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NAME OF SUPERVISOR/NOM DU DIRECTEUR DE THÈSE Thomas K. Peucker

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TOPOLOGICAL RANDOMNESS OF GEOMORPHIC
SURFACES

by

David Michael Mark

B.A., Simon Fraser University, 1970

M.A., University of British Columbia, 1974

A THESIS SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE DEGREE OF
DOCTOR OF PHILOSOPHY

in the Department

of

Geography

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Simon Fraser University

March 1977

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APPROVAL

Name: David Michael Mark

Degree: Doctor of Philosophy

Title of Thesis: Topological Randomness of Geomorphic Surfaces

Examining Committee:

Chairman: Frank F. Cunningham

Thomas K. Peucker
Senior Supervisor

Edward J. Hickin

Michael C. Roberts

Michael Church
Assistant Professor
University of British Columbia

Ronald L. Shreve
External Examiner
Professor of Geology and Geophysics
Department of Earth and Space Sciences
University of California, Los Angeles

Date of Approval: MARCH 11, 1977

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ABSTRACT

Randomness is an important principal in the Earth Sciences in general and in geomorphology in particular. Originators of this concept emphasized constraints placed on randomness by geomorphic processes; most subsequent work has, however, concentrated on properties of one aspect of surface form (the drainage net) in the absence of structural control. The present study investigates other network properties of surfaces, primarily ridge patterns, and emphasizes the constraints which must be placed on randomness in order that a surface's topological properties will approximate those of geomorphic surfaces. The study is largely theoretical, and all data are derived from medium scale topographic contour maps.

Various concepts of a "random" surface are examined. One apparent constraint is that the variance spectrum of the terrain must approximate a power function. "Random, mature, fluvially-eroded" terrain is further constrained by an absence of large, closed depressions. Attention is at first concentrated on topological properties of ridge patterns. A simple model, in which magnitude-N ridge networks are simulated by forming minimum spanning trees of sets of N points (representing peaks), distributed randomly within an ellipse, provides good predictions for magnitude-6 ridge topological class frequencies;

modelled frequencies are generated using the Monte Carlo approach, with 5000 trials. When goodness-of-fit (measured by Chi-square) is plotted against the elongation ratio of the ellipse, the shape of the plot indicates the degree of ridge anisotropy present within the topography. In areas of heterogeneous geology but without overt structural control, the curve shows approximately equal fit for all elongations between a circle and a 2:1 ellipse, beyond which point the fit rapidly becomes very poor. In contrast, areas of flat-lying, homogeneous geology show curves which decline noticeably as one goes from the circle to an optimum fit at an elongation of about 0.5. This indicates a significant within-network ridge anisotropy in these landscapes, which is attributed to local topographic control imposed by the major tributaries of master streams. The model also fits areas with overt structural control (ridge-and-valley topography), but with an optimum elongation ratio of about 0.1. Surface trees, which represent some three-dimensional aspects of surface topology, are completely determined by ridge graph topology and pass elevation ranks. Their topologically-distinct classes are not predicted well if pass elevations are assumed to be independent, but require the assumption of pass height autocorrelation. All passes (and most topographic complexity) are found to be concentrated above the mean elevation of the terrain. Although topological features exclude explicit references to geometry and

scale, they are nevertheless influenced by these factors. Fundamental scales of topographic surfaces are discussed. The ridge model fits networks of magnitudes from 4 to 7, but the shapes of the goodness-of-fit plots indicate that degree of anisotropy varies with magnitude in some landscapes; this is a scale effect.

It is difficult to relate the ridge topology research presented in this study to previous stream topology work; reasons for this difficulty are discussed. It is concluded that further research should attempt to resolve these difficulties, and also should concentrate on constraints on topological randomness, rather than on the randomness itself.

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This thesis is dedicated to:

My Parents

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CHAPTER 1: INTRODUCTION

"Anyone who has looked can hardly deny the presence of a random element in natural landscapes. It is impossible to predict exactly what will be around the next bend or over the next ridge."

R.L. Shreve (1975, p. 527).

"It may be visualized that the study of the landscape is the study of constraints imposed by geologic structure, lithology, and history. To identify those constraints is the purpose of much of geologic enquiry; to evaluate the effects of constraints upon the landscape is an object of quantitative geomorphology."

L.B. Leopold and W.B. Langbein (1962, p. A11).

1.1: Randomness in Geomorphology

Leopold and Langbein (1962) suggested that "the distribution of energy in a river system tends toward the most probable state", and, by analogy with thermodynamics, termed this concept "entropy". Their work implied that geomorphic patterns tend to be probabilistic, rather than deterministic, emphasizing the importance of a random or apparently random element in landscapes. This approach was applied by Scheidegger and Langbein (1966), and has found its most successful expression in the writings of Shreve (1966, 1967, 1969). Shreve (1975) and Smart and Werner (1976) have recently reviewed the many quantitative properties of drainage systems which have been "explained" by this probabilistic-topologic approach.

Leopold and Langbein (1962) developed models for the longitudinal profiles of rivers, for hydraulic geometry, and for the planimetric patterns of drainage networks, all based on the entropy concept. Perhaps their most important point, illustrated in their discussion of longitudinal profiles, is that, if entropy considerations are assumed to apply, "the absence of all constraints leads to no solution-- an obvious but not a trivial result" (Leopold and Langbein, 1962, p. A11). The other important point, however, is that one or a few very simple constraints, together with the entropy principle, lead to "solutions" (most probable states) which closely approximate reality. In the case of the longitudinal profile, the simple constraints of a base level and a headwater elevation lead to the commonly observed exponential profile form. Similarly, random walks constrained in such a way as to exclude circuits or loops give rise to simulated drainage nets which obey "Horton's Laws" (Horton, 1945) of drainage composition (Leopold and Langbein, 1962; Milton, 1966). One must, of course, be aware of the possibility of convergence of forms, and it is dangerous to assume that river patterns which obey "Horton's Laws" are necessarily random. It is, however, generally far more dangerous to interpret as meaningful those patterns which may reasonably be ascribed to randomness.

Given that "Horton's Laws" may be due to random branching and the nature of the ordering system, one faces two alternatives: one may conclude, as did Milton (1966, 1967), that Horton's Law of Stream Numbers is "irrelevant to geomorphology" (1967, p. 83); Scheidegger and Langbein (1966), Shreve (1966), and most other geomorphologists have, however, drawn the other conclusion, namely that random or apparently random processes are very important in geomorphology. The question has arisen in the geologic literature as to whether the "random" component in geologic processes is truly random or whether it only appears to be random "due to a large number of deterministic events involved" (Mann, 1970, p. 402). Simpson (1970, p. 3184) stated that events may be unpredictable because (a) not all predictors are known, or (b) not all predictors are knowable, or (c) even if all predictors are known, there is an unresolvable stochastic component. Simpson believed that no macro-geomorphic processes fall into class (c). Watson (1969) and Smalley (1970) also entered the debate, but I agree with Scheidegger and Langbein (1966) and Shreve (1975) in their conclusion that the distinction between "real" and "apparent" randomness is largely irrelevant to any practical analysis or evaluation of data in geomorphology, at least at the physiographic or "landscape" scale.

In the discussion of the implications of their work for geomorphology, Leopold and Langbein (1962, p. A17) stated that "in a sense, then, much of geomorphology has been the study of the very same constraints that we have attempted to express in a mathematical model". In responding to the criticism that his probabilistic-topologic approach lacks physical content, Shreve (1975, p. 528) echoed Leopold and Langbein, stating that "the geomorphology, in other words, consists in choosing the right basic postulates"; these "postulates" represent some of the "constraints" mentioned above. To a large extent, however, subsequent researchers have ignored Leopold and Langbein's emphasis on constraints, and have concentrated their efforts on fluvially-eroded terrain in the absence of structural control.

Topological properties of drainage nets, which ignore or at least de-emphasize geometric properties such as lengths, areas, and angles, have been an important area of concern for geomorphologists since the seminal paper of Horton (1945).

Among other things, a concentration on topology allows for a simplification of systems, making "explanation" more tractable. Once these properties have been explained (as by Shreve, 1966, 1967, for stream patterns), it may be possible to re-introduce geometric properties (such as stream lengths; Shreve, 1969).

The probabilistic-topologic approach has thus far been applied almost exclusively to planimetric properties of the drainage

net, the exception being Werner's (1972a, 1972b, 1972c, 1973) work on divide patterns. The principal objective of the present study is to go beyond the drainage net and to examine other network representations of geomorphic surfaces, such as those used in "macrogeography" by Warntz (1966) and in computer cartography by several authors. A recurring theme will be an examination of the constraints which must be placed on the randomness of surfaces in order that their topological properties approximate those of real geomorphic surfaces.

The study is organized into four main chapters. Chapter 2 introduces some basic terms and concepts from graph theory which will be useful for the examination of topological properties, and reviews two lines of research into surface networks. The next two chapters apply the random approach to predicting Pfaltz graph (ridge) topology (Chapter 3) and surface trees (Chapter 4). In Chapter 5, the effects of scale, both topologic (by varying the magnitudes of systems) and geometric, on these networks will be described. Graph-theoretic terms, data sources and error evaluations, and additional computer cartographic applications of the networks developed and discussed, are contained in three appendices. In the balance of this Chapter, the important question: "What is a random surface?" will be explored.

1.2: What is a Random Surface?

The concept of "randomness" often is used to provide a "null hypothesis" for various statistical tests; with respect to surface analysis, this gives rise to the question which heads this section. First, it is important to note that the unqualified term "random" has essentially no meaning. It is only meaningful when it represents objects or events which have been randomly selected from some population, and that population, be it uniform, Gaussian (normal), or other, must be defined. Such a definition, however, when applied to geomorphic surfaces, constitutes the first in a series of constraints which contain the "physical content" of the random approach in geomorphology.

Perhaps the first concept of a "random surface" to occur would be to assign a randomly chosen (from some population) elevation to each datum location. On such a surface, adjacent points would have independent elevations, and this is the two-dimensional equivalent of "white noise" (Figure 1.1A). Segner (1969) called this the "random roughness model", and studied the drainage nets which would develop on such surfaces, with and without the further constraint of an inclined plane added to the white noise. Generally, however, this "model" does not produce surfaces which resemble most natural terrain in its

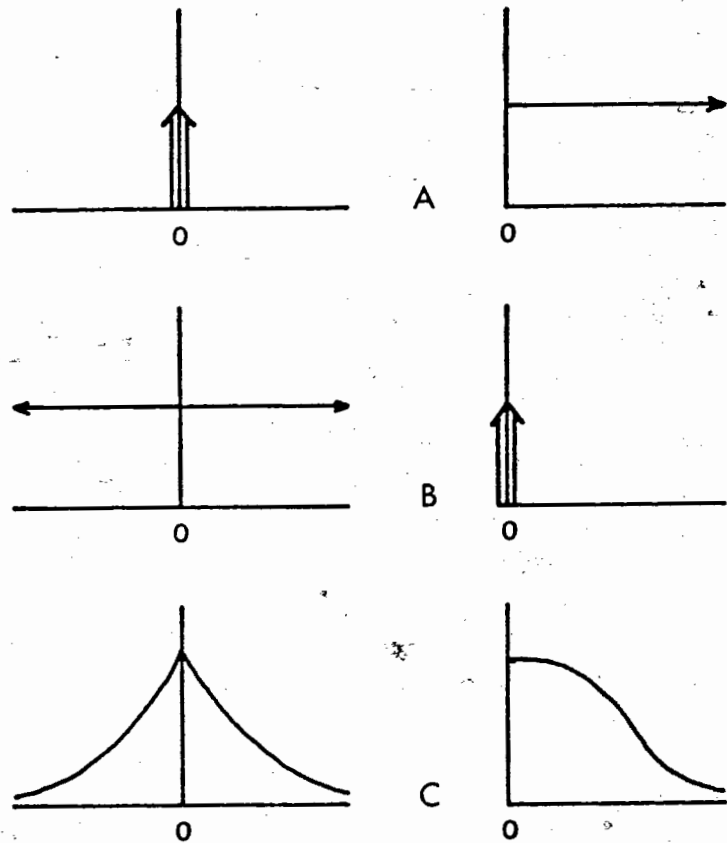


Figure 1.1: Autocorrelation functions (left) and associated power spectra (right) for three types of "random" processes. A: White noise; B: Brownian or Wiener-Levy process over a finite interval; C: Stationary Gaussian Markov process. All scales are linear.

autocorrelation and general appearance, since there is no continuity. In contrast to this approach, Mandelbrot (1975) produced a "Brownian Surface" based on the two-dimensional generalization of Brownian motion. This bears a much closer resemblance to natural terrain than does the white noise surface, and even has well developed "ridges" and "valleys" which are solely the consequence of the continuity constraint imposed by the generating process. The probability distribution for ridge orientation on a Brownian surface is isotropic, but Mandelbrot suggested that the surface could be made to resemble some terrain even more closely by introducing a preferred orientation (anisotropy) into the ridge directions by deforming the coordinate system to "stretch" the plane. Mandelbrot (1975, p. 3827) stated that these surfaces have a continuous power spectrum, with spectral density proportional to frequency to the -2 power. Papoulis (1965, p. 293); however, stated that the autocorrelation between $f(t_1)$ and $f(t_2)$ of a Brownian (or Wiener-Levy) process is a constant times the smaller of (t_1, t_2) . For any finite-length Brownian series, the autocorrelation plot would thus be a horizontal line (see Figure 1.1B), and the limit of the autocorrelation plot as the series length goes to infinity is undefined; for a finite series, the power spectrum is just a spike at zero frequency. A visual inspection of a block diagram presented by Mandelbrot would suggest that the spectrum is of the power function form, and

certainly not simply such a zero-frequency spike; whether the spectral properties change dramatically when one generalizes the Brownian concept to more than one dimension, or whether Mandelbrot's surface generation procedure is in fact not a correct generalization of Brownian motion, is not known.

Much of the rather limited amount of research into autocorrelations and power spectra of geomorphic surfaces has concentrated on "atypical" terrain such as lunar surfaces and their analogues (Jaeger and Schuring, 1966; Marcus, 1967; Rozema, 1969). McDonald and Katz (1969) studied sea floor topography, while Pike and Rozema (1975) are among the few to examine terrestrial "macro"-relief. Jaeger and Schuring (1966), Rozema (1969), and Pike and Rozema (1975) plotted power spectra on double-logarithmic graph paper, and all found the spectra to plot as approximate straight lines (indicating power function relations between variance and frequency), with slopes around -2 to -3 (see Figure 1.2). Marcus (1967) and McDonald and Katz (1969) reported their results as autocorrelation plots, which were approximately exponential in form.

The observed spectra and autocorrelation plots are distinct from white noise, and much more closely approximate the exponential form shown in Figure 1.1C. This form is characteristic of a stationary Gaussian Markov process (Bendat and Piersol, 1966, p.

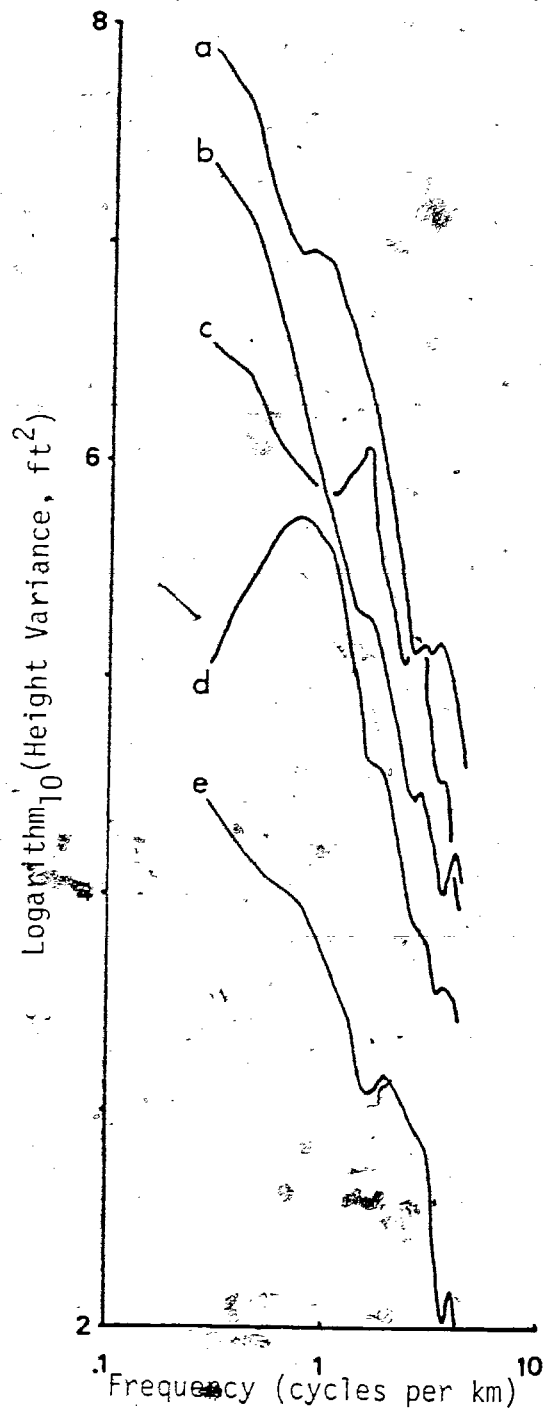


Figure 1.2: Logarithm (base 10) of variance against frequency

(cycles per kilometre) for five terrain profiles, after

Pike and Rozema (1975, p. 510). a: alpine glacial;

b: ridge-and-valley; c: mature fluvial; d: dunes;

e: ground moraine.

90), but the similarity does not, of course, indicate that geomorphic surfaces are formed by such processes. The power spectrum in Figure 1.1C has a tail which approximates a power function with exponent -2 ; this is also the exponent of Mandelbrot's (1975) surface, while an exponent of -3 indicates "uniformity of topographic slope in all features in an area regardless of size" (Pike and Rozema, 1975, p. 512). In any case, the autocorrelation function (or power spectrum) represents a constraint on the randomness of terrain surfaces.

All of these surfaces, including those with exponential autocorrelation, have equal expected numbers of peaks (maxima) and pits (minima). The almost complete absence of large pits (closed depressions which would appear on medium-scale contour maps) is, however, perhaps the most important constraint on the possible randomness of mature fluvial topography. Given this, together with a further constraint on the branching patterns of stream networks (namely, that stream patterns are "trivalent planted plane trees"; see Chapter 2 for an explanation of this term), Shreve (1966, 1967, 1969) developed the probabilistic-topologic approach to stream networks. It is important, however, to recognize that this simple model has built-in constraints. The present study will concentrate on the concept of "randomness" as applied to topological properties of

other network representations of surfaces, primarily the ridge networks, together with the constraints necessary to provide an adequate representation of these aspects of geomorphic surfaces.

CHAPTER 2: SURFACE NETWORKS AND GRAPH THEORY CONCEPTS

Since a map may in many ways be considered to be a special case of the mathematical concept termed a "graph" (see below), graphs have always been an important part of Geography. With the relatively recent development of the branch of Mathematics called "Graph Theory", geographers have available a theoretical framework for the study of maps and networks (see Haggett and Chorley, 1969, for an extensive review).

In the mid-nineteenth century, an approach to the topology of continuous smooth surfaces was developed by Reech (1858), Cayley (1859), and Maxwell (1870). This approach, which identifies points of equilibrium and their interrelationships, was "re-discovered" by Warntz (1966), and placed in a more formal graph-theoretic framework by Pfaltz (1976). So far, this approach has received only very limited attention in a geomorphic context (Woldenberg, 1972; Warntz, 1975).

In the mid-1960's, another method for describing some aspects of the topology of continuous smooth surfaces was developed in computer science (Boydell and Ruston, 1963; Morse, 1965). This "contour tree" approach would appear to have considerable potential for characterizing the topology of terrain, but has not yet been applied to that problem.

After presenting some basic graph-theoretic terms and concepts, this chapter will review and develop these two lines of research; their potential applicability to the investigation of geomorphic "randomness" will be discussed in later chapters.

2.1: Graph Theory Concepts and Terminology

Graph theory is a relatively new branch of mathematics, and suffers from a lack of standardized terminology. This study will in general follow the definitions and terms of Harary (1969), but since this is relatively new to many geographers, and since some changes and additional concepts are necessary, a brief review seems appropriate here. The terms listed and defined below are repeated in alphabetical order in Appendix A for ready reference; some of the terms are illustrated in Figures 2.1 and 2.2.

A graph can be defined as a set V of p points (usually termed vertices), together with a set E of q edges connecting distinct pairs of vertices in V . Such a graph is usually denoted $G(V;E)$, and may also be referred to as a (p,q) graph. Customarily, a graph is represented by a diagram; the graph is, however, the mathematical relation underlying the diagram. A graph is labelled if there is a name or label associated with each vertex. It is edge-labelled if each edge is so named. A rooted

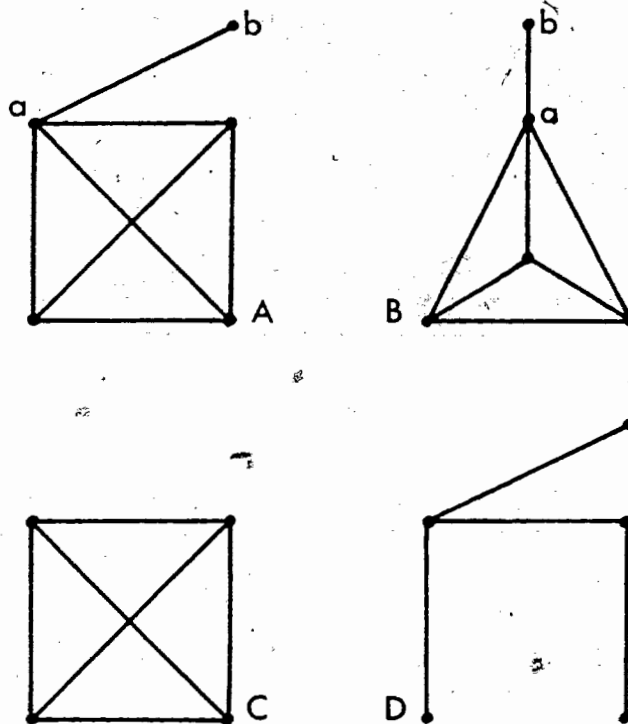


Figure 2.1: Some graph-theoretic terms and concepts:
 points a and b are adjacent; A and B are isomorphic;
 C and D are subgraphs of A (and of B); C is the
 complete graph K_4 ; D is a spanning subtree of A (and
 of B); B and D are plane graphs; all four graphs are
 planar.

