A FIRST PASSAGE TIME MODEL FOR COUNTERPARTY DEFAULT

by

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Abstract

Counterparty risk is becoming an important issue for over the counter trades. However, valuation of counterparty risk is still in its early stage. The previous studies often ignore the wrong way risk which is a key aspect of counterparty risk valuation. In this paper, we develop a tractable first passage time model, which is able to capture the correlation between the default and market risk factor, particularly interest rates. We derive closed form solutions for survival and marginal default probability in the presence of correlation between the default event and interest rate based on the model assumptions. The numerical results for pricing credit default swap illustrate that the closed form solutions are exact when the volatilities for default driver and risk factor are constant and a good approximation when the volatilities are piece-wise constant. We also showed how sensitive the CDS spread is to different model parameters. In particular, we found the correlation between the default and market factors has a significant impact on the default probability and CDS spread.

Keywords:

Counterparty risk, credit default swap, first passage time model, wrong way risk, Monte Carlo simulation

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Table of Contents

APPROVAL ii
Abstract iii
Acknowledgements iv
Table of Contents v
List of Figures vi
List of Tables vii
1. Introduction 1
1.1. Counterparty Credit Risk 1
1.2. Wrong Way Risk 2
1.3. Modeling Firm Default 3
1.4. Credit Default Swap Valuation 5
2. Model Assumptions and Formula Derivation 7
2.1. Counterparty Default Driver 8
2.2. Hull White Interest Rate Model 9
2.3. Derivation of Approximate Formulas 10
3. Numerical Results 15
3.1. Comparison of the Model Implementation by Monte Carlo and the Closed Form Formulas 15
3.2. Sensitivity of CDS Spread to Different Parameters 19
4. Conclusions 20
5. Future Research 21
5.1. Modeling Recovery Rate 21
5.2. Valuation of other Credit Derivatives 22
5.2.1. Foreign Currency CDS 22
5.2.2. CDS Options 23
5.2.3. Valuation of Counterparty Risk by CVA 24
5.2.4. Contingent CDS 25
Bibliography 27

List of Figures

Figure 1 CDS diagram 6

Figure 2 The mechanics of a C-CDS with an IRS as the underlying derivative 26

Figure 3 CDS's sensitivity to x0..37

Figure 4 CDS's sensitivity to the drift term 38

Figure 5 CDS's sensitivity to sigma σ 39

Figure 6 CDS's sensitivity to correlation 40

Figure 7 Comparison of survival probabilities from formula and Monte Carlo..41

Figure 8 Comparison of 5 year CDS spreads from formula and Monte Carlo 42

List of Tables

Table 1 Input parameters 28

Table 2 Survival probabilities that tau bigger than five years 29

Table 3 Default probabilities $4 < \tau < 5\ year$ 30

Table 4 Default probabilities $4 < \tau < 5\ year$ with piece wise constant interest rate volatility..31

Table 5 CDS spreads (in bps) with 5 year tenor 32

Table 6 CDS spreads (in bps) with 4 year tenor 33

Table 7 CDS spreads (in bps) with 3 year tenor 34

Table 8 CDS spreads (in bps) with 2 year tenor 35

Table 9 CDS spreads (in bps) with 1 year tenor 36

Table 10 Run time in seconds..36

1. Introduction

The motivation for this study is to derive an approximate closed form solution to price the counterparty risk for over the counter (OTC) trades with credit risky counterparties. Valuation of counterparty risk becomes an active research area especially since the financial crisis in 2007. A key aspect of counterparty risk valuation is to capture the wrong way risk (WWR), which occurs when the default has adverse correlation with risk factors driving the underlying asset value. Current commercial implementations of counterparty risk often ignore the correlation between counterparty default and the risk factors. However, recent research has showed that the correlation between default and market risk factors can have a significant impact on counterparty risk valuation (Brigo & Pallavicini, 2008). The traditional first passage time model is not particularly well suited for models involves correlated market and credit risk variables. Here we outlined an approach to modeling wrong way risk by a new first passage time model based on Merton's structural model (Merton, 1974). By making a few adjustments to Merton's model, we find it possible to capture market/credit correlation while retaining much of the analytical simplicity of the original model.

1.1. Counterparty Credit Risk

Counterparty credit risk is the risk that the counterparty does not make payments as promised in a trade such as a bond, credit derivative or credit insurance. It is also known as default risk since the event of failing to pay as scheduled is called a default. Because many financial contracts are traded over the counter, the credit quality of the counterparty can be relevant. During the most recent financial crisis, many OTC derivatives defaulted because of the collapse of financial institutions, such as Lehman Brothers. The Basel Committee on Banking Supervision increases counterparty credit risk capital requirements and required more related risk management practices in Basel III framework. In order to mitigate counterparty risk, new credit default swap contracts are being introduced into the international credit market and the use of central clearing entities for some credit derivatives has been proposed.

1.2. Wrong Way Risk

Counterparty risk can be affected by wrong way risk, namely the risk that different risk factors be correlated in the most harmful direction. Wrong way risk occurs when exposure to counterparty is adversely correlated with the credit quality of that counterparty. An example is that the buyer of counterparty risk enters an interest rate swap (IRS) receiving floating rate from counterparty, the value of IRS increase when interest rate goes up. In this case the correlation of default of counterparty with interest rate becomes an important factor. The contract covering the counterparty risk is worth more if the counterparty is less likely to default when the trades have positive value to the counterparty risk buyer.

WWR is further divided to two categories: specific WWR and general WWR. The specific WWR arises through poorly structured transactions, for example those collateralized by own or related party shares; and general or conjectural WWR is that the credit quality of the counterparty wrong way risk for non-specific reasons correlated with a macroeconomic factor which also affects the value of derivatives transactions. For example that corporate credit quality might have a correlation with the interest rates.

1.3. Modeling Firm Default

A key step to correctly price the counterparty risk is to model the firms default probability under the risk neutral measure. The default models can be divided into two main categories: reduced form models and structural models.

Reduced form models are also called intensity models, which model the firm default by Poisson process with deterministic or stochastic intensity (Jarrow & Turnbull, 1995). The default time is the first time that the jump happens. Default is triggered by unobservable exogenous component, which is independent of all the default free market information. The first event of a Poisson counting process which occurs at some time τ with a probability defined as

$$\Pr[\,\tau < t + dt |\, \tau > t] = \lambda(t)dt$$

Which is the probability of a default occurring within the time interval [t, t+dt) conditional on surviving to time t. This probability is proportional to sometime dependent function $\lambda(t)$, known as the hazard rate, and the length of the time interval dt. In the initial models, the hazard rate process is assumed to be deterministic and independent of interest rates and recovery rates. Now there are more models trying to capture the correlation between the hazard rate with interest rate and recovery rates. The advantage of reduced-form models is that it is easy to calibrate to Credit Default Swap (CDS) data (Brigo & Alfonsi, 2005).

They can also be extended to price more exotic credit derivatives. The disadvantage of these models is that there is no economic rationale behind default.

Counterparty risk under correlation between interest-rates and default using reduced form model was studied (Brigo & Pallavicini, 2008). In this model, a stochastic intensity model with possible jumps is adopted for the default event; the interest-rates are modelled according to a short-rate Gaussian two factors process. The Brownian motions in the two processes are correlated. It has been showed that correlation between interest-rates and default has a relevant impact on the counterparty risk value. This model can be only implemented by Monte Carlo simulation and needs very intensive computation.

Structural models, also called first passage time models, are based on the work by Merton (Merton, 1974). They assume that the firm value follows a geometric Brownian motion just like stocks price and can be observed from the market. The payoff for equity holders is similar to a call option on the firm. In first passage time models, the default time is the first time that the firm value hits either a deterministic or a stochastic barrier (Black & Cox, 1976). Therefore, these models use the same mathematics of barrier options pricing models. The default process can be monitored based on the information from the market. In this case there is an economic rationale behind the default. The structural models are the basis for methodologies such as CreditMetrics4 and related proprietary methods developed by risk analytics vendors such as Moody's KMV (Crosbie, Pr; Bohn, J, 2003). However, these models cannot be calibrated exactly to CDS market data (Brigo & Tarenghi, 2005). A complex numerical calibration process involving Monte Carlo simulation has to be used to do the calibration when a time-varying default threshold is used (Arvanitis & Gregory, 2001).

1.4. Credit Default Swap Valuation

A credit default swap is a type of derivative contract that protects the lender in the event of default, which includes bankruptcy, failure to pay, restructuring. It is used to transfer the credit risk of the reference entity from one party to another. Therefore, CDS contract is considered as a form of insurance policy. However, when entering into a CDS, both the buyer and seller of CDS take on counterparty risk, which needs to be managed too. One of the major differences between a tradition insurance policy and a CDS is that people who don't own the underlying asset can still buy a CDS.

Figure1 is a diagram of CDS contract. In a standard CDS contract, the buyer of the CDS contract makes a series of payment, known as the premium leg, to the insurance company and, in exchange, receivers a payoff, known as a protection leg, if the underlying asset in the contract defaults. The size of these premium payments is calculated from a quoted default swap spread in basis points (bp) paid quarterly, semi-annually or annually based on the face value of the contract (O'kane, D; Turnbull, S, 2003).

