REVERSE CENTRALITY QUERIES IN COMPLEX NETWORKS

by

Brittany Nielsen

B.Sc. (Hons.), Simon Fraser University, 2007

A Thesis submitted in partial fulfillment
of the requirements for the degree of
Master of Science
in the School
of
Computing Science

SIMON FRASER UNIVERSITY
Fall 2009

APPROVAL

Name: Brittany Nielsen

Degree: Master of Science

Title of Thesis: Reverse Centrality Queries in Complex Networks

Examining Committee: Dr. Janice Regan
Chair

Dr. Jian Pei, Associate Professor
Computing Science
Simon Fraser University
Senior Supervisor

Dr. Ke Wang, Professor
Computing Science
Simon Fraser University
Supervisor

Dr. Oliver Schulte, Associate Professor
Computing Science
Simon Fraser University
SFU Examiner

Date Approved: Dec 15, 2009

Declaration of
Partial Copyright Licence

Abstract

The increasing availability of complex network data from social networks and other sources provides new opportunities for exploration and analysis. In this thesis, we introduce the *reverse centrality query*, a novel query for complex networks. For a query node q, the reverse centrality query returns a locally maximal induced subgraph R, where $q \in R$, such that q dominates R according to a centrality index C. Many centrality indices have been introduced to describe the relationships between nodes in complex networks. We focus on degree, graph, and closeness centrality indices and their respective reverse graph centrality queries. The theoretical properties of these queries, together with heuristic variants, are explored. Algorithms for solving these queries are given and experimental results are provided on three real world datasets. The experiments demonstrate reverse centrality queries to be a useful tool for social network analysis.

To Geoff

"Ohne Fleiß, kein Preis."

— *German proverb*

Acknowledgments

I would particularly like to thank my senior supervisor Dr. Jian Pei for his dedication and creativeness in helping me pursue this thesis through from initial idea, through many iterations, to its final completion.

I would like to express my gratitude to Dr. Ke Wang for serving as my supervisor and taking great care in reading and critiquing my thesis.

Many thanks to Dr. Oliver Schulte for serving as examiner on my committee.

Thanks to Kate Tsoukalas, Ming Hua, and Crystal Xing and the other members of the lab for generous feedback, help and suggestions throughout my masters program.

I would like to give special acknowledgement to NSERC for providing funding which directly supported my graduate studies.

Thank you to my family and friends who accepted my occasional absence due to commitments to my graduate studies.

Finally, a special thank you to Geoff for offering suggestions, encouragement, and support throughout the writing process.

Contents

Approval **ii**

Abstract **iii**

Dedication **iv**

Quotation **v**

Acknowledgments **vi**

Contents **vii**

List of Figures **x**

List of Algorithms **xii**

1 Introduction **1**

1.1 Contributions 4

1.2 Outline 4

2 Problem Definitions and Related Work **6**

2.1 Preliminaries 6

2.2 Complex networks 8

2.2.1 Definition of a complex network 8

2.2.2 Examples of complex networks in real life 8

2.2.3 Structural properties of complex networks 10

2.3 Queries in networks 11

2.4 Clustering and community finding . . . 11
2.4.1 Related work on node queries . . . 13
2.5 Centrality indices . . . 14
2.5.1 Degree-based centrality indices . . . 14
2.5.2 Distance-based centrality indices . . . 15
2.5.3 Path-based centrality indices . . . 16
2.6 Problem definitions . . . 18
2.6.1 Reverse centrality queries . . . 18
2.6.2 Local centrality dominance . . . 18
2.6.3 Reverse degree centrality query . . . 19
2.6.4 Reverse closeness centrality query . . . 21
2.6.5 Reverse graph centrality query . . . 22
2.6.6 Constrained queries . . . 23

3 Reverse Centrality Queries: Algorithms 25
3.1 Reverse degree centrality query . . . 27
3.1.1 One-phase incremental algorithm . . . 28
3.1.2 Two-phase incremental algorithm . . . 29
3.1.3 Degree centrality neighbourhood dominance query . . . 30
3.2 Reverse closeness centrality query . . . 31
3.2.1 Incremental algorithm . . . 33
3.2.2 Heuristics . . . 37
3.3 Reverse graph centrality query . . . 38
3.3.1 Incremental algorithm . . . 40
3.3.2 Heuristics . . . 42

4 Experimental Results and Discussion 43
4.1 Datasets . . . 43
4.2 Implementation . . . 44
4.3 Experimental results . . . 45
4.3.1 Reverse degree centrality query results . . . 45
4.3.2 Reverse closeness centrality query results . . . 47
4.3.3 Reverse graph centrality results . . . 54
4.3.4 Reverse centrality queries versus global centrality . . . 61

4.4 Summary 61

5 Conclusion and Future Work **63**

5.1 Summary 63

5.2 Future work 64

5.2.1 Heuristic improvements 64

5.2.2 Feature vector networks 65

5.2.3 Advanced search methods 65

5.2.4 Variants on reverse centrality query answering 65

5.2.5 Local Centrality Measures 66

A Appendix 1 **67**

A.1 The All-Pairs Shortest Path (APSP) Problem 67

A.1.1 Changing Distances: Dynamic All-pairs Shortest Path 68

Bibliography **70**

List of Figures

1.1 Visualization of the *Les Miserables* complex network 2
1.2 Example of a reverse centrality query result 3

2.1 Visualization of the Zachary karate club network 9

3.1 Visualization of a star graph with 20 nodes 26
3.2 Reverse degree centrality query result for Cosette 30
3.3 Toy example proving non-dominance 32
3.4 Reverse closeness centrality query result for Cosette 37
3.5 Reverse graph centrality query result for Cosette 42

4.1 Degree Distribution of Sample Datasets 44
4.2 Result Size v. Time (Degree Centrality) 46
4.3 Degree of q v. Result Size (Degree Centrality) 46
4.4 Result Size v. Time (Closeness Centrality) 47
4.5 Global Centrality v. Result Size (Closeness Centrality) 48
4.6 Candidates Tested v. Time (Closeness Centrality) 48
4.7 Candidates Tested v. Successful Candidates (Closeness Centrality) 49
4.8 Modularity v. Result Size (Closeness Centrality) 50
4.9 Transitivity v. Result Size (Closeness Centrality) 50
4.10 Result Size v. Time (Closeness Centrality): 1 Chance 51
4.11 Result Size v. Time (Closeness Centrality): Sorting 52
4.12 Comparison of result sizes for variants (Closeness Centrality) 53
4.13 Global Centrality v. Result Size (Graph Centrality) 54
4.14 Modularity v. Result Size (Graph Centrality) 55
4.15 Result Size v. Time (Graph Centrality) 55

4.16 Transitivity v. Result Size (Graph Centrality) 56
4.17 Candidates Tested v. Time (Graph Centrality) 56
4.18 Candidates tested v. Successful candidates (Graph Centrality) 57
4.19 Result Size v. Time (Graph Centrality): 1 Chance 58
4.20 Result Size v. Time (Graph Centrality): Sorting 59
4.21 Comparison of result sizes for variants (Graph Centrality) 60

List of Algorithms

1 One-phase incremental algorithm for reverse degree centrality 29
2 Incremental algorithm for reverse closeness centrality: Initialization 35
3 Incremental algorithm for reverse closeness centrality: Growth 36
4 Incremental algorithm for reverse graph centrality: Initialization 40
5 Incremental algorithm for reverse graph centrality: Growth 41

Chapter 1

Introduction

Imagine that a detective has infiltrated a crime syndicate and collected information about the web of connections between members of the syndicate. As an investigator, you want to utilize this information to gain a deeper understanding of the relationships between members. One natural question to ask is: for a targeted member, what is the relationship between this member and those to whom he is connected? This thesis describes a novel means for understanding the local relationships between an individual and his neighbours.

The advent of widespread data storage and information processing has led to an information explosion. For instance, the Internet is predicted to double in size every 5.3 years [53]. The increasing availability of information provides analysts with new opportunities to evaluate data to generate new conclusions. One area of significant growth is in social networks, where data is most naturally represented using a graph structure.

Graph data is a format where entities are stored as nodes and relationships between the entities are represented by links in a graph. Many domains have natural datasets which may be represented as graphs. Datasets as diverse as social networks, protein co-expression data, co-authorship data from peer-reviewed journals, and bank transaction records may all be viewed as graphs. The lack of a regular order and natural complexity of the structure of these graphs leads to the common term *complex networks*.

Figure 1.1 shows a small complex network, illustrating co-appearances of characters in a Victor Hugo novel. In this small example, we can see the complex network of relationships between individuals. Within the context of this example, our interests lay in understanding the role of the individual character within the larger context of the network in which he appears. The *Les Miserables* network will be examined in more detail in Chapter 4.

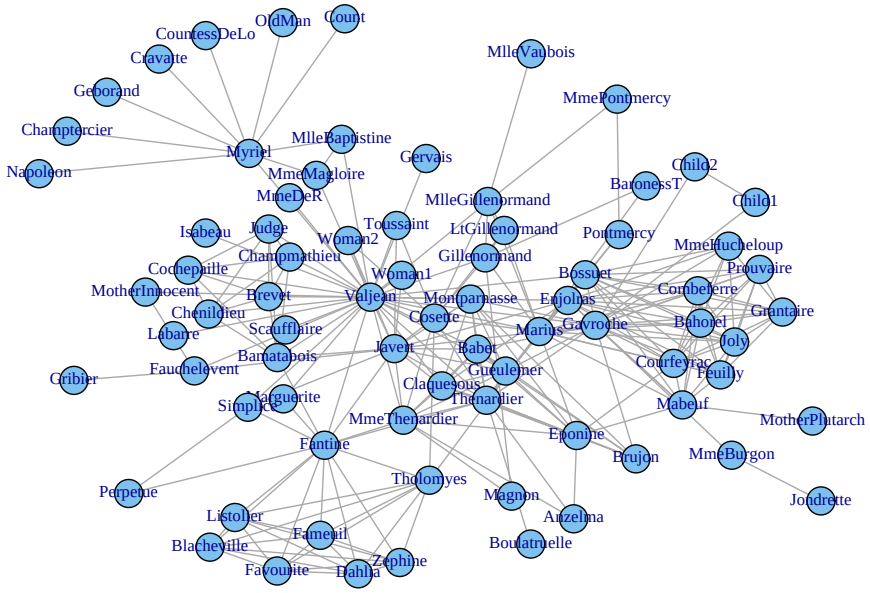


Figure 1.1: Example of a complex network: Co-appearances of characters in *Les Miserables* by Victor Hugo. [32]

The field of *social network analysis* provides tools for the analyst's arsenal for exploring complex networks. Social network analysis is a relatively young field that was initially pioneered by sociologists who had access to small hand-collected datasets. Amongst the tools originally developed in the realm of sociology are centrality measures. *Centrality measures* provide a quantitative tool for evaluating the importance of a node in a graph relative to the other nodes in a graph. Centrality measures can be simple, such as *degree centrality*, which is a measure of the number of links a node has within a graph, or more complex, such as the average distance from a node to all other nodes which is known as *closeness centrality*. Centrality measures form the basis for the queries described within this thesis.

Imagine you are a member of a social network and you wonder: "In what group of individuals are you the most popular member with respect to that group?" Popularity in social networks is often defined by how many direct friendship links an individual has. By this definition of popularity, you may want to determine a large group of connected people where you are the most popular. This kind of query is also known as the *reverse degree centrality query* and will be explored in further detail in the following chapters.

In another application, a biologist is investigating a particular protein X within a

protein-protein interaction network, in particular, she is interested in understanding and exploring the relationship of that protein with others in the network. One interesting query may be to find the group of proteins that the protein X is most central to, where it has the shortest average distance to all other proteins within the group. This group of proteins is centered on the query protein X, and as such provides a more meaningful description of the vicinity of X than the set of all neighbours of X or the set of all proteins within distance 2. This type of query is answered by the *reverse closeness centrality query*, which is explored in further depth within the thesis.

In this thesis, we are interested in determining a region of dominance for a query node, where the query node q is not outranked by any other node according to a given centrality measure. The node q is called *locally dominant* in the region. By exploring this problem, we provide a new avenue for the exploration of complex networks.

Due to the exponential number of induced subgraphs in a graph, the search space for this problem is massive, providing an efficient solution to the reverse centrality query problem will not be trivial.

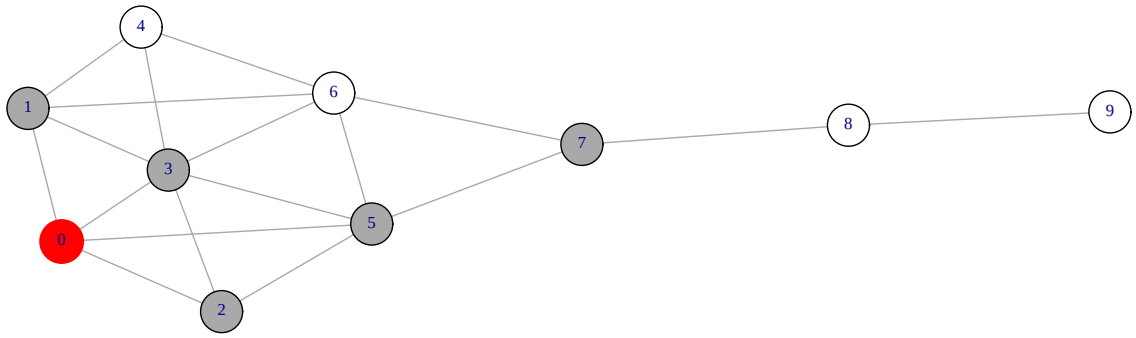


Figure 1.2: Example of a graph centrality query result: In the Krakhardt Kite dataset [33] for query node 0 under closeness centrality. Grey nodes indicate inclusion in the result set.

We see a conceptual illustration in Figure 1.2 of what a query result looks like for a reverse centrality query. We see that the reverse centrality query result indicates that node 0 dominates a relatively large region that actually includes three nodes of higher global closeness centrality: nodes 3, 5, and 7. This demonstrates that in the context of a query node, local dominance (behaviour in the induced subgraph) may differ significantly from behaviour in the larger graph.

1.1 Contributions

The main contribution of this thesis is the introduction of a new type of query for complex networks: the *reverse centrality query*. In particular, three specific types of reverse centrality queries are explored, based on degree centrality, closeness centrality, and graph centrality, respectively. We present practical incremental algorithms for finding *locally maximal* regions of dominance for these three types of the reverse centrality query. Following this, we give experimental results on real-world datasets, showing the application and behaviour of the implemented algorithms. Finally, we offer future directions for reverse centrality queries for exploration and development.

Furthermore, we provide a detailed analysis of centrality measures and their relationships to one another. We explore and describe the existing algorithms for global centrality measure calculations. In addition, we provide an introduction to complex networks and social network analysis for those unfamiliar with the field.

In more detail, the contributions of this thesis include the following aspects. We provide the general framework for reverse centrality queries: a framework that allows for the definition of a reverse centrality query irrespective of the centrality measure used. We give detailed formal problem definitions for reverse centrality queries based on three different centrality measures: degree, graph and closeness centrality. We provide a detailed practical incremental algorithm for each query type and discuss the expected characteristics of each algorithm. For each query type, formal proofs of the behaviour of the query are given. We describe and explore practical algorithmic solutions for these queries using real world graphs and provide detailed analysis of the results. Applications of these novel query types are given for several domains, including marketing, social network analysis, and computer network analysis. In addition, we provide and analyze several heuristics to improve query speed and result quality. Finally, we explore limitations of the methods given in this thesis and provide suggestions for future work on reverse centrality queries.

1.2 Outline

A brief outline of the contents of the following chapters:

- Chapter 2 contains related work in social network analysis and definitions used for the rest of the thesis. Following that, we give formal problem definitions for reverse

centrality queries.

- Chapter 3 presents algorithms for implementing three types of reverse centrality queries. In addition, we present several variations on the basic methods to improve query time as well as query quality.
- Chapter 4 provides experimental results for three real world datasets showing the query behaviour and applicability of the reverse centrality queries. The behaviour of several algorithmic variants is also explored and analyzed.
- Chapter 5 gives the limitations of the work presented in this thesis, options for future work and a summarization of the contents of the thesis.
- Appendix 1 describes related work in the all-pairs shortest path problem from graph theory that is related to calculating reverse centrality queries.

Chapter 2

Problem Definitions and Related Work

In this chapter, we introduce complex networks and several areas of research in social network analysis related to this study. Furthermore, community detection and cluster finding algorithms are explored and described. We describe several families of centrality indices, based on degree, distance and paths, and give a brief description of related work.

We also introduce reverse centrality queries and then describe several constraints that may be placed on queries. Formal problem definitions for the reverse centrality queries are provided for three centrality indices: degree, graph and closeness centrality. We also describe several applications for each of the introduced reverse centrality queries.

2.1 Preliminaries

In this section, we introduce some of the preliminary definitions that are required for formalizing the reverse graph centrality problem.

Definition 2.1.1. A *graph*, $G = (V, E)$ is a mathematical structure consisting of a set V of *vertices* and a set E of *edges*, where an edge connects a pair of vertices. These vertices and edges are alternatively known as *nodes* and *links*. In an *undirected graph*, edges have no direction, so the edge (a, b) indicates a link between a and b and visa versa.

Definition 2.1.2. Given a graph $G = (V, E)$ and two vertices, $s, t \in G$, a *geodesic path* is the shortest path in the graph from s to t, which is measured by the number of edges

contained in the path. The *network distance* is defined as the length of a geodesic path between two vertices. The notation used for the distance between vertices s and t is $d(s,t)$.

