Embodied Curiosity in the Mathematics Classroom Through the Affordance of *The Geometer's Sketchpad*

by

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Abstract

This dissertation examines the role of curiosity in understanding the process of mathematical meaning-making. I argue that human curious behaviour coupled with the affordances of digital technology are instrumental in the way students construct mathematical meanings and that the body plays an important role in this curiosity-technology relationship.

I use data collected from two secondary schools in Jamaica to examine how curiosity could be exploited in the mathematics classroom. The students who participated in this study were between thirteen and fifteen years old and followed the Jamaican Grade 9 curriculum. The data analysis is qualitative in nature and is based on selected pairs of students' interactions involving digital technology and circle geometry theorems.

To frame this research, I designed a theoretical framework, which I named Embodied Curiosity, that is grounded in theories of embodied cognition and draws on Andrew Pickering's (1995) conception of agency. The main idea around this framework is the reconceptualization of curiosity (trait-curiosity), to relational-curiosity (the agential relationship between the students' curiosity and digital technology).

The broader aim of this study is to respond to the limited research in the mathematics education field around the affective dimension of learning and the integration of digital technology in the mathematics classroom. However, the specific goal is to identify the physical markers of curiosity and to investigate the extent to which Embodied Curiosity fosters the construction of mathematical meanings. In addition, this research seeks to find out how the potentialities and affordances of *The Geometer's Sketchpad* contribute to the Embodied Curiosity process.

This study accentuates the significance of considering Dynamic Geometry Environments (DGEs) as essential tools for stimulating curiosity. It also presents pedagogical implications for teaching circle theorems and fostering deeper understandings about how the attributes of a circle connect to each other. Furthermore, this research allows me to understand that mathematics teaching and learning should not be concerned solely with the nature of mathematics but also the nature of human beings.

Keywords: curiosity; embodied cognition; digital technology; affordances; agency; *geometer's sketchpad*

Dedication

This work is dedicated to my family (Lenford, Zahmayne and Roshaad) and in loving memory of my best friend Georgia Hall-McLeary (June 1, 1973 – February 10, 2020).

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Thank you to all the Faculty members of the mathematics education department for their thoughtful, inspiring and exceptional leadership throughout my courses, book clubs, conferences and seminars. My indebtedness also goes out to my seminar group whose continuous feedback on my work helped to shape the thoughts and ideas that emerged in my dissertation. These experiences have truly been one of the most important reasons for my success.

I would like to express gratitude to the Jamaican Ministry of Education – Research and Information Centre for allowing me to conduct my research in the selected schools. A special thank you to the schools' administrators, participating teachers and students whose commitment and support were instrumental in my work coming to fruition. Also, thank you to McGraw-Hill for providing free *Sketchpad* licenses to my participating schools. This significant gesture has contributed to my data collection process and beneficial to both students and teachers at the participating schools.

My support system was deep and wide, I could not do justice to name every single person here. However, I would like to express gratitude to a few friends who played a pivotal role in my development: Jason Forde, Dr. Megel Barker, Dr. Vinu Subashini Rajus and Dr. Mina Sedaghat Jou for the invaluable discussions about mathematics, my research interest and the various components of the dissertation. Their stimulating discussions provided deeper conceptual understanding of my topic and was beneficial in providing clarity when things seemed fuzzy.

Thank you to my remarkable family (Lenford, Zahmayne, Roshaad) for taking this journey (all of it) with me and to my sisters Dee, Tiya, Jean and Lori, for their understanding when I needed time and space to think things through. Although we were far apart, their love and support kept me socially connected and served as a reminder that determination and resilience are the core components of our upbringing.

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ANT	Actor Network Theory
CAPE	Caribbean Advanced Proficiency Examination
CPU	Computer Processing Unit
CSEC	Caribbean Secondary Education Certificate
DGE	Dynamic Geometry Environment
DGS	Dynamic Geometry Software
EDPM	Electronic Document Preparation Management
e-Ljam	e-Learning Jamaica
ESTP	Education System Transformation Programme
GSAT	Grade Six Achievement Test..
ICMI	International Commission on Mathematical Instruction
ICT	Information Communication Technology
IT	Information Technology
IWB	Interactive Whiteboards
LMS	Learning Management Systems
MoE	Ministry of Education
NEG	Negro Education Grant
NEI	National School Inspectorate
NSF	National Science Foundation
PEP	Primary Exit Profile
TFER	Task Force on Educational Reform
URL	Universal Resource Locator

Chapter 1. Introduction

> *"Curiosity in children is but an appetite after knowledge and should be encouraged in them" (John Locke, In J.W. Adamson, 1922)*

John Locke's quotation sums up what this dissertation is about – searching for ways to support, encourage and promote curiosity. For a long time, the field's focus has been on cognition in mathematics education. For instance, Sarah Lubienski and Andrew Bowen (2000) collected mathematics education research from the ERIC database between the period 1982 and 1998. The predominant themes related to societal issues, topics in mathematics and general educational. Their research findings revealed that most of these researches were about students' learning and cognition. Furthermore, for decades, mathematics education research has focused on the nature of mathematics (Begg, 1994; Dossey, 1992; Presmeg, 2002) and throughout the 20th century, the Cartesian mind–body divide permeated the landscape of mathematics education research. However, more recent research has focused on embodiment (Lakoff and Núñez, 2000), which further sparked research that disrupts the mind-body binary in mathematics learning (de Freitas and Sinclair, 2014, Hall and Nemirovsky, 2012; Radford, Edwards and Arzarello, 2009; Sinclair and Heyd–Metzuyanim, 2014). Other non-dualistic research emerged about the way the sensorimotor system shapes mathematical thinking (Nemirovsky, Rasmussen, Sweeney and Wawro; 2012), as well as, the role of the affect in mathematics education (Hannula, 2012). Despite the increased interest in research on embodiment, it was not clear to me how these theories of embodiment took into consideration the affective domain of learning.

My interest is at the intersection of embodiment (which is relatively new in mathematics education) and the affective (curiosity) domain of learning. I want to understand how the body reacts to curiosity when students interact with digital technology (*The Geometer's Sketchpad*). This interest originated from three experiences: growing up as a child, my own educational journey and my experience as a mathematics educator. Since curiosity seemed to be a recurring theme in all three of these experiences, my dissertation has evolved with a main focus on curiosity in mathematics education.

In the first three chapters of my dissertation, curiosity, which has a biological function (Kidd & Hayden; 2015) and allows us to seek out knowledge, is portrayed and discussed as trait-curiosity or, simply, curiosity. However, in Chapters 6 and 7, curiosity takes on a new meaning, which involves as a connection between trait-curiosity and digital technology with the body playing a mediatory role in connecting them. This relationship is referred to as relational-curiosity and it is the foundation of my study and is mainly featured in the process of data analysis and the discussion of results. In Chapter 8, I revisit trait-curiosity to reinforce its transformation to relational-curiosity and show how the two types of curiosities differ from each other.

1.1. Background and Context

I grew up in a rural community in Jamaica where farming was predominantly the main source of income for many families. My siblings and I were adventurous and enjoyed life immersed in activities that involved discovering the physical and material world. As a result, most of our upbringing involved traversing the hills and valleys, climbing trees and exploring the rivers and beaches in our community. During this period of my life, I expressed naively, yet genuinely, a need to know and understand how things around me worked.

For instance, having a copious number of fruit trees in my community, I can recall at about eight years old one of my ‘favourite’ things to do while walking home from school was to taste as many different fruits as possible along my path. My intention, then, was to establish similarities and differences not only in the flavour of each fruit, but also to differentiate between their outer textures, size and colour. I can still remember being warned by my older siblings to desist from this practice because there was no way of deciding whether or not a particular fruit was edible. I remembered how anxious I was to continue this unwise activity. Little did I know that this anxiety reflected an innate desire “to know”, a concept I now understand as curiosity.

As I grew older, and my home experience became more dynamic, I remembered spending summer time with my grandmother in another community far away from home, but with similar demographic features. Spending time with my grandmother contributed significantly to my development as a contemplative individual who not only wondered about *why* things work, but also about *how* they work. In this sense, my curiosity was nurtured and developed in ways which influenced my view of people and how I make sense of the world today. This development was possible through my grandmother’s teaching. Her dialogues were inundated with “Jamaican Proverbs”[1] a literary device which was most commonly used informally in the Jamaican language (The Jamaican Creole)[2]. These Jamaican Proverbs became instrumental to the way I interact and perceive the world today. For instance, one Jamaican Proverb which she used frequently was, “Yuh cyaan hab yuh cak an eat it tu”, which translates to English as, “You cannot have your cake and eat it at the same time”. This Jamaican Proverb means that it is difficult to have things both ways – you cannot be happy and sad at the same time. As a little girl, my understanding was merely conceptualized around being the owner of a literal cake. I can recall every time my grandmother used this particular Proverb the following dialogue would ensue:

> *Me: Why mama, why, why can’t you have your cake and eat it, if it is your cake?*
>
> *Grandma: Because that is how it is.*
>
> *Me: So, who should eat it mama?*
>
> *Grandma: I really don’t know, my child.*
>
> *Me: So, who can I ask?*

As an adult, I easily could see how a routine such as this could have driven my grandmother to total frustration, particularly because it seems she did not have an immediate response to my quandaries. But today, as a researcher, I see the emergence of curiosity as a possibility for meaning-making (in this case, an understanding of who should eat the cake). A reflection on my early development as a child allowed me to question two things: [1] How can our innate abilities be identified and exploited at an early age; and, [2] What role does the education system plays in nurturing these abilities? In order to find responses to my questions, I examined the structure of education in Jamaica – partly to identify whether or not there were deliberate attempts to cater to the affective needs of children, especially now that I am reflecting on my own experience. This I believe

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[1] Jamaican Proverbs are usually one sentence long. They tell life’s stories and convey ideas about human behaviour in the environment.

2 Jamaican Creole is the native language of the Jamaican people. It is an English-based creole language influenced by West African culture, particularly with Akan origins.

would provide context for my research interest, which would later be important in the methodological decisions I have made (discussed in Chapter 4). An account of the structure of the education system in Jamaica also provided insights into the possible limitations of my study, some of which relate directly to the structural differences within the schools that participated in this research.

1.2. Education in Jamaica

The structure of the Jamaican education system dates back to the period of colonization when Jamaica was once ruled by England from 1655. According to Sherry Keith (1978), and corroborated by Rae Davis (2004) in the Task Force on Educational Reform (TFER) report, the Jamaican education system was based on the British system. Initially, education in Jamaica existed to serve the children of plantation owners, but, according to the TFER report, in 1835 Jamaica received financial assistance under the Negro Education Grant (NEG) from the British government to educate ex-slaves. Keith (1978) suggests that, during this period, “Education was viewed as a means of teaching the freed slave population to submit to the conditions of wage labor when the threat of violent coercion to work was eliminated by abolition” (p. 39). The mandate of the NEG was therefore given to religious organizations to administer these grants, while the legislative directives were given by colonial bodies.

As a result, a segregated system of two different types of schools emerged, one for the children of the higher social class called “elite schools” and the other for children of the working class, who went to publicly funded schools. The curriculum in the elite schools was intended to prepare students for traditional careers (lawyers, doctors, teachers, nurses…), while the publicly funded schools framed their curriculum to cater to the needs of skilled and unskilled careers (cane cutters, craftsmen, carpenters, …). Traces of this class-segregating system are still evident in the education system of Jamaica today, with a few churches maintaining their presence in the administrative structure of some schools, while vast majority owned and operated by the Government of Jamaica. The churches that are currently active, provide guidance to the schools’ operation, serve as Chaplin and, most noticeably, some of these schools retain the names of the church members who were given authority to establish the schools.

In 1958, Jamaica began to acquire steady sovereignty in the management of its affairs closer to the period of independence from the British government. This led to the establishment of various government ministries with a specific directive to manage certain aspects of the country's development. The inception of government ministries later gave rise to the formation of the Ministry of Education (MoE). The task of this ministry was to manage the educational system and institutions of the country. The MoE is also the body responsible for the development of policies governing education in Jamaica. It is credited with transforming the education system from one that reflected the colonial inequality to a network of agencies focused on addressing the recommendations about the state of education in Jamaica put forth by the TFER report. The Jamaican education system, through the MoE, now offers education to its citizens through the following four levels: early childhood, primary, secondary and tertiary, as shown in Table 1-1.

Table 1-1 The Education System in Jamaica

Education	Age Range (years)	Grades
Early Childhood	2-4	K1 - K3
Infant/Basic	4-6	K3 - K5
Primary/Elementary	6-12	Grade 1 - 6
Junior High	12-14	Grade 7 - 9
Secondary	14-18	Grade 10 - 14
Tertiary	18 and above.	

In keeping with the dual-track system from which the Jamaican education system evolved, secondary schools have transformed into what are now called "traditional" and "non-traditional" high schools. This system still subtly reflects the elitist and publicly funded types of schools respectively. The most noticeable shift in the structure of secondary education in Jamaica today comes from an increase of opportunities for more children to gain access to education and more children from the working class to enter traditional high schools. This is a major achievement for the MoE, whose vision is to provide equal educational opportunities for all children.

In the Jamaican education system, students are rigorously assessed at the primary level through the following national examinations:

Grade 1: Readiness inventory

Grade 3: Assessment test in mathematics and language arts

Grade 4: Literacy and numeracy test

Grade 6: Grade Six Achievement Test (GSAT)

The GSAT examination, which was replaced by the Primary Exit Profile (PEP) in 2019, was then used to place students into secondary schools based on their academic performance. The top achievers were usually awarded places in traditional high schools, while the others were assigned places either in non-traditional high schools or at the junior high school level (see Table 1-1). At the end of secondary schooling (at Grade 11), students are assessed through a regional examination body, namely, the Caribbean Secondary Education Certificate (CSEC). After successful completion of this examination, the top achievers then move on to Grade 12 and 13, where they pursue the Caribbean Advanced Proficiency Examination (CAPE) which is geared towards filling the gap between secondary and tertiary education. CAPE is usually seen as the platform which places students who are in good academic standing in position for tertiary education regionally and globally. Those who did not proceed along the CAPE pathway embark on skilled training or apply for job opportunities in the workforce.

For the purpose of this research, the two participating schools were unintentionally selected from both traditional and non-traditional streams. This categorization was not taken into consideration for data analysis or presentation of findings. However, I believe it was noteworthy to provide the historical context of the origin and categorization of the school system, in order to provide a context for administrative differences and to provide a possible response to any differences in the way my phenomenon of interest may have developed in each school. In addition, I believe that the categorization of schools could have implications on the validity of my research.

1.3. Curiosity and My Development

Throughout my educational journey, from as early as primary school, there were limited opportunities to support my desire to learn. Traditional teaching methods and teacher-centred approaches were dominant features in the classrooms at the time. This created a disunity between how I came “to know” at home and how I did at school. School, at that time, aroused mixed feelings within me. On the one hand, I found the social interactions engaging, while, on the other, I had several misunderstandings about the things I was learning. I often wondered about different real-life situations, but the answers or practical opportunities which were provided at school had limited impact on my curiosity. I felt as if what I was learning at school did not coincide with what I was learning at home. It is within this imbalance I now see the need to examine how one's curiosity relates to one's learning experiences.

As far back as I can remember, school mathematics was an isolated activity divorced from the environment and saturated with laws, formulas and principles that must be followed not only in a fixed manner but also the way the teacher says it should go. The higher up I moved along the educational system the more abstract and less engaging mathematics learning has become. The focus was on developing computational skills rather than conceptual ones. There was little opportunity for collaboration, investigation, exploration and, even worse, less connection to the real world. Curiosity in my experience then was more about inquisitiveness about the world and never about understanding mathematics. As I transitioned from learning mathematics in the former years to teaching the subject, things became worrisome, especially because I felt that I was perpetuating the vicious cycle of teaching the way I myself had been taught.

During my experience in Jamaica, first, as a secondary mathematics teacher, and then as a school administrator and school inspector, it became apparent to me that there was a need for continuous professional development around and research into the way children learn mathematics. This was most evident in my role as a mathematics teacher, which was usually a frustrating experience. This was because I lacked the necessary knowledge that was required to address the many mathematical needs and issues relating to the teaching and learning of the subject. However, at the heart of my teaching was always a sense that students' under-performance and further national deterioration in the subject had to do with more than fear or anxiety about the subject. After all, 'fear' is an emotional construct that refers to the belief that something is harmful and will cause pain. I could not see the discipline of mathematics in this way. As a result, the desire to understand deeply how children learn mathematics became a recurring theme in my educational pursuit and professional development.

While I had a deep sense of instinctual judgement about what was going on in my classroom, I was unable to present an argument to my superiors and peers about what I was feeling and how much I desired a better understanding about what was hindering my students to progress in mathematics. The first step towards reconciling this tension was to reflect on my own practice in order to identify my own shortcomings. This led me to various roles within the Jamaican Ministry of Education. I worked on a project as a mathematics specialist within the Education System Transformation Programme (ESTP)[3] and as a school inspector in the National School Inspectorate (NEI)[4]. These two agencies were created as a direct response to the recommendations from the TFER and provided me with better insights into mathematics teaching and learning in general. I had the opportunity to examine mathematics teaching and learning from a wide cross-section of classrooms, both from the elementary and secondary level of the education system. By this time, it became apparent that my frustration was common among many mathematics teachers, which propelled me to seek a better understanding.

Humbled by these roles, I became more aware that although emphasis was placed on the cognitive aspects of learning, there was little focus on the affective (my own feeling of curiosity, as well as the students'). That is, I became conscious of the disregard for children's emotions in the learning process which I believed, at the time, was an important factor in how children learn. This eventually led me to the Ph.D., where I would immerse myself in theoretical, pedagogical and epistemological understandings of mathematics education. Again, my own curiosity had been piqued during my Ph.D. program due to my experiences both with contemporary theories in mathematics education and dynamic geometry environments (DGE). My phenomenon of interest was developed out of a need to understand how curiosity and integration of technology played a role in mathematics learning.

1.4. Theory Development

According to Arnone, Small, Chauncey and McKenna's (2011) research on children's curiosity, interest and engagement; children's curiosity levels diminish as they move from one grade level to the next. These authors further argue that this is as a result of the

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[3] The Education System Transformation Programme is the response to an overall assessment of the performance of Jamaica's education system as documented by the Task Force on Educational Reform in 2004.

[4] The National Education Inspectorate is an independent body which seeks to address the issues identified outlined by the TFER and to implement changes which coincides with the transformation of the education system according to the MoE's vision.

emphasis in curriculum and assessment practices; as I believe was the case for me in elementary and high school. Furthermore, my experience as a mathematics educator has led me to believe that it could also be as a result of teachers' unawareness of how to identify and exploit curiosity in the mathematics classroom. I recognise the possibility that both issues combined may have contributed to the low levels of curiosity as children progressed along the educational system. Therefore, I used this opportunity (writing my dissertation) to reflect on my own awareness of the way children learn, the role curiosity plays in learning and how curiosity might be evoked. This has led me to a deep aspiration towards understanding theoretical frameworks in mathematics education and how they work.

It was obvious to me that one of the missing components of my experience as a mathematics teacher was a deeper understanding of how my students learn. This lack of understanding was exposed when I began studying about contemporary theories in mathematics education. I became intrigued by the ways in which theoretical frameworks provided analytical lenses, not only to assess critically how children learn, but also to determine what contributes to their learning. Furthermore, I found that the theories that I was acquainted with in mathematics education research primarily draw on constructivism, which assumes that it is the student that is the autonomous subject (Brooks & Brooks, 1993). Because of this, I was a little concerned that the field of mathematics education had only limited theoretical frameworks that spoke both to technology integration and to the affective dimension of learning. What was even more concerning was that little precedence was given to the material in a people-things interaction. As a result, I became interested in ways I could find a construct to respond to how children learn mathematics using digital technology in a unified way.

Indeed, I recognized that, in recent times, there has been an influx of research on technology use in mathematics education, most of which explored the influence of technology on generating certain kinds of behaviour from children or on how children use technology tools to foster mathematics learning. However, there was little research which focused on the relationship between children's' emotions and technology tools, particularly in geometry. In fact, I was puzzled about how the interplay between children and digital technology could develop mathematical meanings without giving precedence to one or the other.

In order to develop a theoretical framework adequate to my phenomenon of interest, I first examined two known theoretical frameworks: the theory of semiotic mediation and the theory of instrumental genesis. Both of these theories focus on tool use in mathematics education. For the purpose of understanding how theoretical frameworks were utilized in mathematics education, I applied them during the coursework stage of my Ph.D. program. I found that neither of these theories responded to the internal questions I had about the relationship between curiosity and digital technology. While I found both to be useful in understanding how mathematical meanings could be developed, both failed to address the role curiosity played in the learning process. Furthermore, both theories privileged the human in the human-tool interaction, which seemed to me to run against my own experience, both as a learner and as a teacher. And neither seemed to adequately account for the role of the body in mathematical meaning-making. These ideas will be further developed and discussed in details in Chapter 3 where the focus is on how Embodied Curiosity[5]– namely the theoretical framework that informs my data.

1.5. Movement of the Body

From a Jamaican cultural perspective, movement and dance are powerful ways of accessing and expressing one's identity. In that context, many educators use rhythm, movement and dance to teach various curriculum disciplines (Minott, 2008; Hanna, 1999). Having students write a descriptive piece in English Literature from a dance choreography, or use their bodies to show how atoms of various particles operate when exposed to excessive heat or cold, is simpler than teaching them to use their bodies to demonstrate the rationality of numbers. However, many writers have proposed that the body is more than skin-bound and plays an important role in learning. For example, Gol Tabaghi and Sinclair (2013) give a comprehensive overview of the role of the body in mathematics learning. In their analysis, they show that learning takes place when speech, body

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[5] Embodied Curiosity is not a novel term. In fact, it first appeared in Fridman (2013) who, at the time, was seeking a way to explain personal “w{o|a}nderings” and a place to store things discovered by her curiosity in nature. Fridman did not explore the idea beyond its origins and, to the best of my knowledge, it is still left untheorized. In a recent search for the website proved unsuccessful. My work on Embodied Curiosity is not a continuation of Fridman’s work; nor is it grounded there. Instead, I am simply acknowledging that the term itself is not new. I have chosen to express the term with capital letters throughout this dissertation to add some level of originality to it. Since embodied cognition is sometimes referred to as EC, I did not want to shorten the term and add any ambiguity to its use.

movement, gestures and material work together in a harmonious relationship. They emphasize that, although many researchers pay attention to the visual, aural and tactile nature of embodiment, there is value in the kinesthetic nature of the human experience.

Drawing on the cognitive scientist Jay Seitz's (2000) three core cognitive abilities, Gol Tabaghi and Sinclair show that kinesthetic memory allows us to think in respect to movement and claims that kinesthetic memory relies heavily on dynamism. Within these claims, I found a key component that I believe links curiosity to mathematics learning, which was body movement. I propose that body movement connects curiosity to the digital technology and it is within this connection that mathematical meanings are constructed. In my research, technology is portrayed in two ways: one, to stimulate curiosity and two, to provide an outlet for curiosity to be developed.

1.6. Research Aims and Questions

The purpose of this qualitative study is to explore how curiosity, embodiment and digital technology relate to the construction of mathematical meanings when grade nine students interact with digital technology in a circle geometry lesson. My aim is first to, realign curiosity as a relationship with the digital tool and second, to show that within this relationship mathematical meanings are constructed. A broader aim is to formulate Embodied Curiosity as a theoretical framework for mathematics education and, with it, to show the centrality of students' emotion in mathematics learning. Bearing these overarching aims in mind; my research seeks to address the following research questions:

> [1] What are the physical markers of curiosity in the secondary mathematics classroom?
> [2] To what extent does Embodied Curiosity foster the construction of mathematical meanings?
> [3] How do the potentialities and affordances of *The Geometer's Sketchpad* evoke Embodied Curiosity?

Since curiosity cannot be seen with the naked eye, Question 1 serves as a foundation for my research to differentiate between examples and non-examples of when curiosity emerges. Question 2 acts as the nucleus of my study and my response provides an account of the possibilities for mathematics learning to take place. Question 3 examines the role *Sketchpad* plays in understanding the relationship between embodiment and curiosity.

1.7. Structural Overview of the Dissertation

This dissertation consists of eight chapters. Chapter 1 contains an overview of how my upbringing and school experiences provided an outlet to understand the development of my own curiosity. I present a historical description of the Jamaican education system for clarity about my distinctly different research sites and for context of my ‘underdeveloped’ curiosity. In addition, I give an account of my initial motivation for generating a theoretical framework which speaks to the significance of the affective domain and the intertwining relationship between students and technology in constructing mathematical meanings. Furthermore, I express the importance of the body in shaping the mind and illustrates my interest in the use of digital technology, specifically *The Geometer's Sketchpad* as the preferred tool for geometry.

Chapter 2 outlines the literature relevant to my research. It focuses on the development of curiosity within different disciplines and shows the problematic way in which the concept has been conceptualised over time. A historical landscape of digital technology is also analysed with the intent to show not only that the advancement of technology is as a result of our constant need for knowledge, but also to show that its development may be linked to curiosity. In addition, the literature review focuses on the dynamic geometry environments (DGEs) and their impact on mathematics education today, while exploring previous research conducted with *Sketchpad*. This approach was taken because I was uncertain about the ways in which curiosity and digital technology were aligned to provide answers for my research questions.

In Chapter 3, I present Embodied Curiosity, a theoretical framework which seeks to address the relationship between the affective domain (more specifically, with curiosity) and digital technology. I focus on embodied cognition as the overarching principle which guides Embodied Curiosity, but draw on Pickering's (1995) “mangle of practice” to account for how human, material and disciplinary agencies interact in learning. I also describe ways in which Berlyne's (1978) curiosity dimension model was useful in helping me both identify and reimagine curiosity.

Chapter 4 describes my research methodology. It outlines the research design, data source, data collection and analysis procedures, as well as, outlining the strategic decisions I made while conducting this research.

Chapter 5 details an understanding of the issues surrounding the teaching and learning of circle geometry theorems, focusing on some of the properties of the circle as they are expressed in a DGE. It also illustrates the rationale for exploring the impact of dynamic geometry on the teaching and learning of circle theorems.

In Chapter 6, I describe students' interactions with *Sketchpad,* teachers' interactions with students and how Embodied Curiosity was evident in each interaction. I also demonstrate how relational-curiosity emerged, as well as, how curiosity was identified in the mathematics classroom. In addition, Chapter 6 contains an analysis of the curiosity-digital technology relationship and presents arguments about when and how mathematical meanings were constructed.

Chapter 7 presents a discussion of the research results. It addresses the research questions and provides insights into the ways Embodied Curiosity influences how children construct mathematical meanings. In this chapter, I also provide an explanation for a better understanding about how children become curious and how this should be taken into consideration in mathematics education.

In my concluding chapter (Chapter 8), I bring together the impact of curiosity on digital technology, my professional development and students learning. I make suggestions, based on my findings, about the future of Embodied Curiosity, as well as, the way children's emotion and digital technology could provide new insights into the way mathematical meanings are constructed. I also make suggestions about the important role of the body in shaping thinking and address implications for the teaching and learning of geometry. Towards the end of chapter eight, I offer an emotional reflection of my journey through this research with the hope that, in addition to filling a gap in the academic community, my research might highlight and emphasize the importance of the affective domain in learning.

1.8. Summary

The central focus of this research is on the role of curiosity and embodiment on digital technology in the mathematical meaning-making process. In order to address this issue, my point of entry is first to examine the role curiosity played in my own life as a child and later as a mathematics educator. Feeling bewildered by the influx of technology and noticing the little attention to its integration in the mathematics classroom (from a Jamaican context), curiosity has led me to explore new ways of learning, teaching, and thinking about mathematics. Using *Sketchpad* and circle geometry theorems with grade nine students in Jamaica, I investigated the relationship between curiosity (trait-curiosity) and digital technology, and further looked at the role of the body in this relationship. Out of a need to understand how the curiosity–embodiment–digital technology relationship influences mathematics learning, the theoretical framework – Embodied Curiosity – was developed and used to observe how children construct mathematical meaning.

Chapter 2. Review of Literature

> *"The important thing is not to stop questioning. Curiosity has its own reason for existing" (Albert Einstein, in Miller 1955.)*

In the quotation above, Einstein suggests that curiosity is a never-ending experience and challenges us to keep questioning things. This reflects my journey with this chapter, as a never-ending process of reflecting, rethinking and reconsidering. The chapter provides me with a deep sense of thinking about my phenomenon of interest, while simultaneously allowing me to reflect on my past and present role as a mathematics teacher. Being able to engage with the work of other researchers is more than merely demonstrating that I have read and understood the available literature around my area of interest; it also provides me with fresh ideas and new ways of thinking about mathematical concepts and issues relating to mathematics education. These new insights broadened my horizons and provides opportunities to improve my practice.

In this chapter, I discuss the development of curiosity from two different perspectives: from a philosophical standpoint and through the lens of psychology. My reason for this is to provide context for the way in which curiosity is operationalized and reconceptualized in this research. I also believe that these two perspectives provided me with a deeper sense of understanding, which enabled me to examine any possible relationship(s) between the evolution of curiosity and the development of digital technology. Furthermore, considering my interest of the embodied nature of curiosity and its influence on mathematics meaning-making, I examine previous mathematics education research on embodiment. This understanding, I believe, has shed light on the way in which curiosity may be situated as an essential component of children's mathematical meaning-making processes.

Since it is my belief that digital technology plays an important role in how curiosity is evoked, I also examine the historical landscape of digital technology. This allows me to understand how changes in the affordances of digital technology can have an impact on mathematics learning. In addition, having established my interest in how the body reacts to curiosity in Chapter 1, I explore previous research on the importance of sensorimotor activity on mathematics learning and report on this in the latter part of this chapter. The reason for this is to establish a need for understanding not only of why curiosity plays a significant role in developing mathematical meanings but also to provide a response to how this can become possible.

Despite the importance of mathematical meanings to this study and its involvement in the Embodied Curiosity process, I have chosen to use the term in its colloquial sense. In Chapter 6, I describe the two types of mathematical meanings that emerged from my data. In addition, section 7.4 provides more detail about how mathematical meanings were constructed.

2.1. Curiosity

> *Curiosity is the most superficial of all affections; it changes its object perpetually; it has an appetite which is very sharp, but very easily satisfied; and it has always an appearance of giddiness, restlessness and anxiety. (Burke, 1757)*

Many academic accounts show that curiosity is immeasurable, intangible and undefinable. For example, in the field of psychology, curiosity is seen as an internal motive which influences human behaviour and fosters active learning (Oudeyer, Gottlieb and Lopes, 2016), while in the field of philosophy curiosity is widely contemplated as a moral virtue (a good habit) or a vice (a bad habit). For example, Elias Baumgarten (2001) describes curiosity as, “a character trait or desire” (p. 257) which becomes a bad habit when “curiosity is unguided” (p. 268). This view of curiosity usually stems from or is stimulated by greed. Burke’s quotation above captures the divisive way in which curiosity is perceived – sharp, yet restless.

However, towards the turn of the 20th century, George Loewenstein (1994), who is well known for his work in economics and psychology, ‘tamed’ curiosity to focus on it as an information gap. His arguments are based on the views of philosophical and theological thinkers. Loewenstein’s work gained recognition in two streams of activity. The first stream happened in the 1960s, when emphasis was on the psychological possibilities of curiosity. The second stream occurred between the 1970s and 1980s when the emphasis was mainly on finding ways to ‘measure’ curiosity. Loewenstein argues that, in both streams, there were shortfalls in terms of the theoretical framing of curiosity. Among these shortfalls was the fact that adequate explanation was not offered about why people voluntarily seek out curiosity, as well as the insufficient attention to its definition. These limitations allowed him to present a new account of curiosity. Since my interest is aligned with Loewenstein’s information-gap perspective, I present a trajectory of the development and evolution of the concept based on Loewenstein's account. In this way, I will establish a need for a proper delineation of the term and a rationale for why I chose to reimagine the term.

At first, *curiositas* (the Latin word meaning "curiosity" or "inquisitiveness") appeared in the writing of Cicero (1345/1914), who viewed curiosity in a dualistic way: on the one hand, he saw it as "an innate love for learning and knowledge" (p. 48), suggesting that curiosity is necessary for the pursuit of knowledge. On the other hand, it was viewed as an immoderate and excessive desire for unsuitable or inappropriate knowledge. At the time, Cicero believed that if the natural state of humanity was to seek knowledge, but its excess made curiosity a bad thing. Then St. Augustine, in *The Confession, Book X*, in the version translated by Sheed (1943), referred to curiosity as a "temptation" or "a certain vain desire" (p. 54), suggesting that the senses of the body (to see, touch, smell, hear, and taste) are used in the pursuit of truth about anything. This is regarded as a need for pleasure. However, St. Augustine claimed that this view of curiosity is usually disguised under a need to gain knowledge since the eye (seeing) is the primary entry into gaining new knowledge. For St. Augustine, curiosity should be more than a desire for things of pleasure, and instead should be a need for new experiences through the body. An example of this may occur in mathematics, when a child experiences an understanding of a right angle through a gesture of the arm in an upright position, signalling that a ninety-degree angle is present.

Interestingly, the etymology of the word "curiosity" has also shown that there has been a back-and-forth shift in its meaning between being something good and something bad. Emerging in the 14th century *curiositatem* (Latin) meant "desire of knowledge, inquisitiveness" akin to cara ("care") which implies something good, but in Middle English "curiosity" meant "a desire to know or learn, inquisitiveness; prying, idle or a vain interest in worldly affairs", which positions curiosity as a bad thing. By the 17th and 18th centuries, David Hume (1777/1888) divided curiosity into two subgroups; "a love for knowledge" (the good type) and "a passion derived from a quite different principle [and involves] an unsatisfiable desire for knowing the actions and circumstances of the neighbours", which is seen as a bad thing.

Conversely, a more contemporary author such as Daniel Berlyne (1978) described curiosity as "a virtue and as one of the prime aims of education" (p. 99), while Hans

Blumenberg (1966/1985) referred to it as “an appetite for intellectual knowledge” (p. 430) – both of whom positioned curiosity as something good. This gives curiosity a post-enlightenment point of view, where we can consciously know and pursue knowledge. However, with this shift and constant change in the definition of curiosity, Loewenstein (1994) claimed that curiosity had lost any prospect for stability over the decades. Drawing attention to how problematic it has been to identify the origins of curiosity, he explained that these definitions did not capture certain significant characteristics of curiosity, such as its intensity (the pain of not having information) or its transience (meaning it is short lived).

As a result, Loewenstein examined the underlying causes of curiosity through three theoretical contexts (Gestalt psychology, behavioural decision theory and social psychology), for the purpose of shedding light on the shortcomings of previous conceptualisations of the term and to provide a rationale for the ‘new’ position he took. Firstly, his claim is that curiosity is derived from an urge to satisfy a specific need and that it has a motivational force which can be stimulated by both internal and external factors. Secondly, he argues that violated expectations are important instigators of curiosity, implying that curiosity arises when there is an imbalance between an anticipated response and how things in the world actually work. According to him, these perspectives position curiosity in a new paradigm, “as a form of cognitively induced deprivation that arises from a gap in knowledge or understanding” (p. 75). In this sense, he sees curiosity as a desire for a specific piece of information due to some realization of a doubt or uncertainty. This view suggests that, curiosity is a mixture of both cognition and motivation. Loewenstein’s new views on curiosity forms the foundation of my research and as such I have adopted his information-gap definition.

