Tusi, Mathematician, Mathematics Educator and Teacher, and the Saviour of the Mathematics

by

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Abstract

Searching for mathematicians who had contributed to the improvement of mathematics education during Islamic Era in Persia became an interest of the author of this thesis during the Master's Program at Simon Fraser University. It was found that Nasir-al-Din Tusi has been an icon in the mathematics education of the Islamic Era.

The purpose of this study was to investigate Tusi's contributions to the improvement of mathematics education. This goal required a summary of the political and educational situation of Persia from the start of Arab control to the Mongol rule and then a focus on Tusi's particular contributions. The method for collecting the information was to initially research books written by historians and mathematicians in North America. Then the search was continued to obtain the opinions of Iranian historians and mathematicians. The sources include Iran's National Library, the library of universities, peer reviewed books available in Iran's mathematics education books market, and the proceedings of conferences about Tusi held in Iran.

It was found that Tusi compiled series of mathematics books and treatises, wrote redactions to many of them, and kept them in a library at the location of an observatory and school he established in Maragheh, the capital of the Mongol Empire. The results of the study revealed that Tusi trained mathematics teachers, taught mathematics. He collected data from twelve years of observation with his team of mathematicians and astronomers in Maragheh. He compiled a calendar from the data. Tusi also recast the entire collection of astronomy books from Euclid's "Elements" to Ptolemy's "Almagest."

The principal conclusion is that Tusi had both influenced and preserved the scholarly life of mathematicians and astronomers. Another conclusion is that he planned and defined a reformed mathematics education by combining written books and translated books into a quality terminology which helped educators teach mathematics with greater ease. He also made academic courses separated from religious studies, and offered them to the candidate from public instead of only from affluent families.

Keywords: Tusi; mathematics education; mathematics books;

Dedication

To my mother, Farrah, who had dreamed of my graduate education since I was a little boy.

To my wife, Laya, for her patience, understanding, and encouragement.

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Chapter 1.

Mathematics Education during Islamic Era

Steps of the improvement of mathematics education from Greek Era to the modern time are interconnected like a chain. Knowing the quality and characteristics of mathematics education during the Islamic Era is a key component of the recognition and appreciation of the improvement of the mathematics education as a whole. The mathematics knowledge of different topics has passed hand-to-hand and mind-to-mind from one mathematician to another to reach to this century. Mathematics education strategies and improvements of them also have followed step-by-step progress to this day. That is why mathematics educators will benefit from a decent understanding of the history of mathematics education, schooling, and mathematics books and textbooks written during the Islamic Era as one of the influencing periods of the time in which teaching and learning mathematics has been improved significantly.

Motivation

During the history course in the Masters of Education Program at Simon Fraser University, the development and improvement of mathematics books, textbooks, and schooling from the point of view of a mathematics teacher were discussed. Contributions of mathematicians from the Greek Era, the Islamic Era, and the Modern Era in mathematics education were reviewed. I was enjoying my time in the class, learning about the mathematicians of different countries in different centuries, and exploring how the mathematics that I teach today has come to this stage. The topics addressed included mathematics curricula, textbooks, grading, schooling, courses, specific topics in mathematics and more. For each topic, the professor provided us with informative articles regarding the chain of educational events and moments in the lives of mathematicians that helped them with the progress of the mathematics toward what we

experience today. The improvement of schooling as a whole was one of the most important aspects of the course. An important aspect of schooling is the quality of books and textbooks including the terminology used within. This was discussed as well as the progression of the development of mathematics textbooks and curriculum, another core topic area of the course. During the course I noticed that the documents recognizing and supporting the achievements and contributions of the Islamic Era's mathematicians were limited to those that were available in North American sources, and to those that were accessible from European on-line resources. I also realized that to deeply study the contributions of Islamic Era's mathematicians, researchers should either spend time in the Middle East, or have access to the libraries and books of historians and mathematicians of the Middle East. As a person who has finished an undergraduate degree in Iran, I knew, many of the original mathematics books and treatises in Farsi and Arabic have never been translated to English. I thought, perhaps, that numerous quantity of information and facts concerning the history of mathematics education in the Islamic Era had never been discussed or translated in English. However, I became interested in searching for the facts about the contributions of Persian mathematicians regarding the topics that were discussed during the course.

Vision

I could see myself searching for some of the facts and information concerning the contributions of Persian mathematicians of the Islamic Era to the scholarly life of mathematicians, teaching mathematics, writing mathematic books, and schooling. I chose only Persian mathematicians to narrow down my research. Writing a master's thesis seemed a good vehicle to take me to the point that I could ensure my research was being conducted systematically while I was benefiting from being supervised by professors of the Education faculty of Simon Fraser University. As a native Farsi speaker, I hoped that I could access better sources and choose between options of Farsi books written by Persian historians and mathematicians. I could see myself travelling to Iran to closely explore different aspects of the contributions of Persian mathematicians to mathematics education. I discussed the matter of my interest with historians, mathematicians, and mathematics educators that I knew inside and outside of Simon Fraser University, and discovered that it was better for me to narrow down my research

even further. Researching Persian mathematicians of the Islamic Era was too extensive a topic to undertake. A historian suggested that I write a historiography. Historiography, shortly, is describing the history of a period of time by focusing on the life of a person living in that period of time. The lives of other people living in that period could also be included, if required. My thesis was not intended to be a history of mathematics discribing the life of a mathematician, but somehow I had to focus on the life of one mathematician in order to discuss the contributions of Persian mathematicians. Perhaps this would be something similar to historiography in some aspect. I chose Tusi, a mathematican I had always desired to know more about. Tusi lived during the Islamic Era and made contributions to mathematics education.

Planning

I had two main interests with my research, Tusi's contributions to mathematics education as a whole, and a more detailed exploration of his life as a Persian mathematician and astronomer during the "Golden Ages" of the Islamic Era. To the Western world, Tusi is known as a mathematician and astronomer, but he is not really recognized as a mathematics teacher and educator. I saw a few articles that each each pointed to one or two aspects of the life of Tusi, mostly discussing his contributions to astronomy, but I had hard time finding, for example, articles about Tusi's contributions to schooling. Checking the Internet in Farsi, I found a couple of articles offered at conferences in Iran about Tusi's style of Tahrir (redaction on other peoples' work) in English, but they were only published in Iran.

Reading Tusi's life history, I understood the difficulty of being a responsible mathematician and scholar at the time he lived. Tusi's achievements and contributions are sometimes introduced as the achievements of an Arab, instead of either a Persian or Islamic Era scholar. Introducing Tusi as an Arab to the Western world is a mistake. In doing so, some of his contributions to mathematics education would be overlooked. His approach toward education and mathematics education in particular, is connected to the time in which he lived, the place he lived, and the expectations he had for Persians. Whenever Tusi was able, he wrote his ideas in Farsi. His Farsi books, after the end of Arab control, are a sign of his interest in the freedom of language. I decided to explain

and clarify this by reviewing the history of Persia at the time of the “Golden Age” of Islamic world. I also decided to examine his philosophy of living as a Muslim Persian, and the place his occupation held within his personal life. In general, it was a desire for me to find out about Tusi’s journey as a mathematician and educator, and to share this information with the North American readers.

Execution

Understanding the influence of Tusi’s domestic life and his scholarly life had on each other was one of my main goals. I remembered from my secondary studies that Tusi had been introduced as a religious person to Iranians. However, I always saw him as a free mind and free soul of any kind of dependency; a scientist who valued scientific approaches as well. I wanted to learn more about Tusi. As such, I began to do research on his role as a hero for Persians, an open-minded scientist, who wanted the nation to be equipped with the best knowledge of the time. Being free from fenatism was and still is against the way that extremist governments and rulers want the people to live. Tusi’s philosophy of living opposes this style of living.

I decided to consider priorities for my research. Tusi, to graduate students of Iran, is well known as a polymath and philosopher who wrote several works in different areas and disciplines including mathematics. I decided to focus on searching for the portion of his mathematical work that was related to his being a mathematics teacher, mathematics educator, and mathematics author. In addition to these sections, I wanted to introduce some of his social and political approaches he had to employ in order to achieve his goals. It seemed difficult to separate Tusi’s scholarly activities from his political and social responsibilities; this is true in the life of a scholar in our time, too. I chose to search and clarify interconnections between the political and scholarly life of Tusi. I knew that social, historical, and religious aspects of his life had also influenced Tusi’s scholarly life as a mathematician and educator. In fact, in my searching I discovered that Tusi had to be much more than a mathematician in order to live the life he lived. Being a mathematician was and is a journey beyond reading and writing mathematics books such that I believe teachers’ responsibility goes beyond finishing the curriculum of the subjects they teach.

My plan was to travel to Iran for the summer of 2011. For thirty-five days I stayed on task of the collection of data for the working hours of the day. I started most of my days at The National Library of Iran (NLI), where only people with a bachelor or degree are allowed to use the facility. I met counselors of NLI and became familiar with the facilities and buildings where books and other sources of information were classified. The computer system of NLI was a useful tool for saving time in finding sources of information that I was looking for. There were many more philosophical and religious books written about Tusi than mathematical ones. I found numbers of sources with specific information regarding his contribution to mathematics books. Most of the books that I chose to look at were supposed to be read at designated reading areas of the NLI. The staff of NLI was in charge of making copies if it was allowed. Soon I noticed that I needed some direction in order to make my search at the NLI more meaningful and effective. So, I started visiting universities that possess mathematics education departments. I was hoping to find a thread to lead me to the desired books. Graduate students who were introduced to me by family and friends were kind and helpful. They connected me with a few mathematics students who had better historical background. I realized that knowing the Arabic language would be an advantage for candidates who are searching mathematics of the Islamic Era. After all, I found a few related books at the libraries of universities that I could use them for the history of mathematics and history of schooling in Persia. Graduate students of the universities offered fresh information about the new books in the market, but when I checked for the new books, I noticed they were either general history of mathematics, life history of Tusi, his achievements and written books as pure mathematics, or they discussed religious philosophical and political aspects of Tusi's life. I met the Mathematics Department Head of Tarbiat Modarres University (TMU). As the topic of this thesis was new to the Mathematics Department Head I offered him a partnership in this project wherein a gradute student from TMU and I would work together on this thesis. He appreciated the opportunity but couldn't find a graduate student immediately ready to work on this thesis. The main reasons he gave for this were the summer vacation and my limited time of stay. I found the book market of Tehran, capital city of Iran, a fantastic place for me because professional publishers helped me to contact the right people and writers, and to find the right books as soon as was possible. Most of the books that I have used for writing my thesis were purchased at the book market. I also received two books as gifts

from a retired professor of Tehran University. Each day usually finished with a couple of new items, and I studied part of each at night to see to what extent they were useful. The number of the on-line Farsi books and websites that provide information about the history of mathematicians has increased in recent years. University students or graduates who are running scientific web sites administered most of these sites. Similarities between the sites' materials about Tusi in general demonstrated that the origins of the information they use are limited. Some of these websites are not peer reviewed, and I made very limited use of them. Some are prepared and supervised by university faculties and citations are systematically used.

Chapter 2.

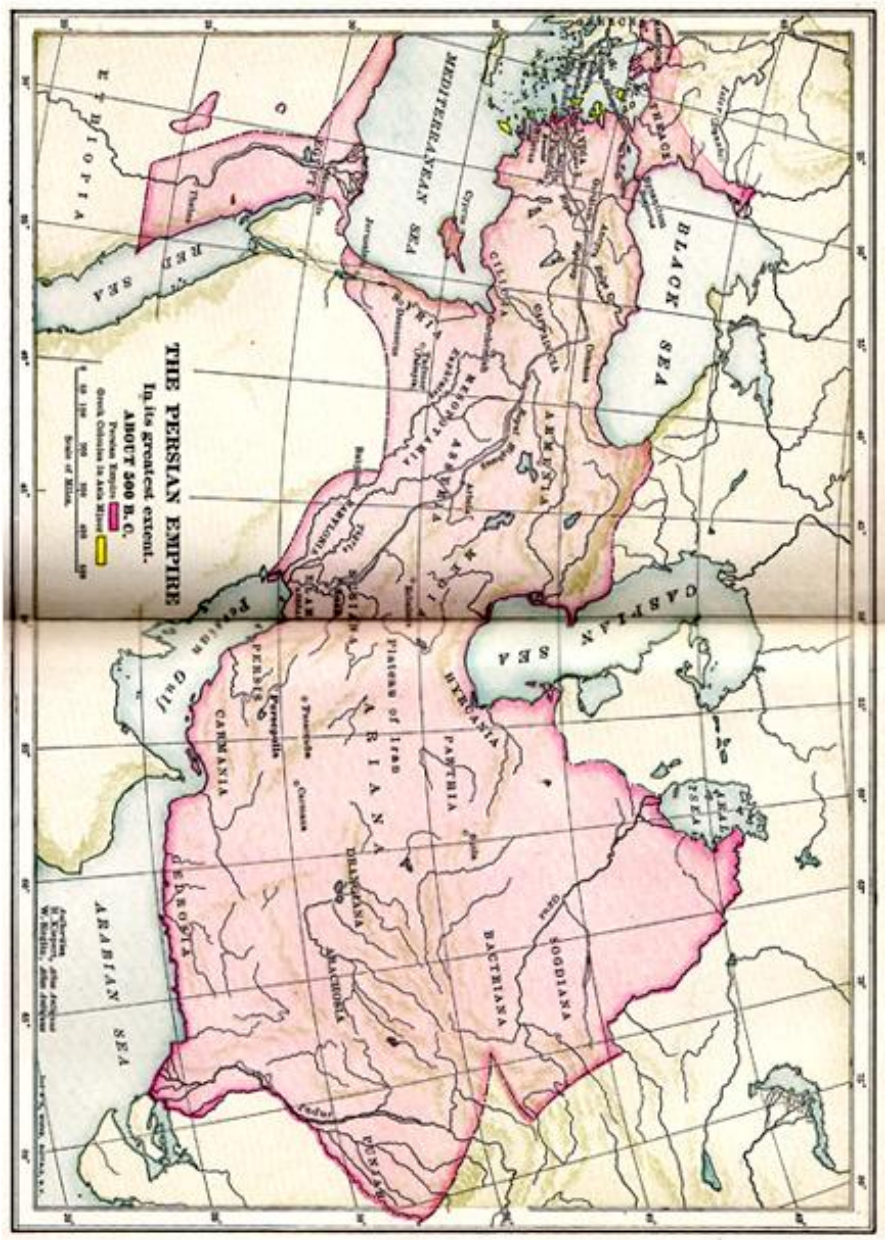
A General History of Persia from Arab's Conquest to the Fall of the Mongols

Tusi's scholarly life was influenced by the social and political situation of his country, Persia. To better understand his life and the options he had for making a living as a mathematician, mathematics teacher, and mathematics educator, reviewing a summary of the history of Persia from the time of Arab's control to the time and during the time of the Mongol's rule is a must. Only after doing this, could one fully appreciate Tusi's contributions and achievements. Being adequately familiar with the culture of Middle East, especially Persia, and being able to read and comprehend written materials in Farsi, are two extra advantages.

Figure 1 shows the map of the great empire of Sassanid where the current Kuwait, Iraq, Syria, Jordan, Lebanon, and Turkey on the western side of the Middle East were once all part of the Persian Empire. Arabs of the Saudi Arabia Peninsula first attacked Persia's south-western border, the place that is now called Iraq, outside of the Saudi Arabia Peninsula, in AD 633. They defeated the Sassanid Empire's army several times, but their complete occupation of Persia took 18 years. The last Sassanid Empire ended in AD 651 retreating to the north-eastern borders of Persia (Balaami History, 1073, translated by author).

According to several historical documents, Arab's main excuse for attacking Persia was to offer Islam to Persians by force, after the King of Persia rejected Arab's so called peaceful invitation of Islam.

Figure 1. Map of Sassanid Empire, Persia, AD 633



Source: www.bible-history.com/map

During the first couple of centuries after Arabs conquered Persia, Persians generally had a difficult time as a defeated nation. Persian states being introduced to Islam were undergoing social challenges and evolution. People were distracted by the

new rules and regulations of Arab authorities. Many were undecided between practicing Islamic laws and continuing with Zoroastrian laws. Several historians have recognized that there existed a complicated social conflict where a number of different social ideals were in competition (Marlow, 2002). Those who accepted Islam or were forced doing so were still influenced by their cultural background or previous religious beliefs such as Zoroasterianism. Although questioning Arab authorities was a suicidal action, people's acceptance or forsakenness of Islam was a question of the mind for individuals.

Umayyad[1] Dynasty (AD 661 – 750)

From the beginning years of Arab control that brought the immediate introduction of Islam, people of Persia observed the messages of the Prophet Mohammad into their culture of well-being that they had learnt from Zoroaster[2]. Although Persians were moving forward naturally within their new religion, the dictating centre of Islam in Damascus, the capital of Umayyad, constantly asserted its superiority. The barbarian culture of nomadic Arabs being fed to Persians, as the new nation of the Islam, was no match with the rich culture of Persians. As Sunni Muslims[3], Umayyad was pushing new Muslims to follow the faith of Sunnism instead of allowing Persians to integrate newly introduced Islam with their background and then adopt the result as their religion. Umayyad had obtained the control of Islamic states including Arab states and Persian states few years after the death of his holiness the prophet Muhammad. This was the beginning of the period that Montgomery (2000) pointed to Islam's own time, "a time of

1 Muawiya b. Abi Sufyan, a clan of Uthman, the third orthodox ruler after Prophet Muhammad, was given the governorship of Syria by Uthman. Upon the accession of Ali to the caliphate, Muawiya refused to pay him allegience. In 658 he gained the control of Egypt. Following Ali's assassination in 661, he subdued Iraq and then formally established himself as caliph. (Princeton.edu)

2 Zoroaster is the name of the founder of Zoroastrianism, and early religion pitting good against evil that has influenced Judaism, Christianity, Islam, Gnosticism, and Buddhism, and perhaps, pre-Socratic philosophy. (N. S. Gill, About.com)

3 Sunni is one of the two main branches of Islam. Most fundamental Islamic beliefs and articles of faith are the same between the two. (Author)

Jahiliyya- ignorance, uncultured ways, and the over-reliance of the powers of the spirit and the body at the expense of those of the mind" (p. 95).

Soon, Umayyad's rule extended to Persia. Umayyad assassinated their opposition, including many of the Prophet's main followers, his holiness Ali who was the Prophet's cousin and son-in-law, and Ali's[4] sons and their successors. Persians, who had come to have deep interest in his holiness Ali's life and philosophy, became discouraged by the action of Umayyad. Meanwhile, Persians felt the pressure of living in their own country as second-class citizens in comparison with Arabs who had migrated to Persia. Arab soldiers and their families who were only a few percent of the population of Persia at that time, were in control of the country. Arab authorities ruled Persians in the name of Islam, but with a harsh, dogmatic interpretation of Islamic laws. As a result, about 8th century AD, a new movement of independence started developing in Persia, mostly in Khurasan Province, against Umayyad.

Independence Movements

Umayyad rulers were not sharing opportunities. Their thirst of power and wealth along with their exclusiveness bothered Persians from the beginning of their governance. Persians endured discriminations in their own country by Arabs who had left the desert of Saudi Arabia, and migrated to the geographically advanced region of Persia around Alborz and Zagros mountain ranges. Although Arab soldiers were clearly instructed by the Prophet to respect and care about nations which are brought to Islam, they acted rude and harsh. For a century, what Persians remembered from Islam was its divine messages in its book, but combined with the terror, poverty, disgrace, and eradication which Arab soldiers and authorities had imposed on them. Persians

[4] Holiness Ali Ibn Abi Talib, March 17, 599 – January 27, 661 was the cousin and son-in-law of the Islamic prophet Muhammad. Ahmad Ibn Hanbal writes that, "There are not as many verses and traditions in the praise of any other companion of Muhammad as there are in the praise of Ali." Ibn Abbas narrates, "three hundred verses of the Holy Koran have been revealed in favour of Ali." (www.ismaili.net/histoire/history03/history336)

gradually noticed the difference between the true values and morals introduced by Islam and its book, the Quran, and those by which the rulers of Umayyad in Damascus were living and governing. Equity, caring, and truth were the words of the mouths of Arab rulers, not goals for them. It became a nightmare for new Muslims of Persia to live under the control of nomadic Arabs who were exploiting the country's human and natural resources. Gradually, Persians strengthened their dream of independence. Finally, the discord between Arabs in the capital of Islam, Damascus, provided the opportunity for Persians to seek their freedom.

Persians sought their freedom with the help of Arabs who for several reasons opposed their own Arabic regime, Umayyad. As it was almost impossible to initiate a cultural, religious, or political independency without the help of concerned Arabs, Persians planned for a joined operation with unsatisfied Arabs. A young Persian born in Middle Persia, known as Abu Muslim Khurasani[5] invited the people of Khurasan and many Arab residents of Persia to stand up for a movement against the central government in Damascus. Abu Muslim gathered a huge army of Persians who desired to live in honour and peace with their new religion, and also of Arab citizens of Persia who had become accustomed to their life style in Persia and were looking to change the Arab regime. He led the first major and organized liberal movement against the Umayyad Dynasty and defeated their army several times in different regions. He finally attacked the last Umayyad ruler and defeated him. The retreat of Umayyad from the Middle East toward Northern Africa happened after about a century of conquest and terrible civil war. Islamic states began to change to an enlightened civilization where the past was employed as a resource (Montgomery, 2000). Islam's, so called "Golden Age" was yet to arrive, but the fall of Umayyad in 750 AD was the first step toward it.

[5] Kurasani mean from Khurasan. His original name was Behzadan, prior to his father Vandad Hurmoz's conversion to Shia Islam, who adopted the name of 'Muslem' for himself. His birthplace remains obscure, though the oldest historical reference, the 11th century *Al-Mahasin al-Isfahan* written by Mafzal Ibn-Sa'd Maforukhi Esfahani, claims he was born in the town of Fereidan in the central Iranian province of Isfahan.
(Zarinkoob, Two Centuries of Silence)

Abbasid Dynasty (AD 750 - 1258)

The Abbasid Dynasty rulers and their court, who ruled from 8th to 13th century AD, came mainly from the east of Persia, “where a cosmopolitan culture had existed for centuries, [and] should not be undervalued in terms of the stimulus given to the intellectual efflorescence that took place beginning in the late eight century” (Montgomery, 2000). It is mentioned in *Akhbaro Al-ddoleh,* by Allah Akbari (2012) that Abbasid were Arabs descended from one of the Prophet’s cousins, Ibn Abbas. They had tried to stay away from the tension between Umayyad and other Muslims including Shias (p. 200). That is why for centuries they were living almost out of the reach of Umayyad rulers by living in more secure areas of Persia. Instead, they had spent time in promoting themselves in Khurasan and other provinces. They finally won the support of Shia[6] Muslims against the Umayyad, as Umayyad was the common enemy of the two groups of Shia and Abbasid. They were helped by Persians under the leadership of Abu Muslim Khurasani. In other words, Abbasid appealed to non-Arab Muslims.

The first change the Abbasid made was to move the Empire's capital of Islam from Damascus, in current Syria, to Baghdad in current Iraq. This was to appease the Persians who had helped them, and also to get closer to the region that had more support and influence of existing Persians’ history and culture. It was also part of the Persians’ demand for decreased Arab dominance in the Islamic Empire. Baghdad was soon transformed from a village to a city.

The vision of Mansur, the first Abbasid ruler, “differed considerably from the type of society associated with the Umayyad, whose origin lay in the less cultured region of Arabia. Indeed it seems far more in the line with the kind of cosmopolitan and intellectualism pursued by Sassanid kings of Persia, notably Anushirwan” (Montgomery, 2000).

[6] Shia Islam account for approximately 15 percent of the total Muslim population in the world. Shiism has the greatest influence in the contemporary world in Iran where nearly 90 percent of the Muslims are Shia. (patheos.com/Library/Shia-Islam)

According to Ibn Khaldun,[7] the Eastern Roman Emperor in response to the request of Mansur, the ruler of Baghdad, sent copies of Euclid and other biology books to him, and Euclid was translated at the time of Mansur to the Arabic for the first time (De Young, 2005). But as Abbasid caliphs became careless about the nations of Islam including Persians, the scholarly activities of Baghdad's Library and its branches slowly dwindled. As a result, after about four centuries from the beginning of the Islamic Era one could distinguish "a distinct decline toward the close of the eleventh century with a notable decrease in the number of mathematicians of the first rank" (Kokomoor, 1994). This was about the time the Mongols attacked eastern Perisa.

Persians in Abbasid Empire

Most of the counsellors of the Abbasid government in the beginning years of their rule were Persian. Persians had agreed to move their educational centre from southern Persia, where the capital of Sassanid was located, to Baghdad. This movement was in balance with Abbasid's changing of the capital to Baghdad. With the direct help of Persians, Baghdad's famous library, "House of Wisdom," and its school were designed and established as a copy of Gundishapur[8] University of southern Persia. Hamilton describes this as follows:

> *By the middle of the 8th century the Muslim world became an intellectual center for science, philosophy, medicine and education as the Abbasid championed the cause of knowledge . . . in Baghdad; where both Muslim*

[7] Ibn Khaldun was a Tunisian. He is known for being a historiographer who is viewed as one of the forerunners of modern historiography. He is best known for his *Prolegomenon*. (www.norrmags.com).

