DOWNSIDE-RISK PERFORMANCE MEASURES AND HEDGE FUNDS

by

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ABSTRACT

This paper delves into performance measures which have recently emerged in an attempt to circumvent the well recognised flaws of conventional measures derived solely from mean and standard deviation. I use several of these measures to rank both equity indices as well as hedge fund strategies based on their likelihood of achieving a particular return level, relative to the downside risk associated with that target return. This method makes intuitive sense since one of the key characteristics of hedge funds is to seek to capture most upside while protecting the downside.

While the conclusions clearly point to the superiority of hedge funds at all return thresholds, with the equity indices improving their rankings from worst to middle of the pack at the higher threshold levels, the use of the downside measures is not clear-cut and can be fraught with some ambiguity in the interpretation of rankings which they yield.

DEDICATION

I dedicate this paper to:

My parents, thank you for everything. The road may have been sinuous, but the experience necessary.

Malini and Druhan, thank you for your patience, love and unflinching support during these last twenty months. You make it all worthwhile.

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GLOSSARY

Downside-Risk   The risk that an investment will suffer a decline in value, and the magnitude of such loss.

MAR   The Minimum Acceptable Return required by an investor. Downside-risk is implicitly measured relative to a specific MAR.

LPM   Lower Partial Moment is a general type of risk measure which calculates only the moments below a target return.

1 INTRODUCTION

Many of the difficulties we encounter in performance measurement and attribution are rooted in two over-simplifications. The first is that mean and variance fully describe the distribution of returns. The second is that the risk-reward characteristics of a portfolio may be described without reference to any return level aside from the mean return. It is a generally accepted fact of empirical finance that returns from investments are not distributed normally. Thus in addition to mean and variance, higher moments are required for a complete description. It is likewise clear that a return at the level of the mean may be regarded as a gain by one investor and as a loss by another and that the "risk" of a return far above the mean has a different impact than that of one far below the mean. In a 1973 paper in the Psychological Bulletin, psychologist John Payne noted that standard deviation accounts for very little of people's perception of risk. The most relevant factors were downside frequency and the magnitude of the possible loss.

There are situations in which an investor's primary concern is earning a return in excess of a particular loss threshold or minimum acceptable return (referred to as MAR[1] throughout). An MAR set at the rate of inflation would protect investors from any erosion in their purchasing power. For some, guaranteeing the principal is all that matters, translating into an MAR of zero. Pension plan sponsors and managers need to earn at least the actuarial rate of return required so they could meet future liabilities. Such examples reveal the asymmetry of the nature of conventionally defined risk, since returns below the MAR are indeed risky but those above are highly desirable. Hence, describing risk in terms of portfolio mean and variance may lead to sub-optimal decisions.

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[1] The MAR is the investment return floor which separates the good outcomes from the bad.

The issues cited above are especially exacerbated when we are faced with return distributions that have been known to display significant higher moments, namely those of hedge funds. The third and fourth moments (skewness and kurtosis) of this alternative asset class have been well documented and are not the subject of this paper. However, the fact that they make the M-V toolkit somewhat inadequate is. This is where the downside perspective of risk and risk measures steps in. However, as many proponents of this approach to risk claim, downside measures are not needed simply to overcome the skewness of non-normal distributions of security returns. They are needed because they most closely mirror how investors actually behave in investment situations, namely as explained by utility theory.

What this paper does not do or attempt to do is to explore portfolio optimisation problems within a downside risk framework as opposed to using the conventional M-V toolbox. Although this is of primary importance and should be the goal of all serious practitioners who believe in the superiority of downside risk over mere standard deviation or variance, I leave this task to others after me. One possibility would be to set up a limited number of portfolios with allocations to the different asset classes in addition to hedge funds. Each portfolio would have a fixed allocation to hedge funds ranging from say 0% to 30% in increments of 5%, with allocations to the other asset classes obviously decreasing proportionately. The idea is to have portfolios that target investors with differing risk aversions. For each portfolio, the same downside measures used in this paper to compare assets on a peer-to-peer basis, could then be applied at different threshold returns to evaluate which is the optimal portfolio. The optimal allocation could then be judged against the traditional M-V allocation.

2 LITERATURE REVIEW

Since the subject of this paper is the application of downside risk measures to hedge fund returns, the literature review focuses on the measures themselves. Their usage in hedge fund academia is sparse except for demonstration in the papers of the authors of the measures outlined below.

2.1 The Early Years[2]: Reward-to-Variability and Semi-Variance

In reviewing the historical academic literature on downside risk measures, my focus will be skewed, no pun intended, towards the last fifteen years or so since the search for more effective ways of capturing risk really took off over that time frame. Even then we still need to go over the seminal work that provided the basis for these more recent measures. And 'seminal' in the investment world, more often than not, traces back to Harry Markowitz's paper in 1952. Downside risk measures are no exception in the sense that it was Markowitz who in the first place developed the notion of risk as a crucial investment criterion besides return. Governed by a quadratic utility function, investors make a trade-off between risk and return, where risk in Markowitz's framework, was captured by the variance of returns. We shall come back to Markowitz again, but only after mentioning another paper published in the same year to which we owe more in terms of the foundation of a downside approach to risk.

Indeed, motivated by his disbelief in the possibility of deriving a mathematical utility function for an investor and, in any case, the impracticality of the notion of maximizing expected utility, Roy (1952) was the first to come up with the idea that investors prefer safety of principal

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[2] My review of the early literature on risk measures and more specifically downside risk measures is based on the work of David Nawrocki from Villanova University.

first and thereafter set some minimum acceptable return that will conserve that principal. He called the minimum acceptable return the disaster level and the resulting technique is the Roy safety first technique. Roy stated that the investor would prefer the investment with the smallest probability of going below the disaster level or target return.

Markowitz(1959) recognized the importance of this idea. He realized that investors are interested in minimizing downside risk because, firstly, only downside risk or safety first is relevant to an investor and, secondly, security distributions may not be normally distributed. Markowitz shows that when distributions are normally distributed, both the downside risk measure and the variance provide the correct answer whereas only the downside risk measure provides the correct answer in case of non normality. Markowitz then provided two suggestions for measuring downside risk: a semivariance computed from the mean return or below-mean semivariance (SVm) and a semivariance computed from a target return or below-target semivariance (SVt).

$$SV_m = (1/K) \bullet \Sigma^K_{\tau=1} \; Max[0, (E - R_T)]^{\wedge}2$$

$$SV_t = (1/K) * \Sigma^K_{\tau=1} \; Max[0, (t - R_T)]^{\wedge}2$$

where R_T is the asset return during time period T, K is the number of observations, t is the target rate of return and E is the expected mean return of the asset's return.

Coining these measures partial or semi-variances for obvious reasons, Markowitz actually states in his treatise that "the semideviation produces efficient portfolios somewhat preferable to those of the standard deviation". The reason he stayed with the variance measure was simply because it was computationally simpler to calculate mean-variance portfolios rather than mean-semivariance ones. The semivariance optimization models using a cosemivariance matrix require more manipulation than the variance model because of the added step of having to search for and convert excess returns to zero. As Nawrocki puts it, "With the lack of cost-effective computer power and the fact that the variance model was already mathematically very

complex, this was a significant consideration until the 1980s with the advent of the microcomputer."

Because Markowitz's variance and semivariance performance measures depend on a normal distribution, researchers started to question them in the 1970s. Studies by Klemkosky(1973) and by Ang and Chua(1979) showed that these measures could provide incorrect rankings and suggested the reward to semivariability (R/SV) ratio as an alternative. For greater details of these and subsequent studies in semi-variance, I would refer the reader to David Nawrocki(1999).

2.2 Lower Partial Moment

The development of the Lower Partial Moment (LPM) risk measure by Bawa (1975) and Fishburn (1977) marked a turning point in the search for better downside risk measures. Nawrocki qualifies moving from the semivariance to the LPM as equivalent to progressing from a silent black and white film to a wide screen Technicolor film with digital surround sound! The justification for such a daring parallel is grounded in the fact that LPM liberates the investor from a constraint of having only one utility function, which is fine if investor utility is best represented by a quadratic equation (variance or semivariance). Lower Partial Moment represents a significant number of the known Von Neumann-Morgenstern utility functions. Furthermore, LPM covers the whole spectrum of human behavior from risk seeking to risk neutral to risk aversion.

The seminal work on LPM is attributable to Vijay Bawa (1975). He defined the relationship between lower partial moment and stochastic dominance. Bawa (1975) was the first to define lower partial moment (LPM) as a general family of below-target risk measures, one of which is the below-target semivariance. The LPM describes below-target risk in terms of risk tolerance. Given an investor risk tolerance value **a**, the general measure, the lower partial moment, is defined as:

$$\texttt{LPM (a,t)} = \texttt{(1/K)} \bullet \Sigma^{\texttt{K}}_{\tau=1} \texttt{Max[0,(t - R}_{\texttt{T}}\texttt{)]}\texttt{^a}$$

where **K** is the number of observations, **t** is the target return, R_T is the return for the asset during time period **T**, and **a** is the degree of the lower partial moment. It is the **a** value that differentiates the LPM from the SV_t. Instead of squaring deviations and taking square roots as we do with the semivariance calculations, the deviations can be raised to the **a** power and the **a** root can be computed. Another point is that **a** can take on any value including non-integers. It is appropriate to draw the parallel here with the *r*-th moment, μ_r, with respect to the mean of a random variable X taken from a Taylor-series expansion for X. μ_r is the expected value of $(X-\mu)^r$, i.e.

$$\mu_r = E[(X - \mu)^r] = \sum_x (x - \mu)^r \cdot f(x)$$

Another pioneer in this area, Fishburn (1977) extends the general LPM model to the (**a, t**) model, where **a** is the level of investor risk tolerance and **t** is the target return. Fishburn provides the unlimited view of LPM with fractional degrees as well. Given a value of the target return, **t**, Fishburn demonstrates the equivalence of the LPM measure to stochastic dominance for all values of **a** > 0. Fishburn also shows that the **a** value includes all types of investor behavior. The LPM value **a**<1 captures risk seeking behavior. Risk neutral behavior is **a** = 1, while risk averse behavior is **a** > 1. The higher the **a** value is above a value of one, the higher the risk aversion of the investor.

Thus, while the variance and semivariance only provide us with one utility function, LPM provides us with a whole rainbow of utility functions that represent the whole range of human (investor) behaviour. The **a**-degree LPM can explain complex and seemingly contradictory behaviour in the way individuals compartmentalize their utilities according to different financial situations and stages in their lifecycle. This is the source of the superiority of the LPM risk measure over the variance and semivariance measures. It can better explain how investors behave in reality as opposed to how they are supposed to behave. Matthias Unser (2000)

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effectively confirmed that in an experimental study published in the Journal of Economic Psychology.

Finally, it is to be noted that the LPM concept captures several other measures, some of which will be discussed shortly. If n=0, LPM gives the probability of shortfall. If n=1, we obtain expected shortfall and n=2 yields the semi-variance.

2.3   Shortfall Measures

Another measure of risk proposed by Leslie Balzer is the probability of shortfall. Balzer, a mechanical engineer by training, made his name in finance with a 1994 *Journal of Investing* article that attacked William Sharpe's standard deviation theory. "If standard deviation is used as a risk measure, overperformance relative to the mean is penalized just as much as underperformance," he wrote. Standard deviation measures uncertainty, he argued, not necessarily risk.

Among others, Balzer proposed the probability of shortfall measure.

Shortfall probability = Probability (R < B)

This measures the chance that returns (R) from the investment may fall below some reference point(B). The reference point is often set at zero, but it could be set at any other level to reflect the minimum acceptable return. The major shortcoming of this measure is that it gives the probability that an undesirable event might occur but gives no clue as to how severe it might be. This is overcome by the expected shortfall measure which incorporates both the probability and magnitude of the potential shortfall if it does occur.

Expected shortfall = E (R-B), over the range where R-B < 0

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The expected shortfall is given by the product of the magnitude of the shortfall and the probability of it occurring. This measure is influenced by the entire downside portion of the probability distribution and is a more complete measure of downside risk than just the probability of shortfall itself. However, its major flaw is that it treats a large probability of a small shortfall as equivalent to a small probability of a large shortfall. This is problematic when we know that investors view the consequences of large losses per unit differently from small losses.

2.4 Upside-Potential and Sortino Ratios

The Upside-Potential Ratio was developed jointly by Sortino, Van der Meer, Plantinga and Forsey and first published in the Fall 1999 issue of the Journal of Portfolio Management. It measures the extent to which a manager has been able to achieve upside potential relative to downside risk, with the MAR as the reference point. The expression for the U-P Ratio is given below:

$$\frac{\int_{mar}^{+\infty} (R - MAR)^1 f(R)\, dr}{[\int_{-\infty}^{mar} (R - MAR)^2 f(R)\, dr]^{1/2}}$$

The U-P ratio thus incorporated a measure of risk relative to the MAR that will achieve that goal and provided then a new perspective on risk-return tradeoffs that seemed well suited to investors seeking the highest consistent performance above their MAR, subject to the risk of falling below. The measurement allows to choose strategies with growth that is as stable as possible for a given minimum return

The Sortino Ratio is different from the U-P ratio and is defined as:

$$S = \frac{\mu - \tau}{\sqrt{\int_{-\infty}^{\tau} (\tau - R)^2\, dF(R)}}$$

where μ is the vector for the periodic returns and τ the threshold return (or MAR) or else a vector of the benchmark returns over the corresponding periods. The numerator is thus the average of the active return and the denominator is the square root of LPM_2 (τ) commonly referred to as the downside risk since it is equal to the volatility of negative active returns.

