OPTIMAL TRANSMISSION POLICIES FOR A STAR WIRELESS SENSOR NETWORK

by

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Abstract

Composed of wireless sensors directly or indirectly connected to the information sink, a wireless sensor network is typically employed to survey a coverage area. Sensors directly connected to the information sink form a star structure. Regardless of network topology, the central star structure shoulders the burden of relaying all information that reaches the sink. Elevated traffic in this region increases mutual interference and probability of congestion, which in turn accelerate battery depletion and ultimately limits the total amount of data that reaches the sink.

It is the goal of this dissertation to provide novel insight into factors that affect communication in a star wireless sensor network. The data received at the sink until the last sensor expires, referred to as the *data volume*, is introduced as a suitable performance criterion for networks of limited energy sensors. Data volume takes into consideration both the network capacity and the period in which it is available.

With an information theoretic approach, optimum transmit powers and scheduling policies that maximizes the data volume for a star wireless sensor network are analytical derived. While accounting for sensor processing powers, solutions for both the stationary and time-varying wireless channels are presented. Parallel solutions derived without considering processing power in the sensor model are used to emphasize its importance. The effects of processing power, initial battery energy and network size on the data volume and the activity duration of the network are discussed.

Once the optimum solution is known, various methods are introduced to reduce the computational complexity of calculating it. Less computation allows calculations for larger networks and networks with more energy. Calculation shortcuts do not compromise the optimality of the solution and simulations show their significant effects.

Suboptimal transmission policies that produce data volume close to the optimum scheme but require significantly less computation are also discussed.

Allowing sensors that are not transmitting to enter low-activity cycles is regarded as an energy saving measure. An analytical derivation for the optimum transmission and sleep policy is presented. Simulations show that allowing sleep cycles extends sensor lifetimes and increases data volume.

Keywords: wireless sensor network (WSN); star network; cross layer optimization; processing power; mutual interference; data volume; static channels; time-varying channels; water-filling; computation reduction; suboptimal policies; sleep scheduling

To mum and dad, and Neil :)

"Equations are more important to me, because politics is for the present, but an equation is something for eternity."

- Albert Einstein

In Memoriam Albert Einstein (1956)

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Abbreviations

ADC	Analogue to Digital Converter
ASK	Amplitude Shift Keying
BPSK	Binary Phase-Shift Keying
CDMA	Code Division Multiple Access
CRT	Contention Resolution Tree
CSMA-CA	Carrier Sense Multiple Access with Collision Avoidance
CSMA-CD	Carrier Sense Multiple Access with Collision Detection
CSS	Chirp Spread Spectrum
DARPA	Defense Advanced Research Projects Agency
DSP	Digital Signal Processor
DSSS	Direct Sequence Spread Spectrum
FDMA	Frequency Division Multiple Access
FFD	Full Function Device
FSK	Frequency Shift Keying
GPS	Global Positioning System
HART	Highway Addressable Remote Transducer
IETF	Internet Engineering Task Force
IP	Internet Protocol
IPv6	Internet Protocol version 6
ISM	Industrial, Scientific and Medical
LEACH	Low Energy Adaptive Clustering Hierarchy
MAC	Medium Access Control
MEMS	Micro Electro-Mechanical Systems

MTO	Microsystems Technology Office
OCDM	Orthogonal Code Division Multiplexing
OFDMA	Orthogonal Frequency Division Multiple Access
OSI	Open Systems Interconnection
O-QPSK	Offset Quadrature Phase-Shift Keying
PAN	Personal Area Network
PCP	Personal Computer Peripheral
PHY	Physical Layer
POS	Personal Operating Space
PSSS	Parallel Sequence Spread Spectrum
RF	Radio Frequency
RFD	Reduced Function Device
SNR	Signal to Noise Ratio
SINR	Signal to Interference and Noise Ratio
S-MAC	Sensor-Medium Access Control
SoC	System on Chip
TDMA	Time Division Multiple Access
TSMP	Time Synchronized Mesh Protocol
UWB	Ultra Wide Band
WiFi	Wireless Fidelity
WPAN	Wireless Personal Area Network
WSN	Wireless Sensor Network

Symbols

P	Sensor Power Consumption
P_{tx}	Transmit Power
P_p	Processing Power
T_i	Time Slot i (Stationary Channel)
T	Length of Each Time Slot (Time-Varying Channel)
N	Number of Sensors in the Network
B	Transmission Bandwidth
N_o	Noise Power Spectral Density
x	Ratio of Transmit Power to Processing Power
$x_{i_{opt}}$	Optimum Normalized Transmit Power for Sensor i (Stationary Channel)
$x_{it_{opt}}$	Optimum Normalized Transmit Power for Sensor i Time t (Time-Varying Channel)
γ_p	Fictitious Processing Power SNR
γ	Transmit SNR
$\gamma_{i_{opt}}$	Optimum Transmit Power for Sensor i (Stationary Channel)
$\gamma_{it_{opt}}$	Optimum Transmit Power for Sensor i Time t (Time-Varying Channel)
ξ	Channel Power Gain
z	Fictitious Processing Power SNR at Reception
D	Normalized Battery Energy
E	Initial Battery Energy
L	Maximum Sensor Lifetime (Stationary Channel)
$\mathcal{L}$	Maximum Sensor Lifetime in Time Slots (Time-Varying Channel)
n	Sensor Lifetime (Time-Varying Channel)

C_i	Sum-Rate Capacity for Time Slot i
V_j	Data Volume for a j-Sensor Network
$V_{j_{max}}$	Maximum Data Volume for a j-Sensor Network
$\mathcal{V}$	Data Volume When Processing Power is Not Modeled
$\mathcal{Q}_i$	Set of Sensors that Expire After Sensor i
α_i	Ratio Defined to Simplify Data Volume Representation
G	Normalized Battery Energy When Processing Power is Not Modeled
Loss_j	Per-Sensor Data Volume Loss Due to P_p in a j-sensor Network
μ	Water Level
λ	Water Level When Processing Power is Not Modeled
$\mathcal{U}_i$	Time Slots in which Sensor i Transmits
$\mathcal{I}_i$	Time Slots in which Sensor i Transmits (Processing Power Not Modeled)
P_s	Power Consumption in Sleep State
χ	Ratio of Sleep State Power Consumption and Processing Power
ν	Number of Time Slots the Sensor is Active (Time-Varying Channel)

Chapter 1

Introduction to Wireless Sensor Networks

A wireless sensor is an autonomous node capable of sensing, processing and communicating data. The low-power, low-cost and limited computation properties of these sensors is what sets them apart from other wireless nodes. It is not intended for these sensors to carry high data rates, nor are they meant to perform very complicated data processing, although data rates and costs are terms fairly relative to the current technology. The purpose of a sensor is to collect data from the network coverage area and communicate it to the sink or sinks in the network.

Sensors are distributed in large numbers, usually densely and at random, in an area intended for monitoring, such as a factory or an ocean bed. The sensors autonomously set up a network, gather information, and convey it to an information sink. The sink is not a sensor node itself and is thus not restricted by limited energy or computational power. It can process the information, connect to other networks, or store the information for long periods of time. Fig. 1.1 shows a typical layout for a wireless sensor network.

A wireless sensor network (WSN) is a very general description and covers a broad range of networks that can be deployed in various environments. For example the network can be composed of just a few sensors monitoring a fish tank or it can be composed of hundreds of sensors monitoring a volcano; it can convey any data such

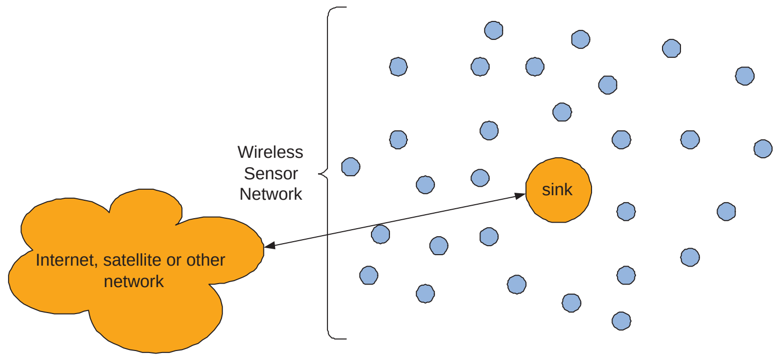

Figure 1.1: Typical layout of a wireless sensor network.

as temperature, speed, location and vibration. The network topology is usually very dynamic since the network may or may not be homogeneous, sensors may move, deplete or be blocked from the network. Some networks are even built to serve more than one purpose; for instance, monitor elements that lead to forest fires and in the event of a fire risk or flare transmit location data and information required for first responders. Different aspects of wireless sensor networks can be used to classify them; examples are, the size of the sensors, the cost per sensor or whether or not the network is homogeneous.

1.1 Early Days

Despite claims that the concept of sensor networks was first born in the 1964 science fiction novel *The Invincible* by Stanisław Lem, the Smart Dust project supported by the DARPA/MTO [1] MEMS program carried out by Dr Kristofer Pister at the University of California Berkeley from 1998 to 2001 was most likely the starting point for sensor networks. The idea behind the Smart Dust project was to build a cubic millimeter sensor that could later be part of a sensor network. The last report on the Smart Dust website states that the final product will be around 4 cubic millimeters and weigh 10 milligrams [13].

TinyOS is the operating system designed for the Smart Dust project which is

[1]Defense Advanced Research Projects Agency/Microsystems Technology Office

based on an event driven programming model, meaning that when an event occurs TinyOS reacts by calling a module to handle the event. The outcome of the Smart Dust project was later commercialized through Dust Networks Inc. where Dr Pister is the Chief Technology Officer. Some of their recent products will be discussed in Section 1.5.

The Smart Dust project ignited the idea of a network of small nodes. Although it was mainly intended for military purposes, it did not take long for researchers to realize that the flexibility of these networks allows a wide range of civilian applications. This led to big companies such as Intel and BP investing in wireless sensor research; and the next generation of sensor networks, employed in everything from agriculture to monitoring gas pipes, were developed. Some of these applications will be discussed in Section 1.4.

1.2 Wireless Sensor Networks vs. Ad Hoc Networks

The closest wireless networks to sensor networks are ad hoc networks. In the early days, researchers worked on extending algorithms and ideas from ad hoc networks to sensor networks. However, there are major differences between these two types of wireless networks and in many cases sensor networks require new solutions.

The most significant difference between sensor networks and ad hoc networks is that while ad hoc networks move information from any node to another, the final aim of all communication in a wireless sensor network is to get the information to the sink, or sinks, in the network. This in turn creates a strong directional flow of data in sensor networks and presents congestion issues that are not widely experienced in ad hoc networks.

The limitations imposed on sensors in terms of cost, power, memory and computation set them apart from the nodes in an ad hoc network. A sensor should be cheap to manufacture, since sensors are deployed in large numbers, and it should be energy efficient because the battery on a sensor is usually not replaced or recharged and the network should be able to fulfill its duty using the batteries available on the sensors. This combination of restrictions rarely applies to ad hoc networks; the nodes in an ad hoc network are usually precious enough to be built so the battery can be recharged and/or replaced allowing the node a theoretically infinite lifetime.

Sensor networks are generally composed of many more nodes than ad hoc networks. The impact of this is that it may no longer be practical for one sensor to keep track of the location of all the sensors in the network, as is the case with nodes in ad hoc networks. The larger size of the network necessitates more localized control in sensor networks and also suggests that scalability in the protocols and algorithms designed for these networks is an important issue. In [62] the author argues that power, bandwidth and computation limitations, and the dynamic nature of sensor networks, require the design of network layer protocols that meet certain criteria under a general localized paradigm.

Another distinction between sensor networks and ad hoc networks is that sensor networks are more prone to failures. Environmental changes or discharge of a sensor's battery may happen at any time and the network should be designed to continue working despite these problems. In other words, the sensor network does not rely on any single node, it is the connectedness that is important. As long as data can still be gathered and relayed to the sink, the sensor network will remain functional despite sensor failures.

As discussed before, the low cost, low energy consumption and flexibility of sensor networks opens up many possibilities in terms of applications. Of all the wireless networks, sensor networks are employed in the most diverse application scenarios.

Since sensor networks and ad hoc networks both transmit over a wireless channel where propagation characteristics are uncertain and variable with time, they do share some properties. The strength or quality of the wireless communication link may change with subtle changes in the environment or location and its integrity is affected by many factors based on where the network is deployed. A sensor network monitoring a fish tank will experience channel variations based on the location of the fish in the tank, but few other factors will influence the channels. On the other hand, a sensor network used for monitoring the migration path of fish in their natural habitat will experience channel variations due to many more factors, e.g. vessels passing by, changes in the riverbed, mud slides, etc.

1.3 The IEEE 802.15.4 Standard

The IEEE 802.15.4 standard, known as the low-rate short-range wireless personal area network (WPAN) standard, provides guidelines for the implementation of the Physical (PHY) and Medium Access Control (MAC) layer of sensor networks. It defines protocols for short-range radio frequency communication for low-rate, low-power and low-complexity nodes [57]. In this section the IEEE 802.15.4 is referred to as "the standard".

The standard was first drafted in 2003 and then revised in 2006. The revision was designed to clarify the standard in hopes that industrial alliances such as ZigBee would lean more towards following it. In 2007 an amendment was made to the IEEE Std 802.15.4-2006 which specified alternate PHY layers to those already specified [58]. Table 1.3 provides some information about the channels defined in the standard and the amendment. It however excludes the UWB and CSS bands since the chip rates and symbol rates can not be displayed in a concise form. This information can be found in [58].

Two other amendments for recognizing alternative PHY layer bands have since been discussed, one to support one or more of the Chinese 314-316 MHz, 430-434 MHz, and 779-787 MHz bands and another to support the Japanese 950 MHz band. Both drafts have yet to be approved by the IEEE 802.15 working group.

In the subsequent text some of the specifications of the low-rate short-range WPAN standard are discussed.

Channels

16 channels in the 2450 MHz band (worldwide), 30 channels in the 915 MHz band (North America), and 3 channels in the 868 MHz band (Europe) are the options available for transmission in the 2006 revision of the standard. In all geographic

regions these channel fall within the unlicensed frequency range. The 2007 amendment adds 14 overlapping chirp spread spectrum (CSS) channels in the 2450 MHz band, and 16 channels in three UWB bands (500 MHz and 3.1 GHz to 10.6 GHz).

Channel Access

The 2006 revision of the standard suggests carrier sense multiple access with collision avoidance (CSMA-CA), discussed in Section 2.3.1, as means to access the multiple access channel. The 2007 amendment adds ALOHA as an option for channel access in ultra wide band (UWB).

Data Rates

The 802.15.4 standard was amended to present 5 different communication rates for the sensor network, 851 Kb/s, 250 Kb/s, 110 Kb/s [2], 40 Kb/s, and 20 Kb/s. As the name suggests these rates are low compared to the usual Megabit data rates in, for example, the 802.11 standard. The CSS frequencies work with a slightly higher data rate of 1 Mb/s and allow an optional rate of 250 Kb/s. The UWB frequencies also allow optional data rates of 110 Kb/s, 6.81 Mb/s, and 27.24 Mb/s.

Fig. 1.2 shows data rates for the WSN standard in comparison with other wireless IEEE standards that operate in the ISM (Industrial, Scientific and Medical) band.

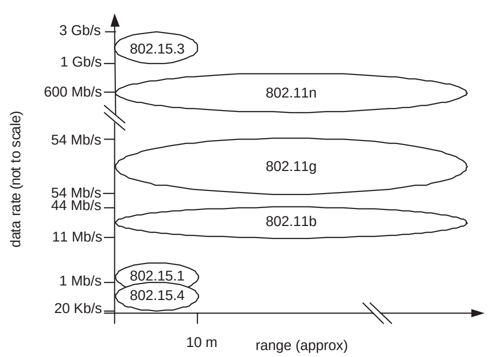

Figure 1.2: Data rates for IEEE standards operating in the ISM band.

[2] This rate was initially defined as 100 Kb/s in the standard but was amended to 110 Kb/s.

Modulation and Spreading

The standard requires that all devices have the ability to operate in the 868/915 MHz BPSK band. Direct Sequence Spread Spectrum (DSSS) is applied to the bits before modulation. The spreading code, pulse shape and power spectral density limits are also described in the standard.

The optional PHY layers in the 868/915 MHz band employ Orthogonal Code Division Multiplexing (OCDM), which is also referred to as Parallel Sequence Spread Spectrum (PSSS), with ASK modulation or 16-ary Quasi-Orthogonal spreading and O-QPSK modulation. The 2400 MHz band uses the latter for transmission.

Allocation	Freq. Band (MHz)	Data Parameters			Spreading Parameters		# of Channels
		Bit Rate (Kb/s)	Symbol Rate (Ksym/s)	Symbol	Chip Rate (Kchips/s)	Modulation	
868/915 MHz mandatory	868 - 868.6	20	20	Binary	300	BPSK	1
	902 - 928	40	40	Binary	600	BPSK	10
868/915 MHz optional	868 - 868.6	250	12.5	20-bit PSSS	400	ASK	1
	902 - 928	250	50	5-bit PSSS	1600	ASK	10
868/915 MHz optional	868 - 868.6	110	25	16-ary Orth.	400	O-QPSK	1
	902 - 928	250	62.5	16-ary Orth.	1000	O-QPSK	10
2450 MHz	2400 - 2483.5	250	62.5	16-ary Orth.	2000	O-QPSK	16

Table 1.1: Frequency bands and data rates for the IEEE 802.15.4 standard (excluding the UWB and CSS bands) [57, 58]

Transmit power

The standard states a maximum transmit power for sensors. For the 900 MHz and 2400 MHz bands that are used in North America this maximum is 1000 mW. The minimum transmit power is assumed less than or equal to -32 dBm. The standard anticipates that devices will operate with transmit powers between -3 dBm and 10 dBm, with 0 dBm being typical. The range for these devices is estimated at around 10m.

Devices

Two types of devices are recognized in the standard, the full function device (FFD) and the reduced function device (RFD). The standard defines a wireless personal area network (WPAN) as two or more devices in a personal operating system (POS) that operate in the same physical channel where at least one of the devices is an FFD. A full function device can communicate with another FFD or an RFD; however, a reduced function device can only communicate with an FFD - note that the sink can also be classified as an FFD in this regard. A reduced function device is only intended for very simple applications.

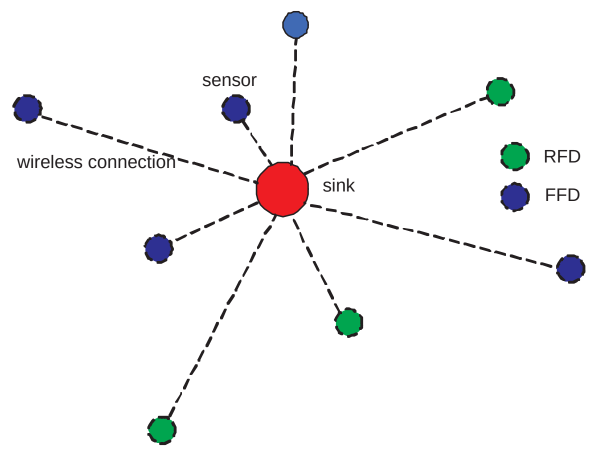

Figure 1.3: Star topology as recognized by the IEEE 802.15.4 standard.

Network structure

With the FFD and RFD the standard recognizes two network configurations, the star layout illustrated in Fig. 1.3, where sensors are connected to the sink and there is no connection between them, and the peer-to-peer connection where sensors are connected to other sensors and the sink, Fig. 1.4. In the standard, the information sink is called a personal area network (PAN) coordinator.

The standard suggests that applications such as home automation, personal computer peripherals (PCPs), toys and games, and personal health care will most likely benefit from the star topology. The peer-to-peer connection topology is slightly more complex; the standard suggests applications such as industrial control and monitoring, asset and inventory tracking, intelligent agriculture, and security.

Architecture

Of the seven layers of the open systems interconnection (OSI) model, revisited in Section 2.5, the IEEE 802.15.4 standard describes the first two layers: the PHY layer and the MAC layer. The PHY layer is the link between the MAC layer and the radio channel. The duties of the PHY layer are to select frequency channels, assess whether the channel is clear for transmission in the CSMA-CA protocol, measure energy and link quality, and transmit and receive packets. The MAC layer manages communication and security.

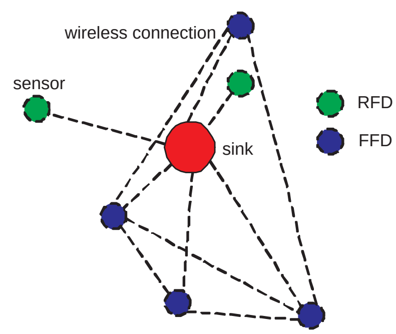

Figure 1.4: Peer-to-peer topology as recognized by the IEEE 802.15.4 standard.

1.3.1 Coexistence: Spectrum and Interference

The 802.15.4 standard features an appendix on the coexistence of the WSN standard and other IEEE wireless standards. The document discusses coexistence issues between the 802.15.4 standard and the 802.11b, 802.15.1 and 802.15.3 standards in the 2400 MHz ISM band. The conclusion is that from the view point of a 802.15.4 device the relatively wideband signals from the 802.11b (WiFi) and 802.15.3 devices are detected as white noise. On the other hand, interference from a 802.15.1 (Bluetooth) device looks like a narrowband signal to the wider spectrum of the 802.15.4. It is also worth noting that since the 800/900 MHz bands in the standard employ DSSS, they are fairly immune to interferers.

From the view point of other standards, the power spectrum of a 802.15.4 device only partially overlaps with the 802.15.1 spectrum. With the frequency hopping exercised by Bluetooth devices interference is only visible in about 4% of the frequency hops. Owing to the wider spectrum of 802.11b devices, a 802.15.4 device looks like a narrowband interferer.

[49] employs measurements from a 802.15.4 device to calibrate simulations and provide more accurate results. With two 802.15.4 devices and two 802.11b/g devices placed diagonally with a distance of 5m the authors report that there should be a minimum of 7 MHz separation between the operating frequencies of the devices to allow for a frame error rate close to zero.

1.3.2 Other Influential Bodies

Parallel to the IEEE, a few other groups have produced guidelines and/or standards for wireless sensor networks. The major players are introduced below.

ZigBee

Zigbee is an international commercial alliance based on the IEEE 802.15.4 standard. Many of the big names in electronics and wireless devices such as Philips, Samsung, Texas Instruments, GE, LG, Honeywell, Siemens and even British Columbia's BC Hydro are members of this group. The main goal of this alliance is to promote their open standard, build it to be the market leader, and control the evolution of the wireless sensor technology. The alliance has also produced white paper, testing and measurement reports on the coexistence of ZigBee and other wireless technologies to encourage adoption of the standard.

IETF

The Internet Engineering Task Force (IETF) is a web-based open group founded in 1986. Their website describes their field of work as *evolution of the Internet architecture and the smooth operation of the Internet*. They formed an active working group on the Internet Protocol version 6 (IPv6) over low-power WPANs in 2007. This protocol is designed to work over the IEEE 802.15.4 MAC layer and is the best candidate to provide means by which wireless devices with different standards can communicate.

HART Communication Foundation

HART (Highway Addressable Remote Transducer) is a master-slave communications protocol, designed based on the Bell 202 Frequency Shift Keying (FSK) standard, for wired devices which allows simultaneous transmission of digital and analogue signals. The slave is polled by the master device and can only communicate in response. The protocol allows a maximum of two master devices in the system but multiple slaves. This protocol has been around since the 1980's but has be revised and updated to accommodate the needs of today's industry. The HART communication foundation is the technology owner and standards setting body for the HART Protocol [15].

The wirelessHART device, based on the HART 7 standard released in 2007, operates on the 2.4 GHz band. HART 7 employs IEEE 802.15.4 radios, DSSS and channel hopping over the 16 channels defined in the standard. To avoid collisions in transmission, communication between devices is based on Time Division Multiple Access (TDMA). The controller (master) is connected to the wirelessHART gateway (network sink) through which it polls the wirelessHART device. The network is structured to allow for a mesh-like configuration that relays information from any device to the gateway.

HART 7 is designed to be backward compatible, meaning that wired HART devices can be retrofit with a wirelessHART adapter and operate as a wirelessHART device. The standard claims that it can coexist with wireless devices even those that do not comply with any of the 802 standards. Many of the big sensor network manufacturers such as Crossbow, Dust Networks and Honeywell are members of the foundation and produce HART-compliant devices.

IO-Homecontrol

IO-Homecontrol with the slogan "*your home takes care of you*" encourages homeowners to install a wireless sensor network in their homes for comfort, security and energy saving. The group is an alliance of ASSA ABLOY, one of the major market share holders in door opening solutions, Honeywell, which alone covers more than 100 million homes and buildings worldwide, Hrmann, a building products company, and a few other firms. IO-Homecontrol accounts for more than 50% of the market share in home control devices [12].

1.4 Applications

Since sensors are so flexible in the type of data they can manage, wireless sensor networks are employed in a wide scope of applications. Some of the many ways sensor networks are utilized are introduced in this section.

Surveillance

As mentioned before wireless sensor networks were initially studied for military purposes, specifically, monitoring friendly and enemy troops and equipment. Since they can perform as event driven devices they can operate with little power and avoid being detected. Today many personal or vehicle surveillance systems employ wireless sensor networks.

Wildlife and Environmental Monitoring

One of the first habitat monitoring projects implemented with WSNs was the Great Duck Island project [43]. In monitoring birds, it is crucial that there be no disturbance to their nests. Being both small and autonomous, sensor networks were the best option for this project. Data was collected and reported to researchers in real time via the internet.

Sensor networks have been used in many geophysical monitoring projects. One experimental deployment was the monitoring of an active volcano in Ecuador [71]. During a 19-day period the sensor network relayed time-stamped topology information, along with humidity, temperature, light and voltage values and recorded 229 earthquakes, eruptions, and other seismo-acoustic events. Their main aim was to evaluate how suitable WSNs are for geophysical applications. They found that timing was a challenge and anticipated that with longer deployment periods providing adequate battery life will also be difficult. Despite these issues the team concluded that sensor networks, once tailored more towards their application, are very suitable for volcano monitoring.

In other projects sensor networks have been used for tracking wildlife , cattle [52] and fish, monitoring trees [67], early flood [3] and forest fire detection, and precision farming. [50] provides some early applications for sensor networks. The reader is reminded that this paper was published in 2004 and details such as the size and cost of the sensors have improved greatly over the years.

Healthcare

Equipping patients with health monitoring sensors will allow them to move around in the hospital, care facility or their home more freely. In addition, in case of an emergency the location and the condition of the patient can be known. The term *body sensor networks* is used to refer to such wearable wireless sensors.

While the sensors that make up the sensor network are implanted under the skin or fixed on the body, the information sink is not necessarily located on the body. For example, readings of a patient's blood pressure and blood glucose levels can be collected every time they walk past a certain point in the facility.

The sensor network can also control the delivery of drugs and their effects. Devices for managing acute diabetes, treating epilepsy and monitoring cardiac diseases are already in use. The same technology can be used for monitoring equipment and devices in hospitals and labs.

"Smart pills" are an example of a small but extremely important and lucrative sensor network. Proteus, a start-up company, has developed pills that can relay data to physicians after the patient has taken the pill [2]. Stomach fluids activate the communication device which transmits data to a body patch worn on or just underneath the skin. The patch then passes the data to the sink. The sink can be a cell phone (technically infinite energy since it is rechargeable) or any internet portal. These pills record time, adverse drug reactions, etc. It is estimated that hospitalizations due to not following prescribed drug dosages costs $100 billion a year in America.

Industry

Employing sensor networks to control the product quality in manufacturing plants and the operation of machinery in all industries can significantly improve the final product and reduce costly downtime. In addition, information gathered from the performance of machinery can be used to schedule maintenance more efficiently and produce statistics for the developer.

Many industries such as automobile manufacturers already employ a large number of sensors in their plants, many are even connected wirelessly. The ability to wirelessly convey strategic information along the production line helps optimize operations in logistics, supply chain and manufacturing.

Mining and Energy

The ability to monitor and control devices remotely eliminates the need for worker presence in many cases. In dangerous work environments such as mining and oil fields this can significantly reduce workplace related accidents. In addition, sensors installed in clothing can monitor environmental conditions and issue warnings when necessary.

Home Automation

Smart home devices are not a very recent development. For years home appliances and garage doors have been built with sensors to improve the consumer experience. However, networking the sensors in these smart devices and allowing them to control the home environment is a new concept. Automated homes are more efficient in energy management and continuous surveillance of the appliances, water, heat, humidity and gas systems ensures a longer lifetime for those systems and protects the habitants. Home entertainment has also become much more integrated with the introduction of sensor networks. Many gaming devices are also developed using sensor networks. Alliances such as ZigBee and IO-Homecontrol, discussed in Section 1.3.2, are now pushing markets towards automated homes.

Traffic

Vehicle, passenger and road safety are all areas that have greatly benefited from the development of WSNs. Bridges, highways, cross roads and traffic lights are now monitored with these systems. Public transit systems and emergency departments utilize sensor networks to clear the path for their vehicles. Novax is a commercial company located in Delta, British Columbia that produces transit priority systems and traffic controllers based on wireless sensor networks

1.5 Commercial Products

Sensor networks have now moved from the research phase to commercial applications. More and more of the applications introduced above are developed into products and offered for daily use. Below, a few producers of sensor networks along with market trends are discussed.

1.5.1 Manufacturers

With the growth in market size, more and more companies are entering the wireless sensor market. A few companies are introduced below.

Crossbow

Crossbow was one of the first companies to produce sensor networks. MICA2 in the 868/916 MHz band, MICAz in the 2400 MHz and the Imote2 designed by Intel that also functions in the 2400 MHz band are the company's better known products and are ZigBee compliant. The Mica motes, produced in earlier years, are about the size of a matchbox, have a range of about 100m outdoors and are claimed to form large scale networks of more than one thousand nodes. Imote2, being a later development, is 36mm $\times$ 48mm $\times$ 9mm, has a line of sight range of about 30m and is geared towards digital image processing, general monitoring and seismic and vibration monitoring. The RF power for these motes is -20 dBm to 0 dBm. Most of the products available by

Crossbow are designed to function as plug and play devices which makes it convenient for the end user.

Crossbow has also developed an open, integrated, standards-based software platform called MoteWorks, currently at version 2.0. The software allows choices to be made for topology, power profile and bandwidth. The company entered precision agriculture with the introduction of its newest product, eKo Pro, last year. The information gathered by eKo Pro is browsable through an internet interface.

Dust Networks

Although it seems unlikely that the predictions Dr Pister made after finishing the Smart Dust project about the size and cost of sensors in sensor networks in 2010 will become a reality any time soon, the company founded based on the outcome of the project, Dust Networks, has done well. Their earlier products, such as the SmartMesh-XD, were based on a protocol they designed for their mesh systems called the Time Synchronized Mesh Protocol (TSMP) and functioned in the 900 MHz and/or 2.4 GHz 802.15.4 standard bands. Currently, the company produces wireless sensor mesh networks that are compatible with the wirelessHART standard, Section 1.3.2. The size of these devices is small, a few centimeters long and wide and only a few millimeters thick, and they are claimed to last for a decade on two AA batteries. The company is currently testing sensor networks with IETF compatible Internet Protocols

Texas Instruments

Texas Instruments established its line of Chipcon products for short range low-power ZigBee compliant devices. The first product of this line, the CC2420, was the first ZigBee compliant 2.4 GHz single-chip RF transceiver. They later developed the CC2431 which is a system-on-chip (SoC) sensor with a location engine. It estimates the location with a resolution of 0.5m using 3 to 8 reference nodes. The location error is less than 3m and it takes less than 40μs. In current literature, accurate position information is regularly assumed and is generally obtained via GPS (Global Positioning

System). Placing GPS devices on sensors causes significant increase in the cost and battery drain. Chipcon products look like a reasonable low power alternative to GPS.

Others

Sensor Wireless Inc. (a Canadian company), Electrochem, Arch Rock and Ember are a few of the other major players in the production of sensor networks.

1.5.2 Market Interest

Google Trends and Google Insights for Search are tools powered by Google that allow users to browse the frequency with which a word or strings of words[3] is searched. The numbers that appear as search traffic for a word or phrase in each geographical area are equal to the number of searches for the word or phrase in that area normalized by the total traffic. Geographical areas are determined by IP (Internet Protocol) addresses. In Google Insights for Search the data is scaled to fit in a range of 0% - 100% and in Google Trends it is scaled so the average is 1.0. In presenting the information considerations such as discarding repeat searches from a single user over a short period of time to eliminate artificial impacts have been made. The information is currently updated daily.

In [1] the author refers to a recent article published on the official Google research blog, [5], that claims there is a correlation between the number of web searches for a service or product and the relevant economic statistics. The Google research article coauthored by Hal Varian, Google's chief economist, concludes that including relevant Google Trends variables in prediction models improves the accuracy of economic statistic predictions, in some cases up to about 20%. Economic statistics such as sales and sales trends are available with a time lag; however, precise estimates of current values would be invaluable to industries. For this reason the information made available through Google Trends and Google Insights for Search is receiving much attention.

[3] In the case of a string of words, all combinations of the phrase are included in the count unless otherwise instructed.

There is very little information available to the public on market shares and company sales, especially broken down in terms of specific products. Thus, based on the correlation between web searches and market interest, in this dissertation the data[4] from Google Insights for Search is used as a general trend illustrator for sales in wireless sensor networks. However, web search numbers are used only as a general indication of interest and are interpreted with great caution.

