

Impact of Abolishment of Mandatory Retirement on BC Employment Income

**by
Olga Strizhkova**

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Approval

Name: Olga Strizhkova
Degree: Master of Science
Title of Thesis: *Impact of Abolishment of Mandatory Retirement on BC Employment Income*

Examining Committee:

Chair: Dr. Tim Swartz

Dr. Gary Parker
Senior Supervisor

Dr. Cary Tsai
Supervisor

Dr. Joan Hu
External Examiner

Date Defended/Approved: December 15, 2011



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Abstract

Amendments to the provincial Human Rights Code effectively abolished mandatory retirement in BC in 2008. Additionally, the Canadian population is aging. The seniors, 65 years of age or older, constitute the fastest growing population group. These facts are expected to have an impact on the insurance and pension industry as well as on social programs. In this project, we study the total employment income in BC. The total income in BC is projected, first, using pre-legislation retirement rates and secondly, using post-legislation retirement rates that are based on a survey of Canadian workers. The workforce is projected using a two-decrement model, with death and retirement as the two causes of decrement. Average annual salaries by age are then applied to the projected workforces to get the total income in BC. Prediction intervals are calculated and sensitivity analysis is performed for some of the key assumptions.

Keywords: Mandatory retirement; aging population; multiple-decrement model; sensitivity analysis

Dedication

To my Lord, Jesus Christ, and my beloved family.

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1. Introduction

Effective January 1, 2008 amendments were introduced to the BC Human Rights Code (Legislative Session, 2007) to extend the protection against age discrimination to those aged 65 and over. Under the new legislation, employers must not refuse to employ someone on the basis of age. Additionally, Statistics Canada reported that the Canadian population is aging. The proportion of seniors within the population has been steadily growing since 1960, while the group of those under 15 has been decreasing. These two facts and higher salary, morbidity and mortality attributed to older ages affect a whole range of economical, social and medical issues, which are of great importance for many companies and government institutions nowadays.

The abolishment of mandatory retirement and demographic changes also affect insurance products, such as health care, group and pension plan benefits. The objective of the project is to estimate the impact the new legislation and demographic changes will have on the total income of the population in BC. The estimated total income can be used as the basis for calculating contributions to the Canada Pension Plan and estimating the cost of providing Health Care and other benefits to the current and projected (future) workforce.

It is noteworthy to state here that the methodology used in this project was developed for a real insurance company providing health and life insurance benefits. For the sake of confidentiality, the methodology was applied to publicly available data with some required changes and assumptions.

2. Data collection and assumptions

The data used for the BC population and their employment income is provided by British Columbia Detailed Taxation Statistics (Canada Revenue Agency, 2010). Due to the fact that data is given in 5 year bands, values between data points were calculated by formulas presented in Appendix C.

2.1. Distribution of exposure by age

In order to better understand the impact of different components to the final results, the distributions of BC's taxable employment income and population by age and gender during the period of 2004-2008 have been analyzed.

In 2008, the age distributions of the population, for both males and females, have two modes, one in the mid-twenties and one in the late forties. These age distributions reveal the fact that the male BC workforce around age twenty was only slightly lower than that of the largest male working group (45-49). For the female workforce, there is a more considerable difference between the two modes in 2008. We also note that for the twenties age group, both male and female workforce populations are about the same size, whereas there are more forty year old working females than males. The groups of male and female employees indicate a consistent increase in size throughout the period of 2004-2008.

The number of BC employees by age and gender for the period of 2004-2008 is provided in the following 2 graphs.

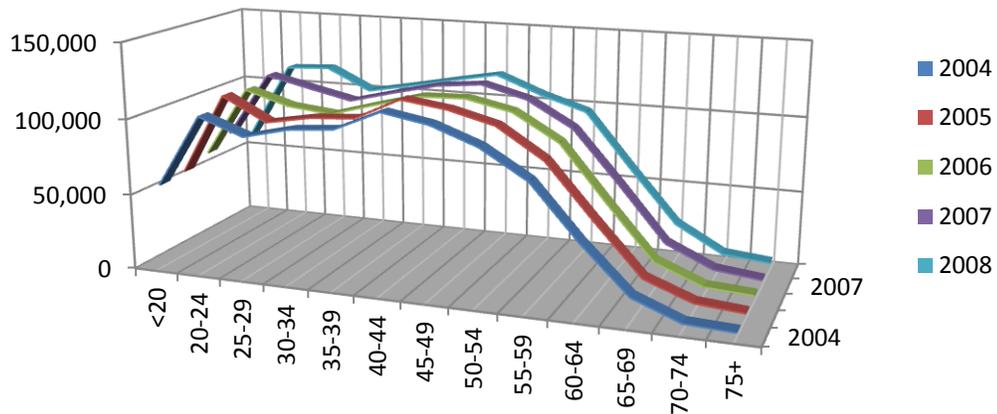


Figure 2.1. BC Male Workforce over 2004-2008 Historical Period

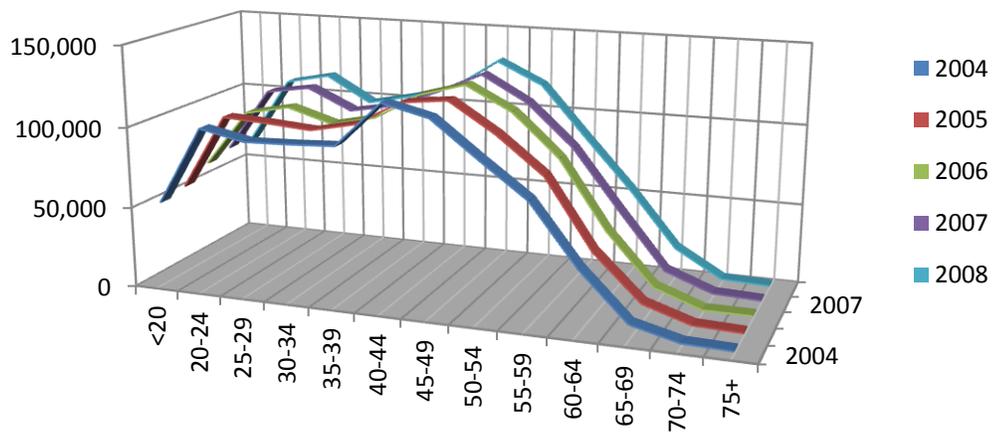


Figure 2.2. BC Female Workforce over 2004-2008 Historical Period

2.2. Employment Income

The data for employment income indicates that income differs by age, gender and calendar year. Employment income has increased over the period of 2004-2008. In general for each gender, the employment income increases with age and reaches its peak around late forties, early fifties, followed by a gradual decrease to 0 at age 75+.

is also observed that men have higher employment income than women for all age categories.

In order to perform a projection of the total income of the BC workforce, the average employment income was calculated by dividing the employment income by the corresponding number of employees. All the calculations are performed for each age group and gender taken separately. The calculated average employment income for males and females in 5 year bands is presented in Appendix C.

It is very important to analyze both population and employment income distributions due to the fact that their combination under different scenarios may provide different impact on the total income of BC population. For example, a 1% increase in the size of the younger population will bring less dollar amount change in the total income than the same increase in the senior population due to the higher average salary range at older ages.

The following graphs represent BC employment income for male and female workforce population.

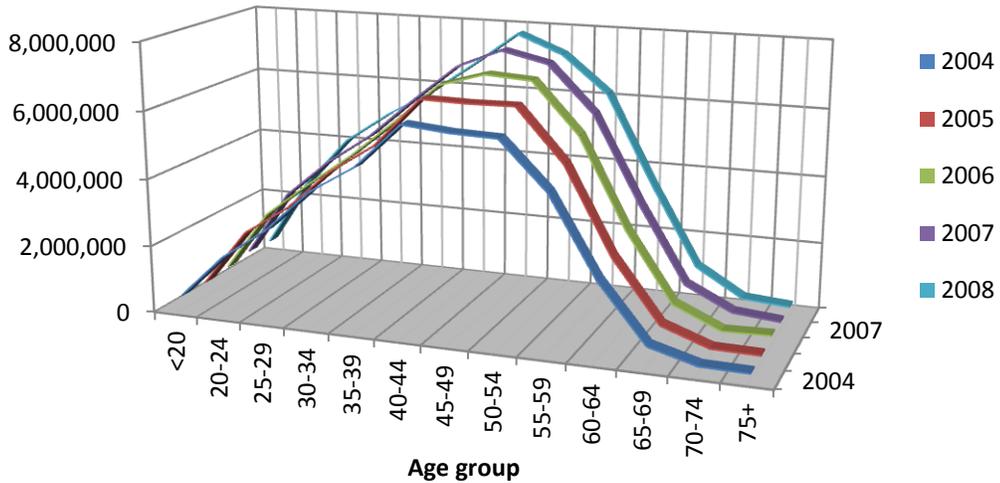


Figure 2.3. BC Employment Income for Male Workforce by Age Group

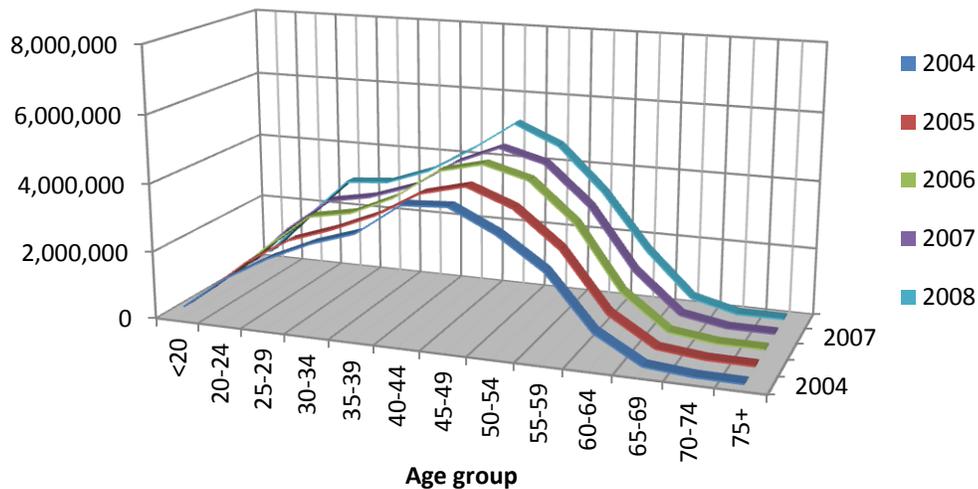


Figure 2.4. BC Employment Income for Female Workforce by Age Group

2.3. Retirement rates

Retirement rates are considered in order to make annual projections of the workforce population for the next 10 years.

Retirement rates were calculated by dividing the number of workers currently in the labor force planning to retire by the corresponding total number of employees. The data for the number of people in the workforce and those planning to retire is taken from Pignal (2010). All the calculations are made for each age group taken separately. The retirement rates used are presented in Appendix D.

It is interesting to note that certain retirement ages are more attractive than others due to, for example, Public Pension Plan Policy. As seen from the following graph these ages are 60, 65 and 70.