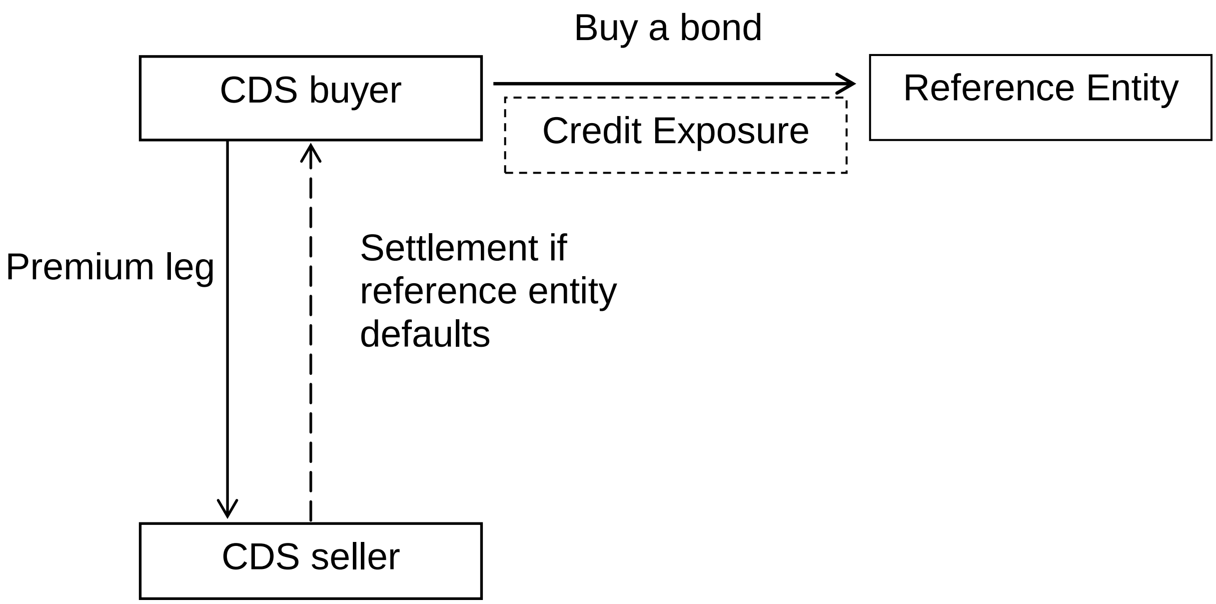

Figure 1 CDS diagram

The greater the credit risk the reference entity has, the higher the credit default spread. The payments are made until a credit event occurs or until maturity, whichever occurs first. In a credit event, the settlement of a CDS involves either a cash payment or physical delivery of the underlying asset.

To value a CDS, a term structure of default swap spreads, a recovery rate assumption and a model are needed. In Hull and White's paper (2000), it assumed that the default events, interest rates and recovery rates are mutually independent. The CDS value is the difference between the present value of the premium leg and the protection leg. If the premium accrued is ignored, the approximation of the mark to market CDS value is as following:

$$CDS = PV\ of\ premium\ leg\ coupons - PV\ of\ default\ leg\ payments$$

$$CDS \approx X \sum_{j=1}^{N} \delta_j \, DF(0,T_j) \mathbb{Q}^N \left(1_{[\tau > T_j]} \right) - (1-R) \sum_{j=1}^{N} \delta_j \, DF(0,T_j) \mathbb{Q}^N \left(1_{[T_{j-1} < \tau \leq T_j]} \right) \quad (1.1)$$

Where

$\mathbb{Q}^N$ is the expectation under the risk neutral measure

δ_j is the day count fraction between premium dates

$DF(0, T_j)$ is the discount factor from valuation date to premium payment date.

X is the default swap spread

R is the recovery rate

When the CDS contract is entered, the CDS spread is the value that makes this expression zero.

In Hull and White (2000), a reduced form model is built to price the CDS. It is assumed that the probability density function is piece wise constant. However, the study from J.P.Morgan indicated that the market practice acts slightly different from this. The hazard rate, $\lambda(t)$, rather than the default probability density is usually assumed to be piece wise constant function.

2. Model Assumptions and Formula Derivation

With an intention to derive a closed form formula to price counterparty risk with correlation between the firm default and risk factor such as interest rate, a new first passage time model is proposed. And the default probability is modeled under the forward measure instead of risk neutral. The approximation of CDS becomes

$$CDS \approx X \sum_{j=1}^{N} \delta_j \, DF(0,T) \mathbb{Q}^T \left(\frac{1}{DF(T_{j,} T)} \, 1_{[\tau > T_j]} \right) - (1-R) \sum_{j=1}^{N} \delta_j \, DF(0,T) \mathbb{Q}^T \left(\frac{1}{DF(T_{j,} T)} \, 1_{[T_{j-1} < \tau \le T_j]} \right) \quad (2.1)$$

The notation in this formula is the same as (1.1). The only difference is that $\mathbb{Q}^{\mathrm{T}}$ is the expectation under the corresponding forward measure T. Consequently, the discount factor becomes $DF(0,T)$ rather than $DF(0,T_j)$.

Just for simplicity, the accrued premium is ignored and the discretization for the protection leg is the same as the payment frequency. The remaining part of this section is to show how we derive closed form formulas for $\mathbb{Q}^T \left(\frac{1}{DF(T_{j,} T)} \, 1_{[\tau > T_j]} \right)$ and $\mathbb{Q}^T \left(\frac{1}{DF(T_{j,} T)} \, 1_{[T_{j-1} < \tau \le T_j]} \right)$.

We model both counterparty default drivers and interest rates as stochastic processes. We assume the correlation between interest-rates and counterparty is constant.

2.1. Counterparty Default Driver

Classical first passage time models postulate that the firm value has a lognormal dynamic similar to a Geometric Brownian Motion process. Crouhy's paper showed that this log normality assumption is quite robust (Crouchy, Falai, &

Mark, 2000). KMVs' empirical studies also demonstrated that actual data conform quite well to this hypothesis. However, it is hard to find an analytical formula for a default process which follows the Geometric Brownian dynamic and has a correlation with the risk factors. Here we use a credit driver $x(t)$ rather than the firm value to model default. And this credit driver is assumed to follow a Gaussian process as following

$$x(t) = x(0) + m(t) + \int_0^t \sigma(s) dW(s)$$

$$m(t) = u \int_0^t \sigma(s)^2 ds$$

$m(t)$: a deterministic drift function, where u is constant

$\sigma(t)$: a deterministic, piecewise continuous volatility function

W: a univariate Brownian motion with respect to a long forward measure $\mathbb{Q}^T$

The counterparty defaults at the first time τ that the univariate process falls to zero or other specific barrier. We also impose an artificial structure on the drift $m(t)$ of the counterparty default driver.

2.2. Hull White Interest Rate Model

The interest-rates are modelled according to Hull-White model, where the dynamics of the short rate $r(t)$ is governed by the following equations:

$$r(t) = (\emptyset(t) - ar(t))dt + \nu(t)dW$$

Where

$r(t)$: the short rate at time t

$\emptyset(t)$: time dependent drift for short rate

a: the mean reversion factor

$\nu(t)$: the volatility parameter

Then the terminal value of $\frac{DF(t,T_i)}{DF(t,T)}$ is a martingale under the forward measure $\mathbb{Q}^T$

$$\frac{DF(t,T_i)}{DF(t,T)} = \frac{1}{DF(T_i\,,T)} = \frac{DF(0,T_i\,)}{\mathrm{DF}(0,\mathrm{T})} \exp\left[\int_0^{T_i} v_i\,(s)\cdot dZ(s) - \frac{1}{2}\int_0^{T_i} \|v_i(s)\|^2 ds\ \right] \quad (2.2)$$

$$for\ \ 0 \le t \le T_i$$

for a deterministic (vector) function $v_i(t)$ and a (vector) Brownian motion Z(t) under $\mathbb{Q}^T$. Here "· " denotes the relevant dot product of vectors.

2.3. Derivation of Approximate Formulas

We further assume that the default driver and interest rates follow multivariate normal distribution. If the correlation of default driver and interest-rates remain constant as ρ_{WZ} , the covariance matrix of default driver and interest rates at time t can be expressed as following

$$\sum(t) \equiv \begin{pmatrix} \int_0^t \|v_i(s)\|^2 ds & \int_0^t \sigma(s) v_i(s)\cdot \rho_{WZ} ds \\ \int_0^t \sigma(s) v_i(s)\cdot \rho_{WZ} ds & \int_0^t \sigma(s)^2 ds \end{pmatrix} \qquad (2.3)$$

When $t = T_j$, the random vector $\left(\int_0^{T_j} v_j(s) \cdot dZ(s), x(T_j)\right)$ is normally distributed with mean $(0, x(0) + m(T_j))$ and covariance matrix $\sum(T_j)$. Given $x(T_j)$, the random variable $\int_0^{T_j} v_j(s) \cdot dZ(s)$ is normal with mean

$$\frac{\sum_{12}(T_j)}{\sum_{22}(T_j)}\left(x(T_j) - x(0) - m(T_j)\right)$$

and variance

$$\sum_{11}(T_j) - \frac{\sum_{12}(T_j)^2}{\sum_{22}(T_j)}$$

Define $$f_j\left(x(T_j)\right) \doteqdot \mathbb{Q}^T\left(\exp\left[\int_0^{T_j} v_j(s) \cdot dZ(s)\right] \middle| x(T_j)\right) \qquad (2.4)$$

