A COMPARISON OF LOCATION EFFECT IDENTIFICATION METHODS FOR UNREPLICATED FRACTIONAL FACTORIALS IN THE PRESENCE OF DISPERSION EFFECTS

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Abstract

Unreplicated fractional factorial designs are usually used to identify location effects and dispersion effects in screening experiments. Various methods for identifying active location effects have been proposed during the last three decades. All of these methods depend on the assumption of no dispersion effects. Meanwhile most dispersion-identification methods rely on first identifying the correct location-effect model. The presence of dispersion effects induces correlation among location effect estimates. If location-effect identification methods are sensitive to this correlation, then finding the correct location model may be more difficult in the presence of dispersion effects. The primary aim of this project is to compare the robustness of different location-identification methods - Box and Meyer (1986), Lenth (1989), Berk and Picard (1991),and Loughin and Noble (1997) - under the heteroscedastic model via simulation studies. Confounding of location and dispersion effects has also been investigated here. The first three methods perform fine with respect to error rates and power, but the last one loses control of the individual error rate when moderate-to-large dispersion effects are present.

Key Words: Heteroscedastic model; Lenth (1989); Berk and Picard (1991); Box and Meyer (1986); Loughin and Noble (1997); Correlation; IER; EER; Power; Simulation.

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Chapter 1

Introduction

1.1 Overview

In the 1950s and 1960s, Japanese goods were known for being cheap but low quality, but since the 1970s their quality started to achieve very high levels, especially in industrial products. A number of highly successful quality control and process improvement techniques were invented by the Japanese during this time (see for example the papers from Taguchi 1980). One distinctive feature of these techniques is the use of statistical experimental design. Loss of Japanese markets caught the attention of manufacturers and researchers in Western Europe and the U.S. and spurred them to return to the enormous potential of statistical experimental design for product improvement. See "Juran's Quality Handbook" (Godfrey, 1999) for more details about quality control and process improvement techniques.

In the early phases of the research process, experimental studies based on factorial designs are often used as screening experiments. In screening experiments, it is of interest to test the effects of a large number of factors that may have an impact on the responses. Once the most important factors are identified, our limited resources - time or budget - can be concentrated on those factors in the follow-up research process. If each of k factors included in an experiment has 2 levels, and the 2^k factor-level combinations are run in a completely random order, the design is referred to as a full 2^k factorial design. If k is large, 2^k is huge, so full factorial designs are rarely used in practise with large k. For operational restrictions or economic reasons, only a subset or fraction of full factorial designs may be used. For the same reasons, replications are often not available either. That is why much statistical research has been done on unreplicated fractional factorial designs during the last

30 years.

Historically, screening experiments have focused on identifying “location effects”. These are factors and interactions that influence the mean of a response. Identifying location effects is important when products or manufacturing processes have specific target values that must be achieved. More recently, attention has turned to also identifying “dispersion effects”. These are factors and interactions that influence the variance of a response. Identifying dispersion effects is also important because reduction of the variability in performance of products or manufacturing processes is also crucial to achieve consistently high quality.

One might think that it is impossible to try to find dispersion effects in unreplicated experiments, because analysis of location effects exhausts all the degrees of freedom, and the error terms cannot be estimated in a usual way. Identifying dispersion effects when variances can’t be estimated would seem hopeless. It can be done, however, provided that certain conditions are met by the experiment. These conditions are represented by the following three empirical principles which are commonly used in analysis of unreplicated fractional factorial designs:

1. **Effect Sparsity** (Box and Meyer, 1986): Only a few of the factorial effects are active.

2. **Effect Hierarchy** (Box and Hunter, 1978): Lower order effects are more likely to be important than higher order effects.

3. **Effect Heredity** (Hamada and Wu, 1992): An interaction can be active only if one or both of its parent effects are also active.

Most analysis methods for unreplicated fractional factional designs are highly dependent on these principles, especially the ‘effect sparsity’ and ‘effect hierarchy’. If only a small fraction of lower order effects really have substantial location effects, then when the design is projected onto those active factors, it has replication that can be used to study the variability. However, these untestable assumptions might be violated in some applications.

1.2 Usual Statistical Model for the Analysis of Unreplicated Factorial Designs

For simplicity, the full 2^k factorial is considered, although similar results are equally applicable to a $2^{(k-p)}$ fractional factorial design. For the analysis of 2^k unreplicated factorial designs, let $n = 2^k$ denote the number of experimental runs and let $\mathbf{X}_{n\times n} = [\mathbf{x}_0, \mathbf{x}_1, ..., \mathbf{x}_{n-1}]$ be the design matrix for a 2^k factorial experimental design, where $\mathbf{x}_0 = (1, ..., 1)'$ and $\mathbf{x}_j = (\pm 1, ..., \pm 1)', j = 1, ..., n-1$, are pairwise orthogonal. The usual statistical linear model is

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon} \tag{1.1}$$

where, $\mathbf{Y} = (y_1, y_2, ..., y_n)'$ is the vector of observations, or responses (possibly transformed to fit model assumptions), and $\boldsymbol{\beta}_{n\times 1}$ is a vector of unknown parameters. Finally, $\boldsymbol{\epsilon} = (\varepsilon_1, ..., \varepsilon_n)'$ is the vector of random error terms. The typical assumptions for the errors are:

(a) $\varepsilon_i, i = 1, ..., n$, are independent normal random variables with expectation zero.

(b) ε_i have common variance σ^2.

It is easy to derive that in the orthogonal case under assumptions (a) and (b), the estimate of $\boldsymbol{\beta}$ is

$$\hat{\beta}_i = \mathbf{x}_i'\mathbf{y}/(\mathbf{x}_i'\mathbf{x}_i) = \mathbf{x}_i'\mathbf{y}/n, i = 0, ..., n-1$$

which is the best linear unbiased estimate for β_i, and $\hat{\beta}_i \sim N(\beta_i, \sigma^2/(\mathbf{x}_i'\mathbf{x}_i))$ with $Cov(\beta_i, \beta_j) = 0, \forall i \neq j$. Further details of this derivation are in Wu and Hamada (2000).

This model is typically used in the identification of “active” location effects; that is, finding a subset of factors or interactions that have important effects on the mean response. A variety of methods have been proposed to accomplish this. Daniel (1959) suggested that the absolute values of the $(n-1)$ independent effects be graphed on a half-normal plot. Significance is declared by noticing if some of the points on the plot deviate from a rough straight line roughly. The interpretation of the resulting plot is subjective.

Motivated by Daniel's work, a large number of objective procedures has been proposed; see, e.g., Holms and Berrettoni (1969), Zahn (1975), Seheult and Tukey (1982), Box and Meyer (1986a), Voss (1988), Benski (1989), Lenth (1989), Bissell (1989, 1992), Juan and Pena (1992), Dong (1993), Schneider *et al.* (1993), Venter and Steel (1996), Loughin and Noble (1997), Hamada and Balakrishnan (1998), Voss and Wang (1999), McGrath and Lin (2001, 2002, 2003). As pointed out by Hamada and Balakrishnan (1998), most of the existing methods rely heavily on the assumption of effect sparsity. This empirical principle may be violated in some applications. Also the statistical model assumes the variance of responses is constant across all factor combinations. In many industrial experiments, this may not be an appropriate assumption, either; see, e.g., Box and Meyer (1986b), Fuller and Bisgaard (1996). Furthermore, understanding of dispersion effects may be of interest in its own right. So it is necessary to extend the usual statistical model to a more general setting.

1.3 Heteroscedastic Statistical Model

The same as the usual statistical model, we still assume that the observations are independently normally distributed, but the variance is no longer constant:

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}, \qquad \epsilon \sim \mathcal{N}(0, \Sigma) \tag{1.2}$$

where Σ is a diagonal matrix that may depend on the factors. There are many ways to relate the covariance matrix to the factors. Most of literature has concentrated on two specific families: the additive model and the multiplicative model. Rao (1970), Bergman and Hynen (1997), and Brenneman and Nair (2001) utilize the additive model, $\sigma_i^2 = \gamma_0 + \sum_{j=1}^{n-1} x_{ij}\gamma_j$, where γ_j represent the unknown dispersion effects. Cook and Weisberg (1983), Davidian and Carroll (1987), and McGrath and Lin (2001, 2003) consider a variance model of multiplicative form in their work. In this project, a multiplicative variance model similar to McGrath and Lin's definition is applied. This Σ has entries

$$\sigma_i^2 = \sigma^2 \prod_{j=1}^{n-1} \Delta_j^{x_{ij}/2} \tag{1.3}$$

where σ^2 and $\Delta_j, j = 1, ..., n-1$ are unknown parameters. The measure of the dispersion effect of a variable represented by column d of $\mathbf{X}$ is defined as follows:

$$\Delta_d = \sigma_{d+}^2 / \sigma_{d-}^2 \tag{1.4}$$

where $\sigma_{d+}^2 = Var(\varepsilon_i | x_{id} = +1, \Delta_{j^*} = 1 \text{ for } j^* \neq d)$ and $\sigma_{d-}^2 = Var(\varepsilon_i | x_{id} = -1, \Delta_{j^*} = 1 \text{ for } j^* \neq d)$. A factor d is said to have an active dispersion effect if Δ_d is apparently different from 1.

When the response variables have unequal variances, this causes the estimators of location effects to be correlated. Also, estimates for location and dispersion effects can be confounded. As observed by McGrath and Lin (2001, P.132): "(1) Failing to include a pair of location effects created a spurious dispersion effect, or (2) Failing to account for a dispersion effect created two location effects". More detailed discussion of the confounding will be given in Chapter 2.

Thus, a problem in the analysis for location effects arises: as mentioned previously, most of existing identification methods for location effects rely on the assumption of *no* *dispersion effects*. What has not been adequately discussed is how those methods perform in the presence of dispersion effects.

1.4 Objective and Outline

The primary aim of this project is to investigate the robustness of different methods for location-effects identification under the assumption of existence of dispersion effects in the unreplicated 2^k factorial design. We first examine the relationship between location effects and dispersion effects, and then compare various methods for identifying location effects in the presence of one or more dispersion effects via simulation studies. Chapter 2 provides the discussion on the confounding of location effects and dispersion effects. In Chapter 3, we review some of the proposed methods for identifying location effects and choose four of them to examine further in the simulation study: Box and Meyer (1986), Lenth (1989), Berk and Picard (1991) and Loughin and Noble (1997). Calibration of these methods and the design of the simulation study is described in Chapter 4. In Chapter 5, simulation results are presented and discussed. Chapter 6 concludes the whole project and suggests further work.

Chapter 2

Confounding of Location and Dispersion Effects

As mentioned in Section 1.3, if variance effects are present in a model, correlations are induced among the estimates of location effects represented by $\hat{\beta}_i$. Most of the proposed methods of identifying dispersion effects are based on first identifying significant location effects and residualizing with respect to those factors. So identifying the correct location model is therefore vital for subsequent estimation of dispersion effects. Pan (1999) indicated that even small or moderate location effects that are missed in the location model can seriously impact subsequent identification of dispersion effects. Unfortunately, methods of identification for location effects rely on the assumption of no dispersion effects. It would be useful to study the robustness of existing location identification methods in the presence of dispersion effects. In the next two sections, we will discuss the nature of location effect estimates under heteroscedasticity and give the exact correlation of the location effect estimates for 2^4 factorial designs when one or more dispersion effects are present.

2.1 Induced Heteroscedasticity Under General Statistical Model

Firstly, we give the mathematical definition of location effects and dispersion effects. The location effect of a factor, say A, is half of the difference between the average response in the experiment at the high (+) level of A and the average response value at the low(-) level of A:

$$\text{Location effect}(A) = \frac{\sum_{A+}\{y_i\}}{n/2} - \frac{\sum_{A-}\{y_i\}}{n/2}$$

where $A+$ represents high level of A, $A-$ represents low level of A. It is also called a Contrast Effect in some literature. It is easy to see that the location effect of A is just two times of the OLS estimate $\hat{\beta}_A$. So in the remaining discussion, without loss of generality we will use $\hat{\beta}_i$'s to stand for the magnitude of location effects.

The measure of the dispersion effect of A is the ratio of the variance of response values at the high (+) level of A to the variance of response values at the low ($-$) level of factor A. There is no dispersion effect in factor A if the ratio equals 1. Notation follows Equation (1.4) in Section 1.3:

$$\Delta_A = \frac{\sigma^2_{A+}}{\sigma^2_{A-}}$$

If we use the usual model (1.1) and assume iid errors, the $2^k - 1$ estimated coefficients $\hat{\beta}_i, i = 1, ..., n-1$ are distributed independently as $N(0, \sigma^2/2^k)$. But if we are working with the general model without constant variance, it can be verified that the estimated coefficient $\hat{\boldsymbol{\beta}}$ is distributed as multivariate normal with mean $\mathbf{0}$ and covariance matrix

$$\sigma^2(X'\Sigma X)/2^{2k}$$

The diagonal entries of the covariance matrix of $\hat{\boldsymbol{\beta}}$ are identical, so that $\hat{\beta}_i$s have identical marginal distributions. However, they are no longer independent; they have non-zero correlation.

In order to better illustrate the correlation structure, we consider the specific case of a 2^4 design in next section. The results are easily generalized to 2^{k-p} design.

2.2 Correlation in the 2^4 Design with Dispersion Effects

In the following descriptions, column d of $\mathbf{X}$ refers to the factor which produces a dispersion effect. By this it refers to either a main effect associated with a single factor, say A, or an interaction effect associated with combinations of some factors, say AB or BCD. Note that in an n-run 2^k design, for any column d there are $\frac{n}{2}$ pairs of columns that satisfy $i \circ j = d$ (referred to as 'alias pairs' later), where $i \circ j$ denotes the contrast obtained by elementwise multiplication of the columns of +1s and −1s for columns i and j. The full 16-run factorial design is presented in Table 2.1, which shows the experimental factors and the column allocations for the 16 runs.

Table 2.1: Column Allocation for the 16-Run Two-Level Design Matrix. In the following of the paper, x_i denotes the vector of column $i, i = 1, ..., 16$, and x'_r denotes the vector of row $r, r = 1, ..., 16$.

Run (r)	Column (i)															
	x0	x1	x2	x3	x4	x5	x6	x7	x8	x9	x10	x11	x12	x13	x14	x15
1	1	-1	-1	-1	-1	1	1	1	1	1	1	-1	-1	-1	-1	1
2	1	1	-1	-1	-1	-1	-1	-1	1	1	1	1	1	1	-1	-1
3	1	-1	1	-1	-1	-1	1	1	-1	-1	1	1	1	-1	1	-1
4	1	1	1	-1	-1	1	-1	-1	-1	-1	1	-1	-1	1	1	1
5	1	-1	-1	1	-1	1	-1	1	-1	1	-1	1	-1	1	1	-1
6	1	1	-1	1	-1	-1	1	-1	-1	1	-1	-1	1	-1	1	1
7	1	-1	1	1	-1	-1	-1	1	1	-1	-1	-1	1	1	-1	1
8	1	1	1	1	-1	1	1	-1	1	-1	-1	1	-1	-1	-1	-1
9	1	-1	-1	-1	1	1	1	-1	1	-1	-1	-1	1	1	1	-1
10	1	1	-1	-1	1	-1	-1	1	1	-1	-1	1	-1	-1	1	1
11	1	-1	1	-1	1	-1	1	-1	-1	1	-1	1	-1	1	-1	1
12	1	1	1	-1	1	1	-1	1	-1	1	-1	-1	1	-1	-1	-1
13	1	-1	-1	1	1	1	-1	-1	-1	-1	1	1	1	-1	-1	1
14	1	1	-1	1	1	-1	1	1	-1	-1	1	-1	-1	1	-1	-1
15	1	-1	1	1	1	-1	-1	-1	1	1	1	-1	-1	-1	1	-1
16	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
factor	I	A	B	C	D	AB	AC	AD	BC	BD	CD	ABC	ABD	ACD	BCD	ABCD

First, suppose that factor A (column x_1) produces a dispersion effect Δ according to equation (1.4). The estimated regression coefficients computed by OLS have common variance

$$\mathrm{Var}(\hat{\beta}_i) = \sigma^2(\sqrt{\Delta} + \frac{1}{\sqrt{\Delta}})/2^5 \tag{2.1}$$

It can be shown that $\hat{\beta}_1$ is uncorrelated with all other effects, but we have the following correlation pattern for the remaining effects:

$$\rho(\hat{\beta}_i, \hat{\beta}_j) = \begin{cases} \dfrac{(\sqrt{\Delta} - \frac{1}{\sqrt{\Delta}})}{(\sqrt{\Delta} + \frac{1}{\sqrt{\Delta}})} & \text{if } i \circ j = A \\ 0 & \text{if } i \circ j \neq A \end{cases}$$

For the example of Table 2.1, the pattern is:

$$\rho(\hat{\beta}_2, \hat{\beta}_5) = \rho(\hat{\beta}_3, \hat{\beta}_6) = \rho(\hat{\beta}_4, \hat{\beta}_7) = \ldots = \rho(\hat{\beta}_{14}, \hat{\beta}_{15}) = (\sqrt{\Delta} - \frac{1}{\sqrt{\Delta}})/(\sqrt{\Delta} + \frac{1}{\sqrt{\Delta}}) \quad (2.2)$$

For multiple dispersion effects, things get more complicated. If there are two dispersion effects corresponding to factors A and B, with magnitudes Δ_A and Δ_B respectively, an extra dispersion effect is induced in their interaction column AB. Calculating it using the definition (1.4), we have

$$\Delta_{AB} = \frac{1 + \Delta_A \Delta_B}{\Delta_A + \Delta_B}$$

We may call $\{A, B, AB\}$ a '*dispersion triple*'. As with a single dispersion effect, the estimated $\hat{\boldsymbol{\beta}}$'s have common marginal variance

$$\text{Var}(\hat{\beta}_i) = \sigma^2 (\sqrt{\Delta_A} + \frac{1}{\sqrt{\Delta_A}})(\sqrt{\Delta_B} + \frac{1}{\sqrt{\Delta_B}})/2^{k+2} \quad (2.3)$$

The correlation pattern relates to the dispersion triple $\{A, B, AB\}$:

$$\rho(i, j) = \begin{cases} \dfrac{(\sqrt{\Delta_A} - \frac{1}{\sqrt{\Delta_A}})}{(\sqrt{\Delta_A} + \frac{1}{\sqrt{\Delta_A}})}, & \text{if } i \circ j = A \\ \dfrac{(\sqrt{\Delta_B} - \frac{1}{\sqrt{\Delta_B}})}{(\sqrt{\Delta_B} + \frac{1}{\sqrt{\Delta_B}})}, & \text{if } i \circ j = B \\ \dfrac{(\sqrt{\Delta_A} - \frac{1}{\sqrt{\Delta_A}})(\sqrt{\Delta_B} - \frac{1}{\sqrt{\Delta_B}})}{(\sqrt{\Delta_A} + \frac{1}{\sqrt{\Delta_A}})(\sqrt{\Delta_B} + \frac{1}{\sqrt{\Delta_B}})}, & \text{if } i \circ j = AB \\ 0 & \text{otherwise} \end{cases} \quad (2.4)$$

Thus when dispersion effects exist, the assumption that effect estimates are iid no longer holds. Instead, they have identical marginal distributions and some of them are pairwise correlated. In particular, any location effect estimates, other than $\hat{\beta}_A$, $\hat{\beta}_B$ and $\hat{\beta}_{AB}$, are pairwise correlated with three other location effect estimates. For example, $(\hat{\beta}_C, \hat{\beta}_{AC}, \hat{\beta}_{BC}, \hat{\beta}_{ABC})$ form a '*correlation quadruple*'. $(\hat{\beta}_0, \hat{\beta}_A, \hat{\beta}_B, \hat{\beta}_{AB})$ form a correlation quadruple, although we are not generally concerned with testing the intercept, so practically speaking this correlation group is defined by the dispersion triple.

From Equation (2.2), even if Δ is not huge, the correlation coefficient is pretty close to 1. For example, if $\Delta = 5^2$, then $\rho = 0.98$. So intuitively, if we suspect dispersion effects may be present, correlation is induced among location effect estimates, and therefore those estimates should not be studied independently. The individual power of a location effect test may not be affected by correlation, but the joint power of testing two location effects is. These ideas will be further investigated via simulation study in Chapter 4.

Chapter 3

Overview: Identifying Active Location Effects

Various methods have been proposed in the past 30 years for identifying active location effects in unreplicated fractional factorial designs. A general overview and comparison of most of these methods - Daniel (1959), Zahn (1975), Seheult and Tukey (1982), Box and Meyer (1986), Johnson and Tukey (1987), Benski (1989), Bissell (1989), Lenth (1989), Berk and Picard (1991), Dong (1993), Juan and Pena (1992), Venter and Steel (1996) - was given by Hamada and Balakrishnan (1998) under the usual statistical model (1.1).

Among all these location identification methods, three were chosen for our simulation based on their performance in Hamada and Balakrishnan (1998) and on their theoretical structure. The Lenth method, Berk and Picard method and Box and Meyer method test the individual effects directly. Lenth method standardizes the contrasts by the estimated *pseudo standard error* (PSE). The method from Berk and Picard (1991) approximates an error mean square by pooling a fixed number of the smallest sums of squares of estimated location effects. Box and Meyer method uses individual posterior probabilities based on Bayesian inference. Finally, Loughin and Noble (1997) suggested a non-parametric permutation method. It was not studied in Hamada and Balakrishnan (1998), so it would be worthwhile to compare it with others. In what follows, we will review the four location identification methods in detail. These methods will be calibrated and compared in next three chapters with the help of computer simulations.

3.1 Box and Meyer's Method

Box and Meyer (1986) suggested a Bayesian method based on the empirical principle of effect sparsity that only a few of the factorial effects are active. It is assumed that $\beta_i = 0$ for inactive location effects and $\beta_i \sim N(0, \sigma^2_{active})$ for active location effects, and an effect $\beta_i, (i = 1, \ldots, n-1)$ is active with probability α_{active}. Thus estimated contrasts corresponding to inactive effects have distribution $N(0, \sigma^2/2^k)$ while estimated contrasts corresponding to active effects have distribution $N(0, \kappa^2\sigma^2/2^k)$, where $\kappa^2 = (\sigma^2 + 2^k\sigma^2_{active})/\sigma^2$. Thus $\hat{\beta}_1, \ldots, \hat{\beta}_{n-1}$ are iid from scale-contaminated normal distribution $(1 - \alpha_{active})N(0, \sigma^2/2^k) + \alpha_{active}N(0, \kappa^2\sigma^2/2^k)$. The parameter κ denotes the inflation factor of the standard deviation which is produced by an active effect.

For each effect, the marginal posterior probability of being active is computed by calculating the following integral through numerical integration:

$$p_i = \int_0^\infty p_{i|\varrho} \cdot p(\varrho|\hat{\boldsymbol{\beta}}) d\varrho \tag{3.1}$$

where

$$p(\varrho|\hat{\boldsymbol{\beta}}) \propto \varrho^{-n} \prod_{j=1}^{n-1} [(1 - \alpha_{active}) exp\{\frac{-\hat{\beta}_j^2}{2\varrho^2}\} + \frac{\alpha_{active}}{\kappa} exp\{\frac{-\hat{\beta}_j^2}{2\kappa^2\varrho^2}\}]$$

$$p_{i|\varrho} = \frac{\frac{\alpha_{active}}{\kappa} exp\{\frac{-\hat{\beta}_i^2}{2\varrho^2\kappa^2}\}}{\frac{\alpha_{active}}{\kappa} exp\{\frac{-\hat{\beta}_i^2}{2\varrho^2\kappa^2}\} + (1 - \alpha_{active}) exp\{\frac{-\hat{\beta}_i^2}{2\varrho^2}\}}$$

where $\varrho^2 = \sigma^2/2^k$.

Box and Meyer (1986) recommended that the effects whose marginal posterior probability p_i exceeds 0.5 be declared active. In order to estimate α_{active} and κ, they examined the results of ten published data sets of unreplicated fractional factorial designs as prior information. The estimated values for α_{active} and κ are (0.13-0.27) and (2.7-18) with averages of 0.2 and 10, respectively. They showed that "the conclusions to be drawn from analysis are usually insensitive to moderate changes in α and κ" (P.13), and recommended 0.2 and 10 for α_{active} and κ, respectively.

3.2 Lenth's Method

Lenth (1989) proposed a quick and easy analysis for identifying location effects. It is classified among the best procedures examined in the simulation studies by Hamada and Balakrishnan (1998). Lenth (1989) considered a robust estimator of the contrast standard error τ based on the argument that if all effects are inactive, the normality of the independent random errors implies that $\hat{\beta}_i \sim N(0, \tau^2), i = 1, \ldots, n-1$. The "pseudo standard error" (PSE) is defined as follows:

$$PSE = 1.5 \cdot \text{median}_{\{|\hat{\beta}_j| < 2.5 s_0\}} |\hat{\beta}_i| \tag{3.2}$$

where

$$s_0 = 1.5 \cdot \text{median}_{\{i=1,\ldots,n-1\}} |\hat{\beta}_i|$$

In Lenth's method, the robust standard error estimate is calculated by trimming those effects that are large. Then active effects can be identified as those that are "large" among all standardized effects. The natural approach is to divide each effect by PSE and compare the standardized statistics against critical values from a reference distribution, for which Lenth (1989) recommended $t_{\alpha,d}$ where $d = (n-1)/3$. For example, $t_{0.975;d}$ is suggested to control marginal error (the average type I error rate of the $n-1$ individual contrasts) for $\hat{\beta}_i$ with 95% confidence while $t_{\gamma;d}, \gamma = (1 + 0.95^{1/(n-1)})/2$ is used to control simultaneous marginal error with 95% confidence. Those two critical values are based on comparing the empirical distribution of PSE to chi-squared distributions.

According to the simulation study given by Haaland and O'Connell (1995), however, the differences between simulated critical values and Lenth's approximate values from the t distribution are great enough so that Lenth's critical values are not recommended for practical use. The calibration of critical values will be discussed in Chapter 4.

3.3 Berk and Picard's Method

Berk and Picard (1991) proposed an ANOVA-based method using a trimmed mean square error (TMSE). Similar to Lenth's method, they also considered a robust scale estimator used for significance test. The TMSE is formed by pooling a fixed number h of the smallest contrast sum of squares into a pseudo-error term assuming they correspond to inactive effects. Effects with larger sums of squares are then tested using the ratio of their sums of square (SS) to the TMSE:

$$\frac{SS_{(l)}}{\sum_{i=1}^{h} SS_{(i)}/h} \tag{3.3}$$

where $SS_{(l)}$ is the lth smallest contrast mean square, and h is the fixed number for pooling. Berk and Picard (1991) suggested that 60% of the smallest mean squares be reserved for construction of TMSE. That is to say, in a 2^4 design, $60\% \cdot 15 = 9$ smallest mean squares are pooled to construct the TMSE.

Berk and Picard (1991) obtained critical values based on a numerical study. The critical values given in Table 1 of their paper were computed for samples of sizes $N = 8, 12, 16, 20, 32$. Berk and Picard's method controls individual error rate (IER) exactly at 0.05. This will be further discussed in Chapter 4.

3.4 Loughin and Noble (1997)

Loughin and Noble (1997) introduced a nonparametric permutation test based on Birnbaum's (1961) test statistic. The Loughin and Noble test is a sequential procedure that may test up to $n-2$ effects. A very brief illustration for identifying a single effect is given here.

Let $|\hat{\beta}_{(1)}| \geq |\hat{\beta}_{(2)}| \geq \ldots \geq |\hat{\beta}_{(n-1)}|$ be the ordered absolute estimates (OAE) of location effects. Suppose we want to test the null hypothesis

$$H_{0(1)}: \quad \beta_{(1)} = 0 \tag{3.4}$$

where $\beta_{(1)}$ is the location effect corresponding to $|\hat{\beta}_{(1)}|$.

If $H_{0(1)}$ is true, we believe the remaining effects are inactive, too. So the elements of the response are exchangeable under the null hypothesis. This allows us to get the distribution for the test statistic

$$W_{(1)} = |\hat{\beta}_{(1)}| \tag{3.5}$$

by computing (3.5) for all possible permutations of $\mathbf{y}$, $\mathbf{y}^*$, and setting $w^*_{(1)} = |\hat{\beta}^*_{(1)}|$. The nonparametric cumulative sampling distribution is given by:

$$G(w^*_{(1)}|\mathbf{y}) = P[W^*_{(1)} \leq w^*_{(1)}|\mathbf{y}] \tag{3.6}$$

The observed significance level

$$P_{(1)} = 1 - G(W_{(1)}|\mathbf{y}) \leq \alpha \tag{3.7}$$

becomes the test statistic for $H_{0(1)}$ as described further below. For the remaining $(n-2)$ effects, things get complicated and won't be explained here in detail. See Loughin and Noble (1997) for details.

The algorithm of the Loughin and Noble test can be written as follows:

1. Compute $\hat{\beta} = [\hat{\beta}_1, \hat{\beta}_2, \ldots, \hat{\beta}_{n-1}]'$ from $\mathbf{y}$, and order the location effects $|\hat{\beta}_{(1)}| \geq |\hat{\beta}_{(2)}| \geq \ldots \geq |\hat{\beta}_{(n-1)}|$.

2. At step $s = 1, \ldots, n-2$, let
$$\hat{W}_s = |\hat{\beta}_{(s)}|$$
and obtain
$$\tilde{\mathbf{y}}_s = \mathbf{y} - \hat{\beta}_{(1)}\mathbf{s}_{(1)} - \ldots - \hat{\beta}_{(s)}\mathbf{s}_{(s)}$$

3. Repeat B times (e.g., $B = 5,000$):
   (a) Obtain $\tilde{\mathbf{y}}_s^*$ through a random permutation of $\tilde{\mathbf{y}}_s$.
   (b) Compute $\tilde{\beta}^* = [\hat{\beta}_1^*, \hat{\beta}_2^*, \ldots, \hat{\beta}_{n-1}^*]'$.
   (c) Compute
$$W_s^* = \left(\frac{n-1}{n-s}\right)^{1/2} |\tilde{\beta}_{(1)}^*|$$

4. Compute the p-value of the test as
$$P_{(s)} = 1 - \left[\frac{\# W_S^* \leq \hat{W}_s}{B}\right]^{(n-s)/(n-1)} \tag{3.8}$$

5. Repeat steps 2-4 for as many effects as desired.

The effect with the smallest $|\hat{\beta}_{(J)}|$ for which $P_{(J)} < P_0$, where P_0 is the critical value, is declared active. Also, all the $(n-J)$ larger contrasts are declared active, regardless of their respective values of $P_{(s)}$. The critical values P_0 for different experiment sizes and error rates are given in Table 1 in their paper.

Chapter 4

Simulation Study to Compare the Methods

In this chapter, a simulation study is presented comparing the robustness of the four location effect identification methods - Box and Meyer (1986), Lenth (1989), Berk and Picard (1991) and Loughin and Noble (1997) - in the presence of one or more dispersion effects. The induced correlation and the confounding of location effects and dispersion effects are also examined.