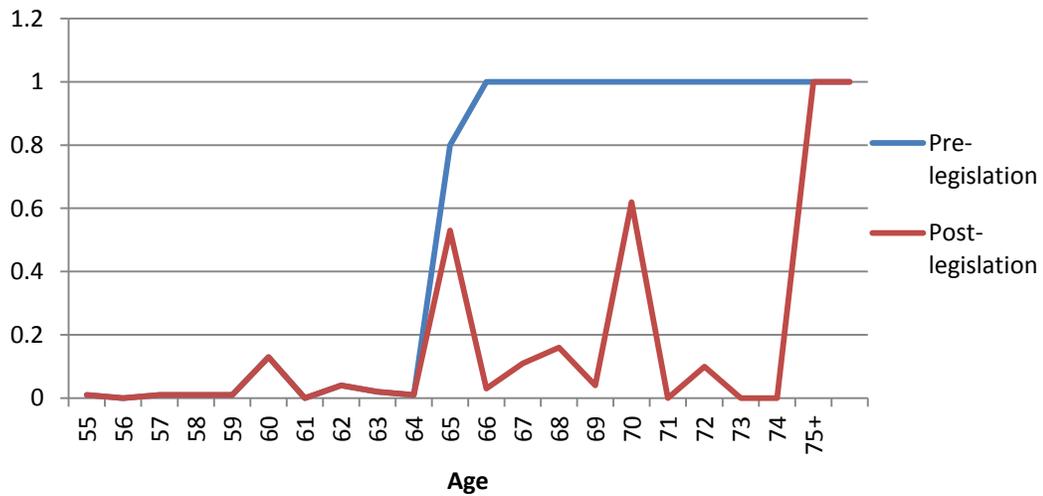


Figure 2.5. Retirement Rates in Pre- and Post-legislation Periods

The data for retirement rates by gender is not publicly available, so to make further calculations possible, it is assumed that there is no difference in retirement rates between males and females.

Assuming that the new legislation would have no impact on retirement rates before age 65, pre- and post-legislation retirement rates for these ages are assumed to be the same. Pre-legislation retirement rate for age 65 is arbitrarily set to 0.8, and to 1 for ages 66+. Post-legislation retirement rates for ages 65-74 are taken from aforementioned Statistics Canada data. Retirement rates for ages 75+ are assumed to be the same for pre- and post-legislation periods and equal to 1, which means that every working person is planning to retire at age 75. For instance, about 53 out of 100 working employees are planning to retire at the age of 65 since the new legislation came into effect, as opposed to 80 out of 100 before the legislation.

2.4. Mortality rates

The mortality rates for males and females for years 2000-2002 were retrieved separately (Statistics Canada, 2006). This report contains life tables constructed based

on mortality rates observed in Canada for each age and gender. The mortality rates are assumed to improve over the projection period at the rates presented in the annual mortality improvement rates table (Montambeault & Menard, 2010). More precisely, mortality rates for 2008, for example, were calculated by multiplying 2000-2002 mortality rates by one minus the annual mortality improvement rates for each subsequent year based on age and gender.

The annual mortality improvement rates vary not only by age and gender, but also by calendar year. Montambeault and Menard (2010) propose 3 sets for mortality improvement rates, for calendar years 2005-2009, 2010-2028 and 2029+, respectively. The improvement mortality rates for 2002-2004 calendar years were estimated based on historical experience, smoothed and applied to mortality rates described in the aforementioned Statistics Canada report. The resulting mortality rates are found to be similar to mortality rates for Canada population (Human Mortality Database, 2007, May).

The following table shows the mortality improvement rates found in Table 5 of Montambeault & Menard (2010).

Table 1. Assumed Annual Mortality Improvement Rates

Age	Males		Females	
	2005–2009	2010–2028	2005–2009	2010–2028
0	2.3%	1.5%	1.8%	1.2%
1–14	3.7%	2.2%	3.8%	2.3%
15–44	2.8%	1.7%	1.6%	1.1%
45–64	2.0%	1.4%	1.4%	1.0%
65–84	2.0%	1.4%	1.2%	0.9%
85–89	0.8%	0.7%	0.5%	0.6%
90–94	0.5%	0.4%	0.3%	0.3%
95+	0.0%	0.2%	0.0%	0.2%

3. Multiple decrement model

3.1. Introduction to multiple decrement model

Multiple decrement models consider the survivorship of an individual subject to multiple contingencies. These models are extensions of standard mortality models with exposure to several causes of termination from a given status at the same time. The termination from a given status is called **decrement**.

There are different areas where these models can be implemented. As an application of this extension, life insurance contracts often pay different benefits depending on the cause of termination, e.g. death or withdrawal. An insurance contract may provide different coverages for disability and death. The cause of death may affect the benefit, e.g. accidental death benefits are typically higher than non-accidental ones. As another example, pension plans provide distinct benefits for death, disability, employment termination and retirement.

To accurately estimate the cost of such plans the model should take into account both the time until decrement and the cause of decrement. In survival analyses this type of model is called **competing risk model** (David and Moeschberger, 1978).

The following scheme represents competing risk model with states “alive” and “dead by cause j ”, which is relevant for cases with different causes of death or termination.

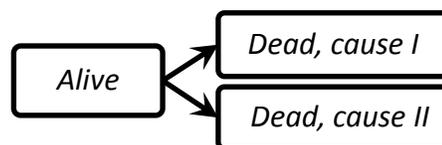


Figure 3.1. Competing Risk Model

3.2. Stochastic survivorship group

Following the notations of Bowers et al. (1997), let $T(x)$, or T , denote the continuous random variable for the period of time until the decrement, and $J(x)$, or J , the discrete random variable representing the cause of decrement.

In order to built the model, the joint distribution of T and J and the corresponding marginal and conditional distributions need to be found. We denote by $f_{T,J}(t, j)$ the joint probability density function of random variables T and J , and by $f_j(j)$ and $f_T(t)$ the marginal probability density functions of J and T , respectively.

The probability density function $f_{T,J}(t, j)$ can be used to calculate the probability of events defined by J and T . For example, for m causes of decrement ($J = 1, 2, \dots, m$),

$$\int_0^t f_{T,J}(s, j) ds = \Pr \{ (0 < T \leq t) \cap (J = j) \} \quad (3.1)$$

is the probability of decrement due to cause j before time t , and

$$\sum_{j=1}^m \int_a^b f_{T,J}(t, j) dt = \Pr \{ a < T \leq b \} \quad (3.2)$$

is the probability of decrement due to all causes between time points a and b .

The probability of decrement due to cause j before time t is denoted by ${}_tq_x^{(j)}$, so

$${}_tq_x^{(j)} = \int_0^t f_{T,J}(s, j) ds, \quad t \geq 0, j = 1, 2, \dots, m. \quad (3.3)$$

According to the definition of the marginal distribution, the probability of decrement due to cause j at any time in the future is

$$f_j(j) = \int_0^\infty f_{T,J}(s, j) ds = {}_\infty q_x^{(j)}, \quad j = 1, 2, \dots, m, \quad (3.4)$$

and the marginal density and distribution functions of T are

$$f_T(t) = \sum_{j=1}^m f_{T,J}(t, j) \quad (3.5)$$

and

$$F_T(t) = \int_0^t f_T(s) ds. \quad (3.6)$$

Using the superscript τ , to refer to all causes of decrement, we can find the probability of decrement due to all causes

$${}_tq_x^{(\tau)} = \Pr(T \leq t) = F_T(t) = \int_0^t f_T(s)ds, \quad (3.7)$$

and the probability of survival until time t is

$${}_tp_x^{(\tau)} = 1 - {}_tq_x^{(\tau)}. \quad (3.8)$$

The total force of decrement at age x is denoted by $\mu_x^{(\tau)}$, and equal to

$$\mu_x^{(\tau)} = \frac{f_T}{1-F_T} = \frac{1}{{}_tp_x^{(\tau)}} \frac{d}{{}_tp_x^{(\tau)}} {}_tq_x^{(\tau)} = \frac{-1}{{}_tp_x^{(\tau)}} \frac{d}{{}_tp_x^{(\tau)}} {}_tp_x^{(\tau)} = -\frac{d}{dt} \log {}_tp_x^{(\tau)}. \quad (3.9)$$

Consequently,

$${}_tp_x^{(\tau)} = e^{-\int_0^t \mu_x^{(\tau)}(s)ds}. \quad (3.10)$$

The force of decrement due to decrement j can be similarly defined as

$$\mu_x^{(j)}(t) = \frac{f_{T,J}(t,j)}{1-F_T(t)} = \frac{f_{T,J}(t,j)}{{}_tp_x^{(\tau)}} = \frac{1}{{}_tp_x^{(\tau)}} \frac{d}{{}_tp_x^{(\tau)}} {}_tq_x^{(j)}. \quad (3.11)$$

Hence,

$$f_{T,J}(t,j) = {}_tp_x^{(\tau)} \mu_x^{(j)}(t). \quad (3.12)$$

So, the probability of decrement between t and $t + dt$ due to cause j equals the probability of staying in the group until time t , times the conditional probability of decrement takes place between time t and $t + dt$ due to cause j , provided that decrement has not occurred before time t .

It follows from formulas (3.3), (3.5) and (3.7) that

$$\begin{aligned} {}_tq_x^{(\tau)} &= \int_0^t f_T(s)ds = \int_0^t \sum_{j=1}^m f_{T,J}(s,j)ds \\ &= \sum_{j=1}^m \int_0^t f_{T,J}(s,j)ds = \sum_{j=1}^m {}_tq_x^{(j)}. \end{aligned} \quad (3.13)$$

Accordingly,

$$\mu_x^{(\tau)}(t) = \sum_{j=1}^m \mu_x^{(j)}(t). \quad (3.14)$$

The total force of decrement is equal to the sum of forces of decrement due to all possible causes.

3.3. Deterministic survivorship group

Another approach is to consider the total force of decrement as the total nominal annual rate of decrement rather than conditional probability density, according to Bowers et al. (1997).

Let $l_x^{(\tau)}$ represents the expected number of survivors to age x from the original group, and $d_x^{(j)}$ represents the expected number of lives who leave the group between ages x and $x + 1$ due to decrement j .

It is conventional to denote the total number of lives leaving the group, for all causes, between ages x and $x + 1$ by $d_x^{(\tau)}$ i.e.

$$\sum_{j=1}^m d_x^{(j)} = d_x^{(\tau)}, \quad (3.15)$$

where m is the total number of possible causes of decrement. Therefore,

$$d_x^{(\tau)} = l_x^{(\tau)} - l_{x+1}^{(\tau)}. \quad (3.16)$$

The probability that a person aged x will leave the group within one year due to decrement j is

$$q_x^{(j)} = d_x^{(j)} / l_x^{(\tau)}. \quad (3.17)$$

The probability that a person aged x will leave the group within one year is

$$q_x^{(\tau)} = d_x^{(\tau)} / l_x^{(\tau)} = \sum_{j=1}^m d_x^{(j)} / l_x^{(\tau)} = \sum_{j=1}^m q_x^{(j)}. \quad (3.18)$$

The probability that a person aged x will stay in the group for one year is

$$p_x^{(\tau)} = 1 - q_x^{(\tau)}. \quad (3.19)$$

3.4. Associated single decrement model

For each of the decrement in a multiple decrement model, a single-decrement model that depends only on one particular decrement can be defined. Each single decrement table represents survivorship of a group of lives subjected to only one cause of decrement. The probabilities of decrement in the associated single-decrement model are called **absolute rates of decrements**, and denoted by ${}_tq_x^{(j)}$. The term rate is used here to emphasize that ${}_tq_x^{(j)}$ is not a probability, since it is not necessary to have

$$\lim_{t \rightarrow \infty} {}_tp_x^{(j)} = \lim_{t \rightarrow \infty} (1 - {}_tq_x^{(j)}) \neq 0, \text{ for all } j \quad (3.20)$$

Let us consider an example of a group of employees that can leave the company due to causes of death or retirement only during a year. Suppose we are given the probabilities of death and retirement while employees were exposed to both types of risks. In order to find the absolute rates of death, we need to assume that no retirement can occur, so there is no competing risk during the year.