Then we can get that

$$f_j\left(x(T_j)\right) = \exp\left[\frac{\sum_{12}(T_j)}{\sum_{22}(T_j)}\left(x(T_j) - x(0) - m(T_j)\right) + \frac{1}{2}\left(\sum_{11}(T_j) - \frac{\sum_{12}(T_j)^2}{\sum_{22}(T_j)}\right)\right] \quad (2.5)$$

Step 1: derivation of the expected survival probability

$$\mathbb{Q}^T\left(\frac{1}{DF(T_j, T)} 1_{[\tau > T_j]}\right)$$

$$= \mathbb{Q}^T\left(\frac{DF(0, T_i)}{DF(0, T)} \exp\left[\int_0^{T_i} v_i(s) \cdot dZ(s) - \frac{1}{2}\int_0^{T_i} \|v_i(s)\|^2 ds\right] 1_{[\tau > T_j]}\right) \approx$$

$$\mathbb{Q}^T\left(\frac{DF(0, T_i)}{DF(0, T)} \exp\left[-\frac{1}{2}\int_0^{T_i} \|v_i(s)\|^2 ds\right] f_j\left(x(T_j)\right) 1_{[\tau > T_j]}\right) \qquad (2.6)$$

$\frac{DF(0,T_i)}{DF(0,T)} \exp\left[-\frac{1}{2}\int_0^{T_i} \|v_i(s)\|^2 ds \right]$ is deterministic. So we only need to get the closed form for

$$\mathbb{Q}^T\left(f_j\left(x(T_j)\right) 1_{[\tau > T_j]}\right)$$

Since we assume that the firm defaults when the credit driver x hits zero, the transition density function G for x has the same form as the down and knock in barrier option.

$$\mathbb{Q}^T\left(f_j\left(x(T_j)\right) 1_{[\tau > T_j]}\right) = \int_0^{\infty} G(x_0, 0, \theta; y) f_j(y)\, dy$$

Where transition density function

$$G(x_0, 0, \theta_1; y) \triangleq \emptyset\,(x_0, 0, \theta; y) - e^{2uy} \emptyset\,(x_0, 0, \theta; -y)$$

And $\emptyset\,(x_0, 0, \theta; y)$ is the normal density function with variance θ and mean $x_0 + u\theta$ because of our assumption 1.

$$\theta = \sum_{22}(T_j)$$

$$\emptyset\,(x_0, 0, \theta; y) \triangleq \frac{1}{\sqrt{2\pi\theta}} \exp\left[-\left(y - (x_0 + u\theta)\right)^2 / 2\theta\right]$$

Substitute $G(x_0, 0, \theta; y)$ and $f_j\left(x(T_j)\right)$ to the equation and do the integration, we can get

$$\mathbb{Q}^T\left(f_j\left(x(T_j)\right) 1_{[\tau>T_j]}\right)$$

$$= e^{\Sigma_{11}(T_j)/2}\left[N\left(\frac{x_0+\Sigma_{12}(T_j)+u\Sigma_{22}(T_j)}{\sqrt{\Sigma_{22}(T_j)}}\right)\right.$$

$$\left. - e^{-2(\Sigma_{12}(T_j)/\Sigma_{22}(T_j)+u)x_0} N\left(\frac{x_0+\Sigma_{12}(T_j)+u\Sigma_{22}(T_j)}{\sqrt{\Sigma_{22}(T_j)}}\right)\right] \quad (2.7)$$

Plug the equation (2.7) and valuation in matrix (2.3) into equation 2.6, we can get

$$\mathbb{Q}^T\left(\frac{1}{DF(T_j,T)} 1_{[\tau>T_j]}\right) = \mathbb{Q}^T\left(\frac{DF(0,T_i)}{DF(0,T)}\exp\left[-\frac{1}{2}\int_0^{T_i}\|v_i(s)\|^2 ds\right] f_j\left(x(T_j)\right) 1_{[\tau>T_j]}\right) \approx$$

$$\frac{DF(0,T_i)}{DF(0,T)}\left[N\left(\frac{x_0+\Sigma_{12}(T_j)+u\Sigma_{22}(T_j)}{\sqrt{\Sigma_{22}(T_j)}}\right) - e^{-2(\Sigma_{12}(T_j)/\Sigma_{22}(T_j)+u)x_0} N\left(\frac{x_0+\Sigma_{12}(T_j)+u\Sigma_{22}(T_j)}{\sqrt{\Sigma_{22}(T_j)}}\right)\right] \quad (2.8)$$

Step 2: derivation of the expected marginal default probability in the interval $[t_{j-1,} t_j]$

$$\mathbb{Q}^{\mathrm{T}}\left(\frac{1}{DF(T_{j-1},T)} 1_{[\mathrm{T_{j-1}}<\tau\leq\mathrm{T_j}]}\right)$$

Using the law of iterated expectations, this expectation can be written as

$$\mathbb{Q}^{\mathrm{T}}\left(\frac{1}{DF(T_{j-1},T)} 1_{[\mathrm{T_{j-1}}<\tau\leq\mathrm{T_j}]}\right) = \mathbb{Q}^{\mathrm{T}}\left(\frac{1}{DF(T_{j-1},T)} \mathbb{Q}^{\mathrm{T}}\left(1_{[\mathrm{T_{j-1}}<\tau\leq\mathrm{T_j}]}\middle| \mathcal{F}^{\mathrm{W,Z}}_{\mathrm{T_{j-1}}}\right)\right)$$

$$= \mathbb{Q}^T\left(\frac{DF(0,T_{j-1})}{DF(0,T)}\exp\left[-\frac{1}{2}\int_0^{T_{j-1}}\|v_i(s)\|^2 ds\right] f_{j-1}\left(x(T_{j-1})\right)\mathbb{Q}^{\mathrm{T}}\left(1_{[\mathrm{T_{j-1}}<\tau\leq \mathrm{T_j}]}\middle|\mathcal{F}^{\mathrm{W,Z}}_{\mathrm{T_{j-1}}}\right)\right) \quad (2.9)$$

where $\{\mathcal{F}_{\mathrm{t}}^{\mathrm{W,Z}}\}_{\mathrm{t}\geq 0}$ is the filtration generated by both Brownian motions W and Z.

Let's define that

$$\mathbb{Q}^T\left(f_{j-1}\left(x(T_{j-1})\right)\mathbb{Q}^{\mathrm{T}}\left(1_{[\mathrm{T_{j-1}}<\tau\leq \mathrm{T_j}]}\middle|\mathcal{F}^{\mathrm{W,Z}}_{\mathrm{T_{j-1}}}\right)\right) = \int_0^\infty G'(x_0,0,\theta_1;y) f_{j-1}(y)\,dy$$

The transition density function

$$\begin{aligned} G'(x_0,0,\theta_1;y) &= \left(\frac{1}{\sqrt{2\pi\theta_1}} exp\left[-\frac{(y-x_0-u\theta_1)^2}{2\theta_1}\right] - \frac{1}{\sqrt{2\pi\theta_1}} e^{2uy}\, exp\left[-\frac{(y+x_0+u\theta_1)^2}{2\theta_1}\right]\right)\left(N\left(\frac{-y-u\theta_2}{\sqrt{\theta_2}}\right) + e^{-2uy} N\left(\frac{-y+u\theta_2}{\sqrt{\theta_2}}\right)\right) \end{aligned}$$

Where $\theta_1 = \sum_{22}(T_{j-1})$ and $\theta_2 = \sum_{22}(T_j) - \sum_{22}(T_{j-1})$

After the integration and substitute the variance term, the equation (2.9) becomes

$$\mathbb{Q}^{\mathrm{T}}\left(\frac{1}{DF(T_{j-1},T)}1_{[\mathrm{T_{j-1}}<\tau\leq \mathrm{T_j}]}\right) =$$

$$\mathrm{N}\left(\frac{B_1-A_1}{\sqrt{\sum_{22}(T_j)}}\right) - \Phi_2\left(-\frac{A_1}{\sqrt{\sum_{22}(T_{j-1})}}, \frac{B_1-A_1}{\sqrt{\sum_{22}(T_j)}}; \sqrt{\frac{\sum_{22}(T_{j-1})}{\sum_{22}(T_j)}}\right) +$$

$$e^{-2u(x_0+\sum_{12}(T_{j-1}))}\left[N\left(\frac{B_2-A_2}{\sqrt{\sum_{22}(T_j)}}\right) - \Phi_2\left(-\frac{A_2}{\sqrt{\sum_{22}(T_{j-1})}}, \frac{B_2-A_2}{\sqrt{\sum_{22}(T_j)}}; \sqrt{\frac{\sum_{22}(T_{j-1})}{\sum_{22}(T_j)}}\right)\right] -$$

$$e^{-2(u+\sum_{12}(T_{j-1}))\,x_0}\left[N\left(\frac{B_3-A_3}{\sqrt{\sum_{22}(T_j)}}\right)-\Phi_2\left(-\frac{A_3}{\sqrt{\sum_{22}(T_{j-1})}},\frac{B_3-A_3}{\sqrt{\sum_{22}(T_j)}};\ \sqrt{\frac{\sum_{22}(T_{j-1})}{\sum_{22}(T_j)}}\right)\right]-$$

