# ASSESSING THE MODIFIED MERTON DISTANCE TO THE DEFAULT MODEL WITH CDS PRICE

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# Abstract

This paper provides a way that a Merton-model approach can be modified to develop measures of the probability of default of companies indexed in Standard & Poor's 500 Index (S&P 500) after a financial crisis. It also examines the accuracy and contribution of the modified Merton Distance to default model based on Merton's (1974) bond pricing model. Credit Default Swap (CDS) spreads as a plausible indicator of default risk are used in the assessment. The tests are implemented by modeling results' correlation with data obtained from 2008 to 2017. The sample is based on 112 firms indexed in S&P 500 and is selected according to the availability of outstanding CDS contracts between the test periods.

It is found that the results generated by the modified Merton-style approach is consistent with the spreads of credit default swaps. Then it can be concluded that although the modified KMV Merton model fails to generate a sufficient result for the probability of default, it still can be used as a reference for default estimate.

**Keywords:** Merton Model, Probability of Default, Credit Default Swaps

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# 1. Introduction

In the last decade, with the start of the financial crisis in 2007 and the European recent debt crisis, investors, regulatory agencies and financial institutions have been paying more attention to credit risk in the financial markets. Besides, regarding to the great volume of over-the-counter derivatives traded and the rapidly developing markets of credit-sensitive financial products, the importance of credit risk modeling is further addressed.

Credit risk modelling underpins a theoretical structure to demonstrate the relationship between the borrowing party's characteristics and its probability of bankruptcy. Currently, there are two primary streams of credit risk modeling approaches: structural and reduced form models. This paper tends to focus on structural models. One of the popular methods for assessing credit risk within this class is Merton's model which is firstly introduced in 1974. Later, the Merton distance to default (DD) model is developed to estimate default in a more straightforward way. Besides, by alternating the inputs as well as assumptions, more complex and sophisticated models such as the hazard model and the reduced form model are developed so as to capture better predictive properties.

In this paper, a modified Merton-style approach (structural approach) is employed to estimate default probability for companies indexed in S&P 500 and assess the accuracy of those estimates using various techniques.

## 2. Literature Review

2.1 The Original Merton Model

Merton (1974) proposes a firm model that provides an approach to indicating credit risk based on firms' capital structure known as the structural model based on the assumption that a company only has two types of issued securities: debt and equity. For debt, it is simplified that debt is a zero-coupon bond which will become due at a future time T. Therefore, debt is a pure discount bond where the principal is repaid at time T. As for equity, equity holders receive no dividends, while the principle of Merton's model is that the company will default if the value of its assets is less than the debt payable at time T. As an extension of the Black-Scholes (1973) option pricing framework, the equity issued by this company can be seen as a European call option on the assets of the company with maturity T and a strike price which is equal to the face value of the debt (Hull, Nelken, & White, 2005).

The Merton model has the advantage of connecting credit risk to the financial fundamental of companies, and provides the intuitive economic interpretation and endogenous illustration of credit default with implementation of option pricing methods (Wang, 2009). There are also some issues with Merton's model, mainly lying in the difficulty of application. For instance, the assumption that the assets of the company can be traded in the frictionless market, is unrealistic. In addition, Wang (2009) sates that in the Merton model, the underlying value of the firm and the volatility of this value cannot be directly observed in the market. Furthermore, the original Merton model fails to capture the features of corporate bonds traded in the market in empirical studies which show that

the Merton model underestimates credit risk, particularly credit risk in the short term for traded bonds with high quality (Kulkarni, Mishra, & Thakker, 2008).

Overall, despite the disadvantages mentioned above, the model not only facilitates the security valuation, but also provides ground for the development of credit risk modelling.

2.2 The Merton Distance to Default Model

Regarding to the drawbacks of the original Merton model, extensions to the Merton model are proposed mainly tackling problems resulting from simplified assumptions Merton and Black-Scholes make. One of the extensions is the Merton Distance to Default model, also known as the Merton DD model that adopts the framework of Merton (1974), in which the equity of the firm is a call option on underlying value of the firm with a strike price equal to the face value of firm's outstanding debt. In the Merton DD model (Benos & Papanastasopoulos, 2007), it is recognized that neither the underlying value of the firm nor its volatility can be directly observed in the market. According to the assumptions of this model, those two can be inferred from the stock price of the company and the volatility of stock prices as well as other variables which are directly observable by using an iterative procedure to solve a system of nonlinear equations. According to the model, the probability of default calculated using iterative results is the normal cumulative density function of a z-score depending on the firm's underlying value, the firm's volatility and the face value of the firm's debt (Tudela & Young, 2005).

