CROSS-SECTIONAL AND MULTIVARIATE TESTS OF THE CAPM AND FAMA-FRENCH THREE-FACTOR MODEL

by

Yuanyuan Liang
Bachelor of Arts, Nankai University 2003

Project submitted in partial fulfillment of
the requirements for the degree of

MASTER OF ARTS

In the
Department
of
Economics

SIMON FRASER UNIVERSITY

July 2004

APPROVAL

Name: **Yuanyuan Liang**

Degree: **M. A. (Economics)**

Title of Project: **Cross-sectional And Multivariate Tests Of The CAPM And Fama-French Three-Factor Model**

Examining Committee:

Chair: Ken Kasa

Robert Grauer
Senior Supervisor

Terry Heaps
Supervisor

Robbie Jones
Internal Examiner

Date Approved: Thursday, July 29th, 2004

ABSTRACT

This project tests the Sharpe(1964)-Lintner(1965)-Black(1972) Mean-Variance Capital Asset Pricing Model (CAPM) and Fama-French's (1993) Three-Factor Model using the cross-sectional and multivariate tests. Four different time periods of American stock market returns ranging from 1933 to 2003 are examined. Although both models are rejected by the multivariate tests, Fama and French argue that the Three-Factor Model fits better in the 1963-1993 period. The results in this paper covering different time periods from 1933 to 2003, however, do not unambiguously support Fama and French's conclusion.

ACKNOWLEDGEMENTS

First and foremost I would like to thank my committee: Professor Robert Grauer, Professor Terry Heaps and Professor Robert Jones for their guidance along the way. Also I would like to express my gratitude to my friends who have supported my study and research in Simon Fraser University during the past year.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

1 INTRODUCTION

The Capital Asset Pricing Model (CAPM) has been a pillar of the modern theory of finance ever since Sharpe (1964), Lintner (1965) and Black (1972) developed its single-period Mean-Variance (MV CAPM) version. The MV CAPM assumes that the market is MV efficient in the sense of Markowitz (1959), and identifies systematic risk (beta) to explain the cross section of expected returns. This relationship is known as the Security Market Line (SML).

Over the last four decades a number of competing models have been developed, including: Merton's (1973) Intertemporal CAPM (ICAPM), Rubinstein's (1974) single-period Linear Risk Tolerance Model (LRT), Ross'(1976, 1977) Arbitrage Pricing Theory (APT), Breeden's (1979) Consumption-based CAPM (CCAPM), and Fama and French's (1993) Three-Factor Model.

Built on empirical observations, the Three-Factor Model explains the asset pricing anomalies with the size effect defined by Banz (1981) and the value effect defined by Stattman (1980) and Rosenberg, Reid and Lanstein (1985). In addition to the market risk, it includes a size premium (SMB) reflecting the difference between the small and big stocks, plus a value premium (HML) reflecting the difference between the high and low book-to-market equity (BE/ME). Fama and French (1993, 1995, 1996, 1997, 1998) test their model and contend that it is an empirical success based on a theoretical equilibrium. Because these variables can be easily measured, the Three-Factor Model is also highly acclaimed by practitioners and extensively tested by researchers.

One fundamental problem associated with testing the MV CAPM is that the SML states an ex ante relationship between the expected returns and betas, while all that we can observe is a time-series of ex post returns. The most natural testing approach is to run the cross-sectional regression of ex post average returns on ex post betas. Various adaptations are proposed, as in Miller and Scholes (1972), Black, Jensen, and Scholes (1972), Blume and Friend (1973), and Fama and MacBeth (1973). Unfortunately the evidence indicates that the estimated slope of the CAPM is too flat and the intercept too large. To the extreme, Fama and French (1992, Page 464) contend: "We are forced to conclude that the [simple Mean-Variance] model does not describe the last 50 years of average stock returns." However, a body of literature raises doubts whether these procedures are truly tests of CAPM if the true market portfolio is not observed, see Roll (1977, 1978), Grauer (1978, 1999), Roll and Ross (1994), Kandel and Stambaugh (1995).

A second approach employs time-series regressions to find pricing errors relative to either the CAPM or Three-Factor Model. One important methodological refinement is the multivariate test. Rather than testing the pricing errors one by one, it tests whether all the pricing errors are simultaneously equal to zero. Gibbons (1982), Jobson and Korkie (1982, 1985), Mackinlay (1987), and Gibbons, Ross and Shanken (1989) develop it under the assumptions of the multivariate normality and independence of asset returns, and utilize an F-statistic to justify whether the intercepts across regressions are jointly zero. The testing result is accurate for finite samples and does not rely on the asymptotic theory. Subsequent research applies it to test the factor-based asset pricing models like the CAPM and Three-Factor Model, as in Kandel and Stambaugh (1995), Fama French (1996), and Grauer and Janmaat (2004).

The basic idea of this project is to extend Fama and French (1996) and Grauer and Janmaat (2004) to see how the seemingly innocuous changes in the sample periods can affect the results the cross-sectional and multivariate tests. Three versions of the cross-sectional tests are discussed, namely Black, Jensen and Scholes (BJS) test, Fama and MecBeth (FM) method, and Grauer and Janmaat (GJ) approach. The monthly data of 70 years' excess returns are examined in three sub-groups (1933-1963, 1963-1993, and 1993-2003) and the full 1933-2003 period.

The project proceeds as follows. Section 2 derives the CAPM and Three-Factor Model under the constrained portfolio selection problem. Section 3 describes the methodology of tests and Section 4 explains the data. Section 5 reports the results and compares the performance of models across different tests in four time periods. And Section 6 concludes.

2 MODELS

2.1 The MV CAPM

The underlying premise of the MV CAPM is that the rational investors maximize their MV utility given a budget constraint. The derivation of the model follows Grauer and Janmaat (2004), who in turn formulate the MV problem as in Markowitz (1959), Sharpe (1970, 1991), and Best and Grauer (1990).

The market portfolio is the optimal portfolio solution to

$$\max L = [T_m(\mu'x + R_f x_f) - \frac{1}{2}x'\Sigma x] + \lambda(1 - \iota'x - x_f), \tag{1}$$

where x, μ and ι are n-vectors containing portfolio weights, unity plus expected rates of return, and ones, respectively; R_f is unity plus the risk-free rate of interest, x_f is the weight invested in the risk-free asset; Σ is the $n \times n$ variance-covariance matrix of asset returns; T_m is the risk tolerance parameter of the representative investor, and λ is the Lagrange multiplier for the budget constraint.

The optimality condition generates the MV CAPM pricing equation:

$$\mu = \frac{\lambda}{T_m}\iota + \frac{1}{T_m}\Sigma x_m, \tag{2}$$

or in scalar notation:

$$E(R_j) = R_f + \frac{E(R_m) - R_f}{\sigma_m^{\;2}} \operatorname{cov}(R_j, R_m), \tag{3}$$

where R_j is the j-th element of μ, R_m is the return of the optimal portfolio, x_m and σ_m is the weights of the market portfolio and the standard deviation of its return, and $\operatorname{cov}(R_j, R_m)$ is the j-th element of the vector Σx_m.

If we define the systematic risk as $\beta = \frac{\operatorname{cov}(R_j, R_m)}{\sigma_m^{\;2}}$, the Security Market Line (SML) gives as the relationship between the risk and return for individual assets:

$$E(R_j) = R_f + (E(R_m) - R_f)\beta_j. \tag{4}$$

2.2 The Three-factor Model

The Fama-French (1993) Three-Factor Model contends that asset pricing is determined by three factors instead of the systematic risk alone. The two new factors are the size premium SMB (Small minus Big, the difference between returns on small-stock and big-stock portfolios), and the value premium HML (High minus Low, the difference between returns on high book-to-market stock and low book-to market stock portfolios).

Fama and French (1993, 1996) argue that Three-Factor Model can be viewed as a multifactor version of Merton's (1973) ICAPM or Ross'(1976, 1977) APT when the market is

Multifactor-Minimum-Variance (MMV) efficient. Following Grauer and Janmaat (2004), the market portfolio is the optimal solution to

$$\max L = [T_m(\mu' x + R_f x_f) - \frac{1}{2} x'\Sigma x] + \lambda(1 - \iota' x - x_f) + \lambda_{SMB}(\sigma_{mSMB} - x'\Sigma x_{SMB}) + \lambda_{HML}(\sigma_{\sigma HML} - x'\Sigma x_{HML}), \quad (5)$$

where x_{SMB} and x_{HMB} are n-vectors containing the weights invested in the portfolios of SMB stocks and HML stocks; $\sigma_{mSMB} = \text{cov}(R_m, R_{SMB}) = x'_m \Sigma x_{SMB}$, $\sigma_{mHML} = \text{cov}(R_m, R_{HML}) = x'_m \Sigma x_{HML}$; and λ, λ_{SMB} and λ_{HML} are Lagrange multipliers associated with budget constraint, SMB constraint, and HML constraint.