Considering all the possible viewpoints of curiosity, Loewenstein considers curiosity as part of the environment that opens up the possibility for us not only to rethink how curiosity might depend not just on the person, but also the environment—and might not be restricted to being a human experience. William James (1890/1950) suggests that, in viewing curiosity as more than an intrinsic value, something previously unrecognized can be detected. In these perspectives it is clear to me that curiosity demonstrates the ability to generate and develop new meanings and as such is potentially linked to the teaching and learning of mathematics. However, it is not clear how this can be achieved and, as a result, I examine the potential link between curiosity and embodiment.

2.1.1. Curiosity Dimensions

Berlyne (1954), in research about the theory of human curiosity, suggests that there are two forms of curiosity which informs not only how one becomes curious, but also, why we become curious. On the one hand, there is a *perceptual* (a drive which is awakened by a stimulus) dimension of curiosity and on the other hand there is *epistemic* (a desire for knowledge) dimension. These two dimensions were further broken down into two categories; *specific* (a desire for a specific piece of information) and *diversive* (seeking stimulation due to boredom). He elaborates these types of curiosity as:

- Perceptual – Diversive: the individual who demonstrates this form of curiosity is usually stimulated by many environmental cues (sights, sounds, smells and texture), which leads to exploration rather than a given answer.

- Perceptual – Specific: this individual is more curious about the desire for new sensations (sight, sounds, textures)

- Epistemic – Diversive: A form of curiosity with the urge to learn new information, which is exploratory.

- Epistemic – Specific: This form of curiosity is the desire for information or knowledge gearing towards a particular answer.

In Berlyne's (1954) curiosity dimension model, he suggests that these dimensions are interrelated and an individual may not show all four forms.

In another piece of research Marilyn Arnone, Ruth Small and Patricia McKenna (2011) advance a theoretical model that examines curiosity, interest and engagement in a technology learning environment. They point out that children's curiosity level diminishes as they move from one grade level to another and suggest that this could be as a result of the emphasis on curriculum and assessment (rather than being an innate tendency). Based on my learning experience, which was discussed in Chapter 1, I am also of the view that teachers' unawareness of how to identify and exploit curiosity within the classroom could be a contributing factor. Since curiosity is a multi-dimensional concept with no single definition, I take interest in Berlyne's four dimensions as being of possible methodological interest in seeking to identify instances of student curiosity.

2.2. Embodiment

The word ‘embodied’ – the act of giving a physical attribute to something that is abstract – appeared many times in my readings. Interestingly, my first encounter with the word was through an embodied cognition lens. My initial reaction to the word when used with cognition was one of bewilderment, because I was used to thinking of cognition as being about mental processes, not bodily ones. Needless to say, I wanted to know more about the meaning of the term, especially because at the time, the word was appearing frequently in mathematics education research. To my dismay, I came across other terms such as ‘embodied learning’, ‘embodied self’, ‘embodied knowledge’ and ‘embodied bilingual language’. I thought to myself that I was experiencing the very type of curiosity which Loewenstein described as "a form of cognitively induced deprivation that arises from a gap in knowledge or understanding” (p.76). That is, there was a gap in my understanding of the word ‘embodied’, which was further compounded by it being used as a verb to describe an engagement with something. Therefore, my initial desire to know and understand the meaning of the word arose out of a need to satisfy my own deficiency.

‘Embodiment’ (the noun), like curiosity, emerged from the disciplines of psychology and philosophy and came to prominence in the work of Maurice Merleau-Ponty (1945), who suggests that the human body is much more than its physicality and biological functions. With this perspective, he distinguishes between two functions of the human body. The objective body (the physical body with attributes such as height and weight) and the subjective body (the body as a mediator through which we feel and move). In the latter view of the body Merleau-Ponty positions embodiment as an important component of human existence which links the body to the world around it.

Julian Kiverstein (2012), in his research on the meaning of embodiment, presents three different perspectives which captures the conflicting views of embodiment in the literature. He labels these three perspectives body-functionalism, body-conservativism and body-enactivism. Body-functionalism, according to Kiverstein, takes the view that “the body is understood as playing a role in implementing the computational machinery that underpins our cognitive capacities” (p. 740). The body-conservativism view, takes the body as an input–output information medium for the brain, while the body-enactivism view takes the body as the primary source of meaning.

Rafael Núñez's (1999) contribution to the meaning of embodiment seems to be in line with Kiverstein's body-functionalism perspective. In his definition Núñez suggests that it is "the peculiarities of the living human brain and the body, along with the bodily experiences they sustain, are essential ingredients of human sense-making and conceptual systems" (p. 41). With this view, he describes embodiment as "a living phenomenon in which the primacy of bodily grounded experience (e.g., motion, intention, emotion […]) is inherently part of the very subject matter of the study of the mind", (p. 41). This implies that Núñez sees the mind as fully embodied and embodiment as the process which involves a relation among cognition, the mind and living bodily experiences. As a result, Núñez's perspectives on embodiment have implication for mathematics education.

2.2.1. Embodiment and Mathematics

In an effort to understand the human origin of mathematics, George Lakoff and Rafael Núñez's (2000) book presents some fundamental assumptions about how individuals understand, create conceptual systems and bring them into existence. Sparked by theoretical ideas of embodied cognition (which will be further discussed in Chapter 3), they use cognitive linguistic methods (language reveals characteristics of cognition) to claim that mathematics is the product of human imagination. In making this claim, they argue that conceptual metaphors do more than promote mathematical understanding, but are also essential in generating it. They see conceptual metaphors as "mappings that preserve the inferential structure of a source domain as it is projected onto a target domain" (p. 52). In other words, conceptual metaphors embrace the understanding of one idea in terms of another. For example, the understanding of time in terms of money, as in, "I spent time with my family".

Lakoff and Johnson (1980) are adamant that almost all of our concepts originate in this way and that we learn something new only by understanding it in terms of a familiar thing that already exists in our experience. They further suggest that this is made possible through bodily experiences. According to Núñez, Edwards and Matos (1999), conceptual metaphors are not arbitrary, because they are motivated by our everyday experiences, especially the bodily ones. This implies that mathematics is created when individuals are able to use everyday experience to map an understanding about one concept onto another, as demonstrated in the example above. Furthermore, Lakoff and Núñez (2000) claim that metaphors link mathematics to sensorimotor experience and it is within this link that mathematics is grounded.

Sinclair and Schiralli (2003), in a review response to Lakoff and Núñez's (2000) book, highlight three basic assumptions that are crucial to mathematics education. These include the premise that human knowledge comes from sensorimotor experiences, that humans understand through conceptual metaphor and that parts of human thinking are inaccessible to awareness. However, Sinclair and Schiralli also refer to the shortcomings of these claims, pointing out that reviewers' criticisms have shed light on some fuzziness within Lakoff and Núñez's framework. In their analysis, they suggest that building on the salient claims could prove more viable in providing clarity to ideas which seem ambiguous. In doing so, they proposed a deeper examination of the word "mathematics" and made a distinction between 'conceptual mathematics' (as implied in Lakoff and Núñez) and 'ideational mathematics', to show that mathematical ideas can be explained through the use of different metaphors. Sinclair and Schiralli clarify that conceptual mathematics refers to mathematics as a discipline and involves processes such as exploration, investigation, connections, representation, patterns and their manipulation, while ideational mathematics focuses mainly on the ideas formed by individuals influenced by their environment and by genetics. As a result, conceptual mathematics represents meanings in an external space, while ideational mathematics are formed internally. With this distinction, Sinclair and Schiralli position mathematics as a double-edged sword with distinct features that may influence mathematical understanding.

I find this distinction intriguing because, firstly, I believe that an important part of understanding mathematical meaning-making is through a clear understanding of the very nature of mathematics itself, or at least, through the difference in various context in which it is used. Secondly, in looking at the possible relationship(s) between curiosity and mathematical meaning-making, this distinction becomes important in my ability to identify the type of mathematics children are attending to when curiosity emerges. In addition, this distinction helps me to understand better the function of the body in the construction of mathematical meanings. That is, do children move their bodies in different ways when they think about conceptual mathematics or ideational mathematics? Or, further, what function of the body is more dominant when conceptual mathematics or ideational mathematics are at play? Is the body objective or is it subjective? The split between conceptual and ideational mathematics may also provide greater clarity about whether or not students' use of bodily experiences represents curiosity about mathematical ideas, or curiosity about the affordances of the technology. Furthermore, this dichotomous separation between conceptual and ideational mathematics may provide an opportunity to distinguish between when students are curious about mathematics as opposed to being attentive.

Radford, Arzarello, Edwards and Sabena's (2017) research about the multimodal material mind and embodiment in mathematics has also shed light on the new ways of understanding human cognition. In this research, they used data collected from a grade five mathematics class to illustrate the multimodality of the mind. First, they propose that mathematical meanings appear in the teaching learning process through multimodal means. That is, elements such as the body, languages, and material artefact (including digital technology) are considered as central components of how students and teachers think mathematically. In making this claim, they suggest that a first step in understanding the body-material-language relationship, is to have a clear interpretation of the role of humans' tactile-kinesthetic bodily experiences of the world. Again, this is implying that the body is closely connected to the material world but in this case, language is the precursor in connecting both. Drawing reference to the many interpretations of cognition, Radford et al. highlight the cognitive linguistic approach, the materiality of cognition and a more phenomenological approach as three possible ways of thinking about mathematics. Despite these varying interpretations, they remind us that, in these variations, there is a clear understanding that "meaning and cognition are deeply rooted in physical, embodied existence" (p. 701). Consequently, it is within this connection that they suggest mathematical meaning-making is initiated and thoughts become relevant to action, emotion, and perception.

To make a more salient point, they examined how children process knowledge through various epistemological lenses. First, they did so from the rationalist epistemology, where knowledge is perceived as emanating only through intellectual means and second, from the empirical epistemology, which positions knowledge as originating through sensuous experiences. With this in mind, they highlighted the Kantian epistemology as a combined perspective of both the rationalist and the empirical epistemology. In Kant's (1781) perspective, knowledge is obtained by both intellectual and sensuous means. In this regard, it does not mean that the intellect and the senses operate individualistically, but rather in shared ways. In the Kantian epistemological approach, the senses in and of itself are not perceived as formulating concepts, but rather help to evoke or trigger intuitions about the concept to be learned. This sensual–intellectual-concept relationship has some implications in the way in which I view the role of curiosity in mathematical meaning-making. In fact, I position curiosity as the element that triggers the senses to act in certain ways, thus, a possible relationship would be curiosity–sensual–intellectual–mathematical meaning.

2.2.2. Perceptuomotor Integration in Mathematics Learning

Nemirovsky, Kelton and Rhodehamel (2013) investigated the development of perceptuomotor integration (an intertwining of the perceptual and motor aspects of using a tool[6]) and its role in mathematics teaching and learning. They argue that mathematics learning occurs similarly to the way one learns to play a musical instrument. In order to illustrate this, they assumed that mathematical thinking is a product of bodily activity, and used two case studies to show that embodiment manifests itself when a learner engages with a tool. They suggest that, "the development of tool fluency entails the interpenetration of the perceptual and motor aspects of activity, allowing the performer to act…" (p. 373). This implies that, when children develop confidence in engaging with a tool, they automatically act in a holistic way, with reliance on interpretations through the senses, which lead to some form of performance by the human muscle.

The results from the case studies show that mathematics learning takes place through physical manipulations, eye gazes, gestures and speech, especially when these occur alongside each other. In addition, the research shows that the learners in their study were able to visualize and produce mathematical ideas about things which were not previously introduced, based on previous perceptuomotor integration. The implication of these findings to mathematics teaching and learning, is that consideration should be given to the multiple modes of learning and the non-polarizing ways of using tools in the mathematics classroom. That is, emphasis should be placed on the role of the perceptual and motor relationship in the teaching and learning of mathematics.

[6] For the purpose of my study, I take 'tool' to mean digital technology tools.

2.2.3. Sensorimotor Schemes and Mathematics Learning

Luis Radford (2013) examines the human subject through the lens of embodiment and takes into consideration the relationship among cognition, the body, sense, sensation and matter. He argues that cognition is conceived as conceptual, embodied and material all in one. Because of this, he suggests that the cognitive domain can only be understood from "a culturally and historically constituted sentient form of creatively responding, acting, feeling, transforming and making sense of the world" (p. 145). This view positions human cognition (similar to the body), in a relationship with one's environment. He coins the term 'sensuous cognition' to refer to a non-dualistic view of the mind, where cognition is seen as a feature of living material bodies characterized by a capacity for responsive sensation and the mind is seen as a property of matter. In making this claim, Radford presents sensation as "a phylogenetically evolved feature of living organisms through which they respond to, reflect or act on their environment" (p. 144). In other words, he sees sensation as something evolving from a broader group of organisms and that the environment plays an important role in how these sensations evolve.

Furthermore, Abrahamson and Bakker (2016), in an embodied design for mathematics learning, suggest that there is a relationship between physical actions and conceptual learning. They argue that this is made possible by the role of sensorimotor activity in human learning. Drawing on Varela et al.'s (1991) view that, "cognitive structures emerge from the recurrent sensorimotor patterns that enable action to be perceptually guided" (p. 173), Abrahamson and Bakker situate their ideas in a classification of two types of movement: the proximal movement, which involves the physical movement itself, and distal movement, which reflects a technological extension of the physical movement. They further suggest that this classification involves three types of character: (1) distal movement as the effect of physical movement on the environment mediated by technological instruments; (2) proximal movement as the physical movements that handle the instruments; (3) sensorimotor schemes that organize the performance of the task. This means that sensorimotor activities play an integral role in how children move their bodies and that the technology acts as a mediator agent between activities of the senses and body movement.

In this section on Embodiment, I found usefulness in Lakoff and Núñez's idea that mathematics is the product of human imagination and that it can be understood and generated from sensorimotor experiences (in this case of my research, through sight). Sinclair and Schiralli, suggestion about conceptual and ideational mathematics also resonate with me as this distinction provides deeper understanding of the types of mathematical meanings that can be generated during classroom interactions. Here I contemplate whether or not both conceptual and ideational can be generated simultaneously. For the interest of my research Lakoff and Núñez's perspective on embodiment is of particular interest to me, as well as, Abrahams and Bakkar idea that there is a relationship between physical actions and the development of meanings. While I do agree that sensorimotor activities play an important role in body movement and that the technology is instrumental in mediating the process. The question here, however, is: In what way does the technology mediate this process?

2.3. Digital Technology

'Digital technology' is an umbrella term used to describe a variety of computer-based programs. These include, but are not limited to, web-based applets, touchscreen applications, interactive whiteboards, interactive games and computer-based software programs. Sarah Howard and Adrain Mozejko (2015) gave a historical account of the landscape of digital technology in education in three "ages" – the pre-digital, the digital and the connected digital. Their reason for this distinction was to provide an understanding of the social trends and changes in beliefs about computer-aided learning in education generally, over the past decades.

Although I find Howard and Mozejko's historical landscape useful, my primary concern is not with the social trends or changes in beliefs about learning with technology, but rather with how digital tools, their affordances and capabilities have improved over time. Furthermore, despite the general way in which these scholars wrote about the development of digital technology in regard to education, my specific interest is on the development of digital technology as it relates to mathematics education. As a result, I have mirrored the three ages of digital technology with mathematics education research during each period, showing how the advancement of digital technology influences research in mathematics education, and vice versa.

2.3.1. A Historical Overview

In regard to Howard and Mozejko's three ages of digital technology, they explained that in **the pre-digital** age, which saw the introduction of film, radio and television between the late 1890s and the mid-1950s in schools was nothing but a "hype" (p. 2), because although many perceived those technologies as ground-breaking in education, there was a tension between the potential of the machines in schools and the educational policies of many education systems. During the mid 20th century, however, a need to expand school space and provide greater educational opportunities with technology became a priority in many countries. As a result, a greater demand on the use of film, radio and television became substantial as educators sought ways to provide opportunities for more students to gain access to knowledge. Despite this demand, integration of these digital technology tools into school curricula was slow due to affordability, accessibility and logistical challenges to organize them in the physical classroom space.

Mathematics was also playing an integral role in the development of digital technology after the pre-digital age. By 1966, a team of educators – Seymour Papert, Daniel Bobrow and Wallace Feurzeig – began exploring ways to introduce children to computers and, as a result, LOGO[7] was born. Further, the publication of Papert's *Mindstorms: Children, computers and powerful ideas*, in 1980, which was a direct response to research done with young children using LOGO, helped to sensitize the connection between digital technology and the learning of mathematics. In addition, the interest in personal computers was heightened due to the sensitization of LOGO at the time. Powell (2017), in an account of Papert's legacy to education, argues that during this period of "intense interest in personal computers […] Papert saw LOGO and the turtle as an opportunity for children to engage in novel interactions with the computer" (p. 153). For Papert, this was a way to enable children to unlock mathematical ideas that were seen as too challenging for them. With the development of LOGO came the interest of computer manufacturers; Apple II was one of the few to capitalize quickly on the potential of the programming language, which brought about greater need for accessibility and affordability of desktop computers.

[7] LOGO is a programming language developed by Seymour Papert, Wally Feurzeig and Cynthia Solomon at Massachusetts Institute of Technology (MIT). This programming language was used to control the movements of a turtle on the screen.

In **the digital** age, the introduction of desktop computers materialized between the 1970s and 1980s and, like the pre-digital age, there was an expectation that desktop computers would bring about an upheaval. Similar to the role of pre-digital technologies, the focus of digital technology in education was on effectiveness, as well as, to provide opportunities for a wider audience of students, but digital technology also provided accountability and productivity, and the drive to measure learning outcomes became more evident. In addition, more money was spent on equipping schools with computers, because of the obvious economic benefit of having tech-educated students. In this way, more children were able to obtain desktop computers and this gave rise to the need for adaptation of a student-centred approach to teaching. But how did this influence change in mathematics teaching and learning?

Despite the increase of desktop computers and a more student-centered approach to teaching, mathematics education was slow to produce research on and theorizing about what the computers could offer students and teachers. Michael Thomas (1996) examined the landscape of computers in the mathematics classroom and found that while computer availability increased in schools, accessing them was a challenge. This, he surmised, was due to the teachers' attitude towards the use of the technology. Cuban, Kirkpatrick & Peck (2001) offered a possible paradox about the correlation between technology accessibility and how they were used. Although their research highlighted classrooms in general, I believe that similar issues were evident in the mathematics classroom as well. They suggested that while the technology improved in accessibility, there was minimal use in the classroom. They further reported that despite ease of access most teachers were occasional or nonusers and that when the technology was in use, usually the teaching practices remained teacher-centred.

According to Michèle Artigue (2013), mathematics education played an important role in the advancement of digital technology towards the latter part of this era. She provided an account for the teaching and learning of mathematics during the 1980s where her focus was on the challenges and perspectives surrounding the proliferation of digital technology tools. She approached this discussion from two perspectives: her experience in the field of mathematics education and from a historical review of technology in mathematics education. According to Artigue, the relationship between digital technology and mathematics education historically arose out of the field of computer science due to the role mathematicians played in shaping the emergence of computer science as a discipline. Again, this is an illustration of the long-standing link between mathematics and technology as reported in the pre-digital era.

This relationship became more noticeable in the production of the first International Commission on Mathematical Instruction (ICMI) study published in 1985 with the title, “The influence of computers and informatics on mathematics and its teaching”. Artigue claimed that, when the 1st ICMI study was introduced, there was already a perception that the evolution of digital technology would be meaningful to the teaching and learning of mathematics. This perception came out of LOGO’s influence and a need for better ways to teach mathematics. Furthermore, she pointed out that, at the time of the introduction of the 1st ICMI study, the most dominant technologies which were featured in the study were scientific calculators and mathematics computer software which were designed for use with desktop computers. She further lamented that only a few contributions investigated the affordances of these computers.

Moreover, while the contributions of the 1st ICMI study focused on the specific digital technology tools, which were available at the time, there was also an interest on three general themes in mathematics education. The themes included the effects of computers and informatics on mathematical ideas, the impact of computers and computer science on the mathematics curriculum, as well as, computers as an aid for teaching and learning mathematics. While the introduction of the 1st ICMI study mirrored the relationship between mathematics education and digital technology during the digital age, there were also implications of the significant influence of digital technologies on the way mathematics was taught and learned.

The beginning of **the connected digital** age in the 1990s saw the appearance of the internet, where information and knowledge were not only connecting people through information sharing (emails, online chatting, search engines, …), but also connecting sources of information through the use of hyperlinks and universal resource locators (URLs). By the early 2000s, the internet became dynamic, and for more than two decades since the inception of the first ICMI study, the 17th ICMI study came to fruition with the title, “Mathematics education and technology – Rethinking the terrain”. This was published in 2010, with a specific focus on the diversity of the available hardware (the physical component of the computer) and software (the programs and operating systems of the computer) to mathematics education. The primary focus was on the role they played in the development of mathematics curricula. At the heart of this diversity was the rapid generation and utilization of digital tools in understanding ways children make meaning of mathematical ideas.

According to Hoyles and Lagrange (2010), “digital technologies were becoming even more ubiquitous and their influence touching most if not all education systems” (p. 2). They argued that the mathematics education research community came into a frenzy and an increased number of studies began to present findings around the use and impact of these hardware and software tools in the mathematics classroom. In the past decade, the main shift in focus has been on teaching and the teacher’s role in integrating digital technology into the mathematics classroom. This interest takes into consideration epistemological, pedagogical and theoretical perspectives on the contribution of digital technologies in mathematics education.

Drijvers (2013), through a historical landscape of research studies in mathematics education, examined the influence of digital technology on mathematics education. The specific focus was on whether or not integrating them in the mathematics classroom would be meaningful. In doing so, an analysis of six cases taken from mathematics education research was carried out. The findings revealed that some crucial factors contributing to effective integration of digital technology were design of the digital tools and design of the corresponding task which would support the pedagogical potential of the tools. Although all six cases were relevant to the influence of digital technology on mathematics education, I found that Drijvers’ findings of Cases 1 and 2 were pertinent to my research interest. Hence, I present a brief description of these two, in order to illustrate where my interest lies.

Case 1 was taken from research conducted by Kathleen Heid (1988) towards the end of the digital age. In this research, a concept-first approach was used in resequencing a calculus course for university students. In this approach, the digital technology (graphical and symbolic-manipulation computer software) was introduced as a tool for routine manipulation of calculus concepts to college students. The findings suggest that the students who received treatment (using digital technology) outperformed those who did not. Drijvers found this research useful not only because it demonstrated that the use of digital technology enhances mathematics performance, but also most importantly, that using the digital technology for pedagogical purposes enriched mathematical concept development. This provides evidence that digital technology is connected to the development of mathematical meanings – a connection to which my research is dependent upon.

In Case 2, Drijvers drew on research conducted by Doerr and Zangor (2000), with a focus on pre-calculus students use of the hand-held graphing calculator. The purpose for integrating this particular digital technology tool was to understand better the concept of function. Similar to Case 1, this research focused on concept development, but was more concerned with the development of “students’ curiosity and motivation…” (Drijvers, 2013, p. 5). Results from this study showed that teachers played an important role in establishing and reinforcing different use of the digital tool. In other words, Drijvers’ use of this case in particular implies that there is a relationship with curiosity and motivation, as Loewenstein suggested and was reported in a previous section. I found this case interesting because the findings suggest that teachers are important in orchestrating the use of the technology. However, although the research focus was on concept development and concerned with students’ curiosity, there was no indication, in the results, that mathematics concept development influenced curiosity or that curiosity played any role in how mathematical concepts were developed. Instead, the focus shifted to the teacher’s role in the interaction and the digital tool emerged as the subject of study. During this technology era, the focus of research has been primarily on what the technology can offer students. This case became important for my phenomenon of interest because it exposed a potential gap in mathematics education research – not only what the technology can offer students, but also what the students can offer the technology.

2.4. Where Are We Today?

Today, the integration of digital technologies still confronts those who are directly involved in mathematics education, because the never-ending changes have pushed mathematics teaching and learning into another realm. Now there is a constant need for the ‘doing’ of mathematics and for children to collaborate and engage in mathematics in ways they have never been able to do before. Currently, the focus is on what deeper thinking can take place when there is an interaction with children and digital technology. The response to such an issue lies within pedagogical considerations and rests on theoretical choices about what it means to learn mathematics. Noteworthy as well is the consideration of

DGEs in providing opportunities for collaboration and new ways of thinking about mathematics. In light of this, my research seeks to capitalize mainly on the theoretical consideration of how learning takes place in a collaborative way – without overlooking any opportunity to examine pedagogical implications or the affordances of the technology. My major focus is not solely on collaboration with humans (teachers and peers), but also on collaborations in a dynamic environment where the person, the material and the mathematics all contribute to the learning process. But first, there must be a clear understanding of what dynamic geometry environments offer mathematics.

According to Goldenberg and Cuoco (1998), the first account of the term "dynamic geometry" was originally coined by Nick Jackiw and Steve Rasmussen meaning motion and change. Olive and Makar (2010) defined the term dynamic geometry environment as, "any technological medium (both hand-held and desktop computing devices) which provides the user with tools for creating the basic elements of Euclidean geometry" (p. 147). The rapid advancement of digital technology software, however, gave rise to other forms of geometric interactivity. Soon after *Cabri* and *Sketchpad,* which were both aimed at secondary school geometry, in the late 1980s and early 1990s, there was an influx of other similar dynamic geometry software materials available across the K–12 mathematics curriculum. This includes algebra, calculus and 3D geometry.

In describing the effects of interaction in a DGE, Jackiw (2006) claims that the central part of any dynamic geometry experience rested on "idealized and friction-free mathematical physics" (p. 146). This means that manipulation of an object does not experience resistance when one aspect moves relative to another and as a result different objects are formed. In Jackiw's estimation, this feature allows for varying representation of a constructed figure and the figure is a new mathematical object which maps a "temporal past of specification and definition onto a present graphical configuration and onto a future potential for manipulation" (p. 146). In other words, a manipulated object goes through a sequence of events that produces something new. Aside from aesthetic, visual and symbolic capabilities, one of the most essential details about exploration in DGEs is the opportunity they provide for investigation of the dynamic actions of a construction by moving it (simply by dragging).

In a research analysing students' dragging practices when they engage in activities with a DGE, Arzarello, Olivero, Paola, & Robutti (2002) show how the affordances of the digital technology can mediate relationships within an interaction. In this research, they claim that students' dragging practices occur in a hierarchy of functions and that these functions have cognitive features which are further categorized into two modalities ascending (explorations which seek invariancy) and descending (validating and refuting conjectures). When students transition from one modality to the next, the opportunity arises for them to make meaningful conjectures. The following is a list of the dragging modalities proposed by Arzarello et al.:

[1] Wandering Dragging: aimlessly moving the points on the screen.
[2] Bound Dragging: moving a point that is linked to an object.
[3] Guided Dragging: dragging the basic points to give it a particular shape.
[4] Dummy Locus Dragging: dragging a point to maintain a previously discovered property
[5] Line Dragging: drawing new points to keep the shape of the figure.
[6] Linked Dragging: connecting a point to an object and attaching it to the object.
[7] Dragging test: moving draggable points to test if the shape main its properties.

These modalities provide greater insights not only into the constructions per se, but also into geometry more broadly. They became useful to my research in that *Sketchpad,* like Cabri, does offer these dragging capabilities. In my research, dragging allows a user to move their mouse in ways that trigger feelings of excitement and indicate emerging curiosity. This is of interest to me in relation to my construct of Embodied Curiosity.

Designing DGEs with capabilities such as dragging fosters a connection with the students and the technology which means that the designs of the software should be considered. Jones, Mackrell and Stevenson (2010) found that, in designing dynamic software environments, certain decisions must be taken by the developer. They examined how the use of DGEs shapes mathematical ideas of children with a focus on two geometries, namely, Euclidean Geometry and Geometry of Co-ordinate Systems. A further exploration of the two geometries within a two-dimensional and three-dimensional space was done. This research not only epitomized the complexity of digital technology in mathematics, but also highlighted that the design of the DGE should take into consideration the relationship between these two geometries. The relationship between the two geometric spaces (two-dimensional and three-dimensional) should also be taken into consideration. This research highlights the possible for a relationship between digital technology and mathematics – a connection in which I have an interest for my research.

In addition, Jones et al.'s research underscores how digital technologies have evolved to take on a more multifaceted role in mathematics education: that is, to expose connections within geometric spaces. This has implications for the need of deeper understanding into the role of DGEs in mathematics learning. In addition, there is implication for the role of software developers in designing programmes that will enable students to explore and identify the connected nature of mathematical concepts. In the case of *Sketchpad* (my specific tool of interest), Jackiw (2006) expressed that, "I am fascinated by dynamic geometry by the nature of its appeal […] and I am intrigued by the way in which variations of *The Geometer's Sketchpad* arose semi-independently" (p. 1471). It cannot be ignored then that, quintessentially, the background from which dynamic geometry software (at least for *Sketchpad*) is designed, should be through a tripartite relationship, developer–mathematics–technology. In this sense, the developer's 'fascination' and source of intrigue (indication of curiosity) may inspire the capabilities of the digital tool, which could further influence how children generate and explore mathematical meanings.

2.5. The Geometer's Sketchpad

The Geometer's Sketchpad[8] is a computer software program which combines technology and mathematics and is one of the first DGEs to revolutionize the teaching of geometry in schools. Its history can be traced back to the late 1980s in a research project[9] at Swarthmore College in the United States of America. "*Sketchpad* was originally conceived simply as a program to draw accurate, static figures from Euclidean geometry" (Scher, 2000, p. 44), but it too has undergone tremendous changes over the decades. After its first launch in 1988 (the latter part of the digital age) from an NSF project, *Sketchpad* experienced a transformation during the early stage of the connected age. In 1995 it included analytic geometry. In 2001, the software's potential was extensively upgraded to include learning of algebra and calculus and later, *Sketchpad* became capable of creating, exploring and analyzing a wide range of mathematical concepts in algebra, geometry and calculus among other areas (Bakar, Tarmizi, Ayub & Yunus, 2009).

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[8] The Geometer's Sketchpad was developed and distributed by Nicholas Jackiw in 1998.

9 The US National Science Foundation (NSF) research project under the direction of Dr. Eugene Klotz and Dr. Doris Schattschneider at Swarthmore College.

Sketchpad is designed to encourage students to discover new ideas through constructions and explorations. The range of explorations which *Sketchpad* makes possible varies from simple to highly complex. Because of its dynamism, construction and exploration allow students to compare examples and non-examples on the same screen at the same time. With this affordance students are able to identify patterns, make conjectures and come up with proofs at a quicker rate than they would in static environments. Through manipulating and observing the ways in which geometrical shapes are related, students develop their own meanings for mathematical concepts and gain better insights about how they are related. *Sketchpad* also enables students to visualize and analyse geometrical shapes in a three-dimensional manner and in recent times there have been numerous research studies on the ways Sketchpad has influenced mathematics education.

In a research study conducted with young children (Grades 1 to 2), Ng and Sinclair (2015) used *Sketchpad* to negotiate ways in which children reason about reflective symmetry. In this instance, they used two of the theories suggested by Radford (semiotic and cognitive linguistic), described in a previous section in this chapter (sensorimotor and mathematics learning), to examine how children's thinking about reflective symmetry changes from engagement in a static (paper-and-pencil) environment to that of a dynamic one (*Sketchpad*). The children were asked to use ready-made constructions in *Sketchpad* to examine properties of reflective symmetry. Afterwards, they were asked to make their own symmetric designs. They were able to differentiate between movement of symmetrical and asymmetrical shapes through the use of words, gestures and diagrams. Evidence from this research also revealed that the ways in which children spoke about reflective symmetry in a static environment changed when they used *Sketchpad*. A fundamental finding of this research suggested that *Sketchpad* has more to offer than developing new mathematical ideas through exploration and manipulation, but also provides opportunities for the development of new bodily movement (in the form of gestures). This speaks directly to the embodied nature of mathematics and, specifically to the role *Sketchpad* plays in this embodied process. This understanding resonated with my research interest and hence played an important role in the methodological decisions pertaining to the software choice.

The decision to use *Sketchpad* for this research is multifaceted. Apart from my personal interest in its capabilities, I was aware that it was not widely used in Jamaica and in the wider Caribbean region due to accessibility and affordability. As a result, findings informed by data collected in the region could align children with equal opportunities for teaching and learning not only in geometry but mathematics in general. In addition, based on the existing literature, there is no evidence of research connecting *Sketchpad* to curiosity, despite Jackiw's (2010) claim that, "a key ingredient of the *Sketchpad* experience is to motivate mathematical curiosity about shapes or spatial relationships" (p. 145). In addition, Sketchpad played an integral role in some of the methodological decisions which I have described in Chapter 4

2.6. Summary

In summary, curiosity and embodiment have both experienced a problematic past in terms of securing a proper definition. The terms have been a subject of debate for decades ('curiosity' to a lesser extent) and are now gaining prominence in mathematics education research. In this chapter, I discussed the development of curiosity from a philosophical and psychological perspective. On the one hand, curiosity is interpreted as something good and on the other as a bad habit. It also emerges as a motive with intrinsic abilities while others imply that there are some external benefits to curiosity. My research, is aligned with Loewenstein's information-gap definition which positions curiosity as something triggered when there is a disequilibrium in one's knowledge and understanding of the world. This definition positions curiosity (an intrinsic motive) with connections to the physical environment.

In terms of embodiment, the discussion in this chapter touches on how embodiment is utilized in mathematics education as a way of making sense about the construction of knowledge. I position my research on the idea that the brain, the body and bodily experiences through perceptual and sensory means all contribute to the way we make sense of the world.

I have provided a historical trajectory of both curiosity and digital technology. This approach indicates that the advancement in technology is as a result of our constant need for knowledge which links both curiosity and technology as driving forces for knowledge. In addition, I provided a description of *The Geometer's Sketchpad* (the DGE of choice) and highlighted how its functionalities create opportunities for students to explore geometric concepts, but, most importantly, I provided possibilities for a relationship between curiosity and technology.

Chapter 3. Theoretical Consideration

> *"Curiosity takes ignorance seriously and is confident enough to admit when it does not know. It is aware of not knowing and it sets out to do something about it." (Alain de Botton, 2013)*

Alain de Botton (2013), in the quotation above, captures the essence of curiosity as the need to fill an information gap, but also implies the very core of this dissertation – setting out to do something about my past wonderings. In this chapter, I present the theoretical consideration for my research – Embodied Curiosity – which was introduced in Chapter 1. Here, I provide a more detailed account of its origins, assumptions and how it is used in my research to analyse children's construction of mathematical meanings. An even more detailed account of the theory is given in Chapter 6 (my data analysis chapter), with an accompanying diagram. I embark on this approach because the data was instrumental in the way the theory was developed. This chapter is by far my most challenging because the constructs that I work with (curiosity, embodiment, and agency) are all difficult to conceptualize individually, at least for me, let alone in coordination. Nevertheless, I acknowledge that the theoretical framework may not be as sound as I would like it to be. However, I see opportunities for it to evolve beyond this dissertation.

Researchers and educators have been theorizing about mathematics education for as long as mathematics education has been considered within the scientific field (Drijvers, Kieran, Mariotti, Ainley, Andresen, Chan & Meagher, 2010). Bharath Sriraman and Lyn English (2010) presents a survey of theories and philosophies in the field of mathematics education, which speaks to the need for theory development to be regulated. This they suggest will improve research and practice in the field. As advancement in technology continues to grow at a rapid pace, the teaching and learning of mathematics has become more complex. In order to address this complexity, the need for robust research to explain how things work, and the ways in which the field of mathematics education can be broadened, becomes important, not only to mathematics education researchers, but to the mathematics teachers as well.

