MICROWAVE FLUX-FLOW IMPEDANCE MEASUREMENTS OF TYPE-II SUPERCONDUCTORS

by

Zhou Xiaoqing

M. S., Wuhan Institute of Physics, Chinese Academy of Sciences, 2002

THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF Doctor of Philosophy IN THE DEPARTMENT OF PHYSICS

© Zhou Xiaoqing 2009
SIMON FRASER UNIVERSITY
Fall 2009

All rights reserved. However, in accordance with the Copyright Act of Canada, this work may be reproduced, without authorization, under the conditions for Fair Dealing. Therefore, limited reproduction of this work for the purposes of private study, research, criticism, review, and news reporting is likely to be in accordance with the law, particularly if cited appropriately.
<table>
<thead>
<tr>
<th><strong>Name:</strong></th>
<th>Zhou Xiaoqing</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Degree:</strong></td>
<td>Doctor of Philosophy</td>
</tr>
<tr>
<td><strong>Title of Thesis:</strong></td>
<td>Microwave flux-flow impedance measurements of Type-II superconductors</td>
</tr>
</tbody>
</table>
| **Examining Committee:** | Dr. Karen Kavanagh, Professor, Physics, 
Simon Fraser University (Chair) |

---

Dr. David Broun, Senior Supervisor, 
Associate Professor, Physics, 
Simon Fraser University

---

Dr. Jeff Sonier, Supervisor 
Professor, Physics, 
Simon Fraser University

---

Dr. Malcolm Kennett, Supervisor 
Assistant Professor, Physics, 
Simon Fraser University

---

Dr. J. Steven Dodge, Internal Examiner 
Associate Professor, Physics, 
Simon Fraser University

---

Dr. William Atkinson, External Examiner 
Professor, Physics, Trent University

| **Date Approved:** | October 13, 2009 |
Declaration of Partial Copyright Licence

The author, whose copyright is declared on the title page of this work, has granted to Simon Fraser University the right to lend this thesis, project or extended essay to users of the Simon Fraser University Library, and to make partial or single copies only for such users or in response to a request from the library of any other university, or other educational institution, on its own behalf or for one of its users.

The author has further granted permission to Simon Fraser University to keep or make a digital copy for use in its circulating collection (currently available to the public at the “Institutional Repository” link of the SFU Library website <www.lib.sfu.ca> at: <http://ir.lib.sfu.ca/handle/1892/112>) and, without changing the content, to translate the thesis/project or extended essays, if technically possible, to any medium or format for the purpose of preservation of the digital work.

The author has further agreed that permission for multiple copying of this work for scholarly purposes may be granted by either the author or the Dean of Graduate Studies.

It is understood that copying or publication of this work for financial gain shall not be allowed without the author’s written permission.

Permission for public performance, or limited permission for private scholarly use, of any multimedia materials forming part of this work, may have been granted by the author. This information may be found on the separately catalogued multimedia material and in the signed Partial Copyright Licence.

While licensing SFU to permit the above uses, the author retains copyright in the thesis, project or extended essays, including the right to change the work for subsequent purposes, including editing and publishing the work in whole or in part, and licensing other parties, as the author may desire.

The original Partial Copyright Licence attesting to these terms, and signed by this author, may be found in the original bound copy of this work, retained in the Simon Fraser University Archive.

Simon Fraser University Library
Burnaby, BC, Canada
Abstract

The electrodynamic properties of type-II superconductors have been studied in strong magnetic fields using microwave spectroscopy to access the flux-flow regime of vortex motion. The measurements give insight into two important physical quantities: flux-flow resistivity, which is intimately related to dissipation from electronic states near the vortex cores; and pinning forces, from the collective interactions of the vortex lattice with material defects.

A notable aspect of this work is the ability to accurately separate contributions from viscous and elastic forces. This has been made possible by the development of microwave apparatus that allows the separate measurement of in-phase and out-of-phase components of surface impedance, and through the use of well-controlled, high quality samples.

Measurements on conventional superconducting systems Nb, NbSe$_2$ and V$_3$Si have been used to test both the experimental technique and the subsequent data analysis, with the results in good accord with established data on these materials. In addition, some interesting new features have been observed, likely associated with the multiband nature of these systems.

The primary focus of this work is on the flux-flow dynamics of cuprate high temperature superconductors. Two material systems, YBa$_2$Cu$_3$O$_{6+x}$ and Tl$_2$Ba$_2$CuO$_{6+x}$, have been used to carry out measurements that span the entire superconducting region of the cuprate phase diagram. For each sample, the pinning force constant and flux-flow resistivity have been extracted across the superconducting temperature range. As has been previously reported by our group, there is an anomalous logarithmic upturn in flux-flow resistivity at low temperatures, previously attributed to localization physics. The main discovery of this work is that the logarithmic behaviour persists across the phase diagram, even in highly overdoped Tl$_2$Ba$_2$CuO$_{6+x}$ samples in which the normal state is known to be a metallic, conventional Fermi liquid. This suggests,
Abstract

for the first time, that the resistivity upturns in cuprates are in fact intimately connected to the presence of vortices, which in turn has strong implications for the nature of the underdoped cuprate normal state.
Dedication

to Zhen and Xuanzhi.
Acknowledgments

The first one I would like to thank is my supervisor Dr. David Broun. Things I learned from Dave can easily stand alone as chapters, or even a book. Here I just present an outline of what I want to thank Dave for:

First, he blows my mind with the beauty and fun of experimental physics. Before I met him I naively regarded theoretical physics to be superior and more creative; after working with him I just wish I could be as good an experimentalist as him some day.

Second, he sets a good model (not a model with parameters, mind you) for a tireless researcher and an enthusiastic teacher.

Last and most important, he keeps me happy during my struggle in the Ph.D program, as he promised the first day. For he is also a caring brother and a joyful friend, aside from his usual role.

I also want to thank my fellow gamer Wendell Huttema, who is always with me and helps me sort things out. I cannot remember how many times I borrowed Wendell’s brain and muscle in my game - what I remember is that they are very, very powerful. There is also Dr. Ben Morgan, whom I have never met in person but am so grateful to his presence in his well written programs and thesis.

Our sample makers deserve my thanks here. Dr. Ruixing Liang and Dr. Darren Peets, without their mysterious alchemy power we will just be out of business. Dr. Jeff Sonier and Dr. Chris Bidinosti provided us some good quality samples, too. There are also people who offered helping hands when we were trapped in an equipment loophole, Dr. Jeff Sonier and Dr. Jennifer Thewalt, for lending us their valuable superconducting magnet. Later on the experiments was carried out on a magnet borrowed from Dr. Walter Hardy and Dr. Doug Bonn. Their generosity is highly appreciated.

Aside from physics, I would like to express my gratitude to my friends here, for all
Acknowledgments

the good things they brought to me. In the lab there are Patrick Turner’s jokes, Nigel David’s tea, Ricky Chu’s friendlies, Christina Kaiser’s singing, Taras Chouinard’s soccer interest and Colin Truncik’s gift..... At home there are my roommate Wenjie Li and others.

I will save the last sentence for my family: my wife, my sister and my parents, for consistently supporting me pursuing what I want.
# Contents

Approval iii

Abstract iii

Dedication v

Acknowledgments vi

Contents viii

List of Figures xii

List of Tables xvii

## 1 Introduction

1.1 Superconductivity ........................................ 1

1.1.1 Two Fluid Model and the BCS Theory ................. 2

1.1.2 The London Theory ..................................... 3

1.1.3 The Ginzburg–Landau Theory, Vortices and the Mixed State . 5

1.2 Flux-flow Resistivity ...................................... 8

1.3 Pinning ..................................................... 12

1.4 Cuprate Superconductors ................................ 17

1.4.1 The Crystal Structure .................................. 17

1.4.2 The Superconducting State ............................ 19

1.4.3 The Normal State ...................................... 20

1.4.4 Fluctuating Superconductivity ....................... 22

1.4.5 The Competing Orders ............................... 25

1.4.6 The Ando–Boebinger Effect and the Metal-insulator Crossover 26
## CONTENTS

1.4.7 Flux-flow Dynamics in Cuprate Superconductors .................. 27
1.5 Previous Work by Our Group ....................................... 29

2 **Theoretical Background** ........................................... 32
   2.1 Experimental Observables ........................................ 33
   2.2 The Bardeen–Stephen Model ...................................... 35
      2.2.1 Limitations of the Bardeen–Stephen Model ................. 37
   2.3 The Rosenblum–Gittleman Model .................................. 40
   2.4 The Waldram Model ................................................. 41
   2.5 Other Models ........................................................ 46
      2.5.1 The Coffey–Clem Model ...................................... 46
      2.5.2 The Brandt Model .............................................. 48
      2.5.3 Summary of the Flux-flow Models ............................. 49
   2.6 Model Independent Interpretation of Our Measurements .......... 50

3 **Microwave Measurements** ........................................... 54
   3.1 The Cavity Perturbation Technique ............................... 54
   3.2 Structure of the Experimental Apparatus .......................... 57
   3.3 Upgrading of the Apparatus ....................................... 62
   3.4 COMSOL Simulation of the Resonant Modes ........................ 64
   3.5 Data Acquisition .................................................... 66
      3.5.1 Determination of Resonator Constant ....................... 68
      3.5.2 Determination of Absolute Surface Resistance ............. 69
      3.5.3 Determination of Absolute Surface Reactance ............. 70
   3.6 Calibrating the Apparatus Performance ........................... 71
      3.6.1 Stability against Signal Drift .............................. 71
      3.6.2 Signal Induced by Paramagnetic Impurities ................. 73
      3.6.3 Signal Induced by Thermal Expansion ....................... 78
      3.6.4 The $c$-axis Current Contribution ............................ 80
   3.7 Summary of the Data Analysis Procedures ........................ 82

4 **Measurements on Conventional Superconductors** ....................... 84
   4.1 Nb and Pb .......................................................... 84
      4.1.1 Zero Field Measurements ..................................... 86
      4.1.2 In-field Measurements ........................................ 89
**CONTENTS**

4.1.3 Depinning Frequency and Pinning Constant .......................... 91
4.1.4 Viscosity Coefficient and Flux-flow Resistivity ..................... 93

4.2 V$_3$Si ........................................ 94
4.2.1 Zero Field Measurements ........................................ 95
4.2.2 Mixed State Measurements ..................................... 98
4.2.3 Depinning Frequency and Pinning Constant ...................... 100
4.2.4 Viscosity Coefficient and Flux-flow Resistivity ............... 103

4.3 NbSe$_2$ ...................................... 105
4.3.1 Zero Field Measurements .................................... 106
4.3.2 Mixed State Measurements .................................. 108
4.3.3 Depinning Frequency and Pinning Constant ................... 109
4.3.4 Viscosity Coefficient and Flux-flow Resistivity .......... 112

4.4 Summary of Results on Conventional Superconductors ............... 115

5 Measurements on YBa$_2$Cu$_3$O$_{6+x}$ Samples .................. 117
5.1 Underdoped YBa$_2$CuO$_{6.333}$ ................................ 119
5.1.1 Measurements in Zero Field ................................ 120
5.1.2 Mixed State Measurements .................................. 121
5.1.3 Depinning Frequency and Pinning Constant .................. 123
5.1.4 Viscosity Coefficient and Flux-flow Resistivity ........ 125
5.2 Ortho-II YBa$_2$CuO$_{6.52}$ .................................. 127
5.2.1 Zero Field Measurements ................................... 128
5.2.2 Mixed State Measurements .................................. 131
5.2.3 Depinning Frequency and Pinning Constant .................. 133
5.2.4 Viscosity Coefficient and Flux-flow Resistivity ........ 136
5.3 Overdoped YBa$_2$CuO$_{6.993}$ .................................. 139
5.3.1 Zero Field Measurements ................................... 139
5.3.2 Mixed State Measurements .................................. 142
5.3.3 Depinning Frequency and Pinning Constant .................. 143
5.3.4 Viscosity Coefficient and Flux-flow Resistivity ........ 145
5.3.5 Comparison with Other Experiments .......................... 146
5.4 Summary of Observations on YBa$_2$Cu$_3$O$_{6+x}$ ............ 148

6 Measurements on Tl$_2$Ba$_2$CuO$_{6+x}$ Samples ............... 150
6.1 $T_c = 74$ K Tl$_2$Ba$_2$CuO$_{6+x}$ ................................. 153
## CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1.1</td>
<td>Surface Impedance</td>
<td>154</td>
</tr>
<tr>
<td>6.1.2</td>
<td>Depinning Frequency and Pinning Constant</td>
<td>156</td>
</tr>
<tr>
<td>6.1.3</td>
<td>Viscosity Coefficient and Flux-flow Resistivity</td>
<td>157</td>
</tr>
<tr>
<td>6.2</td>
<td>( T_c = 45 \text{ K} ) Tl(_2)Ba(<em>2)CuO(</em>{6+x})</td>
<td>158</td>
</tr>
<tr>
<td>6.2.1</td>
<td>Surface Impedance</td>
<td>158</td>
</tr>
<tr>
<td>6.2.2</td>
<td>Depinning Frequency and Pinning Constant</td>
<td>161</td>
</tr>
<tr>
<td>6.2.3</td>
<td>Viscosity Coefficient and Flux-flow Resistivity</td>
<td>162</td>
</tr>
<tr>
<td>6.3</td>
<td>( T_c = 24 \text{ K} ) Tl(_2)Ba(<em>2)CuO(</em>{6+x})</td>
<td>164</td>
</tr>
<tr>
<td>6.3.1</td>
<td>Surface Impedance</td>
<td>164</td>
</tr>
<tr>
<td>6.3.2</td>
<td>Depinning Frequency and Pinning Constant</td>
<td>167</td>
</tr>
<tr>
<td>6.3.3</td>
<td>Viscosity Coefficient and Flux-flow Resistivity</td>
<td>168</td>
</tr>
<tr>
<td>6.4</td>
<td>Summary of Observations on Tl(_2)Ba(<em>2)CuO(</em>{6+x})</td>
<td>169</td>
</tr>
</tbody>
</table>

### 7 Discussion of Flux-flow Resistivity

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.1</td>
<td>Global View of Flux-flow Resistivity</td>
<td>172</td>
</tr>
<tr>
<td>7.2</td>
<td>Viscosity Spectra for ortho-II YBa(_2)Cu(<em>3)O(</em>{6.52})</td>
<td>174</td>
</tr>
<tr>
<td>7.3</td>
<td>Possible Microscopic Origins</td>
<td>179</td>
</tr>
</tbody>
</table>

### 8 Discussion of Pinning

<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.1</td>
<td>Temperature Dependence of the Pinning Constant</td>
<td>180</td>
</tr>
<tr>
<td>8.2</td>
<td>Vortex Matter Phase Diagram</td>
<td>185</td>
</tr>
</tbody>
</table>

### 9 Conclusions

<table>
<thead>
<tr>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>187</td>
</tr>
</tbody>
</table>
List of Figures

1.1 Magnetic field and the order parameter profile near a vortex .................. 7
1.2 $B$-$T$ phase diagram and typical mixed state pattern of a superconductor 8
1.3 Simple sketch of flux-flow in a vortex unit cell ................................. 9
1.4 Empirical law of flux-flow resistivity ........................................... 11
1.5 The $V$-$I$ plot in the DC flux-flow experiment .................................. 13
1.6 The Rosenblum–Gittleman experiment .............................................. 14
1.7 Thermal creep in a random pinning potential ..................................... 15
1.8 Doping-temperature phase diagram for cuprates ................................. 16
1.9 The cuprate crystal structure for YBa$_2$Cu$_3$O$_{6+x}$ and Tl$_2$Ba$_2$CuO$_{6+x}$ 18
1.10 Vortex pancakes in the layered structure of the cuprate system ............. 18
1.11 The $d$-wave energy gap on a round Fermi surface ............................. 20
1.12 A schematic phase diagram of the fluctuating superconductivity scenario 22
1.13 Two phase fluctuation possibilities: spin wave and vortex-antivortex pair 23
1.14 The experimental scheme and typical Nernst data ............................... 24
1.15 A possible diagram for the competing order interpretation .................... 25
1.16 The Ando–Boebinger effect ............................................................. 27
1.17 The metal-insulator crossover ......................................................... 28
1.18 Possible mixed state phase diagram .................................................. 29
1.19 The pinning constant for optimally doped YBa$_2$Cu$_3$O$_{6.95}$ ................. 30
1.20 The flux-flow resistivity for optimally doped YBa$_2$Cu$_3$O$_{6.95}$ ............. 31

2.1 A sketch of the microwave response near the surface ......................... 34
2.2 A sketch of the electric field solution in the Bardeen–Stephen model .... 37
2.3 The free flux-flow limit and the pinning limit .................................... 42
2.4 Relaxation parameter and thermal creep factor .................................. 48
LIST OF FIGURES

2.5 The induced electric field distribution calculated from the order parameter of s-wave superconductor .......................... 51

3.1 Magnetic field lines with sample out of and sample in the cavity ... 55

3.2 Frequency shift and bandwidth shift before and after introducing the sample in the cavity ........................... 56

3.3 The structure of the microwave perturbation probe .......................... 58

3.4 A sample mounted at the end of the sapphire rod ............................. 61

3.5 The sketches of cavity configuration B and C ................................. 63

3.6 Simulated field profiles for the TE_{011} mode and the TE_{013} mode ... 64

3.7 Comparison of the simulated field profiles with the observed ones along the cavity axis ................................. 67

3.8 A typical power sweep data set .................................................. 70

3.9 Typical bandwidth drift and frequency drift for the TE_{011} mode ................................. 73

3.10 \( R_s(T) \) and \( X_s(T) \) with a bare sapphire rod in the cavity .................... 74

3.11 Comparison of the bare sapphire rod signal with the sample signal ................................. 75

3.12 Temperature sweep and field sweep of frequency shift for the sapphire rod ................................. 76

3.13 Field sweep data for the bare sapphire rod .................................. 77

3.14 Bandwidth and frequency shift data for the bare sapphire rod ................................. 79

3.15 Typical \( \sigma \)-axis surface impedance data ................................. 81

4.1 Photos of the Pb and Nb sample .................................................. 85

4.2 Zero field \( Z_s(T) \) of Pb and Nb .................................................. 86

4.3 \( Z_s(T) \) of Pb measured in higher order modes ................................ 87

4.4 Complex conductivity of Nb ...................................................... 88

4.5 In-field \( Z_s(T) \) of Nb ...................................................... 90

4.6 \( \rho_1(T) \) of Nb ...................................................... 91

4.7 \( f_p(T) \) of Nb ...................................................... 92

4.8 \( \alpha_p(T) \) of Nb ...................................................... 93

4.9 \( \eta_{ff}(T) \) and \( \rho_{ff}(T) \) of Nb ...................................................... 94

4.10 Photos of the V_3Si sample ...................................................... 95

4.11 Zero field \( Z_s(T) \) of V_3Si ...................................................... 96

4.12 The normalized superfluid density of V_3Si ................................ 97

4.13 Mixed state \( R_s(T) \) and \( \rho_1(T) \) of V_3Si ................................ 98
LIST OF FIGURES

4.14 Mixed state $X_s(T)$ and $\Delta X_s$ of V$_3$Si .................................. 99
4.15 $f_p(T)$ of V$_3$Si .................................. 100
4.16 Pinning constant of V$_3$Si .................................. 101
4.17 $\eta_{fl}(T)$ and $\rho_{ff}(T)$ of V$_3$Si .................................. 102
4.18 $H_{c2}(T)$ of V$_3$Si .................................. 103
4.19 $\rho_{ff}(B)$ of V$_3$Si .................................. 104
4.20 Photo of the NbSe$_2$ sample .................................. 105
4.21 $Z_s(T)$ of NbSe$_2$ in zero field .................................. 106
4.22 Normalized superfluid density of NbSe$_2$ .................................. 107
4.23 Mixed state $R_s(T)$ and $\rho_1(T)$ of NbSe$_2$ .................................. 108
4.24 Mixed state $X_s(T)$ and $\Delta X_s$ of NbSe$_2$ .................................. 109
4.25 $f_p(T)$ of NbSe$_2$ .................................. 110
4.26 $\alpha_p(T)$ of NbSe$_2$ .................................. 111
4.27 Phase diagram of NbSe$_2$ .................................. 112
4.28 $\alpha_p(B)$ of NbSe$_2$ .................................. 113
4.29 $\eta_{fl}(T)$ of NbSe$_2$ .................................. 114
4.30 $\rho_{ff}(T)$ and $B_{c2}(T)$ of NbSe$_2$ .................................. 115
4.31 $\rho_{ff}(B)$ of NbSe$_2$ .................................. 115

5.1 Oxygen ordered phases of YBa$_2$Cu$_3$O$_{6+x}$ .................................. 118
5.2 Normal state electrical resistivity of YBa$_2$Cu$_3$O$_{6+x}$ .................................. 119
5.3 Zero field $Z_s(T)$ of underdoped YBa$_2$Cu$_3$O$_{6.333}$ .................................. 120
5.4 Zero field $\sigma(T)$ of underdoped YBa$_2$Cu$_3$O$_{6.333}$ .................................. 121
5.5 Mixed state $R_s(T)$ and $\rho_1(T)$ of underdoped YBa$_2$Cu$_3$O$_{6.333}$ .................................. 122
5.6 Mixed state $X_s(T)$ and $\Delta X_s$ of underdoped YBa$_2$Cu$_3$O$_{6.333}$ .................................. 122
5.7 $f_p(T)$ and $\alpha_p(T)$ of underdoped YBa$_2$Cu$_3$O$_{6.333}$ .................................. 124
5.8 $\eta_{fl}(T)$ and $\rho_{ff}(T)$ of underdoped YBa$_2$Cu$_3$O$_{6.333}$ .................................. 126
5.9 $\rho_{ff}(B)$ of underdoped YBa$_2$Cu$_3$O$_{6.333}$ .................................. 126
5.10 Photos of the ortho-II YBa$_2$Cu$_3$O$_{6.52}$ sample .................................. 128
5.11 Zero field $Z_s(T)$ of ortho-II YBa$_2$Cu$_3$O$_{6.52}$ .................................. 129
5.12 Zero field $\sigma_1(T)$ of ortho-II YBa$_2$Cu$_3$O$_{6.52}$ .................................. 130
5.13 Zero field $\sigma_2(T)$ of ortho-II YBa$_2$Cu$_3$O$_{6.52}$ .................................. 131
5.14 Mixed state $Z_s(T)$ of ortho-II YBa$_2$Cu$_3$O$_{6.52}$ .................................. 132
5.15 $\Delta X_s(B,0)$ for ortho-II YBa$_2$Cu$_3$O$_{6.52}$ .................................. 132
5.16 $f_p(T)$ of ortho-II YBa$_2$Cu$_3$O$_{6.52}$ .................................. 133
<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.17</td>
<td>$\alpha_p(T)$ of ortho-II YBa$_2$Cu$<em>3$O$</em>{6.52}$</td>
</tr>
<tr>
<td>5.18</td>
<td>The relation between $T_0$ and $\omega$</td>
</tr>
<tr>
<td>5.19</td>
<td>$\eta_{fl}(T)$ of ortho-II YBCO</td>
</tr>
<tr>
<td>5.20</td>
<td>$\rho_{ff}(T)$ and $\rho_{ff}(B)$ of ortho-II YBa$_2$Cu$<em>3$O$</em>{6.52}$</td>
</tr>
<tr>
<td>5.21</td>
<td>Photo of the overdoped YBa$_2$Cu$<em>3$O$</em>{6.993}$ sample</td>
</tr>
<tr>
<td>5.22</td>
<td>Zero field $Z_s(T)$ of overdoped YBa$_2$Cu$<em>3$O$</em>{6.993}$</td>
</tr>
<tr>
<td>5.23</td>
<td>Zero field $\sigma_1(T)$ of overdoped YBa$_2$Cu$<em>3$O$</em>{6.993}$</td>
</tr>
<tr>
<td>5.24</td>
<td>Normalized superfluid density of overdoped YBa$_2$Cu$<em>3$O$</em>{6.993}$</td>
</tr>
<tr>
<td>5.25</td>
<td>Mixed state $Z_s(T)$ of overdoped YBa$_2$Cu$<em>3$O$</em>{6.993}$</td>
</tr>
<tr>
<td>5.26</td>
<td>$\Delta X_s(B, 0)$ of overdoped YBa$_2$Cu$<em>3$O$</em>{6.993}$</td>
</tr>
<tr>
<td>5.27</td>
<td>$f_p(T)$ of overdoped YBa$_2$Cu$<em>3$O$</em>{6.993}$</td>
</tr>
<tr>
<td>5.28</td>
<td>$\alpha_p(T)$ and $J_c(T)$ of overdoped YBa$_2$Cu$<em>3$O$</em>{6.993}$</td>
</tr>
<tr>
<td>5.29</td>
<td>$\eta_{fl}(T)$ and $\rho_{ff}(T)$ of overdoped YBa$_2$Cu$<em>3$O$</em>{6.993}$</td>
</tr>
<tr>
<td>5.30</td>
<td>Existing results on $\eta_{fl}(T)$ and $\alpha_p(T)$</td>
</tr>
<tr>
<td>5.31</td>
<td>$\rho_n(T)$ measured in a DC experiment</td>
</tr>
<tr>
<td>6.1</td>
<td>The amplitude of the energy gap for Tl$_2$Ba$<em>2$CuO$</em>{6+x}$</td>
</tr>
<tr>
<td>6.2</td>
<td>The quantum oscillation experiment on Tl$_2$Ba$<em>2$CuO$</em>{6+x}$</td>
</tr>
<tr>
<td>6.3</td>
<td>$\rho_n(T)$ of Tl$_2$Ba$<em>2$CuO$</em>{6+x}$</td>
</tr>
<tr>
<td>6.4</td>
<td>Zero field $\sigma_1(T)$ of Tl$_2$Ba$<em>2$CuO$</em>{6+x}$</td>
</tr>
<tr>
<td>6.5</td>
<td>Photos of Tl$_2$Ba$<em>2$CuO$</em>{6+x}$-sample 1</td>
</tr>
<tr>
<td>6.6</td>
<td>$Z_s(T)$ of Tl$_2$Ba$<em>2$CuO$</em>{6+x}$-sample 1</td>
</tr>
<tr>
<td>6.7</td>
<td>$f_p(T)$ and $\alpha_p(T)$ of Tl$_2$Ba$<em>2$CuO$</em>{6+x}$-sample 1</td>
</tr>
<tr>
<td>6.8</td>
<td>$\eta_{fl}(T)$ and $\rho_{ff}(T)$ of Tl$_2$Ba$<em>2$CuO$</em>{6+x}$-sample 1</td>
</tr>
<tr>
<td>6.9</td>
<td>Photo of Tl$_2$Ba$<em>2$CuO$</em>{6+x}$-sample 2</td>
</tr>
<tr>
<td>6.10</td>
<td>$Z_s(T)$ of Tl$_2$Ba$<em>2$CuO$</em>{6+x}$-sample 2</td>
</tr>
<tr>
<td>6.11</td>
<td>$\rho_1(T)$ for Tl$_2$Ba$<em>2$CuO$</em>{6+x}$-sample 2</td>
</tr>
<tr>
<td>6.12</td>
<td>$\Delta X_s(B, 0)$ of Tl$_2$Ba$<em>2$CuO$</em>{6+x}$-sample 2</td>
</tr>
<tr>
<td>6.13</td>
<td>$f_p(T)$ of Tl$_2$Ba$<em>2$CuO$</em>{6+x}$-sample 2</td>
</tr>
<tr>
<td>6.14</td>
<td>$\alpha_p(T)$ of Tl$_2$Ba$<em>2$CuO$</em>{6+x}$-sample 2</td>
</tr>
<tr>
<td>6.15</td>
<td>$\eta_{fl}(T)$ of Tl$_2$Ba$<em>2$CuO$</em>{6+x}$-sample 2</td>
</tr>
<tr>
<td>6.16</td>
<td>$\rho_{ff}(T)$ of Tl$_2$Ba$<em>2$CuO$</em>{6+x}$-sample 2</td>
</tr>
<tr>
<td>6.17</td>
<td>Photo of Tl$_2$Ba$<em>2$CuO$</em>{6+x}$-sample 3</td>
</tr>
<tr>
<td>6.18</td>
<td>$Z_s(T)$ of Tl$_2$Ba$<em>2$CuO$</em>{6+x}$-sample 3</td>
</tr>
<tr>
<td>6.19</td>
<td>$\rho_1(T)$ of Tl$_2$Ba$<em>2$CuO$</em>{6+x}$-sample 3</td>
</tr>
<tr>
<td>Figure</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>6.20</td>
<td>$\Delta X_s(B,0)$ of Tl$_2$Ba$<em>2$CuO$</em>{6+x}$-sample 3</td>
</tr>
<tr>
<td>6.21</td>
<td>$f_p(T)$ of Tl$_2$Ba$<em>2$CuO$</em>{6+x}$-sample 3</td>
</tr>
<tr>
<td>6.22</td>
<td>$\alpha_p(T)$ of Tl$_2$Ba$<em>2$CuO$</em>{6+x}$-sample 3</td>
</tr>
<tr>
<td>6.23</td>
<td>$\eta_{fl}(T)$ of Tl$_2$Ba$<em>2$CuO$</em>{6+x}$-sample 3</td>
</tr>
<tr>
<td>6.24</td>
<td>$\rho_{fl}(T)$ of Tl$_2$Ba$<em>2$CuO$</em>{6+x}$-sample 3</td>
</tr>
<tr>
<td>7.1</td>
<td>Global view of the flux-flow resistivity in cuprates</td>
</tr>
<tr>
<td>7.2</td>
<td>Zero field $\sigma_1(T)$ and mixed state $\eta_{fl}(B,T)$ of ortho-II YBa$_2$Cu$<em>3$O$</em>{6.52}$</td>
</tr>
<tr>
<td>7.3</td>
<td>Zero field conductivity spectra for ortho-II YBa$_2$Cu$<em>3$O$</em>{6.52}$</td>
</tr>
<tr>
<td>7.4</td>
<td>Examples of viscosity spectra</td>
</tr>
<tr>
<td>8.1</td>
<td>Global view of pinning constant</td>
</tr>
<tr>
<td>8.2</td>
<td>Global view of pinning constant in semi-log plots</td>
</tr>
<tr>
<td>8.3</td>
<td>The interpretation of the critical current density data on optimally doped YBa$_2$Cu$<em>3$O$</em>{6.95}$</td>
</tr>
<tr>
<td>8.4</td>
<td>Phase diagrams of ortho-II YBa$_2$Cu$<em>3$O$</em>{6.52}$</td>
</tr>
<tr>
<td>8.5</td>
<td>$B$-$T$ phase diagram of Tl$_2$Ba$<em>2$CuO$</em>{6+x}$-sample 2</td>
</tr>
</tbody>
</table>
List of Tables

3.1 Comparison between simulated resonator frequencies and observed resonator frequencies. ............................................. 65

4.1 Fit parameters for the functional form $\alpha_p(t) = \alpha_0 (1 - t)^\beta$ ........................................... 101
4.2 Fit parameters for the functional fit $\alpha_p(B) = \alpha_0 B^\beta$ ........................................... 102
Chapter 1

Introduction

In this work, microwave spectroscopy has been used to study the electrical transport properties of superconductors in applied magnetic fields. These properties are dominated by the dynamics of trapped magnetic flux inside the superconductor, which has attracted much research interest.

This chapter gives a brief introduction to the background of our work. Section 1.1 illustrates the basic ideas of superconductivity, with a focus on the concept of trapped flux lines. Section 1.2 presents the definition of the most important quantity acquired in this study, the flux-flow resistivity, while Section 1.3 describes another aspect of the dynamics, the pinning. Section 1.4 introduces the main material system on which this study was carried out, the cuprate superconductors. Section 1.5 introduces the work of my predecessor, Dr. Ben Morgan.

1.1 Superconductivity

Superconductivity is a macroscopic quantum phenomenon characterized by zero direct current (DC) electrical resistivity and exclusion of internal magnetic field (the Meissner effect[1]) below a phase transition temperature $T_c$. Originally discovered in 1911, it remains one of the most challenging subjects in modern physics[2]. Over the past 100 years it has been described in successively more complete detail by the generalized two fluid model, the London theory, the Ginzburg–Landau (GL) theory, the Bardeen–Cooper–Schrieffer (BCS) theory, and field theories of superconductivity[3]. These theoretical works, especially the BCS theory, have been very successful in describing conventional low temperature superconductors.
Introduction

What remain as open problems are unconventional superconductors, especially the cuprate high temperature superconductors. This work is intended to contribute to the understanding of these materials.

1.1.1 Two Fluid Model and the BCS Theory

A good starting point to explain the zero DC resistivity in superconductors is the generalized two fluid model[4]. The two fluid model proposes that two types of charge carrier fluid coexist in the superconductor: the normal fluid, which behaves approximately like regular conducting electrons, and the superfluid, which has no viscosity and carries no entropy. These two fluids conduct in parallel, so the DC electrical resistivity is short-circuited by the superfluid.

This idea of the superfluid current is partially inherited by the BCS theory[5], which gives a microscopic description of the superfluid. Starting from a Fermi sea, the BCS theory proposes that due to electron–phonon–electron coupling, below $T_c$ there is a weak attractive interaction that pairs two electrons with opposite spin and momentum (a Cooper pair). These Cooper pairs behave like bosons and can undergo Bose–Einstein–Condensation (BEC) into the same quantum state. At zero temperature, the condensation is energetically favorable as it opens up a zero temperature BCS energy gap $\Delta_k(T = 0 \text{ K})$ at the Fermi surface. In this case the system is approximately in the BCS ground state

$$|\Psi_G\rangle = \prod (u_k + v_k e^{i\theta} c^*_{-k_1} c_{k_1}) |0\rangle,$$

(1.1)

where $u_k$ and $v_k$ are occupation amplitudes which satisfy $|u_k|^2 + |v_k|^2 = 1$, $c_{k_1}^*$ and $c_{-k_1}^*$ are the electron creation operators, and \( \theta \) is the phase. The supercurrent can be understood as a Cooper pair condensation with a finite center-of-mass momentum. At finite temperature, thermal excitation breaks some of the pairs to form fermionic quasiparticles (normal fluid), and the excitation energy spectrum follows

$$E_k^2 = \epsilon_k^2 + \Delta_k^2,$$

(1.2)

where $\epsilon_k$ is the energy relative to the normal state Fermi surface (equivalently, the maximum energy of a quasiparticle in the ground state of the normal state system).

It needs to be stressed that this scenario has a major difference to the classical BEC systems. In the superconductor the condensation happens cooperatively, as the size of the energy gap $\Delta_k(T)$ depends on how many pairs have already condensed. At
finite temperature, the energy gap shrinks as more pairs are broken. The gap is closed at the transition temperature $T_c$, which depends on the size of the zero temperature energy gap

$$\Delta_0 \propto k_B T_c.$$  \hspace{1cm} (1.3)

Above $T_c$ there is no pairing potential so the Cooper pairs are unstable and superconductivity is lost. On the other hand, below $T_c$ the energy gap ensures the existence of a nonzero number of the Cooper pairs. As the center-of-mass motions for all the pairs should be same (BEC), the many-body wave function for the superfluid $\Psi$ can be written as a coherent state for bosons:

$$\Psi = \sqrt{n_p} e^{i\theta},$$  \hspace{1cm} (1.4)

where $n_p$ is the effective number of pairs, and $\theta$ is the phase. It needs to be emphasized that $n_p$ is just an expectation value at which the number of pairs peaks. The exact number of pairs is not well defined, as in a coherent state the number operator $\hat{n}$ and the phase operator $\hat{\theta}$ do not commute (so $\Delta n \Delta \theta \approx 1$). On the contrary, the phase usually is well defined, as a macroscopic superfluid requires long range phase order. The expression of the superfluid current $\vec{J}_s$ in zero field can be derived from the complex wave function $\Psi$:

$$\vec{J}_s = \frac{ie\hbar}{2m_e} (\Psi^* \vec{\nabla} \Psi - \Psi \vec{\nabla} \Psi^*) = -\frac{n_p e \hbar}{m_e} \vec{\nabla} \theta = 2en_p \vec{v}_s,$$

where the phase $\theta$ determines the velocity $\vec{v}_s$ of the superfluid while $n_p$ is called the superfluid density. In the presence of a magnetic field (vector potential $\vec{A}$), the supercurrent is modified as

$$\vec{J}_s = -\frac{n_p e \hbar}{m_e} \vec{\nabla} \theta - \frac{2e^2 n_p}{m_e} \vec{A}.$$  \hspace{1cm} (1.6)

### 1.1.2 The London Theory

The supercurrent density is related to the electromagnetic field distribution by the London theory[6], which provides a good explanation for the Meissner effect. Starting from the electric potential $\phi$ and vector potential $\vec{A}$, the gauge invariant expressions for the electric field and the magnetic field are

$$\vec{E} = -\partial \vec{A} / \partial t - \vec{\nabla} \phi,$$  \hspace{1cm} (1.7)

$$\vec{B} = \vec{\nabla} \times \vec{A}.$$  \hspace{1cm} (1.8)
Introduction

Inside a solid, it is the gradient of the electrochemical potential \( \mu \) that drives the electrons, and the effective field \( E_{\text{eff}}^\tau \) becomes

\[
E_{\text{eff}}^\tau = -\partial \vec{A}/\partial t - \vec{\nabla} \mu/e. \tag{1.9}
\]

As for the supercurrent density, it can be obtained from Equation 1.4 and Equation 1.6:

\[
\Lambda \vec{J}_s = -\frac{\hbar}{2e} \nabla \theta + \vec{A}, \tag{1.10}
\]

which includes the London parameter \( \Lambda = m_e/2n_pe^2 \). Since \( \hbar \nabla \theta \) is the local pair momentum, we have

\[
\hbar \partial \theta /\partial t = -2\mu. \tag{1.11}
\]

The effective electric field \( \vec{E}_{\text{eff}}^\tau \) can be related to \( \vec{J}_s \) by the first London equation, which is obtained as the time derivative of Equation 1.10:

\[
\frac{\partial (\Lambda \vec{J}_s)}{\partial t} = -\partial \vec{A}/\partial t + \vec{\nabla} \mu/e = E_{\text{eff}}^\tau, \tag{1.12}
\]

which is an acceleration equation. The curl of Equation 1.10 gives the second London equation:

\[
\nabla \times (\Lambda \vec{J}_s) = -\vec{B}. \tag{1.13}
\]

Combined with the Ampere's law, this equation can be rewritten as

\[
\nabla^2 \vec{B} = \frac{\vec{B}}{\lambda_L^2}, \tag{1.14}
\]

where \( \lambda_L \) is the London penetration depth with \( \lambda_L = \sqrt{\frac{\Lambda}{\mu_0}} \). This quantity describes the characteristic length that a magnetic field penetrates into the superconductor, which depends on the superfluid density of the screening current. Usually \( \lambda_L \) is much smaller than the dimensions of the superconductor, so effectively the magnetic flux lines are expelled – this is the Meissner effect.