Similar to the MV problem in equation (1), taking the first order condition to equation (5) yields the Fama-French pricing equation

$$\mu = \frac{\lambda}{T_m}\iota + \frac{1}{t_m}x_m + \frac{\lambda_{SMB}}{T_m}\Sigma x_{SMB} + \frac{\lambda_{HML}}{T_m}\Sigma x_{HML} \quad (6)$$

Expressed in scalar notation, the equation (6) is equivalent to:

$$E(R_j) = R_f + (E(R_m) - R_f)\beta_j + E(SMB)s_j + E(HML)h_j \quad (7)$$

where β_j, s_j and h_j are the portfolio's sensitivities to the market, SMB and HML.

3 TESTING METHODS

3.1 Time-Series and Multivariate Tests

The univariate time-series test is the prerequisite for the multivariate test. For the CAPM, the time-series test for individual portfolios is conducted as:

$$R_{jt} - R_{ft} = \alpha_j + (R_{mt} - R_{ft})\beta_j + e_{jt} \tag{8}$$

where R_{jt}, R_{ft} and R_{mt} are the returns on security j, risk-free asset and the market portfolio at time t. If it is for the mutual funds, the intercept is labelled as Jensen's alpha, and reflects the capability of fund managers to generate abnormal returns. In tests of the CAPM, however, the alpha is usually referred to as the pricing error. In the efficient market, pricing errors should be zero.

The multivariate statistics test the CAPM to see whether the individual portfolio's intercepts from the time-series tests are jointly zero. With $e_{it} \sim MVN(0, \Sigma)$, the multivariate test decides whether $\alpha = 0$ from a J-statistics under the central F distribution as in Jobson and Korkie (1982, 1985). In case the normality assumption is not satisfied, the ex post squared Sharpe ratio of the market will not be zero, Gibbons, Ross and Shanken (GRS) further constitutes a Wald-statistics under the non-central F distribution. Power of the multivariate test relies on two assumptions that asset returns are both multivariate normal (MVN) and independent of the model errors. Violation of these assumptions will cause too frequent rejection of the original model.

To test whether the pricing error is jointly zero for various portfolios, the multivariate test constructs the J and W statistics as in equation (9) and (10):

$$J_{CAPM} = \frac{T-N-1}{N}\left(1+\frac{\hat{\mu}_m^2}{\hat{\sigma}_m^2}\right)^{-1} \hat{\alpha}\Sigma^{-1}\hat{\alpha} \sim F_{central,N,T-N-1} \quad (9)$$

$$W_{CAPM} = \frac{T(T-N-1)}{N(T-2)}\left(1+\frac{\hat{\mu}_m^2}{\hat{\sigma}_m^2}\right)^{-1} \hat{\alpha}\Sigma^{-1}\hat{\alpha} \sim F_{noncentral,N,T-N-1,\lambda}$$
$$[\lambda = \frac{T}{1+(\mu_m/\sigma_m)^2}(\hat{\alpha}\Sigma^{-1}\alpha)] \quad (10)$$

where T is the number of observations, N the number of portfolios; and the noncentrality parameter λ is actually the Sharpe's Ratio.

Fama and French (1993, 1996) conduct both the time-series and multivariate tests. The time-series regression for the Three-Factor Model follows:

$$R_{jt} - R_{ft} = \alpha_j + (R_{mt} - R_{ft})\beta_j + SMB_t s_j + HML_t h_j + e_{jt} \quad (11)$$

where SMB_t and HML_t are the size premium and value premium at time t.

And the J and W statistics for the multifactor models change to:

$$J_{3-factor} = \frac{T-N-k}{N}\left(1+\mu_k'\Omega^{-1}\mu_k\right)^{-1} \hat{\alpha}\Sigma^{-1}\hat{\alpha} \sim F_{central,N,T-N-k} \quad (12)$$

$$W_{3-factor} = \frac{T(T-N-k)}{N(T-2)}\left(1+\mu_k'\Omega^{-1}\mu_k\right)^{-1} \hat{\alpha}\Sigma^{-1}\hat{\alpha} \sim F_{noncentral,N,T-N-1,\lambda}$$
$$[\lambda = \frac{T}{1+(\mu_m/\sigma_m)^2}(\hat{\alpha}\Sigma^{-1}\hat{\alpha})] \quad (13)$$

where μ_k is a k-vector of factor means, Ω is the $k \times k$ variance-covariance matrix of factor means, and k equals to 3 for the Three-Factor Model.

From the above equations we can see that the J-statistics and W-statistics do not make much difference. This project conducts the central F tests and reports J-statistics as in Fama and French (1996).

3.2 The Cross-Sectional Tests

There are various ways to conduct the cross-sectional tests and this project discusses three of them. The Black, Jensen and Scholes test (BJS) regresses the average portfolio returns on the time-series estimation of beta. The Fama and MacBeth method (FM) corrects the t-statistics using cross-sectional regressions for each month. The Grauer and Janmaat approach (GJ) drops out the intercept from the BJS test to see how this affects the slope of the SML.

If CAPM holds, at least two conditions have to be satisfied. There is a linear relationship between the expected returns and market betas of securities. And market betas are the only measures of risk needed to explain the expected returns. Therefore, most cross-sectional tests examine the intercept and slope of the SML, to see whether the abnormal return and systematic risk take the proper value.

Black, Jensen and Scholes (1972) conduct a cross-sectional test:

$$\overline{R}_j - \overline{R}_f = \gamma_0 + \gamma_1 \hat{\beta}_j + e_j \tag{14}$$

where $\hat{\beta}_j$ is the systematic risk for each portfolio estimated in the time-series regression equation (8); and $\overline{R}_j - \overline{R}_f$ is the average rate of excess portfolio return across time. For the CAPM to be valid, γ_0 in the equation (14) has to be zero and γ_1 should be the average access market return.

However, both the heteroscedasticity and cross-sectional correlation of the error terms make the t-statistics in BJS test inaccurate. Fama and MacBeth (1973) get around this problem by running the cross-sectional regressions T times for the monthly portfolio excess returns on the full period $\hat{\beta}_j$

$$R_{jt} - R_{ft} = \gamma_{0t} + \gamma_{1t}\hat{\beta}_j + e_{jt} \tag{15}$$

They then form the corrected t-tests of the γ_j's:

$$t(\overline{\gamma}_j) = \frac{\overline{\gamma}_j}{s(\gamma_j)/\sqrt{T}} \tag{16}$$

where $\overline{\gamma}_j$ and $s(\overline{\gamma}_j)$ are the average and standard deviation of the γ_{jt} estimated by the equation (15), and T is the number of time periods.

Grauer and Janmaat (2004) argue that the inclusion of an intercept affects the estimate of the slope. A better estimate of the slope is given when the intercept is set to its theoretical value of zero. With the estimated $\hat{\beta}_j$ for each portfolio, the GJ approach runs one cross-sectional regression:

$$\overline{R}_j - \overline{R}_f = \gamma_1 \hat{\beta}_j + e_j \tag{17}$$

where $\hat{\beta}_j$ and $\overline{R}_j - \overline{R}_f$ are defined identical to those in equation (8).

We also conduct the cross-sectional tests for the Three-Factor Model. BJS test performs as:

$$\overline{R}_j - \overline{R}_f = \gamma_0 + \gamma_1 \hat{\beta}_j + \gamma_s \hat{s}_j + \gamma_h \hat{h}_j + e_j \tag{18}$$

where $\hat{\beta}_j$, $\hat{s}_j$ and $\hat{h}_j$ are the portfolio's sensitivity to the market risk, size premium and value premium estimated in the time-series regression equation (11), and $\overline{R}_j - \overline{R}_f$ is the average rate of excess portfolio return.

The FM method generates the proper values of t-statistics through:

$$R_{jt} - R_{ft} = \gamma_{0t} + \gamma_{1t} \hat{\beta}_j + \gamma_{st} \hat{s}_j + \gamma_{ht} \hat{h}_j + e_{jt} \tag{19}$$

where the notations and t-statistics of $\bar{\gamma}_j$ (j=1, s, and h) follow the formula (16).

We also carry out the Grauer and Janmaat (2004) non-intercept regression.

$$\overline{R}_j - \overline{R}_f = \gamma_1 \hat{\beta}_j + \gamma_s \hat{s}_j + \gamma_h \hat{h}_j + e_j \tag{20}$$

where $\overline{R}_j - \overline{R}_f$, $\hat{\beta}_j$, $\hat{s}_j$ and $\hat{h}_j$ follows the definition in equation (18).

4 DATA

An updated version of Fama and French (1993) twenty-five portfolios of NYSE, AMEX, and NASD stocks sorted by size and book-to-market equity is employed. All the raw data (including the excess return on the market, the returns on 25 portfolios, and the SMB and HML portfolios) except for the risk-free rates are published on Dr. Kenneth French's website: *http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/index.html* And the risk-free rates are the monthly U.S. Treasury bill rates supplied by Professor Robert Grauer.