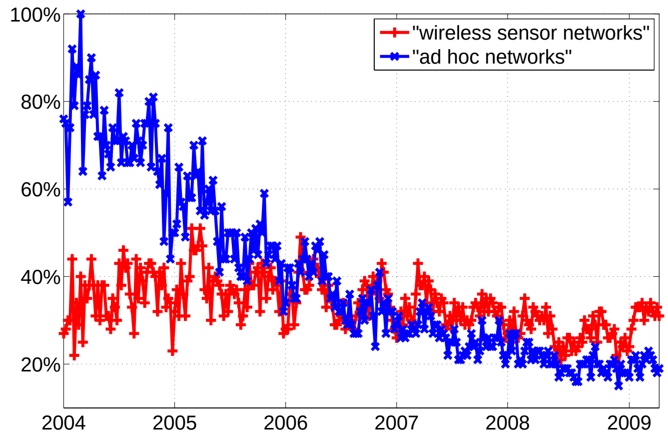

Figure 1.5: Relative search frequency for phrases *wireless sensor networks* and *ad hoc networks* based on data from Google Insights for Search. Data for both terms is normalized by total search traffic in each geographical area and then scaled to a range of 0% and 100%. The average is 36% for *wireless sensor networks* and 43% for *ad hoc networks* over this period.

Fig. 1.5 shows the search frequency for the two word strings *wireless sensor networks* and *ad hoc networks* based on data from Google Insights for Search. Note that the two curves should only be compared in terms of popularity in search phrases and not as absolute traffic. In other words, the downward trend for *ad hoc networks* doesn't mean the absolute traffic for this phrase has decreased but that its popularity has gone down. As the graph illustrates, the interest in *wireless sensor networks* has held a fairly steady value, which indicates it is as popular as before. Since the total

[4]Based on the Terms of Use for Google Trends and Google Insights for Search data from these tools may be used for educational and research purposes.

search traffic is ever increasing, we can conclude that interest in sensor networks has increased at almost the same rate.

To give a better perspective on the search popularity of these queries, their search frequency is compared to a few more familiar terms in Table 1.5.2. For this table Google Trends is employed. The average search frequency of *wireless sensor networks* is set to 1.0 and other averages are scaled accordingly. The data is for the time period January 2004 to May 20, 2009.

Word or Phrase	Scaled Search Average
"wireless sensor networks"	1.0
"wireless networks"	4.65
sensor	92.0
802.15.4	0.5
802.11	15.6
"ad hoc networks"	1.22
loan	268
"stanley park"	3.0
"maple tree"	3.5
canucks	13.0

Table 1.2: Average relative search frequency scaled so that the average for the phrase *wireless sensor networks* is equal to 1.0 for the period January 2004 to May 20, 2009. Source: Google Trends.

It is reminded that the popularity of a search query is to some level correlated to the market interest and the numbers produced here should only be interpreted with that caution.

1.6 Dissertation Outline

This dissertation embodies the following contributions:

Chapter 2 After presenting sensor, network and channel models a new performance criterion tailored specifically towards sensor networks is introduced.

While extending many algorithms and ideas from other networks to wireless sensor networks, in most previous work, the performance criterion has also been borrowed. Since sensor networks represent a new and unique set of properties, it is reasonable to design a performance evaluation metric specific to them.

Chapter 3 The analytical derivation of the optimum transmission policy (transmit power and schedule) for a star wireless sensor network with static channels is presented. Optimization is performed based on the novel criterion presented in Chapter 2.

Chapter 4 A more complex and more realistic channel model is introduced. The optimum transmission policy for a star sensor network with time-varying channels is analytically derived.

Chapter 5 Computation and organization solutions are introduced to eliminate performing tedious and unnecessary calculations in determining the optimum solution for a star wireless sensor network with time-varying channels. It is mathematically shown that the shortcuts in computation do not compromise the optimality of the outcome.

Chapter 6 This chapter embodies variations to the optimum transmit policy. In the first part of the chapter suboptimal substitutes for the optimal transmit policy produced in the previous two chapters are introduced. These suboptimal methods perform within a close margin of the optimal scheme and require less calculation. The methods are attractive in systems where optimality can be sacrificed for computational simplicity. After all, one of the most important roles of optimal solutions is providing performance benchmarks for evaluating suboptimal solutions.

In the second part of the chapter, to conserve energy, sensors are instructed to enter low-activity states when not transmitting. The optimum transmit policy with low-activity states and the effects of such a scheme on the network performance and sensor lifetimes are discussed.

Chapter 2

Models, Structures and Performance

The characteristics of a network depend on the nodes that construct it. In this chapter the building blocks of a sensor network, the sensors, the many ways they can be connected to form a network, the wireless channel they transmit over and the criterion by which their performance is judged are reviewed.

2.1 The Sensor

A wireless sensor, also referred to as a mote or a sensor node, has four major components: power, ability to sense, to process and to communicate. Fig. 2.1 illustrates these components and their relation.

2.1.1 Power

Since sensors are generally battery powered and very few are equipped with rechargeable batteries, low power consumption is an important characteristic of both sensors and their wireless networks. This property sets them apart from many other wireless nodes.

Sensor power consumption, P, is calculated as the sum of the all power used for

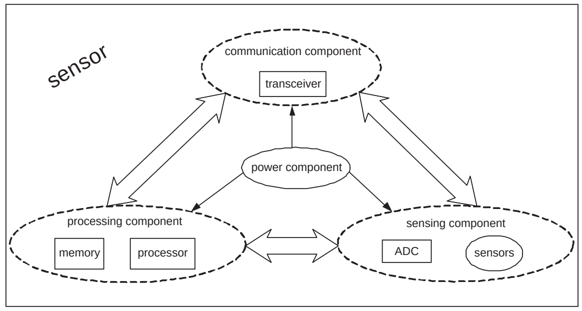

Figure 2.1: Major sensor components: power, sensing, processing and communication.

sensing, processing and communication. For modeling purposes, in this dissertation, the power consumed for transmission, P_{tx}, is separated from other sources of power consumption. The power used for sensing, processing and signal reception is lumped together and termed processing power, P_p.

$$P = P_{tx} + P_p \tag{2.1}$$

Processing power is consumed even when there is no transmission; thus the battery power drain is offset by the processing power. Fig. 2.2 illustrates this effect.

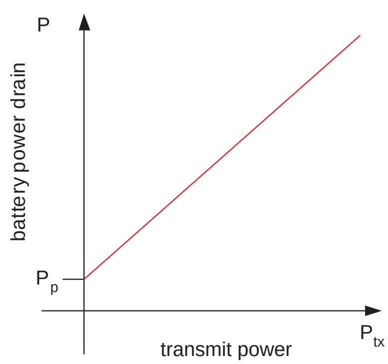

Figure 2.2: Power consumption model for sensors. The battery drain is equal to the sum of the transmit power and the processing power, $P = P_{tx} + P_p$.

In much of the literature related to sensor networks an assumption is made that processing power is a small fraction of the sensor power consumption and can therefore be omitted from the sensor model. However, the effect of discarding the processing power can be substantial since, as discussed in [20], processing power can be a rather significant and sometimes the dominant portion of battery drainage. The effect of eliminating processing power from the sensor model on determining optimal transmission policies is discussed in Chapters 3 and 4. For more details on power consumption measurement and modeling the reader is referred to [32].

Unlike the sensors, the sink or sinks in a wireless sensor network are connected to an infinite source of power. Since battery drainage is not an issue in these devices, their power consumption pattern does not affect the network performance (irrespective of the criterion) and is therefore not discussed.

2.1.2 Sensing

Sensors, i.e. the sensing devices, usually produce an analog voltage output which is a function, but not necessarily a linear function, of the measured phenomenon. They can measure anything from temperature and gas density to vibration and tension. The analogue signal is then digitized by an Analogue to Digital Converter (ADC) and forwarded to the processor, refer to Fig. 2.1.

Common sensors work with voltages in the range of 2 V - 12 V, draw a current in the range of 0.5 mA - 2 mA and have sampling periods of 30 μs to 400 ms. Since sensor nodes in the sensor network are to be small, cheap and low-powered, these properties should be taken into account when selecting a sensing device for the sensor node. As noted before, the wide range of sensors available allows a broad scope of applications for sensor networks. [65] provides a detailed list of the sensors currently available off-the-shelf.

The coverage area of the sensor, the area in which it can accurately record the phenomenon of interest, defines a minimum node density. However, the coverage area of the sensing device is not the only factor influencing the minimum node density of the network. The data accuracy required, based on the reliability of sensors, may necessitate redundancy in data collected. And of course, the range of the wireless transceiver installed on the sensors defines yet another restriction on the maximum separation between sensors.

2.1.3 Processing

Microcontrollers seem to be the choice of processors for sensor networks. Low power consumption is the selling point for these devices. In selecting the processor it should be noted that a sensor node is not expected to handle very complex or high rate data processing, so for most applications a DSP would be excessive.

2.1.4 Communication

The communication standard for sensor networks was studied in Section 1.3. The standard defines frequency bands and data rates as well as signal spreading and modulation techniques. The power consumed for communication, which constitutes a fair share of the sensor power expenditure, also depends on the range of communication and the power efficiency of the devices in use. As discussed above, since sensors should be within the signal range of one another to maintain the connectivity of the network, the coverage area of the transceiver will limit the maximum separation between sensors and is therefore a constraint on the minimum node density.

2.2 Network Structures

Once sensors have been deployed in the field, they are connected via a wireless network. Based on the sensors, networks are divided into two categories: homogeneous networks and heterogeneous networks. The nodes in a heterogeneous network may or may not be different in physical structure. In other words, heterogeneity many refer to the duties of sensors in the network; for example hierarchical networks, introduced below, are heterogeneous networks. Homogeneous networks are employed in presenting design guidelines for sensor networks in [63].

The IEEE 802.15.4 standard recognizes two types of networks: the star network and the peer-to-peer network, refer to Section 1.3. This categorization is based on the type of connections between the sensors which in turn determines the network topology. Similar to the WSN standard, in this dissertation, networks are divided into two categories based on their topology, star networks and mesh networks.

2.2.1 Star Networks

A star network consists of sensors that are directly connected to the sink. There is no communication among the sensors although their transmissions may interfere. A star network topology can consist of homogeneous or heterogeneous sensors. Fig. 1.3 illustrates a star sensor network.

2.2.2 Mesh Networks

A mesh network implies that there is at least one sensor-to-sensor, or as the standard calls it peer-to-peer, connection in the network. Similar to a star configuration, a mesh network can consist of homogeneous or heterogeneous nodes. A typical mesh network need not have any specific structure; so any sensor can be connected to any other sensor or the sink or sinks in the network. However, with a large number of sensors, it is usually desirable to structure the network to determine the best path for the flow of information and design energy efficient routing algorithms. Fig. 2.3 shows a mesh network and Fig. 2.4 shows a structured mesh network.

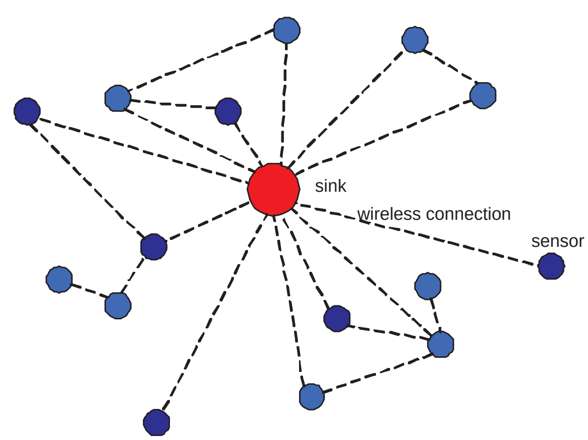

Figure 2.3: A wireless mesh sensor network.

Structured mesh networks are arranged as multi-hop or clustered networks. In a multi-hop network each sensor passes its message to the next sensor which relays it to the next and so on until the information reaches the sink. The pathway that the sensor signal travels to the sink is the subject of much research and debate with respect to conservation of power and reliability of data transfer. In simplistic terms the signal can either travel short hops between sensors in the immediate vicinity or long hops between sensors further apart. In the latter case the signal takes fewer jumps to get to the sink, but at what price?

With shorter hops less transmit power is used per participating sensor; however, more sensors are involved compared with the longer hop approach. Also, with the shorter hops and consequently lower transmit powers fewer sensors are affected by transmissions from each sensor. On the other hand, with longer hops fewer sensors are required for transmission so, depending on conditions, the total network power expenditure to deliver messages may be less. And, with fewer sensors involved there is less probability of failure.

[21] lists 18 reasons why the decision between long hops and short hops is not as simple a decision as it is assumed in many publications. [56] discusses the optimum number of hops for various SNR values assuming a single source, a single sink and all other sensors (or more generally nodes) acting as relays placed on a line with equal distances between them. With this layout and a TDMA transmission schedule, they search for the number of hops between the source and sink that would require the minimum total transmit power and conclude that in some cases taking fewer longer hops is possible with less total power expenditure. In search of longer hops, [19] computes the maximum transmission distance possible while staying below a specified outage probability.

Clustering, on the other hand, is much better suited to sensor networks with more sensors. Since sensors are meant to operate with low power, in many cases it is efficient to run localized algorithms for gathering data from a certain area in the network and then transmitting the aggregated data from that cluster of sensors on a path towards the sink.

In a clustered network, some sensors only communicate within the cluster and some communicate with sensors both inside and outside the cluster or the sink; hence the network is considered heterogeneous. Using clustering allows for better resource management and power control and is thus employed in many other wireless networks such as cellular systems [36] and ad hoc networks [61]. A few examples of methods available for clustering in wireless sensor networks can be found in [22, 33].

Hierarchical Clustering

There are many ways to cluster sensors, connect the clusters, and direct the information to the base. One of the more recent algorithms developed for sensor networks is Low Energy Adaptive Clustering Hierarchy (LEACH) [23]. In this proposed algorithm sensors are clustered around a "*cluster head*" and so form a two-level hierarchical structure. The cluster head gathers and aggregates all the data from the sensors in the cluster then transmits them directly to the sink. Fig. 2.4 is an example of a network structured by LEACH.

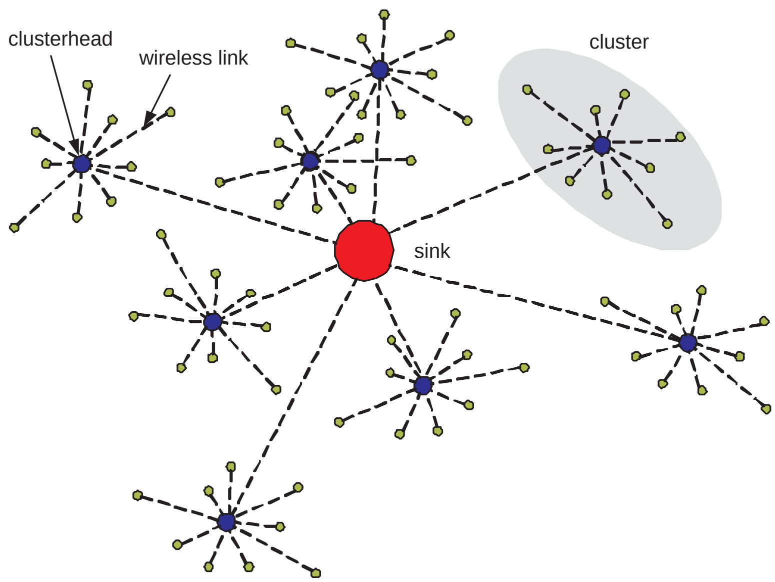

Figure 2.4: A hierarchical wireless mesh sensor network.

The cluster head is critical for cluster operation and cluster failure compromises the network; so it has been argued that the whole network integrity is determined by the first cluster head to fail. Furthermore, the cluster head is more likely to deplete as its role is more power intensive compared to neighboring sensors. The authors in [23] also propose cluster head rotation to prevent draining one sensor. It is shown in [60] that unequal clusters, referring to the number of sensors in each cluster, may be a good solution to a more uniform energy consumption pattern in the network.

2.3 The Wireless Channel

The wireless channel limits the range, volume and speed at which information can be communicated among wireless devices. Wireless devices, depending on their application, experience various aspects of these limitations more than others. For example the low power requirement in wireless sensors limits the range of communication, therefore there is a minimum node density necessary to maintain connections, conversely in a cellular system the user can increase its power up to a threshold to make sure its signal reaches the base station.

In most applications, sensors are in fixed locations, although there are exceptions such as wildlife monitoring. In this, they differ from cellular and mobile radio systems. However, even if the sensors are stationary, changes in the surrounding environment can cause fading and shadowing in the received signal. Thus, the rate of changes in the wireless channel for a wireless sensor will depend strongly on the speed of change in the field where is deployed. Take for example a WSN that is deployed on top of a silent volcano to collect vibration and gas density information. Since communication within the sensor network is short range and there are very few live beings around, except for changes in weather, not much will affect the wireless channel. In such cases, based on the period the network is active, it may be reasonable to assume that the channel does not change much in time. On the other hand, take a sensor network deployed in a bird habitat. The constant movement of birds and elements such as the wind blowing leaves and dust onto the sensors can interfere with transmissions and result in a time-varying channel.

2.3.1 Multiple Access

In a wireless star network, the destination for all sensor transmissions is the central node, the sink or the cluster head in the case of a cluster e.g. Fig. 2.4. As a result the wireless channel of a star network becomes a multiple access medium where simultaneous transmissions from sensors interfere. In conventional wireless systems, there are many ways to deal with multiple nodes accessing the same medium. In this section a few multiple access techniques and some collision resolution algorithms are discussed.

Resource Division Multiple Access - Channel Based

In Time Division Multiple Access (TDMA), time is divided into slots and each slot is allocated to one transmitter. By doing so interference due to simultaneous transmissions is eliminated. Frequency Division Multiple Access (FDMA), including Orthogonal Frequency Division Multiple Access (OFDMA), and Code Division Multiple Access (CDMA) do the same in the frequency and code domains. Dividing the resource is usually performed by a central controller, the sink in the case of a wireless sensor network. Of course enforcing centralized control over all the network will practically restrict the maximum size of the network, at minimum due to the delay in receiving control signals for sensors that are far from the sink.

Multiple Access Protocols - Packet Based

If the nodes in a network have the ability to deal with collisions that may occur during transmissions, scheduling transmissions will no longer be a limiting factor to the size of the network since centralized control is not required. These methods can be regarded as locally controlled time division techniques.

ALOHA Protocols: In ALOHA nodes are allowed to transmit data as they wish. In case of a collision the packets are useless and the nodes will have to re-transmit the data. Slotted ALOHA, where the start of transmissions are synchronized at the beginning of time slots, improves the throughput but requires synchronization. ALOHA-type techniques are very energy intensive since they require transmission of data repeatedly until it is received at the destination correctly. For this reason these techniques are not suitable for wireless sensor networks where batteries are the major system limitation. The 802.15.4 standard allows ALOHA as an optional protocol for UWB.

Collision Avoidance Protocols: It is much more energy efficient for a sensor to be aware of the on-going transmissions in the system before transmitting data. Protocols in which the node listens for transmissions to assess the state of the channel before transmitting are called Carrier Sense Protocols. Carrier sense protocols vary based on how the node reacts to the information from the channel.

Carrier Sense Multiple Access - Collision Avoidance (CSMA-CA) is the recommended protocol by the 802.15.4 standard for sensor networks. In this protocol when the sensor has data to transmit, after waiting for a random period of time, it senses the channel. If the channel is idle the sensor starts transmitting its data. If the channel is busy, the sensor waits a random period of time and listens to the channel again. Acknowledgment frames are sent from the recipient but without the use of CSMA-CA.

Despite assessing the channel before transmission, if two sensors start transmitting simultaneously their signals will collide. Under such circumstances nodes running the Carrier Sense Multiple Access - Collision Detection (CSMA-CD) protocol would immediately refrain from further transmissions. Terminating transmission abruptly saves both time and transmission power since the information from the two sources is not separable at the receiver. Retransmission will be attempted with the same protocol, usually after a random delay to avoid repeating the collision.

2.4 Performance Criteria

After reviewing the sensor, the sensor network, and the wireless channel, it is time to determine how to measure the performance of the wireless sensor network as a whole.

2.4.1 Previous Criteria

Before introducing a performance metric that has been tailored to best suit the characteristics and purpose of sensor networks, previously employed performance measures and their shortcomings are reviewed.

Lifetime

In previous research, in many cases, a lifetime measure has been used as the performance criterion. One example is the depletion of the first sensor, which is the most commonly used criterion [42, 78, 35, 70]. [60] considers the expiration of the first cluster head as the network lifetime.

Defining the lifetime as the expiry of the first sensor assumes all nodes are of equal importance to the network and its operation. Algorithms and optimizations that determine the network lifetime as the expiry of one sensor and work to maximize the lifetime tend to arrange communications such that sensors expire close to one another in time. In other words, they work to distribute energy consumption with regard to initial battery energy in the network. Nonetheless, this will not necessarily maximize the amount of data that can be communicated to the sink with the total energy available in the network and therefore may not be the best use of the sensor batteries.

The disconnection of a percentage of sensors from the network [41], either the sensor expires itself or cannot reach the sink due to the failure of relaying sensors, or more simply the depletion of a percentage of sensors [46], are other definitions of network lifetime that have been used in the literature. However, since the depletion of an individual sensor or a group of sensors in the network does not necessarily reflect the period in which it is feasible to gather data from the coverage area of the network, neither of these measures reflects the actual network lifetime.

There is no benchmark for the lifetime of a sensor network and various research groups choose different definitions making it challenging to compare the outcome of their propositions. As is apparent from the above examples, owing to the complex structure of the network, assessing the system lifetime is ambitious if not impossible.

Capacity

Capacity is a commonly utilized performance parameter for networks of various types and it is an appropriate measure for continuously powered nodes. In wireless sensor networks, however, the limited battery energy places a constraint on the duration in which the capacity is available. Thus, in the absence of a measure of duration, capacity is inadequate for quantifying the performance of a wireless sensor network although it has been used in some research e.g. [26].

Energy Efficiency

Energy is the major limiting factor in sensor networks. In essence, all performance measures are appraising the energy efficiency of the network in optimizing a performance criterion. In [9] the authors measure the network performance with the total energy required to transmit one bit of information while satisfying throughput and delay requirements. They take into account the signal processing circuitry required for transmission of the signal but fail to include the energy consumed for processing purposes. Since processing requires energy regardless of transmission taking place or not minimizing the energy spent on transmission will not minimize the energy consumed in the network.

Placing a latency constraint on data delivery, [77] employs an iterative numerical technique to minimize the energy used in the network. For their real-time scheme they do so by identifying the sensor that would save the network the most energy if it were allowed to continue transmitting for an additional period of time while taking into consideration the latency constraint. Doing so iteratively, the real-time protocol will converge once the latency constraint is reached. Since their approach provides a best next step solution, in most cases it will not provide the most energy efficient solution for the network.

Data Delivery

The authors in [6] define the ratio of total packets received by the sink to the total packets sent from all sensors as the data delivery ratio. The percentage of packets that make it to the sink is a reliability measure of the network; therefore it will give a measure of the quality of service the network provides. However, this criterion can only be used to evaluate transmission protocols in system models that assume losses in transmission due to collision, fading or other environmental measures e.g. it does not measure transmission powers and therefore is not an indication of the network performance in terms of energy efficiency. In other words, data delivery ratio does not provide a full picture of the network and can only be used in conjunction with other performance metrics to evaluate the system.

Throughput

In [36] a dynamic clustering technique is introduced for wireless sensor networks and the end-to-end throughput of the network, measured through Monte Carlo simulations, is used as the performance criterion. Throughput is commonly employed in wireless systems to evaluate the performance. However, since sensor networks depend solely on batteries to operate the network, any system measure should evaluate how well the batteries are utilized to deliver information. Throughput, or more precisely average throughput, presents an average snapshot of the information delivery but, similar to capacity, fails to convey the length of time for which this throughput can be achieved.

Delay

Network efficiency is measured in terms of the end-to-end delay in [38]. Optimizing the performance of a sensor network in terms of the time it takes information to travel from one end to another, or in the case of [37] to the sink, will only guarantee a maximum or average latency in the system but will not address the amount of information that gets through. Therefore, available battery energies can theoretically be spent on the fast delivery of little information to the sink.

Cost to Progress Ratio

Unlike performance measures presented above, cost over progress is a local performance parameter implying that it is a measure by which each sensor can evaluate its own performance or the performance of itself and close neighbors. Such criteria are used in localized protocols to determine routing paths. It can be argued that cost over progress will provide solutions in close proximity of the overall network optimum. However, employing such performance measures will, in many cases, drain certain critically positioned sensors and cause disconnect in the network.

To design a parameterless scheme [62] introduces the cost to progress ratio as a performance criteria for choosing the best route to the sink. To optimize the cost over progress ratio each node is assumed to know the position of itself, its neighbors, the link cost plus the location of the sink. Examples for the cost function are power, hop count, and delay. An example for progress is reduced distance to the sink. As noted above the shortcoming of such local measures is that they tend to drain critical sensors that present low cost to progress. To solve this issue it is necessary to incorporate a cost parameter that aside from other parameters depends on the popularity of the node.

Cost over progress ratio is presented in this dissertation as a general representation of local performance criteria that are typically used in routing. Based on the cost and progress parameters chosen, the cost to progress ratio criterion will take the form of known local performance measures. For instance, if the goal is to minimize the time required to send a message to the sink, the cost is the delay experienced if the message is not forwarded to a neighboring sensor and the progress is the reduced distance to the sink.

2.4.2 Proposed Criterion

In most cases, to evaluate the functioning of sensor networks, a performance criterion has been borrowed from other network types. For instance capacity or throughput in a wireless cellular system are important performance measures and lifetime, as defined for sensor networks, is used to assess ad hoc networks. Sensor networks have unique characteristics and should therefore be evaluated with a performance criterion that recognizes these properties.

In this dissertation, an alternative information-theoretic measure as appropriate to networks of limited-energy nodes is proposed. The author first introduced this measure in [30]. Since the objective of the system is to gather data at the sink(s), the amount of information received at the sink(s) from the network until all sensors are depleted of energy is introduced as a suitable performance measure. This criterion is termed the *data volume* and is measured in nats. In other words, data volume is the time integral of the sum-rate capacity which itself is measured in nats/sec. Mathematical representation of this relation can be found in Section 3.4.

Data volume is the limited-energy counterpart to capacity which is used for continuously powered or rechargeable nodes. Unlike channel capacity or mean channel capacity, data volume takes into account the duration for which the capacity is available, making it a better fit for limited-energy systems.

2.5 Cross Layer Design

To set the scene for presenting the problems and solutions this dissertation deals with, one more topic needs to be discussed. The introduction and rationale behind cross layer design is the conclusion to the background material.

Traditionally a protocol that governs the functioning of a network is thought of as a layered structure modeled as the OSI stack, Fig. 2.5. In this regard [66] states:

> *To reduce their design complexity, most networks are organized as a series of layers or levels, each one built upon the one below it ... in all networks, the purpose of each layer is to offer certain services to the higher layers, shielding those layers from the details of how the offered services are actually implemented.*

The author also notes that the boundaries for the responsibilities of each layer should be defined so that the flow of information is minimized across layers.

Wireless sensor networks, unlike some more traditional networks, do not agree well with the tight flow of information across layers. Because of the limited energy available to the sensors in the network, the highly directional flow of information to the sink and volatile links, further information sharing among layers can reduce energy expenditure and increase network efficiency in delivering information to the sink.

The introduction of cross layer design enables information exchange between layers, greatly enhancing the system performance without the need for redesigning the multi-layer protocol stack. Other networks that share some of the above properties with sensor networks also benefit from cross layer design. [18] argues that cross layer design is the correct solution for ad hoc networks.

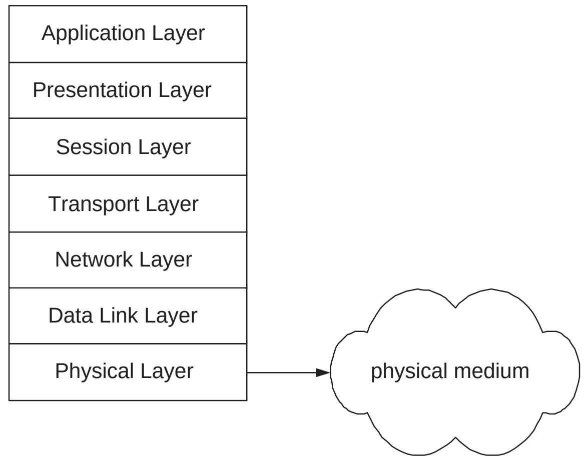

Figure 2.5: OSI seven layer protocol stack connected to the physical medium.

The IEEE 802.15.4 standard provides information about the PHY layer and MAC layer of the OSI system model, see Section 1.3. The MAC layer is a sublayer of the Data Link layer and controls access to the shared physical medium. It allocates available resources, resolves conflicts that arise from sharing a communication medium among multiple users and deals with damaged and lost information.

The OSI model delegates transmission-related issues to the PHY layer. The Physical layer deals with the physical medium over which transmission takes place; in essence, it listens to the channel and determines the power and duration of transmissions.

As the MAC layer and the PHY layer functions are closely related the free exchange of information will most likely result in energy savings. One example of such energy savings is tracing the source of error in delivering information. The physical channel information is available at the Physical layer, whereas channel access is controlled by the MAC layer. Good communication across layers will accelerate the discovery of the error source.

More recent research in sensor networks has shown great tendency towards cross layer design. [42, 44] and [70] employ cross layer design using the Physical layer and the Data Link layer which includes the MAC layer. [6, 35] deal with the routing aspect of network design in sensor networks. Since routing is controlled by the Network layer, also at times referred to as the Routing layer, the authors design protocols involving the Physical layer, Data Link layer and the Network layer.

2.6 Models and Objectives

In this dissertation the new performance measure introduced for sensor networks, data volume, is employed to evaluate network performance. Accounting for power consumed as processing power, and assuming a star wireless sensor network, optimum transmission policies are developed that maximize the data volume for the network in question. These optimum policies employ cross layer design by allowing the MAC layer access to the channel state information available at the Physical layer and determining the transmission schedule (Data Link layer) in combination with transmit powers (Physical layer).

In the following two chapters the analytical solution for a star network with static and time-varying channels is introduced. The effects of various system parameters such as network size on the maximum data volume are discussed. Further, the optimum analytical solution for the sensor model with no processing power is developed and compared to the more accurate model with processing power. This comparison shows the error in computation that will occur as a result of ignoring the processing power.

Chapter 5 is dedicated to simplifying computation of the optimum scheme but Chapter 6 returns to the subject of developing transmission policies. With the optimum scheme as the benchmark, two suboptimal transmission policies with close to optimum performance are introduced. The optimum transmission policy for a star wireless sensor network with time-varying channels that instructs sensors to enter low-activity states when not transmitting is also developed and compared to the maximum data volume obtained with no low-activity states.

Chapter 3

The Static Channel

In the few years that sensor networks have been the focus of research in wireless networks, many aspects of these networks have been studied in detail. The key feature of these networks is their limited battery energy and directional flow of data. Therefore, much attention has been paid to optimizing transmission scheduling and power control.

In contrast to the consensus that communication over the physical medium is key to performance enhancement, hence the attention to transmit power and scheduling optimization, little effort has been made to accurately model the communication channel and design protocols that acknowledge the characteristics of the wireless multiuser channel. In taking a step back and reexamining the communication channel, modeling mutual interference and energy expenditure for means other than transmission, it is the aim of this dissertation to provide a fresh point of view for protocol design in sensor networks.

This chapter assumes a wireless channel that does not vary in time. Although this is a commonly used model, there are a few scenarios for which this model is realistic. The static channel model is mainly used as a simplified model and is later improved to better reflect the properties of the wireless channel. In this dissertation the solution for the static channel will facilitate transition to the solution for the time-varying channel. In addition, comparison between the static channel and time-varying channel solutions highlights the effect of channel models on optimum transmission policies.

3.1 System Model

The network consists of a number of sensors and a single sink. The system model is introduced in two parts, how the sensors and the sink are modeled and how their network is modeled.

3.1.1 The Elements

Most features related to the sensor model have already been discussed in Chapter 2. In the interest of brevity only specific characteristics are discussed here.

Sensors are assumed to function on non-rechargeable batteries and are thus limited in their total power expenditure. On the other hand, the sink has access to an infinite source of energy. Processing power (Section 2.1.1) is accounted for in modeling the sensor power consumption and is assumed to be comprised of the power used for sensing, processing and signal reception.

The majority of communication in the sensor network is from the sensors to the sink. In addition, messages from the sink to the sensors can be assumed equal in size and frequency. Hence, the power consumed at each sensor for signal reception is included in the processing power as a predetermined value. To simplify equations and concentrate on the effect of other factors, the power consumed for sensing and processing is also assumed constant regardless of whether or not the sensor is transmitting data. Thus, processing power is assumed to be a fixed known value for each sensor. That being said, processing power is assumed to vary among sensors so as to generate a more general solution. Aside from manufacturing differences, processing power may differ among sensors when different types of sensors or sensors with different duties form a network.

To highlight the effects of processing power, the general solution for sensor energy expenditure obtained in this chapter will be modified to remove the processing power drain. Doing so will allow an appraisal of the effect of processing power on the data volume produced by each sensor.

As for means of communication, each sensor and the sink are equipped with a single antenna. As stated above, the wireless communication channel for each sensor is assumed static over time. It is also assumed that sensor - sink communication channels are independent among sensors. In other words, sensors are considered to be spaced farther than one or two wavelengths apart to allow independent channel gains to the sink.

In this study only the uplink channel, the link from the sensor to the sink, is discussed. It is assumed that downlink communications, links from the sink to the sensors, are in a time or frequency slot that do not interfere with uplink transmissions. As mentioned, the energy consumed for downlink communication is accounted for in the processing power.

The sink provides scheduling to the sensors and oversees communications in the network. To do so it has knowledge of the battery energy, processing power, and channel gains for the uplink connection of all sensors. Since the channel is static and processing powers are fixed for each sensor, it is sufficient for the sensor to communicate this information to the sink once during start up.

3.1.2 The Network

The network is modeled as a number of arbitrarily scattered sensors that are connected to the sink as a star network - refer to Section 2.2.1. The arbitrary placement of the sensors presents a more realistic layout for the network; however, it is not uncommon to assume that sensors are scattered uniformly or even placed on a grid-like structure [6].

The star structure is one of the two topologies introduced by the IEEE 802.15.4 standard - refer to Section 1.3 - and has also been used in many research publications, e.g. [44]. Since all sensors in a star network require direct access to the sink, the coverage area of the network is limited by the sensors' transmission range. Current industry products have ranges of up to about 100 meters; a star sensor network can therefore be deployed as a body sensor network, in an office, home or factory.

