MARKET RISK PREMIUM: IMPROVING THE DCF MODEL USING VWAP AND BOLLINGER BANDS TO ESTIMATE GROWTH

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Abstract

Market risk premium is one of the most important parameters in finance. Various estimation methods are used with the aim of accurately estimating market risk premium. Business and industry professionals rely and depend on accurate estimations of MRP. In this paper, we will propose a variation of the discounted cash flow model by Harris and Marston (1999) used to estimate MRP. Our model will seek to estimate a growth factor using volume weighted average price and Bollinger Bands, as opposed to using analysts' forecasts as a proxy for growth. Our results show that our model is able to produce statistically significant results that capture market trends, while also eliminating the risk of analysts' bias.

Keywords: Market Risk Premium; Discounted Cash Flow; Volume Weighted Average Price; Bollinger Bands

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List of Acronyms

DCF	Discounted Cash Flows
EPS	Earnings Per Share
FAF	Financial Analysts' Forecasts
GGM	Gordon Growth Model
IBES	Institutional Brokers' Estimate System
MAE	Mean Absolute Error
MRP	Market Risk Premium
VWAP	Volume Weighted Average Price

Glossary

Bollinger Bands	Price envelopes derived at a certain standard deviation level above and below a moving average price.
Discounted Cash Flow (DCF)	A model to estimate market return using dividends, closing price, and a growth factor.
Financial Analysts' Forecasts (FAF)	Expectations made by analysts about the future performance about individual company or group of companies.
Market Risk Premium (MRP)	The return required by investors over a risk-free asset.
Volume Weighted Average Price (VWAP)	Volume Weighted Average Price is a method of calculating the price of an asset which incorporates both price and volume traded during a given time period.

Chapter 1. Introduction

Market risk premium is one of the most widely used parameters in finance. It defines an expected amount of compensation for the risk provided by capital markets. Market risk premium is the incremental return over the yield on treasury bills. Its application in how capital markets are valued is crucial, and ensuring a proper estimation is key. There are several methods to estimate market risk premium, each with their own advantages and disadvantages. In this paper, we will propose a discounted cash flow (DCF) model for estimating market risk premium based on the work of Harris and Marston (1999).

Market risk premium is a key parameter in the capital asset pricing model (CAPM), which is widely used by practitioners today. Lally and Marsden (2003) explain how CAPM is used by managers to assist them in making asset allocation decisions, as well as helping companies calculate their cost of equity. Since market risk premium is key for the CAPM, it is of great importance to ensure that estimates are as accurate as possible. One of the most common and simple ways to estimate market risk premium is through the use of DCF models. A major disadvantage of DCF models lies in their sensitivity to how future dividends are estimated. Harris and Marston (1999) built a DCF model based on the Gordon Growth Model (GGM) to estimate market risk premium using expectational analysts' forecasts to reach a growth factor. In GGM model, one of the following conditions should be satisfied (Harris, R., & Marston, F., 1992):

1- A finite horizon of dividend growth at rate g, plus an assumption about the asset price at the end of the year.

2- An infinite horizon of dividend growth at a rate g.

In this paper we make sure that over a finite horizon the asset price grows at a compound rate of g (Harris, R., & Marston, F., 1992). A drawback of their model is that analysts' forecasts are influenced by inherent biases, so their application may not be fully appropriate. We will suggest a revision to the model that uses the same parameters but differs in how the growth factor is estimated.

We have used volume weighted average price (VWAP) and Bollinger Bands in conjunction with one another to formulate the growth factor. VWAP is a metric that incorporates price and volumes traded to reach an average price. It is graphically represented by a line, similar to how a simple moving average is interpreted. Bollinger Bands are price envelopes that set upper and lower boundaries for index movement at a standard deviation above and below a moving average price. Together, both will be used to provide buy/sell signals that will influence the growth factor.

Our results show that estimating market risk premium using a formulated growth factor not only helps eliminate problems caused by using analysts' forecasts, but improves the models ability to adjust to market trends. Our revision of the model produced estimates that were more in line with actual market risk premium than the original model by Harris and Marston (1999). For the period of

1982-1998, we estimated a value for market risk premium of around 8.29%. During the same period, the Harris and Marston (1999) model estimated a value of 7.14%.

Chapter 2.1 gives a brief overview of historical market risk premium models, outlining some categories with their respective pros and cons. Chapter 2.2 explains the DCF model built by Harris and Marston (1999). Chapters 2.3-2.4 go over VWAP and Bollinger Bands. Chapter 3 describes the methodology used in this study. Our model is described in Chapter 3.1, with the application of VWAP and Bollinger Bands along with the strength factor outlined in Chapters 3.2-3.4. Results and discussions are provided in Chapter 4, closing with a conclusion in Chapter 5.

Chapter 2. Literature Review

2.1 A Brief Review of Market Risk Premium Models

Duarte and Rosa (2015) appropriately explain what market risk premium is, and historical models used to estimate it. In its essence, MRP is the amount of return required by investors for holding a risky market portfolio over a risk-free bond. MRP is derived from expectations regarding future stock market performance, as these returns are not directly observable. Mathematically, MRP can be represented by the following

$$MRP_t(k) = E_t[R_{t+k}] - R^f_{t+k}$$

where $E_t[R_{t+k}]$ is the expected return of the risky market portfolio for the period of $t + k$, and R^f_{t+k} is the return on risk-free bonds over the same investing horizon.

There are five categories for MRP models based on their assumptions. Four of these types of models are built to allow investors to use information available now to reach an estimated MRP value. Within each category, models often reach very similar MRP estimations. Duarte and Rosa (2015) go in depth into each model category, describing their pros and cons. We will go over three common MRP model categories and explain the advantages and disadvantages of each. Understanding what each of these models does well and were they fall behind will allow us to identify key factors needed to reach an efficient value for MRP.

2.1.1 Historical average of realized returns models

These models are arguably the simplest methods used to estimate MRP. They use historical averages for the market return over risk-free bonds. Due to the simplicity of these models, there is not much that could be added to improve them. The major disadvantage of these models is that they are completely reliant on historical data, with the assumption that the past influences the future.

2.1.2 Dividend discount models

The major underlying assumption behind these models is that stock prices are purely a representation of the future cashflows to shareholders. These models discount future expected dividends using a rate based on MRP and the risk-free rate to reach current stock prices. In order to derive MRP, values are plugged into the dividend discount model, and an implied value for MRP is reached in order for discounted future cash flows to equal current prices. Advantages of these models include their forward-looking view and their ease of application. A major drawback is its sensitivity to how expected dividends are calculated.

2.1.3 Time-series regression models

These models assume that the value of MRP is derived from a direct relationship between stock returns and key economic factors. Duarte and Rosa (2015) discuss that in order to derive MRP, a predictive linear regression of excess returns is run on lagged key economic factors. Essentially, future stock returns are being derived from select economic factors. Advantages of these models include their simplicity and need for little assumptions. However, the key to accurately derive MRP using time-series regressions lies in the selected key economic factors.

With this in mind, we can begin to set guidelines for what makes an accurate and effective model. A model needs to be forward-looking and simple to apply, with minimal assumptions to reduce errors and inconsistencies.

2.2 Harris and Marston (1999) DCF Model

Harris and Marston (1999) propose a variation of the common DCF model for estimating market return where growth is represented by analysts' forecasts. The DCF model produces an estimate for market return, which could then be used to derive market risk premium. The DCF model by Harris and Marston (1999) uses a rearranged variation of the Gordon Growth Model. The following equation represents the DCF model used by Harris and Marston (1999)

$$k = \left(\frac{D_1}{P_0}\right) + g$$

where k represents market return, D_1, is the expected next year's dividend, P_0 is the current closing price, and g is the expected growth rate of dividends.

They go on to explain that the primary obstacle in using a DCF model is determining the growth factor. For their model, they opted to use financial analysts' forecasts (FAF) of long-run growth in earnings as a proxy for growth. The mean value of analysts' forecasts of five-year EPS growth is estimated from data provided by IBES, and then used in the model. They found that an advantage of using analyst forecasts is that it can be representative of modern concerns and views on equity performance. This gives the model a strong forward-looking edge when compared to the commonly used historical average methods. This DCF model could be used for single stocks or any portfolio of companies. For the purposes of their paper, all their results were done using the S&P 500 index as the input data.

A drawback in this version of the model is the inherent biases that lie in analyst forecasts. While using a large selection of analysts reduces this risk, it is still present. To eliminate the risk of these biases, our paper will present a method for estimating the growth factor that relies on empirical market data. We will also be using the S&P 500 as our input data to simplify comparisons with the original model. The next sections will focus on the methods used to estimate the growth factor, with an overall view of the improved DCF model and its results to follow.

2.3 Overview of Volume Weighted Average Price (VWAP)

VWAP is a benchmark used by professionals in the finance industry. VWAP can be used to determine and identify trends in the market. Madhavan (2002) explains that an advantage of VWAP models is their computational simplicity and ability to incorporate trading volume into average price. The following is a basic mathematical interpretation of VWAP

$$VWAP = \frac{\sum_j P_j \times Q_j}{\sum_j Q_j}$$

where P_j and Q_j refer to the price and quantity at transaction j.