$$e^{-2(u+x_0/\sum_{22}(T_{j-1}))\sum_{12}(T_{j-1}))}\left[N\left(\frac{B_4-A_4}{\sqrt{\sum_{22}(T_j)}}\right)-\Phi_2\left(-\frac{A_4}{\sqrt{\sum_{22}(T_{j-1})}},\frac{B_4-A_4}{\sqrt{\sum_{22}(T_j)}};\ \sqrt{\frac{\sum_{22}(T_{j-1})}{\sum_{22}(T_j)}}\right)\right] \tag{2.10}$$

where

$$A_1 = x_0 + u\sum_{22}(T_{j-1}) + \sum_{12}(T_{j-1}) \quad ; \quad B_1 = -u\left(\sum_{22}(T_j) - \sum_{22}(T_{j-1})\right)$$

$$A_2 = x_0 - u\sum_{22}(T_{j-1}) + \sum_{12}(T_{j-1}) \quad ; \quad B_2 = u\left(\sum_{22}(T_j) - \sum_{22}(T_{j-1})\right)$$

$$A_3 = -x_0 + u\sum_{22}(T_{j-1}) + \sum_{12}(T_{j-1}) \quad ; \quad B_3 = -u\left(\sum_{22}(T_j) - \sum_{22}(T_{j-1})\right)$$

$$A_4 = -x_0 - u\sum_{22}(T_{j-1}) + \sum_{12}(T_{j-1}) \quad ; \quad B_4 = u\left(\sum_{22}(T_j) - \sum_{22}(T_{j-1})\right)$$

3. Numerical Results

3.1. Comparison of the Model Implementation by Monte Carlo and the Closed Form Formulas

In the previous section, we have showed the closed form formulas for survival probability and marginal default probabilities are exact based on our assumptions given constant volatility for both credit driver and interest-rates. We expect these formulas can also give us a good approximation when the volatilities are piece wise constant for both process. In this section, we provide the numerical results by comparing survival probabilities, marginal default probabilities and the CDS spreads from Monte Carlo simulation and the formulas.

The initial value for parameter credit driver x_0, the credit driver volatility σ, the volatility for interest rate ν, the drift term u, the risk free interest rate r and recovery rate R are listed in table 1. These are the arbitrary numbers which give us reasonable CDS rate.

First we compared the survival probabilities that firm survives after 5 years with the parameters listed in table 1 from both Monte Carlo simulation and formula. 5,000 steps and 100,000 paths of random numbers are simulated for Monte Carlo. The results from both methods are very close with different correlations (table 2). The percentage difference is calculated by taking the difference between the two survival probabilities and dividing it by the results from Monte Carlo. The percentage differences are less than 1% for all correlation tested. And these differences do not increase while the correlation increases. We observe that results from Monte Carlo get closer to the results from the formula when step number N increasing from 1000 to 5000 with the same paths 100000 (Figure 7). We believe the difference between two methods can be smaller if more paths or steps Monte Carlo simulation is done. Interestingly from both methods, we found that the higher the correlation between the credit driver and interest-rates the bigger the survival probabilities are.

The default probabilities for $4\,year < \tau < 5\,year$ are calculated also using the parameters listed in table 1. The results are shown in table 3. The percentage differences between Monte Carlo and formula are less than 1% except for correlation -0.6 and 0.4 for which the percentage difference is slightly higher than 1%. The standard errors for Monte Carlo with different correlations are listed in the column labelled Se. And the 95% confidence intervals are listed in column conf1 and conf2. All the default probabilities from formula are within the 95% confidence interval of Monte Carlo results. The default probability in interval $4\,year < \tau < 5\,year$ is around 4.9% when no correlation is considered. It decreases as the correlation increases.

As mentioned earlier, the formulas are exact when the volatility for interest rate is constant. However, in practice, the volatility is changing during the time. In our model, we are going to calibrate the interest-rates volatility to the market yield curve using Hull White model, in which the volatility is piece wise constant. In our test, we assign the initial value 0.1 to interest rate volatility and let it increase 0.05 every half year to 0.55 by the end of five years. In practice, the interest rate volatility won't be as high as 0.55. The purpose for giving such wide range of interest rate volatility is to check how big the error we can get from the formula approximation. The values for other parameters are the same as the table 1. The marginal default probabilities for $4\,year < \tau < 5\,year$ are shown in table 4. The percentage differences are calculated as described before. The results from both methods are very close when the correlation is between -0.4 and 0.6. The data shows that higher the absolute value of the correlation, the higher the percentage difference.

In order to compare the CDS price by using Monte Carlo and the formula, CDS spreads with tenor from one year to five years (table 5 to 10) and quarterly payments are calculated based on the parameters in table 1. Accrued premium is ignored for this calculation and the discretization for the protect leg is quarterly.

We can see from the table 5 to 10 that the differences between the CDS spread from both methods are around 2% for most correlations tested. The reason that we got greater percentage differences for CDS spreads than survival probabilities and marginal default probabilities is because we have done less discretization for the CDS spreads calculation. And the difference between Monte Carlo and formula from each survival and marginal default probabilities cumulates when CDS spreads are calculated. The difference doesn't increase when correlation increases (Figure 8). In general, the spreads from formula are greater than the spreads from the Monte Carlo. When the correlation between the credit driver and interest rate is positive, the CDS spread is smaller than that without correlation. These results are consistent with the survival probability and marginal default probability results. Therefore, we can conclude that the higher the correlation between credit driver and interest rate, the higher the survival probability the firm will have, the lower the marginal probability will be for each period and the lower the CDS spread.

One motivation for this research is to find a faster way to price CDS than Monte Carlo simulation. The run time from both methods are compared in table 10. The run time for CDS spread is the time to calculation a five year CDS spread. 5,000 steps and 100,000 paths Monte Carlo simulation is done for each test. The run time for marginal default probability is about three times of the survival probability because the formula for default probability is more complicated than the survival probability. It takes about the same amount of time to calculate the survival probability, marginal default probability and CDS spread for Monte Carlo because the same simulation is used for the computation. It only takes 0.00628 seconds for the formula to compute one CDS spread but 85 seconds for Monte Carlo. So for one CDS spread, the formula is about 13 thousand times faster than the Monte Carlo.

3.2. *Sensitivity of CDS Spread to Different Parameters*

In this section, we test the sensitivity of CDS spreads to the input parameters. The same five year CDS described in the last section is used. The CDS spread is calculated for different value of model parameters. For testing the CDS spread sensitivities to the initial value of credit driver, the drift term, and volatility of credit driver, the correlation between the credit driver and interest-rate is set to 0.2. The values of other parameters are the same as in table1. The sensitivity of CDS spread to the initial value of credit drive is showed in the figure 3. The sensitivity of CDS spread to the drift term is showed in the figure 4. Figure 5 and 6 are the sensitivity of CDS spread to the credit driver volatility σ, and the correlation between credit driver and interest-rates respectively. We also compared the CDS spread sensitivities to different parameter with and without correlations between credit driver and interest-rates. As we expected, the higher the initial value of credit driver and the drift term, the lower the CDS spread will be. The higher the volatility of credit driver, the higher the CDS spread will be. There is no significant difference in the curve shapes whether the correlation is present or not for these scenarios. We observed that the CDS spread is significantly affected by the correlation between the credit driver and interest rate. Keeping all other parameters constant, the CDS spread can go up to 491bps if the correlation is -1; and goes down to 190 bps if the correlation is 1 comparing to 318 bps if correlation is zero (Table 7).

4. Conclusions

Valuation of counterparty risk has become a hot topic recently. The correlation between the default and market risk factors is ignored in most valuation models. Currently, full Monte Carlo simulation or other numerical methods are used to pricing the counterparty risk with the correlation. These methods are computation intensive and slow. In this paper we proposed a new model derived from the traditional first passage time model which can give us closed form formulas for survival and marginal default probability capturing the correlation between default and risk factors. In this model, every counterparty is assigned a stochastic process, a credit driver. The first time that this process falls to some threshold level, the counterparty is deemed to have defaulted. We assume that the risk factor, particularly interest rate here, follows a stochastic process in Hull White model. Together, credit driver and discount factor are coupled with a constant correlation between their Brownian motions. Based on these assumptions, we derived closed form formulas for counterparty survival probability and default probability in the interval $[t_{j-1}, t_j]$. Consequently, we are able to price CDS with the consideration of correlation between default event and interest-rate. The numerical results support that the formulas are exact when the volatility for the credit driver and interest rate are constant. And the formulas gave a good approximation when the volatilities are piece wise constant.

The sensitivity analysis shows that the CDS spread can be quite sensitive to all the input parameters. In particular, we found the correlation between the credit driver and interest rate has a great impact on the CDS spread. When the correlation between the credit driver and interest-rates is positive, the counterparty is less likely to default than the case that correlation is negative. The CDS spread is smaller in this case, and vice versa.

5. Future Research

5.1. Modeling Recovery Rate

Our current model assumes that the recovery rate is constant and independent of default. In reality, recovery rate is usually not deterministic and could be negatively correlated with default rate. For valuation of the CDS spread, this maybe not a problem because the sensitivity of the mark to market of a default swap to the recovery rate assumption is very low for low spread levels (O'kane, D; Turnbull, S, 2003). For credit value adjustment (CVA) valuation, without the consideration of stochastic recovery rate could underestimate the counterparty credit risk in a CDS contract (Li, 2010). However, it has been shown that the effect of the negative correlation between default and recovery rate does increase the CVA noticeably but is not as strong as the wrong way risk. This impact maybe even smaller for counterparty risk valuation of other derivative contracts, whose value aren't driven by the recovery rate.