Studying the star network is also important to networks of larger scale that do not have a star structure. Unlike other wireless networks, in a sensor network the majority of information flows from the sensors to the sink creating a strong directional current

[45]. In a large network where information from sensors further out in the field has to reach the sink via other sensors, the volume of information in links gradually increases as they get closer to the sink.

The concentration of traffic in the area surrounding the sink elevates the mutual interference among sensors to levels above what is experienced in other parts of the network. Increased interference, and thus congestion, in turn accelerate battery depletion in these sensors and reduce data volume - see Section 2.4.2.

Ultimately, the sensors that provide the last link to the sink will have the burden of carrying all the information from the network to its destination. Therefore, the performance of this central star network is critical to the overall performance of the network regardless of its structure. With this insight, the solutions for a star sensor network can be used to shed light on factors affecting communication at the central core of any network.

A star network is recognizable at the central core of homogeneous networks as well as the cluster head mesh of a multi-level network such as the hierarchical model presented in [23]. The star structure can also be found within the clusters of a clustered network, Fig. 2.4. However, not all results from studying a star network can be readily applied within a cluster. Factors such as the limited battery of the cluster-head should be taken into account before extending the results to clusters.

It is important to recognize that the network is only intended as a means to collect data in a coverage area, hence data communicated is neither time or delay sensitive.

3.1.3 Assumptions Made

A wireless channel is a difficult structure to model perfectly. In many cases approximations are made in modeling the communication medium, some of which are employed in this dissertation. Below, some of these assumptions are discussed.

Time Slots and Indexing

It is assumed that time is divided into slots; however, the slots are not equal in length. The boundaries of time slots correspond to the expiry of sensors. In other words, a time slot is the period of time between the consecutive depletion of two sensors and so the number of sensors that can transmit is reduced by one when each time slot ends.

Time slots are numbered in reverse order. The last time slot is T_1 and the second last is T_2. To simplify referencing T_i is used to represent both the name and the duration of the i^{th} slot. The time slot index i is initialized to the number of sensors in the network and will at all times represent the number of sensors that have not expired. Time slot T_1 will end when the last sensor expires. Fig. 3.1 exhibits an example. The purpose of this numbering is to simplify equations.

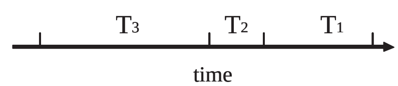

Figure 3.1: The time axis as used in this chapter. With this representation the time index also indicates the number of sensors that have not yet expired.

The assumption of synchronized time slots for the sensors and the sink requires perfect timing throughout the network. Synchronization is required so sensors know when to start transmitting. Asynchronous networks are not discussed in this dissertation.

Sensors are also numbered and all parameters are indexed with the sensor number and, if necessary, with a time index. For example, the transmit power for sensor 3 in time period 5 is indexed 35.

Error Free Transmission

Throughout this dissertation all transmission is assumed error free. Thus, if information is sent from a sensor, it is assumed to be received with no errors at the sink. For this reason acknowledgment or retransmission schemes are ignored. This assumption is made to reduce the number of factors affecting the data volume and allow more fundamental aspects of the network to be studied.

Transmit Power Constraints

There are no constraints assumed on the maximum (or for that matter minimum) sensor transmit powers. The only restriction is that the sum of all energies consumed for transmit and processing powers for each sensor should be equal to the sensor's battery energy. Although allowing transmit power to vary in a large range is not very realistic - at least not with low cost sensors - it is widely accepted for early studies of sensor networks.

Modulation and Pulse Shaping

Taking an information theoretic approach, details of modulation and pulse shaping have been avoided in this work. The physical layer is treated as an abstraction that is represented solely in terms of transmit power and channel gain.

Path Loss and Shadowing

In a wireless channel, the effect of path loss is modeled as a constant dividend. Since this dissertation deals with a star network, the distance between sensors and the information sink varies in a rather small range compared with other networks such as cellular networks. Thus, it is not unreasonable to assume that the effect of path loss is relatively similar for all sensors.

Shadowing is a random phenomenon produced by obstacles blocking the communication path between the transmitter and receiver. This is a factor that will affect a wireless sensor network almost regardless of where it is deployed. However, in a smaller network the impact is less dominant.

Similar to transmission errors and caps on maximum transmit powers, path loss and shadowing are elements that can easily be added to the system model. However, since this is an initial study on the workings of the star sensor network, models need to be simplified such that they are a reasonable representation of the network without incorporating too many variables. The exclusion of these factors will in turn allow clearer conclusions to be drawn about the effects of factors that are modeled.

Availability of Data

For ease of analysis it is assumed that sensors always have data available for transmission. This assumption removes the need to deal with queuing as an addition parameter in the system model. So whenever a sensor is scheduled to transmit, data is sent to the sink.

If data is continuously collected from the coverage area, sensors will always have information to send to the sink. Based on the data volume being transmitted, the sensor can decide what information and with how much detail to convey.

3.2 Prior Research

There has been limited analytical work on performance optimization in wireless sensor networks. In this section the most relevant work to the material presented in this chapter is reviewed. Some of the references also relate to the work presented in Chapter 4.

In [76] the joint source coding, routing and resource allocation optimization problem for a sensor network is decomposed into two distinct parts: the source coding problem, and the resource allocation and routing problem. The objective of the optimization is to minimize rate distortion with constraints on sensor transmit powers. The authors take an information theoretic approach to formulating the question and deal with rate distortion, capacity and sensor power restrictions. All parameters are assumed constant over time. They suggest an *iterative numerical* solution for which they prove convergence.

With the goal of minimizing the energy spent for collecting data, [77] employs rate adaptation to balance energy and latency in a tree-structure sensor network. The authors present a tree-like structure where only the leaves on the tree gather data and the remaining sensors act only as relays that aggregate the data along the path. The communication channel is static and assumed to experience no collisions or interference; however, the method employed to do so is not specified. The authors allow for sensors to shut down completely and be woken up by low power radios with negligible power expenditure. In addition, only power consumed for transmission purposes is modeled. The authors present a centralized offline algorithm and an online distributed algorithm that relies solely on local information to make transmission time determinations. Both methods are iterative and numerical.

[59] relies on convergence to a global optimum to derive transmission rates and power levels which maximize the net utility of the network. In this work the net utility is defined as the sum of the log-utility, logarithm of the data rate, over all sensors that produce data minus the accumulative power consumption during transmission.

A tree-like structure similar to [77] where only the leaves collect data is assumed. The channel over which transmission takes place is static and the path data takes from each leaf to the sink is fixed. The cross layer optimization algorithm is modified to allow distributed processing at the leaves where data is generated.

In [6] the authors resort to dividing the network into collection trees that utilize different sleep patterns to solve congestion issues, especially around the sink, and improve the energy efficiency of gathering data. Sleeping patterns are scheduled so that active periods for sensors do not overlap. They rely on the idea that neighboring nodes do not need to communicate if a certain quality of packet delivery can be guaranteed and can therefore be located on different collection trees and sleep with different patterns. The authors point out that the solution to this problem is NP hard and resort to simulations to evaluate the performance of the proposed scheme and the effect of different numbers of trees.

The channel in [6] is assumed static and therefore collection trees are formed once and never updated. The authors also account for an idle power consumption equal to the power consumed for signal reception and half that of the transmit power. Without mentioning the maximum length of the queue, the authors assume that some packets are dropped due to bounded queue lengths. Based on the simulations presented in the paper the data delivery ratio is improved by up to 40% by allowing additional trees in the network. In another graph, it is shown that the average sensor energy consumption is reduced by about 30% with the increase in the number of collection trees. The authors associate the reduction in energy consumption with the reduction in transmission collisions.

[70] presents a solution to maximizing network lifetime, defined as the time until the first sensor expires, by controlling power, link access, and routing in the network. The communication channel is assumed static throughout time and TDMA is implemented as the channel access method. An analytical solution is developed for a linear single source, and multiple source network. A linear network is a network where all the sensors are arranged in a single line of communication and a source refers to a sensor that generates information or in other words collects data. For more realistic network layouts where sensors are not necessarily arranged in a row the authors present an iterative suboptimal method.

As previously stated, Section 2.4.1, [42] also defines the expiry time for the first sensor in the network as the network lifetime. With the aim of maximizing the network lifetime by determine routing, transmission schedules and transmission powers, the authors formulate a non-convex problem with maximum rate and transmission range constraints. Assuming fixed link schedules, they employ convex rate-constraint approximations, which in turn result in a convex optimization problem. Trading optimality for computational simplicity, the authors opt for an iterative suboptimal solution that alternates between computation of rates and powers, and determining the link schedule. It should be mentioned that for a special case, where link schedules are limited to TDMA schemes, the authors obtain the exact optimal transmission scheme.

Since simulations in [42] are only presented for very specific topologies, i.e. linear and rhombus, and with very few sensors in the network, it seems that the performance improvement claims cannot be extended to other network topologies or networks with many sensors. However, it is worth noting that the system model employed is more advanced than many other research publications. First, the authors account for mutual interference during transmissions and allow that to affect scheduling decisions. Second, processing power is included in the power consumption model of sensors.

Much of the performance optimization literature in sensor networks, especially that of cross layer optimization, is not analytical in nature, which makes it difficult to compare to the results obtained in this chapter. The lack of analytical work published in this area also emphasizes the importance of the analysis presented in this dissertation. The outline of the problem solved in this chapter is presented in the next section.

3.3 Problem Statement

Employing data volume, as introduced in Section 2.4.2, the problem statement is summarized as below:

> ***Given a star wireless sensor network with static channels, find the optimum transmission policy that maximizes the data volume.***

The transmit policy defines the transmission schedule and transmit powers of the sensors. In other words this policy determines how many sensors, which sensors and with how much power should transmit. The optimization is a cross layer optimization since the MAC layer employs information from the Physical layer to determine the transmission schedule (Data Link layer) in combination with transmit powers (Physical layer).

3.4 Notation

The total number of sensors in the system is denoted by N and as mentioned, time slot i is depicted with T_i. The transmission bandwidth and the noise power spectral density, which in the interest of simplicity have been assumed equal for all sensors, are denoted by B and N_o respectively. For ease of presentation sensor indexes have been eliminated in the notation introduction.

In Section 2.1.1, the processing power, P_p, and transmit power, P_{tx}, were introduced. The ratio of the sensor's transmit power to its processing power is x

$$x = \frac{P_{tx}}{P_p}. \tag{3.1}$$

If power equal to the sensor's processing power were used for transmission the transmit SNR would be γ_p, where

$$\gamma_p = \frac{P_p}{BN_o}. \tag{3.2}$$

Denoting the channel power gain with ξ, this signal would be received at the sink with a reception SNR of z

$$z = \gamma_p \xi = \frac{P_p \xi}{BN_o}. \tag{3.3}$$

These fictitious SNRs are presented to help simplify the equations. Based on the notation introduced, the SNR[1] at the base is

$$xz = \frac{P_{tx}}{P_p} \times \frac{P_p \xi}{BN_o} = \frac{P_{tx} \xi}{BN_o}. \tag{3.4}$$

D is the normalized battery energy for the sensor so

$$D = \frac{EB}{P_p} = BL \tag{3.5}$$

where E is the initial battery energy and L is the maximum sensor lifetime achieved when the sensor does not transmit any data. The initial value of D can also be interpreted as the maximum number of complex signaling dimensions available to the sensor.

Although E is used only to represent the initial battery energy, the general definition of the normalized battery energy D, current battery energy multiplied by the bandwidth divided by processing power, is valid throughout the sensor's lifetime. For example D_{23} is the normalized battery energy for sensor 2 at the beginning of time slot 3. For this reason D is indexed with both the sensor and the time index.

The set of sensors that expire after sensor i is expressed as $\mathcal{Q}_i$. The value of this notation will become apparent when dealing with the data volume of a large number of sensors. The sum-rate capacity in nats/sec is represented by C and only indexed

[1] Transmit SNR for the sensor is denoted by γ which will be used in the solution where processing power is not modeled.

with time. Data volume is denoted by V and indexed with the number of sensors in the network. The relationship between data volume and the sum-rate capacity assuming N sensors in the system is

$$V_N = \sum_{j=1}^{N} C_j T_j \tag{3.6}$$

where assuming sensor m is the j^{th} sensor to expire

$$C_j = B \ln(1 + \sum_{k \in \{m, \mathcal{Q}_m\}} x_k z_k). \tag{3.7}$$

Since time slots mark the depletion of sensors, the lifetime of the network is also N time slots. The last notation to be discussed is the root function. In this chapter, $root_x[equation]$ is used to represent the root of the *equation* with respect to x.

3.5 Brute Force Algorithm

The aim of this optimization is to determine transmit powers and schedules for the sensors in the network so that the data volume is maximized. While doing so, the lifetimes of the sensors are also determined. A brute force approach to solving the optimization problem would be to compute the maximum data volume produced by each possible combination of sensor lifetimes and compare them to find the optimal solution. This approach is itemized below.

1. find the maximum sensor lifetime for all sensors, L_i where $i \in [1, N]$
2. based on maximum lifetimes determine all the orders in which sensors could expire
3. for each order of expiry calculate the maximum data volume it can produce
4. choose the depletion order that produces the highest data volume as the optimum solution which in turn presents optimum transmit powers and sensor lifetimes

With the above approach sensor schedules, transmit powers and lifetimes are determined. In the subsequent text the analytical solution to determining the maximum data volume for given lifetimes, Step 3 of the brute force search, is developed. In the following section, Section 3.7, the solution is presented for the case where processing power is not included in the sensor model. This section, in contrast with Section 3.6, highlights the importance of accounting for processing power in the sensor model.

3.6 Maximum Data Volume Given Expiry Orders

In the brute force algorithm present above, there is a need to determine the maximum data volume for each order of sensor depletion. An inductive approach is taken in the presentation of this analytical solution; the solution for a network of one, two and then N sensors is presented. The work presented in this section is based on the author's analytical solution published in [30].

3.6.1 One-Sensor Network

With a network comprised of a single sensor i and the information sink

$$E_i = P_i T_1 = (P_{p_i} + P_{tx_{i1}}) T_1 \tag{3.8}$$

where P_i is defined in (2.1). Multiplying both sides by B/P_{p_i}

$$D_{i1} = B T_1 (1 + x_{i1}). \tag{3.9}$$

The above equation shows how the proposed notation simplifies both the equations and the indexing; it can be rewritten as

$$T_1 = \frac{D_{i1}}{B(1 + x_{i1})} \tag{3.10}$$

Data volume for the single-sensor network is

$$V_1 = C_1 T_1 = B T_1 \ln(1 + x_{i1} z_i) \tag{3.11}$$

with the condition that

$$x_{i1} \geq 0. \tag{3.12}$$

Substituting from (3.10) into (3.11)

$$V_1 = \frac{D_{i1}}{1 + x_{i1}} \ln(1 + x_{i1} z_i). \tag{3.13}$$

The transmit power that leads to the maximum data volume, $x_{i_{opt}}$, is obtained by differentiating V_1 with respect to x_{i1}. The conclusion is

$$x_{i_{opt}} = root_x \left[\frac{z_i(1 + x)}{1 + x z_i} - \ln(1 + x z_i) \right] \tag{3.14}$$

which can be numerically evaluated with known channel gains and is independent of the sensor battery energy. From the equation above it is known that the transmit power that achieves the maximum data volume satisfies

$$\frac{z_i(1 + x_{i_{opt}})}{1 + x_{i_{opt}} z_i} = \ln(1 + x_{i_{opt}} z_i). \tag{3.15}$$

Assuming the sensor transmits with the optimum transmit power and substituting the logarithmic term (3.13) with its equivalent from (3.15), the maximum data volume for a single-sensor network is

$$V_{1_{max}} = \frac{D_{i1} z_i}{1 + x_{i_{opt}} z_i}. \tag{3.16}$$

The parameter α_i is defined as follows to simplify the presentation of equations.

$$\alpha_i = \frac{1}{1 + x_{i_{opt}} z_i} \tag{3.17}$$

The maximum data volume can be rewritten as

$$V_{1_{max}} = D_{i1} z_i \alpha_i. \tag{3.18}$$

3.6.2 Two-Sensor Network

The next step for the inductive solution is a network of two sensors. Assume sensors, p and q, form a star network with the sink and also that sensor p expires before sensor q. The data volume is

$$V_2 = \sum_{j=1}^{2} C_j T_j = BT_2 \ln(1 + x_{p2} z_p + x_{q2} z_q) + BT_1 \ln(1 + x_{q1} z_q) \tag{3.19}$$

conditioned that

$$x_{q1} \geq 0 \ , \ x_{q2} \geq 0 \ , \ x_{p2} \geq 0. \tag{3.20}$$

To constitute a meaningful argument it is assumed

$$z_p > 0 \text{ and } z_q > 0. \tag{3.21}$$

If any of z_p or z_q were zero the system would be equivalent to a single-sensor network and the discussion presented here would be unnecessary.

Based on (3.18) the maximum data volume produced by sensor q after sensor p has expired is known and can therefore be substituted into the equation.

$$V_2 = BT_2 \ln(1 + x_{p2} z_p + x_{q2} z_q) + D_{q1} z_q \alpha_q. \tag{3.22}$$

Note that since the remaining battery energy for sensor q at the beginning of time slot T_1, and thus D_{q1}, depends on how much energy has been used in time slot T_2, and therefore on x_{q2}, the second term in equation (3.22) depends on x_{q2}. The equation below represents energy usage for sensor q during T_2.

$$D_{q2} - D_{q1} = BT_2(1 + x_{q2}) \tag{3.23}$$

Equation (3.23) follows from the fact that time slot T_2 is the time until sensor p expires and is arrived at with the same procedure as (3.10).

$$T_2 = \frac{D_{p2}}{B(1 + x_{p2})} \tag{3.24}$$

After substituting the equivalent of T_2 from (3.24) into (3.23) the equation becomes

$$D_{q2} - D_{q1} = \frac{D_{p2}(1 + x_{q2})}{1 + x_{p2}} \tag{3.25}$$

Substituting the value of T_2 from (3.24) and D_{q1} from (3.25), the data volume equation can be rewritten as

$$V_2 = D_{p2}\frac{\ln(1 + x_{p2}z_p + x_{q2}z_q) - (1 + x_{q2})z_q\alpha_q}{1 + x_{p2}} + D_{q2}z_q\alpha_q. \tag{3.26}$$

The value of x_{q1} and therefore α_q is determined using (3.14), without the need for knowing battery charges. D_{q2} and D_{p2} are the initial battery charges for p and q which are known. The only values that need to be determined are the transmit powers for T_2, x_{q2} and x_{p2}. The fraction containing these powers is separated and represented as a function of the transmit powers, $f(x_{q2}, x_{p2})$, which needs to be maximized in order to maximize the network data volume.

$$f(x_{q2}, x_{p2}) = \frac{\ln(1 + x_{p2}z_p + x_{q2}z_q) - (1 + x_{q2})z_q\alpha_q}{1 + x_{p2}} \tag{3.27}$$

For this maximization the following test is employed.

Unconstrained Extremum Test

> Assume $g(x, y)$ and its first order derivatives are continuous for $x \in [a, b]$, $y \in [c, d]$. An extremum of such a function can only occur at stationary points or at points where the first order derivatives are not completely defined.

A stationary point refers to a local maximum, minimum or a saddle point[2]. And the first order derivatives are completely defined everywhere except the boundaries of the region.

It is a condition on the problem that the transmit powers be positive (3.20) so the region defined by the transmit powers is

[2] Since this dissertation does not deal with the details of these concepts the reader is referred to any basic optimization theory reference for more information

$$x_{q2} \in [0, \infty) \text{ and } x_{p2} \in [0, \infty). \tag{3.28}$$

The function $f(x_{q2}, x_{p2})$ is continuous within this region and its first order derivatives are

$$\frac{\partial f(x_{q2}, x_{p2})}{\partial x_{q2}} = \frac{\frac{z_q}{1+x_{p2}z_p+x_{q2}z_q} - z_q\alpha_q}{1 + x_{p2}} \tag{3.29}$$

$$\frac{\partial f(x_{q2}, x_{p2})}{\partial x_{p2}} = \frac{\frac{z_p}{1+x_{p2}z_p+x_{q2}z_q}(1 + x_{p2}) - (\ln(1 + x_{p2}z_p + x_{q2}z_q) - (1 + x_{q2})z_q\alpha_q)}{(1 + x_{p2})^2} \tag{3.30}$$

which are also continuous within the region. Therefore $f(x_{q2}, x_{p2})$ meets the criteria of the Unconstrained Extremum Test. Based on meeting the conditions of the Extremum Test any extrema on the function is either a point where both first order derivatives are zero or a point on the region boundaries. In the following it is determined whether the first order partial derivatives are simultaneously zero at any point inside the region, i.e. $x_{q2} \in (0, \infty)$ and $x_{p2} \in (0, \infty)$.

Allowing (3.29) to equal zero, with the knowledge that the channel gains are greater than zero

$$x_{q2}z_q = \frac{1}{\alpha_q} - 1 - x_{p2}z_p. \tag{3.31}$$

Setting (3.30) equal to zero and substituting (3.31) into the equation yields

$$\ln(\frac{1}{\alpha_q}) = (z_p + z_q + \frac{1}{\alpha_q} - 1)\alpha_q. \tag{3.32}$$

Rewriting (3.15) with only z_q and α_q,

$$\ln(\frac{1}{\alpha_q}) = z_q\alpha_q(1 + x_{q1}) = z_q\alpha_q(1 + (\frac{1}{\alpha_q} - 1)\frac{1}{z_q}) = (z_q + \frac{1}{\alpha_q} - 1)\alpha_q. \tag{3.33}$$

Knowing that $\alpha_q > 0$ and substituting (3.33) into (3.32) the conclusion is

$$z_p = 0. \tag{3.34}$$

However, to constitute a meaningful discussion it is assumed $z_p > 0$ (3.21). Therefore, the first order derivatives of $f(x_{q2}, x_{p2})$ cannot be zero simultaneously. Employing the Unconditional Extremum Test it is concluded that any extrema can only be on the region boundaries. In other words, if there exists a maximum for $f(x_{q2}, x_{p2})$ it occurs when one of the transmit powers is equal to zero, $x_{q2} = 0$ or $x_{p2} = 0$ i.e. the optimum occurs when only one sensor is transmitting.

As previously mentioned, $f(x_{q2}, x_{p2})$ is the only term in the data volume that is determined by the transmit powers in T_2. Hence, the above arguments can be extended to the data volume and the transmit powers that maximize $f(x_{q2}, x_{p2})$ will also maximize the data volume. To obtain a better understanding of the structure of the function, its general form is studied below.

The first order derivatives of $f(x_{q2}, x_{p2})$ are presented in (3.29) and (3.30); the second order derivatives are as follows.

$$\frac{\partial^2 f(x_{q2}, x_{p2})}{\partial x_{q2}^2} = \frac{-z_q^2}{(1 + x_{p2}z_p + x_{q2}z_q)^2(1 + x_{p2})} \tag{3.35}$$

$$\begin{aligned}\frac{\partial^2 f(x_{q2}, x_{p2})}{\partial x_{p2}^2} =& \frac{-z_p^2}{(1 + x_{p2}z_p + x_{q2}z_q)^2(1 + x_{p2})} \\ & \frac{-2\left[\frac{z_p(1+x_{p2})}{1+x_{p2}z_p+x_{q2}z_q} - \ln(1 + x_{p2}z_p + x_{q2}z_q) + (1 + x_{q2})z_q\alpha_q\right]}{(1 + x_{p2})^3}\end{aligned} \tag{3.36}$$

As is apparent from $f(x_{q2}, x_{p2})$ and its derivatives, the function is smooth and well behaved and infinitely differentiable. It is also worth noting that (3.35), the second derivative with respect to x_{q2}, is always negative indicating that the function is concave with respect to x_{q2}. Since $f(x_{q2}, x_{p2})$ can be sketched knowing only z_p and z_q - from (3.14) α_q is calculated knowing only z_q - Fig. 3.2 illustrates the general shape of the function.

Since the surface is a 3D shape which may not show very well in print, Fig. 3.3 presents the curves on the boundaries of $f(x_{p2}, x_{q2})$. The maximum is also marked.

Thus far it has been shown that the maximum, if it exists, can only exist on the boundaries where one of the two sensors is not transmitting. To determine which of the two sensors should transmit and whether or not this induces a maximum for

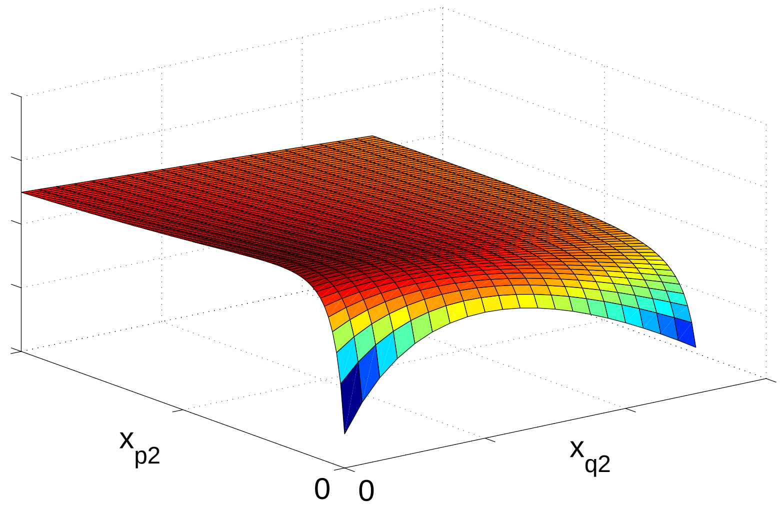

Figure 3.2: General shape of the function $f(x_{p2}, x_{q2})$. To plot the surface it is assumed $z_p = 0.5$ and $z_q = 0.7$. However, other than the origin of the axis the numbers have been omitted to preserve the generality of figure.

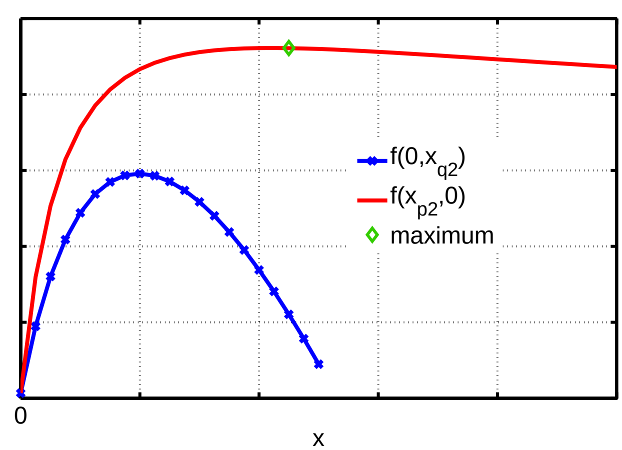

Figure 3.3: The boundaries of $f(x_{p2}, x_{q2})$, as presented in Fig. 3.2, and the maximum of the function.

$f(x_{q2}, x_{p2})$ and in turn the data volume, each of the possibilities is examined. It is noted that $f(x_{q2}, x_{p2})$ becomes a single variable function on the boundaries. To reduce notation, in the subsequent text, f represents this single value function.

First, the boundary where sensor p is silent. The derivative at this point becomes

$$\frac{\partial f(x_{q2})}{\partial x_{q2}} = \frac{z_q}{1 + x_{q2} z_q} - z_q \alpha_q. \tag{3.37}$$

With $z_q > 0$, when the partial derivative is set to zero

$$\alpha_q = \frac{1}{1 + x_{q2} z_q}, \tag{3.38}$$

which in comparison with the definition of α_q yields

$$x_{q1} = x_{q2}. \tag{3.39}$$

So $x_{q2} = x_{q1}$ is a stationary point for $f(x_{q2})$. Since the second derivative with respect to x_{q2} is negative everywhere, based on the Second Derivative Test[3], $x_{q2} = x_{q1}$ is a local maximum for f. At this point the value of the second derivative is

$$\frac{\partial^2 f(x_{q2})}{\partial x_{q2}^2} = \frac{-z_q^2}{(1 + x_{q2} z_q)^2} = -z_q^2 \alpha_q^2 \tag{3.40}$$

With a little manipulation the local maximum data volume is calculated as

$$V_2 \Big|_{x_{p2}=0} = D_{q2} z_q \alpha_q \tag{3.41}$$

Now the other boundary where $x_{q2} = 0$ and in turn f is only a function of x_{p2}. The first derivative with respect to x_{p2} is

$$\frac{\partial f(x_{p2})}{\partial x_{p2}} = \frac{\frac{z_p}{1+x_{p2} z_p}(1 + x_{p2}) - \ln(1 + x_{p2} z_p) + z_q \alpha_q}{(1 + x_{p2})^2}. \tag{3.42}$$

When this is set to zero solving the equation with respect to x_{p2} produces the desirable transmit power

[3]Second Derivative Test: For a twice differentiable function g at a stationary point t_o, if the second derivative is positive, t_o is a minimum; if it is negative, t_o is a maximum and if it is zero t_o is a possible inflection point.

$$x_{p2} = root_x \left[\frac{z_p(1+x)}{1+xz_p} - \ln(1+xz_p) + z_q\alpha_q \right]. \tag{3.43}$$

For this point the second derivative is

$$\frac{\partial^2 f(x_{p2})}{\partial x_{p2}^2} = \frac{-z_p^2}{(1+x_{p2}z_p)^2(1+x_{p2})^3}. \tag{3.44}$$

The second derivative is always negative on this boundary, regardless of the value of x_{p2}, suggesting that the function is concave and that the transmit power calculated from (3.43) is a local maximum.

Defining α_p similar to (3.17), the optimum value for x_{p2} should fit in the following equation.

$$\ln(1+x_{p2}z_p) = \frac{z_p}{1+x_{p2}z_p}(1+x_{p2}) + z_q\alpha_q \tag{3.45}$$

And the data volume will then be

$$V_2 \Big|_{x_{q2}=0} = D_{p2}z_p\alpha_p + D_{q2}z_q\alpha_q. \tag{3.46}$$

So far two local maxima of the function $f(x_{q2}, x_{p2})$ are known. Comparing the value of $f(x_{q2}, x_{p2})$ at these two points will yield the global maximum of the function. Another alternative would be to compare the values of the data volume at these points since the data volume shares all its stationary points with $f(x_{q2}, x_{p2})$. From (3.41) and (3.46) it is deduced that

$$V_2 \Big|_{x_{q2}=0} > V_2 \Big|_{x_{p2}=0}. \tag{3.47}$$

In other words, the global maximum for the data volume is produced on the boundary where $x_{q2} = 0$ and at the point defined by (3.43). To summarize, the maximum data volume produced by two sensors p and q is

$$V_{2_{max}} = D_{p2}z_p\alpha_p + D_{q2}z_q\alpha_q \tag{3.48}$$

where the optimum transmit power for sensor p during the second time slot is

$$x_{p_{opt}} = root_x \left[\frac{z_p(1+x)}{1+xz_p} - \ln(1+xz_p) + z_q\alpha_q \right] \tag{3.49}$$

and α is defined as in (3.17). As with the single-sensor case the optimum transmit power does not rely on the battery energy.

Since the initial assumption was that sensor p expires at the end of time slot T_2, the transmission schedule can be summarized as: sensor p starts to transmit with $x_{p_{opt}}$ and once it expires sensor q starts to transmit with $x_{q_{opt}}$ defined similar to (3.14). In practice, $x_{q_{opt}}$ should be determined first since $x_{p_{opt}}$ depends on the value of α_q. The stationary channel allows for all calculations to be done before transmission commences.

Note that the order of calculating transmit powers is a result of the sensor expiry order assumption - Step 3 Section 3.5. However, the optimality of the single-sensor transmission policy is a general conclusion.

3.6.3 N-Sensor Network

Having presented the solution for a single-sensor network and a two-sensor network, induction is employed to derive the analytical solution to maximizing the data volume for an N-sensor network.

Assume the solution for an $N-1$-sensor network is known to be single-sensor transmission, and the maximum data volume and transmit powers in each time slot are derived from the following equations.

$$V_{(N-1)_{max}} = \sum_{i=1}^{N-1} D_{i(N-1)} z_i \alpha_i \tag{3.50}$$

$$\alpha_i = \frac{1}{1+x_{i_{opt}} z_i} \tag{3.51}$$

$$x_{i_{opt}} = root_x \left[\frac{z_i(1+x)}{1+xz_i} - \ln(1+xz_i) + \sum_{j\in\mathcal{Q}_i} z_j\alpha_j \right] \tag{3.52}$$

The optimum transmission scheme is for sensors to transmit one after the other and for each sensor to transmit until its battery expires. The order of transmission, and in turn expiries, which attains $V_{(N-1)_{max}}$ is found by considering all the $(N-1)!$ possibilities through the brute force algorithm in Section 3.5.

Based on the maximum data volume for $N-1$ sensors the maximum data volume for an N-sensor network, assuming sensor m expires at the end of time slot T_N, has the form

$$V_N = BT_N \ln\left(1 + \sum_{i=1}^{N} x_{iN} z_i\right) + \sum_{i \in \mathcal{Q}_m} D_{i(N-1)} z_i \alpha_i \tag{3.53}$$

conditioned that

$$\forall i \in \{m, \mathcal{Q}_m\} : x_{iN} \geq 0 \text{ and } z_i > 0. \tag{3.54}$$

Knowing that sensor m expires at the end of time slot T_N,

$$T_N = \frac{D_{mN}}{B(1 + x_{mN})} \tag{3.55}$$

and

$$\forall i \in \mathcal{Q}_m : D_{iN} - D_{i(N-1)} = \frac{D_{mN}(1 + x_{iN})}{1 + x_{mN}}. \tag{3.56}$$

Substituting T_N from (3.55) and $D_{i(N-1)}$ from (3.56) into the data volume equation,

$$V_N = D_{mN} \frac{\ln(1 + \sum_{i=1}^{N} x_{iN} z_i) - \sum_{i \in \mathcal{Q}_m} (1 + x_{iN}) z_i \alpha_i}{1 + x_{mN}} + \sum_{i \in \mathcal{Q}_m} D_{iN} z_i \alpha_i. \tag{3.57}$$

Similar to (3.26), the data volume depends on the normalized battery energy of all the sensors at the time the network is set up, D_{iN}, and the transmit powers of all the sensors.

