CRITICAL THINKING

COURSE MANUAL

2023 Spring

This document is still being written and revised.

- If you are the first to report a **minor typo** (a spelling/grammar/numbering mistake, a formatting error, etc.), you will get a 0.5 mark bonus in the course.
- If you are the first to report a **major typo or error**, you will get a 1 mark bonus in the course.
- If you suggest a way to **make a sentence or diagram more clear** and I use your suggestion, you will get a 0.5 mark bonus in the course.

CONTENTS

INTRODUCTION

Critical thinking means different things to different people. Here it means tools and skills for thinking clearly and reasoning well. Some of these skills are analytic: understanding, representing, and using patterns and methods of reasoning. Other skills are evaluative: judging the quality of that reasoning.

This course is about how to think and understand others' thinking, not *what* to think. It will not give you a bunch of facts to memorize, and the exams will mostly not test you on facts. Facts matter! The ultimate purpose of clear thinking and good reasoning is to find, understand, and communicate truth. But our goal is to develop thinking skills you can use to get true beliefs and avoid false ones, in all areas of your life.

We'll always need some true beliefs to begin with. If we start with too many false beliefs, reasoning skills alone will not be enough. But if we care about truth and are willing to learn about the world so that we have some facts to reason with, the tools of critical thinking are very powerful. Developing them requires hard work, but it's worth the effort. These tools empower us to better understand the world around us, to recognize when we should change our beliefs, to find and criticize errors in reasoning, and to clearly organize, express, and defend our thoughts. So get ready to think!

Vocabulary: Important terms appear in **bold** when they are first introduced and defined in the text.

Examples: The examples and practice questions throughout this text use a mix of true, false, and completely made-up claims and numbers. Their only purpose is to help explain the concepts.

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ARGUMENTS

Several units in this course develop the critical thinking skills of reading, writing, and evaluating arguments, so a general overview of arguments is a good place to begin.

i. STATEMENTS

To understand what an argument is, we need the more basic concept of a *statement*.

Three Types of Sentences

Here are two types of sentences: **question** (asking for information); **command** (telling someone to do something).

Question 1.1: Why did the Maya civilization decline?

Command 1.2: Walk backwards with your eyes closed.

A third type of sentence is a **statement**. It asserts (says) that something is a fact.

Statement 1.3: There are twelve continents.

Statement 1.4: Airplane business class seating is nicer than economy seating.

Every statement has a **truth value**: either true or false. This makes statements different from questions and commands, which are neither true nor false. Compare these sentences:

It's true that *why did the Maya civilization decline*. [It's true that QUESTION.]
It's true that *walk backwards with your eyes closed*. [It's true that COMMAND.]
It's true that *there are twelve continents*. [It's true that STATEMENT.]

Only the last sentence makes sense because only the statement has a truth value (false).

ii. ARGUMENTS

We often hear that two people "had an argument" or that someone "won an argument". In ordinary language, an argument is a dispute or disagreement. However for us, an argument is not a dispute, and it's not something that anyone wins or loses.

An **argument** is a set of statements: one or more **premise** statements given as support for (reasons to believe) a **conclusion** statement. The conclusion may be anywhere in the argument. The conclusion is <u>underlined</u> in Arguments 1.5-1.6.

Argument 1.5: <u>Flying in an airplane is more dangerous than riding in a car</u> since airplanes fly 12 km up in the sky whereas cars stay on the ground.

Argument 1.6: The death penalty is very cruel and does not prevent crimes, so <u>it should be abolished</u>.

To make an **inference** is to reason from premises to a conclusion. There are many words and phrases – **inference indicators** – that help us notice an inference. Argument 1.5 says "[Conclusion] since [Premise]"; Argument 1.6 says "[Premises] so [Conclusion]".* There are many others, such as *because*, *consequently*, *therefore*, and *for that reason*. Some arguments have no inference indicator.

Argument 1.7: Social networks (such as Facebook) are where most citizens get their news now. They should be controlled by laws ensuring the accuracy of news stories.

The premise "Social networks are where most citizens…" supports the conclusion "They should be controlled…".

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* **Vocabulary**: Of course the words *so* and *since* have other meanings besides indicating inferences. E.g. *so* also means "this much" or "very"; *since* also means "from that time until now".

iii. RECONSTRUCTING ARGUMENTS

Standard Form

Writing an argument in **standard form** puts its premises above a line and its conclusion below. We can **reconstruct** an argument by reading (or hearing) it carefully and putting it in standard form.

Argument 1.8: The price of gold will go up soon, since investors are getting nervous about the market. They usually sell stocks and buy gold when they get nervous.

Argument 1.8 (standard form):

- Investors are getting nervous about the market.
- Investors usually sell stocks and buy gold when they get nervous about the market.

The price of gold will go up soon.

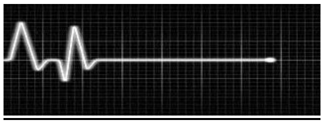
Argument 1.9 is the same argument as Argument 1.8 but includes some **extra information**. These statements may introduce or elaborate on statements in the argument. They are neither premises nor conclusions and should be left out of the standard form reconstruction.

Argument 1.9: Even though the price of gold is already nearly as high as it has ever been, it will go up soon, since investors are getting nervous about the stock market. They usually sell stocks and buy gold when they get nervous. People seem to believe that just because gold is shiny, it is a safe investment.

Implicit Statements

People often do not say everything they think when giving an argument. A statement that someone thinks of as part of their argument, but does not write or speak, is **implicit**.

Argument 1.10: We can’t save him now. He hasn’t had a heartbeat for 5 minutes.



There is a premise in Argument 1.10 that is not stated. Its exact wording and meaning are not completely clear, but the general idea is clear enough. It must be something like:

No one who hasn’t had a heartbeat for 5 minutes can be saved.
If someone hasn’t had a heartbeat for 5 minutes then they cannot be saved.
Very few people who haven’t had a heartbeat for 5 minutes can be saved.
If someone’s had no heartbeat for 5 minutes then it’s very unlikely they can be saved.

Some such statement must be part of the speaker’s meaning in Argument 1.10. It’s implicit.

Extended Arguments

An **extended** argument has a **main argument** with a **final conclusion**, and at least one **sub-argument**. The sub-argument has an **intermediate conclusion**, which is a premise in the main argument. Starting in Unit 4, to make reconstructions efficient, we’ll use statement numbers (①, ②, etc.) for premises, conclusions, and extra information.

Argument 1.11: ① We should buy gold. ② It’s a good investment, since ③ everything shiny is a good investment, and ④ gold is very shiny. And ⑤ we should buy it if it’s a good investment.

Main Argument	Sub-argument
②	③
⑤	④
———	———
①	②

iv. EVALUATING ARGUMENTS

To evaluate an argument is to judge its value, to say whether it is good or bad. We ask two questions: 1) Are the premises true? 2) Do the premises support the conclusion? With a good argument, we can answer *Yes* to both questions. We'll do this a bit in Units 2-3 and more in Units 8-10.

Are the premises true?

Argument 1.12: All paintings are by Picasso, and *Guernica* (1937) is a painting. Hence *Guernica* is by Picasso.

(Image: Wikimedia)

The premises of Argument 1.12 support its conclusion. But its first premise is not true: not all paintings are by Picasso. So it is a bad argument (even though the conclusion is true).

Do the premises support the conclusion?

Premises **support the conclusion** when: *If* the premises are true, they are a reason to believe the conclusion.

Argument 1.13: All Winter Olympics have skiing events, and the 2018 Olympics in PyeongChang, Korea, had skiing events. Therefore the 2018 Olympics was a Winter Olympics.

The premises of Argument 1.13 are true. But its premises do not support its conclusion. So it's a bad argument (even though the conclusion is true).

Perhaps you aren't sure that Argument 1.13 is bad in this way. Or perhaps you are sure but you're not sure how to show or explain it. That's fine. Unit 2 covers this kind of argument. There we'll learn some tools for analyzing and evaluating arguments like this one.

Do we know any relevant background information?

We’ll call an argument good if it has true premises that support its conclusion. But knowing that an argument is good does not automatically make it rational to believe its conclusion. This is because *support* does not always mean *guarantee*. In some cases it does. We’ll look at arguments like that (deductive) in Units 2-3. In other cases it does not. We’ll look at those other types of arguments in Units 4 and 8-10. With these arguments, it can be very important to consider **background information** in addition to the premises.

UNIT 1 SKILLS

You must be able to:

- Recognize and write examples of questions, commands, and statements.
- Identify inference indicators, conclusions, and premises in an argument.
- Reconstruct an argument in standard form.

QUICK TEST QUESTIONS

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1) The conclusion of an argument…

a) …briefly summarizes the premises.
b) …is supported by one or more premises.
c) …supports one or more premises.
d) …comes at the end of an argument.

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2) How is an argument written in standard form?

a) It uses a standard inference indicator such as *because*.
b) It's written in a form that meets a very high standard.
c) It's written as a paragraph with the conclusion stated clearly.
d) The premises are written above a line with the conclusion below it.

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3) An implicit statement is…

a) …part of the argument and is written.
b) …not part of the argument but is written.
c) …not part of the argument and is not written.
d) …part of the argument but is not written.

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4) An extended argument always contains…

a) …an intermediate conclusion and a final conclusion.
b) …an intermediate conclusion and a final premise.
c) …an implicit premise and an intermediate conclusion.
d) …an intermediate premise and a final premise.

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5) To evaluate an argument is to judge whether…

a) …its conclusion is true or false.
b) …the argument is true or false.
c) …the argument is good or bad.
d) …its conclusion is good or bad.

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6) Which things are true or false?

a) Premises and arguments; not conclusions.
b) Premises and conclusions; not arguments.
c) Premises, conclusions, and arguments.
d) Arguments and conclusions; not premises.

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7) What are the features of a good argument?

a) The premises are true; the conclusion is true.
b) The argument is true; the premises support the conclusion.
c) The premises are true; the premises support the conclusion.
d) The conclusion is true; the conclusion supports the premises.

PRACTICE QUESTIONS

1) Is the sentence a question, a command, or a statement?

a) Every person who's gone to the Moon was a woman.
b) Please fly me to the Moon.
c) We should send humans to Mars instead of the Moon.
d) Could humans live on Mars without getting bored?
e) Why does the Moon have craters on it?
f) Humans first landed on the Moon in 1785.
g) Stop spending money on stupid things like sending humans to the Moon.
h) The volcano near our city might explode this year.
i) You are allowed to use your phone during the exam.
j) Why does our instructor always have a headache?
k) Quickly hand me the screwdriver that fits these screws.
l) The deepest part of the ocean is in English Bay, Vancouver.
m) Just get rid of that gross leftover take-out food in the refrigerator.
n) How does a representative democracy like Canada work?
o) The end of the Bronze Age may have been caused by climate change.

2) Identify the conclusion and inference indicators if there are any.

a) College courses could be taught by robots instead of humans. Robots can click PowerPoint slides and mark multiple choice exams just as well as humans can.
b) Since cheetahs are beautiful wild animals that belong in their home in Africa, it would be good to have better enforcement of the laws that prohibit capturing them for pets.
c) It would be better for people to use transit instead of driving. Just consider the fact that, with millions of cars on the road, no one can get where they're going anyway.
d) Obviously K2 is a more challenging climb than Mount Everest. Serious mountain climbers admire people who climb K2 more than they admire people who climb Mount Everest.
e) The Parthenon Marbles in the British Museum were taken from Greece in the 19th century, and now Greece wants them back, so Britain has no right to keep them.
f) The IUD (Intrauterine Device) is the most reliable form of contraception [birth control]. As well as having a low failure rate, IUDs don't depend on women taking a pill every day.
g) In our society there are millions of "soft" (unprotected) targets for terrorists who are willing to die in an attack; consequently we cannot stop terrorism without restricting people's freedoms.
h) In 2009, the cable news channels reported a crazy, false story about a little boy who floated away in a homemade balloon. The people at the cable news channels are idiots.

i) Earth is only about 10,000 years old. The Bible says so, and the Bible was written by the creator of the universe.

j) If the new guy is working today, some customer will get their order screwed up. And a customer did get their order screwed up. So the new guy is working today.

k) Taxes are really just a kind of theft: the government robs people of their money and calls it a tax so that it doesn't sound so bad. That's why taxation is wrong.

l) Sending humans to explore Mars would be far more expensive than sending robots. Unlike robots, humans need protection from the cold vacuum [lack of air] and dangerous radiation of outer space, and a spaceship that can do that would cost a lot.

m) Buddy's Pizza and Bobby's Pizza are both cheap, greasy pizza restaurants, and Buddy's pizzas are gross. Therefore Bobby's pizzas will be gross, too.

n) Sending humans to explore other planets would be very expensive, and most of that money would be spent on machines and equipment whose only purpose is to keep the humans alive. So we should send robots to explore outer space instead, as they can take photographs and do science experiments without the air, food, and warmth that humans need.

o) Since the American government is evil and wanted an excuse to invade Iraq for its oil, probably it planned the 2001 World Trade Center attack and made it look as though terrorists did it.

p) Canada's economy will be affected by a drop in oil prices due to the fact that Canada exports oil from the tar sands of Alberta, and it is so expensive to extract oil from tar sands that it is profitable only if the price of oil is high.

q) Even though they come to the surface for air to breathe, whales have fins and tails for swimming and live their whole lives in the ocean, and anything that swims and lives in the ocean is a fish. It follows that whales are fish.

r) Those herbs that my friend gave me must have cured my arthritis [joint pain]. The proof is that my arthritis went away shortly after I started eating them every day.

ANSWER KEY

QUICK TEST

1) b **2)** d **3)** d **4)** a **5)** c **6)** b **7)** c

1)

a) Statement
b) Command
c) Statement
d) Question
e) Question
f) Statement
g) Command
h) Statement
i) Statement
j) Question
k) Command
l) Statement
m) Command
n) Question
o) Statement

2) Conclusion is underlined; inference indicator is **bold**.

a) College courses could be taught by robots instead of humans. Robots can click PowerPoint slides and mark multiple choice exams just as well as humans can.

b) **Since** cheetahs are beautiful wild animals that belong in their home in Africa, it would be good to have better enforcement of the laws that prohibit capturing them for pets.

c) It would be better for people to use transit instead of driving. **Just consider the fact that**, with millions of cars on the roads, no one can get where they're going anyway.

d) **Obviously** K2 is a more challenging climb than Mount Everest. Serious mountain climbers admire people who climb K2 more than they admire people who climb Mount Everest.

e) The Parthenon Marbles in the British Museum were taken from Greece in the 19th century, and now Greece wants them back, **so** Britain has no right to keep them.

f) The IUD (Intrauterine Device) is the most reliable form of contraception [birth control]. As well as having a low failure rate, IUDs don't depend on women taking a pill every day.

g) In our society there are millions of "soft" (unprotected) targets for terrorists who are willing to die in an attack; **consequently** we cannot stop terrorism without restricting people's freedoms.

h) In 2009 the cable news channels reported a crazy false story about a little boy who floated away in a homemade balloon. The people at the cable news channels are idiots.

i) Earth is only about 10,000 years old. The Bible says so, and the Bible was written by the creator of the universe.

j) If the new guy is working today, some customer will get their order screwed up. And a customer did get their order screwed up. **So** the new guy is working today.

k) Taxes are really just a kind of theft: the government robs people of their money and calls it a *tax* so that it doesn't sound so bad. **That's why** taxation is wrong.

l) Sending humans to explore Mars would be far more expensive than sending robots. Unlike robots, humans need protection from the cold vacuum [lack of air] and dangerous radiation of outer space, and a spaceship that can do that would cost a lot.

m) Buddy's Pizza and Bobby's Pizza are both cheap, greasy pizza restaurants. And Buddy's pizzas are gross. **Therefore** Bobby's pizzas will be gross, too.

n) Sending humans to explore other planets would be very expensive, and most of that money would be spent on machines and equipment whose only purpose is to keep the humans alive. **So** we should send robots to explore outer space instead, **as** they can take photographs and do science experiments without the air, food, and warmth that humans need.

o) **Since** the American government is evil and wanted an excuse to invade Iraq for its oil, probably it planned the 2001 World Trade Center attack and made it look as though terrorists did it.

p) Canada's economy will be affected by a drop in oil prices **due to the fact that** Canada exports oil from the tar sands of Alberta, and it is so expensive to extract oil from tar sands that it is profitable only if the price of oil is high.

q) Even though they come to the surface for air to breathe, whales have fins and tails for swimming, and they live their whole lives in the ocean, and anything that swims and lives in the ocean is a fish. **It follows that** whales are fish.

r) Those herbs that my friend gave me must have cured my arthritis [joint pain]. **The proof is that** my arthritis went away shortly after I started eating them every day.

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CATEGORY LOGIC

i. UNIVERSAL GENERALIZATIONS

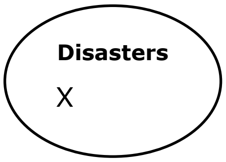
Categories and Individuals

A **category** is a type of thing. We'll picture it as a container, a circle. To show that an individual thing is a member of the category (an item of that type), we'll show the individual inside the circle. We'll label categories with ***A***, ***B***, etc. and an individual thing with X. With these symbols we'll show the **statement pattern**. On the right we'll show a category diagram.[*]

Statement 2.1: The Halifax Explosion (1917) was a disaster.

Pattern: X is an ***A***.

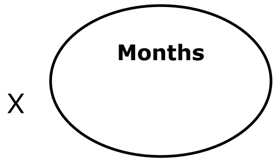
A: Disasters
X: The Halifax Explosion



Statement 2.2: Wednesday is not a month.
(Wednesday is a Non-month.)

Pattern: X is not an ***A***.
(X is a Non-***A***.)

A: Months
X: Wednesday



The X means "something exists here". A category may or may not have any members. Disasters and Months have many members not shown in the diagrams. And some categories are empty. There is no X we could truthfully draw in their circle. For example, the category Canadian $3 Bills has no members.

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[*] The diagrams are called Euler diagrams. This and the following sections present a simplified version of the category logic taught in a formal logic course. Several things are omitted or simplified to help us focus on just a few important skills.

Universal Generalizations

A **universal generalization** relates two categories; *universal* means “not even one exception”. We’ll use the **universal quantifiers** *All*, Only, and *No*, plus alternatives such as *Every* and *None* that function logically as *All* and *No*.

Universal Generalization

[Universal Quantifier] ***A*** are ***B***.

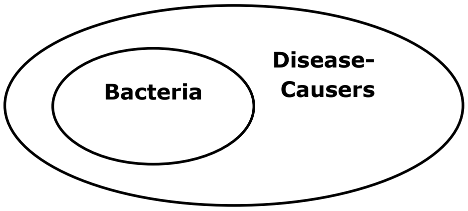
In this statement pattern, ***A*** is the **subject category** and ***B*** is the **predicate category**. The categories are written as nouns (things). Here are a few examples.

Statement 2.3: All bacteria cause disease.

Pattern: All ***A*** are ***B.***

A: Bacteria

B: Things that Cause Disease / Disease-Causers

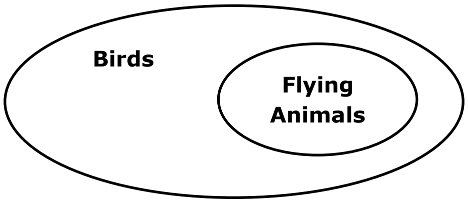


Statement 2.4: Only birds fly.

Pattern: Only ***A*** are ***B***.

A: Birds

B: Flying Animals

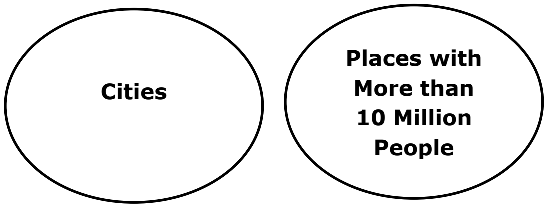


Statement 2.5: No cities have more than 10 million people.

Pattern: No ***A*** are ***B***.

A: Cities

B: Places with More than 10 Million People



Statement 2.3 shows that often there is more than one way to write a category label. Statement 2.4 shows that we must use common sense in thinking of the appropriate category. Common sense tells us that Statement 2.4 is probably not referring to Flying Things or Fliers generally. Probably it’s discussing animals, so its predicate category is Flying Animals.

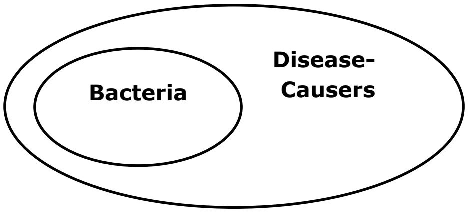
Logical Equivalence

When two categorical statements are **logically equivalent**, they have exactly the same categories diagram. Since they give the same "picture" of the world, it's impossible for one to be true and the other false. Either they are both true or they are both false.

An *All* statement can be converted into a logically equivalent *Only* statement, with a single diagram.

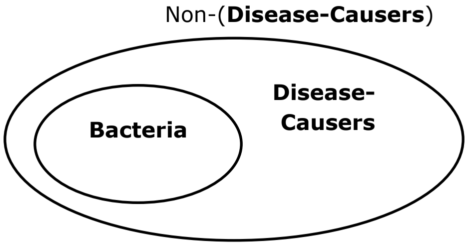
Statement 2.3: All Bacteria are Disease-Causers.
Only Disease-Causers are Bacteria.
Pattern: All ***A*** are ***B***.
Only ***B*** are ***A***.

A: Bacteria
B: Disease-Causers



An *All*… statement can also be converted into a logically equivalent *No*… statement. The *No* quantifier applies to regions that are separated from each other. This diagram shows: Bacteria is separated from Non-(Disease-Causers).

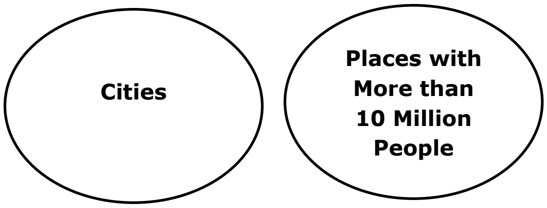
Statement 2.3: All Bacteria are Disease-Causers.
No Bacteria are Non-(Disease-Causers).
Pattern: All ***A*** are ***B***.
No ***A*** are Non-***B***.



Since we can convert *All*… to *No*…, we can also convert *No*… to *All*….

Statement 2.5: No Cities are Places with More than 10 million People.
All Cities are Non-(Places with More than 10 million People).
Pattern: No ***A*** are ***B***.
All ***A*** are Non-***B***.

A: Cities
B: Places with More than 10 Million People

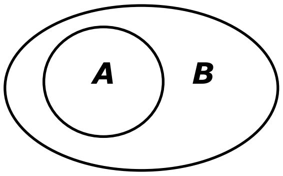


Logically Equivalent Universal Generalizations

All ***A*** are ***B*.**

Only ***B*** are ***A***.

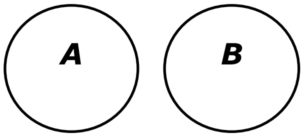
No ***A*** are Non-***B***.



No ***A*** are ***B***.

All ***A*** are Non-***B***.

Only Non-***B*** are ***A***.



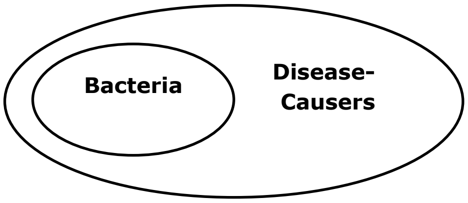
An *All* statement can also be *combined* with a different (non-logically equivalent) *Only* statement. The combined statements have a combined diagram.

Statement 2.3: All bacteria cause disease.

Pattern: All ***A*** are ***B***.

A: Bacteria

B: Disease-Causers



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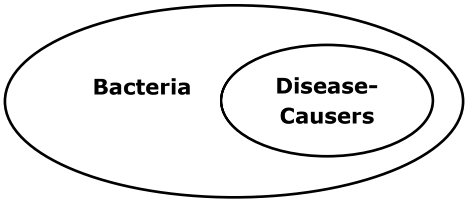
Statement 2.6: Only bacteria cause disease.

Pattern: Only ***A*** are ***B***.



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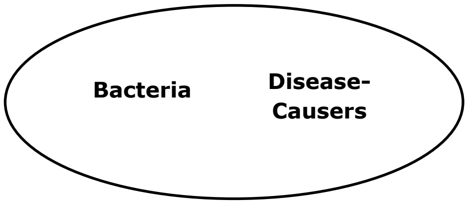
Statement 2.7: All and only bacteria cause disease.

Pattern: All and only ***A*** are ***B***.



All and only… says that the two categories perfectly overlap. They contain exactly the same things.

ii. DEDUCTIVE ARGUMENTS

Logic is the study and use of a family of arguments called deductive arguments. **Deductive arguments** are supposed to be valid. The premises of a **valid** argument support the conclusion perfectly: they absolutely guarantee it.[*]

A valid argument is truth-preserving: If we put truth "in" (the premises are true), we get truth "out" (the conclusion is true). This special quality means that it's impossible for a valid argument to have true premises and a false conclusion.

Since different statements can have the same pattern, different arguments can have the same pattern. Here's an example valid **argument pattern** and two arguments that both have this same pattern.

All ***A*** are ***B***. X is an ***A***. Therefore X is a ***B***.

Argument 2.8 (VALID): All disasters are earthquakes. The Halifax Explosion was a disaster. Therefore the Halifax Explosion was an earthquake.

A: Disasters
B: Earthquakes
X: The Halifax Explosion

Argument 2.9 (VALID): All bacteria cause disease. SARS-CoV-2 is a bacterium. Therefore SARS-CoV-2 causes disease.

A: Bacteria
B: Disease-Causers
X: SARS-CoV-2

Arguments 2.8 and 2.9 have different statements but the same pattern. Since that argument pattern is valid, both arguments are valid. (In the next section, we'll see why the pattern is valid.) These arguments contain several false statements. Validity does not mean that the premises or the conclusion are true! It means: if the premises are true, the conclusion definitely is true.

Short deductive arguments like Arguments 2.8 and 2.9 are often called **syllogisms**. In the next section we'll look at some different syllogisms that have a universal generalization premise.

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[*] **Vocabulary**: In ordinary language, *valid* has other meanings: true, relevant, important, etc. A scientific test is valid if it measures what it's supposed to measure. Deductive validity is different from all of these.

iii. CATEGORICAL SYLLOGISMS

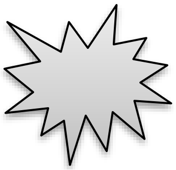
Argument Patterns with *All*

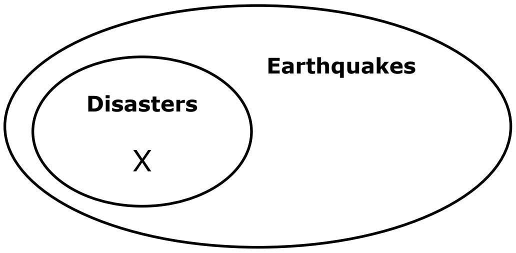
Here is a valid pattern and a similar but invalid pattern (**logical fallacy**), with examples below.

• All ***A*** are ***B***. • X is an ***A***. ——— X is a ***B***.	• All ***A*** are ***B***. • X is a ***B***. ——— X is an ***A***.
VALID	INVALID

Argument 2.10 (VALID): All disasters are earthquakes. The Halifax Explosion was a disaster. Therefore the Halifax Explosion was an earthquake.

A: Disasters
B: Earthquakes
X: The Halifax Explosion

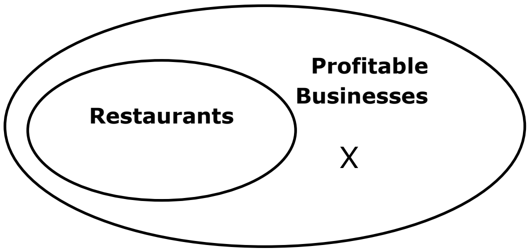




We can see in the diagram why Argument 2.10 is valid: If the premises are true (Halifax Explosion (X) inside Disasters), the conclusion <u>must</u> be true (X inside Earthquakes).

Argument 2.11 (INVALID): Every restaurant is profitable. McDonald's is profitable. Therefore McDonald's is a restaurant.

A: Restaurants
B: Profitable Businesses
X: McDonald's



We can see in the diagram why Argument 2.11 is invalid: Even if the premises are true (McDonald's (X) inside Profitable Businesses), the conclusion <u>could</u> be false (X outside Restaurants).

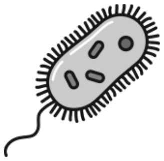
More Argument Patterns with *All*

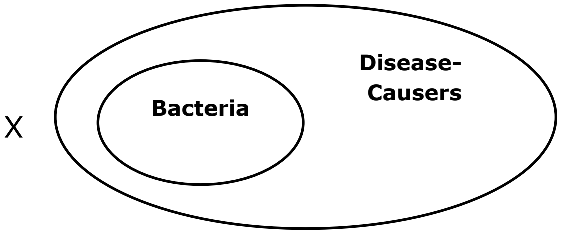
Here is another valid pattern and a similar but invalid pattern, with examples below.

• All ***A*** are ***B***. • X is not an ***A***. X is not a ***B***.	• All ***A*** are ***B***. • X is not a ***B***. X is not an ***A***.
INVALID	**VALID**

Argument 2.12 (VALID): All bacteria cause disease. *Yersinia pestis* does not cause disease. Therefore *Yersinia pestis* is not a bacterium.

A: Bacteria
B: Disease-Causers
X: *Yersinia pestis*

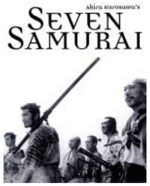




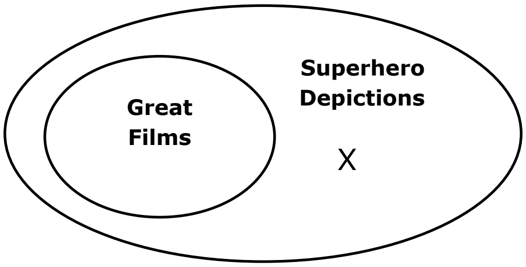
We can see in the diagram why Argument 2.12 is valid: If the premises are true (*Yersinia pestis* (X) outside Disease-Causers), the conclusion must be true (X outside Bacteria).

Argument 2.13 (INVALID): All great films depict superheroes. *Seven Samurai* is not a great film. Therefore *Seven Samurai* does not depict superheroes.

A: Great Films
B: Superhero Depictions
X: *Seven Samurai*



(Image: Criterion Collection)



We can see in the diagram why Argument 2.13 is invalid: Even if the premises are true (*Seven Samurai* (X) outside Great Films), the conclusion could be false (X inside Superhero Depictions).

Argument Patterns with *No*

Here are two valid patterns and two similar but invalid patterns, with examples below.

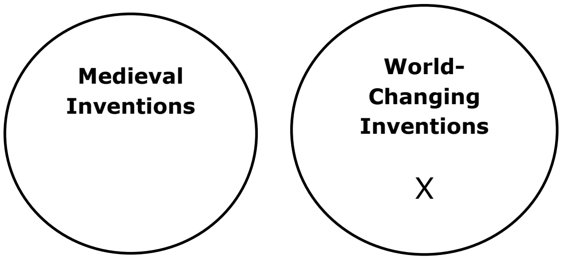
• No ***A*** are ***B***. • X is an ***A***. X is not a ***B***.	• No ***A*** are ***B***. • X is a ***B***. X is not an ***A***.	• No ***A*** are ***B***. • X is not an ***A***. X is a ***B***.	• No ***A*** are ***B***. • X is not a ***B***. X is an ***A***.
VALID	**VALID**	INVALID	INVALID

Argument 2.14 (VALID): No medieval inventions changed the world. The mechanical clock changed the world. Therefore the mechanical clock wasn't a medieval invention.

A: Medieval Inventions

B: World-Changing Inventions

X: The mechanical clock



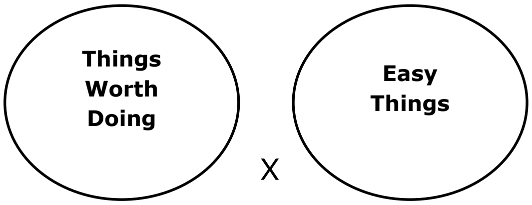
We can see in the diagram why Argument 2.14 is valid: If the premises are true (the mechanical clock (X) inside World-Changing Inventions), the conclusion <u>must</u> be true (X outside Medieval Inventions).

Argument 2.15 (INVALID): Nothing worth doing is easy. Juggling chainsaws is not easy. Therefore juggling chainsaws is worth doing.

A: Things Worth Doing

B: Easy Things

X: Juggling chainsaws



We can see in the diagram why Argument 2.15 is invalid: If the premises are true (juggling chainsaws (X) outside Easy Things), the conclusion <u>could</u> be false (X outside Things Worth Doing).

iv. COUNTEREXAMPLES

A universal generalization is false if there is even one exception to it. (A true universal generalization has no exceptions.) An exception to a generalization is called a **counterexample**.

A ***B***	***A*** Non-***B***
Non-***A*** ***B***	Non-***A*** Non-***B***

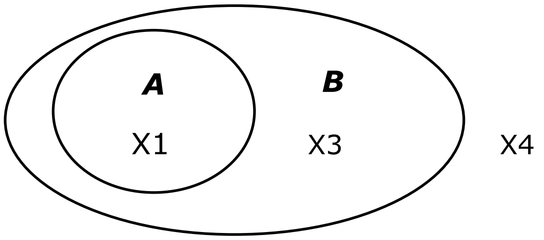
With two categories, anything in the world has one of four possible descriptions.

But the categories diagram for a universal generalization has only three regions. A counterexample is something that doesn't fit into that diagram. It has the one description that has no corresponding region in the diagram. If this thing exists, the generalization is false (the diagram is wrong).

Counterexample to:

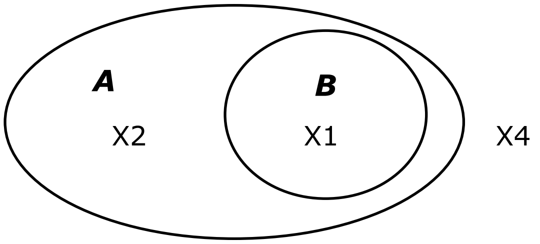
All ***A*** are ***B***.

A ***B*** X1	***A*** Non-***B*** X2
Non-***A*** ***B*** X3	Non-***A*** Non-***B*** X4



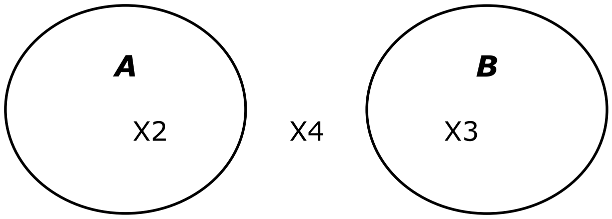
Only **A** are **B**.

A ***B*** X1	***A*** Non-***B*** X2
Non-***A*** ***B*** X3	Non-***A*** Non-***B*** X4



No **A** are **B**.

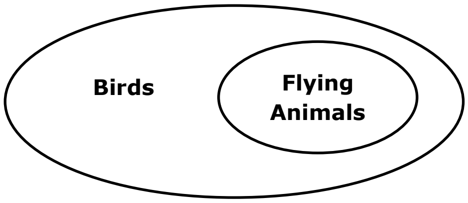
A ***B*** X1	***A*** Non-***B*** X2
Non-***A*** ***B*** X3	Non-***A*** Non-***B*** X4



Case #1: "Only birds fly."

Statement 2.4: Only Birds are Flying Animals.
Pattern: Only ***A*** are ***B***.

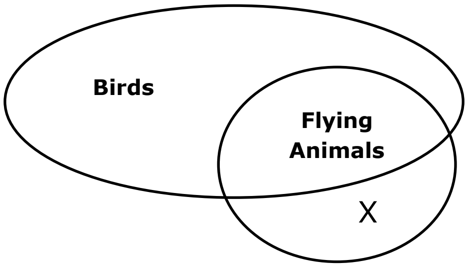
A: Birds
B: Flying Animals



Statement 2.4 is false. A counterexample is a bat (a Flying Animal but not a Bird). A bat (X) doesn't fit into the statement's diagram. It must be corrected. We can write this with *Not only* or with the *Some* quantifier.

Bird **Flying Animal**	**Bird** Non-(**Flying Animal**)
Non-**Bird** **Flying Animal** X: 	Non-**Bird** Non-(**Flying Animal**)

Correction Diagram

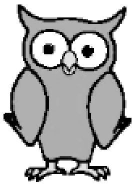


Counterexample (X): a bat
Counterexample Description: Non-Bird [Non-***A***] Flying Animal [***B***]

A bat shows that:

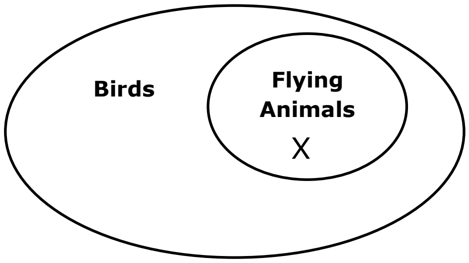
Statement 2.16: Not only Birds are Flying Animals.
Some Flying Animals are Non-Birds.
Pattern: Not only ***A*** are ***B***.
Some ***B*** are Non-***A***.

Before looking at Case #2, let’s see how nothing with any of the other three possible descriptions is a counterexample to “Only birds fly.”



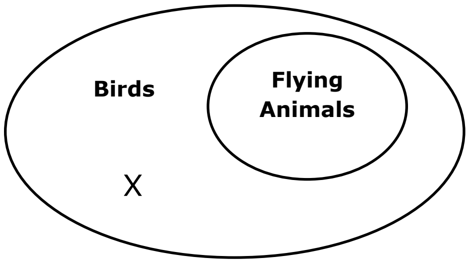
What about an owl? No, an owl has the wrong description: a Flying Animal but also a Bird. It fits into the diagram.

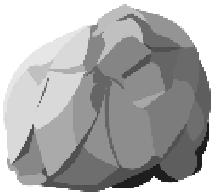
Bird **Flying Animal**  X:	**Bird** Non-(**Flying Animal**)
Non-**Bird** **Flying Animal**	Non-**Bird** Non-(**Flying Animal**)



What about a penguin? No, a penguin has the wrong description: a Bird but not a Flying Animal. It fits into the diagram.

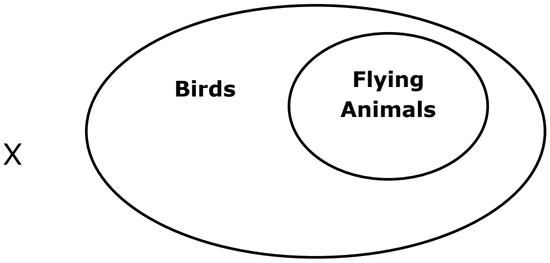
Bird **Flying Animal**	**Bird** Non-(**Flying Animal**)  X:
Non-**Bird** **Flying Animal**	Non-**Bird** Non-(**Flying Animal**)



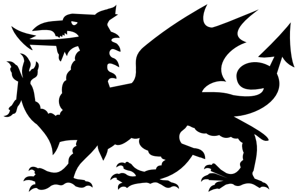


What about a rock? No, a rock has the wrong description: not a Bird and also not a Flying Animal. It fits into the diagram.

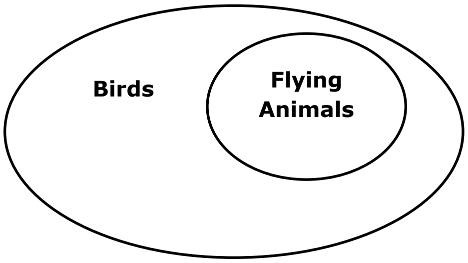
Bird **Flying Animal**	**Bird** Non-(**Flying Animal**)
Non-**Bird** **Flying Animal**	Non-**Bird** Non-(**Flying Animal**)  X:



What about a dragon, a giant, flying lizard? A dragon has the right description: a Flying Animal but not a Bird. It would be a counterexample because it would not fit into the diagram. But it doesn't exist (it's mythical). A dragon doesn't make Statement 2.4 false.



Bird **Flying Animal**	**Bird** Non-(**Flying Animal**)
Non-**Bird** **Flying Animal**  X:	Non-**Bird** Non-(**Flying Animal**)



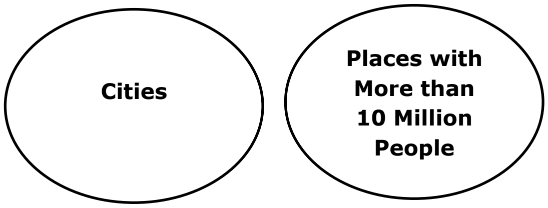
Case #2: "No cities have more than 10 million people."

Statement 2.5: No cities have more than 10 million people.

Pattern: No ***A*** are ***B***.

A: Cities

B: Places with More than 10 Million People

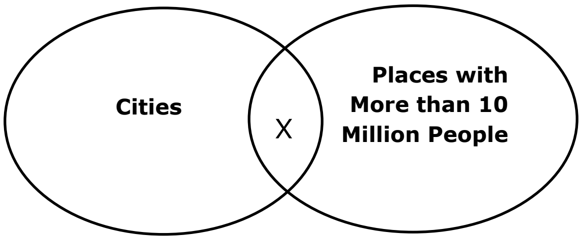


(Image: Freepik.com)

Statement 2.5 is false. A counterexample is Istanbul (a city with more than 10 million people). Istanbul (X) doesn't fit into the statement's diagram. It must be corrected.

City **Place with More…** X: 	**City** Non-(**Place with More…**)
Non-**City** **Place with More…**	Non-**City** Non-(**Place with More…**)

Correction Diagram



Counterexample (X): Istanbul

Counterexample Description: City [***A***] Place with More Than 10 Million People [***B***]

Istanbul shows that:

Statement 2.17: Not no Cities are Places with More Than 10 Million People.[*]
Some Cities are Places with More Than 10 Million People.

Pattern: Not no ***A*** are ***B***.
Some ***A*** are ***B***.

---

[*] This sentence ("Not no…") is not standard accepted English! We're writing it here to make the example clear and complete.

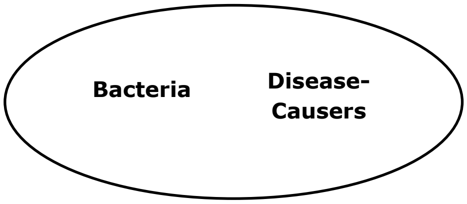
Case #3: "All and only bacteria cause disease."

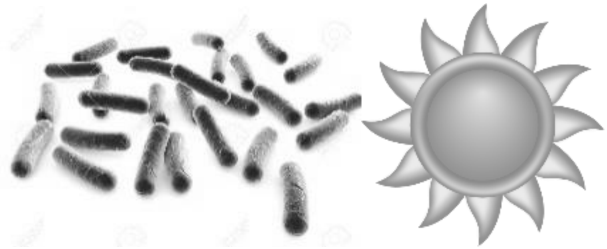
Statement 2.7: All and only bacteria cause disease.

Pattern: All and only ***A*** are ***B***.

A: Bacteria

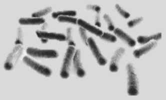
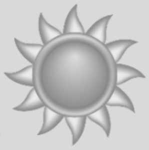
B: Disease-Causers



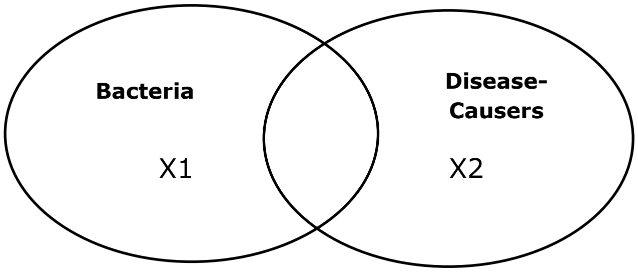


(Image: drmicrobe)

Statement 2.7 is doubly false. There are counterexamples of two types: 1) soil bacteria (bacteria that don't cause disease), and 2) UV radiation from the Sun (non-bacteria that causes disease). Soil bacteria (X1) and UV radiation (X2) don't fit into the statement's diagram. It must be corrected.

Bacteria **Disease-Causer**	**Bacteria** Non-(**Disease-Causer**) X1: 
Non-**Bacteria** **Disease-Causer** X2: 	Non-**Bacteria** Non-(**Disease-Causer**)

Correction Diagram



Counterexample (X1): Soil bacteria

Counterexample Description (X1): Bacteria [***A***] Non-(Disease-Causer) [Non-***B***]

Counterexample (X2): UV radiation

Counterexample Description (X2): Non-Bacteria [Non-***A***] Disease-Causer [***B***]

Soil bacteria and UV radiation show that:

Statement 2.18: Not all and not only bacteria cause disease.

Disagreeing About Universal Generalizations and Counterexamples

Good reasoners may disagree by constructing valid arguments with different premises.

In the disagreement pattern below, Abby and Bob agree that X is not a ***B***. Since Abby is most sure that all ***A*** are ***B*** (her Premise 2), she concludes that X is not an ***A***. Since Bob is most sure that X is ***A*** (his Premise 2), he concludes that not all ***A*** are ***B***. Bob thinks X is a counterexample; Abby disagrees.

Abby and Bob (agreeing)

- [Premise 1] X is not a ***B.***

Abby (disagreeing)	**Bob** (disagreeing)
• [Premise 2] All ***A*** are ***B***.	• [Premise 2] X is an ***A***.
[Conclusion] X is not an ***A***.	[Conclusion] Not all ***A*** are ***B***.

Here's a serious example of this type of disagreement involving a controversial concept: terrorism. In 2017 the worst American mass shooting to date happened in Las Vegas. As far as anyone knows, the gunman had no political or religious motive. A disagreement may begin with agreement on that fact.

Abby and Bob

- The Las Vegas mass shooting was not violence with a political or religious motive.

Abby	**Bob**
• All terrorism is violence with a political or religious motive.	• The Las Vegas shooting was terrorism.
The Las Vegas shooting was not terrorism!	Not all terrorism is violence with a political or religious motive!

A: Terrorism
B: Violence with a Political or Religious Motive
X: The Las Vegas mass shooting

UNIT 2 SKILLS

<u>You must be able to</u>:

- Write the two categories in a universal generalization and diagram it.
- Convert a universal generalization into logically equivalent forms using other quantifiers.
- Construct two syllogisms (valid deductive arguments) from a universal generalization premise.
- Use a categories diagram to prove the invalidity of an argument.
- Name and describe a counterexample.
- Write a "Some…" statement for a false universal generalization and draw a new categories diagram showing the counterexample(s).
- Construct a disagreement (valid argument against valid argument) about a universal generalization and a counterexample.