As such the Sortino ratio measures whether the portfolio's return in excess of a specified benchmark was sufficient to cover the downside risk inherent in the investment. The calculation is similar to calculating tracking error, except that the positive active returns are set equal to zero and still included in the standard deviation calculation. Setting the benchmark return or MAR to zero will indicate whether the portfolio's positive returns were sufficient to cover the risk of negative returns. It is therefore an indicator of capital preservation in nominal terms. The benchmark return can also be equated to the inflation benchmark so that the ratio then indicates whether real returns were sufficient to cover the risk of under-performing inflation. This is an important indicator of a fund's ability to match inflation-adjusted liabilities.

2.5 Omega

The first of the fairly recent measures that we will cover is the Omega measure, coined by Keating and Shadwick (2002). The Omega measure attempts to overcome the two flawed assumptions that riddle conventional performance measures, namely the applicability of the M-V framework to distribution analysis and, secondly, the irrelevance of a reference return, which would depend on investors' risk tolerance, from which risk would then be measured.

Omega for a given threshold return of r is defined as:

$$\Omega(r) := \frac{\int_{r}^{b} [1 - F(x)]\, dx}{\int_{a}^{r} F(x)\, dx}$$

where x is the random one-period rate of return on an investment, $F(y) = \Pr\{x \leq y\}$ is the cumulative distribution return of the one-period return, r is a threshold selected by the investor depending on her risk tolerance, and (a,b) represent the lower and upper bounds of the distribution respectively.

For instance, the probability weighted ratio of gains to losses relative to a return of 70 basis points above the benchmark mean return is $\Omega(70)$. This is illustrated in the diagram below taken from the authors' seminal paper. Omega can be obtained by taking the ratio of the area delineated in black to that outlined in red.

Figure 1:   Graphical representation of Omega
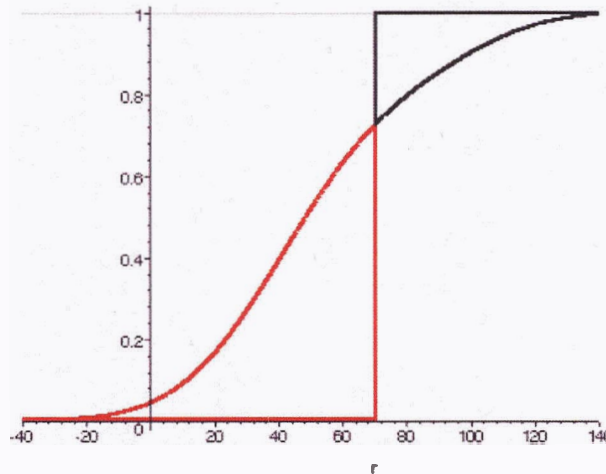
As we see from its expression, Ω is a function of the return level, like the cumulative distribution return curve. It is a smooth monotone decreasing function from (a,b) to $(0,\infty)$; independent of the returns distribution, Ω takes the value 1 at the distribution's mean μ. Keating and Shadwick stress and demonstrate that Ω is, in a mathematically precise sense, equivalent to the returns distribution. The authors call it "a natural feature of the underlying probability distribution".

Some of the characteristics of this measure that make it intuitively appealing are listed below:

- First, Ω does not assume that returns are normal. It incorporates fully in one number all the higher individual moments affecting the distribution, without any need to estimate them.

- Ω takes into account an investor's risk tolerance by measuring the ratio of gains to losses relative to a threshold return level. It can thus be measured relative to any risk threshold in the distribution range.

- Managers can be ranked on the basis of the magnitude of their omegas, without the need to bring into the picture utility functions. The only requirement is that the integrals that constitute the numerator and denominator of omega exist, which poses no problem in reality since discrete return observations are what are observed.

Finally, it is to be noted that when omega is less than one, higher volatility increases the value of the Omega measure since it increases the likelihood of earning a return above the mean.

2.6   From Omega to Sharpe-Omega

Three researchers from the Centre for International Securities and Derivatives Markets take Shadwick and Keating's simple concept further. Kazemi, Schneeweis and Gupta first prove that Omega is essentially the ratio of a call price to a put price, and is as such not an entirely new concept!

$$\Omega (r) = C(r) / P(r),$$

where C(r) and P(r) are the prices of one period call and put options with strike r written on the investment. From there on, the authors come up with a variation on the Omega measure which they call the Sharpe-Omega ratio. This risk measure is more intuitive because of its resemblance to the ubiquitous Sharpe ratio.

$$\text{Sharpe} - \text{Omega} = (E(R) - r) / P(r)$$

where r is a chosen threshold return level, E(R) is the expected return on the investment and P(r) is the put price at the strike r. The put price is computed using a generalisation of the Black-Scholes formula in order to account for non-normalities in the asset return. Also, another caveat is that Blacke-Scholes works in a risk neutral world, with the mean return of the underlying asset substituted for the riskless rate in the option pricing formula. However, the put price that appears in both Omega and Sharpe-Omega has to be calculated in a risk-averse world and therefore the true mean return of the investment must be used. The authors point out that this will result in a difference between the put price observed in the market and that which should be used to calculate the performance measures.

Some of the main properties of the Sharpe-Omega ratio are:

- Sharpe-Omega ranks assets according to the best return divided by the cost to protect this return. Assuming a threshold r higher than the asset expected return E(R), the Sharpe-Omega ratio is negative. In that case, a higher put price, due to higher volatility for instance, translates into a better investment (although the absolute value of the Sharpe-Omega ratio decreases). This is similar to the case of omega being less than one. When the expected return is greater than the threshold return, the ratio is positive and the opposite reasoning holds.

- Sharpe-Omega is shown to be proportional to $\Omega - 1$ and as such will always give the same rankings as its 'cousin' Omega. However, as opposed to Ω, the authors lay claim to the superiority of Sharpe-Omega in terms of its intuitiveness as a return/risk measure. The rationale is that, not only does it factor in the shape of the distribution below the threshold return similarly to Ω, but additionally it captures the riskiness of an investment via the cost - the put price - of protecting the investment from returns below the threshold.

2.7   Kappa: The Mother of All Downside Performance Measures!

This part is entitled thus because Kaplan, of Morningstar fame, and Knowles (2004) effectively show that each of the aforementioned measures, namely the Sortino Ratio, Omega and Sharpe-Omega measures, are simply special cases of Kappa, a generalized risk-adjusted performance measure. A single parameter of Kappa determines whether the Sortino Ratio, Omega, or another risk-adjusted return measure is generated.

The expression for Kappa is:

$$K_n(\tau) = \frac{\mu - \tau}{\sqrt[n]{LPM_n(\tau)}}$$

where μ is the expected periodic return, τ is the investor's minimum acceptable or threshold periodic return and the denominator is the nth root of the nth lower partial moment function.:

$$LPM_n(\tau) = \int_{-\infty}^{\tau} (\tau - R)^n \, dF(R)$$

Substituting different values for n in the expression for Kappa yields the various measures aforementioned. Thus:

- $K_1(\tau)$ = Sharpe-Omega ratio

- $K_2(\tau)$ = Sortino ratio

- Kaplan and Knowles show that $\Omega(\tau) = K_1(\tau) + 1$, which is in line with the findings of Kazemi, Schneeweis and Gupta referred to previously.

The lower partial moment required to calculate Kappa can be estimated in two ways:

The discrete LPM calculation method is done from a sample of actual returns by treating the sample observations as points in a discrete return distribution, as follows:

$$LPM_n(\tau) = \frac{1}{T} \sum_{t=1}^{T} \max[\tau - R_t, 0]^n$$

with T being the sample size and R_t the tth return observation.

The alternative method is to assume that returns follow a particular continuous distribution and calculate the integral in the LPM equation accordingly. For example, we have seen how Sortino [2001] and Forsey [2001] assume that returns follow the three-parameter lognormal distribution when estimating LPM2(τ). The three parameters of the distribution can be set so that the first three moments, mean (μ), standard deviation (σ), and coefficient of skewness (s) of the distribution match a given set of values.

Kaplan and Knowles then find that return rankings for hedge fund indexes are, in general, little affected by the choice of Kappa estimation methodology. They conclude tentatively that, for the purpose of evaluating competing investment alternatives, the parameter-based method of estimating Kappa is robust and, where efficiency and simplicity are important, preferable to the more complex calculation based on discrete return data.

3 DATA AND METHODOLOGY

The data used for the empirical work is the monthly returns on the HFRI hedge fund

indices obtained from the HFR database at www.hedgefundresearch.com. The different strategy

indices are considered to be a good proxy for the actual underlying funds. The period covered is

the entire sixteen-year horizon from the inception date of the indices (January 1990) to January

2006. The performance of hedge funds was compared to the performance of the S & P 500 Total

Return index and the NASDAQ. S & P data was downloaded from the Standard and Poors

website while NASDAQ data was obtained from Yahoo! Finance.

I have ranked the different HFRI strategy and fund of fund indices using the measures of

Probability of Shortfall and Expected Shortfall, Omega (which is equal to one plus Kappa at n=1)

and Kappas from n=2 to n=4. The Sharpe-Omega measure is not shown since, as Kaplan and

Knowles show, it is simply equal to Kappa at n=1 and will therefore always yield the same

ranking as Omega. Kappa is calculated using the discrete method discussed previously. This

should give a sufficiently accurate picture since we will not be using Kappa as an optimization

metric, for stress testing in a quantitative portfolio construction or in an asset allocation process in

which instances Kaplan and Knowles had cautioned that differences from the integral method

might be material. Evidently implicit in this ranking procedure is the ability to borrow or lend at

the risk-less rate, such that an investor would simply be choosing between the risk-free asset or

the asset with the highest risk-adjusted return as given by the downside measures employed here.

On the other hand, when calculating the Upside Potential (U-P) and U-P ratio, the curve-

fitting method of Forsey and Sortino is employed. This involves fitting a curve to the monthly

histogram of data to create a continuous distribution and then using integral calculus to calculate

the downside downside risk. However, Forsey and Sortino make two major improvements that enhance the accuracy of downside risk calculations.

First they point out that simply selecting monthly returns randomly and calculating the deviations below some monthly MAR is not optimal. The data must first be annualized using a boot strapping procedure whereby thousands of annual returns are generated from the sixteen years of monthly data. The bootstrap procedure used is that proposed by Efron and Tibshirani (1993) at Stanford University and involves compounding a random sample of 12 of the monthly returns taken from the available HFRI dataset. Each random draw once taken is replaced into the set so that it could be randomly selected again within that year's returns. The rationale is that such bootstrapping allows us to see what could possibly happen, and not constrain ourselves to simply what has happened so far in order to predict what could happen.

Forsey and Sortino's second improvement is to assume that returns follow the three-parameter lognormal distribution defined by Aitchinson and Brown [1957]. It is well-known that the standard lognormal can be defined by the two parameters which are the mean and the standard deviation or variance. The three-parameter distribution on the other hand is less utilized in the financial arena (though very commonly employed in the biological and sociological areas) and needs some clarification. We are here concerned with a variate X such that a simple displacement of X, say $X' = X - \tau$, and not the variate itself follows a lognormal model defined by the usual mean and variance. The range of X is thus $\tau < x < \infty$, and the two-parameter distribution is then the special case for which $\tau = 0$. Aitchinson and Brown call the third parameter τ the threshold of the distribution since it defines a lower bound to the range of values of the variate X. As evident here, τ allows us to shift the distribution but the latter is still positively skewed like the conventional lognormal. A slight modification provides the possibility of treating negatively skewed distributions as well. In that case $\tau - X$ is lognormally distributed, restricting X to the range $-\infty < x < \tau$. In other words the third parameter is effectively imposing an upper bound on

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the distribution and allowing us to flip it. This third parameter can be set so that the first three moments, mean (μ), standard deviation (σ), and coefficient of skewness (s) of the distribution match a given set of values, with the fourth moment, the coefficient of kurtosis (κ), then being a function of s. In their case, Forsey and Sortino choose an extreme value for the third parameter. This is estimated by first calculating the minimum and the maximum of the sample and then taking the one closest to the mean. The extreme value is obtained from this value by moving it four standard deviations further from the mean. This extreme value together with the mean and variance will determine the skewness and kurtosis (third and fourth moments) of each member of the family. If the estimates of the parameters are good, fitting a three parameter lognormal distribution to the bootstrapped data will be more accurate for calculating downside risk than using the normal or the standard lognormal. Although not a perfect fit, it is far better than the normal since it allows for skewness. Also, it has very few blank spaces in the left tail, which would cause underestimation, and it allows for more pointiness (kurtosis) than the normal. I have used the free software that comes with Sortino and Satchell's book, Managing downside Risk in Financial Markets, to apply the Forsey-Sortino method. The important mathematical formulas for the procedure detailed above and used by Forsey and Sortino are provided in the appendix.

