Resource Allocation in Wireless Relay Networks

By

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ABSTRACT

The idea of using cooperating relays has been much discussed in the past decade. In a wireless communication system, relaying techniques can offer significant benefits in the throughput enhancement and range extension. On the other hand, cognitive radio is an interesting concept for solving the problem of spectrum availability by reusing the underutilized licensed frequency bands. In a cognitive radio network, relays can be particularly useful for reducing the transmission power at the source and thus reduce the interference to the primary users. In this thesis, we study resource allocation problems for cognitive radio networks that employ relays. In this work, the transmission power of the nodes (users and relays) is the resource that we wish to allocate. The power allocation problems are formulated as non-convex non-linear programs and they do not have a structure that could guarantee the quality of the solution. We present a method of transforming the proposed optimization problems to a new formulation so that ε-optimal algorithms can be designed. In general, the transformed problem exhibits certain properties, which enable us to solve the optimization problems to a desirable accuracy by applying known global optimization techniques. We note that the global optimization techniques require significant computations to solve our proposed optimizations. Therefore, we propose low complexity heuristics that provide suboptimal solutions to the given optimization problems. The simulation results show that the performance of the heuristics is close to their respective optimal solutions.

Keywords: Cognitive radio, Cooperative communication, Two-Way Relay, Multi-Way Relay, Global Optimization

DEDICATION

To my family

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LIST OF ABBREVIATIONS

AF	Amplify-And-Forward
CCS	Cooperative Communication System
CRS	Cognitive Radio System
DSA	Dynamic Spectrum Access
FCC	Federal Communications Commission
GeSI	Global e-Sustainability Initiative
GPAMF	Greedy Power Allocation Max-Min Fairness
GPASM	Greedy Power Allocation Sum-Capacity Maximization
GTSPA	Greedy Two-Step Power Allocation
IEA	International Energy Agency
ITU	International Telecommunication Union
ICT	Information And Communication Technologies
JSRMF	Joint Source and Relay Power Allocation with Max-Min Fairness
JSRSM	Joint Source and Relay Power Allocation with Sum-Capacity Optimization
MO	Monotonic Optimization
PU	Primary User
SU	Secondary User
WEO	World Energy Outlook
WRAN	Wireless Regional Area Network

CHAPTER 1: INTRODUCTION

1.1 Research Motivation

The idea of using cooperating relays has been much discussed in the past decade [1], [2]. In a wireless communication system, relaying techniques can offer significant benefits in the throughput enhancement, and range extension [1]. Relaying protocols for one-way half-duplex relays are considered in [1]. Recent research on cooperative communications has shown that half-duplex two-way relaying is spectrally more efficient [2] than the conventional half-duplex one-way, [1], relaying. In two-way relaying two users can exchange data with each other using a relay in two time slots. Recently, the idea of two-way relay channel has been extended to multi-way relay channel in [3]. In this model, multiple users (≥ 2) can exchange information with each other using a relay terminal in two time slots.

Cognitive radio [4], [5], [6] is an emerging technology intended to enhance the utilization of the radio frequency spectrum. A combination of cognitive radio with cooperative communication can further improve the future wireless systems performance. However, the combination of these techniques raises new issues in the wireless systems that need to be addressed. In particular, in this thesis, we address the issue of allocating transmission power to the nodes (users and relays) subject to the constraints on the nodes' transmit powers. We provide low complexity heuristics to the proposed optimizations and compare them with their respective optimal solutions. We also study the rate and power allocation problem in a wireless relay network employing multi-way relaying and provide an optimal solution to the proposed optimization.

1.2 Background

In this section, we provide a brief overview of cooperative communication and cognitive radio.

1.2.1 Cognitive Radio System

Formally, a cognitive radio is defined as [7]

"*A radio that changes its transmitter parameters based on the interaction with its environment*"

The cognitive radio has been mainly proposed to improve the spectrum utilization by allowing unlicensed (secondary) users to use underutilized licensed frequency bands [8] [9] [10]. In reality, unlicensed wireless devices (e.g., automatic garage doors, microwaves, cordless phones, TV remote controls etc.) are already in the market [11] [12]. The IEEE 802.22 standard for Wireless Regional Area Network (WRAN) addresses the cognitive radio technology to access white spaces in the licensed TV band. In North America, the frequency range for the IEEE 802.22 standard will be 54–862 MHz, while the 41–910MHz band will be used in the international standard [9]. Table 1.1 shows the IEEE 802.22 system parameters, e.g., frequency range, bandwidth, modulation types, maximum transmit power ratings, multiple access schemes, etc. [13].

In the context of cognitive radio, the Federal Communications Commission (FCC) recommended two schemes to prevent interference to the television operations due to the secondary (unlicensed) users. These are listen-before-talk and geo-location/database schemes [11] [12]. In the listen-before-talk scheme, the secondary/unlicensed device senses the presence of TV signals in order to select the TV channels that are not in use. In geo-location/database scheme, the licensed/unlicensed users have a location-sensing device (e.g., GPS receiver etc.) The locations of primary and secondary users are stored in a central database. The central controller (also known as spectrum manager) of the secondary/unlicensed users has the access to the location database.

Table 1.1 IEEE 802.22 system parameters.

Parameters	Specification	Remarks
Frequency range	54-862 MHz	TV band
Bandwidth	6 MHz, 7 MHz, 8 MHz	
Modulation	QPSK, 16-QAM, 64-QAM	
Transmit power	4W	For USA, may vary in other regulatory domains
Multiple access	OFDMA	

The main functions of cognitive radio to support intelligent and efficient utilization of frequency spectrum are as follows:

1.2.1.1 Spectrum sensing

Spectrum sensing determines the status of the spectrum and activity of the primary users [9] [4]. An intelligent cognitive radio transceiver senses the spectrum hole without interfering with the primary users. Spectrum holes are the frequency bands currently not used by the primary users. Spectrum sensing is implemented either in a centralized or distributed manner. The centralized spectrum sensing can reduce the complexity of the secondary user terminals, since the centralized controller performs the sensing function. In distributed spectrum sensing, each mobile device (secondary user terminal) senses the spectrum independently. Both centralized and distributed decision-making is possible in distributed spectrum sensing [9]. The central controller (spectrum manager), based on the spectrum sensing information, allocates the resources for efficient utilization of the available spectrum. One major role of the central controller is to prevent overlapped spectrum sharing between the secondary users [7] [9] [10].

1.2.1.2 Dynamic Spectrum Access

Dynamic spectrum access (DSA) is defined as real-time spectrum management in response to the time varying radio environment − e.g., change of location, addition or removal of some primary users, available channels, interference constraints etc [7] [10]. There are three DSA models in the literature, namely, exclusive-use model, common-use model and shared-use model [10]. Fig. 1.1 shows a hierarchal overview of DSA.

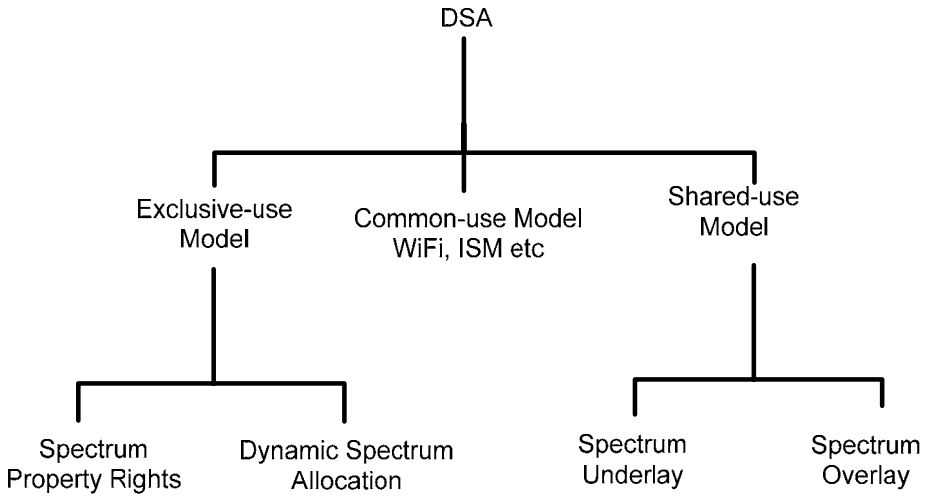

Fig. 1.1 Dynamic spectrum access strategies.

The exclusive-use model has two approaches, spectrum property rights and dynamic spectrum allocation. In spectrum property rights, owner of the spectrum can sell and trade spectrum; and is free to choose the technology of interest. Dynamic spectrum allocation improves spectrum efficiency by exploiting the spatial and temporal traffic statistics of different services [10]. The European Union funded DRiVE (Dynamic Radio for IP Services in Vehicular Environments) project is a classical example of dynamic spectrum allocation [14]. It uses cellular (e.g., GSM, GPRS, and UMTS) and broadcast technologies (e.g., Digital Video Broadcast Terrestrial, Digital Audio Broadcast) to enable spectrum efficient vehicular multimedia services.

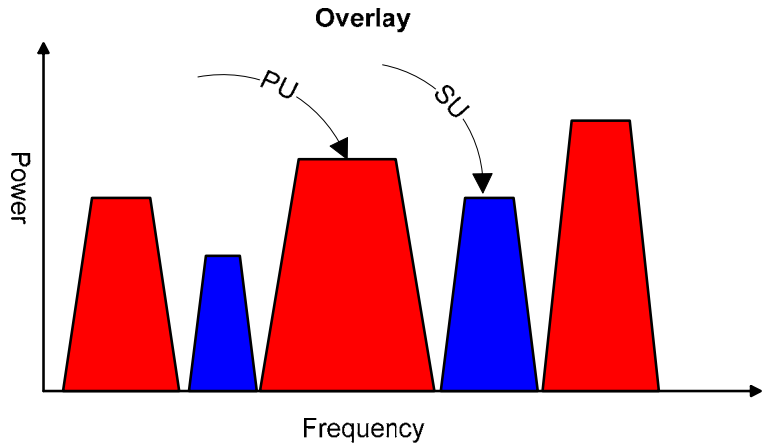

Fig. 1.2 Overlay spectrum access.

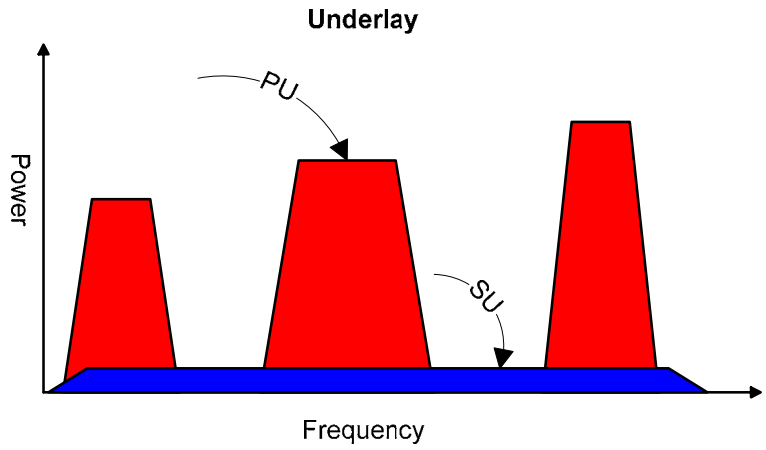

Fig. 1.3 Underlay spectrum access.

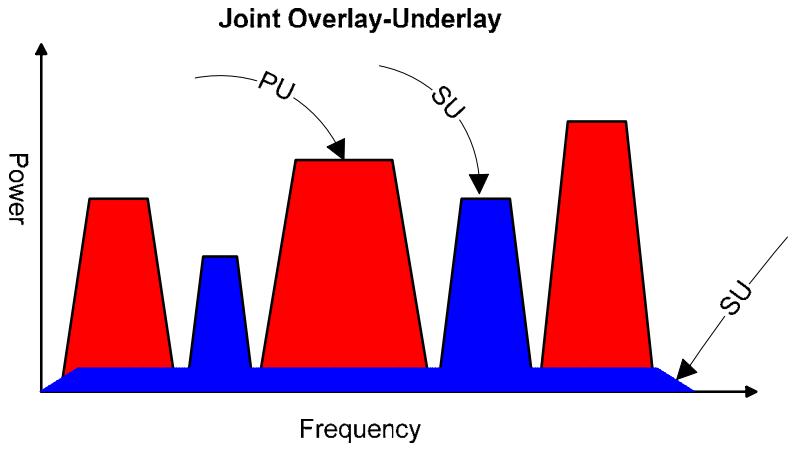

Fig. 1.4 Joint overlay and underlay spectrum access.

The common-use model is an open sharing regime in which spectrum is accessible to all users. The ISM (industrial, scientific and medical) band and Wi-Fi are examples of the commons-use model. Spectrum underlay and overlay approaches are used in the shared-use model [9] [10]. Spectrum overlay or opportunistic spectrum access is shown in Fig. 1.2. In spectrum overlay, the secondary users first sense the spectrum and find the location of a spectrum hole (vacant frequency band). After locating the vacant frequency bands, the secondary users transmit in these frequency bands. In spectrum underlay technique, the secondary users can transmit on the frequency bands used by the primary users as long as they do not cause unacceptable interference for the primary users. This approach does not require secondary users to perform spectrum sensing, however the interference caused by the secondary user's transmission must not exceed the interference threshold. Fig. 1.3 shows the spectrum underlay model.

In [15], a joint spectrum overlay and underlay method is proposed for better spectrum utilization. An illustration of joint spectrum overlay and underlay is shown in Fig. 1.4. In joint spectrum overlay and underlay approach, the secondary users with the help of spectrum sensing first try to find a spectrum hole. If there is a spectrum hole then the secondary users can use the spectrum overlay technique. If there is no spectrum hole then the secondary users will use spectrum underlay technique.

1.2.2 Cooperative Communication

We now provide a brief background of cooperative communications. Recently the idea of cooperative communication has gained much attention. The cooperative communications exploit the broadcast nature of wireless channels. The basic idea is that the relay nodes can assist the transmissions of the source node by relaying a replica of the transmission of the source node that in turn exploits the inherent spatial diversities. Recent research in wireless communication systems shows that relaying techniques can offer significant

benefits in the throughput enhancement, and range extension [1]. A number of relaying schemes e.g., amplify-and-forward (AF), decode and forward (DF), incremental relaying etc. for improving the performance of the wireless networks are in the literature e.g., [1], [16]. In a simple AF relaying scheme, a relay amplifies the received signal and forwards it to the destination. In decode and forward relaying scheme, a relay first decodes the received signal and then transmits the re-encoded signal to the destination. Table 1.2 shows a simple cooperative communication protocol. In this protocol, conveyance of each symbol from the source to the destination takes place in two phases (two time slots). In the first phase, the source transmits its data symbol, and the destination and the relay(s) receive the signal carrying the symbol. In the second phase, the relay(s) forwards the data to the destination.

Recent research on cooperative relaying has shown that half-duplex two-way relaying is spectrally more efficient [2] than the conventional half-duplex one-way, [16], relaying. In two-way relaying, a pair of users exchanges information with each other using a relay using two time slots. The key idea is that a user can cancel the interference (generated by its own transmission) from the signal it receives from the relay to recover the transmission of other terminals. Table 1.3 illustrates the two-way relaying.

Half-duplex protocols for two-way relaying using multiple relays have been discussed in [17] and [18]. In this work, we study the power allocation problem for different types of two-way relaying protocols. In one of the protocols, which we will refer to as orthogonal amplify-and-forward (OAF) relaying, the transmissions of the relays are separated in time by allocating each relay a different time-slot band. In the other relaying protocol, which we will refer to as shared-band amplify-and-forward (SAF) relaying, the relays share the same medium for their transmissions. We study the problem of jointly allocating power to the users and the relays for both OAF and SAF relaying protocols. Tables 1.4 and 1.5 show OAF and SAF bidirectional relaying protocols.

Recently, the idea of two-way relaying has been extended to the case of multiple users sharing a single relay that is referred to as multi-way relaying [8]. In this thesis, we consider multi-way relaying where relay performs amplify and forward relaying. In this relaying scheme, in the first time-slot, the users broadcast their data and in the second time-slot, the relay broadcasts the amplified data. The capacity and achievable rates for different relaying techniques (amplify-and-forward (AF), decode-and-forward and compress-and-forward) for multi-way relay channel are discussed in [3]. Table 1.6 presents the multi-way relaying communication protocol.

Table 1.2 Half-Duplex One-Way Relaying.

Time T_1	**Time T_2**
$S \rightarrow D$, $S \rightarrow R$	
	$R \rightarrow D$

Table 1.3 Half-Duplex Two-Way Relaying.

Time T_1	**Time T_2**
S_1, $S_2 \rightarrow R$	
	$R \rightarrow S_1$, S_2

Table 1.4 OAF Two-Way Relaying.

Time T_1	**Time T_2**	**Time T_3**	**Time T_4**	**…**	**Time T_{L+1}**
S_1, $S_2 \rightarrow (R_1, R_2, .., R_L)$	$R_1 \rightarrow (S_1, S_2)$	$R_2 \rightarrow (S_1, S_2)$	$R_3 \rightarrow (S_1, S_2)$	…	$R_L \rightarrow (S_1, S_2)$

Table 1.5 SAF Two-Way Relaying.

Time T_1	**Time T_2**
S_1, S_2 $\rightarrow$(R_1,R_2,..,R_L)	
	(R_1,R_2,..,R_L) $\rightarrow$ S_1, S_2

Table 1.6 Multi-Way Relaying.

Time T_1	**Time T_2**
(S_1, S_2...,S_K) $\rightarrow$R	
	R $\rightarrow$ (S_1, S_2...,S_K)

1.3 Thesis Overview

The main objective of this thesis is to determine power allocation in wireless relay networks. This thesis discusses three problems: 1) Power allocation in OAF relay networks, 2) Power allocation in SAF relay networks and 3) Power allocation in multi-way relay networks. In all three problems, we examine the effect of different system parameters (e.g. maximum transmit power interference threshold level, the number of primary users, the number of secondary users, relay power levels, etc.) on the performance of the proposed algorithms.

1.3.1 Power allocation in OAF relay networks

In this work, we study a cognitive radio network (comprising multiple primary users) in which a pair of sources communicates with each other through multiple relays that employ two-way amplify-and- forward relaying. The relays use orthogonal channels to transmit their data. We formulate optimization problems to adequately decide the transmission powers of the nodes in such networks. We consider two optimization problems. In one problem, we consider maximizing the minimum among the two sources' capacities. In the other

problem, we consider maximizing the sum capacity of the system. Our formulated optimization problems turn out to be non-convex and non-linear.

For theoretical interest in dealing with these non-convex optimization problems for deciding the power levels, we show in this work that these optimization problems have a set of properties that guarantees ε-convergence if the monotonic optimization techniques are used. Then, we apply the monotonic optimization [19], [20] algorithm to obtain the optimal solution of the proposed non-linear non-convex programming problems. Although the monotonic optimization algorithm can guarantee convergence to a solution that has a performance arbitrarily close (within an arbitrary number ε) to the optimal performance, our experimentation with the monotonic optimization applied to this relay power allocation problem shows rather slow convergence and heavy computational load. Therefore, we propose a low-complexity heuristics, which we name as Greedy Power Allocation with Max-Min Fairness (GPAMF) and Greedy Power Allocation with Sum Capacity Maximization (GPASM). The simulation results show that the proposed heuristics perform well in comparison with the respective optimal solutions and have much lower computational complexity than the monotonic optimization algorithm.

1.3.2 Power allocation in SAF relay networks

In this work, we consider a cognitive radio network (comprising multiple primary users) in which a pair of sources communicates with each other through multiple relays that employ two-way amplify-and-forward relaying. The relays employ SAF relaying, i.e., in the first slot the users broadcast their data and in the second slot, the relays simply re-scale and re-transmit the received signal.

We study the problem of allocating powers to the users and relays such that the minimum among the users' SNRs is maximized subject to the constraints on the transmit power of the nodes. The formulated optimization problem is non-convex non-linear program and we do not see a special structure that could guarantee the quality of a solution. We observe that we can transform the

problem into an equivalent problem (although still a non-convex non-linear program) which exhibit a special property. In the transformed problems, we observe that the objective function and the constraints are increasing function of each optimization variable when other variables are fixed. This property enables us to determine the global optimal solution to our optimization problems by applying the concepts of monotonic optimization algorithm [19]. Monotonic optimization algorithm guarantees convergence to a solution that has a performance arbitrarily close (within an arbitrary number ε) to the optimal performance. However, the application of monotonic optimization techniques to solve the optimization problem requires significant computations. Therefore, we propose a low-complexity heuristic. We perform simulations to examine the quality of the solution obtained from the proposed heuristic and benchmark its performance using the optimal solution obtained by using monotonic optimization techniques.

1.3.3 Power allocation in Multi-Way relay networks

In this work, we consider a multi-way relay channel comprising multiple users and a single relay. We assume that the relay terminal uses AF relaying. For such systems, we study the problem of allocating power to the users and the relay terminal such that the minimum among the users' transmission rates is maximized subject to the constraints on the transmission power of the nodes. The formulated optimization problem is a non-convex non-linear program and does not have a structure to guarantee the quality of a solution. We observe that we can transform the problem into another equivalent problem (although still a non-convex non-linear program) which could be solved to global optimality by applying the concepts of monotonic optimization algorithm [19].

1.4 Literature Review

This section contains a literature review for resource allocation strategies in wireless communication system. In particular, we review the literature

corresponding to the relay assignment and power allocation in relay assisted wireless networks.

Table 1.3 summarizes the literature review for the relay assignment strategies (RAS) in the wireless communication systems. The first column in Table 1.3 lists the objective functions as defined in the literature. The succeeding columns show the relaying type (one-way or two-way relaying), cognitive radio capability, protocol type (e.g. centralized, distributed or decentralized) and power allocation capability respectively. There are three major classes of resource allocation in cooperative communications. The classes are, centralized resource allocation [21-24] [28] [33-32], distributed resource allocation [25] [34], and decentralized resource allocation [26] [27] [33].

In [21], joint bandwidth and power allocation strategies for a Gaussian relay network are investigated. Orthogonal and shared-band AF and DF schemes are analyzed for joint bandwidth and power allocation. The main objective of joint bandwidth and power allocation is to maximize the signal-to-noise ratio at the receiver using AF and DF schemes. The study in [22] proposes a centralized framework that selects multiple relays for transmission in a two-hop network. The aim of the multiple relay selection is to maximize the SNR at the destination using binary power allocation at the relays. An optimal relay assignment and power allocation in a cooperative cellular network is discussed in [23]. Using the sum-rate maximization as a design metric, the authors proposed a convex optimization problem that provides an upper bound on performance. A heuristic water-filling algorithm is also suggested to find a near-optimal relay assignment and power allocation. In [24], a linear-marking mechanism is investigated for relay assignment in a multi-hop network with multiple source-destination pairs. The aim of the proposed linear-marking mechanism is to maximize the worst user capacity.

A distributed nearest neighbour relay selection protocol and its outage analysis are presented in [25]. For the relay assignment in a multiuser communication system, decentralized protocols are discussed in [26] and [30].

The decentralized framework in [26] uses decode and forward relaying and assigns relays without exercising power control. In [30], decentralized amplify and forward protocol is used for joint relay assignment and power control. The scheme maximizes a harmonic mean-based approximate expression for the instantaneous received signal-to-noise ratio. The relay assignment and selection schemes described in [21] − [27] and [30] are not applicable in the CRS because the interference caused by the relays to the primary users can exceed the prescribed interference limit.

The relay selection scheme for a cognitive radio network has been considered in several recent works [28]−[34]. In [28], a mathematical formulation is proposed with the objective of minimizing the required network-wide radio spectrum resource for a set of user sessions. The proposed formulation is a mixed-integer non-linear program. The authors proposed a lower bound for the objective by relaxing the integer variables and using a linearization technique. A near-optimal algorithm is presented that is based on a sequential fixing procedure, where the integer variables are determined iteratively via a sequence of linear programs. In [31], relay selection in multi-hop CRS with the objective of minimizing the outage probability is proposed. The power allocation problem is solved using standard convex optimization techniques for both AF and DF protocols under Rayleigh fading conditions. A joint relay selection, spectrum allocation and rate control (JRSR) scheme in CRS is proposed in [21]. A three-stage sub-optimal algorithm is proposed to address the JRSR problem. A non-cooperative game based decentralized power allocation for primary and secondary users is considered in [33]. The two kinds of links, one of which includes the primary users and their relay, the other includes the secondary users and their relay, are treated as players of the non-cooperative game. Each player competes against the other by choosing the power allocation strategy that maximizes its own rate, subject to the QoS threshold of the primary system. A relay-assisted iterative algorithm is proposed to efficiently reach the Nash equilibrium. In [34], authors proposed both centralized and distributed power allocation schemes for multi-hop wideband CRS. The main objective is to

maximize the output signal-to interference plus noise ratio (SINR) at the destination node of the CRS.

The aforementioned work on relay networks cannot be applied to the bidirectional relay networks. Reference [35], [36], [37] discussed power allocation for two-way relay networks with single relay. In [35], a network comprising a single relay and multiple source-destination pairs is considered, and an optimization problem to decide transmission power of the relay was studied. Reference [35] does not consider jointly allocating power to the sources and the relay. In [36], a cognitive radio network comprising one source-destination pair, a single relay, and a single primary user is considered. For this system, the problem of allocating power to the source and determining optimal beam-forming vectors of the relay is considered. In [37], a two-way relay network comprising a pair of users and multiple relays is considered and the problem of selecting a single relay and allocating power to it is considered.

Power allocation for OAF bidirectional relaying is considered in [17]. Reference [17] considers a two-way relay network comprising a pair of users and multiple relays and performs joint source and relay power allocation. However, reference [17] presents a simpler sub-optimal power allocation by maximizing the lower bound that serves as a good approximation only at the high-SNR region. Power allocation for NAF bidirectional relaying is considered in [38]-[42]. Reference [39], [42] and [43] consider the optimization of only relays' transmit powers. Joint users' and relays' transmission powers allocation is considered in [10], [31], [34] where optimization problems are formulated to determine the transmission power of the users and the beam-forming vectors of the relay. However, the optimizations in [10], [31], [34] consider only a single sum-power constraint and the obtained solutions cannot be applied to the optimization problems that has both sum-power and multiple individual power constraints.

Table 1.3 Literature Survey.