QUICK TEST QUESTIONS

---

1) Which is not a universal quantifier?

a) No
b) All
c) Only
d) Some

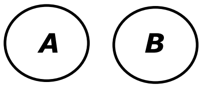
---

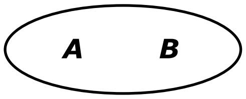
2) Which statement is logically equivalent to "Only ***A*** are ***B***"?

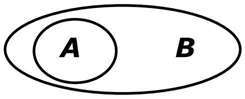
a) Only ***B*** are ***A****.*
b) No ***B*** are ***A***.
c) All ***A*** are ***B***.
d) All ***B*** are ***A***.

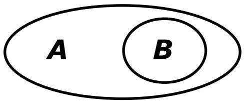
---

3) Which diagram shows that all and only ***A*** are ***B***?

a) 

b) 

c) 

d) 

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4) Which is not logically equivalent to "All ***A*** are ***B***"?

a) Only ***A*** are ***B***.
b) Only ***B*** are ***A***.
c) No ***A*** are Non-***B***.
d) No Non-***B*** are ***A***.

---

5) Describe a counterexample to "Only ***A*** are ***B***".

a) ***A*** ***B***
b) Non-***A*** Non-***B***
c) Non-***A*** ***B***
d) ***A*** Non-***B***

---

6) A true universal generalization has how many counterexamples?

a) Not enough info.
b) ≥1
c) 1
d) 0

---

7) Abby thinks that X is a counterexample to "All ***A*** are ***B***". Bob disagrees. What could Bob say to disagree with Abby?

a) "You're wrong, X is a ***B***."
b) "You're wrong, X is not a ***B***."
c) "You're wrong, X is an ***A***."
d) None of these.

---

8) Which is impossible with a valid argument?

a) True premises; false conclusion.
b) False premises; true conclusion.
c) False premises; false conclusion.
d) True premises; true conclusion.

PRACTICE EXAMS

PRACTICE EXAM 1

1) Complete two valid syllogisms with different argument patterns. For each argument, use the given universal generalization as Premise 1 and complete Premise 2 and the Conclusion using the given subject. Below the arguments, draw a single diagram showing the premises of both arguments true (two circles, X1 and X2).

- [Premise 1] Every legitimate election offers voters mail-in ballots.
- [Premise 2] Azmakia's election (X1)…

  ____

  ____

[Conclusion] ______________________________

____

- [Premise 2] Pseudorica's election (X2)…

  ____

  ____

[Conclusion] ______________________________

____

Diagram

2) Draw a labeled diagram that proves the given argument is invalid and give a brief explanation of how it proves this.

- Only coffee shops that follow public health rules stay open.
- Constant Coffee follows public health rules.

Constant Coffee stays open.

Diagram

3) For the given universal generalization statement, write the two categories used in the statement and convert the statement into logically equivalent statements using the given quantifiers. Once you have assigned letters (***A***, ***B***) to the categories, you may use either the full labels or just the letter labels for the rest of your answer.

No industries extract natural resources from the ground.

Categories

A: ____

B: ____

All…____

Only…____

Name a counterexample to the statement above. Describe it (say what makes it a counterexample). Draw a diagram of the original generalization. Then draw a corrected diagram that includes your counterexample (X).

Counterexample (X): ______________________________________

Description of X: ______________________________________

X proves that: Some…______________________________________

Diagram of Original Generalization

Diagram of Correction (including X)

PRACTICE EXAM 2

1) Complete two valid syllogisms with different argument patterns. For each argument, use the given universal generalization as Premise 1 and complete Premise 2 and the Conclusion using the given subject. Below the arguments, draw a single diagram showing the premises of both arguments true (two circles, X1 and X2).

- [Premise 1] Only Biosafety Level 4 medical labs can safely study samples of Ebola virus.

- [Premise 2] The Gotham Bioresearch Institute's lab (X1)…

  ____

  ____

[Conclusion] ______________________________

____

- [Premise 2] Azmakia's National Virology Lab (X2)…

  ____

  ____

[Conclusion] ______________________________

____

Diagram

2) Prove that the given argument is invalid using a labeled diagram with a brief explanation of how it proves this.

- None of the icebergs that break off from an Antarctic ice shelf float into tropical waters.
- Iceberg B24c didn't float into tropical waters.

Iceberg B24c broke off from an Antarctic ice shelf.

Diagram

3) Write the two categories from the statement. Name two counterexamples with different descriptions. Draw a diagram of the original generalization. Then draw a corrected diagram that includes your counterexamples (X1 and X2). Once you have assigned letters (***A***, ***B***) to the categories, you may use either the full labels or just the letter labels for the rest of your answer.

All and only museums display artworks.

Categories

A: ____

B: ____

Counterexample (X1): ______________________________________

Description of X1: ____

X1 proves that: Not **all** | **only** museums display artworks.

Some…____

Counterexample (X2): ______________________________________

Description of X2: ____

X2 proves that: Not **all** | **only** museums display artworks.

Some…____

Diagram of Original Generalization

Diagram of Correction (including X1 and X2)

PRACTICE EXAM 3

1) Complete a disagreement between Abby and Bob with a valid syllogism on each side. Abby thinks a universal generalization is true; Bob thinks he knows a counterexample. Below each argument, draw a diagram showing the premises of the argument (two circles and X).

Abby and Bob (agreeing)

- [Premise 1] Wackadoodle Industries (X) has a high-school drop-out CEO.

Abby (disagreeing)

- [Premise 2] No companies that are good to invest in…______________________

  ____

[Conclusion] __________________________

____

Diagram

Bob (disagreeing)

- [Premise 2] Wackadoodle Industries…

  ____

  ____

[Conclusion] __________________________

____

Diagram

2) Prove that the given argument is invalid using a labeled diagram with a brief explanation of how it proves this.

- Only tree-planting programs can prevent climate change.
- The UN's Trillion Tree Initiative is a tree-planting program.

The UN's Trillion Tree Initiative can prevent climate change.

Diagram

3) For the given universal generalization statement, write the two categories used in the statement and convert the statement into logically equivalent statements using the given quantifiers. Once you have assigned letters (***A***, ***B***) to the categories, you may use either the full labels or just the letter labels for the rest of your answer.

Every wildfire starts from a lightning strike.

Categories

A: ______________________________

B: ______________________________

Only…______________________________

No…______________________________

Name a counterexample to the statement above. Describe it (say what makes it a counterexample). Draw a diagram of the original generalization. Then draw a corrected diagram that includes your counterexample (X).

Counterexample (X): ______________________________

Description of X: ______________________________

X proves that: Some…______________________________

Diagram of Original Generalization	Diagram of Correction (including X)

PRACTICE EXAM 4

1) Complete two valid syllogisms with different argument patterns. For each argument, use the given universal generalization as Premise 1 and complete Premise 2 and the Conclusion using the given subject. Below the arguments, draw a single diagram showing the premises of both arguments true (two circles, X1 and X2).

- [Premise 1] No one over age 24 truly understands what it’s like to be a teenager today.
- [Premise 2] My archery instructor (X1)…

  ____

  ____

[Conclusion] ________________________

____

- [Premise 2] This podcast host (X2)…

  ____

  ____

[Conclusion] ________________________

____

Diagram

2) Prove that the given argument is invalid using a labeled diagram with a brief explanation of how it proves this.

- Every one of the great civilizations of the Bronze Age (3300-1200 BCE) Near East believed in the supremacy of Marduk.
- The Zazakadian Empire was not one of the great civilizations of the Bronze Age.

The Zazakadian Empire did not believe in the supremacy of Marduk.

Diagram

3) For the given universal generalization statement, write the two categories used in the statement and convert the statement into logically equivalent statements using the given quantifiers. Once you have assigned letters (***A***, ***B***) to the categories, you may use either the full labels or just the letter labels for the rest of your answer.

Only multiplex movie theatres regularly seat large audiences of young people.

Categories

A: ____

B: ____

All…____

No…____

Name a counterexample to the statement above. Describe it (say what makes it a counterexample). Draw a diagram of the original generalization. Then draw a corrected diagram that includes your counterexample (X).

Counterexample (X): ____

Description of X: ____

X proves that: Some…____

Diagram of Original Generalization	Diagram of Correction (including X)

PRACTICE EXAM 5

1) Complete two valid syllogisms with different argument patterns. For each argument, use the given universal generalization as Premise 1 and complete Premise 2 and the Conclusion using the given subject. Below the arguments, draw a single diagram showing the premises of both arguments true (two circles, X1 and X2).

- [Premise 1] Only encrypted messaging apps give the people of Pseudorica access to international journalism sources.
- [Premise 2] Whisper (X1)…___________

  ____

  ____

[Conclusion] ___________________________

____

- [Premise 2] Tapper (X2)…___________

  ____

  ____

[Conclusion] ___________________________

____

Diagram

2) Draw a labeled diagram that proves the given argument is invalid and give a brief explanation.

- No western country has managed to get its smoking prevalence below 10%.
- Azmakia is not a western country.

Azmakia has managed to get its smoking prevalence below 10%.

Diagram

3) For the given universal generalization statement, write the two categories used in the statement and convert the statement into logically equivalent statements using the given quantifiers. Once you have assigned letters (***A***, ***B***) to the categories, you may use either the full labels or just the letter labels for the rest of your answer.

All mental illnesses begin during the teenage years.

Categories

A: ______________________________

B: ______________________________

Only…______________________________

No…______________________________

Name a counterexample to the statement above. Describe it (say what makes it a counterexample). Draw a diagram of the original generalization. Then draw a corrected diagram that includes your counterexample (X).

Counterexample (X): ______________________________

Description of X: ______________________________

X proves that: Some…______________________________

Diagram of Original Generalization

Diagram of Correction (including X)

ANSWER KEY

QUICK TEST

1) d **2)** d **3)** b **4)** a **5)** c **6)** d **7)** a **8)** a

PRACTICE EXAM 1

1) Answer

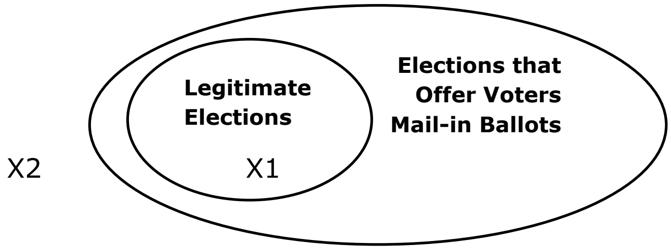
- [Premise 1] Every legitimate election offers voters mail-in ballots.

- [Premise 2] Azmakia's election (X1) is a legitimate election.

[Conclusion] Azmakia's election offers voters mail-in ballots.

- [Premise 2] Pseudorica's election (X2) does not offer voters mail-in ballots.

[Conclusion] Pseudorica's election is not a legitimate election.



Explanation (not required for exam answer)

Both arguments are valid but each has a different pattern:

- All ***A*** are ***B***.
- X1 is an ***A***.

X1 is a ***B***.

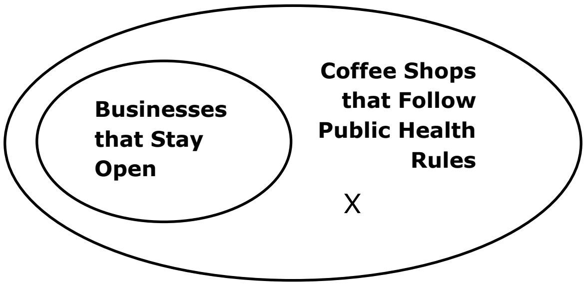
- All ***A*** are ***B***.
- X2 is not a ***B***.

X2 is not an ***A***.

A: Legitimate Elections
B: Elections that Offer…
X1: Azmakia's Election
X2: Pseudorica's Election

In the diagram, the circles show Premise 1 true and the X shows Premise 2 true. Since the arguments are valid, showing the premises true automatically shows the conclusion true.

2) Answer



X: Constant Coffee

The diagram imagines the conclusion false when the premises are true. That possibility proves this deductive argument is invalid.

Explanation (not required for exam answer)

The invalid argument pattern is:

- Only ***A*** are ***B***.
- X is an ***A***.

X is a ***B***.

A: Coffee Shops that Follow…
B: Businesses that Stay Open
X: Constant Coffee

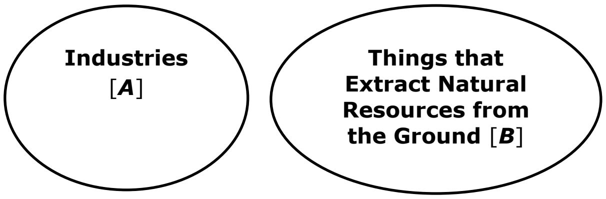
3) Answer

A: Industries
B: Things that Extract Natural Resources From the Ground

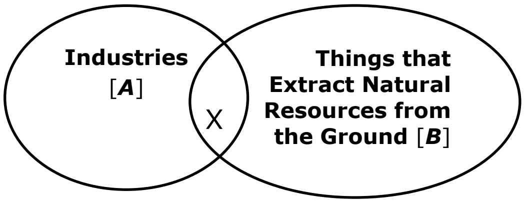
All ***A*** are Non-***B***. [Also correct: All ***B*** are Non-***A***.]
Only Non-***B*** are ***A***. [Also correct: Only Non-***A*** are ***B***.]

Counterexample (X): mining
Description of X: Industry [***A***] Thing that Extracts Natural Resources from the Ground [***B***]
X proves that: Some industries extract natural resources from the ground. [Some ***A*** are ***B***.]

Original Generalization



Correction (including X)



Explanation (not required for exam answer)

The predicate begins on the verb *extract*. The category could be more specific than Things that Extract…. It could be Activities that Extract…, Projects that Extract…, etc. Mining, or some particular type of mining, is an obvious counterexample but other answers could be correct. This question is not a test of factual knowledge. For example, Bob digs a well and then sells glasses of water. Is the water really a natural resource? Is the business really an industry? It doesn’t matter for this question. The answer shows that you’ve done the logic correctly.

The corrected diagram shows the minimum correction – each category partially but not completely overlapping the other – that allows placement of X. A diagram showing complete overlap (containment) is more than necessary but still correct.

PRACTICE EXAM 2

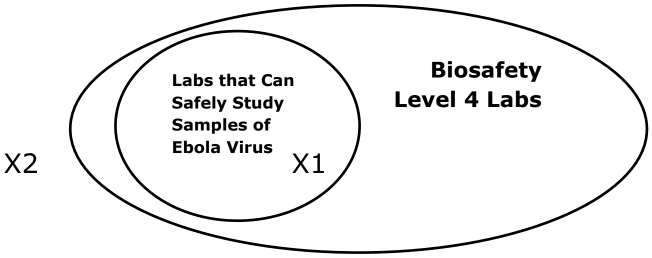
1) Answer

- [Premise 1] Only Biosafety Level 4 medical labs can safely study samples of Ebola virus.
- [Premise 2] The Gotham Bioresearch Institute's lab (X1) can safely study samples of Ebola virus.

[Conclusion] The Gotham Bioresearch Institute's lab is a Biosafety Level 4 lab.

- [Premise 2] Azmakia's National Virology Lab (X2) is not a Biosafety Level 4 lab.

[Conclusion] Azmakia's National Virology Lab cannot safely study samples of Ebola virus.



Explanation (not required for exam answer)

Both arguments are valid but each has a different pattern:

- Only ***A*** are ***B***.
- X1 is a ***B***.

X1 is an ***A***.

- Only ***A*** are ***B***.
- X2 is not an ***A***.

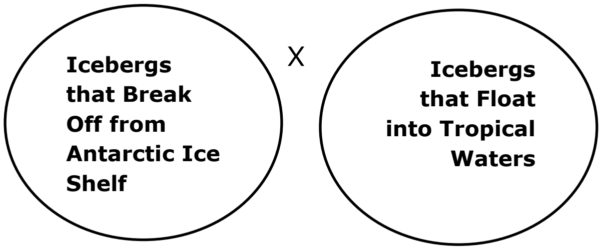
X2 is not a ***B***.

A: Biosafety Level 4 Labs
B: Labs that Can Safely…
X1: Gotham BI lab
X2: Azmakia NV lab

In each diagram, the circles show Premise 1 true and the X shows Premise 2 true. Since the arguments are valid, showing the premises true automatically shows the conclusion true.

2) Answer

X: Iceberg B24c



The diagram shows how it's possible for this argument's conclusion to be false even if its premises are true. An argument like that isn't valid.

Explanation (not required for exam answer)

The invalid argument pattern is:

- No ***A*** are ***B***.
- X is not a ***B***.

X is an ***A***.

A: Icebergs that Break Off…
B: Icebergs that Float into…
X: Iceberg B24c

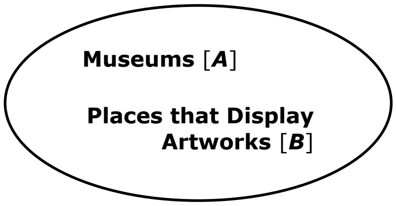
3) Answer

A: Museums
B: Places that Display Artworks

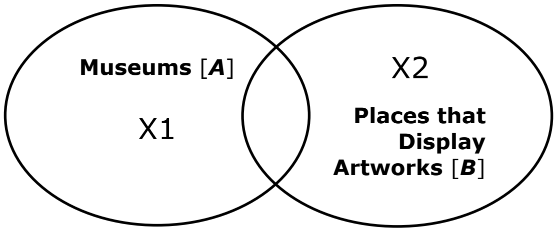
Counterexample (X1): natural history museum
Description of X1: Museum [***A***] Non-(Place that Displays Artworks) [Non-***B***]
X1 proves that: Not **all** museums display artworks.
Some museums do not display artworks.

Counterexample (X2): a restaurant with a painting on the wall
Description of X2: Non-Museum [Non-***A***] Place that Displays Artworks [***B***]
X2 proves that: Not **only** museums display artworks.
Some places that display artworks are not museums.

Original Generalization



Correction (including X1 and X2)



Explanation (not required for exam answer)

The predicate is "display artworks". The predicate category could be more specific, e.g. Buildings that Display Artworks. The category you give could affect which things are counterexamples. For example, a sidewalk (outdoor) art fair is a counterexample if the category is Places that Display Artworks but not if the category is Buildings that Display Artworks.

The corrected diagram shows the minimum correction – each category partially but not completely separated from the other – that allows placement of X1 and X2. A diagram showing complete separation is more than necessary but still correct.

PRACTICE EXAM 3

1) Answer

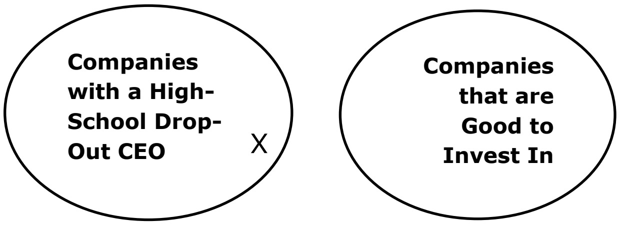
Abby and Bob

- [Premise 1] Wackadoodle Industries (X) has a high-school drop-out CEO.

Abby

- [Premise 2] No companies that are good to invest in have a high-school drop-out CEO.

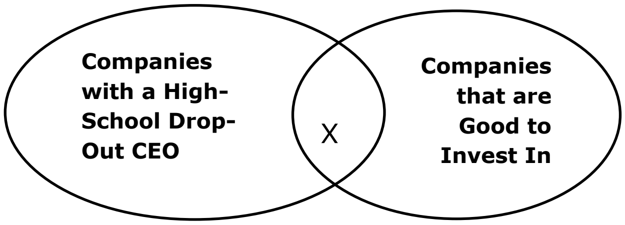
[Conclusion] Wackadoodle Industries is not a good company to invest in.



Bob

- [Premise 2] Wackadoodle Industries is a good company to invest in.

[Conclusion] Some companies with a high-school drop-out CEO are good to invest in.



Explanation (not required for exam answer)

Both arguments are valid but each has a different pattern:

- X is an ***A***.
- No ***B*** are ***A***.

X is not a ***B***.

- X is an ***A***.
- X is a ***B***.

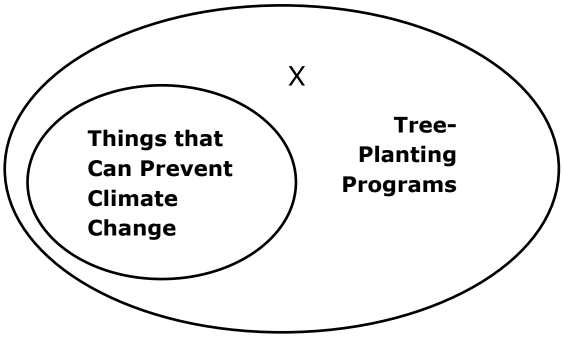
Some ***A*** are ***B***.
(Not no ***A*** are ***B***.)

A: Companies with a High-School Drop-Out CEO
B: Companies that are Good to Invest In
X: Wackadoodle Industries

In each diagram, the circles show Premise 1 true and the X shows Premise 2 true. Since the arguments are valid, showing the premises true automatically shows the conclusion true.

2) Answer

X: UN's Trillion Trees Initiative



The argument is invalid because the X shows a way to make the conclusion false and the premises true. That's impossible with a valid argument.

Explanation (not required for exam answer)

The invalid argument pattern is:

- Only ***A*** are ***B***.
- X is an ***A***.

X is a ***B***.

A: Tree-Planting Programs
B: Things that Can Prevent…
X: UN’s Trillion Trees Initiative

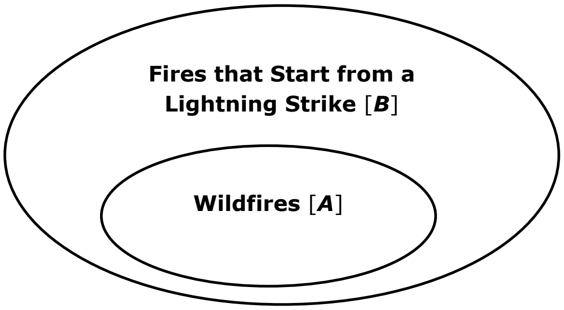
3) Answer

A: Wildfires
B: Fires that Starts from a Lightning Strike

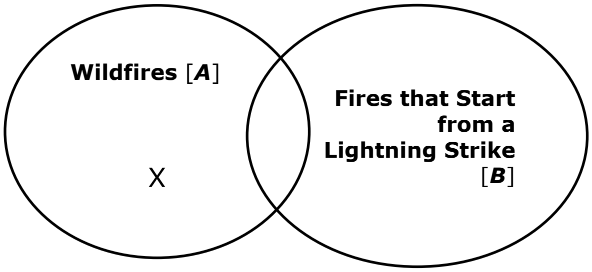
Only ***B*** are ***A***.
No ***A*** are Non-***B***. [Also correct: No Non-***B*** are ***A***.]

Counterexample (X): wildfire that starts from a tossed cigarette
Description of X: Wildfire [***A***] Non-(Fire that Starts from a Lightning Strike) [Non-***B***]
X proves that: Some wildfires do not start from a lightning strike. [Some ***A*** are not ***B***.]

Original Generalization



Correction (including X)



Explanation (not required for exam answer)

The predicate begins on the verb *starts*.

The corrected diagram shows the minimum correction – the categories partially but not completely separated – that allows placement of X. A diagram showing complete separation is also correct.

PRACTICE EXAM 4

1) Answer

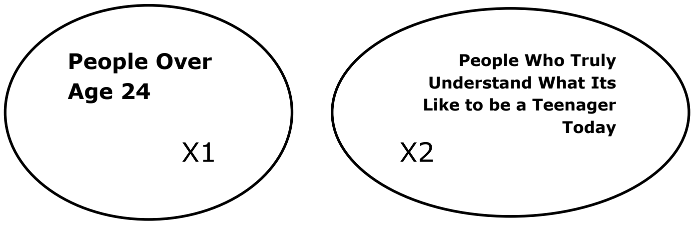
- [Premise 1] No one over age 24 truly understands what it's like to be a teenager today.

- [Premise 2] My archery instructor (X1) is over age 24.

[Conclusion] My archery instructor does not truly understand what it's like to be a teenager today.

- [Premise 2] This podcast host (X2) truly understands what it's like to be a teenager today.

[Conclusion] This podcast host is not over age 24.



Explanation (not required for exam answer)

Both arguments are valid but each has a different pattern:

- No ***A*** are ***B***.
- X1 is an ***A***.

X1 is not a ***B***.

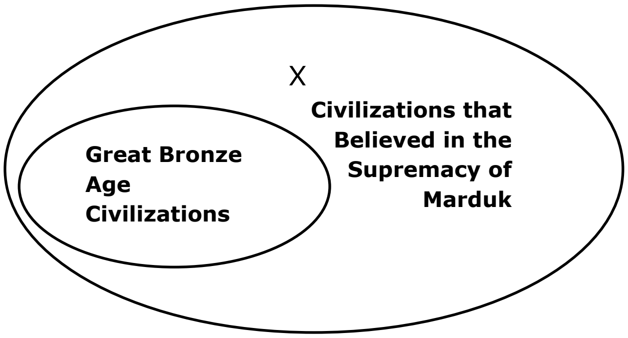
- No ***A*** are ***B***.
- X2 is a ***B***.

X2 is not an ***A***.

A: People Over Age 24
B: People Who Truly…
X1: My archery instructor
X2: This podcast host

In the diagram, the circles show Premise 1 true and X1 and X2 show Premise 2 (left) and Premise 2 (right) true. Since the arguments are valid, showing the premises true automatically shows the conclusion true.

2) Answer



X: Zazakadian Empire

The location of the X makes the conclusion false and the premises true. Any deductive argument for which you can do that is invalid.

Explanation (not required for exam answer)

The invalid argument pattern is:

- All ***A*** are ***B***.
- X is not an ***A***.

X is not a ***B***.

A: Great Bronze Age…
B: Civilizations that Believed…
X: Zazakadian Empire

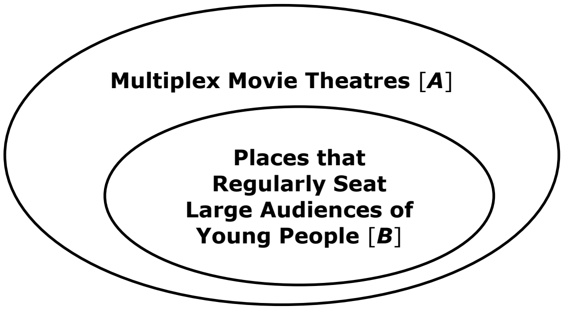
3) Answer

A: Multiplex Movie Theatres
B: Places that Regularly Seat Large Audiences of Young People

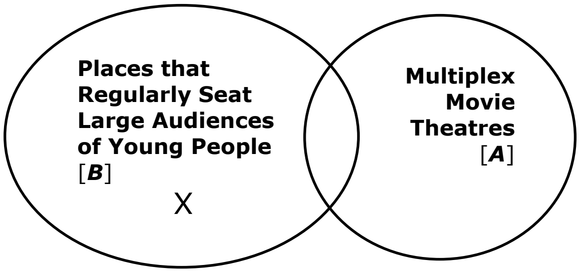
All ***B*** are ***A***.
No ***B*** are Non-***A***. [Also correct: No Non-***A*** are ***B***.]

Counterexample (X): school lecture hall
Description of X: Non-(Multiplex Movie Theatre) [Non-***A***] Place that Regularly Seats Large Audiences of Young People [***B***]
X proves that: Some places that regularly seat large audiences of young people are not multiplex movie theatres. [Some ***B*** are not ***A***.]

Original Generalization



Correction (including X)



Explanation (not required for exam answer)

The predicate begins on the adverb *regularly*. The category could be more specific than Places that Regularly…. It could be Building that Regularly…, etc. A school lecture hall is an obvious counterexample but anything that could reasonably be described the same way would be correct.

The corrected diagram shows the minimum correction – the categories partially but not completely separated – that allows placement of X. A diagram showing complete separation is also correct.

PRACTICE EXAM 5

1) Answer

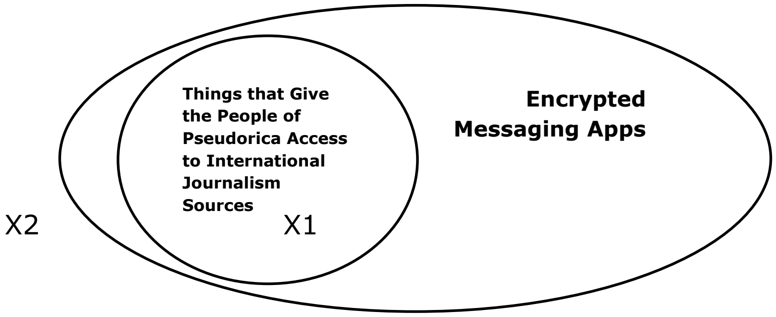
- [Premise 1] Only encrypted messaging apps give the people of Pseudorica access to international journalism sources.

- [Premise 2] Whisper (X1) gives the people of Pseudorica access to international journalism sources.

[Conclusion] Whisper is an encrypted messaging app.

- [Premise 2] Tapper (X2) is not an encrypted messaging app.

[Conclusion] Tapper does not give the people of Pseudorica access to international journalism sources.



Explanation (not required for exam answer)

Both arguments are valid but each has a different pattern:

- Only ***A*** are ***B***.
- X1 is a ***B***.

X1 is an ***A***.

- Only ***A*** are ***B***.
- X2 is not an ***A***.

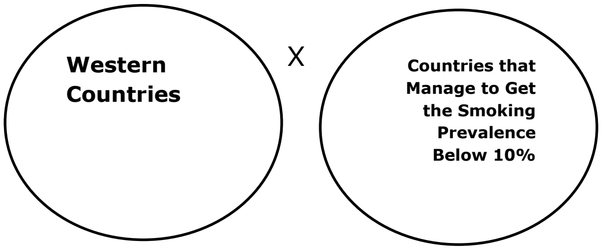
X2 is not a ***B***.

A: Encrypted Messaging Apps
B: Things that Give…
X1: Whisper
X2: Tapper

In each diagram, the circles show Premise 1 true and the X shows Premise 2 true. Since the arguments are valid, showing the premises true automatically shows the conclusion true.

2) Answer

X: Azmakia



We can imagine this deductive argument’s conclusion false and its premises true at the same time. That’s possible only if it’s invalid.

Explanation (not required for exam answer)

The invalid argument pattern is:

- No ***A*** are ***B***.
- X is not an ***A***.

X is a ***B***.

A: Western Countries
B: Countries that Manage…
X: Azmakia

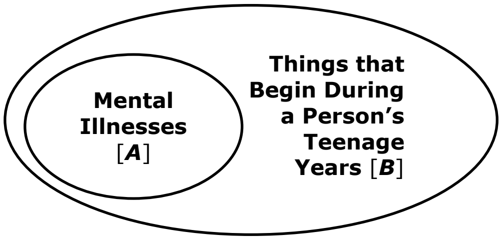
3) Answer

A: Mental Illnesses
B: Things that Begin During the Teenage Years

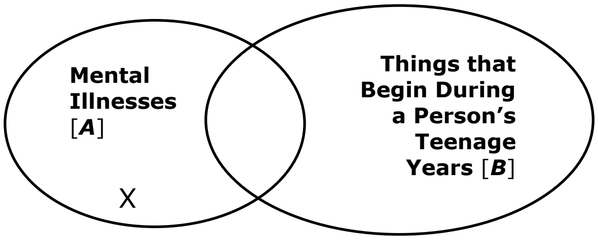
Only ***B*** are ***A***.
No ***A*** are Non-***B***. [Also correct: No Non-***B*** are ***A***.]

Counterexample (X): post-traumatic stress disorder from adult war experiences
Description of X: Mental Illness [***A***] Non-(Thing that Begins During the Teenage Years) [Non-***B***]
X proves that: Some mental illnesses do not begin during the teenage years. [Some ***A*** are not ***B***.]

Original Generalization



Correction (including X)



Explanation (not required for exam answer)

The predicate begins on the verb *begin*. The predicate category could be more specific than Things that Begin…, e.g. Illnesses that Begin…. Anything that could reasonably be described the same way as this counterexample would be correct.

The corrected diagram shows the minimum correction – the categories partially but not completely separated from each other – that allows placement of X. A diagram showing complete separation is also correct.

~ 3 ~

STATEMENT LOGIC

i. LOGICAL OPERATORS

Simple statements (represented by letters: **P**, **Q**, etc.) may be modified or connected with a **logical operator**, which attaches to the statements and "operates" on their truth values. This forms a new statement whose truth value is determined by the truth value(s) of the simple statement(s).

For example, if **P** is true, its **negation** ¬**P** is false; if **P** is false, its negation ¬**P** is true. **P** is logically equivalent to the negation of its negation: **P** ≡ ¬¬**P**.

Negation
¬**P**
Says: NOT **P** (**P** is false.)

The **conjunction** and **disjunction** operators create conjunction and disjunction statements, **compound statements** that connect two simple statements.

Conjunction
P-AND-**Q**
Says: BOTH **P** and **Q** are true.

Disjunction	
Inclusive	**Exclusive**
P-OR-**Q**	**P**-XOR-**Q**
Says: AT LEAST ONE of **P** and **Q** is true.	Says: EXACTLY ONE of **P** and **Q** is true.

A four-line **truth table** shows the four possible truth value combinations for two simple statements. Along each line, it shows the resulting truth values of negation, conjunction, and the disjunctions.[*]

	P	**Q**		¬**P**	**P**-AND-**Q**	**P**-OR-**Q**	**P**-XOR-**Q**
1	True	True		False	True	True	False
2	True	False		False	False	True	True
3	False	True		True	False	True	True
4	False	False		True	False	False	False

---

[*] Our way of writing the operators is not standard because it mixes two different symbol systems. We'll use the logical symbol ¬ (instead of NOT) because it's short and convenient. We'll use AND and OR (instead of ∧ and ∨) because they're clearer to people who are seeing logic for the first time.

Conjunction 3.1: Flowering plants are pollinated by flying insects <u>and</u> by pixies.

Pattern: **P**-AND-**Q**

P: Flowering plants are pollinated by flying insects.
Q: Flowering plants are pollinated by pixies.

Disjunction 3.2: Flowering plants are pollinated by flying insects <u>or</u> by pixies.

Pattern: **P**-OR-**Q**

(Image: depositphotos.com)

Conjunction 3.1 is false since only one, not both, of the statements is true. Disjunction 3.2 is true since at least one statement is true.

The word *and* means AND, as do other words such as *but* and *although*. Words in other languages also mean AND: *et* (French), *y* (Spanish), 也 (Mandarin), *và* (Vietnamese), ਅਤੇ (Punjabi), etc.

In some cases it's not clear whether the word *or* means OR or XOR. We'll read disjunctions in ordinary language as OR, and we'll mainly use this inclusive disjunction.

De Morgan's Laws

De Morgan's Laws are a pair of logical equivalence rules for false compound statements: false AND and false OR. We can see the two rules by looking in the truth table where AND and OR are false.

	P	**Q**		**P**-AND-**Q**	**P**-OR-**Q**
1	True	True		True	True
2	True	False		False	True
3	False	True		False	True
4	False	False		False	False

<u>**P**-AND-**Q** is false</u> on lines 2-4.
These lines are where **Q** is false (2) or **P** is false (3) or both **P** and **Q** are false (4). Therefore:

<u>**P**-OR-**Q** is false</u> on line 4.
That line is where both **P** and **Q** are false. Therefore:

¬(**P**-AND-**Q**) ≡ (¬**P**)-OR-(¬**Q**)

¬(**P**-OR-**Q**) ≡ (¬**P**)-AND-(¬**Q**)

Logical Equivalence 3.3: Flowering plants are <u>not</u> pollinated by flying insects <u>and</u> by pixies. ≡ Flowering plants are <u>not</u> pollinated by flying insects <u>or</u> they're <u>not</u> pollinated by pixies.

Pattern: ¬(**P**-AND-**Q**) ≡ (¬**P**)-OR-(¬**Q**)

P: Flowering plants are pollinated by flying insects.
Q: Flowering plants are pollinated by pixies.

Logical Equivalence 3.4: Flowering plants are <u>not</u> pollinated by flying insects <u>or</u> by pixies. ≡ Flowering plants are <u>not</u> pollinated by flying insects <u>and</u> they're <u>not</u> pollinated by pixies.

Pattern: ¬(**P**-OR-**Q**) ≡ (¬**P**)-AND-(¬**Q**)

ii. DISJUNCTIVE SYLLOGISM

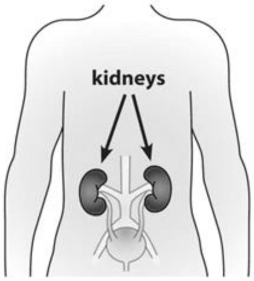
In a **disjunctive syllogism**, one premise says that one of two **disjuncts** is true, and the other premise denies that one of them is true. The conclusion is that the other is true.

Disjunctive Syllogism (**DS**)

- **P**-OR-**Q**
- ¬**P**

Q

Argument 3.5 (VALID): Insulin (a hormone that allows the body to use sugar for energy) is released into the bloodstream from the kidneys <u>or</u> from the liver. But it does<u>n't</u> come from the liver. Therefore it certainly comes from the kidneys.



(Image: NIDDK)

To evaluate this argument as valid or invalid, we need to see its pattern. We can show the pattern using letters we assign to its simple statements and the disjunction and negation operators.

Simple Statements	Argument 3.3 Pattern
P: Insulin is from the kidneys. **Q**: Insulin is from the liver.	• **P**-OR-**Q** • ¬**Q** ――――― **P**

VALID (DS)

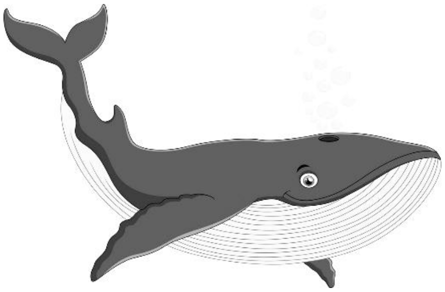
The argument has a false conclusion (insulin comes from the pancreas) even though it's valid. That's possible because it has a false premise. (The premise **P**-OR-**Q** is false because both **P** and **Q** are false.)

Does the disjunction in Argument 3.5 seem exclusive (XOR) or inclusive (OR) to you? Ordinary language often doesn't make this clear. But for DS arguments such as Argument 3.5, it doesn't matter. Both types of disjunction say "one of these two statements is true". So if one is false, the other is true. DS is always valid.

iii. CONDITIONAL

A **conditional** is an *If…then…* compound statement. Unlike conjunction and disjunction, the truth value of a conditional is not determined by the truth values of the statements within it.* A conditional is true when the truth of its **antecedent** statement is **sufficient** for (guarantees) the truth of its **consequent**, and the truth of its consequent is **necessary** (required) for the truth of its antecedent.

Conditional
If [Antecedent] then [Consequent].
Says: [Antecedent] sufficient for [Consequent]. [Consequent] necessary for [Antecedent].



Conditional 3.6: <u>If</u> whales are fish <u>then</u> they can breathe underwater.

Pattern: If **P** then **Q**.

P: Whales are fish.
Q: Whales can breathe underwater.

Often people put a question or command into an *If…then…* sentence instead of a consequent statement. A sentence like this can easily be re-written as a proper conditional.

Conditional 3.7: <u>If</u> there are other technologically advanced beings in the galaxy <u>then</u> where are they?

<u>If</u> there were other technologically advanced beings in the galaxy <u>then</u> we would have encountered them.

Conditional 3.8: <u>If</u> you want to call yourself a male feminist, demand that your female colleagues be paid the same as you are paid.

<u>If</u> you want to call yourself a male feminist, you should demand that your female colleagues be paid the same as you are paid.

(Image: bizjournals.com)

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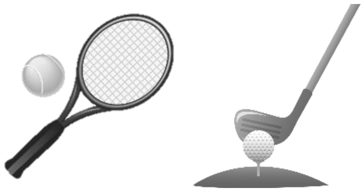
* In a course on formal symbolic logic, there is a logical operator for *If…then…*. This type of conditional works a bit differently from the one we've described here.

Writing Conditionals

Conditionals are often expressed in one of three different forms. All three say: antecedent sufficient; consequent necessary. But they emphasize (stress) one part of this meaning more than the other.

Conditional Form	Emphasis
If [Antecedent] then [Consequent].	Antecedent Sufficient
[Consequent] if [Antecedent].	Antecedent Sufficient
[Antecedent] only if [Consequent].	Consequent Necessary



Conditional 3.9: Tennis was invented before golf if the ancient Egyptians played tennis.

Pattern: **P** [Consequent] if **Q** [Antecedent].

P: Tennis was invented before golf.

Q: The ancient Egyptians played tennis.

Conditional 3.10: Bob will get his passport in time for his trip only if he pays the extra fee for fast processing.

Pattern: **P** [Antecedent] only if **Q** [Consequent]

P: Bob gets his passport.

Q: Bob pays the fee.

Only if in Conditional 3.10 emphasizes that paying the fee is necessary for Bob to get his passport.

Sentences written with *when* instead of *if* also form conditionals. The logical meaning is the same.

Unless

An *unless* sentence is also a conditional. "**P** unless **Q**" says that **P** is true if **Q** is false: **P** if ¬**Q**.

Statement 3.11: You'll hurt yourself exercising unless you stretch first.

You'll hurt yourself exercising if you do not stretch first.

iv. CONDITIONAL ARGUMENTS

Hypothetical Syllogism

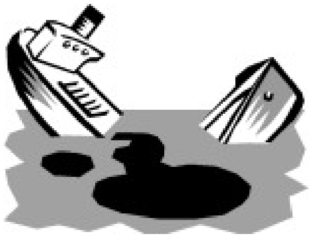
A **hypothetical syllogism** has two conditional premises. These overlap on a simple statement that is antecedent in one conditional and consequent in the other. The conclusion is the conditional we get by cutting out the overlapping simple statement.

Hypothetical Syllogism (**HS**)

- If **P** then **Q.**
- If **Q** then **R.**

If **P** then **R.**

Argument 3.12: There will be a huge oil spill if that oil tanker crashes, and if there's a huge oil spill then many of the animals that live in this area will not survive. Therefore many of the animals that live in this area will not survive if that oil tanker crashes**.**



Simple Statements	**Argument 3.12 Pattern**
P: That oil tanker crashes. **Q**: The animals will survive. **R**: There is a huge oil spill.	• R if P. • If R then ¬Q. ——— ¬Q if P.

VALID (HS)

Affirming Antecedent (AA) and the AC Fallacy

"If **P** then **Q**" says **P** is sufficient for **Q**. It does *not* say **Q** is sufficient for **P**. This means that:

Affirming Antecedent (AA)[*]	Affirming Consequent (AC)
• If **P** then **Q**. • **P** ——— **Q**	• If **P** then **Q**. • **Q** ——— **P**
VALID	INVALID (Fallacy)

(Image: Food & Beverage Insider)

Argument 3.13 (AA): If food prices rise, people will riot. And food prices are rising. Therefore people will riot.

Argument 3.14 (AC Fallacy): People riot when food prices rise. And people are rioting. Therefore food prices have risen.

(Image: Nosyrevy)

Simple Statements	Argument 3.13 Pattern	Argument 3.14 Pattern
P: Food prices rise. **Q**: People riot.	• If **P** then **Q**. • **P** ——— **Q**	• **Q** if **P**. • **Q** ——— **P**
	VALID (AA)	INVALID (AC)

Argument 3.14 is a logical fallacy; it is invalid because its conclusion could be false even when its premises are true. For example, perhaps there's a riot because a local sports team lost a big game instead of food prices.

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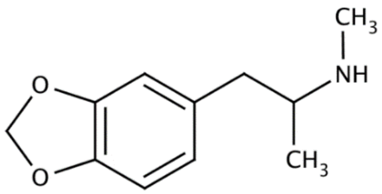
[*] The traditional Latin name for Affirming Antecedent, used in most logic courses, is *modus ponens*.

Denying Consequent (DC) and the DA Fallacy

“If **P** then **Q**” says **Q** is necessary for **P**. It does *not* say **P** is necessary for **Q**. This means that:

Denying Antecedent (DA)	**Denying Consequent** (**DC**)[*]
• If **P** then **Q**. • **¬P** ――――― **¬Q**	• If **P** then **Q**. • **¬Q** ――――― **¬P**
INVALID (Fallacy)	**VALID**

Argument 3.15 (DC): Recreational use of MDMA (Ecstasy) would be legal unless the public believed that it’s a sign of bad personal character. But MDMA is not legal. Therefore the public does believe that.



(Image: South Park)

Argument 3.16 (DA Fallacy): Recreational use of MDMA would be legal only if the public didn’t believe that it’s a sign of bad personal character. And MDMA is not legal. Therefore the public does believe that.

Simple Statements	**Argument 3.15 Pattern**	**Argument 3.16 Pattern**
P: Recreational MDMA is legal. **Q**: Public believes MDMA is sign of bad character.	• **P** if ¬**Q**. • ¬**P** ――――― **Q**	• **P** only if ¬**Q**. • ¬**P** ――――― **Q**
	VALID (DC)	INVALID (DA)

Argument 3.16 is invalid because its conclusion could be false even when its premises are true. Perhaps the public doesn’t believe that using MDMA is a sign of bad character even though it’s illegal.

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[*] The traditional Latin name for Denying Consequent, used in most logic courses, is *modus tollens*.

DC for Objections

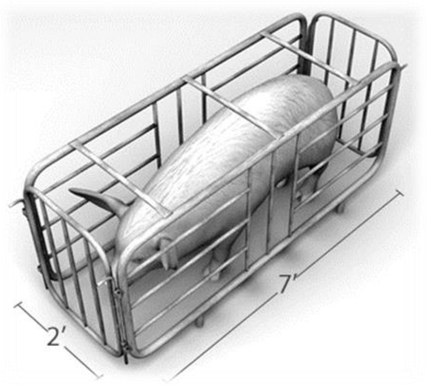
Bob says that eating mammals is morally OK. Abby disagrees. She objects to (against) his statement with a DC argument. To make this argument, she needs a conditional with three features:

1) Bob’s statement is the antecedent.
2) The whole conditional is true, so the conditional premise of the argument is true.
3) The consequent is false, so the DC premise of the argument is true.

$$\overbrace{\text{Eating mammals is morally OK}}^{\textbf{Objection against}}\text{ only if }\underbrace{\text{they do not suffer in factory farms.}}_{\text{False Consequent}}$$

Following the DC pattern, Abby concludes that the antecedent (Bob’s statement) is false.

