

Diverse Perspectives on Teaching Math For Teachers: Living the Tensions

by

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ABSTRACT

While many post-secondary institutions offer mathematics content courses for prospective elementary school teachers within mathematics departments (MFT courses), very little is known about the nature of these courses or the instructors who teach them. This dissertation begins to fill this gap in the research literature through a qualitative analysis of interviews with 10 MFT instructors, all mathematicians in mathematics departments at universities or colleges in British Columbia, offering insight into their diverse interpretations of the course and, particularly, the tensions they experience. This research goes beyond considerations of what MFT courses should be, to begin to build a picture of how they actually are.

Research methods were informed by constructivist grounded theory and hermeneutic phenomenology, while examination of interview transcripts involved thematic coding, constant comparative analysis, and techniques of discourse analysis. Activity theory and positioning theory facilitated the interpretation of results.

The data presented supports the view that for MFT instructors, the experience of teaching MFT courses is very different from their experiences in other mathematics courses. Furthermore, the course as offered is highly dependent on individual instructor decisions related to course priorities, classroom methods, and emphases with respect to conceptions of mathematics, and degree of orientation towards teacher preparation. Personal, practical, and socio-cultural factors are identified that contribute to this diversity. Three levels of tensions experienced by MFT instructors are also articulated, including personal tensions related to instructor identity, internal tensions related to setting course priorities and standards, and systemic tensions related to societal uncertainties with respect to the role of MFT courses. Factors that contribute to these tensions are identified, including indications of influential norms at work in post-secondary mathematics instructional contexts.

While tensions cannot always be resolved, their identification offers avenues for positive change. The dissertation concludes with a number of recommendations, including calls for closer lines of communication between mathematicians and mathematics education researchers, clearer definition of the role of MFT courses, and further research into the knowledge needed by mathematicians to be effective MFT instructors.

Keywords: Elementary teacher preservice; mathematics content courses; instructor tensions; post-secondary mathematics instructors; positioning theory; activity theory

for Mom and Dad

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TABLE OF CONTENTS

Approval	ii
Partial Copyright Licence	iii
Abstract	iv
Dedication	v
Acknowledgements	vi
Table of Contents	viii
List of Figures	xii
List of Tables	xiii
1: Introduction	1
1.1 Math For Teachers.....	1
1.2 Learning to teach MFT	2
1.3 Teaching MFT—encountering tensions.....	3
1.4 Serendipity	5
1.4.1 The Math for Elementary Education Subcommittee	5
1.4.2 Graduate studies	7
1.5 The research questions.....	9
1.6 Overview.....	10
2: Review of the literature	11
2.1 Effective teaching.....	12
2.2 What characteristics should teachers have?	13
2.2.1 Knowledge.....	14
2.2.2 Beliefs	21
2.2.3 Attitudes	24
2.2.4 Other needs.....	25
2.2.5 Listing what teachers need	26
2.3 Teacher education	27
2.3.1 Mathematics content courses	29
2.3.2 The Challenge	31
2.3.3 Priorities	31
2.3.4 Form over content	34
2.4 The Instructors	35
2.5 Summary.....	38

3: Theoretical influences	40
3.1 Constructivist grounded theory	40
3.2 Activity theory.....	43
3.3 Hermeneutic phenomenology.....	45
3.4 Sensitizing concepts	47
3.4.1 Sociomathematical norms	47
3.4.2 Tensions.....	48
3.4.3 Positioning.....	51
4: Collecting and engaging with the data	55
4.1 The search.....	55
4.2 The main study	55
4.2.1 The instructors.....	56
4.2.2 The interviews	58
4.3 The case study.....	60
4.3.1 Motivation, opportunity and intent	60
4.3.2 The conversations with Simon	61
4.3.3 Following up	61
4.4 Analysing the data.....	62
4.4.1 Applying grounded theory	62
4.4.2 Operationalising tensions	66
4.5 Continuing the search through writing	67
4.6 Moving forward	68
5: Experiencing MFT: Common ground.....	69
5.1 MFT students	70
5.1.1 Student attitudes to the course	70
5.1.2 Student attitudes to mathematics.....	72
5.1.3 Emotionality and mathematics anxiety	72
5.1.4 Student knowledge	73
5.2 Resources.....	74
5.2.1 Textbooks.....	74
5.2.2 Colleagues	75
5.2.3 Mathematics education research and faculty	76
5.2.4 School curriculum	77
5.3 The MFT course.....	77
5.3.1 Communication.....	78
5.3.2 Evaluation.....	78
5.3.3 Sociomathematical norms	79
5.3.4 Course format, content, and objectives.....	80
5.4 Not just another math course	81
6: Experiencing MFT: Diversity	83
6.1 Setting priorities	85
6.1.1 Mathematics first	85
6.1.2 Affect first	89
6.1.3 Conceptions of mathematics first.....	92

6.2	In the classroom.....	93
6.3	Images of mathematics	97
6.3.1	Socio-cultural-historical significance	98
6.3.2	Structure.....	98
6.3.3	Mathematicians and the mathematical experience	100
6.3.4	Problem solving.....	102
6.4	Preparing future teachers.....	104
6.4.1	SCK or PCK?.....	104
6.4.2	Teaching about teaching	108
6.4.3	Manipulatives	110
6.4.4	Math Fairs	112
6.5	From diversity to tensions	113
7:	Experiencing MFT: Tensions.....	115
7.1	Personal tensions.....	116
7.1.1	In the classroom	116
7.1.2	Being a mathematician	118
7.2	Internal tensions.....	122
7.2.1	Setting priorities.....	123
7.2.2	Setting standards.....	131
7.3	Systemic tensions	134
7.3.1	Content vs. pedagogy.....	135
7.3.2	The mathematics preparation of elementary teachers	138
7.4	Summary.....	146
8:	Simon's experience	149
8.1	Ideals and intentions	149
8.1.1	Course priorities	150
8.1.2	In the classroom	152
8.2	Tensions arising.....	153
8.2.1	Course priorities	154
8.2.2	In the classroom	159
8.3	The second time around.....	162
8.3.1	Time management.....	163
8.3.2	Role of textbook.....	164
8.3.3	Choice of class activities.....	167
8.4	Living with the tensions	170
9:	Reflections and reverberations.....	172
9.1	The research process	172
9.1.1	Choices	173
9.1.2	Dual roles	174
9.1.3	Evaluating qualitative analysis	176
9.2	Teaching MFT	177
9.2.1	The students.....	177
9.2.2	The course	178
9.2.3	Resources	180
9.3	Diversity revisited	181

9.3.1	Points of divergence	181
9.3.2	Contributing factors to diversity.....	183
9.4	Tensions revisited	185
9.4.1	Contributing factors to tensions	186
9.4.2	Managing tensions	188
9.5	Concluding remarks	188
9.5.1	Seeking resolution	189
9.5.2	Personal reflections	191
9.5.3	In closing	192
Reference List		193
Appendices.....		203
Appendix A:	MFT Subcommittee Report	204
Appendix B:	Annotated Interview Protocol.....	207
Appendix C:	Coding Sample	210
Appendix D:	Coding Summary Sample	211
Appendix E:	Coding Results.....	213

LIST OF FIGURES

Figure 1. Engeström's activity triangle	44
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LIST OF TABLES

Table 1. Summary of Tensions.....	186
Table 2. Coding Sample.....	210
Table 3. Coding Summary Example	211
Table 4. Coding Results	213

1: INTRODUCTION

1.1 Math For Teachers

For the past 20 years, I have been a mathematics instructor at a two-year college in British Columbia, Canada, and for most of those years, I have been involved with teaching a mathematics content course for prospective elementary school teachers. Teaching this “Math for Teachers” (MFT) course is both challenging and rewarding. The students generally have weak mathematics backgrounds and suffer from high anxiety; they need and demand a great deal of support. At the same time, many of them are very motivated, and it is often possible to witness positive changes throughout the term in both their mathematical abilities and their affective states—changes that have the potential to reach beyond and through these particular MFT students, to influence the mathematical experiences of their future pupils.

Mathematics content courses for prospective elementary teachers are taught at most post-secondary institutions in the province, yet among the nine universities that offer professional teacher certification programs, only two of them specifically require an MFT course for admission. For the others, any university-level mathematics course (with some restrictions on statistics or business math courses) can be used to meet admission requirements, though an MFT course is recommended by a further three of the programs.

Although the course names vary, the different versions of the MFT course offered at the various institutions are generally considered as “equivalent” for the purposes of transferring credit from one institution to another. Their content lists are similar, typically covering topics from arithmetic (number systems and operations), geometry (shapes and measurement), and possibly some probability and statistics, or functions and graphing. In their broadest sense, they are intended to address mathematics content that will be required by future elementary school teachers.

Yet, despite their similarities in target audience, broad mandate, and general content descriptions, casual conversations with colleagues both within and beyond my

home institution reveal a certain amount of divergence. There are differences in emphasis or inclusion of specific course topics, and some differences in course structure (e.g. class size, number of hours/meeting times per week), as well as diversity in modes of delivery (e.g. lectures, group work) and approaches to particular topics. On the surface, there is nothing surprising about this. Institutional constraints and instructor preferences unavoidably affect the delivery of any particular course. Over the years, these presumably superficial differences did not trouble me—until quite recently. To get to the heart of my concern, it is necessary to go back to the beginning of my experiences with the MFT course, to retrace the steps that have led me to the focus of this dissertation.

1.2 Learning to teach MFT

I felt, at one time, that my preparation for teaching MFT was as good as it could be. I had a Masters Degree in Mathematics and my provincial teaching certificate. My first experiences with MFT had been as a teaching assistant in graduate school. There, mathematicians¹ who worked in the Mathematics Department but cared passionately about education, and in particular the importance of preparing elementary teachers, introduced me to the course. I was encouraged to sit in on the course lectures; I learned to grade homework for the quality of explanation, not just the numerical solutions; I tutored students in the drop-in centre, and for a while struggled with helping distance education students with mathematics problems over the phone.

Shortly after this, when I was established as an instructor in my own institution, I modelled our course after the one I had been involved with at university. I had the luxury of being able to adapt the course to smaller class sizes, but also made some adjustments to accommodate our slightly weaker students, increasing the in-class hours from four hours per week to six. Ensuring that the local universities would accept our course as “equivalent” to theirs was crucial, as our students would plan to use the course to both count for credit towards their university degrees, as well as to satisfy pre-requisites for entry into teacher-training programs.

¹ The label “mathematicians” within this dissertation will generally be used in place of “instructors or professors within mathematics departments”. It does not carry implications of any particular degree qualifications nor a career focussed on mathematics research.

Even in those early days of my mathematics-teaching career, I knew that the MFT course was different in a very fundamental way from the other mathematics courses that I taught. As I tried to construct the first course outline, I listed the course content and methods of evaluation in the usual way, only to feel that there was still something missing. The course objectives, in keeping with the recommended practice at my institution, were listed in terms of measurable outcomes. But I quickly realised that not all of the goals that I had come to have for the course were measurable. Knowing that others at my institution would be using this document to guide them in teaching the course one day, I felt compelled to add the following:

NOTE TO INSTRUCTORS:

While teaching [this course], the instructor's objectives should be:

- *to spark and nurture a positive attitude towards mathematics;*
- *to help students to reach a level of mathematical competence which will allow them to function effectively as mathematics teachers in an elementary school setting;*
- *to expose students to the beauty of mathematics, along with its fun and creative sides.*

Armed with my experience at the university, my course outline, and my ideals, I looked forward to the task of having a significant influence on the mathematics preparation of future elementary school teachers.

1.3 Teaching MFT—encountering tensions

Over the years, my teaching of MFT “improved”. I learned to anticipate and pre-empt some of the students’ difficulties and stresses. I grew better at justifying my content choices, at organising class tasks, at giving clear instructions. Some students did very well. By the book, I was doing my job: covering the content, sharing my love of math—yet I was dissatisfied. I continue to be dissatisfied.

The primary sources of my discontent were/are² the demands of the course relative to the students' preparation, my own teaching practices, and the extent to which my version of the course is effective (or not) in preparing future teachers of mathematics. These concerns are not disjoint. As mentioned, the students come into the course with weak mathematics backgrounds. This makes it extremely difficult for students to learn the large amount of mathematics covered³ in the course deeply enough to be effective mathematics teachers. Forcing the pace in order to “get through” the material leaves some, maybe many, students behind—they are not able to learn the mathematics and worse, their anxiety is increased, their negative attitudes are reinforced, and their lack of confidence in their ability to do mathematics is confirmed. Providing the students with opportunities to engage in investigative activities would give them opportunities to delve into the mathematics more deeply and facilitate understanding and connections of ideas. It would also model the kind of pedagogy that I expect them to practice as future teachers. But this approach to teaching takes time and some might argue that it is not part of the course. Although I do as much of this as I feel I can, I do not do as much as I feel I should.

When students fail the MFT course, I feel they have been let down somehow; yet at the same time, I feel a sense of relief that these students who are not ready will not be teaching mathematics anytime soon. And for those who pass the course, especially those who are borderline, I am anxious that they are still not as comfortable with the most fundamental mathematics concepts as I would like future teachers to be. With no guarantee that some of these students will ever set foot in a mathematics course again, my concern is that despite my best efforts some of them will pass on their misconceptions and even aversions to their future pupils, perpetuating a vicious cycle of negative attitudes towards mathematics.

My experience in teaching the MFT course has been rife with tension: I have been torn between what I want to do and what I feel I can do, while not being entirely certain about what I should do. I have wanted to make changes, but have often been held back by concerns about course transferability, time constraints, my own comfort

² I struggle here with the tense: past or present? These concerns have been present in me for a long time, and were instrumental in moving me towards the research described in this dissertation. They still exist, hence the present tense. I comment on this again in the final chapter.

³ Use of the passive voice here was spontaneous, but is significant as an indicator of my tensions and will be addressed in Chapter 8.

level with different teaching styles, and the demands of the students themselves. My teaching of the course has been evolving, but the changes are incremental and do not seem to alleviate the tensions. Improvements in one aspect of the course often correspond to losses of other positive aspects.

Living with these tensions was/is part of the experience of teaching the MFT course for me. My own love of mathematics, my belief that elementary teachers can play a crucial role in influencing children's attitudes towards mathematics, and my further belief that this MFT course offers a too brief, yet critical opportunity to shape these teachers, have all been factors that have compelled me to seek a resolution to these tensions.

1.4 Serendipity

Four years ago, I had the opportunity to work towards improving my experience of teaching the MFT course in two very different ways. The first was practical and direct: I became a member of a subcommittee whose task, in broad terms, was to define what the MFT course should be. The second was personal and more indirect: I registered in a PhD program in Mathematics Education. My motives for going back to school were not specifically related to the MFT course, but reflected a general desire to learn more about mathematics education and to become a better teacher. Yet, observations and realisations that arose during my work with the subcommittee, coinciding with my experiences and opportunities offered in the PhD program, led me to ask certain questions that became the basis for my research.

1.4.1 The Math for Elementary Education Subcommittee

In 2007, I joined a subcommittee of the provincial post-secondary articulation committee, the BC Committee on the Undergraduate Program in Mathematics and Statistics (BCcupms), whose mandate was to articulate a core curriculum for MFT courses. This subcommittee was spearheaded by the same university where I had received my early training with this course. Its impetus was concerns about increasing numbers of MFT courses being developed and delivered across the province. The newly proposed courses increasingly diverged from this university's expectations, and even those courses, including my own, which had been developed on the university's model, had failed to keep in step with its evolving visions of what the course should be.

By gathering a group of individuals who represented a variety of institutions, and inviting input from mathematics education experts and elementary school teachers, the committee hoped to draft common guidelines for the course that would ensure future teachers received adequate preparation, and would facilitate decisions on equivalence for course transfer between institutions. A similar project had been undertaken in British Columbia a few years earlier, called the Core Calculus Project. The work of this latter committee resulted in a concise list of common-core content for first-year Calculus courses that all post-secondary institutions across the province were happy to agree to. The MFT subcommittee's task turned out to be much less straightforward.

It quickly became apparent that within the MFT subcommittee there were almost as many different visions of the course as there were people seated around the table: each spoke passionately about what content *must* be included, or what approaches *need* to be taken, in order to prepare future elementary school teachers for the task of teaching mathematics. There were similarities, of course, and as mentioned above, the existence of some differences was to be expected. However, the differences went beyond institutional constraints around hours of instruction, or instructor preferences for modes of delivery—they were far from superficial. They arose from differing fundamental views on what it takes to be an effective mathematics teacher, and called into question the role this one course can and should play in preparing teachers of mathematics. As various committee members spoke, their visions represented diverse perspectives that were often not compatible with each other. And yet as each spoke, it seemed that the view presented was reasonable and appropriate: all cared deeply about the development of effective elementary mathematics teachers. There was much debate, though notably, there was extremely little, if any, use of evidence-based research literature to support the diverse viewpoints.

Despite, and perhaps at the root of the dissension, was one point that committee members agreed on: there is simply not enough time in a one-semester course to transform the significant number of math-phobic individuals with minimal arithmetic skills, low confidence, and an aversion to mathematics, often stemming from deep-rooted negative past experiences, into effective elementary mathematics teachers. But given that there is insufficient time to “do it all”, what topics should be given priority? How many can be squeezed in? Is quality of instruction/learning more important than the quantity of material covered? It was taken as given that it was not within the power of

those around the table to change the system that only requires future elementary teachers to take one mathematics course.

The end-result was a compromise. Unlike for the Calculus Project, any listing of specific content could not be sufficient for delineating the parameters of the course: any such list of realistic length would leave out vital topics. Furthermore, topic lists would fail to capture important affective goals, just as I had found in creating my first MFT course outline at my institution. At the same time, the committee felt an obligation, stemming from their original mandate, to set some ground rules. Eventually, they agreed upon a global “manifesto”, intended to support course designers and instructors in the seemingly inevitable choices they would need to make (see Appendix A).

I left the committee in the end, enlightened by the discussion and debate, but also perplexed. Clearly, both affective and cognitive goals for future teachers were important, but which takes priority when choices need to be made? It was somewhat comforting to know that others wrestle with the same problems, and all feel that the single MFT course is insufficient, but it was apparent that we all choose to manage this challenge in different ways. I was surprised by just how diverse the perspectives I encountered were. I wondered if there is a “best” way, and on what basis I could make (or already was making) my choices for the MFT course, even within the recommended guidelines. The tensions I had experienced in teaching MFT remained.

1.4.2 Graduate studies

Not long after the work of the subcommittee began, I enrolled in the Mathematics Education PhD program at Simon Fraser University. Although I had had some education training during my professional accreditation program (PDP), it had been many years since I had taken any formal education courses. I looked forward to re-entering this world, this time with the benefit of my teaching experiences against which to juxtapose the new theory. It was within this graduate program that I began once again to read about, and be exposed to, the contemporary wisdom on the preparation of elementary mathematics teachers.

Some of what I read was discouraging. Some research reported that preservice teacher beliefs about teaching mathematics are resistant to change, despite interventions, having been so strongly established through years of their own school experiences (Borko et al., 1992; Wilson & Ball, 1996). Some of it was quite exciting,

including the progress that was being made in identifying and defining *mathematics-for-teaching* (e.g. Ball, Thames, & Phelps, 2008).

I looked eagerly to the research to find guidance for my MFT course. As it turned out, there was no shortage of recent discussion on the mathematics preparation of elementary teachers. In response to ongoing concerns about the preparation of teachers, a number of organisations in the U.S. had begun to publish recommendations. Their message was clear: more mathematics content courses are needed. I found myself reading some of the same views that had been expressed by those around the table at the MFT subcommittee meetings.

Then I was struck by the following statement in Greenburg and Walsh (2008): “While we heartily endorse high-quality research on how best to prepare elementary teachers, the reform of current preparation programs cannot be postponed until research results are definitive” (p. 10). The recommendations were not, in fact the authors asserted that they *could not be*, research-based given the current lack of knowledge with respect to the requirements for developing effective elementary mathematics teachers. But they were also clearly not based on any research into the realities of the MFT classrooms: the instructors, the students, the resources available, the institutions themselves.

My re-immersion into the world of academia allowed me to look at my situation as an instructor of MFT with new eyes. As a college mathematics instructor, I felt the responsibility of playing a part in supporting the mathematical development of prospective elementary teachers. As a graduate student, I was exposed to literature that offered the potential for evidence-based theory that could inform my practice. But my encounters with the research fell short of providing me with the practical answers I sought. Given the time that I have with these prospective teachers, their needs, and the resources at my disposal, how do I go about doing my part in preparing them to become effective elementary school mathematics teachers? What is my part?

Both the subcommittee work and the research literature had both failed to help me resolve the tensions; however, the opportunity for research within my graduate program offered another avenue of pursuit.

1.5 The research questions

I had many questions, but needed to narrow the scope of my interests in order to delineate a reasonable research project. To begin, a deeper engagement with the extant literature on my part was required: I wanted to investigate what was known about the knowledge, attitudes, and beliefs needed by effective teachers of mathematics, and about the role the MFT course is intended to play in helping prospective teachers achieve these attributes. But even if such a role could be well defined, how would I be able to reconcile such an “ideal” with the diverse perspectives on the course that I had encountered in my work on the MFT subcommittee? Why was there such diversity for MFT, but not for Calculus? My experience strongly suggested that the MFT course “as delivered” is highly and non-trivially dependent on the instructors, and my literature search soon revealed that very little is known about the post-secondary mathematics instructors who typically teach the MFT courses.

As I considered the potential impact that these different instructors, with their variety of approaches and perspectives, might have on the preparation of future teachers, I became more and more interested in the experience of teaching the MFT course—my own and that of others. As an MFT instructor myself, I was well placed to talk to them, to attempt to document and come to understand the extent of the diversity between their experiences, including whether or how they experience and manage any tensions. Understanding how they approach the teaching of the course could offer insight into the role the course plays in teacher development in practice, a theoretical concern, and perhaps reveal ways to reduce or eliminate the (my?) tensions, a practical concern.

These considerations led me, ultimately, to the following research questions:

1. How do instructors interpret and experience the teaching of MFT courses? What factors contribute to the diversity of these interpretations and experiences?
2. In particular, what are the major tensions they experience? What factors contribute to these tensions and how are they managed?

1.6 Overview

In the following chapters, I present the results of my investigations. Chapter 2 elaborates on my search of the literature, providing an overview of current research on the knowledge, beliefs and attitudes needed by effective teachers of elementary school mathematics, on the role that Math for Teachers courses are meant to play in teacher preparation, and on what is known about the instructors who teach these courses. Chapter 3 describes the theoretical influences that have shaped my approach to the research questions and my interpretation of the data, while Chapter 4 summarises the methods applied in data collection and analysis. A brief contextual outline of the experience of teaching MFT (Chapter 5) is followed by two chapters that delve into the diversity (Chapter 6) and the tensions (Chapter 7) revealed. Chapter 8 provides an in-depth look at the experience of tensions of one particular instructor, and finally, in Chapter 9, the research questions are addressed, along with a consideration of the implications of these results and avenues for future research.

2: REVIEW OF THE LITERATURE

It was once thought that all one needed to teach elementary school mathematics was the ability to do arithmetic and to manage a classroom (Shulman, 1986). Since that time there has been an ever-increasing awareness of the complexity of the tasks that face elementary school mathematics teachers and a corresponding concern with their preparation. There continue to be many myths and assumptions, despite a proliferation of research and papers on what elementary school teachers need to teach mathematics effectively. What exactly does the research tell us? What does it mean to be an *effective* mathematics teacher? What knowledge, beliefs and attitudes are needed? Where and how should teachers acquire/develop these traits? In this chapter, I begin with an exploration of these broader questions and then move to consider more closely the implications of the research on one particular aspect of elementary teacher preparation: “mathematics content” courses for preservice elementary teachers. In particular, I examine what the research tells us about the role these courses play in supporting the development of effective mathematics teachers. What can these courses hope to achieve, and what do we know about the people who teach them?

What knowledge, beliefs and attitudes do elementary teachers need to teach mathematics effectively? This seemingly simple question is problematic at every turn. To begin we need to consider what is meant by “effective”—its criteria influence what teachers will need. Next it will be necessary to pick our way through the tangled web of literature on teachers’ knowledge, beliefs and attitudes. What types of knowledge are relevant to mathematics teaching? Which are associated with “effective” teaching? How do we uncover teachers’ beliefs and attitudes and how are these related to what teachers do in the classroom? And to confound the issues even further, where do we draw the lines among knowledge, beliefs, and attitudes, or are these all, in fact, intertwined, simultaneously affecting and being affected by teachers’ actions? The review of the literature in this chapter is not, cannot, be exhaustive. Research on teacher education has been prolific, having grown rapidly since 1986 (Connelly, Clandinin, & He, 1997), and theories about what is needed to teach elementary school mathematics abound. Inevitably, simply through the choices of literature made, this

review of the literature will reflect only one possible perspective in this very complex domain.

2.1 Effective teaching

In order to elucidate what is meant by “effective”, it is informative to consider how effectiveness is measured. There are two main approaches: one group of studies uses student achievement as the measure, while others, recognising that many factors other than teacher performance can affect students’ test scores, look for specific teacher behaviours or routines that are conducive to student learning (Wilson, Floden, & Ferrini-Mundy, 2001; Berliner, 1989). These behaviours are often inferred from studies that report on exemplary classrooms where practice is based on particular pedagogical stances (e.g. Lampert, 1986; Cobb & Yackel, 1996). Brown, Askew, Baker, Denvir, and Millett (1998) found some agreement amongst international studies that such desirable behaviours include:

the use of higher order questions, statements and tasks which require thought rather than practice; emphasis on establishing through dialogue, meanings and connections between different mathematical ideas and contexts; collaborative problem-solving in class and small group settings; more autonomy for students to develop and discuss their own methods and ideas (Creemers, 1997; Bell, 1993; Cobb & Bauersfeld, 1995; Yackel & Cobb, 1996; Wood, 1996; Stigler & Hiebert, 1997; Boaler, 1997; Askew et al., 1997a). (p. 373)

Kilpatrick, Swafford, and Findell (2001) describe characteristics of effective mathematics teachers, which they capture under the term “teacher proficiency”. This label adds “versatility” to more traditional conceptions of teacher effectiveness, encapsulating:

a conceptual understanding of the core knowledge required in the practice of teaching, fluency in carrying out basic instructional routines (which includes dealing with student misconceptions and with challenging student questions), strategic competence in planning effective instruction and solving problems that arise during instruction, adaptive reasoning (i.e. reflective practice), a productive disposition (which consists of coherent beliefs about mathematics, teaching practice, students, as well as the capacity to learn). (p. 391)

The “core knowledge” here refers to strong, highly connected subject and pedagogical knowledge.

There are difficulties inherent in both of these methods of measuring effectiveness. Research that relies on student achievement as the measure faces the challenge of taking into account the other contextual factors that affect student learning aside from the teacher. Those that judge effectiveness by examining teacher behaviour and comparing it to a check-list of attributes are necessarily pre-supposing what those attributes should be. A more detailed examination of this issue is beyond the scope of this dissertation, but given that studies on teacher effectiveness are used to support so much of the advice on teacher preparation, it is worthy of note.

2.2 What characteristics should teachers have?

There is no shortage of opinions on what characteristics elementary teachers should have in order to teach mathematics effectively. A readily available source of these views is to be found in policy documents and reports produced by both provincial/state Education Ministries and mathematics teacher organisations. Often motivated by concerns that students of mathematics are not measuring up to international standards, these documents seek to inform teachers and teacher educators of the needs of teachers, providing lists of attributes and proficiencies. Some of these lists are content-based, listing what teachers need to know, beginning with the elementary school curriculum and pushing somewhat beyond this to ensure teachers will have some perspective on the topics that they teach (e.g. National Council of Teachers of Mathematics, 1991). Since the reform movement of the '90s, they typically emphasise the need for teachers' subject knowledge to be deep and connected. Others are more practice-based (e.g. Kilpatrick et al., 2001), providing recommendations in the form of activities that teachers must be proficient in, for example:

appraising and adapting instructional materials, representing content honestly and accessibly, planning and conducting instruction, assessing students' learning, hearing/seeing expressions of students' mathematical ideas and designing appropriate responses, interpreting written work, analysing reasoning, seeing the mathematical possibilities of a task, and developing and assessing learning trajectories. (p. 370)

In their more comprehensive survey of the prominent U.S. policy documents, Ball, Lubienski, and Mewborn (2004) report finding both general (not mathematics-specific) calls for knowledge that is "deep, connected, and conceptual", which "include[s] knowledge of the epistemology of the field" (p. 441), as well as identifications of specific

mathematics topic listings which they observe use an approach that is “rooted primarily in policy deliberations and often does not reflect research evidence” (p. 441).

In the *No Common Denominator* report of the National Council on Teacher Quality, the authors freely admit that many of their recommendations are not based on research (Greenberg & Walsh, 2008), but maintain that the need to improve the mathematics preparation of elementary teachers is too urgent to wait for definitive research results. The recent report of the National Mathematics Advisory Panel (2008) identifies a number of questions requiring further research (including some of the questions addressed in this dissertation). In particular, more research is needed to determine exactly what effective teachers do to enhance student learning, to define and measure teachers’ knowledge, and to identify the types of teacher preparation that work best.

So, the short answer to the question of what knowledge, beliefs, and attitudes teachers of elementary mathematics need to have to be effective, is that from a research perspective, we do not exactly know. Researchers have made a start towards answering these questions, but the issues are still muddled. In what follows, I will try to unravel the many threads by addressing what the literature tells us about the knowledge, beliefs and attitudes of mathematics teachers separately, to the extent possible given their interconnectedness.

2.2.1 Knowledge

No research is needed to tell us that teachers’ knowledge is an important factor in mathematics teaching. As observed by Ball (1991), “philosophical arguments (e.g., Buchmann, 1984), as well as common sense, have already persuaded us that teachers’ knowledge of mathematics influences their teaching of mathematics.” Fennema and Franke (1992) concur that, “no one questions the idea that what a teacher knows is one of the most important influences on what is done in classrooms and, ultimately, on what students learn” (p. 147). However, we are far from having a consensus on what precise form this knowledge takes (Ball, Lubienski, & Mewborn, 2004; Fennema & Franke, 1992).

Classifications of Knowledge

A number of researchers have turned their efforts toward the fundamental task of defining exactly what types of knowledge are relevant to mathematics teaching. These definitions tend to go beyond mere subject knowledge. Strategies for approaching this include theoretical work (Shulman, 1986); comparative studies across continents (L. Ma, 1999); and practice-based approaches (Ball & Bass, 2000, 2003). I will consider each of these more closely in turn.

Shulman's (1986) framework has been very influential. His initial classifications of subject content knowledge (which includes both substantive and syntactic structures of a subject), pedagogical content knowledge (which involves the integration of subject knowledge with the practical matters of conveying that knowledge to another) and curricular knowledge, draw attention to the idea of specialised subject knowledge for teaching that goes beyond mere subject knowledge. Particularly prominent is his notion of pedagogical content knowledge (PCK). This builds on subject content knowledge, but specifically addresses "the ways of representing and formulating the subject that make it comprehensible to others" (p. 9). Mathematics is rich in varieties of representations and models, which the expert teacher should have at her disposal. In this category, Shulman also includes "an understanding of what makes the learning of specific topics easy or difficult" (p. 9).

Under curricular knowledge Shulman includes:

the full range of programs designed for the teaching of particular subjects and topics at a given level, the variety of instructional materials available in relation to those programs, and the set of characteristics that serve as both the indications and contraindications for the use of particular curriculum or program materials in particular circumstances. (p. 10)

In later work (Grossman, Wilson, & Shulman, 1989; Thompson, Philipp, Thompson, & Boyd, 1994), these organising principles of the discipline of mathematics are subsumed under subject content knowledge.

For her research, Ma (1999) compared U.S. elementary school teachers to their Chinese counterparts, asking teachers in both groups to engage in specific teaching activities: explaining subtraction with regrouping, analysing student errors in multi-digit multiplication, creating a story or model for division by fractions, and evaluating a student's geometric "discovery". The Chinese teachers consistently out-performed the

American teachers, and Ma sought to identify the characteristics that distinguished the two groups. She found that the very best teachers exhibited what she dubbed a “profound understanding of fundamental mathematics” (PUFM) (p. xxiv). This notion encapsulates specific content knowledge along with “awareness of the conceptual structure and basic attitudes of mathematics inherent in elementary mathematics and the ability to provide a foundation for that conceptual structure and instil those basic attitudes in students” (p. 124). This characterisation highlights the importance of deep and connected knowledge of mathematics, and mathematics teaching and learning.

Ball, along with a number of collaborators (Ball & Bass, 2000; Ball, Thames, & Phelps, 2008) has made significant contributions to building a theory of mathematics-for-teaching. Their approach is to build this theory from the ground up, by examining teacher practice in mathematics classrooms and identifying the knowledge demands that arise. In some of their most recent work, through qualitative analysis of video-taped third-grade classes, Ball, Thames and Phelps (2008) have identified two subdomains of pedagogical content knowledge, which they describe as “knowledge of content and students” and “knowledge of content and teaching”, as well as a category of “specialised content knowledge”, which is part of subject content knowledge but is distinct from the mathematics needed by non-teachers (p. 389).

“Knowledge of content and students” (KCS) involves being able to anticipate students’ difficulties, knowing what will interest and motivate them, being able to work with students’ developing conceptions of math, and being aware of common misconceptions. “Knowledge of content and teaching” (KCT) involves knowing how to sequence content and examples effectively, knowing the potentials of various representations, and being able to deal in-the-moment with student comments and questions in order to achieve teaching goals. Teachers’ “specialised content knowledge” (SpCK) includes being able to quickly analyse students’ errors, evaluate alternative solutions for correctness and general applicability, provide rationales for mathematical procedures, choose examples strategically to bring out the important mathematical concepts, and “unpack” mathematical concepts. This specialised mathematics knowledge is distinguished from “common content knowledge” (CCK), which is the mathematics that is in the curriculum, the mathematics that their students are expected to be able to do. It includes problem solving, executing procedures, providing correct definitions and using notation appropriately.

This highlights only three of the many attempts at pinning down the knowledge required for mathematics teaching. Some of the other distinctions between forms of knowledge that researchers have made include: “knowledge of” vs. “knowledge about” (Ball, 1990); procedural knowledge vs. principled knowledge, or “knowing that” vs. “knowing why” (Lampert, 1986); theoretical knowledge vs. practical knowledge, or “knowing that” vs. “knowing how” (Ernest, 1989).

Another recent contribution is that of Silverman and Thompson (2008), who build on Simon’s (2006) concept of “key developmental understandings” (KDUs). KDUs are a construct intended to focus research and discussion of students’ understanding on those concepts “that are critical to the development of important mathematical ideas” (Simon, 2006, p. 363). In their work, Silverman and Thompson (2008) observe that teachers need more than these KDUs themselves, they must be able to develop appropriate learning trajectories to allow their students to build KDUs. It is this latter ability that, in their view, is characteristic of possessing mathematics-knowledge-for-teaching.

Another perspective on this conversation is provided by those who problematise the distinctions made, in particular the distinctions between mathematics subject knowledge and pedagogical content knowledge. Davis and Simmt (2006) try to capture the complexity of the interrelationship between these types of knowledge by describing “mathematics-for-teaching” as a structure with four intertwining categories, where subjective understanding is situated within a classroom collectivity, which is set within curriculum structures, which, in turn, lie within mathematical objects.

Furthermore, Goulding, Rowland, and Barber (2002) observe that:

Students who have multiple representations for mathematical ideas and whose mathematical knowledge is already richly linked will be able to draw upon these both in planning and in spontaneous teaching interactions. In such cases we would argue that the students’ subject matter knowledge is ripe for exploitation and that in turn the experience of teaching will feed back into and enrich subject matter knowledge. (p. 691)

This conveys not only a close interrelationship between knowledge of mathematics and pedagogical content knowledge, but draws attention to the connections between these forms of knowledge and practice. In the same vein, Cochran, DeRuiter and King (1993) see the knowledge required for teaching as dynamic, replacing Shulman’s (1986) notion of pedagogical content knowledge with “pedagogical content knowing (PCKg)”, which

they define as: “a teacher’s integrated understanding of the four components of pedagogy, subject matter content, student characteristics, and the environmental context of learning” (p. 266).

Alongside and concurrent with efforts to categorise teacher knowledge, there is literature that seeks to be more prescriptive about the knowledge teachers need to have.

Recommendations for mathematics subject knowledge are often supported by research that demonstrates particular deficiencies in teachers (e.g. Martin & Harel, 1989, regarding conceptions of proof; Graeber, Tirosh, & Glover, 1989, regarding multiplication and division; Zazkis & Campbell, 1996, regarding divisibility and multiplicative structures). These studies do not establish links to student success, but operate under the assumption that these deficiencies will directly affect both teaching and learning. As Graeber, Tirosh and Glover (1989) state, “If preservice teachers hold these misconceptions [related to multiplication and division], they are not likely to recognize the related errors students make. In fact, their instruction might inadvertently contribute to perpetuating the misconceptions” (p. 95).

Ma’s (1999) work is also based on demonstration of teacher deficiencies, but makes recommendations based on a comparison with relatively effective teachers. As exemplified by the Chinese teachers, mathematics teachers at any level need to be able not merely to see the connections between mathematical ideas, but also to see how the learning of certain mathematical concepts is linked to the learning of other concepts. This understanding enables teachers to appreciate the relative importance of topics, providing them with information needed to know when to slow down, or even backtrack, and to identify the concepts that are essential to future success.

Ball and Bass (2000) also make recommendations for the knowledge that teachers should have, but rather than starting from teacher deficiencies, they draw their conclusions from their observations of mathematics teachers in practice. For example, some of the activities of effective mathematics teachers that they identify include “unpacking familiar mathematical ideas, procedures, and principles”, “understanding the role of definitions and choosing and using them skilfully, knowing what constitutes an adequate explanation or justification, and using representations with care”, as well as “mathematical problem solving both like and unlike the problem solving done by mathematicians or others who use mathematics in their work” (p. 13), some of which are subsumed in the descriptions of KCS, KCT and SpCK above (Ball, Thames, & Phelps,

2008). Once lists of proficiencies have been identified, they go on to construct assessment tools to measure the extent to which teachers have these abilities, and compare these scores to student performance. Ball, Hill and Bass (2005), report a positive correlation between teachers' scores on their measures of mathematical knowledge-for-teaching and student achievement gains for a large cohort (approximately 700 teachers, and 3000 students) in grades one and three. Even when the data was controlled for other factors that might affect student gains, such as socio-economic status or absenteeism, the teachers' scores were still a good predictor of those gains (Hill, Rowan, & Ball, 2005).

Does subject knowledge matter?

Despite the apparent consistency in the recommendations above and such confident declarations as "Knowledge of the content to be taught is the cornerstone of teaching proficiency" (Kilpatrick et al., 2001, p. 372), the findings are far from conclusive as to the extent that subject content knowledge contributes to the effectiveness of elementary mathematics teachers. In his oft-cited survey of research on the effects of teacher variables on student performance, Begle (1979) came to the conclusion that teachers' mathematical understanding is not a significant factor in improved student performance (as cited in Ball, Lubienski, & Mewborn, 2004).

Many report finding limited or no connection between possession of mathematics knowledge and student achievement. Kukla-Acevedo (2009), drawing on a large state administrative data set that allowed the matching of students to teachers over time, found that only the teachers' overall GPA was consistently predictive of grade 5 students' mathematics achievement across student groups. In fact, they found that teachers' mathematics GPA was initially negatively correlated with students' mathematics achievement, though this eventually became positive after the teacher had been practicing for 5 years. In their comparison of teacher background qualifications, attitudes, and instructional practices to mathematics achievement gains of their first-grade students, Palardy and Rumberger (2008) also confirmed that teacher background qualifications are not associated with mathematics achievement gains; and Askew, Brown, Rhodes, William, & Johnson (1997), in a study of 90 teachers of 5- to 11-year-olds, found that although some of the teachers had difficulties with their understanding of the school subject knowledge, they did not find a significant impact on their

effectiveness, as measured by pre- and post-tests of their students' mathematical performance.

There is also evidence that more mathematics content knowledge has no impact on teacher practice. From a study of 59 post-graduate primary education students who were tested for subject knowledge, Bennett, Carré, and Dunne (1993) conclude "there is virtually nothing to distinguish [the teaching performance of] mathematicians and others in teaching mathematics" (as cited in Henderson & Rodrigues, 2008, p. 96). Simply having the knowledge is insufficient to guarantee that teachers will make appropriate use of it. Leinhardt (1989) observed that despite having good subject knowledge, novice teachers "did not seem to access that knowledge during teaching" (p. 73).

On the other hand, some researchers (including Ball, Hill, & Bass (2005) and Hill, Rowan, & Ball, (2005), cited above) have found positive correlations between teacher knowledge and teacher performance. Goulding, Rowland and Barber (2002) found that poor school mathematics knowledge of preservice elementary teachers (as measured by an assessment tool) was associated with weaknesses in their planning and teaching. However, the researchers stopped short of affirming a causal link, observing that good subject knowledge was not sufficient for good teaching. Leinhardt, Putnam, Stein, and Baxter (1991) conclude that subject knowledge has the potential to impact teaching in several ways: for example, teachers' mental plans for lessons were dependent upon their familiarity with the content to be taught (Borko, Livingstone, McCaleb, & Mauro, 1988), as were the questions they asked and the explanations they offered to pupils (Bennett et al., 1993). Millett and Johnson (1996) observe: "Feelings of inadequacy over subject knowledge have also been shown to lead to over-reliance on a commercial scheme" (p. 375). Mullens, Murnane, and Willett (1996) found that "third-grade students in Belize learn more mathematics when their teachers have a strong command of the subject", where teacher knowledge was ranked according to levels of teacher qualification.

One of the possible reasons for these conflicting results lies in the methods used to measure teachers' subject knowledge. Often this knowledge is evaluated by considering levels of credentials, number of courses, or grades in mathematics courses. It is not at all clear whether these can provide an accurate reflection of teachers' knowledge, particularly of the mathematics that they teach. In fact, in a study involving a school mathematics assessment of 80 preservice teachers, Henderson and Rodrigues

(2008) found that participants with higher levels of prior mathematics education did not perform better on their assessment of school mathematics. Other researchers (Ball, Hill, & Bass, 2005; Hill, Rowan, & Ball, 2005) make use of self-designed or existing assessment tools to measure teachers' subject knowledge. However, it is not clear that these tools are measuring the "right" sort of knowledge (Goulding, Rowland, & Barber, 2002). In many cases (e.g., Bennett et al., 1993; Askew et al., 1997), the assessments used are tests of what Ball et al. (2008) refer to as common content knowledge (CCK, see above), and not mathematics knowledge-for-teaching.

2.2.2 Beliefs

There has been a growing interest in mathematics teachers' beliefs over the last 30 years, in particular with respect to how those beliefs influence what happens in the classroom (see Thompson, 1992, and Philipp, 2007, for a review). Ernest's (1989) theoretical paper offers one possible framework to characterise the types of beliefs that have relevance for the teaching of mathematics. He separates teachers' beliefs into those about the nature of mathematics (math as dynamic and problem-driven, math as static and structured, and math as a collection of facts and rules), beliefs about the teaching and learning of mathematics (e.g. investigational/problem-based discovery model, conceptual understanding model, mastery of skills model), and general principles or values with respect to education. He notes that beliefs are sometimes held implicitly, but have the power to affect teaching, particularly in regards to the choices of teaching methods and the degree of autonomy given to students. His conclusions are based on the strong connection observed between teachers' conceptions of the nature of mathematics and their models for the teaching and learning of mathematics. He comments that these different philosophies have been reflected in the various curriculum trends over the years.

Following the research on teachers' beliefs is even more complex than that on teacher knowledge. A number of factors contribute to this. The first is that there is often a very blurry boundary between beliefs and knowledge, and beliefs and attitudes/emotions. Second, there are many different types of beliefs that may influence teaching and learning in the mathematics classroom. These include (but are not limited to) beliefs about mathematics, beliefs about the teaching of mathematics, beliefs about the learning of mathematics, beliefs about students, beliefs about teachers own ability to

do mathematics, to teach mathematics, etc. Third, how are teachers' beliefs to be measured or captured? There is a substantial amount of literature on the inconsistencies between teachers' espoused beliefs, enacted beliefs, actual beliefs, and the attributed beliefs that the researchers assign to them (e.g. Liljedahl, 2008; Hoyles, 1992; Speer, 2005). Finally, how can the impact of those beliefs be measured? In only a few cases have researchers attempted to show a connection between teachers' beliefs and students' achievement. In most cases, the effects of beliefs on students are assumed to be mediated: the beliefs influence teachers' actions, in particular classroom practice, which in turn affects students. Recommendations for beliefs that teachers should hold are inevitably tied to theoretical perspectives on teaching and learning.

A number of researchers have investigated the relationship between teachers' beliefs and teacher practice, with mixed results (see Thompson, 1992, for a review), though studies more consistently find a relationship between teachers' beliefs about mathematics and their practice than between their pedagogical beliefs and their practice (e.g. Raymond, 1997). In a case-study involving three junior high school teachers, Thompson (1984) found that "teachers' beliefs, views, and preferences about mathematics and its teaching played a significant, albeit subtle, role in shaping their instructional behaviour" (p. 105). The teachers' conceptions of mathematics were strongly related to how they chose to present mathematics topics, but she also noted that the teachers' beliefs that were non-specific to mathematics often interfered and sometimes took precedence. In a more extensive quantitative study of 140 volunteer preservice and practicing teachers, Schwartz and Riedesel (1994) found teachers' pedagogical beliefs were significantly correlated with teachers' professed teaching practices. Other studies, such as Sosniak, Ethington and Varelas (1991, cited in Hoyles, 1992) found serious inconsistencies between the beliefs (about mathematics and pedagogy) and practices of American eighth-grade mathematics teachers. Such inconsistencies have been attributed to flaws in research methodology (e.g. Speer, 2005) or a failure to consider the realities of the classroom context (e.g. Skott, 2001).

Askew et al. (1997) provide one of the few examples of research that looks at how teachers' beliefs are related to student achievement. Out of an initial pool of 90 teachers, whose students' mathematics skills were tested at the beginning and the end of the study, 18 were identified as effective based on peer and administrator recommendations. These teachers were then interviewed to probe their beliefs about

students and learning, and their classes were observed with a focus on organisation/management, teaching styles, teaching resources, and student responses. Using qualitative research methods, they identified three orientations of pedagogical beliefs: connectionist, transmission, and discovery (which roughly correspond to Ernest's (1989) three categories above). They found that "teachers with a strong connectionist orientation were more likely to have classes that made greater gains over the two terms than those classes of teachers with strong discovery or transmission orientations" (p. 5). They caution that links between beliefs and the teachers' practice can only be hypothesised from their data, acknowledging that individuals can take similar actions without the same intent, and even without the same result.