Note that the absolute rate of decrement is always greater than or equal to the relative rate, since

$${}_tq_x^{(j)} = \int_0^t {}_sp_x^{(j)} \mu_{x+s}^{(j)} ds \geq \int_0^t {}_sp_x^{(\tau)} \mu_{x+s}^{(j)} ds = {}_tq_x^{(j)}. \quad (3.21)$$

In real life issues, we seldom observe a system with only one cause of decrement. Therefore, the relationships between the probability of decrement and the absolute rate of decrement should be established.

The basic relationship is that the force of decrement due to cause j in a multiple decrement model, $\mu_x^{(j)}$, equals the force of decrement due to cause j in associated single decrement model, $\mu_x^{\prime(j)}$, due to the fact that the force of decrement represents the instantaneous failure rate. So,

$$\mu_x^{(j)} = \mu_x^{\prime(j)}, j = 1, 2, \dots, m. \quad (3.22)$$

Since

$${}_tp_x^{(\tau)} = \exp\left(-\int_0^t \mu_x^{(\tau)}(s) ds\right) = \exp\left(-\int_0^t \sum_{j=1}^m \mu_x^{(j)}(s) ds\right), \quad (3.23)$$

it follows that

$${}_tp_x^{(\tau)} = \prod_{j=1}^m {}_tp_x^{\prime(j)}. \quad (3.24)$$

Some assumptions regarding the time of the decrement emergence within a year should be made to perform these calculations. There are two major assumptions commonly made regarding the distribution of decrement within a year, namely constant force and uniform distribution.

4. A Two-Decrement Model

This chapter describes a two-decrement model that will be used for estimating the total income of population in BC.

Consider the following scheme representing two possible decrements for an employee.

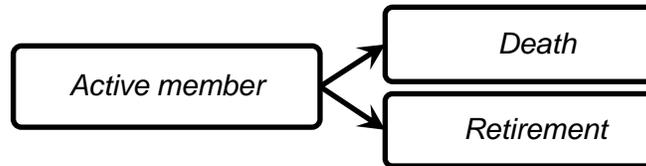


Figure 4.1. Multiple Decrement Model for BC Employees

We use these two possible decrements, death and retirement, with the assumptions that retirements occur at the end of the year, while the time of death is uniformly distributed over each year. Since retirement occurs at the end of the year, the absolute rate of decrement from cause of death is denoted by $q_x^{(d)}$, and equal $q_x^{(d)} = q_x^{(d)}$, where $q_x^{(d)}$ is the probability of death described in Section 2.4. The absolute rate of decrement due to retirement at age x is denoted by $q_x^{(r)}$. The retirement rates, $q_x^{(r)}$, used for the projection are described in Section 2.3.

The probability of decrement over one year from all causes is denoted by $q_x^{(\tau)}$, and it follows that

$$p_x^{(\tau)} = 1 - q_x^{(\tau)} = p_x^{(d)} * p_x^{(r)}. \quad (4.1)$$

Let Z_x^t be a random variable representing the number of employees aged x in year t . Given n_x^t employees at age x in year t , Z_{x+1}^{t+1} has a Binomial distribution with parameters $(n_x^t, p_x^{(\tau)})$, where $p_x^{(\tau)}$ is the probability of surviving for any reason.

Consequently, the expected value of the working population at age $x + 1$ in year $t + 1$ equals

$$E(Z_{x+1}^{t+1}) = n_x^t * p_x^{(\tau)}. \quad (4.2)$$

The variance of the number of employees of age $x + 1$ in year $t + 1$ equals

$$Var(Z_{x+1}^{t+1}) = n_x^t * p_x^{(\tau)} * q_x^{(\tau)}. \quad (4.3)$$

Let TI^t be a random variable representing the total income at the projection year t . Then the expected value for total income at year t is

$$E(TI^t) = \sum_x Sal_x * E(Z_x^t), \quad (4.4)$$

where Sal_x is the annual salary of a person aged x .

The variance of the total income is

$$Var(TI^t) = \sum_x Sal_x^2 * Var(Z_x^t). \quad (4.5)$$

For a large number of trials, the Binomial distribution can be approximated by a normal distribution. In this project we use the normal approximation to estimate the 95 % confidence interval of the Total Income of BC employees. Since the critical value for a 95% confidence interval is 1.96, then the 95% confidence interval for the projected total income in year t , TI^t , is

$$(E(TI^t) - 1.96 * \sqrt{Var(TI^t)}, E(TI^t) + 1.96 * \sqrt{Var(TI^t)}). \quad (4.6)$$

5. Closed group projection of the number of employees

For a closed group projection it is assumed that no new employees can join the workforce and that no withdrawal except for death and retirement can occur over the projected period.

5.1. Base scenario

Under the base scenario, the main focus of the projection is to investigate the impact of the demographic changes and that of the new legislation. To estimate the effect of these two factors, the average employment income per person in 2008, by age and gender, has been calculated and applied to the projected workforce population without considering either trend or inflation.

For each age category, the total number of BC employees in 2008 has been projected annually for the next 10 years. The projections were performed with pre-legislation and post-legislation retirement rates separately. This allowed us to analyze the impact of the new legislation.

To make a one-year projection, survival and retirement rates are applied to the number of employees in BC in 2008. One should refer to formula (4.2) in Chapter 4. The same procedure is repeated for 10 years with the following two assumptions.

Assumption 1.

The data for 2008 employees has non-zero values for ages 75+. Under pre- and post-retirement scenarios, employees are forced to retire at age 75. This produces a larger reduction in projected number of employees for 2009 than one would normally expect. This fact was taken into account and influenced all the following projections which are compared to exposures in 2009 from now on. So, the first projection year is considered to be 2010.

Assumption 2.

Since there are no new employees joining the study group under a closed group projection and people aged 17 or younger in 2009 have not entered the workforce yet, the projected number of employees at young ages will be distorted. Without new young employees, the exposure for those aged 18 will become zero in projection year 1 and this will continue until the age of 27 in the 10th projection year. This will skew our calculations and consequently our final results. In order to deal with that, it is assumed that BC will have the same number of employees at the age of 17 for each projection year. This assumption is consistent with a stationary working population.

5.2. Results for base scenario in closed group projection

If we were to assume that people retire with pre-legislation retirement rates after 2008, then the projected total number of employees decreases over time due to mortality and retirement for both male and female employees. In contrast, under the post-legislation scenario the projected total number of employees starts to increase in the first projection years and decreases thereafter. This moderate increase can be explained by the fact that the number of people leaving the group because of death is less than the number of people staying in the workforce because of the new legislation.

The increase under the post-legislation scenario is more significant for the female workforce than for the male one. This difference can be explained by the peculiarities of the workforce distributions by age and gender. As it was described in Section 2.1, the workforce distribution by age for males has 2 modes of similar height, while for females the height of the first mode is lower than the second one. So, after applying the same post-legislation retirement rates for males and females past age 65, there is a larger proportion of females staying in the workforce compared to males, due to the fact that there are more females in the second mode of the workforce distribution. Also, mortality rates are lower for females than males.

The following chart represents the projected number of male and female employees under the pre- and post-legislation scenarios.

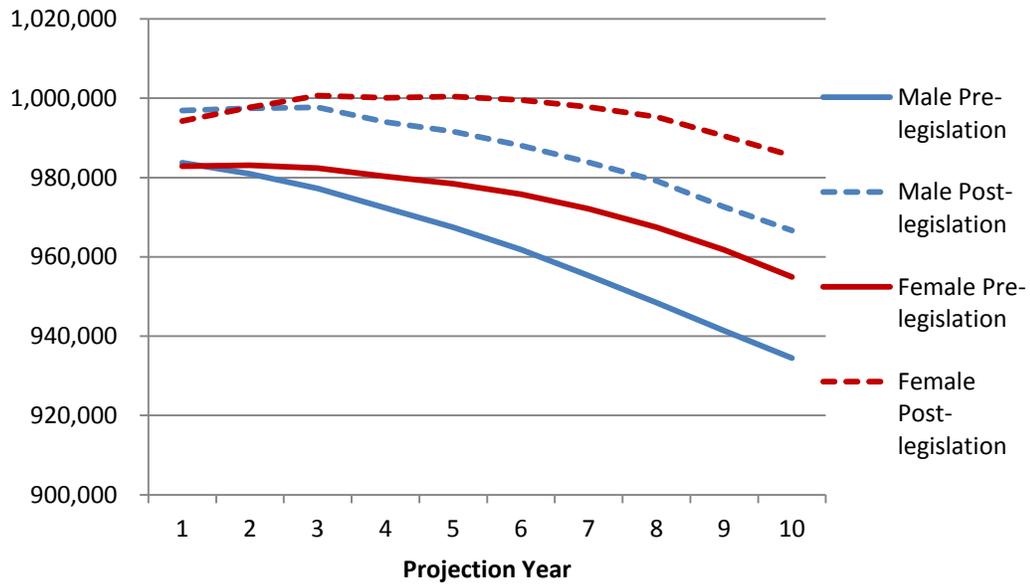


Figure 5.1. Projection of the BC Workforce by Gender

The width of the 95% prediction interval increases with the number of projection years, but still remains very narrow. Accordingly, the prediction bounds are very close to the mean over the whole projection period.

Table 2 Mean and 95% Prediction Interval for the First, Fifth and Tenth Projection Years for Male Employees with Pre-legislation Retirement Rates

Projection year	Pre-legislation scenario. Male employees.		
	1 st	5 th	10 th
Expected Number of employees	983,700	967,472	934,500
95% Prediction Interval (CI)	(983,510; 983,890)	(967,188; 967,757)	(934,183; 934,817)
Width of CI	381	569	634

Table 3 *Mean and 95% Prediction Interval for the First, Fifth and Tenth Projection Years for Male Employees with Post-legislation Retirement Rates*

Post-legislation scenario. Male employees.			
Projection year	1st	5th	10th
Expected Number of employees	996,891	991,617	966,609
95% Prediction Interval (CI)	(996,671; 997,110)	(991,265; 991,969)	(966,202; 967,015)
Width of CI	439	704	813

Table 4 *Mean and 95% Prediction Interval for the First, Fifth and Tenth Projection Years for Female Employees with Pre-legislation Retirement Rates*

Pre-legislation scenario. Female employees.			
Projection year	1st	5th	10th
Expected Number of employees	982,841	978,423	954,935
95% Prediction Interval (CI)	(982,672; 983,011)	(978,163; 978,682)	(954,629; 955,240)
Width of CI	340	519	611

Table 5 *Mean and 95% Prediction Interval for the First, Fifth and Tenth Projection Years for Female Employees with Post-legislation Retirement Rates*

Post-legislation scenario. Female employees.			
Projection year	1st	5th	10th
Expected Number of employees	994,236	1,000,441	985,505
95% Prediction Interval (CI)	(994,039; 994,433)	(1,000,117; 1,000,765)	(985,114; 985,896)
Width of CI	394	649	782

If we were to assume that people retire with post-legislation retirement rates after 2008, then the total decrease in the workforce in 10 years would be 14,460 employees, where a loss of 18,664 comes from the male workforce and a gain of 4,204 from the female one. The change in workforce depends on two factors: mortality and the new legislation. For males, mortality would reduce the workforce by 50,773, while the new legislation retains 32,109 employees. For females, a decline of 26,366 employees and an increment of 30,570 employees would emerge through mortality and the new legislation, respectively.

Figure 5.2 shows the impact mortality and the new legislation have on the projected workforce population in 10 years from calendar year 2009.