The Merton Distance to Default model with little estimation towards model inputs, uses iterative methodology to get implied parameter values. However, the inputs of this iterative approach have been criticized. As the market value of equity drops, the

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probability of the default increases and this is under the assumption that the capital market

is sufficiently efficient to reflect the information fully and timely (Severinsson & Wedin,

2013).

## 3. Data & Summary Stats

3.1 Credit Default Swap (CDS)

A credit default swap is a kind of credit derivative contract designed to transfer the credit exposure from one party to another, and it is like an insurance contract against credit events. The CDS purchaser pays a series of premiums to the seller depending on the CDS horizon until the expiration of the contract or the credit event occurs, whichever comes first. If the credit event occurs before the contract expires, the seller has an obligation to compensate the CDS buyer based on the contract. The premium that the buyer needs to pay to the seller is known as the CDS spread (Yu, 2006).

Credit default swaps were originally created in the mid-1990s to transfer the credit exposure for commercial loans. The CDS market developed extremely fast from about $900 billion in 2000 to $45 trillion by the end of 2007. However, there was only the $25 trillion bond in the market in 2007, so it means that about $20 trillion were speculative "bets" on the possibility of credit events. It is unnecessary for the buyer to own the entity for buying the CDS, unlike insurance, and it is also the reason why a number of people use CDS as a speculative tool (Zabel, 2008).

3.2 Date Source

All the data used in our paper were obtained from the Bloomberg Terminal. For the CDS Spread, CMAN is chosen as our vendor. We obtained 1-year, 3-year and 5-year monthly CDS spread of companies listed on S&P 500 in June 2017 from January 2008 to June 2017, since these CDSs are the most actively quoted and traded in the market. We drop the companies, for which CDS data are not available and also exclude those who do not have CDS maturity of 1, 3, or 5 years. Eventually, 112 companies are chosen

as our research companies. Then, the daily stock price (closing price), market share outstanding and the quarterly book value of current liabilities and non-current liabilities for these 112 companies are downloaded. Besides, the annualized daily US Treasury risk-free rate from January 2008 to June 2017 is also downloaded.

The raw data is converted into the format we need to fit the Merton DD model using the following equations

$$Market\ Capitalization = Share\ Price * Market\ Share\ outstanding$$

$$Stock\ Return = \ln(\frac{P_{t+1}}{P_t})$$

$$\sigma_E = \sqrt{\frac{\sum_t^T (R_t - \bar{R})}{T - t - 1}}$$

$$Debt = Current\ Liability + a * Non\text{-}Current\ Liability$$

$$Leverage = \frac{Current\ Liability + Non\text{-}Current\ Liability}{Market\ Capitalization}$$

where P is the daily stock price; R is the stock return; a is a constant based on tenor estimate.

As only the quarterly data for the companies' current liability and non-current liability can be obtained and most companies' debt experiences an insignificant change in a short time, the quarterly data is converted to monthly data by taking the weighted average, such as, $D_{February} = \frac{2}{3} * D_{January} + \frac{1}{3} * D_{April}$.

When the daily data is converted to monthly data, for matching the debt reporting date and accuracy, the data is converted at the end of the according month.

3.3 Summary Statistics

     Table 2 shows summary statistics for all the variables used in the Merton DD model. From the table, it is easy to observe our data ranging from small-size companies to large-size companies, from the company with zero short-term debt to the company with billions of long-term debts. With the increase of the estimated length, the distance to default keeps going down, while the CDS spread keeps going up.

# 4. Methodology

4.1 The KMV-Merton Model

The KMV-Merton model was developed by KMV Corporation in the late 1980s, and it uses the assumption in the Merton model (Merton, 1974) as its base. In the KMV-Merton model, it is assumed that the firm promises to pay B to the bondholders at maturity T. If this payment is not met, or in other words, if the value of the firm is less than B, the bondholders take over the company, and the shareholders receive nothing, which means the company goes to default. To calculate the default probability, the model uses an estimate of the firm's market value subtracts the face value of the firm's debt, and then divides this difference by an estimate of the volatility of the firm's asset. The result is known as the distance to default, and then the result is fit into a cumulative density function to calculate the probability that the firm value will be less than its face value of debt. In this paper, the distance to default is used as our main result to make further investigation, since it is more intuitive compared with the default probability.

The Merton model has two significant assumptions. The first assumption is that the total value of a firm follows a geometric Brownian motion,

$$dV = \mu V \, dt + \sigma_v V \, dW \tag{1}$$

where V is the total value of the firm; $\mu$ is the expected continuously-compounded return on V; $\sigma_v$ is the volatility of the firm value and $dW$ is a standard Wiener process.