Another dimension of teachers' beliefs to consider is teachers' self-efficacy, their belief in their ability to teach mathematics effectively. (This sits on the boundary between beliefs and attitudes as it incorporates emotional factors, i.e. confidence and anxiety.) For Swars, Smith, Smith, and Hart (2009):

Teachers' possessing a strong sense of efficaciousness is of critical importance as teacher efficacy has been linked with classroom instructional strategies and willingness to embrace educational reform (Riggs & Enochs, 1990), commitment to teaching (Coladarci, 1992), and student achievement (Anderson, Green, & Loewen, 1988). (p. 50)

However, other studies (Ashton & Webb, 1986, as cited in Rowan, Chiang, & Miller, 1997) found only mixed support for the view that teacher efficacy has an effect on students' achievement. Interestingly, Swars et al. (2009) note that if teachers' efficacy beliefs are connected to the traditional teacher-centred teaching approaches, they will be in tension with the constructivist philosophies of current reform curricula. So if teacher efficacy matters at all, we need to ensure that it is associated with "appropriate" pedagogical beliefs.

A major challenge to articulating and supporting specifications of the beliefs that elementary teachers need to be effective teachers is the inter-relatedness of knowledge and beliefs about mathematics and pedagogy. Ernest's (1989) framework is useful, but fails to capture this complexity. Wilson and Cooney (2002) warn of the dangers of separating knowledge and beliefs, citing this as a cause of incomplete descriptions of beliefs, which in turn can lead to apparent inconsistencies between beliefs and practice. Ball (1991) espouses the view that "in teaching, teachers' understandings and beliefs about mathematics interact with their ideas about the teaching and learning of

mathematics and their ideas about pupils, teachers, and the context of classrooms” (p. 19). Peterson, Fennema, Carpenter and Loef (1989) have pursued this by studying the relationship between pedagogical content knowledge (PCK) and a construct they term “pedagogical content beliefs” (PCB), which are beliefs about student learning. Thirty-nine first-grade teachers and their students participated in their study. The teachers’ PCBs were classified using both interviews and beliefs surveys, their PCK was tested, and their students were tested on both number facts and on problem solving. Teachers whose PCBs were classified as “cognitively based” had higher PCK scores, demonstrating both a wider knowledge of strategies and having better knowledge of their students. They also found strong connections between all of: PCK, PCB, teaching strategies, presentation of content, and student performance on the problem-solving portion of the test (though not on the number facts portion).

2.2.3 Attitudes

Common sense indicates that teachers should have positive attitudes towards mathematics and their own ability to learn and teach it. Again, the body of research is considerable, and I touch on only some of the main ideas here.

Prospective teachers need to have positive attitudes towards mathematics for their own sakes. Meta-analyses conducted by Ma and Kishor (1997) and X. Ma (1999) indicate a significant relationship between students attitudes towards mathematics and their achievement in mathematics, and a negative correlation between mathematics anxiety and mathematics proficiency (as cited in Philipp, 2007). Many studies confirm that low confidence and relatively high mathematics anxiety are common amongst preservice elementary teachers (e.g. Ball, 1990; Hembree, 1990). In a large-scale study of preservice teachers whose feelings about mathematics were assessed using both a survey instrument and through closely observed interviews, Ball (1990) found a close connection between teachers’ attitudes and their performance on a task involving modelling a representation of fraction division:

The teacher candidates’ knowledge, ways of thinking, beliefs, and feelings jointly affected their responses. Their approaches to figuring out problems were shaped by their self-confidence, their repertoire of strategies, what they were able to remember about related concepts, as well as what they believed about the fruitfulness of trying to figure out a problem in the first place. (p. 461)

Negative attitudes towards mathematics can interfere with teacher learning. Unfortunately, these negative attitudes can be very difficult to change in adults (Wedega, 1999; Evans, 2000).

It is less clear whether teachers need good attitudes for their students' sakes. Palardy and Rumberger (2008) found that teacher efficacy and teacher expectations were the only attitudes that were positively correlated with student achievement. Although Schofield (1981) found that "high achievement and high attitudes in teachers were each significantly related to high achievement in pupils", they "were also related to the least favorable pupil attitudes toward the subject" (p. 462). In his review of the literature, Aiken (1976) reports mixed results both on studies that attempt to measure the relationship between teachers' attitudes and students' attitudes towards mathematics, and in research linking teachers' affect and teaching.

Research on the relationship between teachers' attitudes and teacher practice are rare (Philipp, 2007). In her cross-cultural study, L. Ma (1999) noted that the American teachers frequently failed to exhibit basic mathematical attitudes, including the value of mathematical justification and consistency, the importance of understanding the "why" as well as the "how", and the benefits of approaching the same problem in multiple ways. She felt that their basic attitudes towards mathematics along with their lack of confidence in their own abilities affected their willingness to engage in mathematical problem solving with their students. Ernest (1988) found some indications that attitudes towards teaching mathematics were more influential in teachers' practice than their attitudes towards mathematics.

Still other desirable attitudes of elementary school mathematics teachers that have been discussed in the literature are: curiosity (Simmt, Davis, Gordon, & Towers, 2003), high motivation for success for themselves and their students (Rowan, Chiang, & Miller, 1997; Kukla-Acevedo, 2009), as well as appreciation for the elegance of solutions and for a "good" problem (Ball, 2002).

2.2.4 Other needs

So far, the recommended "needs" of effective elementary mathematics teachers have been discussed with respect to knowledge, beliefs and attitudes. However, a survey of the literature reveals a number of other recommendations that though related to knowledge, beliefs and attitudes, defy categorisation as they cut across all three. I will

touch on these very briefly. Theoretically, all are desirable to promote in teachers, though to date, perhaps due to their novelty or their complexity, have not been the focus of studies on teacher effectiveness.

Kilpatrick et al. (2001) call for teachers to have a “productive disposition”. Teachers that have this disposition have integrated knowledge and beliefs about mathematics, teaching practices, and how their students think. Their learning has reached a point where it becomes “generative”: they perceive themselves as in control of their own learning and see themselves as lifelong learners who can learn from studying curriculum materials, from analyzing their practice and from their interactions with students (paraphrased from Kilpatrick et al., 2001, p. 384).

Embedded in this description is the notion that teachers will be disposed towards reflective practice. Schön (1987) promoted “reflection-on-action” in order to support “reflection-in-action”, the ability to respond in-the-moment while teaching. This requires:

knowledge of the greatest possible number of methods, the ability of inventing new methods and, above all, not a blind adherence to ONE method but the conviction that all methods are one-sided, and that the best method would be the one that would answer best to all the possible difficulties incurred by a pupil. (p. 4)

Kilpatrick et al.’s (2001) mention of integrated knowledge and beliefs about mathematics, teaching and learning, is also reminiscent of calls for teachers to have an appreciation for and an ability to draw on connections. This includes: connections between procedural and conceptual ways of knowing (Lampert, 1986; L. Ma, 1999); connections between knowledge of established mathematics and its historical emergence (Davis & Simmt, 2006); connections between various representations of mathematics concepts (Davis & Simmt, 2006); and connections between the knowledge of mathematics, teaching, and learning, with practice (Ball, Lubienski , & Mewborn, 2004).

2.2.5 Listing what teachers need

Rowland, Huckstep and Thwaites (2005) warn: “Whilst we see certain kinds of knowledge to be *desirable* for elementary mathematics teaching, we are convinced of the futility of asserting what a beginning teacher, or a more experienced one for that matter, *ought* to know” (p. 257, their emphasis). Making a list of what teachers need to teach mathematics effectively is not only extremely difficult (perhaps impossible), it also

brings with it dangers. Askew (2008) expresses concern that if such lists are adhered to too closely, teacher training will become too prescriptive. Since any list of mathematical and pedagogical knowledge and beliefs will be tied closely to the curriculum, there may be a danger that curriculum will become entrenched. He suggests that mathematics educators focus on developing “a mathematical sensibility” in teachers “that would enable them to deal with existing curricula but also be open to change” (p. 22). The key elements of this sensibility include precision (being accurate and valuing precise language), generalisation (generalising from examples, but also within a particular situation), and romance (caring about the mathematics and its origins).

Mason (2008) agrees that there is danger in providing checklists, as they draw us away from what is most essential in preparing teachers of mathematics, developing teachers’ awareness of the possibilities and potentials that are available to them in their practice. He affirms, “...teachers need to have access to a range of examples and be aware of different obstacles to trying to learn from examples” (p. 311), but their attention needs to be focussed not on lists, but on noticing: “The aim of teacher education is to prepare the ground so that novice teachers will find themselves increasingly sensitised to noticing possibilities for initiating, sustaining or completing actions which they might not previously have had come to mind” (p. 317). Easily said, but how is this to be achieved?

2.3 Teacher education

Armed with at least a sense of what teachers need, I turn now to consider where they might acquire these attributes and skills.

It is often assumed that they will learn the mathematics content, the curriculum that they are going to teach, during their own K – 12 schooling. However, significant concerns have been raised about the level of preservice teachers’ knowledge of school mathematics (Ball, 1990). Even if they do have fairly sound procedural knowledge, their conceptual understandings often fall short of even our ideals for the students who will be in their classes (cf. L. Ma, 1999). Furthermore, the specialised content knowledge as described by Ball, Thames, and Phelps (2008) is unlikely to be learned during their years at elementary or high school. Beliefs and attitudes will be acquired throughout their schooling, but the massive amount of research on assessing and evaluating teacher beliefs and attitudes testifies to the concerns that preservice teachers enter their

programmes with beliefs and attitudes that are less than optimal for both their own learning and their future as teachers (Philipp, 2007).

Ideally, teachers should go on learning throughout their careers, and much of their learning (some argue all) will occur in practice. However, the responsibility for bringing potential teachers from their state at the end of their high school mathematics to a state where they are ready to begin working in classrooms falls to the post-secondary institutions, to the mathematics departments and the education departments. Though it can depend on students' choices and on local requirements, often students will take only one mathematics content course from the mathematics department and one methods course from the education department. The superficial view is that the mathematicians will teach the mathematics in the content course, and the education faculty will teach the pedagogy in the methods course. But what mathematics should the mathematicians teach?

Research by Henderson and Rodrigues (2008), cited above, supports the growing conviction that prospective elementary school mathematics teachers are not improved by general university mathematics courses (see also Foss, 2000; Fennema & Franke, 1992; Matthews & Seaman, 2007). What they need is specialised mathematics courses that have been developed to meet the perceived needs of future elementary school mathematics teachers (Williams, 2001). With these specialised courses in place, the division of labour between the mathematics and education departments moves from the superficial view to the naive view: mathematicians will teach the "subject content knowledge" (SCK) while education experts will teach the "pedagogical content knowledge" (PCK) (from Shulman's (1986) framework).

This view is naive in that it assumes a relatively clear division between these domains, which is not at all the case. This attempt at division between content and pedagogy is considered to be unfortunate by many. A purpose of Shulman's (1986) original framework was to draw attention to the need for greater integration of content and pedagogy in the preparation of teachers. Ball and Bass (2000) comment that this separation leaves teachers to integrate content and pedagogy on their own once they are in practice, a task for which they are underprepared. Davis and Simmt (2006) recommend integration of content and pedagogy, in order to "help educators develop the conceptual sophistication needed, for example to interpret idiosyncratic student responses, prompt multiple interpretations, trace misinterpretations and structure rich

learning experiences” (p. 293). In other words, they see integration as necessary in order to facilitate development of knowledge of mathematics-for-teaching.

The implication of this is that it is less than clear what learning outcomes are expected to be achieved in the mathematics content course vs. what is expected to be achieved in the mathematics methods course. In the next subsection I will focus on the mathematics content course, considering what outcomes are called for by policy makers and what strategies are recommended by researchers.

2.3.1 Mathematics content courses

Policy documents and reports provide the clearest mandate for specialised mathematics content courses. The No Common Denominator report (Greenburg & Walsh, 2008) calls for prospective elementary teachers to take no less than three such courses as part of their certification programme. Their expectation is that these courses will “deal explicitly with elementary and middle school topics” (p. i), seeking to develop deep conceptual understanding. They go so far as to recommend the number of hours these courses should spend on each of four topic strands: numbers and operations, algebra, geometry and measurement, and data analysis and probability. In their Final Report, the National Mathematics Advisory Panel (2008) makes recommendations about school mathematics curriculum, but also the preparation of elementary teachers:

teachers must know in detail and from a more advanced perspective the mathematical content they are responsible for teaching and the connections of that content to other important mathematics, both prior to and beyond the level they are assigned to teach. (p. xxi)

The recommendations are primarily content-focused, leaving implicit that these courses are also expected to contribute to developing the beliefs and attitudes (as described above) that these students will need to become effective mathematics teachers.

The latest report is a venture supported by the American Mathematical Society and the Mathematics Association of America, a document entitled *The Mathematical Education of Teachers* (Conference Board of the Mathematical Sciences, 2010). It is aimed at a wide audience, but particularly those who design programs and set standards for teacher certification. It echoes many of the recommendations of the earlier documents with respect to calls for more mathematics courses for elementary teachers,

especially those focused on fundamental ideas of elementary school mathematics. It bases the need for more sophistication in mathematics on the part of elementary school teachers on the increasingly demanding quantitative and reasoning skills called for in modern society. It goes further than the other documents in that it makes quite detailed recommendations for school teacher preparation at all levels, listing specific content and offering particular examples that demonstrate some of the deep conceptual understandings they expect teachers to have and to develop in their students. Notably, it recommends that mathematicians teach these courses, asserting that, “Mathematicians are particularly qualified to teach mathematics in the connected, sense-making way that teachers need” (p. xiii).

These reports provide the most specific recommendations that can be found regarding the outcomes for these mathematics content courses. Wilson and Berne’s (1999) survey found that “few studies have been successful in pinpointing an appropriate mathematics ‘curriculum’—whether it be purely mathematical, grounded in practice, or both—that can provide teachers with the appropriate mathematics to help students learn” (as cited in Ball, Hill, & Bass, 2005, p. 16).

Much of the research reports on various approaches to content courses that are successful in increasing not only content knowledge, but “mathematics-for-teaching”, beliefs, and attitudes, as well. Ball, Lubienski and Mewborn (2004) cite a number of studies where researchers report gains in deep conceptual content knowledge and such things as: willingness to try new things, conceptions of students, self-efficacy, and their ability to engage and support mathematics learning in their pupils (Swafford, Jones, & Thornton, 1997; Carpenter, Fennema, Peterson, & Carey, 1988; Fennema & Franke, 1992). The approaches taken in all of these cases involved situating the mathematics to be learned in the context of teaching. Ball and Bass (2000, 2003) are particularly strong advocates for this tactic, recommending that:

Practice in solving the mathematical problems they will face in their work would help teachers learn to use mathematics in the ways they will do so in practice, and is likely also to strengthen and deepen their understanding of the ideas. (Ball & Bass, 2003, p. 13)

Some further recommendations for mathematics teacher educators include: taking advantage of the diversity of knowledge, beliefs and perspectives that the preservice teachers bring to both their content and methods courses (Oliveira &

Hannula, 2008), and the importance of building a community where preservice teachers can engage in mathematical discussions, critically evaluating their own, and each others' mathematical explanations (Simon & Blume, 1996).

2.3.2 The Challenge

Beyond the general calls for deep, connected, knowledge of the elementary curriculum, and some of the methodological considerations, the literature offers no specific recommendations for priorities of outcomes for a specialised mathematics content course for teachers. No priorities are given, even within the list of school content knowledge—they need to know it all!

There is evidence to support concerns that the students coming into these courses often have poor understanding of the elementary school mathematics topics; Ball's (1990) study of 252 preservice teachers revealed "understandings that tended to be rule-bound and thin" (p. 449). Regarding their beliefs, the elementary preservice teachers in the group "tended to see mathematics as a body of rules and facts, a set of procedures to be followed step by step, and they considered rules as explanations" (p. 464). Preservice elementary teachers often suffer from mathematics anxiety (Hembree, 1990), and some are only enrolled to "fulfil a requirement rather [sic] to learn more mathematics" (Kessel & Ma, 2001, p. 477). They also come with "baggage". Ball (1988) observes: "Long before they enrol in their first education course or math methods course, they have developed a web of interconnected ideas about mathematics, about teaching and learning mathematics, and about schools" (p. 40). They have much to learn, and to unlearn. As noted above, the latest recommendations (Greenburg & Walsh, 2008; Conference Board, 2010) recommend three content courses to accomplish what they feel is needed. The reality in this province is that prospective teachers will only take one. How can designers of this course set their priorities?

2.3.3 Priorities

Although the research provides no specific advice on which particular learning outcomes are most important, there is considerable literature engaged in the debate over whether teacher preparation should focus first on knowledge, beliefs or attitudes, either mathematical or pedagogical, or on some combination thereof.

Though some researchers make a case for the priority of strong mathematics knowledge, pointing out that such knowledge can both boost confidence and make teacher practice (i.e. the implementation of teachers' pedagogical beliefs) more effective (Goulding, Rowland, & Barber, 2002; Schwartz & Riedesel, 1994), a large number advocate for an emphasis on teachers' beliefs in specialised mathematics content (and methods) courses. This is the case for Liljedahl, Rolka, and Roesken (2007) who argue that teacher education needs:

to focus on reshaping the beliefs and correct misconceptions that could impede effective teaching in mathematics (Green, 1971). More specifically, teacher education programs need to explicitly facilitate the reshaping of beliefs about mathematics *as part of* the learning of mathematics. (p. 2, their emphasis)

Their rationale is that without appropriate beliefs, teachers' knowledge cannot be applied appropriately in the classroom. Kessel and Ma (2001) concur, also drawing attention to the importance of beliefs in the prospective teachers' own learning: "Prospective teachers' beliefs about mathematics and its teaching and learning may influence their learning in teacher preparation courses, and later their teaching (see, e.g., Borko & Putnam, 1996; Zazkis, 2000)" (p. 467).

Mizell and Cates (2004) agree that beliefs matter most in the development of teachers, but found these beliefs to be very resistant to change in mathematics content courses. They studied two groups of preservice teachers, one of which had two more specialised mathematics courses than the other group, and found no significant differences in their beliefs on mathematics or on pedagogy. Although many concur that it is difficult to change beliefs (McLeod, 1992; Nathan & Koedinger, 2000), others claim to have found some success. Liljedahl et al. (2007) found that challenging the students, involving them as learners of mathematics in a constructivist setting, and providing opportunities for "AHA!" experiences had a positive effect on preservice teachers' beliefs. Kajander (2007) also found that providing opportunities for preconceptions to be challenged during a mathematics methods course affected beliefs, but expresses uncertainty about whether these changes in beliefs will persist once teachers move into practice.

In their classic case study, Borko et al. (1992) promote the view that students' knowledge *and* beliefs need to be challenged in teacher education programmes. They describe the case of one student teacher who, despite espousing pedagogical beliefs

that placed priority on concrete representations and applicability of mathematics, was unable to teach division of fractions effectively. A clear barrier was her lack of conceptual understanding of the content (supporting the priority of the precedence of knowledge); however, she was also not particularly troubled by her inability to provide an explanation to her students and failed to remedy this lack of knowledge after the failed lesson was over. Her attitude calls into question how deeply held her beliefs about teaching and learning mathematics were. The researchers suggest that her beliefs about how she should learn mathematics may have also been at issue, suggesting that a belief that teaching will improve with practice (without need for other interventions) is a common and detrimental misconception. In this case, weak subject/pedagogical content knowledge interfered with her ability to implement espoused pedagogical beliefs, but beliefs about her own learning of mathematics were ultimately what prevented her from overcoming her deficiency.

In a mixed-methods study of students in a preservice methods course, Swars et al. (2009) also explored the interdependence of knowledge, beliefs, and attitudes, by analysing the relationship between specialised mathematics content knowledge (SpCK), cognitively oriented pedagogical beliefs and self-efficacy. Their comparison of two groups of students, who differed in the number of specialised mathematics content courses they had taken, revealed a positive correlation between SpCK and both cognitively oriented beliefs and self-efficacy, leading them to suggest that perhaps:

as prospective teachers develop their understanding of the mathematical content knowledge needed to teach in the elementary grades, they become better able to understand and embrace more cognitively oriented pedagogical beliefs and become more confident in their skills and abilities to teach mathematics effectively. (p. 63)

Although the quantitative portion of their study brought out the importance of developing both mathematics knowledge and mathematics knowledge-for-teaching, in the qualitative portion Swars et al. (2009) found students' beliefs to be the biggest obstacle in their acquisition of knowledge. They observe:

the challenge of improving prospective teachers' knowledge of mathematics in teaching continues to be influenced by their beliefs, which are rooted in their experiences as students of mathematics and often reinforced by traditional teaching practices in university mathematics content courses and field experiences in schools. (p. 48)

This suggests that especially when it comes to mathematics content courses it may not be so much the “what” that matters as the “how”.

2.3.4 Form over content

The work of Swars et al. (2009) (and others) suggests that the form of these courses matters more than their specific mathematics content. As Ernest (1989) observes:

The acquisition of models of teaching and learning mathematics will largely occur through the modes of instruction experienced and witnessed. Views of the nature of mathematics will likewise depend on the modes of instruction and types of experience through which mathematics is learned. Attitudes to mathematics and its teaching will represent the teacher’s personal reactions to experiences in these areas, compounded with other influences. (p. 30)

Ultimately, it is the experiences that prospective teachers have in these courses that will contribute to their development. In her ICMI study Working Group Report, Williams (2001) lists a number of key mathematical experiences that her group concluded preservice teachers should have regardless what types of math courses they take. They recommend that preservice teachers be given opportunities to “reflect on their own learning and understanding of mathematics as well as reflecting on the approaches used by their teachers to introduce and discuss topics” (p. 451) and that these approaches include:

developing mathematical thinking (reasoning, proving); solving problems (unknown/open/non-routine); using knowledge in new (non maths) situations/contexts; modelling; creating ‘new’ (for themselves) mathematical knowledge; contact with historic and contemporary mathematics; communicating mathematics—reading, writing, talking, listening; connecting mathematical ideas; recognising cross-curricular links. (p. 451)

In the end, despite all of the advice to be found in policy reports and the research, the priorities for what is and will be taught in these specialised mathematics content courses for teachers is decided upon by the instructors of these courses. I turn now to consider what we know about them.

2.4 The Instructors

Whether prospective teachers opt to take a specialised or general mathematics course, their post-secondary “content” course requirement will usually be taught by a “content” expert, in other words, a member of the mathematics department of the institution. What do we know about post-secondary mathematics teachers? The research does not provide much information. There is a general lack of research on post-secondary instructors and, with only a few exceptions, research into higher mathematics is primarily focused on understanding student difficulties with particular concepts and the effectiveness of particular interventions (see Artigue, 2001, for a review). Though research is not helpful in providing information about this group, it is not difficult to paint the stereotype.

As is the case for such things, fact mixes freely with assumptions and presumptions. Mathematics instructors at post-secondary institutions have advanced degrees in mathematics (fact). They teach large classes of students (often, but not always true). Their lecture style is traditional “stand and deliver”, with very little interaction between students and instructors. (This is not an unreasonable assumption given that teachers of mathematics often teach the way they have been taught (Holton, 2001) and this has been the traditional university mode of instruction.) They have no formal training in pedagogy (which is also not unreasonable given that no teacher qualifications are required to obtain a post-secondary teaching position). Furthermore, as mathematicians they are extremely intelligent, elitist (i.e. only interested in students who show promise of becoming mathematicians or physicists), and are often unable to and uninterested in making mathematics comprehensible. This last statement goes too far, of course, but represents one of the prevalent views.

There is some support for concern about the quality of teaching provided by post-secondary mathematics instructors, at least in some contexts. Motivated by a concern for the beliefs and attitudes acquired by secondary mathematics teachers during their (much more extensive) undergraduate mathematics courses, Goulding, Hatch and Rodd (2003) provided free response surveys to 173 British student teachers who had just entered a one-year teacher certification programme after having completed their undergraduate degrees with concentrations in mathematics. Of the 40 students who chose to comment on the teaching they experienced at their various universities, 30 were labelled as negative. However, in contrast to impressions that their mode of

instruction is primarily lecture, Strickland (2008) observed a variety of teaching approaches in her examination of the practices of university mathematics teaching, including: “small groups, technology, lecture, whole class conversation, student presentations, pair-share, inquiry, problem solving, proving, connections, communication, representation, communities of practice, and alternative assessments” (p. 201). Teaching styles and characterisations of mathematics varied considerably from instructor to instructor. Indeed, their diversity is the only thing we can be fairly certain about.

Certainly, at their worst, mathematicians as instructors can easily alienate students who are already insecure in their abilities and not positively disposed towards mathematics. But, as mathematicians themselves will profess, at their best they have much to offer future teachers, even at the elementary school level (Hodgson, 2001; Williams, 2001; Schoenfeld, 1990). Jonker (preprint) describes mathematicians in mathematics departments as “stewards of their discipline”, passionate about mathematics, “eager to share their excitement with students, and concerned about the place of mathematics in the world.” As evidence for this he points to their involvement in conferences, the funding they make available to undergraduate students, and their support of math contests, fairs and camps. He believes that “ensuring high quality mathematics instruction in elementary schools is probably the best way to ensure that mathematics is valued and understood by the average citizen”, though he admits that not all of his colleagues are as inspired by the level of mathematics that is addressed in specialised mathematics courses for elementary teachers as he is.

Hodgson (2001) observes that the extent to which mathematicians can become interested and involved in teacher preparation is largely influenced by the support they receive from their departments. He notes that activities related to improving teacher education are not as valued as pure and applied mathematics research, as is evidenced by promotions and the awarding of grants. He also acknowledges that most do not have any formal education training. Nevertheless, he believes that by virtue of being mathematicians they have a unique perspective on mathematics to offer to future teachers: they can offer preservice teachers a glimpse into some of the “deep ideas at the heart of math” (p. 507). Their expertise puts them in a position to not only investigate “elementary mathematics from an advanced standpoint” (as recommended

by others above), but to look at “higher mathematics from an elementary point of view” (p. 516).

Given their own past experiences with mathematics teaching, and their lack of education training, there are legitimate concerns about the ability of mathematicians to convey the requisite specialised mathematics-for-teaching (Ball & Bass, 2003). In a study of the links between mathematics content knowledge and pedagogical reasoning in mathematicians, mathematics-method professors, and school teachers, von Minden, Walls, and Nardi (1998) found that “although university mathematicians possessed integrated content-knowledge structures, they tended to represent teaching as transmission of knowledge and learning as accumulation of knowledge” (p. 339). Furthermore, it is not clear to what extent, if at all, mathematics-for-teaching is a subdomain of mathematics (Kajander et al., 2010). In an extreme case, in their examination of university mathematics courses (that were not specialised for future teachers), Moreira and David (2008) found “examples which indicate that, at least to some extent and related to some particular topics, academic mathematics provides a kind of knowledge that *conflicts* with school teaching oriented forms of knowing mathematics” (p. 24, my emphasis). However, these concerns need to be held in contrast with the fact that mathematicians have in the past, and are continuing to, make valuable contributions to teacher preparation. Hodgson (2001) draws attention to the long tradition of involvement of mathematicians in education, pointing to the existence of ICMI, and naming several well-known mathematicians (Euler, Klein, Polya and Freudenthal) who have made an impact on the mathematics education landscape.

Even concerns that mathematicians will be unable to meet recommendations to situate learning mathematics in the context of teaching are muted by instructors of MFT courses like Jonker (2008). His content course is modelled on a practice-based approach in which his students have opportunities to work with elementary school pupils, and benefit from peer discussions of conceptions/misconceptions as they try to develop the in-depth understandings of mathematics that they will need. Williams (2001) comments that, “Perhaps what needs to be made more overt is an articulation of the distinctive contribution that mathematicians and mathematics educators can make to this training” (p. 452).

In Hodgson’s (2001) view, “the idea that mathematicians should see the mathematical education of primary school teachers as part of their responsibilities clearly

appears as an emerging trend” (p. 510), as evidenced by statements found in the National Policy Statement of the American Mathematical Society (AMS, 1994) and in the Mathematical Association of America’s *A Call for Change* (Leitzel, 1991). This is confirmed in the more recent *No Common Denominator* report, along with the advice that they “consider cooperation and coordination between content and methods instructors to be essential” (Greenburg & Walsh, 2008, p. 12). If this is the case, then mathematicians, at least those involved in teaching specialised mathematics courses aimed at future teachers, will “have mathematics to learn, and new problems to learn to solve, even as they also contribute resources” (Ball & Bass, 2003, p. 13). Learning about mathematics-for-teaching will not only help them support teacher education students, but may also enrich their teaching of their other mathematics classes (Cohran, DeRuiter, & King, 1993), perhaps even helping to attract more teachers to the discipline.

2.5 Summary

The literature tells us that teachers of elementary school mathematics should be equipped with deep, connected conceptual knowledge of all of the mathematics of the elementary curriculum, embedded within a specialised mathematics knowledge-for-teaching, and general pedagogy. They need to be initiated into this knowledge in the context of teaching and learning, in an environment that permits (both knowledge and belief) preconceptions to be challenged, allowing them to develop beliefs (about teaching and learning) and attitudes (related to themselves and their students) that are conducive to effective teaching. What is abundantly clear is that no teacher preparation programme can do it all, and in particular, it cannot all be accomplished in the mathematics content course for future elementary teachers.

It may be that the best advice an instructor of these mathematics content courses can take, is to focus on the big picture. Kilpatrick et al. (2001) recommend, “The overriding purpose of a course like this is to provide prospective teachers with ample *opportunities* to learn fundamental ideas of school mathematics, how they are related, and how students come to learn them” (p. 388, my emphasis). The hope is that students will “...learn mathematical ideas in ways that will equip them with mathematical resources needed in teaching” (p. 389). As well, Borko et al. (1992) advocate providing students with experiences that will:

prepare teachers who can identify the constraints of their beliefs, the limits of their knowledge, and the restrictive demands of their situations, and who are equipped with the tools and attitudes that support the independent development of beliefs, knowledge, thinking, and actions long after the conclusion of their teacher preparation programs. (p. 221)

Perhaps, in time, research will provide more specific answers as to exactly how this is to be done.

As can be seen, a review of the literature provides a wealth of suggestions and recommendations on what *should* be done in order to help prepare effective elementary school mathematics teachers. A common criticism of research in general is that it often leads to idealisations that are far removed from practice. In this case we see that, in part because of the overwhelming list of teacher “needs”, and in part because of lack of clear guidelines on priorities, much is left in the hands of the individual instructors to decide. At the same time it is evident that although the research is intended *for* instructors (or for course/program designers), there is very little research *about* the instructors. When it comes to bridging the gap between theory and practice, in this context there is a lack of documented research about what happens in ordinary mathematics content courses for preservice teachers, ones that are not undergoing studies for particular interventions.

If we hope to improve the mathematics preparation of our elementary teachers, based on recent recommendations, it is important that we have a sense of the current realities. We need to understand the current activity, its dynamics and its tensions, in order to see how it can be developed. This dissertation will attempt to address this, at least in part, by capturing the experiences of instructors of the MFT course. It will offer us a glimpse of this course and its challenges from the instructors’ point of view, a perspective that has been relatively neglected.

3: THEORETICAL INFLUENCES

The literature search described in Chapter 2 set the backdrop for my research: it brought out what is known about what the MFT course should be and, not surprisingly, the information it yielded was at the same time both vast and incomplete. It confirmed that much is left to be decided by individual instructors (or their institutions), and that little is known about these instructors, bringing me back once again to my research questions:

1. How do instructors interpret and experience the teaching of MFT courses? What factors contribute to the diversity of these interpretations and experiences?
2. In particular, what are the major tensions they experience? What factors contribute to these tensions and how are they managed?

It would be false to pretend that these questions began as they are formulated here, although they still encapsulate the original concerns that brought me to this project. The process of this research project has been truly emergent, as considerations of a variety of theoretical perspectives influenced both how I expressed and sought answers to my questions, and more particularly, affected what I saw and was able to see in the data I collected.

In this chapter, I will briefly describe the most significant of these theoretical viewpoints and how they shaped my research, namely: constructivist grounded theory, activity theory, and hermeneutic phenomenology. As well, I will elaborate on the notions of “sociomathematical norms”, “tensions”, and “positioning”, all of which arose within, and contributed to, the analysis of the data.

3.1 Constructivist grounded theory

The earliest incarnations of my research questions above were both personal and naive: What was happening in other MFT courses other than my own? Were we (MFT instructors) really doing things so differently from each other? How were others making their decisions on what to cover, and how to teach it? I felt early on that my best

starting point, given that I was an MFT instructor myself, was to talk with other MFT instructors directly, to hear about their intentions for the course and the choices they were making. Given that the phenomenon I was interested in was not easily quantifiable, I began to investigate qualitative research methods, which led me to grounded theory, and eventually to Charmaz's (2006) constructivist grounded theory.

The grounded theory approach appealed to me for several reasons. The first was that I was intent on studying a group of individuals in a context that had not to my knowledge been studied before. It was not clear to me that any existing theories that I had encountered to this point were a good fit. At the same time, I was attracted to the notion of engaging in an emergent, yet systematic, process. I could interview and listen to the accounts of the MFT instructors; initial coding, constant comparative analysis, further coding, and thematic analysis (Creswell, 2008) would allow me to describe what I had begun to think of as the "process of teaching the MFT course". The methodology offered a route to building a thick description (Corbin & Strauss, 2008) of the instructors' experiences, which was what I hoped to capture and grow to better understand.

Further research into grounded theory led me to Charmaz's (2006) constructivist grounded theory. This framed grounded theory within philosophical perspectives that struck a chord. As described by Charmaz, this version of grounded theory is consistent with (though not necessarily tied to) symbolic interactionism (Blumer, 1969) and pragmatism. Symbolic interactionism is "a theoretical perspective that assumes society, reality, and self are constructed through interaction and thus rely on language and communication" (Charmaz, 2006, p. 7). The pragmatist perspective is characterised by:

a problem-solving approach, a view of reality as fluid, an assumption of a situated and embodied knowledge producer, a search for multiple perspectives, a study of people's actions to solve emergent problems, and a view of facts and values as co-constitutive and truths as conditional. (Paraphrased from Charmaz in Morse et al., 2009, p. 139)

This resonated with the situation that I found myself in as a researcher. My data would be the words of the instructors, communicated to and interpreted by me, about their circumstances as they were relating them in the moment. The object of study (as I understood it at the time), the process of teaching the MFT course, is fluid, multi-dimensional and completely embedded in both cultural and temporal contexts. How the instructors would reply to the interview questions would be affected by many things, including events that occurred just prior to the interview, and the phrasing of the

questions themselves. The very views that I would be trying to capture could change during the course of the interview itself, let alone a year later. The theory helped to sensitise me to the limitations of my data and results.

As well, it offered a way not only to manage, but even to recognise as an advantage, a concern I had about engaging in this research project, namely, objectivity. How would the fact that I am an MFT instructor affect the research process and outcomes? Reflexivity is an important consideration, and although it is required by other research methodologies, constructivist grounded theory goes a step further, taking “reflexivity into explicit and continuous account” (Charmaz, in Morse et al., 2009, p. 133). It brings to the fore the influence that the researcher will have on the research process as she views and interprets the data through the lens of her own context and experience. As Charmaz (2006) describes, “Just as the methods we choose influence what we see, what we bring to the study also influences what we *can* see” (p. 15, my emphasis). It is understood and expected that the researcher is not a blank slate. In fact, the more exposure to the literature, and the closer to the field of study, the more potential the researcher has to see what is going on.

Because I was so close to the field of study, I would need to exert great care in interpreting the data, ever maintaining an awareness of how my experiences might be shaping my understandings. At the same time, my personal involvement offered tremendous advantages. During the interviews I was not only interviewer, but also sympathetic colleague, a role that I believe helped to put the participants more at their ease and put them in a position of being able to discuss a shared experience with an insider, as opposed to with an outsider who might judge them. The sensitivity that arises out of my own experiences may have also helped me recognise issues arising in instructor comments in-the-moment, allowing me to probe further and “go deep into the phenomenon” (Charmaz, in Morse et al., 2009, p. 146). Of course, it also presented challenges during data collection and analysis that I will discuss in more detail in Chapter 4.

In the end, through my encounter with constructivist grounded theory, I adopted its philosophical stance and initial approach to analysing data. In particular, I took to heart the reminder that my interaction with the interview transcripts would be reciprocal, with emergent themes affected by my own readings and experiences as well as by the participants’ words. However, I did not follow the theory through to its intended result,

that of building theory or fully describing the process of teaching MFT. The grounded theory methodology provided me with an overview of the landscape of teaching MFT from the instructors' perspective, but from this vantage point, I could see that a thick description of the process in its full complexity was beyond the scope of what I could manage within this project. At the same time, application of the methodology brought several prominent aspects within the landscape into view. These prompted deeper forays into the literature, which in turn led to richer interpretations of aspects of the interview data. In particular, I was drawn in turn to both activity theory and hermeneutic phenomenology as possible frames for discussing the results of my research.

3.2 Activity theory

During my initial application of grounded theory, I had already begun to think of the teaching of the MFT course as a process; it was a small step to reconceptualise it as an "activity". This led me to consider activity theory⁴, which takes "object-oriented, collective, and culturally mediated human activity" (Engeström, Mietinen, & Punamäki-Gitai, 1999, p. 9) as its central unit of analysis, as a promising theoretical framework for this project. The teaching of the MFT course and, more generally, the preparation of mathematics teachers, is without question such an activity. The theory advocates an examination of activity that embraces the social, historical and cultural contexts in which it takes place, and brings out the connections between the various elements that have an influence on it. This was compatible with my social constructivist point of view: I was acutely aware that I was studying a situation that was in flux, changing even while (and because) it was being studied. As well, it seemed to offer a way to consider the various influences on the instructors that were becoming evident through the coding.

A central model of activity theory, Engeström's (1999) activity triangle, identifies the essential elements of influence within an activity system and represents their interrelationships (see Figure 1). These basic components are: the subject of the activity, the object (related to a specific outcome), the community within which the activity takes place, the mediating artefacts (signs and tools) employed in the activity, the explicit and implicit rules that govern what occurs, and the division of labour amongst

⁴ Activity theory has its origins in the cultural-historical approach to psychology of Vygotsky (1978) and his students Leont'ev (1978) and Luria (1976). It has continued to be developed, and has been increasingly referenced in Western research literature over the last 30 years (Roth & Lee, 2007).

the participants. The system is described from the perspective of the subject, and is seen to be in a state of constant transformation, with each component influencing and being influenced by every other. The activity system does not sit in isolation, but is considered part of a complex network of activity systems in which different aspects of a component play the same or different roles in other activity systems.

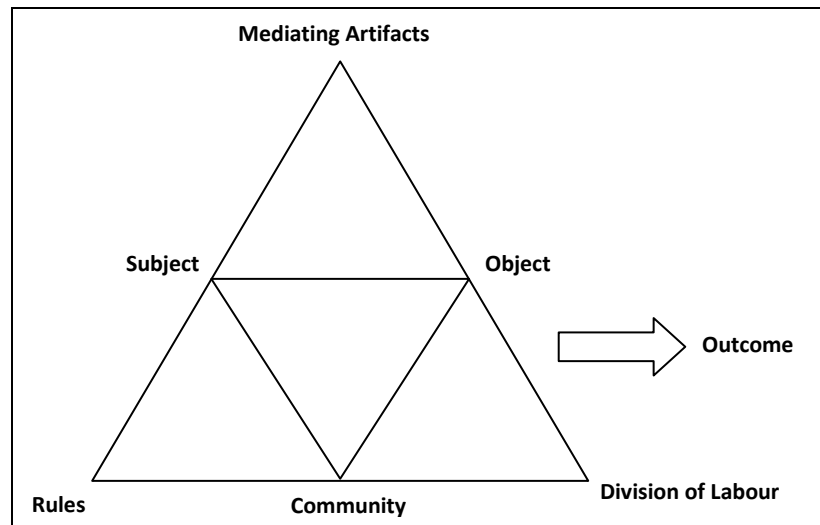


Figure 1. Engeström's activity triangle⁵

The initial appeal of over-laying this framework on my research data⁶ was short-lived, but nonetheless efforts to apply the theory yielded two valuable insights. The first was the observation that the object of the activity, i.e. preparing future elementary school teachers within the context of the MFT course, is not clearly defined. This is suggested in the review of the literature in Chapter 2, and also arises in the analysis of the data. This observation will be revisited more thoroughly in Chapter 7. The second was an appreciation for the significant role of “tensions” within an activity system.

Coding of the interview data revealed “challenges”, “barriers”, and “doubts”, all of which I came to see as “tensions”. Through the lens of activity theory “the internal tensions and contradictions of [...] a system are the motive force of change and development” (Engeström et al., 1999, p. 9). Tensions and contradictions within and

⁵ Note: Adapted from Engeström, 1999, p. 31.

⁶ The research data includes coded transcripts of interviews with 10 MFT instructors, as will be described in more detail in Chapter 4.

between networked activity systems represent opportunities for change and development as participants work to resolve them within a constantly transforming system (Engeström et al., 1999). This conception, along with the themes emerging from the coding, led to a refinement of my research questions to focus more explicitly on tensions (see subsection 3.4.2 below).

Though influential, I ultimately rejected activity theory as an over-arching theoretical framework for the interpretation of the data within this research project. One of the reasons for this was challenges around the place of students within the activity model. With instructors seen as the subjects, it was unclear where to locate the students. Are they members of the community, where they have a voice in negotiating the rules of the activity? Are they mediating artefacts, considered as the raw materials that are to be shaped by the activity? Or are they in some way the object, although the object is generally intended to be understood more in terms of the objective? None of these possible locations was satisfying. All failed to capture the central role that students play within this activity system. It would have been possible, and fruitful, to consider the students as subjects in their own activity system, linked to the one under study here. However, to introduce this would have been to take the study beyond my original focus, to consider the interaction of diverse activity systems within the MFT classroom. The students play a significant role in this study, but it is not the students themselves that feature, but rather the instructors' subjective impressions of their students.

These considerations brought me to notice that, although the other components of the activity triangle were tremendously important, my focus was not the activity system itself, but rather the instructors. Attention to the other components of the activity triangle would enhance what I could observe in my data, but what I was most interested in capturing was the instructors' *experience*, or rather, their *interpretations* of their experience of teaching the MFT course—this led me to a consideration of hermeneutic phenomenology.

3.3 Hermeneutic phenomenology

van Manen (1997) describes phenomenology as “the descriptive study of lived experience (phenomena) in the attempt to enrich lived experience by mining its meaning” (p. 37) and hermeneutics as “the interpretive study of the expressions and

objectifications (texts) of lived experience in the attempt to determine the meaning embodied in them” (p. 37). Hermeneutic phenomenology combines these traditions. Here, once again, I encountered a theory that was compatible with social constructivism, and that acknowledged the interpretive nature of the work I would be doing in analysing the interview transcripts.

Closer study of hermeneutic phenomenology revealed it to be a methodology that taken to its full extent, was too grandiose for the project at hand. The phenomena it typically seeks to elucidate are experiences shared by many, whose instances are embedded deeply in human social and psychological contexts, experiences such as “love” or “parenthood” or “identity”. The experience of teaching the MFT course seemed to fall short of these near-universal notions. However, I did not leave this theoretical perspective without taking something from it: it validated my use of personal experience and interviews as sources for coming to grips with the research questions, and it oriented me towards attention to language in my analysis of the transcripts.

van Manen (1997) observes that “a phenomenological description is an example composed of examples” (p. 122), where the variety of examples provides opportunities to discover what is common and what is distinct, and recommends that the researcher begin with his or her own experience: “To be aware of the structure of one’s own experience of a phenomenon may provide the researcher with clues for orienting oneself to the phenomenon and thus to all the other stages of the phenomenological research” (p. 57). It also advocates the use of conversational interviews in order to “gather lived-experience material” (p. 63). Burch (1990) observes, “the full meaning of experience is not simply given in the reflexive immediacy of the lived moment but emerges from explicit retrospection where meaning is recovered and reenacted, for example, in remembrance, narration, meditation, or more systematically, through phenomenological interpretation...” (p. 134). These considerations confirmed the use of my own personal experiences, combined with the accounts of others, as valid points of entry for my research questions. More than this, this approach to research showed me the interpretive potential of the data I had before me, and as a result helped shape the language used in the final formulation of the research questions: the terms “experience” and “interpretation” were introduced.

Language also became a more conscious focus in my interactions with the data: “The phenomenological method consists of the ability, or rather the art of being

sensitive—sensitive to the subtle undertones of language, to the way language speaks when it allows the things themselves to speak” (van Manen, 1997, p. 111). This became an important aspect of my analysis, leading me to reflection on the participants’ choice of words, considerations of their possible meaning beyond the literal, and brought me in particular to an awareness of “positioning” (discussed in subsection 3.4.3 below).

Exposure to constructivist grounded theory, activity theory, and hermeneutic phenomenology helped me to articulate my research questions more clearly, and appreciably influenced both my philosophical and practical approaches to these research questions. These approaches in turn sensitized me to particular concepts that played a pivotal role in my final analysis.

3.4 Sensitizing concepts

The use of the term “sensitizing concepts” invokes Blumer (1954) who observed that certain concepts in the social sciences defy explicit definition: “Whereas definitive concepts provide descriptions of what to see, sensitizing concepts merely suggest directions along which to look” (p. 7). Within this research project, three such concepts emerged: sociomathematical norms, tensions, and positioning⁷. These were “sensitizing” in Blumer’s sense, in that “what we are referring to by any given concept shapes up in a different way in each empirical instance” (p. 8): I encountered these concepts as I perceived them in the context of the words of MFT instructors. At the same time, they were “sensitizing” in a personal sense: they came to my attention (each in different ways), and once they had drawn my notice became the bases for further noticing. In this section, I give a brief account of the emergence of these concepts and some of the literature that supported my understanding of them.

3.4.1 Sociomathematical norms

At the time that I was beginning to code the earliest interviews, I happened to read an article on sociomathematical norms (Yackel & Cobb, 1996). Sociomathematical norms are normative understandings, negotiated through the interaction of teacher and students, which relate specifically to mathematical activity. It includes such things as

⁷ Forms of “mathematical knowledge”, “beliefs”, and “attitudes” could also be considered to be sensitizing concepts within this project, however because they were discussed in considerable detail in Chapter 2, they will not be readdressed here.

expectations for homework, conventions for writing problem solutions, and even understandings of what is or is not mathematical activity. Because of exposure to this notion, I suddenly found myself aware of instances in the transcripts where instructors appeared to be referring to such norms. It provided a vivid example of the subjectivity of the coding process.

I was already attuned to expressions of “attitudes” and “beliefs”; sociomathematical norms offered the potential for more nuanced observation. Integral to Yackel and Cobb’s (1996) emergent perspective is the view that sociomathematical norms are reflexively related to the individuals’ mathematical beliefs and values:

With regard to sociomathematical norms, what becomes mathematically normative in a classroom is constrained by the current goals, beliefs, suppositions, and assumptions of the classroom participants. At the same time these goals and largely implicit understandings are themselves influenced by what is legitimized as acceptable mathematical activity. (p. 460)

References to beliefs about the nature or activity of mathematics could suggest the sociomathematical norms the instructors hope to establish⁸ in their MFT courses, and vice versa, offering a point of reference for identifying diversity (or commonality) between the instructors.

As an extension of this, through the analysis I became aware of certain norms that seemed to be particular to the MFT course, not exactly sociomathematical, but perhaps socio-mathe-pedagogical, related to particular values and expectations the instructors tried to foster because their students hope to be teachers of mathematics one day. The differences between regular mathematics courses and the MFT course became pertinent in the analysis (see especially subsection 5.3.1).