Definition 2.1.3. Given a graph $G = (V, E)$, the *induced subgraph* of $S \subseteq V$ is defined by the vertex set S and the set of edges $E(S)$ where $E(S)$ is defined to be $\forall (u,v) \in E$ such that $u \in S \wedge v \in S \wedge (u,v) \in E(G)$. As a convention for brevity, we use the term *region* synonymously with *induced subgraph*.

Definition 2.1.4. The *degree* of a vertex v in an undirected graph G is the number of edges in $E(G)$ that contain v.

Definition 2.1.5. The *modularity* [39] of a division of a graph into partitions is a commonly used tool for determining the quality of a community finding algorithm. Modularity is defined as

$$Q = \frac{1}{2m} \sum_{p \in P} \sum_{i,j \in p} (A_{ij} - \frac{d_i d_j}{2m}) \tag{2.1}$$

where $P = \{p_1, p_2, \ldots, p_k\}$, such that $G(V) = p_1 \cup p_2 \cup \cdots \cup p_k$, and $\forall p_i, p_j \in P, p_i \cap p_j = \emptyset$, that is, P is a partition of G into communities, m is the total number of edges in G, A_{ij} is the adjacency of i to j (1 if adjacent, 0 otherwise), and d_i is the degree of the node i. If modularity is positive, the number of links between members of the same partition is higher than is expected by chance.

Definition 2.1.6. Given a graph $G = (V, E)$, we define a *node scoring function* $S(v)$ as some function that returns a constant score for each node $v \in V$.

Definition 2.1.7. The *region of dominance* of a node q is defined as an induced subgraph R, where $q \in R$ and $\forall n \in V(R), S_R(n) \leq S_R(q)$, where $S(q)$ is some node scoring function.

Definition 2.1.8. The *k-neighborhood* of a node q in a graph G is the set of all nodes N in $V(G)$ such that $\forall n \in N$, $d(q,n) < k$. That is, it is the set of all nodes reachable from node q within a distance of k.

Definition 2.1.9. A *clique* is a set of nodes C belonging to a graph G such that $\forall u, v \in C, u \neq v, (u,v) \in E(G)$. The resulting induced subgraph of a clique is a complete graph.

2.2 Complex networks

In this section, we define and explore *complex networks*, and we give examples of real-world complex networks. Finally, we provide a brief introduction into the properties of social networks, a subset of complex networks.

2.2.1 Definition of a complex network

There are many variations on the definition of a complex network that have been used in various studies [38]. The term complex network is used to refer more generally to graphs which represent real-world networks that display complex topological features, and encompasses social networks as well as other types of networks. For this thesis, we will focus on complex networks as defined in Definition 2.2.1.

Definition 2.2.1. A *complex network* is defined to be an undirected graph G with a set of edges E and a set of vertices V. The *vertices*, also referred to as *nodes*, represent individuals or entities in the network. The *edges*, also referred to as *links*, represent relationships between nodes in the network.

Additional modifications can be made, allowing for directed links, weighted links, weighted nodes, heterogeneous nodes, and other variations [38]. A directed network can be reduced to an undirected network by ignoring directionality or by only including bidirectional links.

Throughout this thesis, definitions and algorithms could be easily extended to include directed graphs, but in this thesis we will focus solely on undirected graphs.

As a convention, throughout the rest of this work, for a graph $G = (V, E)$, we will use the notation $|V| = n$ and $|E| = m$ for brevity.

2.2.2 Examples of complex networks in real life

Complex networks occur in many different contexts and disciplines. In bioinformatics, examples of complex networks are the protein-protein interaction networks constructed with nodes representing individual proteins and links representing direct chemical interactions between proteins, such as the yeast protein interaction network [42]. Another example of a complex network is a road network [20], where links are defined by roads and nodes are intersections in the road network.

The most familiar examples of complex networks for most people are social networks found on the online social networking sites that have flourished in recent years. Examples of this kind of network include Facebook[1], MySpace[2], and LinkedIn[3]. The proliferation of digitized records of the underlying social network has opened the door for a new type of data analysis. Previously, work to collect social network data in sociology was small-scale and painstaking. The well-known social network derived from Zachary's karate club study [52], with links between 34 members of a university club, illustrates the scale that manual data collection limits researchers to. Figure 2.1 illustrates the Zachary karate club network. However, the scale of the largest social networking sites is staggering, with Facebook having over 68 million unique visitors in January 2009[4].

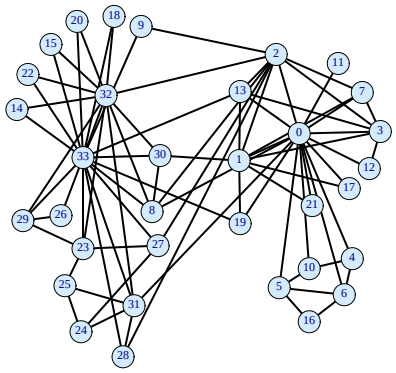


Figure 2.1: The Zachary karate club dataset is a classic example of a small social network. [52]

Major tasks in social network analysis include identifying communities or clusters in the networks, detecting fraud, understanding structural similarities between networks, identifying leaders and understanding growth and change in social networks.

Social network analysis has implications in many fields. In commerce and advertising, there is considerable interest in methods for monetizing social network data for use in

[1] http://www.facebook.com

[2] http://www.myspace.com

[3] http://www.linkedin.com

[4] http://news.cnet.com/8301-13577_3-10160850-36.html

targeted advertising [51]. In criminology, criminal social networks are analyzed to reveal leaders and gatekeepers [50]. In sociology, social network analysis provides a set of formalized tools for exploring group dynamics [49].

The proliferation of large amounts of previously unavailable network data creates a need for new analysis techniques and methods. Network data presents a unique challenge which requires new ways of querying and exploring that data. This thesis presents a novel class of queries that allow researchers and users to explore social networks by examining the relationships between individual nodes and their surrounding regions from a new angle.

2.2.3 Structural properties of complex networks

Social network analysis has often focused on the structural properties that are unique to social networks and not observed in random graphs [38]. The three structural properties that are most commonly associated with social networks are *scale-free*, *small world* and a high *clustering coefficient* [28].

Definition 2.2.2. A *scale-free* network is one where the vertex degree distribution follows a power-law distribution. That is, the number of vertices with degree equal to d is proportional to d^{-k} for some fixed constant k.

Scale free networks exhibit robustness to node removal and can also be characterized by the presence of hub nodes that have a very high degree relative to other nodes in the network.

Definition 2.2.3. The average path length, $L = \frac{1}{n*(n-1)} \sum_{i,j} d(i,j)$, in a network exhibiting the *small-world* property scales logarithmically to the number of nodes, so that $L \approx log(n)$.

The small-world property means that communication is very effective in social networks. This was most famously explored by Stanley Milgram whose experiments led to the phrase "six-degrees of separation" being introduced into the public discourse.

Definition 2.2.4. The *clustering coefficient*, CC, is an expression of the probability given links (a,b) and (b,c) that (a,c) is also a link in the network. The clustering coefficient is also known as the *transitivity* of a graph. Formally for a graph G, $CC(G) = \frac{3*ThreeCliques}{ConnectedTriples}$ where *ThreeCliques* is the number of 3-cliques in G and *ConnectedTriples* is the number of *connected triples* in G. A connect triple is any three vertices $a,b,c \in V(G)$ where $(a,b) \in E(G)$ and $(b,c) \in E(G)$.

These properties play an important role in the complex behaviour that is seen with reverse centrality queries later in Chapter 4. In particular, the great variation in query results results directly from the unique properties of these networks.

2.3 Queries in networks

In social network analysis, queries that allow exploration of individuals or regions in the graph are powerful tools in understanding the graph structure. There are several different types of queries, including graph-level, regional, and individual-level queries. Graph level queries answer questions about the graph itself, potentially in relation to another graph, such as querying the structural similarity between two graphs. Regional queries answer questions about specific regions, either partitions or induced subgraphs, in a single graph, such as identifying dense subgraphs within a graph. Individual queries answer questions about individual nodes in a single graph, such as identifying the important neighbours of a query node. In this thesis, we will focus on individual-level queries.

Queries that focus on the individual node often ask the question of a node's relative importance in relationship to the graph. These individual-level queries can be termed leadership queries, where the query is meant to evaluate the role of the query node in relation to all other nodes in the network. Simple individual-level queries include "Who are my most important neighbours?" and "Which nodes are connected to many of my neighbors but are not directed connected to me?". These queries provide user-level details about the network. This can be particularly useful in exploring large networks that are not easy to understand as a whole or in networks that contain multiple communities.

Understanding the network at the group level is often accomplished using community detection or clustering algorithms that either return a localized community containing a node or provide a global partitioning of nodes into clusters or communities. Community finding algorithms will be explored in more depth in the following section.

2.4 Clustering and community finding

A large body of work in social network analysis and graph mining focuses on either partitioning the network into *communities* or finding dense regions called *clusters*.

On the side of communities, the work often focuses on the delineation of a boundary

between one community and another, with or without regard to the internal link structure.

Newman [39] defines a community as a subset of nodes which have more internal links amongst themselves than external links extending from one of the member nodes to an external node. This definition of community can be neatly quantified using Newman's measure of modularity. A formal definition of modularity is given in Definition 2.1.5. Using the modularity score as the objective function for an agglomerative clustering method provides a parameter free method for discovering the intrinsic community structure within a graph, and as a method, will produce a graph partition that breaks down the graph into non-overlapping communities.

Flake *et al* [18] use a network-flow approach to extract a local community that surrounds a seed query. By using artificial links, the community threshold can be changed to produce communities of varying size and cohesiveness. This work focuses on the directed web graph and includes several examples of communities found using the algorithm.

Other work, such as that by Backstrom *et al.* [2], focuses on analyzing explicit community growth where users self-identify with a group, rather than a *community* as commonly defined by other authors. In particular, Backstrom *et al.* [2] focuses on identifying structural characteristics that make it likely that a user will join a group as well as the growth and change in group membership over time.

Gibson *et al.* [21] focus on the heuristic extraction of dense subgraphs of a very large graph. This technique has applications, in particular, for web graph analysis. No strict definition of a community or cluster is defined here; rather a loose definition of a dense bipartite subgraph exists where *most* of the links of the corresponding bipartite clique exist. This method finds regions of the graph that are densely intra-connected, however, it does not partition the graph nor does it find a localized community.

Recent work by Mishra *et al.* [36] has approached the clustering problem in a manner which allows for overlapping communities to be found. This method finds induced subgraphs where the separation between internal density and external sparsity is sufficient to meet a fixed parameter. *Colibri* by Tong *et al.* [47] is another method that provides cluster detection via a low rank approximation of the adjacency matrix.

Van Dongen [48] presents the Markov Cluster Algorithm (MCA) that is based on simulated flow across the network. This is achieved by simulating random walks across the networks through iterative application of mathematical operators to the Markov matrix representing the graph. MCA is useful in providing a fast clustering method that works well

on large datasets with a small set of explicit parameters and no notion of a seed set.

There are a wide variety of community finding algorithms, with no clear favorite. Each method has strengths and drawbacks but all share a common focus on the graph at large and in that sense, may be considered graph-level queries.

The social network analysis community has also tackled the problem of modeling and understanding the underlying processes in community formation, growth, and evolution. Backstrom *et al.* [2] provide a through experimental analysis of communities in the dynamic DBLP and LiveJournal datasets. At the node level they analyze the properties that influence a node's probability of joining a community, and at the group level they analyze the merging of communities and movement in community interest is explored.

The relationship between community finding and the reverse centrality query is that they provide two different approaches for identifying regions within graphs. Community finding finds regions defined by their boundaries and internal link density. Reverse centrality queries find regions centered on a query node where all nodes are dominated by the query node.

2.4.1 Related work on node queries

Kempe *et al.*'s work [29] on maximizing the spread of influence through a social network is somewhat related to the reverse centrality query. The goal is to identify the most influential individuals in a social network using a model network that has directed, weighted edges representing the level of influence one individual has over another together with some threshold associated with each node. The most influential members in a network may be akin to the nodes found to have large region of dominance by the reverse centrality queries introduced later in this chapter. However, the reverse centrality queries do not rely on specifically weighted edges and can be directly applied to only a subgraph. In addition, the focus of the reverse centrality query is for a given node, identify a region, whereas Kempe *et al.* focus on, for a given graph, identifying a node or set of nodes.

In addition, there are standard individual queries that are used in exploring the region surrounding a query node. The most prominent of these queries is the k-neighborhood query that returns all nodes reachable within a distance of k of the query node. This query provides no guarantees on relationship between the query node and the result region other than this distance guarantee.

2.5 Centrality indices

Various centrality indices have been introduced in the past half century to explain the relative importance of individuals in social networks. Centrality measures attempt to quantitatively capture the relative importance of the individual in relation to the group. There are many measures of centrality including: degree centrality [38], betweenness centrality [19], closeness centrality [41], bridging centrality [25], and Eigenvector centrality [6].

Centrality indices can be categorized based on the information used to calculate their scores. There are three broad categories including vertex-based, distance-based, and path-based centralities.

Discussion here is limited to centrality indices for vertices, although many naturally extend to centrality indices for edges as well.

2.5.1 Degree-based centrality indices

Centrality indices based on degree include both the simple concept of *degree centrality* as well as more sophisticated indices like PageRank [9] and HITS [31].

Definition 2.5.1. *Degree centrality* for a node v is defined as the number of links connecting that node to other nodes in the graph G. Degree centrality, also known as *vertex centrality* [38], can be expressed as:

$$C_D(v) = \frac{degree_G(v)}{n-1} \tag{2.2}$$

A node with the highest degree centrality in a graph has the maximal number of neighbors in the graph.

The idea of *degree centrality* is closely related to the idea of popularity in social networks, which is often measured informally as the number of links, or direct connections, an individual has within the network.

More sophisticated measures are possible when graphs are directed, with both in-degree and out-degree being considered. For the extent of this work, only undirected networks are considered, and so the most relevant centrality index is degree centrality.

A family of degree-based centrality measures is described by Bonacich [6], where a parameter β determines the weighting of local versus global structure and a scaling parameter α normalizes the score. His centrality formula is given as $C(\alpha, \beta) = \alpha(I - \beta A)^{-1}(A * 1)$, where I is the identity matrix, A is the graph adjacency matrix, and 1 is a matrix of ones.

Setting β to 0 is equivalent to degree centrality. The underlying idea is that the power or centrality of an individual is related in turn to the power or centrality of the nodes that the individual is connected to. Another related measure in Bonacich's family of measures is power or *eigenvector centrality*, where the centrality of a node is given by the eigenvalue of that node in the adjacency matrix.

Many ranking algorithms used in determining website importance can be viewed as degree-based centrality scores. *PageRank* [9] is a heuristic approximation of eigenvector centrality [6] based on random walks. The *Hubs and Authorities* scores found in the HITS algorithm [31] are centrality indices also based on calculating the eigenvector of slight variants of the adjacency matrix.

The concept of local centrality for degree-based centrality has been explored to by [43] where local centrality is simply the degree of the node, or a local relative centrality that compares the degree to the potential total degree in the network. However although it mentions the concept of a node that is locally central, it does not formalize the problem of identifying locally central nodes.

2.5.2 Distance-based centrality indices

Several centrality indices are based on quantifying the importance of a node using distances from that node to other nodes in the network. *Closeness centrality* and *graph centrality* are two indices that rely on distances.

Definition 2.5.2. *Closeness centrality* is defined for a node v in a graph G to be:

$$C_C(v) = \frac{1}{\sum_{t \in V(G)} d(v,t)} \tag{2.3}$$

A node with the highest closeness centrality in a graph has the shortest average distance to all other nodes [41]. Closeness centrality can be thought of as a representation of the average communication time of each node to all other nodes in the network.

Although we will focus on connected graphs, it is worthwhile to mention that there is some flexibility in this definition to support disconnected graphs. In disconnected graphs, the choice of distance for unconnected nodes has a large impact on the resulting closeness centrality scores. One common choice is to set $d(s,t) = n$ when s and t are disconnected, where $n = V(G)$ is the length of the longest possible simple path in G.

Latora and Marchiori [34] present a measure termed *efficiency* that is based on the mean distance between any two vertices in a network. The graph efficiency is very similar to closeness centrality, as it is the inverse of closeness centrality, C_C, divided by the number of nodes in the graph. Closeness centrality and efficiency are most meaningful in connected graphs.

Definition 2.5.3. *Graph centrality* is defined for a node v in a graph G to be:

$$C_G(v) = \frac{1}{\max_{t \in V(G)} d(v,t)} \tag{2.4}$$

A node with the highest graph centrality in a graph has the shortest worst-case distance in the graph [23].

Definition 2.5.4. The *eccentricity* E_v of a vertex v is defined as $E_v = max_{u \in G}(dist(u,v))$. A vertex v where $E_v = min_{u \in G}(E_u)$ is called a *graph center* for G. The minimum eccentricity of a graph is called the *radius* of the graph.

Graph centrality is closely related to the graph theory concept of *graph centers*, which are defined as the nodes in a graph with minimum *eccentricity*, where eccentricity is defined as the longest shortest path to another node in the graph. The concepts of *graph centers* and graph centrality itself are only meaningful in connected graphs, since in a disconnected graph, all nodes would have the same graph centrality.

Distance-based centrality indices require calculation of all-pairs shortest paths (APSP) to determine the *distance matrix*. The methods for APSP are explored in detail in Appendix 1.

Definition 2.5.5. Given a graph $G = (V, E)$, the distance matrix D contains the pairwise distances for all nodes in the graph, where each entry $D[i, j]$ is equal to the distance between nodes $i, j \in G$. The distance matrix is calculated by an APSP algorithm.

In distance matrix terms, the closeness centrality of a graph vertex is the inverse of the sum of the row representing the vertex. The graph centrality is the inverse of the maximum value in the row representing the vertex.