4.1 General Settings and Evaluation Standards for Simulations

The compared methods are denoted as follows:

BM86 Box and Meyer method,

LENTH89 Lenth method,

BP91 Berk and Picard method,

LN97 Loughin and Noble method.

All simulations are done under a 2^4 design, because 16 runs is an appropriate number which is neither too small to investigate both location effects and dispersion effects nor too large to complicate the whole study. Four-factor experiments also seem to be the ones that are most commonly discussed in various literature. The usual letter identifiers (A, B, C, D, AB,..., ABCD) are used to label column effects. For each specified scenario of combination of location and dispersion effects, 1825 simulated data sets of 16 responses were generated with normally distributed errors using function NORMAL in SAS/IML. This number ensures simulation precision such that the standard error of the rejection rate is ≈ 0.005 if an effect is not active and $\alpha = 0.05$. The simulations were programmed in SAS/IML, and run on a 32MB Windows XP computer.

In order to compare different methods, it is necessary to provide some evaluation standards. Error rate and power are the most-used criteria in the literature, but their definitions are not unique when multiple tests are considered. Here are the definitions of evaluation standards for reference in this project:

- **IER** (Hamada and Balakrishnan, 1998). Individual error rate (IER) is the average probability of inactive effects declared active.
- **EER** (Hamada and Balakrishnan, 1998). Experimentwise error rate (EER) is the probability that at least one inactive effect is declared active.
- **Power** (Hamada and Balakrishnan, 1998). Power here is the average probability of active effects declared active.
- **RR I** RR I is defined for each effect as the probability of rejecting its null hypothesis. Note that this corresponds to Type I error rate if the effect is not active.
- Number of effects declared active in each simulated data set.

The evaluation standards have been commonly applied in the literature. For example, EER and IER have been used by Loughin and Noble (1997), Hamada and Balakrishnan (1998), and Ye *et al.* (2001). Power has been used as criterion by Dong (1993), Haaland and O'Connell (1995), Loughin and Noble (1997), Hamada and Balakrishnan (1998), McGrath and Lin (2001, 2002). There are several other evaluation standards proposed according to various needs, but we choose for simplicity of interpretation to use those described above.

4.2 Data Generation with Dispersion Effects

The data were generated by using heteroscedastic model (1.2) under different scenarios. More specifically, the following model was used for data generation:

$$y_i = \beta_0 + \beta_A X_{Ai} + \beta_B X_{Bi} + \beta_C X_{Ci} + \beta_D X_{Di} + \ldots + \beta_{ABCD} X_{ABCDi} + e_i, i = 1, ..., 16 \quad (4.1)$$

$$e_i \sim \mathcal{N}\left(0, \quad \exp\left(\frac{\delta_0}{2} + \frac{\delta_A}{2} X_{Ai} + \frac{\delta_B}{2} X_{Bi} + \ldots + \frac{\delta_{ABCD}}{2} X_{ABCDi}\right)\right)$$

where, for example, X_{Ai} denotes the ith element of vector X_A. δ_0 is assumed to be zero here. The corresponding matrix form would be the same as equation (1.2)

$$\mathbf{Y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}, \qquad \epsilon \sim \mathcal{N}(0, \Sigma)$$

where $\boldsymbol{\epsilon}$ is a vector of independent normal random variables $\boldsymbol{\epsilon}' = (e_1, ..., e_n)$. We can write $\Sigma = \text{Diag}(\exp(\frac{x_r'\boldsymbol{\delta}}{2}))$, where x_r is the rth row in Table 2.1. The magnitude of location effects in the simulation is controlled through $\boldsymbol{\beta}$: if factor i has location effect, then $\beta_i \neq 0$, otherwise $\beta_i = 0$, $i = (A, B, ..., ABCD)$. Similarly, the magnitude of dispersion effect is controlled through $\boldsymbol{\delta}$: if factor j has impact on dispersion, then $\delta_j \neq 0$, and if not, then $\delta_j = 0$, $j = (A, B, ..., ABCD)$.

The following example illustrates how to control the location effects and dispersion effects through the model (4.1).

Example

We suppose, without loss of generality, that factor A has location effect, and suppose that both factor A and factor B have significant impact on dispersion. Model (4.1) can be simplified to

$$y_i = \beta_A X_{Ai} + e_i, \quad e_i \sim \mathcal{N}\left(0, \exp\left(\frac{\delta_0}{2} + \frac{\delta_A}{2} X_{Ai} + \frac{\delta_B}{2} X_{Bi}\right)\right), i = 1, ..., 16$$

It is easy to derive that:

$$\text{E}(y_i) = \beta_A X_{Ai} + \text{E}(e_i) = \beta_A X_{Ai}$$
$$\text{Var}(y_i) = \exp\left(\frac{\delta_0}{2} + \frac{\delta_A}{2} X_{Ai} + \frac{\delta_B}{2} X_{Bi}\right)$$

Given the definitions of measure of the location and dispersion effects in Section 2.1, the location effect of A should be:

$$\text{Location effect(A)} = \frac{8\beta_A}{8} - \frac{-8\beta_A}{8} = 2\beta_A$$

And the dispersion effect of A is:

$$\begin{aligned}\Delta_A &= \frac{\sigma^2_{A+}}{\sigma^2_{A-}} \\ &= \frac{8\exp(\delta_0/2)\exp(\delta_A/2+\delta_B/2) + 8\exp(\delta_0/2)\exp(\delta_A/2-\delta_B/2)}{8\exp(\delta_0/2)\exp(-\delta_A/2+\delta_B/2) + 8\exp(\delta_0/2)\exp(-\delta_A/2-\delta_B/2)} \\ &= \exp(\delta_A)\end{aligned}$$

similarly, the dispersion effect of B is $\Delta_B = \exp(\delta_B)$.

The magnitude of dispersion effects was choosen as $\Delta_j = 1^2, 2^2, 3^2, 5^2, 10^2, 20^2, 50^2$, $j = (A, B, ..., ABCD)$ which covers from no dispersion effect ($\Delta_j = 1$) to extremely huge dispersion effect ($\Delta_j = 50^2$).

Because standard errors for location effect estimates depend on the magnitudes of dispersion effects (see Equation (2.1) and Equation (2.3)), care must be taken to make sure the location effects added into the simulation are comparable among different assignments of $\boldsymbol{\delta}$. Instead of using fixed values as the magnitude of location effects, '*effect powers*' (EPower) were designed at small, medium, and large level to measure the location effects. The concept of EPower was discussed in McGrath and Lin (2003). The magnitude of a location effect is defined so that the probability that it is declared active with σ-known Z test is the desired individual power: small (0.2), medium (0.5) or large (0.9) with $\alpha = 0.05$.

Example (cont.)

To illustrate this, the previous example which contains one location effect (A) and two dispersion effects (A and B) is considered again. Also, the correlation patterns for this example are investigated based on a simulation using 1825 samples.

According to the different levels of EPower and the pre-specified dispersion effects A and B, the magnitude of location effect A can be calculated by

$$\Phi\left(q_{.95} - \frac{\beta_A}{Se}\right) = 1 - \text{EPower}$$

where standard error (Se) of β_A can be estimated from Equation (2.3). For example, if $\Delta_A = 2^2$ and $\Delta_B = 5^2$, $Se = 0.4507$ and the location effects of factor A will be $\beta_A = 0.3620, 0.7413, 1.3189$ corresponding to EPower 0.2, 0.5, 0.9 respectively. Table 4.1 gives the simulation results showing the simulated power of testing factor A with σ-known Z test. There is no surprise to see that the simulated powers are pretty close to the desired effect power.

Table 4.2 gives the correlation coefficient matrix when the EPower of A is 0.9, and $\Delta_A = 3^2$ and $\Delta_B = 5^2$. As indicated in Chapter 2, two factors are correlated only if their interaction is one of the dispersion effects or one of the interactions of dispersion effects. For example, $\hat{\rho}_{A,B} = 0.74$, $\hat{\rho}_{A,AB} = 0.923$, $\hat{\rho}_{B,AB} = 0.799$, and $\hat{\rho}_{A,AC} = -0.009$ in Table 4.2. According to Equation (2.4), the corresponding theoretical correlation coefficients should be 0.738, 0.923, 0.8 and 0. The simulation study supports the discussion of the correlation pattern in Chapter 2 very well .

Table 4.1: Simulation results of the power of declaring factor A active when the magnitude of dispersion effects A and B are specified. The EPowers of A are designed at small (0.2), medium (0.5), large (0.9) level. Simulation=1825

Δ_A	Δ_B	Simulated Power of A[1]	EPower of A[2]
2^2	2^2	0.2038	0.2
	3^2	0.1885	0.2
	5^2	0.1978	0.2
2^2	2^2	0.5058	0.5
	3^2	0.4948	0.5
	5^2	0.4910	0.5
2^2	2^2	0.8871	0.9
	3^2	0.8942	0.9
	5^2	0.8955	0.9

[1] The proportion of simulations that declare factor A active.
[2] The effect power of A.

Table 4.2: Simulated correlation coefficient table when the EPower of A is 0.9, $\Delta_A = 3^2$ and $\Delta_B = 5^2$

Factor	A	B	C	D	AB	AC	AD	BC	BD	CD	ABC	ABD	ACD	BCD	ABCD
A	1.00	0.74	0.00	0.01	0.92	-0.01	0.01	0.00	0.01	0.00	-0.01	0.01	0.00	0.01	0.01
B	0.74	1.00	0.00	0.00	0.80	-0.01	0.00	0.00	0.00	0.00	-0.01	0.00	-0.01	0.00	-0.01
C	0.00	0.00	1.00	0.01	0.00	0.80	0.00	0.92	0.01	0.01	0.74	0.00	0.01	0.01	0.01
D	0.01	0.00	0.01	1.00	0.01	0.01	0.80	0.01	0.93	0.00	0.01	0.74	0.01	0.00	0.00
AB	0.92	0.80	0.00	0.01	1.00	-0.01	0.01	0.00	0.01	0.01	-0.01	0.01	0.00	0.01	0.01
AC	-0.01	-0.01	0.80	0.01	-0.01	1.00	0.00	0.74	0.00	0.03	0.92	0.00	0.02	0.02	0.02
AD	0.01	0.00	0.00	0.80	0.01	0.00	1.00	0.00	0.74	0.00	0.01	0.93	0.00	-0.01	0.00
BC	0.00	0.00	0.92	0.01	0.00	0.74	0.00	1.00	0.00	0.01	0.80	0.00	0.01	0.01	0.01
BD	0.01	0.00	0.01	0.93	0.01	0.00	0.74	0.00	1.00	0.01	0.00	0.80	0.01	0.00	0.00
CD	0.00	0.00	0.01	0.00	0.01	0.03	0.00	0.01	0.01	1.00	0.02	0.00	0.80	0.92	0.74
ABC	-0.01	-0.01	0.74	0.01	-0.01	0.92	0.01	0.80	0.00	0.02	1.00	0.00	0.02	0.02	0.02
ABD	0.01	0.00	0.00	0.74	0.01	0.00	0.93	0.00	0.80	0.00	0.00	1.00	0.01	0.00	0.00
ACD	0.00	-0.01	0.01	0.01	0.00	0.02	0.00	0.01	0.01	0.80	0.02	0.01	1.00	0.74	0.92
BCD	0.01	0.00	0.01	0.00	0.01	0.02	-0.01	0.01	0.00	0.92	0.02	0.00	0.74	1.00	0.80
ABCD	0.01	-0.01	0.01	0.00	0.01	0.02	0.00	0.01	0.00	0.74	0.02	0.00	0.92	0.80	1.00

4.3 Initial Comparison and Calibration

First, we check the performance of each analysis method using the settings recommended by the respective authors. An initial set of 1825 simulations is created under model (1.1) assuming no location or dispersion effects. The results are given in in Table 4.3. Clearly, no two methods have exactly the same performance.

Table 4.3: Off-the-shelf performance of the four methods. The values below the column '0' - column '≥ 7' = proportion of simulations that declare i effects as active under the model with no location effects and no dispersion effects for 2^4 design.

Method	Number of Declared Active Effects								IER	EER
	0	1	2	3	4	5	6	≥ 7		
LENTH89	0.748	0.145	0.059	0.028	0.010	0.005	0.003	0.000	0.029	0.252
BM86	0.737	0.185	0.043	0.015	0.010	0.004	0.005	0.001	0.027	0.263
BP91	0.541	0.264	0.116	0.055	0.018	0.005	0.002	0.000	0.051	0.459
LN97	0.726	0.140	0.037	0.032	0.012	0.008	0.014	0.039	0.049	0.274

The different off-the-shelf performance could be explained partly by the different ways in which those methods were designed. BM86 calculates the marginal posterior probability for each effect. Neither IER nor EER is controlled on purpose. LENTH89 attempts to control IER at 0.05, but the critical values from an approximate t distribution with $(n-1)/3$ degrees of freedom are quite inaccurate. BP91 and LN97 control IER exactly at 0.05 but they can't control EER at the same level at the same time.

So in order to fairly compare the four methods, we have to calibrate them somehow. Similarly to the simulation study of Hamada and Balakrishnan (1998), the IER for each method in the project is to be controlled at 0.05 under the assumption of no location effects and no dispersion effects.

4.3.1 BM86

Box and Meyer (1986) recommend that parameters α_{active} and inflation factor κ are $(\alpha_{acitve}, \kappa) = (0.2, 10)$, and effect i is considered active if the calculated marginal posterior probability $p_i > 0.5$. In this case, IER is about 0.027. To control it at 0.05, the threshold value for p_i was calibrated. The simulation method is similar to the one used to obtain the Lenth method critical values by Ye and Hamada (2000):

1. Generate a set of 15 estimated effects from a normal distribution without location effects and dispersion effects.

2. Calculate and save the marginal posterior probabilities $\{p_1, p_2, \ldots, p_{15}\}$ from the 15 estimated effects.

3. Repeat 500000 times to obtain a set of $N = 500000 \times 15$ posterior probabilities.

4. Approximate the threshold value by the $(\alpha \times N)^{th}$ largest of the N posterior probabilities.

5. Approximate the $(1 - \beta) \times 100\%$ confidence interval by $(P_{(r)}, P_{(s)})$ where:

$$r = N \times \left(1 - \alpha + Z_{\beta/2}\sqrt{\frac{\alpha(1-\alpha)}{N}}\right)$$
$$s = N \times \left(1 - \alpha - Z_{\beta/2}\sqrt{\frac{\alpha(1-\alpha)}{N}}\right) + 1$$

   r is rounded down to the nearest integer and s is rounded up to the nearest integer, and $\alpha = 0.05$.

Part of the simulated results are given in Table 4.4.

Table 4.4: Calibrated IER Threshold Value $\hat{C}_\alpha$ for BM86

$IER = \alpha$	$\hat{C}_\alpha$	95% Confidence Interval	
		Lower	upper
0.01	0.7630	0.7615	0.7645
0.05	0.3187	0.3168	0.3204
0.1	0.1722	0.1718	0.1725

So in this project, the threshold value is 0.3187 ($IER = 0.05$), and the effect i is declared active if the posterior probability $p_i > \hat{C}_\alpha = 0.3187$.

4.3.2 LENTH89

For **LENTH89**, we use the simulated critical value which was presented by Loughin (1998), instead of the original critical value based on t-distribution given by Lenth (1989). In order to control the IER at 0.05 when there are no location effects and no dispersion effects, the critical value should be $\hat{C}_{Lenth} = 2.152$. That is to say, the effect i is declared active in 2^4 design if:

$$\frac{|\hat{\beta}_i|}{PSE} > \hat{C}_{Lenth} = 2.152$$

The PSE can be obtained from Equation 3.2.

4.3.3 BP91

In Berk and Picard (1991), the IER was controlled within simulation error of 0.05, so calibration is unnecessary here. In 2^4 unreplicated design, the critical value $\hat{C}_{BP91} = 18.93$, and the effect l is declared active if:

$$\frac{SS_{(l)}}{\sum_{i=1}^{9} SS_{(i)}/9} > \hat{C}_{BP91} = 18.93$$

4.3.4 LN97

As in Loughin and Noble's (1997) study, we use $B = 5,000$ permutation runs in step 3 of the testing procedure. The critical value $\hat{C}_{LN97} = 0.169$ is selected to control IER at 0.05. The testing procedure examines $P_{(s)}$ (Equation (3.8)) in order from the smallest of the absolute estimated effects to the largest, and declares all effects to be active which are larger than the smallest effect for which $P_{(s)} \leq \hat{C}_{LN97} = 0.169$.

In summary, in order to compare these methods on a fair basis, calibrations have been done as described above, so that the IER could be controlled at 0.05 when there are no location effects and no dispersion effects. Table 4.5 shows that these calibrations seem to have achieved this goal.

Table 4.5: Performance of the four methods after calibration. The values below the column '0' - column '≥ 7' = proportion of simulations that declare i effects as active under the model with no location effects and no dispersion effects for 2^4 design.

Method	Number of Declared Active Effects								IER	EER
	0	1	2	3	4	5	6	≥ 7		
Lenth89	0.586	0.200	0.099	0.067	0.028	0.009	0.008	0.003	0.051	0.414
BM86	0.526	0.292	0.098	0.042	0.019	0.010	0.009	0.004	0.051	0.474
BP91	0.541	0.264	0.116	0.055	0.018	0.005	0.002	0.000	0.051	0.459
LN97	0.726	0.140	0.037	0.032	0.012	0.008	0.014	0.039	0.049	0.274

4.4 Simulation Comparison of the Four Methods

Several simulation studies comparing some of these location-identification methods for unreplicated factorial designs have been published by, e.g., Berk and Picard (1991), Haaland and O'Connell (1995), Hamada and Balakrishnan (1998). However, all those comparisons were done under the assumption of constant variance (no dispersion effects). Besides that, none of them included the permutation method proposed by Loughin and Noble (1997).

In our study, numerous simulations were performed to compare the four tests in the presence of one or more dispersion effects. Before describing the various scenarios studied in the simulation, we first clarify some notation. Let $\mathfrak{L}$ and $\mathfrak{D}$ denote the set of location effects and the set of dispersion effects respectively. For example, suppose factor A has a location effect and factor B has a dispersion effect. Then $\mathfrak{L} = \{A\}$ and $\mathfrak{D} = \{B\}$.

The following scenarios are studied here:

Scenario 1: One dispersion effect, no location effect

We assume, without loss of generality, that factor A is the dispersion effect. Then 1825 sets of 16 responses were generated under the scenario: $\mathfrak{L} = \emptyset, \mathfrak{D} = \{A\}$. The IER, EER, RR I, and numbers of contrasts declared active were recorded when $\Delta_A = \{1, 2^2, 3^2, 5^2, 10^2, 20^2, 50^2\}$.

In this case, the investigation focused on the correlation patterns among fifteen factors and their interactions. Note that calibrated off-the-shelf performance is included in this scenario when $\Delta_A = 1$ (see Table 4.5).

Scenario 2: One dispersion effect, one or more location effects

In this case, we examine the influence of induced correlation among factors on the properties of different tests with the assumption of one dispersion effect and one or more location effects. Since the labels of factors are irrelevant, we assume that factor A has dispersion effect, which is $\mathfrak{D} = \{A\}$. Again, $\Delta_A = \{1, 2^2, 3^2, 5^2, 10^2, 20^2, 50^2\}$

In order to exhaust all important configurations of $\mathfrak{L}$ and $\mathfrak{D}$, the choices of location effects were done as shown in Table 4.6. EPowers were designed at 0.2, 0.5, and 0.9 for each active location effect. When there are two location effects, not only were two location effects of the same sign investigated but also two of the opposite signs.

Table 4.6: The arrangement of $\mathfrak{L}$ and $\mathfrak{D}$ with the assumption of one dispersion effect and one or more location effects.

$\mathfrak{D}$	$\mathfrak{L}$	Explanation
A	A	Location effect is dispersion effect
	C	Location effect is different from dispersion effect
A	B, C	Neither of location effects is dispersion effect
	A, B	One of location effect is dispersion effect
	B, AB	The interaction of location effects is dispersion effect

Scenario 3: Two dispersion effects, no location effect

Without loss of generality, we assume that factor A and factor B have dispersion effects, $\mathfrak{L} = \emptyset, \mathfrak{D} = \{A, B\}$. According to the analysis of Chapter 2, in this case we will examine the induced extra dispersion effect AB, and the correlation patterns among rejected null hypotheses.

Scenario 4: Two dispersion effects, one location effects

Next we study the scenario of two dispersion effects and one or more location effect. Similarly to the second scenario, Table 4.7 shows the arrangement of $\mathfrak{L}$ and $\mathfrak{D}$ here. We'd like to compare the performances of the four testing methods and to investigate the confounding pattern of location effects and dispersion effects.

Table 4.7: The arrangement of $\mathfrak{L}$ and $\mathfrak{D}$ with the assumption of two dispersion effects and one location effect.

$\mathfrak{D}$	$\mathfrak{L}$	Explanation
$\{A, B\}$	A	Location effect is one of the dispersion effects
$\{A, B\}$	C	Location effect isn't associated with dispersion effects
$\{A, B\}$	AB	Location effect is the interaction of the dispersion effects

Chapter 5

Simulation Results and Discussion

In this section, we compare the power curves, IER, EER, and RR I curves of the four methods for different scenarios described previously.

5.1 Simulation Results for the IER and the EER

The IER was controlled at 0.05 when there is no location effect or dispersion effect, while the EERs are greater than 0.25 for the four methods, which may be considered large. However, in industrial research where replication is often not possible, power to detect real effects is often low. It is reasonable to sacrifice some degree of control over EER in exchange for maintaining power.

Scenario 1: $\mathfrak{L} = \emptyset$, $\mathfrak{D} = \{A\}$:

Figure 5.1 displays the IERs and EERs for LENTH89, BP91, BM86 and LN97 under various magnitudes of a single dispersion effect. Note $\Delta_A = 1$ represents the calibrated 'off-the-shelf' performance which was presented in Table 4.5. Three of the methods - LENTH89, BP91 and BM86 - are comparable for various sizes of dispersion effects. IERs of these three methods decrease a little bit as the size of the dispersion effect increases from 1 to 3^2, then increase slightly when dispersion effect goes to extremely huge 50^2. None of these IER's are ever appreciably above .05. In contrast with these, the IER of LN97 is substantially impacted by the dispersion effect, rising to almost 0.129 at $\Delta = 50^2$. In addition, LENTH89, BP91 and BM86 have similar patterns for EER, decreasing as the dispersion effect increases. LN97 shows the opposite behavior.

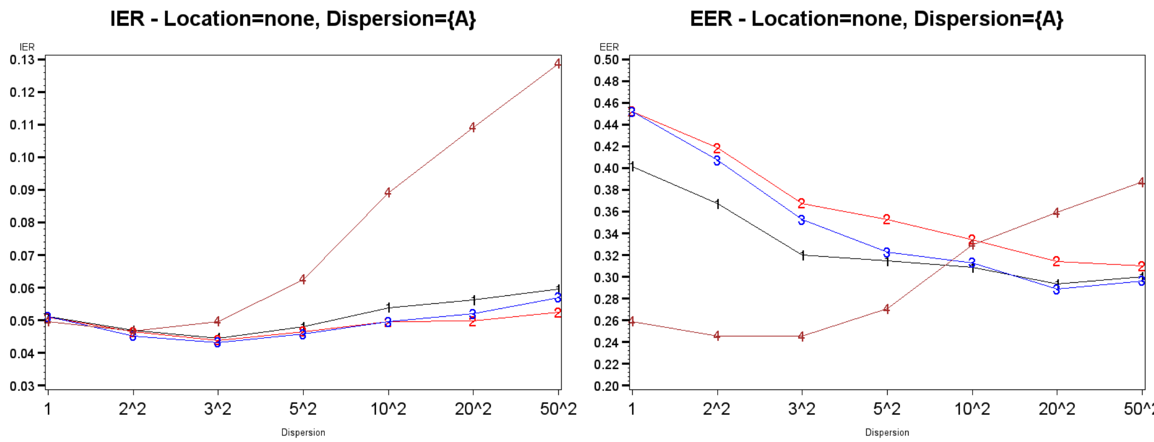

Figure 5.1: IER, EER vs size of dispersion effect for scenario 1 '$\mathfrak{L} = \emptyset$ and $\mathfrak{D} = \{A\}$'. The four methods - Lenth89, BP91, BM86 and LN97 - are labeled as 1,2,3,4 respectively.

Scenario 2: one or two location effects, $\mathfrak{D} = \{A\}$:

First, the IERs of the four methods are displayed in Figure 5.2 assuming one dispersion effect on factor A and one location effect on factor A or factor C.

For small and moderate magnitude of dispersion effects, the IERs of the four tests are around 0.04 and 0.05, and there is no apparent difference between the case of $\mathfrak{L} = \{A\}, \mathfrak{D} = \{A\}$ and the case of $\mathfrak{L} = \{C\}, \mathfrak{D} = \{A\}$. For huge dispersion effects, say 20^2 and 50^2, IERs of the case $\mathfrak{L} = \{C\}, \mathfrak{D} = \{A\}$ are slightly smaller than the case $\mathfrak{L} = \{A\}, \mathfrak{D} = \{A\}$ for LENTH89 and LN97. Again, the IER of LN97 increases dramatically as the dispersion effect increases.

The EERs of the four methods are displayed in Figure 5.3. As with scenario 1, LENTH89, BP91 and BM86 follow similar decreasing patterns in EER which are opposite to the pattern of EER for LN97.

The situation of 'two location effects and one dispersion effect' is described in Table 4.6. In Figure 5.4, there are three different configurations: $\mathfrak{L} = \{B, C\}$, $\mathfrak{L} = \{A, B\}$ and $\mathfrak{L} = \{B, AB\}$. Results for different combinations of EPower for the two location effects are fairly consistent, so we show only one of the combinations of location effects as an example here: Epower of (Location1,Location2)=$(0.5, 0.9)$. Regardless of whether the two location effects are of the same sign or of opposite signs, the presence of a dispersion effect doesn't have substantial influence on the IER for LENTH89, BP91 and BM86. Generally the size of IERs are $0.03 - 0.05$, with some of IERs for BP91

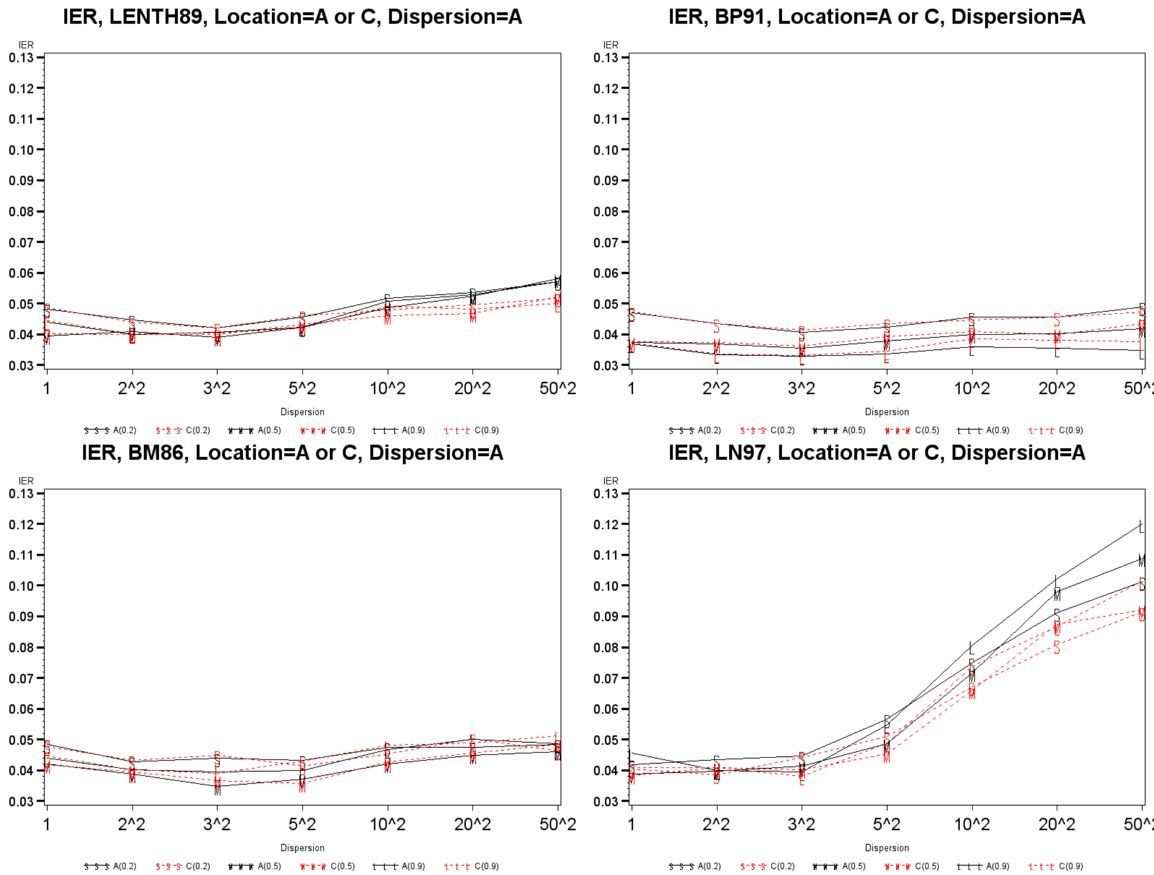

Figure 5.2: IER vs sizes of dispersion effect for scenario 2 ‘$\mathfrak{L} = \{A\}$ or $\{C\}$ and $\mathfrak{D} = \{A\}$’. The labels ‘S’, ‘M’ and ‘L’ denote EPower at 0.2, 0.5 and 0.9 level. Solid lines represent case (1): $\mathfrak{L} = \{A\}$, and dotted lines represent case (2): $\mathfrak{L} = \{C\}$