To construct the returns on the SMB and HML portfolios, Fama and French adopt the following procedure. At the end of June in each year, NYSE, AMEX, and Nasdaq stocks are separated onto small or big groups (S or B) after comparing their June market equity (ME, stock price multiplied by shares outstanding) and the median ME for NYSE stocks. Meanwhile, NYSE, AMEX and NASDAQ stocks are allocated into three book-to-market equity (BE/ME) groups (low, medium, or high; L, M, or H) based on the breakpoints for the bottom 30 percent, middle 40 percent, and top 30 percent of the values of BE/ME for NYSE stocks. The intersection of the two ME and three BE/ME groups defines six size-BE/ME portfolios (S/L, S/M, S/H, B/L, B/M, B/H), and value-weighted monthly returns on these portfolios are calculated from July to the following June. SMB is the difference between the average returns of three small-stock portfolios (S/L, S/M and S/H) and that of three big-stock portfolios (B/L, B/M, and B/H) each month. And HML is the difference between the average returns of two high-BE/ME portfolios (S/H and B/H) and that of two low-BE/ME portfolios (S/L and B/L) each month. In a similar way the 25 size-BE/ME portfolios are constructed, except that the quintile breakpoints for ME and BE/ME for NYSE stocks are adopted instead of their absolute values.

5 RESULTS

Summary Statistics are shown first. Then the testing results for both the CAPM and Three-Factor Model are provided in three sections. Section 5.1 contains the univariate time-series tests similar to those of Fama and French (1996). Section 5.2 contains the multivariate tests as in Gibbons, Ross and Shanken (1989). And Section 5.3 contains the results for the three cross-sectional tests.

Four time periods (1933-1963, 1963-1993, 1994-2003, and 1933-2003) are investigated. This project focuses on the sub-periods 1933-1963 and 1963-1993, because they are at the same length but tell us contradictory stories. The latter one is exactly the period discussed by Fama and French (1996).

Both the CAPM and Three-Factor Model need the portfolios' excess returns, which require subtracting the risk-free rates from the raw numbers of their investment returns. Table 1 in Appendix B summarizes the statistics of the excess returns of 25 portfolio sorted by size and BE/ME for the U.S. stock market from January 1933 to December 2003.

From Panel A in Table 1, the means of excess returns for the time period 1933-1963 are noticeably higher than any other sub-periods especially for the small and low-BE/ME stocks. And Panel B reveals that the standard deviation of the portfolio excess returns is much smaller through 1963 to 1993.

5.1 Results of the Time-Series Tests

Table 2 in Appendix B examines the time-series tests for the CAPM. The estimations and t-statistics of the abnormal return and systematic risk, as well as the R-square and Standard Error of Residuals are reported. In compliance with the CAPM, the intercepts (variously known as Jensen's alphas or the pricing errors) should be zero. The results in Table 2 demonstrate that the slopes range from 0.63 to 1.80 and are all significantly non-zero at any reasonable level. The intercepts range from –0.72 to 1.15. There are several statistically significant pricing errors in each time period. They will be discussed below.

Fama and French (1996) conduct a similar time-series test for their Three-Factor Model. They run the OLS regressions for the excess portfolio return on the excess market rate, SMB and HML in the period 1963-1993, and assert that their model out-performs the CAPM based on the smaller absolute pricing errors of the Three-Factor Model. Table 3 in Appendix B extends their method into four time periods. The panel B of Table 3 is a replication of the Table 1 in Fama and French (1996). The estimates deviate slightly from those of Fama and French (1996) because the data have been updated. Note that the summary statistics of the period 1963-1993 (Panel B of Table 1 in Appendix B) also differs from the Panel A of Table 1 in Fama and French (1996) for the same reason.

Fama and French (1996) assert that the CAPM is at a disadvantage because its absolute pricing errors are three to five times those of the three-factor model. However, after taking a close look at the statistics of the pricing errors displayed in the Figure 1 of Appendix A, their statement appears to be premature. Through three sub-periods (1963-1993, 1994-2003, and 1933-2003), Figure 1 supports the conclusion of Fama and French (1996): the averages of the absolute values of the pricing errors for the CAPM are larger than those of the Three-Factor Model. The maximum difference occurs in the 1963-1993 period. But in the 1933-1963 period, the CAPM is actually superior to the Three-Factor Model by a small margin.

On the other hand, Figure 2 in Appendix A illustrates the frequency of statistically significant pricing errors. The more frequent statistically significant pricing errors indicate the worse performance. Perhaps surprisingly, the CAPM performs better during 1933-1963 and 1993-2004, while the Three-Factor Model outperforms in the other two periods. But one needs to test all the pricing errors simultaneously in order to draw firm conclusions.

5.2 Results of the Multivariate Tests

The P-values of the multivariate tests for the CAPM and the Three-Factor Model are revealed in Table 4 and Table 5 in Appendix B, respectively. And Figure 3 in Appendix A compares the results. The null hypothesis that the pricing errors are jointly zero is rejected in both models for all four periods at the significance level of 5 percent. But at the 1 percent level, the CAPM could be accepted in the 1933-1963 period, while the Three-Factor Model could be accepted in two sub-periods (1933-1963, 1963-1993). The CAPM has higher P-values in the periods 1933-1963 and 1994-2003 while the Three-Factor Model excels during the rest periods.

5.3 Results of the Cross-Sectional Tests

The results of the cross-sectional tests using the Black, Jensen and Scholes (BJS) test, Fama and MacBeth (FM) method, and Grauer and Janmaat (GJ) approach are summarized in Table 6 of Appendix A. The BJS tests reject the CAPM at 5 percent significance level in the sub-periods 1963-1993 and 1994-2004, because the intercepts are significantly positive and the slopes are negative. But the estimation for the 1933-1963 period looks better with an insignificant intercept and positive slope. The intercept and slope in the full-time period (1933-2003) both are positive. The FM method does not change the conclusion on whether to reject or accept the CAPM, although the corrected t-statistics are a little bit lower in general. The GJ approach, however, provides another perspective of the model performance. After dropping out the intercepts, all the slopes are significantly positive. Figure 3 in Appendix A compares the "estimates of the SML" with or without the intercepts. The negative slopes of BJS and FM tests during the periods 1963-1993 and 1994-2003 are reversed in the GJ approach. Through the other two sub-periods, the slopes are steeper.

The cross-sectional tests for the Three-Factor Model are summarized in Table 7 of Appendix B. Again the estimations of the BJS and FM tests are very similar. The market premiums (GammaB) in both tests are negative through four periods. And the premiums of the size effect (GammaS) are mostly smaller than the value effect (GammaH), indicating that portfolios are priced largely on their book-to-market equities. After the FM method corrects the t-statistics, the size premiums and value premiums are rejected at 5 percent significance level in the periods 1933-1963 and 1994-2004, which makes the Three-Factor Model approximate the CAPM.

Without the intercepts, the market risk premiums become positive in the GJ approach, and both the size premiums and value premiums are higher. The premiums of all three factors are significant. This illustrates Grauer and Janmaat's point that the premium of a unit-weight portfolio is affected by the inclusion of the intercept term, while the premiums of the zero-weighted portfolios (SMB and HML) are not.

6 CONCLUSIONS

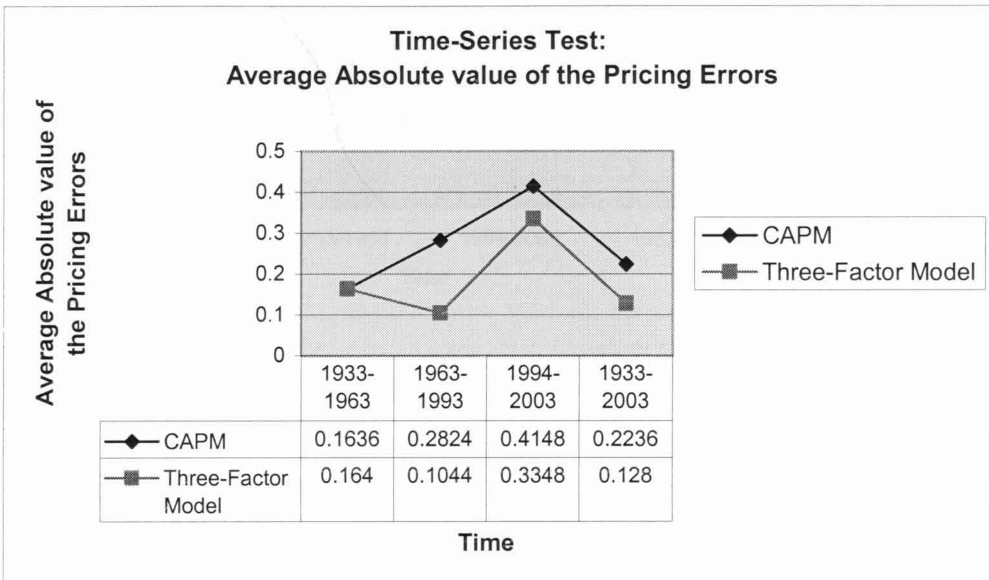
After calibrating the CAPM and Three-Factor Model through different testing methods and time periods, it is interesting to compare the results of the cross-sectional tests with those of the multivariate test. The multivariate test rejects the null hypothesis that the pricing errors are jointly zero for both models, but shows that the CAPM performs better during the periods 1933-1963 and 1994-2003. When an intercept is included in the cross-sectional regressions, the slope is negative in the post-1963 period, but positive in the 1933-1963 period and the 1933-2003 period. But after dropping out the intercepts, the slopes of the SML turn positive in the 1933-1963 period and the 1933-2003 period, which proves that the inclusion of the intercepts affects the estimate of the slope.

