_B(s'',t) \leq 0$ for some s'', and thereby $c(s,t) < c(s'',t)$ if dirty technology capital is underutilized in state s''. By Lemma 3.2 investment does not occur in state s, that is, $x_{ij}(s,t) = 0$ all i and j. Investment input $x_{ij}(s,t)$ remains zero and consumption $c(s,t) = \chi_B a_B(t) + \chi_C(s)a_C(t)$ increases if $\chi_C(s)$ increases by a small amount. A unique consumption amount $c(s',t) = \varphi(\hat{\tau}(t))$ solves $w(s',t) = 0$. The productivity $\chi_C(s)$ takes the critical level such that the consumption amount in the state s is $c(s',t)$. *Q.E.D.*

Consumption may be the same in all subperiods (I-1) or fluctuate (I-2) when capital is fully utilized in all states. In the subcase (I-1) investment may or may not occur



Note: Capacity $\chi(s)a$ is depicted by points on solid curve, consumption $c(s)$ is minimum distance between dashed line and solid curve, and investment $x(s)$ is measured by length of dotted line.

Figure 3.1: *Allocations with some variation in consumption.*

in all states. In (I-2) investment does not occur in all states. Investment absorbs the fluctuation in clean technology output in states in which consumption is equal.[10] The left panel in Figure 3.1 shows the pattern of consumption and investment in the case (I-2) when consumption varies over states. Let $\chi(s) = (\chi_B(s)\,\chi_C(s))$ and $a = (a_B; a_C)$. The aggregate capacity $\chi(s)a$ connected by the solid lines increases in the productivity of clean technology capital. Consumption is the minimum distance between the horizontal axis and the dashed line and the aggregate capacity. The dashed line designates maximum consumption. The dotted lines measure investment $x = x_B + x_C$. Firms earn a positive net revenue from using dirty technology in each state. Note that states can occur in any order and multiplicity over time. Consumption is low in states with low productivity of clean technology. This follows from the fact that investment does not occur if consumption is not maximized and the property that dirty technology capital is fully utilized in all states. In the right panel in Figure 3.1 dirty technology use earns its normal profit only in states

[10]Instead, consumption cannot be equal across subperiods if capital is fully utilized in all subperiods, storage of output is infeasible, and output were only used for consumption, for example, in a partial equilibrium setup with environmental motive, that abstracts from the source of new capital, or in a general equilibrium model in which output cannot be feasibly used for investment. Firms only invest in dirty technology if utilization of dirty technology capital is profitable in at least one state. Thus in such an economy either dirty technology capital is fully utilized in all states so that consumption comoves with clean technology output, or dirty technology capital is underutilized in some states and consumption is not maximized when dirty technology capital is fully utilized.

when there is no investment, because dirty technology capital is underutilized in some states. This is an example of case (II) of Proposition 3.1. Insurance through excess supply of dirty capacity in a state of high productivity of clean technology can be desirable if clean technology is risky in terms of productivity. The right panel in Figure 3.1 shows such a case. Consumption $c(s,t)$ varies positively with clean technology productivity $\chi_C(s) \leq \chi^*$. Output is not used for investment in these states. Consumption is at unique maximum level in the other states. The input use for investment is arbitrarily distributed over the subperiods of these states. In two states dirty technology capital is underutilized. Dirty technology capital could be underutilized in all of the states in which capacity is greater than maximum consumption, because the profit of using dirty technology capital is zero in all these states. In Figure 3.1 investment absorbs the fluctuation in all subperiods with maximum consumption except if this consumption level equals aggregate capacity, because investment occurs when the opportunity cost of investing, which is proportional to $\partial U/\partial c(s,t)$, is smallest.

One may ask whether consumption varies discontinuously with the state when there is a continuous time span in each period. Suppose that consumption is maximized at the smallest productivity χ^* such that the net benefit is zero. If there was a continuum of states then the net benefit does not need to jump to a strictly positive level as $\chi_C(s)$ and consumption decrease. Condition (3.11) tells that the sum of net benefits in states of maximized utilization, yet not the net benefit in all of these states, is strictly positive. Consumption would vary continuously with clean technology productivity. If there is a discrete number of states, then there may be only one state with positive net benefit.

The following section shows that contingent utilization can be optimal, and derives the emissions price that implements a Pareto optimum with contingent or noncontingent utilization, and policies in the other regimes that implement Pareto optimal allocations in which dirty technology capital is equally utilized in all states. One further equilibrium condition is useful for this. The household sets the intertemporal rate of substitution equal to the marginal rate of return on investment, expressed in nominal terms as

$$\psi(t) = \beta\psi(t+1)\hat{R}(t+1), \tag{3.15}$$

where $\hat{R}(t+1) = (q_i(t+1) + d_i(t+1))/q_i(t)$ is the nominal gross return to savings in period $(t+1)$ all i of which households hold positive assets, $\alpha_i(t+1) > 0$.

3.1.2 Welfare

A planner maximizes $J = \sum_{t=0}^{\infty} \beta^t [\sum_{s \in \mathcal{S}} U(c(s,t)) - \Psi(Z(t))]$ subject to feasibility constraints. By Lemma 3.1 the planner can view the aggregation of the law of motion (3.2) of capital over firms. Then the feasibility constraints are the laws of motion

$$a_j(t+1) = \gamma_j(1 - \max_s \mathrm{u}_j(s,t))a_j(t) + \varepsilon_j \sum_{s \in \mathcal{S}} x_j(s,t) \tag{3.16}$$

of aggregate capital all $j \in \mathcal{J}$ and (3.7) of pollution subject to emissions amount $E(t) = \sum_j \sum_{s \in \mathcal{S}} (d_j \chi_j(s) \mathrm{u}_j(s,t) a_j(t) + \rho_j x_j(s,t))$ all $t \geq 0$, the constraint

$$\bar{a}_j \geq a_j(t+1) \tag{3.17}$$

all $t \geq 0$, and the resource constraint

$$c(s,t)/B + \sum_j x_j(s,t) \leq \sum_j \chi_j(s) \mathrm{u}_j(s,t) a_j(t) \quad \forall s \in S \tag{3.18}$$

all $t \geq 0$. The planner selects a policy of consumption, investment, and utilization rates $(c(s,t), x(s,t), \mathrm{u}(s,t)) \in \mathbb{R}_+^3 \times [0,1]^v$ all $s \in \mathcal{S}$ and $t \geq 0$, where v is the number of positive capital values at t. At least one of the stocks $a_B(0)$ and $a_C(0)$ in the given triple $(a_B(0), a_C(0), Z(0))$ is positive. In a Pareto optimum the shadow price ϵ of the constraint (3.7), the marginal welfare benefit of pollution reduction, takes a specific value in each period that follows the difference equation $\epsilon(t) = \beta \partial \Psi / \partial Z(t+1) + \beta(1 - \partial A / \partial Z(t+1))\epsilon(t+1)$ all $t \geq 0$.[11] The necessary optimality condition with respect to capital $a_j(t+1)$ is

$$\begin{aligned} \beta \Big[& \sum_{s \in \mathcal{S}} \chi_j(s) \mathrm{u}_j(t+1) \left\{ B \partial U / \partial c(s,t+1) - d_j \epsilon(t+1) \right\} \\ & + \gamma_j(1 - \max_s \mathrm{u}_j(s,t+1)) \phi_j(t+1) - w_j(t+1) \Big] \leq \phi_j(t), \end{aligned} \tag{3.19}$$

[11] The term $\partial \Psi / \partial Z(t+1)$ is replaced by the sum of marginal utility effects in these subperiods and the shadow price ϵ is noncontingent if the period-utility function specified utility effects of pollution in multiple subperiods of given productivity.