5.2. Valuation of other Credit Derivatives

The ultimate goal for this research is to price a variety of credit derivatives, including foreign currency CDS, Contingent Credit Default Swap (Contingent CDS or C-CDS), CDS options and Credit Value Adjustment (CVA). For these credit derivatives, the correlation between the market and credit quality of a reference entity is important for pricing. Our model and formula will be more powerful if it is able to price these credit derivatives.

5.2.1. Foreign Currency CDS

A foreign currency CDS is a CDS whose payouts or payments or both are denominated in a foreign currency. Pricing foreign currency CDS is more complex than regular CDS because the dynamic of exchange rate has to be modeled in addition to the model processes for pricing regular CDS. For a foreign currency CDS in which both premium leg and default leg are dominated in the same foreign currency, the pricing formula (2.1) must be modified to

$$CDS \approx X \sum_{j=1}^{N} \delta_j \, DF(0,T)\mathbb{Q}^T\left(\frac{FX(T_j)}{DF(\,T_j, T)}\; 1_{[\,\tau > T_j]}\right) - (1-R)\sum_{j=1}^{N} \delta_j \, DF(0,T)\mathbb{Q}^T\left(\frac{FX(T_{j-1})}{DF(\,T_j, T)}\; 1_{[T_{j-1}<\tau\le T_j]}\right) \quad (5.1)$$

Where $FX(T_j)$ is the spot FX rage at time T_j.

Further assumption about the currency exchange rate dynamic is needed. We believe, after a few adjustments, the foreign currency CDS is able to be priced with correlation between the default and interest-rates taking into consideration.

5.2.2.CDS Options

CDS options, similar to equity options, provide users with the right to buy or sell protection at a specified time and spread level off the CDS indices. Currently CDS options are traded off the two main families of CDS indexes -- the U.S.-based CDX and the European-based iTraxx. The CDS option has the European style and is knocked out if the reference entity defaults during the life of the option.

With a full term structure of CDS spreads is known, the valuation of a CDS option depends on two unobservable variables, the expected recovery rate and the volatility. Hull and white described a CDS option valuation methods similar to that of Jamshidian(1997) for valuing a European swaption (Hull & White, 2003). In this model, there is a probability measure M under which all security prices are martingales when the present value of payments g on the CDS is the numeraire. Define V as the value of a European CDS option on a CDS with spread K lasting between times T and T^*. The variable V/g is the ratio of two traded securities and is a martingale under the probability measure M. So that

$$\frac{V_0}{g_0} = E_M\left[\frac{V_T}{g_T}\right] \quad (5.2)$$

Where V_0 and g_0 are the values of V and g at time zero, the V_T and g_T are their values at time T.

Because $V_T = g_T \max(S_T - K, 0)$,

$$V_0 = g_0 E_M[\max(S_T - K, 0)]$$

It is further assumed that the CDS spread is log normal with standard deviation $\sigma_s\sqrt{T}$. The formula for valuing the European CDS option becomes

$$V_0 = g_0[F_0 N(d_1) - KN(d_2)] \quad (5.3)$$

Where

$$d_1 = \frac{\ln\left(\frac{F_0}{K}\right) + \frac{\sigma^2 T}{2}}{\sigma\sqrt{T}} \; ; \; d_2 = d_1 - \sigma\sqrt{T}$$

And $F_0$is the current forward CDS spread.

5.2.3.Valuation of Counterparty Risk by CVA

Credit value adjustment is defined as the difference between the risk-free portfolio value and the true portfolio value that takes into account the possibility of counterparty's default. Therefore, CVA is the market value of counterparty credit risk.

CVA only matters in the case that the portfolio is positive to the investor; if the counterparty defaults at such a time, the investor loses the non-recoverable portion of the portfolio. Most time, the CVA is calculated assuming that the investor is default-free. Let $V_t^0(t,T)$ denote the portfolio value from investor's point of view when the counterparty is assumed to be default-free. Let $V_t(t,T)$ denote the portfolio value that are subjected to counterparty default. Then we have:

$$\boldsymbol{E}_t[V_t^0(t,T)] = \boldsymbol{E}_t[V_t(t,T)] - CVA \qquad (1.2)$$

where CVA is given by:

$$CVA = E\left[\sum_{i\geq 0} DF(t_i)V(t_i)^+ LGD(t_i)1_{\left[t_{i<\tau\leq t_{i+1}}\right]}\right] \qquad (1.3)$$

τ: the default time of counterparty

$V(t)$: the net portfolio value to the investor at time t

$LGD(t)$: the loss given default at time t

5.2.4. Contingent CDS

A Contingent CDS is an insurance contract that isolates counterparty risk from an underlying derivative, while CDS usually has the bond as the underlying asset. The underlying derivative can be an interest rate swap (IRS), a currency swap or a commodity swap. Unlike the traditional CDS, which has a fixed notional, CCDS uses a variable notional that equals that prevailing mark to market of the reference OTC derivative contract. Figure 2 shows the mechanics of a CCDS contract when the underlying derivative is an IRS. There are two scenarios in the event of a counterparty default: either the MTM of IRS contract is positive to the CCDS buyer or negative to the CCDS buyer. If the value of IRS is negative to the CCDS buyer, in other words, that the CCDS buyer owes to the counterparty, the CCDS contract just expires. Only when the counterparty owes to CCDS buyer and defaults, the settlement of the CCDS happens. Assume that the counterparty is only able to pay the MTM of the IRS at the recovery rate R. The CCDS seller will pay the (1-R)*MTM to the CCDS buyer. In return, CCDS buyer delivers the underlying derivative instrument to the CCDS seller.

If the counterparty only defaults when the underlying derivative is negative to the CCDS buyer, the CCDS is worthless. In contrast, CCDS has value in the case that the counterparty might defaults when the underlying derivative is positive to the CCDS buyer. Therefore, correlating interest rate, currency or commodity markets to the credit quality of a reference entity has a significant impact on the value of CCDS contracts. CCDS assumes exactly the same mathematics form as the part in the CVA valuation problem.

Figure 2 The mechanics of a C-CDS with an IRS as the underlying derivative

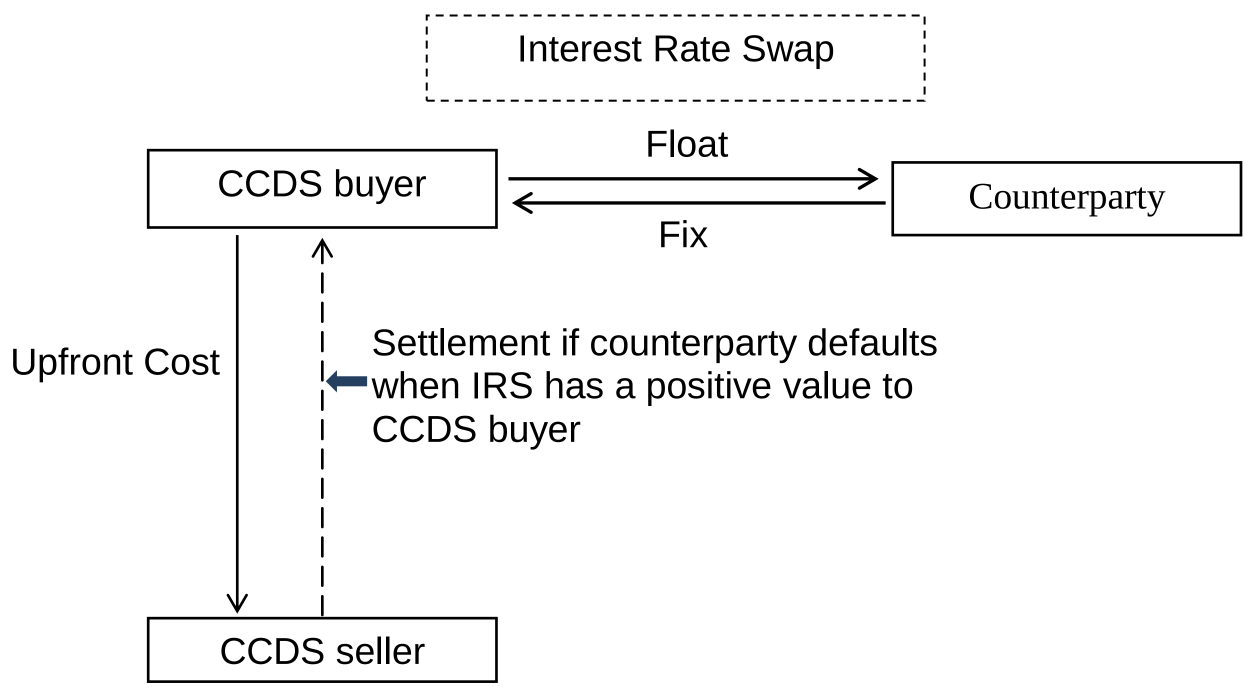

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Table 1 Input parameters

parameter	value
$\boldsymbol{x0}$	1
$\boldsymbol{\sigma}$	0.4
$\boldsymbol{\nu}$	0.2
$\boldsymbol{u}$	0.1
$\boldsymbol{r}$	0.05
R	40%