Objective	**Relaying**	**CR**	**Protocol**	**Power Control**	**Ref.** **[Ref Num, Name]**
Maximize the SNR of AF/DF shared bandwidth schemes	One-Way	No	Centralized	Yes	[21, I. Maric et. al]
Select the multiple relays to maximize the SNR in shared bandwidth AF scheme	One-Way	No	Centralized	Binary Power Control	[22, Y. Jing et al.]
Sum-rate maximization	One-Way	No	Centralized	Yes	[23, Kadloor et al.]
Maximize the minimum capacity	One-Way	No	Centralized	No	[24, Y. Shi et al.]
Protocols and outage analysis	One-Way	No	Distributed	No	[25, Sadek et al.]
Average sum-capacity	One-Way	No	Decentralize	No	[26, P. Zhang et al.]
Maximize the instantaneous received SNR	One-Way	No	Decentralize	Yes	[27, G. Farhadi et al.]
Minimize the total bandwidth	One-Way	Yes	Centralized	No	[28, T. Hou et al.]
Closed-form expressions of detection probability	One-Way	Yes	----	No	[29, J. Zhu et al.]
Cooperation between primary user and secondary user.(secondary user act as relay for primary user)	One-Way	Yes	----	Yes	[30, R. Manna et al.]
Minimize outage probability	One-Way	Yes	Centralized	Yes	[31, Jayasinghe et al.]
Maximize Average throughput	One-Way	Yes	Centralized	Yes	[32, H. Chun et al.]
Maximize the rate utility function	One-Way	Yes	Decentralize	Yes	[33, Xiaoyu et al.]
Maximize SNR of RD link	One-Way	Yes	Distributed	Yes	[34, Mietzner et al.]
Sum-Capacity Maximization for AF/DF relaying	Two-Way	No	Centralized	Yes	[35, Chen et al.]

Sum-Capacity Maximization for OAF two-way relaying	Two-Way	No	Centralized	Yes	[17, Zhang et. al]
Sum-Capacity Maximization for SAF two-way relaying	Two-Way	No	Centralized	Yes	[39, vaze et. al]
Beamforming in SAF Two-Way relaying	Two-Way	No	Distributed	Yes	[38], [40],[42]-[43]
Beamforming in SAF Two-Way relaying	Two-Way	Yes	Centralized	Yes	[36]
Single relay selection and power allocation	Two-Way	No	Centralized	Yes	[37], [41]

1.5 Organization of Thesis

The structure of the thesis is as follows. Chapter 2 describes the power allocation in two-way relay assisted cognitive radio networks where the relays employ OAF relaying. Chapter 3 describes the power allocation in two-way NAF relaying subject to individual and sum-power constraints. Chapter 4 describes power allocation in multi-way relay channel.

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CHAPTER 2: POWER ALLOCATION IN ORTHOGONAL TWO WAY RELAY ASSISTED COGNITIVE RADIO NETWORKS

The conventional one-way relaying protocols suffer from a loss in spectral efficiency due to the half-duplex nature of the terminals. In order to increase the spectral efficiency of such a relay network, a bidirectional (two-way) relay assisted communication protocol was suggested in [3]. In this chapter, we study a cognitive radio [1] network (comprising multiple primary users) in which a pair of sources communicates with each other through multiple relays that employ two-way amplify-and-forward relaying. The multiple relays transmit in orthogonal channels that can be achieved by assigning each relay a non-overlapping time-slot band for their respective transmissions as in [4]. We formulate optimization problems to adequately decide the transmission powers of the nodes in such networks. We consider two optimization problems. In one problem, we consider maximizing the minimum among the two sources capacities. In the other problem, we consider maximizing the sum capacity of the system. Our formulated optimization problems turn out to be non-convex and non-linear.

Power allocation in one-way relay networks has been well studied. Reference [6] presents a distributed single and multiple relay selection schemes with relay power allocation. In [7], a one-way wireless relay network comprising multiple relays and a source-destination pair is studied and optimal relay power allocation is determined. Joint source and relay power allocation is studied in [8]. Reference [8] performs joint optimization of the source and the relay power by using a high SNR approximation of the SNR at the destination. Unfortunately, the power allocation schemes developed for one-way relays cannot be directly applied to the two-way relays, as the optimizations are very different.

Reference [4] deals with joint source and relay power allocation in two-way relay networks where the relays use orthogonal channels. In [4], a high SNR approximation of the SNR at the two users is used to jointly determine the sources' and relays' transmit powers. However, the formulation in [4] is not relevant to cognitive radio network. Bidirectional relaying in the context of cognitive radio has been considered in [5]. In [5], a cognitive radio network comprising one source-destination pair, a single relay, and a single primary user is considered. For this system, the problem of allocating power to the source and determining optimal beam-forming vectors of the relay is considered. In this work, we provide an optimal joint source and relay power allocation. We also consider different cost functions (max-min and sum-rate) and we provide low-complexity heuristics to determine sub-optimal solutions to our optimizations.

For theoretical interest in dealing with these non-convex optimization problems for deciding the power levels, we show in this chapter that these optimization problems have a set of properties that guarantees ε-convergence if the monotonic optimization (MO) algorithm [9], [10] is used. We observe that our formulated optimizations have specific structures that enable us to reduce the number of optimization variables to only two variables. The resulting optimization problems are still non-convex. However, we show that they can be transformed to equivalent optimizations that could be solved to global optimality by using the methods of MO algorithm. Although the MO algorithm can guarantee convergence to a solution that has a performance arbitrarily close (within an arbitrary number ε) to the optimal performance, our experimentation with the MO algorithm applied to this relay power allocation problem shows rather slow convergence and heavy computational load. Therefore, we propose a low-complexity heuristics, which we name as Greedy Power Allocation with Max-Min Fairness (GPAMF) and Greedy Power Allocation with Sum Capacity Maximization (GPASM). The simulation results show that the proposed heuristics perform well in comparison to the respective optimal solutions and have much lower computational complexity than the MO algorithm.

Table 2.1 Notations used in chapter 2.

Symbol	Definition
M	Number of primary users
L	Number of relays
s1,s2	Source 1, source 2
f_l	Channel gain from source 1 to the *l*th relay
g_l	Channel gain from source 2 to the *l*th relay
$h_{l,m}$	Channel gain from *l*th relay to the *m*th primary user
$h_{s1,m}$	Channel gain from *s1* to the *m*th primary user
$h_{s2,m}$	Channel gain from *s2* to the *m*th primary user
p_{s1} p_{s2}	Transmission power of source 1, souece2
p_l	Transmission power of *l*th relay
p_l^{max}	Maximum allowed transmission power of the *l*th relay
I_m^{max}	Maximum allowed interference at *m*th primary user
$\mathbf{R}_+^n \left(\mathbf{R}_{++}^n\right)$	Set of non-negative (positive > 0) integers

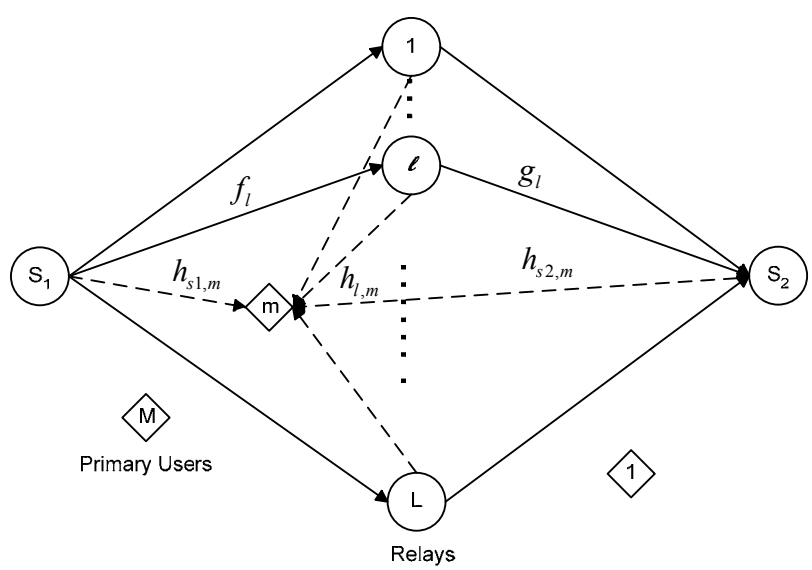

Fig. 2.1 Two-way relay assisted Cognitive Radio network

2.1 System Model

We consider a two-way relay system with two sources and *L* relays. Our system model also includes M primary users, for which the transmission power of the cognitive radio nodes (secondary users) must be limited. Figure 2.1 depicts our system model. The two sources will communicate with each other with the help of relays that use amplify-and-forward relaying. The relays, source 1 (*s*1), and source 2 (*s*2) are equipped each with a single antenna. We assume that the transmission channels of the relays are orthogonal to one another, which can be achieved by assigning the relays to non-overlapping frequency bands or time slots. We denote by f_l the channel gain from s1 to the *l*th relay, g_l the channel gain from s2 to the *l*th relay, $h_{s1,m}(h_{s2,m})$ the channel gain from s1 (s2) to the *m*th primary user and by $h_{l,m}$ the channel gain from *l*th relay to the *m*th primary user. For gaining simple insights to the system, in this chapter we assume that each channel between a source and a relay is symmetric. It is also assumed that transmissions from all nodes are perfectly synchronized. Let p_{s1}, p_{s2} and p_l denote, respectively, sources s1's and s2's transmission powers per dimension.

We consider a two-step two-way amplify-and-forward (AF) scheme, as given in [4] for cooperative communication which we will refer to as orthogonal amplify and forward (OAF) relaying. In the first step the users (sources) send their data to the relays. The signal received by relay *l* (*l* = 1, 2,.., *L*) is

$$y_l^n = \sqrt{p_{s1}} f_l \, X_{s1} + \sqrt{p_{s2}} g_l \, X_{s2} + Z_l \tag{2.1}$$

where complex-valued random variables X_{s1} and X_{s2} represent the transmitted symbols and are normalized such that $E\left(\left|X_{s1}\right|^2\right) = E\left(\left|X_{s2}\right|^2\right) = 1$. Z_l is the complex-valued white Gaussian noise at relay *l* with $E\left(\left|Z_l\right|^2\right) = N_o$. In the subsequent *L* time-slots, relays 1,2 , … , *L* each transmits in its own non-overlapping time-slot/frequency band. Without loss of generality, we define our indexing such that the *l*th relay transmits at the *l*+1 time slot and the sources transmit at the first time

slot of the frame. The *l*th relay amplifies the signal received from sources s1, s2 and transmits the amplified signal.

The received signal at s1 and s2 from the *l*th relay can be written as

$$\begin{aligned} y_{s1} &= \left(\sqrt{p_{s1}} f_l X_{s1} + \sqrt{p_{s2}} g_l X_{s2} + Z_l\right)\sqrt{\beta}_l f_l + Z_{s1,l} \\ y_{s2} &= \left(\sqrt{p_{s1}} f_l X_{s1} + \sqrt{p_{s2}} g_l X_{s2} + Z_l\right)\sqrt{\beta_l}\, g_l + Z_{s2,l} \end{aligned} \tag{2.2}$$

where Z_{s1}, Z_{s2} are the i.i.d complex valued white Gaussian noise at s1 and s2 with $E\left(\left|Z_{s1,l}\right|^2\right) = E\left(\left|Z_{s2,l}\right|^2\right) = N_o \forall l$ and β_l is the amplification gain of the *l*th relay. The amplification gain, β_l, is chosen such that the transmit power of the *l*th relay is p_l, [20] i.e.

$$\beta_l = \frac{p_l}{p_{s1}\left|f_l\right|^2 + p_{s2}\left|g_l\right|^2 + N_o} \tag{2.3}$$

where p_l is the transmission power of the *l*th relay. Notice that in (2.2), the received signal at s1 consists of a self-interference term $\sqrt{p_{s1}} f_l X_{s1} \sqrt{\beta}_l f_l$. Since s1 knows the signal it transmitted and assuming perfect knowledge of the corresponding channel gains, s1 can subtract the interfering signal from its received signal. Similarly, s2 can also subtract the interfering signal from its received signal. The received signals at s1, s2 from the *l*th relay after self-interference cancellation can be written as

$$\begin{aligned} y_{s1} &= \left(\sqrt{p_{s2}} g_l X_{s2} + Z_l\right)\sqrt{\beta}_l f_l + Z_{s1,l} \\ y_{s2} &= \left(\sqrt{p_{s1}} f_l X_{s1} + Z_l\right)\sqrt{\beta_l}\, g_l + Z_{s2,l} \end{aligned}$$

After self-interference cancellation [3]-[4], the received signals at *s*1 and *s*2 from the relays *l* = 1, 2, .., *L* can be written as

$$
\begin{aligned}
y_{s1} &= \begin{pmatrix} g_1 f_1 \sqrt{\beta_1} \\ \vdots \\ g_l f_l \sqrt{\beta_l} \\ \vdots \\ g_L f_L \sqrt{\beta_L} \end{pmatrix} \sqrt{p_{s2}} X_{s2} + \begin{pmatrix} \sqrt{\beta_1} f_1 Z_1 + Z_{s1,1} \\ \vdots \\ \sqrt{\beta_l} f_l Z_l + Z_{s1,l} \\ \vdots \\ \sqrt{\beta_L} f_L Z_L + Z_{s1,L} \end{pmatrix} \\
y_{s2} &= \begin{pmatrix} g_1 f_1 \sqrt{\beta_1} \\ \vdots \\ g_l f_l \sqrt{\beta_l} \\ \vdots \\ g_L f_L \sqrt{\beta_L} \end{pmatrix} \sqrt{p_{s1}} X_{s1} + \begin{pmatrix} \sqrt{\beta_1} g_1 Z_1 + Z_{s1,1} \\ \vdots \\ \sqrt{\beta_l} g_l Z_l + Z_{s1,l} \\ \vdots \\ \sqrt{\beta_L} g_L Z_L + Z_{s1,L} \end{pmatrix}
\end{aligned}
\tag{2.4}
$$

After maximal ratio combining (MRC) SNRs at *s*1 and *s*2 are written as

$$
\begin{aligned}
\gamma_{s1} &= \sum_{l=1}^{L} \frac{p_{s2} p_l |f_l|^2 |g_l|^2}{N_o \left(p_l |f_l|^2 + p_{s1} |f_l|^2 + p_{s2} |g_l|^2 + N_o \right)}, \\
\gamma_{s2} &= \sum_{l=1}^{L} \frac{p_{s1} p_l |f_l|^2 |g_l|^2}{N_o \left(p_l |g_l|^2 + p_{s1} |f_l|^2 + p_{s2} |g_l|^2 + N_o \right)}
\end{aligned}
\tag{2.5}
$$

2.2 Problem Formulations

In this section, we present our optimizations for allocating power to the sources and the relays. We consider two optimization problems. In one problem, we consider maximizing the sum capacity of the system. In the other problem, we consider maximizing the minimum among the two sources capacities. We shall refer to aforementioned optimizations as joint sources' and relays' powers optimization with sum capacity maximization (JSRSM) and joint sources' and relays' powers optimization with max-min fairness (JSRMF).

2.2.1 Joint Sources' and Relays' Powers Optimization with Sum-Capacity Maximization

In this sub-section, we consider the optimization problem in which the sources' and relays' transmit powers that together maximize the sum capacity of the system are sought. We denote by **p** the vector $(p_1, p_2, .., p_L)$. We formulate the optimization problems as minimizing the cost function

$$f\left(p_{s1}, p_{s2}, \mathbf{p}\right) \equiv \frac{1}{L+1}\log\left(1+\gamma_{s1}\right) + \frac{1}{L+1}\log\left(1+\gamma_{s2}\right) \tag{2.6}$$

which is the sum capacity (in bits per degree of freedom) . The optimization problem is formulated as

$$\begin{aligned}
&\max_{\{p_{s1}, p_{s2}, \mathbf{p}\}} f\left(p_{s1}, p_{s2}, \mathbf{p}\right), \\
&\text{subject to} \\
&C1: p_l \left|h_{l,m}\right|^2 \le I_m^{max}, \forall m, l \\
&C2: 0 \le p_l \le p_l^{max}, \forall l \\
&C3: \left(p_{s1}, p_{s2}\right) \in Y, \text{ where} \\
&Y \equiv \left\{ \left(p_{s1}, p_{s2}\right) \middle| \begin{array}{l} p_{s1}\left|h_{s1,m}\right|^2 + p_{s2}\left|h_{s2,m}\right|^2 \le I_m^{max}, \ \forall m, \\ 0 \le p_{s1}, p_{s2} \le p_s^{max} \end{array} \right\}
\end{aligned} \tag{2.7}$$

In the above optimization problem, constraint *C*1 ensures that the threshold of the relays' interference to each PU is not exceeded. *C*2 represents each relay's maximum transmission power constraints. *C*3 limits the sources' transmission powers both from their own limitation p_s^{max} and from the threshold of sources' interference to every primary user. It should be noted that the objective function is not convex with respect to the variables (p_{s1}, p_{s2}, $\mathbf{p}$). Thus, convex optimization techniques cannot be applied to determine the global optimal solution. In the sequel, we will refer to this problem as Joint Sources' Powers and Relays' Gains Optimization with Sum Capacity Maximization (JSRSM).

2.2.2 Joint Sources' and Relays' Powers Optimization with Max-Min Fairness

In this sub-section, we consider the optimization problem that determines the sources' and relays' transmit powers so that the lesser of the two source's (s1 or s2) communication capacities is maximized under the interference constraint to the primary users. The optimization problem is formulated as

$$\begin{aligned}
&\max_{\{p_{s1},p_{s2},\mathbf{p}\}} g\left(p_{s1},p_{s2},\mathbf{p}\right),\\
&\text{subject to}\\
&C1: p_l\left|h_{l,m}\right|^2 \le I_m^{max}, \forall m,l\\
&C2: 0 \le p_l \le p_l^{max}, \forall l\\
&C3: \left(p_{s1},p_{s2}\right) \in Y,\\
&\text{where}\\
&Y \equiv \left\{ \left(p_{s1},p_{s2}\right) \middle| \begin{array}{l} p_{s1}\left|h_{s1,m}\right|^2 + p_{s2}\left|h_{s2,m}\right|^2 \le I_m^{max},\ \forall m,\\ 0 \le p_{s1},p_{s2} \le p_s^{max} \end{array} \right\}\\
&g\left(p_{s1},p_{s2},\mathbf{p}\right) \equiv \min\left\{\gamma_{s1},\gamma_{s2}\right\}
\end{aligned} \tag{2.8}$$

In the above optimization problem, constraint *C*1 ensures that the threshold of the relays' interference to each PU is not exceeded. *C*2 represents each relay's maximum transmission power constraints. *C*3 limits the sources' transmission powers both from their own limitation p_s^{max} and from the threshold of sources' interference to every primary user. It should be noted that the objective function is not convex with respect to the variables (p_{s1}, p_{s2}, $\mathbf{p}$). Thus, convex optimization techniques cannot be applied to determine the global optimal solution. In the sequel, we will refer to this problem as Joint Sources' Powers and Relays' Gains Optimization with Sum Capacity Maximization (JSRMF).

2.3 Proposed Approach to a Solution

Both the optimization problems (2.7) and (2.8) have two sets of decision variables: relay transmission power represented by $\left\{p_1, p_2, \ldots p_L\right\}$, and the

source's transmission power represented by p_{s1}, p_{s2}. We first note a special structure of the optimization problem (2.7) and (2.8). The only constraints on the variables $\{p_1, p_2, \ldots p_L\}$ are $p_l \left|h_{l,m}\right|^2 \le I_m^{max}, \forall m, l$ and $0 \le p_l \le p_l^{max}, \forall l$, which can be simplified to $0 \le p_l \le \min\left\{ p_l^{max}, \frac{I_1^{max}}{\left|h_{l,1}\right|^2}, \frac{I_2^{max}}{\left|h_{l,2}\right|^2}, \cdots, \frac{I_M^{max}}{\left|h_{l,M}\right|^2} \right\} \forall l$, and variable p_l do not appear in any other constraints in (2.7) and (2.8). That is, the interval constraint $0 \le p_l \le \min\left\{ p_l^{max}, \frac{I_1^{max}}{\left|h_{l,1}\right|^2}, \frac{I_2^{max}}{\left|h_{l,2}\right|^2}, \cdots, \frac{I_M^{max}}{\left|h_{l,M}\right|^2} \right\} \forall l$ of p_l, is decoupled from all other constraints in (2.7) and (2.8). Therefore, problem (2.7) (and also (2.8)) can be rewritten as:

$$\max_{p_{s1}, p_{s2}} \left[\begin{array}{l} \max_{\mathbf{p}} f\left(p_{s1}, p_{s2}, \mathbf{p}\right) \\ \text{subject to} \quad C1: \ p_l \left|h_{l,m}\right|^2 \le I_m^{max}, \forall m, l \\ \qquad\qquad\quad C2: \ 0 \le p_l \le p_l^{max}, \forall l, \end{array} \right] \tag{2.9}$$

$$\begin{array}{l} \text{subject to} \\ C3: \left(p_{s1}, p_{s2}\right) \in Y, \text{ where} \\ Y \equiv \left\{ \left(p_{s1}, p_{s2}\right) \middle| \begin{array}{l} p_{s1} \left|h_{s1,m}\right|^2 + p_{s2} \left|h_{s2,m}\right|^2 \le I_m^{max}, \ \forall m, \\ 0 \le p_{s1}, p_{s2} \le p_s^{max} \end{array} \right\} \end{array}$$

or equivalently,

$$\max_{p_{s1},p_{s2}} \left[\begin{array}{l} \max_{\mathbf{p}} f\left(p_{s1},p_{s2},\mathbf{p}\right) \\ \text{subject to:} \\ 0 \le p_l \le \min\left\{ p_l^{max}, \frac{I_1^{max}}{\left|h_{l,1}\right|^2}, \frac{I_2^{max}}{\left|h_{l,2}\right|^2}, \cdots, \frac{I_M^{max}}{\left|h_{l,M}\right|^2} \right\} \forall l \end{array} \right] \tag{2.10}$$

subject to

$C3:\left(p_{s1},p_{s2}\right) \in Y$, where

$$Y \equiv \left\{ \left(p_{s1},p_{s2}\right) \middle| \begin{array}{l} p_{s1}\left|h_{s1,m}\right|^2 + p_{s2}\left|h_{s2,m}\right|^2 \le I_m^{max},\ \forall m, \\ 0 \le p_{s1}, p_{s2} \le p_s^{max} \end{array} \right\}$$

For any choice of sources' transmission power $\left\{p_{s1},p_{s2}\right\}$, the inner maximization has a nice structure. The objective function is monotonically increasing function of $\left\{p_l, l=1,2,..,L\right\}$ and the constraints set is a box (Appendix A). Therefore, for any sources' transmission powers $\left\{p_{s1},p_{s2}\right\}$, the maximizing *l*th relay's transmit power is $p_l = \min\left\{ p_l^{max}, \frac{I_1^{max}}{\left|h_{l,1}\right|^2}, \frac{I_2^{max}}{\left|h_{l,2}\right|^2}, \cdots, \frac{I_M^{max}}{\left|h_{l,M}\right|^2} \right\}$. We denote the optimal transmit power of *l*th relay as

$$\hat{p}_l = \min\left\{ p_l^{max}, \frac{I_1^{max}}{\left|h_{l,1}\right|^2}, \frac{I_2^{max}}{\left|h_{l,2}\right|^2}, \cdots, \frac{I_M^{max}}{\left|h_{l,M}\right|^2} \right\} \tag{2.11}$$

With $p_l = \hat{p}_l\ \forall l$, the optimization problem (2.7) (and also (2.8)) is reduced to

$$
\begin{aligned}
&\max_{p_{s1},p_{s2}} \ \hat{f}\left(p_{s1},p_{s2}\right) \\
&\text{subject to} \\
&C1:\left(p_{s1},p_{s2}\right)\in Y, \text{ where} \\
&Y\equiv\left\{\left(p_{s1},p_{s2}\right)\middle|\begin{array}{l} p_{s1}\left|h_{s1,m}\right|^2+p_{s2}\left|h_{s2,m}\right|^2\le I_m^{max},\ \forall m, \\ 0\le p_{s1},p_{s2}\le p_s^{max}\end{array}\right\} \\
&\hat{f}\left(p_{s1},p_{s2}\right)\equiv\frac{1}{L+1}\log\left(1+\hat{\gamma}_{s1}\right)+\frac{1}{L+1}\log\left(1+\hat{\gamma}_{s2}\right) \\
&\hat{\gamma}_{s1}=\sum_{l=1}^{L}\frac{p_{s2}\hat{p}_l\left|f_l\right|^2\left|g_l\right|^2}{N_o\left(\hat{p}_l\left|f_l\right|^2+p_{s1}\left|f_l\right|^2+p_{s2}\left|g_l\right|^2+N_o\right)}, \\
&\hat{\gamma}_{s2}=\sum_{l=1}^{L}\frac{p_{s1}\hat{p}_l\left|f_l\right|^2\left|g_l\right|^2}{N_o\left(\hat{p}_l\left|g_l\right|^2+p_{s1}\left|f_l\right|^2+p_{s2}\left|g_l\right|^2+N_o\right)}
\end{aligned}
\tag{2.12}
$$

Similar analysis can be applied to the optimization in (2.8) and the resulting optimization problem can be written as

$$
\begin{aligned}
&\max_{p_{s1},p_{s2}} \ \hat{g}\left(p_{s1},p_{s2}\right) \\
&\text{subject to} \\
&C1:\left(p_{s1},p_{s2}\right)\in Y, \text{ where} \\
&Y\equiv\left\{\left(p_{s1},p_{s2}\right)\middle|\begin{array}{l} p_{s1}\left|h_{s1,m}\right|^2+p_{s2}\left|h_{s2,m}\right|^2\le I_m^{max},\ \forall m, \\ 0\le p_{s1},p_{s2}\le p_s^{max}\end{array}\right\} \\
&\hat{g}\left(p_{s1},p_{s2}\right)\equiv\min\left\{\frac{1}{L+1}\log\left(1+\hat{\gamma}_{s1}\right),\frac{1}{L+1}\log\left(1+\hat{\gamma}_{s2}\right)\right\} \\
&\hat{\gamma}_{s1},\hat{\gamma}_{s2} \text{ as defined in (2.12)}
\end{aligned}
\tag{2.13}
$$

Optimizations in (2.12)-(2.13) are simpler than (2.7)-(2.8) and have two optimization variables. Further, the optimization problems (2.12)-(2.13) have only linear constraints on the optimization variables. However, the cost functions in (2.12)-(2.13) are neither concave nor monotonically increasing/decreasing functions of the optimization variables. However, we note that we can determine the optimal solutions to the optimizations in (2.12)-(2.13) by using methods of monotonic optimization [9]. In the following section, we discuss the preliminaries

of monotonic optimization. In the subsequent section, we present our solutions based on monotonic optimization techniques.

2.4 Preliminaries of Monotonic Optimization

We first present some definitions related to monotonic optimization. Then, we present the standard monotonic optimization problem and the polyblock outer approximation algorithm, which gives the *ε*-optimal solution to the standard monotonic optimization problem. The material in this section is taken from [9], [10]

2.4.1 Definitions and Concepts of Monotonic Optimization

<u>*Definition 1:*</u> For any two vectors $\mathbf{x}, \hat{\mathbf{x}} \in \mathbf{R}^n$, we write $\mathbf{x} \geq \hat{\mathbf{x}}$ and say that $\mathbf{x}$ *dominates* $\hat{\mathbf{x}}$ if $x_i \geq \hat{x}_i, \forall i = 1, 2, .., n$. We write $\mathbf{x} > \hat{\mathbf{x}}$ and say that $\mathbf{x}$ *strictly dominates* $\hat{\mathbf{x}}$ if $x_i > \hat{x}_i, \forall i = 1, 2, .., n$.