Regarding the MARs chosen for this paper, I need to point out that I have deliberately not used any benchmark or the risk free rate as the threshold return. This is in line with the absolute return philosophy of hedge funds which strive to achieve alpha irrespective of market conditions. I have set the MARs in increments of ±5%, starting at 0% (capital preservation in nominal terms) up to ±10%. I also chose an MAR of ±20% in order to gauge how the indices fare at such relatively extreme threshold return levels.

It would be a crucial oversight not to point out that hedge fund databases suffer from well-documented biases such as the backfill bias (whereby managers only start reporting their

returns once they have achieved a respectable initial track record, and then these past positive results are 'backfilled' into the database together with current results) and survivorship bias (whereby hedge funds that have collapsed or have stopped reporting their results due to poor performance are not included in the database). Various researchers have come up with varying degrees of the impact of this bias. The latest in that line were Malkiel and Saha (2005) who constructed a bias free database and showed that these biases significantly overstated returns by between 375 and 500 basis points. One way of accommodating this flaw in this present paper would be by 'penalizing' the mean return of the HFRI indices by a certain amount in order to counter the aforementioned upward bias. However, I have opted not to do this for lack of a sufficiently solid decision rule in the choice of the penalty to be applied. Also, Hedge Fund Research does try to limit the effect of survivorship bias. For instance, when a fund liquidates or closes, its performance remains in the indices as of its last performance report date. Also, only the current month and the prior three months are left as estimates and are subject to change. All performance prior to that is locked. As such, biases that have crept in will be less important.

4   DISCUSSION

4.1   Forsey-Sortino Method

The results of this method outlined in section 3 are to be found in Table 1. The actual

distributions are shown in appendix A. Our focus is the Upside Potential ratio which is the ratio

of the Upside Potential and Downside Risk. The empty spaces in the top part of the table mean

that the probability of beating the MAR is simply 100% and the U-P ratios are too large to be

meaningfully ranked.

U-P consistently ranks the equity indices in the bottom spots, except for the highest MAR

where the S&P climbs up to 6th spot and Nasdaq to the 3rd. At that threshold return level, ENH

and EM share the top prize. RVA outperforms at the 0% and 5% thresholds. Among the hedge

fund indices, the non-directional strategies like EMN, CA and FI perform worst at the higher

MARs of 10% and 20% . More about a possible explanation will be given in the next section.

4.2   Discrete Rankings using Omega, Kappa and LPM

Table 2 gives the results at the six MARs for rankings effected using the discrete

calculation methodology. It must be noted that Omega and Kappa$_1$ will always yield the same

outcomes, since Omega is mathematically equal to one plus Kappa$_1$. Also Omega has been

calculated by the continuous integrand method as well but these results are only identified at the

-20% threshold level since the rankings provided by this method are very close to those

calculated by direct summation. Also, the Sortino ratio and Kappa$_2$ are one and the same figure as

noted by Kaplan and Knowles. Finally, I use the Shape ratio with a risk-free rate set at zero

(referred to as Pseudo-Sharpe) as the conventional comparison performance measure. The top

part of Table 2 also contains rankings by skewness and kurtosis as well as the third and fourth moments, backed out from the former similar to what Brulhart and Klein (2005) did. As they had noted, the actual moments show a very different picture, drastically demoting the S&P and Nasdaq to last positions.

Although I have used the shortfall measures, these will not be dealt with extensively since they are quite simplistic and do not embody as much the distribution characteristics as do the Omega, Sortino and other Kappa measures. Suffice it to say that the Probability of Shortfall, equivalent to LPM1, can and does often contradict markedly rankings given by LPM2 or the Expected Shortfall. Where they do seem to agree overwhelmingly is in ranking the Short Selling and equity indices at the 14th and 15th spots most of the time. From here onwards, I shall refer only to the four downside measures given by Kappa for n=1 to 4.

Conversely to the LPMs, the higher the Kappas, the better the investment and the higher its rank. A general observation is that the rankings for these downside measures agree more at higher MARs. Also, the non-directional strategies such as Convertible Arbitrage, Relative Value Arbitrage, Fixed Income Arbitrage and Equity Market Neutral perform much better than the opportunistic ones for MARs at or below 0%. This means that risk averse investors who are for example simply seeking capital preservation in nominal terms, and most probably seeking the diversification benefits of hedge funds, are better off staying with those funds. Investors with very high required returns at 20% or more will do better with ENH, EM or the Nasdaq for that matter. This finding does make intuitive sense since the arbitrage and market neutral funds are by their very mandate and investment philosophy engineered to capture small returns consistently as opposed to making bets that yield very high returns. RVA is the best strategy for an MAR up to 5%. These results are in line with the results yielded by the boot-strapping technique of Forsey and Sortino.

Appendix C contains the plots of Omega (log-values) for each strategy against the S&P TR Omega. . An observation that reinforces the point raised in the above paragraph concerning the non-directional strategies can also be seen from their Omega curves which span a narrow return range, are steeper over that range and dip more rapidly towards the S&P curve. It can be seen that the Omega functions of the hedge fund indices dominate the equity index for monthly returns of less than around 2% in most instances, but then the situation reverses. EM is a notable exception as well as ENH whose curve dominates but is very close to the S&P one. At the highest MAR of 20%, the ENH strategy outperforms, in line with the finding using Forsey-Sortino's statistics

The Sortino ratio also agrees with Omega for the most part. What is interesting is that the rankings provided by the third and fourth Kappa variants - which depend on LPM_3 or "semi-skewness" and LPM_4 or "semi-kurtosis" if such terms could indeed be used –seem to agree with Brulhart and Klein's (2005) findings by sanctioning the equity indices for their higher moments. However, at the highest MAR of 20% , this picture changes with the equity rankings improving sensibly. More research has to be done there in order to understand fully what is occurring at the higher returns. Actually, Kaplan and Knowles themselves had remarked that it is hard to ascribe a specific direct relationship between the higher semi-moments and the higher Kappas since all Kappa variants are sensitive, to some degree, to the first four as well as other moments of the return distribution.

5 Conclusion

"Risk, like beauty, is in the eye of the beholder." *Leslie A. Balzer*

What the plots and Omega rankings in Table 2 as well as the Forsey-Sortino U-P results demonstrate is that the hedge fund indices consistently dominate the equity indices up to about 25% annualized returns! The latter are definitely penalized when their higher moments and downside risk are taken into consideration.

We can also conclude that the technique developed by Forsey and Sortino of successively boot-strapping and fitting a 3-parameter lognormal curve yields results that are not very dissimilar to those obtained discretely using the Kappa statistics. It would probably be of much added value though when we have to rank funds which have been in existence only a few years.

With the increasing use of financial instruments with asymmetric pay-offfs incorporated into trading or portfolio management strategies, investment return distributions are increasingly predisposed to be asymmetric. Hedge funds have traditionally fallen in this category and will continue to by their very mandates. Hence, investors and advisors need to understand and use performance measures that help to "select and reward not only managers who produce higher returns, but also those who produce asymmetric distributions of value-added above benchmark with enhanced upside and curtailed downside", to use Leslie Balzer's words.

The new measures outlined and used here such as Omega, Kappa are welcome additions to the arsenal of both risk professionals and sophisticated investors alike, provided they are not viewed in isolation. Otherwise, erroneous and costly decisions could result from relying on one preferred measure. No single risk measure should be consistently and blindly applied. Whether

these newer measures gain popularity in the industry will depend on how practitioners perceive the added value they provide versus traditional measures. I believe that the main hurdle to adopting Omega, Kappa or other downside risk statistics springs from their very forte. The fact that they can be customised so as to give a picture of performance relative to a specific investor's required threshold return or MAR also renders them incomparable when calculated using different MARs. Greater comparability can nevertheless be achieved by levering or delevering following the astute method of Brulhart and Klein(2005) who sought to compare hedge fund and equity indices by matching the second and fourth moments. We shall then be in presence of the measures evaluated at similar threshold returns. However, it could also be advanced that such a method could pose a problem since not all investors have the mandate to add leverage to magnify the expected returns on their portfolios. Pension fund sponsors are one such example.

An additional area of research would be to conduct a similar performance measurement exercise using the measures used in this paper on a universe of actual hedge funds benchmarked against their respective strategy indices, and then rank the different funds accordingly. For instance, the historical monthly realized returns of a macro fund would be benchmarked against the monthy returns on the HFRI Macro index and downside risk statistics calculated from the annualised data obtained by bootstrapping these excess returns. Such results would be of tremendous practical value for investors who will be able to gauge hedge fund performance relative to the performance of a universe of managers pursuing similar strategies, similarly to what goes on in the traditional equities world, but all the while taking into account the unique distribution characteristics of hedge funds through the use of the different downside measures used throughout this paper.

Table 1:   Forsey-Sortino Calculation of U-P ratio

	FWC	FOFC	CA	DS	EM	EH	EMN	ENH	ED	FI	MAC	MA	RVA	SS	S&P TR	NASD
									FORSEY-SORTINO MODEL							
Mean Return	14.50	10.10	10.10	15.20	17.20	17.70	9.20	17.30	14.80	10.50	16.10	10.30	12.00	3.80	11.50	14.20
SD	8.00	6.10	3.90	7.10	17.40	10.40	3.40	16.70	7.60	3.80	9.40	4.70	4.00	22.00	16.10	28.70
Extreme Value	-45.00	-34.00	38.90	-38.20	-105.20	-52.70	-15.20	-98.60	69.20	-17.10	-50.30	41.20	-19.90	-139.70	-101.50	-184.30
MAR = -20%																
P > MAR					99.40			99.60						86.60	98.70	89.10
Downside Risk					0.40			0.30						4.10	0.60	4.60
Upside Potential					37.30			37.50						24.80	31.40	35.10
U-P Ratio					92.89			131.42						6.08	52.90	7.61
Ranking					13			12						16	14	15
MAR = -10%																
P > MAR					95.60			96.30	99.70					72.30	92.10	79.70
Downside Risk					1.40			1.20	0.30					7.50	2.00	7.60
Upside Potential					27.50			27.30	24.80					17.00	21.90	26.80
Upside Potential Ratio					19.01			22.52	97.59					2.26	10.73	3.55
Ranking					13			12	11					16	14	15
MAR = -5%																
P > MAR	99.80	99.80	99.90	100.00	90.90	99.50	100.00	92.10	98.90	100.00	99.60	99.70	100.00	63.20	85.40	73.90
Downside Risk	0.10	0.10	0.10	0.00	2.50	0.20	0.00	2.10	0.50	0.00	0.20	0.20	0.00	9.80	3.30	9.40
Upside Potential	19.60	15.00	15.00	20.30	22.70	22.60	9.10	22.70	19.80	10.50	21.10	15.30	12.10	13.50	17.60	23.10
Upside Potential Ratio	234.83	249.95	223.19	998.4	9.23	111.28	622.5	10.78	37.58	685.64	128.18	89.71	1586.32	1.38	5.41	2.46
Ranking	3	2	4	1	10	6	#N/A	9	11	#N/A	5	7	#N/A	13	11	12
MAR = 0%																
P > MAR	97.90	96.50	98.80	99.30	84.20	97.00	100.00	85.40	96.40	100.00	97.10	97.50	100.00	53.60	75.40	67.50
Downside Risk	0.40	0.40	0.30	0.20	3.80	0.70	0.00	3.40	1.00	0.00	0.60	0.50	0.00	12.50	5.00	11.50
Upside Potential	14.50	10.20	10.10	15.20	18.40	17.70	9.10	18.30	14.90	10.50	16.20	10.30	12.10	10.50	13.50	19.70
Upside Potential Ratio	35.25	23.2	35.84	85.08	4.91	27.08	622.5	5.42	14.25	685.64	27.76	19.32	1586.32	0.84	2.69	1.71
Ranking	6	9	5	4	13	8	3	12	11	2	7	10	1	16	14	15
MAR = 5%																
P > MAR	89.00	79.70	89.60	93.80	74.70	90.00	90.40	76.10	89.60	93.80	89.10	86.70	97.10	45.00	63.30	59.80
Downside Risk	1.30	1.60	1.00	0.80	5.60	1.50	0.50	5.10	2.00	0.40	1.50	1.40	0.30	15.20	7.30	14.10
Upside Potential	9.80	5.70	5.30	10.30	14.30	13.10	4.30	14.20	10.30	5.60	11.50	5.70	7.00	8.20	9.90	16.20
Upside Potential Ratio	7.42	3.52	5.23	13.38	2.57	8.48	8.68	2.82	5.2	13.69	7.56	3.9	27.12	0.54	1.37	1.14
Ranking	7	11	8	3	13	5	4	12	9	2	6	10	1	16	14	15
MAR = 10%																
P > MAR	69.20	47.90	53.30	76.20	64.20	76.10	38.00	65.00	75.10	52.10	73.10	55.30	67.70	36.20	51.20	52.90
Downside Risk	3.10	4.00	2.90	2.20	7.70	3.10	2.70	7.20	3.50	2.30	3.20	3.40	1.60	18.50	9.80	16.70
Upside Potential	5.70	2.50	1.60	6.10	11.00	8.90	1.00	10.70	6.10	1.80	7.40	2.00	2.80	6.10	7.20	13.40
Upside Potential Ratio	1.86	0.61	0.55	2.78	1.43	2.83	0.36	1.5	1.75	0.77	2.34	0.6	1.69	0.33	0.73	0.8
Ranking	4	12	14	2	8	1	15	7	5	10	3	13	6	16	11	9
MAR = 20%																
P > MAR	22.80	6.20	0.10	23.50	41.10	38.60	0.30	41.10	26.40	1.30	31.90	0.80	3.30	21.50	27.80	39.50
Downside Risk	9.10	11.60	10.60	8.00	13.30	8.30	11.30	12.70	8.80	10.30	8.80	10.80	8.90	25.80	16.40	22.60
Upside Potential	1.20	0.20	0.00	1.10	5.70	3.20	0.00	5.50	1.00	0.00	2.20	0.00	0.10	3.20	3.20	8.90
Upside Potential Ratio	0.13	0.02	0	0.14	0.43	0.38	0	0.43	0.12	0	0.25	0	0.01	0.12	0.19	0.4
Ranking	8	11	13	7	1	4	13	1	9	13	5	13	12	9	6	3

