INCORPORATION OF CONDITIONAL VALUE-AT-RISK INTO MEAN-VARIANCE OPTIMIZATION FOR PORTFOLIOS OF HEDGE FUNDS

by

Fuling Wang,
B.Ec, Nankai University, 2016

and

Judy Korzeniowski
BGS, Simon Fraser University, 2005

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Supervisory Committee:

____

Dr. Peter Klein
Senior Supervisor
Professor

____

Dr. Ying Duan
Second Reader
Assistant Professor,

Date Approved: ____

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Abstract

In this paper we aim to search for a systematic optimization model that can properly measure hedge fund risks and can optimize capital across Canadian hedge fund portfolios that can cater to investors' risk appetites. As the characteristics of hedge funds returns impose different layers of risk from traditional equity and bond investments, the conventional mean-variance optimization would not accurately capture the risk associated with non-normal distributions and negative skewness. The process requires a different approach that modifies the drawback of a mean-variance optimization to take non-normal and asymmetric distributions into consideration. The research of this process leads to a Mean-Conditional Value-at-Risk (CVaR) optimization. CVaR measures the mean expected short fall between value-at-risk and excess losses that reflect the risks of kurtosis and negative skewness. Combining the cluster analysis to overcome variation of correlation issue and Mean-CVaR optimization, we found the Mean-CVaR optimization model that will serve the requirements of guiding investors' capital allocation among hedge fund strategies.

Keywords: hedge funds; skewness; kurtosis; optimization; cluster analysis; skew gap;

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1: Introduction

It has been noticed that there has been increasing interest in hedge fund investments from institutional investors and high net worth individuals since the early 90s, as hedge funds act as an effective diversifier in traditional portfolios. However, the unlimited downside risk of hedge funds discourages some conservative investors from allocating more weight to hedge funds. How to control the downside risk while diversifying traditional portfolios with hedge funds is a topic that interests portfolio managers and researchers.

Much research has been done on global hedge funds, while very little has been done for Canadian hedge funds due to the lack of reliable data. BULLWEALTH hedge fund indices, which consist of over 300 distinct Canadian hedge funds, have been used for this research. The risk-return characteristics are analyzed and higher points of risk are discussed in detail. In addition, we examine the varying correlations between different strategies, which inhibits a consistent portfolio construction.

To solve the problem of inconsistency among various strategies, RSG groups are constructed. With a more consistent classification, we then move on to portfolio optimization. On top of Mean-Variance Optimization, we adopt Mean-CVaR portfolio optimization to account for the tail risk. The optimization results from the two approaches are compared and discussed. Also, different optimized combinations are proposed to cater to different risk appetites. As a result of our optimization, less weight will be allocated to equity RSG group if an investor aims to reduce the tail risk.

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2: Literature Review

The literature on hedge funds shows that hedge funds generally outperform traditional portfolios and provide diversification benefits. Consequently, there has been increasing interest in funds from institutional investors and high net worth individuals since early 90s. Brooks and Kat (2002) shows in 2002 there were estimated 6,000 hedge funds with an estimated $500 billion in capital and $1 trillion in total assets, and 80% of the hedge funds were smaller than $100 million and 50% were smaller than $25 million. According to the Hedge Fund Research Institute (2017) the Third quarterly industry report, the total hedge fund industry capital has rose to $3.15 trillion, and an increase of $50 billion over the previous quarter. The superior returns and diversification benefits of hedge funds have attracted steady growth over 6.3 times global-wide over the past 15 years.

We begin from the idea of how to allocate capital across different hedge fund strategies or funds of hedge funds and possibly incorporate hedge fund allocations into traditional portfolios. Unlike traditional equity and bonds capital allocation, there is little consensus or shared common methods on how portfolio managers should optimally allocate capital across different hedge fund strategies. De Souza and Gokcan (2004) indicate that there have been very few publications on how to construct a robust multi-strategy portfolio in a systematic method for allocation of capital among hedge fund strategies.

De Souza and Gokcan (2004) provide some insights of why systematic investment methodology within hedge fund strategies is not commonly available. Four primary issues have been associated with and have contributed to the limited development of the systematic allocation method.

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- Hedge fund index performance data is questionable due to illiquid of some strategies and subject to definitional (discretionary of index management) and survivorship biases.
- The return distributions of hedge fund strategies usually exhibit non-normal return distributions, especially significant negative skewness and high kurtosis.
- Correlations among hedge fund strategies vary widely across them with mixed signs
- Overly smooth return data. Due to some illiquid hedge fund strategies, fund administrators rely on "old" or observed transaction prices for similar but more liquid assets, which create the so-called "smooth" returns.

A combination of the characteristics of hedge fund returns and the procedure valuation errors results in a serial correction in monthly returns and underestimation of hedge fund true standard deviations. Beside the above issues, the mentality of fund manager selection being the key return driver of hedge fund investments further diverts attention away from the development of a systematic allocation methodology.

Among some published research on funds of funds portfolio selection and capital allocation, we discovered that Davies, Kat, & Lu (2008) offer the PGP optimization method. This model takes hedge funds' non-normal return distributions and negative skewness and high kurtosis into consideration. According to Davies et al. (2008), the PGP optimization model "balances multiple conflicting and competing hedge fund allocation objectives: maximizing expected return while simultaneously minimizing return variance, maximizing skewness and minimizing kurtosis" (p. 92). The model provides guidance on how to best allocate capital among different hedge fund strategies while incorporating the investor's goals. This model also compares "like for like" hedge funds that construct portfolios with the same number of funds for each hedge fund strategy to capture the possibilities of small size

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and large size investments. The purpose of this approach is to simulate constraints of small portfolios that have a significant barrier to diversification in reality because of the minimum investment amount requirements in funds of funds. In contrast, large funds of funds can usually be spread among a relatively large number of managers to achieve diversification.

De Souza and Gokcan (2004) utilize Mean-CVaR optimization method, a model that also deals with non-normal distributions and negative skewness and kurtosis but does so using a different measure from the PGP model. This approach to multi-strategy allocation methodology takes a similar traditional mean-variance optimization and considers the hedge fund as an asset class. The critical pre-assumption of this approach is that hedge fund strategies are uncorrelated to each other in order to construct useful portfolio optimization. This model is what we are interested in and will pursue our study to incorporate hedge fund strategies into capital allocation. This approach optimizes portfolios and takes higher moments into consideration by maximizing expected return and minimizing conditional value-at-risk (CVaR). This is similar to a mean-variance optimization among hedge fund strategies but instead of using standard deviation as the measure of risk, it utilizes CVaR as a risk measure. CVaR is a weighted average between the value at risk and losses exceeding the threshold, so it helps to account for skewness and kurtosis to measure mean expected shortfalls. Therefore, it's a more appropriate measure of hedge fund risks that usually exhibit negative skewed distributions. The model also incorporates the optimization by grouping highly correlated hedge fund strategies together and separating low correlated strategies into several clustered groups, the so-called "Rational Strategies Group". This is intended to solve the high correlation issues among hedge fund strategies by maximizing intra-clustering correlation and minimizing inter-clustering correlation. This is discussed further in the RSG

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section. Once the correlation issue is dealt with in a satisfactory way, then the model will be useful to provide insight into the Mean-CVaR optimization.