Argument 3.17: Eating mammals is morally OK only if they do not suffer in factory farms. But certainly they do suffer in factory farms. Therefore eating mammals is not morally OK.



(Image: Animal Outlook)

Simple Statements	Argument 3.17 Pattern
P: Eating mammals is OK. **Q**: Mammals suffer in factory farms.	• **P** only if ¬**Q**. • **Q** ――――― ¬**P**

VALID (DC)

Abby’s objection is a valid argument. If Bob continues to believe that eating mammals is morally OK, he must reject one of her premises. To reject the conditional premise, he must say that the moral OK-ness of eating mammals really does *not* require that they not suffer in factory farms. To reject the DC premise, he must say that mammals do *not* suffer in factory farms.

AC and DA in Everyday Language

Once you are aware of the AC and DA patterns, you will notice them everywhere.

> **Argument 3.18**: I think our restaurant won't be able to stay in business. It would be able to if customers took selfies that attracted their social media followers, but people just eat and talk and don't take selfies.

Argument 3.18 has the pattern of an invalid DA fallacy. But that may not be the most reasonable way to read it. It may have an implicit **biconditional** premise: "Our restaurant would be able to stay in business <u>if and only if</u> customers took selfies that attracted their social media followers." Read this way, the argument is an implicit DC argument.

Or the argument may not be deductive at all. It may not be intended as valid. The speaker may understand that the restaurant *could possibly* be able to stay in business even without customer selfies, but they think the lack of customer selfies makes it *unlikely* that it will.

Similarly, many examples of the AC pattern in everyday language are not most reasonably read as invalid deductive arguments. They may have an implicit biconditional or they may be intended as a non-deductive pattern that we will learn in Unit 4.

People do make logical errors! But in these short examples with no context, it can be impossible to determine the most reasonable way to read an argument. So the questions in this unit ask you to construct and use valid patterns, but not to identify invalid ones.

UNIT 3 SKILLS

You must be able to:

- Recognize and write negations, conjunctions, and disjunctions.
- Use De Morgan’s Laws to write the negations of conjunctions and disjunctions.
- Recognize and write conditionals in three forms, plus *unless*.
- Construct the valid syllogisms DS, HS, AA, and DC.
- Construct an objection against a statement using the DC pattern.

QUICK TEST QUESTIONS

1) What does a conjunction of two statements say?
- a) Neither statement is true.
- b) Both statements are true.
- c) One statement is true and the other is false.
- d) At least one statement is true.

2) What does a disjunction of two statements say?
- a) Neither statement is true.
- b) One statement could be true.
- c) One statement is true.
- d) Both statements are true.

3) "If **P** then **Q**" says:
- a) **Q** is the antecedent and **P** is the consequent; **P** is sufficient for **Q**.
- b) **P** is the antecedent and **Q** is the consequent; **P** is sufficient for **Q**.
- c) **P** is the antecedent and **Q** is the consequent; **Q** is sufficient for **P**.
- d) **Q** is the antecedent and **P** is the consequent; **Q** is sufficient for **P**.

4) Which conditional form emphasizes that **Q** is necessary for **P**?
- a) If **P** then **Q**.
- b) **Q** if **P**.
- c) **Q** when **P**.
- d) **P** only if **Q**.

5) "**P** if **Q**, but ¬**Q**. Therefore ¬**P.**" This pattern is:
- a) DA
- b) AA
- c) DC
- d) AC

6) Which are valid syllogism patterns?
- a) DC and AC
- b) AC and DA
- c) AA and DA
- d) AA and DC

7) Which pattern is particularly well suited to making an objection against a statement?
- a) DC
- b) AA
- c) DS
- d) DA

PRACTICE EXAMS

PRACTICE EXAM 1

1) Use De Morgan's Laws to write the negation of the given statement.

The Toosla Model Z is very fast and it's very reliable.

2) Choose the most reasonable way to complete the sentence.

A candle is **sufficient | necessary** to read a (paper) book at nighttime.

3) Complete AA and DC syllogisms using the given Premise 1. You may change tense (past/present/future) in your answers.

- [Premise 1] The president resigns only if the people do not still support her.

- [Premise 2] ______________________

____

[Conclusion] ______________________

____

- [Premise 2] ______________________

____

[Conclusion] ______________________

____

4) Complete a hypothetical syllogism (HS) using the given Premise 1. For full marks, write reasonable premises.

- [Premise 1] If Netflix stops people from sharing passwords then we'll have to watch YouTube.
- [Premise 2] ____

[Conclusion] ____

5) Write a DC argument as an objection against the statement. For full marks, make the objection reasonable. You may change the tense (past/present/future) of the statement.

The revolution succeeds.

PRACTICE EXAM 2

1) Use De Morgan's Laws to write the negation of the given statement.

The movie will be promoted heavily this fall or it will be delayed a year.

2) Choose the most reasonable way to complete the sentence.

Deathtrap nightclub's new hiring policy states that a first aid certificate is **sufficient | necessary** for being employed at the club.

3) Complete two different disjunctive syllogisms (DS) using the given Premise 1. You may change tense (past/present/future) in your answers.

- [Premise 1] Either nuclear fusion is a viable energy source or civilization as we know it does not have a long future on Earth.

- [Premise 2] ______________________
____

[Conclusion] ______________________
____

- [Premise 2] ______________________
____

[Conclusion] ______________________
____

4) Complete a hypothetical syllogism (HS) using the given Conclusion. For full marks, write reasonable premises.

- [Premise 1] ____
- [Premise 2] ____

[Conclusion] Bob will get a different apartment if he's afraid of heights.

5) Write a DC argument as an objection against the statement. For full marks, make the objection reasonable. You may change the tense (past/present/future) of the statement.

Video games should not be an Olympic sport.

PRACTICE EXAM 3

1) Use De Morgan's Laws to write the negation of the given statement.

The insurance policy does not cover floods and it's impossible to add coverage for floods.

2) Choose the most reasonable way to complete the sentence.

Wackadoodle Industries has made a new policy stating that sexual harassment is **sufficient | necessary** for being terminated from the company.

3) Complete AA and DC syllogisms using the given Premise 1. You may change tense (past/present/future) in your answers.

- [Premise 1] Gun sales go up whenever people don't trust the government to maintain social order.

- [Premise 2] ______________________

____

[Conclusion] ______________________

____

- [Premise 2] ______________________

____

[Conclusion] ______________________

____

4) Complete a hypothetical syllogism (HS) using the given Premise 1. For full marks, write a reasonable Premise 2.

- [Premise 1] Free speech is worth protecting unless people say things that make me angry.
- [Premise 2] ____

[Conclusion] ____

5) Write a DC argument as an objection against the statement. For full marks, make the objection reasonable.

Hamsters are the best pets.

PRACTICE EXAM 4

1) Use De Morgan's Laws to write the negation of the given statement.

South Australia has most of the world's known opal deposits or it's a popular destination for film shoots.

2) Choose the most reasonable way to complete the sentence.

To get consumers to buy electric cars, it will be **sufficient | necessary** to build a network of charging stations across the county.

3) Complete AA and DC syllogisms using the given Premise 1. You may change tense (past/present/future) in your answers.

- [Premise 1] There is not a World Cup this year unless the virus does not keep spreading.

- [Premise 2] ______________________
____

[Conclusion] ______________________
____

- [Premise 2] ______________________
____

[Conclusion] ______________________
____

4) Complete a hypothetical syllogism (HS) using the given Conclusion. For full marks, write reasonable premises.

- [Premise 1] ____
- [Premise 2] ____

[Conclusion] Humans will travel to Mars only if nations are not preoccupied with war and disasters.

5) Write a DC argument as an objection against the statement. For full marks, make the objection reasonable.

People will never get bored of superhero movies.

ANSWER KEY

QUICK TEST

1) b **2)** c **3)** b **4)** d **5)** a **6)** d **7)** a

PRACTICE EXAM 1

1) **Answer**

The Toosla Model Z is not very fast or it’s not very reliable.

Explanation (not required for exam answer)

The given statement is a conjunction **P**-AND-**Q**. Its negation is logically equivalent to (¬**P**)-OR-(¬**Q**).

2) **Answer**

A candle is **sufficient** to read a (paper) book at nighttime.

Explanation (not required for exam answer)

Under normal circumstances (e.g. the person can read, they are not blind), a candle would be enough (make it possible) to read a book at nighttime. But it’s not required since there are other sources of light would make reading a book possible.

3) **Answer**

- [Premise 1] The president resigns only if the people do not still support her.

• [Premise 2] The president resigns. [Conclusion] The people do not still support her.	• [Premise 2] The people do support the president still. [Conclusion] She doesn’t resign.
[AA]	[DC]

Explanation (not required for exam answer)

Since the consequent “The people do not still support the president” already contains the NOT operator, its negation in Premise 2 (right argument) removes the word *not*.

The argument patterns are:

- **P** only if ¬**Q**.
- **P**

¬**Q**

- **P** only if ¬**Q**.
- **Q**

¬**P**

P: President resigns.
Q: People support the president.

4) **Answers** (examples)

- [Premise 1] If Netflix stops people from sharing passwords then we’ll have to watch YouTube.
- [Premise 2] If Netflix profits go down then they will stop people from sharing passwords.

[Conclusion] If Netflix profits go down then we’ll have to watch YouTube.

- [Premise 1] If Netflix stops people from sharing passwords then we’ll have to watch YouTube.
- [Premise 2] If we have to watch YouTube then we’ll be forced to watch lots of ads.

[Conclusion] If Netflix stops people from sharing passwords then we’ll be forced to watch lots of ads.

Explanation (not required for exam answer)
Since the given conditional Premise 1 contains two simple statements, an answer must contain a third simple statement to make an HS argument. In the first answer, Premise 2 connects with the antecedent of Premise 1 (“Netflix stops people from sharing passwords”); in the second answer, Premise 2 connects with the consequent of Premise 1 (“We’ll have to watch YouTube”).

5) **Answers** (examples)

- The revolution will succeed only if the government doesn’t shut down the internet that revolutionaries use to coordinate their protests.
- The government is shutting down the internet that revolutionaries use to coordinate their protests.

The revolution will not succeed.

- The political prisoners would have been released by now if the revolution had succeeded.
- The political prisoners still have not been released.

The revolution failed. [see explanation below]

- If the revolution were succeeding, the army would be joining with the protestors against the government.
- The army is not joining with the protestors against the government.

The revolution is not succeeding.

Explanation (not required for exam answer)
The argument is an objection to the statement "The revolution succeeds", in the future, past, or present tense. For the DC argument pattern, this statement is the antecedent of the conditional premise. The conclusion denies this statement: The revolution does <u>not</u> succeed. This situation is imaginary, but it's reasonable to think the consequent might be necessary for the antecedent.

(Note: Sometimes there is a word that clearly means the negation of another word. In the second answer here, *fail* simply means "not succeed". In cases like this where the meaning is completely clear, a correct answer can use a word substitution instead of adding or removing *not*.)

PRACTICE EXAM 2

1) Answer

The movie will not be promoted heavily this fall and it will not be delayed a year.

Explanation (not required for exam answer)
The given statement is a disjunction **P**-OR-**Q**. Its negation is logically equivalent to (¬**P**)-AND-(¬**Q**).

2) Answer

Deathtrap nightclub's new hiring policy states that a first aid certificate is **necessary** for being employed at the club.

Explanation (not required for exam answer)
It would be a very strange or desperate hiring policy that guaranteed a job to anyone at all who has a first aid certificate. At any normal nightclub, the certificate would not be sufficient for employment. It would be much more reasonable for the certificate to be a requirement (necessary).

3) Answer

- [Premise 1] Either nuclear fusion is a viable energy source or civilization as we know it does not have a long future on Earth.
- [Premise 2] Nuclear fusion is not a viable energy source.

[Conclusion] Civilization as we know it does not have a long future on Earth.

- [Premise 2] Civilization as we know it has a long future on Earth.

[Conclusion] Nuclear fusion is a viable energy source.

Explanation (not required for exam answer)

Since the disjunct "Civilization as we know it does not have a long future on Earth" already contains the NOT operator, its negation in the Conclusion (left argument) and Premise 2 (right argument) removes the word *not*.

The argument patterns are:

- **P** OR ¬**Q**.
- ¬**P**

¬**Q**

- **P** OR ¬**Q**.
- **Q**

P

P: Nuclear fusion is viable.
Q: Civilization has a long future.

4) Answer (example)

- [Premise 1] Bob isn't able to go near the window in his apartment if he's afraid of heights.
- [Premise 2] Bob will get a different apartment if he isn't able to go near the window in it.

[Conclusion] Bob will get a different apartment if he's afraid of heights.

Explanation (not required for exam answer)

Since the given conditional Conclusion contains two simple statements, an answer must contain a third simple statement to make an HS argument. In the HS pattern, both the antecedent and the consequent of the conclusion are used in the premises. Premise 1 connects with the antecedent ("Bob's afraid of heights"); Premise 2 connects with the consequent ("Bob will get a different apartment").

5) Answer (example)

- Video games should not be an Olympic sport only if including them in the Olympics will lead to demands for a bunch of crazy new Olympic sports (whistling, flying paper airplanes, etc.).
- Including video games in the Olympics will not lead to demands for a bunch of crazy new Olympic sports.

Video games should be an Olympic sport.

Explanation (not required for exam answer)

The argument is an objection to the statement "Video games should not be an Olympic sport". For the DC argument pattern, this statement is the antecedent of the conditional premise. The conclusion denies this statement: Video games should be an Olympic sport. This situation is imaginary, but it's reasonable to think the consequent might be necessary for the antecedent.

PRACTICE EXAM 3

1) Answer

The insurance policy covers floods or it’s possible to add coverage for floods.

Explanation (not required for exam answer)

The given statement is a conjunction (¬**P**)-AND-(¬**Q**). Its negation is logically equivalent to (**P**)-OR-(**Q**). This answer treats *possible* as the negation of *impossible* since the *im-* prefix means NOT. The double negation *not impossible* would also be correct: (**P**)-OR-(¬¬**Q**).

2) Answer

Wackadoodle Industries has made a new policy stating that sexual harassment is **sufficient** for being terminated from the company.

Explanation (not required for exam answer)

It would be a very strange company policy that said sexual harassment is necessary for termination. That would mean that nothing else an employee does – not stealing money, not burning the building down – could get them fired. At any normal company, sexual harassment might be enough (sufficient) to get fired but would not be necessary.

3) Answer

- [Premise 1] Gun sales go up whenever people don’t trust the government to maintain social order.

- [Premise 2] People don’t trust the government to maintain social order.

[Conclusion] Gun sales are going up.

[AA]

- [Premise 2] Gun sales are not going up.

[Conclusion] People trust the government to maintain social order.

[DC]

Explanation (not required for exam answer)

Since the antecedent "People don't trust the government to maintain social order" already contains the NOT operator, its negation in the DC Conclusion removes the word *not*.

The argument patterns are:

- **P** if ¬**Q**.
- ¬**Q**

P

- **P** if ¬**Q**.
- ¬**P**

Q

P: Gun sales go up.
Q: People trust the government to maintain social order.

4) **Answer** (examples)

- [Premise 1] Free speech is worth protecting unless people say things that make me angry.
- [Premise 2] People don't say things that make me angry unless they make jokes about my religion.

[Conclusion] Free speech is worth protecting unless people make jokes about my religion.

- [Premise 1] Free speech is worth protecting unless people say things that make me angry.
- [Premise 2] I'll go to the protest rally downtown unless free speech is not worth protecting.

[Conclusion] I'll go to the protest rally downtown unless people say things that make me angry.

Explanation (not required for exam answer)

Since the given conditional Premise 1 contains two simple statements, an answer must contain a third simple statement to make an HS argument. In the HS pattern, both the antecedent and the consequent of the conclusion are used in the premises. In the first answer, Premise 2 connects with the antecedent of Premise 1 ("People don't say things that make me angry"); in the second answer, Premise 2 connects with the consequent of Premise 1 ("Free speech is worth protecting").

5) **Answer** (examples)

• If hamsters were the best pets, they would fetch a stick and bring it back to you. • Hamsters won't fetch a stick and bring it back to you. ——— Hamsters are not the best pets.
• Hamsters are the best pets only if they don't bite little children. • Hamsters do bite little children. ——— Hamsters are not the best pets.

Explanation (not required for exam answer)

The argument is an objection to the statement "Hamsters are the best pets". For the DC argument pattern, this statement is the antecedent of the conditional premise. The conclusion denies this statement: Hamsters are not the best pets.

PRACTICE EXAM 4

1) Answer

South Australia does not have most of the world’s known opal deposits and it’s not a popular destination for film shoots.

Explanation (not required for exam answer)

The given statement is a disjunction (**P**)-OR-(**Q**). Its negation is logically equivalent to (¬**P**)-AND-(¬**Q**).

2) Answer

To get consumers to buy electric cars, it will be **necessary** to build a network of charging stations across the county.

Explanation (not required for exam answer)

A network of charging stations might be sufficient if many other things are also true, but the most reasonable thing to say without any other information is that it’s necessary.

3) Answer

- [Premise 1] There is not a World Cup this year unless the virus does not keep spreading.

- [Premise 2] The virus will keep spreading.

[Conclusion] There will not be a World Cup this year.

[AA]

- [Premise 2] There was a World Cup this year.

[Conclusion] The virus did not keep spreading.

[DC]

Explanation (not required for exam answer)

The conditional uses *unless* = *if not*, so the antecedent is the double negation “The virus does not not keep spreading” = “The virus keeps spreading”.

The argument patterns are:

- **P** if ¬¬**Q**.
- **Q**

P

- **P** if ¬¬**Q**.
- ¬**P**

¬**Q**

P: There is a World Cup this year.
Q: The virus keeps spreading.

4) **Answer** (example)

- [Premise 1] Humans will travel to Mars only if national space programs get funded.
- [Premise 2] National space programs will get funded only if nations are not preoccupied with war and disasters.

[Conclusion] Humans will travel to Mars only if nations are not preoccupied with war and disasters.

Explanation (not required for exam answer)

Since the given conditional Conclusion contains two simple statements, an answer must contain a third simple statement to make an HS argument. In the HS pattern, both the antecedent and the consequent of the conclusion are used in the premises. The premises connect with the consequent of Premise 1 and the antecedent of Premise 2 (“National space programs get funded”).

5) **Answer** (example)

- People will never get bored of superhero movies only if creators can think of more interesting plots.
- Creators cannot think of more interesting plots.

People will get bored of superhero movies.

Explanation (not required for exam answer)

The argument is an objection to the statement “People will never get tired of superhero movies”. In this statement, *never* functions as the NOT operator. For the DC argument pattern, this statement is the antecedent of the conditional premise. The conclusion denies this statement: People will get bored of superhero movies.

~ 4 ~

EXPLANATION AND CONFIRMATION

i. SCIENTIFIC REASONING

Scientific reasoning is not just for scientists doing research. It is for anyone using observations to gain knowledge. Often it begins with an attempt to **explain** some observed fact, to say why or how something happens. A proposed explanation is one type of **hypothesis**, a statement that we're not yet ready to fully believe. A new observation may **confirm** (support) or **disconfirm** the hypothesis. **Evidence** is any observation, past or new, that helps us judge the likely truth or falsity of a hypothesis.

A Story of Scientific Reasoning

Abby has pizza for dinner, and some is left over. She goes out for a while and when she comes back, a piece of pizza is gone. "Maybe my cat, Mr. Business, ate that piece of leftover pizza", she thinks. Then she thinks of another explanation: "Maybe my roommate, Zelda, ate the pizza."

(Image: @RealGrumpyCat)

Mr. Business has taken human food from the table before, but never anything as big as a whole piece of pizza. Abby knows that Zelda likes pizza. It makes more sense that Zelda ate it.

But she wants to test her first idea. "If my cat ate the pizza then probably he won't want his cat food tonight." She prepares Mr. Business' usual cat food and waits to see if he wants it.

ii. INFERENCE TO THE BEST EXPLANATION

To explain an event, we think of a hypothesis that would make us expect that event: H explains E (after we observe E) if we could have used H to predict E (before we observe E). Abby observes that the pizza is missing. Her hypothesis ("the cat ate it") explains the missing pizza because it would lead us to expect the pizza to be gone. Often an explanation is a **causal** hypothesis, a story about what caused the event we're explaining. The cat eating the pizza would cause the pizza to be gone.

Abby thinks of different possible explanations and chooses the best one. This is **inference to the best explanation** (IBE). IBE is non-deductive, or **inductive**, reasoning. Inductive arguments are not supposed to be valid – instead they are evaluated as **strong** or **weak**. In a strong argument, the premises support the conclusion well but don't guarantee it.

Why is IBE inductive rather than deductive? There are a few reasons:

- There is no logical method for determining that one explanation is better than another.
- Maybe there's a better explanation that we've not thought of.
- Maybe the best explanation is not true. (The truth may be surprising!)

Competing Explanations

In IBE we choose a single "winning" explanation from among **competing explanations** (alternative hypotheses). Competing explanations are inconsistent: if one is true, the others are false.

In the story, Abby considers two competing explanations.

Hypothesis 4.1: Her cat, Mr. Business, ate the leftover pizza.

Hypothesis 4.2: Her roommate, Zelda, ate the leftover pizza.

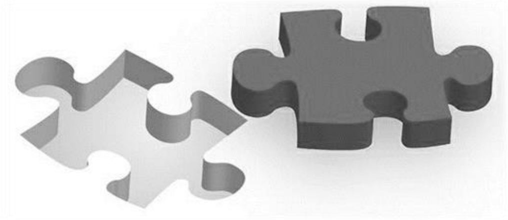
Common sense tells us that if her cat ate the pizza then her roommate did not eat it. Of course it's possible to imagine a weird story in which they both ate the pizza (e.g. they each ate half of it). Many of our examples of competing explanations will be like this, very unlikely though perhaps not impossible that they're both true.

The Best Explanation (That We've Thought Of!)

It can be difficult to decide which explanation is best. There are many things we could look for in a good explanation. Here are two important ones.

1) Fit with Background Information

An explanation is better when it **fits** better with background information: it's less surprising, based on what we already know.

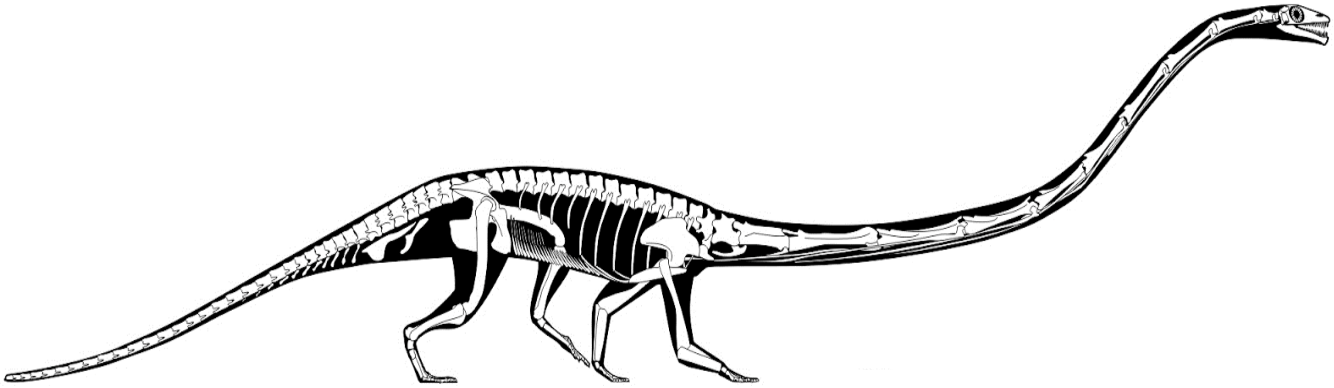


2) Simplicity

A simple explanation is better than a complicated one. One hypothesis is **simpler** than another if it's "easier" to state, if it explains the observation with fewer assumptions or it has fewer parts. As with fit, usually there is no way to precisely measure simplicity. We normally can't say that, for example, one hypothesis is twice as simple (half as complicated) as another. Simplicity is an informal concept.

Example (Real)

A very strange reptile called *Tanystropheus* lived 242 million years ago. Fossils show that about half the animal's length was its neck! Why did it have this bizarrely long neck?



(Image: markwitton.com)

Observation 4.3: *Tanystropheus* had an extremely long neck.

Competing explanations say that its neck was an evolutionary adaptation for different hunting styles.

Hypothesis 4.4: *Tanystropheus*' neck was for reaching fish while it stood on land.

(Image: markwitton.com)

Hypothesis 4.5: *Tanystropheus*' neck was for sneaking up on fish while it hid in water.

(Image: Emma Finley-Jacob)

Perhaps Hypothesis 4.4 is somewhat simpler. In 2017, it also fit better with background information. Fossils showed that *Tanystropheus*' limbs and tail were not suitably shaped for swimming. Many paleontologists thought that Hypothesis 4.4 was the best explanation.

(Image: Emma Finley-Jacob)

But in 2018-20, new fossil studies showed that *Tanystropheus* had legs that would be good for jumping forward from an underwater hiding place[*] and a head with breathing holes on top of a flat snout[+], similar to crocodiles, which hide in water to hunt. Hypothesis 4.4 fits much worse with all this new information. By 2020, Hypothesis 4.5 was the best explanation.

---

[*] Renesto, S. and Saller, F. (2018) Evidences for a semi-aquatic lifestyle in the Triassic diapsid reptile *Tanystropheus*. *Research in Paleontology and Stratigraphy* **124**(1), 23-34.

[+] Spiekman, *et al*. (2020) Aquatic habits and niche partitioning in the extraordinarily long-necked Triassic reptile *Tanystropheus*. *Current Biology*, Aug 6 on-line.

Inference to the Best Explanation

Main Argument

- EVIDENCE (observation)
- HYPOTHESIS is the best explanation of EVIDENCE.

____

HYPOTHESIS is the true explanation of EVIDENCE.

Sub-argument

- HYPOTHESIS fits better with background information than ALTERNATIVE HYPOTHESES.

OR

- HYPOTHESIS is simpler than ALTERNATIVE HYPOTHESES.

____

HYPOTHESIS is the best explanation of EVIDENCE.

> **Argument 4.6**: [2017] ① Many animals of the Triassic period (252–201 million years ago) were reptiles. ② One of them, *Tanystropheus*, had a bizarrely long neck that was about half of its body length! Probably ③ this was an adaptation for reaching fish while it stood on land. ④ That seems simpler than the other popular explanation, that a land animal moved to the water and evolved a long neck to sneak up on fish. ⑤ It also seems to fit better with what else we know about *Tanystropheus* from fossils: its body, limbs, and tail were not well shaped for swimming.

The intermediate conclusion is implicit: ⑥ "'*Tanystropheus*' neck was an adaptation for reaching fish while it stood on land' is the best explanation of its long neck.", in other words, "③ is the best explanation of ②." This argument also has a small additional sub-argument, beyond the extended pattern, in which background information supports the "fit" claim.

Main Argument

② [Observation]
⑥ [H is the best explanation.]
____
③ [H is the true explanation.]

Sub-argument

④
⑤
____
⑥

As we saw above, this argument was stronger in 2017 than it is now. Based on the information from the newer fossil studies, ⑤ and ⑥ now seem to be false; ③ is no longer well supported.

iii. CONFIRMATION AND DISCONFIRMATION

Confirmation

Not every hypothesis is an explanation. In this section, we'll start to consider hypotheses, such as hypotheses about the future, that do not explain anything in particular, as well as others that do.

In scientific reasoning, **confirmation** just means support, not proof. To confirm a hypothesis, we make and check a **prediction**, a statement that the hypothesis leads us to expect to be true. To be useful, the prediction must be something whose truth we can check sooner or more easily or more directly than the hypothesis. If the prediction is true, this confirms the hypothesis.

A conditional premise connects the hypothesis (antecedent) to the prediction (consequent).

Confirmation

- If HYPOTHESIS then EVIDENCE (prediction).
- EVIDENCE (prediction true)

HYPOTHESIS is true.

Argument 4.7: If mindfulness meditation (sitting quietly, focusing on your breathing) works then Abby will feel relaxed after she does it. Later, after meditating, she feels relaxed. So mindfulness meditation works.

This argument looks like the deductive fallacy AC! But it's not. Like IBE, confirmation is inductive. It's supposed to be strong, not valid.

Disconfirmation and Background Assumptions

Abby does mindfulness meditation but doesn't feel relaxed afterwards. This is simple **disconfirmation**, a regular DC argument:

- If HYPOTHESIS then EVIDENCE.
- EVIDENCE is false.

HYPOTHESIS is false.

We'll construct a more useful version of this argument that recognizes a **background assumption**, a belief we haven't doubted before now, and which we've used to make the prediction. The easiest way to notice our background assumptions is to think of ways to complete a more complex conditional sentence: If HYPOTHESIS then EVIDENCE – unless…

Conditional 4.8: If meditation works then Abby will feel relaxed after meditating – unless Abby drank coffee.

We saw in Unit 3 that *unless means* "if not".

Conditional 4.9: If meditation works then Abby will feel relaxed after meditating – if Abby did not drink coffee.

This shows us the background assumption: Abby did not drink coffee.

Conditional 4.9 has two antecedents, the hypothesis and the background assumption. It says these are *together* sufficient for the predicted evidence. The AND operator gives us a way to write this. Conditional 4.9 is logically equivalent to:

Conditional 4.10: If meditation works and Abby did not drink coffee then she'll feel relaxed after meditating.

Conditional 4.10 says: If HYPOTHESIS AND BACKGROUND ASSUMPTION then EVIDENCE.

The prediction is false in our story. So we get a DC argument:

- If (H-AND-BA) then E.
- ¬E

¬(H-AND-BA)

In Unit 3 we learned De Morgan’s Laws for the negations of AND and OR. We’ll use one of them here.

¬(H-AND-BA) = (¬H)-OR-(¬BA)

This will be a more useful way of writing the conclusion of our disconfirmation argument since it directly reminds us that our background assumption might be false instead of the hypothesis.

Disconfirmation (with a Background Assumption)

- If H then E unless ¬BA. / If H-AND-BA then E.
- ¬E

(¬H)-OR-(¬BA)

Argument 4.11: If mindfulness meditation works and Abby didn’t drink coffee then she will feel relaxed after meditating. But later, after meditating, she doesn’t feel relaxed. So either mindfulness meditation doesn’t work or she drank coffee.

UNIT 4 SKILLS

<u>You must be able to</u>:

- Think of, or identify in a story, alternative (competing) explanations for an observation.
- Judge the probabilities of hypotheses by their simplicity and fit with background information.
- Reconstruct an IBE argument.
- Recognize hypotheses and predictions and think of background assumptions.
- Construct a disconfirmation argument that includes a background assumption.

QUICK TEST QUESTIONS

---

1) In an IBE argument…

a) …a prediction explains an observation.
b) …an observation explains a hypothesis.
c) …a hypothesis explains an observation
d) …a hypothesis explains a prediction.

---

2) IBE is an inference to the best among explanations that…

a) …are simple.
b) …have been true in the past.
c) …have high probabilities.
d) …compete.

---

3) When is an explanation better (more probable)?

a) It is confirmed by observations and it is supported by predictions.
b) It is supported by alternative hypotheses and it is simple.
c) It fits our background information and it is confirmed by observations.
d) It is simple and it fits our background information.

---

4) A hypothesis can be confirmed only if what?

a) It is simple.
b) We can make a prediction from it.
c) It is plausible.
d) We can observe it somehow.

---

5) A confirmation argument has the same pattern as which deductive fallacy?

a) DA
b) AA
c) AC
d) DC

---

6) How is a background assumption BA included in the conditional of a disconfirmation argument?

a) If H-AND-BA then E.
b) IF H then E-OR-BA.
c) If H then E-AND-BA.
d) If H-OR-BA then E.

---

7) Our prediction is false. What does this tell us?

a) Our hypothesis is false.
b) Some background assumption is false.
c) (a) and (b).
d) (a) or (b).

PRACTICE EXAMS

PRACTICE EXAM 1

1) Identify the alternative hypotheses (which compete with the best explanation). Write the implicit statement and give it a number. Reconstruct the argument in standard form as an extended IBE argument.

> ① In the Shanghai Pigeon Association's annual race, pigeons are released far from their home, and the first to return wins a cash prize for its owner. ② Some pigeons have great speed and endurance, but ③ this year two returned home in an incredibly short time – the fastest pigeons ever recorded. ④ The owner claims the breeder bred super-pigeons!
>
> ⑤ It's an outrageous plan but ⑥ the best explanation appears to be that the pigeons' owner recaptured them after the race began and took them back to Shanghai on the high speed train. ⑦ The train makes more sense than a car, with which it would be nearly impossible to drive to Shanghai fast enough in the dense traffic. ⑧ And the super-pigeon idea, in which a breeder figured out how to break laws of biology, is way more complicated than the train scheme. ⑨ It's amazing the things that people will cheat at.

2) Answer the questions based on the story.

In the 10th century, the Cadronian Kingdom dominated the Eastern Middle region. In the year 978, Princess Yesenia was in line to become the Cadronian queen when her father the king died. All she had to do to become queen was marry Prince Rastupin. But instead she fled the castle, giving up her royal status, and lived the rest of her life as a common person.

Historians who study the Cadronian Kingdom have an idea why Yesenia did this: she was influenced by the ideas of a subversive thinker named Flasgar, who led a movement that rejected rule by royalty and urged royals to give up their status in solidarity with the common people. A few of them did. Now the historians are thrilled by the discovery of a scroll that appears to be a letter Yesenia wrote to her sister in 977, shortly before she fled. They expect that if their idea is right, the letter will tell her sister her plan to join Flasgar's movement.

a) What observation do the historians try to explain?
b) What is their hypothesis (explanation)?
c) Think of an alternative hypothesis.
d) What is their prediction?
e) Think of a background assumption for this prediction.

They open the scroll and read the letter. It says nothing about Flasgar's anti-royalty movement. Yesenia mostly complains about her royal duties and gossips about other royals.

f) Write a disconfirmation argument that includes the background assumption from (e).

PRACTICE EXAM 2

1) Identify the alternative hypotheses (which compete with the best explanation). Write the implicit statement and give it a number. Reconstruct the argument in standard form as an extended IBE argument.

> ① The idea of abduction (kidnapping) by aliens has been part of our culture for decades. ② Most famously, in 1961 Betty and Barney Hill claimed they had been briefly abducted from their car by a UFO (alien spaceship). But think about it: ③ aliens flying across the galaxy just to abduct two humans for an hour is a far more complicated explanation of the Hills' claim than another theory, that the Hills were delusional (hallucinating or confused).
>
> And although ④ Betty seemed credible at first, ⑤ delusion makes more sense when you consider that she spent the next twenty years claiming that UFOs were following her, which even other UFO believers thought was crazy. Moreover ⑥ the fact that no one has ever produced any physical evidence of alien abduction makes it hard to believe their story. The conclusion is inescapable: ⑦ the Hills were delusional.

2) Answer the questions based on the story.

Gotham's train system connects downtown to surrounding areas. The Ministry of Transportation is reviewing ridership data and discovers that despite upgrades to the system, ridership has declined. They think that perhaps people are now using the ridesharing service Uber instead of the subway.

They want to find out, since an underused train and more street traffic is bad for Gotham. So they get political support to add an extra fee to Uber rides. The new fee begins. They assume that if the train ridership decline was because of Uber, next month's data will show increased ridership. (The ridership data is based on ticket sales records.)

a) What observation does the Ministry try to explain?
b) What is their hypothesis (explanation)?
c) Think of an alternative hypothesis.
d) What is their prediction?
e) Think of a background assumption for this prediction.

After a month of the new fee on Uber rides, the Transportation Ministry gets new data on train ridership. Oddly, the data shows no increase in subway ridership.

f) Write a disconfirmation argument that includes the background assumption from (e)**.**

PRACTICE EXAM 3

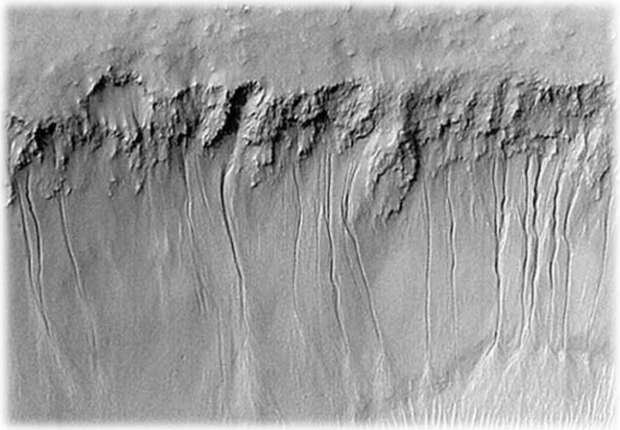
1) Identify the alternative hypotheses (which compete with the best explanation). Write the implicit statement and give it a number. Reconstruct the argument in standard form as an extended IBE argument.

① TV manufacturers used to make 3D TVs that worked with special glasses. But ② the manufacturers stopped making these TVs. ③ The reason this happened is debated. ④ Maybe TV stores didn't promote and demonstrate the 3D technology. ⑤ It's even possible that filmmakers who hate 3D organized a campaign to get manufacturers to drop 3D technology out of respect for traditional film.

⑥ But the best explanation seems to be that TV buyers just found the 3D glasses too annoying. ⑦ This is more sensible considering the fact that the stores had a big profit incentive to sell the new products, and ⑧ the fact that most TV buyers watch TV with their families in living rooms where the 3D glasses make everything in the room look blurry. And obviously ⑨the glasses explanation is just more straightforward than a campaign by angry filmmakers!

2) Answer the questions based on the story.

Scientists at the National Organization for the Study of Planets and Moons (NOSPAM) are studying Mars, looking for liquid water, which is very important for life. They are excited when their telescope shows wavy patterns on Mars. These could be a sign of running water. It's difficult to be sure, but this is high enough that it's worth investigating.



(Image: Encyclopædia Britannica)

The European Space Agency will send a space probe to Mars in a few years. The NOSPAM scientists have an idea. They calculate the amount of water vapour that would be created in the Mars air by running water. Based on their calculation, they design a new instrument, the Water Vapour Detector (WAVD), to attach to the probe. They think: "The WAVD will detect water vapour in the Mars air if those wavy patterns were made by running water."

a) What observation do the NOSPAM scientists try to explain?
b) What is their hypothesis (explanation)?
c) Think of an alternative hypothesis.
d) What is their prediction?
e) Think of a background assumption for this prediction.

The probe flies to Mars and lands safely. NOSPAM activates the WAVD and it begins measuring gases in the air. The scientists are disappointed when the WAVD sends its report back to Earth: no water vapour in the Mars air.

f) Write a disconfirmation argument that includes the background assumption from (e).

PRACTICE EXAM 4

1) Identify the alternative hypotheses (which compete with the best explanation). Write the implicit statement and give it a number. Reconstruct the argument in standard form as an extended IBE argument.

> ① Sparrows are little birds known for the males' birdsong. ② The dominant song across Canada has had a triplet pattern, while a variant song on the West coast has had a doublet pattern instead. Probably ③ the doublet pattern is more attractive to female sparrows. That's suggested by a surprising recent event: ④ the doublet variant spread across the country, replacing the triplet pattern.
>
> ⑤ One idea, that bird lovers have inadvertently trained the sparrows to sing the doublet pattern by giving more birdseed to doublet singers, is obviously far too complicated to take seriously. ⑥ A more reasonable explanation, that the doublet version works better for establishing territories, is not well supported by other observations: birds singing the doublet version don't establish territories more often or quickly. ⑦ Perhaps in the future there'll be some new sparrow song!

2) Answer the questions based on the story.

Abby notices that Bob has not said anything about plans for her upcoming 21st birthday. She thinks maybe this is because Bob is planning a surprise party for her. This seems like a good explanation. Then she thinks: "Bob knows how much I love karaoke. If I'm right about his plan, I bet when I call my favourite karaoke bar, Katie's Karaoke, they'll tell me they have a big reservation on the night of my birthday".

a) What observation does Abby try to explain?
b) What is Abby's hypothesis (explanation)?
c) Think of an alternative hypothesis.
d) What is Abby's prediction?
e) Think of a background assumption for this prediction.

Abby calls Katie's Karaoke and asks them about reservations on the night of her birthday. They tell her that they have no reservations on that night.

f) Write a disconfirmation argument that includes the background assumption from (e).

ANSWER KEY

QUICK TEST

1) c **2)** d **3)** d **4)** b **5)** c **6)** a **7)** d

PRACTICE EXAM 1

1) Answer

Alternative hypotheses:

- The owner drove the pigeons back to Shanghai.
- The breeder bred super-pigeons.

Implicit statement: ⑩ The pigeons' owner recaptured them and took them back to Shanghai on the high speed train.

Main Argument	Sub-argument
③ [Observation]	⑦
$\boxed{\text{⑥}}$ [H is the best explanation.]	⑧
———	———
⑩ [H is true.]	$\boxed{\text{⑥}}$

Explanation (not required for exam answers)

The alternative hypotheses are not numbered statements in the paragraph. They are mentioned or referred to in statements that say something about the hypotheses (e.g. that one is simpler). The role that each statement has in the IBE main argument is given in the answer in [brackets].

2) Answer

a) Princess Yesenia fled the castle and lived the rest of her life as a common person.

b) Yesenia was influenced by the ideas of the subversive thinker, Flasgar.

c) Example answers:
   - Yesenia didn’t want to marry Prince Rastupin.
   - Yesenia wanted to be free of royal duties.
   - Yesenia got a strange brain parasite that made her go crazy.

d) The scroll letter will tell her sister her plan to join Flasgar’s movement.

e) Example answers:
   - Yesenia was not worried the letter would be intercepted by royal spies.
   - Yesenia trusted her sister not to reveal her plan.
   - The letter is not written in a secret code that Yesenia shared with her sister.

f)

- If Yesenia was influenced by Flasgar then the scroll letter will tell her sister her plan to join Flasgar’s movement unless she was worried the letter would be intercepted by royal spies.

  or

  If Yesenia was influenced by Flasgar and she was not worried the letter would be intercepted by royal spies then the scroll letter will tell her sister her plan to join Flasgar’s movement.
- The scroll letter does not tell her sister her plan to join Flasgar’s movement.

Yesenia was not influenced by Flasgar or she was worried the letter would be intercepted by royal spies.

Explanation

For answer (f), there are two correct patterns for the conditional premise of the argument. Choose one of them. Either will get full marks on an exam. Both patterns use an example background assumption, so other answers are correct. Logical operators and conditionals are underlined to help you read the answer, but you do not need to do this for the exam answer.

PRACTICE EXAM 2

1) Answer

Alternative hypothesis:

- Aliens flew across the galaxy to abduct two humans for an hour.

Implicit statement: ⑧ Delusion is the best explanation of the Hills' claims.

Main Argument	Sub-argument
	③
② [Observation]	⑤
⑧ [H is the best explanation.]	⑥
⑦ [H is true.]	⑧

Explanation (not required for exam answers)

The alternative hypotheses are not numbered statements in the paragraph. They are mentioned or referred to in statements that say something about the hypotheses (e.g. that one is simpler). The role that each statement has in the IBE main argument is given in the answer in [brackets].

2) Answer

a) Subway ridership has declined despite system upgrades.

b) People are using Uber instead of the subway.

c) Example answers:
  - People are riding bicycles.
  - People are working from home.
  - (Also correct: The computers that count subway riders are broken and missing riders.)

d) Subway ridership will increase after the new Uber fee.
(Also correct: Their data will show an increase in ridership after the fee.)

e) Example answers:
  - The new fee is large enough to change people's travel behaviour.
  - People do not switch from Uber to Lyft.
  - (Also correct: The computers that count subway riders are not broken.)

f)

- If people are using Uber instead of the subway then subway ridership will increase after the new Uber fee unless the new fee is not large enough to change people's travel behaviour.

  or

  If people are using Uber instead of the subway and the new fee is large enough to change people's travel behaviour then subway ridership will increase after the new Uber fee.
- Subway ridership did not increase after the fee.

People are not using Uber or the new fee was not large enough to change behaviour.

Explanation

The "also correct" answers are not just other example answers, but a different kind of answer. They concern the information, or the observation itself, rather than the fact observed. For example, in (c), the first two example hypotheses explain the fact of reduced ridership. The third hypothesis explains *mistaken information* about reduced ridership (the ridership is not reduced).

For answer (f), there are two correct patterns for the conditional premise of the argument. Choose one of them. Either will get full marks on an exam. Both patterns use an example background assumption, so other answers are correct. Logical operators and conditionals are underlined to help you read the answer, but you do not need to do this for the exam answer.

PRACTICE EXAM 3

1) Answer

Alternative hypotheses:

- TV stores didn’t promote and demonstrate 3D technology.
- Filmmakers who hate 3D coordinated a successful campaign to pressure manufacturers into dropping the technology out of respect for traditional film.

Implicit statement: ⑩ TV buyers found the 3D glasses too annoying.

Main Argument	Sub-argument
	⑦
② [Observation]	⑧
$\boxed{\text{⑥}}$ [H is the best explanation.]	⑨
———	———
⑩ [H is true.]	$\boxed{\text{⑥}}$

Explanation (not required for exam answers)

The alternative hypotheses are not numbered statements in the paragraph. They are mentioned or referred to in statements that say something about the hypotheses (e.g. that one is simpler). The role that each statement has in the IBE main argument is given in the answer in [brackets].

2) Answer

a) There are wavy patterns on Mars.
(Also correct: Images from telescopes show wavy patterns on Mars.)

b) There is running water on Mars.

c) Example answers:
- Wind made the wavy patterns.
- Aliens drew the wavy patterns as a giant art project.
- (Also correct: There is a wavy shaped smudge on the lens of the telescope.)

d) There is water vapour in the Mars air.
(Also correct: The WAVD will detect water vapour in the Mars air.)

e) Example answers:
- The water vapour has not escaped into space.
- (Also correct: NOSPAM’s calculations were correct.)
- (Also correct: The WAVD works as designed (is not damaged in its flight to Mars).)

f)

- If there is running water on Mars then there is water vapour in the Mars air unless the water vapour has escaped into space.

  or

  If there is running water on Mars and the water vapour has not escaped into space then there is water vapour in the Mars air.
- There is no water vapour in the Mars air.

There is not running water on Mars or the water vapour has escaped into space.

Explanation

The “also correct” answers are not just other example answers, but a different kind of answer. They concern the information, or the observation itself, rather than the fact observed. For example, in (c), the first two example hypotheses explain the fact of wavy lines on Mars. The third hypothesis explains *mistaken information* about wavy lines (there are no wavy lines).