To this end, Jeremy Kilpatrick (2010) argues that theories of mathematics education should provide clarity in their ontology, methodologies and epistemology. Implicitly suggesting that the field is not yet synchronized in terms of theoretical development. Drijvers et al. (2010) also claimed that, "the body of theoretical knowledge is still growing

[and] now that the issue of integrating technological tools into the teaching and learning of mathematics has become rapid and urgent, one can wonder what the existing theoretical perspectives have to offer” (p. 3).

Acknowledging the gap in theoretical frameworks surrounding technology integration in mathematics, I set out firstly to understand the influence of the affective and embodiment on mathematics. In doing so, I placed emphasis on the role of the digital technology in this relationship. In this chapter, I present a theoretical framework called Embodied Curiosity, which is grounded in embodied cognition. My aim is to address how mathematical meaning is generated when students’ trait-curiosity is triggered by a digital technology tool and the role the body plays in mathematical meaning-making. In developing this framework, I examine embodied cognition from a cognitive neuroscience perspective to locate its relevance to mathematics education.

In addition, I draw on Andrew Pickering’s (1995) assumptions of human and non-human agency and discuss how his views shift the perspective of knowledge generation as being solely a function of the mind. In his theory, knowledge is generated across human, material and the disciplinary agencies. I used his concept of “The Mangle” to express the tripartite relationship among students and their trait-curiosity, digital technology and mathematics[10]. Towards the latter part of the chapter, I present some basic assumptions about the framework to illustrate how it might be of benefit to mathematics education. Within this framework, trait-curiosity manifest itself through the wondering questions and students’ manifestations of surprise, which indicate the presence of doubt or uncertainty.

3.1. Embodied Cognition

In trying to understand the philosophy behind the learning and doing of mathematics, researchers have sought answers from cognitive neuroscience. According to Lawrence Shapiro (2007), the traditional view in cognitive science is that thinking is a process of symbol manipulation. In his account of the traditional view of cognition, he likened the human brain to that of a computer processing unit (CPU) and claims that, “cognition is computation and that minds are programs that run on brain hardware” (p. 339). In my view, this perception positions cognition as a software program (usually the instructions used to

[10] In using Pickering’s view, I have adopted mathematics as a scientific discipline.

operate a computer) and the brain as a hardware device (usually the tangible components of the computer). Since in computer science, software and hardware play two different roles, Shapiro's idea subtly suggests that cognition begins with input and output of information; a continuous process which takes place in the brain.

Margaret Wilson (2002) presents six views of embodied cognition and states that "traditionally, the various branches of cognitive science have viewed the mind as an abstract information processor, whose connections to the outside world were of little theoretical importance" (p. 625). Her claim is that, with views similar to this, there was little or no emphasis on the role or contribution of the psychomotor functions (movement associated with mental activity) on learning. She argues that embodied cognition shows diversity in its claims but one central view that stands out is that "cognitive processes are deeply rooted in the body's interaction with the world" (p. 625). This includes a view that cognition is situated, meaning that cognition takes place in the context of real-life experience. There is also the view that cognition is time-pressured, which Wilson refers to as an understanding of cognition in terms of how it functions under pressure with real-time interaction with the environment. She further explains that cognition is recognised as a source of action, in the sense that the purpose of the mind is to guide active movements. Yet the most understudied claim of embodied cognition is that, even when the mind is not engaged with the environment, learning takes place through sensorimotor processes.

According to Wilson, there was a radical shift which placed more emphasis on sensorimotor and perceptuomotor modes of learning, and speaks to "a growing commitment to the idea that the mind must be understood in the context of its relationship to a physical body that interacts with the world" (p. 625). Despite the success in support for this view of embodied cognition, there is still a longstanding presence of dualism in human philosophy attributed to René Descartes during the seventeenth century. This dualism positions the mind and the body as two distinctly different things which is implied in Shapiro's account in the previous paragraph. According to Descartes, the mind is non-extended and the body is extended and that makes it impossible for one to exist with the other. However, this dualism has met opposition, since many cognitive scientists do not believe that mind and body are so distinct. For example, Andy Clark and (1998) claims that, "Biological brains are first and foremost the control systems for biological bodies. Biological bodies move and act in rich real-world surroundings" (p. 506). This was later reiterated by de Freitas and Sinclair (2014), who claimed that "learning involves the body, the brain, after all, is a part of the human body" (p. 16), implying that the Cartesian dichotomous view about the process of thinking is false.

With these contrasting oppositions about embodied cognition, advocates within the cognitive science discipline such as George Lakoff and Rafael Núñez (1997/2000), conceptualize embodied cognition in mathematics learning. They suggest that the human body must be given a fundamental role in shaping the mind and earlier Lakoff and Johnson (1980) present as an essential view, that human beings made use of metaphor in their conception of the world and that abstract concepts are based on metaphors for bodily physical concepts. They also propose that there are some basic concepts that human beings can understand without relying on metaphorical reasoning such as up, down, front, back, in, out, near and far which are more aligned to the functions of one's body.

Drawing on Lakoff and Núñez's (1997/2000) view that mathematical ideas are shaped by our everyday experience and that our sensorimotor system of the brain and body plays an important role in how knowledge is structured, I positioned curiosity as an element that helps us to seek information from our environment. With this in mind, I see curiosity as essential to the meaning-making process and the body plays an important role in how meanings are distributed. This means that curiosity is at the heart of the relationship between an individual and the environment. It acts as the agent which drives individuals to explore their surroundings. In so doing, curiosity triggers the senses (eyes, ears, hand…) to react and move in certain ways. It is difficult to distinguish between when body movement is triggered by curiosity or when it is not. This is because curiosity cannot be observed with the naked eye. However, students' wondering (out loud) can be an indicator that curiosity is present or getting ready to emerge, which comes in the form of 'what ifs' and 'suppose' questions and can be detected when there is a moment of surprise. Due to the challenge of quantifying curiosity, I see the need to reimagine curiosity (trait-curiosity as we know it) with a focus on the relationship with the material world. This relationship is made possible by the ability of both the human and non-human to act freely and creatively through an idea I now understand as agency.

START

3.2. Agency

Candia Morgan (2016) writes about the two dichotomous positions of the origins of mathematics, which she suggests have become the centre of controversy in mathematics education. On the one hand, there is the formalist perspective, that positions mathematics conception out of axiomatic and logical deductions, while on the other, there is the perception that the foundation of mathematics rests on problem posing and human intuition. Out of these contrasting views, Morgan argues that this form of discussion, "proposes mathematical objects themselves and the relationships between them as the agents in the generation of new knowledge" (p. 123). Similar to the debate about the origins of embodiment and that of curiosity, there is also a debate about human agency and its role in the development of mathematical knowledge. Indeed, there is advocacy for human agency in the relationship between students and mathematics teachers (Herbel-Eisenmann & Wagner, 2007), but there is a more important aspect to this. Morgan (2016) claims that the discussion should also be around "how students relate to mathematics as a discipline and, in particular, whether they are able to see it as a human practice" (p. 123). Researchers have explored agency as an ontological approach to mathematics knowledge (Boaler, 2002; Barad, 2007; Ingold, 2011). However, in this research, my focus is on Pickering's (1995) interpretation of agency because of the role each of his case studies plays in providing a deeper understanding of the relationship between the human (the students) and the non-human (digital technology).

3.2.1. Pickering's Theory of the Human and Non-human

Andrew Pickering (1995) presents a model of human and non-human agency that seeks to give an overarching account for the emergence of science and technology. Pickering finds himself in a predicament because he felt that humans were presumed to have too much autonomy in and authority over the world. In response to this problem, he takes into consideration the relationship between human and the environment. In doing so, he considers science as a performative practice where the focus is on "doing" rather than acquiring, which is aligned with the participationist views of learning. His central tenet is an ontological view that regards thinking and agency as central to performance (Pickering, 2017). In his view, humans are not the only ones with agency in scientific activity; non-human players such as machines and the discipline are accorded agency as well. As a result, he proposes that agency has to do with the influence that one thing has on another. Pickering (1995) provides a decisive analysis of scientific practice via four case studies: Glaser and the bubble chamber, Morpurgo and the search for quarks, Hamilton and the quaternions and the computerized machine tools at a General Electric (GE) factory. He uses these case studies to show the interplay between human and non-human agencies. In my analysis of Pickering's work, I take the field of mathematics as a scientific practice and, hence, apply Pickering's ideas to mathematics activity.

3.2.2. The Mangle of Practice

Pickering launches his theory – The Mangle – with an emphasis on performance rather than cognition. In this way, his focus is on what humans can do, what the material does and how these performances intertwine or 'mangle' with each other. He proposes that, in people's desire to understand the world around them, they are led to do certain things, thus encountering resistance from various sources, including material objects. Resistance usually hinders the smooth running or data collection of a process. In order for humans to accomplish the task, they must first make accommodations to overcome or circumvent these resistances. This interplay among the human, the scientific discipline (in this case, mathematics) and the non-human is what Pickering called "the dance of agency".

The most important claim that Pickering has made is that, the mangling process encapsulates two things. One, it is "temporally emergent", which means the forms of human and non-human agencies are not known in advance, they emerge during scientific practice, and two, that the mangling process is "posthumanist"[11], meaning that within the process, human agency is not given precedence over any other form of agency (material or disciplinary). However, emphasis is on the intertwining relationship between them; they are equally productive to each other.

Using the case study with Donald Glaser's building of the bubble chamber (a radiation detector), Pickering demonstrates that an individual will face challenges when interacting with a machine and that the outcome may be different from the one intended in the first place. As a result, the human will need to re-configure the process, see what the machine can do after re-configuring and then react to that again. This repetitive back-and-forth

[11] See Sean Chorney (2014), From agency to narrative: Tools in mathematics learning for further details.

between the human and the non-human results in a functional apparatus. In this example, Pickering describes the imbricated relationship between people and things.

In the case of Hamilton's construction quaternions (a number system which extends the complex numbers), Pickering details how concepts are developed when actions are taken as a result of following standard ways of doing things. In the discipline of mathematics, one has to follow the structural rules in order to achieve results. In making this claim, he shows that Hamilton extended the rules of algebra and geometry into three-dimensional space, which led to the creation of the quaternions. The structural nature of mathematics and the need to maintain this structure in order to develop a 'new' concept is what Pickering refers to as disciplinary agency.

In his account of Hamilton's work, the focus is on the development of conceptual theory through the mangling of the human with the rules of mathematics (the discipline), hence, there is 'technically' no relationship identified with material agency. I say technically because the human can also be seen as material and hence possess material agency. However, in my research, I treat both things (human and material) as separate entities. I find the decentralization of material agency in Pickering's analysis of the development of the quaternions intriguing because I believe that while the human and the discipline were caught up in web of agency, the environment also played a role in how mathematics evolved – in Hamilton's case, the pencil-and-paper forms of inscription, for example. Similarly, by extending Pickering's research to the context of learning mathematics, I assert that the environment will also play a role. This is particularly true for geometry because of the its connection to physical shapes and actions. In this sense, I see agency playing a significant role not solely in the mangling of mathematics and the human, but also with the material as well. This mangling plays an important role in my conceptualization of Embodied Curiosity in that, curiosity is what links the human to the material, which makes curiosity an agent in this relationship.

In the case of technology, Pickering shifts his attention to the "mangling of the social" (p.157), in order to shed light on social performances and relationships. He draws on David Noble's study of numerically controlled machines to project the argument that machines are built to perform specific tasks. However, machines cannot perform the task on its own and therefore need some form of collaboration with "skilled operators to channel their agency in desired directions" (p. 158). This perspective does not negate Pickering's move to decentre humans; what it suggests, however, is that there is a human-machine pairing that exist and it is through this pairing that social relationships are built.

3.2.3. Agency and Curiosity

Pickering's process of the mangle is important to my research in multiple ways. Firstly, I argue that there is no telling what will emerge when a learner engages with a mathematical task using *Sketchpad*, which speaks to the temporal nature of agency. The unpredictable nature of the interactions provides authentic learning experiences, but most importantly, provides opportunities for curiosity to be triggered. This has potential not solely to respond to research question two about the role of Embodied Curiosity in constructing mathematical meanings, but also to illustrate that the element of surprise is a signal for the inception of meaning-finding. By this, I mean, since the actions are not known prior to engagement, students will demonstrate surprise when something unfamiliar occurs. Secondly, the posthumanist nature of the mangle has implications for the most salient aspect of my research: that is, mathematics learning should not be solely dependent upon the nature of mathematics (attributes of the discipline) or cognitive development. It should also take into consideration the affective nature of the learner (attributes of people) and the role of the digital technology (and its affordances). The technology tool should be seen as more than a tool to aid learning, but also as a tool of learning. Furthermore, the posthumanist position of the mangle provides an opportunity to examine ways in which equal importance is given to the role of curiosity and the potential of the technology tool.

Using the bubble chamber case study as an analogy, I argue that the students and *Sketchpad* are in a constant back-and-forth arrangement with each other and that the body is the vessel through which this arrangement is made possible. In the quaternions case study, I find resonance in Pickering's consideration of the structured way in which science is developed. In the discipline of mathematics, and the use of *Sketchpad* itself, certain structural rules must be followed in order for results to be achieved. When Pickering references the social interaction between people and things, I see a connection between how students utilize *Sketchpad* and the way *Sketchpad* responds to them.

3.3. Surprise and Wonder: Meaning-finder and Meaning-seeker

Although surprise is often associated with events in social and cultural context, like, "I was surprised to see my friend", surprise does play an important role in the way human beings behave and learn. Andrew Barto, Marco Mirolli and Gianluca Baldassarre (2013) argue that surprise is widely instrumental to human behaviour and is closely linked to sensory processing. They also suggest that "surprise is an emotion arising from a mismatch between an expectation and what is actually observed or experienced" (p. 2). Nitsa Movshovits-Hadar (1988) suggests that "A person is surprised when something occurs unexpectedly, when it is in contradiction to expectation" (p. 34). Furthermore, Movshovits-Hadar presents surprise within a cognitive realm as "intellectual surprise", implying that surprise may be a critical factor for learning. She describes intellectual surprise as giving one "a sense of fulfillment, an appreciation of some wisdom, a joy from its wittiness, and a drive to find some more" (p. 35).

Despite the spurt of writing about surprise in mathematics learning in recent times (Nunokawa, 2001; Stanley, 2002), there has yet to be a clear understanding about its explicit role in the learning of mathematics. For example, Kazuhiko Nunokawa (2001), in his description of surprises in mathematics lessons, portrayed surprise as a 'thing' that is tangible – something that one can touch and feel, as in, "it can be found that there are many lessons which seem to incorporate this surprise element" (p. 43). Additionally, Darren Stanley (2002), in his critical analysis of Nunokawa's work, suggested that surprise "is an emergent phenomenon that arises in different levels" (p. 15).

In my research, I see surprise in a sense through Stanley's lens as something emerging as a result of a satisfaction of a desire. As a result, surprise is closely connected to curiosity as an indicator that, one, the desire has been satisfied, hence surprise is seen as a meaning-finder, and two, as a physical marker that curiosity was present. Surprise as a physical marker is examined in my research through the body movements (gasp, eye movement, hand gestures …) of students. Therefore, one way of knowing that students have developed mathematical meaning is through the act of surprise.

Paul Opdal (2001) conceptualizes wonder in total contrast to curiosity. He believes that curiosity is a "motive that can move a person to do all kinds of research, but within an accepted framework […] wonder, on the other hand is not a motive, but an experience or state of mind signifying that something that so far has been taken for granted is incomplete or mistaken" (p. 342). Zazkis and Zazkis (2014) provide fuzziness to Opdal's claim by interjecting a linguistic point of view. They position wonder in two distinctly different linguistic forms; "wonder" as noun which is connected to admiration, awe and surprise, and "wonder" as a verb which is connected to interest and curiosity. The fascinating feature about this distinction is the fact that "wonder" in this dichotomous sense is not independent. In fact, Zazkis and Zazkis imply that there is a relationship between "wonder-noun" and "wonder-verb" and suggest that "surprise" about an experience may lead a person to curiosity about its cause, while interest in an experience could lead an individual to a state of awe. Zazkis and Liljedahl (2009) offer a definition for wonder as "a kind of emotional memory of what we have lost […] an engine of intellectual inquiry" (p. 18), while I suggest that it is more than what we have lost but also what we have never experienced.

Sinclair and Watson (2001) also offer an argument that seeks to operationalize wonder. They argue that wonder is predominantly marked by surprise, due to its unexpected and sudden nature. They also provide a distinction of the different types of wonder (wonder-why, wonder-how and wonder-at). For wonder-why and wonder-how, they claim that surprise terminates wonder when an answer is reached and that curiosity is satisfied but wonder-at will not be resolved by an investigation because it signifies bewilderment even though there is knowledge of how it works. They also suggest that a focus only on wonder-at will reduce in priority, the other types of wonders.

These perspectives of wonder provide my research with more than just a wonder–curiosity–surprise relationship, but also shed light on the ways in which curiosity plays an important role in the learning of mathematics. Moreover, I see wonder as the initial trigger of curiosity, which positions it as the agent of meaning seeking. As a result, students' "what if", "whys", "suppose" and "I wonder if" statements signify the emergence of curiosity, as well as certain non-verbal communication cues in body movement (moving toward, or away from).

3.4. Other Theoretical Underpinnings

Since my work is rooted in the role of digital technology in this embodiment–curiosity relationship, it stands to reason that I examine some of the existing research relating to technology integration in the mathematics classroom.

First, I draw attention to instrumental genesis which originates in the work of Pierre Verillon and Pierre Rabardel (1995) on the complexity of converting tools into mathematical artefacts. Their aim is to find out the relationship between thought and construction of knowledge when there is mediation through an artefact (a physical object that is structured to fulfill a particular purpose like that of a geometric compass). In doing so, Verillon and Rabardel distinguish between artefact and instrument, suggesting that, artefact is a human-made object while, the instrument is a mental construct that represents activities of the artefact. Instrumental genesis is construed to involve two processes. The instrumentalisation process involves movement from people towards the physical tool (artefact). In this process, the attributes and affordances of the artefact are enhanced by individual users. The other process is called instrumentation, which takes into consideration movement from the artefact towards people. In this process, the user is forced to realign activity based on the potential and constraints of the artefact. It is through this process that knowledge is constructed based on the user's experience with the artefact. While instrumental genesis offers a good analytic lens for the interaction between people and things, it does not address the emotional aspect of the people–tool interaction, which is key to my development of Embodied Curiosity.

Another theoretical perspective that is widely adopted in mathematics education is semiotic mediation. Semiotic mediation is rooted in Vygotskian principle – that the use of signs and artefacts (to include cognitive artefacts) mediate the actions of people and this is the main source of learning. Mariolina Bartolini Bussi and Maria Mariotti (1999) argue from a Vygotskian lens that, despite the widespread recognition of the role artefacts play in cognition, many theorists keep a separation view of technological artefacts and people. This, I believe, could be attributed to the longstanding perception of the physical benefit of technology as was evident in the three ages of technology development discussed in Chapter 2. However, Bartolini Bussi and Mariotti find resonance in Vygotsky's perspective of an analogous relationship between them and examine these principles along the lines of mathematics education. In semiotic mediation Bartolini Bussi and Mariotti explain that when a student accomplishes a task using an artefact shared signs are usually generated. This process may lead to two outcomes: [1] the artefact contributes to accomplishing the task and [2] the artefact may have a connection towards the content that is being mediated, which, in this case, is mathematics. They suggest that the former is easily recognized by the different systems of signs but not so for the latter. It is within these two-pronged conceptualizations that meanings are constructed. While the mediating implications of this theory could be utilized to understand the relationship between the students and the technology, it did not give me satisfaction about the role of both students and the technology. It seems to me that the students are more likely the orchestrators of what the technology does, which again ascribes control to humans.

I find that inclusive materialism detailed by Elizabeth de Freitas and Nathalie Sinclair (2014) aligns well with my research interest, but my aim is to focus on curiosity and inclusive materialism did not offer that main aspect of my work. Drawing on Châtelet's perspective that diagrams and gestures are historical representations of mathematics, de Freitas and Sinclair introduce inclusive materialism. They depart from the mind/body dualism and insist that the body is essential in shaping the mind. In presenting this claim, de Freitas and Sinclair focus on reimagining the body and agency. In doing so, they draw on the idea of distributed agency and, in particular, Karen Barad's idea of intra-action, to propose that agency belongs to relations (the student-tool relation, for example), rather than to the related. While inclusive materialism offers agency, actions between human and the non-human, the embodiment component and a heterogeneity of actors, it did not speak explicitly to the role of the attribute of people (feelings of desire, curiosity, interest…), and the attribute of things (shape, texture, size, weight, ...) intertwined with each other: that is, what role does the emotions play in this assemblage? And how does curiosity, in particular, influence the way actors operate in the assemblage? With this in mind, I designed Embodied Curiosity with the aim of responding to some of these gaps.

3.5. Embodied Curiosity

In keeping with embodied cognition grounding the mind in the details of sensorimotor embodiment, I am proposing a theoretical framework called Embodied Curiosity as a possible way to evaluate how mathematical meanings are constructed when students and digital technology interact with each other. The need for such a framework arises out of the limited attention that is given to the affective dimension of experience in research on the use of digital technology. A corollary to this aim is to find ways to address the growing need for effective integration of technology in the mathematics classroom. Drawing on Lakoff and Nùñez's (2000) and Lakoff and Johnson's (1980) versions of embodied cognition, which conceptualize knowledge in relation to the way human bodies operate in the environment, I argue that the agency of the human, the material and that of mathematics intertwine in certain ways to trigger curiosity. My first assumption is that trait-curiosity emerges when students ask wondering questions. This is an indicator that there is uncertainty. These uncertainties are signs that there is an information gap (in Loewenstein's terms), that needs to be filled. My second assumption is that when there is uncertainty, students working with digital technology will use it to offer some sort of clarity. In doing so, their body movements act as agents in transmitting signals to the technology and the functionalities of the technology in turn reacts to the students. In this interplay, both the students and the digital technology are locked in an exchange that allows them to take on alternate passive – active roles and the students become engaged in **curious dragging**[12]. It is within this interplay that Pickering's dance of agency becomes applicable.

I argue that the body, in this type of interaction, extends beyond its physical function and serves as an indicator that curiosity is indeed present. When body movements connect students' wonderings with the digital technology, a new way of thinking about curiosity emerges. I reconceptualize this 'new' curiosity as **relational-curiosity** and suggest that this type of curiosity only occurs within a student-technology interaction. It is important to point out that this type of curiosity is not controlled by the human (students), but instead relies heavily on both students and technology. Therefore, relational-curiosity serves two main purposes: [1] to provide a physical marker that makes curiosity identifiable; and, [2] to provide the foundation for the mathematical meaning-making process to take place.

Due to the displacement of the students from the central position, Embodied Curiosity is **temporal and emergent**. This means it is unplanned and emerges in real time. As a result of these features, I also propose that the constructs of wonder and surprise, being closely connected to curiosity, are infused in the process as meaning-seeker (wonder) and meaning-finder (surprise). Since Embodied Curiosity is first developed in this research, I

[12] Curious dragging is an extension of Ferdinando Arzarello (2002) dragging modalities and will be discussed in greater details in Chapter 6.

will revisit its development in Chapter 6 (data analysis) to provide a more detailed account of how it came about, how it operates and how the relationships are formed.

3.6. Summary

This chapter presents the theoretical consideration of my research. The theory – Embodied Curiosity – is deep-rooted in theories of embodiment and seeks to address how curiosity, embodiment and digital technology relate to each other, as well as how they work hand-in-hand to foster mathematics learning. The theory recognizes agency from Pickering's perspective as an important part of how people perform actions and the mangle of practice as a metaphor for the student–technology interaction. Using Pickering's case studies, I show how his argument for human, material and disciplinary agencies are taken into consideration when students interact with technology. I draw on Pickering's posthumanist perspective to situate Embodied Curiosity as a theoretical framework that disrupts the mind/body dualism. In doing so, I examine some established theoretical frameworks in mathematics education that focus on the use of technology and identify some gaps which were not pertinent to me. Although each of the theories that I report on utilizes digital technology, they did not give an account for the relational way I perceived the elements of Embodied Curiosity. For example, one theory did not give an account for the emotional consideration of my phenomenon of interest, while the other failed to address the specific role both students and technology play within an interaction. This provides an opportunity for me to introduce Embodied Curiosity as the framework that is best suited for my research.

Embodied Curiosity emerges as a relational framework that shows connections among curiosity, digital technology, body movement and mathematical meaning. I argue that wonder and surprise are two important constructs of curiosity because they indicate when there is a need to find knowledge and when knowledge is achieved. In addition, I introduce some basic assumptions of Embodied Curiosity, first, suggesting a new type of curiosity which I call relational-curiosity. This type of curiosity represents the interaction between the student's trait-curiosity and the digital technology. In addition, I suggest that within this interaction students engage in curious dragging, which is an extension of other dragging modalities. The most important point I have made in this chapter is that Embodied Curiosity materializes in real time: that is, it is emergent and temporal.

Chapter 4. Methodology

> "All children are curious and I wonder by what process this trait becomes developed in some and suppressed in others." (Marston Bates,1990)

Research methods involve more than a systematic way of recording, analyzing and interpreting data. The critical decisions which a researcher makes prior to, during and after conducting research shapes the meaning of the research findings. Decision-making throughout the research process also has an extensive impact on how the findings of a research study is perceived within the academic community. Therefore, this chapter provides an understanding of the qualitative nature and the strategic decisions I have made in conducting this research. Bates' quotation above reflects the mood I was in while I worked on this chapter. What were the processes involved in student's learning? I take into consideration the nature of Embodied Curiosity and the ways I have come to understand how knowledge is created. This helps to provide a deeper sense of how curiosity can support the development of mathematical meanings.

I chose Jamaica as my data source for two compelling reasons. Having over fifteen years of experience as an educator at various levels of the Jamaican education system has shed light on a need for effective integration of technology in the teaching and learning of mathematics. These issues were not solely theoretical ones, but also empirical as well. This work experience triggers my own curiosity, which is more than merely wondering about why things are the way they appear. Instead, my curiosity involves moving towards identifying issues and offering possible solutions. In other words, I wanted to understand the situation and implement changes. Another reason for my decision to conduct data collection in Jamaica was the dynamic ways in which technology has made knowledge more accessible in principle, but in reality, hardly at all in the area of education. The motivation for collecting data in my home country was to put forth viable solutions for using digital technologies to enhance the teaching and learning of mathematics. My research would be a source of evidence for why it is important to place priority on technology integration in the mathematics classroom.

The schools that participate in my research were deliberately chosen due to past knowledge about their resources. They share many similarities but provide a different experience throughout the data collection process. It is important to note that the categorizing of Jamaican secondary schools into “traditional” and “non-traditional” high schools (as discussed in Chapter 1) did not play a role in my selection process, despite the stark differences in the structure of the research process. Many decisions were taken only because of differences between the schools in terms of administrative policies and ethical issues, the latter being the bedrock of my research.

I therefore present in this chapter an overview of the research questions which guided my research, as well as the research sites, participants and the procedures which were taken in conducting my research.

4.1. Research Questions and Designs

In order to satiate my curiosity about human–material interactions in the mathematics classroom, I explore the relationship between the research questions and the literature review along with the theoretical consideration of my study. This relationship serves firstly as a catalyst for the methodological decisions I had to make and, secondly, as the point of reference for developing deeper understanding in the way things work throughout my study. As a result, I begin this chapter with the central questions (that are also in Chapter 1) that form the pillars of my dissertation. In particular, my research seeks to respond to:

(1) What are the physical markers of curiosity in a secondary mathematics classroom?
(2) To what extent does Embodied Curiosity foster the construction of mathematical meanings?
(3) How do the potentialities and affordances of *The Geometer’s Sketchpad* evoke Embodied Curiosity?

As mentioned in Chapter 3, my main focus is on how trait-curiosity and the meditative role of digital technology tools co-ordinate with each other to bring about specific elements of body movement. This co-ordination, I believe, has the potential to develop mathematical meanings and is well aligned to offer a response for my first research question. The second research question is directly concerned with the way in which technology tools offer possibilities for mathematical meaning-making (Noss & Hoyles, 1996), with a specific focus on *The Geometer’s Sketchpad*. Research question three addresses concerns or ambiguities about being able to recognize whether or not there is curiosity present in a situation.

Although there are numerous studies on embodiment and the use of technology in the mathematics classroom, there are equally as many on curiosity. The gap my research aims to fill lies within the intersection of these three ideas. This research, therefore, provides a path not for a collision among them, but rather for a mutually harmonious relationship with benefits to mathematics education. As such, this chapter outlines details about the methodological approach I undertake in order to respond to how curiosity is involved in mathematics teaching.

4.2. Research Sites

My research sites were located in Jamaica, in the Caribbean. Four types of data were collected; video-recordings of classroom observations, audio recordings of semi-structured interviews, field notes and students' work. These data were collected in two co-educational secondary schools which I refer to as School X and School Y. The schools were deliberately selected because of my previous work experience in Jamaica. I was aware that both schools were well equipped with computer labs through the e-Learning Jamaica[13] (e-Ljam) program, but students' performance in mathematics at both schools continued to be dissatisfying. Also, having the opportunity to work alongside the regional co-ordinator, administrators and teachers of School Y, I became conscious of the need for appropriate interventions in effective integration of technology in the mathematics classroom. For this reason, I give a brief description of the research sites to provide context for my choices.

School X is a government-run secondary high school located in one of Jamaica's popular cities. The school has a long history of high academic achievement particularly in the science disciplines, but has seen dissatisfying performances with regard to mathematics in recent years. Additionally, students who attend School X are from varying socio-economic backgrounds. Similarly, School Y is also operated by the government of Jamaica under the auspices of the Ministry of Education and is located in one of Jamaica's

[13] e-learning Jamaica Company Limited is an agency of the Ministry of Science and Technology with a mandate to transform the national learning outcomes using information and communication technologies (ICT), through integration and infusion of technology in the education system.

most historic farming communities. Contrastingly, most students are typically from low socio-economic backgrounds and the school is known mainly for its long-standing historical traditions and its excellence in sporting activities rather than in academics.

4.2.1. Participants

The student participants (ages ranging from 14 – 15 years old) were randomly selected from a pool of five and three grade 9 classes at School X and School Y, respectively. The teachers of both selected classes, whom I will call Sammy and Andrew for the purpose of anonymity, also formed part of the data sample. Sammy is a trained mathematics teacher and she has been teaching mathematics at the secondary level for seventeen years, most of this experience includes teaching from grade 7 (13 years old) to grade 11 (18 years old) and, most recently, to grade 12. Andrew is a trained information technology teacher with a minor in mathematics. He has eleven years' experience in the teaching of mathematics, but has been teaching at School Y for four years. In Table 4-1, I have summarized the data from the participating teachers about the composition of the total population.

Table 4-1 Research population

Name of schools	Name of teachers	Name of grades	Number of girls	Number of boys	Total
School X	Sammy	9 X	13	10	23
School Y	Andrew	9 Y	18	17	35
Total			31	27	**58**

The grade 9 classes were also selected because the geometric topic of circle theorems forms part of the grade 9 mathematics curriculum and it was also due to be taught in the semester of my data collection. Furthermore, in keeping with the functionalities of *Sketchpad*, the teachers and I believed that circle theorems would be most suited to trigger curiosity and stimulate students' interest in exploring geometric relationships in a non-static way. This, I believe, was significant since both the teachers and the students were using the software for the first time. The students whose work is featured in this dissertation were all assigned pseudonyms which were known only to me and the classroom teachers. Also, the episodes used in my data analysis were specifically selected because of the evidence they provided about all four elements (curiosity, body movement, digital technology and mathematics meaning) of Embodied Curiosity. There were instances when student's interactions revealed two or three elements. For example, there were examples where relational-curiosity was present but it did not lead to any explicit mathematical meaning.

One of the most significant decisions I made during the data-collection process was to decide on which role to take in the research field. Initially, my intention was to be an observer because I would be better able to identify ways the digital technology and curiosity related, as well as, to keep a written record of things that the video-recording may have missed. I had to rethink that decision because of complications in the way video-recording was done. In one school, not all the students consented to video-recording, even though they consented to participate in the research. In this circumstance, the video cameras could not remain in a static position at all times. In addition, I chose to be an active participant because of how the tasks were being introduced. This occurred when the teachers used a prescriptive teaching approach, rather than an investigative one.

Being a participant – observer was not void of challenges. One difficulty I found was that, since both Andrew and Sammy were not familiar with the constructs of Embodied Curiosity, they were unable to zoom in on specific interactions between pairs of students at times when I was a participant. Another challenge I came across, especially in reviewing the data, was that there was an inconsistency in capturing students' interactions. This was due in part to the sudden transition from one interaction to another or from a swift movement of the video-recorder around the room. These challenges resulted in me having to track particular pairs of students throughout the entire video-recording.

Towards the latter part of the month (February, 2018), I offered an explanation to students of the purpose and rationale of the study, along with a detailed description of the procedures I would have to take in order to investigate my interest. They were given assent[14] forms which were to be signed and returned by a set time while their parents were given consent[15] forms to accept their child/ward's participation. No student was hindered from taking part in class activities because it was regular classes. However, the sample for my research constitutes only those students who signed the assent forms and

[14] To denote agreement with an opinion and is usually given by children under the age of 18.

[15] Denoting agreement to let something happen. Usually given by a parent or guardian when the participant is not of the legal age or capable of doing so.

whose parents also permitted them to participate. In addition, the two teachers were given consent forms to indicate their willingness to participate in my research.

Table 4-2 Student sample

Name of school	Number of students enroll	Assenting Students	Consenting Parents	Total
School X	23	23	23	23
School Y	35	28 (7 did not assent)	18	18
Total	58	51	41	**41**

Of the 23 students registered in 9X, all students and parents granted permission to participate in my research. However, of the 35 students registered in 9Y, 28 assented while only 18 of the students who assent received consent from their parents. In School Y the 7 students who did not assent along with the 10 students who assented but did not receive consent from their parents were not excluded from classroom activities. However, they were not video-recorded and did not form part of the sample. Therefore, 41 students in total (23 from school X and 18 from school Y) participated in the research, as shown in Table 4-2 above.

4.3. The Data Collection Process

From January to April 2018, I commenced the data-collection portion of my research in three phases. In **Phase 1** (January), which I considered the initial phase, I co-ordinated with administrators and teachers in both schools making the appropriate introductions and welcomes. During this phase, I installed twenty-five temporary *Sketchpad* licenses in one of the information technology (IT) labs at School X, while a lab technician installed twenty-five licenses at School Y. In addition, I collected information from the teachers on the demographics of the participants and the research site. I also planned the timelines and schedule for data collection, as well as trained the teachers in the use of *Sketchpad*. The purpose of the training sessions was to help teachers become familiar with the functionalities of *Sketchpad*, provide an alternative approach to the teaching of geometry and ensure commonality and consensus on the mathematical knowledge associated with concepts relating to circle theorems.

Training of teachers lasted over a three-week (January – February) period for eight hours in School X and three hours in School Y. More time was spent on teacher training in School

X than School Y for two reasons. First, teacher training in School X also involved other teachers of the mathematics department who were not part of the research sample. This was not the case in School Y – and although Andrew was using the software for the first time, his expertise in IT allowed him to navigate *Sketchpad* more proficiently than the other teachers. Therefore, most of the time was spent on preparing tasks and discussing the relationships we thought would be more beneficial to students at the grade 9 level. The second reason was that, School X has twice as many mathematics teachers as School Y. During the teacher-training process, we also examined some of the anticipated errors students might display and came up with a consensus on how to respond to them should they arise. For example, it is customary for students to confuse the radius of a circle with its diameter, so, in a situation such as this, we decided to have students use *Sketchpad* to examine how these two concepts relate to each other.

