Spectral Analysis and Dynamical Behavior of Complex Networks

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Roadmap

- Introduction
- Internet topology and the BGP datasets
- Collection of BCNET BGP traffic
- Power-laws and the Internet topology
- Spectral analysis of Internet graphs
- Dynamics in complex networks
- Conclusions, future work, and references

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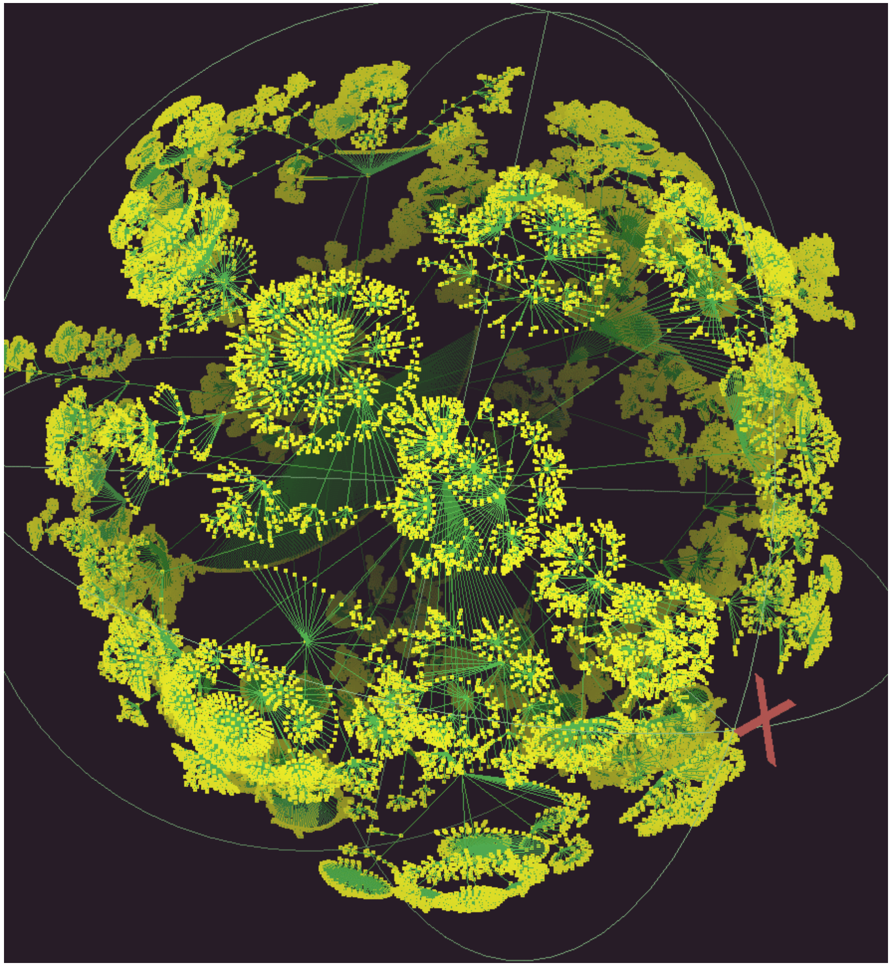
lhr (535,102 nodes and 601,678 links)



http://www.caida.org/home/

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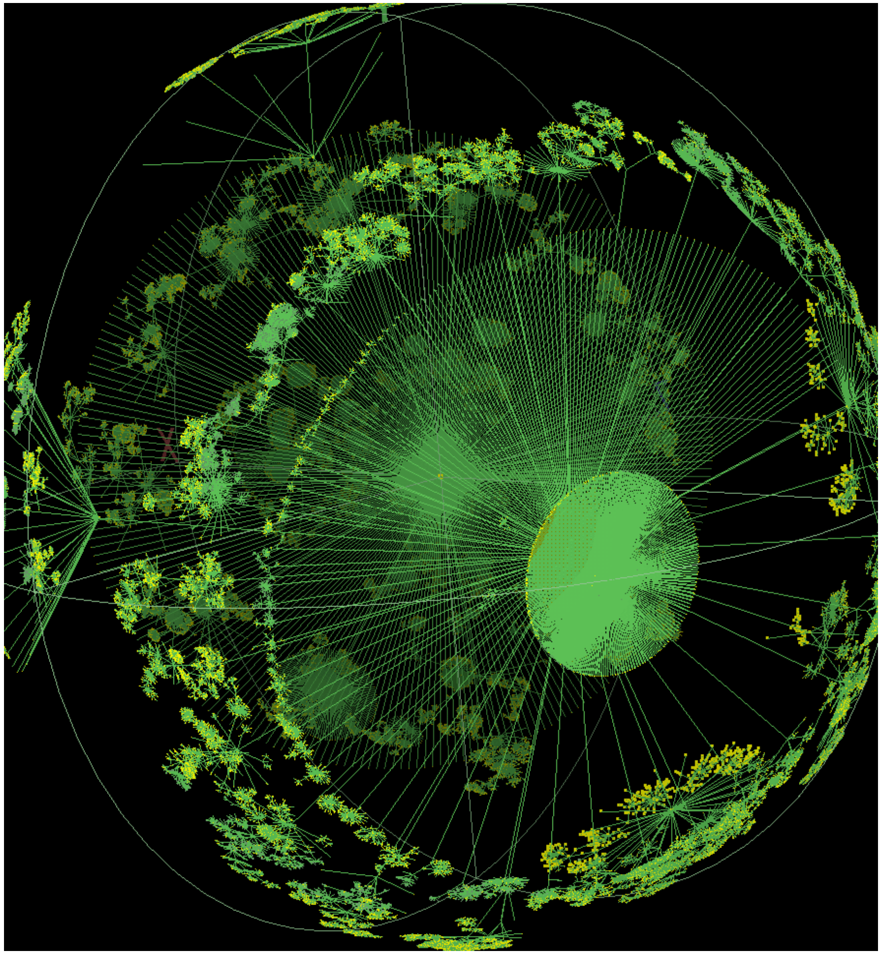

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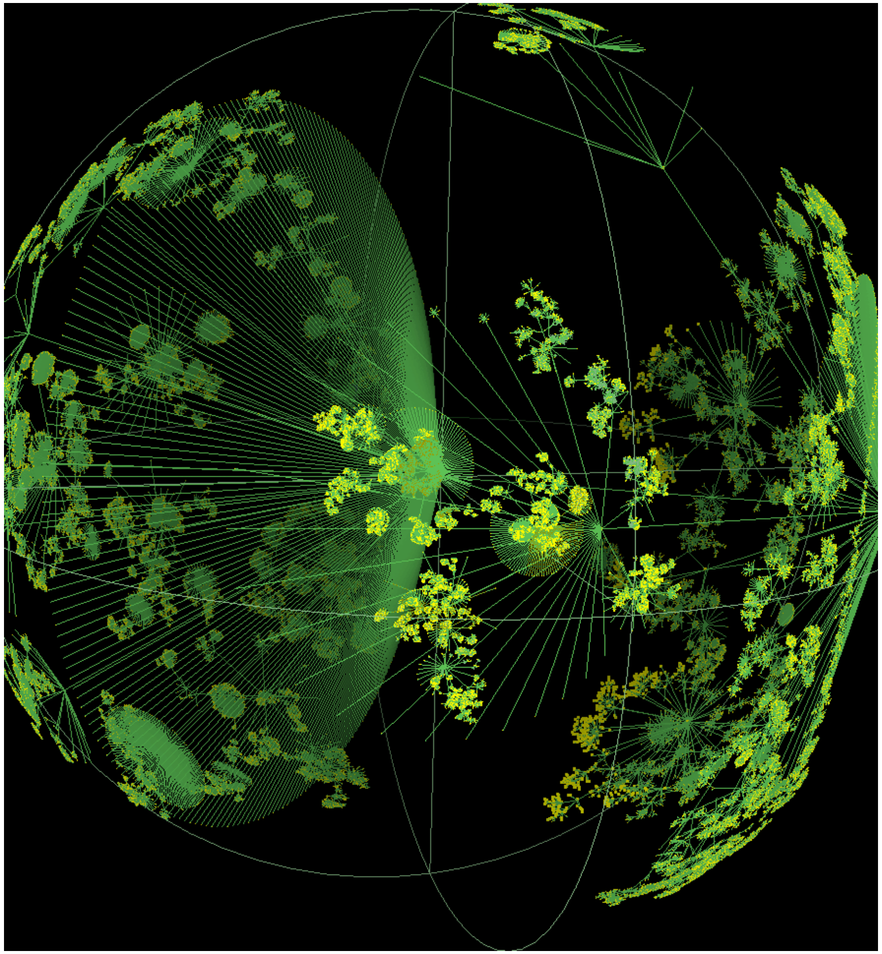

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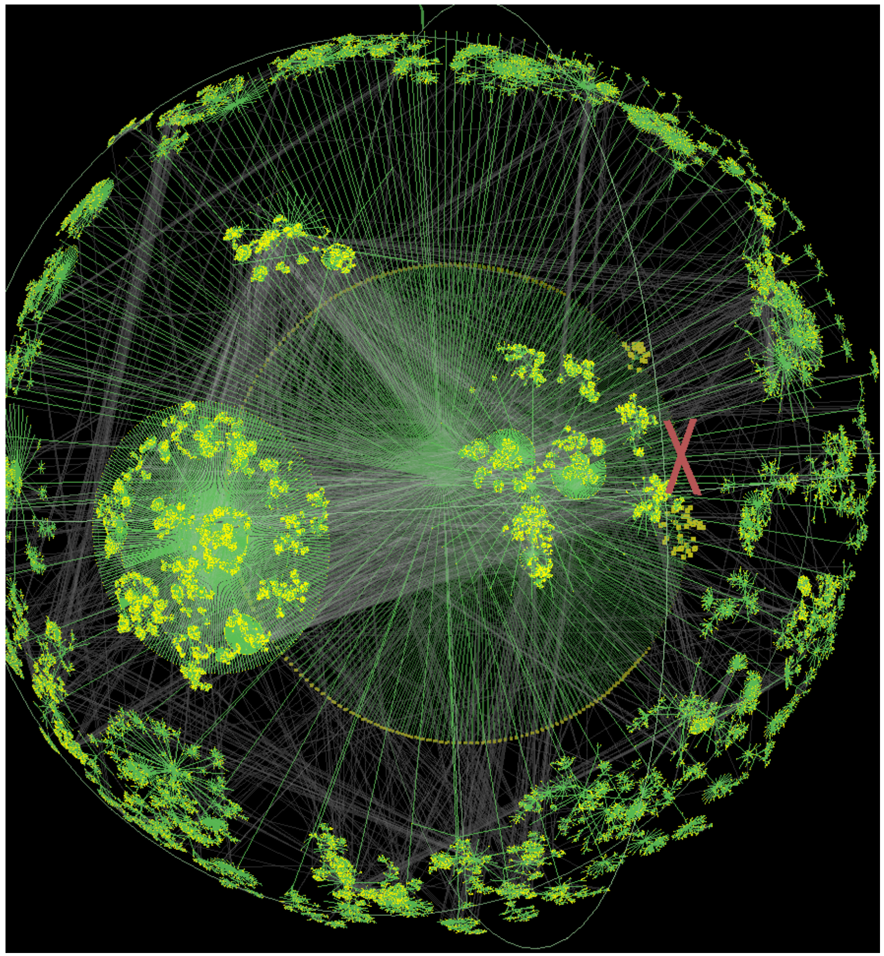

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Introduction

- Random graphs:
  - nodes and edges are generated by a random process
  - Erdős and Rényi model
- Small world graphs:
  - nodes and edges are generated so that most of the nodes are connected by a small number of nodes in between
  - Watts and Strogatz model
- Scale-free graphs:
  - graphs whose node degree distribution follow power-law
  - rich get richer
  - Barabási and Albert model

Introduction

- Many complex networks have universal characteristics:
  - small-world (Watts and Strogatz, 1998)
  - scale-free (Barabasi and Albert, 1999)
- Analysis of complex networks:
  - discovery of spectral properties of graphs
  - constructing matrices describing the network connectivity

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Internet graphs

- Internet is a network of Autonomous Systems:
  - groups of networks sharing the same routing policy
  - identified with Autonomous System Numbers (ASN)
- Autonomous System Numbers: http://www.iana.org/assignments/as-numbers
- Internet topology on AS-level:
  - the arrangement of ASes and their interconnections
- Analyzing the Internet topology and finding properties of associated graphs rely on mining data and capturing information about Autonomous Systems (ASes).