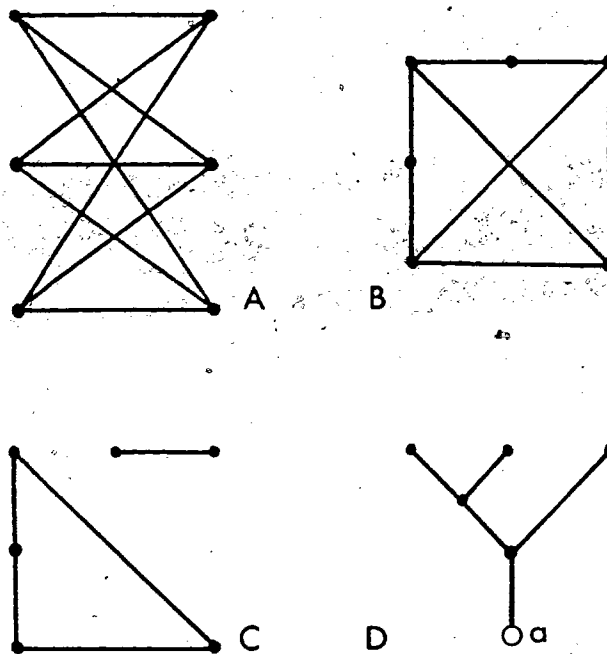


Figure 2.2: Additional graph-theoretic terms and concepts:

A is the complete bipartite graph $K_{3,3}$; B is homeomorphic to Figure 2.1C; C is a disconnected subgraph of B; D is a trivalent planted tree, rooted at a; C and D are plane graphs; B is planar but A is not.

graph is one in which a particular vertex is identified as the "root", a uniquely identified reference point.

If two vertices share an edge, they are said to be adjacent. The set of all vertices adjacent to a given vertex are said to be the neighbours of that vertex, while edges which share a vertex are termed adjacent edges. The number of edges incident on (connected with) a vertex is the degree of that vertex. Vertices of degree one are termed end-points, those of degree zero isolated vertices. If every vertex is adjacent to every other, the graph is complete, and is usually denoted K_n , where n is the number of vertices. If a graph has no edges, it is said to be a null graph. A graph must have at least one vertex; a graph with only one vertex is termed "trivial".

A subgraph of $G(V;E)$ is a graph having all of its vertices in V and its edges in E . A spanning subgraph contains all the vertices of V . If the graph can be divided into two subgraphs such that every vertex is in a subgraph and such that each subgraph is a null graph, the total graph is said to be bipartite. The complete bipartite graph (or, bigraph) with m vertices in one subgraph and n in the other is designated $K_{m,n}$. If the graph can be divided into k null subgraphs such that each vertex is in exactly one subgraph, the graph is k -partite.

Two graphs are said to be isomorphic if there exists a one-to-one correspondence of their vertices which preserves adjacencies. Two graphs are homeomorphic if they can be made isomorphic by adding or deleting vertices of degree two. If a graph is drawn on (embedded in) a plane without any intersecting edges, it is a plane graph. A graph which is isomorphic to some plane graph is said to be planar, and two plane graphs which are isomorphic and whose points can be made to correspond through a continuous deformation in the plane are said to be topologically identical.

If there exists a function which assigns a non-negative weight or length to each edge, the graph is termed a network. In a geometric graph, every vertex has a position in Euclidean (Cartesian) space. One network associated with a geometric graph would have as its edge weights the Euclidean distances between adjacent vertices. Two networks are said to be isomorphic if their graphs (ignoring edge weights) are isomorphic. Properties of the graph of a network are called topological properties of the network.

A walk is an alternating series of adjacent edges and vertices; a path is a walk in which all vertices, except possibly the first and last, are distinct. If there exists at least one path between every pair of vertices, the graph is connected. The

topological length of a path is the number of edges it contains. A cycle is a path leading from a vertex to itself. A connected graph having no cycles is termed a tree, and denoted $T(V;E)$. A tree with p vertices must have $q=p-1$ edges. A disconnected graph containing no cycles is a set of trees and is termed a forest. A tree all of whose vertices are of degree 1 or k is said to be k -valent. A tree rooted at an end-point is said to be planted.

An elementary cycle of a plane graph is one which has no points or lines within it. Such an elementary cycle encloses or bounds a face or cell. A plane map is a connected plane graph, together with all its faces. Faces are adjacent if they share at least one edge; such faces are neighbours. The geometric dual of a plane map is formed by placing a new vertex in each face and connecting these new vertices if their faces are adjacent. This dual is also a plane map, and its dual is the original map.

2.2: "Normal" Surfaces and "Critical" Points

I will herein restrict the term "surface" to a continuous, single-valued function of two variables, $z=f(x,y)$ (the term could, of course, be extended to N dimensions). Discussion will be further restricted to geographic surfaces (where the

independent variables, x and y , denote position in geographic space), and to surfaces which are "closed". A closed surface is generally that part of a surface which occupies the portion of a plane bounded by a single closed contour, a line of constant z -value on the surface. Unless otherwise noted, surfaces will furthermore be assumed to be "smooth", that is, to be continuous in at least the first derivative.

Reech (1858) demonstrated a theorem concerning certain "critical" points on a closed surface--these are equivalent to points where the first derivative of the surface is zero. Reech identified three types of such points, which Warntz (1966) termed peaks, pits, and passes (Warntz actually distinguished two types of saddle points as "passes" and "pales"; this distinction, while important, is not relevant to the present discussion). Reech then placed these points in the context of equilibrium theory: peaks are points of unstable equilibrium, pits of stable equilibrium, and passes of mixed equilibrium (for a more complete development of these concepts and their extension to N dimensions, see Morse, 1925, 1949). Reech (1858) further demonstrated that on a sphere, the number of peaks plus pits minus passes is always exactly two. Within a closed contour, peaks plus pits number one more than passes, and the area outside the closing contour is considered to be the "extra" critical point, either a peak or a pit, depending on the

direction of surface slope at the bounding contour.

An interesting consequence of the assumption of smoothness is that the contour which passes exactly through a pass must cross itself there. For cartographic reasons, such self-crossing contours are never portrayed on contour maps; nevertheless, their existence may be proven by considering the limit of the shape of the contours in the neighbourhood of the pass (see Peucker, 1972, p. 46).

The work of both Reech (1858) and Morse (1925, 1949) ignored the possibility that three or more areas of elevation (or of depression) might meet at a single "multiple" pass. In such a case, the self-crossing contour crosses itself three or more times. Maxwell (1870), however, presented a more general relation for numbers of critical points which allowed for such multiple passes. Such features are extremely rare in most geographic surfaces (especially in topographic surfaces), and so the present study will follow Pfaltz (1976) in examining only "normal surfaces", which are defined as smooth surfaces all of whose passes are of the simple "single" form of most saddle points. A related assumption is that no possible contour passes exactly through more than one pass, that is, no two passes have identical elevation. These assumptions do not unduly restrict the results, since for any geographic surface, there exists a

normal surface which is everywhere within an arbitrarily small epsilon of the original surface (Pfaltz, 1976). Some of the rather diverse terminology which has been applied to critical points is listed in Table II.1.

2.3: Critical Lines, Networks, and Graphs

Cayley (1859) defined a network of critical lines which indicate relationships among the critical points. A slope line is defined as a line in the direction of steepest slope on the surface; such lines must everywhere be orthogonal to the contours. On a normal surface, every point which is not a critical point lies on exactly one contour and one slope line. Peaks and pits have no contour lines (or, the contour line is reduced to a single point) and an infinite number of slope lines; passes have exactly two slope lines (or pairs, if each is divided at the pass) and two contour line segments (both, of course, parts of the same self-crossing contour). Although all other slope lines lead from a peak to a pit, one of the pairs of slope lines through a pass leads from a peak to another peak (rarely, to the same peak), and the other pair from pit to pit (rarely, the same pit). The smoothness assumption forces one to assume that the slope line coming down from a peak to a pass continues on to the other peak, rather than down to one of the pits. The line joining the peaks is designated as two ridge

TABLE II.1: CHARACTERISTICS AND TERMINOLOGY OF CRITICAL POINTS

	0	1	2
instability (Morse, 1925)			
vergency (Warntz, 1975)	divergent	mixed	convergent
terminology:			
Reech (1858)	"sommet" (summit)	"ithmes" (isthmus)	"fond" (bottom)
Cayley (1859)	summit	knot	immit
Maxwell (1870)	summit	pass	bar
Morse (1965)	cap	pass (saddle)	bar
Warntz (1966)	peak	pass (knot)	pale
			pit

lines, while the one joining the pits is termed two course lines. The critical points can thus be seen as the vertices in a network having the ridge lines and course lines as edges. Such a network, when "embedded" in a geographic surface, is herein termed the "Warntz Network" of that surface (see Figure 2.3), after William Warntz, who revived this interesting theoretical framework after a century of neglect (Warntz, 1966). The terminology which has been applied to the lines and areas of Warntz networks is contained in Tables II.2 and II.3, respectively.

Recently, Pfaltz (1976) placed this work firmly in a graph-theoretic context. His approach ignores the geometrical properties of Warntz Networks, and represents their topology in a tripartite graph which will herein be referred to as the "Pfaltz Graph"; Pfaltz's term "Surface Network" is considered to be too general, as it could easily refer to any of the representations discussed in this chapter. Pfaltz (1976, p. 84) listed five necessary conditions for a tripartite graph $G(V_0, V_1, V_2; E)$ to be realizable, that is, to be isomorphic to some possible Warntz network, where the V_i 's are the sets of all pits, passes, and peaks, respectively. These conditions are repeated below, with slightly altered notation and terminology:

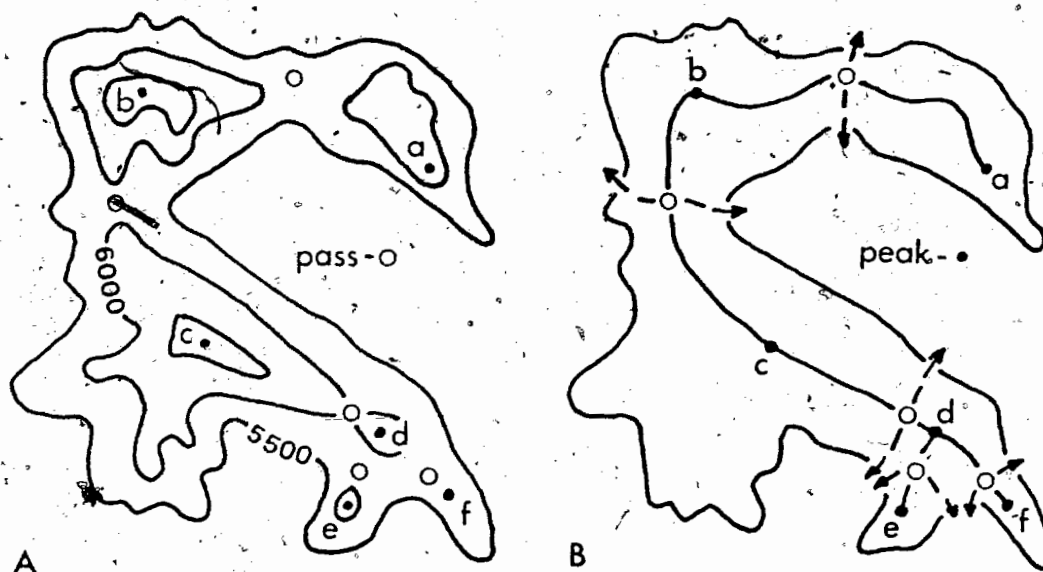


Figure 2.3: A: Simplified contour map of a magnitude-6 hilltop, with critical points shown; B: Critical lines forming the Warntz network of the surface (ridge lines are solid; course lines are dashed, and all lead to some pit or pits outside the bounding contour).

TABLE II.2: CHARACTERISTICS AND TERMINOLOGY OF CRITICAL LINES

vergence (Warntz, 1975)	divergent	convergent
terminology:		
Cayley (1859) Warntz (1966)	ridge line	course line
Maxwell (1870) Morse (1965)	watershed	watercourse
Warntz and Waters (1975)	ridge line	trough line

TABLE II.3: CHARACTERISTICS AND TERMINOLOGY OF CRITICAL AREAS

vergency (Warntz, 1975)	divergent	mixed	convergent
terminology:			
Maxwell (1870) Morse (1965)	hill	natural district	basin or dale
Warntz (1966)	hill	territory	dale
Warntz and Waters (1975)	high	territory	low

- (1) The graph must be connected;
- (2) The number of peaks plus pits minus passes must be two (the result of Reech, 1858);
- (3) Each pass must be a vertex of degree four, and be of degree two in each of the bipartite subgraphs $C(V_0, V_1; E)$, the course graph, and $R(V_1, V_2; E)$, the ridge graph;
- (4) If a particular peak and pit are both singly connected to a common pass, they must be singly connected to at least one other common pass;
- (5) A pass is on a circuit in the course graph if and only if it is not doubly connected to a single peak; it is on a circuit in the ridge graph if and only if it is not doubly connected to a single pit (A double connection is itself considered to be a circuit).

Pfaltz (1976, p. 84) stated that "it is not known whether these properties are sufficient to guarantee the realizability of G ". In fact, they are not. First, the connectivity requirement (1) must be increased; specifically, the subgraphs C and R must each be connected. Furthermore, two other conditions must be fulfilled:

(6) The graph must be bipartite; specifically, the subgraph $H(V_0, V_2; E)$ must be a null graph, that is, have no edges (Pfaltz's examples clearly illustrate that he was aware of this condition, but he failed to list it explicitly);

(7) The graph must be planar, since to be realizable, it must be isomorphic to some Warntz Network which is embedded in the plane without intersecting edges.

An interesting special case arises when a closed surface has no pits, except for the one representing the area outside the bounding contour. Such a closed surface with no pits will be termed a "hilltop". This is of particular interest in the geomorphic context, since pits large enough to appear on medium-scale topographic maps are very rare in fluvially-eroded terrain. Since V_0 then consists of a single point, each pass ~~must~~ be doubly connected to it in C . Then, by property 5, the ridge graph contains no circuits, and since it is furthermore connected (property 1, as modified), it must take the form of a tree. Each pass is of degree 2 in R , and thus it is convenient to represent the ridge graph by a tree whose vertices are only the peaks, and with peaks which share a pass connected to each other; this tree, which can be termed the "ridge tree", is homeomorphic to R , and completely determines the Pfaltz graph of a hilltop, up to isomorphism. Each edge in the ridge tree can

be seen to "represent" a pass. The number of non-isomorphic distinct types of trees with exactly one to ten vertices are 1, 1, 1, 2, 3, 6, 11, 23, 47, and 106 (for the continuation of this series, see sequence 299 in Sloane, 1973, p. 53). Due to the aforementioned homeomorphism, this sequence also enumerates topologically-distinct Pfaltz graphs and Warntz networks for areas with no pits. Figure 2.4 illustrates the six Pfaltz graphs and ridge trees for magnitude-6 hilltop areas.

2.4: Complexity of Warntz Networks

Warntz and Waters (1975) applied several measures of network complexity from Kansky (1963) to Warntz networks for atmospheric pressure surfaces. They suggested (p. 488) that there may be some relationship between these measures and certain meteorological measures relating to atmospheric physics.

Actually, Warntz networks are connected in a very specific way, as outlined by Pfaltz (1976) and above, and thus the purely topological Kansky measures used by Warntz and Waters (1975) are largely fixed by these constraints and could not possibly relate to physical concepts. Since Warntz networks are necessarily planar, the planar versions of Kansky's measures should be employed. Table II.4 indicates that, for large, complete Warntz networks, the alpha index must be approximately one half, the beta index two, and the gamma index two thirds, regardless of

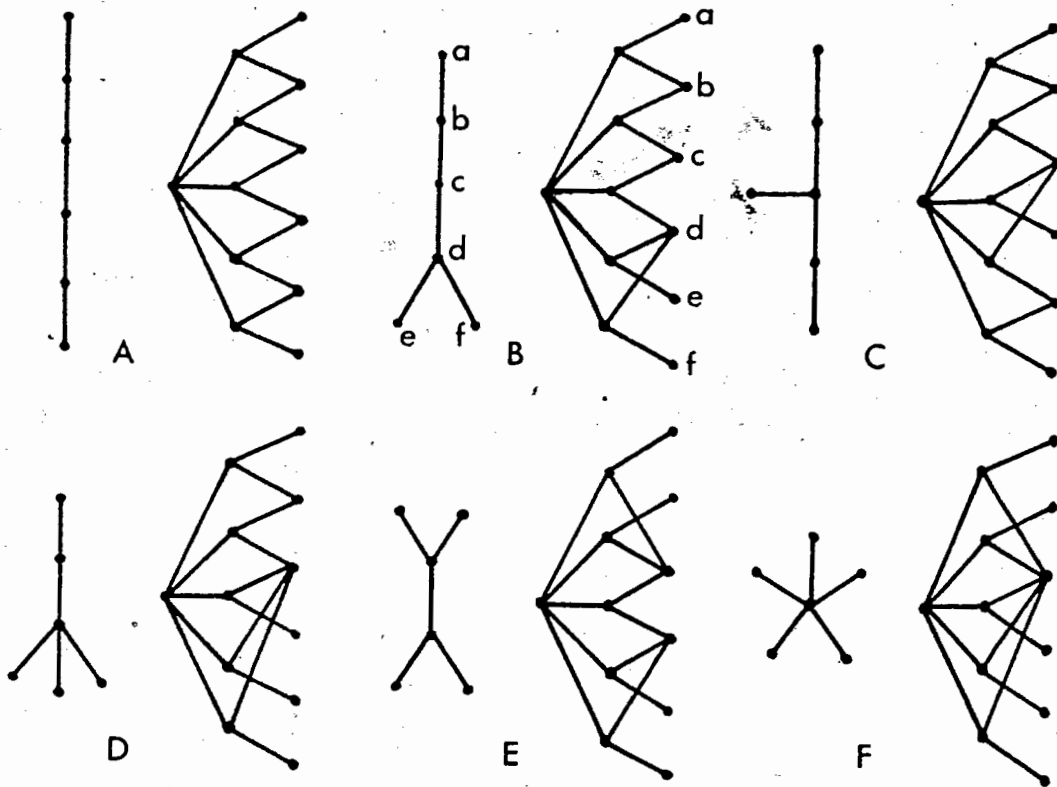


Figure 2.4: The six topologically-distinct magnitude-6 Pfaltz graphs, represented as tripartite graphs (right) and as ridge trees (left). For clarity, the double connections of all passes to the single ("outside") pit are not shown. B is isomorphic to the network in Figure 2.3B; the letters indicate the correspondence.

TABLE II.4: KANSKY'S MEASURES FOR
WARNTZ NETWORKS

M = "magnitude" of the network
 p = number of vertices
 q = number of edges

$$p = 2M - 1$$

$$q = 4(M-1)$$

$$\begin{aligned} \alpha &= (q - p + 1) / (2p - 5) \\ &= 2(M - 1) / (4M - 7) \\ &= 1 / 2 \quad (\text{as } M \rightarrow \text{infinity}) \end{aligned}$$

$$\begin{aligned} \beta &= q / p \\ &= 4(M - 1) / (2M - 1) \\ &= 2 \quad (\text{as } M \rightarrow \text{infinity}) \end{aligned}$$

$$\begin{aligned} \gamma &= q / (3(p - 2)) \\ &= 4(M - 1) / 3(2M - 3) \\ &= 2 / 3 \quad (\text{as } M \rightarrow \text{infinity}) \end{aligned}$$

the details of the network. For the ridge and course networks examined separately, these three measures depend only upon the relative numbers of peaks and pits. It is possible that the Kansky indices based on network flows (θ , ι) may have relationships to physical aspects of surfaces, but at present it would appear that the Kansky indices are not useful for characterizing the topology of Warntz networks. Warntz and Waters (1975) suggested several measures of the relative importance of particular nodes, but no useful measures of overall network complexity.

2.5: The Surface Tree

Another way of representing some aspects of the topology of a geographic surface is the surface tree, a new representation based on the "contour tree" developed in computer cartography. The idea of representing adjacency relations of a set of contours by a tree was apparently first published by Boyell and Ruston (1963). Their "enclosure tree" was based on the fact that a contour loop can enclose any number of other contour loops, but can itself be immediately enclosed by only one. Thus each such loop can be represented by a vertex in a tree (that is, a connected graph containing no circuits). This is topologically identical to representing a set of disjoint rings in a plane (Berge, 1962, p. 161). Berge directed each edge

toward the enclosed ring, and called such a directed tree an "arborescence"; Harary's (1969, p. 201) term "out-tree" is to be preferred, since "arborescence" has been used in graph theory in a different context by other workers.

The "contour tree" concept was re-invented, apparently independently, by Morse (1966, 1969). In his "graph of a contour map", sets of mutually adjacent contours (which bound an inter-contour area) were identified as vertices in a tree whose edges were equivalent to those contours shared by adjacency sets. Since each adjacency set contains only one "bounding" contour which encloses the others, this definition is isomorphic to Boyell and Ruston's. Freeman and Morse (1967) drew the analogy between the contour tree and the geometric dual of a planar graph, in which a vertex is placed in each region, and adjacent regions' vertices are joined. It is, however, only an analogy, since a contour map, having no vertices, is not strictly a graph. Boehm (1967) introduced the preferred term "contour tree" for these contour enclosure graphs. Figure 2.5 shows the contour trees for Figure 2.3, but with two different contour intervals.

Given this concept of a contour tree, we can define the surface tree of a geographic surface. It was already assumed above that no two passes have identical heights; we can then imagine that

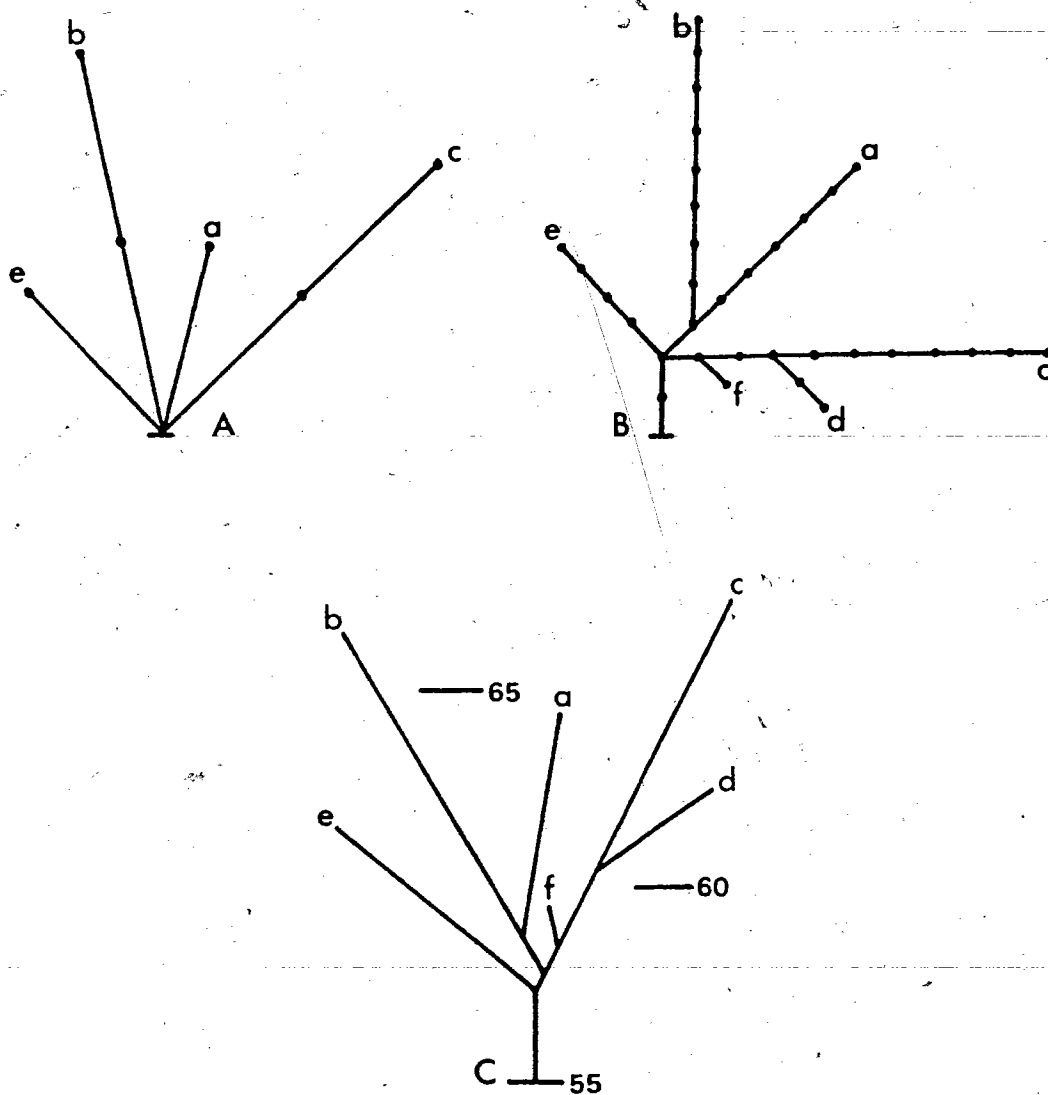


Figure 2.5: A: Contour enclosure tree for the area depicted in Figure 2.3; B: As in A, but for a 100 unit contour interval; C: The surface tree of the area, with elevations of critical points plotted to scale (figures in hundreds of units). The lower-case letters indicate the correspondence with Figure 2.3.

the contour interval is decreased until no inter-contour area contains more than one critical point. All points in the contour tree of such a map will then be of degree one (representing inter-contour areas containing peaks or pits), two (if the area contains no critical point), or three (if the area contains a pass). Finally, we remove all vertices of degree two (by a series of "homeomorphic contractions") to obtain the surface tree.