What has been studied in this work is a more intriguing case, in which the flux lines are trapped in the superconductor. Deep inside the superconductor there should be no screening supercurrent and no accelerating field. So, for a flux line, \( \Phi \) is conserved inside the superconductor, as it satisfies

\[
-\frac{d\Phi}{dt} = 0 = \oint \vec{E}_{\text{eff}}^\tau \cdot dl, \tag{1.15}
\]
where the integration is taken along a loop that encircles the flux line. Since $\theta$ is
the phase angle, it has to be changed by a multiple of $2\pi$ as one moves along this
loop. This results in the trapped flux line being quantized. For a loop deep inside
the superconductor where $\mathcal{J}_s = 0$, the line integral for Equation 1.10 gives
\[
\oint \mathbf{A} \cdot d\mathbf{l} = \Phi = \frac{\hbar}{2e}2\pi n,
\]
where $n$ is an integer. Due to the resemblance of the quantized flux line and its
screening current to a vortex in the superfluid $^4\text{He}$, the flux line is usually referred as
"vortex" in superconductivity literature. Similarly, an "antivortex" can be defined as
a "vortex" with reverse phase winding. Vortex physics is an important branch in the
study of superconductors, to which our work belongs.

1.1.3 The Ginzburg–Landau Theory, Vortices and the Mixed
State

Another important theory for superconductivity is the Ginzburg–Landau theory\[7\],
on which the earliest prediction of vortices in superconductors is based\[8\]. Devel­
oped before the microscopic BCS theory, the Ginzburg–Landau theory focuses on the
phenomenology of the phase transition in general. In this theory, physical properties
are governed by an abstract complex quantity called the order parameter $\Psi$, which in
superconductivity corresponds to the BCS wave function with $\Psi^*\Psi = n_p$. The second
order phase transition from a normal conductor to a superconductor is the result of
minimizing the Ginzburg–Landau free energy, which has the following form:
\[
F = F_n + \int_V [\alpha(T)\Psi^*\Psi + \frac{1}{2}\beta(T)(\Psi^*\Psi)^2 + \epsilon|\nabla\Psi - \frac{2e}{i\hbar}\mathbf{A}\Psi|^2]dV + \int_V \frac{1}{2\mu_0}B^2dV, \quad (1.17)
\]
where $\alpha$ and $\beta$ are parameters, $F_n$ and $\epsilon$ are constants, and $B$ is the magnetic field.
It can also be rewritten as
\[
F = F_n + \int_V \frac{1}{2\mu_0}B^2dV + \int_V [\alpha(T)n_p + \frac{1}{2}\beta(T)n_p^2]dV + \int_V \epsilon n_p|\nabla\theta - \frac{2e}{i\hbar}\mathbf{A}|^2dV, \quad (1.18)
\]
in an equilibrium (time independent) case with uniform amplitude of $\Psi$. In this form,
the first integral is the magnetic field self-energy, the second integral is determined by
the amplitude of the order parameter, and the third integral is related to the phase
of the order parameter. Clearly, $n_p$ determines the energy cost of imposing a phase
gradient and is often referred as the phase stiffness, a quantity that measures the
superfluid robustness against phase fluctuations. As $n_p$ decreases, phase fluctuations become more important.

The Ginzburg-Landau theory contains two important characteristic length scales for a vortex. The first is the penetration depth:

$$\lambda = \sqrt{m\beta/4\mu_0e^2|\alpha|}, \quad (1.19)$$

which reproduces the London expression for the penetration depth as the Ginzburg-Landau theory has $n_p = -\alpha/\beta$.

Another quantity is the coherence length $\xi$:

$$\xi = \sqrt{\hbar^2/2m|\alpha|}, \quad (1.20)$$

which is the characteristic length scale of spatial changes in $\Psi$. This length scale is missing in the London theory, and is interpreted as the size of the Cooper pair in the BCS theory. It plays an important role in the description of the vortex, as the order parameter $\Psi$ or equivalently the energy gap $\Delta$ would be suppressed over a distance of $\xi$ near the flux line. The vortex core can be approximately treated as “normal” within a radius of the order of $\xi$. For conventional superconductors, the vortex “core” can be regarded conceptually as a textbook quantum well with dimension $\xi$ and potential $\Delta$, which traps the quasiparticle excitations in bound states. The energy level spacing $\epsilon_0$ of these bound states depends on $\xi$:

$$\epsilon_0 \propto \mu\Delta/k_F\xi, \quad (1.21)$$

where $\mu$ is the angular momentum quantum number and $k_F$ is the Fermi vector. Since in conventional superconductors $\epsilon_0$ is much smaller than the thermal energy $k_BT$, the excitation spectrum can be regarded as continuous, and the scattering process with an impurity is similar to that of a normal metal. There is, however, a subtle difference to the quantum well analogy: as an electron-like excitation approaches the vortex boundary, it can be reflected back as a hole and contribute $2e$ to the superfluid outside the core, or vice versa. This is the famous Andreev reflection[3], which allows the bound states to couple with the superfluid and carry a supercurrent.

The screening supercurrents flow around the normal core up to a characteristic length $\lambda$, which screens the magnetic field distribution $h(r)$, as shown in Figure 1.1.

A vortex takes energy to form. As a magnetic flux penetrates into the superconductor, there is a reduction in the field self energy $\int_V \frac{1}{2\mu_0} B^2 dV$ due to screening,
which is proportional to the penetration depth \( \lambda \). At the same time the closing of the energy gap inside the vortex core results in an energy increase, which is proportional to the coherence length \( \xi \). The net energy can be regarded as a boundary energy between the “normal” core and the superconducting background. The sign of the boundary energy depends on the ratio of these two characteristic lengths, leading to the Ginzburg–Landau parameter \( \kappa \):

\[
\kappa = \frac{\lambda}{\xi}.
\]  

When \( \kappa < 1/\sqrt{2} \) the boundary energy is positive, so flux lines will always be excluded from the bulk of the superconductor. This is true for many pure elements like mercury, and since they were discovered first they were defined to be the type-I superconductors. In contrast, when \( \kappa > 1/\sqrt{2} \) the boundary energy becomes negative, so it would be energetically favorable for the superconductor to contain internal flux lines. Such a system is defined to be a type-II superconductor, for which a typical \( B-T \) phase diagram is presented in Figure 1.2. Below the lower critical field \( B_{c1} \), flux lines are excluded from the superconductor, and the system is in the Meissner state. As the magnetic field becomes higher than \( B_{c1} \), vortex “normal” cores can be trapped inside the superconductor. To minimize the boundary energy, vortices are usually divided into single flux quanta and separated by the superconducting background. Such a state is called the mixed state (see Figure 1.2), in which the majority of our experiments have been carried out. At stronger magnetic fields, the intervortex spacing becomes shorter and shorter, and the vortex cores eventually
Introduction

overlap each other as the field reaches the upper critical field $B_{c2}$. Superconductivity is then suppressed globally and the system reaches the normal state. In conventional superconductors, this means no vortices exist above $B_{c2}$.

The electromagnetic response from vortices in the mixed state has great importance. On the practical side, superconducting devices like magnets or electrical cables may operate in the mixed state, and their performance will be limited by the presence of vortices. On the fundamental side, the mixed state can be used as a tool to study superconductivity[10], which is the approach taken in this work. Specifically, we present accurate measurements of two aspects involved in vortex dynamics: friction and pinning.

1.2 Flux-flow Resistivity

Our work focuses on the low field region of the mixed state, where the vortices are well spaced. As a starting point vortices are assumed to be independent of each other. For simplicity, an isolated vortex is treated as a rigid object and discussions are limited
Introduction

to 2D. It will be shown later that such treatments are good approximations in our work.

Let us start with the first aspect of the vortex dynamics, the frictional response. If a transport current with current density \( \mathbf{J} \) flows through a superconductor, it will apply a Lorentz force per unit length \( \mathbf{F} \) to the flux line (see Figure 1.3):

\[
\mathbf{F} = \Phi_0 \times \mathbf{J}.
\] (1.23)

Figure 1.3: Simple sketch of flux-flow in a unit cell. The vortex core is represented by the circle (red) in the superconducting background (blue), \( J_R \) is the screening current flowing around the vortex, \( J_T \) is the transport current, and \( v_R \) is the vortex velocity.

If there is no other force to hold the vortex in position, or if such force has been overcome, the flux line is free to "flow". For the frictional response, a steady mechanical solution can be established:

\[
\mathbf{F} = \eta \mathbf{v}.
\] (1.24)

where \( \eta \) is defined as the viscosity per unit length, and \( \mathbf{v} \) is the vortex velocity. From an electrodynamic point of view, a moving flux line induces an electric field which counters the flux motion and creates viscosity. The frictional response can be understood as the energy dissipated by this induced electrical field. B. D. Josephson[11] proved that the average effective induced fields for the vortices would be

\[
\mathbf{E} = \mathbf{B} \times \mathbf{v}.
\] (1.25)
In the case $\vec{J} \parallel \vec{E}$, an electrical resistivity can be defined:

$$\rho_{ff} \vec{J} = \vec{E},$$

(1.26)

where $\rho_{ff}$ is called flux-flow resistivity, and is the most important quantity extracted from our measurements.

For conventional superconductors, microwave measurements of the flux-flow response have provided powerful insights. An early microwave experiment of the flux-flow response on Nb$_3$Sn taken by B. Rosenblum and M. Cardona[12] in 1964 had revealed a correlation between the mixed state response and the normal state response:

$$R_m(T)/R_n = B/B_{c2}(T),$$

(1.27)

where $R$ is the microwave surface resistance, which will be defined in Chapter 2. Subscripts $m$ and $n$ correspond to the mixed state and the normal state respectively. This correlation suggested the existence of a normal region inside the mixed state superconductor, which consists of the vortex cores.

During the same period of time, an alternative method of moving the vortices[13] was developed – the application of a strong DC current through the superconductor. The flux-flow resistivity $\rho_{ff}$ was directly measured, and from its comparison to the normal state resistivity $\rho_n$ (see Figure 1.4) a similar empirical law was established by Y. B. Kim et al.[13]:

$$\frac{\rho_{ff}}{\rho_n} = f\left(\frac{T}{T_c}\right) \frac{B}{B_{c2}},$$

(1.28)

where $f\left(\frac{T}{T_c}\right)$ is a weakly temperature dependent scaling factor. Empirically it is often reduced to the temperature independent unity, because $\rho_{ff} \rightarrow \rho_n$ is a constraint when $B \rightarrow B_{c2}$. For most conventional superconductors, this approximation is valid.

Equation 1.28 is usually referred as the Bardeen-Stephen law, as it was well explained by a microscopic model developed by J. Bardeen and M. J. Stephen[15] in 1965. The microwave measurements mentioned above can also be understood in the context of this model, as the flux-flow resistivity happens to dominate the surface resistance signal.

In the Bardeen-Stephen model, the flux-flow resistivity originates from scattering of the charge carriers inside the vortex core, where the physics is assumed to be similar to that of the normal state. As a result, the flux-flow resistivity can be understood as a “fractional normal resistivity”. This law was quantitatively confirmed.
Figure 1.4: Empirical law of flux-flow resistivity [14]. At low temperature, a linear field dependence is shown in the ratio $\rho_H / \rho_n$. The departure from the linear field dependence at high temperature is predominantly due to the temperature dependence of $B_{c2}$.

in other conventional superconductors [14, 16, 17], and has been cited by most flux-flow studies. It builds a connection between the normal state properties and the flux-flow measurements, which is crucial for our study. This model will be discussed in more detail in Chapter 2.

However, it remains controversial whether the Bardeen-Stephen model can be applied to cuprate superconductors. A few theoretical attempts have been made to extend the Bardeen-Stephen model beyond its limitations [18–22], or to take a different approach for the vortex electrodynamic response, but in general they have not been completely established by experiments. On the other hand, measurements on cuprate superconductors have shown deviations from the linear field dependence of the flux-flow resistivity, and agreement has not been reached on how the field dependence in the Bardeen-Stephen law should be modified [23–25].

Aside from the flux-flow resistivity, there are other effects involved in the flux-flow picture. The first is the Hall effect. Given another electric field component $E_\perp$ that is perpendicular to the transport current $J_\parallel$, the Hall coefficient can be defined:

$$R_H = \frac{E_\perp}{J_\parallel B}.$$  \hspace{1cm} (1.29)
Introduction

Compared with the flux-flow resistivity, the Hall effect contribution is usually 100 times smaller[26]. Although it also attracts much attention, in our measurements it only plays a minor role and will be neglected.

There is also the magneto-thermoelectric effect in the flux-flow picture. Since the flux line core carries entropy $s$ per unit length, in the presence of a thermal gradient $\nabla T$ it will feel an additional force per unit length:

$$F_i = -ns^*\nabla T.$$  (1.30)

This effect has been studied in the Nernst effect experiments[27], which will be introduced in more detail in Section 1.4. In our experiment, the thermal gradients are vanishingly small in the sample. In addition, there is no thermal gradient that oscillates at the microwave frequency. As a result, this effect will also be ignored in our study.

1.3 Pinning

In a realistic superconductor there is always pinning, and it comprises the other half of the flux-flow response. This results in flux lines being “pinned” in equilibrium positions, allowing current to be transported without incurring energy dissipation from the flux-flow. By definition, equilibrium position means the local minimum of free energy for the vortex.

For simplicity, let us start with an isolated vortex. Pinning is often described as a harmonic potential which contains the vortex, and for small displacement the pinning force can be regarded as a linear restoring force:

$$\vec{F}_p = \alpha_p \vec{x},$$  (1.31)

where $\alpha_p$ is defined as the pinning constant, and $\vec{x}$ is the displacement of the vortex to its equilibrium position.

$\alpha_p$ is important as it determines the threshold values for a superconductor to conduct current without dissipation. In the DC transport case it relates the critical current density $\vec{J}_c$ to the maximum displacement $\vec{x}_0$:

$$\vec{J}_c \times \vec{\Phi}_0 = \alpha_p \vec{x}_0.$$  (1.32)

Below the critical current density a vortex is merely shifted slightly to a different equilibrium position $x$ so there is little dissipation, while above the critical current...
Figure 1.5: The \( V-I \) plot in the DC flux-flow experiment\[13\]. A constant resistance \( dV/dI \) can be observed above a threshold value of current \( I \).

density \( J_c \), the vortex can move out of the pinning potential and be "depinned", resulting in dissipation from the flux flow. This behavior has been observed in DC flux-flow resistivity measurements, as shown in Figure 1.5. If \( x_0 \) is treated as a constant then \( \alpha_p \) and \( J_c \) should have the same temperature and field dependence.

In microwave experiments the transport current is usually very small so the critical current density is less important; instead, that role is replaced by another parameter called depinning frequency. It was first observed by B. Rosenblum and A. I. Gittleman\[28\] in 1965, as shown in Figure 1.6. At low frequency there is almost no dissipation, but after passing through a crossover frequency \( f_0 \), the dissipation jumps to a constant dissipation at high frequency. They developed a simple model to interpret their data, which defined the depinning frequency as the ratio between the pinning constant and the viscosity coefficient:

\[
f_p = \frac{\alpha_p}{\eta}.
\] (1.33)

The depinning frequency contains equivalent information to the pinning constant once \( \eta \) is known. This model has great importance to our study and will be discussed in more detail in Chapter 2.

Let us now consider an entire piece of vortex matter instead of an isolated vortex. In this case, \( \alpha_p \) or \( f_p \) represent the average pinning felt by the vortex matter, and there are three factors that need to be considered.

The first factor is the repulsive interaction between the flux lines, which has the
Figure 1.6: Power dissipation as a function of frequency[28]. $f_0$ is the depinning frequency.

tendency to maintain a translationally invariant vortex lattice (Abrikosov lattice), as suggested by Abrikosov[8]. In analogy to a mechanical system that can sustain shear force, this crystal-like structure allows a pinning force or a driving force to act on the entire vortex matter instead of a single vortex. The second factor is the presence of local material defects, which are called “pinning sites”. While the driving forces and the repulsive interaction are rather homogeneous in this work, the locations and pinning strengths for the pinning sites are usually randomly distributed. In many cases the repulsive interactions are much stronger than the strength of the pinning sites, and one may expect the random distribution of the pinning sites to result in zero net pinning. However, this contradicts the real case, in which there is always a finite net pinning.

Regarding the competition between these two factors, A. I. Larkin and Y. N. Ovchinnikov developed a collective pinning theory in 1979[29], which is conceptually similar to mechanical friction[30]. The vortex matter behaves predominantly like a rigid 2D crystal, but each vortex can be shifted slightly off its equilibrium position by the local pinning force. This configuration corresponds to a deformation of the vortex matter, which destroys the long range translational order but allows many pinning sites to act collectively rather than competitively in pinning. The vortex matter now consists of small patches of a snowflake-like structure, in which a vortex maintains a topological order with its 6 nearest neighbours. This is referred to as the “Bragg glass” state[31] and has been experimentally established.
Another variation of the vortex matter phase, the "vortex glass" state\cite{32}, has also been suggested in the case that the strengths of the pinning sites are stronger than the repulsive interactions. The vortex matter structure would then be dominated by the spatial distribution of the local pinning sites, and the ordered structure would be completely destroyed. Inhomogeneity of the vortex density is likely to arise, depending on the location of the pinning sites.

The third, and probably the most important factor is thermal fluctuations, which affect pinning in two ways. The first way is called thermal creep\cite{33}, in which thermal fluctuations temporarily move a vortex out of its local free energy minimum, and relocate it to a neighbouring site (see Figure 1.7). In the presence of an external force (in our case, the Lorentz force), it is as if the vortex is jumping in a tilted potential. The average "hopping" is controlled by the thermal excitation energy scale $k_B T$ and the energy barrier $U$ between two nearest local minima:

$$\tau_0/\tau = e^{-U/k_B T},$$

where $\tau$ is the time scale of the thermal "hopping", $\tau_0$ is the characteristic attempt time for the system, and $U$ can be regarded as the amplitude of the pinning potential. Effectively, pinning is reduced as thermal creep becomes stronger.

Figure 1.7: (a) Thermal creep in a random pinning potential. The possibility of vortex "hopping" is represented by the dashed line. (b) Thermal creep under an external force.

Another way that temperature affects pinning is the smoothing of the effective local pinning potential. The idea is that thermal fluctuations can cause a phonon-like motion of the vortices, so the vortices sample the pinning potential over an extended spatial region. In other words, the displacement $\vec{x}$ for a vortex becomes a function of temperature and time. As a result, the average critical current density measured in a DC experiment is reduced as temperature becomes higher.
Introduction

In some sense, what thermal fluctuations do to the vortex matter is similar to what they do to a genuine solid. It plays a minor role at very low temperature, but becomes more important at higher temperatures. On approach to $T_c$ it may overcome the other two pinning factors and the “vortex solid” can undergo a phase transition into the “vortex liquid” state, in which the vortices are free to flow. The “vortex lattice melting” [34], or more broadly the vortex matter phase transition is an interesting topic that will be addressed by our measurements of the pinning constant. Experimentally, a vanishing critical current density has been observed at a temperature lower than the superconducting transition temperature $T_c$, indicating the existence of a “vortex liquid” state.

As a summary, these three factors may greatly change the structure of the vortex matter, and the $B-T$ phase diagram shown in Figure 1.2 is just a sketch illustrating the basic concepts. The detailed vortex matter phase diagram for the systems we have worked on will be discussed in the next section.

![Figure 1.8: Doping-temperature phase diagram for hole-doped cuprates. The doping range that corresponds to highest $T_c$ is referred as “optimally doped”; to the left of “optimally doped” is called “underdoped”, and to the right is called “overdoped”.](image-url)
1.4 Cuprate Superconductors

The main systems studied in this work are cuprate superconductors, compounds with CuO$_2$ planes in their crystal structures. In these materials superconductivity can be achieved by manipulating the doping level of the parent compounds. Figure 1.8 shows a universal doping-temperature diagram for the hole-doped cuprates, to which our cuprate samples belong.

Compared with conventional materials, the cuprate superconductors can have much higher transition temperatures, upper critical fields and critical current densities – properties that are highly desired for applications like electrical cables or superconducting magnets. Ever since their discovery in 1986[35], great efforts have been made exploring these compounds, yet the origin of the high temperature superconductivity remains uncertain.

1.4.1 The Crystal Structure

The crystal structure of cuprates determines their properties. Examples of crystal structure in cuprate compounds like YBa$_2$Cu$_3$O$_{6+x}$ (YBCO) and Tl$_2$Ba$_2$CuO$_{6+x}$ (Tl2201) are shown in Figure 1.9. The common feature is the existence of parallel CuO$_2$ sheets in the $\hat{a}\hat{b}$ plane, into which the electron or hole doping is introduced. These CuO$_2$ planes are generally believed to be crucial to cuprate superconductivity[36]. As a result, a variety of cuprate compounds can be represented by the universal phase diagram shown in Figure 1.8.

The highly anisotropic structures give cuprate superconductors anisotropic properties. For instance, the ratio between the penetration depths in the $\hat{c}$ axis and the $\hat{a}\hat{b}$ plane ranges from $10^1$ to $10^2$[38]. Similar anisotropies have been observed in the electrical transport properties[39, 40]. Mobile charge carriers mainly move in the CuO$_2$ planes, and coupling between different layers is very weak. In practice, cuprates are often regarded as quasi-2D systems with weak coupling between CuO$_2$ planes.

The strong two dimensionality manifests itself in our vortex study, as the screening current of a vortex is confined in the CuO$_2$ planes. Unlike the straight flux lines in the conventional case, vortices in these systems are more like “vortex pancakes”[41], as shown in Figure 1.10. Since our work focuses on the averaged response from the $\hat{a}\hat{b}$ plane, as a good approximation the flux-flow dynamics can be modeled in 2D and the structure along the $\hat{c}$ axis can be ignored.
Introduction

Figure 1.9: The cuprate crystal structure for (a) YBa$_2$Cu$_3$O$_{6+x}$ and (b) Tl$_2$Ba$_2$CuO$_{6+x}$, reprinted with the kind permission of Dr. D. Peets [37]. YBa$_2$Cu$_3$O$_{6+x}$ has CuO chains in the $c$ direction while Tl$_2$Ba$_2$CuO$_{6+x}$ does not.

Figure 1.10: Vortex pancakes in the layered structure of the cuprate system. On each layer, the vortex is pinned almost independently, resulting in bending or tilting of the flux line.
1.4.2 The Superconducting State

Although the cuprate system is widely regarded as "unconventional", in the superconducting state it can be described reasonably well in terms of the BCS and the Ginzburg-Landau theories. It has been established that superconductivity still involves condensation of the electron pairs, as vortices in this system carry the flux quantum $h/2e$[42].

Nevertheless, the superconducting state in cuprate superconductors has a few distinct differences from the original BCS theory. First, the Ginzburg-Landau parameter $\kappa$ in cuprate superconductors is usually large(of the order of 10 to 100)[43], corresponding to a small coherence length, usually a few nanometers[44]. The vortices are small, so it takes less energy to create them. Second, cuprate superconductors have small charge carrier densities, especially on the underdoped side. These correspond to small superfluid densities, which enhance the importance of the phase fluctuations.

Another major difference between the original BCS theory and cuprates is the strong anisotropy of the order parameter. Unlike the isotropic superconducting energy gap in the original BCS theory, the gap $\Delta_k$ in the cuprates system has a $d$-wave symmetry and takes approximately the following form:

$$\Delta_k = \Delta_0 (\cos k_x - \cos k_y),$$  \hspace{1cm} (1.35)

where $\Delta_0$ is the gap maximum. Along the directions of $k_x = \pm k_y$, there are nodal points in the energy gap, as shown in Figure 1.11. The quasiparticle excitation spectrum near the nodal points dominates the thermodynamic and transport properties at low temperature, as it takes little energy to create an excitation. At finite temperature, the superfluid density is depleted more effectively by thermal fluctuations at those nodal points[45, 46], which further enhances the importance of phase fluctuations. The importance of phase fluctuations will be discussed later in this section. In addition, along the nodal line the coherence length $\xi(k)$ becomes diverging, which results in quasiparticles been delocalized far away from the vortex core.

The small vortex core size or the high anisotropy of the order parameter may have substantial impact on transport properties in the mixed state. The small core size calls into question the normal metal analogy of the vortex core, while the gap anisotropy leads to the Volovik effect[47], which predicts that the low temperature transport properties are dominated by quasiparticle excitations outside the vortex.
1.4.3 The Normal State

As superconductivity is an instability of the normal state, it is crucial to understand the ground state from which superconductivity arises. However, the normal state properties of the cuprate superconductors are far more confusing than those of the superconducting state.

What we understand best is the overdoped region, where there is a consensus that the system behaves like a Fermi liquid[36]. In particular, as doping increases towards the overdoped end, the low temperature normal electrical resistivity approaches a typical Fermi liquid form $\rho_n = A + BT^2[48]$. Strong magnetic field has also been used to suppress superconductivity in the overdoped region, and this Fermi liquid behaviour remains[49]. A number of experiments, such as quantum oscillation measurements[50], confirm the existence of a large Fermi surface, which is consistent with the predictions of conventional band theory. In addition, it has been suggested that $T_c$ is related to the opening of an energy gap[51] that is BCS-like. So far, few deviations from Fermi liquid theory have been reported on this side. In other words, the system is surprisingly normal.
Introduction

However, at the opposite end of the phase diagram, there is a much poorer understanding of the normal state. At zero doping, the parent compound is a Mott insulator: a system that should be conducting under conventional band theory, but is insulating due to electron-electron repulsion. At light doping, the system is still characterized by strong antiferromagnetic order. Although the antiferromagnetism itself is well understood, its interplay with superconductivity may imply a deviation from the original BCS theory, where the interactions are relatively weak. It has been suggested that high temperature superconductivity can be understood as doped Mott physics[52], and it is evident that the magnetic order persists outside the antiferromagnetic phase. For instance, μSR studies have revealed magnetic order in the vicinity of the vortex cores on the underdoped side[53], while STM experiments have revealed antiferromagnetic order inside the vortex cores on the overdoped side[54].

As the system is tuned away from the antiferromagnetic phase, at finite temperature there is the so-called “pseudogap” region lying between the antiferromagnetic phase and superconducting phase. First discovered by nuclear magnetic resonance (NMR) spin relaxation rate measurements[55], Knight shift measurements[56], spin susceptibility measurements[57] and later identified by spectroscopies like angle resolved photoemission spectroscopy (ARPES)[58], the pseudogap is characterized by a partially formed energy gap above $T_c$ and below a crossover temperature $T^*$. Although agreement has not been reached on the exact values of $T^*$, the “normal state” in this region demonstrates consistently unusual properties. In particular, the “normal state” at low temperature has been reached under a high magnetic field of 60 T, and deviations from Fermi liquid theory have been observed. These include violation of the Wiedemann–Franz law[59], the Hall coefficient anomaly[60] and a low temperature upturn in the electrical resistivity[61]. The last effect is the well cited Ando–Boebinger effect and is particularly relevant to our study of microwave flux-flow resistivity.

The pseudogap is the most controversial part of the phase diagram, and its interpretation is crucial to understand high temperature superconductivity. So far, two major schools of thought have been established[62]. The controversy is whether the pseudogap represents another ordered state that competes with superconductivity on the same Fermi surface, or is a precursor to superconductivity. It is worth noting that these two points of view may not necessarily exclude each other, and the correct explanation may contain some elements of each.
1.4.4 Fluctuating Superconductivity

A major school of thought regarding the pseudogap is termed "fluctuating superconductivity". In this scenario, the pseudogap is regarded as the same as the superconducting gap, which implies the existence of preformed Cooper pairs above $T_c$ and below $T^*$. In the pseudogap region, the amplitude of the order parameter remains finite but the ability to form a global superfluid is destroyed by phase fluctuations.

![Phase Diagram](image)

Figure 1.12: A schematic phase diagram of the fluctuating superconductivity scenario, reproduced with the kind of permission of Dr. M. Norman[63].

The importance of phase fluctuations was first pointed out by V. I. Emery and S. A. Kivelson[64]. As they showed, global superconductivity required two conditions: the existence of Cooper pair condensation, and a stiffness to phase fluctuations. In clean conventional superconductors, the temperature scale to break the phase coherence is much higher than thermal energy at $T_c$. Cooper pair formation is then the weaker link, and $T_c$ is marked as the closing of the energy gap. In the cuprate system, due to the $d$-wave symmetry of the order parameter, the low charge carrier density and the short coherence length, the energy scale of phase fluctuations becomes comparable to that of the Cooper pair binding energy. As mentioned earlier, the importance of phase fluctuations increases as phase stiffness decreases. So, on the underdoped side it is the reverse situation to the conventional case: $T_c$ is the onset for complete depletion of phase stiffness, or the loss of long range coherence; $T^*$ on
the other hand is interpreted as the thermal energy scale that suppress the amplitude of the order parameter.

In this scenario, “real” superconductivity emerges with the coexistence of the phase stiffness and the superconducting gap, which have opposite dependences on doping. The phase diagram in this view is sketched in Figure 1.12. On the overdoped side $T_e$ is limited by the size of the energy gap, while on the underdoped side it depends on the phase stiffness. Experimentally, strong phase fluctuations in the pseudogap region have been suggested by terahertz spectroscopy[65] and high resolution susceptibility measurements[66]. Correlation between the phase stiffness and $T_e$ has been reported on the underdoped side[67, 68], and a correlation between the energy gap and $T_e$ has been pointed out on the overdoped side[51].

![Figure 1.13: Two phase fluctuation possibilities (a) spin wave and (b) vortex-antivortex pair. Phase angle is represented by the orientation of the arrow.](image)

How does our study impact on this debate? It is because phase fluctuations may create vortices above $T_e$. As shown in Figure 1.13, phase fluctuations may behave either like a spin wave or as vortex-antivortex pairs. In the latter case, in the presence of a transport current, the vortex and antivortex should move in opposite directions and generate dissipation, which should be reflected in electrical transport measurements. Experimentally, the existence of vortices above $T_e$ has been mainly supported by Nernst effect experiments[27], which may also involve flux flow and are hence related to our study.
The left panel of Figure 1.14 shows the experimental setup for studying the Nernst effect signal. Thermal gradients have been applied to the sample in an applied magnetic field, and the electric field in the form of a voltage has been measured perpendicular to the thermal gradient. The Nernst signal is defined to be electrical field per unit thermal gradient $e_N(H, T) = E/|\nabla T|$. In the vortex liquid state, the vortices can be moved by the applied thermal gradient, and such vortex motion will induce an electrical field. This vortex response dominates the Nernst signal, and as temperature goes above $T_c$ its contribution is expected to reach zero if there are no vortices. This should result in a very small Nernst signal in the normal state. However, as first discovered by Ong’s group [69], the Nernst signal is surprisingly large in underdoped cuprates at temperatures above $T_c$. This is referred as the “giant Nernst effect” and has been confirmed in other cuprates[27]. As shown in the right panel of Figure 1.14, the so-called “tilted hill” profile of the Nernst signal remains qualitatively unchanged as temperature rises above $T_c$. Since in the superconducting state the Nernst signal mainly comes from flux flow, such continuity has been very suggestive of the existence of vortices above $T_c$. The existence of vortices has also been supported by
measurements of a diamagnetic signal by the same group[70].

However, a recent observation of large Nernst effect signal on the conventional semimetal bismuth[71] has raised some doubts about the vortex interpretation of the Nernst signal. The Nernst signal in bismuth is found to be of an order of magnitude larger than would be expected for a correlated metal, and clearly does not come from flux-flow but from the small charge carrier density and a long mean free path. This result implies that the giant Nernst signal might come from sources other than fluctuating vortices. Using a microwave fields instead of a thermal gradient to drive vortices, our study provides additional insight into this topic.

It is worth mentioning that the vortex fluctuating regime identified by the Nernst experiments resembles the original superconducting dome, and does not cover the entire pseudogap phase.

1.4.5 The Competing Orders

![Diagram](image)

Figure 1.15: A possible diagram for the competing order interpretation, reproduced with the kind of permission of Dr. M. Norman[63]. The quantum critical point is represented by the red dot.

An alternative point of view is the competing states scenario, which treats the pseudogap as a phase that competes with superconductivity on the same Fermi surface. The phase diagram is shown in Figure 1.15.
Although agreement has not been reached about the nature of the competing state, the existence of a quantum phase transition is a common feature in many different proposals [72, 73]. Unlike a classical phase transition which is tuned by the temperature, a quantum phase transition is driven by tuning certain parameters other than temperature (pressure, doping, sample thickness, disorder densities, applied magnetic field, etc.) in the Hamiltonian, so the system can evolve between distinct quantum ground states.

In the doping-temperature phase diagram the tuning parameter is the charge carrier doping. A quantum critical point at intermediate doping [74] \((x \approx 0.19)\) has been proposed, which separates the two different ground states. Near the critical point, the physics would be governed by low energy excitations from these ground states, which should be reflected in the electrical transport properties. To the right side of the quantum critical point, the ground state is a Fermi liquid, and resistivity should decrease with decreasing temperature. On the underdoped side, the system seems to demonstrate insulating behaviour with persisting local antiferromagnetic correlation, but a more definite confirmation is needed.

A number of experiments have been carried out to locate and classify the quantum critical point deep inside the superconducting phase. In particular, this critical point has been associated with a metal–insulator crossover [75] in the normal state resistivity. If the vortex cores in cuprate superconductors can still be treated as “normal”, flux-flow resistivity measurements should allow us to probe such a crossover at low temperature.

1.4.6 The Ando–Boebinger Effect and the Metal-insulator Crossover

Of the numerous experimental observations in the pseudogap region, the Ando–Boebinger effect will receive particular attention, as it is intimately related to our experiment.

First observed by Ando et al. in 1995 [61] on an underdoped \(La_{2-x}Sr_xCuO_4\) sample, then by Boebinger et al. in 1996 [75] on the same kind of sample with a wide range of dopings, the Ando–Boebinger effect is characterized by a logarithmic temperature dependence of the “normal state” resistivity at low temperature, as shown in Figure 1.16. The “normal state” is accessed by using a strong magnetic field (60 T) to
Introduction

Figure 1.16: The Ando–Boebinger effect[61]. Both the \( ab \)-plane and the \( c \)-axis resistivity show a logarithmic temperature dependence.

suppress superconductivity. It has been suggested that such logarithmic temperature dependence is confined to the underdoped side and evolves into metallic behaviour on the overdoped side, as shown in Figure 1.17. This behavior is referred to generically as the metal-insulator crossover, has been supported by other experiments[76, 77], and has been related to the quantum critical point inside the superconducting domain[78].

However, since \( B_{c2} \) is known to be exceptionally high in cuprates, it is question­able whether 60 T is sufficient to completely suppress pairing. Another concern is that such a strong magnetic field might induce additional competing orders in the superconductor. Although a similar logarithmic temperature dependence has been observed in zero field at a doping slightly away from the superconducting dome[79], at intermediate dopings more experimental evidence is required. One of our initial motivations was using flux-flow resistivity to investigate the Ando–Boebinger effect, which relies on the validity of the Bardeen–Stephen law to build the connection. Our scheme has its own advantage, as it requires a much smaller magnetic field to generate vortices.

1.4.7 Flux-flow Dynamics in Cuprate Superconductors

The investigation of the flux-flow resistivity in cuprate superconductors is far from easy. In conventional superconductors, the flux-flow resistivity can be directly measured using a strong enough current. In cuprate superconductors, the critical current
density is usually so high that this technique is only practical at extreme dopings or near $T_c$. As an alternative, microwave spectroscopy has been used, but most experiments have been troubled by low resolution. In general, few have been able to probe vortex dynamics deep inside the cuprate superconducting domain at very low temperature, and a complete experimental study of the flux-flow resistivity in cuprates is lacking. Filling this gap is one of the primary goals of this work.

Pinning in cuprate superconductors should be another interesting topic, as it is expected to differ from that in conventional materials. In conventional superconductors, repulsive interactions between flux lines dominate the pinning, and a well ordered vortex lattice is favorable. The vortex matter is usually an Abrikosov lattice or Bragg glass, and vortex lattice melting should happen at a temperature very close to $T_c$. As will be shown later, in conventional superconductors, this corresponds to a mixed state resistive transition that is almost as sharp as the zero field one. On the contrary, in cuprate superconductors, the system is often much more sensitive to disorder and thermal fluctuations, as the vortices are much smaller and $T_c$ is much higher. The strong electrical anisotropy of cuprates also plays an important role, as the vortices are pinned as “pancake vortices”. Vortex lattice melting can happen at a temperature
Introduction

Figure 1.18: Possible mixed state phase diagram for: (a) a high temperature superconductor with little disorder (b) a highly disordered high temperature superconductor. In both cases, $B_{cd}(T)$ becomes a crossover, and phase transition happens between vortex solid and vortex liquid state at melting line $B_m(T)$ or phase transition line $B_{pt}(T)$.

farther away from $T_c$, and a field dependent broadening of the resistive transition has been observed[80]. There are also other possibilities, like a transition from the Bragg glass state to the vortex glass state[41]. In general, the vortex matter phase diagram is very complicated and has not been completely resolved yet. Figure 1.18 shows examples of vortex matter phase diagrams[41] for cuprate superconductors with different levels of disorder. Since in our study clean samples and weak magnetic fields are used, the vortex matter phase transition is expected to happen near $T_c$, although the possibility of a continuous transition cannot be excluded. The different phases of the vortex matter are expected to leave strong signatures in the pinning constant $\alpha_p$, and our study may contribute to this understanding.

1.5 Previous Work by Our Group

This work inherits many legacies from people before me. The experimental protocol we used can be traced back to my supervisor's supervisor's supervisor, Prof. A. B. Pippard. The theoretical background to interpret the microwave spectroscopy data
Figure 1.19: Pinning constant for optimally doped YBa$_2$Cu$_3$O$_{6.95}$ studied by Ben Morgan[81] at a resonant frequency of 5.47 GHz and applied magnetic fields of 1 T (red), 2 T (blue), 3 T (brown), 4 T (green), 5 T (pink) and 6 T (black). A peak is observed near $T_c$.

was provided by my supervisor's supervisor, Dr. John Waldram. The experimental apparatus was designed and built by my supervisor, Dr. David Broun and his coworker, Dr. Ben Morgan. In 2001 and 2003 the flux-flow study was pioneered by Ben Morgan[81] on two optimally doped YBa$_2$Cu$_3$O$_{6.95}$ samples, and some very interesting discoveries have been made.

As shown in Figure 1.19, Ben Morgan found the pinning constant to be exponentially temperature dependent at low temperature. He proposed that this behavior was related to thermal creep, which contains an exponential activation thermal energy term. What is more, he found that near $T_c$ there were peaks in the pinning constant, presumably related to phase transitions in the vortex matter.

A more exciting discovery was on the flux-flow resistivity. As shown in Figure 1.20, Ben Morgan found that the flux-flow resistivity showed a logarithmic temperature dependence at very low temperature. Using the Bardeen–Stephen "fractional normal resistivity" picture, he related this behavior to the Ando–Boebinger effect. He then drew a conclusion that the normal state resistivity for optimally doped YBa$_2$Cu$_3$O$_{6.95}$ was also logarithmically temperature dependent, which seemed to correspond to an
Figure 1.20: The flux-flow resistivity measured by Ben Morgan at a resonant frequency of 5.47 GHz and applied magnetic fields of 1 T (red), 2 T (blue) and 4 T (green), on optimally doped YBa$_2$Cu$_3$O$_{6.95}$[81].

...insulating ground state and support the initial interpretation of the Ando–Boebinger effect.