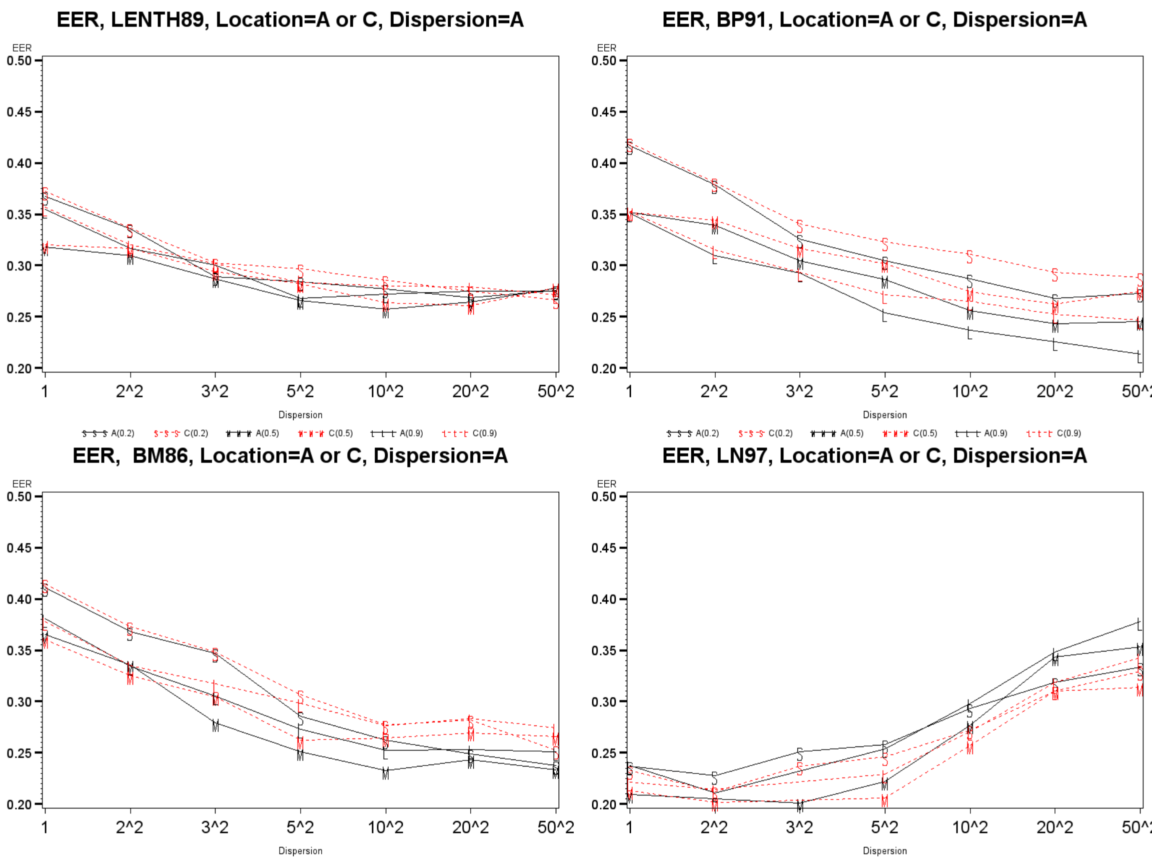

Figure 5.3: EER vs sizes of dispersion effect for scenario 2 '$\mathfrak{L} = \{A\}$ or $\{C\}$ and $\mathfrak{D} = \{A\}$'. The labels 'S', 'M' and 'L' denote EPower at 0.2, 0.5 and 0.9 level. Solid lines represent case (1): $\mathfrak{L} = \{A\}$, and dotted lines represent case (2): $\mathfrak{L} = \{C\}$

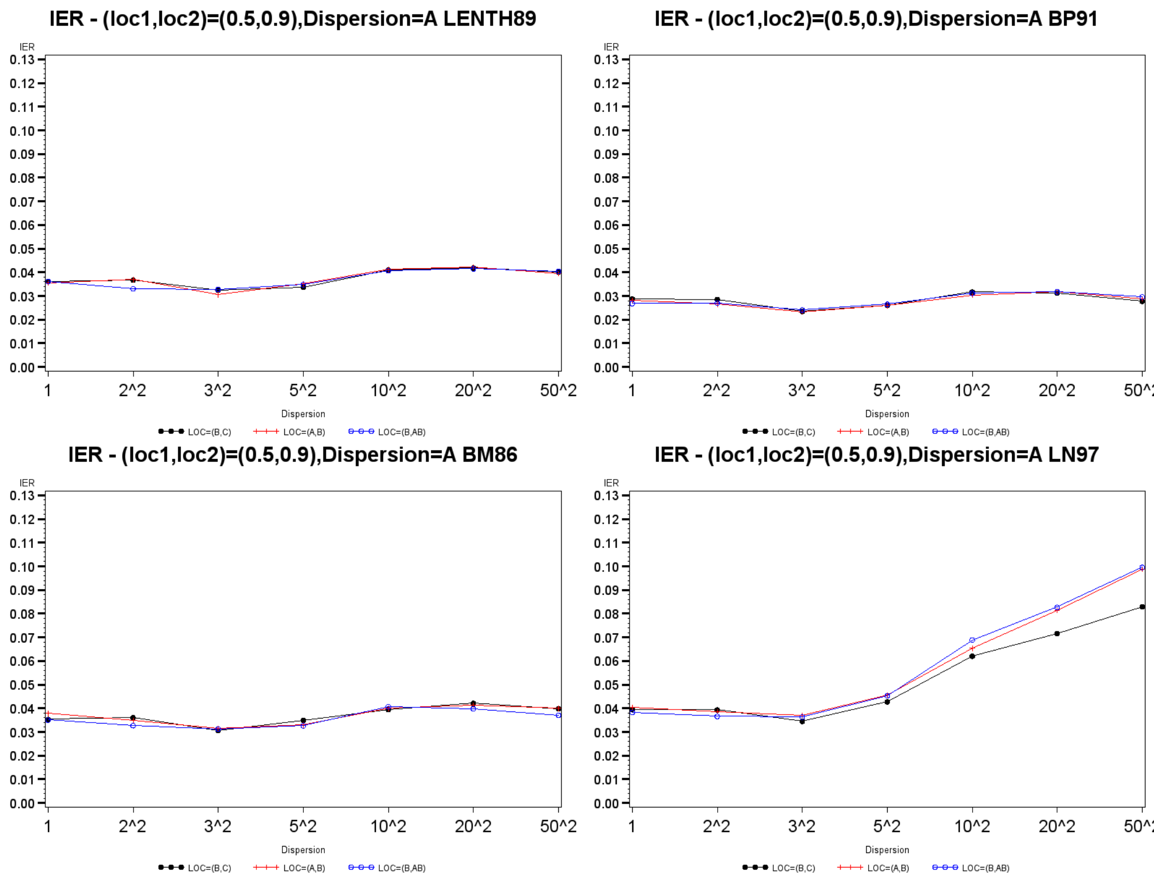

Figure 5.4: IER vs sizes of dispersion effect for the scenario 2 'two location effect and one dispersion effect'. There are three cases: (1) '•' - $\mathfrak{L} = \{B(0.5), C(0.9)\}$, (2) '+' - $\mathfrak{L} = \{A(0.5), B(0.9)\}$, (3) '○' - $\mathfrak{L} = \{B(0.5), AB(0.9)\}$.

are smaller than 0.03. But the IER of LN97 is out of control, increasing to 0.09 - 0.12 as the magnitude of dispersion effects increase. For more details, see tables in Appendix B.

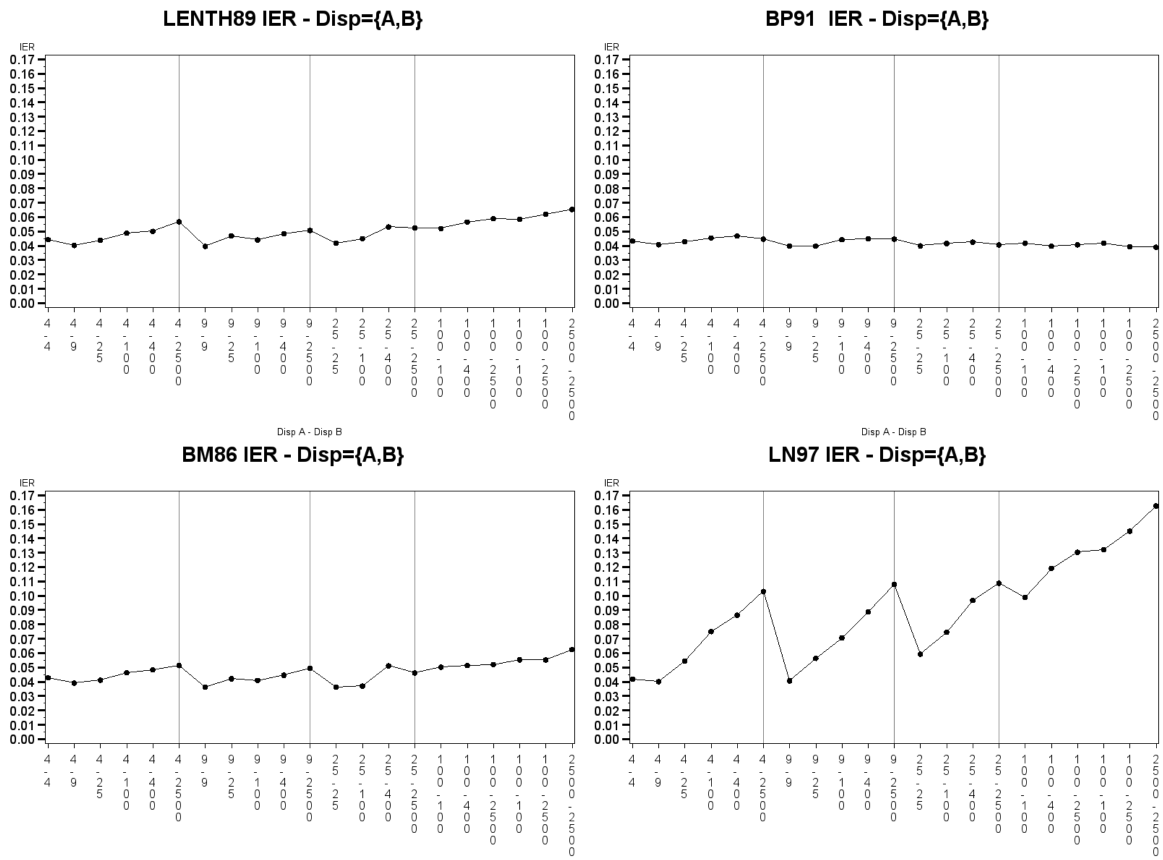

Figure 5.5: IER vs sizes of dispersion effects for the scenario 3 '$\mathfrak{L} = \emptyset$, $\mathfrak{D} = \{A, B\}$'. The horizontal axis gives the combinations of different magnitudes of dispersion effects. Within each segment, the magnitude of the smaller dispersion effect is held fixed while the larger one increases.

Scenario 3: $\mathfrak{D} = \{A, B\}$, $\mathfrak{L} = \emptyset$:

In this scenario, we examine the IERs and EERs when there are no location effects and two dispersion effects. The IER plots under this condition are shown in Figure 5.5. For LENTH89, BM86 and LN97 in each segment, the IER increases when one of the dispersion effects increases. The IER of LN97 increases dramatically, but the increases for both LENTH89 and BM86 are relatively small, controlled between the level of 4% and 6%. The IERs of BP91 appears stable at the level of 4% in the presence of dispersion effects.

Scenario 4: $\mathfrak{L} = \{A\}$ or$\{C\}$ or $\{AB\}$, $\mathfrak{D} = \{A, B\}$

The IERs of scenario 4 are given in Figure 5.6. Similarly to scenario 3, the IERs increase slightly when the dispersion effects increase in each segment of plots of LENTH89 and BM86. Again, BP91 is the most stable one, and LN97 is dramatically impacted by the dispersion effects. For small and moderate dispersion effects, there is no substantial difference between the three cases: $\mathfrak{L} = \{A\}$, $\mathfrak{L} = \{C\}$ and $\mathfrak{L} = \{AB\}$. But when the dispersion effects become huge, for LENTH89, the IERs of case $\mathfrak{L} = \{A\}$ and case $\mathfrak{L} = \{AB\}$ increase faster than that of case $\mathfrak{L} = \{C\}$. It seems that, for LENTH89, the dispersion effects have more impact on the IER if the location effect is one of the dispersion effects or the interaction of the dispersion effects. However, for the other three methods, this pattern is not obvious.

5.2 Simulation Results for the Power

The ability of each location-identification method to detect active location effects is investigated here under the different scenarios. There are no powers for scenario 1 or scenario 3, so we will only discuss scenario 2 and scenario 4.

Scenario 2: one or two location effects, $\mathfrak{D} = \{A\}$:

We first study the power of the four tests in the scenario of one location effect and one dispersion effect. There are two cases: (1) $\mathfrak{L} = \{A\}$, $\mathfrak{D} = \{A\}$; (2) $\mathfrak{L} = \{C\}$, $\mathfrak{D} = \{A\}$. The power curves are displayed in Figure 5.7. For LENTH89, BP91 and BM86, it appears that the existence of a small or moderate magnitude of dispersion effect doesn't have substantial influence on the power of these tests, whether the dispersion effect and the location effect are at the same factor or not. Along with the increase of the dispersion effect, the power of the four methods slightly increases.

Next, we study the power of the four tests in the scenario of two location effects and one dispersion effect. Both the same sign and the opposite signs of the two location effects are considered. It turns out that the simulation results of power are pretty similar regardless of the signs of the location effects. As an example, the simulated results of LENTH89 are given in Table 5.1. For all powers of the four methods, see Appendix B.

However, considering the correlation induced by the dispersion effect, we are also

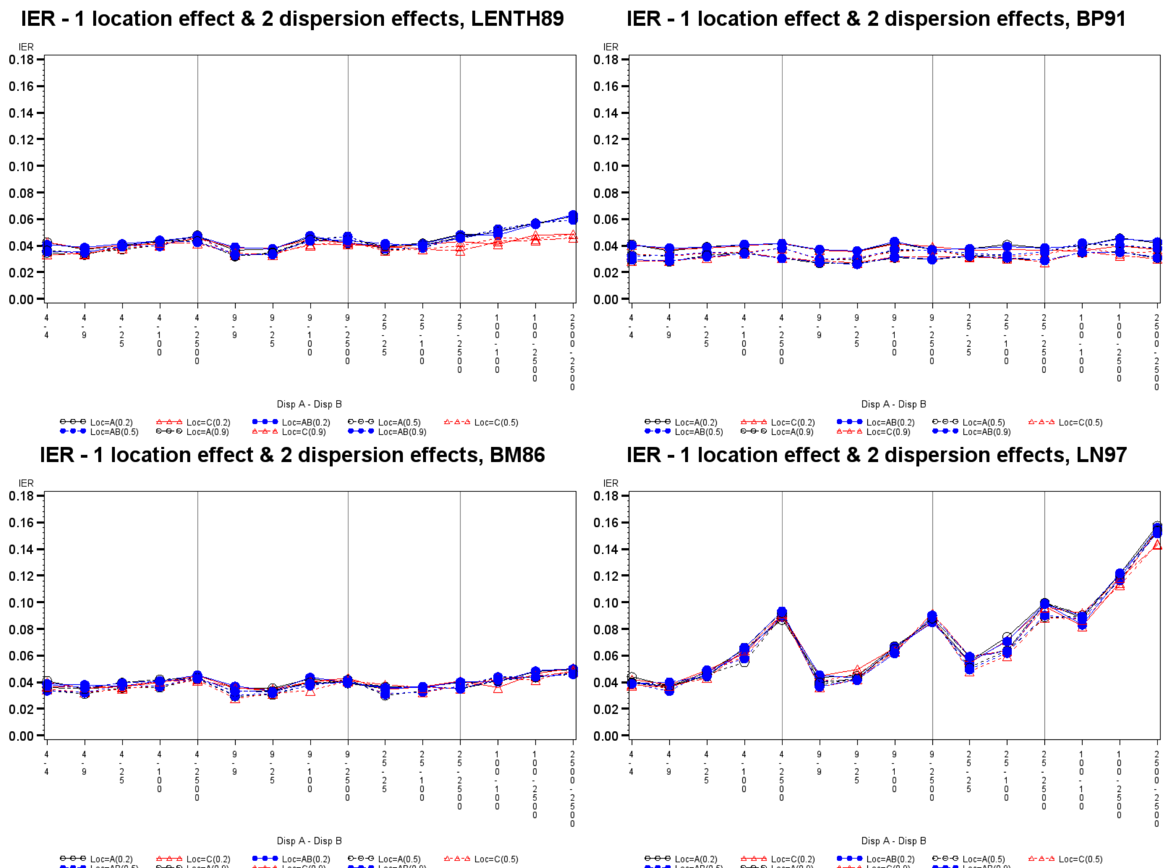

Figure 5.6: IER vs sizes of dispersion effects for the scenario 4 'one location effect and two dispersion effects $\{A, B\}$'. There are three cases for location effect: (1) '$\circ$' - $\mathfrak{L} = \{A\}$, (2) '$\triangle$' - $\mathfrak{L} = \{C\}$, (3) '$\bullet$' - $\mathfrak{L} = \{AB\}$. Each location effect has three levels of EPower: 0.2,0.5, and 0.9 denoted by solid lines, dotted lines and dot-dash lines respectively.

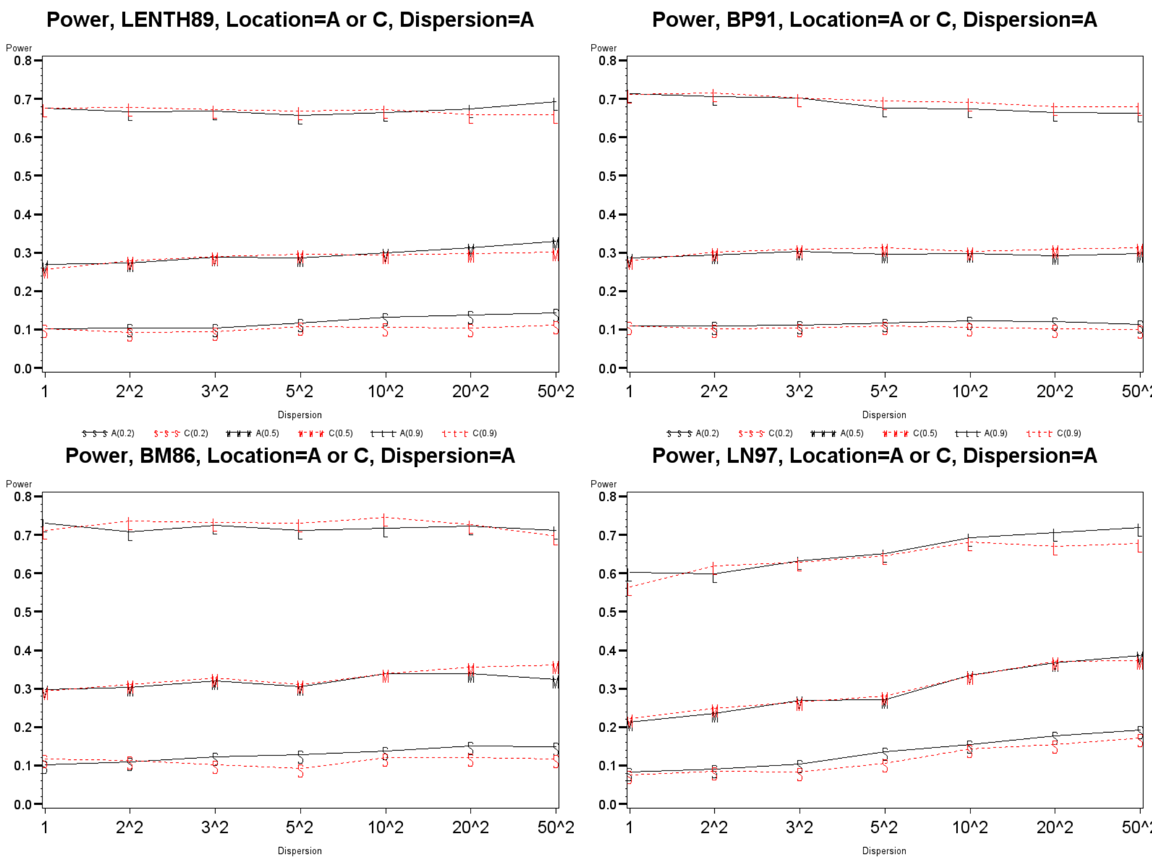

Figure 5.7: Power curves vs sizes of dispersion effect for the scenario 2 '$\mathfrak{L} = \{A\}$ or $\{C\}$ and $\mathfrak{D} = \{A\}$'. The labels 'S', 'M' and 'L' denote EPower at 0.2, 0.5 and 0.9 level. Solid lines represent case (1): $\mathfrak{L} = \{A\}$, and dotted lines represent case (2): $\mathfrak{L} = \{C\}$

Table 5.1: Power of the LENTH89 in the condition of two location effects and one dispersion effects. The EPower of the two location effects are 0.5 and 0.9.

Δ_A	Power of LENTH89[1]					
	$\mathfrak{L}$ (same sign)			$\mathfrak{L}$ (opposite signs)		
	$\{B, C\}$	$\{A, B\}$	$\{B, AB\}$	$\{B, C\}$	$\{A, B\}$	$\{B, AB\}$
1	0.446	0.448	0.444	0.447	0.447	0.442
4	0.452	0.446	0.447	0.454	0.452	0.437
9	0.456	0.451	0.446	0.448	0.453	0.432
25	0.446	0.445	0.433	0.444	0.445	0.425
100	0.445	0.450	0.449	0.442	0.449	0.429
400	0.468	0.462	0.451	0.471	0.445	0.450
2500	0.444	0.418	0.423	0.451	0.436	0.429

[1] EPower of the two location effects are (loc1, loc2)= $(0.5, 0.9)$.

interested in the joint power of testing two location effects. Figure 5.8 shows the joint power of LENTH89 for testing two active location effects at the same time. In case $\mathfrak{L} = \{B(\pm 0.9), C(0.9)\}$ and case $\mathfrak{L} = \{A(\pm 0.9), B(0.9)\}$, the signs of location effects don't have significant impact on the joint power when the two location effects are uncorrelated. But in case $\mathfrak{L} = \{B(\pm 0.9), AB(0.9)\}$, the joint power increases when the two location effects are in the same sign and decreases when they are in opposite signs.

So the presence of dispersion effects may have no influence on the average power of the tests regardless of the signs of the location effects. But when the location effects form a triple with the dispersion effect, for example when $\mathfrak{L} = \{B, AB\}$ and their interaction falls exactly on the dispersion effect A, it does impact the joint power of detecting both active location effects or neither of them, depending on the signs of the location effects.

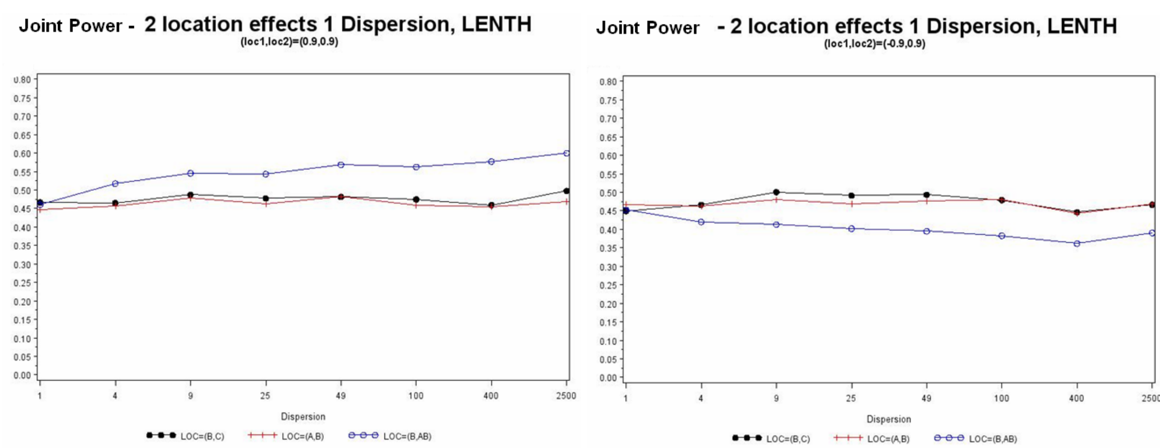

Figure 5.8: Power curves vs sizes of dispersion effect for LENTH89 in the condition of two location effects and one dispersion effect $\mathfrak{D} = \{A\}$. There are three cases: (1) '•' - $\mathfrak{L} = \{B(0.9), C(0.9)\}$, (2) '+' - $\mathfrak{L} = \{A(0.9), B(0.9)\}$, (3) '◦' - $\mathfrak{L} = \{B(0.9), AB(0.9)\}$.

To illustrate this property, Figure 5.9 and Figure 5.10 show observed distributions of $\hat{\beta}_B$ and $\hat{\beta}_{AB}$ in case 3, and $\hat{\beta}_A$ and $\hat{\beta}_B$ in case 2. The 'same signs' case is plotted on the left while 'opposite signs' on the right. Also shown is the 'theoretical bound' for the respective rejection regions, which is based on the true (expected) values of the parameter estimates and their standard errors. Figure 5.9 shows that the strong correlation between location effect estimates changes the joint power depending on the combination of the direction of the correlation and the position of the true values relative to the joint rejection region. Figure 5.10 shows that when there is no correlation, the estimated effects are identically distributed relative to joint rejection regions regardless of the true signs.

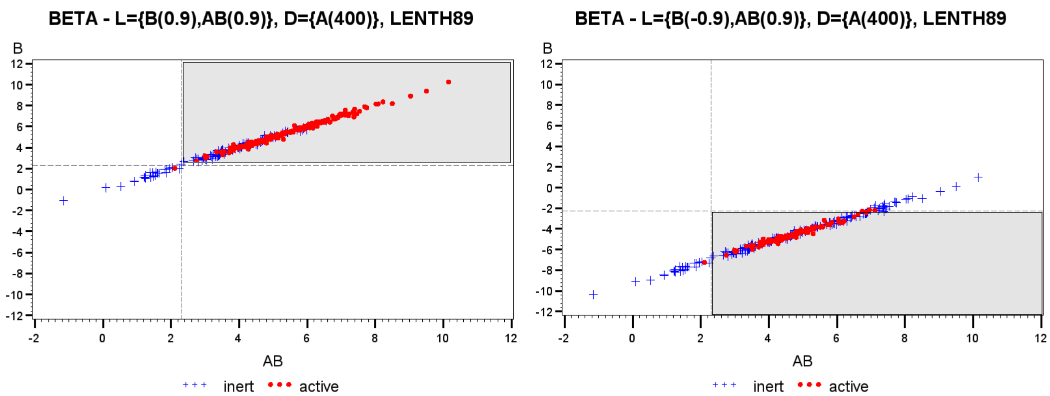

Figure 5.9: $\hat{\beta}_B$ vs $\hat{\beta}_{AB}$ under the condition $\mathfrak{L} = \{B(\pm 0.9), AB(0.9)\}$ and $\mathfrak{D} = \{A(20^2)\}$. The dash vertical line is the theoretical bound to reject $H_0 : \beta_{AB} = 0$. Similarly, the dash horizontal line is the theoretical bound to reject $H_0 : \beta_B = 0$. '•' represents that both B and AB are declared active, '+' otherwise.

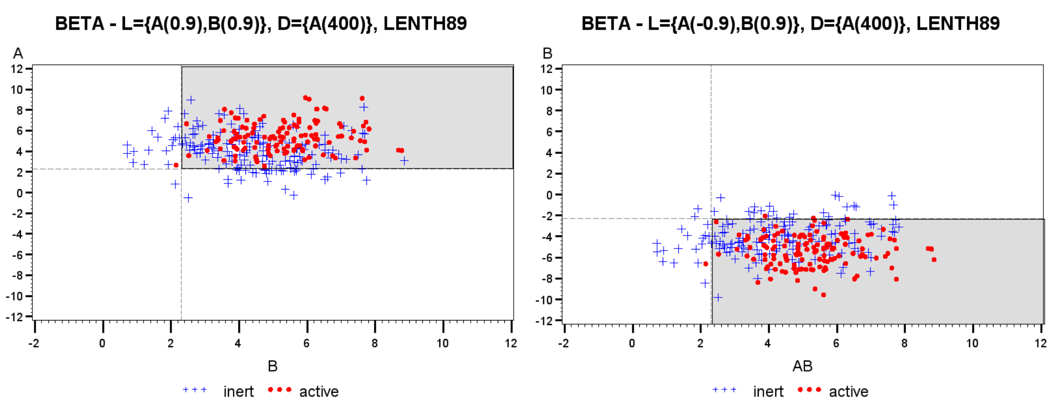

Figure 5.10: $\hat{\beta}_A$ vs $\hat{\beta}_B$ under the condition $\mathfrak{L} = \{A(\pm 0.9), B(0.9)\}$ and $\mathfrak{D} = \{A(20^2)\}$. The dash vertical line is the theoretical bound to reject $H_0 : \beta_B = 0$. Similarly, the dash horizontal line is the theoretical bound to reject $H_0 : \beta_A = 0$. '•' represents that both B and AB are declared active, '+' otherwise.

Scenario 4: $\mathfrak{L} = \{A\}$ or $\{C\}$ or $\{AB\}$, $\mathfrak{D} = \{A, B\}$

Figure 5.11 shows the power for scenario 4. Similarly to scenario 2, for LENTH89, BP91 and BM86, the power is not impacted by small or moderate dispersion effects. For LN97, the power increases drastically when dispersion effects increase. Again, BP91 has the most stable performance here.

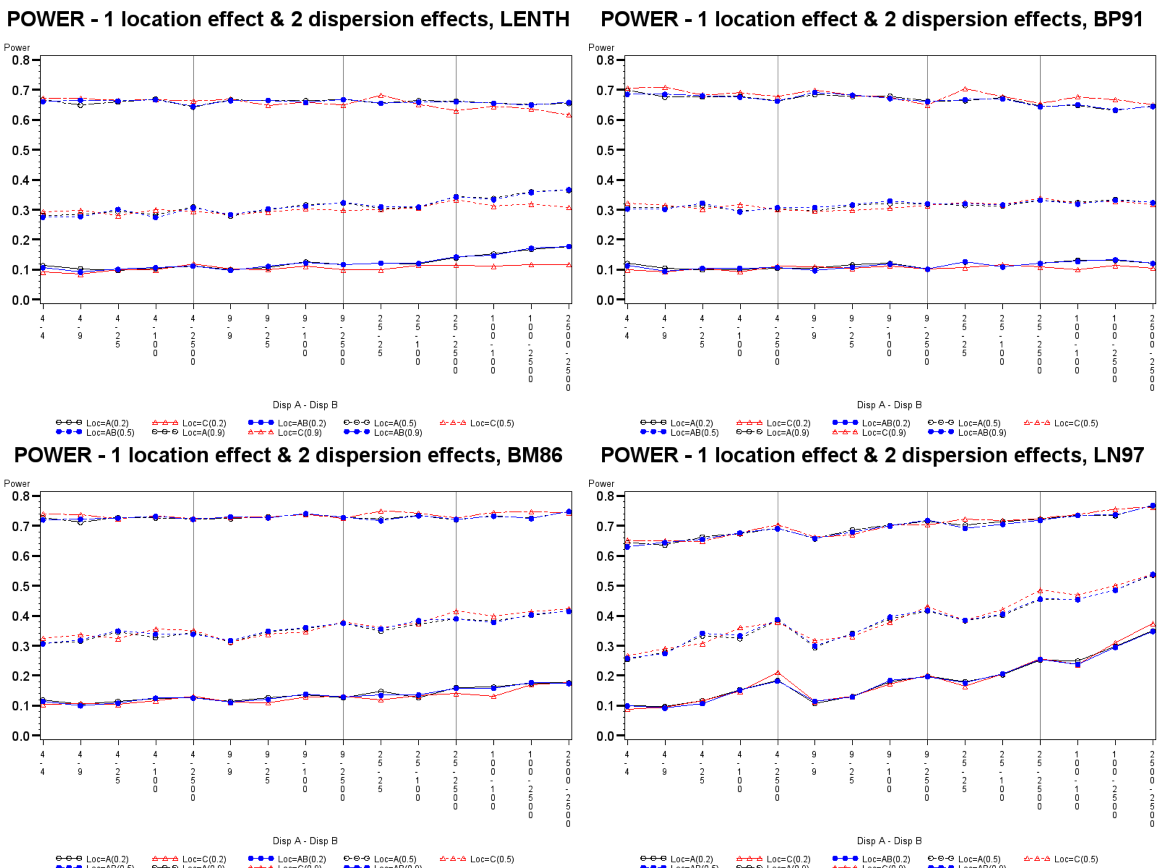

Figure 5.11: Power curves vs sizes of dispersion effects for scenario 4 'one location effect and two dispersion effects'. There are three cases for location effect: (1) '$\circ$' - $\mathfrak{L} = \{A\}$, (2) '$\triangle$' - $\mathfrak{L} = \{C\}$, (3) '$\bullet$' - $\mathfrak{L} = \{AB\}$. Each location effect has three levels of EPower: 0.2,0.5, and 0.9 denoted by solid lines, dotted lines and dot-dash lines respectively.

