

**CONVERSATIONS HELD AND ROLES PLAYED
DURING MATHEMATICS TEACHERS'
COLLABORATIVE DESIGN: TWO DIMENSIONS OF
INTERACTION**

by

Armando Paulino Preciado Babb
M.Sc., Universidad Nacional Autónoma de México, 2000
B.Sc., Universidad Nacional Autónoma de México, 1997

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Approval

Name: Armando Paulino Preciado Babb
Degree: Doctor of Philosophy (Mathematics Education)
Title of Thesis: Conversations held and roles played during mathematics teachers' collaborative design: Two dimensions of interaction

Examining Committee:

Chair: Dr. Cécile Sabatier
Associate Professor

Dr. Peter Liljedahl
Associate Professor
Senior Supervisor

Dr. Nathalie Sinclair
Associate Professor
Member

Dr. David Pimm
Professor Emeritus
Member

Dr. Stephen Campbell
Associate Professor
Internal/External

Dr. Cathy Bruce
Associate Professor
External Examiner
Trent University

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Abstract

The focus of this study is on interactions among teachers, and other participants, in the collaborative design of mathematics teaching and learning artefacts. There is a variety of modalities of collaborative design of these artefacts around the world, and research has shown the benefit of this activity for students' learning and teachers' professional growth. My purpose in conducting this study was to understand what happens inside these teams of collaborative design in terms of participants' interactions and activities. I decided to take a social approach in researching these interactions.

This research was conducted in three stages differing in the types of data sources and data generation. The first stage consisted of the study of a single case in which I participated as a member in a team of collaborative design. I analysed the conversations and actions held during the design process identifying two emerging themes: (1) the focus of the conversations and actions, and (2) the roles held by the participants of collaborative design. I characterized interactions using these two themes, which I consider as dimensions of interactions in teachers' collaborative design. In the second stage of this study I looked at other cases of collaborative design. Participants from three different modalities were contacted in order to identify resonances and dissonances with the case analysed in the first stage. Lastly, in the third stage, three pieces of literature served as second-hand data to explore large-scale modalities of teachers' collaborative design. Considering all the cases included in the second and third stages, I refined and extended the characterization for interactions among participants in teachers' collaborative design. The resulting characterization for interactions serves as a language that acknowledges the diversity of both the settings in which collaborative design can be conducted and the participants' roles played in each case. Such characterization has implications for both practitioners and researchers in mathematics education interested in teachers' collaborative design and professional development.

Keywords: Mathematics teachers; teachers' collaborative design; teacher professional development; design of teaching and learning artefacts.

I dedicate this study to my daughter,

Paulina,

who is always in the core of my heart.

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Lesson study has served as much as a strategy as an inspiration for this work; I thank Natasa and Melania for introducing me to lessons study, as well as being a motivating part of my cohort in the doctoral programme of mathematics education in SFU.

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My graduate studies in mathematics education changed my life in more than one sense. My perspectives on teaching and learning mathematics have change as much as my perspectives on mathematics. I feel lucky, and thankful, of have taken courses with Rina, Sen, David and Peter during these studies.

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Chapter 1 Introduction

Notwithstanding the lack of agreement among researchers and educational stakeholders on what it means or entails, *teacher professional growth* is a common term used in mathematics education. Identifying the type, and level, of knowledge that teachers at the elementary and secondary school levels need in order to improve students' mathematics learning has represented a broad debate in mathematics education (Askew, 2008; Linares & Krainer, 2006; Ponte & Chapman, 2006). This debate also includes the problem of how teachers should acquire such knowledge. Beliefs and knowledge, however, are not the only factors influencing the performance of a teacher: local contexts, such as curriculum and school level policies, shape teachers' practices as well (Gates, 2006; Skott, 2008). In order to conceive teacher's professional development beyond change in beliefs and acquisition of knowledge, social structures that form a part of the local context should be taken into account.

Collaborative work among teachers, prospective teachers, and mathematics educators represents an option for teacher professional development that includes a social structure. In particular, the collective enquiry about students learning processes, as well as the designing of learning situations, has been applied in several countries (Jaworski, 2006; Lewis, Perry, & Hurd, 2009; Ling & Runesson, 2007; Marton & Tsui 2004; Slavit, Nelson, & Kennedy, 2009; Stigler & Hiebert, 1999). The collaborative design of such learning situations among teachers and educators has a relevant impact on both teacher professional development and curriculum improvement. In this study I was interested in exploring the interactions that teachers, and other educational actors, engage in while designing a mathematics lesson or other mathematics teaching or learning artefacts.

In this introductory chapter, I detail the motives that drove me to conduct this research study. For this purpose, I present my background in mathematics education and provide some of the assumptions for this study.

1.1 Personal Background

Describing my personal background has a double purpose. Firstly, an historical account would explain my interest in teachers' professional development. Secondly, this background affected the way I saw and interpreted phenomena (Bowers & Schatzman, 2009; Charmaz, 2009) and thus shaped many of the decisions I made for, and during, this research study.

As a teacher, I have taught a variety of mathematical courses at the high school, college, and undergraduate levels. Working as a mathematics high school teacher in an educational system in Mexico City¹, I had the opportunity to collaborate in the revision of its mathematics curricula. As a means to conduct such a revision, many teachers had to meet and discuss, propose, and agree on the changes required in order to improve the curriculum. From those meetings, I started to inquire into the purpose of teaching mathematics at the upper high school level, as well as the role of the teacher in the learning of mathematics. During my final six months working for that educational system, I had the opportunity, as a consultant, to interact with many mathematics teachers, across all the schools of the system. I realized that, as teachers, we could benefit by learning from each other: There should be spaces that facilitate teachers' exchanges of experiences and knowledge, and foster the discussions of issues in mathematics education.

Once I was in the doctoral programme in the Faculty of Education at Simon Fraser University, I participated as a research assistant in a project focused on mathematics teachers' change of beliefs. Additionally, I taught undergraduate courses in the mathematics department, and courses in the education faculty for prospective teachers, at both the elementary and secondary levels. As a consequence, I developed an interest in mathematics teacher development for both prospective and practising

¹ This educational system was the upper high school level *Instituto de Educación Media Superior del DF* that had, in 2006, sixteen schools across Mexico City.

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teachers. Such an interest was reflected on the literature I read during my studies in the doctoral programme.

During my doctoral studies in mathematics education I was invited to participate in a *Lesson Study* group (Fernandez & Yoshida, 2004; Hart, Alston, & Murata, 2011; Stigler & Hiebert, 1999) at an institute partnered with several universities. As a means to keep the confidentiality of research participants' identity, I use SIGMA as a pseudonym for this institute. Therein, I met other teachers and educators and started participating in the design of mathematical lessons².

A closer contact with lesson study was possible by attending the *Lesson Study Immersion Programme 2007* where I observed and participated with teachers undertaking lesson study in Japan for two weeks. At that juncture I was also interested in Wenger's (1998) *communities of practice* notion and in the research in mathematics education concerning mathematics, and its teaching, as a social practice. Thus, I started experiencing the collective work of lesson study, and conceptualizing a collaborative practice among teachers and educators at the same time.

In the Fall of 2007, I led, for one semester, three teams of teachers conducting lesson study in Mexico³. Teachers described their experience as very valuable and profitable, as well as enjoyable (Preciado & Liljedahl, 2008). So, I decided to explore the effects of lesson study on teachers when engaged in this collaborative activity.

Having an academic background that includes both a Masters degree and doctoral studies in pure mathematics, I always showed my passion for the subject as a teacher. However, my conception about mathematics changed as I moved towards education and read about the history and philosophy of mathematics. Influenced by Hadamard (1996) and Lakatos (1976), among other authors, I have changed from a Platonic perspective about mathematics to a more socially constructed and continuously evolving point of view of both mathematics and mathematical activity. This shift means a

² Although we did not implement lesson study as has been used in Japan, we went through the process of selecting goals, designing a lesson, implementing and observing the lesson, and having a debriefing afterwards.

³ Again, we were far of having the whole context of Japanese lesson study. The work during this semester included: determining goals, designing lessons collaboratively, implementing the lessons, and debriefing the results of the lessons.

stronger focus on students' mathematical activity which goes beyond procedural operations: for instance, the use of problem solving as a means to introduce a mathematical concept. This new perspective about mathematics learning, and related pedagogical practices, as a social activity based on discovery, permeated not only my teaching, but also my participation as a member of lesson study teams.

1.2 Assumptions and Significance

I am convinced about the benefits of implementing lesson study, or other forms of teachers' collaboration, for students and for the teacher—as individuals and as a community. Being in different educational systems, and being informed by the related literature, made me aware of some of my assumptions about education in mathematics. Making explicit these assumptions is important because they informed my decisions during data generation and analysis (Bowers & Schatzman, 2009; Charmaz, 2009); therefore, I consider them as both a ground from which I started and an influence during the research. I will outline three basic assumptions about mathematics, mathematics education, and teachers' activities behind this study.

1. Mathematics at school is a worthwhile activity for the benefit of individuals and society. Although it seems obvious that school contributes to citizen's development, the school system has been criticized for being detrimental to society (Illich, 1970), and promoting a type of slavery among the lower classes (Freire, 1970). The outcomes of teaching cannot be taken for granted, as it is shown in the interesting case of Benny (Erlwanger, 1973) who, having a conceptualization about fractions which differed from the mathematical one, was able to obtain good grades in multiple choice answers tests. This case shows how student's 'school learning' might be very different than the intended one from the curriculum. Another example that challenges the role of school comes from the study of non-schooled vendor children in Brazil (Saxe, 1988) who outperformed schooled children in arithmetical tasks using their own procedures. I believe that teachers must be aware of possible limitations and constraints of the school system and reflect on our own practice in order to avoid not only

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mathematical misconceptions, but also side effects of the school system on the individuals.

2. Mathematics is a historical-cultural construct. Individuals participate in making sense of their own understanding of the mathematical activity based on a social practice. Borne in historical and cultural processes, mathematics have been evolving over time rather than being a fixed object: there is no one definition of mathematics that all mathematicians ascribe themselves to (Davis & Hersh, 1998; Lakatos, 1976; Larvor, 2008; Sfard, 2008). This assumption is actually more general: The individual makes sense, and shapes, his or her own reality where social interactions are quintessential. Teachers holding different perspectives about mathematics can still learn from each other if they are aware of those differences.

3. Improving teachers' mathematical understanding, as well as developing an open perspective about learning might provide their classroom environments with a deeper and more meaningful mathematical activity for students. Teachers' lack of mathematical understanding prevents students from having a better environment to develop their mathematical thinking. However, the opposite is not necessarily true: the students of teachers with a greater mathematical background do not necessary obtain better results (Askew, 2008).

During my experience working collaboratively with other teachers in the design of teaching artefacts, such as lessons, mathematical tasks, and assessment instruments, I have learned with them and from them. I also found a space for reflection about the purpose of mathematics education and the outcomes of formal education. I had a chance to learn new mathematics, and in many cases new ways of seeing the mathematics I already knew, while interacting with teachers in a collective endeavour such as mathematical curriculum discussion and review. Teaching strategies have been often discussed when I had the chance to talk to other teachers. Having benefited myself from this contact with my peers, I believe that collaboration among teachers has great potential to improve mathematics teaching and learning.

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Teachers can work collaboratively in several ways and with a variety of purposes (Fernandez & Yoshida, 2004; Marton & Tsui, 2004; Slavit, et al., 2009). Because of my experience conducting lesson study, I have focused in a particular way of collaboration which has as its purpose the design of a teaching artefact, such as a lesson or a mathematical task. I am convinced about the benefits, for both teachers and students, of this way of collaboration. However, besides being able to say that teachers have an opportunity to learn from each other and reflect on their practice, I wanted to identify those factors that would impact on teachers' practices during the collaborative design of a teaching artefact. I was also interested in understanding the types of interactions that happen during the design of such artefacts: What do teachers talk about? How and what do they learn during the design of the artefacts? What is the impact of this learning on their teaching practices?

Although I focused my attention on the collaborative work among teachers that involves the design of teaching artefacts, the range of possibilities with this type of collaboration is still wide. Besides, following the same strategies of collaborative design from one place to another may have different effects. Cultural aspects and local settings vary from one place to another. Then, it makes sense to ask the following questions: What other types of collaborative design of teaching artefacts have been implemented? What are the similarities and differences among different instances of collaborative design?

A comparison of instances of this type of work among teachers has a practical value. Contrasting different cases of collaborative design of teaching artefacts would inform people interested in the implementation or the improvement of collaboration among teachers. However, I do not think that just copying one model of collaborative design from one place to another is desirable, or even possible in a general case. Local contexts must be considered and models can be adapted or partially adopted in order to implement collaboration among teachers in the designing of teaching artefacts.

The comparison and analysis of instances of collaborative design among teachers has, nevertheless, another purpose. From the descriptions of resonances and differences of several instances, it is possible to construct concepts that would give a

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grounding, and a language, which allows us to theorize about the factors that contribute to the improvement of mathematics teacher's practice. In this dissertation I attempted to conceptualize social interactions among participants in the collaborative design of teaching artefacts. Such conceptualization should fit several cases, providing a language to talk about teacher's interactions during collaborative design and helping to identify factors that potentially contribute to teachers professional development in this particular setting. Identifying those factors has the potential to allow the implementation of more profitable instances of teacher's collaborative design, as well as other modalities of mathematics teacher professional development.

Chapter 2 Teachers' Collaborative Design

This chapter contains a literature review on collaboration among teachers in mathematics education, as well as the research questions for this study. In the first section I elaborate on several aspects of mathematics teacher education, and research, that served as a motivation for the study. The second section presents an overview of different forms of collaboration among mathematics teachers. In the third section of this chapter I introduce the concept of *teachers' collaborative design*, including some examples such as *lesson study*, *learning study* and *communities of inquiry*. Then, the research questions are stated and a description of this study is presented in the fourth section. Finally, I summarize the content of this chapter relating the research questions and my findings in this study with my original interest in understanding those factors that promote professional development in teachers' collaborative design.

2.1 Motivation

The academic motivation for this study stems from several interests. One of them was my interest in contributing to the literature of mathematics education taking a qualitative approach. The Organization for Economic Co-operation and Development (OECD) in its "Orientations for policy" (OECD, 2009) has advocated for a particular form of research in education, the *use-inspired basic research*. Governments not only dictate policy about education, but also influence teachers' education and even research on education. For example, the National Council of Teachers of Mathematics (NCTM) Research Committee (2009) criticized the U.S. policy on research in education.

Recent federal educational policies and reports have generated considerable debate about the meaning of 'scientific research' in mathematics education. ... Concentrating on the critical problem of determining which educational programs and practices reliably improve students' mathematical achievement, these policies and reports focus almost exclusively on experimental/quasi-experimental research. ... these

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policies and reports advocate an extremely narrow view of 'scientific' research. This view excludes many high-quality research methods and majority of extant mathematics education research studies that can and should be used in efforts to guide and improve mathematics education
(p. 216)

Such a *narrow* view of *scientific* research is present in the OECD perspectives of research in education. There is a constant stress on the *use-inspired basic research* in education (OECD, 2003), which entails the use of quantitative methodologies. The current perspective of the OECD in education considers that educational research and development do not give the needed support to effect change and promote innovation by arguing that "despite the key role of knowledge-based innovation in education, the country reviews of educational R&D have confirmed the following features as commonly (thought not universally) characterising OECD systems: ... Generally low levels of research capacity, especially in quantitative research" (OECD, 2009, p. 88). However, due possibly to the complexity of the learning phenomena, particularly in teacher education, a great part of the research on teacher professional development bears on qualitative methodologies and mixed methods (Creswell, 2008). In this study, I use one of such qualitative methodologies, grounded theory, as a means of understanding teachers' interactions while participating in the collaborative design of an artefact to be implemented inside the classroom. In doing so, I intended to contribute to the research in mathematics education by joining the call of the NCTM, Research Committee (2009), of using alternative methods to *use-inspired research*.

The description and characterization of the knowledge that teachers must have in order to improve mathematical instruction has been a polemic issue among mathematics educators. Teachers' knowledge, as categorized by Shulman (1986), is described as *subject matter knowledge*, *pedagogical content knowledge*, and *curricular content knowledge*. Such categorization is broadly used among researchers in mathematics education—one hardly would find a paper on mathematics teachers' knowledge making no reference to Shulman's categorization. However, while many researchers have used, refined, or redefined such characterization (Graeber & Tirosh, 2008; Rowland, Hukstep and Thwaites, 2005; Thames & Ball, 2010), others have argued against it. For instance, Askew (2008) claimed that "trying to draw distinctions between content knowledge and

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pedagogical content knowledge may no longer be helpful" (p. 15). Based on a situated perspective of learning, he argued that "there is no mathematical discipline knowledge that can be removed from the way that it has been studied and looked at separately. There is no content knowledge separate from pedagogic knowledge" (p. 29). This perspective extends the question of '*What should teachers know?*' to '*How should teachers hold their knowledge?*' and then to '*How should teachers learn mathematics and learn to teach mathematics?*' These questions also motivated my interest for this study.

The increment of teachers' mathematical content knowledge is not enough to improve students' mathematical learning. For the case of secondary school-level, Monk (1994) concluded that student attainments and the number of teachers' completed undergraduate mathematics courses are associated for five or less courses; for more than five courses there was no significant difference in student attainment. In the case of elementary school-level teachers, conclusions were even more surprising as Askew (2008) mentioned, "the higher the level of [teacher's] qualifications, the lower the gains pupils made" (p. 17). A possible reason for this correlation might be that teachers, as they take more mathematics courses, reinforce the idea of mathematics as a set of procedures to be repeated: "Observations of lessons conducted by teachers with higher formal mathematical qualifications did tend to be more procedural in their content" (p. 17). A common issue for prospective and practising teachers is how their background as students and first experiences as teachers shape their teaching. Proulx (2007) explained that teachers' mathematical knowledge is usually procedural and is rooted in their experience first as a learner, and later as a teacher reinforcing such a perspective about mathematics with their years of experience; he referred to this as "the cycle of reproduction creating mathematics as a set of techniques and facts" (p. 13). Breaking this cycle is not easy as teachers may have to relearn mathematics differently from how they have been taught. Moreover, Davis (2008) highlighted that teachers' mathematical knowledge must be different from the mathematical knowledge for other professions, and proposed *concept study*, which "simultaneously inquires into how individuals learn mathematics, how mathematics is taught, and how disciplinary mathematics arises" (p. 90). Research has shown that important teacher learning takes place in spaces for

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collaborative design and inquiry (Goldsmith, Doerr, & Lewis, 2009; Groth, 2011; Hart, et al, 2011; Jaworski, 2006, 2008, 2009; Lewis, et al. 2009; Slavit, et al., 2009), and this has been another motivation to conduct this research.

In addition to the mathematics teacher's knowledge, research studies in mathematics education have focused on beliefs and attitudes toward mathematics and learning mathematics (Leder, Pehkonen, & Torner, 2002; Llinares & Krainer, 2006; Maaß & Schlöglmann, 2009; Ponte & Chapman, 2006; Sullivan & Wood, 2008). Identifying the knowledge, beliefs, and practices desired for mathematics teachers represents one aspect of the research in mathematics education; another problem is how to promote teachers', and prospective teachers', learning accordingly. Researchers have focused, for more than twenty years, on teachers' beliefs and values, as well as how to change them in order to improve mathematical instruction (Hackett & Betz. 1989; Maaß & Schlöglmann, 2009; McLeod & Adams, 1989; McLeod & McLeod, 2002; Ozder, 2011; Pajares & Miller, 1994; Philippou & Christou, 1998; Thompson, 1992). Furthermore, the assumption that beliefs are enough to change teacher's practice has been challenged (Liljedahl, 2009; Skott, 2008). My research is also the result of an interest in the design of teacher professional development programmes that impact on teachers' beliefs and practice.

There is a varied set of academic requirements, according to different countries or provinces within a country, that teachers at the elementary and secondary levels must fulfil before they start teaching. Nevertheless, teachers' professional duties are also based on the perspective that each country has towards the teacher's job. One example is Japan where teachers participate in collaborative programmes, with other teachers and researchers, that have an impact on the development of textbooks and curriculum (Stigler & Hiebert, 1999; Watanabe, 2007). Another example is Finland where teachers are required to develop curriculum at the school level (Pehkonen, Ahtee, & Lavonen, 2007). These two countries conceive teachers' duties beyond working with students inside the classroom: Teachers participate in the process of improving and designing curriculum at school.

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In contrast to the expected role of teachers in Japan and Finland, other countries do not consider teachers' participation for curricular decisions. For instance, Brown and McNamara (2005) conducted two studies of elementary teachers' education in the U.K. at a time of major changes to curriculum and regulative policies for the teaching of mathematics took place. They stressed the limited lifetime of the curriculum, due to the change in government administration, and warned about the possible implications to the teachers' role in the educational system.

Curriculum packages have a limited shelf life and it would be worrying if all training were directed at conforming to just one current model, resulting in a proliferation of civil servants of a time-specific governmental truth. ... It would be unfortunate if the prevailing conception of teacher development reached further towards the preference of providing new rules, with the teacher understanding their own professional development in terms of following those rules more effectively. ... It seems that we need to enable teachers to participate in developing understandings of how we might see mathematics in the classroom—rather than receiving a curriculum as something to be implemented, constructed by people outside of the classroom. (p. 167)

Government policies concerning teacher certification, or evaluation, are common in some countries. Tests and other means of teachers' certification-evaluation are intended to assure quality in teaching; however, an unintended message might be perceived by teachers making them believe that: (1) the format of the assessment serves as a model of assessment to be used with their students, and (2) "mathematical competence is demonstrated by quick solutions to routine mathematical problems" (Hill, Sleep, Lewis, & Ball, 2007, p. 150). Although means of teacher certification and evaluation may not be a part of a teacher professional development programmes, they shape teachers' practices. Fostering a conception of teachers as contributors instead of simple implementers of the mathematics curriculum, was also a motivation for this study.

In the US, one decade ago, teachers were assumed to be competent once they had completed their professional development programmes, whereas in Japan "participation in school based professional development groups is considered part of a teacher's job" (Stigler & Hiebert, 1999, p. 110). I see that the former view of teachers' preparation, fixed and terminal upon completion of the corresponding development

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programme, does not help to incorporate innovative instruction in schools; many teachers might not want to change their practice. A motivation for this study was to conceive of mathematics teachers as learners through their whole career, not only during pre-service professional development. The collaboration among teachers at school provides a space for such activities where knowledge for teaching mathematics can be shared and developed (Bruce & Ross, 2008; Cobb, Confry, diSessa, Lehrer, & Schuabale, 2003; Lerman & Zehetmeier, 2008; Llinares & Krainer, 2006; Turner, Warzon, & Christensen, 2011).

In addition to the individual teacher's knowledge, we can consider the knowledge of a community as a whole—taking advantage of members' expertise in specific areas. Boaler (2002) argued in favour of representing knowledge "not as an individual attribute, but as something distributed among people, activities and systems of their environment" (p. 42). For the case of mathematics teachers, forming communities of teachers that collaborate in their own practice and professional development represents a means of improving instruction at school. In a study of 90 primary school teachers, Askew, Brown, Rhodes, William, and Johnson (1997) identified a school where students had consistently higher results in an assessment of numeracy. Not all the teachers at that school demonstrated a strong discipline knowledge; however, there were two teachers who shared responsibility of mathematics across the school. Whereas one had a strong mathematical background, the other had been involved for many years in professional development and had studied the psychology and pedagogy of primary mathematics. This is an example of how shared knowledge in a community of teachers can be used in order to improve mathematics instruction, as well as teachers' professional practices.

Professional development programmes aimed to prepare new teachers in mathematics are not enough for assuring teaching competency. New teachers lack teaching experience, whereas experienced teachers may not be up to date on educational issues. Furthermore, teachers' mathematical misconceptions might be reinforced through time (Proulx, 2007). New educational reforms have been informed by research on mathematics education and programs for teacher professional development situate the teacher as a learner (Llinares & Krainer, 2006). Additionally, in order to introduce innovative practices in the educational system, such as those related to the

collective generation of knowledge with the use of technology, Organizational structures at school may constrain teachers' collective reflection on their practice (Chan, 2011). Moreover, "the ability of teachers to shift, change, and refine the innovation " must be considered (p. 151). For these reasons, I am convinced that in order to assure teachers' competency, it is important to both conceive mathematics teaching as a lifelong learning profession and find mechanisms that support continuous teacher professional development in collaboration with other teachers and educational stakeholders.

2.2 Collaboration Among Teachers in Mathematics Education

The collaborative work among teachers has been described in different forms and it is often referred to as *teachers' professional communities* (Nickerson & Moriarty, 2005; Secada & Adajian, 1997), *communities of practice* (Wenger, 1998), and *professional learning communities* (Lin & Ponte, 2008; Servage, 2008). In this section I describe a variety of modes of collaboration among mathematics teachers and educators. My main focus here is on practising teachers; however, much of the referenced literature also involved prospective teachers. Although I present these modes under different subtitles, professional development programmes often include elements of more than one mode. The level and type of collaboration varies from mode to mode. For instance, while some modes of collaboration only entail teachers participating in group discussion on a case study (e.g. Chapman, 2008), other situations entail an active participation in the design of assessments of students' mathematical thinking (Fukawa-Connelly & Buck, 2010; Suurtamm, Koch, & Arden, 2010). Moreover, additional benefits for students, and teachers, in mathematical learning have been documented as informal professional development (Nickerson & Moriarty, 2005) and informal teachers' learning (Hom, 2010) in schools where teachers work in collaboration within a community, The modes of collaboration among mathematics teachers described in this section are mostly oriented to formal professional development. However, in some cases the collaboration among teachers had other purposes, such as the assessment of students' mathematical thinking.

Problem Solving and Concept Study

Lead by the assumption that “for teachers to become competent in mathematics it was necessary for them to learn mathematics in the same way as they were expected to teach it” (Llinares & Krainer, 2006, p. 440), many programmes for teachers professional development have used problem solving, and other mathematical inquiry tasks, as a strategy for collaborative work among teachers that encourage reflection on their experience of coping with such tasks (Boero & Guala, 2008; Davis, 2008; Fernandes, Koehler, & Reiter, 2011; Fired & Amid, 2005; Murray, Olivier, Human, 1999; Schifter, 1993; Whitenack, Knipping, Novinger, Coutts, Standifer, 2000). This mode of collaboration enables teachers to see different approaches to the given tasks constructing a common understanding of the different solutions to the tasks, as well as a reconstruction of the mathematical concepts in a social setting (Davis, 2008; Whitenack, et al., 2000). The mathematical tasks posed to, or the mathematical concepts to be elaborated by, the teachers are, more often than not, aimed at a range of grade levels, instead of being specific to one grade level.

Analysing Students' Mathematical Thinking

Learning to listen and analyse student's mathematical thinking is an asset for teachers who encourage students to solve mathematical problems, or engage in mathematics inquiry using their own approaches and communicating their thinking (Empson & Jacobs, 2008). Teachers develop responsive listening to students' thinking through “(a) discussions of children's written work, (b) discussion of videotaped interactions with children, and (c) opportunities for teachers to interact with children and then reflect on those experiences with other teachers” (p. 257). Empson and Jacobs argued that listening to children's mathematics is important to: (a) improve children's understandings, (b) provide formative assessment, (c) increase teachers' mathematical knowledge, and (d) engage teachers in generative learning. The collaborative analysis of student's mathematical thinking has been used as a means of teacher professional development (Fennema, et al., 1996; Kazemi & Franke, 2004; Lerman & Zehetmeier, 2008; Llinares & Krainer, 2006; Tirosh, Stav, & Tsamir, 2001; Whitenack, et al., 2000). More recently, teachers have also collaborated in the design of assessment instruments

for students' mathematical thinking (Fukawa-Connelly & Buck, 2010; Suurtamm, et al., 2010).

Reflecting on Teaching Practice

Using collaborative reflection of teaching practice has been a common tool for mathematics teacher professional development. For instance, Learman & Scott-Hodges (1991) reported a positive impact on prospective and in-service teachers' practices after sharing ideas and reflecting on their own writings about 'critical classroom incidents.' Writing these type of incidents, or other stories, has also been a professional development strategy which includes "narrative inquiry through peers" (Chapman, 2008, p. 34) in which teachers share and reflect on stories of related personal experience. Additionally, "video-stimulated recall has been used with a variety of participants ... as a way to understand their pedagogical thinking and actions" (Muir, Beswick, & Williamson, 2010, p. 131). Groups of teachers have created videos of their own interaction with students in mathematics lessons and used them for group discussion as part of professional development (Jacobs, Ambrose, Clement, & Brown, 2006). However, this use of video has the risk of teachers perceiving the discussions of the video as a personal attack. "Critical collegueship" (Lord, 1994) might be fostered among teachers before using self-recorded video for group discussion (Empson & Jacobs, 2008; Jacobs, et al., 2006; Muir, et al., 2010). A culture of constant teaching reflection supports the conception of the teacher as a lifelong learner: "it is assumed that reflection is a means by which teachers continue learning about teaching and about themselves as teachers" (Llinares & Krainer, 2006, p. 442).

Constructing and Analysing Cases

The use of case study in mathematics teacher education has been a consistent and effective strategy widely documented in the literature (e.g. Barnett, 1998; Barnett, Goldenstein, & Jackson, 1994; Markovits, 2008; Merseth, 1991, 1996, 2003; Schifter, Bastable, & Russell, 1999, 2002, 2007). Cases have been written and re-written with the help of teachers and educators allowing "teachers to share perspectives and ways of translating their theories into classroom practice" (Llinares & Krainer, 2006, p. 443).

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Other cases have been presented in the form of video and used for group discussion as part of teacher professional development (Griffin, 1999); Seago, Mumme, Branca, 2004; Sherin, & van Es, 2009; Whitenack, et al., 2000). The use of cases in mathematics teacher education provide “realistic context” (Markovits & Smith, 2008, p. 40) with the potential for helping teachers to: “develop skills of analysis and problem-solving, gain broad repertoires of pedagogical technique, capitalize on the power of reflection, and experience a positive learning community” (Merseth, 1999, p. xi).

Virtual Communities and Networks

Virtual communities of mathematics teachers and online networks have been used for collaborative professional development in the form of masters level courses and workshops (Borba & Gadanidis, 2008; Heng-Yu, Akarasriworn, Glassmeyer, Mendoza, & Rice, 2011; Llinares & Valls, 2010; Renninger, Cai, Lewis, Adams, & Ernst, 2011), as well as a form of continuous teacher support by collaboration among peers and educators (Alvermann, Friese, Beckmann, & Rezak, 2011; Barah, Schatz, & Scheckler, 2004; Barab, MaKinster, & Scheckler, 2003; Borba & Gadanidis, 2008; Dalgarno, & Colgan, 2007). Virtual communities and networks not only help to subsume geographical barriers, but also represent alternative forms of communication—e.g. asynchronous, graphical, and media based (Renninger, et al., 2011). Moreover, Borba and Gadanidis explored the “role of virtual environments and tools both as factors *mediating teacher collaboration* and as *co-actors in the collaborative process*” (p. 182, italics in original). Virtual communities have been used for collaboration in the modes previously described in this section such as: approaching and analysing mathematical tasks and concepts (Lachance & Confrey, 2003; Lee, Chauvot, Plankis, Vowell, & Culpepper, 2011), analysing students mathematical thinking and reflecting on teaching practice (Chinnappan, 2006), and analysing cases (Chieu, Herbst, & Weiss, 2011).

Action Research

Collaboration among teachers has also included reflecting on their own professional growth and determining their own research agenda (Adler, 1997; Brown & Jones, 2001; Mousley, 1992). In action research “teachers are regarded as

professionals who systematically aim at investigating their own practice” (Llinares & Krainer, 2006, p. 443). In other words, “one might describe action research as reflection-on-reflection-on-action” (Lerman & Zehetmeier, 2008, p. 135). Reflection on teaching practices is a component of action research. Additionally, teachers can develop their awareness of their own mathematics by reflecting on mathematics students' learning process (Stephens, 2006). Interestingly, virtual communities have also taken an action research approach in mathematics education (Thang, Hall, Murugaiah, & Azman, 2011).

Designing for Learning

One mode of collaboration among teachers and other educators is the collaborative designing of teaching/learning artefacts. Known examples of collaborative design are *Lesson study* (Hart, et al., 2011; Lewis, et al., 2009; Fernandez & Yoshida, 2004), including some of its variations such as *learning study* (Elliott & Yu, 2008; Marton & Tsui, 2004) and *action education* (Huang & Bao, 2006). This mode has an impact on both teachers' continuous professional development at school and the development and improvement of curriculum and textbooks (Clarke, 2008; Watanabe, 2007). Other examples of collective design are the *communities of inquiry* proposed by Jaworski (2006, 2009), *collective teacher inquiry* (Slavit et al., 2009), and *professional learning communities* that Servage (2008) described as critical and transformative practices by and for teachers. Collaborative design for learning among teachers has also been conducted online (Alvermann, 2011; Lee, et al., 2011). The collaborative design for learning among teachers is the main focus of this study and some of the examples mentioned in this paragraph will be elaborated in the next section.

2.3 Teachers' Collaborative Design

The collaborative work among teachers and educators which I focus on in this study entails the design of mathematics teaching and learning artefacts. I use the word 'artefact' instead of 'lesson' or 'mathematical task' in order to encompass other teaching and learning 'tools' that teachers design collaboratively—such as an assessment rubric or a class project. The collaborative design among teachers and educators addressed in this study includes the following three steps: (1) designing a teaching learning artefact

intended to approach previously selected goals; (2) implementing the artefact in a classroom, or several classrooms; and (3) debriefing the implementation and refining the task. Although the artefacts can be of different types, they include the following: (a) a specific purpose in terms of teachers' interest, prescribed learning outcomes, or goals of the educational program; (b) an indication of its implementation in the classroom, such as a lesson plan or instructions for the teacher; and (c) a specific mathematical content reflected in the topics, examples, exercises, or problems for the artefact. Despite the fact that these artefacts include a description of their implementation, teachers often adapt them to their own classroom according to specific students' needs or situations—therefore there is some flexibility in the format of the implementation. I call the collaborative activity described in this paragraph *teachers' collaborative design*—or just *collaborative design* for brevity. I present below three models of teachers' collaborative design that have been broadly applied. Although many other models of collaborative work among teachers have been used for professional development, those presented here are spread in many regions around the world and I used them as second-hand data for this study (Chapter 8).

Lesson study (*jogyou kenkyuu* in Japanese) is a teachers' way of working at virtually all elementary schools and many middle schools in Japan⁴, and widely adopted in several countries (Hart, et al., 2011; Lewis, et al., 2009; Motoba, Crawford, & Sarkar Arani, 2006), that consists of the following steps (Stigler & Hiebert, 1999, p. 112-116): (1) teachers define the learning goals they want to approach in the lesson; (2) the team of teachers plans the lesson which will be implemented by one of them; (3) the members of the design team observe and video record the implementation of the lesson; (4) a debrief of the results of the lesson is conducted by this team; (5) based on the observations and criticism coming from the debrief, the lesson is redesigned; (6) the lesson is taught again, observed not only by the team's members, but also other members of the school faculty are invited—and possibly external people are involved as well; (7) a debrief of the lesson is conducted, with all observers participating in the

⁴ Lesson Study is the most popular form of in-service professional development model in Japan conducted in elementary schools and many middle—lower—schools; "in contrast, very few Japanese high schools carry out this activity today or have ever engaged in the past" (Fernandez and Yoshida, 2004, p. 16).

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discussion and criticism; and (8) the lesson and its results are published in order to share the experience nation-wide. Although it is not compulsory in Japan, teachers value lesson study and consider it as a quasi-obligatory in-service professional development programme.

Learning study has been developed from lesson study and stresses the importance of *variation* as a necessary condition for learning. In learning study the unit of analysis is students' learning, as opposed to analysing a lesson as is the case in lesson study. Marton and Tsui (2004, p. 192) described the learning study cycle in six steps: (1) choosing and defining educational objectives; (2) exploring students' capabilities or values prior to beginning teaching; (3) designing a lesson or series of lesson accordingly; (4) teaching the lesson; (5) evaluating the lesson; and (6) documenting the aim, procedures, and results. Learning study has been adopted in several places around the world, particularly in Hong Kong where large-scale projects within the educational system had utilized this model of teachers' collaborative design (Elliott & Yu, 2008; Marton, et al., 2004).

The number of countries applying lesson study or learning study has been increasing, as can be seen at "The World Association of Lesson Studies" that has been holding conferences annually in Hong Kong since 2005, and Canada is not an exception. For instance, the Pacific Institute for the Mathematical Sciences at the University of British Columbia has served as a host for meetings of teachers conducting, or interested in conducting, lesson study. Additionally, the Institute of Child Study Laboratory School at the Ontario Institute for Studies in Education, University of Toronto, has been conducting lesson study as teacher professional development for more than a decade.

Another example of teachers' collaborative design are *communities of inquiry*. Jaworski (2006) used *design experiments* (Cobb, et al., 2003) as a methodology in order to propose "*inquiry* as a fundamental theoretical principle and position," and suggested that the "*use of inquiry as a tool* can lead to developing *inquiry as a way of being ...* when practised as part of a community, in which members collaborate, as learners ..., to develop their practice" (p. 187). Based on Wells (1999), Jaworski described

communities of inquiry as communities of practice (Wenger, 1998) where their members—e.g. teachers and researchers—engage in the community through *critical alignment*—as opposed to just alignment. Jaworski identified three forms of inquiry in these communities: (1) inquiry on mathematics, (2) inquiry in teaching mathematics, and (3) inquiry in research which results in developing the teaching of mathematics. In communities of inquiry mathematics teachers and educators engage in collective inquiry-based research, where teachers select their inquiry goals and conduct research on those goals. "Such research might involve action or design cycles in which practitioners plan their activity, observe it critically during action, reflect analytically on their observations and feed back to further cycles of activity" (Jaworski, 2009, p. 111). In the planned activities, inquiry is also used in order to implement, or design, mathematical tasks for students (Jaworski, 2008).

Even though teachers in many cases are willing to engage in collaborative inquiry, Nelson and Slavit (2008) argued that often "complex layers of support" (p. 99) are required. They provided a theoretical framework, the *inquiry cycle*, for the *supported collaborative teacher inquiry*.

In this inquiry cycle, teachers determine a focus for the inquiry, then proceed through stages of developing a plan for action, carrying out the plan while collecting and analyzing data, and determining the implications of their findings as they relate to their collective and individual situation.
(Nelson & Slavit, 2008, p. 100)

The stages of an inquiry cycle were described by Slavit and Nelson (2010) as follows: (1) *focus*, where common areas of improvement are identified, common goals and values are developed, an inquiry focus is selected, and a inquiry question is developed; (2) *implement*, where a common action is planned including data collection, and a common action is implemented, and thus data is collected; and (3) *assess*, where collected data is analysed, implications for practice are derived, and findings are disseminated. An inquiry cycle does not always follow this trajectory, they often "involve 'doubling back' periods of readjustment. ... a number of mini-cycles can occur that, collectively, constitute an overall inquiry process" (p. 202). Slavit and Nelson examined

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an example of a common action in a group of secondary mathematics teachers consisting of the "use of carefully chosen and implemented mathematical tasks" (p. 201).

Lesson study and learning study can be also considered as communities of inquiry, or collaborative teacher inquiry, where the practice consists of collaborative design and implementation of a lesson, or a mathematical task. The analysis of students' work is included in all these cases, which in turn informs the refinement of the artefacts and serves as an occasion for teachers' reflections on both their practices and students' mathematics learning processes.

Collaborative design has the potential to impact on teachers' knowledge in several aspects. The debriefing of an implemented artefact helps to make explicit not only what it is that teachers have learnt through teaching (Groth, 2011; Leikin, 2006), but what they learn is also increased by the collective contribution and sharing by teachers and researchers. Moreover, participant teachers and their students are not the only beneficiaries of teachers' collaborative design. In addition to teachers' development, lesson study also contributes to the development of mathematical subject matter knowledge and pedagogical content knowledge of coaches for schools (Knapp, Bomer, & Moore, 2008), who in turn contribute to improve teachers' knowledge. By collectively designing lessons, or other artefacts, teachers also change the way they hold their beliefs/knowledge toward mathematics and mathematics learning (Liljedahl, 2007). Kennedy (2002) distinguished the sources of knowledge as *craft*, *formal*, and *prescriptive* according to the level of systematic study. Books, journals, studies at universities or colleges, and the official settings are sources of teachers' formal knowledge. In the case of communities of inquiry, craft teachers' knowledge becomes more formal as it comes from their own research and experience and is reported for its dissemination. Therefore, the impact of collaborative design on teacher development is not limited to the type of knowledge, it also includes the source and the condition of knowledge.

Despite the fact that the benefits of collaborative design have been documented (Goldsmith, et al., 2009; Lewis, et al., 2009; Ling & Runesson, 2007; Minori, 2009), a focus on the teachers' interactions in this context barely appears in the literature.

Communities of practice (Wenger, 1999), as well as cultural-historical activity theory (Engeström, 2008), have been used by researchers (Jaworski, 2009; Davis, 2008; Minori, 2009) as social theoretical frameworks in order to describe interactions among teachers and educators when engaging in collaborative work. Nevertheless, these frameworks neither fully describe the interactions during the design process in teachers' collaborative design, nor do they explain how these interactions influence teachers' practice in their own classrooms. Recently, attempting to “construct a collaborative, interactive model of teacher change,” Kaasila and Lauriala (2010) described one case of “student teachers' collaboration and its relevance to the change of their beliefs and practices” (p. 855). It seems plausible that researchers and practitioners interested in teachers' collaborative design could benefit from having a conceptual framework for interaction among participants in teachers' collaborative design.

2.4 Description of the Study and Research Questions

The purpose of this study is to explore the interactions among team members in teachers' collaborative design in mathematics. I was interested in conceptualizing such interactions without using pre-established frameworks (Engeström, 2008; Wenger, 1999) that were not specific to mathematics education. Additionally, although teachers' professional growth was a motivation for this study, I decided to avoid using a specific characterization for teacher's knowledge as a framework for analysing such interactions. This decision was made based on two reasons. First, what mathematics teacher should know, and how they should learn it (Askew, 2008; Graeber & Tirosh, 2008; Rowland, et al., 2005; Shulman, 1986; Thames & Ball, 2010), has been debated without arriving at a general consensus. And second, a theory of individual knowledge would not account for the distributed knowledge (Askew, et al., 1997; Boaler, 2002) that may be shared in, or developed by, a team of teachers and other educators. The fact that I focused on interactions among mathematics teachers and teacher professional growth might suggest the use of the notion of *identity* for this study. However, this term “has been criticized for its being pervasively unclear and undefined” (Sfard, 2008, p. 290), and I decided to start my study with as few theoretical assumptions as possible in order to

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foster the emergence of a conceptualization, grounded deeply in the data and in the context of teachers' collaborative design.

This research study was conducted in three stages, each one having different sources of data. In the first stage I focused on describing in detail the participants' interactions during the meetings held for the design of artefacts in one particular case, the Lougheed team—details below. The research questions in this first stage were:

RQ 1. *How can we characterize the participants' interactions during collaborative design in the case of the Lougheed team? How can we identify factors that promote teacher professional growth in such interactions?*

My experience in both conducting teachers' collaborative design in Mexico (Preciado & Liljedahl, 2008) and participating in the SIGMA lesson study group guided me to form a team of collaborative design for my research. I knew about the challenges involved in organizing the sessions for collaborative design. It is hard to meet with teachers after their working hours, especially when they need commute to a different place to hold meetings for the design of the lesson. Having teams of collaborative design from different schools would imply that at least some teachers have to dedicate after hours and commute to the location of the meetings. In contrast, when a collaborative design team meets at the same school where teachers work, it is easier to set up a time for the meeting either during their working hours, or close to their schedule so they do not need to dedicate too much extra time and do not need to commute to a different place. As I was interested in lesson study—inspired—activities in this first stage of the research, observing the implementation of the designed lessons was a part of the collaborative work. Having teachers from different schools travelling to observe the implementation often entails finding teachers' replacements for the time they would be absent from their classroom, which is difficult. For this reason, I looked for a group of teachers from a single school. One of the participants of the lesson study group at SIGMA served as a contact to other mathematics teachers at Lougheed Secondary School (pseudonym); there, three mathematics teachers and myself formed a team and conducted a collaborative design project which was inspired by lesson study.

As a result of the research in this first stage I developed a characterization of the interactions of the Lougheed team members while designing artefacts—two lessons and one assessment rubric. The meetings, sixteen in total for the collaborative design and 2 for group interviews, were recorded in audio and video as a means to generate data. Individual interviews were conducted in order to capture participants' perceptions corroborating and expanding my preliminary findings.

The analysis of the Lougheed team was enlightening with respect to understanding interactions among its members. However, I needed to look at other cases in order to expand, refine, and maybe modify, the findings from the first stage, as well as verify the relevance of the developed characterization. With the purpose of exploring other cases of collaborative design I started the second and third stages of this study approaching the following research questions:

***RQ 2.** Does the generated characterization from the Lougheed team describe participants' interactions in other cases of collaborative design? What can be expanded from such a characterization by analysing other cases of collaborative design?*

In the second stage of the research I conducted eight interviews with people in British Columbia that have participated in teachers' collaborative design: four teachers, one prospective teacher, two mathematics educators, and one facilitator. When comparing all the data regarding the role of the different participants, I found that several features present in the school district case were absent in the Lougheed team. Then, I developed descriptions of the roles in the different settings of collaborative design explored in this study. Additionally, particular local settings influenced the design process—for instance, having a timetable for collaborative work as part of teachers' duties at school facilitated collaborative design.

For the third stage of the research, in which I used second-hand data, I selected the following literature sources, each one describing a large-scale project of collaborative design:

•*Lesson Study: a Japanese Approach to Improving Mathematics Teaching and Learning*, by Fernandez and Yoshida (2004). A detailed description of the design process of a team in Tsuta primary school, in Japan, is presented in this book. This

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description includes transcriptions of teachers' conversations and examples of how the lesson under design evolved during a lesson study cycle.

• *Learning Studies as an Educational Change Strategy in Hong Kong: An Independent Evaluation of the 'Variation for the Improvement of Teaching and Learning (VITAL) Project,'* by Elliott and Yu (2008). This report is based on several interviews with teachers, principals and other stakeholders that participated in a three-year project involving 120 schools—including primary, secondary and special education sectors.

• *Collaborative Teacher Inquiry as a Tool for Building Theory on the Development and Use of Rich Mathematical Tasks,* by Slavit and Nelson (2010). This paper describes a case of *supported collaborative of teacher inquiry* where a group of high school mathematics teachers are interested in “increasing students' engagement and problem solving in the classroom” (p. 201). Transcriptions of the discussions among participant teachers are presented, as well as a description of the role of the facilitator.