This is where the current story begins. In Chapter 2, theoretical models for flux-flow response are discussed. In Chapter 3, microwave experimental details are introduced. Chapters 4, 5, and 6 focus on three conventional superconductors, Nb, NbSe$_2$ and V$_3$Si, three YBa$_2$Cu$_3$O$_{6+x}$ samples, and three Tl$_2$Ba$_2$CuO$_{6+x}$ samples. Chapter 7 analyzes the key results of the flux-flow resistivity, while Chapter 8 focuses on discussion of pinning. Chapter 9 is the conclusion.
Chapter 2

Theoretical Background

This chapter focuses on the answers to two questions: what is directly measured in our experiment? And how does the flux-flow dynamics affect the measured quantities?

The answer to the first question is presented in detail in Section 2.1. In short, we measure the electrical transport properties in the presence of vortices. In particular, the energy dissipation from the coupling between the quasiparticle excitations and the local electric fields is directly measured. We stress that phenomenologically our results are essentially comparable to those measured in the DC experiments, regardless of their microscopic origins. This ensures the connection between our results and those of many relevant works, which have only been measured at DC.

The rest of the chapter is dedicated to answering the second question. Section 2.2 introduces the Bardeen–Stephen model, which focuses on the microscopic origin of the flux-flow dissipation. As specific assumptions about the vortex core have been made in this model, its validity is discussed in Section 2.2.1. Section 2.3 presents the Rosenblum–Gittleman model, which focuses on extracting the flux-flow resistivity and the pinning constant from microwave measurements. Section 2.4 presents a unified version of the above two models by Dr. John Waldram, which has been used by Ben Morgan and I to interpret our data. Section 2.5 compares our model to other notable models, from which the data extraction method is shown to be model independent.

We end this chapter with Section 2.6, in which a more general and model independent description of our measurements is presented. The pinning and the viscosity are expressed in terms of a conductivity profile of the vortex unit cell, without specific assumptions made about the vortices.
Theoretical Background

2.1 Experimental Observables

In our experiment, we measure the electrodynamic response of the superconductor at microwave frequencies. The directly accessible quantity is surface impedance $Z_s$:

\[ Z_s = R_s + iX_s = E||/H||, \tag{2.1} \]

where $R_s$ is the surface resistance, $X_s$ is the surface reactance, and $E||$ and $H||$ are the tangential electric field and magnetic field components at the sample surface. In the local electrodynamic limit, the electronic mean free path $\ell$, the coherence length $\xi$ and the skin depth $\delta$ satisfy $\xi \ll \ell \ll \delta$, and the surface impedance can be directly related to either the complex resistivity $\rho = \rho_1 + i\rho_2$, the complex conductivity $\sigma = \sigma_1 - i\sigma_2$ or the complex skin depth $\delta = \delta_1 - i\delta_2$:

\[ Z_s = i\mu_0\omega\delta = \sqrt{i\mu_0\omega\rho} = \sqrt{i\mu_0\omega \sigma}, \tag{2.2} \]

which is usually valid for our cuprate samples. Surface reactance $X_s$ is proportional to the real part of the effective skin depth or penetration depth $\delta$, while $R_s$ approximately reflects the energy dissipation due to coupling between the quasiparticle excitations and the local electric fields. An illustration of these relations is shown in Figure 2.1. Assuming a uniform microwave field $H||e^{i\omega t}$ near the sample surface, from Faraday’s law we have:

\[ \oint E \cdot dl = -d\Phi/dt. \tag{2.3} \]

The magnetic field inside the sample has the form $B(z) = \mu_0 H||e^{-z/\delta}e^{i\omega t}$, where $\delta$ is the skin depth. For the loop in Figure 2.1 we have:

\[ |E|| = |\mu_0\omega\delta H||, \tag{2.4} \]

which gives $X_s = \mu_0\omega\text{Re}(\delta)$. In the superconducting state, this means $X_s \propto \lambda$ and $E \propto \lambda$, where $\lambda$ is the effective penetration depth. In zero field, the effective penetration depth is dominated by the superfluid density and converges to the London penetration depth in the limit $\omega \rightarrow 0$ and $\sigma_1 \rightarrow 0$.

The energy dissipation, $P = \frac{1}{2} \int \sigma_1 E^2 dV$, is reflected by the surface resistance as

\[ P = \frac{1}{2} R_s \int H^2 dS. \tag{2.5} \]

Perturbatively this form results in $R_s \propto \sigma_1 \cdot \delta^3$. Microscopically, $\sigma_1$ is a function of location, and what we measure is the averaged dissipation in the real space. If $\delta$
can be treated approximately as a constant, then $R_s$ is proportional to the averaged real part of the conductivity $\sigma_1$. It depends on the energy density of states $N(E)$ and the quasiparticle scattering time $\tau$, and in the self-consistent t-matrix approximation it can be written as[82]

$$\sigma_1 = \frac{e^2}{m^*} \left\langle N(E) \frac{\tau(E)}{1 + i\omega\tau(E)} \right\rangle_E,$$

which is averaged over the energy space. Thus $\sigma_1$ can be affected by vortex dynamics in two ways: the presence of vortices either modifies the quasiparticle energy spectrum; or the quasiparticle scattering dynamics; or both.

Although these observables are measured in a different frequency range, our measurements can be directly compared to more common DC transport measurements. For length scales, the microwave wavelength is usually much larger than the sample size, the skin depth or the mean free path, so in this aspect it is no different from a DC experiment. For time scales, if the microwave frequency is much larger than the quasiparticle scattering rate, the inertial term dominates and the response is purely reactive. On the other hand, if the microwave frequency is much smaller than the scattering rate, $\sigma_1(\omega)$ should be close to the DC conductivity. At low temperature, this condition would be satisfied for the cuprate samples.

As to $\sigma_2$, the contribution of the superfluid density needs to be considered. In
Theoretical Background

zero field a two fluid Drude model[83] can be introduced:

\[ \sigma_1(\omega) - i\sigma_2(\omega) = \frac{ne^2}{m} \left[ \frac{f_s}{i\omega} + \frac{f_n}{1/\tau + i\omega} \right]. \]  

(2.7)

Here \( f_s \) and \( f_n \) correspond to the fraction of superfluid and normal fluid respectively, and follow the oscillator strength sum rule \( f_s + f_n = 1 \). The real part of the conductivity \( \sigma_1 \) depends on the normal fluid density, while \( \sigma_2 \) is dominated by the superfluid at low temperature. In an applied field, \( \sigma_2 \) should also contain a term that is related to pinning, but it is hard to evaluate its magnitude in the presence of a substantial superfluid contribution. Disentangling the pinning contribution from the superfluid contribution thus requires detailed modeling, which will be introduced later in this chapter.

2.2 The Bardeen–Stephen Model

Let us turn to specific models of flux-flow dynamics. In regard to the microscopic origin of the flux-flow resistivity, the Bardeen–Stephen model[15] is the first and probably the most successful one. Originally developed for conventional superconductors with \( s \)-wave symmetry, it is still the basis for many flux-flow studies on cuprates.

As mentioned in Chapter 1, the Bardeen–Stephen model was originally developed to explain the results of the DC flux-flow experiments, in which pinning has been overcome. Discussion is limited to 2D for simplicity. In addition, this model assumes that \( B_{c1} \ll B \ll B_{c2} \), so that the intervortex spacing \( a \) follows \( \lambda \gg a \gg \xi \). In this case, an “unit cell” containing a single vortex is studied.

The most important assumption is the description of the vortex core. The core is assumed to have a cylindrical symmetry with a radius \( r_0 \), which is related to the coherence length \( \xi \). A local model for the vortex is used, in which the superconducting gap drops abruptly to zero at the vortex boundary, rather than decreasing continuously to zero at the vortex center. The core is treated as in the normal state, surrounded by the superconducting background.

A uniform transport current \( \vec{J}_T \) with charge carrier velocity \( \vec{v}_T \) flows in the \( \hat{x} \) direction near the vortex. The screening current density flowing near the vortex is of the order of \( 10^6 \) A/cm\(^2\), while the transport current density is usually much smaller and can be treated as a weak perturbation. The transport current applies a Lorentz force that moves the vortex with velocity \( \vec{v}_L \). Consequently, the screening
Theoretical Background

current should move along with the vortex with velocity $\vec{v}_L$ without disturbing the current distribution. The change in the superfluid momentum should correspond to an “accelerating” force (electric) field in the London theory, which follows the first London equation:

$$\frac{d\vec{J}_s}{dt} = \frac{2n_pe^2}{m_e} \vec{E} = (\vec{v}_L \cdot \nabla) \vec{J}_s.$$ (2.8)

If the Hall effect contribution can be ignored, the vortex motion should be perpendicular to the transport current. The screening current density $\vec{J}_s$ for an $s$-wave vortex can be found from the second London equation:

$$\nabla \times \vec{J}_s = -\frac{2n_pe^2}{m_e} \vec{B},$$ (2.9)

which takes the form:

$$\vec{J}_s = \frac{2n_pe^2}{m_e} \frac{\Phi_0}{2\pi r} \hat{\theta}.$$ (2.10)

Combined with the first London equation, it turns out that the induced electric field has a dipole form outside the core:

$$\vec{E}(r, \theta) = -\frac{\Phi_0v_L}{2\pi r^2} (\hat{r} \cos \theta + \hat{\theta} \sin \theta),$$ (2.11)

where $r$ and $\theta$ are the polar coordinates with the origin at the center of the core.

Inside the core, there is no supercurrent, and hence no spatial change of the electric field, but continuity at the boundary must be maintained. Both the tangential component of the electric field and the normal component of the transport current density must be continuous on the vortex boundary. These two constraints comprise the Bardeen–Stephen boundary conditions, from which we can find a uniform electric field solution inside the core:

$$\vec{E}_c = \frac{v_L\Phi_0}{2\pi r_0^2} \hat{x}.$$ (2.12)

A sketch of the electric field solution is presented in Figure 2.2. The uniform electric field drives the transport current directly through the normal core, and causes dissipation. A concern here is that the normal component of the electric field is not continuous on the boundary, which would imply the existence of static charges. As Bardeen and Stephen pointed out, this is merely an artifact created by the unrealistic local model and can be neglected. By integrating the dissipation inside and outside the core, Bardeen and Stephen found the viscosity to be a function of the vortex core size $r_0$:

$$\eta = \frac{2\pi n_r^2}{m_e} \left( \frac{\hbar}{2r_0^2} \right)^2 \tau(1 + br_0^2).$$ (2.13)
Theoretical Background

Figure 2.2: A sketch of the electric field solution in the Bardeen–Stephen model. The electric field has a dipole form outside the vortex core (represented by the yellow circle), and is uniform inside the core.

where \( n \) is the quasiparticle number, and \( b = H/\Phi_0 \) is the flux line density or equivalently the inverse of the vortex unit cell area. By relating \( r_0 \) to \( H_c^2 \) and \( H \) as

\[
\frac{\Phi_0}{4\pi r_0^2} = H_c^2 - \frac{H}{2},
\]

a field independent viscosity is obtained as

\[
\eta = \Phi_0 H_c^2 \sigma_n,
\]

where \( \sigma_n \propto n\tau \) is the normal state conductivity. This field independent viscosity corresponds to a linear field dependence in the flux-flow resistivity

\[
\rho_F = \rho_n \frac{H}{H_c^2},
\]

which reproduces the empirical law[13].

2.2.1 Limitations of the Bardeen–Stephen Model

In the Bardeen–Stephen model, the simple treatment of the vortex has proved to be very insightful for conventional superconductors. However, as mentioned in Chapter 1, the breakdown of the Bardeen–Stephen law in cuprates has been reported in a
few experiments, in which a power-law field dependence of the flux-flow resistivity has been observed at high fields. The relation between the normal state resistivity and the flux-flow resistivity on the other hand, has not been experimentally challenged so far.

It is natural to think that the oversimplified treatment of the vortex core is the cause for this deviation, and it might be inaccurate in the more complicated $d$-wave case, for a number of reasons.

First, the Bardeen-Stephen model assumes a “dirty” superconductor, in which the quasiparticle scattering time $\tau$ and the energy spacing $\epsilon_0$ between the bound states satisfy $\epsilon_0 \tau \ll \hbar$. This condition ensures that the quasiparticles are scattered by impurities in the same way as they would be in a normal metal, without the need to consider the details of the core states. In the opposite clean limit, the core structure as well as nonlocal effects might play important roles. With such considerations, Bardeen himself extended his model to the low $\kappa$, clean limit case[84]. His conclusion was that the dissipation still came from the current flowing through the core, but due to the shrinkage of the core size $r_0 \propto \xi(T/T_c)$ (the Kramer-Pesch effect[85]) at low temperature, the flux-flow resistivity was reduced (not enhanced, as in the case of Ben Morgan’s observation) by a logarithmic factor $\ln(\xi/r_0)$. As to the high $\kappa$ case, the viscosity coefficient of vortex motion in a moderately clean, high $\kappa$, layered superconductor has been studied by M. A. Skvortsov et al.[18]. Their conclusion confirms that the Bardeen-Stephen expression is insensitive to the detailed core structure.

Second, the “normal core” notation is based on a quasiclassical description of the vortex core. As mention in Chapter 1, the quasiclassical (continuous) approximation for the bound states energy spectrum is reasonable for conventional superconductors, but for cuprate superconductors $\xi$ can be so small that the energy spacing between the bound states becomes comparable to the size of the energy gap. The bound states are discrete, and some argue that there is no true bound state inside the core[86] due to the small core size. A. I. Larkin and Yu N. Ovchinnikov[87] have developed a model based on the discrete energy spectrum in the superclean limit, and their conclusion is that the flux-flow conductivity is enhanced at low temperature, compared with its quasiclassical counterpart.

Last, but not least, in the presence of vortices there is the “Volovik effect”[47] due to the anisotropy of the energy gap. As G. E. Volovik pointed out, the screening supercurrent of a vortex adds a Doppler shift energy term $e\vec{v}_k \cdot \vec{A}$ (where $\vec{v}_k$ is the
Theoretical Background

normal state velocity) to the quasiparticle excitation spectrum. In an isotropic s-wave system this term is negligible, but near the nodal points of a d-wave vortex it becomes comparable to the energy gap. Consequently depairing can be invoked at a diverging distance along the nodal directions, which results in extended quasiparticles outside the core. The low temperature thermodynamic properties would be dominated by these extended quasiparticles instead of the trapped ones, which makes the intervortex spacing or equivalently the size of the unit vortex cell a more important length scale than the vortex core size. Intuitively, this could be a challenge to the Bardeen–Stephen idea, in which the core size is emphasized.

The interplay of the intervortex spacing modifies the scaling law of many thermodynamic properties in an applied field. A power law field dependence instead of a linear one is often predicted, which has received some experimental support from heat capacity measurements[88]. Less success has been achieved for other quantities like the thermal conductivity, and there are no experimental indications of how the Volovik effect might change the electrical conductivity in the absence of flux-flow. A theoretical work by Volovik himself[20] predicts the linear field dependence of the Bardeen–Stephen law to remain valid in the d-wave case, except for in the superclean limit $\varepsilon_0 \tau \gg \hbar$. A modified version of this work, in combination with the anisotropy of the d-wave vortex, has been used to interpret the power law field dependence of flux-flow resistivity at high field[89]. However, agreement has not been reached on the exact form of the power law. To my knowledge, such breakdown of the Bardeen–Stephen law remains a open question.

Aside from these extensions of the Bardeen–Stephen model, an alternative mechanism for the flux-flow dissipation has been suggested. From a Ginzburg–Landau point of view[90], in the Bardeen–Stephen model the dissipation comes from changes in the phase-related term in the free energy. A flux-flow dissipation model based on changes in the amplitude-related term was proposed by M. Tinkham[91], at about the same time when the Bardeen–Stephen model was developed. In this model, the amplitude of the order parameter varies with time as a vortex moves, so quasiparticles are forced to be paired and depaired. If the quasiparticle relaxation time is smaller than the characteristic time for the vortex motion, dissipation will arise. Nevertheless, this mechanism received much less attention, as the Bardeen–Stephen mechanism seems to fit the experimental observations well. Given that the amplitude-related energy term is relatively small in cuprate superconductors, the impact of this possibility
Theoretical Background

should be negligible.

2.3 The Rosenblum–Gittleman Model

The Bardeen–Stephen model provides a good microscopic explanation for the origin of flux-flow resistivity, but it ignores pinning. The model works well in traditional DC flux-flow resistivity measurements, in which pinning has been overcome. In a microwave experiment, since the microwave transport current density is usually more than $10^6$ times smaller than the typical critical current density at low temperature, the pinning can only be overcome at a frequency beyond the depinning frequency $f_p$. Unfortunately, the depinning frequencies for cuprate superconductors are usually comparable to or higher than our working frequencies, so our signal depends on both the flux-flow resistivity and the pinning. Here we introduce the first and simplest model for extracting the flux-flow resistivity and the pinning constant from microwave measurements, which was developed by B. Rosenblum and J. I. Gittleman[28].

Unlike the Bardeen–Stephen model, which focuses on a microscopic description of the flux-flow resistivity, this model focuses on disentangling the pinning contribution and the flux-flow contribution as phenomenological parameters from the mixed state data. For simplicity, it treats a vortex as a rigid point object and ignores its microscopic structure. This should be a valid treatment, as long as the vortex structure is not distorted by the perturbative vortex motion.

As for pinning, it is treated as a harmonic potential. The experimental geometry is the same as for the Bardeen–Stephen model, without the Hall effect. Taking into account the viscosity of the vortex and the Lorentz force, for an isolated flux line, the force equation can be written as:

$$m_a \frac{d^2 y}{dt^2} + \eta v + \alpha y = J_T \Phi_0,$$

and includes an effective vortex mass $m_a$. The average effective transport current $J_T$ is time dependent: $J_T = \text{Re} \{ J_0 e^{i\omega t} \}$, where $\omega$ is the microwave frequency. The origin and the role of the effective mass is another interesting topic, but in microwave experiments it can be neglected[26], since it only makes a significant contribution at far-infrared frequencies.

This force equation describes a driven damped oscillator, which has the steady
Theoretical Background

state solution:

\[ y = \frac{J_0 B}{\alpha + i\eta \omega} e^{i\omega t}, \quad v = \frac{J_0 B}{\alpha + i\eta \omega} i\omega e^{i\omega t}. \tag{2.18} \]

As mentioned in Chapter 1, the effective average electric field is proportional to the magnetic field and the flux line velocity:

\[ E = B \frac{dy}{dt}, \tag{2.19} \]

so the effective resistivity would be

\[ \rho_{\text{eff}} = \frac{E_{\text{eff}}}{J_T} = \frac{\omega B \Phi_0}{1 - i\frac{\omega p}{\omega}}, \tag{2.20} \]

where the depinning frequency \( \omega_p = \alpha/\eta \) is defined to be the ratio of the pinning constant \( \alpha \) to the viscosity coefficient \( \eta \). It is given this name because

\[ \text{as } \frac{\omega_p}{\omega} \to 0, \quad \rho_{\text{eff}} \to \rho_{\text{eff}}; \tag{2.21} \]

\[ \text{as } \frac{\omega_p}{\omega} \to \infty, \quad \rho_{\text{eff}} \to 0. \tag{2.22} \]

This simple model provides the definition of the depinning frequency, and can be used to interpret the microwave mixed state data. Although it uses an oversimplified treatment of the vortex and the transport current, its validity will be confirmed later in this chapter.

A potential drawback of this approach, as well as the Bardeen-Stephen model, is that the contribution of the superconducting background has not been considered. This will be discussed in more detail in the Waldram model.

### 2.4 The Waldram Model

The Waldram model is a high frequency version of the Bardeen-Stephen model. Aside from the transport current being alternating (AC) rather than direct (DC), the basic assumptions of this model are the same as the Bardeen-Stephen model, which include: treating a vortex as a normal cylinder submerged in a superconducting sea; imposing a nearly uniform drive current that moves the vortex in a perpendicular direction; enforcing boundary conditions on the boundary of the core; and considering an isolated vortex unit cell.

Instead of dealing with velocity or momentum of charge carriers as in the Bardeen-Stephen model, the Waldram model focuses on finding a self-consistent solution for
Theoretical Background

electric field $E$ and transport current $J$, which is equivalent to the charge carrier momentum. In the local electrodynamic limit, $E$ and $J$ are related by the conductivity $\sigma$. Inside and outside the core, the conductivity profiles are known to be different, and their contributions to the resistive and reactive response depend on the exact form of the local electric field.

Since in microwave experiments the vortex moves like an oscillator rather than with constant speed, the Waldram model starts with two limits: the free flux-flow limit $\omega \to \infty$ and the pinning limit $\omega \to 0$, as shown in Figure 2.3.

![Diagram of vortex core and transport current](image)

**Figure 2.3:** (a) Free flux-flow limit and (b) pinning limit. The vortex core is represented by the gray cylinder; the screening current is represented by the blue lines; the transport current is represented by the red lines.

In the free flux-flow limit, vortices are free to flow, and the Waldram model is essentially the same as the Bardeen–Stephen model. In fact, as the vortex moves in phase with the drive current $J_{\text{ff}}$ (the subscript "ff" means flux-flow), the time dependence term of the microwave current, $e^{i\omega t}$, is not even introduced in the calculation. The only difference is that the Waldram model includes the contribution from the superconducting background, which has a uniform conductivity profile $\sigma_s$.

Outside the core, assuming the transport current flows uniformly in the $\hat{x}$ direction, the electric potential $\phi$ should be $-J_{\text{ff}} r \cos \theta / \sigma_s$ in a polar coordinate system with origin at the center of the core. Inside the core, the uniform transport current
Theoretical Background

gives electric potential \(-J_{\Pi} r \cos \theta / \sigma_n\), where \(\sigma_n\) is the normal conductivity.

There is then the motion of the vortex, which generates an additional dipole potential \(\phi_{fl}\) outside the core as

\[
\phi_{fl} = \frac{\Phi_0 v}{2\pi r} \cos \theta. \quad (2.23)
\]

Enforcing the Bardeen-Stephen boundary conditions we have

\[
\frac{\Phi_0 v}{2\pi r_0^2} + \frac{J_{\Pi}}{\sigma} = \frac{J_{\Pi}}{\sigma_n}. \quad (2.24)
\]

At low temperature \(\sigma_s \gg \sigma_n\), so

\[
J_{\Pi} \approx \frac{\Phi_0 \sigma_n v}{2\pi r_0^2}, \quad (2.25)
\]

which relates the transport current \(J_{\Pi}\) to the vortex velocity \(v\). The effective resistivity can be calculated from the averaged dissipation over the entire unit cell, giving

\[
\rho_{\text{eff}} = f_n \rho_n + (1 - f_n) \rho_s, \quad (2.26)
\]

where \(\rho_n = 1/\sigma_n\), \(\rho_s = 1/\sigma_s\), \(f_n\) and \(f_s\) correspond to the area fraction of the normal core and the superconducting background respectively. Therefore, in the free flux-flow limit, it is as if the superconducting background and the normal core are conducting in series. As \(\sigma_s \gg \sigma_n\), the superconducting background contribution to the dissipation is almost negligible, and Equation 2.26 is almost the same as the Bardeen-Stephen law.

The pinning limit, \(\omega \to 0\) is, on the other hand, like the DC case, only with much smaller transport current. As a good approximation, at infinite time \(t \to \infty\), the vortex can be treated as static. Consequently there is no dipole field from the vortex motion. The current distribution should have no contribution to the electric potential, which satisfies Laplace’s equation

\[
\nabla^2 \phi = 0. \quad (2.27)
\]

Due to the cylindrical symmetry, the solution takes the following form:

\[
\phi = \sum r^{\pm n} \left(A_n \cos(n\theta) + B_n \sin(n\theta)\right), \quad (2.28)
\]

where \(A_n\) and \(B_n\) are constants. At infinite distance, the electric field equals the external field source:

\[
\phi_\infty = -E_\infty r \cos \theta, \quad (2.29)
\]
Theoretical Background

so only the $n = \pm 1$ terms are kept. Inside the core, as $r \to 0$, the electrical potential has to be finite, and the $n = -1$ term is ruled out. The consistent solution for the electric potential is

$$\phi_{st} = \left( \frac{A_{-1}}{r} - E_\infty r \right) \cos \theta$$

outside the core, and

$$\phi_{st} = -E_c r \cos \theta$$

inside the core. Applying the Bardeen–Stephen boundary condition, we have

$$E_c = \frac{2\sigma_s}{\sigma_n + \sigma_s} E_\infty,$$

$$A_{-1} = \frac{\sigma_n - \sigma_s}{\sigma_n + \sigma_s} r_0^2 E_\infty.$$  

(2.32) \hspace{1cm} (2.33)

The overall conductivity can be calculated to be

$$\sigma_{st} = \frac{f_n \sigma_n E_c + (1 - f_n) \sigma_s E_s}{(1 - f_n) E_s + f_n E_c},$$

where the subscript “st” represents the static case. As $B \ll B_{c2}$, $f_n \ll 1$. Together with $\sigma_n \ll \sigma_s$, Equation 2.34 can be simplified to

$$\sigma_{st} \approx (1 - 2f_n)\sigma_n + 4f_s \sigma_s.$$  

(2.35)

So, it is as if the superconducting background and the normal core are connected in parallel, and most transport current is short-circuited by the superconducting background. There is little dissipation, as expected.

For the frequency region that lies in between these two limits, the solution can be obtained from a superposition of these two limiting cases. In this situation, the electric potential $\phi$ becomes

$$\phi_{tot} = - \left( J_{st} \frac{2}{\sigma_n + \sigma_s} + \rho_n J_{ff} \right) r \cos \theta$$

outside the core, and

$$\phi_{tot} = - \left[ J_{st} \frac{r_0^2 \sigma_n - \sigma_s}{r^2 \sigma_n + \sigma_s - 1} - J_{ff} \left( \frac{1}{\sigma_n} \frac{r_0^2}{r^2 + \frac{1}{\sigma_s}} \right) \right] r \cos \theta$$

inside the core. The effective resistivity is

$$\rho_{eff} = \frac{E_{ff} + E_{st}}{J_{ff} + J_{st}}.$$  

(2.36) \hspace{1cm} (2.37) \hspace{1cm} (2.38)
Theoretical Background

The only obstacle is to find the weighting factor between $J_f$ and $J_{st}$. This is provided by the force equation borrowed from the Rosenblum-Gittleman model:

$$\eta v + \alpha x = J_T \Phi_0. \quad (2.39)$$

In this equation, the work done by the transport current consists of two parts: one against the viscous force, and another against the elastic pinning force. Equivalently, these two parts can be regarded as the work done from the fraction of current that passes through the normal core, and the fraction that flows around the core, respectively. The ratio of these two energies provides the weighting factor

$$\frac{J_f}{J_{st}} = \frac{\eta v}{\alpha x} = \frac{i \omega \eta}{\alpha}. \quad (2.40)$$

The final effective resistivity can be obtained to be

$$\rho_{\text{eff}} = \left[ f_n \rho_n + (1 - f_n) \rho_s \right] \left( \frac{i \omega \eta}{\alpha + i \omega \eta} \right) + \frac{1}{(1 - 2f_n) \sigma_s + 4f_n \sigma_n} \left( \frac{\alpha}{\alpha + i \omega \eta} \right). \quad (2.41)$$

At low temperature, $f_n \ll 1$ and $\rho_s \ll \rho_n$, this form can be simplified to

$$\rho_{\text{eff}} = \rho_s + \frac{\rho_{\text{ff}}}{1 + i \omega_p/\omega}, \quad (2.42)$$

which gives us a similar result to the Rosenblum-Gittleman model[28] if we ignore the superconducting background contribution. Even though in our experiment the vortex is shaken by a weak microwave current, the dissipation in the free flux-flow limit can be extracted as

$$\rho_{\text{ff}} = \frac{|\rho_{\text{eff}} - \rho_s|^2}{\text{Re}(\rho_{\text{eff}} - \rho_s)}. \quad (2.43)$$

Likewise, the pinning constant can be extracted as

$$\alpha_p = B \Phi_0 \omega \frac{\text{Im}(\rho_{\text{eff}} - \rho_s)}{|\rho_{\text{eff}} - \rho_s|^2}. \quad (2.44)$$

and so are the depinning frequency and the vortex viscosity.

A simplified analogy is that the effective resistivity behaves as though the resistive part (the free flux-flow limit) and the reactive part (the pinning limit) are connected in parallel with each other, and are then connected in series with the superconducting background.
2.5 Other Models

Although the Waldram model is intuitive and simple, it has not been published yet. However, its results are similar to and compatible with other models. A good review of other models used by microwave experimentalists has been given by N. Pompeo and E. Silva[26], who conclude that the data extraction methods are rather model independent at low temperature, and the same as the Waldram model (see Equation 2.43).

At higher temperature, thermal creep becomes important and differences between these models begin to appear. Here we just give a brief review of the key elements for these models and focus on the common features.

2.5.1 The Coffey–Clem Model

The most cited microwave flux-flow resistivity model was developed by M. W. Coffey and J. R. Clem[92], who studied the AC complex penetration depth \( \lambda_{AC} \), which is equivalent to the complex resistivity \( \rho_{eff} = i \mu_0 \omega \lambda_{AC}^2 \). This model can be regarded as a refined version of the Rosenblum–Gittleman model.

It is modeled in 3D, in which the depth \( z \) below the superconductor surface is included. From the second London equation

\[
\nabla \times \vec{J}_s = -\frac{2n_p e^2}{m_e} \vec{B},
\]

we have \( J(z) = J_0 e^{-z/\lambda} \), where \( \lambda \) is the London penetration depth. The transport current induced by the microwave field is no longer treated as uniform, but decays exponentially into the sample. Such a treatment differs from the Waldram model, in which the variation in the \( z \) direction is ignored. Like the Rosenblum–Gittleman model, the Coffey–Clem model treats a vortex as a rigid object, without including the details of the vortex core structure.

Since the vortex motion induces an electric field and consequently a magnetic field, this model includes the internal magnetic field induced by the “primary” vortex motion, which is driven by the microwave transport current. The primary microwave field \( \vec{b} \) contributes to the driving force:

\[
\vec{b} = \vec{b}_0 e^{-z/\lambda} e^{iwt},
\]

\[
\vec{f} = \vec{z} \frac{1}{\mu_0} (\nabla \times \vec{b}) \times \vec{\Phi}_0.
\]
The theoretical background

The secondary magnetic field caused by the vortex motion is a higher order term which adds perturbatively to the force $\delta f$ and consequently to the vortex motion $\delta v$. The secondary motion modifies the induced internal field as $O(b)$ and the force term as $O(\delta f)$, and so on. This process is infinite, but can be solved self-consistently. The total perturbation $\delta b$ is

$$
\delta b = B \frac{z}{\lambda_{AC}} \frac{e^{i\omega t}}{1 - \lambda^2 / \lambda_{AC}^2} (e^{-z/\lambda_{AC}} - e^{-z/\lambda}),
$$

where $\lambda_{AC}$ is the complex penetration depth, which is modified by the final solution of the vortex motion. The magnetic field perturbation results in a current density perturbation

$$
\delta j = -\frac{1}{\mu_0} \frac{\partial (\delta b)}{\partial z}.
$$

Using the Rosenblum–Gittleman force Equation 2.17

$$
\eta \nu(z) + \alpha_p x(z) = (j(z) + \delta j(z)) \Phi_0,
$$

the self-consistent solution for complex effective penetration depth is obtained as

$$
\lambda_{AC}^2 = \frac{B \Phi_0}{\mu_0 (\alpha_p + i\omega \eta)} + \lambda^2.
$$

As the complex penetration depth is related to resistivity $\rho$, this result can be rewritten as

$$
\rho_{\text{eff}} = \rho_s + \frac{\rho_{\Pi}}{1 + \epsilon \omega / \tau},
$$

which is the same as the result in the Waldram model.

The next step is to include thermal creep, which is the major difference from the Waldram model. Thermal creep is modeled as Brownian motion in a periodic pinning potential, which adds a random force to the right side of the Rosenblum–Gittleman Equation 2.17. As a result, the complex resistivity is modified as

$$
\rho_{\text{eff}} = \rho_s + \frac{\epsilon + \omega^2 \tau^2 + i(1 - \epsilon) \omega \tau}{1 + \omega^2 \tau^2},
$$

where $\epsilon$ is the thermal creep factor, and $\tau$ is the relaxation time. Here $\epsilon$ and $\tau$ take the following forms:

$$
\epsilon = \frac{1}{I_0(\nu)^2},
$$

$$
\tau = \frac{\eta I_0(\nu)^2 - 1}{\alpha_p I_1(\nu) I_0(\nu)}.
$$
Theoretical Background

where \( I_0(\nu) \) and \( I_1(\nu) \) are modified Bessel functions, and \( \nu = U_0/2k_B T \) is the ratio between the pinning potential strength \( U_0 \) and the thermal fluctuation energy \( k_B T \). The parameters \( \epsilon \) and \( \omega_p \tau \) as functions of \( \nu \) are shown in Figure 2.4. At low temperature, \( U_0 \gg k_B T \), \( \epsilon \ll 1 \) and \( \omega_p \tau \approx 1 \), the effective resistivity reduces to the result of the Waldram model. At high temperature, \( \epsilon = 1 \), and \( \rho_{\text{eff}} \) is dominated by \( \rho_\parallel \), which corresponds to the vortex liquid state.

Figure 2.4: The relaxation parameter \( \tau / \eta \kappa_p \) (here \( \kappa \) is a different notation for \( \alpha_p \)) and the thermal creep factor \( \epsilon \) as a function of \( \nu = U_0/2k_B T \), the ratio between the pinning energy and the thermal energy.

This model contains at least 3 independent parameters \( \alpha_p \), \( \eta \) and \( \epsilon \), while in a microwave experiment at one fixed frequency, there are only two independent observables, which correspond to the in-phase and out-of-phase component of surface impedance. For our work, this model can only be used reliably at low temperature where \( \epsilon \to 0 \).

2.5.2 The Brandt Model

At around the time the Coffey–Clem model was published, a model by E. H. Brandt[93] was developed and also received considerable attention. The idea of this model is very similar to the Coffey–Clem model and will not be discussed in detail. Its main result is

\[
\chi_{AC}^2 = \chi^2 + \frac{B \Phi_0}{\mu_0 (\alpha_p + i \omega \eta)},
\]

(2.56)
Theoretical Background

which has exactly the same form as the Coffey–Clem model.

The only noticeable difference from the Coffey–Clem model is the thermal creep factor. Thermal creep is described as weakening of effective pinning:

$$\frac{\alpha_p(t)}{\alpha_p} = e^{-t/\tau},$$

(2.57)

where $\alpha_p(t)$ is the time dependent pinning constant, $\alpha_p$ is the pinning constant without thermal creep, and $\tau$ is the time scale for thermal creep. Thermal creep modifies the pinning constant, so the effective resistivity becomes

$$\rho_{\text{eff}} = \rho_s + \rho_{\text{ff}} \frac{\epsilon + i\omega \overline{\tau}}{1 + i\omega \overline{\tau}},$$

(2.58)

where $\epsilon = \frac{1}{1+i\omega \tau}$, $\overline{\tau} = \epsilon \tau$, and $\omega_p \tau = e^{U_0/k_BT}$. At low temperature $\epsilon \to 0$, the effective resistivity is reduced to the result of the Waldram model. At the upper limit, we have $\epsilon < 1/2$. Thus this model works best for describing situations with low thermal creep rate.

Compared with the Coffey–Clem model, the Brandt model does not require any assumption about the pinning potential and is purely phenomenological. This difference is not a concern for us, because the thermal creep factor $\epsilon$ remains as an independent variable that cannot be directly measured.

2.5.3 Summary of the Flux-flow Models

As pointed out by N. Pompeo and E. Silva[26], the results of all these models can be written as a universal form:

$$\rho_{\text{eff}} = \rho_s + \rho_{\text{ff}} \frac{\epsilon + i\omega \tau_{\text{eff}}}{1 + i\omega \tau_{\text{eff}}},$$

(2.59)

where $\epsilon$ is the thermal creep factor and $\tau_{\text{eff}}$ is a time constant related to the depinning frequency $\omega_p$. At low temperature, this is equivalent to the result of the Waldram model, as $\tau_{\text{eff}} = 1/\omega_p$. At high temperature, when $k_BT \approx U_0$, the contribution of $\epsilon$ may need to be considered. As a summary, the Waldram model works best at low temperature, when $\epsilon$ is vanishingly small and can be neglected.
2.6 Model Independent Interpretation of Our Measurements

After reviewing all these models, it is clear that the extraction of the flux-flow parameters itself should be model independent at low temperature. On the other hand, the Bardeen–Stephen or the Waldram approach gives a microscopic picture of the flux-flow dynamics, for which a more general and model independent version can be constructed. Instead of assuming an artificial normal core, a more general conductivity profile $\sigma$ should be used to represent the coupling of the quasiparticle excitations to the induced electric field.

Let us start with the Rosenblum–Gittleman equation, which should be valid as it only requires the vortex structure to remain unchanged under the perturbative vortex motion. Assuming a transport current with current density $J_T$ in the $x$ direction, the vortex then moves in the $\hat{y}$ direction, with the force equation

$$\eta v + \alpha y = J_T B = J_0 B e^{i\omega t}.$$  \hfill (2.60)

This is a damped, driven oscillator, with steady-state solution

$$y = \frac{J_0 B}{\alpha + i\eta \omega} e^{i\omega t}, \quad v = \frac{J_0 B}{\alpha + i\eta \omega} i\omega e^{i\omega t}. \hfill (2.61)$$

The induced field can be calculated from the first London equation[2]:

$$\frac{dJ_s}{dt} = \frac{1}{\Lambda} E_{\text{eff}} = \frac{dJ_s}{dy} v, \hfill (2.62)$$

and the power dissipation would be

$$P = \int \int \sigma_1 |E|^2 dxdy = \int \sigma_1 \Lambda^2 \left(\frac{dJ_s}{dy}\right)^2 v^2 dxdy, \hfill (2.63)$$

where $\sigma_1$ is the real part of the complex conductivity. Under the Bardeen–Stephen assumptions this expression will provide the dipole field solution.

The first step to move beyond the Bardeen–Stephen assumption of a hard-walled cylindrical vortex core, is to use a more realistic model of the core, in which the gap continuously shrinks to zero at the vortex center rather than at the vortex boundary. This extension is made below using a more realistic order parameter solution for an isotropic $s$-wave superconductor. As shown in Figure 2.5, the induced electric field is still similar to a dipole field, and inside the core the electric field is almost uniform.
Theoretical Background

The resulting effective resistivity should not be qualitatively changed. Also, in this case no boundary condition is needed, since the Bardeen–Stephen boundary condition arises from the assumption of a hard-walled vortex core.

Figure 2.5: The induced electric field distribution calculated from the order parameter of an isotropic s-wave superconductor. The vortex core boundary is represented by the circle.

For a more general extension we cannot continue following Waldram’s method, as the exact solution for electric field and transport current cannot be obtained with an arbitrary conductivity profile. Instead, the importance of the vortex velocity phase has to be emphasized:

\[ v = \frac{J_B}{\alpha + i\eta \omega} i\omega e^{i\omega t}. \]  

(2.64)

No matter what the current or conductivity distribution is, the phase of the induced electric field always depends on the phase of the velocity alone. Let us start with the pinning limit and the free flow flux limit.