VWAP is simple in nature and can be used in applications similar to moving averages. Madhavan (2002) describes how VWAP is most commonly used in an intraday period, allowing traders to get an idea about their performance relative to the market for a given day. This means that the VWAP calculation above would be used on a daily basis, for specific stocks or the market as a whole. For example, traders are happy to see a closing VWAP higher than the price paid during the day.

VWAP gives professionals another perspective on the price of a stock using trade volume. During a given day, a stock's price can vary drastically between transactions. Some stocks are highly traded, while other stocks see little trades on a day-to-day basis. Guéant and Royer (2014) highlight the bias of looking at closing prices alone, since they ignore trading volume, which can give deeper insights into the value of a stock. VWAP aims to resolve the bias for closing trades on a given day.

In the industry, VWAP benchmarks can be supplemented by trading strategies. VWAP benchmarks give investors insight into their performance relative to the market. Investment banks may use trading strategies centered around VWAP benchmarks. Some of these strategies used by investors as outlined by Madhavan (2002) include direct access, forward VWAP cross, and guaranteed principal VWAP bid. These strategies differ in terms of trading horizon, placement strategy, and venue. For example, the guaranteed principal VWAP bid strategy directs traders to pursue their desired positions over several days, as to ensure that the purchase is not made during an unfavourable VWAP period. A major concern of this strategy is opportunity cost, since it limits when traders can sell and capitalize on favourable prices. Each trading strategy has its respective costs that need to be considered. All these strategies use VWAP benchmarks calculated on a daily basis to help investors make decisions and gauge their performance.

2.4 Overview of Bollinger Bands

Bollinger Bands are price envelopes plotted above and below a simple moving average depending on a certain standard deviation level. Typically, a level of one standard deviation is used in order to calculate these upper and lower bands. Graphically, Bollinger Bands are represented by three lines: the top band, middle band, and bottom band. The middle band is representative of a simple moving average price. The following shows how these bands are calculated in Sierra Chart

$$Top\ Band: TB_t^{(B)}(X, n, v) = SMA_t(X, n) + v \times \sigma_t(X, n)$$

$$Middle\ Band: MB_t^{(B)}(X, n, v) = SMA_t(X, n)$$

$$Bottom\ Band: BB_t^{(B)}(X, n, v) = SMA_t(X, n) - v \times \sigma_t(X, n)$$

where X is input data, n is length, v is standard deviation, and t is the index or asset.

John Bollinger (1992) explains how traders can apply Bollinger Bands to identify when certain stocks are overbought or oversold. When a stock's price hits the top band, traders consider the stock to be overbought. Prasetijo et al.

(2017) explain that when a stock's price hits the bottom band, it can be considered oversold. These identifiers allow for more informed decisions.

Chapter 3. Methodology

3.1 A Variation of the DCF Model for Risk Premium

The following is a non-linear, multivariate model used to estimate market risk premium. Our model uses a DCF model similar to that of Harris and Marston (1999) in order to estimate market return. The following formula is used to find market return

$$E(R) = \frac{D_0(1+g)}{P_0} + g$$

where D_0 is the dividend per share expected to be received this year, g is the expected growth rate in dividends per share, and P_0 is the S&P 500 closing price this year.

Where this model builds on the standard DCF approach is in how g is estimated. We have opted to incorporate VWAP and Bollinger Bands in tandem with one another to reach an estimations of future expected growth. VWAP and Bollinger Bands are both used to identify market trends. Market trends are a driving force for market performance. Hence, using VWAP and Bollinger Bands to capture market trends will allow us to estimate the expected effect on market performance. The following equation is used to derive g

$$g = X_1 + \beta X_2 + \gamma X_3 + \epsilon$$

$$X_1 = -\log\left(\frac{Close\ Price_{(t0)}}{VWAP\ Price_{(t0)}}\right)$$

$$X_2 = -sign\,(\frac{\#\,of\ weeks\ per\ year\ closing\ near\ Top\ Bollinger\ Band}{\#\,of\ weeks\ per\ year\ closing\ near\ Bottom\ Bollinger\ Band})^{sign}$$

$$X_3 = \frac{\#\,weeks\ that\ price\ abpve\ VWAP}{\#\,weeks\ that\ price\ below\ VWAP} \times \frac{\#\,P\ above\ BBL\ band}{\#\,P\ beloww\ BBl\ band} \times \frac{Yearly\ closing\ Price}{VWAP}$$

where X_3 is the combination of three different dependent variables.

Hidalgo and Goodman (2013) explain multivariable regression models as equations containing multiple variables used to forecast an outcome. Our model uses multiple variables in order to forecast an outcome for market return.

$$E(R) \sim (\mathrm{P}, \mathrm{D}, X_1, X_2, X_3)$$

Archontoulis and Miguez (2015) describe how non-linear regression models can be used and applied. We found that a non-linear regression fit our model best as logarithmic and multifactor equations were used in the calculation of independent parameters (X_1 and X_3). Furthermore, the model in its essence estimates market return using what Rhinehart (2016) describes as a “modified hyperbola”, which can be seen below

$$Y = \frac{aX}{(1 + bX)}$$

where Y is the response variable, X is the explanatory variable, a and b are parameters that determine the shape of the curve and the magnitude of the Y value.

The X_1 variable represents assumptions calculated using VWAP. When the closing price is greater than the closing VWAP for a given year, there is a negative impact on the growth factor. When the closing VWAP is greater than the closing price for a given year, there is a positive influence in the growth factor. The X_2 variable represents signals produced by Bollinger Bands. If the number of weeks per year closing near the top band is greater than the number of weeks per year closing near the lower band, a sign of +1 will be used. In the opposite case, a sign of -1 will be used instead. Buy signals introduce a positive bias, while sell signals introduce a negative bias. This captures the signals produced by Bollinger Bands for a given year. The X_3 variable is a strength factor incorporating the two methods simultaneously, which helps interpret strong or unusual years of data.

Once g is computed, it can be used to complete the DCF model and reach an estimation for market return. MRP could then be estimated as follows

$$MRP = E(R) - Rf$$

3.2 Application of VWAP in Forecasting Risk Premium

Our model incorporates VWAP to gain more insight into the price of a stock. For market data, we will be using the S&P 500 index. Sierra Chart is a trading and charting platform for financial markets. Sierra Chart also supports technical analysis using real time and historical data for a variety of international markets. Since this market data is part of the platform, using it for analytics becomes simple and straightforward. Our aim is to calculate VWAP for the S&P 500 using Sierra Chart, identify trends in the market, and collate this data as part of our process for estimating the growth factor.

To calculate VWAP for the S&P 500, we have decided to use weekly data as opposed to daily data. This will help simplify our model, while also correcting for certain daily spikes that might occur. For this, the following formula was used to calculate the weekly VWAP of the market

$$VWAP(P,n) = \frac{\sum_{i=1}^{n} P_i \times Q_i}{\sum_{i=1}^{n} Q_i}$$

where n refers to the number of trading days used for consideration, which in our case will be 5.

The weekly VWAP prices will be plotted and compared with weekly market prices in order to generate buy and sell signals (Figure 1 and Figure 2). Traders using VWAP benchmarks will try to buy when the price is below the VWAP, or sell when the price is above it. Coles and Hawkins (2011) describe how this action can help drive the price back toward the average instead of away from it.

Therefore, when the weekly VWAP is higher than the weekly market price, a positive signal will be generated, and vice versa. These weekly indicators will be accumulated to generate a signal for an entire year (Figure 3). To do so, we anchored the VWAP line so that it resets at the beginning of every year. This means that on January 1st, both VWAP and S&P 500 prices start at the same point. Resetting the VWAP every year prevents old historical data from distorting assumptions and estimations.

In this paper, for the first variable (X_1) we have used a ratio of closing market price over the closing VWAP price.

$$Price\ over\ VWAP = \frac{Closing\ Market\ Price}{Closing\ VWAP\ Price}$$

When the market price is above VWAP, it will generate a sell signal which has a diminishing effect on the growth factor, and vice versa. As illustrated in the model, we have used a negative logarithmic sign to capture this impact. When the price is greater than the WVAP, a negative sign will be generated, and when the closing price is below the VWAP, the outcome is a positive number which boosts the growth factor.