Whereas I found strong resonance between the characterization for the conversations in collaborative design, developed from the Lougheed team, and the related literature (Fernandez & Yoshida, 2004; Goldsmith et al., 2009; Lewis et al., 2009; Slavit et al., 2009), the role of the participants, as well as the possible settings for collaborative design had a wide range of variability. I found that both of these factors, the settings for the collaborative design and the participants' role, influenced the interactions in the meetings. Therefore, I decided to identify the different settings and actors, including their roles, involved in teachers' collaborative design. I came up with the following research question:

RQ 3. *What are the possible roles of participants in different cases of teachers' collaborative design and how do they influence the interactions within the teams?*

The data from the three stages of the research were compared in order to identify and classify the roles of participants in collaborative design. The influence of these roles in the interactions of the teams was analysed using the two emerging themes developed in the first stage of the research.

Improving Mathematical Instruction

Though I had a genuine interest in teacher professional growth, in this study I did not attempt to show that teachers' collaborative design has a positive impact on mathematics teaching. This positive impact on teachers' practices has been documented elsewhere (Fernandez & Yoshida, 2004; Jaworski, 2006; Lewis et al., 2009; Ling & Runesson, 2007; Marton & Tsui 2004; Slavit et al., 2009; Stigler & Hiebert, 1999). However, the design process offers an occasion for teachers' professional growth while engaged in collaborative design which is congruent to the theoretical model of lesson study proposed by Lewis et al. (2009), who identified “three pathways through which lesson study improves instruction: changes in teachers’ knowledge and beliefs; changes in professional community; and changes in teaching–learning resources” (p. 285). During the data analysis of this study I was mindful of identifying those factors that provided teachers with an opportunity to grow professionally vis-a-vis these three pathways. My intention was not to prove that these factors promote professional development. Rather, these factors served as indicators of teacher professional growth.

2.5 Summary

This study has been conducted in three stages: (1) the detailed analysis of a single case, the Lougheed team; (2) the analysis of other three cases of collaborative design and; (3) the three cases reported in the literature as second-hand data. As a result, I developed a characterization of participants' interactions consisting of two dimensions: (1) the conversations and actions during the design process, and (2) the roles involved in collaborative design. This characterization describes the dynamics of conversations and actions during collaborative design, across all the cases analysed in the data, in a way that it is possible to identify moments in which teachers: (1) learnt about mathematics and mathematics teaching, and (2) self-reported the incorporation of new practices in their teaching. These are potential moments for teachers learning and instruction improvement and I purposefully identified them when analysing the data (Chapters 6, 7, and 8). Identifying those moments is not the same as showing that they are instances of teacher learning. As indicated before, the benefits of collaborative design have been described elsewhere. My concern in this study was to be able to

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capture and understand those moments in the interactions among participants of teachers' collaborative design.

Chapter 3 Methodological Considerations

... neither data nor theories are discovered. Rather, we are part of the world we study and the data we collect. We construct our grounded theories through our past and present involvements and interactions with people, perspectives, and research practices.
(Charmaz, 2006, p. 10)

In order to analyse participant's experiences while conducting collaborative design, I decided to take a social approach within the tradition of *symbolic interactionism* (Blumer, 1969). According to this approach, people, including the researchers, make sense of the experienced—lived—world in a constant process of interpreting social interactions. Such a process is captured in the epigraph which emphasises that perspectives about theory and the way researchers make sense of reality influence research decisions and findings. In this chapter, I justify the use of grounded theory methodology for this study and elaborate on my role, and position, as a researcher describing the theoretical perspectives adopted in this dissertation. These perspectives have epistemological and methodological implications that include not only the way I constructed and analysed data, but also the influence of my personal background on this research.

3.1 Selecting a Methodology

The selection of a methodology for this dissertation was based on three factors. Firstly, the research was meant to be an exploratory analysis of the design process of a teaching artefact in terms of participants' interactions in different modalities of teachers' collaborative design. Such an exploratory analysis would allow me to identify emergent themes during the research and to compare different cases of collaborative design. Secondly, in order to characterize the interactions among participants in collaborative design from the specific data used for this research, I would try to make as few

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assumptions as possible. Often, the research on mathematics teachers' interactions in collaborative work has been framed within general social theories such as activity theory (Engeström, 2008) and communities of practice (Wenger 1998). These theories, however, are general and may not provide accounts for the specific case of mathematics teachers working collaboratively. A fresh perspective might afford characterization of participants' interactions based on empirical observations instead of deducing facts from pre-established theories. And thirdly, the access to data sources was a limitation. As a doctoral student I had contact with some teachers and educators who had been conducting lesson study. My doctoral supervisor included collaborative design in workshops and courses for practising teachers, which represented another source of data. The literature, as second-hand data, was a promising option to complement the research looking at other cases of collaborative design.

The characterization of the interactions among members of a team of collaborative design that I was looking for could not be obtained by a quantitative method. Even though I was interested in identifying factors that promote teacher professional growth, in this research I focused only on characterizing the interactions within this model of collaborative work. No correlation would be useful at this moment—and might not be useful in the future, either. Additionally, I was not interested in finding predominant patterns in several cases: one single case among many others could provide insight in the characterization I was looking for. Therefore, a qualitative analysis was more suitable for this research.

The purpose of this study, the access to data sources, and their different types, were factors that influenced the selection of the methodology for the research. Creswell (2008) distinguished among five qualitative research designs in education: (1) grounded theory, (2) ethnographic, (3) narrative research, (4) case study, and (5) action research. Additionally, he describes two other qualitative traditions, (1) biography, and (2) phenomenology (Creswell, 1998). In order to understand interactions among participants in a team of collaborative design, it would be helpful to consider individual perspectives of each participants in the study. Among the research methodologies indicated before, both grounded theory (Charmaz, 2006, Corbin & Strauss, 2008) and phenomenology (Creswell, 1998; Moustakas, 1994) might serve this purpose. For instance, in

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constructivist grounded theory researchers “aim for an interpretative understanding of the empirical phenomena in a theory that has credibility, originality, resonance, and usefulness, relative to its historical moment” (Charmaz, 2009, p. 139), whereas McCaslin and Scott (2003) claimed that “phenomenology is described as the study of the shared meaning of experience of a phenomenon for several individuals” (p. 449). I did not take a phenomenological perspective in this study because: (1) I planned to include several cases in this study and it was not clear whether participants from different settings for collaborative design would experience the same phenomena; (2) I was interested in exploring the interactions, the phenomena itself, and not the shared meaning among participants; and (3) my perspective on research takes into account the background of the researcher (Lather, 1986a, 1986b), instead of attempting to “eliminate everything that represents a prejudice, setting aside presuppositions, and reaching a transcendental state of freshness and openness” (Moustakas, 1994, p. 41). Although these qualitative research designs share several features, their purposes are different. Among these designs, grounded theory not only allows the researcher to explore social processes and develop conceptualizations with a strong attachment to the data, but also affords the analysis of multiple types of data (Charmaz, 2006, Creswell, 2008). Thus, this methodology was appropriate for my study because of: (1) the nature of the general questions of this study, (2) my interest in developing a characterization without imposing other social theories of social interaction, (3) the limitations regarding my access to data sources, (4) and my perspective on the role of the researcher's background.

Grounded theory methodology bears traditionally on written documents as research data consisting mainly of interview transcriptions, field notes and texts; however, new approaches also use electronic media as both a data source and a tool for analysis (Corbin & Strauss, 2008). A main set of steps in grounded theory consists of the systematic classification and comparison of data segments, as well as the development of categories and concepts (Charmaz, 2006; Corbin & Strauss, 2008; Glaser & Strauss 1967). As the analysis is conducted strictly from the data, the resulting categories and concepts are valid within such data. If the results of a research afford a new understanding of a phenomenon under study, then such results are relevant within the area. In this research I aimed to generate categories based on the data, as opposed to

the use of already established categories that may not fit the data in the particular case of teachers' collaborative design. In doing so, I could provide a language that acknowledges the differences and similarities among different cases of collaborative design.

Although grounded theory has been described as "a systematic, qualitative procedure used to generate a theory that explains, at a broad conceptual level, a process, action, or interaction about a substantive topic" (Creswell, 2008, p. 432), there is a strong debate among different researchers using this methodology (Glaser, 2009; Morse et al., 2009). Corbin and Strauss (2008) recognize grounded theory as "a specific methodology developed by Glaser and Strauss (1967) with the purpose of building theory from data" (p. 1) and at the same time use the term grounded theory in "a more generic sense to denote theoretical constructs derived from qualitative analysis of data" (p. 1). The research for this dissertation falls within the latter case. Additionally, the epistemological perspectives about the research and the theory described by Charmaz (2006) are assumed in this study: (1) there exist many multiple realities according to different individuals' perspectives; (2) the researcher constructs categories based on her/his own values, priorities, and positions; (3) generalizations are partial, conditional, and contextual; and (4) the researcher must engage in reflexivity and must consider participants' views and voices. My role as a researcher had, thus, a particular relevance for this study.

3.2 Researcher's Role

I believe that all research has a certain level of subjectivity (Lather, 1986a, 1986b). The researcher shapes the data collection and frames the way of seeing the observed phenomena (Charmaz, 2006, 2009). The use and development of theory as a means to understand and predict phenomena was dominated by a perspective claiming that there is one reality that researchers can describe by discovering theories using a 'scientific method' (Berger & Luckmann, 1966; Kuhn, 1970). However, there is a strong resistance against this perspective mainly, but not exclusive, from the social sciences. Some researchers using qualitative methods, particularly grounded theory, have adopted and stance against this 'one' reality: "I realize there is no one 'reality' out there

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waiting to be discovered. ... However, I do believe there are external events" (Corbin & Strauss, 2008, p. 10). Corbin also referenced Schawndt (1998) who claims that "one can reasonably hold that concepts and ideas are invented (rather than discovered) yet maintain that these inventions correspond to something in the real world" (p. 237). Rather than focusing on the debate of whether these 'external events' correspond to the 'reality,' I acknowledge that people invent concepts and theories based in their own experiences and the meaning they give to such experiences. Researchers in education that have adopted a symbolic interactionism approach recognised that theory is "actually constructed in the researcher's head but is rigorously checked and rechecked against the ongoing data" (Woods 1992, p. 383). I agree with the authors cited in this paragraph in that theories and concepts are not mirrors of the reality, but rather interpretations and ways of seeing certain phenomena. The conceptual categories that I have developed— invented—through this study help to describe and look at some details of the interactions among participants of teachers' collaborative design. These categories serve as a language that differentiates and identifies features of collaborative design within the data presented and analysed through this study and can be used to analyse other cases of collaborative design in further research.

The use and purpose of pre-established theoretical frameworks in order to conduct research *respecting the empirical world* has been debated in the symbolic interactions tradition: "Respecting the empirical world means making as few assumptions in advance of the study as possible" (Woods, 1992, p. 349). Some researchers using *grounded theory* suggest to avoid established theoretical frameworks in initial steps of the research (Charmaz, 2006; Corbin & Strauss, 2008; Glaser & Strauss, 1967) in order to generate a theory that better explains the particular phenomena under observation without a biased point of view. However, it has been argued that personal background and interests affect the way we see and interpret phenomena (Bowers & Schatzman, 2009; Charmaz, 2009). Assuming a world of multiple lived realities, or multiple perspectives, entails not only looking at others' perspectives and trying to understand their points of view, but also taking into account the researcher's perspectives and background (Lather, 1986a, 1986b). For this reason, I found it important to incorporate my personal background in Chapter 1.

3.3 Interaction and Meaning

Blumer's (1969) symbolic interactionism is based on three main premises: (1) "human beings act toward things on the basis of the meanings that the things have for them" (p. 2); (2) "the meaning of such things is derived from, and arises out of, social interactions" (p. 2); and (3) "these meanings are handled in, and modified through, an interpretative process" (p. 2). These premises concede a central role to the individual in making sense of situations and making decisions as opposed to acting according to established psychological or social structures. The influence of factors such as social structures, culture and history in individuals' perceptions and acts, is not neglected under the interactionist tradition. Rather, the meaning that the individual makes of a certain situation or objects is considered as a dialectical process with these factors. For instance, in a team of collaborative design consisting of teachers and one educator, the way that each teacher interacts with the educator might vary. While one teacher would see the educator as an expert from whom they can learn, another teacher may perceive the same educator as an authority to obey or a supervisor. The interactions between the teachers and the educator, based on these different perceptions, reflect the position (Langenhove & Harré, 1999) of each member in the team. In this example the social structure is a context in which teachers make sense of the educator's actions and act accordingly. The meaning that every teacher ascribes to the words and actions of the educator may change after interacting over time.

Status, Role, and Position

Kaasalia and Laurilia (2010) considered that "in social situations a person must adopt a social role, which refers to a set of expectations of how a member of a special group or community is expected to act in his/her position" (p. 855). Blumer (1969), however, argued that "social interaction is obviously an interaction between people and not between roles; the needs of the participants are to interpret and handle what confronts them ... and not to give expression to their roles" (p. 75). The role of a person in a particular group also depends on the competency level of each individual. Kaasalia and Laurilia understand the status of a person according to a competence level.

A person's status characteristic is associated with his/her performance expectations, i.e., with a belief about how a member having a given characteristic is expected to perform. ... Status and role are defined on the basis of competence: The higher the status and role a member of a group has, the bigger contribution other members of the group expect he/she to have in solving the task. So the members who have a higher status are expected to be more active than the members having a lower status. (p. 855)

The status and the role of participants in collaborative design shape the interactions of the team and may change over time. Social interactions, however, take part in specific contexts which frame the actions and expectations in a group. The notion of settings will be used in this dissertation referring to the factors of the context in which collaborative design is conducted—such as some economical support, or the physical arrangement of facilities. Thus, in order to describe the members' roles in a collaborative design team, the settings must be described as well.

The interactions of the members in a team of collaborative design are also influenced by other factors such as: (1) the ongoing storyline developed on time, (2) the norms of the team, (3) the interests of each member to participate in the endeavour of collaborative design, and (4) the perceptions of the role and status that each participant has toward other members of the team. The concept of 'positioning' proposed by Langenhove and Harré (1999) as a "dynamic alternative to the more static concept of role" (p. 14) can be used in order to consider these factors. "A position in a conversation ... is a metaphorical concept through reference to which a person's 'moral' and personal attributes as a speaker are compendiously collected" (p. 16). The positions of participants in a conversation serve to understand the interactions that are taking place. For instance, the same sentence uttered by different members of a team can be interpreted in a different way according to the position of the speaker.

Artefacts as Objects

Objects exist according to the meaning that individuals have, and act, toward them (Blumer, 1969). As mentioned before, a person acts towards an object according to the meaning that this person has to such an object, or symbol. Consequently, the

meaning that an individual has of an object is not intrinsic to the object, it arises from how this individual is prepared to act toward it. Objects are socially produced, formed and transformed by social interaction and "people are prepared or set to act toward objects on the basis of the meaning of the object for them" (p. 68).

In teachers collaborative design participants engage in the creating—or recreating—different artefacts such as mathematical tasks, lesson plans, assessment rubrics. Those artefacts are objects that have specific meaning for each teacher. In particular, the way a teacher implements in class a designed artefact varies more or less on the level of instructions provided by the very same artefact. However, there is always room for differences in the implementations. Some authors distinguish artefacts from instruments—which might be considered as more prescriptive in the sense that teachers should follow strict directions. For instance, Gueudet and Trouche (2009) represented this distinction by the formula "*Instrument = Artifact + Scheme of Utilization*" (p. 204). The way I am using the term artefact includes instruments, but it also includes teaching tools with a less prescriptive set of instructions for their implementation.

As teaching artefacts are objects in social interactions, they have some specific meaning for teachers. As an instance, Liljedahl (2007) used "reification" (Wenger 1998) to explain the meaning that teachers ascribe to a mathematical designed task resulting from a social process of designing, piloting, refining and reflecting. The meaning ascribed to an object is also based on social interaction. In this case, the mutual experience of designing an artefact collectively provides a specific social context from which meaning is made. Additionally, other factors such as a recent curricular reform may impact on the interactions among members of a team of collaborative design, as well as the meaning that people ascribe to objects and collective situations.

3.4 Summary

The theoretical assumptions used for this study were based on symbolic interactionism traditions (Blumer, 1969). The individual's perceptions of the role, position and status of each member in a group of collaborative design were relevant when analysing and generating data during this study, specially in the Lougheed team in which

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I played the double role of a participant and a researcher. The artefacts in collaborative design are objects subjected to individual interpretation even though they are designed collaboratively.

The discussion and rationale for the selection of the methodology in this research was based on three factors: (1) the exploratory nature of the research's purposes, (2) the use of as few assumptions as possible, and (3) the available sources of data. Under these factors, I found that the best approach was grounded theory, as described by Corbin and Strauss (2008), in a more generic sense. The purpose of the use of grounded theory in this study is not the generation of a predictive theory, but rather the development of a characterizations of interactions among participants of teachers' collaborative design.

The emergent nature of this research is reflected in the research questions stated in Chapter 2. While the first research questions focus on a single case, the Lougheed team, the second and third research questions sprang from the ongoing analysis of the first questions. The data used for this study were generated from different sources. The pros and cons of each source of data is discussed in the following chapter which focuses on the methods used for the study.

Chapter 4 Method

This research study, devoted to categorizing the participants' interactions in teams of collaborative design, was conducted in three main stages (see Table 4.1). In the first stage I focused on the study of one case, the Louegheed team. In the second and third stages I explored other cases of collaborative design based on the findings from the first stage. My focus during the first stage was on the type of actions and interactions among the members of only one team. As a result of the analysis of this case I developed: (1) a categorization of the conversations among members while designing artefacts, and (2) a description of the roles that members of the team played during collaborative design.

Table 4.1: Data Description by Stage in the Study

	RQ #1	RQ #2 & RQ #3	
	First Stage	Second Stage	Third Stage
Cases	(1) The Lougheed team	(1) The professional Independent programme (2) The school district initiated (3) The independent lesson study group	(1) The lower grade group (2) The Madrid group (3) The VITAL project
Sources of Data	Video and audio recordings Interviews Field notes	Interviews A conversation Field notes	Literature

As described in Chapter 2, the outcomes of the first stage of the research shaped the elaboration of the further research questions, as well as the subsequent generation and analysis of data in the second and third stages. In order to extend, verify and refine, the results of the first stage I analysed other instances of collaborative design. In the second stage I looked at three cases of collaborative design contacting and interviewing eight participants in total. In the third stage I considered three cases reported in the

literature. Each stage of the research used different sources of data. As discussed in Chapter 3, the differences in the data sources, my accessibility to them, and the emergent nature of the research were decisive factors in the selection of a grounded theory approach for this study (Charmaz, 2006). This chapter contains the description of the data and methods of analysis for each stage of the research.

4.1 First Stage: The Lougheed Team

The initial step of the first stage of the study was to find a team of teachers eager to participate in my research. From my experience conducting collaborative design, I was aware that getting teachers to meet outside school and after hours is challenging. Forming teams of teachers from the same school and working within their job schedule—or at least taking as few after hours as possible—is a strategy that I had used before (Preciado & Liljedahl, 2008) which facilitates collaborative design. So, I looked for a team of teachers from the same school. I feel myself lucky to have met Sofia⁵ in the lesson study group at the SIGMA institute. She was the liaison for other teachers at Lougheed Secondary School. After visiting her at this school and talking with other fellow teachers, two more people agreed to participate in a lesson study⁶ project, Arnold and Brad, and we formed a team of collaborative design (Table 4.2). I will refer to this project and this team as the Lougheed project and the Lougheed team, respectively.

Some visits to Lougheed Secondary School had to be made before starting the project. During these visits, I invited teachers to join the project, obtained permission from the school district, and organized—with the volunteer teachers—sessions for the collaborative design. Finally, we decided to meet once a week and design one mathematical lesson in a first round of collaborative design, from September to December 2008. A second round was held from January to April 2009 designing one mathematical lesson and one assessment rubric. All the meetings were held in Arnold's classroom at 7:00 AM, before students arrived at school. As per Lesson Study (Fernandez & Yoshida, 2004), in each round we defined the goals for each artefact—the

⁵ In order to assure the participants' confidentiality, I use pseudonyms instead of original names of teachers and school.

⁶ Strictly speaking the way we worked was inspired by lesson study. We missed video recording of the implemented lessons and second open implementation.

lessons and the rubric—we designed them, implemented them, and conducted a debriefing thereafter.

Table 4.2: Participant Teachers' Backgrounds

	Arnold	Brad	Sofia
Years teaching mathematics	7	15	6
Years teaching	7	20	6
Also teaching	Chemistry and Science	Physics and Science	-
Undergraduate Specialization	Ecology	Physics	Mathematics and Child studies

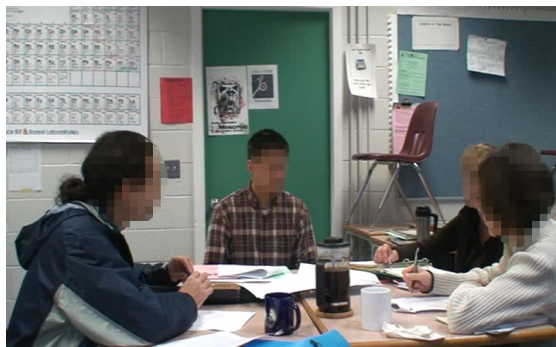
During the Lougheed project I played the double role of participant and researcher. This double role entailed some tensions between my interest as a member of a team of collaborative design and my interest as a researcher looking at teachers' interactions during the project. For instance, in one meeting teachers engaged in a discussion related to the purpose of mathematical instruction at the high school level. During this conversation teachers expressed their beliefs about mathematics and its use in society: something that was interesting to me as a researcher on mathematics teacher education. However, this discussion deviated from the original purpose of designing a mathematical lesson. I tried to balance these two, sometimes competing, interests: the research on teachers' interactions during collaborative design, and the opportunity to listen to teachers' beliefs about mathematics. Fortunately, having several sessions for the designing of the artefacts allowed room for some off-track discussions.

The Lougheed project had specific circumstances which differed from the other cases I analysed in the second and third stages of the research. First, participant teachers knew that this was a research project that focused on teachers. Had this project been conducted without a research purpose, or without a research focus on teachers, teachers may have behaved differently. Second, as a part of the team, I was completely involved in the design process. Thus, I had first hand information about this process. Third, I was an outsider to the community of teachers at Lougheed Secondary School: the discussions and the actions held during the project might be different if the

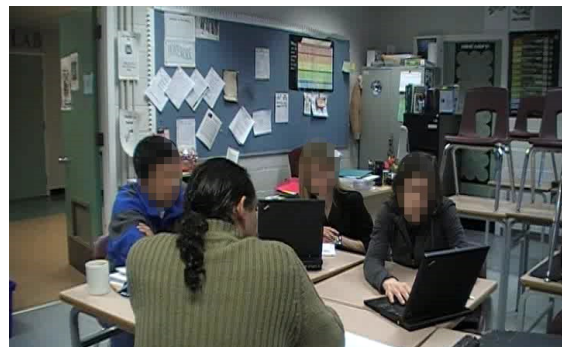
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team were formed only by teachers. And fourthly, collaborative design was not a part of teachers' duties at Lougheed Secondary School; the three participant teachers had not conducted collaborative design at that school before the project. Under these circumstances I had, on the one hand, the advantage of being involved directly with the team I was studying, and on the other hand, the limitation of a non-natural situation: teachers at Lougheed conducting collaborative design. I was able to generate detailed data of a single case which I created for this study in this stage of the research.

In order to register what went on during the Lougheed project, all the meetings were recorded. The first meeting was recorded in audio, and then, further meetings were recorded in video. The video, as opposed to just audio recording, allowed me to capture the conversations during the meetings, as well as to have an account of gestures or actions which would not be possible to track otherwise. The camera was fixed in such a way that all the team members were visible and no operator was required (Figure 4.1). Curiously, without making any explicit decision, we sat in the same position in every meeting during the project. After a review of the data generated in December 2008—at the middle of the project—I changed the location of the camera in an attempt to better capture every teacher's face. In addition to the video recordings, I took some pictures of written work and documents used in the sessions. I also wrote field notes during the meetings.



First round



Second round

Figure 4.1: Position of the Video Camera During the Lougheed Project

The meetings held during the Lougheed project served different purposes, as described in Table 4.3. The recordings of each meeting were coded as soon as possible, usually the same day of the meeting. This initial coding was an open coding (Glaser & Strauss, 1967; Charmaz, 2006; Corbin & Strauss, 2008) that represented my first approach to organize and analyse the data.

Table 4.3: Lougheed Team's Meetings

First round September to December 2008	Second round January to April 2009
9 meetings total 5 for designing a lesson 1 for debriefing the lesson 1 for a group interview 2 planning next round	9 meetings total 6 for designing a lesson 1 general discussion about assessment 1 for debriefing the lesson 1 for a group interview

Each recording was split into small segments giving each one a code that included a description of what was happening therein, and registering the position in time into the recording. Codes were tentative and evolved after further data analysis (Charmaz, 2006). The descriptions of the segments included the transcriptions of some conversations. In some cases a *memo* (Charmaz, 2006) was added to the register. This coding method allowed me to have further access to a specific segment of the recording, if necessary.

The coding was conducted using the spreadsheet of the StarOffice software (see Figure 4.2). This allowed me not only to register all the information, but also to compare instances of the same code along all the data. With the filter tool of the spreadsheet I could see all the instances with the same code or similar description.

During this open coding, I described all the moments in the recordings—e.g. the parts where we did not start the meeting yet. Some of these moments included the sharing of some teaching material or discussions about teaching some particular mathematical content. I was specially interested in such moments because they represented an occasion for teachers' learning. Figure 4.2 shows an example of this open coding during the first 15 minutes of the meeting held in October 14, 2008.

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DATE	CODE	DESCRIPTION	POSITION
2008_10_14	Sharing square numbers	Arnold shares activities about square numbers.	00:00:00
2008_10_14	Books	Brad is reviewing books: He comments on attracting titles in books.	00:02:10
2008_10_14	Food Preference	Sofia talks about vegetarian food	00:03:40
2008_10_14	Meeting for work	Teachers meet on Friday and held a discussion about the lessons and Arnold's schedule, the sequence in the unit of the book, and the introduction of using variables and problem solving	00:04:50
2008_10_14	Coffee talk	Brad talks about coffee.	00:07:10
2008_10_14	Reviewing curriculum	Armando reviews curriculum and discussed it with Arnold's course plan, which is the same as the book. Arnold uses a district format, which follows textbook units.	00:07:53
2008_10_14	Scheduling according to textbook	The team has suggested to use the cube problem at the end of the unit, and the other lesson in the middle.	00:11:00
2008_10_14	Students possible approachers	Sofia redirects the discussion to what students have to do first. How students will solve the problems. Arnold looks for necessary skills.	00:12:45
2008_10_14	Teaching strategy	Brad reflects on which methodology we want to use as teachers: 'discovery' versus 'guided discovery.'	00:15:50

Figure 4.2: Example of Registers of the Initial Open Coding

The initial open coding conducted on the recordings provided a direction for further generation of data and afforded the emergence of one of the themes I focused on in the research. According to grounded theory methodology the data collection, or generation, in subsequent steps of the research is based in a large part on the analysis of the data generated in the previous steps when the main themes emerged—theoretical sampling (Charmaz, 2006; Corbin & Strauss, 2008; Glaser & Strauss, 1967). My first attempt to select a theme to focus on was based on the review of the open coding up to the implementation of the first lesson. At that moment I identified from the data instances where teachers used theoretical statements regarding students' learning processes. Those theoretical statements were interesting for me because they reflected teachers' beliefs and sources of knowledge and I considered them as a possible theme to explore in the research. After the implementation of the first lesson I decided to conduct the first group interview showing to the teachers some segments of the video recordings and my interpretations of how they were using theoretical statements. At that moment I also thought about participants' roles as a possible theme and decided to explore this aspect in the first group interview: participants of the Lougheed team wrote a description of the role of each member and we took turns explaining those roles. However, before

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conducting this activity during the interview Arnold commented on my role as researcher in the team: I, as a researcher, could not pretend to be as another teacher in the team, I did not belong to the community of teachers at the Lougheed Secondary School. This episode was the key moment when I decided to focus on the roles of participants in collaborative design as a main theme.

For the second round of the project I kept conducting the open initial coding to the recordings of all the meetings. At the end of the project I conducted a second group interview, where general aspects of the process were explored (Table 5.3). Having coded all of the recordings in this way, I had a general description of the whole project. At this moment I decided to concentrate in the two emerging themes: (1) the focus of the conversation, and (2) the roles of the participants. The analysis of the conversation included the actions and conversations held during the meetings. These two themes seemed to me equally important and I chose them as core categories (Corbin & Strauss, 2008; Creswell, 2008; Glaser 1978,). Another important decision made at this time was limiting the study to the design process. The selection of goals and the debriefing of the implementation were important and also provided evidence of teachers' learning. However, the whole process of design was rich in such evidence. Additionally, this single process was so complex that I found it worth focusing on it exclusively. A study of the negotiation of goals and the debriefing of the implementation was beyond the scope of this study, but promises to be fruitful in future analysis. Once I decided to focus on these two emerging themes, I re-analysed the data—*focused coding* (Charmaz, 2006; Corbin & Strauss, 2008; Glaser & Strauss, 1967).

In order to focus on the interactions during the design process of the artefacts in the Lougheed project, I decided to code again those recordings of the meetings designated for this purpose (see Table 4.3). The recordings of the meetings in the first round were re-coded several times comparing and redefining segments, splitting and collapsing categories—called the *constant comparative analysis* (Charmaz, 2006; Corbin & Strauss, 2008; Glaser & Strauss, 1967). Several aspects were considered at the same time during further coding. These aspects were recorded in the spreadsheet in additional columns: who speaks, to whom, about what, topic, properties of the code. Knowing who spoke to whom would provide some insight about the position (Langenhove & Harré,

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1999) of the speaker within the conversation. Verbs in gerund form were used to identified actions, as suggested by Charmaz (2006). The use of the filter tool of the spreadsheet allowed a constant comparison across all codes and instances (see Figure 4.3). An important distinction of the segments was whether the conversations and activities were focused on the designing of the artefacts or had deviated to something else. This distinction of the conversations and actions was classified initially as *lesson* and *else*, and changed later to *on-task and off-task*, respectively. Additionally, sometimes the team was talking about something that happened during the week, when I was not present in the school, as a consequence of participating in the project. I used the words *inside* and *outside* to distinguish those moments. A detailed description of this classification is presented in Chapter 6.

New code	Property	CATEGOR	SPEAKE	ACTION	WHOM	TOPIC	DESCRIPTION	POSITIC	MEMOS
anticipating	low expectation	else outside	Brad	anticipating		students	I don't know if this gays will ever get the algebraic expression"	00:24:50	
anticipating	good expectation	else outside	Sofia	anticipating	Brad	students	Sofia responds to Brad: "I'm sure they can. They are doing in elementary school."	00:24:56	Originally this was "else inside." However, Sofia is commenting on Brad's conversation which is outside.
choosing problems	suggesting to increase the difficult of the problems	lesson inside	Armando	proposing		problem	Armando mentions to recall the use of patterns to students. We can increase the level of the lesson. Mixed thing in order to may be the formula come from two or three linear relations.	00:25:00	
anticipating	Students harder problems for the "speedies"	lesson inside	Sofia Armando	agreeing		problem	Sofia Agrees with Armando suggestions.	00:26:41	

Figure 4.3: Example of the Coding Registers in Further Data Analysis

Once the meetings dedicated to the designing of the the lesson in the first round were classified, the on-task and off-task parts of the recordings were re-coded and analysed separately. The results of such analysis consisted of a categorization for each part. Such results are elaborated in Chapter 6. For the on-task part of the recordings, the re-coding gave as a result a list of eleven new codes, summarized in Table A.1 in Appendix A. There were some relationships among these codes which suggested a complex set of codes rather than unrelated topics that can be separated in the analysis of the conversation. Finally, I developed a framework that comprises all of these new codes, which I called the *design braid*. At this moment of the research I started synthesizing the data into categories and subcategories identifying relationships among

them. This process is called *axial coding* (Charmaz, 2006; Corbin & Strauss, 2008). The developed categories are considered as temporal and may change after subsequent coding, or analysis of further data. Axial coding and open coding usually take place simultaneously; however, the result of the axial coding is often presented graphically (Charmaz, 2006). Axial coding took place in this study at different moments at which categories and themes, and the relationships among them, were developed. The results of axial coding are commonly represented with diagrams in this study.

The off-task part of the meetings during the first round of the project were re-coded giving also a new set of codes described in Table A.2 in Appendix A. In contrast to the on-task part of the meetings where I developed interwoven categories, in the off-task part of the conversation the generated categories were less interrelated. The new codes used in order to describe the conversation were synthesized into four broader categories.

The resulting categorizations of the on-task and off-task parts of the recordings were used to code the design process in the second round of the project. In this way more instances of the generated codes were identified and compared against those in the first stage. The initial categorization was enough to code all of the moments in this second part. However, a few new properties were added to the developed categories.

Having developed the categorization for the conversations during the process of collaborative design, and keeping my interest in exploring the roles of the participants in the team, I decided to meet the teachers again; this time individual interviews were conducted with each teacher. In these interviews (Table 5.4) participants described their perceptions about the whole process of the project, the goals, the activities, and the relevant moments. They were asked to report changes in their teaching practice as a result of participating in the project. All the interviews were transcribed and coded in a similar way as the open coding of the recordings of the meetings.

The interviews represented data which differed in nature from the recordings of the sessions dedicated to designing the artefacts. As in any interview, I had to consider that teachers have varied perspectives and interests. However, what they said could be compared with the data from the meetings. This provided a way of validating some of my

findings and, at the same time, discovering nuances in teachers' interests and perspectives during the project. As an additional means of verifying my findings, I tried to contact participant teachers during the final part of this research. They were asked to read my findings of the project, which are presented in Chapters 5 and 6. Two of the three teachers responded mentioning that they agreed with the descriptions of the events presented in this study, and the corresponding findings made sense to them. I could not get in contact with the third teacher.

In summary, the interactions among the members of the Lougheed team were categorized with respect to the two emerging themes. The analysis of the meetings for collaborative design was used mainly to develop the characterization of the focus of the conversation, whereas the analysis of the interviews was used to both verify some of the findings from the design of the artefacts and explore the perception of the roles of each member in the team. Once I had explored the interactions of participants in the Lougheed project, it was time to look at other cases of collaborative design.

4.2 Second Stage: Other Cases of Collaborative Design

The categorization developed in the first stage of the research served to describe participants' interactions in the Lougheed project. However, in order to develop a conceptualization of collaborative design with more generality, it was important to look at other cases. In this way, common features and domains of variability would be identified. In the second stage of this study, other three cases of collaborative design were analysed. These cases were different from each other due to the settings in which collaborative design was conducted.

One case is an independent Lesson Study group at the SIGMA institute. This group met at least six times a year in a general meeting where teachers from a variety of schools and educational institutes gathered together for three hours. In this group, small teams were formed in order to design and implement a mathematical lesson based on the Japanese Lesson Study model (Fernandez & Yoshida, 2004; Stigler & Hiebert, 1999). I had participated in this group at the general meetings and designing lessons with some teams. This situation allowed me to contact people who have conducted

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collaborative design where I was not a team member—as opposed to the Lougheed project. From this case I interviewed one instructor, one organiser, who is also a mathematics teacher in secondary school, and one prospective mathematics teacher.

Another two cases of collaborative design studied in this research were identified from the workshops and programmes aimed at teacher professional development that the Faculty of Education of Simon Fraser University offers to practising mathematics teachers: (1) an initiative of implementing collaborative work among teachers from a school district, and (2) workshops and programmes that include collaborative design. From the school district initiative I interviewed a professional development coordinator who was in charge of organizing the collaborative work in the district, and two teachers participating in this initiative. In the case of the workshops and professional development programmes, I interviewed the professor who has been the instructor for these programmes and two teachers who had partaken in some of the workshops.

The interviews in this second stage of the research were audio recorded, or conducted by e-mail. Transcripts of these interviews were coded at least twice: (1) open coding, and (2) focused coding based on the categorization developed during the first stage of the research. The main themes developed in the first stage were a focus during these coding processes. The topics and categories were compared against the Lougheed team. Chapter 7 contains the details of these cases, as well as the findings in this stage of the study.

The data generated in the cases previously described were based only on the interviews. I did not have the chance, as it was the case of the Lougheed team, to look closely at the design process. However, from these interviews I realised that collaborative design could be conducted in very different ways from the lesson study projects I had participated in before. Then, I was able to compare different settings as well as the way that participants engaged in the process. Particularly, new roles, compared to those found in the first stage, played by participants of collaborative design were found expanding my findings about this theme. In contrast to the first stage of the research, I did not create the situations for the collaborative design, on the one hand, and I had no direct access to the interactions among participants during the designing of

the artefacts, on the other hand. In addition to these aforementioned cases, I used the literature as a source of data for the third stage of the study.

4.3 Third stage: Literature as Second-Hand Data

The second and third stages of the research were conducted simultaneously. In this third stage I compared cases reported in the literature with the Lougheed project. This comparison allowed me to identify common features across several instances of collaborative design using the literature as second-hand data. I found three relevant cases: (1) the lower grade group, a lesson study case conducted in Japan and reported by Fernandez and Yoshida (2004); (2) the Madrid group, a case of supported teacher collaborative inquiry reported by Nelson and Slavit (2010); and (3) the Variation for the Improvement of Teaching and Learning (VITAL) project, a three-year large-scale project conducted in Hong Kong (Elliot & Yu, 2008). The referenced literature for these three cases include transcriptions of interviews and segments of the meetings for the designing of the artefacts. For this reason, I decided to integrate these pieces of literature as data. The coding in these cases was different than the coding of the previous data. Instead of using the computer during the coding process, I used sticky notes to codify segments of the texts (books and article) and pencil to write some notes in the margin. The coding for the second-hand data was focused on topics already identified in the generated data and served as a means to verify and expand my findings.

The literature used in this study as second-hand data served to support, modified and expanded the developed characterization from the first stage. These data were generated with a purpose which differs from my goals in this research. However, I was able to make comparisons of both the focus of the conversation and the different roles played in other instances of collaborative design. At the end of the second and third stages of the research, categories of the two main themes of the research were revisited, expanded, and modified in order to generate a characterization of the interactions among participants in collaborative design.

4.4 Landscape of the Research

In this chapter, I have described the methodology for the three stages of this research. Whereas in the first stage a detailed analysis of a single case was conducted, including recordings of the meetings for collaborative design and interviews, in the second stage another three cases were analysed by means of interviewing engaged participants. The role of the literature was also important, three pieces of literature on mathematics education were used as second-hand data in third stage of the research in order to compare, contrast, and amplifying my understanding of the interactions among participants of collaborative design. Two emerging themes were developed in the first stage of the study: (1) the focus of the conversation during the process of collaborative design, and (2) the roles played by the people involved each case. These themes were the focus of the study in the cases analysed in the second and third stages.

The first stage of this study is elaborated in the next two chapters: Whereas Chapter 5 presents parts of the data generated from the Lougheed project, Chapter 6 describes the analysis and its resulting categorizations. Chapter 7 contains descriptions of the other three cases of collaborative design studied in the second stage of the research. The cases analysed from the literature in the third stage are described in Chapter 8. The second and third stages of the study were conducted during the same period of time—2009 to 2010. The findings across all the cases studied in this research are synthesized in Chapter 9. Finally, Chapter 10 contains the conclusions of this study.

Chapter 5 The Lougheed Project

Interested in characterizing teachers' interactions while participating in collaborative design, I started this research study analysing one case, the Lougheed project. As a result of the analysis of this case, two themes emerged as characterizations for interactions in teachers' collaborative design: (1) the focus of the conversations during the design process, and (2) the roles that participants played during this process. As explained in the previous chapters, this research is divided into three stages. Whereas the first stage consisted of categorizing participants' interactions in the Lougheed project, in the second and third stage I explored other cases of collaborative design through such characterization. This chapter presents part of the data generated for this study during the first stage.

The Lougheed project consisted of two rounds of collaborative design: the first taking place in the Fall of 2008 and the second in the Spring of 2009. The analysis of the first round of the project and the first group interview played an important role in the research. The categorization developed in this stage of the research arose mainly from the first round of the project—the second round extended properties of the categories already identified. The first group interview, conducted at the end of the first round, proved a key moment for the research, after which I decided to focus on the roles of the team members. This chapter contains accounts of this project with an emphasis on both the design process during the first round and the first group interview.

The purpose of this chapter is to present a view of the whole design process during the first round of the Lougheed project in order to have a sense of the chronological path of events. The excerpts included in this chapter are representative instances of the characterization of participants' interactions developed during this stage of the research. This characterization is elaborated in the next chapter based mainly on the data presented in the following sections.

When referring to myself in the recordings of the Lougheed project, I used my first name Armando. The reason for this distinction is to differentiate between myself as a researcher analysing the data of the project and writing this document, and myself as a member of the Lougheed team who appears in the recordings. My intention is not to present the "I" and "Armando" as two different persons, but rather to refer to myself at two different moments in time. Accordingly, I use the third person to refer to Armando.

5.1 The Team and the Project

The Lougheed team was made up of three mathematics teachers and myself, Armando. These teachers, Arnold, Brad and Sofia⁷, had full-time teaching positions at Lougheed secondary school at the time of the project. Teachers' background is summarized in Table 4.2. Brad, the only male teacher in the team, had a considerable number of teaching-experience years compared with both Sofia, who had only taught mathematics for six years, and Arnold, who had taught mathematics and science for seven years. In contrast to Arnold and Brad, who both had a background in science, Sofia had received professional education in mathematics and child development. Sofia was acknowledged as more informed about current mathematics education issues than Arnold and Brad. She also had had previous experience of collaborative design with the lesson study group⁸ which met at the SIGMA institute. During the project, Armando not only was a researcher and facilitator, but also a member of the team partaking in the designing of the artefacts. For this reason he was considered as a participant-researcher inside the team.

One main obstacle at the beginning of the project was to find the time and a place for meetings. However, after having discussed the benefits of collaborative design for teachers' practice, the participants concluded that planning was already something they had to do at school: meeting to design, implement and debrief mathematical lessons might help in their planning. So, the team decided to hold weekly meetings in

⁷ For the Lougheed project, teachers chose their own pseudonyms as a means of confidentiality. The only male teacher chose Brad as a pseudonym and one female teacher, in order to have some connection with notorious Hollywood actors, chose to be Arnold, regardless of the male gender of this pseudonym. The third teacher was Sofia, the younger female teacher.

⁸ This group is the independent lesson study group studied in Chapter 7.

Arnold's room from 7:00 AM to 8:00 AM—when the teachers were already at school, but had not started teaching yet.

Table 5.1: Differences between the Two Rounds of the Lougheed Project.

First round September to December 2008	Second round January to April 2009
Focus on one lesson.	Focus on whole unit, assessment, as well as one lesson.
One lesson was planned	Plan of one lesson, and an assessment rubric were planned.
Grade 9.	Grade 8.
Goal: to get students to translate from verbal to algebraic expressions	Goals: to understand that ratios and proportions involve multiplicative rather than additive comparisons; to have several strategies to solve proportions; to recognize proportional relationships in a variety of situations; and to understand part-part and part-whole relationships
Language: English.	Language: French.
Teacher: Arnold.	Teacher: Sofia.
Students presentation of their solutions in teams.	Teacher presented students' work—posters—on the board, in order to analyse and compare different solutions.
Five meetings to design the lesson	Six meetings to design the lesson and the assessment rubric.

During this project, two rounds of lesson design and implementation were conducted, sharing the following similarities: (1) problem solving was used as a learning strategy, and (2) students were provided with worksheets in the designed lessons, worked in teams, and presented their work to the class. However, there were important differences also, as summarized in Table 5.1. In contrast to the first round, which focused on only one lesson, in the second round the team decided to focus on the whole unit and included the design of an assessment rubric. The grade levels and the language for the lessons in each round were different; the first was a grade nine mathematics lesson taught in English by Arnold and the second was a grade eight mathematics lesson taught in the French immersion programme by Sofia. Although in both lessons students' work was presented to the class, the format for the presentations

differed. In the first round, students presented their work in teams in front of the group, whereas in the second round the teachers presented and compared students' solutions on the board.

Another important difference between the two rounds was the specific goals that the team chose to focus on. In the first round, the team focused on the particular issue of getting students to translate from verbal to algebraic expressions. This was suggested by Brad and the team found it worth pursuing such a goal. For the second round, the goals were varied and included: having several strategies to solve problems, and recognising proportional relationships in a variety of contexts. These goals were suggested by Sofia and discussed before choosing them.

Based on the experience from the first round, the team planned the second round of the project. The process of designing the lesson during the first round involved recurrent discussions about the problems to be used in the lesson, the way they would fit into Arnold's course, and how the lesson would contribute to the specified goal. As a consequence, a broader approach was taken by the team during the second round focussing not only on one lesson, but also on the whole unit and an assessment instrument.

Despite the differences of the two rounds, after the recurrent coding processes I found that it was possible to state the results of the analysis using mainly the first round, because such results were consistent with the analysis in the second round. For this reason, I decided to provide accounts of the design process during the first round only. The following section provides these accounts.

5.2 The Designing of a Lesson

In this section, I present a chronological account of the first round of the Lougheed project as a means to describe two processes: (1) the design of a lesson by the team, and (2) the emergence of themes for this research. This account includes some transcripts and descriptions of relevant events which provide a background for the analysis presented in Chapter 6. This section emphasizes the design process that took place during this first round: accounts of the debriefing meeting after the implementation

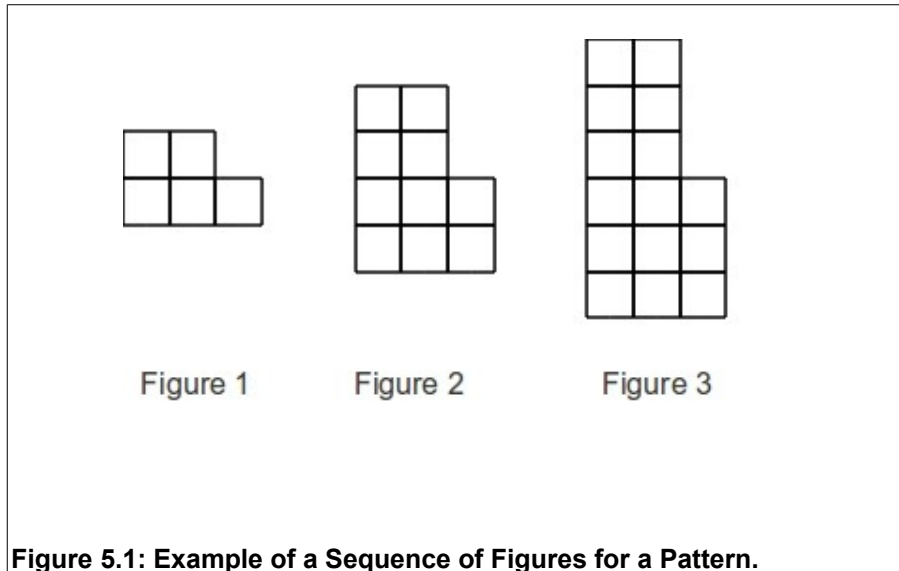
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of the designed lesson, as well as details of the discussions held in order to plan the second round, are omitted.

