

**ESSAYS ON TIME PREFERENCE ANOMALIES,  
INTERTEMPORAL CHOICE, INSURANCE, AND STATUS**

by

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# Abstract

The goal of my dissertation is to analyze individuals' behavior when they make choices over time and within a group. The first chapter is devoted to explaining some key time preference anomalies which are inconsistent with the standard discounted utility model. In the second chapter, I focus on how inter-personal comparisons would affect people's intertemporal choices. Finally, the last chapter studies how the concern for status affects the optimal risk sharing across individuals.

The first chapter studies some key time preference anomalies. These include the time preference reversal characteristic of hyperbolic discounting, the magnitude effect and the extreme sign effect. I propose a simple explanation of discounting that accounts for these three anomalies simultaneously, within the context of the expected utility model with uncertainty, risk aversion and preference for precautionary saving.

The second chapter develops an intertemporal model in which individuals care about consumption not only for its own sake but also for the status it implies. By putting an additive status term into the utility function, I show that the level of inequality in the initial wealth distribution affects individuals' saving and consumption behavior. The direction of the distortion in intertemporal choice relative to the standard model without status depends on the elasticity of intertemporal substitution in the utility from absolute consumption. I also analyze how changes in the initial wealth distribution affect saving.

In the third chapter we develop a series of optimal social insurance models in which people care about both consumption per se and the status it implies. We show that the concern for status does impact the optimal contract under various information structures. Particularly, under complete information without commitment problem, the optimal contract may assign all the society resources to the minority group if the status term is convex enough. Under the limited enforcement regime, compared to the optimal allocation in the pure consumption model, it is optimal to transfer

more resources to high income people when the status term is convex. Under moral hazard, the relatively lower status resulting from the higher effort level may make implementation of high effort level more difficult.

**Keywords:** Time preference anomalies; risk attitude; status; intertemporal choice; inequality; insurance

# Dedication

*To my parents, Tongke and Shufang.*

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# Chapter 1

## A Simple Explanation of Some Key Time Preference Anomalies<sup>1</sup>

*Individual time discounting behavior experimentally exhibits important anomalies that are inconsistent with the standard discounted utility model. These include the time preference reversal characteristic of hyperbolic discounting, the magnitude effect and the extreme sign effect. I propose a simple explanation of discounting that accounts for these three anomalies simultaneously, within the context of the expected utility model with uncertainty, risk aversion and preference for precautionary saving.*

### 1.1 Introduction

A strong empirical regularity occurs as animals and humans value future payoffs less than current ones, other things equal. This so-called time discounting behavior has a number of key features. Of interest here is “hyperbolic discounting,” the key implication of which is that, if a smaller payoff realized sooner is compared to a larger payoff realized later, people prefer the larger-later payoff at first but switch to the smaller-sooner payoff over time. Another feature of interest is the “magnitude effect”, where larger payoffs are discounted less than smaller ones, for a fixed time delay. The final feature is the "sign effect". Generally, this is seen when gains are discounted more than losses, and in extreme cases, an individual may prefer to incur a fixed loss

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<sup>1</sup>This chapter is based on the work published in *Canadian Journal of Economics*. I would like to thank Arthur Robson, Alex Karaivanov, Daniel Monte and David Andolfatto for the continuous encouragement and insightful suggestions. The valuable comments from Yoram Halevy, two referees and seminar participants at SFU and FUR XIII conference in Barcelona are very appreciated.

immediately rather than delay it. Frederick, Loewenstein and O'Donoghue (2002) provide an excellent survey of these observations.

None of the features are consistent with the standard model of discounted utility, where the rate of time discounting is constant and independent of the size and the sign of future payoffs. Thus the above features are termed “time preference anomalies”. Within the framework of expected utility with uncertainty, risk aversion, and preference for precautionary saving, I provide a simple model that simultaneously accounts for all of these “anomalies”. Consequently, these anomalies are perfectly consistent with each other.

In this paper, I take the view that the crucial difference between future payoffs and present payoffs is that the former are associated with uncertainty. Hence, the evaluation of such payoff uncertainty determines the time preference. Individuals are equipped with a certain attitude about risk for contemporaneous payoffs, and this hardwired risk attitude leads them to behave “anomalously” with respect to future payoffs. Specifically, in the model, a reward is perceived to be associated with uncertainty with regards to its magnitude. The uncertainty increases with time into the future, and for risk-averse individuals with a concave utility function, it follows that future rewards are discounted because of the increasing uncertainty. The magnitude effect is then a consequence of the convexity of marginal utility. As a result, when offered a smaller-sooner payoff and a larger-later payoff, individuals may experience the preference reversals characteristic of hyperbolic discounting. Finally, the extreme case of the sign effect, when individuals prefer taking the losses right away, is an immediate consequence. Throughout the paper, I assume that the only source of time discounting is the uncertainty about the magnitude of the future reward; however, this does not deny the possibility of other sources. Indeed, the pure time preference can be incorporated into the model and the main conclusions will not be altered.

Recently there is emerging empirical literature that shows people's risk attitudes and time preference are related. For example, Dohman et al. (2007) found that more risk averse people tend to discount future payoffs more. In the theoretical literature, an interrelationship between intertemporal choice and choice under uncertainty has long been discussed by economists. For example, a number of authors (Prelec and Loewenstein (1991), Loewenstein and Prelec (1992), Quiggin and Horowitz (1995), and Rubinstein (2003)) draw analogies between the problems inherent in a model with a constant rate of time preference and those inherent in the one shot expected

utility model (see Kahneman and Tversky (1979) for a description of the expected utility anomalies). As summarized by Halevy (2008), “these authors borrowed tools developed for the uncertain environment to analyze the separate problem of temporal decision making under certainty.” Halevy himself links hyperbolic discounting and the certainty effect<sup>2</sup>. I also adopt the perspective that time preference can be derived from individuals’ attitude to risk. In any case, the present paper derives key time preference “anomalies” in a simple expected utility model. No anomalous expected utility behaviour is seen here, which is then not necessary for resolving time preference utility anomalies. Bommier (2006) takes a similar view in arguing that hyperbolic discounting results from hyperbolic absolute risk aversion with respect to length of life, but does not consider other time preference anomalies.

Other authors have developed the link between time preference and uncertainty. For example, both Sozou (1998) and Dasgupta and Maskin (2005) focus on how future uncertainty can account for hyperbolic discounting. Sozou supposes that the rate at which the future payoff disappears is constant but uncertain. Individuals use Bayes Theorem to update their knowledge about this unknown rate, so the posterior rate decreases over time. As Dasgupta and Maskin point out, this explanation only applies to comparisons between a smaller-sooner payoff and a larger-later payoff, on one hand, and the same two payoffs with the two delays prolonged by the same time length, on the other. It cannot account for time inconsistent choices, that is, reversals of choice over time, a phenomenon which is usually the crucial implication of hyperbolic discounting. Dasgupta and Maskin themselves suppose that the realization time of future payoffs is uncertain. As time passes by, the probability of early realization decreases, which may cause a preference reversal. But they do not reconcile hyperbolic discounting with the two other time preference anomalies mentioned before. In this paper, the model not only explains the inherent consistency among these three anomalies but also generates "hyperbolic discounting" that is not subject to Dasgupta and Maskin’s criticism since it leads to time inconsistent behavior.

In this paper, Section 1.2 presents the basic model and its analysis, and Section 1.3 presents the conclusions.

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<sup>2</sup>It is an anomaly in the expected utility model that an event has greater impact when it turns impossibility into possibility, or possibility into certainty, than when it merely makes a possibility more or less likely.

## 1.2 Model

Denote by  $(x, t)$  a future payoff of magnitude  $x$  to be received after  $t$  periods of delay. The time delay  $t$  is continuous on the support  $[0, \infty)$ , and  $x$  can be any real number. Let  $V(x, t)$  denote the value of this delayed payoff. It is assumed to be continuous and twice differentiable.

Before the model analysis, it is convenient to formally define the time discounting behavior and the three time preference anomalies mentioned above.

**Definition 1** *The individual exhibits positive time discounting behavior if the following holds:*

$$\frac{\partial V(x, t)}{\partial t} < 0 \quad (1.1)$$

*Thus,  $V(x, t)$  is strictly decreasing in  $t$ .*

**Definition 2** *For the two payoffs  $(x_1, t_1)$  and  $(x_2, t_2)$  with  $x_1 < x_2$  and  $t_1 < t_2$ , the individual exhibits the time preference reversal characteristic of hyperbolic discounting if there exists a time length  $l > 0$  such that the following holds:*

$$V(x_1, t_1) > V(x_2, t_2) \text{ and } V(x_1, t_1 + l) < V(x_2, t_2 + l) \quad (1.2)$$

This definition has two distinct interpretations. One weaker, static interpretation is that the individual compares two payoffs  $(x_1, t_1)$  and  $(x_2, t_2)$ , on one hand, and compares two payoffs  $(x_1, t_1 + l)$  and  $(x_2, t_2 + l)$ , on the other. The preferences reverse across the comparisons. The other stronger, dynamic interpretation is that the individual first prefers the larger-later payoff  $(x_2, t_2 + l)$  to the smaller-sooner payoff  $(x_1, t_1 + l)$  but after  $l$  periods prefers the smaller-sooner payoff  $(x_1, t_1)$  to the larger-later payoff  $(x_2, t_2)$ . This is time inconsistent behavior.

**Definition 3** *Time preference exhibits the magnitude effect if the individual discounts the larger payoff at a lower rate. More precisely,*

$$\frac{\partial^2 V(x, t)}{\partial t \partial x} > 0 \quad (1.3)$$

As in Definition 1,  $\partial V(x, t)/\partial t$  is the rate at which the individual discounts the future payoff. Then the cross partial derivative  $\partial^2 V(x, t)/(\partial t \partial x)$  measures how this

rate of discounting depends on the magnitude of the payoff  $x$ . The positive sign of it means the larger magnitude, the lower rate, that is, there is a magnitude effect.

**Definition 4** *The extreme case of the sign effect is represented as  $V(x, t_1) > V(x, t_2)$  with  $x < 0$  and  $t_1 < t_2$ .*

In the model, the individual is assumed to react to a future payoff of magnitude  $x$  as if it were a gamble, denoted as  $x + \epsilon$ , where  $\epsilon$  is the realization of a random variable that is the departure of actual value from the nominal value  $x$ . The level of this uncertainty is assumed to increase with the time delay.<sup>3</sup>

More specifically, Hadar and Russell (1969, 1971), and Hanoch and Levy (1969) introduced the notion of second order stochastic dominance, to characterize the riskiness of different distributions. Following their approach, I assume that, if the payoff is delayed by time  $t$ , then  $prob(\epsilon \leq P) = F(P, t)$ , where  $P \in (-\infty, \infty)$ . Here,  $t$  is the index of the riskiness of the corresponding distribution. For any two time delays  $t_1$  and  $t_2$ , the cumulative density function  $F(P, t)$  satisfies the following conditions:

$$\int_{-\infty}^y F(\epsilon, t_1) d\epsilon \leq \int_{-\infty}^y F(\epsilon, t_2) d\epsilon, \quad \forall y \in (-\infty, \infty), \text{ and } t_1 < t_2 \quad (1.4)$$

and the above inequality holds strictly for at least one  $y \in (-\infty, \infty)$ . Under this condition, we say that the distribution  $F(\epsilon, t_1)$  second order stochastically dominates the distribution  $F(\epsilon, t_2)$ . These two distributions cross at least once in the domain  $(-\infty, \infty)$ . The mean of  $F(\epsilon, t_1)$  is no less than the mean of  $F(\epsilon, t_2)$ .

Then, an increase in  $t$  represents an increase in risk. The individual's expected utility for a payoff at time  $t$  is:

$$V(x, t) = \int_{-\infty}^{\infty} u(x + \epsilon) dF(\epsilon, t) \quad (1.5)$$

where  $u(\cdot)$  is a bounded and strictly increasing Bernoulli utility function with continuous derivative of order one and two.

I first make the following standard assumption:

---

<sup>3</sup>This is taken as given for the present purpose, but is a truism in the learning literature, where people gather more information about the environment over time. For example, Wiener processes with variance that is linearly increasing with time are used to capture the idea that uncertainty is increasing in time.

**Assumption 1.1** *The individual is risk averse. More precisely, the second derivative of the Bernoulli utility function  $u''(\cdot) < 0$  so that  $u(\cdot)$  is strictly concave.*

Such concavity is widely assumed<sup>4</sup>. It generates the following result for time preference. For the proof, see the appendix. (All proofs for this paper are in the appendix.)

**Proposition 1.1** *Under Assumption 1.1, there is time discounting behavior. That is,*

$$\frac{\partial V(x, t)}{\partial t} < 0 \tag{1.6}$$

*Hence,  $V(x, t)$  is strictly decreasing in  $t$ .*<sup>5</sup>

Because the individual is risk averse, the expected utility for a future payoff will decrease as the time delay increases. Indeed, this holds regardless of the sign of  $x$ . That is, regardless of whether a gain or a loss is involved, the individual prefers earlier resolution. This is the extreme version of the “sign effect”. An immediate loss is bad, but a future loss is even worse because of the increased uncertainty.

Much empirical evidence shows the importance of the precautionary saving motive (see Carroll and Kimball (2007) for a survey). I therefore make the following additional assumption about the hardwired attitude towards risk.

**Assumption 1.2**  *$u'''(\cdot) > 0$ , so that marginal utility is strictly convex.*<sup>6</sup>

As shown in Proposition 1.2, this assumption gives rise to the magnitude effect of time discounting.

**Proposition 1.2** *Under Assumption 1.1 and 1.2, time discounting exhibits the magnitude effect. That is, the cross partial derivative of  $V(x, t)$  with respect to  $x$  and  $t$  is positive:*

$$\frac{\partial^2 V(x, t)}{\partial t \partial x} > 0 \tag{1.7}$$

---

<sup>4</sup>Concavity can also be shown to be optimal in an evolutionary sense. See Robson (2001) for a formal model.

<sup>5</sup>The extreme case of the "sign effect" follows from (1) directly.

<sup>6</sup>This property is linked to precautionary saving.

This proposition states that a larger payoff will be discounted more slowly than will a smaller one, as in the “magnitude effect”. As the time horizon increases, risk increases, so the expected value from either the larger payoff or the smaller payoff decreases. Nevertheless, marginal utility decreases at a lower rate for the larger payoff, so the increased risk level has a smaller negative effect.

What do Propositions 1.1 and 1.2 imply for an individual choosing between a smaller-sooner payoff  $(x_1, t_1)$  and a larger-later payoff  $(x_2, t_2)$ , with  $x_1 < x_2$  and  $t_1 < t_2$ ? It is clear that he would always prefer the one with higher expected utility, i.e., he would prefer the smaller-sooner payoff  $(x_1, t_1)$  to the larger-later payoff  $(x_2, t_2)$  if  $V(x_1, t_1) > V(x_2, t_2)$ , and vice versa. Propositions 1.1 and 1.2 imply that two competing effects are occurring in this comparison. On one hand, the higher level of uncertainty for the larger-later payoff leads to a higher level of discounting; but on the other hand, the larger payoff is discounted at a lower rate. If the former effect dominates the latter, the individual will prefer the smaller-sooner payoff; if the latter dominates, the individual will prefer the larger-later one. For an appropriate pair of smaller-sooner payoff and larger-later payoff, the utility discounted value of the payoffs will cross at some point of time. The individual prefers the smaller-sooner payoff before the crossing point and prefers the larger-later payoff after that point. Thus, the individual experiences the preference reversals characteristic of hyperbolic discounting. The preference reversals work for two interpretations of hyperbolic discounting, as mentioned in Definition 2 above. According to the first interpretation, the preferences reverse because people perceive the future according to second order stochastic dominance. According to the second interpretation, individuals may make time inconsistent choices. The time inconsistency is optimal in the sense that the individuals’ knowledge about the environment implicitly changes over time. Given the little knowledge about the future environment they have at first, people may prefer the larger-later payoff. Nevertheless, they may switch to the smaller-sooner payoff later as they accumulate more information over time. This reversal results from the implicit way in which people learn information about the future payoff over time. This implies that uncertainty about the magnitude of the payoff decreases over time in the sense of second order stochastic dominance. Note it is different from the Bayesian updating process as assumed by Chew and Epstein (1992) and Halevy (2005). Therefore, the time preference reversal generated here does not contradict their result that an expected utility decision maker who updates his belief using Bayes rule will behave consistently over time.