at equality if $a_j(t+1) > 0$, given shadow prices $\phi_j(t)$ of (3.16) and $w_j(t+1)$ of (3.17). Let $\hat{S}_j(t)$ be the set of states in which the utilization of capital $a_j(t) > 0$ is maximized. Denote by $\hat{\mathcal{S}}_j(t)$ the corresponding list of occurrences. Then capital utilization satisfies

$$\mathrm{u}_j(s,t) \left\{ \begin{array}{l} = 1 \\ \in (0,1) \\ = 0 \end{array} \right\} \implies \chi_j(s)\{B\partial U/\partial c(s,t) - d_j\epsilon(t)\} \left\{ \begin{array}{c} \geq \\ \geq \\ \gtreqless \end{array} \right\} 0, \quad s \in \hat{S}_j(t);$$

$$\chi_j(s)\{B\partial U/\partial c(s,t) - d_j\epsilon(t)\} \left\{ \begin{array}{c} \text{not defined} \\ = \\ \leq \end{array} \right\} 0, \quad s \in S \setminus \hat{S}_j(t), \tag{3.20}$$

at date $t \geq 0$. In addition,

$$\max_s \mathrm{u}_{ij}(s,t) \left\{ \begin{array}{l} = 1 \\ \in (0,1) \\ = 0 \end{array} \right\} \implies \sum_{s \in \hat{\mathcal{S}}_j(t)} \chi_j(s)\{B\partial U/\partial c(s,t) - d_j\epsilon(t)\} \left\{ \begin{array}{c} \geq \\ = \\ \leq \end{array} \right\} \gamma_j\phi_j(t), \tag{3.21}$$

holds in a Pareto optimum. The investment satisfies

$$\varepsilon_j\phi_j(t) \leq B\partial U/\partial c(s,t) + \rho_j\epsilon(t), \tag{3.22}$$

at equality if $x_j(s,t) > 0$. The types (I-1), (I-2), and (II) of equilibrium allocations in Section 2.1 characterize an optimal allocation if the emissions tax internalizes the externality.

Noncontingent policy.—Consumption and the utilization rates of capital are equal in each state, $c(s,t) = c(s',t)$ and $\mathrm{u}_j(s,t) = \mathrm{u}_j(s',t)$ all $s \neq s'$, as a constraint. There may be no contingent markets or contracts because electricity use is metered only once per period.[12] Clean technology capital $a_C(t) > 0$ is fully utilized in all states in period t. Capital $a_B(t) > 0$ is fully utilized for $t \geq 0$ if dirty capacity $\chi_B a_B(t)$ is not too large. Otherwise a Pareto improvement is possible through less investment. Thus a

[12]Consistently, government policy cannot be contingent. Then only allocations with equal utilization rates in a given period are implementable. The planner finds these allocations in constrained optimum.

noncontingent policy maximizes welfare in the case when (I-1) describes an optimum, and the constrained optimal noncontingent policy does not maximize welfare in the cases when (I-2) or (II) characterize an optimum.

The next section shows that a uniform emissions tax for all sources implements a Pareto optimum.

3.1.3 Emissions tax

A government policy implements a Pareto optimum if equilibrium conditions subject to this policy satisfy all necessary optimality conditions of the planner problem.

Proposition 3.2 *The emissions tax $\hat{\tau} = \epsilon/\psi$ and lump-sum transfer tr equal for all households implement a Pareto optimum.*

Proof. The marginal welfare of the industry constraint (3.3) is its dollar value multiplied by the marginal value of income: $w_j(t) = \psi(t)\hat{w}_j(t)$. This value $w_j(t)$ equals the shadow price of the planner's constraint (3.17) at a solution to the planner's problem when the equilibrium allocation is Pareto optimal. The marginal welfare of aggregate capital, $\phi_j(t) = \psi(t)\upsilon_{ij}(t)$, is the marginal value of income multiplied by the dollar value of individual capital. An additional unit capital is worth $\phi_j(t)$ to the planner in current terms at a maximum of J when the equilibrium allocation is Pareto optimal. Let $\lambda(s,t)$ be the shadow price of constraint (3.18). In a Pareto optimum $B\partial U/\partial c(s,t) = \lambda(s,t)$. The firms' condition $B\hat{p}(s,t) = p(s,t)$ and the households' condition (3.14) yield $B\partial U/\partial c(s,t)/\psi(t) = p(s,t)$ all $s \in S$. To obtain (3.19) substitute $q_j(t)$, $q_j(t+1)$, $w_j(t+1)$, and $\epsilon(t+1)$ into (3.10). Then

$$\begin{aligned} &\frac{1}{\hat{R}(t+1)}\Bigg[\sum_{s\in\mathcal{S}}\chi(s)\mathrm{u}_{ij}(s,t+1)\left\{B\frac{\partial U/\partial c(s,t+1)}{\psi(t+1)} - d_j\frac{\epsilon(t+1)}{\psi(t+1)}\right\} \\ &\quad + \sum_{s\in\hat{\mathcal{S}}_j(t+1)}\gamma_j(1-\mathrm{u}_{ij}(s,t+1))\frac{q_j(t+1)}{\psi(t+1)} - \frac{w_j(t+1)}{\psi(t+1)}\Bigg] \leq \frac{q_j(t)}{\psi(t)} \end{aligned} \tag{3.23}$$

at equality if $a_{ij}(t+1) > 0$ some i all $j \in \mathcal{J}$. Utilization of (3.15) and $\hat{\tau} = \epsilon/\psi$ delivers (3.19). Conditions (3.20)-(3.22) follow analogously. *Q.E.D.*

The marginal welfare of pollution, $\psi(t)\hat{\tau}(t)$, that society incurs is the marginal value of income multiplied by the dollar value of emissions, because emissions are measured in the same units as pollution. Thus, avoiding an additional pollution unit is worth $\epsilon(t) = \psi(t)\hat{\tau}(t)$ to the planner in current terms at a maximum of J when the equilibrium allocation subject to the dollar price $\hat{\tau}(t)$ of emissions is Pareto optimal. The emissions tax rate $\hat{\tau}(t)$ is the product of the contingent price $\hat{p}(s,t)$ of the consumption good and the cost of polluting $\theta(s,t) = \epsilon(t)/(\partial U/\partial c(s,t))$. The marginal value of income $\psi(t)$ is determined by normalizing the contingent price $p(s',t)$ in some state s'. All other prices $p(s,t)$, $s \neq s'$, are then determined through the allocation. The assumption of aggregate certainty implies that the emissions tax rate that implements a Pareto optimum is deterministic.

The following section examines noncontingent subsidies.

3.1.4 A role of clean output subsidy?

The price of energy fluctuates in allocations of the types (I-2) and (II) because consumption fluctuates. Proponents of a subsidy to clean renewable electricity production reason the usefulness of this policy instrument in stabilizing the price and thereby inducing investment that is efficient yet would otherwise not occur. This section shows that there is a contract with a stable price given an information asymmetry but government intervention is not needed to stabilize the price. The asymmetry is between clean energy producers that do not know the state and clean energy users that know the state. For example, energy producers do not have access to techniques that predict wind speed or solar radiation while energy users have equipment that responds to the price of energy in each subperiod. The role of the government is to internalize the externality. Second the government can internalize the externality through a subsidy to clean energy output (for example, in the form of a feed-in premium for grid-distributed electricity) in combination with an energy tax at the date of energy sale or in combination with a tax on the purchase of capital that in the future produces energy. Then clean technology users receive a rebate through the subsidy that compensates the increase in the cost of energy related to using capital, which dirty technology users fully bear. The tax on investment goods accounts for the emissions in the investment sector. Thus, a system with an energy tax has an investment goods tax given that there are emissions in the production of investment goods.

