THE SEMANTICS OF VAGUENESS:
SUPERTRUTH, SUBTRUTH, AND THE COOPERATIVE
PRINCIPLE

by
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Abstract

Experimental results are used to assess the predictions of supervaluation, subvaluation, and many-valued logic with regards to the problem of vagueness. Supervaluation predicts a truth-value gap in the borderline range of a predicate, thus assigning the predicate neither true nor false as value, subvaluation predicts a truth-value glut, where the predicate is both true and false, and many-valued logics assign at least three truth-values to propositions in their domain. The results of the experiment are analyzed and shown to oppose each of these frameworks, and instead to favor an approach in which the predicate and its negation are false in the borderline range, but where their conjunction is true.
To my loving family, and to Mayo.
“Contradiction is not a sign of falsity, nor the lack of contradiction a sign of truth.”

— BLAISE PASCAL (1623-1662).
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Chapter 1

Introduction

A fundamental rule of deduction in any classically-abiding system of logic is the rule of \( \wedge \)-elimination. The rule is based on the intuition that a conjunct can be proven true if the conjunction in which it appears is true (\( \Phi \wedge \Psi \vdash \Phi, \Psi \), and \( \Phi \wedge \Psi \vdash \Phi, \Psi \), by soundness). It is then expected that from a contradiction like \( \Phi \wedge \neg \Phi \), the conjunct \( \Phi \) and its negation \( \neg \Phi \) follow. But, as is shown by the experimental results described in the present work, when \( \Phi \) denotes a gradient property, \( \Phi \wedge \neg \Phi \) approaches truth when \( \Phi \) is applied to a variable \( x \) if \( x \) is borderline-\( \Phi \), that is, if \( x \) falls in the penumbra of \( \Phi \)'s extension. But as \( x \) approaches the borderline range, the predicate \( \Phi \) and its negation \( \neg \Phi \) fail to hold true of \( x \), resulting in the combination seen in (\( * \)):

\[
\begin{align*}
[\Phi(x)] &= F \\
[\neg \Phi(x)] &= F \\
[\Phi(x) \wedge \neg \Phi(x)] &= T.
\end{align*}
\]

Throughout the remainder of this thesis, I shall use (\( * \)) to refer to the pattern where individual contradictory conjuncts are considered false, and where their conjunction - the contradiction - is considered true.

That the behavior of gradient predicates presents challenges to the rules of classical logic comes as no surprise. Vagueness has, over the past 40 years or so, inspired many logicians to argue in favor of a variety of non-classical logical systems, such as, to name a few, 3-valued logics, fuzzy logics, supervaluations, and subvaluations. I intend to illustrate, however, that none of these systems can account for the results in (\( * \)).
CHAPTER 1. INTRODUCTION

My plan is to discuss in sufficient detail the assumptions that underlie each of the frameworks mentioned above, and to go over some of the theoretical complications and criticisms that surround them. I undertake this task in Chapter 2, in which I begin with a brief outline of the history of the problem of vagueness, and then focus on the theoretical proposals that I wish to evaluate. In Section 2.1 I briefly discuss the epistemic view of vagueness, and in the remaining parts of the chapter I discuss many-valued logics (Section 2.2) and super- and subvaluations (Section 2.3). Sections 2.2 and 2.3 are divided, respectively, in two and three parts. The first half of Section 2.2 is devoted to a discussion of fuzzy logic (2.2.1), and the second to 3-valued logics (2.2.2). Section 2.3, which focuses primarily on supervaluations, includes two subsections: 2.3.1 presents subvaluations, the paraconsistent variant of supervaluations, and 2.3.2 presents a dynamic semantic interpretation of the supervaluationary solution to vagueness. I end Chapter 2 with an outline of experimental results that point in the direction of truth-value gaps, which are theoretically predicted in supervaluations.

In Chapter 3 I discuss my experimental findings. My exposition of the results is focused mainly on showing evidence for truth-value gaps and on giving sufficient motivation for considering (*) seriously. I begin the chapter with a description of the experiment and the recruited participants (Section 3.2) and then move, in Section 3.3, to a detailed review of the results, the most important parts of which are Sections 3.3.2 and 3.3.3, which include, respectively, the motivation for truth-gaps and the evidence for (*).

In Chapter 4 I offer further evaluation of multi-valued logics and super-/subvaluations in light of my experimental results (Sections 4.1-4.3). In Section 4.4 I digress, temporarily, to illustrate the ambiguity that negated vague expressions lend themselves to, paying special attention to the distinction between predicate negation and predicate-term negation, or weak and strong negation, respectively. In Section 4.5 I offer my reconciliation between truth-gap and truth-glut theories using Gricean pragmatic principles, and subsequently discuss the consequences of my solution with respect to the semantics of comparatives (Section 4.6.1), dynamic semantics (4.6.2), and multi-dimensional adjectives (4.6.3).

The theoretical problems surrounding vague expressions are numerous and very challenging. My modest contribution in this work, if any indeed, consists of a proposal in which two seemingly conflicting logical systems are shown to be compatible and in fact simultaneously needed in order to account for (★), which, I hope to show, is experimentally evident.
Chapter 2

Background

The earliest known formulation of the problem of vagueness is usually attributed to Eubulides of Miletus, due to his alleged portrayal of what is known as the sorites paradox\(^1\) (from the Greek ἱππος for “heap”.) In its modern form, the paradox results from induction on premises like the following:

(1) 1,000,000 grains of sand make a heap.

(2) if \(n\) grains of sand make a heap, then \(n - 1\) grains of sand also make a heap.

The acceptance of both these premises inductively leads to concluding that 1 grain, and even 0 grains of sand make a heap. Similar, seemingly harmless, sets of assumptions can lead to statements of equal absurdity: Larry Bird is tall, and a tall man will remain tall if a millimeter is taken away from his height, therefore, Danny DeVito is tall; Yul Brynner is bald, and since adding a single hair to bald man’s head will still render him bald, Jerry Garcia (in his prime) is bald\(^2\). Expressions that are most obviously susceptible to sorites are the ones involving the so-called adjectives of degree: gradable adjectives like ‘tall’, ‘bald’, color predicates, etc., but other examples, such as quantifiers like ‘most’ and ‘many’, modifiers like ‘almost’, and even nouns like ‘mountain’ and ‘cloud’ can serve in constructing soritical arguments. Gottlob Frege, in his quest for the language of mathematical reasoning,

---

\(^1\)That Eubulides was the true inventor of this puzzle is not certain. Nor is his purpose behind it known. See Hyde (Fall 2005) and Williamson (1994).

\(^2\)Baldness was another example that was used by Eubulides in his *falakros* paradox: the bald man. (Black, 1963; Goguen, 1969). The Yul Brynner-Jerry Garcia series is adapted from Shapiro (2006).
regarded vagueness as a deficiency of natural language and suggested that a proper remedy
would be to replace vague expressions with precise ones. According to Williamson (1994),
Frege thought that vague expressions had senses but no referents, and because of this, they
could not be used in deriving meanings of larger expressions compositionally (Williamson,
1994, pgs. 40-41). Recall that Frege's distinction between sense and reference applied even
to sentences and predicates: the referent of a sentence is either one of the values 'true' and
'false', while its sense is its "mode of presentation" (Frege, 1977). The sense of a predicate
can likewise differ from its referent, the latter being a function that maps individuals to
truth-values, and the former being a way of presenting that function to the speaker.

Bertrand Russell's predilection for the precise inspired similar sentiments in him. He
writes that "Logic takes us nearer to heaven than most other studies ... [T]hose who dislike
logic will ... find heaven disappointing" (Russell, 1923). But he did not find vagueness as
dispensable as Frege did, perhaps owing to the former's greater interest in natural language.
On his view, vagueness was ineliminable from representations, be they linguistic or not:

Vagueness in our knowledge is, I believe, merely a particular case of a general law
of physics, namely the law that what may be called the appearances of a thing
at different places are less and less differentiated as we get further away from the
thing ... From a close-up photograph it is possible to infer a photograph of the
same object at a distance, while the contrary inference is much more precarious
... Therefore the distant appearance, regarded as a representation of the close-up
appearance, is vague according to our definition. I think all vagueness in
language and thought is essentially analogous to this vagueness which may exist
in a photograph.

During the 20th century, vagueness came to be accepted as an inherent feature of natu-
ral language, and because of the growing interest in finding appropriate logics for the
phenomenon, many proposals were put forward by logicians and philosophers as solutions.
In addition to susceptibility to sorites, two other features were recognized as characteristics
of vague predicates: the admission of borderline cases, and context-dependence\(^3\). Russell
spoke of the connexion between vagueness and borderline cases, but the earliest explicit
mention of this relationship that I am aware of was put forth by Charles Sanders Peirce:

\(^3\)These criteria are borrowed from Keefe and Smith (1997) and Kennedy (2007).
A proposition is vague when there are possible states of things concerning which it is intrinsically uncertain whether, had they been contemplated by the speaker, he would have regarded them as excluded or allowed by the proposition. By intrinsically uncertain we mean not uncertain in consequence of any ignorance of the interpreter, but because the speaker's habits of language were indeterminate. (Peirce, 1902, pg. 748).