While most hedge fund studies have been focused on global hedge funds indices such as TASS index and HFR Index, we would like to explore the Canadian hedge fund indices and examine whether the Canadian hedge fund indices provides similar or better results than its global counterparties. Klein, Purdy, and Schweigert (2009) compare and contrast the risk and returns of a broader Canadian hedge fund, the KCS Composite index, with its global hedge fund counterparties from January 2005 to June 2009. The paper indicates that the KCS Composite index and sub-indices have the highest returns in all cases. Although the standard deviations of the KCS indices are usually higher than of the global counterparties, the Sharpe ratios are still better. Klein, Purdy, and Schweigert (2009) further suggest that because of the Canadian hedge funds' unique characteristics of being smaller in asset size, inefficient due to its size, a relative lack of international investors compared to its global counterparties, and the Canadian hedge fund manager's home market advantage in addition to its less correlated to US and global equities, investors can greatly benefit from allocating a diversified Canadian hedge fund into their existing portfolios.

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3: Data, Methodology and Statistical Properties

3.1 Data and Methodology

For this study, we retrieved data from BULLWEALTH (BW) Indices. The BW Index is one of the Canadian hedge fund Indices. BULLWEALTH has developed hedge fund indices that are published monthly through the BW Canadian Hedge Fund Index. The BW Canadian Hedge Fund Index provides a comprehensive overview of the performance of the Canadian hedge fund universe, which consists of 9 sub-index strategies. In addition, the index is based on a database of returns for over 300 distinct Canadian hedge funds. The inception date of the Index and Sub-Indices is January 2003. [1]

For the analysis, we used 7 out of 9 strategy Indices after considering the overlap between some strategies. These strategies include BW Multistrategy Index, BW Equity L/S Index, BW Event Driven Index, BW Fixed Income Index, BW Equity Market Neutral Index, BW Global Macro Index and BW Managed Futures Index. We also noted that the BW Equity L/S Index contains BW Equity Hedge Sub-Index, and BW Equity Directional Sub-Index.

We looked at the monthly returns from January 2003 to September 2017. Our dataset holds data collected during 2008 financial crisis as we believe that including the data gives a more accurate picture of the hedge fund performance.

[1] http://bullwealth.com/analysis.php

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Table 3.1 Summary of Statistics for 7 Indices

	BW Multistrategy Index	BW Equity Market Neutral Index	BW Fixed Income Index	BW Event Driven Index	BW Equity L/S Index	BW Global Macro Index	BW Managed Futures Index
annl.Indices Ret	9.28%	3.67%	8.35%	13.49%	10.27%	2.22%	7.30%
annl.Indices std	7.87%	3.26%	4.30%	10.88%	10.29%	8.70%	8.83%
Sharp ratio	0.93	0.53	1.49	1.06	0.81	0.03	0.61
Skewness	-0.94	-0.55	-1.52	-0.83	-0.88	0.37	0.24
Kurtosis	5.93	4.24	9.00	6.72	5.52	5.91	3.93

According to the statistics in the above table, BW Event Driven Index has the highest monthly-compounded annualized return while BW Global Macro Index has the lowest monthly-compounded annualized return. Also, BW Event Driven Index has the highest annualized volatility and Market Neutral Index has the lowest annualized volatility. BW Fixed Income Index has the highest annual Sharpe ratio, while BW Global Macro Index has the lowest. The statistics may not reflect a true picture of the risk and return characteristics of each strategy index due to the survivorship bias and the smoothing effect of indices.

Except for BW Global Macro Index and BW Managed Futures Index, the others all show negative skewness. Among all negatively skewed indices, BW Fixed Income Index has the most significant amount of negative skewness, while Market Neutral Index is slightly skewed left, with a skewness of -1.515 and -0.545 respectively. All of the seven strategy indices exhibit excess kurtosis, among which BW Fixed Income Index has the most significant leptokurtic, 9.002. With such a high kurtosis, it might be inappropriate to assume normal distribution.

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3.2 Serial Correlation and Unsmoothing

One good thing about the index is that it diversifies individual funds. With the diversification within each strategy index, the serial correlation becomes less significant. Ljung-Box Q-Test is used in our analysis, 4 strategy indices reject the null hypothesis that the residuals of the returns are not auto-correlated, including BW Multistrategy Index, BW Equity L/S Index, BW Event Driven Index, BW Fixed Income Index, while the other three indices fail to reject the null hypothesis. We decided to use the original index data rather than the unsmoothed one, considering that not all strategy indices exhibit significant serial correlation and the unsmoothing will introduce noise.

A similar observation of unstable correlation issues presents in Canadian hedge fund indices as well. Table 3.21 exhibits the 12-month rolling average correlation of seven strategies. Equity L/S has two highest correlation with Multistrategy and Event Driven strategies of 0.834 and 0.823 respectively. Interestingly, Fixed Income strategy also exhibits high correlation coefficient with Multistrategy, Event Driven and Equity L/S strategies of 0.695, 0.578 and 0.712 respectively. The rest of other strategies have relatively low correlation with other strategies, especially Global Macro and Managed Futures. Global Macro strategy particularly appears mostly negative correlation coefficient with other strategies from -0.089 to -0.009 except with Equity Market Neutral and Managed Futures 0.069 and 0.381. Table 3.22 shows the spread of maximum and minimum 12-month Moving Average Correlation between two strategies. The average spread among strategies is 1.28 with highest spread of 1.655 and lowest spread of 0.591. This indicates unstable correlations. Table 3.23 provides the volatility of 12-Month Moving Average Correlations among the

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hedge fund strategies and further suggests the instability of the correlation among hedge fund strategies. The average volatility is 0.317 with the highest volatility of 0.515 for Global Macro and Fixed Income and the lowest volatility of 0.122 for Equity L/S and Event Driven strategies. The low correlation between equity and bond asset classes is common understanding; however, the lack of correlation among some hedge fund strategies is a new discovery. This suggests that it is even more attractive to add portfolios of hedge fund strategies into traditional asset classes. However, we have to handle the volatility of correlation between some hedge fund strategies to satisfy the pre-condition of mean-variance optimization.