Table 2: Discrete Calculation of Shortfall, Omega and Kappa

	FWC	FOF	CA	DS	EM	EH	EMN	ENH	ED	FI	MAC	MA	RVA	SS	S&P	NASD
								Rankings								
Skewness	10	5	14	11	13	3	2	9	15	6	1	16	13	4	8	7
Kurtosis	9	11	8	14	10	6	1	2	12	13	3	16	15	7	4	5
Third Moment (%^{3})	10	7	8	9	15	3	4	14	12	5	2	11	6	1	13	16
Fourth Moment (%^{4})	8	6	2	7	14	11	1	12	9	3	10	5	4	15	13	16
Pseudo-Sharpe(R_f=0)	8	11	4	5	13	9	3	12	7	2	10	6	1	16	14	15
Monthly MAR = -1.84 % (-20% Annualized)																
LPM$_0$(Prob of Shortfall)	10	6	3	7	12	11	1	13	8	4	9	5	2	16	14	15
LPM$_1$(Expected Shortfall)	7	5	3	6	13	10	1	12	9	2	8	7	4	15	14	16
Omega Discrete (=Kappa$_1$ + 1)	5	4	2	6	12	10	N/A	11	9	1	8	7	3	14	13	15
Omega (Continuous)	5	4	2	6	12	10	N/A	11	9	1	8	7	3	14	13	15
Sortino Ratio (=Kappa2)	8	5	1	6	12	7	N/A	11	10	2	4	8	3	15	13	14
Kappa$_3$	9	7	1	8	13	5	N/A	11	10	2	4	6	3	15	12	14
Kappa$_4$	9	7	1	8	13	5	N/A	11	10	2	4	6	3	15	12	14
Monthly MAR = -0.87 % (-10% Annualized)																
LPM$_0$(Prob of Shortfall)	9	8	6	5	12	11	3	13	7	2	10	4	1	16	14	15
LPM$_1$(Expected Shortfall)	9	7	4	6	13	11	1	12	8	3	10	5	2	15	14	16
Omega (=Kappa$_1$ + 1)	10	7	4	5	13	11	1	12	9	3	8	6	2	16	14	15
Sortino Ratio (=Kappa2)	9	7	3	5	13	8	1	12	11	2	6	10	4	16	14	15
Kappa$_3$	9	8	3	7	13	6	1	12	11	2	5	10	4	16	14	15
Kappa$_4$	9	10	2	7	14	6	1	12	11	3	5	8	4	16	13	15

Monthly MAR = -0.43 % (-5% Annualized)

LPM$_0$(Prob of Shortfall)	8	9	6	5	12	10	2	13	7	1	11	4	3	16	14	15
LPM$_1$(Expected Shortfall)	9	7	4	6	12	11	1	13	8	3	10	5	2	15	14	16
Omega (=Kappa$_1$ + 1)	10	7	4	5	12	11	2	13	8	3	9	6	1	16	14	15
Sortino Ratio (=Kappa$_2$)	9	8	4	5	13	7	1	12	11	2	6	10	3	16	14	15
Kappa$_3$	8	9	3	7	13	6	1	12	11	2	5	10	4	16	14	15
Kappa$_4$	8	11	3	7	13	6	1	12	10	2	5	9	4	16	14	15

MAR = 0%

LPM$_0$(Prob of Shortfall)	8	8	4	6	12	10	5	13	7	1	11	3	2	16	14	15
LPM$_1$(Expected Shortfall)	9	7	4	6	12	11	2	13	8	3	10	5	1	15	14	16
Omega (=Kappa$_1$ + 1)	9	11	5	4	12	10	3	13	7	2	8	6	1	16	14	15
Sortino Ratio (=Kappa$_2$)	8	11	4	5	13	7	1	12	9	3	6	10	2	16	14	15
Kappa$_3$	8	10	3	7	13	6	1	12	9	2	5	11	4	16	14	15
Kappa$_4$	8	11	3	7	13	6	1	12	9	2	4	10	5	16	14	15

Monthly MAR = 0.41 % (5% Annualized)

LPM$_0$(Prob of Shortfall)	7	12	3	6	10	8	8	13	5	4	11	1	2	16	14	15
LPM$_1$(Expected Shortfall)	9	7	4	6	12	11	3	13	8	2	10	5	1	15	14	16
Omega (=Kappa$_1$ + 1)	9	11	8	2	12	5	6	13	4	3	7	10	1	16	14	15
Sortino Ratio (=Kappa$_2$)	7	11	9	2	13	4	6	12	8	3	5	10	1	16	14	15
Kappa$_3$	8	11	7	6	13	1	2	12	9	5	3	10	4	16	14	15
Kappa$_4$	8	12	7	6	13	2	1	10	9	4	3	11	5	16	14	15

Monthly MAR = 0.80 % (10% Annualized)

LPM$_0$(Prob of Shortfall)	3	14	5	2	3	5	15	5	1	11	13	8	9	15	9	12
LPM$_1$(Expected Shortfall)	9	8	4	6	12	11	3	13	7	2	10	5	1	15	14	16
Omega (=Kappa$_1$ + 1)	5	13	14	1	8	2	15	7	3	10	4	12	6	16	11	9
Sortino Ratio (=Kappa$_2$)	5	13	14	2	8	1	15	7	4	10	3	12	6	16	11	9
Kappa$_3$	5	13	14	3	8	1	15	6	4	10	2	12	7	16	11	9
Kappa$_4$	5	13	14	3	8	1	15	6	4	10	2	12	7	16	11	9

Monthly MAR = 1.53 % (20% Annualized)

LPM$_0$(Prob of Shortfall)	5	11	14	8	2	4	15	1	7	15	9	12	12	10	6	3
LPM$_1$(Expected Shortfall)	8	9	4	3	12	10	7	13	6	2	11	5	1	16	14	15
Omega (=Kappa$_1$ + 1)	8	11	16	9	2	4	15	1	10	14	5	13	12	7	6	3
Sortino Ratio (=Kappa$_2$)	9	11	14	8	2	4	16	1	7	15	5	12	13	10	6	3
Kappa$_3$	9	11	14	8	1	4	16	2	7	15	5	12	13	10	6	3
Kappa$_4$	9	12	14	8	1	4	16	2	7	15	5	11	13	10	6	3

APPENDICES

Appendix A
Basic Mathematical Formulas for the Three-Parameter Lognormal

The three basic parameters estimated from the sample are Mean, the sample mean, SD, the sample standard deviation and τ, the extreme value computed as described in the Data and Methodology section.

Some auxiliary parameters

$$Dif = |\ Mean - \tau\ |$$

$$\sigma = \ln (\ (SD/Dif\ )^2 + 1\ )$$

$$\mu = \ln (Dif) - \sigma^2$$

$$\alpha = 1 / ((2\pi)^{1/2} . \sigma)$$

$$\beta = -1/(2\ \sigma^2)$$

Formula for the lognormal curve f(x)

If the extreme value is a minimum and x is greater than the extreme value then

$$f(x) = (\alpha / (x - \tau)\ ) . \exp (\beta . (\ln(x - \tau) - \mu))$$

If the extreme value is a maximum and x is less than the extreme value then

$$f(x) = (\alpha / (\tau - x)\ ) . \exp (\beta . (\ln(\tau - x) - \mu))$$

Formula for the lognormal cumulative distribution function F(x)

If the extreme value is a minimum and x is greater than the extreme value then

$$F(x) = 1 - (\ erfc(\ln(x - \tau) - \mu) / (2^{1.5}). \sigma)$$

If the extreme value is a maximum and x is less than the extreme value then

$$F(x) = 1 - (\ erfc(\ln(\tau - x) - \mu) / (2^{1.5}). \sigma)$$

Where *erfc* is the complementary error function.

Appendix B
Forsey-Sortino 3-Parameter Lognormal Bootstrapped Curves
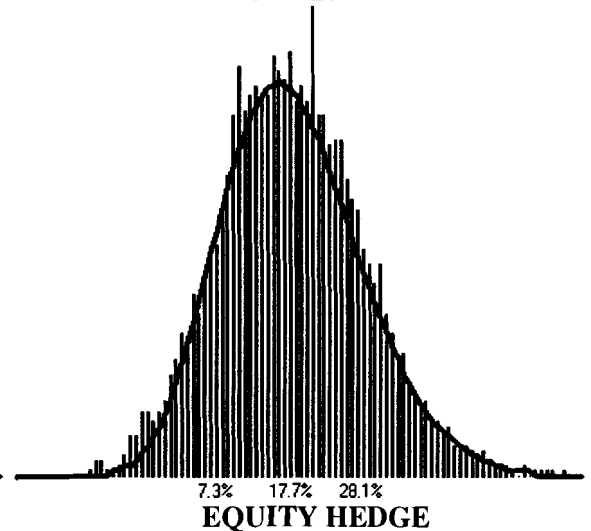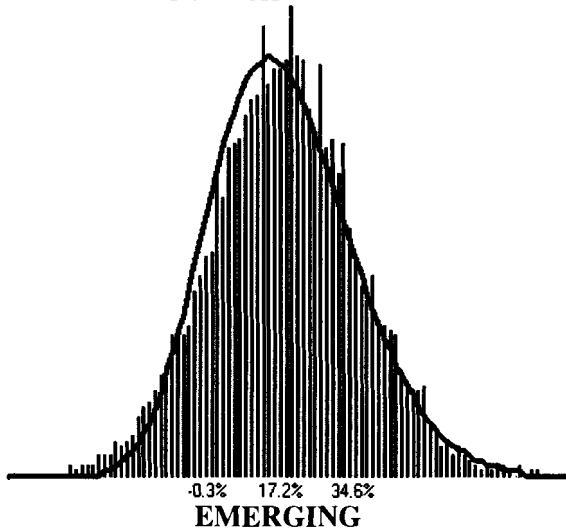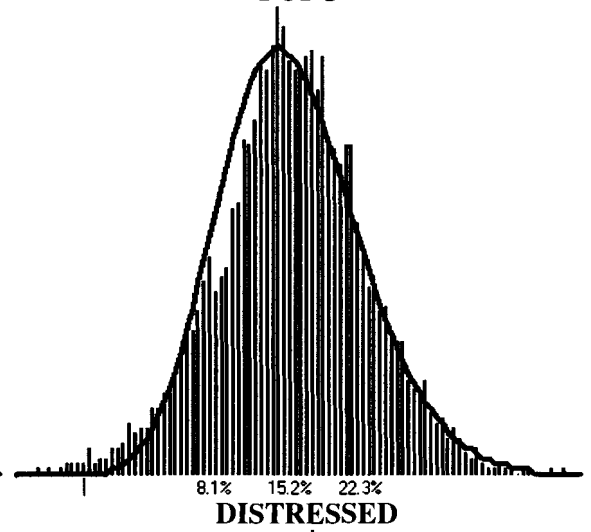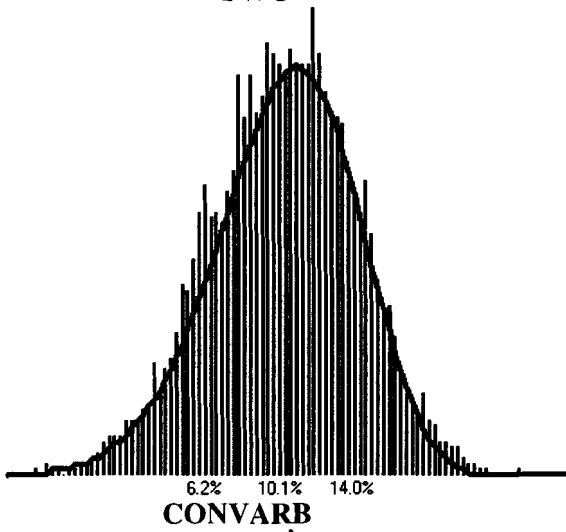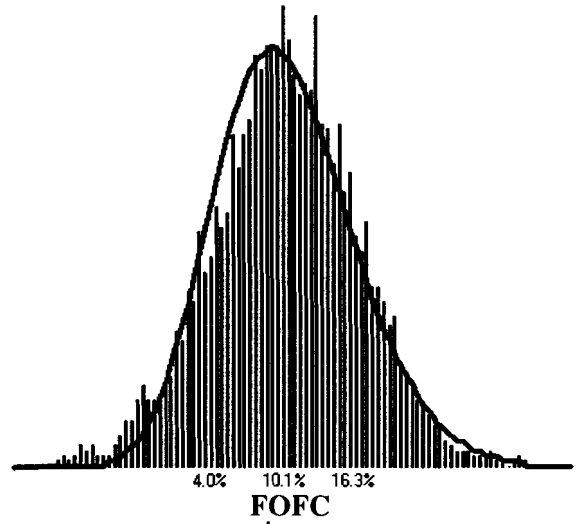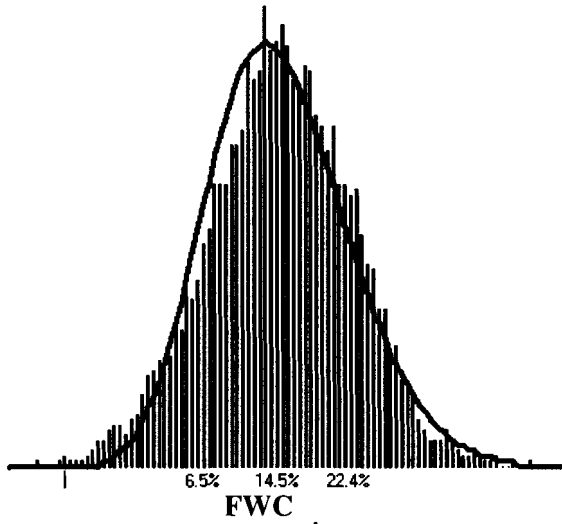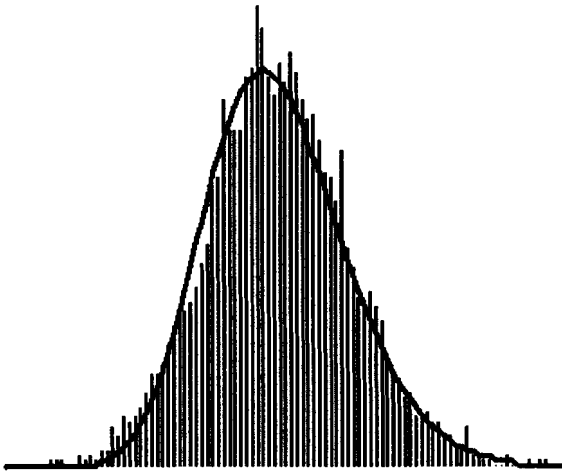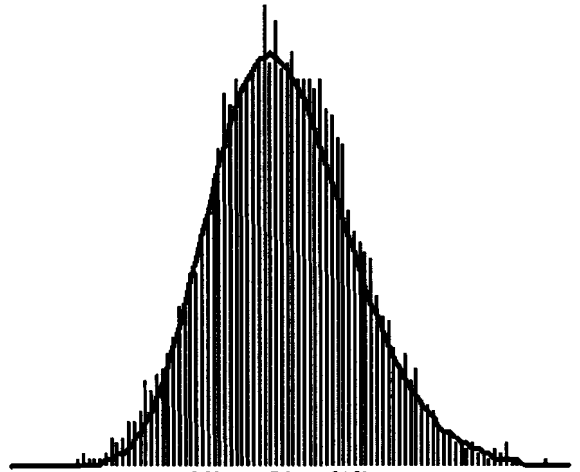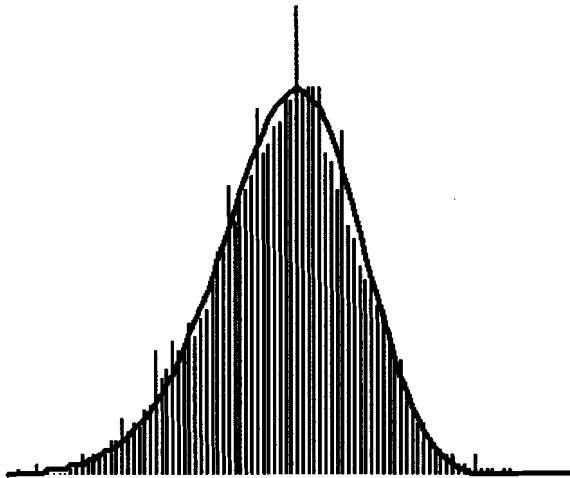
6.5% 14.5% 22.4%
FWC