The second assumption of the Merton model is that the firm only issued zero-coupon bonds maturing in T periods. Under these two assumptions, the payoffs to the bondholders can be seen as a call option on the value of the firm with the strike price equal to the face value of firm's total debt outstanding and the time to maturity is T.

Therefore, the option pricing function of Black and Scholes (Black & Scholes, 1973) can be used to estimate the value of the option and the underlying probability of default. From this thought, the Merton Model derives the company's market capitalization that satisfies

$$E = VN(d_1) - e^{-rT}DN(d_2) \tag{2}$$

where E is firm's market capitalization (market value of the firm's equity); V is the firm value; D is the face value of the firm's debt; r is the instantaneous risk-free rate; $N(\cdot)$ is the cumulative standard normal distribution function, and $d_1$ is given by

$$d_1 = \frac{\ln\left(\frac{V}{F}\right) + (r + 0.5\,\sigma_V^2)T}{\sigma_V\sqrt{T}} \tag{3}$$

$$d_2 = d_1 - \sigma_V\sqrt{T} \tag{4}$$

Equation (2) is one of the two significant equations that the KMV-Merton model uses. The second significant equation is the relation between the volatility of the firm's value and the volatility of the firm's equity. According to equation (1), the value of equity is a function of the value of the firm and time, and therefore, it must follow Ito's lemma that

$$\sigma_E = \left(\frac{V}{E}\right)\frac{\partial E}{\partial V}\sigma_V \tag{5}$$

From equation (2), $\frac{\partial E}{\partial V} = N(d_1)$ can be seen, so equation (5) can be converted to

$$\sigma_E = \left(\frac{V}{E}\right)N(d_1)\sigma_V \tag{6}$$

where $d_1$ is defined in equation (3).

4.2 The Solving Method

Two nonlinear equations (2) and (6) can be used to imply the probability of default. In the KMV-Merton model, the firm's total debt outstanding, market capitalization, debt outstanding and the volatility of stocks are easy to be obtained from the market and have

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been mentioned in the previous part, but the value of the firm and the volatility of the firm's asset cannot be directly observed, which means that they must be implied.

For matching the CDS spread data, the estimating horizon of one year, three years and five years is chosen. For different time horizons, different face values of the firm's debt are assumed as follows:

$$D = Current\ Liability \qquad\qquad\qquad\qquad\qquad\qquad\qquad T = 1$$

$$D = Current\ Liability + 0.3 * Noncurrent\ Liability \qquad\qquad T = 3$$

$$D = Current\ Liability + 0.5 * Noncurrent\ Liability \qquad\qquad T = 5$$

It is easy to solve equation (2) and equation (6) simultaneously, but according to Crosbie & Bohn (2003), "In practice the market leverage moves around far too much for equation (6) to provide reasonable results." Crosbie & Bohn also mentioned an iterative producer, a more complicated but accurate method, to solve these two equations. For the accuracy, we decide to use the iterative producer in this paper. An initial value of $\sigma_V = \sigma_E * \frac{E}{E+D}$ and $V = E + D$ is proposed, and then all the data are put into equation (2) to get the first implied firm value with the Newton iterative method (Weisstein, 2005). Besides, the implied firm value is employed to calculate the implied volatility of firm assets, and the new volatility is adopted to imply the new firm value. Furthermore, these two steps are repeated more times until $\sigma_V$ converges. For both iterations, the settings of tolerance are $10^{-6}$.

Once both equations are solved at the same time, the last step is to calculate the distance to default using the following equation

$$DD = \frac{\ln\left(\frac{V}{F}\right) + (\mu - 0.5\sigma_V^2)T}{\sigma_V\sqrt{T}} \tag{7}$$

where $\mu$ is an estimate of the expected annual return of the firm's assets. In this paper, $\mu$ equal to the risk-free rate is assumed.

4.3 Data Analysis

4.3.1 Data Testing

For time series analysis with regression, the analysis is conducted by inspecting whether the CDS spread and the distance to default are stationary. "In general, regression models for non-stationary variables give spurious results" (Nielsen, 2005). Firstly, as figure 1 – figure 3 demonstrate, the average CDS spread for different tenors is plotted virtually. It is easy to observe that it had a higher volatility in the beginning of the series. The 2008 financial crisis may lead to the dramatic increase in the spread, and we wonder the data could have a random walk. Then, we conducted "Adftest" for the CDS spread and the distance to default of 1-year, 3-year and 5-year data in the Matlab. The results show that our data are stationary, which means that we can input our data into the regression function directly without any further processing.

As mentioned above, the financial crisis in 2008 had a huge effect on the CDS spread. Thus, we decide to divide our regression into two different tenors. One is from January 2008 to June 2017 and the other is from January 2010 to June 2017, as any financial model may temporarily lose its effectiveness during the crisis period. Moreover, dividing the data into two different periods can also help us better verify the following regression model.

