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Equilibria and Teleological Statics

> The most characteristic feature of the work of our generation of economists is probably the general endeavour to apply the methods and results of the pure theory of equilibrium to the elucidation of more complicated 'dynamic' phenomena.... The realistic significance of the tendencies towards a state of equilibrium, traditionally described by pure theory, can be shown only when we know what the conditions are under which it is at least conceivable that a position of equilibrium will actually be reached.
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> Friedrich Hayek [1933/39, pp. 350 and 353]

> [M]icroeconomic theory is primarily about positions of equilibrium. The plans of agents ... are taken together, and certain variables – usually prices – are assumed to take on values that make those plans mutually consistent. Comparative static analysis then proceeds to compare equilibria corresponding to different values of underlying parameters.
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> ... The view that equilibria are the points of interest must logically rest on ... underlying properties about the dynamics of what happens out of equilibrium. ... If the equilibrium approached depends on the adjustment process, this needs to be studied....
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> In brief, the question of what, if any, disequilibrium stories have equilibrium endings like those assumed ab initio by economic theorists is a question of paramount interest for such theorists especially if the world is stable.
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> Franklin M. Fisher [1983, pp. 3–5]

It is unlikely that economic theorists today who depend so heavily on mathematical techniques will ever consider the limitations of calculus or calculus substitutes to be obstacles in the way of explaining disequilibrium economics in terms of equilibrium analysis. It is even doubtful whether we should ever want to explain disequilibria without equilibrium analysis as almost anything goes in such explanations. The interest in disequilibrium microeconomics stems mostly from the appreciation of Paul Samuelson's 'Correspondence Principle between comparative statics and dynamics' [Samuelson, 1947/65, p. 5]. If the implications of a claimed equilibrium are to be used to explain the behavior of the individuals participating in that equilibrium, then the claimed equilibrium must be 'stable'; that is, the idea of an equilibrium is necessarily a question of dynamics. For example, it is not enough to identify the market equilibrium with an equality between demand and supply, since it is possible for such an equality to exist accidentally in an 'unstable' market. We need to recognize always that the idea of an equilibrium claims that there are reasons for why, if the market is not clearing, there will be a convergence to market clearance. Those reasons are discussed under the topic of market stability or stability analysis. Unfortunately, the topic of stability analysis is too often mistakenly treated as a separate, secondary issue of interest only to sophisticated analytical theorists.

It would seem that the questions of stability analysis are precisely the questions of disequilibrium microeconomics. To explain how the market converges to a point where demand equals supply, we must consider points where demand does not equal supply – that is, we must consider so-called disequilibrium points. While disequilibrium analysis may be more concerned with why we are at such a state of disequilibrium, stability analysis is concerned with why we would move from there towards the equilibrium point. Thus, stability analysis is concerned with the logical adequacy of any equilibrium explanation of the behavior of individuals or the economy.

In the next chapter we will return to questions of stability analysis and the completeness of equilibrium explanations. The remainder of the present chapter will be devoted to a more fundamental question. Consider an equilibrium model for which the structure ensures that any defined equilibrium state is 'stable' (i.e. any endogenous movement from a disequilibrium point is always *towards* an equilibrium point). For any such model we ask: Is the movement towards an equilibrium fast enough to expect achievement of the equilibrium in a realistic amount of time? Should the answer be negative there is a serious question of the appropriateness of the model should it be used to form the basis of an equilibrium explanation. The only case where the answer is obviously intended to be affirmative is where the speed of adjustment is assumed to be instantaneous. But, the affirmative answer would, as noted above, amount to assuming an infinite speed of adjustment and, as always, the term 'infinite' really means 'impossible'.

1. Exogenous Variables and Teleological Comparative Statics

We will, for now, accept the stability of any model in question. We wish to discuss models for which the equilibrium converges to the well-defined endogenous point when the exogenous variables change. Stability, however, is not enough – 'convergence must take place relatively quickly' [Fisher, 1983, p. 3]. Unfortunately, stability theorists do not always make clear what is meant by *relatively* quickly. But, with reference to comparative states, Fisher does say that 'reacting to a given parameter shift, [the system must get] close to the predicted new equilibrium before parameters shift once more' [Fisher, p. 3]. In one sense this sounds arbitrary, since comparative statics is an artificial method of explanation – that is, any change in the 'parameters' (i.e. exogenous givens) is merely a thought experiment. However, if we keep Samuelson's correspondence principle in mind, we can note that when we state how long the exogenous parameters will hold their value, the question of convergence is not arbitrary. But whenever we fail to specify the durability of the exogenous givens, the empirical content of the equilibrium explanation is surely in doubt.