In the free flux-flow limit, assuming \( \alpha \to 0 \), the velocity is in phase with the drive current, and current will flow in phase with the electric fields to cause dissipation. The resultant rate of dissipation will be

\[ P = \int \int \sigma_1 A^2 \left( \frac{dJ_y}{dy} \right)^2 \frac{J_B^2 B^2}{\eta^2} \cos^2(\omega t) dxdy, \]  

(2.65)
where $\omega \rightarrow 0$. Averaged over a period, $\cos^2 \omega t$ is equal to $1/2$, so Equation 2.65 gives half the same dissipation as in a DC case. Since by definition $P = \eta \nu^2$, the viscosity coefficient should be

$$\eta = \int \int \frac{1}{2} \sigma_1 \Lambda^2 \left( \frac{dJ_s}{dy} \right)^2 dxdy,$$

which gives the origin of the viscosity. Strictly speaking, this form cannot be applied to the Bardeen–Stephen model, in which the electric field solution is not calculated from $J_s$ inside the core. Nevertheless, if we assume $J_s$ to be proportion to $r$ inside the core, and continuous on the core boundary, the Bardeen–Stephen law will be reproduced.

In the pinning limit, assuming $\alpha \rightarrow \infty$ or $\omega \rightarrow 0$, the velocity is effectively zero. That implies the work done should be out of phase with the drive current and there should be no dissipation. These facts apparently do not depend on the details of the vortex core structure.

If we gradually turn on the viscosity, the velocity would gradually change, and there will be a relatively small fraction of electric field that is in phase with the averaged drive current. The rate of dissipation would be

$$P = \int \int \sigma_1 \Lambda^2 \left( \frac{dJ_s}{dy} \right)^2 \frac{J_0^2 B^2}{\eta^2} \frac{1}{1 + \omega_p^2/\omega^2} \cos^2(\omega t) dxdy,$$

which is $\frac{1}{1 + \omega_p^2/\omega^2}$ times the dissipation in the free flux-flow limit. The dissipation should be proportional to the real part of the complex resistivity, and the ratio $\omega_p/\omega$ would be determined from the ratio of the out-of-phase component to the in-phase component of resistivity $\rho_2/\rho_1$, which correspond to the stored energy and the dissipated energy. Regardless of the current and conductivity profile, the flux-flow resistivity can be extracted from the effective resistivity:

$$\rho_{\Pi} = |\rho_{\Pi}^2|/\text{Re}(\rho_{\Pi}),$$

which shows that the original form in the Rosenblum–Gittleman model remains valid. If we then take the superconducting background into consideration, this form should be modified into the Waldram form (see Equation 2.43).

As a summary, we can extract the following frictional and pinning parameters from the complex resistivity:
Theoretical Background

the depinning frequency \( f_p \)

\[
f_p = \frac{\omega_p}{2\pi} = \frac{f}{\rho_{m2} - \rho_2},
\]

(2.69)

the pinning constant \( \alpha_p \)

\[
\alpha_p = B\Phi_0 \omega \frac{\rho_{m2} - \rho_2}{|\rho_{m} - \rho_s|^2},
\]

(2.70)

the flux line viscosity \( \eta_{fl}(T) \)

\[
\eta_{fl} = B\Phi_0 \frac{\rho_{m1} - \rho_1}{|\rho_{m} - \rho_s|^2},
\]

(2.71)

and the flux-flow resistivity \( \rho_{ff} \)

\[
\rho_{ff} = \frac{|\rho_{m} - \rho_s|^2}{|\rho_{m1} - \rho_{s1}|},
\]

(2.72)

where \( \rho_m = \rho_{m1} + i\rho_{m2} \) is the mixed state resistivity, and \( \rho_s = \rho_1 + i\rho_2 \) is the zero field resistivity.
Chapter 3

Microwave Measurements

As shown in Chapter 2, to evaluate the flux-flow resistivity and the pinning constant, the complex surface impedance must be measured precisely. In our study, this has been achieved using a carefully designed apparatus based on the microwave cavity perturbation technique.

This chapter introduces the basic aspects of our experiment configuration. Section 3.1 introduces the idea of cavity perturbation. Section 3.2 presents the main structure of the experiment apparatus, and Section 3.3 describes the modifications to it. Section 3.4 includes relevant theoretical simulations. Section 3.5 focuses on data acquisition, while Section 3.6 contains the calibration of the apparatus performance and the error analysis. This chapter ends with Section 3.7, which summarizes the data processing procedure.

3.1 The Cavity Perturbation Technique

In our study, all measurements are based on the cavity perturbation technique[94–96], which is one of the most precise ways to measure electrical transport properties. It allows us to measure the surface impedance, which can be converted into complex resistivity for the extraction of flux-flow parameters.

The simplest example of a cavity perturbation apparatus is a textbook hollow cavity, usually made from a good conductor like copper. Microwave fields form standing waves (resonant modes) inside the cavity, as those conducting walls impose boundary conditions to the Maxwell equations. A single resonant mode has two characteristic parameters: the resonant frequency $f_0$, and the bandwidth $f_B$. The resonant fre-
Figure 3.1: Magnetic field lines with (a) sample out of and (b) sample in the cavity. The size of the sample has been exaggerated for clarity. The presence of the sample leads to shifts in the resonant frequency $f_0$ and bandwidth $f_B$. Results were calculated using the finite element analysis software COMSOL.

Frequency depends on the geometry of the cavity, while the bandwidth, which is inversely proportional to the quality factor $Q$, is related to how much energy is dissipated per cycle. Combined with a microwave input and an output, the system is in analogy to a resistor-inductor-capacitor (RLC) circuit, in which the total "impedance" of the cavity is directly related to the shifts $f_0$ and $f_B$. In practice the resonator configuration is usually more complicated than an empty cavity, but the idea remains the same.

We then make a perturbation. In our case this is the presence of a sample inside the cavity. Figure 3.1 shows the basic idea of cavity perturbation. In general, analytically solving the field distribution with a sample inside the cavity is impossible, but if the size of the sample is much smaller than the cavity, its effect can be treated perturbatively. Away from the sample the field profile remains unchanged, while at the vicinity of the sample the field lines are distorted. The simplest example is a perfectly conducting sample inside the cavity. Flux lines will be excluded from the sample, resulting in a reduced effective volume of the cavity, which corresponds to a positive frequency shift $\Delta f'_0$ but no bandwidth shift. If we now imagine that the sample resistivity is gradually turned on, the field lines will penetrate into the sample,
resulting in a slightly increased effective volume, or another resonant frequency shift $\Delta f_0$. At the same time, part of the microwave energy will be dissipated by the induced current on the sample surface, resulting in a shift in the quality factor, or resonance bandwidth $\Delta f_B$, as shown in Figure 3.2.

![Figure 3.2: Frequency shift and bandwidth shift before (black line) and after (blue line) introducing the sample in the cavity. The presence of the sample increases the resonant frequency and the bandwidth.](image)

A detailed discussion about the frequency and bandwidth shifts can be found in reference[94], in which the central result is the cavity perturbation relation:

$$\Delta f_0 + i \Delta f_B / 2 = \left\{ \frac{i}{2\pi} \int_S \Delta Z_s H_1 \cdot H_2 dS - f_0 \int_V [\Delta \mu H_1 \cdot H_2 + \Delta \epsilon E_1 \cdot E_2] dV \right\} / 4U,$$

(3.1)

where $U$ is the energy stored in the resonator, subscripts 1 and 2 correspond to the configurations before and after the sample is introduced, and $S$ and $V$ correspond to the surface and the volume of the resonator. The surface impedance $Z_s$ defined in Chapter 2, is related to the frequency shift $\Delta f_0$ and the bandwidth shift $\Delta f_B / 2$. 

56
Microwave Measurements

The cavity perturbation relation can be further simplified in our configuration, in which the sample experiences a uniform field in its vicinity. As a good approximation, the volume integral in the cavity perturbation relation can be ignored once the sample position is fixed. Thus the cavity perturbation relation becomes

$$\Delta f_B/2 - i\Delta f_0 \approx \Delta Z_s/\Gamma,$$  \hspace{1cm} (3.2)

where $\Gamma = 8\pi U/\int_S H_1 \cdot H_2 dS$ is called the resonator constant, or filling/scaling factor. As mentioned in Chapter 2, the complex resistivity $\rho = 1/\sigma$ can be related to the surface impedance by

$$Z_s = \sqrt{i\mu_0 \omega (\rho_1 + i\rho_2)} = \sqrt{\frac{i\mu_0 \omega}{\sigma_1 - i\sigma_2}}.$$  \hspace{1cm} (3.3)

Since all of our samples are in the local electrodynamic limit, their complex resistivities or complex penetration depths can be extracted from the frequency and bandwidth measurements.

In the normal state, where $\rho_2 \ll \rho_1$, the surface impedance becomes:

$$R_s = X_s = \sqrt{\frac{\mu_0 \omega \rho_n}{2}},$$  \hspace{1cm} (3.4)

which is the classical skin effect.

### 3.2 Structure of the Experimental Apparatus

In principle, the technique of microwave cavity perturbation should have enabled people to investigate the flux-flow resistivity of the cuprates on the first day that samples were available. Yet for a long time such experiments were relatively few and the results were not satisfying by modern standards. One of the main challenges is the resolution, because in the superconducting state $R_s$ is vanishingly small. On the other hand, high resolution also amplifies certain systematic errors, which need to be minimized to ensure meaningful results. Other requirements include the ability to operate in an applied field and the ability to reach sufficiently low temperature.

These requirements have been met with the carefully designed structure of our apparatus, which is shown in Figure 3.3. The core of the apparatus is a dielectric resonator in the middle of a cylindrical cavity, which ensures a high $Q$ for the resonant
Figure 3.3: The structure of the microwave perturbation probe. The key component, a rutile resonator, is introduced in the middle of a copper enclosure, to form resonant modes with high quality factors. The sample, mounted on the end of a sapphire hot-finger, can be moved into the cavity to introduce a perturbation.
Microwave Measurements

modes without the use of superconducting coatings. The quality factor $Q$ is determined by the fraction of energy dissipated per cycle, and the majority of dissipation $P$ comes from the metal walls:

$$P = \frac{1}{2} \int_S R_{\text{wall}} H_{\text{wall}}^2 dS,$$

where $R_{\text{wall}}$ is the surface resistance of the metal walls, and $H_{\text{wall}}$ is the magnetic field at the wall surfaces. A common way to reduce the dissipation is to use highly conductive materials to reduce $R_{\text{wall}}$. In our case, copper is used, which should ensure an intrinsic $Q$ around $10^4$ in an empty cavity at room temperature. To further improve the quality factor, superconducting coatings are often applied to the walls. Nevertheless, this is not a practical option in our case, as the superconductivity would be suppressed by a magnetic field.

What the rutile ($\text{TiO}_2$) dielectric resonator does is to alter the field profile, and to reduce $H_{\text{wall}}$. The resonator has a cylindrical shape to match the cavity symmetry, with an axial hole to contain the sample. Because of rutile’s high dielectric constant (> 100), the microwave energy density is largely concentrated around it, and consequently the fraction of energy dissipated in the copper walls is reduced. The intrinsic $Q$ of good rutile material is larger than $10^8$, and quality factors of $10^6$ to $10^7$ are typically achieved in the mode used for microwave spectroscopy.

In the absence of the rutile resonator, due to the cylindrical symmetry of the boundary condition, numerous $\text{TE}_{mnp}$ modes form inside the cavity, where the subscripts $m$, $n$ and $p$ correspond to the number of nodes along the $\hat{\theta}$, $\hat{r}$ and $\hat{z}$ directions respectively. Our focus is on modes with $m = 0$ and odd $n$. When $m = 0$, the TE modes have no electric field along the $\hat{z}$-axis, minimizing the impact of any potential dielectric component near the axis, such as a sample holder. When $n$ is odd, the magnetic field along the $\hat{z}$-axis has an anti-node at the center of the cavity. The platelet sample is located at the anti-node, and experiences a strong and nearly uniform magnetic field over the small size of the sample, usually millimeters in dimension.

In general, introducing the rutile resonator into the empty cavity would drastically change the structure of resonant solution. In our case, the rutile resonator also has cylindrical shape, and is located at the center of the cavity, so the rotational symmetry and reflectional symmetry above the mid-plane of the resonator are not broken. The cavity resonant modes hybridize with the rutile resonant modes to form new $\text{TE}_{mnp}$ modes, and their quality factors become substantially higher and their frequencies
Microwave Measurements

much smaller, since the majority of the displacement current now flows within the rutile dielectric. Strictly speaking, the integer $p$ component would be replaced by $p - \delta$, because the boundary condition in the top and bottom walls of the rutile is not abrupt. Nevertheless, the $\text{TE}_{mpn}$ notation will be kept in this study for convenience.

Ideally, there is no breaking of reflection symmetry along the $\hat{i}$ axis, from introducing the rutile itself. Nevertheless, the rutile resonator has to be mechanically supported by a round, thin sapphire plate, which then has to be mechanically supported by a small copper "step" underneath. In addition, there are openings on the top and bottom of the copper enclosure. Therefore, the boundary conditions are not simple, and reflection symmetry along the $\hat{z}$-axis is broken. Empirically, the field distributions along the $\hat{z}$-axis for most of the modes are not substantially altered from the symmetric case, as will be shown in Section 3.4.

The sample is mounted at the end of a sapphire rod. Sapphire is chosen because it is almost transparent to the microwave fields, is a very good thermal conductor, and has very little thermal expansion at low temperature. Although not perfect, these properties greatly reduce potential systematic errors. As shown in Figure 3.4, the sample will be oriented so that its $\hat{c}$-axis is parallel to the $\hat{z}$-axis, allowing surface impedance for current flowing in the $ab$-plane to be measured. It needs to be mentioned that such arrangement introduces a demagnetization field: the total effective magnetic field is enhanced near the edge of the sample, and the cavity perturbation resonant constant is modified. Nevertheless, the cavity perturbation relation remains valid, as long as the perturbation is viewed as arising from a change in sample temperature.

During experiments, the entire cavity is submerged in a liquid helium bath, which ensures a stable cavity temperature of 4.2 K. Prior to this, the cavity should be pumped to a pressure of $10^{-6}$ torr at room temperature. The vacuum is further improved by a sorption pump mounted below the resonator, resulting in excellent thermal isolation between the cavity and the sample. The sapphire rod serves as a hot finger\cite{97, 98} to thermally connect the sample to a small sapphire thermal platform outside the resonator, at which a thermometer and a resistive heater are attached. The temperature of the sapphire platform and consequently the sample are regulated by a Cryocon 62 AC resistances bridge, with a precision of $10^{-4}$ K typically achieved at low temperature. The sapphire platform is weakly thermally connected to a small 1-K pot by a thin-walled quartz tube. The 1-K pot is initially filled with
overpressurized helium gas, which is then liquefied at a temperature slightly higher than 4.2 K by thermal contact with the liquid helium bath. By pumping on the 1-K pot, a temperature as low as 1.0 K is subsequently achieved. The sample can be moved into or out of the cavity by the mechanical motion of a drive rod, operated from room temperature. In our experiment, the drive rod is made of low thermal conductivity G10 fiberglass.

A pair of coaxial lines is located on each side of the cavity (side coupling). Both of them are terminated in loops, which serve as the microwave input and output to the cavity by converting the time dependent magnetic flux $d\Phi/dt$ into a voltage signal. This signal is processed by a vector network analyzer, and the data taking protocol is controlled by a LabVIEW program.

The cavity is positioned inside the bore of a superconducting magnet, capable of providing magnetic fields as high as 8 T. The cavity axis is aligned with the cylindrical axis of the magnet coils, and the sample is located near the midpoint of this axis, where the magnetic field is highly uniform. We are always careful to field cool samples before taking any measurements, so that the internal flux will be close to its equilibrium distribution.
3.3 Upgrading of the Apparatus

The original apparatus was built around 2001, and has functioned well up to the present with some minor modifications.

The biggest modification made in this work was a change of the dimensions of the dielectric resonator and its enclosure. Initially, a smaller rutile resonator and enclosure were used, the dimensions of which can be found in Ben Morgan’s thesis[81] or in Wendell Huttema’s paper[99]. A TE\(_{011}\) mode with a resonant frequency of 5.47 GHz was used and a quality factor of \(10^6\) was achieved. Data for our first sample, the underdoped \(\text{YBa}_2\text{Cu}_3\text{O}_{6.333}\) sample, were taken in this configuration. From now on, we call it configuration A for convenience.

Later on, two larger resonators, made from similar rutile material, were acquired. Both are 10 mm in height and diameter, with a hole 3 mm in diameter along the cylindrical axis. Since rutile is electrically anisotropic, the orientation of the optical axis is very important, and ideally it should match the cylindrical axis. Of the two resonators, only one satisfied this condition and was used for most of our measurements. This large resonator was first adapted into a large enclosure to get better resolution than that in configuration A, and a quality factor 4 times larger was achieved in the main working TE\(_{011}\) mode at a resonant frequency of 2.64 GHz. We call this configuration B for convenience. A sketch of this configuration is presented in the left panel of Figure 3.5. Because the positions of the microwave coaxial lines remained unchanged, the microwave coupling loops were loaded directly into the larger enclosure from the top, instead of outside the smaller enclosure (see Figure 3.3). Such an arrangement was later found to cause strong microwave direct coupling between the input and the output. While this can be minimized by carefully adjusting the orientation of the coupling loops, this process is very time-consuming. In addition, the higher order modes data were found to be very sensitive to the apparatus vibration. Therefore, this configuration was only briefly used for measuring our second sample, the ortho-II \(\text{YBa}_2\text{Cu}_3\text{O}_{6.52}\) sample.

The final configuration (called configuration C) was set up with the large rutile resonator inside a small enclosure, the dimensions of which are shown in the right panel of Figure 3.5. Microwave coupling loops were positioned outside the enclosure, the same as in Figure 3.3. This resulted in smaller quality factors but still provided reasonable resolution. Most of our data have been taken in this configuration, and a total of 12 resonator modes have been used. The majority of our results have been
Microwave Measurements

extracted from data taken in the TE_{011} mode, which has a resonant frequency of 2.79 GHz and a quality factor of 10^5.

Figure 3.5: (a) The sketch of cavity configuration B, in the \( \hat{r} - \hat{z} \) plane. (b) The sketch of cavity configuration C[100] is reprinted with the kind permission of Paul Carriere. A notable feature in this configuration is the large "width" of the "step" below the sapphire plate, which is negligible in configuration B. In both figures, the cavity axis is represented by the dashed line.

Another modification was made in 2007, when we had a major vacuum leak near the end of the 1-K pot. It was determined that this was probably due to the tension between different parts of the drive rod when it underwent thermal contraction at low temperature. We took this opportunity to change several other parts during the repair, including:
1. Replacing the stainless steel drive rod by a fiberglass one, of a much smaller thermal conductivity;
2. Replacing the soldered stainless steel foil in the 1-K pot by a carefully machined stainless steel tube as the thermal weak link;
3. Chemically etching the copper cavity to remove potential mechanical damage, reducing the energy loss at the copper walls.

These modifications improved the low temperature performance. Originally, the 1-K pot achieved a lowest temperature of 1.2 K, but after the upgrade it could reach
Microwave Measurements

1.0 K. The performance has also been verified to be far more stable to apparatus vibration.

### 3.4 COMSOL Simulation of the Resonant Modes

In our configuration, numerous modes exist in the cavity, including the TE modes, the TM modes and more complicated modes with azimuthal number \( m \neq 0 \). The main concern with these other modes is that they might interfere with our desired TE modes. This becomes more and more troublesome as frequency increases, because the density of background modes is very high at high frequency, making the prediction and identification of the desired modes difficult. Exact electrodynamic solutions in our resonator geometry are impossible to obtain, and visualization of the field profile is highly desired. A finite element analysis software package called COMSOL has been used to obtain a numerical solution to this problem.

![Simulated field profiles](image)

**Figure 3.6:** Simulated field profiles for (a) the TE\(_{011}\) mode and (b) the TE\(_{013}\) mode. The rutile resonator is represented by the yellow rectangles, sapphire by the blue rectangles, and the sample shown as a black dot. In this work, the samples are usually not spherical, but are platelets.

Since the TE modes are our focus, we take advantage of the rotational symmetry of the cavity to reduce the 3-D simulation to a 2-D one. The cavity walls have been
Microwave Measurements

treated as perfect conductors, and the openings on the top and the bottom of the cavity, as well as the opening for the microwave coaxial lines on the sides have been neglected. The step (see Figure 3.5) below the sapphire plate has been included in the simulation, but it seems to have little impact to the simulated results. An isotropic in-plane dielectric constant $\varepsilon_r$ of 106.8 has been used for rutile, and $\varepsilon_r = 9.4$ used for sapphire. Examples of the simulated field profiles are shown in Figure 3.6.

Table 3.1: The comparison between simulated resonator frequencies and observed resonator frequencies. The modes that do not agree well with the simulations are marked with “??”.

<table>
<thead>
<tr>
<th>Modes</th>
<th>Simulated Frequency (GHz)</th>
<th>Observed Frequency (GHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>TE$_{011}$</td>
<td>2.7905</td>
<td>2.7902</td>
</tr>
<tr>
<td>TE$_{013}$</td>
<td>4.5582</td>
<td>4.5579</td>
</tr>
<tr>
<td>TE$_{021}$</td>
<td>5.7853</td>
<td>5.7800??</td>
</tr>
<tr>
<td>TE$_{015}$</td>
<td>6.9612</td>
<td>6.9460</td>
</tr>
<tr>
<td>TE$_{023}$</td>
<td>7.1360</td>
<td>7.1330</td>
</tr>
<tr>
<td>TE$_{025}$</td>
<td>9.1098</td>
<td>9.0943</td>
</tr>
<tr>
<td>TE$_{031}$</td>
<td>9.3325</td>
<td>9.2982</td>
</tr>
<tr>
<td>TE$_{033}$</td>
<td>10.3650</td>
<td>10.3680??</td>
</tr>
<tr>
<td>TE$_{035}$</td>
<td>11.9729</td>
<td>11.9596</td>
</tr>
<tr>
<td>TE$_{041}$</td>
<td>13.1560</td>
<td>13.1240</td>
</tr>
<tr>
<td>TE$_{043}$</td>
<td>14.0145</td>
<td>13.9902</td>
</tr>
<tr>
<td>TE$_{055}$</td>
<td>15.3068</td>
<td>15.2602</td>
</tr>
<tr>
<td>TE$_{051}$</td>
<td>17.1046</td>
<td>17.0390</td>
</tr>
<tr>
<td>Unknown</td>
<td>17.7497</td>
<td>17.7330</td>
</tr>
<tr>
<td>Unknown</td>
<td>18.9210</td>
<td>18.9320</td>
</tr>
</tbody>
</table>

From the simulated field profile, modes can be identified by counting the nodes and anti-nodes along the $\hat{z}$-axis and the $\hat{r}$-axis. Experimentally, these modes have been confirmed by a bead-pull technique, which measures the bandwidth and frequency shift signals as a function of sample position along the $\hat{z}$ axis. Since $\Delta f_B$ is expected to be proportional to $|H|^2$, this technique measures the field profiles, which can then be compared with those of the simulation. The simulated resonant frequencies turn
Microwave Measurements

out to agree well with the empirical observations, as shown in Table 3.1. These are the results obtained under configuration C, but agreement between simulations and observations has also been confirmed on configuration B.

Comparisons between the simulated and the observed field profiles along the cavity axis are shown in Figure 3.7. For most modes the agreement is excellent, with the exception of the TE$_{021}$ and TE$_{033}$ modes. While it is possible that the mode identification is incorrect, the suspect TE$_{021}$ mode empirically provides reasonable data. In addition, at 5.78 GHz there are not many background modes that can interfere with the TE$_{021}$ mode, giving further confidence that this is a correct identification.

From the simulation it can be concluded that the lowest mode is the most stable one to small shifts in sample position, which agrees with experimental observation. The higher order modes are not as stable, but they may be used in the investigation of the frequency dependence of the surface impedance. Data have been taken from a maximum of 9 different modes at once.

It is worth mentioning that although my work is limited to TE and TM modes, investigation of modes with $m \neq 0$ has been made by Paul Carriere[100], whose theoretical simulations agree with the experimental observations. This required the field equation to be cast in a very different form, and the ability to do this at all for $m \neq 0$ is a relatively recent development[101].

3.5 Data Acquisition

This apparatus enables us to acquire the sample surface impedance as a function of temperature and applied field, using the cavity perturbation relation

$$
\frac{\Delta f_B(B, T)}{2} - i \Delta f_0(B, T) = \frac{\Delta Z_s(B, T)}{\Gamma},
$$

where $\Delta$ corresponds to a change from the background signal. The background is defined to be an experimental configuration in which all the physical parameters are fixed as constants, including the applied field, sample position, microwave field strength, cavity temperature, sample temperature and so on. To be specific, our cavity perturbation process can be divided into two steps, and each of them corresponds to a different background. The first step is to introduce a sample at temperature $T_0$ and magnetic field $B_0$ into the empty cavity, and the background is the empty cavity. For the second step, we sweep the temperature of the sample to measure a
Microwave Measurements

Figure 3.7: Comparison of the simulated field profiles with the observed ones along the cavity axis. Position means the distance along the $z$-axis from the cavity center, and the energy densities have been normalized.
Microwave Measurements

temperature dependent signal, which is called the temperature sweep process. In this case, the background is the cavity with a sample at temperature \( T_0 \). Equation 3.6 can be rewritten as

\[
\left[ \frac{f_B(B_0, T)}{2} - \frac{f_B(B_0, T_0)}{2} \right] - i [f_0(B_0, T) - f_0(B_0, T_0)] = \frac{Z_s(B_0, T) - Z_s(B_0, T_0)}{\Gamma},
\]

where \( T_0 \) is the background temperature, usually chosen to be 5 K. It can be seen from Equation 3.7 that if we know the resonator constant \( \Gamma \), the absolute bandwidth shift \( f_B(B_0, T_0) \) and absolute frequency shift \( f_0(B_0, T_0) \), the absolute surface impedance \( Z_s(B_0, T) \) can be derived.

3.5.1 Determination of Resonator Constant

To obtain the surface impedance from the observables (bandwidth shift and frequency shift), the resonator constant \( \Gamma \), which depends on the geometries of the sample and cavity, needs to be known. A common way to obtain the resonator constant is by comparing the raw normal state data with known normal resistivity. From Equation 3.2 and Equation 3.4 we have

\[
\Gamma = \frac{\sqrt{2\mu_0\omega\Delta\rho_n}}{\Delta f_B}.
\]

In general, the resistivity includes a residual term which is sample dependent. Therefore, the inelastic part of the resistivity is compared, which is not generally sample dependent. For cuprates, the inelastic part of the resistivity is also doping dependent, and it is very difficult to find reliable resistivity data for a sample with exactly the same doping as that of our sample. Most of time, we use the data of a sample with a doping close enough to that of our sample. The resulted uncertainty of the resonator constant has been estimated to be less than 2.5% in this case[99].

For some samples, there may be no published resistivity data available, so a replica sample made from materials with known resistivity has to be used. Since the resonator constant mainly depends on sample geometry, as long as the sample and the replica have the same shape and size, their resonator constants will be the same. The resonator constant for our ortho-II \( \text{YBa}_2\text{Cu}_3\text{O}_{6.52} \) sample was obtained this way, using a Nb replica sample. The inaccuracy to exactly reproduce the sample geometry in this case has been confirmed to have little impact to the obtained resonator constant, as verified in Section 5.2.
3.5.2 Determination of Absolute Surface Resistance

The bandwidth shift of introducing a sample into the cavity gives us the absolute surface resistance at temperature $T_0$ and applied field $B$:

$$R_s(B, T_0) \approx \Gamma \frac{f_B(\text{in}) - f_B(\text{out})}{2}, \quad (3.9)$$

where “in” and “out” correspond to the sample location. In practice, the bandwidth shift between the “sample in” and “sample out” cases at a given temperature and an applied field is usually measured in a power sweep process. Higher input microwave power can give us stronger signal, but it can also drive nonlinearities in the sample or resonator, arising from self-heating; spin susceptibility and weak links acting as Josephson junctions. Typical power sweep data are shown in Figure 3.8. At high power, there is an upturn in bandwidth at high power, implying that nonlinearities are substantial. At lower power, uncertainties in the data increase, due to the weaker signals. To optimize the trade-off between signal-to-noise ratio and accurate temperature measurement, we sweep the input power to find an optimal value for our experiment, with other conditions held fixed. During the temperature sweep process, the microwave output power is held fixed at the optimal value, which guarantees that the microwave field strength in the resonator remains fixed. Under the same power, the absolute surface resistance can be obtained. Most of the time, the part of the signal power dependence due to the sample is comparable to the statistical noise.

Adding $R_s(B, T_0)$ to the $\Delta R_s(B, T)$ signal obtained in the temperature sweep process gives the absolute $R_s(B, T)$. As a consistency check for $R_s(B, T_0)$, $R_s(B, T)$ should have little dependence on the static magnetic field in the normal state. This is because the magnetoresistance of the cuprates is usually small.

It needs to be emphasized that the accuracy of the absolute surface resistance measurement is vital to our experiment. For example, in one Tl$_2$Ba$_2$Cu$_{6+x}$ sample, the surface resistance was very small, even in the mixed state. In this case, extracted results were extremely sensitive to the small uncertainty of the absolute surface resistance, and incorrect estimation affected the qualitative form of our conclusions. Even with our high resolution, the data for that particular sample were not as reliable as others and will not be presented in this work.
Figure 3.8: A typical power sweep data set. The difference between the “sample in” data and the “sample out” data is the absolute bandwidth shift.

3.5.3 Determination of Absolute Surface Reactance

The absolute surface reactance cannot be obtained in the same way as the absolute surface resistance, because even a perfectly conducting sample with $X_s = 0$ has a finite volume that will shift the resonant frequency by a positive $f_0'$. It is impossible to get absolute $X_s(B_0, T_0)$ by subtracting the empty cavity signal from the “sample in” signal, as the latter contains an additional term $f_0'$ which cannot be directly measured.

Instead, a common way of obtaining the absolute $X_s$ is to match the zero field $X_s$ to the zero temperature penetration depth $\lambda_0$, since these two are directly related at low temperature:

$$X_s(T \to 0) \equiv \mu_0\omega_0\lambda_0.$$  \hfill (3.10)

Our data have mainly been offset this way. The main concern about this approach is that accurate measurements for the penetration depth are extremely challenging, and are known to be weakly sample dependent. The uncertainty in penetration depth can be as large as 20% and surpass all other forms of systematic error. Thus our results should be regarded as somewhat qualitative in this respect, and particular conclusions should be verified to be insensitive to our choice of penetration depth. We stress that this is a robust approach used by microwave experimentalists – for
Microwave Measurements

instance, the most decisive early evidence for d-wave superconductivity in cuprates, was provided by showing a qualitative linear temperature dependence in superfluid density, at different choices of $\lambda_0[46]$.

For some samples, there are no reliable $\lambda_0$ data available, and an alternative approach has been used. For the optimally doped YBa$_2$Cu$_3$O$_{6.95}$ sample studied by Ben Morgan[81], the absolute $X_s$ was obtained by matching $R_s$ and $X_s$ in the normal state:

$$R_s = X_s = \sqrt{\mu_0 \omega_0 / 2 \sigma_{DC}}.$$  (3.11)

This approach has also been used for the underdoped YBa$_2$Cu$_3$O$_{6.333}$ sample in this study[102]. It has the advantage of obtaining the absolute value for $\lambda_0$, but requires the c-axis current contribution (see Section 3.6.4) to be quantified, which is almost impossible for many samples.

Once we have the absolute zero field $X_s(0,T)$, the in-field $X_s$ can be offset by matching $X_s(B,T)$ and $X_s(0,T)$ in the normal state, as $X_s$ should have no strong field dependence in the normal state. The signal induced by the magnetoresistance is removed by subtracting $R_s(B,T) - R_s(0,T)$ from the effective $X_s(B,T) - X_s(0,T)$, as $R_s = X_s$ in the normal state.

3.6 Calibrating the Apparatus Performance

In our experiment, notable error sources include signal drift, the effect of paramagnetic impurities in the resonator and sample holder, the thermal expansion of cavity components and the sample, as well as the c-axis current contributions arising from imperfect sample alignment.

3.6.1 Stability against Signal Drift

A common error in microwave experiments is the signal drift. It may originate from the change in the liquid helium bath level, and consequently its influence on the microwave cables. This results in drifts in the effective frequency shift and bandwidth shift. Since the resonant bandwidths are of the order of several kHz, while the frequency is of the order of several GHz, the frequency drift is usually much larger than the bandwidth drift. At the same time the magnitudes of the surface resistance and the surface reactance are usually comparable, which makes frequency drift a far more
Microwave Measurements

serious problem than the bandwidth drift. Frequency drift is not predictable and hard to counter, and has been a major challenge in earlier microwave flux-flow resistivity studies. In some cases, due to the huge fluctuations in $f_0$, $X_s$ had to be assumed as zero in the data extraction, and resulted in misleading conclusions.

This kind of error has been minimized by our apparatus design: the sample is well isolated from the rest of the cavity by vacuum, so changes in sample temperature have little impact on the cavity; the rutile resonator has inherently high dimensional stability; the cavity itself is in direct contact with a liquid helium bath, so after it reaches thermal equilibrium with its environment, there should be little temperature fluctuation. Nevertheless, as our resolution is very high, these sources of uncertainty still need to be quantified.

To monitor the background drift, we took temperature sweep data with respect to a reference temperature. Every ten data points or so, we returned to the background temperature (usually 5 K) and took a data point there. Figure 3.9 shows typical background drifts following a helium transfer. The bandwidth drift was not larger than $10^{-1}$ Hz per hour and could safely be ignored. The frequency drift, on the other hand, was initially large, but eventually became much smaller (about 1 to 10 Hz per hour) after several hours. It is very likely that this frequency drift was due to the slow approach of the system to thermal equilibrium. The resonator then remained relatively stable for more than 10 hours, until the helium bath level approached the top of the cavity. In this case, the cavity warmed up, and resonant frequency drifted again, and the helium bath had to be refilled. To reduce the impact of frequency drift, most of our data were taken when the system was relatively stable.

The signal drift as a function of time was subtracted from the raw data. The background signal is obtained by interpolating between the background data points. Because the temperature dependence of the sample signal is usually very weak at low temperature, to reduce systematic error from the signal drift interpolation, most of the low temperature data have been taken in the most stable regime, where the drift is comparable to the statistical noise. In addition, background data have been taken more frequently in this temperature range. At high temperature, the signal is much larger and the time it takes to return to the background temperature becomes longer. So, as a compromise, less background data have been taken. Typically, at high temperature, background data have been taken every 10 minutes, while at low temperature every 3 or 4.
Figure 3.9: Typical (a) bandwidth drift and (b) frequency drift for the $\text{TE}_{011}$ mode. Sample temperature was held at 5 K.

In this case, the error from signal drift has been estimated to be of the order of 0.1 Hz/K, which can be tolerated. And it is not too much of a problem in the mixed state data, either. In an applied field, the raw frequency shift data is typically of the order of 10 to 100 Hz/K in the superconducting state, so the background drift can safely be ignored.

### 3.6.2 Signal Induced by Paramagnetic Impurities

Our apparatus includes several dielectric components: the rutile resonator, the sapphire rod and the sapphire plate. Ideally they should all be transparent to the microwave signal, but in reality they all contain small amounts of paramagnetic impurities, which contribute to the effective signal. The contribution of these impurities needs to be considered.

Of the signals induced by the paramagnetic impurities, what we are most concerned about is the temperature dependent and magnetic field dependent part. The contribution from impurities inside the rutile and the sapphire plate that supports the rutile resonator is less of a problem, because they are held at constant temperature. Their contribution comprises part of the temperature independent background and will be subtracted out. On the other hand, the sapphire rod has the same tempera-
Microwave Measurements

ture as the sample, so its temperature dependent contribution needs to be quantified. Figure 3.10 shows data taken with only the bare sapphire rod in the cavity, for 8 different resonant modes.

![Figure 3.10](image)

Figure 3.10: (a) $R_s(T)$ and (b) $X_s(T)$ with a bare sapphire rod in the cavity. Data were taken at zero field and multiple frequencies. All data sets have their bandwidth shifts at 5 K offset to zero for comparison.

With the bare sapphire rod in the cavity, there is a notable temperature dependent shift in the resonance frequency. For the TE$_{021}$ (5.78 GHz) and the TE$_{035}$ (11.959 GHz) modes there are also some slightly discernible absolute bandwidth shifts. As shown in Figure 3.11, the temperature dependence of the bandwidth shift is very small, almost negligible compared with the signal from the sample. However, the temperature dependence in frequency shift cannot be ignored. Although the low temperature upturn or downturn only has a magnitude of 10 to 100 Hz, it can be comparable to the sample signal in certain samples.

These upturns and downturns have a strong field dependence. They can be weakened, and eventually suppressed by an increasing magnetic field, as shown in the left panel of Figure 3.12. In the temperature sweep data these upturns and downturns fit well to a $1/(T + T_0)$ temperature dependence of the Curie Law, which implies the existence of electron spin resonance (ESR) transitions. Additional evidence can be found in the field sweep data, where we held the sapphire rod at 5 K and varied
Figure 3.11: Comparison of the bare sapphire rod (a) bandwidth and (b) frequency shift signal with the sample (which has been mounted on the sapphire rod) signal.

the applied fields, as shown in the right panel of Figure 3.12. Signatures of the ESR transitions are clear.

A common solution to get rid of these spurious effects is taking the sapphire rod data at different fields and subtracting its contribution from the data. It is nevertheless better to understand the underlying physics.

ESR transitions in cuprates have been well studied in microwave experiments. Dr. A. Janossy[103] and the UBC superconductivity group have both done a lot of work on ESR spectroscopy in cuprate superconductors, and detailed information has been presented by Patrick Turner[104]and Tamar Pereg-Barnea[105]. As to ESR on impurities in sapphire, researchers in the University of Western Australia[106–108] have studied it extensively. In short, for paramagnetic impurities inside these materials, the susceptibility is modified in the presence of ESR transitions, and thus the resonant frequency is shifted:

$$\frac{\Delta f}{f_0} \approx \sum \Gamma_\omega \chi_{0j} g(\omega, \omega_j),$$

(3.12)

where the subscript $j$ corresponds to the $j$th ESR mode, $\Gamma$ is the resonator constant, and $\chi_{0j}$ is the DC paramagnetic susceptibility, which is proportional to the impurity concentration and follows the Curie law. The frequency dependence of the Lorentzian
shaped \( g(\omega, \omega_i) \) is given by

\[
g(\omega, \omega_i) = \frac{\omega_j (\omega_j - \omega) \tau_j^2}{1 + (\omega_j - \omega)^2 \tau_j^2},
\]

where \( \tau_j \) is the spin-spin relaxation time, and \( \omega_j \) is the \( j \)th ESR frequency. Clearly \( g(\omega, \omega_i) \) can cause upturns for cavity modes at frequencies lower than \( \omega_j \), and downturns for modes with higher frequencies. Although a number of ESR transitions may exist, the dominant term will generally come from the ESR mode that is closest to the cavity mode. In our data, such a dominant ESR mode is apparently located between 11.96 GHz to 13.99 GHz.