Table 3.21 Correlation of Hedge Fund Strategy Indices

12 mon_rolling_Correlations: Original 7 Strategies	Multistrateg y	Equity Market Neutral	Fixed Income	Event Driven	Equity L/S	Global Macro	Managed Futures
Multistrategy	1.000	0.378	0.695	0.743	0.834	-0.009	0.092
Equity Market Neutral	0.378	1.000	0.302	0.135	0.364	0.069	0.161
Fixed Income	0.695	0.302	1.000	0.578	0.712	-0.040	-0.035
Event Driven	0.743	0.135	0.578	1.000	0.823	-0.089	0.043
Equity L/S	0.834	0.364	0.712	0.823	1.000	-0.054	0.090
Global Macro	-0.009	0.069	-0.040	-0.089	-0.054	1.000	0.381
Managed Futures	0.092	0.161	-0.035	0.043	0.090	0.381	1.000

Table 3.22 Spread Between Maximum/Minimum 12-Month Moving Average Correlation: BW Hedge Fund Strategy Indices

spread: Original 7 Strategies	Multistrategy	Equity Market Neutral	Fixed Income	Event Driven	Equity L/S	Global Macro	Managed Futures
Multistrategy	0.000	1.609	0.806	0.814	0.849	1.523	1.445
Equity Market Neutral	-	0.000	1.431	1.655	1.636	1.517	1.028
Fixed Income	-	-	0.000	1.026	0.658	1.615	1.555
Event Driven	-	-	-	0.000	0.591	1.307	1.434
Equity L/S	-	-	-	-	0.000	1.569	1.527
Global Macro	-	-	-	-	-	0.000	1.279
Managed Futures	-	-	-	-	-	-	0.000

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Table 3.23 Volatility of 12-Month Moving Average Correlations: BW Hedge Fund Strategy Indices

Volatility of 12 Month Rolling Correlation: Original 7 Strategies	Multistrategy	Equity Market Neutral	Fixed Income	Event Driven	Equity L/S	Global Macro	Managed Futures
Multistrategy	0.000	0.347	0.178	0.145	0.158	0.388	0.392
Equity Market Neutral	-	0.000	0.295	0.352	0.310	0.367	0.256
Fixed Income	-	-	0.000	0.250	0.180	0.515	0.451
Event Driven	-	-	-	0.000	0.122	0.346	0.437
Equity L/S	-	-	-	-	0.000	0.415	0.448
Global Macro	-	-	-	-	-	0.000	0.307
Managed Futures	-	-	-	-	-	-	0.000

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3.3 Rational Strategy Groups

One of critical pre-requisition requirements of mean-variance optimization is low correlation among asset classes. To solve the issue of varying inter-strategy correlations of hedge fund strategies, Rational Strategy Groups (RSGs) are created to identify groups of underlying similarity or whose distribution functions and correlation structures have some degree of stability. We follow De Souza and Gokcan's RSG analysis method. The following is the process of determining RSGs, the so-called "cluster analyst":

1. To isolate the similar elements: It is an attempt to group data so as to minimize intra-group variation while maximizing inter-group variation. We performed the cluster analysis by using the correlation of 7 of the BW Canadian Hedge Fund Strategy Indices: Multistrategy Index, Equity Market Neutral Index, Fixed Income Index, Event Driven, Equity L/S Index, Global Macro Index, and Managed Futures Index.
2. The results of the cluster analysis show five distinct clustering groups: The degree of similarity within a clustering is measured by its squared correlations (r-squared) with strategies among the group. It was noted that the lowest r-squared is 0.552 within a cluster group and the highest r-squared is 0.507 from the next closest external cluster group. Although the Fixed Income Index exhibits a slightly higher correlation (0.507) with Equity L/S Strategy index than what is desired, it exhibits a correlation lower than 0.5 with the other two strategies (Multi-strategy and Equity Market Neutral strategies) within the Equity Group and significant lower correlation with the rest of strategies. Hence, we separate Fixed Income Index as a stand-alone RSG cluster. The other three strategies definitely exhibit significant low correlation r-squared along with the others. Therefore, we separated Market Neutral, Global Macro and Managed Futures as three distinct RSG clusters.

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Overall, the RSG analysis indicates fairly internal consistency within each clustering and a relatively high degree of dissimilarity among clusters and the outcome satisfies the requirement of low correlation for optimization. Based on the above-described clustering analysis, we have separated these strategies into five RSG clustering groups:

Table 3.31 Cluster Analysis

Cluster Analysis			**r-squared with**	
			Own Cluster	**Next Closest**
1.	Equity Group	BW Multistrategy Index	0.552	0.483
		BW Event Driven Index	0.678	0.334
		BW Equity L/S Index	0.696	0.507
2.	Fixed Income	BW Fixed Income Index	1	0.507
3.	Market Neutral	BW Equity Market Neutral Index	1	0.143
4.	Global Macro	BW Global Macro Index	1	0.008
5.	Managed Futures	BW Managed Futures Index	1	0.145

RSGs are constructed by equally weighting underlying strategy indices. For example, Equity Group RSG allocates equal weight to equity L/S, Event Driven and Multistrategy strategies. The statistical summary of the RSGs is shown below.

The characteristics of the skewness and kurtosis remain similar based on the clustering analysis. These RSGs all have non-normal distribution. The Managed Futures Group shows less fat tail than the other groups while the Fixed Income group is on the other end of spectrum of non-normal distribution. The Global Macro & Managed Futures clusters both have positive skewness while Equity, Fixed Income and Equity Market Neutral cluster groups have negative skewness. Interestingly, the Equity Cluster is slightly more negatively

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skewed (-1.00) than each of the three strategies before forming the group (Multi-strategy (-0.95), Even Driven (-0.84) and Equity L/S (-0.89).

Table 3.32 RSG Statistics

	Equity Group	Fixed Income Group	Equity Market Neutral Group	Global Macro Group	Managed Futures Group
Compound Rate of Return	0.878%	0.670%	0.301%	0.183%	0.589%
Annualized CRR	11.058%	8.349%	3.669%	2.218%	7.295%
Maximum Monthly Return	6.939%	2.997%	2.614%	11.874%	8.024%
Minimum Monthly Return	-11.592%	-6.415%	-3.386%	-8.393%	-7.750%
Max - Min Spread	18.531%	9.412%	6.000%	20.267%	15.774%
Annualized std	9.205%	4.304%	3.255%	8.703%	8.825%
Sharpe Ratio	0.990	1.488	0.530	0.031	0.606
VaR	3.26%	1.55%	1.21%	4.02%	2.84%
CVaR	5.79%	2.69%	1.88%	5.36%	4.61%
MaxDD	28.61%	11.39%	6.03%	18.43%	17.83%
Skewness	-1.0032	-1.5155	-0.5450	0.3687	0.2394
Kurtosis	6.3904	9.0021	4.2414	5.9125	3.9286

In terms of inter-cluster correlation, ideally, the lower correlation is preferred for the purpose of portfolio diversification. The result is fairly consistent with the finding that there is a low correlation among strategy groups--the average correlation is 0.18 lowered from 0.29. The correlation results are summarized below. Like De Souza and Gokcan's analysis, we show the spread between maximum and minimum 12-month rolling correlations and the standard deviation of 12-month rolling correlations for the RSGs pair in the tables below. The mean spread is 1.37 and the mean standard deviation is 0.23 lowered from 0.32 at the strategy level.