Now it is safe to cast some doubts on the advantage of the Three-Factor Model. Although the R-squares are higher and the average absolute pricing errors are generally lower in the Three-Factor Model, there is not a consensus across varied times and tests. The time-series test favours the Three-Factor Model except for the period 1933-1963, the multivariate test rejects both models, and the cross-sectional tests can only support the Three-Factor Model during 1963-1993 to a lesser extent.

APPENDICES

Appendix A: Figures

Figure 1 Average Absolute Values of the Pricing Errors



	1933-1963	1963-1993	1994-2003	1933-2003
CAPM	0.1636	0.2824	0.4148	0.2236
Three-Factor Model	0.164	0.1044	0.3348	0.128

Figure 2 Frequency of Statistically Significant Pricing Errors

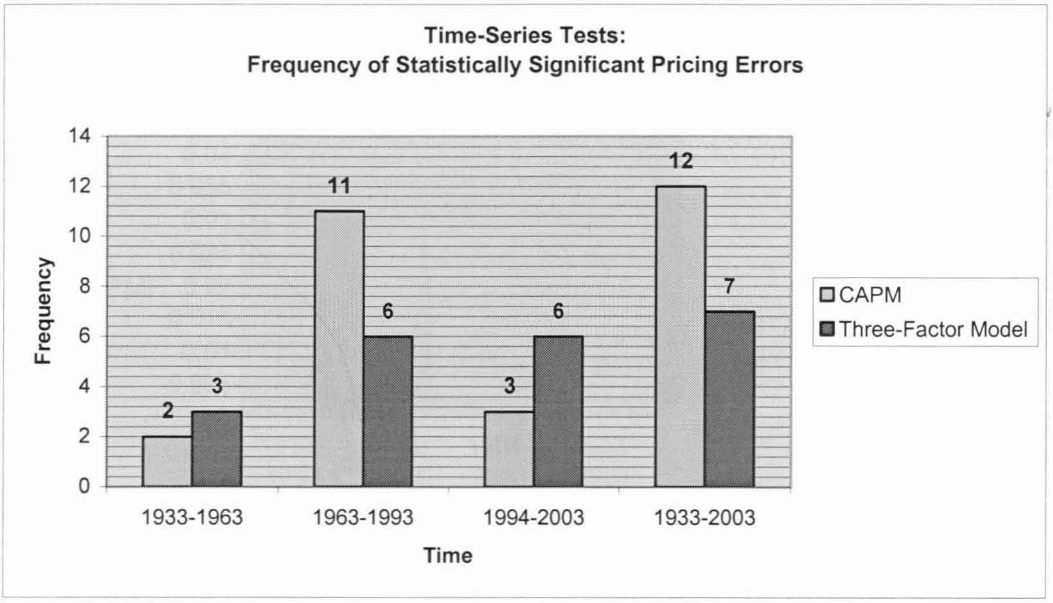


Figure 3 Multivariate Test P-values for the CAPM and Three-Factor Model

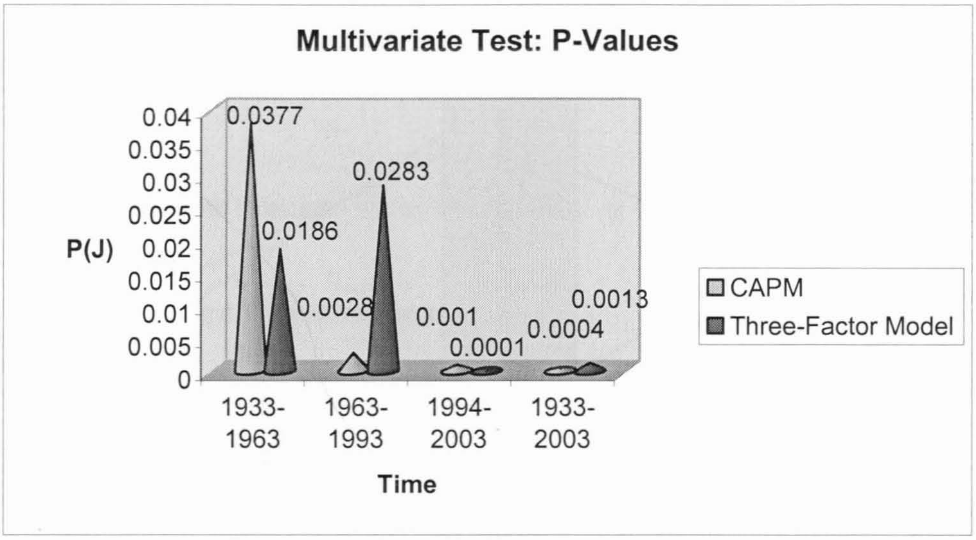
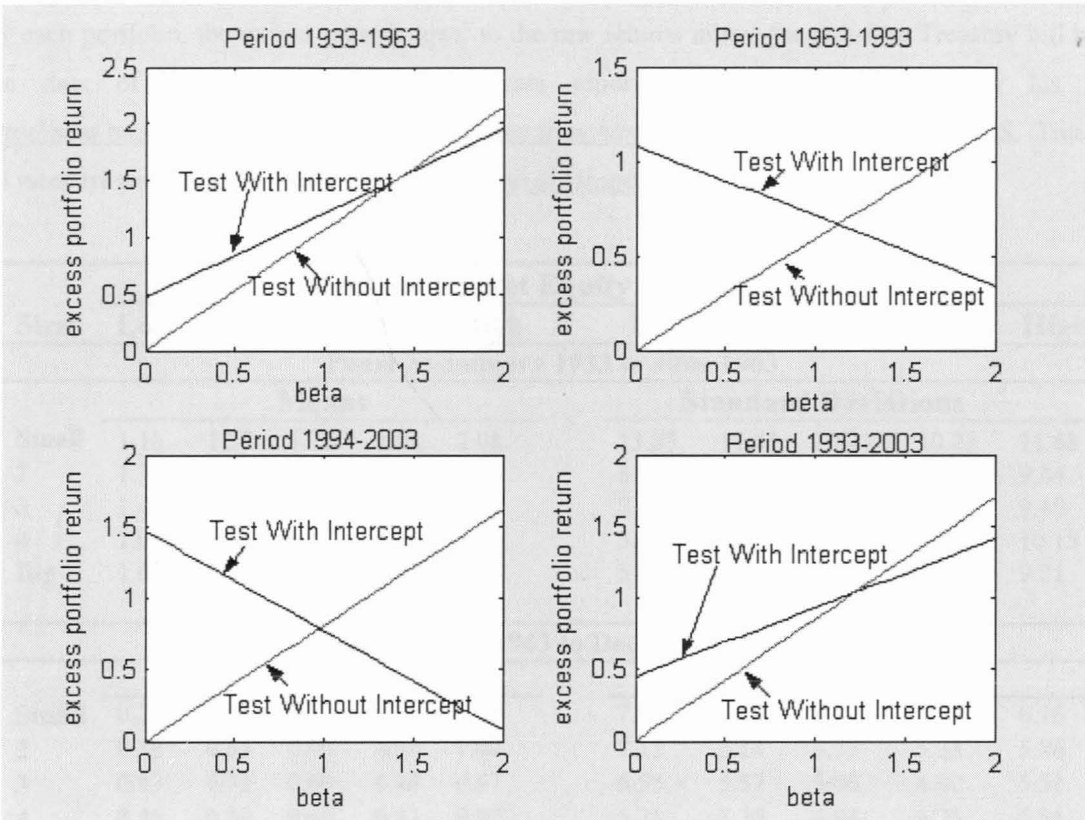


Figure 4 Estimates of the SML in the Cross-Sectional Tests With or Without Intercepts



Appendix B: Tables

Table 1 Summary Statistics
for the Excess Returns of 25 Portfolios Sorted By Size and Book-to-Market Equity

For each portfolio, the excess returns equal to the raw returns minus the risk-free Treasury bill rates. The data of monthly portfolio returns are reported by Kenneth French on his web (*http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/index.html*) and the monthly U.S. Treasury bill rates are published by CRSP. (*http://gsbwww.uchicago.edu/research/crsp/*)