Since the solution for a network of $N-1$ sensors is known, all the α_i values in the equation are also known. The only unknowns are the transmit powers in time slot T_N, x_{iN}. Following the same steps as the case of two sensors, the fraction which depends on these transmit powers and therefore should be maximized is separated and studied.

$$f(x_{1N}, x_{2N}, \ldots, x_{NN}) = \frac{\ln(1 + \sum_{i=1}^{N} x_{iN} z_i) - \sum_{i \in \mathcal{Q}_m} (1 + x_{iN}) z_i \alpha_i}{1 + x_{mN}} \tag{3.58}$$

Noting that the Unconstrained Extremum Test can be extended to the case of a multi-variable function, the first order derivatives of the new function f are examined. For any sensor $k \neq m$

$$\frac{\partial f(x_{1N}, x_{2N}, \ldots, x_{NN})}{\partial x_{kN}} = \frac{\frac{z_k}{1+\sum_{i=1}^{N} x_{iN} z_i} - z_k \alpha_k}{1 + x_{mN}} \tag{3.59}$$

and when set equal to zero, noting that $z_k > 0$ (3.54)

$$\alpha_k = \frac{1}{1 + \sum_{i=1}^{N} x_{iN} z_i}. \tag{3.60}$$

The Unconstrained Extremum Test states that at any extremum point , with values in the range of $(0, \infty)$ for transmit powers, the first order derivatives with respect to all the transmit powers will be zero simultaneously. Allowing all the derivatives, except the first order derivative with respect to x_{mN}, to equal zero results in

$$\forall i \neq m : \alpha_1 = \alpha_2 = \ldots \alpha_i \ldots = \alpha_N. \tag{3.61}$$

However, based on its definition (3.51), the value of α for each sensor is derived from its z and the value of $z\alpha$ of sensors that expire after it, and is therefore unique. In other words, so far it is concluded that for all sensors except m the first derivative with respect to no two transmit powers can be zero simultaneously and so it is not optimum for any two to transmit simultaneously.

The only case not excluded from simultaneous transmission is when a sensors transmits alongside sensor m. As is obvious, the N-sensor problem has now degenerated into a 2-sensor problem for which it is known that the solution is for only one sensor to transmit[4]. Now that it has been established that the optimum policy is for

[4]Another way to prove that sensor m transmitting simultaneously with another sensor is not optimal is to calculate the first order derivative of $f(x_{1N}, x_{2N}, \ldots, x_{NN})$ when m and another sensor are transmitting. The derivative is always positive and thus cannot equal zero. It is therefore concluded that such a point on the function is not an extremum.

only one sensor to transmit in any time slot including T_N, the question that remains is *of the N sensors which sensor should transmit in time slot T_N?*
If sensor $k \neq m$ is the non-silent sensor, i.e.

$$\forall i \neq k : x_{iN} = 0, \tag{3.62}$$

allowing the first derivative with respect to x_{kN} to equal zero yields

$$\alpha_k = \frac{1}{1 + x_{kN} z_k}. \tag{3.63}$$

Similar to the two-sensor case $f(x_{1N}, x_{2N}, \ldots, x_{NN})$ is a well behaved smooth infinitely differentiable function and since the second order derivative with respect to x_{kN} at this point is

$$\frac{\partial^2 f(x_{kN})}{\partial x_{kN}^2} = \frac{-z_k^2}{(1 + x_{kN} z_k)^2}, \tag{3.64}$$

$(0, 0, \ldots x_{kN} \ldots, 0)$ is a local maximum. Assuming sensor k expires in time slot $R < N$,

$$x_{kN} = x_{kR}. \tag{3.65}$$

And after some manipulation, the data volume becomes

$$V_N \left|_{x_{kN} \neq 0} = \sum_{i \in \{\mathcal{Q}_k + \{k\}\}} D_{iN} z_i \alpha_i + \sum_{i \in \{\mathcal{Q}_m - \mathcal{Q}_k - \{k\}\}} \left(D_{iN} - D_{mN}\right) z_i \alpha_i. \right. \tag{3.66}$$

If sensor m transmits in time slot T_N the first order derivative with respect to its transmit power becomes

$$\frac{\partial f(x_{mN})}{\partial x_{mN}} = \frac{\frac{z_m(1+x_{mN})}{(1+x_{mN} z_m)} - \ln(1 + x_{mN} z_m) + \sum_{i \in \mathcal{Q}_m} z_i \alpha_i}{(1 + x_{mN})^2}. \tag{3.67}$$

When set to zero this yields

$$x_{mN} = root_x \left[\frac{z_m(1+x)}{1 + x z_m} - \ln(1 + x z_m) + \sum_{j \in \mathcal{Q}_m} z_j \alpha_j \right]. \tag{3.68}$$

The second order derivative at this point equals

$$\frac{\partial f(x_{mN})}{\partial x_{mN}} = \frac{-z_m^2}{(1 + x_{mN} z_m)^2 (1 + x_{mN})^3}. \tag{3.69}$$

suggesting a local maximum. The data volume with x_{mN} determined by (3.68) is

$$V_N \mid_{x_{mN} \neq 0} = \sum_{i=1}^{N} D_{iN} z_i \alpha_i. \tag{3.70}$$

Comparing (3.66) and (3.70) it is apparent that sensor m transmitting in time slot T_N, i.e. $V_N \mid_{x_{mN} \neq 0}$, achieves the global maximum for V_N.

To conclude, the solution for an N-sensor network is for sensors to transmit consecutively. Once a sensor is scheduled to transmit, it does so until it expires. The transmit power for each sensor is determined via (3.52), again independent of any battery energies. The resulting data volume is

$$V_{N_{max}} = \sum_{i=1}^{N} D_{iN} z_i \alpha_i. \tag{3.71}$$

The above provides the analytical solution for calculating the maximum data volume for a certain order of expiry for an N-sensor network - Step 3 of the brute force algorithm presented in Section 3.5. Once the maximum data volume for all the possible sensor expiry orders has been calculated, they can be compared and the highest chosen as the optimum solution which determines sensor lifetimes, transmit powers and scheduling - Step 4 Section 3.5.

3.7 The Effect of Processing Power

The aim of this section is to highlight the deviation of results when processing power is not modeled, as is the case in most of the sensor network literature. The comparison provides additional justification for the emphasis on the importance of modeling processing power. Since the model with no processing power is not the main focus of the dissertation, and the general procedure is similar to the case with processing power, some details are not discussed.

In the subsequent text, where processing power is not modeled, data volume is denoted by $\mathcal{V}$. Transmit SNR values are represented by γ.

3.7.1 One-Sensor Network

For the case of a single sensor i, its lifetime is,

$$T_1 = \frac{E_i}{P_{tx_{i1}}}. \tag{3.72}$$

Similar to (3.11) the data volume is

$$\mathcal{V}_1 = C_1 T_1 = B T_1 \ln(1 + \gamma_{i1}\xi_i) \tag{3.73}$$

where

$$\gamma_{i1} = \frac{P_{tx_{i1}}}{BN_o} \geq 0. \tag{3.74}$$

Substituting from (3.72) and using the definition of G given below

$$G_{i1} = \frac{E_i}{N_o} \tag{3.75}$$

$$\mathcal{V}_1 = \frac{BE_i}{P_{tx_{i1}}} \ln(1 + \gamma_{i1}\xi_i) = G_{i1} \frac{\ln(1 + \gamma_{i1}\xi_i)}{\gamma_{i1}}. \tag{3.76}$$

Differentiating the data volume with respect to γ_{i1} and setting the derivative to zero

$$\ln(1 + \gamma_{i1}\xi_i) = \frac{\gamma_{i1}\xi_i}{1 + \gamma_{i1}\xi_i}, \tag{3.77}$$

the solution is $\gamma_{i1} = 0$ which suggests

$$\mathcal{V}_{1_{max}} = G_{i1} \lim_{\gamma_{i1} \to 0} \frac{\ln(1 + \gamma_{i1}\xi_i)}{\gamma_{i1}} = G_{i1} \lim_{\gamma_{i1} \to 0} \frac{\xi_i}{1 + \gamma_{i1}\xi_i} = G_{i1}\xi_i \tag{3.78}$$

using L'Hopital's rule. Thus, the solution for a single sensor with no processing power is to transmit with close to zero transmit power, theoretically, for eternity. This is clearly very different from the normalized optimum transmit power defined in (3.14).

To assess the effect of modeling processing power on the maximum data volume, the two values with and without processing power, (3.18) and (3.78), are compared. Writing $V_{1_{max}}$ from (3.18) in terms of the variables that are used in this section yields

$$V_{1_{max}} = G_{i1}\xi_i \frac{1}{1+\gamma_{i_{opt}}\xi_i} \tag{3.79}$$

which shows that processing power introduces a loss factor of

$$\text{Loss}_1 = \frac{1}{1+\gamma_{i_{opt}}\xi_i} = \alpha_i. \tag{3.80}$$

3.7.2 Two-Sensor Network

Since there is no processing power, unless a sensor transmits, its battery will stay intact. Thus, unlike the previous case where processing power would deplete the battery, there is no maximum lifetime for the sensors. This being the case, there are two orders of expiry for the two sensors p and q where

$$\xi_p > 0 \ , \ \xi_q > 0 \tag{3.81}$$

$$\gamma_{p2} \geq 0 \ , \ \gamma_{q2} \geq 0. \tag{3.82}$$

Assuming sensor p expires first, it remains to be determined whether sensor q should also transmit in time period T_2 or not. Since p expires first

$$T_2 = \frac{E_p}{P_{tx_{p2}}} \ , \ G_{q2} - G_{q1} = G_{p2}\frac{\gamma_{q2}}{\gamma_{p2}} \tag{3.83}$$

$$\mathcal{V}_2 = G_{p2}\frac{\ln(1+\gamma_{p2}\xi_p+\gamma_{q2}\xi_q) - \gamma_{q2}\xi_q}{\gamma_{p2}} + G_{q2}\xi_q. \tag{3.84}$$

Defining $\mathcal{F}(\gamma_{p2}, \gamma_{q2})$ similar to $f(x_{p2}, x_{q2})$ in (3.27) as the term that is to be maximized, its first order derivative with respect to the transmit powers are

$$\frac{\partial \mathcal{F}(\gamma_{p2},\gamma_{q2})}{\partial \gamma_{p2}} = \frac{\frac{\xi_p\gamma_{p2}}{1+\gamma_{p2}\xi_p+\gamma_{q2}\xi_q} - \ln(1+\gamma_{p2}\xi_p+\gamma_{q2}\xi_q)+\gamma_{q2}\xi_q}{\gamma_{p2}^2} \tag{3.85}$$

$$\frac{\partial \mathcal{F}(\gamma_{p2},\gamma_{q2})}{\partial \gamma_{q2}} = \frac{\frac{\xi_q}{1+\gamma_{p2}\xi_p+\gamma_{q2}\xi_q} - \xi_q}{\gamma_{p2}}. \tag{3.86}$$

When set to zero (3.86) yields

$$\gamma_{p2}\xi_p + \gamma_{q2}\xi_q = 0. \tag{3.87}$$

Since $\xi_p > 0$ and $\xi_q > 0$, and the transmit powers are non-negative, the solution is

$$\gamma_{p2} = 0 \text{ and } \gamma_{q2} = 0. \tag{3.88}$$

Fig 3.4 shows the general form of $F(\gamma_{p2},\gamma_{q2})$.

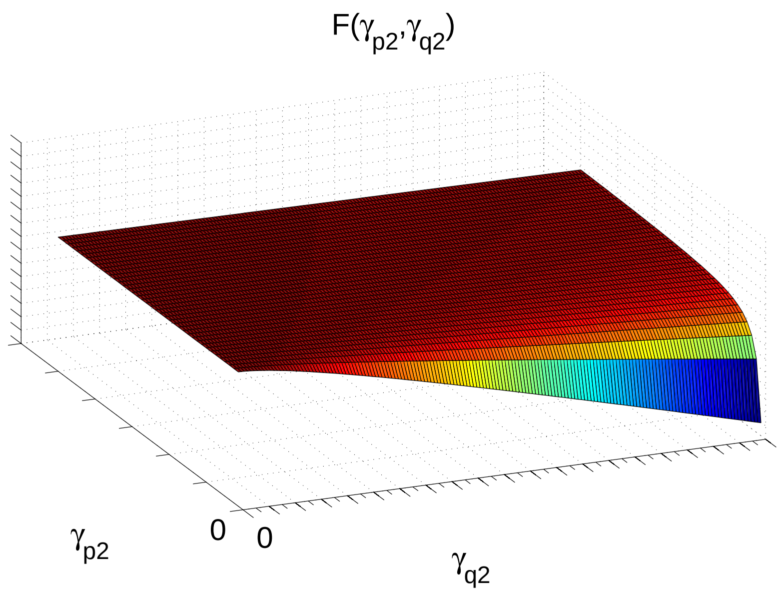

Figure 3.4: General form of $F(\gamma_{p2},\gamma_{q2})$. To plot the surface it is assumed $\xi_p = 0.5$ and $\xi_q = 0.7$. However, other than the origin of the axis the numbers have been omitted to preserve the generality of figure.

Given the optimum transmit powers, (3.88), the maximum data volume is

$$\mathcal{V}_{2_{max}} = G_{p2}\xi_p + G_{q2}\xi_q. \tag{3.89}$$

The maximum data volume when processing power is modeled can be rewritten in the form of

$$V_{2_{max}} = \frac{G_{p2}\xi_p}{1+\gamma_{p_{opt}}\xi_p} + \frac{G_{q2}\xi_q}{1+\gamma_{q_{opt}}\xi_q}. \tag{3.90}$$

As is apparent, the data volume loss factor due to processing power for any sensor i in a 2-sensor network is

$$\text{Loss}_2 = \frac{1}{1+\gamma_{i_{opt}}\xi_i} = \alpha_i. \tag{3.91}$$

3.7.3 N-Sensor Network

Up to this point optimum transmission schedules and transmit powers have been for a single-sensor and a 2-sensor network with no processing power. In this subsection the case of N sensors is studied using induction. To do so it is assumed that the maximum data volume for an $N-1$-sensor network is

$$\mathcal{V}_{N-1_{max}} = \sum_{i=1}^{N-1} G_{iN-1}\xi_i. \tag{3.92}$$

and that

$$\forall i : \gamma_{iN} \geq 0 \text{ and } \xi_i > 0. \tag{3.93}$$

If sensor m is the last sensor to expire the data volume for N sensors, based on the maximum data volume for a network of $N-1$ sensors, is

$$\mathcal{V}_N = G_{mN}\frac{\ln(1+\sum_{i=1}^{N}\gamma_{iN}\xi_i) - \sum_{i\in\mathcal{Q}_m}\gamma_{iN}\xi_i}{\gamma_{mN}} + \sum_{i\in\mathcal{Q}_m} G_{iN}\xi_i. \tag{3.94}$$

Defining $\mathcal{F}(\gamma_{1N}, \gamma_{2N}, \ldots, \gamma_{NN})$ similar to (3.58), to find its maximum the first order derivatives are set to zero. For any sensor $k \neq m$ the result is

$$\sum_{i=1}^{N} \gamma_{iN}\xi_{iN} = 0. \tag{3.95}$$

Based on (3.93) it is concluded that

$$\gamma_{iN} = 0 \tag{3.96}$$

is the optimum transmit power for any sensor i and the limit for the maximum data volume is

$$\mathcal{V}_{N_{max}} = \sum_{i=1}^{N} G_{iN}\xi_i. \tag{3.97}$$

Comparing the above data volume to (3.71) it is deduced that the data volume produced by sensor i is reduced by a factor of

$$\text{Loss}_N = \frac{1}{1+\gamma_{i_{opt}}\xi_i} = \alpha_i \tag{3.98}$$

when processing power is modeled compared to when it is not modeled regardless of the size of the network. $\gamma_{i_{opt}}$ represents the optimum transmit power for sensor i when processing power is modeled.

3.8 Simulations

As (3.71) shows, for a certain order of sensor expiry, when processing power is modeled the maximum data volume depends on the normalized battery charges, D, and the receive processing SNR, z, of all the sensors. The optimum transmit power for each sensor being independent of battery charges depends on its own processing SNR γ_p and channel gain ξ, and that of the sensors that expire after it. For each sensor γ_p and ξ in effect determine the receive processing SNR z. Figures 3.5 and 3.6 display these dependences. Note that all data volume curves represent the optimum N-sensor solution determined after studying all possible orders of sensor expiry. Throughout the simulations it is assumed that $B = 1$ and $N_o = 1$. Channel gains are selected independently from an exponential distribution, to model static Rayleigh fading.

To show the trend in which the optimum transmit power changes with the increase in channel gain, Fig. 3.5 shows this relation for the last sensor that expires in the network. The last sensor is chosen since its optimum transmit power and therefore α have no dependence on other sensors. As the graph shows, at higher values of z the optimum transmit power is not very sensitive to changes in the value of z.

In addition, α which represents the loss in data volume for each sensor due to processing power (3.98), is also plotted versus z on Fig. 3.5. MatLab estimates this curve as linear with a slope of -0.199.

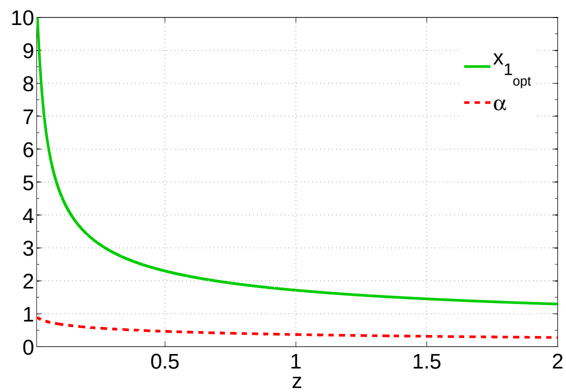

Figure 3.5: Optimum transmit power and loss in data volume due to processing power for the last sensor to expire.

Fig. 3.6 represents the average maximum data volume for a network of 5 identical sensors, equal processing SNR and initial battery energy, over 200 simulations. The top curve shows the increase in data volume with D and constant γ_p. The approximation curve shows that the increase in data volume with the increase in D, for 5 sensors and $\gamma_p = 0.5$, is almost linear with a slope of 0.4745.

There is a simple explanation for the linear correlation of the maximum data volume and D. The maximum data volume is calculated in accordance with (3.71). For any network given the channel gains, and therefore z_i's, optimum sensor transmit powers are calculated, independent of D's, employing equation (3.52). If the normalized battery charge D for all sensors increases, all elements except D remain constant in (3.71). Therefore the maximum data volume increases linearly with D, which can also be interpreted as the maximum number of dimensions available to the sensor. Since Monte Carlo simulations average out the effect of channel gain variations, the average maximum data volume curve shows linear dependence on D.

The lower curve in Fig. 3.6 demonstrates the dependence of data volume on processing SNR where the normalized battery charges are fixed, which means that the unnormalized battery charge grows proportional to the processing power. The data volume curve is roughly logarithmic as one would predict from (3.6) and (3.7).

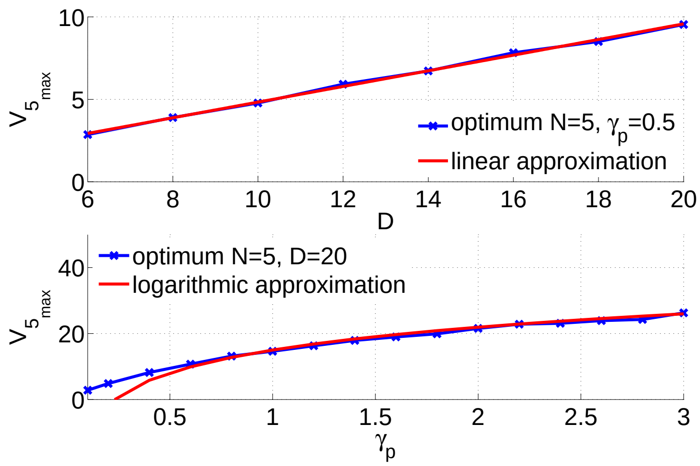

Figure 3.6: Maximum data volume in nats for a network of 5 identical sensors with a Rayleigh static channel averaged over 200 Monte Carlo simulations.

In the two following subsections the data volume and activity duration of the optimum policy is compared to two other scheduling policies.

3.8.1 Data Volume

Data volume is the performance criterion chosen in this dissertation to evaluate the performance of wireless sensor networks. The data volume produced by the optimum policy is compared to three other scheduling policies, the *strongest* sensor policy, a *random* order policy and the *fixed power* policy. In all cases only one sensor transmits in each time period. For the strongest and random policies the transmit power is determined via (3.52). For the fixed power policy the order of transmission is optimized with the fixed transmit powers.

The fixed power policy is implemented to show that it is power optimization more than scheduling that affects the data volume. The greatest optimum transmit power for the maximum data volume policy is averaged over all Monte Carlo simulations to determine the greatest fixed transmit power. The second greatest is used for the second fixed transmit power and so on. The fixed values are determined by rounding the average transmit powers for the maximum data volume policy to the closest integer since this makes it easier to deal with the numbers.

For the fixed power policy each transmit power is assigned to a sensor and the transmission order that produces the maximum data volume is found. Since sensors are assumed identical there is no need to experiment with transmit power to sensor assignments. The case which produces the maximum data volume is chosen as the solution and the order of transmission is noted as optimal.

In all transmit policies each sensor continues transmission until its battery is depleted. The only difference between the strongest and random methods, and the optimum policy is that they do not choose the optimum transmission order for the sensors. The strongest sensor policy gives priority to sensors with a greater channel gain, and the random policy ranks sensors in a random order for transmission.

Fig. 3.7 shows the maximum data volume for a star network in comparison with the non-optimum scheduling policies. The small gap between the random scheme and the optimum scheme shows that applying optimum transmit powers rather than the transmission order is the main factor in achieving high data volume. The poor performance of the fixed power scheme reenforces this conclusion.

As is obvious, with the increase in the number of sensors in the network the gap between the optimum scheduling policy and the alternative methods increases. However, the strongest and random policies remain close suboptimal alternatives. If achieving a close to optimum solution is acceptable either of these policies can be used as a low computation alternative. Note that, on average, the random policy performs slightly better than the strongest policy.

It is worth noting that regardless of the transmit policy the data volume grows linearly with the number of sensors in the network. For the values employed here, each additional sensor in the network increases the maximum data volume by approximately 1.4345 nats.

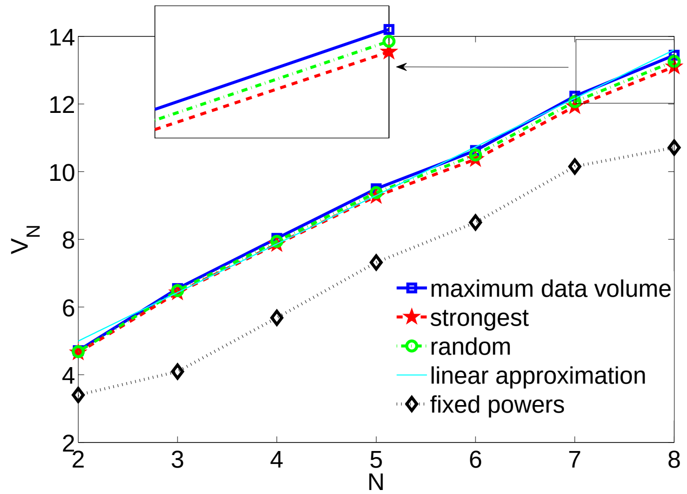

Figure 3.7: Data volume for N identical sensors with $D = 50$ and $\gamma_p = 0.5$. The curves show average data volume in nats for the maximized data volume, a linear approximation of the maximum data volume curve, the strongest channel, a random order of expiry, and fixed powers, over 400 Monte Carlo simulations.

3.8.2 Activity Duration

Since lifetime is a popular performance measure for sensor networks, the duration of network activity for the optimum transmission policy is compared to the alternatives introduced above. This duration is measured as the sum of the duration individual sensors transmit,

$$T_{\sum} = \sum_{i=1}^{N} T_i. \tag{3.99}$$

Similar to the data volume, the activity duration of the network increases with the number of sensors in the network. For the optimum policy, the network lasts roughly 2 time units more due to each additional sensor. It is interesting to point out that the random policy performs significantly better in terms of time compared to

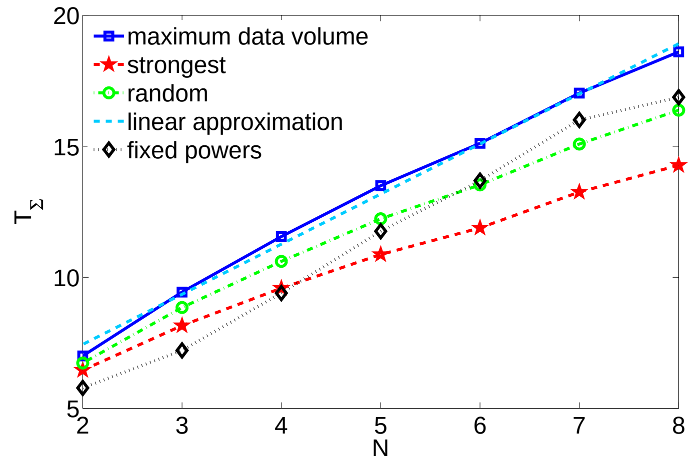

Figure 3.8: Activity duration for the optimum transmit policy, the strongest channel policy and the random policy, for a network with identical sensors using the same set of data as Fig. 3.7.

the strongest sensor policy. In addition, the gap between the optimum policy and the random policy goes from about 4% to 12% where as the gap for the strongest channel policy and the optimum solution grows from 8% to 23%, almost twice as wide.

Fig. 3.7 shows that fixed powers do not generate a high data volume even compared to suboptimal methods that employ optimum transmit powers. However, in the above graph, the network activity period for fixed powers is shown to surpass those suboptimal methods as the number of sensors in the network grows. The fixed powers policy optimizes the order in which sensors transmit. Because transmit powers are fixed, the transmit order that maximizes the data volume is one that pushes sensor transmission times to the maximum. With more sensors in the network, the effect of maximizing transmit times is greater and therefore the fixed power policy shows superior performance with more sensors compared to other suboptimal schemes.

3.9 Conclusions

With the introduction of data volume in Chapter 2, it is the aim of this chapter to derive the maximum data volume produced by an N-sensor static-channel star network analytically. With an information theoretic approach an analytical expression for the optimum transmit power for each sensor, the resulting data volume, and finally the tallied data volume of the network is developed.

It is demonstrated that to achieve the maximum data volume sensors should transmit consecutively. The fact that the optimum scheduling scheme is time division multiple access is intriguing since TDMA is assumed in many of the papers with static channel sensor networks, examples are [26, 35, 36, 42, 46, 70, 77]. However, this work is the first to show that time division multiplexing is optimal.

The optimum order in which sensors transmit is determined based on their battery charges and channel gains. As the simulations highlight, giving sensors priority based on channel gains, which is what one might expect, does not produce the optimum solution. In fact this policy is shown to produce lower data volume than a random transmission order.

Further it is determined that in the optimum transmission scheme, given the assumptions, each sensor should transmit until its battery is fully depleted, immediately after which the next sensor in line should start transmission. The optimum transmit power for each sensor depends on its processing power and channel gain and that of the sensors that expire after it.

This analysis is general in the sense that it does not limit the number of sensors in the network, their processing powers, or their battery charges, and is meant to serve both as a stepping stone to the time-varying channel solution and as a benchmark for performance in static channels. Unlike much of the work published until now the results presented in this chapter are analytical and optimum and do not rely on the convergence of iterative numerical solutions.

Processing power is included in the sensor model throughout this dissertation. Aside from presenting the optimum transmission policy for the star sensor network, this chapter also presents the solution that would arise if processing power were omitted from the sensor model. It is shown that the maximum data volume each sensor produces is reduced by a predetermined ratio due to processing power. This ratio depends on the sensor's channel gain and processing power, and that of the sensors that expire after it i.e. on the sensor's optimum transmit power when processing power is modeled.

From the simulations it is deduced that the maximum data volume produced by a network increases linearly with the increase in the normalized battery energy and the number of sensors in the network, and logarithmically with the processing power SNR when all other values are constant and sensors are identical. At minimum, this observation provides an easy to obtain estimate of the maximum data volume produced by a network of sensors.

It was also shown through simulations that allowing sensors to transmit in a random order does not degrade the data volume produced by the network much as long as transmit powers are determined using the optimum transmit power equation. However, it should be noted that the time until the last sensor expires is significantly shortened when employing suboptimal scheduling schemes; so the network as a whole is active for a shorter period of time.

Chapter 4

The Time-Varying Channel

Even if the sensors in a network are stationary, changes in the surrounding environment will cause the quality of the wireless link between the sensor and sink to change over time. Hence, assuming a time-varying channel model for wireless sensor networks is much more realistic than assuming a static channel.

In the previous chapter, the communication policy for a star wireless sensor network with static channels was optimized to produce the maximum data volume. Formulas for determining the optimum transmit power and calculating the resulting data volume were also derived. In this chapter, the channel model is upgraded to a more realistic time-varying model for which the optimum transmission policy is derived. Some of the work presented in this chapter has been published in [31].

4.1 System Model

The system model employed in this chapter is very similar to that of Chapter 3. Thus, only distinctions with the previous system model are discussed in the subsequent text.

4.1.1 The Elements

The wireless communication channel for each sensor is assumed to vary over time. For each sensor, channel gains vary from one time slot to the next independent of other sensors and previous channel gains. As discussed before, independence of sensor-sink connections is attained by assuming sensors are spaced more than one or two wavelengths apart. Since the information sink is responsible for scheduling communications, it is assumed aware of the sensor-sink channel quality.

4.1.2 Assumptions Made

Although time-varying channels are a more realistic channel model for wireless sensor networks, as will be seen in reviewing previous research in the following section, very little analytical work has been published employing this model. This shortage is mostly due to complications that arise with such channel models. To allow initial analysis on the time-varying channel for sensor networks, in addition to the assumptions mentioned in Section 3.1.3, the following assumptions are made.

Non-causality and Channel State Information

It is assumed that along with perfect knowledge of the battery charge and processing power of each sensor, the information sink has prescient knowledge of channel power gains for sensors throughout their lifetime. With non-causal knowledge of channel gains it is possible to determine the maximum data volume attainable by the network of sensors.

Developing the non-causal solution will demonstrate network performance limitations and provide a benchmark for the performance of the causal network. Once the non-causal solution and performance restrictions on the star network are fully understood, the next step would be to design a causal system that performs as close as possible to the non-causal optimum. It is the aim of this dissertation to provide the initial understanding and performance benchmarks.

Assuming prescient knowledge of channel gains is equivalent to assuming perfect channel predictions for the lifetime of all sensors. This assumption is not unusual when dealing with time-varying channels, and appears in many other publications e.g. [48].

Time Slots and Indexing

In this chapter, unlike the static channel case, time is divided into equal length slots T during which the channel power gain is assumed constant. It is also assumed that the channel power gain varies from every time slot to the next independent of all previous channel gains. The assumption that channel gains are constant within a time slot and change exactly as the system transfers to the next slot is an approximation. In reality channel gains vary continuously. To remedy this discrepancy, the duration of time slots is chosen short enough for the gap between the physical reality and the analytical model to be small. In other words, it is assumed that time slots are short enough to capture all channel changes and are therefore determined in relation to the correlation time of the channel.

Time slots are numbered starting at 1 going forward in time as demonstrated in Fig. 4.1. The difference in indexing time slots compared with the static channel case, although superficial, has a very big impact on the clarity and presentation of equations.

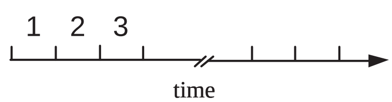

Figure 4.1: Time axis and indexing for the time-varying channel.

4.2 Prior Research

Publications listed in Chapter 3 are also relevant to the work presented in this chapter. However, references listed here account for a more sophisticated channel model compared to those introduced in the previous chapter.

Employing a star topology, [55] presents two approaches to minimizing energy consumption in a DS-CDMA based wireless sensor network through joint transmit power and transmission duration optimization for individual sensors. One suggested approach is to numerically compute the optimum solution by studying all possibilities. The second approach is to solve analytically an approximation of the problem by decoupling the joint optimization problem into two sequential problems: the transmit power optimization with transmission times factored in as parameters, and the transmission time optimization. An iterative method of alternating between the two optimizations is employed to reach the final solution. With a star network and assuming a static channel but accounting for shadowing, path loss, and static fading in the channel, and also processing power in the sensor model, the authors present numerical results that show the approximate analytical solution minimizes the network energy consumption to close proximity of the optimal numerical solution. Although the authors claim that their method can be implemented in real time, it seems that the computational burden required for each sensor would be overwhelming.

In a TDMA system, [35] presents a solution similar to [55]. Looking to maximize the network lifetime defined as the expiry of the first sensor, the authors take into account the unreliability of channel links by accounting for a failure probability in the channel model. It is noted that the acknowledgments of received signals are assumed to be sent at the highest power level and are thus not perturbed by failures.

The channel is affected by log-normal shadowing and sensors report channel statistics (the mean determined by distance and the variance of shadowing) recorded during the set up of the network. The sensor model accounts for electronic circuitry power consumption. However, sensors are assumed to enter sleep mode and not consume energy when they are not transmitting.

Based on the channel statistics, the sink determines routing paths for packets and either transmit powers (from one of the possible discrete values) or retransmission limits for sensors. When retransmission is allowed in the model, the problem is simplified so that all sensors transmit at the same power level and in turn the retry limit is chosen to minimize energy consumption per sensor. The general idea is that the energy consumed by each sensor is minimized by controlling its transmit power (or retry limit) while guaranteeing a minimum reliability constraint. The overall network lifetime is maximized by constricting usage of routing paths based on their total power consumption. Through numerical results it is shown that the retransmission method is not very energy efficient for dense networks.