For answer (f), there are two correct patterns for the conditional premise of the argument. Choose one of them. Either will get full marks on an exam. Both patterns use an example background assumption, so other answers are correct. Logical operators and conditionals are underlined to help you read the answer, but you do not need to do this for the exam answer.

PRACTICE EXAM 4

1) Answer

Alternative hypotheses:

- Bird lovers have trained sparrows to sing the doublet pattern by giving more birdseed to doublet singers.
- The doublet version works better for establishing territories.

Implicit statement: ⑧ The greater attractiveness of the doublet pattern to females is the best explanation of it spreading across the country.

Main Argument	Sub-argument
④ [Observation]	⑤
⑧ [H is the best explanation.]	⑥
———	———
③ [H is true.]	⑧

Explanation (not required for exam answers)

The alternative hypotheses are not numbered statements in the paragraph. They are mentioned or referred to in statements that say something about the hypotheses (e.g. that one is simpler). The role that each statement has in the IBE main argument is given in the answer in [brackets].

2) Answer

a) Bob has said nothing about her upcoming birthday.

b) Bob is planning a surprise birthday party for Abby.

c) Example answers:

- Bob can’t afford to buy her a present and is hoping that she will forget her own birthday.
- Bob is so busy studying philosophy that he forgot about her birthday.

d) Katie’s Karaoke will tell Abby that there is a big reservation on the night of her birthday.

e) Example answers:

- Bob is not planning a surprise party somewhere else instead.
- Bob has not instructed Katie’s Karaoke to lie to Abby.

f)

- If Bob is planning a surprise party for Abby then Katie’s Karaoke will tell Abby that there is a big reservation on that night unless Bob instructed Katie’s to lie to Abby.

  or

  If Bob is planning a surprise party for Abby and he has not instructed Katie’s Karaoke to lie to Abby then Katie’s will tell Abby that there is a big reservation on that night.
- Katie’s does not tell Abby that there is a big reservation on that night.

Bob is not planning a surprise party for Abby or he has told Katie’s to lie to Abby.

Explanation

For answer (f), there are two correct patterns for the conditional premise of the argument. Choose one of them. Either will get full marks on an exam. Both patterns use an example background assumption, so other answers are correct. Logical operators and conditionals are underlined to help you read the answer, but you do not need to do this for the exam answer.

~ 5 ~

CONFIRMATION BIAS

Confirmation bias is a bad use of evidence that makes a hypothesis seem better supported than it really is. This bias is a natural human tendency. We all instinctively look for evidence that supports the beliefs we already have. We might do this when we want our belief to be true (**wishful thinking**), but we do it even with beliefs we wish were false. While it's very difficult to suppress confirmation bias, we can try to counter it by remembering to look for evidence that we're *wrong*.

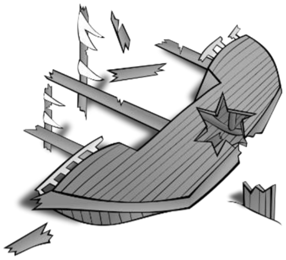
Here are four ways confirmation bias can happen.

---

1) **Ignoring Alternative Explanations** (of confirming evidence)

An observation supports a hypothesis when the hypothesis explains it. If there are *other* explanations, the observation is weaker evidence for the hypothesis, since we can explain the observation without believing the hypothesis. So one form of confirmation bias is ignoring alternative explanations.

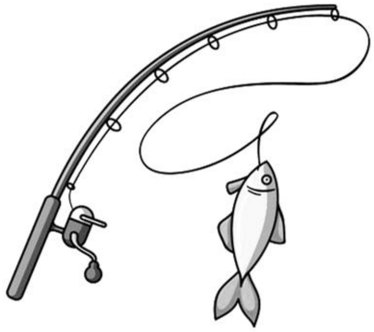
> **Confirmation Bias 5.1**: Bob is walking around his city's harbour. Past the nice part of the harbour, near an old unused dock, there's a broken boat stuck in the sand. Bob has read stories about sea monsters reported throughout history. He thinks: "I bet this boat was broken by a sea monster! A sea monster would probably crash into boats and leave them broken like this."



When Bob finds evidence (the broken boat) that's explained by a sea monster, he constructs a confirmation argument that supports that hypothesis. But there are alternative – and better! – explanations. The boat is near an old dock where there's little activity. The boat could have been smashed in a storm or by a larger boat, or just rotted without maintenance, etc. By ignoring these explanations, Bob makes the boat seem to be stronger support for the sea monster than it really is.

2) Biased Search for Evidence

Confirmation bias can occur in the way that someone looks for, collects, or remembers evidence. They select evidence that confirms their hypothesis and ignore, or filter out, evidence that disconfirms it. We should judge the hypothesis based on our total evidence, and we should actively search for disconfirming evidence to counter our natural inclination to search for confirming evidence.



Confirmation Bias 5.2: "If there's a sea monster," Bob thinks, "probably it eats a lot of fish and people catch fewer fish here than normal." He asks fishing people about their catches. He talks to some in the harbour who tell him they catch a normal amount. He asks others returning on a fishing boat and they caught fish just as easily as they do elsewhere. He asks a man fishing north of the harbour who tells him that fishing is a waste of time there – he catches very few fish. "Aha", Bob thinks, "The sea monster has been hungry!"

Bob uses the small fish catch of one person to confirm his sea monster hypothesis but ignores other people's normal fish catches. He selects evidence to support the hypothesis he already believes.

---

3) *Ad Hoc* Explanation (of disconfirming evidence)

If disconfirming evidence is too obvious to ignore, someone with confirmation bias might give an ***ad hoc* explanation** instead. This is an excuse someone makes up to "protect" their hypothesis from disconfirmation. They have evidence that should make them think their hypothesis might be false, but they're convinced it's true, so they think of some other way to explain the disconfirming evidence.

Confirmation Bias 5.3: Bob writes letters to the local news demanding that the city government search for the sea monster. The news publishes an article. The city puts an underwater camera in the harbour for a few weeks. When they announce that the camera found no sea monster, Bob thinks: "Probably the government doesn't want citizens to panic and flee the city. They're keeping the sea monster a secret."

Bob would accept an announcement that the city had a photo of a sea monster as confirmation of his hypothesis, so he should accept the announcement of no photo as disconfirmation. But he doesn't. He's still sure that there's a sea monster. To protect his hypothesis from disconfirmation, he explains the announcement by saying that the city doesn't want people to panic, though he has no good reason to think that.

---

4) Imprecise Prediction

[This error is not usually called confirmation bias. However the effect of making the error is that confirmation becomes "too easy" and people may overestimate how well a hypothesis is supported, so we'll include it as a form of confirmation bias.]

An **imprecise prediction** is not specific enough for strong confirmation. The less precise it is, the more possible ways it could be true. But in many of these ways, the hypothesis will be false. A prediction is supposed to be a test of the hypothesis, but this test is too easy! Passing it shows almost nothing. An imprecise prediction is another way of protecting a hypothesis from disconfirmation.

Confirmation Bias 5.4: "A sea monster is a big creature that would cause some disturbance," Bob thinks. He spends the week looking around and talking to people. At a cargo loading dock, workers tell him that a container full of TVs somehow fell into the water last month. "Wow, that's certainly a disturbance," Bob thinks. "This harbour has a sea monster!"

Bob's prediction is "some disturbance" around the harbour. Many different things could count as a disturbance. This prediction is not very precise (specific). So it's not surprising, and not strongly confirming, that the prediction is true. There would likely be a disturbance of some sort even if there's no sea monster. It's too easy for the sea monster hypothesis to pass this test.

UNIT 5 SKILLS

You must be able to:

- Recognize and describe examples of confirmation bias in a story.

QUICK TEST QUESTIONS

1) We'll overestimate the support for our hypothesis by ignoring alternative explanations of what?

a) Confirming evidence.
b) Disconfirming evidence.
c) Our prediction.
d) Our test.

2) Which is <u>not</u> an example of a biased search for evidence?

a) Ignoring false predictions.
b) Looking only for confirmation.
c) Remembering only supporting evidence.
d) Testing only true hypotheses.

3) What is explained by an *ad hoc* explanation?

a) Confirming evidence.
b) Disconfirming evidence.
c) A true hypothesis.
d) A false hypothesis.

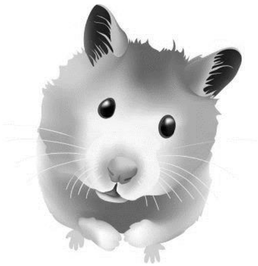
4) An imprecise prediction is bad because it's more likely to be _____ when the hypothesis is _____.

a) …true… true
b) …true… false
c) …false… true
d) …false… false

PRACTICE EXAMS

PRACTICE EXAM 1

Identify the hypothesis. Then evaluate the reasoning. Describe two forms of confirmation bias in the story, referring to specific details.



Bob is buying a hamster. The pet store owner points to one. "That's a Mexican hopping hamster. These evolved to be very active because they're hunted by snakes." Bob buys it and names her Kalypso. After several days of chatting to her, he gets a weird feeling. "I think that Kalypso understands me!"

"This is amazing", he thinks. "I wonder why the pet store owner never noticed this. It'll be great to have a new friend. I hope she stays interested in my life." One day he says "You know, Kalypso, I work really hard at my job and I deserve a raise. Do you think I should ask for one?" She hops up and down in her cage. "She thinks I should! What a nice and clever little furry friend I have!"

The next day after feeding her, he decides to test Kalypso to make sure that he's right about her abilities. He puts three blocks in her cage: a square, a circle, and a triangle. He says: "OK, Kalypso, show me how smart you are. Go sit on the square block." She wanders over to the triangle and grooms her fur. "Huh – I bet she's probably extra hungry today and not in the mood for tests."

He's now convinced and starts posting about Kalypso on Snapagram for his friends to read.

PRACTICE EXAM 2

Identify the hypothesis. Then evaluate the reasoning. Describe two forms of confirmation bias in the story, referring to specific details.

(Image: Vecteezy.com)

Abby is alarmed about her town's new mayor, Ladnod Prumt. She suspects that Prumt is corrupt (that he uses his political power for his own personal gain). She wants the media to expose him as corrupt so that he's removed from office.

Abby figures that if Prumt really is making secret illegal deals (money or gifts in exchange for his votes on city council), he'll have to meet people. She follows him for a few weeks, watching for meetings. One day he meets another man in a coffee shop, and they talk for half an hour. "Gotcha", Abby thinks. "I am going to expose your corrupt ways, Ladnod." She takes a photo.

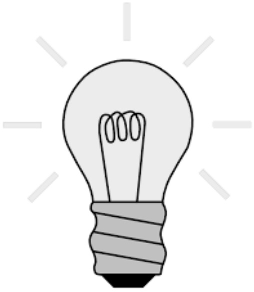
Of course the main thing to look for in a corrupt politician is fancy expensive possessions or activities. She checks Prumt's house. It's pretty normal and needs some new paint. He drives a six year old Honda Civic. Later she spies on him shopping for a new suit at Suits4Less. One day she hides in a bush while he's getting his mail and spies an Omega watch on his wrist. She Googles the price: $7,500.

Abby starts writing emails to the media to alert them that Prumt is corrupt.

PRACTICE EXAM 3

Identify the hypothesis. Then evaluate the reasoning. Describe two forms of confirmation bias in the story, referring to specific details.

Venty Capital is a venture capital firm looking to invest in an innovative startup company. They're interested in Wackadoodle Industries. Venty thinks that Wackadoodle's bold, outside-the-box thinking on how to streamline product development may shake up the industry and make Wackadoodle the next Amazon or Tesla.



One thing Venty looks at in a startup is whether it's building relationships with potential customers who are experts about the industry. That could indicate that Wackadoodle is a good investment. So Venty is excited to learn that Wackadoodle is in talks with Lambast Technologies, whose chief operations officer is the Wackadoodle founder's uncle. Lambast gets government contracts and awards many sub-contracts. "Lambast would recognize a solid company!" Venty thinks.

Venty feels good about Wackadoodle, but still wants a tour of their headquarters. When they arrive, some of Wackadoodle's employees have just put out a fire they accidentally started in the workshop. Many offices are empty and Venty eventually finds everyone crowded into one office playing a PlayStation 5. Down the hall, two engineers are drawing diagrams on a whiteboard. "Hey, look at that," Venty says. "These Wackadoodle folks seem to be real pros!"

They arrange to transfer $10 million to Wackadoodle.

PRACTICE EXAM 4

Identify the hypothesis. Then evaluate the reasoning. Describe two forms of confirmation bias in the story, referring to specific details.

Vaping is popular in Gotham and thousands of people have lung illnesses. The main problem is sarsaparilla flavour vape juice, which is dangerous and addictive, so Gotham banned it. Prohibition led to a black market. The Eastside Sarsaparilla Gang sells it illegally on the street.

The ESG is afraid the Gotham Police Department is watching them. They meet at their headquarters and the leader says: "Hey everyone, if the GPD is watching us, there will probably be something unusual around here. Let's look." They walk around the block and notice a parked van that isn't normally there. They start to panic. "The GPD is going to arrest us!"

The ESG is making deliveries and one says: "There's a helicopter flying around. Let's get a look at it. I bet we'll see GPD markings. Then we'll know for sure we're being watched." They go to a bridge to look at the helicopter with binoculars. On the side it says *NEWS 1120 Traffic Chopper*. "Wow, the GPD is clever! They've disguised their helicopter to look like it's for a news station!"

They look at each other and don't need to say anything else. They race back to the headquarters, grab the cash, and get the hell out of Gotham.

ANSWER KEY

QUICK TEST

1) a **2)** d **3)** b **4)** b

PRACTICE EXAM 1

Answer

Hypothesis: Kalypso understands Bob.

Bob shows confirmation bias in supporting his hypothesis.

1) Ignoring alternative explanations of confirming evidence. Kalypso's hopping seems to be confirming evidence, but she is a Mexican hopping hamster. Maybe she jumps around when Bob asks about the raise because she's naturally hoppy, not because she understands him.
2) *Ad hoc* explanation of disconfirming evidence. When Kalypso fails to show that she understands a simple command, Bob explains his false prediction by thinking that she's extra hungry and not in the mood. But he had no other reason to think this. It looks like an excuse he gives to protect his hypothesis from disconfirmation.

Alternative answers (worth part marks)

1) Imprecise prediction. Although it's not explicit, Bob seems to have predicted that Kalypso will show excitement when he asks her a question. But this prediction is imprecise. Many different hamster actions (hopping, running, quivering, squeaking) might count as showing excitement, so this prediction had a high chance of being true even if Kalypso doesn't understand Bob.
2) Biased search for evidence. Although Bob doesn't ignore the disconfirming evidence (he explains it), he also doesn't seem to be seriously interested in disconfirming his hypothesis. Once he gets the idea that Kalypso understands him, he is interested only in evidence that supports this.

PRACTICE EXAM 2

Answer

Hypothesis: Prumt is corrupt.

Abby shows confirmation bias in supporting her hypothesis.

1) Imprecise prediction: “Meet people” is not very precise. Meeting anyone, anywhere, anytime? There are so many ways this could happen, it’s very likely to be true even if Prumt is not corrupt.
2) Biased search for evidence: She seems to ignore or discount the disconfirming evidence that Prumt’s house, car, and clothes are not fancy or expensive. Instead she selects and focuses on the confirming evidence of his high-end wristwatch.

PRACTICE EXAM 3

Answer

Hypothesis: Wackadoodle is a good startup company to invest in.

Venty shows confirmation bias in supporting their hypothesis.

1) Ignoring an alternative explanation of confirming evidence. Lambast Industries' chief operations officer is the Wackadoodle founder's uncle. Maybe Lambast's interest in Wackadoodle is because of nepotism or family loyalty, not because Wackadoodle is a good company.
2) Biased search for evidence. Venty seems to ignore or discount the signs of incompetence (starting fires) and bad work ethic (playing video games) at Wackadoodle. Instead they select and focus on the confirming evidence of the engineers drawing diagrams.

PRACTICE EXAM 4

Answer

Hypothesis: The GPD is watching the ESG.

The ESG shows confirmation bias in supporting their hypothesis.

1) Imprecise prediction. "Something unusual" is not very precise. Anything at all unusual? An object? An event? There are so many ways for something unusual to happen, this is very likely even if the GPD is not watching them.
2) *Ad hoc* explanation of disconfirming evidence. They predict GPD markings on the helicopter, but when there are none, they explain the false prediction by thinking that the GPD disguised the helicopter. Instead of lowering their confidence in their hypothesis, they make an excuse to protect it from disconfirmation and then are even more convinced.

Alternative Answers (worth part marks)

1) Ignoring an alternative explanation of confirming evidence. Although there are no specific details in the story that suggest an alternative explanation, the confirming evidence (the parked van) is a very ordinary thing. It is easy to think of many explanations other than the ESG's explanation, e.g. the van belongs to a contractor doing repair work for someone.
2) Biased search for evidence. Although the ESG doesn't ignore or dismiss the disconfirming evidence (they explain it), they also don't seem to be seriously interested in disconfirming their hypothesis. Once they get the idea that the GPD is onto them, it seems that nothing will reassure them.

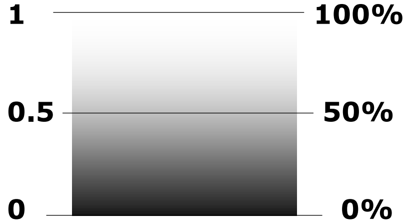
~ 6 ~

PROBABILITY

i. PROBABILITY

The Concept of Probability

A **probability** is a number from 0 to 1 that is assigned to an event or statement. It expresses the likelihood that the event happens or that the statement is true.



The extremes of 1 (100%) (certainly happens) and 0 (0%) (certainly does not happen) apply in special cases, but not in most real-world situations where it's possible (a non-zero chance) for some very strange things to happen. A probability of 50% says that an event is equally likely to happen and to not happen.

Sometimes a probability can be directly calculated based on the details of a situation. For example, we flip a coin. There are two possible outcomes: *Heads* and *Tails*. Since *Heads* is one of two possible outcomes, the probability of *Heads* – ***P***(*Heads*) – is $^1/_2$.

Basic Probability Calculation (for equally likely outcomes)

$$\boldsymbol{P}(\text{A}) \quad = \quad \frac{\text{A-Outcomes}}{\text{Total Possible Outcomes}}$$

This calculation works with a **fair** coin that is equally likely to land *Heads* and *Tails* and can do nothing else (e.g. land on its edge, explode in the air, etc.). This gives the same fraction that we'd get if we flipped the coin again and again, counting the total flips and the *Heads* flips: $^1/_2$. That probability would be based on the frequency of the *Heads* event, how often *Heads* happens when flipping a coin.

Unit 6 examples use these sorts of probabilities. In Unit 7 we'll also see examples of another sort.

Chance and Odds

Chance is often just another word for *probability*. As we saw above, we can express the chance of A as a fraction: [A outcomes]**-in-**[Total outcomes]

Odds is a different way to give a probability.*
Odds is like the score in a sports match:
[A outcomes]**-to-**[B outcomes] or
[A outcomes]**-to-**[Non-A outcomes].

A outcomes + Non-A outcomes = Total outcomes. So we can convert odds into chance.

Odds of A	**Chance of A**
[A outcomes]-to-[Non-A outcomes]	[A outcomes]-in-[Total outcomes]

2:1 = $^2/_3$

2-to-1 = 2-in-3 (2-in-(2+1))

Conditional Probability and Independence

A **conditional probability** ***P***(A**|**B) is the chance that A happens given (assuming) that B happens. When B happens, does that change (raise or lower) the chance of A?

Independent	**Not Independent**
P(A) = ***P***(A**|**B)	***P***(A) ≠ ***P***(A**|**B)
B does not change the chance of A.	B changes the chance of A.

Coin flips are independent. Imagine flipping a coin nine times and getting nine *Heads*.

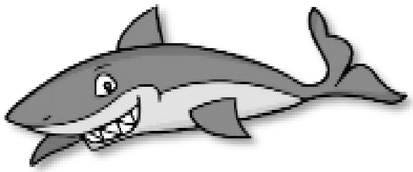
---

* **Vocabulary**: Sometimes people say "odds" when they mean chance (e.g. "the odds are 1-in-100") or "chance" when they mean odds (e.g. "there's a 50-50 chance").

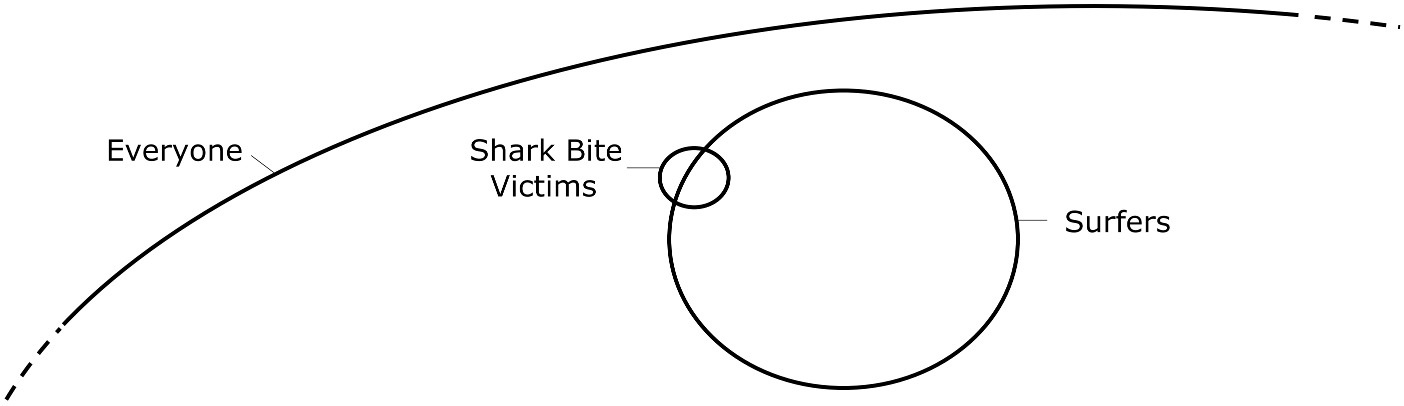
Does it *feel* that *Heads* is less likely (*Tails* is more likely) now? If you're like many people, it does. But the previous nine flips don't matter. ***P***(*Heads*|9*Heads*) = ***P***(*Heads*) = $^{1}/_{2}$. The **gambler's fallacy** is the mistaken belief that an event is less likely when events like it happened recently, or more likely when events like it did not happen recently, though in fact the event is independent of its history.

Some events are *not* independent.

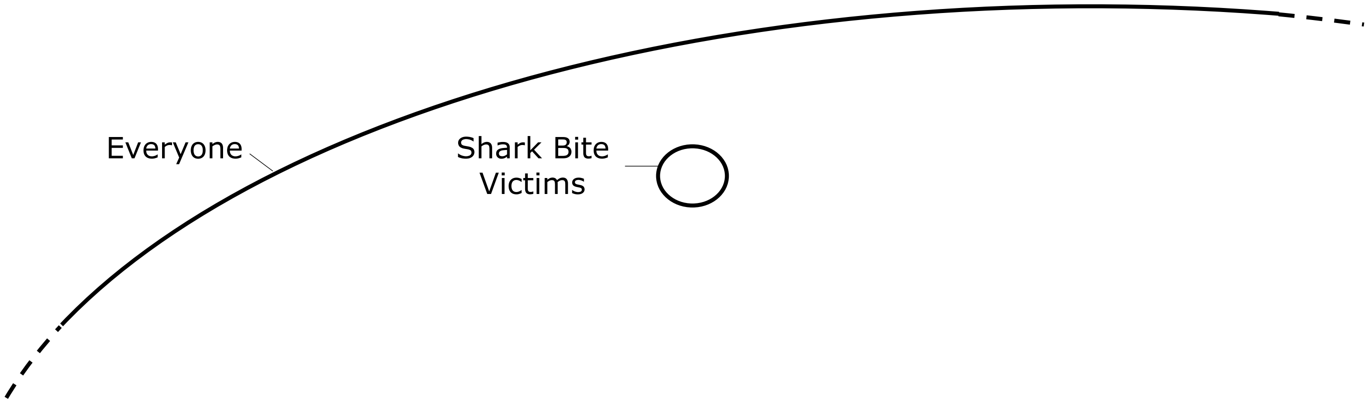
Surf: Abby surfs.
SharkBite: Abby is bitten by a shark.



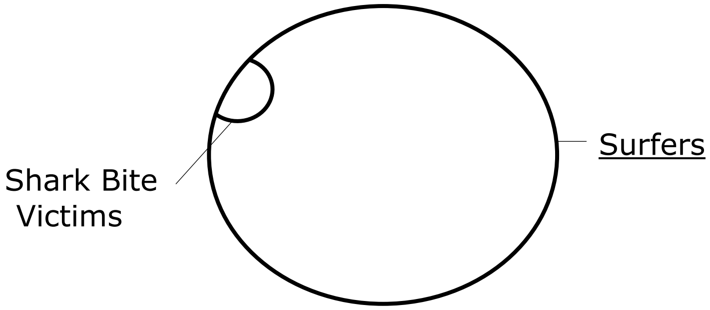
P(SharkBite|Surf) ≠ ***P***(SharkBite). Shark bites are more common among surfers than among people generally (everyone). If we know that Abby surfs, it's more likely that she's bitten by a shark: ***P***(SharkBite|Surf) > ***P***(SharkBite).



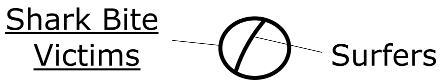
Shark bites are extremely rare among everyone: ***P***(SharkBite) = extremely low.



Shark bites are more common (although still rare) among surfers. The chance of SharkBite, given that Abby surfs, is low: ***P***(SharkBite**|**Surf) = low.



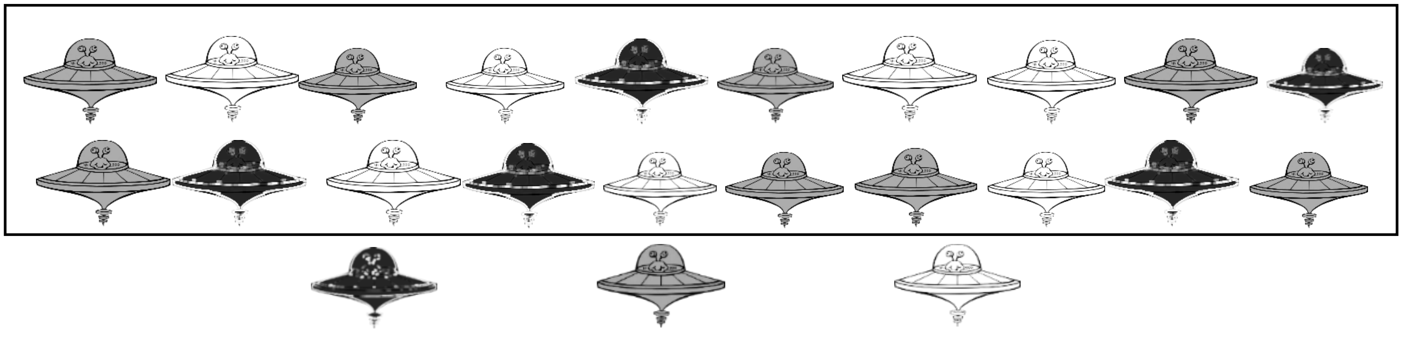
Suppose we know that Abby was bitten by a shark. What's the chance that she surfs? This is a *completely different* conditional probability. Most shark bite victims are surfers! The chance that Abby surfs, given that she was bitten by a shark, is high. ***P***(Surf**|**SharkBite) = high.



ii. **DISJUNCTION**

Disjunction Rule (Special Case: Mutually Exclusive Events)

Twenty UFOs invade Earth: 5 from planet Xenu, 8 from Zargon, and 7 from other planets.

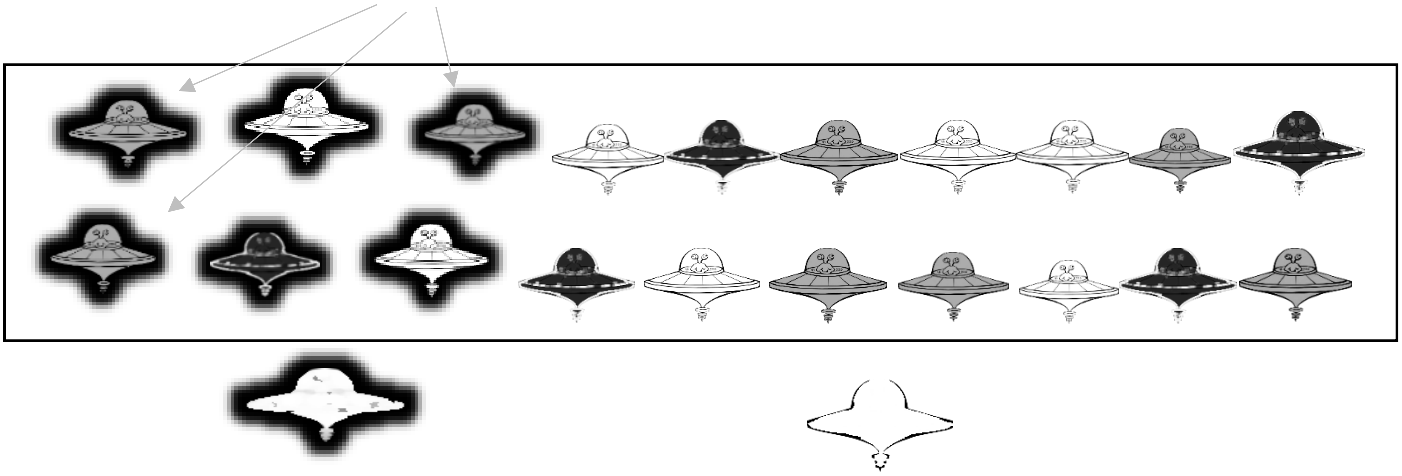


Disjunction uses an addition rule. ***P***(Xen-OR-Zar) = ***P***(Xen) + ***P***(Zar) = $^{5}/_{20}$ + $^{8}/_{20}$ = $^{13}/_{20}$.
This simple rule works here because this is a special case where the events (possibilities) are **mutually exclusive**: a UFO cannot be from more than one planet.

Disjunction Rule

Six UFOs are flown by aliens who plan to abduct (kidnap) humans to do weird experiments on them.

The chance that a UFO is from Zargon OR it's Abducting cannot be ***P***(Zar) + ***P***(Abd) = $^{8}/_{20}$ + $^{6}/_{20}$. That addition double counts 3 Abducting Zargon UFOs.



To correct this, we subtract the **joint probability** ***P***(Zar-AND-Abd), the chance of both events happening. ***P***(Zar) + ***P***(Abd) – ***P***(Zar-AND-Abd) = $^{8}/_{20} + ^{6}/_{20} - ^{3}/_{20} = ^{11}/_{20}$.

Disjunction Rule

P(A-OR-B) = ***P***(A) + ***P***(B) – ***P***(A-AND-B)

In the previous example, the special case of the disjunction of the mutually exclusive events, the subtraction doesn't matter since ***P***(Xen-AND-Zar) = 0.

Event A either happens or it doesn't happen. One of A and ¬A must be true. So ***P***(A-OR-(¬A)) = 1.

The disjunction rule tells us that ***P***(A-OR-(¬A)) = ***P***(A) + ***P***(¬A) – ***P***(A-AND-(¬A)) = 1.

Since A and ¬A are mutually exclusive, ***P***(A-AND-(¬A)) = 0. Therefore: ***P***(A) + ***P***(¬A) – 0 = 1. Therefore ***P***(A) + ***P***(¬A) = 1. Therefore:

Negation Rule

P(A) = 1 – ***P***(¬A)
P(¬A) = 1 – ***P***(A)

iii. CONJUNCTION

Conjunction Rule (Special Case: Independent Events)

We just saw that to calculate the chance of a disjunction, we sometimes need to know the chance of a conjunction, a joint probability. We flip two fair coins, A and B. What's the chance that both coins land *Heads*? Counting possible outcomes, we see the answer: 1-in-4.

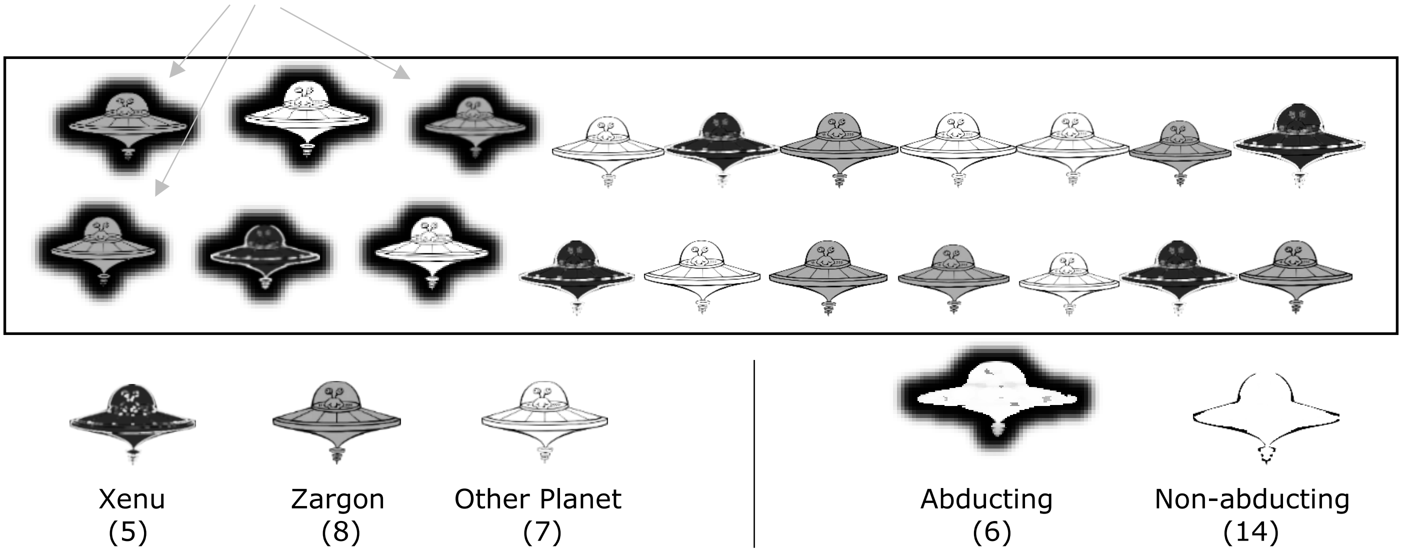

{A*Heads*, B*Heads*}
{A*Heads*, B*Tails*}
{A*Tails*, B*Heads*}
{A*Tails*, B*Tails*}

Conjunction uses a multiplication rule. ***P***(A*Heads*-AND-B*Heads*) = ***P***(A*Heads*) x ***P***(B*Heads*) = ${}^{1}/_{2}$ x ${}^{1}/_{2}$ = ${}^{1}/_{4}$. This simple rule works here because this is a special case where the events are independent.

Conjunction Rule

What is ***P***(Zar-AND-Abd)?

P(Zar) = ${}^{8}/_{20}$ and ***P***(Abd) = ${}^{6}/_{20}$. Multiplying these gives us: ${}^{8}/_{20}$ × ${}^{6}/_{20}$ = ${}^{3}/_{25}$. This is wrong. As we saw in the previous section, there are:
3-in-20 Abducting Zargon UFOs



The problem is that Zargon and Abducting are not independent. We must multiply with:

$\boldsymbol{P}(\text{Abd} \mid \underline{\text{Zar}})$ to show how <u>Zargon</u> changes the chance of Abducting.

$$\boldsymbol{P}(\text{Abd} \mid \underline{\text{Zar}}) \times \boldsymbol{P}(\text{Zar})$$

$$ {}^{3}/_{8} \times {}^{8}/_{20} = {}^{3}/_{20} $$

$\boldsymbol{P}(\text{Zar} \mid \underline{\text{Abd}})$ to show how <u>Abducting</u> changes the chance of Zargon.

$$\boldsymbol{P}(\text{Zar} \mid \underline{\text{Abd}}) \times \boldsymbol{P}(\text{Abd})$$

$$ {}^{3}/_{6} \times {}^{6}/_{20} = {}^{3}/_{20} $$

Conjunction Rule

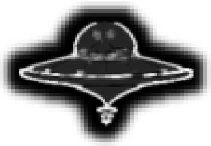
$$\boxed{\boldsymbol{P}(\text{A} \mid \text{B}) \times \boldsymbol{P}(\text{B}) \;=\; \boldsymbol{P}(\text{A-AND-B}) \;=\; \boldsymbol{P}(\text{B} \mid \text{A}) \times \boldsymbol{P}(\text{A})}$$

Disjunction of Conjunctions

We can combine the conjunction and disjunction rules.

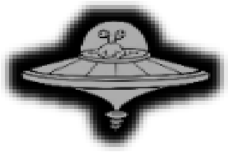
A UFO lands and aliens come out. What's the chance they plan to abduct us? There are three ways this could be an abducting UFO:

Xenu
AND
Abducting



OR

Zargon
AND
Abducting



OR

Other Planet
AND
Abducting

The conjunction rule gives us the chance of each possibility. Since these are mutually exclusive (a UFO cannot be from more than one planet), we add them according to the disjunction rule. The total or overall probability $\boldsymbol{P}(\text{Abd})$ is the sum $\boldsymbol{P}(\text{Xen-AND-Abd}) + \boldsymbol{P}(\text{Zar-AND-Abd}) + \boldsymbol{P}(\text{OP-AND-Abd})$.

$$
\begin{aligned}
\boldsymbol{P}(\text{Abd}) &= \big(\boldsymbol{P}(\text{Abd} \mid \text{Xen}) \times \boldsymbol{P}(\text{Xen})\big) + \big(\boldsymbol{P}(\text{Abd} \mid \text{Zar}) \times \boldsymbol{P}(\text{Zar})\big) + \big(\boldsymbol{P}(\text{Abd} \mid \text{OP}) \times \boldsymbol{P}(\text{OP})\big) \\
&= ({}^{1}/_{5} \times {}^{5}/_{20}) \quad + \quad ({}^{3}/_{8} \times {}^{8}/_{20}) \quad + \quad ({}^{2}/_{7} \times {}^{7}/_{20}) \\
&= {}^{1}/_{20} \quad + \quad {}^{3}/_{20} \quad + \quad {}^{2}/_{20} \\
&= {}^{6}/_{20}
\end{aligned}
$$

We can see in the picture that this answer is correct: 6-in-20 UFOs are abducting.

Three Common Confusions

1) **Is the | symbol in *P*(A|B) a mathematical instruction?**

   No! (A|B) means "A given (assuming) B". The **|** symbol does *not* mean "divided by", "plus", or "multiplied by". A and B are events (or statements), not numbers!

2) **Are joint probability and conditional probability the same?**

   No! They are completely different. We use a conditional probability to calculate a joint probability.

Joint Probability	Conditional Probability
P(A-AND-B)	***P***(A|B)
Chance of <u>two</u> events: A and B	Chance of <u>one</u> event: A

3) **Are mutual exclusion and non-independence the same?**

   No! Mutual exclusion is a special case of non-independence. *Non-independent* means that when one event happens, the chance of the other event changes. *Mutually exclusive* means that when one event happens, the chance of the other event changes *to 0*.

iv. EXPECTED VALUE

Sometimes when deciding what to do, a rational rule is: maximize expected value. We must know the probability and the value (in some units) of each possible outcome of the actions we are considering. An action's **expected value** is the probability-**weighted average** of the values of its possible outcomes. Like a disjunction of conjunctions, this is a "sum of products" calculation.

Expected Value (of Action with Possible Outcomes A and B)

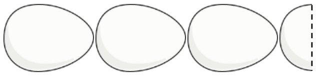
$$\boxed{\text{Expected value} \;=\; (\text{A-Value} \times \boldsymbol{P}(\text{A})) + (\text{B-Value} \times \boldsymbol{P}(\text{B}))}$$

Bob gets a chicken. He wants the most eggs per week. What colour should he paint her coop? He reads about how chickens respond to different colours. In a blue coop, there's a 60% chance his chicken will be happy and lay 5 eggs/week, but a 40% chance she'll be unhappy and lay 1 egg/week. With green, there's an 80% chance of 3 eggs/week and a 20% chance of 2 eggs/week.

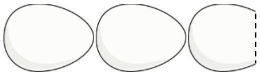
(Image: Big_Ryan / Getty Images)

Assuming that maximum eggs/week is all that matters*, each possible outcome of each action (colour) has a known value (in units of eggs/week). And it has a probability. So Bob can calculate an expected value for each colour and choose the colour that maximizes expected value.

Blue: (5 eggs/wk × 0.6) + (1 egg/wk × 0.4) = 3.4 eggs/wk



Green: (3 eggs/wk × 0.8) + (2 eggs/wk × 0.2) = 2.8 eggs/wk



Bob should paint the coop blue to maximize the expected eggs/wk.

---

[*] This is not realistic! Maybe Bob is happy with 4 eggs/week and a 5th egg has no value to him. Maybe he needs at least 2 eggs/week and 1 egg would be a disaster. Maybe the chicken's happiness has value that should be counted (a moral consideration). Maximizing expected value in real life is complicated.

v. AT LEAST ONE

Chance of At Least One (Equally Probable Cases)

We roll some dice. What's the chance of rolling any *3*s (at least one *3*)? Since *any* means "not none", the chance of any equals 1 minus the chance of none. With more and more dice, the chance of rolling no *3*s gets closer and closer to 0 and the chance of rolling any *3*s gets closer and closer to 1.

***P*(At-Least-1 A)** (with *N* equally probable cases)

$$\boxed{\boldsymbol{P}(\text{AL1-A}(N\,\text{Cases})) \quad = \quad 1 - \boldsymbol{P}(\neg\text{A})^{N} \quad = \quad 1 - (1 - \boldsymbol{P}(\text{A}))^{N}}$$

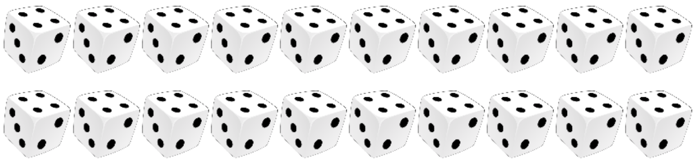
For $\boldsymbol{P}(\text{AL1-Roll3}(N\,\text{Dice}))$ we put an exponent *N* on $\boldsymbol{P}(\neg\text{Roll}3)$ for the number of dice. That tells us to multiply the chance of $\neg$Roll*3* by itself for every dice we roll. With one dice, the exponent is 1, and the chance of rolling at least one *3* (AL1Roll*3*(1Dice)) is simply the chance of rolling *3*: $^{1}/_{6}$.

$$\boldsymbol{P}(\text{AL1-Roll}3(1\text{Dice})) \quad = \quad 1 - \boldsymbol{P}(\neg\text{Roll}3)^{1} \quad = \quad 1 - (1 - {}^{1}/_{6})^{1} \quad = \quad 1 - {}^{5}/_{6} \qquad \approx \quad 0.167$$

The solution is the same for more than one dice. We just adjust the exponent. For 2 dice:

$$\boldsymbol{P}(\text{AL1-Roll}3(2\text{Dice})) \quad = \quad 1 - \boldsymbol{P}(\neg\text{Roll}3)^{2} \quad = \quad 1 - (1 - {}^{1}/_{6})^{2} \quad = \quad 1 - {}^{25}/_{36} \qquad \approx \quad 0.306$$

The solution works the same for any number of cases. For 20 dice, the numbers in the fraction are large, but the structure of the solution is the same.



$$\boldsymbol{P}(\text{AL1-Roll}3(20\text{Dice})) \quad = \quad 1 - (1 - {}^{1}/_{6})^{20} \quad = \quad 1 - {}^{95,367,431,640,625}/_{3,656,158,440,062,976} \quad \approx \quad 0.974$$

Since the chance of rolling no *3*s never reaches 0, the chance of at least one *3* never reaches 1.

UNIT 6 SKILLS

<u>You must be able to</u>:

- Recognize and write probabilities, including conditional probabilities in the chance and odds forms.
- Distinguish independent and non-independent events; recognize the gambler's fallacy.
- Use the disjunction and conjunction rules separately and together (disjunction of conjunctions).
- Calculate the chance that an event happens at least once.
- Determine a correct decision based on expected value.

QUICK TEST QUESTIONS

1) "More likely to happen than not to happen" means what probability?

a) 100%
b) $> 50\%$
c) $\geq 50\%$
d) $> 0\%$

2) Convert "5-to-3 odds" into a chance.

a) 5-in-3
b) 3-in-5
c) 2-in-3
d) 5-in-8

3) What does $\boldsymbol{P}(A|B)$ mean?

a) Chance of A times chance of B.
b) Chance of A divided by chance of B.
c) Chance of A assuming B.
d) Chance of A and chance of B.

4) Do $\boldsymbol{P}(\text{A-AND-B})$ and $\boldsymbol{P}(A|B)$ mean the same thing?

a) Yes.
b) No, but close enough that you don't need to worry about the difference.
c) No.
d) *NO!!!*

5) Which is true?

a) If A and B are mutually exclusive, they are independent.
b) If A and B are mutually exclusive, they are not independent.
c) If A and B are not independent, they are mutually exclusive.
d) A and B are mutually exclusive if and only if they are not independent.

6) Which says "the probability that A happens given that B happens"?

a) $\boldsymbol{P}(A|B)$
b) $\boldsymbol{P}(A)|\boldsymbol{P}(B)$
c) $\boldsymbol{P}(A)|\boldsymbol{P}(B)$
d) $\boldsymbol{P}(B|A)$

7) When are two events A and B independent?

a) $\boldsymbol{P}(A) = \boldsymbol{P}(B|A)$
b) $\boldsymbol{P}(A) \neq \boldsymbol{P}(B|A)$
c) $\boldsymbol{P}(A) \neq \boldsymbol{P}(A|B)$
d) $\boldsymbol{P}(A) = \boldsymbol{P}(A|B)$

8) In the gambler's fallacy, someone mistakenly thinks the chance of an event depends on _________.

a) nothing
b) its future
c) its history
d) a bet that they have made

9) "Few A are B." What probability does this give us?

a) ***P***(B|A) is low.
b) ***P***(A|B) is low.
c) ***P***(A|B) is high.
d) (a) and (b).

---

10) When are two events mutually exclusive?

a) They cannot both happen.
b) They can both happen.
c) Neither of them can happen.
d) If one happens, the other must happen.

---

11) What is ***P***(A-OR-B)?

a) Between ***P***(A) and ***P***(B).
b) ≤ ***P***(A)
c) ≥ ***P***(A)
d) ***P***(A) minus ***P***(B).