3.4.2 Tensions

As mentioned above, through my study of activity theory, I came to conceptualise some of the challenges faced by MFT instructors as “tensions”. This led me to seek out prior mathematics education research that had also engaged this concept. Tensions, often expressed as “dilemmas”, have been recognised as an integral part of teaching practice, dating back at least to the early 1980s. In their seminal work, Berlak and

⁸ Without student interviews or classroom observations I would not be privy to the negotiation of the norms, only the intentions the instructors would bring to the negotiation.

Berlak (1981) examined the complex and sometimes contradictory behaviours of teachers in responding to the curriculum within socio-cultural contexts. Their use of the language of dilemmas was taken further by Lampert (1985), who emphasised the personal and practical aspects of dilemmas, and Adler (1998, 2001) who combined the perspectives of the Berlaks and Lampert in her examination of dilemmas in the teaching of mathematics in multilingual classrooms. Although much has been written about the tensions that arise in teaching, often implicitly, the works of Lampert and Adler, along with a more recent contribution to the research on tensions by Berry (2007), have been particularly relevant and influential in shaping my approach to studying tensions.

Most significantly, from Lampert I have come to think of tensions as problems to be managed, rather than solved. Lampert (1985) characterises teachers as “dilemma managers” who find ways to cope with conflict between equally undesirable (or desirable but incompatible) options without necessarily coming to a resolution. In the face of a teaching dilemma, the teacher must take action:

from the teacher’s point of view, trying to solve many common pedagogical problems leads to practical dilemmas. As the teacher considers alternative solutions to any particular problem, she cannot hope to arrive at the “right” alternative.... Even though she cannot find their right solutions, however the teacher must do something about the problems she faces. (p. 181)

However, “facing a dilemma need not result in a forced choice” (p. 182); rather the teacher must find a way to respond to the particular situation, even while the “argument with oneself” (p. 182) that characterises the dilemma remains. For Lampert, the ongoing internal struggles presented by the tensions arise from and contribute to the developing identity of the teacher, and as such have value in themselves.

In contrast to other approaches to understanding the practice of teaching, from Lampert’s (1985) perspective the admission that some of the conflicts encountered in teaching are not resolvable is not a weakness. While others have sought to solve teaching problems, either by “altering the way education is organized and conceptualized by society” (p. 191) or by suggesting that teachers simply need to implement research-based recommendations, Lampert promotes the view that the teacher seen as a “dilemma manager” “accepts conflict as endemic and even useful to her work rather than seeing it as a burden that needs to be eliminated” (p. 192).

Adler (2001) also takes the view that dilemmas in teaching are often managed rather than solved. As well, she adopts Lampert's (1985) "practical and personal" (Adler, 2001, p. 51) approach to understanding dilemmas, agreeing that their instances arise in the context of teaching, and that the recognition and management of the dilemmas is tied to personal biography. However, along with these considerations, she integrates the Berlaks' socio-cultural perspective, which emphasises the importance of the wider context, beyond the classroom situation. Like Adler, I too found it necessary to consider all three facets—the practical, the personal, and the contextual—in examining the tensions that arose (see Chapter 7)⁹.

One further text on tensions in teaching drew my attention because of the particular context in which the research took place. Berry (2007), who had been a teacher and made the transition to the role of teacher educator, conducted a self-study of her efforts to improve her practice in her new role. She found that:

the notion of tensions offered a useful way of describing teacher educators' experiences of their practice....It captured well the feelings of internal turmoil experienced by teacher educators as they found themselves pulled in different directions by competing pedagogical demands in their work and the difficulties they experienced as they learnt to recognize and manage these demands. (p. 119)

Because MFT instructors tend to be mathematics instructors in mathematics departments, it struck me that at some point the participants in my study would have also undergone, or perhaps not, a transition in their role from teaching future *users* of mathematics, to teaching future *teachers* of mathematics. This raised my awareness to a potential source of tensions for the MFT instructors.

In her report, Berry presents a list of tensions that mainly reflect discrepancies between instructors' intentions and the realities/demands of their students and the classroom. For example, she identifies the tension of "telling and growth" as being "embedded in teacher educators' learning how to balance their desire to tell prospective teachers about teaching and providing opportunities for prospective teachers to learn about teaching for themselves" (p. 120). While inconsistencies between reported intentions and reported actions became an important mechanism for identifying tensions,

⁹ I note here that my use of the term "practical" has more general connotations than occurs in the works of either Lampert or Adler. Both use the label "practical" to refer to tensions arising in the context of dealing with students in the classroom, while my study was not framed to include observations of teachers in the act of teaching.

I did not make use of the particular tensions identified by Berry. All of the tensions she lists are described in pairs, (e.g. “action and intent”), however I found that this representation was not suitable for describing the complexity of many of the tensions experienced by the MFT instructors. Yet Berry does note that the “tensions do not exist in isolation from each other” (p. 120) and she found it useful to consider their interconnections in coming to understand the practice of teaching teachers. I found this to be useful as well.

Berry’s study alerted me to a possible source of tensions, Adler’s work pointed me towards consideration of the practical, the personal, and the contextual in my analysis, but Lampert’s discussion made me thoughtful about what I hoped to accomplish with my research. By investigating the tensions, was I hoping to find ways to resolve them, either in the research literature or in the practices of others? The conception of teachers as dilemma managers opened up the possibility that the tensions might not be resolved, and that perhaps I should not expect them to be.

All three researchers underscore the value in studying dilemmas or tensions. For Berry (2007), “tensions offer a way of reframing traditional notions of knowledge development in teacher education and so create new and different possibilities for understanding and improving practice” (p. 133). Lampert comments: “Our understanding of the work of teaching might be enhanced if we explored what teachers do when they choose to endure and make use of conflict” (p. 194), and Adler (2001) asserts: “The value in identifying key teaching dilemmas and naming them is that they can then become objects of reflection and action” (p. 49). With this in mind, my objective in studying the instructors’ interpretations of their experience, with a special focus on their expressions of tensions, was shifted away from ultimately seeking a resolution to those tensions and towards identifying them in detail, in order to expose their potential as objects of reflection for informing practice.

3.4.3 Positioning

The final sensitizing concept I will discuss here is that of positioning. Hermeneutic phenomenology had raised my awareness of the importance of language in the analysis of the transcript data (see section 3.3), and in my search for relevant techniques of discourse analysis, I came across “positioning theory”. A paper by Wagner and Herbel-Eisenmann (2009) on positioning in mathematics classrooms led me

to Harré and van Langenhove's (1999) foundational book on positioning theory. My understanding of this notion has been influenced by both of these sources.

Harré and van Langenhove (1999) offer positioning theory as "one possible conceptual apparatus that allows for social constructionist theorizing based on a dynamic analysis of conversations and discourses" (p. 2), with position defined as:

a complex cluster of generic personal attributes, structured in various ways, which impinges on the possibilities of interpersonal, intergroup and even intrapersonal action through some assignment of such rights, duties and obligations to an individual as are sustained by the cluster. (p.1)

The theory provides a way to study interactions between individuals and come to a deeper understanding of the dynamics and potential meanings inherent in the situation. The analysis of an interaction takes into account the "moral position" of the speakers (i.e. the social role they take on), the story-line of the conversation (including what is being said when, and what has been said before), and "the actual sayings with their power to shape certain aspects of the social world" (p. 6). The theory also draws attention to who is doing the positioning, the speaker or some other individual or collective, and to whom the positioning is being applied. In a conversation, participants take on various roles (e.g. teacher/student), position others (e.g. as expert/novice), and respond to the positions adopted or imposed (e.g. by shifting position or refusing to accept the imposed position). Positioning is described as "fluid" (p. 17), and interpretation of the utterances cannot be done without attention to the immanent story and social context.

In Harré and van Langenhove's book, Davies and Harré (1999) suggest several dimensions that should be considered when analysing positioning within a conversation. These include:

1. the images and metaphors used by the speaker;
2. the way the speaker is speaking, his choice of words, and what it says about the assumptions he makes about his context;
3. the variability of how the context may be viewed by different participants, which in turn will affect how they interpret the utterances made;
4. the non-linearity and possible inconsistency of the personal stories constructed by the individuals as the conversation transpires; and

5. the extent to which the positions can be conceived of in terms of shared understandings of known roles or characters.

[paraphrased, pp. 35-36]

Through attention to these aspects, I was able to notice shifts in positions that instructors attributed to themselves and others: apparent inconsistencies offered insights into possible tensions. Word choices of the instructors, particularly, “hedgies” (Rowland, 1995), modal verbs, and shifts in pronouns, also proved to be helpful in this regard¹⁰. Furthermore, I became conscious of my position as an “insider” (sometimes taken on by me, sometimes attributed to me by the interviewee), and the effect this had on the interview and my analysis of it. For example, it sometimes led to presumptions of shared understandings (see subsection 4.4.2).

In their paper, Wagner and Herbel-Eisenmann (2009) elaborate on their understanding of positioning theory, offering a number of comments, including useful insights into the interplay between the immanent and transcendent, and stressing the contingent nature of the analysis of positions. They observe that while Harré and van Langenhove focus on positioning that occurs in the moment of interaction, it can also be useful to consider how the speaker is positioned in a broader context that transcends the specific utterance. This observation opened the door, within my work with the MFT instructors, to considerations of how the interview subjects and I were positioning ourselves and each other at the time of the interview, as well as to how they were positioning themselves with respect to the course, their institutions, the mathematics, etc.

With respect to contingency, they warn that “one can interpret any situation with different storylines” and that “there is no way of establishing the correct storylines or positionings” (p. 9). Within the circumstances of this study, when I was a participant in the conversation, I would have been interpreting and responding to the positioning taking place within the dialogue. These interpretations in the moment would represent just my perspective on the situation as it unfolded. This contingency is further manifest in my act of re-interpreting the transcripts, as I attempt to look at the words exchanged through the eyes of an outsider. The “I” who participated in the interview is not the same “I” who analysed the transcripts; the words of the “other” that I responded to in the moment have

¹⁰ These are discussed in more detail in Chapter 4.

been transformed by the act of transcription, and my understanding of them is shifted by subsequent experience, and by the act of the analysis itself.

Although my use of this theory is not pervasive, considerations of positioning became a powerful tool for my data analysis. Sensitized to the concept of “position”, I was oriented towards looking for clues in the language used by me and by the participants in the study. This perspective primarily provided ways for me to notice potential tensions and to attend to issues of reflexivity. It is invoked as needed where it offers the opportunity to shed light on the research questions. Further details of its use in my analysis will be described in Chapter 4.

This brings to a close the exposition of the theoretical perspectives and constructs that were instrumental in shaping every aspect of this investigation. Constructivist grounded theory was my starting point, providing me with a philosophical orientation that affected the collection and coding of my data. Activity theory offered useful insights into the elements that might influence MFT instructors’ experiences and raised my appreciation for the role of tensions, which became a central theme. Furthermore, the tradition of hermeneutic phenomenology confirmed my research methods as appropriate and inspired my approach to data analysis, leading me to a heightened appreciation for the subtleties of language and ultimately to considerations of positioning. These perspectives, along with the literature on sociomathematical norms (as well as forms of mathematical knowledge, beliefs, and attitudes, as described in Chapter 2), laid the groundwork for my interpretation of the experiences and tensions of MFT instructors. In the following chapter, I describe in more detail the application of the methods that developed from these theoretical considerations.

4: COLLECTING AND ENGAGING WITH THE DATA

4.1 The search

I now move from the theoretical to the practical: this chapter tells the story of my search to reach a deeper understanding of the experiences of MFT instructors. My search began with my own personal experiences, the broad strokes of which were described in Chapter 1. Inevitably, these experiences informed every aspect of this investigation, and their impact will be noted as appropriate throughout. I begin with an exposition of my primary sources: single interviews with ten different MFT instructors (referred to as “the main study”), and a sequence of conversations with one of these instructors (referred to as “the case study”). This is followed by an account of my analysis of the data, including the early application of grounded theory methods and the eventual application of hermeneutic techniques to operationalise tensions.

4.2 The main study

As observed by Bean (2006):

Data gathered from face-to-face interviews for qualitative research have the potential to yield a gold mine of insights into the people’s lives and situations. There is no substitute for prolonged and focused conversations between trusted parties to discover what is important to the interviewees and how respondents understand key elements in their own lives. (p. 361)

Face-to-face interviews seemed an appropriate point of entry for investigating the experiences of MFT instructors, and so I sought to set up such “focused conversations between trusted parties”, in order to provide opportunities for these instructors to share their thoughts, concerns and aspirations with respect to the MFT course. Situated as I was, as an MFT instructor at a college and a member of the provincial mathematics articulation committee, I had easy access to MFT instructors at other post-secondary institutions in British Columbia—either I knew them directly, or I knew people who could refer me to the appropriate individuals.

I began with convenience sampling (Creswell, 2008), choosing instructors who were close to me geographically, and whom I knew would be interested and available to discuss the course, but quickly moved to theoretical sampling (Creswell, 2008) to achieve, most importantly, a variety in type of post-secondary institution represented, then varying degrees of experience in teaching MFT, and finally gender balance. I doubted that gender would be significant¹¹, but I did anticipate differences based on years of experience and institutional context. As part of the grounded theory approach (Charmaz, 2006), I did not predetermine the number of instructors to interview (see section 4.4.1), but in the end spoke with ten. Only one instructor, whose name was provided to me by the department chair, did not respond to my invitation to participate in the study. Details of the ten participants and the interviews are described below.

4.2.1 The instructors

Five men and five women participated in the study: Andrew, Al, Alice, Bob, Brooklyn, Harriet, Karen, Maria, Matthew, and Simon (all pseudonyms, reflecting gender but not ethnicity). In the interest of delimiting the range of contexts that they presented, I provide a description of their educational backgrounds, their teaching experience, and their institutional situations. However, as the pool of individuals who have taught the MFT course within the province of British Columbia is relatively small (certainly fewer than 100), in order to preserve their anonymity I will describe these aspects in terms of the characteristics of the cohort in general, rather than attach them to specific individuals. I hope that this will allow the reader to identify *with* the instructors, without being able to identify them. Particular characteristics become attached to the appropriate pseudonym later in the document, as these characteristics become pertinent to the phenomena under discussion.

Educational Background

All of the participants in the study have degrees in Mathematics: three have Bachelor's degrees, six have Master's degrees, and one has a Ph.D. Since a Master's degree is generally a minimum requirement for instructors at the post-secondary level,

¹¹ In the end, gender did not become a factor in the analysis. However, it is interesting to note that in order to achieve gender balance it was necessary for me to make a point of selecting a male instructor at a given institution if it was possible. This reflects the fact that within British Columbia there are more female instructors of MFT than male, despite the larger representation of males in mathematics departments.

those with only Bachelor's degrees in Mathematics hold Master's degrees in other related areas.

With respect to formal study of Education, only three of the instructors reported having had any teacher training: one has a Bachelor's degree in Education, and two who have only Bachelor's degrees in mathematics have Master's degrees in Education and Educational Technology, respectively.

Teaching Experience

As part of my sampling, I deliberately sought out instructors with a wide range of experience in teaching the MFT course, but I did not pay deliberate attention to the number of years of general teaching experience. As it turned out, all of the instructors interviewed are experienced teachers, each with 10 to 30 years of experience. All of this experience has been in post-secondary mathematics classrooms, with the exception of two of the instructors who reported having taught in high schools at one point in their careers, one for 1 – 2 years and the other for 6 years.

With respect to the Math for Teachers course, three were in the process of, or had just completed, teaching it for the first time at the time of the interview. One was teaching the course for the first time at her current institution, but had also taught the course once previously at another. One had taught the course four times, two had taught it six times, one nine times and the remaining two had taught the course more than 20 times each.

Institutional Contexts

While all of the instructors were teaching at post-secondary institutions in British Columbia, they were selected in order to capture a range of different institutional contexts. Two of the instructors teach at research universities, two at teaching universities, three at urban colleges in or near Vancouver, and three at colleges outside of the Lower Mainland. Only two participants were drawn from the same institution, one who had taught the course for many years, and one who had just taught the course for the first time.

The Math for Teachers course in British Columbia falls under the aegis of Mathematics Departments, and so all of the instructors interviewed belong to Mathematics, as opposed to Education, Departments. Because of this, their primary

teaching responsibilities include teaching a range of mathematics courses. Two participants specifically mentioned experience teaching Adult Basic Education courses, though it is likely that most of the college instructors would have some experience teaching university prep-level mathematics courses. Two mentioned having had opportunities to teach in on-line or hybrid learning situations, and two mentioned experiences with Native Education programs. Only one has worked with teachers in an in-service setting.

4.2.2 The interviews

As noted above, my goal was to create opportunities for “*focused* conversations between *trusted* parties”, and so I conducted semi-structured interviews with each of the instructors that lasted between 60 and 90 minutes. These were semi-structured in the sense that the questions asked were guided by an established protocol, but did not adhere to it strictly, as will be described. One interview was conducted over the phone, while all others were conducted face-to-face.

Establishing trust was important. All but two of the participants were known to me in a professional capacity before the study began. I had met each of those I knew, some several times, on various occasions at articulation meetings or at professional development events. The other two were referred to me by members of their departments. Whether I knew them or not, the fact that I was a fellow instructor of the MFT course put me in a position (see section 3.4.3) where I was able to offer a truly sympathetic ear as they described their experiences. I was also clearly not in a position of power with respect to any of them—there was no apparent reason for any of them to be less than frank in their comments.

I tried very deliberately to approach the interview questions in a spirit of genuine inquiry, with no intent to judge. My opening stance was that the teaching of the MFT course is problematic and my study was aimed at gaining a better understanding of what the course could or should be. Whether or not they agreed that the course was problematic, this opening set the tone for a joint investigation into the experience of teaching the course.

The semi-structured interviews began with a list of questions that helped to provide the needed focus, but the format was kept deliberately informal, both to put the participants at ease and to allow them opportunities to talk about what was most on their

minds with respect to the teaching of the MFT course. Such an open-ended (“clinical”) approach is advocated by Ginsburg (1981) in situations where discovery or identification/description of a phenomenon is the objective. This type of interview “is intended to facilitate rich verbalization” (p. 7), and allow the interviewer opportunity “to check verbal reports and clarify ambiguous statements” (p. 7). True to this approach, most but not all of the scripted questions were asked of all of the participants. Sometimes questions were omitted completely if the intent of the question seemed to have already been met. I often asked additional questions in order to request clarification or elaboration on items of interest as they arose. These “items of interest” were often comments that stood out for me because they were different from my own experiences, or resonated with ideas that I had been reading about, or were reminiscent of comments made by other research participants.

The guiding protocol that I used opened with questions to establish facts about the instructors’ background and the nature of the MFT course offered at their institutions. Later questions sought to elicit their conceptions of the course, by examining their goals, describing the approaches they take, comparing the teaching of MFT with teaching of other mathematics courses, and reflecting on the challenges and the successes they experience. The comparison of their experience teaching the MFT course with that of teaching their other mathematics courses seemed to be a logical avenue of investigation: I was trying to see how they experienced teaching the MFT course in particular, and hoped to make a distinction between that and their other teaching experiences. At the end of the interviews, I turned to questions that asked the instructors to reflect on the overall effectiveness of the course, its content and what it achieves. I knew that I was dissatisfied with my course; with this last group of questions I hoped to gauge their level of satisfaction, and solicit their views on how they felt the course could be improved. An annotated list of the protocol is included in Appendix B.

All of the interviews with this group of instructors were recorded using a digital voice recorder and transcribed in full by me. Although some of the instructors may have been somewhat self-conscious, particularly at the beginning of the interviews, because they were being recorded, my sense was that they soon forgot about the microphone and felt quite at ease in speaking to me about their experiences with the MFT course. As will be seen, they spoke very freely about their approaches to the course and the

challenges they face. Details of the analysis that I engaged in with the transcripts are described in section 4.4 below.

4.3 The case study

Along with the ten interviews, my data also includes a series of more in-depth conversations with one of these MFT instructors. The decision to interview one of the participants more intensively was an opportunistic one, wholly consistent with the emergent nature of the project. I describe here the circumstances that gave rise to this component of the research, and the details of what followed.

4.3.1 Motivation, opportunity and intent

The first two instructors that I interviewed, Harriet and Bob, were both seasoned mathematics instructors who had taught MFT many times. Alice was third and a novice to MFT, though not to teaching mathematics. I was struck by how vivid her descriptions were of how the course had both exceeded and fallen short of her expectations. The dilemmas she encountered in trying to make decisions about course content and teaching methods seemed to be very fresh in her mind—nothing about the course was as yet routine. At that stage, I noted that it would be useful to include a number of these “fresh” perspectives in my pool of participants.

Around this time, I became aware that there was an instructor, Simon, who was preparing to teach the course for the first time. It occurred to me that it might be fruitful to arrange to interview Simon before he began the course, to see what he expected to encounter in the course, and to gain insight into the resources that he accessed in preparing to teach this new type of mathematics course. But on further contemplation, I realised that I also had before me an opportunity to speak to someone who was living the experience of making the transition from teaching other mathematics courses to teaching the MFT course. I arranged to speak with Simon, not just once but several times, roughly once every two weeks throughout the term in which he was teaching MFT for the first time. Here was a chance to see, as they emerged, what problems he encountered, what surprised him about the course, and how he adapted his initial conceptions.

4.3.2 The conversations with Simon

During the 15 weeks of the semester, Simon and I spoke 9 times for approximately one hour each time. These conversations were not audio-recorded, but detailed field notes were taken, and were annotated and consolidated immediately following each exchange. Because these conversations were conducted by phone, it was easier for me to take more extensive notes than it would have been face-to-face. The final notes of all of our “meetings” were sent to Simon at the end to provide him with the opportunity to verify their accuracy.

Positioning was an important consideration here as it set the tone for our conversations. Throughout I was playing two roles, as a researcher and as expert-colleague, but I tried to push the latter to the fore. Simon agreed to participate in the study, but I could tell that he was, quite understandably, uncomfortable with feeling as if he was the object of study. While I took notes diligently, and asked questions that were pertinent to my research, understating my role as researcher helped to put Simon more at ease and allowed our conversations to be less formal and more natural. We talked about the day-to-day challenges of teaching the course as they emerged.

It also meant that I abandoned any pretence of maintaining neutrality. I was aware that even if I tried to avoid offering advice or opinions, Simon’s views on the MFT course would be affected by the questions I would ask and how I asked them. At the same time, accepting my role as expert-colleague freed me to fulfil the moral obligation I felt to support Simon when I could by commenting on certain situations, or by sharing my own experiences. Such exchanges are not unusual between instructors, especially when one is new to teaching a course. Any advice that is given can be taken as is, adapted, or rejected. Since it was the struggles and dilemmas themselves, along with Simon’s responses to the situations that arose that were of interest in the study, I was not concerned about biasing results.

4.3.3 Following up

A month after the end of his first semester teaching MFT, I interviewed Simon again, using a modified version of the interview protocol described in subsection 4.2.2 above. This interview was included as part of the main study, and as such was audio-recorded and transcribed. As well, I conducted a follow-up interview (an unstructured conversation) with Simon a full year later, after he had taught the course the second

time. This gave me an opportunity to follow up with Simon on some of the comments he had made the previous year, ask him about the changes he had planned to make to his MFT course, and to see how (or whether) his tensions had evolved or been resolved or how they had been managed over time. This final interview was also audio-recorded and transcribed.

4.4 Analysing the data

I now turn to an account of my engagement with the data. Consistent with the approach of constant comparative analysis (Creswell, 2008), the analysis began early, while data was still being collected. This affected my choices of interview subjects (e.g. deciding to choose a mix of instructors with a wide range of experience teaching MFT courses), influenced my noticing and prompting in subsequent interviews, and helped me to determine how many instructors to interview (see subsection 4.4.1).

My efforts to theorise the data involved many layers of analysis before I was able to construct meaningful answers to my research questions. It began with the grounded theory approach as described by Charmaz (2006), up to the point of thematic analysis, which provided me with an overview of the experience of teaching MFT. After this, techniques from hermeneutic phenomenology, in particular, attention to language and discourse analysis, along with positioning theory allowed me to delve more deeply into the phenomenon of “tensions”.

4.4.1 Applying grounded theory

I began my analysis by applying an iterative process of coding, as recommended by Charmaz (2006), which calls for examination of the data through increasing levels of abstraction. This involved initial coding (detailed and unrestricted), focused coding (to identify concepts and categories (or themes)), and theoretical coding (to determine relationships between categories).

For the initial coding, I considered each phrase uttered by the interview subject and attached a code that was intended to capture in brief the content of the phrase. When I first began, I simply attached descriptive labels to each phrase in an *ad hoc* manner, but early on, I came across Charmaz’s (2006) recommendation of coding with gerunds and found this to be useful, at least at the outset. This can be difficult to do at times, but is intended to help prevent the researcher from jumping immediately to a

classification or conceptual label, something that should really occur at a later point in the analysis. For example, at the early stages of my data analysis, the excerpt “They need to know how to do things in more than one way” was coded as PCK (pedagogical content knowledge). However, with gerund coding the initial code for this data became “knowing multiple methods/approaches”. With my previous method, I was moving too quickly to generalisations based on the research literature. Coding with action-oriented terms forced me to take a step back and allow for other, potentially new, concepts to emerge. This was helpful in keeping my analysis grounded in the data.

While it was helpful, coding with gerunds also had major drawbacks—in particular, the loss of the subject and the loss of verb tense. For example, a fragment was coded as “deciding which method works better”, but from this code it is not clear if this refers to the instructor, or to one or more of her students. By coding with gerunds, the subject was no longer apparent. Since the subjects of the actions were, variously: the instructors, the students, both together, mathematics, other instructors, other students, other courses, the course itself, the text, or an institution, I found that I could manage this by arranging the initial codes in columns that corresponded to these subjects. Loss of verb tense markers was a related issue. Sometimes the reference was to the past, sometimes the present, and other times to aspirations for the future; “being afraid of mathematics” might have referred to a student in the past, at the beginning of the course, or at the end. I found that despite having the initial codes, I very often needed to refer back to the original text in order to confirm the context. These distinctions were no longer required when I moved to focused codes—subject and tense were encapsulated in the new labels¹².

Focused coding involved sorting and synthesising the initial codes to identify concepts and categories or themes. I approached this in two stages: during the first, I recoded the gerund phrases with codes that identified the concepts the phrases appeared to reference; during the second, I re-organised these codes under thematic headings. I found the first of these stages to be the most challenging. It was at this stage that I became most aware of the subjectivity of the coding process. Readings on attitudes and beliefs, forms of mathematical knowledge, and sociomathematical norms

¹² As an example: the code for “deep content knowledge” (DCK) under the theme of “desired student knowledge”, placed students as the subject, with the “end of the course” as the appropriate time frame.

all affected what I noticed and how I imposed order on the data, but this is to be expected in this constructivist approach.

I found it impossible to assign only one concept code to each phrase. For example “not taking the course seriously” was a description of student attitudes to the course (coded SAC), but it was mentioned in the context of explaining why students do poorly in the course, and hence represented a barrier for student success (so also coded B). Many phrases were coded with two codes, sometimes more, and I found the process much easier and more practical once I allowed myself to code in this way.

The constant comparative analysis (Creswell, 2008) was diligently applied at this focused coding stage. This involved continually going back to recode earlier interviews, to make use of new codes that had emerged, and to reconsider former code assignments. This process also provided an indicator for when sufficient data had been collected. By the time I had completed the focused coding for the tenth interview, only a very few new (and relatively insignificant) concept codes were emerging, suggesting a saturation of the data. The process of coding and recoding of the interviews continued until the concept codes became stable¹³.

At this stage, in order to more thoroughly examine and compare the concept codes, I decided to build what I refer to as “coding summaries”. This involved extracting all segments that were coded under each concept code across all of the interviews. For example, some phrases were coded as descriptive of “teaching methods” (TM). All of these phrases were consolidated on one page of a spreadsheet, and were further sorted and compared to get a better understanding of the types of items that had been included under this conceptual label (e.g. class discussion, homework, techniques they avoid). This allowed for a more detailed examination of each of the codes, permitting a view of all of the implicit properties within each code. This resulted in many refinements in the coding, including reduced redundancy and increased consistency, allowing similar ideas to be recognised and grouped together more accurately.

This process led very naturally to the emergence of themes. As each concept code was analysed in the building of the “coding summaries”, I began to group together concepts that appeared to be related. For example, although I had a code for “teaching methods”, I also had codes for “manipulatives”, “team work”, and “math fairs”. The latter

¹³ A brief excerpt of one transcript together with its initial (gerund) codes and concept codes is displayed in Appendix C.

were all clearly methods instructors used, but had occurred frequently enough to warrant their own codes. These were grouped together under the theme of “Instructor Activities/Methods”, along with others that seemed to fit under this heading. Segments that were labelled with multiple codes arose multiple times in this process, making the connections between codes more apparent—I developed a very deep appreciation for the complexity of the phenomena. Some codes were collapsed, even at this late stage as they were observed to be subsets of other codes. This organisation and reorganisation of the data allowed for a more in-depth observation of trends, commonalities and differences within and between the concepts and the themes that emerged¹⁴.

I note here that my notes for the conversations with Simon were coded in the same manner as the other interviews. Although Simon was interviewed multiple times, the experiences we were discussing were the same as those of the other interviews, only more detailed. It seemed appropriate that the codes from the main study should inform, and be informed by, the codes from the case study.

At the conclusion of this process, there were 83 concept codes, grouped under 12 categories or themes, which I found could be easily distributed among three broad categories: themes related to the instructor, to the students, and to the MFT course. One additional theme, that of drawing comparisons between the instructors, students or course at the participant’s institution with those at other institutions overlapped with all three broad categories. The codes, themes, and categories are displayed in Appendix E.

While I stopped short of completing a full theoretical analysis of the data, I did begin the process by engaging in an extensive memo-writing exercise in order to construct a sketch of the landscape of the experience of teaching the MFT course. Memo writing is an integral part of the grounded theory process (Creswell, 2008). It took many forms for me during this research project, ranging from margin notes during the coding phases, journal entries, and short papers exploring some of the concepts or themes. However, at the end of the coding process, I undertook a final memo-writing exercise, writing short expositions on each of the themes, carefully reconstructing my interpretation of the various aspects of the MFT course as revealed through the concepts that had emerged. This consolidated my data and provided me with a vantage

¹⁴ One of the concept summary pages has been provided as an example in Appendix D.

point from which to view the landscape. It was from this that I was able to notice some of distinguishing aspects of the experience of teaching MFT and draw out points of commonality and diversity amongst the instructor perspectives I had gathered (to be discussed in Chapters 5 and 6). It also helped me to begin the process of identifying the tensions that were part of the experience.

4.4.2 Operationalising tensions

Tensions arose from the data analysis as described above, and were most evident within the theme of “Instructor Concerns” which included such concepts as instructor “priorities”, “wishes” and “doubts”, as well as under such concept codes as student “barriers” and “resistance”. These codes helped me to locate places in the transcripts where instructors explicitly addressed aspects of teaching the course that they found to be challenging.

It was at this point that I found hermeneutic techniques, in particular, discourse analysis and positioning theory, to be helpful, allowing access to a deeper understanding of the phenomenon of “tensions”. I returned to the texts themselves, to locations identified through the coding, to interpret the instructors’ experiences by close attention to the words they spoke, and the positions indicated by them.

With respect to word choices, a dimension of analysis recommended explicitly by Davies and Harré (1999)¹⁵, consideration of three word types proved to be especially useful: “hedges”, modal verbs, and pronouns. Rowland (1995) describes “hedges” as “linguistic pointers to uncertainty” (p. 327). Phrases like “I think”, “maybe”, or “probably”, often indicate that the speaker has some doubt about the associated assertion. Noticing their use offered the opportunity to consider the sources and the implications of such uncertainties, and as a result was helpful in locating tensions. Furthermore, the use of modal verbs, like “might”, “could”, or “should”, offered possible insights into whether instructors felt empowered to make changes or not, as did their shifts in pronouns, which gave indications as to whether actions described were being attributed to instructors themselves or to institutions.

Noting how the instructors positioned themselves as they spoke, and noticing shifts in those positions that were sometimes contradictory, provided evidence to support

¹⁵ See item 2, page 77.

interpretations of the existence of tensions. But positioning theory also heightened my awareness of how I was positioning myself during the interviews and how I was being positioned by the participants in the study. Much of the time, the instructors addressed me as an “insider”, someone who understood the context well. As a result, there were times when things were left unsaid—there was a presumption of shared understandings. Because I accepted this position as “insider”, I often didn’t press for clarification or elaboration, and so it became important for me to consider the nature of these shared understandings in my interpretation of the experiences being described. This became an essential consideration throughout.

4.5 Continuing the search through writing

Conducting and transcribing the interviews, taking the data through many iterations of analysis, sorting and resorting the information to construct meaning, and applying analytic tools of discourse analysis and positioning theory, were all significant stages in my search to understand the experience, and in particular the tensions, of these instructors and myself as we teach the MFT course. But the search did not conclude once this analysis was done—it continued throughout the process of writing.

I borrow this notion from the tradition of hermeneutic inquiry in which “writing is closely fused into the research activity and reflection itself” (van Manen, 1997, p. 125). He goes on to observe:

Writing involves a textual reflection in the sense of separating and confronting ourselves with what we know, distancing ourselves from the lifeworld, decontextualizing our thoughtful preoccupations from immediate action, abstracting and objectifying our lived understandings from our concrete involvements (cf. Ong, 1982), and all this for the sake of now reuniting us with what we know, drawing us more closely to living relations and situations of the lifeworld, turning thought to a more tactful praxis, and concretizing and subjectifying our deepened understanding in practical action. (p. 129)

The demands of the practical action of writing, its call for careful reflection, logical organisation of ideas, considerations of meaning in choices of words, and the processes of writing and re-writing, all contributed to my (re)construction of the experience of teaching the MFT course and its tensions. It forced me to consider carefully what I know, and what the data before me offered. Noticing, discovering, and critical

examination of assumptions, continued throughout the writing process. In the remaining chapters, I share my reconstruction, the culmination of this search.

4.6 Moving forward

The process of coding and analysing the data as described above was tremendously useful as an organisational tool to help manage the large quantity of data. It helped to bring to light the various topics and issues raised by the instructors in our discussions, allowing me to consolidate them under meaningful concept codes and themes. However, for the task at hand, that of describing the experience of teaching the MFT course, confining the description to summaries of discrete categories would be too restrictive—it would inhibit conveying the experience in its full complexity. As a result, my description of the instructor experience will not involve a systematic exposition of the concepts or themes.

Instead, I will make use of an organising scheme that will cut across the grain of the categorisations, drawing out what is distinctive about the experience of teaching MFT with special attention on the diversity and tensions revealed in the instructor interviews. I begin, in Chapter 5, with a look at the “common ground”, the aspects of the course (and its students) on which the instructors generally shared a common perspective, focusing on those features which distinguish teaching MFT from teaching other mathematics courses. I then move, in Chapter 6, to consider the points of greatest diversity that emerged, and their sources, which will lead to a detailed analysis of the tensions, in Chapter 7.

5: EXPERIENCING MFT: COMMON GROUND

It seems necessary, when trying to describe any experience whatever, to describe that experience as it relates to, or differs from, some other “taken-as-understood” experience. We often proceed by analogy, e.g. “grief is like a numb emptiness”, and/or by contrast, e.g. “peace is the absence of stress”. In either case, there must be some starting point for the description, some experience shared between participants in the discussion, from which the explication of what distinguishes this experience can begin.

So what is the starting point here for the description of the experience of teaching the MFT course? This was not explicitly addressed with participants in the study, but is implicit in my choice of interview subjects; all are instructors in mathematics departments who teach the MFT course. Between us there was an assumption of a shared experience, that of teaching mathematics at a post-secondary institution. It was a point of entry for my questioning: “What is different about how you prepare for, teach, engage with, the MFT course, compared to your other courses?” It is also the starting point here for my description of the experience of teaching MFT. ***Teaching MFT is like teaching other post-secondary mathematics courses, except....*** Through the interviews, different ways to complete this phrase emerged, all of which offer insight into how teaching the MFT course is (can be) experienced.

With this starting point in mind, in this chapter, I will focus on what I refer to as the “common ground”. These are the features of the MFT course experience that *most* of the instructors in the study agreed on. As it turned out, there is very little strictly common ground, but there are a few areas where the perspectives offered by the instructors were more common than not, namely, perceptions of their students, the resources accessed by the instructors in teaching the course, and certain general aspects of the course itself.

Those readers who are very familiar with the MFT context may find relatively few revelations within this chapter, however elaboration on these points of commonality will serve multiple purposes: it will begin the process of describing the experience of

teaching MFT; it will lay the groundwork for understanding where and why the perspectives and approaches of the instructors diverge; and as well, it will set the context for some of the tensions.

Since the descriptions in this chapter are of shared experience, of both the instructors and me, I frequently adopt a declarative tone, and only occasionally reference particular instructors. Nevertheless, the discussion is grounded in the data. The results reported here emerge from the coding and thematic analysis, with the most relevant codes indicated at the beginning of each section (see Appendix E for their descriptions).¹⁶ In later chapters, as I explore the diversity and tensions, I will call on the various individual voices of the instructors to “speak” for themselves.

5.1 MFT students (STC)

Teaching MFT is different from teaching other mathematics courses because the students are different. Most obviously, they differ demographically: there is a much higher proportion of female students, and many are “mature” students, who have not taken mathematics courses for a number of years. However, the more significant differences between MFT students and other mathematics students relate to affective issues (attitude to the course, attitude to mathematics, emotionality and math anxiety), as well as their mathematics knowledge.

5.1.1 Student attitudes to the course (SAC, B, RE)

The fact that the course is a required course for potential future elementary teachers influences the type of student who enrolls in the course. When the instructors made comparisons between their MFT students and their other students, their comments brought out the very best and the very worst of how MFT students could be.

On the positive side, MFT students display a “refreshing naïveté”; “there’s a spirit of the people that are there because they want to work with young kids” (Bob). There can be a willingness to play and share experiences. They are more “on the same wavelength” (Karen), because of their shared purpose, which can make them friendlier and more focused. Because many of them have had opportunities to work with children,

¹⁶ The codes are included solely as a measure of accountability, signalling the connection between the data as collected and the exposition—knowing what the abbreviations stand for is not needed in order to follow the discussion.

either their own or through volunteer work in schools, these experiences help them to appreciate the applicability of some of the course content.

On the negative side, the MFT students are seen to be much more emotional than other mathematics students (see subsection 5.1.3 below). Several of the instructors mention aspects of stubbornness or closed-mindedness that they sometimes encounter in the form of resistance to new ways of doing arithmetic, and to learning multiple approaches. These practices may not be what the students are expecting to encounter in a mathematics class (see subsection 5.3.3 below).

Instructors make a distinction between students taking the MFT course to become teachers and those who take it for other reasons. Some are there in order to fulfill requirements for electives in other programs. Unlike other mathematics courses, because its content appears to be “elementary school mathematics”, often these students are under the impression that the course will be an easy credit. Sometimes English-as-a-Second-Language students take the course, expecting that as a Mathematics course, it will have very little reading or writing. Generally, these students tend to be surprised by the course content and difficulty, especially the emphasis on communication skills (see subsection 5.3.1). The proportion of such students in a given class influences the class dynamics.

Students who are taking the course to become elementary teachers come in with their own expectations of what the MFT course will be. These students often expect and hope that the course will teach them *how to teach* elementary school mathematics. Karen described, “They confuse this course. They think I’m there to teach them how to teach math. I said, “No! I’m teaching you *Math!*” Other instructors mention this as well, and stress the importance of dealing with this misconception early on. Not only is this mismatch of student expectations and course intent unique to the MFT course, it also foreshadows a systemic tension related to the degree to which MFT courses should make connections to elementary school contexts (see subsection 7.3.1). Furthermore, the extent to which instructors accommodate student expectations to learn about the elementary context is a major point of divergence, as will be seen in section 6.4.

Student attitudes to the course are also a factor in the negotiation of sociomathematical norms in the MFT course, but this will be taken up in subsection 5.3.3.

5.1.2 Student attitudes to mathematics (SA, FT)

Not unlike in other mathematics courses where students are required to take a course to satisfy the requirements of a non-science program, many students in the MFT course have negative attitudes to and narrow conceptions of mathematics. Though some MFT students enjoy mathematics, a majority of them will claim they dislike it. They see it as a chore, consisting of numerical calculations that involve memorisation of rules and application of recipes or tricks. It is a black or white endeavour: either you know it or you do not, answers are either right or wrong, and there is only one correct approach to any problem. They generally do not believe that they are good at it, and often avoid taking MFT for as long as possible in their undergraduate careers.

Although this happens in other mathematics courses, it seems to worry instructors in the MFT course especially. They are concerned about what these negative attitudes towards mathematics might mean for those students who go on to be teachers. They express fears that students who continue with these attitudes might: simply avoid teaching the mathematics that they do not like, show intolerance towards future pupils' alternative solution methods, be unable to make math accessible to their pupils, pass on their negative attitudes to their pupils, or even use mathematics as a punishment.

5.1.3 Emotionality and mathematics anxiety (SE, MA)

For one of the instructors, AI, the most notable distinction between his MFT students and his other mathematics students is their heightened degree of emotionality. Indeed, in the various interviews, instructors bring up many examples of students experiencing anger, frustration, and even having "melt-downs". One of the factors that likely contributes to this is that the MFT course is required for admission into some of the province's professional accreditation programs. This means that students who hope to be teachers have an added pressure that can make them very anxious about their grades and afraid of failing.

Another contributing factor is that a significant number of MFT students are afraid or anxious about mathematics. This seems to occur more in the MFT course than in other mathematics courses. All of the instructors commented on this, observing that this anxiety is most evident when students are working on problem solving, fractions, conjectures and proofs, and even when faced with having to do basic arithmetic

calculations. In some instances, the anxiety is severe enough to impede student learning. Instructors attribute some of this anxiety to bad experiences in students' past encounters with mathematics and/or weak skills (see below). Just as for the negative attitudes, there is concern that this anxiety will affect students' future ability to teach mathematics effectively.

5.1.4 Student knowledge (CK)

Another common experience of the instructors is the poor mathematics knowledge and weak arithmetic skills of the MFT students coming into the course. This came as a surprise to some instructors the first time they taught the course, though exceptionally, not to Matthew, who found they were stronger than he had been led to believe they would be.

Of course, it is expected that the students are familiar with elementary school arithmetic from their own past experiences. The prerequisite for the course is grade 11 mathematics, though many students at the research universities have also taken a grade 12 course. In fact, some of the MFT students are quite strong mathematically but most, including the weaker students, seem to remember the mechanical procedures, like the base-10 arithmetic algorithms, the best. Procedural knowledge is not always accompanied by understanding, and in some cases students who are able to do the calculations easily are reluctant to learn alternative procedures, or do not appreciate the need for deeper analysis. Dealing with the wide variations in background and abilities is a challenge for instructors in teaching the course (see subsection 7.2.2).

Although it is not uncommon to hear instructors complain about the mathematics skills of the students in any of their mathematics courses, there seems to be a different quality about the complaints in the context of the MFT course. This may be in part because of the level of mathematics being studied—there is an expectation that students who have completed a grade 11 mathematics course *should* be able to deal with, say, fractions, a common problem area. Another factor is that because of the kind of work that is done in MFT courses, specifically the degree of explanation required (see subsection 5.3.1), instructors become much more explicitly aware of students' deficiencies in conceptual understanding than they might in other mathematics courses.

It is important to note that the above description reflects an interpretation of how the instructors in this study positioned their students and is not offered as a

representation of how MFT students are. While they described their students as exhibiting a wide range of dispositions and abilities, including positive characteristics, their perceptions of negative attitudes to mathematics and weak mathematics knowledge and skills arose often as particular concerns in their experience of teaching the MFT course. As will be seen in Chapter 7, these concerns come to the fore in discussions of the instructors' objectives for the course and in their conceptions of their students as future elementary school teachers. How they try to meet the perceived needs of their students in their individual courses is a point of divergence that will be discussed in section 6.1.

5.2 Resources

Teaching MFT is different from teaching other mathematics courses because of the resources available and how they are used. These resources include the course textbooks, colleagues, mathematics education research or faculty, and the school curriculum. Textbooks and colleagues are common sources of support in teaching almost any mathematics course, however there are aspects of both that are particular to the MFT context. In contrast, the other two sources, mathematics education research or faculty and school curriculum are rarely, if ever, consulted by post-secondary mathematics instructors. Although these may seem obvious resources to access for teaching MFT, interestingly, the common ground here is their relative lack of use.

5.2.1 Textbooks (TX)

No formal comparison between MFT textbooks and other college-level mathematics books was undertaken as part of this study, however instructor comments during the interviews brought to light distinctive aspects of the MFT texts. One of the most striking of these is that the textbooks include not just mathematics, but also pedagogical theory. Instructors in the study appreciated that this could provide students with access to expertise that they did not feel they had. However, at least one instructor, while recognising that the pedagogical content is useful for the students, commented that it “interfered with the mathematics”, making the particular text used at his institution “turgid” and as a result, difficult to read. The “interference” referred to a perceived lack of mathematical rigour and conciseness.

As well, although instructors sometimes learn new things from course textbooks, because the content of the MFT course is so different from the usual college mathematics that they have studied themselves, this happens frequently within the MFT course. Some specifics mentioned by the instructors interviewed included learning about different manipulatives, Egyptian unit fractions, models for division, and even new algorithms.

In terms of textbook use, many of the instructors, at least initially, rely very heavily on the textbook for the planning of their course and preparation of their lectures. Following the textbook closely provides structure for their students, as it does in other courses, but it also provides much needed guidance to the instructors in terms of specific content and appropriate types and levels of questions (see subsection 5.3.2). One instructor observed that the MFT textbook displayed a notable absence of repetitive or drill exercises, and made inferences from this about the relative unimportance of skill development in the course. The text is an important tool for passing on course expectations from one instructor to the next, all the more so given that the expectations for MFT are not as clear as they might be for other mathematics courses (see section 2.3 and section 7.3).

Interestingly, despite the described heavy reliance on the book, another fairly common experience was dissatisfaction with the text. Instructors complained about such things as: ordering or treatment of topics (e.g. negative integers, polyhedra, fractions); overly strong connections to the US NCTM Standards; lack of worked examples; poor exercise sets; or the need for supplemental material (e.g. on angles, symmetry, and transformations). In three cases, the instructors were seriously considering not using a text at all, freeing themselves to use materials they had developed on their own over many years or to shift the course from a focus on mastering material chapter by chapter to developing broader mathematical problem-solving skills through activities¹⁷.

5.2.2 Colleagues (COI)

Although all of the instructors mention being supported by or seeking support from their mathematics colleagues in the delivery of the MFT course, there was a considerable range in the amount of support they received. At one end of the spectrum,

¹⁷ The role of the textbook with respect to instructor tensions is discussed in subsection 8.3.2.

when they first started teaching the course some instructors had mentors who provided them with course materials and talked to them about the course objectives and the nature of the students. They also provided detailed lesson schedules that helped them organise timely coverage of the course content. At the other extreme, instructors merely inherited textbook selections and general course descriptions from previous instructors of the course.