[8] The Academy of Gundishapur was a renowned academy of learning in the city of Gundishapur, the intellectual center of the Sassanid empire. It offered training in medicine, philosophy, theology and science. The faculty were Persians (Frye, Richard Nelson. The Golden Age of Persia). According to *The Cambridge History of Iran*, it was the most important medical center of the ancient world during the 6th and 7th centuries. In addition to systemizing medical treatment and knowledge, the scholars of the academy also transformed medical education; rather than apprenticing with just one physician, medical students were required to work in the hospital under the supervision of the entire medical faculty. There is even evidence that graduates had to pass exams in order to practice as accredited Gundishapur physicians (as recorded in an Arabic text, the *Tārīkh al-ḥukamā*). Gundishapur

> *and non-Muslim scholars sought to translate and gather all the world's knowledge into* Arabic*. It is well established that the Abbasid modeled their administration on that of the* Sassanid *of* Persia*. (Hamilton, 1982)*

Baghdad, gradually, became a big city and a centre of education and accomplishment where many scholars, including Persians, enabled their talent to flourish by joining the educational environment of the "House of Wisdom[9]." It is important to remember that many research and translation activities in Baghdad were completed by Persians, particularly whenever a book was translated from Pahlavi (the language of Middle Persia) or Indi (the language of Middle India) into Arabic. In fact, Persians were the main administrators of the Library until Arabs dominated it. An Abbasid ruler, Al-Ma'mun, whose mother was Persian, had said:

> *The Persians ruled for a thousand years and did not need us Arabs even for a day. We have been ruling them for one or two centuries and cannot do without them for an hour. (Bertold, 1995)*

Although Shia Muslims, in particular Persians, were hurt by Abbasid, the overall situation for them was significantly better than the time of exploitation of Umayyad rulers. Economical opportunities were more shared, nobody was prohibited from education, Persian scholars were allowed to seek opportunities for research, and people lived with their culture of choice. Abbasid stopped pushing Persians to practice Sunni faith, and

- also had a pivotal role in the history of mathematics. After the Sassanid Dynasty fell to Muslim Arab armies, Caliph al-Ma'mūn founded the famous House of Wisdom where the methods of Gundishapur were emulated; indeed, the House of Wisdom was staffed with graduates of the older Academy of Gundishapur. (Frye, Richard Nelson. The Golden Age of Persia). Will Durant has lauded the Iranian civilization for having built such an academy. Einstein has praised his disciple, Professor Hesabi, for having belonged to a country where an academy had been built 1,700 years ago.

9 The *House of Wisdom* was a library and translation institute established around 800 AD in the Abbassid era, Baghdad, Iraq. It was a key institution in the Translation Movement and considered to have been a major intellectual centre during the Islamic Golden Age. *House of Wisdom* is a calque of the Middle Persian term for a "library" and it was modelled on the libraries of the Sassanid Empire. It had the dual purpose of translating books from Middle Persian to Arabic and also of the preservation of translated books. (Meri & Bacharach, 2006)

allowed Shia Muslims to practice Islam as they wish. Their interest in science and knowledge had satisfied the scholars and intellectuals of the entire Islamic states. Several schools were established throughout the Islamic states. The most famous chain of schools, Nezamieh[10], was directed by Nezamumulk[11], the Persian Prime Minister of Seljuk Dynasty which was ruling in parallel with Abbasid. Nezamieh was the most organized and systematic schooling method for the public at that time, and copies of it were established in Baghdad, too. Aghazadeh (2010) stated "Nezamulmulk had spent two hundred thousand Dinar from his assets to build Baghdad's Nezamieh School, and established markets, public baths, stores, etc. around it, and every year spent fifteen thousand Dinar for the financial aid of instructors and pupils" (translated by author, p. 12). The second biggest Nezamieh belonged to Neishapur of Khurasan within which several scholars lived. By the time of the end of Seljuk Dynasty, there were more than 400 000 books in Neishapur's Library from all languages, but mostly in Arabic and Pahlavi.

Resistance Movements against Abbasid

"While the Abbasid originally gained power by exploiting the social inequalities imposed to non-Arabs in the Umayyad Empire, ironically during Abbasid rule the empire rapidly Arabized as knowledge was shared in the Arabic language throughout the empire" (Ochsenwald, 2004). Once completely in power, the Abbasid embraced Sunni Islam and in a dirty political way disavowed any support for Shia Muslims and for Shia beliefs. They also executed the direct successors of the Prophet Muhammad who were

[10] See footnote 36.

[11] Nezamulmulk, the famous Iranian politician (AD 1018-1092) was born in Khurasan. He served as Minister of two of strongest Seljuk Kings for 29 years. He had great achievements of record such as the foundation of Nezamieh schools. He was the author of *Siasatnameh*, a famous book on state politics. (fouman.com/history/Iranian_History_1092)

also the Shia Imams (religious leaders), including Imam Jafar Sadiq[12], and other respected nobles. In disrespecting Shia Muslims and their beliefs, Abbasid rulers became similar to Umayyad's. They also assassinated Abu Muslim Khurasani, who had helped them to defeat Umayyad, and was a hero to Persians. These cowardly actions injured Persians, and rooted as one of the main reasons that Persians had difficulty trusting Abbasid.

In the end, Abbasid destroyed their reputation in many ways for reasons similar to those of Umayyad's. People in different areas stood for their civil and Islamic rights, and sought independency. Smaller movements came together and created bigger independency phases, which the central government in Baghdad either stopped and suppressed or had to live with in a diplomatic way. During this period (AD 800 – 1200) Baghdad stayed intact as the Abbasid were eager to keep their territory as a political centre at any price, even by giving up Persia to Persians. Abbasid collected little federal tax so governments of the dynasties ruling concurently with Abbasid were able to hold onto their wealth. The only expectation was to respect Baghdad as the centre of Islamic beliefs and practices, and to consider it as a centre of trade. As a result, Baghdad's "House of Wisdom" continued with its scientific activities including translations and the creation of mathematical treatises. This was of great importance for the community of scholars and mathematicians of Persia, as they travelled to and from Baghdad for the communication, sharing, and development of their knowledge, regardless of the current ruler or dynasty, or in what territory they lived. It was about the end of this era when Tusi was born in Persia.

[12] Jafar Ibn Muhammad was a descendant of Prophet Muhammad and a prominent Muslim jurist. Although he is perhaps most famous as the founder of Shi'a Islamic laws, known as Ja'fari jurisprudence, he had many other accomplishments. He was a polymath: an astronomer, chemist, Islamic scholar, Islamic theologian, writer, philosopher, physician, physicist and scientist. (Encyclopedia Britannica Online)

Mongol's Conquest of Persia

The Mongol attack began in Central Asia near Persia in the early 13th century. Mongols burnt cities, and demonstrated their abilities and capabilities with regard to massacre when they occupied Bukhara city and Samarkand city in north-east Persia in 1220 AD, and Kharazm city, Balk city, Merv city, and Neishapur city, all from the great province of **Khurasan** in 1221 AD by Ginghiz[13] Khan's Army (History Research Group of the University of Calgary, 1998). It was during this time that Tusi, who was 19 years old, had to flee Neishapur to go to a safer place. According to Peernia[14] (1992) Ginghiz burnt all wooden houses of Bokhara. Only bigger structures such as mosques survived as they were made from blocks (translated by author, p. 419). Kharazm was attacked by Ginghiz's son, who after the fall of the city "moved over 100 000 technicians and trade people toward the eastern regions where the Mongols had a more stable situation, but the children, women, and the rest of the men were executed by Mongol soldiers. It is written that for every Mongol soldier there were twenty-four lives to take" (Peernia, 1992, translated by author). Neishapur had the worst situation as "the number of killed people in there is written as 1,750,000." Ginghiz's daughter demanded that Neishapur be destroyed so that "they could perform cultivation; they even killed the cats and dogs of the city. Soldiers flooded the city for seven days then planted oats" (Peernia, 1992, translated by author). Disappointment and discouragement were common feelings among the people.

The Mongols became very amazed and occupied with their exploitation of Khurasan as they had never seen such a large population, schools, libraries, industries,

[13] Ginghiz Khan, born Temujin was the founder and Great Khan (emperor) of the Mongol Empire, which became the largest contiguous empire in history after his death. He came to power by uniting many of the nomadic tribes of northeast Asia. After founding the Mongol Empire and being proclaimed "Ginghiz Khan", he started the Mongol invasions that resulted in the conquest of most of Eurasia. (De Hartog, Genghis Khan: Conqueror of the World)

[14] Hassan Peernia (AD 1912-1975) came from a spiritual family. He completed studying law in Mosko, and became a Prime Minister of Iran. The first political school and political course in Iran was planned by him. His history book is a main source of information for graduate students in Middle East. ("Hundred years, hundred faces" by M. Toluee)

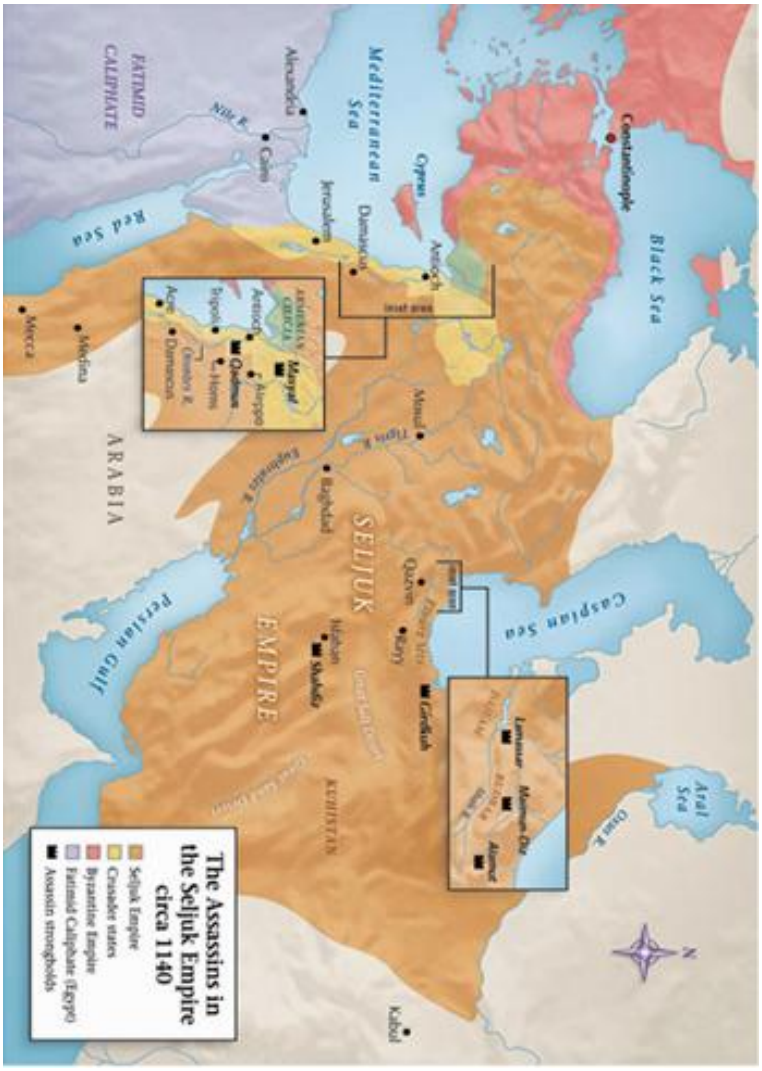
judging system, citizenship and so on. The quality of life in the region, in addition to many smaller resistances, forced them to slow down on their way toward western Persia. It took more than a decade before they continued their exploration toward western Persia. There are documents to support that Hulagu, Ginghiz's grandson, didn't want to attack more cities toward the western states, but Baghdad's acts had forced and encouraged him to teach a lesson to Abbasid of Baghdad. On the way toward the western states of Persia and Baghdad, the Persian city of Isfahan fell in AD 1237. Then toward western Persia, Mongols encountered the Muslim group known as the Assassins[15], an Ismaili sect that practised an extreme version of Shiism.

Mongol and Ismaili

At the beginning of 13th century the Ismailies or Assassins sect was the chief power in Northern Persia and Hassan Sabbah, the Grand Master, resided in the mountain fortress of Alamut, some distance to the north-west of the present capital of Iran (Shephenson, 1923). They had several followers who were emotionally and economically disappointed with the Arab invasion. Many Persians who had lost their family members in defence against the Arabs joined the Assassins or supported them. Assassins terrorized neighbouring cities and rulers of the main government by assigning young men on suicide missions to assassinate (Frazier, 2005). Assassins, as will be discussed further in Chapter 3, kidnapped Tusi and held him in quarantine for decades. Figure 2 shows the territories in which Assassin had a strong hold during the Seljuk Dynasty. Figure 3 depicts a scheme of the Alamut fortress. Tusi unwillingly lived with Ismailies for long years until Mongols reached Alamut.

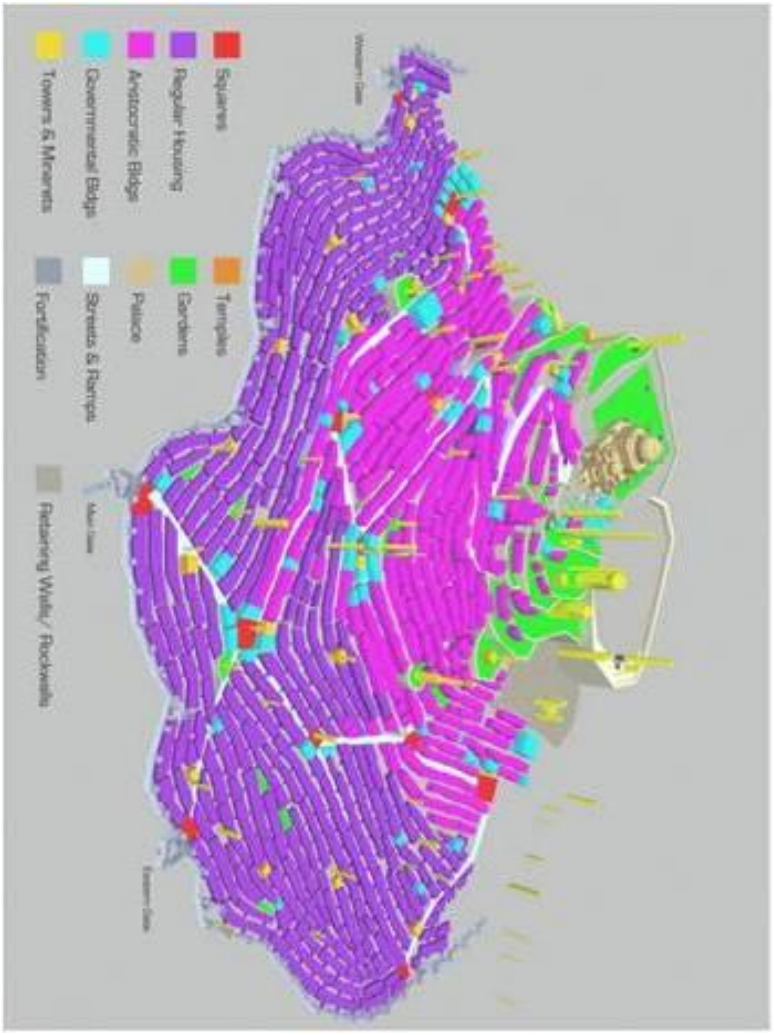
[15] The Assassins were an order of Nizari Ismailis, particularly those of Persia (and Syria) that existed from around 1092 to 1265 AC. Posing a strong military threat to Sunni Saljuq authority within the Persian territories, the Nizari Ismailis captured and inhabited many mountain fortresses under the leadership of Hassan-i Sabbah. Hassan-i Sabbah liked to call his disciples Asasiyun, meaning people who are faithful to the Asās, meaning 'foundation' of the faith. Later the word changed to 'assassins' to indicate the method of their attack to their enemies. (Daftary, A Short History of the Ismailis: Traditions of a Muslim Community)

Figure 2. Ismaili Strongholds during Seljuk Dynasty



Source: map-of-ismaili-castles.png

Figure 3. Scheme of Alamut Fortress



Seljuk had tried diminishing this stronghold for over 100 years, but they were not successful; perhaps because they could not have terrorized Assassins. Mongols, however, were capable of terrorizing their oppositions in battle and they made the fullest use of the terror, with no mercy, inspired by their physique, their ugliness, and their stench (Frazier, 2005).

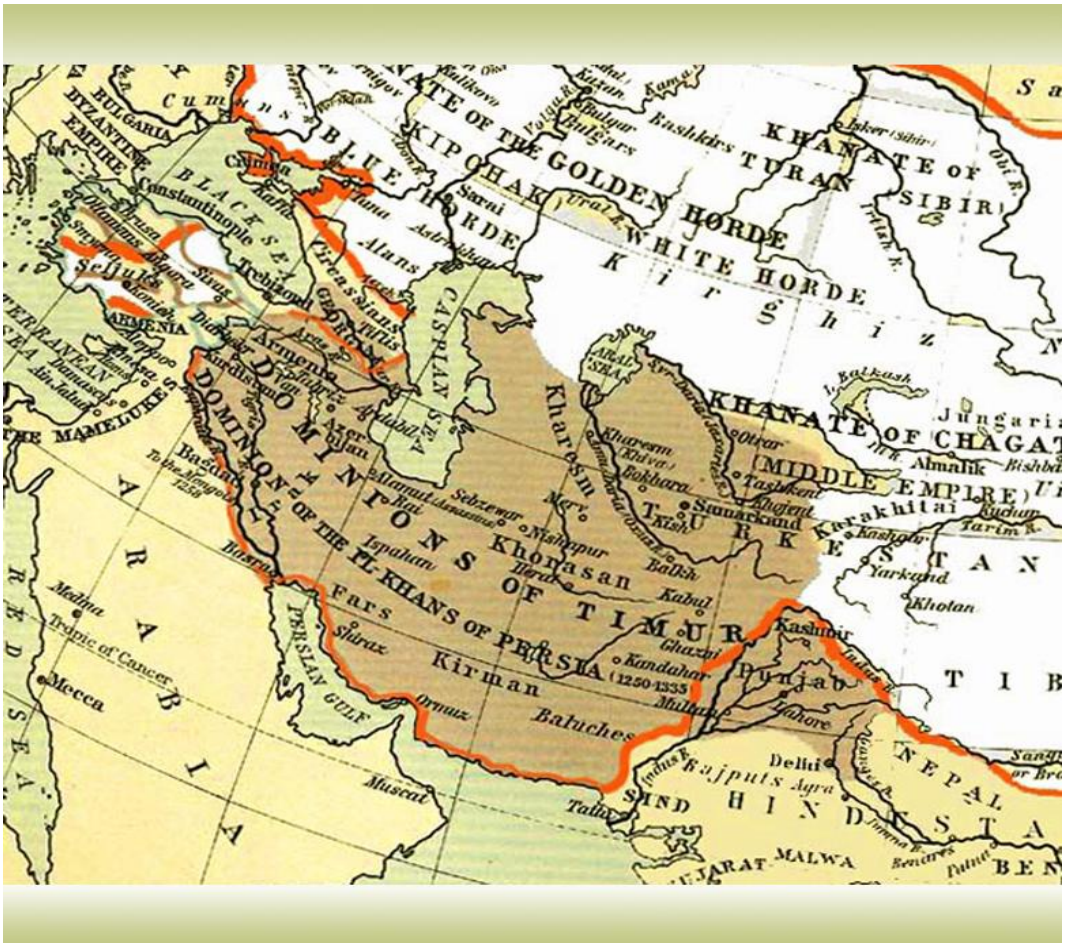
When Hulagu reached the point beyond the gate of Alamut, Tusi was there. Hulagu completed his goal of defeating the Ismailies without bloodshed as Tusi encouraged the Ismaili leader to surrender.

Fall of Baghdad

“In early A.D. 1258, Hulagu took and plundered Baghdad and ... so at least thus it seemed then put an end to the [Abbasid] caliphate as it had been known for hundreds of years” (Amitai-Preiss, 1996). The Abbasids were at a disadvantage although theoretically they had a large army to compete with the Mongols. Historians have shared several reasons for the quick occupation of Baghdad by the Mongols, which can be found in Appendix B of the thesis. Figure 4 shows the Mongol empire at their peak of power. They established the Ilkanate Dynasty in Persia and chose Maragheh, in the northwest of current Iran, as their capital.

After the death of Hulagu, Abagha succeeded his father with the help of his Christian mother. Abagha and his successors who later ruled in Persia had little relation with the Mongol rulers of their home country, Mongolia. “They are considered an independent regime of Persia, not a subdivision of the rule of Mongolia. They followed the routine of Persian emperors” (Peernia, 1992, translated by author). Abagha’s son succeeded after his father became Muslim, and little by little more Mongol turned into Islam. From the time of Hulagu’s rule to AD 1378 which the Ilkhanate regime continued to control their dynasty in Persia, sixteen successors of Hulagu sat on the throne, of which thirteen of them became Muslims, and lived and died in Persia.

Figure 4. Mongol Empire In their Days Of Glory



Source: karakalpak-karakalpakstan.blogspot.com

Chapter 3.

A Summary of the History of the Life of Tusi

Just before the Mongol attacked Persia's eastern border, Tusi was born. The Mongol attack pointed to the end of the glory of education during the Seljuk Dynasty. Living conditions and their safety and security became the main concern of scholars, and Tusi was no exception. In this chapter more details of the historical and social aspects of the life of Tusi will be discussed. His achievements inspite of all social and economical problems will be examined.

Tusi's Background

Muhammad, the son of Muhammad, the son of Hasan, nicknamed Nasiruddin, and known as Tusi, was born on a Saturday, in February of 1201 AD in Tus, a town in Khurasan state of current Iran (Modarres Razavi, 2008, translated by author). Some historians believe his family was from Jahrood, a town in central Iran, but the family had moved to Tus, a city in Khurasan, before Tusi was born. His father who was a learned man with several students was a jurisprudent of Twelver Shia[16] who taught Quran, Hadith[17], discourse and the literature of Pahlavi (the ancient Farsi) to Tusi in his early years. Tusi continued learning Hadith from Borhan-al-Din Hamedani Ghazvini who was a

[16] Twelver or Imami Shia Islam is the largest branch of Shia Islam. They believe in twelve divinely ordained leaders, known as the Twelve Imams. They also believe that the Mahdi is the Twelfth Imam that disappeared and is believed by Twelvers to be in occultation. All twelve Imams were ancestry of Ali Ibn Abi Talib and Fatima the son in law and the daughter of the Prophet Muhammad. (Encyclopedia Britannica Online)

[17] The term Hadith is a saying ascribing validity or invalidity of an act according to the Prophet Muhammad's life. Hadith are regarded as important tools for understanding the Quran and in matters of jurisprudence. (Oxford English Dictionary)

source of imitation. Tusi benefited from the class of Nasiruddin Abutaleb in Neishapur and completed his religious studies. But, Tusi received his first religious permit from his uncle, Noor-al-Din. Tusi's documented permission in Islamic laws is from his teacher, Moeen-al-Din Mesri, and the orginial permission is available at The Library of Parliament of Iran (Modarres Razavi, 2008, translated by author). Tusi was one of the famous students of Ghotb-al-Din Mesri, great physician and theologian of Moraco, who settled in Neishapur. This was during the time Tusi was away from his family and right before Mongol attacked Kurasan. Neishapur, the centre of Khurasan, was one of the most important scientific centres in Islamic states with over a million residents.Tusi benefitted from the presence of Sheikh Abu Saadat in Neishapur. He completed levels of philosophy with his master. Perhaps, the most famous of the several masters of Tusi among Iranians is Farid-al-Din Damad who was born around Neishapur and within four accounts was a student of Avecina, the famous scholar of Persia (Modarres Razavi, 2008, translated by author). It is said that Tusi also studied music and mathematics with Farid-al-Din Damad (personal communication, December 1, 2012). In few years Tusi grew to become an icon in several popular sciences. When Tusi was only 19 years old, the Mongol were threatening the region, and many people were leaving the cities for safer shelters, where they could survive the Mongol brutal attacks. During the rule of Seljuk and Kharazmshahian[18], Mongols savagely attacked eastern Persia, and mercilessly massacred many of the residents of Khurasan's cities. The slaughter of many scientists and the demolition of science centres, in one stroke, "caused continued suffering of science, knowledge, businesses, and even hope. Scholarly life became a dream for scholars. Even those scholars that fled to western regions didn't have the required mental focus and tranquility to keep working on scientific matters" (Sultanzadeh, 1995, translated by author). Kasayi (1999) in his *History of Great Islamic Universities* cites from writing of Ibn-e-Khaldun, a great historian of Tunisia (May 27, 1332 AD – March 19, 1406 AD), that "prosperity and the success of sciences occurred

[18] A dynasty established in eastern Persia parallel to the Seljuk Dynasty in central and western Persia. The Mongols reached the region that Kharazmshahian ruled first on their way to the western Middle East. (Author)

so long as civilization and development was happening in Persia and its regions, such as Khurasan, Iraq[19], and Macedonia. But when the cities were destroyed, civilization and reformism, which serves to generate knowledge and industry left the territory" (translated by author, P. 29). Tusi, who saw himself in danger, had to leave Neishapur to reach a safer location that also provided him with the chance of continuing his education. Some of the causes that forced Tusi to leave Neishapur include but are not limited to the anarchical situation in east of Persia, violent actions of the Mongols, prejudices of Sunni leaders, and persecution of Shia Muslims and scholars by both the Sunnis and the Mongols.

Tusi's Migration to Alamut

"Tusi was already an accomplished scholar when he left Neishapur" (Daftary, 2005, translated by author), and was welcomed everywhere he wanted to migrate. Historians have different reasons explaining Tusi's migration from Neishapur. Ismaili writers believe that he was about to change his faith from Twelver Shia to Ismaili Shia at the time he decided to leave Neishapur. Therefore he decided to stay in the court of Ismailies in Quhistan[20]. Some historians have suggested that he was forced by Ismailies to move out of Khurasan, and to stay in Quhistan. Modarres Razavi Khurasani[21] (2007) believed that "perhaps in the beginning Tusi voluntarily went to Quhistan after receiving

[19] Iraq, Syria, and Turkey were part of Persia and later Uthmani Turks, as far back as history remembers. After the First World War and fall of Uthmani in AD 1923, the Uthmania area was divided into three countries by the political interference of UK. One of them became Iraq (Author).