---

12) Which is always true?

a) ***P***(A) × ***P***(¬A) = 1
b) ***P***(A) + ***P***(¬A) = 1
c) ***P***(A) + ***P***(¬A) ≤ 1
d) ***P***(A) + ***P***(¬A) = 0

---

13) How is ***P***(A-AND-B) calculated?

a) Multiply probabilities.
b) Average probabilities.
c) Find the highest probability.
d) Add probabilities.

---

14) What is ***P***(A-AND-B)?

a) ≥ ***P***(A)
b) Not enough information to know.
c) Between ***P***(A) and ***P***(B).
d) ≤ ***P***(A)

---

15) What is the joint probability of two mutually exclusive events?

a) 100%
b) 0%
c) 50%
d) Not enough information to know.

---

16) How is the chance of a disjunction of conjunctions calculated?

a) Sum of products.
b) Sum of sums.
c) Product of sums.
d) Product of products.

---

17) How many is "not at least 1"?

a) All
b) > 1
c) None
d) 1

PRACTICE EXAMS

PRACTICE EXAM 1

1) Answer the questions based on the story. Show your calculation.

The FRAZ-3 virus is spreading across Azmakia and the country is racing to develop a vaccine that is both safe and effective. Azmakia's labs have been developing several vaccines and now 5 are ready for human trials. Based on past trials, the Health Department knows that a trial vaccine has a $^3/_4$ chance of being safe and a $^1/_3$ chance of being effective.

a) What's the chance that the first vaccine is safe or effective?
b) What's the chance that any of the vaccines are effective?
c) What's the chance that all the vaccines are safe?
d) They test 4 vaccines and none are effective. The Health Department says: "Well, after that, this last vaccine is sure to be effective." Evaluate the Health Department's reasoning.

2) Evaluate the reasoning.

Azmakia has been battling an epidemic of the new FRAZ-3 virus. The country finally has a vaccine and the government has been making good progress getting the population vaccinated. Most people have been vaccinated. But now the government gets some shocking data from the hospitals: 80% of infected people are vaccinated (an infected person is 80% likely to have been vaccinated)! The government is upset – they think that the vaccine doesn't work well.

3) Answer the questions based on the story. Show your calculation.

Abby and Bob have kept ratings of their meals at two restaurants: The Lemon and Antique Land. Today they want the best meal. Which restaurant should they choose?

At The Lemon, 10% of their meals are 1 star, 20% are 2 stars, and 70% are 3 stars, and none are 4 stars. At Antique Land, 20% of their meals are 1 star, 30% are 2 stars, 10% are 3 stars, 30% are 4 stars, and the other times it's closed without notice (0 stars).

PRACTICE EXAM 2

1) Answer the questions based on the story. Show your calculation.

Two viruses begin to spread in Azmakia: FRAZ-3 and GLOP-2. The Health Department wants to prioritize a vaccine for the virus they expect to infect more people. What should they do?

With any virus, an infected person could be a Superspreader. With FRAZ-3, the chance of being a Superspreader is 5%; with GLOP-2, the chance is 10%. FRAZ-3 Superspreaders infect an average of 25.5 other people, while Normals infect 1.6 others. GLOP-2 Superspreaders infect 10.9 others, while Normals infect 2.1 others.

2) Answer the questions based on the story. Show your calculation.

Wackadoodle Industries has a great new product design and they're concerned that a member of the development team will betray the company by selling the design to Wackadoodle's competitor. They want to assign the product's development to the team with the lowest chance of betrayal. Which team should the designers choose?

Team A is loyal but large: each member has a 0.25% chance of betrayal, but there are 60 members. Team B is small but less loyal: 22 members, each with a 0.75% chance of betrayal.

3) Evaluate the reasoning.

There's a large tropical storm in the West Pacific. Probably it'll become a typhoon. Although historically only about 1-in-3 of the storms like this one have developed into a typhoon, it's been several years since there was a typhoon in the West Pacific, and there's normally a typhoon there every year.

4) Answer the questions based on the story. Show your calculation.

Bob and Abby are going to the movies. Bob will choose the movie. What's the chance that Abby falls asleep during the movie?

There's a 30% chance Bob will choose a comedy, a 50% chance he'll choose an action film, and a 20% chance he'll choose a drama. There's a 20% chance Abby will fall asleep during a comedy, a 40% chance she'll fall asleep during an action film, and a 10% chance she'll fall asleep during a drama.

PRACTICE EXAM 3

1) Write the conditional probabilities that correspond to the statistical generalizations. Draw a diagram with two labelled circles that shows (imagines) both statements true.

Most things that make the world better (TTMWB) are charities (C).

P(**|**) = high

Few charities make the world a better place.

P(**|**) = low

2) Answer the questions based on the story. Show your calculations.

Abby is a public health inspector for the Gotham Health Department. This week she will randomly pick 8 restaurants from the database and do surprise inspections. Normally when the GHD does surprise inspections, they find food safety code violations in 15% of restaurants.

a) What's the chance that both of the first 2 restaurants Abby inspects have a code violation?
b) What's the chance that Abby finds any code violations in her inspections this week?

Restaurants in Gotham serve cuisine from several different countries. The most popular food is Azmakian: 26% of Gotham restaurants. Azmakian restaurants are typical for food safety: no more and no less likely to have a code violation.

c) What's the chance that the first restaurant Abby inspects is Azmakian or has a code violation?

Other kinds of restaurants are not typical for food safety. Kazamni restaurants (less popular: only 9% of Gotham restaurants) are worse. Abby knows from her past inspection data that a restaurant with a code violation has a shocking 2/5 chance of being Kazamni.

d) What's the chance that the first restaurant Abby inspects is Kazamni and has a code violation?

e) Abby visits 7 restaurants and all of them have code violations. She thinks: "This is crazy! Well after that I can be pretty sure my 8th restaurant won't have a code violation." Evaluate Abby's reasoning.

ANSWER KEY

QUICK TEST

1) b **2)** d **3)** c **4)** d **5)** b **6)** a **7)** d **8)** c **9)** a **10)** a **11)** c **12)** b **13)** a **14)** d
15) b **16)** a **17)** c

PRACTICE EXAM 1

1)

a) **Answer**

$$\begin{aligned} \boldsymbol{P}(\text{Safe-OR-Effective}) &= \boldsymbol{P}(\text{Safe}) + \boldsymbol{P}(\text{Effective}) - \boldsymbol{P}(\text{Safe-AND-Effective}) \\ &= {}^{3}/_{4} + {}^{1}/_{3} - ({}^{3}/_{4} \times {}^{1}/_{3}) \\ &= {}^{9}/_{12} + {}^{4}/_{12} - {}^{3}/_{12} \\ &= {}^{10}/_{12} \quad = \quad {}^{5}/_{6} \ \text{[Also correct: 0.833]} \end{aligned}$$

Explanation (not required for exam answers)

Disjunction rule for non-mutually exclusive events. Subtraction of the joint probability corrects for overcounting with addition. Since we have no information that effectiveness and safety are non-independent (nothing says that either event changes the chance of the other), we must treat them as independent and multiply their probabilities for the joint probability.

b) **Answer**

$$\begin{aligned} \boldsymbol{P}(\text{AL1-Effective(5Vaccines)}) &= 1 - \boldsymbol{P}(\neg\text{Effective})^{5} \quad = \quad 1 - (1 - \boldsymbol{P}(\text{Effective}))^{5} \\ &= 1 - (1 - {}^{1}/_{3})^{5} \\ &= 1 - {}^{32}/_{243} \\ &= {}^{211}/_{243} \ \text{[Also correct: 0.868]} \end{aligned}$$

Explanation

Any makes this an *At Least 1* probability. The chance of any effective equals 1 minus the chance of none effective. “None effective” is the conjunction of 5 ineffective vaccines. Since every vaccine has the same chance of being ineffective (they are independent for effectiveness), we can use an exponent for the multiplication.

c) **Answer**

$\boldsymbol{P}(\text{Safe(5Vaccines)}) = \boldsymbol{P}(\text{Safe})^5$

$= ({}^3/_4)^5$

$= {}^{243}/_{1,024}$ [Also correct: 0.237]

Explanation

Conjunction rule. Vaccines are independent for safety: every vaccine in a human trial has the same chance of being safe. So we can use an exponent for the multiplication.

d) **Answer**

This reasoning is the gambler's fallacy. The vaccines are independent. The chance of effectiveness is ${}^1/_3$ for every vaccine. It does not go up after several ineffective vaccines.

2) **Answer**

Basic answer: The government is confused about conditional probabilities. Most infected people were vaccinated (***P***(Vaccinated|Infected) is high). This does not mean that most vaccinated people get infected (***P***(Infected|Vaccinated) is high). Maybe (hopefully) very few vaccinated people get infected.

Extended answer: As more and more people get vaccinated, the percentage of infected people who were vaccinated must rise since there are fewer and fewer unvaccinated people left. What's important (for public health) is that the number of infected people goes down even if the percentage of them who were vaccinated goes up.

3) **Answer**

Expected meal rating = (0 Stars × ***P***(0Stars)) + (1 Star × ***P***(1Star)) + (2 Stars × ***P***(2Stars)) + (3 Stars × ***P***(3Stars)) + (4 Stars × ***P***(4Stars))

The Lemon: (0 × 0) + (1 × 0.1) + (2 × 0.2) + (3 × 0.7) + (4 × 0) = 2.6 stars

Antique Land: (0 × 0.1) + (1 × 0.2) + (2 × 0.3) + (3 × 0.1) + (4 × 0.3) = 2.3 stars

They should choose The Lemon for lunch.

Explanation

Expected values. The units of value are rating stars. For each restaurant, the expected rating is a sum of products, the frequency-weighted average of the possible ratings. The weights add to 1. The solution here is fully detailed for clarity. On the exam, you may omit 0-weighted terms.

PRACTICE EXAM 2

1) **Answer**

Expected new infections from each infected person = (Normal Infections × ***P***(Normal)) + (Superspreader Infections × ***P***(Superspreader))

FRAZ-3: (1.6 × 0.95) + (25.5 × 0.05) = 1.52 + 1.275 = 2.795 infections

GLOP-2: (2.1 × 0.9) + (10.9 × 0.1) = 1.89 + 1.09 = 2.98 infections

The Health Department should prioritize vaccine development for GLOP-2.

Explanation

Expected values. The units of value are new infections. For each virus, the solution is the sum of products, a probability-weighted average of the possible new infections. The weights must add up to 1. Since each infected person is a Normal or a Superspreader, ***P***(Normal) = 1 – ***P***(Superspreader).

2) **Answer**

$$\boldsymbol{P}(\text{AL1-ABetray}) = 1 - (1 - \boldsymbol{P}(\text{ABetray}))^{60}$$
$$= 1 - (1 - 0.0025)^{60} \approx 0.139$$
$$\boldsymbol{P}(\text{AL1-BBetray}) = 1 - (1 - \boldsymbol{P}(\text{BBetray}))^{22}$$
$$= 1 - (1 - 0.0075)^{22} \approx 0.153$$

Wackadoodle should assign the product development to team A.

Explanation

Wackadoodle's competitor gets their new product design if *any* team member betrays the company. That's the probability that Wackadoodle must consider. So this is an *At Least 1* question. Since they want the lowest chance of betrayal, team A is the better choice.

3) **Answer**

This looks like the gambler's fallacy. If there is no reason to believe that a recent lack of typhoons raises the chance of a typhoon this year, the way this storm develops is independent of the recent weather. The statistics give every storm the same 1-in-3 chance of becoming a typhoon.

4) **Answer**

$$\boldsymbol{P}(\text{FallAsleep}) = (\boldsymbol{P}(\text{FA}|\text{Com}) \times \boldsymbol{P}(\text{Com})) + (\boldsymbol{P}(\text{FA}|\text{Act}) \times \boldsymbol{P}(\text{Act})) + (\boldsymbol{P}(\text{FA}|\text{Dra}) \times \boldsymbol{P}(\text{Dra}))$$
$$= (0.2 \times 0.3) + (0.4 \times 0.5) + (0.1 \times 0.2) = 0.28$$

Explanation

Conjunction and disjunction rules combined. The solution is the sum of products, a probability weighted average of the probabilities of the possible movie choices. The weights add up to 1, as they must. The answer, the average of the *Fall Asleep* probabilities, is between the high (40%) and low (10%) probabilities.

PRACTICE EXAM 3

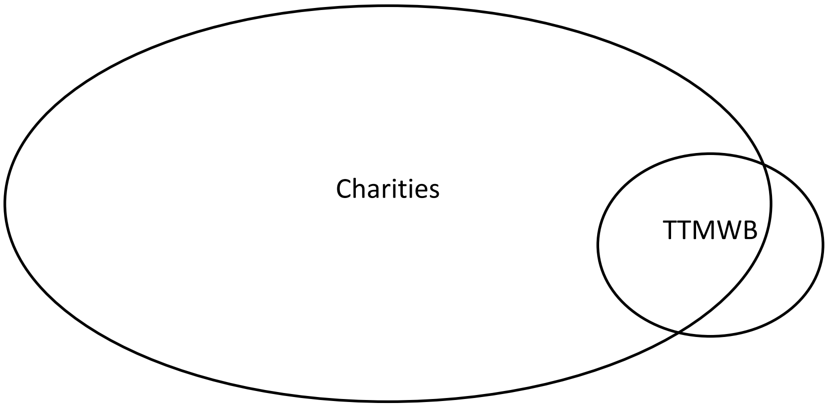
1) Answer

Most things that make the world better (TTMWB) are charities (C).

$\boldsymbol{P}(\mathrm{C} | \mathrm{TTMWB}) = \text{high}$

Few charities make the world a better place.

$\boldsymbol{P}(\mathrm{TTMWB} | \mathrm{C}) = \text{low}$



2)

a) Answer

$$\begin{aligned} \boldsymbol{P}(\mathrm{CV}(2\mathrm{Restaurants})) &= \boldsymbol{P}(\mathrm{CV})^2 \\ &= (0.15)^2 \\ &= 0.0225 \end{aligned}$$

Explanation (not required for exam answers)

Conjunction rule. Restaurants are independent for code violations: every restaurant has the same chance of a violation. So we multiply 15% × 15% or use an exponent of 2.

b) Answer

$$\begin{aligned} \boldsymbol{P}(\mathrm{AL1\text{-}CV}(8\mathrm{Resturants})) &= 1 - (1 - \boldsymbol{P}(\mathrm{CV}))^8 \\ &= 1 - (1 - 0.15)^8 \approx 0.728 \end{aligned}$$

Explanation

Any makes this an *At Least 1* probability. The chance of any violations equals 1 minus the chance of none. “None” is the conjunction of 8 violations. Since every restaurant has the chance of a violation (they are independent for food safety), we can use an exponent for the multiplication.

c) **Answer**

$$\begin{aligned}\boldsymbol{P}(\text{CV-OR-Az}) &= \boldsymbol{P}(\text{CV}) + \boldsymbol{P}(\text{Az}) - \boldsymbol{P}(\text{CV-AND-Az})\\ &= 0.15 + 0.26 - (0.15 \times 0.26)\\ &= 0.41 - 0.039\\ &= 0.371\end{aligned}$$

Explanation

Disjunction rule for independent events. Subtraction of the joint probability corrects for overcounting with addition. Since CV is independent of Azmakian, we multiply their probabilities for the joint probability.

d) **Answer**

$$\begin{aligned}\boldsymbol{P}(\text{Kaz-AND-CV}) &= \boldsymbol{P}(\text{Kaz}\,|\,\text{CV}) \times \boldsymbol{P}(\text{CV})\\ &= {}^{2}/_{5} \times 0.15\\ &= 0.06\end{aligned}$$

Explanation

Conjunction rule. Kazamni and code violation are not independent – a violation raises the chance that a restaurant is Kazamni. We need the conditional probability of Kazamni given violation. There are two solutions for joint probability. The other solution is $\boldsymbol{P}(\text{CV}\,|\,\text{Kaz}) \times \boldsymbol{P}(\text{Kaz})$. However, although we have $\boldsymbol{P}(\text{Kaz})$, we don’t know $\boldsymbol{P}(\text{CV}\,|\,\text{Kaz})$ (the chance that a Kazamni restaurant has a violation), $\boldsymbol{P}(\text{CV}\,|\,\text{Kaz})$, so this solution is not usable.

e) **Answer**

This reasoning is the gambler’s fallacy. The restaurants are independent. The chance of a violation is 15% for every restaurant. It does not go up after several restaurants with violations.

~ 7 ~

BAYES' RULE

i. BAYES' RULE

Two things matter when we judge the probability of a hypothesis based on some new evidence:

1) Prior Probability of the Hypothesis

When new evidence E gives H a new probability, what is that new probability? This depends on how likely H was *before* the new evidence, its **prior probability**. This is the **plausibility** (believability) of H, based on background information, or the **base rate** or **prevalence** (how often H is true in this kind of situation).

$$\boxed{\boldsymbol{P}(H)}$$

2) Strength of the Evidence

Evidence strength is the power of the evidence to change the probability of the hypothesis. Evidence strength is the ratio of two probabilities:

$$\frac{\text{Chance of Evidence E if Hypothesis H is }\underline{\text{true}}}{\text{Chance of Evidence E if Hypothesis H is }\underline{\text{false}}} \qquad \boxed{\frac{\boldsymbol{P}(E|H)}{\boldsymbol{P}(E|\neg H)}}$$

Evidence strength > 1: E **confirms**. E raises the chance of H.

Evidence strength < 1: E **disconfirms**. E lowers the chance of H.

We need a tool to **update** the probability of a hypothesis from its prior probability based on the strength of some new evidence. This tool is **Bayes' rule**. Although the rule is mathematical, it teaches some general lessons that are valuable even without a calculation. We've just learned the first two lessons:

BAYES LESSON #1: Consider the prior probability.

BAYES LESSON #2: Confirming E: more likely if H is true, less likely if H is false.

Bayes' Rule

Bayes' rule for updating the chance of H based on E combines the AND and OR rules we learned in Unit 6. The rule has a short form and an expanded form.

The conjunction rule for Hypothesis-AND-Evidence can be written two ways:

$$\boldsymbol{P}(H \mid E) \times \boldsymbol{P}(E) = \boldsymbol{P}(E \mid H) \times \boldsymbol{P}(H)$$

To get **P**(H|E), we divide both sides by **P**(E). On the left side, **P**(E) cancels.

$$\frac{\boldsymbol{P}(H \mid E) \cancel{\times \boldsymbol{P}(E)}}{\cancel{\boldsymbol{P}(E)}} = \frac{\boldsymbol{P}(E \mid H) \times \boldsymbol{P}(H)}{\boldsymbol{P}(E)}$$

This is the short form of Bayes' rule.

$$\boldsymbol{P}(H \mid E) = \frac{\boldsymbol{P}(E \mid H) \times \boldsymbol{P}(H)}{\boldsymbol{P}(E)}$$

If E happens, it happens in one of two situations: H true or H false. To show the overall chance of E, we can expand the **P**(E) denominator using the AND (multiplication) and OR (addition) rules.

E: (E-AND-H) - OR - (E-AND-(¬H))

$$\boldsymbol{P}(E) = (\boldsymbol{P}(E \mid H) \times \boldsymbol{P}(H)) + (\boldsymbol{P}(E \mid \neg H) \times \boldsymbol{P}(\neg H))$$

The expanded form of Bayes' rule uses this expanded denominator.

Bayes' Rule

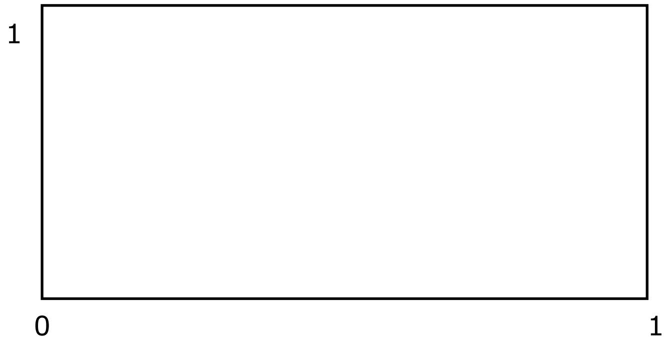
$$\boxed{\boldsymbol{P}(H \mid E) = \frac{(\boxed{\boldsymbol{P}(E \mid H)} \times \boxed{\boldsymbol{P}(H)})}{(\boldsymbol{P}(E \mid H) \times \boldsymbol{P}(H)) + (\boxed{\boldsymbol{P}(E \mid \neg H)} \times \boldsymbol{P}(\neg H))}}$$

In the denominator, **P**(E|H) and **P**(H) are copied from the numerator; and **P**(¬H) equals 1 – **P**(H), so it's also based on the numerator. As with the short form of Bayes' rule, we need three probabilities: $\boxed{\boldsymbol{P}(E \mid H)}$ and $\boxed{\boldsymbol{P}(E \mid \neg H)}$ for the evidence strength, and the prior probability $\boxed{\boldsymbol{P}(H)}$.

How to Draw a Bayes Box

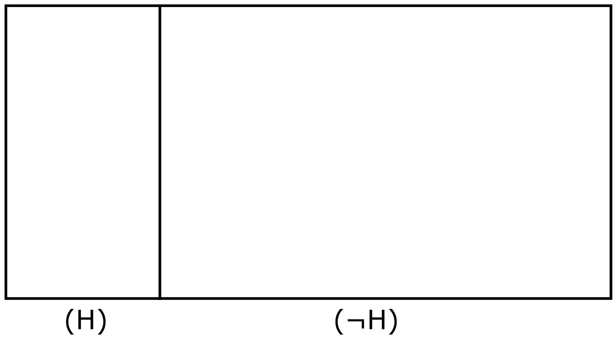
A **Bayes box** is a diagram constructed with the numbers used in Bayes' rule. It's a simple way to show an updated probability as an odds ratio.

Each side of the box equals 1 (100%).
We'll put 0 on the left, at the bottom.

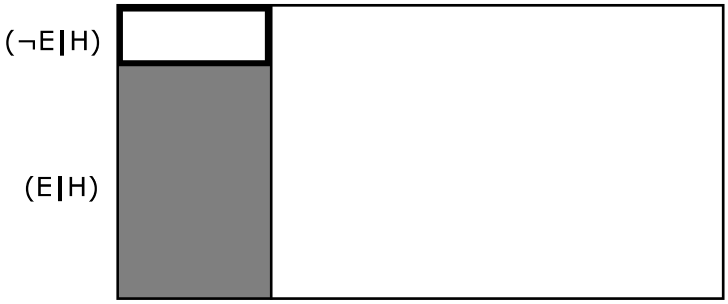


Here is an example Bayes box.
We'll fill in the box in three steps.

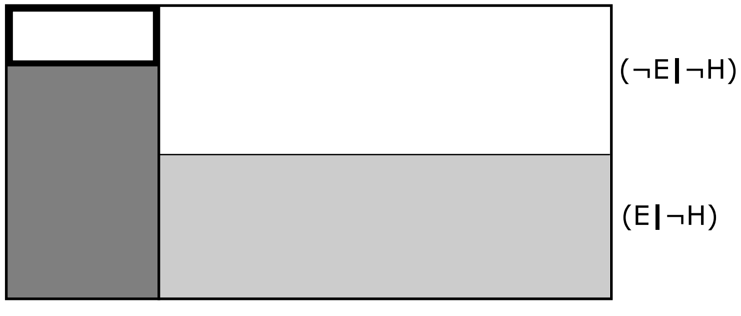
STEP 1
What are the prior odds of H?
(Divide the box between ***P***(H) and ***P***(¬H).)



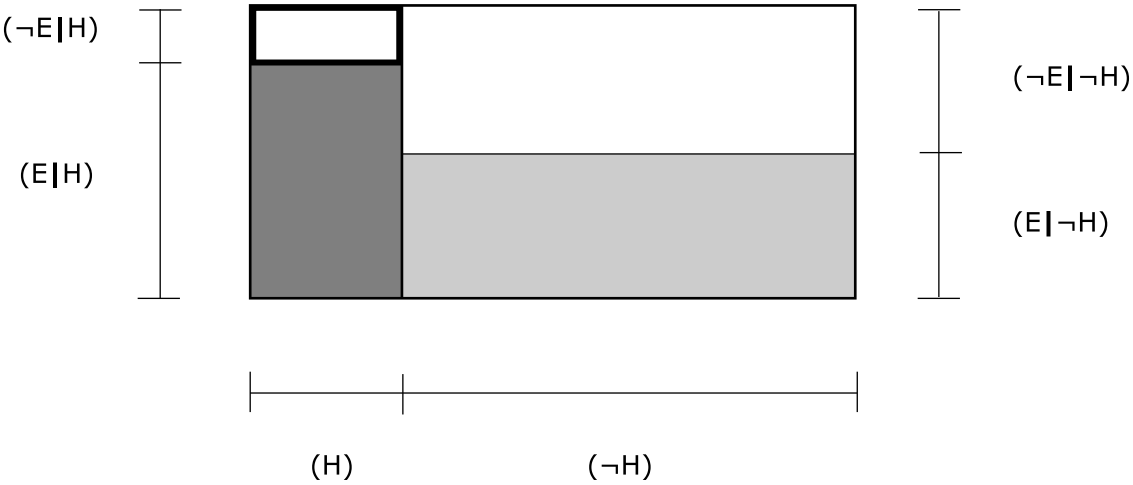
STEP 2
What's the chance of E if H is <u>true</u>?



STEP 3
What's the chance of E if H is <u>false</u>?

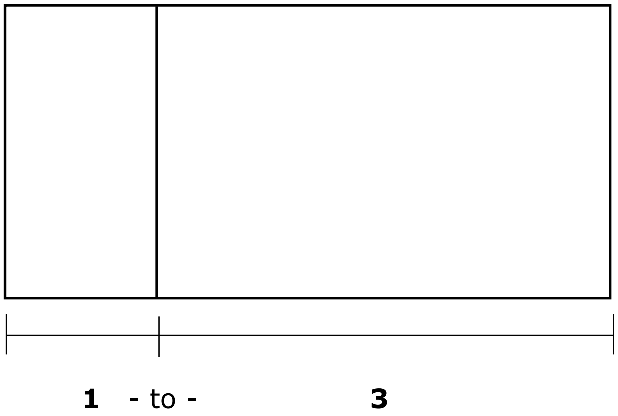


Example Bayes Box



How to Look at a Bayes Box

The example Bayes box above is drawn to show prior odds of H of 1-to-3.



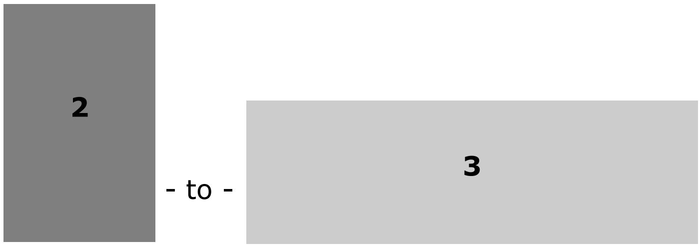
So the prior probability of H is 1-in-(1+3) = $^1/_4$.

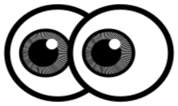
When the box is filled in with the Evidence probabilities ***P***(E|H) and ***P***(E|¬H), we get two pairs of rectangles: a shaded pair for E, and an empty (unshaded) pair for ¬E. We'll first compare the E rectangles to update H on E; then we'll compare the ¬E rectangles to update H on ¬E.

1) Hypothesis given Evidence

The Bayes box shows the updated odds of Hypothesis given Evidence. We compare the size (area) of the H rectangle to the size of the ¬H rectangle.

Updated odds of H =

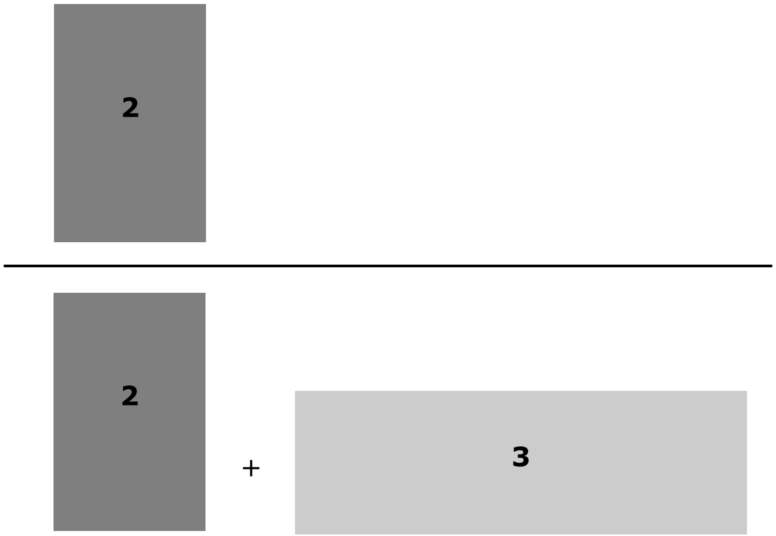




After updating H on E, our visual estimate gives approximately 2:3 odds.

Bayes’ rule converts these updated odds into a chance: divide the H rectangle by the total (sum).

$\boldsymbol{P}(H|E) \approx$



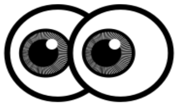
$= {}^{2}/_{5}$

This is confirming evidence because $\boldsymbol{P}(E|H) > \boldsymbol{P}(E|\neg H)$. E raises the chance of H from ~${}^{1}/_{4}$ (prior) to ~${}^{2}/_{5}$ (updated).

A quickly sketched Bayes box is a simple way to double-check a calculated answer. It should also give you a feel for how Bayes’ rule works. Each probability in the rule is one dimension of a rectangle. The H rectangle size compared to the ¬H rectangle size is the updated odds.

2) Hypothesis given ¬Evidence

The same Bayes' box also shows the updated odds of H given ¬E. We compare the two empty (unshaded) rectangles.



Updated odds of H = $\boxed{\mathbf{1}}$ - to - $\boxed{\mathbf{6}}$

A visual estimate gives us approximately 1:6 odds.

Bayes' rule converts the updated odds into a chance: divide the H rectangle by the total (sum).

$$\boldsymbol{P}(H \,|\, \neg E) \approx \frac{\boxed{\mathbf{1}}}{\boxed{\mathbf{1}} + \boxed{\mathbf{6}}} = {}^{1}/_{7}$$

This is disconfirming evidence because $\boldsymbol{P}(\neg E|H) < \boldsymbol{P}(\neg E|\neg H)$. It lowers the chance of H from ~$^{1}/_{4}$ (prior) to ~$^{1}/_{7}$ (updated).

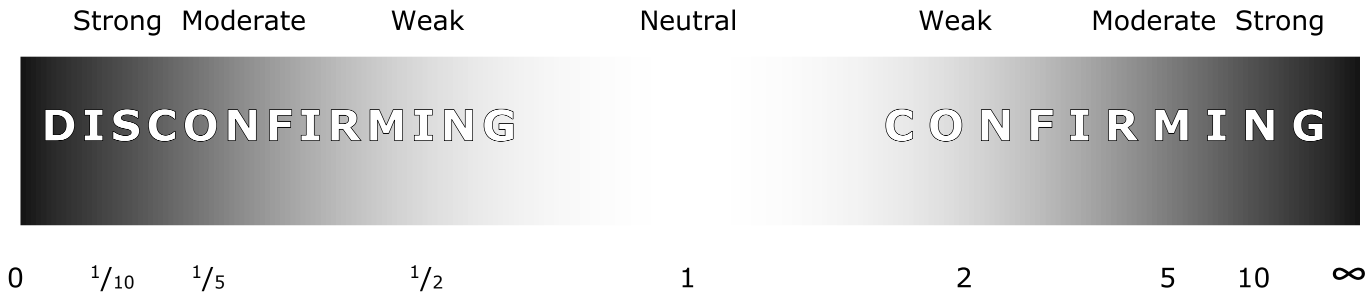
With these two updates, we see:

BAYES LESSON #3: If E confirms H then ¬E disconfirms H.

In Unit 5 we learned about confirmation bias. This bias is a failure to understand Bayes Lesson #3 – in particular, when someone gives an *ad hoc* explanation of disconfirming evidence. They accept E as confirmation but don't accept ¬E as disconfirmation.

Evidence Strength

It’s convenient to have a word as well as a number for evidence strength. Terms like these will be suitable for our examples:



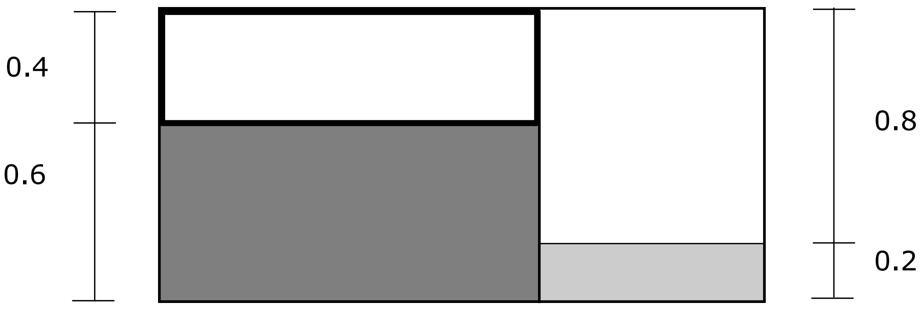
If ***P***(E**|**H) (numerator) is 0, the value of the ratio is 0: maximum disconfirmation.

$$\frac{\boldsymbol{P}(E|H)}{\boldsymbol{P}(E|\neg H)}$$

As ***P***(E**|**¬H) (denominator) goes to 0, the value of the ratio goes to infinity: maximum confirmation.

We learned (Bayes Lesson #3) that if E confirms then ¬E disconfirms. However this does *not* mean that the confirming strength of E must be equal to the disconfirming strength of ¬E. For example, in the case below, E confirms more strongly than ¬E disconfirms.

Disconfirming strength of ¬E: $^{0.4}/_{0.8} = {}^{1}/_{2}$



Confirming strength of E: $^{0.6}/_{0.2} = 3$

Tests

A test asks: Is this hypothesis true?

A *Yes* answer is a **positive** result; an incorrect *Yes* is a **false positive** ("false alarm").

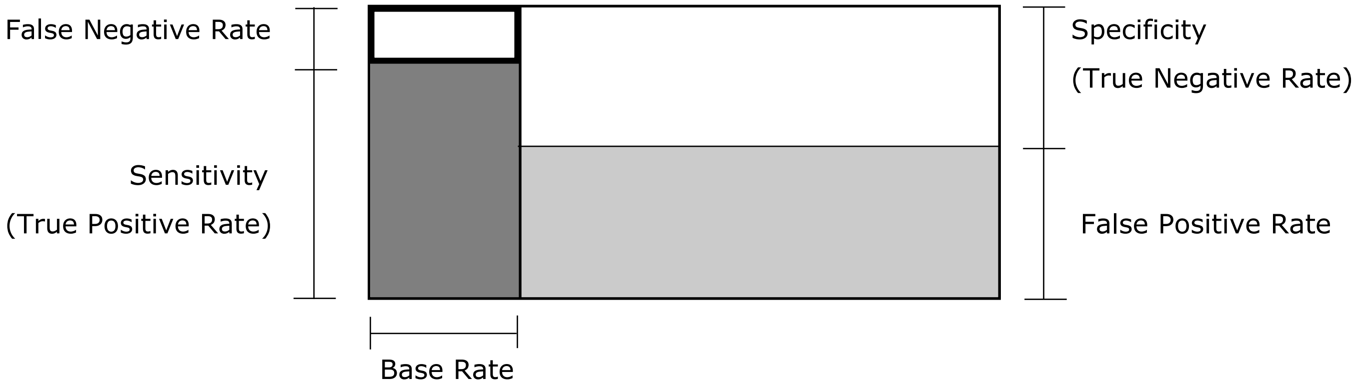
A *No* answer is a **negative** result; an incorrect *No* is a **false negative** ("miss").

Sensitivity is the true positive rate, how well the test detects when the hypothesis is true.

Sensitivity = 1 – false negative rate

Specificity is the true negative rate, how well the test detects when the hypothesis is false.

Specificity = 1 – false positive rate



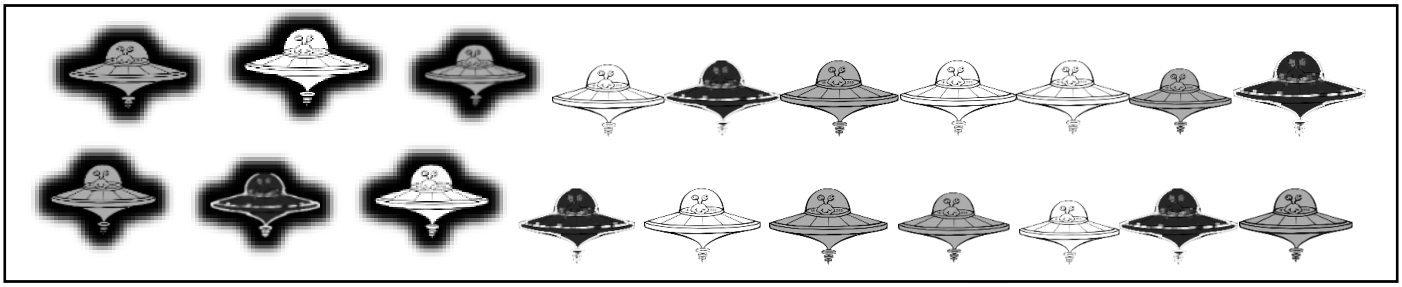
A test that is both sensitive and specific is also **accurate** – most of its results are true.[*] A perfect test would have zero errors of either kind, so its results (positive or negative) would always be true.

---

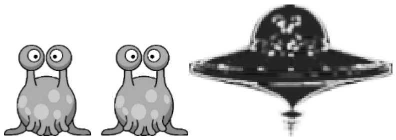
[*] The **accuracy** – the percentage of all results that are true – is somewhat dependent on the base rate, but any test that has high sensitivity and high specificity has at least fairly high accuracy. In contrast, the **positive predictive value** (**PPV**) – the chance that H is true given a positive result (the updated probability of H) – is very dependent on the base rate (the prior probability of H). A test can have high accuracy and low PPV if, due to a low base rate, most results are true negatives but most *positive* results are false positives.

ii. BAYESIAN UPDATING – VISUAL EXAMPLE

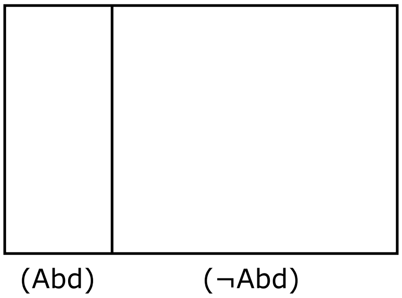
This section uses the 20 UFOs from Unit 6 for an example where we can see the answers in a picture.



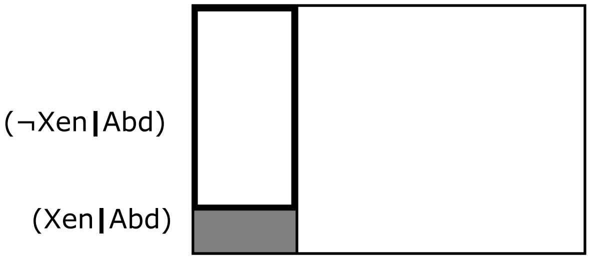
A UFO lands. The aliens tell us “We’re from Xenu.” Will they abduct us? What is the updated probability ***P***(Abduct|Xenu)?



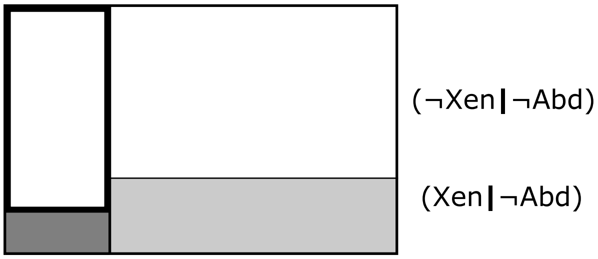
STEP 1
The prior probability the aliens plan to abduct us: $^{6}/_{20}$



STEP 2
The chance the UFO is from Xenu if they plan to abduct us: $^{1}/_{6}$

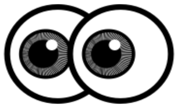


STEP 3
The chance the UFO is from Xenu if they do not plan to abduct us: $^{4}/_{14}$



The first thing to notice is that the Xenu is <u>disconfirming</u> evidence: $\mathbf{P}(\text{Xen}|\text{Abd}) < \mathbf{P}(\text{Xen}|\neg\text{Abd})$. We expect $\mathbf{P}(\text{Abd}|\text{Xen})$ (updated) < $\mathbf{P}(\text{Abd})$ (prior).

Updated odds of Abduct = $\boxed{1}$ - to - $\boxed{4}$



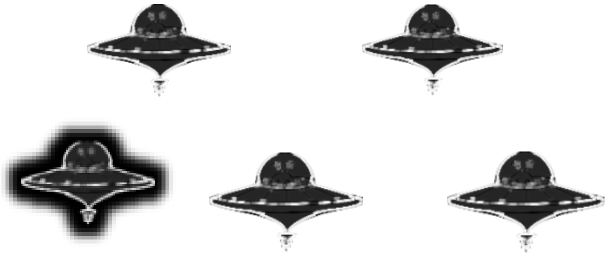
A visual estimate gives us updated odds of approximately 1:4.

Bayes' rule converts the updated odds into a chance: divide the Abduct rectangle by the total (sum).

$$\mathbf{P}(\text{Abd}|\text{Xen}) \approx \frac{\boxed{1}}{\boxed{1} + \boxed{4}} = {}^{1}/_{5}$$

$$\mathbf{P}(\text{Abd}|\text{Xen}) = \frac{\overset{({}^{1}/_{6} \times {}^{6}/_{20})}{(\mathbf{P}(\text{Xen}|\text{Abd}) \times \mathbf{P}(\text{Abd}))}}{\underset{({}^{1}/_{6} \times {}^{6}/_{20}) + ({}^{4}/_{14} \times {}^{14}/_{20})}{(\mathbf{P}(\text{Xen}|\text{Abd}) \times \mathbf{P}(\text{Abd})) + (\mathbf{P}(\text{Xen}|\neg\text{Abd}) \times \mathbf{P}(\neg\text{Abd}))}} = {}^{1}/_{5}$$

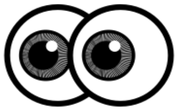
The calculated probability matches the visual estimate. Probably we calculated correctly. We can also see the correct answer in the original picture: 1-in-5 Xenu UFOs abduct.



The updated probability can help us decide how to act. Suppose we think it would be very cool to meet space aliens, but we're afraid of being abducted. We think we should stay and meet the aliens if and only if the chance of abduction is less than 25%. If it's at least 25%, we should run away. Based on the Xenu evidence, the chance is now less than 25%. We should meet the aliens!

Suppose instead that the aliens come out of the UFO and say "We're not from Xenu." This is confirming evidence: $\boldsymbol{P}(\neg\text{Xen}\,|\,\text{Abd}) > \boldsymbol{P}(\neg\text{Xen}\,|\,\neg\text{Abd})$. We expect $\boldsymbol{P}(\text{Abd}\,|\,\neg\text{Xen}) > \boldsymbol{P}(\text{Abd})$.

$$\text{Updated odds of Abduct} = \boxed{\mathbf{1}} \text{ - to - } \boxed{\mathbf{2}}$$



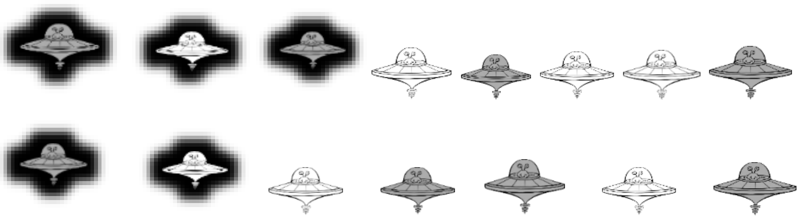
A visual estimate gives us updated odds of approximately 1:2.

Bayes' rule converts the updated odds into a chance: divide the Abduct rectangle by the total (sum).

$$\boldsymbol{P}(\text{Abd}\,|\,\neg\text{Xen}) \approx \frac{\boxed{\mathbf{1}}}{\boxed{\mathbf{1}} + \boxed{\mathbf{2}}} = {}^{1}/_{3}$$

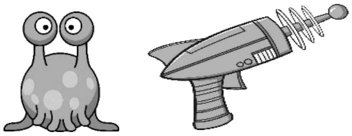
$$\boldsymbol{P}(\text{Abd}\,|\,\neg\text{Xen}) = \frac{\overset{({}^{5}/_{6} \;\times\; {}^{6}/_{20})}{\left(\boldsymbol{P}(\neg\text{Xen}\,|\,\text{Abd}) \times \boldsymbol{P}(\text{Abd})\right)}}{\underset{({}^{5}/_{6} \;\times\; {}^{6}/_{20}) \;+\; ({}^{10}/_{14} \;\times\; {}^{14}/_{20})}{\left(\boldsymbol{P}(\neg\text{Xen}\,|\,\text{Abd}) \times \boldsymbol{P}(\text{Abd})\right) + \left(\boldsymbol{P}(\neg\text{Xen}\,|\,\neg\text{Abd}) \times \boldsymbol{P}(\neg\text{Abd})\right)}} = {}^{1}/_{3}$$

The calculated answer matches the visual estimate and the picture: 5-in-15 (${}^{1}/_{3}$) non-Xenu UFOs abduct. Since this updated probability of abduction is at least 25%, we should run away!



iii. KEEP UPDATING!

A UFO lands and it's from Zargon. We think the aliens may plan to abduct us. We know other information that allows us to test our hypothesis: the space gun test. Do the aliens have space guns?

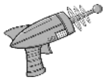


The test is fairly sensitive: 70% of abducting aliens have space guns to zap humans. Only 30% do not (the test's false negative rate). The test is also quite specific: 90% of non-abducting aliens do not have space guns. Only 10% of non-abducting aliens have space guns (the false positive rate).