But note that a proposition cannot be deemed vague only because it fails to hold (or not hold) in a given scenario, contrary to what Peirce suggests in his definition. The existence of borderline cases in the domain of a predicate does not necessarily make it vague, for the predicate could hold of certain individuals, not hold of other individuals, and be borderline of yet other individuals, without there being any penumbral regions between its positive extension and its borderline extension, or between its borderline extension and its anti-extension. (By 'penumbral regions' I mean regions in which the transition, as between extensions or judgements of applicability, is not clear-cut. I use the term extension to mean the set of individuals to which the predicate applies or does not apply. I use the term extension to mean the set of individuals to which the predicate applies, and the negative extension, or the anti-extension, is the set of individuals to which the predicate does not apply.) Suppose, for example, that an apartment is considered accommodating if and only if it has at least one bedroom for each of its occupants, not accommodating if and only if its total number of rooms - including living rooms - is less than the total number of its occupants, and borderline otherwise. For 3 people, then, apartments containing 3 or more bedrooms are in the positive extension of accommodating, 1-bedroom apartments and studio suites are in the negative extension of accommodating, and 2-bedroom apartments are borderline, since one occupant could occupy the living room. Here we have a scenario in which a predicate does allow borderline cases, but is not vague since the boundaries between the positive, borderline, and negative extensions are sharp. Borderline-ness is thus possible without vagueness. Susceptibility to sorites and borderline-ness are both necessary in the definition of gradability.4

Context-dependence is another defining feature of vague terms. 'tall', for example, differs in its use between a context in which basketball players are discussed and another where chartered accountants are. The point at which an individual ceases to be tall for a basketball

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4See Keefe and Smith (1997) for further discussion.
player can differ markedly from the cut-off point of tallness for being a chartered accountant. This feature, like borderline-ness, must be accompanied by sorites susceptibility in order to provide a comprehensive depiction of vague expressions. Consider, again, the apartment example from the previous paragraph. If the context were fixed such that the predicate accommodating is used for apartments for 3 people, the extension of the predicate would differ from a context in which 5 people were concerned. In either case, the predicate itself is sharp, despite the role played by what is known in context.

Lewis (1979) provides a very brief (and rather informal) treatment of vagueness, in which he argues that cut-off points are fixed in context in order to maintain conversational accommodation. One example he uses is the word 'flat' (borrowed from Unger (1975).) Since nothing is absolutely flat, as Unger suggests, Lewis claims that the word itself becomes a utility and not a precise description of the state of affairs under discussion. This is fairly obvious, but it leads to a serious conclusion if considered more carefully: most gradable adjectives are in fact unattainable. It is difficult, if not impossible, to think of what it takes to be absolutely 'tall' or 'red'. Gradient notions exist not only because of deficiency in our perception, but also, I think, because there does not seem to be a practical way of introducing a precise linguistic alternative. The more accurate language is, the less discernible the difference between the precise substitutes. If 'tall' were, say, replaced by a group of adjectives, tall₁, tall₂, . . . , tallₙ, then as n becomes larger, the discriminability between any tallᵢ and either of its neighbors diminishes, leading to increasing difficulty in their application. ⁵

Wright (1975) argues that it is incoherent to think that language possesses the two characteristics summarized in what he calls 'the governing view'. The governing view is a conjunction of two statements. The first is that the semantics of language is rule-governed, and the second is that it is possible to describe these rules by reflecting on our linguistic intuitions. In the same paper, he introduces the concept of tolerance, a notion used in his later works (e.g. Wright (1976) and Wright (1987)) and in much of the literature that was later written on vagueness and the semantics of degree. A predicate \( P \) is tolerant if there can be a degree of change not large enough to affect its applicability. Color predicates like

---

⁵ Actually, greater perceptual accuracy changes nothing, for even if we could detect the difference between 180 cm and 181 cm, as in a man's height, and call the first tall₁ and second tall₂, then the sorites question arises again if one asks how the addition of a micrometer to a man's height changes the applicability of any of these adjectives.
CHAPTER 2. BACKGROUND

‘red’, for example, are tolerant because if ‘red’ is judged to be true of an object, then a slight change in the object’s chromatic appearance will not suffice to render its redness false. Since our intuitions will lead to ascribing tolerance to vague predicates, we are compelled to accept the application of a vague predicate (and its negation) to every object in its domain because of sorites. This derives from the inductive proof on (1) and (2) and, Wright argues, shows that the governing view is inherently incoherent. Consequently, at least one of the assumptions it contains must be rejected: either the semantics of language is not governed by systematic rules, or the rules are not available for investigation upon introspection.

2.1 The Epistemic View of Vagueness

This view has been defended by Timothy Williamson (e.g. 1992; 1994) and Roy Sorensen (e.g. 1988; 2001). Epistemicists argue that sharp boundaries do exist, that is, that there really is a precise shade of color at which an object makes the switch from being red, e.g., to becoming orange, and there is a precise degree of height at which a person crosses the tall/not-tall border. The phenomenon of vagueness, they maintain, results from our ignorance of where these boundaries lie. One celebrated advantage of epistemicist claims is the full preservation they allow of classical logic. Metaphysically, the epistemic account of vagueness requires no alternative logics, since a logical representation of vague properties - as they manifest themselves in the world - will fully abide by such classical principles as bivalence and non-contradiction if it were indeed the case that concepts had precise boundaries.

Regardless of the auspiciousness of this approach in dealing with vagueness in the real world, it seems to provide no solution to vagueness if considered linguistically. Speakers seem to have full command of the use of imprecise terms, and it seems that to no speaker can such terms be used in full precision, irrespective of whether that term does correspond to a sharp-bounded notion. The embarrassment presented by the sorites paradox stands in support of my claim, for our acceptance of the basic and the inductive steps of the paradox shows that we use these terms without having sharp boundaries in mind. If there is to be a linguistic theory of vague terms (whether objects or predicates,) the epistemic view simply cannot be invoked, for in language vagueness is inescapable.6

6Williamson argues that there is no reason to suppose that speakers have full command of vague expressions, and therefore that ignorance, in this case, can manifest itself linguistically (see his 1994, pg. 208).
2.2 Many-valued Logic

2.2.1 Fuzzy Logic

The foundations of fuzzy logic are found in Zadeh's (1965) postulation of fuzzy sets, which were not intended for use in natural language semantics but later inspired future expansions into logic and, thererafter, applications in the study of vagueness and imprecision. With the developing interest in the semantics of vague terms, a number of logicians adopted Zadeh's approach and argued in favor of the fuzzy technique. Examples include Goguen (1969), Lakoff (1973), Machina (1972, 1976), Sanford (1975), Zadeh (1975), Peacocke (1981), and Edgington (1997).

Fuzzy logic is an extension of Łukasiewicz's implementation of three-valued logic. A truth value can be any real number in the interval $[0,1]$, where 0 denotes complete falsity and 1 complete truth. As far as I know, the most popular way of defining negation, conjunction, and disjunction are as follows (borrowed from Machina (1976) and based on Łukasiewicz (1970)):

\[
\begin{align*}
\neg \phi &= 1 - \phi \\
\phi \land \psi &= \min(\phi, \psi) \\
\phi \lor \psi &= \max(\phi, \psi)
\end{align*}
\]

Fuzzy logic is criticized for its failure to preserve some of the laws of classical logic, namely, the Law of Excluded Middle (LEM: $\vdash \phi \lor \neg \phi$) and the Law of Non-contradiction (LNC: $\vdash \neg (\phi \land \neg \phi)$.) Suppose that the truth value of a proposition $\phi$ is 0.5. According to the definitions of the connectives, the truth value of $\phi$'s negation, $\neg \phi$, will equal $1 - \phi$, which is 0.5. Now the truth value of the conjunction $\phi \land \neg \phi$ is 0.5 ($\min(\phi, \neg \phi) = \min(0.5, 0.5) = 0.5$.) Note that, in this case, the value of $\phi \lor \neg \phi$ will also be 0.5, since disjunction is defined as being as true as the truest of its disjuncts ($\max(\phi, \neg \phi) = \max(0.5, 0.5) = 0.5$.). In classical logic neither of these results is permissible; contradictions are always false, and disjunctions of the form $\phi \lor \neg \phi$ are always true.

Another objection to this formulation of fuzzy logic is lucidly made in Edgington (1997). Suppose Norm is indisputably average in height, so that $[\text{tall(norm)}] = 0.5$. Imagine also that Paul is slightly taller than Norm, making $[\text{tall(paul)}] = 0.6$. If the definitions will not address this issue here.
above are adopted, then \([-\text{tall}(\text{paul})]\) = 1 - 0.6 = 0.4, and \([\text{tall}(\text{norm}) \land \neg\text{tall}(\text{paul})]\) = \(\min(0.5,0.4) = 0.4\). But this certainly should not be the case. \([\text{tall}(\text{norm}) \land \neg\text{tall}(\text{paul})]\) says that Norm is tall and that Paul is not tall. While each conjunct can conceivably be assigned a value on the truth scale, the entire conjunction must remain completely false because there cannot be a truth assignment that allows Norm to be tall and simultaneously disqualifies Paul from being so. If Norm is tall to a positive degree, then Paul must be tall also, and if Paul is not tall, then it must be the case that Norm is not tall either. Edgington fixes the problem by proposing a degree-theoretic framework in which the connectives are not defined truth-functionally, but probabilistically. Her truth values correspond to what she calls “verities”: degrees of closeness to complete truth. The connectives, except for negation, are probabilistically defined as functions on the verities of their arguments. \([\phi \land \psi]\), for example, is equal to the probability of \(\psi\) given \(\phi\), multiplied by the probability (or verity) of \(\phi\).\(^7\) With this approach, the problem discussed above is solved, since \([\text{tall}(\text{norm}) \land \neg\text{tall}(\text{paul})]\) = \([-\text{tall}(\text{paul})]\) given \([\text{tall}(\text{norm})]\), multiplied by \([\text{tall}(\text{norm})]\), but since \([\text{tall}(\text{paul})]\) given \([\text{tall}(\text{norm})]\) = 0, the entire conjunction is false regardless of the verity of \(\text{tall}(\text{paul})\).

Another drawback of the degree-theoretic approach is that the assignment of fuzzy truth-values to propositions that include vague predicates restricts the use of comparatives. If an individual \(a\) whose height is 7'0" is tall to degree 1, then anyone above that height will also be tall to degree 1 (call him \(b\).) Since both \(a\) and \(b\) are tall to the same degree, it is impossible to use the truth-values to evaluate statements like \(a\) is taller than \(b\). To save the fuzzy approach one must ensure that no statements - no vague statements at least - be true to degree 1, which results in an unintuitive “asymptotic” distribution of truth-values (see (Keefe, 2000, pgs. 92-93).)