2.5.3 Path-based centrality indices

Path-based centrality indices are those measures that rely on calculating explicit shortest path information about the graph. Unlike distance-based centrality indices, where the

distance matrix is required for computation, path-based indices require explicit knowledge of what nodes are involved in which shortest paths.

Definition 2.5.6. Betweenness centrality BC_v for a node v is defined as

$$CC_v = \sum_{t \neq u \neq v \in G} (\eta_{tu}(v)/\eta_{tu}) \tag{2.5}$$

Where $\eta_{tu}(v)$ is the number of shortest paths between t and u that include v and η_{tu} is the total number of shortest paths between t and u. Betweenness centrality is the fraction of all shortest paths that pass through the node v [19].

Betweenness centrality is an expensive index to calculate for large graphs. Before Brandes [7] introduced an algorithm with $O(nm)$ time complexity and $O(n+m)$ space complexity, the best known algorithm had cubic time complexity. In the naïve implementation, storage of all shortest path representations could be within the order of $O(n^3)$. Various approximation methods have been suggested for betweenness centrality, including variants on betweenness that are easier to calculate [38]. Betweenness centrality is distinct from other centrality indices discussed here because a node of high betweenness centrality does not necessarily indicate that the node is an important node in the graph. A node of high betweenness means that upon deletion, the shortest paths of the network are heavily affected.

Beyond betweenness centrality, other path-based centrality indices have been defined including *stress centrality* [45], which is an absolute count of the number of shortest paths that the node participates in.

Due to the complexity in calculating and maintaining all shortest paths in an incremental fashion, we will not explore path-based centrality variants of reverse centrality queries in this thesis.

2.6 Problem definitions

In this section, we give a general problem definition for the *reverse centrality query*, the novel query introduced by this thesis. More detailed problem definitions are provided for individual reverse centrality query types that are described and explored further in this thesis.

2.6.1 Reverse centrality queries

The novel class of queries called *reverse centrality queries* are leadership queries, where an individual node is evaluated in terms of some centrality index, but where the result is in terms of a region where the node dominates. This class of queries ties together the idea of global centrality measures with that of local behavior of a query node.

The query result is a region, an induced subgraph, where the query point is a local leader. There are potentially an exponential number of results if the query node q globally dominates every subgraph which contains it. In order to reduce the size of the potential answer, a single result is returned. The result is a locally optimal choice according to the search algorithm.

2.6.2 Local centrality dominance

Reverse centrality queries seek to find an induced subgraph surrounding a query vertex that is *dominated* by that query vertex.

Definition 2.6.1. A query vertex $q \in G$ *dominates* the connected induced subgraph $S \subseteq G$ if and only if $q \in S$ and $\forall v \in S, centrality_S(q) \geq centrality_S(v)$. That is, for some centrality measure, $centrality_S$, calculated over the induced subgraph S, the vertex q has maximal centrality. We may also refer to q as *locally dominant* in S. A vertex q is called *globally dominant* if it dominates G.

Notice that the dominance defined in Definition 2.6.1 is not strict. There may be other nodes in the subgraph S that are equally dominant with q. This looser definition is used to allow for the many real world cases of symmetry where two or more nodes will share similar or identical local structure.

Definition 2.6.2. A *maximally dominant* induced subgraph is dominated by the query node q, where for all induced subgraphs with $V(A) \cup V(S)$ where $V(A)$ is some set of nodes

in $V(G)\backslash V(S)$, q does not dominate. In other words, no larger induced subgraph containing the nodes in $V(S)$ is dominated by q.

Definition 2.6.3. A *1-maximally dominant* induced subgraph S is dominated by the query vertex q, but $\forall u \in S, v \notin S, (u, v) \in E(G)$, the induced subgraph $v \cup S$ is not dominated by q. This means that no single node may be added to S such that we obtain a larger connected induced subgraph where q dominates. These subgraphs may also be referred to as *locally optimal* subgraphs.

To limit the size of the query answer, results are limited to those called *maximally dominant*. In addition, it is only meaningful to explore the space of connected induced subgraphs, as the centrality measures are most meaningful when applied to connected graphs.

The definition of *maximally dominant* induced subgraphs corresponds to the notion of a local maximum. Where *1-maximally dominant* regions correspond to a local maximum where no single neighbour of the induced subgraph S not yet in S can be added to S while keeping q as a dominant node.

Definition 2.6.4. A *locally central* node q is a node that dominates, according to a centrality measure, an induced subgraph H of the graph G where $q \in V(H)$.

Definition 2.6.5. A *local centrality* measure is a theoretical measure that would provide a means of quantifying the behaviour of a node q according to some centrality measure C in induced subgraphs containing q. A node with high local centrality would be *locally central* to a relatively large region, whereas a node with low local centrality would remain peripheral even in induced subgraphs.

In Chapter 4, we use the size of the query result of a reverse centrality query as a proxy for the local centrality of each node.

2.6.3 Reverse degree centrality query

The reverse degree centrality problem returns an induced subgraph S where the query node q has a degree equal to the maximum degree observed in the subgraph S.

Definition 2.6.6. The *reverse degree centrality query* on a query node q returns a connected induced subgraph S that is dominated by q. In the induced subgraph S, q has maximal degree. In addition, we restrict S to be *maximally dominant*.

The degree centrality index itself is relatively simple, being given by the degree of the vertex, yet is still non-trivial when applied to finding a dominated subset that satisfied the reverse degree centrality query. Determining whether an induced subgraph is dominated by the query vertex is non-trivial because the degree of a vertex v in an induced subgraph depends on the number of neighbours of that are members of the induced subgraph, thus the degree centrality of nodes will change as additional nodes are added to the region.

Degree centrality neighbourhood dominance query

This section provides a constrained version of the general reverse centrality query. This definition may prove useful for exploring the physical distribution of vertex degree within a graph. This query is not explored further here, but this query provides an example of an extension that integrates well with the existing study of complex networks in terms of degree distribution.

Definition 2.6.7. The degree centrality neighbourhood dominance query returns the distance, d, to the nearest node which dominates q with respect to the graph G. The $(d-1)-neighbourhood$ contains all nodes closer than d to q and therefore the $(d-1)-neighbourhood$ must be dominated by q.

The neighbourhood dominance query is a special constraint on the general reverse degree centrality query, where the returned set S must contain exactly the largest complete neighbourhood graph of q where q dominates.

Applications of reverse degree centrality

Degree centrality identifies nodes with high degree as central, as a result the centrality measure relies on an assumption that nodes with many connections are more important than those on the periphery. Degree centrality defines prominent nodes to be nodes with many connections [49].

The reverse degree centrality problem then provides a region over which the query node q is prominent by this definition. In many social network contexts, popularity is equated with the number of friends, or relationships, an individual has. The reverse degree centrality query would allow a user to determine a group of people (which may be considered a community or social group) in which they are the most popular (or tied for most popular) individual, as determined by node degree.

In the context of complex networks representing businesses as nodes and client relationships between businesses as links, the reverse degree centrality query may also prove useful. The query result for a queried business would result a group of businesses within which the queried business has the most connections (or a tie for most) within that group. If the extension for weighted edges is used and weight is used to represent sales volume, the query result becomes a group of connected businesses where the queried business has the highest sales volume.

2.6.4 Reverse closeness centrality query

The *reverse closeness centrality query* returns a locally maximal induced subgraph where the query vertex q has the minimal average distance to the rest of the induced subgraph.

Definition 2.6.8. The *reverse closeness centrality query* on a query node q returns a connected induced subgraph S that is dominated by q according to closeness centrality. In the induced subgraph S, q has minimal mean distance to other nodes in the graph. In addition, we restrict S to be *maximally dominant*.

The reverse closeness centrality query can be applied to communication networks. In this application, the reverse closeness query calculates, with respect to a query node q, an induced subgraph surrounding q where q has minimal mean distance to all other nodes. In this context, q would be a good choice as a leader node over S in a communication protocol that requires that messages be sent to a leader for redistribution.

The reverse closeness centrality query result is a connected induced subgraph centered on q, where the center is defined by average distance to all other nodes. This corresponds somewhat to the facility location problem using an optimizing function of average distance. [8]

The definition of closeness centrality used here could be easily expanded to handle weighted graphs, where edges have weights associated with them by modifying the distance used in the definition to be the sum of edge weights rather than the number of edges in the shortest path.

Applications of reverse closeness centrality queries

The reverse closeness centrality query provides a practical network analysis tool in several different contexts.

In the context of communication networks, the returned region provides a subgraph over which the query node q is a suitable candidate for a leader, where a leader should minimize round-trip communication costs to all other nodes.

In the context of social networks, the returned region represents a group of individuals where the query node q plays a central role: they are, the closest node, on average, to the rest of the graph. The resulting region could be used to provide a meaningful cluster of individuals centered on the query node. This result could provide an innovative interface for individuals exploring their own social networks from a user's perspective. Rather than showing friends and friends of friends, this region may contain those connected at greater distance, but still those who are well connected to the individual.

2.6.5 Reverse graph centrality query

Hage and Harary [23] originally defined the problem of graph centrality, a centrality measure based on the graph theory notion of a graph center. The reverse graph centrality query seeks induced subgraphs where the query vertex q has the best worst-case distance to any other node. In other words, where q is a graph center.

Definition 2.6.9. The *reverse graph centrality query* on a query node q returns a connected induced subgraph S that is dominated by q according to graph centrality. In the induced subgraph S, q has minimal worst-case distance to other nodes in the graph. In addition, we restrict S to be *maximally dominant*.

An induced subgraph where the query point q is a vertex center will be centered about q. Using the *graph centrality* criterion, an induced subgraph S is dominated by the query point q if and only if q is a *graph center* of S.

Applications of reverse graph centrality queries

In the application of social networks, the result of a reverse graph centrality query on a query node q would return the induced subgraph S of individuals over which q is most *central*, where a central individual has fewest maximum hops required to reach anyone else in S. Graph center-based approaches for energy efficient communication protocols on wireless sensor networks as described in [35] show the applicability of graph centers to a real world problem.

By enabling the extraction of the cluster that forms around a single vertex (the query vertex), the user is able to explore the area of local dominance for the query vertex.

The reverse graph centrality region represents a centered region around the query node where the query node is the graph center. This centered region will not include other nodes that are more prominent than the query node and in that sense will focus on the area of dominance for the query node.

This type of query would be useful for social network exploration, allowing individuals to explore their social connections in a fashion the focuses on those they are most closely connected to. Given the small world property of social networks, providing the neighborhood (those who are perhaps 2, 3 or 4 links away from an individual) would prove to be an increasingly unwieldy region. Rather than multiple neighbourhood queries, we return a centered region using a parameter free method and that is guaranteed to be centered on the node.

2.6.6 Constrained queries

Extensions to the problem definitions for reverse centrality queries can be made by placing additional constraints on the desired answer. Constraints include inclusion and exclusion of other nodes in the result, and constraints on the size of the desired induced subgraph.

Although these definitions are not explored further within this thesis, the formalization of these variants may prove useful. We provide these definitions because they are the most natural extensions to the reverse centrality queries defined above. Algorithmic modifications to implement these constraints would be fairly simple, but the analysis of these constraints is best left to a dataset-specific study.

Follower constraints

Follower constraints place additional constraints on the induced subgraph result for a reverse centrality query by requiring either the inclusion or exclusion of nodes other than q from the result S.

Definition 2.6.10. Let F be a set containing one or more nodes in G. For a reverse centrality query with query node q and the resulting induced subgraph S, a *follower inclusion* constraint requires that $F \subseteq V(S)$, so S must contain the followers in F. Similarly, a *follower exclusion* constraint requires that $V(S) \cap F = \emptyset$, so S contains none of the followers in F.

Follower constraints provide more opportunity for the user to modify the potential query result and also help the user to intuitively explore the space of potential induced subgraph results. These constraints also allow the user to answer queries of the form, "In what group, as defined by an induced subgraph, is q the leader, but s the follower?".

Neighbourhood constraints

In very large graphs, neighborhood constraints may improve speed by isolating the search space to a smaller graph.

Definition 2.6.11. For a reverse centrality query with query node q and the resulting induced subgraph S, a *neighbourhood* constraint requires that S only includes nodes within a specified distance of q, or in other words, within the k-neighbourhood of q, where k is a fixed parameter, $k > 0$.

Neighbourhood size constraints allow for quicker result calculation and may be used to help the user gauge the importance of a node locally without potentially returning an induced subgraph equal to the entire graph, which is possible if q were also globally dominant.

Chapter 3

Reverse Centrality Queries: Algorithms

The algorithms presented in this chapter are methods for finding locally optimal solutions to reverse centrality queries. We naturally approach these problems using a breadth first search method to incrementally add nodes until a local maximum is reached. The query results are *locally optimal subgraphs*, as defined in Definition 2.6.3, where no neighbor node of the current subgraph could be added to produce a larger induced subgraph dominated by the query point q.

One limitation of these algorithms is that they are all input dependent, they will possibly produce different answers if the dataset is permuted. As defined now, they will produce one locally maximal induced subgraph in response to a query. This could be expanded to allow k randomized trials to potentially produce up to k different answers. From the randomized trials, the largest result could be returned.

For any reverse centrality query, one trivial answer is always available. For the induced subgraph defined by $V(S) = \{q\}$ where q is the query node, it follows trivially that q is the leader of the induced subgraph S. This means that in the most basic case, we will always have a query result. Although, as we prove later in this chapter, we can guarantee that the query result will be more than just q.

Detailed algorithms for the reverse closeness, graph and degree centrality queries are presented. For each query type, proofs are given for basic properties of the algorithm. In addition, heuristic optimizations for each of the algorithms presented are provided.

Justification for returning a single region

The problem of finding *all* induced subgraphs in G where the query node q is a dominating node is a very challenging problem due to the potentially exponential number of induced subgraphs satisfying this condition. For a given induced subgraph, the method for determining whether the query node q is a dominating node is polynomial. However, the size of the output for this problem is potentially exponential in terms of the size of the input.

It is possible that all induced subgraphs in the graph G containing q will have q as a dominating node. We can clearly see this exponential behaviour in the star graph that consists only of edges connecting the query node q to all other nodes in G, as illustrated in Figure 3.1 with query node 0. In this example, for any induced subgraph containing q, q will be a dominating vertex. In fact, it will be the only central vertex in connected induced subgraphs of size $k \geq 3$. Since the number of connected induced subgraphs containing q is 2^{n-1}, where n is the size of V, it follows that the size of the solution for this query returning all induced subgraphs of G satisfying the condition that Q is a dominating node is potentially exponential in the size of the input.

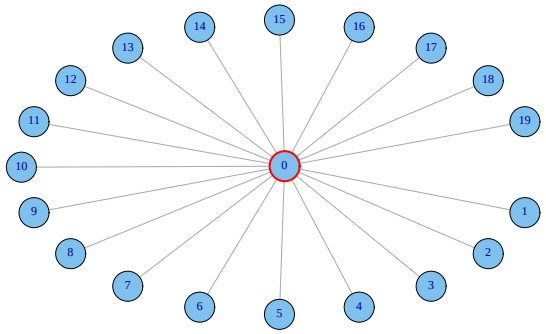


Figure 3.1: The star graph example illustrated here demonstrates a worst case scenario where the number of potential subgraphs where the query node 0 dominates is exponential with respect to the size of the graph.

The algorithms presented in this chapter all return only a single region that is locally maximal. By returning a single induced subgraph, the size of the resulting answer is polynomial instead of potentially exponential. In addition, the single locally maximal region is understandable from the user perspective.

Optimality of Result

We are unable to guarantee a globally optimal query result due to the difficulty of the reverse centrality query problem. A globally optimal result may be defined as the largest possible region containing q that is dominated by q, where size is determined by the number of nodes in the region.

In Chapter 2, the problem definitions given for the reverse centrality queries are based on the idea of *maximally dominant* regions. While theoretically this would be a high quality result to return, it would be prohibitively expensive to satisfy this definition algorithmically.

For practical purposes, we will provide algorithms that find a *1-maximally dominant* induced subgraph as the search space of ensuring that all supergraphs of S are not dominated is potentially exponential. By this definition, no single node in the one-neighbourhood of S can be successfully added to S while maintaining q as the dominant node.

3.1 Reverse degree centrality query

The reverse degree centrality query provides an induced subgraph, or region, where the query node dominates according to degree centrality. In terms of a social network, this means that the query returns a group of individuals within which the query individual has the greatest number of relationships within that group compared to other members of the group.

Degree centrality differs from the other centralities explored for reverse centrality queries in that it is a degree-based centrality index that relies only on local information.

The static degree centrality algorithm is trivial, as the vertex degree of each node in the global graph is either known (in graph representations where edges are stored with respect to each vertex) or easy to calculate (in graph representations with separate edge lists). With knowledge of the degree of each node, the ordered list that determines the ranking according to degree centrality can be established in $O(n \log n)$ time.

As is explored in greater detail in the following sections, it is easy to construct a fast incremental algorithm to produce a reverse degree centrality query answer for a given query node.

3.1.1 One-phase incremental algorithm

The incremental algorithm begins with the 1-neighbourhood of the query node q and expands by adding neighbors to the result set until no more additions are possible. The resulting induced subgraph is *locally maximal*, where no neighbor to the region can be added while maintaining q's dominance.

We begin by proving the validity of starting with the 1-neighbourhood.

Theorem 3.1.1 (1-neighborhood dominance)**.** A node dominates its 1-neighborhood according to degree centrality.