[20] Quhistan or Kohistan (mountainous land) was a region of medieval Persia, essentially the southern part of Greater Khorasan. The area tended to be ruled either as part of Khurasan or as a province with its capital at Herat. It did, however, experience some periods of autonomy. (C. Edmund Bosworth. *The Ismai'ilis of Quhistan* in Farhad Daftary. Medieval Isma'ili History and Thought, 2001)

[21] Muhammad Taghi Modarres Razavi Khurasani (1895 -1991 AD) was a researcher, editor, educator, and professor with more than ten highly cited books and editions on the history of Iran and Iranian scholars. (Author)

an invitation from Mohtasham, the chief of Quhistan fortress" (translated by author, p. 11). The suggestion that is best aligned with the political and social situation of Persia at the time of the Mongol attack is that offered by Modarres Razavi Khurasani above. What all historians agree is that Tusi's first choice in the crucial time of the Mongols' conquering was to migrate to Baghdad, and that he had tried to, but it didn't occure. Historians confirm that Arab scholars of Baghdad's Library didn't want Tusi to move there. Staying in Neishapur was not a choice for Tusi after all. Perhaps Tusi had found possible peace of mind with Ismailies who lived in strong holds out of the reach of the Mongol. Chief Nasiruddin Abi Mansur, the head of Quhistan Fortress in southern Neishapur, was a virtuous person, and had heard about Tusi's skills and knowledge in several sciences. He wanted Tusi in his sight, and respected and welcomed him. Tusi stayed for a long time with Abi Mansur, studied lots of books, and wrote several books and treatises including a treaties on astronomy. Ismailies believe that Tusi moved to Alamut Fortress, later, when the Mongols got closer to Quhistan Fortress. Historians who believe Mansur[22] had kidnapped Tusi from Neishapur, also believe Abi Mansur later forced Tusi to serve the Grand Master of Ismailies at Alamut (Shephenson, 1923). Although the Ismaili Master in Alamut also respected Tusi, from Tusi's "Sharh-e-Esharat" it is understood that "Tusi in Ismaili strong holds was offended and displeased, and had not resided by his will, but with reluctance and because of lack of alternative. He, in fact, had been a prisoner" (Modarres Razavi Khurasani, 2008, translated by author).

> *The oriental court [of Alamut] was wont to derive lustre from the poets and philosophers who frequented it, and the Chief of the Assassins was apparently desirous not to be behind other potentates of the time in the patronage of learning. Tusi remained at Alamut, an honoured if unwilling guest.* *(Shephenson, 1923)*

Tusi's complex character makes it difficult to identify moral and political aspects of his decision to live in Alamut unless his past and future is studied as an aggregation.

[22] The chief of the closest Quhistan fortress to Neishapur (Author).

According to Daftary[23], whether cited in Tusi's "defence or against, the assumption of ethical positions based entirely on sectarian or abstract moral principles neglects a fundamental aspect of his character as a Persian philosopher and vizier, fully present and active in the social and political context of his time" (Dabashi, 1996). Figure 5 shows a residence in Alamut for higher ups.

Figure 5. A Residence for Higher Ups in Alamut

Note: This residence is partially carved in Alamut mountain. Light is added to the facility in recent years.

Tusi lived with his family in mountains and under the supervision of Ismailies for years during which he created some of highest levels of philosophical and mathematical works of his time. He stayed alive and active to connect the education and science of the past to the future. Compiling *Mutiwassitat*, a series of astronomy books that are

[23] Educated in Iran, Europe and the United States, Dr. Farhad Daftary received his doctorate from the University of California at Berkeley in 1971. He has held different academic positions, and since 1988 he has been affiliated with The Institute of Ismaili Studies, where he is Co-Director and Head of Department of Academic Research and Publications. (Institute of Islmaili Studies, http://www.iis.ac.uk/view_person.asp?ID=8&type=auth)

necessary to be studied between Euclid's *Element* and Ptolemy's *Almagest* is one of his achievements in that time. *Mutiwassitat* will be discussed in further detail in Chapter 5. An outcome of Tusi's academic research about astronomy was preparing plans required for running an astronomical observation. The observation will be discussed Chapter 6. Tusi was also running on-going mathematics classes in different levels, including secondary, and was compiling a series of proper mathematics books to be used for teaching purposes. This topic will be addressed in Chapter 4. Tusi's life under the control of Ismaili court was a journey that allowed him to recognize and value the rich culture of Persians who studied and survived under pressure. He identified the weaknesses of the nation against foreign forces such as those of Arab and the Mongol, and decided to educate people for a better and brighter future. This matter also will be discussed further in Chapter 7.

The Fall of Alamut

Hulagu, the Mongol aggressor, made his mind to attack the Ismaili strong-hold, Alamut, in AD 1256. At this time Tusi was over 50 years old, and had been living under Ismaili control for over 30 years of his life. Ismailies had become weak in both of their pillars, military and ideology. Their courage in training assassins had faded, and fewer youth were joining them to serve their Master by offering their life in assassination missions. Tusi knew that Hulagu would be able to break Ismaili's resistance, and following that would burn down everything, including books, as Hulagu had done in other cities such as Neishapur. Therefore, he decided to encourage the last Ismaili Master to surrender quietly rather than get involved in a bloody war with Hulagu. After several back and forth negotiations between the two armies, mediated by Tusi, the 170 year old government of Ismaili faded to a quiet end, and Tusi was respected, by Hulagu, for his efforts in finishing the matter in a diplomatic way (Modarres Razavi, 2008, translated by author). Tusi's conservative act saved and secured thousands of lives of Ismailies residing in Alamut and neighbouring cities. He also saved a tremendous number of books and treatieses which he had collected during his residency in Alamut.

Tusi and Fall of Baghdad

A year after the fall of Alamut Hulagu decided to attack Baghdad and finish Abbasid's rule. One of the Hulagu's trusted Mongol advisors was against this plan, and warned him of consequences that he had foreseen in the appearance and the karma of whoever would overthrow Abbasid. On the contrary, Tusi ensured Hulagu that nothing dramatic would happen if Hulagu attacked Baghdad, and that he would become victorious. It is obvious that Tusi was not pleased with the way Arabs under the rule of Abbasid had treated Persians. Tusi, like many other Persians, never liked Arabs who misused their power, and exploited Persia, but some critics have a different opinion. James Brown (1793 – 1841), who was a lawyer and not a historian, somehow blamed Tusi as he believed Tusi "persuaded Hulagu that no heavenly vengeance was likely to fall on him if the Caliph [Abbasid ruler] were put to death" (Shephenson, 1923). Some other Arab historians also have called Tusi guilty for helping Hulagu to attack Baghdad. This argument from the point of view of an Arab might be considered natural, but as a Persian who was aware of the Arabs' deep cultural attack **to** Persia; this was an opportunity for Tusi to get even with Arabs. Modarres Razavi (2008) cited from Ghazi Noorollah Shooshtari[24] that "Tusi was the main motivator for the decision of Hulagu to put an end to the Abbasid's Dynasty as Tusi knew the fanaticism of the Abbasid ruler in advocating Sunni religion" (translated by author, p. 17). Tusi also was aware of the torture and persecution of Shia Muslims by the ruler.

In the end, Tusi didn't stop Hulagu from reducing the dominance of Abbasid because later Abbasid rulers had acted unfair towards Persians, were overly protective of the assets of Baghdad Library including texts and resources, Arabized the efforts of non-Arab scholars who worked inside and outside of Baghdad, and limited access to the Library for research activities lead by non-Arabs. The aformentioned conditions had

[24] Ghazi Noorollah Shooshtari was born in AD 1549 in a Shia family in Shushtar, current Iran. He studied in Mashhad, Khurasan. He was a historian, judge, and politician who spent much of his life in India, and played a great role in introducing Shiism to Indians. (www.noormicrofilmmedia.com)

started after the rule of Mansur and Mamun of Abbasid, who were more free-minded personalities and had respected Persians for their input into the governing the region. Mamun had chosen Imam Reza, a Shia Imam, as his first minister, and had moved the capital of his government to Sanabad (Mashhad)[25] in Khurasan.

Being aware of Hulagu's most likely decision to attack and destroy Baghdad, Tusi saw it was better to stay close to Hulagu in order to rescue as many scholars, books, and resources as possible. This was a wise decision by Tusi, and he tried his best, as history confirms, to protect both people and the library of Baghdad.

The Years of Peace and Results

After suppressing the government of Baghdad, Hulagu, satisfied of his decision and actions, gave a modicum of authority to Tusi, and asked him to plan the building infrastructure of his capital, Maragheh. Tusi's encouragement caused Hulagu to keep Tusi in Iran, otherwise Hulagu was supposed to send Tusi to help the Khan in China with a project there. Tusi "carried out the work of a minister for Hulagu without any embezzlement. He dominated the mind of Hulagu to such an extent that the latter would never ride a horse or would go on a journey without Tusi's approval" (Jafarian, translated by author, p. 14). Modarres Razavi (2007) stated that "some historians believe that Tusi, for years, was thinking about building an observatory, and was waiting for the right time for it; so, he mentioned his idea to Hulagu" (translated by author, p. 40). Hulagu who was a superstitious person asked for the benefit of an observatory. Tusi knew that Hulagu would not believe anything until he saw something as a proof. Therefore he asked permission to show the benefit of having an observatory through an example. Modarres Razavi (2007) brings Tusi's lesson for Hulagu from Ibn Shaker, Arab historian:

[25] Mashhad is the current name of the city that is the capital of the Khurasan province of Iran. Mashhad means 'the location of sacrificing your life in the way of God' as Imam Reza was assassinated by Sunnis and is buried in there. (Author)

> *Tusi asked soldiers to throw a big empty copper bowl downhill from the neighbouring mountain without warning people. The sound of the bowl rolling down scared the people; some passed out, and many felt terrified. But Tusi and Hulagu who knew about it were comfortable. Then Tusi mentioned to Hulagu that whoever is aware of astronomy [by the use of observation] will not be afraid of any natural phenomena or disasters that makes unlearned people frightened.*
>
> *(Modarres Razavi, translated by author, p. 41)*

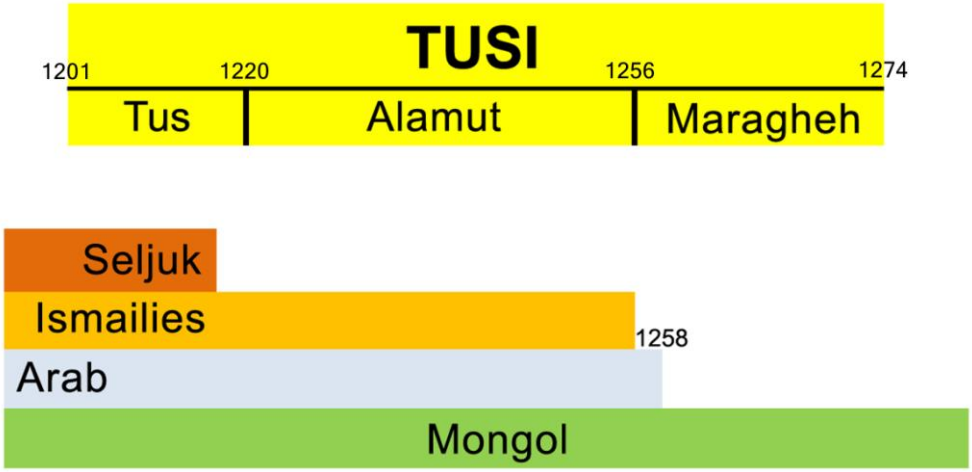
In AD 1259 Tusi received permission for establishing an observatory in Maragheh. Obtaining the permission from Hulagu was in itself a difficult task that Tusi intelligently achieved. Modarres Razavi (2008) stated that Tusi chose a tall hill on the north-western side of Maragheh, and requested an enormous amount of money that was enough for the Observatory expenses (translated by author, p. 40). Although Tusi had planned to spend thirty years for observation in several phases, Hulagu was old by that time, and was in rush to see the result of the observations (Modarres Razavi, 2008, translated by author). He died before the observation process was completed. It took about three years for the building to be constructed and the equipment to be assembled. While the rulers were occupied with the glory and the title of the Observatory in their capital, and bureaucrats arising from Hulagu were working on possible financial opportunities around the Observatory's results, Tusi was mostly looking forward to seeing the promising future that the Observatory was bringing to the region. He supported the growing opportunities for the researchers and students through enrichment activities concerning the project and by establishing a high-end school attached to the Observatory. Both Maragheh Observatory and Maragheh School were of the highest quality of their kind in both scientific and educational aspects. Chapter 6 of this thesis is devoted to the effect of Maragheh Observatory and Maragheh School.

Tusi's last seventeen years was the peak of his involvement and influence in scientific activity and research. He led a great group of mathematicians, engineers, and scholars in a top administrative and technical level. His two sons were also part of the scientific team. Once he observed that his group in the Observatory was capable of running projects, Tusi, occupied his remaining years with travel around the cities, identifying talented candidates. He not only wanted the education to have a strong centre, but also wanted to make it a permanent program for the centre, other cities, and people.

Tusi died on one of these trips to Samarah, a town in current Iraq, in 28 June 1274, when he was about seventy-four years old. In addition to books on mathematics, there are numbers of well-known philosophy books and treatieses, a few poems and some musical works that he left behind.

Following this general review of the history of the life of Tusi, it is safe to say that his life started in a difficult time, and he lived in unusual conditions for most of his life. Tusi experienced life-threatening incidents. He experienced wars, massacre, demolishing of agricultural assets, irreparable damage of human resources, gross ignorance of religions and beliefs, the destroying of culture and countless other atrocities. It is natural to expect a dedicated person like Tusi to stand for saving the nation in one of the possible ways. Education was Tusi's expertise, and he decided to help the nation by reforming several aspects of mathematics education in the region. In the following chapters, the goal is to identify, and to approach areas in which Tusi initiated an educational reform or improvement. His leadership in the mathematics education during Islamic Era will be also examined.

Figure 6. Tusi's Life-Time



Note: Tus, Alamut, and Maragheh are cities Tusi had lived in three intervals of his life. The interval called Tus is, in fact, the life he has had at both Tus and Neishapur until he was 19 years old.

Chapter 4.

Tusi Developed a Series of Updated Mathematics Books

Studying the history of the education of a country informs researchers of problems, obstacles, restrictions, and shortcomings in the way of reforming education in that country. Religious restrictions, gender equity, cultural background, and lack of interest in education are some of the possible problems. “Studying the history also identifies the ups and downs of the country and the challenges it faced during reform” (Aghazadeh[26], 2010, translated by author). Such challenges could include financial situations, nationwide war, domestic power struggle, and others. Nothing in education comes without paying a price for it. Mathematics is not an exception. Years of sacrifice and dedication are needed for mathematicians to write, edit, and publish adequate mathematics books that might be used for preparing mathematics textbooks, and consequently curriculums of mathematics. Tusi is one of the icons of the reformism in writing mathematics books with consistent terminology, and for the purpose of being used as teaching material. He started writing books after years of reading works from different mathematicians. He wrote books and treatises, and wrote tahrirs[27] on the work other mathematicians. Tusi encouraged mathematicians and mathematics students of his time to continue studying mathematics when the community of mathematitians in Persia was distracted. He found it necessary to generate a series of mathematics books

[26] Dr. Ahmad Aghazadeh is a faculty member of Allameh Tabatabayee University. He has five books on the philosophy of teaching and learning with a focus on the history of education and modern approaches of European countries toward education and schooling, four translations, and five education and research projects.in training teachers. (http://www.motarjem-mm.blogfa.com/post-94.aspx)

[27] Tahrir, in short, is a redaction, but its definition is discussed in this chapter. Tahrir is used as noun and verb in this thesis. (Author)

that were easier to read and understand, and inclusive of the opinions of different mathematicians.

A New Beginning for the Mathematics Community

Tusi had noticed that designing a public education, and training mathematics educators would be meaningless without providing this community with proper mathematics books. Therefore, he started compiling the mathematics knowledge from Greece, Egypt, Alexandria, Tunisia, and India, and continued this research for over twenty years when he was in the Alamut Fortress. He studied and reviewed books and treatises, noticing the advantages and disadvantages of the styles of written books, from his point of view, perhaps as a mathematician and mathematics teacher. He identified complications and lack of proper communication of the content to the readers. He also noticed the shortcomings in presentations and visualizations of the content of the topics. Instead of ignoring the problems and using the existing books and treatises, the practice that some mathematicians had followed before Tusi, he generated edited books out of them. Topics became more meaningful and connected, and learning outcomes were more likely to be achieved for readers. The details of how Tusi came up with these improvements are discussed in the following sections.

Overcoming Lack of Proper Connection Between Existing Books

Tusi clearly expressed, in a couple of the introductions to his works that he was struggling to understand mathematical content because of the lack of proper sequencing in the work of former mathematicians. His concern seems reasonable because:

- It is reasonable to assume that not all works of ancient mathematicians were available for Tusi to study. As a result, he was unable to experience the proper connection of the total available knowledge of mathematics.
- Even if connected mathematics content existed in the books of a region, there was a disconnection between it and the works of mathematicians of other regions. Tusi was experiencing this disconnection as he had collected books from almost everywhere.

- Although the great collection of Greek mathematics was available to the mathematics community, many identified areas of mathematics, which needed to be studied, were untouched. At the time of Tusi, centuries after Greek Era, many of these topics were required to be researched and discussed deeply.
- The works of Islamic mathematicians, in the four-hundred years of the Islamic Golden Era before Tusi, were added to the volume of the total knowledge of mathematics, but had limited consistent and similar terminology in comparison with the works of Greek mathematicians. Even schools of the time of Tusi, such as Nezamieh and Rashidieh, had a difficult time finding connected textbooks. Only a few mathematics topics were widely taught from pure mathematics books by individual teachers.

Tusi was challenged when he started compiling the books, as topics of different disciplines were missing, or lots of incomplete topics required different depths of research.

Gaps Between the Topics

Many books were translated into Arabic from Greek, Middle Persian, and Middle Indi just as a bulk of knowledge. The connections between these books and other scholarly books or treatises were either vague or incomprehensive. As a result, for many topics there was a disconnection between the topics and disciplines. Making mathematically adequate connections between available books and treatises required one to have a great deal of knowledge and understanding of mathematics, plus a continuous commitment to filling the gaps by writing complementary books and treatises. Tusi identified the following problems for readers within existing books.

Similar Original Books and Similar Treatises

Often more than one original book was available for a particular topic. Therefore, parallel discussions were written on similar topics by different mathematicians. Choosing between these original discussions as a reference was sometimes troublesome, and consequently, Tusi had to consider several factors in order to pick the best between them. The seniority of the writer was an important factor, as some mathematicians were so influential that the mathematics community expected their book to be used or to be quoted. Even if a better-fit work for a series of discussions was available, it was necessary to come up with a strong argument to substitute the better-fit work of a less

famous mathematician for the work of a well-known mathematician. Clarity and terminology of the written books was also a factor in making the decision to choose book or a book of a series in books. The more a book was similar to Tusi's style of writing and method of reasoning, the more likely that the book would be chosen to consider within the series. Tusi was in favour of works that were easier to read and understand; so, between two similar discussions, he chose the one that was more explanatory and had more quality figures. The fact that Tusi tried to soften the literacy involved in the reading and comprehension of mathematics topics was an important consideration for his decisions between books. His choices of vocabulary were simple and consistent, between two similar books; the one with simpler sentence structure was more appropriate for teachers and students.

Similar Translations of One Book

In addition to the problem of choosing between parallel arguments by former mathematicians that was discussed above, Tusi was sometimes faced with the dilemma of choosing between similar translations of one book by different translators. In this case he usually studied the available versions, and chose the one that was more adequately translated. Tusi's mathematics knowledge was a key tool in identifying adequate translations. Again, the reasoning, terminology, and writing style by which the translator shared the knowledge were important factors in choosing the best-fit translation for filling the gap in series. If no one's translation was technically adequate, or if some editing of the terminology or reasoning were required, Tusi wrote a tahrir on one of the better translations to make it fit with the rest of the books of a series. An example of a translation that Tusi had to write a tahrir on is his Tahrir on Archimedes' "On the Sphere and Cylinder" when he says in the introduction:

> *I was looking to learn some of the problems with the sphere and cylinder of Archimedes for a long time, as I needed it to understand the noble content of geometry, until I got a copy of the famous book that Sabet-Ibn-Qurra had translated. Some content of that was missing as the result of the lack of complete understanding of the translator, and some was incomplete due to the author's written material, which made it untrue in many parts. I understood only the first article of it. Later I noticed the translator had omitted some of Archimedes' writings that were necessary for understanding the book. I was surprised, and felt thirstier for understanding the book.*

> *After a while I found a notebook that had a copy of the explanation of Hypsicles on this book that had explained most of the problems correctly. Another copy was the one that Essac-ibn-Hanin had translated and altogether I learned what I wanted to know, and decided to tahrir the book.* *(Tusi, translated by author, p. 2)*

In all cases mentioned above, Tusi's knowledge of Greek, Middle Persian, Middle Indi, and Arabic, and his expertise in mathematics was a major advantage in compiling the books.

Challenging Topics

Some topics in mathematics were new and/or somehow difficult to understand. It usually took years for mathematicians to show interest in working on these topics. Lack of background knowledge of mathematicians, absence of application of mathematics topics, and lack of interest in general were some of the reasons why these topics stayed in the margins of the main stream of the mathematics discussions. Whenever Tusi recognized the necessity of these topics for the completion of a set of related books, either he arranged a team to work on them, or worked on them by him. An example of these attempts is Tusi's work on trigonometry as a discipline separate from astronomy. This had come to Tusi when his team in Maragheh was working on observation, and he found that many questions were brought to him that trigonometry held the key to answer. Astronomy was not necessarily the only target, and the result could be applied in other branches of mathematics. He had realized that trigonometry should be an individual discipline from which all mathematics topics could benefit. According to Pak (2010), and brought from Kendy "Tusi in his famous trigonometry book for the first time examined trigonometry as a pure branch of mathematics" (translated by author, p. 14). In this book, *Kitab Shakl al-Qitaa* or *Form*, Tusi, first, summarizes the ideas of former

mathematicians, from Abulwafa[28] untill Biruni,[29] regarding trigonometry. Mosaheb[30] believes “this book is the first textbook in trigonometry that is independent from astronomy” (Pak, 2010). It helped “Saccheri (AD 1667-1733), the Italian mathematician, who was famous in using Khayyam’s and Tusi’s works on Euclid” (Pak, 2010, translated by author).

Sometimes in the middle of writing books, Tusi came to difficult supporting topics that he preferred to cover only as much as was required to continue with the main topic. He usually stated that he was not completely satisfied with the result of his written materials about the supporting topic, and that more discussion was possible, and recommended. A sample of such a statement that Tusi made is ‘this topic should be reviewed by future mathematicians but at this time it is the best that we can do for it, or we need.’

Out of Norm Topics

Tusi was not afraid of attacking that was seen as out of norm disciplines such as non-Euclidean geometry. In his *Al-risala al-shafiya'an al-shakk fi'l-khutut al-mutawaziya* (*Discussion Which Removes Doubt about Parallel Lines*), written in 1250, Tusi wrote detailed critiques of the parallel postulate on Khayyam's attempted proof a century

[28] Abulwafa Buzjani (AD 940 – 998) was a Persian mathematician and astronomer who worked in Baghdad. Two of his survived books concern applications of mathematics. Negative numbers are known to be used first in Abulwafa’s calculations. His book on *Almagest* of Ptolemy perished, and he is known to have created a calendar. He made important innovations in spherical trigonometry. He is also credited for compiling tables of Sines and Tangents at 15' intervals. He introduced the secant and cosecant and studied the interrelations between the six trigonometric lines associated with an arc. (http://www.persian-man.ir/senior/mathematician)

[29] Abu Rayhan Biruni (AD 973 – 1048), known as Alberonius in Latin was an Iranian Muslim scholar and polymath. He was one of the greatest scholars of the medieval Islamic era. His *Chronology* was about geology and geography. His book, *Shaddows* was about the tangents and shadows. Biruni was well versed in mathematics, astronomy, physical and natural sciences. (http://www.mathhouse.org)

[30] See footnote 46.

earlier. He attempted to derive a proof by contradiction of the parallel postulate (Katz, 1993). He was also one of the first to consider the cases of elliptical geometry and hyperbolic geometry for which he discussed, clarified, and gave a more comprehensive treatment of parallel lines on them. For example, "Khayyam's postulate had excluded the case of [parallel lines in] the hyperbolic geometry whereas Tusi's postulate ruled [it] out [in] both the hyperbolic and elliptic geometries" (Youschkevitch & Rosenfeld, 1996).

New Topics and Applications

Tusi did not hesitate to introduce new topics, or to research the development of one. It is fair to say that his focus during his leadership in Maragheh Observatory was more on the application of the mathematical topics and engineering developed from them. When European coeval institutes, during the 12th and 13th centuries, considered all of their abilities for carrying the heritage of past generations, and translation was the main activity of them, “Tusi and his team generated as many as texts and treatises they could to improve on the works of former mathematicians or to introduce new topics and applications of them” (Pak, 2010, translated by author). The importance of the Maragheh scientific research institute, which was a modern scientific research organization, was in its innovation and new scientific activities. Investigating new topics or searching for new applications of them was the main activity in Maragheh University where Tusi was leading the team of engineers and mathematicians. Tusi was responsible for writing the results of innovations.

Trigonometry, as it was discussed, was a discipline attached to astronomy, and previously was never discussed as an independent discipline from astronomy. Tusi introduced trigonometry as a separate subject by writing his famous trigonometry book *Kitab Shakl al-Qitaa* or *Form*. This subject then became, and still is, one of the courses of the curriculum of mathematics that students in secondary and post secondary mathematics programs take. Cara de Faw explains that Tusi’s book of trigonometry was translated into Latin, French and English and remained a major reference for the Europeans for many centuries (Haouaria, 2003).