The next proposition gives a sufficient condition for the preference reversals to happen at most once. The intuition for this condition follows later.

**Proposition 1.3** *A sufficient condition for time preference reversal characteristic of hyperbolic discounting to happen at most once is*

$$\frac{\partial^2 V(x, t)}{\partial t^2} > \frac{\partial^2 V(x, t)}{\partial t \partial x} \frac{\partial V(x, t) / \partial t}{\partial V(x, t) / \partial x} \quad \forall x, t \quad (1.8)$$

When comparing a smaller-sooner payoff to a larger-later payoff, it is possible that individuals never reverse their preferences over time. For example, an individual now may prefer one million dollars in one month to one dollar in two days, and still choose one million dollars over one dollar after two days have passed. The condition in Proposition 1.3 guarantees that once individuals change their decisions over time, they would change them as represented by hyperbolic discounting, and this reversal of preferences can happen only once. Roughly speaking, this condition ensures that the time path of the expected utility of the larger-later payoff is flatter than that of the smaller-sooner payoff. As a result, individuals first prefer the larger-later payoff and then switch to the smaller-sooner payoff, as time passes.

From Inequalities (1.6) and (1.7), we know the right hand side of the condition,  $[\partial^2 V(x, t) / (\partial t \partial x)] [\partial V(x, t) / \partial t] / [\partial V(x, t) / \partial x]$ , is negative. Therefore, a sufficient condition for Inequality (1.8) to hold is that the left-hand side of the condition,  $\partial^2 V(x, t) / \partial t^2$ , is positive, which means  $V(x, t)$  decreases in  $t$  at a decreasing rate. Intuitively, the reversal arises at most once when this condition is satisfied. Two effects are at work here, making a larger-later payoff discounted at a smaller rate (or the time path of expected utility is flatter) than would a smaller-sooner payoff: One effect is that the larger-later payoff has a higher  $t$ , and is therefore discounted at a lower rate on this account; the second effect is that it has higher  $x$ , which implies discounting at a lower rate on this account as well (the magnitude effect). These two effects reinforce each other to generate hyperbolic discounting.

The more interesting case occurs when  $\partial^2 V(x, t) / \partial t^2$  is negative, which means that  $V(x, t)$  decreases in  $t$  at an increasing rate. In this case, the magnitude effect plays an important role in generating the preference reversal. Specifically, now the two effects mentioned above compete with each other. The larger-later payoff has higher  $t$ , and is therefore discounted at a higher rate; and it has higher  $x$ , which implies discounting at a lower rate. The latter must dominate the former for the preference reversal to

happen, that is,  $\partial^2 V(x, t)/\partial t^2$  cannot be too negative, or the magnitude effect must be big enough to offset the faster discounting for the later payoff.<sup>7</sup>

This condition involves the individual's utility function  $u(\cdot)$  and the cdf  $F(P, t)$ . For a given utility function, the way in which riskiness increases with time delay determines whether or not a preference reversal will arise. In contrast to Sozou (1998) who assumes that the discount rate is uncertain and that people Bayesian update the distribution of it, I generate hyperbolic discounting by assuming the uncertainty on the magnitude of the payoff decreases over time according to second-order stochastic dominance. By taking into account the magnitude effect, this condition provides a weak sufficient condition for the time reversal behavior to happen at most once. Next, I present a simple example to demonstrate this.

Example 1.1 Suppose the individual's utility function  $u(\cdot)$  is logarithmic and that the uncertainty at time delay  $t$  is defined by a two-point distribution:  $\epsilon = f(t)$  with probability  $\frac{1}{2}$ , and  $\epsilon = -f(t)$  with probability  $\frac{1}{2}$ , where  $f(t)$  is an increasing function in  $t$ . For simplicity, assume  $f(t) > 0$ . The individual's expected utility for a payoff at time  $t$  is:

$$V(x, t) = \frac{1}{2} \ln(x + f(t)) + \frac{1}{2} \ln(x - f(t)) \quad (1.9)$$

A preference reversal will exist for any  $x > f(t)$  if  $f''(t) < -f'(t)^2/f(t)$ .

In fact, the condition  $f''(t) < -f'(t)^2/f(t)$  can be rewritten as  $d \ln f'(t)/d \ln f(t) < -1$ , which states that, if  $f(t)$  increases by 1 percent, then  $f'(t)$  must drop by more than 1 percent to generate hyperbolic discounting.

The results in this section show that it is possible to accommodate time preference anomalies in the framework of expected utility. In particular, the individual's perception of future risk is hypothesized to be crucial to yield preference reversals. Of course, this does not deny the possibility of other sources of time discounting. Indeed, the pure time preference can be incorporated into the model and it will not change the main conclusions of the paper. But more strict assumptions are needed. With the existence of pure time preference, the new discounted value for a future payoff

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<sup>7</sup>In fact, this condition implies that  $V(x, t)$  must be a quasiconvex function. The proof is as follows. From Inequality (1.6), we can get the slope of the level curve of  $V(x, t)$  is  $dx/dt = -[\partial V(x, t)/\partial t]/[\partial V(x, t)/\partial x] > 0$ . Taking derivative of  $dx/dt$  with respect to  $t$ , we can get  $d^2x/dt^2 = \{[\partial V(x, t)/\partial t][\partial^2 V(x, t)/\partial t \partial x] - [\partial^2 V(x, t)/\partial t^2][\partial V(x, t)/\partial x]\}/[\partial V(x, t)/\partial x]^2$ . When Inequality (1.8) holds,  $d^2x/dt^2 < 0$ . The worse set is convex. Therefore,  $V(x, t)$  is quasiconvex.

$(x, t)$  can be written as  $\widehat{V}(x, t) = e^{-\rho t}V(x, t) = e^{-\rho t} \int_{-\infty}^{\infty} u(x + \epsilon)dF(\epsilon, t)$ , where  $\rho$  is the discount rate due to pure time preference. It is straightforward to show  $\widehat{V}(x, t)$  is strictly decreasing in  $t$  under the assumption that  $u(\cdot) > 0$  on the support. For Proposition 1.2, an assumption stronger than  $u'''(\cdot) > 0$  is needed to generate the magnitude effect. The intuition is that now the larger payoff is discounted more due to pure time preference compared to the model in the paper, so it needs to be compensated by a stronger effect in the opposite direction. Incorporating pure time preference necessarily complicates the analysis of the model, but the main message of the model will not change: For a larger-later payoff and a smaller-sooner payoff, the difference in the magnitude may be crucial to generate the time preference reversal.

### 1.3 Conclusions

In this paper I argue that time preference may be determined by the individual's underlying attitude towards risk. In other words, time preference and risk attitude evolve simultaneously as they are both shaped by an uncertain environment. Of course, this does not deny there may be other sources for time discounting behavior. Within the standard expected utility model with uncertainty, risk aversion, and the preference for precautionary saving motive are sufficient to account for several "time preference anomalies": the extreme version of the sign effect, the magnitude effect, and hyperbolic discounting. It is consistent with and provides a plausible mechanism for the result of Noor's (2009) axiomatic proof that hyperbolic discounting and the magnitude effect jointly contradict the standard model of discounted utility. Of course, as documented by Frederick, Loewenstein, and O'Donoghue (2002), other discounting anomalies also exist. A unified theory to simultaneously explain more of these anomalies is beyond the scope of the current model, but will motivate future research.

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## 1.5 Appendix

### 1.5.1 Proof of Proposition 1.1

It follows from Hadar and Russell, and Hanoch and Levy, who show that  $V(x, t)$  decreases as risk increases. The details are as follows.

It is sufficient to show  $V(x, t_1) > V(x, t_2)$  for any two time delays  $t_1$  and  $t_2$  with  $t_1 < t_2$ .

By definition of expected utility, we have

$$V(x, t_1) - V(x, t_2) = \int_{-\infty}^{\infty} u(x + \epsilon) d[F(\epsilon, t_1) - F(\epsilon, t_2)] \quad (1.10)$$

Integrating by parts gives

$$V(x, t_1) - V(x, t_2) = \{u(x + \epsilon)[F(\epsilon, t_1) - F(\epsilon, t_2)]\} \Big|_{-\infty}^{\infty} - \int_{-\infty}^{\infty} u'(x + \epsilon)[F(\epsilon, t_1) - F(\epsilon, t_2)] d\epsilon \quad (1.11)$$

where  $u'(x + \epsilon) = \partial u(x + \epsilon) / \partial \epsilon$ . Since  $F(\infty, t_1) = F(\infty, t_2) = 1$  and  $F(-\infty, t_1) = F(-\infty, t_2) = 0$ , it is clear the first term on the right hand side of Equation (1.11) is equal to zero. So we get

$$V(x, t_1) - V(x, t_2) = - \int_{-\infty}^{\infty} u'(x + \epsilon)[F(\epsilon, t_1) - F(\epsilon, t_2)] d\epsilon \quad (1.12)$$

For the right hand side of Equation (1.12), integrating by parts we can get

$$\begin{aligned} & - \int_{-\infty}^{\infty} u'(x + \epsilon)[F(\epsilon, t_1) - F(\epsilon, t_2)] d\epsilon \\ &= \{-u'(x + \epsilon) \int_{-\infty}^{\epsilon} [F(\delta, t_1) - F(\delta, t_2)] d\delta\} \Big|_{-\infty}^{\infty} \\ & \quad + \int_{-\infty}^{\infty} \{u''(x + \epsilon) \int_{-\infty}^{\epsilon} [F(\delta, t_1) - F(\delta, t_2)] d\delta\} d\epsilon \\ &= -u'(x + \infty) \int_{-\infty}^{\infty} [F(\delta, t_1) - F(\delta, t_2)] d\delta \\ & \quad + \int_{-\infty}^{\infty} \{u''(x + \epsilon) \int_{-\infty}^{\epsilon} [F(\delta, t_1) - F(\delta, t_2)] d\delta\} d\epsilon \end{aligned} \quad (1.13)$$

First, we know the first term of the right hand side of this result is nonnegative

because of the positivity of  $u'(\cdot)$  and second order stochastic dominance assumption. Second, we know the second term is positive because of the second order stochastic dominance assumption and the negativity of  $u''(\cdot)$ . Therefore,

$$\begin{aligned}
& V(x, t_1) - V(x, t_2) \\
&= -u'(x + \infty) \int_{-\infty}^{\infty} [F(\delta, t_1) - F(\delta, t_2)] d\delta \\
&\quad + \int_{-\infty}^{\infty} \{u''(x + \epsilon) \int_{-\infty}^{\epsilon} [F(\delta, t_1) - F(\delta, t_2)] d\delta\} d\epsilon \\
&> 0
\end{aligned} \tag{1.14}$$

### 1.5.2 Proof of Proposition 1.2

The derivative of Equation (1.5) with respect to  $x$  is given by:  $\partial V(x, t)/\partial x = \int_{-\infty}^{\infty} u'(x + \epsilon) dF(\epsilon, t)$ . Hence, given that  $u(\cdot)$  is concave and  $u'(\cdot)$  is convex, it follows that  $\partial^2 V(x, t)/(\partial x \partial t) > 0$ , as in Hadar and Russell, and Hanoch and Levy. The details are as follows.

It is sufficient to show  $\partial V(x, t_1)/\partial x < \partial V(x, t_2)/\partial x$  for any two time delays  $t_1$  and  $t_2$  with  $t_1 < t_2$ .

Using the same argument as in the proof for Proposition 1.1, it is straight forward to show

$$\begin{aligned}
& \frac{\partial V(x, t_1)}{\partial x} - \frac{\partial V(x, t_2)}{\partial x} \\
&= -u''(x + \infty) \int_{-\infty}^{\infty} [F(\delta, t_1) - F(\delta, t_2)] d\delta \\
&\quad + \int_{-\infty}^{\infty} \{u'''(x + \epsilon) \int_{-\infty}^{\epsilon} [F(\delta, t_1) - F(\delta, t_2)] d\delta\} d\epsilon \\
&< 0
\end{aligned} \tag{1.15}$$

where the inequality follows from the second order stochastic dominance assumption, the negativity of  $u''(\cdot)$  and the positivity of  $u'''(\cdot)$ .

### 1.5.3 Proof of Proposition 1.3

Suppose at time 0, the individual knows there is a future payoff  $(x_0, t_0)$ , where  $x_0$  is the magnitude of the payoff, and  $t_0$  is the length of time delay. Consider another payoff

$(x', t')$ , such that at some point of time  $T_0$  with  $0 < T_0 < \min\{t', t_0\}$  the expected utility of  $(x', t')$  is equal to that of  $(x_0, t_0)$ , that is,  $V(x', t' - T_0) = V(x_0, t_0 - T_0)$ . For a given  $t'$ , there exists a unique  $x'$  that satisfies this equality since  $V(x, t)$  is continuous, strictly increasing in  $x$ , and strictly decreasing in  $t$ . Then we can write  $x'$  as  $x'(t', x_0, t_0, T_0)$ . By the Implicit Function Theorem, we get

$$\frac{dx'}{dt'} = -\frac{\partial V(x', t' - T_0)/\partial(t' - T_0)}{\partial V(x', t' - T_0)/\partial x'} > 0 \quad (1.16)$$

Hence  $x'$  is strictly increasing in  $t'$ .

For the future payoff of magnitude  $x'$ , the expected utility of it with time delay  $t \in [0, \infty)$  is given by  $V(x'(t', x_0, t_0, T_0), t)$ . Then the slope of  $V(x'(t', x_0, t_0, T_0), t)$  with respect to  $t$  is given by

$$\frac{\partial V(x'(t', x_0, t_0, T_0), t)}{\partial t} < 0$$

In the above notation  $x'(t', x_0, t_0, T_0)$  is fixed. Accordingly, at time  $T_0$  (in other words, when  $t = t' - T_0$ ) the slope of  $V(x'(t', x_0, t_0, T_0), t)$  with respect to  $t$  is written as  $\partial V(x'(t', x_0, t_0, T_0), t)/\partial t |_{t=t'-T_0}$  or  $\partial V(x'(t', x_0, t_0, T_0), t' - T_0)/\partial(t' - T_0)$ .

The next step is to show that  $\partial V(x'(t', x_0, t_0, T_0), t' - T_0)/\partial(t' - T_0)$  is increasing in  $t'$  if Inequality (1.8) holds.

By Equation (2.7), we get

$$\begin{aligned} & \frac{\partial^2 V(x'(t', x_0, t_0, T_0), t' - T_0)}{\partial(t' - T_0)\partial t'} \\ = & \left[ \frac{\partial^2 V(x'(t', x_0, t_0, T_0), t' - T_0)}{\partial(t' - T_0)\partial x'} \frac{dx'}{dt'} + \frac{\partial^2 V(x'(t', x_0, t_0, T_0), t' - T_0)}{\partial(t' - T_0)^2} \right] \\ = & -\frac{\partial^2 V(x'(t', x_0, t_0, T_0), t' - T_0)}{\partial(t' - T_0)\partial x'} \frac{\partial V(x', t' - T_0)/\partial(t' - T_0)}{\partial V(x', t' - T_0)/\partial x'} \\ & + \frac{\partial^2 V(x'(t', x_0, t_0, T_0), t' - T_0)}{\partial(t' - T_0)^2} \end{aligned}$$

Given Inequality (1.8), the following inequality holds:

$$\frac{\partial^2 V(x'(t', x_0, t_0, T_0), t' - T_0)}{\partial(t' - T_0)\partial t'} > 0 \quad \forall x', t'$$

That is, at time  $T_0$  the slope of  $V(x'(t', x_0, t_0, T_0), t)$  with respect to time delay  $t$  is



strictly increasing in  $t'$ . For the two payoffs whose expected utilities are equal at time  $T_0$ , the expected utility of the payoff with the longer time delay decreases more slowly with time delay (or increases more slowly with time) in the neighborhood of  $T_0$  than that of the payoff with the shorter time delay. And we also know  $x'$  is strictly increasing in  $t'$ . Therefore, the expected utility of the larger payoff decreases more slowly with time delay (or increases more slowly with time) in the neighborhood of  $T_0$ . To make the expected utility of two payoffs equal at time  $T_0$ , the expected utility of the larger payoff must be greater than that of the smaller payoff before time  $T_0$  and become less than that of the smaller payoff after this point of time. Since  $T_0$  is arbitrarily chosen, a preference reversal characteristic of hyperbolic discounting then arises but this can happen at most once.