The goals for the first round were already selected during the first visits to Loughheed school when I invited teachers to participate in the project and the time and place for the meetings were negotiated. These first visits did not form part of the data because ethic permission forms were not already signed by participants and school board at that time. Therefore, when the first recorded meeting for the designing of a lesson took place, the team had already selected a goal for the lesson and had started to think about problems that could be included in this lesson.

September 23rd

It took some time to get focused on the designing of the lesson during the first meeting: Arnold was interested in learning more about lesson study and asked for the reference of the book *The Teaching Gap* by Stigler and Hiebert (1999). Once the team focused on the design of the lesson, Brad started updating Armando about the problems and the approaches teachers were thinking about for the lesson before this meeting. As the goal for this lesson was already selected, teachers started to think about the lesson in advance. Brad also commented on how such discussions would be useful for Armando's research. Then, he asked a question unrelated to the design of the lesson: "What is the difference between the words 'math' and 'mathematics'?" A short exchange about the difference in these words in some countries took place. After some minutes, Sofia redirected the discussion towards designing the lesson. However, Arnold showed an article stressing the need for getting students prepared for the topics teachers will teach, and Sofia responded with a criticism about the article's perspective on assessment.



Thereafter, the team focused on the design of the lesson and some mathematical problems were analysed. Sofia commented on the problems they were discussing during the week before this meeting. After looking at some problems in a textbook and discussing which ones could be used for the lesson, the team started to discuss students' possible approaches. The problems involved the recognition of patterns in the number of objects needed to form a series of figures (e.g., Figure 5.1). The following transcript provides an instance of these discussions.

Brad: What kind of ideas can we expect kids to come up with in place of $5n$ or $3n+2n$? What might they say is the pattern?

Sofia: Well, I would say that they will start with the recursive thing. ...

Possible complications that students may have to contend with when solving the problems were discussed. In the next excerpt, Sofia remarked upon the difficulty of the recursive notation for a formula when using patterns.

Sofia: The recursive formula is easy, but I find the notation—even writing the recursive formula—is really difficult for students. But then it is really difficult to come up with explicit formulas for these [sequences].

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Difficulties for specific students were mentioned as well. Arnold was concerned about the non-algebraic thinking students and was selecting problems that would allow them to start working without algebra.

Arnold: I selected that one cause I thought: well, it will allow all the non-algebraic thinkers to participate with good house-keeping skills.

The discussion also included the need to have students prepared for the type of mathematical tasks for this lesson. Sofia mentioned that teachers were discussing, during the week before this meeting, the use of some problems as preparation for the lesson.

Sofia: We wanted to be sure that they [students] know what we expect them to do for this pattern. So, we thought, we can give them an example, or doing an example briefly of a simple pattern.

Arnold added to Sofia's comment by making reference to the textbook from which they were selecting the problems. She was interested in teaching students a particular set of skills mentioned in the book.

Arnold: I like this particular book because it says what you are suggesting. Like here are some strategies that good problems solvers will do... I want to teach that particular skill set before I get there.

Brad proposed to introduce the use of patterns in each of their classrooms in order to see if students would figure out some easy cases. However, he was uncertain about whether students would come up with an algebraic representation for the numbers involved in the patterns. In contrast to Brad, Sofia had higher expectations for how students would perform while working with the patterns.

Brad: I don't know if these guys will ever get the algebraic expression.

Sofia: I'm sure they can. They were doing it in elementary school.

In the last part of the meeting, the conversation deviated from designing the lesson. Arnold commented that being observed while teaching would represent an opportunity for receiving feedback from other teachers. Then, the conversation changed to school and curriculum issues and the team kept away from the task of designing the lesson up to the end of the meeting (approximately 20 minutes). This included a long discussion of the difficulties completing the courses according to the curriculum. Discussion of general issues in the learning of mathematics were addressed, such as

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the importance of affect in mathematics learning, and teachers' expectation for students. The use of mathematics outside school, including the mathematical requirements for entry to some university programmes, was also debated by the team.

September 30th

Sofia had not yet arrived at the meeting when the other two teachers started talking about the use of pictorial representations—images of apples and oranges—for addition and subtraction in a book that Arnold brought that morning. Brad was interested in the use of pictorial representations of the product, and the square, of different quantities.

Brad: Does it represent a square quantity? Apple squared minus orange squared....

I'm not a mathematician by training, as you know. Is there an analogy for that expression: a squared minus b squared? ...

When you look at this, ... this is a pictogram ... Apple apple minus orange orange equals... I don't know. That makes more sense than apple squared.

Brad was concerned with a pictorial representation for the square of a number, because it made no sense to him using the same representation for addition and subtraction as it was presented in Arnold's book. The conversation changed suddenly when Arnold remembered a game, the 'animal game,' which can be used by students in order to memorize facts by means of repetition of words or sentences.

Sofia arrived putting a coffee jug in the middle of the table. After a few minutes of casual conversation, a summary of the research project and the previous meeting followed. Possible dates for the implementation of the lesson were discussed. Arnold followed the textbook's order of contents for her course, which was important for her in deciding the best date for the implementation of the lesson. Although a goal for the lesson was already set, and this goal was relevant to the course, it was difficult to fit the lesson into Arnold's class; however, no decision for the date of the implementation was made at this moment.

The team decided to proceed with the selection of the problem for the lesson. Arnold proposed a problem from a book. She had been concerned with scaffolding

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students in the lesson and anticipated that some students would not use a table to represent their data. Brad pointed out a page from the textbook with some tables and shapes for problems involving patterns.

Brad: If you haven't worked on this page on functions, there are some patterns, tables, and shapes that might help them to get started, even for the weaker kids.

Brad also stressed that students do better if they are exposed to familiar material in the textbook they use for the course. In order to have students working with familiar material, the team considered including problems from the textbook, or at least problems with a similar format.

As the team planned to observe the implementation of the lesson in Arnold's classroom, the role of the observers during this implementation was also discussed: Are observers limited to watching or can they interact with students? The team decided that observers should have a chance to interact with students, to clarify possible questions. The discussion then focused on the selection of a problem for the lesson and Sofia proposed the '**cube**' problem.

Sofia: If you have a cube, let's say we start with 3 by 3. Then paint the exterior surface, so then we can ask them the question how many of the [unitary] cubes in this 3 by 3 cube have three sides painted?, and the answer is always 8. ... And then two is linear, and then the next one is one face painted.

The team found the problem very interesting and started discussing how to introduce it with the aid of visual representations, such as a Rubik's cube. The conversation then focused on analysing how to pose the question, as well as possible students' approaches and potential struggles. Students' knowledge required for this problem was also a concern for the team—such as the idea of volume and area—as well as getting students used to working with patterns. However, Brad was not sure that the task would serve to achieve the original goal stated for the first round of the project.

Brad: I don't think we can get to this yet [to have students writing algebraic expressions from word problems].

Sofia: I think that our goal for this lesson was to get students to describing their understanding ... Once they describe it in words, I

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don't think it is a huge deal to describe it with an algebraic expression.

Pointing at the textbook, Brad commented on some problems, as well as the use of variables in some exercises. This comment triggered a discussion about the difference between variables and unknowns. Sofia explained the possible use of letters in algebra: (1) as variables defining a function, and (2) as unknowns for an equation. The page that Brad was pointing to used letters as unknowns, whereas the team had been using variables in order to describe functions for the general term for the sequences—particularly for the cube problem. The discussion of whether the use of patterns will help to reach the lesson's goal—to get students to translate from verbal to algebraic expressions—continued.

Sofia: You don't think that what we are doing with patterns addresses this difficulty?

Brad: No, I don't think so. What we were doing is just developing their own expressions, their own solutions. I don't think we addressed how this is working [pointing out to the page of the textbook referring to translating words into algebraic expressions].

Armando: I think we must conclude this lesson ... with some algebraic expression, but the idea is that this algebraic expression should come from the students' words.

Brad was concerned with covering the topics in the curriculum, which were the same as in the textbook, and how to relate them to the use of patterns in the lesson. The team decided to keep the cube problem as a task for the lesson. Thoughts about using this problem would be shared by e-mail before the team met again.

October 7th

After Arnold showed, at the beginning of the meeting, a book that approaches the problem of language in mathematics, Brad mentioned they had had a meeting the day before and Sofia commented on their work. Two lessons were being considered now: one in which the cube problem would be used, and a prior lesson to prepare students for this problem.

Sofia: Yesterday we looked at the problem of the cubes, and looking at how we'll introduce it. So the previous lesson, we thought, we can look at a few patterns without necessarily coming up with the

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formula, but looking at the sequence of triangular numbers ... square numbers, cube numbers. But, just so that they will be better trained to recognise the square numbers ... in the next lesson.

A draft of a handout was already designed with some examples of sequences and some questions. Armando proposed a further question that asked students for a more efficient way of finding the elements of the sequences.

In order to pilot some of the problems that the team was thinking to use for the lesson, Arnold asked her son to solve one of them.

Arnold: I tried with my son yesterday, and he did this,... but he did it in like about two minutes. ... He did it quite differently. I just was curious what the kid was thinking, was watching and breaking them up visually.

Brad thought that some students may not start reading the problem right away and would wait for the teacher to explain the instructions, and proposed to give time for students to do so.

Brad: I have students that can't follow written instructions... . I think they can, but just ... prefer not to, because they think: OK, you are going to explain it to me anyways, why should I follow them? So that may take a little longer for those students ... So, may be 15 minutes?

As a way of describing the students' level of understanding, Arnold referred to an assessment scale proposed by Marzano (2007, p. 107) consisting of four major levels—with some additional sub-levels in between. The lowest score value on the scale is a 0.0, which represents no knowledge of the topic: even with help, the student demonstrates no understanding or skill relative to the topic. A score of 1.0 indicates that with help the student shows partial knowledge of the simpler details and processes as well as the more complex ideas. At a score of 2.0, the student independently demonstrates understanding of, and skill at, the simpler details and processes, but not of the more complex ideas and processes. A score of 3.0 indicates that the student demonstrates skill and understanding of all the content—simple and complex—that was taught in class. A score of 4.0 indicates that the student demonstrates inferences and applications that go beyond what was taught in class. Arnold pointed to the level one on the rubric as

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a means to indicate that some particular students would need for help in solving the problems the team was proposing for the lesson.

Arnold: In other words, ... students will be in the one category because they will need help, guided help.

The team discussed also how to introduce the problems in the prior lesson considering possible student struggles and approaches, as well as the format of students' work, which would be based on team-work. Arnold suggested a particular way of team-work described in a rubric she already had used for problem solving; however such a rubric did not suit the structure of the lesson as it was already designed.

After discussing the format of the team-work of the students, Brad claimed that he was unsure how the lesson would help to achieve the lesson's goal.

Brad: What I am trying to do is [this]: I'm fitting in what I have to teach in that section, where they translate words into algebra, with the activities here. So, I was thinking: let's say they come up with "there is two more than three times the stage." Well, that is good because then we now can express algebraically two more than three times the stage.

Brad, perhaps talking more to himself than to the team, explained a possible course of action in which students may write algebraic expressions from the problem.

Brad: So, if I would do this in my first class, I would say: "O.K. let's take a look at all your answers down here. Is there a way to simplify that? Is there a way to make it easier to write instead of all the words? You know, it took two lines or three lines to explain your answers. Is there a simpler way, an easier way to do that?" And they will say "let's write that algebraically." Or they may not call it algebraically, "let's write that in an easier way, and see what it looks like."

But one group may say "two more than twice the number" or one group may say "add one to the stage and double it." Well, let's say they are the same, "let's work it out algebraically and see they are the same. Why not use algebra instead of all these words?"

After Brad's comments on how students would go from verbal to algebraic expressions, the team continued to focus on the first lesson by discussing the number of problems, as well as the difficulty and the order to present them to students. For instance, Brad proposed to start with easier problems.

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Brad: Maybe we can start with something easier, so that they [students] can come up with descriptions that you can easily write algebraic expressions for.

The format of students presenting their solutions was also one part of the discussion. However, while discussing the difficulty of the problems, compared with the cube problem, Sofia wondered whether to keep using it for the lesson. After comparing the context of the course, and the goals of the lesson, some doubts arose.

Sofia: I do think it is a great problem [the cube problem], but I'm not sure that it is what is necessary; what we want for this lesson.

The team decided to keep working on both lessons. The problems already proposed were analysed and other problems were discussed. The date of the implementation was discussed again without making a final decision. Before finishing the meeting the team distributed the labour: Sofia would type the lesson, and Armando would graphically design the figures.

October 14th

A casual conversation was held at the beginning of the meeting: books and food preferences. Then, Sofia started commenting that they met before, when Armando was not present, to talk about the lesson. They were trying to fit the lesson into the unit that Arnold would be teaching at the moment of the implementation coherently, so they were looking at the proper time to deliver the lesson. In the following excerpt, teachers explained what they were doing during when they met before this session.

Arnold: We went through the calendar and decided what works and what is the time amount and the teacher-talk amount. What should the lesson look like?

Sofia: We wanted to look at where this fits into the unit, and where we should go in the sequences of some other things, and we decided that it will go well just at the beginning of the unit, as an introduction to using variables.

While Sofia was pressing the top of the French press coffee maker, Brad changed the focus of the conversation commenting about the difference between Canadian and French preferences of preparing coffee. But then, Armando redirected the conversation by looking at the prescribed learning outcomes from the curriculum and

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how the team was approaching them in the lesson, and how the lesson fitted into the curriculum. The team looked at the school district outline for the course, as well as the textbook chapters.

Sofia: They [students] will do the bulk of this in that lesson [the first lesson], and then the cube problem may be at the end of chapter ten.

Then the conversation focused on mathematical concepts that students should know before the lesson, as well as other patterns Arnold might use in order to get students used to working with patterns in prior lessons. Then, Brad raised a question about the way the lesson would be implemented.

Brad: It's almost that which methodology we want to use to teach this. I mean, we can say OK here we go, discover, or we can say, you know, this is guided discovery. We are going to lead them somehow. So, I think that depends on you [looking at Arnold].

Arnold: I would like to use a method that has been researched and has shown to be effective as opposed to something that I would just come up with it.

A brief conversation about the use of theories in education took place. Armando commented on constructivism and variation theory. Arnold was concerned with the skills that students would need for the tasks of the lesson.

Arnold: I was just thinking that it would be good to decide today what are the necessary things that we would like students to have ...

She also commented on some textbook problems that students usually struggle with in the course. The team analysed these problems and compared them with the sequences proposed for the lesson. The discussion focused then on what students would be required to know prior to the lesson, and when would be the best time to implement the lesson according to the curriculum and Arnold's course. Then, concerns about the use of the cube problem for the lesson arose again.

Sofia: I am just wondering if this fits well into the class... I think it is a good lesson [the one with the cube problem]. ... But I don't think we should do it where it doesn't make sense to do it.

The lesson of the cube problem was dismissed and the team decided to proceed with the other lesson which would include patterns of figures. The conversation focused

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on selecting the problems for the lesson and identifying students' skills and knowledge required for such a lesson. The proper wording of the questions and the presentation of the patterns were discussed.

The team started to talk about a visual method to multiply two numbers, deviating the conversation for a short period of time. Then the team focused again on the design of student worksheets for the lesson.

After Armando asked whether students will present their answers to the whole group, Arnold explained that what students were used to doing in her classroom was to put the work on the board. The team decided to keep working on the student worksheet through the week by exchanging e-mails.

October 21st

Brad was not present at this meeting. The session started by reviewing the work done during the week prior to this meeting. A draft of the worksheet was already designed and the discussion focused on improving the wording of the questions.

Variations of the problems proposed for the lesson in the previous sessions were discussed. Arnold explained that she had been solving the problems and thinking about the space allowed for students' work on the worksheet.

Arnold: I tried to do them [the problems] yesterday, some of them, but I was thinking, we need more space for them [students] to work.

Arnold explained her solution to one of the problems, and Armando showed two other ways to solve the same problem. The team kept generating possible students' approaches and struggles, and figuring out the time the students should spend working on the task. Concerned with the space for students to write on the worksheets, the team decided to include a table as a guide for students to write their findings for the first ten steps in each sequence.

The discussion also focused on the way students would share their solutions with the class. Sofia suggested looking at students' solutions in order to decide who would present to the class.

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Sofia: We could suggest also, ... we might go around and say: "Oh, I like the way you worked that out, could you present that on the board?"

Arnold: Would we use the word *like*, or we just say people would find this useful, like "this is a different approach"?

While Sofia was suggesting a way of selecting students who would present their work to the class, Arnold was concerned with using a better sentence to invite students to present their work.

The team continued working on both the format of the worksheets and the format of the lesson. The discussion included aspects such as: the size of the paper, the boxes containing information, questions and space for working, the table where students would write their steps, and some drawings of the shapes of the sequences. The format for the lesson included a discussion of whether students would solve the first problem and then share ideas with the whole group before going on to the other problems.

After making some changes to the worksheet, the discussion focused on the role of the teacher: What kind of answers and instructions will the teacher give to students?

Arnold: If your goal or your role [as a teacher] is to ... help them ... to go from all of these to an equation. How would you role play that? What would you say? ...

Armando: I would like to see some of the different solutions, and the words for those solutions, and then we might help them to write from the words the [mathematical] expression.

Sofia: And I would say: ... "You have explained how to get the perimeter if you know the stage. So, what if we call the stage....What if we give a name to the stage? ... If we call the stage n , how do we write...?"

Arnold: Yeah, but is it that that question? (using *the stage*?) ... Oh! That's what you would then say if they don't get to this one!

The team kept discussing the role of the teacher by considering possible students' approaches. Arnold was concerned with the use of proper words during the lesson and she triggered a discussion about mathematical terms. She wondered whether the right words were used for both the students' worksheet and the instructions from the teacher. The team discussed the concepts of *functions*, *equations*, *mathematical expressions*, and *inequalities*, as well as whether students would be

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familiar with the word *function*. As a result of this discussion, the team decided to use the word *formula* rather than *function* in the instructions for students in the lesson.

For a few minutes the conversation moved toward the causes of some students' difficulties counting objects or doing mathematics. Arnold mentioned a conversation with Brad, some days before, about whether some student's problems with mathematics arise from mathematical development or the biological process—or situation—that hampers students mathematics learning.

After finishing the discussion of the format of both the lesson and the worksheet, the discussion focused on what to place attention on during the lesson as observers. Arnold gave some information about the lesson such as the room number and time for the lesson.

This was the last meeting for the design of the lesson during the first round of the Lougheed project. The lesson was implemented on October 22nd and debriefed in the next team meeting on October 28th. As already mentioned, the first experience conducting collaborative design in this team informed the planning for the second round. Such planning took part during November and December of 2008.

Final Comments on the Design Process

As mentioned in Chapter 4, I conducted an open coding on all the recorded sessions of the Lougheed project from which the two main themes emerged: the focus of the conversation during the design process, and the roles taken by the team members during this process. As I focused on the design process, I am not providing details of the sessions dedicated to the debriefing of the implementation of the lesson in the first round, nor details of the planning for the second round.

The accounts presented in this section were selected as representative moments of the characterization that I have developed in this research, which is detailed in Chapter 6. In such accounts I stressed several aspects relevant for this characterization. First, there were different topics of the conversations held during the process, changing constantly during each session. The team held, for instance, casual conversation about coffee and food preferences, as well as discussions of the role of mathematics in

society. These conversations deviated from the original task of designing a lesson. However, they might represent occasions for teachers' learning, as well as fostering coherence within the team. Second, when focussing on designing the lesson, the team held conversations that included: students' possible struggles and approaches to the tasks, as well as possible teacher response; difficulty of the mathematical problems, as well as their pertinence to Arnold's course; and verification that the lesson would serve to achieve the selected goal of making students translate from verbal to algebraic expressions. Third, the type of interventions of team members presented certain patterns. Arnold often brought resources such as books, articles, or rubrics to the discussion. She also took special care in choosing the language for the lesson. Brad was concerned with how the lesson would serve to achieve the stated goal. He also asked questions about mathematics, and teaching mathematics, that were not specifically related to the task of designing the lesson. Sofia was a problem poser who suggested the cube problem, as well as other problems for the lesson. Fourth, teachers performed different activities related to the project outside the regular meetings. The conversations held during the recorded sessions included descriptions of these activities. These outside activities might represent a change in teachers' practices, which could be evidence of the impact of collaborative design on the activities within the school.

The focus of the conversation and the roles of the team members during the design process, as emerging themes, were explored in the interviews conducted with the teachers of the Loughed project. The interviews not only provided further insight on the four aspects described in the previous paragraph, but also served to identify individual perceptions of the roles played by each member of the team. The next section is dedicated to presenting the interviews.

5.3 Interviews

Teachers of the Loughed project participated in three interviews: one group interview after the first round of the project, one group interview at the end of the second round, and individual interviews after the end of the project. The first group interview represented an important moment of the research because it triggered my focus on the

roles that participants played during the project. Although I already was interested in exploring these roles when conducting this interview, I had not considered the influence of my position as a researcher inside the team before. The second group interview and the final individual interviews supported findings obtained from the recordings; however, still more aspects related to teachers' interests and roles in the project emerged out of these interviews. The following subsections present relevant aspects of these interviews.

5.3.1 First Group Interview

As a means to verify some of my insights on the design process with the team, the teachers read a paper explaining my preliminary findings prior to the first group interview (Appendix B), which was held on November 18th, 2008. At that time, my research focus was on the way teachers used theories of mathematics learning during the discussions in the meetings. However, during this interview Arnold challenged Armando's role as a member of the team and as a researcher. Since the beginning of the Loughheed project, I tried to include myself as another member of the team, rather than as a researcher; however, this perspective was not the same for the other participants, as Arnold noticed:

Arnold: Because I think you [Armando] want to be an insider, but, can you? Cause you are doing this ethnography and it's really hard to become fully immersed.

My presence within the team was as an active participant: I took part in the discussions and contributed to the design of the lesson. However, I was an outsider from the community of the teachers at Loughheed Secondary school. Arnold's perception of my role as a member of the design team was based on my *credentials* as an instructor from Simon Fraser University and as a researcher. In contrast to Arnold's position, Sofia had not considered these *credentials* to be a relevant issue. The following excerpt shows part of the conversation during this interview.

Arnold: But, I think though there is a very special place as a researcher and as you [looking at Armando] become published; that always will set [you] outside of this community.

Sofia: I don't think publishing gives any more respect or any more trust to what you are saying—just because you are published. Just because it is written doesn't mean is any more true.

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Arnold: But it does exist in a professional literature, and so is a privileged location.

For Arnold, Armando was not only an outsider, but also a researcher representing an authority within the team. Nevertheless, this authority had not been perceived in the same way by Sofia.

Arnold: You [Armando] clearly have to have more authority on what you say. ... I would perhaps give more weight to what you say just because in theory you have more background knowledge. ... You are becoming a professional in this area. So, in theory you should know more.

Brad: Like you are the supervisor. You have your own supervisor and you are the supervisor of us—kind of.

In this excerpt, Brad positioned Armando's role as a supervisor, which is a different role from the one Arnold stressed as a professional, or an authority, in the area.

In the second part of the interview, members of the Lougheed team described the role that each participant played during the project. Everyone wrote their descriptions before taking turns explaining their perceptions of the roles to the team. The written description for each member's role is presented in Table 5.2. The rows of the table present the descriptions written by each member, and the columns correspond to the descriptions made to the role of each participant.

Table 5.2: Perceptions of each Member's Role in the Lougheed Team

Participants	Descriptions of the roles as perceived by each participant			
	Sofia	Arnold	Brad	Armando
Sofia	Suggest problem tasks	Bring resources (articles, books, etc) general + specific to our focus topic	Focus us on students difficulties Question "what we teach and what students will learn?"	Bring in research, theoretical frameworks Support, validate our ideas, and add to them, Suggest problems
Arnold	Creative, able to generate interesting approaches to problems posed by group Thoughtfully and insightly ideas of how to improve students interest of success with a task	Chatty, off topic, lots of ideas Needs help with how best to structure lesson	Clear, rational, has a clear understanding of what needs to be taught, and how this might be done. Excellent understanding of classroom dynamics and problems teaching weaker students	Great ideas, excellent resource vis a vis How to help students and (all) Formative assessment
Brad	A math expert Suggesting ideas to try Trying things out	Data base expert Suggesting ideas to try background research	Talk too much Focus on translating math words to expressions	Participant/researcher Re-focusing the group
Armando	Focusses the team on designing lesson Designing worksheets	Resources Language improvement Inquiry	Inquirer: students, research, mathematics Critique about the lesson	Sharing experience Trigger discussions

Although there were similarities in the descriptions of these roles, particular individual roles were perceived slightly different among the team. The following paragraphs contain descriptions of these perceptions of the roles of each team member.

Arnold's Role

Arnold always brought resources such as books, papers, or details of websites. Although she did not mention this for herself, it was generally accepted in the team as seen from Brad's perception:

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Brad: [Arnold], as you guys mentioned, is sort of the data-base expert. We come up with something, [Arnold] will have something in the locker or in the cabinet or somewhere. She pulls it out of the air: all this background and research so rich in terms of information.

For Armando and Sofia, Arnold also contributed in an important way by suggesting better words for the students' worksheets and teacher's instructions. The descriptions uttered by the team coincide with the written descriptions presented in Table 5.2.

Brad's Role

Brad was often wondering whether students would learn what was intended through the designed lesson. This position contributed to the discussion forcing a consideration of the real impact of the lesson on the students' learning. He also proposed the goal that the team pursued in the first round of the project.

Sofia: [Brad] focused on difficulties students might have questioning. And came up with the question we were working on now. ... The question of how we teach and how students will learn it.
...

Arnold: [Brad] building on what [Sofia] said, I think it is true: that excellent understanding of class dynamics, problems of how weaker students are going to express on that topic. Clear understanding of what needs to be taught and how this might be done.

From these excerpts, it is possible to see that Arnold considered Brad as an experienced teacher with a good understanding of what has to be taught, and knowledge about weak students' troubles in learning the mathematical content. This perception coincides with her written description of Brad's role.

For Armando, Brad also played an important role in asking questions related to the research project and contributed to it by commenting on the teachers' activities outside of the meetings.

Armando: You are so concerned with the research. You are always asking me about my research, or telling me, for example, that you are working outside these meetings ... and how they may impact on my research.

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Brad described himself as being focused on the original goal for the lesson which was for students to translate word problems into algebraic expressions.

Sofia's Role

The team's perceptions about Sofia's role included: contributing to mathematical tasks for the lesson, and refocusing the team on-task when the conversation deviated from designing the lesson. Additionally, Arnold and Brad perceived in her a level of expertise in mathematics and mathematics learning.

Arnold: [Sofia] you are creative, and able to generate interesting approaches to problems posed by the group. Thoughtful and insightful ideas of how to improve students interest and success with a task.

...

Brad: I saw [Sofia], in this context, as the mathematics expert. She was coming with all this terminology So, I was learning new things from you. And you [Sofia] are always doing the puzzles. I would be sitting and watching you actually figuring out the patterns and coming up with the expressions. So, you were taking a much more active role in the sense that you were trying out and I just sit and watch.

Sofia's background in mathematics and child studies is consistent with the description of her role by Arnold and Brad. Whereas Brad saw her as an expert in mathematics, Arnold acknowledged her ideas to improve students' interest and success with a task.

Armando's Role

Armando's role as a researcher within the team was strongly represented in the teachers' perspectives. The teachers perceived him as an expert in the area of education, as can be seen in the excerpt that follows.

Sofia: And [Armando] brought in research and kind of the theoretical framework. And support or validate our ideas ... and add up to our ideas

Arnold: [Armando], I thought you had excellent ideas and in particular this information of informative assessment and looking to assessment rubrics and how to move students along a continuum [referencing

a rubric Armando showed as an example of alternative assessment]

Brad: And [Armando] is the participant slash researcher.

Armando's role was also shaped by Arnold's interests, who in addition to seeing him as a researcher, also asked for assessment strategies. Again, the spoken descriptions coincided with the written descriptions in Table 5.2.

The descriptions of the roles presented in this section go beyond a pre-established set of duties or responsibilities for the team. These descriptions were based mostly on the type of contributions that each member made during the design process in the first round of the project. Roles such as the mathematics expert, Sofia, or the database expert, Arnold, reflect the type of contributions made by particular members. Additionally, Armando's role as a participant-researcher was perceived differently by each member of the team: as an expert, as an authority, and as a supervisor. This role was challenged by Arnold when she said "you want to be an insider, but can you?" When I, Armando, started participating in the project, I did not mean to be an insider, a part of the teachers of Lougheed school; however, I wanted to be seen as another mathematics high school teacher, which I realised was not the case after conducting the first group interview. Certainly, the descriptions of the role included a position (Langenhove & Harré, 1999) such as being a researcher within the team, and such a position had an effect on the interactions of the team. For this reason, I decided to focus on the role of the participants during collaborative design. These roles were not limited to what people did or said during the meetings, but also included different perceptions of the role and position played by each member of the team.

5.3.2 Final Interviews

The final part of the Lougheed project consisted of a group interview, conducted in April 3rd, 2009, and individual interviews conducted during June in the same year. The timing of these interviews gave me some time to analyse the data and to prepare the questions. The interviews were semi-open and included the questions listed in Tables 5.3 and 5.4. The individual interviews were conducted in isolation from other teachers. Part of the purpose of these final interviews was to corroborate initial findings

on the roles of each participant and the characterization of the conversations during the process of lesson design. However, new information came out of these interviews.

Table 5.3: Questions for the Final Group Interview

About this team
1. What did we do well and what could we improve? 2. In addition to the work done in these meetings, what did you do as a consequence of participating in this project? 3. If we have a chance to work again in September, how would you like to work? 4. What further topics would you like to approach in a lesson study team?
About Lesson Study
5. What does lesson study provide that others professional development programs do not provide? 6. What would you say in order to convince someone to participate in Lesson Study? 7. What would you say for someone who is already signed on a lesson study team?
About the facilitator
8. If I were to start another lesson study team in [this city], would you like to observe the designed lessons? 9. Would you participate in another lesson study team without a facilitator? 10. Suggestions for me as a facilitator in lesson study teams? 11. Is there any question you want to ask me?

When teachers were asked, in the final group interview, about their motivations for participating in this particular research project, they agreed that Sofia had played a crucial role in their decisions. Brad's choice of participating in the project was based on the fact that Sofia invited them and that Arnold also had accepted.

Brad: Like you were saying, if it is a colleague you tend to believe more. If I just had a letter "[Armando] from SFU wants teachers to volunteer in a lesson study project,"... I don't know this guy, I don't know anything about lesson study. I probably put it aside some where. But, because it was [Sofia]'s initiative, and then [Arnold] was on board ...

The teachers mentioned that they usually trust their colleagues, sometimes even more than an external expert. This was a factor that made Arnold and Brad accept Sofia's invitation to join the project. During this group interview, the teachers commented that they: (a) appreciated the collaborative work during the project; (b) found it valuable having an expert, or facilitator, close to them, as opposed to an expert coming for one day as part of a professional development programme; (c) implemented in their

classrooms not only the designed artefacts during the project, but also the ideas discussed during the meetings, particularly, the use of problem solving in class; and (d) would be eager to keep doing teachers' collaborative design, but having a facilitator would be important.

Table 5.4: Individual Interview to the Lougheed Team Members

Team process
<ol style="list-style-type: none"> 1. Describe, as a process, what happened in this project (as a team). 2. How did it get started? 3. How did it started? 4. What were the goals as a team? 5. Which activities were part of the project? 6. Identify relevant moments of the whole process. 7. What differences between the two designed lessons do you find?
Individual process
<ol style="list-style-type: none"> 8. Describe what happened to you as a consequence of participating in this project. 9. What were your goals and expectations during the project? 10. Identify relevant moments for you in the project. 11. What were the most significant moments for you in the project? 12. What have you included in your practise as a consequence of participating in this project? (provide examples). 13. What have you learnt? 14. What will/would you do, or stop doing, in your own practise in the future as a consequence of participating in the project? 15. Which factors did contribute to mathematics teachers' professional growth in this project? 16. In there anything else you think might be relevant and I didn't ask you so far?

The interests in and motivations for teachers participating in the project were explored in the individual interviews. Although the teachers agreed that they wanted to improve their teaching by participating in this project, there were other motivations. For instance, Brad was eager to contribute to a research study.

Brad: Having gone through a master degree, I understand [that] ... people are needing participants in their research. So, I wanted to help people with their research.

In the case of Arnold, she was interested in learning more about lesson study and mathematics instruction in Japan.

Arnold: I wanted to learn more about lesson study in particular, and then so just in terms of an academic or curiosity. And that was my

original interest: learn more about it. And also I was really interested to see the Japanese method of instruction.

Sofia was interested in planning lessons on problem solving and contributing to general discussions with her colleagues.

Sofia: I wanted to improve my delivery of the lesson. I wanted to work on the planning of a lesson on problem solving, and contribute to generating discussion, good discussion around the problem.

Teachers described relevant moments during the project in the individual interviews. According to them, the out-of-track conversations were important and contributed to their learning as much, or more, than those conversations focused on the designing of the lessons, and other artefacts, during the project. Examples of their comments on these conversations can be read in the following transcripts from the interviews.

Brad: Being not a trained mathematics teacher, I did not know things like square numbers or some of other patterns you know about in mathematics. In a way, then, I learned a lot, just for my own sake: I learnt a lot about mathematics and, at the same time, I was learning about how to teach mathematics. So, the meetings themselves were the most useful, the most relevant for me.

Arnold: And then also what I really liked, the most important parts, which we would be able to talk about different things that were not necessarily related to the plan. ... For some reason we needed to talk about these other ideas. And they became more interesting to us than the actual act of delivering or preparing a bunch of lessons.

Sofia: I think we realized that we were doing a lot of side discussions. But those were important as part of the process.

Arnold mentioned a remarkable comment about the most relevant moments of the Loughheed project. She considered the first group interview, when teachers read the paper with some preliminary findings, to be the most interesting part because of the reaction of the teachers after the interview.

Arnold: So, for me, that was the most interesting part, because then people were aware: "Oh we are participating in a study group, how what would you say or do is being commented on." whether I

disagree or agree with you is different, but just how people responded, it was sort of amusing.

This last excerpt reflects teachers' awareness of how what they had said would be commented on as part of the research project. Teachers were influenced by the fact that they were participants in a research study. This suggests that the interaction of the team might be not the same in a different scenario where they would not be participating in a research study.

In summary, during the final interviews the teachers described the whole project in their own words. They explained how it started including Sofia's role as a liaison between Armando, as a researcher, and the school. Descriptions of the design process of the lessons were commented on, as well. The 'side discussions,' in which the conversations deviated from the lessons that were being designed, were mentioned as relevant parts of the project where teachers reported important learning, even more than the moments when the team focused on the design of the lessons.

5.4 Emerging Themes

In this chapter, chronological accounts of the Lougheed project have been presented with an emphasis on the first round of the project and the first group interview. The development of the categorization of the focus of the conversations during the process of collaborative design took place mainly from this round, and the focus of the roles that team members played during this process was identified as an emerging theme after the first group interview. The chronological account of events presented in this chapter helps to stress two processes: One is the design of the lesson during the first round of the project and the other is the emergence of the themes for this study.

From the passages presented in this chapter, it is clear that the meetings were characterized by having discussions about a variety of topics in a flow of constant change, and redirection, of the focus of the conversations. The story presented here explains how the Lougheed team moved from considering a single lesson at the beginning, to considering two lessons, then discarding the lesson that included the cube problem, and ending up with a single lesson which used the mathematical tasks that were originally thought as the preparation to the cube problem. Deciding the date for

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implementation was problematic in order to fit the lesson into Arnold's planned lessons. Additionally, there were several off-task moments where the conversation deviated from the design of the lesson, and moments when teachers reported activity related to the project outside of the recorded meetings. As a result, in order to describe participants' interactions during the collaborative design, I decided to characterize the focus of the conversations during the design process. With this characterization, it is possible to identify those moments that would represent occasions for teachers' learning, as well as instances of the change in teachers' practices within the school.

After the first group interview, I realised the influence of participants' roles on the interactions of the team members during the project. The comment that Arnold made regarding my double role as researcher and member of the team in the project was a triggering point. The notion of *role* needed to be extended, as Langenhove and Harré (1999) suggested, to the notion of *position*. A characterization of participants' roles, including interests and positions, is presented in the next chapter.

Chapter 6 Lougheed Team—Findings

The Lougheed project represented not only the initial stage of this research, but also the most extensive one. During this project I was interested in characterizing teachers' interactions while participating in collaborative design. As a result of the open coding process conducted on the data in first stage (see Chapter 4), I decided to further develop two emerging themes: (1) the focus of the conversation during the design process, and (2) the participants' roles in the project. Due to the extensive amount of data in this stage of the research, I decided to present and discuss the results of this part of the study using mainly the data from the first round of the Lougheed project, which is chronologically described in Chapter 5. In this way, the resulting categorizations are explained within the whole context of this round. The interactions in the second round were consistent with those found in the first one and provided a small number of instances of new codes. Complementary instances from the second round of the Lougheed project are included in this chapter, as well as excerpts of the interviews conducted during the project.

The two main themes that I decided to focus on in this study are presented in the first two sections of this chapter. For the first theme, the focus of the conversation during the design process, I analysed mainly the recording of the meetings held by the Lougheed team. For the second theme, the roles of the participants, I analysed both the recordings and the interviews conducted with the team during the project. I consider these two themes as dimensions of interactions among members of the Lougheed team. These dimensions are related: The role and position of the members of the team shaped the conversation during the process of collaborative design. In this chapter I also explain how the conversations during collaborative design reflect the positions and roles of the team members.

6.1 Focus of the Conversation

During the second coding process of the data generated from the Lougheed project, I classified segments of the conversations and activities held during the meetings dedicated to the design of the artefacts (Chapter 4). As shown in Chapter 5, the focus of these conversations and activities changed constantly and went considerably beyond the design of the artefacts. Teachers' collaborative design has a double purpose: professional development and curricular design. While some of the conversations that went beyond the design of the artefacts might have an impact on teachers' professional growth, they deviated from the design of curricular material, the artefact, and the three teachers mentioned such 'side conversations.' As a consequence, I decided to classify the activities and conversations held during the design process into *on-task* and *off-task*. The former classification includes the actions and conversations that made explicit reference to the designing of the artefacts. For instance, on September 23rd teachers were wondering about the ideas and formulas that students may come up with in a task related to a sequence of shapes which was proposed for the lesson (p. 56). The off-task classification includes the conversations that were not explicitly related to the design of these artefacts. In some cases, the off-task conversations were linked to the artefacts under design; however, the discussion deviated from the original purpose, changing the focus of the conversation. For instance, the conversation about pictorial representations held at the beginning of the session on September 30th (p. 58). This classification of on-task and off-task moments is both a result of the study and part of the methodology: Once I split the data into these two classifications, I conducted a detailed analysis of each.

There were a few moments during the design process when it was difficult to determine whether it was an on-task or off-task moment. Those cases were, nevertheless, very short in time duration and I decided to dismiss them as a change of the conversation. For instance, at minute 68 of the meeting held on October 21st, while discussing the final details for the lesson, Arnold explained that she needed to leave the meeting soon. This 16-second explanation was not related to the designing of the lesson and after that the team kept focused on final details for the students worksheet to be used in the lesson.

Another important distinction I made when classifying the data was whether team members talked about something that was happening outside of the meetings as a consequence of participating in the project—such as implementing some of the discussed ideas in their current teaching. I divided both classifications, on-task and off-task, into *inside* and *outside*. Some instances of the outside sub-classification, both on-task and off-task, provide evidence of teachers' activities outside the recorded meetings as a consequence of participating in the project. These instances might represent evidence of the teacher's professional growth as they incorporated new practices at school. These tentative categories and subcategories are summarized in Table 6.1, which includes examples of each subcategory.

Table 6.1: Description of the Focus of the Conversation

Categories	Subcategories	Examples	
On-task. Conversations and actions taken toward the designing and implementation of the artefact.	Inside. Everything that was discussed inside the meeting about the planning of the artefact.	The team was selecting the problems for the artefacts.	Team members were sharing a previous teaching experience that might contribute to the discussion in designing the artefacts.
	Outside. Participants reported some activity outside the meetings intended to plan the artefact.	Conversations outside the meetings in order to design the artefacts.	Teachers implemented the designed lesson, or a variation of it, in their classroom without observers.
Off-task. The intent of the discussion is not focused on planing the artefact.	Inside. Everything else that was discussed inside the meeting.	A casual conversation about weather or food was held.	Some team member criticized the mathematics curriculum.
	Outside. Activities outside the meetings that participants reported as consequences of being a part of the project.	Teachers incorporated problems discussed in the meetings into their own classroom.	Teachers incorporated the use of assessing rubrics into their course.

The focus of the conversation during the meetings for the design of the artefacts can be represented in time-line graphs, as shown in Figures 6.1 and 6.2. These graphs

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allowed me to show the constant change in the focus of the conversation within the team.

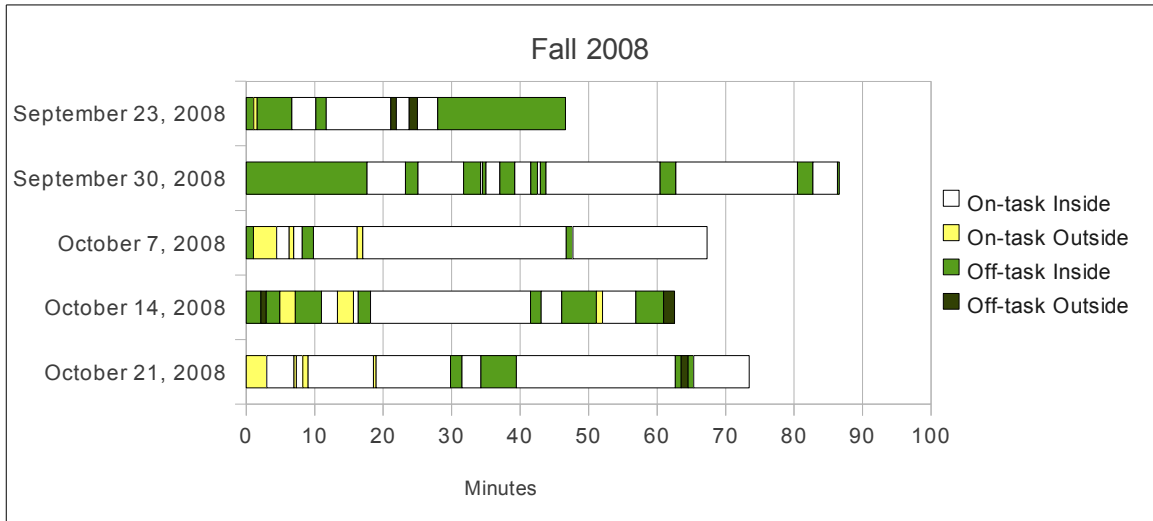


Figure 6.1: Focus of the Conversation During the Design Process, Fall 2008

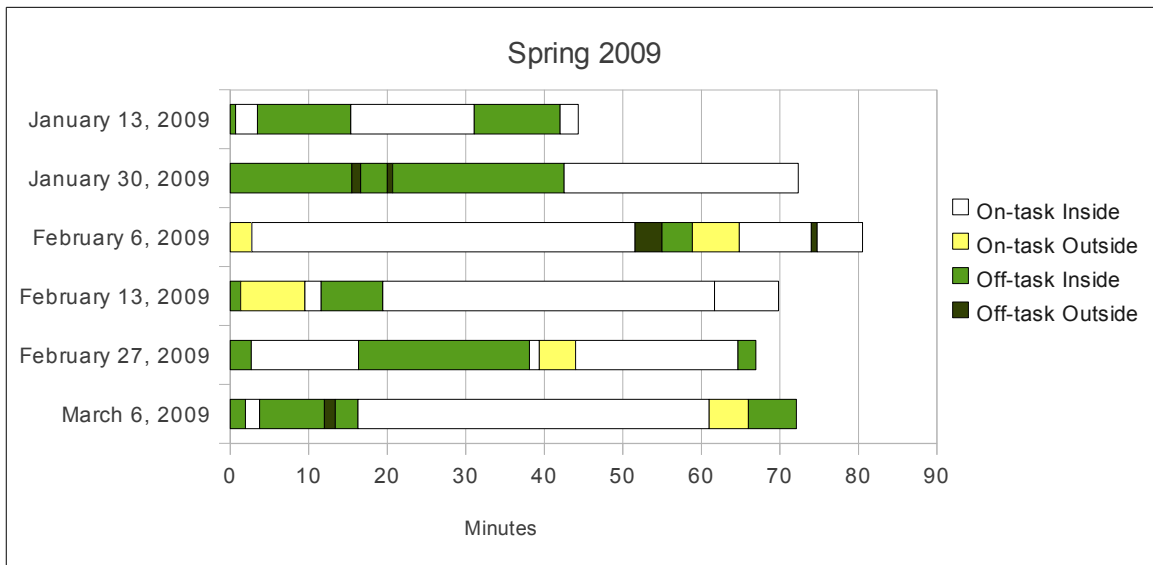


Figure 6.2: Focus of the Conversation During the Design Process, Spring 2009

The conversations of the Lougheed team were often off-task, going back and forth between designing the lesson and talking about something else. The off-task moments of the design process may contribute to the design of the artefacts; however, no direct reference was made to the artefact during these moments. It can be seen from

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Figures 6.1 and 6.2 that those off-task segments lasted a long time in some cases. During the meetings on September 23rd and September 30th, 2008 of the first round, as well as January 30th and February 27th, 2009 of the second, there were long off-task conversations. In contrast, during the meetings on October 7th and 21st, 2008, in the first round, as well as the meetings on February 6th and 13th, and March 6th, 2009, of the second round the conversations were more focused on designing the artefacts.

Teachers' engagement in collaborative design went beyond the activities inside the meetings; the outside moments of the conversations serve as evidence of teachers' outside-activities related to the project. In Figures 6.1 and 6.2, it is possible to identify some of the moments in which participants made reference to activities related to the collaborative design projects that took place outside of the meetings. Examples of these moments were when Brad, in the first meeting on September 23rd (Chapter 5, p. 55), explained that teachers met before this meeting and started to think about the problems and the approaches for the first lesson. Another example is on October 7th, when Sofia explained that teachers met the day before and decided to plan two lessons (p. 60). As it can be seen in Figure 6.1, this moment happened at the beginning of the meeting and lasted around three minutes. Having these outside discussions about the design of the lessons was not planned at the beginning of the project. Teachers incorporated such discussions in their activities within the school. Teachers' practices in the classroom can be identified in some of the off-task conversations when referring to activities outside of the meeting. For example, the following comments on September 23rd show how the project had an impact on teachers' classes.

Arnold: Every time I come ... I try to listen carefully, and try maybe two or three things that came up here in my class.