Since each remaining node represents exactly one critical point, the surface tree has the same vertex set as the Pfaltz graph. Furthermore, we can associate the elevations of the critical points with the equivalent vertices in the surface tree, and plot the tree with the elevations to scale (Figure 2.5C).

The surface tree should have a familiar look to most geomorphologists, since it is a trivalent planted tree. Stream networks are generally assumed to be trivalent planted plane trees (Smart, 1972, p. 306), but in the surface tree there is no "right" or "left" branch at each vertex, and hence the surface tree is not a "plane" graph. In fact, each topologically distinct surface tree is isomorphic to all members of some "ambilateral class" (Smart, 1969) of stream networks. By analogy with stream network conventions, the magnitude of a surface tree will be the number of end-points (peaks plus pits),

excluding the "root" (representing the area outside the bounding contour, and topologically equivalent to the "mouth" of a stream network). Non-plane trivalent planted trees are counted by the "Wedderburn-Etherington Numbers" (sequence 298, Sloane, 1973, p. 53). The first ten numbers in this sequence are 1, 1, 1, 2, 3, 6, 11, 23, 46, and 98; Figure 2.6 shows the six possible surface trees of magnitude six.

The contour tree has considerable utility in computer cartography, and this extends beyond applications to the profile search problem (Freeman and Morse, 1967) and line-of-sight calculations (Boehm, 1967). As part of the present study, it was found that if the contour tree is determined for another type of digital terrain model before contours have been drawn, it should theoretically be able to improve the efficiency of contouring programs. The surface tree may also be of value in surface generalizations. These computer-cartographic applications represent an area of potential practical utility, but are peripheral to the main thrust of the present thesis. For this reason, a detailed exposition of these applications has been relegated to Appendix B.

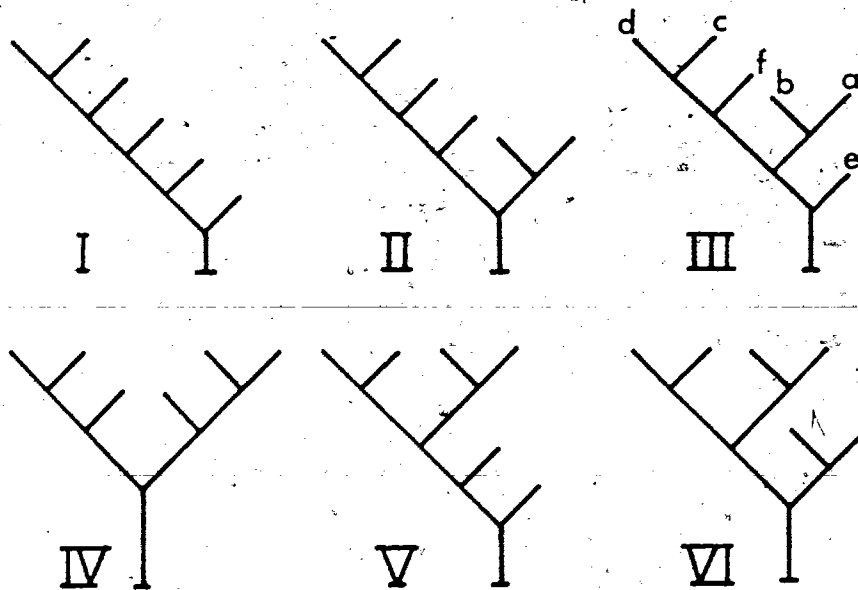


Figure 2.6: The six possible surface trees of magnitude-6 hilltops with no pits. III is isomorphic to Figure 2.5C, as indicated by the letters.

CHAPTER 3: TOPOLOGICAL RANDOMNESS OF TWO-DIMENSIONAL RIDGE NETWORKS

Werner (1972a, 1972b, 1972c, 1973) and Goudie (1969) represent perhaps the only geomorphologists who have previously examined topological aspects of ridge networks. Although the topological representations of geographic surfaces discussed in Chapter 2 have been cited in a geomorphologic context ("Warntz Networks" by Woldenberg, 1972, and Warntz, 1975; "contour trees" by Evans, 1972, and Mark, 1975a), the possibility of applying a random topology model to geomorphic surfaces through these graphs has apparently gone unnoticed until now. Before presenting this new research, previous work on ridge topology will be briefly reviewed.

Goudie (1969) applied "Horton's Laws" to dune ridges, and found that the "Laws" provided a good fit. He also showed that ridge junctions in his area had a Poisson random distribution in space. "Ridges" were not formally defined, but were morphological features not directly related to passes as are the "ridge lines" of a Warntz network. Goudie did not indicate how a "root" of the graph, topologically equivalent to the mouth of a drainage net, was (or could be) identified. Werner (1972a, 1972b, 1972c, 1973) has provided a more firm basis for examining the planimetric topology of morphological ridges. Ridges were

neither formally nor operationally defined in the Werner papers, but were based on contour crenulations in a manner analogous to the extension of the drainage net (C. Werner, pers. comm., 1976). Werner (1972b) proposed dividing ridge systems at all passes, and studying the resulting sub-networks, which represent all the morphological ridges within a single "hill" (Maxwell, 1870). Using Werner's (1972b) approach, his frequencies for various magnitudes in the same area of eastern Kentucky were confirmed; in contrast, hill sub-units for an area of the Cascade Mountains of southern British Columbia had much lower average magnitudes, suggesting that this approach may not be useful for all topographic textures. Furthermore, there is a considerable element of subjectivity involved in deciding which contour crenulations should be interpreted as ridges. Conversely, the ridge lines in the Warntz network can be defined easily and unambiguously, as outlined above in section 2.3.

Werner (1972a, 1972c, 1973) also related ridge patterns to stream topology within drainage basins. While ridge patterns appear to be topologically random when analyzed in isolation (Werner, 1972b), as do stream networks, neither pattern is random once the other has been specified. In particular, Werner (1972a) found that pairs of adjacent first order streams were separated by exactly one first order ridge in 88 of 119 cases; a random (Poisson) model would predict only 44 such occurrences if

the distributions were independent. Werner noted that the relationship is stochastic, rather than deterministic, and only approximates a geometric duality. He later (1973), however, assumed the duality to be exact in order to proceed to further hypotheses regarding ridge topology. In Warntz networks, the duality between the ridge and course networks is exact.

For reasons of reproducibility and applicability to a wider range of terrain types, it would appear that Warntz networks and Pfaltz graphs represent a better approach for examining ridge topology. It must, however, be remembered that Werner's ridges bear a much closer relation to features of local topographic (geometric) significance; the Warntz network only indicates relations among critical points. In practice, Warntz ridge lines largely coincide with the more important geometric ridges in fluvially-eroded (and in alpine glacial) terrain, but not, for example, in karst.

3.1: Topological Randomness and Pfaltz Graphs

Two basic approaches to the examination of topological randomness have been employed in stream network research (Smart, 1972): one involves detailed examination of finite networks of necessarily small magnitude, while the other investigates average properties of infinite, topologically random networks

(Shreve, 1967). Because of a number of mathematical difficulties (chiefly, the absence of a simple closed expression for the number of surface trees of a given magnitude; Etherington, 1937), this study adopts the former approach. Magnitude-6 systems were selected for analysis, since the number of possibilities is rather small for lesser magnitudes but rapidly becomes unmanageable for larger graphs.

In order to provide an initial test for hypotheses concerning topological randomness of Pfaltz graphs, three areas were selected. The first sample consists of all 230 magnitude-6 hilltops in an area of eastern Kentucky between the Levisa and Tug Forks of the Big Sandy River (see Appendix C). This area was selected because "the area, while not perfect, appears to be a good example of a mature landscape developed in the absence of structural control" (Krumbein and Shreve, 1970, p. 4). This area has been extensively used to test various aspects of the probabilistic-topologic approach to landscapes, both for streams (Shreve, 1969; James and Krumbein, 1969; Krumbein and Shreve, 1970; Werner, 1975; Dacey and Krumbein, 1976; Smart and Werner, 1976) and for ridges (Werner, 1972a, 1972b, 1973c, 1973). None of these studies found strong evidence of any major non-random element in those aspects of this landscape which they examined, although discrepancies between observed and expected frequencies of certain topological patterns led to some refinements of the

random topology model. These data will form the primary basis for discussions of topological randomness of hilltops.

Samples were also obtained from an area of the middle California Coast Ranges (65 hilltops) and from southern British Columbia (103 hilltops) (see Appendix C). Each area has a heterogeneous geology; the latter area was heavily glaciated during the Pleistocene Epoch, while the former has been very heavily faulted, but in such a way that structural control is not apparent.

The simplest hypothesis for Pfaltz graphs of closed surfaces without pits is that they are all equally likely to occur, that the probability of each topologically-distinct Pfaltz graph in the absence of pits is the inverse of the number of arbitrary trees with as many vertices as the graph has peaks.

As indicated by the frequencies listed in Table III.1, this hypothesis must clearly be rejected; a Chi-square test confirms the obvious.

3.2: The Simulation Model for Pfaltz Graph Frequencies

A somewhat more complex but still conceptually simple model to explain the Pfaltz graph frequencies was suggested by D. G.

TABLE III.1: OBSERVED FREQUENCIES OF PFALTZ GRAPH CLASSES

PFALTZ GRAPH	A	B	C	D	E	F	Total
Kentucky	100	65	56	2	7	0	230
Middle California	31	16	13	2	2	1	65
British Columbia	41	30	23	4	5	0	103
Northern California	23	15	16	3	1	0	58
Northern Pennsylvania	30	17	13	2	1	0	63
Southern Pennsylvania	57	4	2	2	1	0	66

Kirkpatrick (pers. comm., 1976). This involved forming minimum spanning trees (Kruskal, 1956) of sets of randomly-distributed points, and observing the frequencies of the various topological classes. To implement this suggestion, it was necessary to define a field over which the points would be distributed. A circular field was chosen to simulate an isotropic distribution of possible ridge orientations within a hilltop, and this was deformed into ellipses of varying elongation in order to produce different degrees of anisotropy (see Mandelbrot, 1975, p. 3827). This should in turn influence Pfaltz graph frequencies. The assumptions of the model are:

- 1) A random distribution of points (representing peaks);
- 2) An elliptical field with variable elongation ratio;
- 3) An approximation to a minimum spanning tree by ridge networks.

It is well known that no single test can discriminate between random and non-random point patterns in all cases. Assumption 1 was therefore tested for an area 9 by 12 km in the centre of the Kentucky sample area, using three approaches: a visual examination, a Poisson-quadrat test, and nearest neighbour analysis. None of the tests indicated a significant non-random

element in the spatial distribution of peaks in that area (the nearest neighbour ratio was 0.98, not significantly different from 1.0). Assumption 2 leads to further hypotheses relating geometry and topology, which will be examined in Chapter 5; it is assumption 3, however, which is primarily under test.

Modelled frequencies were generated using Monte Carlo methods with 5000 trials; Figure 3.1 shows the variation in Pfaltz graph frequencies with changing elongation of the elliptical field. Exact modelled frequencies are given in Appendix D.

Because of constraints on the geometry of minimum spanning trees for points in Euclidean space (for example, Black, 1970, has shown that all angles must be greater than or equal to 60 degrees), it is not unreasonable that Pfaltz graph class F did not occur at all in the predicted frequencies. Since the five remaining class frequencies vary together, there is no a priori reason to expect that the model would fit observed frequencies well for any of the elongation values. Figure 3.2 plots the Chi-square value of the difference between the observed and modelled frequencies for the aforementioned three sample areas. Usually, the desired outcome in statistical hypothesis testing is the rejection of the null hypothesis; the researcher is thus being conservative by choosing a small value of alpha (5 or 1 per cent) in testing the significance of results. When testing the output of a simulation model, however, one generally wishes

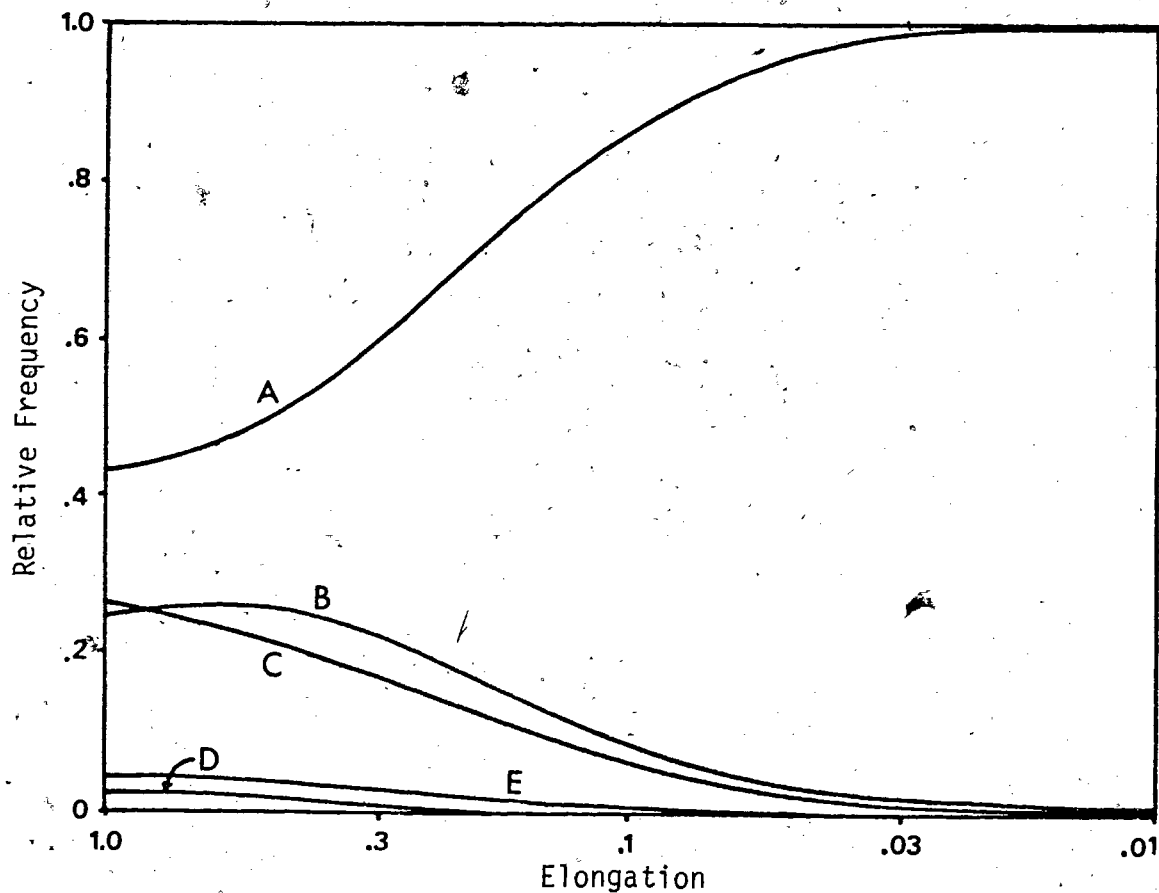


Figure 3.1: Relative frequencies of the six possible magnitude-6 Pfaltz graphs, as predicted by the random minimum spanning tree model, plotted as functions of the elongation of the bounding ellipse. The letters indicate the classes shown in Figure 2.4; occurrence of class F was not predicted by the model.

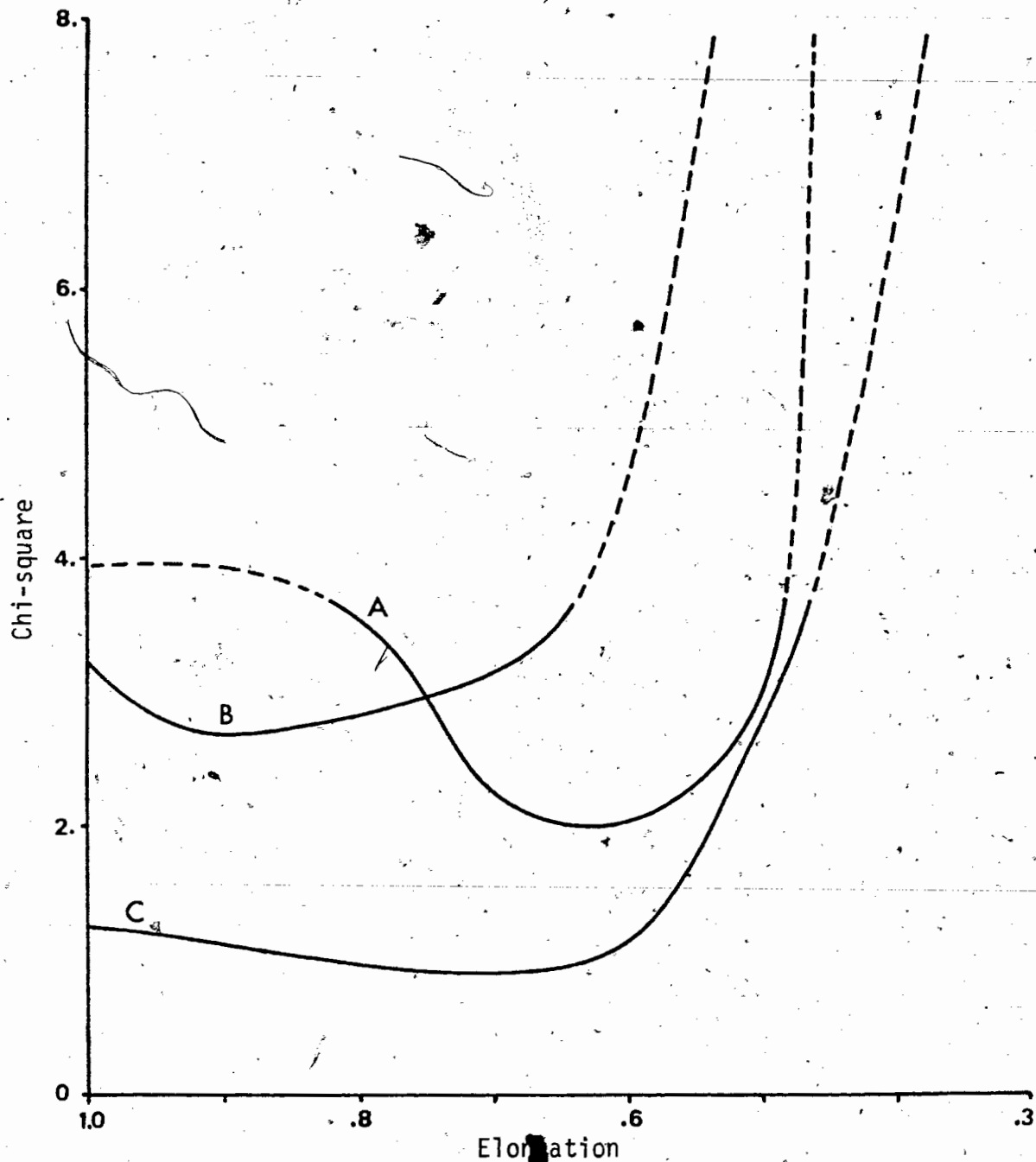


Figure 3.2: Chi-square of the difference between observed and modelled Pfaltz graph class frequencies, plotted against elongation. A: Kentucky sample; B: British Columbia sample; C: Middle California Coast sample. Solid lines indicate no significant difference at the 30 per cent level, dashed at the 5 per cent level.

to accept the null hypothesis of "no significant difference" between the modelled and observed values. In this case, a conservative researcher should use much higher alpha levels; Blalock (1960, p. 162) suggested 10, 20, or "even" 30 per cent. At this 30 per cent level, the results of the present model do not differ significantly from any of the three samples, for any elongation parameter between about 0.81 and 0.64. Indeed, the circle (elongation ratio 1.0, representing isotropic ridge orientations) cannot be rejected at the 30 per cent level for either the British Columbia or California sample.

In the Kentucky sample, there is a clear preferred elongation parameter of about 0.6. In contrast, the California sample shows no particular preferred elongation, with approximately equal fit for all elongation values between 1.0 and about 0.6; the British Columbia sample shows a similar shape but a generally poorer fit. At first, these results seem strange: the area with very uniform geology and horizontal structure (Kentucky) shows a definite preferred elongation (anisotropy), whereas the area of heterogeneous geology and complex structure (California Coast Ranges) does not. A possible explanation for this has been suggested by M. Church (pers. comm., 1976): in an area of horizontal, uniform beds of similar lithology, the presence of master streams and their major tributaries would be likely to produce elongated interfluves, within which ridges

might be expected to be anisotropic and to tend to parallel the interfluve. On the other hand, such relatively straight interfluves would be less likely in areas of complex geology and structure (but without strong structural control).

In order to provide a qualitative test for this hypothesis, samples of Pfaltz graph frequencies were obtained from an area further north in the California Coast Ranges and from part of the Allegheny Plateau in north-central Pennsylvania (for details, see Appendix C). It was hypothesized that the Allegheny area (flat-lying sandstone) would show anisotropy similar to that of the Kentucky sample, but that the second California sample (heterogeneous geology) would not. Chi-square was plotted against elongation for each of these samples (Figure 3.3), and the results seem to confirm Church's hypothesis.