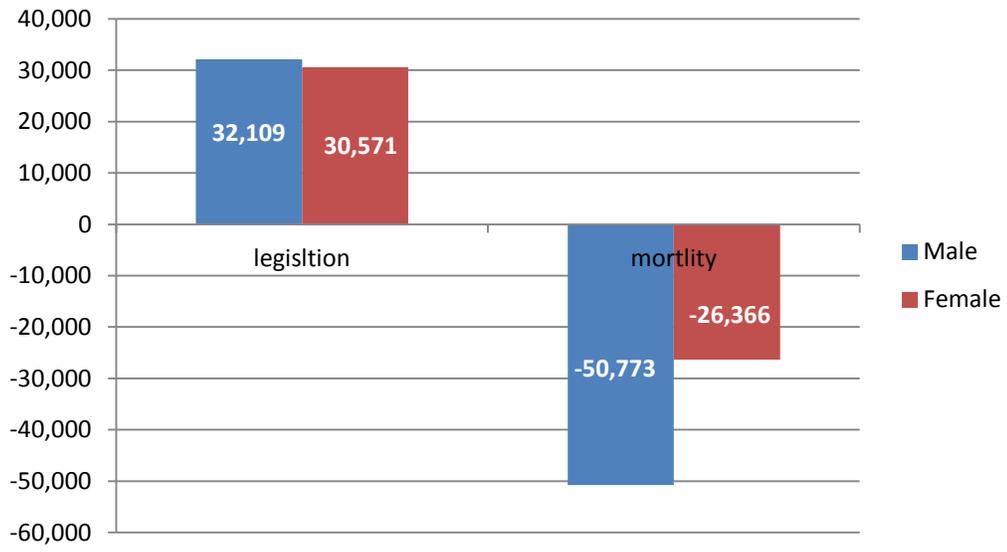


Figure 5.2. Contribution to the Difference in Total Workforce in 10 years by Gender

By applying the 2008 average employment income to the projected workforce one can get the projected total income without considering either trend or inflation.

In order to enhance our understanding of the source of changes let us take a closer look at one of the subgroups of the total working population.

Figures 5.3 and 5.4 represent the results of the projection for the number of male employees and their total income under the post-legislation scenario.

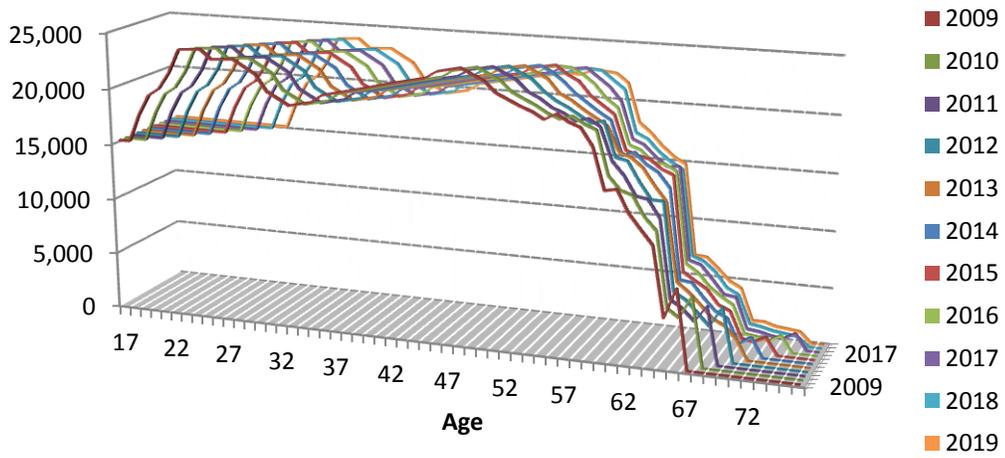


Figure 5.3. Projected Number of Male Employees under the Post-legislation Scenario

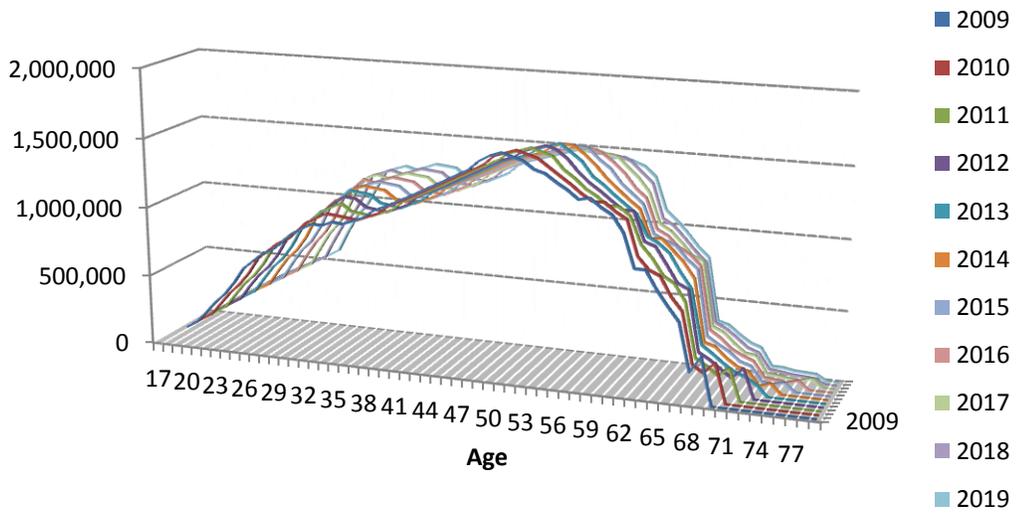


Figure 5.4. Projected Total Income of Male Workforce under the Post-legislation Scenario (in million \$)

From these charts we note that the distribution of the average income in 2009 is increasing with age and reaching a peak at the age of 48 followed by a decline to 0 by age 67. For the following projection years the total income distributions have similar patterns but are shifted by 1 year. Unlike the workforce distribution, the distribution of total income has only one mode. So, people aged 40-60 contribute the most to the total income.

The graphs below show the expected value and the 95% prediction interval over the projection period for the total income of male and female workforce with pre- and post- legislation retirement rates.

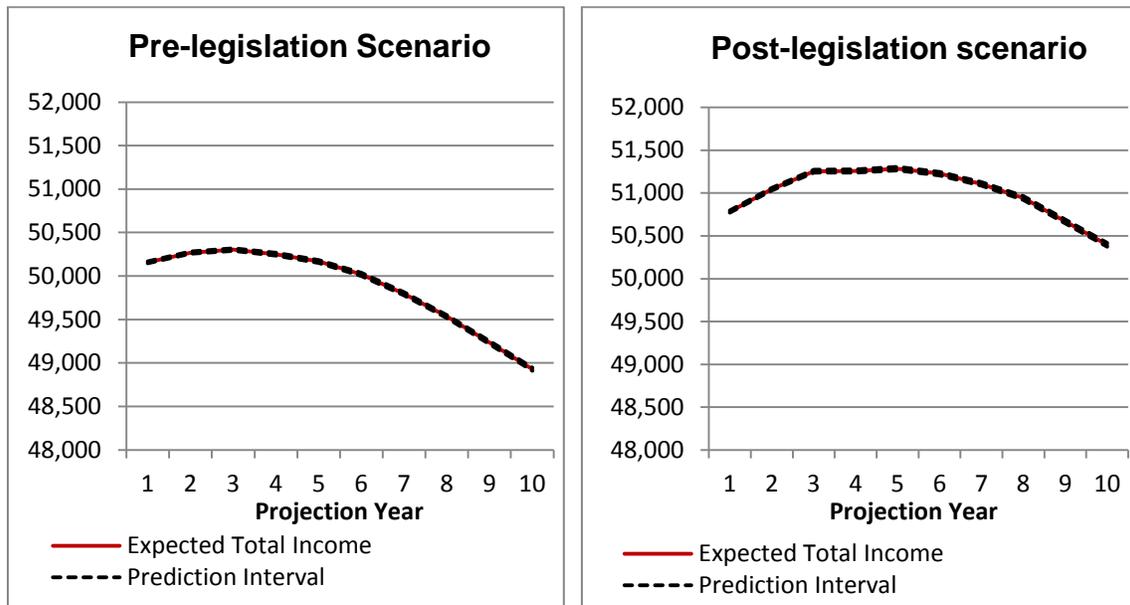


Figure 5.5. Projected Total Income and Prediction Intervals of Male Workforce over the Projection Period (in million \$)

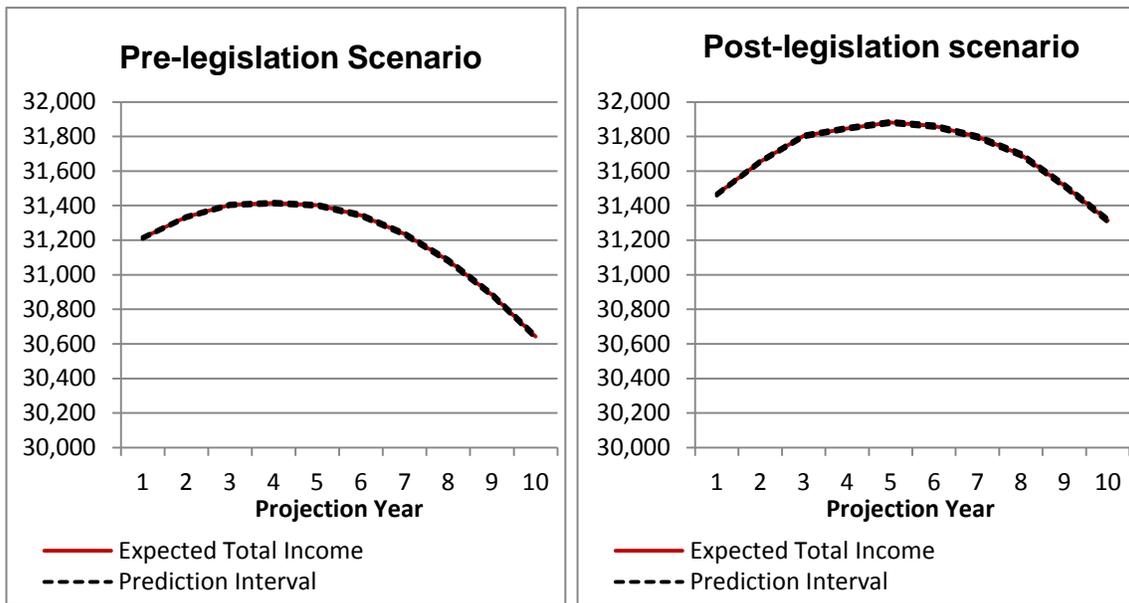


Figure 5.6. Projected Total Income and Prediction Intervals of Female Workforce over the Projection Period (in million \$)

In order to see more clearly, the expected value and the 95% prediction interval are shown for the first five projection years for the total income of male and female employees with pre- and post- legislation retirement rates.

The width of the prediction interval for the expected total income increases with the number of projection years, but remains very narrow.