Brad: Last class we did toothpicks problems [Brad and his students]... Today, having this discussion meeting here, I'm going to extend on that. Now using the toothpicks idea, if I make this pattern, and then another pattern and another pattern, ... but connecting with what they [Brad's students] have already done from the previous class.

As it can be seen from the transcript, the outside moments also provided evidence of change in teaching practice during the project. The time-line graphs, as well

as the filter tool in the spreadsheet I used to code the data, served to identify these moments within the recordings.

The on-task and off-task classification served to characterize the focus of the conversations held during the design process of the Loughheed project. The time-line graphs in Figures 6.1 and 6.2 show the dynamics of the conversations during this process: constant change and refocus. After classifying the data into on-task and off-task, I analysed each part separately. In the next two subsections I elaborate on each part.

6.1.1 On-Task: Designing an Artefact

Once I focused on the on-task moments of the recordings dedicated to the design of artefacts during the Loughheed project, the process of data coding continued until I came up with a broader characterization of these moments. I generated new codes and indicated who was talking, and to whom, in each segment. The filter tool of the spreadsheet helped to compare different moments having the same codes (Figure 4.3). As a result of this comparison among all the on-task segments, I generated 11 new codes, which are summarized on Table A.1 in Appendix A. I tried to synthesise these codes into less broad categories—axial coding. However, after several attempts to generate isolated categories, I realised that it was possible to use four interrelated categories. Finally, I developed the *design braid*, a characterization of the conversation which consists of four categories. Every moment during the on-task parts of the design process is included in this characterization.

The conversations and actions taking place in the on-task moments focused mostly on talking about topics such as students' attempts to solve the related tasks, mathematical content, and teachers' actions. I developed three categories of these conversations and activities: (1) *anticipating*, (2) *achieving goals*, and (3) *pursuing coherence*. The other on-task moments were dedicated to the organization of the team itself, such as discussing and negotiating an agenda for the project, and distributing labour among team members; these conversations comprise the fourth category which I call (4) *team organisation*.

Anticipating

Anticipating refers to the moments in which members of the design team predicted students' performance during the implementation of the artefact, such as students' possible approaches, struggles or successes, to the corresponding mathematical tasks. This category also comprised teachers' proposed responses in accordance to such predictions. Instances of this category can be identified easily in sentences structured as questions for students performance or trouble, or through the use of the verbs indicating a future potential action. Examples of these sentences, shown in Chapter 5, p. 56, are: (1) "What kind of ideas can we expect kids to come up with in place of $5n$ or $3n+2n$?" and (2) "Well, I would say that they will start with the recursive thing." However, instances of anticipating were also present in other linguistic forms. Teachers sometimes brought their own experience as a means to anticipate students performance. An example of this is when Sofia was explaining students' difficulties coming up with a recursive formula for a given pattern.

Sofia: The recursive formula is easy, but I find the notation, even writing the recursive formula, is really difficult for students.

Another example of sentences that characterize anticipating, and are not structured as a question or do not use a verb in future tense, is when Brad commented on those students who do not follow written instructions (Chapter 5, p. 61).

Once I characterized the 'anticipating' moments during the process of design, I also found that it is possible to split this category into two subcategories depending on whether the team focused on student performance or focused on remediating students' possible trouble during the lesson. Using the weather forecast as a metaphor, I designated *forecasting* as a subcategory of anticipating that refers to all that participants say or do in order to predict students' behaviour or performance. The previous examples are instances of forecasting in which the team discussed students' possible struggles, successes, failures, and approaches in order to find algebraic expressions for the involved numbers in the patterns. When forecasting, members of the Loughed team often used their experience as teachers, as Brad did when indicating that some students do not follow written instructions, in order to predict such students' behaviour or performance (Chapter 5, p. 61). As the lesson of the first round was to be implemented

in Arnold's class, she was concerned with students who may have trouble in the lesson (p. 62). Some hypothetical statements were also used for forecasting. For instance, during the first round of the Loughheed project, the team assumed that it would be easier for students to write verbal descriptions for the numbers involved in each sequence of figures in order to come up with an algebraic expression (p. 59 and 62). The literature was used as a means to refer to these hypothetical statements about students' performance. For example, as mentioned before, Arnold was concerned with students that might need guided help. She used Marzano's (2007) scale in order to describe student performance. Another way of forecasting student performance during the lesson was piloting the problems with someone else. For instance, Arnold asked her son to solve one of the tasks for the lesson (p. 61), and Brad suggested that teachers might try using patterns in their own classes and see what students could do (p. 57).

Table 6.2: Subcategories of Anticipation

Anticipating	
Forecasting —as predicting	Commitment —as preventive/responsive action
Anticipation of students performance including approaches, struggles and solutions.	Remmediation of forecasted students' struggle including lesson format or teacher response.

Commitment is the complementary subcategory of anticipating and refers to all that participants proposed in order to remEDIATE, or approach what they have forecasted. This subcategory includes discussions on the format and content of the artefact, as well as possible actions taken by the teacher before and during the implementation. Examples given in Chapter 5 include: (a) the inclusion of tables in students' worksheets as a means to guide their process of finding a pattern in the corresponding numbers of the sequences of figures (p. 65); (b) the response of the teacher to some forecasted student's question or struggle when they would have to write algebraic expressions (p. 66); and (c) the use of patterns in sequences of figures in prior lessons, so students would be familiar with this type of mathematical task (p. 60).

The discussions during forecasting and commitment were often of different types. At moments of forecasting, participants made use of both their teaching experience and

available resources in the discussions. In contrast, commitment was the practical part of anticipating and an active debate was usually present. It was the moment where teachers negotiated the activities and tasks for, and before, the use of the artefact. However, there were also cases in which forecasting and commitment were together in a single sentence. For instance, on October 7th, Brad proposed to start with "something easier," so that students could come up with descriptions that can easily be written with an algebraic expression (Chapter 5, p. 63).

Pursuing Coherence

While working on designing the artefacts, the Lougheed team members set the mathematical content and student activities in a larger context. The outside instance of commitment, reported by Sofia, in which teachers discussed the use of sequences of figures in prior lessons (Chapter 5, p. 60), also represents an instance of another broad category of the on-task conversation, *pursuing coherence*. In this example teachers were concerned with students' lack of ability to recognize square numbers when approaching the cube problem, and then they were committed to introduce a set of mathematical sequences in prior lessons in order to prepare students for the 'cube' problem. This idea of having a sequence of lessons, or activities within a set of lessons, reflects the need for coherence of a lesson within a course, Arnold's course in this case. Pursuing coherence refers to the efforts of the team to properly sequence mathematical tasks, activities, or lessons in order to facilitate students' learning. This category includes the order in which mathematical concepts must be presented to students, or the order in which different tasks would be presented according to their level of difficulty, as well as the class environment, or micro-culture. For instance, although the Lougheed team agreed that problem solving can be used as a learning strategy, students might need to get used to approaching problems by themselves as opposed to waiting for a teacher's answer.

In pursuing coherence teachers not only talked about a single lesson, or any other artefact, but also how it would fit in the course, the curriculum, and the particular group—including some particular students. As commented in Chapter 5, the selection for the date of the implementation of the lesson in the first round was problematic because it

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had to fit into Arnold's course plan. On October 14th, during an outside moment, teachers reported in the meeting their decision of implementing the lesson at the beginning of the unit, as an introduction to the use of variables. In this meeting, the team also discussed the student's prior knowledge necessary for the the tasks in the lesson. As mentioned before, this lesson was aimed originally as an introductory lesson for the cube problem, and became the actual designed lesson when this problem was dismissed (p. 64). Another example of pursuing coherence is the following transcript in which the team was discussing the mathematical skills students should have before the lesson.

- Armando: For example in this case, the use of a chart is a key thing. If you are working, I don't know, with fractions or whatever, you can start looking at some problems.
We are grounding the knowledge of the students, but also we can see: okay, they have a lot of struggle with this and then we can rethink before the first lesson.
- Sofia: I think that it is really important. In the larger picture, what I eventually want to have is a good lesson, ... but also good sequences. ... So, at the end of the year ... they have seen patterns in a lot of different ways.
- Arnold: I like this particular book because it says what you are suggesting ... Here are all these strategies that good problem solvers would have, like making a drawing.
And so, I want to teach that particular skill set before I get to there. Then they have a series of, you know, they are making a drawing, visual representation, strategies every day, life problems, and then applying it.

The use of patterns proposed by Sofia was not limited to one single unit or section of the course. She proposed the use of mathematical problems related to patterns through the whole year. Additionally, Arnold was concerned with having the students prepared for the problems with the particular 'set of skills' required for the lesson.

Achieving Goals

The Lougheed team selected the goals that guided the design process for each round, *achieving goals* category refers to the actions oriented to match those pre-

selected goals. Decisions about the type of student tasks and format for the artefact, had to be made in order to achieve the desired goals. For instance, at the beginning of both rounds of the Lougheed project, some problems or tasks were selected from the textbooks, or previous teachers' course material, in order to achieve the specified goals.

Achieving goals includes team debates on whether some teaching approaches would serve to achieve the goals of the lessons. For instance, Brad was sceptical about the use of patterns of figures as a means to have students translate word problems into algebraic expressions. An example of these debates is presented in Chapter 5 during the meeting on September 30th (p. 60). Later on, after discussing how the use of patterns would serve to achieve the selected goal, Brad explained how it made sense to him during the meeting in October 7th (p. 62).

Team Organization

The fourth category for the on-task conversations and actions during the design process of the Lougheed team is *team organization*. This category refers to the moments in which the team organized itself for the project. An example of this category is the distribution of labour in order to design student handouts for the first round of the project at the end of the meeting on October 7th: Sofia typed the lesson and Armando worked on the graphic design of the figures (Chapter 5, p. 63).

Relationships Among Categories of On-Task Conversations

In the first three categories of the on-task moments, anticipating, achieving goals and pursuing coherence, different aspects of the collaborative design are stressed; nevertheless, they are interwoven through the conversation mixing in such a way that some times one is easy to be identified alone, and other times two or three appear together. An instance of anticipating and pursuing coherence together is when Sofia, during the meeting on September 23rd, explained that they wanted to assure that students would know how to approach problems related to patterns, and then suggested giving them an example first (Chapter 5, p. 57). Teachers were proposing an introduction to the use of patterns before the task of the lesson, so students could do what the team

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was expecting. This is an instance of commitment which involves a sequence of activities in a coherent way.

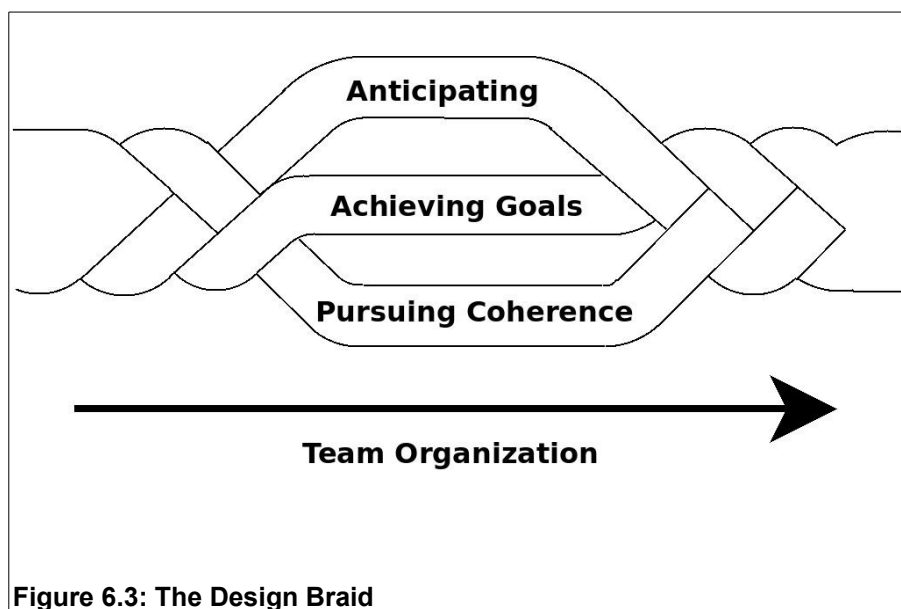
An instance of achieving goals which includes anticipating was on October 21st when Arnold was wondering about helping students to come up with a formula to represent the corresponding numbers related to the sequences of figures used in the lesson (Chapter 5, p. 66). Sofia suggested that the teacher might advise students to give a *name*, or a *variable*, for the stage in the sequence of figures: This would help students to come up with an algebraic representation. This example of commitment includes the efforts for having students write algebraic expressions, which was the goal of the lesson in the first stage of the project.

Achieving goals and pursuing coherence appeared together as well. An example showing these two categories together was on October 14th, when Arnold was concerned with the necessary skills students would need in order to approach the tasks for the lesson (Chapter 5, p. 64). These identified skills were focused on problem solving and included: (1) being able to apply surface area and volume, (2) introducing and naming variables, and (3) writing algebraic expressions to represent quantities or measurements. In this discussion, Sofia also commented that the cube problem may not fit into Arnold's course because of the sequence of topics and the content of the unit which was strongly focused on solving equations, as opposed to using algebraic expressions to represent formulas, or functions—which would be required in order to find the general term related to the patterns proposed for the lesson. The cube problem might be used as a helpful lesson in order to achieve the desired goal; however, the mathematical requirements for this problem might make it hard for Arnold's students to translate the problem into an algebraic expression.

The three categories, anticipating, achieving goals and pursuing coherence, appeared at the same time in a few cases. For instance, On October 7th Brad was making sense of how the use of patterns would help students write algebraic expressions, which was the selected goal for the lesson in the first round of the project (Chapter 5, p. 62). He situated himself as teaching the first lesson in which students would be exposed to easy problems related to patterns as a preparation for the cube

problem—at that point, the team was proposing two lessons. He anticipated also student performance and the possible way he would answer some students' questions in order to help them to write algebraic expressions in the first lesson considered at that moment of the design.

While anticipating, pursuing coherence, and achieving goals were strongly interrelated, team organization was separate from the previous three categories, guiding the activities of the team. These four categories encompass all the activities and conversations of the Lougheed team during the on-task moments. In Figure 6.3, the three interwoven concepts are represented as strands of a braid, *the design braid*, which is directed by team organization.



The outside moments of the on-task conversations indicated teachers' discussions related to the design of the artefacts. These discussions are also represented by the design braid. The example given before of anticipating and pursuing coherence together is actually an outside moment. In this example Sofia explained what teachers discussed before the meeting held on September 23rd (Chapter 5, p. 57). Similarly, on October 7th, Sofia reported that teachers discussed the introduction of a few patterns in a lesson previous to the lesson that would included the cube problem (p. 60). This moment was used before as an example of commitment. However, this

moment also represents an instance of pursuing coherence, as the teachers considered a sequence of two lessons. Another example is the outside moment when Arnold explained how her son solved one of the problems that was being considered for the lesson (p. 61), which is an example of forecasting.

The use of the braid (Figure 6.3) as a metaphor to characterize conversations and actions during the design process serves to emphasise two aspects of collaborative design. On the one hand, there are strong connections among anticipating, achieving goals and pursuing coherence taking place, at times, at the same instant, on the other hand, all the on-task conversations and activities were included in these categories. Additionally, I believe that, being interwoven, these categories enhance the improvement of teachers' practice and the quality of the designed artefacts. In the achieving goals category, teachers reflected on whether the selected tasks for an artefact would serve to achieve the intended goals. In the anticipating category, teachers discussed students' possible reactions and trouble while approaching mathematical tasks and planned ways to support and guide them during the implementation of the artefacts. When teachers attempted to achieve coherence, they took into account several factors that might affect students' performance. These factors included: required students' knowledge and skills for the particular tasks included in an artefact, being used to working in particular settings, and the relevance of the artefact for the whole unit or course. By taking the three strands together, several factors involved in students' mathematics learning were considered and contextualized at the same time. Metaphorically speaking, the braid is strong because it is made out of these three strands.

6.1.2 Off-Task Conversations

After developing the categorization of the on-task conversations, I focused on the off-task conversations. From the coding process I developed in this case a list of 12 new codes, summarized on Table A.2 in Appendix A. I classified these codes in four broader independent categories summarized in Table 6.3: (1) *Teachers' practice*, (2) *Mathematics and educational context*, (3) *Collaborative work*, and (4) *Casual conversation*. This categorization of the off-task conversation is exhaustive: Every off-

task moment in the recordings of the design process during the Lougheed project was considered within these four categories.

Table 6.3: The Off-Task Category.

Categories	Description	Examples
Teachers' practice	Conversations about teachers' knowledge and resources of mathematics and mathematics education, as well as how they would use that for their own teaching. Actions taken by participants included: sharing, asking, discussing, and implementing.	Resource review
		Mathematics concept discussion
		Sharing a teaching strategy
		Students performance
		Teachers implementing ideas discussed in the meetings in their own classrooms
Mathematics and educational context	Discussions about what is, and what should be taught in mathematics courses at school according to social and academic needs. It also includes situations that help/hamper mathematics teaching.	Teacher's concerns about the curriculum
		Mathematics curriculum
		Mathematics outside school
Collaborative work	Decisions about the way the teachers could work in other instances of collaborative design.	Talking about this research project
		Collaborative work among teachers
Casual conversation	Personal comments and discussions of topics outside of mathematics education at high school level	Sharing a personal experience
		Talking about the coffee or food preferences

At some moments, it was clear that the focus of the conversation was far away from the designing of the artefact; however, in some other cases the discussed topics were connected to the task of designing the artefacts, but this connection was not explicitly mentioned. Casual conversation was the most distant from being on-task, whereas some moments where the focus of the conversations was on teachers' practice, some potential connections to the design of the artefacts might occur. For instance, on October 17th (Chapter 5, p. 63) the team held a conversation about coffee and food preferences which had no connection to the design of the lesson. In contrast, the conversation held at the beginning of the meeting on September 30th (p. 58) focused on

the use of pictorial representations in mathematics education. Although no connection of this discussion with the design of the lesson was made at that moment, pictorial representation of quantities might be used in the design of the mathematical tasks for the particular artefacts that the team was designing which was aimed at getting students to write algebraic expressions from word problems.

Teachers' Practice

The *teachers' practice* category encompasses the conversations in which teachers shared, discussed and reported experiences, knowledge and resources that might have an impact on their teaching practice. During the off-task conversations curricular mathematics topics were addressed, such as the distinction between variables and unknowns held on September 30th (Chapter 5, p. 60). Resources such as books and articles were discussed in these moments. For example, in the instant mentioned before when Brad wondered about pictorial representation, the discussion was based on a book that Arnold brought to the meeting. This category also includes comments that teachers made about implementing some ideas discussed in the meetings into their own classroom, as Arnold and Brad did on September 23rd (Chapter 6, p. 84), as well as teachers' expectations for their students: for instance, On September 23rd Brad and Sofia commented on her expectations about students' performance on finding algebraic expressions to general terms involved in sequences of figures (Chapter 5, p. 57). At the end of this meeting, the team also conversed about: difficulties that teachers face completing the courses according to the curriculum, the importance of affect in mathematics learning, and teachers' expectations for their students (p. 57). Within this category, conversations also include the sharing of teaching and learning strategies, such as the animal game suggested by Arnold, at the beginning of the meeting on September 30th, as a strategy to memorize facts (p. 58).

Sometimes a comment on students' common particular troubles triggered an off-task discussion. For instance, on February 27th, during the second round, the team was focused on the design of an assessment rubric for the unit of proportion. The plan was to involve students in the generation of this rubric by proposing descriptions for its categories. The wording in the headers of the rubric was being discussed and Sofia was

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editing the document on her computer during the session. By minute 9 of the meeting, Arnold commented about an activity related to similarity among right triangles, that she did the day before with her students, and brought to the table some triangles and a student worksheet for this activity. She commented on the possible descriptions for the rubric in the context of this activity. At minute 15 of the session a double conversation took place. Although it was not possible to completely understand the two conversations from the video recording, it was possible to see that Brad and Arnold were discussing the task of similar triangles while Armando and Sofia kept working on the design of the rubric. In minute 16 the conversation turned to off-task after Brad made a comment related to the use of calculators in trigonometry.

Brad: For trigonometry, I have students who just cannot figure out the sequence of [keys] to push on their calculator. So, I gave them a table, instead, that [Sofia] gave to me. ... They find that a table is easier to use.

Arnold: I think that even the calculator is cool.

Brad: They just can't.

Armando: The calculator is cool once you understand what you are doing. ... May be as 'step' [slope], if you start with a table, they would know exactly what they are doing. ... Otherwise, you can get lost with the calculator, just drilling a process which you [as student] don't know exactly why it works.

From this comment, a discussion on introducing students to trigonometry was triggered. Armando shared a strategy consisting of making students use paper with a millimetric grid in order to measure the values of the trigonometric functions from the unit circle. Sofia continued working on the editing of the rubric for two more minutes, and then the whole team focused on the topic of trigonometry. Brad commented that he was planning on starting the topic soon in his classroom. Sofia described an activity using a protractor and a paper to explain how to come up with that table. Additionally, she showed an Excel file on her computer which she had used for this topic. The conversation included experiences in the classroom as well as the use of different software such as Geometer's Sketchpad and Cabri. This conversation extended until Sofia redirected the conversation to the design of the rubric. The whole off-task

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conversation took place from minute 16 to minute 38 of the session (see Figure 6.2, February 27th).

This is not only an example of sharing a strategy, but also is an instance that shows how these off-task discussions were initiated. Brad commented that he was starting the topic of trigonometric functions and Sofia also shared another strategy using the protractor as a means to introduce the concept of the sine function. This conversation was a learning occasion as Brad might implement the suggested ideas right away into his teaching practice.

Mathematics and Educational Context

In the *mathematics and educational context* category team members held discussions related to the broader context of mathematics and education. In contrast to *teachers' practice*, in this category the conversations were focused on aspects that went beyond teachers' practice at their schools. The last part of the conversation on September 23rd included instances of this category (Chapater 5, p. 57), such as the discussion about the use of mathematics as a filter in admission exams to university level institutions. In this part, Brad discussed how algebra is being taught and used by younger students. He wondered about the purpose of the mathematical content at high school level.

Brad: We are always passing on information necessary ... to survival to the next generation. I don't feel that with algebra, I don't feel it with a lot of the math that we teach. It is not important to survival. ... Trigonometry, you know? How often do you find cosine or tangent?

Sofia: Survival is no longer ... [interrupted by Brad]

Brad: I know, I am just saying ... the concepts of algebra: was that taught just because you wanted to teach it? And then now we are pushing it down further and further to younger younger sections of the population.

Arnold: There was this amazing study, and they always talked about the public school sector and then the private sector in terms of the cost of training the a work force. And then administration in the US and then ... That was the first time in a long time where, because of the fact that the economy was so strong, the private sectors started training their work force in terms of mathematics

skills in order to accomplish what they need to do for the industry. So, I think that if you have a very strong industrial base and a very sort of strong economy, then the private sector is much more willing to ... because clearly there is many application in business using algebra.

From this comment we can see that Brad did not find many of the topics taught at school useful for survival, or life. This is a very practical view about mathematics, which is debatable by people holding different perspectives. Arnold's comment was based on a study and she focused on the need of the private sector of training their work force in terms of mathematical skills. This conversation was related to the purpose of mathematics education, the topic of the conversation was well beyond the design of the lesson on algebra. The rest of the session was off-task (see Figure 6.1)

Collaborative Work

The *collaborative work* category refers to comments about possible ways of conducting, and researching on, collaborative design. Conversations about lesson study fall within this category. Arnold was particularly interested in lesson study and asked for the reference for the book by Stigler and Hiebert (1999) on September 23rd (Chapter 5, p. 55). In the final individual interview she claimed that one of the reasons she participated in the Lougheed project was to learn more about lesson study (p. 76). Comments about the research project were also included in these interviews. For instance, the commentaries that Brad made on September 23rd about how the discussions that teachers held outside the meetings would be interesting for this research (p. 55). This comment is consistent with his motivation to participate in the project, as he explained in the individual interview (p. 76). This category represents conversations that were very particular to the Lougheed project as a research project inspired by lesson study.

Casual Conversation

The *Casual conversation* category refers to conversations that were not related to mathematics, mathematics education, the research project, or participants' activities as teachers or educators. These conversations tended to happen at the beginning of the

meetings and included comments about the weather or some news in the media. Participants' personal aspects of their life also appeared in this category, such as food preferences and family topics. Therefore, in casual conversations team members had the opportunity to learn about each others' life.

Occasions for Teachers' Learning

Although off-task moments were frequent during the design process of the Lougheed project, as it can be seen from Figures 6.1 and 6.2, these moments represented occasions for teachers to learn, especially in the category of teachers' practice. The books and articles that Arnold brought to the meetings were resources that teachers could use for their own practice. Additionally, the outside conversations provided evidence of teachers implementing new strategies in their classrooms. For instance, in Chapter 5, (p. 84) Arnold and Brad made comments about listening and implementing discussed ideas in their own classrooms. By listening to a teaching strategy, teachers might adapt different ideas in their own classroom. Sharing a teaching strategy might happen in contexts different than teacher's collaborative design. However, during the Lougheed project, sharing resources and teaching strategies was a common practice.

The mathematics and educational context category included examples in which teachers made explicit some of their values and perspectives about teaching mathematics. For instance, in the meeting held on September 23rd, Brad commented about feeling that many mathematical subjects taught in school are not “passing on information necessary ... to survival to the next generation.” This type of comment had the potential to generate a debate on the purpose of mathematics instruction at school within the team. This debate, however, did not take place within the Lougheed team. Instead, other aspects about the use of curricular mathematics in some careers were mentioned, including the use of mathematics as an entrance filter to higher level educational institutions.

6.1.3 Summary of the Focus of the Conversations

In order to characterize interactions during the design process of the Lougheed team, I classified the conversation in each segment of the recordings of the meetings dedicated to the design of the artefacts. This classification consisted of on-task and off-task moments according to whether the focus of the conversation was explicitly on the artefact under design. The conversations held by the Lougheed team were constantly changing between on-task and off-task during the process of design; the time-line graphs in Figures 6.1 and 6.2 serve to illustrate these changes.

The on-task and off-task classifications were analysed and presented separately. While the on-task conversations were categorized by interwoven categories, the off-task categorization consisted of four independent categories. Although these categorizations were different, in both of them there was a special sub-classification into inside and outside conversations. The outside classification served to identify teachers' activities outside of the meetings which were related to the Lougheed project. Some of these moments suggest evidence of change in teachers' practices as a consequence of their participation in the project, representing occasions for teacher's professional growth.

The on-task moments of the conversations were characterized metaphorically using the design braid (Figure 6.3), which included three interwoven categories: anticipating, achieving goals, and pursuing coherence. These categories referred to all the conversations and actions explicitly focused on the artefacts for the Lougheed project. The fourth category of the on-task moments, team organization, referred to the moments when the team talked about its own organization for the collaborative work.

Off-task conversations included occasions for teacher learning, especially during moments within the teachers' practice category. During the individual interviews, Brad claimed that he learned from these moments and Arnold found many of these moments more interesting than the actual on-task conversations (Chapter 5, p. 77).

The interactions within the Lougheed team were also influenced by the roles and positions of its members within the context of the Lougheed project. This project was a study on teachers conducting lesson-study-inspired collaborative design, and interactions were also shaped by this situation. For instance, Arnold, interested in

learning more about lesson study, asked questions about this type of collaborative work. Brad's questions about the research, as well as his comments on teachers' conversations outside of the regular meetings, represent examples of how the situation of being involved in a research project shaped interaction. The type of participations during the design process of the Lougheed team had particular patterns for each member. For instance, Arnold brought very often articles and books to the meetings. She made reference to the literature in order to participate in the discussions held by the team. Marzano's (2007) rubric for problem solving (Chapter 5, p. 61) and the article about assessment mentioned on September 23rd (Chapter 5, p. 55) are instances of this type of participation. In the next section I elaborate on the roles and positions held by the members of the Lougheed team.

6.2 Members' Roles

The second theme that I found relevant for characterizing the interactions of the Lougheed team members was the roles played during the project. During the first round, I identified different roles that participants were playing within the team and tried to explore this theme in a group interview. This interview, held on November 18th, brought more than simple descriptions of the role of each participant: As Arnold argued in this interview (Chapter 5, 70), Armando's role within the team shaped the way teachers interpreted his comments. After the interview, I considered looking at roles as an emerging theme for this research.

The final group interview and individual interviews provided more information about the roles of teachers in the Lougheed project. The descriptions of the roles that each participant held during the project were congruent with the first individual interview. However, when teachers described the motivation to participate in the project, I realised the importance of Sofia's role for the initiation of the project. Arnold's interest in lesson study was reflected by her questions and comments during the meetings. Brad's interest in participating in a research project was also reflected by his comments and questions during the design process. I coded the transcriptions of the interviews focusing on the role of each member of the Lougheed team. The roles that I have identified during this coding are described in this section.

Three Factors that Shaped Interactions

After comparing the codes of all the interviews and looking again into the codes of the design process, I found that the Lougheed project was conducted in a context which included three factors that shaped participants' interactions. The first factor was the collegiality that characterised the relationship among the three participant teachers. Arnold, Brad and Sofia were teachers from the same school and they knew each other before the project started. Moreover, Sofia was the liaison between myself and the other teachers. As such, she played an important role initiating the project. This is seen in Brad's comment during the second group interview where he stressed that teachers would tend to trust more in a colleague than in an outsider (Chapter 5, p. 75).

The second factor was the fact that the Lougheed project was actually a research project. This project was not only an instance of teachers' collaborative design, but also research which focused on the members of the team. Arnold, when asked for the most relevant moments of the project in the individual interview, claimed: "We are participating in a study group, how what would you say or do is being commented on" (Chapter 5, p. 77). Even though teachers knew from the beginning the purpose of the study, being able to read the preliminary findings, as a preparation for the individual interview, made them reflect that what they were saying was being analysed and commented on as part of the research. These findings had the potential to impact on teachers' comments during the interview, as well as further meetings for collaborative design. As such, the conversations held during the team meetings might not be the same if the study were not focused on the teachers as participants.

And the third factor was that participant teachers had personal interests in participating in the project. These interests, described by them in the final individual interviews, can be identified in some of the interventions during the meetings. Brad claimed, for instance, that he wanted to contribute to research and for him that was a reason to participate in the Lougheed project (Chapter 5, p. 76). As described on page 55, he mentioned the conversations that teachers held outside the meetings, and how those conversations would impact on my research. He also asked questions about my research during the project. Arnold was interested in learning more about lesson study

and the Japanese educational system, and that was a reason for her to participate in the project. The Lougheed team held conversations about these topics and Arnold asked, for instance, for the reference of Stigler and Hiebert's (1999) book (p. 55). Sofia, interested in planning lessons on problem solving, was constantly refocusing the conversation to be on-task, to focus on the designing of the artefacts.

These three factors serve as a context to understand why certain conversations took place during the Lougheed project. Before looking at the roles I have identified in the team, it will be useful to recall participants' background and some of their contributions to the conversations held within the meetings. Brad was, by far, the most experienced teacher in the team: He had been teaching mathematics for 15 years, whereas Arnold had 7 years and Sofia had 6 years of teaching experience in mathematics. Brad used this experience to contribute to the designing of the artefacts. For instance, based on his experience, he proposed to use problems similar to those in the textbook, This way, students would be more familiar with them (Chapter 5, p. 59), and they would have more time to read the written instructions (p. 61). This teaching experience was acknowledged by Arnold who claimed in the first group interview that Brad had an "excellent understanding of class dynamics" and "clear understanding of what needs to be taught and how this might be done."

Descriptions of the Roles

Despite being the most experienced teacher, Brad described himself as a non-expert in mathematics, as well as a learner inside the team. This description is consistent with different moments during the meetings and the interviews. For instance, on September 30 he used the sentence "I'm not a mathematician by training, as you know ..." to ask a question regarding the use of pictorial representation for the arithmetic operations. He also stressed, in the second group interview, that he was not a trained mathematics teacher, and that he did not know about square numbers and the other patterns used during the project (Chapter 5, p. 77).

In contrast to Brad, Sofia was considered an expert in mathematics. She proposed several problems (e.g. the cube problem in Chapter 5, p. 59) and engaged in

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solving several mathematics problems during the project. Brad perceived her as an expert. In the first group interview he mentioned, "I would be sitting and watch you [looking at Sofia] actually figuring out the patterns and coming up with the expressions." Sofia's contributions to the discussions during the meetings included the explanation of the use of letters in mathematics: as variables and as unknowns (p. 60).

A factor that shaped Arnold's interactions during the project was her perception about the literature in mathematics education. She ascribed an important value to published works, as she claimed during the first interview regarding Armando's role as a researcher and the status of being published: "I think, though, there is a very special place as a researcher and as you become published." This perception about the literature was not shared by Sofia who argued that "just because it is written doesn't mean [it] is any more true." Arnold's contributions to the discussion were based often on books and articles about mathematics teaching, such as Marzano's (2007) assessment rubric or some problems she referred to in different books. She often brought a resource such as a book or an article to the meetings. She was more interested in using methods from the literature for the lesson than coming up with something new, as she explained: "I would like to use a method that has been researched and has shown to be effective as opposed to something that I would just come up with it." Arnold's perception of the research literature shaped what she did and said during the project.

Considering the factors that shaped the team conversations described before and the identified patterns of actions and conversations of each participant, I present descriptions of the roles that I identified from the Loughheed project in the following paragraphs. Although the perceptions of roles in the team varied, it is possible to identify some of these roles as characteristic of particular members, as well as roles that changed from person to person.

Researcher. Armando took this role on the team, a role that team members considered in different ways. Sofia mentioned that he did 'support or validate,' and 'add up to' teacher ideas. For Arnold, Armando's role as educator was also as a provider of alternative ways of assessment—the use of a rubric. Additionally, she referenced to this role as an authority (Chapter 5, p. 69). For Brad, Armando was a type of expert and

supervisor. Although every teacher had a different perception, the role, and position, of Armando as a researcher was acknowledged by every participant and distinguished him from the other members of the team.

Facilitator. This was a role that Armando also took on during the project. It entailed not only bringing ideas and material to the team, or organizing the meetings, but also how teachers considered the project. Teachers were highly motivated and worked outside the meeting having discussions and preparing themselves for the regular meetings. The fact that this case of collaborative design was a research project was a factor that shaped teachers' perceptions of the role of the facilitator. Armando's role as a facilitator was not only shaped by what he did, but also by what teachers did as participants on the project.

The role of facilitator was also held by Sofia and Arnold in some cases. As a means to facilitate the design of the artefacts, writing the lesson plans was an important part of the project. Sofia facilitated the collaborative design in this regard as she typed up the two lesson plans designed during the project. Arnold brought materials and literature to the discussion, which is one way of facilitating by enriching the discussion with this material.

Promoter. This was the role played by Sofia by serving as a liaison between myself and the other teachers from Lougheed secondary school. This role was strongly related to the collegiality that the teachers of the team had before the project started (Chapter 5, p. 75).

Sceptical voice. This was the role that Brad played by constantly wondering whether, and how, the selected tasks would contribute to the proposed goals for the artefacts. This role was consistent with Brad's position as a learner: He needed to understand why, and how, the use of patterns would contribute to making students write algebraic expressions during the first round of the project. Sofia also played this role. She wondered about the use of the 'cube' problem (Chapter 5, p. 63), and actually proposed to dismiss it from the lesson that was planned during the first round (p. 64).

Expert. Members of the Lougheed team showed different areas of expertise. One was the *data base expert*, played by Arnold, who not only had access to resources

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such as books and journals, but was also up to date and could bring those resources to the discussions in the meetings. Arnold's role as a database expert is consistent with her interest in, and perspective about, the literature. Another type of expert was the *mathematical expert*. Brad perceived Sofia as an expert in mathematics who usually solved the problems, and Arnold perceived her as the one who proposed most of the problems for the artefacts. This is consistent with the passages recorded from the meetings; for instance, when she proposed the cube problem (Chapter 5, p. 59).

In the previous descriptions of the roles it is possible to identify three features. First, the roles are, for the most part, the result of an ongoing storyline. With the exception of the role of researcher, the roles were identified from the types of contributions—e.g. comments and activities—of each participant. Second, the personal interest of each member shaped their contributions, and thus their roles. For instance, Arnold gave a very important value to the literature and was interested in learning about Japanese instruction and lesson study during the project. She asked questions about lesson study and constantly brought articles and books to the meetings: Her contributions often included a reference to a book, a study or a literature source. And third, some roles depended on individual perceptions. For example, Armando's role as researcher was not perceived in the same way for everyone. Likewise, Sofia was a mathematics expert in Brad's eyes; nobody else described Sofia's role in this form. In order to acknowledge these features as possible characteristics of the roles within the Lougheed team, I use the concept of *position* (Langenhove & Harré, 1999). In this way, the role not only includes a set of pre-established tasks that someone has to perform, but also the storyline among participants, their interests, and their individual perceptions of others' roles. As shown in this section, such roles and positions shaped what people said and did during the project, and therefore, influenced the conversations and actions of the team. For example, the off-task conversation triggered by Brad on February 27th (this chapter, p. 96) shows not only his positioning as a learner, but also the roles of Armando and Sofia as experts giving advice of how to approach the topic of trigonometry. The role not only shaped what particular participants said, but also had an influence on how the other members perceived what was being said. This was clearly stressed by Arnold in her comment during the first group interview “I would perhaps give

more weight to what you say just because in theory you have more background knowledge. ... You are becoming a professional in this area“ (Chapter 5, p. 70). I, Armando, wanted to be seen as a colleague, as another high school teacher. After this first group interview, I knew that this was not the case.

6.3 Conclusions

The results presented in this chapter represent my findings during the first stage of this study. In this stage I approached the first two research questions: How can we characterize the participants' interactions during collaborative design in the case of the Lougheed team? How can we identify factors that promote teacher professional growth in such interactions?

Two main themes were developed during this study using mainly the data presented in Chapter 5. I see these two themes as dimensions that describe interactions within the Lougheed team during collaborative design. The first theme was the focus of the conversations and actions during the process of collaborative design. The classification of on-task and off-task moments in this process showed the constant change of the focus of conversations within the team, as it can be seen from the graphs shown in Figures 6.1 and 6.2. For the on-task moments, four categories were used to describe the conversations and actions of the Lougheed team: (1) anticipating, (2) pursuing coherence, (3) achieving goals, and (4) team organization. I used the metaphor of a braid to describe how these categories related to each other. The on-task moments can be described as a braid made of the first three categories as strands (Figure 6.3) and the fourth category, team organization, as a direction to the activities and conversations of the team.

The off-task moments were classified into four categories: (1) teachers' practice, (2) mathematics and educational context, (3) collaborative work, and (4) casual conversation. Although it is not clear whether off-task conversations would contribute to the designing of the artefacts, the first two categories often represented learning opportunities for team members. The outside parts of both on-task and off-task conversations are evidence of teachers' activities in the school during the project. They

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not only held discussions outside the regular meetings of the team, but also started to incorporate, as Arnold and Brad claimed (Chapter 6, p. 84), some practices in their teaching as a consequence of participating in the project. Thus, the categorization for the teachers' conversations and actions presented here served not only to classify moments of the meetings for the designing of the artefacts, but also to identify occasions for professional growth in participant teachers. As Brad mentioned in the individual group interview, he learned about mathematics, and teaching mathematics (Chapter 5, p. 77), from both the on-task and the off-task moments of the design process (e.g. Chapter 6, p. 96).

The second emerging theme explored in this chapter was the roles that the members of the Lougheed team played during the project. I found that these roles, and the positions taken by the team members, were influenced by three factors: (1) the collegiality of the team, (2) the fact that this was a research project focusing on participant teachers, and (3) the interests that each teacher had to participate in the project. On the team there were some fixed roles, such as the role of Arnold and Sofia as teachers who implemented the lessons in their classrooms. Armando's role as a researcher was not perceived the same by the each of the teachers: (a) for Arnold, he had a special position as a researcher because he would end up contributing to the literature; (b) for Brad, Armando was an authority similar to a supervisor; and for (c) Sofia, he brought support and validation of their ideas. Regardless of these different perceptions, Armando was distinguished as the researcher within the team. The fact that people perceived others' roles in different ways suggests that the *role* is made by all the perceptions—possible different—of how a member is expected to act in a particular situation. The concept of *position* (Langenhove & Harré 1999) serves to extend the notion of *role*. The descriptions of the roles generated by the data in the Lougheed project share some features with this concept: (1) the ongoing storyline developed in time, (2) the individual interests of each member to participate in the project, and (3) the varied perceptions of the role of some team members. For this reason, in what follows I often use *position* often in conjunction with *role*.

The two emerging themes explored in this chapter are not isolated. Rather, they correspond to two dimensions of interactions among members of the Lougheed team:

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(1) the conversations and actions, and (2) the roles and positions of the participants. The former dimension provided details of what was said and done during the design process of the artefacts in the Loughheed project, whereas the later focused on who said what, and how that was interpreted by the others in terms of the position held by, and perceived from, each participant.

Once I have described the interactions among participants in the Loughheed project it remains to be seen what these two dimensions of interaction would look like in other cases of collaborative design. Such exploration corresponds to the second and third stages of the research and is reported in the following two chapters.

Chapter 7 Second Stage: Three More Cases of Collaborative Design

In this research study I was interested in characterizing interactions among members in teams of teachers' collaborative design in different modalities. Getting access to these teams was not easy and the Loughheed project represented a great opportunity to study, and participate in, a collaborative design project for the first stage of the research. The recordings from this case allowed me to conduct a detailed analysis of the conversations held during the design process, as presented in Chapters 5 and 6. In the second and third stages of the research, I intended to study other modalities of teachers' collaborative design. However, my access to participants and the meetings for the design of the corresponding artefacts differed significantly from the first stage. Instead of participating as a member of a team, and recording the sessions held for the design process, in the second stage of the research I conducted interviews and surveys with people participating in collaborative design, and used the literature, in the third stage, as a second-hand data. This chapter contains my findings from the interviews conducted in the second stage of the study. The source of data in this case represented a limitation: In contrast to the Loughheed team in which I had direct access to the sessions for the designing of the artefacts, in the cases present in this chapter data were limited to the perceptions and memories of the interviewees. Although this type of data was different from the type of data used in the first stage, I was able to identify resonances and dissonances of the Loughheed team with each of the different modalities examined in this chapter.

The resulting categorization developed in the first stage of the study was used as a frame to analyse the three cases presented in this chapter—focused coding. This analysis was conducted with two main purposes. On the one hand, I was interested in determining the extent to which the characterization developed from the Loughheed project describes participants' interactions in other cases of collaborative design. On the

other hand, identifying resonances and dissonances among these cases and the Lougheed project served to expand such a characterization. This chapter is devoted to the analysis of these three cases.

The initial collection and analysis of data during the second stage of the research informed the collection of further data. In the first section of this chapter, I present a chronological account of the process of accessing research participants, as well as the designing of the corresponding questions for surveys and interviews. The results of analysing the generated data are presented in the subsequent sections, which correspond to each modality of collaborative design included in this chapter.

7.1 Contacting Participants and Generating Data

The three cases studied in this second stage of the research (Table 7.1) consisted of: (1) a professional development programme, (2) a school district initiative, and (3) an independent lesson study group. The professional development programme case corresponds to Masters courses in mathematics education, as well as workshop for mathematics teacher professional development given by a professor from the Faculty of Education at Simon Fraser University. The requirements of both the courses and the workshops included the collaborative design of mathematical tasks. The school district case was an initiative to improve students mathematical performance in a whole district in British Columbia. This initiative used collaborative design as a strategy for both teacher professional development and curriculum design—in the form of mathematical tasks and assessment instruments. These two cases had a strong relationship. When I initiated my search for more cases of collaborative design after the first stage of the study, I had access first to participants in the professional development programme because the Instructor was a professor in the same faculty where I conducted my graduate studies in mathematics education. From this case I contacted people from the school district initiative. The third case was a lesson study group at the SIGMA institute in British Columbia consisting of teachers, mathematicians, and mathematics educators, from different schools and universities, interested in lesson study. Although SIGMA institute supported this group, most of the participants were not affiliated with this institution: participation was voluntary. General meetings of the group six times a year.

Smaller teams design lessons and report to the group in the general meetings. The access to participants in the third case was easier as I formed part of this group.

Table 7.1: The Three Cases in The Second Stage

	Professional development programme	School district initiative	Independent lesson study group
Participants	Teacher 1 Teacher 2		Organizer (teacher) Professor Prospective (teacher)
	Teacher 3 Instructor	Coordinator	
Description	Consisting of a Masters course in mathematics education, or professional development workshop for practising teachers. The courses and workshops were given by the same Instructors which is a professor in the Faculty of Education in Simon Fraser University.	Consisting of an initiative for collaborative design of (1) assessment instruments and (2) learning communities among teachers in one school district in British Columbia.	This was a lesson study group at the SIGMA institute in British Columbia. General meetings of the group six times a year. Smaller teams design lessons and report to the group in the general meetings.

The data for this second stage of the research were generated by semi-open interviews, internet surveys and my field notes from conversations with one of the participants, the Instructor of the professional development programme. The design of the interviews and survey questions was based on: (1) the conversations during the design process and the participants' roles, which are the emerged themes from the first stage (Chapter 6); (2) the responses to initial surveys and interviews; and (3) the role and position of the person answering the survey or being interviewed. The first set of data consisted of a survey that the *Instructor* from the case of the professional development programme answered by e-mail. He was an instructor for the Masters in Mathematics Education at Simon Fraser University, and an instructor in workshops for teacher professional development in mathematics. In both cases collaborative design was a requirement. The questions are presented in Table 7.2. These questions focused on the process of design and the roles held by participant teachers. I also asked for potential participants for my research.

Table 7.2: Survey for the Instructor of the Professional Development Programme

TCD stands from Teachers' Collaborative Design of teaching/learning artefacts intended to be implemented at classroom and having specific goals. Please comment on the following items according to your own experience.

1. What is your experience as a researcher and facilitator in TCD?
2. Describe your role in TCD.
3. What kind of TCD have you participated in?
4. Describe the design process of an artefact in a typical case of TCD.
5. Describe the different roles you find participants in TCD play during the design process.
6. Which roles promote/hamper participants' learning in this process?
7. How does TCD impact in participant teachers' practice?
8. What do you think are the teachers' interests and motivation to participate in TCD?
9. Who else do you think I could interview in order to collect further data?
10. Is there any thing else you find interesting in TCD that I didn't ask before?

The questions in the first e-mail survey, Table 7.2, were designed from the themes identified in the first stage of the research. The first two questions focus on the role of the Instructor. Questions 3 to 6 were aimed at exploring the process of, and roles held during, the design of the artefacts. Questions 7 and 8 were intended to identify occasions for professional growth in collaborative design, as well as teachers' interests and motivation to participate in this type of work. The last two questions served to identify potential research participants and unexpected topics. The Instructor sent written responses to the survey and we met for a follow up conversation about these responses.