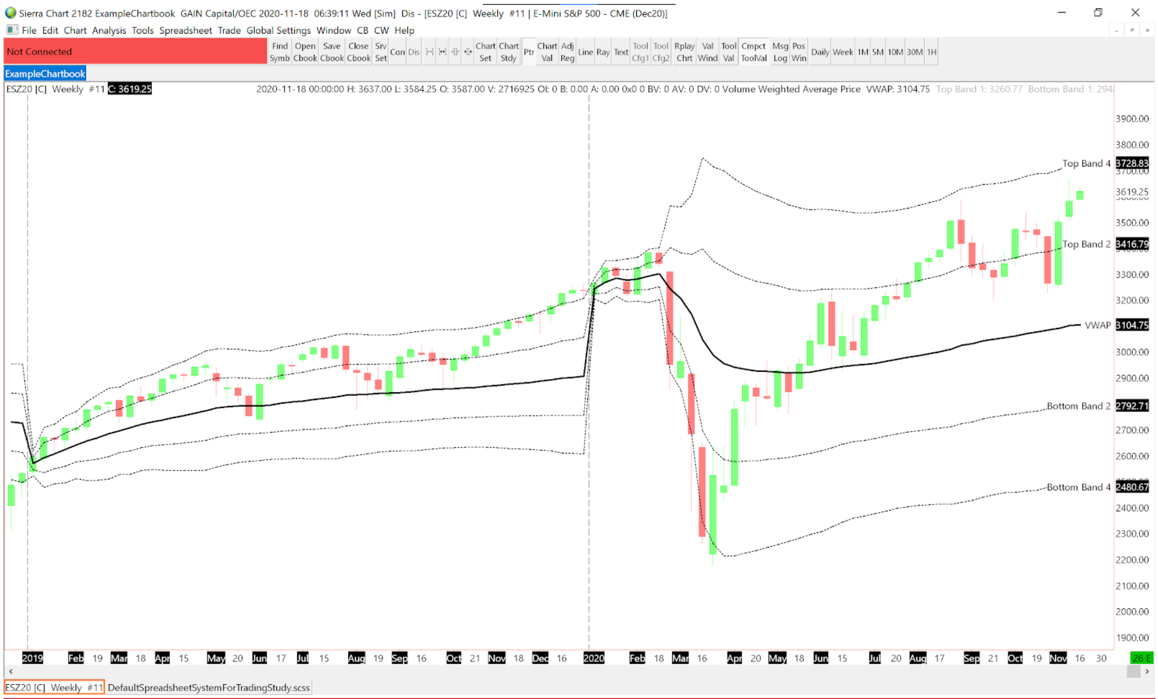


Figure 1 - Two-year weekly VWAP (2019-2020). VWAP is anchored to the start of the year and includes the volumes and price of transactions over the year.

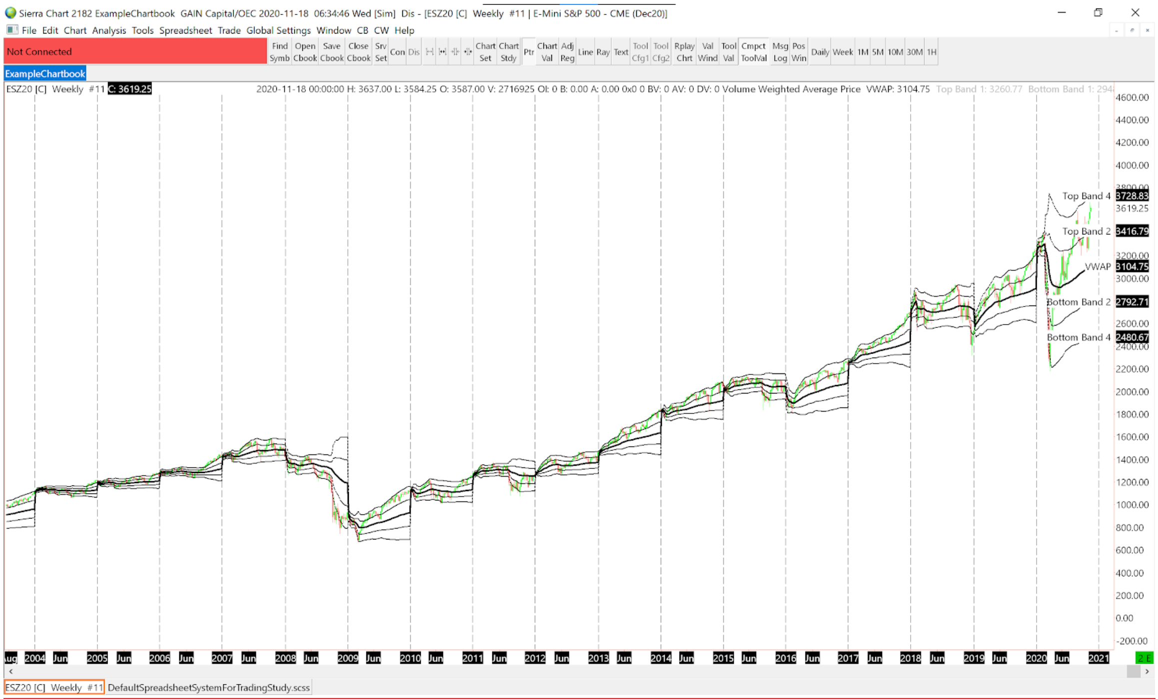


Figure 2 - Seventeen-year weekly anchored VWAP (2004-2020). A similar concept for calculation of anchored VWAP for a single year can be applied to all years where S&P 500 volume data is available.

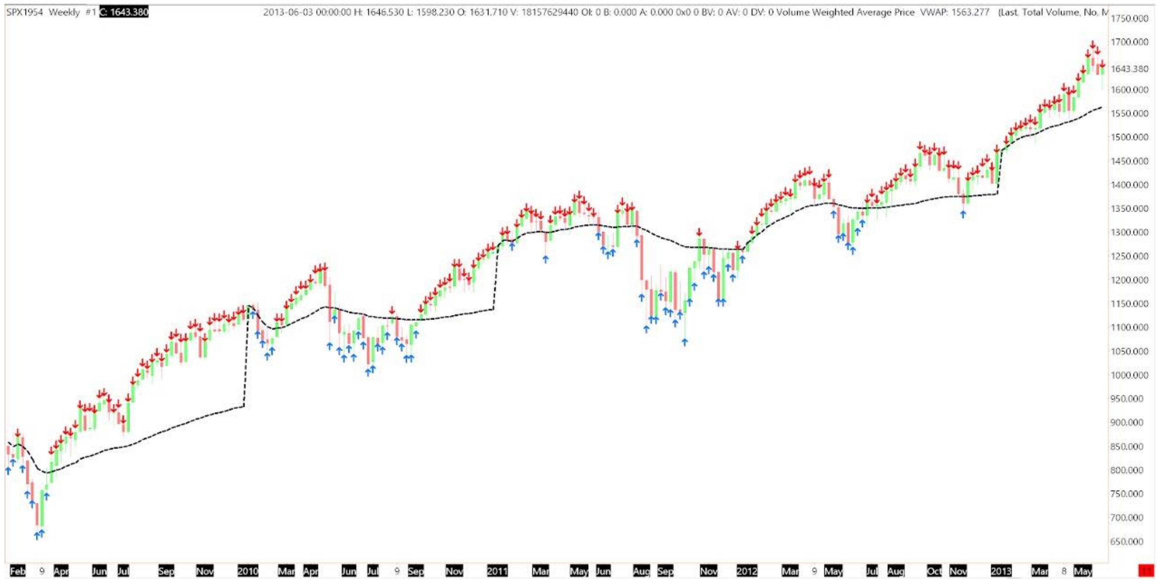


Figure 3 - VWAP Application in Forecasting Risk Premium. This graph shows S&P 500 index weekly closing prices above or below VWAP. Red and blue arrows indicate weekly close above and below VWAP respectively.

3.3 Application of Bollinger Bands in Forecasting Risk Premium

To incorporate Bollinger Bands into our model, we began by using a twenty weeks moving average of the S&P 500 index. We then calculated top and bottom bands at a level of two standard deviations above and below the moving average price. Then by using Sierra Chart, we tracked the number of weeks per year where the closing price was near the top band (between 2 and 1.5 standard deviations). When the closing price was near the top band, that would be interpreted as a negative indicator, and vice versa (Figure 4).

Furthermore, we realized that the larger the number of weeks near the upper band, the more negative the effects on the growth factor, and vice versa. Due to this, in the second variable (X_2), we used the ratio of the number of weeks that the price was near the upper and lower Bollinger Bands, so that when the number of weeks where the price near the upper band was greater than the lower band, X_2 would be:

$$X_2 = -\left(\frac{\#\ of\ weeks\ per\ year\ closing\ near\ Upper\ Bollinger\ Band}{\#\ of\ weeks\ per\ year\ closing\ near\ Lower\ Bollinger\ Band}\right)$$

As illustrated above, the higher ratio will lead to a higher diminishing effect on the growth factor.

On the other hand, when the number of weeks where the price near the lower band was greater than the upper band, we have a buy signal, and we are expecting a positive impact on the growth factor. In this situation, X_2 would be:

$$X_2 = \frac{\#\ of\ weeks\ per\ year\ closing\ near\ Lower\ Bollinger\ Band}{\#\ of\ weeks\ per\ year\ closing\ near\ Upper\ Bollinger\ Band}$$

The higher ratio will lead to a more positive effect on the growth factor.

