

ASSET CORRELATION AND CREDIT QUALITY

by

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Abstract

In this paper, the estimation procedure developed by Jones and Zanganeh (2011) is expanded to a multifactor structure. That is, maximum likelihood estimates of parameters of blockwise equicorrelated Wiener processes observed at discrete time intervals are presented. The estimation procedure then is used to provide a likelihood ratio test for the relation between asset correlation and default probability which is assumed to be negative in Basel II Accord. Using monthly stock prices (December 2002 to March 2011) of North America Oil & Gas, Technology and Industrials companies, we find this relation tends to be positive. We also observe some systematic impacts since the financial crisis on the behaviour of stock prices. Volatility and correlation have substantially increased from the second quarter of 2008 which is followed by a subsequent decline toward the end of the period.

keywords: Asset correlation; Default probability; Equicorrelation; Maximum likelihood estimate; Basel II; Credit Quality

To my family

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1

Introduction

Average asset correlation plays a critical role in determining regulatory capital requirements in the Basel II *Internal Rating-Based* (IRB) approach suggested by Basel Committee on Banking Supervision (BCBS) (2005). Basel committee assumes a negative relationship between average asset correlation and probability of default. In spite of its critical importance in determining the capital requirements, few studies have tried to investigate the empirical validity of this assumption. The first goal of this paper is to provide a statistically legitimate test for this relationship and shed light on the empirical validity of the Basel II assumption.

Correlation modeling lies at the heart of portfolio selection, portfolio risk management, and pricing of the derivatives whose value depend on more than one underlying variable (e.g. dispersion trading positions, CDOs, n-th to default swaps and index tranches). Tractable correlation structures and matching estimation methods are essential in this wide range of financial applications. The second purpose of this paper is to contribute in this area by expanding Jones and Zanganeh (2011) framework to a multi-sector structure. They present maximum likelihood estimators for the parameters of symmetric, correlated Weiner processes observed at discrete intervals. We derive the likelihood function for a more general form by allowing for multiple sources of correlation. This structure allows for a more careful study of correlation between financial assets. As an application, we use the estimation procedure to build a likelihood ratio test for the relationship between asset correlation and credit quality.

The paper is organized as follows. In section I, the relationship between asset correlation and probability of default in the context of Basel II Accord and the existing empirical literature on this relationship is briefly reviewed. The candidate correlation structure in

continuous time is set out, and its discrete time form and likelihood function corresponding to feasibly observable data is obtained in section II. Asymptotic standard errors are also obtained in this section. Simulation testing is performed in section III. Section IV applies the proposed procedure on a data set of stock price returns to investigate the empirical relationship between asset correlation and credit quality. Concluding remarks come in the last section.

2

Asset correlation and probability of default in Basel II

In the context of Basel II *Internal Rating-Based* (IRB) approach suggested by BCBS (2005), average asset correlation plays a critical role in determining capital requirements for banks loan portfolios. For corporate, sovereign and banking exposures, average asset correlation is calculated as

$$\bar{\rho} = 0.12 \times \frac{1 - e^{-50 \times p}}{1 - e^{-50}} + 0.24 \left[1 - \frac{1 - (1 - e^{-50 \times p})}{1 - e^{-50}} \right] \quad (2.1)$$

where $\bar{\rho}$ and p are average asset correlation and probability of default, respectively. Each borrower is assigned a rating grade and the average default probability p for a grade (which could be calculated via certain methods) is used for all the borrowers sharing the same grade. Then $\bar{\rho}$ refers to the average asset correlation of the borrowers in the same rating grade. This formula implies a negative relationship between average asset correlation and probability of default. Average asset correlation then is used in calculation of capital requirements

$$K = \left(LGD \times N \left[\frac{N^{-1}(p)}{\sqrt{1 - \bar{\rho}}} + N^{-1}(0.999) \times \sqrt{\frac{\bar{\rho}}{1 - \bar{\rho}}} - p \times LGD \right) \cdot \frac{1 + (M - 2.5)b}{1 - 1.15b} \right. \quad (2.2)$$

where K is capital requirement, LGD is loss given default, M represents effective maturity and b is the maturity parameter. Lee et al. (2009) show that the negative relationship in equation (2.1) has a dampening impact on the capital requirement specially at higher levels of default probability. In spite of the critical importance of equation (2.1) in determining the capital requirements, few studies have investigated the empirical validity of this

relationship.

Lopez (2004), as the most cited study in this area, calibrates average asset correlations using Moody's KMV Portfolio ManagerTM software for different portfolios created by Expected Default Frequency (EDF), asset size or both. He then observes the pattern of calibrated $\bar{\rho}$ across the portfolios which reveals to be declining as EDF (as a measure of probability of default) increases. However, he does not test the statistical significance of the observed pattern.

Dietsch and Petey (2004) use a one-factor model to estimate default probabilities and asset correlations for French and German small and medium-size enterprises (SMEs), and to investigate the empirical relationship between asset correlation and probability of default. What they find is that asset correlation generally increases with probability of default, unlike Basel committee assumption. Lee et. al. (2009) provide empirical evidence and economic arguments against this assumption. Using Moody's KMV asset returns proprietary database and Expected Default Frequency (EDF), they find little evidence of a negative relationship between asset correlation and default probability for Corporate, Commercial Real Estate and Retail exposures. From the economic perspective, they argue that if default is caused by a systematic negative shock, firms' asset values may show higher correlation causing higher default correlation. Therefore, firms close to default are not necessarily more subject to idiosyncratic shocks.

In the context of the structural approach to default modeling, due to Merton (1974), default correlation between any two firms is modeled based on correlation between their asset values. In their basic form, these models characterize default as the event that asset value falls below the firm's obligations at maturity. Asset values are assumed to follow correlated Geometric Brownian motions, with common factors being the source of correlation. Comovement of asset values causes correlation in default events. E.g., if the realization of the common factor is bad, firms' asset values tend to move down causing the default probability of each firm and their joint default probability¹ to rise. Therefore, the higher is the correlation between asset value of any pairs of firms, the higher will be their joint default probability, so the default correlation. In this sense, Basel II assumption would imply a negative relationship between probability of default and default correlation.