Price stabilization.—The marginal real rate of return on investment is equal for all agents in the economy and agents are not credit-constrained. Thus it makes no difference if a user of physical capital buys or leases this capital. I assume that users buy capital. In an equilibrium in which producers know the state and acquisition of investment goods is not taxed, the necessary conditions (3.10)-(3.13) imply that

$$\begin{aligned} &\frac{1}{\hat{R}(t+1)}\varepsilon_C \sum_{s\in\mathcal{S}} \chi_C(s)\pi_C(s,t+1) \left\{ \begin{array}{c} \geq \\ = \end{array} \right\} \min_s p(s,t) + \rho_C \hat{\tau}(t) \\ &\text{if} \quad \sum_i a_{iC}(t+1) \begin{cases} = \bar{a}_C \\ \in (0, \bar{a}_C) \end{cases} \end{aligned} \tag{3.24}$$

for $t \geq 0$. A competitive industry can develop contracts that yield a certain rate of return from renewable energy investments when their productivity fluctuates. There are competitive distributors that buy energy from energy producers and sell energy to energy users. Clean technology users fully utilize capital, $\mathrm{u}_{iC}(s,t+1) = 1$ all s. Define $Q_j = \varepsilon_j \sum_{s\in\mathcal{S}} \chi_j(s)$. an equilibrium in which users of technology C do not know the state the term $\varepsilon_C \sum_{s\in\mathcal{S}} \chi_C(s)\pi_C(s,t+1)$ is replaced by $Q_C\pi_C(s,t+1)$ where $\pi_C(s,t+1)$ is the same in all states. For example, in policy regime (i) distributors make zero profit paying

$$\pi_C(s,t) = \left[\sum_{s\in\mathcal{S}} p(s,t)\chi_C(s)\right] \Big/ \sum_{s\in\mathcal{S}} \chi_C(s)$$

per unit of output all s agreed upon in a contract with each energy producer using technology C. Distributors may pay a stable price to any producer i that utilizes capital $a_{ij}(t) > 0$ of technology $j \in \mathcal{J}$ equally in all states in t. This holds in an equilibrium with a Pareto optimal allocation. There is a stable price that meets expectations of private agents and induces investment in clean technology, in the same vein as here under aggregate certainty, if the states differed over the possible sequences of productivity in a given period. The information symmetry does not bring about a role of stable clean technology subsidies. A prerequisite for Pareto optimal investment in clean technologies is that the government sets incentives for private agents to internalize the externality.

In the regimes (ii) and (iii) clean technology subsidies implement Pareto efficient investment. Policy instruments not mentioned are zero in what follows.

Energy tax and subsidy.—The excise subsidy may be interpreted as a premium that makes it viable for producers to invest in clean technology and then feed its output to the market. The following proposition shows the energy tax and subsidy rates in the regime (ii) that do this efficiently.

Proposition 3.3 *The dirty and clean energy tax $\tau = d_B\hat{\tau}$, investment goods tax $\hat{\tau}_j = (\rho_j/\varepsilon_j)\hat{\tau}$ all $j \in \mathcal{J}$, and clean energy subsidy $\tau_C^* = \tau$ implement a Pareto optimum.*

Proof. The net revenue $\pi_j(t,s) = p(s,t) - \tau(t) + \tau_j^*$ for an energy producer equals $\pi_j(t,s)$ under the emissions tax for $\tau(t) - \tau_j^* = d_j\hat{\tau}(t)$ all $t \geq 0$. The investment goods tax follows analogously. *Q.E.D.*

The energy tax and subsidy correct the externality associated with using capital that produces energy. The investment goods tax internalizes the externality of emissions in producing such capital. The excise taxes in Proposition 3.3 can be written in terms of ad valorem rates. These rates are contingent regarding energy. Clearly, a noncontingent ad valorem rate cannot induce underutilized capital below the maximum utilization rate in a given period, because such a tax rate cannot vanish the earnings on energy in some state and retain a profit from the energy sale in another state. The rate $\nu(t)$ that satisfies

$$\frac{1}{1+\nu(t)} \sum_{s\in\mathcal{S}} \chi_j(s)\{p(s,t) + (1+\nu(t))\tau_j^*(t)\} = \sum_{s\in\mathcal{S}} \chi_j(s)\{p(s,t) - d_j\hat{\tau}(t)\} \tag{3.25}$$

equalizes the equilibrium and optimal sum of net benefits for energy producers for a constant utilization of capital $a_j(t) > 0$. The next proposition shows how ad valorem goods taxes implement optimal allocations.

Proposition 3.4 *(a) The ad valorem rates $\nu(s,t) = \tau(t)/(p(s,t) - \tau(t))$ and $\hat{\nu}_j(t) = \hat{\tau}_j(t)/(p_j(t) - \hat{\tau}_j(t))$ for the goods taxes in Proposition 3.3, all $j \in \{B, C\}$, and clean energy subsidy in Proposition 3.3 implement a Pareto optimum. (b) The noncontingent ad valorem rate $\nu(t)$ for energy that solves (3.25) for $j = B$, and $\hat{\nu}_j(t)$ all $j \in \{B, C\}$ for investment goods from (a), and clean energy subsidy τ_C^* that solves (3.25) given $\nu(t)$ implement a Pareto optimum in which dirty technology capital $a_B(t) > 0$ is equally utilized, $\mathrm{u}_B(s,t) = \mathrm{u}_B(s',t)$ all $s, s' \in S$.*

Proof. The contingent rates follow from their definition using the excise taxes in Proposition 3.3. The noncontingent rates follow from the fact that $\mathcal{S}$ comprises the set of all states with equal utilization of dirty technology capital. *Q.E.D.*

Allocations with underutilized dirty technology capital in all states complete the catalogue of allocations. Proposition 3.1 ruled these out by assumption. In a type (III) allocation dirty technology capital $a_B(t) > 0$ is underutilized at the same rate in all states, $\mathrm{u}_B(s,t) = \max_{s'} \mathrm{u}_B(s',t) < 1$ all $s \in S$, and in a type (IV) allocation dirty technology capital $a_B(t) > 0$ is underutilized in all states and utilized at different rates in at least two states, $\mathrm{u}_B(s,t) < \max_{s'} \mathrm{u}_B(s',t) < 1$ for some $s \in S$. The contingent energy tax rates equal ∞ in states s in which dirty technology capital $a_B(t) > 0$ is underutilized below the maximum utilization rate, $\mathrm{u}_B(s,t) < \max_{s'} \mathrm{u}_B(s',t)$, to implement a type (II) or (IV) allocation. In these states the government efficiently takes away all revenue from the sale of energy using dirty technology, and rebates the revenue to clean technology users. Type (II) allocations can be optimal in the long-term. Type (IV) allocations can be optimal in early periods if there is sufficiently large dirty capital in the initial period. Allocations of type (I-1), (I-2) or (III) are implementable through noncontingent ad valorem goods taxes.

There are both a tax of clean output and a subsidy of clean output. This makes sense, even if both their rates were dollar amounts or ad valorem rates, for two practical reasons. In the model producers pay the output tax and all energy is traded on a market. To attain an optimal allocation users of clean technology should need to claim the subsidy. A producer would not be able to claim the subsidy for the self-consumed amount of its energy output. Second, the output tax can be administered as a tax that consumers (firms in the model, and firms and households in practice) pay. Then the price of energy is $p - \tau$. One goods tax rate is sufficient because there is one dirty technology. The effciently set energy tax rate would vary among these technologies if there were multiple dirty technologies.

Capital gains tax and capital tax.—Emissions that the use of dirty technology creates are a function of capital use. The dirty goods tax thus can be written as a capital gains tax of financial investments in dirty production that Sinn (2008) proposes to internalize greenhouse gas pollution.[13] Analogously to the goods tax and subsidy system then a general capital gains tax coupled with a clean capital gains subsidy implements an optimum. In line with the vehicle tax in Fullerton & West (2002) it is easy to see that a tax $\tau(t)$

[13]Sinn (2008) uses a model with substitutable capital and emissions to support such a tax, which might not implement an optimum there despite claiming it on p. 384.

per unit of capital implements an optimum with noncontingent utilization and noncontingent dirty technology productivity. The unit net revenue on the energy market is then $\pi_j(s,t) = p(s,t) - \tau(t)/\chi_j(s)\mathrm{u}_j(s,t)$. A noncontingent capital tax cannot internalize the externality from using contingent underutilized dirty technology capital. The reason is that the unit net earnings from the energy market depend on the utilization rate.