To abridge this effect in our model, we defined X_2 as:

$$X_2 = -sign\ (\frac{\#\ of\ weeks\ per\ year\ closing\ Price\ near\ Upper\ Bollinger\ Band}{\#\ of\ weeks\ per\ year\ closing\ Price\ \ near\ Lower\ Bollinger\ Band})^{sign}$$

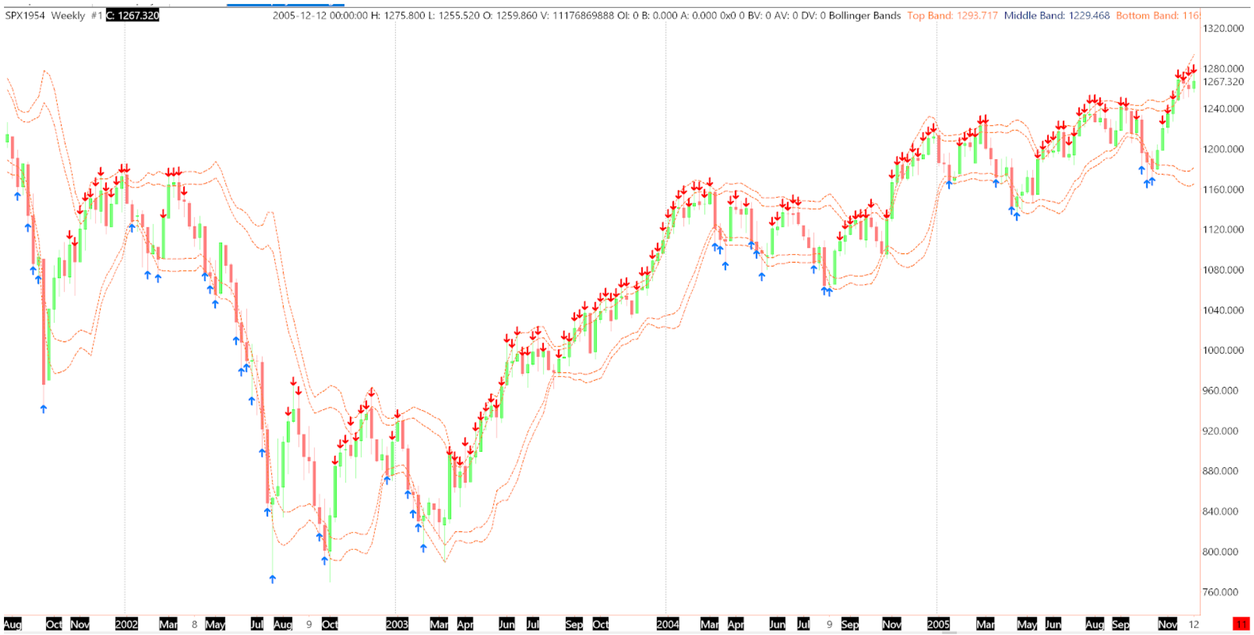
where $sign$ is +1 if

# of weeks Price near Upper Bollinger Band > # of weeks Price near Lower Bollinger band.

and $sign$ is -1 when

# of weeks Price near Upper Bollinger Band < # of weeks Price near Lower Bollinger band.

We can also see that there is a relationship between raw Bollinger Band signals and future market performance (Figure 5). When the number of negative signals outweighed the number of positive signals for a given year, the next year's yield is expected to be lower than the current year's yield. We will be combining interpretations from VWAP and Bollinger Bands in our model in order to reach a value for estimated growth.



We ran our model for the period of 1979-2020 and estimated market return.

Figure 4 - Bollinger Band Application in Forecasting Risk Premium. The following graph shows weekly closing prices above or below one and a half standard deviations of a moving average of the S&P 500 index using Bollinger Bands. Red and blue arrows indicate weekly closing prices above and below the bands respectively.

For the following set of estimates, we used coefficients that were estimated using the same period, meaning that the estimates are not out of sample. **Table 1** shows the estimated market return predicted using our model represented by “Return (Forecast)”. This value can then be converted into market risk premium by subtracting the respective year’s risk-free rate from the estimated market return. The “Market Risk Premium (Forecast)” column lists the implicit market risk premium estimated this way.

Table 1 - Forecasted market return using our model (using coefficients based on the period of 1979-2020).

Year	Return (Forecast)	Risk Free Rate	Market Risk Premium (Forecast)
2020	0.0356	0.0206	0.015
2019	0.2181	0.0194	0.1987
2018	0.2532	0.0093	0.2439
2017	-0.0548	0.0032	-0.058
2016	0.1159	0.0005	0.1154
2015	0.1817	0.0003	0.1814
2014	0.0177	0.0006	0.0171
2013	0.1536	0.0009	0.1528
2012	0.0698	0.0005	0.0693
2011	0.1314	0.0014	0.13
2010	0.0129	0.0015	0.0114
2009	-0.0609	0.0137	-0.0745
2008	0.2113	0.0435	0.1678
2007	0.1822	0.0473	0.1349
2006	-0.0063	0.0315	-0.0378
2005	0.137	0.0137	0.1233
2004	0.0938	0.0101	0.0837
2003	-0.0662	0.016	-0.0823
2002	0.0371	0.0339	0.0032
2001	-0.0956	0.0582	-0.1537
2000	-0.0526	0.0464	-0.099
1999	0.0062	0.0478	-0.0416
1998	-0.0122	0.0506	-0.0628
1997	0.2025	0.0501	0.1525
1996	0.212	0.0549	0.1571
1995	0.1662	0.0425	0.1237
1994	0.2383	0.03	0.2083
1993	0.0809	0.0343	0.0466
1992	0.117	0.0538	0.0632
1991	0.0349	0.0749	-0.0401
1990	0.2755	0.0811	0.1944
1989	-0.011	0.0667	-0.0777
1988	0.1451	0.0578	0.0873
1987	0.1449	0.0598	0.0851
1986	0.0443	0.0748	-0.0305
1985	0.2858	0.0952	0.1905
1984	0.3083	0.0861	0.2222
1983	0.1463	0.1061	0.0402
1982	0.1888	0.1403	0.0485
1981	0.0466	0.1143	-0.0677
1980	0.028	0.1007	-0.0727
1979	0.241	0.107	0.134

In order to gauge our model results, we will be comparing them to the original model by Harris and Marston (1999). It's important to note that for the following comparisons, the coefficients used in our model are estimated using the period of 1979-2020. Our model was able to predict MRP more accurately than the original DCF model by Harris and Marston (1999). A comparison of estimated market risk premium using our model and the traditional DCF model for the period between 1982-1998 (the period originally used by Harris and Marston (1999)) suggest that our model had produced estimates which followed market trends. When we compared our model returns, actual returns, and traditional DCF returns, figures suggest that our model captured trends in the market while the traditional DCF model remained flat (Figure 6 and Figure 7).

When comparing the error in prediction of the S&P 500 actual return for the period between 1982-1998, the figure suggests that our model performed better on average than the traditional DCF model (Figure 8). In the seventeen-year period, our model errors were higher than those of traditional DCF models only on three accounts. When looking at residuals, we can also see that our model had fewer residuals than the traditional DCF model (Figure 9).

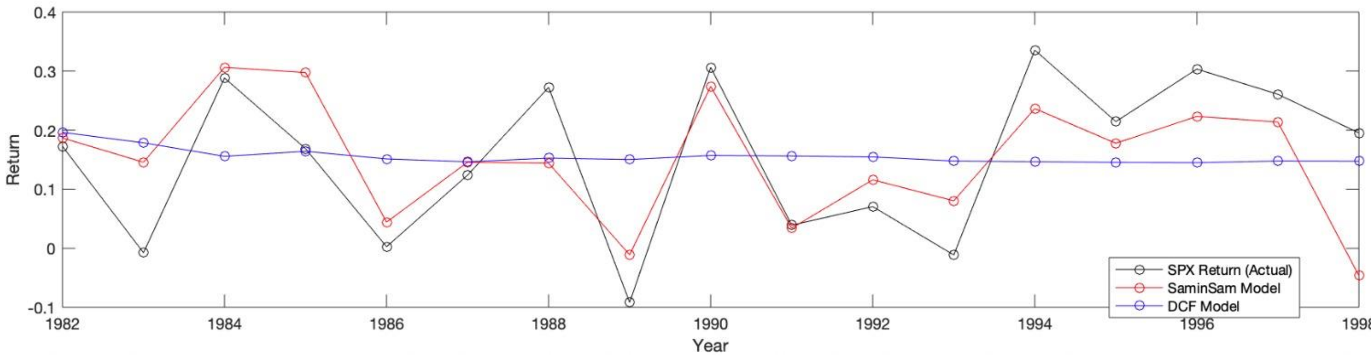


Figure 5 - Our Model Estimates vs. Actual Returns vs DCF Model. The graph shows the estimated market return predicted by our model (using coefficients based on the period of 1979-2020) and the actual S&P 500 returns for the period between 1982-1998. As we can see, the model was able to accurately identify market trends, and accordingly adjust predictions.

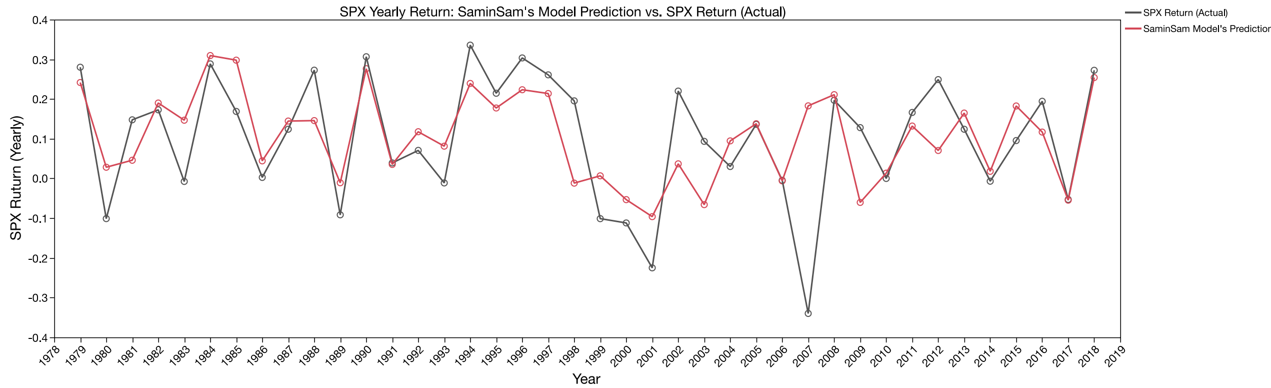


Figure 6 - Our Model Estimates vs. Actual Returns. The graph shows the estimated market return predicted by our model and the actual S&P 500 returns for the period between 1979-2019. As we can see, the model was able to accurately identify market trends, and accordingly adjust predictions.