Using historic default data, Lucas (1995) finds that default correlation generally decreases as credit rating improves. Similar results are obtained by Zhuo (2001) based on a

¹Default events are independent conditional on the common factor, implying the joint probability of default being equal to the product of marginal default probabilities.

first-time-passage model. Using a large portfolio of residential subprime loans and computing a block diagonal equicorrelation matrix, Cowan and Cowan (2004) also show that default correlation increases as the internal credit rating declines. We contribute in this debate by providing a likelihood ratio test for the decreasing relationship assumed in Basel II Accord.

3

Candidate equicorrelation structure

In this section, the continuous time (single-factor) equicorrelation structure and the estimation procedure presented by Jones and Zanganeh (2011) is expanded to a multi-factor case. The resulting structure represents a continuous time blockwise equicorrelation model with multiple sources of correlation. Suppose asset value of firm $i = 1, \dots, n_j$ belonging to sector $j = 1, \dots, m$, $x_{ij}(t)$ evolves in continuous time according to the following stochastic differential equation

$$dx_{ij} = \alpha_{ij}(x, t)dt + \sigma_{ij}(x, t)d\bar{z}_{ij} \quad i = 1, \dots, n_j \text{ and } j = 1, \dots, m \quad (3.1)$$

in which α_{ij} and σ_{ij} are drift and volatility functions, respectively, and $d\bar{z}_{ij}$ are increments in standard Weiner processes with correlations $\rho_{ij}(x, t)$ between them.

This general specification imposes $\sum_j n_j (\sum_j n_j + 3)/2$ parameters (in drift vector and covariance matrix) to be estimated. Estimation of the above structure requires observations on complete set of assets over large enough number of time intervals. The difficulty arises when the number of firms, $\sum n_j$, is large with incomplete time series for individual firms. We thus assume that firms are grouped so that the drift and volatility functions, α_{ij} and σ_{ij} , are the same for all firms in the same sector. Some structure should also be imposed on the correlation between different firms.

There are strong reasons to believe in a higher degree of comovement between firms from the same sector (which could be defined as countries, industries, rating categories, etc) than between those from different sectors. This could be addressed by introducing a general and some sector specific common factors in (3.1). Specifically, we consider three types of factors: (i) z_o which is common to all firms x_{ij} . This common factor gives rise to correlation between the movements of all x_{ij} ; (ii) z_j ($j = 1, \dots, m$) which are common

within each sector. These factors give rise to correlation between the movements of x_{ij} within sector j ; and (iii) z_{ij} which are idiosyncratic to firm ij . Therefore, the following particular structure is considered

$$dx_{ij} = \kappa_j(\mu_j - x_{ij})dt + \sigma_j(\rho_0^{1/2}dz_0 + \rho_j^{1/2}dz_j + (1 - \rho_0 - \rho_j)^{1/2}dz_{ij}) \quad (3.2)$$

in which the $z_0(t)$, $z_j(t)$, $z_{ij}(t)$, $i = 1, \dots, n_j$, $j = 1, \dots, m$ are *independent* standard Weiner processes. The parameters $\kappa_j, \mu_j, \sigma_j, \rho_0, \rho_j$ are constants and $\rho_0 + \rho_j, \rho_0$ each in $[0, 1]^1$. This structure represents a mean-reverting case, and constant and zero drift models obtain as its special cases. This will be clarified in the next section.

Discrete time likelihood function

In this section the discrete time form of candidate processes (3.2) and likelihood function are derived to correspond with feasibly observable data. Assume the N -vector $x(t)$ follows a linear continuous time, constant coefficient process given by

$$dx = (Kx + c)dt + dz \quad \text{with} \quad E(dzdz') = \Omega dt \quad (3.3)$$

K and c are an $N \times N$ matrix and a column N -vector of constants ($N \equiv \sum n_j$), respectively. Following Wymer (1972) the exact discrete time process for x is given by

$$x(t+h) = e^{hK}x(t) + K^{-1}[e^{hK} - I]c + \eta_t \quad \text{where} \quad \eta_t \sim N(0, \int_0^h e^{\tau K} \Omega e^{\tau K'} d\tau) \quad (3.4)$$

I denotes the $N \times N$ identity matrix. According to equation (3.4), the distribution of $x(t+h)$ conditional on $x(t)$ is joint normal. For the model given in equation (3.2) with $x(t)$ denoting the stacked vector of $x_{ij}, i = 1, \dots, n_j, j = 1, \dots, m$, the above components are given as

$$K = \begin{pmatrix} -\kappa_1 I_{n_1} & 0 & \dots & 0 \\ 0 & \ddots & & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & -\kappa_m I_{n_m} \end{pmatrix} \quad c = K \begin{pmatrix} \mu_1 e_{n_1} \\ \mu_2 e_{n_2} \\ \vdots \\ \mu_m e_{n_m} \end{pmatrix} \quad (3.5)$$

¹This last condition on the correlation parameters is made to ensure that the covariance matrix is positive definite. However, the covariance matrix stays positive definite even if correlation parameters are somewhat negative. E.g., for the two sector case, we just need to have $\bar{\rho}_j \in (\frac{-1}{n_j-1}, 1), j = 1, 2$ and $\rho_0 \in \left(-\left(\frac{(\bar{\rho}_1(n_1-1)+1)(\bar{\rho}_2(n_2-1)+1)}{n_1 n_2} \right)^{1/2}, \left(\frac{(\bar{\rho}_1(n_1-1)+1)(\bar{\rho}_2(n_2-1)+1)}{n_1 n_2} \right)^{1/2} \right)$ where $\bar{\rho}_j \equiv \rho_0 + \rho_j$.