	Book-to-Market Equity (BE/ME) Quintiles									
Size	**Low**	**2**	**3**	**4**	**High**	**Low**	**2**	**3**	**4**	**High**
	Panel A: January 1933 to June 1963									
	Means					**Standard Deviations**				
Small	1.16	1.37	1.78	1.98	2.08	13.83	11.41	10.71	10.23	11.68
2	1.30	1.69	1.61	1.68	1.86	8.39	8.53	8.57	8.68	9.84
3	1.41	1.45	1.52	1.56	1.64	7.65	7.08	7.50	7.47	9.40
4	1.07	1.31	1.49	1.30	1.66	5.66	6.59	6.49	7.37	10.15
Big	1.02	0.95	1.29	1.32	1.46	5.30	5.03	5.50	7.32	9.21
	Panel B: July 1963 to December 1993									
	Means					**Standard Deviation**				
Small	0.27	0.70	0.74	0.92	1.09	7.63	6.69	6.12	5.79	6.16
2	0.38	0.65	0.90	0.94	1.07	7.15	6.14	5.57	5.23	5.86
3	0.43	0.72	0.69	0.88	0.97	6.55	5.57	5.06	4.80	5.51
4	0.46	0.39	0.65	0.81	0.92	5.78	5.30	4.94	4.75	5.51
Big	0.32	0.36	0.40	0.52	0.64	4.79	4.59	4.27	4.21	4.76
	Panel C: January 1994 to December 2003									
	Mean					**Standard Deviation**				
Small	0.22	1.21	1.27	1.62	1.43	10.12	8.25	5.92	5.26	5.31
2	0.41	0.63	0.89	1.00	0.92	8.59	5.95	4.85	4.98	5.44
3	0.37	0.63	0.77	0.80	1.16	7.94	5.33	4.64	4.75	5.06
4	0.71	0.83	0.93	0.93	0.77	7.14	4.88	4.74	4.49	5.03
Big	0.73	0.80	0.79	0.73	0.35	5.05	4.66	4.63	4.55	5.13
	Panel D: January 1933 to December 2003									
	Mean					**Standard Deviation**				
Small	0.64	1.06	1.26	1.47	1.57	11.02	9.20	8.39	7.96	8.88
2	0.78	1.10	1.20	1.26	1.39	7.91	7.25	6.95	6.90	7.78
3	0.84	1.02	1.06	1.16	1.28	7.25	6.23	6.19	6.09	7.39
4	0.76	0.85	1.05	1.04	1.22	5.94	5.84	5.64	5.99	7.80
Big	0.68	0.67	0.84	0.89	0.95	5.05	4.80	4.90	5.80	7.07

Table 2 Time-Series Tests for the CAPM

The regression function is $R_{jt} - R_t = a_j + b_j(R_{mt} - R_t) + e_{jt}$, where R_{jt} is the monthly portfolio return; R_{mt}, the market rate is the value-weighted return on all stocks in the size-BE/ME portfolios; and the risk-free rate R_t is the monthly Treasury bill rate published by CRSP.

	Book-to-Market Equity (BE/ME) Quintiles									
Size	Low	2	3	4	High	Low	2	3	4	High
Panel A: January 1933 to June 1963										
	a					t(a)				
Small	0.73	0.63	0.09	0.23	0.23	-1.32	-1.96	-0.30	0.76	0.58
2	-0.15	0.15	0.05	0.08	0.12	-0.62	0.69	0.24	0.37	0.46
3	-0.03	0.07	0.06	0.15	-0.11	-0.16	0.58	0.43	0.96	-0.49
4	-0.05	0.02	0.21	-0.13	-0.20	-0.54	0.15	2.08	-0.95	-0.80
Big	-0.04	-0.06	0.22	-0.09	-0.19	-0.68	-0.82	2.31	-0.64	-0.80
	b					t(b)				
Small	1.70	1.80	1.68	1.57	1.66	16.86	30.82	30.38	28.10	22.96
2	1.31	1.38	1.40	1.44	1.56	29.52	34.40	35.96	38.08	31.61
3	1.29	1.23	1.32	1.27	1.57	41.66	53.30	55.96	43.36	39.37
4	1.00	1.17	1.15	1.29	1.67	63.95	63.54	62.18	53.15	36.85
Big	0.96	0.90	0.96	1.27	1.49	82.23	73.36	55.54	50.64	34.12
	R-square					s(e)				
Small	0.44	0.72	0.72	0.68	0.59	10.37	6.02	5.70	5.76	7.48
2	0.71	0.76	0.78	0.80	0.73	4.56	4.14	4.02	3.89	5.09
3	0.83	0.89	0.90	0.84	0.81	3.19	2.39	2.42	3.01	4.11
4	0.92	0.92	0.91	0.89	0.79	1.62	1.90	1.91	2.49	4.68
Big	0.95	0.94	0.89	0.88	0.76	1.20	1.27	1.79	2.58	4.50
Panel B: July 1963 to December 1993										
	a					t(a)				
Small	-0.36	0.16	0.23	0.45	0.61	-1.57	0.78	1.31	2.57	3.07
2	-0.25	0.11	0.41	0.48	0.58	-1.42	0.77	3.01	3.66	3.55
3	-0.17	0.22	0.24	0.46	0.50	-1.24	1.91	2.14	4.18	3.33
4	-0.08	-0.11	0.20	0.38	0.45	-0.81	-1.24	2.10	3.59	3.16
Big	-0.12	-0.08	0.01	0.15	0.25	-1.30	-1.03	0.16	1.47	1.73
	b					t(b)				
Small	1.42	1.25	1.15	1.07	1.09	27.65	27.92	28.89	27.11	24.39
2	1.43	1.23	1.11	1.04	1.12	36.45	36.92	36.21	35.07	30.57
3	1.36	1.16	1.03	0.98	1.06	44.63	45.70	40.98	39.89	31.59
4	1.23	1.13	1.04	0.97	1.08	53.63	57.31	49.52	40.32	34.23
Big	1.01	0.99	0.87	0.84	0.87	49.08	59.35	41.04	36.76	26.62

	Book-to-Market Equity (BE/ME) Quintiles									
Size	Low	2	3	4	High	Low	2	3	4	High
	R-square					s(e)				
Small	0.68	0.68	0.70	0.67	0.62	4.34	3.78	3.37	3.33	3.80
2	0.78	0.79	0.78	0.77	0.72	3.32	2.82	2.60	2.50	3.10
3	0.85	0.85	0.82	0.81	0.73	2.58	2.15	2.14	2.07	2.85
4	0.89	0.90	0.87	0.82	0.76	1.94	1.68	1.78	2.03	2.68
Big	0.87	0.91	0.82	0.79	0.66	1.74	1.41	1.80	1.94	2.78

Panel C: January 1994 to December 2003										
	a					t(a)				
Small	-0.72	0.49	0.75	1.15	0.95	-1.07	0.85	1.83	3.17	2.65
2	-0.48	0.02	0.40	0.52	0.41	-0.98	0.05	1.36	1.63	1.15
3	-0.48	0.04	0.28	0.35	0.67	-1.09	0.14	1.07	1.14	2.06
4	-0.12	0.27	0.44	0.46	0.31	-0.36	1.15	1.60	1.77	0.90
Big	0.09	0.26	0.29	0.32	-0.10	0.64	1.23	1.16	1.03	-0.29
	b					t(b)				
Small	1.20	0.88	0.68	0.63	0.71	10.51	9.26	9.56	9.60	10.18
2	1.20	0.88	0.75	0.75	0.81	13.71	13.12	12.53	11.10	10.73
3	1.13	0.94	0.84	0.79	0.87	14.52	16.63	13.89	10.80	11.08
4	1.15	0.94	0.89	0.82	0.88	10.51	9.26	9.56	9.60	10.18
Big	1.02	0.97	0.93	0.82	0.91	13.71	13.12	12.53	11.10	10.73
	R-square					s(e)				
Small	0.48	0.42	0.44	0.44	0.47	7.30	6.30	4.46	3.96	3.89
2	0.61	0.59	0.57	0.51	0.49	5.35	3.81	3.19	3.50	3.89
3	0.64	0.70	0.62	0.50	0.51	4.78	2.93	2.87	3.38	3.56
4	0.75	0.73	0.61	0.60	0.47	3.58	2.54	2.97	2.86	3.69
Big	0.91	0.76	0.66	0.44	0.44	1.48	2.28	2.71	3.42	3.87

Panel D: January 1933 to December 2003										
	a					t(a)				
Small	-0.54	-0.08	0.21	0.50	0.55	-2.00	-0.45	1.26	2.98	2.67
2	-0.25	0.13	0.28	0.35	0.40	-1.72	1.04	2.33	2.87	2.63
3	-0.16	0.14	0.19	0.34	0.31	-1.41	1.66	2.18	3.40	2.32
4	-0.09	0.00	0.25	0.21	0.20	-1.19	0.03	3.38	2.32	1.40
Big	-0.06	-0.03	0.15	0.11	0.06	-1.19	-0.49	2.18	1.15	0.45
	b					t(b)				
Small	1.57	1.52	1.39	1.29	1.35	28.60	40.20	40.47	38.06	32.67
2	1.37	1.28	1.22	1.21	1.31	46.48	50.31	49.72	49.17	42.83
3	1.32	1.17	1.15	1.09	1.29	58.68	69.55	64.51	54.50	48.08
4	1.12	1.12	1.07	1.10	1.34	72.56	81.88	72.19	61.66	46.12
Big	0.98	0.93	0.91	1.04	1.17	94.09	87.18	66.37	54.10	41.15
	R-squre					s(e)				
Small	0.49	0.66	0.66	0.63	0.56	7.87	5.41	4.91	4.84	5.92
2	0.72	0.75	0.74	0.74	0.68	4.21	3.64	3.52	3.52	4.38
3	0.80	0.85	0.83	0.78	0.73	3.23	2.41	2.55	2.87	3.83
4	0.86	0.89	0.86	0.82	0.71	2.22	1.96	2.11	2.56	4.17
Big	0.91	0.90	0.84	0.77	0.67	1.50	1.52	1.97	2.75	4.09

Table 3 Time-Series Tests for the Fama-French Three-Factor Model

The regression is $R_{jt} - R_t = a_j + b_j(R_{mt} - R_t) + s_j SMB + h_j HML + e_i$, where r_{mt} and r_t are defined the same as Table 2, SMB is the difference between the average returns of three small-stock portfolios and that of three big-stock portfolios each month. And HML is the difference between the average returns of two high-BE/ME portfolios and that of two low-BE/ME portfolios each month.