[48] establishes network capacity regions for a wireless network of power constrained nodes with time-varying channels and adaptive transmission rates. The publication presents a routing and power allocation policy with which the system is stabilized and the average delay can be bounded when the arrival rates are within range. The solution requires solving an optimization problem for each time slot and is presented as both a centralized and decentralized scheme.

It is assumed that the channel is non-causally known to the network controller or sensor when determining the transmit power and in turn the transmission rate. In the decentralized implementation of the scheme users, through the exchange of information with neighbors, attempt to maximize the weighted sum of data rates for the network iteratively.

The work in [48] is presented as an optimization for any power constrained wireless network, however, it is only implemented both as a centralized scheme and a decentralized scheme for an ad hoc network. Through derivation of delay bounds it is shown that for ad hoc networks the bounds grow asymptotically with N/ξ, where N is the number of users and ξ represents a measure of distance between the arrival rates and the capacity region boundary.

Similar to the optimization presented for a static-channel star sensor network in Chapter 3 and what will be presented in this chapter for time-varying channels, [14] addresses joint scheduling and power optimization but for ad hoc networks. With assumptions such as known user locations, predetermined routing paths and centralized control, the authors take an iterative approach to selecting users who are permitted to transmit and the power with which they should transmit to satisfy the SINR (Signal to Interference and Noise Ratio) constraints at the receiver. The communication channel is static with path loss.

The idea behind their solution is that users spaced far enough apart have limited effect on each others SINR values and can therefore be permitted to transmit simultaneously. Based on this, in the first step of the process a group of users are granted permission to transmit. In the second step respective transmit powers are determined such that the SINR constraint is satisfied at the receiver and energy expenditures are kept to a minimum. Assuming that each user only transmits to one of its neighbors and accounting for path loss, the procedure converges to a final solution by alternating between the two steps. The authors also present heuristics that simplify implementation of their proposed algorithm.

Implementing a power control algorithm [44] attempts to minimize the energy required for successfully delivering each symbol. Using a star wireless sensor network as their network model, accounting for processing power, modeling the channel as static with shadowing and path loss, accounting for fading by allowing a 20dB small scale fading margin, and using DSSS with BPSK modulation, the authors implement an iterative method for determining the transmit power in a system affected by multiple access interference.

Interferences from out of cluster sensors are modeled as additional thermal noise for each sensor. For interference from sensors within the cluster the queuing method introduced in [51] is employed. It is shown that the complexity of computing transmit powers via the proposed algorithm is no more than computing transmit powers for a system that only accounts for noise in the received signal and does not model interferences from other sensors.

4.2.1 Unlimited Energy Counterpart

The analytical work by Knopp and Humblet [34] is the unlimited energy counterpart to the solution presented in this chapter. This subsection summarizes their work which is compared to the results obtained in this chapter in Section 4.8.1.

In [34] the authors determine the optimum transmit power and scheduling policy that maximizes the capacity in the uplink of a single-cell multiuser communication system. The layout of the system constitutes a star network, the channel is time varying and affected by fading, and the average user transmit power is limited. Fading gains are continuous values with a known statistical distribution. Each user and the base station have one antenna.

It is analytically shown that the optimum scheduling scheme for this network is for only the user with the strongest channel gain to transmit while all other users stay silent. The Lagrangian multipliers determined during this optimization, with knowledge of the joint probability distribution function of the channel gains, constitute optimum sensor water levels. Thus, the optimum transmit power for each sensor is determined via water-filling given its channel gain and optimum water level.

4.3 Problem Statement

The problem statement for this chapter is very similar to that of Chapter 3.

> ***Given a star wireless sensor network with time-varying channels, find the optimum transmission policy that maximizes the data volume.***

4.4 Notation

There are only a few changes to the notation introduced in Section 3.4. Since time slots are now defined with equal length and numbered in time sequence, T represents the length of the time slots and is not indexed. In turn, the total number of time slots has no relation to N the number of sensors in the network. A new parameter, n, is introduced to represent sensor lifetimes and is indexed with the sensor number.

With the new time-varying channel model, (3.5) for any sensor i is rewritten so the normalized battery energy is

$$D_i = \frac{E_i B}{P_{p_i}} = BT\mathcal{L}_i \tag{4.1}$$

where $\mathcal{L}_i$ is the maximum number of time slots before the sensor's battery depletes. It is treated as an integer. $T\mathcal{L}_i$ is therefore the maximum sensor lifetime. The sensor achieves its maximum lifetime when it does not transmit any data, therefore possible values for n_i are

$$n_i \in [1, \mathcal{L}_i]. \tag{4.2}$$

Also note that the normalized battery energy D is only indexed with the sensor number, since the time index is not necessary for this solution, and represents the initial battery energy for the sensor. The data volume V_N, assuming sensor m is the last sensor to expire, is

$$V_N = T \sum_{t=1}^{n_m} C_t \tag{4.3}$$

where C_t is the sum-rate capacity in time slot t.

4.5 Brute Force Algorithm

The general approach towards establishing the optimum transmission policies for a time-varying channel is very similar to that of stationary channels. The outcome, however, is not. To determine optimum transmit powers and schedules, and in turn sensor lifetimes, the brute force approach is taken where the maximum data volume is determined for all possible sensor lifetime combinations and the greatest is chosen as the final solution. Once the properties of the solution are established in this chapter, methods for significant computation reduction are presented in Chapter 5, and used to revise the brute force algorithm, Section 5.3.3.

The brute force approach to determining the maximum data volume for an N-sensor star wireless network are as follows:

1. find the maximum sensor lifetime for all sensors, $\mathcal{L}_i$ where $i \in [1, N]$
2. determine all combinations of sensor lifetimes; for sensor i possible lifetimes are $n_i \in [1, \mathcal{L}_i]$
3. for each combination of sensor lifetimes calculate the maximum data volume it can produce, by optimizing transmit power levels and transmission schedules
4. choose the combination of lifetimes that produces the highest data volume as the final solution - the solution also includes transmit power levels and transmission schedules

In the next section, the analytical solution to Step 3 of the above brute force algorithm is presented.

4.6 Maximum Data Volume Given Lifetimes

As with the stationary channel, the solution for a sensor network of any size is derived using induction. This approach also underscores the pattern in the mathematical formulas. The reader is reminded that this derivation is for one combination of sensor lifetimes and is to be repeated for all possible combinations.

4.6.1 One-Sensor Network

With only sensor i connected to the information sink if the assumed lifetime is n_i

$$V_1 = T\sum_{t=1}^{n_i} C_t = BT\sum_{t=1}^{n_i} \ln(1 + x_{it}z_{it}) \tag{4.4}$$

where

$$\forall t : x_{it} \geq 0 \tag{4.5}$$

and

$$E_i = T(\sum_{t=1}^{n_i} P_{tx_{it}} + n_i P_{p_i}). \tag{4.6}$$

or equivalently,

$$D_i = BT(\sum_{t=1}^{n_i} x_{it} + n_i). \tag{4.7}$$

which can be rewritten as

$$\sum_{t=1}^{n_i} x_{it} = \frac{D_i}{BT} - n_i. \tag{4.8}$$

It is apparent from (4.4) that maximizing the data volume for a fixed n_i is analogous to maximizing the sum of capacities over n_i parallel and independent channels with non-negative fixed-sum transmit powers. The solution to this problem is known to be water-filling in time [8] and is derived by writing out the Lagrangian to maximize

(4.4) given (4.7) and differentiating it with respect to x_{it} where (4.5) holds. With $\mu_i > 0$ the Lagrangian is

$$\begin{aligned} Lag &= V_1 + \mu_i \left(D_i - BT(\sum_{t=1}^{n_i} x_{it} + n_i) \right) \\ &= BT \sum_{t=1}^{n_i} \ln(1 + x_{it} z_{it}) + \mu_i \left(D_i - BT(\sum_{t=1}^{n_i} x_{it} + n_i) \right). \end{aligned} \tag{4.9}$$

Therefore, the optimum transmit power for sensor i is

$$\forall t \leq n_i : x_{it_{opt}} = \left(\frac{1}{\mu_i} - \frac{1}{z_{it}} \right)^+ \tag{4.10}$$

where the plus sign indicates that this is the value for x_{it} if the value inside the parentheses is positive, otherwise, $x_{it_{opt}} = 0$. The fraction $1/\mu_i$ is the water level and transmission only takes place in time slots where the water level is greater than the inverse channel gain. The water level is selected to satisfy (4.7) or equivalently (4.8). Using (4.10) the data volume equation can be written as

$$V_{1_{max}} = BT \sum_{t \in \mathcal{U}_i} \ln(\frac{z_{it}}{\mu_i}) \tag{4.11}$$

where $\mathcal{U}_i$ is the time slots where transmission takes place.

Applying the brute force algorithm to the single-sensor network (4.4), or (4.11), and (4.10) are employed to calculate the maximum data volume for all possible sensor lifetimes. The greatest data volume is chosen as the solution and the lifetime n_i and water level $1/\mu_i$ associated with it represent the optimum lifetime and water level for sensor i.

4.6.2 Two-Sensor Network

For a network of two sensors p and q connected to the information sink, the data volume for any combination of sensor lifetimes n_p and n_q assuming sensor p expires before sensor q, i.e. $n_p \leq n_q$, is

$$V_2 = T\sum_{t=1}^{n_q} C_t = BT\sum_{t=1}^{n_p}\ln(1 + x_{pt}z_{pt} + x_{qt}z_{qt}) + BT\sum_{t=n_p+1}^{n_q}\ln(1 + x_{qt}z_{qt}). \qquad (4.12)$$

If $n_p = n_q$ the second term is zero. The following conditions also apply to the two-sensor network:

$$D_p = BT(\sum_{t=1}^{n_p} x_{pt} + n_p) \qquad (4.13)$$

$$D_q = BT(\sum_{t=1}^{n_q} x_{qt} + n_q) \qquad (4.14)$$

$$\forall t : x_{pt} \geq 0\ ,\ x_{qt} \geq 0. \qquad (4.15)$$

Given the above and assuming

$$\mu_p > 0\ ,\ \mu_q > 0 \qquad (4.16)$$

the Lagrangian is

$$Lag = V_2 + \mu_p\left(D_p - BT(\sum_{t=1}^{n_p} x_{pt} + n_p)\right) + \mu_q\left(D_q - BT(\sum_{t=1}^{n_q} x_{qt} + n_q)\right). \qquad (4.17)$$

In the subsequent text, by showing that simultaneous transmission is not optimum for this network, it is proven that the optimum scheduling policy for the two-sensor time-varying system is for at most one sensor to transmit in each time slot. To show that simultaneous transmission of the two sensors is not optimum it is shown that the first order derivatives of the Lagrangian with respect to the transmit powers of p and q cannot be zero simultaneously.

For both sensors to transmit simultaneously in time slot m, where

$$1 \leq m \leq n_p \leq n_q, \qquad (4.18)$$

it would require

$$x_{pm} > 0 \text{ and } x_{qm} > 0. \tag{4.19}$$

Setting the first order derivatives of the Lagrangian with respect to these transmit powers to zero results in

$$\frac{z_{qm}}{1 + x_{pm}z_{pm} + x_{qm}z_{qm}} = \mu_q \tag{4.20}$$

$$\frac{z_{pm}}{1 + x_{pm}z_{pm} + x_{qm}z_{qm}} = \mu_p \tag{4.21}$$

It is therefore concluded that for the two sensors to transmit simultaneously

$$\frac{z_{qm}}{z_{pm}} = \frac{\mu_q}{\mu_p}. \tag{4.22}$$

The probability of the ratio of fictitious receive SNR values for the two sensors in time slot m, z_{qm} and z_{pm}, equaling a fixed value is zero since z depends on the channel gain of each sensor in time slot m and is therefore not predictable. In turn, it is concluded that since the first order derivatives of the Lagrangian cannot equal zero simultaneously, the optimum transmit policy that maximizes the data volume is for at most one sensor to transmit in each time slot.

What is now left is to determine the transmit powers. If the transmit power for sensor q was greater than zero in time slot $1 \leq t \leq n_p$ sensor p would not transmit in this time slot, i.e. $x_{pt} = 0$, and the optimum transmit power for q would satisfy

$$\frac{z_{qt}}{1 + x_{qt_{opt}} z_{qt}} = \mu_q \tag{4.23}$$

which results from setting the first order derivative of the data volume with respect to x_{qt} to zero. Rewriting the above equation and accounting for the positivity constraint on transmit powers

$$\forall t \leq n_p : x_{qt_{opt}} = \left(\frac{1}{\mu_q} - \frac{1}{z_{qt}} \right)^+. \tag{4.24}$$

For any time slot t after sensor p expires, $n_p < t \leq n_q$, the solution for sensor q is that of a single-sensor network with water level $1/\mu_q$. Thus, the transmit power for sensor q is determined by water-filling over all time slots it has access to

$$\forall t \leq n_q : x_{qt_{opt}} = \left(\frac{1}{\mu_q} - \frac{1}{z_{qt}}\right)^+ . \tag{4.25}$$

For any time slot $1 \leq t \leq n_p$ in which sensor p transmits the optimum transmit power satisfies

$$\frac{z_{pt}}{1 + x_{pt_{opt}} z_{pt}} = \mu_p \tag{4.26}$$

or with the positivity constraint added

$$\forall t \leq n_p : x_{pt_{opt}} = \left(\frac{1}{\mu_p} - \frac{1}{z_{pt}}\right)^+ . \tag{4.27}$$

To maximize the data volume, the optimum transmit policy is for at most one of the sensors to transmit in each time slot and the optimum transmit power for a two-sensor network is determined via water-filling in time over their respective lifetimes. The brute force approach to deciding which sensor has access to each time slot is to calculate the data volume produced by all possible assignments of time slots to sensors and choose the highest resulting data volume as the winning combination.

Knowing that no simultaneous transmissions occur the data volume produced by the network is equal to the sum of the data volumes produced by individual sensors

$$\begin{aligned} V_{2_{max}} &= BT \sum_{t=1}^{n_p} \ln(1 + x_{pt_{opt}} z_{pt}) + BT \sum_{t=1}^{n_q} \ln(1 + x_{qt_{opt}} z_{qt}) \\ &= BT \sum_{i \in \{p,q\}} \sum_{t=1}^{n_i} \ln(1 + x_{it_{opt}} z_{it}). \end{aligned} \tag{4.28}$$

Using (4.25) and (4.27) the maximum data volume for lifetimes n_p and n_q can be written as

$$V_{2_{max}} = BT \sum_{i \in \{p,q\}} \sum_{t \in \mathcal{U}_i} \ln(\frac{z_{it}}{\mu_i}). \tag{4.29}$$

From the brute force algorithm, Section 4.5, it is known that to find the optimum network solution the maximum data volume computation is to be performed for all possible n_p and n_q combinations.

4.6.3 N-Sensor Network

Continuing with the inductive method, to determine the solution for an N-sensor network, it is assumed the optimum transmit policy for an $(N-1)$-sensor network is to avoid simultaneous transmissions and that transmit powers are determined by water-filling over the optimum time slots assigned to each sensor. The resulting data volume is

$$V_{(N-1)_{max}} = BT \sum_{i=1}^{N-1} \sum_{t=1}^{n_i} \ln(1 + x_{it_{opt}} z_{it}) \tag{4.30}$$

$$= BT \sum_{i=1}^{N-1} \sum_{t \in \mathcal{U}_i} \ln(\frac{z_{it}}{\mu_i}) \tag{4.31}$$

with

$$\forall t \leq n_i : x_{it_{opt}} = \left(\frac{1}{\mu_i} - \frac{1}{z_{it}} \right)^+ \tag{4.32}$$

for the optimum time slot allocation. For an N-sensor network assuming sensor r is the first sensor to expire and based on the $(N-1)$-sensor solution

$$V_N = BT \sum_{t=1}^{n_r} \ln(1 + \sum_{i=1}^{N} x_{it} z_{it}) + BT \sum_{i \in \mathcal{Q}_r} \sum_{t=n_r+1}^{n_i} \ln(1 + x_{it} z_{it}). \tag{4.33}$$

The following equations also hold for all sensors:

$$\forall i : D_i = BT(\sum_{t=1}^{n_i} x_{it} + n_i) \tag{4.34}$$

$$\forall t \wedge \forall i : x_{it} \geq 0. \tag{4.35}$$

Employing the above and given that

$$\forall i \in [1, N] : \mu_i > 0, \tag{4.36}$$

the Lagrangian is

$$Lag = V_N + \sum_{i=1}^{N} \mu_i \left(D_i - BT(\sum_{t=1}^{n_i} x_{it} + n_i) \right). \tag{4.37}$$

To maximize the data volume the first order derivatives of the Lagrangian with respect to transmit powers x_{it}'s are set to zero. In time slot m, where

$$1 \leq m \leq n_r, \tag{4.38}$$

for any sensor k this yields

$$\frac{z_{km}}{1 + \sum_{i=1}^{N} x_{im} z_{im}} = \mu_k. \tag{4.39}$$

Similar to the two-sensor case, for any two sensors, if simultaneous transmission is to be optimum the ratio of their fictitious receiver SNR values z's should be fixed. This, again, is an event with probability zero. Thus, in the optimum transmission scheme for the N-sensor network, no two sensors can transmit simultaneously.

After sensor r expires, $t > n_r$, and $N-1$-sensor network remains where optimum transmit powers are determined via water-filling over time and the water level is equal to $1/\mu_i$ for any sensor i. Taking into account the positivity constraints on transmit powers for all sensors (4.35), the optimum transmit power for any sensor in an N-sensor network is determined by

$$\forall i \wedge (\forall t \leq n_i) : x_{it_{opt}} = \left(\frac{1}{\mu_i} - \frac{1}{z_{it}} \right)^+. \tag{4.40}$$

No overlapping transmissions allows the simplification of the maximum data volume equation to the sum of individual data volumes

$$V_{N_{max}} = BT \sum_{i=1}^{N} \sum_{t=1}^{n_i} \ln(1 + x_{it_{opt}} z_{it}) \tag{4.41}$$

$$= BT \sum_{i=1}^{N} \sum_{t \in \mathcal{U}_i} \ln(\frac{z_{it}}{\mu_i}) \tag{4.42}$$

where $\mathcal{U}_i$ is the time slots in which sensor i transmits. To determine which time slot is best assigned to which sensor, all possible time slot to sensor assignments are checked and the case producing the highest data volume is chosen as the optimum solution for the lifetime combination n_1 to n_N. This exhaustive procedure is computationally infeasible, but is presented to show the properties of the optimum solution. Chapter 5 will introduce a number of algorithms for greatly reducing the computational complexity.

As with the single-sensor and two-sensor networks, according to Section 4.5, maximum data volume calculations are performed for all possible lifetime combinations to determine the optimum transmission policy.

4.7 The Effect of Processing Power

The solution for an N-sensor network where processing power is not included in the power expenditure model and the channels are time-varying is based on the solutions in Sections 3.6 and 4.6. Since the general procedure has been introduced before, fewer details are discussed in this section.

4.7.1 One-Sensor Network

For a single sensor i with no processing power and lifetime n_i, the data volume is

$$\mathcal{V}_1 = BT \sum_{t=1}^{n_i} \ln(1 + \gamma_{it} \xi_{it}) \tag{4.43}$$

with

$$\gamma_{it} = \frac{P_{tx_{it}}}{BN_o} \geq 0. \tag{4.44}$$

Defining

$$G_i = \frac{E_i}{N_o} \tag{4.45}$$

the equation that balances energy becomes

$$G_i = BT \sum_{t=1}^{n_i} \gamma_{it}. \tag{4.46}$$

Given the above condition the Lagrangian is formed,

$$\begin{aligned} Lag =& \mathcal{V}_1 + \lambda_i \left(G_i - BT \sum_{t=1}^{n_i} \gamma_{it} \right) \\ =& BT \sum_{t=1}^{n_i} \ln(1 + \gamma_{it}\xi_{it}) + \lambda_i \left(G_i - BT \sum_{t=1}^{n_i} \gamma_{it} \right). \end{aligned} \tag{4.47}$$

To determine the optimum transmit power in time slot t the Lagrangian is differentiated with respect to γ_{it} and the outcome is set equal to zero. For any t this produces

$$\lambda_i = \frac{\xi_{it}}{1 + \gamma_{it}\xi_{it}}. \tag{4.48}$$

Coupled with the positivity constraint for the transmit powers, the optimum transmit power for sensor i and the maximum data volume are found to be

$$\gamma_{it_{opt}} = \left(\frac{1}{\lambda_i} - \frac{1}{\xi_{it}} \right)^+ \tag{4.49}$$

$$\mathcal{V}_{1_{max}} = BT \sum_{t=1}^{n_i} \ln(1 + \gamma_{it_{opt}}\xi_{it}) = BT \sum_{t \in \mathcal{I}_i} \ln(\frac{\xi_{it}}{\lambda_i}) \tag{4.50}$$

where $\mathcal{I}_i$ is the set of time slots where transmission takes place. The composition of $\mathcal{I}_i$ brings about an important discussion.

When processing power is not modeled, sensors do not consume energy unless they are transmitting. Thus, their batteries can theoretically last infinitely long. In addition, if transmission takes place over an infinite duration of time, the water level and transmit powers will be close to zero and the data volume will be infinitely large. If the channel length is finite, the maximum data volume depends on the strongest channel gains, but not when they occur, unlike the case with processing power.

The loss factor due to modeling processing power is the ratio of the data volumes with (4.11) and without (4.50) processing power.

$$\mathrm{Loss}_1 = \frac{\sum_{t \in \mathcal{U}_i} \ln(\frac{\gamma_{p_i} \xi_{it}}{\mu_i})}{\sum_{t \in \mathcal{I}_i} \ln(\frac{\xi_{it}}{\lambda_i})} \tag{4.51}$$

If a slightly different loss ratio is defined,

$$e^{\mathrm{Loss}_1} = \frac{e^{\sum_{t \in \mathcal{U}_i} \ln(\frac{\gamma_{p_i} \xi_{it}}{\mu_i})}}{e^{\sum_{t \in \mathcal{I}_i} \ln(\frac{\xi_{it}}{\lambda_i})}} = \frac{\Pi_{t \in \mathcal{U}_i} \frac{\gamma_{p_i} \xi_{it}}{\mu_i}}{\Pi_{t \in \mathcal{I}_i} \frac{\xi_{it}}{\lambda_i}} \tag{4.52}$$

where γ_{p_i}, previously introduced in (3.2), is

$$\gamma_{p_i} = \frac{P_{p_i}}{BN_o}. \tag{4.53}$$

The data volume loss factor due to processing power depends on the the optimum water levels with and without processing power. If the definition of (4.52) is used, based on the number of time slots in $\mathcal{U}_i$ and $\mathcal{I}_i$, the loss ratio may depend on the ratio of water levels with and without processing power.

4.7.2 Two-Sensor Network

With two sensors p and q, while $n_p \leq n_q$, the data volume is

$$\mathcal{V}_2 = BT \sum_{t=1}^{n_p} \ln(1 + \gamma_{pt}\xi_{pt} + \gamma_{qt}\xi_{qt}) + BT \sum_{t=n_p+1}^{n_q} \ln(1 + \gamma_{qt}\xi_{qt}). \tag{4.54}$$

It is simple to show that for any value of t, given the conditions, the first order derivatives of the Lagrangian with respect to γ_{pt} and γ_{qt} cannot be zero simultaneously, and therefore conclude that the optimum schedule is for sensors to avoid simultaneous transmission. Similar to Section 4.6.2 the optimum transmit power for both sensors is found by water-filling over time and the maximum data volume is of the form

$$\mathcal{V}_{2_{max}} = BT \sum_{i\in\{p,q\}} \sum_{t=1}^{n_i} \ln(1+\gamma_{it_{opt}}\xi_{it}) = BT \sum_{i\in\{p,q\}} \sum_{t\in\mathcal{I}_i} \ln(\frac{\xi_{it}}{\lambda_i}) \tag{4.55}$$

where λ_p and λ_q are the inverse optimum water levels. The data volume loss per sensor due to processing power for any sensor i in a 2-sensor sensor network is the same as the single-sensor case,

$$\text{Loss}_2 = \frac{\sum_{t\in\mathcal{U}_i} \ln(\frac{\gamma_{p_i}\xi_{it}}{\mu_i})}{\sum_{t\in\mathcal{I}_i} \ln(\frac{\xi_{it}}{\lambda_i})}. \tag{4.56}$$

The definition of (4.52) can also be used.

4.7.3 N-Sensor Network

To complete the inductive proof, the $N-1$-sensor solution is assumed known. Accordingly, the data volume for the N-sensor network assuming sensor r is the first sensor to expire is

$$\mathcal{V}_N = BT \sum_{t=1}^{n_r} \ln(1+\sum_{i=1}^{N}\gamma_{it}\xi_{it}) + BT \sum_{i\in\mathcal{Q}_r} \sum_{t=n_r+1}^{n_i} \ln(1+\gamma_{it}\xi_{it}) \tag{4.57}$$

Similar to previous derivations, it is easy to show that the first order derivatives of the data volume with respect to transmit powers can not be zero simultaneously. This in turn implies that single-sensor transmission is the optimum scheduling scheme. Following the same steps as when processing power was accounted for, it can be shown that optimum transmit powers are determined via water-filling in time while avoiding simultaneous transmissions. In other words, after trying out all the possible time slot to sensor allocations and finding the one that maximizes the data volume, the transmit power for sensor i is determined by

$$\gamma_{it_{opt}} = \left(\frac{1}{\lambda_i} - \frac{1}{\xi_{it}}\right)^+ \tag{4.58}$$

in the optimum time slots, $\mathcal{I}_i$, it is allocated. The maximum data volume is

$$\mathcal{V}_{N_{max}} = BT\sum_{i=1}^{N}\sum_{t=1}^{n_i}\ln(1+\gamma_{it_{opt}}\xi_{it}) = BT\sum_{i=1}^{N}\sum_{t\in\mathcal{I}_i}\ln(\frac{\xi_{it}}{\lambda_i}) \tag{4.59}$$

The data volume loss per sensor due to processing power for any sensor i in the N-sensor network is

$$\text{Loss}_N = \frac{\sum_{t\in\mathcal{U}_i}\ln(\frac{\gamma_{p_i}\xi_{it}}{\mu_i})}{\sum_{t\in\mathcal{I}_i}\ln(\frac{\xi_{it}}{\lambda_i})} \tag{4.60}$$

or similar to (4.52)

$$e^{\text{Loss}_N} = \frac{\Pi_{t\in\mathcal{U}_i}\frac{\gamma_{p_i}\xi_{it}}{\mu_i}}{\Pi_{t\in\mathcal{I}_i}\frac{\xi_{it}}{\lambda_i}} \tag{4.61}$$

which implies that, regardless of the number of sensors in the network, per-sensor data volume loss due to processing power depends on optimum water levels with and without processing power.

4.8 Conclusions

In this chapter the transmission policy that maximizes the data volume for a star wireless sensor network with time-varying communication channels is discussed. The optimum policy reveals sensor transmit powers, transmission schedules and sensor lifetimes.

A brute force approach is taken towards finding the optimum solution: the maximum data volume produced by each possible sensor lifetime combination is calculated and the highest overall data volume is chosen as the final solution. For each sensor its possible lifetimes are determined based on its maximum lifetime derived when it does not transmit any data.

The maximum data volume for a combination of sensor lifetimes is derived analytically using induction. It is shown that in the optimum scheduling scheme sensors avoid simultaneous transmission. Once again, a brute force approach is taken in assigning time slots to sensors. All possible combinations are tried and the assignment that produces the highest data volume is chosen as the solution for this combination of sensor lifetimes. It is also shown that optimum transmit powers are determined via water-filling in the optimum time slots assigned to each sensor. Since there are no overlapping transmissions in the optimum schedule, network data volume is calculated as the sum of individual sensor data volumes.

The effect of processing power on the maximum data volume is assessed. It is shown that with processing power the data volume produced by individual sensors is reduced by a fraction that depends on the sensor's processing power and optimum water levels with and without processing power. This fraction represents the excess data volume calculated when processing power is not accounted for in the sensor model and can be accessed as a pre-calculated error factor.

4.8.1 Unlimited Energy Counterpart

The optimum transmission policy, that maximizes capacity in a star network with time-varying channels and unlimited energy nodes, is for only the node with the strongest channel gain to transmit in any time slot [34]. The optimum transmit power is determined via water-filling in the time slots assigned to each node with the optimum water level calculated based on the joint probability distribution function of channel gains.

Although the optimum transmit policy for the wireless sensor network is single-sensor transmissions and the optimum transmit powers are determined via water-filling in time slots designated to each sensor, the assignment of time slots to sensors is not so clear cut. It depends on both the processing power and remaining battery charge pertaining to each sensor, factors that do not influence unlimited energy nodes. In addition, the optimum water level is determined based on channel gains and battery charges of individual sensors and not the probability distribution function.

There is a simple explanation to why assigning a time slot to the sensor with the strongest channel gain is not always optimum for the wireless sensor network. If the sensor with the strongest gain does not transmit in the time slot assigned to it, no data volume is produced in that time slot. In addition, all other sensors consume processing power during that period. Thus, energy is consumed in all the network and no data volume is produced. This is of great concern to a wireless sensor network for whom assigning the time slot to a sensor that would transmit data is a more energy efficient policy. In the case of unlimited energy nodes, energy is not a limitation to the network and therefore nodes that do not have the strongest channel gain have the luxury of waiting for a time slot with desired specifications to which they do have access. In this sense unlimited energy nodes behave like sensors with no processing power.

4.8.2 Static Channel

The optimum scheduling scheme for a star sensor network regardless of whether the channel is static or time-varying is for sensors to avoid simultaneous transmissions. In the static channel each sensor transmits until its battery expires and then the next sensor in line begins transmission. In the case of the time-varying channel the solution is more complicated. The sensor that transmits in each time slot is determined by finding the optimum assignment.

Optimum transmit powers for the static channel are determined by finding roots to equations which depend on all the sensors still alive in the network and are independent of the remaining battery energies. Optimum transmit powers for the time-varying channel are determined via water-filling over the time slots assigned to each sensor. For this reason, transmit powers for the time-varying channel have a strong dependence on the battery charge. Each sensor's transmit power relies on other sensors in the sense that they affect the time slots in which it can transmit.

Throughout this chapter all solutions are found by taking the most obvious path, the brute force approach. Despite being straight forward, brute force approaches are computationally expensive. In the next chapter, based on the properties of the solution, a few methods are presented that unload the burden of checking all possible sensor lifetime combinations and all time slot to sensor assignments. Other ideas that further simplify the process are also introduced.

Chapter 5

Computing the Data Volume

To determine the optimum transmit policy for an N-sensor wireless network the brute force approach is to calculate the maximum data volume for each N-sensor lifetime combination and choose the highest value as the final solution. However, this requires extensive computation. In this chapter two sets of solutions are used to remedy this problem.

In Chapter 4 it is shown that the data volume for a given combination of lifetimes is maximized when the transmission schedule contains no simultaneous transmissions. The brute force approach is to calculate the data volume produced by all possible time slot to sensor allocations and choose the candidate with the greatest data volume as the solution. In other words, since optimum transmit powers are determined via water-filling, for each possible allocation perform water-filling over all time slots assigned to each sensor, tally individual data volumes to obtain the network data volume and compare the outcomes. This is an exhaustive and computationally expensive method. In this chapter a simple low-computation solution - low-computation compared to the brute force approach - is introduced for finding the maximum data volume along with optimum transmit powers for given sensor lifetimes.

Any reduction in computation for one combination of lifetimes is multiplied by the number of combinations that require such calculations. Therefore the effect of computation reduction is significant and will grow with the size of the network. If computing the final solution is less burdensome, with the computational power available, it is possible to compute the solution for larger networks and sensors with greater battery energy.

The second set of methods introduced in this chapter omit the need to perform calculations for all lifetime combinations. These methods are used to revise the brute force algorithm and drastically reduce its computational burden. It is proven that the simplifications preserve the optimality of the solution. Simulations are employed to emphasize the effectiveness of these schemes.

5.1 The Contention Resolution Tree (CRT)

In the next two sections where the Contention Resolution Tree is discussed, unless otherwise stated, the term "*optimum*" is used to refer to the "*optimum solution with given lifetimes*" i.e. the scheduling and transmit powers that produce the maximum data volume for the lifetime combination in question. In general "*optimum solution*" is used to refer to the "*optimum network solution*" which accounts for optimum sensor lifetimes in addition to optimum scheduling and transmit powers.

The Contention Resolution Tree (CRT) is based on the idea that it is more efficient to allow sensors to choose time slots they would like to transmit in and work to resolve conflicts rather than try out all possibilities of time slot to sensor allocations, many of which are not even close to the optimum allocation. By employing the CRT, only a fraction of the time slot to sensor allocations need to be investigated.

The CRT only deals with time slots which are of interest to more than one sensor. In other words, only cases of conflict appear on the tree, since there is no need to discuss possible allocations of a time slot for which there is no competition. Each leaf on the resolution tree constitutes one option for resolving the conflict and it is shown in Section 5.2 that the optimum solution is among the leaves.

Given the lifetime for each sensor, single-sensor water-filling is employed to determine the time slots each sensor prefers for transmission. Ignoring all other sensors in the network, water-filling produces the highest data volume possible for individual sensors given their lifetimes - proven in Section 4.6.1. At this point the sum of individual data volumes is not equal to the network data volume since there are simultaneous transmissions in sensor schedules. The overlaps in transmission resulting from this exercise are used to construct the first branches of the CRT.

To facilitate the understanding of the construction technique, this section is continued in the form of an example.

5.1.1 An Example

To demonstrate how the tree is formed an 8-sensor network is assumed. Single sensor water-filling is performed over the lifetime of each sensor independently and the time slots in which each sensor would like to transmit in are colored in lime green in Fig. 5.1. As the figure illustrates there is more than one sensor interested in transmitting in time slots 3, 7 and 13.