Based on the data presented, we can conclude that RSGs approach has satisfied the objectives of maximizing the correlation within an RSG cluster and minimize the correlation among inter-clusters. Hence, it assures allocation based on RSGs will produce more stable results than of that based on a strategy level. Performing allocation at these broader RSGs

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should generate more robustness than the individual strategy statistics which tend to vary over time in different market environments. Hence, the allocation will reflect the general risk and return characteristics of the RSG clusters rather than that at the strategy level.

Table 3.33 Correlation Analysis of RSGs

RSGs Correlation	Equity Group	Fixed Income Group	Equity Market Neutral Group	Global Macro Group	Managed Futures Group
Equity Group	1	0.698	0.293	-0.057	0.073
Fixed Income Group		1	0.302	-0.040	-0.035
Equity Market Neutral Group			1	0.069	0.161
Global Macro Group				1	0.381
Managed Futures Group					1

Table 3.34 Spread Between Maximum/Minimum 12-Month rolling Correlations: RSGs

RSGs Spread	Equity Group	Fixed Income Group	Equity Market Neutral Group	Global Macro Group	Managed Futures Group
Equity Group	0	0.693	1.688	1.460	1.478
Fixed Income Group		0	1.431	1.615	1.555
Equity Market Neutral Group			0	1.517	1.028
Global Macro Group				0	1.279

Table 3.35 Volatility of 12-Month Rolling Correlations: RSGs

Volatility of 12-Month Rolling Correlations: RSGs	Equity Group	Fixed Income Group	Equity Market Neutral Group	Global Macro Group	Managed Futures Group
Equity Group	0	0.347	0.178	0.145	0.158
Fixed Income Group		0	0.295	0.352	0.310
Equity Market Neutral Group			0	0.250	0.180
Global Macro Group				0	0.122

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Unlike De Souza and Gokcan using HFR Indices from January 1990 to October 2002 excluding second half of 1998, we use BW Canadian Hedge Fund Indices from January 2003 to September 2017 including 2008 data. It's interesting to compare and contrast the RSGs generated from the two different indices of different periods and in different markets. We noticed that the RSG groups from the two indices not only have different names of RSGs but also reflect different characteristics of each. Some strategies could have been developed to cater to different time periods or market requirements. For example, in contrast to BW Managed Futures Strategy Index, there wasn't a similar one in HFR indices during the 1990 to 2002 period. However, even a similar name such as Fixed Income strategy appears in both indices as a stand-alone RGS, so it shows different correlations with other strategy indices.

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Table 3.36 Comparison & Contrast of RSGs from HFR Indices & BW Indices:

		RSGs from HFR Indices	**RSGs from BW HF Indices**
1	Equity & Macro Strategies	Clustered as one RSG with r-squared correction of 0.7882 & normal distribution with positive skewness	Very low correlation between the two with r-squared correction of 0.03
2	Market Neutral & Statistic Arbitrage	Clustered as one RSG with r-squared correction of 0.7448. Normal distribution with positive skewness	Market Neutral strategy as stand-alone RSG due to low correction with other strategies. R-squared with next closest is 0.143. Non-normal distribution & negative skewness
3	Event Driven Strategy	No Event Driven Strategy but group Convertible Arbitrage, Distress Securities & Merger Arbitrage as one RSG, so-called Event Driven strategy. Negative skewness & highest kurtosis	Event Driven is grouped with Multistrategy & Equity L/S as one RSG with negative skewness and high kurtosis. These strategies seem to be quite different from its counterparty's Event Driven strategy.
4	Managed Futures Strategy	Not Applicable	Stand-alone RSG with positive skewness and relatively low kurtosis
5	Fixed Income	Fixed Income Arbitrage: very low correlation with other strategies, next closest r-squared is 0.0146 Comp. annual return 8.30% Annual. Std. 6.97% Skewness: -0.84 Kurtosis: 8.13	Fixed Income Strategy: relatively high correlation with Equity L/S strategy, next closest r-squared is 0.507 Comp. annual return 8.35% Annual. Std. 4.30% Skewness: -1.52 Kurtosis: 9.00

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4: Portfolio Construction with Mean-Variance and Mean-CVaR Optimization

After we have dealt with the unstable correlation issue of hedge fund strategies by grouping similar and dissimilar RSG groups, we can make sure that the optimization process will generate meaningful results. The next step is to exam the non-normal and asymmetric return distributions of hedge fund indices and solve these problems. We construct portfolios using Mean-Variance and Mean-CVaR optimization and compare and contrast the results of the two optimization methods. The objective is to better capture the risks of skewness and kurtosis that are not properly considered under the Mean-Variance optimization method.

In this section, we utilize both parametric and non-parametric methods for portfolio optimization. Parametric method only accounts for mean and variance and is based on the assumption of normal distribution, while non-parametric method accounts for higher moments, such as skewness and kurtosis. Not surprisingly, we find a gap between the efficient frontier generated using the two methods. However, the gap indicates that there exists positive skewness in optimized portfolios. The two optimizing methods are explained in this section, and then possible reasons are given regarding the positive skewness gap between the two efficient frontiers.

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4.1 Mean-Variance Portfolio Optimization

To produce an efficient frontier, we must first apply a parametric optimization: Mean-Variance optimization. The idea of mean-variance optimization is to find the highest expected return for a given level of risk. Here we use variance to represent portfolios' risks. For practical purposes, we limit the minimum weight to be 10%. More specifically, the goal is to find the allocation of portfolios such that

$$\underset{weights}{\text{Max}}\ E(R_p)$$

subject to $Var(R_p) = Target$ and $w_i \geq 0$

where $E(R_p)$ is the expected return of the portfolio. $Var(R_p)$ is the variance of the portfolio returns, and w_i is the weight allocated to asset i.

The inputs required for portfolio optimization include portfolio mean return and variance-covariance matrix. In addition, RSG groups are utilized to make a more consistent portfolio construction, since the correlation between RSG groups is more stable, which is indicated by the smaller standard deviation.