Casual conversations with colleagues about the course, just as for their other mathematics courses is also the norm, however a distinguishing feature of the MFT course is that, given its specialised nature, there are often not many colleagues to consult. Half of the instructors interviewed were the *only* MFT instructors at their institution. Advice is often sought from colleagues at other institutions, especially in the case of course designers who need to ensure that their course will meet requirements for course transfer. Provincial articulation meetings also provide a forum for mutual support and discussion of issues pertaining to the MFT course across institutions.

5.2.3 Mathematics education research and faculty (ME)

Use of mathematics education research or consultation with mathematics education faculty seems to be rare. In the interviews, when mathematics education is mentioned as a discipline at all, there is evidence of a sense of separation, an “us vs. them” perspective. Alice, Andrew and Matthew all laughingly confided that the “mathematics education experts” would not agree with or approve of some of the things they do or say in class. Karen described consulting with Education faculty in preparing her course for the first time, but took away a “don’t tramp on our territory” message. Maria mentioned having worked closely with Education faculty in designing an MFT course at another institution, but this type of connection was not mentioned by any of the instructors with respect to their current institutions.

It is important to observe here that most of the instructors in this study do not teach at institutions that have an Education Department, so their opportunities to interact with education experts would be limited. For most, their closest contact with education research is likely their textbook for the course. A few of the instructors do mention reading articles or books related to mathematics education and math anxiety, motivated by their own personal interests.

As the extent to which they consulted mathematics education sources was not a specific research question, too much should not be inferred by the limited number of references. However, the separation between the two disciplines seems to be a genuine phenomenon. In reference to local education faculty, one instructor lamented, “there are really none of them who are mathematicians” and because of this they “don’t really know what mathematicians care about”, adding, “there’s this big distrust really, between people in math education and people in mathematics departments. [At] most universities, they don’t come anywhere near each other—there’s no common ground at the research end of things, which is too bad”. This is a serious concern that will be revisited in subsection 9.2.3.

5.2.4 School curriculum (SC)

References to the school curriculum were even more rare than references to mathematics education resources. The school curriculum is only mentioned by three of the instructors; two of these mention it in the context of children that they know who are in elementary school. Despite the fact that Simon has a copy of the school curriculum on his desk, he admits that he does not make use of it, even though he would like to tie the course more closely to what his students will need as elementary school teachers. This is indicative of Simon’s experience of the systemic tension, mentioned earlier, related to the extent to which connections to elementary contexts should be made in the MFT course, which will be discussed in subsection 7.3.1.

5.3 The MFT course

Teaching MFT is different from teaching other mathematics courses because the nature of the course is different. It is still mathematics, but the added goal of preparing future teachers distinguishes it from the other courses. In particular, there is a greater emphasis on developing communications skills, and instructors are called upon to use different methods of evaluation than they are accustomed to. Furthermore, instructors are faced with negotiating a different set of sociomathematical norms. With respect to course format, and broad course objectives, there is also some common ground, but it does not extend far.

5.3.1 Communication (CM, MN, MI, RE)

Improving the students' communication skills is an objective that was mentioned by almost all of the instructors and yet is outside of, or extends beyond, what is expected in other mathematics courses. Students are pushed to provide clear written explanations, or problem solutions, that not only demonstrate their conceptual understanding to the instructor, but would allow someone off the street to follow and learn from what they have written. Along with written homework assignments with requirements for extended responses, students are also often asked to do project write-ups and journal assignments. There is a sense that it is particularly important to develop good communication skills in future teachers. Brooklyn observed, "It really doesn't matter what you know, if you can't get it across."

The instructors commonly found that they encounter resistance from their students, at least initially, as they promote this emphasis on communication. Being required to write in a mathematics class goes against the students' image of what mathematics is supposed to be. They expect to be doing calculations, not writing explanations or responding to journal prompts. As a result, instructors need to renegotiate this sociomathematical norm, which is not easy. Despite repeated reminders to explain their reasoning, students will often still respond to examination questions like: "Use an appropriate model to demonstrate $2 \div \frac{1}{2}$ ", by simply computing the quotient. Students who are serious about becoming teachers tend to find it easier to buy into the new norm.

Further comments on how the instructors experience the negotiation of this particular norm will be reserved until the more general discussion of sociomathematical norms in subsection 5.3.3 below.

5.3.2 Evaluation (EV)

Not unrelated to the emphasis on developing communication skills are the unique forms of evaluation instructors are called upon to use when teaching the MFT course. Instructors sometimes find it challenging to incorporate methods of evaluation that they have not used before. Projects and/or class presentations are commonly assigned in MFT courses, and several instructors remarked on the difficulties they had deciding on how to do such things as: evaluate group work; assign fair grades to projects that may have been mathematically correct, but represented a wide range of effort; or assess oral

presentations. The subjective nature of project marking is foreign to most mathematics instructors. Many of the instructors in this study found themselves experimenting with peer evaluation schemas and working with rubrics for the first time in their careers for the MFT course.

Another challenge related to evaluation faced especially by some of the new(er) instructors is establishing standards for the course. It is not unusual for it to take some time to get a feel for the appropriate type and level of question for examinations in a newly taught course, but it is more difficult in the MFT course, in part because of the (lack of) support available (see section 5.2), and in part because of ambiguities with respect to the types of knowledge that instructors should be measuring. Should they be testing for proficiency in elementary school mathematics, for deeper content knowledge, for pedagogical content knowledge, or for all three? Except as already noted above, with instructors generally expecting full and clear explanations to accompany procedures, not mere solutions, this is a point of divergence for the instructors and as such will be discussed further in subsection 6.4.1. It also exemplifies one of the internal tensions, related to setting standards for the MFT course, as described in subsection 7.2.2.

5.3.3 Sociomathematical norms

Another common distinguishing characteristic of the MFT course is the need for instructors to work hard to get buy-in for sociomathematical norms that are particular to the MFT context. These include expectations for more writing of higher quality, as mentioned in subsection 5.3.1 above, as well as for an openness and even preference for multiple approaches, and a shift in emphasis from solution-finding to process-analysis. Particular tasks mentioned as points of resistance included such things as learning multiple models for arithmetic operations, considering alternative solutions to word problems, working on arithmetic in other bases, and needing to explain reasoning.

Negotiating these norms with the students who hope to teach one day is easier than with those who have taken the course with hopes of an easy math credit. Students who do not intend to teach are described by the instructors as undesirable members of the class: “they’re not in the right space”, “they’re not there in the spirit of the course”, and “a group like that can poison the whole class”. Since these norms are negotiated,

having a large number of students in the course who do not want to be teachers can make the negotiation difficult.

In contrast, for those who do hope to be teachers, some instructors mention taking advantage of these students' interest in teaching to motivate and get buy-in for these expectations. If students accept that as teachers they will need to provide deeper explanations, offer varieties of approaches, and make connections between ideas, it is easier for them to understand why expectations in the MFT course may differ from what they have been accustomed to in other mathematics courses.

Interestingly, it is sometimes the students with the strongest mathematics skills who find it hardest to adapt to the new norms. At some of the institutions, steps are taken to prevent mathematics majors from taking the MFT course without special permission.

Not only do instructors need to persuade students that writing is a legitimate activity in a mathematics classroom, they have to negotiate the level of rigour and quality of the written responses. Instructors bring their own experience of the norms of mathematical writing and proof into this negotiation. One instructor specifically mentioned her desire for not only correct, but concise explanations¹⁸. Precise use of the language of mathematics is also expected. Demands for high degrees of mathematical rigour can be difficult for students to accept, especially if the mathematically rigorous explanations no longer seem appropriate for use in an elementary school classroom. This contributes to one of the personal tensions, experienced by some of the instructors, that will be discussed in subsection 7.1.2.

5.3.4 Course format, content, and objectives (CC, CD, IR, T, CK, DCK, PCK, SAC, SA, MA, C, AP, EM)

Finally, with respect to the course itself, there is some obvious common ground around course format, and general content and objectives.

The format and logistical details of the MFT course do not distinguish it from other mathematics courses, but are included here to clarify the context of the course within British Columbia. MFT is typically offered as a one-semester (13 – 15 weeks) course, with 4 – 6 hours of class meetings, though occasionally it is offered in an online

¹⁸ Maria: "I pointed out as well to them, one goal, as a mathematician, is to write succinctly, and to be a minimalist. We like short notations!"

format. Class-sizes vary somewhat according to the institution. At the colleges and teaching universities, the class sizes are between 30 – 40 students, while at the research universities, the class sizes range from 60 – 150.

The topics of the MFT course are different from other mathematics courses, focussed as they are on the foundations of elementary school mathematics. Most versions of the course cover a large subset of the following: numeration systems, set theory, number systems, arithmetic algorithms, alternative bases, arithmetic with integers and rationals, logic and critical thinking, probability and statistics, number theory, algebra, and geometry, including shapes and measurement, symmetries, transformations, and tessellations. Beyond the listing of course topics, most of the instructors also mentioned such general goals as trying to improve students' mathematics skills and understanding of mathematics, as well as their attitudes, specifically to mathematics and their ability to do it.

One of the most significant differences between MFT and other mathematics courses related to the course content is that it is not a pre-requisite for any further mathematics courses. Because there is not a clearly defined skill set that the students will be required to apply in a “next course”, in some respects this makes covering the specific topics less important than it might be for a precalculus course. One would think that this might take some pressure off instructors, but in fact, concerns about “time” arose frequently in the interviews. A common experience for instructors of the MFT course, at least for those in the study, is that there is too much to cover in too little time. As will be seen in Chapter 7, instructors feel that they must make difficult choices about what to cover, given the long list of topics, their non-trivial general goals, the described course format, and the instructors' perceptions of the needs of their students (see section 5.1). These perceived time constraints and the choices instructors feel compelled to make also contribute to the differences between the MFT classes of different instructors, as will be described in Chapter 6.

5.4 Not just another math course

Indeed, it is at this point that it becomes more and more difficult to discuss aspects of the experience of teaching MFT that can be considered “common ground”. For the most part, instructors reported a shared experience of their students and the extent to which they use (or do not use) the various resources available. They also had

similar expectations with respect to general course content, communication skills, and particular sociomathematical norms. However, even in describing these aspects, some hints of diversity have started to emerge.

What stands out most from the analysis to this point is that instructors' experience of teaching MFT differs considerably from teaching their other mathematics courses—the students are needier, the stakes are higher, the content is more “elementary”, but the objectives and standards are less clear. Instructors are faced with making decisions about these objectives and standards, yet the resources that might provide them with useful information are often not accessed. All of these factors contribute to both the diversity amongst the MFT instructors and the tensions that they experience. These will be explored in the following chapters.

6: EXPERIENCING MFT: DIVERSITY

The previous chapter laid out the common ground of reported instructor experience in teaching the MFT course. Specifically, it described how instructors generally see their students, how they make use of available resources, and some of the approaches they take; it also outlined aspects of the format and content of the MFT course. However, despite being classified as “common ground”, even within the various points addressed there were traces of differences. For example, not all students in the MFT course dislike mathematics, not all instructors had mentors when they began teaching MFT, not all versions of the course have face-to-face meetings. These types of differences were glossed over, not judged significant enough to be recognised as “diversity”. At what point did the differences that emerged become different enough to warrant special discussion?

The points of diversity discussed in this chapter have been selected because they are peculiar to the MFT course, or exhibit a wider range than would be expected of other post-secondary mathematics classes. Both of these criteria are highly subjective, and hence I rely heavily on my own experience as a mathematics instructor in order to make these judgments. I do not discuss in detail differences between MFT courses that arise because of instructors’ special hobbies or interests, or even policies around such things as calculator use, which contribute to diversity across mathematics courses generally. Instead, I will focus here on the aspects that stood out most for me during the coding of the data, those that offer insight into the particularity of the experience of teaching MFT: instructors’ priorities for the course, their classroom methods, the image of mathematics they seek to convey, and the extent to which they focus on preparing teachers.

Up to this point, I have merely summarised the data gathered, for the most part amalgamating the comments of the instructors. Because the experiences described so far have been common or shared, there has been little need to distinguish between individuals. This is no longer the case. As we proceed in this chapter, and the next, I will begin to refer to particular instructors as I try to convey my interpretation of their

comments about the course, and where appropriate I will allow them to speak in their own words.¹⁹

At this point it seems appropriate to re-iterate that biographic details of the interview subjects are deliberately not provided. In part, this is to protect the anonymity of the participants; however, this is not the only consideration. This research is not meant to provide characterisations of individuals, but rather to attempt to convey the range of possibilities of experience among instructors of the MFT course within the described context. Exploration of how particular aspects of personal history and orientations correlate with particular experiences of the MFT course would make an interesting study, but is beyond the scope of this work.

The differences that will be identified within this chapter take one of two forms: sometimes they are differences in instructor dispositions, how they feel about the course, their students, and their own abilities; sometimes they are differences in instructor actions. Often these are closely related and a close analysis of the words of the instructors will offer insight into their possible explicit and implicit motivations. It is here that positioning theory and the tools of discourse analysis mentioned earlier will begin to be helpful (see subsections 3.4.3 and 4.4.2), occasionally augmenting what can be seen through thematic analysis alone.²⁰ Diversity in perspectives and interpretations of the MFT course can lead to different actions, and the different actions can lead to different experiences of the course, both for the instructors, and for their students.

I need to caution here, that although through the discussion certain instructors will be associated with certain viewpoints, the reality is that these viewpoints are merely possible interpretations of the instructors' words, which were freely given at a particular moment in time, within a particular context. The positions taken by the instructors are changeable and ever-changing, and this could be observed even for the same instructor within a single interview. As noted above, the discussion here presents possible positions, with examples taken from the transcripts, but none of this should be read as a description of these particular instructors as how they *are*—rather it is a description of how an MFT instructor *might* be.

¹⁹ The instructor quotes are edited to remove stutters, interjections, repetitions, and some pauses, in order to facilitate readability, though these were captured in the transcription.

²⁰ These techniques were found to be most useful in the analysis of tensions in Chapters 7 and 8.

6.1 Setting priorities

In section 5.1, instructors' common perceptions of their students' needs were described. In brief, their students come into the course with weak skills, poor attitudes towards mathematics, and low confidence in their own abilities. All of these are a concern for all of the instructors, especially given that these students hope to be teachers one day. However, among the different instructors who participated in this study, a range of perspectives was represented with respect to whether knowledge goals or affective goals should be given priority in the MFT course. This is consistent with the debate in the literature over this issue (see subsection 2.3.3).

During the interviews, participants were asked about their objectives for the course after they had already listed the course content. This was intended to help the instructors think beyond the content, to consider more deeply what they try to achieve through the teaching of this course. Their responses to this prompt offer insight into what was foremost in their minds with respect to priorities. Based on their comments, in the sections that follow, I loosely group instructors according to these priorities; however, I caution that the grouping is a fuzzy one. Instructors were not always consistent throughout their interviews with respect to how they positioned themselves. When it occurred, this inconsistency was taken to suggest underlying tensions, which will be discussed in Chapter 7. Despite the fuzziness of the categorisations, a look at the various viewpoints expressed will offer insight into some of the beliefs, particularly with regard to the role of the MFT course, that motivate the instructors and distinguish their individual versions of the course.

6.1.1 Mathematics first (CK, DCK)

Four instructors, Maria, Andrew, Bob, and Brooklyn, all made statements about their objectives in the course that suggested they placed their highest priority on cognitive aspects, i.e. teaching the mathematics. Each of the examples offers a slightly different perspective on the value placed on developing students' mathematics skills and understanding, some of which tie the cognitive goals very closely to affective ones.

Maria responded to the query about her main objective for the course by reflecting on her students' aspirations to be teachers.

Initially for me, my objective was for the students to get a stronger math background. [...] ²¹ I would like to see, particularly elementary teachers, have a stronger math content and understanding of it. Not to the level they have to teach, but beyond. Because I don't think you can teach at that level without understanding where it's going. Because then you do not know why it's important, and the emphasis that you need to place, and directions you need to, or CAN come at to this topic. So my goal was, primarily, sort of, more content....

Maria believes that deep content knowledge that goes *beyond* merely knowing the school mathematics itself is a *necessary* condition of being an effective teacher of mathematics. She sees that this knowledge will empower the future teachers to judge what is important, to make connections, and know what is possible²². She is quite fervent as she expresses these words. What is not said here is that Maria is aware that students will likely not take much further mathematics beyond this course, making it even more crucial that she support them in building their understanding during the MFT course. At the same time, parenthetically, it is important to note her use of past tense, the qualifier, “initially” at the beginning, and the hedging language “sort of” at the end of the excerpt. These foreshadow second thoughts about placing priority on the cognitive, and offer a glimpse into a tension she experiences (discussed in subsection 7.2.1).

Andrew also asserted that he emphasises teaching the mathematics content, commenting, “I tended to focus on the mathematical content, making sure that they were comfortable with the basic ideas.” As emerged in his interview, this focus for Andrew was influenced by how he interprets the course—it is a mathematics *content* course, and so his emphasis is on teaching the mathematics. This is what his expertise is, and this is what he has been charged with. At the same time, the use of the word “comfortable” here is interesting. It carries with it an affective dimension; “comfortable” implies “free of anxiety”, and there is also a sense of familiarity—you can “call on” those you are comfortable with. This might suggest that for Andrew, there is a very tight association

²¹ When non-trivial content was removed, the symbol [...] was inserted. Any text enclosed in square brackets [like this] indicates an insertion made by myself for clarity or grammatical consistency.

²² These are all characteristics of teachers who possess Ma's (1999) “profound understanding of mathematics” (PUFM).

between the cognitive and affective: to be proficient at mathematics is to be comfortable with it, to be able to use it and enjoy it.

In her opening statement in the excerpt below, Brooklyn uses language very similar to Andrew's, but her words carry a much more explicit connection to concerns about student affect. She describes:

My main objective, I think, is to make students comfortable with mathematical concepts. I think [this] is probably my main objective. Many of them suffer from math anxiety, math phobia. They say, "Oh, you know, I've never done well at math", and these are people we're sending out into the school system, and so I think, the main priority has to be that they're, I guess, that they feel better about math going out than they do coming in.

Just like Andrew, she uses the term "comfortable" as a relationship individuals can have with concepts. If the first sentence were to be taken on its own, she could be seen to share Andrew's perspective on the course. However, taken in context, her words more clearly bring her concerns about student affect to the fore. She explains her position by talking about her students' math anxiety. For Brooklyn, making her students "comfortable" with the concepts is explicitly associated with helping them "feel better about the math"—an affective aim.

She is also explicit about the responsibility she feels. With her use of the pronoun "we", she positions herself as a member of a group, post-secondary instructors who play a role in the community in the preparation of teachers. "We" are sending these students out into the system as teachers, and it is important that "we" build their confidence. The qualifier, "I guess", in the last line presages a slight modification of her goals. At the outset of the excerpt she wants to make the students comfortable with the concepts, but by the end she settles for improving them. They should at least feel better on leaving the course than they did before.

This could seem as if Brooklyn's goals are primarily related to affect, but I have placed Brooklyn here with those who place a higher priority on the cognitive. This is clearly debatable. I place her here because she mentions the mathematics concepts first (barely), and the anxiety second. This may suggest that for Brooklyn, helping her

students understand the concepts will lead to reducing their anxiety, as opposed to the other way around.

For Bob, cognitive and affective goals are also closely intertwined. When asked what his main objective for the course is, Bob replies, “Well, of course to deliver the contents so that students are able to appreciate it”, but he quickly adds:

but a secondary by-product of what you do in the classroom is to get the students to enjoy it. So that they, in particular with this group, because you want [them] to be enthusiastic individuals about math, and be able to do the same for their students.

The phrase “delivering the contents” is an interesting one. It positions Bob, defining how he sees his role as teacher. Much like Andrew, Bob sees himself as a *mathematics* instructor, and as such is charged with teaching a particular list of mathematics topics as prescribed in the curriculum for the course, just like for any other mathematics course. It is because this is the first thing Bob says, that I place him here as one whose priority is the mathematics. Additionally, near the end of his interview, when Bob was asked about what his students get from the course he responded: “probably it’s the technical skills that they have solidified the most”, again suggesting an emphasis on the cognitive.

However, to leave it there would be incomplete. Bob talks about delivering the contents so that students “appreciate it”. The word “appreciate” could invoke affective goals, but when Bob uses the term in other places in the interview, he seems to use the term in a cognitive sense. He describes the course as focusing “on a very sound fundamental ability to appreciate [...], in a theoretical way, why things work”. So the appreciation Bob refers to seems more akin to a “deep understanding of the content and its connections”.

But Bob’s longer excerpt above clearly mentions affective goals. He speaks about “enjoyment” and “enthusiasm”, both of which he sees as important for future teachers who “you” (or rather, “we” as post-secondary instructors) will send out to influence their future students. The affective component is very important to Bob, but he explicitly mentions it as “a secondary bi-product”—understanding comes first, positive feelings follow. Bob also experiences tension around this as will be seen in Chapter 7.

Even though there appears to be a stronger (though not exclusive) emphasis on the *mathematics* for these four instructors, different possible motivations for this priority

are suggested. For Andrew and Bob it is at least in part by definition—the course is defined to be, first and foremost, a mathematics course. Maria’s and Brooklyn’s comments above display further direct considerations of the importance of strong mathematics subject knowledge to empower teachers and to build confidence, respectively. Affect is important, but there is a sense that attitudes to mathematics can be improved through increased proficiency. Such perspectives on the role of the course, the SCK needs of teachers, and the relationship between proficiency in mathematics and affect, can all contribute to placing a higher priority on cognitive goals.

6.1.2 Affect first (MA, I, C, EN, AP, ENG)

For several other instructors, affective goals seem to take precedence. Harriet, Alice, and Karen, all cite changes in student attitudes as their main objective for the course. There are subtle differences in their specific goals but their perspectives on the role of the MFT course are similar.

For Harriet, developing her students’ proficiency in mathematics is important, but it is secondary to overcoming their negative attitudes.

I want them to develop an attitude towards mathematics that it can be fun, that it need not be intimidating, that it’s to do with solving puzzles, and if you can’t do it the first time that’s perfectly OK, that you can develop ways of helping yourself. And that I very much want because many of them [have had] very bad experiences in math and not very good attitudes toward it, and they’re scared of it. So I want them to have enough solid basic mathematics that they have some confidence, and that they realise they will be seeing things that they have never seen before. Students will ask them questions that they don’t know the answers to, and that it’s OK. They can find out, they can figure out, they can work with that.

As for some of the instructors in the previous section, for Harriet there is a very close connection between the cognitive and the affective. She talks about wanting them to have “enough solid basic mathematics”, clearly a cognitive aim, but this is important because it can lead to confidence. This is not much different from Brooklyn’s stance

above, however for Harriet this confidence is explicitly not associated with having mastered the mathematics. She attempts to build confidence that can withstand the likelihood that they will encounter things “they have never seen before”. Cognitive skills are helpful, but a positive attitude in the face of not-knowing is crucial. So Harriet’s main focus is on the affective goals of improving confidence, and independence, specifically developing an “I can figure this out” attitude.

Alice’s main focus is also on affect:

For me the main thing would be that they would not be afraid of math, and like math, think it’s actually interesting, think it’s fun, and think it’s not about, “oh, no, I can add a negative with a negative” [...] It would be nice if they were better at math, but that they’re not afraid of it, that they don’t just consider it as dry, repetitive, “I’m going to repeat the steps and I’m going to get it right”. So I try to make it fun and playful, because many of them, I believe, are a bit hesitant about the whole math thing. And they postpone it as late as they can. So the goal was to make them feel a little bit improved on their confidence.

She begins by mentioning the mathematics anxiety and her hopes to help students find mathematics interesting. There seems to be a close connection for Alice between her students’ anxiety and their perceptions that mathematics is “dry” and “repetitive”, involving memorisation of procedures. The logic here seems to be that if they can see another side of mathematics, a side that is “fun” and “playful”, their anxiety will diminish and their confidence will grow. “Fun” was a recurring theme in Alice’s interview—she mentions it no less than 16 times.

Alice comments “it would be nice if they were better at math, but...”. This clearly positions cognitive aims as not unimportant, but secondary. When she states that she focuses on “making it fun and playful” *because* of their hesitance and their tendency to avoid taking mathematics for as long as possible, there is an implicit association suggested: if they become interested in math, then they may do more of it, and perhaps this will improve their skills. I note that the perspective espoused by Alice here is not unproblematic for her, and arises again in the discussion of tensions in Chapter 7.

Karen is the most explicit in her assertion that changing attitudes is her main priority:

my objective is to turn them around. I want them to leave the room, maybe not saying “I love math!”, but I want them to have more of an appreciation for it, maybe a bit more confidence that they can do things. Maybe they can’t do things that they’re going to do in school, and you just talk to somebody else there. There’s always somebody with experience you can ask for advice.

She recognises that with the limited time that she has with them, radical change might not be possible, but she hopes for improvement, “*more of an appreciation*”, and “*more confidence*”. “Appreciation” for Karen appears to be used in its more aesthetic sense. The term “maybe” before “a bit more confidence” is a hedge that suggests that she is unsure of how successful her efforts are to build this confidence. The fact that this statement is so quickly followed by an explanation of the consequences of not yet having the mathematics knowledge that they might need suggests that for Karen this confidence is associated with competence. However, she quickly rationalises that even if they do not yet have the knowledge, in the context of their future roles as teachers, it will always be possible for them to ask someone for help.

For all three of these instructors, the negative attitudes and anxieties associated with bad past experiences that they see in their students are serious concerns that need to be addressed in the MFT course. Harriet’s and Karen’s comments show considerations of how improved affect will manifest in future teachers—building students’ confidence will help them to cope with uncertainty, in Harriet’s case, and more specifically, to ask for help when they need it in Karen’s. For Alice, better attitudes towards math may encourage students to take more of it. For all three instructors, building mathematics competence is important, but addressing affect first will prepare students to address any knowledge deficiencies on their own later on. Here again, perspectives on the role of the MFT course along with beliefs about the relationship between the cognitive and the affective contribute to informing priorities, in these cases pushing affective goals to the forefront.

6.1.3 Conceptions of mathematics first (M, MI)

While most of the instructors indicated either improving attitudes or improving mathematics knowledge as primary objectives for the course, two instructors, Matthew and Al, both took a slightly different tack: both highlight the development of students' conceptions of the nature of mathematics as the highest priority in their respective courses.

Matthew, who is an instructor who had just finished teaching the course for the first time, struggled a bit with deciding what his objectives should be.

So I think the, the idea was, you should do something that introduces students to: "What is mathematics? What do people think about it?" And many of the topics should at least connect to mathematics they might teach. So, it's not meant to be a remedial course...although it's probably been treated that way over the years. So I was...I had no idea what to do.

Matthew's uncertainty is evident²³. His assumption is that the students who come into his course should already know the mathematics, and so it should not be "a remedial course", although he realises that it is often the case that elementary school mathematics is actually taught in the course. And yet, it should be connected to the elementary mathematics. Matthew had difficulty deciding on the role that the course should play.

Ultimately, he decided that it is what they learn about the nature of mathematics that is most important, basing this conclusion on his prior personal experience working with practicing elementary school teachers.

This [MFT course] is probably an important experience for them. Maybe the content doesn't really matter, but the mathematical experience matters.

Again, note the hedges, "probably" and "maybe". Matthew decided to place less emphasis on the mathematics content, but is unsure whether that was appropriate or not. There is no hedge attached to the assertion that "mathematical experience matters". He believes that it does; he wants them to "know what mathematics is, beyond

²³ This is discussed as a tension in subsection 7.3.2.

that childhood experience that they had”, particularly “the whole idea of creativity and seeing things in different ways”. He elaborates on this in his description of what his students leave the course with:

I think, a better understanding of what mathematics is. What makes something mathematics, and then what makes it not. You know? What kinds of things do mathematicians care about? I don't just mean topics, but ideas and perspectives, the notion of there being a reason behind things. I think the notion of there being a number of ways to look at something.

How Matthew goes about sharing these ideas with his students is described in subsection 6.3.3.

Al's first priority is also “to broaden [the students'] sense of what mathematical activity is”. To avoid repetition, I will reserve describing what this means for Al in more detail until subsection 6.3.1. It is mentioned here because this goal was a primary one for Al. However, other instructors within the study also commented on the conceptions of mathematics that they choose to promote and their views presented sufficient diversity for this to warrant separate consideration.

Both Al's and Matthew's comments suggest yet another perspective on the role of the MFT course. If the course is seen as an opportunity to orient students to the subject of mathematics in order to prepare them to be enthusiastic advocates of the subject in their future elementary school classrooms, this point of view would support placing an emphasis on building positive conceptions of mathematics and understanding the “mathematical experience”. These varying perspectives on the role of the course not only contribute to the diversity among MFT offerings, but are also indicative of the systemic tensions, which will be examined in section 7.3.

6.2 In the classroom (TM, GW)

One of the places that diversity among MFT instructors is the most apparent is in the approaches that they use in the classroom and the types of activities that they have their students engage in. While the differences in perceptions of the role of the MFT course, as described above, can have an influence, instructors' personal experiences, and how they see themselves and their students, also contribute to their choice of

methods. In this section, I focus on general instructor approaches, ranging from traditional lecture, to mixed lecture/activity, to a strong emphasis on group work. Description of some of the particular activities that are more closely tied to the preparation of teachers will be reserved for later sections of this chapter.

As mentioned in section 2.4, there have been concerns raised in the literature about the tendency of mathematicians, teaching at universities, “to represent teaching as transmission of knowledge” (von Minden, Walls, & Nardi, 1998). While the instructors in this study reported a wide variety of teaching approaches, consistent with observations by Strickland (2008), it was certainly the case that the predominant mode of in-class delivery is the lecture for many of the instructors.

Andrew is the most extreme in this regard, admitting that the traditional lecture is what he is most used to and what is most comfortable for him. In his class, he proceeds relatively formally, working from definitions to develop the theory, and takes great pride in preparing carefully thought out lectures, presenting the mathematics on the board in a way that demonstrates the beauty of the mathematical symbolism. Although this is Andrew’s mode, and he does believe that it is effective, it is notable that he is apologetic about it—this is discussed further (along with supporting transcript excerpts) in section 7.1.1.

Karen, Maria, and Matthew also use a lecture mode for the most part, but bring in activities when they can. Karen starts the course with an ice-breaker puzzle, and expressed the wish that she could spend more time on little class activities than she does. Maria uses Power Point lectures to elaborate on material students are meant to have pre-read in the text, but tries to schedule several special hands-on workshops throughout the term on certain topics. Despite her large class size, she attempted to solicit student feedback on the reading assignments at the beginning of each class at the start of the course, but found she had to give this up as it was taking up too much class time. Matthew describes his approach as story-telling. He is “mindful of trying to engage everyone in the class”, and often proceeds by giving his students warm-up problems to try, and then proceeds to harder problems. He finds this provides him with a better sense of what his students really know.

For the other instructors, in-class activities are a relatively frequent occurrence. Many of them bring in manipulatives from time to time (discussed below in subsection 6.4.3), and allow opportunities for students to play and explore. Al is quite deliberate

about using “advance activities” to establish a “framework for instruction”. Harriet incorporates class discussions of what could be seen as mathematics-for-teaching. She describes: “We spend time on the blackboard putting up three different versions of the same problem. Okay, which one makes sense? Which one’s easiest to follow?”

Alice also involves her students in whole-class discussions of the pros and cons of alternative solutions to problems. This allows her to hold back her own personal judgement on the correctness of solutions and allows the students to take ownership of this themselves. Another significant aspect of in-class activities for Alice is that they contribute to making the course fun, reflecting and supporting her priority of improving student affect.

Brooklyn has taught the course both in a face-to-face and in an online format. One of the benefits of face-to-face is that it offers opportunities for in-class activities. Brooklyn expresses the strongest antipathy to teaching MFT in a lecture format:

Math for Elementary Teachers really isn’t going to work if you just stand up there and lecture. [...] Because, I think, you know, that’s part of their experiences and where things went wrong, was that type of format. So, I think you have to offer a different approach, or else you’re going down that same path again.

Her impression is that students are most accustomed to a lecture mode in their past experience of mathematics. Unlike Andrew, who believes that the lecture mode can be effective, for Brooklyn it is at least in part to blame for students dislike of mathematics, and so it is vital to do things differently. In the online course, she can describe activities for students to try on their own, but it is a challenge for Brooklyn to know whether or not they have actually tried them.

Harriet was a particularly strong advocate for having students work in groups:

I do a lot of work in groups, and I don’t let them always pick their own groups. I try to mix them up as much as possible so they get used to working with different kinds of people, and realising they’ve got colleagues who can help them. I ask them to reflect on the experience of writing an assignment with a partner, and how it’s different, and what sorts of things they can learn from that.

“Working with different kinds of people” is an important skill for future teachers to develop to support interaction with pupils, but Harriet’s use of the word “colleagues” suggests another benefit she would like them to gain from the group work experience. For Harriet, the group work begins a habit of collaboration that she hopes her students will continue throughout their careers. She would like them to come to see their peers, not just as fellow students, but also as a resource for supporting their ongoing learning and future teaching. This is consistent with her affective goals of helping them feel more comfortable in the face of uncertainty.

Encouraging reflection on the group work interactions was not mentioned by any of the other instructors, but reflection was a common theme for Harriet. Reflecting on group work provides her students with an opportunity to consider how working through a problem with a colleague might be helpful, but as well, it provides them opportunities to notice the learning process that takes place, both for themselves and for the others they are working with. This latter aspect of group work was also mentioned by Brooklyn, who observed that “it’s important to have them realise that everybody thinks differently, and I think that’s a big thing for them”.

The instructors who did use group work, either in-class or for project work, did so to varying extents, and even those who spoke about the benefits of it, often complained that they did not do as much of it as they would like. Reasons for not using group work more frequently, and in fact other types of in-class activities as well, included perceiving it as taking up too much time, finding it difficult to evaluate, feeling that some students fail to contribute, and even, in the case of Matthew, having had a personal dislike for group work when he was a student. These reasons reflect considerations of course priorities, instructors’ pedagogical background, beliefs about students, and personal inclinations, respectively, implicating all of these as factors in contributing to diversity among MFT courses.

Certainly, the diversity was evident in the data, with the instructors reporting a range of classroom methods, from very traditional through to extensive use of activities and group work. Along with this, two further points are of note. The first is that, when asked, those who used more participatory methods indicated that they do not use these methods as extensively, and in some cases not at all, in their other mathematics courses, for some of the very same reasons for not doing group work cited above. It appears that those who do engage their students in more active participation find a way

to manage any concerns about time and evaluation in order to provide interactive class experiences within the context of MFT courses. The second is that for some of the instructors in particular, decisions about classroom methods are not taken lightly, and in fact are a source of personal tensions. Further insights into this will emerge in the examination of tensions in section 7.1, and especially in the case of Simon in Chapter 8.

6.3 Images of mathematics (M, MI, MN, PR)

As described in subsection 5.1.2, instructors see their students as coming into the course with not only poor attitudes to mathematics, but very limited conceptions of what mathematics entails. There is concern that if students persist in believing that mathematics is merely routine calculations, this will be the impression they share with their future pupils. All of the instructors mentioned wanting to change these conceptions, even to the extent of making it their highest priority (in the cases of Al and Matthew, mentioned in section 6.1.3). This is not at all surprising, given that all of the instructors had studied mathematics formally, and so at least to that extent could be considered “mathematicians”. It was also evident, as observed by Jonker (2008), that they were passionate about the mathematics and were eager to share their enthusiasm for it with their students.

What is somewhat surprising is that although all of the instructors want to promote mathematics, there is considerable variety in the aspects of mathematics that they focus on. During the interviews, it was analogous to hearing several different people describe what makes a particular city a good place to visit: one might rave about the nightlife, another about the parks, and another about the friendliness of the people. But in one’s efforts to promote the city to others, it is common to consider what they might be attracted to; at the same time it is easiest to promote what one enjoys the most oneself. By examining the different aspects of mathematics that the different instructors accentuated, we get a sense both of those features which they feel would be most appealing and relevant to prospective teachers, as well as to what attracted them to mathematics themselves. Four particular themes will be examined here: socio-cultural-historical significance, structure, problem solving, and the nature of the “mathematical experience”. Instructors exhibited differences in the extent to which they emphasised these themes, and in the case of the latter two, differed in the particular aspects of these themes that they sought to highlight.

6.3.1 Socio-cultural-historical significance (MI)

Several instructors mention sharing historical anecdotes with their students in their MFT course, either because of personal interest, or because it adds a dimension of human interest to their discussions of the mathematics. However, for AI, helping students see the broad social, cultural, and historical significance of mathematics is his primary goal. AI asserts, “what I think is most important about this course, is to give them a broader *sense* of what constitutes mathematical activity” (my emphasis). He wants them to expand their view to see that problem solving, geometry, and probability and statistics are all mathematical pursuits, but more than this, he wants them to see “the place in history and society of mathematics”, to recognise that it is “a human endeavour”.

His reasons for this emphasis are reminiscent of Alice (see subsection 6.1.2). He observes, “almost all of them [his students] come in with a *sense* that doing mathematics is doing numerical computation, and a *sense* that they’re no good at it” (my emphases). The close connection suggested earlier in discussion of Alice’s priorities, between conceptions of mathematics (as routine) and lack of confidence, is made more explicit in AI’s words through his repeated use of “sense”, both in the statement of his main objective, and twice in his description of the students. If he can change their conceptions of what mathematics is, then maybe they will no longer believe that they are not good at it. Unlike Alice, he does not mention “fun” as part of the desired conception. Problem solving and historical and social impact come to the fore, ostensibly as examples of the power and potential of mathematics; he wants them to believe that mathematics matters. If he can convince them of this, they will work harder to improve their mathematics skills and they will work hard to be effective in teaching mathematics later on.

6.3.2 Structure (MI)

For Andrew and Bob, the structure of the mathematics is the aspect that they want most for their students to come to know and appreciate, although for different reasons.

Andrew presents mathematics from a quite formalist perspective. He describes it as “just language, dressed up in symbolism” and hopes that at the end of the course his students will see “the subject is coherent and logical, and well organised”. He describes

mathematics as a hard subject that requires a lot of hard work and concentration. He says, “Of course we try to make it accessible, but it doesn’t really make it any easier!” The challenge, the structure, and the abstraction are what he loves and appreciates about mathematics, and this is what he tries to share.

During his discussion of his approach to teaching fractions, Andrew admits, he’s “not a great fan of converting arithmetical problems into pizza cutting exercises”. For him this goes against the very essence of what mathematics is about. He continues:

Certainly a lot of people think it’s good to give them a context, which makes them feel more comfortable. But my, my basic philosophy is that no, this is the antithesis of what mathematical thought is about. Mathematics is trying to distil what is common to all of these situations, and then deal with all of those situations by the symbol, mechanical symbol manipulations.

Although putting the mathematics in context might help ease students’ mathematics anxiety, for Andrew this is at the expense of conveying the nature of mathematics accurately. At the same time, although a “pure” mathematics approach is Andrew’s preference, he quickly modifies his stance somewhat, observing, “...but, you know, algorithms by themselves, without any comprehension or understanding are not really desirable. Especially in a teacher, you want more than that.” Trying to convey this view of mathematics presents its challenges for Andrew. These will be discussed more fully in subsection 7.1.2.

Bob’s emphasis on structure is not as strong as for Andrew, but he mentions the “structure” of mathematics and its theoretical aspects several times in his interview. In summing up the course, he comments:

they leave having had some sense of the structure of mathematics, because there’s a sufficient amount of that in the course....

For Bob, understanding of the structure should help support his cognitive goals for his students. When he teaches about problem solving, at least initially, he hopes they will learn “to work with a really simple problem-solving structure” (discussed further in subsection 6.3.4). Understanding “all of the structure” will also help them answer

questions that their pupils will ask them in the future. For Bob, conveying the structure is important, not because it is the essence of mathematics, but because it provides a scaffold for the learning and doing of mathematics.

6.3.3 Mathematicians and the mathematical experience (M, MI, SA, PR)

Some of the instructors are explicit about exposing their students to what it means to be “a mathematician” or to behave “mathematically”, but even here there are differences in the specifics that they draw out.

As described in the previous section, Andrew presents mathematics to his students from a formalist perspective, all the time aware that he may be the first “mathematician” they have encountered. He notes, “...maybe they hadn’t even been exposed to the kind of enthusiasm they might see from somebody who had actually studied the subject, for its own sake”. He sees the course as a unique opportunity for students to interact with someone who enjoys mathematics.

In contrast to Andrew, and as noted earlier, Alice’s goals for the course are primarily to improve students’ attitudes towards math, to help them see that it can be playful and fun. In trying to convince them of this, one of her strategies is to “make fun of the whole subject”, drawing their attention to, and playing up her enthusiasm for, mathematical aspects of the work they are doing. She gives an example:

Many times I literally just said “Well, look! Math is fun!” And I would be making myself silly in front of the class. “Isn’t this cute?” You know you check two answers and they agree. One solves [it] one way, somebody solves it another way, we get the same answers. And I’m like, “Well, this is the greatest satisfaction a math class can bring you!” Yeah, so they would laugh. And I’m like “Well, what else did you expect [from a mathematician]?”

Here she is making fun, not just of mathematics, but of mathematicians, through exaggeration. She is sharing a truth about the pleasure a mathematician might experience in seeing different solutions to the same problem, but at the same time, she is trying to put her students at ease.

For Maria and Matthew, it is important for their students to have mathematical experiences. They promote this through explicit class discussion about what

mathematicians do and value, and through particular activities that they have students engage in.

As an example, Maria described asking her students to define a rectangle and make conjectures about it. As it turned out, this was a task that many of her students reacted negatively to; they complained about how difficult it was to come up with conjectures and that it was even harder to prove whether or not they were true. She responded:

Whoa! But this is exactly it! It IS HARD to come up with conjectures! It is too, if you want to be engaged with an object. This is what we do! And, then to show that it's true or false! Ah! But this is our livelihood! This is how we earn our living as mathematicians, right? So, don't be afraid of it! This is how this material works or shapes itself.

Maria portrays mathematics as challenging, and precise, but ultimately rewarding. Speaking as a mathematician, she shares her own personal experiences and approaches with them in class discussions, inviting them into the experience. She advises, if the students (“you”), want to “engage” with mathematics, if you want to see what “we” (mathematicians) do, it will be necessary to accept the challenge, and put fears aside. This approach to dealing with students’ anxieties is problematic for Maria, and is discussed again later in subsection 7.1.2.

As discussed in section 6.1.3, Matthew’s main objective is to expose students to mathematical experiences. He describes:

We were constructing group tables of symmetries, and solving equations, and groups, and so forth. It was, I think, a little bit mind-blowing for them...but...we did RSA, I mean, encrypting things and decrypting things, all very elementary sorts of things. I had them do some proofs—there are infinitely many primes—variations on these things, and they kept seeing, and thinking and understanding what mathematics was.

Here Matthew is describing having his students engage in an activity that is almost certainly completely foreign with respect to any of their prior experiences. He

acknowledges that it was likely “mind-blowing” for the students, perhaps outside of their comfort zone. At the same time, for Matthew, the notions being discussed are “elementary”—not in the sense of elementary school—but in the sense of being accessible to novices. In the end, clearly, Matthew feels the activities were successful. His students “kept seeing, and thinking, and understanding what mathematics was”, and this was Matthew’s goal. As described in subsection 6.1.3, his students had opportunities to experience “creativity”, “different ways to look at things”, and also the “reason behind things”.

6.3.4 Problem solving (PS)

All of the instructors comment on wanting to improve their students’ problem-solving skills, with some particularly mentioning the importance of bringing out the “thinking” aspects of mathematics, like reasoning, noticing, puzzling, and discovering. As a result, problem solving was included in all of the MFT courses, although to varying extents, and with different emphases that often reflected the instructors’ broader goals for the course.

For Harriet, engaging in problem solving in the course can be a vehicle for addressing their anxieties:

They are just terrified, most of them, of a problem they haven’t seen before. And so that [problem set] is to try to help them with, you know, this is a sense of fun, we’re supposed to have fun with this. Yes, it’s worth a few marks, whatever. You may never have seen it; you might not get all the way through it. But what I’m looking for is how far did you get, and how well can you explain what it is that you got. And I think that’s really important for them.

She is aware that they will be afraid, especially if the problem is something that they have not seen before, but she tries to address their fears by taking the focus away from getting the right answer and putting it onto the process itself, working against possible sociomathematical norms, established in previous math courses, that place the highest value on getting correct answers.

For Bob, consistent with his goals for the course, problem solving provides another opportunity to demonstrate how useful the structure of mathematics and

structured approaches can be. He provides his students with problem-solving templates, at least initially:

there's lots of template approaches in terms of showing them different types of scenarios, and just how to work with a really simple problem solving structure, and how, it, regardless of the information that they're given, if they could, focus their approach along the lines of good sound problem solving techniques, that they can put the information, assemble the information in a way that fits into some logical framework, and then they can proceed.

Later on in the course, he allows them opportunities to do "more exploratory types of problems" where he will "leave the framework wide open", but in the beginning he feels they need the guidance and structure to build experience in successfully solving problems. This success is important to Bob, in order to help build students' confidence.

Alice, on the other hand, seems to avoid routine approaches. She states explicitly that mathematics is "not the routine steps [...] It's not memorising something [...] It's the thinking process that's behind it, how you approach a problem". Contrary to the sociomathematical norms of her students' high school experience, she goes so far as to encourage "common sense" over algebraic solutions to problems in an effort to break them of over-reliance on memorised procedures. This is also in contrast to Karen, who hopes her students will notice patterns and "make the leap" to appreciating and using algebraic techniques; for Karen, algebraic methods are preferred.

For Simon, ideally, problem solving should be the focus of the whole course. This is based in part on his perceptions of his students' needs; he feels that the students lack problem-solving skills more than they lack content knowledge. Simon wants his students to learn to think mathematically. He states: "one of the main things I'd like to get across to the students is the ability to just, sort of...look at a question calmly and analytically, and with focus, and work through it". His preference is for hard problems that can engage the students at whatever level they are at.

This calmness in the face of challenge is reminiscent of Maria, who as described in subsection 6.3.3 would like her students to experience mathematics the way a mathematician does. She observes that her students get "frustrated" and "blocked" when they encounter something they don't know how to do. She describes her own

personal dispositions to serve as a model for how her students should approach problem solving. She encourages them to let the frustration go, relax, trust in what they are doing and in their knowledge, and look for connections with past experiences. Successful problem solving involves accepting and going with these feelings, though her students tell her they are afraid that “letting it happen” won’t work for them. This is discussed further in subsection 7.1.2.

6.4 Preparing future teachers (FT, TMD, RF, GN, MO, BI)

A final area of diversity to be addressed here is the extent to which instructors orient their course toward the preparation of *teachers*. The instructors all describe having at least some conversations with their students about them becoming elementary school teachers one day, but they differ in what they feel they can or should offer, and in some of the specific techniques they employ.

As part of the interview, instructors were asked if they do anything special in the course just because the students may be teachers one day. Some, like Bob and Andrew, express doubt that they do anything special beyond using the students’ aspirations to motivate students to learn the mathematics well (Bob), or being more gentle with them to help ease their anxiety (Andrew). Others embrace the orientation towards teacher preparation more fully, through addressing aspects of pedagogical content knowledge or through explicitly teaching about teaching (both to varying extents), and through particular activities like using manipulatives, or organising Math Fairs.