Further we define $\mathbf{R}_+^n \triangleq \left\{ \mathbf{x} \in \mathbf{R}^n \middle| \mathbf{x} \geq 0 \right\}$ and $\mathbf{R}_{++}^n \triangleq \left\{ \mathbf{x} \in \mathbf{R}^n \middle| \mathbf{x} > 0 \right\}$. For $\mathbf{x} \in \mathbf{R}_+^n$, let $I(\mathbf{x}) = \left\{ i \middle| x_i = 0 \right\}$ and denote $\mathbf{K_x} = \left\{ \overline{\mathbf{x}} \in \mathbf{R}_+^n \middle| \overline{x}_i > x_i \forall i \notin I(\mathbf{x}) \right\}$.

<u>*Definition 2:*</u> We define a *box* (hyper rectangle) [**0**, **b**] as the set of all **x** such that $\mathbf{0} \leq \mathbf{x} \leq \mathbf{b}$.

<u>*Definition 3:*</u> A function $f : \mathbf{R}^n \to \mathbf{R}$ is said to be *increasing* on $\mathbf{R}_+^n$ iff $f(\hat{\mathbf{x}}) \geq f(\mathbf{x})$ whenever $\hat{\mathbf{x}} \geq \mathbf{x} \geq \mathbf{0}$.

<u>*Definition 4:*</u> A function $f : \mathbf{R}^n \to \mathbf{R}$ is said to be *increasing* on a box [**0**,**b**] $\subset \mathbf{R}_+^n$ iff $f(\hat{\mathbf{x}}) \geq f(\mathbf{x})$ whenever $\mathbf{b} \geq \hat{\mathbf{x}} \geq \mathbf{x} \geq \mathbf{0}$.

<u>*Definition 5:*</u> A set $G \subset \mathbf{R}_+^n$ is called *normal* iff for any two vectors $\hat{\mathbf{x}}, \mathbf{x} \in \mathbf{R}_+^n$ such that $\hat{\mathbf{x}} \geq \mathbf{x}$, $\hat{\mathbf{x}} \in G \Rightarrow \mathbf{x} \in G$. In other words, if $\hat{\mathbf{x}} \in G$ then all the points in the box, $[\mathbf{0}, \hat{\mathbf{x}}]$, are also in *G*, i.e. $[\mathbf{0}, \hat{\mathbf{x}}] \subseteq G$. The empty set, singleton {**0**} and $\mathbf{R}_+^n$ are some examples of normal sets.

Definition 6: A point $\mathbf{y} \in \mathbf{R}_{+}^{n}$ is called an *upper boundary point* of a bounded closed normal set *G* if $\mathbf{y} \in G$ while the set $K_{\mathbf{y}} = \mathbf{y} + \mathbf{R}_{++}^{n} = \left\{ \hat{\mathbf{y}} \in \mathbf{R}_{+}^{n} \middle| \hat{\mathbf{y}} > \mathbf{y} \right\}$ lies outside *G*, i.e. $K_{\mathbf{y}} \subset \mathbf{R}_{+}^{n} \setminus G$. The set of upper boundary points of *G* is called the *upper boundary* of G and it is denoted as $\partial^{+} G$. Figure 2.2 presents an exemplary normal set with its upper boundary. Note that the normal set in fig. 2.2 is not a convex set.

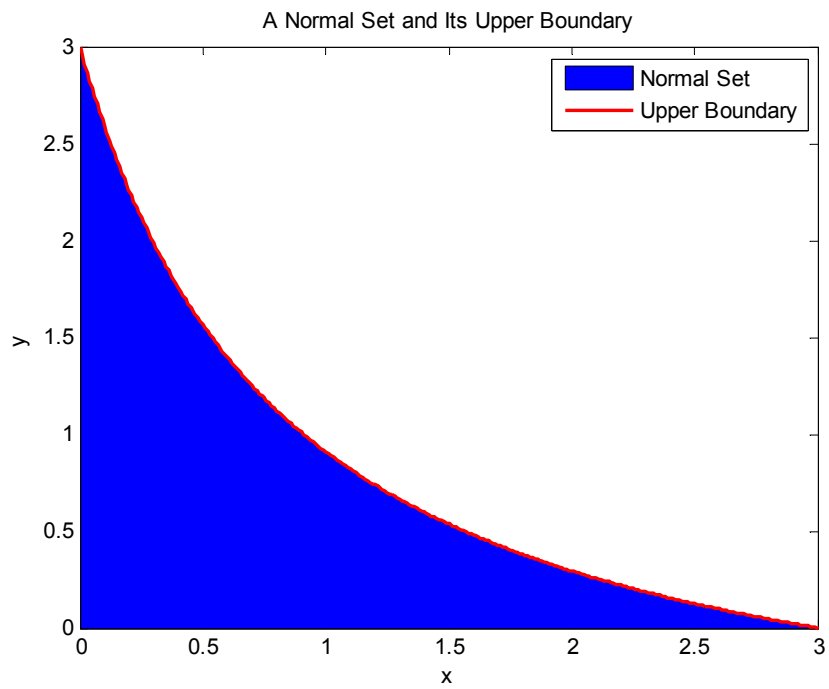

Fig. 2.2 An Exemplary Normal Set and Its Upper Boundary.

Definition 7: Let $G \subset [\mathbf{0}, \mathbf{b}]$ be a compact normal set. For every point $\mathbf{z} \in \mathbf{R}_{+}^{n} \setminus \{0\}$, the half-line from **0** through **z** meets $\partial^{+} G$ at a unique point $\pi_{G}(\mathbf{z})$ which we call as the projection of point **z** onto *G*. The projection is defined as $\pi_{G}(\mathbf{z}) = \lambda \mathbf{z}, \lambda = \max\{\alpha > 0 \mid \alpha \mathbf{z} \in G\}$. Fig. 2.3 presents the projection of the vector **z** on the upper boundary of the feasible set *G*.

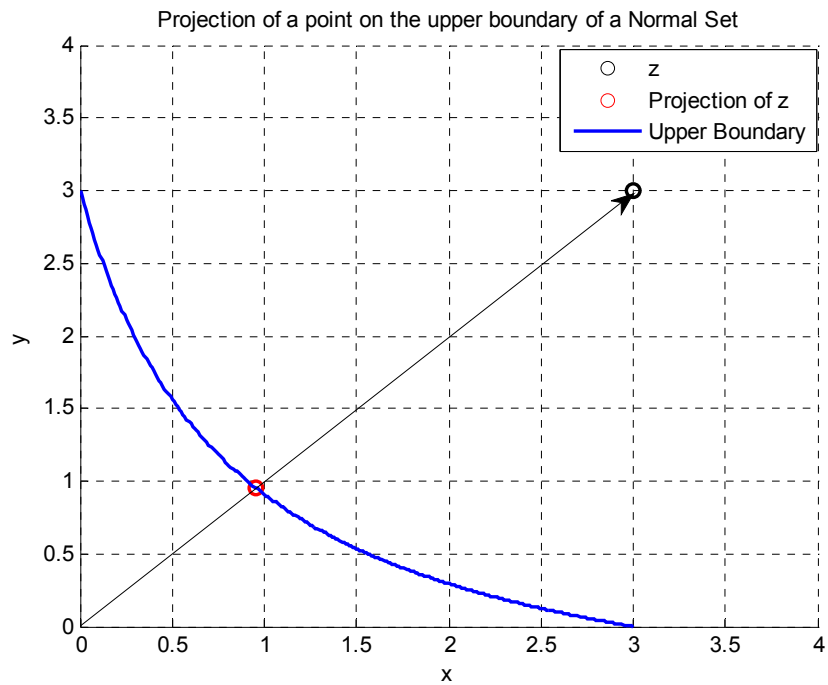

Fig. 2.3 Projection on the Upper Boundary of the Normal Set.

The standard monotonic optimization problem can be written as follows

$$\max_{\mathbf{x}} f(\mathbf{x}) \qquad s.t. \mathbf{x} \in G \tag{2.14}$$

where $G \subset [\mathbf{0}, \mathbf{b}] \subset \mathbf{R}_+^n$ is a compact *normal* set with nonempty interior and *f*(**x**) is an increasing function on $[\mathbf{0}, \mathbf{b}]$.

<u>*Proposition 1*</u>: The maximum of *f*(**x**) on G, if it exists, is attained on $\partial^+ G$.

Proof: See [9].

The approach we follow to determine the optimal solution of the monotonic optimization problems is based on the approximation of the normal sets by simpler sets, called polyblocks (which we will define in the following).

Definition 8: A set $P \subset \mathbf{R}_+^n$ is called a *polyblock* in $[\mathbf{0},\mathbf{b}] \subset \mathbf{R}_+^n$ if $P = \bigcup_{\mathbf{z}\in T}[\mathbf{0},\mathbf{z}], T \subset [\mathbf{0},\mathbf{b}], (|T| < \infty)$. The set *T* is called the *vertex set* of the polyblock *P*.

Definition 9: A vertex $\mathbf{z} \in T$ is called *proper* if it is not dominated by any other vertex $\hat{\mathbf{z}} \in T$, i.e., if $\mathbf{z} \notin [\mathbf{0},\hat{\mathbf{z}}] \forall \hat{\mathbf{z}} \in T \setminus \{\mathbf{z}\}$. The set of all proper vertices of a polyblock is called the *proper vertex set*. A polyblock is fully determined by its proper vertices. Further, it has been shown in [10] that a compact normal set can be approximated as closely as desired by a polyblock. In addition, the maximum of an increasing function over a polyblock is achieved at a proper vertex. Therefore, we can design an algorithm where we construct a nested sequence of polyblocks which are an outer approximation of the normal feasible set *G*, that is we approximate the feasible set *G* as $P_1 \supset P_2 \supset \cdots \supset G$, such that

$$\max_{\mathbf{x}} \{ f(\mathbf{x}) \mid \mathbf{x} \in P_k \} \to \max_{\mathbf{x}} \{ f(\mathbf{x}) \mid \mathbf{x} \in G \} \quad \text{as } k \to \infty \tag{2.15}$$

We now present a method to construct the nested sequence of polyblocks. At iteration *k*, let T_k denote the set of proper vertices of the polyblock P_k and $\mathbf{z}^k = \arg\max_{\mathbf{x}\in T_k} f(\mathbf{x})$. Let $\mathbf{x}^k$ denote the projection of $\mathbf{z}^k$ on $\partial^+ G$. We are interested in constructing a new polyblock $P_{k+1} \subset P_k \setminus \mathbf{z}^k$ and $P_{k+1} \supset G$. According to proposition 17 in [10], the new polyblock P_{k+1} can be obtained from P_k by replacing $[\mathbf{0}, \mathbf{z}^k]$ with $[\mathbf{0},\mathbf{z}^k] \setminus K_{\mathbf{x}^k}$, i.e. $P_{k+1} = \left([\mathbf{0},\mathbf{z}^k] \setminus K_{\mathbf{x}^k}\right) \cup_{\mathbf{z}\in T_k \setminus \{\mathbf{z}^k\}} [0,\mathbf{z}]$. The obtained polyblock, P_{k+1}, satisfies $P_{k+1} \subset P_k \setminus \mathbf{z}^k$ and $P_{k+1} \supset G$. In the following proposition, we provide an explicit method for determining the vertices of the polyblock P_{k+1}. Let T_{k+1} denote the proper vertex set determined by removing improper elements from V_{k+1}.

Proposition 2: The set of vertices (not necessarily proper) V_{k+1} of polyblock P_{k+1} can be determined as follows

$$V_{k+1} = \left(T_k \setminus \left\{\mathbf{z}^k\right\}\right) \cup \left\{\mathbf{z}^k - \left(z_i^k - x_i^k\right) e^i \,\middle|\, i = 1, 2, .., n\right\}, \tag{2.16}$$

where e^i is the ith column of the identity matrix. The polyblock P_{k+1} obtained from T_{k+1} satisfies that $G \subset P_{k+1} \subset P_k$. The proof of proposition 2 is given in [9].

In Fig. 2.4, we provide an example to explain the method of constructing the sequence of polyblocks. Fig. 2.4 consists of two sub-plots. In subplot 1, a polyblock P_1 is given. Using the maximizer $\mathbf{z}^1$ (maximizer of $f(\mathbf{x})$ on $\boldsymbol{p}^1$) and the projection $\mathbf{x}^1$, we compute the new vertices that would determine the polyblock P2 from (2.16). The maximizer $\mathbf{z}^1$ and the new vertices (for P_2) are highlighted in sub-plot 2. Notice that the new generated polyblock P_2 is a subset of P_1 and is a better approximation of the feasible set.

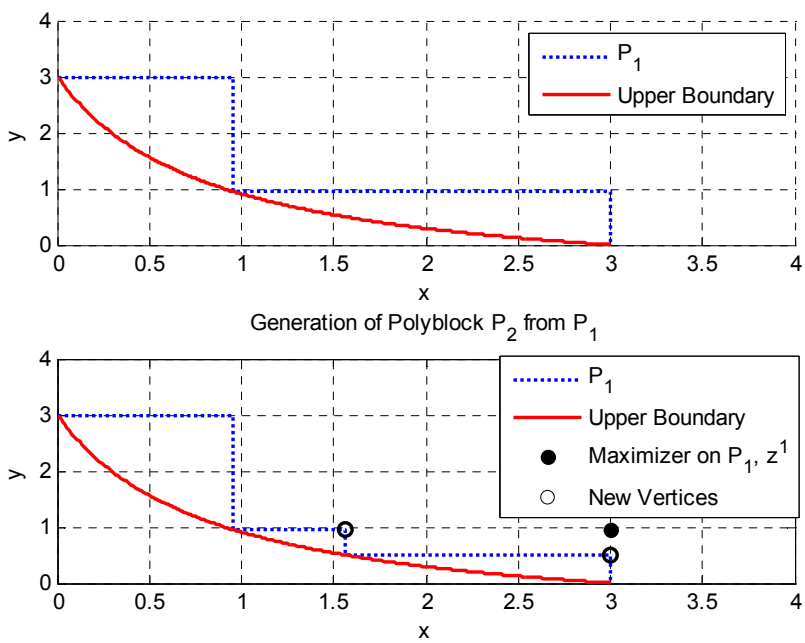

Fig. 2.4 Determination of Polyblock P_2 from P_1.

As the iterations proceed, there are improvements in the outer approximation of the normal set. Fig 2.5, illustrates the polyblock generated in

the 143rd iteration. We see that the polyblock in Fig. 2.5 is a significantly better approximation of the feasible set as compared to the ones given in Fig. 2.4.

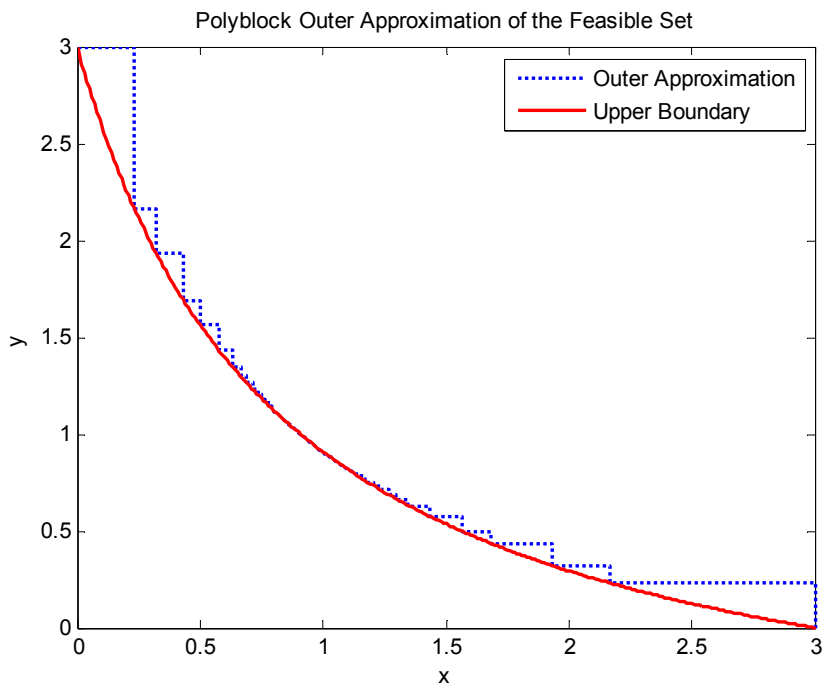

Fig. 2.5 Outer Approximation of the Upper Boundary of the Feasible Set

2.4.2 Polyblock Outer Approximation Algorithm

The pseudocode of the polyblock outer approximation algorithm is given in table 2.2. We now provide a brief description of the algorithm based on the pseudocode in table 2.2. The polyblock outer approximation is an iterative algorithm. At iteration k, we have polyblock P_k with vertex set T_k. The algorithm first determines the maximizer, $\mathbf{z}_k$, of $f(\mathbf{x})$ on the polyblock P_k. If the maximizer $\mathbf{z}_k \in G$, then $\mathbf{z}_k$ solves the optimization in (2.14). Otherwise, the algorithm computes the projection, $\mathbf{x}_k$, of $\mathbf{z}_k$ on the boundary of the feasible set G. If $f(\mathbf{z}_k) - f(\mathbf{x}_k) \le \varepsilon$ then the algorithm terminates with $\mathbf{x}_k$ as an ε-optimal solution. Otherwise, based on proposition 2 (and the steps 10-11 the pseudocode in table 2.2), the algorithm constructs a new polyblock P_{k+1} by using P_k and $\mathbf{x}_k$ such that $G \subset P_{k+1} \subset P_k$. We repeat this procedure until the termination criteria is met, i.e., either we find an

optimal solution or we find an optimal or ε -optimal solution to the optimization in (2.14).

Theorem 1: If the polyblock outer approximation algorithm in table 2.2 is infinite, each of the generated sequences $\{\mathbf{z}_k\}$, $\{\mathbf{x}_k\}$ contains a subsequence converging to an optimal solution.

Proof: See [9].

Table 2.2 Pseudocode of polyblock outer approximation algorithm

***INITIALIZATION*:** Select $\varepsilon \geq 0$, Set $T_1 = \{\mathbf{b}\}$, set $k = 1$, $Set\ CBV$ (current best value) $= -\infty$
1: ***While*** **(1)** 2: $\mathbf{z}_k = \arg\max\{f(\mathbf{x}) \mid \mathbf{x} \in T_k\}$; 3: ***if*** $\mathbf{z}_k \in G$ 4: **x*=z**$_k$ is the optimal solution; 5: ***else*** 6: Compute the projection of $\mathbf{z}_k$ on $\partial^+ G$ as $\pi_G(\mathbf{z}) = \lambda\mathbf{z}, \lambda = \max\{\alpha > 0 \mid \alpha\mathbf{z} \in G\}$; 7: ***if*** $f(\mathbf{z}_k) - f(\mathbf{x}_k) \leq \varepsilon$ 8: $\mathbf{x}^k$ is an ε-optimal solution; 9: ***else*** 10: Compute a new polyblock P_{k+1} with vertices set $V_{k+1} = \left(T_k \setminus \{\mathbf{z}_k\}\right) \cup \{\mathbf{z}_{k,1}, \cdots, \mathbf{z}_{k,n}\}$, $\mathbf{z}_{k,1} = \mathbf{z}_k - \left(\mathbf{z}_k^i - \mathbf{x}_k^i\right) e^i, i = 1, 2, ..., n$ 11: Determine T_{k+1} by removing improper elements of P_{k+1}; 12: ***end*** 13: ***end*** 14: k:=k+1; 15: ***end***

2.5 ε-Optimal Solution based on Monotonic Optimization

In this section, we apply the monotonic optimization techniques to determine the ε-optimal solutions of our optimizations in (2.12)-(2.13). The

optimizations in (2.12)-(2.13) are not the standard monotonic optimization problems as their respective cost functions are not an increasing function of the optimization variables p_{s1} and p_{s2} (see Appendix C). We notice that we can transform our optimizations in (2.12)-(2.13) to the standard monotonic optimization problem given in (2.14). More specifically, we write our optimizations as difference of increasing functions and then with the help of an auxiliary variable, we transform our optimizations to the standard monotonic optimization problems. In the following subsections, we transform the sum-capacity and max-min optimizations in (2.12)-(2.13) to the standard monotonic optimization problem.

2.5.1 ε-Optimal Solution of Sum-Capacity Optimization

The sum-capacity optimization problem in (2.12) is not a standard monotonic optimization problem. However, by using an auxiliary variable, we can transform the optimization in (2.12) to the standard monotonic optimization problem. More specifically, we write the cost function in (2.12) as difference of two functions that are monotonically increasing functions of the optimization variables p_{s1} and p_{s2} and then with an introduction of an additional variable we write the sum-capacity optimization problem in (2.12) as a standard monotonic optimization problem.

We first observe that the SNR at the users s1and s2 can be written as follows

$$\hat{f}\left(p_{s1},p_{s2}\right)=\frac{1}{L+1}\left(\log\left(1+\hat{\gamma}_{s1}\right)+\log\left(1+\hat{\gamma}_{s2}\right)\right)$$

$$\hat{\gamma}_{s1}=\sum_{l}\frac{p_{s2}p_l\left|f_l\right|^2\left|g_l\right|^2}{N_o\left(p_l\left|f_l\right|^2+p_{s1}\left|f_l\right|^2+p_{s2}\left|g_l\right|^2+N_o\right)}$$

$$=\frac{\sum_{l=1}^{L}p_{s2}p_l\left|f_l\right|^2\left|g_l\right|^2\prod_{\substack{j=1\\j\neq l}}^{L}N_o\left(p_j\left|f_j\right|^2+p_{s1}\left|f_j\right|^2+p_{s2}\left|g_j\right|^2+N_o\right)}{\prod_{j=1}^{L}N_o\left(p_j\left|f_j\right|^2+p_{s1}\left|f_j\right|^2+p_{s2}\left|g_j\right|^2+N_o\right)} \tag{2.16}$$

$$\hat{\gamma}_{s2}=\sum_{l}\frac{p_{s1}p_l\left|f_l\right|^2\left|g_l\right|^2}{N_o\left(p_l\left|g_l\right|^2+p_{s1}\left|f_l\right|^2+p_{s2}\left|g_l\right|^2+N_o\right)}$$

$$=\frac{\sum_{l=1}^{L}p_{s1}p_l\left|f_l\right|^2\left|g_l\right|^2\prod_{\substack{j=1\\j\neq l}}^{L}N_o\left(p_j\left|g_j\right|^2+p_{s1}\left|f_j\right|^2+p_{s2}\left|g_j\right|^2+N_o\right)}{\prod_{j=1}^{L}N_o\left(p_j\left|g_j\right|^2+p_{s1}\left|f_j\right|^2+p_{s2}\left|g_j\right|^2+N_o\right)}$$

The capacity of user s1 and s2 can be written as difference of increasing functions as follows:

$$\frac{1}{L+1}\log\left(1+\hat{\gamma}_{s1}\right)=q_1\left(p_{s1},p_{s2}\right)-r_1\left(p_{s1},p_{s2}\right)$$

$$q_1\left(p_{s1},p_{s2}\right)$$

$$=\frac{1}{L+1}\log\left[\prod_{j=1}^{L}N_o\left(p_j\left|f_j\right|^2+p_{s1}\left|f_j\right|^2+p_{s2}\left|g_j\right|^2+N_o\right)+\right. \tag{2.17}$$

$$\left.\sum_{l=1}^{L}p_{s2}p_l\left|f_l\right|^2\left|g_l\right|^2\prod_{\substack{j=1\\j\neq l}}^{L}N_o\left(p_j\left|f_j\right|^2+p_{s1}\left|f_j\right|^2+p_{s2}\left|g_j\right|^2+N_o\right)\right]$$

$$r_1\left(p_{s1},p_{s2}\right)=\frac{1}{L+1}\log\left[\prod_{j=1}^{L}N_o\left(p_j\left|f_j\right|^2+p_{s1}\left|f_j\right|^2+p_{s2}\left|g_j\right|^2+N_o\right)\right]$$

$$
\begin{aligned}
&\frac{1}{L+1}\log\left(1+\hat{\gamma}_{s2}\right)=q_2\left(p_{s1},p_{s2}\right)-r_2\left(p_{s1},p_{s2}\right)\\
&q_2\left(p_{s1},p_{s2}\right)\\
&=\frac{1}{L+1}\log\left[\prod_{j=1}^{L}N_o\left(p_j\left|g_j\right|^2+p_{s1}\left|f_j\right|^2+p_{s2}\left|g_j\right|^2+N_o\right)+\right.\\
&\left.\sum_{l=1}^{L}p_{s1}p_l\left|f_l\right|^2\left|g_l\right|^2\prod_{\substack{j=1\\ j\neq l}}^{L}N_o\left(p_j\left|g_j\right|^2+p_{s1}\left|f_j\right|^2+p_{s2}\left|g_j\right|^2+N_o\right)\right]\\
&r_2\left(p_{s1},p_{s2}\right)=\frac{1}{L+1}\log\left[\prod_{j=1}^{L}N_o\left(p_j\left|g_j\right|^2+p_{s1}\left|f_j\right|^2+p_{s2}\left|g_j\right|^2+N_o\right)\right]
\end{aligned}
\tag{2.18}
$$

Therefore, the sum-capacity of user s1 and s2 can be written as difference of increasing functions as follows:

$$
\begin{aligned}
&\frac{1}{L+1}\log\left(1+\hat{\gamma}_{s2}\right)+\frac{1}{L+1}\log\left(1+\hat{\gamma}_{s2}\right)=q\left(p_{s1},p_{s2}\right)-r\left(p_{s1},p_{s2}\right),\\
&where,\ q\left(p_{s1},p_{s2}\right)=q_1\left(p_{s1},p_{s2}\right)+q_2\left(p_{s1},p_{s2}\right)\\
&\qquad r\left(p_{s1},p_{s2}\right)=r_1\left(p_{s1},p_{s2}\right)+r_2\left(p_{s1},p_{s2}\right)
\end{aligned}
\tag{2.19}
$$

We showed above that the objective function of our optimization in (2.12) can be written as difference of increasing functions. We now transform our optimization to the standard monotonic optimization problem as in (2.14). We introduce a new decision variable that expands the feasible set of the solutions but at the same time allows us to apply the monotonic optimization techniques to determine the optimal solution of sum-capacity optimization in (2.12).