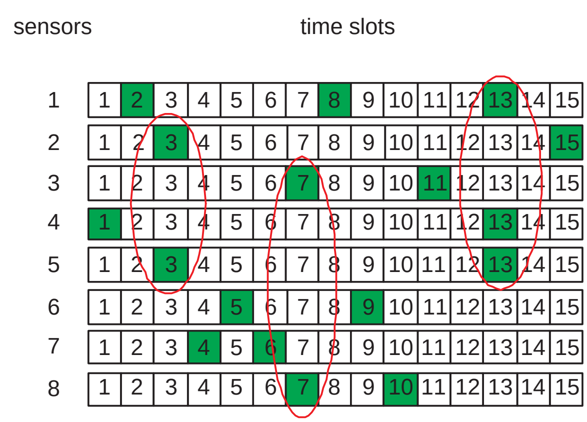

Figure 5.1: An example of time slot selection after single-sensor water-filling for an 8-sensor wireless network . There is conflict in time slots 3, 7 and 13.

Given the time slots in dispute a tree is formed where the first layer of nodes are those who have selected time slot 3 for transmission. Branches representing each of the sensors that want to transmit in time slot 7 are connected to each of the nodes in the first layer. And from each of the second layer nodes there are branches that represent sensors that have selected time slot 13 for transmission. This is the first phase of forming the CRT; the tree appears in Fig. 5.2.

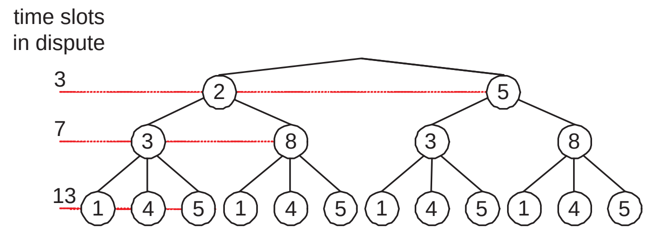

Figure 5.2: First phase of constructing the Contention Resolution Tree i.e. time slot selections of Fig 5.1. Disputed time slots are 3, 7 and 13; contenders are sensors $\{2, 5\}$, $\{3, 8\}$ and $\{1, 4, 5\}$ respectively.

Each of the leaves on the tree in Fig. 5.2 represents a candidate for assigning disputed time slots to sensors who have requested them. When a time slot is assigned to one sensor, other contenders (but not all other sensors) are banned from transmitting in that time slot. In Fig. 5.2, the leftmost leaf represents designating time slots 3, 7 and 13 to sensors 2, 3 and 1 respectively. This in turn means that sensors 5, 8 and $\{4, 5\}$ are banned from transmitting in the time slots they had contended for i.e. time slots 3, 7 and 13 respectively.

Once a time slot is designated to one sensor, all the sensors who no longer have access to the time slot should reallocate the energy they had planned to use for transmission in that time slot. To assure all sensor energies are accounted for, it is necessary to perform another round of water-filling at each leaf. The next round of water-filling may produce new scheduling conflicts which in turn require new branches on the tree. The procedure is repeated until after single-sensor water-filling at the leaf there are no more scheduling conflicts.

The first phase of the constructing the CRT resulted in the tree of Fig. 5.2. In the second phase the leaves are examined to determine whether the tree should be extended. Starting at the leftmost leaf for the purpose of this example, water-filling is performed for all sensors that have been banned from certain time slots by the assignments on the path leading to this leaf, i.e. for sensors $\{4, 5, 8\}$. The slots selected by sensors after this round of water-filling are shown in Fig. 5.3. Note also that the time slots each sensor is banned from is blackened out. As is apparent there are conflicts in time slots 5 and 7. New branches are added to the leftmost leaf on the

CRT the same way the first phase of constructing branches was done. The resulting tree appears in Fig. 5.4.

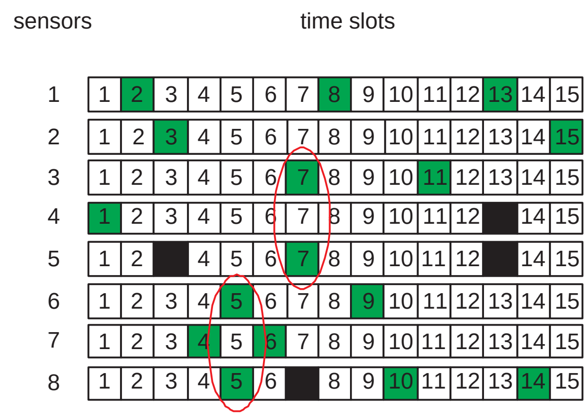

Figure 5.3: Time slot selection after the second round of single-sensor water-filling for the 8-sensor wireless network of Fig. 5.1. There are new conflicts in time slots 5 and 7. Sensors are banned from transmitting in blackened time slots.

Note that a time slot may appear on a path more than once. However, the sequence in which sensors come into contention over a time slot is important and different tree layers discussing the same time slot cannot be combined. A time slot reappears on the tree when a sensor reallocates its energy from a time slot it has been banned from to one that has previously appeared on the tree. Since a sensor has already scheduled transmission in the time slot in question higher on the path, there will be conflict.

For example, in Fig. 5.2, sensor 3 is in contention with sensor 8 over time slot 7 and the option of assigning the slot to either sensor appears on the tree. In Fig. 5.4, on the path that allocates time slot 7 to sensor 3, after sensor 5 is banned from time slot 13 it reallocates some of its energy to time slot 7 and is then in conflict with sensor 3 over this time slot. Yet, the decision for allocating time slot 7 should be made between sensors 3 and 8 first and then 3 and 5. Assuming Fig. 5.4 is the final form of the CRT, i.e. assuming that no conflicts arise when water-filling is performed at any of the leaves on the current tree, it is clear that if time slot 7 is assigned to sensor 8 the conflict with sensor 5 never arises.

Fig. 5.5 shows the transmission schedule at the leftmost leaf of Fig. 5.4. Since

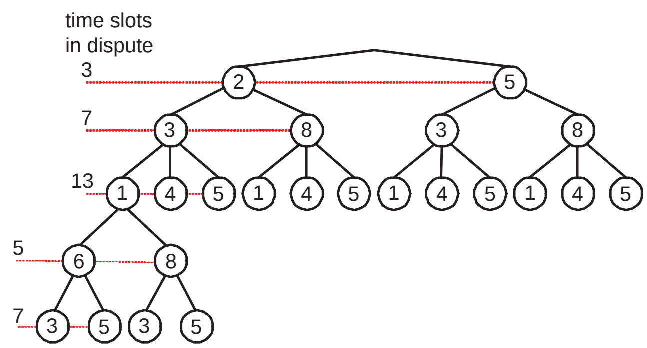

Figure 5.4: Continuing the construction of the Contention Resolution Tree on the leftmost leaf. From Fig. 5.3, disputed time slots are 5 and 7 and contenders are sensors $\{6, 8\}$ and $\{3, 5\}$ respectively.

there are no simultaneous transmissions, this path has reached a final leaf. To reach a leaf on other paths and determine the final form of the CRT the same water-filling process and tree extension described above is repeated for all other leaves. This includes any new leaves that appear on the tree. The growth of each path stops when water-filling at the leaf results in no more conflicts.

When the CRT is complete, paths will have grown to different lengths. If a time slot appears more than once in a path from root to leaf, the sensor assignment to the time slot is that in appearance closest to the leaf. The leaf with the greatest data volume is chosen as the optimum solution that produces the maximum data volume and represents the optimum transmission schedule for the sensor lifetime combination.

In this section constructing the Contention Resolution Tree was discussed. In the subsequent text the procedure of assigning values to each of the nodes on the tree is addressed. In Section 5.2 it is shown that the CRT provides the same final solution as the brute force approach. In other words, it is proven that the leaf on the CRT with the greatest data volume represents the optimum schedule. Based on the values assigned to tree nodes, a method for truncating paths before growing them to full length is presented in Section 5.3.4.

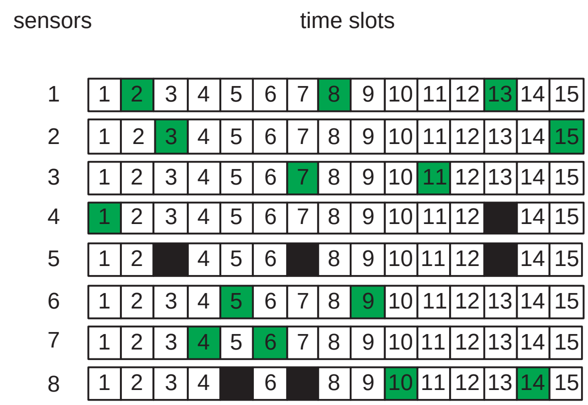

Figure 5.5: Time slot selection for the leftmost leaf on the tree of Fig. 5.4. There are no more schedule conflicts.

5.1.2 Assigning Values to Tree Nodes

In constructing the CRT, the first round of water-filling results in time slot selections presented in Fig. 5.1. Given water levels, the transmit powers and single-sensor data volumes are easily derived from single-sensor equations presented in Section 4.6.1. This data volume is the maximum each sensor could produce given its lifetime and if it had the channel to itself. Since there are overlaps in the transmission schedules for these sensors, the sum of individual sensor data volumes presents an upper bound on the maximum network data volume given sensor lifetimes. This upper bound value is assigned to the root of the CRT.

Because the data volume assigned to the root is an upper bound on the maximum network data volume, and since the leaves on the CRT represent a conflict-free transmission schedule and therefore a valid network data volume, the value at the root is an upper bound on the data volume produced by any of the leaves. As will be proven, one of the leaves on the CRT represents the maximum data volume for the lifetime combination.

The upper bound value assigned to the root is updated for each node on the tree to reflect the decisions depicted along the path that reaches the node. This is done by updating the water-filling process for the sensor(s) that have been affected at each node. Any assignment of a time slot to one of the sensors competing for it prevents other competing sensors from transmitting in the time slot. Sensors deprived of their original choice are forced to reallocated their energy to other time slots. This in turn reduces the data volume they produce as individuals and therefore the tally of single-sensor data volumes. For this reason the value assigned to a parent node is always greater than its children.

Fig. 5.1 represents time slot selections from which schedule overlaps are determined and Fig. 5.2 is formed. To calculate the value assigned to the node where time slot 3 is assigned to sensor 2, top leftmost node, the data volume for sensor 5 which is the only sensor affected by this decision is recalculated. The change for sensor 5 is that it no longer has access to time slot 3. Thus, its transmit powers are determined via water-filling over its lifetime with the exclusion of time slot 3.

The new data volume for sensor 5 is less than its previous data volume, i.e. when it had access to time slot 3, since water-filling with no restrictions produces the greatest data volume for the sensor. The new value replaces the previous value in the sum of single-sensor data volumes that was assigned to the root and the new sum is assigned to the top leftmost tree node with 2 written in it.

Keeping to the leftmost branch on the tree, the next node assigns time slot 7 to sensor 3 and bans sensor 8 from transmitting in this slot. The value assigned to this node is equivalent to the new data volume produced by sensor 8 where it no longer has access to time slot 7 replacing its previous value in the sum assigned to its parent node, the node that has 2 written inside it. For the same reasons presented above, the new data volume for sensor 8 is less than its original data volume. Therefore, the data volume at its parent node is an upper bound on the data volume at this node.

Continuing on this path, the data volume for sensors 4 and 5 are updated to determine the value assigned for the leftmost third level node, i.e. the node with 1 written in it. Given the water-filling updates for the sensors affected by the time slot allocations along the path, all the information required for forming Fig. 5.3 is readily available. In addition, the tree in Fig. 5.2 can be updated to Fig. 5.4. As before, values for the nodes on the new branches are calculated by updating the values assigned to their parent nodes.

The same procedure described above for one path on the tree is repeated for all the paths until on each path a leaf is reached where there are no simultaneous transmission scheduled. At this point the CRT is complete and all nodes have values assigned to them, including the leaves. Since there are no simultaneous transmissions at the leaf, the sum of single-sensor data volumes is no longer an upper bound but the data volume relative to the transmission schedule represented by the leaf. The greatest data volume among the leaves is the maximum data volume for the lifetime combination and the leaf it is associated with represents the optimum transmission schedule.

The reader is reminded that constructing the CRT over contended slots alone and assigning values to the nodes replaces the brute force search of calculating the data volume for all possible time slot to sensor allocations with given sensor lifetimes. In the next section the worst case scenario in which the CRT requires the same amount of computation as the brute force approach is discussed.

5.1.3 Worst Case Computation

The amount of computation required for the Contention Resolution Tree depends on the channel gains, battery energy and lifetimes of sensors. Some measurements of the computational savings provided by the CRT are discussed in Section 5.4. However, discussing the case that would require the same amount of computation for the CRT and the brute force will provide insight. This is the worst case scenario for the tree.

If after water-filling all the sensors choose to transmit in all the time slots, there will be conflict in all the time slots between all the sensors. This case is equivalent to the brute force search of examining all possible time slot to sensor assignments.

For the 8-sensor network example used to explain the CRT, assuming all sensors had a lifetime of 15 time slots, the brute force search, which is equivalent to the worst case scenario for the tree, would require calculating the data volume for 8^{15} cases i.e. over 3.5×10^{13} computations of an 8-sensor data volume. At each node on the CRT the data volume for a few sensors is updated in the sum of single-sensor data volumes. This calculation is less than an 8-sensor data volume calculation, i.e. calculating 8 single-sensor data volumes and summing them. But assuming they were equal, the amount of computation for the brute force search can be compared to the number of nodes on the CRT.

5.2 Equivalence of the CRT and the Brute Force

The aim of this section is to show that the CRT finds the optimum solution that the brute force search of all time slot to sensor allocations achieves. This is done by showing that for any assignment of time slots to sensors the brute force search considers, there is a leaf on the CRT that produces equal or greater data volume. Each leaf on the CRT represents a valid transmission schedule that contains no simultaneous transmissions so that the data volume at the leaf can be calculated by summing individual sensor data volumes.

Having proven that at least one leaf on the CRT produces data volume greater or equal to that of each candidate of the brute force search, and given that the brute force search finds the maximum data volume given sensor lifetimes, it is concluded that the greatest data volume produced at a leaf on the CRT is equal to the maximum data volume for the lifetime combination in question.

To constitute the proof, a wireless sensor network is assumed and its corresponding Contention Resolution Tree and all possible time slot to sensor allocations are formed. Consider any of the time slot to sensor assignments the brute force search recognizes as a candidate. This assignment is referred to as "assignment B" and the data volume it produces is V_B. It is shown below that there exists a leaf on the CRT that produces data volume greater or equal to V_B.

5.2.1 Finding a Leaf to Compare

Starting at the root of the tree, at each level, if assignment B associates one of the sensors competing for the time slot with the time slot in question that branch is chosen. Otherwise, one of the branches is randomly chosen and followed to the next level. This process is continued until a leaf is reached on the CRT. From the description it is clear that there can be more than one leaf on the CRT comparable to assignment B.

As an example, take the tree in Fig. 5.4 and assume the brute force assignment is [1 7 2 4 5 8 5 1 6 7 2 3 8 4 ...], meaning that time slot 1 is given to sensor 1, time slot 2 to sensor 7 and so on. Starting at the top of the tree, since the brute force assignment associates time slot 3 with sensor 2 the left branch is taken. However since it associates time slot 7 with sensor 5, which is not among the contending sensors, any branch can be taken to the next level. At this level if any of the branches that assign slot 13 to sensor 4 or 5 are taken those leaves are used for comparison in terms of the data volume produced. If the branch that assigns 13 to sensor 1 is taken, a random choice for the branch to continue on is made for slot 5. However, for slot 7 the brute force combination picks sensor 5 so the data volume produced by either of the leaves with 5 can be used to compare to the brute force data volume. Regardless of the leaf chosen, the data volume it produces is denoted by V_L.

5.2.2 Proof of $V_B \leq V_L$

Assuming there are t_∞ time slots to be assigned to sensors, t_∞ is the length of assignment B. Time slots that are associated with the same sensors at the leaf as they are in assignment B are numbered $t_1 \ldots t_q$ and are termed the *fixed slots*. Time slots that are assigned to different sensors at the leaf than in assignment B are numbered $t_{q+1} \ldots t_p$ and called the *critical slots*. In other words, in assignment B, these time slots are assigned to sensors that did not compete for them on the tree. The rest of the time slots, $t_{p+1} \ldots t_\infty$, do not appear on the path so the leaf is indifferent to which sensor transmits in those time slots and from its stand point all sensors have access to them.

Since both at the leaf and in assignment B there are no overlaps in transmission, the data volume in both cases is calculated as the sum of individual sensor data volumes. At each leaf, for any sensor, the time slots available for water-filling are the time slots it never contended for along the path, and the disputed time slots that have been assigned to it. Assignment B is a specific time slot to sensor assignment which leaves no shared time slots for water-filling.

To compare the data volume produced by each sensor at the leaf and in assignment B, the time slots available to them for water-filling are compared. For this purpose sensors are divided into three groups based on their access to the critical slots, i.e. time slots in the range $[t_{q+1} \dots t_p]$: one group are sensors that are assigned these times slots by the leaf, the second group are sensors that are assigned these time slots by assignment B, and the third group are the remaining sensors in the network. There may be overlaps in the first two groups of sensors.

group α

Sensors that are assigned at least one of the critical slots at the leaf constitute group α. In Section 4.6.1 it is shown that for a single sensor with known lifetime, water-filling over time provides the optimum transmission schedule that maximizes its data volume. Therefore, if a sensor is not banned from any time slot it produces its maximum data volume. When permission to transmit in a time slot the sensor would like to transmit in is taken away, the data volume is reduced.

The leaf and assignment B agree on the allocation of time slots $t_1 \dots t_q$. The leaf gives all sensors access to time slots $t_{p+1} \dots t_\infty$. In other words, at a minimum the leaf provides these sensors access to all slots allocated to them by assignment B in ranges $[t_1 \dots t_q]$ and $[t_{p+1} \dots t_\infty]$. To outperform the brute force approach the leaf should allocate time slots in the range $[t_{q+1} \dots t_p]$, the critical slots, so that sensors produce more data volume than assignment B.

Of the time slots a sensor in group α is denied at the leaf, none are allocated to it by assignment B. Because if that were the case the branch associated with that allocation would have been followed, this time slot would be among the fixed slots, and a different leaf would be compared to assignment B. In addition, a sensor will only appear on the CRT in relation to a time slot if water-filling renders the time slot a candidate for transmission. Thus, all the sensors in group α are scheduled to transmit in at least one critical slot awarded to them at the leaf.

From the above it is concluded that the leaf, in addition to all the time slots allocated to the sensor by assignment B, provides each sensor in group α with access to at least one of the time slots it prefers for transmission. Otherwise stated, assignment B bans each sensor in group α from at least one time slot it wants to transmit in compared to the leaf. Therefore, the data volume the sensor produces under assignment B is less than what it produces at the leaf. This is true for all the sensors in group α and is therefore true for the sum of their data volumes since there are no simultaneous transmissions in either case. Thus,

$$V_{B\alpha} < V_{L\alpha} \tag{5.1}$$

where $V_{B\alpha}$ is the sum of the data volumes for sensors in group α under assignment B and $V_{L\alpha}$ is the data volume for the same sensors at the leaf. If assignment B made all the same allocations as the leaf for the time slots on the CRT, group α would be null.

group θ

Sensors allocated the critical slots by assignment B make up group θ. The time slots available for water-filling at the leaf for any sensor in group θ are: the fixed time slots they are assigned, time slots $t_{p+1} \dots t_\infty$ and at minimum any critical slot that assignment B assigns to them.

If a sensor in group θ were to appear on the CRT in relation to a time slot it is allocated by assignment B, a leaf with the same assignment would be chosen for comparison. The time slot would then be among the fixed slots and not among the critical slots. Therefore, of the sensors that belong to group θ, none have competed on the CRT for the time slot(s) they are allocated by assignment B. For this reason, at minimum, the leaf provides them access to the critical slot(s) they are assigned to by assignment B.

For these sensors assignment B provides access to fixed time slots they are allocated and one or more time slots from the range $[t_{q+1} \dots t_\infty]$. Since the time slots made available to each sensor in the group by assignment B is a subset of the slots available to it at the leaf, the data volume produced by each sensor under assignment B is less than or equal to its data volume at the leaf. Since the inequality holds for every sensor in the group it also holds for the cumulative data volumes $V_{B\theta}$ and $V_{L\theta}$.

$$V_{B\theta} \leq V_{L\theta} \tag{5.2}$$

If there are overlaps in the sensors that belong to groups α and θ, sensors $\theta \cap \alpha$, when summing data volumes to arrive at the network data volume, the data volume for these sensors will be accounted for twice. Thus, these sensors are removed from group θ and the data volume for the remaining sensors in this group at the leaf is represented by $V_{L(\theta-\theta\cap\alpha)}$. The same holds for assignment B. Given that the data volume for any sensor in group θ at the leaf is greater or equal to its data volume under assignment B,

$$V_{B(\theta-\theta\cap\alpha)} \leq V_{L(\theta-\theta\cap\alpha)} \tag{5.3}$$

holds.

group ϕ

Sensors that are not assigned a critical slot by assignment B or at the leaf belong to group ϕ. To these sensors assignment B allocates: the relative fixed time slots (if any) and some time slots in the range $[t_{p+1} \ldots t_\infty]$. At the leaf water-filling is performed over the same fixed time slots and all time slots in the range $[t_{p+1} \ldots t_\infty]$. Similar to group θ, the time slots allocated to each sensor by assignment B are a subset of those available to the sensor at the leaf, so the data volume produced by the leaf is greater than or equal to the data volume produced by assignment B. Extended to the group data volumes $V_{L\phi}$ and $V_{B\phi}$,

$$V_{B\phi} \leq V_{L\phi}. \tag{5.4}$$

The three groups constitute the network. Summing the data volumes, the network data volume for the leaf and assignment B are

$$V_B = V_{B\alpha} + V_{B(\theta-\theta\cap\alpha)} + V_{B\phi} \tag{5.5}$$

$$V_L = V_{L\alpha} + V_{L(\theta-\theta\cap\alpha)} + V_{L\phi}. \tag{5.6}$$

Based on (5.1), (5.3), and (5.4),

$$V_B < V_L. \tag{5.7}$$

However, if group α is null,

$$\begin{array}{r} V_{B\theta} \leq V_{L\theta} \\ +\underline{V_{B\phi} \leq V_{L\phi}} \\ V_B \leq V_L. \end{array} \tag{5.8}$$

In essences, based on the three groups of sensors, the data volume for any sensor at the leaf is no less than that of assignment B. Since the union of the three sets constitutes the network, it is concluded that the data volume produced at the leaf, V_L, is greater than or equal to the data volume produced by assignment B, V_B.

5.2.3 Wrap Up

In the preceding text it is proven that for any candidate of the brute force search there exists a leaf on the CRT that produces equal or greater data volume. This is true for the time slot to sensor allocation that produces the maximum data volume for the sensor lifetimes. Thus, there exists a leaf on the CRT that produces the maximum data volume the brute force search finds and searching among all candidates of the brute force search produces the same result as searching the leaves on the CRT.

5.3 Expediting the Search

The Contention Resolution Tree presented above is employed to reduce the computation required to determine the data volume for one combination of sensor lifetimes.

In the subsequent text two methods are introduced that can altogether eliminate the need to perform data volume calculations for lifetime combinations that are known not to be optimal. Based on these methods the brute force algorithm is revised. The revision made to the brute force algorithm allows a truncation scheme to be applied to the CRT which further reduces the computational load of calculating the maximum data volume for the sensor network.

5.3.1 Early Discard of Candidates

Similar to the induction method employed to find optimal transmit powers and data volumes, the concept is initially explained for a single-sensor network, then a two-sensor network and then extended to the case of an N-sensor network.

One sensor: if the maximum data volume for sensor i with lifetime n_i is achieved when the sensor does not transmit in the last time slot, i.e. $x_{in_{i\,opt}} = 0$ for a given n_i, n_i cannot be the optimum lifetime for sensor i. This is because sensor i is staying alive for at least one time slot where it is not contributing to the data volume. With a shorter lifetime the energy used for processing purposes in the last time slot can be utilized for transmission and therefore a higher data volume can be achieved.

There is one exception to this rule. If $n_i = \mathcal{L}_i$, i.e. the sensor does not transmit any data throughout its lifetime, the above argument does not stand. Although the argument is not valid, $n_i = \mathcal{L}_i$ is never optimum for a one-sensor network. However, this point is useful when it comes to multiple sensors. To amend the above statement to hold for all cases: if the optimum transmit schedule for sensor i with lifetime n_i is to transmit in any time slot, in order for n_i to be the optimum sensor lifetime, the sensor must also be scheduled to transmit in time slot n_i.

Two sensors : From the above it can be concluded that for lifetimes n_p and n_q to be optimum for sensors p and q respectively, the sensors should transmit in these time slots unless $n_p = \mathcal{L}_p$, in which case sensor p does not have to transmit in n_p, or $n_q = \mathcal{L}_q$, in which case sensor q does not have to transmit in n_q. As a result, if

$$n_p \neq \mathcal{L}_p \ , \ n_q \neq \mathcal{L}_q \tag{5.9}$$

$$n_p = n_q = \eta \tag{5.10}$$

lifetimes n_p and n_q cannot be optimum since both sensors would have to transmit in time slot η. Hence, $n_p = n_q$ can only be optimum if

$$n_p = n_q = \mathcal{L}_p \tag{5.11}$$

or

$$n_p = n_q = \mathcal{L}_q. \tag{5.12}$$

The above property is used to reduce the number of sensor lifetime combinations studied. Assume $\mathcal{L}_p \leq \mathcal{L}_q$, for all $n_p < \mathcal{L}_p$, $n_p \neq n_q$ is a condition of the optimum solution. Therefore, combinations that do not have the condition can be eliminated without any effort to calculate the data volume they produce. For the case of two sensors $\mathcal{L}_p \times (\mathcal{L}_q - 1) + 1$ combinations are investigated instead of $\mathcal{L}_p \times \mathcal{L}_q$ combinations.

N **sensors** : Extending the above argument to an N-sensor network, all lifetime combinations where

$$n_p \neq \mathcal{L}_p \ , \ n_q \neq \mathcal{L}_q \tag{5.13}$$

$$n_p = n_q \tag{5.14}$$

for any two sensors p and q can be eliminated from the list of candidates.

This simple check significantly reduces the number of lifetime combinations for which data volume is calculated. It is easy to calculate the effect. The number of lifetime combinations for N sensors with equal maximum lifetimes $\mathcal{L}$ is

$$\text{total} = \mathcal{L}^N. \tag{5.15}$$

If early discard is applied, the combinations studied are all combinations that have no equal sensor lifetimes, plus all combinations where only sensors with the maximum lifetime $\mathcal{L}$ have equal lifetimes. These combinations occur when, two sensors have lifetimes equal to $\mathcal{L}$, three sensors have lifetimes equal to $\mathcal{L}$, four sensors have lifetimes equal to $\mathcal{L}$ ... and finally when all sensors have lifetimes equal to $\mathcal{L}$. The number of lifetime combinations studied after early discard has been applied is

$$\text{remaining} = \underbrace{\prod_{i=0}^{N-1}(\mathcal{L}-i)}_{\text{no equal sensor lifetimes}} + \sum_{j=2}^{N-1} \underbrace{\mathcal{C}_j^N \prod_{i=1}^{N-j}(\mathcal{L}-i)}_{j \text{ sensors have lifetime } \mathcal{L}} + 1 \qquad (5.16)$$

where $\mathcal{C}_j^N$ counts the number of ways the j sensors with lifetime equal to $\mathcal{L}$ can be chosen from N sensors. The number of possible combinations when only j sensors have lifetimes equal to $\mathcal{L}$ and all other sensors have unequal lifetimes is described by the term $\prod_{i=1}^{N-j}(\mathcal{L}-i)$.

The $+1$ is the combination of lifetimes where all N sensors have lifetimes equal to $\mathcal{L}$. For this case the data volume is zero and this lifetime combination cannot be optimum. However, the early discard method does not eliminate this lifetime combination. Since the sum of individual sensor data volumes, i.e. the upper bound, is zero, this combination will appear as the last combination when ordering sensor lifetimes with the following upper-bounding and ordering scheme and will never be studied.

5.3.2 Upper-bounding and Reordering

Despite eliminating some candidates with the early discard, calculating the maximum data volume for the remaining lifetime combinations can still be computationally overwhelming. The amount of computation required to determine the maximum network data volume grows exponentially with the number of sensors in the system and their maximum lifetime. The simple ordering method introduced in this section will, in most cases, eliminate the need to perform data volume calculations for many lifetime combinations. Note that this method does not jeopardize the optimality of the solution.

Employing the fact that there are no simultaneous transmissions in the optimum transmission schedule, the network data volume is calculated as the sum of individual sensor data volumes. If individual sensor data volumes are calculated solely based on the sensor lifetime and summed before transmission overlaps are eliminated, the outcome is an upper bound on the maximum data volume for the lifetime combination. This is the value assigned to the root of the CRT which then resolves the transmission overlaps. However, before employing the CRT, this upper bound can be used as an easy-to-compute estimate of the maximum data volume for the lifetime combination in question.

If the maximum data volume for combination A of lifetimes is greater than the *upper bound* on combination B of sensor lifetimes, there is no need to calculate the maximum data volume for combination B since the outcome will be lower than that of combination A. With this in mind, the maximum data volume for lifetime combinations that have higher data volume upper bounds are calculated first, in hope that at some point the highest computed data volume is greater than the next upper bound and thus the search can be terminated without the need to examine the remaining cases. Using the two methods introduced above the brute force algorithm of Section 4.5 is revised.

5.3.3 Updated Search Algorithm

Although there are more steps in the revised version of the search algorithm, originally presented in Section 4.5, it requires much less computation. Both the early discard and ordering shortcuts have been incorporated into this algorithm. It should be emphasized that the search still obtains the optimum solution despite requiring much less computation.

1. find maximum sensor lifetimes for all sensors, $\mathcal{L}_i$ where $i \in [1, N]$

2. calculate the data volume produced by each sensor via single-sensor water-filling for all possible lifetimes; for any sensor i possible lifetimes are $n_i \in [1, \mathcal{L}_i]$

3. calculate the upper bound on the maximum data volume by summing single-sensor data volumes for respective lifetimes for all sensor lifetime combinations that pass the early discard test

4. sort the upper bounds in descending order and start at the top of the list

5. calculate the maximum data volume for this combination of sensor lifetimes using the upper bound as the root value for the CRT

6. if the greatest data volume so far is greater than the next upper bound, terminate the search and present the data volume (along with the transmit powers and scheduling regime) as the final solution; else, move to the next combination of sensor lifetimes on the list and go to step 5

5.3.4 Truncating the CRT

The brute force algorithm for finding the optimum transmission policy was revised using shortcut methods and replaced by a less computationally demanding search algorithm. The CRT does the same for the brute force search of finding the optimum time slot to sensor allocation for a given lifetime combination. In this section the updated search algorithm, specifically the ordering method, is used to further reduce the amount of computation performed by the CRT.

Based on the last step of the updated search algorithm, the data volume calculated for each lifetime combination is compared to the greatest previously found data volume and if greater, replaces it. However, if the data volume is less than the highest previously found value it is discarded. In the following, the greatest previously calculated data volume is used as a threshold to terminate paths before reaching a leaf, hence eliminating unnecessary computation. For the remainder of this section *the greatest previously found data volume* is dubbed *the threshold*.

The Method

During the construction of the CRT the data volume upper bound for the lifetime combination, associated with the root, is updated when each new time slot assignment is made. The new value, associated with the tree node representing the assignment, is the data volume upper bound for any leaves branching from this node. If the upper bound value at a node is less than the threshold, extending the path and reaching a leaf will not provide a greater data volume than the threshold. For this reason if at any point the data volume upper bound associated with a node falls below the threshold, the path is abandoned. This proactive measure eliminates the needless computation performed to reach leaves that do not produce data volumes above the threshold.

Implementing the truncation method, many paths do not reach a leaf. It may occur that all the paths on a tree are eliminated before reaching a leaf. In this case, without the need to compare a leaf to the threshold, the next lifetime combination on the descending list of upper bounds is considered.

Generally, as the updated search algorithm progresses to lifetime combinations with lower upper bounds, the number of paths that are eliminated without reaching a leaf increases. However, if more than one path on the CRT reaches a leaf, the leaf that produces more data volume is compared to the threshold to determine whether or not this lifetime combination can provide a greater data volume than what was previously produced.

A Word on Optimality

Truncating the tree does not affect the optimality of the final solution for the sensor network since paths supporting leaves that achieve data volumes greater than the threshold will survive truncation. However, the data volume for each lifetime combination has to be interpreted with diligence.

For each lifetime combination, if its maximum data volume is greater than the threshold, all values assigned to the nodes on the path leading to the leaf with the optimum assignment will also be greater than the threshold. Thus, at minimum this one path will survive all the way to the leaf. In other words, if the greatest data volume produced at a leaf on the CRT exceeds the threshold, this data volume is indeed the maximum data volume for the lifetime combination and time slot assignments at the leaf are optimum.

However, if the maximum data volume for the lifetime combination is lower than the threshold, there is no guarantee that the leaf which produces the maximum data volume for the lifetime combination will survive truncation. For this reason, if neither of the leaves on the CRT produce a data volume greater than the threshold, it is concluded that the maximum data volume for the lifetime combination is less than the threshold. However, it is not certain that either of the leaves reached (if any) produce the maximum data volume for the lifetime combination.

5.4 Simulations

The tools introduced in this chapter are employed to simulate networks and obtain their optimum transmission policy with less computation compared to the brute force approach. The optimum policy is derived in Chapter 4 and the techniques presented here only reduce the computational burden while preserving the optimality of the outcome.

To show the data volume gain in employing the optimum transmission policy, the data volume for the round robin scheduling scheme is presented alongside the optimum solution. In the round robin scheme sensors take turns in transmitting and transmit powers for each sensor are determined via water-filling in its designated time slots. The lifetime that produces the highest data volume for each sensor is chosen as the final solution.

To illustrate the performance improvement due to optimizing sensor lifetimes, data volumes for a case where sensor lifetimes are not optimized but rather set to a fixed value will be presented. This policy will be termed *fixed lifetimes*.