5.3 Simulation Results for RR I

For scenario 1 and scenario 3, (no location effects and one or two dispersion effects), RR I corresponds to Type I error rate for each effect. Since the simulation results for scenario 1 are similar to scenario 3, we only display the plots for scenario 3 - '$\mathfrak{L} = \emptyset$, and $\mathfrak{D} = \{A, B\}$' in Figure 5.12. For more details of scenario 1, see the figures and tables in Appendix A. For LENTH89, BP91 and BM86, the six lines in each plot squeeze tightly, especially BP91. They fluctuate around 0.05. However, we can easily see bulges at factor A and factor B, which coincide with the active dispersion effects. Thus null location effects corresponding to the same factors as active dispersion effects tend to be detected slightly more often than other null location effects. Further, as indicated in Chapter 2, two active dispersion effects will induce an extra dispersion effect in their interaction. Here the rejection rate for the AB location effect bulges out in each plot, too. When the magnitudes of dispersion effects are small or moderate, see lines labeled '1', '2' and '3', the bulges at factors A, B and AB are not far from 0.05 relative to simulation error. For LN97, the Type I error rate increases substantially as the dispersion effects increase. When the magnitudes of dispersion effects are (5^2, 50^2), the Type I error of factor B is as high as 0.156. But for small or moderate dispersion effects, LN97 performs fine.

For scenario 2 and scenario 4, (one or more location effects and dispersion effects), RR I corresponds to Type I error rate if the effect is inactive, and corresponds to the power if it is active. Since the performances of LENTH89, BP91 and BM86 are similar, we only take BM86 as an example to discuss the simulation results. The IER of LN97 is messed up by the dispersion effects, so we won't give further discussion of power here.

First, Figure 5.13 gives the RR I plots of BM86 for three different configurations of location effects relative to dispersion effects in scenario 2 when there are two location effects; see Table 4.6. The EPowers of the two location effects are $(0.5, 0.9)$. Figure 5.13 shows that, in each case, the presence of a dispersion effect doesn't have substantial impact on the power for testing two active location effects individually. However, the plots of case 1 and case 2 show that the correlation induced by the dispersion effect increases the Type I error rate of the factors that form alias pairs with an active location effect. See, e.g., the bugles at AB from alias pair (B, AB) and AC from alias pair (C, AC).

Next, we consider the RR I of BM86 in the scenario 4 when there is one location effect and two dispersion effects. Things get more complicated. To illustrate, the RR I plot of

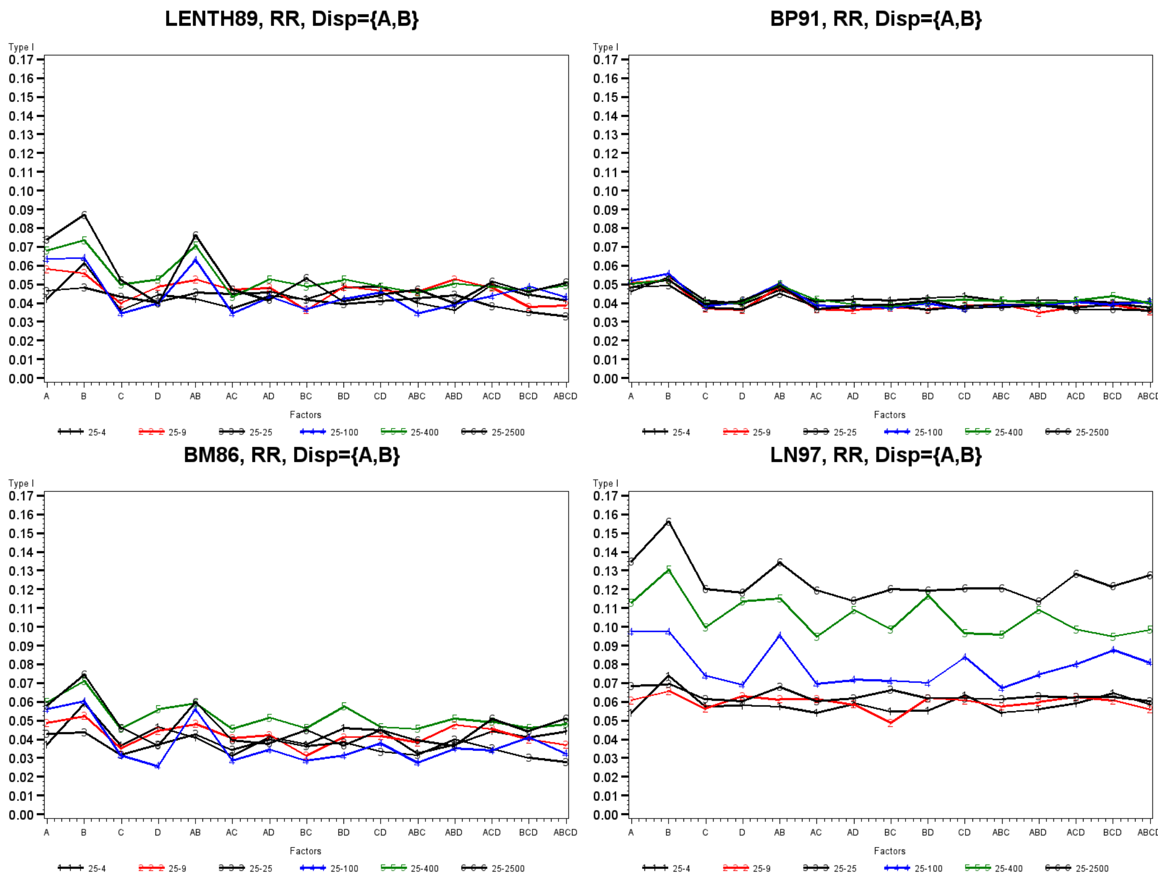

Figure 5.12: RR I vs factors for the scenario 3 'no location effects and two dispersion effects'. The different combinations of magnitudes of dispersion effects are labeled: '1' - (5^2, 2^2), '2' - (5^2, 3^2), '3' - (5^2, 5^2), '4' - (5^2, 10^2), '5' - (5^2, 20^2), and '6' - (5^2, 50^2).

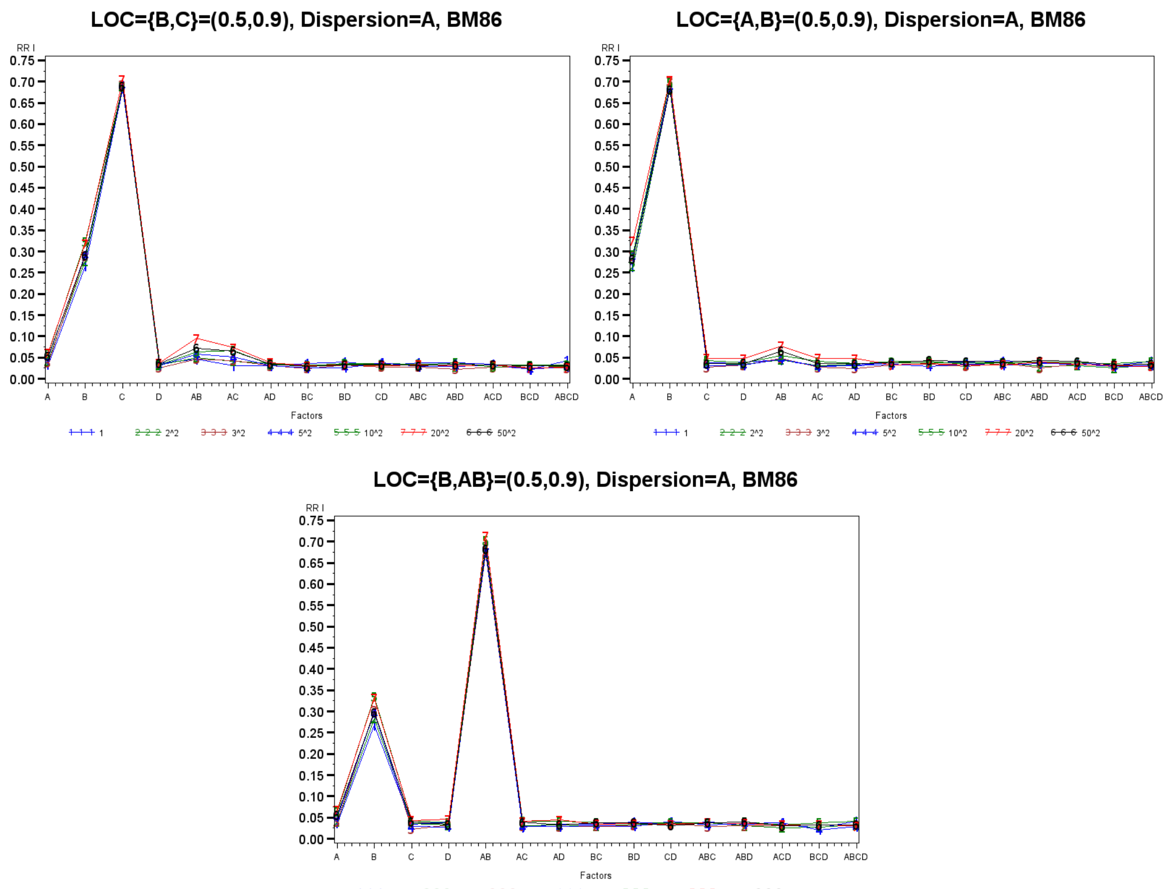

Figure 5.13: RR I vs factors for the scenario 2 'two location effects and one dispersion effects' for BM86. There are seven lines in each plot represented the different magnitudes of the dispersion effects which are labeled as 1,2,...,7.

BM86 under the condition of '$\mathfrak{L} = \{C(0.5)\}, \mathfrak{D} = \{A, B\}$' is given in Figure 5.14. The five lines squeeze tightly, so the magnitude of dispersion effect has little influence on the rejection rate of the test. However, besides the peak at factor C, we can see more bulges at factor A and factor B which are active dispersion effects, and at AC, BC and ABC from the alias pairs (C, AC), (C, BC) and (C, ABC). However the magnitude of change in rejection rate with increasing size of dispersion effect is not very large.

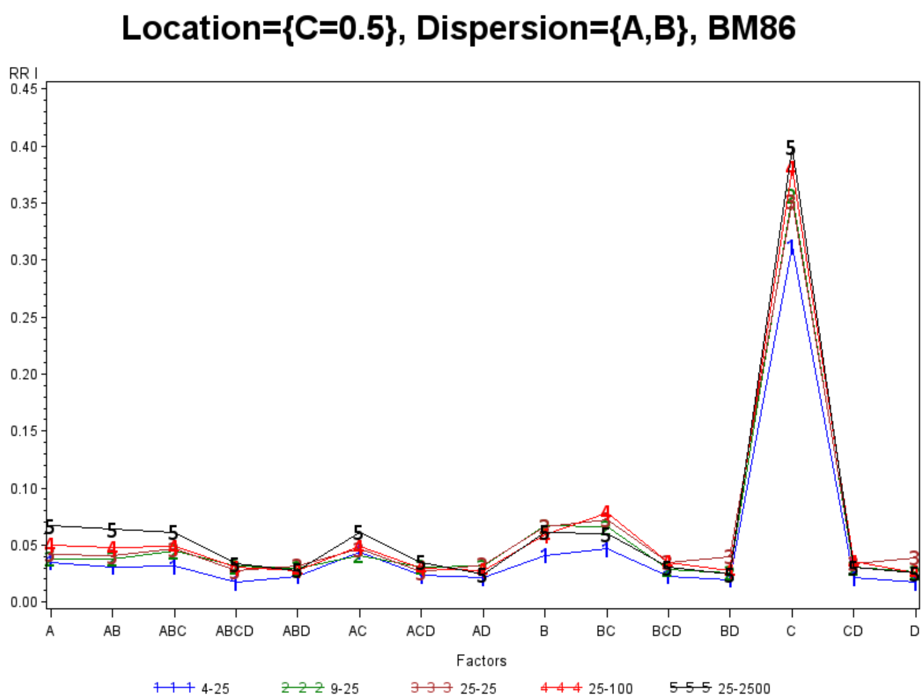

Figure 5.14: RR I vs factors of BM86 under the condition of '$\mathfrak{L} = \{C(0.5)\}, \mathfrak{D} = \{A, B\}$'. The different combinations of magnitudes of dispersion effects are labeled: '1' - (5^2, 2^2), '2' - (5^2, 3^2), '3' - (5^2, 5^2), '4' - (5^2, 10^2), and '5' - (5^2, 50^2).

5.4 Simulation Results for Number of Effects Declared Active

Starting with scenario 4, Table 5.2 gives the proportion of simulations in which values that declare 0 to 7+ effects were declared active under the condition of '$\mathfrak{L} = \{C(0.9)\}, \mathfrak{D} =$

$\{A, B\}$'. Different combinations of magnitudes of dispersion effects are considered including null $(1, 1)$, small $(2^2, 2^2)$, medium $(3^2, 5^2)$ and relatively large $(10^2, 10^2)$. These simulation results provide an insight into the impact of the correlation patterns induced by the dispersion effects.

Table 5.2 shows that, for each method, the proportion of simulations declaring two location effects active together decreases, while the proportion declaring four or five location effects active increases when the dispersion effects increase. Since they are following the same trend, we only take BP91 as an example to do further analysis.

When the magnitudes of the two dispersion effects are $(2^2, 2^2)$, there are 276 tests declaring two active location effects in 1825 simulation sets for BP91. Among them, 261 tests declare factor C as active and the other one is roughly evenly distributed among the remaining 14 factors. For example, there are 19 tests declaring (C,A) active, 21 (C,D), 18 (C,AC), 23 (C, CD), and so on. But along with the increase of dispersion effects, the induced correlations between alias pairs of effects get stronger. For example, when $\Delta_A = 2^2$, and $\Delta_B = 2^2$, then $\rho(AD, BD) = 0.36$. But when $\Delta_A = 10^2, \Delta_B = 10^2$, $\rho(AD, BD)$ increases to 0.961. So if the test declares factor AD active accidentally, it may have a greater chance to declare BD active at the same time.

In this case we have a dispersion triple (A, B, AB). Along with the location effect at C, these effects to be grouped into three situations

$$\begin{array}{lll} \text{case 1 - } C & \text{with DT:} & (C, A, B, AB) \\ \text{case 2 - } C & \text{with } C \times \text{DT:} & (C, AC, BC, ABC) \\ \text{case 3 - } C & \text{with CQ:} & (C, D, AD, BD, ABD) \\ & & (C, CD, ACD, BCD, ABCD) \end{array} \tag{5.1}$$

where DT represents to 'dispersion triple' and CQ represents to 'correlation quadruple', see Chapter 2.2. In case 1, whenever one of (A, B, AB) is declared active along with C, their high correlations for large Δ_A and Δ_B dictate that the other two are likely to be large as well. Similar results hold for the other cases. This is confirmed by the simulation results. Suppose that $(\Delta_A = 10^2, \Delta_B = 10^2)$. For BP91, there are 54 (C, A, B, AB) and 41 (C, AC, BC, ABC) among 111 tests which declare four effects active. The other 16 tests declare effect C active along with three factors from a CQ, like (C, D, AD, BD) or (C, CD, ACD, BCD). And there are 30 (C, D, AD, BD, ABD) and 44 $(C, CD, ACD, BCD, ABCD)$ among 75 tests which declare five effects active. So the proportion of simulations that declare four or five active location effects increases, when the dispersion effects A and B increase.

Table 5.2: Number of declared active effects under the condition of '$\mathfrak{L} = \{C(0.9)\}, \mathfrak{D} = \{A, B\}$'. The values below the column '0' - column '≥ 7' = proportion of simulations that declare i effects as active in 1825 simulation sets.

METHOD	$\mathfrak{D} = \{A, B\}$	Number of Declared Active Effects							
		0	1	2	3	4	5	6	≥ 7
LENTH89	$(1, 1)$	0.271	0.426	0.176	0.070	0.037	0.008	0.008	0.003
	$(2^2, 2^2)$	0.301	0.462	0.126	0.055	0.036	0.008	0.010	0.003
	$(3^2, 5^2)$	0.328	0.459	0.089	0.054	0.038	0.022	0.008	0.003
	$(10^2, 10^2)$	0.336	0.461	0.035	0.045	0.060	0.041	0.014	0.008
BP91	$(1, 1)$	0.236	0.457	0.204	0.074	0.020	0.006	0.002	0.000
	$(2^2, 2^2)$	0.254	0.510	0.151	0.058	0.020	0.007	0.001	0.000
	$(3^2, 5^2)$	0.287	0.524	0.092	0.052	0.030	0.014	0.002	0.000
	$(10^2, 10^2)$	0.303	0.517	0.036	0.040	0.061	0.041	0.003	0.000
BM86	$(1, 1)$	0.215	0.450	0.203	0.076	0.028	0.015	0.009	0.003
	$(2^2, 2^2)$	0.220	0.508	0.162	0.061	0.022	0.013	0.008	0.005
	$(3^2, 5^2)$	0.247	0.518	0.118	0.058	0.025	0.024	0.007	0.003
	$(10^2, 10^2)$	0.241	0.580	0.026	0.031	0.056	0.038	0.012	0.018
LN97	$(1, 1)$	0.397	0.442	0.095	0.033	0.017	0.013	0.012	0.025
	$(2^2, 2^2)$	0.340	0.472	0.085	0.032	0.016	0.012	0.012	0.031
	$(3^2, 5^2)$	0.322	0.488	0.056	0.041	0.025	0.027	0.011	0.031
	$(10^2, 10^2)$	0.248	0.464	0.020	0.019	0.073	0.067	0.012	0.097

Chapter 6

Conclusions

In this project, we have compared the robustness of four location-effect identification methods - LENTH89, BP91, BM86 and LN97 - under the heteroscedastic model in terms of power, IER, EER and RR I. Through simulation results, we have also shown the correlation patterns induced by the presence of dispersion effects and the extra dispersion effect that is created when there is more than one dispersion effects.

To conclude, we summarize briefly the simulation results of the four methods considered:

- LENTH89, BP91 and BM86 control the IER very well for even huge dispersion effects. In particular, they maintain nearly constant IERs as the magnitude of the dispersion effects increases. LN97 performs fine when there is only one small or moderate dispersion effect. But IER for LN97 is out of control when dispersion effects become large.

- For LENTH89, BP91 and BM86, the presence of dispersion effects don't have severe impact on their power which is defined as the average probability of declaring active effects active. However, because of the induced correlations, the joint power of testing, for example, both active location effects together relies on the signs of location effects when their interactions fall on the active dispersion effects. For LN97, the power increases substantially as the magnitude of dispersion effects increases, but this may be only a reflection of its excessive IERs.

- Assuming there are dispersion effects, the factors tend to be declared active as a group, like triple, quadruple, or even quintuple depending on the configuration of location-dispersion effects, see Section 5.4.

Based on this study, some recommendations for choosing location-identification methods in unreplicated 2^{k-p} may be given as follows:

- LENTH89, BP91 and BM86 perform robustly in the presence of one or more dispersion effects, especially BP91.

- However based on the structure of BP91, it is unavailable if there are too many active location effects. For example, in 16-run unreplicated factorials, BP91 as examined here can detect no more than 6 active factors. And the Bayesian method BM86 is time consuming comparing to the others. So LENTH89 would be the reasonable choice if we suspect there may be lots of active effects.

- If we are confident that at least one dispersion effect exists, we should not use LN97 since it is quite sensitive to the dispersion effects.

In the course of this research, there are some limitations and other issues which ask for more study in the future:

- The error terms are generated based on normal distributions. We have not considered comparing the robustness of these methods under nonnormality.

- Although we exhausted all the configurations of one or two location effects and dispersion effects, it may be worth to investigating the properties when the principle of effect sparsity is violated, say with more than 7 active location effects or more dispersion effects.

- Our research focuses only on the 16-run 2^k design. Is there reason to think that something different could occur with smaller or larger designs? We don't know what might happen in other unreplicated designs.

Appendix A

Scenario 1

A.1 IER

Table A.1: IER for scenario 1 - no location effect and one dispersion effect on factor A.

$\mathfrak{D} = \{A\}$	METHODS			
	LENTH89	BP91	BM86	LN97
1	0.051	0.051	0.051	0.050
2^2	0.047	0.046	0.046	0.047
3^2	0.045	0.044	0.047	0.050
5^2	0.048	0.046	0.046	0.063
10^2	0.054	0.050	0.050	0.089
20^2	0.056	0.050	0.054	0.109
50^2	0.060	0.052	0.054	0.129

A.2 RR I

Table A.2: RR I of LENTH89 for scenario 1 - one location effect and one dispersion effect.

EFFECTS	1	2^2	3^2	5^2	10^2	20^2	50^2
A	0.050	0.052	0.060	0.059	0.070	0.076	0.085

B	0.052	0.042	0.040	0.049	0.055	0.055	0.060
C	0.044	0.044	0.044	0.051	0.056	0.050	0.057
D	0.052	0.045	0.046	0.049	0.053	0.054	0.058
AB	0.052	0.048	0.040	0.048	0.053	0.054	0.058
AC	0.050	0.040	0.044	0.051	0.054	0.050	0.058
AD	0.050	0.049	0.048	0.047	0.053	0.055	0.058
BC	0.055	0.049	0.042	0.040	0.045	0.056	0.059
BD	0.049	0.049	0.043	0.049	0.050	0.054	0.052
CD	0.048	0.046	0.042	0.048	0.057	0.057	0.061
ABC	0.058	0.049	0.039	0.041	0.047	0.055	0.060
ABD	0.052	0.046	0.047	0.051	0.049	0.053	0.052
ACD	0.052	0.045	0.040	0.046	0.055	0.056	0.062
BCD	0.051	0.050	0.045	0.044	0.056	0.058	0.059
ABCD	0.053	0.049	0.047	0.045	0.054	0.059	0.060

Table A.3: RR I of BP91 for scenario 1 - on location effect and one dispersion effect.

EFFECTS	1	2^2	3^2	5^2	10^2	20^2	50^2
A	0.049	0.051	0.058	0.059	0.062	0.061	0.057
B	0.049	0.042	0.036	0.047	0.051	0.048	0.051
C	0.048	0.043	0.043	0.046	0.051	0.044	0.051
D	0.045	0.043	0.041	0.051	0.046	0.050	0.052
AB	0.049	0.045	0.039	0.048	0.050	0.047	0.052
AC	0.052	0.042	0.046	0.047	0.050	0.044	0.051
AD	0.048	0.049	0.046	0.049	0.046	0.051	0.052
BC	0.055	0.049	0.041	0.037	0.043	0.048	0.053
BD	0.048	0.049	0.043	0.048	0.048	0.048	0.050
CD	0.051	0.045	0.042	0.048	0.052	0.052	0.054
ABC	0.058	0.053	0.044	0.038	0.044	0.046	0.053
ABD	0.051	0.048	0.044	0.048	0.047	0.047	0.050

ACD	0.052	0.041	0.042	0.043	0.053	0.051	0.056
BCD	0.053	0.051	0.046	0.043	0.052	0.055	0.053
ABCD	0.053	0.047	0.046	0.043	0.049	0.054	0.051

Table A.4: RR I of BM86 for scenario 1 - on location effect and one dispersion effect.

EFFECTS	1	2^2	3^2	5^2	10^2	20^2	50^2
A	0.045	0.046	0.062	0.072	0.068	0.092	0.078
B	0.058	0.035	0.045	0.042	0.048	0.045	0.055
C	0.049	0.048	0.045	0.044	0.051	0.053	0.054
D	0.046	0.044	0.042	0.044	0.049	0.043	0.052
AB	0.049	0.042	0.051	0.042	0.047	0.047	0.052
AC	0.055	0.049	0.041	0.042	0.047	0.054	0.054
AD	0.047	0.046	0.045	0.044	0.052	0.044	0.054
BC	0.055	0.043	0.053	0.050	0.051	0.054	0.049
BD	0.047	0.049	0.048	0.047	0.052	0.052	0.050
CD	0.050	0.050	0.045	0.037	0.044	0.052	0.051
ABC	0.054	0.048	0.049	0.052	0.055	0.059	0.050
ABD	0.048	0.045	0.045	0.050	0.052	0.056	0.053
ACD	0.050	0.056	0.047	0.036	0.044	0.053	0.050
BCD	0.053	0.052	0.042	0.044	0.047	0.056	0.059
ABCD	0.053	0.043	0.048	0.046	0.047	0.058	0.056

Table A.5: RR I of LN97 for scenario 1 - on location effect and one dispersion effect.

EFFECTS	1	2^2	3^2	5^2	10^2	20^2	50^2
A	0.047	0.050	0.057	0.068	0.103	0.125	0.150
B	0.043	0.043	0.043	0.056	0.088	0.105	0.128
C	0.045	0.041	0.046	0.066	0.084	0.101	0.133

D	0.045	0.043	0.047	0.059	0.083	0.107	0.128
AB	0.048	0.042	0.045	0.057	0.088	0.107	0.128
AC	0.049	0.042	0.050	0.061	0.084	0.100	0.133
AD	0.045	0.047	0.048	0.060	0.082	0.106	0.128
BC	0.050	0.046	0.051	0.054	0.077	0.106	0.127
BD	0.042	0.045	0.047	0.061	0.084	0.107	0.121
CD	0.047	0.045	0.045	0.057	0.089	0.102	0.132
ABC	0.053	0.046	0.045	0.054	0.075	0.107	0.126
ABD	0.050	0.044	0.049	0.061	0.085	0.106	0.121
ACD	0.044	0.042	0.044	0.055	0.090	0.102	0.132
BCD	0.051	0.050	0.044	0.059	0.086	0.111	0.122
ABCD	0.050	0.048	0.049	0.058	0.084	0.110	0.122

Appendix B

Scenario 2

Here only attached the cases of two location effects and one dispersion effect.