I pause to note that the stable unit revenue π_j for $j \in \{B, C\}$ and price p of energy across states satisfy

$$\eta\pi_C + (1-\eta)\pi_B = p - (1-\eta)(d_B/B)\hat{\tau}$$

in all previous efficient regulations, where η is the output share of the clean technology given noncontingent utilization, $\mathrm{u}_j(s,t) = \mathrm{u}_j(s',t)$ all $j \in \{B, C\}$. The weighted earnings of dirty and clean technology is smaller than the average price of energy—except if the clean technology is exclusively used, $\eta = 1$.[14]

Investment tax and output subsidy.—Energy producers efficiently invest indirty technology only if the investment goods are immediately—in the period succeeding the period of investment—fully utilized in at least one state. This can be shown using the arguments in Chapter 2 on capital utilization with one subperiod. Then type (III) and (IV) allocations do not follow investment in optimum. Such allocations or type (II) allocations in which capital is underutilized until the date of investment cannot be implemented by taxing investment or subsidizing output. Underutilization is optimal because of environmental cost which is not internalized. The following holds.

Proposition 3.5 *The investment goods tax* $\hat{\tau}_j = (\rho_B/\varepsilon_B)\hat{\tau}(t) + (d_B Q_B/\varepsilon_B)\hat{\tau}(t+1)/\hat{R}(t+1)$ *all* $j \in \{B, C\}$ *and clean output subsidy* $\tau_C^* = (d_B Q_B/Q_C)\epsilon(t+1)/\psi(t+1) + \hat{R}(t+1)(\rho_B\varepsilon_C - \rho_C\varepsilon_B)/Q_C$ *implement a Pareto optimum in which dirty technology capital* $a_B(t) > 0$ *is fully utilized in all states,* $\mathrm{u}_B(s,t) = 1$ *all* $s \in S$*, all* $t \geq 0$.

Proof. In an equilibrium the conditions (3.10)-(3.13) and $v_{ij}(t) \leq p_j(t)$, and the conditions

[14] The average unit revenue is $\pi_j(t) = \sum_{s\in\mathcal{S}} \chi_j(s)\pi_j(s,t)/\sum_{s\in\mathcal{S}} \chi_j(s)$. Competitive distributors would compute the average price $p(t) = (\sum_{s\in\mathcal{S}} p(s,t)\sum_j m_j(s,t))/\sum_{s\in\mathcal{S}}\sum_j m_j(s,t) = (1-\eta)(\pi_B(t) + (d_B/B)\hat{\tau}) + \eta\pi_C(t)$ given utilization $\mathrm{u}_j(s,t) = \mathrm{u}_j(s',t)$. This formula holds if there are multiple clean technologies in set $\mathcal{C}$ letting $\eta_j = \sum_{s\in\mathcal{S}} m_j(s,t)/\sum_j\sum_{s\in\mathcal{S}} m_j(s,t)$ all technologies j and $\eta = \sum_{j\in\mathcal{C}} \eta_j$. The algebra simplifies for noncontingent production, $m_j(s,t) = m_j(s',t)$. Then $\eta\pi_C + (1-\eta)\pi_B = p - (1-\eta)(d_B/B)\hat{\tau}$ holds at arbitrary $\eta \in [0,1]$ because consumption is noncontingent.

(3.14) and $B\hat{p}(s,t) = p(s,t)$, imply that

$$\frac{1}{\hat{R}(t+1)} \sum_{s \in \mathcal{S}} \chi_j(s) \left\{ \frac{B(\partial U/\partial c)(s,t+1)}{\psi(t+1)} + \tau_j^*(t+1) - \hat{w}_{ij}(t+1) \right\}$$
$$\leq p_j(t) \leq \frac{B\partial U/\partial c(s,t)}{\varepsilon_j \psi(t)} + (\rho_j/\varepsilon_j)\hat{\tau}(t) + (\hat{\tau}_j(t) - (\rho_j/\varepsilon_j)\hat{\tau}(t)),$$

at equalities if $a_j(t+1) > 0$ and $x_j(t) > 0$, all $t \geq 0$. The optimality condition (3.19) implies that

$$\frac{1}{\hat{R}(t+1)} \sum_{s \in \mathcal{S}} \chi_j(s) \{\tau_j^*(t+1) + d_j \hat{\tau}(t+1)\} = \hat{\tau}_j(t) - (\rho_j/\varepsilon_j)\hat{\tau}(t)$$

which delivers $\hat{\tau}_B$ for $j = B$. Substituting $\hat{\tau}_B$ and premultiplying by $\varepsilon_B \varepsilon_C$ yields the clean technology subsidy. *Q.E.D.*

The investment goods tax has two components. The first part internalizes the environmental cost from using the investment good. The second part internalizes the externality from building capital. The payment of the investment goods tax by both dirty and clean technology investors motivates the clean technology subsidy. The internalization of the environmental cost of investment explains the second term in the clean technology subsidy if the emission intensity relative to the productivity in manufacturing differs, $\rho_B \varepsilon_C \neq \rho_C \varepsilon_B$. A differentiated investment tax and no subsidy to clean output can implement the same type of allocation. The tax rate on investment in multiple dirty technologies would vary among these technologies.

The next section examines a form of clean output subsidy that the German government practices since 1991, which is fully-funded.[15] The volume of this so-called feed-in tariff was roughly 12 billion EUR in 2012. Other regional and federal governments use similar policies.

3.1.5 Fully-funded feed-in premium or feed-in tariff

Producers of consumption goods and investment goods buy energy at unit price $p(s,t)$. I assume that consumption is constant in all states, albeit it takes away the price stabi-

[15] Stromeinspeisungsgesetz since 1991, Erneuerbare-Energien-Gesetz since 2000 translated as Renewable Resources Act.

lization aspect to proponents of subsidies, to simplify the algebra. Constant consumption is optimal in (I-1), which is likely when the environmental shadow cost is sufficiently low so that dirty capacity is sufficiently high or when the variance of clean technology productivity is sufficiently low. Then $p(s,t)$ is the same in all states, so that $p(t) = p(s,t)$. The distributors pay π_B dollars for one unit of dirty output and π_C dollars per unit of clean output using revenue from the sale of energy to users. Producers and distributors take the feed-in tariff π_C and the surcharge rate τ^* as given.[16] The legal requirement $\eta\pi_C = \tau^*$ of the fully-funded system leads to $(p - \tau^*)$ dollars unit net revenue where p is the average price.[17] Distributors make zero profit if the proceeds of dirty technology users equal the funds net of the surcharge, $(1-\eta)\pi_B = p - \tau^*$. Then the weighted average of unit revenue

$$\eta\pi_C + (1-\eta)\pi_B = p \tag{3.26}$$

is the price in equilibrium. The unit earnings $\pi_j(s,t)$ is the same in all states in a given period t. To suit the discussion of supporting renewables there are multiple clean technologies, for example, through locational differences in productivity $\chi_j(s)$. The emission intensity in creating capital is equal for all technologies at $\rho = \rho_j$ all $j \in \mathcal{J}$ for simplicity. The return on investment is the discounted net earnings divided by the current cost of investment, the left side divided by the right side in the following weak inequality. The necessary equilibrium condition

$$\frac{1}{\hat{R}(t+1)}\varepsilon_j \sum_{s\in\mathcal{S}} \chi_j(s)\pi_j(s,t+1) \left\{ \begin{array}{c} \geq \\ = \end{array} \right\} \min_s p(s,t) + \rho\hat{\tau}(t)$$
$$\text{if} \quad \sum_i a_{ij}(t+1) \left\{ \begin{array}{l} = \bar{a}_j \\ \in (0, \bar{a}_j) \end{array} \right.$$

all $j \in \mathcal{J}$ is analogous to (3.24) for $t \geq 0$ and implies that $Q_B\pi_B = Q_j\pi_C$, assuming that some clean technology $j \in \mathcal{J}$, with $d_j = 0$, equilibriates the return on investment to that of dirty technology. See above for the definitions of the marginal product Q_j all $d_j \geq 0$.