To determine the fixed values, in each Monte Carlo iteration optimum sensor lifetimes are sorted and then averaged over all iterations. Thus, averages produced are the average for the last sensor to expire, second last to expire and so on. These values are rounded and adopted as the fixed lifetimes for sensors. The maximum data volume is determined via single-sensor water-filling over the fixed lifetimes and by employing the CRT, the procedure that is executed for each lifetime combination when searching for the optimum transmission schedule.

The problem of maximizing the sum-rate capacity for unlimited energy users with constrained average powers is studied in [34] and discussed in Sections 4.2.1 and 4.8.1 in this dissertation. It is proven that single-user water-filling provides optimum transmit powers and that only the strongest user should transmit in any time slot. The Knopp and Humblet solution constrains average transmit powers and requires knowledge of channel probability distribution functions. The constraint on average transmit powers is necessary to keep transmit powers within bounds since there are no limitations on the energy available to users. There is no direct way to apply the optimum transmission policy for the unlimited nodes to a sensor network without making assumptions about average sensor transmit powers and a statistical model of the channel gains.

To remedy this, the suboptimal conflict resolution technique tagged as *only strongest channel* inspired by the optimum unlimited energy node transmit policy is introduced. This policy assigns each time slot to the sensor with the strongest channel gain and determines transmit powers via prescient water-filling. Regardless of whether or not the sensor with the strongest channel transmits any data, all other sensors are banned from transmitting in that time slot.

For all the following simulations the channel is modeled as a time-varying Rayleigh fading channel, i.e. for each sensor channel gains are selected independently from an exponential distribution. The sink has prescient knowledge of these gains in all policies examined. Sensors are all assumed identical and the values $B = 1$, $T = 1$ and $N_o = 1$ are employed.

5.4.1 Data Volume

Fig. 5.6 shows the dependence of data volume on network size. Simulations are produced with processing power and normalized battery charge for sensors set to $\gamma_p = 0.5$ and $D = 8$. The maximum data volume is averaged over 1,000 Monte Carlo simulations. The approximation line for the maximum data volume shows an almost linear increase with the increase in the number of sensors. This was predictable. The trend will continue until the saturation point at which additional sensors will not increase the data volume as much.

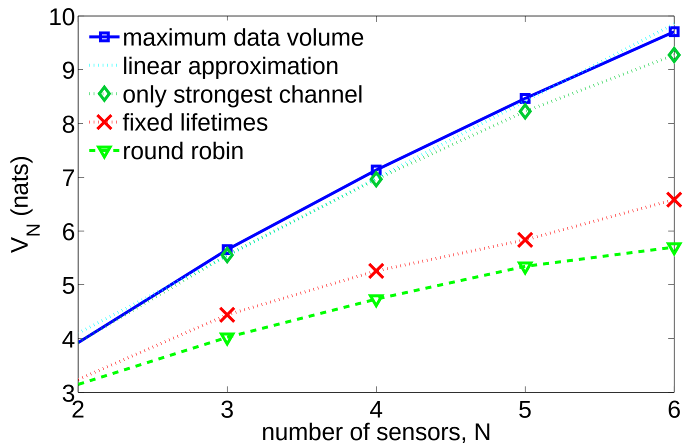

Figure 5.6: Data volume for identical sensors with $D = 8$ and $\gamma_p = 0.5$ for the maximized data volume, a limited-energy adaptation of the Knopp and Humblet solution, fixed lifetimes, and round robin. 1,000 Monte Carlo simulations in a time-varying Rayleigh fading channel.

The figure shows a fairly wide gap between the optimum transmission schedule and the fixed lifetime and round robin policies, about 20% less data volume for the non-optimal schemes for a 2-sensor network. The curves diverge further with more sensors in the system, indicating that both maintaining an optimal transmission schedule and determining optimum sensor lifetimes, or in effect optimum transmit powers, play an integral role in the volume of data that reaches the sink.

As discussed in Section 4.8.1 assigning time slots to the sensor with the strongest channel gain is not always optimum. This is emphasized by the gap between the maximum data volume curve and the *only strongest channel* curve in Fig. 5.6. The curve shows that although the strongest channel assignment is not always optimum, it is a close competitor for the optimum solution. Determining close to optimum transmission policies is one of the important applications of an optimum solution.

Fig. 5.7 plots the maximum data volume versus the normalized battery energy. Data volume curves are presented for networks of 4 and 5 sensors with $\gamma_p = 0.5$. Data volume increases linearly with the increase in normalized initial battery energy.

The two suboptimal techniques, round robin and the limited-energy adaptation of the Knopp and Humblet solution, show performances consistent with Fig. 5.6. The round robin scheme maintains a wide gap with the optimal solution whereas the policy of assigning time slots to sensors with the strongest channel gain follows the optimum solution closely. The wide gap for the round robin scheme exists despite the fact that its transmit powers are determined by single-sensor water-filling over assigned slots. This gap emphasizes the value of scheduling based on channel gains, the optimum transmission schedule, rather than just taking turns in transmission. Both suboptimal policies exhibit linear growth in data volume with the increase in normalized battery energy. The gaps are wider with 5 sensors compared with the 4-sensor network which is again consistent with Fig. 5.6.

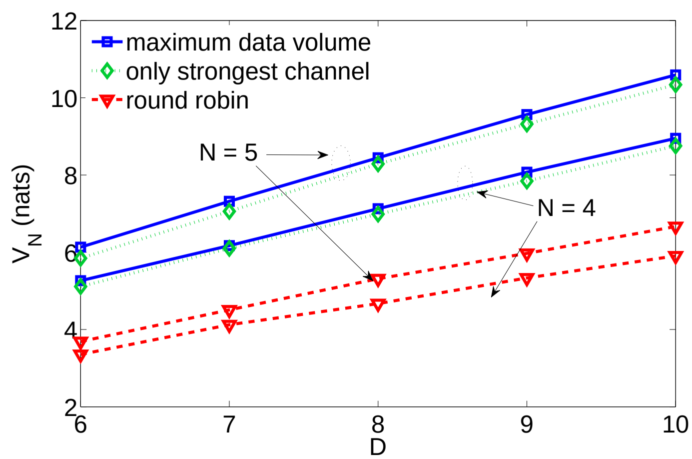

Figure 5.7: Maximum data volume, data volume for a limited-energy adaptation of the Knopp and Humblet solution, and round robin, with results averaged over 1,000 Monte Carlo simulations for various values of D. Simulations are for $N = 4$ and 5 identical sensors where $\gamma_P = 0.5$ and a time-varying Rayleigh fading channel.

Fig. 5.8 presents the data volume for a sensor network of $N = 4$ and 5 identical sensors plotted against the fictitious processing power transmit SNR, γ_p. The general form of the curves, both the maximum data volume and the suboptimal schemes, is logarithmic. For the maximum data volume this relationship can be seen in (4.42) which represents the maximum data volume for an N-sensor network.

Note that the gap between the suboptimal techniques and the optimum policy is wider at a higher γ_p. It is also greater with 5 sensors in the network compared to 4 sensors. Similar to the figures with data volume versus network size and normalized battery, round robin scheduling performs poorly in comparison to the optimum scheme. The policy of assigning time slots to sensors with the strongest channel gain exhibits close to optimal performance.

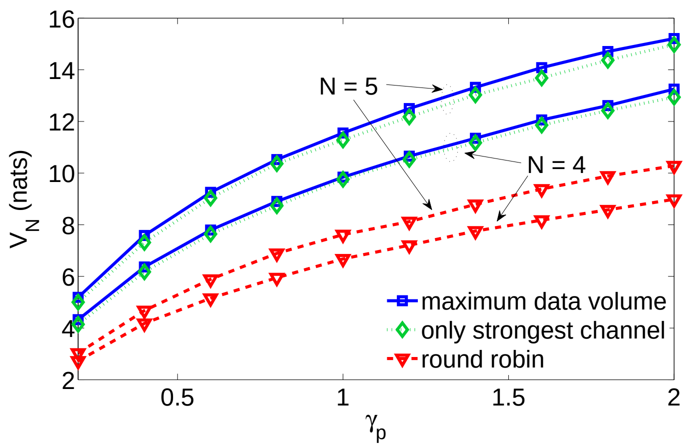

Figure 5.8: The effect of γ_p on the maximum data volume, a limited-energy adaptation of the Knopp and Humblet solution, and round robin transmission, for $N = 4$ and 5 identical sensors with $D = 8$. Simulations are averaged over 1,000 Monte Carlo runs for a time-varying Rayleigh fading channel.

5.4.2 Activity Duration

The network activity duration is defined as the maximum sensor lifetime, i.e. the maximum number of time slots any sensor survives times the duration of a time slot $T \times \max(n_i)$. The activity duration for the optimum transmission policy increases almost linearly with the increase in network size. Of course the curve will saturate at a point where there are too many sensors in the network for all sensors to get a chance to transmit. In effect, with identical sensors, the network activity period reaches its maximum with the first sensor that expires without transmitting any data. Additional sensors that do not transmit will not increase the network activity period.

The limited-energy adaptation of the Knopp and Humblet solution, which assigns time slots to sensors with the strongest channel gain, achieves slightly longer activity periods than the optimum solution. Thus, its loss in data volume compared to the optimum solution translates to a gain in network activity period. The reason for this is that at times when the sensor with the strongest channel gain is assigned a time slot that it does not transmit in, the energy that is not used to produce data volume is consumed by extending the sensor lifetime.

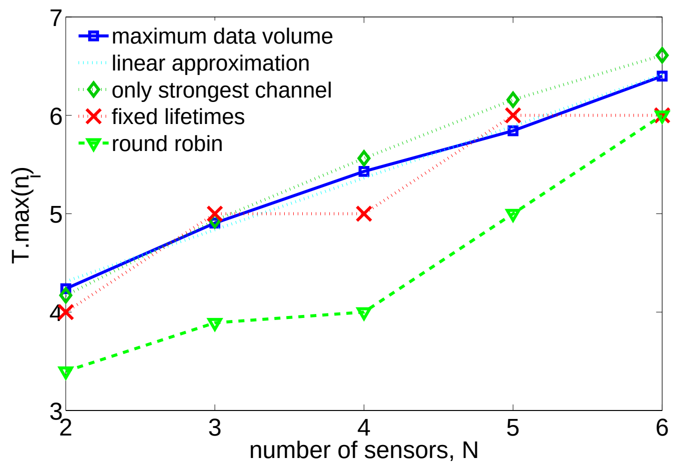

Figure 5.9: Activity duration for the optimum transmit policy, a limited-energy adaptation of the Knopp and Humblet solution, fixed lifetimes, and round robin, with identical sensors using the same set of data as Fig. 5.6.

Activity durations for the fixed lifetime curve are rounded values from the optimum scheme. Since in some cases the value is closer to the larger integer, the lifetime curve for the fixed lifetimes surpasses the lifetime curve for the optimum policy.

The round robin scheme performs much weaker compared to all other techniques in terms of activity duration, except for when there are six sensors in the network. When sensors are only allowed access to time slots placed apart in time, they consume a lot of processing power between two transmission opportunities. Thus, in many cases, it is more efficient for them to use their energy to transmit in a weaker channel earlier in time rather than to consume energy waiting for a stronger channel in the future. This leads to lower data volumes and shortened lifetimes for sensors despite transmitting with optimum transmit powers determined via water-filling. Fig. 5.6 and Fig. 5.9 reflect the poor performance in data volume and activity duration by the round robin scheme. These figures emphasize the importance of optimum, or in the case of *only strongest channel* close to optimum, scheduling for sensors.

The activity duration for the round robin scheme improves with a larger network size because of the diversity added with additional sensors in the network. With six sensors it is more likely that one sensor will not transmit at all, thereby pushing its lifetime to the maximum and increasing the network activity duration. As the activity duration curve for the optimum policy saturates with the growth in network size, the wide gap with the round robin scheme narrows. Fig. 5.6 shows that in terms of data volume the round robin has its worst performance relative to the optimum scheme with six sensors.

5.4.3 Effect of the Expedited Search

Fig. 5.10 illustrates the computational savings made by employing the early discard check. With the assumption that all sensors have an equal maximum lifetime $\mathcal{L}$, the percentage of candidates that survive early discard is calculated by dividing (5.16) by the total number of candidates (5.15). As expected, the figure shows that with equal maximum sensor lifetimes, early discard is more effective with more sensors in the network and shorter maximum lifetimes.

Fig. 5.11 shows the search efficiency introduced by upper-bounding and ordering the candidates (Section 5.3.2) that have survived the early discard (Section 5.3.1). These curves represent the percentage of lifetime combinations after early discard that, in the search for the optimum solution and after applying upper-bounding and ordering, require data volume evaluation. Candidates eliminated by the early discard method and the upper-bounding and ordering technique make up the computation reduction compared to the brute force algorithm of Section 4.5.

The search efficiency is important because, despite the fact that the CRT combined with the truncation technique greatly reduces calculations, evaluating the maximum

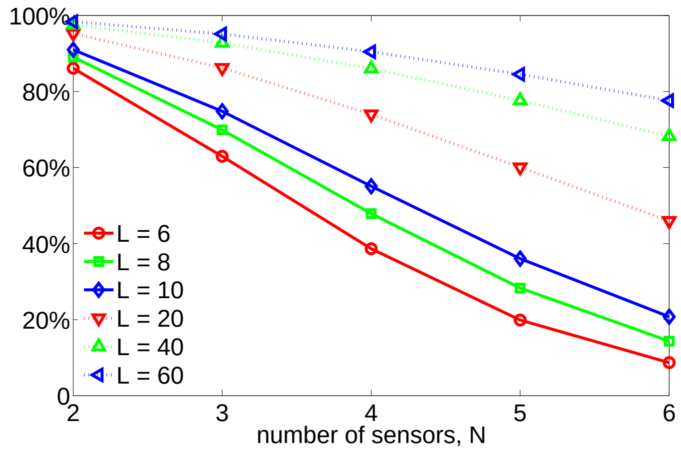

Figure 5.10: Effectiveness of early discard. Percentage of lifetime combinations remaining after early discard is applied, i.e. the ratio between (5.16) and (5.15). All sensors have maximum lifetime $\mathcal{L}$.

data volume for a lifetime combination is a bulky computation. The methods introduced in this chapter collectively reduce the computational burden of calculating the optimum solution and allow the study of larger networks and sensors with greater battery energies.

The curves in Fig. 5.11 show that in most cases very few of the candidates need to be studied after the upper-bounding and ordering technique has been applied - less than 1% in some cases.

For the cases that survive early discard and upper-bounding and ordering, the optimum transmission schedule can be determined via brute force search or using the CRT with truncation. Fig. 5.12 compares the amount of computation required for these two options. It presents the number of single-sensor water-filling procedures required for the CRT as a percentage of those required for the brute force search.

As is apparent from Fig. 5.12 the number of water-fillings required for the CRT with truncation is less than 1% that of the brute force search, regardless of the number of sensors in the network. Fig. 5.12 fails to account for the computation involved in comparing sensor transmission schedules and determining schedule overlaps, which is used to expand the CRT. Further, it does not account for the computational power

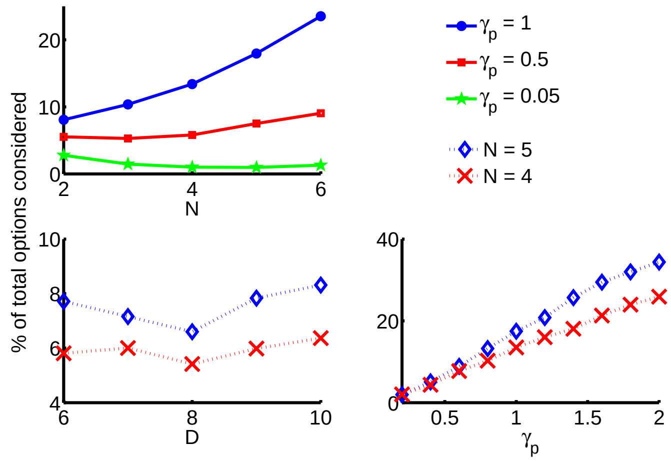

Figure 5.11: Average percentage of lifetime combinations studied in order to determine the optimum solution. Percentage calculated after eliminating some lifetime combinations in accordance with the early discard technique, Section 5.3.1.

consumed for determining all the possible brute force combinations. In general, however, these energy expenditures are negligible compared to the energy spent for water-filling.

The figure shows that with more sensors in the network it is even more efficient to use the CRT in comparison with the brute force search. The reason is simple. The amount of computation for the brute force search grows much faster than the CRT with the increase in the number of sensors. The brute force looks at all combinations of assigning time slots to sensors whereas the CRT only resolves conflicts in their transmission schedules. With two sensors in the network there are fewer brute force combinations to go through so the number of calculations the CRT performs is closer to the brute force scheme. However, with five sensors, although there are probably more conflicts in the transmission schedule to consider there are many more time slot to sensor assignments.

The figure also shows that with more normalized battery energy D per sensor, i.e. longer maximum lifetimes, the CRT finds the optimum solution with an even lower percentage of calculations compared to the brute force method. The general conclusion from this simulation is that the CRT adds greater efficiency in networks

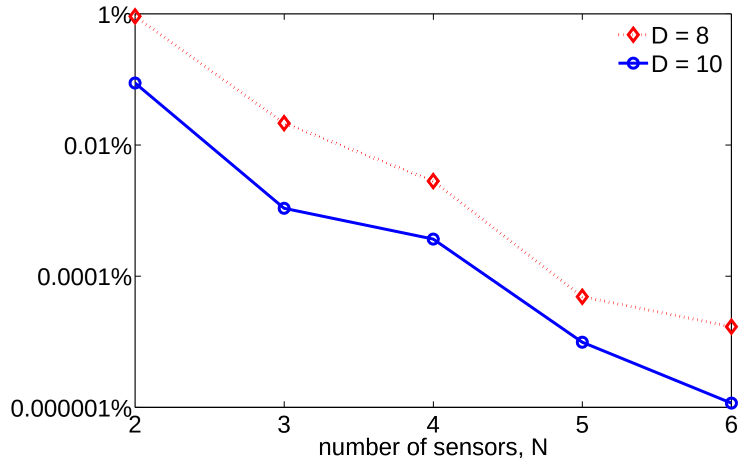

Figure 5.12: The number of water-filling procedures required for the Contention Resolution Tree with truncation as a percentage of those required for the brute force search where $\gamma_p = 0.5$. 1,000 Monte Carlo simulations in a time-varying Rayleigh fading channel.

with more sensors or sensors with more battery energy i.e. where there is more computation to be done.

5.5 Conclusions

In Chapter 4 the optimum transmit policy for a wireless sensor network was derived. Deriving the optimum schedule and transmit powers for any network based on this policy requires the computation and comparison of maximum data volumes for many lifetime combinations. In this chapter, a few methods that avoid the need to perform a considerable share of the calculations while maintaining the optimality of the solution are presented.

One method is to eliminate lifetime combinations that have no chance of being optimum before any calculation is performed. This is introduced as the early discard technique. Another is to employ a simple-to-calculate upper bound to determine the order in which the remaining candidates are studied. The maximum data volume achieved up to any point is used as a threshold to discard candidates with upper bounds below this value. Simulations further emphasize the effectiveness of this upper bound.

Both of the above methods are designed to eliminate as many candidates as possible before the data volume computation is performed for lifetime combinations. The Contention Resolution Tree (CRT) is introduced as a tool to simplify maximum data volume computations for individual lifetime combinations. A pruning technique that takes advantage of the need to compare maximum data volumes for various lifetime combinations, the threshold used to discard candidates with low upper bounds, further reduces the amount of computation required to determine the optimum network solution.

It is mathematically proven that the CRT, in its full capacity and without truncation, finds the optimum solution that a brute force search of all possible transmission schedules yields. It is also argued that the pruning technique does not affect the optimality of the network solution. Simulations show that employing the CRT along with the pruning technique requires less, in many cases significantly less, than 1% of the calculations for the brute force approach. They also show that the CRT is more efficient compared to the brute force approach in larger networks and networks with more energy available to them.

Chapter 6

Optimum Solution Variations

In the last two chapters the transmission policy that achieves maximum data volume for a wireless sensor network with time-varying channels was analytically derived and the computational tools for calculating the solution were established. One of the important applications of the optimum solution is to serve as a benchmark for evaluating the performance of other schemes.

In this chapter two variations to the optimum solution are introduced. One variation is the suboptimal solutions that are lighter than the optimum scheme in terms of computation and produce data volumes close to the maximum data volume. These solutions can be used as substitutes for the optimum policy where data volume can be sacrificed for computational simplicity. The second variation is sleep scheduling which allows sensors that are not transmitting to operate in a low-activity state and thereby conserve energy. Tweaking models and solutions from previous chapters, the optimum transmission and sleep policy is derived for the star sensor network with time-varying channels.

6.1 Suboptimal Solutions

The Contention Resolution Tree (CRT) is a tool that greatly simplifies the search for the optimum time slot to sensor allocation for a lifetime combination. Instead of searching through all time slot to sensor allocations, like the brute force approach, the

CRT only finds the optimum allocation of time slots in which more than one sensor wants to transmit data.

Following the same strategy, this section discusses two contention resolution methods that require less computation than the CRT. These conflict resolution methods do not achieve the optimum time slot to sensor allocation that maximizes the data volume, but maintain a close to optimum performance. Similar to the CRT, for each combination of sensor lifetimes, the suboptimal solutions start by allowing sensors to pick the time slots they would like to transmit in. In the event that more than one sensor schedules transmission for a certain time slot, the suboptimal method assigns the time slot to one of the competing sensors using a given criterion. For sensors that have lost access to a time slot, water-filling is performed to reallocate the unused energy. If new overlaps occur, the procedure is repeated.

One of the schemes randomly assigns the time slot in dispute to one of the sensors competing for it. Performance results for this method are tagged as *random*. The *strongest* method assigns the time slot in dispute to the sensor with the strongest channel gain. It is important to realize that this scheme is different from the limited-energy adaptation of the Knopp and Humblet solution, [34], discussed in the previous chapter. In that case, tagged *only strongest channel*, the transmission policy was based on assigning time slots to sensors with the strongest channel gains regardless of whether or not the sensor would transmit any data in the time slot and without any input from sensors. In the *strongest* method discussed here, channel gains are used to resolve conflict. Data volumes produced by both schemes appear on the figures in the following section.

6.1.1 Data Volume

The following simulations present the data volume produced by the two suboptimal conflict resolution techniques, *random* and *strongest*, in comparison to the maximum data volume and the data volume produced when time slots are strictly assigned to sensors with the strongest channel gain. Sensors are all assumed identical and 1,000 Monte Carlo simulations are used to produce each curve. The time-varying channel is assumed to be a Rayleigh fading channel and the values $B = 1$, $T = 1$ and $N_o = 1$ are employed. All policies have prescient knowledge of the channel gains for all sensors during their lifetimes.

Fig. 6.1 presents the data volume versus the network size. The data volume produced by the three suboptimal schemes is indistinguishable from the maximum data volume for a network of two sensors. As the number of sensors in the network increases the data volume curves diverge slightly. With six sensors the gap is about 10% of the maximum data volume for *random* and 4% for *strongest*.

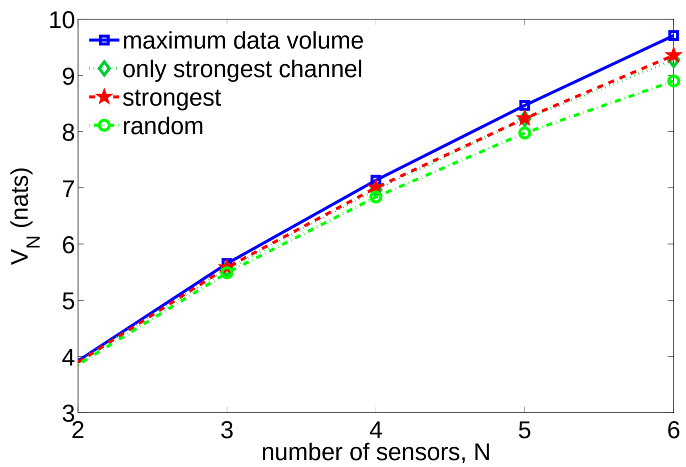

Figure 6.1: Data volume for identical sensors with $D = 8$ and $\gamma_p = 0.5$ for the maximized data volume, a limited-energy adaptation of the Knopp and Humblet solution, the random and strongest conflict resolution schemes. 1,000 Monte Carlo simulations in a time-varying Rayleigh fading channel.

Another interesting observation is that the transmission policy of strictly assigning time slots to the strongest sensor, introduced as the limited-energy adaptation of the Knopp and Humblet solution [34], performs very close to the strongest channel gain conflict resolution scheme and better than the random scheme. The amount of calculation required for determining the transmission schedule and transmit powers for all these methods is comparable, although slightly more for the contention resolution techniques since they may require additional rounds of water-filling for some sensors.

The relative position of the data volume curves in comparison to one another

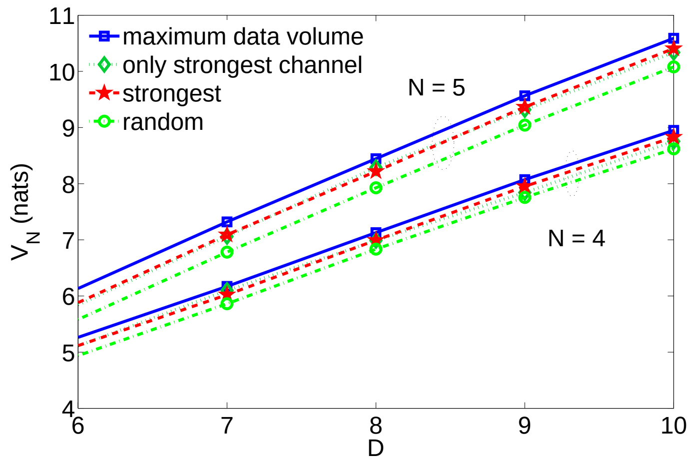

Figure 6.2: Maximum data volume, data volume for a limited-energy adaptation of the Knopp and Humblet solution, the random and strongest conflict resolution schemes with results averaged over 1,000 Monte Carlo simulations for various values of D. Simulations are for $N = 4$ and 5 identical sensors where $\gamma_P = 0.5$ and a time-varying Rayleigh fading channel.

stays the same in all data volume figures presented in this section. The data volume curve for the conflict resolution technique that favors the sensor with the strongest channel gain is the closest to the maximum data volume curve, followed very closely by the Knopp and Humblet adaptation. The data volume produced by the random assignment scheme is a little less than the other two suboptimal schemes. It is also interesting that all three suboptimal curves have the same shape as the maximum data volume curve. The reason is that for a small percentage of cases the suboptimal techniques make non-optimal assignments, causing the their average data volume to fall below the maximum data volume. However, since the majority of their decisions are optimal, the average data volume curve maintains the shape of the optimal data volume curve.

Fig. 6.2 and Fig. 6.3 both reflect the observation from Fig. 6.1 that with more sensors in the network performance gaps between suboptimal schemes and the maximum data volume are wider. However, Fig. 6.2 illustrates that performance gaps are almost constant with a fixed network size and fixed processing power, and are independent of normalized battery energy changes within bounds. Therefore, the data volume loss when employing any of these suboptimal schemes can be estimated from the data volume versus network size curve or fictitious processing power curve when D does not deviate too far from the values used for simulation.

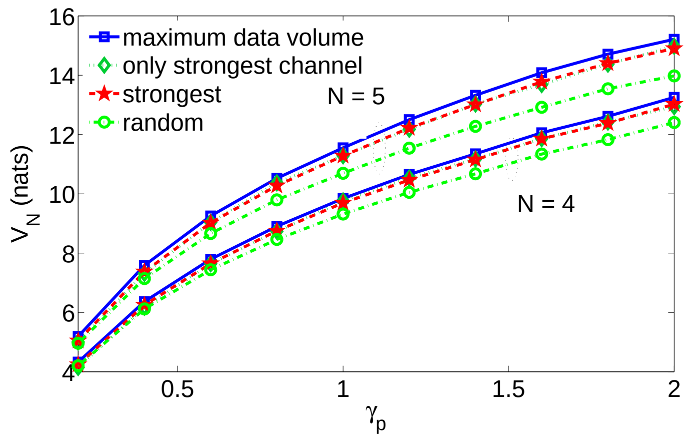

Figure 6.3: The effect of γ_p on the maximum data volume, a limited-energy adaptation of the Knopp and Humblet solution, and the random and strongest conflict resolution schemes, for N =4 and 5 identical sensors with $D = 8$. Simulations are averaged over 1,000 Monte Carlo runs for a time-varying Rayleigh fading channel.

6.1.2 Activity Duration

Fig. 6.4 shows the activity duration, defined as the time until the last sensor expires, for the three suboptimal schemes in comparison to the optimum transmission scheme. As seen with the data volume curves plotted against network size, the activity period curves start very close for the 2-sensor network and diverge with the growth in the size of the network. While the *strongest* and *only strongest channel* schemes have activity periods relatively close to the optimum policy, in larger networks, the random scheme moves farther away. It exhibits an average of over 10% longer activity compared to the optimum scheme with six sensors in the network.

The curves representing activity periods for the four policies are positioned in reverse order compared to the data volume curves with the optimum policy curve being the lowest. This suggests the energy that might have been employed for transmitting data in the random scheme, has instead been consumed to extend sensor lifetimes.

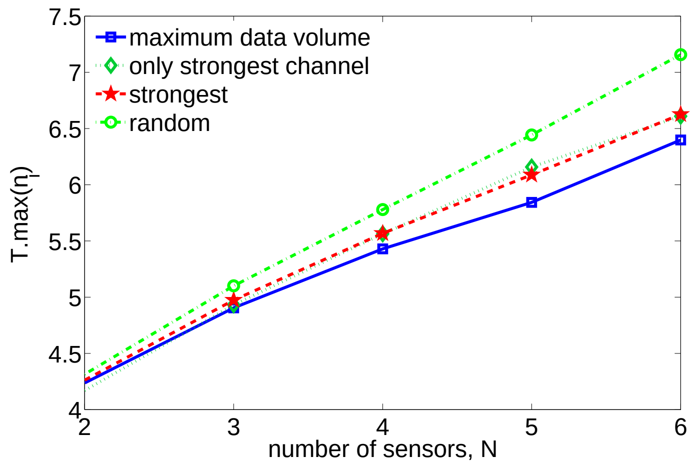

Figure 6.4: Activity duration for the optimum transmit policy, a limited-energy adaptation of the Knopp and Humblet solution, and the random and strongest conflict resolution schemes, with identical sensors using the same set of data as Fig. 6.1.

6.2 Sleep Scheduling

Energy conservation is a major concern in wireless sensor networks. In any sensor network, sensors that are not communicating information consume valuable battery energy to no effect. If sensor schedules are known, even for a short window into the future, sensors that are not scheduled to transmit or receive can slow down their activity and settle into what is referred to in the literature as a sleep state.

There are a few crucial matters to discuss when designing a sleeping scheme for a wireless sensor network, some of which apply to the star network discussed in this dissertation. Based on the application, it may be important to maintain a minimum level of connectivity over the coverage area. In most cases densely deployed networks are much more convenient for sleep scheduling.

Another issue is returning sensors to their active state, which is referred to as waking up the sensors. The more common assumption is that sensors know how long to sleep when they are entering the sleep state and can employ a timer to wake up. Using a wake up call, e.g. [74], requires a low power receiver radio, which for the time being is too costly to install on a large scale.

The transition to and from the sleep state is time and energy consuming. Although most researchers choose to ignore time and energy costs, those who take it into account find that it is more energy efficient to schedule consecutive time slots for sleep in order to avoid transitional energy expenditures. For this reason some authors, e.g. [74], limit the frequency of wake ups permitted for each sensor.

In the subsequent text some of the previous research into sleep scheduling is presented. In Section 6.2.2 the optimum transmission schedule for a star wireless sensor network with time-varying channels is revisited and the solution is updated to accommodate sleeping options for sensors.

6.2.1 Prior Research

[25] investigates the relationship between reduction in sensor duty cycles and the level of data redundancy required for a fixed performance. The performance measure is the probability that any point in the network coverage area is not covered by an active sensor when it could be covered. In other words, they measure the probability of data not being collected from a point because a sensor is asleep.

The authors assume synchronized sensors with low duty cycles, i.e. more sleep time compared to active time, in a time slotted system. They also account for idle energy consumption but do not model energy expenditures during sleep. Two types of sleep schedules are investigated. The random sleep schedules, i.e. no coordination with other sensors, and the coordinated sleep schedule in which a sensor coordinates its sleep pattern with its neighbors taking into account its exact location and remaining battery. In real life, unless sensors are laid out on a grid, obtaining exact location information requires expensive and rather bulky hardware. In addition, remaining battery charge readings are not always accurate.

Judging on the chosen performance measure the authors concluded that with the same density of nodes, despite the overhead in communication, the coordinated scheme provides better data redundancy and allows for more reduction in sensor duty cycles. No discussion is made on the comparison of throughput for these two methods.

Following the conclusion that sleep algorithms should be coordinated within the network, [74] introduces sensor-MAC (S-MAC) as a Medium Access Control (MAC) protocol for an event driven wireless sensor network. Virtual clusters based on common sleep schedules are formed to reduce control overhead and enable traffic-triggered wake up. Each sensor is required to keep a table of its neighbors' sleep schedules. It also broadcasts its own sleep schedule.

S-MAC and all its variations are compared to an 802.11 style MAC protocol in terms of energy consumption, throughput and latency. A 10-sensor network is used for all simulations. In general, all variations of S-MAC consume less energy than the 802.11 style MAC protocol. Understandably, they also take longer to deliver a packet and provide less throughput. To provide a combined measure of energy expenditure and throughput, the authors use the energy-time cost per byte calculated for various traffic loads - the unit being *joules.sec/byte*. Other than in very high traffic loads (arrival times of less than 1.5sec) S-MAC is shown to out perform the case of no sleep cycles.

In an effort to analyze adaptive sleeping schemes in a more abstract light [17] presents the formulation model for sleep schedules as a tunable property of any wireless sensor network. Implementing a mathematical formulation as the basis for simulating network performance, the authors use energy consumption, packet loss and throughput as performance criteria. They show that S-MAC type protocols have a small performance advantage compared to asynchronous protocols.