Strong structural control, such as the ridge-and-valley topography of the eastern Appalachian Mountains, would be expected to show Pfaltz graph frequencies corresponding with a pronounced elongation value (highly anisotropic). A sample of ridge class frequencies was obtained from such an area in south-central Pennsylvania (see Appendix C), and the circle hypothesis (isotropy; elongation = 1.0) was very strongly rejected (Chi-square = 49.67); conversely, the model predictions with an elongation value of 0.1 produced a very good fit

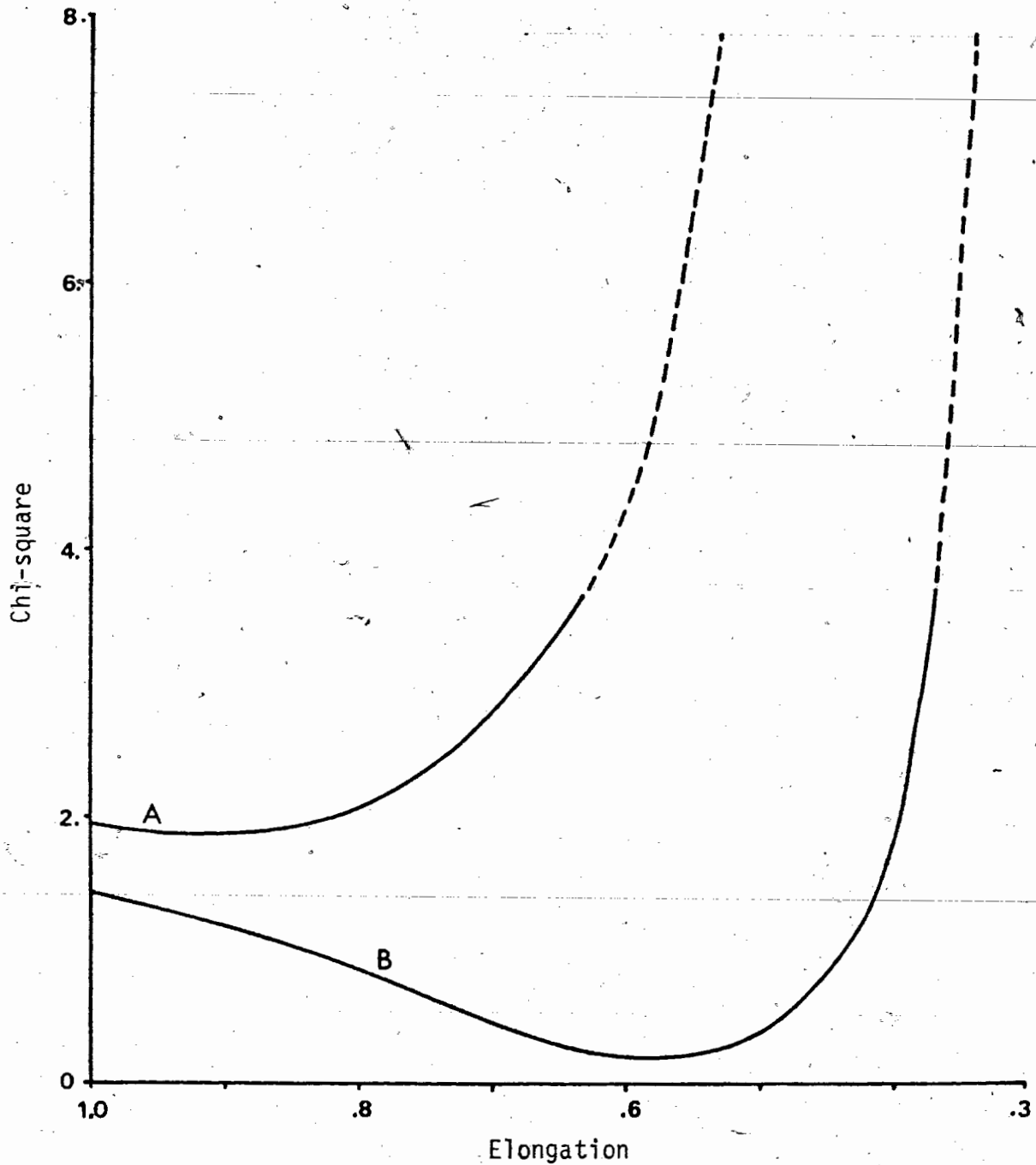


Figure 3.3: As in Figure 3.2, but for the Northern California (A) and northern Pennsylvania (B) samples.

(Chi-square = 1.06, not significant at the 30 per cent level).

3.2.1: Effects of Sampling on Chi-square Plots

Since the shapes of plots of Chi-square against elongation have been used to infer the degree of ridge anisotropy in landscapes, it is in order to examine the sensitivity of these curves to sampling effects. All samples except the Kentucky one were based on rectangular blocks formed by adjacent map quadrangles, and so pairs of quadrangles were selected from the central portion of the Kentucky sample area. Specifically, the 37 hilltops located in the Inez quadrangle were combined with those from each of that map's neighbours to produce four sub-samples of from 55 to 75 hilltops; Chi-square plots for these sub-samples are shown in Figure 3.4. Sampling effects, and perhaps spatial variations in Pfaltz graph class frequencies, produce a considerable variation in overall degree of fit. While three of the curves have shapes similar to the entire set of 230 hilltops (distinct optimum elongation parameters around 0.5 to 0.6), the fourth sample plot has a shape more like the California samples, which were interpreted as being more or less isotropic. Thus while samples of around 60 appear generally to give reliable results, some caution must be applied to their interpretation.

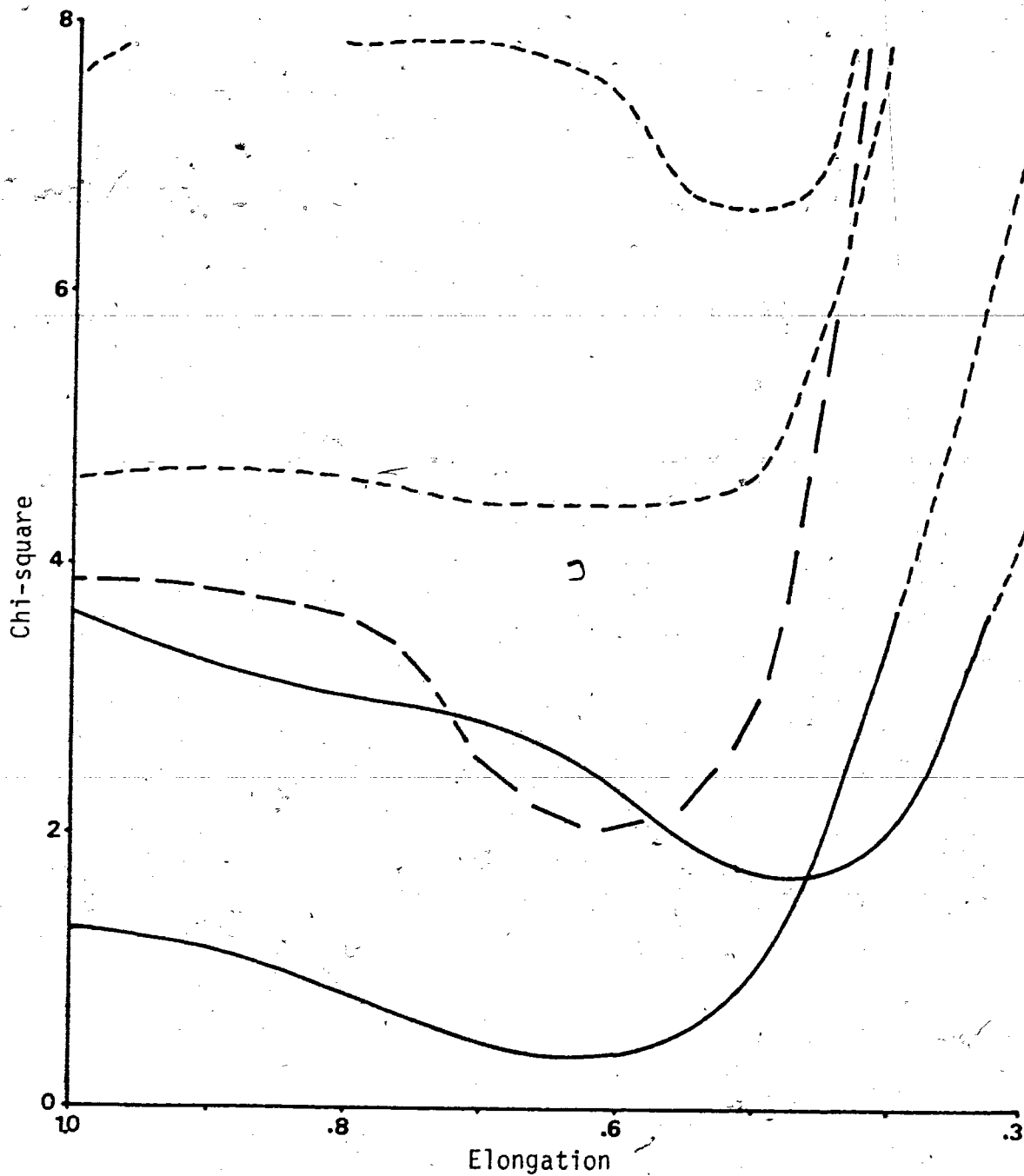


Figure 3.4: As in Figure 3.2, but for four sub-samples of from 55 to 75 hilltops from the Kentucky sample area. The long dashes show the total Kentucky sample (230 hilltops) for comparison.

3.3: Conclusions

A simple geometric model with a random component and with one adjustable parameter (constraint), an elongation ratio which characterizes the degree of anisotropy of ridge orientation within hilltop units, provides excellent predictions of Pfaltz graph frequencies for a variety of areas. Regions with heterogeneous geology and structure but with no overt structural control are best fit by an isotropic ridge distribution. At the other extreme, the overt structural control presented by ridge-and-valley topography produces ridge topology frequencies which are well predicted by the model, but with a very pronounced (10:1) elongation of the elliptical field in which peaks are assumed to be randomly distributed. Somewhat surprisingly, areas of extremely uniform, horizontal geology have hilltops which are best fit by the model with an elongation ratio of roughly 2:1. This may be due to topographic control introduced by master streams and their major tributaries. Even if this is not the cause, the elongation parameter in the model represents a constraint on the "randomness" of at least some topographic surfaces.

CHAPTER 4: SURFACE TREES: A THREE-DIMENSIONAL APPROACH TO GEOMORPHIC RANDOMNESS

The surface tree (see section 2.5) is a topological representation which contains information about the three-dimensional arrangement of a surface. While it is completely determined by the Pfaltz graph of the surface, together with the associated pass height ranks, it portrays this information in such a different way that it is an interesting representation in its own right. This chapter examines surface trees from the point of view of topological randomness in geomorphology.

4.1: Topological Randomness and Surface Trees

For each of the 398 hilltops in the Kentucky, British Columbia, and middle California coastal samples examined in the last chapter, the surface tree class was determined from among the six possible topologies (Figure 4.1). As with Pfaltz graphs, the simplest possible hypothesis, that of equal likelihood of all topologically distinct trees, is untenable, and is strongly rejected using a Chi-square test. Since, as already noted, the surface tree is completely determined by the Pfaltz graph and pass ranks, the question arises: how do the six surface tree classes relate to the Pfaltz graph classes?

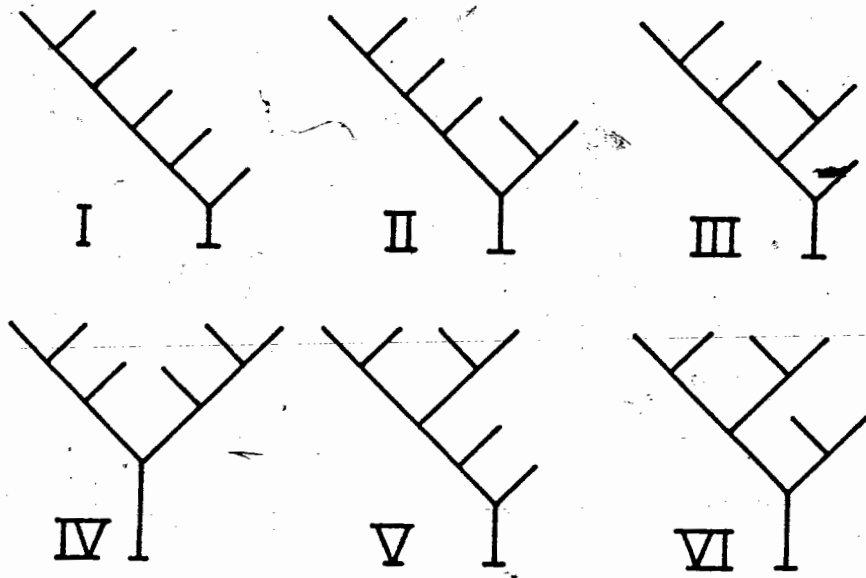


Figure 4.1: The six possible magnitude-6 surface trees.

Assigning the pass elevation ranks to the passes is exactly equivalent to the problem of labelling the edges of the ridge graph with labels $1, 2, \dots, N-1$. In general, the edges of all trees with exactly N vertices can be labelled in exactly N^{N-3} distinct ways (Moon, 1970, p. 6). For $N=6$, there are thus 216 distinct edge-labelled trees, each with its Pfaltz graph class and surface tree class determined. Table IV.1 tabulates these 216 trees-- it can be seen that only 25 of the 36 cells can possibly occur. If the pass elevation ranks are mutually independent, the relative frequencies of surface trees for any particular Pfaltz graph class should be proportional to the entries in the appropriate column of Table IV.1. This independence assumption can then be combined with the expected frequencies of Pfaltz graphs from the last chapter to produce expected surface tree frequencies (Figure 4.2). As elongation varies, contributions from various Pfaltz graph classes vary, but the totals stay almost constant for three classes (II, III, V); classes IV and VI increase steadily with increasing elongation, while only class I declines.

The circle (elongation value of 1.0) provides best predictions in all three areas, but the difference between the expected and observed surface tree frequencies is highly significant for the California and British Columbia samples, and fairly large (but not quite significant at the 5 percent level) for the Kentucky sample (Table IV.2). Basically, all areas show too many surface trees of class I; this would be expected to occur if the pass

TABLE IV.1: NUMBERS OF "PASS-LABELLED" PFALTZ
GRAPHS AND ASSOCIATED SURFACE TREES

PFALTZ GRAPH SURFACE TREE	A	B	C	D	E	F	Total
I	8	15	18	11	7	1	60
II	16	12	16	4	-	-	48
FII	12	15	12	3	3	-	45
IV	12	12	-	-	3	-	27
V	4	6	6	2	2	-	20
VI	8	-	8	-	-	-	16
Total	60	60	60	20	15	1	216

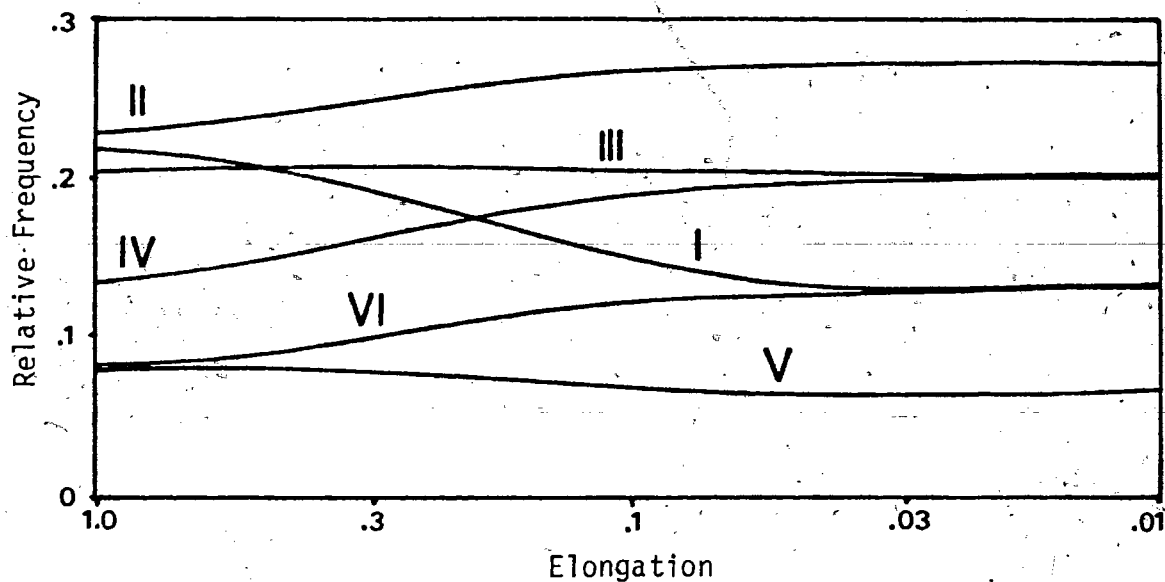


Figure 4.2: Expected relative frequencies of the six surface trees as functions of ellipse elongation, based on the model (Figure 3.1) and assumed pass height independence.

TABLE IV.2: THEORETICAL AND OBSERVED FREQUENCIES OF SURFACE TREES, CIRCLE SIMULATION MODEL

Tree Class	Expected Probability	Kentucky	Observed California	B.C.
I	0.226	67 (0.291)	32 (0.492)	44 (0.427)
II	0.238	62 (0.270)	11 (0.169)	19 (0.184)
III	0.211	41 (0.178)	11 (0.169)	16 (0.155)
IV	0.145	26 (0.113)	7 (0.108)	9 (0.087)
V	0.087	16 (0.070)	1 (0.014)	12 (0.117)
VI	0.093	18 (0.078)	3 (0.046)	3 (0.029)
Totals		230	65	103
Chi-squared		9.43	28.21**	29.11**

** significant, 0.1 per cent level

heights are not independent but rather show a positive "autocorrelation".

4.2: Autocorrelation of Pass Elevations

The continuity of topographic surfaces ensures that points which are very close together in the horizontal (x-y) plane must have very similar heights. Generally, this "autocorrelation" will decline (roughly exponentially; see section 1.2) as horizontal distance increases, and it is not unreasonable to suppose that topologically adjacent passes might have more similar elevations than more distant pairs. If autocorrelation is high over the entire scale of the individual hilltops, minor variations will produce independence of elevation ranks; if, on the other hand, autocorrelation declines significantly over intra-hilltop distances, this should disturb the independence assumption and influence surface tree frequencies.

Surface tree class I results if, and only if, the highest pass is adjacent to the second highest, the third is adjacent to one of these, and the fourth next to one of these three. Any other combination will produce one of the other five surface tree classes. It is thus easy to see that a positive spatial autocorrelation of pass elevations should favour class I over the others, resulting in underprediction of this class following

the independence assumption. All three areas show this underprediction, which is less marked in the Kentucky sample, probably because of the effects of the scale of autocorrelation, as mentioned above. It remains now to demonstrate the presence of autocorrelation statistically.

4.2.1: An Intuitive Approach to Autocorrelation

For each of the three more frequent Pfaltz graph types (A, B, and C), the probability of the highest and second highest pass being adjacent was determined, assuming independence. Furthermore, the conditional probabilities of the other adjacency relations which would lead to surface tree class I were determined (conditional on all previous adjacency requirements being met). These probabilities were then used as null hypotheses in binomial tests of the observed frequencies of various adjacencies for each of the three study areas, and observed frequencies significantly higher than expected are interpreted as evidence of positive autocorrelation. The results of these tests (Table IV.3) clearly indicate that significant positive autocorrelation of lag-1 neighbours is present in all three areas. Departures from the null hypotheses are, in general, most pronounced for the California sample and weakest for the Kentucky area. (This trend is, coincidentally, inversely related to sample size, so that the "significance" of

TABLE IV.3: BINOMIAL TESTS FOR PASS HEIGHT
AUTOCORRELATION

Pfaltz Class		12	Adjacency	
			{12}3	{{12}3}4
A	Expected	0.400	0.500	0.667
	Kentucky	0.520**	0.673**	(0.486)*
	B.C.	0.683**	0.714**	0.800
	California	0.806**	0.520	0.796
B	Expected	0.500	0.600	0.833
	Kentucky	0.600	0.821**	(0.719)
	B.C.	0.700	0.857**	0.944
	California	0.813**	0.923**	(0.667)
C	Expected	0.500	0.733	0.818
	Kentucky	0.732**	(0.732)	(0.719)
	B.C.	0.565	(0.538)	(0.571)
	California	0.732*	0.900	1.000

* significant, 5 per cent level

** significant, 1 per cent level

(-) observed value less than expected

the difference is similar in most cases).

4.2.2: Formal Autocorrelation Analysis

Formally, spatial autocorrelation can be defined as a situation where "the presence of some quality in a county of a country makes its presence in neighbouring counties more or less likely" (Cliff and Ord, 1973, p. 1), where "county" and "country" refer in the general case to areal sub-unit and study area, respectively. Cliff and Ord suggested two general autocorrelation coefficients to quantify these neighbour influences: the first, due to Moran (1950) relates the sums of cross-products of neighbouring cases to the total variance of all cases; the second, suggested by Geary (1954), divides the variance of neighbour differences by total variance. The former coefficient is closely related both to the standard measures of autocorrelation in time-series analysis, and to the product-moment correlation coefficient between values of all neighbour pairs, and differs from the latter only in weightings for the total variance calculation. The product-moment autocorrelation coefficient is intuitively more appealing in its relation to well-known statistical procedures and concepts, and is used in the following analysis.