Table 4.11 Representative Portfolios of Mean-Variance Optimization

	avg return	std	VaR	CVaR	Portfolio Allocations				
					Equity	*Fixed Income*	*Market Neutral*	*Global Macro*	*Managed Futures*
Mean-Variance optimization	0.713%	1.814%	-2.204%	-3.743%	59.919%	10.000%	10.000%	10.000%	10.081%
	0.668%	1.626%	-1.698%	-3.029%	43.380%	10.000%	10.000%	10.016%	26.604%
	0.481%	1.266%	-1.429%	-2.402%	17.977%	15.218%	22.019%	30.409%	14.377%

Three representative portfolios are selected as above. The method is basically to select the top, middle and bottom portfolios from 1000 optimized ones. Each representative

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portfolio represents a unique risk-return characteristic. We observe that the higher the portfolio return, the more weight is allocated to Equity RSG Group, which consists of BW Multistrategy Index, BW Equity L/S Index and BW Event Driven Index. The equity group has the highest averaged return among all five RSG groups, and at the same time, it has the largest maximum drawdown. The most conservative portfolio allocates a comparatively large weight to Global Macro Group, which has the lowest averaged return and a relatively low correlation with other RSG groups. This can be interpreted as the more aggressive an investment strategy is, the higher weight is needed towards to higher return RSG group to achieve the higher expected return, in this case, the Equity Group. The more conservative an investment strategy, the less depends on the high return group.

Mean-Variance optimization has its own limitations. As this parametric optimization is performed under the assumption of normal distributed returns. This model fails to account for asymmetric return distributions and could underestimate the risks. In addition, this optimization method assumes that the co-variation of the returns on different RSG methods is linear,[2] which is not true as the results from the correlation spread matrix indicates.

[2] Morningstar_Asset_Allocation_Optimization_Methodology

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4.2 Mean-CVaR Portfolio Optimization

Considering the limitations of the Mean-Variance optimization, we apply the non-parametric method that is the Mean-CVaR Optimization. Mean-CVaR is more appropriate when assets exhibit skewness or fat tails. Since the historical data captures risks of higher moments compared to limited parameters, using the historical method would be more accurate to estimate the risks. The idea of mean-CVaR optimization is similar to mean-variance optimization, except that mean-CVaR optimization does not require any assumption for normal distribution of returns on assets. The objective of mean-CVaR optimization is to find the highest returns for a given level of risk, so here we use CVaR to represent the risk of the portfolios.

$$\underset{weights}{\text{Max}}\ \text{E}(\text{R}_p)$$

$$\text{subject to CVaR } (\text{R}_p) = \text{Target and } w_i \geq 0$$

where $E(R_p)$ is the expected return of the portfolio. $CVaR(R_p)$ is the conditional Value-at-Risk of the portfolio returns, and w_i is the weight allocated to asset i.

With the historical method, we generate all possible combinations of weights, 316,251 in total, and select the efficient portfolios to plot the efficient frontier. The inputs required for portfolio optimization include time series data of returns on each RSG group and all possible weight combinations.

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Table 4.21 Representative Portfolios of Mean-CVaR Optimization

	avg return	std	VaR	CVaR	Portfolio Allocations				
					Equity	*Fixed Income*	*Market Neutral*	*Global Macro*	*Managed Futures*
Mean-CVaR optimization	0.444%	0.942%	-0.932%	-1.751%	10.000%	16.000%	54.000%	10.000%	10.000%
	0.644%	1.431%	-1.492%	-2.722%	31.000%	26.000%	10.000%	10.000%	23.000%
	0.711%	1.799%	-2.176%	-3.696%	59.000%	10.000%	10.000%	10.000%	11.000%

Three representative portfolios are selected as above using the same method as mean-variance optimization. Each portfolio represents an unique risk-return characteristic. The higher the portfolio return, the more allocation to Equity Group, which is similar to the results from Mean-Variance optimization. The most conservative portfolio allocates the largest weight to Market Neutral Strategy Group, while the most conservative proposed by mean-variance optimization allocates the largest weight to Global Macro Group. The reason for that is that the Market Neutral Strategy Group has the least negative CVaR among all five strategy groups. In the comparison, we notice that the Mean-Variance optimization fails to consider negative CVaR in Global Macro Group and allocates higher weight to it for the conservative investment strategy.

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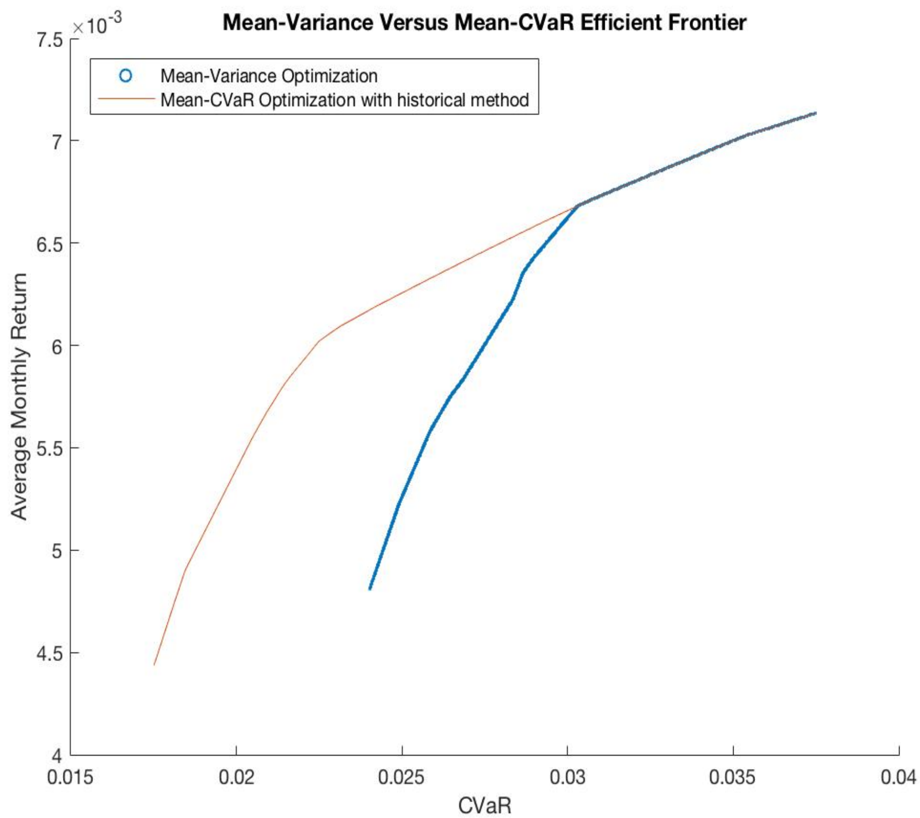
4.3 The Skew Gap