#### 1.5.4 Proof of Example 1.1

The derivative of this expected utility with respect to  $t$  is:

$$\frac{\partial V(x, t)}{\partial t} = \frac{f'(t)}{2} \left( \frac{1}{x + f(t)} - \frac{1}{x - f(t)} \right) < 0$$

and the cross partial derivative of  $V(x, t)$  is:

$$\frac{\partial^2 V(x, t)}{\partial t \partial x} = \frac{f'(t)}{2} \left[ \frac{1}{(x - f(t))^2} - \frac{1}{(x + f(t))^2} \right] > 0$$

And we can show the following:

$$\frac{\partial^2 V(x, t)}{\partial t^2} \frac{\partial V(x, t)}{\partial x} - \frac{\partial^2 V(x, t)}{\partial t \partial x} \frac{\partial V(x, t)}{\partial t} = \frac{x(f(t)^2 - x^2)(f(t)f''(t) + f'(t)^2)}{(x - f(t))^3(x + f(t))^3}$$

Given  $x > f(t)$ , when  $f''(t) < -f'(t)^2/f(t)$ , the following inequality holds:

$$\frac{\partial^2 V(x, t)}{\partial t^2} \frac{\partial V(x, t)}{\partial x} - \frac{\partial^2 V(x, t)}{\partial t \partial x} \frac{\partial V(x, t)}{\partial t} > 0$$

That is,

$$\frac{\partial^2 V(x, t)}{\partial t^2} > \frac{\partial^2 V(x, t)}{\partial t \partial x} \frac{\partial V(x, t)/\partial t}{\partial V(x, t)/\partial x}$$

## Chapter 2

# Status, Inequality and Intertemporal Choice<sup>1</sup>

*This paper develops an intertemporal model in which individuals care about consumption not only for its own sake but also for the status it implies. By putting an additive status term into the utility function, I show that the level of inequality in the initial wealth distribution affects individuals' saving and consumption behavior. The direction of the distortion in intertemporal choice relative to the standard model without a concern for status depends on the elasticity of intertemporal substitution in the utility from absolute consumption. In particular, I prove that, for conventional parameter values of the elasticity, (e.g. CES parameter larger than one) people save less than what they do without the status concern but the magnitude of this decrease is reduced by the concern for future status. It is also possible that people save more than what they do without the status concern. I also analyze how changes in the initial wealth distribution affect saving. For example, when wealth is Pareto distributed, for a reasonable parameterization, the rich save more and the poor save less when society gets more unequal, which implies that inequality is self-enforcing in this economy. Finally, the resulting allocation is Pareto inefficient due to the externalities generated by the concern for status.*

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## 2.1 Introduction

The relationship between inequality and growth is a vitally important question in economics. Nevertheless, little consensus about the relationship exists, either empirically or theoretically. The commonly held view, however, is that saving behavior plays an important role in determining the speed of economic growth. As amply documented in cross-country empirical studies, a strong positive correlation exists between saving rates and real per capita growth (see, for example, Maddison, 1992, and Bosworth, 1993). The usual interpretation is that saving drives growth through the saving-investment link (for example, Levine and Renelt, 1992, and Mankiw, Romer and Weil, 1992).

In this paper, by adding a social status term in the utility function, I directly explain how inequality may affect growth by altering people's saving decisions. Basically, in a hybrid model where people care about both absolute consumption and status, inequality affects saving through the concern for status, which can distort the intertemporal choices relative to the standard model without status. The concern for status therefore may cause people to save more or less, depending on the elasticity of intertemporal substitution in the individuals' utility function for absolute consumption. In the empirically likely case where the elasticity of substitution parameter is greater than one, if the interest rate is less than the discount rate, people save less than what they would without the concern for status. This prediction is consistent with conventional wisdom that conspicuous consumption tends to be too high. Although people may exhibit conspicuous consumption behavior, as in Veblen (1899), compared to a static model, the magnitude of this conspicuous consumption is reduced since part of the status effect is offset by the concern for future status. A startling result occurs if the interest rate is greater than the discount rate: people save even more than what they would without the concern for status. The concern with conspicuous consumption does not necessarily increase the present consumption at all. This can be explained by people caring about not only today's status but about their future status as well. People would save more (consume less) when tomorrow's status is more important, at the margin, than today's status, and vice versa. The reduced, or even reversed, effect of status on consumption should be taken into account in the related empirical work. In either case, the resulting intertemporal allocation is Pareto inefficient due to the externalities generated by the concern for status.

I also analyze how the change in the initial distribution of wealth affects saving.

The results depend on how the marginal utility of consumption, via the status term, responds to the change in distribution. If it increases with the change in the initial distribution, individuals' saving behavior becomes closer to the pure status case (where people care only about their status). Otherwise, individuals' saving behavior is closer to the absolute consumption case (where people care about consumption per se). When wealth is Pareto distributed, for a reasonable parameterization, the rich save more and the poor save less, when society becomes more unequal, which implies that inequality is self-enforcing in this economy.

The most closely related papers in the literature are those of Corneo and Jeanne (2001), and Hopkins and Kornienko (2006). In contrast to Corneo and Jeanne (2001), who assumed status is determined by the relative wealth level, I assume status is determined by the relative consumption level. Indeed, it is plausible that consumption is observed more readily than is wealth. By including relative consumption, and not relative wealth, as an additional argument in the utility function, I obtain different results from those of Corneo and Jeanne (2001). They concluded that more equal wealth distribution leads to more saving, hence a higher growth rate of the economy, whereas, in my paper, more equal wealth distribution does not always encourage saving. While Hopkins and Kornienko (2006) assumed that status depends on relative consumption and that people care about status only when they are young, in this paper people are concerned with status throughout their lifetime.<sup>2</sup> They concluded that more equal wealth distribution diverts resources to wasteful consumption, rather than to productive investment, which leads to lower economic growth, due to the competition for status at the young stage. In contrast, in my paper, since people care about relative consumption not only today but tomorrow as well, I show that the incentive to save for future consumption may overwhelm the incentive to consume now, which enhances the saving rate. Another difference between this paper and the work of Corneo and Jeanne (2001) and Hopkins and Kornienko (2006) is that my conclusions apply to more general function forms of utility for absolute consumption and status, in comparison to the logarithmic functions in both absolute consumption and status terms used in their earlier papers. Corneo and Jeanne (1998) also considered the case where status-seeking endogenously takes the form of conspicuous consumption, i.e., status depends on the relative ranking of consumption. The authors found that

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<sup>2</sup>Frank (1985b) also assumed that people do not care about tomorrow's status. In the empirical literature, as far as I know, no strong evidence suggests people care about status only when they are young. Here, I take the stance that a legitimate assumption is that people can foresee the utility generated by future relative consumption as long as they can foresee the utility generated by future absolute consumption.

status encourages saving if individuals engage in conspicuous consumption when old, but not when young, whereas, in this paper, status may encourage saving even when people have conspicuous consumption motives along their whole lifetime.

This paper proceeds as follows. Section 2.2 reviews the related literature. Section 2.3 describes the model environment. Section 2.4 characterizes the equilibrium saving and consumption decisions. Section 2.5 examines the impact of inequality on equilibrium saving and consumption behavior. The last section concludes.

## 2.2 Related Literature

The idea that utility depends on the comparison of one's own consumption to that of others can be traced back, at the very least, to Veblen (1899). This stance was further developed by Duesenberry (1949) and Frank (1985a). Recently, a large body of empirical literature has appeared to support this view: individuals tend to evaluate their consumption in comparison to the consumption of others (Easterlin, 1974, McBride, 2001, Luttmer, 2005, Johansson-Stenman and Martinsson, 2006, Dyanan and Ravina, 2007, among others.) Moreover, evidence from neurophysiological studies indicates that pleasure felt by individuals is strongly influenced by interpersonal comparisons. In a recent brain scanning experiment, Fliessbach et al. (2007) found that participants who got more money than their co-players showed a much stronger activation in their brain's "reward centre" compared to when both players received the same amount.

This paper is related to the work of Ray and Robson (2009) who analyzed people's intertemporal choice in a model with status. The authors emphasized how persistent gambling might arise naturally in the steady-state. They showed that the pure status model generates a deterministic linear policy function when the production function is convex. The pure status model and a linear production function is behaviorally equivalent to the absolute concern model with a logarithm utility function. Consistent with this, the direction of the status-effect distortion here is independent of the specific form of the status term, though the magnitude of the distortion depends on it.

This paper is also related to earlier work on the effect of status on saving behavior. Duesenberry (1949) proposed that individual saving decisions are guided more by a concern for status, rather than by the absolute standard of living. More recently, a revival has been seen in the theoretical literature that examines the implications

of including a concern for status as an additional argument in the utility function. Among these papers, Frank (1985b) uses a static model that assumes consumption on things other than status is saving and the author concludes that the concern for status might decrease saving. Hopkins and Kornienko (2004, 2009) pay particular attention to the interaction between the status concern and the distribution of wealth in the static environment. They found that people may spend more on conspicuous consumption in more equal and richer societies. Cole, Mailath and Postlewaite (1992), Fershtman, Murphy and Weiss (1996), and Eaton and Eswaran (2009) incorporated status as a social reward in models of economic growth, though they do not emphasize the relationship between inequality and growth.

Following Frank (1985b) and Robson (1992), I define status as the cumulative distribution function of consumption. Of course, this is not the only way to model status. In the literature status is sometimes defined as the difference between the individual's consumption and the average consumption of the whole society. Here, I use the former definition because the distribution function seems a more precise measure of inequality than are deviations from the average. Inevitably, this leads to some conclusions that differ from those in the literature that use the latter definition. For example, Arrow and Dasgupta (2009) also take the view that people care about their status throughout their lifetime. They use the latter definition of status and show that status may not matter if the marginal disutility of average consumption is a constant fraction of the marginal utility of individual consumption over time. In my paper, the elasticity of intertemporal substitution of individual consumption determines how status matters for people's intertemporal choices, as mentioned before. Since Arrow and Dasgupta studied a representative agent model, they cannot talk about the effect of wealth distribution on savings. Eswaran and Oxoby (2008) also applied the latter definition of status and showed that status increases people's impatience. In my paper, though this may not necessarily be true, it holds for some reasonable values of the elasticity of intertemporal substitution.

A stream of literature exists that examines other channels through which inequality may affect people's saving decisions. Some of the work assumes heterogeneous preferences which change with the positions in the wealth distribution (for example, Lewis, 1954, and Kaldor, 1957). Some authors have also proposed a link between inequality and saving through institutional constraint, such as borrowing constraint, political instability, and government policies, etc (for example, Persson and Tabellini, 1994, Alesina and Rodrik, 1994, Alesina and Perotti, 1995, and Perotti, 1996). In

contrast, the analysis in this paper generates different saving behaviors for individuals with identical preferences, and people's behavior is independent of any institutional consideration.

## 2.3 The Model Environment

Suppose there is a continuum of measure one of agents, where each agent is endowed with initial wealth  $w \geq 0$ . The cumulative distribution of the wealth is denoted by  $G(w)$ , assumed to be continuously twice differentiable on the support  $[0, \bar{w}]$ , with  $\bar{w}$  a constant and  $\bar{w} > 0$ . The associated density function is denoted by  $g(w)$ . The agents allocate their wealth over two periods, i.e.,  $t = 0, 1$ . The agents can borrow or save any amount at interest rate  $r$ .

The individuals have identical preferences and the decision problem is given by

$$\begin{aligned} \max_{\{c_t\}} \quad & \sum_{t=0}^1 \frac{(1-\alpha)u(c_t) + \alpha v(F_t(c_t))}{(1+\rho)^t} \\ \text{s.t.} \quad & \sum_{t=0}^1 \frac{c_t}{(1+r)^t} = w \end{aligned} \tag{2.1}$$

where  $\rho$  is the discount rate. The one period utility function consists of two parts. The first,  $u(c_t)$  represents the utility derived from consuming  $c_t$  amount of goods.  $u(\cdot)$  is a standard utility function, with  $u'(\cdot) > 0$  and  $u''(\cdot) < 0$ . The second part,  $v(F_t(c_t))$  represents the utility derived from social status. Status here is the individual's rank in the distribution of consumption, denoted by  $F_t(\cdot)$ , the cumulative distribution function of consumption at time  $t$ .<sup>3</sup> The associated density function is denoted by  $f_t(\cdot)$ . (We will only need to consider differentiable  $F_t(\cdot)$ .) The agent values her status according to some continuously twice differentiable function  $v(\cdot)$  with  $v' > 0$ . The constant  $\alpha \in [0, 1]$  is the weight of the status term.

Since people care about their relative ranking in consumption in the society, the

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<sup>3</sup>There are two notes about this specification of status:

(1). I assume the entire population in the society is the reference group when I make this specification. Recently there is a growing body of literature that focuses on the endogeneity of the reference group, but it is beyond the scope of this paper.

(2). An alternative specification of status is  $\frac{1}{2}F(c_t) + \frac{1}{2}F^-(c_t)$ , as in Robson (1992) and Hopkins and Kornienko (2004), where  $F^-(c_t) = \lim_{c'_t \rightarrow c_t^-} F(c'_t)$  is the mass of individuals with consumption strictly less than  $c_t$ . These two specifications are different only when there are ties in consumption, which happens in measure 0 in the equilibrium discussed in this paper.

individual's intertemporal decision is strategic in the sense that she has to take all the others' decisions into account. The cumulative distribution function of consumption,  $F_t(c_t)$ , emerges as the result of aggregate actions. The individual's objective function in Problem (2.1) is endogenously generated, i.e., the status term,  $\alpha v(F_t(c_t))$ , is endogenous. This endogeneity makes Problem (2.1) different from the standard utility maximization problem.

## 2.4 Equilibrium Analysis

Before we discuss the optimal solution of Problem (2.1), it is helpful to consider two problems, where individuals care only about consumption per se and care only about consumption for status, for comparison purposes.

According to neoclassical economic theory, people only care about absolute consumption. This corresponds to the case that  $\alpha = 0$  in Problem (2.1). The individual's decision problem becomes

$$\begin{aligned} \max_{\{c_t\}} & \sum_{t=0}^1 \frac{u(c_t)}{(1 + \rho)^t} \\ \text{s.t.} & \sum_{t=0}^1 \frac{c_t}{(1 + r)^t} = w \end{aligned} \tag{2.2}$$

This is a standard utility maximization problem and there is a unique solution, denoted by  $c_t^C$ , since  $u''(\cdot) < 0$ .

On the other hand, if people care only about status, then  $\alpha = 1$  in Problem (2.1). The decision problem becomes

$$\begin{aligned} \max_{\{c_t\}} & \sum_{t=0}^1 \frac{v(F_t(c_t))}{(1 + \rho)^t} \\ \text{s.t.} & \sum_{t=0}^1 \frac{c_t}{(1 + r)^t} = w \end{aligned} \tag{2.3}$$



As shown in Ray and Robson (2009), Problem (2.3) is equivalent to

$$\begin{aligned} \max_{\{c_t\}} \quad & \sum_{t=0}^1 \frac{\ln c_t}{(1+\rho)^t} \\ \text{s.t.} \quad & \sum_{t=0}^1 \frac{c_t}{(1+r)^t} = w \end{aligned} \tag{2.4}$$

under the assumption that  $v(G(\cdot))$  is strictly concave.