Omitting Unnecessary Topics

Considering the interest of current mathematicians and outcomes of a particular series of books Tusi omitted unwanted topics. Some sections of the books were deleted by Tusi, usually those presented by pre-Euclidean Greek mathematicians that were outdated in the 12th century. This was because the application of mathematics as a whole was significantly improved during the Islamic Era. In fact, the pathway of the industry and economy, the interest of newer mathematicians, and the demands of society caused these primitive mathematics topics to be unwanted. Consequently, Tusi allowed readers to focus on topics that were practically alive and were in the mainstream of books and industry. An example of this is Tusi's reluctance in putting *optics* inside the series of the books that he considered for teaching students of secondary level because in his opinion it was not a must for secondary grade students. This action of deletion or giving priority to newer topics or applications still occurs when any new mathematics textbook is written for schooling. For instance, in recent decades finding the square root of a number without a calculator, which used to be part of the curriculum, has been eliminated. The elimination has happened perhaps because studying the applications of square root are now more demanding than the procedural skills of finding the square root using an algorithm. Now, technology helps with finding the square root, is faster, and allows teachers to conduct more problem solving in which calculating square root is required. In general, as mathematics was in its developmental era, the number of omissions by Tusi was limited to those that did not fit in a series of books.

What is Tahrir?

Tahrir is a text, essentially the working version of mathematical masterpieces, which has a classical mathematical approach. It is a difficult term to translate exactly, but it can be approximately rendered by 'redaction.' The term is used as both noun and verb in several translations. "In a sense, the concept of the tahrir itself, as developed and applied by Tusi was a somewhat idiosyncratic though extremely useful type of a new scientific composition" (Saliba, 2009) Tahrir is somehow similar to writing a unit plan for a course that is to be taught from a textbook of another language. The planning activity for the instructor would be to study the textbook, and to re-write a new version of it in a

way that is understandable for his readers, including instructors and students. The planning would also rely on other available materials, related to the course, which could be used in complement. The planner would choose the vocabulary which best convey the intended scientific meaning of the original text. If it were detected that there exists improper vocabulary in the book or in its translation, the planner would substitute the vocabulary of his choice. The planner next would "try to improve upon them by avoiding, for example, repetitions, which [are] in some instances quite frequent, and would also update them by using contemporaneous concepts and techniques rather than holding on to [older] concepts and methods"(Saliba, 2009). Whenever impossible to manipulate the statements to make them readable and understandable, a tahrir offers a totally different and independent statement to ensure the proper ideas are communicated to the readers. Tahrir should not be mistaken with commentary. Tahrir of a book is minimized to what the book discusses. In writing commentary, however, writers feel no obligation to limit their argument to those topics of the original book. The main advantage of tahrir for the writers is the freedom it brings to approach the topics by re-writing the sections as they wish while keeping the main structure and argument offered by the original writer.

The Quality of Books

Tusi's style of tahrir and translation was unique. Some details of his approaches of writing educational passages are discussed in Chapter 5 of this thesis, but here some important points of his writing style are reviewed.

Tusi's written materials either as tahrir on others' work or as a book of his own, were easier to understand for the students and teachers of his time than the abstract Greek mathematics materials. A supporting note to this argument is presented as follows:

> *Why virtually all the Persians renditions of the Euclidean corpus looked to the Tahrir of al-Tusi as their primary source rather than the primary transmission that had come down from the Greek by way of the early Arabic translations? Perhaps it was because the translators recognized in al-Tusi's work as attempt to give vitality and renewed meaning to the translation movement of ninth century Baghdad .... Tusi's re-editions which often included insightful and original commentary provided the means by which generations of students of late medieval Islam could*

assimilate the Greek scientific tradition either with or without a teacher.
(De Young, 2008)

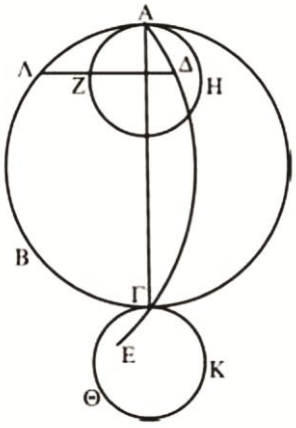
Tusi, by his tahrirs smoothed rough complicated topics. For example, when proofs of theorems were difficult for students and teachers, he included examples, illustrations, or numerical verifications to the original book. When discussions around the topics were greatly narrowed down to special cases or to the special interest of the original author, Tusi eliminated these extra parts and restricted the topic to more general cases, and considered special cases as enrichments. This mostly happened when spacial cases were not important prerequisites for understanding the core argument and content of the related topics. Some Tusi's writing strategies, shared by Saliba (1994), are listed below:

- The development of a series of explanations whenever he recognized extra notes would provide readers with better understanding of the argument. Comparing Tusi's tahrirs with the original written works illustrates the significant difference of the number of small or lengthy notes he added into his tahrirs of books. There is about 180 notes in the tahrir of Ptolemy's *Almagest* that Tusi used to clarify and explain the points of Ptolemy.
- Employing a more practical approach when the sections were complicated. This tactic was basically the result of Tusi's writing style that he had tried to remain consistent with for every book.
- Considering figures or drawings that could help readers to digest the problem more easily. Tusi demonstrated that he didn't hesitate to repeat a figure to show step by step of a geometrical proof, whereas Greek mathematicians used a limited number of figures. Tusi's figures of non-Euclidean geometry are well designed, larger, and completely labelled, so readers have a better visualization of the problems. For instance, Tusi states in his tahrir on Ptolemy's Almagest that "I have used various colors to draw the lines and the numbers of the diagrams, which I have added to the original text, in order that they could be distinguished at a glance without any excessive concern."
(p.146)

Tusi also improved the quality of the figures offered by former mathematicians whenever he wrote a tahrir on their books. For example, in Autolycus' *On the Moving Sphere*, for the following figure, Figure 7, none of the steps of the construction of the circles are either postulated or established through propositions. How to draw a great circle through two given points is not discussed. Also, how to lay out one arc of a great circle equal to another is not mentioned (Sidoli, 2004). In the same picture in Tusi's *Tahrir Korat-ul-*

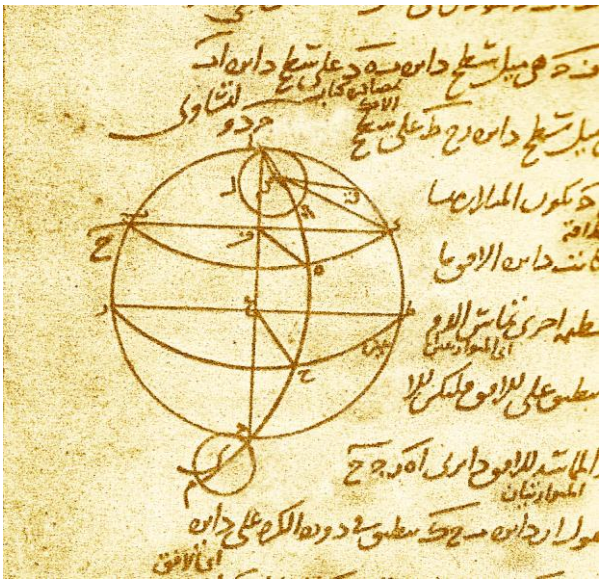
Moteharreke on this book the diagram is more effective and has additional information around it as clarifications. The figure in Tusi's tahrir, Figure 8 bellow, is provided for comparison.

Figure 7. Moving Spheres in Autolycus' Book



Source of the picture is Sidoli's article in *Historia Mathematica.*

Figure 8. Moving Spheres in Tusi's Tahrir of Autolycus' Book

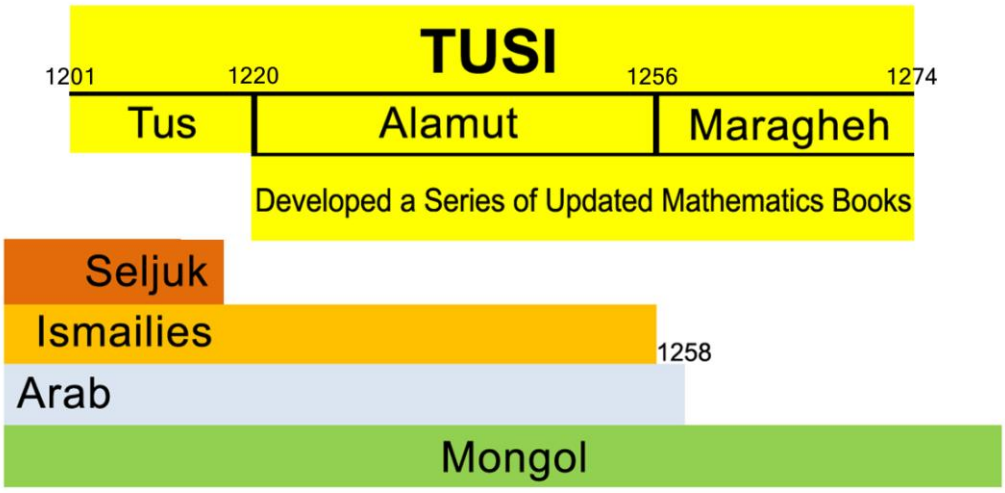


Source: Tusi's *Mutiwassitat*.

Tusi's contribution to the development of mathematics books enabled mathematicians and mathematics instructors of his time to have access to updated

mathematics books of their time. These books became the main sources of information for teaching purposes. Tusi's revolutionary character was the driving force of this lengthy and tedious project. Consequently, for years after his work, Islamic and European education systems benefited from the books he had prepared. The following chapter discusses Tusi's method of writing tahrirs and a compilation of books on Astronomy. Astronomy studies in Europe were influenced by the studies and books of the Islamic Era, particularly those by Tusi.

Figure 9. Development of Mathematics Books within the Life -Time of Tusi



Chapter 5.

Tusi Connected Euclid's *Elements* and Ptolemy's *Almagest* by Bridging the Gap between Them

Astronomy was one of the most important and practical areas of ancient mathematics knowledge from the longest time before the Greek Era to the Islamic Era. A handful of books and treatises were written by Greek astronomers and mathematicians concerning astronomy. The collection of these books had some technical but many educational flaws. An example of technical flaw was Ptolemy's astronomical configuration. The absence of consistent terminology within each book and between books was an example of an educational flaw. Tusi re-wrote the entire collection, and edited it as much as he could. Tusi thus generated an ordered a series of astronomy books through which readers and educators were led from the beginning level of the mathematics of astronomy to the *Almagest* of Ptolemy, the highest level at the time. Of course the thought of compiling this collection is not from Tusi, but this collection, essentially, is a more completed version of what in Alexandrian mathematics is famous as *Small Collection of Astronomy* (Massoumi, 1996, translated by author). *Small Collection of Astronomy* was only a collection of existing books of astronomy that were related to each other. The collection covered lots of astronomical discussions leading the readers to the *Almagest* of Ptolemy which in that time was the most difficult book to study. As studying the astronomy was extremely difficult if starting from studying the *Almagest* of Ptolemy, a proper educational approach to astronomy was to start from Euclid's *Elements*, and to get skills required for understanding the *Almagest* by studying the books of the collection. Tusi decided to connect the books of *Small Collection of Astronomy* together with a consistent terminology where there was a sequential approach to the collection from the *Elements*, considered as the first book to study for astronomy, to the *Almagest*, as the last available book in astronomy. For this reason Tusi had to work on the technical and instructional gaps between Euclid's *Elements* and

Ptolemy's *Almagest*, as he wanted the collection to be used as a resource by interested candidates. This project led Tusi onto a road that took years for him to take. The project of making the series of astronomy books was somehow similar to what Tusi had started with the series of mathematics books in all disciplines that was discussed in Chapter 4. He decided to *bridge* the gap between the two books, *Elements* and *Almagest*, so there was a reasonable sequence starting from *Elements* and finishing with *Almages* covering all the mathematical skills one should know to understand astronomy. Figure 10 shows approximate period of time Tusi worked on this project. He has spent significant period of his scholarly life on the completion of this task. The dates of the completion of the books and tahrirs of this collection demonstrate that they were mostly done during his residency in Alamut, but some reviews were performed years after the books were written. Tusi wrote tahrir on all of them, and the collection became the teaching source and material for a full astronomy course.

Tahrir on the *Elements* of Euclid

A magisterial book on which Tusi wrote a tahrir on is Euclid's *Elements*. Tusi received and studied the Ptolemy's *Almagest* before he found *Elements*. In his "Tahrir of *Elements*, called *Osul Uglidis* among Muslim scholars, he mentions 'I wrote this book after writing *Almagest*"' (Modarres Razavi, 2008, translated by author). Perhaps this was one of the reasons he had found the *Almagest* difficult to understand. Later Tusi wrote a *Sharh*[31] to his own tahrir of *Elements*, called *Albelaagh* to clarify some of its topics with more detailed and explanatory approach.

Perhaps one of the most important books to be translated in the "Translation Era" is *Elements*. The first known translation is done by Hajaj-Ibn-Yusof who wrote two translations of it at two different times for two rulers of Abbasid Dynasty. There exist five different translations and editions of *Elements* in The Library of The Parliament of Iran.

[31] Sharh is to rewrite or retell a book or other material from one's point of view (Author).

Elements was also summarized several times by different mathematicians, the number of which, including those that are available and those that are lost, is about fifty (Sezgin, 1974). Perhaps the most popular tahrir to *Elements* was written by Tusi. His work was the first mathematical work to be printed in Europe in Arabic, and it was printed in Rome by the Medici Press in 1594 (Mohd Zain, 2005). His tahrir is in the rare books collection at The Library of the International Institute of Islamic Thought and Civilization.

Although other well-known researchers and translators such as Mohieddin Maghrebi and Asireddin Abhari provided tahrirs for *Elements*, Tusi's tahrir surpassed all translations and editions (Sezgin, 1974). Perhaps it was because of its vast use in Islamic scientific institutes, and also because of its way of tahrir and terminology. Tusi wrote in his introduction to *Osul Uglidis.*

> *Because I finished the tahrir of Almagest which is an innovation from Ptolemy, I found it appropriate to tahrir the foundations of geometry and arithmetic book that is ascribed to Euclid in a summary that doesn't upset understanding; and impelling the minds on his [Euclid's] tasks with a rope that doesn't bother; and adding whatever beneficial that I have learned from other resources from other scholars of this science; and I found it proper to credit those additions to the place that I have found them from the two translations of original book of Euclid by Sabet and Hajjaj, in a way that original figures are in red and numbers in black, and additions be vice versa.* *(Tusi, translated by author p. 2)*

The History of Science Group of University of Calgary cited (1998) in its web site from De Young that

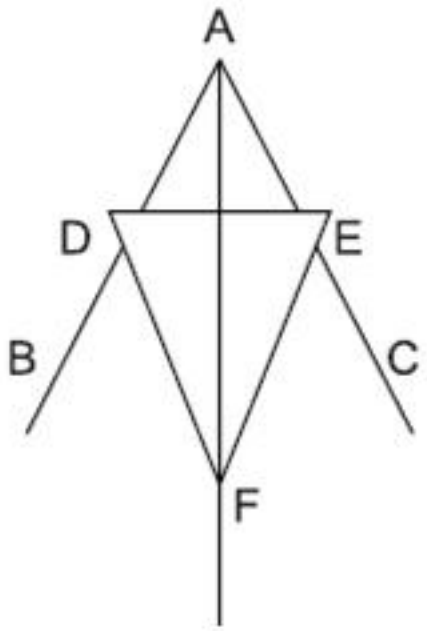
> *Tahrir of* Elements *consists of the summary of Elements propositions within more than two hundreds notes in explaining the content ... ; and about 180 propositions are arranged in this dissertation. These dissertations are written by Tusi.... almost half of the notes are second proofs for propositions, in another important group of notes different cases are added to propositions, and another group of notes are information about the differences between the translation of Hajjaj and re-written work of Sabet from the translation of Hanin, especially from the point of view of order.*
>
> *(The Applied History Research Group)*

An example of the difference between Tusi's tahrir on *Elements* as compared to the work of Euclid is explained by Babayeva and Mezhlumbekova (2006) in page 49 to 51

when they compare Proposition 3 of Book One of Euclid to Tusi's tahrir on it. In bisecting rectilinear angles, Euclid shows the following steps:

Let the angle BAC be given in the Figure 11

Figure 11. Proposition 3, Book I



Note: This figure is taken from the source as it was published. Points D and E should be on the lines AB and AC.

> We take on AB an arbitrary point D, take away straight line AE equaled AD from AC, connect D with E, construct on DE equilateral triangle DEF (Proposition 1). Connect A with F.

Further it is confirmed that the angle BAC is bisected halves by the straight line AF. It is proved on the basis of equality of the triangles ADF and AEF (Babayeva & Mezhlumbekova, 2006).

Tusi's tahrir on the same proposition is reproduced below:

> We take on AB a point D, take away straight line AE equaled AD from the point A. Connect D with E, construct

on DE equilateral triangle DEF. The straight line AF bisects the angle.

Further the proof is similar to one cited by Euclid ... I note that the point F must not necessarily lie between AB and AC. The point F can lie on one of these straight lines or exterior to angle.' Tusi considers the last two cases and gives the proof of impossibility of each of them.

(Babayeva and Mezhlumbekova, 2006)

Another example is in Definition 15 where Euclid states the definition of *circle* as follows:

The circle is a plane figure contained inside of one line on which all straight lines falling from one point inside of figure are equal between each other.

Tusi, in a corresponding place adds, "This point is a center of *circumference*."

It seems that Euclid had no term for "circumference" and used "circle" or "surround" instead, in several places, for example in Book III. The same situation exists for the term "*arc*" that Tusi uses, but Euclid uses "sector surround" instead.

De Young (2005) explains that we find additional opinions on the translation quality of Arabic translations of the original Greek works of Euclid:

Arabic discussions of Euclid's work do not stand in complete intellectual isolation. They are part of a historical transmission of knowledge that crosses both linguistic and social boundaries as it moves forward. The Arabic transmission cannot be fully appreciated unless seen the backdrop of the Greek mathematical tradition where Euclid's work first took shape and where the earliest elaborations on the treatise were produced. The initial transfer to the Arabic–Islamic world was conceptually, and almost certainly formally, a direct reflection of the state of Euclidean scholarship in Byzantium and the Near East. In the course of time, we are not surprised to find the two traditions slowly diverging as Euclidean studies become naturalized into Islamic culture.

(Encyclopaedia of Yahuda, p.135)

This trend shows itself more dominant in Arabic secondary transmission texts such as tahrir that Tusi wrote on *Elements*. Many believe that *Osul Uglidis* was the peak of the quality of communication of ideas that a mathematician could have offered in that

time. “There were nearly two hundred commentary notes in the Tahrir, of which approximately ninety are alternate demonstrations. These alternative demonstrations and constructions were added by Tusi from several sources” (De Young, 2005). One of the important and lengthy commentary notes of Tusi is offered right before Proposition 29, the first proposition that cannot be proved without using Proposition 5, from the first book of “Elements.” Euclid had proved Proposition 29 without proving Proposition 5, but Tusi proved Proposition 5 here before working on Proposition 29. Analysis of Tusi’s notes in tahrir of “Elements” illustrates the change in the conceptual and logical base of geometry as a scientific effort of Tusi (Babayeva *and Mezhlumbekova*, 2006).

The technique utilized by Tusi makes it difficult for educators to experience the difference between Tusi’s interpretation of the content of original texts and the approach of the texts. It requires one to compare the original Greek text with Tusi’s version in order to understand and appreciate the “radical reformulations” that Tusi brought to those texts. This is a necessary task in order to understand the very technical and advanced nature of the developments that Tusi instigated (Saliba, 2009).

Tahrir on the *Almagest* of Ptolemy

The text that best illustrates Tusi’s method of tahrir is his tahrir on Ptolemy’s *Almagest* (Saliba, 1994). *Almagest* was a known book to Persians and Arab astronomers and mathematicians. Several translations and editions were written on this astronomical book before Tusi wrote his tahrir on it (Sezgin, 1974). In many of the editions of Ptolemy’s *Almagest* the author has been criticized for his calculations as well as his major assumptions for the model of the universe. Tusi also criticized Ptolemy’s work, but followed the writing method and pattern of former translators of *Almagest* in his tahrir on *Almagest*. He (1247) wrote in his introduction of the book that “I wrote this book in 13 articles, 141 chapters, and 196 figures, similar with the translation” (Tusi, p.1, translated by author) which was Ishagh-Ibn-Hanin’s translation, edited by Sabet Ibn Gharreh. Tusi (1247) also stated that the reason for writing the tahrir on *Almagest* was the request of friends and the shortage of some necessary points and topics in the available translated versions of *Almagest* (Tusi, p.2, Translated by author). Tusi then added that he had extensively used colors in figures because this book was considered

as a main reference for scholars and mathematicians who gathered around it in their discussions, and referred to specific points in its problems (Saliba, 1994). Tusi wanted to prepare a work that:

- omitted some of the excess theoretical discussions
- kept the chapters in order
- provided a proper arrangement between the articles and figures
- avoided complicated arguments
- stayed short and summarized
- presented the results of the researches and opinions of recent astronomers and mathematicians

"There he replaced the chord calculations that were used by Ptolemy in the earlier chapters of the *Almagest* by their more current trigonometric equivalents that Tusi knew were widely accepted and used in his time" (Saliba, 2009). Tusi's work on plane trigonometry is considered to be the first systematic work that introduced plane trigonometry as an independent discipline. He discussed oblique-angled triangles after the modern manner, instead of reducing the solution of them to that of right-angled triangles (Tropfke, 1921).

Parts added to *Almagest* bring out the services rendered to mathematics in the Middle Ages by Arab or more precisely Arabic-speaking, geometers (Tropfke, 1921). Tusi has added several explanations and editions to the book. An indicator of the sections Tusi added to his tahrir is, Tusi used the term "I say" at the beginning of the sections that he added, and finished with the term "to be replaced." Perhaps the most important additions of Tusi to the translations and editions of "Almagest" are the explanations of the new calendars. In the introduction of the Section 3 of the Article Number 6 he explains and introduces the year length, and the name of the months in Gabti, Yazdgerdi, Hijri, and Hebrew languages.

The *Bridge* was a Must

After gaining the knowledge shared in *Elements* and *Almagest*, Tusi became curious in finding, learning, and writing the mathematics books that could fit between the

two magisterial books. He then started collecting any book or treaty he could, and studied them to answer his questions about the subject knowledge between the two books.

As this process was an open ended road, it was hard for Tusi to satisfy himself with the so-called *Small Collection of Astronomy*. Apparently Tusi had three options:

- Considering a translated book or treatise from Greek mathematicians as one of the books of the *bridge.* This was proper if the book by itself was fit into existing materials.
- Writing a tahrir on a translated book or treatise, and editing or updating the content of it to be fitted to the sequence of the *bridge.*
- Writing a book or treatise from scratch to fill a gap. This was needed if either nobody had attempted to write the required work before, or if it was not available at the time that it was needed.

As will be discussed below, Tusi had to perform all three above items to accomplish the generation and compilation of the set of books that is now known as the *Great Collection of Astronomy*.

Filling the Gaps with Existing Resources

Tusi as the wise-man of the Ismaili ruler, for the time he lived in Alamut, used his authority to collect some Arabic translations of major books including the items of the *Small Collection of Astronomy*. His relationship with the Ismaili ruler, who was willing to educate his followers and companions, was a key element in finding and collecting original books and translations. All of the existing books of the *Small Collection of Astronomy* were needed to create the sequence of the *bridge* between *Elements* and *Almagest*, but Tusi had to write tahrir on them to improve the alignment with the terminology of his tahrirs on *Elements* and *Almagest*. It is also reasonable to believe that the books of *Small Collection of Astronomy* didn't reach Tusi at the same time or in order. So, either Tusi wrote tahrir on them any time he received any of them, or he waited to collect all of them and wrote tahrir on them all at once. The completion dates of composing the books of the *bridge* indicate that the latter possibility is what likely happened. Some of the books or their translations to Arabic were far out of the line of

the terminology of the *bridge.* In these cases, Tusi had to utilize more of his editorial procedures to make them fit into the line of the *bridge*. An example of this type of the books is Euclid's *Phenomena*. Tusi, in his introduction to the tahrir of this book says:

> *From this book, I had only an erroneous copy that had lots of alterations, such that it was not possible to understand the content unless one spent an inordinate amount of the time. After a while I found an explanation to the book by Neirizi which also was full of obvious mistakes. After serious consideration finally I wrote a tahrir on what I had found from the two copies. If one finds my tahrir dissimilar to the original book this is the reason. If I find a correct copy, I will correct the shortcomings.*
> *(Tusi, 1255, translated by author)*

It is clear that lack of access to a good copy of the original or translation has been a reason for writing deeper tahrirs in some cases. Another reason for writing a deeper tahrir was technical difficulties with the method with which the writer communicated mathematical concept. For example, Tusi, in his introduction of the Tahrir on the *Sphaerica* of Menelaus writes

> *I got access to different versions of Menelaus'* Sphaerica *that were different, and one could not come to a good understanding of them. And these versions were the translations and editions of Abu Abdullah Mohammad-ibn-Essa Almahani and Abulfazl Ahmad-ibn-Abi Saeed-alharvi and others. And because some were unfinished, and some were hard to understand or were inaccurate, fixing their problems was not possible, and problems were not solved. Therefore, I was wondering in understanding of some problems ... then I started to Tahrir it.*
> (Modarres Razavi, 2007, translated by author)

Tusi's collection of tahrirs on the work of *Small Collection of Astronomy* plus the two beginning and finishing books is called the *Great Collection of Astronomy* in the East. This collection is listed in Table 1.