4.0% 10.1% 16.3%
FOFC

6.2% 10.1% 14.0%
CONVARB

8.1% 15.2% 22.3%
DISTRESSED

-0.3% 17.2% 34.6%
EMERGING

7.3% 17.7% 28.1%
EQUITY HEDGE

EQ MKT NEUT
5.8%   9.2%   12.6%

EQ NON HEDGE
0.6%   17.3%   34.0%

EVENT DRIVEN
7.2%   14.8%   22.4%

FIXED INCOME
6.6%   10.5%   14.3%

MACRO
6.7%   16.1%   25.5%

MERG ARB
5.6%   10.3%   15.0%

REL VAL ARB

SHORT SELLING

S & P TR

NASDAQ

33
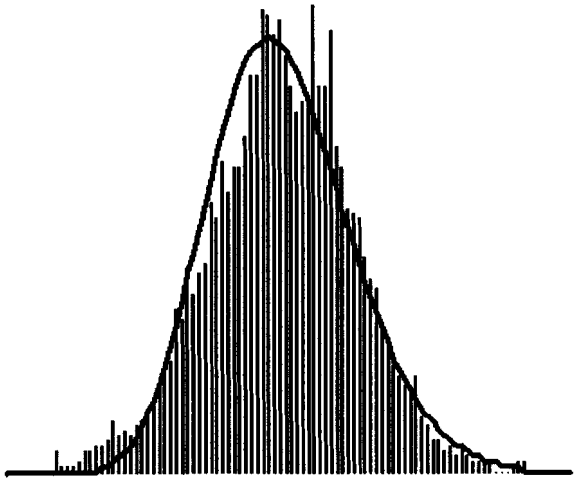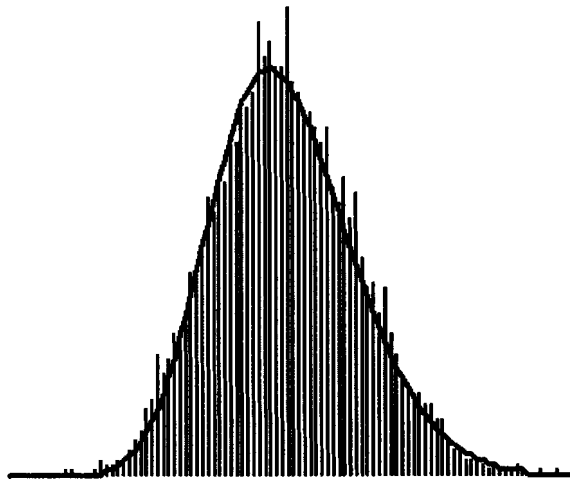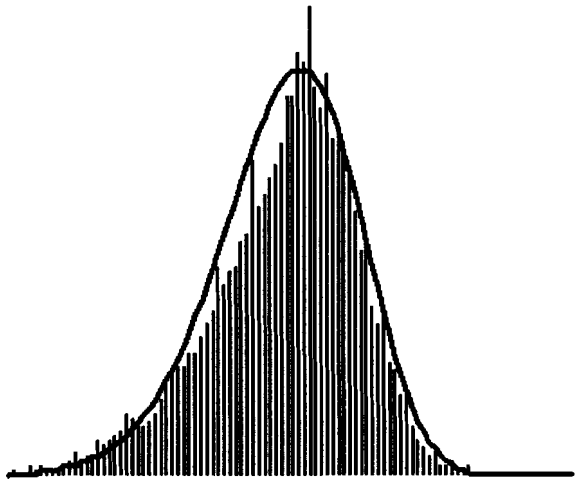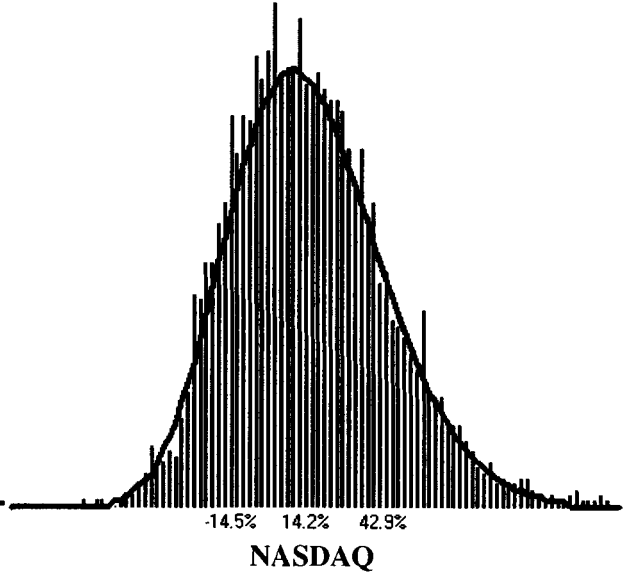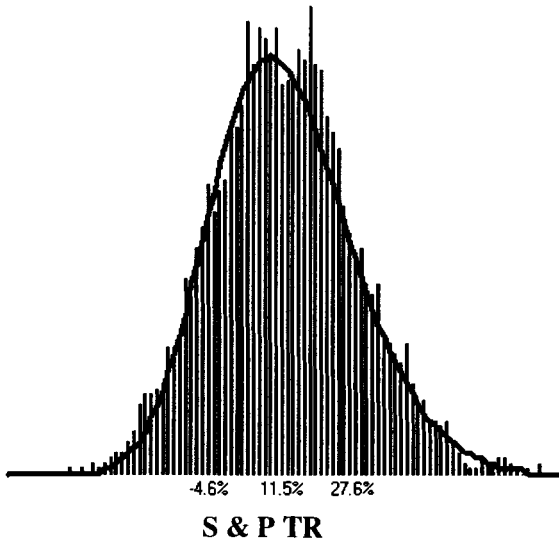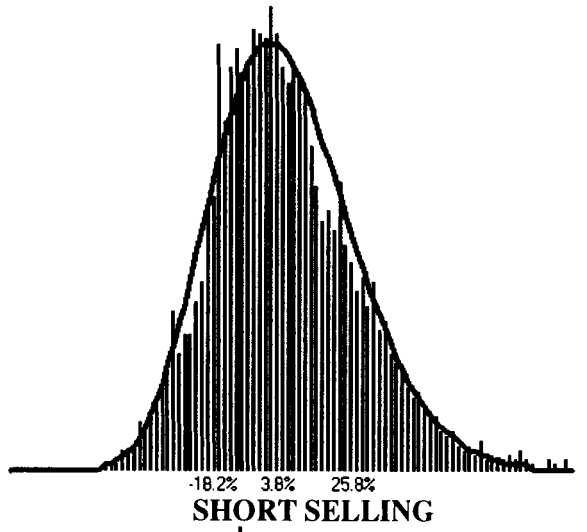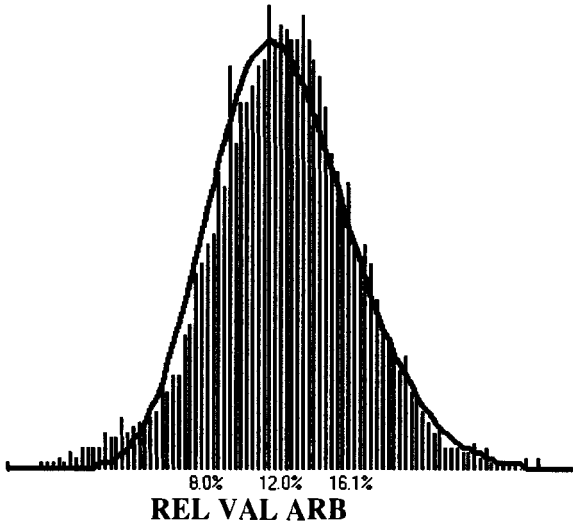
Appendix C
Omega Monotonic Functions

Graphs appear on following pages.