There is one more comment on the degree-theoretic analyses that I would like to make. My objection to the use of fuzzy logic in natural language semantics, and in the semantics of vague predicates in particular, is that gradient truth (if such a thing can be postulated) is not isomorphic to truth as it is expressed in natural language. One may consent to the proposal that tallness, for instance, does indeed correspond to degrees of truth, e.g. that \([\text{tall}(6'6'\prime)] = 1, [\text{tall}(6'2'\prime)] = 0.7, [\text{tall}(6'0'\prime)] = 0.55, \) and so on. But it is clear that such values have no linguistic equivalents. Zadeh (1975) does attempt to match various degrees

\(^7\)This violates the logical independence of atomic formulas in classical and non-classical systems.
of truth to natural language modifiers like *quite*, and *very*, but there are only finitely many modifiers in language, and whichever way they are mapped to points along the truth scale, the sorites question will strike again since no sharp boundaries exists between what may count as *very tall* and what may not.\(^8\) I find that this approach has similar problems to those that support the epistemic nature of vagueness. Either one of them may be applicable to vagueness as a phenomenon in the real world, but neither is satisfactory if vagueness is thought of linguistically.

### 2.2.2 Finitely-valued Logic

I use the term ‘finitely-valued’ to refer to logics in which three or more truth values are recognized, without the number of admissible values being infinite. The oldest documented motivation for them is found in Aristotle’s *De Interpretatione*, where he seems to find reason to dismiss bivalence while maintaining the law of excluded middle in future-tensed statements\(^9\):

> A sea-fight must either take place to-morrow or not, but it is not necessary that it should take place to-morrow, neither is it necessary that it should not take place, yet it is necessary that it either should or should not take place to-morrow. Since propositions correspond with facts, it is evident that when in future events there is a real alternative, and a potentiality in contrary directions, the corresponding affirmation and denial have the same character. (Aristotle, 1941, *De Interpretatione* 19a.30)

The first formal treatment came in the work of Łukasiewicz, who developed the first 3-valued propositional calculus in 1920.\(^10\) Conjunction, disjunction, and negation are defined in Figure 2.1.\(^11\)

The finitely-valued approach is adopted for vagueness in Tye (1994). His truth tables are derived from Łukasiewicz, and he argues that the sorites paradox can be solved by

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\(^8\)The same objection, among others, is raised in Haack (1979).

\(^9\)I assume, for my present purposes, that the translation of this passage is accurate.

\(^10\)See Łukasiewicz (1970). See also Gottwald (Winter 2006) and Malinkowski (2001) for more on Łukasiewicz’s systems.

\(^11\)Not all three-valued systems agree on these definitions. Bochvar (1981), for instance, classifies truth values into two categories, the meaningful and meaningless/paradoxical. The meaningful values are *true* and *false*, and the third value, *U*, is meaningless. Since *U* is meaningless, any compound proposition containing a proposition of value *U*, whether it be a conjunction, disjunction, or conditional, is also meaningless.
accepting the first premise as true, where appropriate, and by assigning the third value, I, to the inductive premise. That is, while it is true that for some \( n \), a man with \( n \) hairs on his head is not bald, it is indefinite that if a man with \( i \) hairs is not bald, then a man with \( i - 1 \) hairs on his head is also not bald.

Here my earlier complaint regarding the correspondence of fuzzy truth values to the linguistic use of truth and falsity is addressed. Not only can borderlineness be modeled, but there is also a (temporary) fix to the way in which truth is expressed in natural language. The problem, however, lies in the transition between what counts as borderline and what counts as true (or false). A sorites paradox can be easily reconstructed between a predicate’s positive extension and its borderline extension, or between its borderline extension and its negative extension. Thus it seems that the system is doomed to precision despite the recognition of borderlineness. However, Tye attempts to fix this by allowing the metalanguage to be vague. Take a sorites series of propositions ‘a man with \( n \) hairs on his head is bald.’ At \( n = 0 \) the proposition is true, but at some value \( k \) the truth value of the corresponding proposition must change, for otherwise the series will result in the truth of ‘a man with 1,000,000 hairs on his head is bald’. But in order to evade the sorites trick, there must be a \( k \) such that the corresponding proposition, call it \( M_k \), is true and \( M_{k+1} \) is not true, which produces a sharp boundary between the true and the borderline extensions of ‘bald’. Tye responds by rejecting the truth/falsity of either of the following statements: (1) ‘There is a \( k \) such that \( M_k \) is true and \( M_{k+1} \) is not true,’ (where ‘not true’ includes both false and indefinite,) and (2) ‘It is not the case that there is a \( k \) such that \( M_k \) is true and \( M_{k+1} \) is not true.’ Instead, he designs his metalanguage such that both (1) and (2) are indefinite, which consequently compels him to claim that, in such a series, “it is indefinite whether there are any sentences that are neither true nor false nor indefinite,” (my italics) for if it were true that there were any such sentences, then Tye is committed to a sharp transition between the truth values, and if it were false, then that necessitates additional

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**Figure 2.1:** Truth-tables in Many-valued Logic

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(a) Conjunction (b) Disjunction (c) Negation
truth-values. It is reasonable to reject the truth/falsity of the italicized statement because of the undesired consequences, but Keefe (2000) notes that if this is used to deduce that the (italicized) statement is indefinite, then that presupposes that every statement must have either one of the values true, false, and indefinite, which is the very claim that Tye thinks should be indefinite.

I add also that finitely-valued logics like Tye’s suffer from the same counter-intuitive consequence as fuzzy logic, i.e. that the truth values of \( p \land \neg p, p \land p, p \lor \neg p, \) and \( p \lor p \) are equal when \( p \) is borderline.

### 2.3 Supervaluations

Suppose that a predicate that lacks sharp boundaries were to be made precise, in a way that permits the retrieval of all the laws of Classical Logic. The question that would immediately follow is just where can the new-found cut-off point be placed? The answer must be indeterminate, for there couldn’t possibly be one unique height, for example, that can be used to sharpen the predicate tall. Instead, the supervaluationist responds by admitting a set of multiple classically-constructed sharpenings, each of which containing its own precise boundary that separates the positive and negative extensions of the originally vague predicate. Each vague predicate \( P \) is then associated with a set of precisifications; alternative definitions of \( P \) in which there is a sharp boundary between what counts as \( P \) and what doesn’t, but in each placing the sharp boundary differently. A possible way of sharpening tall, for instance, is to ascribe the (now sharp) property to anyone above 6’4” and to no one else. One could alternatively place the cutoff point at 6’2”, or at 6’1”. Let us now suppose that these are the only available precisifications we have for the vague predicate tall. In supervaluation, for any vague predicate \( P \), if \( P \) is true of an individual \( a \) in every precisification, then \( P(a) \) is said to be super-true. If in every precisification, \( P \) does not hold of \( a \), then \( P(a) \) is super-false. Otherwise, \( P(a) \) does not have a truth value. In our example, only those whose height is 6’4” or more are considered super-tall, since they are tall according to every available way of making the predicate sharp. Those who are shorter than 6’1” are super-not-tall, and those in between are neither. In order for a proposition to be true, according to supervaluation, it is necessary for it to be super-true: “truth is
super-truth,” according to Fine (1975), “truth from above.”

I now provide the formal details of a supervaluationary framework. This particular model is faithful to Fine’s initial characterization, but I borrow it from Kyburg and Morreau (2000) for its simplicity. Let \( \mathcal{L} \) be a language consisting of constant symbols \( a, b, c, \) etc., variable symbols \( x, y, z, \) etc., and predicate symbols \( P, Q, R, \) etc.. Let \( \mathcal{M} \) be a model consisting of an ordered quadruple \( (\mathcal{U}, \mathcal{P}, \leq, V) \), where \( \mathcal{U} \) is a non-empty set of individuals in the domain of discourse, \( \mathcal{P} \) is a set of specification points \( p, q, r, \) etc., \( \leq \) is a an ordering relation on the specification points, and \( V \) is the valuation function that assigns an individual in the domain - i.e. an element of \( \mathcal{U} - \) to each constant symbol in the language, and assigns each \( n \)-place predicate symbol a unique pair of disjoint sets at any given specification point. The first of these two disjoint sets, \( V^+(R, p) \), is the positive extension of \( R \) at point \( p \), and the second, \( V^-(R, p) \) is the negative extension of \( R \) at \( p \). Specification points are structured in a similar way to a tree: at the terminal points (or nodes), each vague predicate is fully precisified, yielding a completely classical range of predicates. In supervaluationist terms these are called complete points. At any complete specification point \( p \), every predicate \( R \) is such that \( V^+(R, p) \cup V^-(R, p) = \mathcal{U} \), leaving the empty set as the borderline extension. At the base point (the root of the tree) predicates may lack positive/negative extensions, but the base point can be sharpened by specification points that are in turn sharpened by other specification points, and so on. A point \( p \) is said to sharpen a point \( q (q \leq p) \) for some vague predicate \( R \) if the positive/negative extensions of \( R \) in \( q \) are subsets of the positive/negative extensions of \( R \) in \( p \). In other (informal) words, a point sharpens another if the set of undecided (borderline) cases in the former is a subset of the set of undecided cases in the latter:

\[
\forall p, q \in \mathcal{P} [q \leq p \iff \forall R \in \mathcal{L} (V^+(R, q) \subseteq V^+(R, p) \land V^-(R, q) \subseteq V^-(R, p))]
\]

Fine enforces the property of completability on his system, which has the consequence that every specification point is fully sharpened. Note that this need not be satisfied directly; a point \( q \) may sharpen another point \( p \) incompletely, but completability guarantees that there exist another point which ultimately provides a full sharpening of both \( p \) and \( q \). In

\[12\] Though Fine (1975) is considered the classic supervaluationary treatise of vague predicates, the idea finds earlier origins in Mehlberg (1958) and was first formalized in van Fraassen’s work on presupposition (see his 1966; 1968; 1969). Interestingly, the application of van Fraassen’s method to vagueness seems to have been thought of independently by Kamp (1975) and Fine.
the tree analogy, COMPLETABILITY corresponds to saying that every terminal node must be completely sharpened.