Internet routing protocol

- Border Gateway Protocol (BGP):
  - de-facto Inter Autonomous System routing
  - used to exchange network reachability information among BGP systems
  - reachability information is stored in routing tables
  - peer routers exchange four types of messages: open, update, notification, and keepalive
- BGP utilizes a path vector algorithm called the best path selection algorithm to select the best path
- BGP routing tables are publicly available and may be retrieved from the Route Views and Réseaux IP Européens (RIPE)

Internet AS-level data

Source of data are routing tables:

- Route Views: http://www.routeviews.org
  - most participating ASes reside in North America
- RIPE (Réseaux IP européens): http://www.ripe.net/ris
  - most participating ASes reside in Europe
- The BGP routing tables are collected from multiple geographically distributed BGP Cisco routers and Zebra servers.
- Analyzed datasets were collected at 00:00 am on July 31, 2003 and 00:00 am on July 31, 2008.

Analyzed datasets

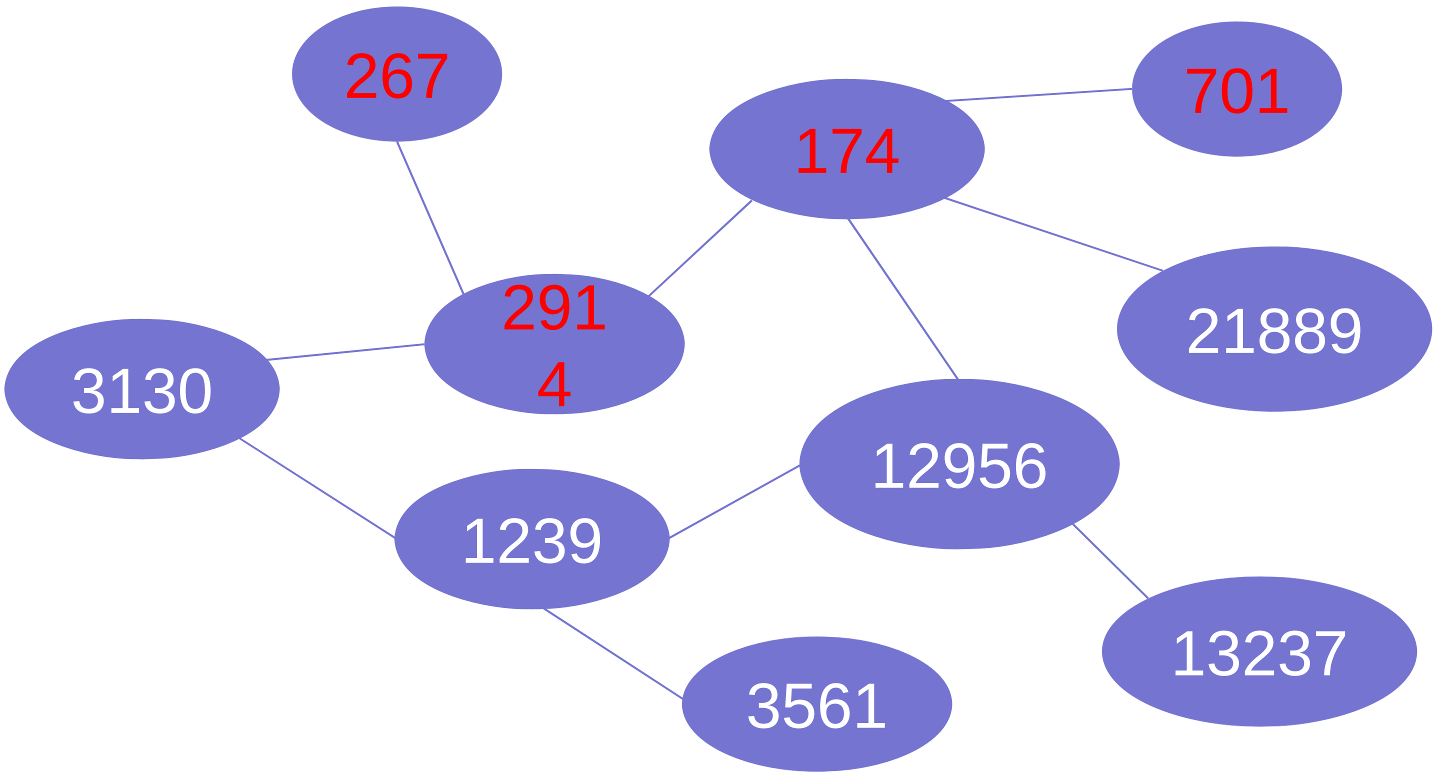
- Sample datasets:
  - Route Views:

    TABLE_DUMP| 1050122432| B| 204.42.253.253| 267| 3.0.0.0/8| 267 2914 174 701| IGP| 204.42.253.253| 0| 0| 267:2914 2914:420 2914:2000 2914:3000| NAG| |
  - RIPE:

    TABLE_DUMP| 1041811200| B| 212.20.151.234| 13129| 3.0.0.0/8| 13129 6461 7018 | IGP| 212.20.151.234| 0| 0| 6461:5997 13129:3010| NAG| |

Internet topology at AS level

- Datasets collected from Border Gateway Protocols (BGP) routing tables are used to infer the Internet topology at AS-level.



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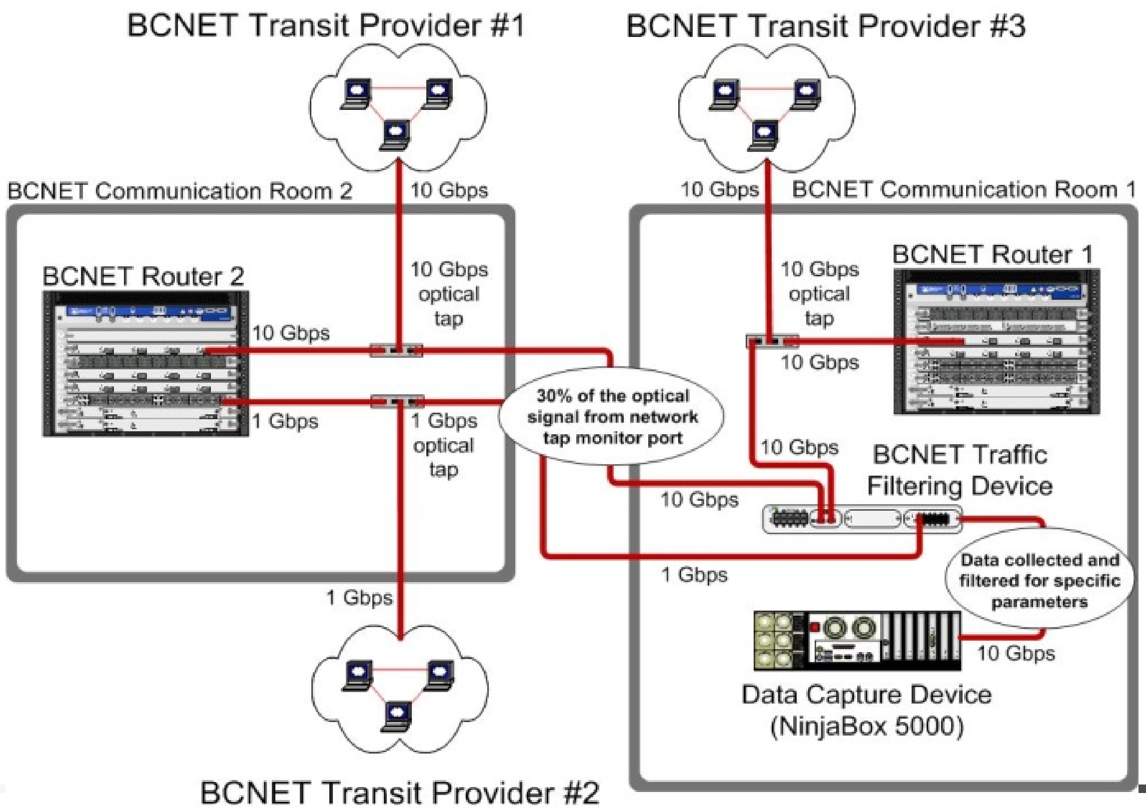
BCNET packet capture: physical overview

- BCNET is the hub of advanced telecommunication network in British Columbia, Canada that offers services to research and higher education institutions

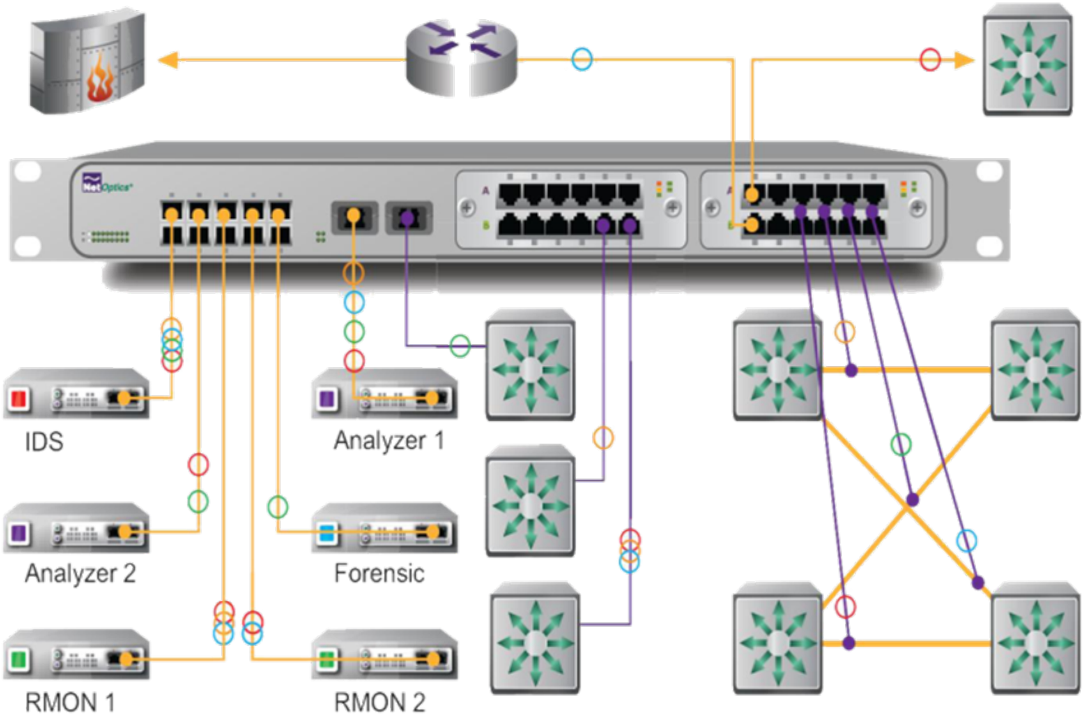


BCNET packet capture

- BCNET transits have two service providers with 10 Gbps network links and one service provider with 1 Gbps network link
- Optical Test Access Point (TAP) splits the signal into two distinct paths
- The signal splitting ratio from TAP may be modified
- The Data Capture Device (NinjaBox 5000) collects the real-time data (packets) from the traffic filtering device

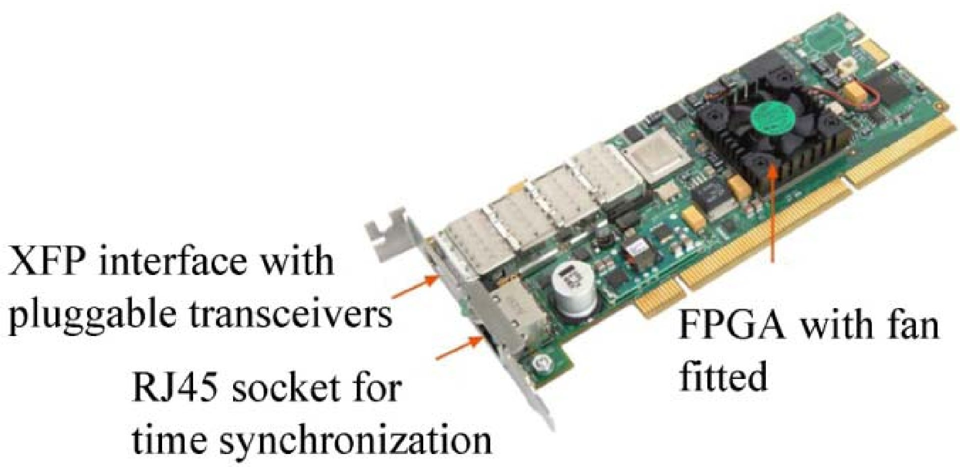
Net Optics Director 7400: application diagram

- Net Optics Director 7400 is used for BCNET traffic filtering
- It directs traffic to monitoring tools such as NinjaBox 5000 and FlowMon