B.1 IER

Table B.1: IER for LENTH89 assuming two location effects and one dispersion effect

Location	$\mathfrak{D} = \{A\}$						
	1	2^2	3^2	5^2	10^2	20^2	50^2
$(B(0.2), C(0.2))$	0.044	0.042	0.044	0.039	0.045	0.047	0.043
$(B(0.2), C(0.5))$	0.038	0.038	0.036	0.034	0.042	0.046	0.040
$(B(0.2), C(0.9))$	0.040	0.037	0.037	0.040	0.043	0.044	0.045
$(B(0.5), C(0.5))$	0.035	0.035	0.035	0.032	0.036	0.043	0.041
$(B(0.5), C(0.9))$	0.036	0.036	0.031	0.034	0.041	0.041	0.040
$(B(0.9), C(0.9))$	0.033	0.033	0.035	0.036	0.040	0.042	0.042
$(A(0.2), B(0.2))$	0.044	0.040	0.043	0.043	0.045	0.045	0.042
$(A(0.2), B(0.5))$	0.041	0.038	0.035	0.037	0.043	0.045	0.044
$(A(0.2), B(0.9))$	0.039	0.040	0.036	0.041	0.043	0.048	0.050
$(A(0.5), B(0.5))$	0.035	0.037	0.034	0.034	0.035	0.041	0.037
$(A(0.5), B(0.9))$	0.036	0.036	0.031	0.035	0.040	0.043	0.041
$(A(0.9), B(0.9))$	0.033	0.032	0.035	0.036	0.038	0.039	0.039

$(B(0.2), AB(0.2))$	0.044	0.044	0.044	0.044	0.047	0.052	0.049
$(B(0.2), AB(0.5))$	0.040	0.036	0.036	0.036	0.041	0.046	0.042
$(B(0.2), AB(0.9))$	0.041	0.038	0.035	0.041	0.042	0.045	0.048
$(B(0.5), AB(0.5))$	0.035	0.033	0.031	0.036	0.037	0.042	0.043
$(B(0.5), AB(0.9))$	0.036	0.034	0.032	0.035	0.041	0.041	0.041
$(B(0.9), AB(0.9))$	0.032	0.033	0.035	0.037	0.038	0.042	0.040

Table B.2: IER for BP91 assuming two location effects and one dispersion effect

Location	$\mathfrak{D} = \{A\}$						
	1	2^2	3^2	5^2	10^2	20^2	50^2
$(B(0.2), C(0.2))$	0.043	0.040	0.042	0.038	0.040	0.042	0.039
$(B(0.2), C(0.5))$	0.036	0.036	0.034	0.032	0.036	0.039	0.035
$(B(0.2), C(0.9))$	0.032	0.030	0.030	0.032	0.034	0.033	0.035
$(B(0.5), C(0.5))$	0.031	0.031	0.029	0.029	0.031	0.033	0.030
$(B(0.5), C(0.9))$	0.029	0.029	0.024	0.026	0.032	0.031	0.028
$(B(0.9), C(0.9))$	0.022	0.023	0.024	0.024	0.025	0.027	0.025
$(A(0.2), B(0.2))$	0.044	0.041	0.042	0.039	0.041	0.042	0.038
$(A(0.2), B(0.5))$	0.038	0.035	0.032	0.033	0.036	0.037	0.038
$(A(0.2), B(0.9))$	0.033	0.031	0.029	0.033	0.034	0.035	0.037
$(A(0.5), B(0.5))$	0.030	0.030	0.028	0.029	0.031	0.034	0.031
$(A(0.5), B(0.9))$	0.028	0.027	0.023	0.026	0.030	0.032	0.029
$(A(0.9), B(0.9))$	0.022	0.022	0.024	0.023	0.025	0.026	0.024
$(B(0.2), AB(0.2))$	0.043	0.043	0.041	0.041	0.042	0.046	0.042
$(B(0.2), AB(0.5))$	0.036	0.035	0.034	0.032	0.037	0.040	0.035
$(B(0.2), AB(0.9))$	0.032	0.032	0.029	0.033	0.033	0.034	0.036
$(B(0.5), AB(0.5))$	0.030	0.030	0.027	0.031	0.032	0.033	0.034
$(B(0.5), AB(0.9))$	0.027	0.027	0.024	0.027	0.031	0.032	0.030
$(B(0.9), AB(0.9))$	0.022	0.022	0.023	0.023	0.026	0.025	0.024

Table B.3: IER for BM86 assuming two location effects and one dispersion effect

Location	$\mathfrak{D} = \{A\}$						
	1	2^2	3^2	5^2	10^2	20^2	50^2
$(B(0.2), C(0.2))$	0.043	0.041	0.043	0.037	0.038	0.045	0.040
$(B(0.2), C(0.5))$	0.037	0.036	0.034	0.032	0.038	0.040	0.041
$(B(0.2), C(0.9))$	0.041	0.039	0.037	0.040	0.043	0.045	0.045
$(B(0.5), C(0.5))$	0.032	0.034	0.034	0.030	0.036	0.039	0.038
$(B(0.5), C(0.9))$	0.035	0.036	0.031	0.035	0.039	0.042	0.040
$(B(0.9), C(0.9))$	0.035	0.036	0.038	0.037	0.040	0.041	0.042
$(A(0.2), B(0.2))$	0.045	0.040	0.041	0.039	0.041	0.043	0.040
$(A(0.2), B(0.5))$	0.040	0.037	0.035	0.034	0.037	0.040	0.044
$(A(0.2), B(0.9))$	0.040	0.040	0.035	0.039	0.044	0.046	0.046
$(A(0.5), B(0.5))$	0.032	0.033	0.032	0.032	0.034	0.038	0.037
$(A(0.5), B(0.9))$	0.038	0.035	0.032	0.033	0.040	0.041	0.040
$(A(0.9), B(0.9))$	0.034	0.033	0.037	0.036	0.037	0.039	0.040
$(B(0.2), AB(0.2))$	0.045	0.042	0.041	0.041	0.042	0.050	0.047
$(B(0.2), AB(0.5))$	0.038	0.036	0.034	0.031	0.036	0.041	0.041
$(B(0.2), AB(0.9))$	0.041	0.037	0.035	0.040	0.041	0.045	0.045
$(B(0.5), AB(0.5))$	0.034	0.032	0.030	0.035	0.036	0.039	0.041
$(B(0.5), AB(0.9))$	0.035	0.033	0.031	0.033	0.041	0.040	0.037
$(B(0.9), AB(0.9))$	0.034	0.035	0.038	0.036	0.038	0.040	0.042

Table B.4: IER for LN97 assuming two location effects and one dispersion effect

Location	$\mathfrak{D} = \{A\}$						
	1	2^2	3^2	5^2	10^2	20^2	50^2

$(B(0.2), C(0.2))$	0.038	0.037	0.041	0.043	0.059	0.070	0.078
$(B(0.2), C(0.5))$	0.036	0.040	0.039	0.041	0.057	0.074	0.077
$(B(0.2), C(0.9))$	0.040	0.038	0.036	0.048	0.062	0.077	0.084
$(B(0.5), C(0.5))$	0.035	0.036	0.038	0.039	0.056	0.075	0.080
$(B(0.5), C(0.9))$	0.040	0.039	0.035	0.043	0.062	0.072	0.083
$(B(0.9), C(0.9))$	0.035	0.037	0.047	0.048	0.057	0.077	0.084
$(A(0.2), B(0.2))$	0.041	0.040	0.041	0.050	0.063	0.083	0.087
$(A(0.2), B(0.5))$	0.040	0.040	0.038	0.045	0.059	0.086	0.096
$(A(0.2), B(0.9))$	0.041	0.039	0.039	0.050	0.073	0.088	0.103
$(A(0.5), B(0.5))$	0.034	0.038	0.037	0.041	0.063	0.080	0.094
$(A(0.5), B(0.9))$	0.040	0.039	0.037	0.046	0.065	0.081	0.099
$(A(0.9), B(0.9))$	0.036	0.035	0.044	0.050	0.062	0.084	0.095
$(B(0.2), AB(0.2))$	0.040	0.041	0.040	0.052	0.071	0.092	0.106
$(B(0.2), AB(0.5))$	0.036	0.040	0.038	0.043	0.064	0.084	0.088
$(B(0.2), AB(0.9))$	0.043	0.041	0.037	0.051	0.066	0.089	0.102
$(B(0.5), AB(0.5))$	0.036	0.039	0.039	0.046	0.064	0.087	0.101
$(B(0.5), AB(0.9))$	0.038	0.037	0.036	0.045	0.069	0.083	0.100
$(B(0.9), AB(0.9))$	0.034	0.037	0.045	0.050	0.066	0.087	0.099

B.2 Power

Table B.5: Power for LENTH89 assuming two location effects and one dispersion effect

Location	$\mathfrak{D} = \{A\}$						
	1	2^2	3^2	5^2	10^2	20^2	50^2
$(B(0.2), C(0.2))$	0.095	0.094	0.094	0.089	0.098	0.096	0.098
$(B(0.2), C(0.5))$	0.178	0.178	0.187	0.174	0.188	0.196	0.191
$(B(0.2), C(0.9))$	0.365	0.373	0.375	0.372	0.377	0.370	0.369

$(B(0.5), C(0.5))$	0.242	0.251	0.265	0.258	0.268	0.293	0.272
$(B(0.5), C(0.9))$	0.446	0.452	0.456	0.446	0.445	0.468	0.444
$(B(0.9), C(0.9))$	0.623	0.647	0.640	0.640	0.626	0.621	0.616
$(A(0.2), B(0.2))$	0.095	0.093	0.103	0.105	0.107	0.108	0.114
$(A(0.2), B(0.5))$	0.171	0.176	0.181	0.187	0.203	0.198	0.193
$(A(0.2), B(0.9))$	0.359	0.376	0.377	0.380	0.372	0.381	0.378
$(A(0.5), B(0.5))$	0.247	0.246	0.262	0.259	0.263	0.272	0.262
$(A(0.5), B(0.9))$	0.448	0.446	0.451	0.445	0.450	0.462	0.418
$(A(0.9), B(0.9))$	0.630	0.634	0.628	0.626	0.604	0.610	0.590
$(B(0.2), AB(0.2))$	0.102	0.083	0.093	0.095	0.092	0.089	0.101
$(B(0.2), AB(0.5))$	0.182	0.164	0.176	0.175	0.176	0.185	0.187
$(B(0.2), AB(0.9))$	0.369	0.373	0.388	0.388	0.380	0.390	0.388
$(B(0.5), AB(0.5))$	0.232	0.240	0.236	0.238	0.249	0.256	0.243
$(B(0.5), AB(0.9))$	0.444	0.447	0.446	0.433	0.449	0.451	0.423
$(B(0.9), AB(0.9))$	0.619	0.623	0.615	0.600	0.566	0.598	0.575

Table B.6: Power for BP91 assuming two location effects and one dispersion effect

Location	$\mathfrak{D} = \{A\}$						
	1	2^2	3^2	5^2	10^2	20^2	50^2
$(B(0.2), C(0.2))$	0.098	0.098	0.095	0.088	0.095	0.088	0.086
$(B(0.2), C(0.5))$	0.183	0.190	0.194	0.184	0.188	0.198	0.185
$(B(0.2), C(0.9))$	0.375	0.384	0.389	0.383	0.380	0.380	0.365
$(B(0.5), C(0.5))$	0.256	0.264	0.279	0.277	0.278	0.283	0.268
$(B(0.5), C(0.9))$	0.463	0.464	0.472	0.461	0.458	0.473	0.445
$(B(0.9), C(0.9))$	0.642	0.656	0.658	0.652	0.637	0.639	0.629
$(A(0.2), B(0.2))$	0.100	0.097	0.107	0.108	0.107	0.107	0.107
$(A(0.2), B(0.5))$	0.182	0.183	0.194	0.190	0.203	0.202	0.198
$(A(0.2), B(0.9))$	0.374	0.390	0.394	0.388	0.380	0.381	0.383

$(A(0.5), B(0.5))$	0.257	0.264	0.277	0.267	0.273	0.285	0.272
$(A(0.5), B(0.9))$	0.466	0.459	0.463	0.456	0.456	0.467	0.435
$(A(0.9), B(0.9))$	0.645	0.653	0.648	0.650	0.624	0.639	0.621
$(B(0.2), AB(0.2))$	0.104	0.090	0.094	0.097	0.091	0.088	0.086
$(B(0.2), AB(0.5))$	0.187	0.173	0.184	0.184	0.174	0.179	0.189
$(B(0.2), AB(0.9))$	0.385	0.385	0.396	0.399	0.386	0.387	0.387
$(B(0.5), AB(0.5))$	0.250	0.258	0.258	0.247	0.256	0.267	0.245
$(B(0.5), AB(0.9))$	0.459	0.468	0.459	0.441	0.452	0.458	0.422
$(B(0.9), AB(0.9))$	0.641	0.639	0.651	0.636	0.598	0.624	0.611

Table B.7: Power for BM86 assuming two location effects and one dispersion effect

Location	$\mathfrak{D} = \{A\}$						
	1	2^2	3^2	5^2	10^2	20^2	50^2
$(B(0.2), C(0.2))$	0.102	0.098	0.101	0.093	0.104	0.105	0.108
$(B(0.2), C(0.5))$	0.184	0.197	0.202	0.197	0.212	0.214	0.224
$(B(0.2), C(0.9))$	0.393	0.406	0.403	0.409	0.416	0.422	0.403
$(B(0.5), C(0.5))$	0.264	0.268	0.288	0.292	0.300	0.324	0.307
$(B(0.5), C(0.9))$	0.474	0.483	0.491	0.491	0.505	0.511	0.490
$(B(0.9), C(0.9))$	0.666	0.682	0.690	0.687	0.678	0.694	0.679
$(A(0.2), B(0.2))$	0.099	0.095	0.110	0.120	0.117	0.124	0.122
$(A(0.2), B(0.5))$	0.181	0.184	0.199	0.205	0.226	0.228	0.230
$(A(0.2), B(0.9))$	0.393	0.407	0.414	0.420	0.417	0.421	0.420
$(A(0.5), B(0.5))$	0.264	0.272	0.284	0.287	0.299	0.313	0.303
$(A(0.5), B(0.9))$	0.474	0.475	0.495	0.488	0.496	0.514	0.482
$(A(0.9), B(0.9))$	0.669	0.671	0.678	0.682	0.660	0.682	0.675
$(B(0.2), AB(0.2))$	0.109	0.082	0.097	0.091	0.087	0.085	0.093
$(B(0.2), AB(0.5))$	0.183	0.178	0.186	0.195	0.193	0.209	0.219
$(B(0.2), AB(0.9))$	0.403	0.406	0.425	0.439	0.437	0.451	0.442

$(B(0.5), AB(0.5))$	0.257	0.253	0.247	0.250	0.244	0.265	0.246
$(B(0.5), AB(0.9))$	0.471	0.486	0.490	0.492	0.519	0.522	0.490
$(B(0.9), AB(0.9))$	0.671	0.652	0.660	0.642	0.599	0.633	0.619

Table B.8: Power for LN97 assuming two location effects and one dispersion effect

Location	$\mathfrak{D} = \{A\}$						
	1	2^2	3^2	5^2	10^2	20^2	50^2
$(B(0.2), C(0.2))$	0.079	0.078	0.081	0.092	0.119	0.125	0.141
$(B(0.2), C(0.5))$	0.143	0.153	0.173	0.175	0.211	0.234	0.241
$(B(0.2), C(0.9))$	0.318	0.339	0.339	0.362	0.387	0.404	0.407
$(B(0.5), C(0.5))$	0.193	0.207	0.238	0.248	0.279	0.325	0.318
$(B(0.5), C(0.9))$	0.375	0.401	0.411	0.431	0.455	0.483	0.480
$(B(0.9), C(0.9))$	0.538	0.564	0.590	0.592	0.600	0.623	0.628
$(A(0.2), B(0.2))$	0.078	0.075	0.092	0.112	0.132	0.156	0.162
$(A(0.2), B(0.5))$	0.136	0.150	0.170	0.191	0.230	0.259	0.272
$(A(0.2), B(0.9))$	0.324	0.336	0.353	0.377	0.391	0.428	0.439
$(A(0.5), B(0.5))$	0.192	0.210	0.232	0.252	0.295	0.328	0.340
$(A(0.5), B(0.9))$	0.384	0.394	0.413	0.435	0.465	0.499	0.484
$(A(0.9), B(0.9))$	0.538	0.555	0.585	0.615	0.610	0.644	0.644
$(B(0.2), AB(0.2))$	0.089	0.070	0.078	0.103	0.130	0.141	0.171
$(B(0.2), AB(0.5))$	0.139	0.146	0.162	0.184	0.214	0.252	0.265
$(B(0.2), AB(0.9))$	0.329	0.349	0.362	0.400	0.412	0.446	0.459
$(B(0.5), AB(0.5))$	0.192	0.210	0.228	0.245	0.288	0.343	0.346
$(B(0.5), AB(0.9))$	0.373	0.406	0.426	0.457	0.501	0.520	0.515
$(B(0.9), AB(0.9))$	0.529	0.567	0.603	0.619	0.617	0.665	0.670

B.3 RR I

Table B.9: RR I for LENTH89 assuming two location effects and one dispersion effect

Factors	$\mathfrak{L} = \{B, C\}$	$\mathfrak{D} = \{A\}$						
		1	2^2	3^2	5^2	10^2	20^2	50^2
A	(0.2,0.2)	0.044	0.041	0.053	0.054	0.054	0.065	0.063
B	(0.2,0.2)	0.099	0.092	0.098	0.091	0.098	0.098	0.100
C	(0.2,0.2)	0.092	0.096	0.091	0.087	0.099	0.094	0.096
D	(0.2,0.2)	0.039	0.037	0.042	0.036	0.041	0.042	0.042
AB	(0.2,0.2)	0.049	0.042	0.052	0.044	0.059	0.055	0.056
AC	(0.2,0.2)	0.041	0.051	0.044	0.048	0.056	0.060	0.052
AD	(0.2,0.2)	0.045	0.042	0.043	0.036	0.041	0.041	0.042
BC	(0.2,0.2)	0.050	0.045	0.044	0.040	0.048	0.044	0.035
BD	(0.2,0.2)	0.038	0.037	0.045	0.034	0.042	0.041	0.044
CD	(0.2,0.2)	0.047	0.031	0.042	0.039	0.039	0.037	0.033
ABC	(0.2,0.2)	0.041	0.047	0.047	0.039	0.045	0.049	0.035
ABD	(0.2,0.2)	0.040	0.043	0.035	0.033	0.036	0.041	0.045
ACD	(0.2,0.2)	0.047	0.048	0.043	0.033	0.039	0.038	0.035
BCD	(0.2,0.2)	0.047	0.039	0.038	0.037	0.038	0.049	0.038
ABCD	(0.2,0.2)	0.043	0.040	0.047	0.034	0.039	0.052	0.037
A	(0.2,0.5)	0.038	0.032	0.058	0.037	0.058	0.062	0.051
B	(0.2,0.5)	0.099	0.079	0.089	0.080	0.092	0.098	0.096
C	(0.2,0.5)	0.258	0.277	0.285	0.267	0.284	0.293	0.285
D	(0.2,0.5)	0.035	0.043	0.035	0.039	0.032	0.045	0.031
AB	(0.2,0.5)	0.035	0.045	0.041	0.044	0.053	0.061	0.056
AC	(0.2,0.5)	0.036	0.049	0.056	0.058	0.064	0.070	0.064
AD	(0.2,0.5)	0.048	0.034	0.035	0.035	0.030	0.044	0.031
BC	(0.2,0.5)	0.035	0.038	0.027	0.024	0.034	0.035	0.037
BD	(0.2,0.5)	0.041	0.036	0.038	0.033	0.031	0.041	0.031
CD	(0.2,0.5)	0.039	0.042	0.029	0.033	0.041	0.039	0.040

ABC	(0.2,0.5)	0.036	0.030	0.030	0.031	0.036	0.036	0.037
ABD	(0.2,0.5)	0.036	0.037	0.038	0.035	0.033	0.041	0.031
ACD	(0.2,0.5)	0.037	0.036	0.030	0.027	0.041	0.037	0.042
BCD	(0.2,0.5)	0.039	0.039	0.031	0.026	0.043	0.045	0.036
ABCD	(0.2,0.5)	0.041	0.038	0.026	0.025	0.043	0.043	0.037
A	(0.2,0.9)	0.044	0.041	0.043	0.048	0.057	0.055	0.061
B	(0.2,0.9)	0.088	0.083	0.090	0.088	0.090	0.104	0.112
C	(0.2,0.9)	0.642	0.662	0.660	0.655	0.663	0.637	0.626
D	(0.2,0.9)	0.042	0.039	0.033	0.030	0.042	0.038	0.038
AB	(0.2,0.9)	0.042	0.039	0.046	0.048	0.058	0.055	0.050
AC	(0.2,0.9)	0.039	0.035	0.052	0.045	0.050	0.066	0.049
AD	(0.2,0.9)	0.043	0.042	0.032	0.031	0.043	0.038	0.039
BC	(0.2,0.9)	0.035	0.038	0.031	0.043	0.034	0.045	0.042
BD	(0.2,0.9)	0.034	0.038	0.037	0.043	0.032	0.035	0.049
CD	(0.2,0.9)	0.043	0.039	0.040	0.042	0.048	0.036	0.040
ABC	(0.2,0.9)	0.043	0.031	0.030	0.039	0.039	0.046	0.044
ABD	(0.2,0.9)	0.039	0.034	0.031	0.041	0.030	0.037	0.050
ACD	(0.2,0.9)	0.042	0.032	0.039	0.044	0.052	0.035	0.039
BCD	(0.2,0.9)	0.044	0.035	0.029	0.033	0.036	0.042	0.042
ABCD	(0.2,0.9)	0.037	0.040	0.031	0.032	0.034	0.040	0.041
A	(0.5,0.5)	0.028	0.035	0.038	0.042	0.044	0.062	0.056
B	(0.5,0.5)	0.237	0.253	0.262	0.255	0.277	0.293	0.270
C	(0.5,0.5)	0.247	0.249	0.267	0.260	0.259	0.293	0.273
D	(0.5,0.5)	0.038	0.036	0.030	0.025	0.024	0.047	0.040
AB	(0.5,0.5)	0.032	0.033	0.047	0.043	0.061	0.064	0.068
AC	(0.5,0.5)	0.035	0.044	0.056	0.049	0.059	0.064	0.066
AD	(0.5,0.5)	0.036	0.037	0.032	0.025	0.028	0.048	0.038
BC	(0.5,0.5)	0.031	0.038	0.030	0.034	0.037	0.044	0.033
BD	(0.5,0.5)	0.037	0.032	0.030	0.032	0.026	0.028	0.033
CD	(0.5,0.5)	0.037	0.037	0.031	0.028	0.030	0.030	0.033
ABC	(0.5,0.5)	0.033	0.034	0.029	0.031	0.035	0.042	0.033

ABD	(0.5,0.5)	0.032	0.027	0.025	0.032	0.025	0.027	0.032
ACD	(0.5,0.5)	0.035	0.038	0.033	0.032	0.032	0.031	0.033
BCD	(0.5,0.5)	0.038	0.028	0.032	0.025	0.030	0.038	0.031
ABCD	(0.5,0.5)	0.039	0.035	0.036	0.023	0.032	0.039	0.030
A	(0.5,0.9)	0.039	0.047	0.038	0.047	0.051	0.057	0.052
B	(0.5,0.9)	0.250	0.255	0.261	0.255	0.278	0.284	0.256
C	(0.5,0.9)	0.643	0.648	0.650	0.638	0.611	0.653	0.632
D	(0.5,0.9)	0.035	0.035	0.030	0.028	0.038	0.036	0.035
AB	(0.5,0.9)	0.042	0.044	0.042	0.048	0.056	0.080	0.062
AC	(0.5,0.9)	0.031	0.038	0.035	0.040	0.059	0.062	0.055
AD	(0.5,0.9)	0.036	0.032	0.031	0.028	0.037	0.038	0.037
BC	(0.5,0.9)	0.037	0.033	0.032	0.029	0.033	0.035	0.037
BD	(0.5,0.9)	0.037	0.039	0.032	0.031	0.041	0.036	0.038
CD	(0.5,0.9)	0.036	0.035	0.030	0.036	0.038	0.033	0.036
ABC	(0.5,0.9)	0.041	0.035	0.030	0.028	0.033	0.035	0.038
ABD	(0.5,0.9)	0.036	0.033	0.027	0.035	0.043	0.036	0.038
ACD	(0.5,0.9)	0.039	0.036	0.028	0.037	0.037	0.030	0.037
BCD	(0.5,0.9)	0.024	0.032	0.029	0.022	0.031	0.029	0.030
ABCD	(0.5,0.9)	0.039	0.033	0.025	0.030	0.033	0.028	0.030
A	(0.9,0.9)	0.032	0.033	0.044	0.045	0.064	0.056	0.056
B	(0.9,0.9)	0.620	0.648	0.647	0.647	0.620	0.633	0.620
C	(0.9,0.9)	0.627	0.645	0.633	0.633	0.632	0.608	0.612
D	(0.9,0.9)	0.039	0.030	0.028	0.035	0.040	0.039	0.037
AB	(0.9,0.9)	0.032	0.037	0.049	0.052	0.051	0.061	0.061
AC	(0.9,0.9)	0.033	0.041	0.048	0.051	0.057	0.058	0.060
AD	(0.9,0.9)	0.028	0.030	0.030	0.037	0.036	0.038	0.037
BC	(0.9,0.9)	0.034	0.031	0.035	0.027	0.027	0.042	0.032
BD	(0.9,0.9)	0.028	0.030	0.022	0.035	0.031	0.032	0.043
CD	(0.9,0.9)	0.036	0.037	0.034	0.028	0.034	0.039	0.035
ABC	(0.9,0.9)	0.034	0.026	0.039	0.031	0.030	0.042	0.033
ABD	(0.9,0.9)	0.027	0.036	0.030	0.039	0.032	0.032	0.041

ACD	(0.9,0.9)	0.037	0.033	0.035	0.033	0.036	0.036	0.035
BCD	(0.9,0.9)	0.035	0.031	0.038	0.030	0.047	0.033	0.037
ABCD	(0.9,0.9)	0.039	0.035	0.030	0.028	0.042	0.033	0.037

Factors	$\mathfrak{L} = \{A, B\}$	$\mathfrak{D} = \{A\}$						
		1	2^2	3^2	5^2	10^2	20^2	50^2
A	(0.2,0.2)	0.094	0.095	0.107	0.110	0.112	0.115	0.117
B	(0.2,0.2)	0.095	0.090	0.099	0.100	0.102	0.101	0.111
C	(0.2,0.2)	0.034	0.041	0.039	0.037	0.048	0.040	0.043
D	(0.2,0.2)	0.039	0.042	0.040	0.041	0.042	0.040	0.041
AB	(0.2,0.2)	0.051	0.032	0.055	0.045	0.059	0.059	0.061
AC	(0.2,0.2)	0.045	0.042	0.036	0.042	0.042	0.042	0.041
AD	(0.2,0.2)	0.039	0.042	0.041	0.043	0.042	0.039	0.042
BC	(0.2,0.2)	0.049	0.040	0.044	0.049	0.048	0.047	0.036
BD	(0.2,0.2)	0.045	0.038	0.045	0.043	0.045	0.041	0.045
CD	(0.2,0.2)	0.041	0.033	0.045	0.043	0.039	0.042	0.036
ABC	(0.2,0.2)	0.041	0.041	0.044	0.048	0.044	0.052	0.035
ABD	(0.2,0.2)	0.049	0.041	0.038	0.043	0.045	0.039	0.045
ACD	(0.2,0.2)	0.042	0.048	0.045	0.039	0.039	0.041	0.036
BCD	(0.2,0.2)	0.051	0.042	0.038	0.043	0.043	0.054	0.044
ABCD	(0.2,0.2)	0.043	0.035	0.046	0.043	0.044	0.054	0.045
A	(0.2,0.5)	0.089	0.092	0.105	0.093	0.117	0.112	0.096
B	(0.2,0.5)	0.253	0.261	0.258	0.280	0.288	0.284	0.291
C	(0.2,0.5)	0.041	0.040	0.035	0.036	0.044	0.041	0.045
D	(0.2,0.5)	0.041	0.043	0.029	0.043	0.043	0.045	0.041
AB	(0.2,0.5)	0.034	0.048	0.060	0.054	0.059	0.076	0.079
AC	(0.2,0.5)	0.039	0.036	0.042	0.036	0.045	0.039	0.045
AD	(0.2,0.5)	0.051	0.035	0.031	0.039	0.040	0.045	0.041
BC	(0.2,0.5)	0.035	0.029	0.024	0.032	0.032	0.041	0.035
BD	(0.2,0.5)	0.039	0.039	0.044	0.036	0.040	0.045	0.036
CD	(0.2,0.5)	0.045	0.043	0.029	0.037	0.047	0.038	0.045
ABC	(0.2,0.5)	0.047	0.031	0.029	0.036	0.031	0.041	0.035

ABD	(0.2,0.5)	0.039	0.041	0.037	0.038	0.042	0.043	0.036
ACD	(0.2,0.5)	0.039	0.032	0.028	0.035	0.048	0.036	0.045
BCD	(0.2,0.5)	0.040	0.038	0.031	0.032	0.044	0.047	0.044
ABCD	(0.2,0.5)	0.039	0.037	0.033	0.030	0.042	0.047	0.044
A	(0.2,0.9)	0.083	0.090	0.103	0.107	0.110	0.119	0.113
B	(0.2,0.9)	0.636	0.662	0.650	0.653	0.633	0.642	0.643
C	(0.2,0.9)	0.040	0.037	0.041	0.039	0.041	0.050	0.045
D	(0.2,0.9)	0.047	0.039	0.038	0.035	0.044	0.044	0.046
AB	(0.2,0.9)	0.040	0.042	0.046	0.050	0.060	0.063	0.070
AC	(0.2,0.9)	0.036	0.038	0.046	0.032	0.039	0.050	0.045
AD	(0.2,0.9)	0.038	0.041	0.029	0.034	0.050	0.044	0.048
BC	(0.2,0.9)	0.036	0.036	0.033	0.044	0.036	0.052	0.047
BD	(0.2,0.9)	0.035	0.040	0.033	0.047	0.036	0.041	0.056
CD	(0.2,0.9)	0.042	0.038	0.041	0.045	0.049	0.042	0.047
ABC	(0.2,0.9)	0.039	0.041	0.031	0.049	0.041	0.050	0.048
ABD	(0.2,0.9)	0.041	0.038	0.024	0.044	0.036	0.043	0.054
ACD	(0.2,0.9)	0.045	0.037	0.035	0.047	0.051	0.042	0.046
BCD	(0.2,0.9)	0.040	0.041	0.031	0.035	0.039	0.049	0.049
ABCD	(0.2,0.9)	0.034	0.046	0.035	0.034	0.042	0.049	0.049
A	(0.5,0.5)	0.242	0.233	0.261	0.254	0.249	0.262	0.255
B	(0.5,0.5)	0.252	0.259	0.264	0.264	0.277	0.283	0.269
C	(0.5,0.5)	0.029	0.037	0.043	0.035	0.036	0.039	0.037
D	(0.5,0.5)	0.040	0.040	0.033	0.029	0.024	0.049	0.037
AB	(0.5,0.5)	0.036	0.034	0.049	0.049	0.066	0.062	0.066
AC	(0.5,0.5)	0.035	0.037	0.036	0.036	0.035	0.038	0.038
AD	(0.5,0.5)	0.032	0.038	0.024	0.031	0.022	0.048	0.037
BC	(0.5,0.5)	0.033	0.036	0.029	0.033	0.041	0.047	0.036
BD	(0.5,0.5)	0.032	0.038	0.030	0.032	0.025	0.032	0.029
CD	(0.5,0.5)	0.036	0.043	0.032	0.032	0.035	0.031	0.037
ABC	(0.5,0.5)	0.036	0.040	0.033	0.036	0.035	0.047	0.038
ABD	(0.5,0.5)	0.032	0.031	0.028	0.036	0.026	0.030	0.029

ACD	(0.5,0.5)	0.037	0.039	0.030	0.031	0.035	0.030	0.036
BCD	(0.5,0.5)	0.038	0.027	0.033	0.031	0.036	0.041	0.033
ABCD	(0.5,0.5)	0.035	0.035	0.039	0.027	0.035	0.041	0.032
A	(0.5,0.9)	0.259	0.248	0.252	0.258	0.267	0.288	0.237
B	(0.5,0.9)	0.637	0.644	0.650	0.632	0.632	0.637	0.598
C	(0.5,0.9)	0.035	0.036	0.030	0.038	0.041	0.048	0.039
D	(0.5,0.9)	0.042	0.035	0.031	0.034	0.036	0.047	0.039
AB	(0.5,0.9)	0.046	0.042	0.039	0.043	0.050	0.073	0.055
AC	(0.5,0.9)	0.026	0.033	0.028	0.031	0.039	0.045	0.039
AD	(0.5,0.9)	0.035	0.031	0.030	0.032	0.038	0.050	0.041
BC	(0.5,0.9)	0.036	0.043	0.037	0.032	0.041	0.036	0.043
BD	(0.5,0.9)	0.038	0.046	0.031	0.032	0.039	0.038	0.040
CD	(0.5,0.9)	0.038	0.034	0.031	0.036	0.039	0.041	0.041
ABC	(0.5,0.9)	0.042	0.037	0.036	0.036	0.043	0.036	0.044
ABD	(0.5,0.9)	0.031	0.032	0.026	0.038	0.042	0.037	0.039
ACD	(0.5,0.9)	0.038	0.032	0.032	0.041	0.040	0.039	0.041
BCD	(0.5,0.9)	0.028	0.027	0.027	0.032	0.036	0.033	0.033
ABCD	(0.5,0.9)	0.030	0.036	0.028	0.035	0.041	0.033	0.034
A	(0.9,0.9)	0.625	0.621	0.611	0.619	0.601	0.606	0.581
B	(0.9,0.9)	0.635	0.648	0.645	0.632	0.608	0.615	0.598
C	(0.9,0.9)	0.033	0.025	0.043	0.039	0.038	0.042	0.037
D	(0.9,0.9)	0.039	0.034	0.031	0.033	0.041	0.034	0.041
AB	(0.9,0.9)	0.036	0.036	0.052	0.053	0.056	0.067	0.063
AC	(0.9,0.9)	0.035	0.028	0.039	0.037	0.037	0.039	0.035
AD	(0.9,0.9)	0.027	0.031	0.031	0.040	0.038	0.034	0.039
BC	(0.9,0.9)	0.028	0.030	0.035	0.027	0.030	0.045	0.031
BD	(0.9,0.9)	0.030	0.033	0.026	0.039	0.029	0.030	0.042
CD	(0.9,0.9)	0.034	0.039	0.034	0.030	0.036	0.042	0.035
ABC	(0.9,0.9)	0.031	0.024	0.036	0.033	0.031	0.043	0.032
ABD	(0.9,0.9)	0.029	0.033	0.032	0.040	0.033	0.031	0.042
ACD	(0.9,0.9)	0.039	0.033	0.036	0.036	0.033	0.041	0.035