4.3.2 Locally Weighted Scatterplot Smoothing (LOWESS)

Before regression analysis on our data is made, the LOWESS method is used to visualize the relation between the CDS spread and the distance to default. LOWESS is a

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popular method adopted in regression analysis. This method creates a smooth line through a time plot or scatter plot to help people know the relationship between variables and foresee trends clearly (Andale, 2013).

LOWESS has a number of advantages in data smoothing. It does not require any specification of a function to fit all the data. Furthermore, LOWESS is also significantly flexible, making it ideal for modeling complex processes. "The simplicity of the LOWESS method makes it one of the most attractive of the modern regression methods (Le , 2016)."

### 4.3.3 Regression

At the beginning, we simply believe that the relation between the CDS spread and the distance to default is linear. When the linear regression result is seen, the independent variables cannot explain the dependent variable well. Then we smoothen scatter data in the distance to default and the CDS spread using the Lowess smoothing method. Figure 4 to figure 6 are the scatter plot of our smoothing result.

After observing the graphs, it is assumed that the CDS spread and the distance to default can have the inverse relation, exponential relation or logarithmic relation. Then the following possible equations that the variables could fit are listed.

$$CDS\ Spread = b + a * k^{DD} \tag{8}$$

where DD is the initial of the distance to default; a and b are constants that need to be estimated in the regression, and k is a number between 0 to 1. We try to fit different values of k when the regression test is conduced.

$$CDS\ Spread =\ b +\ a * \log_k DD \tag{9}$$

where a and b are constants that need to be estimated in the regression; k is also a constant and we try to fit k = e or 10.

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$$CDS\ Spread = k + a * DD^{-1} + b * DD^{-3} + c * DD^{-5} \tag{10}$$

where k, a, b and c are all constants.

After all the possible relations are listed, we start to run the regression and collect the result, and then we choose one that can best explain the relation between the CDS spread and the distance to default.

# 5. Results

5.1 Correlation

As reported in table 1, the correlation between the CDS spread and the distance to default (DD) of different tenors is tested. With the increase of the tenor length, the CDS spread and DD become gradually correlated. The high correlation for 5-year data probably results from the 5-year CDS contract, the most liquid one in the market (Arakelyan, Rubio, & Serrano, 2012).

The correlations of the data excluding the financial crisis are all higher than the correlations of data that include the financial crisis which proves our initial assumption that during the financial crisis period, most of the financial models lost their effectiveness or partial effectiveness. However, only observing the correlation between variables cannot know how they influence each other, and thus, we should carry out more investigations as mentioned in section 4.3.3.

5.2 Regression Results

After all the possible relations mentioned in section 4.3.3 are tested, form (10) has the highest average of $R^2$. In other words, compared with other assumptions, the inverse relation can better explain the relation between the CDS spread and the distance to default. Moreover, the p-values of all the independent variables in the regression are significant under the 95% confidence level.

The regression results of the inverse relation are reported in table 3 and table 4. In table 3, data are regressed during the whole testing period. In table 4, the data are only regressed after the financial crisis (start from 2010). Because of the effect of the financial crisis, it is easy to observe $R^2$ in table 3, and they are all lower than corresponding $R^2$ in

table 4. If $R^2$ is compared by the tenor, $R^2$ of the 5-year CDS becomes double from 0.1385 in table 3 to 0.2796 in table 4, which implies that the financial crisis had huge influence on the 5-year CDS spread and it also proves that the 5-year CDS has the highest liquidity. In addition, the result also demonstrates that the Merton DD model can explain more about the change of the CDS spread during the stable economic period.

# 6. Conclusion

The purpose of this paper is to examine how accurate the modified Merton DD model can estimate the potential default. When looking at the CDS-implied default probability regressions and their correlation, the modified Merton DD model does not appear to be a significant predictor. Therefore, it is concluded that the Merton DD model still can be used as a default estimater, but it is incapable of providing sufficient statistics for default.

Then, only three kinds of possible relations between the CDS spread and the distance to default are tested in this paper based on the smoothing graph we draw, and thus, there could be more complicated relations that may be ignored. We can also substitute the fluctuant volatility for the constant volatility in the future research.