That is to say, the equilibrium consumption time path followed by individuals with pure preference for status is the same as that followed by individuals with a logarithm utility function in absolute consumption. For all possible values of  $\rho$  and  $r$ , initial inequality has no impact on people's intertemporal choices when they care only about relative consumption in the society. Status itself is not sufficient for inequality to affect saving hence growth. Here the solution to Problem (2.3) and (2.4) is denoted by  $c_t^S$ . Next we go back to the hybrid model (2.1).

As discussed in last section, the status term in this model,  $\alpha v(F_t(c_t))$ , is endogenous, which makes Problem (2.1) different from the standard utility maximization problem. Therefore, it is necessary to first establish the existence and uniqueness of a Nash equilibrium for this simultaneous move game. I will focus on this Nash equilibrium throughout the paper.

**Proposition 2.1** *There exists an upper bound  $\bar{\alpha}$  s.t. when  $0 < \alpha < \bar{\alpha}$ , there is a unique Nash equilibrium for Problem (2.1) with the property that it lies in the class of equilibria such that the equilibrium strategy  $c_t^H(w)$  is strictly increasing and differentiable in  $w$ .*

In the equilibrium established in Proposition 2.1, everyone wants to get ahead of the others. However, in the end they end up with the same ranking as in the initial wealth distribution in both time periods. Here I restrict my analysis for small  $\alpha$  mainly for technical reasons. With a small  $\alpha$ , the solution to the first-order differential equation (2.7) (see Appendix) can be shown to satisfy the second-order condition for maximization, hence it is indeed the solution for Problem (2.1).<sup>4</sup> As we will see later, this restriction is also technically useful to simplify the remaining analysis.

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<sup>4</sup>If  $\alpha$  is large, a mixed strategy, i.e., gambling between two periods' consumption may be optimal. But it is hard to characterize such equilibrium strategy in general. Ray and Robson (2009) discuss the equilibrium strategies when  $\alpha$  is large and there are fair bets available in each period.

The result in Proposition 2.1 applies to any standard utility function. For the equilibrium analysis of the hybrid model, however, it is convenient to make the following assumption about the utility function of absolute consumption.

**Assumption 2.1** *The pure consumption term  $u(c_t)$  is CRRA, that is,  $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$ , where  $\sigma > 0$ . When  $\sigma \rightarrow 1$ ,  $u(c_t) = \ln c_t$ .*

For this class of utility functions, the elasticity of intertemporal substitution is constant and equal to  $\frac{1}{\sigma}$ . From the proof of Proposition 2.1, it is easy to verify the equilibrium solution for the hybrid model,  $c_t^H$ , is the same as the equilibrium solution for the pure status model,  $c_t^S$ , when  $\sigma$  approaches 1 (that is, when  $u(c_t) = \ln c_t$ .) In this case the initial distribution of wealth has no impact on the individual's intertemporal choice. This is analogous to Arrow and Dasgupta (2009), who find status may not matter if the marginal disutility of average consumption is a constant fraction of the marginal utility of individual consumption over time. Since people care about status in both periods, the incentive to save for future status exactly offsets the incentive to consume for today's status and hence there is no distortion caused by this relative concern.

My result, however, sharply contrasts with those of Corneo and Jeanne (2001) and Hopkins and Kornienko (2006), in which the initial distribution of wealth impacts on the individual's intertemporal choice when both absolute consumption and status term are logarithmic functions in the respective arguments. This difference is due to the different assumptions on other things: Corneo and Jeanne (2001) assume status depends on relative wealth instead of relative consumption, and Hopkins and Kornienko (2006) assume people only care about status in the first period. In other words, if status in Corneo and Jeanne (2001) was relative consumption, or the individuals in Hopkins and Kornienko (2006) cared about status in both periods, the effect of the concern for status on growth would disappear.

When  $\sigma \neq 1$  things are different. Now the initial distribution of wealth will play a role in the determination of the equilibrium solution. The next proposition gives one property of the optimal solution of the hybrid model (2.1) for this case.

**Proposition 2.2** *Under Assumption 2.1, in the Nash equilibrium established in Proposition 2.1, the following holds for  $0 < \alpha < a$ , where  $a$  is some positive constant: If  $r < \rho$ , the optimal saving is strictly decreasing (increasing) with  $\alpha$  if  $\sigma > 1$  ( $\sigma < 1$ ); If*

$r > \rho$ , the optimal saving is strictly increasing (decreasing) with  $\alpha$  if  $\sigma > 1$  ( $\sigma < 1$ ).<sup>5</sup>

In the proof for this proposition (see Appendix), it shows that in the equilibrium, the status term in the hybrid model,  $v(F_t(c_t))$ , works just like a logarithm utility function in consumption. The intuition is as follows. When  $\alpha$  approaches zero, the equilibrium consumption is approximately linear in wealth due to the CRRA utility function in the absolute term. As a result, the distribution of consumption in each period looks like the squeezed (or spreaded) distribution of wealth. The balance generated by the squeezed (or spreaded) distributions in two periods is roughly the same as that generated by a logarithm utility function. When  $r < \rho$ , if the elasticity of intertemporal substitution for the absolute consumption is lower than one ( $\sigma > 1$ ), adding a status term would give people less incentive to postpone consumption. Therefore people would consume more in the first period and save less than in the pure consumption model. This tendency is reinforced by increasing the weight of the status term,  $\alpha$ . If the elasticity of intertemporal substitution for the absolute consumption is higher than one ( $\sigma < 1$ ), adding a status term would give people more incentive to postpone consumption. Therefore people would consume less and save more than in the pure consumption model. The opposite holds for the case  $r < \rho$ .<sup>6</sup> Consumption competition due to the concern for status does not necessarily lead to undersaving (overconsumption) here. This contrasts with Hopkins and Kornienko (2006) who conclude the presence of a concern with status would decrease saving. In my paper the individuals care about not only first period status but also second period status, so undersaving (overconsumption) is not necessarily optimal as it will decrease the following periods consumption hence status. It is also different from Eswaran and Oxoby (2008) who assume status is the difference between individual's consumption and average consumption in the society and conclude the individual saves less with status than without it. They predict that the instantaneous marginal utility of consumption from the status term is constant over time. Then with the time discounting it is easy to see the individual would save less for the future compared to the pure consumption model. Whereas in this paper the instantaneous marginal utility of the consumption from the status concern is changing over time and its effect

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<sup>5</sup>In the special case that  $r = \rho$ , it is easy to verify that  $c_t^H = c_t^C = c_t^S$ . A concern for status has no impact on people's intertemporal choices.

<sup>6</sup>In fact, a more general result is the optimal consumption for the hybrid model is always in between the solutions for pure consumption model and pure status model, and it is true for any utility in absolute consumption with  $u' > 0$  and  $u'' < 0$ . That is, the optimal solution  $c_t^H$  is strictly increasing with  $\alpha$  if  $c_t^C < c_t^S$ , and strictly decreasing with  $\alpha$  if  $c_t^C > c_t^S$ . And  $c_t^H = c_t^C = c_t^S$  if  $c_t^C = c_t^S$ .

would depend on the specific function forms.

As Proposition 2.2 predicts the changes in saving for all possible parameterization of the model, it is interesting to think more about the empirically relevant ones. Although the exact value of  $\sigma$  in the model remains disputed, most of the literature has estimated it higher than one (for example, Kocherlakota, 1996, Cohen and Einav, 2005, Abel, 1990, and Gali, 1994).<sup>7</sup> If we make the assumption that  $r < \rho$ , the model predicts the individual who cares about status would save less than one who does not care about it. The desire to consume for today's status outweighs the desire to consume for tomorrow's status. This prediction is consistent with conventional wisdom that the conspicuous consumption is too high. People may exhibit conspicuous consumption behavior, as in Veblen (1899). Nevertheless, compared to a static model, the magnitude of this conspicuous consumption is reduced since part of the status effect is offset by the concern for future status. A more startling result occurs if the interest rate exceeds the discount rate: people's incentive to save for tomorrow's status is so strong that they actually save more than what they would do without the concern for status. The concern for conspicuous consumption does not necessarily increase consumption at all. In either case, the main point is that there is an intertemporal trade-off between today's status concern and tomorrow's status concern. The effect of conspicuous consumption becomes reduced or even reversed due to this trade-off. The reduced or even reversed effect of status on consumption should be taken into account in the related empirical work.

The next proposition considers the Pareto efficiency of the resulting allocation.

**Proposition 2.3** *In the Nash equilibrium established in Proposition 2.1, the equilibrium allocation  $c_{t,i}^H(w)$  is weakly Pareto inefficient when  $\sigma \neq 1$ .*

The externalities generated by the status concern distorts individuals' intertemporal decision making and thus the resulting allocation is not Pareto efficient in general. For every individual, the optimal solution for the pure consumption model,  $c_{t,i}^C$ , Pareto dominates the solution for the hybrid model,  $c_{t,i}^H$ . In the special case that  $\sigma = 1$ , the equilibria in all models coincide with each other so the optimal solution in the hybrid model is efficient.

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<sup>7</sup>The first two papers do not take the status concern into account. In fact, according to the model prediction in this paper, the actual value of  $\sigma$  would be even higher in that case.

## 2.5 The Effect of Change in Initial Distribution on Equilibrium

As pointed out in last section, the optimal decision of the hybrid model depends on the initial distribution of wealth,  $G(w)$ , except in the special case  $\sigma = 1$ . In societies with different wealth distributions, individuals with the same level of wealth may decide to save different amounts, even if they have the same utility function. This has important implications for the redistribution policies of government. By redistributing wealth in the whole society, the government makes the individuals change their decisions on optimal saving and hence affects the growth of the economy. The initial inequality matters. The question is: How?

Let's start with a simple example. Suppose the initial wealth distribution in the society is uniform on the support  $[w_0 - \beta, w_0 + \beta]$ , where  $w_0$  and  $\beta$  are some constants with  $w_0 > \beta > 0$ . The distribution becomes more dispersed when  $\beta$  increases, although the mean of the distribution kept constant. The cumulative distribution function is given by  $G(w, \beta) = \frac{w - (w_0 - \beta)}{2\beta}$ , and the corresponding density function is given by  $g(w, \beta) = \frac{1}{2\beta}$ . For simplicity, it is assumed that  $v(G) = G$ . So the individual is risk neutral as to the change in status. Then the government proposes a redistribution policy that reduces inequality or  $\beta$ . Then what is the effect on the individuals with some wealth level  $\hat{w}$ ? It is easy to show that, compared to their optimal saving before the policy change, they will now save at the level closer to optimal in the absolute consumption model if  $\frac{\partial g(\hat{w}, \beta)}{\partial \beta} > 0$ , and save at the level closer to optimal in the pure status model if  $\frac{\partial g(\hat{w}, \beta)}{\partial \beta} < 0$ . That is to say, if density around  $\hat{w}$  decreases, people would behave as if they weigh the status term less relative to the pure consumption term. Otherwise, they will weigh the status term more relative to the pure consumption term. This is consistent with Hopkins and Kornienko (2009), who analyze a static model and find that more social competition encourages consumption. As people get closer, it is easier to overtake others by status, thus it raises the marginal utility of consumption from status. Otherwise, the marginal utility of consumption from the absolute concern is relatively larger. In fact, in this model society, everybody would behave as if they are more motivated by status than before since the density  $g(w, \beta) = \frac{1}{2\beta}$  increases everywhere as  $\beta$  decreases. Then if  $r < \rho$  and  $\sigma > 1$ , people will save less when the distribution is more equal. The redistribution policy increases the social competition and diverts more resources to current consumption. Equality does harm growth in this case. If  $r > \rho$  and  $\sigma > 1$ , people will save more when the

distribution is more equal. The redistribution policy increases the social competition but diverts more resources to future consumption. As a result aggregate saving will go up. Equality potentially enhances growth in this case.

The example above shows a special case how the change in the initial distribution may affect the optimal saving of the individuals. Then how about the general case?

First it is assumed that the initial wealth distribution is given by the cumulative density function,  $G(w, \beta)$ , on the support  $[\underline{w}, \bar{w}]$ . Here  $\beta$  is the index of inequality. Following Rothschild and Stiglitz (1970)'s approach, define the following mean-preserving spread:

$$\int_{\underline{w}}^{\bar{w}} G_{\beta}(w, \beta) d\epsilon = 0, \text{ and } \int_{\underline{w}}^y G_{\beta}(w, \beta) d\epsilon \geq 0, \forall y \in [\underline{w}, \bar{w}] \quad (2.5)$$

where  $G_{\beta}(w, \beta) = \frac{\partial G(w, \beta)}{\partial \beta}$ . As  $\beta$  increases, the distribution is more unequal. The next proposition gives the general conditions for the direction of change in optimal savings.

**Proposition 2.4** *Under Assumption 2.1, for an individual with wealth  $\hat{w}$ ,*

*i). If  $\frac{\partial^2 v(G(\hat{w}, \beta)}{\partial w \partial \beta} > 0$ , in equilibrium her optimal savings level is closer to the optimal savings level in the pure status model when society gets more unequal (e.g., when  $\beta$  increases). Her optimal savings level is closer to the optimal savings level in the pure consumption model when society gets more equal (e.g., when  $\beta$  decreases).*

*ii). If  $\frac{\partial^2 v(G(\hat{w}, \beta)}{\partial w \partial \beta} < 0$ , in equilibrium her optimal savings level is closer to the optimal savings level in the pure consumption model when society gets more unequal (e.g., when  $\beta$  increases). Her optimal savings level is closer to the optimal savings level in the pure status model when society gets more equal (e.g., when  $\beta$  decreases).*

The analysis above shows for a particular wealth level  $w$ , the result depends on  $\frac{\partial^2 v(G(w, \beta)}{\partial w \partial \beta}$ , i.e., how the marginal utility of wealth from the concern for status changes with the index of inequality. When this marginal gain is increasing in  $\beta$ , the optimal solution will be closer to that in the pure status model. The individuals behave as if they put more weight on the status term. Otherwise, the optimal solution will be closer to that in the pure consumption model, so the individuals behave as if they put more weight on the absolute consumption term. This result is very intuitive. There is no uniform pattern for all  $w$  if  $\frac{\partial^2 v(G(w, \beta)}{\partial w \partial \beta}$  changes over different wealth levels. In the special case that  $v(G) = G$ ,  $\frac{\partial^2 v(G(w, \beta)}{\partial w \partial \beta} = \frac{\partial g(w, \beta)}{\partial \beta}$ , as in the example discussed above. Then if density around  $w$  increases, the optimal solution will be closer to that in the pure status model. Otherwise, the optimal solution will be closer to that in the pure

consumption model.