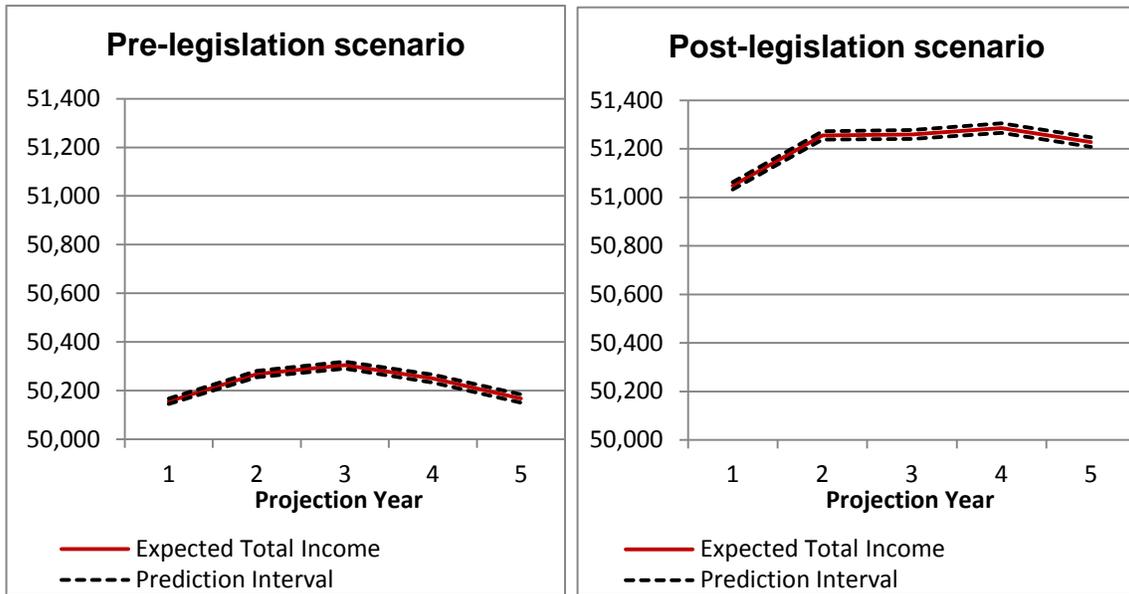


Figure 5.7. Projected Total Income and Prediction Intervals of Male Workforce for the First 5 Projection Years (in million \$)

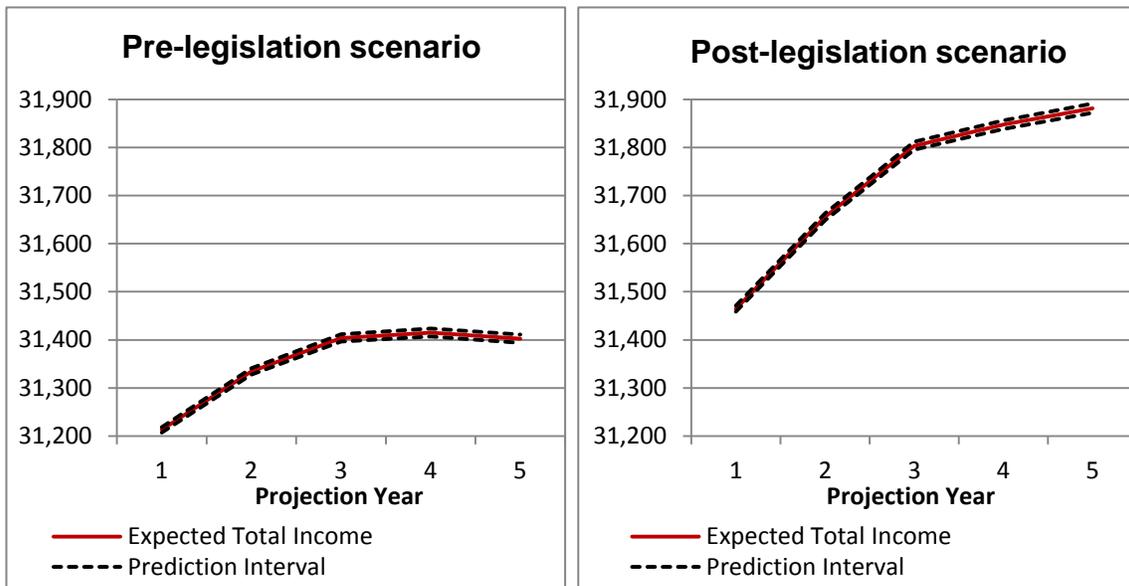


Figure 5.8. Projected Total Income and Prediction Intervals of Female Workforce for the First 5 Projection Years (in million \$)

To estimate the impact of the new legislation, one should compare the total income of BC employees under the pre-legislation and post-legislation scenarios for each projection year.

The following two graphs plot the projected total income of BC workforce under the pre- and post-legislation scenarios for both genders.

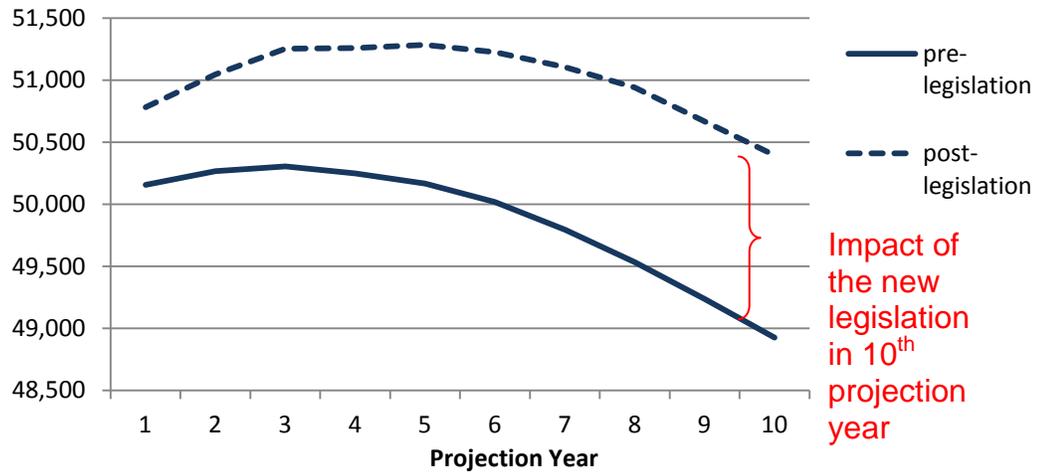


Figure 5.9. Projected Total Income for Male Workforce (in million \$)

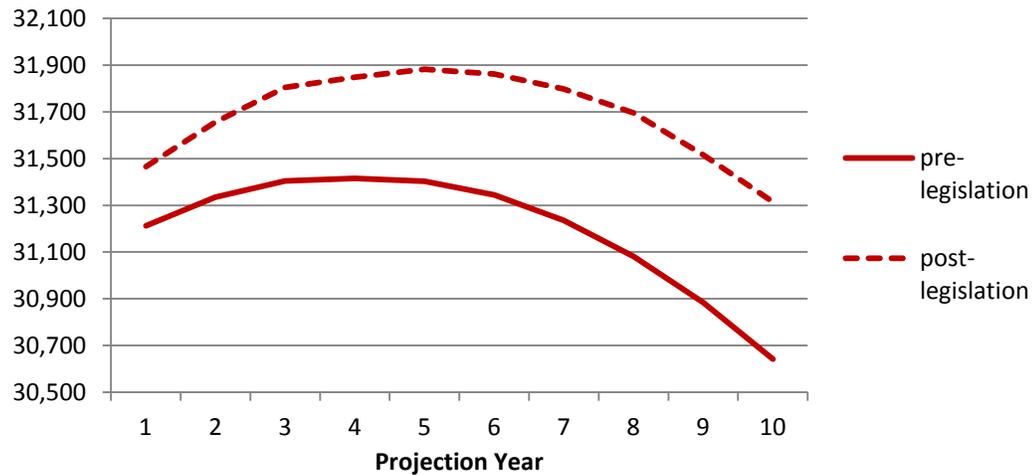


Figure 5.10. Projected Total Income for Female Workforce (in million \$)

The difference due to the new legislation over 10 projected years ranges from 630 million dollars to 1,470 million dollars for males, and from 230 million dollars to 640 million dollars for females.

The following graph represents the change in total income due the legislation for each projection year.

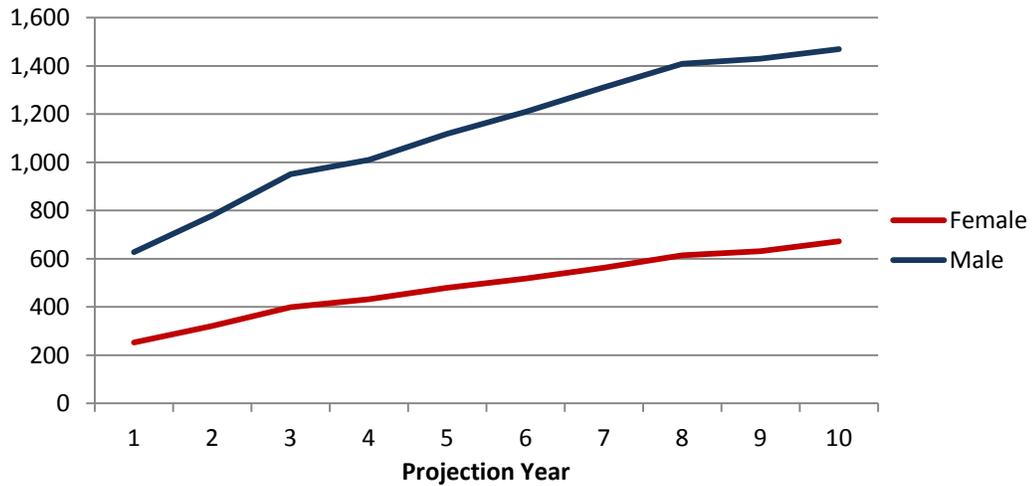


Figure 5.11. Change in Total Income due to the New Legislation (in million \$)

The total projected income in 10 years is expected to be 726 million dollars more as a result of the new legislation, where about 1,417 million dollars loss comes from mortality, and 2,143 million dollars gain is the impact of the new-legislation. The male workforce contributes about 444 million dollars, while females contribute only 282 million dollars to the gain because of the new legislation.

The scheme below represents the components of the change in the projected total income in 10 years.

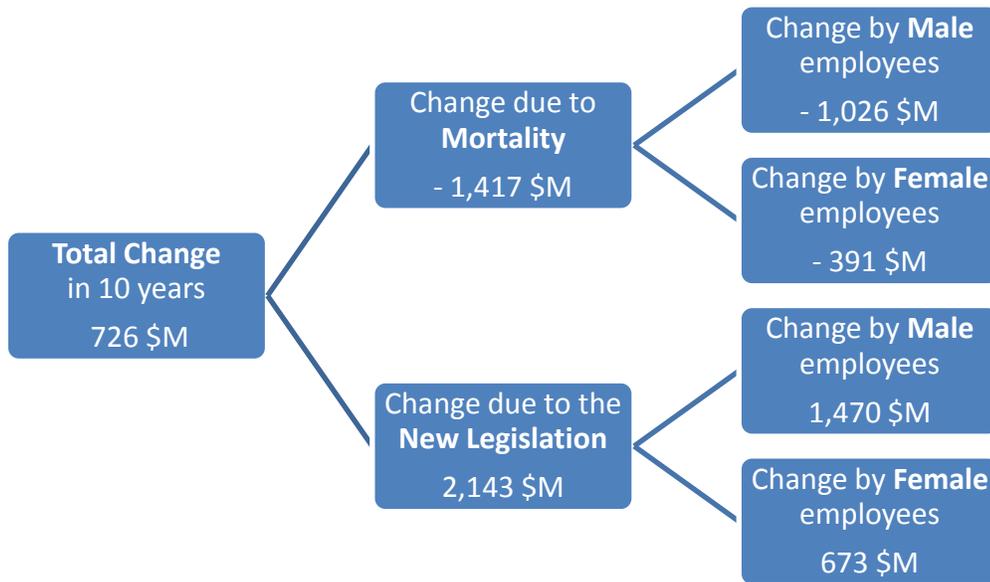


Figure 5.12. Change in Projected Total Income and its Components in 10 years

To recapitulate all described results for the closed group study, assuming that all other characteristics (such as average employment income and mortality) remain unchanged, the projected total income will moderately increase over the first projection years and decrease thereafter. Accordingly, in the tenth projection year the total income of the working population in BC is forecasted to be higher by 726 million dollars.