BCD	(0.9,0.9)	0.032	0.035	0.033	0.033	0.048	0.032	0.038
ABCD	(0.9,0.9)	0.037	0.039	0.034	0.031	0.040	0.034	0.038
Factors	$\mathfrak{L} = \{B, AB\}$	$\mathfrak{D} = \{A\}$						
		1	2^2	3^2	5^2	10^2	20^2	50^2
A	(0.2,0.2)	0.04	0.04	0.05	0.06	0.06	0.08	0.08
B	(0.2,0.2)	0.10	0.09	0.10	0.09	0.09	0.09	0.10
C	(0.2,0.2)	0.04	0.05	0.04	0.04	0.05	0.04	0.05
D	(0.2,0.2)	0.04	0.04	0.04	0.04	0.05	0.05	0.05
AB	(0.2,0.2)	0.11	0.08	0.09	0.10	0.09	0.09	0.10
AC	(0.2,0.2)	0.05	0.05	0.04	0.04	0.04	0.05	0.05
AD	(0.2,0.2)	0.04	0.05	0.05	0.04	0.05	0.05	0.05
BC	(0.2,0.2)	0.06	0.04	0.05	0.05	0.04	0.06	0.04
BD	(0.2,0.2)	0.04	0.04	0.05	0.04	0.05	0.04	0.05
CD	(0.2,0.2)	0.04	0.04	0.04	0.04	0.04	0.04	0.04
ABC	(0.2,0.2)	0.04	0.04	0.05	0.05	0.05	0.06	0.04
ABD	(0.2,0.2)	0.04	0.05	0.03	0.04	0.05	0.04	0.05
ACD	(0.2,0.2)	0.05	0.05	0.04	0.04	0.04	0.04	0.04
BCD	(0.2,0.2)	0.05	0.04	0.04	0.04	0.05	0.06	0.05
ABCD	(0.2,0.2)	0.05	0.04	0.04	0.04	0.05	0.06	0.05
A	(0.2,0.5)	0.04	0.03	0.06	0.04	0.05	0.07	0.06
B	(0.2,0.5)	0.10	0.08	0.10	0.10	0.11	0.12	0.12
C	(0.2,0.5)	0.04	0.04	0.04	0.03	0.04	0.04	0.04
D	(0.2,0.5)	0.04	0.04	0.03	0.04	0.04	0.05	0.03
AB	(0.2,0.5)	0.27	0.24	0.25	0.25	0.24	0.25	0.26
AC	(0.2,0.5)	0.04	0.04	0.04	0.04	0.04	0.04	0.04
AD	(0.2,0.5)	0.05	0.03	0.04	0.03	0.04	0.05	0.03
BC	(0.2,0.5)	0.03	0.03	0.03	0.03	0.03	0.04	0.04
BD	(0.2,0.5)	0.03	0.04	0.04	0.04	0.04	0.05	0.04
CD	(0.2,0.5)	0.04	0.04	0.04	0.03	0.05	0.04	0.05
ABC	(0.2,0.5)	0.04	0.03	0.03	0.04	0.03	0.04	0.04
ABD	(0.2,0.5)	0.04	0.03	0.04	0.04	0.04	0.04	0.04

ACD	(0.2,0.5)	0.04	0.04	0.03	0.03	0.05	0.04	0.04
BCD	(0.2,0.5)	0.04	0.04	0.03	0.03	0.05	0.05	0.04
ABCD	(0.2,0.5)	0.04	0.04	0.03	0.03	0.05	0.05	0.04
A	(0.2,0.9)	0.04	0.04	0.04	0.05	0.06	0.06	0.06
B	(0.2,0.9)	0.09	0.09	0.10	0.11	0.11	0.13	0.13
C	(0.2,0.9)	0.04	0.04	0.04	0.04	0.04	0.04	0.04
D	(0.2,0.9)	0.04	0.04	0.03	0.03	0.05	0.04	0.05
AB	(0.2,0.9)	0.65	0.66	0.67	0.66	0.65	0.65	0.65
AC	(0.2,0.9)	0.04	0.03	0.04	0.03	0.04	0.05	0.04
AD	(0.2,0.9)	0.04	0.04	0.03	0.03	0.05	0.04	0.05
BC	(0.2,0.9)	0.03	0.04	0.04	0.04	0.04	0.06	0.05
BD	(0.2,0.9)	0.04	0.04	0.03	0.05	0.03	0.04	0.06
CD	(0.2,0.9)	0.05	0.04	0.04	0.04	0.05	0.04	0.05
ABC	(0.2,0.9)	0.04	0.04	0.03	0.04	0.04	0.05	0.05
ABD	(0.2,0.9)	0.04	0.04	0.03	0.04	0.03	0.04	0.05
ACD	(0.2,0.9)	0.05	0.03	0.03	0.05	0.05	0.04	0.04
BCD	(0.2,0.9)	0.04	0.04	0.03	0.03	0.04	0.04	0.05
ABCD	(0.2,0.9)	0.03	0.04	0.03	0.04	0.04	0.04	0.05
A	(0.5,0.5)	0.03	0.03	0.04	0.05	0.05	0.07	0.07
B	(0.5,0.5)	0.24	0.24	0.24	0.24	0.25	0.25	0.24
C	(0.5,0.5)	0.03	0.03	0.04	0.04	0.04	0.04	0.04
D	(0.5,0.5)	0.04	0.04	0.03	0.03	0.03	0.05	0.04
AB	(0.5,0.5)	0.23	0.24	0.23	0.24	0.25	0.26	0.24
AC	(0.5,0.5)	0.03	0.03	0.04	0.04	0.04	0.04	0.04
AD	(0.5,0.5)	0.03	0.03	0.03	0.04	0.03	0.05	0.04
BC	(0.5,0.5)	0.03	0.03	0.03	0.04	0.04	0.05	0.05
BD	(0.5,0.5)	0.04	0.03	0.03	0.04	0.03	0.03	0.04
CD	(0.5,0.5)	0.04	0.04	0.03	0.03	0.04	0.03	0.04
ABC	(0.5,0.5)	0.04	0.03	0.03	0.04	0.04	0.05	0.05
ABD	(0.5,0.5)	0.04	0.03	0.02	0.04	0.03	0.03	0.04
ACD	(0.5,0.5)	0.03	0.04	0.03	0.03	0.04	0.03	0.04

BCD	(0.5,0.5)	0.04	0.03	0.03	0.03	0.04	0.04	0.04
ABCD	(0.5,0.5)	0.04	0.04	0.04	0.03	0.04	0.04	0.04
A	(0.5,0.9)	0.04	0.04	0.04	0.04	0.06	0.06	0.06
B	(0.5,0.9)	0.25	0.25	0.28	0.27	0.30	0.29	0.27
C	(0.5,0.9)	0.04	0.04	0.03	0.04	0.04	0.05	0.04
D	(0.5,0.9)	0.04	0.03	0.03	0.03	0.04	0.05	0.04
AB	(0.5,0.9)	0.64	0.64	0.61	0.60	0.60	0.61	0.58
AC	(0.5,0.9)	0.03	0.03	0.03	0.03	0.04	0.04	0.04
AD	(0.5,0.9)	0.03	0.03	0.03	0.03	0.04	0.05	0.04
BC	(0.5,0.9)	0.03	0.03	0.03	0.03	0.04	0.04	0.04
BD	(0.5,0.9)	0.04	0.03	0.04	0.03	0.04	0.04	0.04
CD	(0.5,0.9)	0.03	0.03	0.03	0.04	0.04	0.04	0.04
ABC	(0.5,0.9)	0.04	0.04	0.03	0.03	0.04	0.04	0.04
ABD	(0.5,0.9)	0.04	0.04	0.03	0.04	0.04	0.03	0.04
ACD	(0.5,0.9)	0.04	0.03	0.03	0.04	0.04	0.03	0.04
BCD	(0.5,0.9)	0.03	0.03	0.03	0.03	0.04	0.03	0.04
ABCD	(0.5,0.9)	0.04	0.03	0.03	0.03	0.04	0.03	0.04
A	(0.9,0.9)	0.03	0.03	0.04	0.05	0.07	0.07	0.06
B	(0.9,0.9)	0.63	0.63	0.62	0.60	0.57	0.60	0.57
C	(0.9,0.9)	0.04	0.03	0.04	0.04	0.04	0.04	0.04
D	(0.9,0.9)	0.04	0.03	0.03	0.04	0.04	0.04	0.04
AB	(0.9,0.9)	0.61	0.62	0.61	0.60	0.56	0.60	0.58
AC	(0.9,0.9)	0.03	0.03	0.04	0.04	0.04	0.05	0.04
AD	(0.9,0.9)	0.03	0.03	0.03	0.04	0.03	0.04	0.04
BC	(0.9,0.9)	0.03	0.03	0.04	0.03	0.03	0.04	0.03
BD	(0.9,0.9)	0.03	0.03	0.02	0.04	0.03	0.03	0.05
CD	(0.9,0.9)	0.04	0.04	0.03	0.03	0.04	0.04	0.04
ABC	(0.9,0.9)	0.03	0.03	0.04	0.04	0.03	0.04	0.03
ABD	(0.9,0.9)	0.03	0.04	0.03	0.04	0.03	0.03	0.05
ACD	(0.9,0.9)	0.03	0.03	0.03	0.04	0.03	0.04	0.04
BCD	(0.9,0.9)	0.03	0.03	0.03	0.03	0.05	0.04	0.04

ABCD	(0.9,0.9)	0.04	0.04	0.03	0.03	0.04	0.04	0.04

Table B.10: RR I for BP91 assuming two location effects and one dispersion effect

Factors	$\mathfrak{L} = \{B, C\}$	$\mathfrak{D} = \{A\}$						
		1	2^2	3^2	5^2	10^2	20^2	50^2
A	(0.2,0.2)	0.039	0.043	0.054	0.050	0.047	0.060	0.058
B	(0.2,0.2)	0.104	0.089	0.099	0.089	0.092	0.087	0.086
C	(0.2,0.2)	0.093	0.106	0.092	0.087	0.099	0.090	0.085
D	(0.2,0.2)	0.039	0.039	0.037	0.041	0.037	0.038	0.036
AB	(0.2,0.2)	0.047	0.040	0.047	0.037	0.045	0.047	0.042
AC	(0.2,0.2)	0.041	0.043	0.043	0.043	0.052	0.045	0.046
AD	(0.2,0.2)	0.042	0.039	0.040	0.036	0.038	0.039	0.037
BC	(0.2,0.2)	0.048	0.042	0.040	0.042	0.045	0.044	0.032
BD	(0.2,0.2)	0.038	0.035	0.043	0.035	0.039	0.036	0.041
CD	(0.2,0.2)	0.044	0.035	0.037	0.036	0.032	0.032	0.036
ABC	(0.2,0.2)	0.039	0.042	0.044	0.038	0.044	0.044	0.033
ABD	(0.2,0.2)	0.043	0.038	0.038	0.036	0.034	0.038	0.040
ACD	(0.2,0.2)	0.044	0.047	0.042	0.033	0.032	0.031	0.036
BCD	(0.2,0.2)	0.046	0.041	0.039	0.036	0.037	0.045	0.032
ABCD	(0.2,0.2)	0.047	0.037	0.045	0.035	0.037	0.044	0.032
A	(0.2,0.5)	0.038	0.030	0.052	0.036	0.048	0.050	0.044
B	(0.2,0.5)	0.092	0.085	0.085	0.083	0.087	0.093	0.082
C	(0.2,0.5)	0.275	0.294	0.304	0.285	0.289	0.302	0.288
D	(0.2,0.5)	0.032	0.041	0.034	0.035	0.025	0.038	0.025
AB	(0.2,0.5)	0.034	0.041	0.039	0.035	0.039	0.044	0.042
AC	(0.2,0.5)	0.032	0.046	0.049	0.054	0.052	0.058	0.057
AD	(0.2,0.5)	0.041	0.034	0.034	0.033	0.028	0.042	0.026
BC	(0.2,0.5)	0.031	0.027	0.027	0.024	0.030	0.031	0.029

BD	(0.2,0.5)	0.037	0.033	0.037	0.029	0.030	0.033	0.028
CD	(0.2,0.5)	0.041	0.040	0.027	0.031	0.037	0.030	0.040
ABC	(0.2,0.5)	0.037	0.030	0.029	0.028	0.032	0.032	0.030
ABD	(0.2,0.5)	0.036	0.034	0.036	0.035	0.031	0.032	0.028
ACD	(0.2,0.5)	0.035	0.036	0.026	0.026	0.037	0.031	0.041
BCD	(0.2,0.5)	0.037	0.036	0.029	0.026	0.038	0.043	0.033
ABCD	(0.2,0.5)	0.038	0.039	0.026	0.027	0.039	0.041	0.033
A	(0.2,0.9)	0.035	0.031	0.036	0.038	0.042	0.044	0.047
B	(0.2,0.9)	0.078	0.075	0.083	0.082	0.078	0.093	0.082
C	(0.2,0.9)	0.671	0.692	0.696	0.684	0.681	0.667	0.648
D	(0.2,0.9)	0.033	0.028	0.029	0.025	0.035	0.030	0.030
AB	(0.2,0.9)	0.035	0.029	0.040	0.035	0.043	0.038	0.037
AC	(0.2,0.9)	0.028	0.032	0.045	0.036	0.047	0.050	0.043
AD	(0.2,0.9)	0.026	0.032	0.025	0.024	0.036	0.030	0.032
BC	(0.2,0.9)	0.027	0.029	0.024	0.033	0.028	0.034	0.035
BD	(0.2,0.9)	0.029	0.030	0.030	0.031	0.026	0.026	0.033
CD	(0.2,0.9)	0.037	0.028	0.031	0.038	0.033	0.026	0.027
ABC	(0.2,0.9)	0.037	0.030	0.025	0.036	0.028	0.032	0.035
ABD	(0.2,0.9)	0.030	0.027	0.022	0.030	0.029	0.026	0.033
ACD	(0.2,0.9)	0.038	0.027	0.028	0.035	0.037	0.026	0.029
BCD	(0.2,0.9)	0.035	0.029	0.022	0.031	0.029	0.035	0.034
ABCD	(0.2,0.9)	0.029	0.032	0.027	0.027	0.030	0.034	0.033
A	(0.5,0.5)	0.025	0.031	0.033	0.043	0.040	0.048	0.046
B	(0.5,0.5)	0.250	0.262	0.278	0.275	0.294	0.293	0.261
C	(0.5,0.5)	0.261	0.265	0.279	0.279	0.261	0.273	0.276
D	(0.5,0.5)	0.033	0.032	0.025	0.023	0.022	0.035	0.028
AB	(0.5,0.5)	0.028	0.035	0.043	0.035	0.051	0.050	0.055
AC	(0.5,0.5)	0.032	0.041	0.042	0.047	0.048	0.047	0.047
AD	(0.5,0.5)	0.031	0.028	0.022	0.023	0.024	0.036	0.028
BC	(0.5,0.5)	0.028	0.028	0.028	0.030	0.032	0.031	0.020
BD	(0.5,0.5)	0.029	0.028	0.025	0.024	0.025	0.022	0.024

CD	(0.5,0.5)	0.036	0.032	0.028	0.027	0.026	0.027	0.024
ABC	(0.5,0.5)	0.032	0.030	0.021	0.025	0.027	0.032	0.020
ABD	(0.5,0.5)	0.032	0.024	0.024	0.025	0.025	0.021	0.025
ACD	(0.5,0.5)	0.028	0.038	0.026	0.027	0.028	0.027	0.024
BCD	(0.5,0.5)	0.032	0.027	0.024	0.021	0.030	0.027	0.024
ABCD	(0.5,0.5)	0.036	0.035	0.031	0.022	0.031	0.027	0.024
A	(0.5,0.9)	0.030	0.038	0.032	0.037	0.042	0.041	0.035
B	(0.5,0.9)	0.243	0.254	0.262	0.254	0.269	0.275	0.243
C	(0.5,0.9)	0.683	0.673	0.682	0.667	0.646	0.671	0.647
D	(0.5,0.9)	0.030	0.026	0.019	0.021	0.031	0.031	0.025
AB	(0.5,0.9)	0.032	0.035	0.033	0.037	0.043	0.056	0.041
AC	(0.5,0.9)	0.020	0.032	0.026	0.037	0.050	0.053	0.042
AD	(0.5,0.9)	0.028	0.027	0.022	0.023	0.028	0.033	0.026
BC	(0.5,0.9)	0.029	0.027	0.018	0.018	0.027	0.025	0.023
BD	(0.5,0.9)	0.033	0.029	0.025	0.022	0.031	0.025	0.027
CD	(0.5,0.9)	0.029	0.027	0.024	0.030	0.027	0.024	0.026
ABC	(0.5,0.9)	0.035	0.025	0.021	0.020	0.025	0.024	0.022
ABD	(0.5,0.9)	0.027	0.026	0.019	0.028	0.033	0.024	0.026
ACD	(0.5,0.9)	0.028	0.032	0.024	0.027	0.026	0.023	0.026
BCD	(0.5,0.9)	0.019	0.020	0.021	0.019	0.026	0.023	0.021
ABCD	(0.5,0.9)	0.033	0.026	0.022	0.021	0.023	0.023	0.022
A	(0.9,0.9)	0.021	0.022	0.027	0.031	0.047	0.039	0.038
B	(0.9,0.9)	0.647	0.657	0.663	0.660	0.625	0.648	0.638
C	(0.9,0.9)	0.638	0.654	0.653	0.643	0.648	0.631	0.620
D	(0.9,0.9)	0.031	0.022	0.019	0.025	0.022	0.018	0.021
AB	(0.9,0.9)	0.022	0.031	0.036	0.036	0.033	0.043	0.038
AC	(0.9,0.9)	0.019	0.028	0.033	0.039	0.036	0.043	0.040
AD	(0.9,0.9)	0.021	0.018	0.018	0.028	0.022	0.019	0.021
BC	(0.9,0.9)	0.019	0.022	0.025	0.018	0.018	0.027	0.021
BD	(0.9,0.9)	0.017	0.021	0.015	0.024	0.019	0.024	0.024
CD	(0.9,0.9)	0.019	0.023	0.025	0.016	0.019	0.024	0.022

ABC	(0.9,0.9)	0.022	0.018	0.024	0.018	0.018	0.026	0.021
ABD	(0.9,0.9)	0.019	0.023	0.021	0.024	0.019	0.022	0.023
ACD	(0.9,0.9)	0.025	0.019	0.027	0.021	0.018	0.024	0.021
BCD	(0.9,0.9)	0.024	0.021	0.022	0.019	0.026	0.019	0.019
ABCD	(0.9,0.9)	0.028	0.027	0.022	0.016	0.024	0.019	0.018

Factors	$\mathfrak{L} = \{A, B\}$	$\mathfrak{D} = \{A\}$						
		1	2^2	3^2	5^2	10^2	20^2	50^2
A	(0.2,0.2)	0.100	0.099	0.115	0.117	0.123	0.121	0.116
B	(0.2,0.2)	0.099	0.094	0.099	0.100	0.090	0.093	0.097
C	(0.2,0.2)	0.040	0.041	0.042	0.033	0.043	0.038	0.038
D	(0.2,0.2)	0.037	0.044	0.044	0.044	0.039	0.038	0.039
AB	(0.2,0.2)	0.045	0.038	0.053	0.040	0.049	0.051	0.048
AC	(0.2,0.2)	0.048	0.038	0.033	0.036	0.042	0.035	0.039
AD	(0.2,0.2)	0.043	0.039	0.038	0.038	0.039	0.037	0.038
BC	(0.2,0.2)	0.049	0.037	0.043	0.048	0.045	0.043	0.033
BD	(0.2,0.2)	0.034	0.042	0.044	0.041	0.041	0.036	0.039
CD	(0.2,0.2)	0.042	0.038	0.043	0.037	0.034	0.040	0.034
ABC	(0.2,0.2)	0.045	0.042	0.044	0.041	0.044	0.045	0.033
ABD	(0.2,0.2)	0.045	0.042	0.039	0.040	0.038	0.037	0.039
ACD	(0.2,0.2)	0.042	0.046	0.048	0.037	0.037	0.038	0.033
BCD	(0.2,0.2)	0.050	0.042	0.038	0.038	0.041	0.053	0.037
ABCD	(0.2,0.2)	0.048	0.040	0.041	0.039	0.042	0.054	0.038
A	(0.2,0.5)	0.088	0.090	0.111	0.087	0.108	0.109	0.105
B	(0.2,0.5)	0.276	0.277	0.278	0.293	0.298	0.294	0.292
C	(0.2,0.5)	0.038	0.039	0.031	0.031	0.036	0.037	0.040
D	(0.2,0.5)	0.036	0.041	0.027	0.039	0.036	0.039	0.033
AB	(0.2,0.5)	0.037	0.045	0.053	0.049	0.051	0.064	0.065
AC	(0.2,0.5)	0.032	0.035	0.035	0.031	0.038	0.037	0.040
AD	(0.2,0.5)	0.047	0.030	0.032	0.040	0.036	0.042	0.033
BC	(0.2,0.5)	0.032	0.024	0.021	0.028	0.028	0.032	0.030
BD	(0.2,0.5)	0.036	0.033	0.040	0.029	0.031	0.032	0.030

CD	(0.2,0.5)	0.041	0.040	0.028	0.033	0.041	0.027	0.044
ABC	(0.2,0.5)	0.043	0.030	0.027	0.031	0.029	0.030	0.031
ABD	(0.2,0.5)	0.037	0.033	0.035	0.036	0.035	0.033	0.030
ACD	(0.2,0.5)	0.036	0.033	0.024	0.030	0.040	0.027	0.042
BCD	(0.2,0.5)	0.041	0.036	0.032	0.027	0.035	0.041	0.039
ABCD	(0.2,0.5)	0.039	0.035	0.032	0.030	0.039	0.041	0.037
A	(0.2,0.9)	0.078	0.084	0.096	0.093	0.095	0.102	0.100
B	(0.2,0.9)	0.671	0.695	0.692	0.683	0.665	0.659	0.666
C	(0.2,0.9)	0.034	0.028	0.033	0.030	0.032	0.032	0.035
D	(0.2,0.9)	0.038	0.034	0.032	0.027	0.031	0.031	0.034
AB	(0.2,0.9)	0.038	0.035	0.043	0.042	0.050	0.047	0.052
AC	(0.2,0.9)	0.028	0.032	0.035	0.025	0.028	0.034	0.034
AD	(0.2,0.9)	0.027	0.035	0.024	0.026	0.033	0.033	0.034
BC	(0.2,0.9)	0.025	0.031	0.026	0.035	0.029	0.036	0.039
BD	(0.2,0.9)	0.029	0.030	0.031	0.037	0.030	0.031	0.040
CD	(0.2,0.9)	0.036	0.026	0.035	0.033	0.040	0.032	0.036
ABC	(0.2,0.9)	0.038	0.031	0.027	0.038	0.032	0.037	0.038
ABD	(0.2,0.9)	0.035	0.028	0.016	0.037	0.031	0.031	0.040
ACD	(0.2,0.9)	0.033	0.029	0.026	0.035	0.039	0.033	0.035
BCD	(0.2,0.9)	0.034	0.034	0.026	0.029	0.033	0.037	0.031
ABCD	(0.2,0.9)	0.031	0.033	0.030	0.031	0.035	0.038	0.031
A	(0.5,0.5)	0.258	0.258	0.275	0.262	0.270	0.281	0.272
B	(0.5,0.5)	0.257	0.270	0.279	0.271	0.276	0.290	0.272
C	(0.5,0.5)	0.024	0.030	0.037	0.031	0.031	0.031	0.032
D	(0.5,0.5)	0.038	0.038	0.027	0.029	0.022	0.042	0.029
AB	(0.5,0.5)	0.031	0.032	0.043	0.042	0.054	0.053	0.051
AC	(0.5,0.5)	0.033	0.028	0.026	0.031	0.029	0.031	0.032
AD	(0.5,0.5)	0.028	0.034	0.020	0.025	0.019	0.043	0.029
BC	(0.5,0.5)	0.025	0.027	0.027	0.032	0.037	0.037	0.030
BD	(0.5,0.5)	0.030	0.028	0.026	0.026	0.025	0.025	0.031
CD	(0.5,0.5)	0.031	0.032	0.028	0.030	0.030	0.028	0.026

ABC	(0.5,0.5)	0.035	0.030	0.027	0.030	0.032	0.036	0.031
ABD	(0.5,0.5)	0.026	0.028	0.024	0.026	0.026	0.026	0.032
ACD	(0.5,0.5)	0.028	0.032	0.024	0.026	0.029	0.026	0.027
BCD	(0.5,0.5)	0.036	0.023	0.025	0.026	0.036	0.035	0.027
ABCD	(0.5,0.5)	0.029	0.033	0.032	0.024	0.035	0.034	0.028
A	(0.5,0.9)	0.252	0.241	0.255	0.258	0.256	0.278	0.237
B	(0.5,0.9)	0.680	0.677	0.671	0.655	0.656	0.656	0.633
C	(0.5,0.9)	0.027	0.028	0.024	0.027	0.028	0.036	0.028
D	(0.5,0.9)	0.031	0.026	0.021	0.026	0.031	0.039	0.026
AB	(0.5,0.9)	0.034	0.035	0.033	0.035	0.038	0.052	0.038
AC	(0.5,0.9)	0.020	0.022	0.021	0.022	0.033	0.037	0.027
AD	(0.5,0.9)	0.024	0.029	0.022	0.023	0.026	0.038	0.025
BC	(0.5,0.9)	0.031	0.028	0.024	0.024	0.028	0.026	0.028
BD	(0.5,0.9)	0.033	0.031	0.025	0.027	0.034	0.030	0.031
CD	(0.5,0.9)	0.031	0.024	0.019	0.026	0.030	0.031	0.032
ABC	(0.5,0.9)	0.033	0.025	0.028	0.024	0.031	0.026	0.027
ABD	(0.5,0.9)	0.025	0.026	0.021	0.028	0.034	0.026	0.030
ACD	(0.5,0.9)	0.028	0.024	0.022	0.030	0.030	0.027	0.031
BCD	(0.5,0.9)	0.020	0.021	0.024	0.021	0.025	0.022	0.025
ABCD	(0.5,0.9)	0.028	0.026	0.021	0.025	0.026	0.024	0.026
A	(0.9,0.9)	0.643	0.642	0.635	0.645	0.627	0.633	0.612
B	(0.9,0.9)	0.647	0.664	0.662	0.654	0.620	0.645	0.629
C	(0.9,0.9)	0.020	0.014	0.023	0.026	0.027	0.031	0.025
D	(0.9,0.9)	0.029	0.019	0.019	0.022	0.027	0.018	0.026
AB	(0.9,0.9)	0.025	0.031	0.038	0.032	0.037	0.043	0.036
AC	(0.9,0.9)	0.022	0.021	0.028	0.027	0.028	0.030	0.025
AD	(0.9,0.9)	0.023	0.021	0.020	0.027	0.026	0.021	0.025
BC	(0.9,0.9)	0.017	0.020	0.021	0.020	0.022	0.032	0.019
BD	(0.9,0.9)	0.018	0.020	0.017	0.028	0.023	0.020	0.026
CD	(0.9,0.9)	0.022	0.025	0.023	0.018	0.022	0.025	0.022
ABC	(0.9,0.9)	0.020	0.016	0.027	0.019	0.021	0.031	0.018

ABD	(0.9,0.9)	0.023	0.024	0.022	0.025	0.023	0.022	0.026
ACD	(0.9,0.9)	0.025	0.024	0.026	0.022	0.018	0.024	0.022
BCD	(0.9,0.9)	0.022	0.022	0.023	0.020	0.024	0.021	0.024
ABCD	(0.9,0.9)	0.023	0.026	0.024	0.019	0.025	0.019	0.024

Factors	$\mathfrak{L} = \{B, AB\}$	$\mathfrak{D} = \{A\}$						
		1	2^2	3^2	5^2	10^2	20^2	50^2
A	(0.2,0.2)	0.040	0.042	0.052	0.057	0.052	0.064	0.056
B	(0.2,0.2)	0.101	0.094	0.092	0.095	0.087	0.090	0.087
C	(0.2,0.2)	0.036	0.044	0.037	0.033	0.046	0.039	0.044
D	(0.2,0.2)	0.043	0.041	0.041	0.043	0.039	0.045	0.043
AB	(0.2,0.2)	0.107	0.086	0.096	0.098	0.094	0.085	0.086
AC	(0.2,0.2)	0.050	0.049	0.031	0.032	0.045	0.038	0.044
AD	(0.2,0.2)	0.038	0.045	0.045	0.041	0.036	0.043	0.044
BC	(0.2,0.2)	0.053	0.044	0.039	0.052	0.041	0.048	0.036
BD	(0.2,0.2)	0.036	0.041	0.041	0.039	0.045	0.043	0.043
CD	(0.2,0.2)	0.039	0.042	0.041	0.039	0.034	0.040	0.039
ABC	(0.2,0.2)	0.041	0.043	0.045	0.045	0.044	0.050	0.036
ABD	(0.2,0.2)	0.045	0.042	0.036	0.042	0.044	0.042	0.043
ACD	(0.2,0.2)	0.039	0.047	0.045	0.041	0.034	0.042	0.039
BCD	(0.2,0.2)	0.047	0.041	0.037	0.037	0.044	0.052	0.039
ABCD	(0.2,0.2)	0.047	0.041	0.038	0.035	0.047	0.053	0.039
A	(0.2,0.5)	0.038	0.028	0.053	0.041	0.044	0.056	0.044
B	(0.2,0.5)	0.096	0.082	0.093	0.098	0.096	0.109	0.102
C	(0.2,0.5)	0.033	0.041	0.032	0.035	0.035	0.040	0.040
D	(0.2,0.5)	0.035	0.038	0.033	0.039	0.037	0.043	0.027
AB	(0.2,0.5)	0.277	0.264	0.275	0.270	0.252	0.248	0.276
AC	(0.2,0.5)	0.036	0.037	0.036	0.031	0.035	0.038	0.039
AD	(0.2,0.5)	0.042	0.032	0.035	0.037	0.034	0.045	0.027
BC	(0.2,0.5)	0.026	0.030	0.027	0.027	0.030	0.031	0.035
BD	(0.2,0.5)	0.035	0.034	0.039	0.033	0.032	0.035	0.028
CD	(0.2,0.5)	0.039	0.039	0.031	0.030	0.040	0.033	0.036