At any complete specification point $p$, the truth of a formula $R(a)$ is classical:

$$[R(a)]_p = 1 \text{ iff } V(a) \in V^+(R, p)$$

Otherwise, $[R(a)]_p = 0$. This is enforced in Fine’s system under FIDELITY, which requires that all complete specification points preserve the theorems of classical logic.

At an incomplete point $p$, $R(a)$ is super-valuated: it is true iff it is true at every specification point $q$ that sharpens $p$, false iff it is false at every $q$, and truth-value-less otherwise:

$$[R(a)]_p = 1 \text{ iff } \forall q \in \mathcal{D} [(p \leq q) \rightarrow ([R(a)]_q = 1)];$$

$$0 \text{ iff } \forall q \in \mathcal{D} [(p \leq q) \rightarrow ([R(a)]_q = 0)];$$

undefined otherwise.

This definition evaluates the truth of a formula at a point $p$ based on the truth of the formula in every point $q$ that sharpens $p$. This is reminiscent of the evaluation of a formula that contains the necessarily operator $\Box$ in possible world semantics: $\Box \phi$ is true in world $w$ iff $\phi$ is true in every world $w'$ accessible from $w$. But suppose that $\phi$ was true at a specification point $p$ but false at at least one point $q$ that sharpens $p$. Then $\phi$ can be said to be true at $p$ even though it is false in at least one way of sharpening $p$, which should not happen. The truth of $\phi$ at $p$ should depend on the truth of $\phi$ at every point that sharpens $p$, so if $\phi$ holds at $p$, it must hold at every point that sharpens it. Fine’s imposition of STABILITY guarantees this. If a formula is true at any point $p$, then it must be true at every $q$ that sharpens $p$:

$$\forall p \in \mathcal{D} ([[\phi]^p = 1] \rightarrow \forall q \in \mathcal{D} [(p \leq q) \rightarrow ([\phi]^q = 1)])$$

In summary, supervaluations provide a semantics of vague predicates in which evaluation is made at points that possibly contain non-empty borderline extensions. At the base point, vague predicates are said to hold of individuals iff the predicates hold of those individuals in every point that sharpens the base point. If in some of those points the individual falls in the borderline extension of the predicate, then the predicate is evaluated according to its extension in the points that sharpen those points, and so on. COMPLETABILITY guarantees the complete sharpening of every specification point in the model, so every predicate will be completely sharpened if the tree is traversed to its terminal points, FIDELITY promises
a classical evaluation at every complete point, and STABILITY commits every point to the
sharpenings made in the point it extends.

Before I move on, I will briefly mention that much criticism has been directed at super­
valuations because of its employment of precisifications. Some believe, in effect, that it is
misleading to treat the semantics of vague expressions as if it were possible to make them
precise. This argument has come in different flavors in e.g. Dummett (1975), Sanford (1976),
and Fodor and Lepore (1996), all of whom question the role of, and even the need for, invok­
ing specifications and endorsing the possibility of precisifying language. But note that the
assignment of truth to a vague expression is done only in the meta-language, in which there
is no direct reference to individual precisifications. In other words, even though the system
is built on classical structures, which, individually, misrepresent vague expressions as precise
ones, the semantics of these vague expressions is laid out in terms of whole collections of
specification points, not individual ones. Objections to the very existence of precisifications
are met in Keefe (2000) with a brief comparison to the use of possible worlds in treating
modality in natural language. If one takes issue with the ontological/metaphysical status
of precisifications, then much of the abstraction that proved essential in formal semantics,
logic, mathematics, and even physics, is placed in just as much jeopardy. I side with Lewis
(1986) in his belief in the “plurality of worlds [...] because the hypothesis is serviceable,”
though I suspend judgement on whether this is “a reason to think that [the hypothesis] is
true.” (Lewis, 1986, pg. 3. Emphasis mine.)

One central advantage claimed by supervaluationists is that the inductive premise of
the sorites paradox does not hold (see premise (2) in heap example on pg. 3.) In each
precisification, there exists a sharp boundary that separates the positive and the negative
extensions of a vague predicate, which makes it true in every precisification that there exists
some \(x_i\) such that \(P(x_i)\) and \(\neg P(x_{i+1})\), and that, in turn, falsifies the inductive premise of
the sorites argument. The failure of the inductive premise in every precisification guarantess
its falsity simpliciter since falsity (or truth) in supvaluation requires falsity (or truth) in
every way of making a vague predicate precise.\(^1\)

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\(^1\) A sorites series can alternatively be constructed with a finite (but large) set of assumptions. In a series
of men ranging in tallness from Larry Bird (as the tallest) to Danny DeVito, where the difference in height
between every adjacent pair is 1 mm, the sorites can be formulated as a set of propositions of the form:
"Larry Bird is tall", and "the person next to Larry Bird is 1 mm shorter," (call that person Jim,) therefore
"Jim is tall." In supervaluation, such a collection of propositions is false since there will be a pair of adjacent
men \(a_i\) and \(a_{i+1}\) such that \(\text{tall}(a_i) \land \neg \text{tall}(a_{i+1})\) in every specification, even through the value of \(i\) will differ
from specification to specification.
Additionally, supervaluationary systems preserve the two classical laws violated in multi-valued logics, namely, the Law of Non-Contradiction (LNC) and the Law of Excluded Middle (LEM). The former holds in every precisification because every precisification is classically constructed. If a vague predicate \( P \) is made precise, there will be no instance of \( P \) in any of the sharpenings such that \( P \) will hold and simultaneously not hold of any given constant. \( P \land \neg P \) will therefore be false in every precisification no matter where the boundary is drawn. Contradictions are consequently false in virtue of their falsity in every precisification in a supervaluationary framework. Similarly, because \( P \) is completely sharpened in every precisification, for any constant \( a \) in \( P \)'s domain, \( P(a) \lor \neg P(a) \) will be true in every precisification regardless of the cut-off point between \( P \) and \( \neg P \).

Complications arise in supervaluationary semantics in the presence of the \( D \) operator (read “definitely”), which is introduced in Fine’s paper in preparation for treating higher-order vagueness. (I return to higher-order vagueness in the next section.) The operator is added to the object language \( \mathcal{L} \), and its syntax is such that if \( \Psi \) is a well-formed formula, then \( D\Psi \) is a well-formed formula. Its semantics is similar to the \( \Box \) operator in modal logic: \( D\phi \) is true iff \( \phi \) is super-true, i.e. if \( \phi \) is true in all specification points in \( \mathcal{S} \). Now we have added to the object language a way to describe 1st-order vagueness since it is possible to differentiate between \( \phi \) and definitely \( \phi \).

The problems accompanying the addition of \( D \) become clear when one considers argument validity in supervaluation. In classical logic, an argument is valid iff when its premises \( \Gamma \) are true, its conclusion is also true: \( \Gamma \vdash \phi \) is valid iff whenever the contents of \( \Gamma \) are true, \( \phi \) is true. (The symbol \( \vdash \) denotes logical entailment. To emphasize the difference between classical entailment and entailment in supervaluations, I will use the symbols \( \models_{cl} \) and \( \models_{sv} \), respectively.) Proponents of supervaluation claim that classicality is maintained because every complete specification point is classical, as granted by FIDELITY. So one expects any classically-valid argument to also be valid in supervaluation. But because supervaluation is based on a multiple of admissible precisifications, validity can be defined in two ways, either locally or globally. An argument \( \Gamma \vdash_{sv} \phi \) is locally valid iff in every specification point, if \( \Gamma \) is true then \( \phi \) is true. An argument \( \Gamma \vdash_{sv} \phi \) is globally valid iff if \( \Gamma \) is true in
every specification point, then $\phi$ is true in every specification point. Since validity is usually defined as preservation of truth, and since truth is supertruth, validity in supervaluation must be defined as preservation of supertruth, which is to say that an argument is valid iff whenever its premises are supertrue (true according to supervaluation,) its conclusion is also supertrue. This corresponds to *global* validity. Williamson (1994) claims that a global treatment of validity threatens four classically-valid arguments, namely, contraposition, the conditional proof (the theorem of deduction), argument by cases (i.e. $\lor$-elimination), and *reductio ad absurdum*.

Contraposition rules that, if $\Phi \vDash_{cl} \Psi$ then $\neg\Psi \vDash_{cl} \neg\Phi$. The supervaluationary version of this rule, if valid, would run along the lines of: if the truth of $\Phi$ in every precisification entails the truth of $\Psi$ in every precisification, then the falsity of $\Psi$ in every precisification entails the falsity of $\Phi$ in every precisification. In other words, if the supertruth of $\Phi$ entails the supertruth of $\Psi$, then the supertruth of $\neg\Psi$ entails the supertruth of $\neg\Phi$. Contraposition is invalidated because, even though $\phi \models_{sv} D\phi$ (if $\phi$ is supertrue entails $D\phi$ is supertrue,) $\neg D\phi \not\models_{sv} \neg\phi$ because $\neg D\phi$ could be supertrue without $\neg\phi$ being supertrue; if $\phi$ is borderline, then $\neg D\phi$ is supertrue by the definition of $D$, but neither $\phi$ nor $\neg\phi$ is supertrue. Therefore $\neg D\phi \not\models_{sv} \neg\phi$.

The failure of (this form of) contraposition is taken by Williamson to be a drawback, but I will show that, if it did hold, it would lead to disastrous consequences. From $\phi \models_{sv} D\phi$, which is beyond doubt, it would follow (by contraposition) that $\neg D\phi \models_{sv} \neg\phi$, i.e., that the supertruth of $\neg D\phi$ entails the supertruth of $\neg\phi$. But the supertruth of $\neg\phi$ entails the supertruth of $D\neg\phi$, again by the definition of $D$, leading to the offensive claim that from $\neg D\phi$ we can deduce $D\neg\phi$.