In the present study, "neighbour" was used in the graph-theoretic or topological sense (section 2.1) in the calculation of pass elevation autocorrelation coefficients for the Kentucky and British Columbia hilltop samples. This was done for all possible lags, with "lag-2" or second order neighbours being defined as those passes on edges adjacent to the first order (lag-1) neighbours, et cetera. Obviously, lag-4 neighbour relations occur only in Pfaltz graph class A, while third order neighbours do not occur in classes D and E. Product-moment autocorrelation coefficients were computed for each possible lag and each Pfaltz graph class for each of the two sample areas. When raw elevations were used, the resulting plots of autocorrelation against lag (Figure 4.3) are dominated by inter-hilltop elevation differences, thus emphasizing the similarity of pass elevations within each hilltop, compared with the over-all relief of the study areas. Only intra-hilltop autocorrelation will influence surface tree frequencies, and thus it was deemed necessary to attempt to standardize ("de-trend") the data before analysis. Several different standardizations were applied, and that which set the highest pass to one and the lowest to zero appears best, in that its dimensionless form may reduce scale effects. Figure 4.3 shows the autocorrelation plots for different Pfaltz classes using this transformation. In each area, but especially in the Kentucky sample, Pfaltz graph classes A, B, and C show mutually

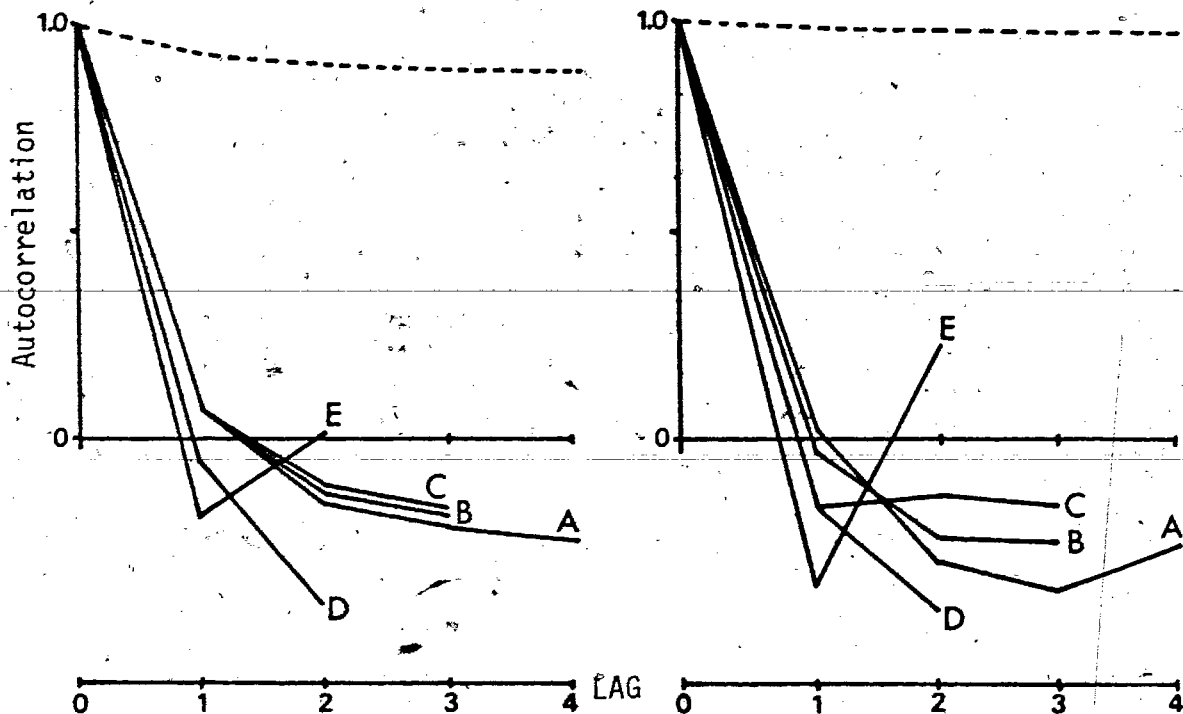


Figure 4.3: Pass height autocorrelation plotted against "lag" (topological distance) for the Kentucky (left) and British Columbia (right) hilltops. Letters refer to the Pfaltz graph classes, and those lines represent transformed heights; the dashed lines indicate untransformed autocorrelation plots.

similar autocorrelation plots. In each area, class D shows the strongest negative autocorrelation; this was for lag-2, and indicates that the one edge not incident on the central (degree four) peak has a pass height which is very distinct from the other edges. On the other hand, Pfaltz graph class E has negative autocorrelation at lag-1 and independence or positive autocorrelation for lag-2. In this class, it would appear that the "central" edge has a distinct (and by inspection, generally lower) elevation. The numbers of occurrences of classes D and E are, however, so low that these trends may not be reliable.

4.3: Surface Trees for Larger Areas

Thus far, attention has been focused on surface trees of small, fixed magnitude (6), in order that all topologically-distinct possibilities could be considered. Now, trees for larger areas will be considered. Surface trees were determined for five larger terrain samples. The first two (Figure 4.4) were computed for 3 by 3 km and 2 by 2 km square sample areas from the Inez quadrangle near the centre of the Kentucky study area. The larger sample was arbitrarily located, but the smaller was chosen so as to minimize the amount of major valley floor. The form of the larger tree (Figure 4.4A) is dominated by two effects: a general accordance of summit elevations, and influences of the study area boundary, which "produced" the two

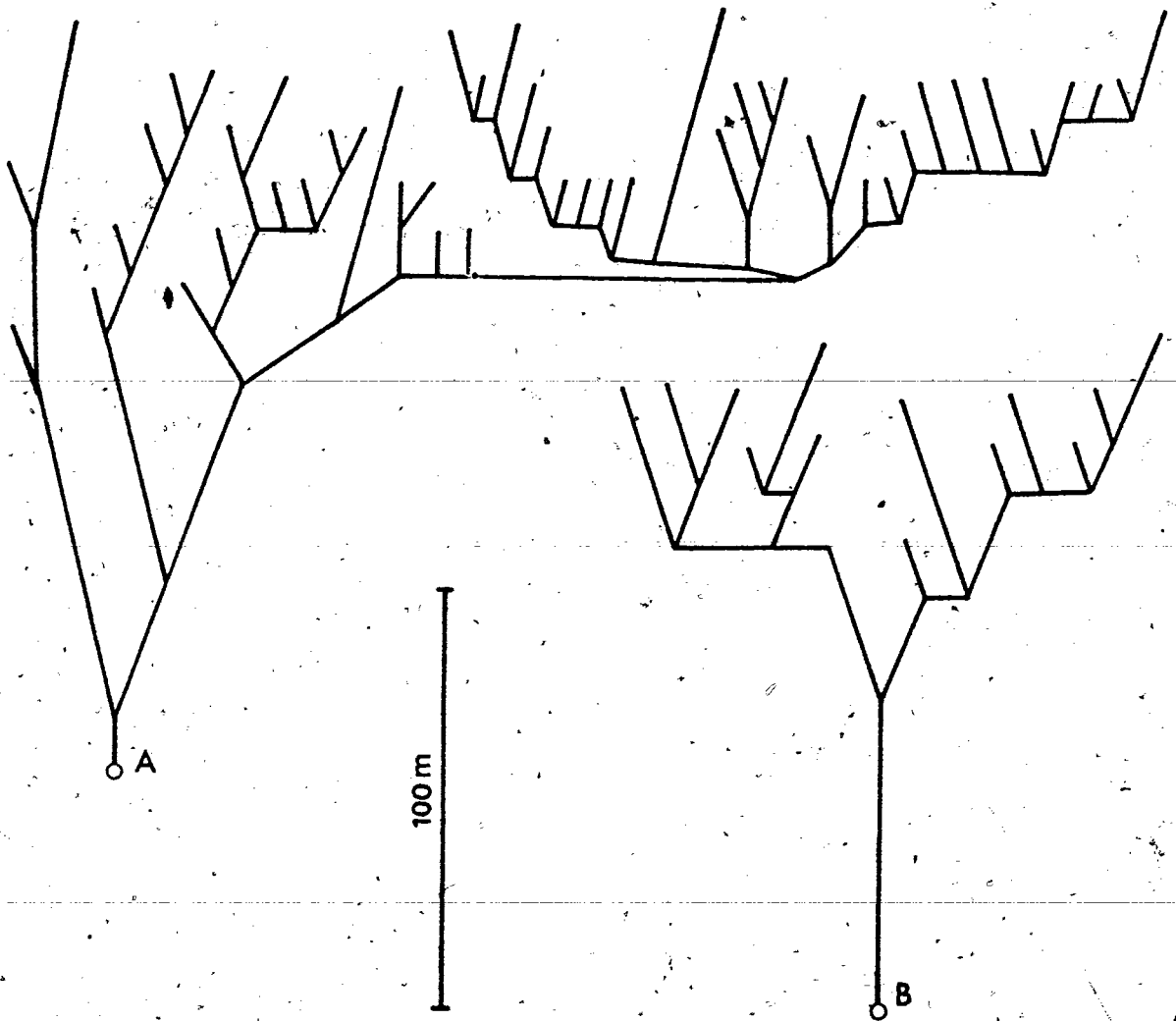


Figure 4.4: Surface trees for two sub-areas of the Inez quadrangle, Kentucky. A: 3 by 3 km area; B: 2 by 2 km area.

lowest passes (which are not passes when the sample area is increased). The smaller tree (Figure 4.4B) owes its simpler form to the selection procedure which avoided the introduction of artificial "edge" passes.

As noted in the introduction (Chapter 1), a feature of either a "white noise" or a "Brownian" random surface would be an equal expected frequency of peaks and pits. Furthermore, the expected elevations of arbitrary points are normally distributed for both of these surfaces (if the white noise is Gaussian), and the passes have the same expected elevation as the arbitrary points, but probably with a smaller variance. Figure 4.5 shows the frequency distribution for all points, for passes, and for peaks, for the area represented by Figure 4.4A (excluding "edge" peaks and passes). All peaks, and more importantly, all passes, lie above the mean elevation of the terrain; this uneven distribution of critical points within the elevation range of the terrain represents a very important non-random element (constraint) on the terrain of this area. This is probably related to Leopold and Langbein's (1962, p. A8) claim that "the higher the landscape above base level, the greater becomes the distribution of possible slopes". This in turn may be a consequence of diffusion models for geomorphic surfaces viewed as transport surfaces (Scheidegger and Langbein, 1966; Luke, 1974). A similar result may be derived from completely

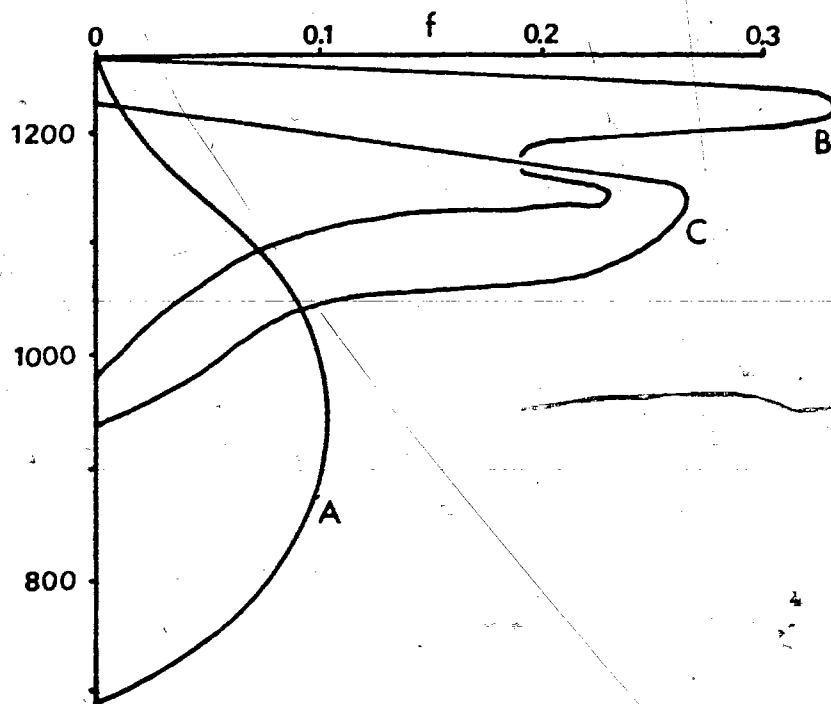


Figure 4.5: Frequency distribution of all points (A), peaks (B), and passes (C), by elevation in feet, for the 3 by 3 km Kentucky area upon which Figure 4.4A was based.

different process considerations, such as the "flooded" random surface of Marcus (1967).

Surface trees were also determined for three much larger terrain samples from the glaciated mountains of western Canada; these are shown in Figure 4.6. C represents a 7 by 14 km rectangle from the Ptarmigan Creek map area (83 D/10), and was generated by the computer from a digital terrain model using the methods described in Appendix II. D and E are based on 7.9 by 9.6 km rectangles from the Lake Louise (82 N/8E) and Manning Park (92 H/2W) map areas, respectively. The Manning Park area was completely overtopped by glacier ice during the Pleistocene Epoch, while the other two areas show sharp "cirque-and-horn" topography in the upper parts of their elevation ranges and smoother forms below. In the trees of these areas (Figure 4.6, C and D), cirques containing closed depressions appear as short branches which slope downward away from the main branches. These pits, and the over-all scale of relief, are really the only features which distinguish these trees from those shown in Figure 4.4, suggesting that surface trees may be of limited value as a physiographic tool. The general accordance of summits in the Manning Park area, which probably represents a pre-glacial erosion surface (Rice, 1960, p. 2), is shown very well in the surface tree of that area (Figure 4.6E), but could have been shown equally well by any of a number of simpler

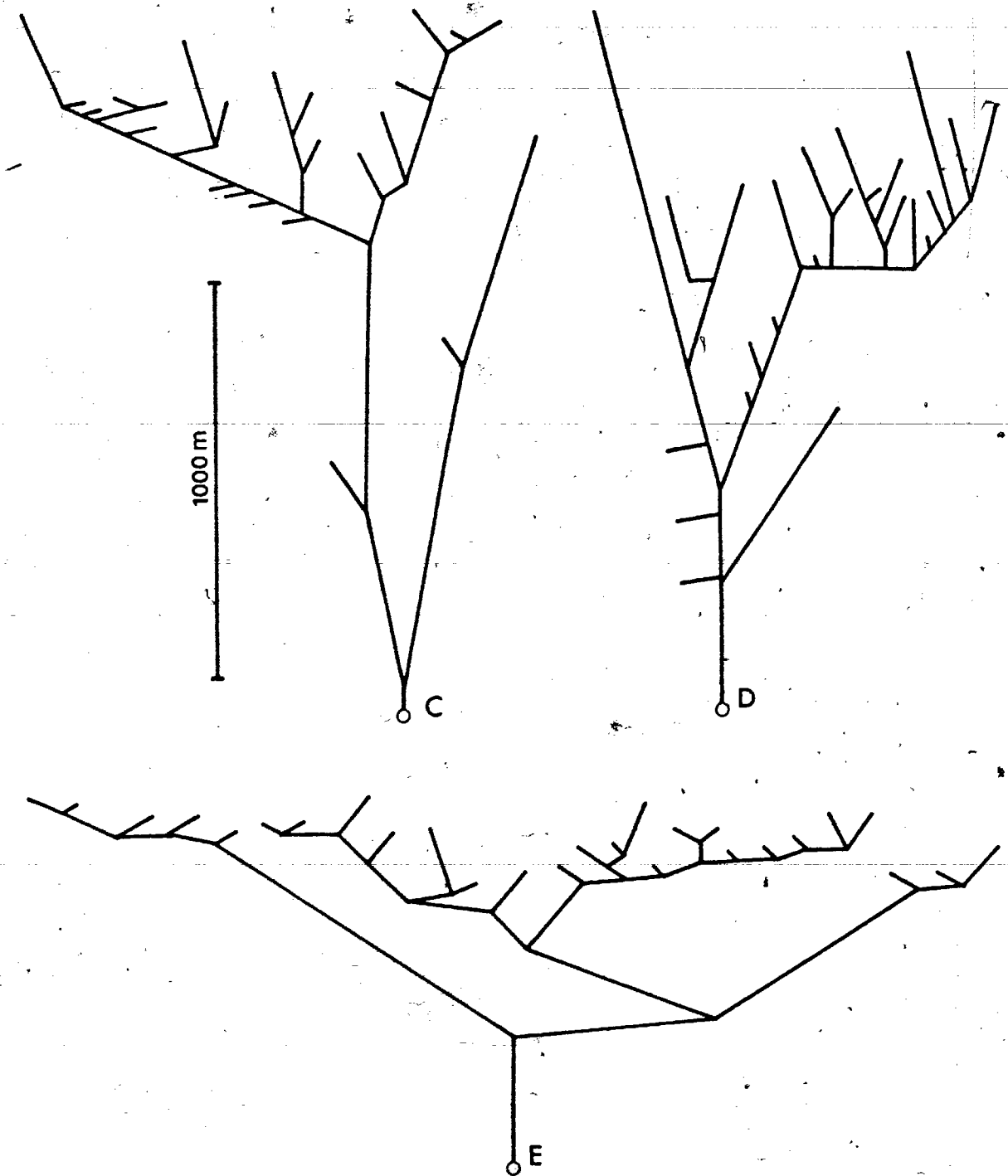


Figure 4.6: Surface trees for three areas of glaciated mountains from western Canada. C: Ptarmigan Creek map area; D: Lake Louise map area; E: Manning Park map area.

graphical techniques.

The surface trees shown in Figures 4.4 and 4.6 have the elevations of their nodes to scale, but their horizontal scales are arbitrary. It is possible (but very tedious using manual methods) to compute the surface area associated with each branch of the tree, and use these values to scale the horizontal lengths of the branches. Figure 4.7 shows the Ptarmigan Creek sample area (already shown in Figure 4.6C), but with the horizontal components of the branches scaled by the square roots of associated areas. The slopes of the branches are indirectly related to average land slopes, and the tree gives an impression of hypsometry, as well as indicating relief, summit accordance, and perhaps trim lines and cirque elevations.

4.4: "Horton Analysis" of Surface Trees

In a frequently used approach, "the ~~first~~ step in drainage-basin analysis is designation of stream order, following a system introduced into the United States by Horton (1945) and slightly modified by Strahler (1952)" (Strahler, 1964, p. 4-43). This step is often followed by a "Horton analysis", in which the logarithms of stream numbers, mean lengths, and mean drainage areas are plotted against order. For most streams, these semi-logarithmic plots produce approximate straight lines; these

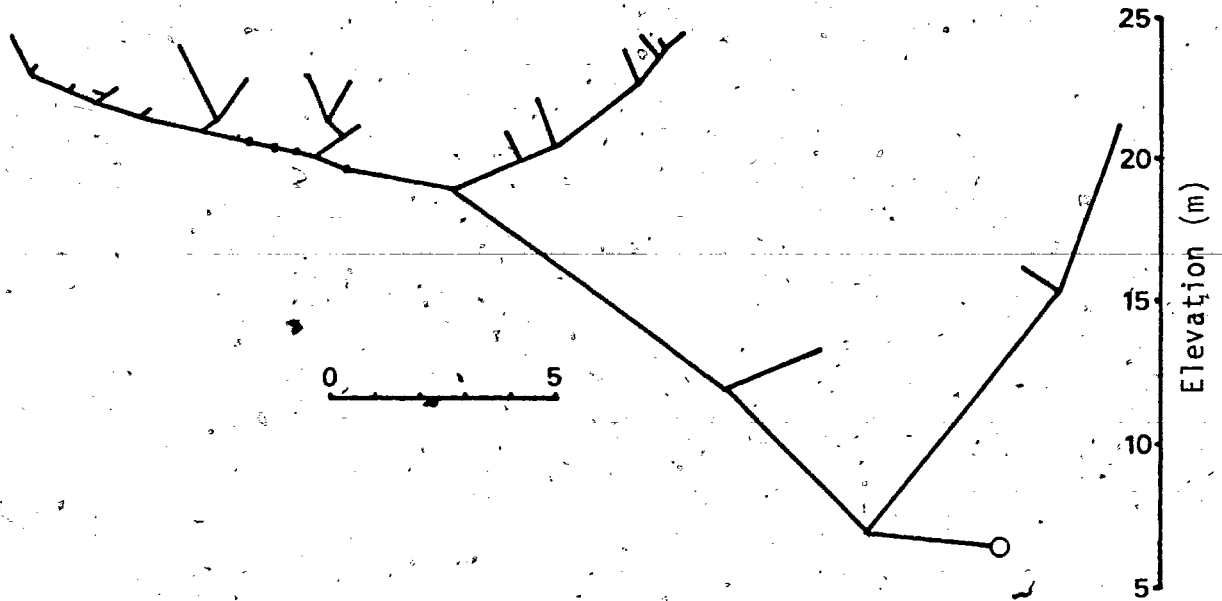


Figure 4.7: Same as in Figure 4.6C, but with horizontal components of branches scaled by the square root of area (in km), and with absolute elevations in hundreds of metres.

are known as "Horton's Laws". Shreve (1966), and others, have shown that these relations represent the most probable states of randomly-branching networks, and that furthermore, the expected slopes of these plots are closely determined. The expected slope of the number plot (the "bifurcation ratio", R_b) is 4 for very large stream systems (Shreve, 1967); the expected value of the length ratio (R_L) is 2 and of the area ratio (R_A) 4 (Shreve, 1967). While Horton's "Laws" do involve "averaging processes" which "obscure the basic qualities" of mean link lengths and drainage areas (Shreve, 1975, p. 529), their application to surface trees provides an interesting descriptive tool for large networks.

First, it is possible to establish the expected bifurcation ratios for magnitude-6 trees under various conditions. Surface tree class I (see Figure 4.1) has a Strahler order of 2, and thus has a bifurcation ratio of 6. For the other five tree classes, the tree is of third order, and the geometric mean bifurcation ratio is 2.45 (square root of 6). For stream networks, Figure 4.1 can be thought of as representing the six ambilateral stream classes, and the number of members in each class can be used to calculate a weighted mean bifurcation ratio. Class I would contain 16 of the 42 distinct magnitude-6 trivalent planted plane trees, and this would produce an average bifurcation ratio of 3.80 (see Table IV.4). When the surface

TABLE IV.4: EXPECTED MEAN BIFURCATION RATIOS FOR
MAGNITUDE-6 TREES

	Class I	All Others	Mean R_b
Geometric Mean R_b	6.0	2.45	-
Tree Frequencies:			
Stream Networks	0.381	0.619	3.80
Circle Simulation	0.226	0.774	3.25
Kentucky	0.291	0.709	3.48
British Columbia	0.427	0.573	3.97
California	0.492	0.508	4.20

tree values for the simulation model (see Figure 4.2) are used as weights, a considerably lower R_b (3.25) results, but this rises with increasing positive autocorrelation of pass heights, so that mean values for observed surface trees are similar to the values for stream networks (see Table IV.4).

The five surface trees shown in Figures 4.4 and 4.6 were ordered using Strahler's system, and the numbers and mean relief values for each order were determined. Figures 4.8 and 4.9 represent Horton-type diagrams for "hill numbers" and "hill mean relief", respectively. The number plots (Figure 4.8) are rather similar to typical stream plots, with geometric mean bifurcation ratios between 3 and 5; as explained above, however, this close approximation to stream ratios is largely coincidental. Mean relief seems to increase geometrically with order (Figure 4.9), but it would appear that the constraint imposed by total available relief means that the relief ratio is not independent of order, as is generally the case for stream networks.