After having a conversation with the Instructor, we agreed that he would contact teachers who had attended the workshops in which he had implemented collaborative design. He sent an e-mail with the questions in Table 7.3. The questions of this survey were negotiated with him and we identified participants from the responses to the survey. Those teachers who agreed to participate where contacted for a personal interview. Two of them agreed to be interviewed and another one accepted to answer the interview questions by e-mail.

Table 7.3: Survey for Participants in the Professional Development Programme

<ol style="list-style-type: none">1. Grades taught during time of collaboration:2. Can you give a brief summary of process that we used in our work together? Please pay attention to the goals of the project, the steps we went through to meet these goals, the challenges we encountered.3. Please detail your contributions to the aforementioned process.4. Please comment on the artefacts that were created.5. How did your participation in this collaboration affect you as a teacher during our time together and after?6. Please comment on the support structures from your school and/or district that were in place in order to make the project successful.7. Finally, and most important, please comment on the nature of our time together. What stands out the most about our meetings together? What was most useful to you as a teacher? Was it our structured time together or our unstructured time together? That is, was it the times we worked on the agenda or was it our side conversations? In essence, what went right? Can you give an example?
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I used the responses of the three teachers who accepted to participate in the project to design of the interview questions. Additionally, my previous experience conducting lesson study informed some of these questions. I knew about the challenges of finding the time and the place for conducting collaborative design (Chapter 1). In order to contextualize the cases presented in this chapter, I decided to include questions about the settings in which collaborative design took place for each case, including the place and the time for the meetings of the teams. The questions for these interviews are presented in Table 7.4. These questions were the same for each participant teacher. In the interview with the teachers I wanted to explore: (1) the nature of the designed artefacts; (2) the settings, history, and type of activities of the collaborative work; (3) the roles of participants; and (4) the off-task conversation.

From the interview with two teachers from the professional development programme I realised that they were participating in another form of collaborative design at their schools, or school district. This caught my attention and I contacted the professional development coordinator of the school district, the *Coordinator*, for an interview. In this way, I generated the data for the second case of collaborative design presented in this chapter, the school district initiative.

Table 7.4: Interview Questions for Teachers Participating in Collaborative Design

Questions	Purpose
<ol style="list-style-type: none"> 1. What were the goals of the artefacts you have designed? 2. What type of artefact have you designed? 3. How were the designed artefacts implemented? 	<p>Nature of the artefacts: to obtain information about the type of artefacts, as well as details about their implementations.</p>
<ol style="list-style-type: none"> 4. Why did you start conducting collaborative design? (causes) 5. How did the collaborative work started? (events) 6. What supports/hampers this work? 7. Who had contributed to the implementation of collaborative design? 8. How often did you meet? 9. Where did you meet? How long? 10. Did you share experiences and have feedback from one team to another? 11. Describe the type of activities/discussions you engaged in while working in collaborative design: both on and off task. 	<p>Collaborative work: to explore information about the settings, history, and type of activities of the collaborative work. The type of conversations during collaborative design are also explored in these set of questions.</p>
<ol style="list-style-type: none"> 12. How did you distribute labour (tasks)? 13. Was there anyone in the team who had some “expertise” or “specializes” on some activities or contributions in the design process? 14. Describe differences among team members. 	<p>Roles: to identify roles of participants during collaborative design.</p>
<ol style="list-style-type: none"> 15. What kind of aside conversations did you have in meetings designated to design an artefact? Provide with some examples. 16. Did someone in a design session ever: a) asked for advice or support in a specific topic or concern; b) had a casual conversation such as talking about the weather, or the family; c) discussed about the curriculum or educational issues in BC, Canada, or the world; d) discussed/solved mathematical problems or talked about history/philosophy of mathematics; e) talked about a problem with certain students or parents; or f) shared issues at your school? 17. Was there any other topic or issue you discussed in these designing sessions? If yes, elaborate on it. 18. Did you think some of those “aside conversations” or “off-task” moments were relevant? Explain why. 	<p>Off-task conversations to obtain information about the off-task moments during the process of design.</p>

When interviewing participants with a specific role, such as the Instructor of the professional development programme or the Coordinator in the school district initiative, I

included questions about the impact on teacher's professional growth as a consequence of participating in collaborative design. For instance, in the particular case of the Coordinator of the school district initiative I designed different interview questions. This interview was targeted for this particular role (see Table 7.5).

Table 7.5: Interview with the Coordinator in the School District Initiative.

<ol style="list-style-type: none">1. What is your experience as a development coordinator in TCD⁹?2. Describe your role in TCD.3. What kind of TCD have you participated in?4. Describe the type of artefacts teachers have designed5. Describe the type of goals for the artefacts that teachers have designed.6. How do teachers choose the goals for the artefacts that they have design?7. Describe the design process of an artefact in a typical case of TCD.8. Do different teams of TCD share experience and results one each other?9. Describe the different roles you find participants in TCD play during the design process.10. Which roles promote/hamper participants' learning in this process?11. How does TCD impact on participant teachers' practice?12. What do you think are the teachers' interests and motivation to participate in TCD?13. Did someone in a designing session ever: a) ask for advice or support in a specific topic or concern; b) have a casual conversation such as talking about the weather, or the family; c) discuss about the curriculum or educational issues in BC, Canada, or the world; d) discuss/solve mathematical problems or talked about history/philosophy of mathematics; e) talk about a problem with certain students or parents; or f) share issues at your school?14. Is there any thing else you find interesting in TCD that I didn't ask before?

The last interviews I conducted in this second stage of the research were with participants from the independent lesson study group. I used the same questions as with the teachers in the previous cases, Table 7.4. Finding people to be interviewed in this case was easier as I was also part of the group. After inviting different people to participate in the study, I contacted three participants from this group: an *Organizer*, a university *Professor*, and a *Prospective* teacher. As the Professor was also involved in mathematics teachers' professional development at the graduate level, I asked her a question about the factors that contribute to professional development in collaborative design.

At the end of this second stage of the research, I contacted at least three participants for each case of collaborative design presented in this case. Two

⁹ Teachers' Collaborative Design.

interviewed teachers were participants in both the professional development programme and the school district initiative. The participants in each of the three modalities are presented in Table 7.1.

The answers to the surveys, the field note from the conversations, and the transcriptions of the interviews were the data which I analysed conducting a focused coding (Chapter 4) process based on the characterization generated from the Lougheed project (Chapter 6). However, I was also looking for new elements not contemplated in the categories developed from the Lougheed team, so that I could extend, or refine, them. As a result, I identified resonances and dissonances between these cases and the Lougheed team. In the following sections I present my findings from this analysis.

7.2 The Professional Development Programme

The professional development programme case consisted of workshops for mathematics teachers given as either a professional development programme or as a part of a masters programme in mathematics education. In both scenarios collaborative design was conducted and teachers implemented the artefacts in their own classrooms. Although participants were mainly from British Columbia, teachers from different provinces also participated in the workshops. These workshops were led by the same instructor (the Instructor) who was a professor and researcher interested in mathematics teacher professional development.

Settings

The Instructor had been conducting these programmes for more than eight years in several school districts across British Columbia. The groups formed by the workshops went from eight to twenty participants and were usually held in one-day sessions, or in afternoon sessions in a few cases. They met again on another day allowing teachers to implement the designed artefacts in their own classrooms. The artefacts included numeracy tasks as well as assessment rubrics. One of the participant teachers described the tasks in the e-mail survey (Table 7.3) as follows.

We looked at developing numeracy tasks (i.e., fair share, planning, estimating) for students that were contextual to their lives and had a low floor, a high ceiling, degrees of freedom, a sense of ambiguity and a "liberating constraint." (Teacher 1)

The type of artefacts in this case differs from the type of artefacts designed by the Lougheed team. Although both cases included the design of assessment rubrics, the focus on a lesson and on a task marked a distinction. The Lougheed project was inspired by lesson-study and the main purpose of the design was to create, implement, observe, and debrief mathematics lessons. In the case of the professional development programme, the focus was on designing numeracy tasks, instead of lessons, that would be implemented by teachers without being observed.

Teachers had their own goals which were not the same as the general goal, as explained in the following excerpt of the interview (Table 7.4).

Teacher 2: Last year with [the Instructor] we were working toward a common goal as a group and so we all have ideas of where did we wanted to go as a group, but still we have individual goals toward our own development.

Although there were general goals for the artefacts, teachers brought their individual goals to the collaborative design. This also was reported by the teachers in the Lougheed team as they had different goals for the project—not for the designed lesson.

Conversations and Activities During Collaborative Design

The workshops included more than only the design of artefacts. During the sessions where participants met together, they were also asked to solve mathematical tasks as part of the Instructors' strategy, who described in the e-mail survey (Table 7.2) the process of design in several steps.

We begin with a common experience of doing a task like the kind we hope to eventually design. We then clearly delineate what we are trying to design. This usually requires the definition of certain terms. We set a time-line for design, field testing, refinement, more field testing, and implementation. Then we begin to design. I allow very rudimentary things to get tested in the field. I find that the variety of such things brings back very good feedback to the group. This feedback then allows the group to develop a clearer picture of what they are trying to do. Once the field

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testing is done, I help the team to write what I call a 'script'. This is a plan for implementation that helps to make concrete the details of enacting that have, until this point, not been treated explicitly. (Instructor)

The process of designing tasks in this case contrasts with the Lougheed team where the artefacts started to be designed after selecting the corresponding goals. In the case of the professional development programme, teachers started "doing" similar tasks to what they hoped to design afterwards and designing "very rudimentary things to get tested in the field" before developing a "clearer picture of what they are trying to do."

In the previous description of the process given by the Instructor it is possible to identify resonances with the on-task moments during the Lougheed project. Setting a time-line for designing, field testing, refining, and implementing entails the organization of the work during the process of collaborative design, which was categorized as team organization in the Lougheed project. The previous script is also evidence of conversations within the anticipating category described in Chapter 6. Field testing in this case resonates with Arnold piloting with her son (Chapter 5, p. 61) or Brad implementing the use of patterns in his classroom (Chapter 5, p. 57).

One of the elementary-level teachers who had participated in the professional development programme case explained in an interview (Table 7.4) the goals and process of generating a rubric aimed at assessing particular students' behaviours.

Teacher 2: [For] the artefacts I worked on last year, some of the goals were to develop some of the behaviours that we want to promote in the classroom. We looked at cooperation, we built a rubric according to the cooperation. We also talked about perseverance, but that was very hard to develop a rubric for. And, we also looked at communication and representation. ... we were trying to see what worked in the rubric and what do not and [what] changes to be made because sometimes it was improper to the task that we were doing.
So we wanted to look at the rubrics to see if they match the task and if they are appropriate to being used and so on.

From this excerpt it is possible to identify that participants in collaborative design had specific goals for the artefacts—rubrics for assessing cooperation in this case. The achieving goals and pursuing coherence components of the design braid (Chapter 6)

were present when the teacher explained that sometimes the rubric was improper for the tasks.

The feedback after field testing, which according to the Instructor was important in developing a clear picture of what they wanted to do, represented a moment for sharing experiences within the group, as mentioned by Teacher 3 when describing the process in an e-mail in response to the survey (Table 7.3).

We always solve a [mathematical] problem or two. We share what we have tried in our classrooms with students. If [the Instructor] is involved he always challenges us to take the next steps. (Teacher 3)

This last excerpt corroborates what the Instructor mentioned about teachers "doing" a mathematical task. The excerpt also mentions the instructor's role within the team challenging teachers to "take the next steps." While it is not clear what these next steps represented to Teacher 3, it is clear that the Instructor had a role within the group.

Roles and Positions

One important factor that resembles the collegiality among participants in the Lougheed team was that teachers felt comfortable working with specific people. When asked about anyone in the team who had expertise or specialization with some activities or contributions in the design process, Teacher 3 did not mention any expert or specialist. Rather he mentioned being comfortable working with some people, as we can read from the e-mail answers to the interview (Table 7.4).

I don't know if we had expertise, rather it was more who we felt comfortable with. (Teacher 3)

Thus, interactions were, for this teacher, shaped by a personal preference for specific people whom the teacher felt more comfortable working with instead of a level of expertise. This has a resonance with the Lougheed team where participants acknowledged their willingness to participate in the project as a consequence of seeing someone else participating—Sofia.

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When asked for experts, the teacher mentioned the assistant and the Instructor as mathematics and mathematics education experts. This distinction of experts is congruent with the Lougheed team.

The role of the Instructor in this case included leading and facilitating the collaborative design. Participant teachers played, in an initial part of the activities, the role of students, as the Instructor described in the following quotation from the e-mail survey (Table 7.2).

I lead and I facilitate. Usually, I get full day sessions so I usually structure some activities that are related to what we are doing. This puts the teachers in the role of students. I set guidelines and deadlines, I lead discussions, and I help. But I also try VERY HARD to build the capacities for the project to continue without me. (Instructor)

The role of the facilitator included structuring those activities that put the teachers in the students' role. Additionally, the Instructor tried to get teachers conducting collaborative design on their own. However, the role of the facilitator seemed to be relevant, as he explained later in the following up conversation: "Teachers lack of confidence in themselves. Without a facilitator, there is no goal or coherence" (Instructor). This comment suggests that the presence of the facilitator in the group influenced teachers' activities. The influence of the facilitator on the teachers interactions was not only affected by what he did, but also by how teachers perceived him. This is an example of the role as position within this case of collaborative design.

When asked about the role of participant teachers, the Instructor identified different roles regarding the way teachers engaged during the collaborative design in the e-mail survey.

Early Adopters – the ones most willing to field test early on. Finishers – the ones most willing to refine the task to implementation ready. Graphic Artists – these are the ones who are willing to create graphics for the task. Scripters – the ones most willing to finalize the script. Info Leaders – often the [artefacts] we create are for broad consumption and they need to be shared out. There are some of the participants who are willing to take on the role of doing this. (Instructor)

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The role of early adopter was present in the Lougheed team as well: For instance, when Brad wanted to try the use of patterns in his class. The roles of Graphic Artists and Scriptor resonate also with the Lougheed team where Armando designed some of the drawings for the worksheets and Sofia wrote the lesson plan. However, the Info Leaders is a new role of collaborative design that extended the roles developed from the Lougheed project.

Regarding the level of expertise, the Instructor only commented, in the follow up conversation after the e-mail survey, about one case where participants attributed a particular status to a member of the team.

Instructor: There was the one who proposes a lot of ideas and strategies and everyone else likes her. She uses some shared jargon probably coming from some in-service professional development program.

With the exception of this case, teachers, according to the Instructor, were not differentiated by a level of expertise. However, a differentiation in the role and position of the participant teachers appeared. As these professional development programmes included teachers from both the elementary and the secondary level, the Instructor was able to identify distinctions of these two types of teachers. In Table 7.6 I summarize these distinctions which were mentioned by the Instructor in the follow up conversation after the e-mail survey.

Table 7.6: Differences between Secondary and Elementary Teachers

<i>Teachers 8-12 Secondary</i>	<i>Teacher K-3 Primary</i>
They say that they teach math.	They say they teach children.
They wonder: How am I going to report this?	They Wonder: How am I going to make this work in the classroom?
Concerned with motivating students.	Motivation is not a concern.
They listen to primary teachers carefully.	They don't care about secondary teachers.

These differences between secondary teachers and elementary level, K to 3, teachers also reflect a different type of interaction and interest during the design process. This distinction suggests another type of role, or position, characterised by grade level.

The design process in this case also involved students' contribution in the process of constructing an assessment rubric. When asked about the design process, Teacher 2 explained how students were involved in designing a rubric for assessment (Table 7.4).

Teacher 2: Quite a process! First I had to develop it with the kids. ... I used a computer and ... projected the rubric on to the board. And the kids in groups, then, came up with what we were looking at: What does representing a mathematical problem would look like?

I have one group working on *not yet meeting*, another group working on *meeting*, and another group working on *fully exceeding*.

And then, from there we discussed as a group and put it up as the rubric: so we have a draft now, which is a living document. And then, from there I looked over and picked up the language that was repetitive and got the stuff they wanted. I brought it back to them again, for their feedback in smaller groups, to see what they like or don't like.

From there, we actually printed it up. The kids used it for self evaluation first. And then, I would use it for whatever task.

And then we see if actually it worked, because some times it is not until you actually use it that you realise that you are still in the same component ... so we had to make changes once again.

Students' participation in the designing of the rubric in this case contrasts with the Lougheed team in which students were not involved as designers. A new component can be added to the roles of participants in the design team, the role of the students as designers.

Off-Task Conversation

When asked for the activities that the teams usually went through in collaborative design, on and off-task, this teacher explained his perception in the following excerpt of an interview by e-mail (Table 7.4).

As for On Task/Off Task, I find that when I work with other teachers there is a good deal of *off-task-on-task* time. We always get talking about tangential topics. Often one statement from a person will send you investigating something related but not truly *on task*. (Teacher 3)

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As we can read in the previous excerpt of the e-mail response, Teacher 3 explained how a tangential topic might trigger the teacher to investigate a topic related to the designing of the artefacts, but not "truly *on-task*." This activity resonates with the outside off-tasks moments of the Lougheed team: The activity was conducted outside of the regular sessions and was not an on-task moment. This teacher also stressed the importance of the off-task moments when asked whether these moments were relevant.

These are often the richest. When one personalizes their learning/development it becomes very rich. This is what occurs often when off-task. The key is not letting the off task drift off into meaningless activities. (Teacher 3)

Finding the off-task moments as the *richest* resonates with Arnold's opinion (Chapter 5, p. 77). For her the most important parts during the meetings for collaborative design were when teachers talked about things that were not necessary related to the lesson plan that was being designed during the Lougheed project.

Off-task conversations were, according to Teacher 2, an important component of the collaborative work related to the collegiality which was also present in the Lougheed team, as we can read from the transcription of the interview (Table 7.2).

Teacher 2: We would go off-task quite a bit, but all the conversations were still meaningful and there were usually about things we have trouble with.

... any time someone else has a trouble with a concept or not necessarily understanding the best way to implement something or not sure about ideas, ... that is where the conversation started happening. Or we had another problem as a group that we did not quite know how to address or didn't realise that somebody else has the same concern, so we just opened up, you know, open up a new direction. ... The information is actually interesting because you walk away [knowing that] you are not the only one may be feeling the same way or the same frustration or have the same questions.

Interviewer: How do you feel when you realise that you are not the only one?

Teacher 2: Oh, it's good. If you are not the only one is distressing ... that people have the same concerns about the programme or the students.

In the Lougheed team teachers also communicated concerns or frustration as part of the off-task conversations. For instance, Brad's comments about the use of calculator in class (Chapter 6, p. 96). In the previous transcript, Teacher 2 explained how they felt after sharing their concerns or problems, which resonated with the Lougheed team in the teachers' practice category.

Teacher's Learning

From the data generated in this case of collaborative design I also found participant's opinions on the impact on teachers after participating in the design process. In the following excerpt, the Instructor comments on two impacts at two different steps of the process.

One of the two impacts comes when they field test. This leads to many implementation discussions.

The other impact is when we first define and delineate the task. This usually is accompanied by a philosophical shift to allow for the task to fit into their conception of teaching or mathematics curriculum. (Instructor)

The Instructor also mentioned that there was another transformative effect not explicit at the beginning. He had in mind hidden goals for the professional development programme. For example, according to the Instructor, in a workshop in which generating numeracy tasks was the announced goal, teachers also started to use more group work in their classroom, and they also changed their assessment strategies.

The three teachers interviewed reported changes in their practice as a consequence of participating in collaborative design. The following excerpt presents the answer to the e-mail survey of Teacher 1.

My approach to teaching math has changed. I now look at the *big ideas* and do my best to engage students in the process of learning. The classroom focus is on the process and a student's ability to represent and communicate their thinking, rather than the product (finding the "right answer"). I have tried and am very interested in continuing to do so.
(Teacher 1)

The changes Teacher 1 mentioned is a focus on students' thinking, as opposed to only focusing on their answers to mathematical tasks. The focus on looking at

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students' thinking, or ideas, and their solutions is also mentioned by Teacher 2, who was also interested in providing feedback using the rubric developed during the workshop.

I preferred using the rubrics that were developed for measuring their explanations or justifications of their solutions/ideas. I found it easier to give immediate feedback when I marked their work.

After the collaboration, I'm trying to expand the samples that I'm using for assessment in class. I'm trying to go beyond the typical quizzes/tests etc. I'm hoping to try and incorporate some of the observable behaviours to better understand the students I'm teaching in the classroom. (Teacher 2)

The previous excerpt also includes a shift from using only tests and quizzes to the use of other methods of assessment in class. Trying to expand the strategies for assessment suggests a personal plan for professional development by incorporating these strategies in her practice. Teacher 3 mentioned something similar in this sense. However, he also indicated a change in his enthusiasm for teaching.

[The] workshops have renewed my enthusiasm for teaching and have given me direction for professional development. (Teacher 3)

An important change I noticed from the data was that teachers became involved in collaborative design at their schools and in their school district. For instance, Teacher 2 described how she worked at school when she was asked to describe the collaborative work during the workshops.

Teacher 2: And that is very similar to that collaboration that we had last night. You come with your goals from the previous session, what steps did you do to try to reach that goal?, and did it work? And did it not? And then you listen to all the other ideas that might support you in to getting it what you want it be, set another goal and then we have to go back and try it. It's quite a process. But it is a valuable one. Because you get to hear different ideas.

The interview with this teacher was conducted approximately one year after participating in the development programme. Therefore, the previous transcription represents evidence of collaborative work done after the programme. This teacher was also working in the case of the school district that initiated collaborative design. This case is presented in the next section.

Summary

The settings in which the professional development programme case of collaborative design was conducted had strong differences from the Loughheed team, such as: the lack of observers in the implementations, and the place and periodicity of the the sessions. However, there are resonances in both cases in terms of the characterization described in Chapter 6.

1. Instances of each of the four categories of the on-task conversation were found in the data in this case.
2. In both cases teachers conducted field testing, or piloting.
3. Some of the roles such as the graphic designer and the scripter, or writer, were present in both cases.
4. There were several off-task moments which teachers found relevant.
5. Teachers felt comfortable working with particular colleagues.
6. Teachers brought their personal goals into the collaborative design, which had specific general goals.

The dissonances that I found from the professional development programme with the Loughheed team consisted of several aspects, which are summarized as follows.

1. The type of designed artefacts were different. Although in both cases rubrics were designed, the focus was on designing lessons in one case, and on numeracy tasks in the other.
2. The process of design included, in the professional development programme, solving tasks in order to put teachers in the place of the student. Although in the Loughheed case teachers solved some of the problems for the designed lessons, the motivation for this was different. Whereas in one case the Instructor tried to put teachers in the students' role before start designing, in the other teachers were already planning a lesson when they solved the mathematical problems.

3. Experts in the professional development programme case do not seem to play a relevant role. Although the Instructor mentioned one case, teachers did not make any reference to a level of expertise from other participants.
4. There was a difference in the interactions during collaborative design which depended on teachers grade level. This case included both elementary and secondary teachers, which is already a difference from the Lougheed team. The Instructor identified a difference in the type of contributions between these two groups during the process of design.
5. The *info leaders*, who shared the results with other teachers, were not present in the Lougheed case.
6. Students in one case of the professional development program were involved as designers of a rubric. This did not happen in the Lougheed project.
7. The Instructor in this case and Armando in the Lougheed project led the collaborative design from two different positions. The former was a Professor in a context of professional development, the latter was a doctoral student in a context of research on teachers conducting collaborative design.

7.3 The School District Initiative

The second case of collaborative design presented in this chapter consisted of a school district initiative for the design of mathematical tasks. As mentioned before, Teacher 1 and Teacher 2 from this school district had participated in the workshops of the professional development programme described in the previous section. In this case teachers participated in collaborative design as part of their duties. The collaborative design was conducted either in their same school or in a different place of the school district. There were two different types of collaborative design in this case: (1) the teams of design for district assessment in mathematics, and (2) the learning teams. For both

cases, there was the same staff coordinator for numeracy from K-12, the Coordinator, who I contacted for an interview.

Settings

One of the purposes of collaborative design in this school district was to create a district wide assessment for mathematics in grades five and eight. This assessment was focused on problem solving, as explained in the following transcription of the interview conducted with the Coordinator (Table 7.5).

Coordinator: I have been doing that for five years. And over those five years we have implemented or constructed teams for various reasons. One of them has been to create a district wide assessment in math for grade five and grade eight with a problem solving focus. Those teachers come together in a team that has been anywhere from five to ten people.

They come together to construct two problems that would be used to conduct district wide assessment and getting district wide data. That uses the BC numeracy performance standards and focuses on the representation and communication strands. So, how students are communicating their thinking as they are solving the problems.

From the previous description I identified a dissonance with the Lougheed project. The size of the teams varied from five to ten teachers. The team changed from one year to another and the constructed problems were used for district assessment. Both the size of the teams and the purpose of the collaborative design contrast with the Lougheed team which had three teachers focused on the design of two mathematics lessons and one assessment rubric.

In addition to the design of artefacts for district wide use, the school district also supported learning teams which might conduct collaborative design. In these learning teams numeracy and problem solving were also a focus for the designed artefacts.

Coordinator: There is a second type; and its foundational in our district. We call them learning teams. It is an action research model ... And so, we would have learning teams around numeracy, usually around problem solving, where teachers would like to do problem solving more in their classrooms. And so they would come together and, if the group is six to eight teachers, I am the facilitator of the group

and those teachers are released for part of the time, and part of the time they use their own time to really investigate an issue they have with their instruction or their assessment.

This second type of collaborative work in the school district resonates with the Loughheed team: teachers met at the same school, and the goals were selected by the teachers. However, in this case the time for collaborative work was extended from the regular working hours. The time was negotiated with the school. The learning teams were also facilitated by the Coordinator when they were six to eight members, which resonate with the Loughheed team in which was facilitated by the researcher, Armando.

Conversations and Activities During Collaborative Design

The goal for collaborative design impacted the activities of participant teachers. For the design of district wide assessment, the goal was to develop assessment for mathematics communication and representation strands of the curriculum standards. In the following transcription, Teacher 1 explained how the selected goal influenced the design process.

Teacher 1 The goal ... is to improve students' ability to represent their mathematical thinking and also improve on their ability to be able to communicate what it is that they are thinking and what it is that they have worked out. We were working together as a team to come up with tasks that basically would be engaging for students. There would be tasks that students can enter into at their own level and take whatever they like. So they have a low floor and a high ceiling: there are all these different entry points for them to be able to work from. And we were also looking at trying to come up with tasks that are varied. So, they are not always doing the same type of task. ... We have done tasks around themes that they would be familiar with like birthday parties, planning school events. ... It's trying to come up with something that teachers feel comfortable with, and like I said, the students will be really engaged in.

From the previous transcription, I identified a resonance with the design braid developed from the Loughheed team. As there was a selected goal, teachers had to make sure that the designed tasks contributed to the achievement of this goal: the *achieving goal* category of the on-task conversations (Chapter 6). Additionally, teachers were trying to create engaging tasks for students. For me, this suggests that there were

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conversations related to the *anticipating* category. In this case of collaborative design there was an explicit focus on students' engagement. The idea of having tasks with a "low floor and high ceiling" means that students with a low mathematical level would be able to approach the problem and, at the same time, students with a high mathematical level would keep working on the problem for a long period of time. The low floor and high ceiling idea also suggests the existence of *anticipating* discussions during the design process. The Loughheed team also discussed entry points for students with a low mathematical level, as Arnold mentioned (Chapter 5, p. 61). Looking at themes that students were familiar with was an instance of *pursuing coherence*. The tasks would be immersed in a broader context in which students have some familiarity.

As this case of collaborative design involved implementing and refining the tasks, testing them in the classroom and discussing the results was part of the process of design. Although piloting was also used in the Loughheed team, in this case the testing in class was conducted systematically including several implementations of the tasks. In the following transcription we can see how Teacher 1 explained this process.

Teacher 1: It is a lot of giving trial rounds with [teachers'] own classes. Finding other teachers that would be willing to let us experiment on their classes, just to work out the bugs and make sure the task is ready to run. ...

So, [the task] will get sent out to schools that are willing to participate in a numeracy task, and then there will be a script for the teachers. The teachers then will implement it with their classes. ... And then we would have a district wide marking day ... we will come in and we will talk about it. ... But essentially it's getting there, out to the school, having the students do the task, and then looking for exemplars and being able to quote the students as to whether they are *not yet meeting*, *minimum meeting*, *meeting* or *exceeding* in the areas.

The use of students work, or *exemplars* was important during this process. As can be seen from the previous transcript, the design not only included the tasks, but also the rubrics for the corresponding assessment, which was the purpose of collaborative design in this case. The exemplars served to refine both the task and the description of the students' performance.

The previous transcription also shows the role of teachers who, without participating during the initial process of design, implemented the tasks in their classrooms. Implementer teachers get involved in the process of task design, but at the stages of field testing and reviewing exemplars. The activities and conversations at this stage of the process included the comments of these implementer teachers.

Roles and Positions

Teachers who implemented the tasks played a particular role during the design process. The Coordinator explained in the following transcript, how these teachers were involved during this process.

Coordinator: Those teachers, they are also involved in leading the marking sessions for that. So they would have some role in helping set exemplars. ... The teachers who conduct the problem ... are released from their classrooms to come and do the marking for their own students.

The role of implementer has a strong dissonance with the Lougheed team in which implementers were part of the design team from the beginning. Another dissonance is the absence of observers during the implementations in the school district case: As the Lougheed case was inspired by lesson study, the implementations of the designed lessons were observed by the team.

The role of the coordinator in this case was multi-layered. When asked about his role, the Coordinator described several features of the activities he conducted in order to support and coordinate the collaborative design within the school district.

Coordinator: I am the coordinator of the team, so I invite teachers to get together. I have connected with [the Instructor] who has been an outside consultant. Most of the staff development that we do involves some kind of outside academic voice to bring in most of the current research on practice.

So, my job is to coordinate the team. Set date, keep the team on track, I do the word processing, I do all of the background piece, to help them get to where they need to go. So, I really facilitate the discussion.

I participate to a certain degree because I don't have a classroom

to pilot it and because I have not necessarily taught the grades that they have come from. I ask the questions that they may not think about. I can be an outside voice to the group because I can ask them if the language that they are using to the prompts of the question is too difficult, too easy ... It is really my job to sort of push the group ahead, facilitate that conversation, so ... I will obviously contribute with pieces.

The previous transcript contains evidence of both the role and the position of the Coordinator. The roles he played included: inviting teachers and expert speakers, keeping the team on track, helping with the language of the artefacts, and doing word processing. The Coordinator described himself as an outsider in the sense that he was not teaching the same courses for which the artefacts were designed. Being an outsider represents a position of the Coordinator within the team.

When asked about the level of expertise and the roles that teachers held during collaborative design the Coordinator did not stress any difference among teachers' expertise in the teams.

Coordinator: Most of my experience is that they are all very much equals. Some people will ask questions, some people will offer solutions. It's very much an equal dialogue. I mean, I am the closest that comes to really be a leader only because I control the time to make sure that they are using the time well and make sure they are not getting off-track: that they are not talking about students behaviour for too long and those kind of stories.

The previous transcript explains that teachers had the same level of expertise. The lack of experts contrasts to the mathematics expert, Sofia, and the data base expert, Arnold, of the Loughheed case (Chapters 5 and 6). However, whereas the expertises in the Loughheed cases were identified by the interviews and compared against the recordings, the previous transcript only reflects the opinion of the Coordinator and some teachers might identify expertise in some of their peers. The Coordinator also played a role of time-controller. He controlled the time and made sure that teachers did not go off-task. Moreover, he suggested that for the case of the learning team the role of the facilitator was vital.

Coordinator: An outside facilitator is vital. We have learnt that when there are learning teams, if they are allowed to facilitate themselves it would not work.

Having a facilitator in the teams of collaborative design resonates with the Lougheed project. In the school district initiative case the Coordinator played a role of facilitator, whereas in the Lougheed team Armando held this role. However, there is a difference in the type of participation of these two facilitators. The reason is that the Coordinator was not an expert in mathematics, as he explains.

Coordinator: To certain degree, I am not a mathematics expert. ... It's very much not about my mathematical background [that he got the position as coordinator], it's my skills as a coordinator-facilitator to help set up professional development. But, I mean, I put in the effort about the curriculum and philosophical changes to be able to help teachers understand why things are changed. Because I read the documents, I know the background, I talked to people like [the Instructor] and other academics, so I can bring that to the table to give them the bigger picture.

From the interview with the Coordinator I identified a new role within teams of collaborative design. In a design team, an administrator might want to take part in the design process, which, however, might hamper the discussion within the team, as explained in the following transcript.

Coordinator: Sometimes administrators being part of the team can hamper what happens. ... Sometimes they are there in a ... supervisory capacity. When I do a learning team and a principal wants to take part in it, I require them to have a classroom that would allow them to apply some learning, too. When I haven't done that, then they are kind of being there to find out what teachers are doing. They are not there to learn, they are there to supervise the teachers. And that definitely shuts down the discussions, it hampers dialogue. There is just a different atmosphere. When those administrators come and they have a classroom where they apply something, like they have a project that they are doing as well, then they just become part of the team, and it's very good. It's actually great for the teachers to hear them learning and struggling, too.

The participation of an administrator in the teams of collaborative design represents another variation in the role and position of the participants which was not present in the Lougheed team. An administrator would have a particular position in the team which might be akin to a supervisor rather than a team member, which might hamper the collaborative work. However, as indicated in the transcript, when an administrator participated in the team and had to implement in his or her own classroom,

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then teachers hear them learning and struggling, which according to the Coordinator, might be beneficial for teachers.

In order to implement collaborative work in the district, economical resources were needed. When describing the collaborative work in the district, Teacher 1 commented on another feature of the role of the Coordinator: the funds seeker.

Teacher 1: We meet ... depends on the year and the funding that [the Coordinator] has. And generally meet about every five weeks, I would say. Five to six weeks. But we have a pretty decent chunk of time to meet with them. It's generally the entire morning.

This very same teacher explained that the presence of the Coordinator did not impact on the performance of the team. She participated in the design of tasks for district wide assessment.

Teacher 1: There have only been a couple of times where [the Coordinator] had to step out for a meeting, or we wanted to meet, but the days we were available didn't match with the day he was available. It's very weird, but when it does happen this still keeps going, which I find pretty impressive. ... There have been other groups that I have been involved with where the facilitators had to step out and it changes. ...

I think it's because for the most part, we all are pretty seasoned as far as this goes, so sort of knowing what the expectations are. We all seem to be pretty goal oriented. we don't really want to waste our time together. We would like to get it done, so we stay on task.

In the cases of the teams of design for district assessment, the presence of the Coordinator was not important in order to have teachers working on-task, as explained in the previous transcript. The presence of the coordinator seemed to be more important in the case of the learning teams, probably because the teams for the district task design were already very focused and the goals were clear and general.

Off-Task Conversation

Off-task conversation were reported in the district initiative case, as indicated by Teacher 1 who had been participating in collaborative design for several years.

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Teacher 1: There have been years where we were really on board committed and dedicated and then we get the job right the way. And there have been years that we had a lot of chatting.

The Coordinator elaborated on the type of off-task conversations in this case, as in the following transcription: teachers talked about students and parents.

Coordinator: We also always have the question about parents. How do we help parents see the value in this? So, it still is connected to the work, but they are thinking more widely. We always have discussions about... it's always stories of certain kids.

Having off-task conversations resonates with the Lougheed team. Casual conversations were held in the school district initiative as well. However, the position of the Coordinator triggered other type of off-task conversations. As he had a position within the school district, he was informed about school policy and had information that teachers might want to know.

Coordinator: Oh, all the time, yes. We talk about families, we talk about kids. Because my role is as a district person, most of them ask about district policy: why is this happening, or what is happening with the budget? Or questions like that.

Discussions about how to involve more teachers in collaborative design were also part of the off-task conversation, as the Coordinator explained.

Coordinator: We often have conversations about how can I get colleagues to buy in, which doesn't necessarily pertained to the task: "How can I create a collaborative culture really back in my school? What can I do? Because none of the other teachers would do this, I am the only one."

I find a resonance of this case with the Lougheed project regarding the off-task discussions triggered by particular positions of members within the teams, as well as the settings in which collaborative design was immersed. The position of the Coordinator triggered conversations about school policy and how to engage more teachers in collaborative design. Likewise, in the Lougheed project Armando's position as the researcher, and the fact that the project was a research study, triggered off-task conversations. For instance, Arnold wanted to know more about lesson study (Chapter 5, p. 55), and Brad explained what teachers did outside of the regular meetings (p. 55).

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Additionally, discussions about teachers problems and education in general were held, as reported by the Coordinator.

Coordinator: The curriculum has been the biggest thing now. I am doing this study in the middle of the biggest curriculum change ... within Canada in twenty years. So, they are constantly talking about resources, what is available, when is something coming, what it would be better to use, what are other people doing? I am asked often in what way can we help parents?

In the school district initiative case off-task conversations were valued to the point that specific time for them was designated in the meetings. In the following transcript the Coordinator described the *five minute write* part of the sessions stressing its importance to the participants.

Coordinator: Now, one of the things that we have done, because we know the people need to have these conversations, is that we start all of the sessions with this five minutes, we call the five minute write.

In the following transcription, Teacher 1 gave more details about the five minute write. Meeting teachers from a different school also influenced the type of off-task conversation, as she explained.

Teacher 1: [The five minutes write is] just an opportunity to get some things off of your chest without breaking any confidences: "I got students in my class this year that I am finding really difficult to work with. Have you had anyone similar?" So it's more sort of helping each other.

When they have an opportunity to do that [side conversations], I think it makes feel better.

I think teachers need opportunity to talk to other teachers. And I think some times it really helps to talk to other teachers that aren't at the same school you are at. Just because they don't have any preconceptions about who you may be talking about them, and then you can talk more in confidence I think they can offer you a fresh perspective, I think that can help sometimes.

Although teachers in the Lougheed team valued off-task conversation, the five minute write component in the school district initiative represents a difference in the ways of working. In the later case off-task conversation were included as part of the meetings with a specific duration.

Teacher's Learning

Teachers involved in the design process were not the only ones who benefited from the collaborative design. Those who implement the resulting artefacts had also changed their practices. In this way, this case of collaborative design also had an impact on the teachers who used the artefacts both during the process and as final products.

Coordinator: It's meant to be professional development for the teacher who creates the problem, but also for the teacher who uses it.

The collaborative work among teachers in the school district was lasting. The Coordinator explained how teachers kept participating in collaborative design activities in their schools.

Coordinator: Well, I guess the one thing I find is that the changes stick. They are lasting changes. ... I find that most of the teachers who have been involved with this, if they have been involved with the learning teams they keep going. Usually the team runs for two or three years, and by then people have made them part of their practice. So, it does: they are lasting impacts.

This situation also contrasts with the Lougheed team which was dissolved after the project finished. The school district support for collaborative design seemed to be crucial in keeping teachers working in this fashion.

The effects of collaborative design on students improvement were also mentioned by the coordinator, who also stressed the importance of incorporating representation and communication in collaborative design.

Coordinator: In five years, it came from fifty six percent to eighty four percent. ... And the grade fives were doing much better ... they were about seventy and now they are about ninety percent. ... [Teachers] changed their teaching practices, the kids have gone better because they are incorporating teaching representation and communication into their math. ... I would say this type of model, although [some teachers] are not part of the team, the effects of the team are the change in [teaching] practice more widely than I think it was anticipated.

Teacher 2 had also participated in the development programmes and participated in collaborative design in the learning teams of the school district. She explained that the organization of the learning teams had been evolving.

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Teacher 2: It wasn't necessarily organized as it is now. I think collaboration itself has gone through a process at our school, I think more teachers are involved now than they were in the past, because of the way it is structured. At first it was informal.... if you have a common interest or common subject that you want to design a unit for, or some lessons for outside of our time-table and this collaboration time and we would get together.

But now we have evolved to having other people from the district coming in and leading the collaboration time ... We had [the Instructor] and [an assistant] who came in for those who were interested in support for their mathematical programmes.

The connection between the school district and the professional development programmes of the previous case is important because teachers received instruction for task design that prepared them to work at their schools, or in their school district in this case. However, the school and district settings also facilitated the collaborative work among teachers.

Teacher 2: We have collaboration built into our time-table as a team. So even as a team we had an opportunity to get together and talk about our programme. And if we were fortunate we had enough time to talk about our curriculum, and our process, and our lessons, and what worked and what didn't and so on. So we have a lot of opportunity to collaborate in our school.

In this case school settings facilitated collaborative design, which contrast to the Loughheed project that was the only opportunity for teachers to conduct collaborate design at their school.

Summary

The school district initiative case presented in this section represents two modalities of collaborative design: the design of tasks for school district wide use, and the learning teams. In both cases interviewed teachers had also participated in the professional development programme described in the previous section. The settings in which collaborative design was conducted differ from the Loughheed team in at least three aspects. First, the purposes of the design were different. In the Loughheed team the purpose was to design and implement two mathematics lessons. In the school district initiative their focus was on numeracy tasks and assessment. Second, teams in the

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cases presented in this section were mainly formed by teachers from different schools, as opposed to the Lougheed case which consisted of teachers from the same school. And third, the school district supported and fostered the collaborative design.

Besides the differences in the settings of the school district initiative and the Lougheed team, I identified resonances among these cases.

1. The elements of the design braid (Chapter 6) seemed to be presented in the case of the school district initiative, especially the category of *anticipating* which relates to the piloting. Field testing of the artefacts under design was an important part of the design process in the school district initiative. Although piloting in the Lougheed team was different from field testing in this case, both activities relate to anticipating students performance. In the school district initiative field testing served to refine the instruments before they were used district wide.
2. In both the school district initiative and the Lougheed project, off-task conversation were valued by the participants. Moreover, in the former case, teams implemented the five minute write at the beginning of the sessions systematically.
3. The Coordinator of the school district initiative and Armando in the Lougheed team were outsiders holding a position that triggered off-task conversations. Due to the position of the Coordinator in the school district, teachers often asked questions to him regarding school policy and collaborative work in the district. In the Lougheed project Armando held the position of researcher which triggered some off-task conversations during the meetings held by the team.

The dissonances that I detected between the Lougheed team and the school district initiative allowed me to identify other possible roles of participants in teachers collaborative design. These new roles are included in the dissonances listed as follows.

1. The implementation of the designed artefacts were different. In the Lougheed case the designed lessons were implemented once with the

members of the team as observers. In the school district initiative the implementations of the tasks were not observed by other participants. These implementations were conducted by teachers who might not participate as designers at the beginning of the process of design. Implementer teachers met to bring exemplars of students work in order to refine the artefacts.

2. The Coordinator in the school district initiative played several roles and was not an expert in mathematics. He served as a liaison with experts in mathematics and mathematics education, invited teacher to participate in collaborative design, did the word design, suggested language for the tasks, controlled the time during the meetings preventing teachers from having many off-task moments, and sought out funding in order to continue the collaborative work within the school district. Although in the Lougheed team Armando played a role of a facilitator, he did not engaged in all the activities of the coordinator just mentioned.
3. In contrast to the levels of expertise identified in the Lougheed team, in this case teachers did not attribute the expertise to other peers, this status was attributed to the external experts such as the Instructor of the professional development programme.
4. In the school district initiative off-task conversation was considered as an important part of the meetings and time for these conversations was deliberately set at the beginning in the form of the five minute write.
5. The impact on teachers' practice after participating in collaborative design was wide and lasting. As mentioned by the Coordinator, those teachers who had participated in collaborative design would keep doing so. The benefits of the collaborative design were not limited to teachers who participated as designers, or teachers who participated as implementers in during the design process. Teachers from the school district implemented the final version of the designed numeracy assessment

tasks into their classrooms, which might represent a change in their practice.

7.4 The Independent Lesson Study Group

The independent lesson study group consisted of a group of mathematics teachers and educators interested in conducting lesson study. This group started in 2006. An institute partnered with several universities, SIGMA¹⁰, has supported the collaborative activity of this group by providing the space and refreshments for the meetings. The group met on Saturdays for three hours around six times a year and teams of collaborative design were formed within the group. The teams worked independently and reported to the whole group about the design of their lessons as well as their findings during the debriefings.

Part of the lessons were implemented in the schools where some of the participant teachers worked. However, as lesson study requires team members to observe the lessons, it was problematic for some teachers from different schools to attend the implementation of the lessons. The SIGMA institute offered special mathematical courses at the elementary and the secondary levels on Saturdays. So, the group started to implement the designed lessons in these courses, making it easier for the group to observe the implementations.

The first sessions of this group were led by a duo consisting of an experienced mathematician and a mathematics educator. The role of the mathematician was to ensure a sound mathematical basis, to point out interesting connections, and to fill in details where needed. The role of the educator was to provide an interpretation from a teacher's perspective and connect it with typical classroom situations. This format of having two experts leading the sessions was not used all of the time, although experts in mathematics and experts in education were always present in the sessions—mathematicians and PhD students in mathematics education were part of the group. Additionally, guest speakers were often invited. Every session started with a mathematical problem that participants first solved and then shared different solutions.

¹⁰ Pseudonym.

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Discussions about the mathematical concepts involved in the problem, as well as the implications for learning inside the classroom, were held afterwards.

Three participants from the independent lesson study group were interviewed: an Organizer, a Prospective teacher, and a Professor. Each one of these interviewees had participated actively in designing mathematical lessons within the independent group. The Organizer started, jointly with other participants, this independent group. She was also a mathematics teacher at the secondary level and had had an active agenda conducting and promoting lesson study in her school, where some of the designed lessons were implemented. As I was also part of this group, I had worked in collaborative design with the three interviewees. However, all of them had also participated in teams where I was not a member.

Settings

The fact that the Lougheed team was inspired by lesson study represents a resonance with the independent lesson study group. However, there were several differences in the settings of each case. As the independent lesson study group had several teams of collaborative design implementing lessons in different places, the settings were varied. For instance, the Organizer reported several cycles of lesson study at her school; however, few of her peers participated regularly in the sessions of the independent group at SIGMA institute. The following excerpt contains the Organizer's descriptions of the lesson study activity at her school including the settings for the collaborative work.

Organizer: [We meet] about once every ten days, I would say, for let's say four to five times for a lesson. And we run two or three cycles in a year. And now we are fragmented actually, so we are having ... three teachers run in one cycle and it doesn't have to be in conjunction with the other part of the school.

We meet in one of the classrooms after school, some times in the library, most of the time in one of the teacher's classrooms.

In the independent lesson study group time and place were often limitations for the collaborative work. Teachers needed to have a strong commitment to the work, as the Organizer explained.

Organizer: But definitely the time is always an issue for teachers to find this common time and meet. So there has to be a big commitment to this work, that's the key.

The Professor had partaken regularly in the sessions of the independent group since the beginning. She had a PhD in education and was an instructor of a teacher professional development programme. Additionally, she was working as a mentor in a school, as she explained in the following excerpt.

Professor: I work as a math mentor at school. I am working with lessons and planning. ... Helping teachers to develop lessons that involve the manipulatives, to the drawing, to the abstract. ... The [independent group] automatically does that but the teachers at school are still *talk and chalk* teachers.