6. Sensitivity Analysis

6.1. Indexed income scenario

Under the indexed income increase scenario it is assumed that employment income or salary will increase at a rate of 2.4% a year. This value originates from the assumption that the average income will grow at the same rate as the inflation rate in BC during the period of August 2010 - September 2011. The information about inflation rate is taken from Statistics Canada (2011, September).

Since the only change is the salary over the projected year, then the workforce projection remains the same as for the base scenario. Assuming that retirement patterns manifest themselves according to our post-legislation scenario, the total projected income of BC employees in 10 years is forecasted to be 23,138 million dollars higher than in 2009. This change is the result of the following factors: mortality with the negative impact of 1,417 million dollars, which is the same as under the base scenario; and the new legislation and salary increase with positive impacts of 2,781 and 21,774 million dollars, respectively.

All factors contributing to the total change in the projected total income in a 10 year period are presented in the following diagram.

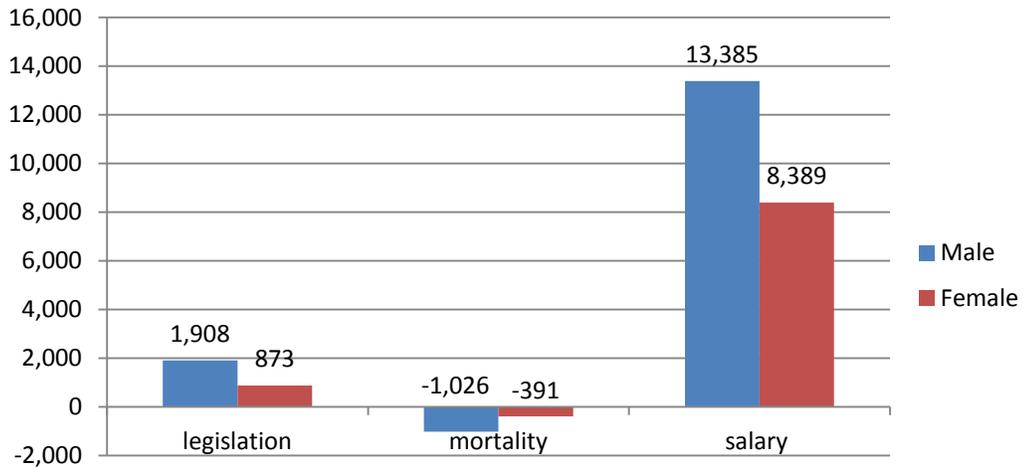


Figure 6.1. Contribution to the Difference in Total Income in 10 years (in million \$)

The total income for male employees in 10 years will increase by 14,267 million dollars, where a gain of 1,908 million dollars is due to the new legislation, 13,385 million dollars is caused by salary increases, and a decrease of 1,026 million dollars happens because of mortality.

Similarly in the female workforce, a decrease of 391 million dollars is the direct result of mortality and increases of 873 and 8,389 million dollars are derived from the impact of the new legislation and salary increase, respectively.

It is worth noting that the impact of the new legislation under the salary increase scenario is higher than under the base scenario. It can be explained by higher salaries earned by those aged 65+ remaining in the workforce after the new legislation came into effect. Also, it is seen from the diagram that the negative impact of the mortality is fully compensated by the positive impact of the new legislation.

So, the total income in the tenth projection year is forecasted to be 65,419 million dollars for males and 40,649 million dollars for females. The total income is predicted to fall into the intervals of (65,390; 65,447) and (40,634; 40,663), expressed in million dollars, with probability of 95%, respectively.

The following two figures plot the projected total income for BC workforce by gender and for each projection year under the pre- and post-legislation scenarios.

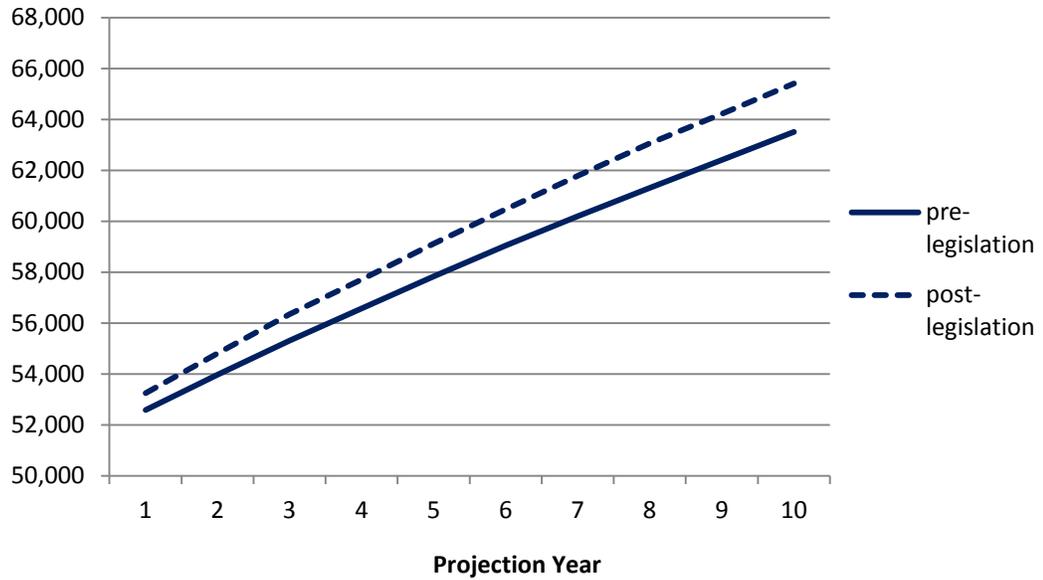


Figure 6.2. Projected Total Income for Male Workforce (in million \$)

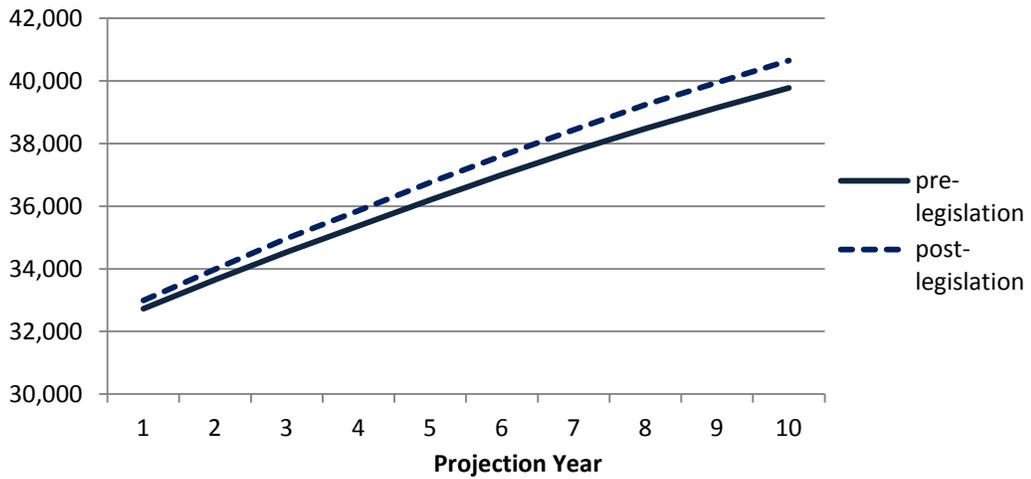


Figure 6.3. Projected Total Income for Female Workforce (in million \$)

The projected total income of BC workforce increases at a linear trend with the coefficients shown in the following table.

Table 6 *Coefficients for a Linear Regression for Male and Female Employees under the Pre- and Post-legislation Scenarios*

	Male		Female	
	Pre-legislation	Post-legislation	Pre-legislation	Post-legislation
Intercept	51,630	52,206	32,142	32,360
Slope	1,208.8	1,348.6	785.8	851.9
R^2	0.998	0.998	0.997	0.997

The slope coefficient illustrates how much the total income is expected to increase over each projection year. The coefficient of determination, R^2 , is the same as the square of the correlation between dependent and independent variables, and indicates how well the model fits the data. When R^2 is close to 1, the model almost perfectly fits the data.

What should the rate of salary increase be in order to compensate the mortality in the closed group projection? To have the total income in 10 years equal the total income in 2009 calendar year under both the pre- and post- legislation scenario, the salary should increase at a rate presented in the following table for each gender and scenario.

Table 7 ***Average Salary Rate Increase Required to Compensate the Impact of Mortality***

Gender	Pre- legislation	Post- legislation
Male	0.405%	0.220%
Female	0.332%	0.188%

In order to compensate both the impact of mortality and new legislation, salaries should increase at a rate of 0.135% for males and a rate of 0.134% for females. The rates of increase are roughly the same for both genders.

6.2. Mortality improvement scenario

Under the mortality improvement scenario, it is assumed that mortality rates will decrease in accordance with Montambeault & Menard (2010), while all the other parameters and methodology stay the same as under the base scenario.

The results for this scenario indicate that mortality improvement has almost no impact on the workforce projection, and, consequently, it makes an insignificant impact on the projected total income of the working population in BC.

The figures below represent the projected number of employees over a ten year period under both pre- and post-legislation scenarios by gender.

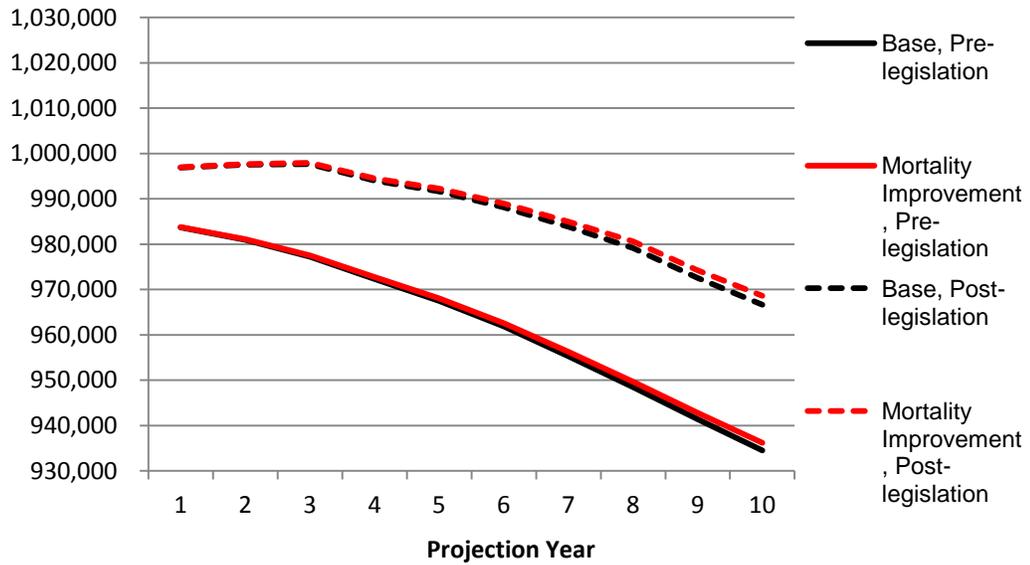


Figure 6.4. Projected Male Workforce under Mortality Improvement and Base Scenarios