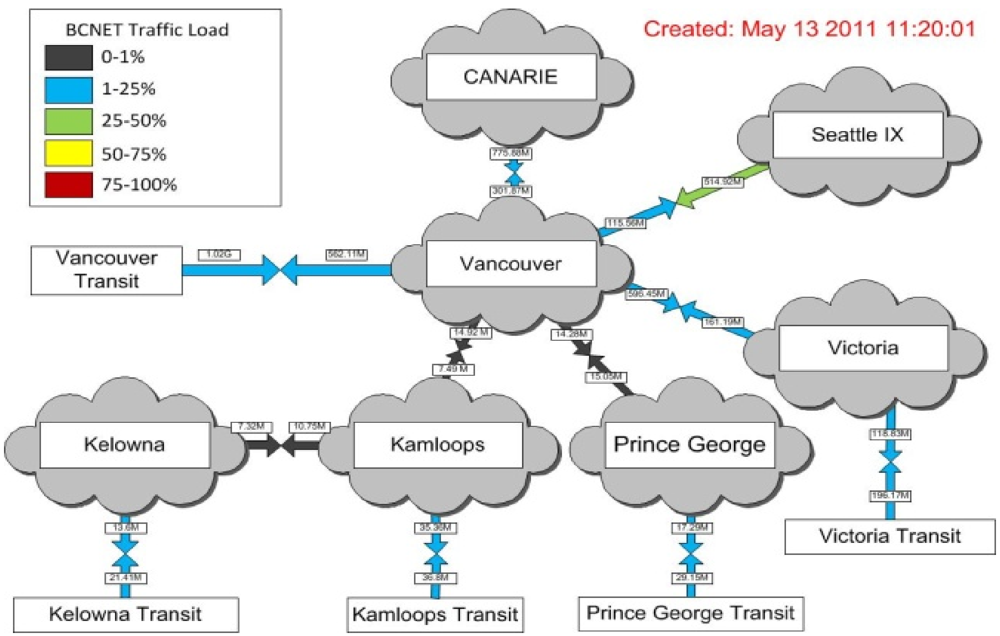
Network monitoring and analyzing Endace card

- Endace Data Acquisition and Generation (DAG) 5.2X card resides inside the NinjaBox 5000
- It captures and transmits traffic and has time-stamping capability
- DAG 5.2X is a single port Peripheral Component Interconnect Extended (PCIx) card and is capable of capturing on average Ethernet traffic of 6.9 Gbps



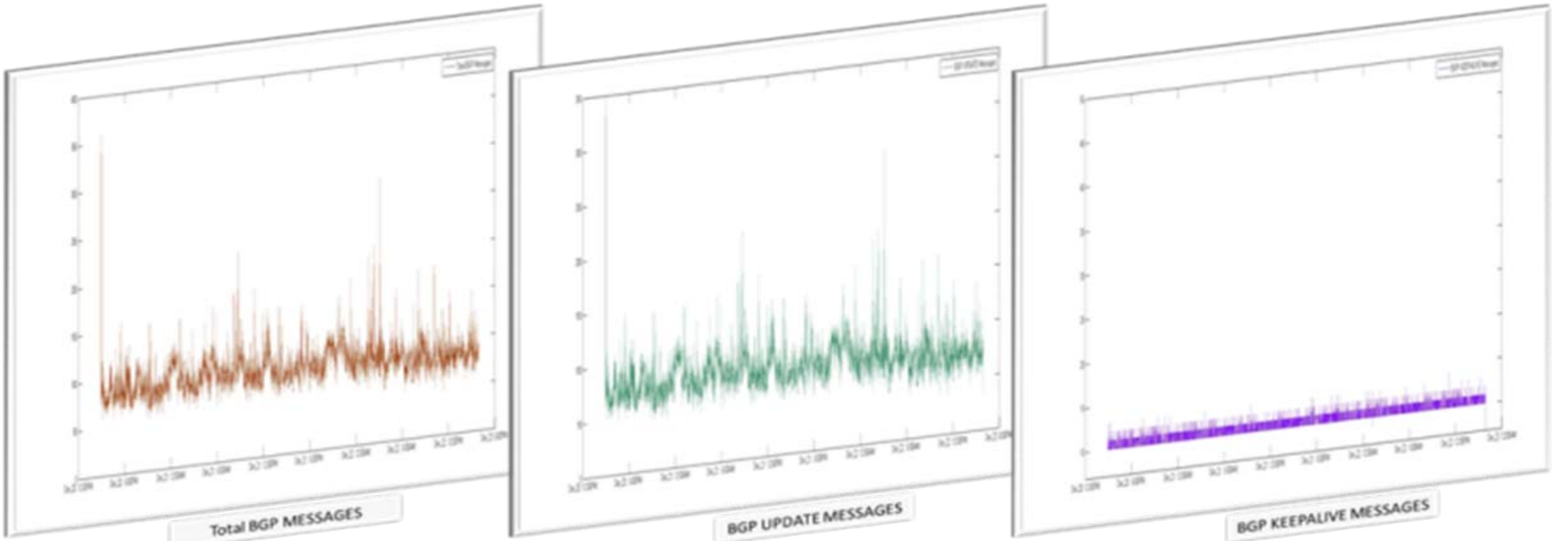
Real time network usage by BCNET members

- The BCNET network is high-speed fiber optic research network
- British Columbia's network extends to 1,400 km and connects Kamloops, Kelowna, Prince George, Vancouver, and Victoria



BGP messages

- Sample of display filters are: bgp.type, bgp.next_hop, bgp.origin, bgp.local_pref, bgp.community_as, bgp.as_path, and bgp.multi_exit_disc
- 88% are BGP update messages
- The remaining are keepalive messages



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Internet topology

- The Internet topology is characterized by the presence of various power-laws observed when considering:
  - node degree vs. node rank
  - node degree frequency vs. degree
  - number of nodes within a number of hops vs. number of hops
  - eigenvalues of the adjacency matrix and the normalized Laplacian matrix vs. the order of the eigenvalues.

M. Faloutsos, P. Faloutsos, and C. Faloutsos, 1999

G. Siganos, M. Faloutsos, P. Faloutsos, and C. Faloutsos, 2003

Internet matrices

- Adjacency matrix A(G):

$$A(i,j) = \begin{cases} 1 & \text{if } i \text{ and } j \text{ are adjacent} \\ 0 & \text{otherwise} \end{cases},$$

where i and j are the graph nodes.

- Normalized Laplacian matrix NL(G):

$$NL(i,j) = \begin{cases} 1 & \text{if } i = j \text{ and } d_i \neq 0 \\ -\dfrac{1}{\sqrt{d_i d_j}} & \text{if } i \text{ and } j \text{ are adjacent} \\ 0 & \text{otherwise} \end{cases},$$

where d_i and d_j are degrees of node i and j, respectively.

Power laws: node degree vs. rank

- The graph nodes v are sorted in decreasing order based on their degrees d_v and are indexed with a sequence of numbers indicating their ranks r_v.
- The (r_v, d_v) pairs are plotted on the log-log scale.
- The power-law implies:

$$d_v \propto r_v^R$$

where v is the node number and R is the node degree power-law exponent.

Power laws: CCDF of a node degree

- The frequency of a node degree is equal to the number of nodes having the same degree.
- The complementary cumulative distribution function (CCDF) D_d of a node degree d is equal to the number of nodes having degree less than or equal to d, divided by the number of nodes.
- The power-law implies:

$$D_d \propto d^{D}$$

where D is the CCDF power-law exponent.

Power laws: eigenvalues

- The eigenvalues λ_i of the adjacency matrix and the normalized Laplacian matrix are sorted in decreasing order and plotted versus the associated increasing sequence of numbers i representing the order of the eigenvalue.
- The power-law for the adjacency matrix implies:

$$\lambda_{ai} \propto i^{\varepsilon}$$

- The power-law for the normalized Laplacian matrix implies:

$$\lambda_{Li} \propto i^{L}$$

where ε and L are the eigenvalue power-law exponents.

Analysis of datasets

- Calculated and plotted on a log-log scale are:
  - node degree vs. node rank
  - frequency of node degree vs. node degree
  - eigenvalues vs. index
- The power-law exponents are calculated from the linear regression lines $10^a x^b$, with segment a and slope b when plotted on a log-log scale.
- Linear regression is used to determine the correlation coefficient between the regression line and the plotted data.
- A high correlation coefficient between the regression line and the plotted data indicates the existence of a power-law, which implies that node degree, frequency of node degree, and eigenvalues follow a power-law dependency on the rank, node degree, and index, respectively.

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Spectrum of a graph

- Spectrum of a graph is:
  - the collection of all eigenvalues of a matrix
  - closely related to certain graph invariants
  - associated with topological characteristics of the network such as number of edges, connected components, presence of cohesive clusters.

- If x is an n-dimensional real vector, then x is called the eigenvector of matrix A with eigenvalue λ if and only if it satisfies:

$$Ax = \lambda x,$$

where λ is a scalar quantity.

Spectrum of a graph

- The number of times 0 appears as an eigenvalue of the Laplacian matrix is equal to the number of connected components in a graph.
- Algebraic connectivity, the second smallest eigenvalue of a normalized Laplacian matrix is:
  - related to the connectivity characteristic of a graph
- Elements of the eigenvector corresponding to the largest eigenvalue of the normalized Laplacian matrix tend to be positioned close to each other if they correspond to AS nodes with similar connectivity patterns constituting clusters.

F. R. K. Chung, 1997

M. Fiedler, 1973

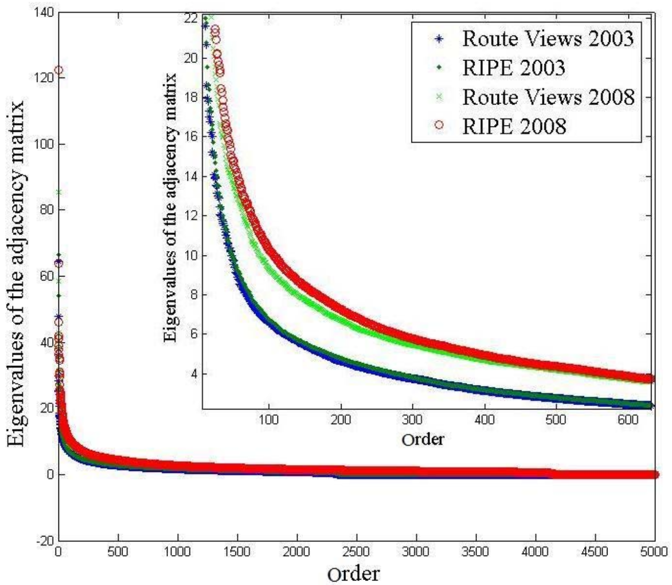
D. Vukadinovic, P. Huang, and T. Erlebach, 2001

Spectrum of a graph

- The eigenvectors corresponding to large eigenvalues contain information relevant to clustering.
- Large eigenvalues and the corresponding eigenvectors provide information suggestive to the intracluster traffic patterns of the Internet topology.
- We consider both the adjacency and the normalized Laplacian matrices.