Pareto first used the Pareto distribution to describe the allocation of wealth in the society since it captures rather well the way a large share of the wealth in society is owned by a small fraction of people. It is still widely used today, especially for the rich end (for example, see Chatterjee, Yarlagadda and Chakrabarti, 2005, for an empirical examination.) Then suppose in the society the initial wealth distribution  $G(w, \beta)$  is Pareto distributed with  $G(w, \beta) = 1 - [\frac{w}{m(1-\beta)}]^{-1/\beta}$ , where  $m$  is some positive constant,  $0 < \beta < 1$  and  $w \geq m(1 - \beta)$ . The mean of this distribution is  $m$  and the inequality increases with  $\beta$ , satisfying Condition (2.5). For simplicity it is assumed that  $v(G) = G$ . It is easy to get  $\frac{\partial^2 v(G(w, \beta))}{\partial w \partial \beta} = -\beta^{-2} g(w, \beta) [\ln m(1 - \beta) + \frac{(2-\beta)\beta}{1-\beta} - \frac{1+1/\beta}{w}]$ . Therefore there exists a cutoff wealth level  $\tilde{w} = \frac{1+1/\beta}{\ln m(1-\beta) + \frac{(2-\beta)\beta}{1-\beta}}$  such that for the richer individuals with wealth  $w > \tilde{w}$ , they will behave as if they weigh the status term more as the society becomes more equal since  $\frac{\partial^2 v(G(w, \beta))}{\partial w \partial \beta} < 0$ . If we take the case that  $r < \rho$  and  $\sigma > 1$  as an example, they will save less when the society gets more equal, or save more when the society gets more unequal. For the poorer individuals with wealth  $w < \tilde{w}$ , they will behave as if they weigh the status term less as the society becomes more equal since  $\frac{\partial^2 v(G(w, \beta))}{\partial w \partial \beta} > 0$ . In the case that  $\sigma > 1$  and  $r < \rho$ , they will save more when the society gets more equal, or save less when the society gets more unequal. The individuals at the two extremes adjust their optimal saving in the opposite directions. As a result, we observe that the rich and the poor's saving decisions are closer when the society becomes more equal, and they diverge when the society becomes more unequal. Equality/inequality is self-enforcing in this sense in this economy. The effect on the aggregate saving will depend on the specific parameter values in the model.

From the analysis above we show how the redistribution may affect individuals' intertemporal choices under certain conditions. Generally the direction of change in the individuals' optimal saving is related to the change in the marginal gain of consumption from status concern and is not necessarily uniform. The total effect on aggregate savings will depend on the full characterization of individuals' preferences and the wealth distribution in the society.

## 2.6 Conclusions

In the framework of the two-period intertemporal choice problem, this paper develops a model where individuals care about consumption in both absolute and relative terms. I show that, in general, the concern for status distorts individuals' optimal saving decisions. The direction and magnitude of this distortion depends on the elasticity of intertemporal substitution in the individuals' utility from absolute consumption. In an empirically plausible case, where the elasticity of intertemporal substitution is less than one and the interest rate is less than the discount rate, people tend to save less, because of their concern for status, but the magnitude of the conspicuous consumption is smaller relative to that of the static model, due to the intertemporal trade-off between today's status and tomorrow's status. A more startling result occurs if the interest rate is greater than the discount rate: people save even more than what they would without the concern for status. A concern with conspicuous consumption does not necessarily increase consumption at all. I also analyze how the redistribution of the initial wealth affects the optimal saving. The resulting change in marginal gain of consumption from the status term will determine how people behave, i.e., whether they get closer to the optimal decision in the pure status model or closer to the optimal decision in the pure consumption model. For a reasonable parameterization, the rich save more and the poor save less, when society gets more unequal, implying that equality/inequality is self-enforcing in this economy. A careful evaluation of parameterization of the whole economy is needed for any policy suggestions. Finally, the resulting allocation is Pareto inefficient due to the externalities generated by a concern with status.



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## 2.8 Appendix

### 2.8.1 Proof of Proposition 2.1

Consider the intertemporal choice of a particular individual facing Problem (2.1) in a population where all other individuals choose  $c_t = p_t(w)$  which is strictly increasing and differentiable in  $w$ , and  $\frac{dc_t}{dw} > 0$ . It follows that  $F_t(p_t(w)) = G(w)$ . Then this individual solves the following problem:

$$\begin{aligned} \max_{\{c_t\}} \sum_{t=0}^1 \frac{(1-\alpha)u(c_t) + \alpha v(G(p_t^{-1}(c_t)))}{(1+\rho)^t} \\ \text{s.t.} \quad \sum_{t=0}^1 \frac{c_t}{(1+r)^t} = w \end{aligned} \quad (2.6)$$

Then F.O.C. is given by the following differential equation:

$$\begin{aligned} (1-\alpha)u'(c_0) + \alpha v'(G(w))g(w) \frac{1}{\frac{dc_0}{dw}} \\ = \frac{1+r}{1+\rho} [(1-\alpha)u'(c_1) + \alpha v'(G(w))g(w) \frac{1}{(1+r)(1-\frac{dc_0}{dw})}] \end{aligned} \quad (2.7)$$

The solution to this first-order differential equation will be shown to be the equilibrium strategy.

First I will show there exists a solution for Equation (2.7).

We can get an expression for  $\frac{dc_0}{dw}$  as follows:

$$\frac{dc_0}{dw} = \begin{cases} \frac{(1-\alpha)[u'(c_0) - \frac{1+r}{1+\rho}u'(c_1)] - \alpha \frac{2+\rho}{1+\rho}v'g \pm \sqrt{M}}{2(1-\alpha)[u'(c_0) - \frac{1+r}{1+\rho}u'(c_1)]} & \text{if } u'(c_0) - \frac{1+r}{1+\rho}u'(c_1) \neq 0 \\ \frac{1+\rho}{2+\rho} & \text{if } u'(c_0) - \frac{1+r}{1+\rho}u'(c_1) = 0 \end{cases} \quad (2.8)$$

where  $M = \{(1-\alpha)[u'(c_0) - \frac{1+r}{1+\rho}u'(c_1)] - \alpha \frac{2+\rho}{1+\rho}v'g\}^2 + 4\alpha(1-\alpha)[u'(c_0) - \frac{1+r}{1+\rho}u'(c_1)]v'g$ .

When  $u'(c_0) - \frac{1+r}{1+\rho}u'(c_1) \neq 0$ ,  $\frac{dc_0}{dw}$  has two expressions, one with the positive sign and the other with the negative sign. We can show either  $\frac{dc_0}{dw} > 1$  or  $\frac{dc_0}{dw} < 0$  if we take the negative sign. Since  $c_t$  is strictly increasing and differentiable in  $w$ , it is impossible to have  $\frac{dc_0}{dw} > 1$  or  $\frac{dc_0}{dw} < 0$  by the budget constraint in Problem (2.6). So we drop the negative sign. Let  $k(c_0, w)$  denote the RHS of Equation (2.8) without the negative sign. (We can show  $0 < k(c_0, w) < 1$  always holds.) Since now  $k(c_0, w)$  is continuous,

by Cauchy-Peano Existence Theorem (Coddington and Levinson (1955, p. 6)), there exists a solution for Equation (2.8) with the initial condition  $c_0^0 = c_0(w_0)$ .<sup>8</sup> This solution is strictly increasing and differentiable in  $w$  since we know  $0 < \frac{dc_0}{dw} < 1$ .

Next, I will show there is a unique global solution for Equation (2.8).

We know  $k(c_0, w)$  is not only continuous but also continuously differentiable. Pick up any pair  $(c_0^0, w_0)$ . Define a compact set

$$B(c_0^0, w_0) = \{(c_0, w) : |c_0 - c_0^0| \leq a, |w - w_0| \leq b, a, b > 0\}$$

Then pick up two points,  $(x, w)$  and  $(y, w) \in B$  (W.L.O.G., assume  $x < y$ ). By the Mean Value Theorem, there exists some  $z \in (x, y)$  such that  $\frac{dk(c_0, w)}{dc_0} \Big|_{c_0=z} = \frac{k(y, w) - k(x, w)}{y - x}$ . Since  $\frac{dk(c_0, w)}{dc_0}$  is continuous on  $B$  and  $B$  is compact,  $\frac{dk(c_0, w)}{dc_0}$  is bounded. Therefore,  $|\frac{dk(c_0, w)}{dc_0} \Big|_{c_0=z}| \leq \sup |\frac{dk(c_0, w)}{dc_0}|$ . That is,  $|k(y, w) - k(x, w)| \leq \sup |\frac{dk(c_0, w)}{dc_0}| |y - x|$ . Therefore,  $k(c_0, w)$  is locally Lipschitz. Then there is a unique global solution for Equation (2.8) (Coddington and Levinson (1955)).

Finally, I will show there exists an upper bound  $\bar{\alpha}$  s.t. when  $0 < \alpha < \bar{\alpha}$ , this unique global solution is the maximum for Problem (2.6).

The first step is to show the second derivative of the instantaneous utility function,  $(1 - \alpha)u'' + \alpha[(v''g^2 + v'g')\frac{1}{(\frac{dc_t}{dw})^2} + v'g\frac{d^2p_t^{-1}}{dc_t^2}]$ , converges uniformly to  $u''$  as  $\alpha \rightarrow 0$ . It is equivalent to show  $\limsup_{\alpha \rightarrow 0} \sup_{c_t} |\alpha[(v''g^2 + v'g')\frac{1}{(\frac{dc_t}{dw})^2} + v'g\frac{d^2p_t^{-1}}{dc_t^2}]| = 0$ . It suffices to show  $(v''g^2 + v'g')\frac{1}{(\frac{dc_t}{dw})^2} + v'g\frac{d^2p_t^{-1}}{dc_t^2}$  is bounded over  $c_t$ . We already know  $v'', v', g, g'$ , and  $\frac{dc_t}{dw}$  are bounded. Next I will show  $\frac{d^2p_t^{-1}}{dc_t^2}$  is bounded. We know  $\frac{d^2p_t^{-1}}{dc_t^2} = \frac{d(\frac{dp_t^{-1}}{dc_t})}{dc_t}$  and  $\frac{dp_t^{-1}}{dc_t} = \frac{1}{\frac{dc_t}{dw}}$ . We also know  $k(c_0, w)$  is continuously differentiable. Then  $\frac{dp_t^{-1}}{dc_t} = \frac{1}{\frac{dc_t}{dw}}$  is continuously differentiable. That is,  $\frac{d^2p_t^{-1}}{dc_t^2}$  is continuous on a bounded set as  $0 \leq c_t \leq w$ . It follows that  $\frac{d^2p_t^{-1}}{dc_t^2}$  is bounded. Therefore,  $\limsup_{\alpha \rightarrow 0} \sup_{c_t} |\alpha[(v''g^2 + v'g')\frac{1}{(\frac{dc_t}{dw})^2} + v'g\frac{d^2p_t^{-1}}{dc_t^2}]| = 0$ . So as  $\alpha \rightarrow 0$ ,  $(1 - \alpha)u'' + \alpha[(v''g^2 + v'g')\frac{1}{(\frac{dc_t}{dw})^2} + v'g\frac{d^2p_t^{-1}}{dc_t^2}]$  converges uniformly to  $u''$ , which is negative. As a result, there exists an upper bound  $\bar{\alpha}$  s.t. when  $0 < \alpha < \bar{\alpha}$ ,

<sup>8</sup>Note  $c_0(0) = 0$  is a singular point for the differential equation because of the Inada condition  $u'(0) = \infty$ . But we can show the following. Suppose we start at some initial condition with  $w_0 = \xi$  with  $\xi > 0$ , and  $c_0^0 = \delta = \frac{1}{2}\xi$ . Equation (2.8) is well-behaved at this point. There exists a solution, say,  $\bar{c}_0(w)$ , for Equation (2.8) with this initial condition. We know  $\lim_{\xi \rightarrow 0} \frac{1}{2}\xi = 0$ . From the budget constraint, we can get  $0 \leq c_0 \leq w$ . That is,  $c_0^*(0) = 0$ . Therefore, we can define the solution of Equation (2.8) with initial condition  $(0, 0)$  as  $c_0^*(w) = \lim_{\xi \rightarrow 0} \bar{c}_0(w)$ .

$(1 - \alpha)u'' + \alpha[(v''g^2 + v'g')\frac{1}{(\frac{dc_t}{dw})^2} + v'g\frac{d^2p_t^{-1}}{dc_t^2}]$  is negative. Then the solution satisfies the second order condition for maximization. Therefore, there exists a unique equilibrium for Problem (1).

## 2.8.2 Proof of Proposition 2.2

Here we copy the F.O.C. as follows:

$$\begin{aligned} & (1 - \alpha)u'(c_0) + \alpha v'(G(w))g(w)\frac{1}{\frac{dc_0}{dw}} \\ &= \frac{1+r}{1+\rho}[(1 - \alpha)u'(c_1) + \alpha v'(G(w))g(w)\frac{1}{(1+r)(1 - \frac{dc_0}{dw})}] \end{aligned} \quad (2.9)$$

Take derivative w.r.t.  $\alpha$  and evaluate at  $\alpha = 0$ , we can get the comparative static<sup>9</sup>

$$\frac{dc_0}{d\alpha} \Big|_{\alpha=0} = \frac{v'g[-\frac{1}{\frac{dc_0}{dw}} + \frac{1}{1+\rho}\frac{1}{1 - \frac{dc_0}{dw}}]}{u''(c_0) + \frac{(1+r)^2}{1+\rho}u''(c_1)} \quad (2.10)$$

When  $u(c_t) = \frac{c_t^{1-\sigma}}{1-\sigma}$ , we can get the following

$$\frac{dc_0}{d\alpha} \Big|_{\alpha=0} = \frac{v'g[1 + (\frac{1+r}{1+\rho})^{\frac{1}{\sigma}}\frac{1}{1+r}][(\frac{1+r}{1+\rho})^{1-\frac{1}{\sigma}} - 1]}{-\sigma[c_0^{-\sigma-1} + \frac{(1+r)^2}{1+\rho}c_1^{-\sigma-1}]} \quad (2.11)$$

Therefore,  $\frac{dc_0}{d\alpha} \Big|_{\alpha=0} > 0$  if  $(\frac{1+r}{1+\rho})^{1-\frac{1}{\sigma}} - 1 < 0$ ;

$$\frac{dc_0}{d\alpha} \Big|_{\alpha=0} < 0 \text{ if } (\frac{1+r}{1+\rho})^{1-\frac{1}{\sigma}} - 1 > 0.^{10}$$

When  $\sigma < 1$ ,  $(\frac{1+r}{1+\rho})^{1-\frac{1}{\sigma}} - 1 < 0$  if  $\frac{1+r}{1+\rho} > 1$ ;

$$(\frac{1+r}{1+\rho})^{1-\frac{1}{\sigma}} - 1 > 0 \text{ if } \frac{1+r}{1+\rho} < 1;$$

When  $\sigma > 1$ ,  $(\frac{1+r}{1+\rho})^{1-\frac{1}{\sigma}} - 1 > 0$  if  $\frac{1+r}{1+\rho} > 1$ ;

$$(\frac{1+r}{1+\rho})^{1-\frac{1}{\sigma}} - 1 < 0 \text{ if } \frac{1+r}{1+\rho} < 1.$$

As in the proof for Proposition 1, we can show  $\frac{dc_0}{d\alpha}$  converges uniformly to  $\frac{dc_0}{d\alpha} \Big|_{\alpha=0}$  as  $\alpha \rightarrow 0$ . Therefore, there exists a upper bound  $a$ , s.t. when  $0 < \alpha < a$ , the following holds:

<sup>9</sup>To derive this expression, I use  $u'(c_0) = \frac{1+r}{1+\rho}u'(c_1)$  as  $\alpha \rightarrow 0$ .

<sup>10</sup>Note, when  $\sigma = 1$ , that is,  $u(c_t) = \ln c_t$ ,  $\frac{dc_0}{d\alpha} \Big|_{\alpha=0} = 0$ .

$\frac{dc_0}{d\alpha} > 0$  when  $r > \rho$  and  $\sigma < 1$  (or  $r < \rho$  and  $\sigma > 1$ );

$\frac{dc_0}{d\alpha} < 0$  when  $r > \rho$  and  $\sigma > 1$  (or  $r < \rho$  and  $\sigma < 1$ ).

### 2.8.3 Proof of Proposition 2.3

It is sufficient to show there exists another allocation  $c_{t,i}^{FB}$  such that

$$\sum_{t=0}^1 \frac{(1-\alpha)u(c_{t,i}^{FB}) + \alpha v(F_t(c_{t,i}^{FB}))}{(1+\rho)^t} > \sum_{t=0}^1 \frac{(1-\alpha)u(c_{t,i}^*) + \alpha v(F_t(c_{t,i}^*))}{(1+\rho)^t}$$

with  $\sum_{t=0}^1 \frac{c_{t,i}^{FB}}{(1+r)^t} = w_i$  (2.12)

Claim: The optimal solution to Problem (2.2),  $c_{t,i}^C$ , satisfies conditions (2.12).