[16] The German feed-in tariffs weakly diminish over time for a given machine—for example be fixed for the first five years since installation and then decrease to a lower level. An argument for this might be the term structure of the repayment of debt that finances the investment.

[17] Transmission cost is excluded here for simplicity. This cost can drive a wedge between dirty and clean technology output prices when dirty technology output requires transmission to users and clean technology output is produced at the location of use.

The marginal rate of substitution of consumption is $R(t+1) = \partial U/\partial c(t)/(\beta\partial U/\partial c(t+1))$ if consumption does not vary with the state. This equals the real marginal gross return to holding assets. The latter condition and (3.26) yield, under the assumption of constant consumption, the real marginal benefit of energy production one period after investment

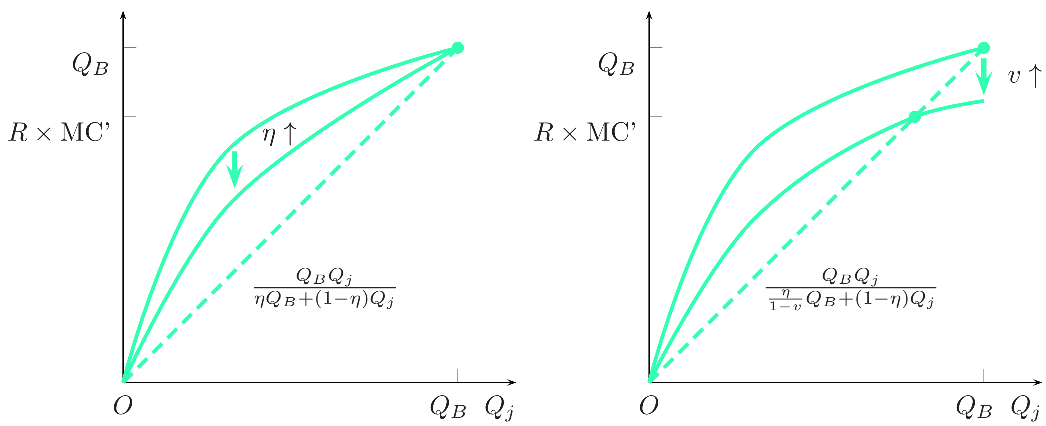
$$\frac{Q_B Q_j}{\eta Q_B + (1-\eta)Q_j} = R(1 + \rho(\hat{\tau}/p)')$$

which the left panel in Figure 3.2 plots as a function of the marginal product Q_j of clean technology for two values of the portion η of clean technology output.[18] Here prime denotes previous period's values and $\text{MC}' = 1 + \rho(\hat{\tau}/p)'$ is the real marginal cost of investment. The curves merge at Q_B. The dashed line depicts the optimal minimum rate of return on investment in clean technologies because

$$Q_j \geq R(1 + \rho(\hat{\tau}/p)')$$

all j with $x_{ij} > 0$ some i given $\hat{\tau}(t) = \hat{p}(t)\epsilon(t)/(\partial U/\partial c(t))$ holds in a Pareto optimum. Investment in clean technology types with lower marginal product than its optimal minimum level is worthwhile to private agents in equilibrium, if dirty technology investment is optimal, $\eta < 1$. Thus there is an overinvestment in clean technologies relative to a Pareto optimum if clean technologies exist with Q_C between the equilibrium level Q_j and the minimum optimal level Q_j. The distortion is smaller the greater the portion η of clean technology output holding it at the same level in an optimal allocation and an equilibrium allocation. As η increases the unit earnings π_C of clean technology users and the price p come closer so that the rate of return on investment better reflects its optimal level $BQ_j/(B + \rho_j\hat{\tau})$, given $\hat{\tau}$ is set efficiently to internalize the marginal effect of emissions of investment on society. Dirty technology investment and output are greater than in optimum if the portion η of clean technology output is not too large relative to its optimal level, and there is overinvestment in clean technologies. Output exceeds its optimal level, and the excess output is distributed over both dirty and clean technologies. This implies excess emissions and long-term pollution. The fully-funded feed-in tariff that implements the optimal share of clean technology output η and the optimal marginal real rate of

[18] The earnings are generally weighted by output. Equal consumption implies that the sum of productivities can be factored out. The real marginal rate of return $R(s, s', t+1)$ is the inverse of $\beta\partial U/\partial c(s', t+1)/(\partial U/\partial c(s,t))$.



Note: The marginal product Q_j for clean technologies, $d_j = 0$, is on the x-axis, the marginal benefit $Q_B Q_j/((\eta/(1-v))Q_B + (1-\eta)Q_j)$ is on the y-axis, at $v = 0$ in the left panel.

Figure 3.2: *Marginal product of clean technologies.*

return R in an economy with a dense set of return rates of a set of clean technologies (for example, in different locations or engineering systems) leads to greater output of both dirty and clean technologies and greater emissions than in optimum. Implementation of the efficient scale of clean technology, through the efficient Q_j, induces a greater marginal rate of return on investment than in optimum. Thus savings is greater, and thereby investment in dirty technology is greater than optimally.

These results seem to hold in case (II) when consumption is not equalized. The algebra is more complicated than above because the price is not constant.

Feed-in premium.—The following describes three schemes used to support renewable energy investment in practice such that producers sell at the market price p and receive a premium to show that these schemes have in common the condition (3.26) and thus premium and tariff induce the same allocations. How the premium emerges depends on the regime. (i) Suppose that the government sets the premium. Since the 2012 amendment the German law allows clean technology producers that sell their electricity on the wholesale market to receive a premium rate that is determined monthly. A portion of the difference between the unit revenues equals the surcharge, $\eta(\pi_C - \pi_B) = \tau^*$, and the unit net revenue of dirty technology equals the net market price, $\pi_B = p - \tau^*$, if a unit premium $(\pi_C - \pi_B)$ is paid to clean energy producers in addition to the net market price $p - \tau^*$. (ii) In a second regime, practiced in Italy and Sweden, τ^* is the market price of a credit. The unit net revenue of a dirty technology producer is $\pi_B = p - \alpha(\eta/(1-\eta)\tau^*$. Each dirty technology producer must hold credits equal to the proportion $\alpha\eta/(1-\eta)$ of dirty technology output. A producer satisfies this requirement through purchasing credits on a market or using clean technology. The regulator endogenously distributes credits to each clean technology producer proportional to output that the latter does not consume. Let there be α credit per clean output unit. Then $\pi_C = p+\alpha\tau^*$ is the net revenue of a unit of clean output. (iii) Another renewable energy support scheme is the procurement of power purchasing agreements through auctions and levy of the incremental cost of energy on all consumers to induce a renewable portfolio standard $\eta > 0$ that the regulator sets. The policy satisfies $\eta(\pi_C - p) = \tau^*$ and $(1-\eta)(p-\pi_B) = \tau^*$ in equilibrium in which the auction winners supply renewable energy offering the lowest surcharge τ^* or the lowest offer price π_C. According to Wiser et al. (2003, p. 37) the states California, Pennsylvania, and New York, have used the former and Northern Ireland, the UK, and France, have used the latter.