Note: S&P Omega curves are labelled B

FWC

FOFC

CONVARB

OMEGA

DISTRESSED

OMEGA

EMERGING

OMEGA

EQUITY HEDGE

OMEGA

37
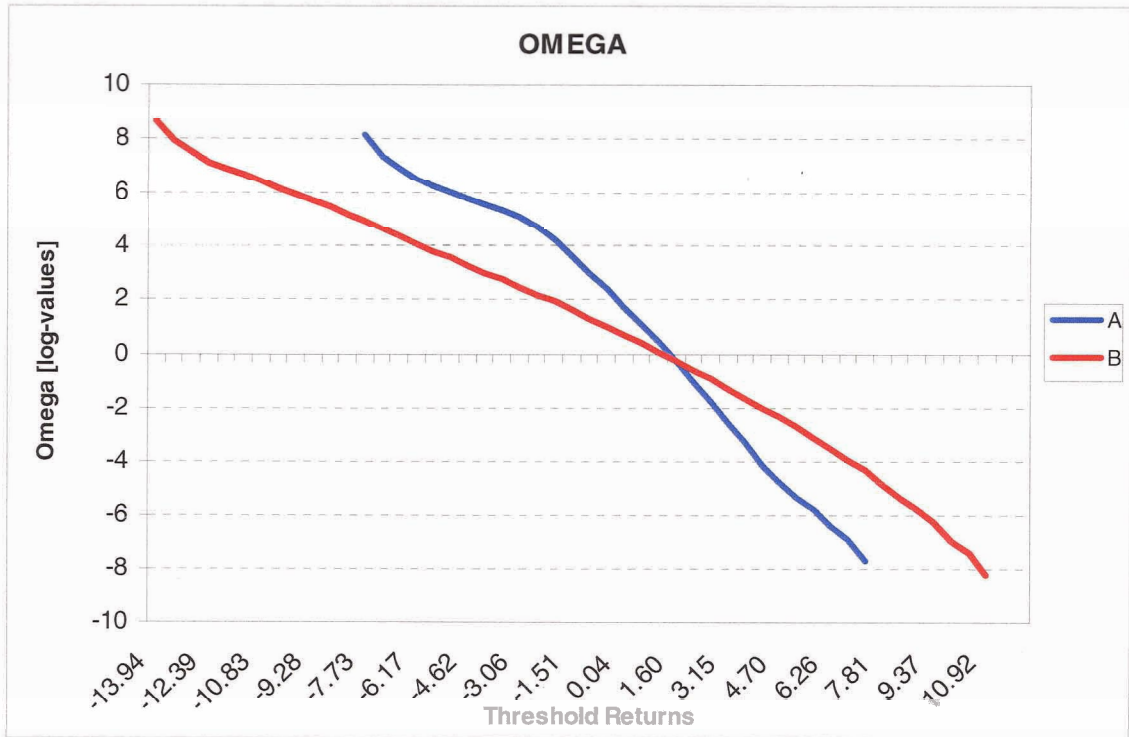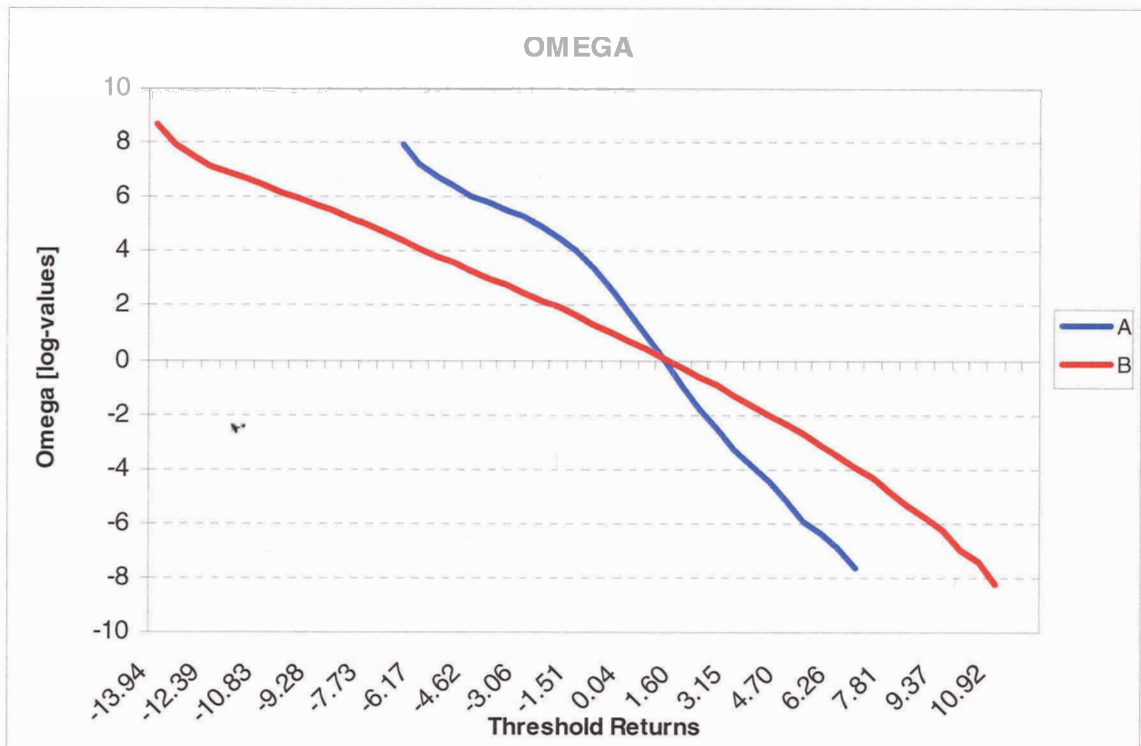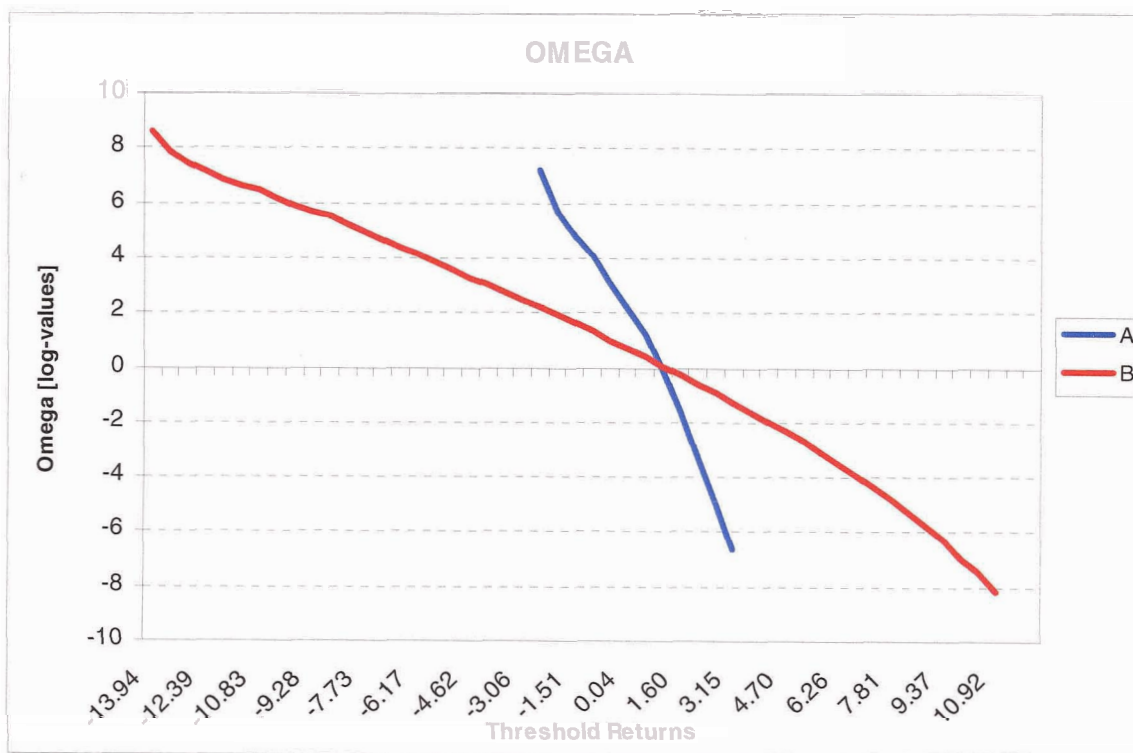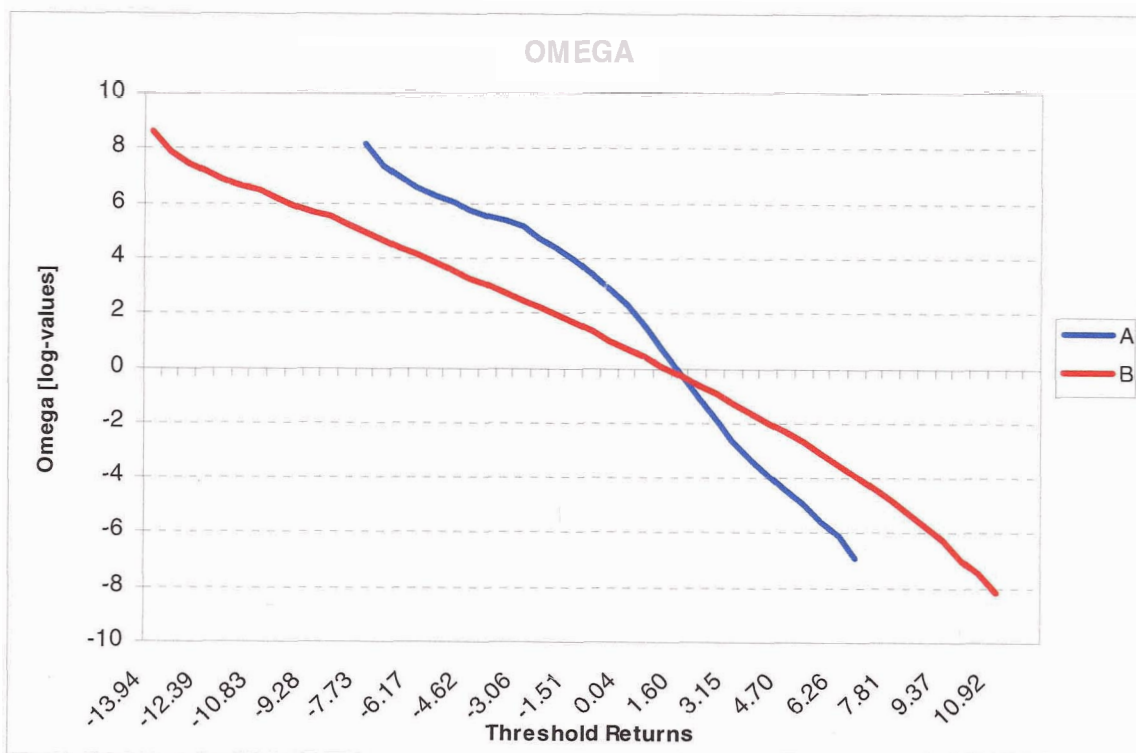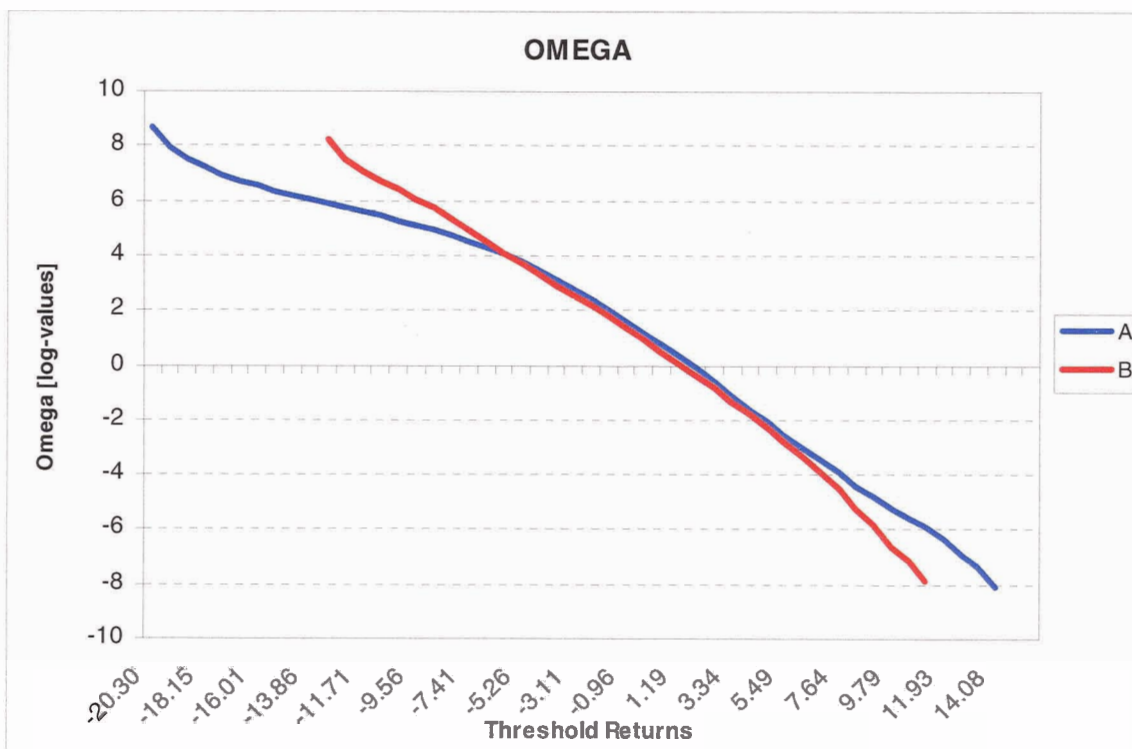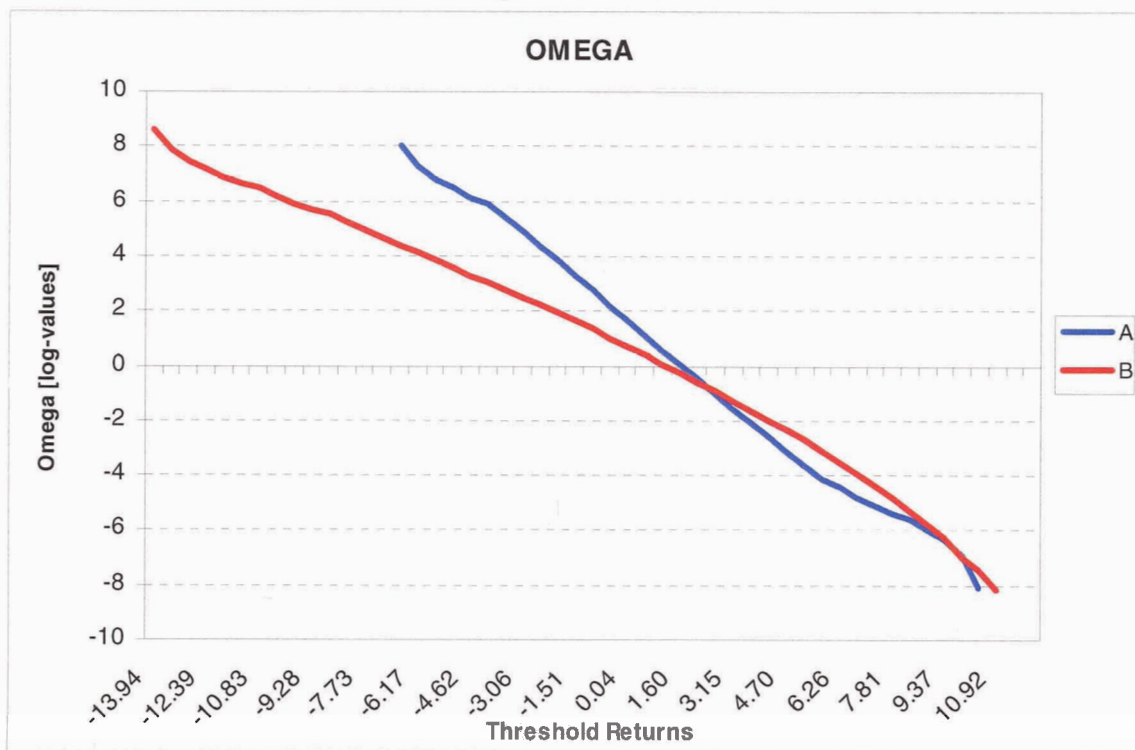
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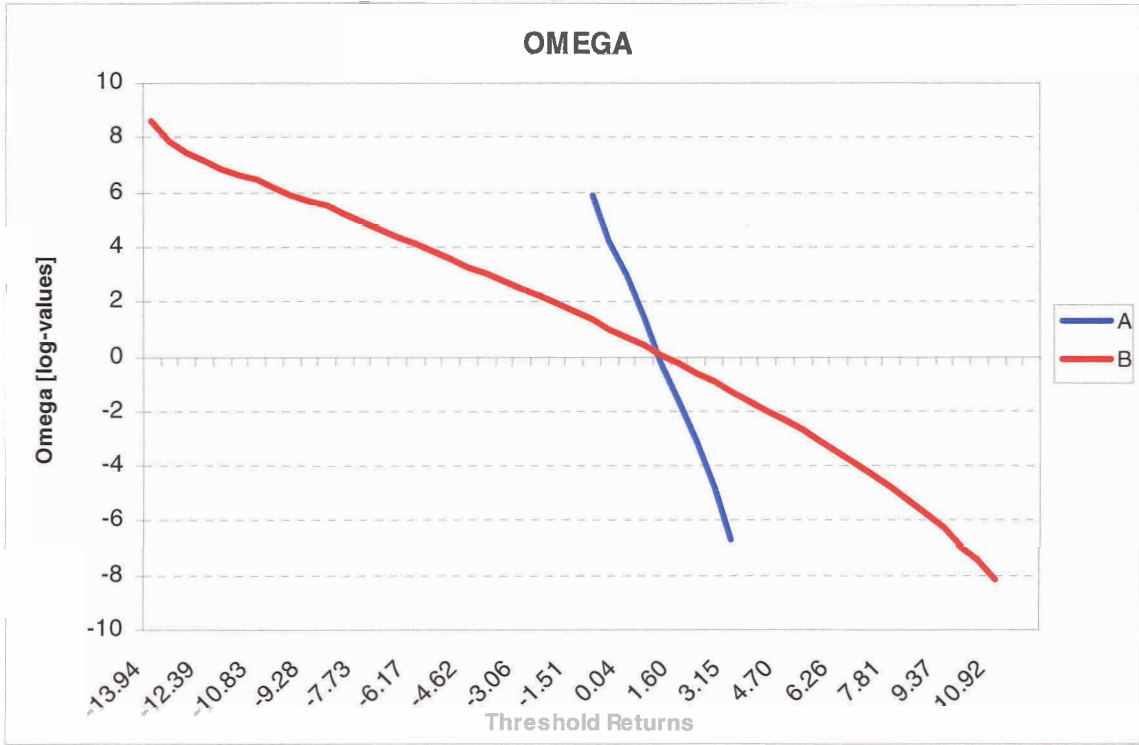
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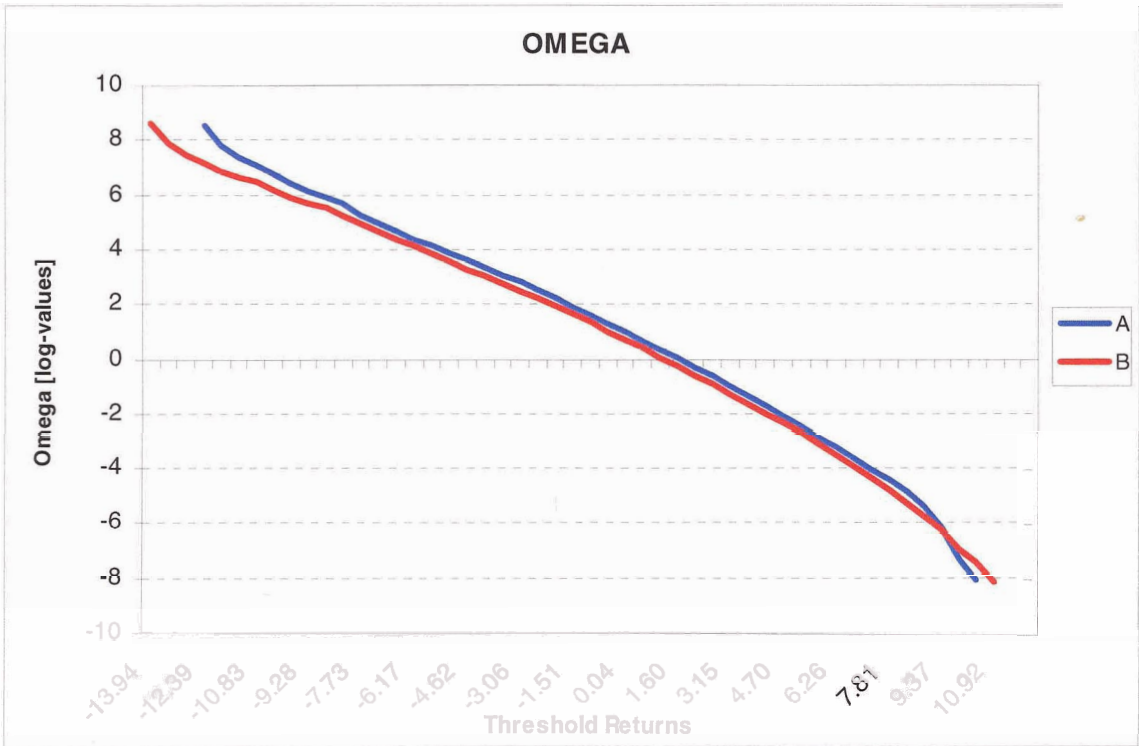
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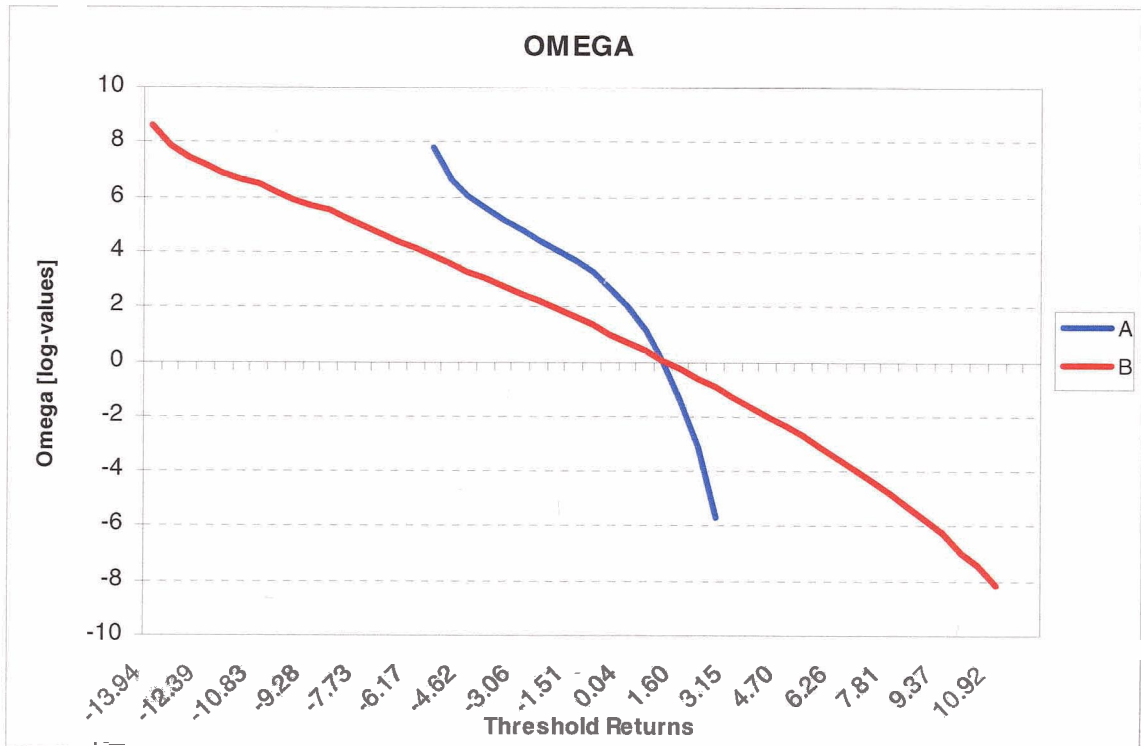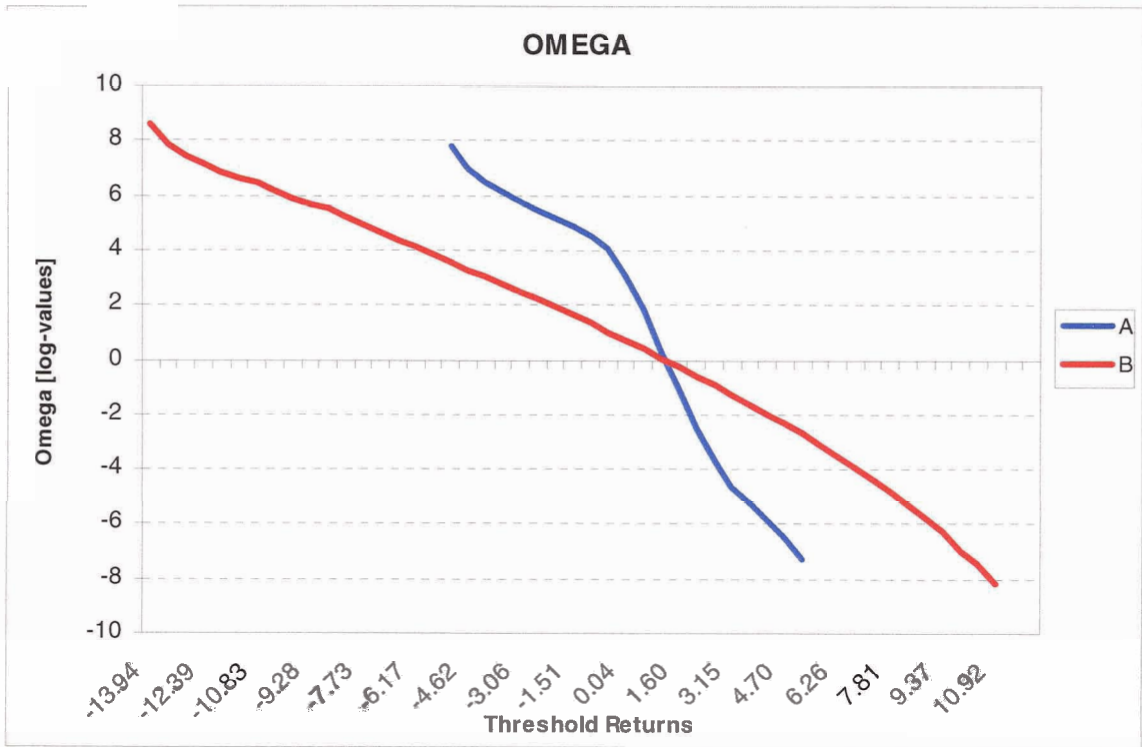FIXED INCOME