where I_{n_j} is the identity matrix of size n_j and e_j is an n_j -vector of ones. Define $\bar{\rho}_j \equiv \rho_0 + \rho_j$, $J_{n_j n_{j'}} = e_{n_j} e'_{n_{j'}}$, $\sigma_{jj'} = \sigma_j \sigma_{j'}$. The covariance matrix Ω is then expressed as

$$\Omega = \begin{pmatrix} \sigma_1^2(1 - \bar{\rho}_1)I_{n_1} & 0 & \dots & 0 \\ 0 & \ddots & & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & \sigma_m^2(1 - \bar{\rho}_m)I_{n_m} \end{pmatrix} + \begin{pmatrix} \sigma_1^2 \bar{\rho}_1 J_{n_1 n_1} & \sigma_{12} \rho_0 J_{n_1 n_2} & \dots & \sigma_{1m} \rho_0 J_{n_1 n_m} \\ \sigma_{21} \rho_0 J_{n_2 n_1} & \ddots & & \sigma_{2m} \rho_0 J_{n_2 n_m} \\ \vdots & \vdots & & \vdots \\ \sigma_{m1} \rho_0 J_{n_m n_1} & \sigma_{m2} \rho_0 J_{n_m n_2} & \dots & \sigma_m^2 \bar{\rho}_m J_{n_m n_m} \end{pmatrix} \quad (3.6)$$

The covariance matrix Ω has a block-wise structure, with blocks $\Omega_{jj'}$ having the following form

$$\Omega_{jj'} = \sigma_j^2 [(1 - \bar{\rho}_j)I_{n_j} + \bar{\rho}_j J_{n_j n_{j'}}] \quad j = j' \quad (3.7)$$

$$\Omega_{jj'} = \sigma_{jj'} \rho_0 J_{n_j n_{j'}} \quad j \neq j' \quad (3.8)$$

The covariance matrix of $x(t+h)$ as in equation (3.4) is given by

$$\tilde{\Omega} \equiv \int_0^h e^{\tau A} \Omega e^{\tau A'} \quad (3.9)$$

with the diagonal blocks ($j = j'$)

$$\tilde{\Omega}_{jj} \equiv e^{-\tau \kappa_j} \Omega_{jj} e^{-\tau \kappa_j} d\tau = \Omega_{jj} \int_0^h e^{-2\tau \kappa_j} d\tau = \underbrace{\frac{1 - e^{-2h\kappa_j}}{2\kappa_j}}_{s_j} \sigma_j^2 [(1 - \bar{\rho}_j)I_{n_j} + \bar{\rho}_j J_{n_j n_j}] \quad (3.10)$$

and the off-diagonal blocks ($j \neq j'$)

$$\tilde{\Omega}_{jj'} \equiv e^{-\tau \kappa_j} \Omega_{jj'} e^{-\tau \kappa_{j'}} d\tau = \Omega_{jj'} \int_0^h e^{-\tau(\kappa_j + \kappa_{j'})} d\tau = \underbrace{\frac{1 - e^{-h(\kappa_j + \kappa_{j'})}}{(\kappa_j + \kappa_{j'})}}_{s_{jj'}} \sigma_{jj'} \rho_0 J_{n_j n_{j'}} \quad (3.11)$$

Substituting these relations in equation (3.4) and rearranging gives

$$x(t+h) - \begin{pmatrix} a_1 I_{n_1} & 0 & \dots & 0 \\ 0 & \ddots & & 0 \\ \vdots & & & \vdots \\ 0 & 0 & \dots & a_m I_{n_m} \end{pmatrix} x(t) - \begin{pmatrix} b_1 e_{n_1} \\ \vdots \\ b_m e_{n_m} \end{pmatrix} \sim N(0, \tilde{\Omega}) \quad (3.12)$$

where a_j and b_j are

$$a_j = e^{-h\kappa_j} \quad b_j = (1 - e^{-h\kappa_j})\mu_j \quad (3.13)$$

Now suppose that we have observations on the $N \equiv \sum n_j$ state variables at equally spaced times t_j , $j = 1 \dots T+1$. Define the N -vector

$$y_j \equiv x(t_{j+1}) - ax(t_j) - be \quad j = 1 \dots T \quad (3.14)$$

Being joint normally distributed, and independent because the x process is Markov and the time intervals do not overlap, the likelihood function for these observations is

$$L = \prod_{j=1}^T \frac{1}{(2\pi)^{N/2} |\tilde{\Omega}|^{1/2}} e^{-y_j' \tilde{\Omega}^{-1} y_j / 2} \quad (3.15)$$

Finally, we choose to work with the two times log-likelihood function given by

$$\Lambda \equiv 2 \ln L = -NT \ln(2\pi) - T \ln |\tilde{\Omega}| - \sum_{j=1}^T y_j' \tilde{\Omega}^{-1} y_j \quad (3.16)$$

The estimation procedure is performed by the numerical maximization of (3.16) with respect to the parameters $a_j, b_j, s_j, \rho_0, \rho_j$. This is equivalent to the maximization of the likelihood function L as given in (3.15). There will be $v \equiv 4m + 1$ parameters to be estimated when the state variables are grouped into m sectors. The continuous time parameters are retrieved from

$$\kappa_j = -\frac{1}{h} \ln a_j \quad \mu_j = \frac{b_j}{1 - a_j} \quad \sigma_j^2 = \frac{2s_j \ln a_j}{h(a_j^2 - 1)} \quad \rho_0 = \rho_0 \quad \rho_j = \rho_j \quad (3.17)$$

Correlation is given by $\bar{\rho}_j = \rho_0 + \rho_j$ for any pairs of x_{ij} from sector j and by ρ_0 for those from different sectors. The zero drift case obtains by setting $a_j = 1$ and $b_j = 0$. The constant drift case obtains by setting $a_j = 1$, estimating b_j , and obtaining the continuous time drift rate from $\mu_j = b_j/h$. In both zero drift and constant drift models $\sigma_j^2 = s/h$.