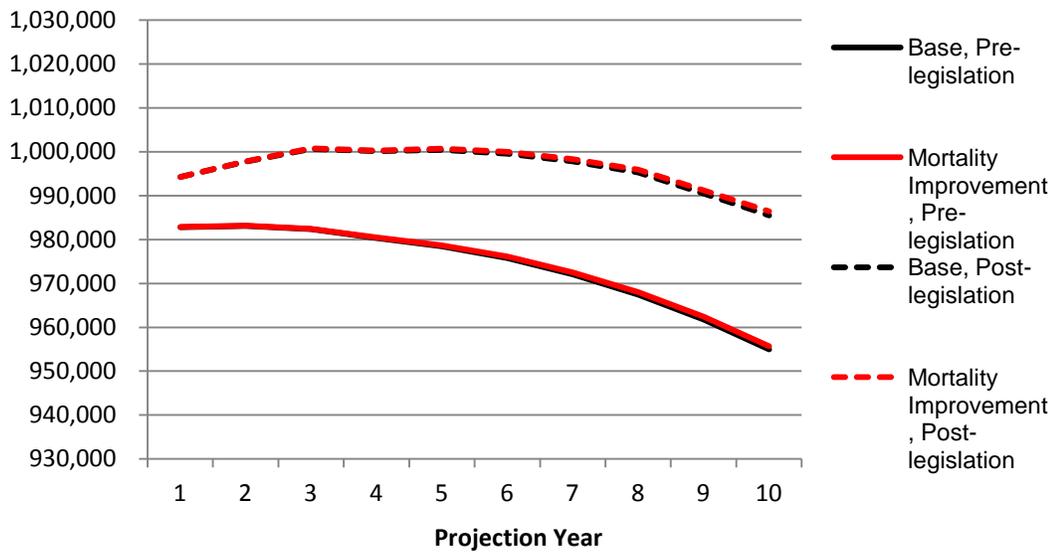


Figure 6.5. Projected Female Workforce under Mortality Improvement and Base Scenarios

In terms of dollar amounts, the total income of BC employees in the tenth year is projected to be 50,511 million dollars for males and 31,345 million dollars for females, which are only 115 and 30 million dollars higher compared to the base scenario.

6.3. Open group projection

In an open group, it is assumed that people can join the BC workforce (e.g. migration from other provinces and countries, younger generation joining the workforce, etc.), and employees can withdraw from the group for causes other than death and retirement.

Since data for inflow and outflow in the workforce is not available, the average net migration in the workforce population by age is estimated based on historical data for 2004-2008 calendar years, presented in Section 2.1.

According to Figures 2.1 and 2.2, the workforce for both genders has a relatively stable population distribution for older ages with a moderate growth for younger generations.

In order to estimate the net migration of employees, first, the change in the number of employees aged x from year t to year $t + 1$ is calculated as the difference between the number of employees aged x in year $t + 1$ and the number of employees aged x in year t . This difference is denoted by $\Delta E_x^{t,t+1}$.

Assuming that over the period of 2004-2008, people died and retired with the mortality and pre-retirement rates described in Chapter 2, then the net migration of employees aged $x + 1$ in year $t + 1$ is represented by M_{x+1}^{t+1} , and can be calculated by the following formula

$$M_{x+1}^{t+1} = \Delta E_{x+1}^{t,t+1} - (E_x^t * p_x^{(\tau)} - E_{x+1}^{t+1}), \quad (6.1)$$

where E_x^t represents the number of employees aged x in year t .

The net migration is calculated for each calendar year from the period of 2004-2008 based on age and gender. The average net migration for employees aged x is

denoted by \bar{M}_x , and equal the average of the net migration over the period of 2004-2008 for each age and gender. Since the historical period is short and data for the net migration is very volatile, the average net migration, \bar{M}_x , fluctuates significantly from age to age. In order to avoid the impact of volatility, the average net migration has been smoothed using a logarithmic regression. The smoothed average net migration is denoted by $\bar{\bar{M}}_x$, and satisfies the equation

$$\bar{\bar{M}}_x = \hat{\alpha} * \ln(x) + \hat{\beta} \quad (6.2)$$

with coefficients presented in Table 8.

Table 8 **Coefficients for a Logarithmic Regression of the Average Net Migration**

Gender	$\hat{\alpha}$	$\hat{\beta}$
Male	-528.8	1,957.3
Female	-580.9	2112.2

For an open group projection it is assumed that the flow of new employees and those who withdraw for causes other than death and retirement will remain the same over the projection period, and be equal to the smoothed average net migration, $\bar{\bar{M}}_x$.

Thus, to estimate the number of employees for an open group projection, we use the next formula

$$E_{x+1}^{t+1} = n_x^t * p_x^{(\tau)} + \bar{\bar{M}}_{x+1}^{t+1}. \quad (6.3)$$

The results for an open group projection reveal that the workforce population will continue to follow the same trend as it did over the historical period.

The following charts represent the projected workforce by age, gender and year of projection for the post-legislation retirement scenario.

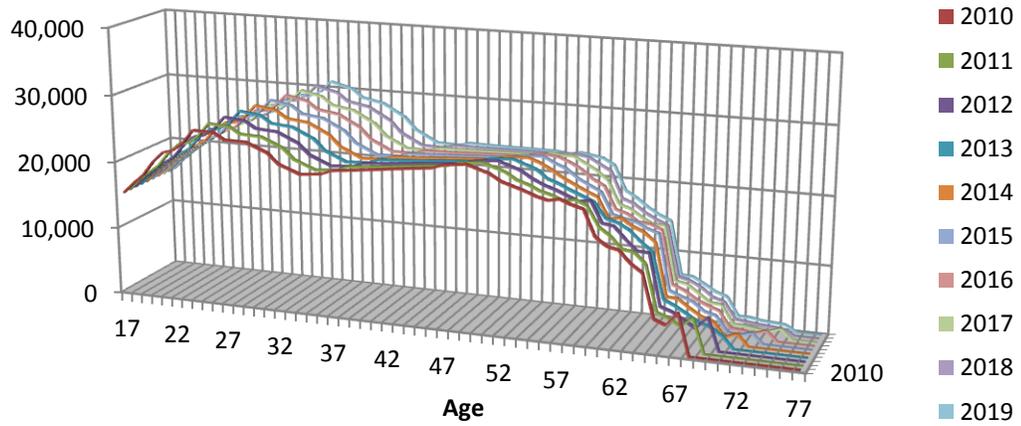


Figure 6.6. Projected Workforce for Males under the Post-legislation Scenario

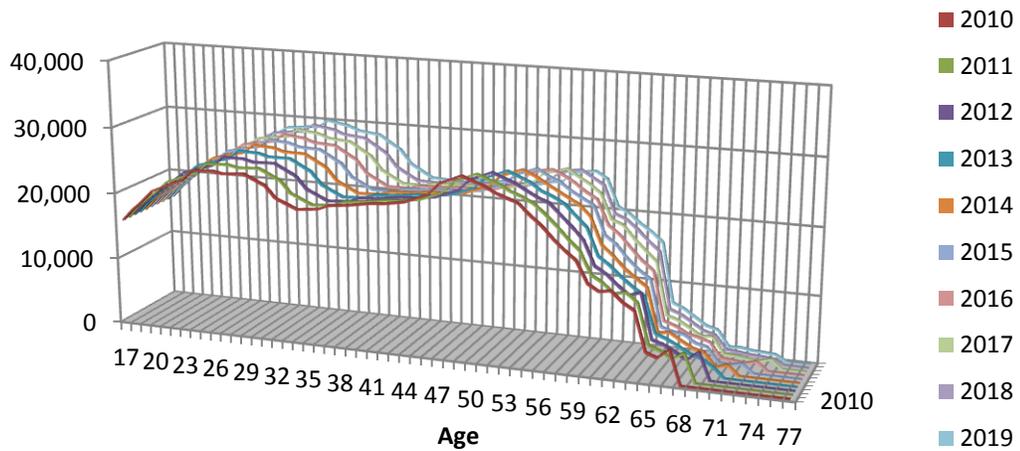


Figure 6.7. Projected Workforce for Females under the Post-legislation Scenario

In total, the projected population of BC employees has an increasing trend over the projection period for both genders.

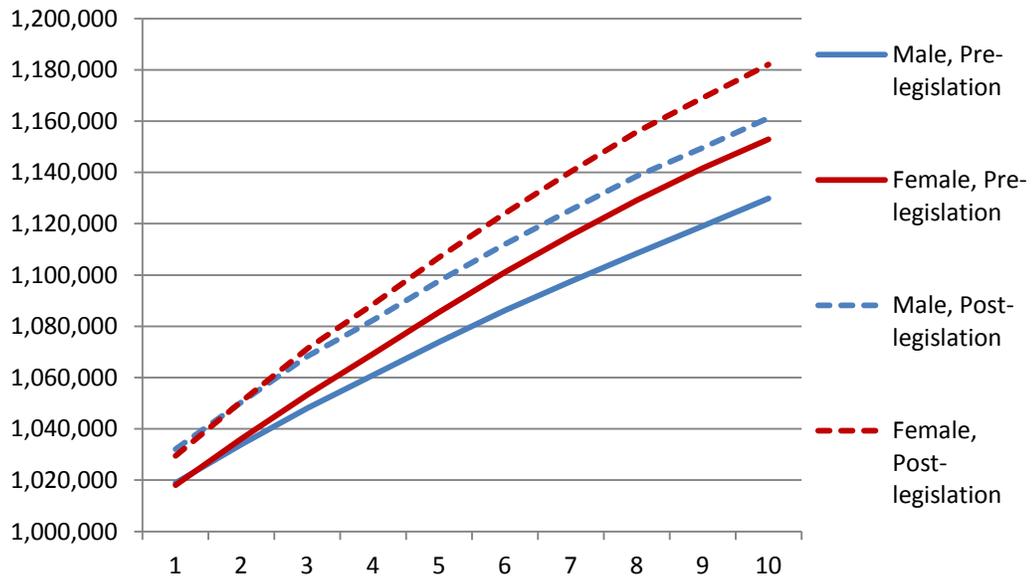


Figure 6.8. Workforce Projection under an Open Group Scenario with Pre- and Post-legislation Retirement Rates by Gender

As a result, the total income of the working population in BC is projected to steadily increase over the next 10 years.

The total income is increasing over the projection period for both genders, with significantly higher total incomes coming from the male workforce. The difference in the projected total income due to the new legislation is in the same range as under the base scenario, ranging from 630 to 1,450 million dollars for males and from 250 to 643 million dollars for females.

The following graph represents the total income for both genders under the pre- and post-legislation scenarios.