Table 1. Great Collection of Astronomy

Tusi's Tahrir Name	Year it was Finished	Original Book	Author	No. of propositions and problems
Tahrir of Elements (Osul Uqlidis)	12/1248	Elements	Euclid	
Tahrir of Data from the translation of Thabit Ibn Quara (Ketab al- Motiat)	1255	Data	Euclid*	95 propositions and problems of Euclidean geometry related to the first 6 Books of the Elements
Tahrir of Sphaerics (Ketab-ul- Akar)	8/1253	Sphaerics	Theodosius of Bithynia	59 propositions and problems of non-Euclidean geometry
Tahrir of Spherics		Spherics	Menelaus	
Tahrir of Moving Sphere (Tahrir Korat-ul-Moteharreke)	1253/54	On the Moving Sphere	Autolycus of Pitane	12 propositions and problems about the circles that are located on a sphere
Tahrir of On habitation (Ketab-ul-Masaken)	1255	On Habitation	Theodosius of Bithynia	12 propositions
Tahrir of Optica (Ketab-ul-Manazer)	12/6/1253	Optics	Euclid	Foundation of optic
Tahrir of Phenomena (Ketab Zaherat-ul-falak)	19/5/1255	Phenomena	Euclid	25 propositions and problems
Tahrir On Days and Night (Ketab Fi-layl Va-Nahar)	1255	On Days and Nights	Theodosius of Bithynia	33 propositions and problems, a study of the apparent motion of the Sun
Tahris of Measurement of the Circle (Taksir al-Dayereh)	1262/63	Measurement of the Circle	Archimedes	
Tahrir of Ketab Marefe Masahat-ul-Ashkal-ul-Basiteh val-Korieh from Ashkal ul Gheta	1255	Knowledge on Measuring Plane and spherical Figures	Beni Musa Ibn Shaker*	18 propositions and problems on the area of triangles
Tahrir on Assumptions	1255	Assumptions	Thabit-Ibn-Quara*	36 propositions and problems
Tahrir of Lemmas (Ketab-ul-Makhuzat fi Osul Hendeseh)	1255	Book of Lemmas	Archimedes	15 propositions and problems on circles include arbelos** and salinon***

Tusi's Tahrir Name	Year it was Finished	Original Book	Author	No. of propositions and problems
Tahrir On the Size and Distances of the Sun and Moon (Tahrir Ketab fi Germ-ul-Neerin va Badihoma)	1259/60	On the Sizes and Distances of the Sun and Moon	Aristarchus of Samos	17 propositions and problems
Tahrir on the Sphere and Cylinder (Ketab fil-Koreh val-Ostovaneh)	1262/63	On the Sphere and Cylinder	Archimedes	98 propositions and problems
Tahrir on the article of Pi (Tahrir ul-Maghaleh fi Taksir-ul-Dayereh)	N/A	Proposition 2 Book XII	Euclid	A treaty on Pi
Tahrir On Risings and Settings (Ketab-ul-Tolu val-Ghorub)	1255	On Risings and Settings	Autolycus of Pitane	36 propositions and problems
Tahrir On Ascension of Stars (Tahrir Ketab fil-Mataleh)	1255	On Ascension of Starts	Hypsicles of Alexandria	3 articles and 2 propositions
Second Tahrir of Conics (Tahrir- e-Makhrutat)	N/A	Conic Sections	Apollonius of Perga	Beni Musa had written a tahrir on this book first then Tusi wrote a tahrir on it again
Tahrir of Almagest (Tahrir al-Majesti)	2/1247	Almagest	Ptolemy	Astronomy

[a] The source of information are Tusi's *Mutiwassitat* and http://www.math-khajenasir.blogfa.com/cat-16.aspx. All books in this collection are written in Arabic.

* Books that were not considered part of the *Small Collection of Astronomy*. It was either not considered as part of the collection in first place or were written long after the period the collection was compiled.

** A plane region bounded by a semicircle of diameter 1.

*** Consists of 4 semicircles.

Filling the Gaps by Writing New Books

Writing books and treatises from scratch became a necessity for Tusi to complete the content of *Great Collection of Astronomy* for following reasons:

- Proper, adequate information and treatises for filling a particular gap were not available in the books of the collection.
- Questions arose from Tusi or his students, but the answers were not found in the books of the collection.

For instance, Tusi wrote a book, *A Remedial on Disbelief on Parallel Lines*, on Euclid's fifth postulate, regarding parallel lines. He argued about Euclid's mistake in the way he discussed parallel lines. Here is the postulate written by Euclid:

> If a line intersect two other lines and the sum of the interior, but not alternate angles, are less than 180 degrees, the two lines intersect at the side that the two angles were less than 180 degrees.

This is taken as definite and indisputable by Euclid, and he has not proven it, but for mathematicians this should be proved in order to be used for other postulates. Tusi first proved it, and then in his lengthy introduction mentioned that Euclid had left the proof of it to those who are coming after him (Modarresi, 1956, translated by author). One would experience Tusi's fair comments on the work of the former mathematicians by writing notes such as the above. Another example of this type of work is Tusi's *A Memorandum on Astronomy*, which offers the revision of the Ptolemy's astronomical model. These books or treatises are not considered as part of the *bridge* but they are complementary works for better understanding of the course of astronomy.

Compiling *Mutiwassitat*

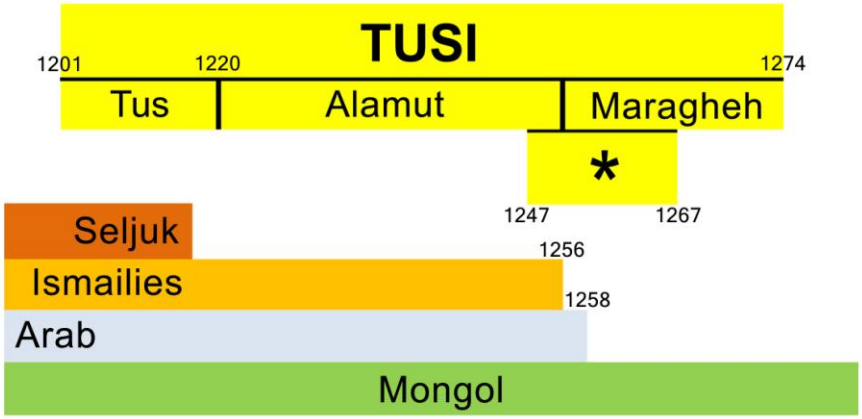
Tusi's main contribution in writing and compiling the *bridge* is into adopting, delicately, a unique theme in preparing the content of all books of the collection, and combining them in such a way that readers experience a unique, connected pathway, just if there was an intention to write a curriculum for the entire astronomy. Tusi compiled the *bridge* from 18 books together in a bigger book that he called *Mutiwassitat*.[32] Several

[32] Tusi started compiling *Mutiwassitat* (mediums), and continued adding to it throughout his scholarly life. After Tusi, other mathematicians added Tusi's other books to *Mutiwassitat*. There are copies of it with 13, 16, 18, and 19 books in it. (Personal communication). According to Dr. Berggren, Mutiwassitat are books that should be studied between *Elements* and *Almagest*. (mirasmaktoob.ir)

copies of it are available in different libraries in Iran, India, and Spain. “In a sense, he seems to have recast the whole library of classical mathematical sciences” (Badakhchani, 2004). In fact, *Mutiwassitat* is the rewritten work of a course of mathematics science, geometry, and astronomy of the time of Tusi. Anyway, it seems that Tusi had put together about thirteen books as *Mutiwassitat* but mathematicians have added other books of Tusi to the list. One can find *Mutiwassitat* with different numbers of books, from thirteen to eighteen, of which they all are either Tusi’s tahrirs or his own works.

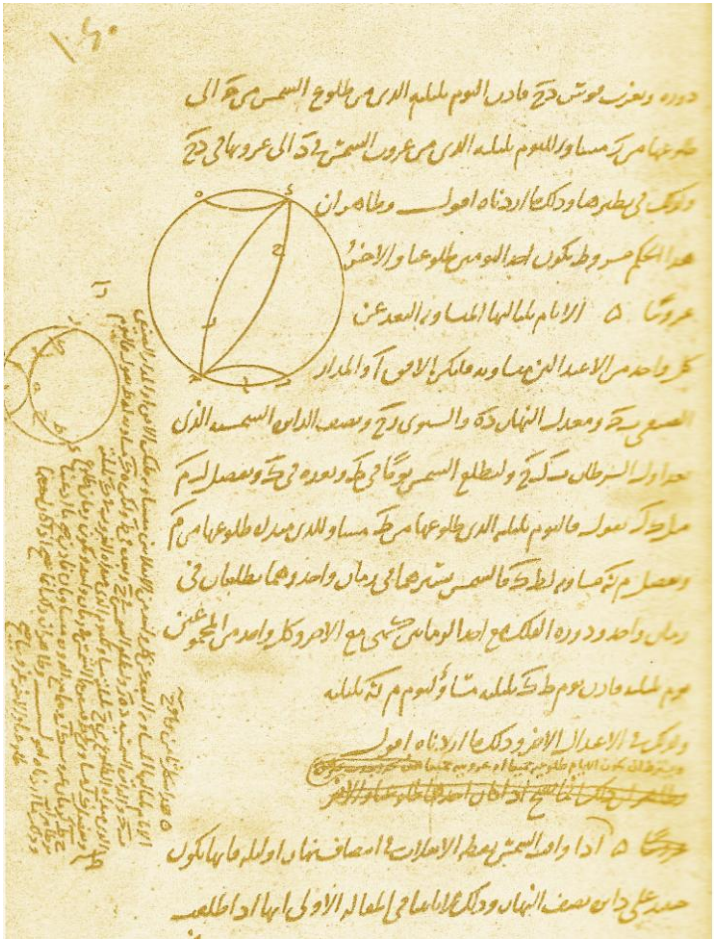
The result of Tusi’s effort in writing *Mutiwassitat* solved many problems of those who were interested in studying astronomy. The collection has a systematic approach toward astronomy. It includes the prerequisite knowledge of astronomy, mostly geometry and other mathematics skills. It also discusses deep astronomical concepts and facts that one should know before studying Ptolemy’s *Almagest*. *Mutiwassitat* became the middle content of the course of astronomy, where students used to study *Elements* as the elementary, and *Almagest* as the advanced education. Tusi continued editing *Mutiwassitat* as long as he was alive. One can see numbers of editions and additions in the margin of the books of *Mutiwassitat*. Figure 12, below, is a picture of half of a page from Tusi’s *Ketab fi-layl Va-Nahar*, which is his tahrir on Theodosius’ (of Bithynia) book, *On Days and Night*.

Figure 10. Connection of Euclid’s Elements And Ptolemy’s Almagest within the Life-Time of Tusi.



* Connected Euclid’s *Elements* and Ptolemy’s *Almagest* by bridging the gap between them.

Figure 12. Tusi's Tahrir is dated AD 1225.

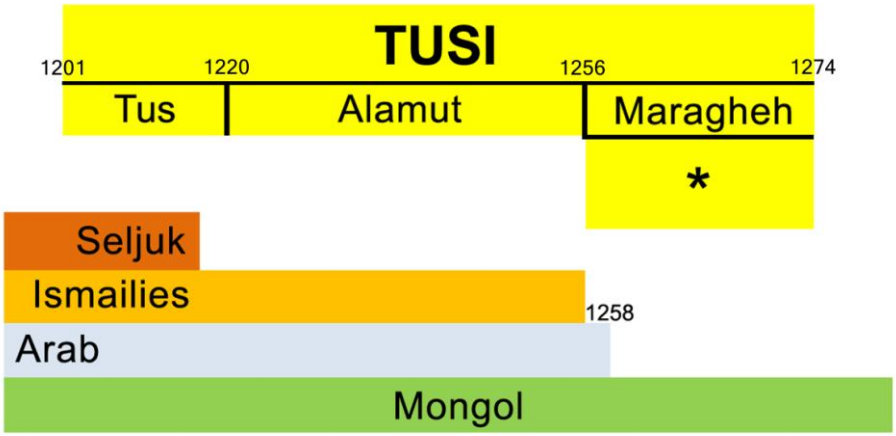


Note: The source is the copy of *Mutiwassitat* printed in 2005 by Institute for humanities and Cultural Studies in Tehran, with an introduction of Dr. Jafar Aghayani Chavoshi. The figure shows only half of a page.

Chapter 6.

Tusi Protected Mathematicians' Scholarly Life

Figure 13. ***Protection of The Life of Mathematicians within The Life-Time of Tusi***



* Protected mathematicians' scholarly life.

Tusi had dreamed of working with other scholars since he was a young scientist. He had dreamed of being able to research freely and communicate effectively with other scholars whose minds were focused and whose lives were stable and safe. Tusi had dreamed of bringing mathematicians together in a secure scientific environment where scientific discussions could occur, and scientific projects could run.

Since the Mongol conquest of Persia, Tusi had experienced a slowdown in the fast pace of the growing knowledge of mathematics of the previous four centuries in the Islamic Era. Tusi wanted to come up with a plan to protect mathematicians' scholarly life from the harsh political and social conditions of Islamic states, in particular Persia. He knew other related matters including mathematics education and schooling would benefit from the protection of mathematicians and scholars.

Muslim “rulers during the Golden Age of ‘Muslim knowledge power’ were very strong top down in sponsoring research and development (R&D) activities” (Hij Salma Bee Hi Noor, 2008). People deeply appreciated education, and encouraged teachers to work full time as teachers. Also, regional governments and business owners, who benefited from educated staff, backed R&D activities that educational institutes of the time were offering to them (ibid., 2008). These conditions were critically changed by the Mongol exploitation of Persia. The critical changes included but were not limited to the insecurity of students, destruction of books and resources, and terrorism of scholars and mathematicians. In regards to the security of scholars, Sunni Muslims had joined Mongol’s anti-resistance unit, for the execution of Shia Muslim scholars.

Modern day Iranian researchers have searched and studied different aspects of the Mongol conquest of Persia. Sultanzadeh has studied and provided educators with an unbiased assessment of the impact of the Mongol presence in Persia and disadvantages that resulted for the nation. A summary of his research printed in AD 1985 reveals that after the Mongols savagely attacked eastern Persia, and mercilessly conducted a massacre of the residents of the cities, the invasion reached to the northern and central regions of current Iran. The slaughter of many scientists and the demolition of science centres, in one stroke, caused science, knowledge, businesses, and even hope to continue to suffer. Scholarly life became a dream for scholars. Even those scholars, who fled to the western regions didn’t have the mental focus and tranquility to continue working on scientific matters. Consequently, a discontinuous theme is experienced in the work of philosophers and in the research of scientists at the beginning of the Mongol Era. Most of the scientific and educational activities ceased for a while. Kasayi[33] (1999) in his *History of Great Islamic Universities* cites from Ibn-e-Khaldun, a great historian of Tunisia (May 27, 1332 AD – March 19, 1406 AD) that prosperity and the success of the sciences occurred while civilization and development was happening in Persia and its

[33] Dr. Noorollah Kasayi was a teacher and faculty member of Tehran University whose degree was in educational psychology. He wrote five books and completed over twenty major educational research projects while he instructed at university.
(http://holybible.blogfa.com)

regions such as Kharazm, Khurasan, Susa, and Mesopotamia. But when the cities were destroyed, civilization and reformism which generate knowledge and industry left the territory (p. 29).

The main concerns of Tusi were combating Mongol savagery, saving the lives of innocent scholars and the establishment of probably the most important centre of learning of his time in Maragheh (Badakhchani[34], 2004). Tusi, taking advantage of his authority in Hulagu's court, decided to bring back prosperity to the country by returning the educational situation back to the state that people remembered during the rule of Seljuk, this time with a significant difference, no Arab control. To accomplish this, he had to buy the trust of the Mongol empire, show enthusiasm to make a big difference in the Capital city, demonstrate and prove the advantage of building an observatory, and justify the cost of it for Hulagu's Ministers who were looking for making money during their time of ruling the country. As Tusi's position in Hulagu's court improved, his influence became so great that he held a noble and spiritual position in the court of the Mongol. Whatever he asked of Hulagu, was carried out immediately, and the expenditure of any amount was provided (Jafarian, translated by author, p. 13). Tusi had convinced Hulagu that for a regime that wanted to govern great lands of the world, in the era of evolvement of the science of far-east and middle-east, and the shift of science and philosophy to a higher level, the strategic placement of the scientific centre of Maragheh was very important. Hulagu believed in Tusi, and Tusi had promised the glory to Hulagu.

Considering the school and observatory Tusi made in Maragheh, it is evident that Tusi's main strategy for preserving mathematics was to protect mathematicians, scholars, researchers, engineers, astronomers, and translators. Compiling the mathematical works of scholars and of former mathematicians was the other important result of the project in Maragheh. Tusi trained as many students as he could in the secure, safe environment of Maragheh. This matter will be discussed in greater detail in Chapter 7 of this thesis.

[34] S. J. Badakhchani has a doctorate in Islamic Philosophy from the University of Oxford, and is a Research Associate at the Institute of Ismaili studies, London. (Institute of Ismaili Studies)

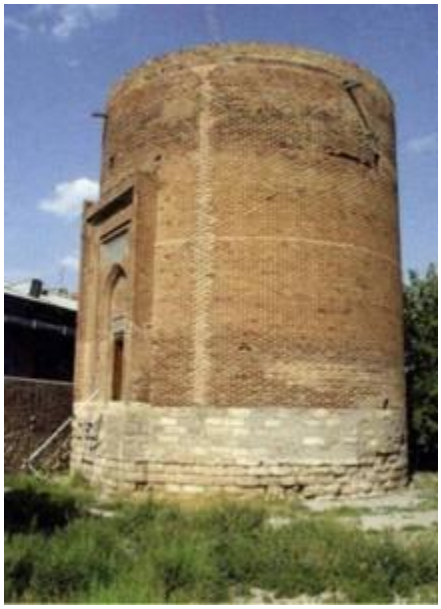
Maragheh Observatory and Calendar

Tusi reinforced and encouraged the study of astronomy and the mathematics relating to it when scholars and scientists were hopeless. He stood for the nation's right to study. He cleverly planned for the construction of a proper research and education facility so a team of scholars and mathematicians could work cooperatively to achieve defined goals. He demonstrated excellent leadership in both education and research.

Observatory

Persians' efforts in the foundation of science became fruitful in the third "Wisdom House" of Islamic civilization in Maragheh. Tusi chose a tall hill on the north-western side of Maragheh and requested an enormous amount of money, enough to construct an observatory and all its facilities including tools and a library (Modarres Razavi, 2007, translated by author). It took about three years for the building to be constructed, and for the equipment to be assembled. Figure 14 shows a section of the Maragheh Observatory.

Figure 14. A Section of Observatory.



Note: Compared to the rest of the Observatory, this section seems to have been maintained over time. Source is (Tayyar Maraghi, 2009).

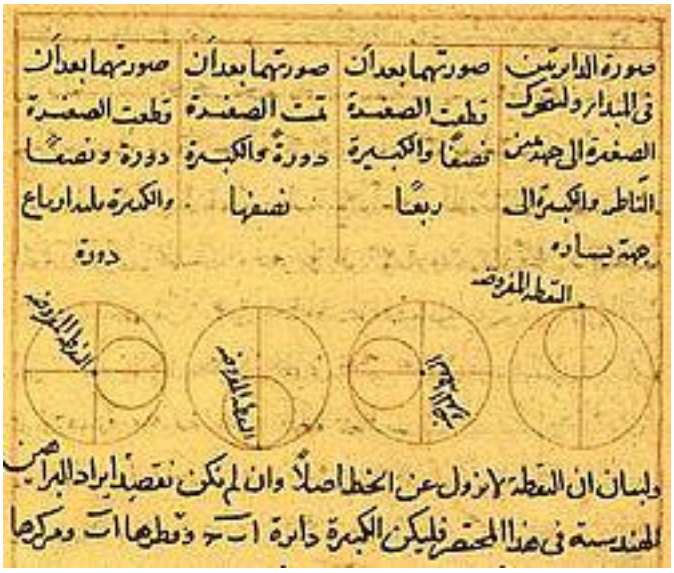
Pak (2010) calls Maragheh School a “scientific research and foundation of science education institute” (translated by author, p. 11), while many others considered it an accredited university among the biggest universities of ancient time, such as Al-Azhar of Cairo, Gundishapur of Susa, and Nezamieh of Bagdad. This observatory is of higher quality than former observatories of the Islamic Era. Observatories in the early Islamic period had been small, and usually linked to only a single astronomer. The major improvement of the Maragheh Observatory was in its teamwork approach to research. It was the first major scientific institution in which a number of scientists, astronomers and mathematicians gathered together to work on astronomy, to collect and record data, and to teach (Ballay, 1990).

The main task of the observatory team was to establish a calendar based on the collected data, but the innovations which they introduced into the Ptolemaic models became the most celebrated scientific achievements of this group. This includes the tools they engineered for the observation. Another accomplishment of this group was to bring the results of practicing astronomy into written materials in book called *Zij* that Tusi wrote in Farsi.

Calendar (Zij), Astrolabe, and Side Products

The Maragheh scientific team “meant to eliminate some of the objectionable features of the Ptolemaic models” (Langermann, p.198). They attempted to solve the problem with the model Ptolemy had introduced as equant. They created an alternative configuration and system that is famous, called Tusi-Couple. Tusi-Couple is a geometrical technique invented by Tusi that generates linear motion from the sum of two circular motions. It is a 2-cusped hypocycloid obtained by rolling a circle of radius *a* inside a circle of radius *2a*. The result is a line segment (Steinhaus 1999). Figure 15 shows a general diagram of Tusi-Couple.

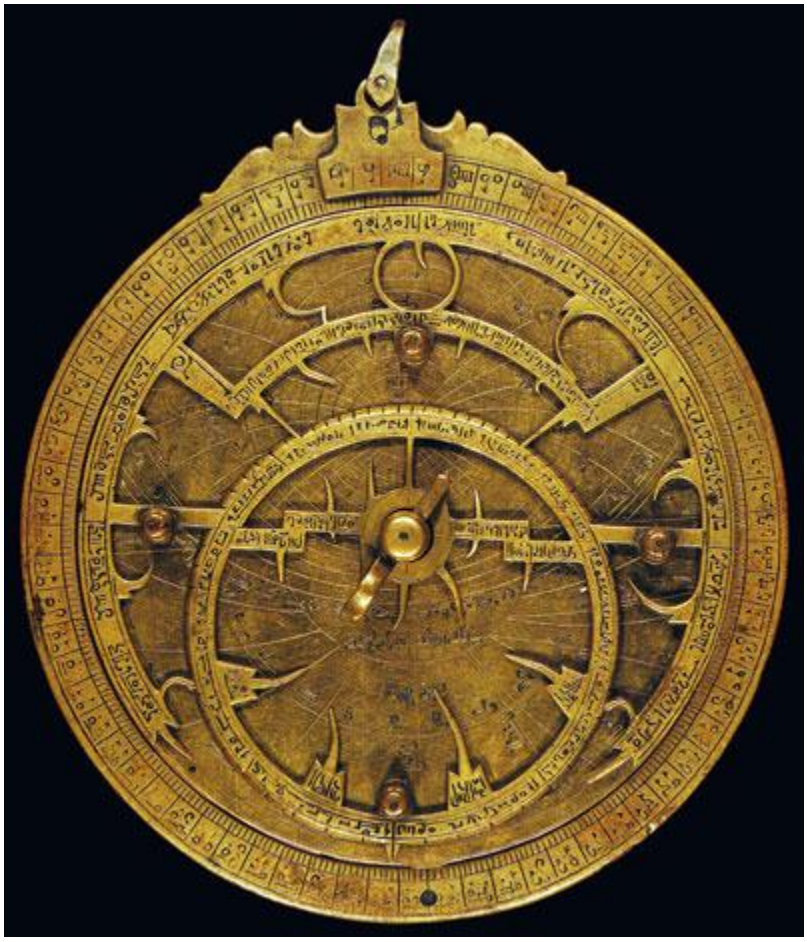
Figure 15. Tusi-Couple



Source: Pearson Education web site, Astronomy Today.

These new configurations, in fact, eliminated the three mathematical constructions that Ptolemy introduced to examine and explain the movement of spatial objects around the Earth. Using Tusi-Couple, Tusi determined the precise value of 51 arcsec (one arc-second = 0.000277777777778 degrees) for the annual precession of the spring (March 20th) and fall (September 22nd) equinoxes. Tusi also contributed to the construction and usage of some astronomical instruments including the astrolabe (an instrument for measuring the altitude of the sun and stars). Figure 16 shows the astrolabe he made that is currently kept in Cambridge Museum.

Figure 16. Astrolabe Made By Tusi



Source: Tayyar Maraghi, 2009

A four-metre wall quadrant made from copper and an azimuth quadrant were also invented by Tusi to help with calculations. Tools that were engineered within the observatory in Maragheh became the main equipment widely used in observatories around the world.

As a result of the mathematical research of the team in Maragheh, the language of astronomy was dramatically changed to represent a new model of planetary position. Empirical observations conducted by the team proved that the new model was able to describe the configuration of the spatial bodies more accurately than former calculations. Calculations of the new configuration became an important scientific revolution in astronomy. Astronomers realized that astronomy should describe the behaviour of

physical bodies in mathematical language instead of remaining a mathematical hypothesis. This was later emphasized by Copernicus. Maragheh astronomers, also, realized and proved that the Aristotelian view of universal motions is a type between circular and linear. In short, the revolution that made Maragheh's studies extraordinary, according to several historians and matheamicians, was the shift away from the philosophical foundations of Aristotelian cosmology and Ptolemaic astronomy towards empirical observation and mathematization of astronomy.