Using the notations in (2.17)-(2.18), we can write the sum-capacity optimization problem as the following:

$$
\begin{aligned}
&\max_{\{p_{s1},p_{s2}} q\left(p_{s1},p_{s2}\right)-r\left(p_{s1},p_{s2}\right),\\
&\text{subject to}\\
&\quad C1:\left(p_{s1},p_{s2}\right)\in Y\\
&Y\equiv\left\{\begin{array}{c}\left(p_{s1},p_{s2}\right):p_{s1}\left|h_{s1,m}\right|^2+p_{s2}\left|h_{s2,m}\right|^2\le I_m^{max}\forall m,\\ 0\le p_{s1},p_{s2}\le p_s^{max}\end{array}\right\}
\end{aligned}
\tag{2.20}
$$

Since $r(p_{s1}, p_{s2})$ is a monotonic function of $\left(p_{s1},p_{s2}\right)$, $r\left(p_{s1},p_{s2}\right)$ $\le r\left(p_{s1}^{max},p_{s2}^{max}\right)\forall(p_{s1}, p_{s2}) \in Y$. As a result, we have $r\left(p_{s1},p_{s2}\right)+t=r\left(p_{s1}^{max},p_{s2}^{max}\right)$. With the help of a new optimization variable, t, we can substitute for $r(p_{s1}, p_{s2})$. The resulting optimization problem can be written as follows,

$$
\begin{aligned}
&\max_{\{p_{s1},p_{s2},t} q\left(p_{s1},p_{s2}\right)+t-r\left(p_{s1}^{max},p_{s2}^{max}\right)\\
&\text{subject to}\\
&\quad C1:\quad r\left(p_{s1},p_{s2}\right)+t\le r\left(p_{s1}^{max},p_{s2}^{max}\right)\\
&\quad C2:\left(p_{s1},p_{s2}\right)\in Y
\end{aligned}
\tag{2.21}
$$

The optimization in (2.21) is equivalent to that in (2.20). Further, the optimization in (2.21) is a standard monotonic optimization problem, which can be solved by using the polyblock outer approximation algorithm presented in previous section. Let us denote by G, the set of feasible solutions of (2.21).

The application of polyblock outer approximation method to find the ε-optimal solution of the optimization in (2.21) is quite straightforward with the exception of a single non-trivial step. In the polyblock outer approximation algorithm, in each iteration, we need to project the maximizer (over the vertex set of the polyblock at that iteration) onto the boundary of the feasible set of the solutions. Let us define the vector $\mathbf{z}^k=\left(p_{s1}^k,p_{s2}^k,t^k\right)$. The projection can be written as $\mathbf{x}^k=\pi_G\left(\mathbf{z}^k\right),\pi_G\left(\mathbf{z}^k\right)=\lambda^k\mathbf{z}^k$, $\lambda^k=\max\left\{\alpha>0\,|\,\alpha\mathbf{z}^k\in G\right\}$. As mentioned above, the computation of projection requires us to solve the following single variable optimization in α, which can be written as follows

$$
\begin{aligned}
&\max_{\{\alpha} \alpha \\
&\text{subject to} \\
&\quad C1: \quad r\left(\alpha p_{s1}^{k}, \alpha p_{s2}^{k}\right) + \alpha t^{k} \le r\left(p_{s1}^{max}, p_{s2}^{max}\right) \\
&\quad C2: \quad \alpha p_{s1}^{k}\left|h_{s1,m}\right|^{2} + \alpha p_{s2}^{k}\left|h_{s2,m}\right|^{2} \le I_{m}^{max} \quad \forall m \\
&\quad C3: \quad 0 \le \alpha p_{s1}^{k} \le p_{s}^{max}, 0 \le \alpha p_{s2}^{k} \le p_{s}^{max} \\
&\quad C4: 0 \le \alpha \le 1
\end{aligned}
\tag{2.22}
$$

Due to the monotonicity of the constraint functions in (2.22), we can use a bisection search based algorithm to solve the optimization above. We first initialize an interval $\left[\alpha^{\min}, \alpha^{\max}\right]$ which contains the optimal solution, *t**, to the optimization in (2.22). In our case α^{min} = 0 and $\alpha^{max} = \min\left(1, \frac{p_{s}^{max}}{p_{s1}^{k}}, \frac{p_{s}^{max}}{p_{s2}^{k}}, \frac{I_{m}^{max}}{p_{s1}^{k}\left|h_{s1,m}\right|^{2} + p_{s2}^{k}\left|h_{s2,m}\right|^{2}}\right)$. We then check if $\hat{\alpha}$ = $\frac{\alpha^{\min} + \alpha^{\max}}{2}$ is a feasible solution to the optimization in (2.22). If $\hat{\alpha}$ is a feasible solution, then due to the monotonicity of the constraint functions we can conclude that the optimal solution to the optimization problem is in the interval $\left[\hat{\alpha}, \alpha^{\max}\right]$. Otherwise, if $\hat{\alpha}$ is not a feasible solution to the optimization in (2.22) then it means that the optimal solution to the optimization problem is in the interval $\left[\alpha^{\min}, \hat{\alpha}\right]$. We formally present in table 2.2 our bisection search based procedure for solving the optimization in (2.22).

The proposed algorithm keeps on iterating until $\alpha^{\max} - \alpha^{\min} \le \varepsilon$. In each iteration, the interval $\left[\alpha^{\min}, \alpha^{\max}\right]$ is bisected in two parts. As the iteration proceeds, the length of the interval $\left[\alpha^{\min}, \alpha^{\max}\right]$ keeps on diminishing. Exactly $\left\lceil \log_{2}\left(\frac{\alpha^{\max} - \alpha^{\min}}{\varepsilon}\right)\right\rceil$ iterations are required before the algorithm terminates.

Table 2.3 Bisection Search Based Projection Computation

Main Algorithm
***Initialization*:** $\alpha^{\max} = 1, \alpha^{\min} = 0, \varepsilon$
While $\alpha^{\max} - \alpha^{\min} > \varepsilon$ 1: $\alpha = \dfrac{\alpha^{\min} + \alpha^{\max}}{2}$; 2: Check if α is a feasible solution to the optimization in (2.22) 3: ***If*** α is a feasible solution 4: $\alpha^{\min} = \alpha$ 5: ***else*** 6: $\alpha^{\max} = \alpha$ 7: ***endif*** ***EndWhile***

2.5.2 ε-Optimal Solution of Max-Min Optimization

The max-min optimization problem in (2.13) is not a standard monotonic optimization problem. However, by using an auxiliary variable, we can transform the optimization in (2.13) to the standard monotonic optimization problem. More specifically, we write the cost function in (2.12) as difference of two functions that are monotonically increasing functions of the optimization variables p_{s1} and p_{s2} and then with an introduction of an additional variable we write the sum-capacity optimization problem in (2.12) as a standard monotonic optimization problem.

Using notations from (2.17) and (2.18), the cost function of the max-min optimization problem in (2.13) can be written difference of increasing functions as follows:

$$
\begin{aligned}
&\min\left\{\frac{1}{L+1}\log\left(1+\widehat{\gamma}_{s1}\right),\frac{1}{L+1}\log\left(1+\widehat{\gamma}_{s2}\right)\right\}\\
&=\min\left\{q_1\left(p_{s1},p_{s2}\right)-r_1\left(p_{s1},p_{s2}\right),\right.\\
&\qquad\left.q_2\left(p_{s1},p_{s2}\right)-r_2\left(p_{s1},p_{s2}\right)\right\}\ \text{from}\,(3.17)-(3.18)\\
&=\min\left\{q_1\left(p_{s1},p_{s2}\right)+r_2\left(p_{s1},p_{s2}\right)-r_1\left(p_{s1},p_{s2}\right)-r_2\left(p_{s1},p_{s2}\right),\right.\\
&\qquad\left.q_2\left(p_{s1},p_{s2}\right)+r_1\left(p_{s1},p_{s2}\right)-r_2\left(p_{s1},p_{s2}\right)-r_1\left(p_{s1},p_{s2}\right)\right\}\\
&=\min\left\{q_1\left(p_{s1},p_{s2}\right)+r_2\left(p_{s1},p_{s2}\right),\right.\\
&\qquad\left.q_2\left(p_{s1},p_{s2}\right)+r_1\left(p_{s1},p_{s2}\right)\right\}-\\
&\qquad r_1\left(p_{s1},p_{s2}\right)-r_2\left(p_{s1},p_{s2}\right)\\
&=q\left(p_{s1},p_{s2}\right)-r\left(p_{s1},p_{s2}\right),\text{where}\\
&q\left(p_{s1},p_{s2}\right)=\min\left\{q_1\left(p_{s1},p_{s2}\right)+r_2\left(p_{s1},p_{s2}\right),\right.\\
&\qquad\qquad\left.q_2\left(p_{s1},p_{s2}\right)+r_1\left(p_{s1},p_{s2}\right)\right\}\\
&r\left(p_{s1},p_{s2}\right)=r_1\left(p_{s1},p_{s2}\right)+r_2\left(p_{s1},p_{s2}\right)
\end{aligned}
\tag{2.23}
$$

We now transform our optimization to the standard monotonic optimization problem as in (2.14). We introduce a new decision variable that although, expands the feasible set of the solutions but at the same time allows us to apply the monotonic optimization techniques to determine the optimal solution of max-min optimization in (2.13).

Using the notations in (2.17)-(2.18) and (2.23), we can write the max-min optimization problem as the following:

$$
\begin{aligned}
&\max_{\{p_{s1},p_{s2}}\ q\left(p_{s1},p_{s2}\right)-r\left(p_{s1},p_{s2}\right),\\
&\text{subject to}\\
&\qquad C1:\left(p_{s1},p_{s2}\right)\in Y\\
&Y\equiv\left\{\begin{array}{c}\left(p_{s1},p_{s2}\right):p_{s1}\left|h_{s1,m}\right|^2+p_{s2}\left|h_{s2,m}\right|^2\le I_m^{max}\forall m,\\ 0\le p_{s1},p_{s2}\le p_s^{max}\end{array}\right\}
\end{aligned}
\tag{2.24}
$$

Since $r(p_{s1}, p_{s2})$ is a monotonic function of p_{s1}, p_{s2}, $r\left(p_{s1},p_{s2}\right)\le r\left(p_{s1}^{max},p_{s2}^{max}\right)$ $\forall(p_{s1},\ p_{s2})\in Y$, where Y is defined in (2.24) . Hence, we have $r\left(p_{s1},p_{s2}\right)+t$

$= r\left(p_{s1}^{max}, p_{s2}^{max}\right)$ for some $t \geq 0$. Hence, with the help of a new optimization variable, *t*, we can substitute $r(p_{s1}, p_{s2})$. The resulting optimization problem can be written as follows,

$$\begin{aligned} &\max_{\{p_{s1}, p_{s2}, t} q\left(p_{s1}, p_{s2}\right) + t - r\left(p_{s1}^{max}, p_{s2}^{max}\right) \\ &\text{subject to} \\ &\quad C1: \quad r\left(p_{s1}, p_{s2}\right) + t \leq r\left(p_{s1}^{max}, p_{s2}^{max}\right) \\ &\quad C2: \left(p_{s1}, p_{s2}\right) \in Y \end{aligned} \tag{2.25}$$

The optimization in (2.25) is equivalent to that in (2.24). Further, the optimization in (2.25) is a standard monotonic optimization problem, which can be solved by using the polyblock outer approximation algorithm presented in previous section. For computation of projections, we use the bisection search based method that we presented in previous sub-section for sum-capacity optimization.

Although the monotonic optimization algorithm can guarantee convergence to a solution to the optimization in (2.20) and (2.24) that has a performance arbitrarily close (within an arbitrary number ε) to the optimal performance, our experimentation with the monotonic optimization algorithm applied to this relay and source power allocation problem shows rather slow convergence. Therefore, we propose low-complexity heuristics that have low computational complexity and perform well in comparison with the respective optimal solutions. In the next section, we present the heuristics to solve the optimizations in (2.12) and (2.13). We name these heuristics as Greedy Power Allocation with Max-Min Fairness (GPAMF) and Greedy Power Allocation with Sum Capacity Maximization (GPASM).

2.6 Proposed Heuristics

In this section, we present low-complexity iterative heuristic algorithms to determine suboptimal solutions to the optimizations in (2.12) and (2.13). The

proposed algorithm uses the simple bounds on the received SNR at s1 and s2. The bounds can be obtained as

$$\begin{aligned}
\hat{\gamma}_{s1} &= \sum_{l=1}^{L} \frac{p_{s2}\hat{p}_l |f_l|^2 |g_l|^2}{N_o\left(\hat{p}_l |f_l|^2 + p_{s1}|f_l|^2 + p_{s2}|g_l|^2 + N_o\right)} \\
&\le \sum_{l=1}^{L} \frac{p_{s2}\hat{p}_l |f_l|^2 |g_l|^2}{N_o\left(\hat{p}_l |f_l|^2 + p_{s1}|f_l|^2 + p_{s2}|g_l|^2\right)} \\
&\le \sum_{l=1}^{L} \frac{p_{s2}\hat{p}_l |f_l|^2 |g_l|^2}{N_o\left(\hat{p}_l |f_l|^2\right)} = \sum_{l=1}^{L} \frac{p_{s2}|g_l|^2}{N_o} \qquad (2.26) \\
\hat{\gamma}_{s2} &= \sum_{l=1}^{L} \frac{p_{s1}\hat{p}_l |f_l|^2 |g_l|^2}{N_o\left(\hat{p}_l |g_l|^2 + p_{s1}|f_l|^2 + p_{s2}|g_l|^2 + N_o\right)} \\
&\le \sum_{l=1}^{L} \frac{p_{s1}|f_l|^2}{N_o}
\end{aligned}$$

Thus, we observe that the SNRs at s1 and s2 depend on the corresponding sources power and the channel gains $|g_l|^2$ and $|f_l|^2$. In the following sub-section, we first describe the GPASM algorithm that provides the sub-optimal solution to the optimization in (2.7). In the subsequent sub-section, we describe the GPAMF algorithm which provides the sub-optimal solution to the optimization in (2.8).

2.6.1 Greedy Power Allocation with Sum-Capacity Maximization

In this sub-section, we present a low-complexity heuristic algorithm to determine a suboptimal solution to the optimization in (2.12). The proposed algorithm uses the simple bounds on the received SNR at s1 and s2 given in (2.26). In this heuristic algorithm, we consider the optimization of the upper bounds subject to the constraints on the source powers (p_{s1}, p_{s2}) as given in C1 (2.24). That is, we solve the following optimization,

$$
\begin{aligned}
\max_{p_{s1},p_{s2}} \quad & \frac{1}{2}\log\left(1+\frac{p_{s2}}{N_o}\sum_l |g_l|^2\right)+ \\
& \frac{1}{2}\log\left(1+\frac{p_{s1}}{N_o}\sum_l |f_l|^2\right) \\
s.t. \;\; & C1: p_{s1}\left|h_{s1,m}\right|^2 + p_{s2}\left|h_{s2,m}\right|^2 \le I_m^{max} \forall m, \\
& C2: 0 \le p_{s1}, p_{s2} \le p_{max}^s
\end{aligned}
\tag{2.27}
$$

The optimization in (2.27) is a simple convex optimization problem in two variables and can be solved using interior point methods. Let us denote the solution to (2.17) as $\left(\hat{p}_{s1}, \hat{p}_{s2}\right)$. The sub-optimal sum-capacity can be written as follows

$$
\begin{aligned}
& \frac{1}{L+1}\log\left(1+\hat{\gamma}_{s1}\right)+\frac{1}{L+1}\log\left(1+\hat{\gamma}_{s2}\right) \\
& \hat{\gamma}_{s1} = \sum_{l=1}^{L} \frac{\hat{p}_{s2}\hat{p}_l |f_l|^2 |g_l|^2}{N_o\left(\hat{p}_l |f_l|^2 + \hat{p}_{s1}|f_l|^2 + \hat{p}_{s2}|g_l|^2 + N_o\right)} \\
& \hat{\gamma}_{s2} = \sum_{l=1}^{L} \frac{\hat{p}_{s1}\hat{p}_l |f_l|^2 |g_l|^2}{N_o\left(\hat{p}_l |g_l|^2 + \hat{p}_{s1}|f_l|^2 + \hat{p}_{s2}|g_l|^2 + N_o\right)}
\end{aligned}
$$

2.6.2 Greedy Power Allocation with Max-Min Fairness

In this sub-section, we present a low-complexity heuristic algorithm to determine a suboptimal solution to the max-min optimization problem in (2.13). In this heuristic algorithm we first consider the optimization of the upper bounds subject to the constraints on the source powers (p_{s1}, p_{s2}) as given in $C3$ in (2.8). More specifically, solve the following optimization problem and determine suboptimal values of source powers $\left(\hat{p}_{s1}, \hat{p}_{s2}\right)$.

$$\max_{p_{s1},p_{s2}} \min\left(\frac{1}{L+1}\log\left(1+\sum_l \frac{p_{s2}|g_l|^2}{N_o}\right), \frac{1}{L+1}\log\left(1+\sum_l \frac{p_{s1}|f_l|^2}{N_o}\right)\right)$$
$$s.t.\ C1: p_{s1}|h_{s1,m}|^2 + p_{s2}|h_{s2,m}|^2 \le I_m^{max} \forall m, \qquad (2.28)$$
$$C2: 0 \le p_{s1}, p_{s2} \le p_{max}^s$$

Due to the monotonicity of logarithm, we can consider optimizing the arguments of the logarithm in the cost function above. More specifically, we can replace (2.28) with the following linear program:

$$\max_{p_{s1},p_{s2}} \min\left(\frac{p_{s2}}{N_o}\sum_l |g_l|^2, \frac{p_{s1}}{N_o}\sum_l |f_l|^2\right)$$
$$s.t.\ C1: p_{s1}|h_{s1,m}|^2 + p_{s2}|h_{s2,m}|^2 \le I_m^{max} \forall m, \qquad (2.29)$$
$$C2: 0 \le p_{s1}, p_{s2} \le p_{max}^s$$

The objective function $\min\left(\frac{p_{s2}}{N_o}\sum_l |g_l|^2, \frac{p_{s1}}{N_o}\sum_l |f_l|^2\right)$ is continuous and the feasible set is compact, so a maximum exists. In fact, we can express the maximizer in closed form as $(\hat{p}_{s1}, \hat{p}_{s2}) = \left(\frac{\hat{p}_{s2}\sum_l |g_l|^2}{\sum_l |f_l|^2}, \hat{p}_{s2},\right)$, where $\hat{p}_{s2} = \min\left(p_{max}^s, \frac{p_{max}^s \sum_l |f_l|^2}{\sum_l |g_l|^2}, \frac{I_m^{max}\sum_l |f_l|^2}{|h_{s1,m}|^2 \sum_l |g_l|^2 + |h_{s2,m}|^2 \sum_l |f_l|^2} \forall m\right)$. This closed form solution is determined by using the fact that for the optimization problem in (2.29) $\exists$ a maximizer $(\hat{p}_{s1}, \hat{p}_{s2})$ such that $\hat{p}_{s2}\sum_l |g_l|^2 = \hat{p}_{s1}\sum_l |f_l|^2$. We establish this fact in *lemma II* given below.

Lemma II: There exists a maximizer $\left(\hat{p}_{s1}, \hat{p}_{s2}\right)$ at which $\hat{p}_{s2}\sum_l \left|g_l\right|^2 = \hat{p}_{s1}\sum_l \left|f_l\right|^2$

Proof: See the Appendix B.

Now that we have established the *lemma II*, we can add the equality constraint to the set of constraints in (2.29) and solve for

$$\begin{aligned}
&\max_{p_{s1},p_{s2}} \min\left(\frac{p_{s2}}{N_o}\sum_l \left|g_l\right|^2, \frac{p_{s1}}{N_o}\sum_l \left|f_l\right|^2\right) \\
&s.t.\ \ C1: p_{s1}\left|h_{s1,m}\right|^2 + p_{s2}\left|h_{s2,m}\right|^2 \le I_m^{max} \forall m, \\
&\qquad C2: 0 \le p_{s1}, p_{s2} \le p_{max}^s \\
&\qquad C3: \frac{p_{s2}}{N_o}\sum_l \left|g_l\right|^2 = \frac{p_{s1}}{N_o}\sum_l \left|f_l\right|^2
\end{aligned} \tag{2.30}$$

Using *C*3 in (2.30), we substitute $p_{s2} = p_{s1}\dfrac{\sum_l \left|f_l\right|^2}{\sum_l \left|g_l\right|^2}$ and solve the following trivial single variable linear program to obtain the closed form solution.

$$\begin{aligned}
&\max_{p_{s1}} \frac{p_{s1}}{N_o}\sum_l \left|f_l\right|^2 \\
&s.t.\ \ C1: p_{s1} \le \frac{I_m^{max}\sum_l \left|g_l\right|^2}{\left|h_{s1,m}\right|^2 \sum_l \left|g_l\right|^2 + \sum_l \left|f_l\right|^2 \left|h_{s2,m}\right|^2} \forall m, \\
&\qquad C2: 0 \le p_{s1} \le p_{max}^s \\
&\qquad C3: p_{s1}\frac{\sum_l \left|f_l\right|^2}{\sum_l \left|g_l\right|^2} \le p_{max}^s
\end{aligned} \tag{2.31}$$

2.7 Results

We present the simulation results of the proposed suboptimal schemes GPAMF, GPASM and compare their performance with the optimal solutions obtained by applying monotonic optimization techniques. In our simulations, we assume same interference constraints at all the PUs and denote it as I_{max}. We denote the total number of relays as *L* and total number of primary users as *M*. The maximum allowed transmission powers of the source and relays are denoted as p^s_{max}, p^r_{max} and their values are fixed to 5 watts and 2.5 watts respectively. For the polyblock outer approximation algorithm, the value of convergence tolerance parameter ε is kept at 0.05.

Fig. 2.6 and 2.7 present the convergence results of the polyblock outer approximation technique applied to the optimizations in (2.7) and (2.8). We present our convergence results for the scenario M = 1, L = 2, I_{max} = 1mw. As discussed earlier the polyblock outer approximation algorithm generates a sequence of nested polyblocks. As the iterations proceeds, the approximation of the feasible set by the polyblocks improves. The maximization of the objective function over a polyblock is an upper bound to the original optimization problem. The corresponding projection of the solution, obtained from the polyblock outer approximation algorithm, on to the feasible set is a lower bound to the original optimization problem. The lower bound is essentially the best feasible solution obtained until the current iteration. As the iterations proceeds the difference between the upper and lower bound decreases and the algorithm terminates in a finite number of iterations when the difference is within a desirable accuracy, ε. The result in fig. 2.6 and 2.7 are in conformance with the theory that we discussed above.

The polyblock outer approximation algorithm converges to within ε of the optimal solution in finite number of iterations. However, the algorithm may take unmanageably large number of iterations for ε-convergence. Thus, in order to avoid the heavy computational burden due to slow convergence, we set an upper bound on the number of iterations for which we run the polyblock outer

approximation algorithm. In the following results, we compare the heuristics with upper bounds and lower bounds obtained from polyblock outer approximation after 2000 iterations.

Figs. 2.8 and 2.9 represent the performance of the heuristics GPAMF and GPASM with respect to the interference threshold. We compare our proposed heuristics with the upper and lower bounds obtained from the polyblock outer approximation algorithm for the scenario M = 1 and 4, L = 3. Figs. 2.4 and 2.5 show that the sum-rate and minimum among the users' rate increase with the interference threshold because the feasible set of the optimization problem with lower interference threshold is a subset of the feasible set of the optimization problem with higher interference threshold. Further, we notice that with the increase in the number of primary users, the sum-rate and minimum among the users' rate decreases as more number of primary users means that the secondary users have more constraints on their transmit powers.

Fig. 2.10 represents the performance of the heuristic GPAMF with respect to the number of primary users. We compare our proposed heuristic with the upper and lower bounds obtained from the polyblock outer approximation algorithm for the scenario L = 3, I_{max} = (0.1, 100) *mw*. The observations are as expected.

Figs. 2.11 and 2.12 represent the performance of the heuristics GPAMF and GPASM with respect to the number of relays. We compare our proposed heuristics with the upper and lower bounds obtained from the polyblock outer approximation algorithm for the scenario M = 1, I_{max} = 1mw, 100mw. We observe that the sum-rate (minimum user's capacity) increases with the increase in number of primary users. This is because with the increase in number of relays there are more options in assigning transmit power to the relays. Similarly, the minimum among the users' rate increases with the relays.