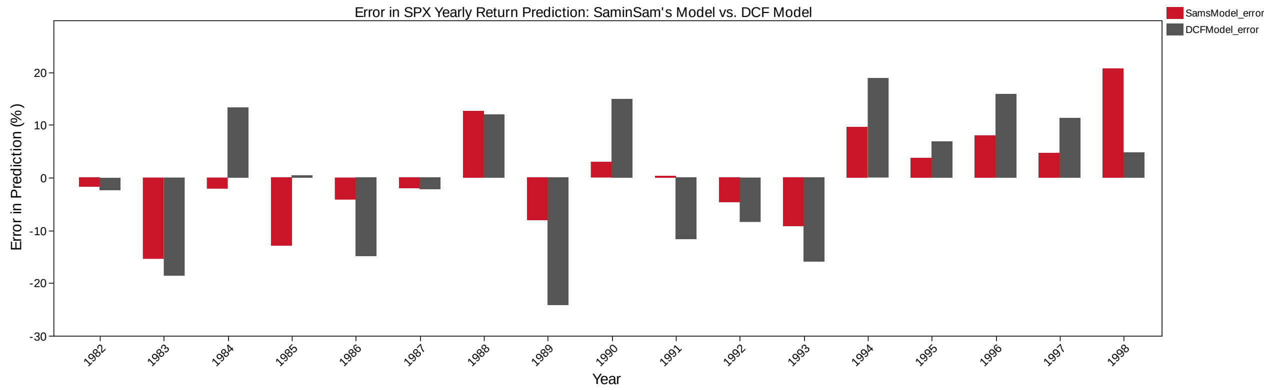


Figure 7 - Comparison of Error in S&P 500 Return Prediction. In order to gauge the accuracy of our model's predictions (using coefficients based on the period of 1979-2020) over the traditional model, we tracked the error in estimations found in both models. We used the same period as Harris and Marston (1999). We found that on fourteen out of the seventeen years, our model was more accurate than the traditional DCF model.

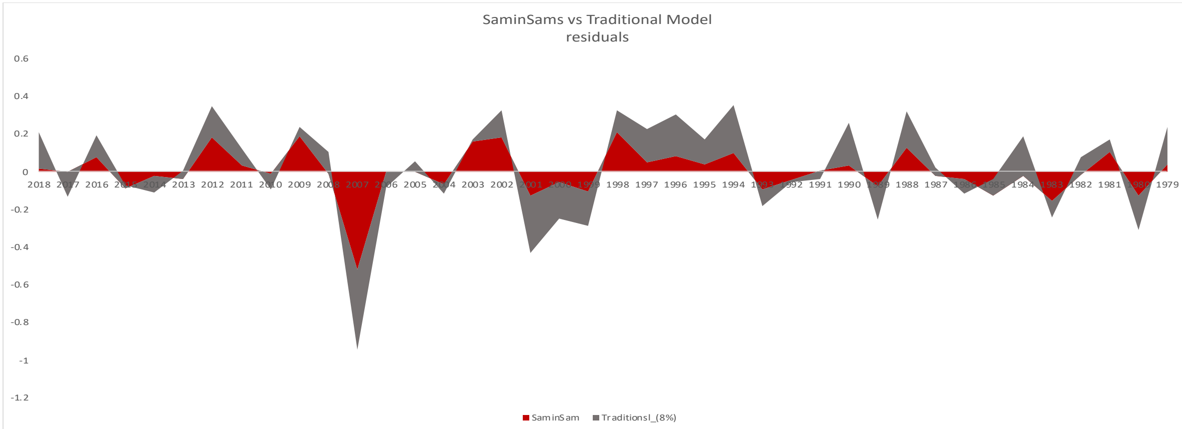


Figure 8 - Comparison of Residuals. The following figure tracks the residuals of our model (using coefficients based on the period of 1979-2020) as well as the traditional DCF model for the period of 1979-2018. This long-time horizon would help test the models' efficiency and reliability over time. Our model was able to consistently produce less residuals than the traditional DCF model over the 39 year period.

The following in sample statistical tests were performed on the coefficients estimated using the period of 1979-2020. These tests aim to showcase the possibility for VWAP and Bollinger Bands to be used in MRP calculations. Table 2 shows the estimates of the coefficients along with a test for their statistical significance. As illustrated in the table, both β and γ (coefficients of Bollinger Bands and the strength factor respectively) had a tStat value higher than two, which shows their statistical significance in the model. Table 3 shows a linear regression between our model's estimated return and the actual return on the S&P 500 for the period of 1979-2020. These results show the stability of our model while using these coefficients.

Table 2 - Model coefficient estimates using the period of 1979-2020.

Model Coefficients Estimated Using the Period of 1979-2020				
	Estimate	**SE**	**tStat**	**pValue**
b1	0.19167	0.035228	5.4409	3.0914e-06
b2	0.22999	0.052767	4.3585	9.2418e-05
Number of observations: 41, Error degrees of freedom: 39 ***Root Mean Squared Error: 1.03*** ***R-Squared: 0.335, Adjusted R-Squared 0.318*** ***F-statistic vs. zero model: 21.2, p-value = 5.93e-07***				
	Estimate	**tStat**	**Statistically Significant**	
	β	*5.22 > 2*	✓	
	γ	*4.03 > 2*	✓	

Table 3 - Regression between S&P 500 returns and our model expected returns for the period of 1979-2020* (using coefficients based on the period of 1979-2020)*.

	Estimate	**SE**	**tStat**	**pValue**
(Intercept)	0.068577	0.014317	4.7899	2.4247e-0
x1	0.37003	0.077969	4.7458	2.7836e-05

Number of observations: 41, Error degrees of freedom: 39

Root Mean Squared Error: 0.0761

R-squared: 0.366, Adjusted R-Squared: 0.35

F-statistic vs. constant model: 22.5, p-value = 2.78e-05

4.1 Further Evaluation of the Model Based on Training and Test Datasets

A key point in evaluating a model is to see how it performs on an unseen dataset, i.e., test datasets. This is a key step in ensuring that the model can consistently provide reliable estimations of the future datapoints. For further evaluation of our model in estimating market risk premium, S&P 500 annual return data was divided into two datasets:

1) *In-sample dataset*, which will be referred to as the *'training dataset'*. Training dataset include the data from 1982-1998. The reason to select this range is to be aligned with what Harris and Martson (1999) have used to develop their DCF model.
2) *Out-of-sample dataset*, which will be referred to as the *'test dataset'*. Test dataset includes data from 1999-2019.

4.1.1 Training Dataset

The model was first developed using the training dataset. The model parameters were then estimated, as shown below. The growth factor, g, was then estimated for each year, followed by S&P 500 return estimation. Figure 10 shows the expected returns estimated using the training dataset for the period of 1982-1998. For the sake of comparison, the actual S&P 500 return and the traditional DCF model estimation are also plotted.

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The error in estimating the expected return using the training dataset is shown in Figure 11. The figure suggests that in fourteen out of the seventeen years, the estimated return by our model was closer to the actual S&P 500 return than that of the DCF model.

Table 4 - Model parameters using the training dataset only. The dataset using the period of 1982-1998 is used for training.

	Estimate	SE	tStat	pValue
b1	0.21488	0.03931	5.4663	6.5038e-05
b2	0.24209	0.046511	5.205	0.00010673
Number of observations: 17, Error degrees of freedom: 15 **Root Mean Squared Error: 1.06** **R-Squared: 0.522, Adjusted R-Squared 0.49** **F-statistic vs. zero model: 30.7, p-value = 4.95e-06**				

	Estimate	**tStat**	**Statistically Significant**	
	β	*5.47 > 2*	✓	
	γ	*5.2 > 2*	✓	

Table 5 - Our model's expected return and the associated error using the training dataset. S&P 500 return and traditional DCF expected return (and its error) are shown for comparison.