Under the conditional proof, an argument of the form $\Phi \vDash_{cl} \Psi$, where $\supset$ denotes the material implication. This fails in supervaluation when $D$ is involved. $\phi \models_{sv} D\phi$ says that the supertruth of $\phi$ entails the supertruth of $D\phi$, which is valid. If the theorem of the deduction held, then $\models_{sv} \phi \supset D\phi$, which says that $\phi \supset D\phi$ is supertrue. But if $\phi$ is borderline, then there are some precisifications in which $\phi$ (the antecedent) holds, and since $\phi$ is borderline, $\neg D\phi$ (the negation of the consequent) is supertrue and thus true in those precisifications. Therefore $\not\models_{sv} \phi \supset D\phi$ even though $\phi \models_{sv} D\phi$.

I shall digress very briefly, again, to show that the conditional proof – like contraposition – is unwanted in supervaluation. If one could indeed derive $\models_{sv} \phi \supset D\phi$ from $\phi \models_{sv} D\phi$, then
the $D$ operator would be redundant, for $\models_{sv} D\phi \supset \phi$ certainly holds because it is supertrue that if $\phi$ is supertrue then $\phi$ is true. But if this is combined with the first conditional then one can prove that $\models_{sv} \phi \leftrightarrow D\phi$, which makes $D$ unnecessary, thereby obstructing the expressibility of supertruth in the object language.

$\lor$-elimination permits the inference from $\Phi \models_{cl} \Theta$ and $\Psi \models_{cl} \Theta$ to $\Phi \lor \Psi \models_{cl} \Theta$. Williamson finds this counterexample in supervaluation: $\phi \models_{sv} D\phi \lor D\neg\phi$, and $\neg\phi \models_{sv} D\phi \lor D\neg\phi$, but $\phi \lor \neg\phi \not\models_{sv} D\phi \lor D\neg\phi$. The supertruth of $\phi$ entails the supertruth of $D\phi \lor D\neg\phi$, and the supertruth of $\neg\phi$ also entails the supertruth of $D\phi \lor D\neg\phi$. Therefore, by $\lor$-elimination we expect that the supertruth of the disjunction $\phi \lor \neg\phi$ entail the supertruth of $D\phi \lor D\neg\phi$, but it does not. $\phi \lor \neg\phi$ is supertrue because it instantiates LEM, which holds in every precisification. However, if $\phi$ is borderline, then $D\phi \lor D\neg\phi$ is superfalsely since neither $\phi$ not $\neg\phi$ is definite.

Finally, reductio ad absurdum (RAA) allows the deduction of $\neg\Phi$ if $\Phi$ is shown to lead to a contradiction: from $\Phi \models_{cl} \Psi$ and $\Phi \models_{cl} \neg\Psi$ one can infer $\models_{cl} \neg\Phi$. RAA is not valid in supervaluation because $\phi \lor \neg D\phi \models_{sv} D\phi$, which holds because if $\phi \lor \neg D\phi$ is supertrue, then $\phi$ is supertrue by $\land$-elimination, and since $\phi$ is supertrue, $D\phi$ is supertrue. $\phi \land \neg D\phi \not\models_{sv} \neg D\phi$ also holds by $\land$-elimination. The conclusions of these arguments are contradictory, so, by RAA, we should get $\models_{sv} \neg(\phi \land \neg D\phi)$, but we don’t if $\phi$ is borderline since $\phi \land \neg D\phi$ will hold in just those precisifications where $\phi$ is true.

Williamson views the failure of these laws as a serious flaw in supervaluation, but Keefe (2000) argues that the failures come up only when the $D$ operator is involved, whose logic is not classical. In order to illustrate where supervaluationary semantics departs from classical logic, she proposes a revision of each of Williamson’s four counterexamples. Contraposition is reformulated as Contrap*, on which it can be argued from $\Phi \models_{sv} \Psi$ that $\neg\Psi \models_{sv} \neg D\Phi$: if the supertruth of $\Phi$ entails the supertruth of $\Psi$, then the superfalsity of $\Psi$ entails the not supertruth of $\Phi$. This solves the problem shown above, because from $\phi \models_{sv} D\phi$ we now argue from Contrap* that $\neg D\phi \models_{sv} \neg D\phi$, which is trivial. The conditional proof is altered to $\supset \Gamma^*$, wherein the supertruth of the premise is included in the conditional: from $\Phi \models_{sv} \Psi$ infer that $\models_{sv} D\Phi \supset \Psi$ (instead of $\models_{sv} D\Phi \supset \Psi$). In words, if the supertruth of $\Phi$ entails the supertruth of $\Psi$, then $D\Phi \supset \Psi$ is supertrue, i.e. ‘if $\Phi$ is supertrue (definite) then $\Psi$’ is supertrue. The problem with $\phi \models_{sv} D\phi$ is now solved, since we can only deduce $\models_{sv} D\phi \lor D\phi$ under $\supset \Gamma^*$. Arguments by cases, or $\lor$-elimination, is modified to $\lor E^*$, which allows the inference from $\Phi \models_{sv} \Theta$ and $\Psi \models_{sv} \Theta$ to $D\Phi \lor D\Psi \models_{sv} \Theta$, instead of $\Phi \lor \Psi \models_{sv} \Theta$, and
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(i.e., it no longer suffices that the disjunction is supertrue. One of the disjuncts must be supertrue in order for the argument to succeed.) This can now be shown to rescue the breach of classicality found by Williamson: from $\phi \models_{sv} D\phi \lor D\neg \phi$ and $\neg \phi \models_{sv} D\phi \lor D\neg \phi$ we infer by $\forall E^*$ that $D\phi \lor D\neg \phi \models_{sv} D\phi \lor D\neg \phi$, which is trivial. Finally, RAA is revised as $\neg I^*$ ($\neg$-introduction,) which from $\Phi \models_{sv} \Psi$ and $\Phi \models_{sv} \neg \Psi$ allows $\models_{sv} \neg D\Phi$; if the supertruth of $\Phi$ entails both the supertruth and the superfalsity of $\Psi$, then instead of inferring that $\Phi$ is superfalse, we infer that $\Phi$ is not supertrue. The problematic counterexample was that $\models_{sv} \neg D\phi \land \phi \land \neg \phi$.

That the abnormalities are found where the $D$ operator is found informs the distinction between classical logic, which governs the individual precisifications, and supervaluationary logic. Recall, for instance, that while supervaluations do maintain LNC and LEM, the (classical) Law of Bivalence no longer holds. (According to Bivalence, a proposition must be either true or false.) Supervaluation shares its maintenance of LEM with classical logic because $\models_{sv} \phi \lor \neg \phi$, i.e., $\phi \lor \neg \phi$ is supertrue. If truth is treated classically, then LEM should find its metalinguistic version in Bivalence, but it does not because truth is supertruth. To say that a proposition must either be supertrue or superfalse would be a misrepresentation of supervaluationary semantics, simply because some propositions can be neither. The object-linguistic representation of this false principle is $D\phi \lor D\neg \phi$, which is not supertrue, and therefore not valid, because $\phi$ can be true in some precisification(s) but false in others.

The failure of bivalence is a feature that must be welcomed in a system that is designed to capture vagueness, for as was shown earlier, the admission of borderline cases is essential in the use of vague predicates.

Higher-Order Vagueness

I begin my discussion of higher-order vagueness with Fine’s definition of the $I$ operator.

Recall that, with introduction of $D$, it became possible to express supertruth and superfalsity in the object language, and because we can express supertruth and superfalsity, we can express ‘not supertruth and not superfalsity’, in other words, borderlineness. Fine defines $I$ as:

$$I\Phi =_{def} \neg D\Phi \land \neg D\neg \Phi$$
Higher-order vagueness is the vagueness of 'vague', the vagueness of 'vaguely vague', and 'vaguely vaguely vague', etc. So far, the supervaluationary truth-values have been divided three ways: true if supertrue, false if superfalse, and neither if neither. A proposition $\phi$ would fall into a truth-value gap if $\neg \phi$ held in one admissible specification point, and $\phi$ was true in every other point in $\mathcal{P}$. If another proposition $\psi$ was true in half the admissible points and false in the other half, then $\psi$ would also fall into a truth-value gap. In the first case $\phi$ is almost true (but borderline nevertheless) and in the second $\psi$ is plain old borderline, but no such distinction is evident in their truth-values because both are borderline, or truth-value-less, in supervaluations. The question is whether it is possible to distinguish between the two in the object language: both $I\phi$ and $I\psi$ hold globally because in every admissible specification point, $\neg D\phi \land \neg D \neg \phi$ and $\neg D\psi \land \neg D \neg \psi$. Perhaps one could say that $\psi$ is definitely borderline, and therefore that $DI\psi$, while $\phi$ is borderline borderline, and therefore $II\phi$. If we break these expressions open, we find that $D(\neg D\phi \land \neg D \neg \phi)$ (by the definition of $I$) and $\neg D(I\phi) \land \neg D(\neg I\phi)$, which (I hate to tell you) is $\neg D(\neg D\phi \land \neg D \neg \phi) \land \neg D(\neg D \neg \phi \land \neg D \neg \phi)$. Since double-negation is eliminable, given the classicality of the points over which $D$ abstracts, the expression concerning $\phi$ can be re-written as $\neg D(\neg D\phi \land \neg D \neg \phi) \land \neg D(\neg D \neg \phi \land \neg D \neg \phi)$, which, you may notice, is a conjunction of two identical propositions. The expression can be simplified to $\neg D(\neg D\phi \land \neg D \neg \phi)$.

So, as desired, the expression concerning $\psi$ (roughly) says 'it is definite that neither $\psi$ nor $\neg \psi$ is definite', i.e., that according to all specification points, it is not the case that $\psi$ holds in every specification point and it is not the case that $\neg \psi$ holds in every specification point. The expression concerning $\phi$, which was closer to truth than $\psi$, says 'it is not definite that neither $\phi$ nor $\neg \phi$ is definite', which is to say that it is not true in every specification point that $\phi$ is not definite and $\neg \phi$ is not definite. This can only hold if there is some specification point in which either $\phi$ or $\neg \phi$ is definite. But if there can be such a point, then either $\phi$ or $\neg \phi$ is supertrue, contrary to our initial setup in which $\phi$ was still borderline, albeit not definitely borderline.