If certain regularity conditions are met, as the sample size T goes to infinity the asymptotic distribution of maximum likelihood estimates of the \mathbf{v} -vector of parameters $\theta \equiv (\mu_1, \dots, \mu_m, \kappa_1, \dots, \kappa_m, \sigma_1, \dots, \sigma_m, \rho_0, \rho_1, \dots, \rho_m)'$ is given as²

$$T^{1/2}\Phi_{\mathbf{v}}^{1/2}(\hat{\theta} - \theta) \xrightarrow{d} N(\mathbf{0}, I_{\mathbf{v}}) \quad (3.18)$$

where $\Phi_{\mathbf{v}}$ is the information matrix of size $\mathbf{v} \times \mathbf{v}$ defined as

$$\Phi_{\mathbf{v}} = -\frac{1}{T}E\left(\frac{\partial^2 \ln L}{\partial \theta \partial \theta'}\right) \quad (3.19)$$

L is the likelihood function evaluated at the true value of θ . We use Berndt, Hall, Hall and Hausmann (1974) (BHHE) method to estimate the information matrix. According to Judge (1985), $\Phi_{\mathbf{v}}$ can be numerically computed as

$$\frac{1}{T} \left[\sum_{t=1}^T \left(\frac{\partial \ln L_t}{\partial \theta} \right) \left(\frac{\partial \ln L_t}{\partial \theta} \right)' \right]_{\theta=\hat{\theta}} \quad (3.20)$$

where L_t denotes the probability density of the one-period observation y_t ³. Standard errors of the parameters are square roots of the diagonal elements of $(1/T)\Phi_{\mathbf{v}}^{-1}$.

²See Hamilton (1994, p.388-389).

³Computing $\Phi_{\mathbf{v}}$ based on numerical evaluation of the second order derivatives of the likelihood function as given in Hamilton (1994) behaves less satisfactorily.

4

Simulation testing

In this section a Monte Carlo simulation exercise is performed to test the estimation method and examine its small sample properties. A mean-reverting case is considered since the constant and zero drift models are obtained as special cases of this more general structure. Without loss of generality a two-industry structure is assumed as it is going to be used to investigate the empirical relationship between asset correlation and credit quality. Benchmark parameter values used to generate data for two industries a and b are $\kappa_a = \kappa_b = 1$, $\mu_a = \mu_b = 5$, $\sigma_a = 1$, $\sigma_b = .5$, $\rho_0 = .3$, $\rho_1 = .2$ and $\rho_2 = .4$, with quarterly observations $h = .25$. Number of diffusions in each industry is set to be 100 and 500 simulations/estimations are performed.

The simulation results are reported in Table 4.1. The average of each parameter estimate agrees closely with the true value used to simulate the data and no estimation bias is observed. Standard errors of the parameters using Berndt-Hall-Hall-Hausmann method along with sample standard deviations of the parameters are presented in Table 4.1. Evaluation of parameters covariance matrix from second derivatives of the likelihood function¹ results in less satisfactory estimates of the standard errors. As can be seen, BHHH standard errors well agree with the sample standard deviations and no obvious bias is observed. Note that the coefficient of variation (defined as the ratio of the standard deviation to the average parameter estimate) is least for κ and μ and highest for correlation parameters.

If one does not separate between within- and across-sector common factors and treat each sector individually, the model is reduced to two one-sector structure. In this case, the simulation/estimation procedure will be basically the same except that estimated correlation for sector j will be an estimate of $\rho_0 + \rho_j$.

¹For this method look at Hamilton (1994) page 388-389.

Table 4.1: Large sample simulation: mean reversion model for two-industry case

	κ	μ	σ_1	σ_2	ρ_0	ρ_1	ρ_2
input value	1.0000	5.0000	1.0000	0.5000	0.3000	0.2000	0.4000
avg. estimate	1.0018	5.0022	0.9985	0.4971	0.2956	0.2010	0.3987
minimum	0.9515	4.7314	0.9192	0.4369	0.1154	0.0665	0.2788
maximum	1.0871	5.2461	1.1578	0.5793	0.4344	0.4207	0.5936
sample st. dev.	0.0223	0.0837	0.0377	0.0230	0.0531	0.0468	0.0470
BHHH st. dev.	0.0237	0.0865	0.0384	0.0264	0.0557	0.0480	0.0496

100 observations of 100 diffusions. 500 Monte Carlo trials.

Now, let's look at the small sample properties of the estimators. To examine how the estimators behave as the number of observation intervals decreases, Monte Carlo simulations are performed for progressively fewer number of observations on 10 diffusions in each sector. The results are presented in Table 4.2. Some features are noticeable as T decreases. First, estimates of drift and mean-reversion parameters do not show any significant bias, while volatilities become mildly biased toward zero. However, estimates of correlations reveal considerable bias, with severe bias toward zero in ρ_0 , and mild upward and downward bias in ρ_1 and ρ_2 , respectively. Second, BHHH standard errors consistently overestimate the sample standard deviations. They rise at an increasing rate as T decreases. Finally, correlation estimates become statistically insignificant as T decreases, that is correlation structure may not be correctly inferred as the number of observation intervals declines.

Table 4.3 reports the simulation results for 30 observations on progressively decreasing number of diffusions per sector. Again, drift and mean-reversion terms do not reveal notable bias as N decreases. Bias in estimated volatility is considerably smaller compared to Table 4.2. One observation worth noting in this experiment is that no systematic bias occurs in estimated correlation as number of diffusions declines. Moreover, BHHH standard error are generally close to the standard deviations with the bias being negligible when $N \geq 10$.