These Zargon aliens have space guns! We can update the probability of Abduct a second time. The original prior probability of Abduct was ${}^{6}/_{20}$. Updated on the Zargon evidence, the chance of Abduct increased to ${}^{3}/_{8}$. That becomes the *new prior* probability for updating on the Space Gun evidence.

$$\boxed{\boldsymbol{P}(\text{Abd} \mid \text{Zar})} = \frac{(\boldsymbol{P}(\text{Z} \mid \text{A}) \times \boldsymbol{P}(\text{A}))}{(\boldsymbol{P}(\text{Z} \mid \text{A}) \times \boldsymbol{P}(\text{A})) + (\boldsymbol{P}(\text{Z} \mid \neg\text{A}) \times \boldsymbol{P}(\neg\text{A}))} = \frac{({}^{3}/_{6} \times {}^{6}/_{20})}{({}^{3}/_{6} \times {}^{6}/_{20}) + ({}^{5}/_{14} \times {}^{14}/_{20})} = \boxed{{}^{3}/_{8}}$$

$$\boldsymbol{P}(\text{Abd} \mid \text{SG}) = \frac{(\boldsymbol{P}(\text{SG} \mid \text{A}) \times \boxed{\boldsymbol{P}(\text{A} \mid \text{Z})})}{(\boldsymbol{P}(\text{SG} \mid \text{A}) \times \boldsymbol{P}(\text{A} \mid \text{Z})) + (\boldsymbol{P}(\text{SG} \mid \neg\text{A}) \times \boldsymbol{P}(\neg\text{A} \mid \text{Z}))} = \frac{(0.7 \times \boxed{{}^{3}/_{8}})}{(0.7 \times {}^{3}/_{8}) + (0.1 \times {}^{5}/_{8})} \approx 0.81$$



Here's our last lesson of Bayesian hypothesis testing:

BAYES LESSON #4: Yesterday's updated probability = today's prior probability.

iv. SUBJECTIVE PROBABILITY

This introduction to Bayes' rule uses probabilities that are directly calculated from a simple picture to make them as clear as possible. But many probabilities cannot be assigned in this way.

What's the chance?

- There is advanced intelligent life elsewhere in our galaxy.
- Qatar bribed FIFA to be made host of the 2022 World Cup.
- Canada's next prime minister is a woman given that they belong to the Conservative party.

(Image: medium.com)

There is no way to calculate probabilities such as these. We cannot count equally likely possible outcomes and there don't seem to be any frequencies that directly give us the answers. (For example, there was only one 2022 World Cup.) But we can give a **subjective probability** that expresses a **degree of belief** (confidence or feeling of certainty).

Even if they aren't mathematically correct or incorrect, subjective probabilities can still be reasonable or unreasonable. We can explain or justify this kind of probability, but normally we'll be explaining why it is, for example, 60% rather than 80% or 40%, not 60% rather than 61% or 59%.

Even though subjective probabilities do not come directly from counting or measuring anything, they are real probabilities! The rules of probability covered in Units 6-7 apply to them just as they do to other probabilities. The rules tell us what degree of belief we *should* have in something given the degrees of belief we have in other things.

What's the chance?

How to assign a probability to H? First think of some commonly used subjective probabilities in different ways, as in the chart below.

Percentage	**Fraction**	**Odds**	
100%	1	1:0	For the smallest benefit, you would bet anything that hypothesis H is true. This probability says that no evidence at all would ever change your mind about the truth of H.
99%	$^{99}/_{100}$	99:1	For many things, we act as though "99% sure" means "no doubt", although for very important things (e.g. my plane will not crash) we often want even higher confidence (e.g. 99.9%, 99.999%...).
95%	$^{19}/_{20}$	19:1	This probability is high enough to be considered significant for some common statistical purposes (see Unit 9).
80%	$^{4}/_{5}$	4:1	
75%	$^{3}/_{4}$	3:1	
66.6%	$^{2}/_{3}$	2:1	
50%	$^{1}/_{2}$	1:1	If you had to guess whether H is true or false, you would just flip a coin to choose your answer.
33.3%	$^{1}/_{3}$	1:2	
25%	$^{1}/_{4}$	1:3	
20%	$^{1}/_{5}$	1:4	
5%	$^{1}/_{20}$	1:19	
1%	$^{1}/_{100}$	1:99	
0%	0	0:1	

Here are two ways to discover your degree of belief in H.

Fair Bets

You can measure your degree of belief by asking yourself what a **fair bet** would be – neither side of the bet has an advantage, so you wouldn't prefer to take one side of the bet more than the other. For example, maybe you feel that this bet is fair: win $1 if H is true; lose $2 if H is false. This means you think the odds of H are **2**-to-**1** (since $1 × **2** = $2 × **1**). Or maybe it seems fair to win $5 if H is true and lose $1 if it's false. Then you think the odds of H are **1**-to-**5** (since $5 × **1** = $1 × **5**).

Imaginary Frequencies

Imagine that the evidence relevant to H exists in many imaginary "worlds", or that many imaginary people have the same evidence that you have. How often in all of these worlds, or for all these people, do you think the hypothesis is true? This frequency tells you your subjective probability.

UNIT 7 SKILLS

<u>You must be able to</u>:

- Write Bayes' rule for a hypothesis and evidence.
- Recognize and use base rates and test error rates.
- Draw a Bayes box and visually estimate the updated probability of H based on E and based on ¬E.
- Calculate a Bayesian update using Bayes' rule.

QUICK TEST QUESTIONS

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1) What is evidence strength?

a) $\boldsymbol{P}(E \mid H) / \boldsymbol{P}(\neg E \mid \neg H)$

b) $\boldsymbol{P}(E \mid H) / \boldsymbol{P}(E \mid \neg H)$

c) $\boldsymbol{P}(E \mid H) / \boldsymbol{P}(\neg E \mid H)$

d) $\boldsymbol{P}(E \mid \neg H) / \boldsymbol{P}(E \mid H)$

---

2) To update the probability of a hypothesis based on evidence, what do we need to know?

a) Simplicity of the evidence.

b) Prior probability of the hypothesis; strength of the evidence.

c) Prior probability of the evidence; strength of the hypothesis.

d) Plausibility of the evidence.

---

3) How many independent probabilities (probabilities that we need to separately learn) are in the expanded form of Bayes' rule?

a) 6

b) 5

c) 4

d) 3

---

4) When does evidence E confirm hypothesis H?

a) $\boldsymbol{P}(E \mid H) > \boldsymbol{P}(E \mid \neg H)$

b) $\boldsymbol{P}(H) > \boldsymbol{P}(E)$

c) $\boldsymbol{P}(H) > \boldsymbol{P}(\neg H)$

d) $\boldsymbol{P}(E \mid \neg H) > \boldsymbol{P}(E \mid H)$

---

5) When is hypothesis H confirmed by evidence E?

a) $\boldsymbol{P}(H) > \boldsymbol{P}(E)$

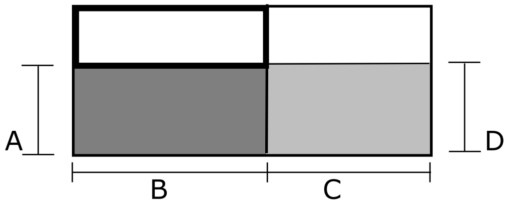
b) $\boldsymbol{P}(H \mid E) > \boldsymbol{P}(H)$

c) $\boldsymbol{P}(H \mid E) > 0.5$

d) $\boldsymbol{P}(E \mid \neg H) > \boldsymbol{P}(E \mid H)$

---

6) Label the Bayes box.



a) A: $(\neg E \mid H)$; B: (H);
C: $(\neg H)$; D: $(\neg E \mid \neg H)$

b) A: (H); B: $(E \mid H)$;
C: $(E \mid \neg H)$; D: $(\neg H)$

c) A: $(E \mid H)$; B: (H);
C: $(\neg H)$; D: $(E \mid \neg H)$

d) A: $(E \mid H)$; B: $(H \mid E)$;
C: $(E \mid \neg H)$; D: $(\neg H \mid E)$

---

7) Choose the correct statement about a test.

a) Sensitivity, specificity, false negative rate, and false positive rate are all independent.

b) Sensitivity = 1 – false positive rate; specificity = 1 – false negative rate

c) Sensitivity = 1 – specificity; false negative rate = 1 – false positive rate

d) Sensitivity = 1 – false negative rate; specificity = 1 – false positive rate

---

8) Which is determined by a base rate?

a) Evidence strength

b) Prior probability

c) True positive rate

d) Test specificity

---

9) Which parts of the Bayes box are compared for updating on ¬E (false prediction/negative result)?

a) P and R

b) Q and S

c) P and Q

d) R and S

---

10) Which probability is determined by the false negative rate of a test?

a) ***P***(E**|**¬H)

b) ***P***(¬E**|**H)

c) ***P***(H**|**¬E)

d) ***P***(¬E**|**¬H)

---

11) When is checking for evidence E an accurate test of hypothesis H?

a) ***P***(E**|**H) = low; ***P***(E**|**¬H) = low

b) ***P***(E**|**H) = high; ***P***(E**|**¬H) = high

c) ***P***(E**|**H) = low; ***P***(E**|**¬H) = high

d) ***P***(E**|**H) = high; ***P***(E**|**¬H) = low

---

12) Which part of a Bayes box comes from the false positive rate of a test?

a) A

b) B

c) C

d) D

---

13) To update on an additional piece of evidence, the previous ______ becomes the new _______.

a) prior probability… updated probability

b) updated probability… prior probability

c) sensitivity… specificity

d) false positive rate… false negative rate

PRACTICE EXAMS

PRACTICE EXAM 1

Answer the questions based on the story.

Crustie’s auction guarantees the authenticity of items they auction. Their policy is to auction an item only if it has at least 15-to-1 odds of being authentic, as determined by their lab.

A seller brings a violin that he claims was made in 1680 by Leonardo Varistradi. In the past, Crustie’s has found that 60% of claimed Varistradi violins are authentic. Another ¼ are made by apprentices in Varistradi’s workshop (less valuable). The rest are forgeries (worthless fakes).

With a microscope, the lab can see what kind of wood a violin is made of. Varistradi usually made violins with Mieffe Valley Cyprus. This can be used as a test for authenticity. The MVC test is 75% sensitive. And the MVC test has false positives: 10% of the inauthentic violins are MVC by workshop apprentices, and an additional 8% of inauthentic violins are MVC by forgers.

1) Write the information for the Mieffe Valley Cyprus (MVC) test for an Authentic Varistradi (AV).

AV base rate: False negative rate: Specificity:

Sensitivity: False positive rate:

2) Draw a labelled Bayes box for the AV hypothesis and the MVC evidence.

3) Visually estimate the updated probability of AV after each possible test result.

Probability of AV after positive result (MVC): Odds:______ Chance:_______

Probability of AV after negative result (¬MVC): Odds:______ Chance:_______

The lab checks the violin. It’s <u>not</u> made of MVC wood.

4) Write Bayes’ Rule (just the rule, no numbers) for the updated probability of AV.

5) Calculate the updated probability of AV. Show your calculation.

STOP. Does your calculated answer match your visual estimate?

6) The lab's analysis **confirms | disconfirms** the AV hypothesis because…

7) Now Crustie's **should | should not** auction the violin because…

8) The chance that the violin is a forgery **does | does not** have to change now. What is the maximum probability of Forgery now?

9) Imagine instead that the violin wood is MVC. Use Bayes' Rule to calculate what the updated probability of AV would be. Show your calculation.

STOP. Does your calculated answer match your visual estimate?

The lab looks inside the violin at the maker's label. It's an unusual bumpy paper that Varistradi liked. This is important because only 10% of authentic Varistradi violins are missing this bumpy label, but the vast majority – 98% – of inauthentic violins do not have it.

10) Based on this new, additional evidence, what is the new updated probability of the AV hypothesis? Show your calculation.

11) Now Crustie's **should | should not** auction the violin.

PRACTICE EXAM 2

Answer the questions based on the story.

Archaeologists are studying the early settlement of the Azmakia region. They think there's a ¼ chance that the first settlers were the Gthylio people. But most likely – they think 70% – the Trameron people arrived first by boat. There are also some other unlikely theories.

There's an ancient painting in the Wazoo cave. Archaeologists know that only about 20% of Trameron paintings are made with ochre (natural paint), but ochre is more common in other ancient cave paintings: about 55% of those.

They send a sample of the cave paint to a lab. They have a small research budget; they decide to use it to search the coast for Trameron artifacts if and only if the odds of the Trameron hypothesis become at least 3-to-2.

1) Draw a labelled Bayes box for the Trameron (Tram) hypothesis and the Ochre (Och) evidence.

2) Visually estimate the updated probability of Tram after each possible lab result.

Probability of Tram given Och: Odds:______ Chance:_______
Probability of Tram given ¬Och: Odds:______ Chance:_______

The lab announces the findings of the analysis. The Wazoo paint sample contains ochre.

3) Write Bayes' Rule (just the rule, no numbers) for the updated probability of Tram.

4) Calculate the updated probability of Tram. Show your calculation.

STOP. Does your calculated answer match your visual estimate?

5) The lab's analysis **confirms | disconfirms** the Trameron hypothesis because…

6) Now the archaeologists **should | should not** use their budget to search the coast because…

7) Imagine instead that the paint had not contained ochre. Use Bayes' Rule to calculate what the updated probability of Tram would be. Show your calculation.

STOP. Does your calculated answer match your visual estimate?

Pre-historic painters sometimes made hand outlines. There's a hand outline test for a Trameron painter. It's not very sensitive since 2-in-8 Trameron paintings never had hand outlines and experts think that another 1-in-8 did have one, but it faded away. However the test is quite specific since the Trameron were almost the only painters who made hand outlines. In 95% of paintings by other pre-historic peoples, the painters never made hand outlines.

8) Write the information for the Hand Outline (HO) test for Tram.

False negative rate: Specificity:
Sensitivity: False positive rate:

Looking carefully around the cave painting, the archaeologists are excited to find a hand outline.

9) Based on this new, additional evidence, what is the new updated probability of the Tram hypothesis? Show your calculation.

10) Now the archaeologists **should | should not** use their budget to search the coast.

11) The chance of the Gthylio hypothesis **does | does not** have to change now. What is the maximum probability of Gthylio now?

PRACTICE EXAM 3

Answer the questions based on the story.

Ozville has a tornado warning siren. There's no tornado tonight but the siren sounds. City officials' best explanation is that a electrical malfunction activated the siren. An alternative is that computer hacker criminals sounded the siren to scare people.

The system has never malfunctioned, and hackers have hacked other emergency systems. Based on this, the officials think that a malfunction is 60% likely and a hack 30% likely.

The city can upgrade the sirens with expensive encryption to prevent hacking. They want to spend the money for that if and only if the odds of a hack are at least 1-to-2.

Hackers would have only a $^1/_4$ chance of breaking the access panel without scratching it. But raccoons sometimes scratch the panel. It's been a while since officials last checked and by now there's about a 35% chance the panel has been scratched by raccoons even if the sirens weren't hacked.

1) Draw a Bayes box for the Hack hypothesis and the Scratched Access Panel (SAP) evidence.

2) Visually estimate the updated probability of Hack (odds and chance) after each possible discovery at the access panel.

Probability of Hack given SAP: Odds:______ Chance:_______
Probability of Hack given ¬SAP: Odds:______ Chance:_______

The officials inspect the siren and find that its access panel is scratched.

3) Write Bayes' Rule (just the rule, no numbers) for the updated probability of Hack.

4) Calculate the updated probability of Hack. Show your calculation.

STOP. Does your calculated answer match your visual estimate?

5) The scratched access panel **confirms | disconfirms** the Hack hypothesis because…

6) The city **should | should not** upgrade the siren system with encryption because…

7) Imagine instead that the access panel had not been scratched. Use Bayes Rule to calculate what the updated probability of Hack would be.

STOP. Does your calculated answer match your visual estimate?

Hackers like to brag in internet forums. A forum post test for a hack is only about 60% sensitive, and its false positive rate is fairly high: in about 1-in-3 cases when emergency systems are activated by a malfunction or other non-criminal reason, someone in a forum falsely claims to have hacked the system just to brag.

8) Write the information for the ForumPost (FP) test for Hack.

False negative rate: Specificity:

Sensitivity: False positive rate:

Searching hacker forums, the officials find a new post by Hackma$ter69 claiming to have hacked the Ozville tornado siren.

9) Based on this new, additional evidence (ForumPost), what is the new updated probability of the Hack hypothesis? Show your calculation.

10) The chance of the Malfunction hypothesis **does | does not** have to change now. What is the maximum probability of Malfunction now?

ANSWER KEY

QUICK TEST

1) b **2)** b **3)** d **4)** a **5)** b **6)** c **7)** d **8)** b **9)** a **10)** b **11)** d **12)** d **13)** b

PRACTICE EXAM 1

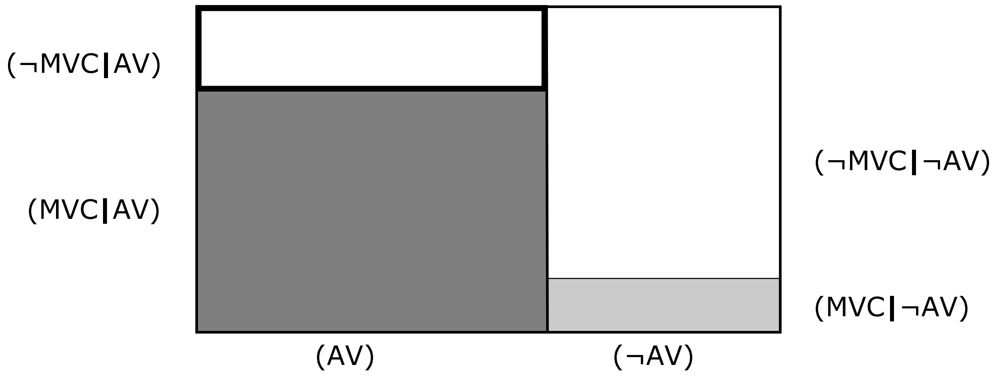
1) Answer

AV base rate: 60%
False negative rate: 25%
Specificity: 82%
Sensitivity: 75%
False positive rate: 18%

Explanation

The question gives us the test sensitivity of 75%. The false negative rate = 1 – sensitivity = 25%. The question gives us the false positive rate in two components (10% and 8%) which we add together for the total false positive rate. Specificity = 1 – false positive rate = 82%.

2) Answer



3) Answer

Visual estimates:

Probability of AV after positive result (***P***(AV❙MVC)): 7:1 = $^{7}/_{8}$

Probability of AV after negative result (***P***(AV❙¬MVC)): 1:2 = $^{1}/_{3}$

Explanation

Positive result (MVC): shaded rectangles. ***P***(AV❙MVC): What percentage of the total shaded region is the shaded AV rectangle? Negative result (¬MVC): empty rectangles. ***P***(AV❙¬MVC): What percentage of the total empty region is the empty AV rectangle? There is no reason the updated probabilities should add up to 1! The probability estimates are *not* for AV and ¬AV (which always add up to 1), but for AV, given two different pieces of information.

4) **Answer**

$$\boldsymbol{P}(\text{AV}\mid\neg\text{MVC}) \;=\; \frac{(\boldsymbol{P}(\neg\text{MVC}\mid\text{AV}) \times \boldsymbol{P}(\text{AV}))}{(\boldsymbol{P}(\neg\text{MVC}\mid\text{AV}) \times \boldsymbol{P}(\text{AV})) + (\boldsymbol{P}(\neg\text{MVC}\mid\neg\text{AV}) \times \boldsymbol{P}(\neg\text{AV}))}$$

Explanation

The MVC test is negative – the wood is not MVC. We update AV on ¬MVC.

5) **Answer**

$$\boldsymbol{P}(\text{AV}\mid\neg\text{MVC}) \;=\; \frac{(0.25 \times 0.6)}{(0.25 \times 0.6) + (0.82 \times 0.4)} \;\approx\; 0.314$$

Explanation

Our visual estimate for AV updated on a negative result (¬MVC) was $^{1}/_{3}$ (0.333), so 0.314 is reasonable.

6) **Answer**

The lab's analysis **disconfirms** the AV hypothesis because ***P***(AV❙¬MVC) < ***P***(AV). (The updated probability is less than the prior.)

[Also correct: …because ***P***(¬MVC❙AV) < ***P***(¬MVC❙¬AV). (The evidence strength is less than 1.)]

7) **Answer**

Now Crustie's **should not** auction the violin because ***P***(AV❙¬MVC) is less than 15-to-1 odds ($^{15}/_{16}$).

8) **Answer**

The chance that the violin is a modern forgery **does not** have to change now. The maximum probability now is 0.686.

Explanation

The question is whether the probability *must* change, not whether it does. Its probability must (logically) change only if there is no longer enough probability available for it to remain the same. The chance of AV went down, so there is more probability (1 – 0.314 = 0.686) available for a competing explanation. Of course it's reasonable to think the chance of forgery is now higher.

9) **Answer**

$$\boldsymbol{P}(\text{AV}\,|\,\text{MVC}) \;=\; \frac{(0.75 \times 0.6)}{(0.75 \times 0.6) + (0.18 \times 0.4)} \;\approx\; 0.862$$

Explanation

We're imagining a positive result with the MVC test. We update AV on MVC. Our visual estimate for AV updated on a positive result (MVC) was $^7/_8$ (0.875), so 0.862 is reasonable.

10) **Answer**

$$\boldsymbol{P}(\text{AV}\,|\,\text{BumpyLabel}) \;=\; \frac{(0.9 \times 0.314)}{(0.9 \times 0.314) + (0.02 \times 0.686)} \;\approx\; 0.954$$

Explanation

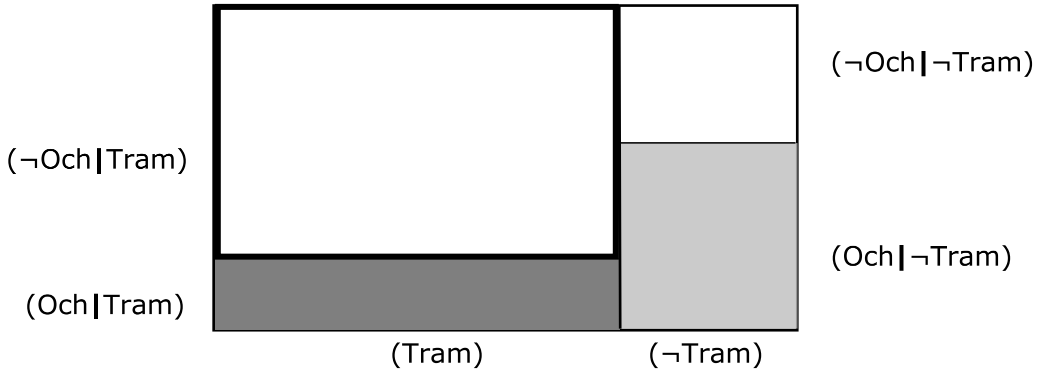
The story continues with a second piece of evidence (the bumpy label). The prior probability of AV when they check the label is the updated probability of AV after the ¬MVC evidence: 0.314.

11) **Answer**

Now Crustie's **should** auction the violin.

PRACTICE EXAM 2

1) **Answer**



2) **Answer**

Visual estimates:

$\boldsymbol{P}(\text{Tram}|\text{Och})$: 5:6 = $^5/_{11}$

$\boldsymbol{P}(\text{Tram}|\neg\text{Och})$: 4:1 = $^4/_5$

Explanation

Ochre: shaded rectangles. $\boldsymbol{P}(\text{Tram}|\text{Och})$: What percentage of the total shaded region is the shaded Trameron rectangle? ¬Ochre: empty rectangles. $\boldsymbol{P}(\text{Tram}|\neg\text{Och})$: What percentage of the total empty region is the empty Trameron rectangle? There is no reason the updated probabilities should add up to 1! The probability estimates are *not* for Tram and ¬Tram (which always add up to 1), but for Tram, given two different pieces of information.

3) **Answer**

$$\boldsymbol{P}(\text{Tram}|\text{Och}) = \frac{(\boldsymbol{P}(\text{Och}|\text{Tram}) \times \boldsymbol{P}(\text{Tram}))}{(\boldsymbol{P}(\text{Och}|\text{Tram}) \times \boldsymbol{P}(\text{Tram})) + (\boldsymbol{P}(\text{Och}|\neg\text{Tram}) \times \boldsymbol{P}(\neg\text{Tram}))}$$

Explanation

The paint contains Ochre, so we update Tram on Och.

4) **Answer**

$$\boldsymbol{P}(\text{Tram}|\text{Och}) = \frac{(0.2 \times 0.7)}{(0.2 \times 0.7) + (0.55 \times 0.3)} \approx 0.459$$

Explanation

Our visual estimate for Tram updated on Och was $^5/_{11}$ (0.454), so 0.459 is reasonable.

5) Answer

The lab’s analysis **disconfirms** the Trameron hypothesis because $\boldsymbol{P}(\text{Tram} \,|\, \text{Och}) < \boldsymbol{P}(\text{Tram})$. (The updated probability is less than the prior.)
[Also correct: …because $\boldsymbol{P}(\text{Och} \,|\, \text{Tram}) < \boldsymbol{P}(\text{Och} \,|\, \neg\text{Tram})$. (The evidence strength is less than 1.)]

6) Answer

Now the archaeologists **should not** use their budget to search the coast because the updated probability of the Trameron hypothesis is less than 3-to-2 odds ($^{3}/_{5}$).

7) Answer

$$\boldsymbol{P}(\text{Tram} \,|\, \neg\text{Och}) \;=\; \frac{(0.8 \times 0.7)}{(0.8 \times 0.7) + (0.45 \times 0.3)} \;\approx\; 0.806$$

Explanation

Our visual estimate for Tram updated on ¬Och was $^{4}/_{5}$ (0.8), so 0.806 is reasonable.

8) Answer

False negative rate: $^{3}/_{8}$	Specificity: 95%
Sensitivity: $^{5}/_{8}$	False positive rate: 5%

Explanation

The story describes two kinds of false negatives (Trameron paintings with no hand outline). We add these rates together for the total false negative rate. Sensitivity = 1 – false negative rate = $^{5}/_{8}$. The question gives us the test specificity. False positive rate = 1 – specificity = 5%.

9) Answer

$$\boldsymbol{P}(\text{Tram} \,|\, \text{HO}) \;=\; \frac{(^{5}/_{8} \times 0.459)}{(^{5}/_{8} \times 0.459) + (0.05 \times 0.541)} \;\approx\; 0.914$$

Explanation

The story continues with a second piece of evidence (the hand outline). The prior probability of Tram when they check for the hand outline is the updated probability of Tram after the Och evidence: 0.459.

10) Answer

Now the archaeologists **should** use their budget to search the coast.

11) Answer

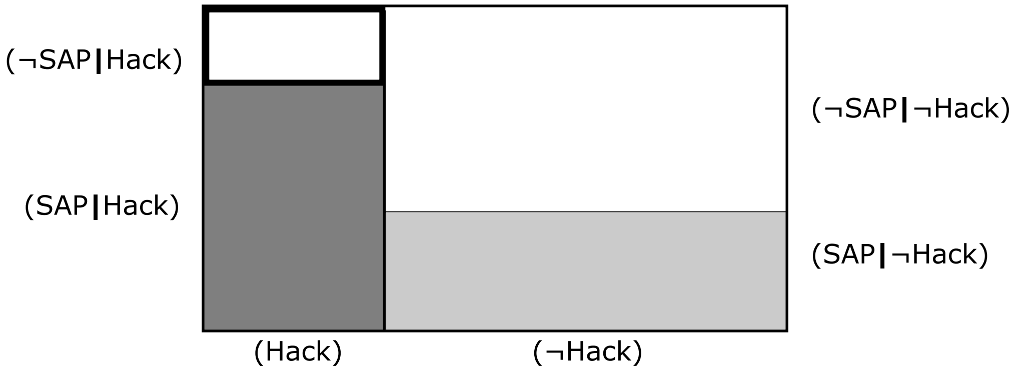
The chance of the Gthylio hypothesis **does** have to change now. Its maximum probability now is 0.086.

Explanation

Its probability must change if there is no longer enough probability available after an update for it to remain the same. By the end of the story, the Trameron hypothesis is more likely than it was at the beginning, and the Gthylio hypothesis can no longer be as likely as it was. The maximum probability (1 – 0.914 = 0.086) is the logical maximum, i.e. if there were no other alternative hypotheses. Of course there still would be, so its actual probability would be less than 8.6%.

PRACTICE EXAM 3

1) Answer



2) Answer

Visual estimates:

P(Hack❙SAP): 1:1 = $^1/_2$

P(Hack❙¬SAP): 1:7 = $^1/_8$

Explanation

SAP: shaded rectangles. ***P***(Hack❙SAP): What percentage of the total shaded region is the shaded Hack rectangle? ¬SAP: empty rectangles. ***P***(Hack❙¬SAP): What percentage of the total empty region is the empty Hack rectangle? There is no reason the updated probabilities should add up to 1! The probability estimates are *not* for Hack and ¬Hack (which always add up to 1), but for Hack, given two different pieces of information.

3) Answer

$$\boldsymbol{P}(\text{Hack} \mid \text{SAP}) = \frac{(\boldsymbol{P}(\text{SAP} \mid \text{Hack}) \times \boldsymbol{P}(\text{Hack}))}{(\boldsymbol{P}(\text{SAP} \mid \text{Hack}) \times \boldsymbol{P}(\text{Hack})) + (\boldsymbol{P}(\text{SAP} \mid \neg\text{Hack}) \times \boldsymbol{P}(\neg\text{Hack}))}$$

Explanation

The access panel is scratched, so we update Hack on SAP.

4) Answer

$$\boldsymbol{P}(\text{Hack} \mid \text{SAP}) = \frac{(^3/_4 \times 0.3)}{(^3/_4 \times 0.3) + (0.35 \times 0.7)} \approx 0.479$$

Explanation

The ¼ chance that hackers would *avoid* scratching the panel means there's a ¾ chance they *would* scratch it. Our visual estimate for Hack updated on SAP was $^{1}/_{2}$ (0.5), so 0.479 is reasonable.

5) **Answer**

The scratched access panel **confirms** the Hack hypothesis because ***P***(Hack**|**SAP) > ***P***(Hack). (The updated probability is greater than the prior.)

[Also correct: …because ***P***(SAP**|**Hack) > ***P***(SAP**|**¬Hack). (The evidence strength is greater than 1.)]

6) **Answer**

The city **should** upgrade the siren system with encryption because the updated probability of the Hack hypothesis is greater than 1-to-2 odds ($^{1}/_{3}$).

7) **Answer**

$$\boldsymbol{P}(\text{Hack}\,|\,\neg\text{SAP}) \;=\; \frac{(^{1}/_{4} \times 0.3)}{(^{1}/_{4} \times 0.3) + (0.65 \times 0.7)} \;\approx\; 0.142$$

Explanation

Our visual estimate for Hack updated on ¬SAP was $^{1}/_{8}$ (0.125), so 0.142 is reasonable.

8) **Answer**

False negative rate: 40%　　Specificity: $^{2}/_{3}$

Sensitivity: 60%　　False positive rate: $^{1}/_{3}$

9) **Answer**

$$\boldsymbol{P}(\text{Hack}\,|\,\text{FP}) \;=\; \frac{(0.6 \times 0.479)}{(0.6 \times 0.479) + (^{1}/_{3} \times 0.521)} \;\approx\; 0.62$$

Explanation

The story continues with a second piece of evidence (the forum post). The prior probability of Hack when they check the forums is the updated probability of Hack after the SAP evidence: 0.479.

10) **Answer**

The chance of the Malfunction hypothesis **does** change now. Its maximum probability now is about 38% (1 – 0.62).

~ 8 ~

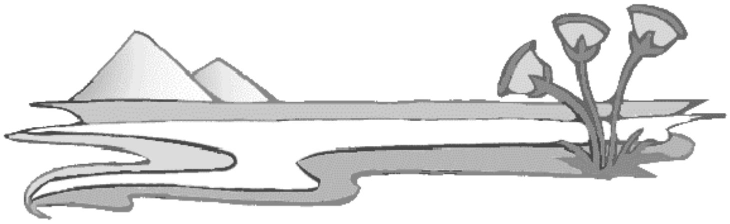
ANALOGIES

i. ARGUING FROM ANALOGY

Analogies and Analogical Arguments

An **analogy** says that one thing is similar to another.

Analogy 8.1: The Amazon River is like the Nile River.



In an extended analogical argument, an analogy is the intermediate conclusion. The sub-argument supports the analogy by noting common (shared) features of the items in the analogy.

Analogical Argument Pattern

Main Argument

- Analogy: **A** is similar to **B**.
- **B** has [Feature].

A has [Feature].

Sub-argument

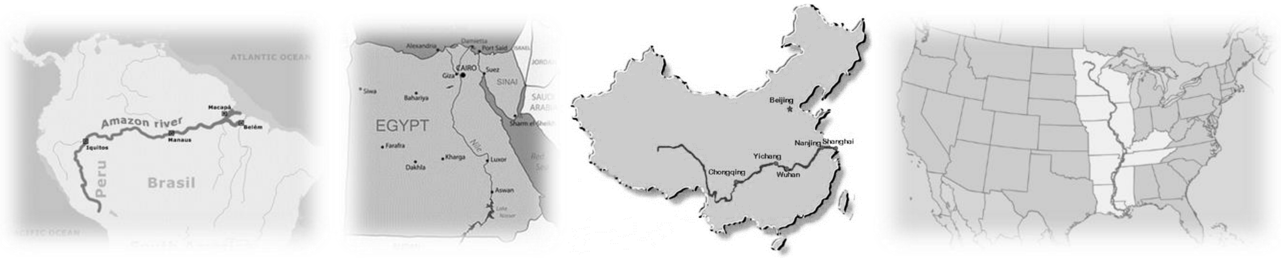
- **A** and **B** both have [Common Feature 1].
- Etc.

Analogy: **A** is similar to **B**.

The argument pattern is inductive. *Similar* does not mean “the same”! Even if **A** and **B** are generally alike, **A** might not have the feature in the conclusion.

Here’s an example using the rivers analogy. The main argument feature is “has several tributary rivers that flow into it.” The Amazon River is compared to three other rivers; the analogy is supported by premises noting three common features.

Argument 8.2: ① The Amazon, Nile, Yangtze, and Mississippi are huge rivers flowing for thousands of kilometres, ② they periodically flood, and ③ they empty into the sea. And since ④ the Nile, Yangtze, and Mississippi rivers have several tributary rivers that flow into them, probably ⑤ the Amazon River has several tributary rivers that flow into it, too.



(Image: worldmapblank.com)

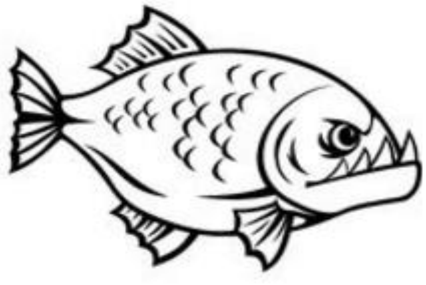
The analogy is implicit: ⑥ “The Amazon River is similar to the Nile, Yangtze, and Mississippi rivers.” The reconstruction below shows the role of each statement in the argument pattern.

Main Argument	Sub-argument
	①
④ [**B, C, D** have Feature.]	②
⑥ [Analogy: **A** is similar to **B**, **C**, **D**]	③
———	———
⑤ [**A** has Feature.]	⑥

ii. WEAK ANALOGIES

Like any argument, an analogical argument needs true premises to be a good argument. To criticize an analogical argument, someone could disagree with any premise. But very often criticism of an analogical argument criticizes the analogy.

Even though an analogy is a statement, analogies are usually said to be strong or weak rather than true or false. An analogy is **weak** if the compared items are significantly different. When is a difference significant? There’s no rule for how to judge this significance. All we can do is consider what we know about the topic and judge if we think the analogy may be misleading.



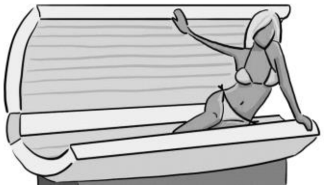
(Image: Nihongonihongo)

The Amazon River has flesh-eating piranha fish but the other rivers don’t. Is this difference significant? Can we think of any “connection” between piranhas and tributaries that would make the conclusion less likely? Maybe not. Maybe the piranha difference isn’t very significant, and the analogy, and argument, are still fairly strong.

iii. ANALOGICAL NORMATIVE ARGUMENTS

An analogy is a common way to support a **normative** statement, a claim that something should (or should not) be done, or that it's good (or bad) or morally right (or wrong). A general rule for normative arguments is that a normative conclusion requires a normative premise. An analogical argument with a normative main argument feature follows this rule.

> **Argument 8.3**: ① It should be illegal for tanning salons to sell their services to minors (people under 18). ② Just like smoking, tanning has proven health risks – it causes cancer (skin cancer). Moreover ③ like smoking, tanning appeals to young people who take their health for granted. ④ There is even evidence that people become addicted to tanning, as they do to smoking. So in fact, ⑤ selling tanning services to minors is really a lot like selling cigarettes to minors.



The analogy is explicit (⑤) but a premise is implicit: ⑥ "It should be illegal to sell cigarettes to minors."

Main Argument	Sub-argument
	②
⑤	③
⑥	④
①	⑤

Does Argument 8.3 convince you that statement ① is true? You might criticize the argument by disagreeing with any premise. Maybe you reject ⑥ because you think selling cigarettes to children should be legal! A more reasonable criticism might be: ⑤ is a weak analogy. There are significant differences between smoking and tanning. For example, there's no way to smoke without cigarettes, but it's easy to tan without a tanning salon by just lying in sunlight.

UNIT 8 SKILLS

You must be able to:

- Reconstruct an extended analogical argument.
- Recognize normative statements.
- Evaluate analogies and analogical arguments.

QUICK TEST QUESTIONS

1) In an extended analogical argument, where is the analogy?

a) The conclusion of the main argument.
b) A premise in the sub-argument and the conclusion of the main argument.
c) A premise in the main argument and the conclusion of the sub-argument.
d) A premise in the sub-argument.

2) What is a weak analogy?

a) The items are completely different.
b) The items are significantly different.
c) The items are different in any way.
d) The items don’t share the feature in the main argument.

3) Which of the following is not a normative statement?

a) F is a good thing to do.
b) F is usually done.
c) F should not be done.
d) F is the right thing to do.

PRACTICE EXAMS

PRACTICE EXAM 1

Write the implicit statement and give it a number. Reconstruct the argument in standard form using statement numbers.

Evaluate the argument by discussing both of the main argument's premises. Suggest a reasonable criticism of at least one of them. This criticism may be your own personal opinion or something you think a reasonable person might say.

> ① Artificial intelligence is rapidly improving at the task of facial recognition, matching an image of a face (e.g. from a security camera) to a face in a database. ② Now privacy supporters around the world want laws banning corporations and police from using facial recognition technology. But these people are misguided: ③ there's nothing wrong with police use of facial recognition systems. ④ Privacy supporters who fear that we'll all soon be living in a "surveillance society" where everyone's movements are tracked and recorded are just being paranoid.
>
> Consider this: ⑤ a "super-recognizer" is someone with highly developed structures in the temporal lobe of their brain that allow them to reliably identify far more faces than normal people can, and ⑥ police departments in the UK employ super-recognizers to identify people in security camera images. Surely ⑦ there's nothing wrong with police using super-recognizers. But ⑧ facial recognition technology, much like the temporal lobe of a super-recognizer, is just a system that's highly effective for a function that normal people already do less well, although ⑨ both systems make mistakes. ⑩ While it's true that citizens don't know about or consent to having their images viewed by facial recognition technology, the same is true of super-recognizers.

PRACTICE EXAM 2

Write the implicit statement and give it a number. Reconstruct the argument in standard form using statement numbers.

Evaluate the argument by discussing both of the main argument's premises. Suggest a reasonable criticism of at least one of them. This criticism may be your own personal opinion or something you think a reasonable person might say.

> ① Of the 6000-7000 distinct languages in the world today, about half are spoken only by small communities, and several hundred are close to extinction, meaning that without active preservation, they will soon disappear as their last speakers die. If you don't see why you should care about this, here's a new way to look at it: while ② it's not often noticed, ③ the extinction of a language is actually a lot like the extinction of a biological species. ④ In both cases, the world loses a wonderful kind of diversity that was created by a long evolutionary process. And as everyone acknowledges, ⑤ it's important to prevent the extinction of biological species.
>
> And there's more. ⑥ The scientific value of languages, based on all the things scientists can learn about the evolutionary and other processes that created them, is similar to the scientific value of biological species, although of course ⑦ scientists are also the first to acknowledge in both cases that having the real thing existing in the world is much more wonderful than just having knowledge about its past existence. ⑧ It's not surprising that many linguists, as well as most speakers of these endangered languages, want greater efforts to save them.

PRACTICE EXAM 3*

Write the implicit statement and give it a number. Reconstruct the argument in standard form using statement numbers.
Evaluate the argument by discussing both of the main argument's premises. Suggest a reasonable criticism of at least one of them. This criticism may be your own personal opinion or something you think a reasonable person might say.

> ① Allowing people to sell their personal data is a terrible idea, although ② that is currently a goal of several companies and politicians. ③ They think that people should legally own – and be able to sell – all the personal data that gets created about them in the medical system, social media, and online shopping. ④ Some of this data is already being collected and sold in a huge data marketplace of which few people are fully aware.
>
> ⑤ It might seem like a positive and empowering change until you reflect on the fact that ⑥ it would be a terrible idea to allow people to sell their organs (e.g. kidneys) for transplant operations. Notice that ⑦ your data and your organs are both created by you just in the course of living your life. And crucially, ⑧ in both cases poor people would feel pressure to sell them if they had the option of doing so – a marketplace would lead to the exploitation of people who are already disadvantaged. I suspect that ⑨ some of these politicians are well-meaning but just haven't thought their plan through.

---

[*] The argument in this exam is based on Jeong, Sarah. (2019 July 5). Selling your private information is a terrible idea. *The New York Times*. https://www.nytimes.com/2019/07/05/opinion/health-data-property-privacy.html

PRACTICE EXAM 4

Write the implicit statement and give it a number. Reconstruct the argument in standard form using statement numbers.
Evaluate the argument by discussing both of the main argument's premises. Suggest a reasonable criticism of at least one of them. This criticism may be your own personal opinion or something you think a reasonable person might say.

> ① Heroin addicts are at risk of deadly infectious diseases and overdoses. ② Society also spends a lot of money putting these people in jail, either for possessing an illegal drug or for stealing things to be able to buy it. ③ Many of these problems could be addressed if we learned to see heroin addiction in a different way. Of course ④ we'd like to cure every heroin addict, but ⑤ it should be legal for doctors to supply the opioid drug heroin to people with incurable addiction.
>
> ⑥ Here's the idea: heroin addiction is really not much different from diabetes (a disease that makes people unable to absorb sugar unless they take the drug insulin). ⑦ Just like diabetes, heroin addiction can be a long-term, life-threatening, incurable condition. Moreover ⑧ like heroin addicts who originally chose to use heroin, many adult diabetics get the disease partly from lifestyle choices (e.g. poor diet). But ⑨ however they got their disease, the fact is that addicts and diabetics can each live fairly normal lives with simple, cheap injections. ⑩ Maybe this perspective on heroin, shocking though it is to many people, will be enough for society to approach the problem in a more humane and effective way.

ANSWER KEY

QUICK TEST

1) c **2)** b **3)** b

PRACTICE EXAM 1

Answer

Implicit statement: ⑪ Facial recognition systems are similar to super-recognizers. (Also correct: Police use of facial recognition systems is similar to police use of super-recognizers.)

Main argument	Sub-argument
	⑧
⑦ [**B** has Feature.]	⑨
⑪ [**A** is similar to **B**.]	⑩
———	———
③ [**A** has Feature.]	⑪

Evaluation [Example Answer]: This analogical argument is fairly strong.

I think ⑦ is true. Super-recognizers are just people who use their talent to prevent crime. It would make no sense to insist that police departments hire only bad recognizers.

I think the analogy in ⑪ is strong enough to provide good support for the conclusion. However I can see why some people might think there are important differences between facial recognition systems and super-recognizers that are not acknowledged in this argument. For example:

- Super-recognizers are people who could decide whether to help police or corporations based on whether doing that seems ethical, but computers can't make those ethical decisions.
- Being recognized by a person is a normal and natural thing but being recognized by a computer is weird and creepy.

Explanation (not required for exam answers)

The function of each statement in the main argument is given in [brackets]. This is not a required part of the answer. The evaluation is an example answer. A good answer could express different opinions of the argument or its premises, or suggest different criticisms of the analogy.

PRACTICE EXAM 2

Answer

Implicit statement: ⑨ It's important to prevent the extinction of languages.

Main argument	Sub-argument
	④
③ [**A** is similar to **B**.]	⑥
⑤ [**B** has Feature.]	⑦
―――	―――
⑨ [**A** has Feature.]	③

Evaluation [Example Answer]: This analogical argument is very weak.

I disagree with ⑤. While there may be some species that it's important to save because they're useful to us, in general it doesn't really matter whether a species continues or not. In fact in many cases (e.g. mosquitos) it would probably be better if they did go extinct.

I also think that ③ is a weak analogy. There are important differences between languages and biological species that are not acknowledged in this argument. For example:

- Languages can isolate people from each other culturally, socially, and economically, but biological species don't isolate people from each other.
- A biological species is group of living creatures trying to survive, but the words and grammar of a language aren't living things and don't try to survive. They are just a way in which people choose to communicate with each other.
- While the disappearance of a biological species could have unpredictable effects on ecosystems, it doesn't seem that there are similar risks with the disappearance of a language.

Explanation (not required for exam answers)

The function of each statement in the main argument is given in [brackets]. This is not a required part of the answer. The evaluation is an example answer. A good answer could express different opinions of the argument or its premises, or suggest different criticisms of the analogy.

PRACTICE EXAM 3

Answer

Implicit statement: ⑩ Selling one’s personal data is similar to selling one’s organs.

Main argument	Sub-argument
⑥ [**B** has Feature.]	⑦
⑩ [**A** is similar to **B**.]	⑧
① [**A** has Feature.]	⑩

Evaluation [Example Answer]: This analogical argument is weak.

The analogy in ⑩ is surprising but actually pretty strong. It isn’t perfect. For example:

- Data is information and organs are parts of your body.
- You can make more data but you can’t regrow organs.

But I don’t think these differences are big enough or important enough to affect this argument. However ⑥ seems totally wrong to me. I think people should be free to do what they want with their bodies so long as it doesn’t hurt anyone else. Not only does selling an organ for transplant not hurt anyone else, but it could even save someone’s life.

Explanation (not required for exam answers)

The function of each statement in the main argument is given in [brackets]. This is not a required part of the answer. The evaluation is an example answer. A good answer could express different opinions of the argument or its premises, or suggest different criticisms of the analogy.

PRACTICE EXAM 4

Answer

Implicit statement: ⑪ It should be legal for doctors to supply insulin to diabetics.

Main argument	Sub-argument
	⑦
⑥ [**A** is similar to **B**.]	⑧
⑪ [**B** has Feature.]	⑨
⑤ [**A** has Feature.]	⑥

Evaluation [Example Answer]: This analogical argument is somewhat weak.