First,  $c_{t,i}^C$  is increasing in  $w_i$ . Given  $c_{t,i}^H(w)$  is increasing too, it follows that

$$F_t(c_{t,i}^C) = F_t(c_{t,i}^H) = G(w_i) \tag{2.13}$$

Then we can get

$$\sum_{t=0}^1 \frac{\alpha v(F_t(c_{t,i}^C))}{(1+\rho)^t} = \sum_{t=0}^1 \frac{\alpha v(F_t(c_{t,i}^H))}{(1+\rho)^t} \tag{2.14}$$

Second, as  $c_{t,i}^C$  is the unique optimal solution to Problem (2.2), and  $c_{t,i}^H \neq c_{t,i}^C$  from Proposition 2.1, the following inequality holds:

$$\sum_{t=0}^1 \frac{u(c_{t,i}^C)}{(1+\rho)^t} > \sum_{t=0}^1 \frac{u(c_{t,i}^H)}{(1+\rho)^t}, \forall i \tag{2.15}$$

That is to say:

$$\sum_{t=0}^1 \frac{(1-\alpha)u(c_{t,i}^C)}{(1+\rho)^t} > \sum_{t=0}^1 \frac{(1-\alpha)u(c_{t,i}^H)}{(1+\rho)^t} \tag{2.16}$$

By summing up Equation (2.14) and Inequality (2.16), we show  $c_{t,i}^C$  satisfies condition (2.12). Moreover,  $c_{t,i}^C$  satisfies condition (2.12) from the budget constraint.

## 2.8.4 Proof of Proposition 2.4

Consider Equation (2.11), take the derivative w.r.t.  $\beta$  and evaluate at  $\alpha = 0$ , so that

$$\begin{aligned} \frac{\partial^2 c_0}{\partial \alpha \partial \beta} \Big|_{\alpha=0} &= \frac{[1 + (\frac{1+r}{1+\rho})^{\frac{1}{\sigma}} \frac{1}{1+r}] [(\frac{1+r}{1+\rho})^{1-\frac{1}{\sigma}} - 1]}{\sigma [c_0^{-\sigma-1} + \frac{(1+r)^2}{1+\rho} c_1^{-\sigma-1}]^2} \{v'g \frac{\partial [c_0^{-\sigma-1} + \frac{(1+r)^2}{1+\rho} c_1^{-\sigma-1}]}{\partial \beta} \\ &\quad - (v''g \frac{\partial G}{\partial \beta} + v' \frac{\partial g}{\partial \beta}) [c_0^{-\sigma-1} + \frac{(1+r)^2}{1+\rho} c_1^{-\sigma-1}]\} \end{aligned} \quad (2.17)$$

As  $\frac{\partial c_t}{\partial \beta} \Big|_{\alpha=0} = 0$ , this reduces to

$$\begin{aligned} \frac{\partial^2 c_0}{\partial \alpha \partial \beta} \Big|_{\alpha=0} &= - \frac{(v''g \frac{\partial G}{\partial \beta} + v' \frac{\partial g}{\partial \beta}) [1 + (\frac{1+r}{1+\rho})^{\frac{1}{\sigma}} \frac{1}{1+r}] [(\frac{1+r}{1+\rho})^{1-\frac{1}{\sigma}} - 1]}{\sigma [c_0^{-\sigma-1} + \frac{(1+r)^2}{1+\rho} c_1^{-\sigma-1}]} \\ &= \frac{v''g \frac{\partial G}{\partial \beta} + v' \frac{\partial g}{\partial \beta}}{v'g} \frac{dc_0}{d\alpha} \Big|_{\alpha=0} \end{aligned} \quad (2.18)$$

If  $\frac{\partial^2 v(G(w,\beta))}{\partial w \partial \beta} > 0$ , that is,  $v''g \frac{\partial G}{\partial \beta} + v' \frac{\partial g}{\partial \beta} > 0$ , we can get:

When  $\frac{dc_0}{d\alpha} \Big|_{\alpha=0} < 0$ ,  $\frac{\partial^2 c_0}{\partial \alpha \partial \beta} \Big|_{\alpha=0} < 0$ .

More unequal distribution will drag the first period consumption down more.

When  $\frac{dc_0}{d\alpha} \Big|_{\alpha=0} > 0$ ,  $\frac{\partial^2 c_0}{\partial \alpha \partial \beta} \Big|_{\alpha=0} > 0$ .

More unequal distribution will pull up the first period consumption more.

If  $\frac{\partial^2 v(G(w,\beta))}{\partial w \partial \beta} < 0$ , that is,  $v''g \frac{\partial G}{\partial \beta} + v' \frac{\partial g}{\partial \beta} < 0$ , we can get:

When  $\frac{dc_0}{d\alpha} \Big|_{\alpha=0} < 0$ ,  $\frac{\partial^2 c_0}{\partial \alpha \partial \beta} \Big|_{\alpha=0} > 0$ .

More unequal distribution will drag the first period consumption down less.

When  $\frac{dc_0}{d\alpha} \Big|_{\alpha=0} > 0$ ,  $\frac{\partial^2 c_0}{\partial \alpha \partial \beta} \Big|_{\alpha=0} < 0$ .

More unequal distribution will pull up the first period consumption less.



# Chapter 3

## Social Insurance and Status<sup>1</sup>

*In this paper we develop a series of optimal social insurance models in which people care about consumption not only for its own sake but also for the status it implies. We show that the concern for status does impact the optimal contract under various information structures. Particularly, under complete information without commitment problem, the optimal contract may assign all the society resources to the minority group if the status term is convex enough. Under the limited enforcement regime, compared to the optimal allocation in the pure consumption model, the social planner may transfer more resources to high income people when the status term is convex. Under moral hazard, the relatively lower status resulting from the higher effort level may make implementation of high effort level more difficult.*

### 3.1 Introduction

People in developing countries face substantial risks in their daily life. There is a large body of evidence that shows that informal insurance via inter-personal loans or reciprocal transfers is an important risk-smoothing mechanism in developing countries where formal insurance markets or access to credit are often very limited or absent. A large empirical literature shows that financial markets in both developed and developing countries do not operate in a way consistent with the full insurance hypothesis: Cochrane (1991), Mace (1991), Townsend (1994), Udry (1994), Attanasio and Davis (1996), Fafchamps and Lund (1998), Grimard (1997), Jacoby and Skoufias (1997), Jappelli and Pistaferri (2006), Albarran and Attanasio (2001), Ligon et al.

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<sup>1</sup>This chapter is based on the work coauthored with Alexander Karaivanov.

(2002), Pistaferri et al. (2004), Dubois, Julien and Magnac (2002, 2008), Attanasio and Pavoni (2007).

In this paper we analyze how concern for status affects optimal risk-sharing across risk-averse agents with stochastic endowments. What makes our model different from all these previous studies is that we allow for a new component in people's preferences: we look at how much mutual insurance can be supported when agents care about consumption not only on its own but also for the status it implies, measured by the difference between one's consumption and average consumption in the group (e.g., village) where informal risk-sharing institutions operate.

We develop several optimal mutual insurance models in which people care about their relative position in society and study how this concern for status affects the design of optimal risk-sharing contracts (defined as the contracts maximizing ex-ante expected utility). This idea is motivated by the following observation. For a particular individual, if other people in the village become richer, on one hand, his welfare would increase because in bad times he can receive more from others (consumption level effect); on the other hand, his welfare would decrease since he becomes relatively poorer e.g., enjoys lower social esteem (status/relative consumption effect). These two effects work in opposite directions and the total effect on individual welfare is unclear. Moreover, the more important question is how these effects intertwine with each other and impact individual behavior and the amount of mutual insurance that can be supported in various environments characterized with information or commitment problems.

The idea that one's utility may depend on the comparison of one's own consumption to that of others can be traced back, at the very least, to Veblen (1899). It was further developed by Duesenberry (1949) and Frank (1985a). Recently, a large body of empirical literature has appeared to support this view: individuals tend to evaluate their consumption in comparison to the consumption of others (Easterlin, 1974; McBride, 2001; Luttmer, 2005; Johansson-Stenman and Martinsson, 2006; Dyanan and Ravina, 2007, among others.) Furthermore, evidence from neuro-physiological studies indicates that pleasure felt by individuals is strongly influenced by interpersonal comparisons. For example, in a recent brain-scan experiment, Fliessbach et al. (2007) found that participants who received more money than their co-players showed a much stronger activation in their brain's reward centre compared to when both players received the same amount.

Following the mutual insurance literature, we analyze three environments with

different information structure and enforcement/commitment ability. Suppose there is a continuum of measure one of agents who have stochastic income that can be either high or low. The income realizations are i.i.d. across agents and time. There are no insurance or credit markets – the only way agents can smooth their consumption is through a mutual insurance scheme in which agents with currently high income make a net transfer to agents with currently low income with the understanding that the roles may be reversed in future periods.

The first setting we study features complete information and full enforcement across the agents. Without a status effect full insurance (everyone consuming the village average income) is optimal. The status term has no effect on the optimal consumption allocation when the agent’s utility function remains strictly concave in one’s consumption level when including both the level and status effects. However, when the status term is convex enough to generate a utility function that is overall convex in one’s consumption, we show that it is optimal to have the minority group (i.e., all agents with high or low income, depending on which group is smaller) consume all available resources – perfect inequality ensues. Intuitively, there is a trade-off between providing status vs. consumption smoothing. The presence of a strong status effect can thus lead to very imperfect insurance (in fact less than what would be observed in autarky). One may thus conclude that the setting is plagued with various information or other problems but actually this unequal allocation is first-best optimal ex-ante.

The second regime we study is one with limited commitment where individuals are free to walk away from the mutual insurance contract (the implied ‘reciprocal obligation’) at any point such as in Coate and Ravallion (1993) or Ligon et al. (2004). In the pure consumption model without status concern, the result is that depending on parameters, only partial insurance may be supported. We find that when the status term is convex in the difference between one’s consumption and average consumption but the utility function is concave in one’s consumption overall, the individual with high income consumes more compared to the solution in the pure consumption model where people do not care about relative position in the society. That is, less insurance can be supported than in the same environment but populated with agents without status concern.

We also consider a setting where individual income is dependent on the amount of effort they put in (increasing the probability of having high income). Individual efforts are not observed, thus a moral hazard problem ensues and full insurance is

not incentive-compatible. When people exert a higher effort level, it increases the average income level thus decreases an individual's welfare through the concern for status. This effect makes the implementation of the higher effort level more difficult in the model with status. As a result, it is more costly to enforce mutual insurance, compared to the pure consumption model.

In this paper we define status as the difference between the individual consumption and average consumption in the reference group. We do this mainly for simplicity, however, there is support for this idea from some empirical studies. For example, Luttmer (2005) finds that an increase in average earnings in one's neighborhood makes one report a lower level of happiness, after controlling for own income. The effects of equal increases in own and average incomes were found to roughly cancel out. Of course, we do not deny that our results may depend on our particular specification of status which however is not uncommon in the status literature.

Relative to most of the papers in the literature on concern for status, we do not have a competitive market setting but look at the optimal allocations that can be supported through cooperation (potentially subject to information or commitment frictions). As a result, the usual externalities generated by the relative concern are fully internalized by the optimal insurance plan. Any distortion caused by the relative concern effect compared to the pure consumption model must come from a different source, namely the interaction between the status effect and the information or enforcement imperfections.

This paper is organized as follows. Section 3.2 describes the model environment. Section 3.3 characterizes the optimal insurance contract under complete information without enforcement problem. Section 3.4 describes the case of limited enforcement. Section 3.5 discusses moral hazard model. The last section concludes.

## 3.2 Model Environment

In a village there are a continuum of measure one ex-ante identical agents who live for infinite number of periods,  $t = \{0, 1, \dots, \infty\}$ . Each agent receives a stochastic endowment stream  $\{y_t\}_{t=0}^{\infty}$ , where for each  $t \geq 0$ ,  $y_t$  is independently and identically distributed according to the two-point probability distribution  $prob(y_t = y^H) = \pi$ , and  $prob(y_t = y^L) = 1 - \pi$ , with  $y^H > y^L$  and  $0 < \pi < 1$ . Following Coate and Ravallion (1993), we focus on the pure insurance arrangements and assume that the agents

can make transfers among each other, but they do not save. Hence, income transfers at any date depends only on the income realized at that date. This precludes possible credit features of informal insurance arrangements in which reciprocal transfers may look like loans that are paid back at least in part at some later date.<sup>2</sup>

All agents have identical preferences defined over their own consumption and the status implied by the relative consumption in the village. For agent  $i$ , the instantaneous utility function at time  $t$  is given by

$$v(c_t^i, \bar{c}_t) = u(c_t^i) + k(c_t^i - \bar{c}_t) \quad (3.1)$$

Above, we assume that preferences are additively separable in the ‘pure consumption’,  $u(\cdot)$  and ‘status / relative concern’,  $k(\cdot)$  terms. Specifically,  $c_t^i$  denotes agent  $i$ ’s own consumption at time  $t$ . Thus,  $u(c_t^i)$  represents the satisfaction derived from consuming  $c_t^i$  units of the consumption good, i.e., the utility due to the intrinsic worth of that good. As usual, we assume that  $u(\cdot)$  is strictly increasing and strictly concave in  $c_t^i$ ,  $u' > 0$  and  $u'' < 0$ . In the second term,  $\bar{c}_t$  is average consumption in the village at time  $t$ . Note that due to our lack of savings assumption we always have  $\bar{c} = \bar{y} = \pi y^H + (1 - \pi)y^L$ . We assume that one’s utility depends on the difference between their own consumption and the average consumption in the village. This term is denoted by  $k(c_t^i - \bar{c}_t)$ . It is assumed that  $k' > 0$ . Hence the agent is better off when his own consumption increases or society becomes poorer on average. Where appropriate, we assume that agents discount future payoffs with discount factor  $\beta \in (0, 1)$ .

### 3.3 First best

We start with the case of full information and perfect enforcement/commitment which we call ‘the first best’. The problem of mutual insurance can be written as the problem of a fictitious ‘social planner’ who maximizes the social welfare function equal to the average ex-ante utility in the society. That is, in each period, the social planner observes the realization of  $y_t$  for each individual and maximize the expected utility,

$$\pi v(y^H + \tau^H, \bar{c}) + (1 - \pi)v(y^L + \tau^L, \bar{c}) \quad (3.2)$$

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<sup>2</sup>In some of the environments we study this assumption can be restrictive, as history-dependent arrangements may increase ex-ante welfare. We rule out such arrangements by no-savings (both private and public) assumption for simplicity and to focus on the pure effect of status on mutual insurance rather than state-contingent credit.

by choice of  $\tau^H$  – the amount of goods transferred to individuals with income  $y^H$  and  $\tau^L$  – the amount of goods transferred to individuals whose income is  $y^L$ . Above  $\bar{c}$  is the average consumption  $\pi c^H + (1 - \pi)c^L$ , where  $c^H = y^H + \tau^H$ , and  $c^L = y^L + \tau^L$ . The maximization is subject to the per-period resource constraint

$$\pi\tau^H + (1 - \pi)\tau^L = 0 \quad (3.3)$$

Plugging the resource constraint (3.3) into the objective function (3.2), the optimal insurance problem is,

$$\max_{\tau^H} \pi[u(y^H + \tau^H) + k(y^H + \tau^H - \bar{y})] + (1 - \pi)[u(y^L - \frac{\pi}{1 - \pi}\tau^H) + k(y^L - \frac{\pi}{1 - \pi}\tau^H - \bar{y})] \quad (3.4)$$

The status term in the utility function causes some complications here. If there were no status term  $k(\cdot)$  in the utility function, the above is a standard utility maximization problem which yields equal consumption allocation,  $c_{FB}^H = c_{FB}^L = \bar{y}$  at the optimum, where  $c_{FB}^H$  denotes the amount of goods consumed by high income individuals in the first best, and  $c_{FB}^L$  the amount of goods consumed by low income individuals in the first best. This full-insurance result is the key implication of the full information model without status – barring any information or enforcement problems it is optimal to perfectly insure everyone. That is, individual's consumption is only function of the aggregate (or average) income level and is independent of one's own income level.