Table 4.2: Varying length of time series with $n_1 = n_2 = 10$

	κ	μ	σ_1	σ_2	ρ_0	ρ_1	ρ_2
input value	1.0000	5.0000	1.0000	0.5000	0.3000	0.2000	0.4000
average: $T = 50$	1.0112	5.0002	0.9981	0.4951	0.2885	0.2051	0.3983
standard dev.	0.0845	0.1302	0.0604	0.0384	0.0788	0.0751	0.0742
BHHH st. dev.	0.0918	0.1330	0.0614	0.0406	0.0908	0.0818	0.0842
average: $T = 20$	1.0209	5.0244	0.9936	0.4862	0.2672	0.2143	0.3977
standard dev.	0.1243	0.2347	0.0984	0.0643	0.1322	0.1198	0.1231
BHHH st. dev.	0.1544	0.2749	0.1214	0.0786	0.1785	0.1598	0.1698
average: $T = 10$	1.0259	5.0750	0.9802	0.4683	0.2305	0.2204	0.3861
standard dev.	0.1955	0.5173	0.1446	0.0922	0.2058	0.1838	0.2044
BHHH st. dev.	0.4239	1.0239	0.3511	0.2198	0.5115	0.4563	0.5126
average: $T = 9$	1.0322	5.0803	0.9792	0.4704	0.2264	0.2213	0.3901
standard dev.	0.2225	0.6144	0.1557	0.0962	0.2309	0.2087	0.2316
BHHH st. dev.	0.5915	1.5213	0.5259	0.3433	0.8079	0.7170	0.8024
average: $T = 8$	1.0324	5.1686	0.9686	0.4588	0.2037	0.2264	0.3783
standard dev.	0.2530	0.8999	0.1640	0.1077	0.2396	0.2126	0.2523
BHHH st. dev.	3.9351	16.3251	2.6781	2.3709	3.6088	3.7705	3.4645

500 Monte Carlo trials (simulations/estimations) are performed for two sectors each containing 10 diffusions. The length of time series is progressively decreasing.

Table 4.3: Decreasing number of diffusions with $T = 30$

	κ	μ	σ_1	σ_2	ρ_0	ρ_1	ρ_2
input value	1.0000	5.0000	1.0000	0.5000	0.3000	0.2000	0.4000
average: $n_1 = n_2 = 50$ standard dev. BHHH st. dev.	1.0089 0.0586 0.0664	4.9961 0.1751 0.1797	0.9980 0.0704 0.0833	0.4918 0.0464 0.0575	0.2850 0.1063 0.1213	0.2044 0.0843 0.1048	0.3959 0.0942 0.1089
average: $n_1 = n_2 = 20$ standard dev. BHHH st. dev.	1.0069 0.0851 0.0956	5.0028 0.1779 0.1852	0.9972 0.0735 0.0855	0.4900 0.0479 0.0568	0.2803 0.1042 0.1242	0.2076 0.0896 0.1090	0.3984 0.0978 0.1125
average: $n_1 = n_2 = 10$ standard dev. BHHH st. dev.	1.0232 0.1019 0.1197	5.0030 0.1739 0.1899	0.9948 0.0790 0.0864	0.4910 0.0497 0.0568	0.2768 0.1056 0.1290	0.2082 0.0978 0.1163	0.3998 0.0970 0.1214
average: $n_1 = n_2 = 5$ standard dev. BHHH st. dev.	1.0227 0.1244 0.1467	5.0176 0.1773 0.1999	1.0019 0.0831 0.0929	0.4883 0.0517 0.0573	0.2903 0.1136 0.1377	0.1989 0.1080 0.1311	0.3864 0.1063 0.1332
average: $n_1 = n_2 = 2$ standard dev. BHHH st. dev.	1.0255 0.1365 0.1794	5.0200 0.1974 0.2119	0.9887 0.1006 0.1056	0.4832 0.0583 0.0578	0.2728 0.1389 0.1657	0.2106 0.1634 0.2067	0.3939 0.1573 0.1872

500 Monte Carlo trials (simulations/estimations) are performed for two sectors. There are 30 observations on progressively fewer number of diffusions.

5

Empirical analysis of asset correlation and credit quality

In this section, the estimation procedure developed above is used to investigate the empirical relationship between credit quality and asset correlation. The main restriction to the estimation procedure is that asset values are not observable. One solution to this problem is to estimate stock price return correlation as a proxy for asset value correlation. This approximation is used by Zhuo (2001) and also adopted by CreditMetrics, an industrial credit risk model. Moody's KMV instead uses the contingent claim approach to the pricing of securities and estimates asset values from the time series of stock prices and balance sheet data. More specifically, KMV assumes that the firm's capital structure is composed of equity, short-term debt which is considered equivalent to cash, long-term debt which is assumed to be a perpetuity, and convertible preferred shares¹.

In this paper, we follow the first solution and use stock price returns as a proxy for asset value returns to estimate a two sector model. Then a likelihood ratio test is built to test the negative relation imposed by Basel committee. The procedure of building the likelihood ratio test is as follows. First, the portfolio is grouped into investment and speculative grade classes, a two-factor model is estimated, and separate equicorrelation parameters are obtained for each category (i.e. $\rho_0 + \rho_1$ and $\rho_0 + \rho_2$, where ρ_1 and ρ_2 represent the within sector equicorrelation parameters for the investment grade and speculative grade firms, respectively). Then, average asset correlation for the whole portfolio is estimated by imposing a single-factor structure (i.e. $\rho_1 = \rho_2 = 0$). Since speculative grade firms have

¹Look at Kealhofer (2003) for a review of KMV methodology and see Crouhy et. al. (2000) for a comparative analysis of credit risk models.