The outcomes are the same in each variant as for the fully-funded tariff since (3.26) holds and all users pay the same price p. Böhringer & Rosendahl (2010) find that a feed-in tariff and tradeable green certificates lead to same outcomes.[19] Novelties here are the universal balancing condition (3.26) and the characterization of output in an equilibrium with optimal output portions in dirty and clean technologies. Uniform surcharges that fund the clean output subsidy induce overinvestment when R and η are at optimal levels.

Price discrimination.—In the following I examine exempting some agents from paying the surcharge. There is an unequal treatment of energy buyers in Germany. For example, large (export-oriented) investment goods producers do not pay the surcharge, and thus spend $(p-\tau^*)$ per unit of energy. The clean output subsidy is the portion $\eta\pi_C = (1-v)\tau^*$ of price times energy output if the fraction v of energy users is exempted from paying the surcharge τ^*. This implies that the marked-up feed-in tariff $\pi_C/(1-v)$ is weighted in the average price, $\eta\pi_C/(1-v) + (1-\eta)\pi_B = p$. In a stationary equilibrium so that $p(t) = p(t+1)$ the marginal cost of investment $(1 + (p/(p-\tau^*))\rho(\hat{\tau}/p)) = Q_B/R(1-\eta)$ is greater than the dirty technology marginal product discounted by R if all investment

[19]Böhringer & Rosendahl (2010) include an "end-user" tax τ in their feed-in tariff. This tax reduces the unit earnings of both dirty and clean technology use. The feed-in tariff is thus greater than π_C by this tax. Then $\pi_B = p - \tau$ so that the budget is balanced if $\tau = \eta(\tau^* + \tau)$ which solves for the tax $\tau = \beta\tau^*$ defining $\beta = \eta/(1-\eta)$ and letting $\alpha = 1$ in the certificate system.

goods producers are exempted and if there is some dirty technology investment, $\eta < 1$. The optimal level of the marginal real rate of return cannot be implemented, if optimal dirty technology investment is unconstrained—(3.3) is slack—and there are no emissions of investment. A comparison of the equilibrium condition under exempted investment goods producers to the equilibrium condition $(1 + \rho(\hat{\tau}/p)) = ((p - \tau^*)/p)Q_B/R(1 - \eta)$ under uniform surcharges for all energy users shows that the investment in dirty and clean technologies is greater given the exempted investment goods producers compared to the latter if the optimal rate of return on investment and the optimal portion of clean technology output are implemented. Under the implementation of the efficient scale of clean technology the exemption of investment goods producers exacerbates the distortion in terms of savings used to invest in dirty technology.

Exempting some households from paying the surcharge reversely affects the efficiency. Let consumption goods producers discriminate buyers in charging $B(p - \tau^*)$ from households that comprise the fraction α in the population, and Bp from other households. If households bought energy, then the sellers of energy would discriminate. Given consumption c' and c'' of these groups, respectively, assume appropriate endowments and define welfare $\alpha J' + (1 - \alpha)J''$, because consumption amounts are heterogeneous. The fraction of exempted agents is $v = \alpha(1 - \hat{s})$ given the savings rate $\hat{s}$ when there is a unit mass of identical energy users. Equation (3.15) that governs savings remains valid so that $R(1 + \rho(\hat{\tau}/p)') = Q_B\pi_B/p = Q_j\pi_C/p$. As a result greater exemption mitigates the distorting effect of the policy on the marginal real rate of return on investment R. The real marginal benefit of energy production

$$\frac{Q_B Q_j}{\frac{\eta}{1-v}Q_B + (1 - \eta)Q_j} = R(1 + \rho(\hat{\tau}/p)') \tag{3.27}$$

is strictly smaller when more households are exempted, $v \in (0, 1)$ increases. There is a critical level $v_j(\eta) < 1 - \eta$ that satisfies the efficiency condition $Q_j = R(1 + \rho(\hat{\tau}/p)')$ for the marginal clean technology j whose investment is optimal, depicted through the curve in the right panel of Figure 3.2. When exempting a fraction of households $\alpha = v_j/(1 - \hat{s})$ from paying the surcharge such that v_j is smaller than the output share of dirty technology the policy implements an optimum with fully utilized capital.[20] The intensity $v_j(\eta)$ of

[20] This level is $v_j(\eta) = (1 - \eta)(Q_B - Q_j)/(Q_B - (1 - \eta)Q_j)$.

the discrimination changes over time if the Pareto optimal levels Q_j and η vary over time, which they generally do. The intensity $v_j(\eta)$ depends positively on Q_j/Q_B and negatively on η. Thus $v_j(\eta)$ decreases on a path with increasing absolute and relative output from clean technology. I discuss in Section 3.3 if discriminating the surcharge between households is politically feasible.

Another possibility to implement an optimum might be a uniform surcharge and a greater tax rate $\hat{\tau}$ than $\hat{p}\epsilon/(\partial U/\partial c)$ for emissions in the production of investment goods.

3.2 Extraction cost

This extension relaxes a price rigidity that appears in partial equilibrium in Ambec & Crampes (2012). Dirty technology capital that is used with fuel in fixed proportion is fully utilized absent emissions pricing, in contrast to Ambec & Crampes (2012). The production of

$$m_{iB}(s,t) = \min[\chi_B \mathrm{u}_{iB}(s,t)a_{iB}(t), r_{iB}(s,t)/\alpha]$$

units of energy in firm i uses the complementary factors physical capital a_B and fuel r_B at some efficiency $\alpha > 0$. Capital is produced one period before its use. The fuel technology is not analogous to the dirty technology because then fuel would need to be produced and delivered within a day. There are two interpretations of the model. (i) Fuel is produced and purchased in the period of use. The law of motion of each firm i's capital in fuel production is

$$a_{iR}(t+1) = \gamma_R(1 - \mathrm{u}_{iR}(t))a_{iR} + y_{iR}(t)$$

all $t \geq 0$ given chosen utilization rate $\mathrm{u}_{iR}(t) \in [0,1]$ and new capital units $y_{iR}(t) \geq 0$. Then emissions in fuel production occur at rate d_R per fuel amount $m_{iR}(t) = \chi_R \mathrm{u}_{iR}(t)a_{iR}(t)$ produced in period t. (ii) Fuel is produced directly using inputs and forwarded into the next period in which it can be used. The fuel stock evolves according to

$$S_i(t+1) = \gamma_R(S_i(t) - m_{iR}(t)) + \chi_R y_{iR}(t)$$

in each firm i defining $S_i(t) = \chi_R a_{iR}(t)$. Old stock net of use, the term in parentheses, is nonnegative and depreciates at rate $(1 - \gamma_R)$. Then the emission intensity ρ_R of energy input in fuel production accounts for emissions of fuel production, for example, in natural gas flaring at extraction sites of petroleum, petroleum refining, underground ventilation of coal mines, and transportation of raw fuel. The endowment of capital in the initial period satisfies $W\sum_{iB}\chi_B a_{iB}(0)=\sum_{iR}\chi_R a_{iR}(0)>0$ so that the maximum fuel demand equals the maximum (unproduced or existing) fuel supply. Dirty technology capital is useful in W subperiods. Thus, in (i) underutilized capital in the fuel technology avoids production of fuel that is not demanded, and in (ii) storing fuel balances fuel supply and fuel demand. I abstract from other variable cost than the input cost x that yields $\varepsilon_R x$ units of new capital or $\chi_R\varepsilon_R x$ units of fuel.