4.3.1 The Skew Gap between Mean-CVaR Efficient Frontier Generated with Two Optimization Methods

Figure 1 Mean-Variance Versus Mean-CVaR Efficient Frontiers



Using the weight derived from mean-variance optimization and the time series of returns, we translate Mean-Variance efficient frontier into Mean-CVaR efficient frontier. Combining both Mean-CVaR efficient frontier and the translation of Mean-Variance efficient frontier, we get figure 1, Mean-Variance versus Mean-CVaR Efficient Frontiers. In the low return / low CVaR area, there is a huge gap between the two efficient frontiers. This is what De Souza and Gokcan (2004) call *the "Skew" Gap.* This gap indicates that the "mean-

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variance" efficient frontier and mean-CVaR efficient frontier give different risks with the same given returns. To be more specific, if there is a negative *Skew Gap*, with given returns, the CVaR value of Mean-CVaR Optimization will be more negative than that of the Mean-Variance Optimization. If there is a positive *Skew Gap*, the CVaR value of Mean-Variance Optimization will be more negative than that of the Mean-CVaR Optimization. Surprisingly, our result shows a positive skew gap between these two efficient frontiers, which is opposite to De Souza and Gokcan's (2004) findings from HFR indices RSG portfolios.

The positive skew gap indicates that when there is an asset with mixed positive and negative skewness in a portfolio, Mean-Variance optimization is not able to capture the benefits of positive skewness of portfolio diversification and tends to rely heavily on more weight from Equity RSG, which has the highest return as well as highest standard deviation. If we look at a given lower return, the CVaR value from the Mean-Variance optimization actually reflects the higher risk than CVaR value from the Mean-CVaR optimization. CVaR value efficiently captures the risk of skewness but Mean-Variance optimization doesn't take into account. Therefore, the portfolios from Mean-Variance optimization have higher CVaR risk, which efficiently measures skewness. As De Souza and Gokcan (2004) suggest although the effect of skew may seem to be small (50 basis point for a 6% return in our case), the impacts of non-index data would be much greater. Looking from a different viewpoint, we can interpret that in the Canadian hedge fund portfolio construction space, investors of Canadian hedge funds can have more room to be conservative to take the advantage of positive skewness that Canadian hedge fund portfolios can offer. For example, at a given return of a 0.65% monthly rate with 1.6% standard deviation, the CVaR value of M-V optimization is 2.96%. On the other hand, at the same level of return (0.65%) with a 1.46%

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standard deviation, CVaR of M-CVaR optimization is 2.77%. It shows that investors can take less risk by 14 basis points to reduce skewness by 0.19%. The non-index data effects apply as well. However, we need to be cautious about the limitations of this practice as the skew gap closes at about a 0.67% return level and CVaR value at 0.03%. It is where the positive skew is exhausted as the requirement of return increases.

The Skew Gap concentrates on the bottom left corner of the graph, as figure 1 shows, where returns and CVaR are at low levels. We selected a comparison group, as shown in Table 4.31, which matches the CVaR level, to better indicate the pattern of allocation results from the two optimization methods.

Table 4.31 Portfolio Allocation Comparison

	avg return	std	VaR	CVaR	Portfolio Allocations				
					Equity	*Fixed Income*	*Market Neutral*	*Global Macro*	*Managed Futures*
Mean-CVaR optimization	0.62%	1.22%	-1.17%	-2.41%	17%	47%	10%	10%	16%
	0.63%	1.34%	-1.36%	-2.59%	25%	35%	10%	10%	20%
	0.65%	1.46%	-1.54%	-2.77%	33%	23%	10%	10%	24%
	0.66%	1.60%	-1.68%	-2.96%	41%	12%	10%	10%	27%
	0.68%	1.64%	-1.82%	-3.14%	46%	11%	10%	10%	23%
	0.69%	1.69%	-2.05%	-3.34%	51%	10%	10%	10%	19%
	0.70%	1.74%	-2.08%	-3.51%	55%	11%	10%	10%	14%
	0.71%	1.80%	-2.18%	-3.70%	59%	10%	10%	10%	11%
Mean-Variance optimization	0.48%	1.27%	-1.44%	-2.41%	18%	15%	22%	30%	15%
	0.56%	1.40%	-1.36%	-2.59%	30%	11%	16%	24%	19%
	0.60%	1.50%	-1.51%	-2.77%	36%	10%	12%	20%	22%
	0.65%	1.60%	-1.65%	-2.96%	42%	10%	10%	12%	26%
	0.68%	1.65%	-1.81%	-3.14%	46%	10%	10%	10%	24%
	0.69%	1.69%	-2.05%	-3.34%	51%	10%	10%	10%	19%
	0.70%	1.74%	-2.12%	-3.51%	55%	10%	10%	10%	15%
	0.71%	1.80%	-2.18%	-3.70%	59%	10%	10%	10%	11%

In Table 4.31, it is easy to see that within a given range of CVaR, between -2.41% and -2.96%, the Mean-CVaR optimized portfolios tend to have a higher return than Mean-Variance optimized portfolios do. Also note that given a similar CVaR, Mean-CVaR

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optimized portfolios allocate more weight to the fixed-income group while Mean-Variance optimized portfolios allocate more weight to global macro group.

To decrease CVaR, or to decrease the tail risk, both optimization approaches reduce the weight of equity of the RSG group. In addition, mean-CVaR optimization approach decreases the weight of managed futures as well. More weight is allocated to fixed income as tail risk is effectively cut down in the mean-CVaR optimization. However, in mean-variance optimization approach, more weight is allocated to market neutral and global macro RSG groups.