However, with the existence of a status term in the utility function, the above results may not hold. As discussed in Robson (1992), putting a status term in the neoclassical utility function may generate a Friedman-Savage concave-convex-concave utility. Becker, Murphy and Werning (2005) also show how a status term may add convexity into the utility function. In line with the literature, it seems reasonable to allow the status term  $k(\cdot)$  to be convex, which implies the marginal satisfaction from the improvement of social status is increasing as an individual climbs up the social ladder. Note that when the convexity of  $k$  is strong enough, the overall utility function  $v(c, \bar{c})$  may become convex in  $c$ , at least in some range. Note that, given our assumptions we only need to make assumptions on the curvature of  $v$  at  $\bar{c} = \bar{y}$  which significantly simplifies the analysis.

The following proposition fully characterizes the optimal mutual insurance contracts when individuals have a concern for status expressed as function of their con-

sumption level relative to the average village consumption level.

**Proposition 3.1** *In a setting with full information and perfect enforcement,*

(a) *If  $v(c, \bar{c})$  is strictly convex in  $c$  at  $\bar{c} = \bar{y}$  then: (i). when  $\pi = \frac{1}{2}$ , it is optimal to transfer all resources either to the individuals with low income  $y^L$  or to the individuals with high income  $y^H$ ; (ii). When  $\pi < \frac{1}{2}$  it is optimal to transfer all resources to the individuals with high income  $y^H$ ; (iii). When  $\pi > \frac{1}{2}$ , it is optimal to transfer all resources to the individuals with low income  $y^L$ ;*

(b). *If  $v(c, \bar{c})$  is strictly concave in  $c$  at  $\bar{c} = \bar{y}$  it is optimal to allocate consumption equally across agents, that is,  $c^H = c^L = \bar{y}$ .*

Proposition 3.1 states that the optimal insurance arrangement with status concern depends critically on the overall shape of the individuals' utility function  $v$ . When the status term is sufficiently convex so that the overall utility become convex in one's consumption, the distribution of income plays an important role. It is optimal to allocate all the resources to the minority group in the village, whether they are the ex-ante high-income or low-income individuals. The intuition is as follows. When the utility function is convex, it is optimal to make the distribution of consumption as unequal as possible to achieve efficiency. Assuming that the lowest possible consumption level is zero, it is optimal to let one type individual have zero consumption, and let the other type have the highest possible consumption. When the majority of the society are low-income individuals, it is optimal to let the high-income agents consume all resources since each high income person receives a transfer from more than one low-income person, compared to the case where the low-income person would receive the transfer from less than one high income person if we let the low income person consume all the resources. The convexity in utility offsets the linearity of the expected utility in the population fractions  $\pi$  and  $1 - \pi$ . The result that the convex utility due to the concern for status leads to unequal ex-ante Pareto efficient consumption allocation is consistent with Robson (1992) and Becker, Murphy and Werning (2005).

### 3.4 Limited Enforcement

In the last section, we assumed that the village can successfully implement the optimal insurance contract every period, that is, no individual can walk away from the initial

arrangement (either perfect-equality or perfect-inequality allocation). This can be interpreted as full ex-ante commitment to the optimal contract or, alternatively, as ability to perfectly enforce that contract.

It should be clear that in the absence of such commitment, the mutual insurance arrangement cannot be sustained in one-period interactions. No matter what is agreed ex-ante, some individuals will have incentive to renege. In repeated interactions, however, such arrangements can be sustained. In this section we shall discuss how concern for status affects the optimal insurance contract when agents are not committed to the contract. They are now free to walk away from the mutual insurance contract at any time and must be induced not to do so by the design of the contract itself.

Specifically, assume that at any point of time, an agent can renege on the agreed transfer scheme and simply consume their endowment. However, if one does so, one must live in autarky from that time on. The per-period value of expected utility in autarky is given by,

$$V_{aut}^S = \pi v(y^H, \bar{y}) + (1 - \pi)v(y^L, \bar{y})$$

At any point of time, having observed his current period endowment, an agent can guarantee himself a present value of utility of  $v(y_t, \bar{y}) + \frac{\beta}{1-\beta}V_{aut}^S$  by consuming his own endowment. Therefore, to sustain any insurance, the mutual insurance contract must offer this person more than that amount at any point of time. Thus, in addition to constraint (3.3), the ‘social planner’ is facing the additional two constraints reflecting the inability of agents to commit or inability of society to punish with more than sending such an agent to autarky forever,

$$v(y^H + \tau^H, \bar{y}) + \frac{\beta}{1-\beta}V^S \geq v(y^H, \bar{y}) + \frac{\beta}{1-\beta}V_{aut}^S \quad (3.5)$$

$$v(y^L - \frac{\pi}{1-\pi}\tau^H, \bar{y}) + \frac{\beta}{1-\beta}V^S \geq v(y^L, \bar{y}) + \frac{\beta}{1-\beta}V_{aut}^S \quad (3.6)$$

where  $V^S = \pi v(c^H, \bar{c}) + (1 - \pi)v(c^L, \bar{c})$ . Here, it is once again key that we allow current transfers to depend only on current income realizations (no private and public savings). The general case, if public access to a technology to carry resources over time is present (e.g., see Thomas and Worrall, 1990) would typically feature a history-dependent optimal contract.

Equations (3.5) and (3.6) are the limited enforcement constraints for individuals who obtain high income and low income levels, respectively. The planner’s problem



is to maximize (3.2) subject to (3.3), (3.5), and (3.6)<sup>3</sup>.

From the discussion in last section, we know the curvature of the utility function  $v$  plays a critical role in determining the optimal allocation. Similarly, here we shall discuss two cases one by one, when  $v(c, \bar{c})$  is strictly concave in  $c$  at  $\bar{c} = \bar{y}$ , and when  $v(c, \bar{c})$  is strictly convex in  $c$  at  $\bar{c} = \bar{y}$ .

### 3.4.1 When $v(c, \bar{c})$ is strictly concave in $c$ at $\bar{c} = \bar{y}$

First, we have the following result about the limited enforcement constraint for low income individuals.

**Lemma 3.1** *If  $v(c, \bar{c})$  is strictly concave in  $c$  at  $\bar{c} = \bar{y}$ , the limited enforcement constraint for the agents with incomes  $y^L$  (3.6) is not binding.*

The result in the Lemma is very intuitive: the individuals with the higher income has a stronger incentive to walk away from the contract because he has a better outside option. Therefore, if ever there is someone who wants to renege on the contract, it must be a high-income person.

We have two possible cases here, depending on whether the constraint for the  $y^H$ -type is binding or not:

Case 1: The limited enforcement constraint for the  $y^H$ -type (3.5) is not binding.

In this case, we easily obtain that the optimal allocation is identical as in the case of full-information without commitment problems:  $c^H = c^L = \bar{y}$ , or  $\tau^H = (1 - \pi)(y^L - y^H)$  as in Proposition 3.1(b). The status term has no effect on the optimal allocation and the ability of agents to mutually insure each other.

Case 2: The limited enforcement constraint for the  $y^H$ -type (3.5) is binding.

Let  $\tau^C$  be the transfer to individuals with  $y^H$  in the pure consumption model, and let  $\tau^S$  be the transfer to high income individuals in the status model. We have the following result.

**Proposition 3.2** *If  $v(c, \bar{c})$  is strictly concave in  $c$  at  $\bar{c} = \bar{y}$ , then the optimal risk-sharing contract under limited enforcement among agents with status concern has the*

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<sup>3</sup>By the folk theorem, we shall assume the discount factor  $\beta$  is large enough to sustain the prescribed optimal allocation as subgame perfect Nash equilibrium outcome for the remaining analysis in this section.

following properties:

(i) The participation constraint for low-income agents is not binding.

(ii). The participation constraint for high-income people may or may not bind. When it is not binding, i.e., when  $\frac{1}{1-\beta}v(\bar{y}) \geq v(y^H, \bar{y}) + \frac{\beta}{1-\beta}V_{aut}^S$ , the optimal allocation is the same as in Proposition 3.1. The concern for status has no effect on it. When the  $y^H$  participation constraint is binding, i.e., when  $\frac{1}{1-\beta}v(\bar{y}) < v(y^H, \bar{y}) + \frac{\beta}{1-\beta}V_{aut}^S$ , and  $k(\cdot)$  is convex, it is optimal to transfer less resources from the individuals with high income, i.e.,  $\tau^S > \tau^C$ .

The intuition for Proposition 3.2 is as follows. When the status term is convex, it adds more convexity into the utility function. Therefore, it is optimal to make the final allocation more unequal than in the pure consumption model to maximize the social welfare. Roughly speaking, while the coinsurance motive and the concave utility function calls for an equal allocation, the need to provide incentive for limited enforcement calls for an unequal allocation and the additional status term strengthens the force in this direction by convexity. It makes the final allocation more biased toward the direction of unequal allocation.

### 3.4.2 When $v(c, \bar{c})$ is strictly convex in $c$ at $\bar{c} = \bar{y}$

Our analysis above was based on the assumption that  $v(c, \bar{c})$  is strictly concave in  $c$  for  $\bar{c} = \bar{y}$ . We next study what would happen if  $v(c, \bar{c})$  is strictly convex in  $c$ . From the discussion in the previous section, we know that now there are three possible cases, depending on the income distribution parameter  $\pi$ .

Case 1.  $\pi < \frac{1}{2}$

Under full information without commitment problems, it is optimal to transfer all the resources to the individual with high income  $y^H$ , that is,  $\tau^{FB} = \frac{1-\pi}{\pi}y^L$ . It is easy to see this allocation satisfies the enforcement constraint for high-income individuals (3.5), but it may or may not satisfy the enforcement constraint for low income people (3.6). We can see that it does so when  $\frac{\beta}{1-\beta}\pi v(\frac{\bar{y}}{\pi}) + [\frac{\beta}{1-\beta}(1-\pi) + 1]v(0) \geq v(y^L) + \frac{\beta}{1-\beta}V_{aut}^S$ . Now the first-best allocation is also the solution for limited commitment problem. When it does not, i.e., when  $\frac{\beta}{1-\beta}\pi v(\frac{\bar{y}}{\pi}) + [\frac{\beta}{1-\beta}(1-\pi) + 1]v(0) < v(y^L) + \frac{\beta}{1-\beta}V_{aut}^S$ , constraint (3.6) will hold with equality at optimum:  $v(y^L - \frac{\pi}{1-\pi}\tau^H) + \frac{\beta}{1-\beta}V^S = v(y^L) + \frac{\beta}{1-\beta}V_{aut}^S$ , and it uniquely defines the optimal solution  $\tau^H$ .

Case 2.  $\pi > \frac{1}{2}$

It is just the opposite of case 1. Now the first-best allocation  $\tau^H = -y^H$  satisfies the enforcement constraint for low-income individuals (3.6). If it does satisfy the constraint for high income person (3.5), that is, when  $(\frac{\beta}{1-\beta}\pi + 1)v(0) + \frac{\beta}{1-\beta}(1 - \pi)v(\frac{\bar{y}}{1-\pi}) \geq v(y^H) + \frac{\beta}{1-\beta}V_{aut}^S$ , the optimal allocation is the first-best. Otherwise, it is determined by (3.5) with equality  $v(y^H + \tau^H) + \frac{\beta}{1-\beta}V^S = v(y^H) + \frac{\beta}{1-\beta}V_{aut}^S$ .

Case 3.  $\pi = \frac{1}{2}$

The first-best allocation is  $\tau^{FB} = y^L$  or  $\tau^{FB} = -y^H$ . The social planner is indifferent between these two options under full information. However, with limited commitment, there may be a difference. Suppose we check  $\tau^{FB} = y^L$  first following the analysis for Case 1, and then similarly for  $\tau^{FB} = -y^H$ . Here we only need to check whether the following two inequalities hold separately:

$$v(0) + \frac{\beta}{1-\beta}[\frac{1}{2}v(2\bar{y}) + \frac{1}{2}v(0)] \geq v(y^L) + \frac{\beta}{1-\beta}V_{aut}^S \quad (3.7)$$

$$v(0) + \frac{\beta}{1-\beta}[\frac{1}{2}v(0) + \frac{1}{2}v(2\bar{y})] \geq v(y^H) + \frac{\beta}{1-\beta}V_{aut}^S \quad (3.8)$$

Inequality (3.7) is the limited enforcement constraint for  $y^L$  type when all the resources are transferred to  $y^H$  type, and Inequality (3.8) is the limited enforcement constraint for  $y^H$  type when all the resources are transferred to  $y^L$  type. We have the following result:

**Proposition 3.3** *If  $v(c, \bar{c})$  is strictly convex in  $c$  at  $\bar{c} = \bar{y}$  and  $\pi = \frac{1}{2}$ , it is always optimal to allocate all or most of the resources to high income agents.*

From the proof of Proposition 3.3, we can see that the symmetry in the proportion of high and low income agents greatly facilitates the analysis. Since the high income individuals have higher outside option, they must consume at least as much as the low income individuals do to make them willing to stay within the insurance arrangement. The asymmetry in the initial endowment breaks down the balance for these two groups, as discussed in the first best scenario. Notice there is subtle difference in terms of the argument from that in the case of concave utility, in which the driving force for the unequal consumption is also due to higher outside option of the high income agents.

### 3.5 Moral Hazard

In this section we study a contract design problem where there is an incentive problem which may preclude full insurance in the first best due to asymmetric information. Assume that, as in Section 3.2, all agents can credibly enter into enduring binding contracts, and everyone can observe individual incomes. Suppose however, that the probability of obtaining the high income  $y^H$  depends on the agent's effort level,  $z$ . There are two action levels:  $z = 0, 1$ . The action affects the probability distribution in the following way:  $\pi = \pi_1$  if  $z = 1$ ;  $\pi = \pi_0$  if  $z = 0$ , and  $1 > \pi_1 > \pi_0 > 0$ . This implies that output realizations are no longer necessarily i.i.d. across individuals – a shirking individual is more likely to have a low income compared to a hard-working one. We maintain the assumption of i.i.d. realizations, *conditional on the effort level*, however.

The cost function of effort is denoted as  $e(z)$ . For simplicity, suppose it costs an agent  $e(z) = z$  to exert effort  $z = 1$ . The cost of putting zero effort,  $z = 0$  is normalized to 0. People in the village cannot observe the individual's effort level. Thus, the optimal insurance arrangement will make contingent payments according to the individual's income realization, which is observable.

Since we have introduced a new variable  $z$ , it is helpful to rewrite the social planner's problem under full information. In the first best, the planner can observe and enforce the desirable effort level  $z$ . The corresponding maximization problem for the first-best risk-sharing contract is,

$$\begin{aligned} & \max_{\{\tau^H, z=0,1\}} \pi(z)[u(y^H + \tau^H) + k(y^H + \tau^H - \bar{y}(z))] & (3.9) \\ & + [1 - \pi(z)][u(y^L - \frac{\pi(z)}{1 - \pi(z)}\tau^H) + k(y^L - \frac{\pi(z)}{1 - \pi(z)}\tau^H - \bar{y}(z))] - e(z) \end{aligned}$$

Since now the individual effort level affects the probability distribution of income, the average income and hence average consumption will change with the effort level. For a particular individual with certain consumption level, his status may thus change with the effort level. Define  $\bar{y}_i \equiv \bar{y}(z = i) \equiv \pi_i y^H + (1 - \pi_i) y^L$ , for  $i = 1, 2$ .