The Observatory team achieved the goal of writing the calendar or *Zij* within twelve years. It consisted of an almanac and astronomical table, figuration, astronomical fact, activities, and explanations which represented the highpoints of Tusi's and his team's research. It was and still is of great importance to the history of Islamic and of the world's astronomy (Ballay, 1990). Tusi wrote *Zij* in Farsi as there was no pressure to write it in Arabic, under Mongol rule. He indicated in his introduction to the *Zij* that their job was not finished, and it was the best they could have come up with in the allotted time. He also stated:

> *Observation for less than 30 years which completes a loop of these seven stars is less likely. And if more than 30 years is considered, it would be better and proper. But our King who demanded to establish the observatory commanded to rush to finish it quickly. And King ordered to finish it in 12 years. We decided to endeavour if circumstances allow us.*
>
> *(Tusi, translated by author, p. 3)*

Recent scientific excavations at Maragheh City, have unearthed the Maragheh Observatory's foundations. After studying the architectural foundation of the Observatory it is now clear that this observatory was the source of inspiration for the 14th century observatory that was established in Samarquand on such a grandiose scale (Ballay, 1990).

Recruiting Scholars

Beginning activities in Maragheh involved inviting several scientists from around the Islamic world and even beyond the Islamic world. This became an ongoing activity as long as the Maragheh Observatory and School were in session. Running instruction

sessions and technical meetings was a routine procedure that depended upon the presenting scholars. Maragheh became a dream come true for many scholars whose interest was to live a scholarly life despite the social condition of the region.

After Tusi received the approval of Hulagu, he started inviting scientists from all Islamic regions and cities to Maragheh to live and help create a strong team. He wanted the team to work on the foundations of the Observatory, and the applications of the data to be collected. Aghazadeh (2010) in his "History of Training and Education of Iran" explains that Tusi with great endeavour and effort could summon many of the scientists and scholars that had escaped the Mongol sword's edge and lived in secret in safe regions (translated by author, p. 15). Tusi used the reputation of Hulagu to invite foreign scientists from China. Hulagu, who became extremely interested in building the Observatory, tried to help the team reach their task. Sarten in his "History of Science" stated that Hulagu brought some Chinese astronomers and scholars with him, that one of them was Fao-Mun-Ji, and that Tusi learned the Chinese astronomy and calendar from them (Modarres Razavi, translated by author, p. 47). Tusi also attracted scholars from the Byzantine Empire. Most notable of them was Gregory Choniades, who later translated the calendar into Byzantine Greek and took it to the Byzantine Empire. He may have been responsible for the transmission of the Tusi-Couple to Europe, where it eventually influenced Copernican heliocentrism. Bartold, a Russian scientist, in his *Historical Geography* mentioned Maragheh and its observatory and stated that the Observatory had the biggest library, expensive tools, and great scientists of Persia and other regions of the East including China (Modarres Razavi, translated by author, p. 49). The names of the most influential members of Tusi's team in Maragheh are summarized in Table 2.

Table 2. Tusi's Team in Maragheh Observatory

Name of the scholar	Home region and expertise
Moaeduddin Orzi (d. 1285)	Damascus, Syria - Geometer and engineer Obs. Instrument
Najmeddin Dabiran Katebi (d. 1296)	Ghazvin, Iran - Metaphysics, discourse, dialectics
Kamaluddin Ghazvini Maraghi	Ghazvin, Iran - Judge, principal of the Ghazi Maragheh school
Fakhreddin Abulfazl Khalati (1208 -1301)	Tbilisi, Georgia - Physician, the chief judge of Tiflis and Armania

Name of the scholar	Home region and expertise
Fakhreddin Muhammad Maraghei (d. 1288)	Musel, Iraq - Mathematician, engineer, astronomer, built the copper tower of Observatory.
Najmeddin Kateb Baghdadi	Baghdad, Iraq - Mathematician, Obs. Instruments, and Phases
Mohyeddin Yahya Maghrebi (d. 1303)	Andalusia, Tunisia - Obs. Reading, Mathematician, engineer. Note taking and writing
Ghotbeddin Shirazi (d. 1331)	Shiraz, Iran - Astronomer. Tusi's assistant in observation
Shamseddin Shirvani	Shirvan, Iran
Farideddin Tusi (d. 1320)	Tus, Iran - Engineer
Kamaleddin Iiji	Iran
Hesameddin Shaami	Damascus, Syria
Najmeddin Shaami	Damascus, Syria
Najmeddin Asterlabi	
Rokneddin Estarabadi (d. 1336)	Estarabad, Iran
Ibn-alfuta	
Sadreddin Ali	Tutsi's son - Astronomer, Poet, Observatory administrator
Asileddin Hasan	Tutsi's son
Fakhreddin Abolreza Yehudashti	Yehudasht, Iran
Karimuddin Mahmud Salmasi	Engineer and inventor. Made a hollow glob for the Observatory.
Kamaleddin Natanzi	Natanz, Iran
Najmuddin Damghani (d. 1301)	Damghan, Iran
Zakaria Muhammad Ghazvini	Gazvin, Iran
Fakhreddin Abumasoud Kazeruni (Sadrieh School teacher)	Kazerun, Iran
Asiruddin Abhari	Abhar, Iran
Abulfaraj Masihi (d. 1303)	
Kamaluddin Reza Abi	
Majduddin Harsani	
Mohyeddin Baghdadi (d. 1285)	Baghdad, Iraq
Amidussin Baghdadi	Baghdad, Iraq
Sing or Shing or Tou-Mi-Tzeu	Gharaghorom, China - Philosopher, Metaphysics
Fakhruddin Zarandi (d. 1313)	Zarand, Iran - Student then the University Education Department member

Name of the scholar	Home region and expertise
Emaduddin Masoud Kashghari	Iran - Student then the University Education Department member
Alaoddin Najjari (d. 1308)	Iran - Student then the University Education Department member
Fakhruddin hakim Ghazvini (d. 1288)	Ghazvin, Iran - Student then the University Education Department member
Ghavamuddin Abdullah Yazari	Iran - Student then the University Education Department member
Ghavamuddin Muhammad Yazari	Iran - Student then the University Education Department member
Kamaluddin Hendi (d. 1290)	India - Student then the University Education Department member
Shamsuddin Orfi Dameshghi	Damascus, Syria - Student then the University Education Department member
Emaduddin Savoji	Savoj, Iran - Student then the University Education Department member
Majduddin Omar Maraghi	Student then the University Education Department member
Emaduddin Mosib Damghani (d. 1308)	Damghan, Iran - Student then the University Education Department member
Ezzuddin Hassan Ibn Muhammad Vaseti Attar	Iran - Student then the University Education Department member
Ezzuddin Toghrol Ibn Sanjar Sahebi	Student then the University Education Department member
Ezzuddin Ghasem Ibn Abdulkareem Sanjari	Sanjar, Iran - Student then the University Education Department member

[a] Source: http://farabiauoob.mihanblog.com/extrapage/150 . (Tayyar Maraghi, 1981)

Tusi's efforts were not limited to preserving the scholarly life of academics. He was, literally, a redeemer for them. Reckoned as a second generation historian of the era, Nakhjavani wrote his history book in AD 1324. In his chapter of Baghdad he mentions that Tusi heard that two scholars were brought in to the court for execution.

> *'Kneeling, according to the Mongol custom, before the Khan, [Tusi] appealed for mercy. ` Khwajah [Tusi] offered himself to be killed instead. Hearing this, Hulagu remarked: `had we wanted to kill you, we wouldn't have let you live until now. ` Then Hulagu ordered that both the condemned men be handed over to him.*
>
> *(Jafarian, translated by author, p. 8)*

Another story of this kind is reported by Ibn Shakir, “the most renowned and proficient calligrapher and scribe not only of the whole Arab world but of all Islamdom” (Pamuk, 2002):

> *Once information reached the Tusi that Hulagu intended to kill Juwayni, theologist, lawyer, and the chair of Nezamieh School of Neishapur, Tusi found it necessary to think of a strategy. Taking his staff, rosary and astrolabe, he set out with someone carrying an incense burner following him towards Hulagu's tents. When Hulagu`s men saw him near Hulagu's tent looking into the astrolabe and burning incense they informed him. Tusi approached Hulagu's men and enquired about Hulagu's welfare. They replied that Hulagu was alright. Tusi told them that he wanted to see Hulagu with his own eyes. Hulagu, who had refused to admit anyone at that time, permitted him in. Tusi told him that an evil event was expected to occur, and that he had recited prayers, burnt incense and beseeched God to deflect the evil from Hulagu. He recommended that Hulagu too should as a good gesture of gratitude free prisoners in the different lands and grants them amnesty. Forthwith, Hulagu ordered that the Tusi’s recommendation be carried out. Thus Juwayni was liberated, and without any specific request by Tusi. (Jafarian, translated by author, p. 8)*

“Ibn Shakir, after quoting this anecdote, comments: Tusi ‘displayed extreme cleverness in achieving his objective and thus saved people from harm. His bringing about the freedom of so many prisoners in all the places is indeed an incomparable achievement’” (Jafarian, translated by author, p. 9).

Tusi had to know his team members’ interests and expertise to assign appropriate responsibilities to all of them, or the human resource could go in the wrong direction and some people might have left the project. He also had to consider wages and accommodations that preserved their scholarly life style. Therefore, Tusi had to hire people that were comfortable with the administrative responsibilities of an educational institute. The administrative components of Maragheh Observatory will be discussed in Chapter 7.

Compiling and Writing Books for the Observatory

The process of compiling scientific and scholarly books was a time consuming process of secrecy, security, bargaining, and investment for the Observatory and its

School. Collecting many books, texts, and notes was a sensitive project that required timely procedures and manners. Some books were in the hands of rulers of Baghdad, Cordova, Egypt or Tunisia, and were either too expensive to buy or not for sale at all. Some books were difficult to identify as they were required the presence of a person who was able to recognize the original copy or a decent translation of the original text from a low quality one. That is why Tusi had to travel several times to different cities for this task. Some books were to be borrowed for a while either to be translated or copied. Usually there were conditions for this kind of lending such as the lender request of extra copies of translation, or request of an extra translation of the book into a language of interest of the lender. Therefore, Tusi had to manage both time and budget properly for collecting resources. At the same time he had to demonstrate a proper and reasonable progress of the investment to Mongol ruler(s) even though he was highly trusted by them.

The process of writing books or translating them also was sensitive and required a timely procedure. Tusi wrote several treatises and books during establishment of the observatory. Some of the reasons for Tusi to be engaged in writing or tahrir a book while also being busy with managerial tasks are:

- To write books that were the direct output of his research components during the observation, including applications of mathematical calculations, and applications of equipment.
- To tahrir on the output of his team's observations, written materials, and answers to the questions arose during collection of data.
- To tahrir on the translations that were important to be studied for answering observation questions, or translations important to be studied parallel to the observation process for development of the Calendar.
- To write books to develop and improve on the team's work as a whole.
- To write books or tahrirs on the reports on generated data of the Calendar, in particular.

Some of the books and tahrirs that Tusi finished during the observation became major sources of future astronomical topics and research. One of these books is a famous astronomy book that became a main source for researchers, including European astronomers, for centuries. Aghazadeh (2010) stated "[Tusi] wrote a book in astronomy that was used by researchers and astronomers of Europe during the Renaissance

(translated by author, p. 15). Dealing with all kind of administrative, managerial, and scientific duties, in addition to criticism was worth doing for Tusi as he knew the team would end up with a strong source of data in Maragheh. At the time the project was finished, about 400 000 textbooks were in the Library of Maragheh University. This became a main source of data for education of both scholars and public in the future of the region. The list of the books/tahrirs Tusi wrote immediately before and during the improvement of the establishment of the observatory are listed in Table 3.

Table 3. Tusi's Mathematics Works other than Mutiwassitat

Original Writer	Tahrir on/Written as	*
Ptolemy	"Tahrir of Almagest" or Tahrir al-Magisti	A
Archimedes	"Measurement of the circle"	A
Euclid	"Tahrir of Elements" or Tahrir Osul Uqlidis	A
Tusi	"Treatise on Secants in the Science of Geometry" or "Removal of the Veil from the Mysteries of the Secants Figure" or Kashf al-Ghena an asrar al-Shekl al-Qetaa	A
Tusi	On Motion of Rolling and Ratio between Straight and Curved Lines	A
Tusi	"Projecting the Sphere onto a Plane"	A
Tusi	"Observation Tools" or Alat e-Rasadi	F
Tusi	"On the Sector Figure" or Al-Resaleh Al-Moeenieh	F
Tusi	Meyar al-Ashaar (on astronomy)	F
Tusi	"A Summary of the Knowledge of Calendar"	F
Tusi	"Fake Sunrise" or Sobh e-Kazeb	F
Tusi	"Commentary of Ptolemi's Tetrabiblos" or Sharh e-Samare e-Batlamyus	F
Tusi	"Thirty Chapter on the Knowledge of Calendar" or Si Fasl Dar Marifat e-Taghvim	F
Tusi	"The Translation of Thirty Chapters on the Knowledge of Calendar"	A
Tusi	"Treaty in Twenty Chapters on the Knowledge of the Astrolabe" or Risala e-Bist Bab Dar Marifat e-Astrolabe	F
Tusi	Muvadhdhih al-Rusum fi 'ilm al-Nujum (on astronomy)	F
Tusi	"Treaty on Astronomy" or Al-Tazkareh al-Nasirrieh	A
Tusi	"Memoir on the Science of Astronomy" or Altadhkira fi ilm al-heyaa	A
Tusi	zobdat al-Heyat (on astronomy)	F
Tusi	Hall e-Moshkelat e-Moyeenieh (on astronomy)	F
Tusi	"Collection of Arithmetic by means of Board and Dust" or Javame al-Hesab fi-Takht v-Altorab	A

Tusi	"On Algebra and Arithmetic Problems" or Jabr v-almoghabeleh	A
Tusi	"Treaty on Salvation from Doubts about Parallel lines" or Risaleh fi Shak e-Khottot al-Movasiah	A
Tusi	Treatise on Proving the Impossibility of a Square Number being the Sum of two Odd Square Numbers to be a Square Number	A
Tusi	"Tahrir of Elements of Geometry of Euclid" or Tahrir Ketab Osul al-Hendeseh	F
Tusi	"On Multiplication and Division" or Hesab al-Zarb v al-Ghesmah	F
Tusi	"Calculations of Inheritance" or Hesab al-Ers	F
Tusi	"Reflection of Beams and Inclinations" (on optic)	F

* Language that Tusi wrote the book or treaty, either "A" for Arabic, or "F" for Farsi.
The known English names of the books are given in quote, and the known original names are Italic.

Tusi had to make sure the terminology of the written books and translations were as consistent as possible to provide a comfortable understanding of content of books for the readers and educators.

Research Literacy and Terminology

During the work in Observatory, Tusi continued writing with the same terminology and theme in which he had written guring hid years in Alamut. Centuries before Tusi, Greek mathematicians had used a terminology for mathematics books, but not many books were written with the same terminology. Tusi had experienced the existence of a definition of terms and explanation of figures as examples of the theme of mathematics books of the Greek Era. Perhaps this quality encouraged him to exert effort into considering and maintaining a terminology that helped readers to understand both translated books and original books. In fact, he improved upon the work of Greek mathematicians in regard to the terminology of mathematics books. His students continued with this legacy as well. The following example, offered by De Young (2008), demonstrates the noticeable difference between Tusi's method of scholarly writing and those of Euclid in the case of using vocabulary and sentence structure.

Euclid's Book V:

Definition 1

A magnitude is a *part* of a magnitude, the less of the greater, when it measures the greater.

Definition 2

The greater is a *multiple* of the less when it is measured by the less.

Tusi's tahrir:

When the smaller of two magnitudes measures the greater, it is its *part* and the greater is its *multiple*.

It is not difficult to compare the two definitions and notice that the one Tusi has written is a combination of the two definitions that is short and effective. Regardless, both definitions are understandable. It might be even noticeable in the word by word translation to English that the language of Tusi is plain compared to Euclid's. Below is the definition offered by Tusi's famous student, Qotbeddin Shirazi, who wrote his books in Farsi. It is clear that Shirazi didn't change the definition much, and tried to follow Tusi's style.

Shirazi's[35] definition:

Whenever the smaller of two magnitudes measures the greater of them, it is *a part of it* and the greater is *a multiple of it*" (De Young, p. 22).

[35] Qutbeddin Shirazi was Tusi's student, his assistant in collecting data during the observations in Maragheh, and according to many historians, one of his best companions. (Author)

Here is another example given by De Young.

Proposition 47 by Euclid:

In right-angled triangles the square on the side opposite the right angle equals the sum of the squares on the sides containing the right angle.

Tusi's tahrir:

In any "right-angled triangle, the square of the chord of the right angle is equal to the square of its (the right angle) two sides" (De Young, 2008). The term *chord* is used instead of *opposite* for clarity and keyword purposes to initiate the rationale behind consistent and specific terminology for the benefit of the educators.

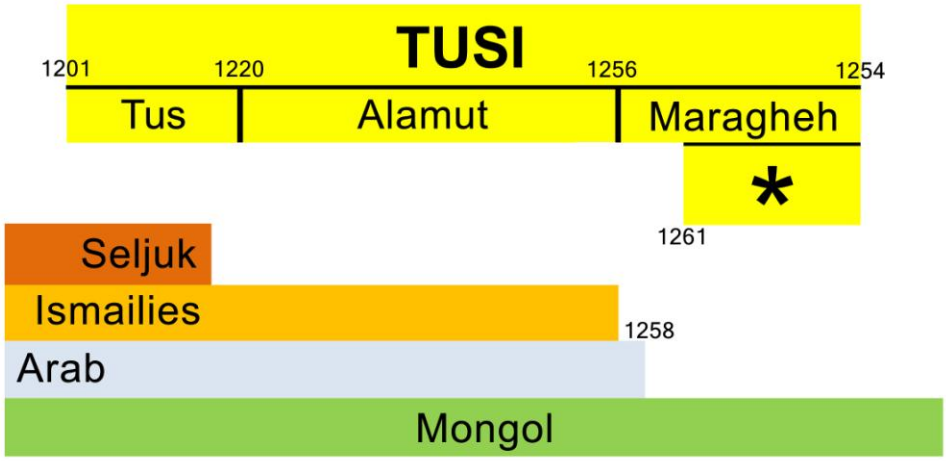
The Observatory in Maragheh and the products and side products of the entire research process were the apex of Tusi's scholarly life. Everything that he had worked on for years came together in Maragheh. Living with and leading a number of mathematicians and philosophers, meeting and discussing with numbers of educational environment managers and investors, and training and working with many talented students, was a dream come true for Tusi. Maragheh also became a place for scholarly work for many interested candidates. Thousands of the books and treatises were collected and complied for the betterment of researchers and educators. Maragheh Library provided the best service of its time to those who studied mathematics and any other science related to mathematics.

In the next two chapters, Tusi's contributions to the reform on mathematics education for public, and his program for training mathematics teacher are discussed. These two topics are among the rare discussions about the scholarly life of Tusi. Very limited researches are conducted around these two topics, or very limited of those were identified by this research. As the result, there are speculations, personal communication such as interviews, and general knowledge of author combined with the collected data that are introduced thoroughout the chapters.

Chapter 7.

Tusi Reformed Mathematics Education for the Public

Figure 17. Reform of Mathematics Education within the Life-Time of Tusi



* Reformed Persia's mathematics education.

Tusi dreamed of creating an education system in which mathematics was a subject to be studied by all Persians of any financial level and any religion. Before the 13th century mathematics was still a subject to be studied only by the children of aristocrats who were able to pay for private tutors. Male children of top religious families

used to attend Nezamieh[36], Rashidieh[37] and other religious schools. The children of trade people, merchants, and families that were well off usually attended the house of dedicated private instructors. The main subjects were discourse and religious jurisprudence. Some sciences, including mathematics, were also taught, but there were few mathematics topics which were widely offered by instructors. There was little consistency in the mathematics topics taught by instructors. The number of mathematics topics taught in Nezamieh and Rashidieh was limited, although these chain schools had a defined mathematics program.

In Tusi's opinion, studying mathematics was a mandatory task for any education system. He had studied mathematics, and had found it necessary for the success of the people and well-being of the nation. He had learned from studying mathematics that one will benefit from it. How much education was proper for the public, from Tusi's point of view, is not the argument of this section, but the direction of Tusi's life, and characteristics of his journey demonstrates that for the following reasons he wanted the nation to gain as deep an understanding of mathematics as possible.

Evolution of Education for the Nation

Tusi, as a Persian, was aware of the history of the life of his ancestors. Studying the history of education and training in ancient Persia reveals that training and education in ancient Persia, during the Archimedes and Sassanid era, were influenced by the social demands of aristocrats. The principles of training and education in that era had a conservative, classical nature. Family training and private tutoring was popular only between royal and rich classes (Aghazadeh 2010, translated by author). This remained

[36] Nezamieh was a chain of schools established by Nezamulmulk, the Persian Minister of Seljuk in several cities. Its main purpose was religious studies, but some academic activities were occurred. (www.lailatolgadr.net)

[37] Rashidieh was a chain of schools started by Rashiduddin Fazlullah Hamedani, Minister and scientist, from Hamedan, Iran. Rashidieh was first established in Maragheh but was opened in other cities. The funding was basically obtained from charity resources. (rasekhoon.net)

the state of education for the public for many years although after Alexander (II) a great deal of knowledge of Greek sciences was shared with Persians, and a door of hope was opened for a dialogue among scholars of the two nations.

A major change occurred after Persia was brought to Islam. The approach toward education and the seeking of it, practiced by Prophet Mohammad and his true followers, changed Persians' approach to the education, dramatically. It is learned from different sources that Prophet Muhammad freed captured enemies of war after they taught reading and writing to soldiers of Prophet's army. After Islam education was for everybody not only rich; it was equally for men and women; it was for any person with any religion or background. This was one of the fundamental ideas of Islam that Persians found encouraging.

Evolution of the Education for Persians after Islam

Prophet Mohammad's message for Muslims was to seek knowledge when he said: "Seeking knowledge is obligatory to every Muslim" (Quran). This was not limited to religious knowledge. By "knowledge", the Prophet meant all kinds of information that help people to live as righteous humans. Prophet Mohammad had stated that according to Islamic faith, one ought to acquire knowledge even if it was necessary to take hard steps such as traveling to China (Hofmann, 2000). Also, the life of Prophet Mohammad and his immediate followers reveals that duty or compulsory sciences that are obligatory for Muslims to study are not limited to religious sciences. It is compulsory for Muslims to seek knowledge of any science upon which Islamic society depends (Almasi, 2007). Studying these sciences were meritorious duties, essential to be performed either by any person himself or by someone on the person's behalf.

> *For example, society needs physicians; therefore, medical practice is meritorious. It means, it is necessary that there are enough physicians in the society, and if there are not, it would be compulsory for people to help with training enough of them. As training physicians requires studying medical science, as a matter of course, medical science is a meritorious duty.*
>
> *(Almasi, 2007, translated by author)*

During the Buyyid Dynasty (AD 934-1055), Persian Muslims came to a strong understanding of the benefit of public education. Some of the Buyyid Amirs[38] were well educated. They invited some Arab scholars to join the scholars of the Khurasan territory to improve the condition of public education to some extent. But a major step toward the improvement of public education was taken during Seljuk Dynasty (AD 1038-1187), whose rulers started their territory by ruling Khurasan, and later expanded their territory to the centre of Persia, where the current Iran is located.

Authorities have different opinions about the condition of public education at the time of Seljuk. Some historians believe that the social and educational condition of Khurasan, as the centre of the educational movement during Buyyid and Seljuk, was the best since the Islamic life of Persians and before the Mongol reached Persia. This group gives the credit of the improvement to the talented, strong Minister of Seljuk, Khajeh Nezamulmulk[39], who established several schools named Nezamieh, after him, throughout the entire Islamic region, including Baghdad. The same schools, for intermediate and advanced education, existed in cities such as Amol, in the northern part of the current Iran, Neishapur and Sabzevar in northeast of the current Iran, Rey, Yazd, and Esfehan in the central region of the current Iran. These schools became models for other schools, and even in the years after Seljuk, Nezamieh was the main practical educational environment that satisfied the expectations of the educators and students. Nezamulmulk was an intolerant Shafei[40] Muslim. He had monopolistic policies that supported his favourite government. His rigorous regulations created stress and exclusiveness in the educational environment so that educational and scientific activities turned into religious axes, in particular Shafei jurisprudence. This practice was against the Prophet's suggestion for compulsory education of the nation for all sciences that are necessary for the betterment of society. In the schools that Nezumulmulk designed,

[38] Amir means chief, Khan in Arabic but is also used in Farsi. (Author)

[39] See footnote 11.

[40] Shafei is one of the Sunni Muslim faiths. Muhammad Idris Shafei who was born in Gaza (current Palestine) was the leader of this faith. This faith, later, found its way in Persia. (www.eslahe.com)

religious courses were far ahead of scientific courses. He had forgotten his responsibility as a minister toward the nation in regard to the sciences by which the nation could make a difference in their living. One reason for “Nezamulmulk, perhaps, to ignore this problem was the fact that during the Seljuk Dynasty several schools were established that were in competition. The main reason for the competition was the different religious faiths of the directors and students” (Abdollah Tootian, personal communication, June 10, 2011). The Sunni faith has four main branches, and each of them had their own schools. Two Shia branches, Twelver and Ismaili, also had their own schools. Sufi’s, who believed in spiritual paths toward God, by devotion and love, had established their schools, too. Christian and Jewish groups who lived in numbers in Persia at that time were actively training students in their schools. Regarding this educational environment and competition, historians cite the names of some known scholars, mathematicians, and philosophers who had the greatest influenced on education in Persia. Three famous former classmates, Omar Khayyam, Nezamulmulk, and Hassan Sabbah, who lived less than a century before Tusi, are among those who contributed to schooling during the Seljuk Dynasty. They were three characters who had encouraged education in different ways at the same time, and had caused the nation to take education more seriously.