The description that the Professor made of the work done as a mentor in the school shows some similarity to teachers' collaborative design. However, it is not clear whether teachers debriefed the results of the designed artefacts. In the previous transcription she compared those teachers that she had been working with as a mentor and the independent group. This differentiation reflects, in my opinion, a different type of conversation and activities between the two groups of teachers.

The Prospective mathematics teacher had participated in at least three lesson study cycles with the group. In the last cycle she was in a team where both the Professor and myself were also members. As mentioned before, the teams formed out of the group met independently, as the Prospective teacher explained in the following excerpt.

Prospective: We meet once or twice or three times a month ... in [Professor]'s office [for] two hours.

The Prospective teacher participated in the lesson study group designing lessons that were implemented with the students that attend the special courses at the SIGMA institute. In this case the place for meeting was the Professor's room at the university.

Conversations and Activities During Collaborative Design

The components of the design braid (Chapter 6) were present in the descriptions of the design process as described by the interviewees. The conversations during collaborative design included anticipating students' performance during the implementation of the class, which is similar to the *anticipating* category of the design

braid. The Organizer stressed how teachers engaged in predicting students' performance during the implementation of the lesson. In the following excerpt she described the type of activities or discussions people engaged in when they were designing a lesson or an artefact.

Organizer: Well, they really get engaged with predicting how something is going to work in the classroom based on their prior experience and sharing their prior experience related to that. ... So, [we] find out that the most engaging part is the task design. Task design and maybe how we are going to know what students actually understand: the focus questions we are going to use as we observe the lessons; we discuss them in advance. That's quite engaging and ... shaping the goal is always very engaging, too. And then, these focus questions are related to the goal. Like how are we going to be aware of actually seeing these goals?

This excerpt contains other elements of the design braid. Members of the design team engaged in anticipating students' performance and were inquiring about how they would observe what students understand from the lesson. This inquiry was related to *achieving the goals* for the lesson. Having focus questions for observers during the implementation serves to assess the achievement of the goals. This discussion about the role of observers resonates with the conversations from the design process in the Loughheed project when the team planned what observers would do during the implementation of a lesson, which falls into the *team organization* category of the on-task conversation (Chapter 5, p. 59).

Roles and Positions

As the independent group was based on lesson study, the designed lessons were implemented in the classroom and members of the team, and the group, observed the implementations. In this case observers had a particular role, which was reflected in the conversations during the process of design (as explained in the previous transcription). The distribution of labour determined also the role of the writer, as shown in the following transcription.

Organizer: [The teacher] who would be teaching the lesson takes charge and writes out the lesson and then sends the first draft and then everybody comments, inputs and so on.

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Regarding the role and status of participants, the Organizer claimed that every one has something to contribute to the design of the lesson, and then everybody has certain expertise, as it is shown in the following excerpt.

Organizer: I think that everybody has some kind of expertise. ... Like everybody is valued for their contribution. ... The way I see it is like every teacher is the expert in their own class. ... This is something that I really listen to ... when they are designing the lesson for another teacher's class.

Although the Organizer acknowledged that every member of the design team had some expertise, she was more specific when talking about a teacher whom she respected and considered as a mentor.

Organizer: Like [the Mentor] is an expert in having the experience ... and teaching. He is an expert master teacher and he is seen that way by all the colleagues. ...

He was the most famous teacher in his country. He wrote textbooks, he trained teams for the international mathematics Olympiads and so on. It's very rare to find teachers like that.

Therefore, even though every member of the design team had some level of expertise, this Mentor, according to the Organizer, had a higher status as a mathematics teacher. This Mentor also played a role in making the conversations very focused when engaged in designing a lesson.

Organizer: If [the Mentor] is in the team we don't have any aside conversation, I can tell you that.

In contrast to the Organizer who stressed that every member had a level of expertise, the Professor made a distinction regarding the participants' level in the school where she worked as a mentor. However, the level in the lesson study independent group was the same according to her.

Professor: Different expertise, depending on the level. ... At [the independent lesson study group] they [members of the group] are at the same level.

For the Prospective teacher the roles of the team members were important depending on the status given by the *credentials* as researcher or professor. For her, this level of expertise, or status, represented a position within the group.

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Prospective: I think you and [the Professor] are the experts. ... I think you are doing your PhD, so that's what you are focused on. And [the Professor] has many years of teaching experience. ...

The difference in perception between the Prospective teacher and the Professor resonates with the individual perceptions of the roles in the Lougheed team, in which the perceptions of the roles were different, especially with the role of the researcher (Chapter 5, p. 69). These perceptions were also based on the position of the participants. The previous transcript is an instance of how the Prospective teacher recognized the statuses of the Professor and the researcher (myself), giving us a status according to the level of expertise.

Off-Task Conversation

When asked about the off-task moments of the conversations during collaborative design, the Organizer explained that those conversations were strongly related to the lesson.

Organizer: Well, it might go in the tangent of describing certain students or how they behave in class or something like that, and teachers would point that out, you know if certain student is, has a very unique way of thinking and would always come up with a different solution, or would come up with a solution very very fast. For him or her there has to be something else prepared, but I don't think this is an aside conversation. I would say we pretty much keep on-task.

Talking about particular students was also a part of the conversations of the Lougheed team as both anticipating for the lesson during on-task moments and as a part of teachers' experience during off-task moments.

Some of the conversation that the Organizer reported also made reference to the way in which participant teachers experienced the collaborative design.

Organizer: I had conversations with teachers doing lesson study, how it is a really fun experience: the process, not the outcome. ... they really enjoy the process. ... There were also comments that it's hard work and it took a long time to actually write out the lesson and stuff like that.

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As the Organizer mentioned, some teachers found lesson study a fun experience. The stress on the process is consistent with the learning that teachers reported from the Lougheed team at the end of the project.

For the Professor, off-task conversations during collaborative design were closely related to the task of designing the lesson. This perception is similar to the Organizer, as shown above.

Professor: Most of the 'side' conversations are triggered by something we have been talking about in the lesson and something that we have seen happen in class when we tried something. ...

I can't say that we got too many side discussions in my group that were not related somehow to the lesson that we were doing.

The Prospective teacher reported that the team was very focused on the task of designing the lesson: which is consistent with the Professor's opinion shown above.

Prospective: We talked about the lesson most of the time in ... these three parts We didn't have enough time to talk. ... We are very focused.

When the Professor was asked about conversations related to the curriculum, or educational issues in BC, Canada, or the world, she mentioned some of these types of conversations.

Professor: Oh, Yes. We discussed the curriculum and the planning of the new curriculum. We discussed the writing of the new textbooks and things like that, when I think about it. But it is all mathematics related.

Additionally, when asked for conversations that include solving mathematical problems, or related to philosophy and history of mathematics, or other topics, the Professor acknowledged having talked about these themes.

Professor: We have talked about people that were involved in the area of study and developing formulas. Like Descartes, we have talked about him a few times. ... With respect to history, talking about how or why certain aspects of mathematics were developed: some just for the fun of it, some because they had some specific goals in mind. ...

I guess it comes out in various conversations about different

aspects of mathematics that we would really like to see in the curriculum, or some that could disappear ...

Interviewer: Do you think some of those side conversations are relevant?

Professor: Are [they] off-task? No. I think they are add-to-the-task and bring more depth to it. Something that is just tangentially related.

The off-task conversation were described as 'tangential' conversations related to the lesson under discussion. The Organizer also used the idea of tangent when mentioning how the conversations, at times, focused on some students.

Teacher's Learning

The interviewees in this case did not report a change in their teaching practice as a consequence of conducting lesson study. However, they mentioned factors from lesson study that contribute to professional growth.

Professor: The willingness to share ideas, multiple viewpoints, willingness to take risk knowing to be supported, we don't have a lot of that in regular schools.

When asked about what supports or hampers collaborative design, this prospective teacher stressed the fact that participants learn from each other.

Prospective: I think we have a group. That [is] not my personal lesson plan, [it's] the group's lesson plan, so we can learn from one each other. That is the support: group work.

Although the Prospective teacher commented that people learnt from each other, it is not possible to identify a change in her practice as she was not working as a teacher at the time of the interview. Similarly, the Professor was not a teacher and made no comments on changes of teachers' practices. In the case of the school in which the Organizer promoted lesson study, the implementation of lessons designed by the teams could be a change of the teaching practices within the school.

Summary

The independent lesson study group and the Lougheed team were cases of collaborative design based on lesson study which, therefore, shared some features. In both cases the purpose was to design mathematics lessons. The implementation of the

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lessons would be observed by members of the team, or group, and possibly by other invited people. After the implementations there were debriefings of the lessons. Besides these common aspects, the setting in which collaborative design took place varied between these two cases. The time and frequency of meetings was not the same in each case. The Organizer explained how teachers worked at her school, having meetings in the library or in a teacher's classroom. The Professor mentioned collaborative work at the school she worked at as a mentor, and the Prospective teacher commented that her team met at the professor's room at the university. Additionally, the Lougheed team lacked general meetings in which experts in mathematics and mathematics education were present and where participants used to solve mathematics problems at the beginning of the sessions.

The parallels I found between the Lougheed team and the independent lesson study group are summarized as follows.

1. The artefacts designed in both the independent lesson study group and the Lougheed team were lessons. Although the Lougheed team also worked on the design of an assessment rubric, as well as a unit plan, the main focus was on the two designed lessons.
2. As these cases were based on lesson study, the implemented lessons were observed, entailing the role of observers.
3. Some of the elements of the design braid were identified in the independent lesson study group. The *anticipating* and *achieving goals* categories of the Design Braid described in Chapter 6 have resonance with the process described by the Organizer in this case of collaborative design. However, in this case the data did not show many of these categories for the on-task conversations.
4. Diversity of perceptions on the roles of certain members of a team was a characteristic of these two cases. The Organizer and the Professor mentioned that people from the independent lesson study group had a similar level of expertise. However, the Organizer also commented on the high status of her mentor within the group and her school. The

Prospective teacher referred to the Professor and the researcher, myself, as experts. She explained this expertise in terms of academic degrees, PhD. These perceptions were similar to the Arnold's perspective on Armando's position in the Lougheed team (Chapter 5, p. 69). Moreover, the role of the researcher, Armando, was perceived differently by the members of the Lougheed team.

5. Conversations related to teachers' practice were present in both cases. The off-task conversations were, as reported by the interviewees, scarce in the independent lesson study group. However, many of those off-task conversations were also related to educational contexts and had often derived from the discussion of the lesson that was being designed. When I classify off-task moments, they might be related to the artefacts under design even if it was not explicitly mentioned. Many of the off-task conversations of the Lougheed team could be related to the designing of the artefacts. These conversations resonate with the tangential conversations of the independent lesson study group, as indicated by the Organizer and the Professor.

The following list contains the distinctions that I identified between the Lougheed team and the independent lesson study group.

1. The numbers of observers varied drastically between the two cases. In the Lougheed team there were four members and no additional observers participated in the implementation of the designed lessons. In contrast, in the independent lesson study group the implementations were observed not only by the design team, but also by other members of the group, or other teachers in the case of the school in which the organiser conducted lesson study.
2. Casual conversations were not mentioned in the independent lesson study group. In fact, interviewees mentioned that there were no off-task conversation. Rather tangential conversations were indicated, which suggest that casual conversations did not took place in this case.

3. The role of the writer was assigned differently in both cases. In the independent lesson study group the writer was the same as the teacher who implemented the lesson, whereas in the Lougheed team the writer was Sofia, who was not an implementer in the first round of the project.

The existence of casual conversations, such as family or other topics unrelated to education or mathematics, was difficult to identify because, unlike the Lougheed project, the data in this case were generated only from interviews.

The nature of the data also made it difficult to identify particular changes on teachers' practice. However, the implementation of a designed lesson may represent a change on teacher's practice, at least in such implementation. However, holding discussions for the collaborative design has the potential to make students learn about mathematics and teaching mathematics. Additionally, teachers who observed the implementation of the lessons might include into their practice new ideas or approaches.

7.5 Conclusions

The purpose of my research was to describe interactions among members of teams of collaborative design. In the first stage of the research I categorized the interactions within one single team, the Lougheed team (Chapters 5 and 6). Using this categorization, I explored, in the second stage of the research, the three cases presented in this chapter. From these cases I identified similarities and differences with the Lougheed team. The similarities served to determine the extent to which the categorization developed from of the Lougheed project served to describe participants' interactions in these cases. The differences were useful to review, extend and modify such categorization so that it can be used to describe other cases of collaborative design (Chapter 9).

The analysis presented in this chapter served to give an answer to the research questions for the second stage of the study. The first of these questions was: does the generated characterization from the Lougheed team describe participants' interactions in other cases of collaborative design? My conclusions regarding this question are presented as follows.

1. The characterization developed from the Lougheed project served to describe interactions in part of the data generated for the second stage of the research. The limitation of the source of data did not allow an exploration in detail of the conversations during the process of design. However, instances of the on-task conversation were found in each case. In the cases of the teacher professional development programme and the school district initiative instances of all the categories of the on-task conversations were found.
2. Filed testing, as reported in both the professional development programme and the school district initiative, is similar to the piloting that Arnold did with her son during the Lougheed project. However, field testing occurred in the classroom, instead of occurring in a single case. Filed testing, as conducted here, has a greater potential of informing the improvement of the designed artefacts. This potential is not only due to the number of individuals who took part in field testing, but also because of the classroom context in which it happened. The anticipating category of the designing braid can be extended including field testing. Moreover, the relationship between roles and conversations has another component for the case of the school district initiative: some teachers did not participate in the initial design of the tasks, they got involved in the filed testing part of the process. The roles of these teachers was related to particular conversations of the process: the analysis of field testing, which included bringing student exemplars to the team of design.
3. The off-task conversations were explored in more detail in the questions of the interviews. Off-task conversations were identified in each of the three cases of collaborative design presented in this chapter. However, most of these conversations were described as tangential, instead of off-task. Such tangential conversations might not explicitly contributed to the design of the artefacts; more specifically, they might fall within either the *teachers' practice* or the *mathematics and educational context* categories of the off-task classifications presented in Section 6.1.2 .

4. Off-task conversations, including casual conversations, seemed to play an important role in the school district initiative in which the Coordinator acknowledged that teachers need these type of conversations. The five minute write in each session was a systematic form of including these conversations.
5. The settings were relevant factors for the description of each case. Explaining the settings in each case helped to understand certain roles as well as their impact on the interactions within the teams of design. Additionally, in the case of the independent lesson study group the description of the settings served to identify a particular limitation: the time and place for collaborative design. The description of the settings also served to identify the support that this group received from the SIGMA institute.
6. The notion of role as a position served to consider different perspectives on the roles and positions of some participants. The levels of expertise did not seem to be relevant in the cases of collaborative design presented here. Only two exceptions were mentioned. One by the Instructor of the professional development programme and the other by the Organizer in the independent lesson study group. However, the Prospective teacher in the latter case made a differentiation regarding the academic status of the Professor and the Researcher. This is consistent with the role, and position, of Armando in the Loughheed team, which was perceived differently by different team members. Probably, the Organizer and the Professor perceived everyone at the same level within the group; however, the Prospective teacher situated the Professor and the researcher, myself, at a higher status. This is a position where she situated herself compared with the previous two statuses.

The second research question approached in this stage of the study was: What can be expanded from such a characterization by analysing other cases of collaborative

design? The following factors can be added to the characterization developed from the Lougheed team.

1. Cycles of implementation form part of the design process. Anticipating in the form of piloting, field testing, or trial rounds was part of the process of design in both the professional development programme and the school district initiative.
2. Tangential conversations can be added as a description of conversation that are strongly related to the task of designing an artefact. These conversations might be related to the artefact under design, but may not contribute to its design.
3. Conversations related to *teachers' practice, mathematics and educational contexts*, and *casual conversations* might be important to the members of the team. The five minutes write allowed space for these conversations.
4. The role of the implementers in the school district initiative was important and allowed feedback from several classrooms on the implementation of the tasks.

Finally, the last research question of this study was also approached through the cases presented in this chapter: What are the possible roles of participants in different cases of teachers' collaborative design and how do they influence the interactions within the teams? The roles played in the teams of collaborative design described here varied significantly. I identified new roles, or characteristics of such roles, which are explained as follows. These explanations include comments on the impact of these roles on the conversations within the teams.

Scribe. The scribe is the one in charge of word processing and graphic design. In the cases of the independent lesson study group and professional development programmes this role was played by teachers, whereas the Coordinator played this role in the school district initiative. This was not a new role, as members of the Lougheed team also held this role. This particular role can facilitate the work of the other members

of the team, as it was the case of the school district initiative in which the Coordinator played the role of scribe.

Observer. This role is particular to the lesson-study-based modes of collaborative design such as the independent group and the Lougheed team. Other cases of collaborative design did not include observers for the implementation of the artefacts. The fact that there were observers in the independent lesson study group entailed conversations about what they should focus on during the implementations of the lessons. In the categorization of the Lougheed team, all the members were observers and this was not a particular role for other participants. However, due to the lack of observers in both the professional development programme and the school district initiative, I decided to include this role in order to acknowledge it as particular of models of collaborative design such as lesson study.

Implementer. As collaborative design entails the creation of artefacts to be used in the classroom, some teachers have to play this role. However, they played this role in different fashions. In the independent group there was usually one implementer in the team; in the professional development programme all the teachers were implementers; and in the school district initiative teachers that did not participate in the design initially implemented the artefacts in their classrooms. In this case, implementers who did not participated as creators of the tasks engaged at a different stage of the design process. So, their role contributed to a specific type of discussion: the comments on students exemplars.

Designer. Because there exists a role for teachers not participating in the design process, it makes sense to also have a clear designation for all those who were. Thus, I have given the name of designer to all those who help with the design of the artefact in some capacity during a considerable part of the design process.

Administrator. This is a position that might hamper the work in a team of collaborative design, as explained by the Organizer in the school district initiative. If an administrator were a part of a team of design, he or she may play the role of the supervisor instead of a designer, which, according to the Organizer, hampers the process. However, having administrators participating in teams of collaborative design

might be good for some teachers because they would hear them learning and struggling as well.

Support seeker. The collaborative design was made possible by different types of support, in terms of economic or other resource. In the independent lesson study group the organizers got support in the form of refreshments and a place for the general meetings. In the case of the school district initiative, the Coordinator was also engaged in finding economical support for the collaborative work within the district. The effects on the interactions in the design process can be seen in the off-task conversations mentioned by the Coordinator.

Facilitator. Although *facilitating* encompasses a broad collection of activities, having a facilitator in a team seemed to be crucial, as indicated by the Coordinator in the school district case and the Instructor in the professional development programme. Activities related to this role included posing key questions to the team as well as organizing the work. A facilitator does not need to be an expert in mathematics or mathematics education, as was the case of the Coordinator.

External expert. The Instructor of the professional development programme was an external expert who supported the collaborative work in the school district initiative. In the Lougheed team Armando also held this role, which I did not identify before the analysis of the data in this chapter.

Not only the roles played by the members of collaborative design teams shaped interaction, participants' positions were also factors. Both roles and positions were related to the setting, such as place, time and format, in which the collaborative work was conducted. In particular, the collegiality among teachers was important. For instance, in the case of the professional development programme, the teachers reported that they work better with someone who they feel more comfortable with, as opposed to working with an expert. When differentiating the types of participation of primary teachers and secondary teachers the Instructor indicated that the grade levels were also a factor that determined a position in the interactions among teams of collaborative design (Table 7.6).

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The resulting expansion to the categorization of interactions among members of teams of collaborative design can be used now to describe the Loughheed case and the three cases presented in this chapter. This categorization describes interactions in two dimensions: (1) the conversations during the process of design, and (2) the roles and positions that participants played within the teams. Using the categorization described so far acknowledges the differences and similarities among the cases. The findings from this chapter not only allowed for expansion of the characterization, but also enabled refinements. For instance, the some off-task conversations were described as tangential conversation: They were closely related to the designing of the artefacts and impacted on teacher professional growth. The off-task classification can be modified in order to include acknowledge the importance of tangential conversations. Modifications to this characterization are presented in Chapter 9, which were also influenced by the findings of the next chapter in which I analyse other cases of collaborative design from particular pieces of literature as second-hand data.

Chapter 8 Third Stage: Three Cases from the Literature

In order to characterize interactions among members of teams of teachers' collaborative design, I decided to analyse cases that represent large-scale modalities of this type of collaborative work among teachers designing mathematics artefacts. The first stage of this study consisted of developing a categorization of interaction among the members of the Lougheed team (Chapters 5 and 6). In the second and third stages I decided to analyse other cases of collaborative design using this categorization. Chapter 7 focuses on three cases in which I contacted participants and generated data from interviews, surveys and conversations. This chapter corresponds to the third stage of the research in which another three cases were explored using the categorization developed in Chapter 6. The sources of data for these cases were particular pieces of literature related to teachers' collaborative design. Using the literature as a source of data represented both an advantage and a limitation for this study. The advantage was that the literature allowed me to consider large-scale modalities of collaborative design. However, I did not select the type and source of data in these cases. For this reason some emerging features from Chapter 6 were hard to observe in these cases. For instance, evidence of off-task conversations was very difficult to identify. The purposes of the analysis in this chapter were, on the one hand, to identify how the categorization developed in the first stage of the research describes interactions in other cases; and on the other hand, to extend the descriptions of the conversations, actions and the roles that take place during collaborative design through a wider range of instances. Similarly to Chapter 7, identifying resonances and dissonances with the Lougheed team served as a means to pursue these purposes.

The cases described here were selected because they provided descriptions of team members' interactions during collaborative design, including conversations and

explanations of particular roles of the participants. Additionally, these cases were representative of a large number of teams of teachers and educators.

The presentation of the three cases of teachers' collaborative design in this chapter is slightly different from the structure of Chapter 7, in which each case is presented using the following headings: (1) settings, (2) conversations and activities during collaborative design, (3) roles and positions, (4) off-task conversation, and (5) teachers' learning. When analysing the data from the literature used in this part of the study, it was hard to identify instances of off-task conversation. Due to this limitation, the presentation of the cases in this chapter does not include this heading. The analysis of these cases and the cases presented in Chapter 7 were conducted simultaneously. However, for the purpose of clarity in the presentation of the findings, I limited to contrast the cases of the literature with the Loughheed project in this chapter. An integration of the results from all the cases of the study is presented in Chapter 9.

The analysis of the data presented in this chapter was conducted by a focused coding of each piece of literature. The categories developed from the Loughheed project were used as a frame for this coding. Written notes in the margins or on post-it notes stuck to the documents were the means by which I coded and compared each piece.

8.1 The Lower Grade Group

The first piece of literature used as second-hand data for this study was the book *Lesson Study: A Japanese Approach to Improving Mathematics Teaching and Learning* by Fernandez and Yoshida (2004). This book contains a description of lesson study and how it is conducted as teacher professional development in Japan. Its authors focused on one case, the lower grade group, as an instance of lesson study conducted in 1993 at Tsuta elementary school. They claimed that "the type of conversations and activities that the lower grade teachers engaged in are very typical of lesson study [in Japan]" (p. 29).

Lesson study, as commonly conducted in Japan (Fernandez & Yoshida, 2004), has been based on long-term school, or district, mission statements or goals. These goals were discussed by teachers and every lesson must contribute to the school's general goal or the school's mission statement. Part of teachers' duties at school has

been to participate actively in professional development programmes, which in the majority of the cases has included lesson study.

Settings

The members of the lower grade group described by Fernandez and Yoshida (2004) were four teachers and the vice-principal of the school. Details of the team members and their teaching experience are shown in Table 8.1. The fact that teachers in this case participated in collaborative design as part of the duties at school represents a dissonance with the Lougheed team in which teachers participated in collaborative design as part of this research project. Another difference is that in the lower grade group an administrator, the vice-principal, was involved in the team. The Lougheed team had no administrator as a member.

Table 8.1: Members of the Lower Grade Group

Members	Position	Grade	Years of teaching experience
Ms. Tsukuda	Teacher (implementer)	First	11
Ms. Nishi	Teacher(implementer)	First	0
Ms. Chijiwa	Teacher	Second	21
Ms. Maejima	Teacher	Second	5
Ms. Furumoto	Vice-principal	-	24

Fernandez and Yoshida (2004).

Teachers in Tsuta elementary school selected a four-year statement of the school: "focusing on problem solving-based learning in mathematics in order to promote students' ability to think autonomously, invent, and learn from each other" (Fernandez & Yoshida, 2004, p. 24). The lower grade group followed this school statement. This long-term goal represents a dissonance with the Lougheed team which had only short-term goals that did not included the rest of the school and were oriented to specific mathematical content (see Table 5.1).

A chronological description of the activities involved during the cycle of lesson study of the lower grade group is presented in Table 8.2. From this description it is possible to identify two dissonant aspects with the Lougheed team. Firstly, whereas the lower grade group worked on a first grade lesson that was implemented twice in

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November 1993, the Lougheed team designed two different lessons that were implemented and observed once at grades eight and nine. The lesson designed by the lower grade group was refined for the second implementation based on the debriefing conducted after its first implementation. The Lougheed team did not implement the lessons for a second time. Secondly, unlike the Lougheed project in which only the team observed the implementation of the designed lessons, in the lower grade case other teachers participated as observers and also provided feedback to the design team. Moreover, all the academic staff at the school observed both lessons and participated in the debriefing of the second implementation, where an external advisor, Mr. Saeki, was invited. The participation of more academic staff as observers in the implementations was also a consequence of having lesson study as an integral part of the teacher's duties at school.

Table 8.2: Chronology of Activities of the Lower Grade Group.

Dates	Activities
October 25 to 30	The two first grade teachers, who were the implementers, developed a preliminary lesson plan.
November 1	The whole lower grade group discussed the lesson plan (2 hours).
November 4	The lower grade group presented their lesson plan to all the staff and received feedback (0.5 hours)
November 5	The lower grade group worked on the lesson considering teachers feedback (1 hour).
November 5 to 14	The two first grade teachers, with help from the other lower grade group members, finalized the lesson plan, created lesson materials, and prepared to teach the lesson.
November 15	First lesson was implemented by Ms. Nishi. All the teachers of the school, the principal and the vice-principal observed the implementation.
November 15	Debriefing of the lesson (1.6 hours). Lower grade teacher and the principal meet at principals' office.
November 16	Debriefing from the previous day continued.
November 16 and 17	Ms. Nishi and Ms. Tsukuda finalized the lesson plan and prepare to teach the second lesson.
November 18	The second implementation was conducted by Ms. Tsukuda. All the teachers, the principal, the vice-principal, and Mr. Saeki, and external advisor, observed the lesson.
November 18	All Tsuta teachers, the principal, the vice-principal and Mr. Saeki participated in the debriefing of the lesson (2 hours and 20 minutes).

Fernandez and Yoshida (2004).

Conversations and Activities During Collaborative Design

The elements of the design braid developed from the Loughheed project (Chapter 6) have strong resonance with the lesson plan of the lower grade group. The three strands of the on-task conversations can be identified in the lesson plan of the lower grade group (Fernandez & Yoshida, 2004, p. 128). This plan detailed the unit on subtraction as well as the unit's content, goals and chronology, including the number and content of lessons, which resonates with the *pursuing coherence* category of the design braid. Moreover, in this lesson plan a column of evaluation was included in which teachers described how to observe the established goals for the lesson during its implementation (p. 76), which resonates with the *achieving goals* category of the braid. The lesson plan also included a subsection entitled "related items," where teachers must describe the connections between the subtraction unit and other units from grade 1 to grade 5 (p. 78). The inclusion of this subsection in the lesson plan represents another instance of *pursuing coherence*. Additionally, descriptions of students' possible reactions and teachers' responses were anticipated, which is described in the *anticipating* element of the design braid.

The elements of the design braid can also be identified in the conversations of the lower grade group, as presented by Fernandez and Yoshida (2004). Instances of *anticipating* were present in this group when teachers discussed the numbers and the story to be used for the lesson. The operation 12 minus 7 was considered for the problem of the lesson for two particular reasons: (1) while breaking 7 into 1 and 6 was easy, breaking it into 2 and 5, according to Ms. Tsukuda, was difficult for first year students; and (2) one specific low achiever student might be engaged by choosing the number 7 for the story problem.

Ms. Tsukuda added that she wanted to use the number 7 because one of her students happened to have seven family members. She said she wanted to choose this student for the story problem because he was a low achiever. She thought that making him the focal point for the lesson might help him gain more confidence. (p. 52)

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A variation of the problem considering the operation 12 minus 9 was discussed as a second practice problem for those students who finished faster (p. 54). The selection of the number 12 for the story problem was based mostly on the mathematics content required for the operation; however, students' particular family context were considered in this decision.

The teachers all felt that 12 was a good choice because regrouping would be needed whenever the number of people in any of the students' families was chosen as the subtrahend. This was because there were no students who had less than three people in their family.

(Fernandez & Yoshida, 2004, p. 53)

The selection of the numbers for the problem in the class was, therefore, a combination of: (a) *anticipation* of the engagement of particular students; (b) *pursuing coherence* of students' family context and the story problem of the lesson, and (c) *pursuing the goal* of performing subtraction using regrouping.

Preparing students for the main task of the lesson by means of introducing them to simpler problems at the beginning was a part of the discussions in this group. This is an instance of both *pursuing coherence* and *anticipating* during the process of collaborative design.

The group also discussed that it would be important for kids to work on specific review problems before tackling the main problem of the lesson. In particular they talked about presenting the problems 10 minus 5, and 12 minus 2.

(Fernandez & Yoshida, 2004, p. 53)

An interesting feature of this group was the rehearsal they performed before the lessons. Ms. Tsukuda and Ms. Nishi carefully went over the use of the blackboard one day before the lesson.

They pasted the story problem on the board and discussed how much space would be needed for the teacher to write when she went over the handout. They considered how much space they should reserve for pasting the students' work on the blackboard.

(Fernandez & Yoshida, 2004, p. 89)

The lower grade group had a long discussion about the use of manipulatives in the lesson. The decision about the use and design of these manipulatives involved

conversations regarding *anticipating* students use of them and *pursuing* the *goals* of getting students using regrouping as a means to subtract whole numbers (Fernandez & Yoshida, 2004, p. 55). For this lesson students used drawing paper and blocks made of cardboard paper as tiles. Teachers decided to use spray glue to make the surface of the paper "sticky so the tiles could be pasted and peeled off easily" (p. 66). Ms. Nishi, Ms. Tsukuda and Ms. Maejima prepared the manipulatives for the lesson (p. 87), which entailed the *organization* of the *team* by distributing labour.

Roles and Positions

Different roles and positions can be identified within the lower grade group. The years of experience of the teachers varies from a novice to a teacher with more than 20 years of teaching experience (Table 8.1). This difference in experience was reflected in the roles and positions that members of this group hold. The two teachers who implemented the lesson, Ms. Tsukuda and Ms. Nishimi, worked initially on a draft of the lesson plan: "As they worked on this lesson plan, the two teachers consulted numerous resources, including their teachers' manuals, other instructional materials, and lesson plans that have been stored over the years in the school's staff room" (Fernandez & Yoshida, 2004, p. 34). As implementers, these two teachers were in charge of writing the lesson plan. However, the role of the writer was taken by Ms. Tsukuda.

Ms. Tsukuda would actually write the lesson plan because she is the one teaching the lesson at the second time ... and had more years of teaching experience. ... In addition, Ms. Nishi was enrolled in mandatory beginning teacher training organized by the government while also teaching full-time, and therefore had a much fuller schedule than Ms. Tsukuda. (p. 34)

The decision of having Ms. Tsukuda type the lesson was not based only on her position as a more experienced teacher; a schedule limitation of Ms. Nishi was a factor to designate the role of the writer in this case. She had this limitation as a consequence of being a novice teacher.

The position of Ms. Tsukuda as a more experienced teacher is also reflected in the contributions she often made to the discussions during the design of the lesson. For instance, she contributed actively proposing the numbers to be used for the problems of

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the lesson, including those problems for early finishers (Fernandez & Yoshida, 2004, p. 52-54).

Ms. Furumoto was a team member with a special role and status in the lower grade group. She was the vice-principal of the school and was the most experienced teacher of the lower grade group. She supported teachers' collaborative design: "Ms. Furumoto behaved as an equal member of the lower group. ... She explained that her role, like that of the principal, was to guide, support, and motivate all teachers to participate in [lesson study as in-service professional development]" (Fernandez & Yoshida, 2004, p. 30). Ms. Furumoto's role included fostering collaboration among teachers in the school. She was aware of her position in the group and explained that "administrators like her needed to be careful not to intervene in teachers' activities. According to her, it was critical for teachers to feel total autonomy in their lesson study work without any sense of being controlled from above" (p. 30). Additionally, Ms. Furumoto served as a liaison with Mr. Saeki, the outside advisor, providing him with details of the collaborative work at the school, and facilitating his advice to the lower grade group when visiting the school.

The contributions of Ms. Furumoto had an important impact on the decisions of the group. A long discussion about the use of manipulatives took place during the design of the lesson within the lower grade group. This discussion not only reflects the level of details at which this group of teachers focused on while preparing the lesson, but also serves as an instance of the role, and position, of the vice-principal in the team.

The teachers also discussed at length the issue of what manipulatives to use during the lesson. This discussion was in part prompted by a comment made by Ms. Furumoto [vice-principal], who mentioned that the range of student solutions obtained during the lesson would depend on the kind of manipulative that the students would work with.

(Fernandez & Yoshida, 2004, p. 55)

The use of manipulatives in the lesson designed by the lower grade group represents a dissonance with the Lougheed team which did not use manipulatives for its artefacts. Moreover, the lower grade group constructed the manipulatives for its

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lesson. Ms. Nishi, Ms. Tsukuda and Ms. Meajima played a role of manipulative-designer in the team.

During the implementation of the lesson there were two specific roles within the observers: the one who kept track of the time during the lesson, and the outside advisor. Ms. Maejima was officially assigned to keep track of time in Ms. Nishi's classroom. Mr. Saeki was the outside advisor during the second implementation of the lesson. This role is important in the context of Japanese lesson study. None of these roles were played in the Loughheed team.

Undoubtedly, enlisting the help of an outside advisor provides a mechanism by which lesson study groups can learn about each others' success and failures, rather than trying to reinvent the wheel. This learning is possible because the outside advisor provides a bridge between the various lesson study group that he or she works with or knows about. (Fernandez & Yoshida, 2004, p. 211)

Teachers who have participated in lesson study several times have a chance to share their learning with their peers. However, the role of Mr. Saeki as a liaison with other schools enhanced this sharing among teachers in the school system in Japan.

Teacher's Learning

Lesson study in Japan offered occasions for teachers' learning in at least three forms. First, those involved in the design of a lesson had a chance to learn from each other and to share whatever they searched for in preparing a lesson. This occasion for learning resonates with the Loughheed team in which teachers reported that they learned by participating in the discussions during the project. Additionally, participants holding the role of external advisor, such as Mr. Saeki (Fernandez & Yoshida, 2004, p. 211), shared the experiences of several lesson studies among different teams and different schools. Second, teachers observed and participated in the debriefing of many lessons. This occasion of learning by observing several lessons was not present in the Loughheed team. Third, refined lesson plan and lesson study written reports were spread among teachers at the regional and the global levels (p. 211). The written reports were offered in the open house implementation where teachers from different schools observed a

lesson. Some of these reports were published as monographs or articles and distributed across the country (p. 212). This part was also missing in the Lougheed team.

Summary

The lower grade group and the Lougheed team were both based on lesson study. This situation implied several similarities among these two cases, included those listed as follows.

1. Teachers met at their school.
2. The designed artefacts were lessons which which implementations were observed by other participants.
3. The on-task conversations during the design process described in the Tsuta elementary school have strong resonance with the Lougheed team. The four components of the design braid were involved: anticipating, achieving goals, pursuing coherence, and team organization. Attention to particular students was a shared feature between the Lougheed team and the lower grade group. Anticipating performance of students that were high or low achieving was part of the discussions.

The school settings for collaborative design were very different from the Lougheed project. The lower grade group was immersed in Japanese lesson study, which entailed appropriate resources and facilities for the collaborative work. The roles and positions of the participants in each case differed. I found the following differences among these cases.

1. In the lower grade group collaborative design was not only encouraged, but also formed a part of teachers' duties and the school provided resources for this activity. In contrast, the Lougheed project lasted eight months and collaborative design was not a part of their duties at school.
2. Having a selected school's goal or general mission statement contrasts to the Lougheed case, where the goal for the first lesson was content focused. Whereas the lesson study activity in Tsuta elementary school

was oriented by a general goal discussed by all the teachers, in the Loughheed project the goal was negotiated only by one team.

3. In the lower grade group teachers rehearsed the lesson, planned the use of the board, and designed the manipulatives. These three features of the on-task actions of the group were not present in the Loughheed team.
4. The grade level and the teaching experience were different, as well as the fashion in which experienced teachers interacted. In the Loughheed team the most experienced teacher, Brad, positioned himself as a learner. He asked questions about mathematics and wondered whether the selected activities would serve to achieve the selected goals for the lessons. In contrast, in the lower grade group, Ms Tsukuda and Ms. Furumoto, both teachers with more than ten year of experience, made important contributions to the decisions during the process of design. Ms. Furumoto's role also included a liaison with Mr. Saeki, who was a bridge among different teams of lesson study. His observations were based, in part, on the experience of participating in other lesson study teams.

8.2 The Madrid Group

The second piece of literature described in this chapter is the article *Collaborative Teacher Inquiry as a Tool for Building Theory on the Development and Use of Rich Mathematical Tasks* written by Slavit and Nelson (2010). This article presents a case of a group of mathematics teachers, the Madrid group, at the secondary level engaged in a one year-long collaborative inquiry activity. Collaborative design of lessons and mathematical tasks are the means by which teachers inquire into specific pre-selected topics (Nelson & Slavit, 2008). Teachers were interested in increasing student engagement and problem solving by choosing and implementing mathematical tasks in the classroom. Some roles of participants in this group, as well as interactions among them, are described in the article. Particularly, teachers building of individual and collective theories of learning and instruction are explored. The supported collaborative inquiry described by Slavit and Nelson (2010) is immersed in a professional

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development project called Partnership for Reform in Secondary Science and Mathematics [PRiSSM]. This was a three-year-long project involving 175 mathematics and science teachers from six school districts in north-western US. This project included several teams of teachers working on collaborative design.

The PRiSSM project aimed at building leadership and research capacity among teachers while creating professional learning communities. Involved schools allowed time for teachers to meet and participate in the project. Other activities were included such as summer and mid-year academies that supported lead teachers' abilities to organize the collaborative work at their schools. Lead teachers formed professional learning communities at their schools and become facilitators of the collaborative inquiry process. A 12-member steering committee composed of researchers and district specialists in mathematics and science education developed and oversaw the project. Members of the committee facilitated the collaborative inquiry at schools conducting visits to the learning communities.

Settings

The Madrid group consisted of eight mathematics teachers from the same school, the Madrid high school. Two of these teachers were facilitators, Camron and Bryce. Whereas Camron had participated actively with the PRiSSM project for two years, Bryce began to engage in collaborative inquiry as a facilitator at the time of the study conducted by Slavit and Nelson (2010). Madrid high school supported the collaborative inquiry delegating "weekly, 30-min times as well as two additional 90-min sessions" (p. 205). The research focus of this group of teachers was captured by the question: "How can the use of rich tasks and group work increase students engagement in the classroom?" (p. 206). The collaborative inquiry work was organized into three mini-cycles. The cycles started with a conclusion of the previous cycle and presenting the new task, which was worked on by the teachers. Then, there was a presentation and negotiation of the lesson plan including task format and instructional approaches. And finally, a debrief of student responses to the tasks was conducted, which included some students work to guide the discussions.

The Madrid group had two types of facilitators: internal and external. Both Camron and Bryce were internal facilitators. A member of the steering committee of the PRiSSM project, Ginny, was the assigned external facilitator and visited the school once every six weeks. She was a regional mathematics specialist and an expert in collaborative inquiry.

The mathematical task designed by the Madrid group is presented in Table 8.3. This task was proposed and used during the second cycle of the group.

Table 8.3: Rich Mathematical Task Used by the Madrid Group.

<p>Every year over 2,000 men and women apply to the U.S. Air Force to become airplane pilots. Applicants are eligible only if they satisfy three conditions: they must have 20-20 vision with glasses or contact lenses; they must have no allergies requiring medication; and they must not get altitude sickness when flying.</p> <p>This year exactly 1,400 people applied, and of those:</p> <ul style="list-style-type: none">570 did not have 20-20 vision798 had allergies65 had altitude sickness120 did not have 20-20 vision and had allergies32 did not have 20-20 vision and had altitude sickness45 had allergies and altitude sickness25 had all three disqualifiers <p>To receive credit, you must answer the following five questions correctly:</p> <ol style="list-style-type: none">1. How many applicants actually qualified?2. What percent of the applicants actually qualified?3. How many applicants only had allergies?4. How many applicants had exactly two disqualifiers?5. What is the probability a person who is chosen at random has exactly one disqualifier? <p>Explain clearly how you solved each of the problems above. You may use words, pictures and/or numbers.</p>
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Note. Adapted from Slavit and Nelson (2010) p. 209.

Conversations and Activities During Collaborative Design

The elements of the design braid (Chapter 6) resonate with the description of the design process presented by Slavit and Nelson (2010). They found that teachers from the Madrid group constructed individual and collective theories related to scaffolding and the delivery of rich tasks. The fact that these theories were mentioned is evidence of on-task conversations which include *anticipating* of students' performance during the mathematical task, as well as *pursuing a coherent* form of posing the task to the

students. Three instructional theoretical perspectives were repeatedly debated during the process of collaborative design:

(a) the degree to which related skills and concepts should be taught prior to the implementation of the task, (b) the degree to which a task should mesh with the current instructional topic or be used as review or foreshadowing of different content ..., and (c) the degree to which a task should be "broken down" into specific steps or presented as a "large problem." (p. 208)

Students at this grade level had to take the 'state achievement test,' and this fact also influenced the design of the task, *making* it *coherent* to the whole context at school. For instance, Carmon commented in one of the sessions: "I think this is a nice problem to do. It is very possible they see a Venn [diagram] in the [state achievement test]" (Slavit & Nelson, 2010, p. 211). Whereas this instance of the pursuing coherence category resonates with the Lougheed project, in the Lougheed team provincial or state achievement tests were not a factor influencing the design of the lessons. The focus on this test represents a dissonance between these two cases.

Although there is no explicit mention of conversation related to *team organization* in the article of Slavit and Nelson (2010), norms for the team work were mentioned. As the Madrid group consisted of eight teachers, norms for participation in the discussions during the meetings were important. The *team organization* included norms generated for this group such as: "turn taking, active listening, and arriving on time" (p. 208).

Teachers from the Madrid group worked on the rich mathematical task proposed for their project (Table 8.3). After a few minutes working on this task, the group started discussing their solutions (Slavit & Nelson, 2010, p. 211). Solving the proposed tasks for a lesson and discussing different solutions resonates with the Lougheed team (Chapter 5, p. 65).

Roles and Positions

A particular role emphasised by Slavit and Nelson (2010) was the one that Camron played as a facilitator and leader of the Madrid group. She "took the lead in establishing the agenda and directing meeting flow as well as providing specific

resources, including the mathematical tasks which framed most of the group activity" (p. 205). Additionally, she served as a mathematics coach at Madrid high school.

Camron proposed the task used by the group in the second cycle of the collaborative inquiry (Table 8.3) and led the discussion for its implementation. After some minutes of independent work, the group of teachers analysed their solutions and started to discuss the problem, as we can read in the following part of the conversation.

Camron: ... So it's a pretty straightforward problem. And I could find a more complicated problem, or we could just expose them to this and see how they think of it. As soon as they come up with the Venn diagram strategy it's pretty easy.

Kris: You want to expose them to some smaller kinds of

Camron: *Integrated 2* last year had exposure to Venn diagrams, I think. If they recognize it, they will know how to do it ...

Camron: So how do we feel about that? What's our gut? I'm thinking about as we get them ready for the [state achievement test], while we're on logical reasoning, do we want to do a Venn diagram? Do we want to do a more difficult task? Do we want to give a different puzzle that's just logical reasoning?

(Slavit & Nelson, 2010, p. 209)

This conversation shows "the manner in which Camron controlled the flow of the discussion in terms of both time and topic" (Slavit & Nelson, 2010, p. 209). This control shaped the focus of the conversations within the group. Camron argued, during the discussions held by the team, that students should figure out by themselves that using Venn diagrams could be useful for this problem; whereas, some teachers wanted to give students a more prescribed direction indicating explicitly the use of the diagrams for the problem. In an interview, Camron commented on the debate about prescribed vs unprescribed instruction held by the group.

We've debated how much the problems should be prescribed or unprescribed, and whether or not they should be connected to what book is doing ... I don't think other teachers will use the task if they don't see a connection to the content they are teaching. (p. 210)

From this last excerpt we can see how Camron was eager to select activities that made sense to other teachers. This is another example of the position that she took as a facilitator of the group by being concerned about teachers' perceptions on the mathematical task.

The external facilitator, Ginny, was busy and could not visit the school as often as she wanted. This fact made her feel as though she was holding an outsider status.

I'm not feeling successful as a facilitator these days. I think that I, or we, still have lots to learn about facilitating PLCs, especially if you aren't a part of the school district community. (Slavit & Nelson, 2010, p. 205)

Ginny, as a regional mathematics specialist, was an external expert. However, she did not have the impact on the group she might have wished. The limited facilitation, among other factors, shaped interaction among teachers that focused more on evaluation than on understanding students' thinking (Slavit & Nelson, 2010).

The role of facilitators in this case represents a dissonance with the role of the facilitator in the Lougheed team. While Camron was a teacher-facilitator who proposed the task used by the teacher, Sofia, as a teacher, was the one who proposed the majority of the mathematical tasks in the Lougheed team. Although she was perceived as a mathematics expert by Brad, she did not hold a role of facilitator as Camron did in the Madrid group. The role of (external) expert in mathematics education in Lougheed team was held by Armando, who participated in all the meetings. This participation contrast with the participation of Ginny in the Madrid group.

Teacher's Learning

Slavit and Nelson (2010) claimed that supported collaborative teacher inquiry has specific and direct links to classroom practice (p. 217). They also explain the opportunities that this model of collaborative design offer to participant teachers.

A [collaborative teacher] inquiry context is inextricably linked to the way teachers interact, form theories, and make use of student work... The teacher interactions presented opportunities for teachers to hear others' similar or dissimilar conjectures and assessments of appropriate scaffolding, and then negotiate personal and collective theories around these issues. (p. 14)

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Creating conjectures and theories about students' thinking was also a part of the discussions in the Lougheed team. For instance, the discussions about how having students write their ideas would help them to write the corresponding algebraic expressions (Chapter 5, p. 53-56).

Being a part of the PRiSSM project provided a context for collaborative design with different types of support. The school supported the collaborative work of the group, members formed part of the project and then attended workshops in the Summer. Additionally, the group was supported by Ginny as the external facilitator. Slavit and Nelson (2010) stressed the impact of supported collaborative inquiry on the teachers' conversations.