# Appendix

## Table 1: Correlation Result

Correlation between dependent variable and independent variable of different tenors.

| Correlation between CDS and DD | | |
|---|---|---|
| Tenor | 2008 - 2017 | 2010 - 2017 |
| 1Y | -0.16727 | -0.18315 |
| 3Y | -0.23361 | -0.26551 |
| 5Y | -0.27248 | -0.32685 |

## Table 2: Descriptive Statistics

Descriptive statistics for the dependent variable CDS Spread of 1Y 3Y 5Y, independent variable Distant to Default of 1Y 3Y 5Y and the input for calculating the Distant to Default Market Cap, Current Debt, Non-Current Debt, Risk free rate and stock volatility.

| | | | Quantiles | | | | |
|---|---|---|---|---|---|---|---|
| Variables | Mean | SD | Min | 0.25 | Median | 0.75 | Max |
| Market Cap (Million) | 750.17 | 938.49 | 41.25 | 257.19 | 443.84 | 807.44 | 6156 |
| Current Debt (Million) | 10374 | 18955 | 0.00567 | 2.3477 | 4.0065 | 249 | 26946 |
| Non-Current Debt (Million) | 19938 | 26946 | 532 | 5846 | 11705 | 23080 | 246925 |
| Risk Free Rate (%) | 0.0049 | 0.0057 | 0.0025 | 0.0025 | 0.0025 | 0.0050 | 0.0300 |
| $\sigma_V$ (%) | 0.0177 | 0.0121 | 0.0032 | 0.0103 | 0.0144 | 0.0208 | 0.1454 |
| Distant to Default 1Y | 28.89 | 18.58 | 0.69 | 15.84 | 24.87 | 37.79 | 278.48 |
| Distant to Default 3Y | 13.76 | 8.66 | -0.75 | 7.67 | 11.97 | 17.84 | 112.48 |
| Distant to Default 5Y | 9.92 | 6.21 | -1.39 | 5.60 | 8.67 | 12.82 | 77.77 |
| CDS Spread 1Y | 0.5644 | 2.6185 | 0.0190 | 0.1000 | 0.2011 | 0.4400 | 130.94 |
| CDS Spread 3Y | 0.8913 | 2.4519 | 0.0496 | 0.2607 | 0.4450 | 0.8599 | 117.05 |
| CDS Spread 5Y | 1.2348 | 2.3993 | 0.0950 | 0.4500 | 0.7196 | 1.3298 | 112.63 |

**Table 3: Regression Result (2008 – 2017)**

We run an OLS between CDS spread and according Distant to Default from 2008-2017.

| Regression Result | | | | | | |
|---|---|---|---|---|---|---|
| 2008 - 2017 | 1Y | | 3Y | | 5Y | |
| | Coefficient | P-Value | Coefficient | P-Value | Coefficient | P-Value |
| Intercept | -41.16 | 4.2E-35 | -16.35 | 7.2E-10 | 59.32 | 7.8E-129 |
| $DD^{-1}$ | 17.19 | 2.7E-247 | 9.24 | 0 | 4.07 | 0 |
| $DD^{-3}$ | 18.04 | 1.5E-17 | -2.37 | 1.2E-219 | -3.83 | 8.2E-308 |
| $DD^{-5}$ | -7.18 | 1.2E-10 | 4.08 | 3.8E-199 | 2.79 | 9.6E-292 |
| R Square | 0.2055 | | 0.2092 | | 0.1385 | |

**Table 4: Regression Result (2010 – 2017)**

We run an OLS between CDS spread and according Distant to Default from 2010-2017.

| Regression Result | | | | | | |
|---|---|---|---|---|---|---|
| 2010 - 2017 | 1Y | | 3Y | | 5Y | |
| | Coefficient | P-Value | Coefficient | P-Value | Coefficient | P-Value |
| Intercept | -1.69 | 2.7E-01 | -8.94 | 4.1E-07 | 12.90 | 6.1E-11 |
| $DD^{-1}$ | 6.39 | 1.5E-95 | 8.01 | 0 | 7.24 | 0 |
| $DD^{-3}$ | 10.14 | 9.0E-185 | -1.76 | 7.9E-290 | -1.26 | 2.4E-277 |
| $DD^{-5}$ | -12.53 | 7.7E-204 | 2.99 | 1.1E-263 | 0.99 | 8.0E-276 |
| R Square | 0.2609 | | 0.2384 | | 0.2796 | |