Omar Khayyam, who is a well-known mathematician, astronomer, and poet to both the East and the West, was a non-religious person who was criticized for his pure, non-religious approach toward education. His classes were stopped by the religious government of the time, but his philosophy of education for the sake of education and better life, became popular among Persians. Khayyam presented his thoughts and philosophy in his famous poems. Khayyam was born and resided in Neishapur where Tusi lived for years. He is still a symbol of freedom of religion and has many followers in the current Iran.

Nezamulmulk, who was introduced earlier in this thesis, became the great Minister of three Seljuk rulers, and established Nezamieh School chain throughout the entire Islamic states. He admired education and educators. His philosophy of education, as to better understand and practice religion and to better serve and please the God, had many followers. “Nezamieh schools disseminated speculative and historic knowledge, and were less interested in academic knowledge” (Agazadeh, 2010, translated by author). According to Agazadeh (2010) Nezamulmulk provided substantial

funding to public education and spent two hundred thousands Dinar (ancient Persian money) from his own asset, and increased the willingness of youth and families to benefit from it (translated by author, p. 12). He was a role model and mentor for education in the eyes of Persians, including Tusi. His religious goals were his first priority. His main enemy was his former classmate, Hassan Sabbah, whose life had become dedicated to the resistance against the exclusiveness that Nezamulmulk had enforced as a Sunni Muslim.

Hassan Sabbah was an extremist Shia who admired education and educators. He needed to educate his followers and youth who joined his team to make them ready for the missions. Hassan attracted many students to his educational environment. Hassan's main enemy was the Seljuk government of which Nezamulmulk was the Prime Minister. Hassan finally assassinated Nezamulmulk. As mentioned in Chapter 3, Hassan's successors invited Tusi to Alamut, and honoured him because of his knowledge. Hassan and his successors were eager to invest anything they had for the purpose of educating the youth who joined their group. Many wealthy Ismaili families used to financially support Hassan and the Assassins. The competition between these three groups ignited a desire for seeking education among the nation, but the situation was not flawless, as some of the educational outcomes were influenced by the religious or political demands and agendas.

Tusi was aware of the history of education and the competition involved with it. He knew that the competition had caused some subjects, such as mathematics, to be in the margins of education. He dreamed of establishing an education system where people could study free of religious and political obstacles and considerations.

A Talented Nation that Deserved to Learn Mathematics

Some of the most successful scientific centres in the history of ancient education, such as Sankore University and Ahmad Baba Centre, have their root in Persia. This indicates that the people of this land cared about knowledge. It also indicates their respect for scholars, and also the rich civilization and culture of their land. Tusi made connecting the mathematics knowledge of the past and future focus of his life, as he

knew the nation would value his effort. But the most significant difference between Tusi's plans and those of scholars such as Nezamulmulk and Hassan Sabbah was Tusi's vision of education. Although Tusi was born and raised in a religious family, he was not in favour of combining religious education with scientific education. Tusi wrote several books in different disciplines but his books on mathematics and astronomy were the primary steps in his goal of education. Tusi's life as a mathematician was separate from his life as a Muslim. He valued both of them, but did not make his life complicated by combining the two. In regard to this so called modern approach toward education, Tusi is one of the pioneers of Islamic Era or perhaps the world.

Persians, excited about the improvement of education after Islam, managed to plan for their children to practice reading, writing, and Islamic laws and regulations. "In many areas, mathematics instructors had dedicated their life to run classes. Instructors were well-respected as the nation had identified the value of the contribution of mathematics teachers" (Abdollah Tootian, personal communication, June 10, 2011). Since the introduction of Islam to Persians until the fall of the Seljuk Dynasty, history remembers numbers of Persian mathematicians and instructors, who walked into the hall of fame of ancient polymaths. Most of them were from Khurasan of Iran. Karaji[41], Omar Khayyam, Abolwafa Buzjani, Ghiaseddin Kashi[42], Abu Saeed Sajzi, and Sharafeddin Tusi[43] (different from Nasiruddin Tusi) are among them. Tusi as another

[41] Abu Bakr Karaji, also known as al-Karaji, was born in Karaj, in current Iran. He was a mathematician and engineer who held an official position in Baghdad. Three of his works are *Glorious on algebra*, *Wonderful on calculation*, and *Sufficient on calculation*. A now lost work of his is a description of what later became known as Pascal's triangle. *Extraction of underground waters* is another famous work of engineering from him. (http://91.98.46.102:8080/Farhikhtegan/details.aspx?id=1346. Retrieved from the web site of The Iranology Foundations)

[42] Ghyaseddin Kashani, also known as Al-Kāshī, was born in Kashan, in current Iran, He completed *Treatise on the Circumference*, a computational masterpiece in which he determined the value of 2π to 9 sexagesimal places. His best-known work is the *Key of Arithmetic*. In his *Treatise on the Chord and Sine*, he calculates the sine of 1° correct to 10 sexagesimal places. He edited *Zij* written by Tusi. (www.daneshnamah.com)

[43] Sharafeddin Mozaffar Tusi is a mathematician and astronomer from Tus in current Khurasan of Iran. He wrote *Plane*, on astronomy, a book on algebra, and a book on conics. (www.irstm.ir)

polymath of Khurasan, was eager and willing to share his mathematics skills with his people. He felt youth's thirst for learning, and planned to improve mathematics education by publicizing mathematics courses.

Mathematics for Youth

For over one century before the Mongol conquer Persia, youth were trained in schools of the Seljuk Dynasty, and had become better decision makers. In fact, this had reduced the impact of the Mongol attack. At the time when the Mongols were applying pressure, youth wanted freedom of speech and thinking. Youth, who were looking for a solution to the miserable condition of the country, saw their freedom in seeking for knowledge more than their dominant ruler. This, practically and mentally, helped youth to cope with the depressing conditions and to hope for a brighter future by educating themselves. Tusi supported their feelings and desires by designing a public education system in which youth studied pure mathematics as well as some popular applications of it in trade and construction.

Tusi who had dedicated his life to boost scientific capacity of his countrymen, targeted youth, and planned a more systematic public education for them. He focused on mathematics education, which at the time was considered one of the two high level academic educations, the other begin medical education. "There was equal opportunity to all people who wanted to pursue higher study. The bright students would get sufficient funds to carry out researches or to produce knowledge" (HSBHN, 2008). Tusi's contribution to the education of youth was a long-life journey. He started caring about youth when he was in Alamut, and this sentiment continued throughout his time of authority in Maragheh. Tusi played an important role when he was the head of the school of Alamut, the stronghold of Ismailies.

> *By constant visits with scholars, and by frequent correspondences, a habit which he developed from a very young ages, Tusi kept his contact with academics who were outside the Ismaili circle. It was surprising enough which he was addressed as* 'the scholar' *from very early ages of his scholarly life.*
>
> *(Badakhchani, 2004, translated by author)*

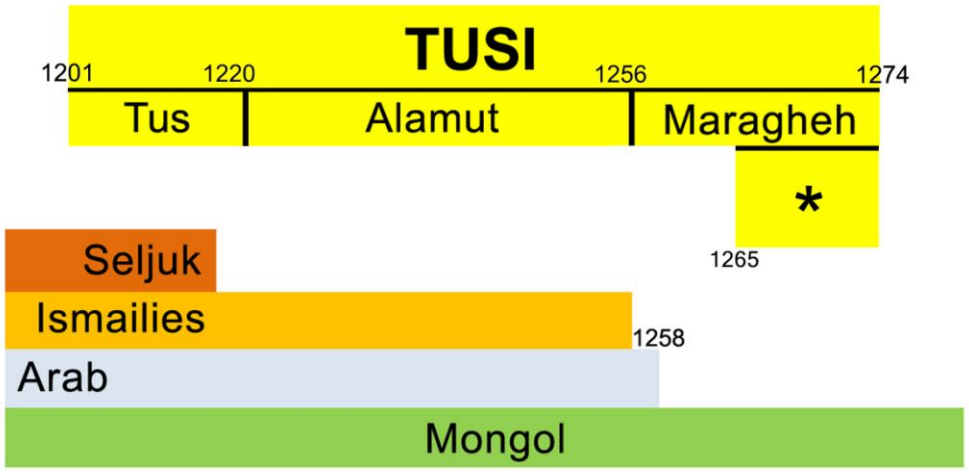
Tusi himself was a young person when he was showing his dedication to helping youth who were living inside of Alamut and outside of it. Youth, including Assassins, living inside Alamut studied mathematics under Tusi's supervision. His course materials were copied and used by other instructors who were teaching mathematics to the youth who lived outside Alamut. Tusi never stopped encouraging youth to participate in educational activities.

Later, after Tusi was running Maragheh School and its dependant schools, he achieved his dream of creating a safe, intellectual environment for youth to study academic subjects. Many families who had gifted children had the opportunity to send their children to Maragheh School. Scholarship and financial aid was considered for the students (Aghazadeh, 2010, translated by author). Very soon, the quality and quantity of the Maragheh School's programs surpassed any other school in the Islamic states. Scholars were invited to join to teach great numbers of students, so many that one would be amazed to see them all in one place. Maragheh School, with a high number of youth studying mathematics, became a model for public education in regions of Persia and other neighbouring lands such as Arab regions, Ottoman provinces in Eastern Europe, India, and southern states of the Russian Empire.

Chapter 8.

Tusi and Training Professional Mathematics Teachers

Figure 18. Training of Mathematics Teachers within the Life-Time of Tusi.



* Trained professional mathematics teachers.

Tusi spent a great deal of time and much thought to help with training mathematics teachers during his life. He also established a system for this training to be continued. In reforming the way of training qualified teachers, Tusi had to employ new strategies to attract candidates, obtain financial support, foresee future opportunities, and design proper training.

Continuing with the Persian's Vigilance Movement

Concerned about the nation's freedom of education after the discouragement incurred by the occupancy of the Mongol, Tusi dreamed of continuing what educators

and mathematicians had started before the appearance of the Mongol. Tusi trained as many people to be mathematics teachers as he could. Knowing that a one man war was not reasonable any more, Tusi wanted to leave his responsibilities as a teacher and mentor on the shoulders of numbers of trained professional mathematics teachers. He operationalized his plan in his time of residency in both Alamut and Maragheh with his main contribution being in Maragheh.

Tusi spent a tremendous amount of time traveling between the cities to seek and invite talented candidates to join Maragheh School to study as future mathematics teachers. Later, “in his last years of life, one of his primary activities was to recruit future teachers for Maragheh School from around the entire Islamic region, including Khurasan, Mesopotamia, and Damascus” (Abdollah Tootian, personal communication, June 22, 2011).

After the death of Hulagu (AD 1265), Tusi helped Abaqan, Hulagu’s son, to ascend the royal throne by supporting him. Thereafter, the influence of Tusi was instrumental in protecting learned men. During the time Hulagu’s son ruled, nearly a hundred scientists, mathematicians, and teachers who had been trained at Maragheh School, as disciples of Tusi, were graduated (Jafarian, 1994, translated by author).

Creating a Community of Educators

Even after the establishment of Maragheh School, it is reasonable to believe a scholar and minister in the level of Tusi could still have felt insecure about the conditions in which scholars and scientists lived. This was because the harsh situation of the past was possible to recur in the future, as the country was unstable. Everything could go suddenly wrong, and the small community of scientists and educators would be easily affected. This can be understood as Tusi tried to make the community of scholars as large and strong as possible while he was acting as an authority. A number of scholars invited to Maragheh and number of Tusi’s students are supporting this assumption. Tusi trained many mathematics teachers while he was in charge, and established a connected community of educators who could stand up for the right of education in case the next ruler was not dedicated enough. Tusi also wanted to ensure that there were

enough mathematics teachers who could travel and/or live around the region to benefit the community. Tusi invited many candidates to come for free education in Maragheh with the condition of returning to their original region after their graduation. He encouraged them to stay connected via trips or messengers.

Caring about the Community of Educators

Tusi was famous for caring about his students even after their graduation. Educators who studied and trained under Tusi's supervision were secure in finding jobs and supporting themselves in life. Many of these educators are known to have been hired by schools that requested teachers from Maragheh University.

As the community was getting stronger and larger, Tusi initiated a dialogue with Abaqan and his Ministers to consider a proper wage for educators. To reach an agreement, Tusi developed plans for improving the economy of the cities through using the knowledge of his students. So his request of continuous fund for educators seemed logically and economically reasonable. It is said that the educators who enrolled in Maragheh's programs were paying tuition fees. Some received financial aid and housing for the duration of their program at a place attached to the university that Tusi had arranged to be built. "Tusi, with his encouraging personality, was a great mentor for mathematics educators. It is known that he had planned further steps for the future of this department before he passed away, but the details of his plan are not known" (Abdollah Tootian, personal communication, June 22, 2011).

Tusi encouraged mathematicians and communities to open schools and to request qualified teachers from Maragheh University. As a result, there were number of schools working actively in Maragheh and other cities of Iran. In return, these schools trained students who joined the University for further education in the mathematics department. As the result of this cyclical process, the quality of the pedagogy of the training courses at the University was systematically and significantly improved. Table 4 provides the names of the known active schools in Maragheh.

Table 4. Active Public Schools in Maragheh

Name of the school	Established by	Expertise of the Principal
Ghazi Mohyeddin	Ghazi Mohyeddin Abu Hassan	Judge
Mohyieh	Ghazi Mohyeddin Abu Hamed	Judge
Atabak	N/A	
Ezzieh	Amir Ezziddin Abu Nasr	Language and Music teacher
Sadrieh	Sadruddin Abubakr	Administer and wise-man
Mojahedieh	N/A	
Ghazi	Amir Ezzuddin Abuhares then Kamluddin Muhammad Ghazvini	Administrator Judge

[a] Source: http://farabiauoob.mihanblog.com (Tayyar Maraghi, 2009)

Tusi reformed the education system such that its educators were reliable and professional. He included programs specializing in mathematics education and in introduced professional development activities.

Training Mathematics Educators as Specialists in Mathematics Topics

Being knowledgeable about different mathematics disciplines, Tusi was able to work on several topics in mathematics including geometry, arithmetic, trigonometry, astronomy, optics and more. He found it diffiuclt for an ordinary person to achieve a deep and complete understanding of all areas of mathematics, and to master application of it as a mathematics educator. Therefore, he preferred the mathematics educators to be trained in specific topics in order to be able to focus and become specialists. This, in fact, happened during the time Maragheh University was growing bigger. The administration of the University prepared course descriptions for each course and considered instructors who were capable of delivering the courses adequately. It is known that some engineering courses were prepared particularly for students who were looking for trade and technical occupations in their future careers.

Professional Development Activities

Tusi's several trips to different cities were productive with regard to his purpose of professional development. "When he visited cities, he arranged for meetings with mathematics teachers, and informed them of the newer discoveries, innovations, and methodologies. He invited instructors to stay connected with Maragheh University" (Abdollah Tootian, personal communication, June 22, 2011). In these sessions the educators of the cities shared their thoughts and opinions. They also discussed the problems involved in delivering lessons and running schools in their regions, and sought help from Tusi. These educators also became informed of the positions available throughout the country.

Tusi died and was buried in Samereh, a town close to Baghdad, during one of trips to Baghdad for the purpose of professional developement.

Chapter 9.

Tusi, Saviour or Traitor?

Tusi's life style has caused the community of historians, critics, and perhaps mathematicians to have different opinions about his character. Some have pointed out that Tusi's actions had merely been pleasing the enemy (ies) of Islamic states, and some others strongly believe that he had been a free man. Critics of Tusi, who are particularly religious Arabs, called Tusi a traitor because he had helped the Mongol to attack Baghdad. Persians, particularly those who are not in favour of Arab's domination of Middle East in both territorial and religious matters, see Tusi as a saviour at the time of the Arab dictatorship. It is recorded that about twenty years before the fall of Baghdad the Mongols had launched small attacks on Baghdad and its neighbouring cities. This demonstrates that Tusi could not have played a major role in instigating the invaders. At least three known historians, Ibn Abi al-Hadid, Ibn al-Fuwati, and Ibn al-Kazeruni, who lived in the same century as Tusi, have stated that the Mongols had attacked Baghdad long before the eventual fall of Baghdad.

Saved Muslims from Further Execution

Tusi knew that the Mongol would continue their legacy of murder until they fulfilled their thirst for blood. He also had experienced that no army or physical obstacle could stop them. Therefore, holding on a middle ground where it was possible to deal with them and to save lives was a smart idea.

The Mongol, with their rigorous and continuous charges had proved that the Persians' sword and military training was no match with their courage and desire for victory. Rough fighting and cruelness was in their blood and nothing could stop them on their journey of destruction, rape, and theft. There had been enough death among

Persian families, and Tusi decided to stop it at any price, even a tactical retreat. That is why when Hulagu was behind the gate of Alamut, Tusi spent about a year in back and forth negotiations between the two armies of the Mongol and Ismaili to ensure the Ismaili leader surrendered peacefully without casualties from the Persian Ismaili citizens of Alamut. Tusi helped the Mongol to achieve their goal more quickly and with fewer casualties. He made Hulagu and his army satisfied with their siege of Alamut, and he turned the Mongol rush to conquer the Ismailies into a diplomatic victory. Since they had begun their conquest of Persia, this was the first war in which the Mongols had long waited to achieve victory and accomplished it without massacre. Consequently, a vast majority of Ismaili left Alamut Fortress unharmed, and safely returned to their cities to resume a regular life after years of quarantine in the mountains. In fact, this was a victory for Persians, in comparison with the losses they had experienced before, as the result of fighting with the Mongol.

Tusi tried to employ the same tactic for Baghdad to ensure the safety of its citizens, both Arab and non-Arab. Tusi, as a true Muslim, was trained that a Muslim's first responsibility is to keep himself safe and sound, educated and spiritual, to benefit his family and Islamic society for his life-time. He had nothing against Arab citizens who were about to lose their lives because of the aggressiveness of their government. From this point of view Arabs and Persians were the same for Tusi. He also was against extreme and uncontrolled acts during which most decisions are made upon madness and revenge. He managed to stop Hulagu behind Baghdad's gates and waited for negotiations between the two armies. His plan could have been to achieve a middle ground as it happened in Alamut. Perhaps the plan was the security of Baghdad as a whole, and the safety of its assets in return for the retreat of Abbasid by stepping down from power. Unfortunately, the egotism of the last Abbasid ruler and his ministers caused the negotiations to go nowhere. They clearly cared less about their people than about their authority. When Hulagu lost his patience, he ordered the attack. As a result, numbers of Arabs in Baghdad lost their lives. It seems that some Arabs have difficulty believing that with or without Tusi, Hulagu would have attacked Baghdad and would have destroyed the roots of Abbasid in there. It is not fair to overlook Tusi's effort to end the terror with fewer casualties behind the closed gates of Baghdad.

In both incidents, Alamut and Baghdad, scholars were among the most vulnerable groups of citizens who would be harmed by Mongol. Tusi had tried his best to save the life of as many scholars as he could. His negotiations regarding the lives of the scholars, with both parties of the war, were an ongoing process during both incidents and are repeatedly recorded in history.

Tusi Saved Non-Muslims of Persia

The conditions under which non-Muslim Persians lived were devastating during the rule of the Arabs. For Persians, facing their newly forced religion was difficult. They had a strong background in their previous religion, Zoroastrian[44]. They were aware of a holy religion, and were educated, for centuries, by its heavenly messages of its prophet and its book. Although in the last years of the leadership of Zoroastrian authorities the people of Persia were experiencing insecurity and injustice from their religious leaders, Persians did not walk blindly into Islam's invitation. Many of them compared the two religions, and accepted Islam into their life as it was promised to provide them with a better life. But many never changed their faith. This portion of Persians, which in that time was not a small portion, but out-numbered all but new Muslims, stayed Zoroastrians, and suffered greately for holding on to their faith. Compared with the Arabs, the Mongols had almost nothing to do with religions, and simply allowed anyone to practice any religion. This, either a political promise or a religious belief by the Mongol, was pleasing for many minorities of Persia. Tusi, by helping remove Arab dominancy, helped the minorities, particularly Zoroastrians, to return to their normal life, and to practice their religion freely. The lives of Christian and Jewish Persians also

[44] Zorostrianism, or Mazdaism, is dated as far back as 1200 BCE. A priest named Zarathustra or Zoroaster, as the Greeks called him, founded the religion. He was born in Iran at the time when the Stone Age was gradually moving into the Bronze Age. The Iranians then, believed in a polytheistic religion. Zoroaster convinced and converted them to monotheism.
(Neil Payne, http://EzineArticles.com/1017450)

changed to a more secure and respected existence after the dominancy of the Arabs faded. Persia, again, became a true country for minorities and stayed in this condition for centuries.

Reinforcing True Islamic Ideology during Mongols' Rule

When Mongols attacked Persia, Tusi saw both Islam and education in jeopardy. However, he noticed that the enemy was fragile when it came to religion and science. The Mongols were a crowd of nomadic, simple people who had been in wars, away from their habitat and family, for several years and had lost their connection with their origins. Inviting them to a true Islam was not only possible but also useful for their settlement in Persia. Tusi decided to introduce them to the Islam that Persians were living with. Facing or practicing the exclusive Islam that in that time was practiced under the rule of Baghdad, as the centre of Islamic government, would make the Mongols feel resentful of Islam and the Islamic states. Islam was dictated by Baghdad's regime during their rule. In fact, by turning many Mongols to Shia Islam, the kingdom of Sunni Islam became much less attractive in Persia, and centuries later in modern Iran. The majority of Mongol soldiers and administrators accepted Shia Islam after being exposed to its divine messages. The Islamic nation of Iran is partially indebted to Tusi for this. From this point of view, Arab Muslims also should be thankful of Tusi as he brought a number of new Muslims to their religion.

Tusi introduced the Islam that he had practiced as a well-educated philosopher. Tusi was clearly against those who practiced Islam dogmatically for the sake of Islam versus practicing for the sake of Muslims. He practiced his religion as a way of living better. According to the philosophical ideas of Tusi, Islam is in the service of its followers not its followers in the service of Islam. Apparently, understanding Tusi's deep beliefs on the definition of Islam has been a big disagreement for extremist Arabs, and that is why they have blamed Tusi for not helping them. The questions are: shouldn't Arabs be thankful that Tusi had brought numbers of barbarian Mongols into their faith? Isn't this the responsibility of a Muslim, to introduce Islam to non-Muslims in the best possible way? Wasn't this the excuse by which Arabs encouraged their soldiers to attack other

nations such as Persians, Egyptians etc? The answer to all above questions from the point of view of trusted historians and trusting Muslims is “yes.”

Freedom from the Exploitation of Arabs

Tusi was a saviour for Persians. Tusi had found the moment had arrived, for once and for all, to get rid of the old enemy who was stealing everything, including the knowledge, from the nation of Persia. Overcoming the power of the Arabs had become a dream for Persians, and the Mongols were the only force that could help to retrieve some of the Persian belongings. Some writers have shut their eyes to the reality, and ridiculously acted defensive towards the liberated, cognisant nature of Tusi in hoping for the freedom of the nation. For instance, Michot (2009) cited that Ibn Taymiyyah (1263–1328 CE), the Arab fundamentalist, who lived at the time of Tusi, wondered,: “Al-Tusi and his like, were they selling well among the associationist Tatars[45] with anything else than astrologers’ lies and tricksters’ artifices that are all contrary to Reason and the Religion?” (p. 10). This graduate of one of the most extremist schools of all time who “is known for his devotion to Jihad as ... the best form of voluntary service man can devote to God” (Rana, Z. J. & Rana, A. J, 2012) blamed Tusi for the help he provided to Hulagu when the Mongols attacked the most culpable Abbasid ruler. Abbasid had turned both Arabic and Persian states into religious mafias of dictatorship and controversy. Ibn Taymiyyah had ignored the anarchy that Arabs had brought to the nations due lack of respect and governing skills of the later Abbasid rulers. The consequences were devastating for the nations that were brought to Islam.

The fact that the Mongol ruler was born from a Christian mother had nothing to do with Tusi’s decision of helping Hulagu to attack Baghdad. Any help from any group to finish Arabs’ imperialism was appreciated and accepted by the Persians as there existed nothing worse than the Arabs’ exploitation of Persia prior to Tusi’s time. Even the conquest of Persia by Alexander II for many Anti-Arab Iranian historians was not as

[45] A Mongol tribe.

harmful as the Arabs' exclusiveness and racism agaist Persians. Considering all factors involved in the decision Tusi made to help Hulagu, a Persian would see Tusi's action as a heroic move that is usually expected from a military general, not a mathematician. Tusi led the country and nation toward a more peaceful period of life.

New Era of Life for Persians

Tusi was a saviour for both Persians and Mongol authorities and soldiers. After the Abbasid were defeated, Persians started feeling calm and restful. It had been years of stress while the Mongol were expanding their territory. The Mongols, too, felt satisfied by the invasion of Baghdad. They began settling down and governing the country by installing Persians in almost all critical positions. Not only the public benefited from Tusi's input and encouragement towards the finishing of the Abbasid by the Mongols, but Mongol administrators also modernized their life style after the Abbasid's influence decreased and Persians started running the country. For example, according to Jafarian (1994) "A comparison of the two periods of Hulagu's life reveals the moderation brought about in him under the influence of the likes of the Khwajah [Tusi]" (translated by author, p. 9). Hulagu, who once was an un-educated dictator, wrote the following poem after years of companionship with Tusi:

> Because of his just rule, fire does not burn silk,
> The deer, too, suckles the lioness's milk
> The people are at peace due to his justice,
> And all the tyrants are wretched and weak.