Training sessions were done in two different forms at School X. On the one hand, two professional development sessions (2 hours per session) were held, which involved all members of the school's mathematics department, and on the other subsequent occasions, Sammy and I interacted after school hours and during "non-contact sessions"[14]. In School Y, training was more informal because Andrew is also an IT teacher and a portion of the technical functions relating to Information Communication Technology (ICT) within the school is a part of his job responsibility. This made it difficult for meetings to be convened at scheduled times. Instead, training sessions were done when time became available during the school day. Interestingly, although both participating teachers were being exposed to *Sketchpad* for the first time, they were intimately involved in the planning and execution process of the training sessions. They were enthusiastic in suggesting ideas about tasks and delighted by the affordances of the software.

During ***Phase 2,*** I met with the students and gave an outline of my research, expressing to them what my research interest was about, in what ways they could help and how I intended to go about doing the research. I met with the students because I thought that adding a voice and a face to an activity (the research) was important. I believed that children would get a better sense of the similarities between conducting research and classroom teaching. I also used the opportunity to inform the participants of their role

[14] A non-contact session refers to a period of time during school hours where the teacher is not scheduled to interact with students.

should they opt to partake in the study. They were made aware of the potential risks and the volunteer nature of their research participation, and I explained that the interactions would be video-recorded and that selected students would be interviewed. In addition, I made it clear to the students that they would not be excluded from any lesson, but if they did not want to be part of the research they would not be video-recorded, nor would their work appear in my research. Having heard these explanations, the participants were asked to assent or dissent. They were also given consent forms to take home for parents or guardians to grant permission. Upon noticing that not all parents had given consent, I devised a strategy for video-recording for the purpose of respecting the wishes of students who dissented or whose parents did not provide consent. Figure 1 below represents the classroom layout at School X. The computers are represented by the squares and the circles indicate the vantage points of the cameras when they were static.

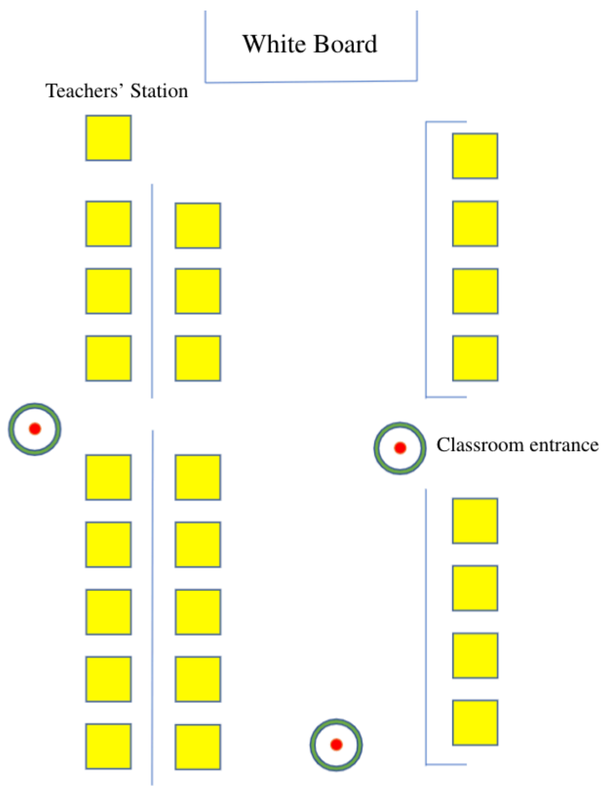


Figure 1: Classroom layout at School X

At School X, since all students assented and their parents granted permission, video-recording did not pose a challenge. However, there were instances when I moved closer to selected students in order to zoom in on their hand gestures, eye gazes and facial expressions.

The experience in School Y was different from that of School X. Ideally, the criteria for participation in my research required student assent and parental consent. However, the fact that assent and consent were only partially provided at School Y, as reported previously, resulted in non-standard ways of video-recording in an effort to prevent compromising the data. Interestingly, in School Y, two different scenarios were presented with respect to students' participation. On the one hand, parental consent was given, but the student did not offer assent, and on the other, student gave assent but the parents did not agree for them to participate. Moreover, there was one instance when the parent gave consent and suggested that the student could be video-recorded, however, the student assented but did not want to be a part of the video-recording. In the event that either assent or consent forms were not signed, those students did not form part of the sample, while the student who did not want to be video-recorded was included in the sample, but not video-recorded or interviewed. Since the IT lab was not the students' home room, I was able to assign them specific seats, so that video-recording was done effectively with minimal challenges. In Figure 2, I show the classroom layout at School Y and how video-recording was done based on the position of the cameras.

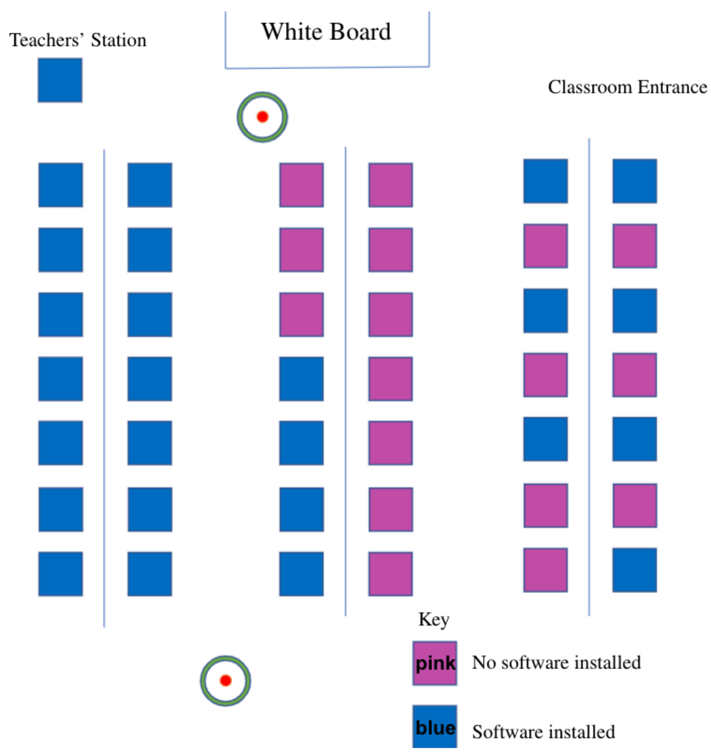


Figure 2: Classroom layout at School Y

The IT lab which was used for data collection at School Y was also used to teach students Computer Science, Information Technology and Electronic Document Preparation Management (EDPM). Due to this, the lab was fully equipped with adequate computers and ICT materials. Since I had only 25 *Sketchpad* licenses, they were installed as shown in Figure 2, taking into consideration the class size, the number of students who participated in the research and the technicalities of video-recording. The non-participating students sat in pairs at the row of computers next to the classroom entrance in pairs and used the computers in blue squares with *Sketchpad* installed.

Student attendance fluctuated throughout the period and, at most, there were 31 students engaging in the tasks. Also, in deciding on this particular layout, Andrew and I thought it was best to allow students who were not participating in the research to work on tasks in a collaborative way, in order to foster meaningful mathematical talk amongst each other. The section of the classroom with multiple pink and blue squares (computers) and no video-recording cameras was designated for those participants who did not want to be video-recorded or who were not a part of the research sample, While the section with the blue squares was designed for those who were participating in the research.

Finally, **Phase 3** (mid-March – April 2018**)** involved the procedures taken to conduct semi-interviews, video-recordings of classroom observations, retrieving students' work and recording field notes. From mid-March to April, I visited Schools X and Y for a combined three weeks to investigate about Embodied Curiosity and mathematics teaching and learning. In both schools. The mathematics curriculum was delivered for forty-five minutes each session. However, School X offers mathematics four times per week while students at School Y are engaged in mathematics lessons for five days of the week. I was accommodated for classroom observations twice per week (90 minutes) in School X and three times per week (135 minutes) in School Y. During the non-research days both teachers engage the students in simple geometry tasks, such as constructing and classifying triangles based on their properties. In this way students became more proficient with the technology while reinforcing knowledge they may have had previously, only this time the experience was with a DGE. While Figures 1 and 2 show the classroom layout and the angles from which video-recording was possible, the process of video-recording took into consideration two important aspects of the day-to-day running of the schools.

High security of the equipment in the schools' labs was a priority, which meant that access to these labs required a special protocol. At School X, the keys for the research room were only available from one of the schools' Vice-Principals and at School Y, from the Head of the IT department. As a result, setting up the video-recorder could only be done approximately five minutes prior to the commencement of each class. Furthermore, the labs were also used by other classes during the school day, which required close co-ordination with the teachers who used the labs before and after the scheduled data-collection period. In this sense, the video recorder was set up before the interactions and data was retrieved from the computers after each session had ended.

Being a participant–observer involved interchanging roles with the participating teachers: that is, at times I facilitated task performance, while in other situations I was engaged in video-recording. Switching between these two roles was decided on prior to the start of each session. However, in some instances, I switch roles spontaneously. Role-switching was mainly evident when participants demonstrated something of interest to me. Despite designated spots from which video-recording was proposed, I oftentimes moved throughout the classroom with the video recorder to ensure that I had not missed crucial data, while at the same time being careful not to record those who had not agreed to be research subjects.

Some participants were selected for semi-structured interviews after a review of their work and video-recording, based on the unusual ways in which their diagrams were constructed and if there were preliminary evidence of the appearance of curiosity. For example, if a student diagram was somewhat 'different' from the others, I would invite the participant to an interview. In the interview they would describe what they were curious about when they produced certain diagrams and how they felt upon noticing something of interest. This process helped me to better understand how the technology helped the participants become curious about things. I performed a total of six student interviews (in pairs) in both schools, talking to 12 participants in total, along with two teacher interviews. Each interview lasted approximately ten minutes for students and fifteen minutes for teachers during which an audio-recording was done.

During the class periods, computers were labelled numerically, which allowed for my identification of participants' work. They were also encouraged to save their work using the format (<last name><first name> <day><computer number>). An example of this could be RodneyShereeDay1Comp4. Once participants completed the tasks, their work was saved to the school's server and then removed by me at the end of each session. Prior to the next video-recording, I reviewed the students' work to determine whether or not I needed to attend to something specific on the subsequent day. A key feature of my research (which constitutes research question one) was being able to identify the emergence of curiosity. I also needed to distinguish between when students were merely gazing for attentiveness or fixated because of curiosity. Reviewing the video-recordings at the end of each helped me to make the distinction between attentiveness and curiosity.

4.4. Data Analysis Process

In developing an understanding of the relationships among the four elements of Embodied Curiosity, I embarked on a three-phase process for the data analysis similar to the data-collection process. **Phase one** involved reviewing and selecting the data, **phase two** included transcribing and **phase three** considered the analysis of the selected episodes and transcripts. This three-phase method was used over an eight-month period beginning in June 2018. The process was done in a continuous, iterative manner, firstly to identify when curiosity was triggered and secondly to observe instances of relational-curiosity. Finally, the data was analyzed to locate scenarios when mathematical meanings emerged explicitly or were implied. My rationale for this recursive method was twofold; I had approximately 625 minutes ($\approx 10\, hours$) of video-recordings and approximately 190 minutes ($\approx 3\, hours$) of audio-recordings, which would require an enormous time investment and I did not want to lose critical data simply due to exhaustion. Also, because trait-curiosity could not be determined with the naked eye, going through the data analysis process iteratively provided opportunities for me to interpret certain factors as signs of emerging curiosity. For example, visual fixation at a diagram does not mean curiosity is present, but if the visual fixation is followed by a wondering question ("what if…"), then that could mean a potential occurrence of curiosity. The final stage involved observing the selected data to locate when relational-curiosity led to the manifestation of mathematical meanings. From this final stage, the richness of the data provided evidence for interpreting Embodied Curiosity and responding to my research questions.

4.4.1. Reviewing and Selecting Data

In June 2018, the ongoing process of reviewing and selecting the data began for the ten hours of video-recordings, which were obtained collectively from both schools. I began with School X, simply because the commencement of the data collection process began at that school. The approach for reviewing and selecting data was to examine the video-recordings as a whole and to decide whether or not I wanted to transcribe the entire interaction or use segmentation of interactions, as Gumperz, and Berenz (1993) suggested. Over a two-month period, the data was examined to identify preliminary evidence of ways in which curiosity was triggered when the interaction involved student–task–*Sketchpad*. After selecting these snippets from the data, I re-examined them to see what mathematical meanings were coming up. A total of nine episodes were identified and tabulated. However, three episodes did not demonstrate a continuous flow of relational-curiosity to mathematical meaning. I call those non-examples of Embodied Curiosity. Six episodes with evidence of all four elements of Embodied Curiosity were selected and transcribed for analysis. From those six episodes I gave descriptions of how the Embodied Curiosity process transitioned from one layer to the other and the others to either describe how mathematical meanings were constructed and the role of the teachers in the process.

Students were involved in a myriad of tasks (see Appendix A) about properties of the circle and their relations.

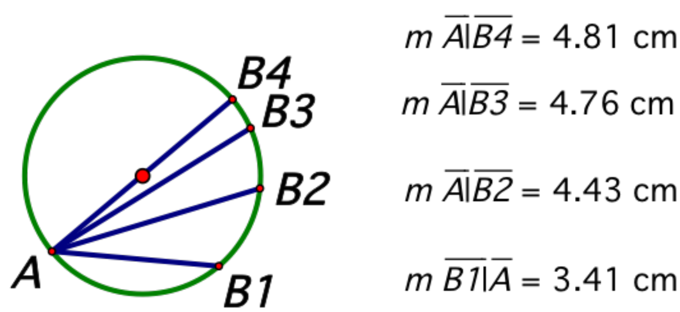


Figure 3: Finding the diameter of the circle

For example, one task (Figure 3) invited participants to construct a chord and explore the diameter of a circle. They were expected to fix one point (A) and drag the other (B1) around the circumference and describe what they noticed about the maximum length of the chord. The purpose for this task was to foster an understanding that the diameter is the longest chord within a circle. In generating this mathematical understanding, I was also interested in how curiosity was triggered and the role *Sketchpad* played in this process. The type of questions students asked and their body movements were also of importance to me. Once students completed the tasks, they would share some of the main ideas they came up with while they worked together. Providing students with this opportunity also generated different types of body movements such as gestures. In addition, I was interested in the role teachers played in the Embodied Curiosity process, since my development of the framework has implications for both teaching and learning.

In keeping with research question one and Berlyne's (1954) curiosity dimension model, which was discussed in Chapter 2, I was able to determine when students demonstrated **perceptual curiosity,** in this case, visual experiences, and to a lesser extent those that were tactile. I was also able to pinpoint **epistemic curiosity,** which is the type of curiosity that relates cognition. In other words, I could deduce that curiosity was emerging based on a combination of visual fixations, eye gazes, facial expressions, gestures and the need to lean forward into the technology when there were uncertainties. Additionally, the type of questions students asked were indicative of their "wonderings" and their desire to acquire new knowledge. Using these two categories, I was able to identify specific sections of the data for possible analysis. This will be further discussed in Chapter 6 and examples offered to show the interconnected nature of Berlyne's two main curiosity dimensions. Therefore, I utilized Berlyne's curiosity model as a method of analysis with the claim that epistemic and perceptual curiosity are not mutually exclusive, but rather work hand-in-hand in the process of constructing knowledge.

After a few weeks had passed, I revisited the data, this time to identify ways in which students' body movements indicated possibilities for mathematical meanings, and finally a third look at the data allowed me to identify snippets of episodes with rich possibilities for Embodied Curiosity. My next step was to listen to the audio recordings of teacher and student interviews. The purpose of this was to obtain preliminary evidence of the ways both teachers and students spoke about curiosity and its embodied nature. At this stage, I was able to identify data through the lens of Berylne's curiosity dimension model sub-categories, that is, **specific** (a desire for a specific piece of information) and **diversive** (seeking stimulation due to boredom). For example, when I asked a student in an interview to explain why the cyclic quadrilateral (a quadrilateral whose four vertices lie on a circle) did not follow the conventional rectangle that most students drew she said, “I wanted to get angles that were not 90°”.

My field notes were in two forms. One set of fieldnotes were collected from students and written on paper at the end of the data-collection process. On this paper, the students wrote what they believed was important in their learning process, their experience with *Sketchpad* and its functionalities as well as how they felt about their new experience learning circle theorems with the computer. The other was my own hand-written record of what had been presented, which could act as a reference later on, along with students' interactions that were not recorded. Students' work contained diagrams and information about the task, the relationships they investigated and a description of the process involved in arriving at their own understandings. Having selected data from the data pool in this systematic way allowed for efficient and effective transcription of the specific data to be used in my dissertation.

4.4.2. Data Transcription

An important aspect of transcribing the data was to associate the ways curiosity emerged with ways mathematical meanings were represented. Since I have an interest in the relationships among the elements of Embodied Curiosity, I used two different transcription formats to capture this relationship. A multi-modal, column-based transcription format was used to illustrate speakers' utterances and actions when they occurred concurrently. In addition, the column-based format was used to capture instances when verbal communication was accompanied by nonverbal acts. For example, when a student suggested that the diameter must go through the centre while using his index finger to demonstrate how it is done. The partiture[16] format was used to highlight the collaborative aspects of the interactions (Edwards & Lampert, 1993). My decision to exploit these formats was to distinguish between students' actions and actions of the digital technology. In addition, I wanted to concentrate on verbal communications that were directly linked to epistemic and perceptual curiosity dimensions.

[16] The partiture format is used to show when turns by different speakers are put on different lines in a manner similar to instruments on a musical score. It shows that discourse is unified and achieved jointly by multiple participants.

In transcribing the data, another critical decision I made was whether or not English orthography was sufficient or whether to preserve nuances of spelling and pronunciation in the Jamaican Creole that was heavily used in the lessons. Despite British English being the official language of Jamaica, I chose to preserve the students' native language as much as possible, because I felt it was significant to the student's identity. I also believed that the native language maintained certain social links within students' interactions. In situations where the Jamaican Creole was used, an English translation is given in bold and placed in squared brackets. This convention is consistently used when the utterances appear within the main body of the analysis or in the transcripts.

I used transcript conventions (see Appendix B) to show where words spoken in the Jamaican Creole were detected and provided the English equivalent for readability of the dialogue between students and teachers. Translating students' spoken words from their native language to standard English was an important part of this phase because the research focus is on the development of mathematical meanings. As such, I believe that interpretation of language could impact how mathematical meanings were construed. When a description was offered by me, about how students perform a specific task and what ensued within the interaction, I used English to provide descriptions, which allowed patterns to become visible and meanings easily identified.

Since non-verbal communication methods played an important role in determining when curiosity was triggered, I paid keen attention to nonverbal cues as they occurred in pairs. A review of the data also showed that students' utterances were usually accompanied by body movements when they interacted with *Sketchpad*. As a result, when body movements and students' utterances occur alongside each other, I represent them in the transcript with in a double-rowed manner within each turn (speech first followed by body movement).

4.4.3. Analyzing the transcript

In this phase, I took into consideration Embodied Curiosity along with Berlyne's curiosity dimension model (as a unified whole) and examined the transcripts from the selected video-recordings to ascertain how each element of the theoretical framework was represented individually and then collectively. The transcript analysis process was on-going because I needed depth in understanding how and why the relationships in which I was interested were developed. My main aim was to discover specific relationships between curiosity, body movement and mathematical meaning-making when technology is infused in the interaction. In order to achieve this, my research questions played a pivotal role in the process of analyzing the transcripts.

With respect to research question one, my aim was to identify, from the transcripts, physical markers of curiosity, specifically when curiosity emerged as a result of using *Sketchpad*. This might include a student rolling her eyes as she contemplates the relationship between a complete revolution and the sum of the interior angles in the cyclic quadrilateral. I interpreted that as a signal of emerging curiosity. Another example occurred with a student visually fixated on the computer screen while performing a task. Having an understanding that fixation does not necessarily negate a desire for knowledge, I also examined the verbal communication acts which students used concurrently with visual fixation. The verbal communications usually appear in the form of questions which is interpreted again as emerging curiosity.

I also examined the transcripts for patterns when students "wondering" led to moments of surprise: that is, when they used interrogative words such as "what", "when", "why" and "how". This would give a clear indication that visual fixation along with certain words ("why", "how", "suppose") and non-verbal communication acts such as facial expressions in the form of surprise, were ways in which curiosity could be identified. Therefore, in analyzing the transcripts I looked for segments of the interactions where body movement and speech acts co-ordinated together as meaning-seeker (wonder) and meaning-finder (surprise). I then matched the verbal and non-verbal communication acts against Berlyne's curiosity dimension model to align each emerging curiosity to a combination of epistemic and perceptual curiosity. In this way, I was able to ascertain from the data tangible ways in which students demonstrated curiosity.

Pertaining to research question two, the focus of analysis was mainly on instances when Embodied Curiosity led to the construction of mathematical meanings. Thus, I highlighted from the partiture transcripts instances where curiosity, technology and body movements intertwined in a harmonious way. Since research question two emphasizes the relationship between Embodied Curiosity and the development of mathematical meanings, I then organized the highlighted segments into two main categories: one, representing the presence of Embodied Curiosity while mathematical meanings were explicitly stated and the other when the meanings were implied.

For the purpose of investigating the role of the potentialities and affordances of *The Geometer's Sketchpad,* which question three sought to address, my analysis took into consideration instances when the technology was active within an interaction or when it was passive. I wanted to know whether or not *Sketchpad* triggered curiosity which further led to continuous exploration and development of mathematical meanings.

4.5. Summary

In an attempt to examine how the dynamic geometry software – *Sketchpad* – prompted students' body movements, I adapted Berlyne's curiosity dimension model (as a method for analysis) along with Embodied Curiosity (as a theoretical lens) to explain relationships that lead to the development of mathematical meanings. In order to examine these relationships, the methodological decisions made in this research followed closely along the methods of a qualitative approach. Four types of data sources, including classroom observations, student's work, interviews and field notes were used in this study. I provide a description of the research sites, the methods used to gain entry, select participants and request participation in my research. Additionally, I describe the data collection process, how episodes were selected and how the data was analysed. I use strategies such as various transcript formats, to identify relations within the data and to ensure and maintain validity of my research findings.

Chapter 5. Circle Geometry with *Sketchpad*

> *"A great discovery solves a great problem, but there is a grain of discovery in the solution of any problem. Your problem may be modest, but if it challenges your curiosity and bring into play your inventive faculties, and if you solve it by your own means, you may experience the tension and enjoy the triumph of discovery." (George Pólya, 1944/2004)*

As I examine this quote by George Pólya, and reflect on the report about students' performance in circle geometry theorem in the Caribbean (discussed below), I wonder about the many students who may have been deprived of opportunities to challenge their curiosity by solving problems through their own means and to experience both joy and anxiety about geometry. This chapter briefly examines the origin of geometry and provides a description of the topic circle geometry theorems. It presents the concepts relevant to some of the tasks used in this research and discuss tensions concerning the problematic way the topic is taught in schools, at least in the Jamaican context. I report on the underperformance of students at the end of secondary school in Jamaica as it relates specifically to the topic circle geometry theorems and examine the implications of learning the geometry in a DGE.

5.1. What is Geometry?

According to Carey (1996), "Geometry is concerned primarily with the spatial properties of objects and with the endless stream of abstract generalizations to which these properties give rise" (p. 1). Geometry has a long-standing history in mathematics and, like curiosity and technology, it has evolved over time. The topic journeyed from Early Geometry with the Egyptians, Babylonians and Indians to Ancient Greek Geometry and then to Modern Geometry. It would be thoughtful of me, in this chapter, to present a historical timeline of the development of Geometry, at least to see whether or not its journey crossed path with curiosity and technology. However, my focus in this chapter, is to present some of the conceptual understandings that form part of the requirement for a study about circle geometry, as well as some of the issues surrounding the teaching and learning of it.

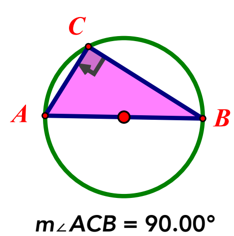
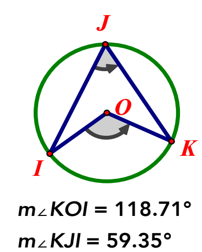
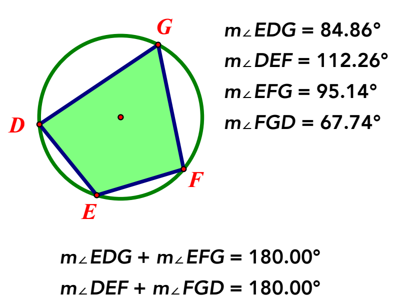
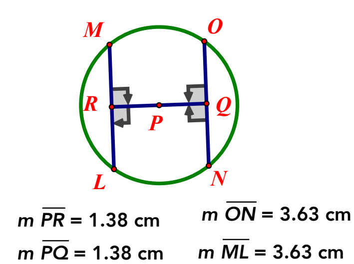
The main purpose for this focus is that, from a Jamaican context, circle geometry has been a problematic topic for teachers to teach and, by extension, a challenge for students to learn. Furthermore, the dominant mode of delivery for this topic is through the use of static environments. I surmise that this experience gives students little opportunity to examine the true nature of the relationships among objects of the circle. For example, in my teaching experience both in Jamaica and Canada, some students are not aware that pi (π) is the ratio or relationship between the circumference and the diameter of a circle, despite their confidence in using the formula to calculate area and perimeter of a circle. This could be as a result of the limited opportunities for exploration in geometry. With this awareness, I set out to investigate circle geometry theorems using the *Sketchpad* as a tool to promote new ways of thinking about the topic. In order to achieve this, I will explore the main concepts surrounding some of the theorems used for data collection in this chapter.

5.2. What is Circle Geometry?

Drawing a triangle or a quadrilateral seems like an easy task to many people. This event requires joining three or four independent line segments together. A circle is even because it requires only a fixed point (the centre of the circle) and a (curved) line made up of points equidistant to the fixed point. Despite the predominant representation of the circle in our everyday environment, the circle has been one of the most difficult shapes in geometry to comprehend. This can be attributed to the many interpretations and definitions of what constitutes a circle. My early memory and first experience of the geometric circle goes as far back as preschool when the circle was seen as a 'basic' shape along with triangles, squares and rectangles. It wasn't until about grade 5 that I recognised that there was something different about the circle (it had curved sides). By then my understanding of the circle was 'a round shape that is closed', and by middle school my understanding morphed into 'a closed two-dimensional curved shape without corners or edges'. At that time, 'two-dimensional' was not clearly understood, but the idea of 'flatness' could suffice. Things got a bit more complex when my understanding of the circle in high school became more distinctly aligned with Euclid's definition as 'a curved plane figure bounded by a line so that all straight lines drawn from the centre are equal'.

A later version of my understanding about the circle emerged during teacher training as 'the locus of a point moving at a constant distance to a fixed point'. Today, my interpretation of a circle involves topological ideas that a square can be continuously stretched and bent into a circle, which makes the square and the circle equivalent by distortion. As a researcher, Chorney's (2017) conception of the circle as, "[it] ought not to be seen as a reproduction of an ideal form, but rather as a meshwork of materials and forces" (p. 45) has put into perspective the fundamental tensions that surround an understanding about the circle. This personal journey with the shape is an illustration of the problematic way in which circles are usually perceived, taught and understood. It seems that the circle in and of itself has transitioned back-and-forth, at least for me, from a 'shape' to an 'object'. With this experience, it thought is important that the concepts pertaining to circle geometry be made comprehensible. I therefore designed tasks aimed to encourage students to explore the circle in depth. These will be discussed throughout the data analysis Chapter, along with some of the main concepts and issues relating to an understanding about them.

Table 5-1 Selected circle geometry theorems

	
[a] Angle in a semi-circle The angle formed in a semi-circle is a right angle.	[b] Angles at the centre and circumference The angle at the centre is twice the size of the angle at the circumference subtended by the same arc.
	
[c] Opposite angles in a cyclic quadrilateral Opposite angles in a cyclic quadrilateral are supplementary (adds up to 180°).	[d] Equal chords equidistant from the centre Chords in a circle which are equidistant from the centre are equal.

The diagrams in Table 5 -1 [a] – [d] represent some of the tasks that students performed during the data-collection process. Example [a] draws on students' conceptualization of the diameter, a semi-circle and a right-angled triangle. This task was designed to allow students to think deeply about how each object (points, lines, angles) of the construction relates to each other and the circle in which it is inscribed. The task in example [b], asks students to examine the attributes of the circle, such as the centre, the radius, chords and the angles to determine the relationship between these angles. The aim of this task is for students to develop a new way of thinking about geometric objects. That is, to view them in a relational way. In this particular task, students should also recognize and articulate the converse case, namely that the angle at the circumference is half the size of the one formed at the centre. Example [c] was one of the most popular tasks featured in this research because more interesting ideas emerged from it. I discuss this task in greater detail in an episode in Chapter 6. The concepts pertinent to task [c] involve ideas such as the meaning of 'inscribed', circumscribed, the concyclic property of a cyclic quadrilateral and the term 'cyclic quadrilateral' in itself. A cyclic quadrilateral is a four-sided polygon that is inscribed in a circle with the circle passing through all four of its vertices.

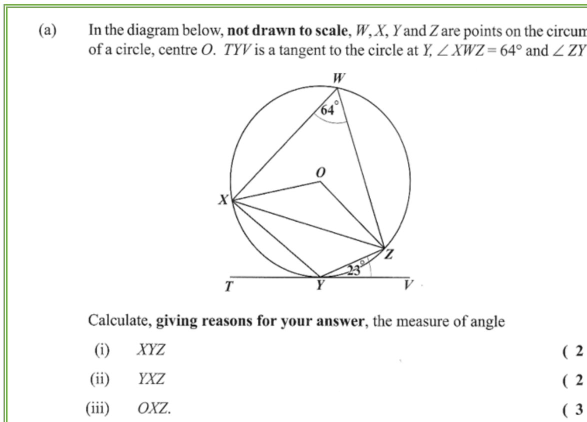
Reflecting on my practice as a mathematics teacher at the secondary level, I noticed that students were more likely to construct a rectangle rather than any other quadrilateral. I could offer many reasons why this is so, but I chose to examine my own teaching practice. I had not provided students with the opportunity to "discover", "challenge their curiosity" or "find joy and tensions" as Pólya suggests in the quotation at the beginning of this chapter. Indeed, the topic was taught in a static environment. Needless to say, it was important for me to experience students learning and thinking about this particular task in a dynamic way.

The example in part [d] was an interesting one. Interesting in the sense that while some students examined this task beyond its boundaries, many students were unable to discuss the perpendicular relationship between the chord and the line connecting it to the centre of circle. This task requires students to think about how the chords in a circle relate to its centre. Again, students are unable to reconcile the tensions they faced in understanding first, the concept of equal chords and second, that in Euclidean geometry it is the shortest distance that give rise to the perpendicular relationship between the two lengths.

5.3. Why Circle Geometry Theorems?

As mentioned in the previous chapter, it is customary for students in the Commonwealth Caribbean region to write the Caribbean Secondary Examination Certificate (CSEC). This exam is equivalent to the Ordinary Level (O-Level) examination that the British system follows and marks the end of five-year secondary level education. While the Caribbean continues to lag behind other regions in mathematics performance, Jamaica has had a reasonable amount of dismal performances. According to Bourne (2019), only 37.70 per cent of the eligible secondary students in Jamaica were able to obtain a passing mark on the exam in the year 2000 and in 2016 only 44 per cent attained a favourable mark. Although this data reflects a slight improvement, it should be noted that this improvement was achieved over approximately two decades. Geometry is considered one of the leading topics contributing to students' poor performance, especially circle geometry theorems. Table 5-2 shows two examples of a typical circle geometry theorem question on a CSEC examination.

Table 5-2 Past CSEC circle geomerty exam questions



(a) In the diagram below, **not drawn to scale**, W, X, Y and Z are points on the circum of a circle, centre O. TYV is a tangent to the circle at Y, $\angle XWZ = 64°$ and $\angle ZY$

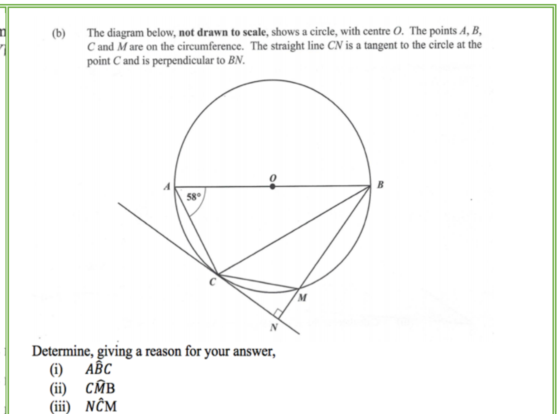
Calculate, **giving reasons for your answer**, the measure of angle

(i) XYZ (2

(ii) YXZ (2

(iii) OXZ. (3

[a] Question 10, May 2011



(b) The diagram below, **not drawn to scale**, shows a circle, with centre O. The points A, B, C and M are on the circumference. The straight line CN is a tangent to the circle at the point C and is perpendicular to BN.

Determine, giving a reason for your answer,

(i) $A\hat{B}C$

(ii) $C\hat{M}B$

(iii) $N\hat{C}M$

[b] Question 9, May 2018

According to the Caribbean Examinations Council, (2014/2020), circle geometry question 10, May 2011, Table 5-2 [a] "tested the candidates' across the Caribbean ability to use the properties of circle and circle theorems to determine the sizes of angles" (p. 349). The report shows that, while the question was attempted by 45 per cent of the students, only one per cent obtained the maximum mark. In addition, the average mark was 1.57 out of 7. The report further suggested that, despite students' knowledge about the size of angle XYZ, they were unable to give a justification for their answer. Furthermore, the report revealed that most of the students were unable to "interpret and apply the alternate segment theorem to correctly find the value of angle YXZ" (p. 349). Similarly, question 9, May 2018 (Table 5-2 [b]) was poorly done by students. According to the same report, question 9's objective served the same purpose as question 10 – to test students' ability to solve geometric problems using properties of circles and circle theorems. This question was geared towards specific application of two circle geometry theorems involving: (1) angles in a cyclic quadrilateral (2) the angle in a semi-circle. In this particular year all the students who sat the exam attempted the question. However, only 1.15 per cent obtained maximum mark while the average mark was 1.62 out of 12. Moreover, the report claimed that "many candidates were unable to quote a correct circle theorem to support their answers" (p. 486). Based on these results, the examining body recommends two things:

- that teachers engage students in more practice on problems associated with circle theorems;
- that teachers should use a systematic approach to provide students with sufficient exposure and practice in solving problems based on circle theorems.

Besides the alarming underperformance of Caribbean students on the circle geometry questions, I find the recommendations disturbing because there seemed to be little or no change in students' performance despite a 7-year difference in the reporting periods. To suggest 'more practice' seems paradoxical to me, considering that students' knowledge of the theorems is not secure. What then will they practice? In addition, it seemed that students' exposure to this topic was concentrated around experience in a static environment (as was the case with my two participating schools) and very little was being done to promote the use of DGEs. As a result, I deliberately chose to introduce the *Sketchpad* as a tool to encourage new ways of thinking about properties and relationships between the objects of a circle but, most importantly, to provide an option that will encourage change.