**Figure 1: 1 Year Average CDS Spread**



1Y Average CDS Spread

**Figure 2: 3 Years Average CDS Spread**



3Y Average CDS Spread

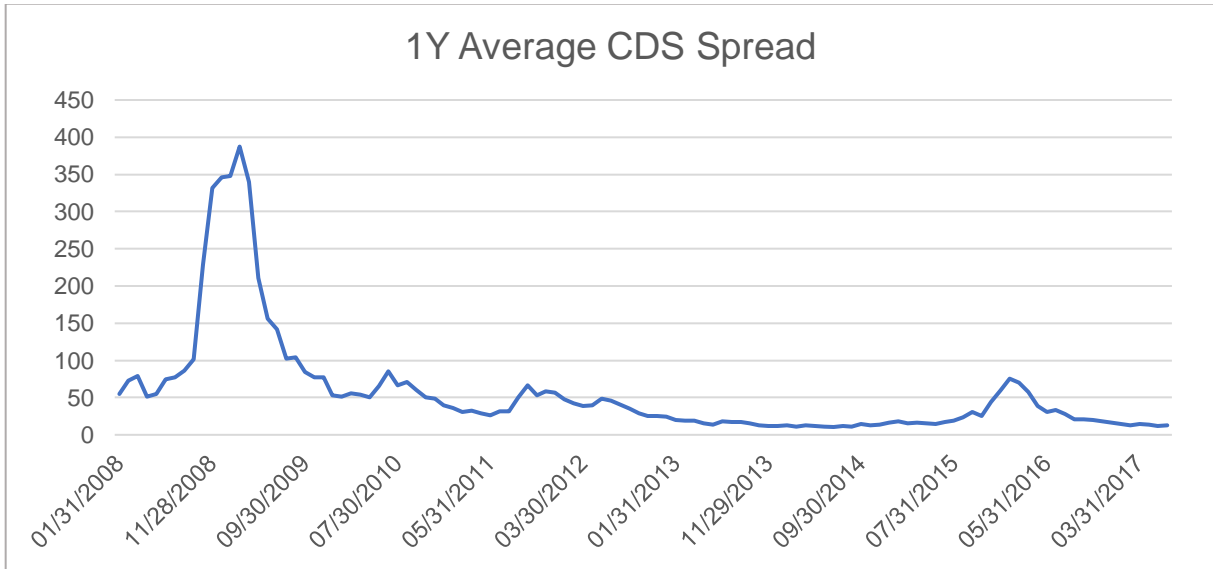**Figure 3: 5 Years Average CDS Spread**
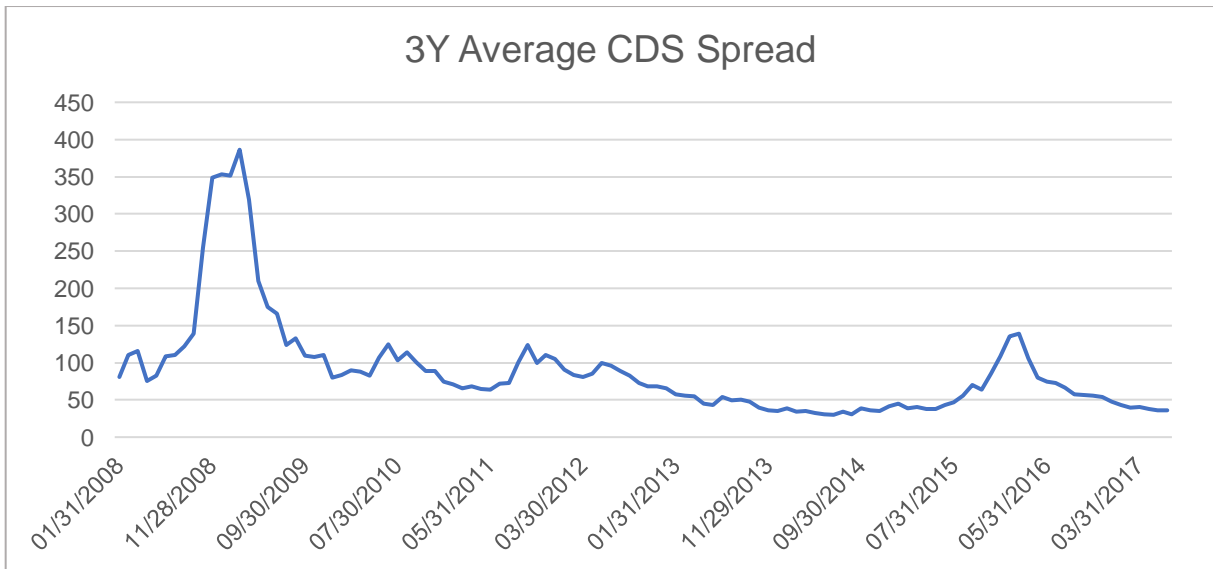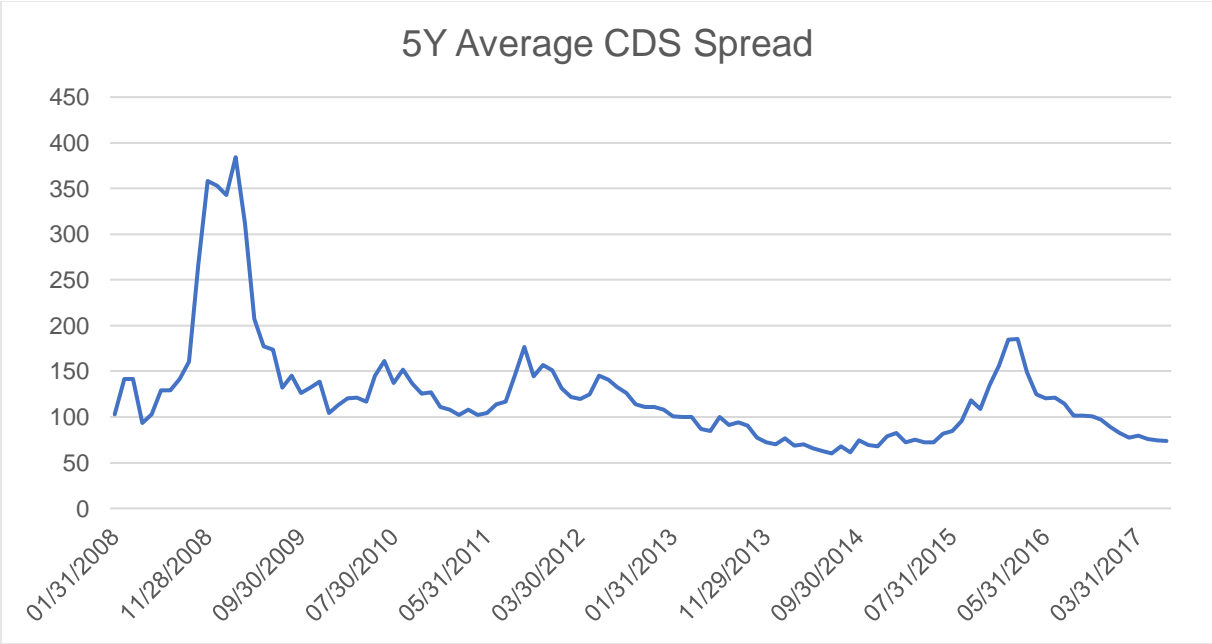