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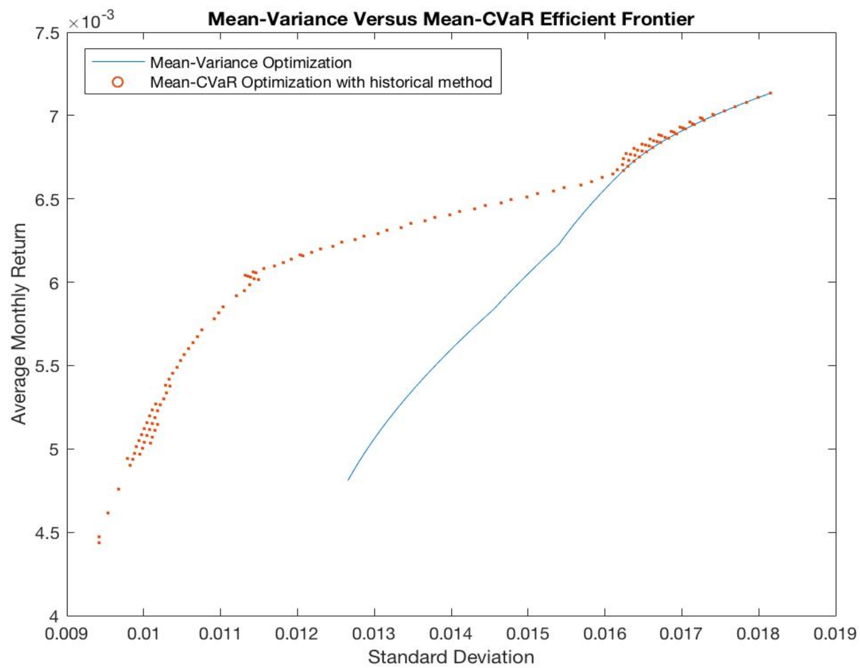
4.3.2 The Skew Gap between Mean-Variance Efficient Frontier Generated with Two Optimization Methods

Figure 4 Portfolio Allocation Comparison



We notice that in figure 2 (return vs standard deviation) the mean-variance efficient frontier has the same positive gap as M-CVaR efficient frontier does, which is quite unusual. This is a very interesting finding as our intuition tells us that Mean-Variance optimization usually underestimate the risk compared to M-CVaR optimization. Because M-CVaR optimization captures higher moments of risks, which are ignored by M-V optimization. If this is the case, with the same return, M-CVaR optimized portfolio will have a higher risk (measured by standard deviation). However, our result turns out that with the same return, M-V has a higher standard deviation, that is, the M-V optimization overestimates the risk compared to the Mean-CVaR optimization.

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Why a Positive Gap?

The M-V optimization underestimates risks under fat tailed distributions. That being said, as our result turns out, the mean-variance optimization overestimated the real risk of proposed portfolios. That is, with a given standard deviation, the CVaR of portfolios proposed by M-V is lower than that of portfolios proposed by M-CVaR. That means, if we want to have the same CVaR, we will have a higher standard deviation under the M-V. In this regard, the standard deviations of portfolios of M-V overestimate the risk (measured by CVaR).

Table 4.32 CVaR for M-V and M-CVaR Efficient Portfolios

std	CVaR (M-V)	CVaR (M-CVaR)
1.275%	-2.419%	-2.497%
1.335%	-2.503%	-2.587%
1.379%	-2.563%	-2.659%
1.431%	-2.641%	-2.722%
1.478%	-2.728%	-2.795%
1.533%	-2.824%	-2.857%
1.582%	-2.919%	-2.930%
1.624%	-3.023%	-3.014%

The difference between M-V and M-CVaR optimizations lies in skewness. To be more specific, the M-CVaR takes skewness and kurtosis into account while M-V ignores risks other than standard deviation. Considering this, we look into the skewness and kurtosis of the RSG groups and the proposed portfolios.

If we take a closer look at the proposed portfolios where the gap exists (portfolios with a standard deviation of 1.6% or lower), as table 4.31 shows, we will see that these two

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optimization methods have different allocations. M-V optimization allocates the greatest proportion to Market Neutral Group (skewness: -0.5450) and Global Macro Group (skewness: 0.3687) while M-CVaR allocates more to the Fixed Income RSG Group (skewness: -1.515). Since the skewness of Market Neutral and Global Macro is slightly positive (close to zero), the M-V could overestimate the risks (measure by CVaR) because it ignores the positive skewness. Therefore, as Figure 2 indicates, given the same returns, M-V has a higher volatility than M-CVaR.

Table 4.33 Portfolio Allocation Comparison

	avg return	std	VaR	CVaR	Portfolio Allocations				
					Equity	*Fixed*	*Market*	*Global*	*Managed*
Mean-CVaR optimization	0.62%	**1.22%**	-1.17%	-2.41%	17.0%	47.0%	10.0%	10.0%	16.0%
	0.63%	**1.34%**	-1.36%	-2.59%	25.0%	35.0%	10.0%	10.0%	20.0%
	0.65%	**1.46%**	-1.54%	-2.77%	33.0%	23.0%	10.0%	10.0%	24.0%
	0.66%	1.60%	-1.68%	-2.96%	41.0%	12.0%	10.0%	10.0%	27.0%
	0.68%	1.64%	-1.82%	-3.14%	46.0%	11.0%	10.0%	10.0%	23.0%
	0.69%	1.69%	-2.05%	-3.34%	51.0%	10.0%	10.0%	10.0%	19.0%
	0.70%	1.74%	-2.08%	-3.51%	55.0%	11.0%	10.0%	10.0%	14.0%
	0.71%	1.80%	-2.18%	-3.70%	59.0%	10.0%	10.0%	10.0%	11.0%
Mean-Variance optimization	0.48%	**1.27%**	-1.44%	-2.41%	18.3%	15.1%	21.8%	30.2%	14.5%
	0.56%	**1.40%**	-1.36%	-2.59%	30.0%	11.3%	15.7%	23.9%	19.2%
	0.60%	**1.50%**	-1.51%	-2.77%	36.4%	10.0%	11.7%	19.7%	22.3%
	0.65%	1.60%	-1.65%	-2.96%	41.9%	10.0%	10.0%	12.4%	25.7%
	0.68%	1.65%	-1.81%	-3.14%	46.2%	10.0%	10.0%	10.0%	23.8%
	0.69%	1.69%	-2.05%	-3.34%	51.0%	10.0%	10.0%	10.0%	19.0%
	0.70%	1.74%	-2.12%	-3.51%	55.2%	10.0%	10.0%	10.0%	14.8%
	0.71%	1.80%	-2.18%	-3.70%	59.0%	10.0%	10.0%	10.0%	11.0%

Mean-CVaR optimization tries to minimize CVaR value while maximize the returns. Therefore, shown in Table 4.31, it allocates more weight to Fixed Income, which has relatively high returns, and low CVaR.

Mean-Variance optimization should have allocated more weight to groups with the higher Sharpe ratio (e.g. fixed income and equity). To the contrary, it allocates more weight to Market Neutral and Global Macro group, which have the lowest Sharpe ratio. It suggests that there are other reasons apart from the counterbalance of risk and return. As Neutral &

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Global Macro have low correlation with other RSG groups, we believe that correlation is key factor behind the outcome of this optimization.

Because of the different allocations, the portfolios of the same standard deviation exhibit different skewness. Since Equity Market Neutral and Global Macro strategy groups are slightly positive skewed while Fixed Income Group is negatively skewed, the portfolio consists of Equity Market Neutral and Global Macro is slightly positively skewed while the portfolio heavily weighted in fixed income is negatively skewed.