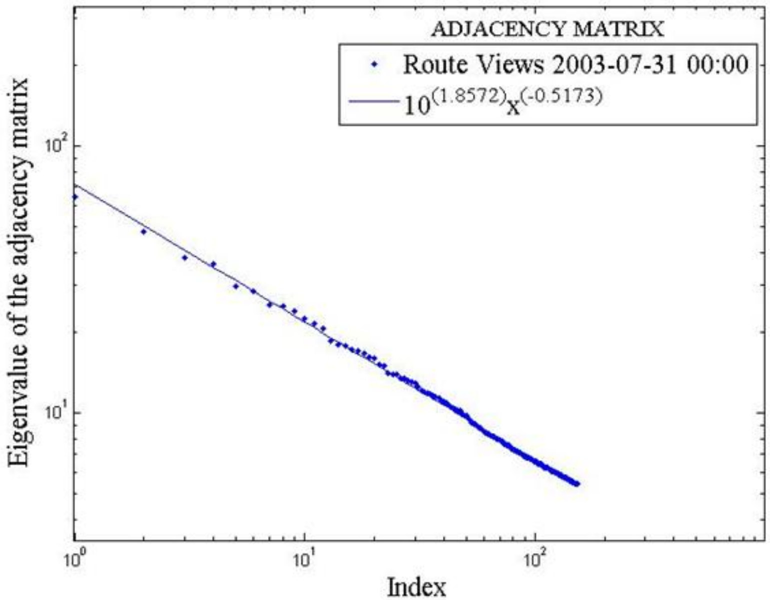
C. Gkantsidis, M. Mihail, and E. Zegura, 2003

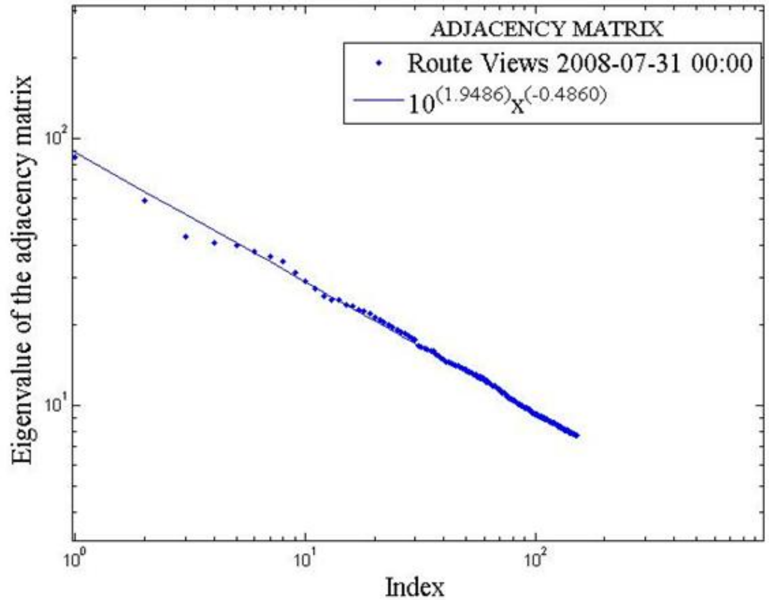
Eigenvalues of the adjacency matrix



order	Route Views 2003	Route Views 2008	RIPE 2003	RIPE 2008
1	64.30	85.43	66.65	122.28
2	47.75	58.56	54.19	63.94
3	38.15	42.77	38.24	46.14
4	36.23	40.85	36.14	41.98
5	29.88	39.69	31.21	41.08
6	28.50	37.85	27.38	38.93
7	25.47	36.21	26.41	37.94
8	25.06	34.66	25.06	36.47
9	24.13	31.58	23.86	35.08
10	22.51	29.34	23.32	34.47
11	21.61	27.40	22.02	30.97
12	20.69	25.69	21.77	30.54
13	18.58	25.00	20.75	29.68
14	17.94	24.82	19.55	27.03
15	17.78	23.89	18.67	25.74
16	17.31	23.69	18.42	25.35
17	16.99	22.81	17.85	24.83
18	16.75	22.46	17.44	24.30
19	16.22	22.04	17.24	24.06
20	16.01	21.36	16.63	24.00

Power laws: eigenvalues vs. index





Adjacency matrix:

- Route Views 2003 datasets: ε= -0.5713 and r= -0.9990
- Route Views 2008 datasets: ε= -0.4860 and r= -0.9982

ε= power-law exponent; r= correlation coefficient

Power laws: eigenvalues vs. index

Adjacency matrix:

- Route Views 2003 datasets: ε= −0.5713 and r= −0.9990
- Route Views 2008 datasets: ε= −0.4860 and r= −0.9982

ε= power-law exponent; r= correlation coefficient

Adjacency matrix:

- RIPE 2003 datasets: ε= −0.5232 and r= −0.9989
- RIPE 2008 datasets: ε= −0.4927 and r= −0.9970

ε= power-law exponent; r= correlation coefficient

Adjacency matrix:

- r > 99% for all datasets

r= correlation coefficient

Power laws: eigenvalues vs. index

Normalized Laplacian matrix:

- Route Views 2003 datasets: L= -0.0198 and r= -0.9564
- Route Views 2008 datasets: L= -0.0177 and r= -0.9782

L= power-law exponent; r= correlation coefficient

Normalized Laplacian matrix:

- RIPE 2003 datasets: L= -0.0206 and r= -0.9636
- RIPE 2008 datasets: L= -0.0190 and r= -0.9578

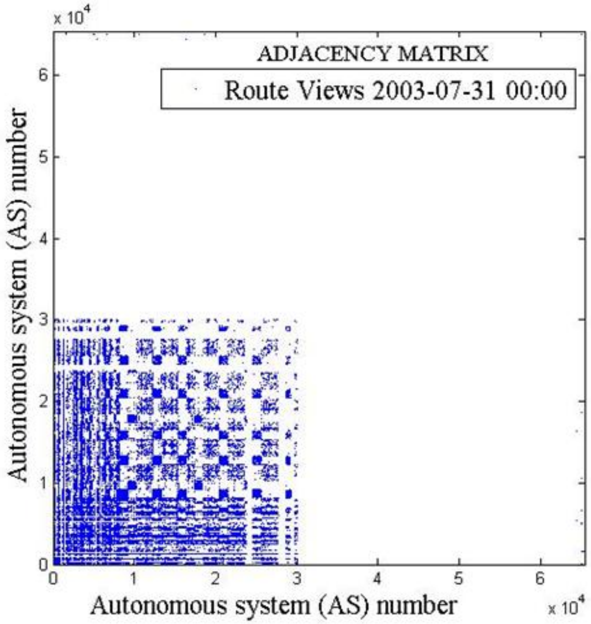
L= power-law exponent; r= correlation coefficient

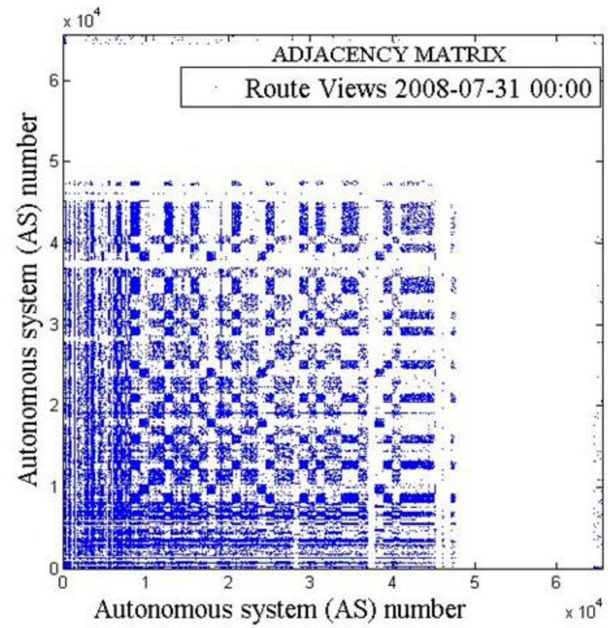
Normalized Laplacian matrix:

- r > 95% for all datasets

r= correlation coefficient

Clusters of connected ASes: Route Views





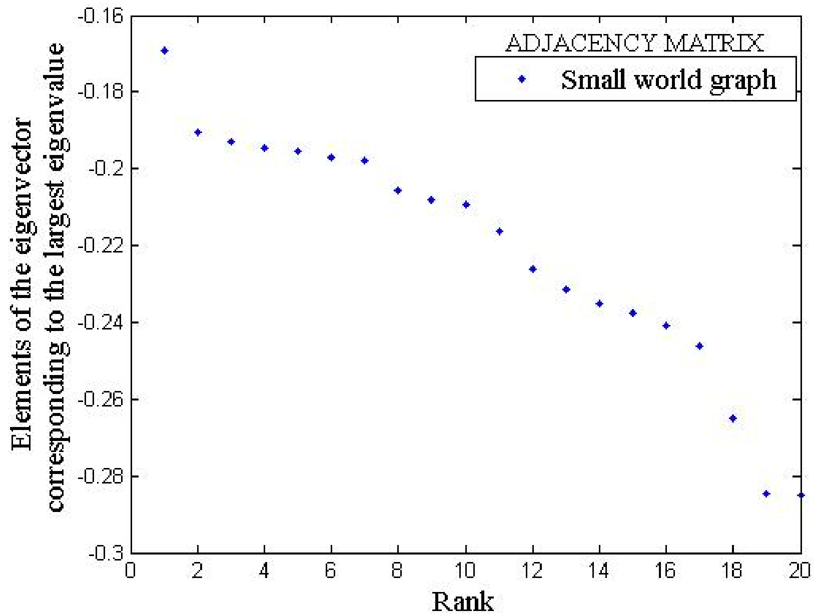
- A dot in the position (x, y) represents the connection patterns between AS nodes.
- Existence of higher connectivity inside a particular cluster and relatively lower connectivity between clusters is visible.
- Similar patterns for Route Views and RIPE 2003 and 2008 datasets

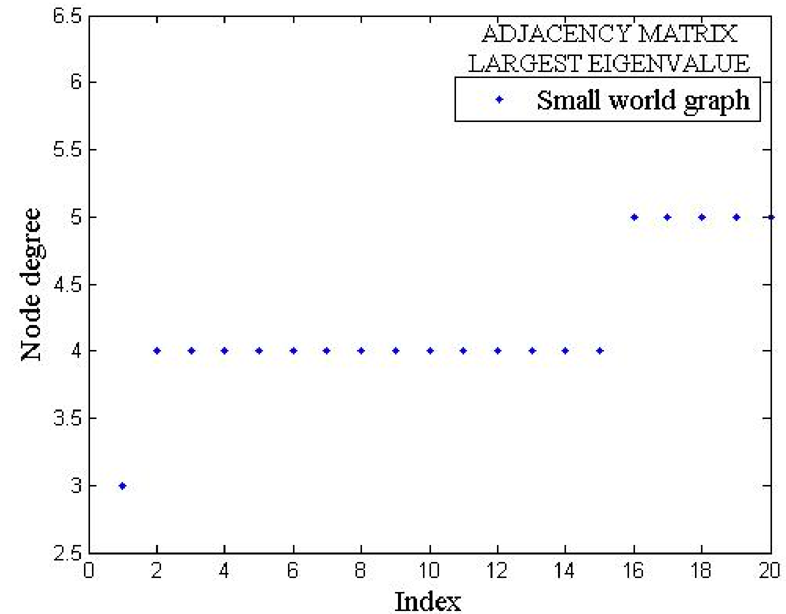
Spectral analysis of Internet graphs

- The second smallest eigenvalue, called "algebraic connectivity" of a normalized Laplacian matrix, is related to the connectivity characteristic of the graph.
- Elements of the eigenvector corresponding to the largest eigenvalue of the normalized Laplacian matrix tend to be positioned close to each other if they correspond to AS nodes with similar connectivity patterns constituting clusters.

C. Gkantsidis, M. Mihail, and E. Zegura, 2003

Clusters of AS nodes: small world network

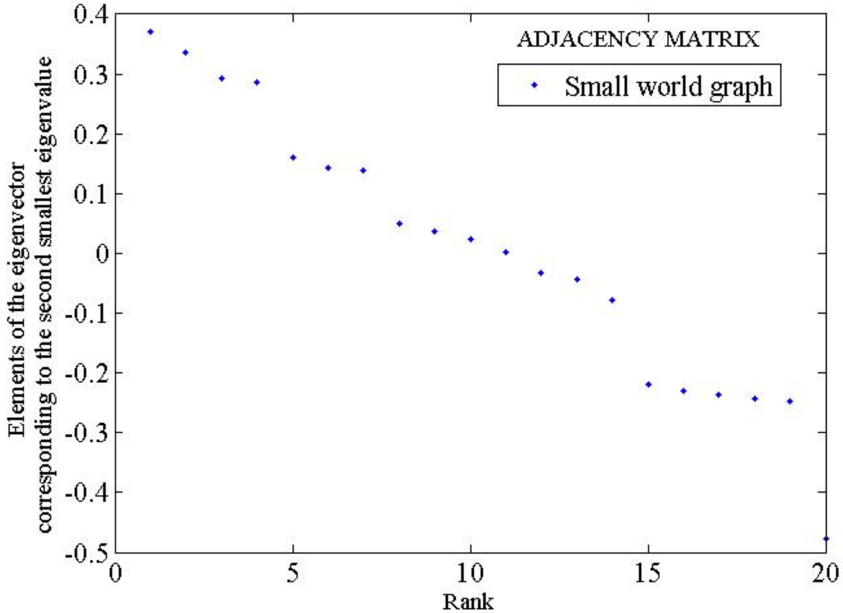


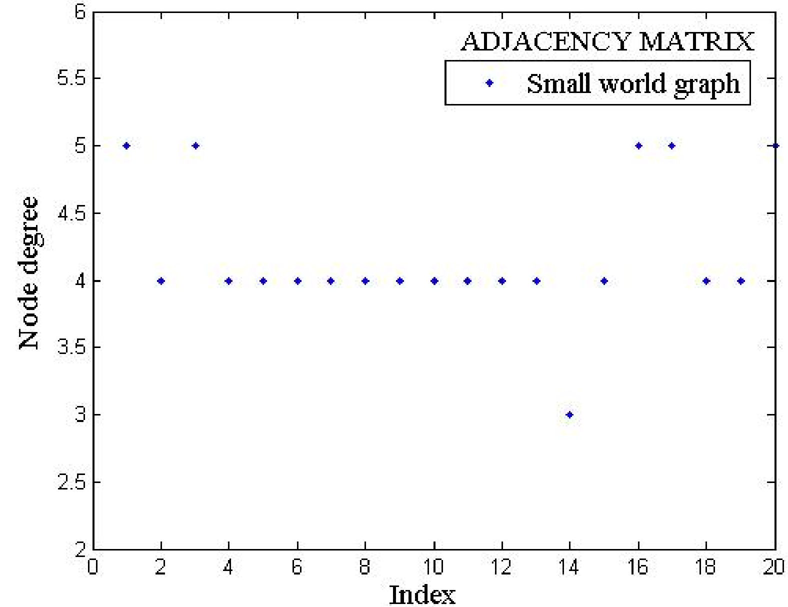


Small world network with 20 nodes:

- nodes having similar degrees are grouped together based on the element values of the eigenvector corresponding to the largest eigenvalue of the adjacency matrix

Clusters of AS nodes: small world network





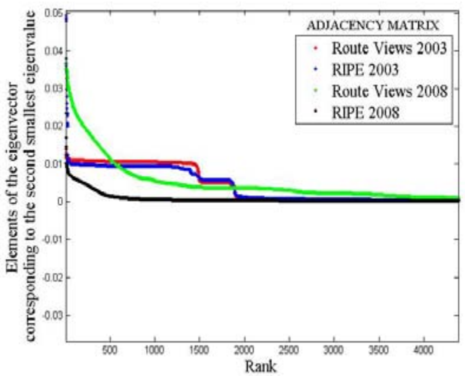
Small world network with 20 nodes:

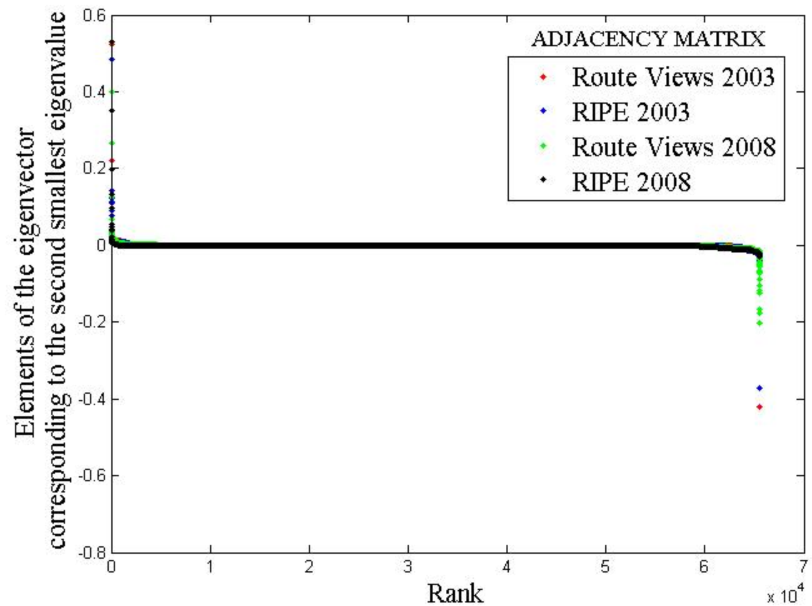
- nodes having similar degrees are not grouped together based on the element values of the eigenvector corresponding to the second smallest eigenvalue of the adjacency matrix

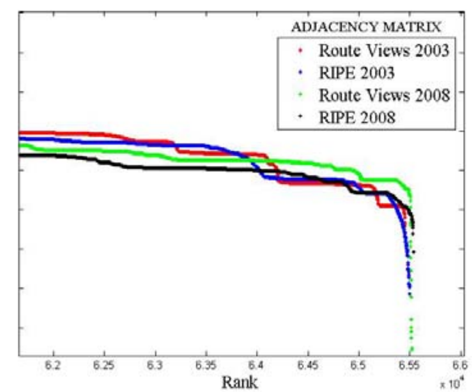
Clusters of AS nodes

- We calculate the elements of the eigenvectors corresponding to the second smallest and the largest eigenvalues of the matrix.
- These elements are sorted in descending order and are plotted vs. the index.
- We then calculate the index of AS node based on the index of the corresponding element of the eigenvector and plot node degree of AS node vs. the index of the AS node.
- We consider both the adjacency and the normalized Laplacian matrices.

Eigenvector: the second smallest eigenvalue



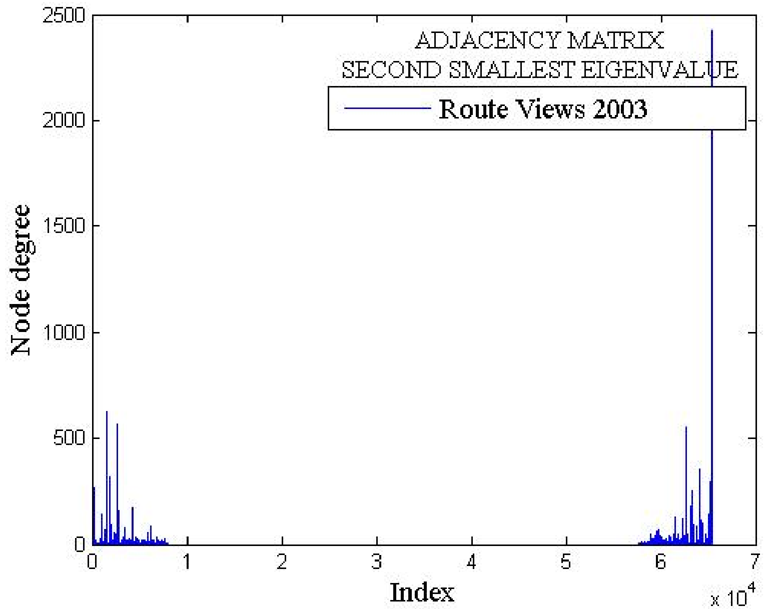


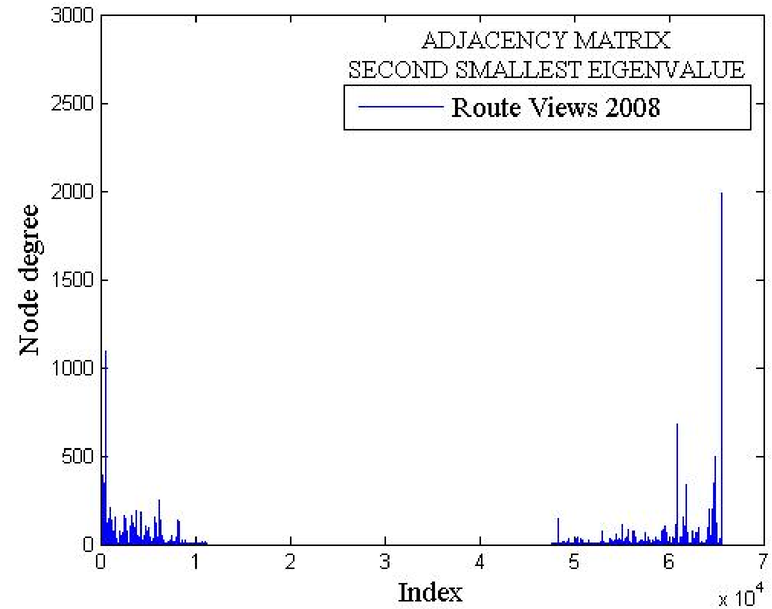


Route Views and RIPE 2003 and 2008 datasets:

- elements of the eigenvectors corresponding to the second smallest eigenvalue of the adjacency matrix

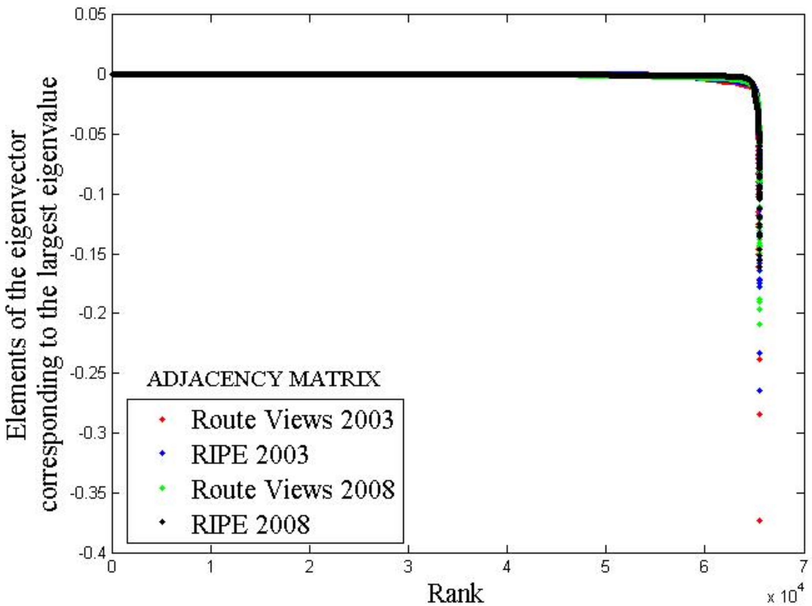
Clusters: Route Views 2003 and 2008 datasets

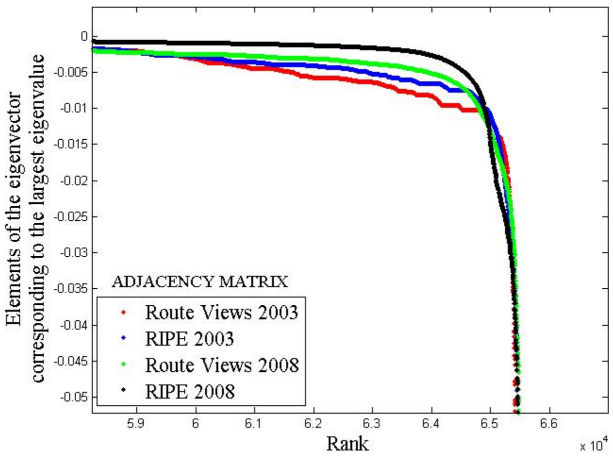




- Element values of the eigenvector corresponding to the second smallest eigenvalue of the adjacency matrix divide nodes into two separate clusters of connected nodes.
- Similar clusters for the RIPE 2003 and 2008 datasets

Eigenvector: the largest eigenvalue

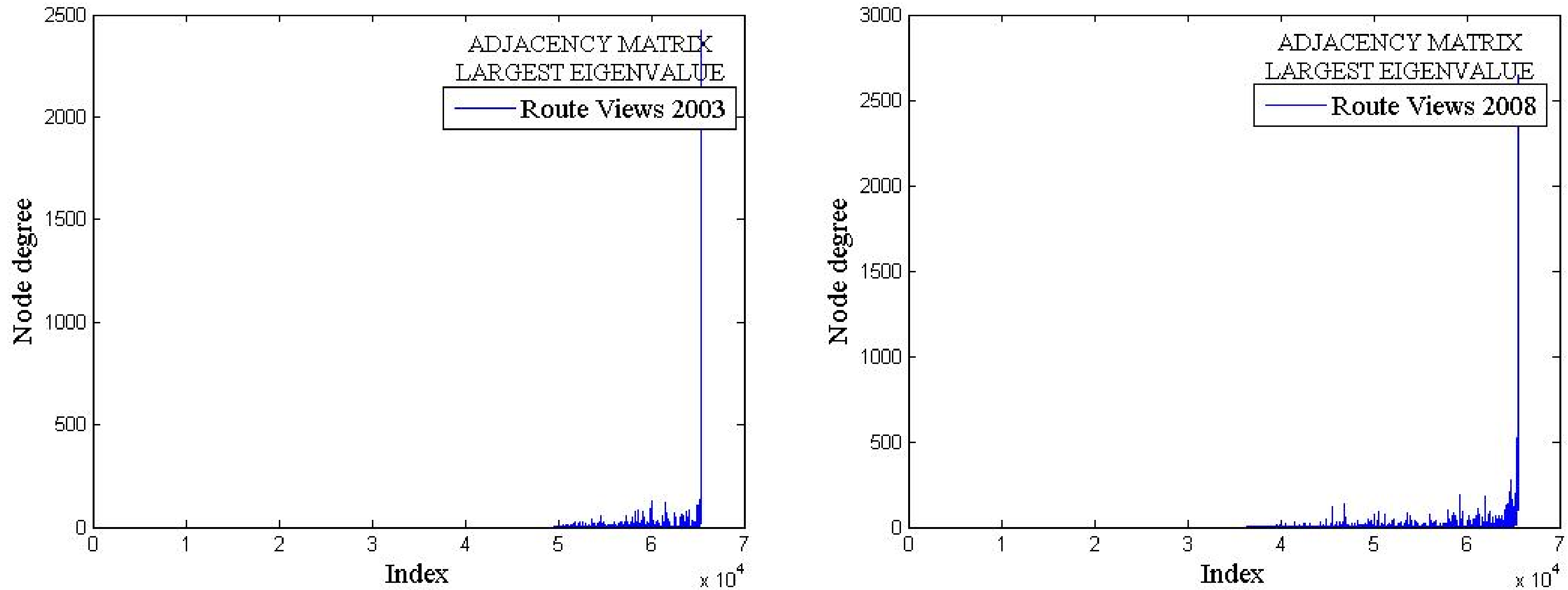




Route Views and RIPE 2003 and 2008 datasets:

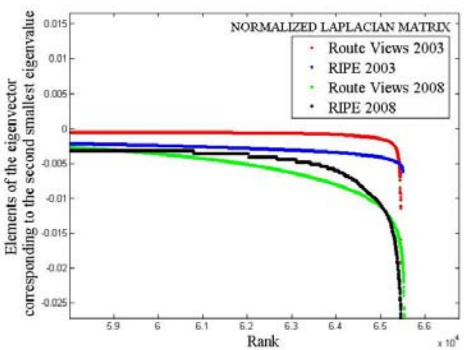
- elements of eigenvectors corresponding to the largest eigenvalue of the adjacency matrix

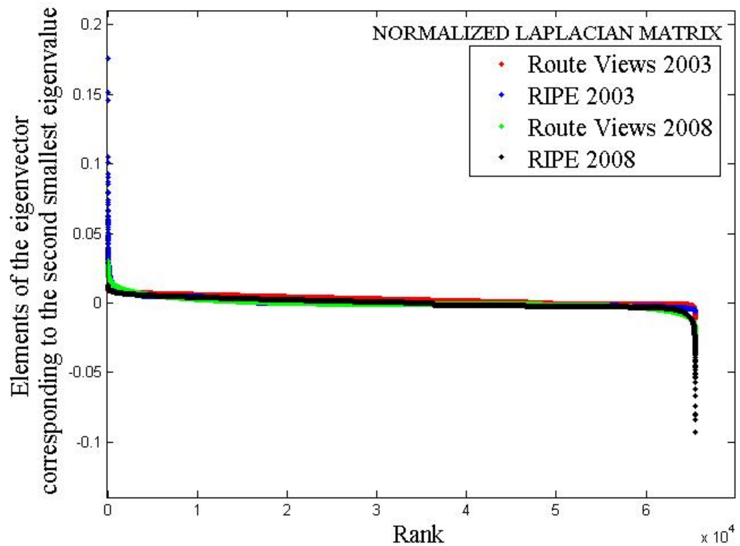
Clusters:
Route Views 2003 and 2008 datasets

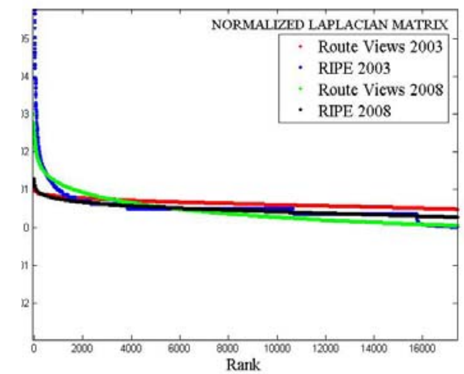


- Element values of the eigenvector corresponding to the largest eigenvalue of the adjacency matrix group nodes into a cluster of connected nodes towards the highest end of the rank spectrum.
- Similar clusters for the RIPE 2003 and 2008 datasets

Eigenvector: the second smallest eigenvalue



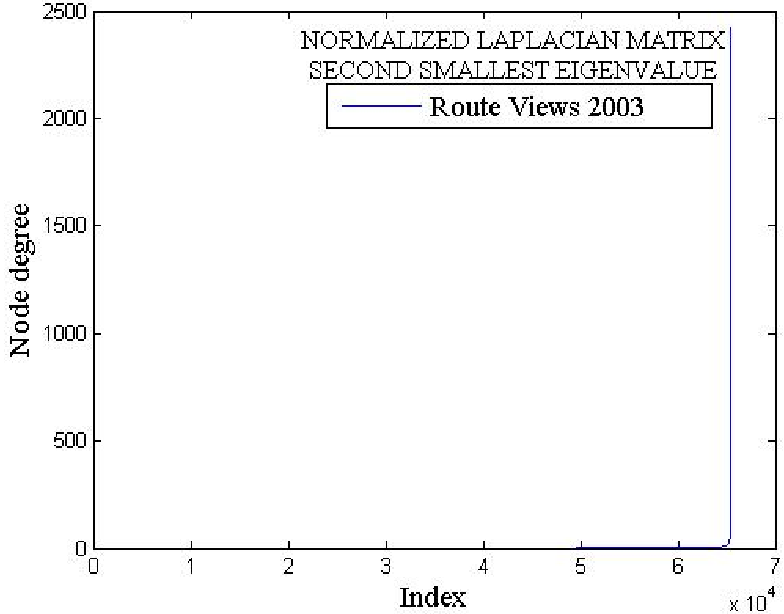


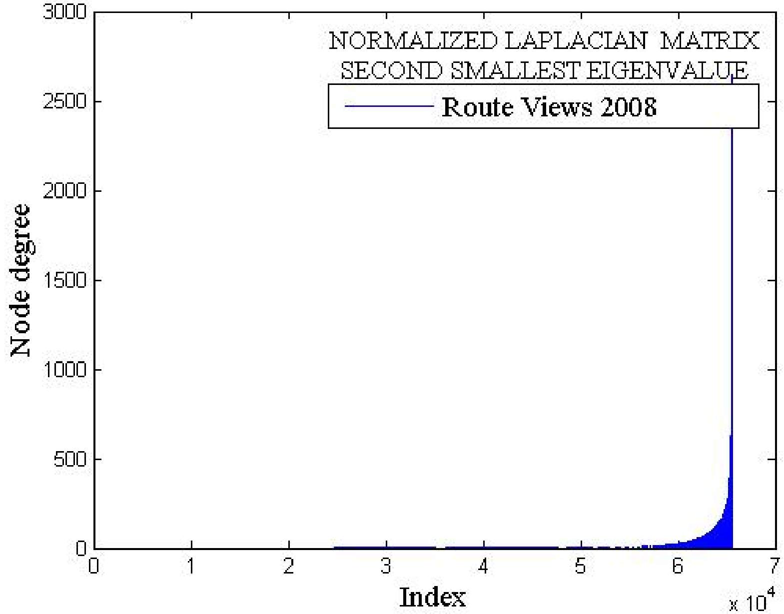


Route Views and RIPE 2003 and 2008 datasets:

- elements of eigenvectors corresponding to the second smallest eigenvalue of the normalized Laplacian matrix

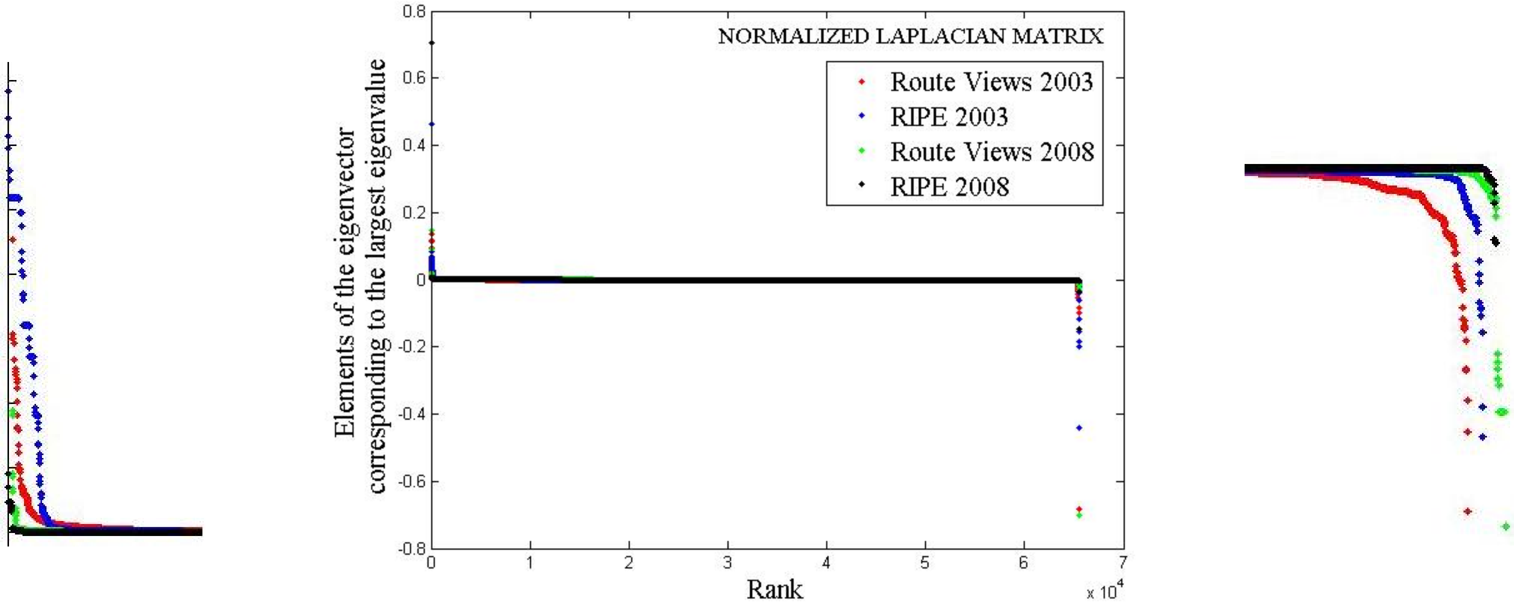
Clusters: Route Views 2003 and 2008 datasests





- Element values of the eigenvector corresponding to the second smallest eigenvalue of the normalized Laplacian matrix group nodes having similar node degrees.
- Similar clusters for the RIPE 2003 and 2008 datasets

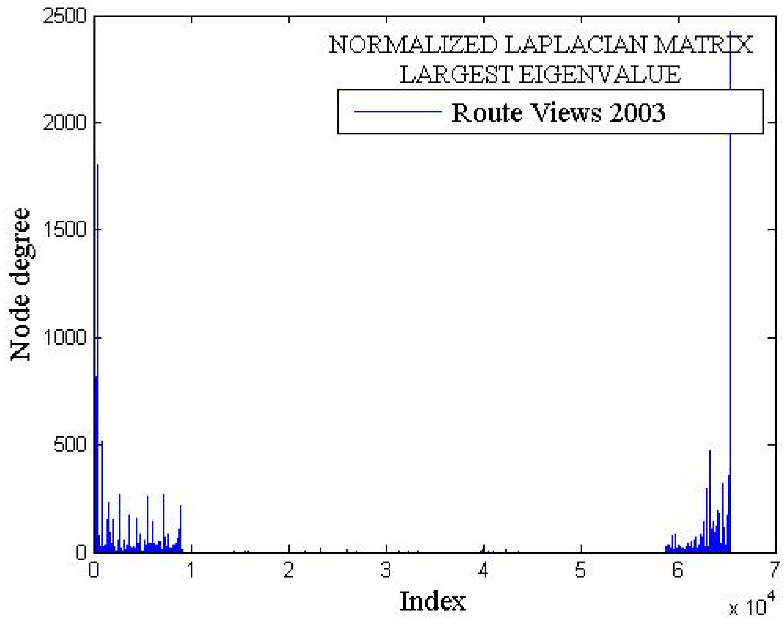
Eigenvector: the largest eigenvalue

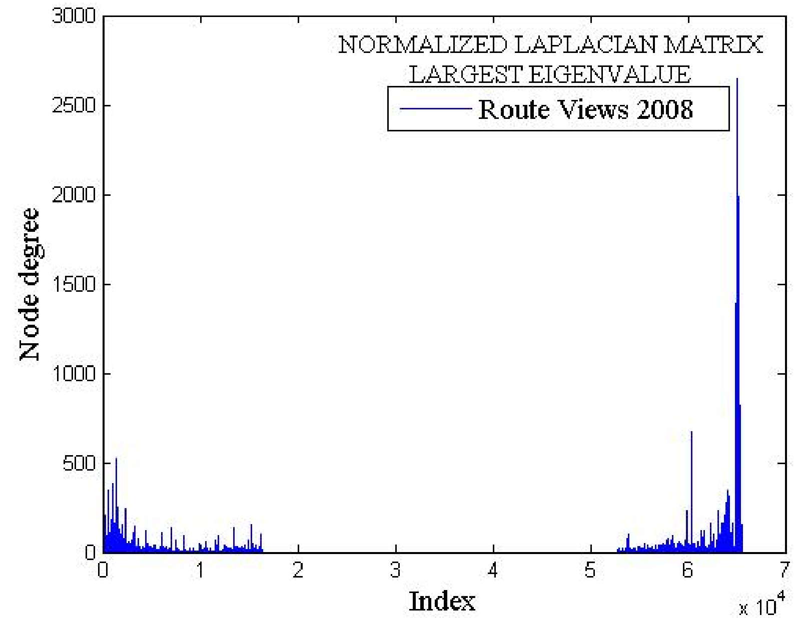


Route Views and RIPE 2003 and 2008 datasets:

- elements of eigenvectors corresponding to the largest eigenvalue of the normalized Laplacian matrix

Clusters: Route Views 2003 and 2008 datasets





- Element values of the eigenvector corresponding to the largest eigenvalue of the normalized Laplacian matrix divide nodes into two clusters of connected nodes.
- Similar clusters for the RIPE 2003 and 2008 datasets

Clusters of AS nodes

- The second smallest eigenvalue of the normalized Laplacian matrix groups nodes having similar node degree:
  - group of nodes having larger node degree follows nodes having smaller node degree.
- Clusters of nodes based on the elements values of the eigenvector corresponding to the second smallest eigenvalue of the adjacency matrix are similar to clusters based on the largest eigenvalue of the normalized Laplacian matrix.
- Clusters the Internet graphs are different from clusters of small world networks.

Roadmap

- Introduction
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Dynamics in complex networks

Early analysis of network dynamics:

- regular networks (Endo and Mori, 1976a, 1976b, 1978)
- synchronism in the lattice, ladder, and ring networks
- each node contained a Van der Pol oscillator
- nodes were connected by resistors or inductors.

Networks with chaotic circuits:

- analyzed by Nishio and Ushida, 1995a, 1995b, 1996, 2002
- star-connected oscillator (Moro, Nishio, and Mori, 1995)
- ring coupling of chaotic circuits (Uwate and Nishio, 2006)
- coupled oscillators networks as cellular neural networks (Moro. Nishio, and Mori, 1997).

Synchronization

Small-world networks:

- small average distance and high clustering
- small-world property does not generally guarantee synchronization in the network (Barahona and Pecora, 2002).

Synchronization

Scale free networks:

- power-law connectivity distribution of the node degree:

$$P(k) \propto k^{-\gamma}$$

where:

P(k) is the probability distribution function

k is the node degree of the network.

- the smaller the parameter γ:
  - the more the network becomes heterogeneous in its connectivity distribution
  - the average network distance decreases
  - synchronization is more difficult to achieve (Nishikawa et al., 2003).

Complex networks

Complex networks:

- each node contains an oscillator or a dynamical system that generates periodic or chaotic oscillations
- network topology is represented by a Laplacian matrix $L(G)$:
  - symmetric and has a single zero eigenvalue for a connected network
- number of edges incident to a node in an undirected graph is called the degree of the node
- two nodes are called adjacent if they are connected by a link.

Graph matrices

- Adjacency matrix $A(G)$
- Diagonal matrix $D(G)$ with row-sums of $A(G)$ as the diagonal elements, indicates the connectivity degree of each node.
- Laplacian matrix: $L(G) = D(G) - A(G)$
- The spectrum of $L(G)$:
  - collection of all eigenvalues
  - contains 0 for every connected graph component

Assumptions

- Consider a network with N nodes:
- Assume that each network node is governed by a self-oscillatory autonomous system with m variables.
- Examples:
  - m = 2: Van der Pol oscillator
  - m = 3: Lorenz system
- Assumption:

  Oscillators are identical with identical coupling to other oscillators.

First step: formulate state equations

Definitions:

- $\mathbf{x}^i$: m-dimensional vector of state variables of the i-th node
- $\mathbf{F}(\mathbf{x}^i)$: the isolated (uncoupled) dynamics for each node
- $\mathbf{H}$: $R^m \rightarrow R^m$: a coupling function

- The dynamics of node i is:

$$\dot{\mathbf{x}}^i = \mathbf{F}(\mathbf{x}^i) + \sigma \sum_{j=1, j\neq i}^{N} G_{ij}\mathbf{H}(\mathbf{x}^j)$$

where σ is a coupling strength.

Dynamics of the network

- Define matrices:

$$\mathbf{x} = (\mathbf{x}^1, \mathbf{x}^2, \cdots, \mathbf{x}^N)$$
$$\mathbf{F}(\mathbf{x}) = [\mathbf{F}(\mathbf{x}^1), \mathbf{F}(\mathbf{x}^2), \cdots, \mathbf{F}(\mathbf{x}^N)]$$
$$\mathbf{H}(\mathbf{x}) = [\mathbf{H}(\mathbf{x}^1), \mathbf{H}(\mathbf{x}^2), \cdots, \mathbf{H}(\mathbf{x}^N)]$$

$\mathbf{G}$: $N \times N$ matrix of coupling coefficients G_{ij}

- Note: **G = –L(G)**

- The dynamics of the network is described as:

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}) + \sigma \mathbf{G} \otimes \mathbf{H}(\mathbf{x})$$

where $\otimes$ s the direct product.

Second step: periodic solutions

Find periodic solutions of the state equation:

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{x}) + \sigma \mathbf{G} \otimes \mathbf{H}(\mathbf{x})$$

Third step: variational equation

- Derive the variational equation from the periodic steady-state in order to investigate the stability of synchronized steady-state or periodic solution

$$\dot{\xi} = [\mathbf{1}_N \otimes D\mathbf{F} + \sigma \mathbf{G} \otimes D\mathbf{H}]\xi$$

where:

ξ^i are variations on node i

$$\xi = (\xi^1, \xi^2, \cdots, \xi^N)^{tr}$$

Master stability equation

- The variational equation becomes the linear differential equation with periodic coefficients combined with the Laplacian matrix.
- By using an appropriate linear transformation, the variational equation can be divided in separate blocks, each block corresponding to an eigenvalue:

$$\gamma_k (k = 0, \cdots, N-1)$$

where N is the number of nodes:

$$\dot{\xi}_k = [D\mathbf{F} + \sigma \gamma_k D\mathbf{H}]\xi_k$$

- Each separate block equation is called the *master stability equation*.

Fourth step: master stability function

- From the variational equation, we compute the maximum Lyapunov exponent

  $$\Lambda_{max}$$

  called the *master stability function.*

- If Λ_{max} is negative, the corresponding periodic steady-state is stable and the variations die out.

Stability regions

- Factor $\alpha \equiv \sigma\gamma_k$, defined as the product of γ_k and the overall strength of coupling parameter σ, is a measure used to express the coupling strength.
- The stability plots of Λ_{max} vs. α (generic coupling factor for nonlinear function and output function at each node) are used to define stability regions.
- The oscillatory systems such as periodic oscillators have a master stability function that has $\Lambda_{max} < 0$ over the interval $(\alpha_{min}, \alpha_{max})$ in these stability plots.
- The generic requirement for the synchronous state to be stable is given by $\sigma\lambda_k \in (\alpha_{min}, \alpha_{max})$ for each *k*.

Stability of synchronization

- This requirement can be equivalently written as

$$\lambda_{max}/\lambda_1 < \alpha_{max}/\alpha_{min}$$

  where λ_1 and λ_{max} are the second smallest and the largest eigenvalues.

- The left-hand side of the inequality is determined solely by the Laplacian matrix while the right-hand side is defined by the master stability function.

- Hence, we can analyze the stability of synchronization and network dynamics by only observing the network topology.

Modeling the Internet dynamics

- Laplacian matrix has distinct real eigenvalues.
- The behavior of the nodes is governed by network transport protocols and queuing algorithms:
  - Transport Control Protocol (TCP) combined with Random Early Detection (RED) queuing algorithm
- Modeling network dynamics:
  - fluid-flow models
  - discrete (one and two dimensional) models

TCP/RED fluid flow model: state variables and parameters

- $w(t)$: averaged instantaneous window size (in packets) of the TCP sources
- $r(t)$: round trip time
- $q(t)$: averaged instantaneous queue length (in packets)
- $x(t)$: filtered queue length after removal of short bursts
- $p(t)$: marking probability
- A: filter resolution ($0 < a < 1$)
- k: a proportionality constant dependent on the implementation of the RED algorithm
- X_{max}: maximum threshold of $x(t)$
- X_{min}: minimum threshold of $x(t)$
- P_{max}: maximum threshold of $p(t)$
- R_0: propagation delay
- C: bottleneck bandwidth in packets/second
- B: maximum physical queue length

TCP/RED fluid flow model

$$\frac{dw(t)}{dt} = \frac{1}{r(t)} - \frac{w(t)}{2}\frac{w(t-r(t))}{r(t-r(t))}p(t-r(t))$$

$$\frac{dq(t)}{dt} = N\frac{w(t)}{r(t)} - C$$

$$\frac{dx(t)}{dt} = C\ln(1-\alpha)(x(t)-q(t))$$

V. Misra, W. B. Gong, and D. Towsley, 2000

TCP/RED fluid flow model

$$p_b(t) = \begin{cases} 0 & 0 \le x(t) < x_{\min} \\ \dfrac{x(t) - x_{\min}}{x_{\max} - x_{\min}} p_{\max} & x_{\min} \le x(t) \le x_{\max} \\ p_{\max} \\ +\dfrac{1 - p_{\max}}{x_{\max}}(x(t) - x_{\max}) & x_{\max} < x(t) \le 2x_{\max} \\ 1 & 2x_{\max} \le x(t) \end{cases}$$

$$p(t) = \kappa p_b(t) \quad \text{and} \quad r(t) = \frac{q(t)}{C} + R_0$$

V. Misra, W. B. Gong, and D. Towsley, 2000

TCP/RED discrete model: one state variable and parameters

- Variables:
  - $\bar{q}_{k+1}$: average queue size in round k+1
  - q_k: average queue size in round k
  - w_q: queue weight in RED
  - N: number of TCP connections
  - K: constant = $\sqrt{3/2}$
  - p_k: drop probability in round k
  - C: capacity of the link between the two routers
  - d: round-trip propagation delay
  - M: packet size
  - *rwnd*: receiver's advertised window size

TCP/RED discrete model

- Dynamical model of TCP/RED:

$$\bar{q}_{k+1} = \begin{cases} (1-w_q)\cdot \bar{q}_k + w_q \cdot \max\left(\dfrac{N\cdot K}{\sqrt{p_k}} - \dfrac{C\cdot d}{M}, 0\right) \ if \ p_k \neq 0 \\ (1-w_q)\cdot \bar{q}_k + w_q \cdot \left(rwnd\cdot N - \dfrac{C\cdot d}{M}\right) \ if \ p_k = 0 \end{cases}$$

TCP/RED model: two state variables and parameters

- Variables:
  - $\underline{q}_{k+1}$: instantaneous queue size in round k+1
  - q_{k+1}: average queue size in round k+1
  - W_{k+1}: current TCP window size in round k+1
  - w_q: queue weight in RED
  - p_k: drop probability in round k
  - RTT_{k+1}: round-trip time at k+1
  - C: capacity of the link between the two routers
  - M: packet size
  - d: round-trip propagation delay
  - ssthesh: slow start threshold
  - rwnd: receiver's advertised window size

TCP/RED discrete model

No-packet loss:

- window size: $W_{k+1} = \begin{cases} \min(2W_k, ssthresh) & if\, W_k < ssthresh \\ \min(W_k + 1, rwnd) & if\, W_k \geq ssthresh \end{cases}$

- average queue size:

$$\bar{q}_{k+1} = (1 - w_q)^{W_{k+1}} \bar{q}_k + (1 - (1 - w_q)^{W_{k+1}}) \cdot \max(W_{k+1} - \frac{C \cdot d}{M}, 0)$$

One-packet loss:

- window size: $W_{k+1} = \frac{1}{2} W_k$

- average queue size:

$$\bar{q}_{k+1} = (1 - w_q)^{W_{k+1}} \bar{q}_k + (1 - (1 - w_q)^{W_{k+1}}) \cdot \max(W_{k+1} - \frac{C \cdot d}{M}, 0)$$

TCP/RED discrete model (cont.)