This is well illustrated by the Kentucky samples. The average local relief in this area is no more than 180 metres, while the mean relief of first order hilltops is consistently about 19 m. In the two samples, the highest order branch has a relief of about 110 m, but due to the sample area size and number of peaks, this branch is third order for the smaller sample and

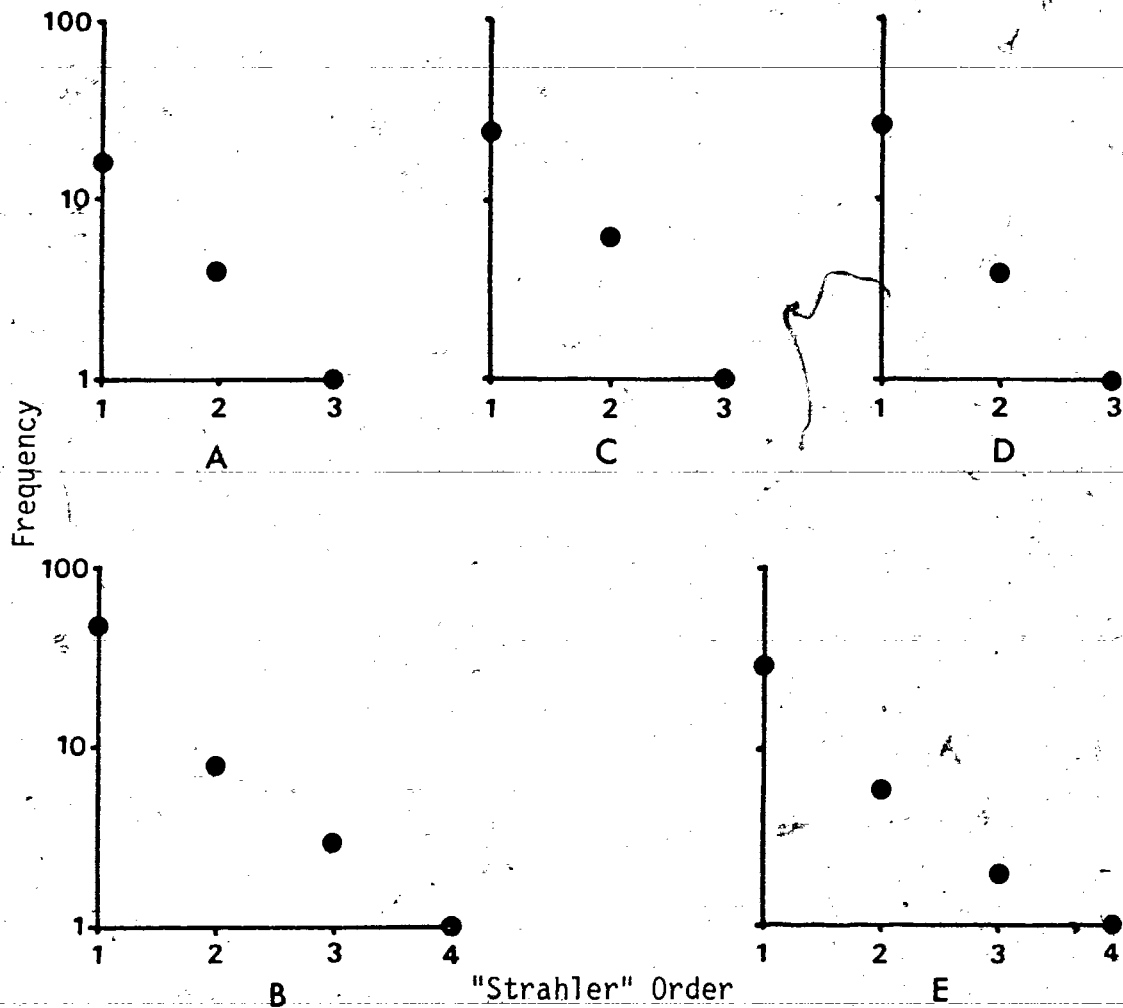


Figure 4.8: "Horton" diagrams plotting numbers of branches against Strahler order for the five surface trees shown in Figures 4.4 and 4.6.

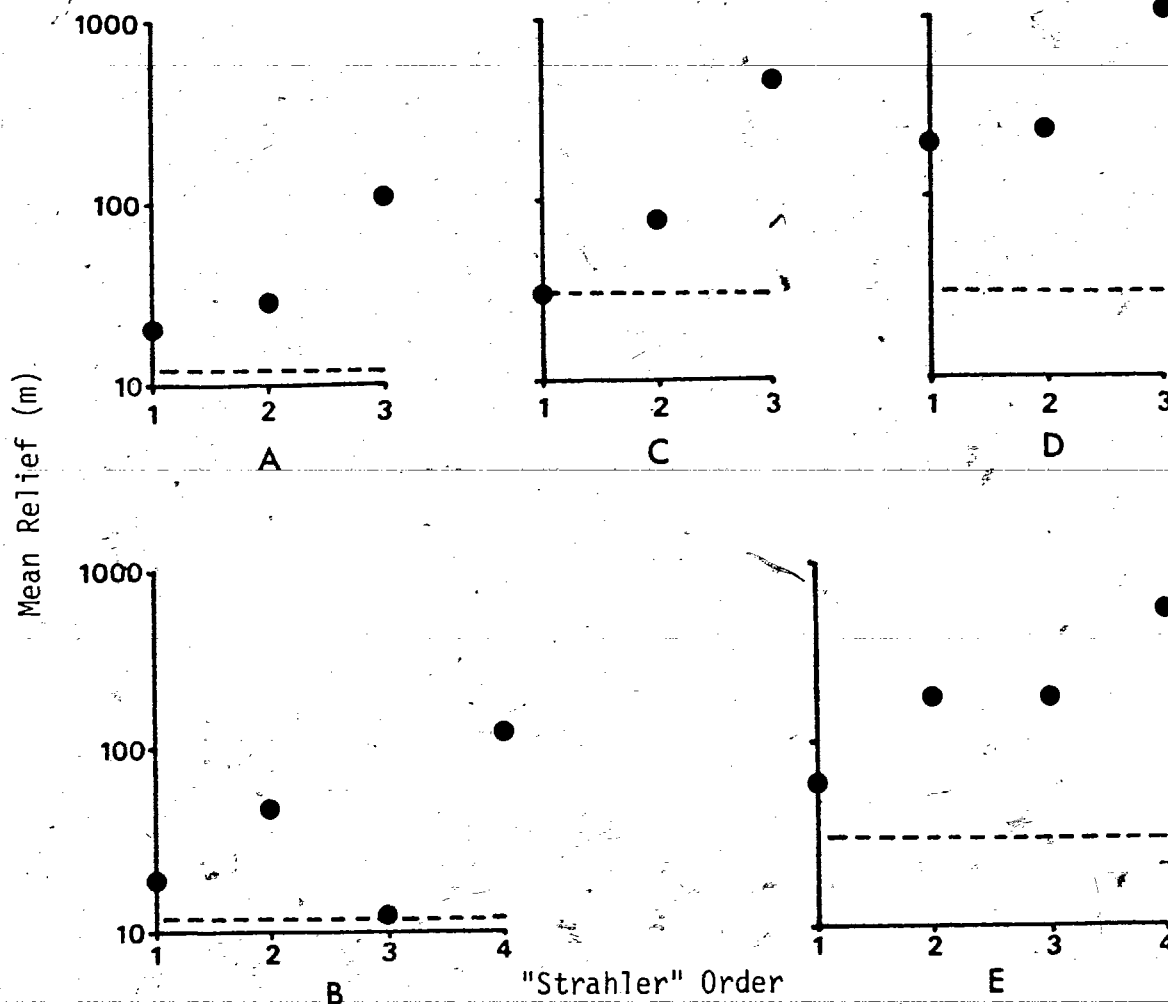


Figure 4.9: "Horton" diagrams plotting mean branch relief (in metres) against Strahler order for the five surface trees shown in Figures 4.4 and 4.6. The dashed horizontal line indicates the contour intervals of the source maps.

fourth for the larger. The Inez quadrangle contains over 400 peaks, and its total surface tree would probably be of order five or six, but the highest order branch would again be around 110-140 m. The geometric mean relief ratio of an order-M tree is computed by taking the (M-1)th root of the ratio of Mth order relief to mean first order relief. Since this ratio is relatively constant once the sample area exceeds the regional grain (Wood and Snell, 1960) (and the 2 by 2 km sample appears to exceed the grain of the Kentucky area), the mean relief ratio will decline with increasing order. Probably, only the mean relief of first order hills is meaningful, and even this is constrained by the contour interval when maps are used as the data source.

4.5: Conclusions

In the last chapter, it was found that the frequencies of Pfaltz graph classes could be well predicted by a model based on a random spatial distribution of peaks, with the ridge network linking these constrained to be of minimum total length. A further constraint of an elongated field for these points was present in some but not all terrain. These frequencies, together with assumed independence of pass heights, produced expected frequencies for magnitude-6 surface trees which were not in agreement with the data. This is because pass elevations

are not independent: the positive autocorrelation of elevations on geomorphic surfaces, which results from continuity, represents a further constraint on the "randomness" of topological representations.

Large surface trees generally obey "Horton's Laws", but mean bifurcation ratios are similar only by coincidence. The mean relief ratio is not independent of order, since the relief of the highest order branch cannot exceed the local relief of the sample area.

CHAPTER 5: EFFECTS OF SCALE ON TOPOLOGY

The scale (size, extent) of landscape elements represents a very important aspect of all geomorphological research, for even if scale is not explicitly specified or investigated, it will be implicit in the delimitation of a study area and of a sampling density or in the resolution of individual measurements. While topological properties are by definition scale-free, they are, however, influenced in various ways by scale effects. For example, fixing the magnitude of a drainage basin or of a hilltop establishes an expected planimetric size for these units, based on drainage, ridge, or peak density. The first section of this chapter will review various measures of landscape scale as they have been (or should have been) employed in geomorphometry. Attention will then turn to scale effects as they relate to hilltops of various magnitudes. Finally, an attempt is made to relate size and shape of hilltops to each other and to ridge topology.

5.1: Fundamental Scales of Geomorphometry

The scale of a geomorphic system cannot be described by any single measure, but rather must be characterized by a suite of basic or "fundamental concepts of geomorphometry" (Mark, 1975b). First among these is the planimetric or horizontal scale, which

itself cannot be adequately represented by a single value, but can only be completely characterized by the two-dimensional power spectrum of the terrain. This can sometimes be summarized by the average one-dimensional power spectrum, or more often by the important or characteristic wavelengths of the terrain. Perhaps the most important of these are the smallest and largest wavelengths of interest, distinguished as "texture" and "grain", respectively (Mark, 1975b), for the former influences the required resolution of measurements and the latter the size of sampling areas. Drainage density and peak density are more commonly used geomorphometric parameters which are related to this horizontal scale, as are the average sizes of fixed-magnitude basins or hilltops, for reasons outlined above. What most previous researchers (including Mark, 1975b) have ignored is that horizontal scales may be different in different directions; that is, anisotropy may be present. Furthermore, this anisotropy may itself vary with scale, being present at some scales and absent at others. For example, Figure 3.2 clearly indicates that an important element of ridge anisotropy is present within magnitude-6 hilltops in the Kentucky sample area. When the orientations of these hilltops are investigated, however, no significant between-hilltop anisotropy is present (Figure 5.1, A, B), despite the fact that magnitude-5 drainage basins in the same area show a significant departure from a uniform distribution (Krumbein and Shreve, 1970, p. 94-101; see

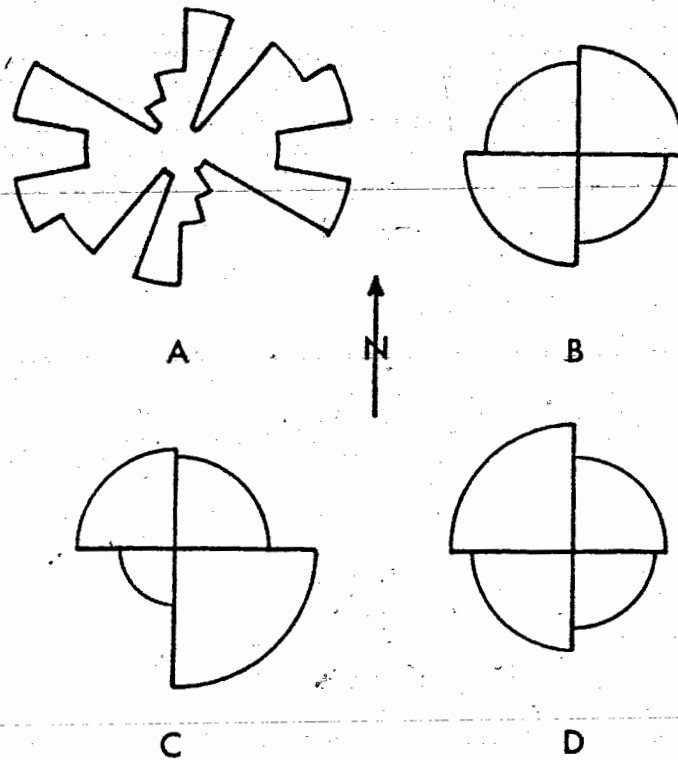


Figure 5.1: Orientation properties of the Inez quadrangle, Kentucky: A: Hilltop trend, 20 degree classes; B: Hilltop trend, 90 degree classes; C: Outlet orientation, magnitude-5 streams; D: Dip of strata in magnitude-5 basins. (C and D after Krumbein and Shreve, 1970, p. 95).

also Figure 5.1). Evidently, the anisotropy which influences magnitude-6 Pfaltz graph topology does not extend to larger landscape elements. This is entirely consistent with the hypothesized reason for the anisotropy, the influence of major streams on local divides, which would not be expected to extend to larger systems.

Vertical scale, commonly termed "relief", represents another basic dimension; correlation and factor analyses have generally shown this to be independent of the horizontal scale (Gardiner, 1975; Mark, 1975b). Another fundamental concept proposed earlier (Mark, 1975b) is the dispersion of orthogonals to the surface. The averaging procedures employed in this calculation, however, obscure the fact that these orthogonals may reveal aspects of landscape anisotropy (see Chapman, 1952). Anisotropy is probably also what is being measured by most basin shape parameters.

5.2: Pfaltz Graph Topology for Varying Magnitudes

In order to test further the model and hypotheses developed in Chapter 3, and also to investigate indirect effects of scale on topology, samples of hilltops of magnitudes 4, 5, and 7 were obtained from the Kentucky and British Columbia sample areas. These hilltops were each classified into one of the 2, 3, or 11

(respectively) topologically distinct Pfaltz graph classes. The minimum spanning tree model was run for each of these magnitudes, using a range of elongation values to produce expected frequencies (see Appendix D).

Figures 5.2 and 5.3 plot Chi-square (goodness-of-fit) against elongation for the four magnitudes in these two areas. Because of low expected values for some Pfaltz graphs, some classes were amalgamated. Magnitudes 4 and 5 each have but one degree of freedom, while magnitudes 6 and 7 have 3 and 4 degrees of freedom, respectively. In the Kentucky area (Figure 5.2), the shape observed for magnitude-6 (interpreted as indicating within-hilltop anisotropy) appears to apply throughout this magnitude range; the second "optimum" for the circle at magnitude-7 suggests that the isotropy at larger horizontal scales in this landscape may be beginning to influence topology at this magnitude. In the British Columbia sample (Figure 5.3), isotropic effects were thought (Chapter 3) to dominate at magnitude-6, and this clearly extends to magnitude-7. The smaller magnitudes, however, show anisotropic shapes similar to the Kentucky sample, suggesting that the boundary between local anisotropy and general isotropic effects for this landscape lies between magnitudes 5 and 6. In contrast, this boundary would appear to lie above, but perhaps not far above, magnitude-7 systems for the Kentucky study area. The difference is perhaps

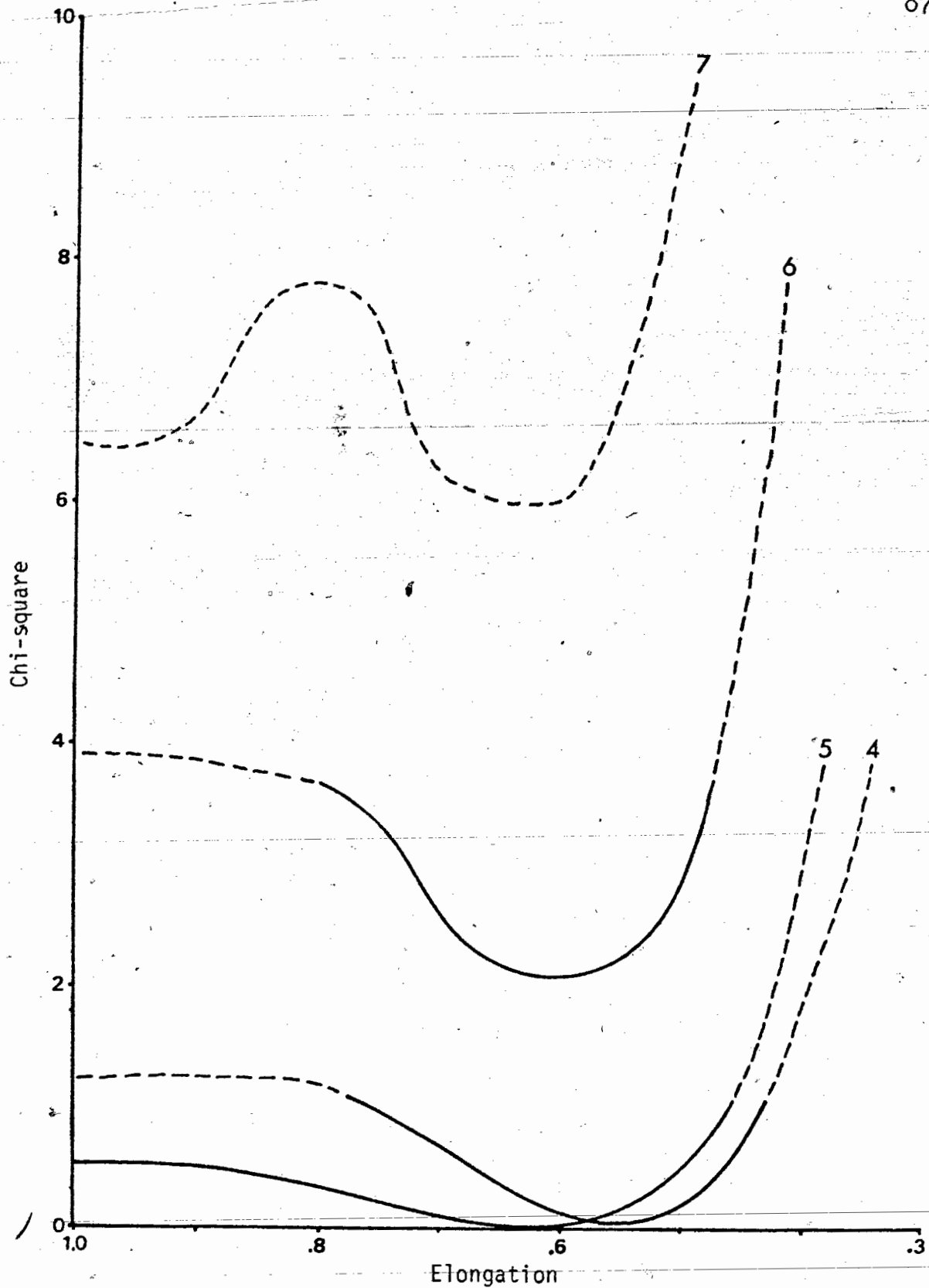


Figure 5.2: Chi-square of the difference between observed and modelled Pfaltz graph class frequencies, plotted against elongation, for different magnitudes in the Kentucky sample. Symbolism as in Figure 3.2.

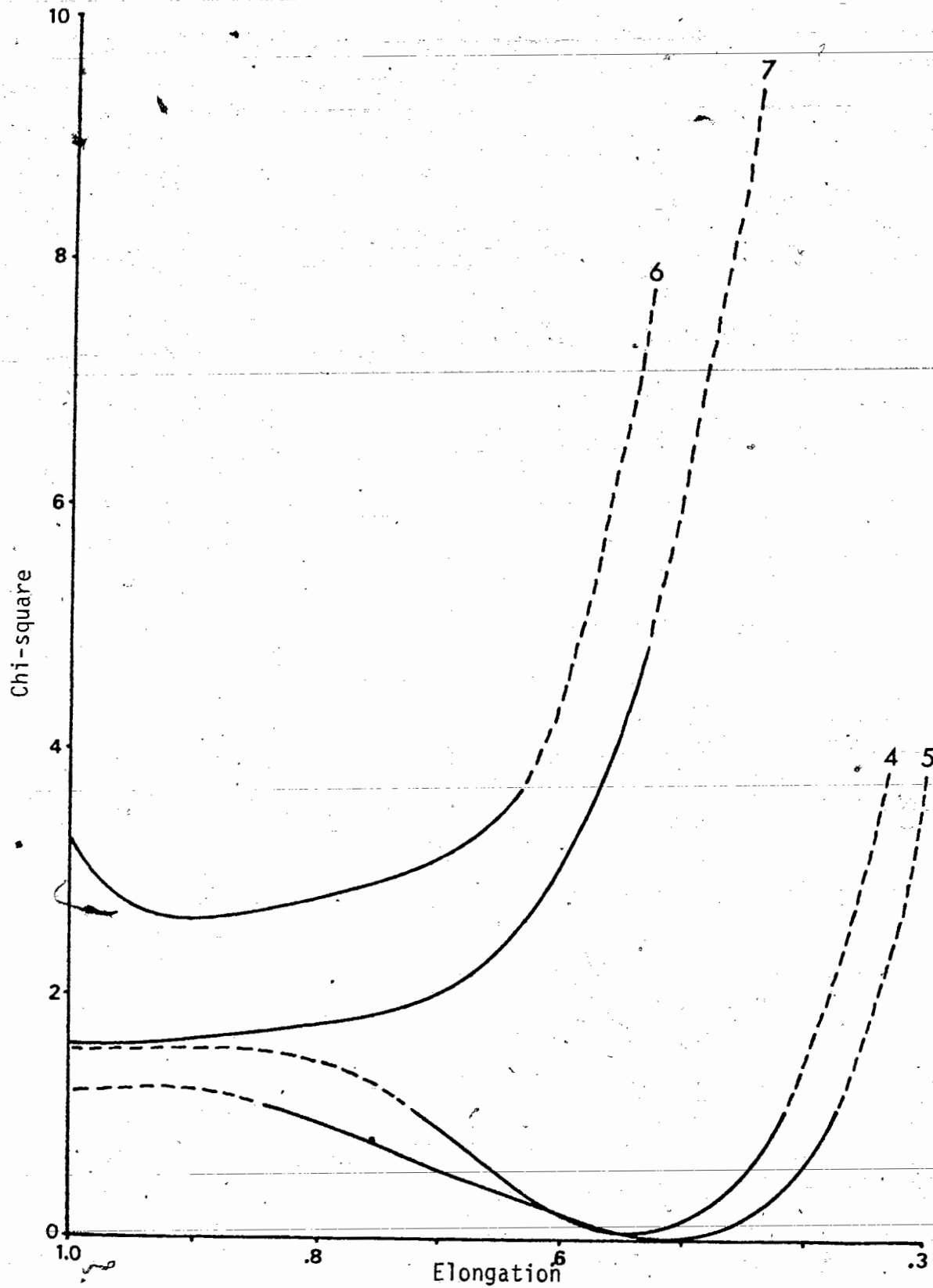


Figure 5.3: As in Figure 5.2, but for the British Columbia sample.

largely a result of different map scales and contour intervals (Kentucky: 1:24,000, 40 feet; British Columbia: 1:50,000, 100 feet). Although areal extents of different magnitudes were not measured, the magnitude-4 systems in British Columbia probably have a similar mean area to the magnitude-7 hilltops of the Kentucky maps. The comparison is not straightforward, however, because there are probably real differences in topographic texture between the two areas. Furthermore, the British Columbia area has been glaciated; it is interesting to speculate that possible scale differences among fluvial, glacial, periglacial, and paraglacial processes may contribute to the differences in the goodness-of-fit plots.

5.3: Allometric Analysis of Hilltop Form

Correlation and factor analyses indicate that the logarithms of area, length of ridges, and radii of the largest inscribed and smallest circumscribed circles for each hilltop are all very closely related, while relief, the remaining size measure, is less closely linked with these. This provides further confirmation of the relative independence of horizontal and vertical scales of topography. Allometric analysis provides a method for examining the structure of morphometric systems in more detail. The allometric approach studies the relationship of size of one part of a system to some measure of the total

system, and was originated in Biology (see Gould, 1966, for a review). The approach was introduced into the Earth Sciences by Woldenberg (1966) and was recently reviewed by Bull (1975). It would appear that area is the best measure of hilltop size. Power functions relating each of the other measures to area were computed using functional analysis (see Mark and Church, 1977), with the error variance ratio for that technique (λ) based on the error variances of the logarithms reported in Appendix C. The results, given in Table V.1, indicate that only the relation between relief and area differs significantly from isometry (the preservation of form with changing size). They also indicate that planimetric shape is essentially independent of size; the relief-area relation suggests that small hilltops tend to have a higher relative relief than larger ones. Mosely and Parker (1972) have discussed the danger of interpreting size-related changes in shape as growth patterns. Such an interpretation would be most inappropriate here, since even if space can be substituted for time, process considerations would indicate that hilltop planform shrinks, rather than grows, isometrically. The model developed in Chapter 3 indicates that hilltop shape alone should be related to topology, and that size should be irrelevant; the allometric analyses confirm the relative independence of shape and size.