6.4.1 SCK or PCK?

As discussed in subsection 2.2.1, Ball and colleagues (Ball & Bass, 2000; Ball, Thames, & Phelps, 2008) have done much work in recent years to distinguish particular mathematics knowledge-for-teaching, building on Shulman’s (1986) notion of pedagogical content knowledge (PCK). The instructors in the study were not exposed to this literature. However, through questions about what made teaching MFT different from teaching other mathematics courses, I hoped to capture moments when they might distinguish between subject content knowledge (SCK), or the school mathematics that is taught to elementary pupils, and pedagogical content knowledge (PCK), the kind of

mathematics knowledge that would support its teaching. This arose to varying extents in the instructor interviews.

Andrew, Karen, Matthew, and Bob spoke the least about items that could be coded as PCK, with Andrew not referring to it at all, and Karen and Matthew only referring to discussion of varieties of arithmetic models and algorithms. Bob also discussed these models for arithmetic operations, but was very oriented towards developing deep content knowledge in his students. There seems to be no distinction for him between the mathematics knowledge that a “user” of mathematics as opposed to a “teacher” of mathematics should have. For Bob, deep content knowledge will empower teachers to answer student questions and “to be able to differentiate whether that’s something that can lead you into a teachable moment”, an aspect of what Ball et al. (2008) refer to as “knowledge of content and teaching (KCT)”.

In Maria’s class, students have opportunities to see video clips of elementary school pupils doing mathematics. These come as a supplement to the text, and she uses them to launch discussions of such things as the dangers of applying invert-and-multiply as a rote rule with no understanding, and the importance of being open to alternative approaches.

The reason why I brought it in is to show to them, future teachers, “If you know how to add numbers, and you know this very well, do not expect that somebody else is going to add this exactly the way that you are adding this. You need to be able to understand how they’re communicating their understanding of addition. And in order for you to do that, you need to be very flexible with your understanding. I need you to try out things in a variety of ways, because you are going to be listening to learners later. In this course you are the learner, but later on this is where you want to be moving”.

Encouraging students to think about how their future learners will be thinking about the mathematics is reminiscent of Ball et al.’s (2008) category of “knowledge of content and students (KCS)”.

For Alice and Simon, the distinction between the different types of mathematics was made in the context of discussing examination questions. With Alice, this arose

spontaneously, with her making the observation that asking students to bisect an angle on an exam would seem “too elementary school”. Also, on her exams she insists that they use pictures and models at all times in answering questions, commenting, “There was nothing on the exam where they would just have to do it. They always had to also do the visual for it.”

In the conversations with Simon, the distinction was deliberately raised by the interviewer.

Interviewer: Are you giving any consideration to which questions will be testing school math and which will be testing what they need to know to teach math?

Simon: [paraphrase]²⁴ I suppose, even a question that tests mastery of arithmetic could be put in terms of correcting or explaining why. I need to make sure that they're competent in elementary school arithmetic, of course. I could have separate sections on the exam, one including questions of basic skill and one for more advanced questions.

This is likely the first time that Simon had considered this question. He does wish to ensure that they have mastered the school mathematics, but in this excerpt is considering how he might also test what he refers to as “more advanced questions”. In a subsequent conversation, when Simon described his exam, it did include questions that asked students to “identify a pattern of mistakes and to do another question in the same way”, to identify which models of subtraction are represented by given examples, and to select the correct solution to a question on the product of numbers with exponents, explaining what was wrong with the remaining solutions offered. These skills are encompassed in Ball et al.'s (2008) category of “specialised content knowledge (SpCK)”.

Harriet stood out from the other instructors, having the most comments coded as “PCK”. When describing the content of her course she mentions varieties of algorithms for arithmetic operations, along with models for their representation, focusing explicitly on “how those can lead into different understandings of what you're doing when you're

²⁴ Excerpts from the sequence of conversations with Simon are all, by necessity, paraphrased, as they were not tape-recorded.

multiplying, or adding...”, trying to help her students see that “what may work well for some situation, or for some student, may not work for some other one”. For Harriet, this is knowledge of the mathematics content that is particularly relevant for her students as prospective teachers.

Connections between mathematical ideas also play a central role in Harriet’s conception of the knowledge content of the course. She explains: “...what you can do with a grade three student, and what you can do with a grade six student are quite different and I want them to see that it’s all interconnected...” Her evident appreciation for these connections echoes Ball and Bass’s (2003) description of knowledge-for-teaching, addressing both the connections within and across grade levels:

I emphasize it [connections between topics] all the way through. I don’t try to plan the course to start from the beginning and go through to the end with an obvious thread, because mathematics is way too big for that. [...] But at all times I connect it, as far as I can, to what goes on at different levels. What you might do with a grade 1 class, how that connects to what they’re going to see in, you know grade 4 or 5 or something like that, how that connects to what they might do in high school and how that connects to what I’m doing in Calculus. Because they’ve got to see how it’s connected, and how we build bigger and bigger, you know, understandings of sets of numbers, or calculations, or whatever.

Harriet does not just pay lip-service to these ideas. She describes assignments and activities for her classes that provide them with opportunities to exercise their pedagogical content knowledge: her students engage in analyses of pupil errors, as well as activities that allow them to compare alternative methods for solving math problems.

The interview data suggests considerable variation among the instructors in both the extent to which, and the types of knowledge for teaching they address. A detailed comparison of their examinations might shed further light on this, but was not undertaken at the time of this study. This is one area where the interview itself may have had an influence on instructors’ future actions. Certainly this was the case for Simon, who was in the position of having to decide how to evaluate his students for the first time. I suggest that raising awareness of the distinctive nature of PCK, can open up

possibilities for instructors that may not have occurred to them before. This will be taken up again in subsection 9.5.1.

6.4.2 Teaching about teaching (TMD)

Aside from the possible variations in types of mathematics knowledge, I turn now to consider the extent to which the instructors attempt to specifically tailor their course to meet other perceived needs of their students as future teachers, particularly those related to pedagogy.

All of the instructors would agree that the MFT course is first and foremost a *mathematics* course, and most emphatically deny any expertise in the area of how to teach elementary school pupils—but some cannot resist engaging with the students in discussions about pedagogical aspects. As Harriet confessed, “I’m not a math methods teacher at all, but I can’t help but talk about things that work and don’t work”. Maria also asserts that she is not an expert, but feels she has had some relevant experiences in the past that she can offer, as does Matthew, who elaborates, “I may not know all the lingo, but I do know what it is to think about how my students learn, and what they’re learning, and how you might achieve a goal, and so forth. So that’s what I share.” These instructors see strong parallels between what they do as post-secondary mathematics instructors and what their students will be doing as elementary school teachers. There is a sense that they are initiating their students into a “community of practice” (Lave and Wenger, 1991), that of mathematics teachers, and within this context they share their expertise.²⁵

Even the instructors who spoke the least about incorporating elementary school concerns into the course, were very conscious of the example they set as instructors in the MFT classroom. Al observed, “I’d like to think that when I teach the course I model constructive pedagogy that they might ultimately use, if not exactly, but in some way”. Simon expressed concern that his teaching may have lacked sufficient variety for a group of potential teachers, although he did try many things with them that he does not normally do in his other mathematics classes. Alice makes a point of bringing in “little fun problems to solve”, with the deliberate intention of shaping the kind of mathematics

²⁵ Whether such a “community of practice” actually exists in the sense intended by Lave and Wenger is doubtful.

her students will share with their pupils one day, so that “they won’t just have to teach the kids later on to add numbers over and over and over.”

Matthew described having students engage in particular activities that are very specifically related to teaching. He is sure that his education colleagues would be distressed by how he handles it, but he asks his students to write a lesson plan, paying attention to learning goals, assessment, and potential difficulties children might experience. He also tries to give them opportunities to teach, either each other or in elementary schools. The fact that he jokes about what his education colleagues might think of this suggests that he is aware he is beginning to teach material that is, or at least that they would think is, in their domain. These activities are not mandated by the course curriculum, so it is Matthew’s choice to incorporate them, suggesting he feels both that the activities are relevant for his students, and that he has something to offer in supporting his students’ development as teachers.

For a few of the instructors the emphasis on teacher preparation is pervasive, and takes the form of encouraging students to reflect on their current learning and experiences of mathematics, and on their role as future teachers. This is a major theme for Harriet, as noted previously. She asks them to write about their experiences working with a partner on a problem:

This gets them starting to think what it’s like to be a teacher, but always aware of how you’re learning. You know, what it is that you’re learning, how the other person is reacting to you. I spend a lot of time focussing on that. It isn’t math, but it’s all around the teaching and learning of math.

Although she does not see it as “math”, she feels that incorporating these opportunities for reflection on teaching and learning is essential for her students, providing motivation in the short term, and supporting their teaching in the long term.

For several of the instructors reflection is encouraged through the writing of journals, although these are handled differently in different courses. For Karen, the journal is used to chronicle students’ experiences as they prepare for a Math Fair (see subsection 6.4.4 below). Alice has her students ponder pedagogical questions, such as the relative merits of alternative questioning techniques (e.g. “What is the next number in the sequence?” vs. “What are the possibilities for the next number?”), as well as more

mathematical questions, such as whether or not multiplication is just repeated addition. Maria builds in a reflection exercise at the end of each lecture, using it to gather feedback from the students on their understandings and difficulties, while giving them the opportunity to reflect on their own learning.

Other specific activities that some instructors use because their students will be teachers one day include using manipulatives and having the students organise Math Fairs. These will be elaborated on in the following sections.

6.4.3 Manipulatives (MP)

Manipulatives are one of the tools that are often brought into MFT classrooms to support the learning of future teachers; they are discussed in virtually all textbooks designed for this course. For the instructors in the study who use the manipulatives there is some common ground with respect to their perceived benefits, but only half of them chose to do so.

The instructors who do talk about using manipulatives all mention that they play a dual role. On one hand, activities with manipulatives give students a chance to practice using tools they will find to be helpful later. On the other hand, the instructors observe that it also helps the students build conceptual understanding of the mathematics. Both roles can be seen in the following comments, made by AI and Maria, respectively:

We have quite a cart of things that we can bring into the class, we use because they might find it useful in their practice, and we hope that they're good in promoting understanding of things.

and

So there I wanted them to have the idea of the relationships between the sizes [of fraction pieces], the sizes themselves. But I also thought this is a good tool maybe for them, later on as a teacher, to come back and think about some manipulatives, something for their group of students.

This dual-purpose use of manipulatives epitomises the multiple layers that some of the instructors are trying to address as they teach the MFT course. They are teaching mathematics while at the same time they are, to varying extents, trying to prepare their

students to be teachers. This is discussed further in subsection 7.3.1. Note that although Al's and Maria's comments both include assertions that students will be applying their knowledge of manipulatives in their future classrooms, the claims are embedded with hedging terms: "they *might* find it useful", "this is a good tool *maybe* for them". The instructors hope that using manipulatives will be helpful to their students as future teachers, but they are not sure that this will happen.

Harriet observes that another advantage of having the students work with manipulatives is that the novelty of the activities helps the students imagine what it is like to learn some of the mathematics from a child's perspective:

We get a chance to work with manipulatives [...] and it may look a bit silly, but I'm trying to get them to do things they haven't thought of before. Using manipulatives so they get a sense of how it is when children do this. You know, like, working with negative numbers, red and blue poker chips.

There was a sense, from both Harriet's comments and those of others, that students sometimes know the algorithms and procedures so well that it is difficult for them to stand back from their prior knowledge to see the mathematics with "fresh eyes".

Her comment that "it may look a bit silly" is a reference to the general perception (misconception) that manipulatives are just for elementary school children. One of the advantages of the fact that the students are planning to teach elementary school is that it presents an excuse to bring toys into the classroom. Others commented on this as well. Manipulatives of any kind are rare in post-secondary mathematics classrooms, but this group is open to the experience. For those who use the manipulatives this is helpful, especially given the instructors' perceptions of their students' weak conceptual understanding. It enables them to use a tool to support their students' learning that they would not feel comfortable using with adults otherwise.

However, as noted, only half of the instructors interviewed mentioned bringing manipulatives into their MFT classrooms. Reasons for not using them at all included not being aware of them or knowing how to use them, not wanting to take the class time, and not being comfortable with that type of class activity. Sometimes the practicality of physically getting the manipulatives to the classroom was a deterrent. These "obstacles" likely were also present to some extent for all of those who did use manipulatives, but for

some instructors the perceived potential benefits of using the manipulatives were such that they found ways to accommodate them.

6.4.4 Math Fairs (MF)

One activity that three instructors mentioned using with MFT students that is clearly aimed at preparing future teachers is requiring students to put on a Math Fair. This involves students creating posters and interactive displays that will engage elementary school pupils in a mathematics puzzle or game. Students usually work on these projects in teams and have an opportunity to present their project at an elementary school. Setting this up requires a fair amount of coordination between the instructors, their home institutions, and the elementary school(s) involved.

Those instructors who are involved in this speak very passionately about it and its benefits, with each bringing out a different aspect of what they find worthwhile in the activity. Alice admits, “You don’t need to know much math to do this”, but for her, the purpose of the event is “to have some play and fun, and [develop a] puzzle solving attitude for the Math”. In contrast to this, Al adds a written component to the Math Fair project that increases the mathematical requirements somewhat. In addition to researching and presenting a puzzle problem, he expects the students to “analyse it more deeply in a paper”. In his view, this in-depth analysis is one of the features that distinguishes the MFT course from the other mathematics courses his students have taken and helps to further Al’s agenda of broadening student conceptions of what it means to do mathematics.

Karen’s focus for the Math Fair is the experience her students will have with the elementary school pupils. She states, “for many of them it’s really their first interaction like that with students. So, I think it’s good for them”. She goes on to observe that “when they see some of the kids at the Math Fair, and they can see how bright some of these kids are, it opens their eyes a bit too, to what’s going to be coming”. Karen sees this interaction with inquisitive elementary school students as a motivating experience— if they want to be able to answer their pupils questions, they will need to put in more effort into learning the mathematics.

Despite the rave reviews that instructors who employ this activity give it, only three out of the ten instructors interviewed do so. Karen offered some insight into why this is the case, observing that many of her colleagues do not want to, perhaps because

of the perceived work involved in coordinating the Math Fair, but mainly because of the challenges of marking. She expressed deep satisfaction that finally, after many years, she had found a method of evaluating the Math Fair projects that is both practical and fair.

Instructor concerns about the challenges of more subjective evaluation were mentioned in subsection 5.3.2. Although the concern is shared by many of the instructors, while the response of some is to avoid activities that require this type of evaluation, others find ways to adapt. For those who do not see connections to the elementary context as part of the course, there is even less incentive to make such adaptations. Again, we see that what happens in the MFT course depends on instructor choices, and that these choices are influenced by their conceptions of the role of the course, as well as by their personal experiences and inclinations.

6.5 From diversity to tensions

In this chapter, we have seen a number of ways in which individual versions of the MFT course, as offered by these instructors, differ from each other. The instructors diverge in their perceptions of the course priorities, with some placing their focus on mathematics content knowledge development, while others choose to attend primarily to affect or conceptions of mathematics. Their classroom methods range from traditional lecture to more interactive approaches, and although the course is nominally a *mathematics* course for prospective *teachers*, there is diversity in both the particular aspects of mathematics they emphasise and the extent to which they orient their course towards teacher preparation.

Differences in priorities trace back to instructors' individual and diverse conceptions of the intended role of the course, their perceptions of the needs of their students, and beliefs about the relationship between cognition and affect. Classroom methods, treatment of mathematics, and orientation to teacher preparation are further influenced by instructors' personal inclinations and past experiences. Instructors are in a position of making course decisions based both on what they believe they *should* do and what they feel they *can* do. The results of these decisions are manifest in the actions they undertake, affecting such things as whether or not they have students engage in group work, use manipulatives, or set up Math Fairs, as well as influencing the types of

class discussion they entertain, the varieties of examination questions they employ, and even the ways they approach problem solving.

The diversity described in this chapter was recognised through the different perspectives and actions described by the instructors, but as noted at the beginning, the positions taken by the instructors, though largely presented as stable through the selection of particular excerpts, were often not. In fact many instructors, some explicitly, some implicitly, expressed uncertainty about some of their goals and some of their methods. When stances shifted for an individual, this was taken as a possible indicator of tensions—it is this aspect of the experience of teaching the MFT course that I turn to next.

7: EXPERIENCING MFT: TENSIONS

Tensions are endemic to the experience of teaching the MFT course. Throughout the description of the commonalities and the points of diversity that arose in the interviews with the instructors, there have been frequent glimpses of some of the challenges they face, as well as some of the uncertainties they feel. Particularly in Chapter 6, through the discussion of diversity, there was an opportunity to see some of the choices instructors make: where there is divergence, there are degrees of freedom of both interpretation and action. By describing these choices, the act of recording them makes them appear fixed and immutable. But often it is the case that instructors take a stand, provide a response to an interview prompt, approach the teaching of MFT in a particular way, because in the moment they must make a decision, they must act in order to move forward. The difficulty is that the encapsulated response no longer reveals the deep thought, and possible tensions, that led to its enactment.

In this chapter, I return now to re-open some of these responses, to reveal some of the tensions within. I will focus on those that emerged most clearly in the analysis of the transcripts, noting that not all of the instructors experienced all of these tensions, and even those that are shared are not experienced in the same way. Three levels of tensions emerged and will be explored here: personal tensions (those that are related to instructor identity, who and how to be); internal tensions (those related to instructor decisions about the course itself); and finally systemic tensions (those that relate to the position of the MFT course within the broader educational system).²⁶ Discussion of these tensions will require a more detailed look at excerpts from particular interviews than has been done to this point, with close attention to language and positioning, in order to locate tensions, to recognise factors that might contribute to them, and to gain insight into how they are managed.

²⁶ Despite this categorisation, these tensions are closely interrelated, making it difficult to describe one without mention of others, as will be seen.

7.1 Personal tensions

Lampert (1985) observes, “Who the teacher is has a great deal to do with both the way she defines problems and what can and will be done about them” (p. 180). For two of the tensions that emerged from the data, this was especially the case. These were tensions that were only expressed by a few of the instructors and were closely tied to their personal identities, particularly their identities as mathematicians and/or mathematics instructors.

The research literature mentions the potential that mathematicians have in teaching the MFT course to share their enthusiasm for, and their insights into, their field (Hodgson, 2001). It also mentions concerns about their capacity to relate the mathematics to the elementary school teaching context (Kajander et al., 2010). Some instructors seem to readily meet this latter challenge (see section 6.4). However, for others there was evidence of an internal struggle, an effort to readjust their thinking in order to adapt to the demands of teaching the MFT course. Two tensions are described here, one that relates to the way mathematics is presented, and the other to the way the mathematics is represented.

7.1.1 In the classroom

In section 6.2, in the discussion of the range of classroom techniques employed by the MFT instructors, Andrew was placed at the near end of a spectrum that ranged from traditional lectures to fully participatory approaches. Tension was evident in his interview, as he admitted exclusive use of the traditional lecture format, while at the same time offering that this might not be the ideal approach for this group of students. A closer look at Andrew’s words offers insights into this tension:

I tended to teach the course, you know, because of my practice from teaching calculus and so on, I tended to teach the course in a somewhat orthodox way, you might say. When I say orthodox, I mean in, like, lectures, and in the way I teach my other courses, and I don’t see that was a good thing! That was partly lack of experience with doing things the other way, partly, perhaps, a little bit of laziness, and comfort with the way I was doing things.

For Andrew, the tension exists between what he knows and is comfortable with, and what he believes is in the best interests of his students. His “practice”, his expertise, is lecturing, explaining calculus and other post-secondary mathematics to students. He owns this practice; it is his (“*my practice*”), and as such it is part of his identity. Lecturing is what he does as a college mathematics instructor, and he describes teaching the MFT course in this way, just as he teaches his other courses. He uses the word “orthodox”, which connotes tradition, correctness, and appropriateness, although he hedges it with the word “somewhat”, suggesting acknowledgement that perceptions of this approach (among teachers?) might not be what they once were. He continues:

Certainly, I think that it would be better to do well-designed activities that, you know, engaged the students [...]. I simply don't have, I'm not very imaginative, I guess, when it comes to that kind of thing, so I didn't do it. I tended to just lecture and hope for the best.

He affirms with “certainty”, modified only by the slight hedge “I think”, that an activity-based approach would be better, especially in order to engage the students. Yet despite this, he sticks with the lecture approach. His statement, that he “tended to just lecture and hope for the best”, is a nice example of living with the tension. It is not the case that Andrew does not care about his students, but rather other factors weigh in that prevent him from taking the course of action he believes he should. With these factors in mind, he does the best he can.

Andrew cites personal attributes, namely, lack of experience and laziness (in the first quote above), and lack of imagination, as factors that contribute to his loyalty to the traditional lecture mode. Given the effort that he describes putting into crafting his lectures, laziness does not seem an accurate portrayal. A more convincing contributor is his belief that the lecture mode is, or at least has the potential to be, effective. He comments: “This is how I was taught, it served me in good stead. So I may have had too much of a tendency to think it would serve them in good stead”.

Note the past tense used in the last sentence: he “may have had too much of a tendency”. With this, he positions himself at the time of the interview as possibly different from the self who taught the MFT course in the past, who at that time rationalised that since he had learned mathematics through lectures, the MFT students

would be able to do so as well. Indeed, Andrew does speculate that if he teaches the course again, he may make greater efforts to incorporate in-class activities. He goes on to observe that he is at a different stage in his teaching career that may allow more time to investigate alternatives. The use of the modal verb “may” in these last two sentences is carried over from Andrew’s words. He is considering whether his past methods have been appropriate and is attracted to the idea of making some changes, but is not fully committed to them.

His comment about having more time now, in order to explore different classroom approaches, implicates time with respect to instructor workload as a possible additional contributing factor for this tension. Andrew’s personal characteristics and inclinations may have been a barrier to his desire to incorporate more novel approaches, but lack of time for professional development may also have been an aggravating factor, or alternatively an excuse, to stay within his comfort zone.

This tension between how individual instructors teach the MFT course, and how they believe they should be teaching it was the most evident in Andrew’s interview, but also arose for Maria and for Simon. For Maria it was less related to her identity as a college mathematics instructor, but rather more to her priorities for the course, and as such will be discussed in subsection 7.2.1. Simon’s tensions are examined separately in Chapter 8.

7.1.2 Being a mathematician

Other instances arose within the interviews where it seemed that an instructor’s identity as a *mathematician*, specifically some of the values, norms, and interests that are often associated with being a mathematician, seemed to lead to challenges in the MFT classroom. Examples of this arose in the interviews with Maria, Bob, Andrew, and Matthew.

Maria’s case has already been mentioned in subsections 6.3.3 and 6.3.4, which respectively describe resistance that Maria encountered in engaging her students in a conjecture-building exercise and her approaches to dealing with student anxiety in problem-solving activities generally. In both instances, Maria tries to convince her students that the difficulties that they experience are simply part of the nature of mathematics and are not to be feared. She uses herself as a model to exemplify

mathematicians' dispositions in the face of such challenges. During the interview, she described explaining the mathematician's approach to her students:

What do we do if we don't understand something? We don't freak out over it. Right? We puzzle maybe, or maybe we let it slide a little bit, and say, "OK, let's give it some time. Let's think about it. Let's try to find some resources. OK, and what can we do about it?" And then we seek out colleagues, or somebody, right? We're not afraid to ask questions.

Maria's use of the pronoun "we" is interesting here. On one level, the "we" could be interpreted as including herself and me, both mathematicians, and both familiar with the stance described when faced with a challenging problem. But within the storyline of the interview, Maria is in a quotative mode. She is recounting what she has said to her students. In this context, her use of "we" positions her within the community of mathematicians, and it is her intention, through modelling the appropriate dispositions, to invite students to position themselves within that community as well.

However, the students often reject the position offered to them. She quotes some of them as complaining, "Well, with me it won't happen!", with the "it" referring to the experience of coming to a resolution of the challenging problem eventually. This is a source of frustration for Maria. She has other strategies for trying to deal with her students' math anxieties, but this offers a nice example of a tension between the dispositions she, as a mathematician, would like to foster in her students, and what they are prepared to accept. She would like them to be like mathematicians, to delve into the problem, ask questions, let the process happen and ideally, to enjoy it as much as she does. However, the gap between her students' dispositions and the mathematical experience she offers is difficult to bridge.

Another point of resistance occurs as instructors work to establish certain mathematical norms. This was briefly mentioned in subsection 5.3.3, at the end of the discussion of sociomathematical norms that are particular to the MFT course. A prime example of this is mathematical standards for proof. Bob spoke about whether his students see the need for mathematical proof:

In most (sigh), you know, to just get sort of a survey by looking at the class, I would say more than half the students see that there's

justification, 'cause you can go through simple examples to explain why certain patterns don't necessarily beget a proof, and so the need for giving other sound reasoning... devices, like a proof would...They can appreciate that to a certain extent, even if they don't like it. I mean, I would say much more than 50%, they're not really crazy about the idea of having to go through some proofs, but I'm hoping that..., I mean, (laughs) I reach as many with their ability to understand that the proof actually works.

Bob's sigh is an indication of his disappointment. He describes his perception that most of his students, although they may ("to a certain extent") understand that there is a need to distinguish between mere examples and more formal proofs, do not enjoy the proofs, not in a way that a mathematician does. Bob lives with this tension between his own interests and those of many of his students, settling for raising their awareness and perhaps building knowledge of how mathematics works. Ideally, he would prefer that they, as potential future teachers of mathematics, buy-in to the mathematical disposition that values proof.

Andrew provides an example of a different mathematical orientation that creates personal conflict for him in the teaching of the MFT course. In section 6.3.2, as part of the description of the aspects of mathematics that Andrew tries to bring out for his students, he is cited as being opposed to "converting arithmetical problems into pizza cutting exercises". I repeat here the excerpt which epitomises his point of view as a mathematician:

Certainly a lot of people think it's good to give them a context, which makes them feel more comfortable. But my, my basic philosophy is that no, this is the antithesis of what mathematical thought is about. Mathematics is trying to distil what is common to all of these situations, and then deal with all of those situations by the symbol, mechanical symbol manipulations.

This excerpt highlights Andrew's love of abstraction, and how he sees it as being in opposition to "what a lot of people think" is good for teaching MFT students. The conflict here is not within Andrew, he is clear on where he stands on this, but Andrew senses it between his ideals as a mathematician and what he perceives as advocated for the

teaching of mathematics within the larger community. It is possible to speculate, but not to know for certain, who is included within this community. It likely includes his colleagues, both at his institution and those he is in contact with through broader professional activities; perhaps he encounters this view in the MFT textbooks, or even in readings he may do related to mathematics education for his own interest.

With his statement about what “mathematics is...”, Andrew captures a classic example of one of the fundamental differences between mathematics and mathematics-for-teaching as identified by Ball and Bass (2003). While in mathematics the goal is to compress knowledge to achieve insight through abstraction, mathematics-for-teaching involves an unpacking of concepts, a making of the implicit, explicit. For Andrew the move from abstract to concrete is a move in the wrong direction. He resists, but at the same time is aware that he is pushing against a contemporary mathematics education norm.

Another instructor, Matthew, also mentions his love of abstraction as a potential source of difficulty in his MFT classroom. He feels the need to protect his students from his mathematicians’ tendencies, and actually has his students use a “safe word” to bring him back when he starts to leave them too far behind. He describes:

There were lots of frustrations, you know. I gave them a key word, I think they chose [...] ‘mayday’, so they would say, ‘mayday’, as if, you know, “We’re way out there right now!” because I’m naturally abstract at some level, and so they would be drawing me back.

He wants to share his world of mathematics with them, but is sensitive to how easily he might try to take them beyond where they are ready to go.

It is somewhat ironic that one of the potentially positive contributions (Williams, 2001) that mathematicians can offer their students, their passion for their subject, can also be a source of tension in the MFT classroom. Once again, Andrew articulates this very well:

And I wanted to try to get them to see that the subject itself is interesting, you know? But, oh well! At least to me! It’s obviously not interesting for everybody, but that there is something there worth thinking about. I, kind of, talk about it philosophically

sometimes with them, you know? Well, you have to be careful about that kind of thing of course, but, you know, this structure of mathematics, the whole way that the mathematicians started in the middle and worked their way back to the foundations, and worked their way forward to analysis. The whole edifice, try to show them, you know, the whole thing, without overwhelming at the same time, you know? I always enjoyed that.

The tension is evident. His lament “But, oh well!”, signals his recognition that he does not reach as many of his students as he would like, but nevertheless feels it is worthwhile to persevere. He knows that he needs “to be careful” and needs to avoid “overwhelming”, but at the same time he feels it is important to reveal “this structure of mathematics”. He needs to walk a line here, between giving them the full picture, and losing them completely.

In the case of Bob, demonstrating proofs is a common aspect of the MFT course, but it is his personal enjoyment in contrast to the ambivalence of his students that accentuates his disappointment, and leads to what is perhaps merely a surface tension. For Maria too, the tension manifests itself as disappointment that her students are not willing or able to embrace the mathematical approach that she is advocating as a remedy for their anxiety. However, it is notable that the tensions suggested by both Andrew and Matthew in their respective examples are completely self-imposed. The course curricula do not require the instructors to show “the whole edifice” of mathematics—it is their personal passion for the subject that drives them, and sometimes leads to the tensions. I note here, parenthetically, that the existence of tensions is not necessarily negative.

7.2 Internal tensions

The personal tensions discussed to this point, while being part of the experience of teaching the MFT course, at least for some of the instructors, are relatively superficial—they do not spring from “an argument with oneself” (Lampert, 1985). Rather, they emerge as conflicts between individuals or groups; i.e., between instructors as mathematicians and the “community” (with respect to teaching styles or the relative merits of concrete vs. abstract representations), or between instructors and students (with respect to norms and dispositions).

I turn now to a consideration of some of the deeper tensions, tensions which exist within individual instructors—arguments that they *do* have with themselves. In some cases these appeared through the coding process, but often the tensions were not revealed until specific passages in the transcripts were examined more closely for language choices and positioning, and sometimes not until various passages within the same transcript were considered in juxtaposition.

I refer to the tensions in this section as “internal tensions”. These arise as instructors strive to make decisions for the content they will teach and the activities they will incorporate into their MFT course, while attempting to address the perceived needs of their students. These tensions stem from uncertainties with respect to the best course of action. Two such tensions emerged from the research, both of which have been foreshadowed in the discussion of diversity in Chapter 6. The first is a tension around setting priorities for the course: both cognitive and affective goals are important, but they are often experienced as being in conflict. The second is a tension around setting standards for the course. This is related to the first, but reflects the need to make decisions around the level and type of mathematics taught.

7.2.1 Setting priorities

The literature review in Chapter 2 outlines the debate in mathematics education research over whether it is more important for elementary school teachers to have strong mathematics knowledge, or to have appropriate beliefs and a good attitude towards mathematics (see subsection 2.3.3). The debate that is occurring at the theoretical level is visible within the local activities of the instructors. In section 6.1, instructors’ objectives for the course are described, with instructors loosely grouped according to whether they place more emphasis on cognitive or affective objectives in their course. Such a grouping facilitates presentation of the results, but masks the underlying conflict that some of the individual instructors deal with in formulating their objectives. Fundamentally, this conflict takes the form of a tension related to student needs: what do they need more—stronger mathematics or improved attitudes? What do they need first? Given that they need both, how do we (as MFT instructors) achieve them both, and is this even possible? Three different perspectives on how this particular tension is experienced can be seen through a closer look at the cases of Bob, Maria, and Alice.

Bob was tentatively grouped with the instructors who emphasise cognitive gains. His focus is on building deep conceptual understanding, though affective considerations are also very important to him. Recall, his course “focuses on a very sound fundamental ability to appreciate [mathematics], in a theoretical way, why things work, as opposed to technical aspects of how do you do mathematics”, along with having “a secondary by-product of what you do in the classroom is to get the students to enjoy it”.

The cognitive and affective are closely related for Bob. His students’ anxieties are at least in part caused by, and at the same time the cause of, their lack of arithmetic skills.

In many cases, some of the very elementary arithmetic operations are, in fact, confused in their minds and so when they hit upon things, in particular when you hit rational numbers, as an example, that’s one place where students have a great deal of anxiety and they would demonstrate poor understanding of ideas.

For Bob, the students’ lack of skills is a major impediment to their developing an appreciation for the mathematics. And so, Bob puts his focus on helping the students learn about mathematics, particularly its structure. Knowing the structure of mathematics will solidify their understanding, giving them confidence, competence, and enjoyment...ideally.

Bob’s comments about his students at the end of the course offer a glimpse into one of his tensions. He believes that they “have improved most in their technical abilities”, along with having gained some problem-solving skills, although these need to continue to be developed. Note that although he stated that the “technical aspects” were *not* the focus of the course (see above), there is no real contradiction here since, consistent with other comments made by Bob, improved technical skills should follow from a deeper understanding of the structure. But he is ultimately disappointed, both in his hopes to build deep theoretical understanding, and in his hopes to increase his students’ appreciation for, and love of, mathematics.

In terms of appreciating some of the more subtle aspects of the theory, I think that’s another thing that they could do better, if they had better basic arithmetic skills, coming in. So...yeah, in terms of what I produce, I guess, in terms of the other goal, for love of

math? Unfortunately, the course is so packed, that in some ways, I think they do get a little bit beaten by the end, and they're just tired.

He does see some success with improving their technical skills, but admits that he is less than successful (by his own standards) in terms of affective gains. He is trying to cover too much, trying to pack too much in, to the extent that his students are overwhelmed. The distance between where they are at the beginning of the course, and where he is trying to take them is too great for some of the students.

A closer look at this passage, with particular attention to pronoun use, offers some further insights. In the first sentence, he ostensibly places the responsibility on the students, "*they* could do better, if *they* had better basic arithmetic skills". However, as Bob is most certainly aware, the prerequisites for the course are not controlled by the students, or set by himself, but are negotiated by the "community", where the community is made up of (at least) mathematics departments, education departments, and teacher accreditation organisations. Whether it is the fault of the community or the students themselves, the lack of student skills coming into the course, is an impediment to his ability to realise his goals for his students.

He then switches to consider what he ("I") produces. Having already mentioned that students have increased their technical abilities, he moves to his "secondary" goal, improving affect. The results here are "unfortunate"; he describes his students as "beaten" and "tired"—not at all what he desired. The phrase, "the course is so packed", is telling. It is offered as an explanation for the students' states of exhaustion; there is too much material in too little time. But it appears from the use of the passive voice that Bob is not in control of the course content; with it he positions himself as unable to remedy this "unfortunate" situation.²⁷ The course, as he is expected to deliver it by his institution, demands too much of the students. At least with respect to some of the students in the course, from Bob's perspective, the instructor is in a position of trying to achieve goals that do not seem to be achievable.

This raises a new factor in considerations of course priorities. Instructors need to make decisions on how to balance building students' mathematics proficiency with

²⁷ To the extent that the course is defined by the community, this foreshadows systemic tensions, described in section 7.3. However Bob may have more control over the course content than his comments suggest.

addressing their affective needs, but Bob's reference to "the course" as a pre-defined entity draws attention to the fact that instructors make reference to a course syllabus, the list of content topics, when designing their course. As will be seen in comments made by Maria (below) and Simon (in subsection 8.1.1), feeling an obligation to "cover the material" is very much a part of the tradition of teaching post-secondary mathematics courses. It is based in more than an ethic of responsibility to the institution as employer. When teaching a course within a sequence (say from precalculus through to calculus and differential equations), there is trust that students coming in will have been exposed to the prerequisite material. It is a serious matter to omit covering a later required topic. Although the MFT course is not a prerequisite course for a later mathematics course, there may be some transfer for instructors of the feeling that the topics listed *must* be addressed. They may treat the MFT course content as just as essential as they do in their other mathematics courses, even though it is not situated within a sequence like these other courses. This can be in tension with *both* cognitive and affective goals for the MFT students.

Bob is not a new instructor of the MFT course, and so has likely lived with this problem for some time. He is stuck in this dilemma. On the one side he has students who are unprepared for the level of mathematics he believes they need in order to "appreciate" the mathematics (both in a cognitive and in an affective sense). On the other side, he has a prescribed curriculum, a textbook, topics he is expected to "cover". He feels a strong responsibility as a mathematics instructor, seeing himself as being charged with "delivering the content" (to use his own words). He does not question the validity of this norm in the context of the MFT course, although he could.

From Bob's perspective, the situation could be improved if the students were stronger coming in. He also would like to see students be required to take two courses instead of one. This would take pressure off the demands of the single MFT course. Neither of these remedies is within his immediate power to adopt. So he manages the best he can, trying to meet the needs of the students and the demands of the institution, never completely satisfied with the outcome.

Similar to Bob, Maria expresses a strong intention to improve students' mathematical understanding. She reiterates several times that the MFT course is a "content course". However, as pointed out in subsection 6.1.1, Maria's description of her primary emphasis on teaching the mathematics is permeated with indications that she is

experiencing second thoughts about her priorities. She used the past tense in describing her goals, even though she was teaching the course in the semester in which the interview took place.

Maria was a first-time instructor of the course at the time of the interview, and was surprised by the needs of her students, not only their weak mathematics skills, but their mathematics anxiety and the barrier to learning it presented.

So my goal was, primarily, sort of more content, and I [...] knew that there would be some issues of, let's describe it as "math phobia" or anxiety, with math. I just [was] still surprised to see it so strong at this level, that it overrides their learning, that it blocks their learning! That's what I discovered, and it surprised me that it would be this strong.

She went into the course expecting that she would be teaching mathematics and would need to deal with some math anxiety, but at some point she realised that, at least for some of her students, the affective issues would need to be addressed *before* they could learn the mathematics. Maria commented that she believed she had lost about a third of her students, and was not sure how to get them back on track. She described her struggling students:

For this group of students at this point, content? Forget it. I need an attitude change. I need [their] perception of math to change. And I can't reach it anymore. It was very high, you know, it was a good high in the beginning of the course, because of what I did, free, sort of, problem-solving, open discussion, everybody let's just... [there was a] fuzzy, cosy atmosphere. But the topic does get difficult, yeah?

Maria feels as if she has missed an opportunity. For this particular group of students, she does not believe it will be possible for them to make progress in learning the content without an attitude change, and this change is not possible to attain "anymore". She speaks nostalgically about a time at the beginning of her course where her approach was different: there was "open discussion", "free" problem solving, and a friendly atmosphere. She takes responsibility for the positive feelings at the beginning of the course; it was good because of "what [she] did", but something changed; her approach

changed, and in this excerpt the reason for the change is the “topic”, i.e. the mathematics, which gets difficult. Her interjection, “yeah”, with the interrogative tone, positions me as an insider, a fellow teacher of the MFT course who knows the nature of the course content. She expects me to know, as I do, that she will leave exploratory topics about patterns, reasoning, and critical thinking, and move on to discuss properties of operations and rational numbers, topics students can find abstract and difficult.

Again, this raises the spectre of the course syllabus. Maria is torn between what she feels she *should* be delivering—the mathematics content—and what her students *need*. Like Bob, she feels a responsibility to cover the listed topics. But there is an additional consideration for Maria that adds to her tension; it is a perception that the MFT course has the potential, if not the responsibility to act as a filter. Her comments with respect to the importance of deep mathematics content knowledge for mathematics teachers (see subsection 6.1.1), reveal a strong commitment to ensuring that she does her part in the preparation of future elementary teachers. She does not want to let them move on to become teachers if their mathematics skills are too weak. But this begins to touch on a systemic tension, discussed in subsection 7.3.2 below.

As well, Maria’s tension between her desire to “cover” the content, as well as to build conceptual understanding and address her students’ affective needs, becomes a tension with respect to teaching methods. Her efforts to complete the course content compel her to reduce in-class activities, such as open discussions of readings and problem-solving sessions, methods that she believes are effective. Time constraints are an aggravating factor here. Early in the term, she allowed time at the beginning of lectures to discuss material the students had pre-read, but students had difficulty keeping up with the readings, and so this fell by the wayside. At the early stages of the course, she felt that she and her students were “walking together”; by the time of the interview, she feels she is “pushing them”. Covering the material is important to her, but it troubles her that she is leaving students behind. Those students still suffer from negative attitudes to math and continue to have weak skills.

Maria is far from resigned to living with this tension. She is still seeking to understand her students better and find methods that will be more effective for them, to find a way to change their attitudes so that the mathematics can be learned. Cutting content was not considered by Maria, unlike for Simon, as will be seen in subsection 8.2.1.

In contrast to Bob and Maria, Alice was less concerned about building mathematics knowledge and much more concerned about affect. Yet this also creates tension for her. This tension is not evident early in the interview. She does complain, “I don’t know how to ease their anxiety”, however she goes on to describe the many ways that she tries to address it, and to build their confidence. She strives for a very relaxed classroom atmosphere where questions and student interaction are encouraged. She claims never to laugh at any student work or comments, or even to ever declare “that’s wrong”. When someone proposes a solution, the class as a whole investigates its viability. She tries to allow the students to discover the merits or flaws in the arguments on their own, which can cause some tension in the moment as occasionally students will lead each other “further into darkness”.

However the main tensions do not emerge until she responds to the question of whether her students are ready to go on to be teachers.

That’s a very good question. That, that’s a very deep question. Because we don’t teach so much math in that class, you know. We don’t drill them on whether they can do those fractions. We kind of believe they have the elementary math, that’s how we let them in [...]. But how much above it should they be? You see they always say that you should be significantly above what you want to teach, because then you have the big picture, you see the troubles and all that. I don’t know that much about that. [...] Many at least will not be afraid to go for it. But I still think there are people who will be afraid. I still think I let people go in there being afraid.

She goes on to comment that those who are still afraid will likely avoid the mathematics as much as possible in their future classrooms, though they may be “wonderful at some other subjects”, and laments the fact that there are not specialist mathematics teachers at the elementary school level.

She begins with the admission that improving students’ mathematics proficiency is not a major objective in her course. This is followed by a justification that students are presumed to come into the course with sufficient mathematics skills, however the hedge “we kind of believe” and other comments in her interview suggest that she realises those

skills are often lacking. She then considers that even if they could do the arithmetic, that perhaps that would not be enough, that teachers of mathematics should have a deeper understanding of the subject they are teaching.²⁸ She even provides reasons for why this deeper understanding might be helpful, but then quickly dismisses this as education theory, something she is not an expert in. She looks to her goal of improving attitudes next, to see if “at least” her students will no longer be afraid of mathematics, but sadly admits that even in this respect, some of her students are not ready.

A careful parsing of this passage reveals some of the different forces contributing to the tensions that Alice operates under. As she thinks aloud, her pronoun use changes from “we” to “they” to “I”. “We” likely represents her institution as she describes what does not happen in the course: there is not much math and no skill drill. Even if she disagrees, the objectives for the course are set by her institution. In the phrase “they always say...”, the “they” seems to point to education experts, or at least to those who have an informed opinion, but she disassociates herself from this group, switching to the pronoun “I”, and denying any expertise in deciding what students need. Ultimately, responsibility for the content and objectives of the course is deferred to others, her institution and/or the “community”.²⁹

Alice believes that the goals for improving students’ attitudes and diminishing their anxiety are important, and this fits well with the course as it is set up at her institution. She does not express the same concern as the others with respect to “covering” the course content, which likely reflects a local institutional emphasis on the affective. But there is still a tension here as she contemplates what she achieves with the course, and what future teachers might need both in terms of mathematics understanding, which she does not address to a great extent, and attitudes towards mathematics, which she tries to address, but does not entirely succeed in. She deals with this tension by deferring authority for deciding these priorities to others at her institution, and suggesting that deficiencies may need to be addressed at the systemic level, with specialist teachers.

Bob, Maria and Alice experience the tensions differently, but all struggle with finding the balance between building mathematics proficiency and fostering positive attitudes, within the parameters set for the course. For Bob, his affective aims are

²⁸ This excerpt also illustrates a second tension that will be discussed in the following section.

²⁹ This is related to the systemic tensions to be discussed in section 7.3.

sabotaged by an emphasis on content that is too much for his students to absorb given their skills coming into the course. He opts in favour of covering the course content, fulfilling his obligation as a post-secondary mathematics instructor, even though this means the students leave the course far less excited about mathematics than he would like. Maria, too, sticks to the course curriculum, even though students are left behind, largely to try to ensure that students who do not have a certain level of understanding will not become teachers before they are ready. For Alice, whose emphasis is already primarily on the affective, there is an uneasiness that what her course provides may not be enough to meet either one of her students' affective or cognitive needs, at least for some of her students.

Not all instructors appeared to experience this tension to the same degree. Sometimes, although clearly not in the case of Bob, those who see a very close connection between the cognitive and the affective are less troubled by their decisions to emphasise one of these aspects over the other. For example, Andrew focuses on building mathematics understanding and proficiency, and believes that from this enjoyment and appreciation will follow. For instructors like Harriet, improved confidence and attitudes will ultimately lead to better mathematics skills, even if they are not fully attained during the course itself. This is not to say these instructors do not experience this tension with respect to course priorities at all, but rather is suggestive of a state of mind that may help instructors live with it more easily.

7.2.2 Setting standards

Alice's excerpt above provides an example of instructors' tensions around trying to balance cognitive and affective goals for the MFT students, but it also reveals a second major tension, one that relates to uncertainties about the level of mathematics that should be the target of an MFT course. As described, Alice struggles with the notion that teachers of mathematics should have knowledge that is "significantly above what you want to teach", especially when she considers this in light of the (lack of) skills that many of her students come into the course with, along with the reality that, at her institution they "don't teach so much math in that class". There is an uneasiness that the course may not provide the level of mathematics that prospective teachers need, and an uncertainty about exactly what the MFT course is expected to achieve. Alice deals with

the tension by following what others at her institution have done for the MFT course in the past, feeling, as noted above, that she is unqualified to suggest alternatives.

Matthew also experiences this tension around the nature of the mathematics that he should be teaching, although he deals with it differently. As cited previously in section 6.1.3, he expressed doubts about what the MFT course should entail, considering that it is “not meant to be a remedial course...although it’s probably been treated that way over the years”. This suggests that there were some pressures (discussed further below), perhaps not too strong, for him to keep the mathematics content of the course on the level of school mathematics. This excerpt is completed by the comment, “I had no idea what to do”, clearly showing his uncertainty. Matthew’s further musings on how to set up his course add to this:

What’s the content of this course look like? Do I just make a list of all the things that are taught in grade 6 and touch on each of them? At some level I felt, you know, I have no idea if I’m right or wrong, depth was better than, than, just, sort of, lightly dancing over everything.

He decides to aim for depth of content knowledge rather than breadth, but his language choices reveal some tension in his decision. He qualifies his belief that depth is preferable to breadth, noting that he feels this “at some level”. He is not certain and states this clearly, commenting, “I have no idea if I’m right or wrong”. He is not aware of it, but his instincts are exactly in line with the recent recommendations of the Conference Board of the Mathematical Sciences (2010) which assert that “the quality of mathematical preparation is more important than the quantity” (p. 7).