Since we cannot vary our cavity resonant frequency continuously, we did not follow the UBC group’s practice of zero field ESR spectroscopy. The method used to identify the ESR transition is by continuously sweeping the applied field to locate the ESR peak. Assuming in zero field we have a ESR frequency:

\[
h\omega_0 = E_1 - E_2,
\]

then after applying a field, Zeeman splitting breaks the degenerate energy levels and
Microwave Measurements

creates new ESR frequencies:

\[ \hbar \omega_n = (E_1 + g\mu_B J_{z1}H) - (E_2 + g\mu_B J_{z2}H), \]  

(3.15)

where \( J_{z1}, J_{z2} \) are the quantum numbers for the magnetic moments in the \( z \) direction. Subtraction of these two equations gives the zero field transition frequency \( \omega_0 \), if \( J_{z1} - J_{z2} \) and the Lande factors \( g(i, j) \) are known:

\[ \omega_0 = \omega_n - \frac{g(i, j)\mu_B H(J_{z1} - J_{z2})}{\hbar}. \]  

(3.16)

The ESR peaks in the field sweep data are shown in Figure 3.13. Without the knowledge of \( (J_{z1} - J_{z2}) \), the identification of the ESR transition has been done by comparing the field sweep data for modes with close frequencies. Assuming that similar peak structures in these data correspond to the same ESR transitions, and assuming a Lande factor \( g = 2 \) (if the electron spin is the only source of angular momentum), the dominant ESR transition is estimated to occur around 12.04 GHz, which is the ESR frequency for Fe\(^{3+} \) ions, a common type of impurity in sapphire[109].

Figure 3.13: Field sweep data for the sapphire rod (a) at high frequency modes; (b) at low frequency modes.

Although we are not certain if other kinds of impurities exist, in principle this identification helps us to qualitatively predict the magnitudes of those upturns and
Microwave Measurements
downturns as a function of the applied field. It has been estimated that an applied field of 1 T can shift the ESR frequency by about 30 GHz, so at high field the dominant ESR peak should be far away from our working frequency. In this case, given the Lorentzian shape of the ESR peak, the ESR contribution to the effective signal should be much less than the statistical noise.

Another implication of the ESR absorption is that it would modify the absolute surface resistance measurements as well, especially for those close enough to ESR peaks. In certain resonance modes (for instance, the TE₀₃₅ mode), the ESR contribution can be comparable to the sample signal. Fortunately, in our main working TE₀₁₁ mode the ESR contribution is comparable to statistical noise and can be neglected. The reason, I believe, is that this mode has a resonant frequency of 2.79 GHz, far enough away from the dominant ESR peak at 12 GHz. With subtraction of the sapphire signal, other modes may be used as well.

3.6.3 Signal Induced by Thermal Expansion

During a microwave experiment, as we heat up the sample, other components which are thermally connected to the sample, especially the sample holder, are heated as well. In our case, the sample holder is the sapphire rod, which is thermally connected to a thin sapphire plate and a weak quartz thermal link. All these components have very small thermal expansion coefficient and weakly temperature dependent dielectric constant, but again, because of our high resolution, their contributions cannot automatically be ignored, especially for the frequency shift. The reason is that the presence of the sapphire rod has little contribution to the bandwidth shift but a more substantial contribution to the frequency shift, as indicated by the cavity perturbation relation:

\[ \Delta f_0 \approx -\frac{f_0}{4U} \int (\epsilon_r - 1)\epsilon_0 E_1 \cdot E_2 dV. \]  

(3.17)

The dielectric constant \( \epsilon_r \), the sapphire rod volume \( V \), as well as the local field distribution \( E \) near the sapphire rod all contribute to the frequency shift and need to be considered. These contributions can be seen in Figure 3.14. A bandwidth shift of the order of 10 Hz at high temperatures does not have any impact on our measurements and can be ignored. As for the frequency shift, the lowest TE₀₁₁ mode seems to be most insensitive to temperature change, but for the higher order modes, the frequency shift is considerably larger and may need to be subtracted.
Microwave Measurements

The subtraction usually does not have any substantial impact on the typical normal state data, where the frequency shift is of the order of $10^6$ Hz. However, in the superconducting state, at intermediate temperature range and in the higher order modes, it might be as large as half of the effective signal from a sample.

![Graphs showing bandwidth and frequency shift data for the bare sapphire rod.](image)

Figure 3.14: (a) Bandwidth and (b) frequency shift data for the bare sapphire rod.

Why is the bare sapphire rod signal so different in the different modes? This fact is very likely tied to their different field distributions along the $\hat{z}$ axis, and it would be interesting to build a complete model of the temperature dependence of this frequency shift. However, since the field profiles for the higher order modes are known to be very complicated, these contributions are quantified perturbatively here.

Take the $\text{TE}_{011}$ mode as an example. At 5 K, the presence of the sapphire rod shifts the frequency by the order of $10^3$ Hz, relative to that of an empty cavity. Using the temperature dependence of dielectric constant found in reference [110], the change in dielectric constant from 4 K to 150 K is estimated to be about $5 \times 10^{-5}$, which should result in a frequency shift of $10^{-2}$ to $10^{-1}$ Hz. It turns out in all the modes, the temperature dependence of dielectric constant only contributes to 1% of the signal at most.

The next to consider is the thermal expansion of the sapphire rod. From 4 K to 150 K, the thermal expansion coefficient for sapphire is[110] is

$$\alpha = 7.5 \times 10^{-13} \ T^3.$$  

(3.18)
Microwave Measurements

The volume change has been estimated to be about 100 parts per million (PPM). Again, this is highly unlikely to be the main cause for the sapphire rod signal.

That leaves us with only one possibility: motion of the sapphire rod. Thermal expansion of the quartz thermal weak-link moves the sapphire rod further into the cavity, and this motion is likely the main cause for the frequency shift. Evidence for this explanation can also be found in Figure 3.14, as there are no other mechanisms that can explain the decrease in the frequency shift for the 5.78 GHz mode. In fact, the decrease implies that the sapphire rod motion has a rotational component in addition to the translational one. Since this effect varies with the field profiles which are very complicated, we use background subtraction to remove its contribution.

Since the lowest TE_{011} mode (2.79 GHz) has the most uniform field distribution, it is regarded as the most reliable one and is our main working mode. Its frequency shift is much smaller than that of other modes, and can be safely ignored. Other resonant modes are also used, but their reliability has to be tested empirically.

Compared with the motion of the sapphire rod, a less significant systematic error source is the thermal expansion of the sample. For the optimally doped YBa_2Cu_3O_{6.95} samples, Ben Morgan has thoroughly modelled and quantified the contribution from the sample thermal expansion in his Ph.D. thesis[81]. However, repeating his analysis for samples used in this study would be extremely difficult, because some of them have no thermal expansion data available, and have irregular shapes that are hard to model. Instead, empirical estimates of the sample thermal expansion are used in this study, based on simplified sample geometries, and measured thermal expansion of related materials. It will be shown that these contributions do not have substantial impact on most of our data in the superconducting state.

3.6.4 The \( \hat{c} \)-axis Current Contribution

Another systematic error source is \( \hat{c} \)-axis currents. This occurs when the microwave magnetic field is not perfectly aligned with the sample \( \hat{c} \)-axis, and magnitudes of the order of 1° are common. To analyze this case, the magnetic field must be resolved into two components, one that is perpendicular to the \( \hat{ab} \)-plane of the sample, and the other that is perpendicular to the \( \hat{c} \)-axis. Microwave current will flow not only in the \( \hat{ab} \)-plane but also along the \( \hat{c} \)-axis, and the contribution to the total signal depends
on the angle. The resulting effective signal contains the $c$-axis surface impedance:

$$Z_s = Z_{ab} \cos^2 \theta + Z_c \sin^2 \theta; \quad (3.19)$$

Figure 3.15: Typical $c$-axis current signal for (a) $R_s$ and (b) the penetration depth $\Delta \lambda \approx \Delta X_s/\mu_0\omega [111]$, to be compared with those along the $a$ and $b$ axis. Data were taken at TE$_{011}$ mode at 22 GHz. Except for the temperature range close to $T_c$, the $c$-axis current signals are not large in the superconducting state.

where the subscripts $ab$ and $c$ correspond to the orientations, and $\theta$ is the angle of the misalignment. To account for the effective $Z_c$, we use the general form

$$Z_{\text{eff}} = Z_s \tanh \left( \frac{i\mu_0\omega t}{2Z_s} \right), \quad (3.20)$$

where $t$ is the sample thickness in the $c$ direction. In the superconducting state, $i\mu_0\omega t / 2Z_s \gg 1$, and $Z_{\text{eff}} \approx Z_s$. Typical $R_c$ and $X_c$ data are presented Figure 3.15. With a misaligned angle of 1°, their contributions to our effective signal should be negligible at temperature far below transition. In the normal state, because the $c$-axis skin depth is usually larger than the thickness of the sample (of the order of $10^{-5}$ m), Equation 3.20 reduces to $Z_{\text{eff}} = i\mu_0\omega t$, so $X_c$ is large while $R_c$ is vanishingly small. This results in a large offset between the absolute effective $R_s$ and $X_s$ in the normal state, but does not affect appreciably in $\Delta R_s(T)$ and $\Delta X_s(T)$ in the superconducting state.
Microwave Measurements

Near sharp edges or corners of the sample, the $c$-axis current can also contribute to the effective signal. This is because near these corners the electromagnetic field distribution is greatly distorted, and the flux penetration becomes rather complicated. Microwave currents will have a small $c$-axis current component near these corners, but it is very difficult to quantitatively model this contribution to the surface impedance. As mentioned above, it should not have a substantial impact on the signal in the superconducting state.

3.7 Summary of the Data Analysis Procedures

For convenience, we summarize here the data analysis procedure:

1. We subtract the background signal drift in the temperature sweep data, which gives us the relative bandwidth shift and frequency shift as functions of temperature. Both are offset to 0 at a background temperature $T_0$.

2. We add the absolute bandwidth shift at the background temperature $\Delta f_B(T_0)$, which is obtained from the power sweep data, to the relative bandwidth shift $\Delta f_B(T)$ in the temperature sweep raw data. This gives us the absolute bandwidth shift as a function of temperature and field.

3. We subtract the temperature dependent background contribution of the sapphire rod. This process is vital to the low temperature part of the analysis, due to the presence of the diverging upturns and downturns in the frequency shift.

4. We estimate the resonator constant to connect to surface impedance. This is usually done by comparing the raw data for absolute bandwidth shift to the known normal state resistivity:

$$\Gamma \approx \frac{\sqrt{2\mu_0\omega\Delta \rho_n}}{\Delta f_B}.$$  \hspace{1cm} (3.21)

The surface impedance $Z_s$ is then obtained from

$$\frac{\Delta f_B}{2} - i\Delta f_0 \approx \frac{\Delta Z_s}{\Gamma}.$$ \hspace{1cm} (3.22)

5. We offset the zero field absolute $X_s$ by the penetration depth at low temperature:

$$X_s = \mu_0 \omega \lambda,$$ \hspace{1cm} (3.23)

and the in-field $X_s$ is offset by matching to the zero field $X_s$ in the normal state.
6. We convert $R_s$ and $X_s$ to the complex resistivity $\rho = \rho_1 + i\rho_2$ using

\[
\rho_1 = \frac{2R_sX_s}{\mu_0\omega}, \\
\rho_2 = \frac{X_s^2 - R_s^2}{\mu_0\omega}.
\]

Equivalently, the complex conductivity can be obtained as $\sigma = 1/\rho$. In particular, in zero field, the superfluid density $\rho_s$ is proportional to the imaginary part of the conductivity in the low frequency limit:

\[
\rho_s(T) = \mu_0\omega\sigma_2(T).
\]

7. We extract the flux-flow parameters, as shown in Chapter 2.
This chapter contains measurements on three different conventional superconductors: Nb, NbSe₂, and V₃Si. In addition, zero field measurements have been carried out on Pb. These materials are mainly used to calibrate and validate our experiment, as their properties are much better understood than those of the cuprates. However, each of these samples holds its own uniqueness and demonstrates interesting properties. It will be shown in this chapter that measurements on these seemingly conventional materials not only confirm the validity of our experiment, but also provide powerful insight into the flux-flow study of the cuprates.

4.1 Nb and Pb

Let us start with the simplest materials first: Pb and Nb. Photos of the samples we used are presented in Figure 4.1.

Pb is so common a metal that it draws little research attention nowadays. It is a typical conventional superconductor with $T_c$ around 7.2 K. Depending on its purity, it may have such a small $\kappa$ that it is a type-I superconductor and is therefore not suitable for our flux-flow study. The properties that interest us are its availability and its large thermal expansion coefficient, which can be used to calibrate the systematic error from the sample thermal expansion. This Pb sample was made by my coworker Wendell Huttema, who used a similar one as the replica sample for the underdoped YBa₂Cu₃O₆.₃₃₃ sample, to obtain the resonator constant. He has also done a good
Measurements on Conventional Superconductors

Figure 4.1: (a) The Pb sample attached at the end of the sapphire rod. It has a spherical shape with a radius of 0.175 mm. (b) The Nb sample. It can be approximately treated as a rectangular thin film 0.682 mm in length and 0.577 mm in width. From its weight the average thickness has been estimated to be 0.032 mm.

correction for the thermal expansion effect[99], which resulted in a well matched $R_s$ and $X_s$ in the normal state: $R_s = X_s = \sqrt{\mu \rho_{DC}/2}$. Since his result came from cavity configuration A, repeating his work allows us to test the reproducibility of our experiment.

Nb is a transition metal and a simple conventional superconductor. In its pure form, it has a $T_c$ around 9.1 K, and a $\kappa$ around 2. Although it has been extensively studied for over 50 years, there is still some ongoing research interest[112]. On the practical side, it is widely used in many applications, including for microwave cavities. On the fundamental side, it can be used to represent the BCS type-II superconductors with low $\kappa$, for the purpose of flux-flow study.

The Nb sample used in this study was cut from an electro-polished high purity Nb foil under the microscope. It was then etched by a mixture of HF and HNO$_3$ to roughly the same shape and size of the ortho-II YBa$_2$Cu$_3$O$_{6.52}$ sample. Measurements on this sample have been taken in cavity configuration B and C. The results taken in these two configurations have been confirmed as consistent. Here only the data taken in cavity configuration C are presented because of the better data quality.
4.1.1 Zero Field Measurements

Zero field measurements have been made for both samples in the TE_{011} mode at 2.79 GHz, the TE_{021} mode at 5.78 GHz, the TE_{031} mode at 9.30 GHz and the TE_{041} mode at 13.99 GHz. For both samples, the frequency dependence $R_s \propto \sqrt{\omega}$ in the normal state has been confirmed: after being scaled by $1/\sqrt{\omega}$, data from different modes overlap each other. As to $X_s(T)$, it has a different form for each mode, presumably due to the systematic errors discussed in Chapter 3. After removing these errors, $R_s(T)$ and $X_s(T)$ should match well in the normal state.

Figure 4.2 shows the zero field surface impedance for both samples in our main working TE_{011} mode. For the Pb sample, Wendell’s results are well reproduced: $R_s(T)$ and $X_s(T)$ match well in the normal state after applying the thermal expansion correction $\Delta X_s^{\text{corr}} = \mu \omega r \frac{\Delta l}{l}$. The difference between $R_s(T)$ and $X_s(T)$ below 25 K is the result of nonlocal electrodynamics, because at this temperature range, the mean free path $\ell$ becomes larger than the skin depth $\delta$. For the Nb sample, it has a platelet form, the geometry of which makes the thermal expansion correction more
Measurements on Conventional Superconductors

complicated. Here an approximate form $\Delta X_s^\text{corr} = \mu \omega \frac{r_{ab}}{3} \frac{\Delta l}{l}$ is used, where $r_{ab} = \sqrt{r_a r_b}$ is the effective sample dimension calculated from the length $r_a$ and width $r_b$ of the sample. As shown in Figure 4.2, the effect of the sample thermal expansion is not large, and is negligible (a few $\mu\Omega$) at low temperature (below 10 K). This is because the Nb has a thermal expansion coefficient an order of magnitude smaller than that of Pb. In addition, below 40 K there is not much difference between $R_s(T)$ and $X_s(T)$, so the nonlocal effect must be small.

The results for Nb allow us to estimate the sample thermal expansion contribution for all the other platelet samples used in this study. Although some of their thermal expansion coefficients remain unknown, they can reasonably be assumed to be of the same order as Nb, if not smaller. Combined with their small dimensions, which are all comparable to or smaller than this Nb sample, the thermal expansion contribution in these samples should be slight.

![Figure 4.3: Surface impedance of the Pb sample (a) in the TE_{031} mode at 9.30 GHz and (b) in the TE_{043} mode at 13.99 GHz. A large difference in $X_s$ before and after applying the thermal expansion correction can be observed.](image)

However, the sample thermal expansion is not the only dominant error source for data taken in the high frequency modes. As shown in Figure 4.3, for the Pb
Measurements on Conventional Superconductors

sample, after applying the thermal expansion correction there are still large differences between $R_s(T)$ and $X_s(T)$, which increase with the resonant frequency. Similar behaviour has also been observed in the Nb sample. Since both Pb and Nb are electrically isotropic materials, the $\hat{c}$-axis current contribution should be negligible and is unlikely to be the cause of the differences. The most likely cause for these differences is sapphire rod motion, as mentioned in Chapter 3. Its contribution is extremely hard to calculate, but from our observations its scale can be estimated. We focus on the Nb sample, because it has a platelet geometry, the same as most of our samples.

![Graphs](image)

Figure 4.4: (a) $\sigma_1(T)$ of the Nb sample. As $\sigma_1(T)$ is extremely sensitive to $R_s(T)$, a constant residual $R_s$ is subtracted from the raw data to get the dynamic $R_s(T)$. A BCS coherence peak[113] just below $T_c$ is seen in all the modes. (b) $\sigma_2(T)$ of the Nb sample, scaled by the resonant frequency $f$. Data from different modes agree well with each other, with a slight frequency dependence that may be due to quasiparticle relaxation contribution.

Data taken in the TE$_{011}$ mode have been confirmed as very reliable and insensitive to motion of sapphire rod, since there is little difference between $R_s(T)$ and $X_s(T)$ in the normal state. In addition, by matching $R_s(T)$ and $X_s(T)$ in the normal state, the zero temperature penetration depth $\lambda_0$ for Nb has been estimated to be 31.7 nm, which agrees well with published results (about 33 nm)[114, 115]. By comparing the raw resistivity data to those obtained by the UBC group, the resonator constant for
Measurements on Conventional Superconductors

the Nb sample has been estimated to be $1.1764 \times 10^{-6} \, \Omega/\text{Hz}$ for cavity configuration C, and $1.1121 \times 10^{-6} \, \Omega/\text{Hz}$ for cavity configuration B. The ratio between these two resonator constants is roughly the same as the ratio of their corresponding resonant frequencies, which suggests a simple, uniform field profile. A similar behaviour in the TE$_{011}$ mode has also been confirmed for the Pb sample.

The complex conductivity, which is more sensitive to these systematic errors, is presented in Figure 4.4 for the Nb sample. The real part of the conductivity $\sigma_1(T)$ demonstrates the "coherence peak" predicted by the Mattis–Bardeen theory[113]. The imaginary part of the conductivity $\sigma_2(T)$, which strongly depends on $X_s(T)$, follows a $1/\omega$ scaling predicted by the same theory. The scaled $\sigma_2(T)$ is proportional to the superfluid density, and its temperature dependence reproduces that of a previous experiment[114]. No impact of the systematic errors is observed in the superconducting state. In addition, the same analysis has been repeated for the Pb sample, and no deviation from these BCS predictions was found. Thus, at least at temperatures below 9.2 K, data from the high frequency modes can be regarded as reliable.

4.1.2 In-field Measurements

Since our Pb sample turns out to be a type-I superconductor, no in-field measurements were taken. From now on we will focus on the Nb sample.

Nb samples have relatively small $\kappa \approx 2$, which will result in a very narrow range of the mixed state. This imposes a challenge in taking the mixed state data, because there is a large uncertainty in the exact values of $B_{c1}$ and $B_{c2}$, while $B_{c2} - B_{c1}$ is small. From the literature, the zero temperature $B_{c1}$ has been estimated to range from 900 to 1700 gauss [116], while $B_{c2}$ has been estimated to be around 4700 gauss [116]. These values may not be accurate for our sample, because they are strongly sample dependent[117]. Another complication is that $B_{c1}(T)$ and $B_{c2}(T)$ decrease with increasing temperature[2], so it is possible that only a fraction of our temperature sweep data were taken in the mixed state.

The in-field temperature sweep data are shown in Figure 4.5. They have been taken at 0.025 T, 0.05 T, 0.1 T, 0.125 T, and 0.25 T to cover a wide field range. According to the phase diagram of Nb[118], the majority of the 0.25 T data should have been taken in the mixed state, while the majority of the 0.025 T data should have been taken below $B_{c1}(T)$. The other data should include the mixed state part at $B > B_{c1}(T)$, and another at $B < B_{c1}(T)$. Note that even in the case $B < B_{c1}(T)$,
Measurements on Conventional Superconductors

Figure 4.5: The in-field (a) \( R_s(T) \) and (b) \( X_s(T) \) for the Nb sample. Data were taken in the TE\textsubscript{011} mode. Vortex dynamics clearly dominate the mixed state signal. And, as a consistency check, \( Z_s \) shows little field dependence in the normal state.

there will still be trapped vortices in the sample. This is because samples are always field-cooled prior to our experiments.

Another complication induced by the small \( \kappa \) is that the assumption \( B_{c1} \ll B \ll B_{c2} \) is not valid in Nb, and what we measure may not be strictly the averaged response from an isolated vortex unit cell (see Chapter 2). This may prevent us from studying the field dependence of the flux-flow parameters in detail, but this seems to have little impact on their temperature dependence. As seen from the surface impedance data, applying a magnetic field does not broaden the resistive transition noticeably but shifts it to a lower temperature, and the onset of the resistive transition approximately corresponds to \( B_{c2}(T) \). This is in agreement with the Nb \( B-T \) phase diagram\cite{118}, as for conventional superconductors the region of the vortex liquid state is known to be very narrow.

The real part of the extracted complex resistivity \( \rho_1(T) \) is presented in Figure 4.6. In the normal state, all data match well with the data taken in a 2 T field, where the superconductivity has been completely suppressed. The normal resistivity is well
Measurements on Conventional Superconductors

described by

\[ \rho_n = (0.398 + 6.843 \times 10^{-6} T^3) \mu\Omega\text{cm}, \]  

where the \( T^3 \) temperature dependence agrees with earlier measurements\[117\], but differs from the typical \( T^5 \) temperature dependence in many other metals. This behaviour probably indicates strong Umklapp scattering process, in which the temperature dependence of the electrical resistivity solely depends on the \( T^3 \) temperature dependence of the phonon density.

![Graph](image)

Figure 4.6: \( \rho_1(T) \) of Nb. Data were taken in the TE\(_{011}\) mode at 2.79 GHz. The normal state data can be fit well by Equation 4.1.

4.1.3 Depinning Frequency and Pinning Constant

Figure 4.7 shows the extracted depinning frequency. At low temperatures, the depinning frequency is comparable to our operating frequency. As the applied field increases, the depinning frequency becomes lower.

The more important parameter is the pinning constant \( \alpha_p \), and its temperature dependence and field dependence are presented in Figure 4.8. At a given field, all temperature dependence data show downward curvatures at low temperature, and approach zero with upward curvatures at temperatures near \( T_c \). At a given temper-
Measurements on Conventional Superconductors

Figure 4.7: The depinning frequency of the Nb sample. Data were taken in the TE\textsubscript{011} mode.

At all temperatures, all data decrease sharply with increasing field at low field, but become less field dependent at high field.

The functional temperature and field dependence of the pinning constant $\alpha_p$, or of the critical current density $J_c$ measured in DC experiments, may provide insight into the pinning mechanism. Although $\alpha_p$ and $J_c$ are different quantities, in our model they are directly related by $\alpha_p x_0 = J_c \Phi_0$, and should have the same functional temperature dependence and field dependence. In conventional superconductors, since $J_c$ is often expected to reach zero at $T_c$, a convenient empirical form $J_c \propto (1 - t)^\beta$ is widely used to fit experimental data of many materials. For this Nb sample, it seems to be more accurate to relate $\alpha_p(t)$ to $H_{c2}(t)$\cite{119}, since for many Nb-based materials $H_{c2}$ is found to be the dominant parameter that determines the temperature dependence of the bulk pinning force $F_p = J_c B$:

$$F_p = J_c B \propto H_{c2}(t)^\beta g(b),$$

(4.2)

where $\beta \approx 2.5$, $b = H/H_{c2}$ and $g(b)$ is a function that incorporates the field dependence of $F_p$\cite{120}. Since for Nb $H_{c2}(t) \propto \frac{1 - t^2}{1 + t^2}$\cite{119}, the fractional power law form $\alpha_p(t) \propto H_{c2}(T)^\beta$ successfully explains the qualitative behaviour of our pinning constant data. In particular, for the mixed state (0.25 T) data, $\beta \approx 2.5$ provides the best fit. This seems to imply that the coherence length is the dominant length scale that determines the temperature dependence of the pinning, since $H_{c2} \propto \xi^{-2}$.
Measurements on Conventional Superconductors

Figure 4.8: (a) The temperature dependence of the pinning constant for the Nb sample. Data have been fit to a fractional power law $\alpha_p = \alpha_0((1 - t^2)/(1 + t^2))^\beta$, where $\alpha_0$ and $\beta$ are fitting parameters. The best fits are obtained by $\beta = 1.47$ (0.025 T), $\beta = 1.84$ (0.05 T), $\beta = 2.21$ (0.1 T), $\beta = 2.47$ (0.125 T) and $\beta = 2.45$ (0.25 T). (b) The field dependence of the pinning constant for the Nb sample. Data were taken in the TE_{011} mode.

It needs to be mentioned that in Equation 4.2, $\beta = 2.5$ has been established with relatively high precision, while the exact form of $g(b)$ varies from sample to sample[120]. $g(b) = b^p(1 - b)^q$ with $p = 0.5$ and $q = 2$ is given by the flux line shear model[121], and is in accordance with the collective pinning theory, but experimental results often deviate from this form. For reasons mentioned in Section 4.1.2, a detailed analysis of $g(b)$ in our Nb sample will not be made.

4.1.4 Viscosity Coefficient and Flux-flow Resistivity

The viscosity coefficient and the flux-flow resistivity are shown in Figure 4.9. The viscosity decreases monotonically with increasing temperature and field. In the Bardeen–Stephen model, this behaviour implies a sublinear flux-flow resistivity. However, as mentioned in Section 4.1.2, the assumption $B_{c1} \ll B \ll B_{c2}$ is not valid due to the small $\kappa$. Validating the linear field dependence in the Bardeen–Stephen law is
Measurements on Conventional Superconductors

![Graphs](a) The viscosity coefficient and (b) the flux-flow resistivity for the Nb sample. Data were taken in the TE\textsubscript{011} mode.

thus a task for superconductors with higher $\kappa$. Here we stress that the flux-flow resistivity decreases monotonically with temperature, and does not contradict the Bardeen–Stephen “fractional normal resistivity” picture.

### 4.2 V\textsubscript{3}Si

We now move to a material which is slightly more complicated than Nb but still conventional: V\textsubscript{3}Si. It belongs to the A15 intermetallic compounds. In general, these compounds have cubic structures, unusual temperature dependences of the normal state resistivity\cite{122, 123}, relatively high $T_c$, and large Ginzburg–Landau parameters $\kappa$. For V\textsubscript{3}Si, $T_c \approx 16$ K and $\kappa \approx 25$. Such properties enable us to investigate the flux-flow dynamics in a temperature and field range that is comparable to that of the highly underdoped cuprate samples.

What is particularly interesting for this study is that V\textsubscript{3}Si has a field-induced vortex lattice phase transition. First observed by a neutron scattering experiment\cite{124} then by an STM experiment\cite{125}, the vortex lattice evolves from hexagonal symmetry under 0.75 T to square symmetry above 4 T. A later $\mu$SR study by Sonier et al.\cite{126} suggests that this transition is driven by the interaction between vortices, which
is reflected by a field dependent vortex core size. It is interesting to test if our measurements are sensitive to these features.

This $V_3Si$ sample has been borrowed from Dr. Chris Bidinosti, while the origin of the sample can be tracked back to Dr. Bernd Seeber. The sample has been cut and polished by Chris Bidinosti to a rectangular thin platelet, 1.12 mm in width along the $\hat{a}$-axis, 1.88 mm in length along the $\hat{b}$-axis, and 0.11 mm in thickness along the $\hat{c}$-axis. Compared with the other samples, this one is much larger and gives stronger signal. Photos of the sample are shown in Figure 4.10.

(a)  
(b)

Figure 4.10: Photos of the $V_3Si$ sample.

4.2.1 Zero Field Measurements

For this sample, data have only been taken in the $TE_{011}$ mode at 2.79 GHz. Figure 4.11 shows the zero field surface impedance. A resonator constant of $3.95 \times 10^{-7} \, \Omega/\text{Hz}$ has been extracted by comparing the normal state resistivity to published data [127]. An absolute penetration depth $\lambda_0$ of 110 nm has been estimated from references [126, 128], and used to offset surface reactance by $X_s = \mu_0 \omega \lambda$ at low temperature. There is a small difference between $R_s$ and $X_s$ in the normal state, which is unlikely to be the $\hat{c}$-axis current contribution, since $V_3Si$ is approximately electrically isotropic. The difference might be due to an inaccurate $\lambda_0$ – which is not our concern, as our qualitative observations have been confirmed to be insensitive to the value of $\lambda_0$.

A sharp transition around 16 K has been observed, and at 20 K there is a residual
measurements on conventional superconductors

resistivity of about 12 μΩcm. Compared with samples used in others experiments, this V₃Si sample seems to be moderately clean.

Right below Tₑ, Xₛ(T) shows a peak. As Zₛ = √iμ₀ω/(σ₁ - iσ₂), the peak occurs when σ₁(T) = σ₂(T). Thus, the simplified assumption σ₂ ≫ σ₁ in the Waldram model is invalid near the peak (see Chapter 2). In addition, for BCS superconductors, the assumption fₙ ≪ 1 should also be violated near Tₑ. So, in this temperature range, the fraction of the superconducting background is vanishingly small, but its dissipation per unit area is becoming comparable to that of the normal core. In this case, our original approach of subtracting the zero field resistivity ρₛ, which takes into account the background dissipation over the entire vortex unit cell, would lead to unphysical outcomes. In particular, it will result in a negative pinning constant near Tₑ, as αₚ ∝ [ρ₂(B,T) - ρ₂(0,T)]. On the other hand, for BCS superconductors the pinning constant and flux-flow resistivity should be constrained by the fact that ρₙ → ρₙ and αₚ → 0 when T → Tₑ. A more reasonable modification is to neglect the superconducting background contribution near Tₑ in Equation 2.70 and 2.71, with
Measurements on Conventional Superconductors

the result that our data extraction formula becomes

\[
\alpha_p = B\Phi_0 \omega \frac{\rho_2}{|\rho|^2},
\]

\[
\eta_{fl} = B\Phi_0 \frac{\rho_1}{|\rho|^2},
\]

showing that \( \alpha_p \) and \( \eta_{fl}(T) \) are proportional to \( \sigma_2(B,T) \) and \( \sigma_1(B,T) \) respectively. In general, at relatively strong magnetic field \( b = B/B_{c2} \), our extracted flux-flow parameters are not substantially altered by such modification. For this V\(_3\)Si sample, such modification removes a spurious zero-crossing in the pinning constant near \( T_c \), but otherwise has little impact (about 2% in magnitude) on the extracted parameters at low temperature.

![Graph](image)

Figure 4.12: (a) Our measured normalized superfluid density of V\(_3\)Si. In the BCS theory, it should be a function of the reduced temperature \( t \) and approach \( T_c \) almost linearly\([2]\). However, a change in slope in the superfluid density is observed at around 0.7 \( t \), an effect that appears in models of superconductors with two coupled order parameters. (b) Published result of normalized superfluid density of V\(_3\)Si\([130]\).

The complex conductivity has been extracted from surface impedance data. The imaginary part of the conductivity \( \sigma_2(T) \) is particularly interesting, as in zero field it is proportional to the superfluid density (see Figure 4.12). Unlike the original BCS
Measurements on Conventional Superconductors

prediction[2], at around 0.7 \( T_e \) there is an upward curvature. This behaviour has been suggested as an indication of coexistence of two different order parameters in the superconducting state[129], and agrees with published results[130].

### 4.2.2 Mixed State Measurements

![Graphs showing mixed state measurements](image)

Figure 4.13: (a) \( R_s(T) \) and (b) \( \rho_1(T) \) of the \( V_3Si \) sample. Applying magnetic field shifts the resistive transition to a lower temperature. Data were taken at TE\(_{011}\) mode at 2.79 GHz.

From published data[128], \( B_{c1} \) and \( B_{c2} \) for \( V_3Si \) at zero temperature have been estimated to be around 0.2 T and 18.5 T respectively. Mixed state measurements have been taken from 0.25 T to 6 T.

Figure 4.13 shows the mixed state \( R_s \) and \( \rho_1 \). Their behaviours are very similar, as \( \rho_1 = 2R_s X_s/\mu_0 \omega \). In the superconducting state, at fields as low as 0.25 T, the vortex response dominates the signal and the background feature has been smeared out. At higher fields, the resistive transition has been shifted to lower temperatures, indicating that the superconductivity is suppressed above \( B_{c2}(T) \). In the normal state little field dependence in \( R_s \) and \( \rho_1 \) has been observed, as data from different fields overlap each other almost perfectly. This confirms the robustness of our absolute \( R_s \)
measurements. The normal state resistivity $\rho_n$ fits well to:

$$\rho_n = (12.34 + 6.54 \times 10^{-4} T^{2.4}) \, \mu\text{Ω} \text{cm}, \quad (4.5)$$

in the temperature range $16 \, \text{K} \leq T \leq 50 \, \text{K}$. The fractional power law temperature dependence is typical for the A15 compounds[122, 123].

Similarly, $X_s(T)$ data under different fields also overlap each other in the normal state, as shown in the left panel of Figure 4.14. This indicates that we have accurately measured the relative field dependence of $\Delta X_s(B,0)$ in the superconducting state, as $\Delta X_s = X_s(B,T) - X_s(0,T)$ is almost a constant in the normal state. The difference $\Delta X_s(B,0)$ is expected to have a smooth field dependence, and can be interpolated by power law functions (see the right panel of Figure 4.14), which may not strictly represent the underlying physics of $X_s$, but provides a useful smoothing of random errors that may lead to spurious field dependence of other quantities. For this V$_3$Si sample, the random error in $\Delta X_s$ is almost negligible.

![Figure 4.14](image)

Figure 4.14: (a) $X_s(T)$ of the V$_3$Si sample in different magnetic fields. (b) The field dependence of $\Delta X_s$ at zero temperature. A power law fit $\propto B^{0.5}$ is used to interpolate the data. The standard error of $\Delta X_s$ is so small that the error bar is smaller than the data point.

Nevertheless, there is also uncertainty in the absolute surface reactance $X_s(0,T)$, as emphasized in Chapter 3. Such uncertainty can be large in some samples, and
would then lead to corresponding uncertainties in the magnitude of the extracted flux-flow parameters. In general, a large uncertainty in $X_s(0,T)$ may prevent us from carrying out a quantitative analysis on the extracted flux-flow parameters, but has little effect on their qualitative temperature dependence. At strong magnetic field or high temperature, the uncertainty in our results is smaller. For this V$_3$Si sample, a 20% uncertainty in $\lambda_0$ would result in 10% uncertainty in the extracted parameters at low temperature and low field, decreasing to $\approx$ 5% in high field or at high temperature.

### 4.2.3 Depinning Frequency and Pinning Constant

Figure 4.15 shows the depinning frequency at multiple fields. At low temperature, our measurements have been made at a frequency of 2.79 GHz, comparable to the depinning frequency.

![Depinning Frequency](image)

Figure 4.15: Depinning frequency of the V$_3$Si sample.

Figure 4.16 presents the temperature and the field dependence of the pinning constant. Following the common practice, the temperature dependent data have been fit to a fractional power law function $\alpha_p = \alpha_0 (1 - t)^\beta$. The fit parameters are listed in Table 4.1. At magnetic fields below 4 T, $\beta \approx 2.5$. Above 4 T $\beta$ becomes smaller. It is possible that this is related to the change in symmetry of the vortex lattice, which may affect the pinning.
Measurements on Conventional Superconductors

While a peak in the critical current density $J_c$ near $B_{c2}(T)$ (the peak effect) has been reported for this material[131], no signature of the peak effect is found in our data. The pinning constant approaches zero smoothly near $T_c$, which seems to imply a simple mixed state phase diagram, in which the vortex lattice is destroyed at $B_{c2}(T)$. This is supported by the resistivity measurements (see Figure 4.13), as at all the fields, the resistive transition is very sharp and demonstrates no other features.

Table 4.1: Fit parameters for the functional form $\alpha_p(t) = \alpha_0(1 - t^\beta)$.

<table>
<thead>
<tr>
<th>Magnetic Field (T)</th>
<th>$\alpha_0$</th>
<th>$\beta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>16642</td>
<td>2.48</td>
</tr>
<tr>
<td>0.5</td>
<td>13170</td>
<td>2.62</td>
</tr>
<tr>
<td>0.75</td>
<td>11187</td>
<td>2.67</td>
</tr>
<tr>
<td>1</td>
<td>9845</td>
<td>2.67</td>
</tr>
<tr>
<td>2</td>
<td>7231</td>
<td>2.48</td>
</tr>
<tr>
<td>4</td>
<td>5190</td>
<td>2.04</td>
</tr>
<tr>
<td>6</td>
<td>4269</td>
<td>1.74</td>
</tr>
</tbody>
</table>

Figure 4.16: (a) The temperature dependence of $\alpha_p$ for the V$_3$Si sample, fits to the function $\alpha_p = \alpha_0(1 - t^\beta)$. (b) A power law fit $\alpha_p = \alpha_0 B^\beta$ for the field dependence of $\alpha_p$. 
The field dependence of the pinning constant can be fit well to a simple power law, the fit parameters of which are presented in Table 4.2. Since the behaviour of the pinning constant is qualitatively field independent, there is no obvious indication of a vortex lattice phase transition. This implies that our measurements might be insensitive to the vortex lattice structure, which is perhaps not surprising, as they probe local properties of the flux line.

![Figure 4.17](image-url)  
(a) The viscosity coefficient and (b) the flux-flow resistivity for the $V_3Si$ sample below the resistive transition.
4.2.4 Viscosity Coefficient and Flux-flow Resistivity

Figure 4.17 shows the extracted viscosity coefficient and the flux-flow resistivity. The majority of the viscosity data decrease monotonically as temperature approaches $T_c$.