There can be no serious doubt about ⑥. It's obviously true the diabetics should get insulin.

The analogy in ⑪ is fairly well supported in the argument but there are some important differences between heroin and insulin that are not acknowledged. For example:

- Without insulin, a diabetic will die; without heroin, addicts will suffer terrible withdrawal symptoms, but they won't actually die.
- The two chemicals treat the two diseases in very different ways. Insulin is naturally made by the body and a diabetic's insulin shots simply replace it. But heroin is completely different from the body's own opioids (such as endorphins). It's not naturally made by the body; it just happens to attach to the same receptors in the brain.

Explanation (not required for exam answers)

The function of each statement in the main argument is given in [brackets]. This is not a required part of the answer. The evaluation is an example answer. A good answer could express different opinions of the argument or its premises, or suggest different criticisms of the analogy.

~ 9 ~

GENERALIZING FROM A SAMPLE

i. POPULATION AND SAMPLE

A **sample** is part of a larger **population** of items. Sampling is a work-saving procedure. We learn about the population by measuring the sample and then generalizing from it to the whole population.

Generalizing from a Sample Argument Pattern

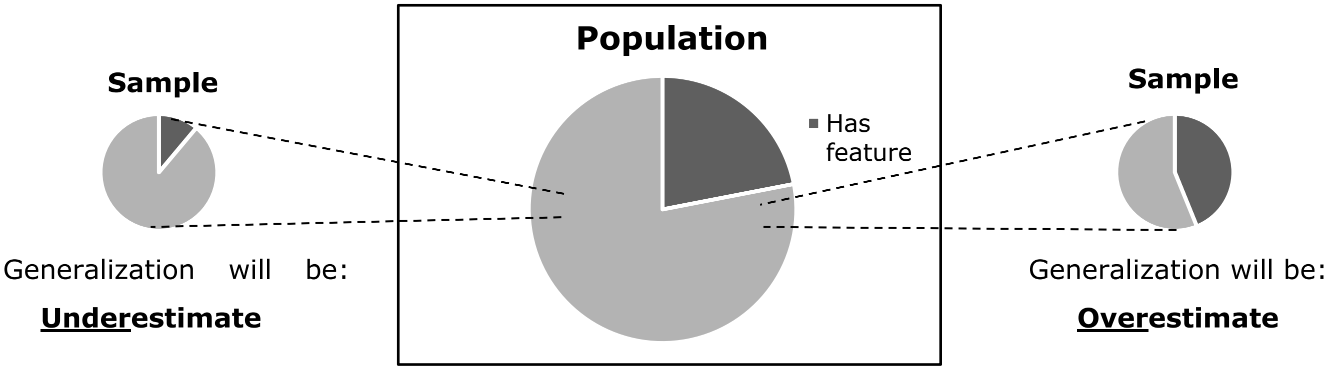
- *N*% of [Sample] has [Feature].

N% of [Population] has [Feature].

Argument 9.1: At Gotham General Hospital, 30% of pregnancies with women aged 20-24 are unplanned, so 30% of pregnancies in women aged 20-24 in Gotham are unplanned.

In Argument 9.1, the population is *pregnancies in women aged 20-24 in Gotham*, the sample is *pregnancies in women aged 20-24 at GGH*, and the feature is *unplanned*.

If the feature is less or more common in the sample than it is in the population, a generalization based on that sample will be an **underestimate** or **overestimate**.



ii. SAMPLE SIZE

Measuring a larger sample gives us a better ability to generalize. Sample size is indicated with n=.

> **Argument 9.2**: At GGH, 30% of pregnancies with women aged 20-24 (n=500) are unplanned, so 30% of pregnancies in women aged 20-24 in Gotham are unplanned.

Precision and Accuracy

People often use the words *precision* and *accuracy* as though they mean the same thing. In fact they are completely different concepts. The **precision** of a statement is its exactness or specificity. The **accuracy** of a statement is how close it is to the truth.

> **Statement 9.3**: The speed of light is 155,286,903 $^{m}/_{s}$.

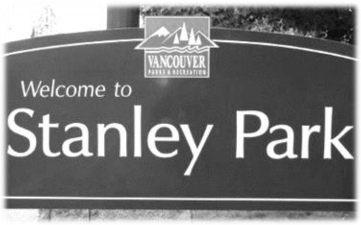
> **Statement 9.4**: The speed of light is 271,306,682 $^{m}/_{s}$.

> **Statement 9.5**: The speed of light is around 300,000,000 $^{m}/_{s}$.

Statements 9.3 and 9.4 are equally precise. Statement 9.5 is less precise because of *around*.

The true speed of light is 299,792,458 $^{m}/_{s}$. That means that each statement is more accurate – closer to the truth – than the previous one.

The concepts of precision and accuracy also apply with non-quantitative examples.



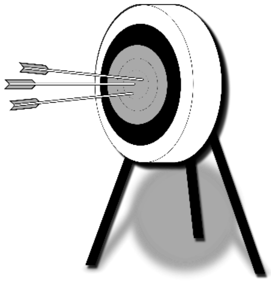
Statement 9.6: Stanley Park is in Canada.

Statement 9.7: Stanley Park is in BC, Canada.

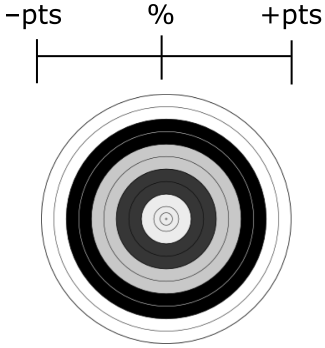
Statement 9.8: Stanley Park is in Victoria, BC, Canada.

Each statement is more precise than the previous. The most precise, Statement 9.8, is inaccurate.

Archery is a helpful analogy. Generalizing is like shooting an arrow at a target. Precision is the target size. Higher precision = smaller target. It's more difficult to hit (be accurate with) a small target.

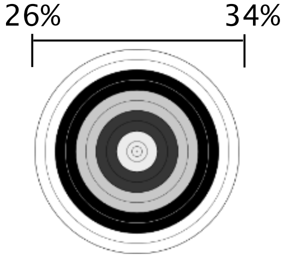


Margin of Error and Confidence Level



A complete statistical generalization includes a **margin of error** (**MoE**), which quantifies the precision. The statistic (*N*%) is the target's centre and the MoE is the distance to either edge. The MoE is given in the units of percentage, **percentage points** (**pts**). Larger MoE = bigger target = lower precision.

Argument 9.9: At GGH, 30% of pregnancies with women aged 20-24 (n=500) are unplanned, so 30% of pregnancies in women aged 20-24 in Gotham are unplanned (±4 pts).



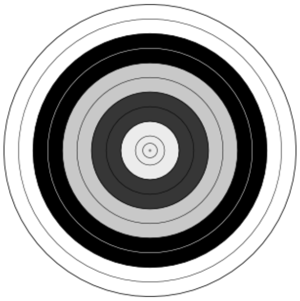
MoE is calculated for the sample size at a chosen **confidence level**, the chance that the correct percentage for the population is within this range. This is the strength of the argument (assuming an unbiased sample – see Sec.iv).[*] A 95% confidence level is common – if it's not stated, as in Argument 9.9, we can assume 95%.[+]

---

[*] MoE also depends on population size. In most of the examples in this unit, population size is the same: huge. This makes it simpler to see the effect of different sample sizes. MoE also depends on the percentage measured in the sample. Often MoE is simply given for a sample size, in which case MoE is calculated assuming a measurement of 50%. This is the *maximum* MoE. For a measured percentage lower or higher than 50%, the MoE is smaller.

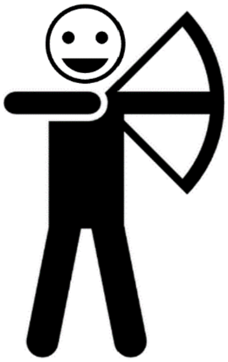
[+] Although 95% is convenient for mathematical reasons, it is a choice. We could choose any confidence level. (And there are other mathematically convenient but less memorable numbers, such as 68% and 99.7%.) A confidence level of 95% is standard because it's easy to remember and picture (a 19-in-20 chance), high enough to take the statistic seriously, but not so high that the margin of error becomes huge.

With a small sample, the archer has bad aim (they're shaky), so it's hard to be accurate. To have high confidence, they must shoot at a large target (low precision/large MoE).



If the target is smaller (higher precision/smaller MoE) but their aim is the same (same sample size), they have lower confidence.

To have high confidence with a small target (high precision), they need better aim (larger sample).



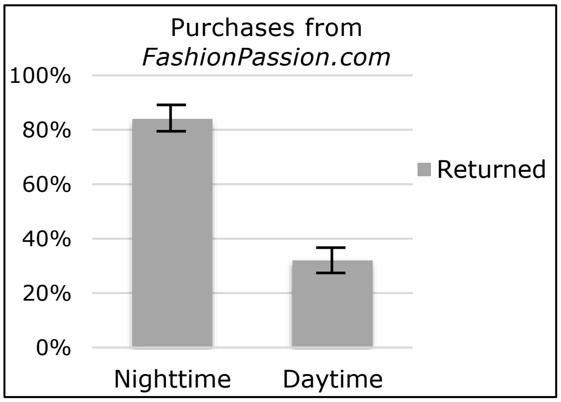
The usefulness of a sample is determined by its size. Compare Argument 9.9 to 9.10:

> **Argument 9.10**: At GGH, 30% of pregnancies in women aged 20-24 (n=500) are unplanned, so 30% of pregnancies in women aged 20-24 in Gotham are unplanned (±5.3 pts; 99% Confidence).

The sample size (n=500) is the same, so it's equally useful. But it's been used *differently*. Argument 9.10 has a higher confidence level (99% vs. 95% in Argument 9.9) – it's stronger. How is that possible, if the higher confidence has not been earned by doing more sampling work? Higher confidence doesn't come for free! Here it's been achieved simply by lowering precision (±5.3 pts vs. ±4 pts).

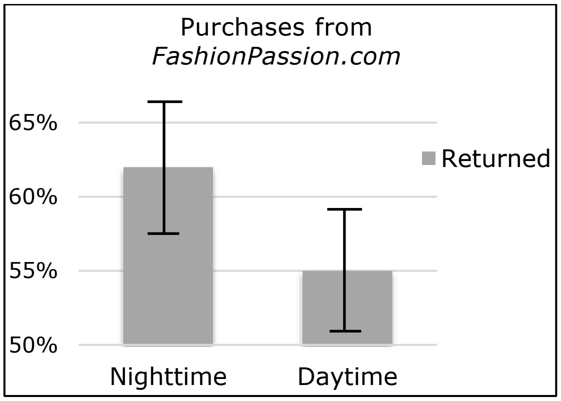
iii. COMPARING STATISTICS

Margin of error can be important when we compare statistics to find a difference.



Argument 9.11: It seems that clothes bought at night from our website *FashionPassion.com* are more likely to be returned. Last month's records (n=250) show that the return rate for nighttime purchases is 84% (±4.5 pts). But the records (n=350) show that the return rate for daytime purchases is only 32% (±4.1 pts).

Argument 9.11 has two sub-arguments. Their conclusions support a final conclusion that asserts a difference ("more likely"). Both statistics have MoE, shown with **error bars** in the chart. Since there's a large gap between error bars, this is strong evidence that there really is a difference between nighttime and daytime purchases.



Imagine instead that the measurements had been 62% and 55%. These statistics are separated by only 7 pts. Now the error bars overlap.

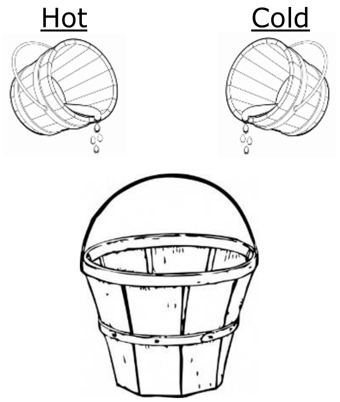
This suggests there's a difference, but we can't be very sure. To be more sure of this small difference, we need smaller MoE; to get smaller MoE (at the same confidence level), we need larger samples.

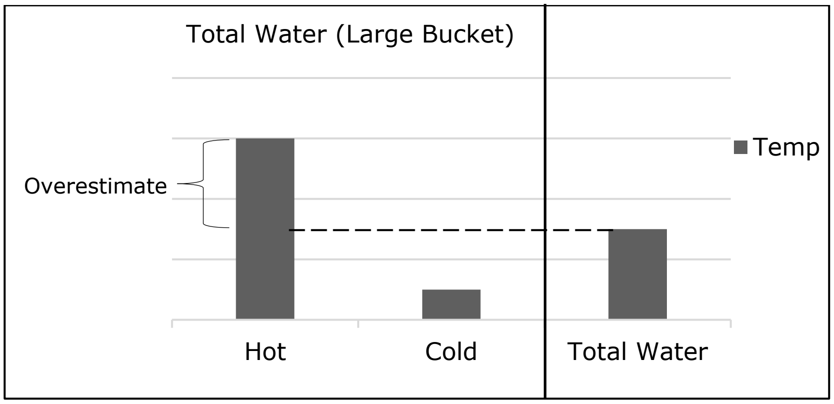
iv. SAMPLE SELECTION

Sample Bias

In a **biased sample**, the measured feature is more (or less) common than it is in the whole population – but not just by bad luck (sampling error). Rather, it’s because the items in the sample have some feature that makes them more (or less) likely to have the measured feature. To understand the sample bias, we must know about this relationship between the measured feature and a sample feature. If we know this background information, we know to expect an overestimate (or underestimate).

Here’s an analogy. We’re pouring hot water and cold water into a large bucket. To find the temperature of the large bucket water (population), we measure the temperature of the hot water (sample).

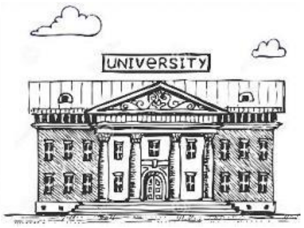




Regardless of the amounts of hot and cold water we combine, the large bucket water temperature must be between the hot and cold temperatures, which means that it’s lower than the hot water we sampled. So measuring the hot water to find the large bucket temperature gives an overestimate.

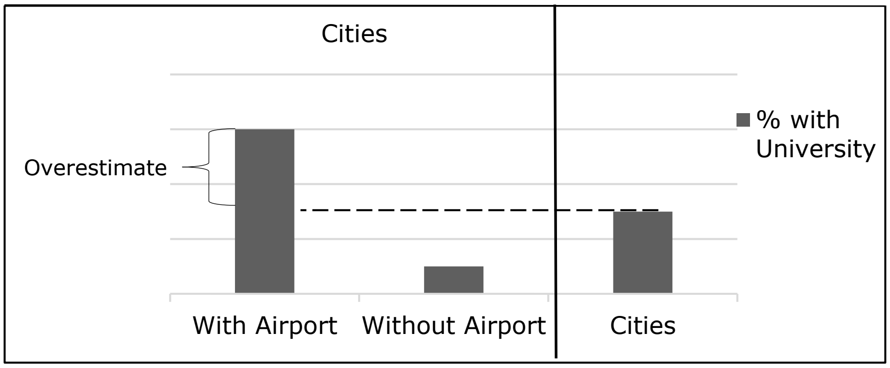
Abby has been travelling around the world by airplane, visiting many cities. She thinks:

> **Argument 9.12**: 60% of the cities I’ve visited have a university (and I’ve visited a lot of cities!). So about 60% of cities have a university.



Can you see the problem here?

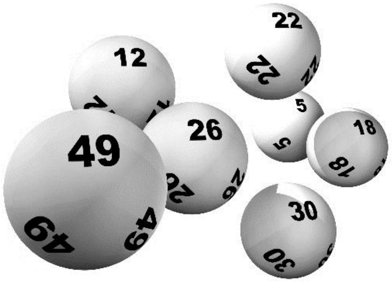
- This is a biased sample.
- Background info: Universities (**measured feature**) are more common among cities (**population items**) with an airport (**sample feature**).
- Probably the statistic is an overestimate.



Convenience Samples vs. Random Samples

Sample bias is a risk when using a **convenience sample**, a sample composed of population items that are available and easy to include. Convenience samples are not necessarily biased and are common in many kinds of research. But we must be aware of the risk of bias and watch out for it.

Ideally we want to use a **random sample** (e.g. selected by lottery) for which every item in the population has the same chance of being selected. This sample could be randomly different from the population, just by bad luck – there's still the problem of sampling error! But we have no reason to expect specifically an overestimate (or underestimate).



UNIT 9 SKILLS

<u>You must be able to</u>:

- Say how precision, MoE, confidence level, or sample size would change when one of the others changes. (NOTE: You do not need to calculate MoE for this course.)
- Recognize and evaluate an argument in which sample size affects argument strength.
- Recognize a biased sample and explain what makes it biased.

QUICK TEST QUESTIONS

1) A statistic that is less precise is more likely to be…

a) …true.

b) …false.

c) …an overestimate.

d) …an underestimate.

2) Margin of error…

a) …is smaller when the sample is smaller.

b) …is smaller when the confidence level is higher.

c) …quantifies the precision of a statistic.

d) …quantifies the confidence level of a generalization.

3) Two statistical generalizations support a difference strongly when…

a) None of these.

b) …the high end of one MoE and the low end of the other MoE include the same numbers.

c) …the difference between them is small compared to their MoE.

d) …their error bars on a bar chart overlap.

4) 10% of A-Ws (Ws with A) have B; 30% of Non-A-Ws have B. What percentage of Ws have B?

a) Not enough information to know

b) More than 30%.

c) Less than 10%.

d) Between 30% and 10%.

5) A sample is random if every item in the…

a) …population had the same chance of being selected for the sample.

b) …sample had the same chance of being selected for the population.

c) …sample is more (or less) likely to have the feature being measured.

d) None of these.

PRACTICE EXAMS

PRACTICE EXAM 1

1) Which argument is stronger? Briefly explain your answer.

a. We tested water quality in buildings around the city (n=150), and 31% of them had water that is too dirty to be safe to drink. So 31% of buildings in the city have dirty water (±7.4 pts).
b. We tested water quality in buildings around the city (n=300), and 31% of them had water that is too dirty to be safe to drink. So 31% of buildings in the city have dirty water (±5.2 pts).
c. (a) and (b) are equally strong.

2) Answer the question based on the story. Explain your answer.

Bob and Abby work at sidewalk food carts downtown. Today Bob got the best (busiest) location, where he will get more customers than Abby. Mostly they sell hot dogs, but about 10% of their customers buy their deep-fried squid-on-a-stick. Today who is more likely to have at least ¼ of their customers order squid-on-a-stick: Abby, Bob, or are they equally likely?

3) Identify the elements of the argument and then evaluate it.

We learned that only 22% of houses in Gotham have smoke detectors with properly charged batteries (±3.5 pts). We checked houses (n=550) in Burnside, the poorest neighbourhood in Gotham and only 119 of them had smoke detectors with properly charged batteries.

a) State the population, sample feature, measured feature, sample size, margin of error, and confidence level.
b) Evaluate the argument. Illustrate your evaluation with a bar chart.

PRACTICE EXAM 2

1) Which argument is stronger? Briefly explain your answer.

a. We asked Hollywood producers (n=250) if they considered themselves bigshots, and 75% did, so 75% of Hollywood producers consider themselves bigshots (±5.4 pts; 95% Confidence).

b. We asked Hollywood producers (n=250) if they considered themselves bigshots, and 75% did, so 75% of Hollywood producers consider themselves bigshots (±3.5 pts; 80% Confidence).

c. (a) and (b) are equally strong.

2) Reconstruct and evaluate the argument.

① Gotham's popular Deathtrap nightclub is not only the coolest club in town but also has the highest success rate for getting patrons to buy the overpriced, watered-down drinks – greater success than Gotham's next most successful club, Candyland. ② 74% of Deathtrap patrons buy the overpriced, watered-down drinks, based on the patrons we surveyed (n=54) recently at Deathtrap. ③ 74% of our sample bought the drinks. But when we surveyed patrons (n=48) at Candyland, ④ only 71% had bought the drinks, so ⑤ Candyland's rate is only 71%.

3) Evaluate the reasoning of the news stories in the following (true) story.

Researchers at Boston University were interested in the link between (American) football and Chronic Traumatic Encephalopathy (CTE), a neurodegenerative disease that causes cognitive and mood problems. It's believed to be caused by mild repetitive brain trauma of the sort that football players normally experience even while wearing helmets. The researchers studied 202 football player brains that were donated to the CTE study by their families when they died. They found CTE in nearly every brain. Many news headlines reported the findings by announcing that almost all football players get CTE.

PRACTICE EXAM 3

1) Which argument is stronger? Briefly explain your answer.

a. At our gadget factory, we tested gadgets coming off the assembly line to see if they work as designed. In our sample (n=400), 88% work as designed. Therefore 88% of our assembled gadgets work as designed (±3.2 pts).
b. At our gadget factory, we tested gadgets coming off the assembly line to see if they work as designed. In our sample (n=400), 88% work as designed. Therefore 88% of our assembled gadgets work as designed (±4.2 pts).
c. (a) and (b) are equally strong.

2) Evaluate the reasoning in the story.

Azmakia is having a referendum next week to decide whether to officially make the dung beetle the national arthropod of Azmakia. Azmakian law requires 3-to-2 support for a referendum proposal to pass. The dung beetle supporters have been campaigning for months and are now sure they have enough support for success in the referendum. They asked 115 people from all around Azmakia who plan to vote and 63% of them support the proposal.

3) Identify the elements of the argument and then evaluate it.

At the Wackadoodle gadget factory, the Quality Control Department needs to determine the defective rate for the gadgets they produce. There are several employees in the factory making gadgets, so QC picks one employee whose gadgets they will monitor. They choose Bob, a new employee. It's his first week on the job. They check Bob's gadgets (n=320) and discover that 16% are defective. "Wackadoodle has a major quality control problem!", they conclude. "16% of our gadgets are defective! (±4 pts)."

a) State the population, measured feature, sample feature, sample size, margin of error, and confidence level.
b) Evaluate the argument. Illustrate your evaluation with a bar chart.

ANSWER KEY

QUICK TEST

1) a **2)** c **3)** a **4)** d **5)** a

PRACTICE EXAM 1

1) Answer

(c) – The arguments have the same (assumed) confidence level: 95%. The larger sample in argument (b) is used to increase precision (smaller MoE) rather than increase confidence.

2) Answer

Abby. Her less busy location gives her a smaller sample of customers that is more likely to have a different composition (at least ¼ who want squid-on-a-stick) than the population of all customers (10% who want squid-on-a-stick).

3) Answer

a)

Population: Gotham smoke detectors [Also correct: Gotham houses]

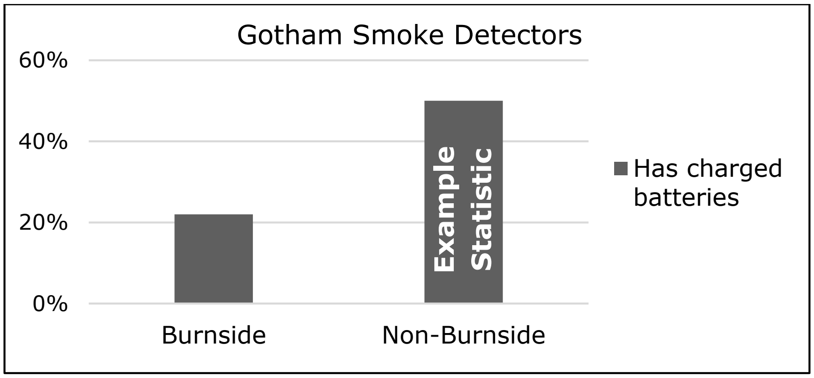
Measured feature: has charged batteries [Also correct: has smoke detector with charged batteries]

Sample feature: in Burnside

Sample size: 550 MoE: ±3.5 pts Confidence level: 95% (assumed)

b)

Evaluation: This argument is bad because it uses a biased sample. Burnside is a poor neighbourhood where people have bigger problems to worry about than replacing smoke detector batteries, and do little house maintenance. We can expect that charged batteries are less common in Burnside Gotham smoke detectors. [Also correct: Burnside Gotham smoke detectors are less likely to have charged batteries.] Probably 22% is an underestimate. The study should have checked randomly selected houses from all over Gotham.



Explanation

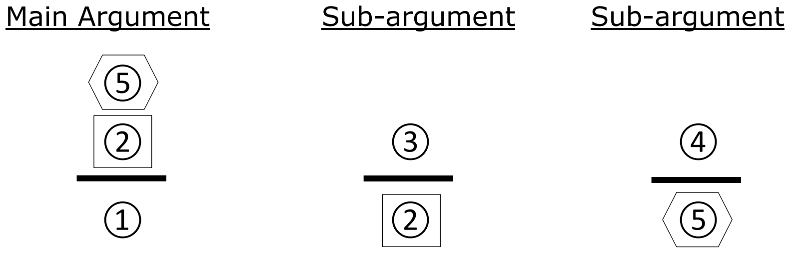
The population is a group of items, the subject of the conclusion. The measured and sample features are not things – they are properties that population items may have or not have. The margin of error is given in pts, not %. The confidence level is not the statistic, but the chance that the true statistic is within the percentage range defined by the margin of error.

PRACTICE EXAM 2

1) Answer

(a) – The arguments have samples of the same size but they do not use the sample in the same way. Argument (b) uses the sample to increase precision (smaller MoE); argument (a) uses it to increase confidence.

2) Answer



Evaluation: This is a weak argument. The conclusion is based on a comparison of two statistical generalizations – the drink-buying rates – that are based on fairly small samples of patrons at the clubs. Using standard confidence levels, these statistics would have margins of error that are large compared to the difference between them (3 pts). In other words, the margins of error would overlap. Patrons at Deathtrap nightclub might be equally or even less likely to buy the drinks than those at Candyland.

Explanation

The two sub-arguments each use the pattern (generalizing from a sample) introduced in this unit. The main argument simply compares two statistics (its premises) and concludes that there is a difference. This is the written version of the two-bar bar chart. In the chart, if margins of error were known, they would be indicated with error bars.

3) Answer

The news stories treat the brains in the CTE study as a sample of football players' brains. But this sample is extremely biased. To be in the CTE study, someone's brain had to be donated by their family. Why would a family do that? In most cases, they did it because the person had cognitive and mood problems, i.e. symptoms of CTE. So probably CTE (measured feature) was more common among donated (sample feature) football player brains, and the statistic ("almost all") is an overestimate.

PRACTICE EXAM 3

1) **Answer**

(b) – Although the confidence levels are unstated and this normally means 95%, it is impossible for both arguments to have the same confidence level. The sample sizes are the same but the statistic in argument (b) is less precise (larger MoE), so its confidence level must be higher.

2) **Answer**

Based on the measurement of their sample, they infer 63% support among planned voters, which is high enough for the proposal to pass. However 63% is only 3 pts higher than the support percentage required (3-to-2 = 60%). At any normal or useful confidence level (e.g. 95%), a sample size of 115 has a margin of error larger than that. Since a failing support percentage (59%) is well within the margin of error of this statistic, the campaign should not be so sure that the dung beetle will be Azmakia's new national arthropod.

3) **Answer**

a)

Population: Wackadoodle gadgets

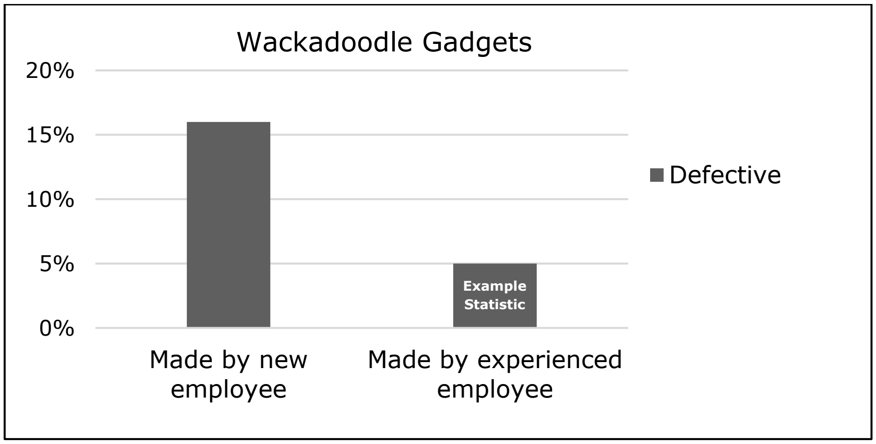
Measured feature: defective

Sample feature: made by new employee

Sample size: 320 MoE: ±4 pts Confidence level: 95% (assumed)

b)

Evaluation: This argument is bad because it uses a biased sample. Bob is a new employee still learning how to do his job well. Probably defects are more common among Bob's gadgets. [Also correct: Bob's gadgets are more likely to be defective.] Probably 16% is an overestimate. The study should have checked randomly selected gadgets from many different employees in the factory.



Explanation

The population is a group of items, the subject of the conclusion. The measured and sample features are not things – they are properties that population items may have or not have. The margin of error is given in pts, not %. The confidence level is not the statistic, but the chance that the true statistic is within the percentage range defined by the margin of error.

~ 10 ~

CAUSE AND EFFECT

i. THE METHOD OF DIFFERENCE

Causal Statements

Determining that one thing causes another is important both in science and in everyday life. A **cause**[*] produces an **effect** (makes it happen or exist). The effect happens after or together with the cause.

Many normal sentences can be restated as explicit causal statements.

Statement 10.1: Abby's smartphone distracts her while she is driving.
Statement 10.1 (explicit causal statement): Abby's smartphone causes her to be distracted while driving.

Many explanations are causal claims.

Statement 10.2: The dog barked because it saw a cat.
Statement 10.2: **(explicit causal statement)**: Seeing a cat caused the dog to bark.

Some causes are **preventions**. The effect they produce is a lack or reduction of something.

Statement 10.3: The HPV vaccine prevents cervical cancer in women.
Statement 10.3 (explicit causal statement): The HPV vaccine causes women to not get cervical cancer.

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[*] **Vocabulary**: The word *cause* is not the word *because*, but *because* often indicates an explanation, and many explanations are causal statements. "**A** because **B**" often means "**B** caused **A**".

The Method of Difference

Causal reasoning often uses the **method of difference**.*

Method of Difference Argument Pattern

Main Argument

- **A** is different when **B** is different.
- There are no other relevant differences besides **A**.

A causes **B**.

Sub-argument

- **C** is the same when **B** is different.
- Etc.

There are no other relevant differences besides **A**.

Argument 10.4: ① Normally Abby doesn't text while driving, and she drives well, but today she texted her friend and she crashed. ② She wasn't driving in the rain for the first time and ③ wasn't trying to avoid hitting an animal on the road. So ④ texting caused her to crash.

To reconstruct Argument 10.4, we need to recognize an implicit intermediate conclusion: ⑤ "There were No Other Relevant Differences (besides texting) when Abby crashed."

Main Argument

① [**A** different when **B** different.]
⑤ [NORD (besides **A**).]

④ [**A** caused **B**.]

Sub-argument

②
③

⑤

Since the method of difference depends on (or works best when) finding the **only relevant difference**, it can be criticized if we notice another difference that seems relevant (a possible cause). What is relevant? That can only be judged based on background information about the effect.

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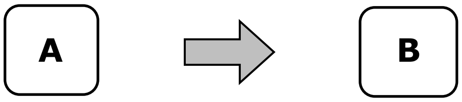
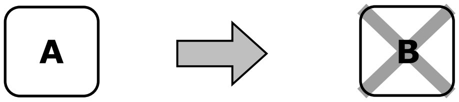
* There is also a method of *agreement*: **A** is the same when **B** is the same, therefore **A** causes **B**. This may be understood as a method of searching for something we expect would be the only relevant difference between **B** situations and non-**B** situations.

ii. CAUSE AND EFFECT

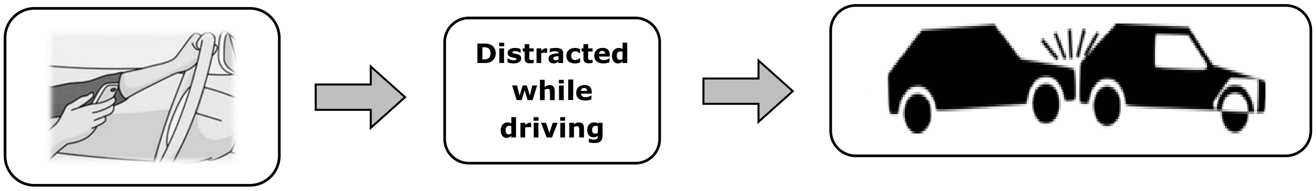
To show a causal explanation (A causes B; B is an effect of A), we'll draw a cause-and-effect diagram.



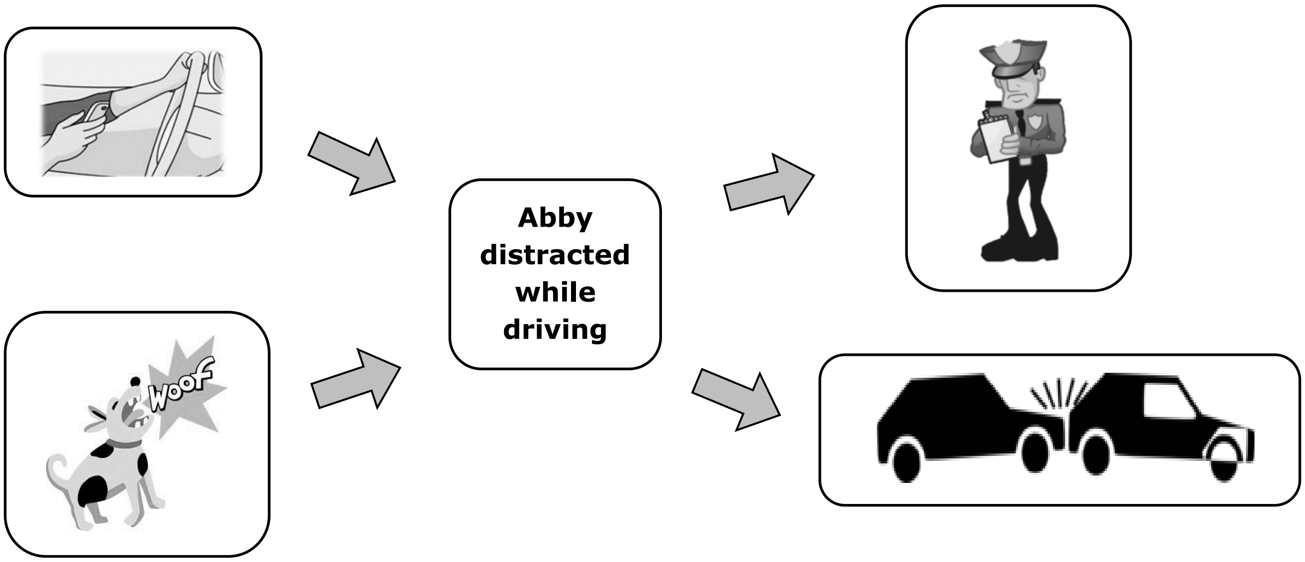
To show prevention (A causes B to *not* happen), we'll cross out B in the diagram.



Multiple causes and effects may be ordered in a **causal chain**.



Most things have more than one cause and more than one effect.



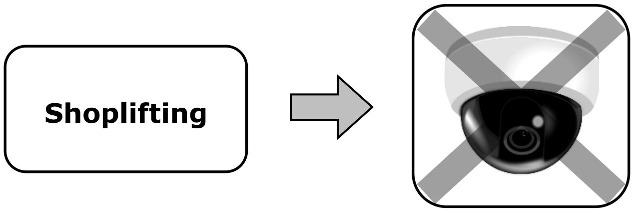
iii. EXPLAINING DIFFERENCE

A statistical difference: **B** is more common among **W** that have feature **A**. Why?

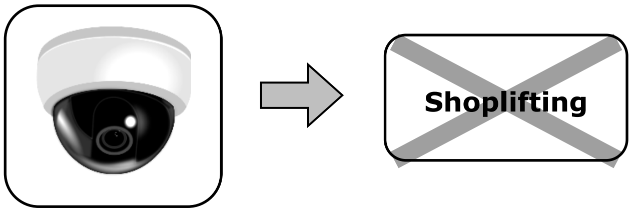
Three Causal Explanations of a Difference

Why is the **B** rate higher in the **A** group?

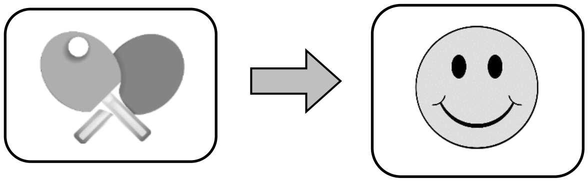
Argument 10.5: Security cameras are less common in stores with lots of shoplifting, so shoplifting prevents security cameras.



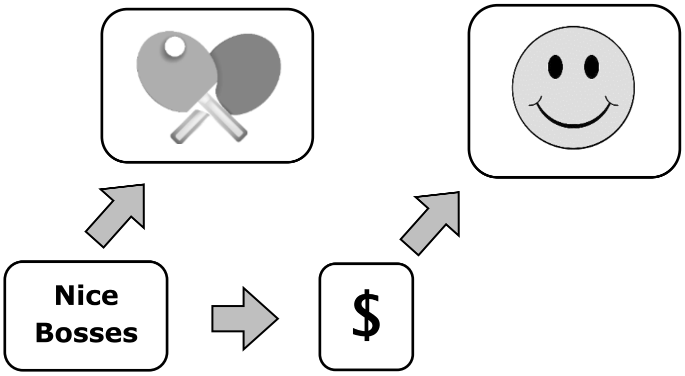
But maybe the true explanation is the reverse: security cameras prevent shoplifting.



Argument 10.6: Offices with ping pong tables are more likely to have happy employees, so ping pong tables make employees happy.



But maybe there is a **common** (shared) **cause**, something that *separately* causes ping pong tables and happy employees. For example, maybe nice bosses buy ping pong tables for the office and make employees happy by giving them raises.

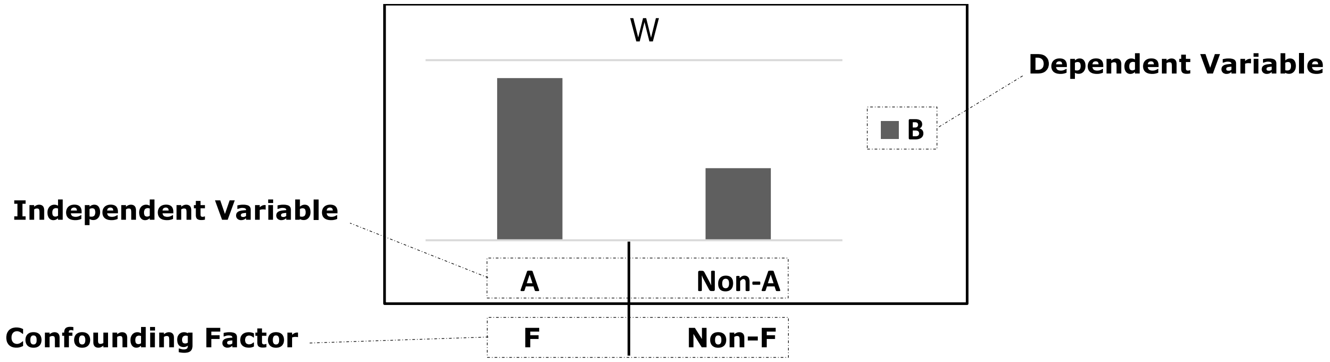


We can easily eliminate the reverse causal explanation (B causes A) when B happens after A. Causes happen before (or with) their effects, never after them.

In cases with no clear time order, and in cases where we think there may be a common cause, eliminating the alternative explanations require an experiment.

iv. CONTROLLED EXPERIMENTS

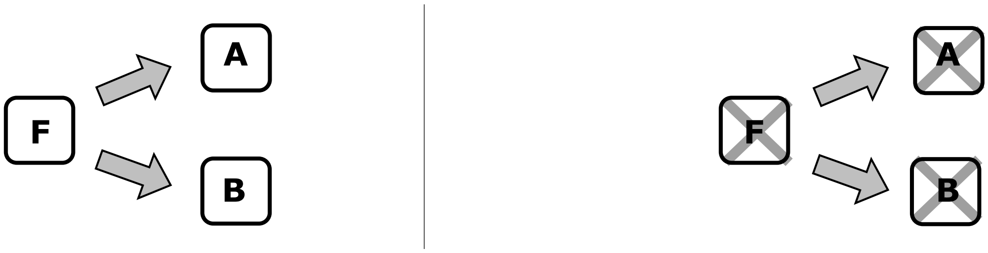
A **controlled experiment** compares two groups: an **experimental group** and a **comparison group**[*]. The **independent** (or manipulated) **variable** (**A**) should be the only relevant difference between the two groups. We measure some feature, the **dependent variable** (**B**), in each group.



Confounding Factors and Alternative Explanations

The chart above shows that **B** is more common with **A**, and **B** is less common without **A**.

The **confounding factor F** is another difference (besides **A**) between the **A** and Non-**A** groups. It could[+] be a common cause of **A** and **B**.



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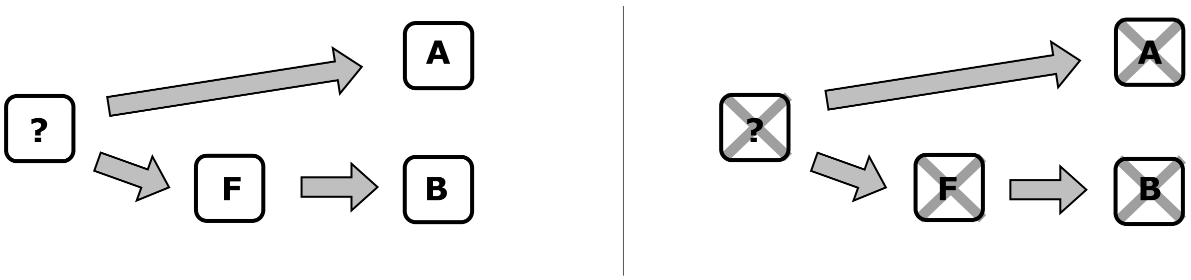
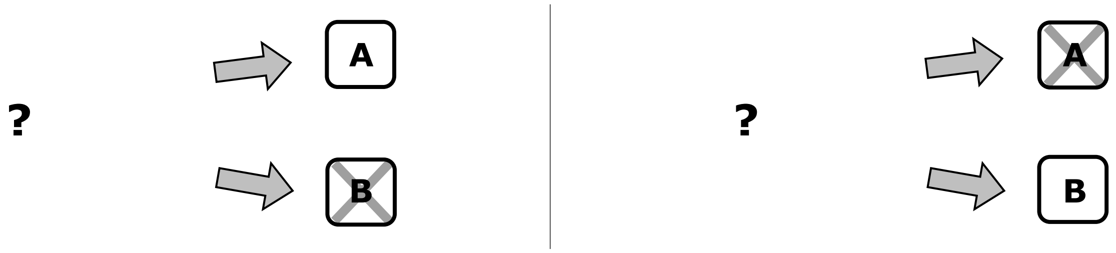
[*] **Vocabulary**: A comparison group is often called a **control group**.

[+] **Vocabulary**: In some scientific fields, *confounding factor* (or *confound*) = common cause. Here it means simply some other difference besides the independent variable. It *might* be a common cause.

Or perhaps some other confounding factor, not yet discovered, is a common cause.

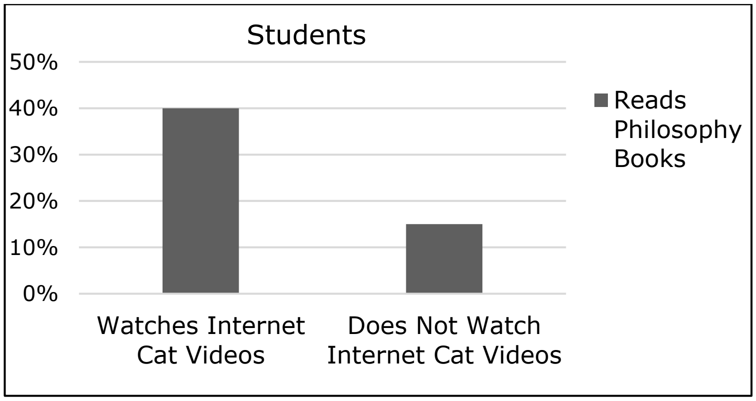


These explanation patterns are the same if **B** is *less* common with **A**, except that one thing is caused, and the other thing is prevented, by some common cause.

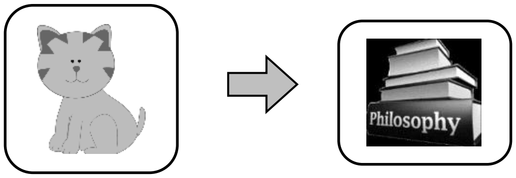


Example

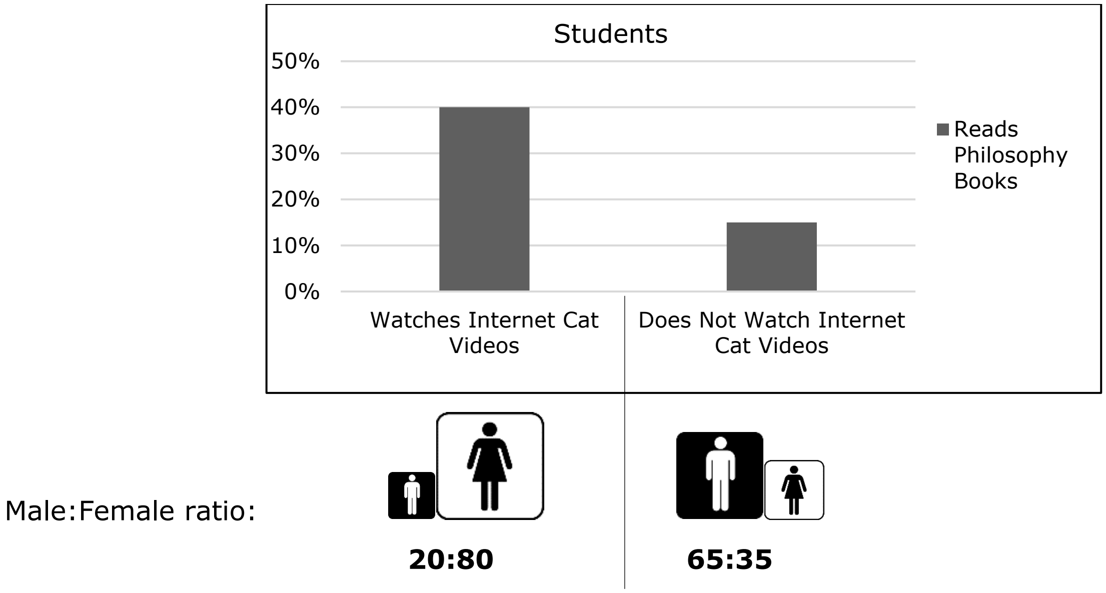
Philosophy book reading is more common among students who watch cat videos.