Table 5.1: Stock log-return sample properties

	N	Mean	Volatility	Avg. Corr.	Min	Max
Oil&Gas whole	73	0.0827	0.3805	0.2897	-1.1352	1.8189
Oil&Gas investment	53	0.0640	0.3380	0.2694	-1.1352	0.7972
Oil&Gas speculative	20	0.1323	0.4931	0.3694	-0.9217	1.8189
Technology whole	51	0.0380	0.4076	0.3071	-1.3480	2.8641
Technology investment	35	0.0461	0.3339	0.2977	-1.0016	2.8641
Technology speculative	16	0.0203	0.5687	0.3505	-1.3480	2.3900
Industrials whole	42	0.0909	0.4068	0.3223	-1.4358	1.8948
Industrials investment	29	0.0717	0.3753	0.3457	-1.4358	1.8948
Industrials speculative	13	0.1338	0.4772	0.2773	-0.7724	0.9065

Sample means and volatilities are annualized.

supposedly higher probability of default, equation (2.1) would receive support from data if: (i) $\rho_0 + \rho_1 > \rho_0 + \rho_2$; and (ii) the likelihood ratio test results in the rejection of the one-factor (restricted) in favour of the two-factor (unrestricted) model. LR test statistic is χ^2 distributed with the degrees of freedom equal to the number of restrictions².

Evidence from stock prices

As previously mentioned, stock price returns correlation is estimated as a proxy for asset correlation. Three samples of the North America companies listed on NYSE are selected for this empirical investigation: Oil&Gas, Technology and Industrials sectors. List of the companies are obtained from NYSE Symbol file³, and monthly stock prices and S&P credit ratings from COMPUSTAT dataset. The sample only includes the companies with monthly stock prices and credit ratings recorded for the full period of December 2002 to March 2011, with no switch between investment and speculative grades during that period. The descriptive statistics of the data is given in Table 5.1. The average asset correlation reported

²The LR test statistics is given by $-2(\ln L_R - \ln L_{UR})$ with L_R, L_{UR} being the maximum value of the likelihood function of the restricted and unrestricted models, respectively.

³The file is available at: <http://www.nyxdata.com/Data-Products/NYSE-Group-Symbols-Package>. List of the sub-industries under each of the three selected industries is given in the Appendix

in the table is the average of the off-diagonal elements of the sample correlation matrix for each sector.

Table 5.2 represents the estimation results of the zero drift model for the three industries. The first and the third lines for each sector report the estimated parameters for the unrestricted and restricted ($\rho_1 = \rho_2 = 0$) models, respectively. The numbers in the parentheses are the BHHH standard errors. Since the estimates of volatility and equicorrelation are quite insensitive to the drift term assumption, we base our analysis on the zero drift model⁴. Estimated volatility and equicorrelation are generally significant. Volatility tends to be higher in the speculative grade companies, a pattern which is quite significant in Oil & Gas and Technology sectors while weaker among Industrials.

The main issue on which we are trying to shed light is whether the negative relationship between probability of default and asset correlation assumed in Basel II Accord gets support from data. Table 5.2 shows the total equicorrelation for investment grade ($\rho_{in} \equiv \rho_0 + \rho_1$) and speculative grade ($\rho_{sp} \equiv \rho_0 + \rho_2$) companies for each industry, along with the Likelihood Ratio test statistic and its P-value. For all the three industries under investigation, ρ_{sp} turns out to be bigger than ρ_{in} . However, the difference between them is only significant for Oil & Gas and Technology sectors.

It is worth pointing out that the difference between average sample correlations of Industrials investment and speculative grade classes (reported in Table 5.1) is quite large, suggesting a statistically significant difference between them. However, this first impression is rejected by LR test. For the two other industries, LR is statistically different from zero at any reasonable significance level.

To investigate the time varying characteristics of volatility and equicorrelation, rolling estimates of the zero drift model are performed and the results are reported in Figure 5.1. For this purpose, a 24-month window is selected which moves forward over 1-month steps. Therefore, the estimated volatility and equicorrelation for each month, say November 2004, are the estimates of the parameters over a 24-month period ending in November 2004. Some patterns are noticeable in the instantaneous volatility and equicorrelation estimates. In Technology and Industrials speculative classes, volatility starts to rise in September 2008. A similar pattern is observed in Industrials investment grade companies. The volatility rise starts earlier in Oil & Gas speculative, while it tends to stay stable in Oil & Gas and Technology investment classes. In summary, except for Oil & Gas and Technology in-

⁴Estimating the constant drift model shows that the drift term is statistically insignificant for all the samples. Moreover, none of major results would change if the constant drift specification is used.

Table 5.2: Estimation results for stock prices

	σ_1	σ_2	ρ_0	ρ_1	ρ_2	Λ
Oil&Gas	0.3578	0.5123	0.2826	-0.0215	0.0411	1.2881e+004
	(0.0056)	(0.0100)	(0.0246)	(0.0089)	(0.0107)	
	0.3593	0.5069	0.2744	-	-	1.2868e+004
	(0.0056)	(0.0081)	(0.0238)	-	-	
		$\rho_{in} = 0.2611$	$\rho_{sp} = 0.3237$	LR Stat = 26		P-value = 0.0000
Technology	0.3658	0.6043	0.2512	-0.0242	0.0516	7.9306e+003
	(0.0041)	(0.0094)	(0.0193)	(0.0118)	(0.0104)	
	0.3678	0.5973	0.2450	-	-	7.9166e+003
	(0.0043)	(0.0061)	(0.0181)	-	-	
		$\rho_{in} = 0.227$	$\rho_{sp} = 0.3028$	LR Stat = 28		P-value = 0.0000
Industrials	0.4054	0.5108	0.2866	-0.0005	0.0171	6.5730e+003
	(0.0061)	(0.0081)	(0.0202)	(0.0170)	(0.0149)	
	0.4058	0.5098	0.2879	-	-	6.5728e+003
	(0.0052)	(0.0058)	(0.0186)	-	-	
		$\rho_{in} = 0.2861$	$\rho_{sp} = 0.3037$	LR Stat = 4		P-value = 0.1353

vestment grade companies, higher volatility during the financial crisis is noticeable which is followed by a subsequent decline toward the end of the period.