The flux-flow resistivity $\rho_{ff}(T)$ decreases monotonically with temperature. This behaviour resembles that of $\rho_n(T)$ and agrees with the normal core description in the Bardeen–Stephen model. To compare the temperature dependence of $\rho_{ff}$ and $\rho_n$ in detail, we extract $H_{c2}(T)$ from

$$H_{c2}(T) = C(H, T) \frac{\rho_n}{\rho_{ff}} H,$$

(4.6)

![Figure 4.18: (a) The extracted $H_{c2}(T)$ from $\rho_{ff}(T)$ and the Bardeen–Stephen law. $\rho_n(T)$ is extrapolated to low temperature using Equation 4.5. (b) The published $H_{c2}(T)$ data for $V_3Si[128]$.](image)

where the function $C(H, T)$ is introduced to contain any field dependence and temperature dependence other than the Bardeen–Stephen mechanism. For conventional superconductors, it is usually assumed to be a constant close to 1. By using $C = 1.75$, the temperature dependence of our extracted $H_{c2}$ approximately agrees with published data (see Figure 4.18), so the correlation between $\rho_{ff}(T)$ and $\rho_n(T)$ is confirmed.
Measurements on Conventional Superconductors

The extracted $H_{c2}(T)$ shows a weak field dependence, which should approximately correspond to a linear field dependence in $\rho_{\text{ff}}$. However, we are constrained by the fact that $\rho_{\text{ff}}$ reaches $\rho_n$ at $H_{c2}$. In this case, the large scaling factor at low fields implies that at higher fields a sublinear field dependence or a small scaling factor has to be introduced; otherwise by extrapolation we would have $\rho_{\text{ff}}(H_{c2}) = 1.75\rho_n$. The flux-flow resistivity $\rho_{\text{ff}}(B)$ thus should contain a downward curvature at high fields, to move the $\rho_{\text{ff}}$ below the low field linear fit.

The flux-flow resistivity $\rho_{\text{ff}}(B)$ is presented in Figure 4.19. At low temperature, the data fit well to linear functions. At higher temperature, when $H/H_{c2}(T)$ moves towards the high field end, $\rho_{\text{ff}}(B)$ moves below the linear fit, which implies a sublinear field dependence. In this aspect, the Bardeen–Stephen law is “violated”.

![Figure 4.19: The Bardeen–Stephen law fit for the flux-flow resistivity. The linear field dependence is obeyed at low fields.](image)

The violation itself is nothing new – in fact, in all the reported violations of the Bardeen–Stephen law, $\rho_{\text{ff}}(B)$ behaves the same way: linear at low field with a large scaling factor, and evolving into a downward curvature at higher field. However, this violation in “conventional” $\text{V}_3\text{Si}$ is unexpected. Such violations have been reported on $\text{UPt}_3[132]$, $\text{YNi}_2\text{B}_2\text{C}[133]$ and $\text{Bi}_{1.74}\text{Pb}_{0.38}\text{Sr}_{1.88}\text{CuO}_x[134]$, the anisotropy of which has been interpreted as the cause – and this is highly unlikely to be the case for the rather isotropic $\text{V}_3\text{Si}$. A more reasonable explanation might come from the same
Measurements on Conventional Superconductors

origin as the MgB$_2$[135], where the importance of multiband superconductivity has been emphasized. V$_3$Si, like most transition metals and transition metal alloys, is also a multiband superconductor.

4.3 NbSe$_2$

Figure 4.20: The surfaces of the NbSe$_2$ sample under a microscope. This sample has an average length of 0.678 mm, a width of 0.462 mm, and a thickness of 0.041 mm.

The transition metal dichalcogenide NbSe$_2$ is a layered, highly anisotropic superconductor, and is often treated as a good quasi-2D system. This makes it favorable for comparison with the cuprates. The layered structure is relatively simple, consisting of triangular prisms with niobium atoms in the center and selenium atoms at the corners. Two forms of the structure exist, and here a 2H-NbSe$_2$ sample has been used. The prefix “2H” denotes the existence of 2 layers in a unit cell.

For our study, NbSe$_2$ has several interesting properties. First, it has $\kappa \approx 30$[136], which allows us to test the Bardeen–Stephen law; second, it has an intriguing vortex matter phase diagram, particularly in the “peak effect” region[137, 138]; last, but not least, it has been shown to be a multiband superconductor with anisotropy in the superconducting gap[139], signatures of which might be reflected in its superfluid density.

The NbSe$_2$ sample we measured was borrowed from Dr. Jeff Sonier, who used it to study the field dependence of the vortex core size[140]. Photos of the sample are
Measurements on Conventional Superconductors

shown in Figure 4.20.

4.3.1 Zero Field Measurements

Figure 4.21: (a) The zero field surface impedance of NbSe$_2$, taken in the TE$_{011}$ mode at 2.79 GHz. A very sharp transition near 7 K, with a transition width of 0.1 K is observed. (b) The peaks in $X_s(T)$ at multiple frequencies, at which $\sigma_1 \approx \sigma_2$.

Figure 4.21 presents the zero field surface impedance of NbSe$_2$. A resonator constant of $2.30 \times 10^{-6} \ \Omega$/Hz has been estimated by comparing to published resistivity data[141]. An absolute zero temperature penetration depth $\lambda_0$ of 125 nm has been estimated from references[142, 143] to offset $X_s$.

In the normal state $X_s$ is larger than $R_s$, but they have very similar temperature dependence. As mentioned in Chapter 3, this is likely the result of the $c$-axis currents, since NbSe$_2$ is highly electrically anisotropic. The $c$-axis current effect would result in a unphysical nonzero $\rho_2$ in the normal state, but would have little impact on the data far below $T_c$. In the superconducting state, both $R_s$ and $X_s$ fall rapidly and are very smooth. The residual resistivity ratio $\rho_{ab}(300 \text{ K})/\rho_{ab}(7.5 \text{ K})$ is estimated to be around 26, indicating the high purity of the sample.

Near $T_c$, $X_s$ shows a peak, the width of which increases with frequency. As mentioned in Section 4.2, near the peak $\sigma_1 \approx \sigma_2$, requiring our data extraction method
Measurements on Conventional Superconductors

Figure 4.22: Our measured normalized superfluid density of NbSe₂. Data at multiple frequencies agree well with each other. Similar to the V₃Si sample, there is an upward curvature in $1/\lambda^2(T)$ near 0.8 $t$, which might be associated with multiband superconductivity. (b) Published tunnel diode measurements of superfluid density of NbSe₂ [139].

to be modified. In this sample, such a modification has an impact on the magnitude of the extracted flux-flow parameters. In our main working TE₀₁₁ mode, its impact is not large ($\approx 5\%$) for the high field (> 1 T) data, but increases substantially with increasing frequency and decreasing field. This impact, however, does not change the qualitative behaviour of our extracted flux-flow parameters.

The superfluid density, which is proportional to $\sigma_2(T)$, is presented in Figure 4.22. Its behaviour shares similarities with that of V₃Si, and is comparable to similar tunnel diode measurements [139]. At about 0.8 $T_c$ there is an upward curvature in the temperature dependence, which may suggest coexistence of two different order parameters. At low temperature our results are similar to published tunnel diode measurements, but close to $T_c$ there is some differences. This might be due to a sample-dependent interband coupling strength.
Measurements on Conventional Superconductors

Figure 4.23: The mixed state (a) $R_s(T)$ and (b) $\rho_1(T)$ of NbSe$_2$. For viewer’s convenience, the data at a few selected fields are used to represent the entire data set. For the $\rho_1$ data taken at a field of 2 T, two temperature scales $T_k$ and $T_r$, are marked by arrows. Data were taken in the TE$_{011}$ mode at 2.79 GHz.

4.3.2 Mixed State Measurements

For this sample, data have been taken in a wide range of fields. In particular, field sweep data were taken from 1 K to 6 K for the TE$_{011}$ mode at 2.79 GHz. From the literature we estimated $B_{c1}$ to be around 0.04 T$^{[144]}$ and $B_{c2}$ to be 4.5 T$^{[136]}$ at zero temperature, so the majority of the data were taken in the mixed state.

The mixed state $R_s(T)$ and $\rho_1(T)$ are presented in Figure 4.23. In the mixed state, the vortex response clearly dominates the signal, as the mixed state response is much larger than that in zero field. Little field dependence is observed in the normal state resistivity, indicating the consistency of our measurements. Below 20 K, $\rho_n(T)$ can be fit to a fractional power law:

$$\rho_n(T) = (8.9 + 7.8 \times 10^{-4} T^{2.8}) \, \mu\Omega\text{cm},$$

which will be used to extrapolate the normal resistivity into the low temperature range. Although an arbitrary fit function has been used here, the residual normal resistivity has little dependence on the fitting methods.
Measurements on Conventional Superconductors

Figure 4.24: (a) $X_s(T)$ of NbSe$_2$ in the mixed state. (b) $\Delta X_s(B, 0)$ fits to a fractional power law $\propto B^{0.4}$. Data were taken in the TE$_{011}$ mode at 2.79 GHz.

The resistive transition is not substantially broadened by the fields, which corresponds to a relatively narrow vortex liquid phase. At the transition, the high field data show a round shoulder, and less obviously a kink. At the kink, a temperature scale $T_k$ can be defined at a local maximum in $d\rho/dT$. Another temperature scale, $T_r$, can be defined by $\rho_1(T_r) = \rho_n$. The meaning of these will be discussed in the next section.

The mixed state $X_s(T)$ and $\Delta X_s(B, 0)$ is shown in Figure 4.24. A power law interpolation is used to fit $\Delta X_s(B, 0)$. It works particularly well in the low field region, but deviations larger than the statistical errors can be seen at high fields.

4.3.3 Depinning Frequency and Pinning Constant

The depinning frequency for NbSe$_2$ is presented in Figure 4.25. The depinning frequencies are smaller than the resonant frequency of our main working mode (2.79 GHz), so the measurements have been taken close to the limit of free flux-flow. An interesting observation is that the high field ($B \geq 1$ T) data show a kink.

This kink is analyzed in terms of the pinning constant. To avoid an unphysical zero-crossing in the pinning constant near $T_c$ (see Section 4.2), the superconducting background contribution is not included in the data extraction model. Such modifi-
Measurements on Conventional Superconductors

Figure 4.25: The depinning frequency of NbSe₂ at multiple fields. Data were taken in the TE₀₁₁ mode at 2.79 GHz.

cation has been confirmed to have no effect on the qualitative temperature and field dependence of our data.

Figure 4.26 shows the pinning constant at multiple fields and frequencies. At low temperature the pinning constant curvature is similar to what we observed in Nb and V₃Si. Like Nb and V₃Si, the pinning constant of NbSe₂ decreases monotonically with increasing magnetic field, independent of the details of our model. As for the frequency dependence, if the superconducting background contribution is not included in our model, \( \alpha_p \) increases monotonically with decreasing frequency. Nevertheless, this behaviour might not be physical, as it depends on whether the superconducting background contribution is included in our model or not (see Section 4.2). We stress that the qualitative temperature dependence of \( \alpha_p \) is not strongly frequency dependent.

At intermediate temperature, the data show a step-like structure, which is independent of the details of our model, and can be observed as the kink in the raw resistivity data. The characteristic temperature of the step, \( T_k \), shows a strong field dependence: it is shifted to lower temperature as magnetic field increases. On the other hand, \( T_k \) shows a very weak frequency dependence: it seems to be a constant. Above \( T_k \), the pinning constant curvatures eventually evolve into nonzero plateaus. These nonzero values for \( \alpha_p \) are merely the results of the \( \hat{c} \)-axis current effect, which
Figure 4.26: The pinning constant at (a) multiple fields and a single frequency (2.79 GHz for the TE$_{011}$ mode) and (b) a single field 2 T and multiple frequencies. Data were extracted using the modified method of Section 4.2. Both plots show a step-like structure. The onset temperature $T_k$ of the step-like structure is marked by arrows in the left panel and dashed line in the right panel.

must have artificially enhanced the apparent pinning constant in the normal state, by causing $R_s \neq X_s$. The reason is that the pinning constant depends heavily on $P_2$, and $X_s$.

According to the published data of $B_{c2}(T)[147]$, $T_k$ should indicate the superconducting transition at upper critical field. However, it does not correspond to the apparent resistive transition in our data (see Figure 4.23). At $T_k$, $\rho_1(T_k)$ is smaller than $\rho_n$, and smoothly reaches a field independent $\rho_n$ at a larger temperature $T_r$. These seemingly contradictory observations have a simple explanation: surface superconductivity[148]. Surface superconductivity means that a superconducting layer with thickness $\xi$ persists on the sample surface above $T_c$ and below the third critical field $B_{c3}(T)$, and thus the effective resistivity can be simplified as

$$\rho_{\text{eff}} \approx \rho_n(1 - p),$$

where $p$ is the fraction of the superconducting region near the surface. At $B = B_{c3}(T_r)$, $p \to 0$, the effective resistivity reaches the normal state resistivity. Similar
Measurements on Conventional Superconductors

behaviour (the reduction of $\rho_1$ below $B_{c_3}(T)$) has been observed in a differential resistance study on NbSe$_2$\cite{145}, and their extracted $B_{c2}(T)$ and $B_{c3}(T)$ are comparable to ours (see Figure 4.27). In contrast, since the thickness of the surface superconducting layer $\xi$ is very small in cuprates, its contribution must be negligible in these materials.

Figure 4.27: (a) Our sketch of the NbSe$_2$ phase diagram, with uncertainties in $B_{c2}$ and $B_{c3}$ included. (b) NbSe$_2$ phase diagram obtained from a differential resistance study\cite{145}.

The field dependence of the pinning constant is presented in Figure 4.28, to be compared with published $J_c(H)$ data\cite{146}. As temperature rises, the field dependence of the pinning constant remains qualitatively unchanged. Each curve shows a minimum at a field scale that agrees with published data of $B_{c2}(T)$. Below $B_{c2}(T)$, the monotonic decrease of the pinning constant with an increasing magnetic field qualitatively agrees with published $J_c(H)$ data\cite{146}. Unlike the $J_c(H)$ data, our pinning constant does not become zero at its minimum. As mentioned earlier in this section, this is likely due to contributions from the $\hat{c}$-axis currents. The peak effect, characterized by a peak in $\alpha_p(H)$ near $T_c$, is not observed in our data. It is possible that it is smeared out by the $\hat{c}$-axis current effect and the surface superconductivity.

4.3.4 Viscosity Coefficient and Flux-flow Resistivity

The temperature dependence of the extracted viscosity coefficient at multiple fields and frequencies is shown in Figure 4.29. All curvatures have similar shapes, but
Measurements on Conventional Superconductors

Figure 4.28: (a) The field dependence of the pinning constant. Data were taken in the TE_{011} mode at 2.79 GHz, using field sweeps. At all the temperatures, the pinning constant curve shows a minimum. This behaviour is qualitatively unaffected by the introduction of the superconducting background in our model. (b) The field dependence of the critical current density\[^{146}\] for NbSe\(_2\) platelet samples of different thickness. Data were taken at 4.24 K.

The magnitude of the viscosity is field dependent at low temperature, which corresponds to a nonlinear field dependence in the Bardeen–Stephen model. At higher temperature, the viscosity becomes field independent, which is as expected in the Bardeen–Stephen model. In the region when \(B/B_{c2}(T)\) is large, viscosity is approximately linear, and can be extrapolated to zero at \(T_c\). In addition, there is only a weak frequency dependence in the viscosity.

The temperature dependence of the flux-flow resistivity and the extracted \(B_{c2}(T)\) are presented in Figure 4.30. \(\rho_{fl}(T)\) becomes flat at low temperature, which bears a resemblance to the normal state resistivity and agrees with the Bardeen–Stephen model. Like we did in V\(_3\)Si, the correlation between \(\rho_{fl}\) and \(\rho_n\) is reflected by the extracted \(B_{c2}(T)\). The extrapolated zero temperature upper critical field ranges from 3.6 T to 4.6 T, close to the initial estimate of 4.5 T.

The field dependence of the flux-flow resistivity has been obtained from a field sweep. It is very similar to the results of a DC experiment\[^{149}\]. The close resem-
Figure 4.29: The temperature dependence of the viscosity in (a) multiple fields and a single frequency (TE_{011} mode at 2.79 GHz) and (b) a single field (2 T) and multiple frequencies. Above $T_c$ the sample is in the normal state and the notion of viscosity loses its meaning.

Balance confirms the connection between our microwave measurements and the DC measurements. Interestingly, our $\rho_{\text{ff}}(B)$ may also be approximately fit to a linear function, as done in the DC experiment. This may not contradict the field dependent viscosity: the empirical law itself, as well as our field sweep data, was obtained with lower resolution and was fit to the linear function approximately, so small deviations may not be apparent.

At sufficiently high field $B_m(T)$, all curves of $\rho_{\text{ff}}(B)$ become rounded. Since $B_m(T)$ agrees with $B_{c2}(T)$, and at $B_m(T)$ all $\rho_{\text{ff}}(B)$ curves reach approximately the same value, the rounded feature should correspond to the onset of the normal state. At $B_m(T)$, $\rho_{\text{ff}}(B) < \rho_n$, which is likely the result of surface superconductivity. At $B_{c3}(T)$ surface superconductivity is suppressed, and $\rho_{\text{ff}}(B)$ converges to the normal state resistivity $\rho_n$. 
4.4 Summary of Results on Conventional Superconductors

Our measurements on conventional superconductors provide very useful information for our flux-flow study. From our measurements on Nb and Pb, the magnitudes...
Measurements on Conventional Superconductors

of possible systematic errors are quantified in the superconducting state, and the reliability of the data from high frequency modes is verified. From our measurements on V$_3$Si, the connection between $\rho_{ff}$ and $\rho_n$ is confirmed, in accord with the Bardeen–Stephen model. On the other hand, a violation of the simple linear field dependence in the Bardeen–Stephen law is observed. From the measurements on NbSe$_2$, we identify the key signatures of surface superconductivity.

Perhaps a more important observation is that our extracted flux-flow resistivity $\rho_{ff}$ is almost identical to that measured in a DC experiment. The functional temperature and field dependence of our extracted pinning constant, in addition, share strong similarities to those of the critical current density $J_c$ measured in DC experiments. Therefore, it can be empirically concluded that our measurements accurately separate the free flux-flow response from the pinning response, and provide at least as much information as the DC experiments.
Chapter 5

Measurements on YBa$_2$Cu$_3$O$_{6+x}$ Samples

This chapter contains flux-flow response data on high quality YBa$_2$Cu$_3$O$_{6+x}$ samples. At oxygen content $x = 0.993$, YBa$_2$Cu$_3$O$_{6.993}$ is a fully oxygenated overdoped superconductor with $T_c = 89$ K. Slightly reducing the oxygen content to $x = 0.95$ provides the optimal doping with $T_c = 94$ K. When $x$ is reduced to a critical value below 0.33, it leaves the underdoped side of the superconducting dome and becomes an antiferromagnetic Mott insulator. YBa$_2$Cu$_3$O$_{6+x}$ is therefore useful for studying the left half of the cuprate phase diagram.

Compared with other cuprates, YBa$_2$Cu$_3$O$_{6+x}$ has several remarkable properties that make it the gold standard for crystalline purity. YBa$_2$Cu$_3$O$_{6+x}$ has high chemical stability and can be made free of secondary phases. The oxygen anions can form well ordered CuO chains and have high mobility, as shown in Figure 5.1. These chains pull electrons out of the CuO$_2$ plane, modifying the hole doping and hence $T_c$. By manipulating the oxygen ordered phases, high homogeneity can be achieved, and random oxygen disorder minimized. Nowadays, samples of high chemical purity (99.995%) and high degree of crystalline perfection are available, most notably, from the UBC group using BaZrO$_3$ crucibles[150].

Transport properties have been extensively studied on these systems. Most relevant to this study is the normal state electrical resistivity, which is shown in Figure 5.2. Near optimal doping, the normal state resistivity decreases monotonically with temperature, but it is then masked by the superconducting transition at high $T_c$. As doping decreases, the normal state resistivity evolves and eventually becomes
Measurements on YBa$_2$Cu$_3$O$_{6+x}$ Samples

Figure 5.1: Oxygen ordered phases of YBa$_2$Cu$_3$O$_{6+x}$, characterized by filled or empty CuO chains.

non-superconducting with a logarithmic upturn at low temperature. We reiterate that this normal state resistivity upturn is in fact universal on the underdoped side at low temperature, as shown by the Ando–Boebinger experiment[61].

As mentioned in Chapter 1, Ben Morgan found the flux-flow resistivity in optimally doped YBa$_2$Cu$_3$O$_{6.95}$ to have a similar logarithmic upturn [81], which is in accord with the Ando–Boebinger effect and the Bardeen–Stephen relation $\rho_{ff} \propto \rho_n$. He thus interpreted his observation as an indication of an insulating ground state. Following on from this, I have continued to investigate this behaviour at other dopings, with a view to determine its origin.

Several high quality YBa$_2$Cu$_3$O$_{6+x}$ samples were obtained from the UBC group for this study. As a starting point, data on highly underdoped YBa$_2$Cu$_3$O$_{6.333}$[152] are presented in Section 5.1, to be compared with the original Ando–Boebinger experiments taken at a similar doping. In Section 5.2 we continue with an intermediate doped ortho-II sample[153], which is still underdoped but has minimal disorders. In Section 5.3 we move to another low disorder system, overdoped ortho-I YBa$_2$Cu$_3$O$_{6.993}$. Together with the optimally doped YBa$_2$Cu$_3$O$_{6.95}$ measured by Ben Morgan[81], this provides a good representation of the left half of the cuprate super-
5.1 Underdoped YBa$_2$CuO$_{6.333}$

The first sample measured is a highly underdoped YBa$_2$Cu$_3$O$_{6+x}$ sample. It allows us to make a direct comparison with the original Ando–Boebinger experiments, which were taken in the highly underdoped region[61]. This sample has an ellipsoidal shape, and was oriented with its $c$-axis parallel to the microwave field. It was measured in cavity configuration A. Only the TE$_{011}$ mode, with a resonant frequency of 5.47 GHz was used, and its quality factor was around $10^6$. It needs to be mentioned that $T_c$ in this kind of sample can be tuned without changing the oxygen content, as the oxygen ordering process can be controlled by annealing at room temperature. Our study on this sample was made when $T_c = 16.5$ K, and a complete set of zero field measurements from $T_c = 17$ K to 0 K can be found in the published work of our group[102].
5.1.1 Measurements in Zero Field

Let us start with the zero field data. A resonator constant of $7.004 \times 10^{-7} \Omega/\text{Hz}$ has been estimated by comparing the normal state resistivity with published data[151]. By matching $X_s$ to $R_s$ in the normal state, the penetration depth has been extensively studied by our group[102]. The zero temperature penetration depth $\lambda_0 \approx 360$ nm is estimated from the matching, since $X_s \approx \mu_0 \omega \lambda$ at low temperature. Uncertainties in $\lambda_0$ rescale the magnitude of our extracted flux-flow parameters but do not change their qualitative behaviour.

![Graph](image)

Figure 5.3: (a) Surface impedance of underdoped YBa$_2$Cu$_3$O$_{6.333}$. $R_s$ and $X_s$ match almost perfectly in the normal state. (b) $\rho_1(T)$ of the underdoped YBa$_2$Cu$_3$O$_{6.333}$ sample.

Surface impedance $Z_s(T)$ and the real part of the resistivity $\rho_1(T)$ are presented in Figure 5.3. A resistive transition around 16.5 K can be observed. The round shoulder in surface impedance and the resistivity is a typical feature for the underdoped YBa$_2$Cu$_3$O$_{6+x}$ sample because of fluctuations[102].

The complex conductivity is shown in Figure 5.4. In $\sigma_1(T)$, a broad peak caused by fluctuations[34] is observed at $T_c$. Below $T_c$, $\sigma_1(T)$ increases monotonically with decreasing temperature. It needs to be emphasized that $\sigma_1$ is extremely sensitive to the noise and residual $R_s$ at low temperature. The accurate measurement of $\sigma_1$ is a well known challenge in microwave experiments, and without samples of the highest
quality and complete knowledge of residual $R_s$, it can be dangerous to discuss $\sigma_1$ quantitatively.

The imaginary part of the resistivity $\sigma_2(T)$ is proportional to the superfluid density, which has been extensively studied by our group [102]. The famous result of the linear temperature dependence in the superfluid density [46] is reproduced, indicating the existence of line nodes in the energy gap.

### 5.1.2 Mixed State Measurements

We are most interested in the mixed state data. Data have been taken at 0.25 T, 0.5 T, 1 T, 3 T, 5 T and 7 T. Since YBa$_2$Cu$_3$O$_{6+x}$ samples have small $B_{c1}$ and very large $B_{c2}$, all data have been taken in the mixed state, and the condition $B_{c1} \ll B \ll B_{c2}$ has been satisfied. The mixed state $R_s(T)$ and $\rho_1(T)$ are shown in Figure 5.5, while the mixed state $X_s(T)$ and the field dependence of $\Delta X_s(B, 0)$ are shown in Figure 5.6.

Except for a very small magnetoresistance, no magnetic field dependence in the normal state is observed, indicating the consistency of our measurements. At the low temperature end, the value of both $R_s(T)$ and $X_s(T)$ in an applied field are much
Measurements on YBa$_2$Cu$_3$O$_{6+x}$ Samples

Figure 5.5: (a) $R_s(T)$ and (b) $\rho_1(T)$ of the underdoped YBa$_2$Cu$_3$O$_{6.333}$ sample. There is a kink at around 10 K in the data below 1 T, while at higher field the kink disappears.

Figure 5.6: (a) $X_s(T)$ and (b) $\Delta X_s(B, 0)$ of the underdoped YBa$_2$Cu$_3$O$_{6.333}$ sample. The standard error is shown by error bars which are smaller than the data point. $\Delta X_s(B, 0)$ is fit by a power law $\propto B^{0.7}$. 

122
larger than their zero field values (double the zero field values at a field of about 0.2 T), and are 10 or 100 times smaller than the normal state $Z_s(T)$. As discussed in Chapter 4, this means that at low temperature, our extracted parameters are qualitatively unaffected by whether the superconducting background contribution is included in the model. The vortex core size in cuprate superconductors is usually smaller than the intervortex spacing, and $B_{c1} \ll B \ll B_{c2}$ should be satisfied over most of the temperature range, so that the Waldram model can be safely used without modification.

Both $R_s(T)$ and $\rho_1(T)$ show a broadening of the resistive transition, which has been regarded as evidence for the existence of a vortex liquid state\cite{41}. We will examine this in terms of the pinning constant.

### 5.1.3 Depinning Frequency and Pinning Constant

The temperature dependence of depinning frequency and the pinning constant is presented in Figure 5.7. At low temperature the depinning frequency is about 30 GHz, which is much higher than the working frequency of 5.47 GHz. This indicates that the data have been taken in the limit of strong pinning over most of the temperature range.

The temperature dependence of the pinning constant in this sample is remarkably different from those of conventional superconductors in Chapter 4. In every pinning constant curve, $\alpha_p(T)$ approaches zero with an upward curvature around a crossover temperature $T_m$, which decreases sharply with increasing field. Below $T_m$ the data slope $d\alpha_p/dT$ is large, and has a strong field dependence. Above $T_m$, both $\alpha_p$ and $d\alpha_p/dT$ approach zero. Since above $T_m$ the broadening of the rounded resistive transition is apparent (see Figure 5.5), this should correspond to a different vortex matter phase, possibly the vortex liquid state. On the other hand, in all our measured conventional superconductors, $T_m$ is not apparent, and the pinning constant curves can be fit to the master forms $\alpha_p \propto B_{c2}(T)^3$ over a large temperature range. This difference is very likely due to the impact of thermal fluctuations, which are dominant in cuprates but very small in conventional superconductors.

As mentioned in Chapter 1, thermal fluctuations can lead to smoothing of the pinning potential and thermal creep, and consequently reduce pinning to zero at a temperature lower than $T_c$. Therefore, in the empirical law of the temperature dependence $J_c \propto (1 - T/T_c)^3$, $T_c$ is often replaced by $T_0[154]$, the temperature scale
Figure 5.7: (a) The depinning frequency and (b) the pinning constant of underdoped $\text{YBa}_2\text{Cu}_3\text{O}_{6.333}$. Data were taken in the TE$_{011}$ mode at 5.47 GHz. In the low temperature data, $\alpha_p$ can be extrapolated to zero at some temperature $T_m$.

of vortex lattice melting. On the other hand, the phase transition line $H_{c2}(T)$ is masked by thermal fluctuations and becomes a crossover. We stress that thermal fluctuations contribute a dynamic term to the functional temperature dependence, while the term $H_{c2}(T)$ is very likely to be a static one. To consistently interpret all results, it is probably more accurate to consider both terms, and the simplest form of the functional temperature dependence should be:

$$\alpha_p \propto H_{c2}(T)^\beta (1 - T/T_0)^\gamma,$$

in which $\gamma$ is a characteristic parameter related to thermal fluctuations, and $H_{c2}(T) \propto (1 - (T/T_c)^2)$ is assumed as starting point. In conventional superconductors, if $T_0 \gg T_c$, the term $H_{c2}(T)^\beta$ dominates. In cuprates, if $T_0$ is smaller than $T_c$, $(1 - T/T_0)^\gamma$ becomes the leading term, while above $T_0$ the thermal fluctuation term may become zero to ensure $\alpha \approx 0$. If $T_0$ is comparable to $T_c$, then $\alpha_p$ is approximately proportional to $(1 - (T/T_c)^2)^\beta (1 - T/T_c)^\gamma$. In underdoped $\text{YBa}_2\text{Cu}_3\text{O}_{6.333}$, we should still be in the weak field limit at 7 T, so $H_{c2}(T)$ should not have a strong field dependence. Thus the field dependent behaviour in the pinning constant curves must depend solely on the thermal fluctuation term. Therefore, $T_m$ is very likely to be $T_0$, and corresponds
Measurements on YBa$_2$Cu$_3$O$_{6+x}$ Samples

to vortex lattice melting. Since the pinning constant is approximately linear below $T_m$ in the high field data (3 T, 5 T, 7 T) and $T_m < T_c$, $\gamma \approx 1$ can be assumed. This might be related to thermal creep, and a more detailed discussion is presented in Chapter 8.

A concern here is that our pinning constant data are small but not zero around $T_m$, deviating from Equation 5.1. A possible explanation is that Equation 5.1 is a phenomenological form appropriate to the DC case $\omega \to 0$, and $\alpha_p$ has a frequency dependence on the scale of GHz.

Another factor that needs to be considered, is that our model does not explicitly take into account the random distribution of pinning potentials and thermal fluctuations. A nonzero $\alpha_p$ in the vortex liquid state can probably be understood, by considering these two factors and the microwave time scale. Assuming that $T_m$ is the vortex lattice melting temperature, then the average thermal energies of the vortices would be comparable to the averaged pinning potential. Given the random distribution of the pinning potential strength, at any given time a fraction of vortices would remain trapped. Nevertheless, a small DC current would effectively "tilt" the pinning potential, and with long enough time the trapped vortices would eventually gain enough velocity to escape the pinning potential. The pinning constant $\alpha_p$ would then be vanishingly small at DC, and viscosity would be the limiting factor for the vortex velocity. On the contrary, in the microwave case, there would always be a fraction of vortices that remain in their pinning sites for the duration of a microwave cycle $1/\omega$. This would result in small but nonzero $\alpha_p$. As temperature increases above $T_m$, the fraction of trapped vortices decreases and reduces $\alpha_p$, but $\alpha_p$ remains finite. A natural conclusion of this is that, in the vortex liquid state, the viscosity coefficient obtained in our analysis would be slightly reduced from its DC value, and would have a weak frequency dependence.

5.1.4 Viscosity Coefficient and Flux-flow Resistivity

The viscosity coefficient and the flux-flow resistivity are presented in Figure 5.8. The viscosity coefficient shows a strong field dependence. This implies a deviation from the linear field dependence in the Bardeen–Stephen law. A broad peak can be observed between 4 K to 6 K. As field increases, the height of the peak increases, while the width of the peak deceases. At low temperature all the viscosity data show a downturn, which corresponds to an upturn in the flux-flow resistivity.
Measurements on YBa$_2$Cu$_3$O$_{6+x}$ Samples

Figure 5.8: (a) The viscosity coefficient and (b) the flux-flow resistivity of underdoped YBa$_2$Cu$_3$O$_{6.333}$. The viscosity shows a broad peak at 2 K to 6 K, the implication of which will be discussed later. At low temperature, the flux-flow resistivity can be fit to logarithmic functions at all fields.

Figure 5.9: The field dependence of the flux-flow resistivity at fixed temperatures, showing marked deviations from linearity.

All extracted flux-flow resistivity data appear to show the same logarithmic tem-
Measurements on $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$ Samples

Temperature dependence as that of optimally doped $\text{YBa}_2\text{Cu}_3\text{O}_{6.95}$ samples[81]. At this doping the normal state resistivity is known to show the same logarithmic behaviour[61], apparently indicating that the usual Bardeen–Stephen connection exists. At the highest field of 7 T, the flux-flow resistivity is about 1/5 of the extrapolated normal resistivity.

The field dependence of the flux-flow resistivity is shown in Figure 5.9. The linear field dependence of $\rho_{ff}$ in the Bardeen–Stephen law is not strictly followed.

5.2 Ortho-II $\text{YBa}_2\text{CuO}_{6.52}$

The second sample measured has an intermediate doping level, and an ortho-II vacancy ordered phase, which is characterized by alternating filled and empty CuO chains along the $\hat{a}$-axis near the CuO$_2$ plane[153]. This system has a very low level of disorder, and is thus an ideal choice for carrying out clean studies on the underdoped side of the phase diagram. Its quasiparticle dynamics in zero field and at microwave frequencies have been extensively studied by the UBC group, the results of which can be compared with ours. In addition, recent quantum oscillation measurements[155] at strong magnetic fields have suggested a Fermi liquid ground state on this type of sample – this would mean that if the Bardeen–Stephen “fractional normal resistivity” picture remains valid, Fermi liquid behaviour might be expected in our extracted flux-flow resistivity.

The sample used here is a rectangular platelet with an average length of 0.632 mm and width of 0.54 mm. It will be referred to as ortho-II $\text{YBa}_2\text{Cu}_3\text{O}_{6.52}$ sample-A for convenience. The photos of this sample are shown in Figure 5.10.

For historical reasons, the data on this sample were initially taken in cavity configuration B, where our main working TE$_{011}$ mode had a resonant frequency of 2.64 GHz and a quality factor of $2 \times 10^6$. Other modes used include the TE$_{013}$ mode at 4.51 GHz, the TE$_{035}$ mode at 9.12 GHz and the TE$_{043}$ mode at 13.97 GHz. Data taken in configuration B were good at most frequencies, but a few exceptions exist. Thus the measurements were later repeated in configuration C. In this configuration, the main working TE$_{011}$ mode had a resonant frequency of 2.79 GHz and a quality factor of $1.6 \times 10^5$. There is some overlap between these two data sets, in particular the zero field measurements, which have been repeated more than 4 times. In the process, the reproducibility of our experiment has been shown to be excellent, and no obvi-
Measurements on YBa$_2$Cu$_3$O$_{6+x}$ Samples

Figure 5.10: (a) The ortho-II YBa$_2$Cu$_3$O$_{6.92}$ sample (black) and the Nb replica sample (silver). (b) The ortho-II YBa$_2$Cu$_3$O$_{6+x}$ sample on top of the sapphire rod. There is some excess vacuum grease on its surface in this photo.

ous disagreement was found. In 2009 the same measurements have been taken on a larger, unpolished sample of the same kind (referred to as sample B), confirming our observations to be sample independent. Here we mostly present data on sample-A, taken in configuration B, as they cover a much wider range of magnetic field and frequency.

5.2.1 Zero Field Measurements

The Nb replica sample (see Section 4.1) has been used to determine the resonator constant for this sample. For our main TE$_{011}$ mode at 2.64 GHz, it is $1.2864 \times 10^{-6}$ Ω/Hz, about the same as the value obtained by comparing our data to published normal state resistivity data[151]. The zero temperature penetration depth $\lambda_0 = 176$ nm has been taken from reference[156] to offset surface reactance. As the uncertainty in $\lambda_0$ is relatively small, it will not be included in our analysis.

Surface impedance is presented in Figure 5.11. Typical features for YBa$_2$Cu$_3$O$_{6+x}$ samples, including the rounded resistive shoulder and the broad “bump” in the superconducting state $R_s[157]$, are all observed, suggesting that this is a high quality sample. The data qualitatively agrees with published results[158], with the exception of the low temperature upturn of $X_s$ (see Figure 5.11 (b)). This is probably due to a small amount of paramagnetic impurities on the sample surface, which is
Figure 5.11: (a) $Z_s(T)$ of the ortho-II YBa$_2$Cu$_3$O$_{6.52}$ sample. Between 30 K to 50 K, $R_s(T)$ shows apparent fluctuations. This is very likely due to tiny steps on the sample surface, a typical by-product of sample polishing. Our later measurements on the larger, unpolished sample confirm that these fluctuations are merely artifacts, and have no substantial impact on our results. (b) A low temperature upturn in $X_s(T)$ due to paramagnetic impurities, likely from nonsuperconducting flux on the surface of the sample.

almost inevitable during sample growth. At strong enough field, this upturn should be suppressed and have no impact on our mixed state data.

To prove that the qualitative behaviour of the data remains unaffected by the surface flux, the complex conductivity is compared with results from the UBC group[158] (see Figure 5.12). Aside from the fluctuations between 30 K to 50 K at 2.64 GHz, the behaviour resembles their measurements on an ortho-II YBa$_2$Cu$_3$O$_{6.52}$ sample [158]. It needs to be emphasized that the UBC results were obtained in a different sample mounting geometry from ours. In their experiment, data were taken with the $\hat{a}\hat{b}$-plane of the sample parallel to the microwave field, which minimizes complications from the demagnetizing factor and nonlocal dynamics. In contrast, in our experiment, in order to obtain $c$-axis vortices, the $\hat{a}\hat{b}$-plane has to be aligned perpendicular to the microwave field, and nonlocal dynamics might make some small contributions to the zero field conductivity at low temperature.

A prominent feature in $\sigma_1(T)$ is the large, broad peak below 15 K. A similar peak
Measurements on YBa$_2$Cu$_3$O$_{6+z}$ Samples

Figure 5.12: (a) Our measurements of the zero field $\sigma_1(T)$ for ortho-II YBa$_2$Cu$_3$O$_{6.52}$. A small residual $R_s$ is subtracted from the raw data. (b) The zero field $\sigma_1(T)$ data of the same kind of sample, from the UBC group[104]. Because $\sigma_1$ is extremely sensitive to residual $R_s$ at low temperature, it is usually very difficult to make quantitative comparison in the presence of impurities. Note also that the sample geometry used in our experiment is different from used by the UBC group, which in particular may act to limit the effective mean free path at low temperature. Nevertheless, both plots show a broad peak at low temperature, the height of which decreases with increasing frequency.

has been observed in the data of sample-B, in which fluctuations between 30 K and 50 K are absent. The origin of the 15 K peak is well understood[157]. It is the result of competition between a decreasing quasiparticle density $n$ and an increasing quasiparticle scattering time $\tau$ on cooling, since $\sigma_1 \propto n\tau$. When temperature decreases below $T_c$, the strong inelastic scattering is quickly suppressed[159], and $\tau(T)$ increases much faster than $n(T)$ decreases, resulting in a rising conductivity. As temperature decreases further, inelastic scattering is completely suppressed, the effective scattering time is limited by elastic scattering, and saturates at some impurity-related limit at the low temperature end. This results in a decreasing in conductivity, and hence forms the peak. The position of the peak increases with the impurity concentration and frequency, while the height of the peak decreases with increasing impurity concentration and frequency.
Measurements on YBa$_2$Cu$_3$O$_{6+x}$ Samples

Figure 5.13: (a) Our measurements of the $\hat{a}\hat{b}$-averaged zero field $\sigma_2(T)$ for the ortho-II sample in the $\hat{a}\hat{b}$ plane. (b) $\sigma_2(T)$ data for the same kind of sample along the $\hat{a}$-axis, from the UBC group[104]. The different magnitude of $\sigma_2(T)f$ is the result of the difference between $\lambda_{a\hat{b}}(0)$ and $\lambda_{a}(0)$, which are used to offset $X_s$.