ABC	(0.2,0.5)	0.038	0.032	0.028	0.029	0.030	0.031	0.035
ABD	(0.2,0.5)	0.037	0.031	0.036	0.032	0.034	0.038	0.029
ACD	(0.2,0.5)	0.037	0.033	0.027	0.023	0.040	0.034	0.036
BCD	(0.2,0.5)	0.037	0.038	0.030	0.030	0.043	0.048	0.039
ABCD	(0.2,0.5)	0.038	0.037	0.030	0.030	0.041	0.044	0.037
A	(0.2,0.9)	0.029	0.033	0.037	0.044	0.045	0.045	0.049
B	(0.2,0.9)	0.079	0.079	0.093	0.098	0.099	0.112	0.104
C	(0.2,0.9)	0.033	0.028	0.033	0.030	0.032	0.030	0.028
D	(0.2,0.9)	0.032	0.030	0.031	0.028	0.035	0.031	0.035
AB	(0.2,0.9)	0.690	0.691	0.699	0.700	0.674	0.662	0.671
AC	(0.2,0.9)	0.030	0.026	0.031	0.025	0.031	0.032	0.027
AD	(0.2,0.9)	0.027	0.033	0.025	0.024	0.038	0.031	0.035
BC	(0.2,0.9)	0.027	0.033	0.030	0.038	0.031	0.042	0.037
BD	(0.2,0.9)	0.027	0.035	0.025	0.032	0.026	0.032	0.039
CD	(0.2,0.9)	0.038	0.028	0.031	0.036	0.035	0.029	0.033
ABC	(0.2,0.9)	0.037	0.032	0.027	0.038	0.034	0.039	0.036
ABD	(0.2,0.9)	0.033	0.037	0.020	0.032	0.025	0.031	0.041
ACD	(0.2,0.9)	0.041	0.030	0.030	0.038	0.036	0.028	0.033
BCD	(0.2,0.9)	0.036	0.032	0.026	0.032	0.032	0.032	0.041
ABCD	(0.2,0.9)	0.029	0.033	0.031	0.032	0.032	0.033	0.040
A	(0.5,0.5)	0.026	0.030	0.033	0.041	0.038	0.046	0.047
B	(0.5,0.5)	0.249	0.259	0.262	0.251	0.257	0.268	0.244
C	(0.5,0.5)	0.027	0.034	0.033	0.030	0.033	0.028	0.035
D	(0.5,0.5)	0.036	0.036	0.028	0.030	0.023	0.041	0.037
AB	(0.5,0.5)	0.250	0.258	0.254	0.243	0.254	0.266	0.246
AC	(0.5,0.5)	0.032	0.031	0.032	0.032	0.031	0.030	0.033
AD	(0.5,0.5)	0.030	0.031	0.025	0.032	0.021	0.042	0.037
BC	(0.5,0.5)	0.027	0.027	0.028	0.035	0.039	0.037	0.033
BD	(0.5,0.5)	0.035	0.025	0.021	0.031	0.031	0.024	0.034
CD	(0.5,0.5)	0.031	0.035	0.025	0.025	0.031	0.028	0.026
ABC	(0.5,0.5)	0.027	0.030	0.028	0.035	0.036	0.038	0.032

ABD	(0.5,0.5)	0.033	0.028	0.019	0.032	0.030	0.024	0.036
ACD	(0.5,0.5)	0.026	0.034	0.022	0.026	0.032	0.027	0.026
BCD	(0.5,0.5)	0.031	0.022	0.024	0.025	0.035	0.035	0.030
ABCD	(0.5,0.5)	0.032	0.031	0.030	0.030	0.036	0.033	0.031
A	(0.5,0.9)	0.032	0.038	0.032	0.037	0.042	0.042	0.034
B	(0.5,0.9)	0.245	0.245	0.270	0.245	0.272	0.267	0.235
C	(0.5,0.9)	0.024	0.028	0.024	0.023	0.033	0.037	0.032
D	(0.5,0.9)	0.026	0.023	0.021	0.022	0.028	0.036	0.028
AB	(0.5,0.9)	0.672	0.690	0.648	0.637	0.633	0.648	0.610
AC	(0.5,0.9)	0.018	0.025	0.022	0.024	0.034	0.034	0.031
AD	(0.5,0.9)	0.026	0.030	0.024	0.025	0.027	0.037	0.027
BC	(0.5,0.9)	0.026	0.028	0.020	0.022	0.032	0.029	0.031
BD	(0.5,0.9)	0.030	0.028	0.028	0.024	0.032	0.032	0.031
CD	(0.5,0.9)	0.028	0.027	0.022	0.036	0.029	0.027	0.031
ABC	(0.5,0.9)	0.032	0.029	0.021	0.026	0.031	0.028	0.031
ABD	(0.5,0.9)	0.027	0.024	0.025	0.026	0.031	0.032	0.028
ACD	(0.5,0.9)	0.027	0.025	0.024	0.034	0.030	0.028	0.029
BCD	(0.5,0.9)	0.021	0.021	0.024	0.021	0.028	0.026	0.026
ABCD	(0.5,0.9)	0.032	0.026	0.025	0.025	0.028	0.025	0.026
A	(0.9,0.9)	0.020	0.022	0.030	0.029	0.044	0.036	0.035
B	(0.9,0.9)	0.648	0.643	0.650	0.638	0.601	0.621	0.613
C	(0.9,0.9)	0.025	0.015	0.024	0.025	0.028	0.032	0.026
D	(0.9,0.9)	0.030	0.021	0.022	0.020	0.024	0.017	0.021
AB	(0.9,0.9)	0.634	0.634	0.651	0.635	0.596	0.627	0.610
AC	(0.9,0.9)	0.020	0.020	0.026	0.025	0.026	0.032	0.027
AD	(0.9,0.9)	0.024	0.020	0.022	0.028	0.021	0.019	0.020
BC	(0.9,0.9)	0.019	0.020	0.025	0.018	0.024	0.025	0.018
BD	(0.9,0.9)	0.016	0.019	0.018	0.022	0.021	0.022	0.028
CD	(0.9,0.9)	0.024	0.026	0.024	0.020	0.027	0.027	0.025
ABC	(0.9,0.9)	0.019	0.018	0.024	0.021	0.025	0.025	0.018
ABD	(0.9,0.9)	0.021	0.028	0.023	0.021	0.019	0.020	0.029

ACD	(0.9,0.9)	0.022	0.021	0.028	0.024	0.019	0.027	0.026
BCD	(0.9,0.9)	0.023	0.022	0.019	0.022	0.028	0.022	0.021
ABCD	(0.9,0.9)	0.026	0.027	0.021	0.020	0.026	0.022	0.020

Table B.11: RR I for BM86 assuming two location effects and one dispersion effect

Factors	$\mathfrak{L} = \{B, C\}$	$\mathfrak{D} = \{A\}$						
		1	2^2	3^2	5^2	10^2	20^2	50^2
A	(0.2,0.2)	0.042	0.042	0.056	0.054	0.047	0.071	0.062
B	(0.2,0.2)	0.102	0.093	0.106	0.092	0.104	0.102	0.111
C	(0.2,0.2)	0.101	0.102	0.095	0.094	0.104	0.108	0.105
D	(0.2,0.2)	0.035	0.042	0.039	0.035	0.034	0.037	0.038
AB	(0.2,0.2)	0.051	0.043	0.055	0.047	0.056	0.061	0.062
AC	(0.2,0.2)	0.043	0.050	0.045	0.047	0.052	0.067	0.061
AD	(0.2,0.2)	0.038	0.037	0.038	0.034	0.033	0.038	0.037
BC	(0.2,0.2)	0.044	0.041	0.048	0.039	0.039	0.039	0.027
BD	(0.2,0.2)	0.038	0.040	0.041	0.031	0.035	0.035	0.041
CD	(0.2,0.2)	0.047	0.036	0.039	0.033	0.031	0.034	0.031
ABC	(0.2,0.2)	0.042	0.039	0.043	0.041	0.036	0.040	0.027
ABD	(0.2,0.2)	0.042	0.039	0.035	0.032	0.032	0.037	0.041
ACD	(0.2,0.2)	0.045	0.044	0.039	0.030	0.031	0.035	0.031
BCD	(0.2,0.2)	0.046	0.042	0.036	0.033	0.035	0.045	0.034
ABCD	(0.2,0.2)	0.046	0.038	0.041	0.031	0.031	0.046	0.035
A	(0.2,0.5)	0.038	0.030	0.053	0.041	0.055	0.062	0.058
B	(0.2,0.5)	0.094	0.087	0.089	0.090	0.100	0.101	0.111
C	(0.2,0.5)	0.274	0.307	0.315	0.304	0.323	0.328	0.337
D	(0.2,0.5)	0.033	0.036	0.033	0.035	0.026	0.038	0.030
AB	(0.2,0.5)	0.031	0.044	0.043	0.041	0.050	0.064	0.065
AC	(0.2,0.5)	0.036	0.049	0.050	0.061	0.071	0.071	0.081

AD	(0.2,0.5)	0.045	0.033	0.033	0.034	0.026	0.038	0.031
BC	(0.2,0.5)	0.037	0.028	0.026	0.019	0.030	0.030	0.033
BD	(0.2,0.5)	0.033	0.039	0.038	0.025	0.029	0.030	0.031
CD	(0.2,0.5)	0.039	0.035	0.026	0.032	0.037	0.028	0.039
ABC	(0.2,0.5)	0.038	0.033	0.030	0.024	0.031	0.028	0.032
ABD	(0.2,0.5)	0.037	0.031	0.033	0.026	0.031	0.028	0.030
ACD	(0.2,0.5)	0.035	0.036	0.021	0.026	0.038	0.027	0.041
BCD	(0.2,0.5)	0.037	0.036	0.028	0.027	0.035	0.041	0.035
ABCD	(0.2,0.5)	0.039	0.042	0.026	0.028	0.034	0.040	0.033
A	(0.2,0.9)	0.042	0.040	0.049	0.058	0.061	0.070	0.065
B	(0.2,0.9)	0.092	0.089	0.094	0.102	0.100	0.128	0.115
C	(0.2,0.9)	0.694	0.722	0.711	0.717	0.732	0.717	0.692
D	(0.2,0.9)	0.043	0.040	0.032	0.027	0.039	0.037	0.038
AB	(0.2,0.9)	0.043	0.042	0.049	0.048	0.063	0.067	0.069
AC	(0.2,0.9)	0.037	0.041	0.054	0.050	0.065	0.076	0.064
AD	(0.2,0.9)	0.041	0.042	0.032	0.031	0.042	0.038	0.037
BC	(0.2,0.9)	0.035	0.037	0.030	0.041	0.036	0.042	0.038
BD	(0.2,0.9)	0.037	0.036	0.036	0.039	0.031	0.035	0.043
CD	(0.2,0.9)	0.045	0.038	0.042	0.039	0.040	0.032	0.037
ABC	(0.2,0.9)	0.049	0.035	0.027	0.041	0.035	0.041	0.038
ABD	(0.2,0.9)	0.040	0.039	0.027	0.038	0.031	0.037	0.042
ACD	(0.2,0.9)	0.041	0.036	0.039	0.040	0.042	0.032	0.036
BCD	(0.2,0.9)	0.049	0.038	0.027	0.035	0.034	0.039	0.041
ABCD	(0.2,0.9)	0.036	0.040	0.036	0.036	0.036	0.041	0.039
A	(0.5,0.5)	0.030	0.032	0.040	0.045	0.054	0.062	0.058
B	(0.5,0.5)	0.260	0.273	0.288	0.292	0.305	0.323	0.295
C	(0.5,0.5)	0.268	0.262	0.288	0.293	0.295	0.325	0.318
D	(0.5,0.5)	0.031	0.033	0.030	0.025	0.024	0.038	0.031
AB	(0.5,0.5)	0.027	0.036	0.056	0.046	0.074	0.070	0.073
AC	(0.5,0.5)	0.033	0.046	0.060	0.056	0.072	0.071	0.069
AD	(0.5,0.5)	0.035	0.032	0.026	0.020	0.022	0.038	0.031

BC	(0.5,0.5)	0.030	0.034	0.029	0.028	0.035	0.033	0.028
BD	(0.5,0.5)	0.036	0.032	0.022	0.026	0.024	0.028	0.028
CD	(0.5,0.5)	0.036	0.037	0.032	0.020	0.030	0.024	0.027
ABC	(0.5,0.5)	0.030	0.031	0.025	0.027	0.030	0.032	0.028
ABD	(0.5,0.5)	0.032	0.028	0.024	0.027	0.026	0.027	0.028
ACD	(0.5,0.5)	0.033	0.034	0.032	0.025	0.030	0.024	0.026
BCD	(0.5,0.5)	0.032	0.028	0.031	0.021	0.025	0.028	0.033
ABCD	(0.5,0.5)	0.039	0.034	0.032	0.019	0.024	0.029	0.032
A	(0.5,0.9)	0.033	0.043	0.038	0.052	0.060	0.061	0.054
B	(0.5,0.9)	0.265	0.278	0.289	0.291	0.323	0.318	0.290
C	(0.5,0.9)	0.684	0.689	0.693	0.690	0.688	0.704	0.689
D	(0.5,0.9)	0.037	0.032	0.026	0.031	0.033	0.037	0.037
AB	(0.5,0.9)	0.045	0.050	0.047	0.058	0.064	0.095	0.072
AC	(0.5,0.9)	0.031	0.042	0.043	0.051	0.067	0.073	0.065
AD	(0.5,0.9)	0.031	0.036	0.031	0.030	0.033	0.038	0.035
BC	(0.5,0.9)	0.037	0.031	0.025	0.026	0.032	0.031	0.029
BD	(0.5,0.9)	0.040	0.037	0.032	0.026	0.035	0.032	0.033
CD	(0.5,0.9)	0.031	0.036	0.028	0.038	0.032	0.031	0.034
ABC	(0.5,0.9)	0.038	0.035	0.028	0.029	0.032	0.033	0.030
ABD	(0.5,0.9)	0.037	0.029	0.022	0.030	0.038	0.030	0.034
ACD	(0.5,0.9)	0.035	0.033	0.027	0.033	0.028	0.032	0.033
BCD	(0.5,0.9)	0.022	0.032	0.026	0.022	0.031	0.026	0.031
ABCD	(0.5,0.9)	0.043	0.033	0.026	0.028	0.030	0.027	0.031
A	(0.9,0.9)	0.031	0.035	0.048	0.050	0.066	0.064	0.063
B	(0.9,0.9)	0.666	0.688	0.695	0.693	0.667	0.698	0.673
C	(0.9,0.9)	0.666	0.676	0.684	0.681	0.689	0.690	0.684
D	(0.9,0.9)	0.040	0.035	0.028	0.036	0.033	0.033	0.035
AB	(0.9,0.9)	0.037	0.046	0.057	0.060	0.064	0.073	0.066
AC	(0.9,0.9)	0.036	0.045	0.056	0.060	0.070	0.071	0.070
AD	(0.9,0.9)	0.031	0.026	0.030	0.035	0.031	0.031	0.035
BC	(0.9,0.9)	0.033	0.035	0.037	0.032	0.027	0.037	0.031

BD	(0.9,0.9)	0.032	0.037	0.025	0.032	0.029	0.027	0.042
CD	(0.9,0.9)	0.037	0.036	0.042	0.028	0.033	0.039	0.029
ABC	(0.9,0.9)	0.037	0.033	0.039	0.027	0.029	0.036	0.031
ABD	(0.9,0.9)	0.031	0.040	0.031	0.038	0.030	0.026	0.043
ACD	(0.9,0.9)	0.037	0.035	0.037	0.031	0.031	0.037	0.031
BCD	(0.9,0.9)	0.038	0.033	0.034	0.030	0.038	0.031	0.032
ABCD	(0.9,0.9)	0.040	0.038	0.028	0.026	0.036	0.030	0.033
Factors	$\mathfrak{L} = \{A, B\}$	$\mathfrak{D} = \{A\}$						
		1	2^2	3^2	5^2	10^2	20^2	50^2
A	(0.2,0.2)	0.100	0.096	0.115	0.130	0.127	0.136	0.129
B	(0.2,0.2)	0.099	0.093	0.106	0.111	0.107	0.111	0.115
C	(0.2,0.2)	0.041	0.042	0.037	0.034	0.038	0.041	0.043
D	(0.2,0.2)	0.041	0.042	0.044	0.036	0.040	0.037	0.039
AB	(0.2,0.2)	0.051	0.039	0.054	0.054	0.067	0.065	0.072
AC	(0.2,0.2)	0.048	0.040	0.034	0.033	0.038	0.040	0.042
AD	(0.2,0.2)	0.036	0.037	0.038	0.036	0.038	0.037	0.038
BC	(0.2,0.2)	0.048	0.038	0.047	0.044	0.037	0.043	0.028
BD	(0.2,0.2)	0.045	0.039	0.039	0.042	0.044	0.037	0.041
CD	(0.2,0.2)	0.042	0.039	0.041	0.036	0.033	0.042	0.036
ABC	(0.2,0.2)	0.043	0.042	0.047	0.042	0.037	0.044	0.029
ABD	(0.2,0.2)	0.044	0.040	0.037	0.041	0.041	0.038	0.041
ACD	(0.2,0.2)	0.044	0.045	0.043	0.034	0.037	0.041	0.036
BCD	(0.2,0.2)	0.053	0.042	0.037	0.039	0.041	0.049	0.038
ABCD	(0.2,0.2)	0.048	0.038	0.041	0.041	0.040	0.050	0.039
A	(0.2,0.5)	0.084	0.084	0.112	0.094	0.118	0.123	0.116
B	(0.2,0.5)	0.278	0.284	0.287	0.316	0.334	0.333	0.344
C	(0.2,0.5)	0.042	0.043	0.034	0.027	0.034	0.035	0.047
D	(0.2,0.5)	0.043	0.040	0.030	0.039	0.035	0.041	0.041
AB	(0.2,0.5)	0.036	0.054	0.068	0.062	0.070	0.081	0.092
AC	(0.2,0.5)	0.038	0.032	0.035	0.028	0.034	0.036	0.047
AD	(0.2,0.5)	0.052	0.033	0.032	0.038	0.034	0.043	0.041

BC	(0.2,0.5)	0.032	0.027	0.025	0.028	0.027	0.034	0.036
BD	(0.2,0.5)	0.037	0.040	0.042	0.030	0.036	0.036	0.035
CD	(0.2,0.5)	0.039	0.035	0.033	0.034	0.039	0.033	0.045
ABC	(0.2,0.5)	0.045	0.031	0.029	0.033	0.029	0.032	0.035
ABD	(0.2,0.5)	0.042	0.038	0.033	0.030	0.036	0.035	0.036
ACD	(0.2,0.5)	0.041	0.033	0.030	0.032	0.041	0.031	0.047
BCD	(0.2,0.5)	0.039	0.035	0.030	0.030	0.036	0.041	0.038
ABCD	(0.2,0.5)	0.038	0.035	0.029	0.031	0.035	0.041	0.039
A	(0.2,0.9)	0.092	0.097	0.109	0.118	0.122	0.125	0.130
B	(0.2,0.9)	0.693	0.717	0.718	0.722	0.712	0.716	0.710
C	(0.2,0.9)	0.042	0.038	0.041	0.038	0.037	0.045	0.039
D	(0.2,0.9)	0.046	0.041	0.035	0.032	0.039	0.042	0.041
AB	(0.2,0.9)	0.041	0.042	0.050	0.052	0.076	0.075	0.081
AC	(0.2,0.9)	0.032	0.043	0.038	0.033	0.042	0.046	0.038
AD	(0.2,0.9)	0.043	0.044	0.027	0.032	0.040	0.043	0.039
BC	(0.2,0.9)	0.035	0.041	0.033	0.042	0.042	0.049	0.043
BD	(0.2,0.9)	0.036	0.039	0.032	0.042	0.036	0.038	0.049
CD	(0.2,0.9)	0.044	0.031	0.039	0.039	0.045	0.038	0.045
ABC	(0.2,0.9)	0.044	0.041	0.029	0.050	0.044	0.050	0.042
ABD	(0.2,0.9)	0.038	0.038	0.024	0.041	0.035	0.041	0.049
ACD	(0.2,0.9)	0.042	0.038	0.033	0.041	0.045	0.041	0.044
BCD	(0.2,0.9)	0.041	0.039	0.032	0.033	0.047	0.043	0.042
ABCD	(0.2,0.9)	0.039	0.041	0.036	0.035	0.042	0.046	0.042
A	(0.5,0.5)	0.268	0.263	0.283	0.277	0.289	0.304	0.298
B	(0.5,0.5)	0.260	0.281	0.285	0.297	0.310	0.322	0.308
C	(0.5,0.5)	0.026	0.031	0.039	0.032	0.033	0.033	0.034
D	(0.5,0.5)	0.032	0.037	0.027	0.028	0.024	0.041	0.034
AB	(0.5,0.5)	0.035	0.039	0.052	0.055	0.078	0.069	0.075
AC	(0.5,0.5)	0.032	0.031	0.033	0.031	0.033	0.035	0.035
AD	(0.5,0.5)	0.030	0.037	0.021	0.027	0.024	0.041	0.033
BC	(0.5,0.5)	0.027	0.031	0.030	0.031	0.037	0.042	0.036

BD	(0.5,0.5)	0.033	0.032	0.030	0.031	0.026	0.029	0.036
CD	(0.5,0.5)	0.032	0.039	0.028	0.027	0.032	0.029	0.031
ABC	(0.5,0.5)	0.036	0.033	0.029	0.035	0.035	0.044	0.036
ABD	(0.5,0.5)	0.030	0.029	0.025	0.031	0.027	0.029	0.036
ACD	(0.5,0.5)	0.036	0.033	0.028	0.028	0.032	0.030	0.032
BCD	(0.5,0.5)	0.034	0.024	0.035	0.030	0.034	0.037	0.035
ABCD	(0.5,0.5)	0.038	0.031	0.036	0.023	0.033	0.035	0.034
A	(0.5,0.9)	0.264	0.265	0.289	0.288	0.293	0.325	0.283
B	(0.5,0.9)	0.684	0.685	0.702	0.688	0.698	0.703	0.682
C	(0.5,0.9)	0.038	0.037	0.027	0.031	0.043	0.048	0.036
D	(0.5,0.9)	0.040	0.035	0.032	0.032	0.038	0.048	0.036
AB	(0.5,0.9)	0.044	0.045	0.048	0.047	0.055	0.077	0.065
AC	(0.5,0.9)	0.030	0.031	0.027	0.028	0.041	0.048	0.036
AD	(0.5,0.9)	0.032	0.035	0.025	0.031	0.037	0.048	0.036
BC	(0.5,0.9)	0.038	0.041	0.033	0.034	0.038	0.032	0.038
BD	(0.5,0.9)	0.038	0.042	0.035	0.030	0.038	0.037	0.043
CD	(0.5,0.9)	0.040	0.034	0.030	0.035	0.038	0.032	0.041
ABC	(0.5,0.9)	0.043	0.038	0.036	0.032	0.038	0.033	0.039
ABD	(0.5,0.9)	0.038	0.028	0.025	0.036	0.039	0.040	0.042
ACD	(0.5,0.9)	0.038	0.031	0.034	0.034	0.038	0.034	0.041
BCD	(0.5,0.9)	0.031	0.026	0.030	0.029	0.036	0.031	0.033
ABCD	(0.5,0.9)	0.042	0.032	0.030	0.031	0.041	0.030	0.035
A	(0.9,0.9)	0.662	0.660	0.671	0.678	0.668	0.679	0.673
B	(0.9,0.9)	0.676	0.681	0.685	0.685	0.652	0.685	0.678
C	(0.9,0.9)	0.030	0.029	0.041	0.038	0.037	0.042	0.038
D	(0.9,0.9)	0.036	0.032	0.035	0.035	0.038	0.033	0.038
AB	(0.9,0.9)	0.042	0.045	0.057	0.061	0.061	0.073	0.073
AC	(0.9,0.9)	0.035	0.032	0.041	0.038	0.038	0.041	0.038
AD	(0.9,0.9)	0.029	0.030	0.033	0.037	0.039	0.034	0.038
BC	(0.9,0.9)	0.028	0.029	0.036	0.031	0.030	0.042	0.031
BD	(0.9,0.9)	0.030	0.035	0.027	0.037	0.030	0.027	0.047

CD	(0.9,0.9)	0.037	0.035	0.037	0.030	0.037	0.039	0.033
ABC	(0.9,0.9)	0.035	0.030	0.039	0.029	0.031	0.043	0.032
ABD	(0.9,0.9)	0.033	0.037	0.033	0.039	0.032	0.027	0.048
ACD	(0.9,0.9)	0.037	0.033	0.038	0.032	0.033	0.039	0.036
BCD	(0.9,0.9)	0.034	0.031	0.038	0.037	0.042	0.036	0.032
ABCD	(0.9,0.9)	0.035	0.036	0.031	0.031	0.036	0.036	0.033

Factors	$\mathfrak{L} = \{B, AB\}$	$\mathfrak{D} = \{A\}$						
		1	2^2	3^2	5^2	10^2	20^2	50^2
A	(0.2,0.2)	0.037	0.042	0.052	0.058	0.061	0.081	0.074
B	(0.2,0.2)	0.106	0.089	0.097	0.088	0.085	0.085	0.093
C	(0.2,0.2)	0.039	0.039	0.041	0.033	0.043	0.043	0.048
D	(0.2,0.2)	0.040	0.042	0.039	0.033	0.043	0.044	0.048
AB	(0.2,0.2)	0.113	0.076	0.097	0.093	0.088	0.084	0.094
AC	(0.2,0.2)	0.048	0.043	0.035	0.036	0.040	0.048	0.047
AD	(0.2,0.2)	0.039	0.039	0.039	0.038	0.041	0.045	0.048
BC	(0.2,0.2)	0.052	0.036	0.049	0.050	0.039	0.050	0.039
BD	(0.2,0.2)	0.044	0.039	0.040	0.041	0.044	0.045	0.050
CD	(0.2,0.2)	0.042	0.044	0.039	0.043	0.037	0.042	0.039
ABC	(0.2,0.2)	0.043	0.043	0.047	0.047	0.036	0.054	0.043
ABD	(0.2,0.2)	0.044	0.043	0.036	0.041	0.042	0.045	0.050
ACD	(0.2,0.2)	0.046	0.047	0.044	0.040	0.038	0.044	0.040
BCD	(0.2,0.2)	0.055	0.045	0.038	0.038	0.045	0.054	0.045
ABCD	(0.2,0.2)	0.052	0.037	0.039	0.034	0.043	0.053	0.045
A	(0.2,0.5)	0.039	0.030	0.058	0.044	0.054	0.071	0.066
B	(0.2,0.5)	0.088	0.084	0.098	0.105	0.114	0.131	0.138
C	(0.2,0.5)	0.036	0.042	0.036	0.029	0.035	0.039	0.045
D	(0.2,0.5)	0.038	0.039	0.031	0.037	0.035	0.041	0.036
AB	(0.2,0.5)	0.278	0.273	0.273	0.285	0.272	0.288	0.300
AC	(0.2,0.5)	0.037	0.038	0.033	0.026	0.033	0.040	0.044
AD	(0.2,0.5)	0.042	0.033	0.031	0.035	0.036	0.043	0.034
BC	(0.2,0.5)	0.028	0.033	0.024	0.026	0.028	0.035	0.038

BD	(0.2,0.5)	0.031	0.042	0.038	0.030	0.033	0.042	0.036
CD	(0.2,0.5)	0.041	0.039	0.030	0.029	0.038	0.033	0.044
ABC	(0.2,0.5)	0.038	0.033	0.031	0.027	0.028	0.036	0.038
ABD	(0.2,0.5)	0.041	0.037	0.032	0.032	0.031	0.039	0.035
ACD	(0.2,0.5)	0.041	0.035	0.030	0.026	0.041	0.032	0.042
BCD	(0.2,0.5)	0.039	0.037	0.030	0.033	0.036	0.044	0.037
ABCD	(0.2,0.5)	0.038	0.037	0.032	0.031	0.038	0.044	0.036
A	(0.2,0.9)	0.037	0.041	0.043	0.061	0.060	0.073	0.070
B	(0.2,0.9)	0.095	0.099	0.113	0.133	0.138	0.162	0.152
C	(0.2,0.9)	0.042	0.032	0.040	0.035	0.035	0.042	0.035
D	(0.2,0.9)	0.044	0.038	0.032	0.028	0.039	0.042	0.038
AB	(0.2,0.9)	0.711	0.714	0.736	0.746	0.736	0.740	0.732
AC	(0.2,0.9)	0.034	0.031	0.043	0.032	0.037	0.043	0.035
AD	(0.2,0.9)	0.042	0.037	0.030	0.033	0.043	0.043	0.039
BC	(0.2,0.9)	0.037	0.039	0.035	0.044	0.040	0.050	0.044
BD	(0.2,0.9)	0.032	0.038	0.031	0.041	0.036	0.042	0.050
CD	(0.2,0.9)	0.050	0.032	0.039	0.047	0.043	0.035	0.041
ABC	(0.2,0.9)	0.051	0.032	0.028	0.048	0.037	0.052	0.043
ABD	(0.2,0.9)	0.038	0.043	0.025	0.034	0.036	0.041	0.052
ACD	(0.2,0.9)	0.044	0.035	0.038	0.041	0.043	0.036	0.040
BCD	(0.2,0.9)	0.044	0.042	0.030	0.037	0.041	0.042	0.047
ABCD	(0.2,0.9)	0.036	0.043	0.036	0.036	0.041	0.044	0.049
A	(0.5,0.5)	0.034	0.037	0.039	0.054	0.055	0.071	0.067
B	(0.5,0.5)	0.256	0.258	0.255	0.250	0.241	0.267	0.247
C	(0.5,0.5)	0.032	0.035	0.041	0.032	0.038	0.033	0.035
D	(0.5,0.5)	0.034	0.036	0.027	0.036	0.029	0.045	0.039
AB	(0.5,0.5)	0.258	0.249	0.239	0.250	0.246	0.264	0.246
AC	(0.5,0.5)	0.036	0.031	0.038	0.032	0.038	0.032	0.035
AD	(0.5,0.5)	0.029	0.027	0.024	0.038	0.028	0.047	0.041
BC	(0.5,0.5)	0.031	0.032	0.028	0.038	0.039	0.042	0.040
BD	(0.5,0.5)	0.036	0.027	0.024	0.036	0.031	0.032	0.042