While a shared vision of practice was not fully developed and the teachers still held a variety of beliefs, multiples theories were being negotiated and a shared sense of each other's practices was emerging. Without the structures afforded by collaborative inquiry, such conversation could not have occurred. (p. 214)

The practice of participant teachers was changed because they were implementing the designed lessons and tasks—such as the rich mathematical task presented in Table 8.3. Being exposed to others' conjectures and theories has a potential to influence their practice within the classroom, too. This change of practice resonates with the teachers who implemented the lessons during the Lougheed project. However, the evidence of this change is limited to the duration of the projects.

Summary

The aspects in which I found resonance when comparing Madrid group with the Lougheed team are mainly in terms of the conversations held during the sessions for collaborative work. However, some aspects of the settings among these two cases were also similar.

1. Participant teachers were from the same school and the meetings were held in their school.
2. Evidence of conversations that fall into the categories of the on-task conversations, the design braid, was present in the Madrid group. The

transcripts of the conversations can be categorised by the three strands from the design braid and the team organization category.

3. Off-task conversations were reported in the Madrid group. In this case some conversations were tangential to the design of the mathematical task and might be productive in terms of teachers' reflections on the purposes of assessment, as well as the standard test. However, this was not the purpose of the inquiry in this cycle, which was focused on rich tasks, group work and student engagement (Slavit & Nelson, 2010).
4. Theories and conjectures about students' learning were present in both cases.

Although teachers worked at school in both the Madrid group and the Lougheed team, other features of the settings of these cases differed. The differences in the settings implied differences in the interactions in both the conversations and the roles of the participants of collaborative design, as presented in the following lists of dissonances between these two cases.

1. The cycles of collaborative inquiry of the Madrid group consisted of designing/selecting mathematical tasks and their corresponding lesson plans, implementing the lessons independently, and sharing and discussing the results with the group. In contrast, the Lougheed team, based on lesson study, designed lessons which were observed during their implementation, and not all of the teachers implemented the lessons in their classrooms.
2. The number of members in the Madrid group, eight, and the limited time for the regular meetings, 30 minutes, influenced the organization of sessions: Norms such as taking turns to speak and arriving on time to the meetings were established. In the Lougheed team such organization of the team was not necessary as there were only four members and the meetings lasted an hour approximately.

3. Facilitators played different roles in each case. In the Madrid group Camron, who was also a teacher, held the role of facilitator. She led the discussions in the meetings and proposed the mathematical task used in the cycles of collaborative inquiry. The role of facilitator held by Armando in the Lougheed team was slightly different. He also proposed mathematical problems, however, the problems used for the lessons were proposed by Sofia, who was considered a mathematics expert. Ginny, the external facilitator, had scarce participation in the Madrid group. In the case of the Lougheed team Armando was also an external facilitator, but he participated actively in the weekly meetings.

8.3 The VITAL Project

The Variation for the Improvement of Teaching And Learning (VITAL) project was conducted in Hong Kong from 2005 to 2008. This project was aimed at bridging the gap between goals of the curriculum reform and students' learning in selected areas which included mathematics. Teacher professional development by collaborative design was a central feature. The project combined Japanese lesson study with Variation Theory (Marton & Tsui, 2004) in the form of learning study. A total of 120 schools including elementary and secondary grade levels participated.

The literature used as second-hand data in this section was the book *Learning Studies as an Educational Change Strategy in Hong Kong: An Independent Evaluation of the 'Variation for the Improvement of Teaching And Learning' (VITAL) Project* (Elliot & Yu, 2008). This book contains an evaluation "focused on the impact of Learning Studies on the school as an organization, on school-based curriculum planning, and on teaching and learning" (p. 2). Surveys, questionnaires, interviews and group discussions were conducted among stakeholders, teachers and some students from 13 participant schools for this evaluation. Perspectives of different participants were collected.

In contrast to the previous two cases presented in this chapter, Elliot and Yu's (2008) book does not focus on a single case. Details of the conversations during the

process of design are scarcely reported in the book. However, different roles and positions held by people involved in collaborative design can be identified in this report.

Settings

The VITAL project was conducted by the Centre for Learning Studies and School Partnership (CLASP) at the Hong Kong Institute of Education (HKIEd). Lesson study was already in use as professional development in some schools, a situation that facilitated the implementation of the VITAL. However, for other schools lesson study, or learning study, was completely new.

Teams consisted of a group of teachers, normally from the same subject, an academic staff member from HKIEd, and a Teacher Development Consultant (Elliot & Yu, 2008, p. 2). The teams met from 6 to 12 times at school. Lessons were video-taped and discussed. Pre and post tests were applied to students in order to inform the teams about the extent of student learning. At the end, the teams presented their experience at CLASP.

An important dissonance between the Loughheed project and the VITAL project is that the former was based on lesson study and the latter on learning study. Pre and post tests were not applied and variation theory was not used in the Loughheed team.

A resonance of the VITAL project with the Loughheed team is that both cases included participants from higher educational institutes. In the Loughheed team Armando was a research from Simon Fraser University, and in the VITAL project the teams included specialists from the CALSP or the HKIEd.

Conversations and Activities During Collaborative Design

Although not much information about the design process is presented in Elliot and Yu's (2008) book, some features of the design braid can be identified. For instance, one teacher understood that "many aspects should be considered when designing a course, including the students actual capabilities and learning needs, and various details are to be taken into account in collaborative lesson preparation" (Elliot & Yu, 2008, p. 44). When considering the planning for the whole course, this teacher was engaged in

pursuing coherence. Considering students' capabilities and needs is a part of *anticipating* when planning lessons. Another comment from teachers illustrated the *achieving goals* feature of the design braid: "collaborative discussions of pre-test and its flow could help the teachers clarify the correlation between the test and the teaching aims: and discussions of teaching contents and activities could provoke teachers to think if the teaching activities matched the teaching objectives" (p. 46).

Roles and Positions

Although principals were not engaged directly in the teams of the VITAL project, they played an important role in initiating and facilitating collaborative design at their schools. They decided to include their school in the VITAL project for several reasons (Elliot & Yu, 2008). First, teachers influenced principals' decision of participating in the project. Those who attended seminars or in-service courses at CLASP reported to the principal the possible benefits of learning study. Second, principals were concerned with the reform changes, or with a specific subject area, and participating in the project would help to implement the changes in their schools. Third, principals were interested in supporting weak students in some specific areas. Fourth, participating in the project would entail obtaining resources for the school. Fifth, principals were interested in external support from education professionals. The role of the principal in initiating the collaborative design represents a dissonance with the Lougheed team in which the principal only authorized the project.

School development officers (SDO) were a key component for the implementation of the VITAL project as well. They were a liaison for the school with the HKIEd (Elliot & Yu, 2008). Schools became involved in the project either by invitation from the SDO or by application from the principal who would have to contact the SDO to explain the reasons for the school to be involved in the project. This liaison role is similar to the role of Sofia in the Lougheed team. She was the liaison between Armando and the teachers at her school. However, the role of the SDOs was not limited to that of a liaison. When principals were asked about the added value of the SDO, they stressed the "importance of front line teachers getting recognition from the wider system for their commitment and effort to improve teaching quality ... along with the appreciation of the

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encouragement, advice and support provided by most of the officers" (p. 29). SDOs were also liaisons between teachers and the principals in that they assessed if the school was ready to participate in the project.

SDOs participated actively with the teams of collaborative design as coordinators and supporters as they: (1) engaged in teachers' actions and took part in the discussions and the meetings, (2) gave suggestions for the lesson and participated during its observation, (3) established a good atmosphere of discussion and put forward many relevant suggestions on teaching, and (4) provided an important role obtaining necessary resources and providing support including technology and equipment (Elliot & Yu, 2008). Teachers also perceived added value to the SDO as government fund-seekers, and giving recognition to the difficulties teachers face in the classroom. Some teachers also mention that SDOs provided spiritual support. SDOs stressed their role as a liaison between teachers and HKIEd in order to conduct the lesson or gain assistance for the school.

Specialists in education supporting teachers at participant schools were an important part of the project. Their role is described in the following quotation of a teacher's comment.

The two doctors from HKIEd really gave us a lot valuable information. In particular, since they have very extensive and broad reading, sometimes when we didn't know how to do something, they were able to give us support on the theoretical side. They also brought their experience from other schools. That was very concrete support. (Elliot & Yu, 2008, p. 48)

Theoretical support was not the only role played by the two doctors mentioned in the previous quotation. Bringing the experience from one team to another was also important. This role served to disseminate results of the learning studies across many teams.

In addition to the external specialists from the HKIEd, some external teachers also participated as teacher development consultants playing the role of subject advisor. In contrast to the Loughheed team whose participants reported that they trusted their colleagues at school more than a external specialist (Chapter 5, p.75), in the VITAL report it is mentioned that "teachers will take more notice of advice from an external

source than from their peers and colleagues. It appears that few teachers can become 'prophets in their own country.'" (Elliot & YU, 2008, p. 49). No reference to a specific level of expertise of participant teachers was mentioned, with the exception of the teacher development consultants who were teachers from a different school—outsider experts.

Teacher's Learning

The interviews conducted with principals and teachers who participated in the VITAL project contain evidence of teachers' change. Comments from interviewed principals included teachers' learning of mathematics content.

The teachers have a more thorough understanding of the concept of "fraction," and the students' errors. They clearly feel that effective lesson preparation and lesson analysis can enhance learning efficiency.
(Elliot & Yu, 2008, p. 42)

Learning mathematical concepts, or mathematical content that might be used in class, also took place in the Lougheed team. For instance, Brad explained that he learned about mathematics during the project (Chapter 5, p. 77). Another resonance with the Lougheed team was the interchange of teaching experiences, as reported in the following quotation of a principal from a participant school in the VITAL project.

Teachers carry out a comparatively deep discussion of their teaching and frequently exchanged their teaching experience. (Elliot & Yu, 2008, p. 42)

In their answers to the questionnaire reported by Elliot and Yu (2008), teachers mentioned specific capabilities they acquired during the VIATL project.

A good exploring approach to understanding the content of the subject and carrying out the teaching aims in the lesson, and it will greatly benefit the teachers' personal carrier development as well. (p. 43)

Collaborations between the colleagues are improved, and their reflections on daily teaching are enhanced, they are able to share their teaching experience and what improvements are needed. (p. 44)

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Exchanges and discussions with experts can stimulate teachers' thinking, improve their teaching skills and enhance the effects of lesson preparation. (p. 44)

Provide a platform for collaborative professional development. Provide and opportunity for experimenting/implanting different styles of instruction. Develop a profound understanding of specific topics through professional discussions. (p. 46)

From these quotations we can identify occasions for teachers' professional development by: (1) understanding of mathematics concepts, (2) being able to share teaching experience, (3) improving teaching skills, (4) experiencing different styles of instruction. Having specialists in the teams and working in collaborative design were important factors as described in the last two quotations. The exchanges with experts were mentioned as stimulating teachers' thinking.

At the end of the project several schools continued conducting lesson study or learning study independently, showing a lasting effect on the implementation of collaborative design. Nevertheless, these implementations were different as many schools reported that learning study in the VITAL project was time consuming. For instance, one of the teachers commented that "to adapt to the arrangements of optimizing lessons, the teachers have to cancel some courses or abandon some topics, which affect the students' study" (Elliot & Yu, 2008, p. 75).

Summary

The number of team members and time duration of the meetings for collaborative design is not specified as constant in Elliot and Yu's (2008) report. As there were 120 schools participating in the project, duration of the meetings and number of members in the teams may have varied according to local situations at each school. For this reason, I was not able to contrast the length and periodicity of the meetings with the Loughheed team. Additionally, no transcripts of the discussions during collaborative design are included in the report, making it difficult to identify the categories for the focus of the conversation developed in Chapter 6. However, I found resonance in the following aspects from the VITAL report with the Loughheed team.

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1. The designed artefacts were lessons in each case.
2. Having a person attached to an educational institute was something in common between these two cases. In the case of the Lougheed team Armando was a researcher from SFU, and in the VITAL project specialists from HKIEd and CLASP supported the teams of teachers.
3. Teachers reported a learning of mathematical content during the project.
4. Teachers shared their experiences during the sessions of collaborative design.

Elliot and Yu's (2008) report allowed me to identify dissonances between the VITAL project and Lougheed project. The aspects in which I found dissonances between the VITAL project and the Lougheed project include the settings, as well as the roles of the participants in the project. Because there were no transcripts from the meetings for design in the report, it was not possible to contrast the focus of the conversations during collaborative design with the Lougheed team.

1. An important difference in the setting of the teams from the VITAL project and the Lougheed case was the use of learning study, as opposed to lesson study. This difference entailed the use of variation theory in the planning and discussion of the implemented lessons in the VITAL project.
2. The notion of expertise in the VITAL project seems to be attributed to the educational specialists from the CLASP or HKIEd or from the external experts. In contrast, in the Lougheed team there were teachers with a level of expertise acknowledged by other team members.
3. The trust among colleagues reported in the Lougheed team, in which teachers tended to trust a peer more than in an external expert, contrasts with the the VITAL project in which teachers tended to take advice from an external expert more than from their peers and colleagues at school.
4. Principals and SDOs held an important role in initiating the VITAL project. The involvement of an administrator in the initiating the collaborative design was not a factor in the Lougheed project.

5. Collaborative design continued, in some schools, after the conclusion of the VITAL project, whereas the Loughheed team dissolved when the project finished.

8.4 Conclusions

In this chapter I used three pieces of literature as second-hand data in order to explore, using the characterization developed in the first stage of the study (Chapter 6), more cases of collaborative design. I selected these cases because each one includes a large number of teams. By identifying resonances and dissonances of each case with the Loughheed team, I identified what categories from the characterization developed in Chapter 6 describe interactions in these large-scale cases, and what can be extended in order to have a characterization of interaction among team members in a variety of cases of collaborative design.

The first research question of the third stage of this study was: Does the generated characterization from the Loughheed team describe participants' interactions in other cases of collaborative design? Such characterization emerged into two themes, or dimensions: the focus of the conversations, and the roles and positions of participants during collaborative design (Chapter 6). Whereas the categorization of the on-task conversations during collaborative design served to describe interactions in the cases presented in this chapter, the identified roles differed to a considerable extent from those detected in the Loughheed team. I summarize my findings regarding this question as follows.

1. On-task conversations, as developed from the Loughheed project, served to describe conversations in two of the cases presented in this chapter: the lower grade group at Tsuta secondary school, and the Madrid group of the PRiSSM project. In both cases the categories of the on-task conversations can be observed from the transcripts included in the corresponding sources (Fernandez & Yoshida, 2004; Slavit & Nelson, 2010). In the case of the VITAL project, Elliot and Yu's (2008) report did not provide much information on the conversations during collaborative

design. However, I identified evidence of the anticipation and achieving goals categories in this project.

2. Few instances of off-task conversation can be identified from the cases present in this chapter. These conversations are barely mentioned in the literature. However, in the Madrid group teachers tended to deviate the conversation toward evaluations and the state achievement test, which can be considered as an instance of the *teachers' practice* category (Section 6.1.2).
3. The roles regarding levels of expertise developed from the Lougheed team were not found in the cases within this chapter. There was no mention of a special expertise level that a teacher might have within the teams of collaborative design, with the exception of the teacher development consultants in the VITAL project, who were teachers from a different school, and therefore were external experts.
4. Teachers' perception of an external expert had a contrasted difference. In the Lougheed team Arnold mentioned that teachers tend to trust in the advice from a peer teacher more than from an external expert. In contrast, a teacher from the VITAL project acknowledged the support received from two doctors from HKIEd, especially on the theoretical side. In the case of the lower grade group the external specialist, Mr. Saeki, was invited to participate in the project and provide advice to the group.

The second research question in this stage of the research was: What can be expanded from such a characterization by analysing other cases of collaborative design? My answers to this question for the cases presented in this chapter are listed below.

1. From the lower grade group at Tsuta elementary school, the *anticipating* category of the on-task conversation can be extended to include new properties: (a) the rehearsal of the lesson, (b) the plan of the use of the backboard, and (c) the design of manipulatives.

2. The *teachers' practice* category of the off-task conversation developed in Chapter 6 can be extended to include conversations about evaluation and state, or provincial, achievement tests.
3. There were people playing an important role in initiating collaborative design. Even though principals did not participate as members of the teams in the case of the VITAL project, they took the decision to participate in the project. SDOs and other teachers also were involved in the initial stages inviting, or suggestion to, the principals to participate in the project.

The last question of my research focused on the roles of participants in collaborative design: What are the possible roles of participants in different cases of teachers' collaborative design and how do they influence the interactions within the teams? Based on the content of this chapter, I present my answer to this question, in this third stage of the research, as follows.

Facilitator. The role of the facilitators consisted mainly of contributing to the design of the artefacts by participating in the meetings with the teams of collaborative design. In the case of the Madrid group there were two internal facilitators and one external facilitator. The contribution of facilitators had an impact on the conversations held by the teams. For instance, Camron proposed the mathematically rich task in the Madrid group and led team discussions. Finding resources for the collaborative design might be also a part of the role of a facilitator. The SDOs of the VITAL project facilitated the collaborative work by not only engaging in the design process, but also by providing resources and different forms of support for the teams.

Disseminator. The role of some specialists was not limited to supporting teachers from the theoretical side, they also shared experience from other schools. For instance, Mr. Saeki in the lower grade group of Tsuta school shared results from other schools conducting lesson study. Likewise, the two doctors from HKIED that brought their experience from other schools in the VITAL project represent another instance of this role.

Administrator. New aspects of the role of participants in teachers' collaborative design appeared in the group of Tsuta elementary school described in this section. Ms. Furumoto, the vice-principal, was concerned with teachers feeling autonomy in their decisions.

Manipulative-designer. Another aspect of the roles was the design of the manipulatives to be used in the lesson. This role was held by Ms. Nishi and Ms. Tsukuda in the lower grade group of the Tsuta school.

Time-tracker. This role is played by one of the observers of the implementation of the lesson in the case of the lower grade group, Ms. Maejima.

One of my goals of describing interaction among members of collaborative design was to identify occasions for teachers' professional growth. I identified some of these occasions in the conversations held in the lower grade group and in the Madrid group. Even though these conversations were not transcribed in the case of the VITAL project, the quotations presented in this chapter are evidence that such conversations took place during the design process. Occasion for professional growth among the three cases presented in this chapter include the following.

1. Those involved in the team of collaborative design shared their teaching experiences, as well as their experiences working with other teams.
2. The discussion of the results of the implementations of lesson or mathematical tasks, also represented occasions for learning from students' performance and results of the implementations.
3. Implementing the designed tasks could represent a change in teachers' practice. If they find that the designed artefacts improve students' learning, they might use them in their future teaching.
4. Mathematical content was included as part of the learning of teachers in VITAL project, as mentioned by a principal and a teacher. The discussion during the process of design afforded this learning.

Chapter 9 Participants' Interactions in Teachers' Collaborative Design

The limits of my language mean the limits of my world.

Wittgenstein (1974, p. 68)

This quotation from the philosopher Wittgenstein encapsulates my interests in characterizing interactions among participants in teachers' collaborative design¹¹. The characterization that I have developed in this study includes categories that serve as a language to talk about this type of collaborative work among teachers and other educators. The use of these categories acknowledges the existence of a variety of modalities for teachers' collaborative design. In the first stage of this study, I developed a characterization of interactions from a single case, the Loughheed team (Chapter 6). Then, I used this characterization as a frame to analyse other cases of collaborative design in the second and third stages (Chapters 7 and 8), identifying resonances and dissonances between the Loughheed team and each of these cases. The resonances served as evidence to the extent of generality of the characterization developed in the first stage, whereas the dissonances served to extend such characterization so many cases can be included in it. Chapters 6, 7 and 8 contain answers to the research questions stated in Chapter 2. This chapter summarizes my findings from all the cases of the study as a whole, including the extension, and refinement, of the characterization of interactions among participants in collaborative design.

During this research I characterized participants interactions with respect to two main emerging themes, or dimensions: (1) the focus of the conversations and actions taken while designing the artefacts; and (2) the roles and positions of the involved

¹¹ I am using Wittgenstein's quotation in isolation from the 'logical positivism' philosophical movement influenced by his work.

participants in each case. Table 9.1 summarizes the findings in each case of collaborative design that I analysed in this study. These findings are elaborated on in the following two sections of this chapter.

Table 9.1: Findings in each Case of Collaborative Design

Stage	Case	Findings
First (Chapter 6)	Lougheed team	<ul style="list-style-type: none"> Two emerging themes: (1) focus of the conversation, and (2) the role-positions of participants in collaborative design
Second (Chapter 7)	Professional development programme	<ul style="list-style-type: none"> Tangential conversations Teachers solved mathematical problems Field testing Teachers did not have a remarkable level of expertise Primary and secondary teachers had a difference in the type of comments Info-leaders Students involved as designers Role of the Instructor as a facilitator: expert in mathematics education Every teacher implements in his or her own classroom.
	School district initiative	<ul style="list-style-type: none"> Field testing Coordinator: (1) not an expert in mathematics education, (2) scribe, (3) his position triggered off-task conversations Five minute write Teachers did not have a remarkable level of expertise Differentiation between designers and implementers Fund-seeker
	independent lesson study group	<ul style="list-style-type: none"> Tangential conversations Participants solved mathematical problems Different level of expertise and academic status among participant teachers Experts in mathematics and mathematics education
Third (Chapter 8)	Lower grade group in Tsuta elementary school	<ul style="list-style-type: none"> Rehearsal of the lesson Plan of the use of the board Design of manipulatives Time-tracker Disseminator
	The Madrid group	<ul style="list-style-type: none"> Deviating the conversation to the provincial standardized test Internal and external facilitators
	The VITAL project	<ul style="list-style-type: none"> Use of Variation Theory Experts in the teams Trusting an external expert Principals and teachers as initiators of collaborative design Disseminator

Descriptions of the settings in which collaborative work took place in every case studied in this research were important as they determined certain roles and shaped the interaction among members of the teams. The influence of these settings on the interactions within the team permeates through this chapter. The third section of this chapter includes the influence of the roles, and positions, on the conversations held by participants of collaborative design in different cases.

One of my interests in conducting this study was to identify occasions for teachers' professional growth while participating in collaborative design. Learning about mathematics and teaching mathematics represents some of these occasions and the characterization for interaction developed in this study serves to identify conversations and actions when this learning might occur. Teachers' reports about their changes of teaching practices during the interviews are also evidence of potential professional growth. Certain people holding particular roles also influenced these occasions for professional growth. In the fourth section of this chapter I elaborate on the potential of these occasions for the improvement of mathematical instruction within schools.

9.1 The Focus of the Conversation

The first characterization that I developed during this study was based on the focus of the conversations held by the Loughheed team. When coding the data from the Loughheed project (Chapter 6) I noticed that the focus of these conversations deviated often from the original task of designing the intended artefact. I decided to classify the conversations into *on-task* and *off-task*. During on-task conversations participants made explicit contributions to the design of the artefact. In contrast, during the off-task conversations there was not explicit mention of the design of the artefact. Although these off-task activities and conversations were often related to the artefact that was being designed, participants did not indicate explicitly any relationship, and it was not clear how these conversations would impact on the design of the artefact. The recordings of the meetings for collaborative design of the Loughheed team allowed me to identify activities that teachers participated in outside of these meetings which were related to the project. I classified these moments as *outside* conversations. The time-line graphs

from Figures 6.1 and 6.2 show the on and off-task dynamic during the meetings for collaborative design of the Lougheed team, including the outside conversations.

As per grounded theory methodology, names of codes and categories are tentative and further analysis may result in their refinement, or change of their names. This was the case of the off-task conversation in the final part of this research. One of the purposes of collaborative design is the professional development of the participant teachers, and I identified many instances of off-task conversations as occasions for teachers' growth. The conversation in these instances deviated from the goal of designing an artefact; however, they were still relevant for the learning of some participants. For this reason, and considering the advice from other researchers, I decided to rename this category as *beyond-task*. A description of the on-task and beyond-task conversations follows in the next paragraphs.

9.1.1 On-Task Conversations

The on-task conversation of the Lougheed team was categorized using what I called the *design braid* (Chapter 6, p 92), made of three strands, *anticipating*, *pursuing coherence*, *achieving goals*, and a fourth element, *team organization*. Anticipating refers to the conversations and actions held in order to: (1) predict students performance during the implementation of the artefact, *forecasting*, and (2) propose or design the corresponding artefact accordingly, *commitment*. Achieving goals consists of the conversations and actions oriented to fulfil the pre-established goals for the artefact. Pursuing coherence consists of all the efforts to design the artefact within the context of the particular course. This includes sequences of topics in the unit and the course, as well as students' work habits in the classroom. Team organizations are those conversations that referred to the organization of the collaborative work. These conversations guided the work during the project. The strands of the braid are not isolated, Section 6.1.1 contains examples of how these categories mix together. These four categories encompass all the on-task conversations and actions identified in the recordings.

In the second and third stages of the research I identified instances of the four components of the braid and extended the descriptions of these categories. The anticipating category was remarkably extended in its two subcategories, forecasting and commitment, when analysing the different activities that members of a team engaged in during collaborative design. One of these activities was *field testing*. Although *piloting* the artefacts was included in the categorization from the Lougheed team, the professional development programme and the district initiative cases from Chapter 7 included field testing in a systematic fashion. Arnold piloted the mathematical tasks to be used in the Lougheed project with her son; in contrast, field testing artefacts with the students was an important part of the process of collaborative design in these other two cases. Field testing entailed conversations that related to both subcategories of anticipating: *forecasting* and *commitment*. Team members looked at what worked well and what needed to be modified in an artefact. However, field testing might be a part of the other strands of the braid. Teachers had a chance to identify the extent to which the goals of the artefact were reached and redefine the artefact for future implementations. Achieving goals is, then, incorporated when reviewing the tested implementations of the artefacts. Pursuing coherence might be also a part of the conversations related to field testing. For instance, in the case of the teacher professional development programme one teacher stressed that the rubric they were designing may not be appropriate for a particular mathematical task at hand (Chapter 7, p. 119). The use of the rubric and the task that this rubric would assess had to match in a coherent way.

The description of the design process of the lower level group (Fernandez & Yoshida, 2004, p. 76) includes details of the process of lesson design in which I found another activity that can be added to the on-task conversations. The lesson plan of the lower grade group contains the three strands of the braid (Chapter 8, p. 163). However, being prepared for the delivery of a lesson included, in this case, more than writing a lesson plan. Additional activities were: the construction of manipulatives, and the rehearsal for the lesson with special attention to the use of the board (Chapter 8, p. 164). Conversations during these activities related strongly, but not exclusively, to the anticipating category of the design braid.

9.1.2 Beyond-Task Conversations

The beyond-task conversations of the Lougheed team were varied and I classified them into four categories (Chapter 6, p. 94): (1) *teachers' practice*, (2) *mathematics and educational context*, (3) *collaborative work*, and (4) *casual conversation*. The teachers' practice category included mathematical topics from the curriculum, as well as strategies for teaching and learning those topics. Additionally, comments on particular students were a part of these conversations. The mathematics and educational context category included discussions about mathematics that should, or should not, be included in the curriculum. The importance of learning mathematics for life fell also within these conversations. The collaborative work category refers to the conversations related to the team work among teachers, including the same project, as well as other cases such as lesson study. The casual conversation category refers to all the conversations unrelated to mathematics, mathematics education, teaching practice or the collaborative work among teachers. These categories encompass all the off-task conversations recorded from the meetings of the Lougheed team.

Beyond-task conversations were mentioned by interviewed people in the three cases presented in Chapter 7. The notion of *tangential* conversation was used by different people suggesting that most of what was being said in the meetings for collaborative design was related to the design of the artefact. Moreover, the Professor in the independent lesson study group referred to these conversations as “add-to-the-task” (Chapter 7, p. 149) Tangential conversations included talking about topics such as curriculum, students, parents, education in general, and school. These conversations can be a part of both the teachers' practice category and the mathematics and educational context category.

In the cases of the professional development programme and the independent lesson study group, participants also solved mathematics problems (Chapter 7). This activity might not be explicitly related to the design of the artefacts in each case, which suggests another type of conversations and actions in collaborative design not found in the Lougheed team: *problem solving*.

The interviewees from the independent lesson study group mentioned that there were no casual conversations in the meetings for collaborative design (Chapter 7). However, in the school district initiative casual conversations were considered as something relevant. The *five minute write* at the beginning of the meetings was intended to get something "off the chest" (Chapter 7, p. 137). Casual conversations not only happened, but also had a particular place within the meetings for collaborative design.

Particular roles of participants in collaborative design, as well as the settings in which the collaborative work took place, had an impact on the type of off-task conversations held in the teams. For instance, the collaborative work category from the Loughheed team was scarcely found in other instances of collaborative design. The conversations in this category were strongly influenced by the fact that the Loughheed project was a study on teachers conducting collaborative design (Chapter 6, p. 102). Another instance of how particular roles and settings influenced off-task conversation was the school district initiative in which the Coordinator had particular information due to his position in the district and teachers asked specific questions about school policy (Chapter 7, p. 136).

Elements of the beyond-task moments of the conversations were not easy to identify in the literature. This, of course, does not mean that such conversations were not held among participants of collaborative design. Similarly, in both the independent lesson study group and the professional development programme casual conversations were mentioned as scarce. However, participants might not pay attention to them. The sources of data in these cases were based either on what authors wrote in the literature or in the memories of interviewees, representing a limitation in identifying these types of conversations.

In summary, I found that the characterization for the focus of the conversation developed in Chapter 6 served to categorize interactions in the other cases of collaborative design presented in Chapters 7 and 8. The activities and conversations that I identified in the second and third stages of the study served to revise and extend the categorization of both the on-task and beyond-task conversations.

9.2 Participants' Role and Position

The notion of role used in my study goes beyond the pre-established activities that members of a team negotiate for the collaborative design. Langenhove and Harré (1999) proposed an alternative and more flexible notion for *role*, the notion of *position*. By taking this notion in mind, I was able to consider not only pre-established activities for members of the Lougheed team, but also how each person perceived the role of the other participants. For instance, Armando's role as a researcher in the Lougheed team was not perceived in the same way for every participant (Chapter 5, p. 70). I also included interests and motivations for participating in the project as part of the notion of role and position. This decision was based on the fact that some conversations in the Lougheed team were triggered by teachers' interests in participating in the project (Chapter 6, p. 102). In this section I describe the roles and positions identified in the study, and discuss these concepts based on the data of the study.

9.2.1 Roles and Positions in Several Cases of Collaborative Design

The context in which the Lougheed project was conducted shaped the roles and positions of the team members. I identified three factors that had an impact on these roles and positions: (1) the collegiality of the participant teachers who had worked together before this project, (2) the fact that the Lougeed project was a research project, and (3) the teachers' particular interests in participating in the project.

The particular roles and positions that I identified from the Lougheed team during the first stage of the research were: the *researcher*, the *facilitator*, the *promoter*, the *sceptical voice*, and the *expert*. The researcher role was played by Armando. Although perceptions of this role varied among participants in the team, there was an agreement about who was the researcher. The role of facilitator was played by different people in the team. Armando brought ideas and material to the meetings and organized the meetings. The presence of Armando as a facilitator motivated teachers to work outside the regular meetings in preparation for these sessions with the whole team. Sofia and Arnold also facilitated the collaborative work by typing the lesson plans and bringing books and articles to the meetings, respectively. The role of the promoter was played by

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Sofia who served as a liaison between the teachers at the Loughheed school and Armando. This role was related to the collegiality of the teachers that already existed before the project. Arnold and Brad decided to participate in the project because Sofia invited them. The sceptical voice was a role played by Brad, who often questioned whether the proposed tasks for the artefacts that were being designed would serve to achieve the selected goals. Sofia also played this role by questioning whether the *cube* problem should be used for the lesson that was being designed. The role and position of the expert was related to the level of expertise of a team member in certain area. Arnold was a *data-base expert* who brought literature and websites to the discussions of the team. Sofia proposed most of the problems for the lessons designed during the project and Brad considered her as a *mathematics expert*.

When analysing the cases of collaborative design described in chapters 7 and 8, I identified a broader set of roles and positions. The particular contexts, or settings, in each case entailed different forms of engagement in collaborative design. For instance, the lesson study independent group and the Loughheed team used observed implementation in which *observers* participated in the debriefing of the lessons. However, the professional development programme and the school district initiative did not conduct observed implementation. The role of *observer* was not present these cases. Finding resonances and dissonances between the Loughheed project and the other cases served to identify roles which were exclusive to certain modes of collaborative design, as well as positions that were not present in the Loughheed project. A list of categories for these settings that differentiate these cases is presented on Table A.3 in Appendix A. The following paragraphs include descriptions of the set of identified roles and positions from the second and third stages of the study in three main categories: (1) the positions within the job, (2) the particular settings for the collaborative work, and (3) the factors that made it possible to work collaboratively and enhance the learning across different teams.

Job position within the school, or an educational institute, defined specific roles such as the *teacher*, the *specialist*, the *facilitator*, and the *administrator*. One member might have multiple roles in a team of collaborative design. A teacher was a member of the team who was teaching a course at either level: elementary or secondary. Teachers

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often made reference to their own students during collaborative design. In most of the cases teams were formed by teachers at the same level. However, grade level seemed to be a factor in the way teachers interacted in the case of the professional development programme (Chapter 7, p. 122), which included teachers from both elementary and secondary levels. Specialists were mathematicians or mathematics educators who support teachers in the collaborative design. Their level of involvement in a team of collaborative design varied from a few visits or consults, such as in the case of the Madrid group (Chapter 8, p. 174), to active participation during the whole process of design, as in the case of the Instructor in the professional development programme. Specialists were often facilitators as well; however, they held official status such as being a professor at a university or holding an academic degree. In contrast, facilitators did not necessarily hold official status. Their involvement in collaborative design also varied. Some facilitators were resource providers, such as the SDOs in the VITAL project (Chapter 8, p. 180). Other facilitators engaged in different ways. For instance, the Instructor in the professional development programme case facilitated the discussion and set learning tasks for the teachers (Chapter 7, p. 121). In the case of the school district initiative the Coordinator was not an expert in mathematics or mathematics education. However, he was an important piece in the organization across the district collaborative work providing resources to the teams (Chapter 7, p. 132). Moreover, he also engaged in writing and graphically designing the artefacts according to teachers' decisions. In this case, as on the case of supported teacher inquiry, the facilitator also played a role of leader in the team. An administrator sometimes took part in the collaborative design. The case of the lower level group included the vice-principal as part of the team (Chapter 8, p. 161). The Coordinator of the school district initiative also commented about including an administrator in the team (Chapter 7, p. 134).

Participants' roles in collaborative design can be also classified according to the activities influenced by particular settings. I identify the following roles which depend on the settings of each modality: *designers*, *implementers*, *observers*, and *scribes*. Every case of collaborative design had designers, those who were in charge of actually constructing the artefact. Designers included teachers and external experts. Moreover, students might be designers, as mentioned by one of the teachers in the school district

initiative. Observers were not present in all the cases. Only those modalities of collaborative design in which implementations were observed had this role. The observer might have an established duty, such as tracking the time during the implementation, as it was the case of the first lesson in the lower grade group (Chapter 8, p. 167). Implementers were those teachers who used the artefact in their classroom. In the case of the school district initiative the implementers may not participate as designers. Scribes were those in charge of writing and designing the actual document for the artefact—such as a lesson plan or a rubric. For instance, in the school district initiative the coordinator supported the collaborative design through being a scribe for the team (Chapter 7, p. 132). In the case of the lower grade group teachers had also to design and make the manipulatives for the lesson (Chapter 8, p. 166). The *manipulative-designer* can be considered as a subcategory of the scribe.

Other type of roles and positions deal with the initiation and sustainment of the collaborative work, including the sharing of ideas and experiences among different teams. The *initiators* were key people in the initiation of the collaborative work. The role of the *promoter* played by Sofia in the Lougheed team can be considered as an initiator (Chapter 6, p. 105). Her position, as a colleague in this group of teachers, was an important factor in initiating the collaborative work. Other examples of people holding the role of initiator were the principals and teachers that made it possible to incorporate their schools in the VITAL project (Chapter 8, p. 179). Related to this is the role of the *fund/resource-seeker*. This last role was played by the Coordinator of the school district initiative (Chapter 7, p. 135). The role of the *disseminator* was played by those people who made it possible to share ideas and experiences among different teams of collaborative design. Examples of people holding this role were the *info-experts* described by the Instructor of the professional development programme (Chapter 7, p. 121), the outside advisor in the lower grade group (Chapter 8, p. 167), and the external experts in the VITAL project (Chapter 8, p. 180).

The use of the *role* and *position* in this study deserves closer attention to the definition and common usage of these concepts. The data from this study can be used to conceptualize role and position within groups of teachers' collaborative design from an empirical basis. Such conceptualization is discussed in the next subsection.

9.2.2 Conceptualizing Role and Position

The original meanings of the words *role* and *position* have evolved into broader social contexts. The word *role* has its origin in the old French word *roule*, which means *roll* and referred "originally to the roll of paper on which the actor's part was written" (Role, 2010). Looking at the current definition in online dictionaries, this word refers to "an actor's part in a play, film, etc" (Role, 2010). However, it is also defined as "the function assumed or part played by a person or thing in a particular situation" (Role, 2010) and "the position or purpose that someone or something has in a situation, organization, society or relationship" (Role, 2011). The latter definition links role with position. The Visual Thesaurus online dictionary (Thinkmap, 2011) includes several definitions of role as shown in Table 9.2. Those definitions have two attributes to role as used in this study: (1) pre-established activities or expectations, and (2) customary activities of behaviour of one person. Roles identified in this study such as designers, implementers, observers, or scribes entail a set of pre-established activities and expectations. In contrast, the roles, or positions, of mathematics expert and data-base expert refer to customary activities and behaviours of particular members in the Loughheed project: Sofia and Arnold.

Table 9.2: Definitions of *Role* and *Position* in a Social Context

Role (Thinkmap, 2011)	Position (Thinkmap, 2011)
<ul style="list-style-type: none"> • normal or customary activity of a person in a particular social setting • the actions and activities assigned to or required or expected of a person or group • an actor's portrayal of someone in a play • what something is used for • any specific behaviour 	<ul style="list-style-type: none"> • the act of positing; an assumption taken as a postulate or axiom • a rationalized mental attitude • the act of assuming or taking for granted • an opinion that is held in opposition to another in an argument or dispute • the post or function properly or customarily occupied or served by another • the relative position or standing of things or especially persons in a society • a job in an organization • (in team sports) the role assigned to an individual player • normal or customary activity of a person in a particular social setting
<p>Image or text from the Visual Thesaurus (http://www.visualthesaurus.com), Copyright ©1998-2011 Thinkmap, Inc. All rights reserved.</p>	

The word *position* has its origin in the Latin word *positio*, or *ponere* which means "to place" (Position, 2010). This concept referred initially to physical space relative to another objects. For instance, position is defined as "a place where someone or something is located or has been put ... a particular way in which someone or something is placed or arranged" (Position, 2010). However, in a social context, this word also means "a person's point of view or attitude towards something" (Position, 2010). The Visual Thesaurus dictionary (Thinkmap, 2011) includes other definitions, as shown in Table 9.2. Among the definitions presented in the table, three attributes can be identified that have been used in this study: (1) assumption; (2) opinion and attitude; (3) customary or specified activities within a group. Note that last definition in the table appears also in the list of definitions of *role*, namely: "normal or customary activity of a person in a

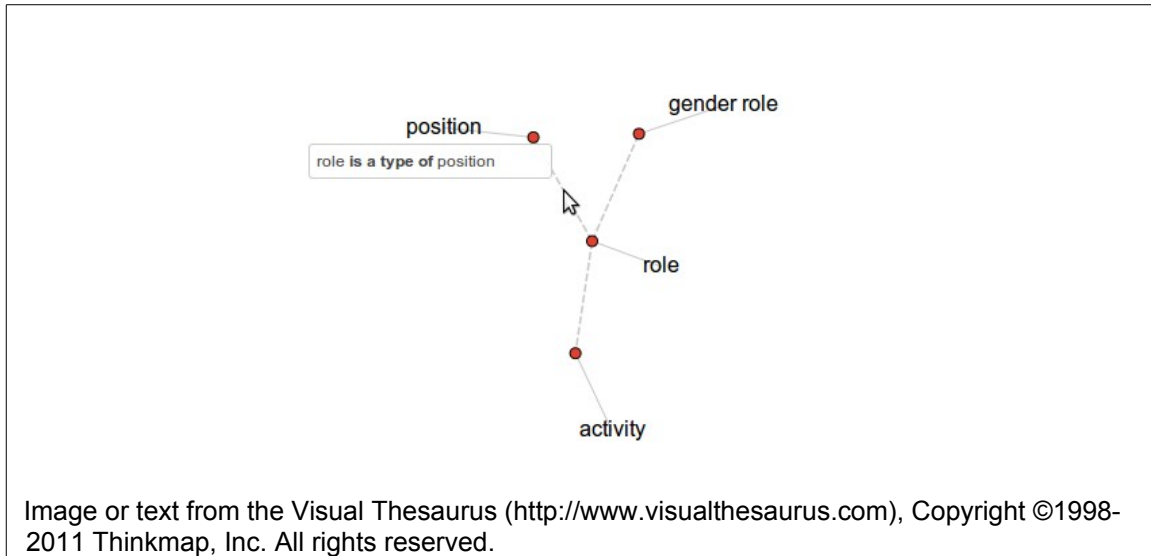


Figure 9.2: Map of the Relationship between Role and Position

In this study I use the word *role* in a broader sense in order to include both pre-established expectations and non pre-established customary performance of a person within a team. Although *role* and *position* have the same meaning in one particular case or context, other aspects of the definitions of *position* were identified in the data. For this reason, it is important to stress that I have identified not only roles, but also positions within the cases of teachers' collaborative design presented in this study.

In addition to the use of role and position described before, there is another aspect that has been considered during the data analysis. The notion of *positioning* described by Langenhove and Harré (1999) referred to the forms in which people position themselves and others through a conversation. They explained how people can follow or reject a position imposed by one who has uttered a sentence in a conversation. In contrast to Langenhove and Harré's, my focus in this study is not on the process of positioning. Rather, I focused on the fact that people perceive the role and position of a person in different manners. This perspective is consistent with the symbolic interactionism main premises (Blumer, 1969), as described in Section 3.3. Under these premises the individual makes sense of situations and decides according to his or her own interpretations instead of just acting according to established norms or expectations. This meaning is constructed in a dialectical process individual-situation. In

the case of this study, roles and positions are understood as individual perceptions in ways that may not coincide from one person to another. The differences of these perceptions may, or may not, be acknowledged by participants; however, a general consensus was enough to work collaboratively. An example of a position was when Brad commented that he had not been mathematical trained (see for example Chapter 5, p. 77). During the first group interview of the Loughheed project, he positioned Sofia as a mathematics expert (p. 73) and himself as a learner within the team.

Another aspect of position adopted, and adapted, from Langenhove and Harré (1999) to this study is the dynamic characteristic of the position: "Conversations have storylines and the positions people take in a conversation will be linked to these storyline" (p. 17). Langenhove and Harré focused on a conversation, whereas I am focusing on the conversations and actions during a longer period of time. The positions of the Loughheed team members were shaped not only during the project, but also before the project, as reflected by the collegiality of the participant teachers.

In summary, I have used *role and position* as a concept that encompasses the definitions of both *role* and *position* in a social context. This concept includes pre-established roles and positions within a team, as well as emerging roles and positions during the process of design. The notion of role as a "set of expectations of how a member of a special group or community is expected to act in his/her position" (Kaasalia & Laurilia, 2010, p. 855) can be extended to include individual perceptions. The set of expectations is not necessarily shared among all the members of the group or community. Different members of the same group can have different expectations for, and positions respect to, the same person.

In this section I have described the notion of *role and position* used in this study since the first stage, as well as several roles played, and positions taken, in cases from the second and third stages. The characterization of the roles and positions developed from the Loughheed project was limited to a single case of collaborative design. I added several descriptions of other roles identified from the cases presented in the second and third stages of the study. The settings in which collaborative design took place were factors that shaped the roles and positions in each case. These settings and the roles of

the members of collaborative design had an influence on the conversations and actions described in the previous section. This influence is described in the following section.

9.3 Relationships among Roles, Settings and Conversations

The two dimensions of interactions among participants in teachers' collaborative design developed in this study have had a significant level of mutual influence, and both took place within particular settings in which collaborative design was conducted. The dimension of the roles and positions can be classified into *formal* and *emergent*. The former classification refers to those roles and positions determined from the beginning such as teacher, researcher, administrator, or specialist. Although perceptions of these roles and positions may change during interactions, they tend to be more stable. In the latter classification roles and positions emerge during the design process. Examples of these are the data base expert or the mathematics expert in the Loughheed team. Such roles and positions are not established from the beginning, they are taken, or adopted, during the interactions among members of the team. In Figure 9.3 the two dimensions are represented within the settings, and the influences between the dimension are represented by arrows. Formal roles and positions that were established during, or before, the beginning of the collaborative design influenced the conversations and actions held within a team. These formal roles and positions were stable during the design process. Actions and conversations had no, or little, impact on the perceptions of these roles or positions. In contrast, the emergent roles and positions were co-evolving with the activities and conversations. They were not established in advance and people took, adopted, or perceived, them during the design process. In this section I explain the relationships between the two dimension for interactions using the data presented in previous chapters.