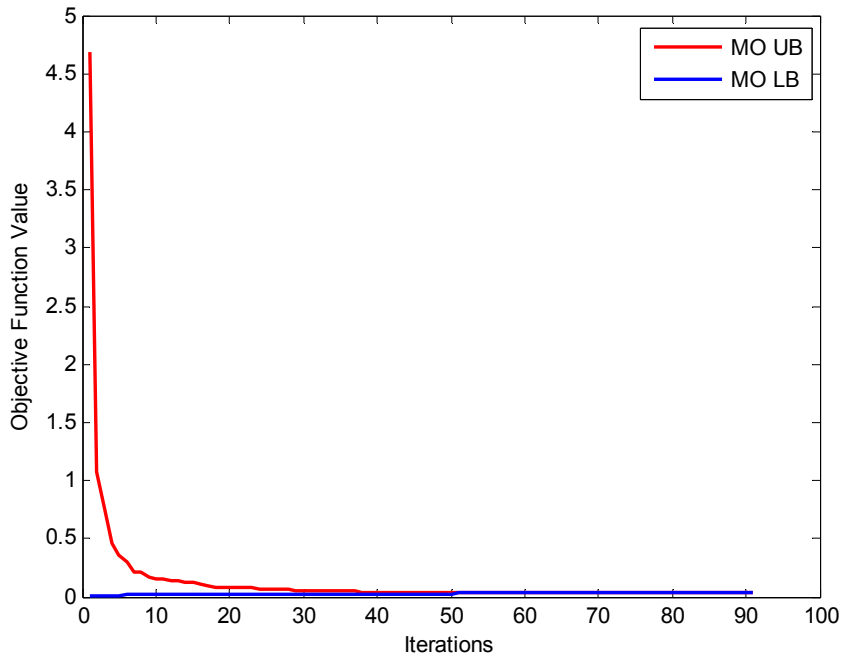

Fig. 2.6 Convergence Results Sum-Capacity Optimization

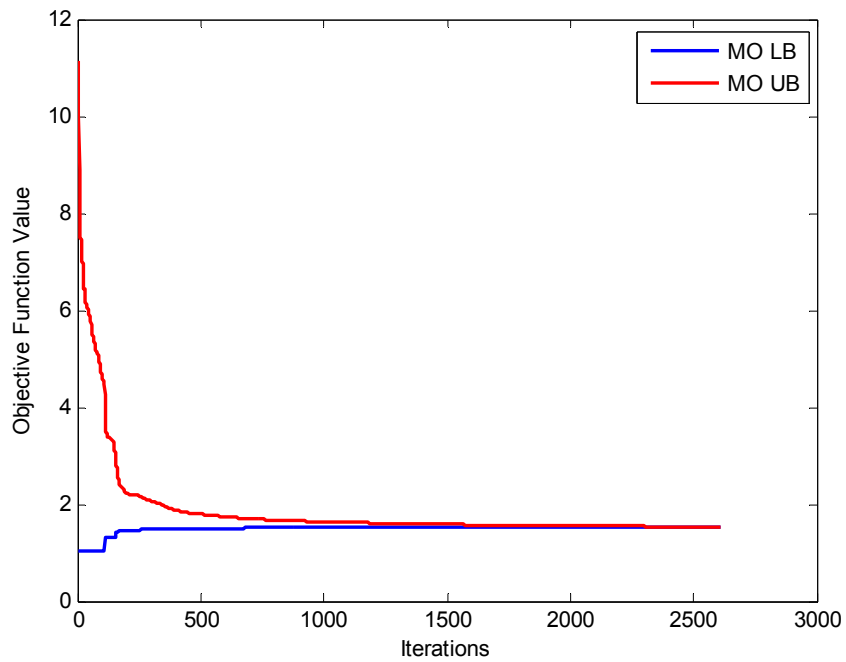

Fig. 2.7 Convergence Results Max-Min Optimization

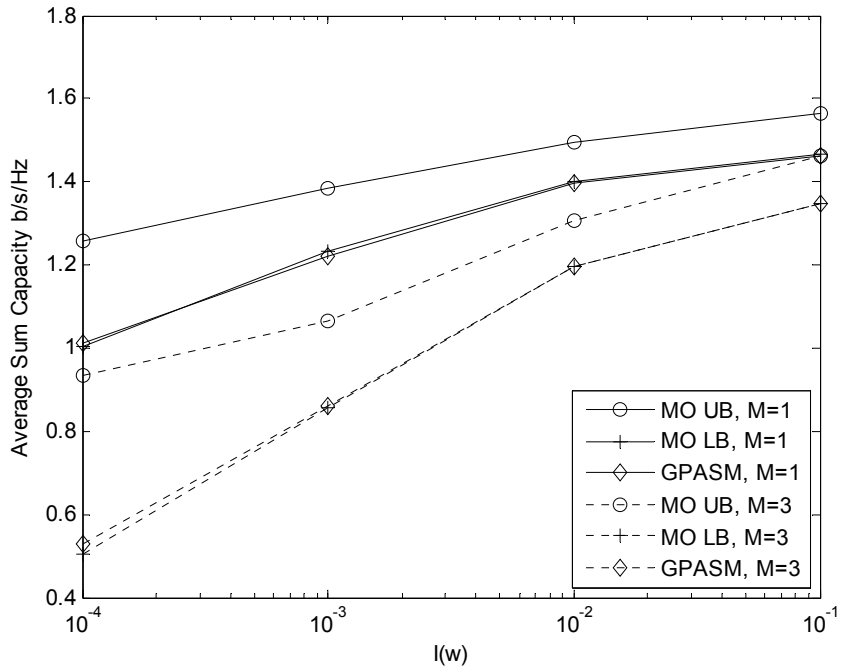

Fig. 2.8 Sum Capacity vs. $I_m^{\max}$, L = 3

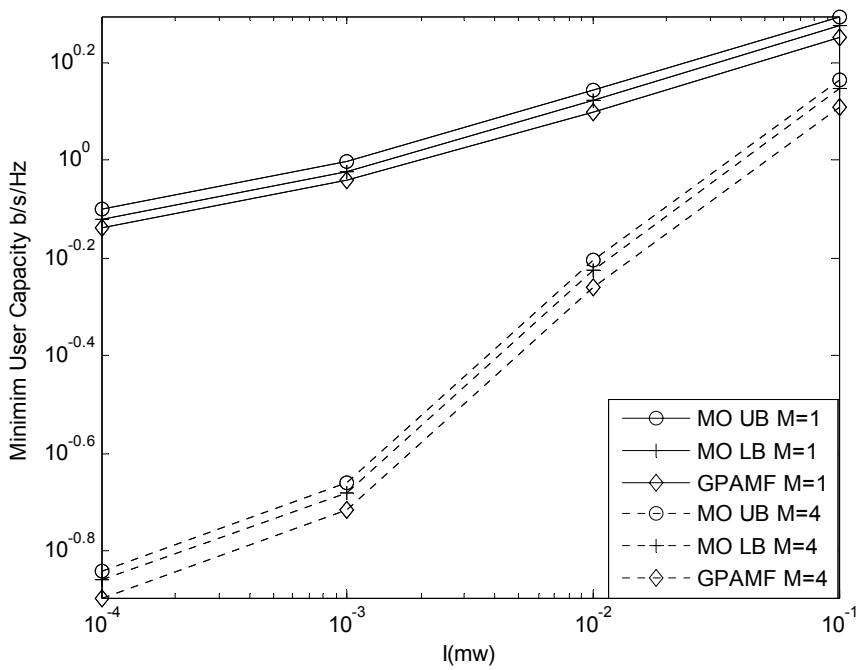

Fig. 2.9 Minimum User Capacity vs. $I_m^{\max}$, L = 3

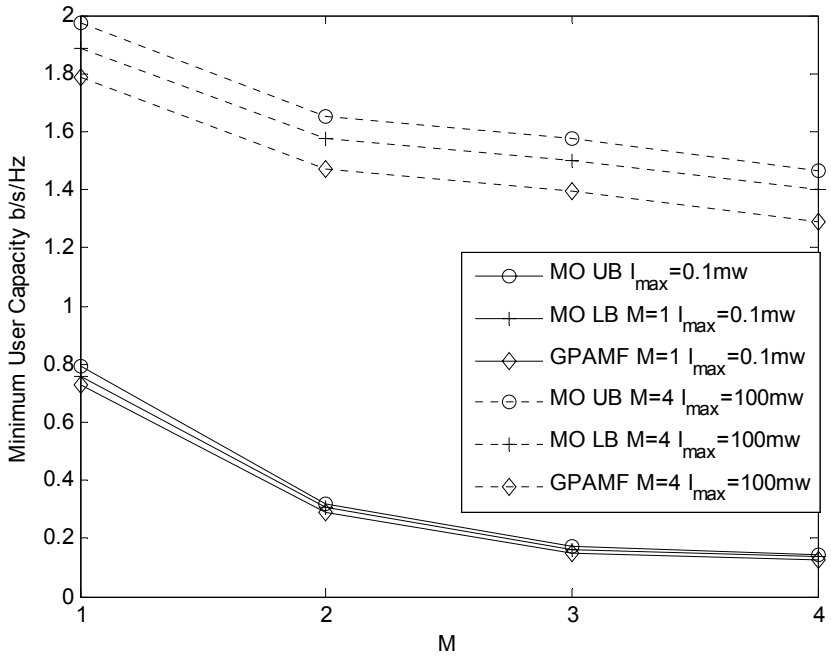

Fig. 2.10 Minimum User Capacity vs.Primary Users, *L* =3

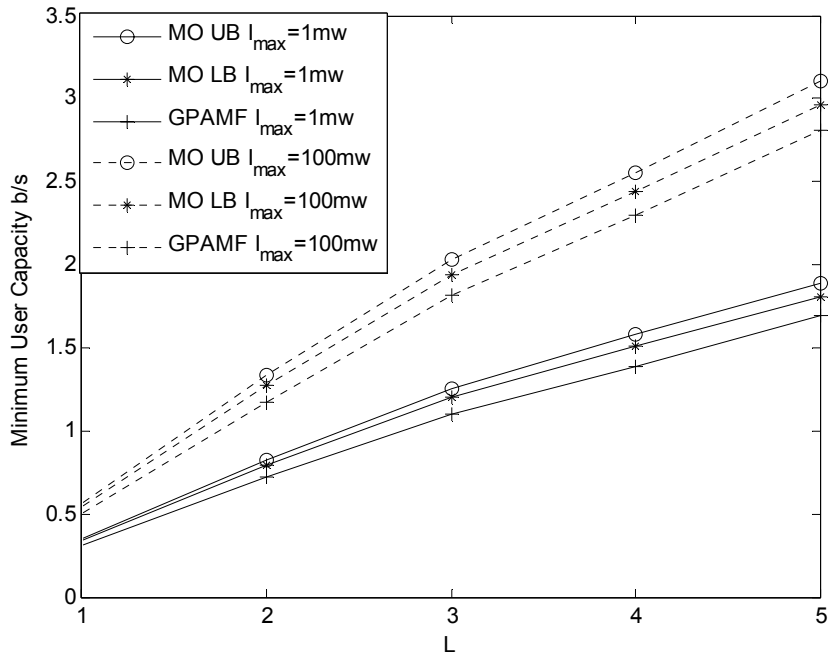

Fig. 2.11 Sum- Capacity vs.Relays, *M* =1

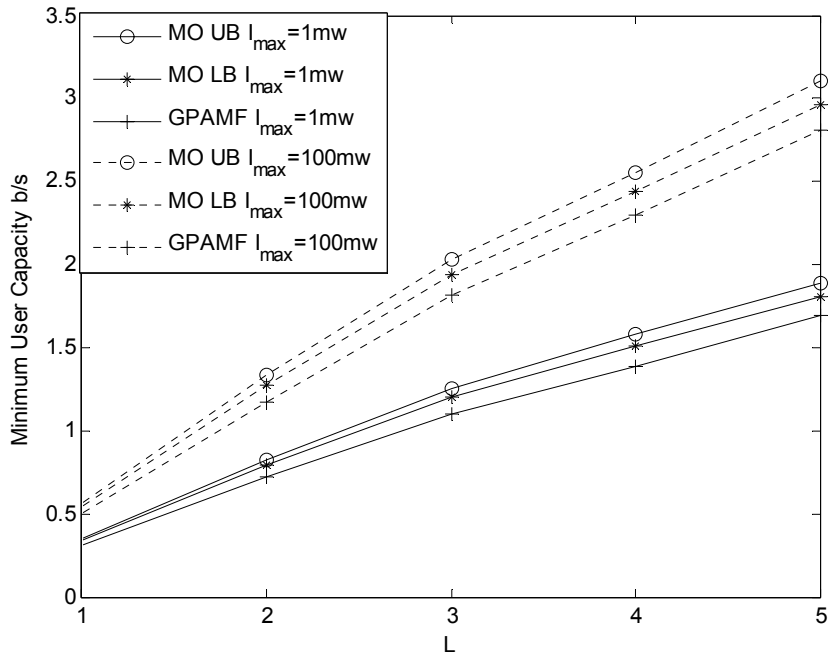

Fig. 2.12 Minimum User Capacity vs.Relays, M = 1

2.8 Summary

In this work, we considered a cognitive radio system comprising a pair of sources and multiple relays. We studied two optimization problems− the sum capacity maximization and max-min capacity. The formulated optimization problems were non-convex and nonlinear in nature. We obtained their optimal solutions by applying the monotonic optimization algorithm. However, our experiments indicated that the computational load of this approach is heavy. For computational efficiency, we proposed low-complexity heuristic algorithms to determine suboptimal solutions of the proposed optimization problems. By comparing the quality of these solutions to that of the solutions produced by monotonic optimization algorithm, we showed that our heuristic algorithms provide good solutions.

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CHAPTER 3: POWER ALLOCATION IN SHARED BAND TWO WAY RELAY ASSISTED COGNITIVE RADIO NETWORKS

In this chapter, we study a two-way relay network comprising a pair of users and multiple relays. The relays employ AF relaying. In AF relaying, in the first time-slot the users broadcast their data. In the second time-slot, the relays simply re-scale and re-transmit the received signal. The received signal (from relays) at each receiver consists of its own transmitted symbol and the transmitted symbol of other user. The receiver can then cancel the effect of its own transmitted symbol and recover the transmitted symbol of the other user. In this work, we consider the relaying protocol where all the relays transmit in the same band at the same time. This communication protocol has been given in [5] for one-way relaying and [4] for two-way relay.

Performance of a cooperative cognitive radio network can be improved by designing efficient resource allocation schemes. Some of the existing work on shared-band amplify and forward relaying can be found in [5], [6], [7], [8], [9] and [10]. Reference [5] and [6] discuss relay power allocation for one-way SAF relaying. In [5], a relay network comprising a source-destination pair and multiple relays is considered and the problem of maximizing the capacity of the SAF relaying scheme is studied. The authors in [5] study the problem of allocating power to the relays subject to a single sum-power constraint on the transmit power of the relays. In [6], the relays' power allocation problem is studied for a SAF relay network comprising a source-destination pair and multiple relays. The optimizations in [6] have a single sum-power constraint and individual power constraints on the transmissions of the relays. The authors in [6] show that the cost functions in their optimizations (although not concave) is a quasiconcave function of the relay powers. The authors in [6], then, derive the optimal relay

power allocation by using the known bisection search procedure for solving quasi-convex programs. Both the references for one-way relay [5]-[6] consider allocating only relay powers. Power allocation and determining beamforming vectors of the relays have been studied in [7]-[10] for the SAF two-way wireless relay network comprising multiple relays and a pair of users. Reference [7], [8] study determining relay beamforming vectors. References [7], [8] optimize sum-rate of the pair of users subject to a sum-power constraint on the relays. The individual rates of the users in SAF two-way relays have a structure similar to that of NAF one-way relay i.e. they are quasiconcave functions of the relay powers. The optimizations in [7], [8] involve sum of two quasiconcave functions (sum-rates) which may not be quasiconcave. To overcome this difficult the authors in [7], [8] consider the optimizations where the user rates are proportional to each other and provide bisection search based solution to their optimizations as done previously in [6]. The joint optimization of source and relay transmit powers unfortunately does not have such structure. Joint source and relay power optimization for SAF two-way relaying is studied in [9], [10]. However, the optimizations in [9], [10] considers only a single sum-power constraint and the solution to the optimizations in [9], [10] cannot be applied to the optimization problems that has both sum-power and multiple individual power constraints.

In this work, we study the problem of allocating power to the sources and the relays subject to individual power constraints on the nodes and the interference temperature constraints on the relays [11]. The cost function of our optimization problem is to maximize the minimum among the sources' capacities. Our formulated optimization problem is a non-convex non-linear program and does not have a structure to guarantee the quality of a solution. We observe that we can transform our proposed optimization problem into another equivalent problem (although still a non-convex non-linear program) which exhibits a special property. In the transformed problem, we observe that the objective function and the constraints are increasing function of each optimization variable when other variables are fixed. This property enables us to determine the global optimal solution to our optimization problem by applying the concepts of monotonic

optimization algorithm [12]. Monotonic optimization algorithm guarantees convergence to a solution that has a performance arbitrarily close (within an arbitrary number ε) to the optimal performance.

Although the monotonic optimization techniques can guarantee convergence to a solution that has a performance arbitrarily close (within an arbitrary number ε) to the optimal performance, our experimentation with the monotonic optimization techniques applied to this relay power allocation problem shows rather slow convergence and heavy computational load. Therefore, we propose a low-complexity heuristics, which we name as Greedy Two-Step Power Allocation (GTSPA). The simulation results show that the proposed heuristics perform well in comparison to the respective optimal solutions and have much lower computational complexity than the monotonic optimization algorithm.

Table 3.1 Notations used in chapter 3.

Symbol	**Definition**
M	Number of primary users
L	Number of relays
s1,s2	Source 1, source 2
f_l	Channel gain from source 1 to the *l*th relay
g_l	Channel gain from source 2 to the *l*th relay
$h_{l,m}$	Channel gain from *l*th relay to the *m*th primary user
$h_{s1,m}$	Channel gain from *s1* to the *m*th primary user
$h_{s2,m}$	Channel gain from *s2* to the *m*th primary user
p_{s1} p_{s2}	Transmission power of source 1, souece2
p_l	Transmission power of *l*th relay
p_l^{max}	Maximum allowed transmission power of the *l*th relay
$I_{m,k}^{max}$	Maximum allowed interference at *m*th primary user
$\mathbf{R}_+^n \left(\mathbf{R}_{++}^n\right)$	Set of positive (strictly positive > 0) integers

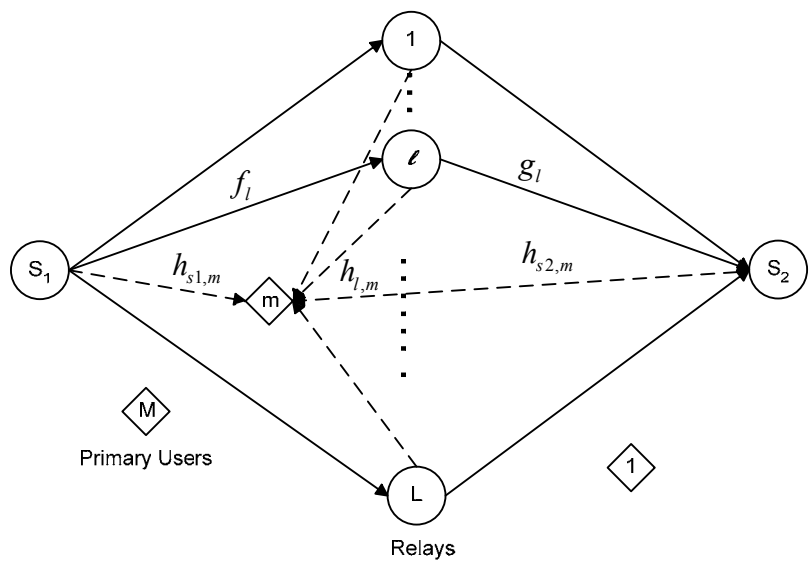

Fig. 3.1 Two-way relay assisted Cognitive Radio network

3.1 System Model

We consider a two-way relay system with two sources and *L* relays. Our system model also includes *M* primary users, for which the transmission power of the cognitive radio nodes (secondary users) must be limited. *M* primary users can mean, as well as primary user devices, *M* geographic locations or regions in which the strengths of the cognitive radio signals must be constrained. Figure 3.1 depicts our system model. The two sources will communicate with each other with the help of relays that use amplify-and-forward relaying. The relays, source 1 (*s*1), and source 2 (*s*2) are equipped each with a single antenna. We denote by f_l the channel gain from s1 to the *l*th relay, g_l the channel gain from s2 to the *l*th relay, $h_{s1,m}(h_{s2,m})$ the channel gain from s1 (s2) to the *m*th primary user and by $h_{l,m}$ the channel gain from *l*th relay to the *m*th primary user. For gaining simple insights to the system, in this chapter we assume that each channel between a source and a relay is symmetric. It is also assumed that transmissions from all nodes are perfectly synchronized. Let p_{s1}, p_{s2} and p_l denote, respectively, sources s1's, s2's and relays' transmission powers per dimension.

We consider a two-way amplify-and-forward (AF) scheme, as given in [4], [7] for cooperative communication which we will refer to as shared-band amplify and forward (SAF) relaying. In the first time-slot the users (sources) send their data to the relays. The signal received by relay *l* (*l* = 1, 2,.., *L*) is

$$y_l = \sqrt{p_{s1}} f_l X_{s1} + \sqrt{p_{s2}} g_l X_{s2} + Z_l \tag{3.1}$$

where complex-valued random variables X_{s1} and X_{s2} represent the transmitted symbols and are normalized such that $E\left(\left|X_{s1}\right|^2\right) = E\left(\left|X_{s2}\right|^2\right) = 1$. Z_l is the complex-valued white Gaussian noise at relay *l* with $E\left(\left|Z_l\right|^2\right) = N_o$. In the second time slot, the relays amplify the received signal y_l, l=1,2,..,L and broadcast it to the users. The transmitted signal from *l*th relay can be written as $x_l = \beta_l y_l$ where β_l is the amplification applied by *l*th relay. β_l is chosen such that the average transmit power of the *l*th relay is p_l. The relay amplification gain can be written as β_l [2], [13], [6]

$$\begin{aligned} \beta_l &= \sqrt{\frac{p_l}{p_{s1}\left|f_l\right|^2 + p_{s2}\left|g_l\right|^2 + N_o}} e^{j\theta_l} \\ \theta_l &= -\arg\left(f_l\right) - \arg\left(g_l\right) \end{aligned} \tag{3.2}$$

The signal received at *s*1 and s2 from the relays can be written as

$$\begin{aligned} y_{s1} &= \sum_{l=1}^{L}\left(\sqrt{p_{s1}} f_l X_{s1} + \sqrt{p_{s2}} g_l X_{s2} + Z_l\right)\beta_l f_l + Z_{s1} \\ y_{s2} &= \sum_{l=1}^{L}\left(\sqrt{p_{s1}} f_l X_{s1} + \sqrt{p_{s2}} g_l X_{s2} + Z_l\right)\beta_l g_l + Z_{s2} \end{aligned} \tag{3.3}$$

where Z_{s1}, Z_{s2} are the i.i.d complex valued white Gaussian noise at s1 and s2 with $E\left(\left|Z_{s1}\right|^2\right) = E\left(\left|Z_{s2}\right|^2\right) = N_o$

After self-interference cancellation [1], the received signals at *s*1 and *s*2 can be written as

$$
\begin{aligned}
y_{s1} &= \sum_{l=1}^{L}\left(\sqrt{p_{s2}}\,g_l X_{s2} + Z_l\right)\beta_l\, f_l + Z_{s1} \\
y_{s2} &= \sum_{l=1}^{L}\left(\sqrt{p_{s1}}\,f_l X_{s1} + Z_l\right)\beta_l\, g_l + Z_{s2}
\end{aligned}
\tag{3.4}
$$

After substituting for β_l, the capacities of *s*1 and *s*2 can be written as the following:

$$
\begin{aligned}
&R_{s1} = \frac{1}{2}\log\left(1+\gamma_{s2}\right), R_{s2} = \frac{1}{2}\log\left(1+\gamma_{s1}\right) \\
&\gamma_{s1} = \frac{p_{s2}\left(\sum_{l=1}^{L}\sqrt{\dfrac{p_l\left|f_l\right|^2\left|g_l\right|^2}{p_{s1}\left|f_l\right|^2 + p_{s2}\left|g_l\right|^2 + N_o}}\right)^2}{N_o + N_o\sum_{l=1}^{L}\dfrac{p_l\left|f_l\right|^2}{p_{s1}\left|f_l\right|^2 + p_{s2}\left|g_l\right|^2 + N_o}}, \\
&\gamma_{s2} = \frac{p_{s1}\left(\sum_{l=1}^{L}\sqrt{\dfrac{p_l\left|f_l\right|^2\left|g_l\right|^2}{p_{s1}\left|f_l\right|^2 + p_{s2}\left|g_l\right|^2 + N_o}}\right)^2}{N_o + N_o\sum_{l=1}^{L}\dfrac{p_l\left|g_l\right|^2}{p_{s1}\left|f_l\right|^2 + p_{s2}\left|g_l\right|^2 + N_o}}
\end{aligned}
\tag{3.5}
$$

3.2 Problem Formulation

In this section, we present our optimizations for allocating power to the sources and the relays. In this work, we study the problem of determining the optimal transmission powers of the users s1, s2 and the relays such that the lesser of the two sources' (*s*1 or *s*2) capacities is maximized subject to the interference constraint to the primary users and the individual transmit power constraints on the sources and the relays. We denote by **p** the vector (p_1, p_2, · · · , p_l, · · · , p_L) . The joint source and relay power optimization can be formulated as the following:

$$
\begin{aligned}
&\max_{\{p_{s1},p_{s2},\mathbf{p}\}} \min\{R_{s1},R_{s2}\} \\
&\text{subject to} \\
&C1: \sum_{l=1}^{L} p_l \left|h_{l,m}\right|^2 \le I_m^{max}, \forall m \\
&C2: 0 \le p_l \le p_l^{max}, \forall l \\
&C3: (p_{s1}, p_{s2}) \in Y, \text{ where} \\
&Y \equiv \left\{ (p_{s1}, p_{s2}) \middle| \begin{array}{l} p_{s1}\left|h_{s1,m}\right|^2 + p_{s2}\left|h_{s2,m}\right|^2 \le I_m^{max}, \ \forall m, \\ 0 \le p_{s1}, p_{s2} \le p_s^{max} \end{array} \right\}
\end{aligned}
\tag{3.6}
$$

In the above optimization problem, constraint $C1$ ensures that the threshold of the relays' interference to each PU is not exceeded. $C2$ represents each relay's maximum transmission power constraints. $C3$ limits the sources' transmission powers both from their own limitation p_s^{max} and from the threshold of sources' interference to every primary user. It should be noted that (as shown in Appendix E) the objective function is not convex with respect to the variables (p_{s1}, p_{s2}, $\mathbf{p}$). Thus, convex optimization techniques cannot be applied to determine the global optimal solution.

3.2.1 Existing methods

Joint source and relay power optimization for SAF two-way relaying is studied in [9], [10]. In the optimizations in [9] and [10], the transmit powers of the sources and the relays are constrained by a single sum-power constraint. The authors in [9] and [10] show that the sum-power constraint is met with equality at the optimal solutions of the optimizations. This observation is then further used to determine the optimal solutions of the optimizations in [9] and [10]. In our formulation in (3.6), we have multiple constraints on the transmit power of the nodes and it is difficult to predict which constraints would be met with equality at

the optimal solution. Therefore, we need to determine other methods to solve our proposed optimizations. In the following section, we provide a ε-optimal solution to the optimization in (3.6).

3.3 ε-Optimal Solution based on Monotonic Optimization

In this section, we apply the monotonic optimization techniques to determine the ε-optimal solutions of our max-min optimization problem in (3.6). The optimization in (3.6) is not of the form of a standard monotonic optimization problem as the cost function is not an increasing function of the optimization variables (p_{s1}, p_{s2}, $\mathbf{p}$). More specifically, the SNRs $\hat{\gamma}_{s1}, \hat{\gamma}_{s2}$ are not an increasing function of the optimization variables $\mathbf{p}$ [15]. We notice that with a change of variables, we can arrive at an optimization problem, which can be conveniently written as difference of increasing functions. We first observe that, from (3.2), the transmit power of lth relay can also be written as $p_l = |\beta_l|^2 \left(p_{s1} |f_l|^2 + p_{s2} |g_l|^2 + N_o \right)$. We substitute $z_l = |\beta_l|$ and substitute for $p_l = z_l^2 \left(p_{s1} |f_l|^2 + p_{s2} |g_l|^2 + N_o \right)$ in (3.6). Let us denote by $\mathbf{z}$, the vector (z_1, z_2,…, z_L). The resulting optimization can be written as the following:

$$
\begin{aligned}
&\max_{\{p_{s1}, p_{s2}, \mathbf{z}\}} \min\{R_{s1}, R_{s2}\} \\
&\text{subject to} \\
&C1: \sum_{l=1}^{L} z_l^2 \left(p_{s1} |f_l|^2 + p_{s2} |g_l|^2 + N_o \right) |h_{l,m}|^2 \le I_m^{max}, \forall m \\
&C2: 0 \le z_l \le \sqrt{\frac{p_l^{max}}{\left(p_{s1} |f_l|^2 + p_{s2} |g_l|^2 + N_o \right)}}, \forall l \\
&C3: (p_{s1}, p_{s2}) \in Y, \text{ where} \\
&Y \equiv \left\{ (p_{s1}, p_{s2}) \middle| \begin{array}{l} p_{s1} |h_{s1,m}|^2 + p_{s2} |h_{s2,m}|^2 \le I_m^{max}, \ \forall m, \\ 0 \le p_{s1}, p_{s2} \le p_s^{max} \end{array} \right\}
\end{aligned}
\tag{3.7}
$$

where

$$R_{s1} = \frac{1}{2}\log\left(1+\gamma_{s2}\right), R_{s2} = \frac{1}{2}\log\left(1+\gamma_{s1}\right)$$

$$\gamma_{s1} = \frac{p_{s2}\left(\sum_{l=1}^{L} z_l \left|f_l\right| \left|g_l\right|\right)^2}{N_o + N_o\sum_{l=1}^{L} z_l^2 \left|f_l\right|^2}, \gamma_{s2} = \frac{p_{s1}\left(\sum_{l=1}^{L} z_l \left|f_l\right| \left|g_l\right|\right)^2}{N_o + N_o\sum_{l=1}^{L} z_l^2 \left|g_l\right|^2}$$

The optimization problem in (3.7) is not a standard monotonic optimization problem. However, by using an auxiliary variable, we can transform the optimization in (3.7) to the standard monotonic optimization problem. More specifically, we write the cost function in (3.7) as difference of two functions that are monotonically increasing functions of the optimization variables p_{s1}, p_{s2} and $\mathbf{z}$. Then, with an introduction of an additional variable, we write the max-min optimization problem in (3.7) as a standard monotonic optimization problem.