5.4. Issues Relevant to Teaching and Learning Geometry

Keith Jones (2002) presents geometry as a beautiful topic to teach. Pointing out the potential for interesting conjectures and surprises, its ability to attract various teaching approaches and “its appeal to our visual, aesthetic and intuitive sense” (p. 122). On the contrary, Jones also suggests that teaching geometry come with challenges. For example, he argues that the topic can be demanding to teach because it encompasses many different aspects of our lives, adding that geometry takes into consideration cultural and historical context. His suggestion is that “presenting geometry in a way that stimulates curiosity and encourages exploration can enhance students learning” (p. 125).

Osta, Laborde, Hoyles, Jones, Graf and Hodgson (1998), like Jones, foresees great potential for computer technology and the teaching of geometry claiming that of all the mathematical strands, it has the most influence on the development of technology. Using this claim as a foundation, they propose two views of technology: one that sees the technology as an essential component of a new culture, involving new ways of work and life, and the other which sees the technology as a tool for modelling not just the physical world but other situations as well.

5.5. Learning Geometry in a Dynamic Environment

In a case study about the computer as part of geometry learning, Colette Laborde (1993) suggests that the availability and advancement of technology has changed the way geometry is taught in schools. In fact, she argues that computers offer more than drawing capabilities that are faster and more efficient than static environments. They also provide an opportunity to think about shapes in different ways. For example, DGEs offers shapes the ability to move, to be distorted and to be decomposed. This provide users with the ability to visualize, explore and make conjectures. However, Laborde believes that technology has changed the status of geometric figures, as well as the behaviour of students when performing tasks on the computer.

Sinclair, Bartolini Bussi, de Villers, Jones, Kortenkamp, Leung & Owens (2017) view the advancement of technology as presenting more challenges to geometry education. In a recent survey, based on research about geometry education published since 2008, they suggest that including the use of new technologies in geometry teaching and learning has not been as fruitful as might have been expected. This is because technology use in geometry is still not sufficiently well understood, especially in relation to teacher practice. They also imply that, despite the ease in access and availability of technology and the modest attention on research involving its use, exploring its capabilities is limited.

However, they propose that if more attention is given to the introduction and design of new technology and the development of more robust theory and methodology to examine various roles of technology, then there should be an improvement in the teaching and learning of geometry. Sinclair et al. also highlight the role of technology in developing learners' spatial abilities as one issue relevant to the use of technology in geometry education.

It is clear that technology integration especially for the teaching of geometry and by extension that of circle geometry, is complex and more work is needed to be done to address its current limited use. My hope is that the findings of my research will not only add to this discussion but will open opportunities for any possible relationship between digital technology and the affective dimension of students' mathematical experiences.

5.6. Summary

In this chapter, I have provided a brief synopsis of the mathematics strand of geometry and speak specifically about why I chose the topic of circle geometry for my research. While researchers have pointed to the beauty in geometry and its potential to stimulate curiosity and encourage exploration, others are of the view that despite the upheaval in technology in recent times there is still a need for a better understanding of how to effectively integration technology in the teaching of geometry. As a result, there is a call for more to be done in terms of theoretical and methodological development. I utilized statistics from the CSEC examining body in the Caribbean to present an argument that students' performance in circle geometry theorem is dissatisfying and needs urgent attention. One way of addressing this issue is by promoting the teaching of geometry in dynamic environments.

Chapter 6. Results

> "The chief Malady of man is restless curiosity about things which he cannot understand; and it is not so bad for him to be in error as to be curious to no purpose." (Blaise Pascal, 1662/1910)

This chapter presents an analysis of students' interactions with and discussions about circle geometry theorems. Using the Embodied Curiosity theoretical framework, as discussed in Chapter 3, the objective of this research is to determine in what ways curiosity, body movement, digital technology and mathematical meanings relate to one another. Furthermore, this research aims to establish how these elements – as a whole – influence the teaching and learning of mathematics. In order to accomplish this, the element of curiosity (in this research context) must be fully understood. As a result, the data analysis also aims to identify physical markers of curiosity in the mathematics classroom. With these goals in mind, my intension is to stimulate discussions about and considerations of the role that human emotions, and specifically curiosity, play in mathematics education. A broader aim is to present new ways of thinking about the teaching and learning of mathematics, with a focus on the affective and digital technology in research.

Subsequent to the methodology described in Chapter 4, using Berlyne's (1954) polarized curiosity dimension model as a methodological instrument became useful only to identify excerpts of classroom video from the data. In order to determine when and how curiosity emerged, I adapted his model towards a more unified approach. I take as a starting point that both perceptual (sensation-seeking) and epistemic (knowledge-seeking) curiosity, in Berlyne's terms, work hand-in-hand when students interact with the digital technology. By adapting this approach, a window was opened to one of Pickering's (1995) main assumptions about agency: that is, human and non-human agency act upon each other in a back-and-forth interplay that decentres the human. This means that analysis of the data in this chapter is based on the premise that curiosity is not assigned solely to the students, but rather emerges when they interact with the technology. Furthermore, since both human and non-human agency participate in the interaction, this implies that curiosity is temporally emergent, which makes it unpredictable. Therefore, the excerpts described and analysed in this chapter are based upon these principles.

As described in Chapter 4, I began by identifying short excerpts from the video-recordings that suggested instances of each element of Embodied Curiosity. For example, when a student used a hand gesture to demonstrate that the diameter extends from one end of the circumference to another and runs through the centre of the circle, I classified those actions as body movement. Also, students' visual fixation, eye gazes and facial expressions while performing a task were classified as body movements. These were also seen as triggers for curiosity. In addition, students' reactions to the draggability, aesthetic appeal and symmetrical potential of *Sketchpad* was selected as instances of the role digital technology plays in the experience. Furthermore, students' explicit and implicit accounts of a 'new' mathematical ideas were selected as the emergence of mathematical meanings.

I took this initial individualized approach because I was interested in locating common themes between each element of Embodied Curiosity. Instead, this individualized approach became useful in establishing two things. Firstly, that there was a need for the identification of physical markers of curiosity, which was pertinent to my research question one. The second point was that looking at each element from an individualistic standpoint helped me to determine that curiosity emerged through a combination of several factors and that these factors were derived from all the elements of Embodied Curiosity. As a result, Embodied Curiosity was seen as an overlaying framework.

6.1. The Layered Structure: How it Works

Although it was evident that relationships exist between the four elements of Embodied Curiosity, I was more interested in how these relationships occurred through a 'stratigraphic' lens, that is, stacking one element onto the other. In the model shown in Figure 4, Embodied Curiosity emerges as an overlaying structure with curiosity considered the main component of the process and is represented by the first layer. The other layered panes represent the remaining elements and relational-curiosity is the entanglement with trait-curiosity, body movement and the digital technology, which was expressed in Chapter 3. The layers are understood to be semi-permeable to accommodate the movement of instances from one layer to another. By "instances", I mean factors such as students' wondering questions ("what ifs?", "suppose…"), visual fixation, eye gazes, facial expressions and curious dragging, among others.

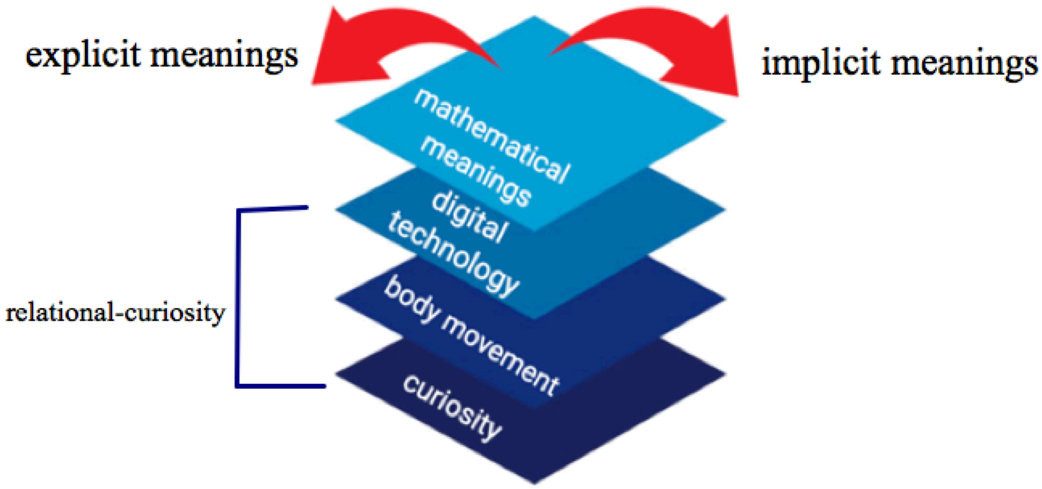


Figure 4: The Embodied Curiosity Model

The semi-permeability of the layers suggests that not all instances are deemed as potential triggers for curiosity. For example, while visual fixation (as a sensation-seeking stimulus) may be considered as one way of triggering curiosity, its movement could be restricted. This is because visual fixation could also be interpreted as attentiveness. However, in order for visual fixation to move from the layer of curiosity to the layer of body movement it must be accompanied by a question demonstrating some sort of uncertainty. Furthermore, visual fixation could also move through the digital technology layer, since students usually sought clarity from the computer. In the final layer (mathematical meaning), there is the students' seeking of a 'new' mathematical meaning. This overlaying of the elements onto each other suggests that they are deeply dependent on one another and that, for Embodied Curiosity to take place, the instances which pass through each layer should percolate through to the final one. In this way, the relationship between the elements are seen as temporal and emerging over time. This makes Embodied Curiosity an on-going, situated process.

Having identified the possible relationships between the four elements of Embodied Curiosity based on the model in Figure 4, I attended to the data using this overlaying technique. In applying it, I examined first the transfer of instances between the layers starting from the layer of curiosity to that of body movement. In doing so, I used visual cues such as actions of the eyes (eye gazes) and of face (facial expressions). An example of the latter can be seen in Figure 5 [a], followed by a description of episodes illustrating how the data was interpreted using the overlaying lens.

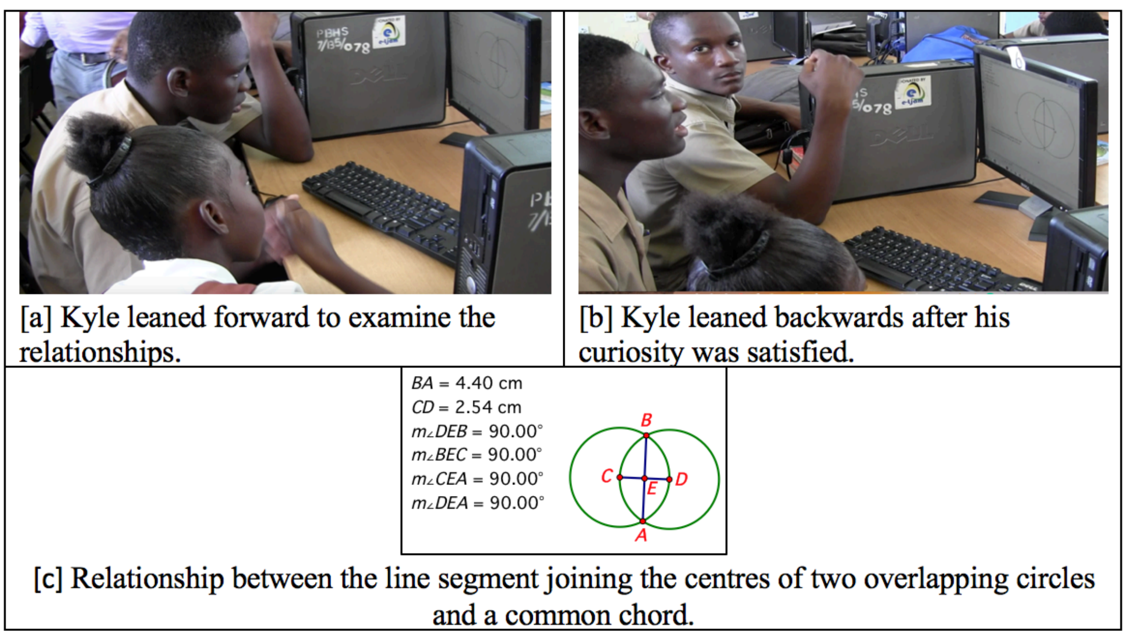
Figure 5: Snapshots of Brea's facial expressions

In Figure 5. [a], Brea, a student at School Y, listened attentively while the teacher (Andrew) gave the task for the day. Her attentiveness in this case could not pass through the semi-permeable layer because being attentive does not always imply that curiosity is activated. However, Andrew gave the task by asking, "What do you notice with the angles in an inscribed quadrilateral?" As soon as he introduced the word 'inscribed', Brea raised her eyebrows (Figure 5. [b]) and then released them into a wide-eyed gaze accompanied by a gasping of her mouth (Figure 5. [c]). When Andrew said the word 'quadrilateral', her facial expression suggested that she was uncertain about what was meant by an 'inscribed quadrilateral'.

This uncertainty was further expressed in her question, "wat a wat sir?" **["What is what, Sir?"],** which suggested that she was unsure about the meaning of the terms. At this point, Brea's curiosity, coupled with her facial expressions and her question, transitioned from the layer of curiosity to the layer of body movements. In this sense, it seemed that her body movements were instrumental in determining her uncertainty. Furthermore, Brea's immediate reaction after her facial expressions was to seek a response from the computer (Figure 5 [d]). This led me to believe that within the overlaying structure, Brea's body became entangled in a relationship with the digital technology, which then created the possibility for relational-curiosity to occur and movement to take place from the layer of body movement to the layer of digital technology. The episode that follows reflects this transition.

In Figure 6. [b], Kyle (the middle student), from School Y, along with his peers Jeff and Eli, illustrated the possibility of movement from the layer of body movement to the layer of digital technology. The group was investigating the relationship between the line segment joining the centres of two overlapping circles and their common chord (Figure 6. [c]). The students managed to complete their construction and began their investigation by first measuring the lengths of the common chord and the line segment connecting the two centres. Despite a few minutes of dragging the end-points of the line segments around, they were still unable to tell whether or not there was a relationship between the measurement of the two line segments.

To scrutinize the diagram further, Kyle leaned forward (Figure 6. [a]) and began to measure the angles formed at the point of intersection (Point E on the diagram). Here, his body movement of leaning forward to study the diagram and then the measuring of angles (rather than the measurement of the lengths) indicates that there was a shift in focus from one attribute of the geometric shape to another. Once he recognized that the angles were all 90˚, he then leaned backwards (Figure 6. [b]) and said, “mi get eh! All a dem a ninety” **[“I got it! All of them are ninety”]**, as if the digital technology had ‘handed’ him the knowledge he was seeking.



[a] Kyle leaned forward to examine the relationships.

[b] Kyle leaned backwards after his curiosity was satisfied.

[c] Relationship between the line segment joining the centres of two overlapping circles and a common chord.

Figure 6: Snapshot of Kyle's body movement while performing a task

In this case, it seems that the digital technology did not solely act as a consolation for disequilibrium, as in Brea's case, but also accentuated the performative nature both of the student and of the digital technology itself. In that, Kyle acted (performed the action of measuring the angles) and the technology 'handed' him the angular measures, which was implied when he said, "I got it! […]". His upper body also played a significant role in this interaction. His forward-and-backward movement or leaning inwards and outwards, implied that curiosity was triggered (leaning inwards) and that the mathematical meaning which he sought (the perpendicular relationship between the two line segments) was satisfied (leaning outwards). Here, the digital technology could be seen as the stimulus that triggered curiosity and the student's body reacted to this stimulus, which further generated mathematical meanings, namely that the angles are ninety degrees. This sets the stage for movement from the layer of digital technology to that of mathematical meanings.

In this next example, I describe the movement from the layer of digital technology to that of mathematical meanings. This was done in a two-pronged approach, bearing in mind the epistemological and pedagogical potential of Embodied Curiosity, which I alluded to in Chapter 3. The focus of this overlaying stage is on the generation both of explicit and of implicit mathematical meanings: that is, I described the kind of instances that were mathematical in nature and were spoken or unspoken. These were the kind of instances with potential to move from the layer of digital technology to that of mathematical meanings. In terms of implicit mathematical meanings, attention was given mainly to the ones which were deemed as 'teachable moments.[17] With this in mind, the example that follows was extracted from the data to demonstrate movement between the last two layers. This interaction was conducted by me and involved two girls (Ali and Joni) from School X. The girls were working on task eight (Appendix A), which asked them to investigate what will happen to the locus of the mid-point of a chord in a circle. In this task, the students were required to fix one end-point of the chord and drag the other along the circumference. With the trace tool activated, the locus of the mid-point of the chord depicted in Figure 7. [a], in red, appeared as a circle.

[17] A teachable moment is an inadvertent circumstance that emerges from an interaction and gives the teacher an opportunity to offer an insight to the students.

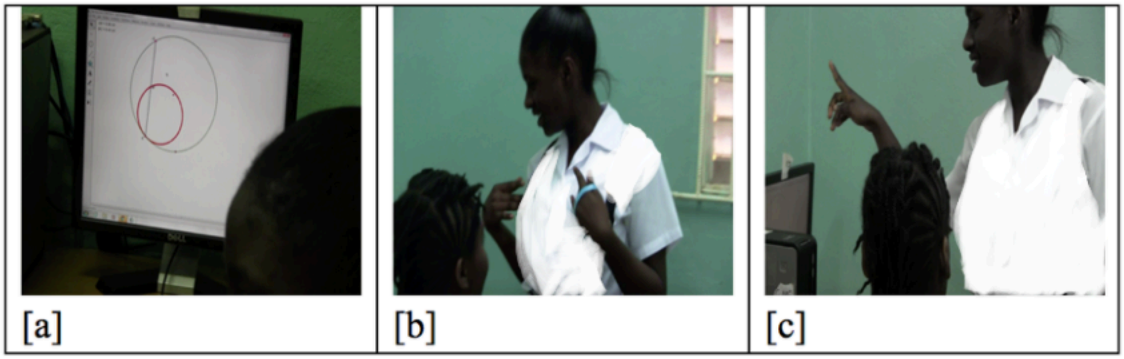


Figure 7: Ali explains the chordal mid-point

Ali volunteered to share her observation with the class (Figures 7. [b] and [c]). In doing so, her explanation, which could be described as animated (due to the heavy use of gestures and facial expressions) illustrated a back-and-forth entanglement with the digital technology. She repeatedly turned towards her audience and glanced back at the computer in a manner that suggested she was seeking validation from the computer. In Pickering's sense, both Ali (her gestures, head movement, turning to and away from the computer and her facial expressions) and the technology were agents acting on each other to bring about stability – an explanation for what happened to the locus of a chordal mid-point. The stability came when she was able to articulate the 'new' knowledge with affirmation from the technology.

In a similar way to Kyle, the technology and Ali entered into a 'partnership': this time 'working together' to unlock the mathematical meanings. This entanglement, it seemed, played an important role in triggering mathematical meanings and, more importantly, positioned this instance of partnership to transition between the layers. In such a circumstance, mathematical meanings were split into two categories: explicit mathematical meanings (relating to students' learning) and implicit mathematical meanings (relating to pedagogical practices). While Ali was able to state explicitly that, "the locus of the mid-point of the chord is a circle", the transcript (see Appendix B for transcript convention) reflects an example of a conversation where mathematical meanings emerged implicitly.

1 Me: What did you noticed? ~ ~ Everybody listen!

2 Ali: Class, what you have noticed, like I have, part of the circle is on the circumference of the bigger circle. unnu undastan weh mi a seh? **[Do you understand what I am saying?]**

3 All: Yes! Yes

4 Me: What she is saying is that there is a point which is shared exactly with the circumference of your original circle. Right?

5 All: Yes Miss!

6 Me: Has anybody ever heard about a word called 'tangency'

7 Ali: Yes Miss! But mi no memba weh eh mean [**Yes Miss! but I don't remember what it means]**

8 Me: Yes! Does that word fit right here?

9 Ali: Miss, ~ like tangent and sine and cosine?

In Turn 2, Ali expressed that, "part of the circle is on the circumference of the bigger circle", implying firstly that the locus of the midpoint is a circle, but also, more importantly, that these two circles shared a point called the point of tangency. My attempt in Turn 6 to mobilize and insert this knowledge into the students' mathematical register was demonstrated in the utterance, "Has anybody ever heard about a word called tangency". By saying "ever heard", I recalled their previous knowledge (Turn 9) and linked their previous knowledge to the new word. This episode not only highlights how instances pass through the semi-permeable layer, but also illustrates the productive potential of Embodied Curiosity. By interjecting the term tangency in the interaction, the overlaying structure could be re-established.

6.2. Data Presentation

The 12 hours of video-recordings, as discussed in Chapter 4, were reviewed, and a selection of potential evidence for Embodied Curiosity was made. Having established the overlaying structure of the theoretical framework from the selected data, I proceeded to record and present the data using a similar overlaying structure. The episodes which formed the corpus for analysis was selected on the sole basis that all four elements of Embodied Curiosity were present and that they illustrated some form of connection. For example, when curiosity was detected, the body was also at play, or when students performed a task with the technology, some new mathematical meaning emerged.

The data also comprised of episodes which were not reported in details as part of the data presentation or analysis (see Table 6-3). I did not offer a detail description of these episodes because my main focus was on Embodied Curiosity and the construction of mathematical meanings, and not non-examples. In the instances where limited evidence existed, I could not distinguish between curiosity and interest or curiosity and attentiveness. As a result, I only selected the episodes that illustrate potential for Embodied Curiosity with evidence of all four layers: those with partial potential were not retained for analysis. In other words, the relationships among curiosity, body movement, digital technology and mathematical meanings could not be activated. For example, in School Y, a student named Chelly used her hand to illustrate that the diameter of a circle passed through the centre point and, while this was as a result of her engagement with the technology, there was no evidence of curiosity at play. As a result, Embodied Curiosity could not be activated. There were also episodes which illustrated the emergence of curiosity, but due to my role as participant observer, video-recording, especially when I was in the role of participant, did not capture some episodes in their entirety. As a result, there were some critical moments that were missed.

I have captured the episodes with the most convincing evidence of the elements of Embodied Curiosity in Table 6-1. I have colour-coded them using the following:

Green – Curiosity

Yellow – Digital Technology,

Blue – Body Movement,

Orange – Mathematical Meanings (bright orange is for explicit meanings and light orange for implicit one)

This was useful in locating each layer of the framework. Table 6-2 was extracted from Table 6-1 because I was concerned both with explicit and with implicit mathematical meanings and I felt the need for a separation of both. In Table 6-2, a further colour-coding was done. I used the bright orange highlighter to represent explicit mathematical meanings and the light orange highlighter to illustrate implicit mathematical meanings, which I considered teachable moments. The explicit mathematical meanings were observable through the diagrams and were spoken by the students, while the implicit ones took into consideration meanings which were observable and unspoken, as well as those which were not observed but were hinted at.

Table 6-1 Video data from both schools showing instances of Embodied Curiosity

Task	Participants	Record of the elements of Embodied curiosity
[1]. Angle in a semi-circle	Dani & Mio (SchX)	Dani seemed uncertain about the diameter when he drew the radius for the diameter and engaged in curious dragging while he examined the behaviour of the angle. He leaned sideways to examine how symmetrical his diameter was, used his left hand to indicate a flip (reflection). Mio leaned forward while tending to the angular measures. He used his index finger to illustrate the diameter. Mio engaged in curious dragging of the angle until the triangle collapsed into a line.
[2].Cyclic Quadrilateral	Ali & Joni (SchX)	Ali had a bewildered look on her face as soon as the task was given. She used her arm to show that a 90^0 angle was formed. She seemed uncertain about what an inscribed quadrilateral was; she asked "should they have the same measurement?" and further engaged in curious dragging in order to place the vertices of the quadrilateral on the circumference of the circle. A collective group of students (including Ali & Joni) used their bodies to show that opposite angles in a cyclic quadrilateral are supplementary.
[2]. Cyclic Quadrilateral	Kami, Gen & Brea (SchY)	Brea seemed uncertain about what an inscribed quadrilateral was. This was based on her facial expressions (raised eyebrows, gasping).
[3]. Cyclic Quadrilateral	Collective group SchX	Students used their bodies to show that the maximum chord in a circle is the diameter.
[4]. Length of a chord in a circle	Mio & Jenny (SchX)	Mio engaged in curious dragging to examine how the task changed when the centre point moved and also to test his conjecture.
[5]. Isosceles triangle in a circle	Kyle, Jeff & Eli (SchY)	Kyle tried to recall the types of triangles, but was unable to differentiate between a scalene and an isosceles triangle. It seemed he was uncertain. Kyle leaned forward to show where the missing side completes the triangle while Jeff and Eli were visually fixated on the screen. Jeff seemed uncertain about how to draw the triangle and he passed the mouse to Kyle.
[6].Chordal mid-point	Roni & Akeel (sch X)	Akeel seemed uncertain of the pronunciation and meaning of the word 'locus'. Roni asked "the one with the t or without?" They used the dictionary to distinguish between 'locus' and 'locust'. Roni engaged in curious dragging to investigate the path of the midpoint and to revisit the meaning of the word 'locus'. The girls leaned forward to examine the meaning of the words and backwards when the meaning was achieved.

Table 6-2 Explicit and Implicit meanings extracted from Table 6-1

Task	Participants	Mathematical Meanings
[1]. Angle in a semi-circle	Dani & Mio (SchX)	Both boys expressed that the angle in a semi-circle is a right angle. Dani used hand gestures to show A flip and a turn. This represents a composite transformation.
[2].Cyclic quadrilateral	Ali & Joni (SchX)	Ali explained that opposite angles in a cyclic quadrilateral add up to 180 and that all the angles in the quadrilateral add up to 360 which is the same measurement of a circle. In re-enacting the task with their bodies, Ali suggested that the students forming the inscribed quadrilateral should hold hands. hinting the concyclic property of the cyclic quadrilateral.
[2]. Cyclic quadrilateral	Kami, Gen & Brea (SchY)	The four angles in a cyclic quadrilateral can be 90˚.When the angles are 90˚ a rectangle is formed in the circle.
[4]. Length of a chord in a circle	Mio & Jenny (SchX)	Jenny explained that the distances from the middle (centre point) to the chords are equal. There is a perpendicular relationship between equal chords and their distance from the circle.
[5]. Isosceles triangle in a circle	Kyle, Jeff & Eli (SchY)	All students were able to explain that the triangle formed using the centre point and two random points on a circle is isosceles. When the angle formed at the centre is 90˚ a right-isosceles triangle is produced.
[6]. Chordal mid-point	Roni & Akeel (Sch X) Ali & Joni (SchX)	Both Roni & Akeel were able to tell that the locus of the mid-point of a chord is a circle. The locus of the mid-point touches the original circle implying that both the locus of the mid-point of the chord and the circle shares a point of tangency.

Table 6-3 below, captures episodes from both schools where some elements of Embodied Curiosity were present. However, the Embodied Curiosity process did not materialize in these episodes due to the absences of one or more elements. For example, in the episode with Shané and Cami I could not detect uncertainty because the students were not asking 'what ifs' questions and their facial expressions could not be observed from the video-recordings. When this occurred, it may have been as a result of a missed opportunity while video-recording was being done as mentioned in Chapter 4 or that the students did not have a need to know something in particular. In addition, the recording could have been done with the student 's back turned to the camera based on seating arrangement.

Table 6-3 Data from both schools showing non-examples of Embodied Curiosity

Task	Participants	Elements of Embodied Curiosity
[7]. Noticing invariance What type of triangle is formed?	Shané & Cami (SchX)	Shané engaged in curious dragging trying to get her triangle to collapse into a line. Cami used her finger to show the position of the point when the circle becomes a line. There was no evidence of uncertainty and they could not tell that the triangle would collapse into a diameter
[8]. Angle in a semi-circle	Kamla & Sam (SchY)	Kamla was excited when she engaged in curious dragging. She turned the upper part of her body (head, neck and shoulder) in a cyclic manner similar to the motion of the diagram on the screen. However, her attention was on the motion of the object but not on the mathematics.
[9] Cyclic Quadrilateral	Mio & Jenny (SchX)	Mio engaged in curious dragging. Body movements was not evident from the video-recording. At the moment of dragging the students were not attending to the mathematics. They were asked to examine the angles but they did not take a measurement for the angles to see while dragging was done.

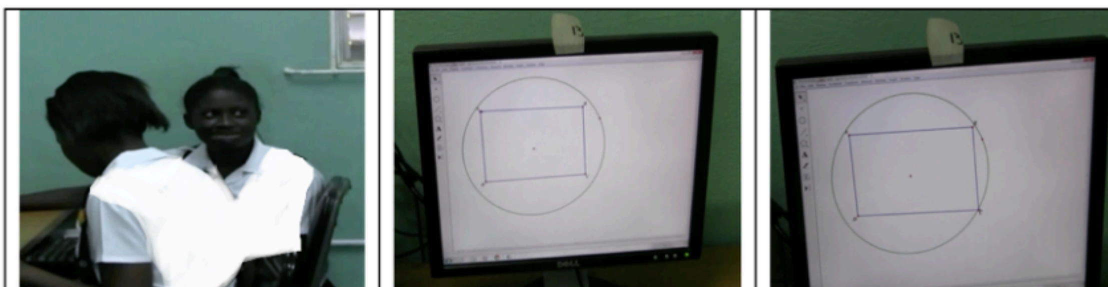
The section which follows is a comprehensive analysis of the data as observed through a unified Embodied Curiosity lens. Given the overlaying nature of the relationships within the Embodied Curiosity framework, and the temporal nature of each element, I present episodes from Tables 6-1 and 6-2 to illustrate how Embodied Curiosity played out in the students' interactions, as well as episodes where the teacher's role was evident.

6.3. Data Analysis

Tables 6-1 and 6 -2 shows that curiosity, body movement, digital technology and the emergent of mathematical meanings were evident when the students engaged with circle geometry theorems. This was seen both in School X and in School Y. To provide evidence and depth to the analysis, I have included pictures and diagrams of the students' work throughout each episode. In addition, the first two episodes give an account of how learning takes place through the lens of Embodied Curiosity, followed by two episodes about the role of the teacher within this framework. The latter two episodes illustrate the potential for Embodied Curiosity to extend beyond geometric tasks into the real world.

6.3.1. Analysis of Students' Learning: Ali and Joni

In this session, Ali and Joni were working together on task two (Table 6.1, Task [2]), which asked them to examine the relationship between the opposite angles in a cyclic quadrilateral. The task further required students to describe what they noticed when one, two or three of the angles remained right angles. Sammy (the classroom teacher) and I interchanged roles, as was discussed in Chapter 4. I was leading the lesson while Sammy was responsible for video-recording. We made this agreement prior to the beginning of the session and students were already familiar with our role-switching strategy. I announced the task orally and encouraged the students first to listen before they started, since the task would not be given in the step-by-step manner to which they were accustomed with their teacher.



[a] Ali's bewildered look. [b] quadrilateral detached from the circle. [c] Ali engaged in curious dragging to get the points onto the circle.

Figure 8: Snapshots of Ali and Joni's curiosity about the cyclic quadrilateral

As soon as the task was given, Ali, with a bewildered look on her face (Figure 8. [a]) asked, "Should they have the same measurement?", showing her first sign of uncertainty. The combination of her facial expressions (bewildered look) and her immediate question signalled an initiation of curiosity. These combined factors seemed to be a physical marker of curiosity and illustrated a possible movement between the layer of curiosity and the layer of body movement again a signal of relational-curiosity. The word 'they' in her question suggested constructing more than one shape. The words “same measurement” implied that, within the relationship between opposite angles in a cyclic quadrilateral, there was a need for angular measurements.

Furthermore, the girls’ initial diagram (Figure 8. [b]), with all four vertices detached from the circumference of the circle, reveals that there was a gap (in Loewenstein’s terms) in their understanding of the cyclic quadrilateral. This special quadrilateral usually requires all four vertices to lie on the circle. Ali briefly glanced at the other computer screens in the room then started to adjust her diagram by dragging each vertex of the quadrilateral onto the circumference. In a sense, Ali’s brief glance (wandering eyes) at the other computer screens, as well as the decision to drag each point of the quadrilateral onto the circumference of the circle (Figure 8. [c]) indicated that the eyes and the computer were engaged in a ‘partnership’: that is, the body (in this case, the eyes) performed the action of glancing at the other screens, which triggered a reaction of the hand (dragging the vertices onto the circumference) to the diagram on her screen.

This was perhaps to maintain consistency with the other diagrams that were visible in the room or to illustrate a better understanding of what the cyclic quadrilateral should look like. In doing so, Ali engaged in curious dragging,[18] that is, curious linked dragging (linking a point to an object and moved it onto that object), but in a slow and cautious manner. This implied that she was either in search of a piece of unfamiliar knowledge or fearful of creating potential problems with the diagram. In the former case, performing curious dragging in such a deliberate manner suggests that Ali’s engagement with the digital technology presented a possibility for movement between the layer of body movement and digital technology. This possibility is described in the upcoming transcript.

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[18] Curious dragging was operationalized in Chapter 2 as an extension of Arzarello et al.’s (2002) dragging modalities (linked, bound, wandering, dummy locus, guided, line and dragging test) to include temporality, speed and emotion.

Table 6-4 Snapshot and multimodal transcript of students re-enacting the task

Turns	ID:	Utterances
1	Me:	Yes, figure it out! <students moving about in the circle trying to position themselves> ~ figure it out!
2	Richie:	<pointing at the students representing the quadrilateral and then to a position on the circumference>. Yuh fi deh deh so, yuh fi deh deh so an yuh fi deh deh so. **[you should be there, you should be there and you should be there]**
3	Ali:	hol on pan im an nuh **[hold on to his hand]**
4	Me:	Why is it important for her to hold on to his hand?
5	Ali:	cause it mus be on the circumference **[because it must be on the circumference]**
6	Me:	Because it must be on the circumference, yes? go ahead, ~ ~ figure it out! figure it out!
7	Richie:	yuh a waan point! Yuh a waan point! an yuh fi deh deh so **[you are a point! you are a point and you are supposed to be there]** <pointing his finger to position each student>
8	Me:	So, hold har han yes, so yuh mus hold onto the circumference **[So hold onto her hand yes, So, you are to hold onto the circumference]**
9	Ali:	Hold on to the circumference!
10	Me:	Alright so we have one side of the quadrilateral here, how can we get the other side of the quadrilateral there? ~ hold on to him, put it on the circumference there yes. Alright so is everybody on the circumference? Everybody? ~~ okay can we step back, step back, step back? <everyone moved backwards>. So, who are the? persons that are opposite
11	Richie:	Dani and Mio!
12	Me:	So, what do we know about these two?
13	Ali:	They add up to one eighty
14	Me:	They add up to one eighty <everyone laughs> So, what is your name? <pointing to a female student>
15	Joni:	Joni!
16	Me:	So, what do you know about Mia and Ashley?
17	Joni:	We add up to one eighty
18	Me:	They add up to one eighty as well.
19	Ali:	Wait! They add up to 360
20	Me:	What about three sixty?
21	Ali:	Miss! A circle adds up to three sixty ~ ~ <rolls eyes as if she was thinking> and the four sides add up to three sixty.
22	Me:	Not the four sides but the four angles

The digital technology in this scenario played a significant role not solely as an agent acting with the student to perform actions, but also as a mediating tool for the re-enactment of the task. The data revealed that it was within this mediating capacity that the emergence of mathematical meanings was most prominent. For example, towards the end of the task, I asked the students to perform a re-enactment using their bodies. The image shown in Table 6-4, along with the accompanying transcript, reflects how mathematical meanings were triggered. The dynamic nature of *Sketchpad* helped the students to see that the position of each vertex on the circumference does not matter, since the quadrilateral is not limited to the rectangle or the square. These were the most dominant figures previously produced by the students on the computer. Instead of fixing the points in a specific position to create a rectangle or square in the re-enactment, students joined the quadrilateral to the circumference by holding onto the hand of a student who was a part of the circumference (Table 6-4). Despite Richie's suggestion in Turns 2 and 7 to position the students at specific places within the circle, his pointing action implied a random location. This action along with Ali's suggestion of holding hands, showed the emergent of the concyclic[19] property of the cyclic quadrilateral.