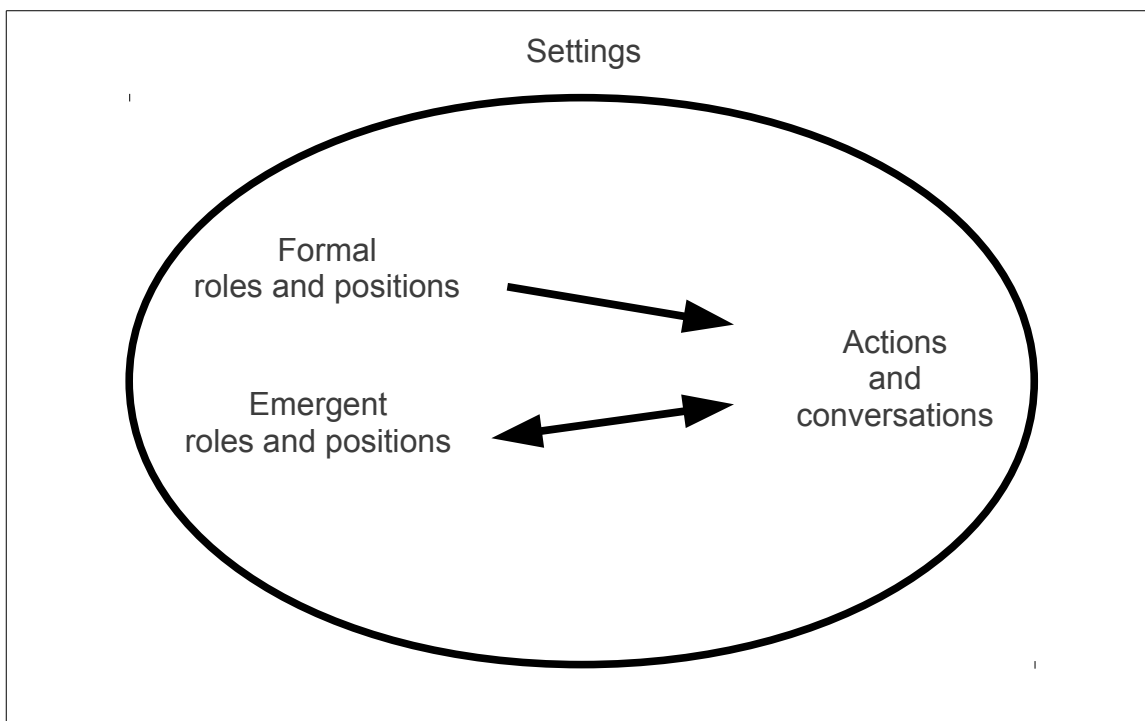


Figure 9.3: Relationships among Roles, Settings and Conversations

Since the first stage of the project, I noticed that the conversations and actions held during collaborative design were influenced by the settings and the roles and positions of the team members. The Lougheed project was a research study based on collaborative design inspired by lesson study, a situation that influenced some of the conversations held in the Lougheed team. Arnold, who wanted to learn more about lesson study asked for references from the literature related with this modality of collaborative design (Chapter 5, p. 76). Brad was interested in contributing to research in education and he reported outside activities that the teachers had done when Armando was not present. He, also asked questions about this research during the meetings for design (p. 76).

The roles and positions of *data-base expert* and *mathematics expert* identified in the first stage of the study were not pre-established (Chapter 6, p. 105). These roles corresponded to what Arnold and Sofia said, and did, during the meetings. As Arnold constantly brought books and articles related to mathematics education to the meetings, she triggered both on and beyond-task conversations in the team. Brad, who positioned

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himself as a learner, took the opportunity to start conversations related to topics he was teaching at the time of the project (Chapter 5, p. 96). These roles and positions resulted from the interactions within the team.

Some participants in the teams of collaborative design were constantly bringing the discussions back to be on-task during the sessions. Sofia redirected the conversation to the design of the artefacts in the Loughheed project (Chapter 5, p. 73). The Organizer in the independent lesson study group mentioned that when her mentor was in the team, they were very focused (Chapter 7, p. 146). Additionally, the Coordinator in the district initiative case also focused the discussion to being on-task (p. 132).

As facilitators, the Coordinator in the school district initiative (Chapter 7, p. 132) and the Instructor in the professional development programme (p. 121) asked questions and lead discussions during the design process. This role of facilitating the discussions in the teams was also indicated in the external expert of the Madrid group (Chapter 8, p. 175).

Having an administrator in the teams of collaborative design might hamper the discussion, as it was the case of the district initiative (Chapter 7, p. 134). However, this effect was opposite in the lower level group where the vice-president of the school was a part of the team (Chapter 8, p. 166). In both cases, the presence of an administrator influenced the conversations within the teams.

The positions of the Coordinator in the school district initiative also influenced beyond-task conversations related to school changes and policy. This contextual situation is similar to the Loughheed team, which was part of a research project. As a consequence, teachers asked questions about this study during the meetings for the collaborative design.

The specialists in mathematics and mathematics education also had an influence in the type of conversations held during collaborative design. These specialists were present in the independent lesson study group, the professional development programme, and the VITAL project. In the case of the school district initiative the Coordinator mentioned that he was not an expert in mathematics and the teams lacked

a specialist in their regular meetings. However, occasionally external specialists were brought to work with the teachers in this case. The lack of a mathematics, or mathematics education, specialists might impact on the conversation within teams of collaborative design. For instance, in the Madrid group the external specialist visited the team a few times and the group focused more on evaluation than on understanding students' thinking, which was the initial goal for the inquiry cycle (Chapter 8, p. 174).

In this section I have described how the roles and positions of participants of teachers' collaborative design influenced, and were influenced by, interactions among team members. Facilitators and specialists in mathematics and mathematics education were relevant roles and positions that influenced the focus of the conversations. Additionally, they provided observed occasions for teachers' learning, as explained in the following section.

9.4 Potential for the Teachers' Learning

The categorization developed in this study serves to identify different occasions for teachers' professional growth. Teachers in the cases presented in this study were exposed to: (1) sharing ideas and experiences, which potentially would widen their repertoire of strategies; (2) reviewing literature on research and other resources related to mathematics and mathematics education; (3) sharing and discussing beliefs about mathematics learning; (4) interacting with peers, promoting in this way the building of a community with its own knowledge; and (5) reflecting on their practise as well as the means and goals of their teaching. These factors resonate with the conclusions given by Lewis et al. (2009), who indicated “three pathways through which lesson study improves instruction: changes in teachers' knowledge and beliefs; changes in professional community; and changes in teaching–learning resources” (p. 285).

Anticipating, as part of the design braid, entails considering students' actions, and thinking, as well as their corresponding teachers' responses. By examining data from three empirical studies, Goldsmith et al. (2009) concluded that “attention to and analysis of student work is an important process within the 'black box' of teacher improvement that deserves principled attention in future research” (p. 103). The design

braid describes how teachers put attention to students' work in the design process of a learning artefact.

Teachers who participate in collaborative design brought their experience and issues to the team. They have the potential to apply in their own classroom what has been designed or discussed in the collaborative design. During the inside and outside subcategory of the focus of the conversation described in Chapter 6, teachers reported innovative practices in their teaching as a consequence of participating in the Lougheed project.

During the Lougheed project the conversations deviated often from the original tasks of designing an artefact. However, such beyond-task moments were usually focused on education and mathematics education. Teachers had opportunities to learn in those beyond-task moments of the design process as they: (a) shared perspectives on mathematics and on education; (b) shared control group strategies; and (c) asked specific questions about teaching a content unrelated with the artefact under design. Beyond-task conversations were often rich in discussions related to teaching and learning mathematics, as mentioned by teachers from the Lougheed team and the professional development programme, providing an occasion for professional development. In this way, the term beyond-task is an apt description, compared to the term off-task, for this classification of the focus of the conversation.

9.5 Discussion

This chapter contains a summary of my findings during this study integrating all the cases presented in the previous three chapters. The characterization for interactions among participants in teachers collaborative design that I have developed consists of two dimensions: (1) the conversations held during the process of design, and (2) the roles and positions held by those involved in collaborative design. For the first dimension I categorized the focus of the conversations and activities during the design process. In the second dimension I developed a list of possible roles and positions held by people involved in collaborative design, as well as their influence on the conversations within the teams. A contextual component important in identifying the roles and activities in

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teams of collaborative design were the settings of each case. The influence of the settings on the interactions among participants of collaborative design was indicated as well.

Notwithstanding the limitations of the sources of data during the second and the third stages of this research, the characterization of possible conversations and roles held in teams of collaborative design presented in this chapter is extensive. During the first stage I participated as a member of the Loughheed team, and the sessions were recorded. For these reasons, I was able to capture many aspects that I found relevant and that would not be possible to register otherwise. In contrast, during the second stage my sources of data were limited to the surveys, interviews, and conversations with participants of three cases of collaborative design, and the used literature in the third stage. Nevertheless, these sources of data allowed me to consider cases which included several teams of collaborative design. The cases used from the literature as second-hand data represented modalities of large-scale collaborative design which encompassed a large number of teams.

Although the characterization for interaction among participants of collaborative design developed in this study is extensive, it cannot be considered as exhaustive. There exist different modalities of collaborative design not included in this study. Additionally, many cases within the modalities presented in Chapters 7 and 8 were not included in the research. However, as I considered representative cases of larger-scale modalities of teachers collaborative design, I expect that this characterization of interaction can be used for other cases.

The categorization of the conversation during teachers' collaborative design that I have developed served to identify potential occasions for teachers' professional growth. Both on-task and beyond-task conversations included topics about mathematics and mathematics teaching. The strands of the design braid entailed conversations in which teachers had to reflect and plan on mathematics teaching. Beyond-task conversation included consults about topics in mathematics and how to teach them, as well as discussion about the curriculum.

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The description of the settings in which each case of collaborative design was conducted not only served to identify contextual factors that influenced conversations and roles held in teams of collaborative design, settings also served to identify constraints on the collaborative work. For instance, in the case of the independent lesson study group finding time for the collaborative work and a place to meet was often a challenge (Chapter 7, p. 144). Observing implementation of the lessons was problematic because participants had to take time from their daily activities. The SIGMA institution that supported this group also offered special mathematics courses at the elementary and the secondary levels on Saturdays (p. 142). The independent lesson study group started to implement their lessons in these courses, making it easier for the members of the group to observe the implementations.

Finally, although the impact on students' improvement on mathematics learning was not a focus of this study, it is worth mentioning that there is evidence of this impact. Japan, being a country with high scores in the international comparisons of mathematics achievements (Martin, Mullis, & Foy, 2008), can be considered as evidence of the impact of collaborative design on mathematics learning by means of lesson study. Additionally, the Coordinator in the school district initiative mentioned a great improvement of students' performance within the district initiative since teachers' collaborative design was implemented (Chapter 7, p. 138). However, more evidence should be generated in order to determine the impact of teachers' collaborative design on students' mathematics performance in a variety of modalities and cases.

Chapter 10 Implications

The characterization for interactions among participants in teachers' collaborative design developed in this study has implications for both practitioners and researchers. Teachers and other stake holders in education can be informed by the results of this study in the implementation of collaborative design at both the school and district levels. The contribution for research of this study includes methodological aspects, as well as a ground for theorizing about interactions among participants of collaborative design. This chapter contains an elaboration of the implications of this study for practitioners and researchers.

10.1 Implications for the Implementation of Teachers' Collaborative Design

Teachers' collaborative design has a potential for a double effect on mathematics instruction. Students' mathematical learning can be improved by means of the implementation of the artefacts designed in this collaborative fashion, as it was reported in the case of the school district initiative (Chapter 7, p. 138). Additionally, teachers have occasions for the improvement of their practice by debriefing the results on the implemented lessons. Those interested in implementing collaborative design can be informed by this study in several aspects. Firstly, this study presents a variety of modalities for collaborative design. People interested in the implementation of this type of collaborative work can adopt, and adapt, features of the cases included within the study.

Secondly, the settings in which collaborative design took place in each case influenced the collaborative work. In order to conduct collaborative design, aspects such as time and a place for the meetings have to be considered. For instance, the independent lesson study group (Chapter 7) had constraints on the time and place for the meetings dedicated for the design process. Moreover, it was difficult for the

participants of this group to observe the implementations of the lessons. Participant teachers might have to leave their classroom in order to observe the lesson. This situation was even more difficult if teachers had to commute to a different school. The implementation of observed lessons has been conducted successfully in Japan because the entire education system facilitates lesson study (Fernandez & Yoshida, 2004). In order to implement lesson-study-type of collaborative design at school, it would be important to make adequate arrangements that facilitate the meetings and implementation of the designed lesson. Other modalities of collaborative design did not include observers, such as the professional development programme and the school district initiative (Chapter 7). Scheduling collaborative design at school within teachers' working hours was a facilitating factor. For instance, the three large-scale cases from the literature used as second-hand data for this study (Chapter 8) included collaborative design as mandatory teacher professional development within school and during working hours. Additional factors in the settings for collaborative design include the number of team members, the time and frequency for the meetings, as well as the norms within the team—e.g. turn talking (Chapter 8, p. 172) and the five minute write (Chapter 7, p. 137). These factors can be also taken into account when conducting collaborative design.

Thirdly, different roles and positions of participants in collaborative design can be considered for its conduction. Facilitators and specialists in mathematics and mathematics education were key roles, and positions, in the data of the study. Dissemination of the results was relevant, as the learning of one group might be extended to more people. The role of the disseminator was played by the 'info-leader,' in the case of the professional development programme (Chapter 7) and by specialists in both the VITAL project and the lower grade group (Chapter 8). Different levels of expertise among participant teachers were found relevant in the Loughheed team (Chapter 6). In other cases few comments about experts in some areas were mentioned. However, recognizing the expertise of some participant teachers in a collaborative design team might inform the goals and design process of the designed artefacts. The teams can take advantage of those who have a level of expertise in certain areas. Shared knowledge, as opposed to "an individual attribute" (Boaler, 2002, p. 42), has the

potential to enhance both teachers' learning and mathematics instruction. An example of the shared knowledge among teachers is the case discussed by Askew et al. (1997) which consisted of a school with high results on an assessment of numeracy. Whereas not all the teachers at that school demonstrated a strong discipline knowledge, two teachers shared responsibility for mathematics across the school: a mathematics expert, and an expert in psychology and pedagogy of primary mathematics.

Fourthly, considering collaborative design as part of teachers' regular practice fosters a perspective on teaching mathematics as a lifelong-learning career. Teachers, as well as school practices, can be up to date on research by having contact with specialist of mathematics and mathematics education who support collaborative design. Additionally, teachers have the opportunity to learn from their own practice through the debriefing of the implementation of the designed artefacts.

And fifthly, collaborative design allows teachers to contribute to the development of curricular material. In the case of the school district initiative (Chapter 7), teachers developed district assessment mathematical tasks. Those artefacts were designed by a small team of teachers, and field tested by many implementers. The final result was an artefact which was implemented in several classrooms, and then refined with the input of these implementations. Thereafter, more teachers used this artefact district wide. The change in teaching practice was not limited to designers involved in the design process, the artefacts were incorporated in the whole district. In this sense, collaborative design has the potential to influence teachers' practice in two forms: (1) becoming a curriculum developer, and (2) implementing the developed artefacts resulting from collaborative design.

The aspects mentioned in this section are supported by the results of this study. Practitioners, including teachers and other stake holders, interested in conducting collaborative design can be advised by them.

10.2 Implications for Research

The implications of this study for research on mathematics education include methodological aspects, as well as a ground for conceptualizing interactions among

participants in teachers' collaborative design. In this section I elaborate on these implications, including possible venues for further research.

10.2.1 Implications for Methodology

In this study I applied a research methodology from social studies to mathematics education. Grounded theory has been used for research on mathematics teachers by a few authors—e.g. Rowland, et al. (2005). This study represents not only another instance of grounded theory in mathematics education, but also a particular way of conducting qualitative research starting from a single case. The first stage of the study consisted of a detailed analysis of the Lougheed project. The recurrent coding of the data, consisting of the recordings of the meetings for collaborative design and the transcribed interviews, afforded the emergence of two themes which became the two dimensions of the characterization of interaction developed in my research. Following Charmaz's (2006) description of open coding and focused coding, this characterization served to generate and analyse further data. In the second and third stages of the study I analysed more cases of collaborative design using the developed characterization for interactions in the first stage. For the second stage I contacted participants of three cases of collaborative design in order to generate more data directly from interviews, surveys and conversations. The literature also played a relevant role during the third stage: three pieces were my source of second-hand data. These pieces comprised three different modalities of collaborative work encompassing a large number of teams. By identifying resonances of the Lougheed project with each of the cases in the second and third stages, I developed a characterization for interaction that includes common features among a variety of modalities. The dissonances served to identify domains of variability among cases and to develop a categorization for conversations and roles held during collaborative design. This categorization can be used as a language that acknowledges the different modalities of collaborative design included in this study.

Three different sources of data were used in my study. During the first stage, when the Lougheed project was conducted, I participated directly with the process of collaborative design. The recordings of the meetings allowed me to analyse in detail each moment of the process of designing the artefacts. Additionally, the corresponding

interviews, both group and individual, served to validate my interpretations of the project, as well as to generate further data for emerging themes. This process of data generation based on the initial findings of the research is congruent with grounded theory methodology as proposed by Charmaz (2006). However, the Lougheed project was a case of collaborative design which was intended for this research since the beginning. This situation was reflected in the beyond-task conversations categorized as collaborative work (Chapter 6, p. 102). The study of other cases of collaborative design in the second and third stages of the research served to mitigate the limitation of studying one case which was designed for this research.

My role as a researcher and participant in the Lougheed team also influenced the interactions during the project. Adler and Adler (1987) classified the researcher's 'membership role' as *peripheral*, *active*, or *complete*. In the active type of membership "researchers participate in the core activities in much the same way as members, yet they hold back from committing themselves to the goals and values of members" (p. 35), and in the complete membership "researchers study their topics from the perspective of full members" (p. 53). I situate my membership role as a researcher in the Lougheed project in between the active and the complete types. While I participated in the core activities of the Lougheed team engaged in the design process, I was not a teacher at the Lougheed school. This fact is reflected, for instance, in the *outside conversations* where teachers reported activities related to collaborative design in which I was not present. Considering the membership role of the researcher in the study entails taking into account researchers' experiences and perspectives (p. 34). An example of how my perspectives and experiences shaped this research was my decision to consider the roles of the participants as an emerging theme after the first group interview of the Lougheed project (Chapter 5).

In the second and the third stages of the research there were two type of sources of data, respectively: (1) interviews, surveys and conversation with participants of three different cases of collaborative design, and (2) three pieces of literature. In the former case I generated the data contacting directly participants of collaborative design. In the case of the independent lesson study group, I was also a member. However, interviewed participants worked with different teams, in which I was not necessary a member. Being

a member of the group allowed me to: (1) include information that was not mentioned in the interviews, and (2) ask questions aimed at issues that I was already aware of within the group, such as the difficulty of implementing observed lessons (Chapter 7). With the exception of this case, I did not have a membership role in the cases of the second stage of the study. The three pieces of literature used as second-hand data were generated with a purpose that differed from my own purpose in this study, yet these data served to find resonances and dissonances with the Loughheed case, extending, and refining, the characterization for interaction among participants of collaborative design as described in Chapter 9.

The diversity of data sources was a key component in this research. My membership role in the study allowed me to consider one case in detail, the Loughheed project. In the second stage of the study I had access to people who had participated in a variety of teams, and used, in the third stage, the literature as second-hand data which included large-scale modalities of collaborative design. Using these forms of data generation afforded the resulting characterization for interactions among participants in teachers' collaborative design in a variety of modalities.

10.2.2 Theoretical Contributions

The characterization for interaction developed in this study can be used as a framework for analysing interactions in other cases of collaborative design. The first dimension for interaction, the focus of conversations and actions, serves as an organizational tool for analysing discussions during professional learning activities. The categories of the on-task classification for the conversations held within teams of collaborative design include teachers' discussions about their practice, students' mathematics learning, and coherence within the curriculum. Potential for teachers' learning during collaborative design can be found in the *outside conversations* where teachers reported activities related to collaborative design which took place outside the regular sessions. Beyond-task conversations, both tangential and non-tangential, included also occasions for teachers' growth. These type of conversations were a side effect of collaborative design that had a potential to impact on mathematics teachers' learning. The characterization that I have developed in this study, however, should not

be considered as complete or universal. Although several cases have been used for its development, other cases of collaborative design in different settings might differ to a considerable extent. When using this characterization as a research framework for other cases it would be important to take a critical stance and be open to contradictions or additional complexities.

Another contribution of this research regarding the classification of the focus of the conversation is the use of graphs (Figures 6.1 and 6.2) to represent the dynamics between the on-task and beyond-task¹² conversations. These graphs allow the researcher to identify: (1) possible patterns across several sessions of collaborative design, (2) moments when the conversations changed, and (3) outside conversations which can be used as evidence of teachers' change of practices. This visual representation of the data may be used to analyse conversations and actions in different teams conducting teachers' collaborative design. Moreover, such graphic representation is not limited to collaborative design, other forms of conversations and actions could be analysed using these type of graphs.

The developed categories for the on-task conversations were represented metaphorically as a braid (Figure 6.3). This metaphor alludes to the entangled nature of the relationships among three categories of the on-task conversation: anticipating, achieving goals, and pursuing coherence. The first category is specially interesting because teacher's beliefs and knowledge can be inferred from the conversations related to anticipating. The two subcategories of anticipation, forecasting and commitment, not only describe anticipating as a process, but also include the reasons why someone foresees a student's performance, as well as the rationale for a possible teachers' response during the implementation of the designed artefact. A fourth category of the on-task conversations was team organization, which serves to identify the way members of a design team organize themselves for collaborative design.

My findings on the second dimension for interactions in this study, the roles and positions held by participants in collaborative design, also have theoretical implications. First, the role and position of a member in a design team may not be perceived the same

¹² In Figures 6.1 and 6.2 I used off-task instead of beyond-task—see Chapter 9, page 191 for an elaboration on the reasons for this change.

by all the participants, as I noticed in the Lougheed team (Chapters 5 and 6). Second, pre-established rules for collaborative work are not enough to describe interactions among participants with different roles. Third, considering the interests and positions held and perceived by these participants serves to better understand why some type of conversations occur during the design process. These implications are considered in Chapter 9 where a conceptualization of role and position within teams of teachers' collaborative design is discussed. Neither activity theory (Engeström, 1998) nor communities of practice (Wenger, 1998) took into account how these roles and positions affect, and are affected by, the interactions within a team. The interactions characterised in this study also include the influence of the setting in which the collaborative work took place on the roles a positions held in the teams.

Finally, the characterization developed in this study may be used as a ground for theorizing about interactions among participants in teachers' collaborative design. *Communities of practice*, (Wegner, 1998) as well as *cultural historical activity theory* (Engeström, 1998) have been used as frameworks by research on teachers working collaboratively. In this study the categorization for conversation among team members in collaborative design, the possible roles and positions taken by these members, and the relationship between these two dimensions of interaction were developed from empirical data based on different modalities of teachers' collaborative design in mathematics. The results of the study can be used to theorize about interactions among collaborative design teams, without assuming broader social theories that may not provide particular accounts, such as: (1) solving mathematical problems, as in the cases of the professional development programme and the independent lesson study group (Chapter 7); (2) selecting the numbers for a task according to the mathematical principle intended to be explored in class, such as the selection for the number 12 in the task designed by the lower grade group (Chapter 8); and (3) talking about teaching of particular topics in mathematics, as the Lougheed team did when the beyond-task discussion about introducing trigonometry took place within the meetings for the design process (Chapters 5, p. 96).

10.2.3 Further Research

The characterization for interaction that I have developed in this study can be extended and refined by analysing more cases of collaborative design. This is a path that seems to be natural to consider for further research. For instance, the graphical representations of the focus on the conversation in Figures 6.1 and 6.2 can be used to identify patterns in other cases of collaborative design and compare them against the results presented in study. Another option would be testing this characterization against data from other cases of collaborative design that support, or contradict, the results presented here, affording extensions and modifications to the developed characterization.

During this study I decided to focus on the design process, leaving the negotiation of goals for the artefact and the debriefing of the implementation apart. Research on these two process would complement the results of this study.

Although I did not focus directly on the benefit of students' mathematics learning as a consequence of conducting teachers' collaborative design, some evidence of improvement in mathematics learning was present in the data. In the school district initiative the Coordinator mentioned statistics that suggest an improvement in students' numeracy (Chapter 7, p. 138). The high ranking of Japanese students in international mathematics tests also suggest an impact of lesson study in mathematical students' learning. However, research focused on the effect of teachers' collaborative design on mathematics learning has to be conducted in order to better understand the benefits of collaborative design. Kullberg (2007) argued that a good lesson design may not have the same results in different implementations.

When the teacher succeeds in replicating the specific pattern of variation and invariance, the effects on learning are replicated also. It is therefore not the lesson design that should be replicated but rather the pattern of variation and invariance of the critical features. (p. 127)

A venue for further research can be the identification of factors of the implementation of artefacts in collaborative design that have a potential to replicate successful results in students' mathematical learning. The on-task conversations classification developed in Chapter 6 can be used to identify interactions among teachers in which they forecast

students' possible performance in class, as well as teachers' responses in order to integrate those factors that replicate success in students' mathematical learning.

Beyond-task conversations identified in this study included discussion on mathematics and mathematics learning that represented occasions for teachers' professional growth. I have not found in the literature research having as a main focus these conversations and their impact on teachers' learning and change of professional practices. Research on this topic would contribute to the understanding of the side benefits of teachers' collaborative design.

The outside conversations, as described in Chapter 6, represent evidence of teachers' innovative practices in the classroom as a consequence of participating in collaborative design. However, there is little evidence that supports change in the long-term; further research may document the impact on mathematics teachers' practice through direct observation.

In this study I showed evidence of the different forms in which the role and the position held by a participant of collaborative design can be perceived by the team members. In the Lougheed team my position as a researcher was perceived in different ways by the participant teachers (Chapter 5 and 6). The notion of role and position, as perceived by different team members, can be explored, including their influence on the interactions among those involved in collaborative design. The roles of facilitators and specialist in mathematics education were indicated as important in the data of my study. These particular roles can be also a focus of further research.

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Appendix A Codes and Categories: Properties and Examples

Table A.1: On-Task, New Codes

New codes	Properties	Examples	
Resources	Literature review: At the beginning of the process in order to search for results in education research and suggestions for activities.	Reading research papers	Looking for useful strategies.
	Use of literature to have a conceptual framework	In order to assess students	In order to describe students process
Anticipating	Students struggles and approaches. Wondering and predicting.	Students possible approaches	Performance: Students failures. Students do not read instructions. They will need help
	Teacher response to students	With questions: "What about the hundred one?"	Suggesting the use of tables and shapes. Giving hints like "square numbers."
	Designing elements to prior lessons	Get students use to problem solving and patterns. Put similar problems before the lesson.	Give mathematical knowledge needed for the lesson. Including the sequencing of PLOs.
	Students previous context: knowledge and skills.	Wondering if students will use a table	Looking at covered topics. Teacher usual practices—like problem solving at the beginning of the lesson.
	Differentiating instruction.	Scaffolding: Point of entry for a non algebraic thinkers. Start with an easy problem. Concrete representation	Speedies. Give more challenging problems

New codes	Properties	Examples	
Anticipating (continued)	Hypothesizing students learning process	If students can come up with the explanations in words, it will be easier to write mathematical expressions.	Proposing to move students from "need help" to work "independently"
Goals	Defining the goal	From previous experience	From PLOs.
	Fitting the goal in the classroom	Fitting the goal in the unit	Fitting the goal in the lesson
	Checking the goals with the lesson	Liking the goal with the unit	The goal does not fit the lesson
	Looking back.	Recalling the original goal	Adjusting the original goal to the context of the lesson
	Expectations for the students in the class	Low expectations doe to students	The can because they have been using patterns since elementary school
Scheduling	The date of the lesson	According to the course —unit, plan topic. The goals. students prerequisites	According to participants' availability and calendar
	The activities for other meetings	Agenda for further meetings	
	Negotiating	Making the lesson fit into the course.	
Observers	Role of th observer	Passive	Supporting students
	Introduction the observers to the class.		
Choosing problems	Analysing the problem	In terms of mathematical difficulty and content.	In terms of the goal
	Selecting Dismissing	By content, or goals. Links with other topics	By difficulty, scaffolding or for speedies.
	Looking for problems	In the literature	Already know by someone
Task	What students will be required to do	Proposing students to come up with their own tasks.	Students would describe with words their thinking.

New codes	Properties	Examples	
Format	Format of the lesson	Team discussion.	Presentation, Comparison of students solutions.
	Posing the problem and task	Format of the worksheet: tables and charts.	Wording the problem. Use of concrete representation
	Link with other topics	Use of units for measure	
	Prior lessons	Do not give answers	
General strategy	Beyond the lesson	Get students use to work with patterns as a powerful tool. Problem solving in general. Assessing strategies	Sequencing of lessons
	Linking the lesson with more topics	In the mathematical course	In science courses.
Piloting	Posing the problems to some else prior to the lesson in order to have some previous feedback	With other classrooms	Whit family: son partner
Recapitulating	Debriefing the lesson as per it is proposed at some specific moment	In order to redirect the discussion to the lesson	In order to clarify and agree.

Table A.2: Off-Task, New Codes

New codes	Properties	Examples	
Resource review	Sharing, discussing, suggesting, criticizing resources such as books, articles, websites, and conferences.	Commenting on a book about L.S. Because of a participant's interest.	Sharing and criticizing a book brought to the meeting related with teaching and learning mathematics
Mathematics concept discussion	Discussing some mathematics concepts or language.	The proper use of the word "mathematics" rather than just "math."	Difference between equation and function.
Interest in the research	Participants involved in the research. It includes all the questions and information given to participants.	Participants ask or suggest something about the research project	Researcher informs the participants about the process.
Sharing a strategy	Participants share or ask for some teaching strategy	Asking for a strategy to assess using portfolio. Use of textbook in the classroom.	Sharing the use of the "animal game."
Collaborative work	Teachers propose or comment on the collaborative work of the project.	Comments on the possible feedback of the observers in the implementation of the lesson.	Proposing to focus on assessment for the next lesson.
Students' learning	Teacher's beliefs and expectations of the students. It also includes learning theories as well. Students problems for learning, and causes of failure, too. It also includes proposing strategies for the improvement of students' mathematics learning.	Teacher's expectations of the students. Comparing students.	Variation theory and Vygotsky
Mathematics curriculum	Commenting and criticizing curriculum	Curriculum is not easy to teach in the course	Mathematics taught at school do not pass "surviving information to the next generation."
Mathematics outside school	Discussing about the use of mathematics outside school and what is taught in school.	Mathematics and workforce	Mathematics as a filter to universities

New codes	Properties	Examples	
Sharing experience	Participants share experience and comment on their own work as collages.	Asking about university courses taught by Armando	Suggesting material for university courses taught by Armando.
Casual conversation	Conversation unrelated to education and mathematics. It includes personal comments	Preference for veggie food	Talking about the tea and coffee.
Teacher's problem	Problems teacher face at their practice	Resource are not available. What to do with certain type of students.	Complete all the topics of the curriculum.
Implementing ideas	This is what teachers reported they were doing, or will do, at their classrooms as a consequence of the meetings	Implementing the use of patterns.	Using an assessing strategy mentioned in the meetings.

Table A.3: Settings for Teachers' Collaborative Design

Category	Properties	Examples	
Condition of participation	In many cases the collaborative work is voluntary, but it seems to be quasi obligatory.	Voluntary	(Quasi) Compulsory
Location	Place for holding meetings and implementing the artefacts.	Inside school	At other institution
Schedule	Time for meetings and implementing the artefacts	At working hours	Weekends
Field testing	The artefacts are tested in some classrooms or with some other people.	Designers pilot the artefacts with family members	Implementers who may not be part of the design team pilot the lesson at their classrooms
Facilities, funds and incentives	Founding has several sources: school, district, government, research projects.	No funds in terms of money, but facilitation of milieu and snacks for meetings.	District funds and incentives for school implementing collaborative design.
Goal choice	The goal for the designed artefacts can be determined by different people.	Individual teacher's goal	School district goal
Attachment to an institution	In many cases there is an educational institution involved in TCD. In other cases there is some research conducted as well.	TCD as part of a professional development program such as a masters degree or a diploma	An educator from an university is invited to give a talk or workshop.
Size of the team	In some cases there is a team that design the artefact and observe the implementation. Other cases there is a small design team that meet in some sessions with more people who contribute to the designing as well.	Two members as designers and implementers	More than ten members in a workshop designing the same artefact
Diffusion	After the implementation and refinement of an artefacts, the results are often shared to other teachers.	No dissemination of the results. Only participants of the team are benefited	Publication of the designed artefact in journals or booklets available for other teachers.
Culture at school	Presence of the teacher at the classroom. Collaborative work as part of the teachers' duties.	Students can stay in classroom working without the teachers	Students would never be working in the classroom without a teacher therein

Category	Properties	Examples	
Use of cognitive theories	Th use of cognitive theories in the design of the instruments. It can be based on both, the literature and the experience of the participants.	There is not an explicit learning theory; however, there are some principles such as the use of "problem solving," and "meaningful learning"	Use of specific learning theory such as Variation theory
Artefact	It is a document containing the students' tasks as well as some explanation for its implementation.	Lesson plan	Assessment rubric
Purpose	Although there is a goal for the artefact under design, there are different purposes of conducting collaborative design additionally to teachers professional growth.	Improve teaching by refining a lesson	Inquiry in some learning issues

Appendix B Preliminary Findings for the First Group Interview

This section contains the document given to the participant teachers of the Lougheed team before the first group interview conducted on November 18th, 2008. At this point of the research I was considering my study as ethnographic because I was immersed in the community of teachers who participated in the project. The methodology, however, was elaborated further and developed as constructivist grounded theory, as explained in Chapter 3.

Connecting Patterns and that Mumbo Jumbo Stuff we have to Teach: A Collaborative Lesson Design

Predicting students' struggles and possible approaches in problem solving is part of Lesson Study strategy. In this paper a team of teachers—including the author—made use of previous experience, knowledge of current students, and some theoretical background from the literature in order to prepare suitable responses in advance to students' questions and thoughts in the designed tasks. While making such predictions, beliefs of mathematics and mathematics learning were discussed and negotiated, and we developed theoretical statements about students' learning process. In conclusion, I argue that predicting such possible students' struggles and approaches not only provides an arena to analyse and negotiate teachers' mathematical and pedagogical knowledge, but also is a critical factor contributing to the improvement to educational systems.

Introduction

This paper is a preliminary result of a wider research project which attempts to analyze teachers' interactions when designing mathematical lessons collaboratively—in particular by conducting lesson study—as described by James W. Stigler and James

Hiebert in their book *The Teaching Gap* (1999). The research focuses on the potential teaching improvement by giving an account of teachers' changes in their beliefs and practices.

Teachers and educators have been using collective lesson design and analysis as part of their professional development. Watanabe (2007) explains that lesson study contributes to improving the curriculum and textbooks design; Marton and Tsui (2004) use learning study, based on variation theory, as means of improving learning. Both learning study and lesson study involve collective lesson design by teachers and educators in a reflexive way. Communities of teachers and educators pursuing learning improvement in a critical way are sustained on social practices of teaching—for instance, Jaworski (2006) argues in favor forming communities of inquiry, and Servage (2008) describes the critical and transformative practices of professional learning communities. This perspective situates the teachers in a more socially-engaged practice, going beyond just the classroom: critical collaborative work among peers is a part of teachers' practice.

It is easy to predict the kind of interactions that members of a lesson design team could have, as well as the knowledge they might share. Subject matter, pedagogical and curricular knowledge—as described by Shulman (1986)—is shared through the discussions teachers have while planning lessons. The reflexive process of teachers and the changes they undergo have been analyzed using narratives (e. g. Brown and Jones, 2001). However, a good understanding of the factors which contribute to professional growth in a community and its members when working in a collaborative way is needed in order to implement and sustain such community' development. Which factors contribute or constrain the collaborative work? What kinds of interactions trigger the teachers' learning in a community? What are the individual and community's learning processes? The aims of this research is to take a look inside a small community in order to observe its members' interactions and learning while participating in a cycle of lesson design, implementation, and refinement.

Among the topics teachers use to discuss in lesson study are: the selected problem for the lesson, including its wording and numbers; anticipated solutions,

thoughts, and responses which students might develop; and the kind of guidance, or questions, that could be given to support students showing some misconceptions in their thinking (Stigler & Hiebert 1999, p. 117). In this paper I show a case where teachers involved in lesson study engage in discussions using theoretical statements, both derived from the literature and developed by their own, while approaching those aforementioned topics.

Theoretical Background

“Communities of practice” (Wenger, 1999) is a useful theory to describe social interactions of people having a common enterprise. The notions of negotiation of meaning and identity serve to describe how knowledge is generated in such a community. In order to describe the process of designing the lesson, and in particular how a team developing theoretical statements about student learning, it is important to consider the collective characteristics of the task. As a community, each participant has a way of engaging with the team. Each individual contributes different ideas and resources according to his or her own perspective. Each participant has an identity, and a role in this particular community. The meaning of the theoretical statements obtained in the discussion of the lesson is negotiated by the team.

I will consider professional development not only as an individual enterprise, but also as a matter of community learning. For instance, if some teachers leave the school, the community of the school keeps a legacy of the former members. From this point of view, improving teachers' practices also improves the school at the same time.

Brown (2001) describes learning not just as adding knowledge, but as a transformative process: “...*knowledge, or at least our state of knowing, can be transformed in many ways; one subtracts from it as well as adds to it . . . one reorganizes so that new things get new meaning*” (p. 84). I use this description of learning for both communities and individuals; the case of learning in a community is described by Wenger (1998). From a phenomenological perspective, Brown (2001) argues that “*it is the individual's experience of the world, of mathematics and social interactions which governs his actions rather than externally defined notion of*

mathematics itself" (p. 138). I consider not only mathematical notions, but also ideas we may have about any other subject; in particular teachers' notions of students' learning process. Teachers and researchers build their knowledge from personal experience, peers discussion, and literature reviews—which in any case are subordinated to personal interpretation. From this point of view, the individual—teacher or researcher—as well as the community, makes meaning of, describes, and predicts certain phenomena—for example, students' performance in a classroom. I will call these individual or collective interpretations *theoretical statements*.

Methodology

In order to give an account of the evolving process of teachers designing a lesson through several meetings, ethnography is a suitable means of describing social interactions and micro-cultural aspects of the community. A team of teachers was video recorded while designing and discussing a mathematical lesson. I split the video records in small segments and write a description according to what was discussed in each moment, generating pre-codes. After focusing on teachers' use of theoretical statements to predict student approaches to the mathematical tasks of the lesson, I selected some segments to transcribe and analyze. My participation in the research is both as a member of the team and as a researcher. The use of video allowed me to focus fully in the lesson design discussion, and observe the meetings later for research purposes.

As I mentioned above, this is a preliminary report of a design experiment (Cobb, Confry, diSessa, Lehrer, & Schuable, 2003) and some steps which will complement and give stronger validation to the study are still missing. Further interviews with participants will be conducted in order to discuss my conclusions, or detect new issues. New interventions in the future will be conducted with a possible shift in the research.

The Study

The team was composed of three secondary mathematics teachers from the same school and me—I have experience teaching at this level, though in another

country. I will refer to myself as Armando when describing and analyzing the video recorded meetings.

The lesson has been designed for a grade nine class in a secondary school in British Columbia. We held five meetings, one a week, before the implementation of the lesson. Teachers selected the goal of the lesson study: for students to write algebraic expressions from word sentences. We decided to use patterns for this purpose in part because it relates to the curricular prescribed learning outcomes students have to achieve.

From the video it is possible to distinguish some differences among the team members. For instance, Arnold (pseudonym) always brings some book, article, or other resource to the discussion; Brad (pseudonym) engages in a critical way in the discussion by questioning whether we will reach the desired goals of the lesson, Sofia (pseudonym) used to redirect the discussion of the meeting when we lost focus, and Armando used to refer to his previous experiences to explain ideas. These differences are instances of each member engaging in the community.

Theorizing in Order to Predict

Since the first meeting, when we decided to use patterns, Armando has hypothesized that students will make meaning of algebraical expression easier if they can verbalize mathematical procedures derived from finding the required number—for example, perimeter, amount of squares—in a sequence of shapes with some linear pattern. In this hypothesis, Armando was theorizing the way they can make sense of algebraic expressions.

When we started to predict possible student approaches and difficulties, theoretical statements were used in the discussion; some of them came from books or other material and some of them were generated by us. I will show first an example of the use of theoretical statements from the literature.

As a way of describing the students level of understanding, Arnold referred to an assessing scale which appears in Marzano (2007) and consists in four major levels—with some additional sub-levels in between.

The lowest score value on the scale is a 0.0, which represents no knowledge of the topic. Even with help, the student demonstrates no understanding or skill relative to the topic . . . A score of 1.0 indicates that with help the student shows partial knowledge of the simpler details and processes as well as the more complex ideas. . . .[with] a score of 2.0, the student independently demonstrates understanding of and skill at the simpler details and processes, but not of the more complex ideas and processes. A score of 3.0 indicates that the student demonstrates skill and understanding of all the content—simple and complex—that was taught in class. A score of 4.0 indicates that the student demonstrates inferences and applications that go beyond what was taught at class (p. 104).

This scale was used by Arnold in the second meeting as both a reference to classifying students as well as a description of how we would like students to move forward.

Arnold: More moving on the scale so that if we model here for students with help, then hopefully the students will be able to move into this area [pointing to the score above 1.0 in the assessing scale].

Further in the same session, Arnold kept using the same scale to describe students' possible paths when we were selecting the problem to be posed in the lesson.

Arnold: If it was up to me, I would introduce that and really try to move them on this continuum to one and then set them independently in something more challenging, and then see where the students can get to [pointing out to the Marzano's scale].

A second case of the use of theoretical statements is the way it helps to make meaning of students' learning process. In the end of the second meeting Brad was questioning whether the use of patterns in the way we were discussing would be effective in making students translate words into algebraic expressions, while Armando argued it was necessary that students come up with the algebraic expressions from their own explanation of how to find the general term in the patterns.

Brad: After doing all those puzzle-solving [problems] and getting their own solutions and writing them down and talking about it, how is that help with specifically this task of translating? [words into algebraic expressions].

....

Armando: We must conclude this lesson.... with some algebraic expressions . But the idea is that these algebraic expression come from the wording of students.

However, Brad is still concern with covering the topics in the curriculum, which are the same as in the book, and how to relate that with the use of patters in the lesson we are designing.

Brad: I'm just trying to find the connection, the link So, this is the class, we now did all this problem solving, and now lets see if they can do this. We still have to teach this, no matter what.... After all these funny games we still have to teach this [pointing to the page in the textbook related to writing algebraic expression from English sentences].

Next meeting Brad made meaning of the students' process of writing their ideas in order to write algebraic expressions. It seems that Brad is thinking and talking, questioning and answering at the same time.

Brad: What I am trying to do is [this], I'm fitting in what I have to teach in that section, where they translate words into algebra, with the activities here. So, I was thinking: lets say they come up with "there is two more than three times the stage". Well, that is good because then we now can express algebraically two more than three times the stage; "is this like this?" and you can write it like that.

In my mind, I am trying to blend in what we are doing here with what we have to teach, or what they have to know how to do in the textbook. I am trying to find that connection between this and those mumbo jumbo stuff they have to do. They say, "Oh this is just two more than the stage." How do we write two more than something?

Arnold: I think that will be amazing if they can verbalize that, because. [interrupted by Brad]

Brad: They can verbalize it down here: you know for the tenth stage you just have to add two more to the previous stage. But, what does it have meant? How do you write that in math instead of just writing it in words?

I try to blend, I'm trying to fit in that section I want to teach with what we were doing here. I try to find the way to do that in my head.

So if I would do this in my first class, I would say O.K. lets take a look at all your answers down here. Is there a way to simplify that? Is there a way to make it easier to write instead of all the

words? You know, it took two lines or three lines to explain your answers. Is there a simpler way, an easier way to do that? And they will say “okay. . . lets write that algebraically, or may not call it algebraically, lets write that in an easier way, and see what it looks like.”

And then Brad agrees with Armando about providing a hypothetical case of two different students' approaches to find the general term in one of the possible sequences we use in the lesson.

Brad: But one group may say “two more than twice the number” or one group may say “add one to the stage and double it.” Well, let's say they are the same. Let's work it out algebraically and see they are the same... Why not use algebra instead of all these words?

After this discussion, Brad started participating in a more enthusiastic way with the lesson design, possibly because Brad has made meaning of the use of the patterns in getting students to translate word sentences to algebraic sentences.

A third case was the use of theory from other sources in order to design our questions for students' task. In the fourth session, Arnold brought a rubric to evaluate communication for students from a binder with many resources which have been used before. The rubric consists in three criteria and four levels for each criterion. Although we did not use this rubric to describe or assess students, it was useful in phrasing one of the questions for the students' task.

Armando: We are missing here the fourth question which is: could you explain how do you get it [the number of the n stage]?

Arnold: How do you phrase that? Explain your ... like it was consistent with this rubric: “my explanations are clear and complete, and easily understood.” [showing the communication evaluating rubric].

We were discussing the point that the textbook presents sentences which students must translate into algebraic expression. These expressions have no context and Sofia, as a theoretical statement, though that it would be problematic for students. Brad agreed with that statement.

Sofia: I'm kind of thinking that when they describe something that is physical, then it's easier to translate it [to an algebraic expression] than just some sentence.

Finally, at the end of the fifth meeting, we came up with a chart of students' possible thought-pathways, struggles, and teachers' responses. This reflects the theoretical statements we came up after meaning discussion and negotiation. These statements provided a framework which was also proposed as a frame to observe students achievements in the lesson while it was being implemented (Table 1).

Student steps and potential gaps	Possible teacher responses
Student draws each pattern and counts number of squares/perimeter.	
Student uses recursive thinking (adding to previous stage) to determine number of squares/perimeter at each subsequent stage.	"Where are you adding lines at each stage?"
Student can predict number of squares/perimeter at any stage (non- recursively). Student can use words to describe how to determine number of squares/perimeter at any stage (non-recursively).	"Is there a more efficient way of adding [the same number] many times?" "For [a particular stage], how many times did you add [the same number]?" "Think aloud."
Student can write a mathematical formula to describe number of squares/perimeter at any stage.	For pattern A, work through this step with the class, using several different examples of student-generated words to come up with (hopefully) a few formulas which can be compared. Use n , as well as $n - 1$. Use the word "previous" in relation to $n - 1$. Use the word "formula". Refer to this example later when students try to find formula for other patterns.

Table 1. Students' steps and potential gaps, and teachers' responses.

After the implementation of the lesson, we could observe that almost all student teams, when presenting their answers to the group, came up with a general formula to describe the patterns in the tasks. This was beyond the expectations of the lesson. We also realized that students didn't have troubles in moving from the drawn figures to the recursive formula. However, they needed some guidance to explain their process. Some other students wrote algebraic general expressions as explanation for their procedures. Although they were correct, explanation with words were missing.

Discussion

The process of anticipating solutions, thought, and responses which students might develop, as well as planning the kind of guidance teachers will give to students, entails teachers' use of theoretical statements—which either come from a known cognitive theory or are developed by teachers. This process challenges teachers' beliefs and assumptions, and triggers an adjustment of both the individual teacher's conceptions and collective meaning.

The idea of generating and refining theories useful for teaching in a collaborative lesson design involving teachers has been applied in design experiments (Cobb, Confry, diSessa, Lehrer, & Schuable, 2003). For instance, learning study uses variation theory as a grounded theoretical framework (Marton & Tsui, 2004). The capability of describing and anticipating phenomena is characteristic of any theory. Therefore, describing and anticipating students' solutions or approaches to posed problems must entail the use or development of theoretical frameworks.

In addition to corroborating and refining teachers' theoretical statements, written reports of implemented lessons contribute to an improvement in the community itself, not only in the teacher. In this way, the educational system can be in a permanent process of enhancement, as Watanabe (2007) mentions with regard to Japanese elementary educational system. Implementing communities of teachers working collaboratively and contributing to next teachers generations in local settings—school or district—will contribute to the learning of such community, not only as a mentor-apprenticeship relation with novice and experienced teachers, but also as generating new knowledge from teacher's practice.