TABLE V.1: ALLOMETRIC RESULTS

X	Y	isometric exponent	observed exponent
A	H	0.5	0.423*
A	SUML	0.5	0.516
A	RC	0.5	0.543
A	RI	0.5	0.515

* significant allometry, 5 per cent level

TABLE V.2: ELONGATION RATIOS FOR PFALTZ GRAPH CLASSES

Class	Mean	t	Mean	t	t
A	0.588		0.588	---	---
B	0.667	---			
C	0.654	---	0.661	---	---
D	0.707	---			
E	0.755	---	0.744	---	---

}- 0.36 N.S. }- 2.64 *
 }- 1.21 N.S. }- 2.98 *

* = significant difference, 1 % level

5.4: Relations Between Topology and Geometry

One intuitively reasonable consequence of the model developed in Chapter 3 is an expected relation between Pfaltz graph class and the shape of the contour bounding the hilltop. If the elongation ratio of the (imaginary) bounding ellipse varies, these ellipses should have different average shapes for the different topological classes, since topology varies with elongation. Based on the contrasts exhibited in Figure 3.1, one would expect no significant differences between the elongation values for classes B and C, nor between D and E. Class A should be more elongated than any other, while B+C should be somewhat more elongated than D+E. Next, it can be shown through another simulation study that average elongation of the bounding contours of hilltops is positively correlated with elongation of the bounding ellipse. Differences between the observed elongations of the bounding contours (as measured by the ratio of the smallest inscribed to largest circumscribed circle) were examined using t-tests; the results (Table V.2) coincide well with the expected consequences of the model outlined above, and provide independent verification for the model.

In order to investigate further the relations between topology of Pfaltz graphs and the geometry of their associated hilltops, an attempt was made to distinguish the Pfaltz graph classes on

the basis of the five measured geomorphometric variables (section 5.3), of ridge density, and of the 10 possible ratios among the basic variables (as measures of shape). This was done, first for each of the sixteen variables alone (using t-tests), and then for all variables together (using stepwise multiple discriminant analysis).

The t-tests indicated that Pfaltz graph classes B and C did not show significant differences for any of the variables, nor did classes D and E. This is not unexpected, since this was found for the elongation ratio already discussed above. For further t-testing, each of these pairs of classes was combined, giving three groups (A, B+C, D+E) in all. Class A was found to have a significantly shorter mean total length of ridges than either of the other classes, and less compact average shape, as measured by two "shape" ratios. The strongest differences were for the ratio of the total length of ridges to the radius of the circumscribed circle ("RD2"). Once again, this is an expected and reasonable result, since geometric considerations require that this measure should be approximately equal to two in a class A (unbranched) hilltop which is relatively straight; in the other classes, side ridges should (and do) increase this length ratio. Groups (B+C) and (D+E) did not differ significantly with respect to any variable.

The multiple discriminant analysis did not produce any new insights, except to confirm that the Pfaltz graph classes show a great deal of overlap with respect to their geometric characteristics. When all five classes were used, the optimum discriminant scheme classified only 37 per cent of the hilltops into the appropriate topological class. When classes B to E were combined, the probability of correct classification rose to 64 per cent. In each case, RD2 was the first variable entered in the stepwise discriminant procedure.

5.5: Conclusions

All geomorphological studies involve scale effects. There are several fundamental scales to describe topographic form: two of the most important are the basic horizontal and vertical scales. Horizontal scale may itself have many components, especially if topography has an anisotropic element; such anisotropy may further be present at some scales and absent at others. For example, between-hilltop orientations for the Kentucky sample area are isotropic, while topological class frequencies for ridge systems of magnitudes 4 through 7 indicate a marked degree of within-hilltop anisotropy. In contrast, the British Columbia sample area shows within-hilltop isotropy for larger magnitudes (6 and 7), but anisotropy for smaller (4 and 5). Exact reasons for these scale effects are unknown.

An allometric analysis confirms the independence of planimetric shape and size for the Kentucky hilltop data. Relative relief is, however, greater on the average for small hilltops than for large. With the exception of the total length of ridges, the topological Pfaltz graph classes are distinguished by their shapes and not by the basic size-related variables. This result engenders further confidence in the model developed in Chapter 3. The geometric properties examined are not, however, so closely related to topology that, for example, topological ridge class can be inferred from hilltop geometry alone.

CHAPTER 6: DISCUSSION AND CONCLUSIONS

6.1: Relating Ridge and Stream Topology

One of the criticisms levelled at the probabilistic-topologic approach in Chapter 1 was that it lacked generality, since although the drainage net is very important, it nevertheless represents only a small fraction of the landscape. The Warntz network and its associated Pfaltz graph include both course lines (related to, but not identical with, stream channels) and ridge lines (related to geometric ridges) in a unified system. As noted in Chapter 3, however, the course graph contains no topological information in an area without pits, such as fluvially-eroded terrain at small and medium map scales, since all courses connect with the same "outside" pit. This presents a paradox: fluvially-eroded terrain, in which stream channels play a dominant role in geomorphic processes, is the very sort of terrain in which the course part of the Pfaltz graph is of no topological interest. This arises primarily because courses and channels are not identical.

In the strictly defined Warntz system, which assumes continuity of the surface itself and of at least the first derivative, course lines may meet only at critical points; exactly two meet at each pass, and any number may meet at a pit. It is obvious,

however, that streams do merge in nature. There are at least three ways of resolving this contradiction. First, one may contend that since topographic surfaces are not continuous in the first derivative, the Warntz system should not be used for such surfaces. For example, Luke (1974, p. 4037-8) has shown that, by assuming surface erosion by continuous fluid flow across surfaces, parabolic troughs would become V-shaped, and thus have a slope discontinuity which would allow course lines to merge. To adopt such a system would, however, require the introduction of several new types of points, including stream and ridge junctions and stream and ridge sources. Leaving aside the problem of defining ridges and courses in such a system, it seems clear that at least some of the topological axioms or properties of Pfaltz graphs would no longer apply.

A second solution was proposed by Warntz (1975): his solution was to retain the strict axioms of these graphs, but also to introduce additional critical points in the neighbourhoods of all stream (or ridge) junctions. Specifically, Warntz introduced a pit on each stream immediately above the junction, and a "triple" pale at the junction itself; the three courses associated with this pale are the two incoming and one continuing streams. Except for the fact that these pits and pales do not exist in nature, this seems at first to be a reasonable solution, but there are additional complications:

the pale also creates three ridges. For simplicity, consider the case of two first order streams merging to form a second order: one of the three ridges is clearly the divide between the first order streams, but the other two lead upward toward the divide which bounds the second order basin. Ridges, however, are not permitted to merge either, and hence triple passes and peaks must be introduced at these points. These, in turn, generate new courses, and so ad infinitum. Warntz (1975) appears to have believed that such an "infinite regress" of ridges and courses actually exists, but that since it is not practical to deal with these complete networks, a threshold below which ridges and courses are ignored must be imposed for analysis. This, however, seems also to be a generally unsatisfactory solution.

The third, and perhaps most promising, avenue for further research is to contend that a stream of high magnitude represents not one course but rather a "bundle" of parallel, infinitesimally close course lines. This would immediately lead to new definitions of stream link ("a reach of a stream having a constant number of course lines") and of link magnitude ("the number of course lines in the bundle forming the link"). One is still faced with the problem that not all channels head in passes, and that there is still no relation between ridge and stream topologies.

If each point where a course leaves a study area is considered to be a distinct pit (rather than a path to a common "outside" pit), the course half of the Pfaltz graph would indicate the drainage basins which cover the area, but would still not indicate in which of the topologically-distinct ways these courses merge (or adjoin each other) in order to form a single stream (course "bundle"). To explain those aspects, it is necessary to use the approach developed by Shreve, which is independent of the Pfaltz graph. Further research in this area might include both Pfaltz graph topology and Shreve's work into a more comprehensive system.

6.2: Conclusions

If a single, main conclusion were to be drawn from the results of this study, it would be that, while there is a very important random element in geomorphic surfaces, this randomness is subject to a considerable number of constraints-- it is the determination of these constraints which should be a primary goal of landscape-scale geomorphology (physiography).

Basically, these constraints can be elucidated using either of two approaches: the researcher can examine geomorphic processes in considerable detail, and use deductive methods to infer what constraints these processes impose on resulting land forms; alternatively, one can study the forms themselves, and determine

what constraints must be placed on geomorphic randomness in order to adequately predict the statistical properties of geomorphic surfaces. The latter approach was adopted in this study.

The following are the major findings and conclusions of this study:

Topological aspects of the drainage net have received considerable attention from geomorphologists, but other aspects of surface topology have largely been ignored. Two topological representations which extend beyond the drainage net, the Pfaltz graph based on nineteenth century research into equilibrium revived by Warntz, and the surface tree based on the contour enclosure tree of computer cartography, were described, and applied to the problem of topological randomness.

A simple model, based on a random distribution of points (representing peaks) within an elliptical field, and on the minimum spanning tree of those points, provided excellent predictions for the frequencies of topologically distinct Pfaltz graph classes for both magnitude-6 and magnitude-5 systems from a variety of terrain-types. The minimum tree represents one constraint on the randomness of the simulated ridge networks, while the elongation ratio of the elliptical field, equivalent

to the degree of anisotropy of within-hilltop ridge orientations, represents another. In three areas of heterogeneous geology and structure but without strong structural control, a circle (ellipse with elongation 1.0) provided optimal or near-optimal predictions of Pfaltz graph frequencies. In contrast, two areas of horizontally-bedded homogeneous geology had best fits provided by the simulation model with an elongation parameter of around 2:1. This may be a result of elongated ridges formed between parallel or sub-parallel major tributaries of master streams in the absence of geological control. (The heterogeneity of the aforementioned three areas could be expected to disrupt such parallelism of streams). The model also fits the strongly structurally-controlled ridge-and-valley topography of south-eastern Pennsylvania, but with a preferred elongation ratio of 10:1.

The surface tree is completely determined by the Pfaltz graph of a surface, together with the pass height ranks. The frequencies of the Pfaltz graph classes (from the simulation model, above) were combined with the assumption of independent pass height ranks to produce expected surface tree frequencies. These were not in agreement with observed frequencies, and the difference would appear to result from an interdependence (autocorrelation) of pass heights which in turn is due to the overall autocorrelation of geomorphic surfaces.

Some larger surface trees were examined. In all areas, peaks and passes were concentrated above the mean elevation. Since, in general, passes on "random" surfaces have the same expected height as arbitrary points, this represents a further constraint on randomness. The trees were ordered using Strahler's system, and the numbers of branches were found to fit "Horton's Laws" well, with bifurcation ratios ranging from 3 to 5. The closeness of the bifurcation ratios to characteristic stream values was shown to be largely coincidental. Branch mean relief values plot against order as approximate straight lines on semi-logarithmic graph paper. The slopes of these "Laws of Hill Heights" are similar, but appear to decrease with increasing tree order; due to the constraints on branch relief imposed by total available relief of a study area.

Scale effects were examined directly, and also indirectly through their influence on topology of Pfaltz graphs of magnitudes 4 through 7. Anisotropy was found to be present for small systems in both landscapes examined, but is only apparent in some landscapes at larger magnitudes. Exact reasons for these scale effects remain unknown. An allometric analysis showed that planimetric dimensions of hilltops vary isometrically, but that relative relief changes with changing size. Some of the Pfaltz graph classes were found to differ significantly in their mean values of certain shape measures,

but there is so much overlap that Pfaltz graph class cannot be predicted adequately on the basis of shape alone.

In the introduction, the question "What is a random surface?" was posed. It may now be seen that this question has no simple answer, since all surfaces are subject to some constraints: "White noise" surfaces, which have a minimum of constraints, are not in general satisfactory, since their statistical properties (especially autocorrelations and power spectra) do not resemble most terrain. If we wished to generate a "random, fluvially-eroded" surface, a great many more constraints, such as absence of pits, presence of a stream network drawn from a topologically-random population, all passes in the upper part of the relief range, ridge patterns approximated by minimum spanning trees among peaks, a (sometimes) anisotropic ridge distribution, and autocorrelated pass heights, would have to be specified. This, however, would hardly be "random" in the normal sense of that word. If landscape scale geomorphometry is to form a useful part of geomorphology, I believe that it will do so through an emphasis on identifying constraints on the randomness of geomorphic surfaces.

APPENDIX A: GLOSSARY OF GRAPH THEORY TERMINOLOGY USED IN THIS
STUDY

adjacent. Vertices which share a common edge; edges which share a common vertex.

bipartite graph. A graph whose vertices can be divided into two subsets such that no points in the same subset are adjacent.

complete graph. A graph in which every vertex is adjacent to every other. A complete bipartite graph is one in which every point in each subset is adjacent to every point in the other.

connected graph. A graph in which there exists at least one path between every pair of vertices.

cycle. A path leading from a point to itself.

degree. The degree of a vertex is the number of adjacent vertices.

dual (geometric, of a plane map). The graph formed by placing a vertex in each face or cell of a plane map, and connecting

those new vertices which lie on faces which share an edge.

edge. A pair of distinct vertices, which are said to be connected.

end-point. A vertex of degree one.

face. A contiguous region of the plane on which a plane graph is embedded, which is bounded by an elementary cycle.

forest. A disconnected graph containing no cycles; a set of trees.

graph. A set of vertices, and a (possibly empty) set of edges.

homeomorphic. Two graphs are homeomorphic if they can be made to be isomorphic by adding or deleting vertices of degree two.

incident. The edges which contain a vertex are said to be incident on that vertex.

isolated vertex. A vertex of degree zero.

isomorphic. Two graphs are isomorphic if there exists a one to one correspondence of their vertices which preserves adjacencies.

labelled graph. A graph in which a label or name (generally the integers $1, 2, \dots, p$) is attached to each vertex. An edge-labelled graph has its edges so named.

map (plane). A plane map is a plane graph, together with all its faces.

neighbour. All vertices adjacent to a particular vertex are its neighbours.

network. A graph, and a function which assigns a non-negative weight or length to each edge.

null graph. A graph with no edges.

path. A walk all of whose vertices, except possibly the first and last, are distinct.

planar graph. A graph which is isomorphic to some plane graph.

plane graph. A graph which is drawn on ("embedded in") a plane without any intersecting edges.

planted tree. A tree rooted at an end-point.

rooted graph. A graph in which one vertex is designated as the root, a uniquely-identified reference point.

spanning subgraph. A subgraph which contains all the points of a graph.

subgraph (of graph $G(V,E)$). A graph all of whose vertices are in the set V and all of whose edges are in the set E .

tree. A connected graph containing no cycles.

trivial graph. A graph containing one point and no edges.

vertex. A point in a graph.

walk. A succession of adjacent vertices.

APPENDIX B: APPLICATIONS OF SURFACE NETWORKS IN COMPUTER
CARTOGRAPHY

The surface networks described in Chapter 2 have a number of applications in computer cartography. For example, Boyell and Ruston (1963), Morse (1965), Freeman and Morse (1967), and Boehm (1967) have all used the contour tree to supply a "neighbourhood function" for a set of digitized contours, which are otherwise difficult to search among, since adjacency relations are not apparent from the contour coordinates alone. Boyell and Ruston (1963) were primarily concerned with the generation of profiles for radar simulation, while Morse and Freeman applied contour trees to the "profile search" problem, the object being to locate a given terrain profile (representing the ground track from aircraft radar) within a contour data-set. Boehm (1967) approached a related problem, namely that of determining inter-visibility for pairs of points on a terrain surface. All three applications have obvious military applications (and indeed were developed in military-supported research projects), but surface networks have, in general, been little used beyond this in computer cartography. This appendix investigates the application of surface networks to two important computer-cartographic problems: contouring, and surface generalization.

B.1: A TOPOLOGICAL APPROACH TO THE CONTOURING PROBLEM

The "contouring problem" may be stated as follows: given either (a) a set of irregularly-distributed points or (b) a regular grid, produce a contour map of the surface which those points represent. Case (a) has generally been approached by first interpolating a dense regular grid, and then using the contouring procedures developed for case (b). In fact, many papers with "contouring" in their titles have dealt only with interpolation and not with the contouring problem *per se* (for example, Pelto et al., 1968; Olea, 1972, 1974). This section, however, will discuss the actual contouring problem, and will assume a given Digital Terrain Model (DTM) which may be either a Triangulated Irregular Network (TIN) like the GDS system (Peucker and Chrisman, 1975), or a regular grid which has been divided into triangular planar facets by inserting either diagonals or additional points in the cell centres.

Batcha and Reese (1964) observed that there are two basic approaches to contouring: "plotting one contour at a time" (also known as contour tracking), and "plotting by grids or segments". They stated that the former method involves "severe" storage requirements and "complex" program language even when grid-based, but that there is a significant saving in plotter time when compared with the "segment" approach. In the latter

method, all contours between two adjacent columns of a grid are found and plotted, and this procedure is repeated for each successive pair of columns. If the only output required is a single contour map, this is a reasonable approach, although as noted above it requires more plotter time. For many purposes, however, the contours must be connected up into chains or loops. The "contour assembly" method, which assembles the output of the segment approach, has been used for regular grids (see Stanger, 1973), and for TIN's (Cochrane, in Peucker, 1973), where in this case contours are determined for each triangle and then assembled. The assembly phase of these procedures is clearly a sorting problem, with the average number of objects in each sort being the mean number of segments per contour level. (To attempt to avoid confusion, contour "level" will refer to all contours of a given height, whether connected or not, while contour "loop" will refer to a connected set of contour segments). The sorting can become very time consuming, and Cochrane (1974) later abandoned the assembly method in favour of a contour tracking approach.

The "standard" method employed in contour tracking programs is, for each contour level, to traverse all the edges connecting neighbouring points in the data-set, checking whether one end is above and the other below the contour being drawn. When this condition is met, the current edge intersects the contour, and

if the edge has not been "flagged", a starting point has been found. There are available good algorithms for tracking (or "following") a contour through either a regular grid or a TIN, given a starting point; these are generally based on the "reference-point - sub-point" method. During the tracking phase, every edge crossed by the contour is flagged, and the routine eventually either returns to the starting point or reaches the boundary of the data area. In the latter instance, one could track along the boundary until the contour entered the area again, and in this way eventually return to the starting point. More usually, however, upon reaching the boundary the routine returns to the starting point and tracks in the opposite direction to the boundary. The checking procedure then continues through the edges, and when all the edges have been checked and the contours found have been tracked, the contour level is complete. The flags are then reset to zero, and the procedure is repeated for each subsequent contour level required. This can become very time consuming, with a great deal of redundant checking, if many distinct contour levels are required, since each contour level will in general intersect only a small proportion of all edges.

Once a starting point has been found, there are good tracking procedures for both grids and TIN's; it is searching for starting points which may require large amounts of computer time. The contouring problem thus centres on finding

contour starting points and assuring that the contours through these points have not already been determined. Any method which finds a set of points, one and only one on each contour loop, should improve contouring speed for problems involving many contour levels. Such a method, based on the Pfaltz graph and surface tree (Chapter 2), is described below.

B.2: METHODS FOR DETERMINING SURFACE NETWORKS AND CONTOUR STARTS

In order to determine the Pfaltz graph of a DTM, each point must be checked to determine whether it is a pass. This is done by a simplified version of Greysukh's (1966) method for characterizing local surface form, which examines the profile of a concentric figure centred at the point in question. Next, one tracks up the two ridges and down the two courses connected to each pass, saving the chains of point numbers representing the ridges and courses, and building the Pfaltz Graph of the surface.

As already noted (Chapter 2), the surface tree of a closed surface can be uniquely determined from the Pfaltz Graph, together with the elevations of the surface specific points (and the assumption that at least all the passes have distinct elevations). The algorithm is relatively simple: First, each

peak is inserted into the tree as a single-node sub-tree. Next, the passes are added in order from highest to lowest, and each pass, as added, is connected to the lowest point in the two sub-trees to which its peaks belong. If both peaks are in the same sub-tree, the pass is actually a pale (see Warntz, 1966), and will only have one edge at this stage. When all the passes have been included, the pits are then added as single-point sub-trees. The algorithm reprocesses the passes from lowest to highest, and whenever a pass points to an unconnected pit, that connection is inserted, until all pits are connected. The surface tree is then complete.

If it is desired to obtain the contour tree for a particular set of contours, it is only necessary to determine points for the corresponding elevations on all edges in the surface tree which have one end above and the other below each successive contour level. If a coarser set of contour levels is used, some complex parts of the tree will be removed if they are not represented by the inserted nodes; these are features which are not represented by the new contours. Exact starting points for the contours can then be determined by reference first to the Pfaltz graph and then to the stored ridge and course chains.

As previously described (Chapter 2), the contour- and surface-tree concepts apply only to sets of closed contours. If

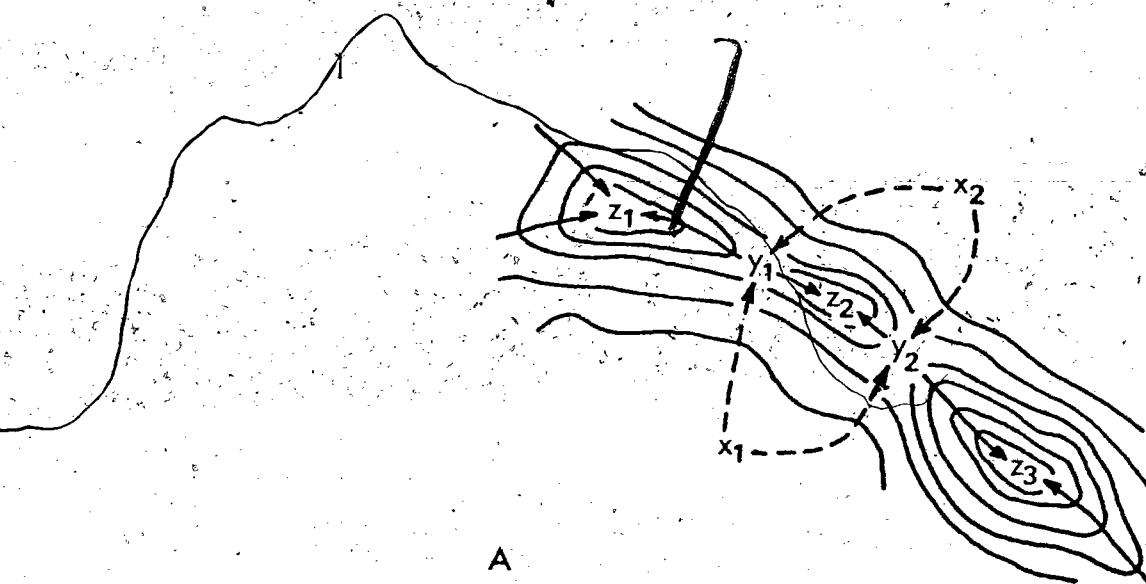
the data-area is arbitrarily-bounded, closure through the boundary must be assumed. Following Freeman and Morse (1967), contours are assumed to close uphill, i.e., around relative maxima, and the area outside the data-area is assumed to be lower and to contain one pit.