Proof. Given a graph $G = (V, E)$ and a query vertex q, the 1-neighbourhood of q is defined as $S = q \cup N_1(q)$, where $N_1(q)$ is the set of vertices with edges to q. The maximum degree of a vertex in the induced subgraph is $|N_1(q)|$, which is exactly equal to the degree of q, it follows that no vertex in S can dominate, by having a strictly greater degree, q, and that q therefore dominates its 1-neighbourhood. □

The update step of adding one vertex v to a region requires incrementally updating the degree values for each node in the induced subgraph which is also a neighbour of v. It is a constant time $O(1)$ operation to update each neighbour vertex and determine whether that vertex now dominates the query vertex q. Adding v to the list of vertices in the induced subgraph takes $O(\log(k))$ time if a sorted list is used, where k is the current size of the induced subgraph. The update operation takes $O(r)$ amortized time where r is the average number of links for a vertex in G. It follows that the total incremental cost is $O(rk+k\log(k))$ for an induced subgraph of size k, assuming no back-tracking.

This algorithm is both cost effective as well as localized, where it can be performed without restriction on massive graphs, since only local information is required to incrementally update.

It is interesting to note that, for a given node, degree centrality is static once the one-neighbourhood of the node is included. This means that once q loses dominance according to degree centrality due to the expansion of the result set S, if S already contains the one neighborhood of q, we are guaranteed that q will not again dominate a larger superset of S. In other words, when S includes the one neighborhood of q, dominance according to degree centrality is monotone.

Algorithm 1: One-phase incremental algorithm for reverse degree centrality

Data: $G = (V, E)$, $q \in G$

Result: Induced subgraph S where q dominates

```
1  V(S) ← N_1(q)
2  CandidateQueue ← N_1(S) \ V(S)
3  UnluckyCandidateQueue ← ∅
4  while CandidateQueue ≠ ∅ do
5      currentCandidate ← CandidateQueue.dequeue
6      TS = currentCandidate ∪ V(S)
7      result ← true
8      forall n ∈ N_1(currentCandidate) ∩ V(S) \ q do
9          if degree_TS(n) > degree(q) then
10             UnluckyCandidateQueue.enqueue(currentCandidate)
11             result ← false
12             break
13     if result = true then
14         V(S) ← V(S) ∪ currentCandidate
15         CandidateQueue.enqueueAll(UnluckyCandidateQueue)
16         CandidateQueue.enqueueAll(N_1(currentCandidate) \ V(S)
```

In Algorithm 1, we consider all nodes that are in the one-neighbourhood of the result set S, but that are not found in S, to be *candidates*. When a node is successfully added to S, its neighbours that are not already in S are appended to the candidate list. In this manner, we iteratively test all nodes found at a distance of one from the candidate set.

3.1.2 Two-phase incremental algorithm

A better two-step algorithm for reverse degree centrality queries can be devised by the observation that there are some nodes which do not require any later observation to determine whether they will affect the dominance of q. The set of all vertices with $degree_G < degree_G(q)$ will always be dominated in an induced subgraph containing $N_1(G)$. Now, rather than testing every vertex as it is added to the set, a two-pass process will be used. First, all neighbour vertices of q with $degree_G < degree_G(q)$ will be added to S, any neighbour

vertices that do not meet this criterion will be added to the candidate list. The process will continue recursively with the newly added vertices until the induced subgraph S contains all vertices globally dominated by q with respect to G that are directly connected to q via a path of other dominated vertices.

The second step is to go through the queue of candidate vertices that are not globally dominated by q and to see which of these vertices can be added to S without challenging the dominance of q, using the same testing procedures as the one-pass algorithm.

The neighbours of successfully added candidate vertices would be added to the back of the queue if they are not already in S. The process would stop when no candidate can be successfully added to S. This process is input-sensitive, in that a different ordering of candidate nodes could produce a different set S when the algorithm finishes.

In Figure 3.2, an illustration of the result of a reverse degree centrality query using the two-phase algorithm is shown.

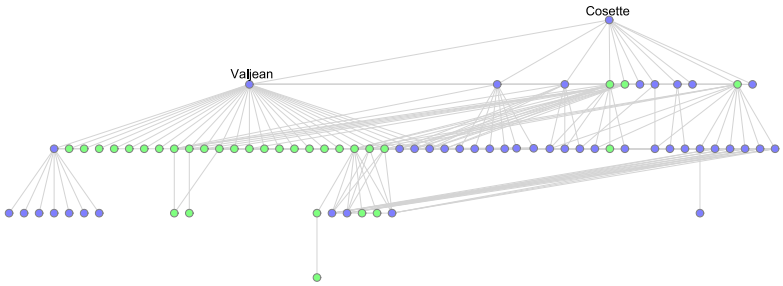


Figure 3.2: The query result for reverse degree centrality for the character Cosette from the Les Miserables dataset shown in Reingold-Tilford form, where each layer of the tree represents distance from the root node, which is the query node Cosette.

3.1.3 Degree centrality neighbourhood dominance query

Definition 3.1.1. The degree centrality neighbourhood dominance query returns the distance, d, to the nearest node which dominates q with respect to the graph G. The $(d-1)$-neighbourhood is by definition guaranteed to be dominated by q.

Given an ordered list of vertices, sorted by degree with respect to G, and a precomputed APSP matrix the neighborhood dominance can easily be calculated for a query point q.

Starting from q, a breadth first search with a search criterion of $degree_G(v) > degree_G(q)$ will provide an answer to minimum distance, d, to a dominating node, since the first node found is guaranteed to be as close as possible. The depth of the search at this point provides the minimum distance to a dominating node. This search takes time $O(r)$ where r is the size of the d-neighbourhood of q. This is potentially expensive if q is the globally dominant vertex, or dominates a large portion of the vertices in G.

Another approach is to look-up the distance of each vertex in the graph known to have a higher degree than q. Assuming that all distances have been precomputed, this takes $O(1)$ time for each comparison, if it has not been precomputed, it will take one single-pair shortest path (SPSP) calculation per node, or one single source shortest paths (SSSP) calculation (whichever is more advantageous) to determine the distance between q and the vertex in question. The node with a greater degree than q at minimum distance takes $O(kp)$ where p is the number of vertices in G with a greater degree and $O(k)$ is the comparison time needed to determine the distance from q to the vertex in question.

Experiments for this variant of the reverse degree centrality query are not provided in Chapter 4, but are left for future experimentation.

3.2 Reverse closeness centrality query

The reverse closeness centrality query returns an induced subgraph (a region), where the query node q has minimal average distance to all other nodes.

It is meaningful to explore the best available methods for closeness centrality in the traditional whole graph. To implement an incremental algorithm for closeness centrality, we will require a dynamic closeness centrality method that allows for the reuse of calculations as the region grows.

For calculating closeness centrality in the static case, Brandes' betweenness algorithm [7] is immediately applicable to the closeness centrality measure as well. It provides a method for calculating closeness centrality in $O(n + m)$ space and $O(nm)$ time where n is the number of vertices and m is the number of edges. Brandes' algorithm relies on a recursive construction of shortest paths where explicit shortest paths are not stored. Unfortunately, this means that it is not possible to modify Brandes' algorithm to provide an incremental algorithm that performs better than repeating the entire algorithm at each step.

Closeness centrality may also be calculated from the distance matrix of the graph G,

which results from calculating all pairs shortest paths, various methods for which are explored in detail in Appendix 1.

Theorem 3.2.1 (Dominance is non-monotone)**.** A region that is locally maximal according to the reverse closeness centrality query may be a proper subgraph of a larger solution subgraph. More formally, it is possible for induced subgraphs $A \subset B \subset C \subseteq G$ where $q \in A$ that q dominates A and C but not B.

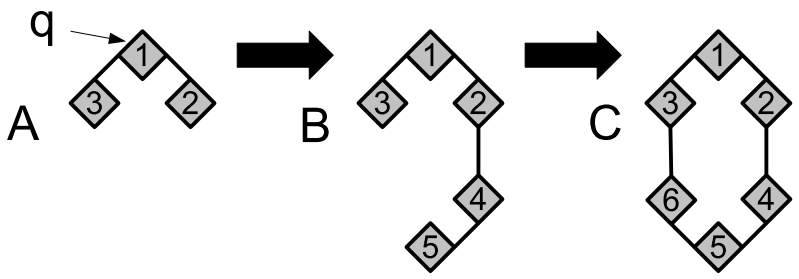


Figure 3.3: An example demonstrating non-monotonicity of both closeness and graph centrality. In A, the query node q dominates. However, in B, the query node is dominated by node 2. With the addition of node 6 in C, q once again dominates (in fact, in C all nodes tie for dominance as all have identical topology.

Proof. Let dominance refer to local dominance according to graph centrality. Given a graph $G = (V, E)$ and a query vertex q, suppose in the induced subgraph $A \subset G$ that q is dominant. Suppose there exists a vertex u such that the induced subgraph $B = A \cup u$ has a shortest path via u which reduces the average distance of some other vertex $x \in B$ below that of q. B no longer has q as a dominant vertex. Now, suppose we introduce another vertex v and expand the induced subgraph to produce $C = B \cup v$ such that C has some new shortest path via v that reduces the average distance of q below that of x, and once again q is dominant. It follows that there is no guarantee of q being dominant or not in proper subsets of an induced subgraph where q is dominant. An example showing this behaviour can be seen in Figure 3.3. □

The non-monotonicity of dominance for closeness centrality tells us that the incremental algorithm can only promise to provide locally optimal results. We are not guaranteed to find the largest possible region by this search process. This also indicates that an optimal greedy algorithm or dynamic programming solution is not possible. In addition, even if we start with a globally dominant node, we cannot guarantee that we will return the entire graph G as the result, as the result may be sub-optimal even in this case.

While the proof that dominance is non-monotone does not prove the case for strict dominance, where the query node is strictly better than any other node in S according to the centrality measure, larger example graphs demonstrating strict dominance may be constructed. Such examples take the general form where the query node q is central to two large, balanced portions of the graph. When only one portion of the graph is contained in S, q does not dominate, the second portion must be added before q again dominates.

Theorem 3.2.2 (1-neighborhood dominance)**.** A node dominates its 1-neighborhood according to closeness centrality.

Proof. Given a graph $G = (V, E)$ and a query vertex q, the 1-neighbourhood of q is defined as $S = q \cup N_1(q)$, an induced subgraph, where $N_1(q)$ is the set of vertices with edges to q. The best closeness centrality achievable is to have an average distance of 1 to other vertices. By definition, q has a distance of 1 to all neighbours. It follows that in S, q has optimal closeness centrality and therefore cannot be dominated by any other vertex in S. Therefore, q dominates its 1-neighbourhood. □

Because a node always dominates its 1-neighbourhood, we can start the incremental algorithm beginning with the 1 neighbourhood of the query node q.

3.2.1 Incremental algorithm

When adding a new vertex v to an induced subgraph S (or graph), only new shortest paths via v will lead to decreased distances. It is this critical observation that we will use when determining updates to the distance matrix in our algorithm.

In order to dynamically update our closeness centrality scores, it is necessary to know the distance from the new vertex v to all other vertices in the graph. We can calculate these distances by determining the single-source shortest paths (SSSP). There are many known algorithms for calculating SSSP including Dijkstra's algorithm. However, because

the current distance matrix has been stored for the region S, a full SSSP calculation is not required to update the distance matrix for $V(S) \cap v$. The distance to all neighbors of the candidate node will be 1 and the distance of a node, v, that is not direct neighbour will be defined by the minimum distance between a neighbour of the candidate node and v, since all paths from the candidate node must pass through a neighbour of the candidate.

Once the shortest distances from v to all other vertices are known, this update algorithm, in time $O(k^2)$ where $k = |V(S)|$, presented in Algorithm 3 can check whether any shortest path via v is shorter than the existing path. This assumes that a shortest distance matrix is being updated and stores explicitly all existing all-pairs distances in the graph. The average distance for each vertex can be updated in $O(1)$ time, for each change in a shortest path that affects that vertex. The algorithm halts when no candidate vertices can be added to S without losing q's dominance.

In the pathological worst case, this algorithm can perform up to $O(n^2)$ candidate node tests, with each test taking $O(k^2)$ time, leading to a worst case complexity of $O(n^4)$. This only happens if the only possible successful candidate is always located at the end of the candidate list and the returned region is equal to the entire graph. The space complexity in all cases is $O(k^2)$ where $k = |V(S)|$ due to the need to store the distance matrix. However, in practice, average complexity is significantly better. Additionally, each distance matrix update is independent, meaning that this portion of the operation could be multi-threaded to greatly increase query speed in practice.

The algorithm is split into two blocks for formatting purposes due to the limitations of typesetting algorithms. These two blocks together define the incremental algorithm for reverse closeness centrality queries. The first block, given in Algorithm 2, contains the initialization of all variables for S when S contains the 1-neighborhood of q. The second block, given in Algorithm 3, defines the growth stage of the incremental algorithm.

Algorithm 2: Incremental algorithm for reverse closeness centrality: Initialization

Data: $G = (V, E)$, $q \in G$

Result: Induced subgraph S where q dominates

```
1 V(S) ← N_1(q)
2 CC                    /* Inverse closeness centralities, initialized to 0 */
3 for i ∈ V(S) do                          /* initialize the distance matrix */
4     d[i, i] ← 0
5     for j ∈ V(S), i < j do
6         if i ∈ N_1(j) then d[i, j] ← 1
7         else  d[i, j] ← 2
8         CC[i] ← CC[i] + d[i, j], CC[j] ← CC[j] + d[i, j]
```

Algorithm 3: Incremental algorithm for reverse closeness centrality: Growth

$CandidateQueue \leftarrow N_1(S) \setminus V(S)$

$UnluckyCandidateQueue \leftarrow \emptyset$

while $CandidateQueue \neq \emptyset$ **do**

  $currentCandidate \leftarrow CandidateQueue.dequeue$

  $TS = currentCandidate \cup V(S)$

  $dCandidate[currentCandidate] \leftarrow 0$

  forall $v \in S$ **do**

    $dCandidate[v] \leftarrow \infty$

  forall $v \in n_1(currentCandidate) \cap S$ **do**

    $dCandidate[v] \leftarrow 1$

    forall $w \in S$ **do**

      if $d[v, w] > 0$ **then** `/* Shortest path exists */`

1   **if** $d[v, w] + 1 < dCandidate[w]$ **then** $dCandidate[w] \leftarrow d[v, w] + 1$

  $CCNew \leftarrow CC$

  forall $v, w \in S, v < w$ **do** `/* Calculate updated distances */`

2     $dUpdate[v, w] \leftarrow 0$ **if** $dCandidate[v] + dCandidate[w] < d[v, w]$ **then**

3       $dUpdate[v, w] \leftarrow d[v, w] - (dCandidate[v] + dCandidate[w])$

4       $CCNew[v] \leftarrow CCNew[v] - dUpdate[v, w]$

5       $CCNew[w] \leftarrow CCNew[w] - dUpdate[v, w]$

6   $success \leftarrow true$

7   **forall** $CCVal \in CCNew$ **do**

8   **if** $CCVal < CCVal[q]$ **then** `/* Value dominates q */`

9       $UnluckyCandidateQueue.enqueue(currentCandidate)$

10      $success \leftarrow false$

11      $break$

12 **if** $success = true$ **then**

13    $V(S) \leftarrow V(S) \cup currentCandidate$

14    $CandidateQueue.enqueueAll(UnluckyCandidateQueue)$

15    $CandidateQueue.enqueueAll(N_1(currentCandidate) \setminus V(S))$

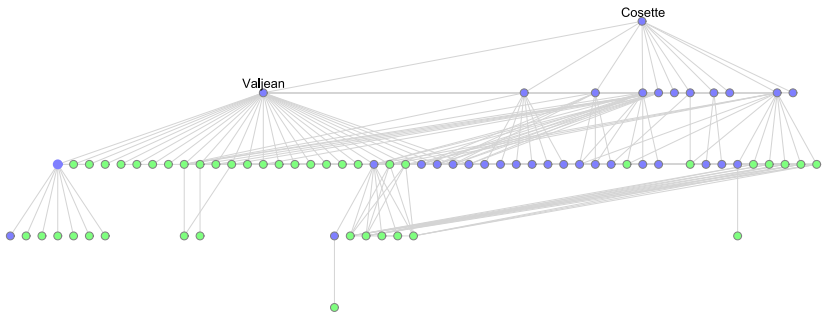


Figure 3.4: The query result for reverse closeness centrality for the character Cosette from the Les Miserables dataset shown in Reingold-Tilford form, where each layer of the tree represents distance from the root node, which is the query node Cosette.

For the character Cosette, the query node in the example given in Figure 3.4, a sizable query result is returned. Nodes in blue belong to the induced subgraph S, nodes in green belong to $G \backslash S$. It is interesting to note the nature of the result, where nodes up to a distance of 3 from Cosette are included. However, the neighbours of Valjean, the left-most node at a distance of 1, are not included. If these nodes were included, Valjean would dominate Cosette. This is an example where the query result contains a globally dominant node, Valjean, the main character of the play, for a less central character, that of Cosette.

3.2.2 Heuristics

Several variants to the basic incremental algorithm may improve query speed or result size. Two of these variants are described below, the first to potentially do both and the second to only reduce query time. The effects of these heuristics are explored in Chapter 4.

Ordering by vertex degree

One simple heuristic to improve the speed of the algorithm may be to sort the queue of candidates by increasing vertex degree. This is based on the heuristic assumption that vertices that have high closeness centrality or strongly affect the connectivity of other vertices will be, on average, of higher vertex degree. This method will not lower time complexity in the worst case, but may improve performance in practice.

This modification would change the order of the *CandidateQueue* found in Algorithm

2, in particular, instead of simply appending the new candidates (lines 14-15), we would modify the method so that the new additions are inserted into the list in order based on vertex degree.

This method will be explored in further detail in Chapter 4.