The capacities of user s1 and s2 can be written as difference of increasing functions as follows:

$$\begin{aligned}
&\frac{1}{2}\log\left(1+\gamma_{s1}\right) = q_1\left(p_{s2},\mathbf{z}\right) - r_1\left(\mathbf{z}\right) \\
&q_1\left(p_{s2},\mathbf{z}\right) \\
&= \frac{1}{2}\log\left[N_o + N_o\sum_{l=1}^{L} z_l^2 \left|f_l\right|^2 + p_{s2}\left(\sum_{l=1}^{L} z_l \left|f_l\right| \left|g_l\right|\right)^2\right] \\
&r_1\left(\mathbf{z}\right) = \frac{1}{2}\log\left[N_o + N_o\sum_{l=1}^{L} z_l^2 \left|f_l\right|^2\right]
\end{aligned} \tag{3.8}$$

$$\begin{aligned}
&\frac{1}{2}\log\left(1+\gamma_{s2}\right) = q_2\left(p_{s1},\mathbf{z}\right) - r_2\left(\mathbf{z}\right) \\
&q_2\left(p_{s1},\mathbf{z}\right) \\
&= \frac{1}{2}\log\left[N_o + N_o\sum_{l=1}^{L} z_l^2 \left|g_l\right|^2 + p_{s1}\left(\sum_{l=1}^{L} z_l \left|f_l\right| \left|g_l\right|\right)^2\right] \\
&r_2\left(\mathbf{z}\right) = \frac{1}{2}\log\left[N_o + N_o\sum_{l=1}^{L} z_l^2 \left|g_l\right|^2\right]
\end{aligned} \tag{3.9}$$

Using notations from (3.8) and (3.9), the cost function of the max-min optimization problem in (3.7) can be written difference of increasing functions as follows:

$$\begin{aligned}
&\min\left\{\frac{1}{2}\log\left(1+\gamma_{s1}\right),\frac{1}{2}\log\left(1+\gamma_{s2}\right)\right\}\\
&=\min\left\{q_1\left(p_{s2}.\mathbf{z}\right)-r_1\left(\mathbf{z}\right),\ q_2\left(p_{s1},\mathbf{z}\right)-r_2\left(\mathbf{z}\right)\right\}\ \text{from}\,(4.8)-(4.9)\\
&=\min\left\{q_1\left(p_{s2},\mathbf{z}\right)+r_2\left(\mathbf{z}\right)-r_1\left(\mathbf{z}\right)-r_2\left(\mathbf{z}\right),\right.\\
&\qquad\left.q_2\left(p_{s1},\mathbf{z}\right)+r_1\left(\mathbf{z}\right)-r_2\left(\mathbf{z}\right)-r_1\left(\mathbf{z}\right)\right\}\\
&=\min\left\{q_1\left(\mathbf{z},p_{s2}\right)+r_2\left(\mathbf{z}\right),q_2\left(p_{s1},\mathbf{z}\right)+r_1\left(\mathbf{z}\right)\right\}-r_1\left(\mathbf{z}\right)-r_2\left(\mathbf{z}\right)\\
&=q\left(p_{s1},p_{s2},\mathbf{z}\right)-r\left(\mathbf{z}\right)
\end{aligned}\tag{3.10}$$

where

$$\begin{aligned}
q\left(p_{s1},p_{s2},\mathbf{z}\right)&=\min\left\{q_1\left(\mathbf{z},p_{s2}\right)+r_2\left(\mathbf{z}\right),\ q_2\left(p_{s1},\mathbf{z}\right)+r_1\left(\mathbf{z}\right)\right\}\text{and}\\
r\left(\mathbf{z}\right)&=r_1\left(\mathbf{z}\right)+r_2\left(\mathbf{z}\right)
\end{aligned}$$

We showed above that the objective function of our optimization in (3.7) can be written as difference of increasing functions. We now transform our optimization to the standard monotonic optimization problem as in (2.14). We introduce a new decision variable that although, expands the feasible set of the solutions but at the same time allows us to apply the monotonic optimization techniques to determine the optimal solution of max-min optimization in (3.7).

Using the notations in (3.8)-(3.10), we can write the max-min optimization problem as the following:

$$
\begin{aligned}
&\max_{\{p_{s1},p_{s2},\mathbf{z}} q\left(p_{s1},p_{s2},\mathbf{z}\right)-r\left(\mathbf{z}\right),\\
&\text{subject to}\\
&\quad C1:\sum_{l=1}^{L} z_l^2\left(p_{s1}\left|f_l\right|^2+p_{s2}\left|g_l\right|^2+N_o\right)\left|h_{l,m}\right|^2\le I_m^{max},\forall m\\
&\quad C2:0\le z_l\ \le\sqrt{\frac{p_l^{max}}{\left(p_{s1}\left|f_l\right|^2+p_{s2}\left|g_l\right|^2+N_o\right)}},\forall l\\
&\quad C3:p_{s1}\left|h_{s1,m}\right|^2+p_{s2}\left|h_{s2,m}\right|^2\le I_m^{max}\forall m\\
&\quad C4:0\le p_{s1},p_{s2}\le p_s^{max}
\end{aligned}
\tag{3.11}
$$

Since $r(\mathbf{z})$ is a monotonic function of $\mathbf{z}$, therefore $\forall(\mathbf{z})$ that satisfies $C1$-$C2$ in (3.11), $r\left(\mathbf{z}\right)\le r\left(\mathbf{z}^{max}\right),\mathbf{z}^{max}=\left(\sqrt{\frac{p_1^{max}}{N_o}},\sqrt{\frac{p_2^{max}}{N_o}},..,\sqrt{\frac{p_L^{max}}{N_o}}\right)$. Hence, we have $r\left(\mathbf{z}\right)+t=r\left(\mathbf{z}^{max}\right)$. Hence, with the help of new optimization variable, t, we can substitute for $r(\mathbf{z})$. The resulting optimization problem can be written as follows,

$$
\begin{aligned}
&\max_{\{p_{s1},p_{s2},\mathbf{z},t} q\left(p_{s1},p_{s2},\mathbf{z}\right)+t-r\left(\mathbf{z}^{max}\right)\\
&\text{subject to}\\
&\quad C1:\quad r\left(\mathbf{z}\right)+t\le r\left(\mathbf{z}^{max}\right)\\
&\quad C2:\sum_{l=1}^{L} z_l^2\left(p_{s1}\left|f_l\right|^2+p_{s2}\left|g_l\right|^2+N_o\right)\left|h_{l,m}\right|^2\le I_m^{max},\forall m\\
&\quad C3:0\le z_l\ \le\sqrt{\frac{p_l^{max}}{\left(p_{s1}\left|f_l\right|^2+p_{s2}\left|g_l\right|^2+N_o\right)}},\forall l\\
&\quad C4:p_{s1}\left|h_{s1,m}\right|^2+p_{s2}\left|h_{s2,m}\right|^2\le I_m^{max}\forall m\\
&\quad C5:0\le p_{s1},p_{s2}\le p_s^{max}\\
&\quad C6:t\ge 0
\end{aligned}
\tag{3.12}
$$

The optimization in (3.12) is equivalent to that in (3.11). Further, the optimization in (3.12) is a standard monotonic optimization problem, which can be solved by using the polyblock outer approximation algorithm presented in

previous chapter. For computation of projections, we use the bisection search based method that we presented in previous chapter for sum-capacity optimization.

Although the monotonic optimization algorithm can guarantee convergence to a solution to the optimization in (3.12) that has a performance arbitrarily close (within an arbitrary number ε) to the optimal performance, our experimentation with the monotonic optimization algorithm applied to this relay and source power allocation problem shows rather slow convergence. Therefore, we propose a low-complexity heuristic that has low computational complexity and perform well in comparison with the respective optimal solution obtained from the polyblock outer-approximation algorithm. In the next section, we present the heuristic to solve the optimizations in (3.7). We name these heuristics as Greedy Two-Step Power Allocation (GTSPA).

3.4 Proposed Heuristic

In this section, we present a low-complexity heuristic algorithm to determine a suboptimal solution to the max-min optimization problem in (3.7). In our proposed algorithm, we try to separate out the optimization of the sources' and relays transmission powers. We first compute some simple bounds on the signal-to-noise ratios at s1 and s2. The bounds that we determine are dependent on the transmission powers of the sources and the channel gains between the sources and the relays. We then optimize the bounds over the constraints C3 in (3.7). The optimization is a simple linear program and we provide a closed form solution to the linear program. In the next step, we substitute the determined source powers into the optimization in (3.7). Although, the resulting optimization is a non-convex non-linear program, it has a special structure, which allows us to efficiently compute its optimal solution.

The proposed algorithm uses the simple bounds on the received SNR at s1 and s2 given in (2.26). The bounds are based on the Cauchy Schwarz inequality. In the Euclidean space, $\mathbf{R}^n$, the Cauchy Schwarz inequality is

$$\left(\sum_i x_i y_i\right)^2 \le \left(\sum_i x_i^2\right)\left(\sum_i y_i^2\right) \tag{3.13}$$

γ_{s1} in (3.7) can be bounded as

$$\begin{aligned}\gamma_{s1} = \frac{p_{s2}\left(\sum_{l=1}^{L} z_l\,|f_l|\,|g_l|\right)^2}{N_o + N_o\sum_{l=1}^{L} z_l^2\,|f_l|^2} &\le \frac{p_{s2}\left(\sum_{l=1}^{L} z_l\,|f_l|\,|g_l|\right)^2}{N_o\sum_{l=1}^{L} z_l^2\,|f_l|^2} \\ &\le \frac{p_{s2}\left(\sum_{l=1}^{L} z_l^2\,|f_l|^2\right)\left(\sum_{l=1}^{L}|g_l|^2\right)}{N_o\sum_{l=1}^{L} z_l^2\,|f_l|^2} \\ &\le \frac{p_{s2}}{N_o}\left(\sum_{l=1}^{L}|g_l|^2\right)\end{aligned} \tag{3.14}$$

Similarly, γ_{s2} can be bounded as $\dfrac{p_{s1}\left(\sum_{l=1}^{L} z_l\,|f_l|\,|g_l|\right)^2}{N_o + N_o\sum_{l=1}^{L} z_l^2\,|g_l|^2} \le \dfrac{p_{s1}}{N_o}\left(\sum_{l=1}^{L}|f_l|^2\right)$.

In this heuristic algorithm, we first consider the optimization of the upper bounds subject to the constraints on the source powers (p_{s1}, p_{s2}) as given in $C3$ in (3.6). Using the obtained suboptimal values of source powers $\left(\hat{p}_{s1}, \hat{p}_{s2}\right)$, we optimize the original objective function over the transmit powers. We formally state the GTSPA algorithm as follows:

<u>Phase 1</u>: Solve the following optimization problem and determine suboptimal values of source powers $\left(\hat{p}_{s1}, \hat{p}_{s2}\right)$.

$$\max_{p_{s1},p_{s2}} \min\left(\frac{1}{2}\log\left(1+\sum_l \frac{p_{s2}|g_l|^2}{N_o}\right), \frac{1}{2}\log\left(1+\sum_l \frac{p_{s1}|f_l|^2}{N_o}\right)\right)$$
$$s.t.\ \ C1: p_{s1}\left|h_{s1,m}\right|^2 + p_{s2}\left|h_{s2,m}\right|^2 \le I_m^{max} \quad \forall m,$$
$$C2: 0 \le p_{s1}, p_{s2} \le p_{max}^s$$

Due to the monotonicity of logarithm, we can consider optimizing the arguments of the logarithm in the cost function above. More specifically, we solve the following linear program:

$$\begin{aligned} &\max_{p_{s1},p_{s2}} \min\left(\frac{p_{s2}}{N_o}\sum_l |g_l|^2, \frac{p_{s1}}{N_o}\sum_l |f_l|^2\right) \\ &s.t.\ \ C1: p_{s1}\left|h_{s1,m}\right|^2 + p_{s2}\left|h_{s2,m}\right|^2 \le I_m^{max} \forall m, \\ &\quad C2: 0 \le p_{s1}, p_{s2} \le p_{max}^s \end{aligned} \tag{3.15}$$

The above optimization is identical to the one given in (2.29). We can express the maximizer in closed form as $\left(\hat{p}_{s1}, \hat{p}_{s2}\right) = \left(\frac{\hat{p}_{s2}\sum_l |g_l|^2}{\sum_l |f_l|^2}, \hat{p}_{s2},\right)$, where

$$\hat{p}_{s2} = \min\left(p_{max}^s, \frac{p_{max}^s \sum_l |f_l|^2}{\sum_l |g_l|^2}, \frac{I_m^{max}\sum_l |f_l|^2}{\left|h_{s1,m}\right|^2 \sum_l |g_l|^2 + \left|h_{s2,m}\right|^2 \sum_l |f_l|^2} \forall m\right).$$

We substitute for $\left(\hat{p}_{s1}, \hat{p}_{s2}\right)$ in (3.7) and solve the resulting optimization problem, which is

$$
\begin{aligned}
&\max_{\{\mathbf{z}\}} f(\mathbf{z}) \\
&\text{subject to} \\
&C1: \sum_{l=1}^{L} z_l^2 \left(\hat{p}_{s1} |f_l|^2 + \hat{p}_{s2} |g_l|^2 + N_o \right) |h_{l,m}|^2 \le I_m^{max}, \forall m \\
&C2: 0 \le z_l \le \sqrt{\frac{p_l^{max}}{\left(\hat{p}_{s1} |f_l|^2 + \hat{p}_{s2} |g_l|^2 + N_o \right)}}, \forall l
\end{aligned}
\tag{3.16}
$$

where $f(\mathbf{z}) \equiv \min \left\{ \frac{\hat{p}_{s2} \left(\sum_{l=1}^{L} z_l |f_l| \, |g_l| \right)^2}{N_o + N_o \sum_{l=1}^{L} z_l^2 |f_l|^2}, \frac{\hat{p}_{s1} \left(\sum_{l=1}^{L} z_l |f_l| \, |g_l| \right)^2}{N_o + N_o \sum_{l=1}^{L} z_l^2 |g_l|^2} \right\}$

The optimization in (3.16) has a convex constraint set but the cost function is not a concave function of the optimization variables **z** [6]. Therefore, convex optimization techniques cannot be applied to determine the optimal solution of the optimization in (3.16). However, we are able to exploit a special structure of the optimization in (3.16). More specifically, we note that the cost function of the optimization in (3.16) is quasiconcave w.r.t to the optimization variables **z**. Thus, optimization in (3.16) is a quasiconvex optimization problem. We further observe that we can efficiently determine the optimal solution to (3.16) by using a bisection method, where in each step we solve a convex feasibility program.

We first show that the cost function in the above optimization is a quasiconcave function of the optimization variables **z**. To do so, we show that the upper level set of the cost function is a convex set.

$$
\begin{aligned}
UL(f,t) &= \{\mathbf{z} : f(\mathbf{z}) \geq t\} \\
&= \left\{ \mathbf{z} : \min \left[\frac{\hat{p}_{s2}\left(\sum_{l=1}^{L} z_l \, |f_l| \, |g_l|\right)^2}{N_o + N_o \sum_{l=1}^{L} z_l^2 |f_l|^2}, \right.\right. \\
&\qquad \left.\left. \frac{\hat{p}_{s1}\left(\sum_{l=1}^{L} z_l \, |f_l| \, |g_l|\right)^2}{N_o + N_o \sum_{l=1}^{L} z_l^2 |g_l|^2} \right] \geq t \right\} \\
&= \left\{ \mathbf{z} : \frac{\hat{p}_{s2}\left(\sum_{l=1}^{L} z_l \, |f_l| \, |g_l|\right)^2}{N_o + N_o \sum_{l=1}^{L} z_l^2 |f_l|^2} \geq t, \right. \\
&\qquad \left. \frac{\hat{p}_{s1}\left(\sum_{l=1}^{L} z_l \, |f_l| \, |g_l|\right)^2}{N_o + N_o \sum_{l=1}^{L} z_l^2 |g_l|^2} \geq t \right\}
\end{aligned}
\tag{3.17}
$$

The individual inequalities in (3.17) can be written as follows:

$$
\begin{aligned}
&\frac{\hat{p}_{s2}\left(\sum_{l=1}^{L} z_l \, |f_l| \, |g_l|\right)^2}{N_o + N_o \sum_{l=1}^{L} z_l^2 |f_l|^2} \geq t \\
&\Leftrightarrow \left(\sum_{l=1}^{L} z_l \, |f_l| \, |g_l|\right) \geq \sqrt{\frac{t}{\hat{p}_{s2}}\left(N_o + N_o \sum_{l=1}^{L} z_l^2 |f_l|^2\right)} \\
&\Leftrightarrow \mathbf{c}\mathbf{z}^T \geq \left\| \mathbf{A}_1 \mathbf{z}^T + \mathbf{b}_1 \right\|
\end{aligned}
\tag{3.18-a}
$$

where $\mathbf{c} = [c_1, .., c_L]$ $c_l = |f_l| \, |g_l|$, $\mathbf{b}_1 = \left[\mathbf{0} \ \sqrt{\frac{tN_o}{\hat{p}_{s2}}} \right]^T$,

$\mathbf{0}$ is a $L \times 1$ null vector

$$
\mathbf{A}_1 = \begin{bmatrix} \overline{\mathbf{A}}_1 \\ \mathbf{0}^T \end{bmatrix}, \ \overline{\mathbf{A}}_1 = diag\left(\sqrt{\frac{t|f_1|^2 N_o}{\hat{p}_{s2}}}, \sqrt{\frac{t|f_2|^2 N_o}{\hat{p}_{s2}}}, \ldots, \sqrt{\frac{t|f_L|^2 N_o}{\hat{p}_{s2}}} \right)
$$

Similarly, the second inequality can be written as follows:

$$
\begin{aligned}
&\frac{\hat{p}_{s1}\left(\sum_{l=1}^{L} z_l \left|f_l\right| \left|g_l\right|\right)^2}{N_o + N_o \sum_{l=1}^{L} z_l^2 \left|g_l\right|^2} \geq t \\
&= \left(\sum_{l=1}^{L} z_l \left|f_l\right| \left|g_l\right|\right) \geq \sqrt{\frac{t}{\hat{p}_{s1}}\left(N_o + N_o \sum_{l=1}^{L} z_l^2 \left|g_l\right|^2\right)} \\
&= \mathbf{c}\mathbf{z}^T \geq \left\|\mathbf{A}_2\mathbf{z}^T + \mathbf{b}_2\right\| \\
&\text{where } \mathbf{c} = \left[c_1, .., c_L\right] \\
&c_l = \left|f_l\right| \left|g_l\right| \ , \mathbf{b}_2 = \begin{bmatrix} \mathbf{0} \\ \sqrt{\frac{tN_o}{\hat{p}_{s1}}} \end{bmatrix}, \mathbf{0} \text{ is a } L \times 1 \text{ null vector} \\
&\mathbf{A}_2 = \begin{bmatrix} \overline{\mathbf{A}}_2 \\ \mathbf{0}^T \end{bmatrix}, \ \overline{\mathbf{A}}_2 = diag\left(\sqrt{\frac{t\left|g_1\right|^2 N_o}{\hat{p}_{s1}}}, ..., \sqrt{\frac{t\left|g_L\right|^2 N_o}{\hat{p}_{s1}}}\right)
\end{aligned}
\tag{3.18-b}
$$

Using (3.18-a) and (3.18-b), we can rewrite (3.17) as

$$
UL(f,t) = \left\{\mathbf{z} : \mathbf{c}\mathbf{z}^T \geq \left\|\mathbf{A}_1\mathbf{z} + \mathbf{b}_1\right\|, \mathbf{c}\mathbf{z}^T \geq \left\|\mathbf{A}_2\mathbf{z} + \mathbf{b}_2\right\|\right\} \tag{3.19}
$$

The upper level set $UL(f,t)$ is a convex set as it is an intersection of a finite number, 2, of second order cones (which are convex sets). Hence, the objective function $f(\mathbf{z})$ is a quasiconcave function of the optimization variables **z**. Further, we have a closed form representation, given in (3.19), of the upper level set of the objective function. Therefore, we can use the methods for quasi-convex optimization given in [14]. More specifically, we use a bisection search based method to solve the optimization in (3.16). We first initialize an interval $\left[t^{\min}, t^{\max}\right]$ which contains the optimal solution, *t**, to the optimization in (3.16). We then

solve the feasibility problem at the midpoint, $t = \frac{t^{\min} + t^{\max}}{2}$, of the interval. In each iteration of the bisection search, we solve a convex feasibility problem. The convex feasibility problem can be written as follows

$$
\begin{aligned}
& find\left(z_1,..,z_L\right) \\
& subject\,to \\
& \quad C1:\ \sum_{l=1}^{L} z_l^2 \left(\hat{p}_{s1}\left|f_l\right|^2 + \hat{p}_{s2}\left|g_l\right|^2 + N_o\right)\left|h_{l,m}\right|^2 = 1,2,..,M \\
& \quad C2: 0 \le z_l \le \sqrt{\frac{p_l^{max}}{\left(\hat{p}_{s1}\left|f_l\right|^2 + \hat{p}_{s2}\left|g_l\right|^2 + N_o\right)}} \quad \forall l = 1,2,...,L \\
& \quad C3:\ \mathbf{c}\mathbf{z}^T \ge \left\|\mathbf{A}_i\mathbf{z}^T + \mathbf{b}\right\| \quad i = 1,2
\end{aligned}
\tag{3.20}
$$

The convex feasibility problem can be solved by using interior-point methods [14]. If we can find a feasible solution to the optimization in (3.20) at *t*, then it means that the optimal solution to the optimization problem is in the interval $\left[t, t^{\max}\right]$. Otherwise, if we cannot find a feasible solution to the optimization in (3.20) at *t* then it means that the optimal solution to the optimization problem is in the interval $\left[t^{\min}, t\right]$. We accordingly bisect the interval by using the results of solving the feasibility problem (3.20). The pseudo-code of the proposed bisection search based method is given in table 3.2. In the proposed BSPA algorithm, the algorithm keeps on iterating until $t^{\max} - t^{\min} \le \varepsilon$. In each iteration, the interval $\left[t^{\min}, t^{\max}\right]$ is bisected in two parts. As the iteration proceeds, the length of the interval $\left[t^{\min}, t^{\max}\right]$ keeps on diminishing. It has been shown in [14] that exactly $\left\lceil \log_2\left(\frac{t^{\max} - t^{\min}}{\varepsilon}\right)\right\rceil$ iterations are required before the algorithm terminates.

Main Algorithm
***Initialization*:** $t^{\max}, t^{\min} = 0, \varepsilon$
While $t^{\max} - t^{\min} \geq \varepsilon$ 1: $t = \dfrac{t^{\min} + t^{\max}}{2}$; 2: Solve the convex feasibility problem in (3.20) 3: ***If*** feasible solution found 4: $t^{\min} = t$ 5: ***else*** 6: $t^{\max} = t$ 7: ***endif*** ***EndWhile***

Table 3.2 Bisection Search Based Power Allocation For SAF Two-Way Relaying

3.5 Results

We present the simulation results of the proposed suboptimal schemes GTSPA algorithm and compare its performance with the optimal solutions obtained by applying monotonic optimization techniques. In our simulations, we assume same interference constraints at all the PUs and denote it as I_{max}. We denote the total number of relays as *L* and total number of primary users as *M*. The maximum allowed transmission powers of the source and relays are denoted as p^s_{max}, p^r_{max} and their values are fixed to 5 watts and 2.5 watts respectively. For the polyblock outer approximation algorithm, the value of convergence tolerance parameter is kept at 0.0005.

Fig. 3.2 presents the convergence results of the polyblock outer approximation technique applied to the optimization in (3.12). As discussed earlier the polyblock outer approximation algorithm generates a sequence of nested polyblocks. As the iterations proceeds, the approximation of the feasible set by the polyblocks improves. The maximization of the objective function over a polyblock is an upper bound to the original optimization problem. The projection

of the maximizer over the polyblocks on to the feasible set is a lower bound to the original optimization problem. The lower bound is essentially the best feasible solution obtained until the current iteration. As the iterations proceeds, the difference between the upper and lower bound decreases and the algorithm terminates in a finite number of iterations when the difference is within a desirable accuracy, ε.

The polyblock outer approximation algorithm converges to within ε of the optimal solution in a finite number of iterations. However, the algorithm may take unmanageably large number of iterations for ε-convergence. Thus, in order to avoid the heavy computational burden due to slow convergence, we set an upper bound on the number of iterations for which we run the polyblock outer approximation algorithm. In the following results, we compare the heuristics with upper bounds and lower bounds obtained from polyblock outer approximation after 4000 iterations.

Fig. 3.3 represents the performance of the proposed heuristics against the number of primary users. We compare our proposed heuristics with the upper and lower bounds obtained from the polyblock outer approximation algorithm for the scenario L = 2, I_{max} = (1, 100) *mw*. The observations are as expected. Fig. 3.3 shows that the minimum among the users' rate increase with the interference threshold because the feasible set of the optimization problem with lower interference threshold is a subset of the feasible set of the optimization problem with higher interference threshold. Further, we notice that with the increase in the number of primary users, the sum-rate and minimum among the users' rate decreases as more number of primary users means that the secondary users have more constraints on their transmit powers.