CD	(0.5,0.5)	0.031	0.036	0.026	0.024	0.038	0.029	0.036
ABC	(0.5,0.5)	0.035	0.031	0.027	0.041	0.035	0.043	0.039
ABD	(0.5,0.5)	0.033	0.026	0.022	0.034	0.028	0.035	0.042
ACD	(0.5,0.5)	0.036	0.030	0.026	0.031	0.035	0.033	0.037
BCD	(0.5,0.5)	0.037	0.028	0.034	0.031	0.036	0.036	0.043
ABCD	(0.5,0.5)	0.036	0.036	0.036	0.027	0.035	0.036	0.042
A	(0.5,0.9)	0.037	0.041	0.041	0.052	0.064	0.068	0.056
B	(0.5,0.9)	0.267	0.283	0.302	0.296	0.334	0.332	0.298
C	(0.5,0.9)	0.034	0.036	0.023	0.029	0.043	0.043	0.040
D	(0.5,0.9)	0.039	0.032	0.033	0.028	0.038	0.045	0.034
AB	(0.5,0.9)	0.674	0.689	0.678	0.688	0.703	0.713	0.682
AC	(0.5,0.9)	0.027	0.031	0.030	0.029	0.042	0.041	0.039
AD	(0.5,0.9)	0.034	0.032	0.029	0.029	0.041	0.044	0.033
BC	(0.5,0.9)	0.035	0.030	0.030	0.031	0.038	0.035	0.038
BD	(0.5,0.9)	0.039	0.035	0.031	0.030	0.036	0.036	0.038
CD	(0.5,0.9)	0.035	0.037	0.036	0.041	0.038	0.034	0.031
ABC	(0.5,0.9)	0.039	0.037	0.028	0.032	0.038	0.038	0.038
ABD	(0.5,0.9)	0.035	0.030	0.030	0.036	0.040	0.037	0.039
ACD	(0.5,0.9)	0.034	0.026	0.031	0.038	0.033	0.033	0.030
BCD	(0.5,0.9)	0.026	0.028	0.033	0.021	0.038	0.033	0.032
ABCD	(0.5,0.9)	0.042	0.032	0.033	0.029	0.041	0.031	0.033
A	(0.9,0.9)	0.036	0.037	0.044	0.056	0.070	0.066	0.069
B	(0.9,0.9)	0.675	0.660	0.655	0.638	0.600	0.632	0.619
C	(0.9,0.9)	0.038	0.026	0.041	0.032	0.036	0.043	0.041
D	(0.9,0.9)	0.037	0.036	0.037	0.035	0.035	0.037	0.040
AB	(0.9,0.9)	0.667	0.645	0.665	0.646	0.598	0.633	0.619
AC	(0.9,0.9)	0.030	0.031	0.041	0.034	0.039	0.043	0.040
AD	(0.9,0.9)	0.029	0.036	0.035	0.038	0.036	0.036	0.041
BC	(0.9,0.9)	0.031	0.037	0.041	0.035	0.035	0.038	0.035
BD	(0.9,0.9)	0.033	0.035	0.035	0.033	0.028	0.032	0.051
CD	(0.9,0.9)	0.036	0.038	0.037	0.030	0.036	0.042	0.036

ABC	(0.9,0.9)	0.034	0.033	0.044	0.033	0.032	0.039	0.033
ABD	(0.9,0.9)	0.034	0.041	0.035	0.042	0.030	0.031	0.050
ACD	(0.9,0.9)	0.036	0.037	0.039	0.034	0.035	0.042	0.036
BCD	(0.9,0.9)	0.036	0.035	0.028	0.037	0.041	0.039	0.036
ABCD	(0.9,0.9)	0.038	0.037	0.036	0.027	0.037	0.037	0.036

Table B.12: RR I for LN97 assuming two location effects and one dispersion effect

Factors	$\mathfrak{L} = \{B, C\}$	$\mathfrak{D} = \{A\}$						
		1	2^2	3^2	5^2	10^2	20^2	50^2
A	(0.2,0.2)	0.038	0.038	0.050	0.053	0.066	0.096	0.098
B	(0.2,0.2)	0.083	0.078	0.083	0.092	0.116	0.122	0.144
C	(0.2,0.2)	0.075	0.077	0.078	0.093	0.123	0.129	0.137
D	(0.2,0.2)	0.035	0.040	0.042	0.043	0.053	0.061	0.080
AB	(0.2,0.2)	0.046	0.045	0.046	0.054	0.073	0.090	0.100
AC	(0.2,0.2)	0.042	0.047	0.047	0.052	0.081	0.088	0.104
AD	(0.2,0.2)	0.039	0.032	0.038	0.041	0.053	0.060	0.080
BC	(0.2,0.2)	0.035	0.038	0.043	0.048	0.065	0.067	0.077
BD	(0.2,0.2)	0.035	0.035	0.047	0.041	0.061	0.067	0.081
CD	(0.2,0.2)	0.042	0.034	0.041	0.043	0.052	0.070	0.073
ABC	(0.2,0.2)	0.041	0.041	0.041	0.049	0.064	0.067	0.077
ABD	(0.2,0.2)	0.037	0.037	0.039	0.042	0.061	0.066	0.081
ACD	(0.2,0.2)	0.042	0.043	0.043	0.045	0.054	0.070	0.073
BCD	(0.2,0.2)	0.037	0.033	0.040	0.039	0.060	0.070	0.075
ABCD	(0.2,0.2)	0.042	0.038	0.041	0.037	0.061	0.071	0.075
A	(0.2,0.5)	0.037	0.037	0.058	0.047	0.072	0.100	0.081
B	(0.2,0.5)	0.079	0.074	0.087	0.083	0.118	0.138	0.150
C	(0.2,0.5)	0.207	0.232	0.259	0.268	0.304	0.330	0.332
D	(0.2,0.5)	0.041	0.039	0.037	0.047	0.049	0.076	0.073

AB	(0.2,0.5)	0.031	0.056	0.049	0.048	0.078	0.098	0.107
AC	(0.2,0.5)	0.033	0.047	0.049	0.060	0.084	0.095	0.109
AD	(0.2,0.5)	0.042	0.038	0.040	0.050	0.050	0.076	0.073
BC	(0.2,0.5)	0.034	0.041	0.036	0.036	0.047	0.068	0.076
BD	(0.2,0.5)	0.038	0.043	0.039	0.040	0.053	0.068	0.074
CD	(0.2,0.5)	0.041	0.042	0.037	0.037	0.062	0.070	0.082
ABC	(0.2,0.5)	0.041	0.042	0.037	0.038	0.048	0.068	0.076
ABD	(0.2,0.5)	0.036	0.044	0.038	0.039	0.052	0.068	0.074
ACD	(0.2,0.5)	0.039	0.041	0.034	0.038	0.064	0.070	0.082
BCD	(0.2,0.5)	0.043	0.044	0.038	0.039	0.060	0.079	0.077
ABCD	(0.2,0.5)	0.040	0.039	0.037	0.040	0.059	0.079	0.077
A	(0.2,0.9)	0.038	0.044	0.049	0.060	0.075	0.095	0.103
B	(0.2,0.9)	0.077	0.077	0.071	0.099	0.123	0.156	0.164
C	(0.2,0.9)	0.559	0.601	0.608	0.626	0.651	0.652	0.651
D	(0.2,0.9)	0.044	0.038	0.034	0.038	0.062	0.067	0.081
AB	(0.2,0.9)	0.040	0.039	0.040	0.057	0.084	0.108	0.108
AC	(0.2,0.9)	0.037	0.041	0.049	0.054	0.077	0.090	0.102
AD	(0.2,0.9)	0.047	0.038	0.035	0.042	0.062	0.067	0.081
BC	(0.2,0.9)	0.036	0.036	0.035	0.050	0.058	0.085	0.090
BD	(0.2,0.9)	0.039	0.042	0.030	0.045	0.056	0.072	0.087
CD	(0.2,0.9)	0.045	0.037	0.040	0.048	0.063	0.072	0.079
ABC	(0.2,0.9)	0.042	0.037	0.031	0.053	0.060	0.085	0.090
ABD	(0.2,0.9)	0.043	0.039	0.026	0.043	0.055	0.072	0.087
ACD	(0.2,0.9)	0.042	0.036	0.039	0.049	0.064	0.072	0.079
BCD	(0.2,0.9)	0.039	0.038	0.036	0.052	0.053	0.075	0.077
ABCD	(0.2,0.9)	0.039	0.040	0.037	0.049	0.054	0.075	0.077
A	(0.5,0.5)	0.036	0.043	0.045	0.049	0.070	0.093	0.088
B	(0.5,0.5)	0.184	0.209	0.238	0.252	0.283	0.326	0.309
C	(0.5,0.5)	0.202	0.204	0.238	0.244	0.275	0.324	0.326
D	(0.5,0.5)	0.039	0.037	0.037	0.037	0.042	0.079	0.084
AB	(0.5,0.5)	0.030	0.033	0.058	0.050	0.077	0.092	0.108

AC	(0.5,0.5)	0.037	0.041	0.053	0.056	0.076	0.101	0.104
AD	(0.5,0.5)	0.029	0.039	0.038	0.037	0.043	0.079	0.084
BC	(0.5,0.5)	0.037	0.038	0.037	0.038	0.055	0.076	0.079
BD	(0.5,0.5)	0.033	0.039	0.031	0.038	0.050	0.073	0.082
CD	(0.5,0.5)	0.042	0.039	0.039	0.035	0.060	0.068	0.081
ABC	(0.5,0.5)	0.036	0.030	0.036	0.040	0.056	0.076	0.079
ABD	(0.5,0.5)	0.033	0.031	0.027	0.042	0.053	0.072	0.082
ACD	(0.5,0.5)	0.041	0.033	0.033	0.037	0.059	0.068	0.081
BCD	(0.5,0.5)	0.040	0.033	0.039	0.033	0.057	0.068	0.073
ABCD	(0.5,0.5)	0.043	0.037	0.040	0.036	0.056	0.067	0.073
A	(0.5,0.9)	0.044	0.047	0.039	0.055	0.073	0.089	0.102
B	(0.5,0.9)	0.198	0.225	0.228	0.260	0.300	0.323	0.316
C	(0.5,0.9)	0.552	0.577	0.594	0.601	0.610	0.644	0.644
D	(0.5,0.9)	0.041	0.036	0.033	0.039	0.054	0.078	0.075
AB	(0.5,0.9)	0.047	0.043	0.047	0.056	0.078	0.106	0.112
AC	(0.5,0.9)	0.037	0.041	0.036	0.052	0.081	0.095	0.097
AD	(0.5,0.9)	0.043	0.037	0.033	0.040	0.052	0.078	0.075
BC	(0.5,0.9)	0.038	0.041	0.037	0.039	0.062	0.071	0.088
BD	(0.5,0.9)	0.039	0.039	0.031	0.038	0.068	0.067	0.089
CD	(0.5,0.9)	0.036	0.041	0.032	0.045	0.050	0.062	0.080
ABC	(0.5,0.9)	0.049	0.043	0.033	0.038	0.060	0.071	0.088
ABD	(0.5,0.9)	0.041	0.042	0.032	0.042	0.069	0.066	0.089
ACD	(0.5,0.9)	0.043	0.043	0.035	0.045	0.050	0.061	0.080
BCD	(0.5,0.9)	0.030	0.033	0.038	0.037	0.068	0.060	0.081
ABCD	(0.5,0.9)	0.044	0.035	0.035	0.042	0.067	0.060	0.081
A	(0.9,0.9)	0.033	0.040	0.053	0.052	0.079	0.090	0.108
B	(0.9,0.9)	0.536	0.564	0.599	0.605	0.592	0.626	0.630
C	(0.9,0.9)	0.541	0.563	0.581	0.580	0.608	0.620	0.627
D	(0.9,0.9)	0.037	0.038	0.047	0.048	0.056	0.077	0.088
AB	(0.9,0.9)	0.037	0.040	0.054	0.064	0.064	0.095	0.106
AC	(0.9,0.9)	0.036	0.044	0.052	0.066	0.072	0.096	0.104

AD	(0.9,0.9)	0.029	0.033	0.041	0.048	0.055	0.076	0.088
BC	(0.9,0.9)	0.035	0.034	0.050	0.039	0.051	0.078	0.085
BD	(0.9,0.9)	0.029	0.039	0.043	0.049	0.048	0.068	0.087
CD	(0.9,0.9)	0.037	0.036	0.053	0.045	0.058	0.077	0.075
ABC	(0.9,0.9)	0.038	0.031	0.054	0.042	0.053	0.079	0.085
ABD	(0.9,0.9)	0.035	0.044	0.045	0.050	0.048	0.068	0.087
ACD	(0.9,0.9)	0.043	0.033	0.056	0.047	0.055	0.078	0.075
BCD	(0.9,0.9)	0.036	0.039	0.044	0.048	0.064	0.073	0.079
ABCD	(0.9,0.9)	0.039	0.038	0.046	0.045	0.063	0.073	0.079

Factors	$\mathfrak{L} = \{A, B\}$	$\mathfrak{D} = \{A\}$						
		1	2^2	3^2	5^2	10^2	20^2	50^2
A	(0.2,0.2)	0.078	0.079	0.096	0.119	0.132	0.161	0.161
B	(0.2,0.2)	0.077	0.071	0.088	0.106	0.133	0.152	0.164
C	(0.2,0.2)	0.042	0.038	0.038	0.044	0.066	0.083	0.088
D	(0.2,0.2)	0.038	0.048	0.042	0.054	0.060	0.081	0.090
AB	(0.2,0.2)	0.043	0.044	0.049	0.060	0.082	0.106	0.123
AC	(0.2,0.2)	0.045	0.045	0.044	0.046	0.067	0.084	0.088
AD	(0.2,0.2)	0.036	0.040	0.039	0.053	0.059	0.080	0.090
BC	(0.2,0.2)	0.047	0.040	0.046	0.054	0.064	0.087	0.079
BD	(0.2,0.2)	0.039	0.037	0.043	0.059	0.070	0.078	0.093
CD	(0.2,0.2)	0.043	0.037	0.043	0.050	0.053	0.088	0.088
ABC	(0.2,0.2)	0.048	0.043	0.046	0.055	0.064	0.088	0.079
ABD	(0.2,0.2)	0.044	0.041	0.041	0.056	0.070	0.078	0.093
ACD	(0.2,0.2)	0.041	0.047	0.045	0.051	0.055	0.088	0.088
BCD	(0.2,0.2)	0.044	0.038	0.039	0.049	0.069	0.092	0.098
ABCD	(0.2,0.2)	0.048	0.038	0.042	0.047	0.070	0.093	0.098
A	(0.2,0.5)	0.068	0.076	0.095	0.091	0.133	0.159	0.171
B	(0.2,0.5)	0.203	0.224	0.244	0.290	0.327	0.359	0.373
C	(0.2,0.5)	0.039	0.048	0.039	0.048	0.063	0.087	0.106
D	(0.2,0.5)	0.042	0.044	0.033	0.047	0.060	0.095	0.102
AB	(0.2,0.5)	0.035	0.054	0.060	0.065	0.078	0.119	0.136

AC	(0.2,0.5)	0.044	0.041	0.040	0.047	0.062	0.087	0.106
AD	(0.2,0.5)	0.051	0.044	0.036	0.048	0.061	0.095	0.102
BC	(0.2,0.5)	0.041	0.032	0.031	0.041	0.050	0.086	0.089
BD	(0.2,0.5)	0.043	0.039	0.047	0.042	0.063	0.091	0.102
CD	(0.2,0.5)	0.041	0.042	0.037	0.043	0.070	0.088	0.103
ABC	(0.2,0.5)	0.049	0.035	0.037	0.045	0.052	0.087	0.089
ABD	(0.2,0.5)	0.041	0.041	0.037	0.045	0.061	0.091	0.103
ACD	(0.2,0.5)	0.039	0.039	0.032	0.046	0.071	0.086	0.103
BCD	(0.2,0.5)	0.039	0.046	0.040	0.043	0.059	0.088	0.098
ABCD	(0.2,0.5)	0.037	0.045	0.039	0.042	0.056	0.088	0.098
A	(0.2,0.9)	0.078	0.074	0.092	0.116	0.132	0.186	0.191
B	(0.2,0.9)	0.570	0.598	0.614	0.637	0.650	0.671	0.687
C	(0.2,0.9)	0.043	0.038	0.044	0.048	0.079	0.090	0.112
D	(0.2,0.9)	0.043	0.041	0.044	0.049	0.072	0.084	0.101
AB	(0.2,0.9)	0.044	0.044	0.045	0.056	0.089	0.110	0.132
AC	(0.2,0.9)	0.040	0.038	0.048	0.047	0.078	0.091	0.112
AD	(0.2,0.9)	0.041	0.041	0.036	0.049	0.071	0.084	0.101
BC	(0.2,0.9)	0.040	0.038	0.041	0.058	0.073	0.096	0.115
BD	(0.2,0.9)	0.037	0.047	0.032	0.052	0.066	0.088	0.113
CD	(0.2,0.9)	0.045	0.036	0.043	0.053	0.080	0.086	0.099
ABC	(0.2,0.9)	0.045	0.043	0.038	0.060	0.075	0.096	0.115
ABD	(0.2,0.9)	0.042	0.037	0.028	0.050	0.068	0.088	0.113
ACD	(0.2,0.9)	0.042	0.039	0.042	0.053	0.081	0.086	0.099
BCD	(0.2,0.9)	0.043	0.041	0.034	0.052	0.076	0.092	0.104
ABCD	(0.2,0.9)	0.047	0.044	0.038	0.044	0.078	0.092	0.104
A	(0.5,0.5)	0.195	0.198	0.220	0.238	0.283	0.312	0.324
B	(0.5,0.5)	0.189	0.223	0.245	0.266	0.307	0.345	0.356
C	(0.5,0.5)	0.030	0.037	0.047	0.038	0.063	0.085	0.098
D	(0.5,0.5)	0.037	0.043	0.038	0.039	0.056	0.088	0.099
AB	(0.5,0.5)	0.034	0.037	0.056	0.061	0.094	0.101	0.132
AC	(0.5,0.5)	0.036	0.035	0.041	0.043	0.062	0.084	0.098

AD	(0.5,0.5)	0.032	0.042	0.035	0.038	0.055	0.089	0.099
BC	(0.5,0.5)	0.037	0.042	0.033	0.043	0.068	0.088	0.099
BD	(0.5,0.5)	0.031	0.040	0.032	0.041	0.060	0.081	0.093
CD	(0.5,0.5)	0.037	0.046	0.035	0.041	0.063	0.078	0.094
ABC	(0.5,0.5)	0.037	0.037	0.037	0.044	0.068	0.087	0.099
ABD	(0.5,0.5)	0.028	0.033	0.027	0.045	0.062	0.079	0.093
ACD	(0.5,0.5)	0.034	0.042	0.037	0.042	0.065	0.078	0.095
BCD	(0.5,0.5)	0.042	0.036	0.041	0.036	0.068	0.072	0.095
ABCD	(0.5,0.5)	0.039	0.040	0.042	0.039	0.067	0.072	0.095
A	(0.5,0.9)	0.207	0.209	0.233	0.256	0.285	0.331	0.317
B	(0.5,0.9)	0.561	0.580	0.593	0.615	0.645	0.667	0.651
C	(0.5,0.9)	0.043	0.042	0.041	0.049	0.067	0.088	0.101
D	(0.5,0.9)	0.044	0.035	0.035	0.043	0.064	0.092	0.098
AB	(0.5,0.9)	0.049	0.037	0.047	0.053	0.078	0.113	0.119
AC	(0.5,0.9)	0.040	0.038	0.039	0.045	0.067	0.087	0.101
AD	(0.5,0.9)	0.036	0.039	0.036	0.044	0.062	0.092	0.098
BC	(0.5,0.9)	0.042	0.044	0.044	0.044	0.068	0.081	0.112
BD	(0.5,0.9)	0.044	0.044	0.035	0.047	0.074	0.083	0.110
CD	(0.5,0.9)	0.037	0.041	0.038	0.049	0.058	0.079	0.098
ABC	(0.5,0.9)	0.048	0.042	0.040	0.042	0.068	0.082	0.112
ABD	(0.5,0.9)	0.038	0.043	0.037	0.048	0.074	0.084	0.111
ACD	(0.5,0.9)	0.044	0.039	0.035	0.050	0.060	0.078	0.098
BCD	(0.5,0.9)	0.036	0.036	0.035	0.044	0.070	0.071	0.103
ABCD	(0.5,0.9)	0.041	0.035	0.032	0.046	0.067	0.071	0.103
A	(0.9,0.9)	0.541	0.544	0.569	0.609	0.609	0.639	0.639
B	(0.9,0.9)	0.535	0.565	0.601	0.621	0.610	0.648	0.649
C	(0.9,0.9)	0.037	0.032	0.043	0.059	0.064	0.087	0.100
D	(0.9,0.9)	0.038	0.037	0.041	0.049	0.065	0.087	0.103
AB	(0.9,0.9)	0.040	0.041	0.051	0.065	0.074	0.105	0.118
AC	(0.9,0.9)	0.036	0.041	0.042	0.057	0.066	0.087	0.100
AD	(0.9,0.9)	0.036	0.032	0.038	0.047	0.063	0.086	0.103

BC	(0.9,0.9)	0.037	0.034	0.044	0.043	0.057	0.092	0.101
BD	(0.9,0.9)	0.035	0.035	0.044	0.057	0.058	0.079	0.105
CD	(0.9,0.9)	0.040	0.031	0.053	0.047	0.065	0.089	0.089
ABC	(0.9,0.9)	0.035	0.035	0.043	0.045	0.056	0.092	0.101
ABD	(0.9,0.9)	0.033	0.039	0.043	0.058	0.057	0.079	0.105
ACD	(0.9,0.9)	0.041	0.037	0.051	0.049	0.063	0.090	0.089
BCD	(0.9,0.9)	0.037	0.037	0.044	0.047	0.071	0.078	0.098
ABCD	(0.9,0.9)	0.041	0.039	0.044	0.043	0.069	0.078	0.098

Factors	$\mathfrak{L} = \{B, AB\}$	$\mathfrak{D} = \{A\}$						
		1	2^2	3^2	5^2	10^2	20^2	50^2
A	(0.2,0.2)	0.038	0.038	0.049	0.070	0.087	0.119	0.131
B	(0.2,0.2)	0.086	0.074	0.075	0.102	0.130	0.140	0.171
C	(0.2,0.2)	0.043	0.047	0.038	0.050	0.079	0.094	0.108
D	(0.2,0.2)	0.041	0.041	0.039	0.048	0.066	0.087	0.115
AB	(0.2,0.2)	0.092	0.066	0.081	0.104	0.129	0.141	0.170
AC	(0.2,0.2)	0.039	0.047	0.037	0.050	0.078	0.095	0.110
AD	(0.2,0.2)	0.041	0.039	0.035	0.047	0.067	0.085	0.115
BC	(0.2,0.2)	0.045	0.042	0.044	0.058	0.078	0.095	0.104
BD	(0.2,0.2)	0.036	0.039	0.042	0.052	0.075	0.098	0.119
CD	(0.2,0.2)	0.041	0.042	0.036	0.055	0.062	0.099	0.109
ABC	(0.2,0.2)	0.047	0.042	0.045	0.059	0.075	0.095	0.103
ABD	(0.2,0.2)	0.044	0.044	0.036	0.055	0.075	0.098	0.118
ACD	(0.2,0.2)	0.047	0.045	0.035	0.054	0.066	0.101	0.110
BCD	(0.2,0.2)	0.044	0.043	0.044	0.048	0.073	0.101	0.112
ABCD	(0.2,0.2)	0.044	0.039	0.047	0.047	0.075	0.101	0.111
A	(0.2,0.5)	0.038	0.038	0.053	0.044	0.081	0.113	0.104
B	(0.2,0.5)	0.077	0.084	0.093	0.108	0.135	0.175	0.195
C	(0.2,0.5)	0.038	0.049	0.044	0.044	0.064	0.076	0.098
D	(0.2,0.5)	0.039	0.042	0.033	0.053	0.058	0.093	0.093
AB	(0.2,0.5)	0.202	0.208	0.231	0.259	0.292	0.329	0.334
AC	(0.2,0.5)	0.034	0.045	0.044	0.045	0.065	0.077	0.098

AD	(0.2,0.5)	0.043	0.040	0.037	0.055	0.060	0.093	0.093
BC	(0.2,0.5)	0.035	0.036	0.038	0.038	0.059	0.087	0.089
BD	(0.2,0.5)	0.033	0.046	0.041	0.045	0.064	0.087	0.085
CD	(0.2,0.5)	0.039	0.042	0.038	0.043	0.077	0.085	0.095
ABC	(0.2,0.5)	0.043	0.041	0.038	0.043	0.062	0.087	0.089
ABD	(0.2,0.5)	0.039	0.041	0.036	0.050	0.061	0.086	0.086
ACD	(0.2,0.5)	0.035	0.042	0.034	0.042	0.079	0.085	0.095
BCD	(0.2,0.5)	0.040	0.045	0.042	0.045	0.067	0.087	0.088
ABCD	(0.2,0.5)	0.035	0.044	0.036	0.045	0.067	0.087	0.088
A	(0.2,0.9)	0.040	0.048	0.046	0.064	0.080	0.112	0.125
B	(0.2,0.9)	0.083	0.084	0.086	0.115	0.138	0.181	0.198
C	(0.2,0.9)	0.048	0.041	0.044	0.047	0.068	0.089	0.104
D	(0.2,0.9)	0.047	0.041	0.035	0.049	0.073	0.087	0.105
AB	(0.2,0.9)	0.575	0.613	0.638	0.685	0.685	0.711	0.719
AC	(0.2,0.9)	0.039	0.039	0.047	0.047	0.069	0.089	0.104
AD	(0.2,0.9)	0.049	0.037	0.036	0.047	0.072	0.087	0.105
BC	(0.2,0.9)	0.041	0.041	0.039	0.059	0.063	0.101	0.107
BD	(0.2,0.9)	0.045	0.045	0.027	0.047	0.061	0.090	0.115
CD	(0.2,0.9)	0.052	0.044	0.037	0.055	0.070	0.087	0.104
ABC	(0.2,0.9)	0.042	0.043	0.035	0.060	0.065	0.101	0.107
ABD	(0.2,0.9)	0.045	0.047	0.030	0.047	0.062	0.090	0.115
ACD	(0.2,0.9)	0.043	0.044	0.039	0.057	0.070	0.088	0.104
BCD	(0.2,0.9)	0.044	0.041	0.039	0.059	0.065	0.089	0.105
ABCD	(0.2,0.9)	0.046	0.045	0.039	0.050	0.065	0.089	0.105
A	(0.5,0.5)	0.039	0.045	0.046	0.064	0.080	0.112	0.128
B	(0.5,0.5)	0.188	0.210	0.230	0.247	0.285	0.343	0.346
C	(0.5,0.5)	0.034	0.038	0.048	0.046	0.071	0.088	0.104
D	(0.5,0.5)	0.044	0.042	0.041	0.049	0.053	0.094	0.109
AB	(0.5,0.5)	0.196	0.210	0.225	0.243	0.291	0.344	0.346
AC	(0.5,0.5)	0.039	0.033	0.044	0.044	0.074	0.088	0.103
AD	(0.5,0.5)	0.028	0.044	0.038	0.044	0.053	0.094	0.111

BC	(0.5,0.5)	0.037	0.040	0.037	0.051	0.065	0.097	0.102
BD	(0.5,0.5)	0.037	0.041	0.037	0.050	0.064	0.083	0.106
CD	(0.5,0.5)	0.035	0.045	0.041	0.037	0.069	0.085	0.106
ABC	(0.5,0.5)	0.038	0.037	0.041	0.052	0.064	0.098	0.102
ABD	(0.5,0.5)	0.037	0.037	0.035	0.055	0.062	0.082	0.105
ACD	(0.5,0.5)	0.038	0.042	0.038	0.041	0.068	0.085	0.106
BCD	(0.5,0.5)	0.039	0.039	0.042	0.038	0.068	0.088	0.101
ABCD	(0.5,0.5)	0.043	0.041	0.044	0.039	0.067	0.088	0.101
A	(0.5,0.9)	0.044	0.047	0.043	0.057	0.083	0.112	0.121
B	(0.5,0.9)	0.200	0.228	0.249	0.275	0.332	0.346	0.344
C	(0.5,0.9)	0.042	0.041	0.037	0.047	0.076	0.093	0.101
D	(0.5,0.9)	0.037	0.036	0.037	0.041	0.070	0.099	0.101
AB	(0.5,0.9)	0.545	0.584	0.603	0.639	0.671	0.695	0.687
AC	(0.5,0.9)	0.036	0.035	0.035	0.044	0.076	0.092	0.101
AD	(0.5,0.9)	0.040	0.037	0.038	0.043	0.068	0.099	0.101
BC	(0.5,0.9)	0.040	0.039	0.037	0.050	0.068	0.083	0.112
BD	(0.5,0.9)	0.042	0.038	0.038	0.043	0.069	0.081	0.112
CD	(0.5,0.9)	0.036	0.039	0.037	0.049	0.066	0.075	0.100
ABC	(0.5,0.9)	0.047	0.043	0.037	0.052	0.066	0.083	0.112
ABD	(0.5,0.9)	0.035	0.036	0.039	0.047	0.070	0.081	0.112
ACD	(0.5,0.9)	0.044	0.037	0.035	0.050	0.066	0.075	0.100
BCD	(0.5,0.9)	0.034	0.033	0.038	0.039	0.073	0.077	0.097
ABCD	(0.5,0.9)	0.042	0.032	0.037	0.040	0.073	0.077	0.097
A	(0.9,0.9)	0.032	0.037	0.053	0.062	0.088	0.105	0.128
B	(0.9,0.9)	0.538	0.574	0.599	0.620	0.616	0.664	0.670
C	(0.9,0.9)	0.039	0.033	0.050	0.056	0.065	0.092	0.108
D	(0.9,0.9)	0.038	0.039	0.044	0.046	0.068	0.090	0.106
AB	(0.9,0.9)	0.520	0.560	0.607	0.618	0.619	0.666	0.671
AC	(0.9,0.9)	0.032	0.038	0.044	0.057	0.066	0.092	0.108
AD	(0.9,0.9)	0.033	0.033	0.041	0.047	0.066	0.089	0.106
BC	(0.9,0.9)	0.031	0.038	0.048	0.045	0.062	0.092	0.102

BD	(0.9,0.9)	0.036	0.037	0.042	0.051	0.064	0.084	0.106
CD	(0.9,0.9)	0.041	0.036	0.049	0.048	0.066	0.090	0.094
ABC	(0.9,0.9)	0.033	0.039	0.047	0.043	0.062	0.092	0.102
ABD	(0.9,0.9)	0.034	0.043	0.044	0.051	0.062	0.083	0.106
ACD	(0.9,0.9)	0.037	0.035	0.052	0.050	0.065	0.090	0.094
BCD	(0.9,0.9)	0.034	0.042	0.043	0.052	0.072	0.093	0.098
ABCD	(0.9,0.9)	0.038	0.044	0.044	0.049	0.071	0.092	0.098

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