Counting chances

Each time a node is selected as a candidate from the candidate list, a counter for the node is incremented. The node only is considered k times, where k is a maximum number of chances that a node will be given to re-enter the candidate list. This decreases the number of possible iterations from $O(n^2)$ to $O(kn)$.

In particular, if there are nodes close to the query node that cannot be added at any step of the incremental process, they will enter the candidate list early and will be repeatedly tested until the entire candidate list fails in the unmodified algorithm. It makes sense that a node that has failed a reasonable number of times is unlikely to ever succeed and that time improvements may be seen by removing such nodes from consideration.

This modification would require the addition of a counter to be kept for each member of the candidate list, initialized to 0 when candidates first enter the list (as neighbours of the region). Each time a candidate is rejected (line 9 in Algorithm 2), the counter would be incremented, and if the counter equals k, the node is permanently rejected, and not included in the $UnluckyCandidateQueue$.

This heuristic means that the query result may not necessarily be a locally maximal region, as we prematurely eliminate nodes from consideration.

3.3 Reverse graph centrality query

The reverse graph centrality query identifies a locally maximal region where the query node is dominant according to graph centrality. That is, the query node has eccentricity equal to the graph radius of the region.

Solutions for calculating eccentricity for graphs have been explored for many specific classes of graphs including chordal [12], 3-cactus [30], and HDD-free [11] graphs. However, the best available solution for general graphs is the all-pairs shortest path (APSP) algorithm. Various solutions for all-pairs shortest path algorithms are explored in depth in Appendix 1.

The method presented includes an incremental all-pairs shortest path algorithm implementation for determining the validity of each update (each potential candidate node). This algorithm may be modified to use other incremental APSP methods.

The method halts when no neighbour (any node in the candidate list) of the current dominated region can be successfully added to the region without eliminating the dominance of the query node.

Theorem 3.3.1 (Dominance is non-monotone)**.** A locally maximal region of dominance found according to the reverse graph centrality query may be a proper subgraph of a larger region of dominance for the query node.

Proof. Let dominance refer to local dominance according to graph centrality. Given a graph $G = (V, E)$ and a query vertex q, suppose in the induced subgraph $A \subset G$ that q is dominant. Suppose there exists a vertex u such that the induced subgraph $B = A \cup u$ has a shortest path via u which reduces the eccentricity of some other vertex $x \in B$ below that of q. B no longer has q as a dominant vertex. Now, suppose we introduce another vertex v and expand the induced subgraph to produce $C = B \cup v$ such that C has a new shortest path via v that reduces the eccentricity of q below that of x, and once again q is dominant. It follows that there is no guarantee of q being dominant or not in proper subsets of an induced subgraph where q is dominant. An example showing this behaviour can be seen in Figure 3.3. □

Because graph centrality dominance is non-monotonic, an incremental algorithm may fail to find a globally optimal solution, so we are limited to guaranteeing a locally maximal induced subgraph.

Theorem 3.3.2 (1-neighborhood dominance)**.** A node dominates its 1-neighborhood according to closeness centrality.

Proof. Given a graph $G = (V, E)$ and a query vertex q, the 1-neighbourhood of q is defined as $S = q \cup N_1(q)$, where $N_1(q)$ is the set of vertices with edges to q. The eccentricity of any node is at least 1 as it is 1 in a complete graph, so the minimum eccentricity in the induced subgraph is 1, which is exactly equal to the eccentricity of q, since q by definition has a distance of exactly 1 to all vertices in $N_1(q)$. It follows that q dominates $N_1(q)$. □

3.3.1 Incremental algorithm

The algorithm for reverse graph centrality queries is closely related to the algorithm presented in Section 3.2.1 for reverse closeness centrality queries. Starting with the one-neighborhood of the query node, the query result is expanded incrementally by attempting to add neighbours to the growing result. When no single node can be added to the growing result, a locally optimal solution has been found and the algorithm will terminate.

The method for updating the distance matrix is shared with the reverse closeness centrality algorithm explored in Section 3.2.1. As a result, this algorithm also has a worst case time complexity of $O(n^4)$ and a space complexity of $O(k^2)$ where $k = |V(S)|$. Although again, in practice, average time complexity is much better than this.

Again, due to typesetting restrictions, the algorithm has been broken into two blocks that describe the same contiguous algorithm. The first block, found in Algorithm 4, consists of the initialization steps required for setting up S as equal to the 1-neighbourhood of q. The second block, in Algorithm 5, consists of the incremental growth algorithm.

Algorithm 4: Incremental algorithm for reverse graph centrality: Initialization

Data: $G = (V, E)$, $q \in G$

Result: Induced subgraph S where q dominates

```
1 V(S) ← N_1(q)
2 Ecc                                   /* Eccentricity, initialized to 0 */
3 for i ∈ V(S) do                       /* initialize the distance matrix */
4     d[i, i] ← 0
5     for j ∈ V(S), i < j do
6         if i ∈ N_1(j) then d[i, j] ← 1
7         else  d[i, j] ← 2
8         if Ecc[i] < d[i, j] then Ecc[i] ← d[i, j]
9         if Ecc[j] < d[i, j] then Ecc[j] ← d[i, j]
```

Algorithm 5: Incremental algorithm for reverse graph centrality: Growth

```
    CandidateQueue ← N_1(S) \ V(S)
    UnluckyCandidateQueue ← ∅
    while CandidateQueue ≠ ∅ do
        currentCandidate ← CandidateQueue.dequeue
        TS = currentCandidate ∪ V(S)
        dCandidate[currentCandidate] ← 0
        eccUpdated[currentCandidate] ← ∞
        forall v ∈ TS do
            dCandidate[v] ← 0
            eccUpdated[v] ← 0
        forall v ∈ n_1(currentCandidate) ∩ S do
            dCandidate[v] ← 1
            forall w ∈ S do
                if d[v, w] > 0 then                          /* Shortest path exists */
1                   if d[v, w] + 1 < dCandidate[w] then dCandidate[w] ← d[v, w] + 1
        forall v, w ∈ S, v < w do          /* Calculate updated eccentricities */
2           dUpdate[v, w] ← 0
3           if dCandidate[v] + dCandidate[w] < d[v, w] then
4               dUpdate[v, w] ← d[v, w] − (dCandidate[v] + dCandidate[w])
5               if eccUpdated[v] < dUpdate[v, w] then
6                   eccUpdated[v] ← dUpdate[v, w]
7       success ← true
8       forall ecc ∈ eccUpdated do
9           if ecc < eccUpdated[q] then                      /* Value dominates q */
10              UnluckyCandidateQueue.enqueue(currentCandidate)
11              success ← false
12              break
13      if success = true then
14          V(S) ← V(S) ∪ currentCandidate
15          CandidateQueue.enqueueAll(UnluckyCandidateQueue)
16          CandidateQueue.enqueueAll(N_1(currentCandidate) \ V(S))
```

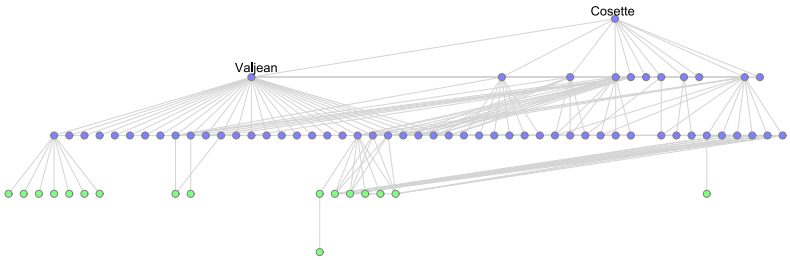


Figure 3.5: The query result for reverse graph centrality for the character Cosette from the Les Miserables dataset shown in Reingold-Tilford form, where each layer of the tree represents distance from the root node, which is the query node Cosette.

In Figure 3.5, we can clearly see that for Cosette, the graph centrality result is exactly equal to the 2-neighborhood of Cosette. This follows from the granularity of this particular centrality index. As is noted in Chapter 4, this particular graph only exhibits a range of three values for graph centrality. In this case, the graph centrality result indicates that there is no node connected to all of Cosette's neighbors and neighbors of neighbors, because otherwise, Cosette would not dominate her 2-neighborhood.

3.3.2 Heuristics

The heuristics that will be experimented with for reverse graph centrality queries are *counting chances* and *vertex sorting*. These were already been explained in detail in Section 3.2.2 and can be applied equally to reverse graph centrality queries as for reverse closeness centrality queries.

Experimental results for these variants will be explored in Chapter 4.

Chapter 4

Experimental Results and Discussion

4.1 Datasets

Zachary Karate Club dataset

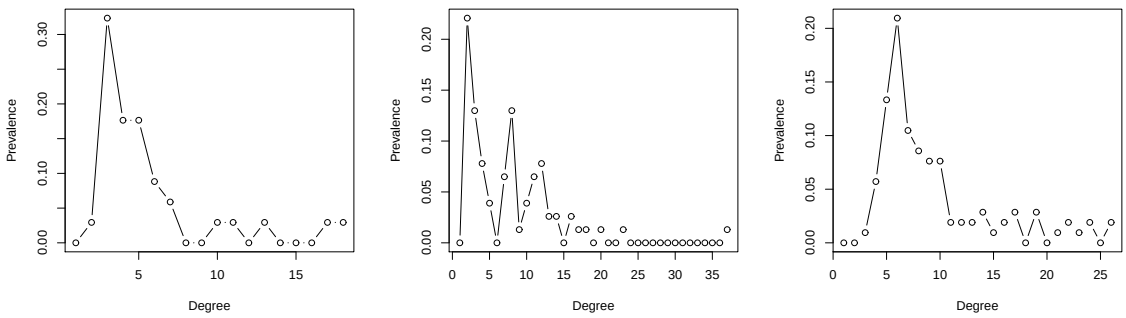
The Zachary Karate Club dataset [52], introduced in Chapter 2, contains 34 nodes from a karate club whose dynamics were studied in depth by W. W. Zachary. Links in this dataset represent friendships or allegiances between members of the university karate club of an unnamed American university. This network displays classical social network properties, with a community clustering coefficient of 0.256, a power-law degree distribution as seen in Figure 4.1a, and an average path length of 2.40. A visualization of this graph is provided in Figure 2.1.

Les Miserables dataset

The *Les Miserables* dataset [32] includes 76 nodes representing characters in the eponymous novel by Victor Hugo. Links in the dataset represent co-appearances in scenes in the novel. This network displays classical social network properties, with a community clustering coefficient of 0.499, a power-law degree distribution as seen in Figure 4.1b, and an average path length of 2.64. A visualization of this graph is provided in Figure 1.1.

Measures	Karate	*Les Miserables*	Political Books
Number of nodes (n)	34	76	105
Number of links (m)	78	254	441
Average path length (L)	2.40	2.64	3.08
Clustering coefficient (CC)	0.256	0.499	0.348

Table 4.1: This table summarizes the graphs that are used in experiments in this chapter.



(a) Zachary (b) Les Miserables (c) Political Books

Figure 4.1: Degree distribution of sample datasets: *Prevalence* is the ratio of all nodes in the dataset with the given degree.

Political Books dataset

The Political Books dataset consists of 105 nodes representing books on U.S. politics published near the time of the 2004 U.S. presidential elections. Links between books indicate frequent co-purchasing of those books according to Amazon.com. The dataset was collected by Valdis Krebs. The basic statistics about this graph include a community clustering coefficient of 0.348, a power-law distribution as seen in Figure 4.1c, and an average path length of 3.08.

4.2 Implementation

For data structures and basic graph algorithms, the igraph Library is used [13]. The implementation of the algorithms for experimentation uses the R scripting language. The strength of R is in the ability to visualize and manipulate graph data easily and to produce statistical graphs for analysis. The timing observed in test runs is impacted by the interpreted

nature of R. If the algorithms were implemented in a compiled language, such as C++, the expectation is that they would run with a much smaller time constant, resulting in quicker queries and feasibility of running the algorithms on large datasets. R is a language and programming environment [40], first developed by Ross Ihaka and Robert Gentleman with continuing development by the R Development Core Team. Variants were implemented as modifications to the original R algorithms.

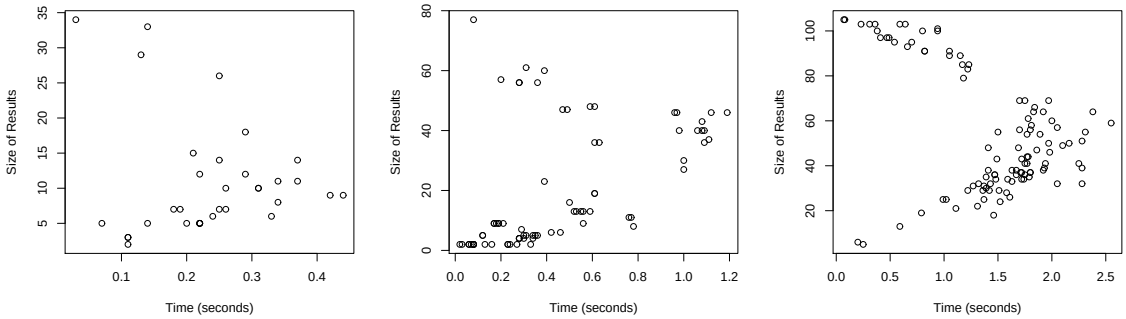
4.3 Experimental results

The reverse centrality query results for closeness centrality and graph centrality were conducted using the same testing framework. For each dataset, the reverse centrality query result was calculated for all nodes in the graph and evaluated using a variety of measures, including the size of the result, query time, and total number of tested candidate nodes. Query time is the total process time required for the algorithm to execute for that specific query node. Linear trends, where shown, are calculated using linear regression fitting to the entire dataset.

The size of results versus query time shows the strong influence the result size has on the running time. Transitivity (also known as clustering coefficient) versus size of results and modularity versus size of results show the structure of the returned results. The behaviour of the growth phase of the query algorithm is explored in graphs that demonstrate the relationship between time and number of trials as well as number of successful candidates versus the number of candidate trials. Finally, the relationship between the size of the result and the global centrality ranking for the query node is shown.

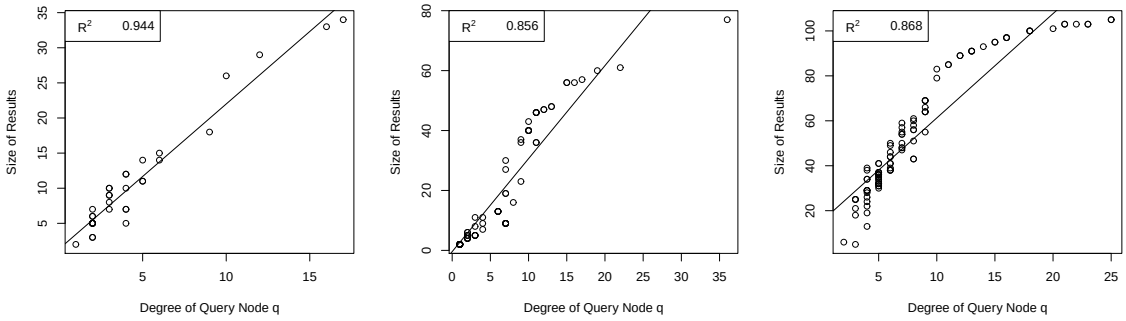
4.3.1 Reverse degree centrality query results

No clear global relationship between query time and size of the returned result set can be seen in Figure 4.2, but the overall speed of the query can be contrasted with the results seen in Figures 4.4 and 4.15. If we look closer though, there are, in fact, two trends that may be observed in Figure 4.2. Both of these trends come from the underlying two-phase algorithm described in section 3.1.2 which was the algorithm implemented and tested. The first trend, seen as the nodes found in the upper left corner, are the results that return very quickly from the first phase of that algorithm, with very few nodes that actually require testing. This trend slopes downward, as the smaller result sets will have required tests of



(a) Zachary (b) Les Miserables (c) Political Books

Figure 4.2: Size of Results versus Time (seconds) with Degree Centrality

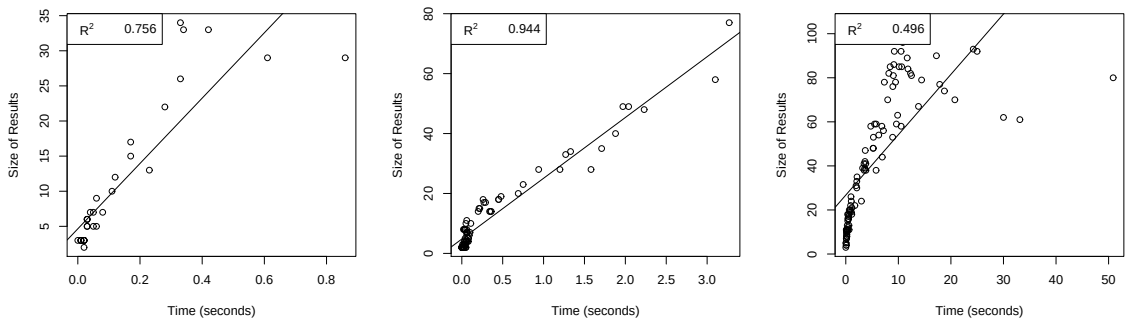


(a) Zachary (b) Les Miserables (c) Political Books

Figure 4.3: Degree of Query node q in G versus Size of Results with Degree Centrality

all remaining nodes to ensure that they could not be added to the result. The second trend observed is seen in the lower right corner as an upward trend, these are the query results that have been dominated by the second phase of the algorithm, and require more testing of candidate nodes and did not benefit significantly from the first phase expansion. These two trends can be observed best in Figure 4.2c.

On the other hand, Figure 4.3 shows a strong, clear linear relationship between size of the result set and the degree of the query node in the original graph. This follows from the possible set of nodes that could be automatically returned as being globally dominated by the query node q, which is directly proportional to the degree of q.