Consider an emissions tax $\hat{\tau}$. Fuel use creates emissions at ratio d_B/α per unit of the fuel input. Then the unit revenue in the profit function (3.6) of dirty energy is $\pi_B(s,t)=p(s,t)-d_B\hat{\tau}(t)-\alpha p^*(s,t)$ given efficient fuel demand $r_{iB}(s,t)=\alpha\chi_B \mathrm{u}_{iB}(s,t)a_{iB}(t)$ at unit price $p^*(s,t)$ of fuel. The unit net revenue of fuel is $\pi_R(s,t)=p^*(s,t)-d_R\hat{\tau}(t)$. One unit of clean energy earns $\pi_C(s,t)=p(s,t)$ dollars. The market clearing conditions (3.8) and (3.9) remain letting $\mathcal{J}=\{B,C,R\}$. In addition, fuel demand cannot exceed fuel supply, $\sum_{s\in\mathcal{S}}\sum_i r_{iB}(s,t)\le\sum_i m_{iR}(t)$ all $t\ge0$. The government budget constraint is

$$\hat{\tau}(t)\left(\sum_i d_R m_{iR}(t)+\sum_{s\in\mathcal{S}}\sum_i\left[(d_B/\alpha)r_{iB}(s,t)+\sum_{j\in\mathcal{J}}\rho_j x_{ij}(s,t)\right]\right)\le tr(t)$$

all $t\ge0$. The left side in the government budget constraint divided by the tax rate $\hat{\tau}$ is the emissions amount in period t.

In the regime (i) with emissions tax the utilization of fuel technology capital satisfies the necessary equilibrium condition

$$\mathrm{u}_{iR}(t)\left\{\begin{array}{c}=1\\ \in(0,1)\\ =0\end{array}\right\}\quad\Longrightarrow\quad\chi_R\{p^*(s,t)-d_R\hat{\tau}(t)\}\left\{\begin{array}{c}\ge\\ =\\ \le\end{array}\right\}\gamma_R\upsilon_{iR}(t)\quad\text{all }\ s\in S\tag{3.28}$$

all $t\ge0$. The result in the next proposition holds in equilibria in which the fuel price is constant or varies over states. The productivities χ_B and χ_R in the dirty and the fuel technology are constant. Define their marginal products $Q_B=W\varepsilon_B\chi_B$ and $Q_R=\varepsilon_R\chi_R$ in converting energy input into energy output and energy input into fuel output, respectively. Furthermore let the rate of fuel per energy attributable to the fuel technology be $\upsilon=Q_R/(Q_R+\alpha_B Q_B)$ as in Chapter 2. Here $\alpha_B=\alpha(\chi_B/\chi_R)$. I assume that growth of consumption is optimal using the dirty technology output until the land required for it is exhausted if there is no environmental cost.

Assumption 3.1 *$\upsilon Q_B > \beta^{-1}$. $Q_R > \alpha$ if $\chi_R > \chi_B$.*

The second condition ensures that in an optimum subject to no environmental cost there is investment in the fuel technology when consumption fluctuates.[21]

The following proposition shows the irrelevance of the fuel technology for utilization of capital in the dirty technology.

Proposition 3.6 *Capital $a_{iB}(t) > 0$ in the dirty technology is fully utilized, $\mathrm{u}_{iB}(s,t) = 1$, in each firm i in all states $s \in S$ all $t \geq 0$, and capital $a_{iR}(t) > 0$ in the fuel technology is fully utilized, $\mathrm{u}_{iR}(t) = 1$, in each firm i all $t \geq 0$ so that $S_{iR}(t) = m_{iR}(t)$ if the emissions tax is zero, $\hat{\tau}(t) = 0$ all $t \geq 0$.*

Proof. Assumption 3.1 is equivalent to $Q_B(Q_R - \alpha_B) > Q_R$. Therefore $Q_R > \alpha_B \geq \alpha_B \gamma_R$. The gross return on using fuel to produce future capacity to produce fuel, Q_R/α_B, is greater than the gross return on storing fuel, γ_R. Thus, the stocks $\sum_i a_{iB}(t)$ and $\sum_i a_{iR}(t)$ have the same ratio at t and $(t+1)$, and all capital is fully utilized at $(t+1)$ if all capital is fully utilized at t. The induction starts in the period $t = 0$, in which the stocks match full utilization. Underutilization of capital $a_{iB}(t) > 0$ in the dirty energy technology in some firm i, $\mathrm{u}_{iB}(s,t) < 1$ in some state s, requires that capital $a_{kR}(t) > 0$ in the fuel technology is underutilized in some firm k, $\mathrm{u}_{kR}(t) < 1$, given that the maximum fuel

[21] Consumption can be increased by lowering the utilization of dirty technology capital and the investment in the fuel technology in any state s' when consumption is maximized holding the input in investment in the fuel technology in all other states and next period's maximum fuel supply constant without affecting aggregate dirty technology investment and the next period's aggregate dirty technology capital stock if $Q_R < \alpha\gamma_R$. Formally, $dc(s',t) = \chi_B d\mathrm{u}_B(s',t) a_B(t) - dx_R(s',t) > 0$ at $\alpha\chi_B(d\mathrm{u}_B(s',t))a_B(t) = \chi_R(d\mathrm{u}_R(t))a_R(t) < 0$ and $\gamma_R(d\mathrm{u}_R(t))a_R(t) = \varepsilon_R dx_R(s',t)$. The first condition implies the second necessary condition in Assumption 3.1 if $\chi_R \leq \chi_B$.

demand matches the aggregate fuel capacity. (i) $\max_s \mathrm{u}_{iB}(s,t) < 1$ some i and s. Then

$$\begin{aligned}
&\upsilon Q_B(\upsilon_B(t) + W\alpha_B\upsilon_R(t)) \\
&\leq (\upsilon\chi_B + (1-\upsilon)\chi_B)\sum_{s\in\mathcal{S}} p(s,t) \quad (\varepsilon_j\upsilon_j(t) \leq p(s,t)\ \ \forall s\in S,\ \upsilon_j \leq p_j = \phi_j,\ (3.13)) \\
&\leq \chi_B\left[\sum_{s\in\mathcal{S}_{iB}} p(s,t) + \sum_{s\in\mathcal{S}\setminus\mathcal{S}_{iB}} \alpha p^*(s,t)\right] \quad (p(s,t) \leq \alpha p^*(s,t)\ \ \forall s\in S\setminus S_{iB},\ \ (3.11),\ \ j=B) \\
&= \chi_B\sum_{s\in\mathcal{S}_{iB}}\{p(s,t) - \alpha p^*(s,t)\} + \chi_B\sum_{s\in\mathcal{S}} \alpha p^*(s,t) \\
&\leq \gamma_B\upsilon_B(t) + \chi_B\sum_{s\in\mathcal{S}} \alpha p^*(s,t) \quad (\max_s \mathrm{u}_{iB}(s,t) < 1,\ \ (3.12),\ \ j=B) \\
&\leq \gamma_B\upsilon_B(t) + \gamma_R W\alpha_B\upsilon_R(t) \quad (\mathrm{u}_{kR}(t) < 1, \quad \chi_R p^*(s,t) \leq \gamma_R\upsilon_R(t)\ \ \forall s\in S,\ \ (3.28))
\end{aligned}$$

for some i and k. This contradicts Assumption 3.1. (ii) $\max_s \mathrm{u}_{iB}(s,t) = 1$ all i. Then underutilized dirty technology capital implies that $\mathrm{u}_{iB}(s',t)$ is below the maximum utilization rate for some i, that is, $S \setminus S_{iB}$ is nonempty. Then

$$\begin{aligned}
&\varepsilon_R\upsilon_R(t) \leq p(s',t) \quad (\upsilon_R \leq p_R = \phi_R,\ (3.13),\ \ j=R) \\
&\leq \alpha p^*(s',t) \quad (\mathrm{u}_{iB}(s',t) < 1,\ \ s' \in S\setminus S_{iB},\ \ (3.11),\ \ j=B) \\
&\leq (\alpha/\chi_R)\gamma_R\upsilon_R(t) \quad (\mathrm{u}_{kR}(t) < 1, \quad \chi_R p^*(s,t) \leq \gamma_R\upsilon_R(t)\ \ \forall s\in S,\ \ (3.28))
\end{aligned}$$

all $s \in S$ and $\gamma_R \leq 1$ imply that $Q_R \leq \alpha$. Assumption 3.1 and $Q_B > 0$ and $Q_R = \varepsilon_R\chi_R > 0$ contradict this. *Q.E.D.*

The extraction of fuel using energy in the technology R is more productive than the conversion of fuel into energy in the technology B, that is, $Q_R > \alpha_B$, if the combined marginal product of energy-to-energy exceeds one, $\upsilon Q_B > 1$. Then investing in energy technology that uses fuel and in fuel technology is more profitable than postponing the use of capital or fuel when emissions pricing is absent.