For comparison purposes, we start with the social planner's problem when people do not care about status:

$$\max_{\{\tau^H, z=0,1\}} \pi(z)u(y^H + \tau^H) + [1 - \pi(z)]u(y^L - \frac{\pi(z)}{1 - \pi(z)}\tau^H) - e(z) \quad (3.10)$$

It is straightforward to verify that the solution for Problem (3.10) is:  $c_{FB}^H = c_{FB}^L = \bar{y}_1$  and  $z_{FB} = 1$  if  $u(\bar{y}_1) - 1 \geq u(\bar{y}_0)$  and  $c_{FB}^H = c_{FB}^L = \bar{y}_0$  and  $z_{FB} = 0$  if  $u(\bar{y}_1) - 1 < u(\bar{y}_0)$ . Similarly, under the assumption that  $v(c, \bar{c})$  is strictly concave in  $c$  for a given  $\bar{c}$ , the solution for Problem (3.9) is:  $c_{FB}^H = c_{FB}^L = \bar{y}_1$  and  $z_{FB} = 1$  if  $v(\bar{y}_1, \bar{y}_1) - 1 \geq v(\bar{y}_0, \bar{y}_0)$ ;  $c_{FB}^H = c_{FB}^L = \bar{y}_0$  and  $z_{FB} = 0$  if  $v(\bar{y}_1, \bar{y}_1) - 1 < v(\bar{y}_0, \bar{y}_0)$ .

Note that the condition  $v(\bar{y}_1, \bar{y}_1) - 1 \geq v(\bar{y}_0, \bar{y}_0)$  is equivalent to the condition  $u(\bar{y}_1) - 1 \geq u(\bar{y}_0)$ , and vice versa. Therefore, the status model yields the same efficient allocation as the pure consumption model under the full information regime. We will assume  $u(\bar{y}_1) - 1 \geq u(\bar{y}_0)$  for the following discussion, i.e., it is socially optimal to implement the high effort level.

Consider now the moral hazard case in which the agent's effort level is no longer observable or contractible to the planner. If the social planner wants to implement  $z = 1$ , he will maximize (3.9) subject to the following incentive constraints:

$$\begin{aligned} & \pi_1 v(y^H + \tau^H, \bar{y}_1) + (1 - \pi_1) v(y^L - \frac{\pi}{1 - \pi} \tau^H, \bar{y}_1) - 1 \\ & \geq \pi_0 v(y^H + \tau^H, \bar{y}_0) + (1 - \pi_0) v(y^L - \frac{\pi}{1 - \pi} \tau^H, \bar{y}_0) \end{aligned} \quad (3.11)$$

The LHS of (3.11) is the individual's expected utility if he puts effort level at 1. The RHS is the individual's expected utility if he puts effort level at 0. The former must be no less than the latter to elicit the high effort level.

**Proposition 3.4** *In the optimal contract for moral hazard, if  $v(c, \bar{c})$  is strictly concave in  $c$  at  $\bar{c} = \bar{y}_1$ , then the social planner would transfer more resources to the individuals with high income in the status model than in the pure consumption model, i.e.,  $\tau^S > \tau^C$  (lower degree of mutual insurance) if  $\pi_0 k(c^H - \bar{y}_0) + (1 - \pi_0) k(c^L - \bar{y}_0) > \pi_1 k(c^H - \bar{y}_1) + (1 - \pi_1) k(c^L - \bar{y}_1)$ , and vice versa.*

The intuition for Proposition 3.4 is as follows. Under the higher effort level, it is more likely to obtain the higher output level hence more utility, but at the same time this increases the average consumption decreasing one's utility holding  $c^i$  constant. When the second effect, i.e., the status effect, dominates the first one, the incentive constraint is more binding than the corresponding ICC constraint in the pure consumption model. Therefore, it is optimal to increase the prize for the higher outcome (i.e., the transfer for the  $y^H$ -type increases). The reverse holds for the opposite case. Now the existence of the concern for status works in the opposite

direction as the coinsurance motive and makes the maintenance of the insurance arrangement more costly. The relevant condition is once again determined by the shape of the utility from status function  $k(\cdot)$ . An intuitive guess would be that  $\tau^S$  is more likely to be greater than  $\tau^C$  (i.e, less insurance in the status model) when the status function  $k(\cdot)$  becomes more convex holding all else constant. This relevance of the shape of the utility from status may look similar as in the case of limited enforcement problem, however, there is an important difference between these two scenarios. Here the agents' action has a direct effect on the average consumption thus change their social status. Therefore the status effect always exists and compete with other forces in the model through the channels other than the shape of the utility function, which is absent in the limited commitment problem (and the full information problem) since the average consumption is fixed over there.

### 3.6 Conclusions

We consider the effects of concern for relative consumption on optimal insurance arrangements in different environments analyzed in the literature. We show that concerns for status expressed as the difference between one's own consumption and average consumption of the reference group changes the optimal risk-sharing contract relative to the case of pure consumption utility. Roughly speaking, how people evaluate the social status, or the shape of the status term, plays an important role and can lead to more or less insurance than in a pure consumption model. Sufficiently convex utility from status can imply optimal allocations characterized with high inequality in consumption which, ignoring the status concern may be interpreted as suggesting the presence of various frictions such as asymmetric information or commitment problems when such are not in fact present. In addition, the concern for status can interact with such frictions in interesting and meaningful ways that may affect the way empirical work on consumption risk-sharing is interpreted.

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## 3.8 Appendix

### 3.8.1 Proof of Proposition 3.1

(a). If  $v(c, \bar{c})$  is strictly convex in  $c$  at  $\bar{c} = \bar{y}$ , it is obvious that we will have corner solution for Problem (3.4). The domain of the choice variable  $\tau^H$  is given by the interval  $[-y^H, \frac{1-\pi}{\pi}y^L]$ . There are two possibilities. If  $\tau^H = -y^H$ , the average social welfare is  $W_{\tau^H=-y^H} = \pi v(0) + (1-\pi)v(\frac{\bar{y}}{1-\pi})$  where we have omitted the second argument of  $v$  to save on notation. If instead  $\tau^H = \frac{1-\pi}{\pi}y^L$ , social welfare equals  $W_{\tau^H=\frac{1-\pi}{\pi}y^L} = \pi v(\frac{\bar{y}}{\pi}) + (1-\pi)v(0)$ . When  $\pi = 1/2$ , it is easy to show that  $W_{\tau^H=-y^H} = W_{\tau^H=\frac{1-\pi}{\pi}y^L}$  so the agents are indifferent between these two allocations when  $\pi = 1/2$ . Denote  $D \equiv W_{\tau^H=-y^H} - W_{\tau^H=\frac{1-\pi}{\pi}y^L}$ . Take derivative of  $D$  w.r.t.  $\pi$  to obtain  $\frac{dD}{d\pi} = 2v(0) - v(\frac{\bar{y}}{\pi}) - v(\frac{\bar{y}}{1-\pi}) + \frac{\bar{y}}{\pi}v'(\frac{\bar{y}}{\pi}) + \frac{\bar{y}}{1-\pi}v'(\frac{\bar{y}}{1-\pi})$ . Given that  $v(\cdot)$  is strictly convex at  $\bar{y}$ , it follows that  $v(0) - v(\frac{\bar{y}}{\pi}) > v'(\frac{\bar{y}}{\pi})(0 - \frac{\bar{y}}{\pi})$ , and  $v(0) - v(\frac{\bar{y}}{1-\pi}) > v'(\frac{\bar{y}}{1-\pi})(0 - \frac{\bar{y}}{1-\pi})$ . This implies  $\frac{dD}{d\pi} > v'(\frac{\bar{y}}{\pi})(0 - \frac{\bar{y}}{\pi}) + v'(\frac{\bar{y}}{1-\pi})(0 - \frac{\bar{y}}{1-\pi}) + \frac{\bar{y}}{\pi}v'(\frac{\bar{y}}{\pi}) + \frac{\bar{y}}{1-\pi}v'(\frac{\bar{y}}{1-\pi}) = 0$ . Therefore,  $D$  is strictly increasing in  $\pi$ . Since  $D = 0$  at  $\pi = 1/2$ , then  $D < 0$  when  $\pi < 1/2$ , and  $D > 0$  when  $\pi > 1/2$  which implies the stated results.

(b). If  $v(c, \bar{c})$  is strictly concave in  $c$  for  $\bar{c} = \bar{y}$ , we have a standard utility maximization problem with an interior solution. Taking first-order conditions we have  $v'(c^H, \bar{y}) = v'(c^L, \bar{y})$  i.e., since  $v'$  is strictly decreasing,  $c^H = c^L$ .

### 3.8.2 Proof of Lemma 3.1

**Proof:** When  $v(c, \bar{c})$  is strictly concave in  $c$  for  $\bar{c} = \bar{y}$ , it is easy to show that  $y^L < c^L \leq c^H < y^H$  as in the pure-consumption case (e.g., Coate and Ravallion, 1993). Then  $v(y^L - \frac{\pi}{1-\pi}\tau^H, \bar{y}) > v(y^L, \bar{y})$  and  $V^S > V_{aut}^S$  since  $v$  is strictly increasing and concave in  $c$ . Therefore, the constraint (3.6)  $v(y^L - \frac{\pi}{1-\pi}\tau^H, \bar{y}) + \frac{\beta}{1-\beta}V^S \geq v(y^L, \bar{y}) + \frac{\beta}{1-\beta}V_{aut}^S$  holds with strict inequality.

### 3.8.3 Proof of Proposition 3.2

(i) See the proof for Lemma 1.

(ii). By plugging  $\tau_{FB}^H = (1-\pi)(y^L - y^H)$  into the limited enforcement constraint for  $y^H$ -type (3.5), it is easy to verify that  $\frac{1}{1-\beta}v(\bar{y}) \geq v(y^H, \bar{y}) + \frac{\beta}{1-\beta}V_{aut}^S$  when the constraint is not binding, and vice versa. The reverse also holds.

We focus on the case where the participation constraint for high-income agents is binding. In the pure consumption model, the binding participation constraint for the  $y^H$ -type is given by

$$u(y^H + \tau^H) - u(y^H) - \frac{\beta}{1 - \beta}(V_{aut}^C - V^C) = 0$$

where  $V_{aut}^C = \pi u(y^H) + (1 - \pi)u(y^L)$  and  $V^C = \pi u(c^H) + (1 - \pi)u(c^L)$ . Then

$$V_{aut}^S = V_{aut}^C + \pi k(y^H - \bar{y}) + (1 - \pi)k(y^L - \bar{y})$$

and

$$V^S = V^C + \pi k(c^H - \bar{y}) + (1 - \pi)k(c^L - \bar{y})$$

The corresponding participation constraint for  $y^H$ -type (3.5) can be rewritten as:

$$\begin{aligned} & u(y^H + \tau^H) - u(y^H) - \frac{\beta}{1 - \beta}(V_{aut}^C - V^C) \\ &= \frac{\beta}{1 - \beta}[\pi k(y^H - \bar{y}) + (1 - \pi)k(y^L - \bar{y}) - \pi k(c^H - \bar{y}) - (1 - \pi)k(c^L - \bar{y})] \\ & \quad - k(c^H - \bar{y}) + k(y^H - \bar{y}) \end{aligned} \quad (3.12)$$

Evaluate equation (3.12) at the optimal solution for the pure consumption model,  $\tau^C$ . The LHS is equal to zero. On the RHS of (3.12), we can see

$$\pi k(y^H - \bar{y}) + (1 - \pi)k(y^L - \bar{y}) - \pi k(c^H - \bar{y}) - (1 - \pi)k(c^L - \bar{y}) \geq 0$$

since  $y^L < c^L < c^H < y^H$  (i.e., the income distribution is the mean preserving spread of the optimal consumption allocation for the pure consumption model) and  $k(\cdot)$  is convex. We also know that  $k(y^H - \bar{y}) > k(c^H - \bar{y})$  since  $k(\cdot)$  is strictly increasing. Therefore, we obtain

$$\begin{aligned} & \frac{\beta}{1 - \beta}[\pi k(y^H - \bar{y}) + (1 - \pi)k(y^L - \bar{y}) - \pi k(c^H - \bar{y}) \\ & \quad - (1 - \pi)k(c^L - \bar{y})] - k(c^H - \bar{y}) + k(y^H - \bar{y}) \\ & \geq -k(c^H - \bar{y}) + k(y^H - \bar{y}) > 0 \end{aligned} \quad (3.13)$$

That is, the RHS of (3.12) evaluated at  $\tau^C$  is strictly positive which is greater than the LHS evaluated at  $\tau^C$  (zero). Hence,  $\tau^S > \tau^C$  if  $k(\cdot)$  is convex. Remember,  $\tau^H$  is negative, so  $\tau^S > \tau^C$  means that there is *less insurance* in the status model compared

to the pure consumption model (the consumption spread,  $c^H - c^L = y^H - y^L + \frac{1}{1-\pi}\tau^H$  is wider).

### 3.8.4 Proof of Proposition 3.3

When Inequalities (3.7) and (3.8) both hold, the social planner is indifferent between the two extreme allocations, as in the first best. It makes sense to say that it is optimal to allocate all the resources to high income agents.

It is readily seen that Inequality (3.7) is more easily satisfied than Inequality (3.8) since  $y^L < y^H$ . In other words, it must be Inequality (3.8) whenever there is one inequality does not hold. Since the first best allocation  $\tau^{FB} = y^L$  is superior to all the other allocations, it is optimal to allocate all the resources to high income agents now.

When neither inequality satisfies, consider a same small amount of deviation from the two extreme first best allocations:  $\tau' = y^L - \epsilon$  and  $\tau'' = -y^H + \epsilon$ , where  $\epsilon > 0$ . Then the constraints corresponding to (3.7) and (3.8) are:

$$v(\epsilon) + \frac{\beta}{1-\beta} \left[ \frac{1}{2}v(2\bar{y} - \epsilon) + \frac{1}{2}v(\epsilon) \right] \geq v(y^L) + \frac{\beta}{1-\beta} V_{aut}^S \quad (3.14)$$

$$v(\epsilon) + \frac{\beta}{1-\beta} \left[ \frac{1}{2}v(\epsilon) + \frac{1}{2}v(2\bar{y} - \epsilon) \right] \geq v(y^H) + \frac{\beta}{1-\beta} V_{aut}^S \quad (3.15)$$

Suppose (3.15) holds for a given  $\epsilon$ , (3.14) must hold, too. But the reverse is not necessarily true. As a result,  $\tau' = y^L - \epsilon$  is at least as good as  $\tau'' = -y^H + \epsilon$ . Therefore, it is optimal to deviate from  $\tau^{FB} = y^L$ , that is, it is optimal to allocate most of the resources to high income agents.

### 3.8.5 Proof of Proposition 3.4

It is well known that (3.11) holds with equality at optimum. For comparison purpose, the counterpart of (3.11) is

$$\pi_1 u(y^H + \tau^H) + (1 - \pi_1) u(y^L - \frac{\pi}{1 - \pi} \tau^H) - 1 = \pi_0 u(y^H + \tau^H) + (1 - \pi_0) u(y^L - \frac{\pi}{1 - \pi} \tau^H)$$

Or it can be written as

$$(\pi_1 - \pi_0)[u(y^H + \tau^H) - u(y^L - \frac{\pi}{1 - \pi}\tau^H)] - 1 = 0$$

This equation defines the optimal transfer in the pure consumption model,  $\tau^C$ .

Similarly, (3.11) can be written as

$$\begin{aligned} & (\pi_1 - \pi_0)[u(y^H + \tau^H) - u(y^L - \frac{\pi}{1 - \pi}\tau^H)] - 1 & (3.16) \\ = & \pi_0 k(c^H - \bar{y}_0) + (1 - \pi_0)k(c^L - \bar{y}_0) - [\pi_1 k(c^H - \bar{y}_1) + (1 - \pi_1)k(c^L - \bar{y}_1)] \end{aligned}$$

When the RHS of (3.16) is positive, to make the equality hold, the LHS must be positive too. It follows that  $\tau^S > \tau^C$ , and vice versa.