Two-packet loss:

- window size: $W_{k+1} = 0$
- average queue size: $\overline{q}_{k+1} = \overline{q}_k$

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Conclusions

- Route Views and RIPE datasets reveal similar trends in the development of the Internet topology.
- Power-laws exponents have not significantly changed over the years:
  - they do not capture every property of graph and are only a measure used to characterize the Internet topology.
- Spectral analysis reveals new historical trends and notable changes in the connectivity and clustering of AS nodes over the years.
- Element values of the eigenvectors corresponding to the second smallest eigenvalue and the largest eigenvalue identify clusters of connected ASes:
  - indicate that clusters of connected nodes have changed over time.

Conclusions

- We consider numerous aspects of the dynamics of complex networks (without necessarily restricting our attention to classical small-world and scale-free networks) .
- We addressed the universal quantification using differential equations combined with graph theory.
- Dealing with dynamics of complex networks with weights imposed on network nodes and edges is essential in understanding various applications of complex networks.

Future work

- Graph of graphs: view each Internet node as a local area network.
- Capture the evolving nature of the Internet topology.
- Model node dynamics by capturing behaviour of network protocols (TCP) and queuing algorithms (RED).
- Include realistic traffic models to include effect of Internet applications (data, voice, images, video).
- Develop effective methods for obtaining synchronous solutions of nonlinear equations with higher dimensions.

References

- CAIDA: The Cooperative Association for Internet Data Analysis
- http://www.caida.org/home/
- Walrus - Gallery: Visualization & Navigation
- http://www.caida.org/tools/visualization/walrus/gallery1/
- Walrus - Gallery: Abstract Art
- http://www.caida.org/tools/visualization/walrus/gallery2/
- BGP capture datasets [Online]. Available: http://archive.routeviews.org.
- Reseaux IP Europens [Online]. Available: http://www.ripe.net/ris.
- Autonomous System Numbers [Online]. Available: http://www.iana.org/assignments/ as-numbers.
- BCNET [Online]. Available: http://www.bc.net.
- Y. Rekhter, T. Li, and S. Hares, "A border gateway protocol 4 (BGP-4)," *IETF RFC 1771*.
- L. Gao, "On inferring autonomous system relationships in the Internet," IEEE/ACM Trans. Networking, vol. 9, no. 6, pp. 733–745, Dec. 2001.

References

- P. Erdős and A. Rényi, "On random graphs I," Publ. Math. Debrecen 6, pp. 290–297, 1959.
- D. J. Watts and S. H. Strogatz, "Collective dynamics of small world networks," Nature, vol. 393, pp. 440–442, June 1998.
- A.-L. Barabasi and R. Albert, "Emergence of scaling in random networks," Science, vol. 286, pp. 509-512, Oct. 1999.
- I. Farkas, I. Dernyi, A. Barabasi, and T. Vicsek, "Spectra of 'real-world' graphs: beyond the semicircle law", *Physical Review E*, vol. 64, no. 2, 026704, pp. 1-12, July 2001.
- F. R. K. Chung, *Spectral Graph Theory*. Providence, Rhode Island: Conference Board of the Mathematical Sciences, 1997, pp. 2–6.
- M. Fiedler, "Algebraic connectivity of graphs," *Czech. Math. J.*, vol. 23, no. 2, pp. 298–305, 1973.

References

- Y. Li, J. Cui, D. Maggiroini, and M. Faloutsos, "Characterizing and modelling clustering features in AS-level Internet topology," *Proc. IEEE INFOCOM 2008,* Phoenix, Arizona, USA, Apr. 2008, pp. 271-275.
- G. Siganos, M. Faloutsos, P. Faloutsos, and C. Faloutsos, "Power-laws and the AS-level Internet topology," *IEEE/ACM Trans. Networking*, vol. 11, no. 4, pp. 514–524, Aug. 2003.
- M. Faloutsos, P. Faloutsos, and C. Faloutsos, "On power-law relationships of the Internet topology," *Proc. ACM SIGCOMM*, *Computer Communication Review*, vol. 29, no. 4, pp. 251-262, Sept. 1999.
- A. Medina, I. Matta, and J. Byers, "On the origin of power laws in Internet topologies," *Proc. ACM SIGCOMM 2000*, Computer Communication Review, vol. 30, no. 2, pp. 18–28, Apr. 2000.
- L. Gao, "On inferring autonomous system relationships in the Internet," *IEEE/ACM Trans. Networking*, vol. 9, no. 6, pp. 733–745, Dec. 2001.
- D. Vukadinovic, P. Huang, and T. Erlebach, "On the Spectrum and Structure of Internet Topology Graphs," in H. Unger et al., editors, *Innovative Internet Computing Systems*, LNCS2346, pp. 83–96. Springer, Berlin, Germany, 2002.

References

- Q. Chen, H. Chang, R. Govindan, S. Jamin, S. Shenker, and W. Willinger, "The origin of power laws in Internet topologies revisited," *Proc. INFOCOM*, New York, NY, USA, Apr. 2002, pp. 608–617.
- H. Chang, R. Govindan, S. Jamin, S. Shenker, and W. Willinger, "Towards capturing representative AS-level Internet topologies," The International Journal of Computer and Telecommunications Networking, vol. 44, no. 6, pp. 735-755, Apr. 2004.
- H. Tangmunarunkit, R. Govindan, S. Jamin, S. Shenker, and W. Willinger, "Network topology generators: degree-based vs. structural," *Proc. ACM SIGCOMM, Computer Communication Review*, vol. 32, no. 4, pp. 147–159, Oct. 2002.
- A. Dhamdhere and C. Dovrolis, "Ten years in the evolution of the Internet ecosystem," *Proc.of the 8th ACM SIGCOMM Conference on Internet Measurement*, Vouliagmeni, Greece, Oct. 2008, pp. 183-196.
- R. Oliveira, B. Zhang, and L. Zhang, "Observing the evolution of Internet AS topology," *ACM SIGCOMM Computer Communication Review*, vol. 37, no. 4, pp. 313-324, Jan. 2007.
- C. Gkantsidis, M. Mihail, and E. Zegura, "Spectral analysis of Internet topologies," *Proc. of Infocom 2003,* San Francisco, CA, Mar. 2003, vol. 1, pp. 364–374.
- S. Jaiswal, A. Rosenberg, and D. Towsley, "Comparing the structure of power-law graphs and the Internet AS graph," *Proc. 12th IEEE International Conference on Network Protocols*, Washington DC, Aug. 2004, pp. 294–303.

References

- V. Misra, W. B. Gong, and D. Towsley, “Fluid-based analysis of a network of AQM routers supporting TCP flows with an application to RED,” *Proc. ACM/SIGCOMM*, Stockholm, Sweden, Aug. 2000, pp. 151–160.
- T. Endo and S. Mori, "Mode analysis of a multi-mode ladder oscillator,“ *IEEE Trans. on Circuits and Systems*, vol. CAS-23, no. 2, pp. 100–113, 1976.
- T. Endo and S. Mori, "Mode analysis of a two-dimensional low-pass multi-mode oscillator,“ *IEEE Trans. on Circuits and Systems*, vol. CAS-23, no. 9, pp. 517–530, 1976.
- T. Endo and S. Mori, "Mode analysis of a ring of a large number of mutually coupled van der Pol oscillators,“ *IEEE Trans. on Circuits and Systems*, vol. CAS-25, no. 1, pp. 7–18, 1978.
- Y. Nishio and A. Ushida, "On a ring of chaotic circuits coupled by inductors,“ *IEICE Trans. on Fundamentals*, vol. E78-A, no. 5, pp. 608–617, 1995.
- Y. Nishio and A. Ushida, "Spacio-temporal chaos in simple coupled chaotic circuits,“ *IEEE Trans. on Circuits Syst. I*, vol. CAS-42, no. 10, pp. 678–686, 1995.
- Y. Nishio and A. Ushida, "Quasi-synchronization phenomena in chaotic circuits coupled by one resistor,“ *IEEE Trans. on Circuits Syst. I*, vol. CAS-43, no. 6, pp. 491–496, 1996.
- Y. Nishio and A. Ushida, "Chaotic wandering and its analysis in simple coupled chaotic circuits,“ *IEICE Trans. on Fundamentals*, vol. E85-A, no. 1, pp. 248–255, 2002.

References

- S. Moro, Y. Nishio, and S. Mori, "Synchronization phenomena in oscillators coupled by one resistor," *IEICE Trans. on Fundamentals*, vol. E78-A, no. 2, pp. 244–253, 1995.
- Y. Uwate and Y. Nishio, "Complex phase synchronization in an array of oscillators coupled by time-varying resistor,“ *Int. Joint Conference on Neural Networks*, 2006, pp. 8345–835.
- S. Moro, Y. Nishio, and S. Mori, "On coupled oscillators networks for cellular neural networks," *IEICE Trans. Fundamentals*, vol. E80-A, no. 1, pp. 214–222, 1997.
- L. M. Pecora and T. L. Carroll, "Master stability functions for synchronized coupled systems,“ *Physical Review Letters*, vol. 80, no. 10, pp. 2109–2112, 1998.
- M. Barahona and L. M. Pecora, "Synchronization in small-world systems,“ *Physical Review Letters*, vol. 89, no. 5, pp. 054101-1–4, 2002.
- T. Nishikawa, A. E. Motter, Y.-C. Lai, and F. C. Hoppensteadt, "Heterogeneity in oscillator networks: are smaller worlds easier to synchronize?,“ *Physical Review Letters*, vol. 91, pp. 014101, 2003.
- C. Hayashi, *Nonlinear Oscillations in Physical Systems*. New York: McGraw-Hill, 1964, pp. 82–97.

References

- S. Lally, T. Farah, R. Gill, R. Paul, N. Al-Rousan, and Lj. Trajković, "Collection and Characterization of BCNET BGP Traffic," *Proc. PACRIM 2011*, Victoria, BC, Canada, Aug. 2011.
- T. Farah, S. Lally, R. Gill, N. Al-Rousan, R. Paul, D. Xu, and Lj. Trajkovic, "Collection of BCNET BGP traffic," *Proc. ITC 2011*, San Francisco, CA, USA, Sept. 2011.
- M. Liu, A. Marciello, M. di Bernardo, and Lj. Trajkovic, "Discontinuity-induced bifurcations in TCP/RED communication algorithms," *Proc. IEEE Int. Symp. Circuits and Systems*, Kos, Greece, May 2006, pp. 2629–2632.
- M. Liu, H. Zhang, V. Vukadinovic, and Lj. Trajkovic, "Modeling TCP/RED: a dynamical approach," in *Complex Dynamics in Communication Networks*, Lj. Kocarev and G. Vattay, Eds., Berlin: Springer, Series: Understanding Complex Systems, 2005, pp. 251–278.
- M. Liu, H. Zhang, and Lj. Trajkovic, "Stroboscopic model and bifurcations in TCP/RED," in *Proc. IEEE International Symposium on Circuits and Systems*, Kobe, Japan, May 2005, pp. 2060–2063.
- J. Chen and Lj. Trajkovic, "Analysis of Internet topology data," *Proc. IEEE International Symposium on Circuits and Systems*, Vancouver, BC, Canada, May 2004, vol. IV, pp. 629–632.
- I. Khalifa and Lj. Trajković, "An overview and comparison of analytical TCP models," in *Proc. IEEE International Symposium on Circuits and Systems*, Vancouver, BC, Canada, May 2004, vol. V, pp. 469–472.

References

- K. Okumura and Lj. Trajkovic, "Spectral analysis and dynamical behavior of complex networks," *Proc. NOLTA 2010*, Krakow, Poland, Sept. 2010, pp.305–308.
- L. Subedi and Lj. Trajkovic, "Spectral analysis of Internet topology graphs," *Proc. IEEE International Symposium Circuits and Systems*, Paris, France, May 2010, pp. 1803–1896.
- M. Najiminaini, L. Subedi, and Lj. Trajkovic, "Analysis of Internet topologies: a historical view," *Proc. IEEE International Symposium Circuits and Systems*, Taipei, Taiwan, May 2009, pp. 1697–1700.
- Lj. Trajković, "Analysis of Internet topologies," *IEEE Circuits and Systems Magazine*, vol. 10, no. 3, pp. 48–54, Third Quarter 2010.

CAS Magazine, vol. 10, no. 3, 2010