5Y Average CDS Spread

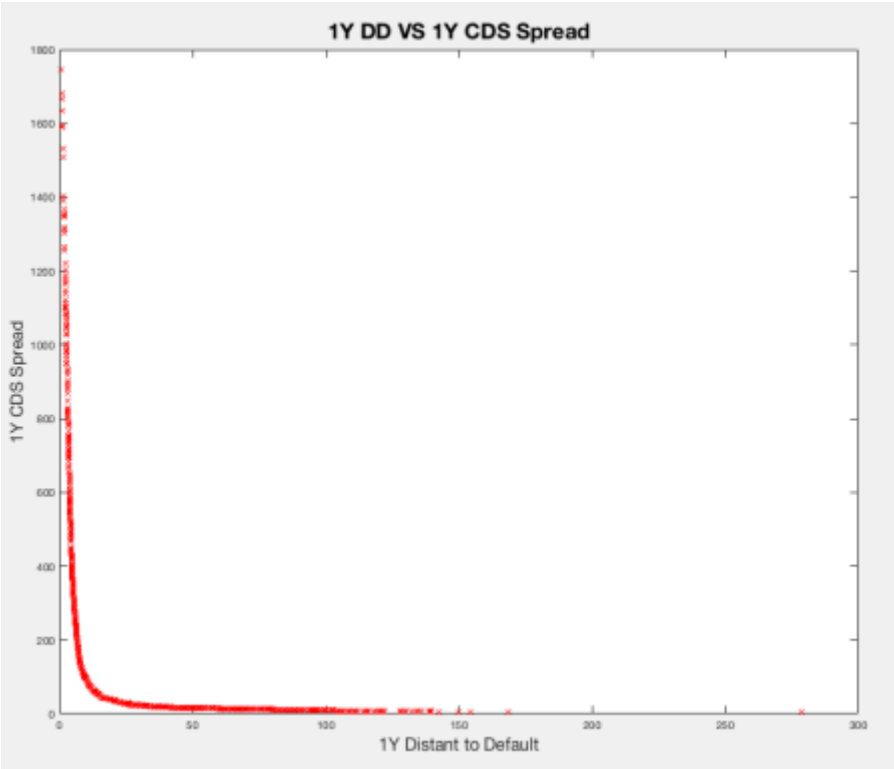**Figure 4: 1 Year Distance to Default VS 1 Year CDS Spread after Smoothing**