The comparison with $\sigma_2(T)$ is shown in Figure 5.13. It is worth noting that our experiments focused on the averaged response in the $\hat{a}\hat{b}$ plane, while the measurements taken by the UBC group were made along the $\hat{a}$-axis. Nevertheless, the temperature dependence of $\sigma_2$ seems to be the same for both cases.

5.2.2 Mixed State Measurements

We now turn to mixed state data, which have been taken at fields ranging from 0.125 T to 7 T. Although impurity-related ESR transitions are a possible concern, a magnetic field of 1 T should shift potential ESR peaks by 30 GHz and suppress the impurity contributions. In addition, if we did not include the zero field data (the superconducting background contribution) in our analysis, it would not have a substantial impact on the qualitative behaviour of the extracted parameters.

The mixed state surface impedance is presented in Figure 5.14. Near $T_c$ the resistive transitions are substantially broadened, implying a wide vortex liquid phase. In the superconducting state both $R_s(T)$ and $X_s(T)$ show field dependent kinks, the
Figure 5.14: (a) $R_s(T)$ and (b) $X_s(T)$ of the ortho-II $\text{YBa}_2\text{Cu}_3\text{O}_{6.52}$ sample. Data were taken in the $\text{TE}_{011}$ mode at 2.64 GHz.

Figure 5.15: $\Delta X_s(B, 0)$ for ortho-II $\text{YBa}_2\text{Cu}_3\text{O}_{6.52}$ (a) sample-A and (b) sample-B. Data were taken in the $\text{TE}_{011}$ mode at 2.64 GHz and 2.79 GHz respectively. A power law of the same form $\propto B^{0.7}$ was used to interpolate the data for both samples.
Measurements on YBa$_2$Cu$_3$O$_{6+x}$ Samples

meaning of which will be discussed in the next section. No signature of ESR transition is observed.

In the normal state $R_s(T)$ and $X_s(T)$ show a weak field dependence, which is not obvious but might affect our determination of $\Delta X_s(B,0)$. Ideally, $\Delta X_s(B,0) = [X_s(B,T) - X_s(0,T)]$ should be temperature independent in the normal state, but it fluctuates with considerable uncertainties for sample-A. This might be associated with the presence of nonsuperconducting flux on the sample surface, and in sample-B the uncertainty in $\Delta X_s(B,0)$ is much smaller.

The interpolations of the averaged $\Delta X_s(B,0)$ for both ortho-II YBa$_2$Cu$_3$O$_{6.52}$ samples are presented in Figure 5.15. Since sample-B appears to have less flux on its surface, its data were interpolated first to obtain a power law $\propto B^{0.7}$. This form was then applied to sample-A to obtain an interpolation with lower uncertainties in the field dependence.

5.2.3 Depinning Frequency and Pinning Constant

![Figure 5.16: The depinning frequency of ortho-II YBa$_2$Cu$_3$O$_{6.52}$. Data were taken in the TE$_{011}$ mode at 2.64 GHz.](image)

The temperature dependences of the depinning frequency and the pinning constant are presented in Figure 5.16. From $f_p$ it can be concluded that our sample is in the pinning limit at low temperature, while it becomes fully depinned close to $T_c$. 

133
The temperature scale of the depinning will be investigated in terms of the pinning constant.

The temperature dependence of the pinning constants at multiple fields and frequencies is presented in Figure 5.17. Over a large temperature range, the pinning constant is approximately exponentially temperature dependent:

\[ \alpha_p \propto e^{-T/T_0}, \quad (5.2) \]

where \( T_0 \) is a characteristic temperature scale that has little field dependence but considerable frequency dependence. This quasiexponential temperature dependence in \( \alpha_p \) or \( J_c \) has been observed in many cuprate samples, including the optimally doped YBa\(_2\)Cu\(_3\)O\(_{6.95}\) samples studied by Ben Morgan\[81\]. It has often been interpreted as the result of the thermal creep, and

\[ T_0 = \frac{U_0}{k_B \ln(1 + \tau/\tau_0)} \quad (5.3) \]

has been proposed\[81\]. Here \( U_0 \) is the effective strength of the pinning potential, \( \tau \) can be treated as \( 1/\omega \), and \( \tau_0 \) is a characteristic time scale for thermal creep. As
shown in Figure 5.18, this form provides a good fit to our $T_0(\omega)$ data. Perhaps coincidentally, from the fit we find $U_0/k_B = 57$ K, close to $T_c = 59$ K. The thermal creep attempt time $\tau_0$ is estimated to be $1.8 \times 10^{-12}$ s, so thermal creep is expected to be frozen below the terahertz range. It needs to be emphasized that Equation 5.3 is a phenomenological form and, to the best of our knowledge, existing thermal creep models have difficulties in consistently interpreting our data. This will be discussed in more detail in Chapter 8.

![Figure 5.18: The relation between $T_0$ and $\omega$. An exponential correlation can be found between $1/T_0$ and $1/\omega$. An uncertainty of about 0.02 K$^{-1}$ is included in $1/T_0$.](image)

At higher temperature, the pinning constant deviates from the exponential fit, and a temperature scale $T_1$ can be approximately defined in the region that the deviation occurs. $T_1$ increases with frequency, indicating a connection to a dynamical effect, possibly thermal creep. Interestingly, $B(T_1)$ approximately coincides with published data of the vortex lattice melting line $B_m(T)$ on the same material[160, 161]. It is possible that $B(T_1)$ is our measured vortex lattice melting line, and that melting is driven by thermal creep.

Above a vortex lattice melting line, a simple vortex liquid phase is usually expected, but as temperature further increases, the high field (> 1 T) data show changes in $d(\ln \alpha_p)/dT$ and form step-like structures. The onset temperature $T_2$ (defined from the kink in $d(\ln \alpha_p)/dT$) approximately corresponds to the kink in $Z_s$. The temperature $T_2$ increases with increasing frequency and decreasing field, and is very likely
related to certain dynamical effects like thermal creep or vortex lattice melting. Above \( T_2 \), \( \alpha_p \) is vanishingly small (\( 10^3 \) times smaller than \( \alpha_p(0) \)). In view of the behaviour of underdoped YBa\(_2\)Cu\(_3\)O\(_{6.333}\) in Section 5.1, it seems reasonable that \( T_2 \) is the onset of the vortex liquid state, while \( T_1 \) is the onset of softening of the vortex lattice. It is also possible that this observation might be related to a two-stage melting transition observed in Bi\(_2\)Sr\(_2\)CaCu\(_2\)O\(_{8+\delta}\)\([162]\), where \( T_1 \) is the temperature scale for intraplanar melting, while \( T_2 \) correspond to interplanar decoupling of vortex lines.

To the best of our knowledge, this step-like structure has not been observed or predicted before. Since it shows a strong frequency dependence, it is possible that it is only apparent in the microwave range. From the frequency dependence data, it can be seen that the pinning constant is almost frequency independent at low temperature, and as temperature increases a marked frequency dependence develops. The step-like structure becomes more flattened at high frequency, suggesting that the dynamical process, possibly thermal creep, is weakened. At sufficiently high frequency the step may disappear. In the opposite limit, at low frequency, the “step” would be steeper, and it is likely that in the DC case \( \alpha_p \) approaches zero at \( T_2 \). A more detailed analysis will be presented in Chapter 8.

### 5.2.4 Viscosity Coefficient and Flux-flow Resistivity

The viscosity coefficient is presented in Figure 5.19 as a function of applied field, temperature and frequency. A striking observation is that all the viscosity data show a broad peak at low temperature, which bears a strong resemblance to the zero field \( \sigma_1(T) \) measured by the UBC group (see Figure 5.12). Surprisingly, the mixed state data may provide better indications of the intrinsic dynamics than our zero field measurements. In a field of 0.5 T, the position of the peak matches that of the zero field \( \sigma_1(T) \) measured by the UBC group. In both data sets, when frequency increases, the position of the peak increases, but the height of the peak decreases. This also resembles the behaviour of the zero field \( \sigma_1(T) \).

This resemblance is likely not a coincidence, since our extraction of \( \eta_{fl}(T) \) is very similar to that of the effective \( \sigma_1(B,T) = \rho_1(B,T) / |\rho(B,T)|^2 \) in the mixed state. As mentioned in Chapter 2, the viscosity is directly related to the conductivity profile by

\[
\eta = \int \int \sigma_1(x,y) \Lambda^2 (\frac{dI_s}{dy})^2 dx dy.
\] (5.4)
Measurements on YBa$_2$Cu$_3$O$_{6+x}$ Samples

Figure 5.19: The field dependence of the vortex viscosity at (a) 2.64 GHz; (b) 4.51 GHz and (c) 13.99 GHz. (d) The frequency dependence of the viscosity coefficient at 0.5 T. At low field the viscosity is very similar in form to the zero field $\sigma_1(\omega, T)$ (see Figure 5.12). At higher fields the viscosity between 10 K to 40 K is substantially enhanced, resulting in a broadening of the peak in $\eta_{fl}(T)$. 
If a uniform conductivity profile is assumed, the viscosity is then proportional to the effective conductivity. In particular, the vortex core size can be estimated from the ratio \( \frac{n}{\sigma_1} \), using Equation 5.4. In the simplest case, if we assume an s-wave vortex and replace \( \sigma_1(B) \) by \( \sigma_1(0) \) (which should be a good approximation at weak field), the vortex core size is found to be of the order of a few nanometers. This is a surprisingly reasonable result, consistent with the core size measured by other techniques, particularly the \( \mu \)SR study on a similar YBa\(_2\)Cu\(_3\)O\(_{6.60}\) crystal\[43\]. Although for a more realistic d-wave case the current distribution and the conductivity profile will certainly be more complicated, this should not have a huge impact on this estimate of the magnitude of the core size. Therefore, the dissipation must still be concentrated near the core, pointing to the validity of our model.

Figure 5.20: (a) The temperature dependence and (b) the field dependence of flux-flow resistivity of ortho-II YBa\(_2\)Cu\(_3\)O\(_{6.52}\). Data were taken in the TE\(_{011}\) mode at 2.64 GHz.

We stress that the resemblance between \( \eta_{fl}(T) \) and the zero field \( \sigma_1(T) \) has a strong implication: that the quasiparticle dynamics reflected by \( \sigma_1(T) \) is not greatly altered by the presence of vortices, and the peak in \( \eta_{fl}(T) \) must still correspond to the competition between the density of states and the quasiparticle scattering time. Consequently, the logarithmic upturn in the flux-flow resistivity \( \rho_{ff}(T) \), which is inversely proportional to \( \eta_{fl}(T) \), would be governed by the same physics.

The central problem is then how to disentangle the impacts of the presence of...
vortices on the density of states and the scattering time, respectively. A detailed discussion will be presented in Chapter 7, after we present the viscosity data for the remaining samples.

The flux-flow resistivity is presented in Figure 5.20. At temperatures below 10 K, all the data show a logarithmic temperature dependence, and their magnitude is larger than that of optimally doped YBa$_2$Cu$_3$O$_{6.95}$. This would seem to agree with the metal-insulator crossover scenario suggested by the Ando–Boebinger experiment.

A linear field dependence is not followed, is also indicated by the field dependent viscosity.

5.3 Overdoped YBa$_2$CuO$_{6.993}$

The overdoped Overdoped YBa$_2$CuO$_{6.993}$ sample is also made by the UBC group. It has an Ortho-I structure, which is characterized by nearly filled oxygen chains. It has very few defects, and is perhaps the cleanest material of all cuprates. As it has a doping near that of the optimally doped sample Ben Morgan measured, we expect its behaviour to be similar to Ben Morgan’s results on optimally doped YBa$_2$Cu$_3$O$_{6.95}$. A photo of the sample is shown in Figure 5.21.

![Photo of the overdoped YBa$_2$CuO$_{6.993}$ sample.](image)

5.3.1 Zero Field Measurements

The zero field data for this sample should be similar to published data from the UBC superconductivity group[156], who used the same technique but different measurement geometry (optimized for measuring the zero field surface impedance) in a
Figure 5.22: Surface impedance of overdoped YBa$_2$Cu$_3$O$_{6.993}$. Measurements were taken in the TE$_{011}$ mode.

Figure 5.23: (a) $\sigma_1(T)$ of overdoped YBa$_2$Cu$_3$O$_{6.993}$. Measurements were taken at a frequency of 2.79 GHz. (b) Published $\sigma_1(T)$ data from the UBC superconductivity group[163]. Note that here we use a more updated $\lambda_0[156]$, which accounts for part of the difference in magnitude.
Figure 5.24: (a) Normalized superfluid density data of overdoped YBa$_2$Cu$_3$O$_{6.993}$. Measurements were taken at a frequency of 2.79 GHz. (b) Normalized superfluid density data from the UBC superconductivity group[163].

similar sample. The absolute penetration depth in the $ab$ plane has been estimated to be 90 nm[156]. A resonator constant was extracted from the comparison of our data to Ben Morgan's optimally doped YBa$_2$Cu$_3$O$_{6.95}$ data, because at high temperature their resistivities should be very close.

The microwave surface impedance is presented in Figure 5.22. A sharp transition around 89 K can be observed. In the superconducting state, the data are smooth except for a small upturn in $X_s$ below 5 K, which seemingly relates to the small amount of paramagnetic materials on the sample surface. Its effect can be ignored after a strong magnetic field is applied (see Section 5.2).

The extracted complex conductivity is compared with earlier work on a sample without contamination on its surface[158]. For $\sigma_1(T)$ in Figure 5.23, both samples show broad peaks at around 25 K at 2 to 3 GHz. For $\sigma_2(T)$ in Figure 5.24, aside from the downturn shown in our data at very low temperature, which is caused by paramagnetic materials on the sample surface, their behaviours are very similar. The comparison suggests that our sample is of the highest quality.
Figure 5.25: (a) $R_s(T)$ and (b) $X_s(T)$ in the mixed state of overdoped YBa$_2$Cu$_3$O$_{6.993}$. As a consistency check, data at different fields match well in the normal state.

Figure 5.26: $\Delta X_s(B, 0)$ of overdoped YBa$_2$Cu$_3$O$_{6.993}$. Errors in the data are vanishingly small.

### 5.3.2 Mixed State Measurements

Mixed state data have been taken from 0.25 T to 4 T in the TE$_{011}$ mode at 2.79 GHz. As mentioned in Section 5.2, a detailed analysis of the ESR correction will not be
applied. Empirically, at a magnetic field of 1 T, the flux-flow response is much larger than the zero field one, and the dominant ESR peaks should be shifted by 30 GHz. Therefore, at 1 T and above, the ESR correction can be neglected.

The mixed state surface impedance is presented in Figure 5.25. Data are similar to those of the optimally doped YBCO sample studied by Ben Morgan[81]. At about 70 K, $R_s(T)$ shows kinks. Below the kinks $R_s(T)$ varies almost exponentially with temperature, and at the low temperature end shows downward curvature.

The field dependence of $\Delta X_s(B, 0)$ is presented in Figure 5.26. A linear function is used to interpolate the data.

### 5.3.3 Depinning Frequency and Pinning Constant

The depinning frequency data are presented in Figure 5.27. Like other YBa$_2$Cu$_3$O$_{6+x}$ samples, at low temperature the depinning frequency is quite large compared with our working frequency, indicating that we are strongly in the pinning limit at low temperature.

![Figure 5.27: $f_p(T)$ of overdoped YBa$_2$Cu$_3$O$_{6.993}$](image)

The pinning constant data are plotted in Figure 5.28, and are compared with published $J_c(T)$ data[164]. Clearly, $\alpha_p(T)$ and $J_c(T)$ have very similar behaviour. They both decrease quasiexponentially as temperature rises, and approach zero at a temperature near $T_c$. As field increases, they both decrease monotonically. This
Measurements on YBa$_2$Cu$_3$O$_{6+x}$ Samples

resemblance is remarkable. It not only confirms the connection between our measurements of $\alpha_p$ and $J_c$ measured in the DC experiments, but also dismisses a potential concern on our data extraction model. As mentioned in Chapter 2, our data extraction model does not directly include a thermal creep factor $\epsilon$ as a free parameter, which might affect our data extraction at high temperature. The resemblance here indicates that the thermal creep factor is incorporated in our pinning constant data, and that our model gives qualitatively the same results as DC experiments.

![Figure 5.28](image)

Figure 5.28: (a) Temperature dependence of the pinning constant, extracted from our overdoped YBa$_2$Cu$_3$O$_{6.993}$ data. (b) Published $J_c$ data extracted from DC magnetization measurements[164]. The exponential fit is represented by a dashed line.

The behaviour of the pinning constant also has a strong resemblance to that of the optimally doped YBa$_2$Cu$_3$O$_{6.95}$ sample measured by Ben Morgan[81], particularly in the exponential temperature dependence at low temperature. Because the doping levels are similar this is expected, but is also an encouraging sign that the measurements are reproducible with no strong sample dependence. As temperature increases, the pinning constant drops below the exponential fit, at about 40 K. In the ortho-II YBa$_2$Cu$_3$O$_{6.52}$ sample such behaviour was thought to be related to vortex lattice melting, but this does not seem to be the case for the overdoped sample.

Following the observations of a step-like structure in the ortho-II YBa$_2$Cu$_3$O$_{6.52}$ sample, we might expect the same behaviour to appear in this sample. It is possible
Measurements on YBa$_2$Cu$_3$O$_{6+x}$ Samples

that such step-like structure exists as a kink around 70 K, but this is not certain. Perhaps 4 T is still much smaller than $B_{c2}(T)$, so a potential structure cannot be distinguished clearly. If the frequency dependence of the pinning constant for this sample is the same as for the case of the ortho-II YBa$_2$Cu$_3$O$_{6.52}$ sample, it is possible that the kink is replaced by a sharp drop of $\alpha_p$ in the DC case as $\omega \to 0$. At our working frequency of 2.79 GHz, the sharp drop in $\alpha$ occurs at a high temperature $T_m$, which seems to coincide the reported vortex lattice melting line[165].

A small peak in the pinning constant is seen in the 2 T, 3 T and 4 T data at around 80 K. This peak feature was observed by Ben Morgan in an untwinned optimally doped YBa$_2$Cu$_3$O$_{6.95}$ sample, and was related to the peak effect[166] and vortex lattice melting. This will be discussed in Chapter 8.

5.3.4 Viscosity Coefficient and Flux-flow Resistivity

![Figure 5.29](image)

Figure 5.29: (a) The viscosity and (b) the flux-flow resistivity of overdoped YBa$_2$Cu$_3$O$_{6.993}$. A logarithmic temperature dependence can be observed at low temperature, for data taken at different fields.

The viscosity coefficient and the flux-flow resistivity are presented in Figure 5.29. All data show a broad peak at 10 K to 20 K, which shares a strong resemblance to that of the zero field $\sigma_1(T)$. From the ratio $\eta/\sigma_1(B = 0)$ (the UBC group's $\sigma_1(T)$
Measurements on YBa$_2$Cu$_3$O$_{6+x}$ Samples

data are used because of their better accuracy), the vortex core size is estimated to be around 2 nm. At low temperature, $\eta$ shows some field dependence, while above 45 K, the viscosity shows little field dependence. A possible explanation is that $B/B_{c2}$ is extremely small in this sample, so the applied field has little effect in the regime where inelastic scattering dominates. The overall behaviour is quite similar to that of optimally doped YBa$_2$Cu$_3$O$_{6.95}$[81]. A detailed analysis of the viscosity will be presented in Chapter 7.

The flux-flow resistivity shows an upturn at low temperature. A logarithmic upturn in $\rho_{\text{ff}}(T)$ is seen at all fields, of similar magnitude to that of optimally doped YBa$_2$Cu$_3$O$_{6.95}$[81].

5.3.5 Comparison with Other Experiments

It is always interesting to compare our results with others. Nevertheless, to the best of our knowledge, at low temperature (below 10 K), published data are only available for YBa$_2$Cu$_3$O$_{6+x}$ samples with similar dopings to this one.

In the microwave range, data are available from Y. Tsuchiya et al. on optimally doped YBa$_2$Cu$_3$O$_{6.95}$[167]. Their technique and data extraction method are very similar to ours, but the resolution seems to be lower. As shown in the left panel of Figure 5.30, the magnitude and the temperature dependence of the viscosity and the depinning frequency obtained in their work are comparable to our results. At low temperature there might also be a downturn in viscosity in their data, corresponding to an upturn in flux-flow resistivity. A later study taken by K. Kinoshita et al. on Zn-doped YBa$_2$Cu$_3$O$_{6+x}$ samples[168] provided similar results, and showed that the viscosity is relatively insensitive to small amount of impurities.

In the terahertz frequency range, there is an interesting experiment carried out by the Orenstein group[169]. By measuring the transmission of picosecond pulses through three YBa$_2$Cu$_3$O$_{6+x}$ samples with $T_c \approx 88$ K, the mixed state complex resistivity was measured in a frequency range from 100 GHz to 500 GHz. From the complex resistivity they extracted the viscosity and the pinning constant in approximately the same way as ours. Their results are presented in the right panel of Figure 5.30.

Their measured viscosity is $1$ to $2 \times 10^{-7}$ Nm$^{-2}$ at 100 to 500 GHz, about 10 times smaller than our results. Since we have already demonstrated that the viscosity decreases with increasing frequency, their results do not contradict ours.
Measurements on YBa$_2$Cu$_3$O$_{6+x}$ Samples

Figure 5.30: (a) Microwave measurements of the viscosity and the depinning frequency on optimally doped YBa$_2$Cu$_3$O$_{6.95}$, carried out by Y. Tsuchiya et al.[167]. (b) Terahertz measurements of the viscosity and the pinning constant on overdoped YBa$_2$Cu$_3$O$_{6+x}$, from the Orenstein group[169].

The width of the peak in viscosity increases with frequency, it is possible that in the terahertz frequency range, the peak in viscosity is flattened.

At low temperature, their pinning constant data are very close to ours. As temperature increases, our pinning constant data decrease faster than theirs and follow some concave curves. Their pinning constant is approximately linear from 10 K to 70 K, while ours has a concave shape. This is probably due to the frequency dependence of thermal creep. As mentioned in Section 5.2, for our ortho-II YBa$_2$Cu$_3$O$_{6.52}$ sample the thermal creep time $\tau$ is estimated to be 1.8 ps. If this time scale is not strongly sample dependent and doping dependent, thermal creep is expected to be frozen in the terahertz frequency range, as $\ln(\tau/\tau_0) \to 0$. Thus the temperature dependence of the pinning constant in their results might indicate how the pinning would behave in the absence of thermal creep.
There is also a DC experiment that measured flux-flow resistivity. It was carried out by M. N. Kunchur et al. [170] on an optimally doped YBa$_2$Cu$_3$O$_{6.95}$ sample. A strong current pulse was used to overcome pinning and the flux-flow resistivity was measured. Their measured flux-flow resistivity is presented in Figure 5.31. Although they claimed that the flux-flow resistivity monotonically decreased at low temperature, the trend of a small upturn might be present in their work. However, without sufficient data and resolution, one cannot draw definite conclusions.

Overall, there are no contradictions between existing results and ours. It is quite possible that in these experiments the flux-flow resistivity upturn was there, only to be hidden by low resolution as well as insufficient data.

### 5.4 Summary of Observations on YBa$_2$Cu$_3$O$_{6+x}$

The flux-flow resistivity and the pinning constant for three YBa$_2$Cu$_3$O$_{6+x}$ samples have been examined in this chapter. In all samples, the vortex viscosity shows a broad peak at low temperature and low field, which shares strong similarities to the zero field conductivity. The left side of the viscosity peak is a downturn, which is
equivalent to an upturn in the flux-flow resistivity. The upturns are found to follow a logarithmic temperature dependence at low temperature, and agree with previous observations made by Ben Morgan. As mentioned in Chapter 1, Ben Morgan has suggested that this logarithmic upturn is an indication of an insulating ground state in the context of the metal-insulator crossover[75], but such an interpretation requires the Bardeen-Stephen relation $\rho_{\text{ff}} \propto \rho_n$ to be valid for the cuprate. Since viscosity and conductivity are intimately related in theory (see Chapter 2) and are very similar in experiment, they can probably be understood in the same way. Therefore, the logarithmic upturn in $\rho_{\text{ff}}$ should probably be interpreted in terms of the quasiparticle scattering, and pair breaking.

As for the pinning, a few characteristic temperature scales can be identified from the temperature dependence of the pinning constant. The underdoped YBa$_2$Cu$_3$O$_{6.333}$ sample shows a linear temperature dependence in the pinning constant at low temperature, and at a crossover temperature $T_m$ the pinning constant becomes vanishingly small. The ortho-II YBa$_2$Cu$_3$O$_{6+x}$ sample data show a step-like structure characterized by temperature scales $T_1$ and $T_2$, which may be related to thermal creep and vortex lattice melting. No peak is observed near $T_c$ in these two samples. On the other hand, a peak near $T_c$ is observed in the overdoped YBa$_2$Cu$_3$O$_{6.993}$ data. These behaviours can be used to map out the complicated vortex matter phase diagram of the cuprates.
Chapter 6

Measurements on Tl$_2$Ba$_2$CuO$_{6+x}$ Samples

Our measurements in Chapter 5 only cover the left half of the cuprate superconducting dome. To complete our study over the entire doping range of the superconducting state, material systems which can be highly overdoped are needed. Tl$_2$Ba$_2$CuO$_{6+x}$ (Tl2201), an intrinsically overdoped cuprate with possibly the simplest structure of these materials, is available for this task. Three samples with different dopings were grown by the UBC group, and the results are reported in this chapter.

A detailed description of the preparation of these Tl$_2$Ba$_2$CuO$_{6+x}$ samples can be found in the Ph.D. thesis of our sample maker, Dr. Darren Peets[37]. These samples are not as chemically perfect as YBa$_2$Cu$_3$O$_{6+x}$, but are still clean enough for high sensitivity transport studies, like the microwave measurements made here. The imperfection is primarily due to Cu-Tl cation cross-substitution, which is likely an important factor in overdoping these materials, and occurs in equilibrium at the growth temperature.

Unlike the underdoped side of the superconducting dome, the normal state of highly overdoped Tl$_2$Ba$_2$CuO$_{6+x}$ is very likely to be a Fermi liquid system, and the emergence of superconductivity is consistent with BCS theory. For instance, a heat conductivity (HC) study[172] has revealed that the amplitude of the energy gap vanishes as $T_c$ approaches zero, as shown in Figure 6.1. To our knowledge, the only notable signature for unusual transport is the Angular Magnetoresistivity Oscillation (AMRO)[173].

The amplitude of the energy gap of Tl$_2$Ba$_2$CuO$_{6+x}$ has been also studied by other
Measurements on Tl$_2$Ba$_2$CuO$_{6+x}$ Samples

Figure 6.1: (a) The amplitude of the energy gap extracted from thermal conductivity measurements[171] and other experiments. (b) An updated diagram of the amplitude of the energy gap[51]. On the overdoped side, the energy gap approaches zero with $T_c$, and is consistent with the BCS prediction.

experiments, including ARPES[174, 175] and inelastic neutron scattering[176] (INS). The results of these experiments agree upon the amplitude of the energy gap being closed when $T_c \rightarrow 0$ on the overdoped side[51]. Therefore, we could conclude that on cooling through $T_c$ superconductivity is achieved by the opening of an energy gap, and that the amplitude of the order parameter is zero above $T_c$. This implies an absence of strong order parameter fluctuations in the normal state, in contrast to the underdoped side of the superconducting dome.

Recent quantum oscillation experiments on the normal state of Tl$_2$Ba$_2$CuO$_{6+x}$ samples have provided direct support to the Fermi liquid picture[50]. In these experiments, quantum oscillations of magnetization, the de Haas–van Alphen effect signal (dHvA), is measured in the normal state, and provides definite signatures of coherent quasiparticles. As shown in Figure 6.2, the dHvA frequency agrees with the Fermi liquid prediction, and a large, complete Fermi surface that covers two-thirds of the first Brillouin zone has been identified.

Fermi liquid behaviour has also been observed in the normal state transport properties. Figure 6.3 shows how the normal state electrical resistivity of Tl$_2$Ba$_2$CuO$_{6+x}$ evolves with an increasing doping. Near optimal doping, $\rho_n(T)$ shows a linear temperature dependence, perhaps associated with proximity to a quantum critical point.
As doping increases towards the right end of the superconducting dome, $\rho_n(T)$ becomes closer to a quadratic Fermi liquid form⁴⁸. Therefore, if the Bardeen–Stephen relation $\rho_{ff} \propto \rho_n$ remains valid for these systems, the same behaviour would be expected to be observed in our flux-flow resistivity measurements, implying that the metal-insulator crossover must occur at some intermediate doping.

As for the electrical transport properties in the superconducting state, zero field microwave measurements have already been taken by D. Broun et al.¹⁷⁷, as shown in Figure 6.4. $\sigma_1(T)$ in the superconducting state is seen to have a broad peak far below $T_c$, similar to that in YBa$_2$Cu$_3$O$_{6+\delta}$ samples, but with a much larger peak width.

It is worth mentioning that on the overdoped side, the superfluid density probed by $\mu$SR study decreases with increasing hole doping⁶⁷, implies that there might be quasiparticles in the ground state. This is suggested as the evidence of a phase separation scenario⁷⁸, in which small patches of normal region coexists with a superconducting background. This scenario also receives some supports from specific heat⁹⁹ and optical conductivity measurements¹⁸⁰. In this scenario, it is possible that our microwave measurements will involve both the response from the normal region and the superconducting background, or the vortices may be trapped by these normal patches as it will be energetically favourable.
Measurements on Tl₂Ba₂CuO₆₊ₓ Samples

Figure 6.3: The normal state resistivity of Tl₂Ba₂CuO₆₊ₓ [48]. As doping increases, Tc is reduced, and the resistivity approaches a Fermi liquid form.

Figure 6.4: Zero field $\sigma_1(T)$ of Tl₂Ba₂CuO₆₊ₓ, measured by D. Broun et al. [177]. A prominent feature is the broad peak below Tc, the height of which decreases with increasing frequency.

6.1 $T_c = 74 \text{ K Tl}_2\text{Ba}_2\text{CuO}_{6+x}$

The first batch of Tl₂Ba₂CuO₆₊ₓ samples have the most visible contamination. To make reliable measurements, only the sample with the sharpest resistive transition...
Measurements on \( \text{Tl}_2\text{Ba}_2\text{CuO}_{6+x} \) Samples

and smoothest signal in the superconducting state was measured. It was then cut under the microscope to remove the part of the crystal with visible flux contamination. Photos of the sample before and after the cut are shown in Figure 6.5. After the cut, the sample has been empirically confirmed to be much cleaner. It will be referred to as \( \text{Tl}_2\text{Ba}_2\text{CuO}_{6+x} \)-sample 1 for convenience.

![Figure 6.5: \( \text{Tl}_2\text{Ba}_2\text{CuO}_{6+x} \)-sample 1 before (a) and after (b) the cutting. The sample "after" is the sharp corner on the left side of the sample "before", which had a more smooth surface and no visible flux contamination. The dimension of the sample "after" can be estimated from a 1 mm graticule in the same figure.](image)

After the sample was cut, measurements had been taken in 2008 at a single field of 4 T and multiple frequencies. To study the field dependence of surface impedance, supplementary measurements were repeated in 2009, at multiple fields and a single frequency of 2.79 GHz. We call these two data sets “Data 08” and “Data 09” for convenience. The data, for the most part, show good reproducibility. In this section, both data sets will be presented.

### 6.1.1 Surface Impedance

The zero field and mixed state surface impedance data are shown in Figure 6.6. In-field measurements have been taken at magnetic fields ranging from 0.25 T to 4 T. Nevertheless, to avoid complications from contaminations on the sample surface, only the data above 1 T are presented.

A resonator constant of \(4.1963 \times 10^{-6} \text{ } \Omega/\text{Hz} \) has been estimated by comparing our results with resistivity measurements of Mackenzie et al.[181]. An absolute penetra-
Figure 6.6: (a) $R_s(T)$ and (b) $X_s(T)$ of Tl$_2$Ba$_2$CuO$_{6+x}$-sample 1 in the mixed state. The 1 T and 2 T data were taken in 2009, while the 4 T data were taken in 2008.

The penetration depth of 176 nm at zero temperature has been estimated from Uemura scaling[67] to offset $X_s$. However, since this law is not quantitatively correct, there is a large uncertainty in $\lambda_0$ and its effect must be kept in mind be when interpreting the flux-flow parameters. In both Data 08 and 09, the sample shows a sharp transition around 74 K, and the signal in the superconducting state is smooth. This confirms the quality of the sample.

We now turn to the reproducibility. The data are mostly reproducible, but some small changes occurred during the eight months. According to Dr. Ruixing Liang, these changes are due to chemical properties of the Tl$_2$Ba$_2$CuO$_{6+x}$ system, as these materials have a tendency to bond water vapour and to rearrange its oxygen ordering. Surface resistance $R_s(T)$ in Data 09 shows a broader transition at $T_c$, indicating that the sample has become less homogeneous. The average difference in $R_s(T)$ below $T_c$ is of the order of 10 $\mu\Omega$, which will have no qualitative impact to the extracted flux-flow parameters. Surface reactance $X_s(T)$ has been offset by the same $\lambda_0$ in both data sets, and the change in $X_s(T)$ is small at low temperature (less than 2% below 20 K), so has little effect on the reproducibility of extracted flux-flow parameters. In both data sets, there is no signature of upturn or downturn in $X_s$, so the ESR transitions should not be a concern.
Measurements on Tl$_2$Ba$_2$CuO$_{6+x}$ Samples

In both $R_s(T)$ and $X_s(T)$, there is a shoulder-like structure between 40 K to 70 K. With increasing magnetic field, this structure is shifted to lower temperature. A similar behaviour has been observed in the resistivity of the YBa$_2$Cu$_3$O$_{6+x}$ sample, and has been interpreted[182] as the onset of the vortex liquid state. This will be examined in our pinning constant data.

6.1.2 Depinning Frequency and Pinning Constant

The depinning frequencies and the pinning constants are presented in Figure 6.7. The depinning frequency is very large, indicating that we are firmly in the pinning limit. All data show similar behaviour, suggesting that the intrinsic properties of this sample are not affected by the chemical changes.

![Graph](image)

Figure 6.7: (a) The depinning frequency and (b) the pinning constant of Tl$_2$Ba$_2$CuO$_{6+x}$-sample 1 in the mixed state. The temperature scale $T_m$ is marked by arrows.

In both Data 08 and 09, a step-like structure is observed in the pinning constant below 50 K, corresponding to the resistive “shoulder” in surface impedance. The onset temperature $T_m$ increases with decreasing magnetic field. Since similar structures have been observed in ortho-II YBa$_2$Cu$_3$O$_{6.52}$, it is possible that the step here is governed by the same physics. In particular, the shape of the pinning constant curve is very similar to that of ortho-II YBa$_2$Cu$_3$O$_{6.52}$ at high field. As for the functional
Measurements on Tl$_2$Ba$_2$CuO$_{6+x}$ Samples

temperature dependence, it does not seem to be exponential at low temperature, but might be approximated by a linear function in the intermediate temperature range.

Above $T_m$, $\alpha_p(T)$ is 100 times smaller than $\alpha_p(0)$. As discussed in Chapter 5, it is possible that $T_m$ is the vortex lattice melting temperature.

### 6.1.3 Viscosity Coefficient and Flux-flow Resistivity

![Figure 6.8: (a) The viscosity of Tl$_2$Ba$_2$CuO$_{6+x}$-sample 1 in the mixed state. (b) The flux-flow resistivity at 4 T, extracted with different values of $\lambda_0$.](image)

The viscosity coefficient and flux-flow resistivity are presented in Figure 6.8. All the viscosity data show a broad peak around 32 K. As shown in Figure 6.4, a broad peak like this has been observed in the zero field $\sigma_1(T)$ data for a Tl$_2$Ba$_2$CuO$_{6+x}$ sample with a similar $T_c$[177]. The height of the peak does not show a monotonic field dependence, but we stress here that the sample properties might have changed over time. Away from the peak, at low temperature, all data seem to join a single trend and show little field dependence. Compared with the YBa$_2$Cu$_3$O$_{6+x}$ samples, the viscosity is an order of magnitude smaller, which has been theoretically predicted[183]. Nevertheless, it might be dangerous to discuss $\eta_{fl}(T)$ quantitatively. In the pinning limit, the real part of the resistive $\rho_1(T)$ is small and sensitive to residual resistivity and uncertainties in $\lambda_0$, which will result in substantial uncertainty in $\eta_{fl}(T)$.
Measurements on Tl$_2$Ba$_2$CuO$_{6+x}$ Samples

The qualitative behaviour, on the other hand, is insensitive to this uncertainty. Even in the presence of a 50% uncertainty in $\lambda_0$, a logarithmic temperature dependence of $\rho_{\|}(T)$ remain unchanged.

6.2 $T_c = 45$ K Tl$_2$Ba$_2$CuO$_{6+x}$

The second batch of Tl$_2$Ba$_2$CuO$_{6+x}$ samples had little visible contamination and provided much better data. Of three available samples, we chose the cleanest one on which our measurements were made. This sample will be referred to as Tl$_2$Ba$_2$CuO$_{6+x}$ sample 2. Photo of this sample is shown in Figure 6.9.

Figure 6.9: Tl$_2$Ba$_2$CuO$_{6+x}$-sample 2. There is little contamination on the surface. A 1 mm graticule is presented for estimation of the sample dimensions.

6.2.1 Surface Impedance

In-field data for this sample have been taken from 0.125 T to 8 T. To avoid spurious effects from possible contamination on the sample surface, only data for fields of 0.5 T and above are presented. Surface impedance data are shown in Figure 6.10. The transition temperature is estimated to be around 43 K, with a transition width about 4 K. There is a kink near the transition, and a foot in the surface reactance, implying some $T_c$ inhomogeneity.

A resonator constant of $1.476 \times 10^{-6}$ in the TE$_{011}$ mode is obtained by comparing resistivity to published data[40]. The absolute penetration depth is estimated to be 220 nm from Uemura scaling[67], with a 50% uncertainty. Surface reactance $X_s(T)$ in the normal state is much larger than $R_s(T)$, indicating a large $c$-axis current effect.
Measurements on Tl₂Ba₂CuO₆₊ₓ Samples

Figure 6.10: (a) $R_s(T)$ and (b) $X_s(T)$ of Tl₂Ba₂CuO₆₊ₓ-sample 2 in the mixed state. Data were taken in the TE₀₁₁ mode.

Figure 6.11: $\rho_1(T)$ for Tl₂Ba₂CuO₆₊ₓ-sample 2. Data were taken in the TE₀₁₁ mode at 2.79 GHz.

As discussed in Chapter 3, such effects are inevitable because of the irregular shape of these samples.

In the mixed state, the vortex response clearly dominates the signal. Therefore,
Measurements on Tl₂Ba₂CuO₆₊ₓ Samples

the residual $R_s$ is less of a concern. The resistive transition temperature decreases monotonically with increasing field, similar to what has been observed before [49]. At 1 T, there is a kink in $R_s(T)$ at intermediate temperatures, while at higher fields this feature goes away.