	Book-to-Market Equity (BE/ME) Quintiles									
Size	**Low**	**2**	**3**	**4**	**High**	**Low**	**2**	**3**	**4**	**High**
Panel A: January 1933 to June 1963										
	a					**t(a)**				
Small	-0.8	-0.68	-0.15	0.17	0.13	-1.81	-3.20	-0.72	1.21	0.81
2	-0.17	0.13	0.01	0.03	0.04	-0.99	1.15	0.13	0.35	0.33
3	-0.03	0.07	0.03	0.12	-0.18	-0.29	0.71	0.40	1.24	-1.44
4	-0.03	0.01	0.2	-0.16	-0.28	-0.43	0.06	2.34	-1.54	-1.88
Big	-0.02	-0.05	0.21	-0.13	-0.27	-0.53	-0.76	2.35	-1.26	-1.75
	b					**t(b)**				
Small	0.99	1.29	1.18	0.98	0.82	10.11	27.62	25.85	31.53	24.19
2	1.01	1.04	1.02	1.03	0.99	27.21	41.46	41.82	50.69	36.95
3	1.09	1.08	1.11	0.98	1.14	43.71	52.87	57.45	45.34	42.24
4	1.03	1.06	1.03	1.08	1.21	59.56	55.09	55.15	48.30	36.74
Big	1.06	0.96	0.93	1.1	1.15	108.78	69.56	48.28	48.71	33.51
	s					**t(s)**				
Small	1.83	1.34	1.12	1.48	1.71	11.94	18.36	15.57	30.27	32.17
2	1.12	1.19	1.04	0.98	0.93	19.32	29.98	27.14	30.62	22.04
3	0.82	0.55	0.52	0.65	0.56	21.06	17.15	17.10	19.08	13.14
4	0.14	0.29	0.3	0.3	0.44	5.21	9.49	10.33	8.50	8.45
Big	-0.04	-0.11	-0.15	-0.07	-0.15	-2.85	-5.04	-4.93	-2.02	-2.79
	h					**t(h)**				
Small	0.58	0.39	0.55	0.5	1.08	3.85	5.38	7.79	10.45	20.65
2	-0.06	0.01	0.27	0.4	0.94	-1.09	0.15	7.19	12.71	22.77
3	-0.11	-0.01	0.17	0.29	0.82	-2.80	-0.29	5.58	8.78	19.74
4	-0.22	0.07	0.12	0.36	1.02	-8.06	2.47	4.05	10.34	20.27
Big	-0.27	-0.09	0.24	0.59	1.16	-17.97	-4.01	8.26	16.96	21.95
	R-square					**s(e)**				
Small	0.64	0.88	0.87	0.93	0.94	8.31	3.97	3.90	2.65	2.89
2	0.86	0.94	0.94	0.96	0.95	3.15	2.14	2.08	1.73	2.28
3	0.92	0.94	0.95	0.94	0.94	2.12	1.74	1.65	1.85	2.30
4	0.93	0.94	0.94	0.93	0.92	1.48	1.64	1.59	1.91	2.80
Big	0.98	0.95	0.95	0.91	0.93	0.83	1.18	1.18	1.63	1.92

---Table 3 continued---

Size	Low	2	3	4	High	Low	2	3	4	High
	Panel B: July 1963 to December 1993									
	a					**t(a)**				
Small	-0.48	-0.14	-0.11	0.05	0.07	-4.53	-1.73	-1.79	0.83	1.01
2	-0.19	0.06	0.14	0.12	0.07	-2.29	-0.97	2.18	1.91	1.04
3	-0.06	0.08	-0.02	0.13	0.02	-0.85	1.13	-0.29	2.00	0.30
4	0.10	-0.18	-0.01	0.08	0.02	1.35	-2.18	-0.07	0.97	0.21
Big	0.15	-0.04	-0.05	-0.09	-0.15	2.31	-0.55	-0.53	-1.25	-1.38
	b					**t(b)**				
Small	1.03	0.97	0.94	0.89	0.95	38.74	49.13	59.93	59.02	58.42
2	1.09	1.01	0.97	0.96	1.06	53.55	60.57	61.27	63.80	65.06
3	1.09	1.02	0.97	0.97	1.06	60.06	56.71	54.32	59.38	52.19
4	1.06	1.08	1.04	1.03	1.14	57.62	52.89	52.64	50.95	46.18
Big	0.95	1.03	0.98	1.00	1.04	57.40	58.45	45.20	53.83	37.87
	s					**t(s)**				
Small	1.43	1.30	1.16	1.11	1.21	37.18	45.47	50.95	51.01	51.21
2	1.03	0.95	0.84	0.72	0.85	34.78	39.08	36.52	32.78	35.95
3	0.72	0.63	0.56	0.46	0.64	27.16	24.02	21.50	19.30	21.77
4	0.32	0.27	0.25	0.21	0.37	11.88	9.11	8.82	7.21	10.28
Big	-0.18	-0.20	-0.26	-0.19	-0.01	-7.59	-7.69	-8.14	-6.91	-0.33
	h					**t(h)**				
Small	-0.27	0.10	0.26	0.38	0.64	-6.34	3.28	10.24	15.75	24.34
2	-0.49	0.01	0.24	0.46	0.69	-14.83	0.33	9.26	18.67	26.03
3	-0.46	0.04	0.31	0.47	0.70	-15.67	1.33	10.69	17.84	21.16
4	-0.47	0.04	0.30	0.52	0.70	-15.65	1.13	9.41	15.93	17.35
Big	-0.46	0.00	0.21	0.54	0.79	-17.28	-0.11	5.94	17.95	17.74
	R-square					**s(e)**				
Small	0.93	0.95	0.96	0.96	0.96	1.97	1.46	1.16	1.12	1.20
2	0.96	0.96	0.96	0.95	0.96	1.51	1.24	1.17	1.12	1.21
3	0.96	0.94	0.93	0.94	0.93	1.35	1.33	1.32	1.21	1.50
4	0.94	0.92	0.91	0.90	0.89	1.36	1.51	1.47	1.49	1.84
Big	0.93	0.92	0.86	0.89	0.82	1.23	1.31	1.60	1.38	2.04

Book-to-Market Equity (BE/ME) Quintiles

---Table 3 continued---

Size	Low	2	3	4	High	Low	2	3	4	High
	Book-to-Market Equity (BE/ME) Quintiles									
	Panel C: January 1994 to December 2003									
			a					**t(a)**		
Small	-0.93	0.29	0.57	0.96	0.70	-2.73	1.09	2.77	4.61	3.52
2	-0.65	-0.19	0.20	0.29	0.14	-3.12	-1.09	1.05	1.31	0.59
3	-0.57	-0.13	0.12	0.15	0.42	-2.67	-0.60	0.53	0.59	1.75
4	-0.14	0.14	0.29	0.33	0.14	-0.74	0.68	1.29	1.43	0.50
Big	0.18	0.21	0.19	0.23	-0.21	1.67	1.37	1.06	1.08	-0.82
			b					**t(b)**		
Small	1.20	0.88	0.68	0.63	0.71	15.84	14.89	14.85	13.65	16.16
2	1.20	0.88	0.75	0.75	0.81	26.07	22.38	18.06	15.57	15.22
3	1.13	0.94	0.84	0.79	0.87	24.02	19.92	16.79	13.92	16.17
4	1.15	0.94	0.89	0.82	0.88	26.71	20.47	17.78	15.87	14.85
Big	1.02	0.97	0.93	0.82	0.91	43.17	28.07	23.66	17.52	16.01
			s					**t(s)**		
Small	1.56	1.41	1.03	0.90	0.92	16.32	18.86	17.76	15.25	16.52
2	1.21	0.92	0.70	0.72	0.80	20.68	18.48	13.41	11.75	11.86
3	0.97	0.50	0.30	0.33	0.40	16.14	8.37	4.76	4.57	5.83
4	0.61	0.22	0.10	0.15	0.03	11.11	3.75	1.55	2.25	0.37
Big	-0.25	-0.20	-0.12	-0.27	-0.28	-8.22	-4.51	-2.36	-4.56	-3.86
			h					**t(h)**		
Small	-0.23	-0.16	-0.01	0.09	0.27	-2.80	-2.50	-0.13	1.84	5.65
2	-0.16	0.14	0.25	0.35	0.40	-3.19	3.45	5.54	6.71	6.98
3	-0.26	0.24	0.35	0.45	0.56	-5.18	4.69	6.52	7.36	9.67
4	-0.26	0.29	0.41	0.34	0.53	-5.73	5.81	7.55	6.10	8.28
Big	-0.15	0.25	0.39	0.46	0.49	-5.78	6.78	9.11	9.13	8.05
			R-square					**s(e)**		
Small	0.87	0.88	0.86	0.82	0.84	3.68	2.88	2.22	2.26	2.14
2	0.93	0.90	0.83	0.78	0.78	2.25	1.91	2.01	2.35	2.59
3	0.92	0.82	0.73	0.67	0.74	2.30	2.31	2.43	2.78	2.63
4	0.92	0.80	0.74	0.70	0.68	2.10	2.23	2.45	2.51	2.89
Big	0.95	0.87	0.83	0.76	0.72	1.16	1.69	1.92	2.27	2.76