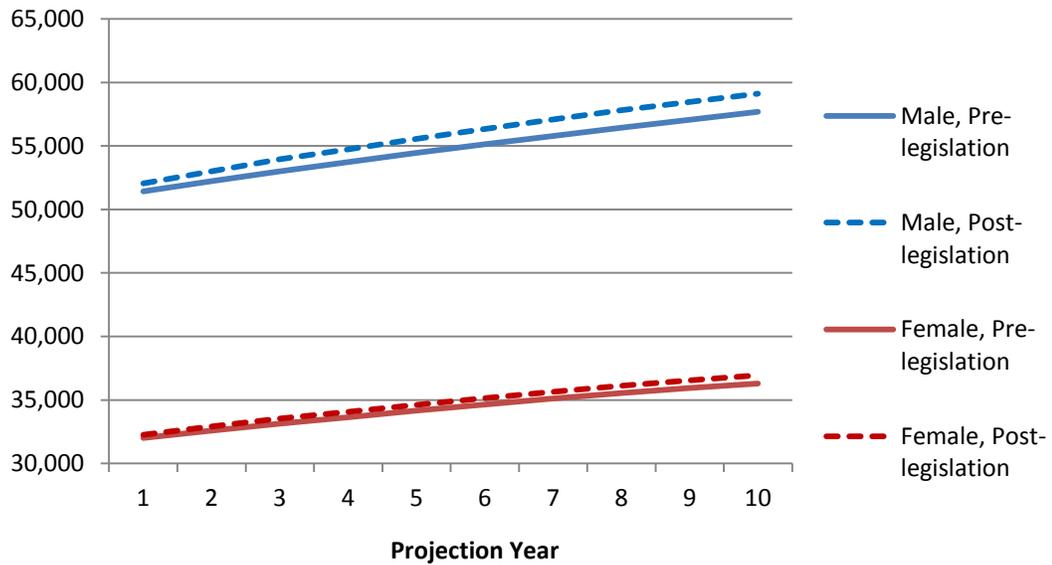


Figure 6.9. Total Income under an Open Group Projection for BC Workforce (in million \$)

The total projected income of BC workforce in 10 years is expected to be 14,084 million dollars higher than in 2009 calendar year, where a gain of 2,079 million dollars happened because of the new legislation.

7. Conclusion

The new legislation contributes positively to the total BC workforce and its employment income.

For a closed group projection, the impact of the new legislation will compensate the impact of the mortality for the first projection years. It can be explained by the fact that less employees leave the workforce due to mortality than those who stay in the workforce due to the new legislation. As time goes on the new legislation will not compensate the impact of mortality under the closed group projection. With the new legislation in force, the workforce in 10 years is projected to decrease by 14,460 employees, while the total employment income is expected to increase by 726 million dollars. The results obtained in this project have very narrow prediction regions.

Additionally, sensitivity analysis is performed to estimate the impact of salary indexation, mortality improvement and open group projection.

Assuming that the salaries increase at a rate of 2.4% per annum, the total employment income is projected to increase steadily over the projection period for both genders. The total income in the tenth projection year is expected to be 23,238 million dollars higher than in 2009. In order to have the same total employment income in 10 years, i.e. compensate the impact of mortality and of the new legislation in the closed group, salaries should increase at an annual rate of 0.135% for males and 0.134% for females.

Our analysis reveals that mortality improvement will have very little impact on the projected total income in 10 years.

The same study was repeated for an open group projection, where the net migration of the workforce was estimated based on the historical data over the period 2004-2008. Projections are done assuming that the future net migration will stay at the

recent historical level. The results reveal that there is a more prominent increase in the number of employees at younger ages, although the employment income for this age group is relatively small. In total, for an open group, the projected workforce and its employment income have an increasing trend over the projection period. The projected total income in the tenth projection year is expected to be 14,084 million dollars higher, where a gain of 2,079 million dollars occurs due to the new legislation.

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Appendix A.

BC Population and Employment Income Data

Table 1

Number of Male Employees by Age Group

Age	2004	2005	2006	2007	2008
<20	55,660	57,130	62,700	69,710	62,670
20-24	103,600	110,830	108,920	113,160	113,290
25-29	92,780	95,470	100,050	106,320	114,850
30-34	99,900	100,880	96,920	99,810	100,760
35-39	102,470	102,840	105,310	107,380	106,490
40-44	115,730	117,350	113,200	114,370	112,510
45-49	110,380	113,220	114,100	117,350	118,150
50-54	99,260	105,410	107,710	109,390	106,470
55-59	80,990	86,090	90,250	92,360	97,090
60-64	46,720	51,340	53,930	57,750	60,720
65-69	16,650	18,040	18,950	21,710	25,820
70-74	4,450	6,280	6,370	7,630	8,650
75+	3,010	3,730	4,470	4,200	5,720

Table 2

Employment Income of Male Employees by Age Group (in '000 \$)

Age	2004	2005	2006	2007	2008
<20	465,728	451,007	517,796	628,452	567,379
20-24	1,774,427	2,127,590	2,324,866	2,596,897	2,633,167
25-29	2,706,433	3,010,930	3,375,465	3,742,813	4,116,725
30-34	3,962,450	4,225,091	4,360,293	4,664,895	5,044,437
35-39	4,791,617	5,107,732	5,478,974	5,843,358	5,821,284
40-44	6,100,776	6,534,055	6,668,887	6,951,078	6,776,249
45-49	5,975,643	6,506,505	7,068,051	7,515,416	7,785,036
50-54	5,931,407	6,530,653	6,980,899	7,214,879	7,255,186
55-59	4,570,556	5,027,697	5,518,937	5,845,239	6,160,677
60-64	2,269,719	2,520,014	2,912,165	3,309,561	3,560,247
65-69	598,256	728,346	890,325	1,045,003	1,177,511
70-74	217,074	232,441	271,609	366,584	357,567
75+	194,566	209,904	307,660	246,648	253,449

Table 3

Number of Female Employees by Age Group

Age	2004	2005	2006	2007	2008
<20	52,370	55,280	63,330	67,050	65,770
20-24	100,730	101,700	98,510	106,520	107,790
25-29	94,950	100,150	104,100	110,980	113,660
30-34	95,800	98,040	95,810	98,280	97,920
35-39	97,190	103,930	101,860	103,560	103,980
40-44	124,750	119,280	118,230	114,300	112,120
45-49	117,960	122,440	126,220	127,150	130,660
50-54	96,260	104,550	111,830	111,640	117,290
55-59	75,380	82,290	86,090	85,740	85,380
60-64	38,640	40,110	44,460	49,210	54,290
65-69	10,000	12,520	13,050	14,150	19,760
70-74	2,760	3,140	2,530	3,200	3,120
75+	2,000	2,120	2,490	2,180	2,050

Table 4

Employment Income of Female Employees by Age Group (in '000 \$)

Age	2004	2005	2006	2007	2008
<20	300,197	347,289	387,222	446,663	436,565
20-24	1,282,379	1,403,807	1,510,575	1,697,448	1,717,057
25-29	2,016,382	2,184,679	2,537,789	2,682,816	2,954,006
30-34	2,596,428	2,635,638	2,780,935	2,925,522	3,060,127
35-39	3,021,681	3,182,616	3,370,002	3,372,775	3,535,467
40-44	3,958,577	3,946,421	4,274,272	4,207,969	4,305,481
45-49	4,034,970	4,272,354	4,589,778	4,762,227	5,199,923
50-54	3,397,021	3,773,619	4,232,546	4,406,658	4,630,201
55-59	2,495,575	2,731,150	3,100,386	3,251,846	3,355,274
60-64	949,602	1,042,400	1,250,852	1,455,855	1,702,052
65-69	181,679	215,113	246,416	310,911	403,786
70-74	61,387	73,763	74,704	76,169	67,028
75+	43,642	56,961	66,031	71,882	68,088

Appendix B.

Average Employment Income by Age and Gender

Age	Male	Female
<20	9,053	6,638
20-24	23,243	15,930
25-29	35,844	25,990
30-34	50,064	31,251
35-39	54,665	34,001
40-44	60,228	38,401
45-49	65,891	39,797
50-54	68,143	39,477
55-59	63,453	39,298
60-64	58,634	31,351
65-69	45,605	20,435
70-74	41,337	21,483
75+	44,309	33,214

Appendix C.

Salaries and Distribution of Workforce by age

Salaries

Let S_n be the average annual salary for a person aged n . According to the data, we are given $S_n = A_n$, for $n = 17, 22, 27, \dots, 77$. Using linear interpolation between data points A_n and A_{n+5} , we obtain salaries for all ages as follows

$$S_{n+i} = i * \frac{(A_{n+5} - A_n)}{5} + A_n, \text{ where } i = 1, \dots, 5. \quad (\text{C.1})$$

Distribution of Workforce

In order to calculate the number of employees in the workforce for each age, a two-step algorithm is applied.

Firstly, interpolation between successive 5-year age groups is used to smooth the distribution of the workforce. Let B_n be the number of employees in the 5-year age group with median age n . According to the data, we are given B_n for $n = 17, 22, 27, \dots, 77$. Interpolated values between successive age groups are calculated as

$$X_{n+i} = i * \frac{(B_{n+5} - B_n)}{5} + B_n. \quad (\text{C.2})$$

Secondly, the interpolated values are rescaled, in order to match the total number of employees for each age group in the original data. The number of employees aged $n + i$ is denoted by X_{n+i}^* , and equals, for $n = 17, 22, 27, \dots, 77$,

$$X_{n+i}^* = B_n * X_{n+i} / \sum_{i=0}^4 X_{n+i}, \quad (\text{C.3})$$

Appendix D.

Pre- and Post-legislation Retirement Rates

Age	Pre-legislation	Post-legislation
55	0.01	0.01
56	0.00	0.00
57	0.01	0.01
58	0.01	0.01
59	0.01	0.01
60	0.13	0.13
61	0.00	0.00
62	0.04	0.04
63	0.02	0.02
64	0.01	0.01
65	0.80	0.53
66	1.00	0.03
67	1.00	0.11
68	1.00	0.16
69	1.00	0.04
70	1.00	0.62
71	1.00	0.00
72	1.00	0.10
73	1.00	0.00
74	1.00	0.00
75+	1.00	1.00

Appendix E.

2000-2002 Mortality Rates for Males and Females

Male				Female			
Age	Mortality rate	Age	Mortality rate	Age	Mortality rate	Age	Mortality rate
17	0.00066	48	0.00304	17	0.00031	48	0.00193
18	0.00072	49	0.00331	18	0.00033	49	0.00210
19	0.00078	50	0.00360	19	0.00034	50	0.00229
20	0.00082	51	0.00394	20	0.00034	51	0.00251
21	0.00085	52	0.00434	21	0.00034	52	0.00276
22	0.00087	53	0.00481	22	0.00034	53	0.00305
23	0.00087	54	0.00533	23	0.00033	54	0.00337
24	0.00085	55	0.00590	24	0.00033	55	0.00372
25	0.00083	56	0.00654	25	0.00033	56	0.00410
26	0.00081	57	0.00726	26	0.00033	57	0.00451
27	0.00080	58	0.00805	27	0.00033	58	0.00494
28	0.00082	59	0.00890	28	0.00035	59	0.00538
29	0.00084	60	0.00982	29	0.00037	60	0.00587
30	0.00088	61	0.01085	30	0.00039	61	0.00641
31	0.00091	62	0.01198	31	0.00042	62	0.00704
32	0.00096	63	0.01321	32	0.00046	63	0.00774
33	0.00100	64	0.01451	33	0.00050	64	0.00850
34	0.00105	65	0.01593	34	0.00055	65	0.00933
35	0.00110	66	0.01752	35	0.00061	66	0.01026
36	0.00116	67	0.01930	36	0.00067	67	0.01131
37	0.00123	68	0.02124	37	0.00073	68	0.01243
38	0.00132	69	0.02329	38	0.00079	69	0.01362
39	0.00141	70	0.02555	39	0.00085	70	0.01493
40	0.00152	71	0.02810	40	0.00092	71	0.01645
41	0.00164	72	0.03104	41	0.00099	72	0.01823
42	0.00178	73	0.03429	42	0.00109	73	0.02019
43	0.00195	74	0.03779	43	0.00120	74	0.02230
44	0.00213	75	0.04165	44	0.00132	75	0.02467
45	0.00233	76	0.04599	45	0.00145	76	0.02742
46	0.00255	77	0.05091	46	0.00160	77	0.03066
47	0.00279	78	0.05631	47	0.00176	78	0.03424