**Figure 5: 3 Years Distance to Default VS 3 Years CDS Spread after Smoothing**
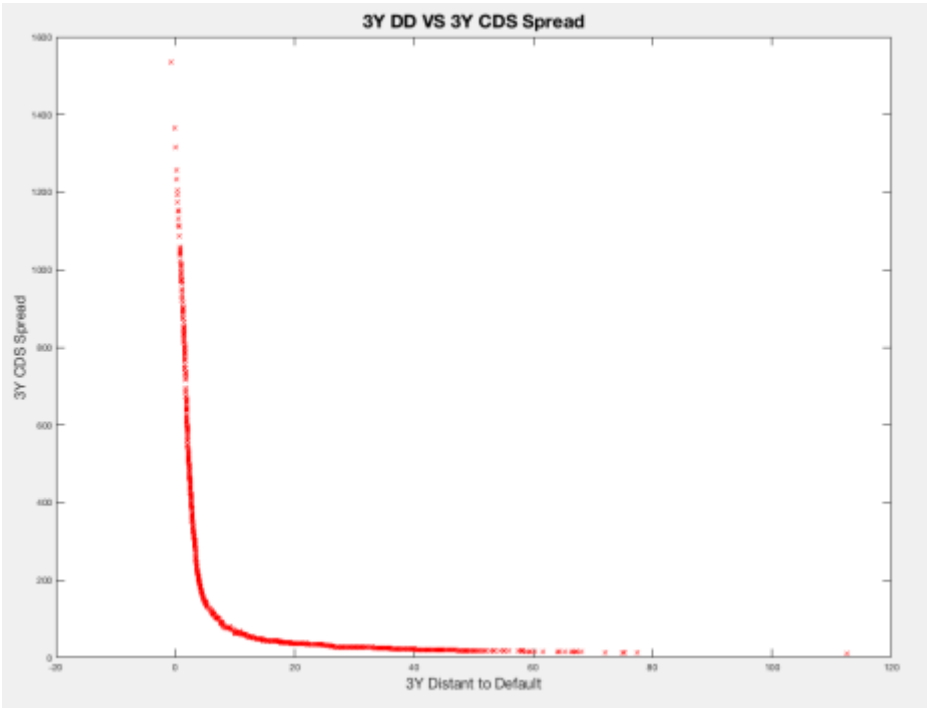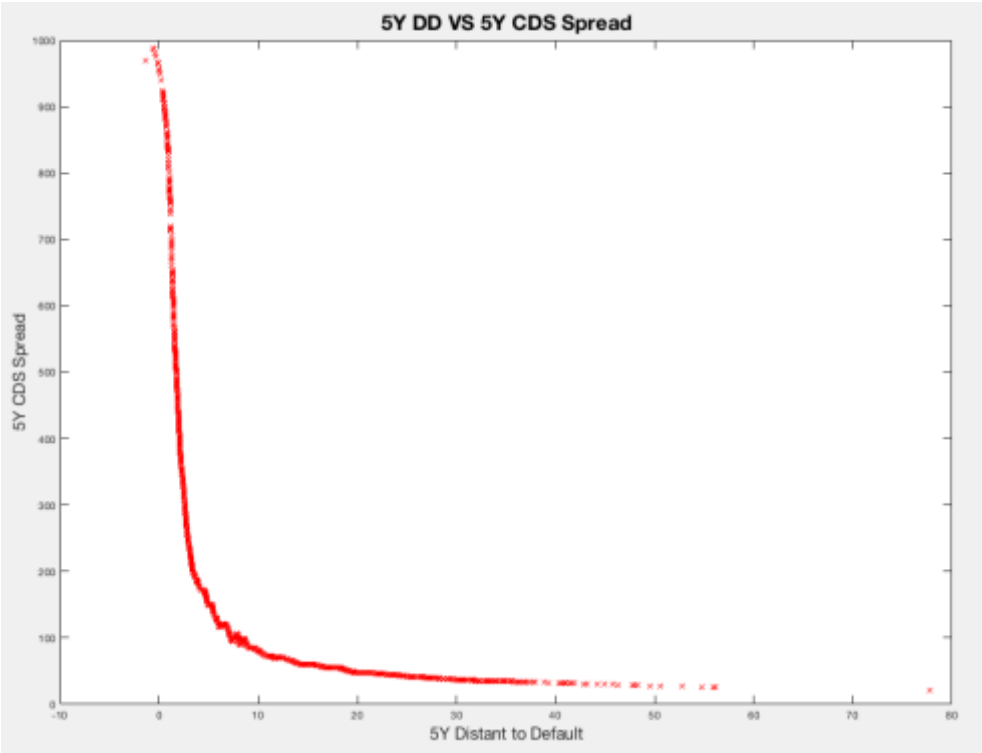
**Figure 6: 5 Years Distance to Default VS 5 Years CDS Spread after Smoothing**

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