Year	SPX Return	Samin-Samer (SS) Expected Return	DCF Expected Return	SS Error	DCF Error
1998	0.1953	-0.0008	0.1962	0.1960	-0.0009
1997	0.2607	0.2209	0.1786	0.0399	0.0821
1996	0.3034	0.2307	0.1757	0.0727	0.1277
1995	0.2145	0.1807	0.1642	0.0338	0.0503
1994	0.3356	0.2613	0.1513	0.0743	0.1843
1993	-0.0113	0.0890	0.1465	-0.1003	-0.1578
1992	0.0706	0.1308	0.1527	-0.0602	-0.0821
1991	0.0390	0.0458	0.1503	-0.0067	-0.1113
1990	0.3064	0.2947	0.1572	0.0116	0.1492
1989	-0.0917	-0.0089	0.1563	-0.0828	-0.2480
1988	0.2725	0.1592	0.1547	0.1133	0.1178
1987	0.1240	0.1335	0.1478	-0.0096	-0.0238
1986	0.0026	0.0451	0.1466	-0.0425	-0.1440
1985	0.1687	0.3074	0.1455	-0.1387	0.0232
1984	0.2884	0.3341	0.1449	-0.0458	0.1435
1983	-0.0076	0.1540	0.1478	-0.1616	-0.1554
1982	0.1727	0.2150	0.1475	-0.0423	0.0252

β	0.21488
γ	0.24209

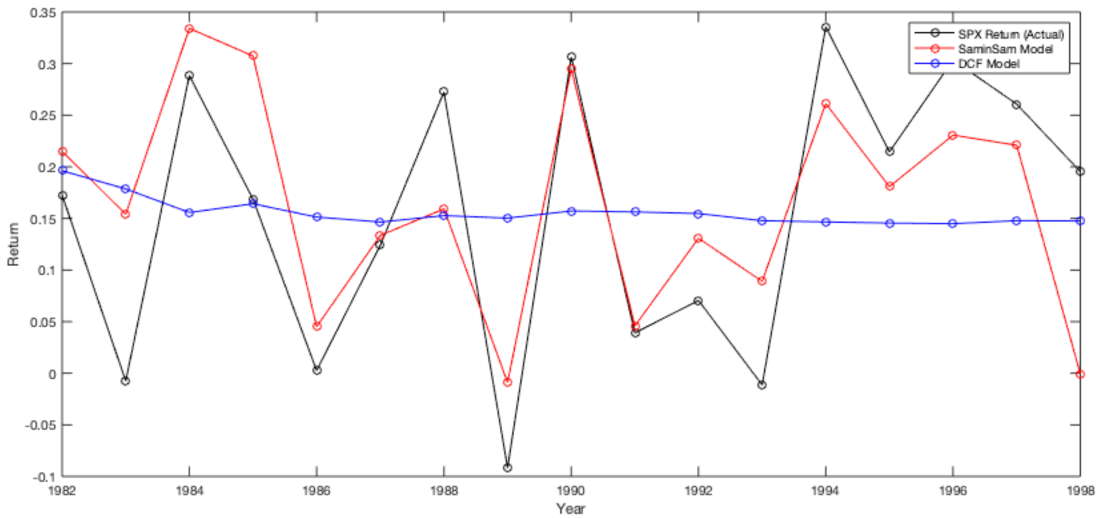

Figure 9 - **Expected Return Estimation Using the Training Dataset.**

The graph shows the estimated market return predicted by our model developed using the training dataset only. For comparison, the actual S&P 500 returns and the DCF estimate for the same period (i.e. 1982-1998) are also shown in the graph. As we can see, the figure suggests that our model was able to identify market trends.

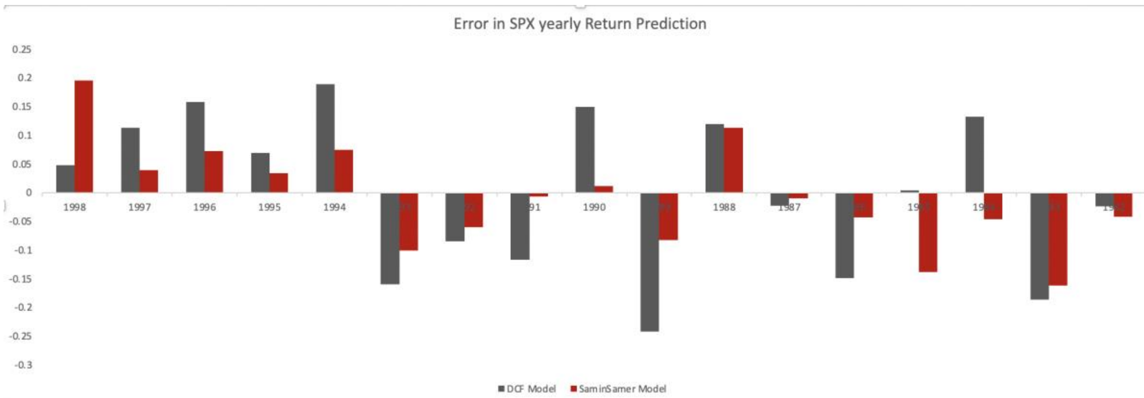

Figure 10 - Error in Expected Return Estimation Using the Training Dataset. The error by the DCF model is also plotted for comparison. Comparison of error in S&P 500 return estimate (using a smaller sample size). For the training dataset, the same period as Harris and Marston (1999) was used. It suggests that on fourteen out of the seventeen years, our model based on the training dataset was more accurate than the traditional DCF model.

Table 6 - Regression between our model using the training dataset and actual S&P 500 returns for the period of 1982-1998.

	Estimate	SE	tStat	pValue
(Intercept)	0.074487	0.0284	2.6226	0.0192
x1	0.57731	0.1402	4.1152	0.00091
Number of observations: 17, Error degrees of freedom: 15 ***Root Mean Squared Error: 0.075*** ***R-squared: 0.53, Adjusted R-Squared: 0.499*** ***F-statistic vs. constant model: 16.9, p-value = 0.000917***				

Model	tStat	pValue	Statistically Significant
Samin-Samer	4.1152 > 2	0.00091< 0.01	✓
Harris and Marston	-0.6035 < 2	0.5551> 0.01	✗

4.1.2 Test dataset overview

In order to examine the ability of the model developed in the previous part using the training dataset to estimate the expected return, the model was tested using a test dataset from 1999-2019. Using the X_1, X_2, and X_3 values for the test dataset and the model parameters obtained from the training dataset, the growth factor was estimated for each year of the test dataset (1999-2019). The expected return was then estimated according to the model outlined in the previous section. Shows the estimated return for the test dataset (red markers). For comparison, the actual return is also plotted (blue markers). Overall, the model developed using the training dataset shows good performance in estimating market return for the test dataset. The model performs poorly in estimating the market return for the following years: 2002, 2007 and 2012.

4.1.3 Evaluation of our model using the test dataset

Makridakis et al. (2018) explain that Mean Absolute Error (MAE) is a metric widely used for evaluating models to see how they perform on test datasets. MAE is the average of absolute difference between the estimation and the observation. It provides a measure of how the estimation deviates from the actual values. Given any test dataset, MAE provides a mean of the absolute error in estimation for all instances of the test dataset. However, it does not give any information on whether the model is overestimating or underestimation the actual data. The F1 score and Area Under Curve (AUC) are some popular metrics that are typically used for model evaluation as well. Makridakis et al. (2018) define MAE as follows:

$$MAE = \frac{1}{N}\sum_{j=1}^{N} |y_j - x_j|$$

where N is the number of points in the test dataset; y and x are estimates and actual test data, respectively.

A comparison of MAEs for the training dataset, test dataset and traditional DCF model is provided in Figure 13. As is shown in the bar chart, MAE for the test dataset is about 0.09, which is greater than that of the training dataset, which is about 0.07. This is expected as the MAE for the test dataset is typically greater than that of the training dataset. However, the MAE for the test dataset is still lower than that of the traditional DCF model, which is about 0.11. This evaluation shows that our model tested for an out-of-sample dataset is still performing better than the traditional DCF model.

Table 7 - Our model's expected return and the associated error using the test dataset. S&P 500 return is also shown for comparison.

Year	SPX Return	Samin-Samer (SS) Expected Return	SS Error
2019	0.1611	0.2386	-0.0775
2018	0.2723	0.2718	0.0005
2017	-0.0530	-0.0531	0.0001
2016	0.1942	0.1364	0.0578
2015	0.0954	0.2001	-0.1047
2014	-0.0069	0.0251	-0.0321
2013	0.1239	0.1666	-0.0428
2012	0.2488	0.0829	0.1660
2011	0.1661	0.1439	0.0222
2010	0.0000	0.0243	-0.0244
2009	0.1278	-0.0490	0.1768
2008	0.1967	0.2020	-0.0053
2007	-0.3399	0.1958	-0.5357
2006	-0.0047	-0.0003	-0.0044
2005	0.1362	0.1548	-0.0186
2004	0.0300	0.1111	-0.0811
2003	0.0933	-0.0598	0.1531
2002	0.2200	0.0289	0.1911
2001	-0.2251	-0.1110	-0.1141
2000	-0.1121	-0.0704	-0.0417
1999	-0.1012	0.0168	-0.1181