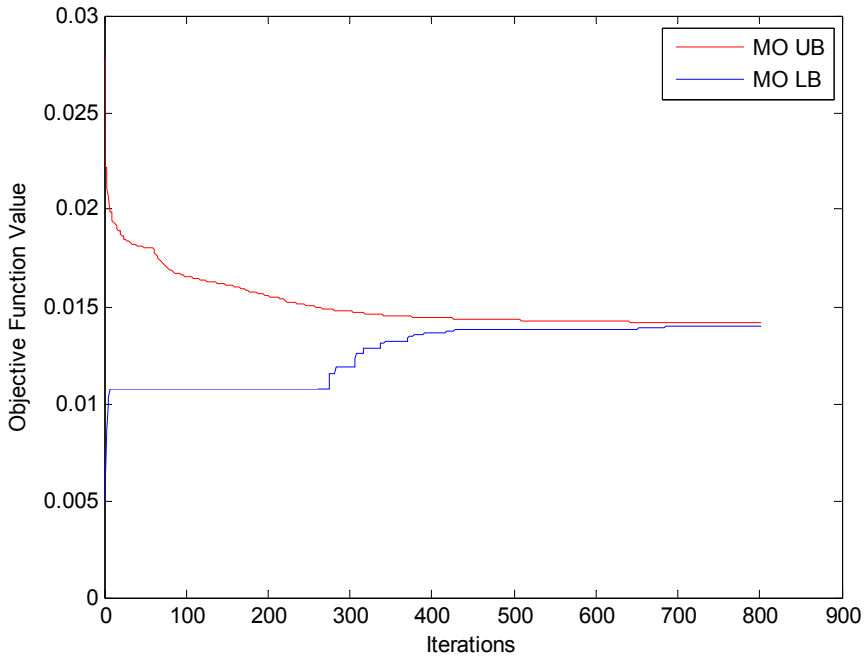

Fig. 3.2 Convergence Results Max-Min Optimization

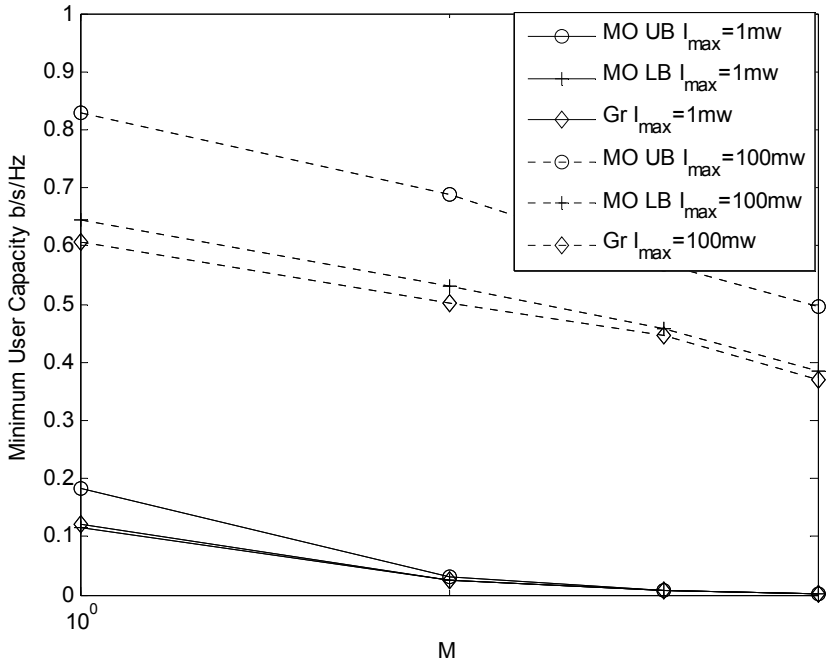

Fig. 3.3 Minimum User Capacity vs.Primary Users, *L* =2

3.6 Summary

In this work, we considered a cognitive radio system comprising a pair of sources and multiple relays. We studied the optimization problem to maximize the minimum among the users' capacities. The formulated optimization problems were non-convex and nonlinear in nature. We obtained their optimal solutions by applying the monotonic optimization algorithm. However, our experiments indicated that the computational load of this approach is heavy. For computational efficiency, we proposed a low-complexity heuristic algorithm to determine suboptimal solutions of the proposed optimization problem. By comparing the quality of these solutions to that of the solutions produced by monotonic optimization algorithm, we showed that our heuristic algorithms provide good solutions.

3.7 Chapter References

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CHAPTER 4: POWER ALLOCATION IN MULTI-WAY RELAY NETWORKS

In the previous chapters, we studied resource allocation in half-duplex two way relay assisted wireless networks. Recently, the idea of multi-way relay channel was discussed in [3]. In this model, there are multiple clusters comprising multiple users. The users in each cluster wish to exchange information with each other using a relay terminal. Various real world communication scenarios can be modelled using multi-way relay channel. Consider, for example, a social network scenario comprising multiple distinct clusters where each cluster consists of users who belonging to e.g. a particular friend group. In such scenario, the users in each cluster may wish to exchange personal information with each other using a relay terminal. Another example could be a sensor network where there are different kinds of sensors observing different physical phenomena. In such scenario, the temperature sensors may form one cluster and exchange their local temperature measurements with each other while there can also be pressure sensors sharing their local pressure measurements by forming another cluster. In [3], different relaying techniques (amplify-and-forward (AF), decode-and-forward and compress-and-forward) for multi-way relay channel are discussed. In this work, we consider a specific case of multi-way relay channel comprising a single cluster consisting of three users and a single relay. We assume that the relay terminal uses AF relaying.

Resource allocation in a cooperative communication system can offer various benefits such as longer network lifetime, better quality-of-service etc. In our model, transmission power of the nodes (users and relay) is the resource that we wish to allocate. In this work, we study the problem of allocating power to the users and the relay terminal such that the minimum among the users' transmission rates is maximized subject to the constraints on the transmission

power of the nodes. The formulated optimization problem is a non-convex nonlinear program does not have a structure to guarantee the quality of a solution. We observe that we can transform the problem into another equivalent problem (although still a non-convex nonlinear program) which exhibits a special property. In the transformed problem, we observe that the objective function and the constraints are increasing function of each optimization variable when other variables are fixed. This property enables us to determine the global optimal solution to our optimization problem by applying the concepts of monotonic optimization algorithm [5]). Monotonic optimization algorithm guarantees convergence to a solution that has a performance arbitrarily close (within an arbitrary number ε) to the optimal performance.

We perform simulations to examine the quality of the solution obtained from the proposed monotonic optimization based scheme. We compare it with a naive scheme, which allocates equal power to all the nodes. The simulation results show that proposed optimal solution is better than the uniform power allocation approach. Our system model and problem formulations are given in section II. Some definitions and concepts related to monotonic optimization are presented in section III. In section IV, we present our solution based on monotonic optimization. The numerical results are given in section V.

4.1 System Model

We consider a system with multiple users and a single relay. The users wish to exchange information with each other using a single relay. There are $K \geq 2$ users in the system. We denote by $I_K = \{1, \cdots, K\}$ as the set of users. The relay and the users are equipped each with a single antenna. We denote by $f_{k,r}$ the channel gain from the *k*th user to the *l*th relay. We assume symmetric channels from users to the relay i.e. the backward and forward channel gains between the users and the relay are identical. We represent by p_k, the transmission power of the user *k* and by p_r the transmission power of the relay. We assume that there is no direct link between the users and all the nodes are in half-duplex mode. Therefore, the transmission occurs in two time slots. In the first time slot, the

users broadcast their information. The signal y_r received at the relay can be written as

$$y_r = \sum_{k=1}^{K} \sqrt{p_k} f_{k,r} X_k + Z_r \tag{4.1}$$

In the received signal at the relay, complex-valued X_k, normalized such that $E(|X_k|^2) = 1$, represent the transmitted symbol, and Z_r is the complex-valued white Gaussian noise at the relay with $E(|Z_r|^2) = N_r$. In the second time slot, the relay amplifies the received signal y_r and broadcasts it to the users. The transmitted signal from the relay can be written as $x_r = \sqrt{\beta} y_r$ where β is the amplification gain of the relay. β is chosen such that the average transmit power at the relay is p_r. Mathematically,

$$\beta = \frac{p_r}{\sum_k p_k \left| f_{k,r} \right|^2 + N_r} \tag{4.2}$$

The signal received at the *k*th user from the relay can be written as

$$y_k = \sqrt{\frac{p_r}{\sum_j p_j \left| f_{j,r} \right|^2 + N_r}} \left(\sum_{j=1}^{K} \sqrt{p_j} f_{j,r} X_j + Z_r \right) f_{k,r} + Z_k \tag{4.3}$$

In the received signal at the *k*th user, Z_k is the complex-valued white Gaussian noise at the *k*th user with $E(|Z_k|^2) = N_k$. After self interference cancellation, the signal received at the *j*th user can be written as

$$
\begin{aligned}
y_k &= \sqrt{\frac{p_r}{\sum_j p_j \left|f_{j,r}\right|^2 + N_r}} \left(\sum_{j=1, j\neq k}^{K} \sqrt{p_j} f_{j,r} X_j + Z_r \right) f_{k,r} + Z_k \\
&= \sqrt{\frac{p_r}{\sum_j p_j \left|f_{j,r}\right|^2 + N_r}} \left(\sum_{j=1, j\neq k}^{K} \sqrt{p_j} f_{j,r} X_j \right) f_{k,r} + \\
&\quad \underbrace{\sqrt{\frac{p_r}{\sum_j p_j \left|f_{j,r}\right|^2 + N_r}} \left(Z_r f_{k,r} \right) + Z_k}_{Noise}
\end{aligned}
\tag{4.4}
$$

As given in [3], at each receiver k, $k \in I_K$ we have a Gaussian Multiple Access Channel (MAC) with K-1 users. In the received signal y_k at user k, the SNR from the jth user, γ_k^j, can be written as

$$
\gamma_k^j = \frac{p_r p_j \left|f_{j,r}\right|^2 \left|f_{k,r}\right|^2}{p_r N_r \left|f_{k,r}\right|^2 + \left(\sum_j p_j \left|f_{j,r}\right|^2 + N_r \right) N_k}
\tag{4.5}
$$

For the kth user to be able to decode the messages in the received signal, the rates $R_1, R_2, \cdots, R_{k-1}, R_{k+1}, \cdots, R_K$ of the users should satisfy the following inequalities (eqn. (8) in [3]).

$$
\sum_{j\in W_k} R_j \leq \frac{1}{2} C \left(\frac{\sum_{j\in W_k} p_r p_j \left|f_{j,r}\right|^2 \left|f_{k,r}\right|^2}{p_r \left|f_{k,r}\right|^2 N_r + N_k \sum_{i=1}^{K} p_i \left|f_{i,r}\right|^2 + N_r N_k} \right) \forall W_k \subseteq I_K \setminus \{k\}
$$

For multi-way relay channel comprising a relay and multiple users, the rate tuple $\mathbf{R} = (R_1, R_2, .., R_K)$ is achievable if it satisfies the following inequalities,

$$\sum_{j\in W_k} R_j \le \frac{1}{2} C\left(\frac{\sum_{j\in W_k} p_r p_j \left|f_{j,r}\right|^2 \left|f_{k,r}\right|^2}{p_r \left|f_{k,r}\right|^2 N_r + N_k \sum_{i=1}^{K} p_i \left|f_{i,r}\right|^2 + N_r N_k}\right), \quad \forall W_k \subseteq I_K \setminus \{k\}, \forall k \in I_K \tag{4.6}$$

In this work, we study the problem of determining the users rates ($R_1, R_2, \cdots, R_K$), the average transmit power of users ($p_1, p_2, \cdots, p_K$) and the transmit power of the relay p_r such that the minimum among the user rates ($R_1, R_2, \cdots, R_K$) is maximized subject to the constraints on the user rates and average transmit power of users and relay. The optimization problem is formulated as

$$\max_{\substack{R_k, k\in I_K \\ p_k, k\in I_K \\ p_r}} \min_{k\in I_K} \{R_1, \ldots, R_K\}$$

subject to

$$\begin{aligned}
&C1: \sum_{j\in W_k} R_j \le \frac{1}{2} C\left(\frac{\sum_{j\in W_k} p_r p_j \left|f_{j,r}\right|^2 \left|f_{k,r}\right|^2}{p_r \left|f_{k,r}\right|^2 N_r + N_k \sum_{i=1}^{K} p_i \left|f_{i,r}\right|^2 + N_r N_k}\right) \\
&\qquad \forall W_k \subseteq I_K \setminus \{k\}, \forall k \in I_K, \\
&C2: \sum_{k=1}^{K} p_k + p_r \le P_{tot} \\
&C3: 0 \le p_r \le P_r^{max} \\
&C4: 0 \le p_k \le P_k^{max}, \ \forall k \in I_K
\end{aligned} \tag{4.7}$$

Constraint *C*1 is the constraint on user rates as discussed earlier. *C*2 requires that cumulative transmit power of all the nodes should be under a specified threshold. The constraints on individual transmit power of the nodes is given in C*3*−C*4*. The optimization problem in (4.7) is a non-convex non-linear program because the constraint set is not convex because of *C*1. In the next section, we present a method to determine the optimal solution to the optimization in (4.7).

4.2 ε-Optimal Solution based on Monotonic Optimization

In this sub-section, for determining, the global optimal solution of the optimization in (4.7), we examine the use of monotonic optimization techniques. We note that the constraint set of the optimization in (4.7) is not a normal set (as shown in Appendix F). Hence, monotonic optimization techniques cannot be applied directly to solve the optimization in (4.7). In the following, we provide a monotonic optimization problem that is equivalent to the optimization in (4.7) in the sense that the optimal solutions of both of the optimization problems are same. In order to arrive at this monotonic optimization problem, we first provide an optimization problem that has reduced number of the optimization variables. The optimization problem, with reduced number of variables, is then transformed to an equivalent monotonic optimization problem.

We now explain the method of reducing the number of optimization variables. We notice that we can associate each feasible vector $\mathbf{x} = \{R_1, \ldots, R_K, p_1, \ldots, p_K, p_r\}$ with another feasible vector $\pi(\mathbf{x}) = \{\hat{R}, \ldots, \hat{R}, p_1, \ldots, p_K, p_r\}$, $\hat{R} = \min_{k \in I_K} R_k$. We also notice that both $\mathbf{x}$ and $\pi(\mathbf{x})$ have the same objective function value. Therefore, we can add the constraint $R_1 = R_2 \cdots = R_K$ to the optimization in (4.7) without compromising the quality of its optimal solution.

Addition of the constraint $R_1 = R_2 \cdots = R_K$ to our formulation in (4.7) results in the following optimization problem:

$$\max_{\substack{R_k, k \in I_K \\ p_k, k \in I_K \\ p_r}} \min_{k \in I_K} \{R_1, \ldots, R_K\}$$

subject to

$$C1 - C4 \, in \, (5.7)$$

$$C5 : R_i = R_{i+1}, i \in I_K \setminus \{K\}$$

We can define a new variable R, such that $R = R_1 = R_2 \cdots = R_K$ and rewrite the above formulation as,

$$\max_{\substack{R, p_k, k \in I_K \\ p_r}} R$$

subject to

$$C1: \; R \le \frac{1}{2|W_k|} C \left(\frac{\sum_{j \in W_k} p_r p_j \left| f_{j,r} \right|^2 \left| f_{k,r} \right|^2}{p_r \left| f_{k,r} \right|^2 N_r + N_k \sum_{i=1}^{K} p_i \left| f_{i,r} \right|^2 + N_r N_k} \right) \tag{4.8}$$

$$\forall W_k \subseteq I_K \setminus \{k\}, \forall k \in I_K$$

$$C2: \sum_{k=1}^{K} p_k + p_r \le P_{tot}$$

$$C3: 0 \le p_r \le P_r^{max}$$

$$C4: 0 \le p_k \le P_k^{max}, \; \forall k \in I_K$$

Although, we have reduced the number of optimization variables by *K*-1, we still do see a structure that we can exploit to determine its optimal solution. We notice that we can omit the variable *R* from the optimization in (4.8) and write the cost function as the point wise minima of multiple functions present in constraint *C*1. To simplify our notations, we define two sets $T_K = \{1, 2, .., 2^{K-1} - 1\}$ and $\bar{W}_k = \{W_k^n : W_k^n \subseteq I_K \setminus \{k\}\}$. The set $\bar{W}_k$ is a collection of all the possible subsets of the set $I_K \setminus \{k\}$. Each element, W_k^n, of the set $\bar{W}_k$ is indexed as $n = 1, 2, .., 2^{K-1} - 1$ and the index $n \in T_K$. The revised formulation can be written as

$$\max_{p_r, p_k, k \in I_K} \min_{\substack{k \in I_K \\ n \in T_K}} j_{n,k}\left(p_r, p_1, ..., p_K\right)$$

subject to

$$C1: \sum_{k=1}^{K} p_k + p_r \le P_{tot} \tag{4.9}$$

$$C2: 0 \le p_r \le P_r^{max}$$

$$C3: 0 \le p_k \le P_k^{max}, \; \forall k \in I_K$$

where,

$$j_{n,k}\left(p_r, p_1, ..., p_K\right) =$$
$$\frac{1}{2\left|W_k^n\right|} C\left(\frac{\sum_{j\in W_k^n} p_r p_j \left|f_{j,r}\right|^2 \left|f_{k,r}\right|^2}{p_r \left|f_{k,r}\right|^2 N_r + N_k \sum_{i=1}^{K} p_i \left|f_{i,r}\right|^2 + N_r N_k}\right), W_k^n \in \overline{W}_k$$

The constraint set of the above formulation is convex. However, the cost function is not an increasing function of the optimization variables $\left\{p_1, \ldots, p_K, p_r\right\}$, e.g. consider $j_{n,k}\left(p_r, p_1, ..., p_K\right)$ which is an increasing function of $p_r, p_j, j \in W_k^n$ and decreasing function of $p_i, i \in I_K \setminus W_k^n$. However, with a simple change of variables, we can easily transform the optimization (4.9) to an equivalent standard monotonic optimization problem. From (4.2), we can also write the relay's transmission power as $p_r = \beta\left(\sum_k p_k \left|f_{k,r}\right|^2 + N_r\right)$. Substituting for p_r in (4.8), we arrive at the following optimization problem:

$$\max_{\beta, p_k, k\in I_K} \min_{\substack{k\in I_K \\ n\in T_K}} r_{n,k}\left(\beta, p_1, ..., p_K\right)$$
$$\text{subject to}$$
$$C1: \sum_{k=1}^{K} p_k + \beta\left(\sum_k p_k \left|f_{k,r}\right|^2 + N_r\right) \le P_{tot}$$
$$C2: 0 \le \beta\left(\sum_k p_k \left|f_{k,r}\right|^2 + N_r\right) \le P_r^{max}$$
$$C3: 0 \le p_k \le P_k^{max}, \ \forall k \in I_K \tag{4.10}$$
$$r_{n,k}\left(\beta, p_1, ..., p_K\right) =$$
$$\frac{1}{2\left|W_k^n\right|} C\left(\frac{\sum_{j\in W_k^n} \beta\, p_j \left|f_{j,r}\right|^2 \left|f_{k,r}\right|^2}{\beta \left|f_{k,r}\right|^2 N_r + N_k}\right), W_k^n \in \overline{W}_k$$

We establish in the following lemma that the optimization (4.10) is an instance of the standard monotonic optimization problem and hence can be solved by the polyblock outer approximation algorithm.

Lemma II: The optimization in (4.10) is an instance of the standard monotonic optimization problem.

Proof: We first show that the set of feasible solution is normal. We refer to proposition 5 in [5] that states that if the constraint functions in an optimization problem are all increasing function of the optimization variables then the constraint set represents a normal set. We note that in our optimization in (4.10), the constraint functions in *C*1 and *C*2 are all increasing function of the optimization variables $(\beta, p_1, ..., p_K)$ and C3 is a simple box constraint on the transmit powers of users. Therefore, the constraint set in optimization (4.10) is normal. We now prove that the cost function is an increasing function of the optimization variables $(\beta, p_1, ..., p_K)$. We observe that the cost function is a pointwise minima of functions $r_{n,k}(\beta, p_1, ..., p_K)$. We know that if the individual functions $r_{n,k}(\beta, p_1, ..., p_K)$ are an increasing function of the optimization variables $(\beta, p_1, ..., p_K)$ then, their pointwise minima is also an increasing function of the optimization variables [5]. We observe that $r_{n,k}(\beta, p_1, ..., p_K)$ is defined in (4.10) as $r_{n,k}(\beta, p_1, ..., p_K) = \frac{1}{2|W_k^n|} C\left(\frac{\sum_{j \in W_k^n} \beta p_j |f_{j,r}|^2 |f_{k,r}|^2}{\beta |f_{k,r}|^2 N_r + N_k} \right)$. Now $C(x) = \frac{1}{2}\log(1+x)$ and due to the monotonicity of logarithm, we can as well establish the monotonicity of its argument to complete our proof. Therefore, we need to establish the monotonicity of $v_{n,k}(\beta, p_1, ..., p_K) = \frac{\beta \sum_{j \in W_k^n} p_j |f_{j,r}|^2 |f_{k,r}|^2}{\beta |f_{k,r}|^2 N_r + N_k}$. We notice that the variables $p_1, ..., p_K$ occur only in the numerator of $v_{n,k}(\beta, p_1, ..., p_K)$. Therefore, $v_{n,k}(\beta, p_1, ..., p_K)$ is an

increasing function of $p_1,...,p_K$ while the partial derivative of $\nu_{n,k}(\beta, p_1,...,p_K)$ with respect to β, computed as $\frac{\partial \nu_{n,k}(\beta, p_1,...,p_K)}{\partial \beta} = \frac{N_k \sum_{j \in W_k^n} p_j \left|f_{j,r}\right|^2 \left|f_{k,r}\right|^2}{\left(\beta \left|f_{k,r}\right|^2 N_r + N_k\right)^2} \geq 0$, is always nonnegative. Therefore, $\nu_{n,k}(\beta, p_1,...,p_K)$ is an increasing function of the optimization variable $(\beta, p_1,...,p_K)$ and thus, the cost function in (4.10) is an increasing function of the optimization variables $(\beta, p_1,...,p_K)$.

The application of polyblock outer approximation method to find the ε-optimal solution of the optimization in (2.21) is quite straightforward and follows the pseudo code given in Chapter 2. Moreover, the step involving the projection of the maximizer (over the vertex set of the polyblock at that iteration) onto the boundary of the feasible set of the solutions is straightforward. As mentioned before (Chapter 2), the computation of projection requires us to solve the following single variable optimization in *α*, which can be written as follows

$$\begin{aligned}
&\max_{\{} \alpha \\
&\text{subject to} \\
&\quad C1: \sum_{k=1}^{K} \alpha p_k^i + \alpha \beta^i \left(\sum_k \alpha p_k^i \left|f_{k,r}\right|^2 + N_r \right) \leq P_{tot} \\
&\quad C2: 0 \leq \alpha \beta^i \left(\sum_k \alpha p_k^i \left|f_{k,r}\right|^2 + N_r \right) \leq P_r^{max} \\
&\quad C3: 0 \leq \alpha p_k^i \leq P_k^{max}, \ \forall k \in I_K \\
&\quad C4: 0 \leq \alpha \leq 1
\end{aligned} \tag{4.11}$$

where $\beta^i, p_1^i,...,p_K^i$ denote the maximizer over the polyblock vertices in the *i*th iteration. Constraint *C*1 and *C*2 have quadratic inequalities and *C*3 represents a linear inequality. We can rewrite the single-variable optimization in (4.11) as follows:

$$
\begin{aligned}
&\max_{\{\alpha} \alpha \\
&\text{subject to} \\
&\quad C1: t_{c1}^{l} \le \alpha \le t_{c1}^{u} \\
&\quad C2: t_{c2}^{l} \le \alpha \le t_{c2}^{u} \\
&\quad C3: \alpha \le \frac{P_k^{max}}{p_k^i}, \ \forall k \in I_K \\
&\quad C4: 0 \le \alpha \le 1
\end{aligned}
\tag{4.12}
$$

where

$$
t_{c1}^{u} = \frac{-\sum_{k=1}^{K} p_k^i - \beta^i N_r + \sqrt{\left(\sum_{k=1}^{K} p_k^i + \beta^i N_r\right)^2 + 4P_{tot}\beta^i \sum_k p_k^i \left|f_{k,r}\right|^2}}{2\beta^i \sum_k p_k^i \left|f_{k,r}\right|^2}
$$

$$
t_{c1}^{l} = \frac{-\sum_{k=1}^{K} p_k^i - \beta^i N_r - \sqrt{\left(\sum_{k=1}^{K} p_k^i + \beta^i N_r\right)^2 + 4P_{tot}\beta^i \sum_k p_k^i \left|f_{k,r}\right|^2}}{2\beta^i \sum_k p_k^i \left|f_{k,r}\right|^2}
$$

$$
t_{c2}^{u} = \frac{-\beta^i N_r + \sqrt{\left(\beta^i N_r\right)^2 + 4P_{tot}\beta^i \sum_k p_k^i \left|f_{k,r}\right|^2}}{2\beta_k^i \sum_k p_k^i \left|f_{k,r}\right|^2}, t_{c2}^{l} = \frac{-\beta^i N_r - \sqrt{\left(\beta^i N_r\right)^2 + 4P_{tot}\beta^i \sum_k p_k^i \left|f_{k,r}\right|^2}}{2\beta_k^i \sum_k p_k^i \left|f_{k,r}\right|^2}
$$

It is clear that $t_{c1}^{l}, t_{c2}^{l} \le 0$. Therefore, the closed form solution to the optimization in (4.12) can be simply obtained as $\alpha^* = \min \ \left\{t_{c1}^{u}, t_{c2}^{u}, 1\right\} \cup \left\{P_k^{max} / p_k^i \ \middle| \forall k \in I_K\right\}$.

4.3 Numerical results

We perform simulations to evaluate the performance of the proposed solution. We consider a system comprising a single relay and three users (denoted as A, B and C). The results, obtained from proposed solution, are compared with a naive scheme where we determine the power of each transmitting node as $p_k = \min\left\{\frac{P_{tot}}{K+1}, P_k^{max}\right\} \forall k \in I_K \ \ p_r = \min\left\{\frac{P_{tot}}{K+1}, P_r^{max}\right\}$. We will refer to this scheme as uniform power allocation (UPA). In our implementation of the

monotonic optimization based solution, we set the accuracy parameter as ε = 0.001 and the maximum number iterations, I^{max} = 5000. Without loss of generality, we set the noise variances N_r = 1, N_A = 1, N_B = 1 and N_C = 1.

In Fig. 4.1, we present the convergence plot of the proposed solution. as discussed earlier the polyblock outer approximation algorithm generates a sequence of nested polyblocks. As the iterations proceeds, the approximation of the feasible set by the polyblocks improves. The maximization of the objective function over a polyblock is an upper bound to the original optimization problem. The corresponding projection of the solution, obtained from the polyblock outer approximation algorithm, on to the feasible set is a lower bound to the original optimization problem. The lower bound is essentially the best feasible solution obtained till the current iteration. As the iterations proceeds the difference between the upper and lower bound decreases and the algorithm terminates in a finite number of iterations when the difference is within a desirable accuracy, ε. The result in fig. 4.1 is in conformance with the theory that we discussed above.