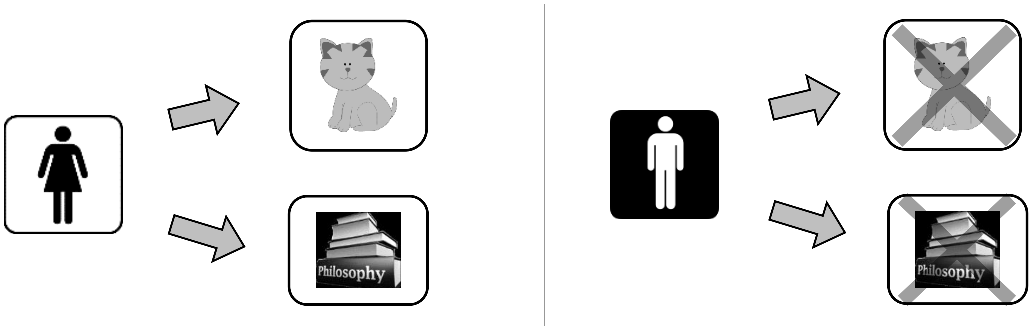
"Watching cat videos causes students to read philosophy books" explains the higher philosophy-reading rate among cat-video watchers. So the difference is evidence for that causal relationship.



The difference is evidence, but not proof. Perhaps gender is a confounding factor.



There is an alternative explanation: Being female is a common cause. Being female separately causes both cat-video-watching and philosophy-reading. (And being male prevents both things.)



We need to **control for** gender: keep the male:female ratio the same in the cat video watcher (experimental) group and non-cat video watcher (control) group. They could be all males, all females, or any other ratio. Then we measure the rates of philosophy-reading again. Is there still a difference?

You're doing an experiment…

PROBLEM **F** is a **confounding factor**. = **F** is another difference.

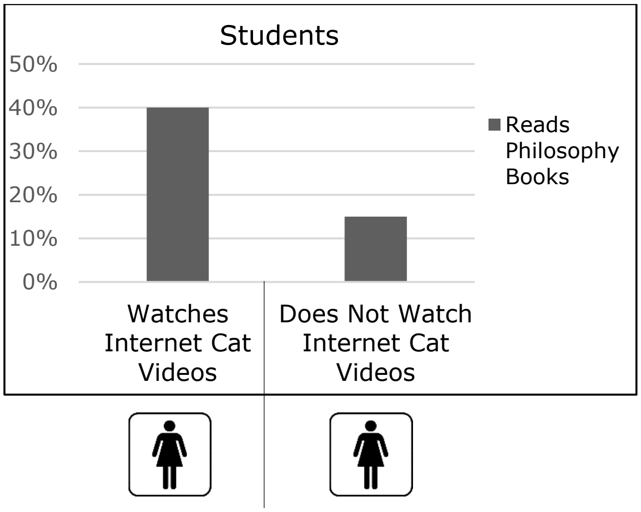
SOLUTION **Control for F**. = Keep **F** the same.

We control for gender: we compare female cat-video-watchers to female non-cat-video-watchers.

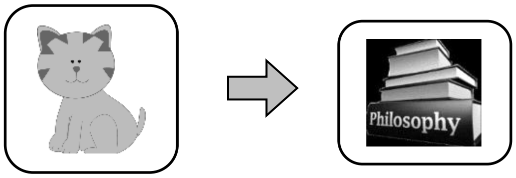
Possible Experiment Outcomes

The philosophy-reading rate is still higher in the cat video group. So gender doesn't matter. This eliminates "being female" as a common cause. We now have better evidence than before that watching cat videos causes philosophy reading.

Difference Remains

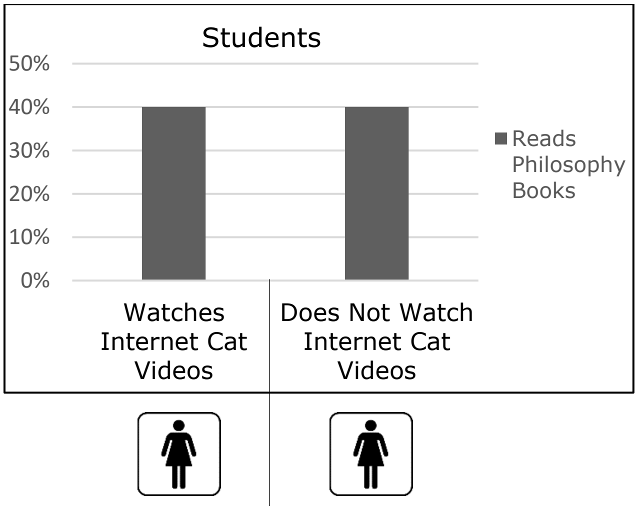


Supports: original explanation

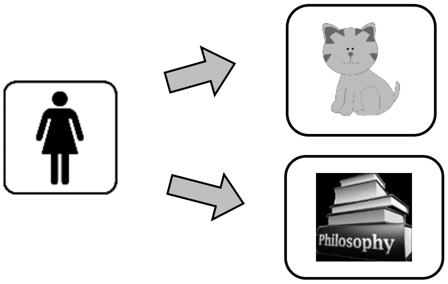


The difference disappears. Gender matters. This supports "being female" as a common cause. Why? Because if it's true, we should expect equal rates of the effect (philosophy-reading) when there are equal rates of the cause (being female).

Difference Gone



Supports: alternative explanation

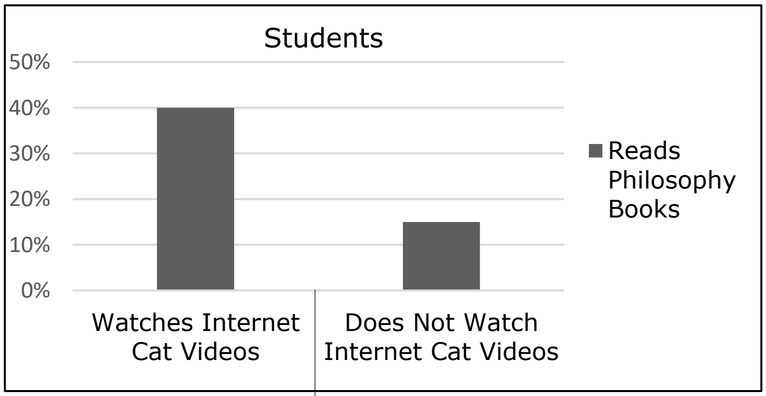


(Comparing males to males, we would see the same *low* rates of philosophy reading instead.)

Randomized Controlled Trials

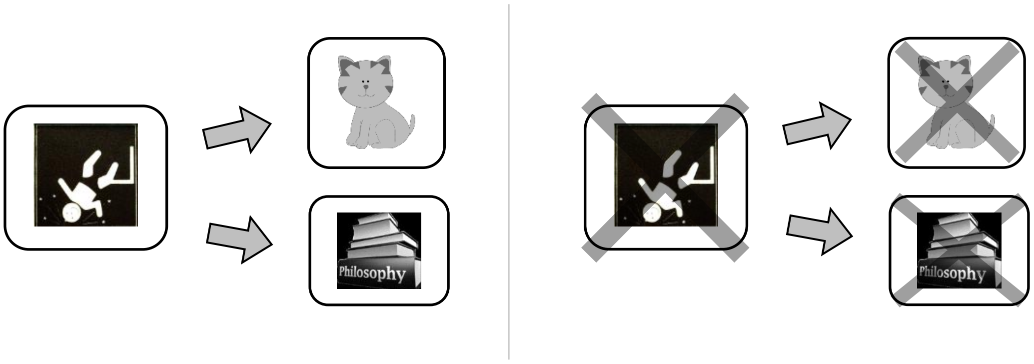
We study 800 students: 300 who watch cat videos and 500 who don't watch them.

Unknown to us, in our study population, 240 students (30%) were dropped on their head as a baby: 225 (75%) of the 300 cat-video-watchers and 15 (3%) of the 500 non-cat-video-watchers.



Dropped:	225	15
Group total:	300	500
Dropped rate:	**75%**	**3%**

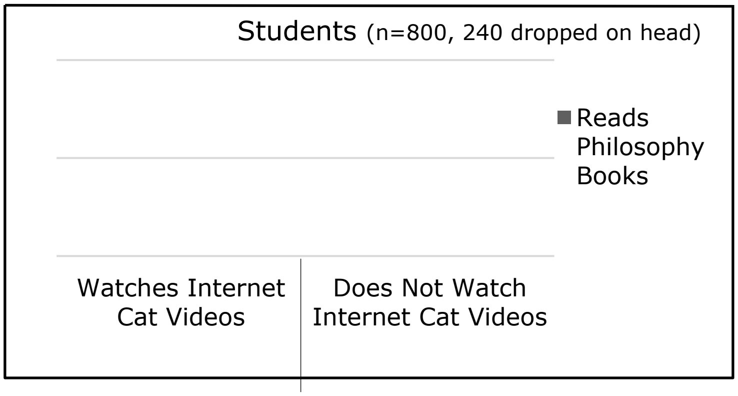
There's a higher "dropped" rate among the cat-video-watchers. Being dropped is a confounding factor. Perhaps it's a common cause of watching cat videos and reading philosophy, and that's why cat-video-watching students are more likely to read philosophy. (This would mean that *not* being dropped *prevents* both things, although saying the explanation this way sounds a bit odd.)



Since we don't know who was dropped on their head as a baby, we can't control for this by directly setting the dropped rate the same in each group. A **randomized controlled trial** (**RCT**) solves this problem. *Random* = every student has 1) an equal chance of being assigned to the experimental group and 2) an equal chance of being assigned the comparison group[*].

With coin-flip (50:50) randomization, we expect each group to get half (400) of the total students (800), and half (120) of the dropped students (240): the same 30% dropped rate in both groups. Randomization automatically controls for being dropped.

Randomization might not produce exactly 30% and 30%, e.g. we might randomly get $^{117}/_{403}$ (29%) and $^{123}/_{397}$ (31%). But we would be very unlucky to randomly get rates as different as 75% and 3%.



Dropped:	120	120
Group total:	400	400
Dropped rate:	**30%**	**30%**

Expected numbers with coin-flip (50:50) randomization

With adequate sample (group) sizes, we can trust randomization to automatically prevent confounding factors. Now is philosophy-reading still more common with cat-video-watchers?

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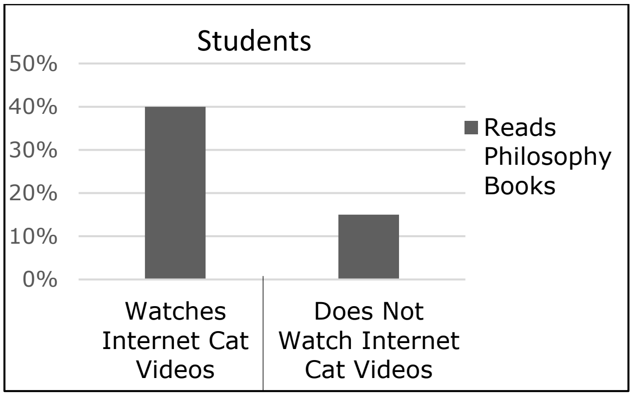
[*] These two chances do not need to be equal to each other – as long as each is the same for every item in the population. So then why coin-flip (50:50) randomization? Why not roll a dice ($^{1}/_{6}$ chance of assignment to the experimental group; $^{5}/_{6}$ chance of assignment to the control group)? Coin-flip randomization produces (roughly) equal group sizes. This is the optimal way to minimize the margins of error on both statistics. That's the only reason for the coin rather than the dice. If the sample (group) sizes are large enough, we could eliminate confounding factors with dice-roll randomization instead.

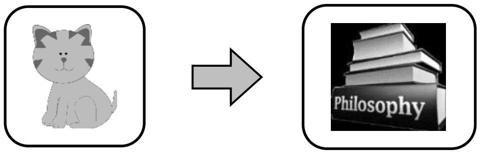
Possible Experiment Outcomes

Difference Remains

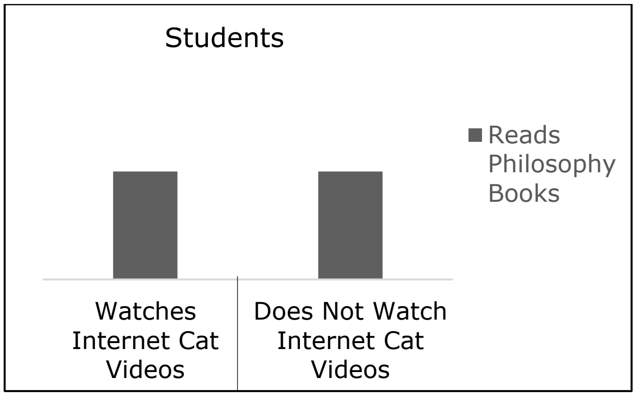


No Confounding Factors

Supports: original explanation

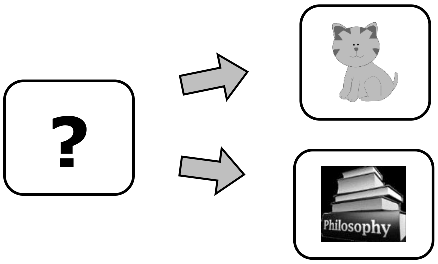


Difference Gone



No Confounding Factors

Supports: alternative explanation



(The rates would be the same, but what rate? That would depend on the rate of the (unknown) cause that is now equally present in both groups.)

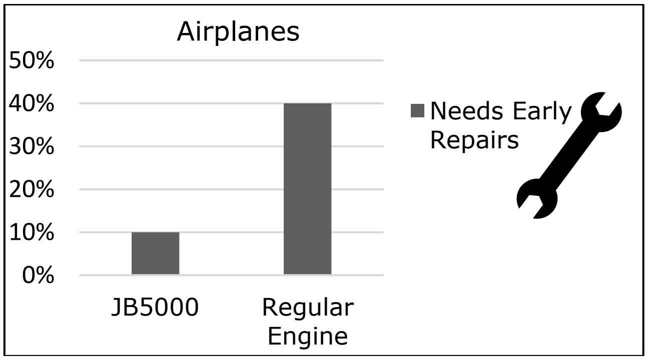
v. NEW CONFOUNDING FACTORS

An experiment may accidentally create a new confounding factor and a new alternative explanation of a difference. In this type of explanation, the common cause may be the experiment itself.

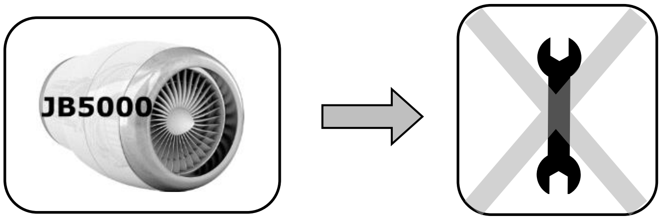
Example

Cloud9 is an airline. Sometimes their airplanes break and need repair earlier than their normally scheduled maintenance. Cloud9 considers upgrading their planes with the new JetBlast5000 engine. They hope the JB5000 is more reliable and will prevent early repair need.

Before upgrading all their planes, they buy some JB5000 engines and do an experiment. To avoid confounding factors, they randomly install JB5000s on some planes and regular engines on others.

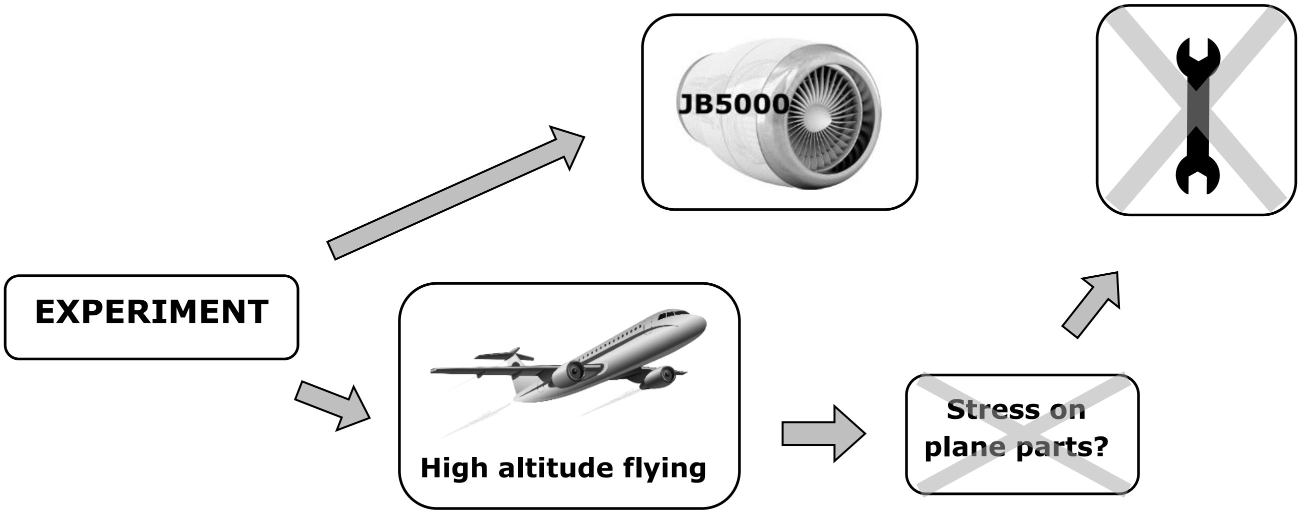


JB5000 planes need early repair less often. Cloud9 thinks that the JB5000 prevents early repair need.



(Image: Angi Reinschmidt)

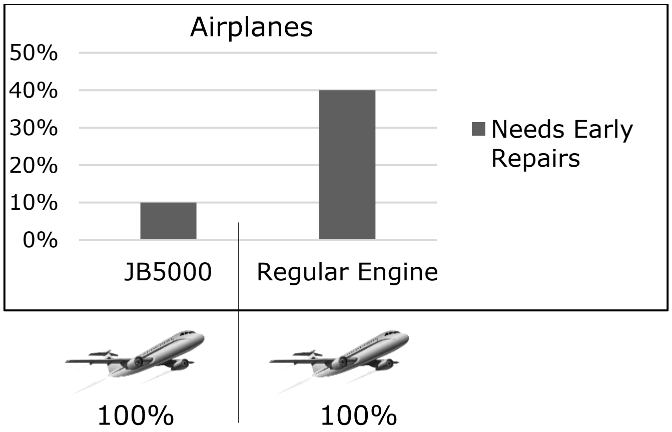
But the experiment may have created a new confounding factor. Perhaps Cloud9 told pilots of JB5000 planes to fly at high altitude because the engines work better there. But the air is less dense. Maybe that prevents stress on the plane parts, preventing early repair needs. If this explanation is right, the JB5000 does *not* prevent early repair needs.



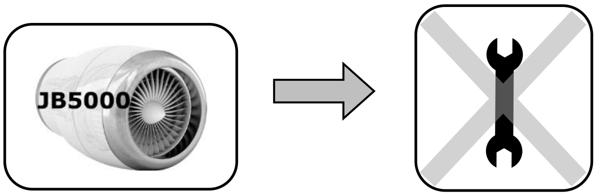
Randomization of the airplane groups did not control for altitude because the experiment itself (after randomization) caused the JB5000 planes to fly at a higher altitude. But the problem can be fixed: control for altitude. Now do JB5000 planes still need early repair less often?

Possible Experiment Outcomes

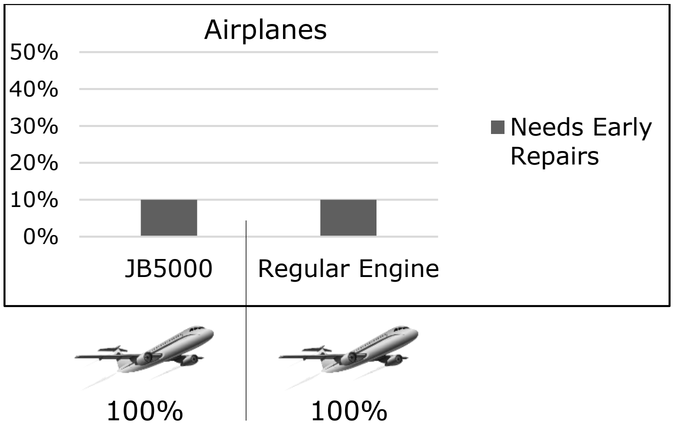
<u>Difference Remains</u>



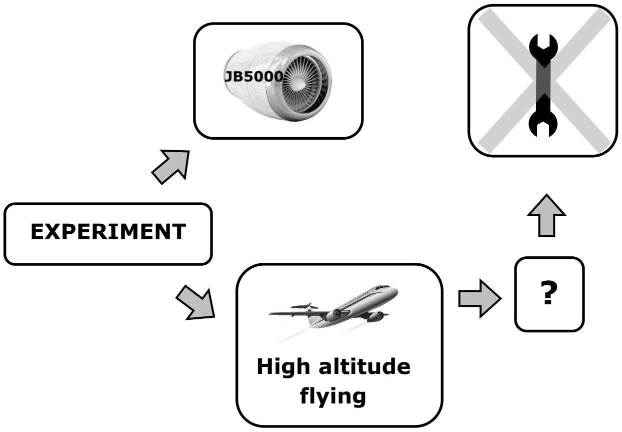
<u>Supports</u>: original explanation



<u>Difference Gone</u>



<u>Supports</u>: alternative explanation



Comparing 0% high altitude JB5000 planes to 0% high altitude regular planes, they would see the same *high* rates of early repair need instead.

UNIT 10 SKILLS

You must be able to:

- Recognize causal statements.
- Recognize, reconstruct, and evaluate method of difference arguments.
- Identify or think of confounding factors in a story.
- Formulate causal explanations and draw cause-and-effect diagrams.
- Describe the randomization procedure of an RCT experiment.

QUICK TEST QUESTIONS

---

1) What does “**A** prevents **B**” mean?

a) **A** does not cause **B** to happen.

b) **A** causes **B** to not happen.

c) **B** causes **A** to happen.

d) **B** causes **A** to not happen.

---

2) In the method of difference, we try to find the only relevant difference between…

a) …one situation when an effect happened and another when it did not.

b) …a cause and an effect.

c) …one possible cause of some effect and another possible cause of it.

d) None of the above.

---

3) Which is possible? **A** causes **B**…

a) …also **A** separately causes **C**.

b) …then **B** causes **C**.

c) …also **C** separately causes **B**.

d) (a) – (c) are all possible.

---

4) Which does <u>not</u> explain a difference in the **B**-rate between **A**-Ws and non-**A**-Ws?

a) **A** and **B** separately cause **C**.

b) **B** causes **A**.

c) **A** causes **B**.

d) (a) – (c) all explain the difference.

---

5) What is a confounding factor?

a) Another difference between the independent and dependent variables.

b) Another difference between the experimental and comparison groups.

c) Another difference between the controlled and uncontrolled experiments.

d) None of the above.

---

6) What does it mean to *control for* **F**?

a) To measure **F** in a controlled experiment.

b) To compare two groups between which **F** is a difference.

c) To compare a group of **F** to a comparison group of Non-**F**.

d) None of the above.

---

7) In a randomized controlled trial, items are randomly assigned to either…

a) …the randomized or the non-randomized group.

b) …the cause group or the effect group.

c) …the experimental or the comparison group.

d) …the independent variable group or the dependent variable group.

PRACTICE EXAMS

PRACTICE EXAM 1

1) Write the implicit statement and give it a number. Reconstruct the argument in standard form using the statement numbers. Evaluate the argument.

Near Gotham there are two crocodile petting zoos: Acme CrocWorld, and Big World o'Crocs. Acme is near the hot springs, where geothermal heat warms the ponds, while Big World is in Shady Valley, where there are no hot springs, but visitors enjoy a cool shady park while they pet the crocodiles. Acme and Big World both breed crocodiles. Crocodiles lay eggs in the mud around their waterholes. The hatch rate at Acme is higher than at Big World. Big World's Board of Directors calls in Zelda, the Zoo Manager, and demands to know why. Zelda says:

① We've looked at the two petting zoos, and we think we know why Acme's hatch rate is better. ② Both zoos give the same amount of water and land space per crocodile, and ③ visitors are kept away from the egg-laying areas where they might accidentally step on the eggs at both parks. However ④ Big World has mainly American crocodiles; Acme has mostly Nile crocodiles, which lay eggs with stronger shells. We think that ⑤ Nile crocodiles' stronger eggs make Acme's hatch rate higher.

2) Answer the questions based on the story.

Cyclists in Gotham mostly use normal locks, but a new bike lock, SuperLock, is on the market. The company does a survey asking cyclists what lock they use and if their bikes was stolen in the past 6 months. SuperLock users had their bikes stolen more often (25%) than normal lock users (15%)! Superlock had hoped their lock prevented bike theft but it seems to *cause* bike theft instead!

a) Draw a bar chart of SuperLock's survey data. Draw a cause-and-effect diagram of the explanation they consider.

The company thinks of an alternative explanation for the shocking survey result: maybe dedication to cycling causes some cyclists to buy the expensive SuperLocks, but dedication also causes those cyclists to have their bikes stolen more often (the company can imagine several different possible causal chains).

b) Draw a cause-and-effect diagram of the alternative explanation that SuperLock imagines. Fill in the causal chain with your own suggestion.

The company will do an RCT experiment. They find 300 cyclists willing to participate.

c) Describe the experiment that the company will do. What are the dependent and independent variables? Assume that 90 cyclists have the feature the company imagined might be a confounding factor in the original survey.

The company has normal locks in November to begin the 6 month experiment. But the SuperLocks have been sent to stores and they cannot start the experiment with those until more are available in February. By August they have all the data. Once again the SuperLock cyclists had their bikes stolen more often! They are depressed.

d) The RCT experiment outcome supports the **original | alternative** explanation.

e) Identify a new confounding factor the company created in their experiment. Draw a cause-and-effect diagram of a new explanation of the difference. The diagram must include the confounding factor you identified, plus one other that you think of.

f) How could the company have fixed the RCT experiment by changing the comparison group?

PRACTICE EXAM 2

[This shorter exam combines the question formats of sections i and ii-v into a single question set.]

Write the implicit statement and give it a number. Reconstruct the argument in standard form using the statement numbers. Then answer the questions.

> The Azmakian military has been fighting in the country's vast northern region against rebels who want to overthrow the government. The government has a network of 40 outposts throughout the northern region. The military's job is to defend the outposts and chase any rebels they can find. Recently the military has acquired some expensive new flying drones to chase rebels. They have only 10 drones, though, so they use them at the outposts that are surrounded by lots of steep mountains that would be dangerous for soldiers to guard on foot.
>
> After a year, the military reviews data from all the outposts.
> They say: "Look at this: ① the outposts protected by drones were attacked less often than the other outposts (protected only by soldier guards). ② We should buy more drones! It seems that ③ they were the only major difference between the outposts. ④ The drone-protected outposts are just as strategically valuable as the other outposts, and ⑤ the fences and lights we have around them are no better than we have at the other outposts. ⑥ The Azmakian taxpayers won't mind that these drones cost 35 million dollars each."

a) Reconstruct the argument in standard form.

b) Draw a bar chart of the military's data. (The exact numbers are not given and not necessary.) Draw a cause-and-effect diagram of the explanation they have.

c) Evaluate their argument. Describe an alternative explanation of their data using details from the story. Show your explanation by drawing in a cause-and-effect diagram.

d) The military didn't think of the explanation you gave above but they want to do a better experiment to be sure that the drones are truly effective before they spend the taxpayers' money [Hahaha…]. Describe the experiment they should do. Suppose that 12 of the outposts have the feature you mention in your answer to question (c). (Hint: Remember that they have only 10 drones.)

e) This year they do the better experiment that you described in your answer to question (d). The drone-protected outposts get attacked less often. Does this fact support the military's original explanation or your alternative?

f) They look at the data and see that 40% of the drone-protected outposts were attacked while about 47% of the others were attacked. Does this information affect the answer to question (e)? Explain why or why not.

PRACTICE EXAM 3

1) Write the implicit statement and give it a number. Reconstruct the argument in standard form using the statement numbers. Evaluate the argument.

To lower its high incarceration rate, Gotham City Council told the city's two prisons to create early release programs. Prison A releases every prisoner when they've served half their sentence. Prison B was more concerned about endangering the community and wanted to do more to stop ex-prisoners from committing new crimes. In their program, prisoners who've volunteered for extra duties such as library attendant are released after half their sentence, but they get a tattoo across their chest:

(backwards so it's readable in a mirror)

After three years, Gotham City Council meets to discuss the programs.

"① The data here is pretty convincing. ② The DON'T COMMIT CRIMES tattoos are the only significant difference between the two groups of early release prisoners. ③ There's no difference in the types of crimes they were convicted of, and ④ they're placed in the same types of job programs to help them re-enter society. Moreover ⑤ conditions in the prisons (presence of gangs, treatment by guards, etc.) are the same. Yet ⑥ 40% of the prisoners from Prison A commit another crime within a year of their release whereas only 10% of those from Prison B do. ⑦ We'll hire a tattoo artist for Prison A."

2) Answer the questions based on the story.

Azmakia Broadcasting Network (ABN) makes comedy shows and is studying people's reactions to them so that they can make more funny shows. They show comedies to focus groups and find a surprising difference between shows with and without a "laugh track" (added recorded laughter): only 40% of the shows with a laugh track are rated "funny" by the focus groups, whereas 75% of the shows without one are rated "funny"! ABN concludes that, contrary to the TV industry's belief, laugh tracks actually prevent shows from seeming funny.

a) Draw a bar chart of ABN's data. Draw a cause-and-effect diagram of the explanation they consider.

But ABN think of an alternative explanation. Perhaps good comedy writers, who write the funniest jokes, also refuse to allow their shows to use laugh tracks (they think the laugh track insults their talent as writers).

b) Draw a cause-and-effect diagram of the alternative explanation that ABN imagines.

ABN will study this with an RCT. They can edit any show to have or not have a laugh track. They will study 42 shows.

c) Describe the experiment that the company will do. What are the dependent and independent variables? Assume that 16 shows have the feature the company imagined might be a confounding factor in the original survey.

The focus group for the shows with laugh tracks comes to the show testing centre and gives their ratings. The next day the focus group for the shows without laugh tracks arrives after the lunchbreak when the centre's receptionist microwaved fish for her lunch. This time ABN finds that shows with laugh tracks are rated funny more often, as the TV industry expects.

d) The RCT experiment outcome supports the **original | alternative** explanation.
e) Identify a new confounding factor the company created in their experiment. Draw a cause-and-effect diagram of a new explanation of the difference. The diagram must include the confounding factor you identified, plus one other that you think of.
f) How could the company have fixed the RCT experiment by changing the comparison group?

ANSWER KEY

QUICK TEST

1) b **2)** a **3)** d **4)** a **5)** b **6)** d **7)** c

PRACTICE EXAM 1

1) **Answer**

Implicit statement: ⑥ There are no other relevant differences between the zoos besides the eggshells.

Main Argument	Sub-argument
④ [**A** different when **B** different.]	②
⑥ [No other relevant differences besides **A**.]	③
―――	―――
⑤ [**A** causes **B**.]	⑥

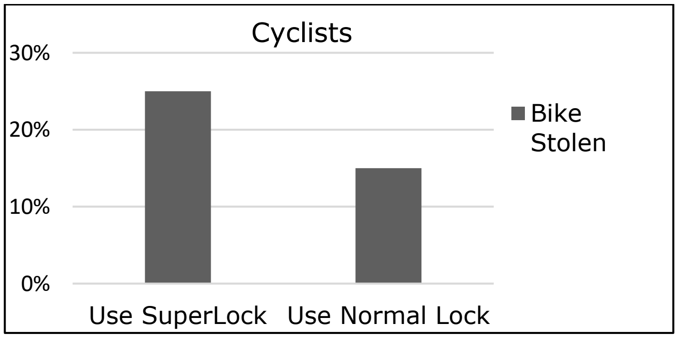
Evaluation: This is a bad method of difference argument. It depends on the claim (⑥) that the crocodiles' eggshells are the only relevant difference between the two zoos. But that statement is false. The zoos are in different kinds of environments. Acme has hot springs but Big World does not. This is another relevant difference, not considered in the sub-argument. Perhaps Acme's hot springs cause the eggs to be healthier which causes the higher hatch rate.

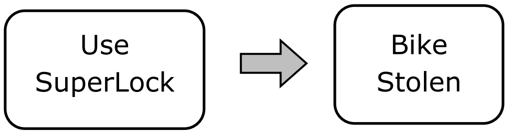
Explanation (not required for exam answers)

The role that each statement has in the method of difference main argument is given in the answer above in [brackets]. This is not a required part of the answer.

2)

a) **Answer**

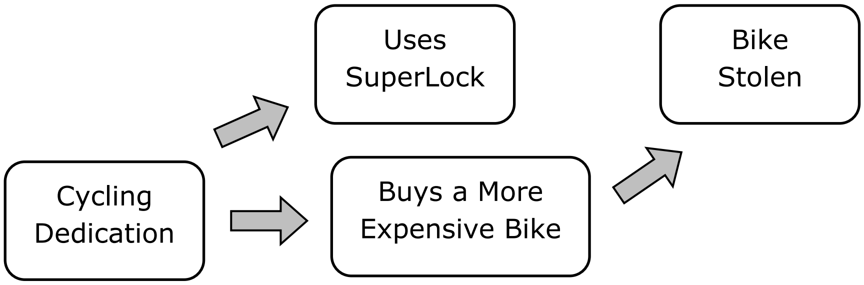




Explanation

Other correct answers are possible for the chart. For example, the population could be Bikes, the independent variable Locked with SuperLock (vs. Locked with Normal Lock), and the dependent variable Stolen.

b) **Answer**



Explanation

The possible confounding factor, Cycling Dedication, is a common cause in this explanation. Another confounding factor, Buys a More Expensive Bike, is in the causal chain to the Bike Stolen effect. This is an *alternative* explanation because there is no causal link between Use SuperLock and Bike Stolen. This diagram refers to the experimental (Superlock) group. The diagram for the control (Normal lock) group would be the same, but with a ✕ over each part.

Other correct answers are possible for the causal chain. For example, perhaps Cycling Dedication causes people to keep their bikes shinier, or it causes them to take their bikes to different areas where bike thieves go.

c) **Answer**

In their randomized controlled trial, the dependent variable is Bike Stolen and the independent variable is Uses Superlock (vs. a normal lock). The company randomly (e.g. with a coin flip) assigns each cyclist to either the experimental (SuperLock) group or the comparison (normal lock) group. With 300 cyclists of which 90 are dedicated, we expect coin-flip randomization to make each group $^{45}/_{150}$ (30%) dedicated cyclists. Randomization automatically controls for dedication.

d) **Answer**

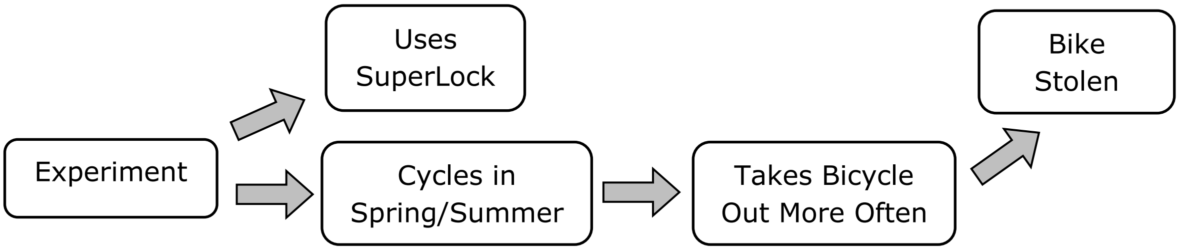
The RCT experiment outcome supports the **original** explanation.

Explanation

The SuperLock group still has a higher theft rate. Since randomization has (we assume) eliminated confounding factors by controlling for dedication and other things, the SuperLocks may be the only difference, and therefore the cause (however see the next answer!).

e) **Answer**

The experiment creates a new confounding factor: time of year. The comparison group (normal locks) is in Winter; the experimental group (SuperLocks) is in Spring and Summer. The nicer weather might cause people to take their bike out more and be targeted by thieves more.



Explanation

Other correct answers are possible for the causal chain. For example, perhaps demand for stolen bicycles is higher in Spring and Summer, so thieves are more active then.

f) **Answer**

The company should run the comparison group later, during the same period as the experimental group, to eliminate seasonal or weather-related confounding factors.

Explanation

Since they don’t have 150 SuperLocks available in November to run the experimental group in Winter, delaying the comparison group is the only option.

PRACTICE EXAM 2

a) **Answer**

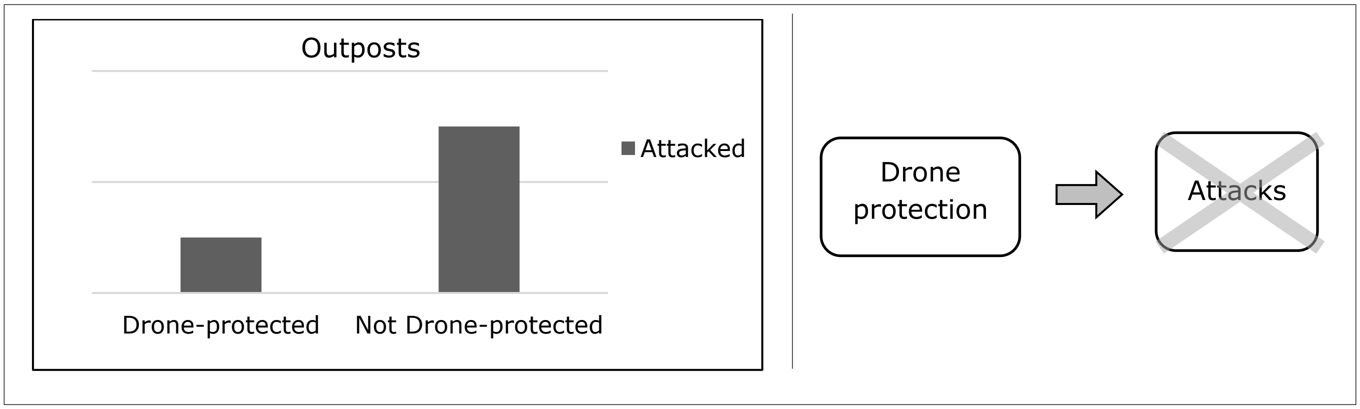
Implicit statement: ⑦ The drones prevented attacks on the outposts.

Main Argument	Sub-argument
① [**A** different when **B** different.]	④
$\boxed{③}$ [No other relevant differences besides **A**.]	⑤
―――	―――
⑦ [**A** prevents **B**.]	$\boxed{③}$

Explanation (not required for exam answers)

The role that each statement has in the method of difference main argument is given in the answer above in [brackets]. This is not a required part of the answer.

b) **Answer**

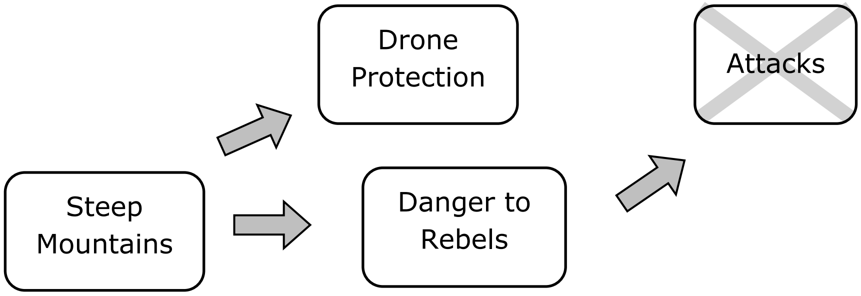


Explanation

The attack rates aren't given so the bar chart simply shows a lower attack rate for drone-protected outposts.

c) **Answer**

Statement ③ is false. There is another relevant difference between the outposts (a confounding factor in the military's experiment): the drone-protected outposts are surrounded by steep mountains. The danger to soldiers of walking in these mountains is the reason why those outposts were chosen for drone protection. But the mountains that are dangerous for soldiers would also be dangerous for rebels and might stop rebels from attacking there. So the steep mountains might be a common cause, causing the drones and preventing the attacks.



d) **Answer**

They should do an RCT. Randomly assign each outpost to an experimental (drone) group or a control (no-drone). Since there are 40 outposts and only 10 drones, they should give each outpost a $^1/_4$ chance (e.g. double heads, flipping two coins) of being assigned to the experimental group – to choose 10. (Each outpost has a $^3/_4$ chance of being assigned to the comparison group – to choose 30.) Since there are 12 outposts with steep mountains, we expect 25:75 randomization to makes groups with steep mountain rates: 3/10 (30%) and 9/30 (30%). Randomization automatically controls for steep mountains.

e) **Answer**

The lower attack rate in experimental (drone) group supports the military's original explanation. This is good evidence that the drones are effective.

f) **Answer**

These percentages affect the answer to (e). Although the attack rate is lower in the experimental group, the difference (7 pts) is fairly small. With the small sample (group) sizes (10 and 30), the margins of error will be so large (at any normal confidence level) that they will significantly overlap. These statistics provide only weak support for the drones' effectiveness.

PRACTICE EXAM 3

1) **Answer**

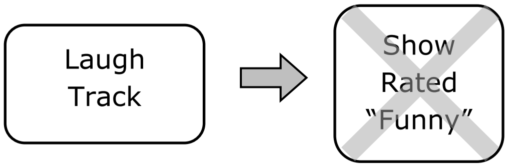
Implicit statement: ⑧ The tattoos prevent early released prisoners from committing new crimes.

Main Argument	Sub-argument
	③
⑥ [**A** different when **B** different.]	④
$\boxed{②}$ [No other relevant differences besides **A**.]	⑤
⑧ [**A** causes **B**.]	$\boxed{②}$

Evaluation: This is a bad method of difference argument. It depends on the claim (②) that there are no other relevant differences besides the tattoos, but that's false. The prisoners in Prison B's program have volunteered for extra duties in the prison, whereas there's no special requirements in Prison A's program. That difference is a relevant because it suggests that the Prison B prisoners are especially motivated to reform and become goods citizen. Maybe that motivation, not the tattoos, causes them to commit new crimes less often.

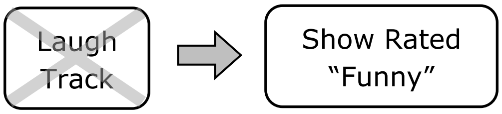
2)

a) **Answer**

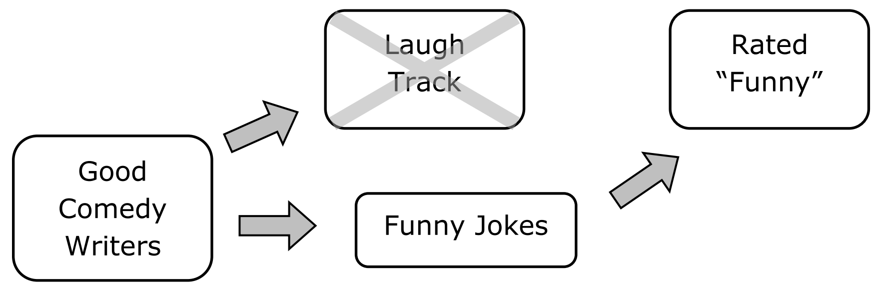


Explanation

The cause-and-effect diagram shows the prevention explanation, referring to the shows in the Laugh Track group. It's important for question (b) to remember that there's a corresponding diagram for the No Laugh Track group (although it would be strange to say "No Laugh Track causes shows to be Rated 'Funny'").



b) **Answer**



Explanation

See the explanation for the answer to question (a). The possible confounding factor, Good Comedy Writers, is a common cause in this explanation. Another confounding factor, Funny Jokes, is in the causal chain to the Rated "Funny" effect. This is an *alternative* explanation because there is no causal link between No Laugh Track and Rated "Funny". This diagram refers to the control (No Laugh Track) group. The diagram for the experimental (Laugh Track) group would be the same but with the ✕ switched to the other factors (No Good Comedy Writers the common cause).

c) **Answer**

In their randomized controlled trial, the dependent variable is Rated "Funny" and the independent variable is Laugh Track (vs. No Laugh Track). The company randomly (e.g. with a coin flip) assigns each show to either the experimental (Laugh Track) group or the control (No Laugh Track) group. With 42 comedy shows of which 16 have good writers, we expect coin-flip randomization to make the Good Writer rate $^{8}/_{21}$ (38%) in each group of shows. Randomization automatically controls for good writers.

d) **Answer**

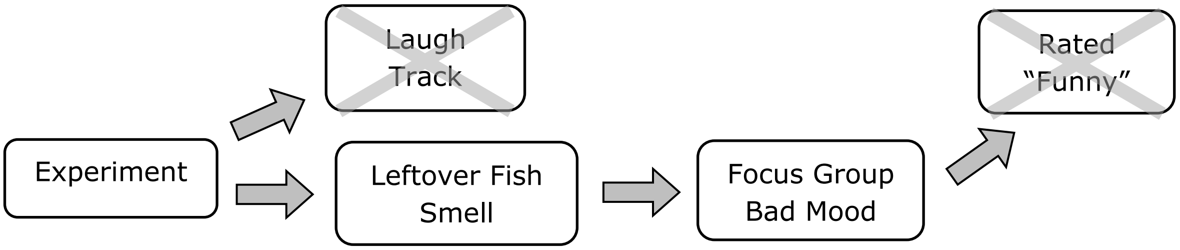
The RCT experiment outcome supports the alternative explanation.

Explanation

The difference in the Rated "Funny" rates reverse when (we assume) randomization eliminates confounding factors. Laugh tracks now make a positive difference. This suggests that the negative difference they originally found in the Rated "Funny" rates was caused by some confounding factor (perhaps good comedy writers), not by the laugh track.

e) **Answer**

The experiment creates a new confounding factor: the microwaved fish. This will make the show testing centre smell of the secretary's leftover fish lunch while the comparison group (No Laugh Track) watches the shows. The leftover fish smell might put the focus group in a bad mood, which prevents them from rating the shows "funny".



Explanation

The diagram refers to the shows in the comparison group.

f) **Answer**

The company should make sure the secretary doesn't microwave her leftover fish lunch before the focus group arrives!