Furthermore, Ali and Joni were able to state explicitly, with conviction, that the "opposite angles add up to one eighty" (Turns 13 and 17). However, the relationship between the sum of the angles in the quadrilateral and the measure of a complete rotation in Turn 19 to Turn 21 was not so convincing. Ali's statements "the circle adds up" and "the four sides add up" (Turn 21) suggest that she was aware of the relationship, but was uncertain about which attribute of the shapes was to be measured. This mathematical connection did not surface when students performed the task on the computer. It seemed, then, that the dynamic capability of *Sketchpad* coupled with the configuration of the diagram which was left on the screen and their body movements acted as a trigger for mathematical meanings. Perhaps Ali connected these two concepts (sum of the interior angles of a quadrilateral and the angular measure of a complete rotation) when I echoed that the other two angles added to one hundred and eighty degrees. After saying "as well" (in Turn 18), she immediately exclaimed, "Wait! They add up to three-sixty" (Turn 19), as if my utterance activated her previous knowledge and triggered a connection between these two concepts. In this scenario, the digital technology was more than a 'partner', as mentioned

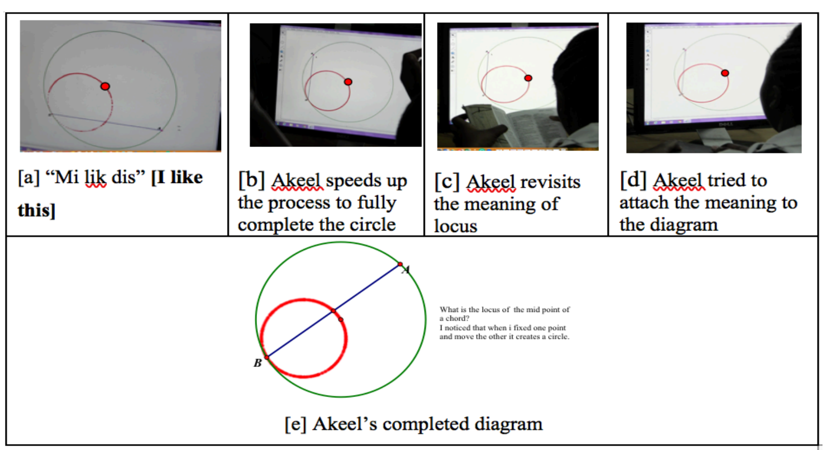
[19] The concyclic property states that a set of points are concyclic if they lie on a common circle.

in the previous episode, but instead acted as an 'instigator', unlocking and inciting mathematical meanings. This further led to the possibility for movement between the layer of digital technology and that of mathematical meaning.

This episode also exposed the problematic way in which the concept of 'a circle' appeared with multiple definitions. Here, the circle was construed to be a complete rotation or a full turn, which implies the definition of a circle as a plane figure bounded by a curved line. However, this conceptualization of the circle was later replaced by the definition of a circle as being a locus of a point equidistant from a fixed point (the centre of the circle). This became more prominent when the students engaged with Task 8, which involved the chordal midpoint. An episode concerning Roni and Akeel shed further insights into the emergent of these two definitions of the circle.

Roni and Akeel

Roni and Akeel, who also attended School X, were investigating the locus of the mid-point of the chord in a circle (Appendix A, Task 8). The aim of this task was to fix one point, making it invariant at the point of intersection between the circumference and the chord, while dragging the other point along the circumference. This task was also conducted by me and was given orally in a similar manner as Task 2 in the previous episode. Unlike Ali and Joni, whose uncertainties associated to their understanding of the mathematical concept of a 'cyclic quadrilateral', Roni and Akeel's initial uncertainty was a linguistic one. The girls were unable to differentiate between the meaning of the words 'locus' and 'locust'.



[a] "Mi lik dis" **[I like this]**

[b] Akeel speeds up the process to fully complete the circle

[c] Akeel revisits the meaning of locus

[d] Akeel tried to attach the meaning to the diagram

[e] Akeel's completed diagram

Figure 9 Roni and Akeel coming to terms with the meaning of locus(t)

In this interaction, I asked the class, “what is the locus of the mid-point of a chord in a circle?” Some of the students were baffled as they exclaimed “What!”, or asked “weh name so?” **[what is that?]** and “wah a wah Miss?” **[What is what, Miss?]**. I surmised that the word ‘locus’ was unfamiliar to them, at least from a mathematical point of view. My speculation was further confirmed from a paragraph the students wrote at the end of the data collection process.

The students were asked to share their experience with *Sketchpad.* One wrote, “I have learnt some new words such as locus and inscribe”, while another student wrote, “Through these sessions I have learnt some new terms like locus (without the ‘T’)”. I offered a spelling, “l-o-c-u-s” and as soon as I called the last letter ‘s’ a student continued and offered ‘t’. This created a conflict and signalled the first indication of uncertainty.

Akeel grabbed her English Dictionary in a bid to distinguish between the meanings of the two words. Both girls leaned forward into the book, as Akeel searched for the meanings. Upon reaching the page on which the words were located, she began to read “a large tropical […]”, which meant she was reading the definition pertaining to the insect – locust. This signalled a second phase of uncertainty as she asked, “which one a dem Miss, di one wid di ‘t’ or widout” **[which of the words miss, Is it the one with the ‘t’ or the one without?].**

At this point, her uncertainty and questioning implied that curiosity was activated and an opportunity was presented for a transition to the layer of body movement. Once this exchange is identified it signals the presents of relational-curiosity. By initiating the use of the dictionary, Akeel demonstrated that there was an initial desire for knowing the meaning of the word. Furthermore, her question implied that it was important in distinguishing between the meaning of ‘locus’ and ‘locust’, which needed to be resolved before the task could be performed. This disunity signalled again that curiosity was triggered; her body movement of leaning forward and backwards suggests that her curiosity transitioned to the layer of body movement.

In addition, Akeel engaged in curious dragging: this time – bound dragging – in a slow and meticulous manner. In this type of dragging, the student moves a semi-draggable point, which is usually linked to the object. The semi-draggable points in this case are points A and B (Figure 9 [e]), which can only be moved on the circle to which they belong. As Akeel fixed point B and slowly dragged point A along the circumference (Figure 9 [d]), she observed that a new circle had emerged (Figure 9 [a]). Upon seeing this, she became excited as she exclaimed “mi lik dis!” **[I like this!]**. Once she recognized that the path travelled by the mid-point of the chord was a circle, as indicated in Figure 9 [e], her dragging action gained momentum and became more vigorous. It seemed that the rationale for this was to produce a completed circle as seen throughout Figures 9 [b] to [e] – one with its circumference completely filled in. This highlighted the emergence of the second definition of the circle – the locus of all points equidistant from a fixed point – and a possible opportunity for a transition from the layer of digital technology to the layer of mathematical meanings.

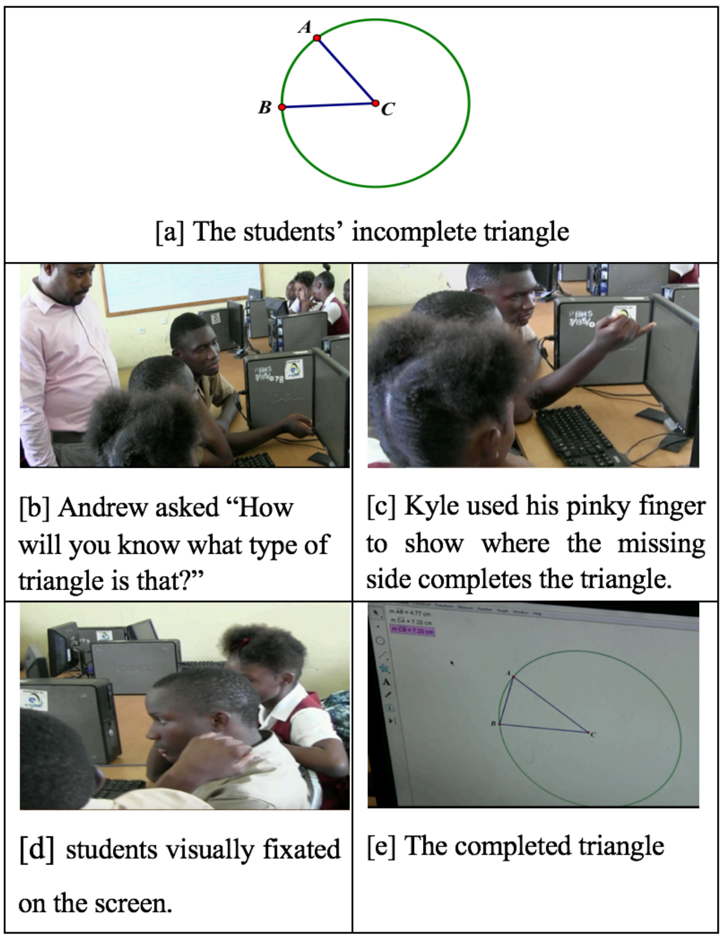
In this scenario, although the student was engaged in bound dragging, being excited could have played an important role in the speed in which the dragging action was carried out. This suggest that her emotion (excitement) was an instrumental factor in initiating curious dragging. Additionally, excitement could be an indicator that she was interested in knowing about the locus of the mid-point of the chord.

With the completed diagram on the screen, Akeel then revisited the meaning of the word ‘locus’ in the dictionary (Figure 9 [c]) and then became visually fixated (Figure 9 [d]), as if she were attaching the meaning in the dictionary to the diagram on the screen. Again, the diagram on the screen could be seen as playing the role of an ‘instigator’, this time prompting a re-examination of the mathematical meaning in the dictionary, while arousing Akeel’s emotion – excitement.

6.3.2. Analysis of the Teacher’s Role: Kyle, Jeff, Eli and Andrew

This next episode highlights the significance of the teacher’s questioning technique in an Embodied Curiosity environment. Although the emphasis is on the teacher’s role, I also chose to give a description of the students’ actions to show that the teacher’s use of questions might have been intentional and that this intentionality triggered the Embodied Curiosity process. In addition, I show that the teacher’s question had a direct link to students’ action and that persistence played an important role in occasioning students’ internal conflicts and uncertainties.

At School Y, Kyle and Jeff, two students who usually worked together in previous sessions, were joined by Eli on this day. They were working on a task together using the shared-computer arrangement outlined in Chapter 4. In this task, they were asked to identify which type of triangle was formed using the centre point of the circle and two random points on the circumference. Andrew (the teacher) was conducting this session and gave the task orally. In performing the task, Jeff began to accomplish this in the following order: the circle; two random points on the circumference; the line segments connecting the centre point to the random points on the circumference. This sequential action produced an incomplete triangle within the circle (Figure 10 [a]).



[a] The students' incomplete triangle

[b] Andrew asked "How will you know what type of triangle is that?"

[c] Kyle used his pinky finger to show where the missing side completes the triangle.

[d] students visually fixated on the screen.

[e] The completed triangle

Figure 10: Students and the incomplete triangle

Kyle called his teacher to their computer station, seeking validation about the diagram on the screen. Kyle asked, "Sir, something like this?". Andrew noticed that the triangle was incomplete (only sides AC and BC were drawn) and responded with a leading question, "But – but it needs one more <pause> that – that's a triangle now?". It seemed that Andrew shifted mid-sentence from a prescriptive question ("it needs one more") to a leading question (that's a triangle now?). He therefore refrained from telling the students that the triangle was missing a side and seemed to want her to notice what was missing herself. Since Andrew's questioning style in the past had tended to be prescriptive, this shift was interesting. After asking this leading question, Kyle then leaned forward and used his little (pinky) finger to perform a tracing action on the computer screen (Figure 10 [c]), indicating where the missing line segment should be drawn. At this point, the other students were visually fixated on the screen (Figure 10 [d]).

Andrew's question seemed to play an important role in drawing the students' attention to the missing side, which activated the Embodied Curiosity process. First, it seemed that the layer of curiosity was activated when Andrew's question triggered an exchange between Kyle and the computer. Kyle's action of leaning forward, then using his pinkie finger to react to the diagram on the computer screen, helped his peers identify the position of the missing side. This action signified that a transition from the layer of curiosity to the layer of body movement, and then subsequently to the layer of digital technology was triggered. In addition, Kyle's action led Jeff to complete the triangle (Figure 10 [e]) by connecting point A to point B. This suggested that a combination of these actions led to a completed triangle within the circle.

With the completed triangle on the screen, Kyle beckoned to his teacher a second time to join them at their computer station. This time Andrew used a probing question, "So how will you know what type of triangle is that?" (Figure 10 [b]). The way in which the task was posed ("What type of triangle […]") indicated that the teacher may have expected the students to recall one specific response – an isosceles triangle. In this case, the teacher's question served as an instrument to prompt the students' previous knowledge.

Besides the strategic and intentional use of the types of questions to recall, lead and probe students' knowledge, Andrew's questions were also repetitive and persistent. For example, after a few minutes had passed and the students had completed the triangle as well as measured the sides, Andrew asked the same question twice. The following transcript represents the conversation between Kyle and his teacher.

1 Andrew: So what type of triangle is that? ~ ~ Weh yuh notiz? **[What do you notice?]**

2 Kyle: Dem side yah equal **[These sides are equal]** < hovering the cursor over the two radii>

3 Andrew: hmm ~ an wen two sides equal wah kinda triangle is dat**? [and when two sides are equal what kind of triangle is that?]**

4 Kyle:<contemplating out loud> equilateral triangle ~ ~ di scalene triangle **[The scalene triangle]** < while using his finger to trace the triangles on top of his desk>.

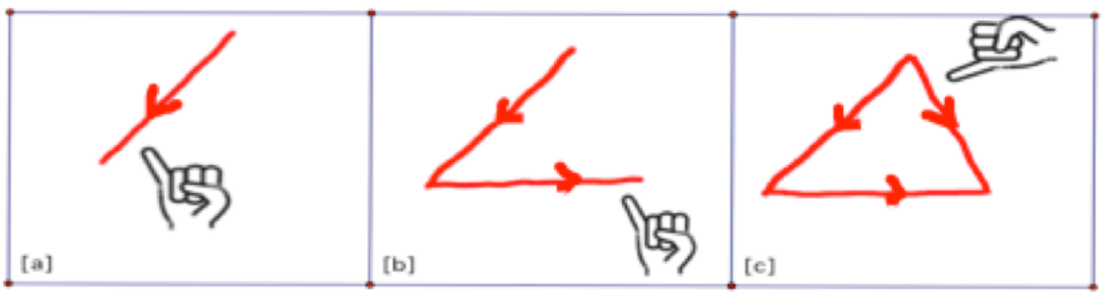


Figure 11: Kyle draws a triangle on the desktop – The scalene triangle

Andrew's repeated leading questioning, "What type of triangle is that?" (Turn 1), along with his probing question "what do you notice?" (Turn 1), helped Kyle re-examine his previous knowledge. This was evident in Turn 4, where he contemplated, out loud, the equilateral and scalene triangles. His body reacted to his wondering and he used his pinky finger to draw two triangles on the desktop, claiming the first to be equilateral (Turn 4) and the second to be scalene (Turn 4).

It seemed that his body movement (finger tracing) and his previous knowledge were entangled in an interplay which led to a conflict between associating the name of the triangle with the property. Andrew's persistence in guiding the students to a specific vocabulary word was evident in Turn 3, when he said, "and when two sides are equal, what kind of triangle is that?". This scenario draws attention to the teacher's shift in focus from a necessary aspect of mathematics (the two sides having the same length) to an arbitrary aspect of mathematics (the vocabulary word) (Hewitt, 1999). It also highlighted the role of the teacher's persistence in bringing to the fore the internal conflicts that students may face when they develop mathematical meanings. The following transcript illustrates how the conflict unfolded and how the teacher's persistence brought the students' conflict to the surface.

5 Kyle: Equilateral, scalene, weh di triangle name agen? **[Equilateral, scalene, what is the name of the triangle again?]**

6 Jeff: Wah? ~ ~ One a dem a scalene! **[What? ~ ~ One of them is scalene]**

7 Kyle: Aah. Uhm! Mi no rememba weh did nex triangle name **[I don't remember the name of the other triangle]** ~ uhm Aaaah ~ if a (sic) equilateral triangle **[If it is an equilateral triangle]** ammm ~ yuh have a scalene

8 Andrew: but no! how yuh know wah kinda triangle it's gonna be? **[but no! How do you know what kind of triangle it is going to be?]**

9 Jeff: Is a scalene triangle! **[It is a scalene triangle]**

10 Kyle: yeah scalene triangle cause two sides dem equal and aam ~**[Yes, scalene triangle, because two sides are equal and]** ~

11 Jeff: One nah go equal. **[One will not be equal]**

In this transcript, the boys tried to remember the name for the different types of triangle by revisiting their properties (Turns 7 and 10). Kyle's argument, that the triangle on the screen must be scalene because the two sides are equal (Turn 7), shows that he correctly identified the relationship, but used the wrong vocabulary term. Although the triangle is not scalene, his utterance in Turn 10, as well as his drawing on the desktop (Figure 11), indicated that there was an uncertainty.

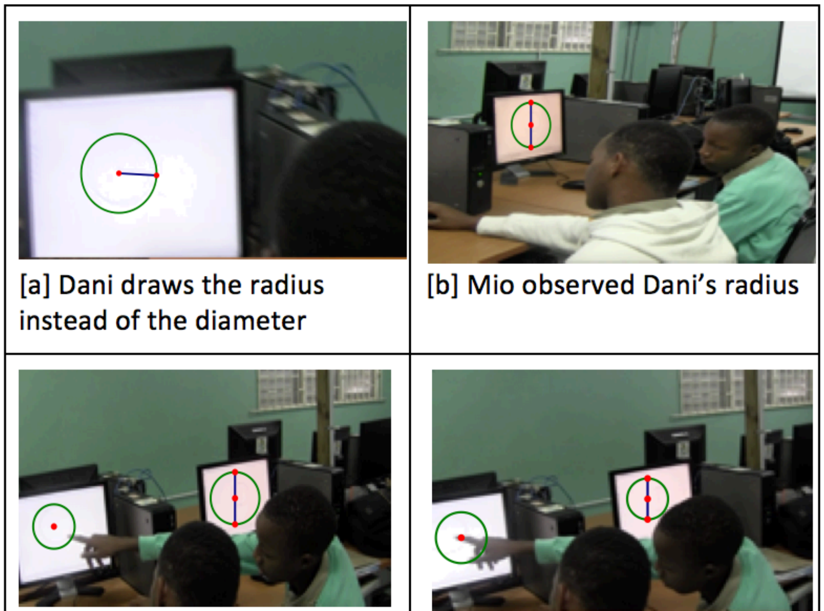
In order to redirect Kyle to the word 'isosceles', Andrew continued to prod him to name the type of triangle (Turn 8). By saying "but no […]" suggested that Andrew's intention was to guide the students to a particular response rather than opening the door to a mathematical meaning. From Andrew's questioning, it seems that what was important was for Kyle to recall and use of the word "isosceles". From my own point of view, it is the noticing of geometric invariants (two sides being the same) that was important.

Mio, Dani and Sammy

In this next episode, I highlight the role Sammy played in an Embodied Curiosity process. Like with Andrew previously, my emphasis is on the role of the teacher, but a description of the students' interaction will be necessary. This is to highlight that the strategies used by the teacher to introduce the mathematical tasks influenced how the Embodied Curiosity process was triggered. In addition, I show that by introducing the task in a step-by-step manner, the possibility exists for the identification of specific scenarios when students were uncertain. However, unlike Andrew, with this episode involving Sammy there were few opportunities for the Embodied Curiosity process to be developed and maintained using this approach.

Mio and Dani both attended School X and were working together on a task using the separate-computer arrangement discussed in Chapter 4. The task for the day was to have students explore what happens when an angle is formed in a semi-circle. Sammy (the class teacher) was in charge of the session and she asked the students first to use the circle tool to construct a circle. Contrary to Andrew, Sammy's approach in introducing the task was to provide the students with step-by-step instructions. This meant that they were given specific microtasks in a sequential manner to create a final product on the screen. First, she said, "You are to use the circle tool to construct a circle".

All the students were able to construct the circle, mainly because they were given both the tool (the circle tool) and the object (the circle). When the follow-up instruction was given, which was to construct the diameter, Sammy said, "Alright, you are to use the line segment tool to construct the diameter of the circle". Once again, the tool (the line segment tool) and the object (the diameter) were given. In this sense, by introducing the tool along with its corresponding object, she guaranteed a constant pace at which students would perform the task. Mio constructed his diameter and soon recognized that, instead of the diameter, Dani had drawn a radius (Figure 12 [a]). It can be inferred that Dani was not aware of the distinction between a radius and a diameter.



[a] Dani draws the radius instead of the diameter

[b] Mio observed Dani's radius

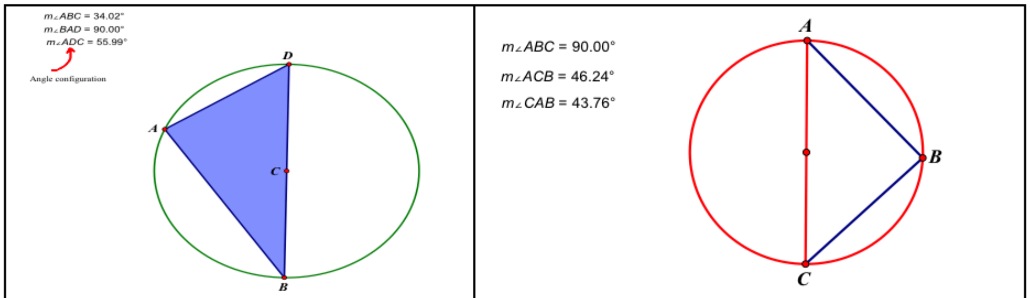
[c] Mio explains that the diameter should touch the circle

[d] Mio moves his fingers inwards to the centre of the circle.

Figure 12: The boys collaborate to construct the diameter

Mio exclaimed, "Dani! a di radius dat" **[Dani! that one is the radius].** Having observed (Figure 12 [b]) that Dani had drawn the radius (Figure 12 [a]), Mio leaned towards Dani's computer to offer support. Instead of telling Dani what to do, Mio quickly used his index finger to show that the diameter could have a possible starting point on the circumference of the circle (Figure 12 [c]). He further moved his index finger inwards (Figure 12 [d]), indicating that the diameter, while having a starting point on the circumference, should also pass through the centre point. While Mio negotiated a possible diameter, Dani was visually fixated on the screen (Figures 12[c] and [d]), which suggests that curiosity may have been triggered.

However, it seemed that by not telling Dani what to do and moving his index finger 'quickly', Mio was trying to avoid having Dani (his frequent partner) lag behind. A rationale for this could be that the nature of the teacher's task-delivery may have played a significant role in highlighting the need for performance in a timely manner. As a result, the boys' focus was shifted from the mathematics itself to getting the task completed in anticipation of a follow-up instruction. It seemed that despite the existing potential for the start of the Embodied Curiosity process (the layer of curiosity) to be activated, the boys' interaction and performance of the task became dependent on the teacher's directive. This likely limited the interaction between the students and the digital technology, as well as the transition between the first two layers of the overlaying framework.



[a]Dani's diagram after construction was done

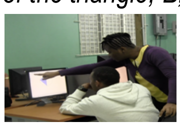
[b] Mio's diagram after construction was done

Figure 13: The boys' completed construction

After a few minutes had passed, Sammy checked to see whether or not the class was functioning relatively at the same pace. She said, "Alright, so I should see your circles with your diameters". She then gave the third instruction, "You are to place a random point on the circumference of the circle and then join all three points" (the end-points of the diameter and the random point on the circumference). This produced an inscribed right-angled triangle within the semi-circle, as seen in the boys' constructions in Figures 13 [a] and [b]. By using the second-person pronouns ('you', 'yours') in her instructions, Sammy seemed to ascribe ownership of the constructions to the boys. I will elaborate further in the upcoming sub-heading "Maintaining meaning and innovation" about how ascribing ownership aided the generation of mathematical meanings.

At this point of the task, Sammy's interest was on angles $\widehat{BAD}$ and $\widehat{ABC}$ (Figure 13 [a] and [b]) respectively. She asked the boys to examine how the angles of the triangle behaved as they dragged the random point along the circumference. They both measured the angles and Dani waited for her to provide the ensuing instruction. None of the boys had noticed that a $90°$ angle was formed. Sammy recognized that Dani was waiting for the follow-up instruction and walked over to him. The multimodal transcript in Table 6-5 shows the interaction between Sammy and Dani

Table 6-5 Sammy and Dani's interaction – angle formed in a semi-circle

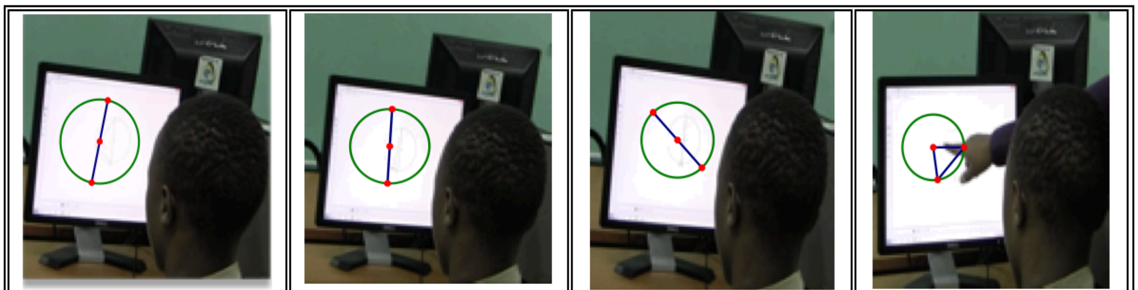
Turns	Utterances
1	Sammy: Move this.
	Points to angle A (see Figure 13 [a])
2	Dani:
	Drags point A along a small portion of the circumference between angles B and D repeatedly
3	Sammy: Weh yuh notice wid angle A? Move it again, which angle is changing? [What do you notice with the angle A? Move it again, Which angle is changing?]
	Dani, Drags point A a bit further
4	Dani: Di fus one miss [The first one, Miss]
5	Sammy: No, it depends on which A you are talking about. The angle you are measuring should be in the middle. So, it's BÂD
	Points to the angle notation shown in the measurements and then points to the vertices of the triangle, B, then A, then D 
6	Dani: But miss di size nah change [But Miss, the size is not changing.]
7	Sammy: eh nah change. [It is not changing]

Again, the pedagogical move made by the teacher in this interaction was to provide Dani with an instruction, commanding him to perform a task (Turn 1). The instruction illustrated the 'what' (point A), and 'how' (move) of the task. This left little room for Dani to engage in independent exploration. As a result, it seemed that he was timid in responding to the task in Turns 2 and 3. Dani engaged in curious dragging (dummy locus dragging[20]) timidly in a back-and-forth manner along a small portion of the arc DAB (Figure 13 [a]) and then a bit further only after the teacher asked him to move the point again. At Turn 3, the teacher's

[20] Dummy locus dragging is one of Arzarello et al.'s (2002) dragging modalities. It involves, "moving a basic point so that the drawing keeps a discovered property and the point which is moved followed a path" (p. 2).

strategy seemed to shift from 'instructing' (making a demand) to 'guiding' (leading the way) towards a particular response – the angle at A remained invariant. This was evident when Sammy asked Dani what he had noticed and specifically, what he noticed about angle A (the $90°$ angle).

Mio's encounter with his teacher was similar, yet produced a different outcome. He seemed to be less worried than Dani, and more confident in his approach towards performing the tasks and using *Sketchpad* in general. Mio had previously been engaged in curious dragging (Figures 13 [a] to [c]) actions, which were independent of his teacher requirements. Upon observing that a completed diagram was on his computer screen, Sammy walked back to the pair and stopped at Mio's desk.



[a] [b] [c] [d]

Figure 14: Mio's focus is shifted

Mio's curious dragging was short-lived, as Sammy pointed to the $90°$ angle (the invariant angle) in Figure 14 [d] and explicitly asked him to examine this specific angle. It seemed that there was an interruption in the flow of Embodied Curiosity, as his focus and his agency were shifted from a temporal-emerging mathematical meaning to a static one. This means that, instead of generating meaning as he continued to explore, the awareness came instantaneously. The multimodal transcript below in Table 6-6 depicts the conversation that transpired between Mio and his teacher at the instant when the shift occurred.

Table 6-6 Transcript showing Mio and his teacher 's interaction

Turn	Utterances
1	Sammy: This angle <inaudible. > so, try to explore it to see what happens
	Pointing to the right angle while Mio drags angle B (Figure Y[d])
2	Sammy: Which angle is changing size?
3	Mio: This one, Miss!
	Pointing to one of the non-right angles
4	Sammy: What about this one?
	Pointing to the right angle again.
5	Mio: It a di same. **[It stays the same]**

This interaction revealed that Mio's engagement with curious dragging was transformed to a single response, as seen in Turn 3. It seemed that the instruction in Turn 1 ("This angle") turned his focus away from the other things that were changing. Namely, changes in the length of the sides, the magnitude of the non-right angle and the size of the circle when the independent object was dragged. As a result, this failed to elicit Embodied Curiosity. An alternative scenario would have involved Mio observing how each object of the construction operates as a unified whole. Instead of pointing to the angle of interest, the teacher would have directed Mio to examine and report on something "unusual" he had observed.

Furthermore, unlike Andrew, Sammy's questions, "Which angle is changing size?" (Turn 2) and "What about this one?" (Turn 4), not only triggered a single response, but also restricted Mio from fully utilizing previous knowledge. Although, in Turn 5, he was able to demonstrate the concept of invariance, the mathematical meaning (the non-right-angles are not invariant) was restricted to a pointed finger accompanied by the utterance, "This one, miss!". Again, this thwarted Embodied Curiosity and restricted movement from one layer to the other: in this instance, from the layer of digital technology to that of mathematical meaning.

In the next section, I elaborate on the idea of ‘ascribing ownership’ by considering some of the diagrams that were produced by the students as they worked on the circle geometry tasks.

6.3.3. Maintaining Meaning and Innovation: A Case of Mio

Mio, a student at School X, who appeared in a previous episode in this chapter, showed consistent innovation with his diagrams throughout the data collection process. In this section, I present two of his diagrams and offer an analysis to show certain possibilities within an Embodied Curiosity environment. Although Dani was Mio’s main collaborative partner throughout the sessions, he also had the opportunity on different days to work with a female student named Jenny. Despite my own curiosity about the role gender (working with a male or a female partner) played in Mio’s innovativeness, my focus is mainly on the diagrams and how the mathematical meanings were maintained. Additionally, my focus will be on how extending the tasks produced a final construction which goes beyond the confines of the task – however, I will focus on the final diagram rather than on the process through which it was constructed.

In Task 4, Mio and Jenny were working together to establish the relationship(s) between the length(s) of each chord in a circle and the distance (shortest) between the chords and the centre of the circle. This session was conducted by me and the task was given verbally. It was Day 5 of the data-collection process and the students were demonstrating some fluency in their use of *Sketchpad*. They were able to use the basic functionalities of the software on different Euclidean objects. Mio and Jenny took turns performing different aspects of the task. For example, the background in Figure 15 [a] was as a result of Mio experimenting with the polygon interior tool in *Sketchpad*, while Jenny contributed to labelling the points. This, she suggested, would provide clarity and guarantee they were speaking about the same object in their conversations.

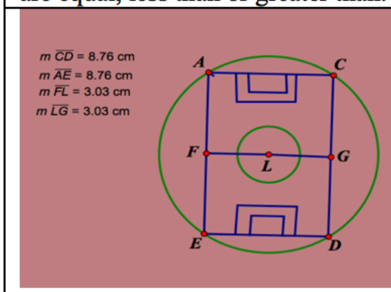
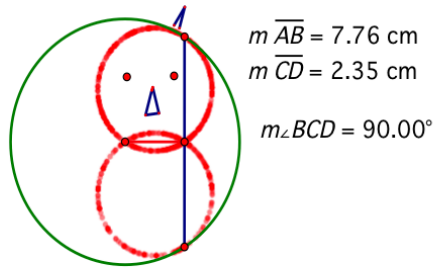
***Task 4*: Length of a chord in a circle** Find the relationship between the length of the chords in a circle, and the distance between the chords and the centre of the circle. Say whether they are equal, less than or greater than.	***Task 10: Perpendicular from the centre*** What can you say about the perpendicular line drawn from the centre of a circle to a chord?
 [a]	 [b]

Figure 15: Mio's superimposed diagram

Both students were engaged in a discussion about relationships, similar to their peers. They were quick in performing the task and were one of the first pairs of students to produce a completed construction. However, after they were satisfied that equal-length chords were equidistant from the centre of the circle, Mio continued with the diagram, while Jenny was visually fixated on the screen.

In extending the task, Mio constructed the line segments AC and ED (Figure 15 [a]) to produce the cyclic quadrilateral ACDE. It seemed that Mio had integrated his understanding about the cyclic quadrilateral (which had been previously done in Task 2) with the relationship between the distance of the chords and their distances from the centre. This implied that there is a possibility for observing mathematical meanings using a similar overlaying structure as the Embodied Curiosity framework. In other words, the stacking of one concept (task 2) onto the other (task 4) can also trigger the emergence of ‘new’ mathematical meanings.

After the diagram was completed, Mio called out to me, “Miss! Is a football (soccer) field mi a bill enno”[21] **[Miss! I am building a football field]**. This utterance is interesting for two reasons: one, Milo had extended the task to incorporate something from his immediate experience (a football field), and two, he took ownership of the diagram by saying “mi a

[21] Enno is a tag word in the Jamaican dialect. It shows affirmation and can be interpreted as “really” or “seriously”.

bill" **[I am building]**, despite having a partner beside him**.** By extending the task, the diagram no longer belonged to the pair, but rather to Dani, whose choice of the football field belonged to his personal experience. It seemed that the diagram shifted from being a geometric construction to a personal football field, which had been built using geometric objects.

The relationship observed between the equal-length chords and, in particular, when the chords are parallel, seemed to trigger the recognition of a shape with which Mio was familiar. When Mio was asked in the interview why he 'build' a football field he said, "Miss-miss, simple! ~~ the idea came to me after we draw the equal chords" he further explained that he was a "fan" of football and that he noticed that mathematics is in everything we do. Perhaps this trigger helped him better understand the structure of a football field. When he was asked to tell something that he found surprising, Mio explained that he was surprised to see that he could colour the 6-yard box and 18-yard box with different colours. Based on his explanation it seemed evident that the functionalities of *Sketchpad* triggered feelings of surprise. This implies that *Sketchpad* acts as an agent in evoking feelings of surprise. Based on these events, I propose that, Embodied Curiosity is not only at play within the confines of the geometric task, but it also breaks through those confines to incorporate elements of Mio's personal life.

In Figure 15 [b], Task 10, Mio's innovativeness was somewhat different. The task asked him to examine the relationship between the line joining the centre of the circle to the mid-point of a chord. Mio performed the task, allowing Jenny to contribute to its construction periodically, and they both found that the line and the chord were perpendicular to each other. However, after the task was completed, Mio took the central position again and added an extension to incorporate the previous task which involved the locus of a chordal mid-point. This produced the two circles seen in his diagram [b]. By incorporating this task, the relationship between the line joining the centre and the mid-point of the chord became entangled with the two overlapping circles. Furthermore, it seemed that this relationship triggered the recognition of something that was not his lived experience – namely a snowman.