Mongol soldiers got married and started a real life in Persia. Many Mongol administrators attended school for the first time, in Persian schools. People felt secure as their new government had a structure of both knowledge and power so they could live without worrying about enemies for long time.

New Era of Life for Scholars

As discussed in Chapter 8, Tusi established an education centre where numbers of scholars, mathematicians, and engineers worked to benefit society, and to improve the education level of the public. What Tusi remembered from his life was the distraction of mathematicians due to the lack of governance of the Abbasid and the conquering by the Mongols. The communication between scholars was lost and scientific activities were put on hold. Tusi, by helping Hulagu to end the Abbasid rule, re-started the similar activities which used to occur in Baghdad's "House of Wisdom" in Maragheh just a year after the fall of Baghdad. In Tusi's plan, scholars found a new building and facility at which to work, a fresh start to target scientific projects, and a team of dedicated members whose approach toward research was not religious. Many of the scholars joining Maragheh University were Arabs whose scholarly life in Baghdad was no longer secure. Tusi was a hero and a new leader for them.

Chapter 10.

Conclusion

Nasiruddin, or Muhammad Abu Jafar, known as Tusi, was a “renowned Persian, mathematician, astronomer, philosopher and theologian.” Tusi, in his *Sayr wa suluk* tells us “that he was born in a family who followed ‘the excoteric aspects of the shari’at’ (the way of life) and whose profession was ‘to promulgate the excoteric sciences’” (Badakhchani, 1999). My research revealed that Tusi is acknowledged throughout the Islamic world, and is titled ‘Great Wisdom.’ “His interest ranged widely as revealed by a prolific literary output which includes studies of algebra, mineralogy, philosophy, as well as several volumes of poetry” (Ballay, p. 1). However, it is his educational achievements in the field of mathematics that are revered in this thesis.

Tusi as a Mathematician

Tusi emerged as a first rank mathematician in the last century of the “Golden Age.” His mathematics skills allowed him to write several mathematics books of the highest scientific grade of his time. His books, words, and strategies in writing mathematics books become a model for his successors. In general, the power of the book in Islamic society brought with it whole array of intellectual needs and demands that could not help but spill out in many directions. Most scholars concur it was the Muslim influence on Europe, through their seminaries, which brought about the Renaissance (Wansbrough, 1963). According to Pak (2010) “The character of Tusi as a universal scientist is a key component to the continuation of the improvement of the foundation of science in the history of world’s science. In other word, this is a part of the share of the Iranian scientists to the world’s science” (translated by author, p.10).

In over seventeen years of his living in Alamut, Tusi wrote several mathematics books and treatises. He also wrote tahrirs on the works of his predecessors. Over one hundred sixty items of these types of works in mathematics and other disciplines are identified as the works of Tusi. He identified some flaws in the work of former mathematicians, partially technical flaws in the original works, and partially problems with translations from other languages to Arabic. He removed the problems, and reviewed his works frequently for further corrections. Tusi found the theme of the books of former mathematicians varied according to the whim of the authors or based on the authors' background knowledge. Therefore, he gave the books of the same discipline of mathematics a consistent theme and terminology to help readers, including teachers and students, with understanding the contents. He also filled the gaps between the content of written materials, by writing treatises, books, and tahrirs whenever he felt it was needed. Tusi offered examples, extra figures, and explanations for complicated contents. He provided a direction to future authors of mathematics books, and they continued writing topics to fulfill the discussions that were suggested by him. Tusi's materials and books became the standard texts and even textbooks in different mathematics disciplines for several centuries (Ragep, 1993). Tusi also used these books to run Maragheh University, one of the biggest schools of Islamic Era. The schools connected and inspired by Maragheh University also used Tusi's books when applicable.

During his journey, one can see that Tusi revisited some of his works to make them better and more efficient, as a true scientist would do. Tusi wrote a series of mathematics books created from work of known mathematicians, and put it in an order that an interested candidate could follow to complete an education degree in mathematics. Consequently, teachers in the Islamic states followed a defined, practiced pathway for teaching mathematics of different grades. "Several schools in Tunisia and Spain used this pathway for two centuries after Tusi" (Abdollah Tootian, personal communication, December 1, 2012).

In the process of making a series of astronomy books, Tusi encountered a bigger challenge. Astronomy was an important and interesting topic for the mathematics community around the world, but by that time a turning point on astronomy researches and books was reached. Tusi studied the *Small Collection of Astronomy*, a collection of books on astronomy that once was compiled by Greek Era mathematicians. He also

studied Ptolemy's *Almagest* and rejected some of the works and some ideas of Ptolemy, including his astronomical table and figurations. Tusi, rejecting Ptolemy's mathematical approach toward astronomical calculations, decided to re-write the entire books of *Small Collection of Astronomy* and to add Euclid's *Elements* and Ptolemy's *Almagest,* respectively, to the beginning and the end of it. He wrote tahrir on all books of the collection written by former mathematicians, and compiled a new collection that is famous as *Mutiwassitat* (medium) or *Great Collection of Astronomy* in the East. "Tusi used arithmetic and trigonometric calculations to solve astronomic problems and to calculate the movement of planets" (Nasr, 1987). Pak (2010) shares a statement from Mosaheb[46] that "Tusi made a classic practical mathematics of astronomy by writing *Mutiwassitat*" (translated by author, p. 13). In fact, *Mutiwassitat* is a *bridge* between the two great books of Euclid and Ptolemy.

> *Tusi "himself in the introduction of his tahrir of Manaleous'* Sphere *that is famous as* Akar *says he wanted to collect books known as* Mutiwassitat*, 'or books prepared to be situated between Euclid's* Osul *and Ptolemy's* Almagest *in mathematics education,' but doesn't mention the number of books included in the collection."*
>
> *(Massoumi, 1996, translated by author)*

Tusi's situation as the head of the Maragheh School allowed his tahrirs to be accepted more easily than other translations and tahrirs of Euclid's *Elements*. Later, when other tahrirs including *Mutiwassitat* were added to Tusi's work, the education system was introduced to a unique set of systematic books with similar approaches and mathematical terminology. This also increased the popularity of Tusi's tahrir on *Elements*. In some instances, Tusi's texts fully supplanted the originals, and as a result,

[46] Gholam-Hossein Mosahab was an Iranian mathematician who is said to be the father of the modern mathematics in Iran. He is also famous in non-mathematical society as the author of The Persian Encyclopedia, a Persian encyclopedia published in Iran. He also wrote the first book in mathematical analysis in Persian. He is the founder of the Institute of Mathematical Research (IMR) which is still known as one of the most important Iranian mathematical centers. The Institute of Mathematical Research started its work in October 1965 under the direction of Mosahab, as a semi-independent institute affiliated to Tarbiat Moaalem University, which is a teacher training university. (Author)

it became harder and harder to locate the original counterparts. One could easily find many more manuscript copies of Tusi's *Tahrir al-majisṭi*, for example, than copies of either of the original Arabic translations of the *Almagest*.

> *When, towards the end of the sixteenth century, the printing of the Arabic version of Euclid's* Elements *was undertaken by the Oriental Medici Press in Florence, the decision was taken not to go back to the sixteen-century Arabic translations of Euclid, although at least two of them were extant, but to opt for a hybrid text of a much later date that embodied Tusi's tahrir instead.* *(Encyclaepeoda Iranica, 2011)*

According to Nasr (1987) the model that Tusi presented for the movement of the universe was exactly the model that Copernicus presented for the Moon (p.104). Documents presented by several astronomers reveal that astronomers of the Renaissance were engaged in the same astronomical activities that the astronomers of the Maragheh Observatory were working on. Being involved on the same project, and using the same set of textual and mathematical equipment, it should be no surprise that Nicolaus Copernicus gained insight into the mathematical calculations of astronomy concerning the motions of Earth and other planets from the work of the astronomers at Maragheh Observatory, including that of Tusi.

As a mathematician, I learned that mathematics is valuable when it can be used practically to solve problems existing in society at the time or in the future. This was not a new understanding for me, as I have been trained as an engineer and have benefitted from applications of mathematics on a daily basis at work. However, the results of this research revealed to me that to approach problems effectively one should develop sufficient knowledge of different sciences. This is how Tusi started his scholarly life and dedicated much of that to continuously improving his scientific and engineering skills. Tusi was a skillful mathematician, and continued improving his skills while he was involved in on-going projects. I learned that I should continue improving my mathematics skills in order to offer something useful to mathematics community. I learnt that dedication and preciseness are two of the important qualities a mathematician needs to generate adequate mathematics books. I am now more determined to continue improving my skills while I am engaged at work, life, and educational projects. Tusi's life also reminded me of the importance of teamwork and networking in writing mathematics

books. Mathematicians who would like to write books would benefit from sharing ideas, strategies, and tactics of mathematics. They would also benefit from sharing writing styles in mathematics books. Receiving feedback from other mathematicians is also a gift for the book writers. Therefore, networking is a key to write proper mathematics books. Based on this research, Tusi was a strong communicator, and he systematically created a network since he was a young scientist. He achieved having a team of mathematician in Maragheh to work on astronomy. I feel that I am now much more prepared to participate in projects on which teams of mathematicians and scientists work together.

Tusi as an Educator

In Tusi's lifetime the social aspects of mathematics education became more crucial, and Tusi was not afraid to implement social characteristics into his teaching and training career. His scholarly life and achievements were influential in the reform of mathematics education of the Islamic Era before and after his death. He noticed that the nation was in jeopardy of losing the connection to mathematics education; so he, as an education leader, reformed the mathematics education of the region, and built an efficient system in which the following were parts of its known outcomes:

1. Saving mathematics books and treatises from being destroyed by collecting them in Maragheh University.
2. Saving mathematicians from the hardships of the critical condition of the region by inviting them to Maragheh Observatory.
2. Providing more opportunities for the public to study mathematics at Maragheh University and related schools.
3. Training mathematics teachers at Maragheh University.

Qualities by which Tusi reached his goals are discussed as follows:

Leadership Qualities

Tusi accepted all kinds of challenges in order to achieve his goals in establishing an adequate educational institute. Tusi's plan for running an institue was a necessity for Persia at the time of distraction due to the Mongols' attack. He saved the results of the

years of commitment of scholars in other places, such as “House of Wisdom” of Baghdad from destruction. Tusi, a mathematician, acted as a wise man for Hulagu, and lead and convinced him to save numbers of books to be tansported to a safer place. This act of Tusi demonstrates that an educational leader may need to get involved in politics in some extent to achieve some educational tasks. This matter might have been crucial centuries ago, but might still be a facot in some extent in our time.

Maragheh School, also known as Maragheh University, was the third greatest of all institutions in the Middle East after Gundishapur University and Baghdad’s House of Wisdom, with one big difference. Tusi had created a great opportunity for educating the public and training mathematics teachers. This had never happened in Middle East or during the Islamic Era (Abdollah Tootian, personal communication, December 1, 2012). Students of the Maragheh School received financial aid and sometimes housing. Tusi trained specialized mathematics instructors in different disciplines of mathematics where the applications of mathematics were targeted. Mathematics for trade, calendar calculations, money exchange, scale, weight, and construction were examples of elementary to high school courses (Abdollah Tootian, personal communication, December 1, 2012). More complicated topics were considered for university level educations. Other Islamic regions such as Egypt, Tunisia, and Spain followed the Maragheh School’s method of teaching to achieve the benefits of systematic mathematics education. Graduated students from Maragheh University were offered jobs in different schools throughout the Islamic states. They were respected and fairly paid. For the first time, teaching mathematics to the public had become a secure career and people were encouraged to register their children for the available courses (Abdollah Tootian, personal communication, December 1, 2012). As an educator, I learnt from Tusi that caring about the mathematics community and members of it is an important factor for being a successful leader. Tusi’s scholarly life also demonstrated that he had a flexible personality. He was able to work with different members of his team in Maragheh - this considering the fact that many different ideas might have been involved. This quality, to me, is the crucial differentiating factor between inclusive leadership and dominant leadership in mathematics education.

Literacy in Mathematics

Tusi's knowledge of Arabic, Greek, Turkish, and Pahlavi helped him to combine the words and phrases of the developed language of Arabic into his Middle Persian writings. Middle Persian written language was not sufficient for explanations and scientific purposes prior to Tusi. It was a rigid, less flexible language where writers had very limited ability to offer explanatory sentences. This was one of the reasons many scholars had decided to write in Arabic as they had found this a much stronger language. Tusi used Arabic vocabulary in Middle Persian grammar. He wrote *Zij* in Farsi using lots of Arabic vocabularies. Combining explanatory phrases, statements, and vocabularies of Arabic in writing scientific books in Farsi was a unique decision and evolvement offered and applied by an educational leader who cared about the quality of work without feeling restrictions on the language. Although many Persians and current Iranians are against existence of Arabic vocabularies in Farsi language but in fact, Arabic has improved the quality of Farsi as a language. The procedure of introducing Arabic words to Farsi is a unique topic for future research.

I also recognized what it takes for a curriculum to be generated. Walking through the pathway Tusi took to combine mathematics books with a steady terminology was the first step in making mathematics textbooks, and perhaps using those books for generating mathematics curriculums. This is likely different now, but it is interesting to see what it took for a mathematician or a team of mathematicians to establish a procedure for generating a curriculum. This thesis showed me the obstacles, challenges, frustrations during the procedure, and the feelings of success.

Inclusiveness and Application Purposes

Tusi wanted to enrich the life style of the society by benefitting from sciences and applications of them. His action supports that he believed in the inclusiveness of the nation when it came to the outcomes of education. The result of this research also proved that he was willing to share his findings with the community of scientists. As a leader, he found himself responsible for documenting and sharing the findings of the team of the Maragheh Observatory. So, he spent a great deal of time to write books such as *Zig* to record the outcomes of the observation at Maragheh. Less than a century after the establishment of Maragheh Observatory, copies were constructed in

Samarkand (Current Uzbekistan), in Istanbul (Turkey), in Fas (Morocco), in Delhi, Geepur, and Ageen (India), and in some Iranian states such as Tabriz and Fars. As an educator, I feel that I am responsible to record the information and knowledge that I collect or earn by research, participation in professional development activities, workshops, projects etc. I also feel responsible for sharing those with my colleagues or the mathematics community as a whole.

Tusi's engineering approaches toward mathematics education and applications of it became a model used for centuries in the classes of Islamic and European countries. Accomplishments of Tusi's "students in Maragheh School prepared the base of the modern astronomy of Europe in the 16th century. His students, Ali-Ibn-Omar Katebi, Qotbeddin Shirazi, and Kamaleddin Shirazi for the first time brought the possibility of the movement of Earth" (Pak, 2010, translated by author).

Saving the Life of Mathematicians and Mathematics Teachers

Tusi's caring for the scholarly life of mathematicians is not limited to physically rescuing them from being hurt by Arab regime, Ismaili assassins, or Mongols. It is true that he invited many scholars to Maragheh and helped them to establish an adequate career in a safe and respectful place and environment. But the more important aspect of the life of mathematicians that Tusi redeemed is teaching and practicing on the fundamentals of education. Tusi saved mathematicians scholarly life by showing them the reason and responsibilities of being a mathematician.

My research clearly and strongly emphasized a lesson that I had learned during my teaching program, which now is one of my fundamental beliefs on teaching. As a teacher, I believe that teachers teach who they are by the way they communicate the content to the students. The subjects teachers deliver to classes are excuses to communicate life and life skills to the students. I believe teachers are better off to live as they expect students to live. Tusi lived according to his beliefs. The purpose of his contribution to mathematics education was for the sake of mathematics and for humanity. In fact, he used mathematics to communicate a more organized and effective life style to his people. This was the way he delivered his lessons to his students, and the way he lived with them at Maragheh. I learnt that I should keep in mind that I am a

role model for my students facing mathematics problems. Since the beginning this thesis, I have put myself, at the same time as my students, in the positive frustration of solving problems in the class to teach them how to cope with problems, how to persist in solving them, and how to celebrate it when we solve problems. This has turned my honor students into a squad of learners who appreciate, enjoy, and celebrate learning mathematics. What is certain in the future of our system is that proper education is a must for everybody. What perhaps will be changed and might be completely revolutionized are the schooling system, subject, and topics.

Tusi as a Redeemer

Tusi was a renaissance man for his country. He dreamed of making a difference in the lives of people when new direction and hope for living a better life was most wanted and needed. He loved his country and his religion, and he was aware of the inequalities existed in Persia, before and after Islam. In fulfilling his destiny, Tusi faced several obstacles, but managed to survive to make a difference in the lives of Persians who had been under the pressure of regimes and rulers including Greeks, Arabs, Turks, and Mongols. Tusi had suffered from sanction, quarantine, and war, and thus felt and valued the freedom that education offers people.

Education for All

Tusi had a range of knowledge of different disciplines. He was a polymath, biologist, chemist, philosopher, physician, physicist, theologian, and a prolific writer. He employed this range of knowledge to help Persians with obstacles he had experienced during his life. This is very different from those scientists who either misused their knowledge or used it for their financial purposes. Tusi dedicated his life to educate people of any background and any life style. While he was in Alamut, he instructed mathematics and wrote different books under the rule of Ismailies, and when he was a minister of the Mongols he trained students and helped improving the capital of the region. Not only did he demonstrate an inclusive approach to the education of residents of Persia in a multicultural atmosphere where Persians, Arabs, and Mongols where living

together, but he also did his best to hold off the values that the majority of Persian residents valued.

The companionship and association of Hulagu and his son with Tusi and other scholars changed their lifestyle and morality. Many Mongol soldiers became new citizens of Persia and started a peaceful life. The Mongols became less dangerous as they took up the domesticated customs of scholars they had ruled (Frazier, 2005). The benefits of the Mongols' changes in morality and manner, in particular the educational developments arising from them, could be the topic of future research. This taught me the fact that we Canadians have the same situation in some extent with those of Tusi's time, and we should design a mathematics education by which majorities and minorities of our society equally take advantage of it, both in the present and in the future.

Academic Education Separated from Religious Education

Another advantage that Maragheh University had over similar schools was its environment. Students of all religions and beliefs studied in a secure atmosphere where science was the only target, and differences in religious ideology had nothing to do with the success of students. Although Tusi was from a religious family, his actions and decisions show he was not in favour of schools where religious training and academic courses were combined. He didn't allow clerics to join schools and influence the academic environment with their historical beliefs while students were engaged with scientific matters.

Further Outcomes

My conclusion of this thesis is prepared from my own point of view, as an engineer trained in Iran, and a teacher trained in Canada. I planned to study and discuss only those aspects of the life of Tusi that had benefited mathematics education. After all, I had limitations of different types when I was collecting the data. Although I traveled to Iran to have access to Farsi resources, I am not sure if I found all of them during the time I spent in Iran. I spent days in The National Library of Iran, visited universities and their libraries, and purchased books from Iran's market. But I feel that there is more work to do in terms of the collection of data. Tusi is a well-round scientist and scholar, and is

popular in neighbouring countries, including former Russia, Turkey, and Syria. Many research projects are conducted about Tusi and his contributions to mathematics in these countries, but most of them are in languages other than Farsi and English. Also, the majority of the articles and projects on Tusi, in Iran and other neighbouring countries, focus on his philosophical opinions, astronomy, and pure mathematics, while I was looking for his contributions to the mathematics education. Consequently, my research parameters were restricted to Tusi's achievements in the context of mathematics education, and only the research, books, and articles that were written either in Farsi or in English. If I decide to perform more research of this type in the future, I would team up with a partner who knows Arabic. Some historic Arabic books and Tusi's own books written in Arabic are better studied in Arabic rather than the translations in any other language. This provides the researchers with a deeper understanding of Tusi opinions and his contributions to mathematics. A Russian-speaking partner, also, would be effective in collecting data when it comes to Tusi's scientific works because I noticed that there are several books and dissertations available in Russian about Tuis and his scientific achivements. With respect to getting to know Tusi and his ideas about living as a Persian and a philosopher, a Persian candidate would be of greatest benefit to the project.

Time, which was not an issue at the beginning of my project, became an issue as I dug deeper into the life of Tusi. His political life was so entwined with his scholarly life that one should spend a longer time getting to know him before coming to a perfect understanding of how he managed to achieve his goals. The more I studied Tusi, the more I understood that my study would be too broad if I wished to cover all aspects of his life. Tusi had lived as a political icon and mathematics education has its roots in all aspects of his life. In several intervals of his life, Tusi had committed tremendous amounts of time developing or improving components of mathematics education while concurrently acting as a wise man, a prisoner, or the administrator of a big project and university.

For further studies on the life of Tusi and his achievements, a review of Tusi's books in detail with respect to terminology would be warranted. Translating his works from Arabic to other languages might be helpful for seeing the differences he made in writing style in comparison with the former mathematicians. An effective study would

search for the translations and writings that Tusi's students offered. More generally, the reform that Tusi offered in the translation and writing of mathematics books through his tahrirs and written books could be studied in detail. The history of schooling in Persia and in particular Maragheh, during Tusi's life and after it, would also be an interesting topic of study. There have been some recent studies about the latter, but there is room to develop a deeper understanding of the infrastructure of the Maragheh Observatory and School.

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Appendices

Appendix A.

Resistance Movements against Arab Rulers

Many of members of these activities were second and third generation Arabs whose families had migrated to the Persia and became accustomed to their citizenship and to the rich culture of Persians. Table 5 summarizes the name, approximate time, and the region of the most important and historically known movements against the dictatorship of Abbasid during the first century of their rule. All of these movements were highly organized by Persian activists who were basically against Arabs' exploitation of Persia, even against exported Islam as a religion.

Table 5. Movements Against Abbasid in the First Century of Their Rule

Date AD	Movement Name	Region and The Reason of Movement
754-758	Sunbath (Sinbad)	He was a Persian cleric from a small village called Ahan near Neishapur, Greater Khurasan, who incited an uprising against Abbasids for the vengeance of the assassination of Abu Muslim
762	Ravandian	In Khurasan. For vengeance of the assassination of Abu Muslim
771	Ostazsis	In Harat, east of Khurasan. For religious reasons against giving free rein to Arab Muslims
780	Moghanne	Hashem, known as Moghanne, was born in Merve, in Greater Khurasan. His movement against Abbasid and in vengeance of the assassination of Abu Muslim was known as In-Whites
783	Mohammareh	Abdolghahhar the chief of Mohammareh was born in greater Khurasan. His movement against Abbasid, in vengeance of the assassination of Abu Muslim was known as In-Reds
795-838	Babak Khurramdin	Babak was born in north of current Iran. His movement was the most extreme anti-Arab movement against Abbasid.
839	Maziar[47]	Maziar was born in Tabarestan, in north of current Iran. He followed Babak's ideas and movement.

* Source: Bagher al-Uloom Research Centre. (www.pajoohe.com)

[47] Said Nafisi, Iranian historian, in his "Babak Khorramdin Delawar-e-Azarbaijan" (Babak Khorramdin, the braveheart of Azarbaijan), brings the actual quote from Maziyar: "I (*Maziyar*), *Afshin Kheydar* son of *Kavus*, and *Babak* had made an oath and allegiance that we re-take the government back from the Arabs and transfer the government and the country back to the family of Kasraviyan (Sassanids)" (Nafisi, 1995)

These movements were not successful for long time as they did not have the support of Arab residents of Persia. More organized movements in which Arab residents of Persia were involved, better survived against the central government of Baghdad. Table 6 summarizes the date, name, capital and the region of the historically known Persian dynasties separated from Abbasid during the five centuries of the rule of Abbasid.

Table 6. Persian Dynasties Separated from Abbasid during the Five Centuries of the Rule of Abbasid

Date (AD)	Dynasty Name	Capital City(ies)	Region of Dynasty
822-881	Tahirid	Neishapur	Khurasan and Sistan from current Iran. Mesopotamia from current Iraq. First independent Persian regime with Arabic background.
866-903	Saffarid	Zarang	Central of current Iran, Afghanistan, Tajikistan, Pakistan. The first independent regime that didn't speak Arabic.
874-1004	Samanid	Balkh and Bokhara	Khurasan, Sistan, and Kerman from current Iran, and Kharazm, Hirkan, Makran in middle of Asia. They established Farsi (Dari) font, but it was amalgamated with Arabic.
1038	Ziarian	Gonbad-e- Kavus	Tabarestan, Deilam, and Roodbar all north of current Iran.
934-1055	Buyid	Shiraz	Central and north (Deylam, Gilan) and west of current Iran, Mesopotamia in current Iraq. Spoke both Arabic and Farsi but embraced Farsi and poetry.
975-1187	Ghaznavid (Turks of Khurasan)	Ghaznin and Lahur	East of current Iran, Afghanistan, Pakistan, western India. Spoke Dari and Arabic
1038-1194	Seljuk (Turks of Kharazm)	Neishapur, Rey, and Esfahan	Current Iran, Samarqand of Tajikistan, Iraq , Georgia, Armenia, Turkey, Syria
1077-1231	Kharazmian	Kharazm	Kharazm of Tajikistan, Khurasan of Iran, and central Asia

[a] Source: http://www.farsinet.com/iran/persian_dynasties.html

Appendix B.

Factors Affecting the Fall of Baghdad by Mongol

Some of the reasons which caused the fall of Baghdad rather quickly are as follows:

- Arab troops had been neglected by their ruler and were not mentally prepared for a hard battle with Mongol. Mongol's earlier invasions had been reported and recorded highly demolishing and frightening, and Arab soldiers were afraid of Mongol.
- Centuries-old rift between the Sunni and Shi'a Muslims had weakened the Arab army. There was a separation between the lines of the army, included high-ranked generals.
- Non-Muslims who lived under the rule of Abbasid were angry with Abbasid. Non-Muslim residents of Baghdad had experienced discriminations by Arab authorities. The business opportunities were not equally distributed between the residents.
- Mongol had the support of non-Muslims, especially Christian community of Baghdad. Many Christians, who used to live in the region, saw the Mongol as saviors.