B.3: SURFACE NETWORKS AND CARTOGRAPHIC GENERALIZATION

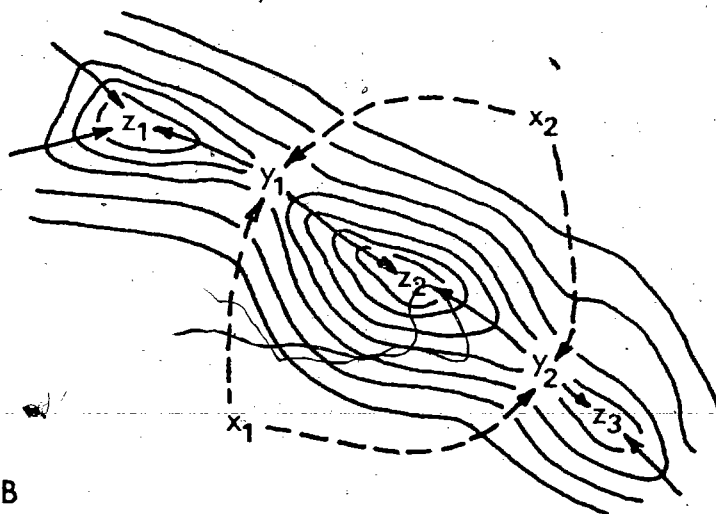
Generalization is a recurring theme in cartography, since any aerial photograph contains far more "information" than can, or indeed should, be portrayed on a map. Much of the literature on cartographic generalization has centred on the portrayal of irregular lines such as coasts and contours, and a good review of traditional (manual) approaches to this problem was given by Pannekoek (1962). More recently, the generalization of lines has been quantified (see Maling, 1963), and a "theory of the cartographic line" has been developed (Peucker, 1976). While these approaches have been rather successfully applied to coastlines (for example, see Douglas and Peucker, 1973), they face certain problems when applied to contours, since if each contour is generalized independently, the resulting lines may touch (see Douglas and Peucker, 1973, p. 119) or perhaps even cross. It would seem to be preferable to generalize the underlying surface first, and then produce contours based on this generalized version.

As Pannekoek (1962, p. 55) pointed out, generalization "consists of two different processes: a) selection of objects to be included in the map, the others being left out, and b) simplification of the shape to be given to the objects chosen for representation". Surface networks would seem to be valuable with respect to the former process.

In discussing applications of a particular type of surface network (which I have called the "Pfaltz graph"), Pfaltz (1976, p. 85) noted that while this graph contains only a very small amount of information about a surface, it may nevertheless contain too much. Pfaltz (p. 86) noted that "on any surface there are (sic) often a multitude of minor peaks, passes, and pits that are of little importance", and that it would be desirable to determine a "macrostructure" of the surface, suppressing (but not discarding) "those passes which are part of a local microstructure"; Pfaltz did not, however, define "importance". He illustrated a proposed graph-theoretic "contraction" procedure, whereby a "minor" pass was associated with a particular sub-graph topology in the Pfaltz graph (Figure B.1A). It would appear, however, that the Pfaltz graph topology alone does not contain sufficient information to indicate either the cartographic or geomorphic "importance" of critical points. This is indicated by the situation shown in Figure B.1B. Here, it is clearly pass y_2 and peak z_3 which should be



A



B

Figure B.1: A generalization procedure proposed by Pfaltz (1976) would delete pass y_1 and peak z_2 in each case; this would be appropriate in A (Pfaltz' example) but not in B.

deleted; the sub-Pfaltz graphs in Figure B.1 are, however, topologically identical, and Pfaltz's contraction procedure would delete z_2 and y_1 . Furthermore, other simple forms, such as the cirque-like structures in Pfaltz's (1976) Figures 11 to 13, are not suppressed by Pfaltz's contraction procedure.

As mentioned in Chapter 2, the surface tree can be plotted with the elevations of critical points to scale. It has already been proposed (Hormann, 1971) that the "importance" of a peak is best measured by its height above its highest adjacent pass (the relative height of a pit can be similarly defined). This information is contained in the "length" of the appropriate branch of the surface tree. Generalization based on the surface tree is conceptually simple: at each successive step, the peak or pit with the lowest relative height (shortest branch), together with its highest pass, is deleted, until no branches shorter than some threshold remain. This generalized surface tree may then be used to guide contouring or other display procedures, leaving out those peaks and pits whose "importance" was below the threshold. This process is illustrated in Figure B.2, using the surface from Figure 2.3. Alternatively, the planimetric area of the hilltop surrounding a peak could be used as the "size" in the above generalization procedure.

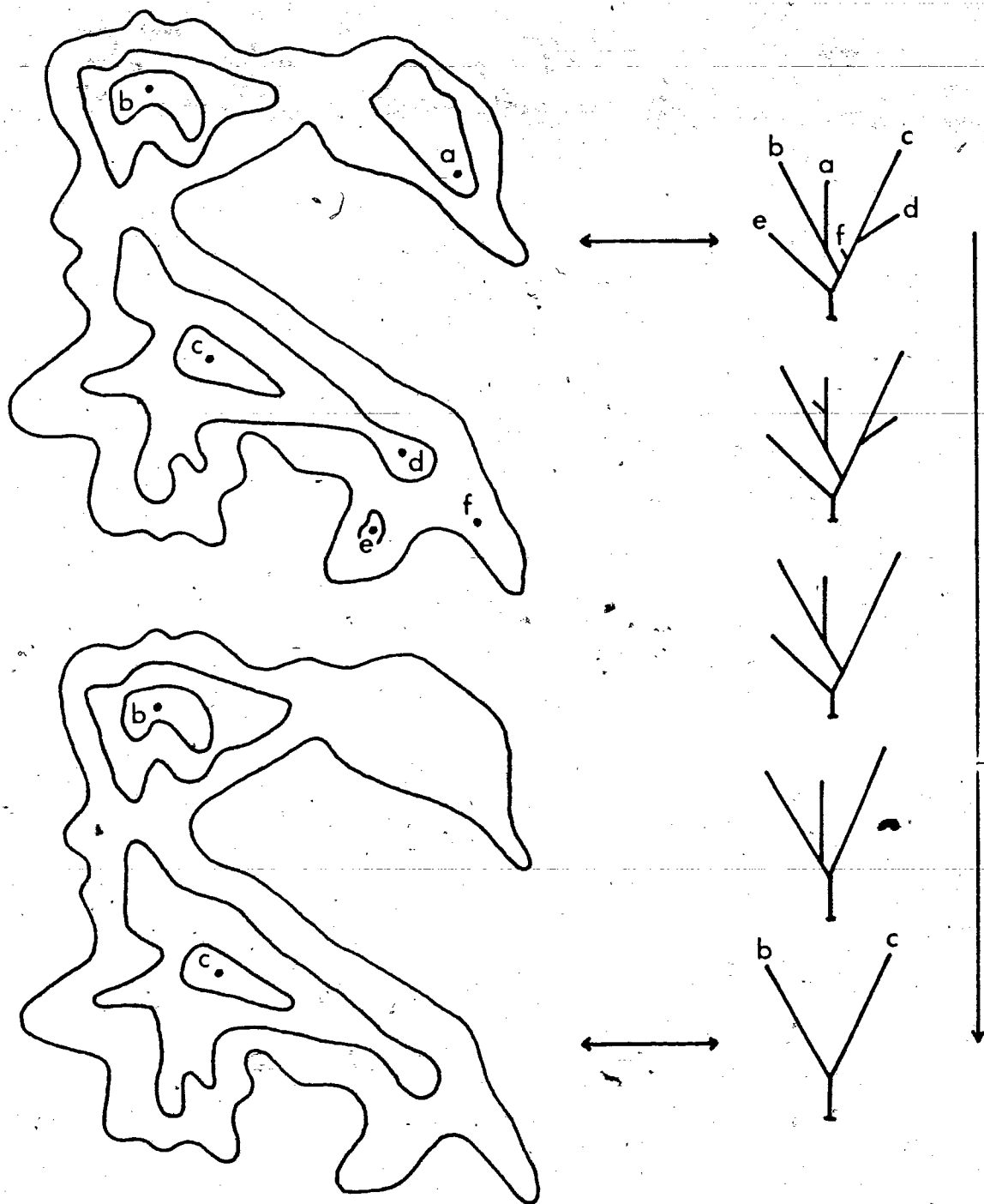


Figure B.2: Illustration of the proposed generalization procedure based on the surface tree.

It would appear that the surface tree can provide a better basis for the generalization of topographic surfaces than can Pfaltz's (1976) procedure.

B.4: CONCLUSIONS

Two of the more important problems in computer cartography, namely contouring and surface generalization, can be approached through surface networks. The solutions proposed have been developed to the degree that they would appear to be workable and of considerable potential, but have not yet been incorporated into fully operational programs. Furthermore, the generalization procedure outlined above must in general be followed by simplification of the forms of the remaining contours. Further research will involve writing of complete "production" programs, and empirical testing to determine optimal surface tree generalization thresholds.

APPENDIX C: DATA SOURCES AND COLLECTION PROCEDURES, ERROR
ESTIMATES, AND NORMALIZATION METHODS

C.1: DATA SOURCES AND COLLECTION PROCEDURES

All data used in this study were derived from medium scale (1:24,000 to 1:62,500) topographic contour maps. All map sheets used for the hilltop samples are listed in C.2 to C.7.

In all but the Kentucky sample, all suitable hilltops (see below) occurring entirely within any of the maps were sampled, but no attempt was made to detect hilltops which spanned the map sheet boundaries. In the Kentucky sample, map edges were compared so as to find any such additional hilltops. While map sheet boundaries were used in the other areas, the Kentucky sample area is bounded by major water courses. The area is roughly triangular, with the eastern and western boundaries formed by the Levisa and Tug Forks, respectively, of the Big Sandy River, and the southern base formed by, from west to east, John's Creek, Brushy Fork, Left Fork, John Young Branch, Middle Fork, Elkins Fork, and Big Creek.

In each study area, all ridge areas were examined, and sets of exactly six adjacent peaks, separated from all other peaks by passes lower than any of their five internal passes, were identified. If an area containing seven such peaks did not have

an acceptable magnitude-6 sub-set, and if at least one peak was represented by only a small single closed loop, the smallest such peak was deleted to produce a magnitude-6 hilltop. All hilltops were then classified into one of the six topologically distinct Pfaltz graph classes. Magnitude 4, 5, and 7 hilltops were later collected in the same way. Y

In the Kentucky, British Columbia, and middle California samples, within-hilltop pass height ranks were assigned to all passes, and the surface tree topological class was determined from these and the Pfaltz graph. Actual pass elevations were recorded for all passes in the Kentucky and British Columbia samples.

Morphometric measures were determined for all hilltops in the Kentucky sample. The elevation of the highest point was found, and from this and the lowest pass elevation, the local relief was calculated. The boundary of each hilltop was drawn as the self-crossing contour through the lowest pass. The radii of the largest inscribed and smallest circumscribed circles were measured to the nearest whole millimetre (representing 24 m), using dividers. Total length of ridges was determined using dividers set at 5 mm (120 m), and areas were estimated using a dot planimeter with a 5 by 5 mm (120 by 120 m) grid.

4 Coordinates of the centre of each hilltop were determined to the

nearest 100 m from the Universal Transverse Mercator grid
printed on the map margins.

C.2: MAP SHEETS (1:24,000 SCALE),
KENTUCKY SAMPLE

Adams	Paintsville
Inez	Prestonberg
Kermit	Richardson
Lancer	Thomas
Louisa	Varney
Milo	Webb
Naugatuck	Williamson
Offut	

C.3: MAP SHEETS (1:50,000
SCALE), BRITISH COLUMBIA
SAMPLE

82 F/3E	92 G/7W
82 F/8W	92 G/8E
82 F/11E	92 G/8W
82 F/15W	92 G/9W
82 G/1E	92 G/15
82 G/2W	92 H/2W
82 G/6W	92 H/3E
82 L/11E	92 H/3W
82 L/13W	92 H/4E
82 N/4E	92 H/4W
83 D/10	92 H/5E
92 F/7E	92 H/5W
92 G/1E	92 O/1E
92 G/1W	103 G/16W
92 G/7E	

C.4: MAP SHEETS (1:62,500 SCALE),
MIDDLE CALIFORNIA SAMPLE

Bryson
Cape San Martin
Junipero Serra
King City

C.5: MAP SHEETS (1:62,500
SCALE), NORTHERN
CALIFORNIA SAMPLE

Calistoga
Healsburg
Kelseyville
Lower Lake

C.6: MAP SHEETS (1:24,000 SCALE), NORTHERN
PENNSYLVANIA SAMPLE

Ayers Hill	Marshlands
Cherry Springs	Oleona
Conrad	Short Run
Galeton	Slate Run
Hammersley Fork	Tamarack
Lees Fire Tower	Young Woman's Creek

C.7: MAP SHEETS (1:24,000 SCALE),
SOUTHERN PENNSYLVANIA SAMPLE

Aughwick	McVeytown
Blain	Newberg
Blairs Mill	Newton Hamilton
Doylsburg	Shade Gap
McCoysville	

C.8: VARIABLES USED IN MULTIVARIATE ANALYSES

ZMAX (Highest elevation)

H (Relief)

A (Area)

SUML (Total length of ridges)

RC (radius of smallest circumscribed circle)

RI (radius of largest inscribed circle)

$D = \text{SUML} / A$

$S1 = \text{RC} / \text{RI}$

$S2 = \text{RC} / \text{Sqrt}(A)$

$S3 = \text{RI} / \text{Sqrt}(A)$

$\text{RD1} = \text{SUML} / \text{Sqrt}(A)$

$\text{RD2} = \text{SUML} / \text{RC}$

$\text{RD3} = \text{SUML} / \text{RI}$

$\text{R1} = \text{H} / \text{Sqrt}(A)$

$\text{R2} = \text{H} / \text{SUML}$

$\text{R3} = \text{H} / \text{RC}$

$\text{R4} = \text{H} / \text{RI}$

C.9: MEANS, STANDARD DEVIATIONS,
COEFFICIENTS OF VARIATION, AND
ESTIMATED ERRORS FOR THE KENTUCKY
MORPHOMETRIC VARIABLES

	Mean	s.d.	c.v.	est. error
ZMAX	0.391	0.041	0.105	0.000010
H	0.046	0.016	0.348	0.000039
A	0.266	0.155	0.583	0.000244
SUML	1.634	0.467	0.286	0.000937
RC	0.678	0.184	0.271	0.000037
RI	0.106	0.034	0.321	0.000037
D	7.250	2.683	0.370	-
S1	0.159	0.040	0.252	-
S2	0.743	0.122	0.164	-
S3	0.214	0.035	0.164	-
RD1	3.326	0.621	0.187	-
RD2	2.428	0.396	0.163	-
RD3	16.109	4.482	0.278	-
R1	0.094	0.025	0.266	-
R2	0.029	0.010	0.345	-
R3	0.069	0.020	0.290	-
R4	0.444	0.109	0.245	-

C.10: ERROR ESTIMATION

As recently noted (Mark and Church, 1977), it is important to report calculated, or at least estimated, errors for all geomorphic variables. Errors in area measurements were estimated using equation 36 of Frolov and Maling (1969, p. 33), while errors in the other planimetric variables were approximated by assuming that the resolution of the measurement represents a five per cent confidence limit (see Mark and Church, 1976). Relief and maximum elevation errors were

estimated in the manner proposed by Thompson and Davey (1953). The estimated error variances of the variables are given in C.9.

Variances of the logarithms of variables can be estimated by dividing the unlogged variances by the squared mean values. This relation was used to estimate the variances of the logarithms for all variables. On the assumption that errors in different variables are independent, the error variance of the logged ratio of two variables was assumed to equal the sum of the individual error variances of the logarithms. The ratio of the total variance to the "real" (total minus error) variance was computed for each variable and compared with the appropriate value of F; only three variables (all relative relief measures) had significant F-ratios, indicating that measurement errors do not constitute an important component of the variance of most of the variables.

ESTIMATIONS OF ERROR VARIANCES OF LOGGED
MORPHOMETRIC VARIABLES

	Total	Variance Error	"real"	F
ZMAX	0.011025	0.000063	0.010962	1.01
H	0.121104	0.018241	0.102863	1.18
A	0.339889	0.003451	0.336438	1.01
SUML	0.081796	0.000351	0.081445	1.00
RC	0.073441	0.000081	0.073360	1.00
RI	0.103041	0.003329	0.099713	1.03
D	0.136951	0.003802	0.133149	1.03
S1	0.063289	0.003410	0.059879	1.06
S2	0.026961	0.000944	0.026017	1.04
S3	0.026749	0.004192	0.022769	1.18
RD1	0.034861	0.001214	0.033647	1.04
RD2	0.026601	0.000432	0.026169	1.02
RD3	0.077412	0.003680	0.073732	1.05
R1	0.070733	0.019104	0.051629	1.37*
R2	0.118906	0.018592	0.100314	1.19
R3	0.084016	0.018322	0.065694	1.28*
R4	0.060268	0.021570	0.038698	1.56**

* significant, 5 per cent level

** significant, 1 per cent level

C.11: NORMALIZATION PROCEDURES

Most statistical procedures assume that all variables have normal distributions. C.11.1 reports the skewness and kurtosis of each variable before transformation; most of the variables depart significantly from the skewness of zero and kurtosis of three which characterize the normal distribution. While a simple logarithmic transformation improves the normality

of most variables, a linear transformation before taking logarithms can further improve the results (Patterson, 1974). C.11.2 indicates the optimum transformations following Patterson's (1974) approach; these transformed variables were used in the various statistical procedures outlined in Chapter 5, which were carried out using the SPSS package. The skewness and kurtosis values after transformation are also listed in C.11.1. Interestingly, the use of transformations did not notably influence the results, although statistical tests can be applied with more confidence.

C.11.1: SKEWNESS, KURTOSIS, AND TRANSFORMATIONS

	Untransformed		Simple Log		Optimum Transformation		
	sk	k	sk	k	sk	k	
ZMAX	0.47**	2.71	-	-	sqrt	0.35*	2.64*
H	0.98**	4.54**	-0.28*	3.87*	log	0.22	3.29
A	2.06**	9.78**	0.19	2.82	log	-0.18	2.87
SUML	0.95**	4.49**	0.08	2.91	log	-0.07	2.90
RC	1.35**	6.35**	0.41**	3.23	log	-0.03	3.07
RI	1.05**	4.50**	0.11	3.01	log	-0.05	3.11
D	1.03**	4.39**	0.01	2.81	log	0.01	2.81
S1	0.64**	3.71**	-0.18	3.23	log	-0.03	3.12
S2	0.10	3.13	-	-	none	0.10	3.13
S3	0.17	2.70	-	-	none	0.17	2.70
RD1	0.31*	3.06	-	-	none	0.31*	3.06
RD2	0.12	3.70*	-	-	none	0.12	3.70*
RD3	0.69**	3.58*	-0.04	2.68	log	0.08	2.68
R1	0.49**	3.59*	-0.51**	4.04**	log	0.09	3.29
R2	0.86**	3.73*	-0.14	3.23	log	0.14	2.91
R3	0.50**	3.21	-	-	sqrt	0.05	3.10
R4	0.37*	3.25	-0.44*	1.22**	log	-0.04	3.05

* significant, 5 per cent level
 ** significant, 1 per cent level

C.11.2: SPSS DATA TRANSFORMATIONS
EMPLOYED

ZMAX = SQRT(ZMAX)
H = LN((H-0.012)*28.1+1.0)
A = LN((A-0.072)*35.5+1.0)
SUML = LN((SUML-0.84)*1.71+1.0)
RC = LN((RC-0.36)*8.15+1.0)
RI = LN((RI-0.048)*29.4+1.0)
D = LN((D-2.851)*0.335+1.0)
S1 = LN((S1-0.067)*10.+1.0)
RD3 = LN((RD3-8.0)*0.089+1.0)
R1 = LN((R1-0.031)*12.+1.0)
R2 = LN((R2-0.008)*60.+1.0)
R3 = SQRT(R3)
R4 = LN((R4-0.169)*1.83+1.0)

S2, S3, RD1, RD2, X, and Y
were not transformed

APPENDIX D: MONTE CARLO SIMULATION RESULTS AS FUNCTIONS OF
THE ELONGATION PARAMETER, 5000 TRIALS

D.1: MAGNITUDE 4

	1.0	0.9	0.8	0.7	0.6	0.55	0.5
1	4045	4043	4051	4092	4174	4208	4260
2	955	957	949	908	826	792	740
	0.4	0.3	0.2	0.1	0.05	0.02	0.01
1	4394	4515	4702	4885	4971	4993	4997
2	606	485	298	115	29	7	3

D.2: MAGNITUDE 5

	1.0	0.9	0.8	0.7	0.6	0.55	0.5
1	3048	3041	3079	3141	3234	3270	3334
2	1913	1920	1878	1822	1741	1709	1647
3	39	39	43	37	25	21	19
	0.4	0.3	0.2	0.1	0.05	0.02	0.01
1	3549	3843	4281	4696	4895	4979	4993
2	1440	1142	718	304	105	21	7
3	11	5	1	0	0	0	0

D.3: MAGNITUDE 6

	1.0	0.9	0.8	0.7	0.6	0.55	0.5
1	2194	2186	2225	2263	2343	2398	2448
2	1200	1219	1230	1264	1282	1268	1280
3	1301	1276	1242	1192	1138	1117	1081
4	84	98	92	83	68	63	52
5	221	221	211	197	169	154	139
6	0	0	0	1	0	0	0
	0.4	0.3	0.2	0.1	0.05	0.02	0.01
1	2717	3087	3626	4389	4769	4938	4981
2	1201	1065	801	366	153	41	14
3	946	778	539	244	77	21	5
4	39	15	9	0	0	0	0
5	97	55	25	1	1	0	0
6	0	0	0	0	0	0	0

D.4: MAGNITUDE 7

	1.0	0.9	0.8	0.7	0.6	0.55	0.5
1	1589	1612	1614	1600	1658	1691	1750
2	721	715	726	800	850	902	941
3	1624	1605	1651	1653	1609	1579	1545
4	279	281	252	244	246	237	227
5	116	109	110	102	103	100	109
6	526	529	504	457	410	382	327
7	60	61	53	48	51	45	44
8	76	77	77	75	61	56	48
9	9	11	13	21	12	8	9
10	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0
	0.4	0.3	0.2	0.1	0.05	0.02	0.01
1	1946	2288	2974	4008	4627	4924	4976
2	968	942	771	411	165	30	11
3	1487	1379	1061	540	204	46	13
4	168	101	53	13	2	0	0
5	98	78	43	10	1	0	0
6	265	173	85	13	1	0	0
7	33	17	5	3	0	0	0
8	30	20	7	2	0	0	0
9	5	2	1	0	0	0	0
10	0	0	0	0	0	0	0
11	0	0	0	0	0	0	0

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