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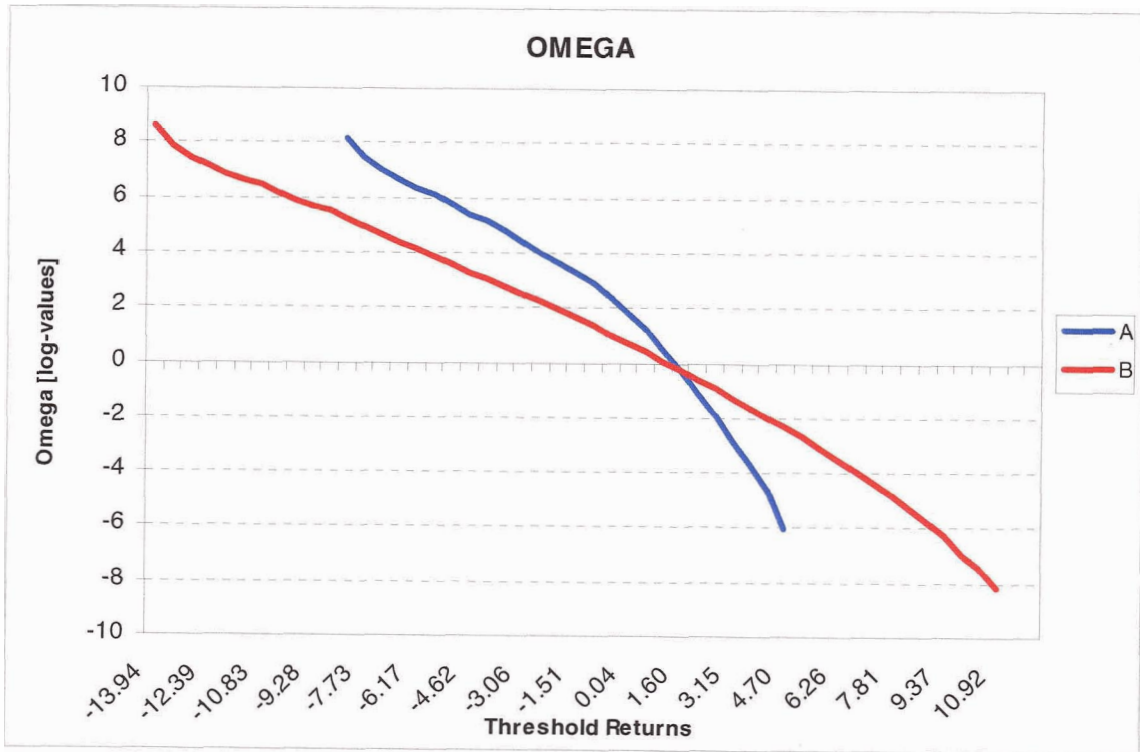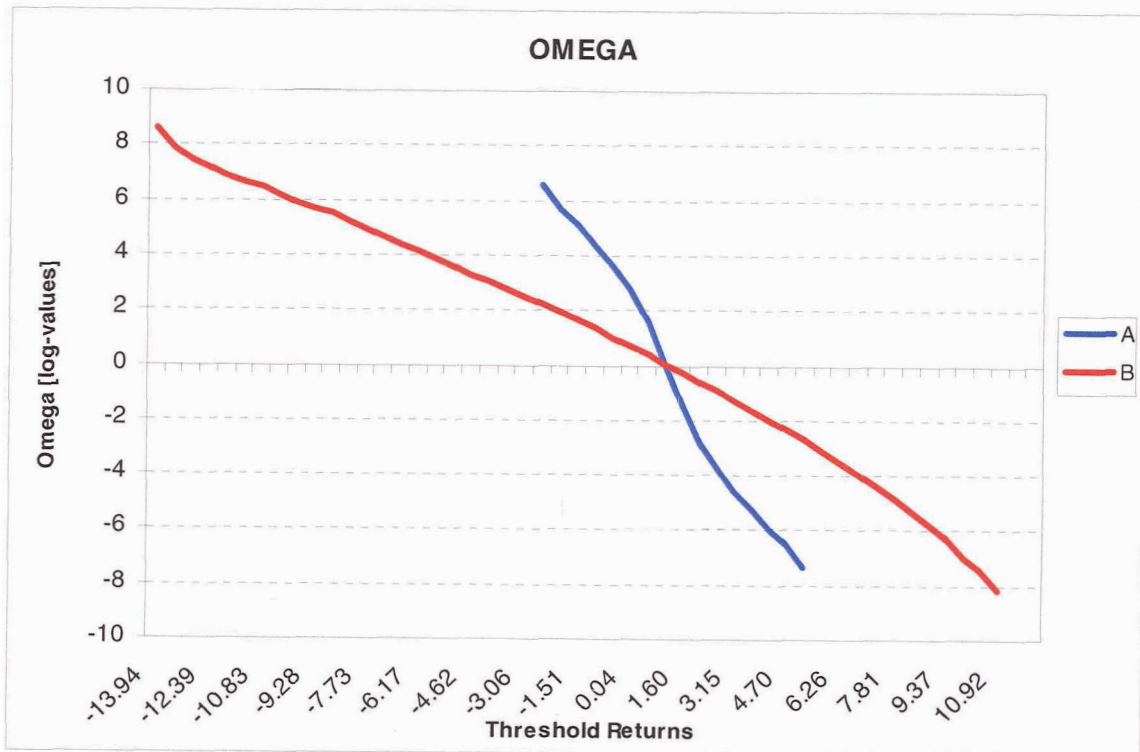
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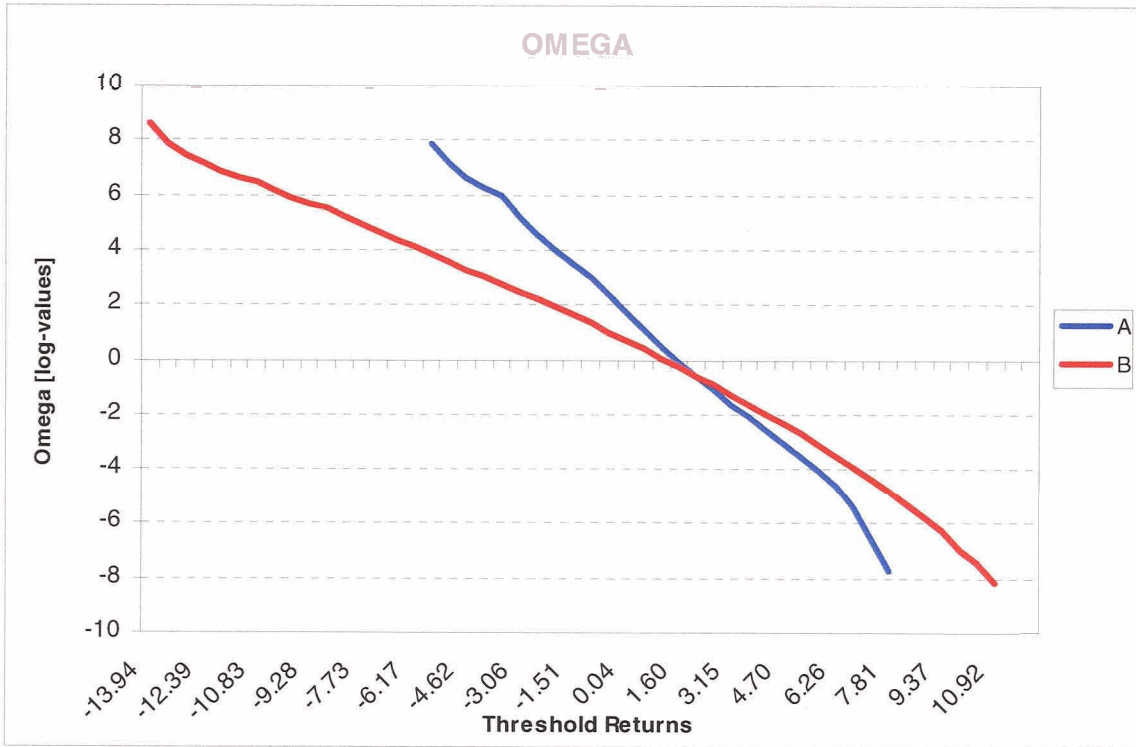
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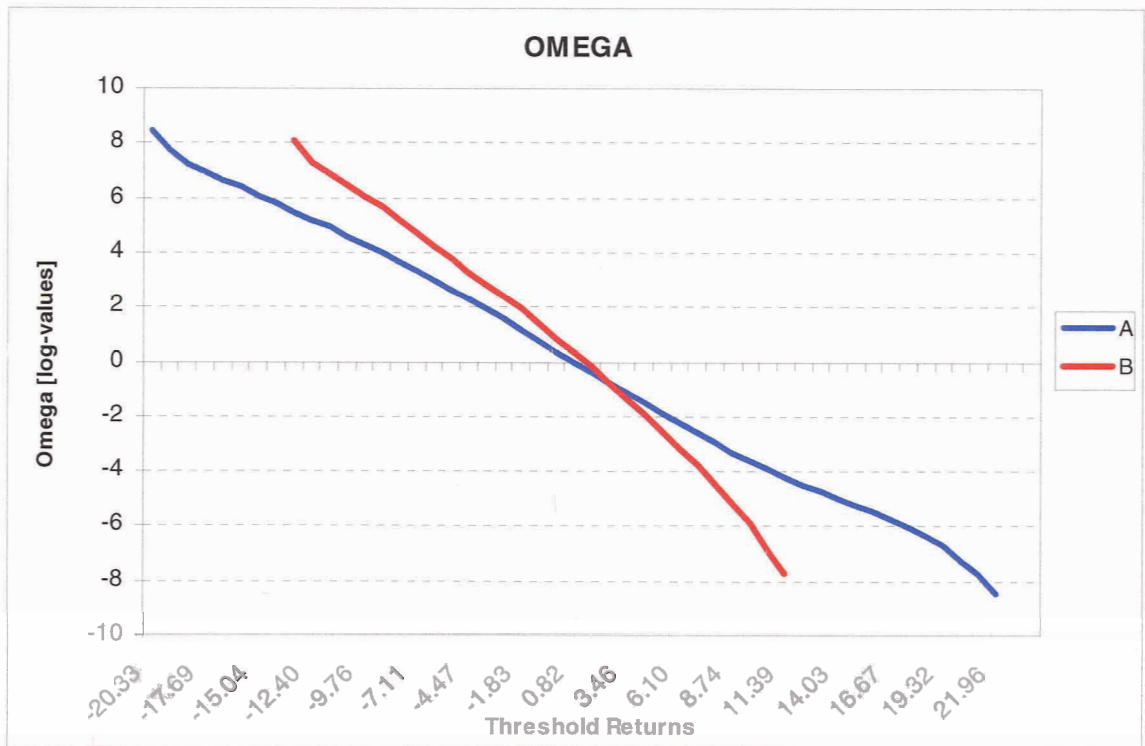
MERGER ARBITRAGE