Rolling estimates of equicorrelation also reveals some interesting points. First, equicorrelation starts to increase in the second quarter of 2008, which is characterized as the time the financial crisis began to hit the stock markets. This is somewhat lagged in the Industrials but occurs at a faster rate. Second, we observe that the relative size of correlation in investment and speculative grade classes changes over time following different patterns in each industry. Moreover, Industrials sector behaves quite differently from the two others. For Industrials ρ_{in} is bigger than ρ_{sp} for the most of the time until July 2008. It reverses then after to the end of the time period. Our conclusion is that Basel II Accord does not generally get support from data, and in fact correlation tends to decrease as credit quality improves.

As can be seen in Figure 5.1, investment grade companies look to be more correlated than the speculative grade ones within some periods. To test if this observation is statistically significant, for each industry the model is estimated over those time periods and LR test is performed. For Oil & Gas the model is estimated over September 2005 to October 2008⁵. The estimates of ρ_{in} and ρ_{sp} are 0.2347 and 0.2094, respectively. However, according to LR test, the difference between ρ_{in} and ρ_{sp} is statistically insignificant (with P-value being 0.4966). Note that the absolute size of the difference is not too small.

Estimating the model for Technology sector over the time period of November 2007 to March 2011, ρ_{in} and ρ_{sp} are obtained as 0.3639 and 0.343, respectively, with the difference being statistically significant (P-value = 0.0012). And finally for Industrials, investment grade category has significantly higher correlation than the speculative class during December 2002 to July 2008, with $\rho_{in} = 0.1653$, $\rho_{sp} = 0.1403$ and P-value = 0.0111.

Our conclusion about the sign of the relationship between asset correlation and default probability (credit quality) is that it changes over time and across industries, and in fact tends to be more of a positive relation. The above findings are generally not supportive of the Basel committee regulatory guideline. While Lopez (2004) finds evidences in support of the Basel committee assumption, there are other studies, like Lucas (1995), Zhuo (2001), Dietsch and Petey (2004), and Lee et. al. (2009), which show asset correlation or default correlation decreases as probability of default increases or credit quality deteriorates. Our findings add to this group of studies.

⁵Note that the parameters estimates recorded for August 2007 in Figure 5.1 is obtained from the sample of September 2005 to August 2007.

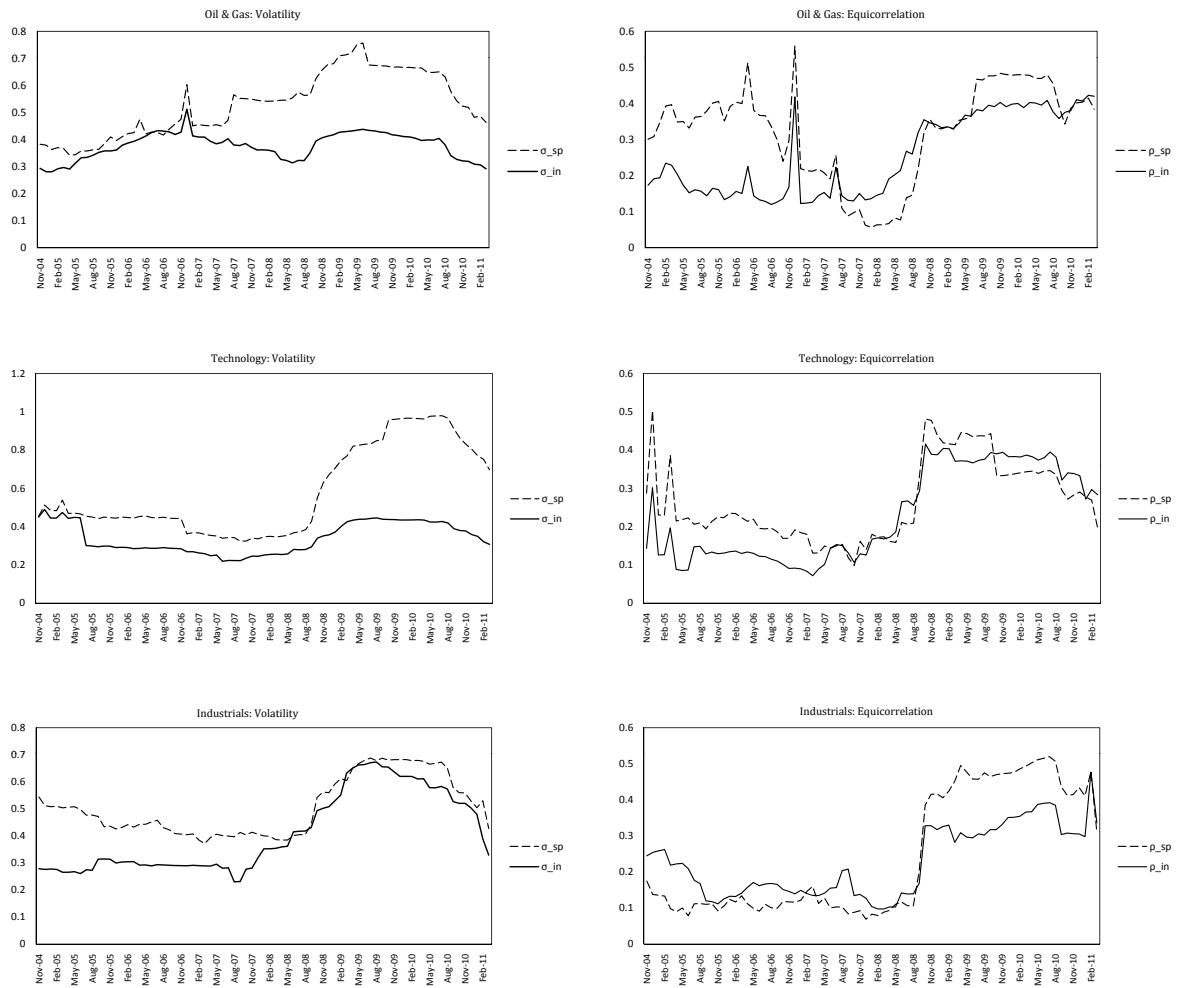


Figure 5.1: Rolling estimation of volatility and equicorrelation

6

Conclusion

This paper expanded the estimation procedure proposed by Jones and Zanganeh (2011) by allowing for multiple sources of correlation. One important benefit from having a more general framework is that it allows for more careful study of comovements of financial assets. The proposed estimation procedure serves variety of applications from portfolio selection to risk management and derivatives valuation. We then used a two-factor version of the model to investigate the empirical relationship between asset correlation and credit quality which is assumed to be negative by Basel committee.