(a) Zachary (b) Les Miserables (c) Political Books

Figure 4.4: Size of Results versus Time (seconds) with Closeness Centrality

4.3.2 Reverse closeness centrality query results

In Figure 4.4, the linear relationship between the size of the returned result and the time to execute the query is clearly shown. Due to the candidate testing phase, the linear relationship between final result size and running time clearly follows. The figures demonstrate that the method is scalable and behaves well on all nodes, whether peripheral or central.

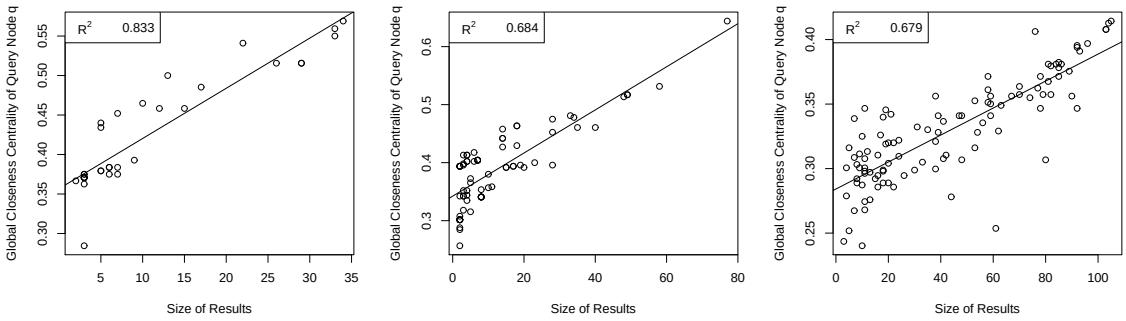
Figure 4.5 clearly demonstrates the strong correlation between size of regional domination under the reverse closeness centrality query and global closeness centrality ranking. One positive result can be noted is that the algorithm successfully returns a result equal to the original graph for the dominant node in the original graph.

The number of tested candidate nodes versus time shown in Figure 4.6 shows a positive correlation between number of candidates tested and time, but the relationship is less pronounced than that between size of result and query time, shown in Figure 4.4.

The total number of candidates tested shows a positive correlation in Figure 4.7a, Figure 4.7b, and Figure 4.7c. Of particular interest are the outliers seen in Figure 4.7c, there are three node queries that result in 1200-1400 tests of individual candidates.

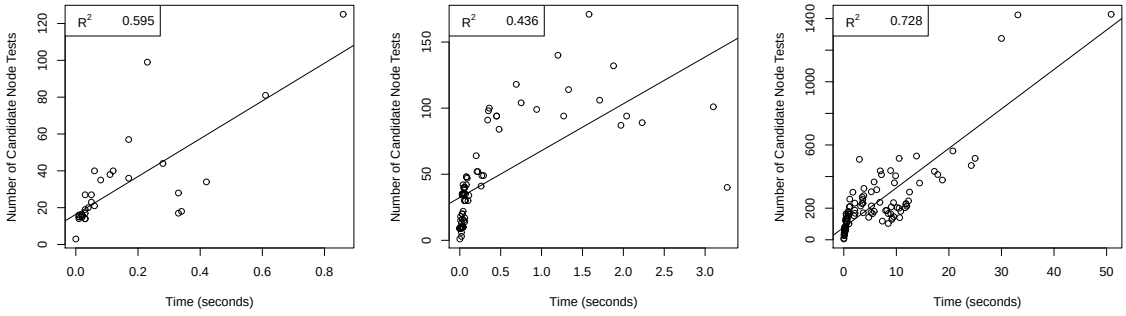
In the most extreme cases, due to the exhaustive search and the high number of possible candidates at the point of termination, $O(|CandidateList|^2)$ trials may be required to terminate. This behaviour can clearly be seen in the far right of Figure 4.7c where nearly 1400 trials are conducted compared to less than 80 successful candidates.

The relationship between modularity and size of result shows no strong correlation as



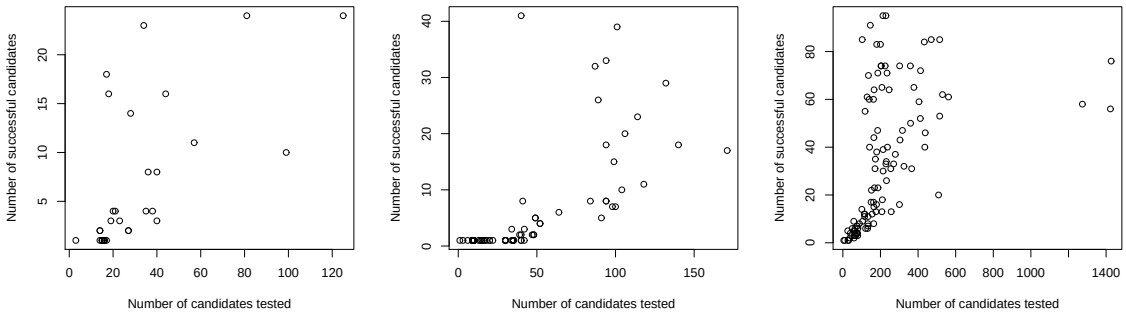
(a) Zachary (b) Les Miserables (c) Political Books

Figure 4.5: Closeness Centrality of Query Node q in G versus Size of Results with Closeness Centrality



(a) Zachary (b) Les Miserables (c) Political Books

Figure 4.6: Number of Candidates Tested versus Time with Closeness Centrality

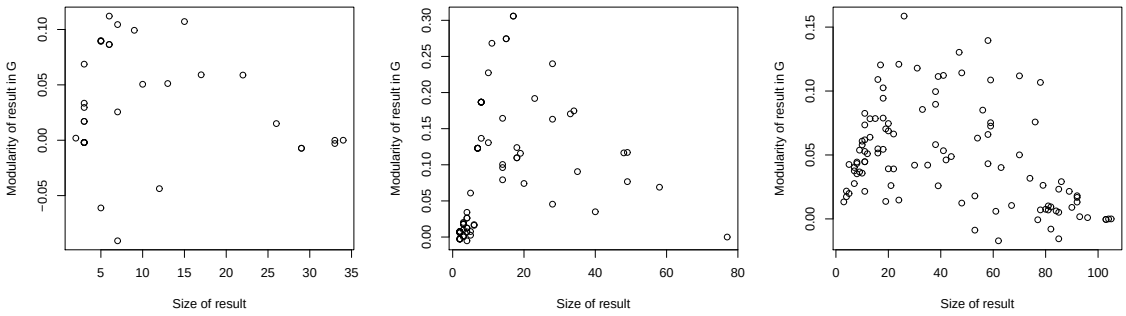


(a) Zachary (b) Les Miserables (c) Political Books

Figure 4.7: Number of Candidates Tested versus Number of Successful Candidates with Closeness Centrality

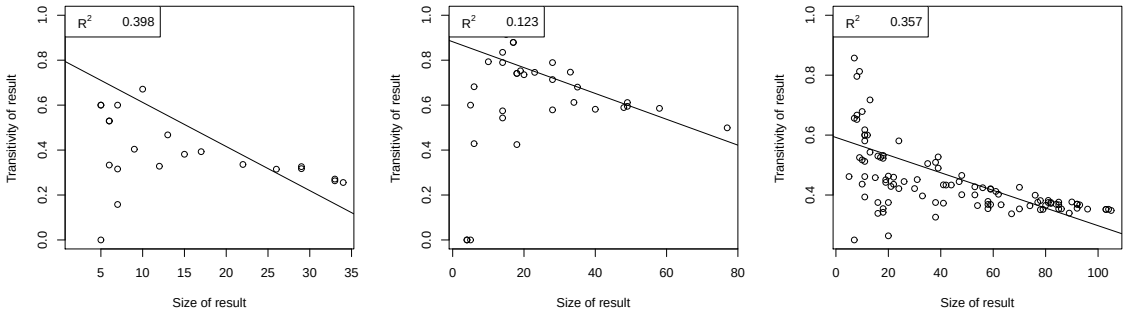
observed in Figure 4.8. These results follow from the root of the algorithm which intends to select a region around a query node q, irrespective of the relationship of the region to the global graph G.

Transitivity, also known as the clustering coefficient, in Figure 4.9 shows a mild negative correlation with the size of the result, although this may simply be a product that small connected subgraphs of a social network are likely to have a high clustering coefficient.



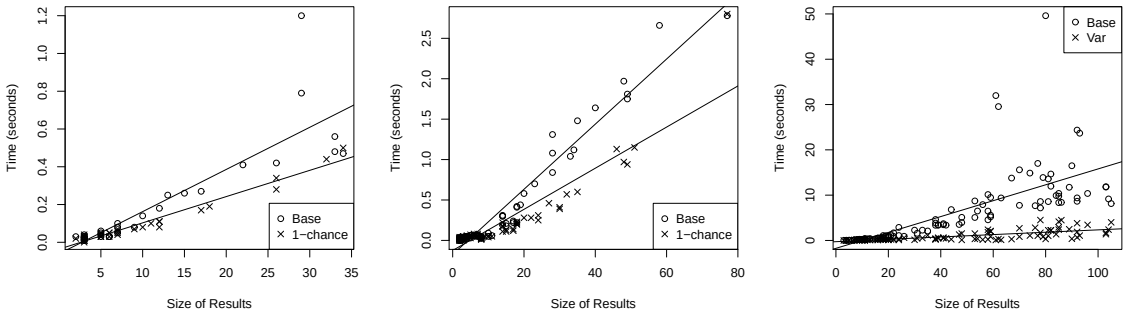
(a) Zachary (b) Les Miserables (c) Political Books

Figure 4.8: Modularity of Results versus Size of Results with Closeness Centrality



(a) Zachary (b) Les Miserables (c) Political Books

Figure 4.9: Transitivity of Results versus Size of Results with Closeness Centrality



(a) Zachary (b) Les Miserables (c) Political Books

Figure 4.10: Size of Results versus Query Time with Closeness Centrality: 1 Chance Variant

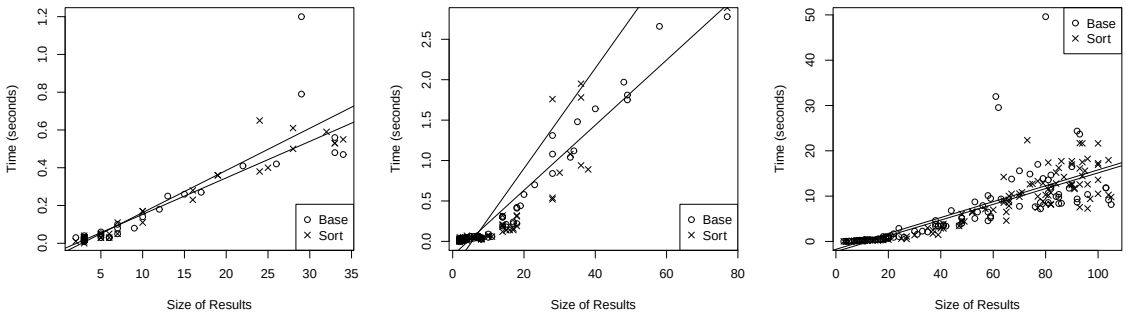
Variant results

This modification involves adding a counter to the CandidateList to track the number of times a node has been tested, on the basis that a significant amount of time is wasted continually retesting nodes in the CandidateList that will never be successfully added, but are repeatedly tested due to the no-failure policy of the base algorithm. Notably, the query response time for a given result size is about twice as fast using the 1-chance variant, according to Figure 4.10.

This modification involves sorting the CandidateList by increasing vertex degree, on the basis that vertices of low degree tend to be peripheral and have little impact on the overall closeness centrality rankings. There is a minor, but potentially insignificant difference in query result time as shown in Figure 4.11.

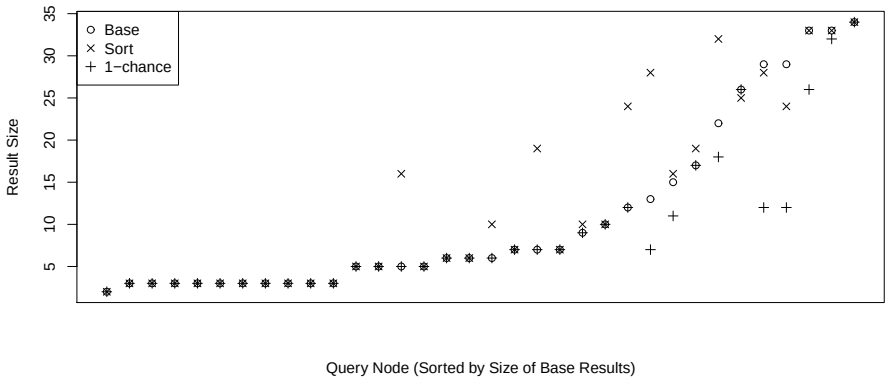
The most revealing results about the variants can be seen in Figure 4.12, here the sizes of results for the same query node are compared across all nodes in the sample graphs. Notably, the order-sensitivity of the algorithm can be seen by the disagreement in result size between the sorting variant and the base algorithm. This indicates that a change that only affects the order of trials has led to a different result, indicating the order sensitivity discussed in Chapter 3.

Also of note, the 1-chance variant occasionally returns results of much smaller size, indicating that the second chance mechanism in the base algorithm is useful for finding larger results.

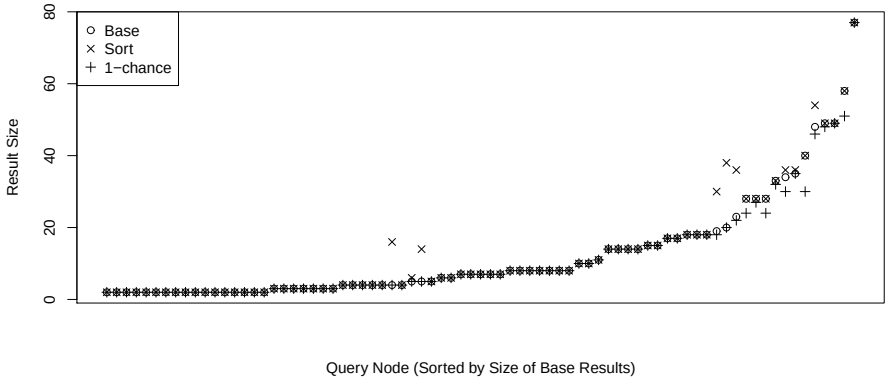


(a) Zachary (b) Les Miserables (c) Political Books

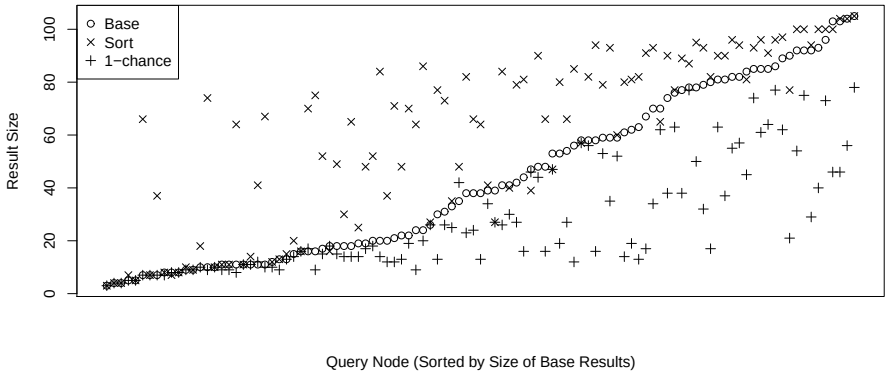
Figure 4.11: Size of Results versus Query Time with Closeness Centrality: Sorting Variant



(a) Zachary

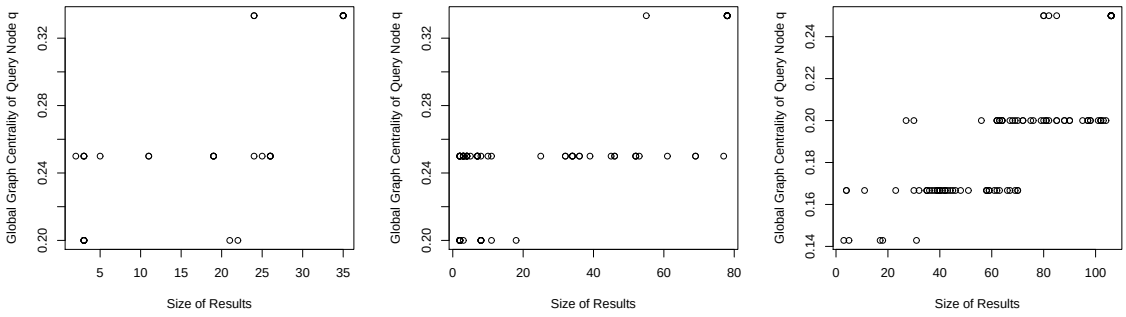


(b) Les Miserables



(c) Political Books

Figure 4.12: Query Node versus Size of Results with Closeness Centrality: Base Algorithm, Sorting Variant and 1-Chance Variant Compared



(a) Zachary (b) Les Miserables (c) Political Books

Figure 4.13: Graph Centrality of query node q in G versus Size of Results with Graph Centrality

4.3.3 Reverse graph centrality results

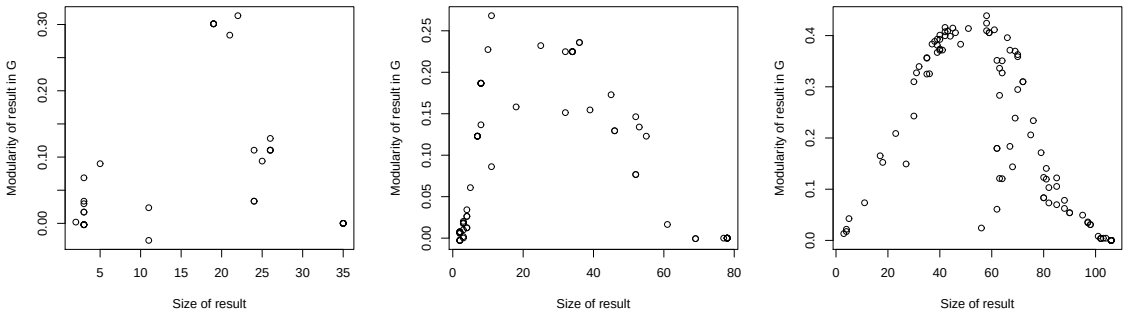
Figure 4.13 emphasizes the granularity of the graph centrality measure: since graph centrality is the inverse of the eccentricity of a node, when this is paired with the small-world property of social networks, the range of expected values is very low. In the case of the Zachary dataset and the Les Miserables dataset, the graph diameter is 5 and the graph radius is 3, leading to three possible options for graph centrality values: 0.2, 0.25, 0.33.