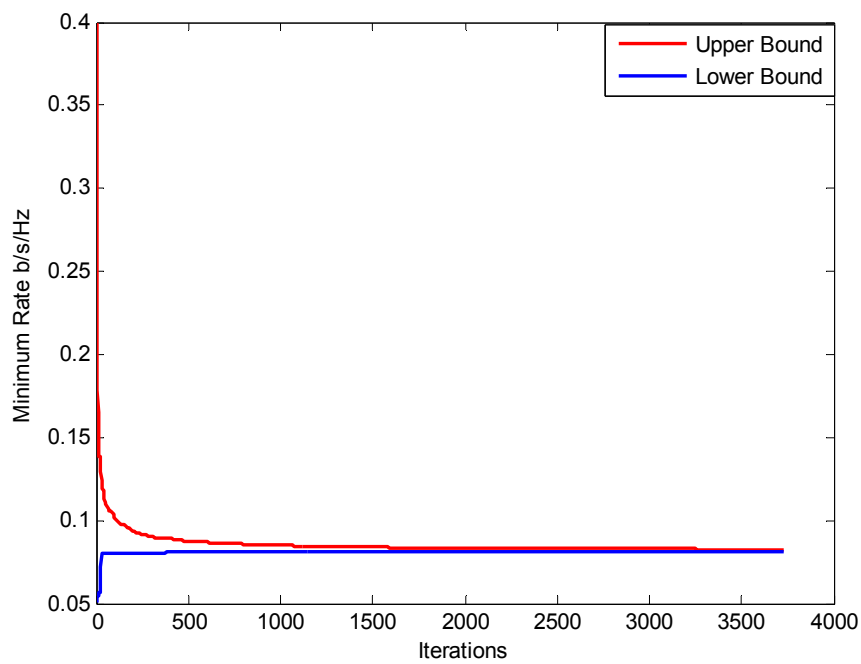

Fig. 4.1 Convergence Results of Max-Min Optimization, $P_{max}^{A} = P_{max}^{B} = P_{max}^{C} = 10^{\frac{P}{10}}, P_{max}^{R} = P_{max}^{A}/2$, P = 1 watt and P_{tot} = 1 watt.

In fig. 4.2, we compare the minimum user rate with the maximum transmission power, *P* (in dB), of the nodes. The maximum transmission power of each node (in watts) is set as $P_{max}^{A} = P_{max}^{B} = P_{max}^{C} = 10^{\frac{P}{10}}, P_{max}^{R} = P_{max}^{A}/2$ and P_{tot} = 1 watts. The results are averaged over 100 different scenario. In each scenario, the channel gains among the users and the relay are randomly generated in accordance with the assumption of independent channel gains drawn from a complex Gaussian distribution. We observe that as transmission power increases, the minimum user rate increases only up to a certain point. This is because at low transmission power the individual constraints on the nodes are active. However, after a certain point the sum-power constraint is active, thus limiting the minimum user rate at higher SNR. Further, we observe that the proposed optimal solution performs much better than the naive Uniform Power Allocation scheme.

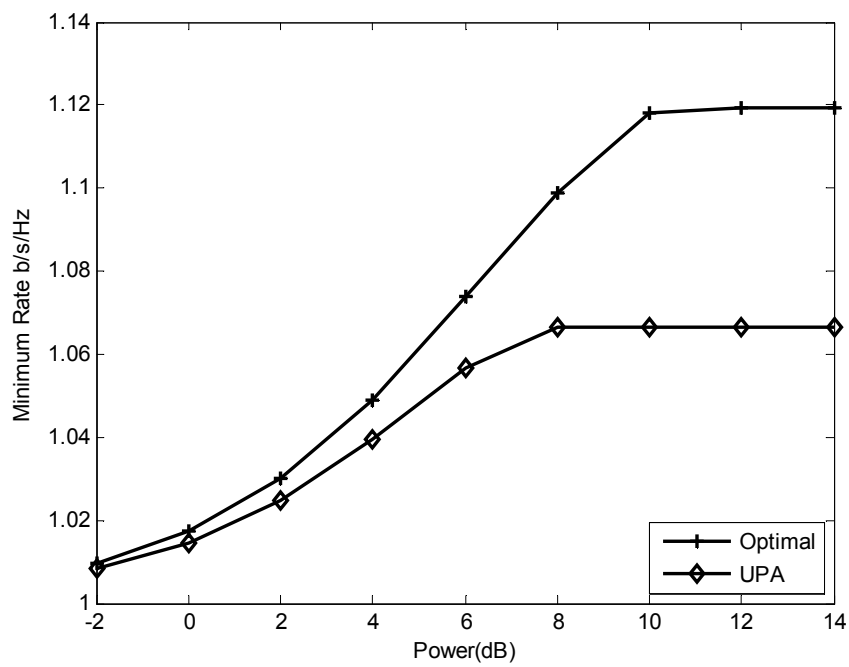

Fig. 4.2 Minimum Rate vs Transmit Power, P_{tot} = 1 watt $P_{max}^{A} = P_{max}^{B} = P_{max}^{C} = 10^{\frac{P}{10}}, P_{max}^{R} = P_{max}^{A}/2$ and *P* is varied from -2 to 14 dB.

4.4 Summary

We considered a multi-way relay channel comprising multiple users and a relay where the users wish to exchange information among them with the help of the relay terminal. The relay used amplify-and-forward (AF) relaying. We formulated the optimization problem of allocating power to the users and the relay such that the minimum among the user's rate is maximized. Then, we presented a method of transforming the problem to a new formulation so that a ε-optimal algorithm can be designed. The formulated problem, although still non-convex, can be solved by the monotonic optimization. In our numerical experiments, we compared the solution obtained from the proposed method with a uniform power allocation method. The numerical results indicated that the proposed method provide excellent solutions.

4.5 Chapter References

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CHAPTER 5: CONTRIBUTIONS AND FUTURE WORK

In this chapter, we present a brief overview of the contributions and discuss open issues that can be addressed in future research.

5.1 Contributions

In this thesis, we studied resource allocation problems in cooperative cognitive radio systems. In particular, we have made following major contributions in this thesis:

5.1.1 Power allocation in Two-Way Relay assisted cooperative cognitive radio networks

We studied power allocation problems for two-way relay assisted cooperative cognitive radio networks employing orthogonal and shared band amplify and forward relaying. The formulated optimization problems were non-convex and nonlinear in nature. We obtained their optimal solutions by applying the monotonic optimization algorithm techniques. Due to the heavy computational burden involved in obtaining the optimal solutions, we compared the bounds obtained from the monotonic optimization algorithm with our proposed heuristics. By comparing the quality of these solutions to that of the solutions produced by monotonic optimization algorithm, we showed that our heuristic algorithms provide good solutions.

5.1.2 Power Allocation in Multi-Way Relaying

We studied joint sources' and relay's transmit power allocation in a multi-way relay channel comprising multiple users and a relay where the users wish to exchange information among themselves with the help of the relay terminal. The

relay used amplify-and-forward (AF) relaying. We formulated the optimization problem of allocating power to the users and the relay such that the minimum among the user's rate is maximized. Then, we presented a method of transforming the problem to a new formulation so that an ε-optimal algorithm can be designed. The formulated problem was then solved using monotonic optimization techniques. In our numerical experiments, we compared the solution obtained from the proposed method with a uniform power allocation method. The numerical results indicated that the proposed method provide excellent solutions.

5.2 Future work

The proposed schemes in this thesis address some aspects of resource allocation in CRS with relaying capabilities. However, there are still many open issues. In the following, we list some important future research directions.

5.2.1 Resource allocation in cooperative CRS with imperfect CSI

For resource allocation, we have assumed that the central controller knows the perfect CSI. However, there always exists some uncertainty in the CSI due to unreliable feedback channel. Therefore, a possible extension of the proposed resource allocation formulation is to analyze the relay assignment schemes with imperfect CSI in multi-hop CRS. Another interesting issue to consider is the effect of quantized CSI on the relay assignment in multi-hop cooperative CRS.

5.2.2 Power allocation in Two-Way Relay assisted cooperative cognitive radio networks

The resource allocation problems were studied for a single user pair. The proposed formulations can be extended to the multi-user, multi-relay assisted CRS. A comparative study of shared band and orthogonal two-way AF relaying can also be done. For the case of shared band two-way AF relaying, we only studied the power allocation with max-min fairness. We can also consider sum-

rate optimization with quality-of-service constraints. Further, we can also extend our power allocation problems with imperfect CSI and propose distributed and decentralized solutions. A direct extension of two-way relaying with single user pair would be to study the case of multi-hop (> 2 hops) cooperative CRS comprising single user and multiple relays An interesting problem to study in the multi-hop cooperative CRS would be to study scheduling of relays' transmissions and power allocation.

5.2.3 Power allocation in Multi-Way Relaying

We studied resource allocation in multi-way relaying with max-min fairness. We can extend our formulation to sum-rate maximization in multi-way relaying. In our max-min power allocation, we noted that there could be a multiple solutions to the max-min optimization problem. Another interesting formulation would to select the solution that has the best sum-rate among these multiple solutions to the max-min optimization. We can also consider the multi-way relaying with multiple users and relays.

APPENDICES

Appendix A

We first show that γ_{s1} and γ_{s2} are increasing w.r.t to p_l in the following.

$$\begin{aligned}
\frac{\partial \gamma_{s1}}{\partial p_l} &= \frac{\partial}{\partial p_l}\sum_{l=1}^{L}\frac{p_{s2}p_l\left|f_l\right|^2\left|g_l\right|^2}{N_o\left(p_l\left|f_l\right|^2+p_{s1}\left|f_l\right|^2+p_{s2}\left|g_l\right|^2+N_o\right)}, \\
&= \frac{\left(p_l\left|f_l\right|^2+p_{s1}\left|f_l\right|^2+p_{s2}\left|g_l\right|^2+N_o\right)p_{s2}\left|f_l\right|^2\left|g_l\right|^2-p_{s2}p_l\left|f_l\right|^4\left|g_l\right|^2}{N_o\left(p_l\left|f_l\right|^2+p_{s1}\left|f_l\right|^2+p_{s2}\left|g_l\right|^2+N_o\right)^2} \\
&= \left(p_{s2}\left|f_l\right|^2\left|g_l\right|^2\right)\frac{\left(p_{s1}\left|f_l\right|^2+p_{s2}\left|g_l\right|^2+N_o\right)}{N_o\left(p_l\left|f_l\right|^2+p_{s1}\left|f_l\right|^2+p_{s2}\left|g_l\right|^2+N_o\right)^2}\geq 0 \\
\frac{\partial \gamma_{s2}}{\partial p_l} &= \left(p_{s1}\left|f_l\right|^2\left|g_l\right|^2\right)\frac{\left(p_{s1}\left|f_l\right|^2+p_{s2}\left|g_l\right|^2+N_o\right)}{N_o\left(p_l\left|g_l\right|^2+p_{s1}\left|f_l\right|^2+p_{s2}\left|g_l\right|^2+N_o\right)^2}\geq 0
\end{aligned} \tag{A.1}$$

Now consider the cost function of (2.7), $f\left(p_{s1},p_{s2},\mathbf{p}\right)=\frac{1}{L+1}\log\left(1+\gamma_{s1}\right)$ $+\frac{1}{L+1}\log\left(1+\gamma_{s2}\right)$. Logarithm is increasing with respect to its argument, in our case γ_{s1} and γ_{s2}. Further in (A.1) we showed that γ_{s1} and γ_{s2} are increasing w.r.t p_l. Therefore, $f\left(p_{s1},p_{s2},\mathbf{p}\right)$ is increasing w.r.t relay transmit powers p_l. Similarly the cost function of (2.8) $g\left(p_{s1},p_{s2},\mathbf{p}\right)=\min\left\{\frac{1}{L+1}\log\left(1+\gamma_{s1}\right),\frac{1}{L+1}\log\left(1+\gamma_{s2}\right)\right\}$ is increasing function of p_l as minimum of two increasing functions is also increasing.

Appendix B

Proof of Lemma II: For notational convenience, we denote $c_1 = \sum_l |f_l|^2, c_2 = \sum_l |g_l|^2$. Suppose that $\left(p_{s1}^*, p_{s2}^*\right)$ is a maximizer and let us assume without loss of generality that $c_2 p_{s2}^* > c_1 p_{s1}^*$. Then, we have $p_{s2}^* > 0$ and the optimal objective function value is $c_1 p_{s1}^*$. We consider the cases of $p_{s1}^* = p_{max}^s$ and $p^*{}_{s1} < p_{max}^s$ separately.

<u>Case 1</u>: $c_2 p_{s2}^* > c_1 p_{s1}^*$ and $p_{s1}^* = p_{max}^s$. In this case, let $\Delta = \left(c_2 p_{s2}^* - c_1 p_{s1}^*\right)/c_2$ and consider a point $\left(p_{s1}^*, p_{s2}^* - \Delta\right)$. Note that $p_{s2}^* - \Delta = p_{s2}^* - \left(p_{s2}^* c_2 - p_{s1}^* c_1\right)/c_2 = c_1 p_{s1}^*/c_2 \geq 0$ and $p_{s2}^* - \Delta \leq p_{s2}^* \leq p_{max}^s$. Also, we have $p_{s1}^* |h_{s1,m}|^2 + \left(p_{s2}^* - \Delta\right)|h_{s2,m}|^2 \leq p_{s1}^* |h_{s1,m}|^2 + p_{s2}^* |h_{s2,m}|^2 \leq I_m^{max}, \forall m$. Thus, $\left(p_{s1}^*, p_{s2}^* - \Delta\right)$ is a feasible point of optimization problem (2.29). Also, $c_2\left(p_{s2}^* - \Delta\right) = c_2\left(p_{s1}^* c_1/c_2\right) = c_1 p_{s1}^*$. Therefore, $\left(\hat{p}_{s1}, \hat{p}_{s2}\right) = \left(p_{s1}^*, p_{s2}^* - \Delta\right)$ is an optimal solution and satisfies $c_2 \hat{p}_{s2} = c_1 \hat{p}_{s1}$.

<u>Case 2</u>: $c_2 p_{s2}^* > c_1 p_{s1}^*$ and $p^*{}_{s1} < p_{max}^s$.

Denote $\alpha \equiv \max\left(\frac{|h_{s1,1}|^2}{|h_{s2,1}|^2}, .., \frac{|h_{s1,m}|^2}{|h_{s2,m}|^2}, .., \frac{|h_{s1,M}|^2}{|h_{s2,M}|^2}\right)$. Consider a point $\left(p_{s1}^* + \delta, p_{s2}^* - \alpha\delta\right)$. Because $p^*{}_{s1} < p_{max}^s$ and $p_{s2}^* > 0$, we have $0 \leq p_{s1}^* + \delta, p_{s2}^* - \alpha\delta \leq p_{max}^s$ for a sufficiently small $\delta > 0$. Also, $\left(p_{s1}^* + \delta\right)|h_{s1,m}|^2 + \left(p_{s2}^* - \alpha\delta\right)|h_{s2,m}|^2 = p_{s1}^* |h_{s1,m}|^2 + p_{s2}^* |h_{s2,m}|^2 +$

$$|h_{s2,m}|^2 \left(\frac{|h_{s1,m}|^2}{|h_{s2,m}|^2} - \alpha\right)\delta \leq p_{s1}^* |h_{s1,m}|^2 + p_{s2}^* |h_{s2,m}|^2 \leq I_m^{max}, \forall m$$

Therefore, for a sufficiently small $\delta > 0$, $\left(p_{s1}^* + \delta, p_{s2}^* - \alpha\delta\right)$ is a feasible point. Also, because $c_2 p_{s2}^* > c_1 p_{s1}^*$, for a sufficiently small $\delta > 0$ we have

$c_1\left(p_{s1}^* + \delta\right) < c_2\left(p_{s2}^* - \alpha\delta\right)$ and thus $\min\left\{c_1\left(p_{s1}^* + \delta\right), c_2\left(p_{s2}^* - \alpha\delta\right)\right\}$ $= c_1\left(p_{s1}^* + \delta\right) > c_1 p_{s1}^*$. This contradicts the optimality of $\left(p_{s1}^*, p_{s2}^*\right)$. Therefore, Case 2 is impossible.

Considerations of Case 1 and Case 2 establishes existence of an optimal solution problem (2.29), $\left(\hat{p}_{s1}, \hat{p}_{s2}\right)$ that satisfies that $c_2\hat{p}_{s2} = c_1\hat{p}_{s1}$. Likewise, suppose that $\left(p_{s1}^*, p_{s2}^*\right)$ is a maximizer and $c_2 p_{s2}^* < c_1 p_{s1}^*$. By following the arguments presented above we can conclude that we can find an optimal solution, $\left(\hat{p}_{s1}, \hat{p}_{s2}\right)$, such that $\hat{p}_{s2}c_2 = \hat{p}_{s1}c_1$. Q.E.D.

Appendix C

In this section, we show that γ_{s1} and γ_{s2} are not increasing w.r.t to p_{s1} and p_{s2} in the following. We provide a counter-example

$$\gamma_{s1} = \sum_{l=1}^{L} \frac{(p_{s2}/N_o)(p_l/N_o)|f_l|^2 |g_l|^2}{\left((p_l/N_o)|f_l|^2 + (p_{s1}/N_o)|f_l|^2 + (p_{s2}/N_o)|g_l|^2 + 1\right)}$$
$$\gamma_{s2} = \sum_{l=1}^{L} \frac{(p_{s1}/N_o)(p_l/N_o)|f_l|^2 |g_l|^2}{\left((p_l/N_o)|g_l|^2 + (p_{s1}/N_o)|f_l|^2 + (p_{s2}/N_o)|g_l|^2 + 1\right)}, \quad \text{(C.1)}$$

For $p_1 = 10N_o, f = 0.0054, g = 18.34$, we have $\min(\gamma_{s1}, \gamma_{s2}) = 0.04952$ when $p_{s1} = 10N_o, p_{s2} = 0.8N_o$ while $\min(\gamma_{s1}, \gamma_{s2}) = 0.02681$ when $p_{s1} = 10N_o$, $p_{s2} = 10N_o$.Therefore, γ_{s1} is not an increasing function of p_{s1}, p_{s2}. Similarly the sum-rate $\log(1+\gamma_{s1}) + \log(1+\gamma_{s2}) = 0.09712$ when $p_{s1} = 10N_o, p_{s2} = 0.8N_o$ and sum-rate equals 0.07852 when $p_{s1} = 10N_o$, $p_{s2} = 10N_o$.

Appendix D

In this section, we show that feasible set of the following optimization problem is not a normal set.

$$\max_{\substack{R_k, k\in I_K \\ p_k, k\in I_K \\ p_r}} \min_{k\in I_K} \{R_1, \ldots, R_K\}$$

subject to

$$C1: \sum_{j\in W_k} R_j \le \frac{1}{2} C\left(\frac{\sum_{j\in W_k} p_r p_j \left|f_{j,r}\right|^2 \left|f_{k,r}\right|^2}{p_r \left|f_{k,r}\right|^2 N_r + N_k \sum_{i=1}^{K} p_i \left|f_{i,r}\right|^2 + N_r N_k} \right)$$

$$\forall W_k \subseteq I_k \setminus \{k\}, \forall k \in I_K, \qquad \text{(D.1)}$$

$$C2: \sum_{k=1}^{K} p_k + p_r \le P_{tot}$$

$$C3: 0 \le p_r \le P_r^{max}$$

$$C4: 0 \le p_k \le P_k^{max}, \ \forall k \in I_K$$

where $C(x) = \frac{1}{2}\log(1+x)$.

In our proof, we consider a multi-way relaying system with three users and a single relay and show that the set of feasible solutions for this system may not always be a normal set. The constraint C1 in (D.1) for this special system can be written as follows:

$$C1.1: R_1 \le \frac{1}{2} C\left(\frac{p_r p_1 \left|f_{1,r}\right|^2 \left|f_{3,r}\right|^2}{p_r \left|f_{3,r}\right|^2 N_r + N_3 \sum_{i=1}^{3} p_i \left|f_{i,r}\right|^2 + N_r N_3} \right),$$

$$C1.2: R_2 \le \frac{1}{2} C\left(\frac{p_r p_2 \left|f_{2,r}\right|^2 \left|f_{3,r}\right|^2}{p_r \left|f_{3,r}\right|^2 N_r + N_3 \sum_{i=1}^{K} p_i \left|f_{i,r}\right|^2 + N_r N_3} \right)$$

$$C1.3: R_1 + R_2 \le \frac{1}{2} C\left(\frac{\left(p_r p_1 \left|f_{1,r}\right|^2 + p_r p_2 \left|f_{2,r}\right|^2\right)\left|f_{3,r}\right|^2}{p_r \left|f_{3,r}\right|^2 N_r + N_3 \sum_{i=1}^{K} p_i \left|f_{i,r}\right|^2 + N_r N_3} \right)$$

$$C1.4: R_2 \le \frac{1}{2} C\left(\frac{p_r p_2 \left|f_{2,r}\right|^2 \left|f_{1,r}\right|^2}{p_r \left|f_{1,r}\right|^2 N_r + N_1 \sum_{i=1}^{3} p_i \left|f_{i,r}\right|^2 + N_r N_1} \right),$$

$$C1.5: R_3 \le \frac{1}{2} C\left(\frac{p_r p_3 \left|f_{3,r}\right|^2 \left|f_{1,r}\right|^2}{p_r \left|f_{1,r}\right|^2 N_r + N_1 \sum_{i=1}^{3} p_i \left|f_{i,r}\right|^2 + N_r N_1} \right)$$

$$C1.6: R_2 + R_3 \le \frac{1}{2} C\left(\frac{\left(p_r p_2 \left|f_{2,r}\right|^2 + p_r p_3 \left|f_{3,r}\right|^2\right)\left|f_{1,r}\right|^2}{p_r \left|f_{1,r}\right|^2 N_r + N_1 \sum_{i=1}^{3} p_i \left|f_{i,r}\right|^2 + N_r N_1} \right) \quad \text{(D.2)}$$

$$C1.7: R_1 \le \frac{1}{2} C\left(\frac{p_r p_1 \left|f_{1,r}\right|^2 \left|f_{3,r}\right|^2}{p_r \left|f_{2,r}\right|^2 N_r + N_2 \sum_{i=1}^{3} p_i \left|f_{i,r}\right|^2 + N_r N_2} \right),$$

$$C1.8: R_3 \le \frac{1}{2} C \left(\frac{p_r p_3 \left| f_{3,r} \right|^2 \left| f_{2,r} \right|^2}{p_r \left| f_{2,r} \right|^2 N_r + N_2 \sum_{i=1}^{3} p_i \left| f_{i,r} \right|^2 + N_r N_2} \right)$$

$$C1.9: R_1 + R_3 \le \frac{1}{2} C \left(\frac{\left(p_r p_1 \left| f_{1,r} \right|^2 + p_r p_3 \left| f_{3,r} \right|^2 \right) \left| f_{2,r} \right|^2}{p_r \left| f_{2,r} \right|^2 N_r + N_2 \sum_{i=1}^{3} p_i \left| f_{i,r} \right|^2 + N_r N_2} \right)$$

Consider a feasible solution vector $\mathbf{x} \equiv \{R_1, R_2, R_3, p_1, p_2, p_3, p_r\}$. Further assume that for vector **x** inequality C1.1 is met with equality. Let us define another vector as $\mathbf{y} \equiv \{R_1, R_2, R_3, \hat{p}_1, p_2, p_3, p_r\}, \hat{p}_1 < p_1$. Notice that $\mathbf{x} \ge \mathbf{y}$. Further notice that right hand side of the inequality C1.1, $\frac{1}{2} C \left(\frac{p_r p_1 \left| f_{1,r} \right|^2 \left| f_{3,r} \right|^2}{p_r \left| f_{3,r} \right|^2 N_r + N_3 \sum_{i=1}^{3} p_i \left| f_{i,r} \right|^2 + N_r N_3} \right)$, is an increasing function of p_1. We initially assumed that $\mathbf{x}$ meets inequality C1.1 with inequality, i.e. $R_1 = \frac{1}{2} C \left(\frac{p_r p_1 \left| f_{1,r} \right|^2 \left| f_{3,r} \right|^2}{p_r \left| f_{3,r} \right|^2 N_r + N_3 \sum_{i=1}^{3} p_i \left| f_{i,r} \right|^2 + N_r N_3} \right)$. We observe that for vector $\mathbf{y}$, $R_1 > \frac{1}{2} C \left(\frac{p_r \hat{p}_1 \left| f_{1,r} \right|^2 \left| f_{3,r} \right|^2}{p_r \left| f_{3,r} \right|^2 N_r + N_3 \left(\hat{p}_1 \left| f_{1,r} \right|^2 + \sum_{i=2}^{3} p_i \left| f_{i,r} \right|^2 \right) + N_r N_3} \right)$. Therefore, vector $\mathbf{y}$ does not satisfy inequality C1.1 in (D.2). Therefore the feasible set of the optimization (D.1) is not a normal set.

Appendix E

In this section, we show that $f(\boldsymbol{\zeta}) \equiv \min(\gamma_{s1}, \gamma_{s2})$ where γ_{s2} and γ_{s2} are given in (E.1) is not a concave function of the variables $\boldsymbol{\zeta} \equiv (p_{s1}, p_{s2}, p_1, .., p_L)$.

$$\gamma_{s1} = \frac{\left(\frac{p_{s2}}{N_o}\right)\left(\sum_{l=1}^{L}\sqrt{\frac{(p_l/N_o)|f_l|^2|g_l|^2}{(p_{s1}/N_o)|f_l|^2+(p_{s2}/N_o)|g_l|^2+1}}\right)^2}{1+\sum_{l=1}^{L}\frac{(p_l/N_o)|f_l|^2}{(p_{s1}/N_o)|f_l|^2+(p_{s2}/N_o)|g_l|^2+1}},$$

$$\gamma_{s2} = \frac{\left(\frac{p_{s1}}{N_o}\right)\left(\sum_{l=1}^{L}\sqrt{\frac{(p_l/N_o)|f_l|^2|g_l|^2}{(p_{s1}/N_o)|f_l|^2+(p_{s2}/N_o)|g_l|^2+1}}\right)^2}{1+\sum_{l=1}^{L}\frac{(p_l/N_o)|g_l|^2}{(p_{s1}/N_o)|f_l|^2+(p_{s2}/N_o)|g_l|^2+1}} \quad \text{(E.1)}$$

For $L = 2, p_1 = p_2 = 10N_o, f_1 = 0.001, f_2 = 0.4838, g_1 = 0.71, g_2 = 11.74$, consider $\boldsymbol{\zeta}_1 \equiv (N_o, N_o, 0, 10N_o)$ and $\boldsymbol{\zeta}_2 \equiv (N_o, N_o, 10N_o, 10N_o)$. Now $f(0.5\boldsymbol{\zeta}_1 + 0.5\boldsymbol{\zeta}_2) = 0.3742 < 0.5f(\boldsymbol{\zeta}_1) + 0.5f(\boldsymbol{\zeta}_2) = 0.379$. Therefore, $f(\boldsymbol{\zeta})$ is not a concave function of $\boldsymbol{\zeta} \equiv (p_{s1}, p_{s2}, p_1, .., p_L)$ for $L = 2, p_1 = p_2 = 10N_o, f_1 = 0.001, f_2 = 0.4838, g_1 = 0.71, g_2 = 11.74$.