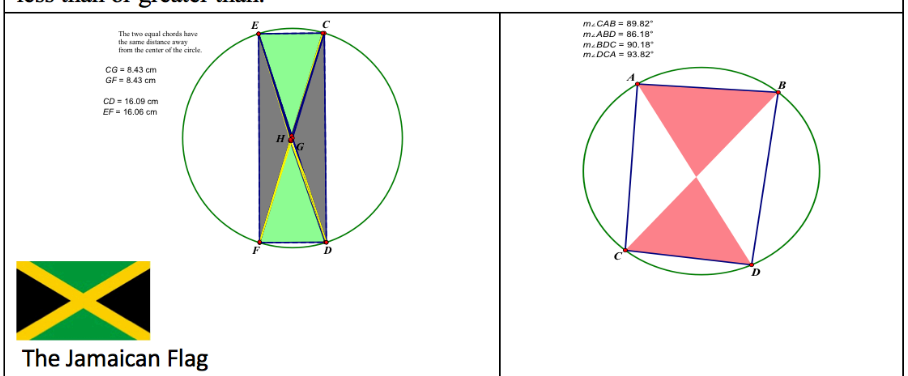
It seemed that the way in which Mio engaged with the geometric tasks illustrated two types of innovating, innovating to real-life and innovating to previous tasks. In previous sections, I have looked at Embodied Curiosity as it occurred in specific tasks. However, in this example, I have shown that one possibility of Embodied Curiosity is to flow outside of the task and into the world outside the mathematics classroom.

6.3.4. Finding the Exceptional Case with Roni and Akeel

Real-world experiences were again at play when students disentangled the exceptional cases in a task. In this episode, I examine how Embodied Curiosity potentially drew attention to the exceptional case in a task. Like Mio's scenario, my focus is not on the process involved in performing the task, but rather on the completed product, which extended beyond the confines of the task. This time, however, I am interested in the role Embodied Curiosity played in identifying and expanding the exceptional cases into the real world.

Roni and Akeel (featured in a previous episode) were working together on Task 4 (see Figure 16). Similar to Mio and Jenny, they wanted to establish the relationship between the length of the chords in a circle and the distance between them and the centre of the circle. This session was conducted by me and the task was, once again, given orally. Roni and Akeel were working together using the separate-computer arrangement. The discussion between them involved their observation while they performed the task. The diagrams in Figure 16 [a] and [b] emerged as a product of their discussion and engagement.

Task 4: Find the relationship between the length of the chords in a circle, and the distance between the chords and the centre of the circle. Say whether they are equal, less than or greater than.



[a] Roni's diagram [b] Akeel's diagram

Figure 16: Roni and Akeel found the exceptional case

In Figure 16 [a], Roni constructed the chords EF and CD and then measured them first to distinguish a relationship between the chords. It seemed that she was attending to the task in a similar manner as the step-by-step approach utilized by her teacher in previous engagements. It appeared that this approach was used despite the oral introduction of the task. Using this step-by-step approach suggested that she may have interpreted the task as two separate things. First, find the relationship between the length of the chords in a circle and then second, find the distance between the chords and the centre of the circle. This fragmented interpretation may have led her to examine the lengths of CG and GF (two radii) rather than the altitude (the perpendicular distance from the apex to the base of the triangle).

Strictly speaking, from a Euclidean perspective, the task required the shortest distance from the centre to the chords. Nonetheless, Roni and Akeel both observed the longest length and were able to tell that equal-length chords are equidistant from the centre of the circle. When asked what they were surprised about when performing the task, Akeel explained that she was surprised to see that the technology could draw and measure the various lengths but most of all, she was surprised when she moved the point, “everything moved!”

Aside from highlighting this exceptional case, Roni pushed the task to a new actualisation of Embodied Curiosity that exceeded the constraints of the task. She suggested that her task could “turn into” the Jamaican flag. She then proceeded to apply the relevant colours in their respective areas. Perhaps she came to this realization when the diagonals ED and CF were drawn. Akeel’s diagram in Figure 16 [b] went through a similar process, but when she was asked whether or not she was creating something in particular, she responded that hers was an hour-glass. It seemed that both girls became aware that their diagrams had the potential of transforming into another object. In this case, objects that extended outside of the mathematics classroom.

6.4. Summary

The aim of my analysis was to show how Embodied Curiosity was conceptualized as an overlaying framework with connections between four fundamental elements (curiosity, body movement, digital technology and mathematical meanings). Using the overlaying framework, I was able to examine the data from two perspectives – one relating to mathematics learning and the other to mathematics teaching. My starting point was first to establish the characteristics of each element that merited transition between each layer of the framework.

I used different scenarios from both schools to illustrate how transitions were possible. Drawing on Berlyne's (1954) curiosity dimension model, I was able to attend to specific sections of the data with potential evidence of emerging curiosity. This opened a possibility not only to identify the physical markers of curiosity, but also to indicate how each element of the framework relates to each other. Pickering's (1995) assumption that human and non-human agency act upon each other – removing sole authority from the human – became a central premise for the data analysis.

The episodes involving Brea, Ali, Kyle and his peers illustrate how certain factors contribute to movement throughout the four layers. My analysis also shed light on how the technology operated in two distinctive roles – as a partner and as an instigator. Brea's episode illustrates how her facial expressions, question and the entanglement of her body and the computer led me to believe that she was uncertain about the mathematical meaning of 'inscribed quadrilateral'. These factors implied a transition between curiosity and the body movement layers.

In the episode with Kyle and his peers, I examined how the transition was made possible from the layer of body movement to the layer of digital technology. I argued that Kyle's hand drawing on the top of his desk and the construction that was on the computer screen revealed both explicit and implicit mathematical meanings. In Ali's episode, I analyzed how she used the digital technology and her body movements to trigger and bring to the fore a 'new' mathematical meaning – tangency.

Having looked at how the transitions were made possible through the layers, I presented the data using a similar overlaying structure and used colour-coding to illustrate the frequency and how each element was observed within different episodes. By doing so, I was able to select episodes that show how students learn in the Embodied Curiosity environment. From Roni and Akeel's engagement, I looked at the problematic way in which the definition of a circle was shifted from one representation (a plane figure bounded by curved lines) to the another (the locus of a point equidistant from a fixed point). In addition, this episode revealed the role linguistic uncertainty plays in the development of 'new' mathematical meaning, as well as the role of emotion in triggering curious dragging.

Ali and Joni's episode reflects how the digital technology and its capabilities along with the students' bodies acted as a trigger for the development of new mathematical meanings. In addition, this episode highlights the way in which the digital technology also emerged as an instigator unravelling and initiating mathematical meanings.

My analysis of the teacher's role in an Embodied Curiosity environment shows that questioning technique, repetition and persistence could play an important role in activating and maintaining the Embodied Curiosity process. In contrast, a non-investigative (step-by-step) approach may limit independent exploration and restricts how curiosity is activated and transitioned through the layers.

Finally, my analysis shows that the diagrams produced by the students (Mio, Roni and Akeel) extend beyond their engagement with the geometric tasks into the real world. Mio's episode shows that not only was his innovativeness with the task extended to his immediate environment, but also from the previous task, which was clearly embedded in his completed diagrams. In Roni's and Akeel's diagrams, real-world experiences also played a significant role, but more so in identifying the exceptional case.

Chapter 7. Discussions and Findings

> *"Train yourself don't wait to be fed knowledge out of a book. Get out and seek it. Make explorations, do your own research work. Train your hands and your mind. Become curious." (Irving Langmuir, 1941)*

In this chapter, I provide a breakdown of the results from Chapter 6. I take into consideration the three research questions and organize the chapter to reflect the ways in which curiosity was triggered from an Embodied Curiosity perspective. I also discuss, in depth, the relationship between the four elements of the theoretical framework, as well as how the digital technology emerged as an embodied tool. Towards the latter part of this chapter, I discuss the potential of Embodied Curiosity and how it shifted the understanding of circle geometry theorem from a Euclidean sense into the real world.

Additionally, I draw on the results to highlight ways in which Embodied Curiosity plays out in student learning and its relation to the teaching of circle geometry theorems. The rationale for this chapter is to provide a coherent combination of the research findings, to respond to my research questions and to inform future work on the role of emotions in the teaching of mathematics, in particular, when expressive digital technology is involved.

7.1. The Emergence of Curiosity

Following the discussion in Chapter 2 about the unstable nature of the definition of curiosity and how technology has evolved over the decades, I found a limited amount of research on the affective dimension of students' learning experiences using digital technology in mathematics education. As a result, I set out first to reconceptualize curiosity as a phenomenon that emerges when students interact with digital technology. In doing so, I suggested that curiosity emerged when students and the digital technology act as co-agents, which is one of the main assumptions of my research.

A close examination of the data in Chapter 6 provides evidence of the presence of relational-curiosity when students and the digital technology engaged in a meaningful interaction. I use 'meaningful' here to refer to the instances when the students were involved in performing geometric tasks. This co-agential relationship led to a response for my first research question, which was about identifying the physical markers or visible signs of emerging curiosity.

The analysis provided evidence that visual fixation, facial expressions, eye gazes (an intense stare), glances (brief looks), leaning forward and backward, as well as asking questions,[22] were all anticipating factors of curiosity. However, these factors seemed to work together for the process of Embodied Curiosity to be activated. For example, in the episode with Brea, her facial expressions accompanied by her question ("What is what, Sir!") signifying that she was uncertain about the meaning of the term 'inscribed quadrilateral'.

Furthermore, in the episode involving Kyle and his peers, visual fixation followed by leaning towards the computer with the diagram on screen implied that curiosity had emerged. His reaction of leaning backwards when his uncertainty was satisfied also played an important role in determining the manifestation of curiosity. This was also evident in the episode concerning Ali, who had a bewildered look on her face while she asked, "Should they have the same measurement?". This led me to conclude that a combination of these anticipating factors acted as physical markers of emergent Embodied Curiosity.

However, it is imperative to note that, on their own, these factors are not sufficient to infer emerging curiosity. For instance, in the episode with Brea, initial visual fixation did not necessarily imply uncertainty, but instead could also be interpreted as simply attentiveness – a sign that there is an interest in something. Furthermore, Brea's facial expression of raising her eyebrows could be interpreted as disdain – refusing to acknowledge the meaning of inscribed quadrilateral because it was unfamiliar. Additionally, in the episode with Jenny and Mio, Jenny's eye-gazing, while Mio extended the boundaries of his construction, could be interpreted as absentmindedness (staring off in the distance without focusing on anything specific). This interpretation could be possible especially with his football field, because it was less of a meaningful experience for Jenny than it was for Mio.

Likewise, a glance could also be interpreted as apprehension, which could also be a sign of boredom. When these factors occurred on their own, movement through the semi-permeable membrane of the layers of Embodied Curiosity was restricted. As a result, my

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[22] The questions I refer to are usually 'wondering' questions that show some sort of uncertainty. They usually begin with interrogative words such as; 'what', 'how' or 'where'. These are the types of questions I refer to throughout this discussion.

conclusion is that Embodied Curiosity can be inferred and that at least two layers are needed for this to occur.

Additionally, in Chapter 3, I argued that curiosity may emerge when there is an integration of Berlyne's (1954) suggested curiosity dimensions. My stance was that perceptual (sensation-seeking) and epistemic (knowledge-seeking) curiosity were not mutually exclusive, as Berlyne had presented them, but rather work together in the emergence of curiosity. The data analysis in Chapter 6 corroborated my interpretation; in seeking knowledge, students usually rely on sensation-seeking stimuli such as eye gazes, glances, visual fixations and wide-eyed-ness, in a way that seems almost to expand their intake of information. These sensation-seeking stimuli were usually accompanied by wondering questions and movement towards the technology to seek answers. This supported my conclusion about the embodied potential of curiosity and suggested that sensation-seeking and knowledge-seeking are integrated processes in mathematics learning.

An example of this integrated relationship occurred when Ali glanced at the diagram on the computer screen, while explaining that the locus of the mid-point and the original circle shared a common point. She further asked if everyone understood her explanation. In a sense, this led me to two possible interpretations of uncertainties: one involved her lack of confidence in explaining the correct meaning, hence, seeking validation from the computer. The other involved whether or not there was a clear understanding of the relationship she had observed. This led to the conclusion that curiosity may emerge from both cognitive and affective pathways.

Since curiosity is the starting point of the Embodied Curiosity process, and the data revealed possible scenarios of when and how curiosity emerged, I then turned my attention to the possible relationships between curiosity and the other three elements.

7.2. The Emergence of a Superimposed Relationship

In response to my second research question, which aims to identify the extent to which Embodied Curiosity fosters the construction of mathematical meanings, I examined each element of the framework independently. My rationale for this was to locate common themes that would make connections between the elements more visible. Not only did the data analysis reveal connections among the elements, but also that these connections appeared as an overlaying structure where each element was superimposed onto the other.

I presented this model metaphorically as a layered structure (Figure 4), with each layer being semi-permeable like the membrane of a biological cell. I argued that mathematical meaning-making can be developed and maintained through a layered structure involving the elements of Embodied Curiosity. In addition, the analysis revealed how this superimposing relationship occurred and in what way the transition from one layer to the other was achievable.

In line with Loewenstein's (1994) definition of curiosity, I sought to address the ways in which a student's knowledge gap or uncertainties might be recognised and satisfied. Suggesting, of course, that the knowledge gap can be narrowed when students generate and develop new mathematical meanings. I argued that a starting point for the development of new mathematical meanings is dependent upon how curiosity is triggered. As a result, the analysis revealed that in the construction of mathematical meanings, there must first be curiosity.

For example, in Brea's episode, which involved her performance on the task about an inscribed quadrilateral, there was evidence that curiosity was triggered and the Embodied Curiosity process was activated. Her uncertainty about the inscribed quadrilateral could be identified when her facial expressions transformed from visual fixation to raised eyebrows, then to wide-eyed-ness, followed by a gasp and a question. In other words, uncertainty is detected through body movements followed by wondering questions. I argued that these factors, as a whole, signified that the body plays an important role in triggering curiosity and, as such, highlights the relationship between curiosity and the body. Once this initial relationship was identified, the Embodied Curiosity process is infiltrated through to the layer of body movement.

Additionally, the episode involving Kyle and his peers, which demonstrated how transition between the layer of body movement and the layer of digital technology was made possible, reflected ways in which uncertainties were evident. From this episode, my conclusion was that, in performing the task, both the body and the technology engaged in a two-way relationship similar to Pickering's (1995) "dance of agency" (discussed in

Chapter 2). In this particular dance, Kyle engaged in a back-and-forth interaction with the construction on the screen, as he tried to ascertain the correct name for the triangle. The analysis revealed that it was within this relationship that Kyle's uncertainty was located. Again, this emphasized that uncertainties may arise in different situations along the Embodied Curiosity process.

The data also revealed that transition from the layer of body movement to the layer of digital technology was made possible in several ways. In one instance, Kyle's body movement of leaning forward for a closer scrutiny of the diagram and subsequently leaning backwards when his uncertainty was satisfied, implied that the technology played a significant role in his meaning-making process. I concluded that the actions of leaning forward to seek information and backwards after it was received, exemplified the embodied nature of the digital technology. It seemed that the technology *offered* or handed him the knowledge he was looking for.

In another situation, Kyle was able to more deeply examine a diagram due to the functionalities of *Sketchpad*. The software's ability to measure and record lengths and angles within the construction provided opportunities not solely for in-depth explorations, but also to satisfy Kyle's uncertainty. The possibility for movement between the layers became apparent once again, when Kyle used his pinky finger to draw the missing side of the triangle on the screen. Again, this illustrated the embodied nature of the technology, where there was a causal effect between the technology and Kyle's finger.

It was evident that Kyle and the technology were imbricated in an experience that prompted his co-partner Jeff to complete the diagram. As a result, I inferred that the digital technology was instrumental in triggering Kyle's previous knowledge, which led him to use his pinky finger to complete the triangle on screen and trace various triangles on the top of his desk. Once these relationships were identified, the process of Embodied Curiosity advanced to the adjacent layer.

Ali and Joni's interaction highlights how transitions were made possible from the layer of digital technology to the layer of mathematical meanings. Ali's uncertainty was evident in her apprehension to offer an explanation without the aid of the technology. The analysis provided evidence that Ali and the technology assumed equal roles in the explanation about the locus of the chordal mid-point. This is in line with Pickering's (1995) assumption that human agency does not precede material (the technology) or disciplinary (the mathematics) agency.

I argued that, in performing the task, the digital technology transformed from being a material object to a more ‘humanized’ one, displaying an affective relationship with the student. Other theories in mathematics education, such as the instrumental genesis (Verillion and Rabardel, 1995), demonstrated a similar transformation. For example, instrumental genesis places emphasis on cognitive structures which involve transformation from an artefact (a material object) to an instrument (a mental thought about the use of the artifact). Similarly, Turkle (2011), in her ground-breaking work on Evocative Objects, drew on Papert’s (1980) idea of the computer as “an object-to-think-with” (p. 11) and introduced the idea that objects serve a dual role. Marking a shift from the cognitive to the affective, Turkle (2011) positioned objects (including computers) as companions to our emotional lives (objects-to-feel-with) as well as to our reasoning (objects-to-think with). However, the analysis of my research showed that in an Embodied Curiosity environment the transformation was from the material (digital technology) to the affective.

An example of this was illustrated when Ali and the computer (with the diagram on screen) entered into a ‘partnership’ to bring about not only meaning but also a clear and concise interpretation of the meaning. Perhaps her intention was to dispel any further uncertainty or perhaps she had developed an intimate relationship with the computer, which influenced the way they communicated with each other. The latter interpretation is in line with Papert’s (1980) vision of intentionally designing computers so that we can learn to communicate with them. Both interpretations are pertinent to my research interest, which is to focus broadly on the affective dimension of digital technology experienced in mathematics education and, more specifically, on the influence of curiosity on mathematical meaning-making.

7.3. Learning in the Embodied Curiosity Environment

Having established the nature of curiosity in this research context, and the ways in which the elements of Embodied Curiosity relate to each other, I sought to determine how learning takes place using this lens. The analysis revealed that once curiosity was noticeable, the body was usually visibly involved. The analysis also showed that in the entanglement with curiosity and the body, students were inclined to utilize the digital technology to satisfy their “induced deprivation” (in Loewenstein’s terms). In other words, the digital technology played an important role in satisfying students’ uncertainties. They did not have to wait for a teacher’s response; and, they could trust the response of *Sketchpad*. I argued that the construction of mathematical meanings is situated within this process. In this section, I discuss how learning takes place from the inception of the Embodied Curiosity process to the construction of mathematical meanings.

The analysis showed that in the episode with Ali and Joni, curiosity was activated when Ali’s facial expression along with her immediate question, prompted her to construct and examine the diagram on the computer screen. In her investigation about the opposite angles in a cyclic quadrilateral, her interest was in establishing two things: one, to know if the cyclic quadrilateral involved more than one shape and two, whether or not the measure of the attributes (angles and the length of sides) were the same. I interpreted this to be the initial stage of Ali’s uncertainty.

Drawing the quadrilateral detached from the circle provided further evidence that Ali was undecided about what is meant by the ‘cyclic quadrilateral’. In order to resolve this ambiguity, her eyes wandered around the room to observe what the other diagrams looked like. It seemed that this signalled an exchange between the layer of curiosity and the layer of body movement. Ali found a difference between her diagram and those produced by her peers. In her construction, the vertices of the quadrilateral were detached from the circumference.

It seemed that her immediate reaction was synonymous with Pickering’s (1995) description of Glaser and the bubble chamber. According to Pickering, the human and non-human act upon each other to bring about some form of stability. In this case, the stability was found in her knowledge of the concyclic property of the cyclic quadrilateral. The digital technology was instrumental in satisfying her uncertainty when she engaged in curious dragging, specifically, linked dragging in a slow and cautious manner. Additionally, in the re-enactment exercise, Ali applied her knowledge of performing the task with *Sketchpad* to make a connection between the sum of the interior angles in a quadrilateral and the measure of a complete revolution.

This episode epitomizes the process of Embodied Curiosity. It began when Ali's facial expression along with her question activated the layer of curiosity and helped me to identify that there was uncertainty – uncertainty which lies in the understanding of the properties of a cyclic quadrilateral. In order to resolve this uncertainty, Ali glanced at her peer's screens and this triggered her to adjust her construction on the screen. In doing so, she engaged in curious dragging which allowed for the construction of new mathematical meaning – the concyclic property of the cyclic quadrilateral. Significantly, in order for Embodied Curiosity to be active, there must be a gap and a desire for knowledge.

The analysis also revealed that a similar process was involved when Roni and Akeel investigated the locus of the mid-point of a chord. However, unlike Ali and Joni's episode where the uncertainty involved a mathematical concept, Akeel and Roni's uncertainty was rooted in linguistic semantics. In other words, it arose from making the distinction between the meaning of 'locus' and 'locust', as well as matching the appropriate spelling to each word.

I interpreted this as something embedded in sociocultural differences and established that the environment played a significant role in the construction of mathematical meanings. Curiosity was activated in this process and the body was also instrumental in detecting uncertainty and triggering curiosity. I found that leaning forward and backwards, along with Akeel's eye movements from the dictionary to the computer screen, both contributed to the way mathematical meanings were developed. Unlike Ali's scenario, where her eye movements were interpreted as an instrument to take information in, it seemed that Akeel's eye movements served not only to take information in, but also to distribute it. I claimed that, by revisiting the dictionary's meaning and then the construction on the screen, Akeel transported the meaning from the dictionary to the diagram. Again, this illustrates the way perceptual and epistemic curiosity became mutually conjoined.

In order to reconcile the uncertainty, Akeel engaged in curious dragging: this time, bound dragging in a slow and meticulous manner. This type of dragging action speaks to the intermediary role that the digital technology plays in meaning-making. Most importantly, the dragging action highlights how agency gets distributed not solely to the student, but also to the digital technology. This is in keeping with Pickering's (1995) assumption that human and non-human agency are interlaced with each other when performing a task. While Akeel was engaged in curious dragging, her emotions shifted from doubt to feelings of excitement, which further led to a shift in the momentum of her dragging action. She was able to express explicitly that the locus of the mid-point of the chord was actually a circle. This suggested that there was a contribution of both students' emotions and digital technology in creating mathematical meaning.

Again, the process of Embodied Curiosity was at play in this episode. It commenced when Akeel asked for a clear spelling of the words and then visited the dictionary, moving her eyes from the dictionary to the screen and leaning forward and backwards for deeper analysis of the construction. Here, the uncertainty was rooted in a distinction between the meaning of the words. Summing-up this episode, in order to satisfy the uncertainty, Akeel engaged in curious dragging which led to a transformation of her emotions – from doubt to excitement. The definition of the circle as a locus of points equidistant from the centre, which was not a part of her prior knowledge, emerged from this process. In this case, Embodied Curiosity highlighted not only a gap and a desire for knowledge, but also how language and culture are infused in learning mathematics.

Although the analysis revealed that uncertainties occurred in two different situations (mathematical conception and linguistic semantic), there was evidence that learning took place through the Embodied Curiosity process. I concluded that mathematical meanings were created when students and the digital technology intertwined with each other and that the body played an important role in this process. In addition, I established that students' feelings of doubt and uncertainty are important in the learning process and that these feelings, when shifted to positive emotions like excitement, were also fundamental to the way students learn. Furthermore, I found that, similar to the mangling process suggested by Pickering (1995), students create mathematical meanings when there is an interplay among the human (students and their body movements), the material (the digital technology and its affordances) and the discipline (in this case, mathematics).

7.4. Mathematical Meaning-Making

At the heart of the Embodied Curiosity framework is the mathematical meaning-making process. The analysis also highlighted the ways in which mathematical meanings developed into two main contrasting categories – explicit and implicit. The analysis showed that verbal and written mathematical meanings were prevalent when students engaged with the tasks and *Sketchpad.*

However, there were many instances when the mathematical meanings were inferred. For example, Mio and Jenny were able to say explicitly that, “The distances from the middle to the chords (sic) are equal”. They referred to the centre point of the circle as ‘the middle’. However, since the distances in question indicate the shortest distance, this implied that a perpendicular relationship exists between the two attributes. This was not explicitly stated, but could be used as a teachable moment (an instance to introduce something new), if the teacher had drawn attention to the perpendicular relationship between the distances.

In another example, Ali was able to say, “the locus of the mid-point of a chord is a circle”. However, the data revealed that the concept of tangency was inferred in her utterance as she concluded that one circle was touching the other. Kyle and his peers also expressed explicitly that the triangle formed by the centre point and two random points on the circumference (creating two radii) was indeed an isosceles triangle. However, hidden in this mathematical meaning was the fact that a right-angled isosceles triangle could be produced when the angle formed between the two radii was 90^{o}. Although this was not written or spoken, the opportunity for it to arise was there, which may have evoked new curiosity and further explorations.

Besides highlighting the two categories in which mathematical meanings were classified, the results of my research show that implied mathematical meanings can be seen as a teaching moment. It is important to note that implied mathematical meanings in an Embodied Curiosity environment cannot be pre-planned, but rather evolved within students’ explorations. Additionally, I argued that by using these implied mathematical meanings as part of the teaching strategy, Embodied Curiosity could become an on-going temporal process. This is because the opportunity exists to develop further uncertainties which prompt additional explorations. I also concluded that the digital technology, with its capabilities, played a crucial role in providing opportunities for the students to attach meanings to their mathematical discoveries.

In addition, I argued that the relationship between the digital technology and mathematical meanings have provided me with a deeper understanding of mathematics learning in a dynamic environment as oppose to a static one. Similar to Ng and Sinclair’s (2015) research finding about children’s reasoning of reflective symmetry, my analysis also highlighted that *Sketchpad* offered more than the development of mathematical meanings.

In both cases, Ng and Sinclair's (2015) account, as well as mine, suggests that using *Sketchpad* can lead to new ways of interpreting movement of the body, and that this can have both conceptual and affective relevance.

7.5. Teaching from an Embodied Curiosity Approach

Considering the connected nature of the elements of Embodied Curiosity and the temporality of curiosity in itself, the teacher's role in this type of environment was critical to mathematics meaning-making. The analysis focused mainly on the questioning style of the teacher and how it related to the prompting of Embodied Curiosity. For example, in the episode involving Andrew, the analysis revealed that the questions he asked contributed to the way in which curiosity was triggered. I argued that the questions *per se* cannot be pre-planned because they are dependent on students' real-time interaction with the tasks and the technology. In other words, the teacher cannot know ahead of time whether or not students would view the construction as a complete triangle with only two line segments and a curved side attached.

Instead, the analysis showed that it was the ***types*** of questions which were important to the Embodied Curiosity process, because it seemed that they were purposefully utilized. It seemed that Andrew deliberately used *recall*, *leading*, and *probing* questions, which aided in the mathematical meaning-making process. A recall question such as, "what kind of triangle is that?" was interpreted as the factor that activated the student's prior knowledge. The probing question, "What do you notice?" evoked further exploration and served as a trigger for curiosity, while the leading question, "But is that a triangle?", led the students to conjecture about the properties of an isosceles triangle, which was the triangle under investigation.

In addition, the analysis revealed that the teacher's persistence contributed not only as a trigger for the students' previous knowledge, but also instrumental in developing mathematical meanings. I argued that persistence was evident through the repetition of the questions and that this allowed the students to utilize their previous knowledge to generate new meanings. From this analysis, I draw the conclusion that the teacher played a fundamental role in activating and sustaining the Embodied Curiosity process.

Furthermore, the analysis also provided evidence that traditional teaching approaches, which are teacher-centred and prescriptive in nature, could be less influential on the emotional aspect of mathematics learning. For example, Sammy's use of a step-by-step approach in announcing the task highlighted the specific doubt that Dani had – the distinction between the radius and the diameter of the circle. Nevertheless, the analysis revealed that this strategy did not provide adequate opportunities for curiosity to be triggered, nor activate the Embodied Curiosity process. I argued that, although there is a potential for the step-by-step approach to reveal specific gaps in students' knowledge, it did not encourage independent exploration. As a result, there was a restriction on how curiosity was triggered.

Embodied Curiosity is conceptualised as an on-going process and the analysis also suggests that the teacher is instrumental in the modulating of this process. In order for this to take place, the teacher must utilize the implied mathematical meanings as teachable moments. This means that the inferred meanings that emerged through the students' interactions should be brought to the forefront and inserted in the learning experience. In this way, curiosity will be a continuously evolving phenomenon and the prime trigger of Embodied Curiosity.

In Chapter 3, I drew attention to the contribution of *inclusive materialism* (de Freitas and Sinclair, 2014) to mathematics education. As mentioned before, *inclusive materialism* emphasizes, "assembling various kinds of agencies rather than a trajectory that ends in the acquiring of fixed objects of knowledge" (p. 52). In other words, agency of the body (both teacher and student), tools (to include digital technology), utterances and the mathematics itself combine to contribute to meaning-making. I argued that, like *inclusive materialism*, the Embodied Curiosity framework also considers agency of the body and of the tool. However, my analysis revealed that curiosity played a significant role in how the agencies were 'assembled' and that the teacher's contribution is on how curiosity was sustained.

The data has revealed that these contrasting pedagogical approaches (questioning vs explicit instructions) yielded different responses and contributed in different ways to the Embodied Curiosity process. While the questioning approach played a significant role in the emergent curiosity, and further activated the Embodied curiosity process, the explicit instruction approach was instrumental in bringing specific uncertainties to the surface. I argued that the type of questions the teacher asked played an important role in activating the students' previous knowledge, in evoking further exploration and in leading students to make their own conjectures. I maintained that, from a teacher's point of view, being persistent created a link between a student's prior knowledge and the development of new mathematical meanings. Since Embodied Curiosity is an on-going process, I argued that the teacher plays an important role in supporting the process through the utilization of teachable moments.

7.6. *The Geometer' Sketchpad* as an Embodied Tool

Research question three sought to establish how the potentialities and affordances of *The Geometer's Sketchpad* evoke Embodied Curiosity. In the literature, I gave an account of the trajectory through which Sketchpad's capabilities have been developed over time. Also, in Chapter 2, I discussed research done by Ng and Sinclair (2015), which positioned *Sketchpad* as a 'negotiating' tool that helped children reason about reflective symmetry. That research implied that the software has the potential to extend beyond its materialistic form. It seemed that by suggesting that *Sketchpad* is 'negotiating', the software has the ability to engage in a discussion with the children, as Jackiw and Sinclair (2010) have argued in relation to *Sketchpad's* discursive interaction design. It was also evident in my research that Sketchpad was far from being a passive object.

The analysis has shown that *Sketchpad* emerged as an embodied tool with human-like interactions similar to the negotiating potential inferred by Ng and Sinclair's study. For example, *Sketchpad* (and the constructions available within it) acted as a partner with Ali as they were both engaged in providing an explanation about the chordal mid-point. Similarly, *Sketchpad* 'worked together' with Kyle to bring about the idea that an isosceles triangle is formed when two random points on the circumference were connected to the centre point.

This anthropomorphising of the technology was further supported in the episode involving Ali and Joni, where the technology unlocked and then provoked mathematical meaning: *Sketchpad* subsumed the role of an instigator (not in a bad way), contributing to the Embodied Curiosity process. In the role of an instigator, *Sketchpad's* contribution was two-fold: first, to arouse the student's uncertainty and second, to persuade an action. In this case, the action was to trigger curiosity. In the re-enactment exercise, Sketchpad again acted as an instigator when it helped Ali to satisfy her doubt that the vertices of the cyclic quadrilateral should be positioned on the circumference. Hence, the rationale for the students to hold hands. Additionally, *Sketchpad* further triggered curiosity when Ali made a connection between the sum of the interior angles of a quadrilateral (as observed in her construction) and the angular measure of a complete rotation.

In another example, *Sketchpad* was perceived as an instigator when Akeel initially found the meaning of the word ‘locus’ in the dictionary and, after the completed construction, she re-examined the meaning. I concluded that *Sketchpad* (the construction on the screen) urged her to re-visit the meaning. Furthermore, *Sketchpad*’s ability to respond to dragged objects resulted in the physical shape on the screen, which was a visual account of the locus as the set of points that share the same distance from the centre point. I argued that, perhaps, the student recognized that the same mathematical meaning could be expressed in two different ways. In addition, it was also possible that the student wanted to verify whether the diagram on the screen was a true representation of the definition. In the former interpretation, I concluded that, by matching the written meaning to the object, there is an isomorphic relationship between the geometric object and the meaning it represents. In the latter interpretation, which I am inclined to believe was the case, the role of *Sketchpad* as an instigator was more noticeable.

Reflecting on how excited Akeel was when she observed the circle coming into existence, and the momentum at which she completely traced the circle, I argued that Sketchpad provided an opportunity for students to gain a new perspective about the circle. Hence, matching the definition with the object could be an instance of a new revelation or a surprise to her. Either way, this justified a need to test if the object is a true representation of the dictionary’s meaning. Moreover, Sketchpad was, in part, responsible for stimulating Akeel’s excitement.

I concluded that, with the development of these two roles, the digital technology should be perceived beyond a mediating tool bounded by material agency: perhaps there is another agency which would be fitting for future research. I found that, in performing geometric tasks, the *Sketchpad*-student pair were interlocked in a role-shifting – each taking on human and material agencies – experience which contributes to the mathematical meaning-making process.

The analysis also revealed that students' dragging actions in an Embodied Curiosity environment were similar to Arzarello et al.'s (2002) dragging modalities. However, I also identified three important elements that had not been previously discussed in the literature: the student's emotions, temporality and speed. It seemed that these elements were instrumental not only to the process which produced the diagrams, but also to how the objects behaved when dragging was done. For example, in Akeel's interaction with the chordal mid-point, a slow dragging action produced a circle with the circumference incompletely filled in. However, a fast and rigorous dragging produced an instantaneous and completed circle, which was a part of the student's existing mathematical register. Although Akeel initially considered this circle as a plane figure bounded by curved lines, this definition later shifted to a locus of points equidistant from a fixed point. The analysis showed that a shift in the definition occurred through a transition from an incomplete circle (with a dotted circumference) to a complete one with a solid circumference.

Bearing the temporality, speed and students' emotions in mind, I extended Arzarello et al.'s modalities to include these three elements and called them curious dragging. When students performed one or more of the dragging modalities that contained these three elements, they were engaging in curious dragging. For example, Ali engaged in curious dragging when she performed linked dragging – in a slow and cautious manner – while showing visible emotions. Perhaps her dragging action was slow because she was attending to something specific on the diagram, and cautious because she was afraid of creating a discord with the objects in her construction.

Likewise, Akeel's engagement with curious dragging (bound dragging with increase momentum while showing excitement) was also indicative of the relationship between her feelings and the way she performed the dragging action. In this case, the speed of dragging produced something on the screen that triggered her excitement. In fact, it was watching how the circumference unfolded in front of her eyes that brought about her excitement. Conversely, Dani's curious dragging, which involved dummy locus dragging in a timid back-and-forth manner, may have triggered another unwanted emotion – reluctance. I concluded that students' curious dragging provided evidence of the affective and cognitive aspects of their interaction with Sketchpad. I also suggested that students demonstrated a shift in their perception of geometric objects. One such shift is in recognizing an alternative definition for a circle,

7.7. Limitless Boundaries

Throughout my research, I mentioned the potential of Embodied Curiosity both from a pedagogical and from an epistemological point of view. In the preceding sections of this chapter, I discussed how Embodied Curiosity presents various possibilities when students interact with the digital technology, as well as how teachers may contribute to the development and sustainability of the process. However, the analysis also provided evidence that students' diagrams can transcend beyond the classroom and connect to everyday life experiences. By this I mean that not only did the final constructions evolve as an object representing a series of actions between the students and *Sketchpad*, but also as a representation of the students' out-of-school lives.