---Table 3 continued---

Size	Low	2	3	4	High	Low	2	3	4	High
				Book-to-Market Equity (BE/ME) Quintiles						
				Panel D: January 1933 to December 2003						
			a					t(a)		
Small	-0.80	-0.34	-0.07	0.18	0.09	-3.86	-3.04	-0.65	2.28	1.03
2	-0.30	-0.03	0.06	0.09	0.04	-3.42	-0.43	0.97	1.47	0.52
3	-0.17	0.04	0.02	0.13	-0.02	-2.48	0.67	0.39	1.95	-0.29
4	0.00	-0.07	0.13	0.02	-0.15	-0.02	-1.23	2.07	0.35	-1.61
Big	0.03	-0.01	0.08	-0.08	-0.25	0.85	-0.28	1.36	-1.31	-2.60
			b					t(b)		
Small	1.13	1.15	1.07	0.94	0.94	25.38	47.81	47.25	53.47	47.51
2	1.10	1.01	0.97	0.97	1.03	58.12	72.51	70.30	75.04	65.82
3	1.13	1.03	1.01	0.94	1.09	77.38	78.74	76.19	67.15	68.39
4	1.07	1.05	1.00	1.01	1.19	86.15	79.80	74.60	70.70	60.46
Big	1.03	0.97	0.94	1.04	1.13	115.73	91.61	74.26	74.54	55.30
			s					t(s)		
Small	1.72	1.42	1.18	1.28	1.45	25.23	38.62	34.42	47.59	48.28
2	1.12	1.03	0.90	0.84	0.94	28.53	39.59	36.70	37.76	31.77
3	0.84	0.54	0.47	0.50	0.59	37.67	27.28	23.23	23.32	24.05
4	0.31	0.25	0.21	0.24	0.39	16.53	12.28	10.34	11.09	12.84
Big	-0.15	-0.20	-0.19	-0.13	-0.05	-10.83	-12.09	-10.01	-6.35	-1.65
			h					t(h)		
Small	0.15	0.26	0.42	0.49	0.83	2.44	7.82	13.59	20.34	30.79
2	-0.24	0.11	0.34	0.48	0.74	-9.28	5.87	17.88	27.29	34.53
3	-0.26	0.10	0.33	0.45	0.76	-12.91	5.61	18.35	23.64	35.04
4	-0.37	0.14	0.28	0.45	0.89	-21.88	7.83	15.39	23.10	32.98
Big	-0.23	0.03	0.27	0.61	0.92	-19.06	2.11	15.36	32.25	33.03
			R-square					s(e)		
Small	0.71	0.88	0.87	0.91	0.91	5.92	3.18	2.99	2.33	2.61
2	0.90	0.94	0.93	0.94	0.93	2.51	1.85	1.84	1.71	2.08
3	0.93	0.92	0.92	0.91	0.92	1.93	1.73	1.76	1.86	2.12
4	0.92	0.91	0.90	0.90	0.89	1.64	1.74	1.77	1.90	2.61
Big	0.95	0.91	0.88	0.90	0.85	1.18	1.41	1.68	1.84	2.71

Table 4 Multivariate Tests for the CAPM

The formula of multivariate tests for the CAPM is

$$J = \frac{T-N-1}{N}\left(1+\frac{\hat{\mu}_m^2}{\hat{\sigma}_m^2}\right)^{-1}\hat{\alpha}'\Sigma^{-1}\hat{\alpha}' .$$

μ_m and σ_m are the average and standard deviation of the market excess return in Table 1. T is the number of time-series observations and N is the number of portfolios. Let $\hat{\alpha} = (\alpha_1, \alpha_2 ... \alpha_n)'$ and $\varepsilon_t = (\varepsilon_{1t,} \varepsilon_{2t} ... \varepsilon_{nt})'$ be n-vectors of the intercepts and residuals of Table1's regressions, and $\Sigma = E(\varepsilon_t \varepsilon_t')$ is the variance-covariance Matrix of the residuals.

Gibbons, Ross and Shanken (1989) shows that for the null hypothesis of $\alpha = 0$, J should follow an unconditionally distributed central F-statistics with N-T-1 degrees of freedom in the denominator and N degrees of freedom in the numerator.

Multivariate Test for CAPM across Time				
	1933-1963	**1963-1993**	**1994-2003**	**1933-2003**
μ_m	1.1119	0.4398	0.6261	0.7547
σ_m	5.3971	4.4372	4.6892	4.9105
$\left(1+\frac{\hat{\mu}_m^2}{\hat{\sigma}_m^2}\right)^{-1}$	0.9593	0.9903	0.9998	0.9769
T	366	366	120	852
N	25	25	25	25
$\frac{T-N-1}{N}$	13.6	13.6	3.76	33.04
$\hat{\alpha}\Sigma^{-1}\hat{\alpha}$	0.139	0.1935	0.8289	0.1003
J	1.8137	2.6056	3.116	3.2385
Prob (J)	0.0377	0.0028	0.0010	0.0004

Table 5 Multivariate Test for The Fama-French Three-Factor Model

The formula of the multivariate tests for the Three-Factor Model is

$$J = \frac{T - N - K}{N}\left(1 + \mu_k' \Omega^{-1} \mu_k\right)^{-1} \hat{\alpha}' \Sigma^{-1} \hat{\alpha}'$$

As estimated in Table 2 and 3, $\mu_k = (\bar{r}_m - \bar{r}_t, \bar{s}_m, \bar{h}_m)'$ is a k-vector of factor mean, and Ω is the $k \times k$ variance-covariance matrix of the factor estimations. T is the number of observations, N is the number of portfolios, and K is the number of factors. $\hat{\alpha} = (\alpha_1, \alpha_2 ... \alpha_n)'$ is the n-vector of the intercepts and $\sum = E(\varepsilon_t \varepsilon_t')$ is the variance-covariance matrix of residuals.

Jobson and Korkie (1985) shows that if there are the k-factors, the J follows a F-statistics with N-T-K degrees of freedom in the denominator and N degrees of freedom in the numerator.

Multivariate Test for Three Factor Model across Time

	1933-1963	1963-1993	1994-2003	1933-2003
$\bar{r}_m - \bar{r}_t$	1.11	0.44	0.63	0.75
$\bar{s}_m$	0.33	0.29	0.27	0.30
$\bar{h}_m$	0.44	0.42	0.14	0.39
$\left(1 + \mu_k' \Omega^{-1} \mu_k\right)^{-1}$	0.9587	0.9436	0.9716	0.9660
T	366	366	120	852
N	25	25	25	25
K	3	3	3	3
$\frac{T - N - K}{N}$	13.6	13.6	3.76	33.04
$\hat{\alpha}' \Sigma^{-1} \hat{\alpha}'$	0.1558	0.1487	2.6523	0.0892
J	2.0194	1.8972	9.4880	2.8388
Prob (J)	0.0186	0.0283	0.0001	0.0013

Table 6 Cross-Sectional Tests for the CAPM

$\bar{r}$ and $\bar{r}_j$ are respectively the time-series average rates of return on Table 1's risk-free Treasury bill and risky portfolio j (j ranges from 1 to 25), and $\hat{\beta}_j$ are the estimated systematic risk (b) in Table 2.

		COEFFICIENT	T-STAT	P-VALUE	R-SQUARE
Panel A: BJS tests $\overline{R}_j - \overline{R} = \gamma_0 + \gamma_1 \hat{\beta}_j + e_j$					
1933-1963	**Gamma0**	0.47	1.93	0.0661	0.4298
	Gamma1	0.74	0.18	0.0004	
1963-1993	**Gamma0**	1.08	2.97	0.0069	0.0524
	Gamma1	-0.37	-1.13	0.2709	
1994-2003	**Gamma0**	1.47	6.46	0.0000	0.2662
	Gamma1	-0.70	-2.89	0.0083	
1933-2003	**Gamma0**	0.45	1.24	0.2256	0.1029
	Gamma1	0.48	1.62	0.1179	
Panel B: FM tests: $R_{jt} - R_{ft} = \gamma_{0t} + \gamma_{1t} \hat{\beta}_j + e_{jt}$					
1933-1963	**Gamma0**	0.48	1.16	0.2477	N/A
	Gamma1	0.74	1.51	0.1313	N/A
1963-1993	**Gamma0**	1.08	2.80	0.0053	N/A
	Gamma1	-0.37	-0.81	0.4138	N/A
1994-2003	**Gamma0**	1.47	2.09	0.0391	N/A
	Gamma1	-0.70	-0.77	0.4406	N/A
1933-2003	**Gamma0**	0.45	1.25	0.2133	N/A
	Gamma1	0.48	1.24	0.2162	N/A
Panel C: GJ tests: $\overline{R}_j - \overline{R} = \gamma_1 \hat{\beta}_j + e_j$					
1933-1963	**Gamma1**	1.07	32.07	0.0000	0.3375
1963-1993	**Gamma1**	0.59	11.54	0.0000	-0.3105
1994-2003	**Gamma1**	0.81	8.09	0.0000	-1.0665
1933-2003	**Gamma1**	0.85	21.05	0.0000	0.0424

Table 7 Cross-Sectional Tests for the Fama-French Three-Factor Model

The variables here follow the estimations of Table 3.