β	0.21488
γ	0.24209

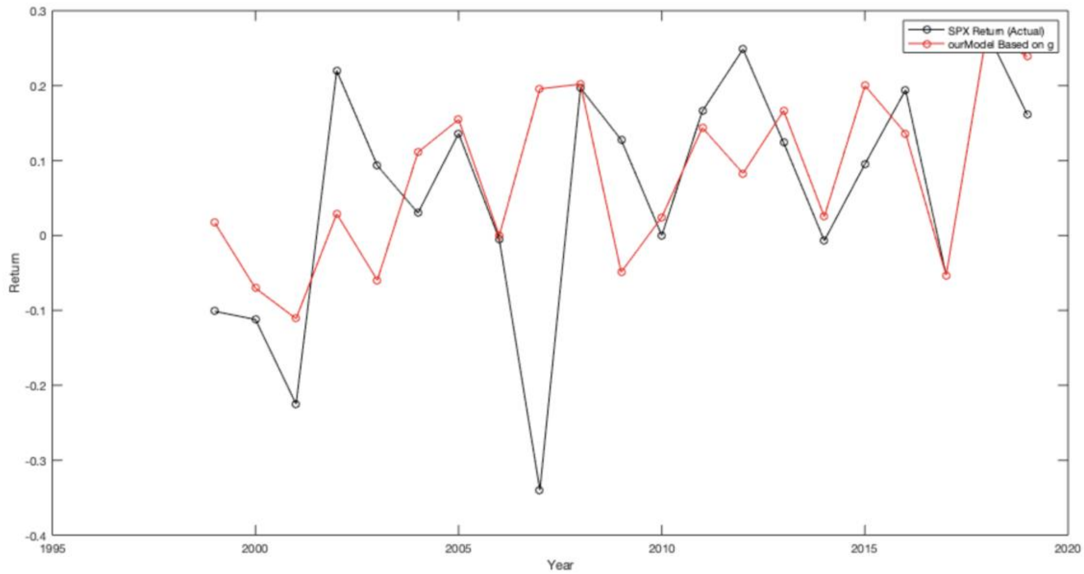

Figure 11 - Finding Expected Market Return for Following Years. This graph shows the expected market return predicted by our model when using coefficients estimated from a smaller sample size. This shows our model's ability to perform out of sample estimations.

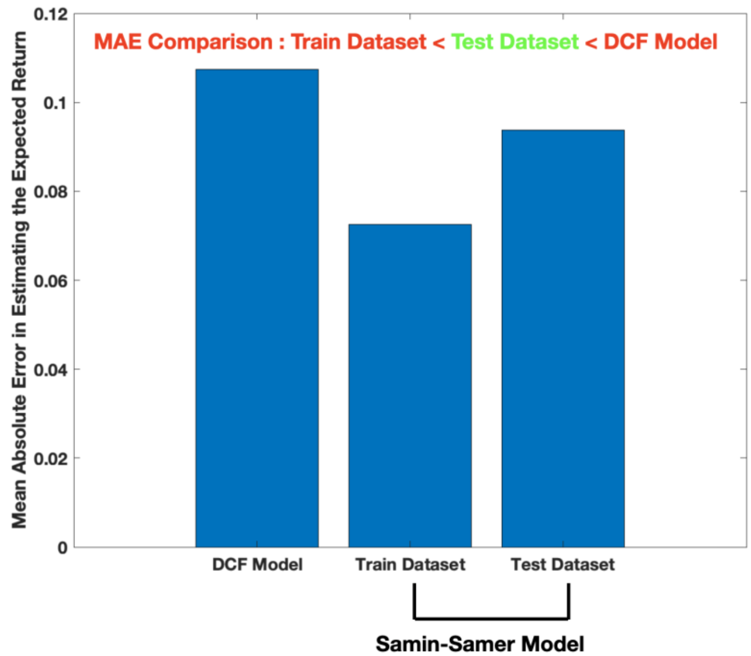

Figure 12 - Mean Absolute Error in Estimating the Expected Return for the Test Dataset. MAE of the test dataset is lower than that of the traditional DCF model.

For further evaluation of the model, distributions of the error in estimating the market return on testing datasets are plotted (Figure 14). A similar distribution for the training dataset is also shown for comparison. The mean and standard deviation for the training dataset are 0.072 and 0.092 respectively. For the test dataset, the mean and standard deviation are 0.093 and 0.097 respectively. From a statistical perspective, this shows that the model performance on the test dataset is comparable to that of the training dataset. The outlier estimation in the test dataset is highlighted using the red marker and is excluded in mean and standard deviation calculations. The overlap of these two distributions is also shown.

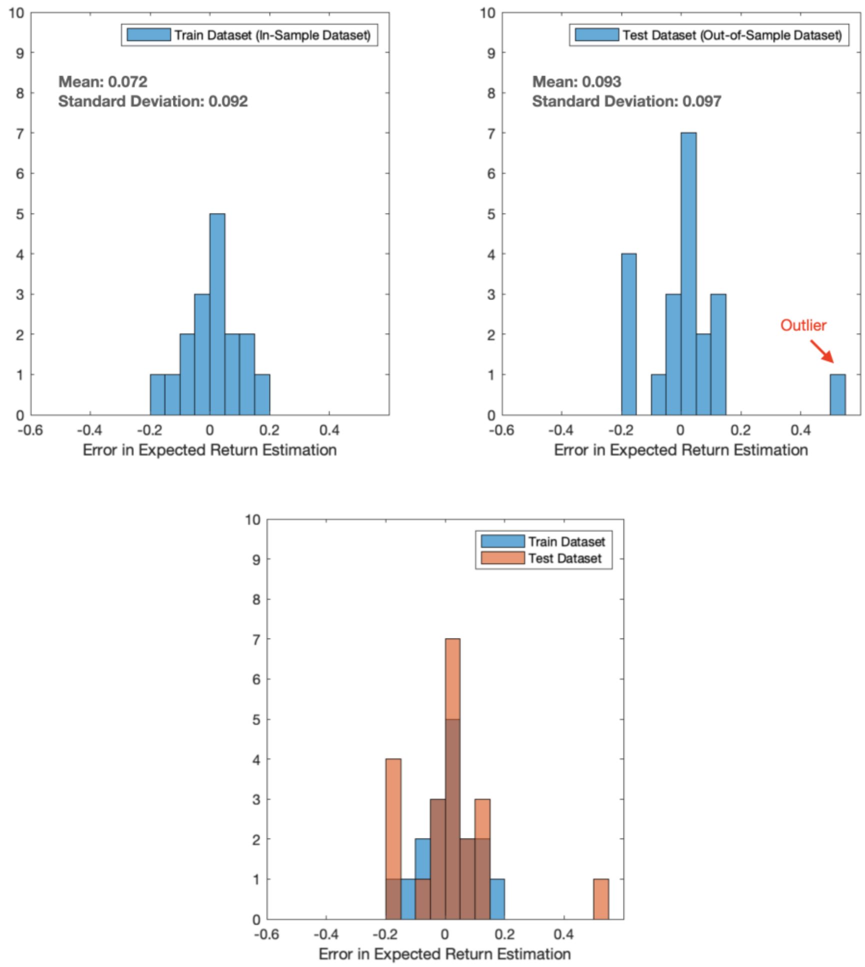

Figure 13 - Distribution of Error in Estimating the Expected Return for Training and Test Datasets. The mean and standard deviation for the training dataset are 0.072 and 0.092 respectively. For the test dataset, the mean and standard deviation are 0.093 and 0.097 respectively.

Table 8 - Regression between our model using the test dataset and actual S&P 500 returns for the period of 1999-2019.

	Estimate	**SE**	**tStat**	**pValue**
(Intercept)	0.0545	0.0233	2.338	0.0304
x1	0.2788	0.1408	1.9798	0.0623

Number of observations: 21, Error degrees of freedom: 19

Root Mean Squared Error: 0.1

R-squared: 0.171, Adjusted R-Squared: 0.127

F-statistic vs. constant model: 3.92, p-value = 0.0624

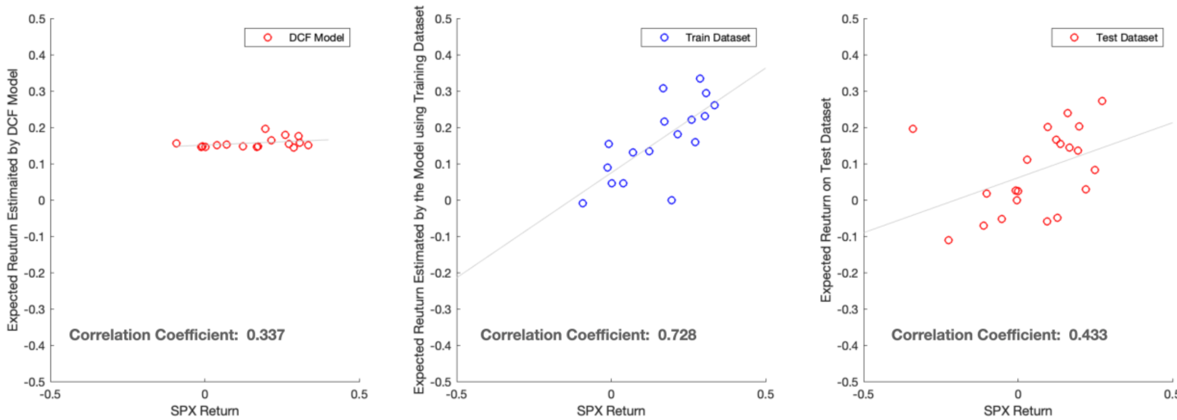

Figure 14 - Illustration of Correlation Between Estimated Return and Actual Return. The correlation coefficient of the test dataset is greater than that of the traditional DCF model. The training dataset shows the highest correlation coefficient among all, as expected.