Whenever we are dealing with long-run equilibrium models, the question of quick convergence would seem to be a serious source of methodological problems. But, again, recall that this is just the kind of problem that the Marshallian strategy of short- vs. long-run perspectives was all about. Certain exogenous variables do not change quickly relative to endogenous ones, such as prices. In this sense, the differences in changeability is a basis for defining exogeneity, but the definition depends on the time period under consideration. Specifically, individuals must take certain variables as exogenous givens whenever they cannot change them during the time period when the endogenous variables are being chosen.

While Marshall may have been satisfied with the distinction between the long and short runs, his method can be a bit troublesome. If we say that the short run is defined on the basis that capital is exogenously fixed, the explanation of short-run variables may make sense. But, if we say only that endogenous labor can change faster than capital, there may never be a short-run equilibrium to discuss. Specifying that, for any finite amount of time, one variable is changing faster than another will still mean that both variables are changing. However, by Marshall's explanatory principles, the determination of the optimum amount of labor depends on a fixed and given amount of capital. Thus, the only equilibrium in this relative 'speed of adjustment' case occurs in a long-run equilibrium, since only when all variables converge into agreement will there be no further reason for labor to change. Unless the quantity of capital has reached its long-run value and thereby stopped changing, labor will never be at a short-run optimum!

Marshallian methods of explanation are not highly regarded these days, so when it comes to the achievement of equilibria it is somewhat questionable what method Fisher has in mind when speaking of converging 'relatively quickly'. If he is not advocating Marshallian methods, the speed of adjustment would seem to be determined by the slowest endogenous variable. But the speed of adjustment must still be relative to the speed of change of the exogenous variables. Even for the Marshallian method, the slowest endogenous variable must still change faster than the fastest exogenous variable. Otherwise, not only is there no short-run equilibrium, there would never be a long-run equilibrium. Before the slowest endogenous variable has changed to the optimum value for the given exogenous variables, the fastest exogenous variable would have changed. These considerations then give explicit meaning to Fisher's term 'relatively quickly'. The equilibrium must converge faster than the fastest changing exogenous variable. Nevertheless, it is still not clear what is meant by this term if it only makes sense in Marshallian terms. If the exogenous variables are always changing, what is an equilibrium explanation?

The Marshallian method of explanation does seem to make sense in terms of disequilibrium dynamics so let us continue to view equilibrium explanations using short-run equilibria and long-run equilibria. Consider again our discussion of Chapters 3 and 4, where the basis for explanation is the individual decision-maker's effect on the state of equilibrium. Consider also the mainstay of neoclassical methodology, namely comparative static analysis. Both considerations are relevant only for states of equilibrium. In the former, the individual is thought to vary his or her choice around the equilibrium value and thereby determine that the equilibrium value is the optimum. In the latter, the theorist is thought to vary the values of any one exogenous variable to show its effect on the next equilibrium. In both cases, there is no explanation without a definable equilibrium which is reached 'relatively quickly'. While the former does not necessarily involve disequilibrium dynamics, the latter does.

Comparative static analysis does not make complete sense in an environment where many exogenous variables are changing. The logic of any comparative static analysis is to calculate all endogenous variables after one exogenous variable is changed from one fixed value to another while all other exogenous variables are fixed. While this may be intellectually interesting to some, it does not seem to correspond to what we can see outside our windows.

Decision-makers in the real world can look outside their windows, too. The key question is: How do we model the decision-maker who is deciding when and how much to invest in a particular project that will be

realized only in the distant future? That is, what is the optimum investment? As we usually explain optimum decisions, something is maximized subject to a specified set of exogenous variables. Unless one already knows the values of the eventual long-run equilibrium exogenous variables, the investment decisions made in the short run are unlikely to be appropriate for the long-run equilibrium values of the exogenous variables. These questions were considered by Hayek many years ago and they may still be worth considering today.

2. Hayek's Contingent Equilibria

While we do not wish to identify with the ideological content of many of Hayek's writings, clearly in the early ones he did recognize the many difficulties inherent in equilibrium models of the economy. For Hayek, the importance of economic theory was not captured in the logical requirements of a state of equilibrium but rather in the process that might, if given enough time, lead to an equilibrium. Just examining an equilibrium at one point of time can be very misleading. At every point in time, decisions must be made concerning investments and the correctness of those decisions is contingent on the fulfillment of expectations about future markets. Even if we adopted the view that today there are markets which deal in future transactions or future deliveries, an equilibrium today is still dependent on the absence of any unexpected changes in the exogenous variables facing future markets. This contingency calls into question the explanatory import of any real-time equilibrium.