While Alice relies on her colleagues who have taught the course before to advise her on the type and level of mathematics that she should focus on, Matthew is much more self-reliant. He uses the pronoun “I” frequently in his description of how his course came together, clearly positioning himself as the one responsible for deciding on the course objectives and content within the loose parameters provided to him. In other excerpts, he also refers to the textbook as providing guidance.

His doubts about whether or not he has chosen the right course are likely aggravated because Matthew is the *only* current instructor of the MFT course at his institution. He did receive some advice from others who had had experience with the

course, but he related that, relative to his experience with other mathematics courses, he felt quite isolated while teaching the MFT course:

This is the most..., you know, it's the first time I've felt disconnected from other mathematicians, in some sense. And I don't know many people who've taught courses like this yet, and what they do. And the ones I do know, and what I've seen so far, it's all very personal.

His comment that "it's all very personal", alludes to the diversity that exists between different MFT courses offered by different instructors.

Matthew commented that it was his impression that the course is often treated as a remedial course. From the interviews with the instructors, it appears that the pressure to keep the course at the level of school mathematics may come from instructor perceptions of the weak skills of their students. Several instructors commented that these weak skills make the course harder for the students than it needs to be. Even instructors who firmly believe that students should leave the course with a deep understanding of the mathematics find their aspirations to be in tension with what the students are able to handle. As Bob notes:

It's unfortunate that this is the case in some ways, that is, that they don't come into the course with enough technical skills to really focus on some of the higher-end thinking, to be able to appreciate some of the more subtle parts of the course.

For Bob, these "more subtle parts of the course" are, without question, part of the course. He does not doubt that they should be there, but he is disappointed with his students' ability to grasp them.

Brooklyn also comments on this in the context of thinking about what she would like to change about the course if she could:

I think that if everybody in the class could do all fraction operations, all operations with ease and accuracy, then we wouldn't have to deal with the mechanics of those things, and focus more on the concepts, and the understanding.

Compounding the tension is the difficulty posed by the wide variety of skill levels the students present. While some students may be ready for the deeper mathematics, others are not. Karen comments the hardest part of teaching the MFT course is managing the different levels of abilities of the students. She observes, “There are some students who are really good, and some students who are so fragile.” Unless instructors can find strategies (such as using manipulatives as described in subsection 6.4.3) that allow students to fill in the gaps in their school mathematics knowledge at the same time as they build a deeper understanding of the concepts, their options are to hold back and focus on the remedial, or to push forward, leaving students behind.

This tension around setting the standards for the level of mathematics taught in the MFT course is a complex one. It became evident as instructors discussed making choices about content (Matthew), or considered whether the course is providing students with the mathematics background they will need to teach one day (Alice). It also appeared as instructors deliberated on student success in the course (Bob and Brooklyn). Many expressed the opinion that teachers of mathematics should have a deep understanding of the mathematics concepts, but there were expressions of doubt about their own ability to judge how deep it needs to be. A desire to have good teachers of mathematics in the schools pulls in the direction of keeping standards high, but the students’ immediate remedial needs (and concerns about affect discussed earlier) pull in the opposite direction. This is further aggravated by beliefs that skills need to be mastered *before* deep conceptual understanding can be developed.

How acutely instructors feel this tension also depends on how they perceive the role of the MFT course in the preparation of teachers. This will be discussed further in the following section.

7.3 Systemic tensions

While the tensions examined to this point have been immanent, part of the instructors’ local experiences, I shift now to considerations of the transcendent (Wagner & Herbel-Eisenmann, 2009). Although the instructors have great freedom in making individual decisions for their own MFT courses, as reflected in the diversity of their approaches, their decisions are informed by their perceptions of the broader context. More specifically, this broader context includes how instructors perceive the role of the MFT course within the larger system whose object is the preparation of elementary

school teachers. When viewed with a wider scope, both the diversity described in Chapter 6 and many of the tensions discussed to this point are indicative of uncertainties with respect to this role, reflecting challenges noted in the review of the literature (see section 2.3). I suggest here that these uncertainties can be seen to point to tensions within the system itself, in particular, tensions related to the extent to which pedagogy is/should be part of the MFT course, and also to questions of the level of mathematics prospective teachers should have both on entering and on leaving the course. Each of these will be discussed in turn.

7.3.1 Content vs. pedagogy

In subsections 5.1.1 and 5.2.4, it was noted that instructors must make decisions with respect to the extent to which they incorporate discussion of pedagogical issues into their MFT course. This leads to diversity in their courses, as described in section 6.4, but does not lead to serious tensions at the local level. For the most part, the instructors in this study seemed to feel comfortable with the level of pedagogy and/or connections to the elementary classroom that they bring into their individual courses. Pedagogy is assumed to be the primary responsibility of those teaching “methods” courses within education programmes that students will take later, although several instructors commented that they were unsure of exactly what these entail, and were concerned to learn that students might avoid taking them on their way to becoming teachers.

At the same time, the extent to which many of the instructors *did* incorporate discussions of pedagogy and activities particularly aimed at teacher preparation shows that description of the MFT course as exclusively a “content course” is an oversimplification. Although instructors do not feel obligated by the syllabus to make these connections, there are other factors that lead them to do so. An examination of an excerpt from Andrew’s interview offers a point of entry for discussion of the systemic tension underlying this phenomenon.

In contrast to the other instructors, Andrew explicitly described finding the balance between content and pedagogy as a dilemma:

I tended not to talk so much about pedagogical issues. [...] I was open to suggestions, and I wasn’t uninterested in such issues, but I didn’t feel any kind of real competence giving too much of my opinion about that kind of thing. [...] I think the course should be

a balance of those things, ideally. Well, I don't know, I mean, maybe later on they're going to take courses in education, when they do their degrees that will help them with that anyway, so it might be that it wasn't such a grave omission, it would come later. But in some ways, it would be appropriate to mix the two together in that course. And the great dilemma for me, and not just for me, I think my colleagues, was, "What should be the balance in this course?"

This might have been offered as an example of an internal tension, representing an argument Andrew has with himself about finding the balance between content and pedagogy in the delivery of his MFT course. However, other parts of Andrew's interview and particular word choices in this excerpt suggest otherwise.

Of all of the instructors interviewed, Andrew is the most focussed on the mathematics—there appears to be no real practical dilemma for him. As has been described, he does not believe he has appropriate expertise in pedagogy; he is comfortable with the mathematics and "traditional" modes of instruction, and sticks to what he knows. But as he states in the above excerpt, he has an interest in pedagogical issues. In asking him whether he does anything in his MFT course particularly because his students hope to be teachers one day, as interviewer I may have placed Andrew in the defensive position of considering why he does not.

In the excerpt, we can follow a train of thought that begins with Andrew's consideration of recalled pressures from students to discuss pedagogy. He is "open to [their] suggestions" and is happy to discuss pedagogy informally with them because of his personal interest. "But" he feels the need to qualify this, explaining that he has no "real competence", likely referring to his lack of formal training. His next words, "I think", closely followed by "ideally", shift him into a philosophical, hypothetical mode (position), as he considers what the implications of increased pedagogy in the course might mean. This is interrupted by a brief consideration of the consequences of his potentially "grave omission" of this aspect. He rationalises that even if he does not cover pedagogy in his course, this "maybe" will be addressed "later on". This helps alleviate any momentary tension he may have felt, and he continues to consider the deeper philosophical question of whether the course would be improved by incorporating "a mix" of both content and pedagogy.

His inclusion of his “colleagues” in dealing with the “great dilemma”, positions him not as a sole instructor making decisions for his course, but as part of a community that needs to clarify the role the MFT course plays, and to consider how it can best attain its goals. This raises this dilemma to the level of a systemic tension.

This tension was foreshadowed in the literature in subsection 2.3.1 through the juxtaposition of policy documents that call for preservice teachers to take more mathematics courses from mathematicians (Greenburg & Walsh, 2008; Conference Board of the Mathematical Sciences, 2010) with research that describes content courses that are successful in building content knowledge, mathematics-for-teaching, as well as positive beliefs and values, when they are taught in the context of how children learn (Ball, Lubienski, & Mewborn, 2004). If the courses are to be taught by mathematicians in mathematics departments, the implication is that these instructors will need to be able to relate the mathematics they are teaching to the elementary context.

There was no evidence that any of the instructors had any direct exposure to this literature, and so would not have felt direct pressure from this source to incorporate more pedagogical aspects into their course. Any suggestion that they should do so would be much more subtle. Several of the instructors, including Andrew, commented on the large amount of pedagogy they found in the texts that they used, likely the result of some of the education research described. As well, there may be some influence through informal discussion amongst fellow instructors of the course, both within and across institutions. Some instructors, like Harriet, make relatively strong efforts to bring the elementary school context into their courses, finding that it helps motivate students. Talking about their successes and satisfaction with this approach with other instructors may also contribute to a sense within the system (at least among the BC post-secondary mathematics instructors) that more of this may be beneficial, despite the fact that doing so places new demands on the instructors who may not have had any formal pedagogical training themselves.

Framed in terms of activity theory (Engeström, 1999), this tension around the extent to which the MFT course should address both content and pedagogy can be understood on two levels. Within the system whose activity is simply the teaching of MFT courses, it appears as confusion with respect to the “object” of the activity: what exactly should the goals of the MFT course be? But this system sits within a larger system whose activity is “preparing elementary teachers”. At this level, the tension can

be seen to arise as the “community” seeks to negotiate the “division of labour” amongst those responsible for the mathematics preparation of elementary school teachers: Who should teach the mathematics pedagogy? The mathematics? Can or should they be separated? The literature review in section 2.3 suggests that these questions are far from being fully answered.

As observed, for the most part, instructors in this study live quite comfortably within this tension at the local level. The MFT course is formally defined to be a “content” course and any connections instructors make to the elementary context are seen as beneficial but not required by the system. However, this is a tension that appears likely to be experienced more immediately by instructors as more evidence is found (and communicated) to support the benefits of teachers learning mathematics in the context of children’s learning, and traditional separation of content and pedagogy continues to be challenged.³⁰ The fact that some of the instructors in this study already make connections to pedagogy and the elementary context to a significant extent suggests that with sufficient support, MFT instructors may be able to do more of this if called upon to do so. However, as time considerations are a significant factor in some of the other instructor tensions within the MFT course, any moves to increase the course demands are more likely to aggravate than to alleviate these tensions.

7.3.2 The mathematics preparation of elementary teachers

While simply defining the MFT course as a “content” course helps many instructors sidestep directly dealing with the first systemic tension, no such option is available for this second tension, which centres on the very nature of this “content”. As will be seen, this tension touches all of the instructors in some way.

The instructors interviewed displayed a wide range of emphasis on developing mathematics proficiency in their students, as described in section 6.1, and as seen in section 7.2, even modest goals for deepening students’ understanding of mathematics often seem to be at odds with affective goals and with the weak mathematics skills of many of the MFT students. As observed, how the instructors manage these tensions,

³⁰ Shulman’s (1986) work criticised this separation between content and pedagogy 25 years ago. His impact is without question and is particularly evident in the research literature, but changes within the teacher education system have been much less apparent.

and the extent to which they experience them at all, is affected by how they perceive the role of the MFT course. Furthermore, the contrast in these perceptions is symptomatic of this second systemic tension, namely, questions around how much mathematics elementary school teachers need to have, and where this mathematics should be learned.

Instructor responses to the question of whether or not their students have the mathematics skills to teach once they are finished the course were particularly useful in revealing this tension. In some cases, especially when the answer was in the negative, the question seemed to put instructors in a defensive position, much like Andrew in the previous section, leading them to consider the role that the course plays within the larger context of preparing elementary teachers. This is a complex tension, as will be seen in the variety of instructor comments.

Karen was very blunt in her response to this question of whether her students' mathematics was strong enough for them to teach elementary school when they left her course:

To be honest, I don't know. I think, in a way, my attitude is this: they're going to get there anyway, even if they just got a D in the course. They will get into the program, they'll get out there. I mean, if they're really bad they'll bomb somewhere else along the way. But they're going to be there, so I'm going to do my best to change, mostly their attitude, and they're going to learn some math.

She begins with an expression of uncertainty, confessing, "I don't know", suggesting she feels ill-equipped to judge what level of proficiency prospective teachers might need. There is a possibility that her students are not prepared. Although her uncertainty exists at a local, personal level, she shifts to a broader perspective, to consider how her course is meant to fit into the larger system. She rationalises, downplaying the role of the MFT course: even the weak students will "get there anyway". In Karen's view, unlike for Maria, the MFT course is not meant to (or cannot?) act as a gatekeeper to weed out students who are not ready to teach. She asserts that the worst students will fail at some other point in their career path, but this can only be a hope, as she does not know if they will take any further mathematics. She can only "do her best", to address mainly

attitudes, and to improve their mathematics skills even if she cannot bring them to the level they may need—it will be the responsibility of others to determine whether or not they are ready mathematically (and otherwise).

Al harbours no illusions about the level of mathematics his students leave the course with, commenting, “I don’t have any expectations of turning them into competent mathematicians”. He adds, “they won’t really learn the stuff until they teach it”. This last comment removes the burden of responsibility on the MFT course, and hence from him as the instructor, in ensuring the mathematical preparedness of those who complete the course. He adjusts his goals for the course to what he believes can be managed given the abilities of his students and the time he has available, managing any tension he might feel with respect to the amount of mathematics he covers in his course. Recall his major emphasis is on changing their conceptions of mathematics.

Despite this, it is apparent that Al is not satisfied with the system as a whole:

I think the requirement of a one-semester content course, as part of a preparation, is woefully inadequate. And as long as that’s all that is going to be required we can’t expect to really improve their mathematical competence. I’m not convinced that we can do much with such little resources.

Taken in light of this comment, Al’s previous moves to defer the responsibility for ensuring the mathematical competence of future teachers appear to be more an effort to manage his tensions around identifying and meeting the cognitive needs of his students than a reflection of how he feels the system should operate.

He identifies his perception of the intended role of the MFT course as a “requirement” that is “*part* of a preparation” for elementary teaching. He positions this requirement as being imposed from outside, beyond his control, which is confirmed by his use of the passive voice in the next sentence. Despite his mention that it is only meant to form “part” of the preparation, he sees it as “woefully inadequate”. He describes the MFT course as a “one-semester content course”, with his emphasis on the short duration of the course pointing towards time as a significant barrier to the course achieving its purpose. Note that he is clear that the object of the course is “content” preparation. With his use of the word “we” in the second sentence, he positions himself and me, within the larger community of MFT instructors who are faced with an

unreasonable task: “We can’t really be expected to improve their mathematical competence”, given the “little resources”. These resources definitely include time resources, but may also include students’ mathematical backgrounds, and perhaps even instructor expertise, though neither of these was explicitly mentioned by AI.

From AI we have an interesting juxtaposition of two different views: there is the suggestion that future elementary teachers need more mathematics than can be provided in a single MFT course, which carries the implication that more courses should be required, alongside the opinion that any deficiencies that they may have are not problematic since they can simply learn what they need as they need it when they teach. These reflect two conflicting perspectives within this tension.

Another nuance in this tension is offered by the instructors who are relatively happy with the mathematics knowledge that their students leave with, especially given that the level of mathematics their students ultimately will be teaching is quite low, and of course, given that a mathematics methods course would follow. This view is expressed by Brooklyn, who observes:

I think they’ve got, by the time they come out, well, I don’t know if they’ve got a complete understanding, but they’ve certainly got a much deeper understanding of math than they did going in. And so, you hope that by the time they take their methods course later on, that they will be able to apply those methods they’ve learned, and remember the concepts that I’ve taught!

In this excerpt, Brooklyn is content that her students improve in their mathematics understanding in her MFT course. Even though their depth of knowledge may not yet be sufficient, she takes comfort in knowing that more time and another methods course still stand between her students and the elementary classroom.

When Brooklyn learned from me, during the interview, that students may opt to not take a mathematics methods course during their teacher-training program, she took a moment to reconsider, but then reconfirmed that “given what’s being taught in the elementary curriculum, I think they probably have the [mathematics] skills”.

Brooklyn’s use of the hedging word “probably” suggests some uncertainty, but Brooklyn’s conviction that her students will be successful teachers is reinforced in Brooklyn’s case by the large number of students that she has seen go on to become

successful elementary school teachers during her many years of teaching the MFT course. Somehow, although the system is not perfect, they manage.

Brooklyn's proviso that her students are likely prepared "given what's being taught in the elementary classroom" suggests a view that elementary school arithmetic is elementary, and as a result, a relatively low level of mathematics knowledge will suffice for elementary school teachers. This is also reflected in one of Andrew's comments: "I think many of them would have been good elementary school teachers, you know, notwithstanding their difficulties with that course". When pressed on this he observes, "I think in terms of mathematics, some of them were ready before they got into the class".

Alongside this view that not much mathematics is needed by elementary school teachers, particularly at the lower grades, is the stereotypical view that those who are stronger in mathematics are often not strong socially, and may have difficulty relating to or communicating mathematics to children. This comes out in Simon's reply to the question of whether or not his students are ready to teach:

It depends on what grade. You know? [Long pause] I don't know. Who, who would I like...? I could see some of them doing alright. I could see some of them doing alright, for sure. You know, you don't have to be an expert to teach it at that grade, but obviously, it would be nicer if you were. But much more important than that is how you are at teaching with kids, you know? I mean, I could have a graduate student in mathematics in number theory go in and teach, you know, grade 5, grade 6, I'm sorry, kindergarten or grade 1 or grade 2, and it could be a huge disaster. So I think there are many of them who would do fine, who have enough confidence, enough enthusiasm, enough interest, and they're not perfect, but they would do fine. I'd rather that I got them for three semesters, like I said, but I don't even know if even after three semesters they'd be ready! Some of them may just not ever be ready to teach math.

The tension is evident in this excerpt as Simon tries to decide whether his students will be effective teachers. He begins with a declaration that the grade level is a factor, with the implication that for lower grades, lower levels of mathematics proficiency are

needed. But he is not sure. His incomplete query “Who would I like...?” and the phrase “I could see”, give the impression that Simon is considering each individual student in his class, imagining each in the role of elementary school teacher. Some clearly fit the role in his mind, as he concludes that some would be alright “for sure”. But now the dilemma starts to emerge as he attempts to defend this position. He offers the view that math expertise would be an advantage, but is not essential at the lower grades. More important is confidence, enthusiasm, and an ability to relate to children. He considers the alternative, accessing the stereotype, choosing a graduate student in number theory as his example of someone who would know the mathematics well, but “could be a huge disaster”³¹. But despite this, even though many of his students would “do fine”, that is, they would be adequate to the task, he feels that it would be better if they had stronger backgrounds and abilities—he would like them to take three MFT courses! The final sentence confirms the importance of at least some minimal proficiency, but also expresses a broader concern: “Some of them may just not ever be ready to teach math.” Coming after his wish that future teachers be required to take more mathematics, the reference to being “ready” is a reference to their mathematics knowledge. The notion that despite more courses some will “not ever be ready” is a pessimistic one, but is also a consideration whose implications would need to be taken into account within the larger system.

The views expressed by Brooklyn and Simon offer insight into the current minimal mathematics requirements for elementary teachers in the province. Perceptions that only a basic level of math is needed to teach “elementary” mathematics, and concerns that many who have personal traits, like compassion and enthusiasm, that suit them well to teaching children, might be excluded from the profession, support keeping the mathematics requirements low.

At the same time, as mentioned in the literature review in Chapter 2, the last few decades have seen increased concern with respect to school teachers’ mathematics knowledge and the effect this has on pupils’ learning of and appreciation for mathematics (L. Ma, 1999; Ball, Lubienski, & Mewborn, 2004). This has led to the policy documents, mentioned in the previous section, which advocate for increased

³¹ Simon’s correction, from grades 5 and 6 down to primary grades, is interesting. He may simply be trying to increase the distance between the level of the teacher and the level of the pupils to make a stronger case, or he may feel that the graduate student might have less trouble with the intermediate pupils.

mathematics education for prospective teachers. The Conference Board (2010) calls for “at least 9 semester-hours on fundamental ideas of elementary school mathematics” (p. 8), which translates to the equivalent of three MFT courses (as suggested by Simon above). Teachers of middle grades and high school are recommended to have even more, with the likely effect that elementary school students would have specialist teachers from the middle grades on.

The option of having specialist teachers for mathematics, even at the early grades is another position within this tension. Several of the instructors, including Alice, Andrew, Simon, and Matthew advocated this approach, sympathising with the challenge presented by the demands on elementary school teachers to be generalists. Andrew also articulated the concern that universal requirements for deeper mathematics knowledge for elementary teachers might preclude some people who would otherwise be well suited to working with young children from becoming teachers. Having specialists, even at the early grades would avoid this.

Other instructors, including Bob and Simon, shared Al's view that at least a second mathematics course should be required. Harriet was also among these instructors, but unlike Al, she did not see the requirement for only one content course as “woefully inadequate”. She sees the MFT course as a good beginning, but sees the potential to take her students further with more time:

I would like them to be further. [...] The people who are confident, who are good at this level, need another course. [...] They need a lot more so that they'd have the confidence to do things like math contests and math fairs, and math enrichment programmes for the kids who love this stuff. [...] So we definitely need more for those people, and we need more for the people who are only now starting to feel a bit more confident and comfortable with mathematical ideas.

For the students who are already strong, the additional course would prepare them to enrich mathematics-learning experiences for their future pupils. As well, those who have just made their first steps towards building confidence and competence need more time to solidify their learning.

The instructors who would like prospective elementary teachers to take more mathematics courses all indicate that the biggest barrier to this is that students are not required to take more mathematics by the system. At some of the institutions, a second MFT course is available, but very few students take advantage of it. As the authors of the Conference Board recommendations are most surely aware, there is a reluctance to add more coursework to the already large demands on elementary teachers, who are expected to be competent in all elementary subject areas. To this end, these policy documents are aimed at the policy makers and program designers in an effort to lobby for greater attention to mathematics in the preparation of teachers, to find a way despite the other demands.

As noted, this tension is a complex one. It manifests at the local level with instructor uncertainties about the level of mathematics proficiency they should strive to develop in their students (as discussed in subsection 7.2.2), but is indicative of these same uncertainties within the larger community. More particularly, what role is the MFT course supposed to play in this preparation? We see a range of opinions, even within this small group of instructors. Some see the course as a gatekeeper, while others presume that later courses will take on that role. For some, more coursework should be required to bring future teachers up to the level they “need”, while for others the learning will take place as they need it at some point in the future. For some, the expectation that all elementary teachers can be effective teachers of mathematics is unrealistic and the best solution is to have specialist mathematics teachers even in elementary schools. The education literature and, notably, the policy documents mentioned, advocate increasing the mathematics preparedness of elementary teachers, but pushing against this is the reality that some members of the community, including some of the instructors themselves, do not believe that there is a problem with the current levels.

From an activity theory perspective, this tension, just as the previous one, can be situated as a debate within the “community” over “division of labour” in the sense that there is uncertainty over where different levels of mathematics proficiency are to be attained by prospective teachers, i.e. at school, in MFT courses, or later in their careers when they are teaching. However, an additional component of the activity triangle is also implicated—the “rules” for adjudicating readiness to teach mathematics are open to question. Also, once again, for the local activity system of teaching MFT courses, the “object” of MFT courses with respect to the level of mathematics is not well-defined. In

order to act within all of this uncertainty, instructors must rely on their own best interpretation of the role of the MFT course, resulting in diversity and often contributing to their tensions.

7.4 Summary

During analysis of the interviews, tensions in the instructors' experience of teaching MFT courses were readily identified, and within this chapter, six of these have been unpacked. The first two were described as personal tensions. These are directly related to instructors' perceptions of themselves as teachers and as mathematicians, emerging as dilemmas with respect to classroom methods in the first instance, and to conflict with students in promoting particular mathematical dispositions and norms, in the second. The next two were classified as internal tensions, namely struggles that instructors have within themselves in trying, first, to decide on cognitive vs. affective priorities for the course, and second, to set appropriate mathematical standards for the course that meet both the perceived current needs of their students and their anticipated future needs as teachers. The final two tensions described were systemic tensions, tensions that are felt by the instructors, but less immediately, as they are situated not at the local level of MFT classrooms, but at the level of policy. These are tensions that exist because of uncertainties within the larger community with respect to where prospective elementary teachers will acquire mathematics knowledge, pedagogical knowledge, and particularly, pedagogical content knowledge, and also with respect to defining just how much and what kind(s) of mathematics elementary teachers need to have.

These tensions are not disjoint. Especially evident is the influence of systemic tensions on instructors' internal and personal tensions. Instructors are participants in the community that is wrestling with the larger questions of the preparation of teachers; the conceptions of the role of MFT courses that they construct for themselves as members of this community shape how they see the course, and in turn affect how they set their priorities and standards. The extent of the diversity between the instructors' offerings of the MFT course, as described in Chapter 6, gives a measure of the magnitude of these systemic tensions, with greater diversity reflecting greater uncertainty within the community.

Numerous additional factors that play a part in the instructor tensions, often at multiple levels, were also identified, including instructor personalities and preferences, as well as beliefs about themselves, their students, and the teaching and learning of mathematics. A perception of a shortage of time to cover the course content, alongside a post-secondary mathematics instructional norm, the expectation that instructors will “cover the content”, aggravated the tensions in some cases, by operating against instructor desires to meet the perceived cognitive and affective needs of students.

Within the description of tensions, a number of strategies for managing them could also be discerned. Some examples included instructors staying with what they are most comfortable with, resting in tradition (as in the case of Andrew), staying true to perceived institutional expectations and norms (Bob), deferring to experts (Alice), relying on personal experience (Matthew), resigning oneself to not being able “to do it all” (Al, Karen and Harriet), trusting the system (Brooklyn), continuing to experiment (Maria), and more generally “doing one’s best” (a common theme).

The names attached to these tension management strategies merely point to examples for illustrative purposes—these strategies are not exclusive to these individuals and the intention here is only to provide a glimpse into the range of possibilities, not to characterise the instructors. It is also important to note the distinction between “managing” tensions and “resolving” them. In all of the examples, the instructors continue to be unhappy with the status quo. These management strategies merely allow them to continue to act, to continue to teach their MFT courses, despite the challenges and uncertainties.

In discussing the systemic tensions, instructors offered a number of strong opinions on how the system for preparing elementary teachers could be improved (e.g. more math courses, higher prerequisites for MFT courses, early education mathematics specialists), but did not position themselves as having any power to make these changes happen. As confirmed by my own experience within our provincial system, responsibility for instigating change is passed up the line, with the post-secondary instructors looking to the universities who offer teacher certification programs to set appropriate mathematics prerequisites. At the universities, the mathematicians look to their education colleagues, who in turn, look to the College of Teachers, which ultimately sets the certification standards. In the meantime, on a local level, the instructors live

with the systemic tensions, doing the best they can to prepare their students mathematically to the extent they can in the time that they have with them.

With some of the major tensions, aggravating factors, and general strategies for managing them laid out, I turn now to take a closer look at the experience of one particular instructor in this study. The case of Simon will allow for a deeper appreciation of how these tensions are lived over time, and will augment our understanding of contributing factors and management strategies.

8: SIMON'S EXPERIENCE

As described in Chapter 4, one of the phases of my research for this dissertation involved multiple interviews with one of the ten instructors who participated in the “main study”: I spoke with Simon several times during his first semester teaching MFT, and then conducted two further interviews, one a slightly modified version of the interview used in the main study (see Appendix B), done right after he had completed the course for the first time, and a follow-up interview with him a year later. These data, collected over a period of 16 months, permit a more detailed look at some of the tensions as experienced by Simon, providing insight into the factors that contribute to them, and giving a sense of their persistence over time.

In this chapter, a brief description of Simon's ideals and intentions for the MFT course sets the stage for an analysis of some of the tensions he experienced during his first offering of the course. In terms of the levels of tensions defined in Chapter 7, those examined here are primarily internal tensions, arguments Simon has with himself, particularly with respect to his course priorities and methods of instruction. This is followed by a look at how Simon continues to live with these a year later, as revealed in his follow-up interview.

For convenience, I will refer to the nine conversations we had throughout the time Simon was teaching the MFT course as “the conversations”. These were not recorded and therefore, by necessity, all excerpts cited are paraphrased.

8.1 Ideals and intentions

During our first conversation, before he met his first MFT class, Simon spoke about his plans for the course, his concerns, his preparation, and his expectations for what would be different. This brought out some of the rationale he was using to make decisions about course priorities and methods of instruction. But this was not the only time we discussed his ideals and intentions. Because of the nature of our conversations and interviews, Simon often spoke in a hypothetical mode, talking about what he hoped to see happen, or would like to see happen in his MFT course, offering further insight

into his point of view. As a result, the excerpts examined here are taken from the conversations that occurred throughout the term, as well as from the main interview.

8.1.1 Course priorities

In our earliest conversations, Simon seemed most concerned about his students' affective needs and their need to build strong communication skills. The latter was a common theme among the MFT instructors (see subsection 5.3.1), but Simon was more concerned about his students' mathematics anxiety than most. He was preparing himself by reading books about it and planned to incorporate particular activities into his course to address it explicitly. With respect to the course content, he commented on the length of his course syllabus, and expressed the view that there was a "crazy amount of material to cover", but there were no overt signs of tension around course priorities at this stage.

There was also no explicit mention of cognitive goals, but later conversation and actions indicated that, in fact, building students' mathematics proficiency and covering the course content were also high priorities for Simon. Consideration of positioning suggests it is likely that Simon did not articulate the importance that he placed on these aspects during our first conversations because he took it for granted, and assumed that I, positioned as a fellow instructor of the course, would know that teaching the students mathematics and completing the course were important. This is supported by a comment made by Simon about halfway through his first offering. In the context of discussing the makeup of an upcoming midterm, he said, "[paraphrase] I need to make sure that they're competent in elementary school arithmetic, of course", with the "of course" implying that ensuring arithmetic proficiency "goes without saying". As well, Simon frequently expressed concern over the time pressure he was feeling while teaching the course, indicating that covering the course content was also a serious concern for him, but this will be addressed in more detail below.

With Simon's affective goals being explicitly expressed, and his cognitive goals being present, though largely implicitly, it is difficult to tell which held priority for him. It seemed at times that he believed the students' cognitive skills needed to be developed first though, at least in part, as a means to affective ends. Early on in the course, he mused, "[paraphrase] In an ideal situation, with more time, I would spend a long time working on arithmetic. If the students were quick with numbers they would feel better

about their ability to do math”, expressing the view that good arithmetic skills would lead to stronger math efficacy beliefs, and hence to a greater enjoyment of mathematics.

However, a close look at an excerpt from Simon’s main interview, done at the end of his first offering of the course, shows that his views on this are not so straightforward. In the context of discussing the importance of problem solving in the MFT course, he commented:

For this level of student, I think [problem-solving skills are] more what they’re missing than information. I don’t think they’re missing... You know they may, it’s true they may not know how to add fractions, but I think it would be better to address that in this kind of way, than... to get them engaged, to get them thinking about it, or otherwise it’s just all more confusing rules that they have to memorise, and don’t...never really understood.

Here Simon is thinking about how he would like to configure the course the second time around, and is explaining his reasoning to me, in response to my prompt that incorporating more problem solving is an “interesting idea”. He was in a position of defending his intentions, and in the excerpt, his anticipation of both sides of an argument is evident.

He begins by hingeing his remarks on “this level of student”, a reference that I, as an insider, am expected to understand: the students are generally weak in skills, but not without prior experience. He weighs their deficiencies and decides that their lack of problem-solving skills is more serious than their lack of knowledge. He begins “I don’t think they’re missing...”, perhaps about to suggest that they are not missing too much in terms of mathematics skills, but stops himself, correcting in response to an anticipated objection (and what he knows), that it is in fact “true” that they are lacking in some fundamental skills.

Despite this, he offers his opinion (“I think”), that a focus on problem solving would be “better”, but at this point there is a subtle shift. He advocates problem solving, not solely for its own sake, but also as a way “to address *that*”, where the “that” is the students’ lack of mathematics skills. The advantage offered by a problem-solving approach is that it will “engage” the students with the mathematics. The alternative evoked is a strict focus on content, which Simon believes his students see as “confusing

rules that they have to memorise”. At the very end, he switches tense, initially saying students “don’t” understand the rules, but changing this to “never really understood”. This suggests the view that his students’ previous mathematics experiences have for the most part been of this confusing, arbitrary nature, and are likely responsible for their current lack of skills, as well as their fears and negative attitudes.

This appears to be a departure from his earlier comment, which suggested that working on arithmetic skills is the key to improving student attitudes to mathematics. He comes close, though does not quite, dismiss his students’ need for arithmetic proficiency, but argues that the development of problem-solving skills is *more* important. This is also, at first glance, a cognitive aim, and Simon does note that the problem solving will support arithmetic development too, but what seems to make it so appealing to Simon is that it addresses affect at the same time. He has found that through this approach, students are more engaged with the mathematics.

Through the conversations and the interviews, Simon expressed goals for the course that included both affective aims (particularly addressing mathematics anxiety), and cognitive aims (building arithmetic proficiency), as well as covering the course syllabus. There is no inherent tension in simultaneously having these goals. Tensions only begin to emerge when choices need to be made that lead to promoting one of them at the expense of any of the others. It is at this point that personal beliefs about the relationships between these goals, and their relative importance, becomes a factor. These tensions will be examined following a description of his intended teaching methods, which were also ultimately affected by his course goals and priorities.

8.1.2 In the classroom

With respect to teaching methods, right from the outset Simon was concerned about how he would deliver the MFT course. During our first conversation, he talked about trying to decide how much lecturing and how much hands-on work he would incorporate. He felt that the course “[paraphrase] should be different from other math courses”, anticipating that there would be a need for more group work, and described searching for appropriate in-class activities that would, ideally, be linked to particular course topics. His ideal would be “[paraphrase] to do an activity each class”.

It is not certain where the notion that the MFT course “should be different” originally came from. It is not uncommon, and was most apparent in instructors’

decisions to employ different teaching approaches and evaluation methods from what they did in their other mathematics courses. Andrew struggled with this explicitly (see subsection 7.1.1) and most of the other instructors interviewed reported incorporating more hands-on activities into MFT than they did otherwise. It is possible that Simon's original motivation to do this came from speaking with other MFT instructors, or from a desire to model teaching methods that these students might use in the future as teachers. It is also possible that it came as a consequence of particular course goals.

It is likely that this latter point was a factor for Simon, although he did not articulate it until after the course was completed, so it is not clear whether this was an original motivation, or if it became a rationale after the fact. Using "hands-on activities", and particularly group problem solving, was an intended method of instruction for Simon right from the beginning, and his commitment to it was voiced even more strongly at the end of his first offering of the course. It was at this point that he articulated his ideal for his next offering, "to focus on trying to really understand a few interesting problems and to work on their problem-solving", since "the issue is actually trying to get them comfortable and able to address math and to do some problem solving"³². Group work, in the form of solving problems together in class, was positioned as a means to an end, a way to meet his cognitive and affective goals.

There is only a hint of personal tensions in our early conversations in the notion that Simon will need to find a "balance" between lecturing and hands-on activities. There is no actual conflict within his ideals or intentions with respect to the classroom approach he would like to take with the MFT course, as these are closely related to his goals for the course. However, as will be seen, the tensions arose during his first offering, as he tried to put his intentions into practice.

8.2 Tensions arising

In Simon's case, the tensions he experiences are most visible when his stresses and reported actions during the course are considered in juxtaposition with his intentions and ideals. One of the dominant codes in Simon's transcripts was that of "time" (T). Almost every conversation we had during the semester began with Simon expressing anxiety over how far behind he was getting in the course syllabus. This concern about lack of time put Simon in a position of having to make choices about what to change in

³² This excerpt is analysed more closely below.

his original plan. As he explained the changes he was making, the tensions came to light, both with respect to his priorities for the course and the classroom methods he hoped to employ.

8.2.1 Course priorities

Simon had a number of options available to deal with his perception that there was not enough time to complete the course syllabus as planned: he could omit some topics altogether, modify them, or persist with the intended plan and race through the full syllabus. He talked about having to “[paraphrase] make *hard choices* and condense some of the material” (my emphasis), and in the end, along with making some modifications, he did end up omitting some material, skipping arithmetic algorithms with other bases, most of the algebra, and much of the geometry he would have liked to cover.³³

Simon seemed to be making these changes in order to preserve his goals for student affect. When he started to think it might be necessary to make these cuts, he reasoned:

[paraphrase] I am trying to make the course as pleasant as I can for them, so that they’ll want to take the next course. *Sacrificing* content may be necessary, if it will mean that they will want to take more math. (my emphasis)

His comment provides insight into a motive for focusing on affective goals in the MFT course. Simon is aware that overloading his students with too much content will make the course a stressful experience, aggravating their anxieties, and reinforcing previous negative experiences with mathematics. In contrast, having a positive experience with mathematics may motivate students to take more of it, a consequence that is understood between us as desirable.

It might appear from this that, unlike other instructors who exhibited a strong commitment to completing the course content (see subsection 7.2.1), Simon was happy to make cuts in order to ensure students had a positive experience. However, his reference to making “hard choices”, and use of the strong word “sacrificing” suggest that

³³ Most of the time in the course was spent on number and operations.

Simon found this difficult³⁴. At the time of the above excerpt, about one month into the course, he was just beginning to consider the changes he might need to make—the modal verb “may” suggests that he was still hopeful that cuts would not be necessary, although this was not to be the case.

His ambivalence is well captured in the following excerpt that arose during his main interview at the end of the course, particularly in the first sentence:

The content in a way is important, but I don't know that the content is really. For example, they can probably use a Finite Math course for the same purpose, you know? Obviously, the content is not the issue here. The issue is actually trying to get them comfortable and able to address math and to do some problem solving.

It is not clear “in [what] way” the content is important, although I might guess that Simon is referring to the generally assumed need for elementary teachers to know the mathematics that they are teaching. However, he immediately asserts that he is not sure that the content is “really” important, suggesting that perceptions that it is necessary may be fallacious. The example he offers in explanation is that students can “use a Finite Math course for the same purpose”. His “you know?” looks to me for confirmation of this, which I likely provided with a nod. The “purpose” referred to here is acceptance into teacher training programs, and the “obvious” message is that if these programs are not concerned about the specific content that potential teachers study in their prerequisite course, then the particular topics are not what matters to accreditation bodies, positioned here as authorities.³⁵ Considerations of what might matter in their stead, lead Simon to formulate his priorities for the course, both with respect to the affective (getting his students “comfortable”) and the cognitive (“do[ing] some problem solving”). The phrase “able to address math” is ambiguous, possibly referring to a positive disposition when faced with doing mathematics.

In this excerpt, Simon is making a case for why the demands of the syllabus should be secondary to considerations of affect. However, this is only one side of the argument that he has with himself. The other side, along with further insights, are

³⁴ Note that although the excerpts in question are paraphrased, these particular expressions were captured as quotes in the field notes.

³⁵ This presents a nice example of the influence of systemic tensions on local decisions.

revealed through his response to a question during his main interview about what aspect of the course he had found to be the most difficult.

I had put too much material into the curriculum, and that was very difficult. I found that I had a hard time keeping up with the pace I had set myself, and didn't really know what to do about it. It feels uncomfortable always, to be making big changes, in the middle of the course. So I ended up more or less sticking to it as much as I could, although trying to cut slack where I could, because, really, I mean, there have been articulation agreements, but I don't think anyone would have fussed if I had had a little less material in it.

Despite Simon's earlier comments about the relative unimportance of the course content, in this excerpt he is sharing the difficulty he experienced coming to the decision to omit topics that had been included in the course syllabus. A close analysis of this excerpt with attention to positioning offers insight into the factors that made it so difficult.

Within these lines, there is ambiguity in Simon's self-positioning with respect to power. His repeated use of "I" at the beginning of the quote seems to show him taking full responsibility for the course content. This is appropriate, as he was the course designer at his institution. *He* was the one who put in too much material, and *he* had set a pace for himself that he could not maintain. And yet, his confession that he "didn't really know what to do about [having put too much in]", suddenly puts him in a position of helplessness. At the end, he describes the actions he ultimately took, but there remains a sense that his power is restricted. This raises the question: if he had the power to set the curriculum at the beginning, what deterred him from adapting to his students needs "in the middle" of the course?³⁶

He explained that, "It feels uncomfortable always, to be making big changes, in the middle of the course". At the time of the interview, as an insider this explanation made sense to me. His sudden switch here, from past tense to a generalised present tense signalled the statement of an accepted truth, a norm of our profession. The use of the word "always" confirmed this. It "always" feels uncomfortable for him, whenever he

³⁶ This echoes my own spontaneous use of the passive voice in the Introduction (see page 5), as I described the amount of material (I) covered in the MFT course. Like Simon, I was the course designer, but in both of our cases, once we had defined the course we felt obliged to stay true to it—we positioned *it* as authoritative.

tries to make such changes, just as it “always” feels uncomfortable for me, as it does for (most of) the others who teach in our context. His sentence was sufficient to evoke in me, stories of student resistance when there was a need to diverge from the agreed upon course syllabus, particularly changes in evaluation criteria. But on further reflection, I also realised that student complaints are rare when the course is adjusted to lighten their load. There was more to the described feeling of discomfort, and more to the norm, than concern about student complaints.

To say initially that certain topics would be covered, and then fail to deliver, could reflect badly on him in his students’ eyes, as an instructor, as a course planner, as an expert who should know what his students will need. But alongside disappointing his students’ expectations is also the possibility of disappointing his own expectations of himself as an instructor. Note that Simon referred to having had difficulties maintaining the pace he had set “himself”, drawing attention to a predetermined personal standard. Furthermore, and connecting once again to our shared experience of being mathematics instructors, these expectations he has for himself are, at least in part, expectations he has, and I have, of instructors of mathematics courses. Making major changes in the middle of the course violates a normative expectation of post-secondary instructors, at least in mathematics: the instructor of a course is expected to teach the content that is prescribed by the course syllabus.

The following excerpt provides further evidence of the existence and influence of this norm:

When you teach calculus, you feel responsible for getting through the bulk of what a calculus course is, and if you didn’t have enough time to get to something like, I don’t know, related rates, you would feel like “Oh, this wasn’t really a calculus course! Something went really wrong there!”

The norm is signalled by Simon’s use of the displaced “you”, and its moral weight is accented by his phrase “you feel responsible”. The consequences of breaching the norm include strong personal recrimination as indicated by his quoted expression of dismay, but the final word “there”, which in a personal instance might have been substituted with “here”, positions the quote as potentially coming from another instructor, looking on in judgment. The content defines the course, so if content was omitted the

course was not delivered, and hence the instructor failed to meet his/her obligations, to self, to students, and to fellow instructors. This is not seen to be a minor problem; it is an indication that “something went *really* wrong”.

Here the norm is attached to the teaching of Calculus, and in the context of the interview is offered as a contrast to how the MFT course *might* be perceived. This was a preamble to Simon’s comment (cited above) that perhaps the specific content in the MFT course is not so important. However, even if the norm might be less applicable in the case of MFT, its influence can still be seen in Simon’s struggles and his decisions, as in the previous excerpt, whose analysis I now continue.³⁷

Simon’s desire to satisfy this norm, compels him to “stick to it” (line 6), but at this point in his response some hedging language is introduced. He sticks to it “more or less”, given that he did make some changes. The “as much as I could” suggests that his efforts were focussed on maintaining the intended course, but then the word “although” signals a subtle shift after which he describes “trying to cut slack where I could”. This time, the “I could” points towards his intention to find ways to alter the course, in line with his priorities, without violating the norm. The tension is evident here in his simultaneous desire to both offer the course as described and to change it.

Simon’s next word is “because”, signalling that he is about to offer an explanation for his actions. He did make cuts in content, and had admitted to me, an insider who is aware of the norm, that he felt the need to breach it. So he moves to justify that breach. The word “really”, as I heard it, translated not as a prelude to a statement of fact, but rather a prelude to a stripping away all but what is essential. It could be paraphrased as “at the end of the day” or perhaps as “the bottom line is”. “There have been articulation agreements” is a reference that Simon knows I will understand. It is a reference to the written artefacts that represent contracts between institutions, describing and promising the course content. In particular, he is referring here to the course syllabus documents that *he* has sent to other institutions describing what he *will* cover in the MFT course. At the end of the day, although he is the course designer, his power is limited by the expectations of others.

³⁷ This norm of “covering the content” was previously described as a factor in both Bob and Maria’s tensions with respect to course priorities and classroom methods, discussed in subsection 7.2.1. As a researcher, I did not recognise it, or its power, until I had done this analysis on Simon’s transcripts. As an “insider”, I was initially too close to the norm to see it and to recognise that its application in the context of the MFT course might be misplaced.

At the same time, the word “but” and the final phrase shows that Simon might see some room to make some changes, perhaps even more than he had already made. Despite the existence of the articulation agreements, he said, “I don’t think anyone would have fussed if I had had a little less material in it”. He expresses the possibility that he could revise the curriculum, to reduce the content, without jeopardising the articulation agreement. This goes back to his earlier speculation that perhaps the expectations are not as rigid for MFT as they are for calculus. But the hedge “I don’t think” is significant here. Simon is not certain that any changes he suggests will be accepted by the larger community, and any reservations he might have are not idle ones³⁸. As a result, although he does make some cuts to course content, he does not make enough to alleviate his perceived time pressures.

A word of clarification is in order here. In this portion of the interview, Simon was considering the choices he had made in light of normative expectations as embodied in the articulation agreement. However, it would be misleading to infer that Simon’s commitment to the course content was solely because of this. Recall again Simon’s uncertainty over whether the content matters “so much” in the MFT course. He is not really sure what knowledge his students will need. He gets an ambiguous message from the posted requirements of teacher training programs, but personally believes that it is important that elementary teachers be “competent in elementary school arithmetic”. So it is likely that his commitment to the course syllabus is also motivated by considerations of his cognitive goals for his students—taken together these are in tension with his affective aims. These tensions are implicated further in his descriptions of the methods he ultimately used in his classroom.

8.2.2 In the classroom

Simon’s time pressures were not eliminated by reducing the course content, and so additional “sacrifices” needed to be made. Contrary to his ideals and intentions with respect to group work and hands-on activities, Simon did not use these methods as much as he had planned. At about the midpoint in the term, faced with time pressures to cover the curriculum, he admitted:

³⁸ One of the other instructors, whose identity will be protected here, expressed consternation over having to modify the MFT course, to increase rigour at the perceived expense of reducing student anxiety, in order to earn or maintain transfer credit to one of the universities.

[paraphrase] I regret not having more time. If I did, I would have tried to do an activity each class. As it is, I've found it impossible to keep activities in the course. I'm doing mostly lectures now, which is disappointing.

Here Simon blames the lack of time for his need to abandon his intended approach. Note the extreme contrast between his ideal of including an activity in each class, with the comment that he found it "impossible to keep activities in the course". Evidence of the experience of tension is strong here: the word "impossible" gives the sense that he felt he had no choice, but as noted above there were other options for managing time. But commitment to completing the course syllabus was strong, and although he cut some content, Simon opted to cut activities as well, deciding that this was his best (his only?) option. Notably, Simon was unhappy with this situation.