The analysis showed that, in Mio and Jenny's interaction, the final diagram emerged as a combination of several previous tasks. I argued that students construct mathematical meanings when they are able to stack concepts onto each other in a similar structure as the Embodied Curiosity model in Chapter 6, Figure 4. I also suggest that a combination of these tasks further converges into student's everyday experiences, and that while doing so, conceptual meanings are maintained. This was evident in Mio's football field where at least three previous circle theorem tasks were embedded in his construction. These were: (1) angle from the centre to the mid-point of a chord; (2) angles in a cyclic quadrilateral; (3) equal chords and their distances from the centre. A combination of these tasks allowed Mio to envision a football field which led him to complement the diagram with other components such as the goal posts, the crossbars, and the centre mark. This produced a completed football field.

As a result, I inferred that Embodied Curiosity extends beyond the boundaries of circle geometry theorems and into the students' immediate environment. This, I argued, can support students' understanding of the applicability of mathematical concepts to contexts outside the classroom. I also concluded that the Embodied Curiosity framework has the potential to foster innovation which may lead students to continuous exploration and curiosity.

Conversely, the analysis also showed that students were able to attach their real-life experiences to the geometric tasks when they probed the exceptional cases. This was evident in the interaction between Roni and Akeel, who affixed the Jamaican flag and an hour-glass respectively onto their constructions. While the students' interpretation of the task may have led them to uncover something intriguing (the exceptional case), they were able to imagine and visualize objects that were part of their immediate environment.

The Embodied Curiosity process has revealed that, although real-life experiences were at play when students were being innovative and probing the exceptional case, the geometric constructions played two different roles. One involved extending the task to real-life situations and the other superimposing real-life experiences onto the geometric shapes.

7.8. Summary

Using my research questions as a guide to organizing the chapter in a logical and coherent manner, I discussed the need for identifiable sources of curiosity. In doing so, I positioned curiosity as a phenomenon that emerged when students interacted with technology and not as a trait ascribed to the students alone. This type of curiosity is essential to the Embodied Curiosity process which provided a window for children to construct mathematical meanings. Furthermore, I reported that students constructed mathematical meanings when there was a gap in their knowledge which needs to be satisfied. In addition, I discussed the four elements of Embodied Curiosity and pointed out that they related to each other in an overlaying structure with each connected by the agency. I described instances when Embodied Curiosity was at play in children's construction of mathematical meanings, as well as, gave an account of the teachers' involvement in such a process.

Additionally, I examined and discussed instances when the technology emerged as an embodied tool, displaying human-like behaviour such as instigating and partnering with students. Towards the end of my discussion, I looked at the potentials of Embodied Curiosity and suggested that part of it is to connect mathematics to the real world, either by extending or superimposing geometric understanding to children's lived experiences.

In the chapter that follows, I recapture the response to my research questions and show how my research findings contribute to mathematics teaching and learning, and how this study has transformed my own perspectives about mathematics.

Chapter 8. Conclusion

> *“Curiosity is the wick in the learning candle.” (William Arthur Ward, In G. Brod & J. Brietwieser, 2019)*

In this chapter, I bring together responses to my research questions which were introduced in Chapter 1 and further elaborated on in Chapter 4. In order to present my responses, I re-evaluate the main results of my research and show that mathematical meanings were constructed and sustained through an embodied process involving curiosity, body movements, digital technology and mathematics. I discuss how this process – Embodied Curiosity – can apply to both the epistemological and the pedagogical aspects of mathematics education.

In doing so, I comment on the need for the Embodied Curiosity framework to promote opportunities that will take into consideration the relationship between emotions and digital technology, more specifically, on curiosity and *Sketchpad*. In addition, I reiterate the main assumptions about Embodied Curiosity and put forward the suggestion that this framework offers a new way of thinking about the role of technology and its implications for the teaching and learning of geometry. Towards the end of this chapter, I reflect upon how trait-curiosity (human nature) led me to relational-curiosity (an agent of mathematical meanings) and provided some provocative thoughts into a way forward with this framework.

8.1. Responses to My Research Questions

In this research, I set out to understand in what ways children’s emotions play a role in the development of mathematical meanings. I was also interested in the role of digital technology in this experience simply because I had a broader goal of integrating technology into the mathematics classroom. This was based on my experience in a Jamaican context, in which there is little incorporation of technology in the mathematics classroom. I recognized that, despite the advancement in technology, I had little or no access and, at times, the technology was not readily available to me. In fact, at times I felt that I was not technologically literate enough to effect changes, not only through pedagogy, but also with my own understanding of mathematics as a discipline.

It turned out that, while I immersed myself in the Ph.D. program, a number of things became important to me. One involved my own wondering about ways in which my feelings impacted the way I learn; how I interact with others; and, how I relate to the students that I am currently teaching. In search of answers to my wondering, I realized that this "desire" to increase awareness was merely a part of who I am, which I have described as trait-curiosity in earlier chapters. As such, my interest in focusing on emotions emerged as one way to improve my understanding of things around me.

Needless to say, I drew on this new-found interest not solely as a topic to research, but also to influence my teaching and learning of mathematics. Being aware of how comprehensive or complex the phenomena of emotions are, I set out to zoom in on something specific. As discussed in Chapter 1, curiosity became an interest through reflection on my childhood days. Based on the literature and my data analysis, curiosity has changed form in my research – going from trait-curiosity to relational-curiosity. Bearing these types of curiosity in mind, the sub-sections which follow offer a deeper assessment and concluding thoughts of my research findings.

8.1.1. What are some physical markers of curiosity in a secondary mathematics classroom?

While emotions such as sadness and happiness are observable, curiosity is not, and this is why the word in itself has gone through an upheaval (discussed in Chapter 2) in terms of pinning down an agreed-upon meaning. As a result, there was a need for me to re-conceptualize curiosity for the purpose of this research. In this process, Loewenstein's (1994) definition and Berlyne's (1954) curiosity dimension model served as a pair of lenses through which I could add a layer of visibility to curiosity. In doing so, I positioned curiosity as something that emerges when students' bodies intertwine with technology and the two act as co-agents with each other. I claim that students' wondering (through the use of questions), along with their body movements and the technology's ability to prompt students to react, was instrumental in my identifying when curiosity was present. Although this was not straightforward at the outset of the data collection, my involvement in the data analysis has shed light on the practicality of seeing curiosity in this way.

In terms of identifying the physical markers of curiosity, my research reveals that students' body movements, such as certain facial expressions, visual fixations, eyes gazes, glances and moving towards and away from the computer screen, signal the emergence of curiosity. However, emerging curiosity does not serve as confirmation that this element of the Embodied Curiosity framework is in full effect. The data also reveals that, in order to pin-point curiosity *per se*, there must be an instance of uncertainty. In many situations, this uncertainty manifests itself as a question from the student accompanying some body movement.

In addition, the findings reveal that, once this uncertainty is satisfied, the students usually lean away from the computer in a manner that suggests the embodied nature of curiosity: that is, curiosity takes on a visible form. By 'leaning away', it seems that the technology had "offered" the student some missing knowledge. I conclude that when students engage in a task using *Sketchpad,* their facial expressions, leaning forward and backwards while asking a question, is one way in which curiosity can be identified. I also argue that, if these components are not taken together, then curiosity, from an Embodied Curiosity perspective, is not present.

In Chapter 7, I alluded to the ways in which curiosity could be misunderstood as interest or attentiveness. Firstly, I reiterate that curiosity, according to my research, does not emanate solely from the students (human) the way interest and attentiveness does, but rather is a result of the collaboration between the human and the non-human, (in this case, the digital technology). Secondly, in order to make the distinction about these terms clear, I suggest that, in identifying curiosity, there must be interaction with the technology, as well as a recognizable doubt about something not readily in the students' experience.

On the other hand, attentiveness, although it could possess some level of doubt, can be identified by a ceaseless stare at the teacher or a peer while an explanation about something is given. This can manifest itself in a student observing a process which does not imply curiosity and may not pertain to the actual performance of a task using the technology. For instance, attentiveness may arise when students are participating in the introduction of the task.

Furthermore, I noticed that although interest is also considered a "desire" in the same way curiosity was presented in its early development, interest does not necessarily imply that there is uncertainty. I draw this conclusion on the basis that the definition of curiosity on which this study is grounded – "a cognitive induced deprivation that arises from the perception of a gap in knowledge" (Loewenstein, 1994, p. 76) – implies that there must be uncertainty (a gap in knowledge) for an instance to be classified as one involving curiosity. Given the opportunity to make this distinction between these similar, yet distinct, emotions, I conclude that aside from the potential for future research, my distinction offers more insights not only to the physical markers of curiosity, but also exposes a gap in Berlyne's (1954) curiosity dimension model – the need for curiosity to be identifiable or operationalizable.

This model also played a crucial role in my understanding of any classification of curiosity, since I recognized the need to be able to identify examples and non-examples. Utilizing this model (perceptual curiosity vs epistemic curiosity), I realized that students' emotions are as impactful as their cognitive experience. Based on the findings of my research, I conclude that relational-curiosity can be identified in the mathematics classroom by a combination of body movements, questions indicating uncertainty and a definite interaction with the digital technology.

8.1.2. To what extent does Embodied Curiosity foster the construction of mathematical meanings?

Results from my study show, firstly, that Embodied Curiosity emerges as a layered framework illustrating how its four elements – curiosity, body movement, technology and mathematics relate to each other. Secondly, the results also reveal that students construct mathematical meanings when interactions transition from one semi-permeable layer to another, and that it is through the maintenance of these transfers that mathematical meanings are developed. Moreover, I construe the role of the teacher as a significant aspect of this process, because the data shows that the teacher's role can be two-fold. One role is to activate the layer of curiosity through questions and persistence, and the other is to rekindle the Embodied Curiosity process, which can be accomplished by imposing implicit mathematical meanings into the process. Taking this into account, the temporal nature of Embodied Curiosity became evident, which reflects how it may contribute to the sustainability of mathematical meanings.

Furthermore, in response to the question about whether Embodied Curiosity contributes to the construction of mathematical meanings, the results clearly show that mathematical meanings are generated both explicitly and implicitly. What is more noticeable is that these two types of meanings were never pre-planned nor orchestrated by students and teachers or by the technology. Instead, they emerged only after students interacted with *Sketchpad.* As a result, I conclude that, in the construction of these two types of mathematical meanings, the human, material and disciplinary agencies were all at play. This aligns Embodied Curiosity with Pickering's (1995) assumption that human agency is not given superiority over any other agency, as in Actor–Network Theory (Latour, 1987), and that it might make more sense to speak of a relational agency or co-agency, as in the theory of inclusive materialism (de Freitas & Sinclair, 2014).

The most fundamental motivating factor of this study is its contribution to the limited research on the affective domain of learning with digital technology in mathematics education. In light of this, preliminary findings do suggest that students' feelings are essential in the construction of mathematical meanings. It is clear that trait-curiosity, on its own, could not determine how and why mathematical meanings are developed. This is because trait-curiosity is immeasurable. However, when it is associated with digital technology through students' interactions, a wide range of possibilities became apparent. I draw this conclusion on the basis that when students ask a question, not only does it indicate that there is indeed an uncertainty, but also that there is a need to seek answers from the technology.

I have highlighted in previous chapters how the dynamic nature and the aesthetic appeal of the technology ignite feelings of excitement in students. These feelings are usually accompanied by students engaging in curious dragging, which propel them to explore ideas on their own. In other words, students work independently with *Sketchpad* when they feel excited and this leads to the development of 'new' mathematical meanings. As a result, relational-curiosity emerged as the type of curiosity that is relevant to the teaching and learning of mathematics.

A corollary to the influence of the affect on the construction of mathematical meanings is the role the body plays in connecting emotions (curiosity) and technology to the way students think about geometry. My research findings suggest that the body is an essential part of the construction of the mathematical meaning process. This is evident in the relationships (Ali–Mio–Akeel–Kyle–Brea–*Sketchpad*) where the students' body movements illustrate doubts. In these examples, the students usually demonstrate uncertainty or, according to Loewenstein, "gaps in their knowledge" through their bodies. Most noticeable are their facial expressions and movement of their upper bodies which chronicle their state of disconcertion when faced with a task that is unfamiliar. These body movements play an important role in how relational-curiosity is triggered, because they are usually accompanied by questions or emerge when an uncertainty is satisfied. Since it is evident from my findings that interpretation of these body movements could mean the manifestation of curiosity, I infer that body movements are important signals for emerging mathematical meanings.

Conversely, digital technology offers dynamic representation, exploration and ways of communicating, which is well-known across the research community in mathematics education. However, my research findings suggest that, in addition to these essential elements, digital technology offers insights into new ways of seeing and thinking about geometric objects. For example, the initial perception of a circle as a round plane figure shifted to a locus of points equidistant from the centre point. Likewise, the cyclic quadrilateral emerged not solely as a four-sided figure inscribed within a circle, but one with all four vertices touching the circumference of the circle. Finally, there is the realization that a right-angled triangle can collapse into a line segment with the 90° angle eventually converging into a 0° angle, but only when the vertex of the triangle shares a common point with the diameter on the circumference.

Taking everything into account, another essential finding of my research is the role Embodied Curiosity (in its entirety) plays in connecting mathematics to the real-world. This research finding leads me to the conclude that the merging of multiple tasks opens up new opportunities for students either to impose their life experiences into mathematics or to extend mathematics into situations where it becomes applicable to their own world. Interestingly, the findings further suggest that these real-world experiences do not necessarily have to be a part of the student's lived experiences or immediate surroundings. This implies that the Embodied Curiosity process is not only possible for relationships between emotions and digital technology, but is also on the role of digital technology on one's imagination. A possible question, then, is: can Embodied Curiosity be extended to the creation of things which are usually deemed unachievable?

8.1.3. How do the potentialities and affordances of *The Geometer's Sketchpad* evoke Embodied Curiosity?

Sketchpad, like many other dynamic geometry environments, offers students the ability to examine relationships among mathematical objects. The draggable feature of the software allows students to investigate and make conjectures about their constructions. Although Arzarello et al.'s (2002) dragging modalities turned out to be useful to my research study, at least to provide a description for students' dragging, I noticed that students' emotions were integrally involved in how they performed dragging. In adding this new dimension to their modalities, and referring to them as 'curious dragging', I show that students' feelings were involved in the way they perform action.

In Pickering's (1995) account of the dance of agency, the emotional aspect of the actor was not taken into consideration. His assumption is mainly on performance (what the human does and what the materials do). From an Embodied Curiosity point of view, performing action requires an acknowledgement that human emotions are closely connected to their experience with the non-human and it is through this connection that new knowledge is generated.

Having established that the functionality of the technology is important in triggering emotions, I observe how curious dragging leads students to extend tasks in ways that would be impossible in static environments. For example, when students construct the typical quadrilateral – the square or the rectangle – in the circle, being able to drag its vertices on the screen reveals a family of quadrilaterals maintaining the same properties. Additionally, when students examine the type of triangle formed in the semi-circle, they realize that the dragging ability of the tool produces right-angled triangles of varying sizes. As a result, I conclude that the draggability of *Sketchpad* fosters deeper explorations and, at the same time, contributes to the development of "new" mathematical meanings. I also draw the conclusion that this draggable feature (and, in particular, curious dragging) contributes to the sustainability of the Embodied Curiosity process because there is a constant need for exploration. This means that curious dragging is another factor that contributes to the temporal nature of Embodied Curiosity.

In Chapter 7, I made mention of the anthropomorphic potential of *Sketchpad* and how it emerges as an embodied tool possessing the ability to act in human-like forms: that is, to 'partner' with, and 'instigate' certain actions from students. This implies that the technology emerges as more than a passive tool and also supports Pickering's (1995) claim that during an interaction the agency of the material de-centres the human. In my research I find that while students create the various circle geometry objects, *Sketchpad* shifts from a mediating submissive tool to a participatory, accommodating one: that is, from being passive to active. I deduce that this is made possible mainly through the student's constant need for change and the need for continuous exploration. This allows them to engage in computational thinking[23] approaches such as decomposing, tinkering and debugging.

I notice that while students' uncertainties were brought to the forefront through the exploratory nature of *Sketchpad*, the technology also prompted the students to seek knowledge. One main strategy in achieving this was through their acting on information which was already known. An example of this occurred when Brea, as well as many other students, constructed the circle first and then inserted the quadrilateral inside when attempting the task about the cyclic quadrilateral. This breaking-down of the task into separate parts, I believe, promotes active learning and supports an environment of collaboration with the technology.

8.2. Implications of Embodied Curiosity to Mathematics Education

Similar to Pickering's (1995) dance of agency, Embodied Curiosity construes the role of agency as the capacity to 'act'. However, it assumes that action does not solely depend on what the human (students and teachers) does or what the non-human (material objects like *Sketchpad* and the computer) do, but rather emphasizes what they do together and what emerges from this interaction. Unlike the dance of agency, these interactions consider the feelings of students as well as their body movements. Like Pickering, I too believe that in constructing knowledge – mathematical knowledge, that is – the focus should be on the doing.

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[23] A term used to refer to the mental problem-solving processes that people are engaged in when they interact with the computer. Wing, J. (2006). Computational thinking. *Communications of the ACM* 49(3), 33–35.

However, Embodied Curiosity departs from this to suggest that performance is not void of human emotions, for the same reason machines and their affordances are not. My argument is that students bring their emotions to the interaction in the same way *Sketchpad* brings its affordances and functionalities. Bearing this in mind, with my general interest on technology integration in the mathematics classroom, I foresee two ways in which this framework has implications for mathematics education: that is, for learning and for teaching.

8.2.1. For learning

First, my overall interest was in finding a construct that could respond to the way children learn mathematics, especially through the integration of digital technology. I was more interested in the relational aspect of the digital tool rather than on its mediating role and, most importantly, on how students' emotion (curiosity) could be considered. In the literature, I found curiosity (what I term trait-curiosity) to be an innate ability, which implies that it is intrinsically motivated and that it is in our nature to be contemplative. Yet, there is little to say about how curiosity can be exploited in mathematics teaching and learning.

Embodied Curiosity, through the reconceptualization of curiosity as something emerging from a relationship with the digital tool (relational-curiosity), offers new ways of thinking about the construction of mathematical meanings. My research findings suggest that learning takes place through collaboration not only with teachers and peers, but also with the technology. The Embodied Curiosity framework suggests that 'new' mathematical meanings can be developed when students' body movement, coupled with a wondering question, prompts them to seek knowledge from the digital technology. However, in the knowledge-seeking process, the role of the digital technology shifted from being passive to active (partnering with and instigating for), signalling the embodied nature of curiosity and its connection to the technology itself.

Second, the affordances of *Sketchpad* played an instrumental role in the development and sustainability of mathematical meanings. The literature has shown that the dynamic potential and the exploratory nature of *Sketchpad* allows students to examine relationships between geometric objects and gives students the opportunity to see these objects as a family of things rather than as a separate entity – which is customary in static environments. Embodied Curiosity suggests that mathematical tasks can also be viewed as a number of different things considered as a single unit. This was seen in the examples which produced the football field, snowman, hourglass and the Jamaican flag, and has implications for task design in mathematics education. In these examples, the students relied on their experience with previous tasks to examine objects in their surroundings.

For a long time, the focus has been on cognition and technology use in mathematics and, more recently, on the body and ways in which the sensorimotor influences the learning of mathematics. Despite the fact that more attention has been given to the affective dimension of learning in recent times, very little still is said about the affective in relation to digital technology. As a result, Embodied Curiosity attempts to bridge the gap on the role of emotions and technology in mathematics education.

The draggable feature of *Sketchpad* allows students to formulate and test conjectures. Embodied Curiosity has shown that this feature triggers certain kinds of feelings (excitement, reluctance, timidity) in the students. In Chapter 6, the data revealed that, when students' emotions are at play, they react to the technology in different ways. They became eager for explorations when they were excited and less engaging when they were timid. In many instances, as suggested in previous chapters, students tend to engage in curious dragging – a term used within the Embodied Curiosity framework to describe the speed, the temporal, and the emotional aspect of students' engagement with their constructions.

While excitement, for instance, may lead to further exploration and a continuous desire for knowledge, reluctance and timidity could suppress the learning process. This awareness promotes the idea that students learn best when they feel excited about doing mathematics. Therefore, this implies that digital technology offers more than "a tool for programming children", as Papert (1980) would agree, but instead provides a pathway that connects mathematics to the body. Additionally, since Embodied Curiosity sets out to reshape the way we think about students' use of digital technology, *Sketchpad*'s draggability implies that Embodied Curiosity may have considerable contributions to the way students communicate and express themselves with technology in the mathematics classroom.

8.2.2. For teaching

In Chapter 4, I alluded to the role of the teacher in designing the tasks for my research, and in the aforementioned sub-section I highlighted the implication of Embodied Curiosity in the way children used the knowledge of their previous tasks to develop new mathematical ideas. This does not have implications solely for the teaching of mathematics, but also for curriculum development, especially as it relates to teaching geometry. Although Embodied Curiosity focuses on the emergence of relationships, it is important to note that geometric tasks should be planned to encourage explorations, foster discussions and generate independent interactions. Without these conditions, students' engagement with the digital technology will be restricted and hinder the way mathematical meanings are developed. Therefore, the teacher plays a crucial role in how Embodied Curiosity is implemented and executed.

Notwithstanding the teachers' role in designing the tasks, the data revealed that another critical function of the teacher was to identify instances when implicit mathematical meanings were present and to infuse them into the interactions. This can be made possible by identifying teachable moments that will guarantee the flow of the process while triggering independent exploration. By extension, these implicit meanings will lead students to uncover 'new' mathematical meanings. It seems, then, that the teacher plays an active role in maintaining the Embodied Curiosity process and being able to maintain Embodied Curiosity requires being able to identify emergent mathematics.

There has been an increase of theoretical perspectives and an urgent need for theoretical framework that focuses on technology integration in mathematics education. For this reason, Drijvers et al. (2010), in their research on theoretical perspectives use in mathematics education, suggest that much focus has been around instrumentation and semiotic mediation when it comes to theoretical perspectives and technology use. Taking this into consideration, they made a call for more integrative theoretical approaches suggesting that the emphasis should be on bringing together different theoretical underpinnings "to guide teaching, understand learning and improve mathematics education" (p. 90).

In contrast to this, I foresee a need for more theoretical frameworks which are grounded in the field of mathematics education and will respond specifically to technology integration in mathematics teaching and learning. My rationale for this is that technology has served more than its physical function. As a result, instead of integrating different theoretical approaches, Embodied Curiosity considers a collection of people and things and how they interact with each other to construct mathematical meanings. This has implications for the teaching of mathematics in two ways: one, it presents a possible new way of thinking about how students learn mathematics and two, it offers a response to the processes involved in the construction of mathematical meanings.

8.3. Relevance

This research has been concerned with identifying new ways of thinking and understanding how students learn mathematics. I draw on my childhood and professional experiences, as well as my experience as a junior researcher, to understand how learning takes place in mathematics. I examined my in-born traits, specifically my curious behaviour towards the world, as a pathway into this understanding. I also reflected on my experience as a mathematics teacher and found that I still had an appetite for understanding how children learn mathematics.

During my experience as a mathematics teacher, I observed how attached students were to their technological tools (cell phones, laptops, tablets, iPads) and, most importantly, how at ease they were in exploring what these tools could do for them. From a Jamaican context, mathematics performance among high-school students was dissatisfying. Yet, there was an upsurge in the interest for technology more for recreational and socialization purposes and less for academic ones. With this background, I wondered about ways in which I could understand the interplay between technology (as a tool then) and the ways students learn mathematics. What made my wonderings more interesting for me was that, at the time, cell phones (which were more dominant) were also banned from many schools due to irresponsible use of the device by students. As a result, the findings of my research have a number of benefits.

Having an awareness of the relationship between emotions and technology use in the development of mathematical meanings could be an influential factor for policy change and decision-making. The advancement of technology propelled mathematics education in a direction that requires a constant need for reform, as well as new ways of thinking about how technology should be used to enhance students' learning. As a result, my research findings could be taken into consideration when policies are being drafted, which should focus on making technology more accessible and affordable to students and teachers.

In addition, my research findings could signal a first step towards implementing programs that seek to promote affective engagement with technology. Furthermore, Embodied Curiosity could also be adapted as a teaching approach which focuses on children doing mathematics rather than acquiring knowledge. Finally, policy change could impact the perception about the use of technology tools and promote effective ways to integrate them in the mathematics classroom.

Based on the context of this research, I believe that these issues are not unique to Jamaica or the Caribbean region *per se*, but also to a wider geographic region. As a result, Embodied Curiosity can play an integral role in curriculum development. Taking the new ways of thinking about the body, technology and curiosity into consideration, mathematics curriculum should reflect the relationships among them and the role the teacher plays in implementing these relationships. This positions teachers as active participants in the process of developing and designing mathematics curriculum.

A broader vision of Embodied Curiosity is to shed light on the on-going dualism between mind and body, and to offer an alternative way of thinking about the relationship(s) between the two. By virtue of this, I see the findings of my research relevant to the on-going discourse and as a means to bridge the divide. Similar to de Freitas and Sinclair's (2014) inclusive materialism, Embodied Curiosity considers the dispersion of agency between both human and material, and positions the body as playing an important role in the learning process. In essence, Embodied Curiosity adds to this reshaping and reconceptualization of the mind and body.

8.4. Limitations of the Research

Like much other research, my study was not devoid of challenges. The most notable challenge was to reconceptualize curiosity as something identifiable. Although Berlyne's (1954) curiosity dimension model served as a benchmark for determining instances of trait-curiosity, it was difficult to pin-point when curiosity was emerging or when it was present. This is because trait-curiosity can be found within an individual. In order to utilize the potential of curiosity effectively, it had to be seen as something that could be visible in the students' actions and movements. As a result, a considerable amount of time was spent revisiting the video-recordings, attending to different things upon each viewing. By doing so, I was able to discard Berlyne's dualism and instead examine how both perceptual and epistemic curiosity related to each other.

Adopting Pickering's (1995) position on agency was useful in describing how the students and digital technology intertwine with each other. However, it was challenging to ascribe curiosity as something that spread across the interaction incorporating the technological tool. After all, how can a tool become curious? It was not until the role of body movements became apparent to me that I was able to rethink curiosity in a relational way: hence, the conceptualization of relational-curiosity. Furthermore, in Pickering's case studies, it was clear that the human and non-human were always performing action with each other. However, it was challenging for me to see how the four elements of Embodied Curiosity would interact in similar ways. As a result, I drew several diagrams trying to decipher how each element would relate to one another. I overcame this challenge after I collated the data in Chapter 6 (Tables 6.1 and 6.2) and realized that each element was connected in a layered format.

Another limitation occurred in the methodology. My initial plan to record students' interactions was to identify a suitable location in each room where the video recorder could be kept stationary and students' on-screen interactions would be captured using the Camtasia studio software. However, I was unable to do this because of the dynamics of student consent as described in Chapter 4, especially at School Y. As a result, recording was done dynamically by myself and the two participating teachers when I changed roles from observer to participant. This had an impact on the quality of some episodes as the video data shows instances when the camera was switched from a pair of students at a time when an idea was emerging

8.5. Concluding Remarks

At the beginning of my dissertation, I characterised my eagerness to learn and to understand how students learn best using my own childhood experiences, learning experiences and my experience as a mathematics teacher. The literature highlights the complexity of curiosity, the advancement of technology and the limited pursuit of research in mathematics education on the affective domain of learning and digital technology. Through the lens of Embodied Curiosity, I was able to show that, when students and technology tools engage with one other, trait-curiosity manifested itself through body movements and wondering questions which signal some form of uncertainty. This implies that there is a relationship among curiosity, body movement and digital technology, and it is within this relationship that I call relational-curiosity, that mathematical meaning-making became possible.

8.6. Did Curiosity Kill the Cat?

Writing the final chapter of my dissertation amidst a global pandemic has produced new perspectives for me about education in general and, more specifically, about mathematics education. The periods that we have come to know as ‘lockdowns’, ‘quarantines’ and ‘isolations’ have been a time of deep reflection for me. My reflection during this time was focused mainly on what it means to teach mathematics and, more broadly, around the inequalities and the purposes of education that the pandemic had exposed. Some of my thoughts were influenced by my involvement in remote teaching and more so by reading the book *Weapons of math destruction* by Cathy O’Neil (2017) during this period. In this final section, I give a detailed account of my general thoughts, not only to provide my readers with insights into how I have evolved, but also as a way of documenting my ‘new’-found feelings about mathematics. I will share how these two experiences have shaped my thoughts and how I foresee myself in future roles as a mathematics educator/researcher.

I tend to refer to this section as ‘uncut’, mainly because my expressions reflect my true feelings and are not aligned with ‘perceived’ norms of what should or should not be written in a concluding chapter, especially for an esteemed document such as a doctoral dissertation. Positively or negatively, my work over the past few years on the role of emotions and technology in mathematics education has taught me that despite the special attention given to the cognitive in mathematics education, the affective is integrally intertwined in learning, teaching, doing and researching. As a result, I feel the need to end this chapter and by extension my dissertation by describing how it was emotionally connected to me.

I will begin by saying that, along the way, I lost interest in teaching mathematics (the practical aspect of it, that is), as well as in Embodied Curiosity and mathematics education more generally. I constantly found myself asking questions like: What is the purpose of education and mathematics education? Why do I teach? What do I want my students to achieve? Is there a disconnect between my expectations and students' intentions? Why is there such disparity between my past and present experiences as a teacher of mathematics? And, most importantly, what is there to be gained from the years of work with Embodied Curiosity? From time to time, one of these questions would surface in my thoughts throughout the journey of writing my dissertation. However, they became worrisome to me when I transitioned to remote teaching due to the pandemic.

I recall my first encounter with undergraduate students taking a geometry course that I was teaching remotely. While I did not anticipate a challenge to transition from in-person to remote teaching at first, my main focus was on maintaining course quality and at the same time creating a fun and engaging classroom culture for my students. I began by asking students to share their thoughts about mathematics. Most of them expressed having a bad experience with the subject or with a teacher who taught them mathematics in earlier years. Unfortunately, they also lamented about having to take the course remotely, fearing that it would not be as meaningful to them as in-person and that they would miss out on the social aspect of school life.

While the pandemic has highlighted for me the importance of eye contact/eye gazes, agency and interaction, my biggest concern at the time was to maintain an active classroom, something that I always aim to achieve. Here I was with an opportunity to understand better the very phenomena that I was studying (eye gazes, agency and interactions), yet my interest was on replicating an 'active' classroom on-line. I realized that perhaps my interest was not about understanding how students learn mathematics, but rather about following perceived norms about what a mathematics classroom should look like.

Undoubtedly, I had two perspectives of ‘active’: one which involves the physicality of students’ interactions and the other which engages actions of teachers, students, materials and even mathematics as a discipline. Furthermore, I became confused about whether or not the focus should be on what the mathematics classroom ‘looks’ like rather than what it ‘feels’ like. I was in a state of melancholy because at this point my research was losing its relevance and I suddenly found my desire for knowing slowly drifting away.

Things took a turn for the worst when I read Cathy O’Neil’s (2017) *Weapons of math destruction*. In this book, O’Neil paints a grim picture of how mathematical models and algorithms are used to marginalize, to disenfranchise and deliberately to exclude vulnerable groups of people. One can understand, as someone who is classified as vulnerable – a black immigrant woman – that, at this point, I wanted no part of this mathematics – something I have spent a substantial part of my life immersed in. I felt helpless because I could not offer my students an alternative experience from the one they shared at the beginning and neither could I continue to pursue Embodied Curiosity, at least not with the passion I had started with. I felt that mathematics education had let me down and more so my students whose journeys were different, yet yielded the same result – an unfavourable feeling towards the discipline.

Adamant that I would not contribute further to the students’ negative feelings about mathematics and remote learning, I drew on another innate ability – empathy: that is, understanding and sharing how my students felt and basically going through the experience with them rather than beyond them. This strategy was a bounce-back moment for me, not only concretizing why a study of the affective of learning was crucial, but also why it was important for me at least to attempt to make a change. This dissertation will be the catalyst of the new change I now desire.

Although my thoughts were in turmoil, one of the most significant accomplishments of my work is not only to rethink, reconceptualize and reshape curiosity as an important entity to meaning-making, but also to realign myself as a mathematics educator and researcher in an infinitely evolving field. In this realignment, I see myself advocating for change, not under the term ‘reform’, but in terms of compassion for mathematics, students and fellow colleagues. I now believe that my biggest take-away from the research conducted here comes from the words of Alvin Day (2003), “If caterpillars can fly, so can I”.

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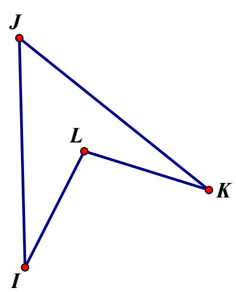
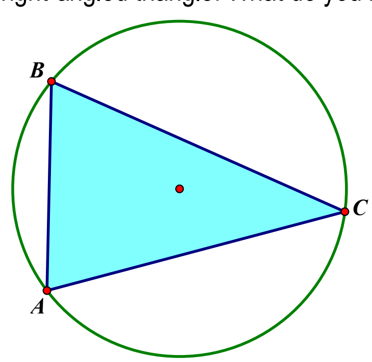
Appendix A.

***Task1:* Angle in a semi-circle** What kind of triangle is formed in a semi-circle?	***Task 2*: Cyclic Quadrilateral** Draw an inscribed quadrilateral with one right angle, two right angles or three right angles. What do you notice?
***Task 3*: Exploring the diameter of circles** Construct a chord in a circle. Fix one point and drag the other along the circumference. What can you say about the maximum length of the chord?	***Task 4*: Length of a chord in a circle** Find the relationship between the length of the chords in a circle, and the distance between the chords and the centre of the circle. Say whether they are equal, less than or greater than.
***Task 5*: Angle at the centre and circumference** 1.Can you find out the places where the angle at I is twice the angle at J?  2.Use the trace function to see what happens with the relationship between the angles in part (1) 3.Construct a circle that goes through J, K and I	***Task 6*: Making circles** How many circles can you make through One point Two points Three points …… ***Task 7*: Noticing invariance: Isosceles triangle in a circle** What kinds of triangles can you make using the centre point and two random points on the circumference or edge of the circle?
***Task 8*: Chordal midpoint** What is the locus of the midpoint of the chord in a circle? Fix one point and drag the other around. ***Task 10:* Centers of touching circles** [a]. What can you say about the line joining the centres of two overlapping circles and their common chord? [b].	***Task 9*: Converse of angles in a semi-circle** Drag points A, B and C so that the triangle ABC is a right-angled triangle. What do you notice. 

Appendix B.

TRANSRIPT CONVENTIONS

Symbols	Meaning	Examples
Speaker ID and Anonymization Turn # Name:	The speaker ID is given at the beginning of each turn with a colon.	1 Sheree:
~	Short pauses (3 seconds)	Miss, ~ like tangent and sine
~ ~	Longer pauses (> 3 minutes)	Go ahead ~ ~ figure it out!
< >	Actions of students	< pointing to the diagram>
[ ]	English translation when students use their native language	Weh yuh notis [what do you notice?]