Table 2 Survival probabilities that tau bigger than five years

Correlation	Monte Carlo	Formula	Percentage Difference
-1	62.6321%	62.0668%	-0.90257%
-0.9	63.9873%	63.6131%	-0.58480%
-0.8	65.4901%	65.1351%	-0.54207%
-0.7	66.9538%	66.6305%	-0.48287%
-0.6	68.3615%	68.0974%	-0.38633%
-0.5	69.8253%	69.5339%	-0.41733%
-0.4	71.2169%	70.9383%	-0.39120%
-0.3	72.6901%	72.3090%	-0.52428%
-0.2	73.9805%	73.6448%	-0.45377%
-0.1	75.2448%	74.9442%	-0.39950%
0	76.5864%	76.2063%	-0.49630%
0.1	77.8002%	77.4302%	-0.47558%
0.2	78.9191%	78.6151%	-0.38520%
0.3	80.0690%	79.7604%	-0.38542%
0.4	81.1895%	80.8657%	-0.39882%
0.5	82.2497%	81.9308%	-0.38772%
0.6	83.3540%	82.9555%	-0.47808%
0.7	84.3553%	83.9398%	-0.49256%
0.8	85.2607%	84.8838%	-0.44206%
0.9	86.1303%	85.7879%	-0.39754%
1	86.9607%	86.6523%	-0.35464%

Table 3 Default probabilities $4 < \tau < 5\ year$

Correlation	Formula	MC	Se.	Percentage Difference	conf1	conf2
-1	6.40241%	6.34534%	0.06462%	0.89940%	6.47199%	6.21869%
-0.9	6.28170%	6.25160%	0.06439%	0.48148%	6.37780%	6.12540%
-0.8	6.15415%	6.11127%	0.06381%	0.70165%	6.23633%	5.98621%
-0.7	6.02029%	5.98386%	0.06335%	0.60880%	6.10802%	5.85970%
-0.6	5.88064%	5.95128%	0.06337%	-1.18697%	6.07548%	5.82708%
-0.5	5.73577%	5.79045%	0.06213%	-0.94431%	5.91223%	5.66867%
-0.4	5.58621%	5.65242%	0.06087%	-1.17136%	5.77173%	5.53311%
-0.3	5.43254%	5.39606%	0.05865%	0.67605%	5.51102%	5.28110%
-0.2	5.27533%	5.27678%	0.05750%	-0.02748%	5.38948%	5.16408%
-0.1	5.11513%	5.09832%	0.05567%	0.32972%	5.20744%	4.98920%
0	4.95251%	4.92249%	0.05367%	0.60985%	5.02768%	4.81730%
0.1	4.78802%	4.76539%	0.05186%	0.47488%	4.86704%	4.66374%
0.2	4.62220%	4.65828%	0.05051%	-0.77453%	4.75727%	4.55929%
0.3	4.45558%	4.48378%	0.04856%	-0.62893%	4.57895%	4.38861%
0.4	4.28868%	4.29692%	0.04638%	-0.19177%	4.38783%	4.20601%
0.5	4.12198%	4.13763%	0.04440%	-0.37824%	4.22465%	4.05061%
0.6	3.95597%	3.96194%	0.04226%	-0.15068%	4.04476%	3.87912%
0.7	3.79110%	3.77762%	0.03993%	0.35684%	3.85588%	3.69936%
0.8	3.62780%	3.60886%	0.03761%	0.52482%	3.68257%	3.53515%
0.9	3.46647%	3.49515%	0.03568%	-0.82057%	3.56508%	3.42522%
1	3.30748%	3.31863%	0.03348%	-0.33598%	3.38425%	3.25301%

Table 4 Default probabilities $4 < \tau < 5\ year$ with piece wise constant interest rate volatility

Correlation	Monte Carlo	Formula	Percentage Difference
1	2.4295%	2.3026%	-5.2254%
0.8	2.8203%	2.7743%	-1.6314%
0.6	3.3069%	3.2878%	-0.5770%
0.4	3.8349%	3.8322%	-0.0696%
0.3	4.0822%	4.1116%	0.7187%
0.2	4.3797%	4.3930%	0.3028%
0.1	4.6463%	4.6741%	0.5996%
0	4.9282%	4.9525%	0.4941%
-0.1	5.1910%	5.2256%	0.6652%
-0.2	5.4699%	5.4906%	0.3788%
-0.3	5.7724%	5.7450%	-0.4747%
-0.4	6.0516%	5.9859%	-1.0855%
-0.6	6.5636%	6.4170%	-2.2335%
-0.8	7.1304%	6.7641%	-5.1370%
-1	7.8123%	7.0105%	-10.2630%

Table 5 CDS spreads (in bps) with 5 year tenor

Correlation	Monte Carlo	Formula	Percentage Difference
-1	480.98	490.99	2.0812%
-0.9	465.454	471.614	1.3234%
-0.8	446.908	452.674	1.2902%
-0.7	429.521	434.177	1.0840%
-0.6	412.663	416.131	0.8404%
-0.5	394.917	398.54	0.9174%
-0.4	378.238	381.409	0.8384%
-0.3	360.043	364.742	1.3051%
-0.2	344.644	348.541	1.1307%
-0.1	329.396	332.809	1.0361%
0	312.98	317.547	1.4592%
0.1	298.266	302.755	1.5050%
0.2	284.8	288.432	1.2753%
0.3	270.721	274.577	1.4243%
0.4	257.217	261.188	1.5438%
0.5	244.121	248.262	1.6963%
0.6	230.707	235.795	2.2054%
0.7	218.394	223.783	2.4676%
0.8	207.208	212.221	2.4193%
0.9	196.446	201.102	2.3701%
1	186.35	190.421	2.1846%

Table 6 CDS spreads (in bps) with 4 year tenor

Correlaion	Monte Carlo	Formula	Percentage Difference
-1	458.9	469.2	2.254%
-0.9	446.2	451.3	1.127%
-0.8	428.3	433.7	1.247%
-0.7	412.9	416.5	0.883%
-0.6	395.7	399.8	1.033%
-0.5	379.3	383.4	1.105%
-0.4	362.7	367.6	1.336%
-0.3	347	352.1	1.457%
-0.2	332.6	337	1.342%
-0.1	317	322.4	1.725%
0	302.9	308.3	1.755%
0.1	290.9	294.5	1.235%
0.2	276.8	281.2	1.577%
0.3	263.9	268.3	1.661%
0.4	251.8	255.8	1.586%
0.5	239.1	243.7	1.943%
0.6	226.5	232	2.458%
0.7	215	220.8	2.709%
0.8	204.7	209.9	2.554%
0.9	194.1	199.5	2.792%
1	184.6	189.4	2.600%

Table 7 CDS spreads (in bps) with 3 year tenor

Correlaion	Monte Carlo	Formula	Percentage Difference
-1	413.41	420.74	1.7733%
-0.9	400.21	405.07	1.2144%
-0.8	386.13	389.76	0.9417%
-0.7	372	374.83	0.7618%
-0.6	355.92	360.28	1.2270%
-0.5	340.73	346.11	1.5784%
-0.4	327.56	332.31	1.4486%
-0.3	314.82	318.88	1.2883%
-0.2	301.07	305.83	1.5790%
-0.1	288.84	293.15	1.4925%
0	276.61	280.83	1.5267%
0.1	264.32	268.89	1.7294%
0.2	253.16	257.31	1.6373%
0.3	241.25	246.09	2.0029%
0.4	230.27	235.22	2.1510%
0.5	218.72	224.71	2.7387%
0.6	208.8	214.55	2.7499%
0.7	200.43	204.73	2.1424%
0.8	190.33	195.24	2.5797%
0.9	181.63	186.1	2.4600%
1	173.21	177.28	2.3498%

Table 8 CDS spreads (in bps) with 2 year tenor

Correlaion	Monte Carlo	Formula	Percentage Difference
-1	310.46	312.96	0.8075%
-0.9	300.22	301.61	0.4643%
-0.8	288.53	290.55	0.6987%
-0.7	275.84	279.78	1.4306%
-0.6	264.32	269.3	1.8841%
-0.5	254.78	259.11	1.7007%
-0.4	244.79	249.2	1.8040%
-0.3	233.98	239.57	2.3891%
-0.2	225.88	230.22	1.9205%
-0.1	215.59	221.14	2.5772%
0	207.76	212.33	2.2036%
0.1	198.72	203.79	2.5524%
0.2	190.34	195.51	2.7184%
0.3	183.48	187.49	2.1861%
0.4	174.78	179.72	2.8241%
0.5	166.99	172.2	3.1181%
0.6	159.64	164.93	3.3080%
0.7	153.62	157.89	2.7795%
0.8	147.06	151.1	2.7473%
0.9	140.17	144.53	3.1149%
1	134.15	138.2	3.0191%