OMEGA

RELATIVE VALUE ARBITRAGE

OMEGA

SHORT SELLING

OMEGA

REFERENCE LIST

Aitchison, J. and J.A.C. Brown, *The Lognormal Distribution*, Cambridge University Press, 1957.

Balzer, Leslie A. 1994. "Measuring Investment Risk: A Review," *Journal of Investing*, v3 (3), pp47-58.

Bawa, Vijay S. 1975. "Optimal Rules For Ordering Uncertain Prospects," *Journal of Financial Economics*, v2 (1), pp95-121.

Brulhart, Todd and Peter Klein. 2005. "Are Extreme Hedge Fund Returns Problematic?" *Working Paper*. Simon Fraser University.

Efron, Bradley and Tibshirani, Robert J. 1993. *An Introduction to the Bootstrap*, Chapman and Hall: London.

Fishburn, Peter C. 1977. "Mean-Risk Analysis With Risk Associated With Below-Target Returns," *American Economic Review*, v67 (2), pp116-126.

Forsey Hal. 2001. "The Mathematician's View: Modelling Uncertainty with the Three Parameter Lognormal," in *Managing Downside Risk in Financial Markets*, Frank A. Sortino and Stephen E. Satchell, eds., Reed Educational and Professional Publishing Ltd.

Kaplan, Paul D., and James A. Knowles. 2004. "Kappa: A Generalized Downside Risk- Adjusted Performance Measure." *Journal of Performance Measurement* 8(3):[no pages].

Kazemi, Hossein., Thomas. Schneeweis., and Raj. Gupta. 2003. "Omega as a Performance Measure." Centre for International Securities and Derivatives Markets, University of Massachusetts, Amherst.

Nawrocki, David N. 1999. "A Brief History of Downside Risk Measures." *Journal of Investing*, v8 (3, Fall), pp9-25.

Nawrocki, David N. 2002. "The Case for the Relevancy of Downside Risk Measures", in *The Handbook of Risk*, IMCA, John Wiley & Sons, Inc.

Payne, J.W. 1973. "Alternative Approaches to Decision-Making Under Risk: Moments vs. Risk Dimensions." *Psychological Bulletin*, 80, pp439-453.

Shadwick, W. and C. Keating. 2002a. "A Universal Performance Measure." Working Paper, The Finance Development Centre, London.

Shadwick, W., and C. Keating. 2002b. "A Universal Performance Measure." *Journal of Performance Measurement*, Spring 2002, pp59-84.

Sortino, Frank A. 2001. "From Alpha to Omega", in *Managing Downside Risk in Financial Markets*, Frank A. Sortino and Stephen E. Satchell, eds., Reed Educational and Professional Publishing Ltd.

Sortino, Frank A. and Hal J. Forsey. 1996. "On The Use And Misuse Of Downside Risk," *Journal of Portfolio Management*, v22 (2,Winter), pp35-42.

Sortino, Frank A. and Robert Van Der Meer. 1991. "Downside Risk," Journal of Portfolio Management, v17 (4), pp27-32.