	COEFFICIENT		T-STAT	P-VALUE	R-SQUARE
Panel A: BJS tests: $\overline{R}_j - \overline{R} = \gamma_0 + \gamma_\beta \hat{\beta}_j + \gamma_s \hat{s}_j + \gamma_h \hat{h} + e_j$					
1933-1963	**Gamma0**	1.49	3.18	0.0045	0.5594
	GammaB	-0.26	-0.60	0.5521	
	GammaS	0.21	2.83	0.0100	
	GammaH	0.36	3.42	0.0026	
1963-1993	**Gamma0**	0.66	1.36	0.1887	0.7534
	GammaB	-0.20.	-0.43	0.6750	
	GammaS	0.20	3.61	0.0016	
	GammaH	0.49	7.17	0.0000	
1994-2003	**Gamma0**	2.77	7.58	0.0000	0.6843
	GammaB	-2.07	-6.07	0.0000	
	GammaS	0.01	0.07	0.9454	
	GammaH	-0.41	-1.66	0.1123	
1933-2003	**Gamma0**	2.01	5.16	0.0000	0.7636
	GammaB	-1.19	-3.18	0.0045	
	GammaS	0.20	4.21	0.0004	
	GammaH	0.46	6.24	0.0000	
Panel B: FM tests: $\overline{R}_j - \overline{R} = \gamma_0 + \gamma_\beta \hat{\beta}_j + \gamma_s \hat{s}_j + \gamma_h \hat{h} + e_j$					
1933-1963	**Gamma0**	1.50	3.70	0.0002	N/A
	GammaB	-0.27	-0.55	0.5814	
	GammaS	0.20	1.08	0.2820	
	GammaH	0.36	1.85	0.0655	
1963-1993	**Gamma0**	0.67	1.94	0.0535	N/A
	GammaB	-0.21	-0.50	0.6185	
	GammaS	0.20	1.30	0.1945	
	GammaH	0.49	3.57	3.5688	
1994-2003	**Gamma0**	2.77	6.84	0.0001	N/A
	GammaB	-2.07	-3.46	0.0007	
	GammaS	0.01	0.02	0.9846	
	GammaH	-0.41	-0.76	0.4511	
1933-2003	**Gamma0**	2.01	6.26	0.0001	N/A
	GammaB	-1.19	-3.41	0.0007	
	GammaS	0.20	1.72	0.0867	
	GammaH	0.46	3.70	0.0002	

	COEFFICIENT		T-STAT	P-VALUE	R-SQUARE
Panel C: GJ tests: $\overline{R}_j - \overline{R} = \gamma_\beta \hat{\beta}_j + \gamma_s \hat{s}_j + \gamma_h \hat{h} + e_j$					
1933-1963	**GammaB**	1.11	15.02	0.0000	0.3471
	GammaS	0.27	3.19	0.0042	
	GammaH	0.33	2.67	0.0141	
1963-1993	**GammaB**	0.44	10.15	0.0000	0.7317
	GammaS	0.22	4.06	0.0005	
	GammaH	0.51	7.37	0.0000	
1994-2003	**GammaB**	0.44	2.87	0.0089	-0.1803
	GammaS	0.41	2.55	0.0181	
	GammaH	1.00	3.28	0.0034	
1933-2003	**GammaB**	0.73	11.30	0.0000	0.4640
	GammaS	0.19	2.64	0.0148	
	GammaH	0.50	4.64	0.0001	

BIBLIOGRAPHY

Black, Fischer, 1972, 'Capital market equilibrium with restricted borrowing', Journal of Business, 45, 444-455

Black, Fischer, Michael C. Jensen and Myron Scholes, 1972, 'The capital asset pricing model: Some empirical evidence', in Michael C. Jensen, ed.: Studies in the theory of capital markets Praeger Publishers, New York.

Blume, Marshall E. and Irwin Friend, 1973, 'A new look at the capital asset pricing model', Journal of Finance 28, 19-33

Fama, Eugene F. and James D. MacBeth, 1973, 'Risk, return and equilibrium: Empirical tests', Journal of Political Economy 81, 607-636

Fama, Eugene F., and Kenneth R. French, 1992, 'The cross-section of expected stock returns', Journal of Finance 47, 427—465

Fama, Eugene F., and Kenneth R. French, 1993, 'Common risk factors in the returns on stocks and bonds', Journal of Financial Economics 33, 3—56

Fama, Eugene F. and Kenneth R. French, 1996, 'Multifactor explanations of asset pricing anomalies', Journal of Finance 51, 55-84

Gibbons, Michael R., 1982, 'Multivariate tests of financial models: a new approach', Journal of Financial Economics 10, 3-28

Gibbons, Michael R., Stephen A. Ross and Jay Shanken, 1989, 'A test of the efficiency of a given portfolio', Econometrica 57, 1121-1152

Grauer, Robert R., 1978, 'Generalized two-parameter asset pricing models: Some empirical evidence', Journal of Financial Economics 6, 11-33 Grauer, Robert R., 1999, 'On the cross-sectional relation between expected returns, betas and size', Journal of Finance 54, 773-789

Grauer, Robert R. and Johannus A. Janmaat, 2004, 'On the power of tests of the CAPM in a Fama-French world', working paper, Simon Fraser University and Acadia University

Jobson, J. D. and B. Korkie, 1982, 'Potential performance and tests of portfolio efficiency', Journal of Financial Economics 10, 433-466

Jobson, J. D. and B. Korkie, 1985, 'Some tests of linear asset pricing with multivariate normality', Canadian Journal of Administrative Sciences 2, 114-138

Kandel, Shmuel and Robert F. Stambaugh, 1995, 'Portfolio inefficiency and the cross-section of mean-variance efficiency', Journal of Finance 50, 157-184

Kandel, Shmuel, 1986, 'The geometry of the Maximum Likelihood Estimator of the zero-beta return', Jounal of Finance 41, 339-346

Lintner, John , 1965, 'The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets', Review of Economics and Statistics, 47, 13-47

Lo, Andrew W. and A. Craig Mackinlay, 1990, 'Data-snooping baises in tests of financial asset pricing models', Review of Financial Studies 3, 431-468

Mackinlay, A. C., 1987, 'On multivariate tests of the CAPM', Journal of Financial Economics 18, 341-372

Mackinlay, A. Craig and Matthew P. Richardson, 1991, 'Using Generalized Method of Moments to test Mean-Variance Efficiency', Journal of Finance 46, 511-527

Markowitz, Harry, 1959, 'Portfolio Selection: Efficient Diversification of Investments', Wiley, New York

Merton, Robert C., 1973, 'An intertemporal capital asset pricing model', Econometrica, 41, 867-887

Miller, Merton and Myron Scholes, 1972, 'Rates of return in relation to risk: A re-examination of some recent findings', in Michael C. Jensen, ed.: Studies in the theory of capital markets Praeger Publishers, New York

Roll, Richard, 1977, 'A critique of the asset pricing theory tests; part I: On past and potential testability of the theory', Journal of Financial Economies 4, 129-176

Roll, Richard, 1978, 'Ambiguity when performance is measured by the Securities Market Line', Journal of Finance 33, 1051-1069

Roll, Richard, 1985, 'A note on the geometry of Shanken's CRT T2 test for mean/variance efficiency', Journal of Financial Economics 14, 349-357

Roll, Richard and Stephen A. Ross, 1994, 'On the cross-sectional relation between expected returns and betas', Journal of Finance 49, 101-121

Rosenberg, Barr, Kenneth Reid and Ronald Lanstein, 1985, 'Persuasive evidence of market inefficiency', Journal of Portfolio Management 11, 9-17

Ross, Stephen A., 1976, 'The arbitrage theory of capital asset pricing', Journal of Economic Theory, 13, 341-360

Ross, Stephen A., 1977, 'Return, risk and arbitrage', in Irwin Friend and James Bicksler, eds.: Risk and Return in Finance, vol. 1, (Ballinger Publishing, Cambridge, MA)

Rubinstein, Mark, 1974, 'An aggregation theorem of securities markets', Journal of Financial Economics, 1, 225-244

Sharp, William F., 1964, 'Capital asset prices: A theory of market equilibrium under conditions of risk', Journal of Finance, 19, 425-442

Stattman, Dennis, 1980, 'Book values and stock returns', The Chicago MBA: A Journal of Selected Papers 4, 25-45