3.3 Conclusion

This chapter has characterized efficient allocations when clean technology productivity fluctuates and there is irreversible investment in dirty and clean technologies. Consumption and investment are distributed such that investment occurs only in subperiods when consumption is maximized. In these subperiods the opportunity cost of investing is minimized. Adjustment costs likely yield investment only in subperiods in which consumption is sufficiently large. In some of these subperiods dirty technology capital may be underutilized because clean technology is very productive. Thus contingent investment efficiently absorbs the fluctuation in clean technology output in states in which consumption is maximized, and investment does not occur in other states so that consumption absorbs it in the subperiods of these states in which dirty technology capital is fully utilized.

Chapter 3 modifies the view of exclusive investment in clean technology in the long-term. Dirty fossil fuel technology is used to back up clean renewable energy production to smooth consumption across subperiods when the renewable energy supply fluctuates because of weather and produced energy cannot be stored at low cost.

The underutilization of dirty technology capital, such as coal power plants, in the long-term is motivated by the fluctucation of clean technology productivity and the disutility of emissions expressed as the environmental shadow cost. Fuel cost does not play a role because the price of dirty output adjusts to the price of fuel.

The environmental shadow cost is constant *over subperiods* in a given period, because emissions in all subperiods have the same marginal effect on pollution and pollution is the same in each subperiod. For this noncontingency it is irrelevant whether the marginal effect of the environment on society occurs in each subperiod or once in any period. The reason for the state history independence of the environmental shadow cost *over periods* is that the frequency of states is certain.

The results on government policy can be summarized in four points. (i) A clean output subsidy can be combined with a tax on output or equipment purchases to implement an optimum. In the former system a tax on capital purchases accounts for the emissions in building capital. The system with output tax is preferred among these variants of a tax-rebate system if dirty technology capital is unequally utilized across states in optimum, because it induces the efficient utilization of capital. The system with taxed investment to internalize both the marginal effects of emissions in using and building capital does not induce underutilized capital. (ii) An excise tax or a contingent ad valorem tax is needed rather than a noncontingent ad valorem tax to implement contingent underutilization through indirect taxes. These tax rates can be expressed in terms of the Pigouvian emissions tax. (iii) Agents that use wind turbines or solar panels to produce electricity may not have access to techniques that forecast wind speed or solar radiation and machines that use electricity can be pregrogrammed conditional on the state. A contract that yields a constant revenue across states in which consumption optimally fluctuates does not hamper efficiency if capital is equally utilized in all states in optimum. Government intervention is not needed to stabilize the price, because distributors offer a stable price. These distributors can be seemingly risk-averse because all states occur with certainty in some subperiod. The role of the government is to internalize the externality. (iv) A clean output subsidy that is funded by a surcharge such that consumers pay a uniform price leads to excessive investment in both dirty and clean technology relative to an optimum if the implemented marginal rate of return on investment is optimal and the implemented portion of clean technology output is not too high relative to its optimal level. The reason is that the revenue of clean technology users is too high relative to the price of its output compared to an optimum of pollution control. This induces too much clean output. The condition on the relative output between dirty and clean technologies implies that there is too much output in dirty technology. For example, the marginal real rate of return on investment is the inverse of the time discount factor when consumption is constant across periods. The optimal marginal real rate of return on investment and the optimal portion of clean technology output are jointly implementable only if there are emissions of investment if only households pay a uniform surcharge, because the emissions tax receives a greater weight relative to the factor price in the marginal cost of investment given this exemption compared to uniform price for all energy users. There is greater output in dirty and clean technologies relative to the allocation in equilibrium with uniform surcharge for all energy users. But an optimum is implementable if some households are exempted from the surcharge because the revenue of clean technology users decreases in the fraction of exempted households for given portion of clean technology output. Discriminating among households as opposed to between households and investment goods producers retains the efficient relative price of output and the investment goods that produce output.

Governments in EU member countries use fully-funded tax-subsidy systems to promote investment in wind and solar renewable energy. In particular, in Germany large electricity users such as investment goods producers are exempted from paying the surcharge. The distortion that exists under uniform surcharges is exacerbated when investment goods producers are exempted from paying the surcharge assuming the real interest rate and the portion of clean technology output are optimal. Examining other situations, for example, nonoptimal marginal rate of return on investment subject to international trade in assets, is a topic for further research. A more detailed analysis of environmental policy under price discrimination is warranted in future research because in practice larger electricity users pay lower prices that are not fully explained by the exemption of the environmental charge.

Usefulness of policies. An emissions tax or a tax-subsidy system with energy tax rates and emissions tax rate in the investment sector can implement all optimal allocations. The latter system requires different tax rates for multiple dirty technology types such as fuel types and vintages that have a different emission intensity of output. This information is needed to administer the emissions tax too if emissions are not measured directly. The carbon dioxide and methane emissions are inferred indirectly, for example, from fuel input use. Thus the information requirement may be the same for these policies.

The capital gains tax and capital tax implement only noncontingent allocations. A capital gains tax or capital tax seems impractible to internalize greenhouse-gas pollution because these policies require to disentangle dirty and clean technology capital in the portfolios of firms and households. The financial balance sheets contain nominal capital amounts so that further accounts must be investigated to differentiate output between fossil-fuel using and renewable energy technologies.

A system with an investment goods tax and an emissions tax rate in the investment sector only implement allocations with fully utilized capital. The information of capital vintages should be collectable at the same cost as the information for an emissions tax on emissions that are not directly measurable. However paying for the emissions not occured yet may pose a legal problem.

Fully-funded renewable energy subsidies implement allocations with full utilization of capital from an environmental perspective. The variant with exempted households seems politically feasible given the large popularity of both feed-in tariffs and feed-in premia and of distributional policies. The identity of households that pay the surcharge does not matter for Pareto efficiency assuming preferences that admit an essential independence between the environment and the distribution of consumption. Thus efficiency can be combined with distributional goals. Beside more differentiated rates than two surcharge levels implement a Pareto optimum and redistribute income. A Mirrleesian approach to redistribution by an incompletely informed government is a topic for further research.

Limitations. Menanteau et al. (2003) argue that increased intermittent capacity creates a need for additional capacity to stabilize the grid. The impact of grid stabilization on capacity building originates in fluctuation of supply that does not arise in absence of intermittent sources. This issue is blended out in the present paper, and may be considered in further work.

Households pay a contingent price if an equilibrium is subject to noncontingent taxes or subsidies and consumption fluctuates. I make the assumption that distributors of energy do not discriminate between energy users that produce consumption goods and energy users that produce investment goods. Thus I restrict attention to economies in which households and producers of consumption goods and investment goods act on contingent markets or make contingent contracts. The inability of electricity users, including households, that produce consumption goods to respond to contingent prices of electricity because it is not metered every day prevents efficiency if consumption should fluctuate. Consumption fluctuation is likely optimal when the environmental shadow cost is sufficiently high so that dirty capacity is sufficiently low or when the clean technology productivity varies much. A question for further research is when the benefit of fluctuating consumption in terms of saved dirty capacity outweighs the cost of devices for daily metering.

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