![Figure 6.12: $\Delta X_s(B, 0)$ of Tl₂Ba₂CuO₆₊ₓ-sample 2, fit to a fractional power law $\propto B^{0.7}$. Data were taken in the TE₀₁₁ mode at 2.79 GHz.](image)

The real part of the complex resistivity $\rho_1$ is shown in Figure 6.11. For $\rho_1$, the normal state data taken in different fields match well. A kink in the resistivity data is observed, which is quickly shifted to low temperature with increasing magnetic field. Above the kink temperature, the resistivity smoothly joins onto the normal state resistivity. Nevertheless, this should not be mistaken as an indication of $B_{c2}(T)$, as there is a strong field dependence of resistivity at the kink temperature. In the normal state, for $B > B_{c2}(T)$, we expect very little field dependence. There might be some small magnetoresistance, but this is beyond the scope of this work. The normal state resistivity is well fit to a linear function:

$$\rho_n = (6.38 + 0.378 T) \mu\Omega\text{cm}, \quad (6.1)$$

which is typical behaviour for Tl₂Ba₂CuO₆₊ₓ in this doping range [184].

Like the cases for other samples, the field dependence of $\Delta X_s(B, 0)$ can be fit to a power law, as shown in Figure 6.12.
Measurements on Tl₂Ba₂CuO₆₊ₓ Samples

6.2.2 Depinning Frequency and Pinning Constant

The temperature dependence of the depinning frequency of Tl₂Ba₂CuO₆₊ₓ sample 2. Data were taken in the TE₀₁₁ mode at 2.79 GHz.

The depinning frequency has been extracted and is presented in 6.13. At low temperature the depinning frequency is an order of magnitude larger than the measurement frequency of 2.79 GHz, and falls quickly to a plateau as temperature rises, eventually reaching zero at a temperature close to Tₑ.

Pinning constant data are presented in Figure 6.14 at multiple fields and frequencies. The pinning constant decreases monotonically with increasing field, or with decreasing frequency. At higher temperature the pinning constant shows a step-like structure, similar to that observed in Tl₂Ba₂CuO₆₊ₓ-sample 1 and in ortho-II YBa₂Cu₃O₆.52. The onset temperature Tₘ, defined from the kink in d(ln αₚ)/dt, decreases quickly with increasing field and decreasing frequency, and coincides with the kink temperature in the resistive transition (see Figure 6.11). This behaviour is suggestive of vortex lattice melting. Above Tₘ, the pinning constant drops to zero on approaching to Tₑ. Near Tₑ a peak can be observed in the pinning constant data. However, this is likely an artifact created by the double transition in the zero field data, rather than a signature of the peak effect.

At low field, the pinning constant data can be approximately fit to exponential functions over a large temperature range (0 to 0.75 Tₘ), which is similar to what was observed for ortho-II YBa₂Cu₃O₆.52. At higher magnetic fields, the exponential
Measurements on Tl$_2$Ba$_2$CuO$_{6+x}$ Samples

Figure 6.14: The pinning constant of Tl$_2$Ba$_2$CuO$_{6+x}$-sample 2, as a function of temperature at (a) a single frequency of 2.79 GHz and multiple fields and (b) at multiple frequencies and a single field of 8 T. The temperature dependences are qualitatively the same for all the fields and frequencies, and the step-like structure (or kink) is marked by open arrows.

The temperature dependence seems to be replaced by a linear one over the temperature range $0 < T < 0.75 \ T_m$. The meaning of this crossover will be examined in Chapter 8.

The pinning constant at fixed field and temperature increases monotonically with frequency. The frequency dependence of the pinning constant is similar to that in the ortho-II YBa$_2$Cu$_3$O$_{6.52}$, and is likely to be governed by the same physics.

6.2.3 Viscosity Coefficient and Flux-flow Resistivity

The viscosity coefficients $\eta_{fl}(T)$ at multiple fields and frequencies are shown in Figure 6.15. Above and below $T_m$ the behaviour is quite different. Above $T_m$ and away from $T_c$, the sample should be in the vortex liquid state, and $\eta_{fl}(T)$ shows a linear temperature dependence, the slope of which seems to be weakly frequency dependent. As discussed in Section 5.1, this might be understood by considering the microwave time scale and the distribution of pinning potentials, and our $\eta_{fl}(\omega)$ should be slightly smaller than the value in the DC case. Below $T_m$, $\eta_{fl}(T)$ shows a broad peak. It should
Figure 6.15: The viscosity coefficient of Tl$_2$Ba$_2$CuO$_{6+x}$-sample 2, as a function of temperature at (a) a single frequency of 2.79 GHz and multiple fields and (b) at multiple frequencies and a single field of 8 T.

Figure 6.16: The temperature dependence of $\rho_{fl}$. Data were taken in the TE$_{011}$ mode.

It should be pointed out that since the uncertainty in $\lambda_0$ has a strong effect on the magnitude of the extracted parameters, it may modify the field dependence of $\eta_{fl}(T)$. Nevertheless, the qualitative behaviour of $\eta_{fl}(T)$ is unaffected by uncertainty in $\lambda_0$. 

163
Measurements on Tl$_2$Ba$_2$CuO$_{6+x}$ Samples

All the $\eta_{fl}(T)$ data show downturns at low temperature, which correspond to upturns in $\rho_{fl}(T)$ (see Figure 6.16). At all fields, these upturns are well described by a logarithmic temperature dependence at low temperature. At high field (8 T), there seems to be a deviation from the logarithmic fit at about 4 K.

Here, the observation of this logarithmic upturn is rather surprising. It seems to suggest that the metal-insulator transition either is not exist, or occurs at even higher doping. This will be further clarified by the data of the next, and most overdoped sample.

6.3 $T_c = 24$ K Tl$_2$Ba$_2$CuO$_{6+x}$

This section contains measurements for the third Tl$_2$Ba$_2$CuO$_{6+x}$ sample. This is a highly overdoped sample with little visible contamination. Its dimensions are considerably larger than the other two Tl$_2$Ba$_2$CuO$_{6+x}$ samples, providing better signal strength. For convenience it will be referred to as Tl$_2$Ba$_2$CuO$_{6+x}$-sample 3. A photograph of it is shown in Figure 6.17.

Figure 6.17: Tl$_2$Ba$_2$CuO$_{6+x}$-sample 3. It has an approximately equilateral triangular shape, with sides of 1 mm in length. There is little contamination on the surface.

6.3.1 Surface Impedance

The surface impedance data are presented in Figure 6.18. The transition temperature is around 24 K. The transition width is around 7 K, indicating inhomogeneity of the sample. A resonator constant of $1.06 \times 10^{-6}$ for the TE$_{011}$ mode at 2.79 GHz is estimated by comparing our normal state resistivity to the data from the Mackenzie
Measurements on Tl$_2$Ba$_2$CuO$_{6+x}$ Samples

group[181]. Data have also been taken in the TE$_{031}$ mode at 9.30 GHz and the TE$_{043}$ mode at 13.99 GHz. An absolute penetration depth $\lambda_0$ of 260 nm at zero temperature has been estimated from Uemura scaling[67], and the effect of a 50% uncertainty will be considered. $R_s(T)$ and $X_s(T)$ match well in the normal state, so $c$-axis current effects are small.

![Graphs](image)

Figure 6.18: (a) $R_s(T)$ and (b) $X_s(T)$ of Tl$_2$Ba$_2$CuO$_{6+x}$-sample 3 in the mixed state. Data were taken in the TE$_{011}$ mode at 2.79 GHz.

An applied field of 4 T seems to shift the resistive transition down to 8 K, while an applied field of 8 T suppresses superconductivity above 3 K. Since our model is more robust at low field, mixed state data were taken at 0.125 T, 0.25 T, 0.5 T and 1 T. Even at fields as low as a 0.125 T, the flux response dominates the signal. As a consistency check, in the normal state, both $R_s(T)$ and $X_s(T)$ at different fields match well. At fields as high as 4 T, instead of a downward resistive transition, there seems to be a low temperature upturn in $X_s(T)$. The meaning of the upturn will be discussed in the next section.

The real part of the complex resistivity $\rho_1(T)$ is presented in Figure 6.19, which fits well to a typical Fermi liquid form:

$$\rho_1(T) = (4.58 + 0.0023 T^2) \mu\Omega\text{cm}, \quad (6.2)$$

and agrees with previous measurements[184].
Measurements on Tl$_2$Ba$_2$CuO$_{6+x}$ Samples

Figure 6.19: $\rho_1(T)$ of Tl$_2$Ba$_2$CuO$_{6+x}$-sample 3. Data were taken in the TE$_{011}$ mode at 2.79 GHz.

Figure 6.20: $\Delta X_s(B, 0)$ of Tl$_2$Ba$_2$CuO$_{6+x}$-sample 3, fits by a fractional power law function $\propto B^{0.5}$.

$\Delta X_s(B, 0)$ is presented in Figure 6.20. A power law function is used to fit the data.
Measurements on Tl$_2$Ba$_2$CuO$_{6+x}$ Samples

6.3.2 Depinning Frequency and Pinning Constant

The temperature dependence of the depinning frequency is presented in Figure 6.21. At low temperature the depinning frequency value is slightly larger than the driving frequency.

![Depinning Frequency Graph](image)

**Figure 6.21:** The depinning frequency for Tl$_2$Ba$_2$CuO$_{6+x}$-sample 3. Data were taken in the TE$_{011}$ mode at 2.79 GHz.

To avoid unphysical results near $T_c$, when extracting the pinning constant we do not include the superconducting background in our analysis. This has been confirmed to have no qualitative impact on the temperature dependence of the pinning constant, and only a slight effect on its magnitude. The temperature dependence of the pinning constant is presented in Figure 6.22, which is very similar to what has been observed in underdoped YBa$_2$Cu$_3$O$_{6.333}$. At low temperature all the pinning constant curves show approximately linear temperature dependence, and have a change in slope at the temperature crossover $T_m$. At stronger magnetic fields the change in slope occurs at a lower temperature, and the slope of the pinning constant becomes much larger. The upturn in $X_s$ in the 4 T data corresponds to nothing but a steep pinning constant curve.

The temperature dependence does not show strong frequency dependence, but the magnitude of the pinning constant increases with increasing frequency. The rounded structure around $T_m$ should correspond to a transition to the vortex liquid state.
Measurements on Tl$_2$Ba$_2$CuO$_{6+x}$ Samples

Figure 6.22: The pinning constant of Tl$_2$Ba$_2$CuO$_{6+x}$-sample 3 at (a) a single frequency of 2.79 GHz and multiple fields and (b) at multiple frequencies and single field of 0.25 T.

6.3.3 Viscosity Coefficient and Flux-flow Resistivity

Figure 6.23: The viscosity coefficient of Tl$_2$Ba$_2$CuO$_{6+x}$-sample 3 at (a) a single frequency of 2.79 GHz and multiple fields and (b) at multiple frequencies and single field 0.25 T.
Measurements on Tl₂Ba₂CuO₆₊ₓ Samples

The viscosity coefficient is presented in Figure 6.23. As we are more concerned about the low temperature behaviour, the superconducting background contribution is included. All data show a broad peak, the height of which increases with increasing magnetic field and decreasing frequency. This shares similarities with the zero field $\sigma_1(T)$, and agrees with our observations in the other cuprates.

![Diagram](image.png)

Figure 6.24: The flux-flow resistivity of Tl₂Ba₂CuO₆₊ₓ-sample 3. Data were taken in the TE₀₁₁ mode at 2.79 GHz.

The downturn in $\eta_{fl}(T)$ at low temperature corresponds to an upturn in $\rho_{ff}$, which is shown in Figure 6.24. Just as in the other samples, a logarithmic temperature dependence clearly exists in all the data. We stress that at this doping the normal state is a Fermi liquid, and the logarithmic upturn is unexpected. As will be discussed in Chapter 7, this leads to a completely different interpretation from our initial motivation.

6.4 Summary of Observations on Tl₂Ba₂CuO₆₊ₓ

The flux-flow resistivity and the pinning constant for three Tl₂Ba₂CuO₆₊ₓ samples are presented in this chapter. Surprisingly, the logarithmic temperature dependence persists to the highly overdoped side, where the normal state is a Fermi liquid. The viscosity coefficient shows a peak that shares a strong resemblance to what has been observed in the YBa₂Cu₃O₆₊ₓ samples, suggesting that the underlying physics re-
Measurements on Tl$_2$Ba$_2$CuO$_{6+x}$ Samples

...mains the same. The meaning of these observations will be discussed in Chapter 7.

As for the pinning, all Tl$_2$Ba$_2$CuO$_{6+x}$ samples demonstrate similar qualitative temperature dependence. A temperature scale $T_m$ can be identified from a change in slope in $\alpha_p$, and may correspond to vortex lattice melting. This may be used to map the vortex matter phase diagram. A detailed analysis will be presented in Chapter 8.
Chapter 7

Discussion of Flux-flow Resistivity

As reported in Chapter 5 and Chapter 6, our measurements of flux-flow resistivity on the cuprate superconductors $\text{YBa}_2\text{Cu}_3\text{O}_{6+x}$ and $\text{Tl}_2\text{Ba}_2\text{CuO}_{6+x}$ reveal a feature that has never been observed or predicted before: that is, a universal logarithmic temperature dependence in the flux-flow resistivity. A global view of this flux-flow resistivity is presented here. We will conclude that the universal logarithmic temperature dependence implies that the Ando–Boebinger effect is dominated by vortex physics, and hence provides strong experimental evidence for the fluctuating superconductivity interpretation of the pseudogap.

It is probably more convenient to analyze this behaviour in terms of the vortex viscosity, which is inversely proportional to the flux-flow resistivity. A striking observation is that the vortex viscosity bears a strong resemblance to the zero field conductivity, implying that the viscosity should probably be understood in the same way as for the conductivity, in terms of $d$-wave quasiparticles.

In light of the intimate connection between viscosity and conductivity, the frequency spectrum of the vortex viscosity is analyzed in Section 7.2 for a specific case, ortho-II $\text{YBa}_2\text{Cu}_3\text{O}_{6.52}$, where detailed conductivity data exist. The analysis suggests that there are two types of quasiparticle states involved: one, the long-lived extended quasiparticle states outside the vortex core, and another, the short-lived quasiparticles localized inside the vortex core.

Possible microscopic origins of this logarithmic temperature dependence are discussed in Section 7.3.
Discussion of Flux-flow Resistivity

7.1  Global View of Flux-flow Resistivity

As shown in Figure 7.1, for cuprates at all dopings, the mixed state $\rho_{ff}(T)$ shows a logarithmic upturn at low temperature, regardless of the applied field or frequency. This unusual temperature dependence of the flux-flow resistivity appears to be universal. It resembles the "normal state" resistivity on the underdoped side, but not the overdoped side.

![Figure 7.1: Flux-flow resistivities for all the cuprate samples we measured, including Ben Morgan's measurements on an optimally doped YBa$_2$Cu$_3$O$_{6.95}$ sample[81]. All data show a logarithmic temperature dependence at low temperature.](image)

As mentioned in Chapter 1, at the beginning of this work, the Bardeen–Stephen
relation $\rho_{ff}(T) \propto \rho_n(T)$ was emphasized. A logarithmic upturn in $\rho_{ff}(T)$ was first observed by Ben Morgan on optimally doped YBa$_2$Cu$_3$O$_{6.95}$, but at that point was thought to be related to the insulating “normal state” resistivity observed by Ando[61] and Boebinger[75], and hence believed to be a signature of an insulating state within the vortex cores. This interpretation is not incompatible with our measurements on underdoped YBa$_2$Cu$_3$O$_{6.333}$, ortho-II YBa$_2$Cu$_3$O$_{6.52}$ and overdoped YBa$_2$Cu$_3$O$_{6.993}$. It would have led to the seemingly reasonable conclusion that the left half of the superconducting dome is the “insulating side” of the metal-insulator crossover[75], if we have not studied the overdoped side.

There are then the measurements on the Tl$_2$Ba$_2$CuO$_{6+x}$ samples. As doping increases further, $\rho_{ff}(T)$ had been expected to “evolve” like the normal state resistivity, which eventually approaches an $A + BT^2$ Fermi liquid form[48]. Surprisingly, the logarithmic upturn persists in all Tl$_2$Ba$_2$CuO$_{6+x}$ samples, and no positive correlation can be found between $\rho_{ff}(T)$ and $\rho_n(T)$. Therefore, the Bardeen–Stephen law must be invalid in this respect, and the “insulating ground state” argument is then not supported by this work.

This violation of the Bardeen–Stephen law has a striking implication. We stress that since superconductivity on the overdoped side is known to be BCS like, this logarithmic temperature dependence is appearing at the same time as the amplitude of the $d$-wave order parameter becomes nonzero. Therefore, the most plausible interpretation of our results is that this logarithmic temperature dependence is intrinsic, and is intimately associated with $d$-wave vortices in cuprate superconductors.

A natural extension of this conclusion is that the original Ando–Boebinger effect is a signature that vortices persist outside the superconducting dome on the underdoped side. As mentioned in Chapter 1, there is a major school of thought in the cuprates that ascribes the unusual normal state properties to this physics. There should be little pinning in this regime, so any electrical transport measurements would inevitably be dominated by the logarithmic flux-flow resistivity at low temperature:

$$\rho_{\text{eff}} = \rho_{\text{bgd}} + \rho_{\text{ff}}. \quad (7.1)$$

Consequently, the insulating behavior in the Ando–Boebinger effect could be due to nothing but vortex physics, and our measurements therefore provide strong support for the fluctuating superconductivity scenario of the pseudogap.
7.2 Viscosity Spectra for ortho-II YBa$_2$Cu$_3$O$_{6.52}$

It is quite informative to analyze this universal logarithmic upturn in terms of vortex viscosity. As shown in previous chapters, in all data a broad peak in vortex viscosity is observed at intermediate temperature, and the data share strong similarities to conductivity data in zero field. And since viscosity is proportional to the spatially averaged $\sigma_1$, both properties should be able to be understood from the perspective of $d$-wave quasiparticle dynamics.

We present and analyze the viscosity spectra for ortho-II YBa$_2$Cu$_3$O$_{6.52}$ in this section. This doping is chosen because here the data have been taken on a very clean sample, and cover a wide frequency and field range. In additional, published zero field conductivity spectra of the highest quality are available for comparison for this kind of sample[104].

Figure 7.2 shows how $\eta_f(T)$ and $\sigma_1(T)$ evolve with increasing frequency and field. At low field and low temperature, $\eta_f(T)$ shows strong similarities to $\sigma_1(T)$, implying that the charge dynamics are not substantially altered.

In zero field, $\sigma_1(T)$ shows a peak around 8 K, the height of which decreases with increasing frequency. The frequency dependence weakens as temperature increases, and above 35 K is not apparent. This behaviour results from the temperature dependence of the effective scattering time. At low temperature, the quasiparticle scattering time is large, and the frequency dependence is strong. At higher frequency and temperature, due to inelastic scattering, the scattering time becomes small, and hence $\sigma_1(T)$ becomes frequency independent.

What is most intriguing and relevant to our study is the downturn to the left of the peak. The downturn approximately follows a linear temperature dependence, the origin of which is still an open problem. In this linear temperature regime, the conductivity spectra have been obtained by Patrick Turner and coworkers at UBC using broadband microwave spectroscopy[185], as shown in Figure 7.3. Their spectra can be fit by a one-component form:

$$\sigma_1(\omega, T) = \sigma_0/[1 + (\omega\tau)^y],$$

where $y = 1.45$ is the result of energy-dependent averaging in the Born limit[104].

In the mixed state, the qualitative shape of $\eta_f(T)$ remains very similar to $\sigma_1(T)$. One small difference is that, at low field, it shows considerable frequency dependence above 35 K. This suggests that the scattering time, either induced or modified by the
Discussion of Flux-flow Resistivity

(a) $\sigma_1(B = 0 \, \text{T})[104]$.  
(b) $\eta(B = 0.75 \, \text{T})$.  
(c) $\eta(B = 1 \, \text{T})$.  
(d) $\eta(B = 2 \, \text{T})$.  
(e) $\eta(B = 3 \, \text{T})$.  
(f) $\eta(B = 4 \, \text{T})$.  
(g) $\eta(B = 5 \, \text{T})$.  
(h) $\eta(B = 6 \, \text{T})$.  
(i) $\eta(B = 7 \, \text{T})$.  

Figure 7.2: Zero field $\sigma_1(T)$ data measured by the UBC superconductivity group, and $\eta_{fl}(B, T)$ data measure by our group for ortho-II YBa$_2$Cu$_3$O$_{6.52}$ samples. The $\eta_{fl}(T)$ data have been taken at resonant frequencies of 2.79 GHz (red), 4.51 GHz (blue), 9.12 GHz (green) and 13.97 GHz (black). Interestingly, the frequency dependence of $\eta_{fl}$ disappears approximately at $T_2$, the onset of the vortex liquid state (see Section 5.2), which is marked by arrows.
Discussion of Flux-flow Resistivity

Figure 7.3: Zero field conductivity spectra for ortho-II YBa$_2$Cu$_3$O$_{6.52}$, studied by P. Turner et al.[185]. Data were taken at 6.7 K (red), 4.3 K (green), 2.7 K (blue) and 1.3 K (black). The inset shows a Drude fit to the conductivity spectrum.

applied field, remains somewhat larger than that in zero field. As field increases, the frequency dependence weakens, and at 7 T there is little frequency dependence above 35 K, indicating a small scattering time in this regime. The temperature at which the frequency dependence disappears coincides with $T_2$, which has been suggested as the onset temperature of the vortex liquid state in Section 5.2. This suggests that there are two types of quasiparticle state involved: one has long scattering time and is dominant at low fields, while the other has short scattering time and is more apparent at high fields.

For simplicity, a two-component Drude model is adopted to interpret the data:

$$\eta_{fl} = \eta_0 + \frac{\eta_1}{1 + (\omega \tau)^2},$$

(7.3)

where $\eta_0$ is the component of viscosity originating from the short-lived quasiparticles, and $\eta_1$ is another fraction related to the long-lived quasiparticles. Our qualitative observation is that $\eta_0$ is similar to our high frequency data, while $\eta_1$ behaves like the low frequency data. Examples of the two-component spectra are presented in Figure 7.4. The shape of the spectra does not change substantially as temperature increases, implying that the distinction between these two branches of scattering is not in energy space. Thus it is very likely that the distinction is in real space, and
Discussion of Flux-flow Resistivity

Figure 7.4: The viscosity spectra at (a) 0.75 T and low temperature; (b) 0.75 T and high temperature; (c) 7 T and low temperature; (d) 7 T and high temperature. The lines are fits by a two-component Drude model.

associated with the spatial inhomogeneity of conductivity in the presence of vortices. It is likely that near the vortex core, the density of states is large, giving rise to a branch of short-lived quasiparticles localized inside the vortex core. Far away from the vortex core, the density of states should be close to that of the superconducting
Discussion of Flux-flow Resistivity

background, and this branch of quasiparticles would have long scattering time. At low field, the intervortex spacing is large, and the vortices are held at the minima of the pinning potentials, so that the extended quasiparticles away from the core contribute substantially to $\eta_{fl}(T)$, and a strong frequency dependence can be observed. At higher fields, the vortex unit cell is smaller. The time scale of the vortex fluctuations has been estimated to be of the order of $10^{-12}$ s in Section 5.2, which is much shorter than the lifetime of the extended states. Therefore, when the vortices start to fluctuate in the vortex liquid state, their motion wipes out the distinction between extended and core states, resulting in a small scattering time and a frequency independent viscosity. On the other hand, at lower temperatures where the thermal fluctuation is weak, the contribution from the extended state would still be apparent.

At intermediate temperature (about 10 K to 40 K), a bump-like structure in viscosity becomes more apparent with increasing field. This bump-like structure indicates the density of states is more rapidly enhanced than the scattering time is reduced, because viscosity increases almost monotonically in this temperature range. Since this bump-like structure is more apparent in the low frequency data, it is likely to arise from the extended states. On the other hand, at sufficiently high temperature, there is the apparent suppression of the extended branch, perhaps as a result of the increase in vortex core size or vortex fluctuation with temperature.

The low temperature part of $\eta_{fl}(T)$, especially the downturns to the left of the peak, are very important, but more challenging to interpret. In zero field, the downturn of conductivity follows a linear temperature dependence. From a theoretical perspective, this is a little confusing. In clean $d$-wave superconductors, the density of states $N(E)$ is linearly energy dependent, and in the limit of weak scattering, the scattering rate should also be linearly energy dependent, resulting in a cancellation in conductivity[186], which is at odds with the linear temperature dependence. To the best of our knowledge, the only theoretical attempt to interpret this linear temperature dependence was made by Hettler et al.[187], who suggested that the linear temperature dependence might be the result of small angle scattering by an order parameters "hole" induced by a point-like impurity.

While this linear temperature dependence might persist in our low field data, as field increases it appears to converge to a $\frac{1}{\log T}$ temperature dependence. From the viscosity data, it seems that the $\frac{1}{\log T}$ is more apparent in the low frequency data. In contrast, in the high frequency data, which are closely related to $\eta_0$ and the short-lived
Discussion of Flux-flow Resistivity

quasiparticles, a linear temperature dependence seems to remain with an increasing field. This suggests that the dynamics of the extended quasiparticle states is the dominant factor in determining the $\frac{1}{\log T}$ temperature dependence.

7.3 Possible Microscopic Origins

There are two broadly different possibilities that may provide the microscopic origin for the logarithmic temperature dependence in $\rho_{ff}$. One is the Mott physics\cite{52}. In particular, an antiferromagnetic vortex core has been proposed on the under-doped side\cite{188}, and evidence of a charge order has been presented by an STM experiment\cite{54}. Strong antiferromagnetic order near the vortex core has also been suggested by high field NMR experiments\cite{189}, and by $\mu$SR experiments\cite{190}. It is possible that these charge or spin orders may interfere with quasiparticle scattering in some ways to result in a logarithmic $\rho_{ff}(T)$. Nevertheless, it is unlikely for them to persist to the highly overdoped side, where the vortex core size is large and divergent, but the logarithmic temperature dependence is still apparent.

Another possibility is tied with the low energy quasiparticle excitations\cite{191}. The presence of vortices is known to have some exotic impacts on the dynamics of $d$-wave quasiparticles, including the Volovik effect\cite{47} and the Andreev bound state\cite{192}, which can be very different from the conventional case. Further measurements on heavy fermion $d$-wave superconductors may provide more insights into this possibility.
Chapter 8

Discussion of Pinning

Another important aspect studied in this work is pinning. The depinning frequency has practical importance for microwave superconductivity applications, while the pinning constant data can be used to map out the $B$-$T$ phase diagram of the vortex matter. We reiterate that our measured pinning constant contains at least the equivalent information to the critical current density measurements in the DC experiments.

The functional temperature dependence is analyzed in Section 8.1. Possible vortex matter phase diagrams are suggested in Section 8.2.

8.1 Temperature Dependence of the Pinning Constant

The pinning constant data of all the samples measured in this thesis are shown in a linear plot Figure 8.1, and in a semi-log plot Figure 8.2. The functional dependence of the pinning constant may provide valuable insights into the pinning mechanism, but it can also be very complicated. Unlike the case of the flux-flow resistivity, the pinning constant does not follow a universal temperature dependent function at low temperature. This section is an attempt to interpret all our data in a single, consistent way, but it is also possible that pinning is sample dependent and needs to be treated on a case by case basis.

We start with the exponential temperature dependence of $\alpha_p(T)$. This exponential dependence has been observed at microwave frequencies by other experimentalists[193], and an equivalent behaviour in $J_c(T)$ has been observed in DC magnetization
Figure 8.1: Pinning constant for all the cuprates sample measured, including Ben Morgan’s results on an optimally doped YBa$_2$Cu$_3$O$_{6.95}$ sample[81]. The pinning constant decreases quickly with increasing temperature.

measurements[194] and vibration reed measurements[195], the latter of which were taken in the kHz frequency range. In the empirical form $\alpha_p \propto e^{-T/T_0}$,

$$T_0 = \frac{U_0}{k_B \ln(1 + \frac{T}{\tau_0})} \quad (8.1)$$

fit has been proposed, and is supported by our data, as shown in Section 5.2. It emphasizes the importance of thermal creep, hence our data may be interpreted by a detailed thermal creep model. Ideally, the model should be able to interpret the
Discussion of Pinning

Figure 8.2: Pinning constant for all the cuprates sample measured, including Ben Morgan’s results on an optimally doped YBa2Cu3O6.95 sample[81]. For the ortho-II YBa2Cu3O6.52 sample, the optimally doped YBa2Cu3O6.993 sample, and the overdoped YBa2Cu3O6.993 sample and the Tl2Ba2CuO6+x-sample 2, an exponential temperature dependence is observed at low temperature.

following observations:
1. The exponential temperature dependence is apparent in the data for ortho-II YBa2Cu3O6.52, overdoped YBa2Cu3O6.993 and optimally doped YBa2Cu3O6.95; perhaps coincidentally, these materials have the least disorders of the samples studied;
2. T0 is weakly field dependent, but the temperature scale T1, below which the ex-
Discussion of Pinning

Exponential temperature dependence is apparent, decreases with increasing field; in addition, \( T_1 \) increases with frequency;

3. The exponential temperature dependence can be observed in \( \text{Tl}_2\text{Ba}_2\text{Cu}_0\text{O}_{6+x} \) sample 2 at low field \(< 1 \text{ T} \), which becomes more like a linear temperature dependence when the applied magnetic field increases.

There are also other interesting observations that may be due to thermal creep. These are:

4. Above \( T_1 \), \( \alpha_p(T) \) in ortho-II \( \text{YBa}_2\text{Cu}_3\text{O}_{6.52} \), overdoped \( \text{YBa}_2\text{Cu}_3\text{O}_{6.993} \) and optimally doped \( \text{YBa}_2\text{Cu}_3\text{O}_{6.95} \) drops faster than the extrapolated exponential function;

5. In underdoped \( \text{YBa}_2\text{Cu}_3\text{O}_{6.333} \), at low temperature, \( \alpha_p(T) \) shows approximately linear temperature dependence, the absolute slope of which increases with field. The \( \text{Tl}_2\text{Ba}_2\text{Cu}_0\text{O}_{6+x} \)-sample 3 data are excluded in this analysis, because the applied field may not be strong enough to overcome potential spurious effects.

The basic concepts of thermal creep have been introduced in in Chapter 1: a small transport current \( \vec{J} \) "tilts" the pinning potential and lets the vortex "hop" with an effective net velocity along the direction of \( \vec{J} \times \vec{B} \). The earliest Kim–Anderson thermal creep model \[33\] predicts a fast decay of the transport current over time, or equivalently the reduction of the effective pinning potential strength

\[
\frac{\alpha_p(T)}{\alpha_{p0}(T)} = 1 - \frac{k_B T}{U_0} \ln(1 + \frac{\tau}{\tau_0}),
\]

in which \( \alpha_{p0}(T) \) is the pinning constant in the absence of the thermal creep. This form might be used to interpret observation 4 and 5 above, but it is strictly valid only when \( \frac{k_B T}{U_0} \ln(1 + \frac{\tau}{\tau_0}) \) is very small. In addition, to interpret observation 5, \( U_0 \) would require a strong field dependence, but observation 2 implies a field independent \( U_0 \).

A refined version, the collective creep model \[196\] gives a more general form

\[
\frac{\alpha_p(T)}{\alpha_{p0}(T)} = (1 + \gamma \frac{k_B T}{U_0} \ln(1 + \frac{\tau}{\tau_0}))^{-\frac{1}{\gamma}},
\]

in which \( \gamma \) is a characteristic parameter depending on the size of the vortex bundle, and the original Kim–Anderson model is the case with \( \gamma = -1 \). When \( \gamma \frac{k_B T}{U_0} \ln(1 + \frac{\tau}{\tau_0}) = \frac{1}{\tau_0} \) is small, Equation 8.2 can be approximated by Equation 5.3, and has been suggested as the explanation \[197\] for the exponential temperature dependence. However, \( \frac{1}{\tau_0} \) extracted from our data is large \((0.06 \text{ to } 0.095 \text{ K}^{-1}) \), and the exponential temperature dependence persists to high temperature – for instance, to 30 K for the
Discussion of Pinning

0.5 T data of the ortho-II YBa2Cu3O6.52 sample. The validity of this interpretation is thus questionable.

A refined approach is to include both the static temperature dependence term $H_{c2}(T)^\beta$ and thermal creep term $\frac{\alpha_p(T)}{\alpha_{p0}(T)}$ in Equation 8.3 to account for the exponential temperature dependence. In addition, it assumes that thermal creep behaves differently in different temperature ranges. This method has been used in a $J_c$ study on an optimally doped YBa2Cu3O6.95 sample[164], and has been qualitatively successful. In that analysis, the exponential temperature dependence region is thought to be the collective creep region where $\gamma$ is positive, and the drop above $T_1$ correspond to the Kim–Anderson region where $\gamma = -1$, as shown in Figure 8.3.

Figure 8.3: The interpretation of the critical current density data on an optimally doped YBa2Cu3O6.95 sample[164]. $H_{c2}(T) \propto (1 - (T/T_c)^2)$ and $\beta = 1.5$ were assumed in the fit. As temperature rises, it evolves from the collective creep region with $\gamma = 1.7$ to the Kim–Anderson region with $\gamma = -1$.

This approach might be applied to our data and can interpret observation 1, 2, and 3 phenomenologically. There are still many questions left, however, such as:

1. What determines the crossover from the collective creep region to the Kim–Anderson region?
2. What are the implications of the collective creep region and the Kim–Anderson region?

Perhaps a more thorough thermal creep model needs to be developed.
8.2 Vortex Matter Phase Diagram

The analysis above can be used to map out the vortex matter phase diagram. We start with data on ortho-II YBa$_2$Cu$_3$O$_{6.52}$, in which two temperature scales $T_1$ and $T_2$ have been identified in Section 5.2. $T_1$ agrees approximately with reported vortex lattice melting line on the same kind of materials, and might be consistent with the collective creep to Kim–Anderson creep interpretation. Nevertheless, the existence of $T_2$ suggests that above $T_1$ the vortex matter is not simply in the vortex liquid phase. Above $T_2$ the pinning constant is $10^2$ to $10^3$ times smaller than its zero temperature value, and it decreases monotonically with temperature. Therefore, it is reasonable to propose $T_2$ as the onset temperature of the vortex liquid state. Although at $T_2$ the pinning constant does not reach zero and shows a strong frequency dependence, it is possible that in a DC case, $\alpha_p(T_2) = 0$ will be satisfied, as shown in Section 5.1.

Using $T_1$ and $T_2$, the phase diagram of the ortho-II YBa$_2$Cu$_3$O$_{6.52}$ sample is sketched in Figure 8.4. The phase boundary $B_1(T)$ and $B_2(T)$ can be fit with fractional power laws $\propto (T_c - T)^v$, with $v = 1.7$ and $v = 1.4$ respectively. It has been proposed[34] that $v$ depends on the form of the model (2D vortex glass, 3D XY model, etc.), but in our case $T_1$ and $T_2$ have large uncertainties, therefore a definite conclusion cannot be reached.

![Figure 8.4: B-T phase diagram of ortho-II YBa$_2$Cu$_3$O$_{6.52}$. The phase boundary $B_{c1}(T)$ and $B_{c2}(T)$ are fit to a fractional power law $(1 - t)^v$. $B_{c2}(T)$ is given by $B_{c2}(T) = 70(1 - t)^2$.](image)

Figure 8.4: $B-T$ phase diagram of ortho-II YBa$_2$Cu$_3$O$_{6.52}$. The phase boundary $B_{c1}(T)$ and $B_{c2}(T)$ are fit to a fractional power law $(1 - t)^v$. $B_{c2}(T)$ is given by $B_{c2}(T) = 70(1 - t)^2$.

The phase diagram of Tl$_2$Ba$_2$CuO$_{6+x}$-sample 2 is sketched in Figure 8.5. This
Discussion of Pinning

can be compared with the work of A. P. Mackenzie[49], in which the resistive critical field $H^*$ shows an unusual temperature dependence with upward curvatures. The vortex lattice melting line is identified from the kink in the pinning constant (see Section 6.2). The exponential to linear temperature dependent crossover in $\alpha_p$ might suggest an ordered to disordered transition (or crossover) line, which would be located between 1 T and 2 T.

Figure 8.5: (a) $B$-$T$ phase diagram of Tl$_2$Ba$_2$CuO$_{6+x}$-sample 2. The phase boundary $B_m(T)$ is fit to a power law $18(1 - t)^2$. Nevertheless, it is also possible for $B_m(T)$ to be diverging at low temperature. (b) The resistive critical field observed by the Mackenzie group[49]. The inset is the low temperature data.

The vortex matter phase diagram for the overdoped YBa$_2$Cu$_3$O$_{6.993}$ sample is slightly more complicated. Aside from the kinks at around 70 K and a sharp drop that is presumably due to the Kim–Anderson creep, there is also a peak in $\alpha_p$ close to $T_c$. In Ben Morgan’s measurements on the optimally doped YBa$_2$Cu$_3$O$_{6.95}$ samples, a similar peak near $T_c$ has been observed. In our data however, such “peak effect” is in general not apparent, even in NbSe$_2$ and V$_3$Si, in which the peak effect has been well studied. A possible explanation is that the peak effect is sample dependent, and for very clean or very dirty samples it is not apparent[131, 137]. For instance, Ben Morgan was not able to observe the peak on a twinned sample with the same doping. We stress that in any case, vortex lattice melting seems to happen near $T_c$, possibly related to the ultra high $B_{c2}(T)$ in this material.
Chapter 9

Conclusions

In this study, we have carried out detailed and accurate measurements of the free flux-flow response and the pinning response of vortices on a variety of superconductors, in particular six cuprate superconductors that span the entire superconducting dome in the hole-doped cuprate phase diagram. This experiment provides useful information on their vortex dynamics in detail, with unmatched resolution, and at a temperature range that is beyond the reach of most other experiments.

The principal achievement of this work is the observation of a universal logarithmic temperature dependence of the flux-flow resistivity in cuprate superconductors. This logarithmic temperature dependence appears with vortices on the overdoped side, persists across the cuprate superconducting dome, and may exist above \( T_c \) on the underdoped side. This observation implies that the “normal state resistivity” measured on the underdoped side is dominated by an intrinsic logarithmic temperature dependence term in the flux-flow resistivity, and consequently provides strong experimental support for the existence of vortices above \( T_c \).

Interestingly, we also find that the vortex viscosity, which is proportional to the inverse of the flux-flow resistivity, shares strong similarities to the zero field conductivity. This suggests that the underlying physics of the logarithmic temperature dependence is intimately related to the enhancement of the quasiparticle density of states, and the reduction of the quasiparticle scattering time.

Another achievement of this work is the accurate measurement of the pinning constant, which contains equivalent information to critical current density data measured in DC experiments. A complete study of the functional temperature dependence of \( \alpha_p \) is presented, and potential vortex matter phase diagrams are suggested, which
Conclusions

might lay the ground work of a more advanced theory for pinning in cuprates.

These are the achievements of this work. As for further experiments, $d$-wave heavy fermion superconductors and the pnictides superconductors are interesting options.
Bibliography

BIBLIOGRAPHY


BIBLIOGRAPHY


BIBLIOGRAPHY


BIBLIOGRAPHY


BIBLIOGRAPHY


200


BIBLIOGRAPHY