These results show that developing the model using the training dataset still results in statistically significant estimates of annual return. Our estimates did change due to the change in coefficients, but the overall conclusions remain the same. Our model estimates were able to adjust to market trends, contained less errors in predictions, and maintained fewer residuals than the traditional DCF model.

We can see from these results that a larger sample size enables our model to perform slightly more optimally. However, an almost twenty-year sample size was still able to produce accurate and statistically significant results that did not vary much from our earlier estimates.

After performing several statistical tests on our model using two sets of samples, we determined that VWAP and Bollinger Bands can be expanded beyond their current use cases. We were able to slightly adjust the DCF model by Harris and Marston (1999) and achieve statistically significant results that captured market trends.

Chapter 5. Conclusion

The market risk premium is one of the most widely used parameters in finance. It's important for practitioners and professionals to have an accurate estimation of market risk premium. While there are several categories for estimating market risk premium, the two most widely used methods involve the use of historical averages and DCF models. Both of these methods are favoured for their simplicity and ease of application.

We set out to adjust the DCF model by Harris and Marston (1999) and improve it by implementing stronger empirical methods for estimating growth in order to eliminate risks that come with using analysts' forecasts. We did so by using a non-linear multivariate model that estimates growth using VWAP and Bollinger Bands. Traditionally, these two methods have been used in order to identify trends and locate buy/sell signals. We applied these methods to reach an assumption for the growth factor in the DCF model.

After performing several tests, we can see that estimating growth using VWAP and Bollinger Bands yields statistically significant results. Our estimations for market risk premium are able to capture market trends while eliminating the risk of analyst biases found in Harris and Marston's (1999) DCF model. Model residuals and errors in prediction were also lower than the original DCF model. This expands the use for VWAP and Bollinger Bands, while also providing a new variation of the traditional DCF model.

References

Archontoulis, S. V., & Miguez, F. E. (2015). Nonlinear regression models and applications in agricultural research. *Agronomy Journal*, 786-798.

Bollinger, J. (1992). Using Bbollinger bands. Stocks & Commodities. 47-51.

Coles, A., & Hawkins D. G. (2011). *MIDAS Technical Analysis: A VWAP Approach to Trading and Investing in Today's Markets.* Wiley.

Duarte, F., & Rosa, C. . (2015). The Equity Risk Premium: A Review of Models. *SSRN Electronic Journal*.

Guéant, O., & Royer, G. . (2014). VWAP execution and guaranteed VWAP. *SIAM Journal on Financial Mathematics*, 445-471.

Harris, R. S., & Marston, F. C. (1999). The Market Risk Premium: Expectational Estimates Using Analysts' Forecasts. *SSRN Electronic Journal*.

Harris, R., & Marston, F. (1992). Estimating Shareholder Risk Premia Using Analysts' Growth Forecasts. *Financial Management*, 63-70.

Hidalgo, B., & Goodman, M. . (2013). Multivariate or multivariable regression? *American journal of public health,*, 39–40.

Lally, M., & Marsden, A. (2003). Tax-adjusted market risk premiums in New Zealand: 1931–2002. *Pacific-Basin Finance Journal*, 291-310.

Madhavan, A. (2002). *VWAP Strategies*. Retrieved from http://www.smallake.kr/wp-content/uploads/2016/03/TP_Spring_2002_Madhavan.pdf

Makridakis, S., Spiliotis, E., & Assimakopoulos, V. (2018). Statistical and Machine Learning forecasting methods: Concerns and ways forward. *PloS one*, *13*(3), e0194889. https://doi.org/10.1371/journal.pone.0194889

Prasetijo, A. B., Saputro, T. A., Windasari, I. P., & Windarto, Y. E. (2017). Buy/sell signal detection in stock trading with bollinger bands and parabolic SAR: With web application for proofing trading strategy. *2017 4th International Conference on Information Technology, Computer, and Electrical Engineering.*

Rhinehart, R. R. (2016). *Nonlinear regression modelling for engineering applications: Modeling, model validation, and enabling design of experiments.* ProQuest Ebook Central.

Appendix A. Input Data and the Forecasted Risk Premium Yield Using Our Model

Input Data and Forecasted Risk Premium Yield using our model.

Year	Closing Price	Dividend	WVAP	Weeks Abobe BB	Weeks Below BB	Weeks Above VWAP	Weeks Below VWAP	Model Return	rf	MRP
2019	3221.29	59.01	2914.2	26	9	50	2	0.218	0.021	0.197
2018	2531.94	55.71	2732.5	23	18	31	21	0.252	0.019	0.233
2017	2673.61	51.68	2450.5	40	3	49	3	-0.055	0.009	-0.064
2016	2238.83	49.29	2079.8	22	19	49	3	0.115	0.003	0.112
2015	2043.94	47.77	2057.5	22	17	30	22	0.181	0.001	0.180
2014	2058.2	43.74	1931.9	32	10	46	6	0.017	0.000	0.017
2013	1831.37	39.1	1647.2	35	1	51	1	0.154	0.001	0.153
2012	1466.47	35.44	1379.9	20	11	44	8	0.069	0.001	0.068
2011	1257.6	30.49	1262.9	21	11	24	27	0.131	0.001	0.130
2010	1257.64	27	1136.9	29	14	29	23	0.012	0.001	0.011
2009	1115.1	27.02	932.42	22	11	43	9	-0.061	0.002	-0.063
2008	931.8	35.16	1185.7	10	26	11	41	0.212	0.014	0.198
2007	1411.63	34.38	1476	28	16	37	15	0.182	0.044	0.138
2006	1418.3	32.11	1312.5	39	10	36	16	-0.007	0.047	-0.054
2005	1248.29	29.39	1207.9	20	15	37	15	0.136	0.031	0.105
2004	1211.92	26.6	1131	22	16	29	23	0.093	0.014	0.079
2003	1108.49	24.56	965.34	37	10	42	10	-0.066	0.010	-0.077
2002	908.6	23.14	982	9	26	6	46	0.037	0.016	0.021
2001	1172.51	23.19	1186.5	15	23	10	42	-0.095	0.034	-0.129
2000	1320.5	24.35	1421.9	18	24	25	27	-0.052	0.058	-0.110
1999	1469.25	25.83	1332.4	31	14	41	11	0.006	0.046	-0.041
1998	1229.23	25.73	1084.8	29	14	41	11	-0.013	0.048	-0.060
1997	975	25.02	877.41	32	8	45	6	0.203	0.051	0.152
1996	748.03	24.46	672.75	27	7	47	5	0.212	0.050	0.162
1995	615.93	23.39	545.3	25	2	51	1	0.166	0.055	0.112
1994	461.17	22.91	460.54	18	17	19	33	0.237	0.042	0.195
1993	466.45	22.46	451.47	30	10	46	6	0.081	0.030	0.051
1992	435.71	22.73	415.82	23	13	31	21	0.116	0.034	0.082
1991	419.34	23.04	377.84	27	12	50	2	0.034	0.054	-0.019
1990	321	23.52	334.2	22	17	20	32	0.275	0.075	0.200
1989	353.4	22.83	323.81	35	3	49	3	-0.011	0.081	-0.092
1988	277.72	21.07	265.76	23	13	45	7	0.144	0.067	0.078
1987	247.09	19.88	286.16	5	11	37	15	0.145	0.058	0.088
1986	246.45	19.51	237.14	31	1	46	6	0.044	0.060	-0.016
1985	210.88	18.82	188.26	33	11	47	5	0.286	0.075	0.211
1984	163.68	18.62	160.92	20	20	23	29	0.307	0.095	0.212
1983	164.93	18.22	160.82	25	9	50	2	0.159	0.086	0.073
1982	140.64	18.33	122.27	22	22	28	24	0.188	0.106	0.082
1981	122.55	18.37	128.31	17	24	15	37	0.047	0.140	-0.093
1980	136.34	18.59	120.15	37	6	40	12	0.028	0.114	-0.086
1979	106.52	19.18	103.61	29	12	39	13	0.240	0.101	0.140

Appendix B. Input Data and the Forecasted Risk Premium Yield using Harris and Marston's Model (1999)

Input Data and Forecasted Risk Premium Yield using Harris and Marston's Model (1999).

Year	Dividend ($)	g	K	rf	rp = K - i
1982	6.89	12.73	19.62	12.76	19.62
1983	5.24	12.6	17.86	11.18	17.84
1984	5.55	12.02	17.57	12.39	17.57
1985	4.97	11.45	16.42	10.79	16.42
1986	4.08	11.05	15.13	7.8	15.13
1987	3.64	11.01	14.65	8.58	14.65
1988	4.27	11	15.27	8.96	15.27
1989	3.95	11.08	15.03	8.45	15.03
1990	4.03	11.69	15.72	8.61	15.72
1991	3.64	11.99	15.63	8.14	15.63
1992	3.35	12.13	15.47	7.67	15.48
1993	3.25	11.63	14.78	6.6	14.88
1994	3.19	11.47	14.66	7.37	14.66
1995	3.04	11.51	14.55	6.88	14.55
1996	2.6	11.89	14.49	6.7	14.49
1997	2.18	12.6	14.78	6.6	14.78
1998	1.8	12.95	14.75	5.58	14.75