Using stock price returns of North America Oil & Gas, Technology and Industrials companies to estimate asset correlation, we could not find support for this assumption. The relative size of asset correlation for investment and speculative grade companies changes over time and across industries. In fact, the relation between asset correlation and default probability (measured by lower credit quality) tends to be more of a positive one. Our findings are in line with the group of studies (e.g. Lee et. al. (2009), Dietsch and Petey (2004), Zhuo (2001), and Lucas (1995)) that show asset correlation (default correlation) tends to increase as probability of default rises. Rolling estimation of the model shows that volatility and equicorrelation have substantially increased from the second quarter of 2008, after the financial crisis hit the stock markets, which is followed by a subsequent decline toward the end of the time period. The rise is less obvious in Oil & Gas and Technology investment grade companies.

Finally, it is appropriate to add some caveats. The main restriction on the empirical investigation of the relationship between asset correlation and probability of default is that market value of a firm's assets is a latent variable. Following Zhuo (2001) and CreditMetrics methodology, we estimate stock price returns correlation as a proxy for asset correla-

tion. Moreover, we have assumed that there is no impact from firm size on the relationship between asset correlation and credit quality. Using properly estimated market value of firms' asset (like Moody's KMV proprietary asset value dataset) and taking into account of the size effect might provide more accurate insight into the topic.

Bibliography

- [1] Berndt, E. K., B. H. Hall, R. E. Hall and J. A. Hausman, “Estimation and Inference in Non-linear Structural Models,” *Annals of Economic and Social Measurement*, 3 (1974), 653-65.
- [2] Basel Committee on Banking Supervision, *International Convergence of Capital Measurement and Capital Standards. A Revised Framework*, Bank for International Settlements, November, 2005.
- [3] Cowan, M. Adrian, Charles D. Cowan, “Default Correlation: An Empirical Investigation of a Subprime Lender,” *Journal of Banking and Finance*, 28 (2004), 753-771.
- [4] Crouhy, Michel, Dan Galai, and Robert Mark, “A Comparative Analysis of Current Credit Risk Models,” *Journal of Banking and Finance*, 24 (2000), 59-117.
- [5] Dietsch, Michel, Joël Petey, “Should SME Exposures be Treated as Retail or Corporate Exposures? A Comparative Analysis of Default Probabilities and Asset Correlations in French and German SMEs,” *Journal of Banking and Finance*, 28 (2004), 773-788.
- [6] Engle, Robert, and Kelly Bryan, “Dynamic Equicorrelation,” Working paper, NYC Stern School of Business, 2009.
- [7] Hamilton, James D., *Times Series Analysis*, Princeton, New Jersey: Princeton University Press, 1994.
- [8] Jones, Robert and Mohammad Zanganeh, “Estimation of Correlated Diffusions from Incomplete Data: An Equicorrelation Structure,” Simon Fraser University, Working Paper, August (2011).
- [9] Judge, George. G., W. E. Griffiths, R. C. Hill, H. Lutkepohl and T. C. Lee, *The Theory and Practice of Econometrics*, New York: John Wiley & Sons, 1985.

- [10] Kealhofer, Stephen, "Quantifying Credit Risk I: Default Prediction" *Financial Analysts Journal*, 59 (2000), 30-44.
- [11] Lee, Joseph, Joy Wang, and Jing Zhang, "The Relationship Between Average Asset Correlation and Default Probability," Moody's KMV Research Paper, July (2000).
- [12] Lucas Douglas J., "Default correlation and credit analysis," *Journal of Fixed Income*, 4(4) (1995), 76-87.
- [13] Lopez, Jose A., "The Empirical Relationship between Average Asset Correlation, Firm Probability of Default, and Asset Size," *Journal of Financial Intermediation*, 13 (2004), 265-283.
- [14] Merton, R.C., "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates," *Journal of Finance*, 2 (1974), 449-470.
- [15] Wymer, C.R., "Econometric Estimation of Stochastic Differential Equation Systems," *Econometrica*, 40(3) (May 1972), 565-578.
- [16] Zhou, Chunsheng, "An Analysis of Default Correlations and Multiple Defaults," *The Review of Financial Studies*, 14(2) (2001), 555-576.

Appendix

List of sub-industries included in the samples

Oil & Gas: Gas Services-Distribution & Integrated Natural Gas Cos / Other Gas Services/ Oil And Gas-Integrated Domestic Refiners/ Oil And Gas-Services And Equipment/ Oil And Gas-Contract Drilling,Exploration/ Oil And Gas-Crude Production/ Gas Services-Natural Gas Transmission Companies/ Oil And Gas-Non-Integrated Refiners/ Oil And Gas-Integrated International Refiners

Technology: Electronics-Semiconductors And Other Components/ Telecommunications/ Computers, Data Systems-Computer Systems/ Computers, Data Systems-Data Processing, Software/ Electronics-Test, Control Instruments And Systems/ Electronics- Telecommunications Equipment/ Electronics-Other Systems And Equipment/ Computers, Data Systems-Peripheral Devices And Supplies

Industrials: Manufacturing/ Industrial Machinery And Equipment-Heavy Machinery/ Motor Vehicles-Parts And Equipment/ Industrial Machinery And Equipment-Transmissions And Engines/ Industrial Machinery And Equipment-Other Industrial Equipment/ Industrial Machinery And Equipment-Measuring And Control Devices/ Industrials/ Motor Vehicles-Auto And Truck/ Industrial Machinery And Equipment-Machine Tools