Despite the discrete nature of the graph centrality measure, a general positive trend between result size and graph centrality score can be seen. However, it interesting to note that there are several communities of low global graph centrality that buck the trend and have unexpectedly large results.

These outliers that have low global graph centrality scores but have large results emphasize that local conditions may differ drastically from global conditions. This emphasizes the usefulness of reverse centrality queries as a meaningful node evaluation tool.

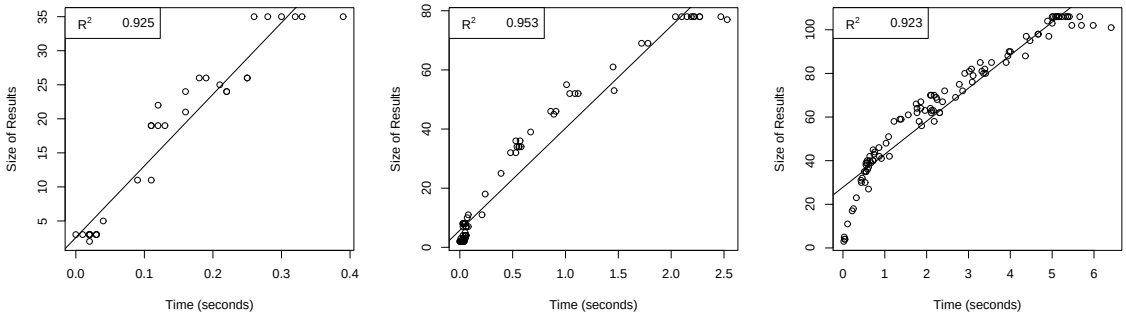
The evaluation of modularity versus result size shows a bell-curve relationship between modularity and the size of the result. At a certain result size, the modularity of the result is generally highest, with lower values seen on either side. This is most clearly seen in Figure 4.13c. It likely follows that some of the returned results correspond to naturally occurring communities in the graph, and smaller and larger results either do not include the entire community or begin to include other communities.

In Figure 4.15, a strong linear relationship between result size and query time can be



(a) Zachary (b) Les Miserables (c) Political Books

Figure 4.14: Modularity of Results versus Size of Results with Graph Centrality



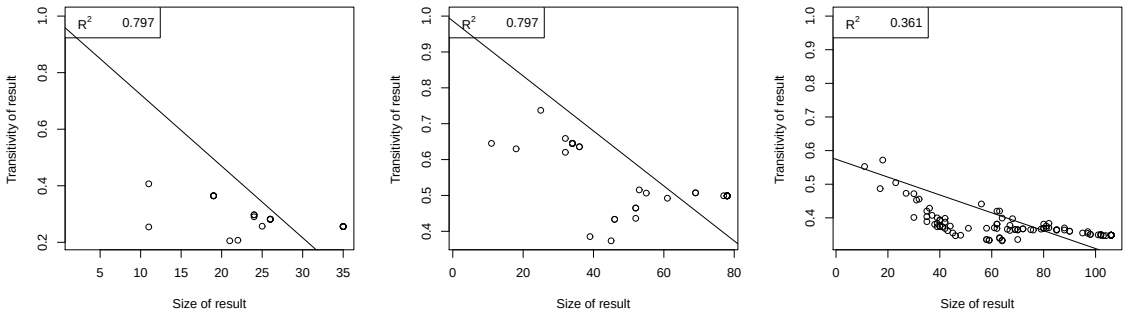
(a) Zachary (b) Les Miserables (c) Political Books

Figure 4.15: Size of Results versus Time (seconds) with Graph Centrality

seen. This is positive as it demonstrates that the growing set of possible candidates as the result grows does not negatively impact the query speed.

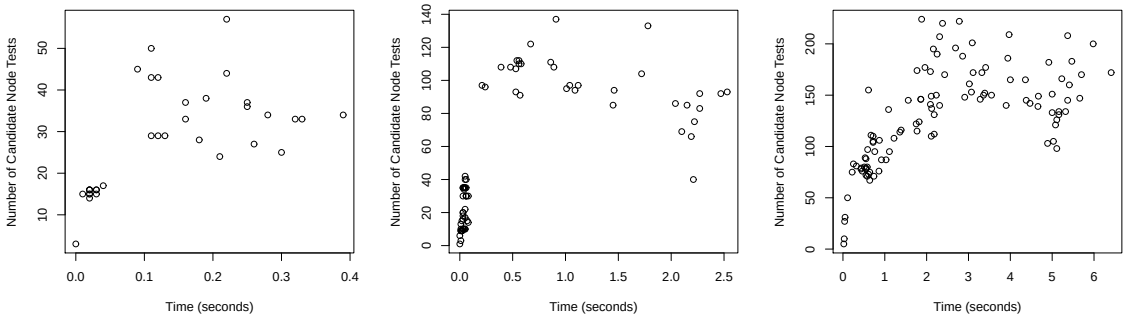
Transitivity, also known as the clustering coefficient, in Figure 4.16 shows an inversely proportional relationship to result size. This indicates that the larger result sets are less cohesive than the smaller results, and this follows from the structure of social networks.

The number of candidate tests performed shows a surprisingly weak relationship to the total query time. In comparing Figure 4.17 to Figure 4.15, the number of successful candidates has a much stronger correlation to the query time than the number of tested candidates.



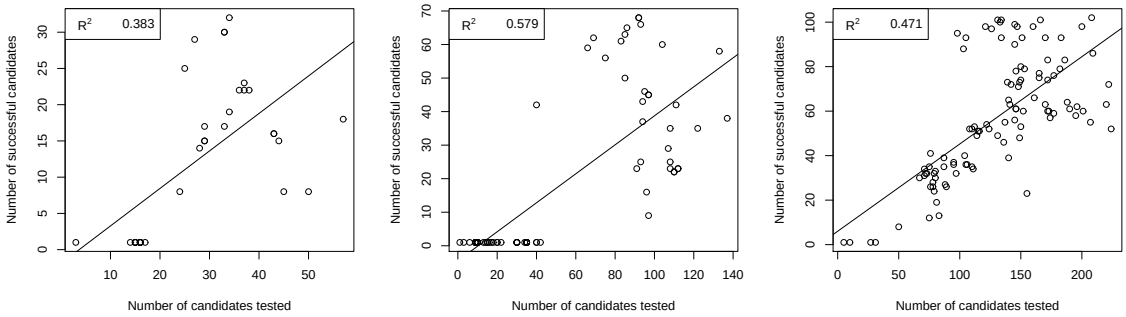
(a) Zachary (b) Les Miserables (c) Political Books

Figure 4.16: Transitivity of Results versus Size of Results with Graph Centrality



(a) Zachary (b) Les Miserables (c) Political Books

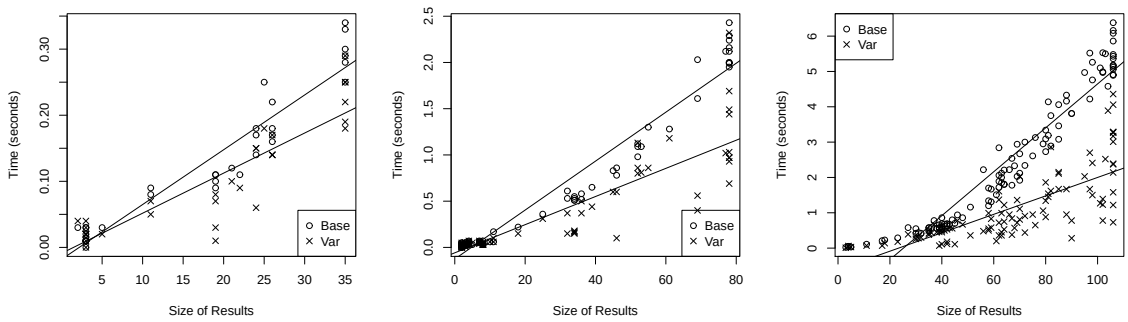
Figure 4.17: Number of Candidates Tested versus Time with Graph Centrality



(a) Zachary (b) Les Miserables (c) Political Books

Figure 4.18: Number of Candidates Tested versus Number of Successful Candidates with Graph Centrality

In Figure 4.18, although not perfect, a general correlation is seen between the number of tested candidates and the number of successful candidates. There are no pathological cases observed where the number of tested candidates approaches $O(|CandidateList|^2)$ as was demonstrated in the reverse closeness centrality query experiments.



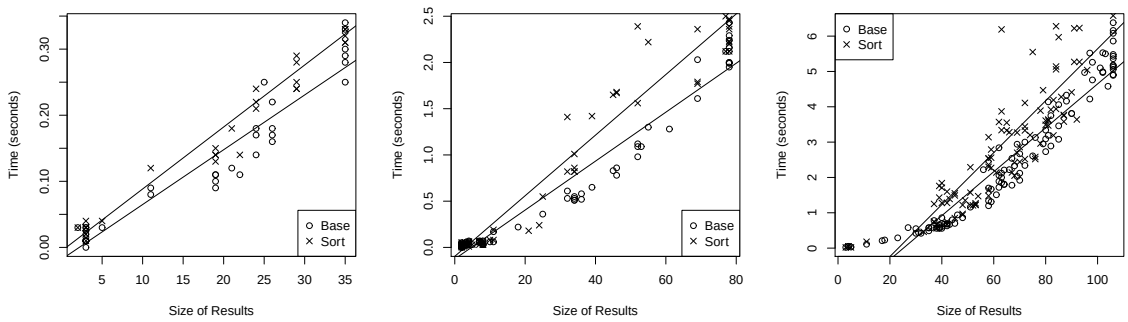
(a) Zachary (b) Les Miserables (c) Political Books

Figure 4.19: Size of Results versus Query Time with Graph Centrality: 1 Chance Variant

Variant results

This modification involves adding a counter to the CandidateList to track the number of times a node has been tested, on the basis that a significant amount of time is wasted continually retesting nodes in the CandidateList that will never be successfully added, but are repeatedly tested due to the no-failure policy of the base algorithm. Notably, the query response time for a given result size is about twice as fast using the 1-chance variant, according to Figure 4.19.

This modification involves sorting the CandidateList by increasing vertex degree, on the basis that vertices of low degree tend to be peripheral and have little impact on the overall closeness centrality rankings. There is a minor, but potentially insignificant difference in query result time as shown in Figure 4.20.



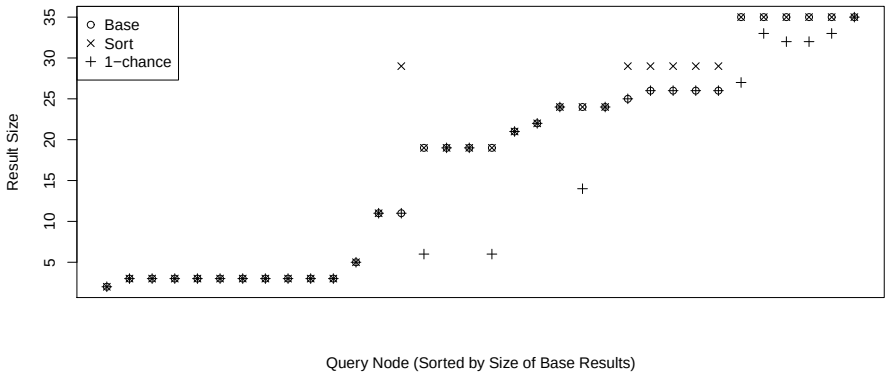
(a) Zachary (b) Les Miserables (c) Political Books

Figure 4.20: Size of Results versus Query Time with Graph Centrality: Sorting Variant

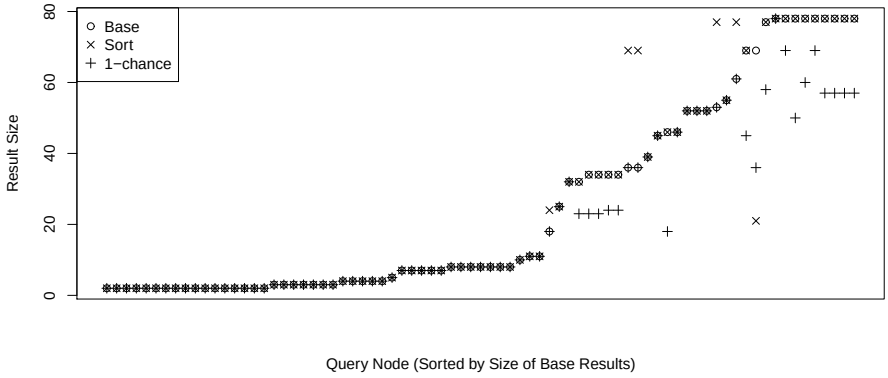
The most revealing results about the variants can be seen in Figure 4.21, here the sizes of results for the same query node are compared across all nodes in the sample graphs. Notably, the order-sensitivity of the algorithm can be seen by the occasional disagreement in result size between the sorting variant and the base algorithm. This is most notable in Figure 4.21c. This indicates that a change that only affects the order of trials has led to a different result, indicating the order sensitivity discussed in Chapter 3. This variation in result size also indicates that the results found are locally, rather than globally, maximal.

In addition, a strong trend can be seen in Figure 4.21c, where the result size for sorting is generally larger than for the base algorithm. This indicates that the sorting variant provides a better searching strategy than the base algorithm.

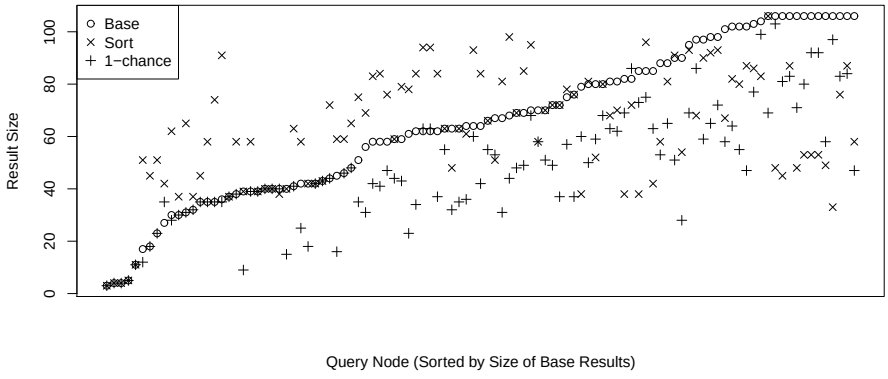
Also of note, the 1-chance variant frequently returns results of much smaller size, indicating that the second chance mechanism in the base algorithm is useful for finding larger results.



(a) Zachary



(b) Les Miserables



(c) Political Books

Figure 4.21: Query Node versus Size of Results with Graph Centrality: Base Algorithm, Sorting Variant and 1-Chance Variant Compared

4.3.4 Reverse centrality queries versus global centrality

Reverse centrality queries provide novel information at the node level, which may significantly differ from the information available with respect to the entire graph. There are cases where a globally insignificant node may play an important part in its own local region. With that, the context of the query node is given greater importance due to local centrality, whereas the same node may seem relatively insignificant at the global level.

4.4 Summary

This chapter began by introducing the three datasets that were used for experimental analysis. The implementation in R used for experimentation was described in Section 4.2. Finally, detailed experimental analysis was provided on the three query types across the three datasets introduced in the beginning of the chapter.

In summary, the major insights we gained from our experimental results are:

- In Section 4.3.1, we established the reverse degree centrality query method provides query results very quickly for all datasets explored. Additionally, we verified the expected linear relationship between result size and query node degree.

- Exploring reverse closeness centrality queries in Section 4.3.2, we saw a general linear relationship between result size and query time, but several pathological cases were noted in the Political Books dataset. The presence of outliers in Figure 4.5 demonstrates that there are cases where nodes of low global closeness centrality can dominate large regions. The counting chances variant was demonstrated to be an effective method for significantly reducing query time, but at the expense of occasionally smaller result sizes. The sorting variant was shown to perhaps marginally improve query times, but more importantly, also occasionally produced larger results than the base method.

- In Section 4.3.3, for reverse graph centrality queries, we established a strong linear relationship between result size and query running time. The counting chances variant proved to be useful in reducing query time, but also occasionally reduced the result size. The sorting variant performed slightly worse than the base method, indicating that sorting by vertex degree is not an appropriate heuristic for graph centrality.

Outliers in Figure 4.13 demonstrate that nodes of low graph centrality, usually nodes considered peripheral to the graph, occasionally return large results, demonstrating strong local dominance under graph centrality.