In his early writings Hayek usually viewed the individual decision-making process as the formation of a 'plan' which is subsequently implemented in the market. Investment decisions are concerned with providing future supply capabilities but are transacted in current markets according to current plans [Hayek, 1933/39]. Let us say the investment is made at time T_0 and the capabilities are delivered at time T_1. And further, let us say that at T_0 the market for investment goods is in equilibrium – that is, the demand based on the planned investments equals the supply of those investments. The contracts are signed at T_0 to supply the demanded investment goods at T_1. If it turns out at T_1 that the future capabilities are not optimal for the actual conditions of the future market, then what does it mean for the market to be in equilibrium at T_0?

This raises the question of why the future capabilities might not be optimum at T_1. But such a question is easy to answer. Every optimum is contingent on the specific values or states of exogenous givens such as weather, resource availabilities, population, tastes, technology, etc.

Investment decisions at T_0 must be based on what is expected to be the values or state of the exogenous givens at T_1. The presumption is that the investment decision is always intended to provide the optimal production capabilities at T_1, and this necessarily must involve a presumption about the actual values and states of exogenous variables at T_1. There is, according to Hayek, no reason to think that at T_0 the individual investor could ever be guaranteed to form true expectations of the values of the future exogenous variables. At T_1, the investment goods are delivered and the capabilities established. If the exogenous variables are not as expected, the capabilities will likely not be optimum and thus the supply of final goods will not necessarily be optimum. While it is easy to say that at any time the production capabilities are exogenous (since it is too late to change them as they were decided at T_0), and at T_1 the supply may still be adjusted to the optimum for the actual values of the exogenous givens, the question is whether the profits were, for the capabilities provided, as expected at T_0. Had the actual values of the exogenous variables at T_1 been known at T_0, a different investment decision would likely have been made. In other words, we cannot always consider both markets to be or to have been in equilibrium. If the expectations at T_0 were wrong, either (1) the market at T_0 was not at the equilibrium (since the demand for investment was not optimum) or (2) the market at T_1 is not at the equilibrium (since the supply of the final good is not optimum).

This situation is a general problem for all general equilibrium models where knowledge (of future exogenous variables) is not perfect and yet there are decisions made whose optimality depends on the state of a future economy. If we opt for (1), the view that despite possible errors in past expectations the market at T_1 can still be in equilibrium in the sense that demanders and suppliers are maximizing for the given situations, then, paradoxically, we call into question that there really is an equilibrium at T_0! This is because, without perfect knowledge, whenever the future market is in equilibrium with what are likely to be sub-optimal capabilities, the prior market was in a false equilibrium, in the sense that the demanders of investment goods were not actually optimizing. The paradox arises because for the future market to be in equilibrium, the prior market must not be; *but* every future market, say for T_1, is eventually a prior market for an even later future market, say for T_2. Whenever we claim that the future market is always in equilibrium, if the market at T_2 is also in equilibrium we have to say the market at T_1 is not in equilibrium. This is contrary to the view held at T_1. For reasons like this, Hayek seemed to think that basing so much of our understanding of economics on the logical properties of equilibrium models is intellectually suspect, at the very least.

3. Calculus of Variations, Dynamic Programming, Control Theory, etc.

While Hayek may have seen the use of equilibrium models as inherently problematic – particularly when discussing decisions involving time and future markets – many other economists have seen this as only a puzzle to be solved. Since the early 1950s economists have been learning to see questions of investment decisions as merely instances of a more general viewpoint which is often called ‘optimal control theory’ [Dorfman, 1969]. The question we will eventually consider is whether this alternative way of looking at the problem of investment resolves any of Hayek’s problems, or whether it merely deceives us by offering a solution to a different problem. For now, let us briefly examine optimal control theory.

To understand optimal control theory we need first to consider the ‘control problem’ [Intriligator, 1971]. Rather than see the question of investment as a single decision made at time T_0, determining the state of the firm’s capabilities at just one later point in time T_1, let us consider a long sequence of such points, T_0, T_1, T_2, ... T_i, ... T_n. The object of concern is the state of capabilities, K_i, at each point in time, T_i. The decision problem is to make investments, X_i, at each point in time to control the time-path or ‘trajectory’ of the K_i such that the optimum capabilities are provided at each point in time, T_i. In effect, the investment decisions ‘control’ the path of the capabilities.