Another factor that weighed into this decision came to light as Simon described one of the few activities he did manage to include. Near the end of the semester, despite the time pressures, Simon had his students engage in an activity related to Escher tilings. He was pleased by how the activity had gone, and shared, "[paraphrase] The time was well spent! It made the students feel good, but maybe it wasn't so good for learning!" With this, he proclaimed the value he sees in "spending" time having the students engage in mathematics, a value that is primarily affective: "it made the students feel good". But his next words suggest that he had doubts about whether or not cognitive aims were met as well, momentarily casting the decision about whether or not to use hands-on activities as a choice between affective and cognitive goals.

However, he quickly reconsidered his statement, retracting it and correcting: "[paraphrase] No! A lot was learned, it just wasn't testable". I did not push Simon to elaborate on what he meant by this. I felt that I understood. During the activity the students would have been engaged in looking for patterns, making conjectures, and proving or disproving them as they tried to convince others in their group of their ideas. They may have learned about over-arching mathematical approaches, like looking for the particular in the general, or the general in the particular. They would have been engaged in mathematical activity, learning about what that entails, and at the same time, potentially, exercising creativity and building confidence. However, all of these approaches and mathematical dispositions are difficult to formulate into exam questions. They are not arithmetic facts, or procedures, definitions or theorems.

This brings up a new consideration that contributed to Simon's tensions. His decisions about classroom methods are linked with his priorities for the MFT course, and to this point those priorities have been shown to include cognitive goals, affective goals, and goals to complete the course syllabus. Now the need to evaluate his students, to assign a letter grade to reflect their level of learning at the end of the course is added to his list of aims. This did not come up in the other interviews in the study, but this was likely more a factor of the questions that I asked, than a reflection that this was not a concern for other instructors as well. It is an example of an activity that all post-secondary mathematics instructors engage in as part of every course we teach, and as such is taken for granted, and would not be mentioned in response to a query about one's aims for a course.

Nonetheless, this requirement has the potential to influence methods of instruction. In post-secondary settings, instruction is very much focussed on producing measurable outcomes, as final grades are used to compare students for scholarships and awards or for entry into programs. In mathematics more particularly, evaluation is almost exclusively quantitative, with grades typically based on written examinations. The difficulty intimated by Simon is that with open-ended class activities, although there is much to be gained from them, the outcomes are not usually as easily controlled or measured as they are in settings that are more traditional. With sufficient time, both measurable and non-measurable objectives could be pursued. But when a choice needs to be made, perceptions that traditional approaches can be more easily evaluated could tip the scales towards convention over innovation. This is a personal/practical consideration and its degree of influence would likely vary considerably from instructor to instructor.

Simon's tension with respect to classroom methods is evident in the blatant contrast between his intention to use hands-on activities frequently and his reality of not doing so. Unlike for Andrew, who felt personally ill-equipped to incorporate activities into the course (see subsection 7.1.1), Simon did not express discomfort with the approach in principle, or indicate that he lacked the time to find and implement activities. More like Maria, his struggles were related to his course priorities (see subsection 7.2.1). Wanting to satisfy affective goals pulled him in the direction of reducing course content and incorporating more activities, while goals for improving arithmetic skills and (more significantly) covering the syllabus within the allotted time pulled him to reduce activities.

Implicit in this latter goal are beliefs that learning through activities is less efficient than learning through lecture, and that learning does happen through lecture, beliefs that were both also expressed by other instructors in the study. Also contributing, in Simon's case, were practical considerations of having to assign a final grade and the perceived challenges of evaluating activity-based learning.

Simon was forced into a position of having to make "hard choices", in-the-moment, in order to carry out the teaching of the course. These choices represented compromises, not resolutions to his tensions. This is evidenced by his expressed disappointment, and also in the resolutions he made during his main interview on how he would run the course the second time. These, along with his reflections on this second offering, reported during his follow-up interview, are discussed in the next section.

8.3 The second time around

At the end of his first offering of the MFT course, Simon discussed his plans and resolutions for what he would like to do differently the second time around during his main interview. In particular, he resolved to put:

more focus on trying to get them engaged in seeking out understanding and information in [an] area, at whatever level I can get them engaged by, and, like I say, try to do it more by problem solving, and that kind of more "hands-on" activity. So less lecture, more problem solving, more activities.

He expressed a renewed commitment to lecture less, to incorporate more activities, and to put a greater emphasis on problem solving. He also hoped "to pare down the curriculum" and focus on depth rather than breadth, to worry less about covering everything covered by the text on a given topic, and worry more about getting students "engaged in seeking out understanding". The new stronger focus on problem solving can be seen as an attempt to manage tensions, offering the potential to simultaneously address both cognitive and affective goals.

Notably, his expectations for the frequency of activities were reduced somewhat, from his original desire to do one activity each class, to having one each week. This adjustment was likely a concession in response to trying to manage time effectively, as

his conviction that hands-on experiences were valuable did not seem to be diminished. As well, he mentioned a further institutional option that he hoped to pursue to alleviate some of the time pressure, namely adding more time to the course.

These were his resolutions, however, when I spoke to Simon a year later, after he had taught MFT the second time, I have to confess I was somewhat surprised by both the changes that he had not made and those that he had. The results lend support to Lampert's (1985) view that tensions are "managed" rather than resolved. I focus here on three aspects of Simon's second MFT offering that each reveal further insights into his tensions: time management, the role of the textbook, and his particular choice of class activities.

8.3.1 Time management

Time pressure continued to be a significant problem for Simon, and again was a dominant theme in his follow-up interview at the end of his second offering of the MFT course.

To help alleviate some of the time pressure, Simon had managed to change the course meetings from two 2-hour classes per week, to two 1.5-hour classes plus one 1-hour class per week. The total number of hours was the same, but his hope was that the students would benefit from more frequent meetings with less content covered at each session. He had originally hoped to add another half-hour per week to the course, but had not yet been successful in negotiating this with his institution, implicating practical, institutional considerations as an aggravating factor in Simon's experience of tensions in the course.

In the follow-up interview, when I asked if the new configuration had allowed him to achieve his goal for incorporating more activities, he responded:

No. I found it was harder to do them in this configuration. Because I didn't have the longer...I don't know, I just...(sigh). I, maybe it was partly the configuration, partly the text, and not...I found myself, sort of, struggling with time a lot more. And maybe it was partly me. But it was... I just found myself struggling with time a lot.

From this, it was evident that once again Simon was disappointed with the extent to which he had incorporated activities into his MFT course. The wording of my question put Simon in a position of connecting the course configuration to his ability to use activities, which he accepts initially, first admitting that he did not do more activities, and then making the link that “it was harder to do them in this configuration”. He begins to formulate a reason for why it was harder, perhaps about to suggest that the shorter class sessions made it less practical, but he abandons this thought partway into the sentence. His expression of “I don’t know” signals a reconsideration of the position I had offered to him: perhaps his ongoing struggle with incorporating activities was not so closely tied to the course configuration. His sigh at the moment of reconsideration possibly suggests both his disappointment with what had occurred, and his acceptance that there had been other reasons for his inability to make the hoped-for changes in his classroom methods.

He briefly considers what other factors might have contributed to his even greater struggle with time during this second offering, suggesting the text, and even himself. His reference to himself is a reference to his own decisions about pacing; he later clarified that “perhaps this time [he] slowed down a little bit” as a reaction to feeling he had rushed through the material the first time. The role of the textbook is more involved and is discussed in detail below. Unstated by Simon, but notable, is that many of the same conditions, including his beliefs, that were present for Simon the first time were also still present.

Whatever the causes, the effects of the time constraints were similar in Simon’s second offering of MFT. Despite his resolutions to increase hands-on activities and group problem solving, and to be less constrained by the content, this clearly was not the case. When asked about it specifically, Simon admitted that he had even had to omit the Escher-tiling activity that he had used and enjoyed in his first MFT offering.

8.3.2 Role of textbook

In subsection 5.2.1, it was observed that the textbook served as an important resource for MFT instructors, with many of the instructors relying on it quite heavily for guidance on content, level of difficulty, and connections to elementary contexts and pedagogy. Simon’s comments with respect to his use of the text reveal that this reliance on the text is also a factor that contributes to his tensions.

In the above excerpt, Simon mentions the textbook as affecting the time pressures he experienced in teaching the MFT course. This can be more clearly understood by comparing Simon's intentions with respect to the text for the second offering of his course, with his description of what actually happened.

After his first offering of the course, he resolved:

I would change that focus, you know, and take a book like the one I use, which was, [name of text], and take it with a grain of salt, if I was going to use it, and just say, "Well, OK, those are the big topics that we want to hit, and if you want background reading on them, there it is! But we're going to focus on trying to really understand a few interesting problems"³⁹ and, to work on their problem solving. I think. You know? I mean it's hard to say. We've got to somehow...

Simon's reference to "that focus" is a reference to an emphasis on content. Here, he is talking about making a shift to a focus on problem solving and how this relates to his use of the textbook. As part of the proposed change he would take the textbook, *if* he would use it at all, "with a grain of salt". Typically, this expression is applied to advice; advice that should not be fully relied upon is taken "with a grain of salt", so in this instance it is the advice (authority) of the text in setting the course content that is called into question. He elaborates on this by switching to a quotative mode, imagining himself speaking to a class of students and explaining how the text will be used. The need for an explanation to the students suggests that what he is about to say is contrary to their expectations, contrary to a norm. In this case, the text will be primarily used as a background resource for the students, not as "the focus". The new focus will be "to really understand a few interesting problems".

Simon is not sure how his rejection of the text will be taken by me. He ends on a note of hesitation, hedging with "I think", seeking confirmation of my understanding with "you know?", following this with another hedge "I mean it's hard to say". There may be concern that rejection of the text, implies rejection of the content as prescribed by the curriculum, and hence the articulation agreements. I am positioned as part of the

³⁹ Placement of the endquote is uncertain, as Simon switches from quoting an imagined address to his class, to again speaking to me about his objectives.

community that expects these agreements to be upheld, but Simon is as well. The concern he projects on me is also within him. As will be seen below, despite the views Simon expresses so strongly at the beginning of this excerpt, it is not easy for him to abandon the text or the course content.

He does not complete the thought “we’ve got to somehow...”, but does carry on to consider the value in problem-solving activities (discussed above). The use of “we” signals a repositioning. At the beginning of the excerpt, the use of “I” positions Simon as an individual proposing a change from the norm. The shift to “we”, positions Simon (and me) within a community of MFT instructors who collectively ought to find a way around the textbook, around the commitment to content, to improve the MFT experience for students.

It appears that Simon sees the textbook as an impediment to addressing student needs, specifically those needs that can be addressed by a problem-solving approach, like having the experience of engaging with mathematics. The tendency of instructors to be guided by the text, to cover everything under a given topic as done in the book, to focus too closely on covering content, takes time away from more important activities. The role of the text as the authority for the content is called into question, but this goes against normative expectations of students, who expect to look to the text to provide the information they are required to learn, and the normative expectations of (many) instructors. Comments of several instructors indicated heavy reliance on the textbook, and Simon seemed to anticipate objections to, or at least appeared uncertain about, rejecting the text as a main guide.

The strength of this “norm” of using the textbook to guide content is illustrated by considering Simon’s comments about his use of the text in his second offering, as described in his follow-up interview. He changed textbooks, on the recommendation of other MFT instructors he had spoken to at other institutions, which resulted in some changes to the course:

The new textbook used different terminology, and covered different material and [the course] had to be adjusted a fair amount. And the types of questions that were asked were different. I’m still covering the same basic stuff, but there was a lot of changes in how it was presented.

The excerpt shows that Simon did not take the new text “with a grain of salt”, but rather allowed it to influence “the types of questions that were asked” and “how [the content] was presented”. At another point he noted, “I covered the sections that looked most like what I had covered in the [previous] textbook”, suggesting a transfer in authority over content from the previous text to the new one. But following the new text once again caused problems for Simon in terms of time management:

I think partly [the reason for running out of time] was working with the new text, and just not really realising how I was going to get into a little trouble with how much time it was taking to get through their material.

Simon was caught by surprise by how much time it would take him to cover the content contained in the text, but the key notion here is that Simon felt compelled “to get through *their* material”. The use of the pronoun “their” shifts the authority for determining how the content of the course is to be delivered away from himself (the course designer and instructor) and onto the authors of the text.

Despite Simon’s expressed dissatisfaction with how he had used the text during his first offering of MFT, he persisted in this behaviour. This may have been because the norm of using the text as authority in this way is strong and difficult to change. It may also have been because Simon hoped that the new text would be more in line with his own priorities, and that using it in the way he did would allow, and maybe even help him, to achieve his goals. This is suggested in Simon’s resolution at the end of his follow-up interview to continue to search for “a good textbook that I can work with”. Either way, Simon’s choice of text for his second offering of MFT, continued to contribute to his tensions by pulling him towards an emphasis on content, contrary to his espoused beliefs.

8.3.3 Choice of class activities

Although the frequency of “hands-on” activities did not increase for Simon, and may have even been reduced, he did make two significant changes in other activities in the MFT course, one an omission and one an addition, that taken together are, at least initially, suggestive of Simon continuing to be in tension with respect to his cognitive and

affective aims for his students. Specifically, he omitted overt attention to dealing with mathematics anxiety, and he added a “skills component”.

At the very beginning of our conversations, one of the priorities that stood out for Simon as compared to other instructors was his very strong commitment to dealing with mathematics anxiety in his students. To this end, he had incorporated particular in-class activities in his first MFT offering to address it explicitly. These were not done in his second offering. He explained:

One thing I did really differently is I didn't talk at all about math anxiety. I basically decided, well, OK, that was an interesting experiment. Let's try not doing that this time. You know, where last time I focussed on it a lot, [...] I wasn't sure, really, how much that it helped. So...especially because it ended up crowding up the course, so I had to rush through everything else, and so you're, on the one hand you're saying, you know, “take it easy, don't get anxious”, only then you're saying “go! go! go! go! go!”

Given his uncertainty about how effective his efforts to address math anxiety had been, and given the time pressures he experienced the first time, he opted to reverse his initial “experiment” and leave out the activities he had previously included. He blames the inclusion of these activities, at least in part, for his former experience of time pressure, observing that “*it ended up crowding up the course*”, causing him to rush. As presented here he was faced with a paradox: doing activities to *reduce* math anxiety created even *more* anxiety as he was put into a position of having to push his students through the other course content more quickly. Of course, this conflict only arises if one is unwilling to reduce expectations for cognitive gains, i.e. if content can be reduced, then affective goals can be pursued without increasing anxiety as described.

From his choice to eliminate attention to mathematics anxiety, it might appear that Simon had reconsidered his course priorities, choosing to reduce attention to affect in order to maintain cognitive goals. However, this is not necessarily the case. When asked, Simon reported no ill effects of not explicitly addressing math anxiety this second time around. His students “seemed very relaxed” and were just as successful. At least in Simon's mind, no sacrifice had been made to his affective aims.

But at the same time, as noted above, Simon also did not experience any relief from the time pressures, despite omitting attention to math anxiety from his course. This may have been in part because of a new activity that he added: quizzes to test proficiency in school mathematics. These were done when he started a new section and tested only “the mechanics”. He explained that their purpose was, “just to get that out of the way, and make sure everyone could do it”, with the “that” referring back to the mechanical skills. He continued, “It wasn’t worth much of the grade, but it was just, sort of an incentive for them to go and review the basics so that we could get on with it, and talk about why it works that way.”

The phrase “so that we could get on with it” is usually used in the context of moving forward after a delay or set-back. It suggests that Simon’s inclusion of these proficiency quizzes was at least in part motivated by the time constraints he had felt the first time around. Instructors in the main study (see subsection 7.2.2) had expressed the view that with stronger skills, students would be better able to tackle “the more subtle parts of the course” (Bob) and “focus more on the concepts” (Brooklyn). In line with this perspective, if Simon could get concerns about his students’ arithmetic proficiency “out of the way”, an obstacle that impeded achieving his deeper cognitive goals would be removed.

Just as for the removal of the math anxiety activities, this change seemed to suggest a shift towards emphasising cognitive aims over affective ones. This seemed particularly the case given that quizzes (or tests of any kind) have the potential to increase students’ stress levels. However, his move to incorporate these quizzes was completely consistent with early comments he made during our conversations, cited previously, that Simon, in an ideal situation, “would spend a long time working on arithmetic”, with the expectation that “if the students were quick with numbers they would feel better about their ability to do math”. As well, his comment above that “it wasn’t worth much of the grade”, carries with it the implication that students would not have been very anxious, given the low stakes of the assessment. Consideration of potential affective benefits of these quizzes (e.g. building confidence) was likely also part of Simon’s decision to add this activity, though he does not mention this explicitly during his interview.

Also not mentioned by Simon was the fact that these quizzes used class time, so necessarily they would have taken away time from having the students engage in hands-

on activities and problem solving that he had hoped to do more of. These specific changes made to the activities in the course, just like the math anxiety activities he tried the first time, are “experiments”, ongoing efforts to find ways to manage the tensions between his multiple aims for the course.

In summary then, these three examples (Simon’s ongoing struggles to complete the course within the time frame, his reliance on the textbook despite its interference with his goals for his students, and his choices of activities and their paradoxical motivations) are illustrative of the persistence of the tensions described in the previous section. They show Simon continuing to live with the tensions, despite his best intentions.

8.4 Living with the tensions

The contrast shown between Simon’s intentions and his reported actions while teaching the MFT course for the first time should not be taken as evidence that his espoused beliefs are not firmly held. He is an experienced mathematics instructor who is passionate about mathematics and is thoughtful about his practice and the impact he will have on his students. In his own words, he was “disappointed” and “regrets” the decisions he felt he had to make. He is living with the tensions, trying to manage them as best he can, trying to meet conflicting demands.

This more detailed look at his experience has helped identify a number of factors that contribute to these tensions, including: simultaneous personal desires to build arithmetic competence and improve students’ attitudes towards mathematics; perceived student deficiencies with respect to both of these; influences of norms that place importance on completing course content (within post-secondary mathematics instruction) and reflect the common practice of reliance on the textbook (at least within MFT); and traditions of teaching/learning through lectures and evaluating students mathematics ability through objective quantitative measures. In Simon’s case in particular, his personal belief that problem-solving activities would greatly enhance the experiences of potential elementary teachers, adds to his tensions, given the barrier to his implementation of such activities that many of these other factors represent. The interconnectedness of these factors was also evident.

When asked during his follow-up interview about further changes he would like to make to the course, Simon again talked about the importance of hands-on experiences

and reconfirmed his desire to find ways to bring more of them into the course. As mentioned, he also reconfirmed his intention to look for a textbook more in line with his priorities. However, given that the factors identified will still be in place, it seems likely that Simon will continue to live with these same tensions for some time—they are an integral part of the experience of teaching the MFT course.

9: REFLECTIONS AND REVERBERATIONS

At the very beginning of this project, naively and optimistically, I hoped to understand what the MFT course should be and resolve the tensions I experience as an instructor of this course. A review of the literature soon inspired me to shift my focus by bringing me to two pivotal realisations. The first was that although the literature contains an abundance of advice on what the MFT course *should* be, little is known about how/what the course *actually is*, or about the instructors who teach it—there was a clear empirical deficit. The second involved a new perception of tensions (e.g. Lampert, 1985; Adler 2001) as not necessarily negative, often calling for management as opposed to resolution.

And so my attention shifted to the instructors themselves. I sought out others who teach MFT, to listen to their interpretations of the course, to study the diversity of their approaches, and to investigate their declared tensions. In this final chapter, following some reflections on the research process itself, I summarise the major findings of this study, considering their place in the research literature, their potential for future research, and their continuing impact on my practice as an MFT instructor, as appropriate.

9.1 The research process

From the interviews and the early application of grounded theory methods, through to the hermeneutic analysis with special attention to language and positioning, and eventually to the writing itself, the research process within this study has also been a learning process for me, fraught with its own challenges and tensions. Before moving on to closing comments on the research results, I pause for a moment here to reflect on some of the issues that arose, including the many choices that needed to be made throughout the process, the benefits and drawbacks of my dual positions as both insider and outsider, and finally consideration of the criteria for assessing qualitative analysis.

9.1.1 Choices

Throughout this project, many, many decisions had to be made. Who to interview? What to ask? What to notice? What to ignore? Sensitised as I am to the concept of “tensions”, I have come to see the choices that I have made as best efforts to manage tensions arising in the research process. At the same time, sensitised as I am to the social constructivist perspective, I am also acutely aware that the choices made matter, each affecting every subsequent decision and ultimately shaping what I experienced and how I represented it.

Chapter 3 tells the story of my search for a theoretical framework through which to look at and interpret the data. The choice was a difficult one as I recognised that while each lens revealed new notions, it obscured others. I managed this tension by borrowing from several theories, taking advantage of aspects of each to come closer to answering my research questions. But many of the roads not taken represent opportunities for future avenues of research. In particular, the study of the interaction of the activity systems of instructors and of students within MFT classrooms stands out as a direction of study that could reveal a great deal about the opportunities for and obstacles to teaching and learning in this context.

In Chapter 4, I describe the challenges of dealing with large amounts of data, the subjectivity in the coding process, and my rationale for my choices of interview subjects, questions, and when to stop collecting data. Throughout the data gathering and the thematic coding, I struggled to deal with the tension between “too much” and “not enough”. Every interview introduced a new perspective; every pass through the data offered opportunities for reclassification of codes. In the decisions made, I was guided by researchers like Charmaz (2006), whose writings provided both direction and reassurance that others had walked this road before; the data did become saturated and the codes stabilised.

However, the tension between “too much” and “not enough” carried through to the writing and the analysis of the instructor tensions. I had to make choices about what to highlight, and what to set aside. I had to set sensible boundaries in order to create a coherent body of work, and to allow for sufficient depth of analysis. As a first broad stroke, I restricted my view to tensions that instructors explicitly addressed in the interviews and were characteristic of MFT, steering away from those commonly experienced when teaching other mathematics courses. I then further focussed my

attention on tensions that resonated with my own experience of MFT, or that arose in interviews of multiple instructors.

Here again, in the tensions not chosen for deeper discussion, I see opportunities for future research. Tensions rejected because they are common in other mathematics courses might still offer insight into the MFT course.⁴⁰ Examples of this category include tensions arising in dealing with students' diversity, the tension between readability and mathematical rigour in the choice of textbooks, and the tension created by the sometime role of mathematics courses as a filter.

One further tension that was only briefly mentioned and not experienced directly by any of the instructors in this study is the tension between teaching and research for university professors. This can make teaching the MFT course undesirable because of the extra work it is perceived to require that does not contribute to tenure (Hodgson, 2001). This could not be addressed within the scope of this study as the instructors interviewed all had teaching, as opposed to mathematics research, as their primary function at their institutions. If questioned, they may have been able to comment on their colleagues' reluctance to teach the MFT course, but these instructors themselves would not have fallen into this group. I believe this to be a significant tension that affects the delivery of the MFT course that is worthy of future investigation.

9.1.2 Dual roles

Throughout this study, I have been called to attend to the challenges and opportunities afforded by experiencing dual roles, in two different respects. In the first instance, I am simultaneously an MFT instructor (an "insider") and a researcher who is studying MFT instructors ("an outsider"). As well, with respect to my roles in a broader context, through my participation in the PhD program and all it entails, I have gradually moved to a position as "insider" in the education community, where I would have once been an "outsider" as a mathematician. I now find myself an "insider" in both worlds, which itself is not without its tensions.⁴¹ Two difficulties that arose during this project, relating to each of these dualities respectively, exemplify their influence on this work.

⁴⁰ Despite the uniqueness of the MFT experience, there are some ways in which it *is* similar to teaching other mathematics courses.

⁴¹ In particular the divide observed between these two communities, discussed in subsection 5.2.3, creates tension as I alternately find myself critiquing or defending one side against the other, but simultaneously provides opportunity to understand the perspectives of both.

My dual role as instructor/researcher was particularly significant in the context of the interviews and during the analysis of the data. For the participants who knew me before the interview, I was already positioned as an insider; for the others I made a point of positioning myself as such, to put them at their ease. In Chapter 4, I discuss these roles in some detail, noting the impossibility of objectivity and the need for continual reflexivity, along with the greater potential for noticing and empathy. However, in addition to these methodological considerations, a further unanticipated consequence of this dual role arose related to the interviews themselves.

As an insider, I was assumed to be well acquainted with contexts and implications, and as a result, many things that might have been made explicit to an “outsider” interviewer, were left unsaid. This became apparent to me as I delved into the hermeneutic analysis. As I applied the principles of positioning theory (Harré & van Langenhove, 1999), and took into account the story-line, the context, and the positions that surrounded the utterances under analysis, I became acutely aware of the extent of these gaps in the uttered text. Forcing myself to problematise the spoken words, to imagine myself as a true outsider, helped me to notice the gaps, and to search out both my underlying assumptions and the aspects of the experience of teaching MFT that I had come to take for granted. A nice example of this was that the strength and implications of the norm of “covering the content” came as a genuine surprise to me (see footnote 37, page 158).

My other dual role, as mathematician/mathematics educator, presented a challenge for me during the act of writing the dissertation. Within education, there are particular teaching approaches that are advocated, particularly those based on constructivist philosophies, and through my induction into this new community I have come to appreciate these approaches. While on a day-to-day level I struggle with personal tensions around implementing my theoretical appreciation in practice, straining against both personal habits and tradition, I know that I have begun to “talk the talk” of one “in education”. During the writing phase, I became concerned at times that in my role as member of the mathematics *education* community, my language choices in my efforts to describe the instructor experiences might position me as sitting in judgment over the actions of my colleagues.

This was not at all my intent. Throughout I hoped only to paint a picture of what the MFT course could be, to portray a variety of perspectives and offer some insight into

the factors that contributed to the approaches and viewpoints expressed. Although the literature to date might suggest that some of the approaches are likely to be more effective than others, the research done on the effectiveness of MFT courses is extremely thin. My position is that these instructors of MFT courses are ultimately “sensible” (Leatham, 2006), each doing the best they can within their respective contexts. Although I was conscious of this tension, I remain uncertain with respect to the extent to which I was successful in realising my intent.

9.1.3 Evaluating qualitative analysis

My long-time history as a mathematician and my new role as a mathematics education researcher also contributed to some initial tensions I experienced around engaging in qualitative research. The emergent qualities of my entire research process and its inevitable subjectivity stood in stark contrast to the tradition of mathematical proof and rigour I had been accustomed to. While I was attracted to qualitative approaches and their potential for building rich description of experience, I found it necessary to adjust my views on the criteria applied in judging their validity.

My initial thoughts were much in line with Schoenfeld (2000) who provides a description of the standards by which mathematics education research and their resulting theories can be evaluated, including: descriptive power, explanatory power, scope, predictive power, rigor and specificity, falsifiability, replicability, and triangulation (p. 646). However, some of these did not seem to make sense within the context of this study. In particular, “predictive power” and “replicability” have positivist connotations, as does “triangulation”. These are problematic when the work is interpretive, and subject to the present context and past experiences of the researcher.

My concerns with the misfit between these criteria and qualitative approaches were eased when I encountered Charmaz’s (2006) somewhat different list of criteria for judging grounded theory, which included credibility, originality, resonance and usefulness. Importantly, both Schoenfeld’s and Charmaz’s criteria seek to ensure results are warranted by the data, that appropriate procedures have been followed, and that sufficient and relevant data has been collected; both value originality and the power to offer insight. However, Schoenfeld’s “predictive power” and “replicability” are replaced by “resonance”. The results in grounded theory “allow for indeterminacy rather than seek causality, and give priority to showing patterns and connections rather than to

linear reasoning” (Charmaz, 2006, p. 126). Despite the fact that my work did not culminate in “theory”, nevertheless Charmaz’s criteria struck a chord with me as appropriate goals for the results of this study. So it is to these that I aspired.

With these criteria in mind, I turn now to a summary of the results, keeping in mind the limitations of this study. To re-iterate, the results as presented here represent my subjective reconstruction of the experience of teaching MFT as interpreted from the words of ten fellow MFT instructors, all teaching the course within the context of post-secondary institutions in British Columbia. The data set, in the form of the instructors’ responses to prompts, is merely a snap-shot of the individual instructors’ point-of-view, reflecting the position they chose to adopt and the concerns they chose to share at the time of the interview. While my history and experience permitted insight and empathy into the instructors’ perspectives, they also affected what I was able to notice in the data and what I bring to the fore. Readers will need to judge for themselves whether the range of possibilities portrayed “rings true”.

9.2 Teaching MFT

There is no compact answer to my initial research question: **How do instructors interpret and experience the teaching of MFT courses?** The question is a broad one, whose responses include considerations of the diversity and the tensions addressed in the remaining questions. The analysis of the interview data, as presented throughout Chapters 5 to 8, offers some perspectives, without pretending to be complete. In this section, I will attempt to summarise some of the key points of the experience of teaching MFT that emerged during this analysis, reflecting where appropriate on how they inform or are informed by the research literature as outlined in Chapter 2. Mention of the diversity and tensions will be inevitable, but I will leave the details of these aspects to be discussed more deeply in subsequent sections. Here, I will comment on what this study shows with respect to instructors’ experience of their students, and the course content, and remark on the instructors’ use (and particularly, non-use) of resources in the teaching of MFT.

9.2.1 The students

As observed in Chapter 2, the literature describes the students in MFT courses as having weak skills and a narrow understanding of elementary school mathematics

(Ball 1990), low confidence and high mathematics anxiety (Hembree, 1990), and holding negative attitudes to mathematics that are resistant to change (Wedegge, 1999; Evans 2000). Kessel and Ma (2001) observe that some students in the course are taking the course only to fill requirements and not to learn about mathematics or mathematics teaching. As well, Ball (1988) describes the “baggage” students bring with them with respect to preconceived notions about mathematics and the teaching and learning of mathematics. All of these student characteristics were confirmed in the instructors’ descriptions of their students.

Notable in this study was that instructors experienced their MFT students as different from their other mathematics students. They reported the students’ weakness in skills as more pronounced and their mathematics anxiety as more prevalent. Although this may be the case, it was also evident that instructors were more concerned about these perceived deficiencies in their MFT students. Given that these students hope to be elementary school teachers one day, there is a sense that the stakes are high. Instructors expressed fears that lack of mathematics proficiency, negative attitudes to mathematics, and mathematics anxiety would prevent their students from becoming effective teachers of elementary school mathematics. There was concern that these negative characteristics could be passed on to future pupils.

As was discussed in Chapter 2, these common-sense views are not firmly supported in the literature, which presents mixed results about both the relationship between teacher beliefs and teacher practice (Thompson, 1992) and between teacher attitudes and student attitudes (Aiken, 1976), as well as casting doubt on the connection between teacher mathematics knowledge and student achievement (e.g. Askew et al., 1997). Although research along these lines continues, it is the common-sense view that was expressed by the instructors in this study. This perception that their students’ deficiencies, if allowed to persist, could have repercussions for future pupils is a significant factor in their experience of teaching the MFT course, setting it apart from their other mathematics courses, and contributing to their experience of tensions.

9.2.2 The course

Also consistent with the research (e.g. Williams, 2001) and the subsequent recommendations (e.g. Greenburg & Walsh, 2008), the MFT course described by the instructors in this study is a specialised mathematics content course, centred around the

mathematics of the elementary school curriculum, not a general university mathematics course. Hence, the nature of the content in itself is sufficient to distinguish the MFT course from other mathematics courses on a superficial level. But analysis of the interviews suggests that there are additional aspects of the course content that set the experience of teaching MFT apart in significant ways.

The first of these is the emphasis on communication, mentioned by all of the instructors in this study. All push their students not simply to perform calculations or solve problems, but also to explain their reasoning clearly. This goes beyond their expectations for their other mathematics students. It is motivated by the expectation that these students will be in a position of having to explain mathematics to others in the future. This emphasis on communication causes challenges for MFT instructors as they find they must work against sociomathematical norms, established through students' years of experience, that mathematics does not involve writing. As well, for some instructors, the emphasis on communication leads to the need to evaluate in ways that differ from the traditional marking of objective quantitative mathematics exams that they use most commonly, pushing them into unfamiliar pedagogical territory.

The second aspect of the MFT course related to its content that sets it apart is that, unlike most undergraduate university mathematics courses, it is not a prerequisite for other mathematics courses. It is, in this sense, a terminal course. One might think that this would be experienced as liberating, taking the pressure off instructors to cover every topic in the syllabus. However, this was not the case. Not only did instructors still express concern about lack of time (discussed under tensions below), this circumstance also led to uncertainties with respect to course standards. Without having the demands of a subsequent course as a point of reference, some instructors found it difficult to determine the level of mathematics proficiency students should achieve to be deemed successful. This was most clearly articulated by the newer instructors of the MFT course.

A further related consideration is that although the MFT course is not a prerequisite for further mathematics study, it is a prerequisite (at many institutions) for becoming an elementary school teacher. In order to make decisions about not only course standards, but also modes of delivery and course emphases, instructors are in the position of having to think about what elementary teachers who will teach mathematics need and to what level. Their varied perspectives on this are a major

factor in the diversity among MFT courses (discussed below), and also raises interesting questions about the resources they use to inform these perspectives.

9.2.3 Resources

With respect to the resources that instructors could use to support their teaching of MFT, the most notable observation is related to what they did *not* use. Despite the role of MFT as a course to prepare future elementary teachers, there was little or no mention of consultation with education faculty, nor with academic research in mathematics education.

The lack of contact with education faculty is, perhaps, not so surprising, given that few of the instructors within this study have education departments at their home institutions. However, this in itself does not preclude interaction across institutions, which occurs frequently among *mathematics* faculty at different institutions, at least as part of the course articulation process. Some comments that were made in the interviews with respect to education faculty suggest that it is more than issues of convenience that preclude greater cooperation between the two disciplines (see section 5.2.3); there is a sense of a divide between mathematicians and mathematics educators. This appears not to be entirely a local phenomenon, as it is recognised in the Conference Board (2010) document, which makes the recommendation that “The mathematical education of teachers should be seen as a partnership between mathematics faculty and mathematics education faculty” (p.9).

From my dual perspectives as mathematician/mathematics education researcher, given the large amount of research literature that is relevant to the mathematics preparation of elementary teachers, and given the uncertainties expressed by the MFT instructors, it seems a shame that there appears to be very little evidence that this research is reaching them. The most likely source of connections between the mathematicians and the research seems to be through the textbooks. Instructors report learning about new algorithms and models, taking cues on appropriate types and levels of exam questions, and approaching topics differently, based on the textbooks they had adopted. This suggests the importance of ensuring that the textbooks used are based on the best research available on the preparation of elementary teachers.

At the same time, while the instructor comments suggested fairly high reliance on the course textbooks, there were also many expressions of dissatisfaction with them.

They criticise their textbooks, for their approaches and ordering of topics, and for their lack of mathematical rigour. Three of the ten instructors were at the point of rejecting the textbook completely, either because they have developed their own materials that seem at least sufficient, if not more relevant, or because being tied to the text keeps the focus on content as opposed to on the doing of mathematics. This suggests that the main line of communication between mathematics educators and mathematicians is a tenuous one.

Notably, this line of communication is also “one-way”. With minimal contact with education researchers, there are limited means for MFT instructors to discuss their concerns about the texts or the course. Certainly greater cooperation between these groups would be beneficial. For the MFT instructors, it could increase their knowledge and build their confidence in understanding what students will need for the context of elementary school teaching, better equipping them to manage their tensions, and improving their experience of teaching MFT.

Another aspect of the MFT course that distinguishes it from other standard mathematics courses is the extent to which the course differs from instructor to instructor. I turn to considerations of this diversity next.

9.3 Diversity revisited

My experience as an MFT instructor and my work on the provincial articulation committee had led me to anticipate that a wide variety of interpretations and experiences of teaching MFT would arise during my investigations. This is reflected in the second half of the first research question, which asks: **What factors contribute to the diversity of these interpretations and experiences?** Here, I highlight some of the main points of divergence and some of the possible contributing factors that emerged through analysis of the interviews.

9.3.1 Points of divergence

From the data, it is evident that what is offered in an MFT course goes far beyond topic listings in a course syllabus, and depends very much on the individuals teaching the course. A wide range of characterisations of the course was captured in the ten interviews, with the MFT course portrayed variously as an opportunity for students:

- to engage with elementary mathematics from the new perspective of becoming teachers (Harriet);
- to deepen their conceptual knowledge of elementary school mathematics (Bob);
- to see that mathematics can be fun and interesting (Alice);
- to broaden their conception of what mathematics is, and the role it plays (Al);
- to learn to communicate in the language of mathematics (Brooklyn);
- to learn to think mathematically (Simon);
- to overcome anxieties and embrace the mathematical experience (Maria);
- to see the beauty and the structure of mathematics (Andrew);
- to build confidence and change their attitudes towards mathematics (Karen);
- to learn what mathematicians do, and what they care about (Matthew).⁴²

Threaded through these various conceptions are three of the four major points of divergence identified in this study, namely, differences in course priorities, in the image of mathematics presented, and in the degree of focus on preparation of elementary teachers. The fourth point of divergence relates to the classroom methods employed by the MFT instructors.

With respect to course priorities, instructors expressed varying degrees of emphasis on affective vs. cognitive goals. This parallels the uncertainty that is evident within the research community, which is described in considerable detail in Chapter 2, and is a factor in the tensions discussed below.

The various aspects of mathematics that instructors sought to convey included its socio-cultural-historical significance, its structure, and its “experience”. Even views on the place of problem solving within the course varied, with some seeing it as a vehicle to address anxieties, to see the utility of the structure of mathematics, to build mathematical intuition and common sense, to think mathematically, and to develop mathematical dispositions of persistence and flexibility. These perspectives suggest instructors may make different choices of problems for students to work on and take very different approaches to teaching problem solving.

⁴² This list provides a glimpse into the wide variety of possible perspectives however, it is important to note that the specific attributions to individuals are not complete or exclusive. I have listed only one or two predominant visions for each—a full picture would reflect some sharing of viewpoints among some instructors.

There was also a range in the extent to which instructors oriented their course to the preparation of elementary teachers. One of the strong messages that emerges from the research literature is that although there is much that remains unknown with respect to the needs of future elementary teachers, there has been evidence of success in building pedagogical content knowledge through providing practice with mathematical problems similar to what they will face in the classroom (Ball & Bass, 2003). This study revealed a range in instructor awareness of issues of PCK, as well as their willingness and ability to relate the mathematics they are teaching to the elementary school context. For some, the use of manipulatives, Math Fairs, and class discussions about pedagogical concerns were integral parts of the course, while others felt that these aspects were beyond their expertise, and outside of the mandate for the course.

Diversity in the classroom methods employed by the various instructors, ranging from traditional “chalk and talk” lectures through to much more interactive approaches, reflected both the stereotypes of university professors and the exceptions observed in research (e.g. Strickland, 2008) as described in section 2.4. This point of divergence between instructors is so closely related to one of the central tensions, that I reserve its discussion for the time being. Instead, I turn now to comment on the factors that contribute to the diversity described.

9.3.2 Contributing factors to diversity

A number of factors contribute to the divergent views expressed by the instructors within this study. Adler (2001) made use of the distinctions among practical, personal, and socio-cultural factors in describing tensions, but here I will apply these same categories differently, using them instead as a frame for describing some of the influences on diversity in MFT. Some notions from activity theory (Engeström, 1999) will also be helpful.

On a practical level, there is diversity among MFT courses because there can be. Despite the existence of articulation agreements whose purpose is to ensure some uniformity of content among courses deemed “equivalent”, the fact remains that the MFT course is not a prerequisite for any other mathematics courses. This allows more leeway than for other standard university mathematics courses in interpretation of what content to deliver and how to deliver it. Furthermore, course syllabi characterised by topic lists and behavioural objectives leave affective goals, including conceptions of

mathematics, as well as teaching approaches unspecified, allowing (forcing) instructors to make decisions on these aspects either on their own, or in consultation with other colleagues at their institution.

The personal factors are much more extensive. Differences in course priorities can be affected by previous personal experiences with school mathematics teachers, with their own learning of mathematics, as well as past teaching experience dealing with math anxious or mathematically weak students. Desires to bring out different aspects of mathematics can be shaped by the characteristics of mathematics that instructors feel personally drawn to (e.g. structure/formalism or creativity/discovery). The degree of incorporating pedagogical content or connections to the elementary school context can be influenced by personal interest in, and life experience with, the mathematics learning of elementary school children. In the case of Harriet, whose interview was permeated with references coded as PCK, opportunities to work with a mentor teacher who had a mathematics education background likely helped shape her particular orientation. And of course, differences in classroom approaches also reflect personal preferences and prior experience.

These personal factors are not in themselves surprising. What does stand out is that although these differences in personal perspectives that trace back to differences in personality and biography are also present as these instructors teach other mathematics courses, the diversity among MFT courses appears to be much more pronounced than among other standard university mathematics courses. This is in part because of the practical factors listed above, which leave more room for instructors' personal interpretations, and also because of socio-cultural considerations examined next.

Instructors base their decisions for how to teach MFT not only on their personal inclinations, but on their understanding of how the MFT course fits into the larger activity (Engeström, 1999) of preparing elementary teachers to teach mathematics. Borrowing here from the language of activity theory, there seems to be some confusion about "division of labour". Specifically, should the students coming into the MFT course know the mathematics already, before they take the MFT course, allowing MFT instructors to work on deepening that knowledge? Should MFT instructors address pedagogical content knowledge, or elementary contexts? Members of the community do not agree on these questions.

The “rules” are also under debate, particularly the criteria for being ready to teach elementary school mathematics. This is reflected in the research literature. Also pertaining to “rules”, but returning again to questions of the place of the MFT course, it is also not clear whether MFT is intended to act as a filter to screen out those not suited to mathematics teaching at the elementary level, or to simply support those wishing to pursue this career by whatever means available. What are the “rules” for successfully completing MFT with respect to the preparation of elementary teachers?

These many unanswered questions with respect to the role of the MFT course leave instructors in a position of having to exercise their best judgment. Taken all together, these practical, personal and socio-cultural factors all contribute to diversity in their offerings, particularly with respect to their priorities, methods, conceptions of mathematics and the extent to which they focus specifically on the preparation of teachers. They also contribute to the instructors’ experience of tensions in the teaching MFT. I turn to these next.

9.4 Tensions revisited

My second research question sought to uncover the tensions experienced by MFT instructors, to identify them and further to answer: **What factors contribute to these tensions and how are they managed?** In all, six tensions were identified, two tied closely to instructor identity, experienced by only some of the instructors in the study, two referred to as internal tensions that reflect instructors immanent uncertainties with respect to course decisions, and two designated as systemic tensions, which are anticipated by the socio-cultural considerations of the discussion in the previous section. These tensions are summarised more specifically in the table below (see Table 1)⁴³.

⁴³ These tensions are described in terms of the questions instructors wrestle with rather than as dichotomies, or one-word captions, as either of these other options would risk oversimplification of the issues involved.

Personal	<ul style="list-style-type: none"> • Should I approach teaching mathematics differently for MFT students compared with my other mathematics students? • To what extent is my personal passion for mathematics, and my desire to share its abstraction and logic, at odds with what my MFT students want and are able to handle?
Internal	<ul style="list-style-type: none"> • How should I set priorities for the MFT course? How can I address both the affective and cognitive needs of my students within the parameters of the MFT course? • What level of mathematics proficiency should MFT students have when they leave the course, given considerations of their skills coming in to the course, and where I would like elementary teachers to be?
Systemic	<ul style="list-style-type: none"> • To what extent should MFT courses address mathematics pedagogy and incorporate elementary school contexts? • To what extent are MFT instructors responsible for ensuring the mathematics preparation of elementary teachers?

Table 1. Summary of Tensions

The instructors' experiences of struggling with these questions are captured in the interview excerpts and the hermeneutic analysis of Chapters 7 and 8. In her study of tensions experienced by novice teacher educators, Berry (2007) observed that "tensions do not exist in isolation from each other" (p. 120), an observation that proved to be relevant in this context as well. Among the many interconnections are the factors that contribute to various subsets of these tensions, as well as some of the ways instructors could be seen to manage them. These are summarised in the following two sections.

9.4.1 Contributing factors to tensions

Once again, Adler's (2001) categories of the personal, the practical and the socio-cultural, will be helpful in describing the factors that contribute to and even aggravate the tensions expressed by the MFT instructors.

Clearly personal factors played a significant role in the "personal tensions" described, so much so that they were only explicitly evident in a few of the instructors' interviews, although it is conceivable that they play a role to a lesser extent in others. Specifically, these personal factors include personal preferences for particular styles of

teaching, and prior experiences with both the teaching and learning of mathematics that might be at odds with current recommended education practice. Also, personal conceptions of mathematics, and what makes it interesting and relevant, can be at odds with students' interests and abilities.

But personal factors also contribute to the internal tensions. Decisions about course priorities and standards are influenced by instructor beliefs about the students' immediate needs as well as their anticipated needs in the future as elementary teachers. These tensions seem strongest when the gap between these perceived needs is the greatest. Furthermore, as mentioned above, these internal tensions can be accentuated by beliefs that the deficiencies in mathematics knowledge or negative attitudes that are not adequately addressed in MFT (or in a later mathematics course) will be passed on to future pupils, perpetuating a negative cycle. These beliefs are shaped by instructors' personal biographies, and as noted above, rarely by encounters with the education research community.

The internal tensions are further influenced by practical factors. Ideals for priorities and standards within the MFT course are challenged by the skill levels of the students coming into the course and by the limited time available to cover the course material. Articulation agreements that dictate the course content and the use of particular textbooks were both cited as limiting instructors' perceived ability to make the changes they would like to make to more closely address the perceived needs of their students.

Although the practical restrictions imposed by the articulation agreements and the textbooks were cited by the instructors, a closer examination revealed socio-cultural factors that contribute to instructors' perceptions of these restrictions. In particular, there were indications of strong norms within the context of post-secondary mathematics instruction related to "covering the content", "following the text" and "assigning a grade". These norms not only contributed to instructors' experience of the internal tensions by limiting the options they felt were available to them in making course decisions around course priorities and standards, but they contributed to the persistence of these tensions over time, as was seen especially in Simon's experience in Chapter 8.

The broader socio-cultural factors already identified as contributing to the diversity among MFT courses, influence both the internal and the systemic tensions identified. As mentioned, the societal uncertainties with respect to the role of the MFT

course in the mathematics preparation of elementary teachers puts instructors in the position of having to make these decisions with respect to standards and priorities. The questions listed in the description of the tensions articulate the unanswered questions within the larger system.

9.4.2 Managing tensions

With respect to managing the tensions, there is no question that all of the instructors do so. Within the parameters of their own capabilities, and the restrictions imposed on the course by their institutions and the larger educational system, they strive to do the best they can. Despite all of the uncertainties, decisions need to be made—the MFT course is taught.

At the same time, the results of this study suggest that instructors used a variety of strategies to help manage these tensions, including: sticking with the familiar, deferring to higher authorities/other colleagues, continuing to adjust and experiment, and resigning oneself to lesser goals. Although these are identified through particular examples that arose in the instructor interviews, it would be unfair to associate any one these as characteristic of any one instructor. It is more likely that any of these could be used by an instructor at any given time. I might venture to hypothesise that novice MFT instructors are more likely to defer to higher authorities, and more experienced instructors are more prone to resignation, but to assert this would require more evidence than this study provides.