Chapter 5

Conclusion and Future Work

5.1 Summary

The rapid growth in the volume and availability of complex network data provides us with many opportunities to define new paradigms for exploring and understanding this data. Reverse centrality queries provide another analysis tool for understanding the complex structure and dynamics of a social network, especially in relation to individual nodes in the network. The methods presented in this thesis provide new methods for answering these novel queries.

We conclude by briefly summarizing the major contributions of this thesis:

- In Chapter 2, we introduced a novel query type for social network analysis called the *reverse centrality query*.
- In Chapter 3, we gave incremental algorithms for finding locally optimal query results for three types of reverse centrality queries.
- In Chapter 4, we provided a detailed analysis of the results of the three query types on three real-world networks and demonstrated the queries provide an effective novel means of graph exploration.
- In Section 5.2, we provide a wide number of future directions in which reverse centrality queries may be improved in the future.

5.2 Future work

In this thesis, we introduced reverse centrality queries, and provided a detailed treatment of several aspects of these queries. There are many potential directions for future work in this area. Further heuristic improvements to the methods presented in Chapter 3 may be possible. Additional modifications to adapt the given methods to run efficiently on large graphs would also be useful. In Chapter 2, we outlined several types of constraints that could be added to the problem definition of reverse centrality queries. A detailed treatment of these constraints, including experimentation and evaluation would be beneficial. The methods of Chapter 3 use a basic breadth first search approach, a more sophisticated approach that incorporates back-tracking is likely to improve the query result by searching more of the problem space. These methods do provide a way to answer several types of reverse centrality queries, but they are limited in their applicability due to time complexity issues. An important future direction is the development of more efficient methods to increase the size of graph that can be queried.

5.2.1 Heuristic improvements

For all algorithms, improvements may be found by using statistics which are pre-computed on the whole graph, or on the connected component of the graph that the query node is found in.

Global statistics

Using global centrality indices such as betweenness centrality, or other statistics such as vertex degree and local density may help improve the efficiency of the algorithm in finding large regions. Other static statistics may also heuristically improve our methods. These statistics would be used to sort the candidate list, changing the search priority of the method.

Positive indicators such as low vertex degree may be used to indicate peripheral nodes in the graph that can very likely be added without disrupting the dominance of the query node. The sorting variant seen in Chapters 3 and 4 for graph and closeness centralities is based on the use of low vertex degree as a positive indicator.

Negative indicators, such as high betweenness centrality may be used to identify influential nodes which are likely to dominate the query node. If a node dominates the query node

in the global graph, it is very likely that that node will not contribute to the dominance region of the query node except possibly as a peripheral node.

Partitioning

Another option is to pre-process the graph into community partitions and only consider nodes within the query point's own partition. This could additionally reduce query time by reducing the size of the set of candidate nodes. In addition, partitioning would be a useful means for reducing the query time on larger datasets. Partitioning could also improve the performance of static statistics used heuristically in node selection in the two methods.

5.2.2 Feature vector networks

An extension that is possible for the future is the adaptation of these methods for feature vector networks, where each node in the network has an associated feature vector. This would allow more powerful queries to be run on the social network which are additionally constrained by vector attributes.

5.2.3 Advanced search methods

The algorithms presented in Chapter 3 use a straight forward breadth first search technique to identify nodes to add to the result set, improved by revisiting all potential candidate nodes. This method could be further modified to allow back-tracking: the removal of successful candidates to explore other search paths.

5.2.4 Variants on reverse centrality query answering

By limiting the expansion of the search to a fixed radius k, we can decrease the number of possible iterations required to find an answer. This is especially useful in large graphs where an answer may potentially be the entire graph and calculating this could be prohibitively expensive.

These same methods can be used to generate multiple answers by varying k from 2 to r, a user defined maximum radius. This parameter will be very small in practice due to the (small world) nature of most complex networks.

Another way to improve query answering time would be to restrict the size of the resulting answer to a fixed number of nodes, where $|V(S)| < h$, and h is a constant. This

technique may be useful in preliminary evaluation to see whether the node dominates a large area or not.

5.2.5 Local Centrality Measures

An idea briefly introduced in Definition 2.6.5 is that of *local centrality*. No available measure that I am aware of properly quantifies this idea. I believe that this is a very fruitful future research direction. The experimental results of this thesis demonstrate that there are indeed nodes that are capable of dominating large regions, while showing only modest centrality in the global graph.

Appendix A

Appendix 1

A.1 The All-Pairs Shortest Path (APSP) Problem

The all-pairs shortest path (APSP) problem involves finding the shortest paths between all pairs of nodes in a graph. The APSP problem is directly related to distance-based and path-based centrality indices. Determining the exact radius of the graph requires knowledge of the longest shortest path (the eccentricity) for each node in the graph. Since the graph centrality problem requires knowing the nodes that have eccentricity equal to the radius of the graph, it follows that we must know the eccentricity of every node in the graph. It follows that the graph center can be trivially calculated from knowing APSP for a given graph. So the problem of determining the graph centrality in the general graph is bounded by the complexity of the APSP problem.

Purpose-built algorithms for finding graph centers have improved on the APSP complexity bound for specific subclasses of graphs, including chordal [12], 3-cactus [30], and HDD-free [11] graphs. These specific algorithms exploit special properties of these classes of graphs to determine at least one central vertex, however, they do not return the full graph center. Additionally, the constraints imposed by these graph classes are too strong to be applied on any real world network.

The APSP problem is well-studied in the field of graph theory. The classic solution of utilizing breadth-first search from every node is $O(mn)$ or $O(n^3)$ since the number of links, m, is bounded by $m \leq n(n-1)$. Speed-ups for this method either use fast matrix multiplication, such as Seidel's algorithm [44] that runs in $O(n^{2.376})$, or methods that give logarithmic improvements over the classic method, such as Feder and Motwani's $O(n^3/\log n)$

method [16] for unweighted, undirected graphs. The static version of APSP is relevant to the reverse centrality query introduced in this thesis.

The recent work by Chan [10] has improved the complexity bound on the problem for static sparse graphs with $m \ll n^{1.376}$. In particular, for graphs where $m \leq n \log\log n$, the method achieves $O(n^2 \log^2 \log n / \log n)$, which is impressive given the naïve method is $O(n^3)$.

Additional work by Boitmanis *et al.* [5] has established an additive approximation algorithm with time bound $O(km)$ with an additive error at most $\frac{2(n-1)}{k+1}$, which leads to an $O(\sqrt{n}m)$ algorithm with at most $O(\sqrt{n})$ additive error. Approximation algorithms use multiple breadth-first searches from k nodes within the graph.

Dijkstra's algorithm [15] can be used to solve the single-source shortest paths problem in time $O(m + n \log n)$ when implemented with Fibonacci heaps.

A.1.1 Changing Distances: Dynamic All-pairs Shortest Path

Most relevant to the graph center problem is the dynamic APSP problem, where vertices are either being added or deleted. An update operation deals with one vertex and any incident edges, an *incremental* operation only involves adding edges and a *decremental* operation only involves deleting edges. The fully dynamic case of APSP is the most challenging, but is beyond the scope of what is required for the reverse centrality query problem.

The fully dynamic APSP problem was addressed in a ground-breaking paper by Demetrescu and Italiano [14] that presents a fully dynamic algorithm based on maintaining locally shortest and historical paths as the graph evolves and achieve updates in $O(n^2 \log^3 n)$.

An exploration of the methods for the incremental and decremental dynamic The APSP problem is relevant to the problem of solving reverse centrality queries. Since we are required to repeatedly re-calculate the vertices with highest centrality as the induced subgraph either grows or shrinks under the searching method. The partially dynamic APSP problem is luckily an easier problem to tackle than the fully-dynamic case.

For the decremental case, Thorup [46] introduces a basic algorithm that handles up to j deletions in $O(j^3 \log j)$ total time. The amortized cost per update for Thorup's method is thus $O(j^2 \log j)$ over j deletions. The decremental algorithm is simplified by the guarantee that any deleted vertex will only increase shortest paths between other vertices whose shortest paths include the deleted vertex along the path.

A simple algorithm for addition updates can be implemented by using Dijkstra's algorithm to calculate the single-source shortest path from the added node to all existing nodes in $O(m + n \log n)$ time and then updating the shortest-path distance matrix in $O(n^2)$ time. This gives an update cost of $O(n^2)$ for additive updates.

Bibliography

[1] Colin Aldridge. The kawachi algorithm: A single-parameter network constructor? In *The 19th Annual Colloquium of the Spatial Information Research Centre (SIRC07)*, 2007.

[2] Lars Backstrom, Dan Huttenlocher, Jon Kleinberg, and Xiangyang Lan. Group formation in large social networks: membership, growth, and evolution. In *KDD '06: Proceedings of the 12th ACM SIGKDD international conference on Knowledge discovery and data mining*, pages 44–54, New York, NY, USA, 2006. ACM.

[3] Albert-Lszl Barabsi and Rka Albert. Emergence of scaling in random networks. *Science*, 1999.

[4] Vladimir Batagelj. Pajek - analysis and visualization of large networks. *Graph Drawing Software*, pages 77–103, 2003. Jnger, M., Mutzel, P., (Eds.).

[5] Krists Boitmanis, Karlis Freivalds, Peteris Ledins, and Rudolfs Opmanis. Fast and simple approximation of the diameter and radius of a graph. In *Workshop on Experimental and Efficient Algorithms (WEA)*, pages 98–108, 2006.

[6] Phillip Bonacich. Power and centrality: A family of measures. *The American Journal of Sociology*, 92(5):1170–1182, March 1987.

[7] Ulrik Brandes. A faster algorithm for betweenness centrality. *Journal of Mathematical Sociology*, 25:163–177, 2001.

[8] Ulrik Brandes and Thomas Erlebach. *Network Analysis: Methodological Foundations*. Springer, March 2005.

[9] Sergey Brin and Lawrence Page. The anatomy of a large-scale hypertextual web search engine. *Comput. Netw. ISDN Syst.*, 30(1-7):107–117, 1998.

[10] Timothy M. Chan. All-pairs shortest paths for unweighted undirected graphs in o(mn) time. In *SODA '06: Proceedings of the seventeenth annual ACM-SIAM symposium on Discrete algorithm*, pages 514–523, New York, NY, USA, 2006. ACM.

[11] Victor Chepoi and Feodor Dragan. Finding a central vertex in an hhd-free graph. *Discrete Appl. Math.*, 131(1):93–111, 2003.

[12] Victor Chepoi and Feodor F. Dragan. A linear-time algorithm for finding a central vertex of a chordal graph. In *ESA '94: Proceedings of the Second Annual European Symposium on Algorithms*, pages 159–170, London, UK, 1994. Springer-Verlag.

[13] Gabor Csardi and Tamas Nepusz. The igraph software package for complex network research. *InterJournal*, Complex Systems:1695, 2006.

[14] Camil Demetrescu and Giuseppe F. Italiano. A new approach to dynamic all pairs shortest paths. *J. ACM*, 51(6):968–992, 2004.

[15] Edsger W. Dijkstra. A note on two problems in connexion with graphs. *Numerische Mathematik*, 1:269–271, 1959.

[16] Tomás Feder and Rajeev Motwani. Clique partitions, graph compression and speeding-up algorithms. In *STOC '91: Proceedings of the twenty-third annual ACM symposium on Theory of computing*, pages 123–133, New York, NY, USA, 1991. ACM.

[17] Gary William Flake, Steve Lawrence, and C. Lee Giles. Efficient identification of web communities. In *Knowledge Discovery and Data Mining (KDD)*, pages 150–160, 2000.

[18] Gary William Flake, Steve Lawrence, C. Lee Giles, and Frans M. Coetzee. Self-organization and identification of web communities. *IEEE Computer*, 35:66–71, 2002.

[19] Linton C. Freeman. A set of measures of centrality based upon betweenness. *Sociometry*, 40:35–41, 1977.

[20] Zhonghua Gao, Zhenjie Chen, Yongxue Liu, and Kang Huang. Study on the complex network characteristics of urban road system based on gis. *Geoinformatics 2007: Geospatial Information Technology and Applications*, 6754, 2007.

[21] David Gibson, Ravi Kumar, and Andrew Tomkins. Discovering large dense subgraphs in massive graphs. In *VLDB '05: Proceedings of the 31st international conference on Very large data bases*, pages 721–732. VLDB Endowment, 2005.

[22] Michelle Girvan and Mark E. J. Newman. Community structure in social and biological networks. *Proceedings of the National Academy of Sciences of the United States of America*, 99(12):7821–7826, June 2002.

[23] Per Hage and Frank Harary. Eccentricity and centrality in networks. *Social Networks*, 17(1):57–63, 1995.

[24] Jiawei Han and Micheline Kamber. *Data Mining: Concepts and Techniques*. Morgan Kaufmann, 2001.

[25] Woochang Hwang, Taehyong Kim, Murali Ramanathan, and Aidong Zhang. Bridging centrality: graph mining from element level to group level. In *Knowledge Discovery and Data Mining (KDD)*, pages 336–344, 2008.

[26] David Jensen and Jennifer Neville. Data mining in social networks. In *Papers of the Symposium on Dynamic Social Network Modeling and Analysis*, Washington, DC, 2002. National Academy of Sciences, National Academy Press.

[27] Donald B. Johnson. Efficient algorithms for shortest paths in sparse networks. *J. ACM*, 24(1):1–13, 1977.

[28] Yuumi Kawachi, Kenta Murara, Shinichiro Yoshi, and Yukinori Kakazu. The structural phase transition among fixed cardinal networks. In *In Proceedings of the 7th Asia-Pacific Conference on Complex Systems*, page 247255, 2004.

[29] David Kempe, Jon Kleinberg, and Éva Tardos. Maximizing the spread of influence through a social network. In *Knowledge Discovery and Data Mining (KDD)*, pages 137–146, New York, NY, USA, 2003. ACM.

[30] Rex K. Kincaid and Oded Z. Maimon. A note on locating a central vertex of a 3-cactus graph. *Comput. Oper. Res.*, 17(3):315–320, 1990.

[31] Jon M. Kleinberg. Authoritative sources in a hyperlinked environment. *J. ACM*, 46(5):604–632, 1999.

[32] D. E. Knuth. *The Stanford GraphBase: A Platform for Combinatorial Computing*. Addison-Wesley, Reading, MA, 1994.

[33] David Krackhardt. Assessing the political landscape: Structure, cognition, and power in organizations. *Administrative Science Quarterly*, 35:342–369, 1990.

[34] Vito Latora and Massimo Marchiori. Efficient behavior of small-world networks. *Phys. Rev. Lett.*, 87(19):198701, Oct 2001.

[35] Chong Liu, Kui Wu, and Jian Pei. An energy-efficient data collection framework for wireless sensor networks by exploiting spatiotemporal correlation. *IEEE Transactions on Parallel and Distributed Systems*, 18(7):1010–1023, 2007.

[36] Nina Mishra, Robert Schreiber, Isabelle Stanton, and Robert Endre Tarjan. Clustering social networks. In *WAW*, pages 56–67, 2007.

[37] Flavia Moser, Rong Ge, and Martin Ester. Joint cluster analysis of attribute and relationship data withouta-priori specification of the number of clusters. In *Knowledge Discovery and Data Mining (KDD)*, pages 510–519, 2007.

[38] Mark E. J. Newman. The structure and function of complex networks. *SIAM Review*, 45(167), 2003.

[39] Mark E. J. Newman. Fast algorithm for detecting community structure in networks. *Physical Review E*, 69:066133, 2004.

[40] R Development Core Team. *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria, 2009. ISBN 3-900051-07-0.

[41] Gert Sabidussi. The centrality index of a graph. *Psychometrika*, 31(4):581–603, December 1966.

[42] Benno Schwikowski, Peter Uetz, and Stanley Fields. A network of protein-protein interactions in yeast. *Nat Biotechnol*, 18(12):1257–1261, December 2000.

[43] John P. Scott. *Social Network Analysis: A Handbook*. SAGE Publications, January 2000.

[44] Raimund Seidel. On the all-pairs-shortest-path problem in unweighted undirected graphs. *J. Comput. Syst. Sci.*, 51(3):400–403, 1995.

[45] Alfonso Shimbel. Structural parameters of communication networks. *Bulletin of Mathematical Biophysics*, 1953.

[46] Mikkel Thorup. Worst-case update times for fully-dynamic all-pairs shortest paths. In *STOC '05: Proceedings of the thirty-seventh annual ACM symposium on Theory of computing*, pages 112–119, New York, NY, USA, 2005. ACM.

[47] Hanghang Tong, Spiros Papadimitriou, Jimeng Sun, Philip S. Yu, and Christos Faloutsos. Colibri: Fast mining of large static and dynamic graphs. In *Knowledge Discovery and Data Mining (KDD)*, 2008.

[48] Stijn van Dongen. Graph clustering via a discrete uncoupling process. *SIAM Journal on Matrix Analysis and Applications*, 30:121–141, 2008.

[49] Stanley Wasserman and Katherine Faust. *Social Network Analysis : Methods and Applications (Structural Analysis in the Social Sciences)*. Cambridge University Press, 1994.

[50] Jennifer Xu and Hsinchun Chen. Criminal network analysis and visualization. *Commun. ACM*, 48(6):100–107, 2005.

[51] Wan-Shiou Yang and Jia-Ben Dia. Discovering cohesive subgroups from social networks for targeted advertising. *Expert Syst. Appl.*, 34(3):2029–2038, 2008.

[52] W. W. Zachary. An information flow model for conflict and fission in small groups. *Journal of Anthropological Research*, 33:452–473, 1977.

[53] Guo-Qing Zhang, Guo-Qiang Zhang, Qing-Feng Yang, Su-Qi Cheng, and Tao Zhou. Evolution of the internet and its cores. *New Journal of Physics*, 10(12):123027 (11pp), 2008.