This shows that it is not possible to express higher-order vagueness in a language that adds only the $D$ operator (as defined) to the usual classical vocabulary. First-order vagueness is accommodated with $D$, but with that alone, according to Fine,

anything that smacks of being a borderline case is treated as a clear borderline case. The meta-languages become precise at some, but no pre-assigned, ordinal level. The only alternative to this is that the set of admissible specifications is
itself intrinsically vague. There would then be a very intimate connection be­
tween vague language and reality: what language meant would be an intrinsically
vague fact.

This is the approach taken by Keefe (2000). Her treatment is not formal, but the idea is to
allow the notion of ‘admissible specification point’ to be vague. For example, suppose $\phi$ is
true in all complete and definitely admissible specification points, but false in a point which
is borderline admissible. Then ‘$\phi$ is true’ is indeterminate because it is not definite that $\phi$
is true in every admissible precisification.

Note that if this move is made, then the system must be modified in order to express
borderline-admissibility, and since the object language makes no mention of specifications
in its vocabulary, the modification must be made in the meta-language, where one might
introduce an operator $D'$ to distinguish the precisifications that are definitely admissible
from those that are borderline admissible. But if the meta-language dictates the truth-
conditions in the object language, what dictates the truth-conditions on admissibility in the
meta-language? There would have to be meta-meta-language, which will itself be vague
if one is to distinguish the precisifications that are definitely-borderline admissible from
those that are borderline-borderline admissible. This requires another operator $D''$, whose
semantics are spelled out in the meta-meta-meta-language, and so on ad infinitum. “[T]he
vague”, Keefe writes, “is not reducible to the non-vague.” (Keefe, 2000, pg. 208).

Whether or not higher-order vagueness is attainable in supervaluations, and whether or
not it is linguistically necessary, remains an open issue. Keefe denies that this approach
produces circularity and insists that the vagueness of a definitely operator (be it $D$, $D'$,
or $D''$, etc.) cannot be expressed using the operator itself, for, as Williamson remarks, ‘it is
like a cloud said to have an exact length because it is exactly as long as itself.’ (Williamson,
1994, pg. 160). Perhaps it is possible to formalize any level of higher-order vagueness,
but it probably suffices to ascend to whatever level is necessary given the requirements of
discourse. With this I end my discussion. For more on higher-order vagueness, see Wright

2.3.1 Subvaluation

A variant that shares the architecture of supervaluationary systems is subvaluation. The
term was coined by Hyde (1997), who took as his starting point Jaśkowski’s (1969) discursive
logic D₂. To Jaśkowski, a proposition is true in discourse iff it is true “in accordance with the opinion of one of the participants in discourse.” (Jaśkowski, 1969, pg. 169, emphasis mine). Hyde uses the supervaluationary structure of specification points, but alters the semantics of vague predicates so that the truth of a formula \( R(a) \) rests on its truth in at least one admissible precisification:

\[
[R(a)]^p = 1 \text{ iff } \exists q \in \mathcal{P} [(p \leq q) \land ([R(a)]^q = 1)]
\]

\[
0 \text{ iff } \exists q \in \mathcal{P} [(p \leq q) \land ([R(a)]^q = 0)].
\]

What results from these semantics is that, if in some complete specification point \( p \), \( V(a) \in V^+(R,p) \) and in some other complete point \( q \), \( V(a) \in V^-(R,q) \), then in \( \mathcal{M} \), both \( R(a) \) and \( \neg R(a) \) hold. But since truth is subtruth, i.e. truth in at least one precisification, the conjunction \( R(a) \land \neg R(a) \) fails.

The subvaluationary method is defended by the proponents of paraconsistent logic, who have long criticized the classical principle of explosion \( \phi, \neg \phi \vdash \psi \). Their systems, such as Jaśkowski’s D₂ and Priest’s LP (1979), are designed to admit explosion-less contradictions. The adaptation of such systems to vagueness produces truth-value gluts in borderline regions.

How is it possible to express higher-order vagueness in subvaluation, and how, in particular, is it possible to express borderlineness? Suppose that we introduce an operator \( \nabla \) such that for any proposition \( \phi \), \( \nabla \phi \) is true iff \( \phi \) is subtrue. Since subtruth is truth in at least one precisification, one can draw the following relation between \( \nabla \) and the supervaluationary \( D \) operator:

\[
\nabla \phi \equiv \neg D \neg \phi
\]

If \( \phi \) is borderline, then we expect \( \phi \) to hold in some precisifications, and \( \neg \phi \) to hold in others. For \( \phi \) to be borderline is thus for \( \nabla \phi \land \nabla \neg \phi \) to be true. We may now investigate the semantics of the operator \( \nabla \). Recall that the supervaluationary definition of validity rested

---

15 The Principle of Explosion is a classical law that permits the inference of any proposition \( \psi \) from a contradiction like \( \phi \land \neg \phi \) (or from an inconsistent set of assumptions \( \{ \phi, \neg \phi \} \) since the conjunction and its conjuncts are interderivable via \( \land \)-introduction and \( \land \)-elimination.) Its proof derives \( \phi \) and \( \neg \phi \) from the premise(s), and permits the inference of \( \phi \lor \psi \) from \( \phi \) and \( \lor \)-introduction. But given \( \phi \lor \psi \) and \( \neg \phi \), Disjunctive Syllogism \( \phi \lor \psi, \neg \phi \vdash \psi \) allows the inference of \( \psi \). Therefore, \( \phi \land \neg \phi \vdash \psi \). In order to prevent their systems from exploding, paraconsistent logicians drop at least one of the deductive rules used in the proof. In subvaluation, \( \land \)-introduction is sacrificed.
on the notion that truth is supertruth. Because truth is supertruth in subvaluation, validity must be, and indeed is, defined as the preservation of supertruth. An argument $\Gamma \vdash_{\text{subv}} \Phi$ is valid just in case whenever the premises are true in some admissible precisification, the conclusions are true in some admissible precisification. At the very start one can see that $\land$-introduction fails, since it is possible for a proposition $\phi$ to be true in some precisification, $\neg \phi$ to also be true in some precisification, but it is impossible that $\phi \land \neg \phi$. So, while in classical logic $\phi, \psi \vdash_{\text{cl}} \phi \land \psi$, in subvaluation $\phi, \psi \not\vdash_{\text{subv}} \phi \land \psi$.

It is interesting to note that the conditional proof does hold in the presence of $\nabla$. We know that whenever $\nabla \phi$ is true, $\phi$ is subtrue, and we know that whenever $\phi$ is subtrue, $\nabla \phi$ is true. I intend to show that the conditional can be introduced in both cases, i.e. that it is possible to infer $\vdash_{\text{subv}} \nabla \phi \supset \phi$ from $\nabla \phi \vdash_{\text{subv}} \phi$, and $\vdash_{\text{subv}} \phi \supset \nabla \phi$ from $\vdash_{\text{subv}} \nabla \phi$.

The only way to falsify the first claim is to find that in every precisification, $\neg (\nabla \phi \supset \phi)$, for otherwise there would be some precisification in which the conditional holds and the conditional would thus be valid. Since the specification points are classical, $\neg (\nabla \phi \supset \phi)$ is equivalent to $\nabla \phi \land \neg \phi$, which can only hold in every precisification if $\phi$ was false in every precisification. But if $\phi$ was false, then $\nabla \phi$ could not be true since there are no specification points in which $\phi$ holds. Therefore, $\neg (\nabla \phi \supset \phi)$ cannot hold in every specification point, so its negation, $\nabla \phi \supset \phi$, is subtrue and therefore valid. The same is true of the conditional $\phi \supset \nabla \phi$, which would be invalidated if it was supertrue that $\neg (\nabla \phi \supset \phi)$, i.e. that $\phi \land \neg \nabla \phi$. But if the first conjunct $\phi$ were true, then surely $\nabla \phi$ would also be true, which makes it impossible for $\neg (\nabla \phi \supset \phi)$ to be supertrue. Therefore, $\phi \supset \nabla \phi$ is valid in subvaluation. The resulting threat from these two laws is the redundancy of the $\nabla$ operator, because $\vdash_{\text{subv}} \phi \leftrightarrow \nabla \phi$. But recall that the introduction of the biconditional in classical logic follows $\land$-introduction, which was already shown to be invalid in subvaluation. Nevertheless, I do think that $\phi \leftrightarrow \nabla \phi$ can be shown to be subtrue, but perhaps without the redundancy as a consequence. In order for it to be valid, the biconditional needs to hold in at least one specification, and I maintain that it does. The reason is that $\phi \supset \nabla \phi$ is not only subtrue, it is supertrue, because the negation, $\phi \land \neg \nabla \phi$, can never be true in any precisification. Since the other conditional, $\nabla \phi \supset \phi$ is true in some (but not necessarily all) precisifications, the biconditional must hold in those same precisifications in which $\nabla \phi \supset \phi$ holds. The biconditional is thus true in some precisifications, therefore subtrue, and therefore $\vdash_{\text{subv}} \phi \leftrightarrow \nabla \phi$. The validity need not, I don’t think, have the consequence of eliminating the contribution of $\nabla$ to subvaluationary semantics, for even though the law holds, it holds only...
in some specification points. In my limited understanding, this shows that, because the biconditional is not universally true, the substitution of \( \nabla \phi \) with \( \phi \) is not always permitted.

Other questions, particularly surrounding the issue of higher-order vagueness, are also, I find, difficult to answer. If admissibility is to be treated as a vague notion, then borderline-admissible precisifications are admissible, and also inadmissible. So if a proposition \( \phi \) is false in every admissible precisification, then without higher-order vagueness, \( \phi \) is false. But if higher-order vagueness is reflected in the vagueness of the metalanguage, and if \( \phi \) is true in some borderline-admissible specification point, then \( \phi \) must be true if borderline-admissible counts as admissible. I leave this issue unattended.