Table 9 CDS spreads (in bps) with 1 year tenor

Correlaion	Monte Carlo	Formula	Percentage Difference
-1	95.7	96.85	1.1984%
-0.9	92.01	93.5	1.6190%
-0.8	88.06	90.24	2.4725%
-0.7	84.72	87.08	2.7947%
-0.6	80.39	84.02	4.5218%
-0.5	78.05	81.05	3.8410%
-0.4	77.6	78.16	0.7215%
-0.3	73.55	75.37	2.4662%
-0.2	70.67	72.66	2.8105%
-0.1	67.69	70.03	3.4472%
0	65.24	67.48	3.4324%
0.1	62.83	65.01	3.4698%
0.2	62.06	62.62	0.9054%
0.3	58.99	60.31	2.2406%
0.4	56.5	58.07	2.7835%
0.5	54.5	55.9	2.5706%
0.6	52.33	53.8	2.8256%
0.7	50.71	51.77	2.0958%
0.8	49.06	49.81	1.5293%
0.9	46.94	47.91	2.0618%
1	45.68	46.08	0.8766%

Table 10 Run time in seconds

	MC	Formula	MC/Formula
CDS	85	0.00628	13568
Survival Probability	83	0.00026	316994
Conditional Default Probability	85	0.00064	132408