The theoretical problem is to find the optimum path of the control variable(s), X_i. What the optimum path is depends on our objectives. We might want to achieve a target value of capabilities in the minimum amount of time, or we might want to maintain a certain level of capabilities at each point of time. Stated this way the control problem is a mechanical engineering problem which only depends on the mechanical relationship of the rate of change of K_i to the control decisions, X_i, and the values of the initial and target capabilities. There are two versions of this problem depending on whether the intermediate values of the capabilities have an effect on the choice of the time-path for the control variable(s). If the intermediate values do not matter, this is called an ‘open loop’ control problem and can be solved at T_0. If the intermediate values matter, it is called a ‘closed loop’ problem and the intermediate values of the control variable(s) will have to be decided at each point in time. Unless there are unknown and changing exogenous factors affecting the outcome of the control decisions at each point in time, there is little reason to distinguish between these two versions [Intriligator, 1971, p. 302].

The basic approach to determining the optimal path for the control variables (and hence for the optimal path for the capabilities) is to consider each conceivable path as an entity and somehow devise a mapping from the set of conceivable paths to a one-dimensional set (i.e. to one point on a line). Recall that the consumer's utility function is also a mapping, one that maps a point in a multi-dimensional set (representing combinations of quantities of goods) to a point in a one-dimensional set (representing levels of utility). The only complexity introduced by the control problem is that, while a point in the consumer's goods-set is just a multi-dimensional singular point, a 'point' in the control theory's path-set is a multi-dimensional line, that is, a sequence of many singular points.

There are many techniques of analysis for dealing with the control problem and the related complexities of mapping paths to scalar points. The 'calculus of variations', 'optimal control theory' and 'dynamic programming' are the most familiar to economists. In the calculus of variations approach no mention is made of the control variable(s). In the dynamic programming approach there is a specific objective function recognized which is presumed to apply regardless of the time it takes to complete the program. Control theory is, in effect, a generalization of the well-known, calculus-based, Lagrangian multiplier technique of representing maximization problems. Control theory is interesting to economists because the control problem can be expressed as one involving an invented variable, much like a Lagrangian multiplier, which can all too easily be interpreted as a short-run optimum discount rate [see Dorfman, 1969].

We do not need to spend time detailing these techniques as there are textbooks that would do them more justice [e.g. Intriligator, 1971]. What is characteristic of all of them is the view that the decision process is a mere matter of mechanical engineering. If we can specify the static and mechanical relationships about the production of investment goods and the static objective or utility function, the problem is merely one of a mathematical analysis of the general solution. Such analysis depends only on the initial and target values of the capabilities. It is a wonder, given such techniques, that anyone would ever think there could be a problem concerning investment decisions.

4. Mechanical Solutions vs. Learning

There are many reasons why control theory and its variants are inappropriate for Hayek's questions. If the technical relationships or the objective functions are not completely known in advance, it will not be easy to apply these techniques. Incomplete knowledge in this regard is not widely recognized by the proponents of these techniques. What is recognized is the uncertainty about future exogenous factors affecting the optimality of decisions concerning the path of the control variables [Intriligator, 1971, p. 302]. Unfortunately this is too easy to transform into a statistical decision problem and thereby mask all the interesting questions concerning learning and knowledge that Hayek was addressing. And, even worse, committing oneself to very special theories of learning – namely, versions of inductive learning – and restricting oneself to specific mathematical forms of production functions and utility functions, it is even possible for one to transform the question of our knowledge of these functions into a question of learning the values of their parameters as part of the process of moving along the optimum path.

Using the sophisticated technique of reducing a dynamic equilibrium problem to one of a static choice of an equilibrium path does not overcome the question of how one deals with errors in one's choice of an optimum time-path. By using the usual techniques of optimization, theorists seem to be always looking for the optimum learning formula, one that precludes the possibility of making mistakes. Unfortunately, such a formula does not make much sense, if theorists think there is really something to learn at each point in time. If Hayek stressed anything about learning, it is that mistakes matter. Understanding the role of mistakes is a central issue in our understanding of the equilibrium process.

The promotion of techniques of analysis such as optimal control theory and dynamic programming unfortunately leads us to think about our problems of decision-making over time in a way that the convenience of mathematical techniques takes precedence over the accuracy of the representation of any problem at issue. What is lost in these mechanical engineering approaches is the recognition that decision-makers learn and may make decisions so as to maximize the possibility of learning. And, in a fundamental sense recognized by Hayek, learning always involves learning from one's mistakes. Decision-makers can always choose time paths that may generate errors in order to learn more about the world in which they are operating.

The mechanical engineering approach of optimal control theory or dynamic programming is too much concerned with successful optimization. When one suspects that one does not possess perfect knowledge, sometimes the best path is to try to generate errors. A classic example of this was an early attempt to place a man-made machine on the Moon. The first American attempt was a failure because the machine successfully landed without any difficulty. The purpose of placing a machine on the Moon was, however, to learn the limits of our understanding, that is, to find out how things might go wrong. Had the machine's landing revealed errors it would have reduced the chance of