2.3.2 A Dynamic Approach

Barker (2002) proposes a dynamic account of vague predicates that is intended to explain not only the dependence of their interpretation upon context, but also the influence they themselves exert on what is accepted in discourse. He argues that a sentence like “Feynman is tall” can be used descriptively: given a set of acceptable standards of tallness, “Feynman is tall” would be roughly equivalent to saying that, under a certain standard of height relevant to the interlocutors in context, Feynman is of a degree of height that is at least equal to that standard. Alternatively, he continues, the statement could be used to modify the standard of tallness in a context. As an example, he offers this scenario: at a party, a newcomer to the country wonders what counts as tall in this part of the world. A long-time resident points to Feynman and says “well, around here, Feynman is tall.”

According to his analysis, a context \( C \) is a set of candidate worlds \( c_1, c_2, c_3, \text{etc.} \), each of which is associated with a unique variable assignment function \( (g_1, g_2, g_3, \text{etc.}) \). The function \( g \) returns an assignment function for each candidate world, e.g. \( g(c_1) = g_1 \), and for some variable symbol \( x \), if \( x \) refers to a in \( c_1 \), then \( g(c_1)(x) = a \). In addition, and following the ideas of Lewis (1970), each candidate world is associated with its own delineation function. A candidate world’s delineation function takes as its argument a gradable adjective and yields a precise standard for that adjective in that world.\(^{16}\) Suppose, for example, that \( c_1, c_2, c_3 \) are associated with the delineation functions \( d_1, d_2, d_3 \), and that the standard of tallness is 5'11" in \( c_1 \), 6'0" in \( c_2 \), and 6'2" in \( c_3 \). Then \( d_1([\text{tall}]) = 5'11", \ d_2([\text{tall}]) = 6'0" \).

\(^{16}\)Barker follows von Stechow (1984) and Kennedy (2001) and treats gradable adjectives as relations from individuals to degrees.
and $d_3([tall]) = 6'2"$. Barker adds to his system the function $d$, which returns for each candidate world a unique delineation function, (in the same way that $g$ returns for each candidate world an assignment function.) $d(c_1) = d_1$, and $d(c_1)([tall]) = 5'11"$.

Each gradable adjective denotes a binary function that takes a degree $s$ and a constant $a$ as input, and maps the pair $(s, a)$ to a set of candidate worlds in which $a$ possesses the property associated with the adjective to at least degree $s$. The adjective *tall*, for example, denotes a function $tall$, such that for any degree $s$ and for any individual $a$, $tall(s, a)$ is the set of candidate worlds in which $a$ is tall at least to degree $s$. Suppose that we model our semantics of the adjective *tall* based only on the function $tall$. In order to do this, we must provide a value for $s$ whenever we expect an output from the function. But if a specific value for $s$ is available, then, contrary to our intuitions, the adjective *tall* will no longer be gradable, since $s$ has a precise numerical value. Instead, the semantics that Barker develops draws on multiple values of $s$ that come from multiple candidate worlds. In order to extract the degree $s_i$ associated with any given candidate world $c_i$ and the adjective *tall*, we apply the function $d$ to $c_i$ to obtain the delineation function associated with $c_i$, and then apply that delineation function to $[tall]$: $d(c_i)([tall]) = d_i([tall]) = s_i$. Now if we want the set of worlds in which $a$ is tall, we need to abstract over $c$ and require only that, whatever the value of $s$ is in any of these worlds, $s$ be less than $a$'s height.

\[
\{c : c \in tall(d(c)([tall]), a)\}
\]

This is the set of worlds whose standards of tallness are low enough for $a$ to count as tall. The set of candidate worlds in context $C$ that make $a$ tall is

\[
\{c \in C : c \in tall(d(c)([tall]), a)\}
\]

The semantic value of the sentence *Paul is tall*, according to Barker, is the function that takes a context $C$ as input and returns an updated context $C'$ that contains only those candidate worlds in $C$ that allow for *Paul is tall* to be true. Formally

\[
[[Paul is tall]] = \lambda C.\{c \in C : c \in tall(d(c)([tall]), p)\}
\]

---

17Barker assumes that degrees, at least for the less abstract adjectives like *tall*, are isomorphic to real numbers, citing Hamann (1991) and Klein (1991) as sources. He also assumes that any adjective's set of degrees $D$ is partially ordered by the relation $\leq$, and that gradable adjectives are monotonic with respect to that relation, i.e. that if an individual $a$ is tall to degree $s$, and $s' \leq s$, then $a$ is tall to degree $s'$. The monotonicity resembles what Fine calls the "penumbral connections" in his supervaluationary system. I also note that Barker's adoption of real numbers as degrees is similar to the use of fuzzy truth-values under a supervaluationary system, an approach taken in Sanford (1993).
Note that the output of \([Paul\ is\ tall]\) will always be a subset of its input. If \([Paul\ is\ tall]\)(C) = C', then C' \subseteq C. Take as an example a context Cα = \{c_1, c_2, c_3\}, where the standards of tallness are 6'1" in c_1, 6'2" in c_2, and 6'4" in c_3. Recall that these numerical values are obtained by applying the function d as shown below. Recall also that, for any individual a and standard of tallness s, tall(s,a) is the set of cs in which a is tall to degree s:

\[
d(c_1)([\text{tall}]) = d_1([\text{tall}]) = 6'1",\ \text{therefore if a is at least 6'1", then } c_1 \in \text{tall}(6'1", a) \\
d(c_2)([\text{tall}]) = d_2([\text{tall}]) = 6'2",\ \text{therefore if a is at least 6'2", then } c_2 \in \text{tall}(6'2", a) \\
d(c_3)([\text{tall}]) = d_3([\text{tall}]) = 6'4",\ \text{therefore if a is at least 6'4", then } c_3 \in \text{tall}(6'4", a)
\]

Let Paul be 6'5" and Danny be 5'7". In Cα, Paul counts as tall and Danny as not tall. The values of Paul is tall and Danny is tall in Cα are computed as follows:

\[
[Paul\ is\ tall](C_\alpha) = \lambda C.\{c \in C : c \in \text{tall}(d(c)([\text{tall}]), p)\}(C_\alpha) \\
= \{c \in C_\alpha : c \in \text{tall}(d(c)([\text{tall}]), p)\} \\
= \{c \in \{c_1, c_2, c_3\} : c \in \text{tall}(d(c)([\text{tall}]), p)\} \\
= \{c_1, c_2, c_3\}.
\]

\[
[\text{Danny in tall}](C_\alpha) = \lambda C.\{c \in C : c \in \text{tall}(d(c)([\text{tall}]), p)\}(C_\alpha) \\
= \{c \in C_\alpha : c \in \text{tall}(d(c)([\text{tall}]), d)\} \\
= \{c \in \{c_1, c_2, c_3\} : c \in \text{tall}(d(c)([\text{tall}]), d)\} \\
= \emptyset.
\]

Assuming that \(\text{is}\) is semantically vacuous, the semantics of \([_\text{vp} \ is \ tall]\) will be equal to \([_\text{AP} \ tall]\), which is definable by \(\lambda\)-abstracting over the individual to which the predicate applies:

\[
[\text{tall}] = \lambda x \lambda C.\{c \in C : c \in \text{tall}(d(c)([\text{tall}]), x)\}
\]

My understanding of this analysis is that the falsehood of a's tallness is formally represented as there being no cs in C in which it is possible to make a tall. That is, if there is no c \in C such that the standard of tallness in c is low enough for a to count as tall, then \([a\ is\ tall](C) = \emptyset\). If every candidate world c in C is associated with a degree of tallness low enough for a to be tall, then \([a\ is\ tall](C) = C\). Finally, if there is some world c that
is too demanding for \( a \) to count as tall, and another \( c \) that allows \( a \) to count as tall, then \( [a \text{ is tall}](C) \) is a non-empty proper subset of \( C \). Suppose, for example, that Norm is 6'3", then in \( C_\alpha \):

\[
[Norm \text{ is tall}](C_\alpha) = \lambda C. \{ c \in C : c \in \text{tall}(d(c)([\text{tall}]), n) \}(C_\alpha)
\]

\[
= \{ c \in C_\alpha : c \in \text{tall}(d(c)([\text{tall}]), n) \}
\]

\[
= \{ c \in \{ c_1, c_2, c_3 \} : c \in \text{tall}(d(c)([\text{tall}]), n) \}
\]

\[
= \{ c_1, c_2 \}
\subset C_\alpha
\]

To account for higher-order vagueness, Barker proposes a dynamic definition of \( \text{definitely} \). The idea is that, for any gradable adjective \( \alpha \), individual \( a \), and context \( C \), \( [\text{definitely } \alpha](a)(C) \) will be an empty set whenever there is a candidate world \( c \in C \) in which \( a \) is not \( \alpha \). The semantic value of any expression including \( \text{definitely} \) in any context \( C \) is either \( C \) itself or the empty set. The outcome is \( C \) just in case \( a \) is \( \alpha \) in every \( c \in C \), and \( \emptyset \) otherwise. The formal definition is:

\[
[\text{definitely}] = \lambda \alpha \lambda x \lambda C. \{ c \in \alpha(x)(C) : \forall d. (c[d/\alpha] \in C) \rightarrow c[d/\alpha] \in \alpha(x)(C) \}
\]

Before I explain how this definition works, a word needs to be said about the notation. In model-theoretic semantics, a quantified expression in which the quantifier binds a variable \( x \) is computed by modifying the assignment function \( g \) so that \( x \) is given a symbol that matches the semantics of the quantifier. If the truth-conditions of the quantifier in \( \forall x \Psi(x) \) are defined as "true iff for every \( d \) in the domain, \( \Psi(d) \) is true," then the assignment function \( g \), which would otherwise assign a constant symbol to the variable \( x \), must be modified in order to return \( d \) for the now quantified variable \( x \). The formalism \( g[d/x] \) represents the modification of the function \( g \) such that \( d \) be the image of \( x \). Barker uses the same notation to modify candidate worlds. In a given context, every world is associated with a precise standard for every vague predicate \( \alpha \) (which, recall, is \( d(c)(\alpha) \)). A candidate world \( c \) that is assigned a new standard \( d \) for an adjective \( \alpha \) is represented as \( c[d/\alpha] \). The semantic value of \( \text{definitely} \) is the update function that returns the set of worlds in \( \alpha(x)(C) \) that would allow \( x \) to count as \( \alpha \) if their standards were changed to any of the available standards of \( \alpha \) in
context $C$. The semantic value of *Norm is definitely tall* in $C_\alpha$ is calculated below.