Table 4.34 Statistics of RSG Groups

	Equity Group	Fixed Income	Equity Market	Global Macro	Managed Futures
mean return	0.878%	0.670%	0.301%	0.183%	0.589%
standard deviation	2.657%	1.243%	0.940%	2.512%	2.548%
Sharpe Ratio	0.990	1.488	0.530	0.031	0.606
CVaR	5.79%	2.69%	1.88%	5.36%	4.61%
Skewness	-1.003	-1.515	-0.545	0.369	0.239
Kurtosis	6.390	9.002	4.241	5.912	3.929

Therefore, the portfolios proposed by different optimization approaches have different skewness. We believe it is the difference in skewness that leads to the gap between two efficient frontiers.

The portfolio with negative skewness has its long tail extends to the far left, as it shows in figure 3, and this to some extent leads to a higher CVaR level.[3]

[3] https://ia600607.us.archive.org/1/items/MathematicsOfStatisticsPartI/Kenney-MathematicsOfStatisticsPartI.pdf Page 106

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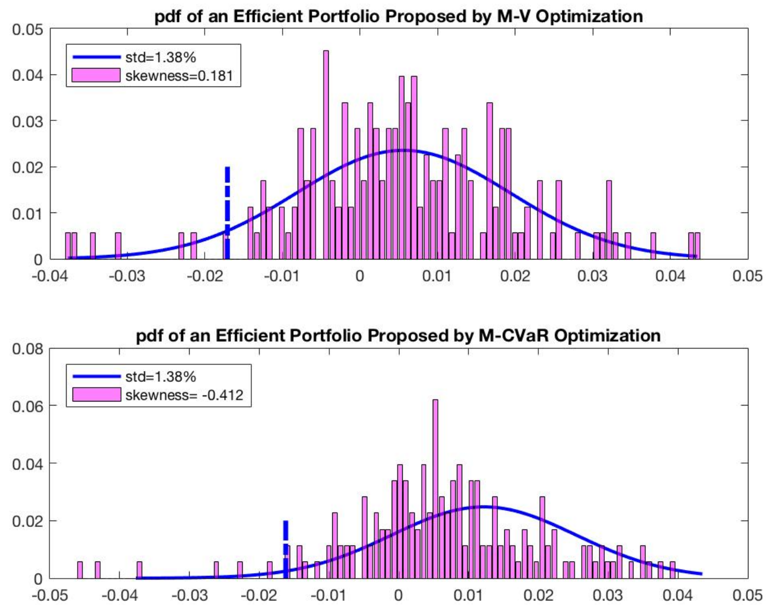
Figure 3 PDF of Two Efficient Portfolios



As table 4.35 shows, having the same volatility, the CVaR of the portfolio proposed by M-CVaR optimization is higher than the CVaR of the portfolio proposed by M-V optimization. For example, an investor's risk tolerance for tail risk (CVaR) is -2.8%, this level of risk can be translated to a standard deviation of 1.533% using M-CVaR optimization. However, this level of risk will be translated a standard deviation of 1.478% if M-V optimization is used instead. In this case, M-V overestimate the risks.

Table 4.35 Efficient portfolios with the same standard deviation

std	CVaR (M-V)	CVaR (M-CVaR)
1.275%	-2.4%	-2.5%
1.335%	-2.5%	-2.6%
1.379%	-2.6%	-2.7%
1.431%	-2.6%	-2.7%
1.478%	-2.7%	-2.8%
1.533%	-2.8%	-2.9%
1.582%	-2.9%	-2.9%
1.624%	-3.0%	-3.0%

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To reinforce our finding of skew gap is indeed attributable to skewness, we also tested the RSG statistics excluding 2007 to 2009 data. The purpose of this exercise is to make sure that the 2008 financial crisis does not influence the positive skew gap on Canadian hedge fund performance. It turned out that we have obtained similar statistics of RSG without 2007 – 2009 data (see Table 4.36 RSG statistic without 2007-2009 data). The outcome reassured us that the financial crisis did not change the positive skewness.

Table 4.36 Statistics of RSG Groups Excluding 2007 – 2009 data

RSG Statistics Excluding 07-09 data	Equity Group	Fixed Income Group	Equity Market Neutral	Global Macro Group	Managed Futures Group
Compound Monthly Return	0.951%	0.708%	0.267%	0.082%	0.562%
Annualized CRR	9.472%	6.973%	2.585%	0.789%	5.500%
Annualized std	7.376%	3.585%	2.918%	9.284%	9.505%
Sharpe Ratio	1.020	1.402	0.219	-0.125	0.374
CVaR	3.60%	1.88%	1.50%	5.73%	5.11%
MaxDD	14.54%	3.53%	6.03%	18.43%	17.83%
Skewness	-0.0969	-0.5079	-0.2521	0.3924	0.2356
Kurtosis	3.4496	4.2345	3.0531	5.6048	3.5842

To summarize, the different focuses of two optimization approaches result in the different allocations. In addition, allocation to Fixed Income makes the portfolio negatively skewed and the portfolio of negative skewness has its long tail extended to the far left. The long tail to some extent means a higher level of CVaR and therefore, with the same level of standard deviation, the portfolio proposed by M-CVaR has a higher level of CVaR.

This positive skew gap is quite counter-intuitive. We hope this study can lead to further research studying Canadian hedge funds and more comprehensive analysis on downside risks for hedge funds.

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5: Conclusion

This article discusses a few issues related to Canadian hedge funds, including the risk-return characteristics of various strategy indices and the unstable correlations between different hedge fund strategies. We dealt with the problem of varying correlations with proposed RSG groups, which result in a lower averaged volatility of 12 month rolling correlations, to allow for a more consistent portfolio construction.

With the RSG groups, this study has presented two different methodologies for hedge funds allocation. Mean-Variance Optimization ignores tail risks, which puts limitation for an accurate picture of hedge fund portfolio allocations. Considering the negative skewness and significant excess kurtosis of hedge funds, this paper adopted Mean-CVaR Optimization on top of Mean-Variance Optimization.

In the end, results from two optimization approaches are compared and representative portfolios are selected to satisfy different risk appetites. In addition, we conclude appropriate ways to limit tail risks for both optimization approaches. As a matter of fact, both approaches reduce the weight on equity RSG group for higher risk-aversion investors. Interestingly, the mean-variance efficient frontiers resulted from two optimization approaches exhibit a counter-intuitive skew gap. We explain the gap with the different skewness of proposed portfolios.

Last but not least, we look forward to further research explaining the skew gap exists in Canadian hedge fund indices and expect further explore in optimized allocations among traditional portfolios and hedge funds.

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