Figure 3 CDS's sensitivity to x0

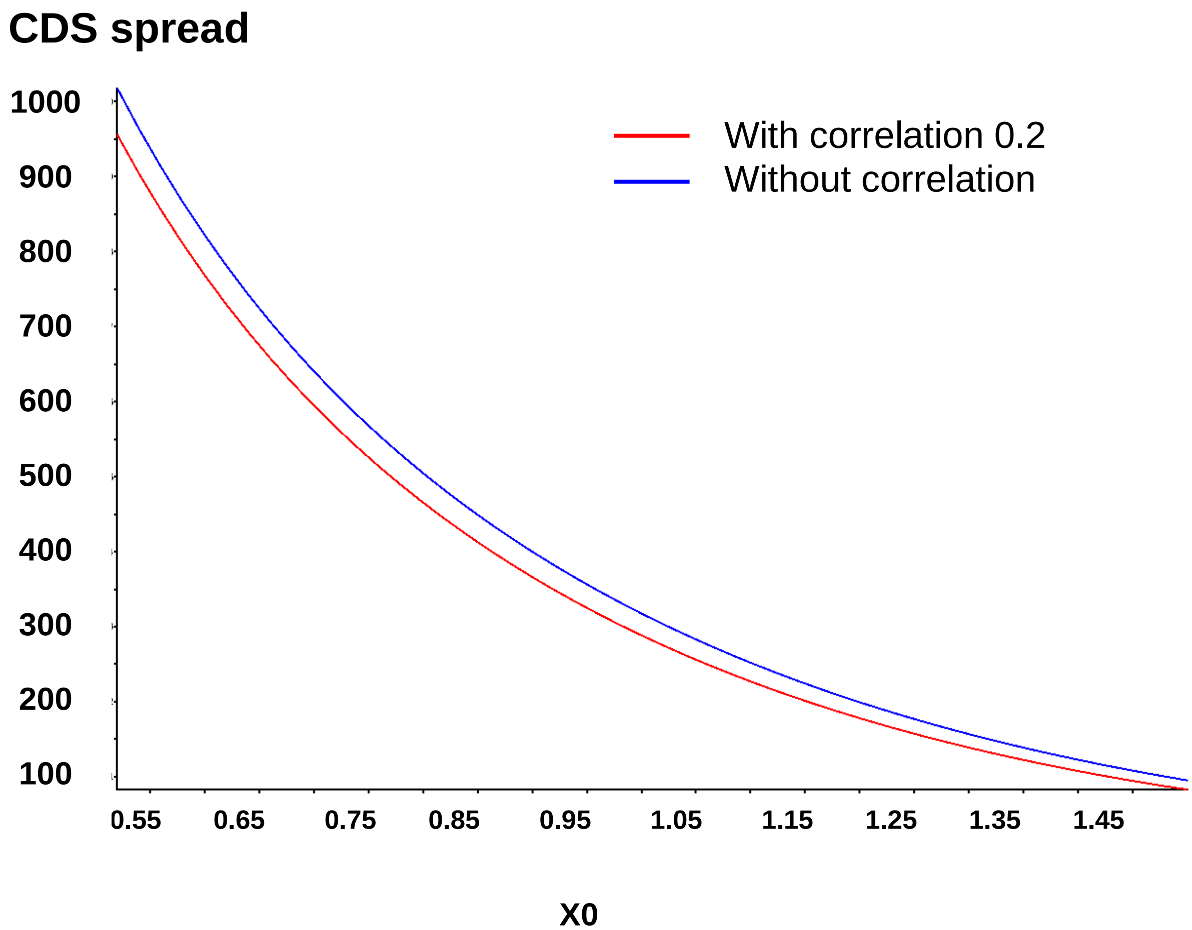

Figure 4 CDS's sensitivity to the drift term

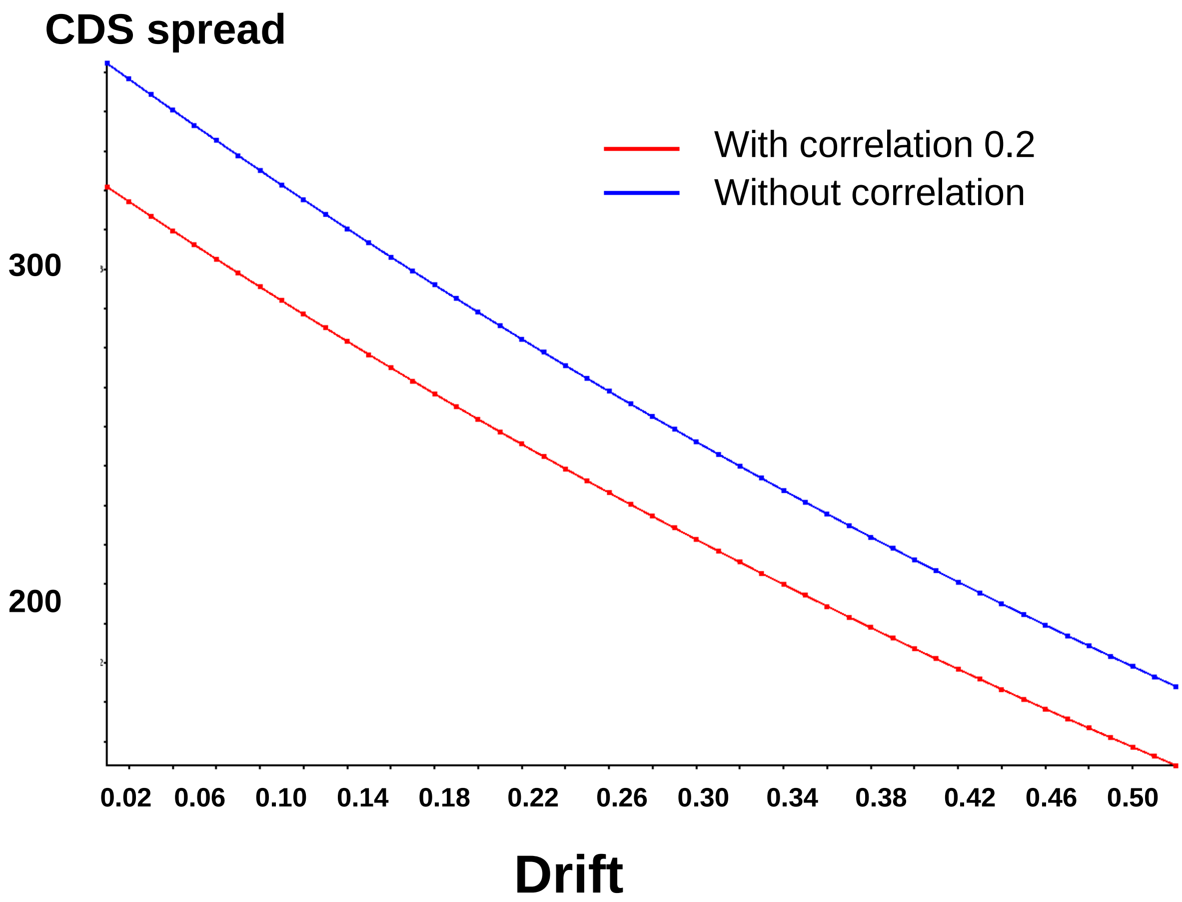

Figure 5 CDS's sensitivity to sigma σ

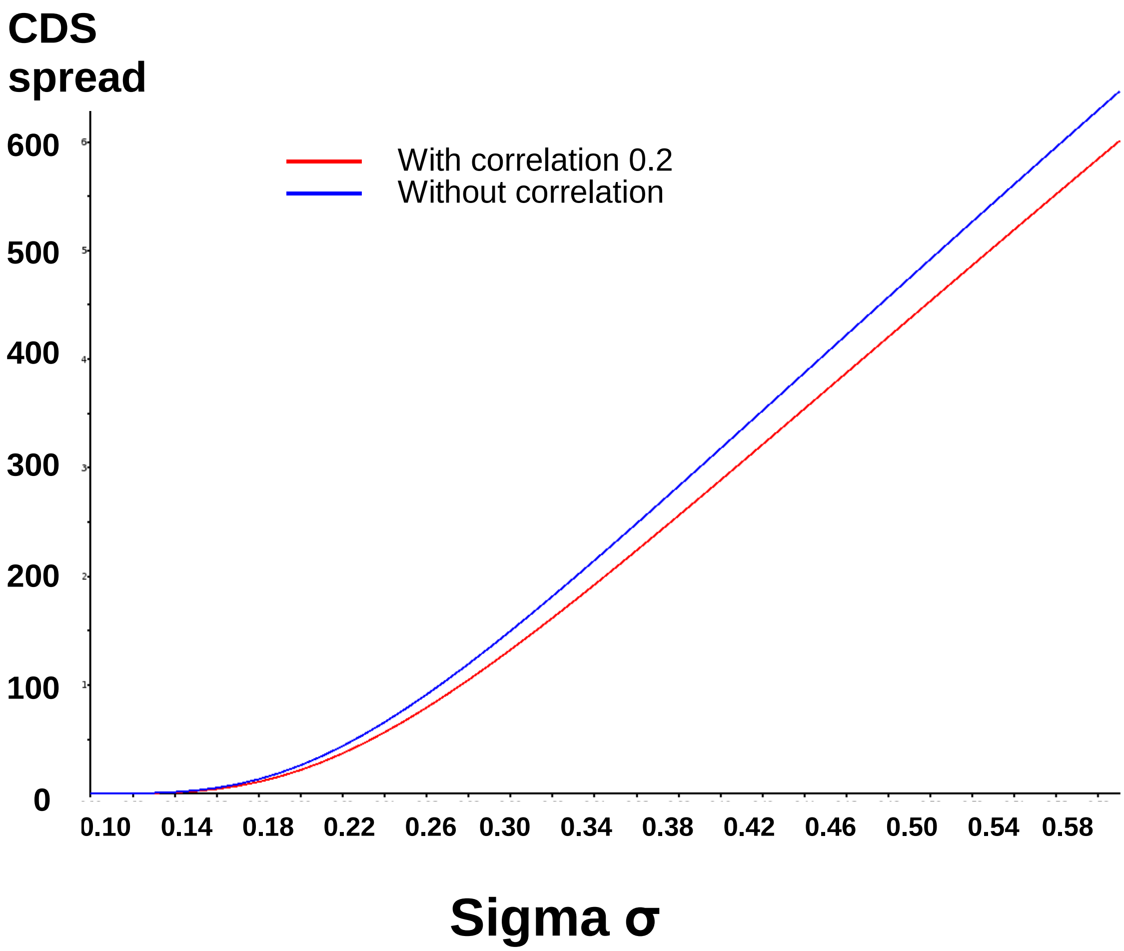

Figure 6 CDS's sensitivity to correlation

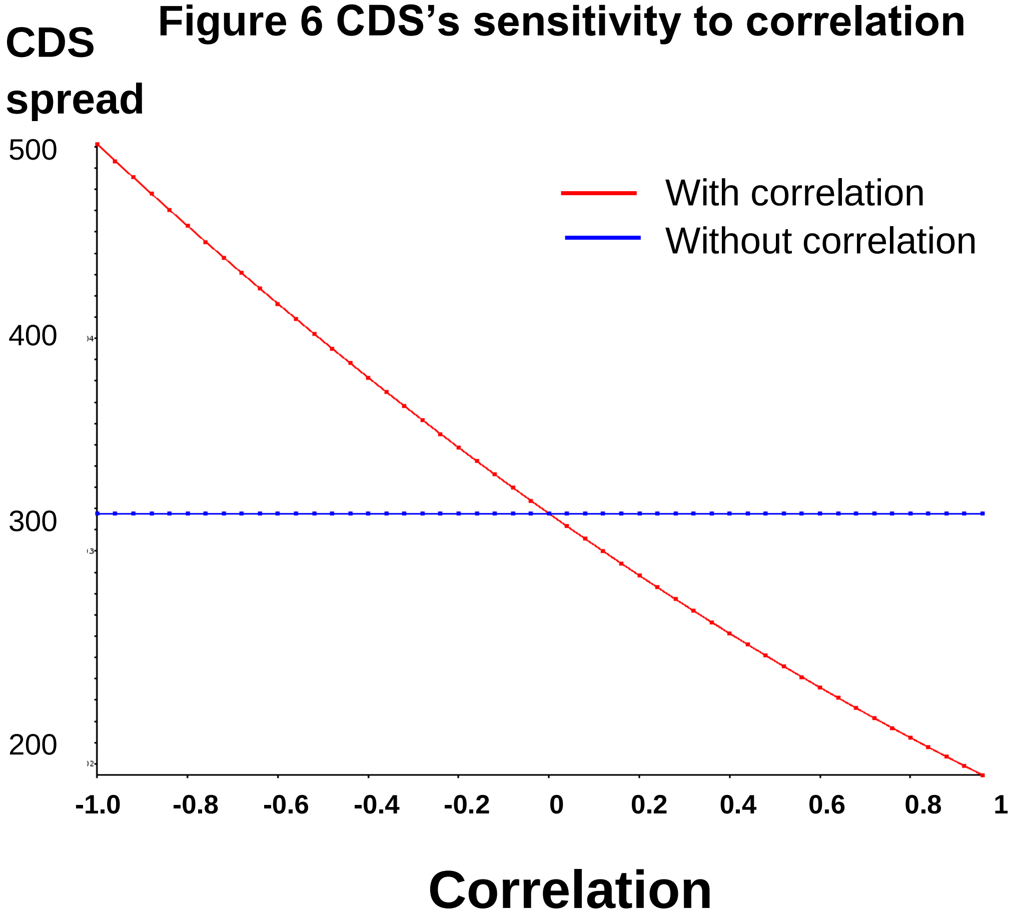

Figure 7 Comparison of Survival probabilities from formula and Monte Carlo

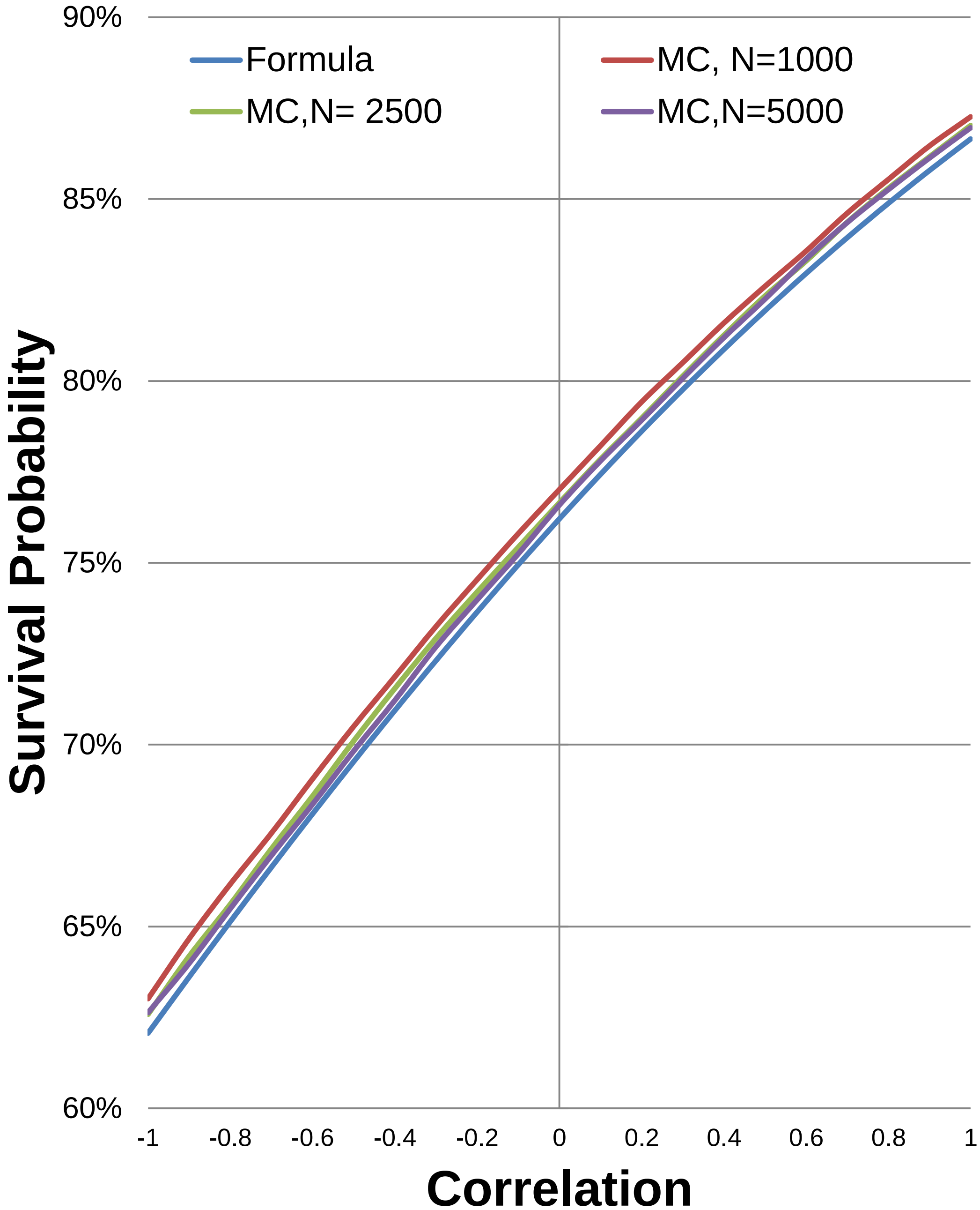

Figure 8 Comparison of 5 year CDS spreads from formula and Monte Carlo