Most adaptive sleeping protocols including S-MAC require tight synchronization which becomes increasingly difficult in larger networks. Thus, showing that asynchronous schemes provide close to S-MAC performance in terms of packet loss, throughput and idle time, encourages the development of these schemes especially since they provide significant reductions in complexity and communication overhead.

In [39] the authors work to minimize the overall network delay due to sleeping sensors while maintaining an average duty cycle requirement for sensors. To measure delay they define the *delay diameter* as the maximum delay along the shortest delay path between two sensors. The duty cycle is assumed fixed, the network is synchronized and the data rate is assumed very low with the pretext that sleep schedules are not as effective in high data rates.

An analytical solution is found for two special cases, a tree layout and a ring layout. For a general topology heuristics are employed. It is concluded, from simulations, that purely localized heuristics perform worse in terms of delay than random sleep scheduling. The centralized scheme is found to provide about 50% delay reduction compared to the random scheme. Note that in order to draw these conculsions average duty cycles for the network, or at times for individual sensors, are fixed. In other words, sensors are required to be awake on average one time slot in every k time slots. Carefully desgined sleeping schemes that allow a sensor more than one wake up are shown to perform significantly better in terms of delay.

To ensure overall network performance targets are met despite variable connectivity due to channel variations, depletion or addition of sensors to the network, and sleep schedules, [69] suggests an adaptive sleep discipline in the homogeneous wireless sensor network it studies. Under the recommended scheme decisions to sleep or stay awake and the duration are formed by individual sensors based on local observations without the need to track their neighbors' sleep patterns. Sensors make decisions on when and how long to sleep based on the number and wait time pertaining to messages that arrived while they were asleep. These messages are kept at neighboring sensors until the sensor is active and can receive them.

The protocol works regardless of topology and does not require much communication overhead. As the authors refer to it, the algorithm is lightweight. It is concluded via simulations that energy savings under this protocol are proportionate to sensor density. However, density numbers presented do not include units. Simulations show that reductions in channel quality, defined as the probability that communication between two sensors is successful, and node density have similar effects on the durations sensors choose to sleep.

[11] partitions the network into groups based on mutual sensor connections. In each group, at any given time, only one sensor needs be active to keep the flow of

information. Defining the network lifetime as the time when the backbone network formed by active sensors is disconnected, the authors use simulations to show that the proposed scheme improves network lifetime compared to geographical partitioning of the network.

Following the introduction of hierarchical clustering in [23], Section 2.2.1, sleep scheduling schemes for the network are studied in [10]. Here, cluster heads make sleep decisions for sensors within their cluster based on a proposed balanced-energy sleep scheme. The scheme is aimed at increasing the network lifetime defined as the expiry of a fraction of sensors.

Two methods are compared to the balanced-energy method. One is random sleeping where sensors are randomly permitted to sleep and the second is distance based scheduling in which sensors closer to the cluster head are less likely to sleep. This is rationalized by the idea that they need to be awake to convey messages from outer sensors. The balanced-energy case is a special case of the distance based scheduling where on average sensor energy consumption is kept the same for all sensors.

Employing simulations the authors show that the balanced-energy method prolongs the network life time by a factor of 1.5 and 0.7 compared to the random and distance based scheduling. In addition, simulations show that the coverage area for all three methods is similar. In [53] and [54] the authors introduce comparable coverage-aware sleep scheduling techniques which claim to out perform the methods introduced in [10] in providing coverage while maintaining the network lifetime.

In [73] the authors propose a *data coverage* scheme based on the ability of active sensors to recover the missing data of sleeping sensors. This data is recovered employing the correlation of data collected at active sensors. The results are compared, via simulation, to a linear distance based sleeping scheme in which sensors are more likely to sleep if they are farther away from the cluster head. The results show that while maintaining the same loss ratio their proposed scheme allows more sensors to sleep and therefore saves more energy.

Focusing on the effect of synchronization errors [72] works to minimize expected energy consumption for a cluster head in a hierarchical network, described in [23], while maintaining a minimum level of message capture probability. The energy consumed at the cluster head for receiving data and idle periods are accounted for and the structure within the cluster is assumed to be a single-hop (star) structure. In the next step, given a requirement for a certain network quality of service, the authors formulate the optimization problem that determines message capture probability for each sensor in the cluster while minimizing total energy consumption for the cluster head. A suboptimal solution is introduced that consumes at most 37% more energy than the optimum scheme. The solution is based on the idea that the collective performance of the cluster is gauged and not the performance of individual sensors.

[16] formulates the optimum sleeping policy for a centrally controlled sensor network whose aim is to track a randomly moving object. Sensors are placed on a grid and their coverage areas do not overlap. With knowledge of the current position of the object and its projected trajectory the central controller determines sleep durations for sensors. Since the sensors that are predicted to be away from the target's path are set to sleep, if the target diverges from the predicted path tracking errors will occur. Thus, the sleep policy design is a trade off between sensor energy consumption and tracking errors.

The analytical state space formulation of the problem in [16] proves difficult to solve for more than a few sensors. In turn, the authors present two suboptimal sleep policies with close to optimum performance and derive lower bounds on the optimal sleep policy.

Allowing sensors to sleep and accounting for the processing power in addition to transmit and reception power and using a fairly elaborate Markov model [7] evaluates the network performance in terms of energy consumption, network capacity and delay. The work accounts for mutual interference and routing. Unlike most of the literature the paper is not presented as an optimization problem but an investigation into the correlation of energy consumption and delay, despite the fact that the routing algorithm favors energy saving over delay reduction. Simulations show that the analytical model behaves very similar to the simulated wireless sensor network.

Defining network lifetime as the time until there are no longer any connections to the sink, [64] derives semi-analytical upper bounds on sensor duty cycles and on the achievable network lifetime for a wireless sensor network with no specific configuration.

Recognizing the energy expenditure of transitioning between active and sleep states [40] proposes to keep sensors in one state for a period of time instead of switching back and forth. This is similar to the assumption of S-MAC which allows only one wake up for each sensor within a fixed period of time. This is done in a system with strict synchronization and TDMA channel access. Central and distributed algorithms are suggested in this work.

6.2.2 Modifications to the System Model

The system model adopted in this chapter is very similar to that of Section 4.1 with the exception that sensors can now reside in one of two activity states, the transmission state or the sleep state. In the transmission state the power consumed, sensor and time subscript omitted, is

$$P = P_{tx} + P_p \tag{6.1}$$

similar to equation (2.1). In the sleep state, the sensor enters a low-activity state and only a small fraction of the processing energy is consumed. For this state the power consumed, having omitted time and sensor subscripts, is

$$P = P_s. \tag{6.2}$$

Generally, sensors would have a third state in which they would be awake but not transmitting. However, to keep the problem formulation simple for the first inquiry into optimum scheduling, coverage issues and transition energies are not discussed. As a result the third state does not occur. If a sensor is not transmitting it enters a low-activity state or in other words sleeps.

Since the sink is responsible for providing transmission schedules to sensors, Section 3.1, it will in effect provide sleep schedules too. Sensors are assumed to use a timer to wake up so no wake up call is required.

6.2.3 Problem Statement

The problem statement is modified to read:

> ***Given a star wireless sensor network with time-varying channels, find the optimum transmission and sleep policy that maximizes the data volume.***

6.2.4 Notation Changes

A few symbols are added to the previously introduced notation. To represent the power consumed when in sleep state P_s was introduced above (6.2). When normalized by the processing power χ represents low-activity power consumption, without sensor and time indices

$$\chi = \frac{P_s}{P_p}. \tag{6.3}$$

Employing (6.1) and (6.2), the energy consumption for sensor i is

$$D_i = BT\left(\sum_{t=1}^{n_i} x_{it} + (n_i - \nu_i)\chi_i + \nu_i\right) \tag{6.4}$$

where ν_i is the number of time slots sensor i is active. This equation is reduced to (4.7) if sleeping is not permitted, in which case $\chi_i = 1$.

6.2.5 Modifications to the N-sensor Solution

With the assumption that each sensor enters a sleep state when it is not transmitting, the solution for the optimum transmission scheme closely follows the solution presented in Chapter 4.

One-Sensor Network

Assuming sensor i has lifetime n_i, the data volume remains unchanged

$$V_1 = BT \sum_{t=1}^{n_i} \ln(1 + x_{it} z_{it}) \tag{6.5}$$

where

$$\forall t : x_{it} \geq 0. \tag{6.6}$$

Equation (6.4) can be rewritten as

$$D_i = BT \left(\sum_{t=1}^{n_i} x_{it} + n_i \chi_i + \nu_i (1 - \chi_i) \right) \tag{6.7}$$

and the transmit power constraint equivalent to (4.8) becomes

$$\sum_{t=1}^{n_i} x_{it} = \frac{D_i}{BT} - n_i \chi_i - \nu_i (1 - \chi_i). \tag{6.8}$$

It is important to note that ν_i is a variable that is directly determined by a modified water-filling process and is therefore not an independent value. The modification to the water-filling process is as follows: During the water-filling process as the water level rises and water pours into a time slot, the value of ν_i is updated. ν_i is at its final value when all the available energy has been utilized for water-filling i.e. when (6.8) is satisfied.

As the layout of the problem suggests, the single-sensor solution with sleep periods is the same as the case with no sleep periods presented in Chapter 4, except for the modification in water-filling that accounts for the change in processing power. Therefore, optimum transmit powers are

$$\forall t \leq n_i : x_{it_{opt}} = \left(\frac{1}{\mu_i} - \frac{1}{z_{it}} \right)^+ \tag{6.9}$$

with the inclusion of the normalized sleep processing power χ_i so as to satisfy (6.8).

Two-Sensor Network

The only change in the equations for a two sensor network is the energy consumption equation for sensors p and q.

$$D_p = BT\left(\sum_{t=1}^{n_p} x_{pt} + n_p\chi_p + \nu_p(1-\chi_p)\right) \tag{6.10}$$

$$D_q = BT\left(\sum_{t=1}^{n_q} x_{qt} + n_q\chi_q + \nu_q(1-\chi_q)\right) \tag{6.11}$$

$$\forall t : x_{pt} \geq 0\ ,\ x_{qt} \geq 0. \tag{6.12}$$

All other equations and therefore the solution that sensors should not transmit simultaneously remain intact. Optimum transmit powers are determined via modified water-filling, satisfying (6.10) and (6.11), over the optimum time slot allocation.

N-Sensor Network

Data volume equations for the N-sensor and $N-1$-sensor network are exactly the same whether or not low-activity states are permitted. The general energy consumption equation for any sensor i is that of (6.7). Since only the energy consumption model for sensors has changed, the optimum N-sensor network solution that no simultaneous transmissions should occur in the optimum transmission scheme is still valid. In addition, similar to the derivations in Chapter 4, optimum transmit powers are determined via single-sensor water-filling over time slots assigned to sensors. However, the modified water-filling process is employed to accommodate the normalized sleep processing power χ and determine ν for all N sensors.

6.2.6 Simulations

The power consumed during the low-activity period is assumed to be a small fraction of the processing power. For simulation purposes this value is set to

$$\chi = \frac{1}{100}. \tag{6.13}$$

With the exception of [7], and to some extent [25], it seems that common practice is to ignore processing power during the active state when designing sleep schedules for sensors. For this reason there are no benchmark numbers available for χ. Authors who account for sleep processing power assume fixed transmit powers and define the power consumed while sleeping as a ratio of the transmit power. However, since transmit powers are optimized in this dissertation, this definition is not useful. In [7] the authors do not provide an explanation for the values they choose, so it is assumed that the numbers are within reasonable bounds but arbitrary.

The data volume attained by optimizing the transmission and sleep policy is compared to the data volumes obtained in Chapter 5 where sleeping is not an option. In other words, the case of $\chi_i = 0.01$ and optimum ν_i and n_i is compared to the case of optimum n_i where $\chi_i = 1$. The main goal of this comparison is to gain a general understanding of the effects of the sleep policy. As will become clear in Section 6.2.7 the results should be interpreted with caution.

In Fig. 6.5 the data volume produced with and without implementing a sleep state are compared. Simulations are performed with parameters similar to those in Chapter 5 and Section 6.1 to allow for easy comparison. The data volume curves show that the sleep state provides a considerable increase in the data volume. Even with only two sensors in the network, implementing a low-activity state increases the data volume by 25%. A network with four sensors that are allowed to sleep provides the same data volume as a 6-sensor network that does not schedule sleep. Of course the number of sensors that provide equivalent data volume in the two scenarios varies based on network parameters, most significantly χ.

Fig. 6.6 provides two sets of curves for each scheduling policy. One curve is the time until the last sensor expires (maximum expiry time) and the other is the average expiry time for all the sensors that make up the network. The maximum expiry is calculated as the average expiry time for the last sensor to expire over all Monte Carlo simulations. These curves highlight a few interesting trends.

The sleep scheduling scheme provides greater expiry times than the no-sleep policy regardless of the number of sensors in the network. The two sleep scheduling curves have the same outline for networks of 2 to 6 sensors. The sudden jump in both the average and maximum lifetime with 7 sensors is a result of the value of the normalized battery energy i.e. $D = 8$. With 7 sensors in the network, in some cases due to the channel gains, the maximum data volume is achieved when one of the sensors does not

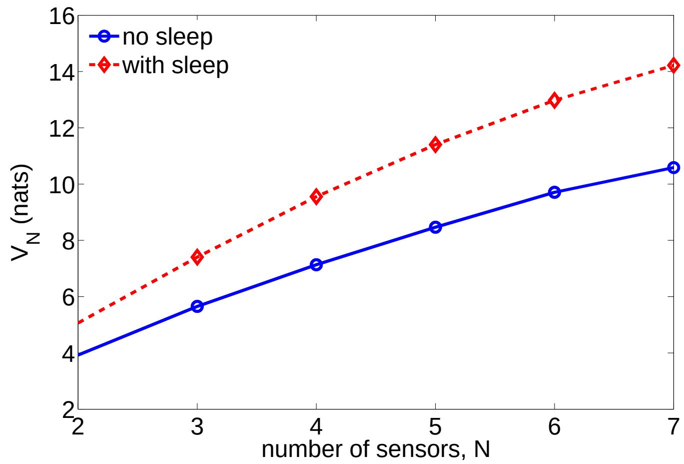

Figure 6.5: Effect of sleep scheduling on network data volume with the increase in network size. Sensors are assumed identical with $D = 8$ and $\gamma_p = 0.5$, $\chi = 0.01$ for the policy with sleep and $\chi = 1$ for the no-sleep policy. Curves are produced by 1000 Monte Carlo simulations. The curve with no sleep is identical to that of Fig. 5.6 and Fig. 6.1.

transmit any data. The sensor that does not transmit any data sleeps until its battery expires, which in turn creates a very long lifetime for the sensor and significantly increases maximum and average sensor expiry times.

The slight decrease in the maximum and average lifetimes for the sleep scheduling policy at around six sensors can also be explained. Since the cost of sleeping until a time when the sensor-sink connection is strong is low, when there are only two sensors in the system each sensor can be instructed to transmit at times when its channel is strongest and to sleep between transmissions. This also brings about extended lifetimes for these sensors. When more sensors are added to the network, compromises are reached in terms of access to time slots for transmission to maximize the data volume. In effect, although the network data volume increases, more conflicts in transmission schedules require sensors to settle for less desirable time slots and their lifetimes are adjusted accordingly. This is why average sensor lifetimes decrease slightly with more sensors in the network.

The maximum and average expiry times for the no-sleep scheme increase steadily as the number of sensors increases. Compared to the sleep scheduling scheme, due to processing power, the sensors in a 2-sensor network with no sleep expire much sooner. To produce their maximum data volume these sensors transmit in the best time slots within their reach. With a larger network there are more sensors available for transmission and some sensors can wait a little longer for a more desirable time to transmit. Therefore, the average and maximum lifetimes increase with more sensors. It should, however, be reminded that the average expiry time for the sleep scheduling scheme is always greater than the no-sleep scheme regardless of the number of sensors since it allows sensors a wider choice of time slots for transmission and the ability to preserve energy to reach the desired time slots.

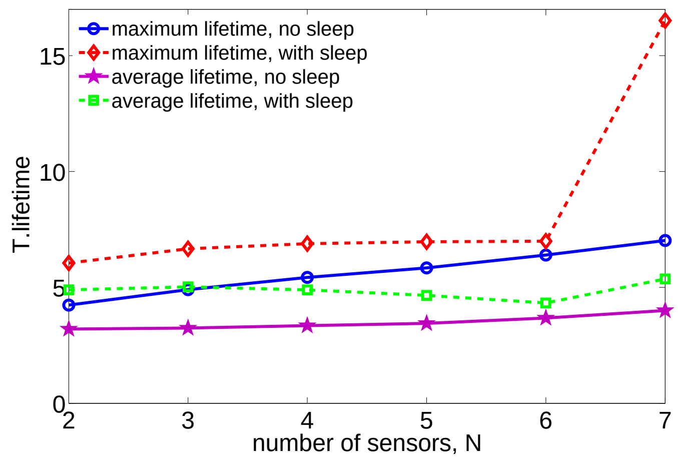

Figure 6.6: Effect of network size on average and maximum sensor expiry times. The maximum expiry is calculated as the average of maximum sensor lifetimes over all Monte Carlo simulations. Data is taken from simulations presented in Fig. 6.5.

In Fig 6.7 data volumes are compared for the two scenarios with and without sleep scheduling as the initial sensor battery energy is increased. A 4-sensor network with sleep scheduling produces more data volume than a 5-sensor network with no sleep scheduling. At $D = 9$, the gap of the 4-sensor network with sleep and the 5-sensor with no sleep schedule is almost equal to that of the 4-sensor and 5-sensor networks with no sleep scheduling. Therefore, it can be said that with $D = 9$ allowing sleep states for sensors is equivalent to an extra sensor in the network in terms of data volume. Again, these comparisons depend on network values such as D and most significantly χ.

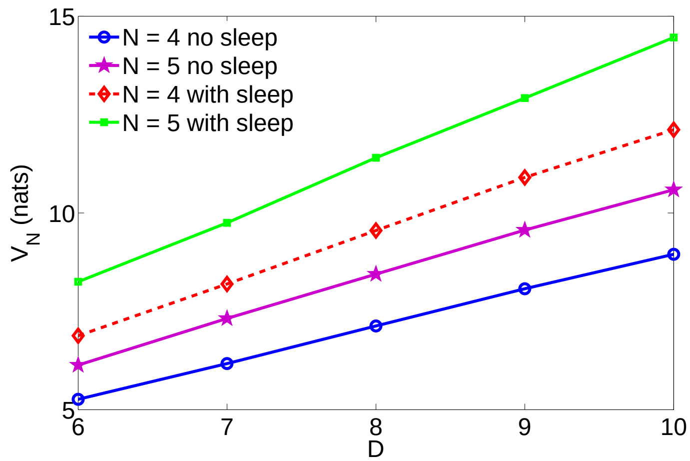

Figure 6.7: Effect of battery energy on a 4-sensor and 5-sensor network with and without sleep scheduling. Sensors are assumed identical with $\gamma_p = 0.5$ and $\chi = 0.01$ for the policy with sleep and $\chi = 1$ for the no-sleep policy. Curves are produced by 1000 Monte Carlo simulations. The curves with no sleep are identical to those of Fig. 5.7 and Fig. 6.2.

6.2.7 Interpretation Limitations

The substantial increase in data volume brought about by implementing sleep states for sensors should be interpreted with caution. The sensor model employed in this chapter assumes sensors can switch back and forth between transmission and sleep states with no time delay or energy costs. This simplistic model allows an initial inquiry into the behaviour of the network and the change in data volume with the introduction of sleep states. However, once the time and energy consumed for switching between states is factored in, it is very likely that the increase in data volume and the frequency of switching between the two states will be reduced.

Presenting numbers for a MICA2 node, [40] suggests that turning on and initializing the radio and switching to the transmission state costs about the same amount of energy as transmission itself. They assume the ratio of the energy consumed during sleep to energy consumed for transmission is 0.0015.

6.3 Conclusions

This chapter of the dissertation studies two sets of variations to the optimum transmission policy for the N-sensor wireless network with time varying channels developed in Chapters 4 and 5. The maximum data volume is a performance benchmark and provides an upper bound for the data volume produced by suboptimal transmission policies.

The first section of the chapter discusses suboptimal solutions with close to optimum performances. In effect, these techniques replace the optimal decision making process of the Contention Resolution Tree, Chapter 5, with suboptimal alternatives that require little computation. The performance of these suboptimal schemes is also compared to another suboptimal technique, the scheduling policy of strictly assigning time slots to sensors with the strongest channel gain, which is applied as the limited-energy adaptation of the Knopp and Humblet solution [34].

To resolve transmission conflicts and to replace the CRT, the time slot in contention can be randomly assigned to one of the sensors competing for it or it can be assigned to the sensor with the strongest channel gain. It is found that in terms of data volume the strongest channel scheduling policy (Knopp and Humblet adaptation) and the strongest channel conflict resolution technique have very close performance. In addition, they maintain a small gap with the maximum data volume regardless of the network size, battery energy or processing power. The performance gap for these two suboptimal schemes relative to the maximum data volume increases slightly with the network size and processing power but is fairly unaffected by normalized battery energy.

Resolving conflict by randomly assigning the disputed time slot to one of the sensors in contention is the least favorable solution. It requires about the same amount of computation as the Knopp and Humblet adaptation and performs worse. In general, after investigating suboptimal solutions, it seems that the Knopp and Humblet adaptation and after that the strongest channel conflict resolution technique are low-computation alternatives to determining transmission policies.

The energy that could have been consumed to produce data volume in the suboptimal schemes is instead consumed to extend sensor lifetimes. Thus network activity duration is greatest for the random conflict resolution scheme, followed by the other two suboptimal schemes. The maximum data volume scheme has the shortest network activity duration.

The second section of the chapter presents the optimum transmission and sleep policy which maximizes the data volume for the N-sensor wireless network. It is assumed that sensors enter the low-activity state when they are not transmitting. Sleeping allows sensors to preserve energy until such time as they need to transmit data, making it more feasible for sensors to take advantage of stronger channel gains further in the future.

The only change in the system model from Chapter 4 is that sensors are assumed to consume less power, a small fraction of the processing power, when in the low-activity state. Optimum scheduling remains unchanged and optimum transmit powers are determined via water-filling. However the sum of energies equation, and therefore optimum transmit powers, reflect the change in the power consumption model. A modification is made to the water-filling process to accommodate the normalized sleep processing power.

Simulations show that allowing sensors to enter a sleep state in between transmissions not only extends sensor expiry times but increases the data volume rather significantly - 25% for a 2-sensor network where the power consumed during sleep is 1/100 of the processing power. It is important to remember that for this first analysis of sleep scheduling, assumptions were kept simple and not all factors were accounted for in the network model, e.g. transition energies to and from sleep. Once accounted for, these considerations will reduce the effects of sleep scheduling.

Chapter 7

Conclusions

The goal of this dissertation was to allow a better understanding of the factors influencing the data volume produced by a star wireless sensor network. This quest begins with the study of optimum transmission policies that maximize the network data volume with various channel conditions. Sleep scheduling is another variation to the network model for which optimum transmission policies are derived. With the knowledge of optimum policies, the maximum data volume is used as a benchmark to suggest and evaluate the performance of suboptimal techniques.

7.1 Contributions

The contributions and novel insights of this dissertation are summarized below.

Chapter 2 Foundation for the Dissertation Including a New Performance Criterion

- Basic sensor, network and channel models are introduced and the main purpose of wireless sensor networks, gathering data at the sink, is brought to light. Along with the introduction of the models employed in the dissertation, inclusion of processing power and mutual interference are emphasized since they have been widely neglected in the literature. In the subsequent chapters the effects of processing power on the data volume are discussed.

- Sensor networks have a new and unique set of properties, particularly that of finite battery charge. It is only reasonable to design a performance evaluation metric specific to them. To this end, data volume is presented as a new performance criterion tailored specifically towards sensor networks.
- Setting the scene with a summary of prior work in the field, the specific questions addressed in the dissertation are introduced.

Chapter 3 Optimum Transmission Policies for a Star Network with Static Channels

- This dissertation is the first to analytically derive jointly optimum transmit powers and scheduling schemes that maximize the data volume for a star wireless sensor network with static channels while accounting for processing power.

  The solution proves that, regardless of the number of sensors in the network, the optimal scheduling policy is for sensors to avoid simultaneous transmissions. Each sensor transmits with the optimum transmit power until its battery expires at which point the next sensor in line starts transmission. The optimum transmission order depends on initial battery charges, processing powers and channel gains. Optimum transmit powers are determined by solving an equation that relies on all the sensors that have not yet expired.
- Closed form equations for the transmit power and resulting data volume are presented. Transmit power equations are independent of normalized sensor battery charges, D, and rely on the fictitious processing SNR, γ_p which is the processing power divided by the transmission bandwidth and noise power spectral density, and channel gains of all the sensors that are still active. Given that the channel is static, transmit powers and optimal transmission orders can be calculated beforehand.
- The effects of variations in processing power, network size and initial battery charge are studied through simulation. It is shown through simulations that with identical sensors the maximum data volume changes linearly

with network size and initial normalized battery charge, and logarithmically with the fictitious processing SNR, γ_p. Comparison to a scheme with fixed powers equal to average optimum values emphasizes the importance of determining optimum transmit powers.

The two suboptimal techniques developed with computation-saving modifications to the optimum solution perform very well and are suggested as close substitutes for the optimum solution when fictitious processing power transmit SNR γ_p is around 0.5 and where data volume can be compromised. In these schemes transmission, and in turn expiry, order is determined randomly or based on channel gains. However, transmit powers are calculated using optimum equations.

- Although the suboptimal techniques exhibit close to optimum performances, on average they show shorter network activity durations, defined as the sum of transmission times for sensors. In other words, sensors expire more quickly when not transmitting in optimal order.
- Introducing the solution with no processing power in the sensor model underlines the effect of processing power and cautions against conclusions drawn from inadequate sensor models. It is analytically shown that, due to processing power, the data volume produced by each sensor is reduced by a fixed amount calculated based on its channel gain and optimal transmit power.

Chapter 4 Time-Varying Channel Model

- The combination of processing power in the sensor model, mutual interference in the network model, use of the data volume criterion, and time-varying channel gains sets this problem formulation apart from previous work. In this chapter the transmission policy for a star sensor network with time-varying channels is analytically optimized. The optimum policy presents jointly optimum transmit powers, transmission schedules and sensor lifetimes.

Similar to the case of static channels it is shown that, regardless of the number of sensors in the system, the data volume is optimized when no simultaneous transmissions are scheduled. However, unlike the static channel solution, there may be time slots where no transmission is scheduled. Sensor schedules and transmit powers are determined via single-sensor water-filling and ensuring no overlaps in transmission. Here, a brute force approach is used in searching for optimum sensor lifetimes and scheduling, i.e. determining optimum time slot to sensor allocations.

- Unlike the optimum transmit power with static channels, the optimum water level and transmit power equations for the time-varying channel rely solely on the sensor's channel gains and battery energy. However, since the time slots each sensor transmits in is determined in combination with other sensors, these values are not independent of the network.

- The effect of processing power on the data volume is studied by deriving the parallel optimum solution where processing power is not modeled. Equations show that per-sensor data volume reduction due to processing power is a function of the sensor channel gain and the water level, optimized with and without processing power.

- The unlimited-energy counterpart problem presented by Knopp and Humblet [34] is discussed. Transmit powers for the Knopp and Humblet solution, with constrained averages, are determined by water-filling with a water level derived using the joint probability distribution of channel gains. Thus, the solution requires a statistical model of channel behavior. Time slots are strictly assigned to sensors with the strongest channel gain.

  The optimum time slot to sensor allocation for the wireless sensor network does not always favor the strongest channel gain. Also, water levels are determined based on the gains in the optimum time slot assignment and the battery energy available to sensors. These contrasts are brought about by restrictions on the energy available to sensors.

- The assumption in deriving the optimum transmission policy for the time-varying wireless sensor network and for all other time-varying policies discussed, including those in Chapters 5 and 6, is that the sink, who is in charge of designing the transmission policy, has prescient knowledge of the channel gains for the lifetime of the sensors.

Chapter 5 Low-Computation Tool to Determine Transmit Schedules

- The Contention Resolution Tree is introduced to eliminate the need for performing tedious and unnecessary calculations in finding the optimum transmission schedule. It is proven that the CRT does not compromise the optimality of the final solution. This technique replaces the brute force search of the optimum time slot to sensor allocation and is shown to require less than 1% of its calculations.
- Two methods for expediting the search among lifetime combinations for the optimum solution are introduced. Early discard of some candidates in the search eliminates them before any computation is performed.

  It is also suggested that upper bounds on the maximum data volume of remaining combinations be used for determining the order in which calculations are performed. Ordering combinations helps find the optimum solution sooner. Once the highest data volume calculated is greater than the upper bound on the next candidate the search is terminated. These upper bound values are used for the CRT and therefore do not add to the computational burden.

  The early discard and upper-bounding and ordering technique are employed to revise the brute force algorithm of finding the optimum solution. The effect of the early discard method can be calculated based on sensor lifetimes; and simulations show the ordering technique reduces the number of lifetime combinations studied after early discard by about 60% to over 99%.
- The highest calculated data volume, used as the cut off threshold for the

upper-bounding technique, is employed to terminate paths on the CRT that will not result in the optimal solution. While improving the search efficiency, the truncation method does not require additional computation except that of comparing values to the cut off threshold.

- Employing the above techniques, simulations are performed to compare the optimum transmission policy to alternative policies such as the round robin and a limited-energy adaptation of the Knopp and Humblet solution in terms of data volume and network activity duration. The effect of processing power, network size and initial battery charge on the maximum data volume are also studied via simulations. Knowing the performance of the optimum technique allows evaluation of suboptimal schemes and outlines how much their performance can be improved.

  The Knopp and Humblet adaptation which assigns time slots to sensors with the strongest channel gain and performs water-filling over the assigned time slots is found to perform close to the optimum solution. The energy it does not use to produce data volume extends sensor lifetimes, leading to a longer maximum sensor lifetime compared to the optimum solution.

Chapter 6 Suboptimal Policies and Sleep Scheduling

- With knowledge of the optimum transmission policy and tools that simplify its calculation, two suboptimal lower-computation techniques are introduced. These techniques replace the CRT in resolving conflict in the transmission schedule. In one scheme the time slot in contention is assigned to a random competitor. In the other, the time slot is assigned to the sensor with the strongest channel gain.

  Simulations show that the strongest channel conflict resolution and the Knopp and Humblet adaptation perform close to the maximum data volume. They are therefore suggested as suboptimal substitutes where optimality can be compromised for computational simplicity. Both methods show network activity durations, defined as maximum sensor lifetimes, greater than the optimal policy. The random conflict resolution scheme performs better than other schemes in terms of activity duration and worse in terms of data volume.

- After reviewing previous publications in sleep scheduling for wireless sensor networks, the optimum transmission (and sleep) policy for the star network with time-varying channels is developed. It is assumed that sensors sleep when they are not transmitting. The novelty of this work is not only that it derives the transmission policy analytically, but that it also accounts for mutual interference and processing power during transmission and sleep.

  The solution for sleep scheduling differs from the previous no-sleep solutions in that the energy expenditure equation now accounts for two processing powers, one for when the sensor is asleep and one for when it is transmitting. Sleep state processing power is a small fraction of the processing power during transmission.

  The optimum transmission schedule is for sensors to avoid simultaneous transmissions. Optimum transmit powers are found employing a modified water-filling technique that accounts for different processing powers during transmission and sleep. It is shown via simulations that the data volume is significantly increased with the introduction of sleep states. As expected, sleeping also extends average and maximum sensor lifetimes.

7.2 Future Research

This dissertation provides initial insight into the workings of a wireless sensor network. However, there are still many avenues to be discovered. Optimal transmission policies for time-varying wireless sensor networks are derived with the assumption of prescient knowledge of channel gains. All other time-varying policies are also simulated with this assumption. In order to take a step closer to implementing the optimum, or close suboptimum, methods in the future there is a need to develop a causal solution.

This may be possible by showing, mathematically, that average data volume is a linear function of the normalized battery energy D. Simulations suggest that this is the case, at least for reasonable changes in D. Given the linear relation, and assuming only a short window of prescient knowledge into the future, the average data volume produced in the future where knowledge of the channel does not exist can be found based on the normalized battery energy that remains after the prescient window. Thus, the optimum water level for transmission in the window can be calculated. This water level is used to determine the optimum transmit power in the current time slot and the prescient window of knowledge is moved one time slot forward. The same process is repeated to determine the optimum transmit power for the next time slot.

Even if the linear correlation of the data volume and D is an estimate, the slope can be used to estimate the average data volume based on the remaining normalized battery energy. This will yield a suboptimal solution since data volume values are average estimates. In either case, the resulting data volume should be compared to the optimum data volume with prescient knowledge.

Another possible research path is to explore the case of multiple antennas at the information sink. [75] extends the work of [34] for unlimited-energy users with average power constraints, known fading distribution and channel state information by adding spacial diversity at both the transmitter and receiver. They find that each user's optimal power allocation is derived by single-user water-filling against the noise and combined interference from all other users. It is believed that this work can serve as a guideline for extending the work of this dissertation to the case of multiple antennas at the sink. However, due to the nature of sensors, it is not recommended that multiple antenna be installed on sensors.

In Chapter 6, when developing the optimum sleep policy for sensors, it is assumed that sensors can switch back and forth between transmission and sleep states with no time delay or energy costs. This simplistic model allows an initial inquiry into the behaviour of the network and the change in data volume with the introduction of sleep states. However, it is not very realistic. Allowing a third state for the sensor, the idle state where the sensor is neither asleep nor transmitting and consumes processing power greater than that of sleep, transitional energies between the sleep and transmission states can be accounted for and the optimum sleep, idle and transmission policy can be derived. Most likely the data volume and the frequency of switching between the sleep and transmission states will be reduced while the sensor is scheduled to spend some time in the idle state.

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