All of these strategies represent ways of “living with the tensions”. Making the choices that need to be made on a day-to-day basis in the teaching of the MFT course does not make those tensions disappear.

9.5 Concluding remarks

In brief, this study has permitted me to take an in-depth look at the experience of teaching MFT through an analysis of the voices of ten instructors of this course alongside reflection on my own practice. It has confirmed that the teaching of MFT is an experience that is distinct from that of teaching other mathematics courses. There is considerable diversity among instructors with respect to their priorities, emphases, and modes of delivery of the course content, and I have come to see this as symptomatic of the underlying tensions that are endemic to their experience. The variations in

interpretation and delivery of the course reflect the choices made as instructors strive to do their best to reconcile their ideals, perceptions of the mandate of the course, and the immediate needs of their students.

In these final sections, I take a moment to consider some of the possible implications of the findings of this study for research and practice, and to reflect on the impact it has had on my own growth as an MFT instructor.

9.5.1 Seeking resolution

Through the process of describing the experience of teaching MFT and examining the factors that contribute to the diversity and the tensions, a number of challenges faced by the MFT instructors have been brought to light. In the face of problems, the mathematician in me has a strong desire to *solve*, to seek resolution of the tensions, although I am aware that in complex situations such as this, set within systems that are under constant transformation, true resolution is not attainable. With this in mind, I offer here four recommendations for changes to practice, or directions for research, that are suggested by the results of this study and the research literature that supports it. While they do not represent solutions, they offer the hope of some positive change.

Recommendation 1: *Prospective elementary teachers should be required to take more than one MFT course.* This recommendation is in line with recent policy documents from the United States (Greenburg & Walsh, 2008; Conference Board of the Mathematical Sciences, 2010) which both recommend that future elementary teachers take *three* MFT courses as part of their preparation. In British Columbia, the single course required falls far short of this, a situation that the local governing and accreditation bodies would be well advised to reconsider. As this study shows, the limited time that MFT instructors have with their students is a major aggravating factor in their experience of tensions in teaching the course, putting them frequently in a position of having to make difficult choices.

Recommendation 2: *Closer lines of communication should be forged between MFT instructors within mathematics departments and mathematics education researchers.* This recommendation follows from the earlier discussion of the resources that MFT instructors are notably *not* accessing to support their teaching of the course. Exposure to the research would equip mathematicians who teach MFT with

additional knowledge to support research-based decisions in the face of the many choices they must make in the teaching of MFT students. Two-way communication between these groups might further facilitate negotiation of the “division of labour” and “rules” needed to more clearly define the role (the “object”) of the MFT course in the preparation of elementary teachers, supporting the next recommendation.

Recommendation 3: *Further research and negotiation between interested parties is needed to more clearly define the role of MFT in the preparation of elementary teachers.* A major factor in the tensions experienced by the MFT instructors was the uncertainty around the place of the MFT course in the larger system. Policy documents like the Conference Board (2010) report makes recommendations that make great strides towards the issue of more clearly delineating the object of teaching MFT courses, but these recommendations are often made in advance of the research to support them. As described in the review of the literature in chapter 2, there is more room for research into what makes an effective mathematics teacher and on priorities for development of these attributes, and beyond these there is a need for thoughtful consideration of where and how these attributes can be best developed.

Recommendation 4: *Further research is needed to investigate specifically what mathematicians need to know in order to be effective teachers of MFT students.* Given many of the challenges MFT instructors face in dealing with their students’ cognitive and affective needs, there seems to be a need for research into “mathematics-for-teaching-mathematics-teaching”, or more broadly, pedagogical content knowledge for the mathematics preparation of teachers. Such work could help identify ways to support MFT instructors.

Stating the recommendations is simple, but it is important to be realistic about what can be accomplished. More time, better lines of communication, more clearly defined objectives, and more information on how to support teacher learning would all help improve the teaching of MFT and the experience of teaching MFT, and as such are worthwhile aims, but....

Although providing more time is the most easily accomplished of the recommendations, no matter how much time is available, difficult decisions will still need to be made in the context of teaching—spending more time on one concept or activity inevitably takes away from another, and it is unlikely that priorities can be prescribed in ways that take all contexts into account. More specifically, no matter how clearly

objectives are identified and the lines of responsibility set amongst mathematicians and mathematics educators, the “lists” that are generated will present their own problems, including lack of flexibility (Askew, 2008). Another consideration is that research into PCK for the mathematics preparation of teachers will only be helpful to the extent that it reaches the mathematics instructors, and to the extent that it takes into account the norms and personal factors that will influence its reception.

Thankfully, however, as Lampert (1985) observes, it is not the resolving of the tensions that is most important—we can learn much simply from identifying them and being aware of how we manage them. They can become “objects of reflection and action” (Adler, 2001, p. 49), and by examining the factors that contribute to them, it is possible to identify opportunities for change and development (Engeström, 1999). The above recommendations represent such opportunities, while the following section offers an example of how reflection on these tensions, even without resolution, can influence practice.

9.5.2 Personal reflections

On a personal note, this research has greatly affected the way I look at how I teach MFT, and has even helped me to understand why my actual practice has not been affected more by my exposure to the mathematics education environment.

I now see that some of the choices I make, to lecture or to run a group activity, to discuss mathematics pedagogy or not, to set my standards for the course as I do, are all efforts to find a balance between the deficiencies I see in my students and what I would like them to have as future teachers of mathematics in elementary schools. I also understand that the uncertainties that I feel about the “rightness” of my choices is an inherent part of teaching the MFT within its current, only loosely defined position in our socio-cultural context.

I had expected that learning more about constructivist teaching approaches through my mathematics education studies would result in big changes in my day-to-day teaching, but through this research I have come to recognise that the norm of “covering the content” that I have been encultured into, impedes such change. My awareness of this now brings me to question the appropriateness of this norm in the context of a course like MFT, and I cautiously anticipate that this will empower me to explore a

greater range of possibilities in my next offering of MFT, particularly with respect to classroom approaches.

But having a greater range of possibilities will not alleviate the tensions I experience. Nor will the greater knowledge I now have based on my study of the research literature. Although I have learned a great deal about what the MFT course “should” be, the list of attributes that characterises effective mathematics teachers is long and my time with these prospective teachers is short. I will still need to set priorities and standards within a context where the role of the MFT course in the larger system is ill-defined, and where the division of labour amongst members of the community that works to prepare teachers is unclear.

At the same time, I take heart in the perspective on tensions that Lampert offers. While as a mathematics education researcher, and a member of the community that participates in the preparation of teachers, I can continue to research, to study, and to advocate, in order to seek solutions that might alleviate some of the tensions identified, as a mathematics instructor I can “embrace the conflict” (Lampert, 1985, p. 190) and “welcome [its] power to influence [my] working identity” (p. 192). I can continue to reflect deeply on my practice and the implications that my choices have, and continue to adjust my priorities and my teaching in response to my students, in the context of my environment, knowing that there are no “right” answers and that it is OK to do my best.

9.5.3 In closing

While the findings of this study and the perspectives I have gained through study of the literature have been of benefit to me personally, it is my hope that this work will also be of help to others. In the world of education research, I hope that it will increase understanding of the context that MFT instructors find themselves in, raising awareness of the avenues of support that might be provided to them by the education community. In the world of MFT instruction, I hope that it will help other MFT instructors place the challenges they face into perspective, making them more aware of the choices they are making and the factors that influence them. With one foot in each world, embracing the tensions that entails, I hope that this work and the further research it leads to, contributes to forging the link between mathematicians and mathematics educators, for the benefit of both—and for the benefit of future MFT students.

REFERENCE LIST

- Adler, J. (1998). A language of teaching dilemmas: Unlocking the complex multilingual secondary mathematics classroom. *For the Learning of Mathematics*, 18(1), 24-33.
- Adler, J. (2001). *Teaching mathematics in multilingual classrooms*. Dordrecht: Kluwer Academic.
- Aiken, L. R., Jr. (1976). Update on attitudes and other affective variables in learning mathematics. *Review of Educational Research*, 46(2), 293-311.
- Artigue, M. (2001). What can we learn from educational research at the university level? In D. Holton (Ed.), *The teaching and learning of mathematics at university level: An ICMI study* (pp. 207-220). New York: Kluwer Academic Publishers.
- Ashton, P. T., & Webb, R. B. (1986). *Making a difference: Teachers' sense of efficacy and student achievement*. New York: Longman.
- Askew, M. (2008). Mathematical discipline knowledge requirements for prospective primary teachers, and the structure and teaching approaches of programs designed to develop that knowledge. In P. Sullivan & T. Wood (Eds.), *Knowledge and beliefs in mathematics teaching and teaching development* (pp. 13-35). Rotterdam, NL: Sense Publishers.
- Askew, M., Brown, M., Rhodes, B., William, D., & Johnson, D. (1997). *Effective teachers of numeracy: Report of a study carried out for the teacher training agency*. London: King's College, University of London.
- Ball, D. L. (1988). Unlearning to teach mathematics. *For the Learning of Mathematics*, 8(1), 40-48.
- Ball, D. L. (1990). The mathematical understandings that prospective teachers bring to teacher education. *The Elementary School Journal*, 90(4), 449-466.
- Ball, D. L. (1991). Research on teaching mathematics: Making subject matter part of the equation. In J. Brophy (Ed.), *Advances in research on teaching: Teachers' knowledge of subject matter as it relates to their teaching practice* (Vol. 2). Greenwich, CT: JAI Press.
- Ball, D. L. (2002). What do we believe about teacher learning and how can we learn with and from our beliefs? *Proceedings of the 24th International Conference for Psychology of Mathematics Education – North American Chapter*, Athens, Georgia.

- Ball, D. L., & Bass, H. (2000). Interweaving content and pedagogy in teaching and learning to teach: Knowing and using mathematics. In J. Boaler (Ed.), *Multiple perspectives on the teaching and learning of mathematics* (pp. 83-104). Westport, CT: Ablex.
- Ball, D. L., & Bass, H. (2003). Toward a practice-based theory of mathematical knowledge for teaching. *Proceedings of the Annual Meeting of the Canadian Mathematics Education Study Group* (pp. 3-14), Edmonton, Canada.
- Ball, D. L., Hill, H. C., & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? *American Educator*, (Fall), 14-46.
- Ball, D. L., Lubienski, S. T., & Mewborn, D. S. (2004). Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge. In V. Richardson (Ed.), *Handbook on research on teaching* (4th ed., pp. 433-456). Washington, DC: American Educational Research Association.
- Ball, D. L., Thames, M. H., & Phelps, G. (2008). Content knowledge for teaching: What makes it special? *Journal of Teacher Education*, 59(5), 389-407.
- Bean, J. P. (2006). Light and shadow in research design. In C. F. Conrad & R. C. Serlin (Eds.), *The Sage handbook for research in education: Engaging ideas and enriching enquiry* (pp. 353-372). Thousand Oaks, CA: Sage.
- Begle, E. G. (1979). *Critical variables in mathematics education: Findings from a survey of the empirical literature*. Mathematical Association of America; National Council of Teachers of Mathematics.
- Bennett, N., Carre, C., & Dunne, E. (Eds.). (1993). *Learning to teach*. London: Routledge.
- Berlak, A., & Berlak, H. (1981). *Dilemmas of schooling: Teaching and social change*. London: Methuen.
- Berliner, D. C. (1989). Implications of studies on expertise in pedagogy for teacher education and evaluation. *New Directions for Teacher Assessment: Proceedings of the 1988 ETS Invitational Conference* (pp. 39-68), New York, NY.
- Berry, A. (2007). Reconceptualizing teacher educator knowledge as tensions: Exploring the tension between valuing and reconstructing experience. *Studying Teacher Education*, 3(2), 117-134.
- Blumer, H. (1954). What is wrong with social theory? *American Sociological Review*, 19(1), 3-10.
- Blumer, H. (1969). *Symbolic interactionism: Perspective and method*. Englewood Cliffs, N.J.: Prentice-Hall.
- Borko, H., Eisenhart, M., Brown, C. A., Underhill, R. G., Jones, D., & Agard, P. C. (1992). Learning to teach hard mathematics: Do novice teachers and their instructors give up too easily? *Journal for Research in Mathematics Education*, 23(3), 194-222.

- Borko, H., & Putnam, R. (1996). Learning to teach. In D. Berliner & R. Calfee (Eds.), *Handbook of educational psychology* (pp. 673-708). New York: Macmillan.
- Borko, W., Livingstone, C., McCaleb, J., & Mauro, L. (1988). Student teachers' planning and post-lesson reflections: Patterns and implications for teacher preparation. In J. Calderhead (Ed.), *Teachers' professional learning*. Lewes: Falmer Press.
- Brown, M., Askew, M., Baker, D., Denvir, H., & Millett, A. (1998). Is the national numeracy strategy research-based? *British Journal of Educational Studies*, 46(4), 362-385.
- Burch, R. (1990). Phenomenology, lived experience: Taking a measure of the topic. *Phenomenology + Pedagogy*, 8, 130-160.
- Carpenter, T. P., Fennema, E., Peterson, P. L., & Carey, D. A. (1988). Teachers' pedagogical content knowledge of students' problem solving in elementary arithmetic. *Journal for Research in Mathematics Education*, 19(5), 385-401.
- Charmaz, K. (2006). *Constructing grounded theory*. London; Thousand Oaks, CA: Sage Publications.
- Cobb, P., & Yackel, E. (1996). Constructivist, emergent, and sociocultural perspectives in the context of developmental research. *Educational Psychologist*, 31, 175-190.
- Cochran, K. F., DeRuiter, J. A., & King, R. A. (1993). Pedagogical content knowing: An integrative model for teacher preparation. *Journal of Teacher Education*, 44(4), 263-272.
- Conference Board of the Mathematical Sciences. (2010). *The mathematical education of teachers*. Providence RI and Washington DC: American Mathematical Society and Mathematical Association of America.
- Connelly, F. M., Clandinin, D. J., & He, M. F. (1997). Teachers' personal practical knowledge on the professional knowledge landscape. *Teaching and Teacher Education*, 13(7), 665-674.
- Corbin, J. M., & Strauss, A. L. (2008). *Basics of qualitative research : Techniques and procedures for developing grounded theory* (3rd ed.). Los Angeles, CA: Sage Publications, Inc.
- Creswell, J. W. (2008). *Educational research: Planning, conducting, and evaluating quantitative and qualitative research* (3rd ed.). New Jersey: Pearson Education, Inc.
- Davies, B., & Harré, R. (1999). Positioning and personhood. In R. Harré, & L. van Lagenhove (Eds.), *Positioning theory: Moral contexts of intentional action* (pp. 32-52). Oxford; Malden, MA: Blackwell.
- Davis, B., & Simmt, E. (2006). Mathematics-for-teaching: An ongoing investigation of the mathematics that teachers (need to) know. *Educational Studies in Mathematics*, 61(3), 293-319.
- Engeström, Y. (1999). Activity theory and individual and social transformation. In Y. Engeström, R. Miettinen, & R. Punamäki-Gitai (Eds.), *Perspectives on activity theory* (pp. 19-38). Cambridge; New York: Cambridge University Press.

- Engeström Y., Miettinen R., & Punamäki-Gitai R. (Eds.). (1999). *Perspectives on activity theory*. Cambridge; New York: Cambridge University Press.
- Ernest, P. (1988). The attitudes and practices of student teachers of primary school mathematics. In *Proceedings of the 12th Annual Conference of the International Group for the Psychology of Mathematics Education (Veszprem, Hungary, July 20-25, 1988)*, (Vol. 1, pp. 288-295).
- Ernest, P. (1989). The knowledge, beliefs and attitudes of the mathematics teacher: A model. *Journal of Education for Teaching*, 15(1), 13-33.
- Evans, J. (2000). *Adult's mathematical thinking and emotions: A study of numerate practices*. London, England; New York: RoutledgeFalmer.
- Fennema, E., & Franke, M. L. (1992). Teachers' knowledge and its impact. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 147-164). New York: Macmillan.
- Foss, D. (2000). Conceptions of mathematics teaching and learning: Middle level and secondary preservice teachers. *Paper Presented at the Annual Meeting of the American Educational Research Association*, New Orleans, LA.
- Ginsburg, H. (1981). The clinical interview in psychological research on mathematical thinking: Aims, rationales, techniques. *For the Learning of Mathematics*, 1(3), 4-11.
- Goulding, M., Hatch, G., & Rodd, M. (2003). Undergraduate mathematics experience: Its significance in secondary mathematics teacher preparation. *Journal of Mathematics Teacher Education*, 6, 361-393.
- Goulding, M., Rowland, T., & Barber, P. (2002). Does it matter? Primary teacher trainees' subject knowledge in mathematics. *British Educational Research Journal*, 28(5, Subject Knowledge and Application), 689-704.
- Graeber, A. O., Tirosh, D., & Glover, R. (1989). Preservice teachers' misconceptions in solving verbal problems in multiplication and division. *Journal for Research in Mathematics Education*, 20(1), 95-102.
- Greenberg, J., & Walsh, K. (2008). *No common denominator: The preparation of elementary teachers in mathematics by America's education schools*. Washington, DC: National Council on Teacher Quality.
- Grossman, P. L., Wilson, S. M., & Shulman, L. S. (1989). Teachers of substance: Subject matter knowledge for teaching. In M. C. Reynolds (Ed.), *Knowledge base for the beginning teacher* (pp. 23-36). New York: Pergamon.
- Harré, R., & van Lagenhove, L. (1999). *Positioning theory: Moral contexts of intentional action*. Oxford; Malden, MA: Blackwell.
- Hembree, R. (1990). The nature, effects, and relief of mathematics anxiety. *Journal for Research in Mathematics Education*, 21(1), 33-46.

- Henderson, S., & Rodrigues, S. (2008). Scottish student primary teachers' levels of mathematics competence and confidence for teaching mathematics: Some implications for national qualifications and initial teacher education. *Journal of Education for Teaching: International Research and Pedagogy*, 34(2), 93-107.
- Herbel-Eisenmann, B., & Wagner, D. (2009). Re-mythologizing mathematics through attention to classroom positioning. *Educational Studies in Mathematics*, 72, 1-15.
- Hill, H. C., Rowan, B., & Ball, D. L. (2005). Effects of teachers' mathematical knowledge for teaching on student achievement. *American Educational Research Journal*, 42(2), 371-406.
- Hodgson, B. (2001). The mathematical education of school teachers: Role and responsibilities of university mathematicians. In D. Holton (Ed.), *The teaching and learning of mathematics at university level: An ICMI study* (pp. 501-518). New York: Kluwer Academic Publishers.
- Holton D. (Ed.). (2001). *The teaching and learning of mathematics at university level: An ICMI study*. New York: Kluwer Academic Publishers.
- Hoyles, C. (1992). Mathematics teaching and mathematics teachers: A meta-case study. *For the Learning of Mathematics*, 12(3), 32-44.
- Jonker, L. (preprint). Prospective elementary school teachers in a mathematics department. Retrieved from <http://www.mast.queensu.ca/~leo/Files/edpapers.html>
- Jonker, L. (2008). A mathematics course for prospective elementary school teachers. *PRIMUS*, 18(4), 325-336.
- Kajander, A. (2007). Unpacking mathematics for teaching: A study of preservice elementary teachers' evolving mathematical understandings and beliefs. *Journal of Teaching and Learning*, 5(1), 33-54.
- Kajander, A., Mason, R., Taylor, P., Doolittle, E., Boland, T., Jarvis, D., & Maciejewski, W. (2010). Multiple visions of teachers' understandings of mathematics. *For the Learning of Mathematics*, 30(3), 50-56.
- Kessel, C., & Ma, L. (2001). Mathematicians and the preparation of elementary teachers. In D. Holton (Ed.), *The teaching and learning of mathematics at university level: An ICMI study* (pp. 467-480). New York: Kluwer Academic Publishers.
- Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). (2001). *Adding it up: Helping children learn mathematics*. Washington, DC: National Academies Press.
- Kukla-Acevedo, S. (2009). Do teacher characteristics matter? New results on the effects of teacher preparation on student achievement. *Economics of Education Review*, 28(1), 49-57.
- Lampert, M. (1985). How do teachers manage to teach? Perspectives on problems in practice. *Harvard Educational Review*, 55(2), 178-194.
- Lampert, M. (1986). Knowing, doing, and teaching multiplication. *Cognition and Instruction*, 3(4), 305-342.

- Lave, J., & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation*. Cambridge England; New York: Cambridge University Press.
- Leatham, K. R. (2006). Viewing mathematics teachers' beliefs as sensible systems. *Journal of Mathematics Teacher Education*, 9(1), 91-102.
- Leinhardt, G. (1989). Math lessons: A contrast of novice and expert competence. *Journal for Research in Mathematics Education*, 20(1), 52-75.
- Leinhardt, G., Putnam, R. T., Stein, M. K., & Baxter, J. (1991). Where subject knowledge matters. In J. Brophy (Ed.), *Advances in research on teaching: Teachers' knowledge of subject matter as it relates to their teaching practice* (Vol. 2). Greenwich, CT: JAI Press.
- Leitzel, J. R. C. (Ed.). (1991). *A call for change: Recommendations for the mathematical preparation of teachers of mathematics: An MAA report*. Mathematics Association of America.
- Leont'ev, A. N. (1978). *Activity, consciousness, and personality*. Englewood Cliffs: Prentice-Hall.
- Liljedahl, P. (2008). Teachers' insights into the relationship between beliefs and practice. In *Proceedings of the 14th International Conference on Mathematical Views (MAVI)*, St. Wolfgang, Austria.
- Liljedahl, P., Rolka, K., & Roesken, B. (2007). Affecting affect: The re-education of preservice teachers' beliefs about mathematics and mathematics learning and teaching. In M. Strutchens & W. Martin (Eds.), *69th NCTM yearbook – the learning of mathematics* (pp. 319-330). Reston, VA: National Council of Teachers of Mathematics.
- Luria, A. R. (1976). *Cognitive development: Its cultural and social foundations*. Cambridge, MA: Harvard University Press.
- Ma, L. (1999). *Knowing and teaching elementary mathematics : Teachers' understanding of fundamental mathematics in China and the United States*. Mahwah, N.J.: Lawrence Erlbaum Associates.
- Ma, X. (1999). A meta-analysis of the relationship between anxiety towards mathematics and achievement in mathematics. *Journal for Research in Mathematics Education*, 30(5), 520-540.
- Ma, X., & Kishor, N. (1997). Assessing the relationship between attitude toward mathematics and achievement in mathematics: A meta-analysis. *Journal for Research in Mathematics Education*, 28(1), 26-47.
- Martin, W. G., & Harel, G. (1989). Proof frames of preservice elementary teachers. *Journal for Research in Mathematics Education*, 20(1), 41-51.
- Mason, J. (2008). PCK and beyond. In P. Sullivan, & T. Wood (Eds.), *Knowledge and beliefs in mathematics teaching and teaching development* (pp. 301-322). Rotterdam, The Netherlands: Sense Publishing.

- Matthews, M. E., & Seaman, W., I. The effects of different undergraduate mathematics courses on the content knowledge and attitude towards mathematics of preservice elementary teachers. *Issues in the Undergraduate Mathematics Preparation of School Teachers*, 1(July 2007). Retrieved from www.k-12prep.math.ttu.edu/journal/contentknowledge/matthews01/article.pdf
- McLeod, D. B. (1992). Research on affect in mathematics education: A reconceptualization. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 575-596). New York: Macmillan.
- Millett, A., & Johnson, D. C. (1996). Solving teachers' problems? The role of the commercial mathematics scheme. In D. C. Johnson & A. Millett (Eds.), *Implementing the mathematics national curriculum: Policy, politics and practice (New BERA dialogue series, 1)* (pp. 54-70). London: Paul Chapman Publishing Ltd.
- Mizell, J. A., & Cates, J. The impact of additional content courses on teacher candidates' beliefs regarding mathematics content and pedagogy. *Issues in the Undergraduate Mathematics Preparation of School Teachers*, 4(May 2004), July 15, 2009. Retrieved from <http://www.k-12prep.math.ttu.edu/journal/curriculum/mizell/article.pdf>
- Moreira, P. C., & David, M. M. (2008). Academic mathematics and mathematical knowledge needed in school teaching practice: Some conflicting elements. *Journal of Mathematics Teacher Education*, 11(1), 23-40.
- Morse, J. M., Stern, P. N., Corbin, J. M., Bowers, B., Charmaz, K., & Clarke, A. E. (2009). *Developing grounded theory: The second generation*. Walnut Creek, CA: Left Coast Press.
- Mullens, J. E., Murnane, R. J., & Willett, J. B. (1996). The contribution of training subject matter knowledge to teaching effectiveness: A multilevel analysis of longitudinal evidence from Belize. *Comparative Education Review*, 40(2), 139-157.
- Nathan, M. J., & Koedinger, K. R. (2000). An investigation of teachers' beliefs of students' algebra development. *Cognition & Instruction*, 18(2), 209-237.
- National Council of Teachers of Mathematics. (1991). *Professional standards for teaching mathematics*. Reston, VA: National Council of Teachers of Mathematics.
- National Mathematics Advisory Panel. (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Washington, D.C.: U.S. Department of Education. Retrieved from <http://wmatthews180.googlepages.com/USAMathAdvfinal-report.pdf>
- Oliveira, H., & Hannula, M. S. (2008). Individual prospective mathematics teachers: Studies on their professional growth. In K. Krainer, & T. Wood (Eds.), *Participants in mathematics teacher education: Individuals, teams, communities and networks* (pp. 13-34). Rotterdam, The Netherlands: Sense Publishers.
- Palardy, G. J., & Rumberger, R. W. (2008). Teacher effectiveness in first grade: The importance of background qualifications, attitudes, and instructional practices for student learning. *Educational Evaluation and Policy Analysis*, 30(2), 111-140.

- Peterson, P. L., Fennema, E., Carpenter, T. P., & Loef, M. (1989). Teacher's pedagogical content beliefs in mathematics. *Cognition & Instruction, 6*(1), 1-40.
- Philipp, R. A. (2007). Mathematics teachers' beliefs and affect. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (pp. 257-315). Charlotte, NC: National Council of Teachers of Mathematics.
- Raymond, A. M. (1997). Inconsistency between a beginning elementary school teacher's mathematics beliefs and teaching practice. *Journal for Research in Mathematics Education, 28*(5), 550-576.
- Roth, W., & Yew-Jin, L. (2007). Vygotsky's neglected legacy: Cultural-historical activity theory. *Review of Educational Research, 77*(2), 186-232.
- Rowan, B., Chiang, F.-S., & Miller, R. J. (1997). Using research on employees' performance to study the effects of teachers on students' achievement. *Sociology of Education, 70*(4), 256-284.
- Rowland, T. (1995). Hedges in mathematics talk: Linguistic pointers to uncertainty. *Educational Studies in Mathematics, 29*, 327-353.
- Rowland, T., Huckstep, P., & Thwaites, A. (2005). Elementary teachers' mathematics subject knowledge: The knowledge quartet and the case of Naomi. *Journal of Mathematics Teacher Education, 8*(3), 255-281.
- Schoenfeld, A. H. (Ed.). (1990). *A source book for college mathematics teaching: A report on the Committee on the Teaching of Undergraduate Mathematics*. Washington, D.C.: The Mathematical Association of America.
- Schoenfeld, A. H. (2000). Purposes and methods of research in mathematics education. *Notices of the American Mathematical Society, 47*(6), 641-649.
- Schofield, H. L. (1981). Teacher effects on cognitive and affective pupil outcomes in elementary school mathematics. *Journal of Educational Psychology, 73*(4), 462-471.
- Schön, D. (1987). Educating the reflective practitioner. *Address to the 1987 Meeting of the American Educational Research Association, Washington, DC*.
- Schwartz, J. E., & Riedesel, C. A. (1994). *The relationship between teachers' knowledge and beliefs and the teaching of elementary mathematics*. (Paper presented at the Annual Meeting of the American Association of Colleges for Teacher Education (Chicago, IL, February 16-19, 1994).)
- Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.
- Silverman, J., & Thompson, P. W. (2008). Toward a framework for the development of mathematical knowledge for teaching. *Journal of Mathematics Teacher Education, 11*(6), 499-511.

- Simmt, E., Davis, B., Gordon, L., & Towers, J. (2003). Teachers' mathematics: Curious obligations. In N. Pateman, B. Dougherty, & J. Zilliox (Eds). *Proceedings of the 2003 Joint Meeting of the International Group for the Psychology of Mathematics Education PME North America* (Vol. 4, pp. 175-182), University of Hawaii.
- Simon, M. A. (2006). Key developmental understandings in mathematics: A direction for investigating and establishing learning goals. *Mathematical Thinking & Learning*, 8(4), 359-371.
- Simon, M. A., & Blume, G. W. (1996). Justification in the mathematics classroom: A study of prospective elementary teachers. *Journal of Mathematical Behavior*, 15(1), 3-31.
- Skott, J. (2001). The emerging practices of a novice teacher: The roles of his school mathematics images. *Journal of Mathematics Teacher Education*, 4(1), 3-28.
- Sosniak, L. A., Ethington, C. A., & Varelas, M. (1991). Teaching mathematics without a coherent point of view: Findings from the IEA second international mathematics study. *Journal of Curriculum Studies*, 23(2), 119-131.
- Speer, N. M. (2005). Issues of methods and theory in the study of mathematics teachers' professed and attributed beliefs. *Educational Studies in Mathematics*, 58(3), 361-391.
- Strickland, S. (2008, Unpublished doctoral dissertation), Michigan State University, USA.
- Swafford, J. O., Jones, G. A., & Thornton, C. A. (1997). Increased knowledge in geometry and instructional practice. *Journal for Research in Mathematics Education*, 28(4), 467-483.
- Swars, S. L., Smith, S. Z., Smith, M. E., & Hart, L. C. (2009). A longitudinal study of effects of a developmental teacher preparation program on elementary prospective teachers' mathematics beliefs. *Journal of Mathematics Teacher Education*, 12(1), 47-66.
- Thompson, A. G. (1984). The relationship of teachers' conceptions of mathematics and mathematics teaching to instructional practice. *Educational Studies in Mathematics*, 15(2), 105-127.
- Thompson, A. G. (1992). Teachers' beliefs and conceptions: A synthesis of the research. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching and learning* (pp. 127-146). New York; London: Macmillan Library Reference USA; Simon & Schuster and Prentice Hall International.
- Thompson, A. G., Philipp, R. A., Thompson, P. W., & Boyd, B. A. (1994). Computational and conceptual orientations in teaching mathematics. In D. B. Aichele, & A. F. Coxford (Eds.), *Professional development for teachers of mathematics* (pp. 79-92). Reston, VA: National Council of Teachers of Mathematics.
- van Manen, M. (1997). *Researching lived experience: Human science for an action sensitive pedagogy* (2nd ed.). London, ON: Althouse Press.

- von Minden, A. M., Walls, R. T., & Nardi, A. H. (1998). Charting the links between mathematics content and pedagogy concepts: Cartographies of cognition. *Journal of Experimental Education*, 66(4), 339-358.
- Vygotsky, L. (1978). *Mind in society: The development of higher psychological processes*. Cambridge, MA: Harvard University Press.
- Wedege, T. (1999). To know or not to know--mathematics, that is a question of context. *Educational Studies in Mathematics*, 39(1-3), 205-227.
- Williams, H. (2001). Preparation of primary and secondary mathematics teachers: A working group report. In D. Holton (Ed.), *The teaching and learning of mathematics at university level: An ICMI study* (pp. 445-454). New York: Kluwer Academic Publishers.
- Wilson, S. M., & Ball, D. L. (1996). Helping teachers meet the standards: New challenges for teacher educators. *Elementary School Journal*, 97(2), 121-138.
- Wilson, S. M., & Berne, J. (1999). Teacher learning and the acquisition of professional knowledge: An examination of research on contemporary professional development. In A. Iran-Nejad & P. D. Pearsons (Eds.), *Review of research in education* (Vol. 24). Washington, DC: American Educational Research Association.
- Wilson, S. M., & Cooney, T. J. (2002). Mathematics teacher change and development: The role of beliefs. In G. Leder, E. Pehkonen, & G. Toerner (Eds.), *Beliefs: A hidden variable in mathematics education* (pp. 127-148). Boston, MA: Kluwer Academic Publishing.
- Wilson, S. M., Floden, R. E., & Ferrini-Mundy, J. (2001). *Teachers preparation research: Current knowledge, gaps and recommendations*. (Research Report for the U.S. Department of Education No. R-01-3). Washington, DC: Center for the Study of Teaching and Policy. Retrieved from <http://depts.washington.edu/ctpmail/PDFs/TeacherPrep-WFFM-02-2001.pdf>
- Yackel, E., & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, 27(4), 458-477.
- Zazkis, R. (2000). Factors, divisors, and multiples. In E. Dubinsky, A. H. Schoenfeld, & J. Kaput (Eds.), *Research in collegiate mathematics education IV* (pp. 210-238). Providence, RI: American Mathematics Society.
- Zazkis, R., & Campbell, S. (1996). Divisibility and multiplicative structure of natural numbers. *Journal for Research in Mathematics Education*, 27(5), 540.

APPENDICES

APPENDIX A: MFT SUBCOMMITTEE REPORT

BCcupms Subcommittee on Mathematics for Elementary Teachers Courses

*Excerpt from the Report submitted
May 16, 2010*

I. Guiding Principles

Math for Elementary School Teachers is the most important course taught in any mathematics department. It has the potential to shape the mathematical knowledge and understanding of those who will influence the attitudes and develop the abilities of future generations.

Mathematics for teaching is different from the mathematics that is used by other professions. A much deeper understanding of concepts is required to teach them successfully than simply to use them. All instructors teaching a MFET course need to be aware of this, and of the need to get this idea across to their students, the future teachers.

The course needs to emphasize conceptual understanding to support procedural knowledge, and broaden students' understanding of what mathematics is. Students need to learn to make explicit their implicit understanding of mathematical concepts. The students in a MFET course and the teacher need to work together to generate insight and understanding.

Throughout all school levels, mathematical concepts need to be presented in a way that supports further development at subsequent levels. This is particularly important in teaching elementary mathematics. Therefore, any MFET course needs to make the students aware of how mathematical ideas and definitions are developed sequentially, and how these ideas have evolved historically.

Everybody can learn to understand and enjoy mathematics at a high level. However, performance at a high level requires more than somebody else explaining the concept well; it requires hard and focused work. Students must share the responsibility for high achievement with the teacher. High achievement and competence together foster confidence in one's abilities.

Given that teachers often teach the way that they are taught, MFET should set a high standard for quality teaching, making use of a variety of different approaches to teaching and learning.

The course cannot be taught properly if there is not enough time for students to reflect on the concepts and ideas. While a two-semester course would be ideal, if a one-semester course is all we can offer, a minimum of 4 hours per week or at least 52 hours is required to cover the material in the spirit of the guidelines.

Given time constraints, the course cannot cover the contents of the entire elementary school curriculum, nor can it prepare future teachers for all future curriculum changes. It is expected, however, that the course will cover the most important concepts and ideas required to teach elementary mathematics curriculum. The course needs to be taught in such a way that the students realize the need for ongoing exploration of mathematical ideas, and develop a desire to continue their mathematical professional development throughout their careers.

II. Suggested Course Content

Mathematical Thinking:

- mental calculation, estimation, number sense
- identification of errors in calculation & reasoning; explanation of the errors
- problem-solving
- algebraic thinking (use of formulae, understanding equal signs)
- concept of symmetry
- appropriate use of the symbols & language of mathematics
- formulation of conjectures & assessment of their reasonableness

Core Math Topics

Numbers & Operations

- strategies for understanding basic operations on integers, fractions & decimals
- properties of numbers and operations on numbers
- number theory concepts: primes, prime factorisation, divisibility, GCF, LCM
- fractions/decimals/percent; conversions and equivalents

Geometry & Measurement

- concepts of unit, measure & dimension
- derivation of/rationale for basic area, perimeter & volume formulae
- strategies for conversion of measurements (especially metric)
- definition & properties of geometric objects
- properties of lines, angles & triangles (intuitive development)

Language & Reasoning

- Venn diagrams, sets as collections of objects
- mathematical definitions (appropriate precision & rigour)

Other topics to choose from (to enhance overall goals)

- arithmetic with different bases
- algorithms (history, variety)
- unsolved problems in number theory
- history of numbers
- precision in measurements/calculations
- other geometries
- topics from probability & statistics
- graphs and interpretation of graphs
- basic concepts of set theory
- the difference between inductive & deductive reasoning

APPENDIX B: ANNOTATED INTERVIEW PROTOCOL

The following annotated list outlines the set of questions that were used as a basis for the interviews, and clarifies their intent:

- 1. How many years have you been teaching mathematics?**
- 2. Have you taken any Mathematics Education courses?**
- 3. How many times have you taught the Mathematics for Elementary Teachers course?**

These first questions were used to establish instructor background and experience teaching the MFT course. As part of this discussion, experience teaching in other than post-secondary settings and additional educational background, including degrees earned, were discussed.

- 4. What percentage of the students in this course intend to become teachers?**
- 5. What topics are covered in this course at your institution?**

These two questions helped to establish the nature of the course at the instructor's institution. To what extent is the course truly a preservice course for prospective mathematics teachers as opposed to a mathematics course simply aimed at non-majors or liberal arts students? Asking about course topics also provided a basis for comparison of the syllabi at the various institutions.

- 6. What are your main objectives as you teach this course?**

This was a key question that was asked of all participants. Asking it following the previous question about course content encouraged the instructors to provide answers that went beyond simply listing topics, and to think about what it is they want their students to gain through taking the MFT course. Order of response was also of interest here to offer potential insights into what was foremost in their minds with respect to the course objectives.

- 7. In what ways is this course different from other mathematics courses that you teach, with respect to your objectives, your students, the methods that you use?**
- 8. Would someone walking into your MFT classroom be able to tell that it wasn't a math course for math, business or science students? Would they be able to tell by looking at the students? By looking at what you are doing? By looking at the board/displayed material?**
- 9. Can you describe any specific activities, methods or approaches that you use with this course particularly because the students are potentially future teachers?**

These three questions (or variations on them) were intended to help the instructors consider what unique situations they experience while teaching the MFT course. It was hoped that they might reveal any differences in how they prepare for classes, how they deal with the students, how they present material, and how they evaluate their students. Question 9 in particular asks the instructors to consider how what they do is related to the intended future careers of their students. It was hoped that if the instructor recognised any types of mathematical knowledge or skill that are needed by teachers (but not necessarily by other users of mathematics), that it might arise in discussion of this question.

- 10. What makes teaching this course difficult for you? What makes it enjoyable/rewarding?**

This more general question sought to probe for rewards and challenges the instructor deals with in teaching the course.

- 11. Is math anxiety a problem? Do you do anything special to help reduce the students' anxiety about mathematics?**

From my own experience as an MFT instructor, I was aware that dealing with math anxious students is always a factor for this course. This question is included in the list, but usually instructors mentioned mathematics anxiety much sooner in the interview, sometimes in response to question 6, 7 or 10.

12. Can you describe how you approach the teaching of division of fractions? What do you emphasise? How do you handle the “invert-and-multiply” rule? How do your students respond?

This question was constructed in order to provide a context in which the instructor could talk about a particular content topic that is a part of most MFT courses, that students traditionally have difficulty with, and that offers a wide variety of approaches. This topic can be taught using manipulatives, using proofs, or by simply teaching a rule. It was hoped that answers to this question might offer insights into the instructor’s overall approach to topics. Students in other college mathematics classes are free to use the “invert and multiply” rule in their work. Would that be true for MFT courses? I expected that it would not be, but that it would be an interesting opportunity to look for differences in instructor perspectives on what future teachers might need to know.

13. What is one thing you would like to cover in the course but don’t? Why don’t you do this? Why would you like to? Would you cut anything to put in it?

This question sought to uncover sources of discomfort with the current course as offered. If the instructor was unhappy about any aspect of the course that had not come up yet, it was hoped it would come up here. Also of interest were the reasons the instructors might offer for why they did not simply make the desired changes.

14. What do your students leave the course with (skills, attitudes, etc.)?

15. Are you satisfied with what students get from the course? If not, what more could they be provided with? Is their math knowledge sufficient for them to be elementary teachers?

These final questions pushed the instructors towards a self-evaluation for their course. Juxtaposed against the early question about their objectives for the course, in this section of the interview they were asked to consider how successful they were in meeting their own objectives. They were then asked to go further: Given that the MFT is possibly the only content course these prospective teachers will take, are they now prepared with their content knowledge?

APPENDIX C: CODING SAMPLE

The following table represents a sample transcript excerpt, together with the initial gerund codes and concept codes assigned. More subject headings were included under Gerund Codes as needed, but only “Instructor”, “Students”, and “Math”, are needed for this excerpt, taken from Harriet’s interview. Definitions of codes can be found in Appendix E.

Line #	Transcript	Gerund Codes			Concept Codes	
		Instructor	Students	Math		
56	I want them to...develop an attitude towards mathematics that	Working on student attitudes: fun, not intimidating			W	SA
57	it can be fun, that it need not be			Solving puzzles	MI	
58	intimidating, that it’s to do with	Reassuring that struggling is alright			MI	
59	solving puzzles, and if you can’t do it the first time that’s perfectly OK,	Wanting them to learn they can help themselves			I	C
60	that you can develop ways of helping yourself. And that I very	Desiring improved attitudes			SA	P
61	much want because they are very (many of them) are/have very bad		Having had bad experiences, having bad attitudes, fears		STC	
62	experiences in math and not very		Having bad attitudes		SA	
63	good attitudes toward it, and they’re scared of it. So I want		Being scared of math		MA	
64	them to have enough solid basic mathematics that they have some	Trying to build confidence through providing solid basic mathematics			C	CK
65	confidence, and that they realise they will be seeing things that they have never seen before. Students		Realising that they will see things they’ve never seen before		N	FT
66	will ask them questions that they		Being asked novel questions by their		DCK	FT

Table 2. Coding Sample

APPENDIX D: CODING SUMMARY SAMPLE

The table displayed on the next two pages shows a small piece of the coding summaries that were generated. In the excerpt below, the details for the code of “doubts” are displayed. Some information has been deleted for formatting purposes: columns labeled with “#” normally contain references to specific line numbers; even though the original summary sheet has columns for each of the ten instructors, only four are included here; two rows have been deleted. In the original, this code of “doubts” appears on the same page as the other codes included under the theme of “Instructor Concerns” (see Appendix E).

Detail for DOUBTS	Harriet	#	Bob	#	Alice	#	Matthew	#	Notes
Lack of training	Methods				Not education, no pedagogy		Disconnected, what do others do		Self-doubts
Personal weakness					Drawing nice pictures		Made lots of mistakes, wanting to solve problems, failing every day		
Quantity of material--See CD	How much to show, too much?, overwhelming				vs quality		Depth over breadth?		Doubts and difficulties wrt resources
Equipment/Supplies					Can't test bisections				
Textbook--See TX							Lack of worked examples, not good at following		
School curriculum--See SC					Why tessellations? When division of fractions?				
Math Ed people--See ME					Wouldn't like what she says		Wouldn't approve of methods		

Table 3. Coding Summary Example

Coding Summary Sample, continued

Detail for DOUBTS	Harriet	#	Bob	#	Alice	#	Matthew	#	Notes
Students			No experience with before, Do they tune out?		As good next time?, Why some were there?, Not reaching potential, some poisoning class		Doing too well, worry some might fail, not successful with all, respond to differences, getting them someplace new, determining what they know, figuring out motivations		Doubts and difficulties with students current and future
Future concerns			Those who leave with a low grade becoming teachers				Legacy, students still need technical things, reading a curriculum		
Next course	What is methods?								
Teaching	About what does or doesn't work in a classroom, memory rhyme				Cancelling by common sense?, managing fractions, misleading by simplification? Discussing pedagogical issues		Lesson planning, RSA, fractions, absurdness of improper fractions, basic ideas, invert and multiply		Doubts and difficulties in the classroom
Classroom methods					Branching off, Disapprove of project(1)--but successful anyway, using both paper and computer submissions, puzzles throughout		Non-expert, projects difficult, nothing in his box, no stories, blank, how to teach, stories, pretests?, "not nice" examples, having a plan, groupwork?, invention tasks		
Tests/Evaluation					What should she test, bisections difficult to test		How many midterms, anxious about first, projects difficult		
This course	Whether how people learn is part of this course?, relevance of topics						Difference between this and Finite Math, what to do, first time, at a loss, orphaned, content, how to cast, why two?, needs rethink, not misleading with half-formed objectives		Doubts and difficulties about the nature of the course
Math content included--See CC					Boxplots, probability				
Knowledge			Knowledge of fractions for teachers?		What teachers need, to judge approaches				

Table 3 continued

APPENDIX E: CODING RESULTS

The table below shows the final results of the coding, including concept codes, themes, and broad categories.

CODING RESULTS		
<p>THE INSTRUCTORS</p> <p>Instructor Knowledge and Experience</p> <p>IBE Instructor Background Education IBW Instructor Background Work IBP Instructor Background Personal PC Personal Characteristics CKI Instructor Content Knowledge PCKI Instructor PCK LP Learning/teaching Philosophy</p> <p>Instructor Activities and Methods</p> <p>TM Teaching Methods MP Manipulatives TW Team Work MF Math Fair EV Evaluation TC Technology</p> <p>Instructor Resources</p> <p>TX Textbook COI Colleagues of the Instructor ME Mathematics Education SC School Curriculum</p> <p>Instructor Relationship with Students</p> <p>R Instructor Role CMI Instructor Communication QS Querying Students US Understanding/Rapport GN Gentleness/Encouragement</p> <p>Instructor Concerns</p> <p>P Priorities W Wishes D Doubts IE Emotions/Expectations RFI Instructor Reflection IL Instructor Learning WLI Workload for Instructor</p>	<p>THEIR STUDENTS</p> <p>Student Identity and Affect</p> <p>STC Student Characteristics SAC Student Attitude to Course SA Student Attitude to Math MA Math Anxiety SE Other Student Emotion B Barriers RE Resistance</p> <p>Desired Student Knowledge</p> <p>CK Content Knowledge DCK Deep Content Knowledge PCK Pedagogical Content Knowledge CRK Curricular Knowledge</p> <p>Desired Student Affect</p> <p>I Independence C Confidence EM Empathy EN Enjoyment AP Appreciation ENG Engagement</p> <p>Student Activities</p> <p>SX Sharing Experiences KU Keeping Up H Seeking help LA Approach to Learning WL Workload AC Other Actions</p>	<p>THE COURSE</p> <p>Structure</p> <p>CC Course Content CD Course Design IR Institutional Realities T Time G Gatekeeper LEG Legacy</p> <p>Mathematics</p> <p>M Math & Mathematicians MI Mathematics Image MN Math & Other Norms PR Proof PS Problem Solving CN Connections N Novelty</p> <p>Preparing Teachers</p> <p>FT Future Teachers TMD Teaching about Methods ET Elementary Teachers CM Communication RF Reflection BI Buy-in MO Motivation LL Later Learning SYS System LNC Links to the Community CO Colleagues for Students PST Past Students</p>
	<p>Comparisons with Others</p> <p>CKO Others Content Knowledge MAO Math Anxiety Other Students OI Other institutions TMO Teaching Methods for Others EVO Evaluation of Others O Other Students</p>	

Table 4. Coding Results