\[[\text{Norm is definitely tall}](C_\alpha)\]

\[= \lambda \alpha \lambda x \lambda C. \{c \in \alpha(x)(C) : \forall d. (c[d/\alpha] \in C) \rightarrow c[d/\alpha] \in \alpha(x)(C)\}(\text{[tall]})(n)(C_\alpha)\]

\[= \lambda x \lambda C. \{c \in \text{[tall]}(x)(C) : \forall d. (c[d/[\text{tall}]]) \in C) \rightarrow c[d/[\text{tall}]] \in \text{[tall]}(x)(C)\}(n)(C_\alpha)\]

\[= \lambda C. \{c \in \text{[tall]}(n)(C_\alpha) : \forall d. (c[d/[\text{tall}]] \in C_\alpha) \rightarrow c[d/[\text{tall}]] \in \text{[tall]}(n)(C_\alpha)\}\]

\[= \{c \in \{c_1, c_2\} : \forall d. (c[d/[\text{tall}]] \in C_\alpha) \rightarrow c[d/[\text{tall}]] \in \{c_1, c_2\}\}\]

\[= \emptyset.\]

The substitutions in the first three lines allow the computation of $\text{[tall]}(n)(C_\alpha)$, which is $\{c_1, c_2\}$ as shown above. Since the semantic value of *Norm is definitely tall* is a set of candidate worlds in $\{c_1, c_2\}$, its value must be a subset of $\{c_1, c_2\}$. For each of the worlds in $\text{[tall]}(n)(C_\alpha)$, if there is a way of modifying that world by changing its standard of tallness and obtain as a result a world that belongs to $C_\alpha$, then that result must also be in $\text{[tall]}(n)(C_\alpha)$. Candidate worlds that do not satisfy this requirement do not survive the update and are thus excluded from the semantic value of the expression. The standards in $C_\alpha$ are 6'1", 6'2", and 6'4". Take the world $c_1$ from $\{c_1, c_2\}$ and modify it using any possible standard of tallness. There are only three values of $d$ that make $c_1[d/[\text{tall}]]$ an element of $C_\alpha$: 6'1", which (vacuously) modifies $c_1$ into itself, 6'2", which modifies $c_1$ and produces $c_2$, and 6'4", which produces $c_3$. In order for $c_1$ to survive the update denoted by *Norm is definitely tall*, each of the modifications just listed must produce a candidate world which belongs to the set $\{c_1, c_2\}$, and this is clearly false, since $c_1[6'4"/d] \notin \{c_1, c_2\}$. Therefore, $c_1$ does not survive the update. $c_2$ does not survive it either because $c_2[6'4"/d] \notin \{c_1, c_2\}$. Therefore, none of the elements of $\{c_1, c_2\}$ pass the restriction imposed in the definition of *definitely*, which makes $\text{[[Norm definitely tall]}(C_\alpha) = \emptyset$.

Consider, on the other hand, the semantic value of *Paul is definitely tall*. Recall that, since Paul is tall in every candidate world in $C_\alpha$, the value of $\text{[tall]}(p)(C_\alpha) = C_\alpha$. In this case, the worlds that survive the update denoted by $\text{[[Paul definitely tall]}(C_\alpha) are those in $\text{[tall]}(p)(C_\alpha)$, i.e. $C_\alpha$, that satisfy the following conditional: if modified with any degree,
then if the result is in $C_\alpha$, then the same result is in $C_\alpha$, which is tautologous:

$$[\text{Paul is definitely tall}](C_\alpha)$$

$$= \lambda C. \{ c \in [\text{tall}](p)(C) : \forall d. (c[d/\text{tall}] \in C) \rightarrow c[d/\text{tall}] \in [\text{tall}](p)(C))(C_\alpha)$$

$$= \{ c \in [\text{tall}](p)(C_\alpha) : \forall d. (c[d/\text{tall}] \in C_\alpha) \rightarrow c[d/\text{tall}] \in [\text{tall}](p)(C_\alpha) \}$$

$$= \{ c \in C_\alpha : \forall d. (c[d/\text{tall}] \in C_\alpha) \rightarrow c[d/\text{tall}] \in C_\alpha \}$$

$$= C_\alpha.$$

Now I consider how Barker's semantics may be extended to handle negation. The simplest way of achieving what I think are desirable results - especially considering the apparent similarities between supervaluationary semantics and Barker's dynamics - is to define the semantic value of $\text{not tall}$ for any constant $a$ so that it requires the exclusion of precisely the candidate worlds in which $a$ is tall. If a sharp standard $s$ is fixed, then the set of worlds in which $a$ is tall to at least degree $s$ is $\text{tall}(s,a)$. The set of worlds in which $a$ is not tall to degree $s$ must therefore be the complement $(\text{tall}(s,a))^\sim$.

$$[\text{not tall}] = \lambda x \lambda C. \{ c \in C : c \notin (\text{tall}(d(c)(\text{tall}), x))^\sim \}$$

This definition allows for obtaining just the opposite of the results seen in the above examples: $[\text{Paul is not tall}]$ yields an empty set in $C_\alpha$, $[\text{Danny is not tall}]$ produces $C_\alpha$ itself, and $[\text{Norm is not tall}]$ produces $\{c_3\}$. The negation of any adjective $\alpha$ can then be defined as follows:

$$[\text{not} \alpha] = \lambda x \lambda \alpha C. \{ c \in C : c \notin \alpha(d(c)(\alpha), x) \}$$

To settle for this definition would not be satisfactory, however, since negation is also applicable to definitely tall. Given the strong similarity between the dynamic approach and supervaluations, a semantics for $\text{not}$ must be formulated so that the supervaluationary interpretation is obtained. So, if in supervaluation $\neg \text{tall}(a)$ is false for a super-tall individual $a$, then in the dynamic approach, $[\text{not definitely tall}](a)(C)$ should return $\emptyset$ if $a$ is tall in every element of $C$. If $a$ is tall in some precisifications of a super-model but not in others, then $\neg \text{tall}(a)$ is true. In the dynamic approach, then, if $a$ is tall in a proper subset of $C$, then $[\text{not definitely tall}](a)(C)$ should return $C$.

18This suggests that definitely tall itself is not vague, which is disputable. However, Barker explicitly
tall](a)(C) as follows:

\[
\text{[not definitely tall]} = \lambda x \lambda C. \{ c \in \text{[tall]}(x)(C) : \neg \forall d. (c[d/\text{[tall]}] \in C) \rightarrow c[d/\text{[tall]}] \in \text{[tall]}(x)(C) \} \\
= \lambda x \lambda C. \{ c \in \text{[tall]}(x)(C) : \exists d. (c[d/\text{[tall]}] \in C) \land c[d/\text{[tall]}] \notin \text{[tall]}(x)(C) \}
\]

Take Paul in \( C_\alpha \) : \([\text{tall}](p)(C_\alpha) = \{c_1, c_2, c_3\} \), which is \( C_\alpha \). Of this set, the update function defined above should preserve only the worlds to which the following holds: there is a modification to that world's standard of tallness that makes it a member of \( C_\alpha \) and not a member of \([\text{tall}](p)(C_\alpha)\). But \([\text{tall}](p)(C_\alpha)\) is identical to \( C_\alpha \), so there can't be any such world. Therefore, no worlds survive the update, and the result, as desired, is \( \emptyset \). But this fails with Danny and Norm. Danny is not tall in any of the worlds in \( C_\alpha \) : \([\text{tall}](d)(C_\alpha) = 0\). Because of this, the semantics of \text{not definitely tall} will only return an empty set, since the restrictor clause denotes an empty set to begin with. But this is not the result we want because it seems true that Danny is not definitely tall. The same problem will arise in the case of Norm, who is tall in \( c_1 \) and \( c_2 \), but not in \( c_3 \). The restrictor clause will contain only the worlds that Norm is tall in, and completely discard \( c_3 \). But to say that Norm is not definitely tall is true in \( C_\alpha \), certainly as true as Paul is definitely tall. So we would like \([\text{not definitely tall}](n)(C_\alpha)\) to return \( C_\alpha \).

The bug, I think, resides in the definition of the restrictor clause itself. Barker's analysis does not mention negation. His work on the semantics of \text{definitely} deals only with its positive use, in which case it produces a non-empty result only with individuals \( a \) that the modified adjective \( \alpha \) holds of in every candidate world in context \( C \). In such cases, the restrictor clause, which is the set of candidate worlds in which \( a \) is \( \alpha \), is equal to \( C \) itself, since \( a \) is \( \alpha \) in every element of \( C \), i.e. \( \alpha(a)(C) = C \). Barker's semantics for \text{definitely} should therefore be:

\[
[\text{definitely}] = \lambda a \lambda \alpha \lambda x \lambda C. \{ c \in C : \forall d. (c[d/\alpha] \in C) \rightarrow c[d/\alpha] \in \alpha(x)(C) \}
\]

And so, I revise my semantics of \text{not definitely tall} as follows:

\[
[\text{not definitely tall}] = \lambda x \lambda C. \{ c \in C : \exists d. (c[d/[\text{tall}]] \in C) \land c[d/[\text{tall}]] \notin [\text{tall}](x)(C) \}
\]

Barker's account of \text{not definitely tall} does not mention negation. His work on the semantics of \text{definitely} are not designed to exhibit higher-order vagueness, comparing its use to \text{indisputably}, which he thinks is also sharp given the meaning of phrases like 'indisputably indisputably tall'. His account of second- and third- order vagueness appears in his treatment of the semantics of \text{very} and \text{clearly}, which I cannot get to here.
Now, the restrictor clause always begins with the full context, $C_\alpha$ in what matters to us. Start with that, and keep every world as long as it can be modified in at least one way and still be in $C_\alpha$, but be missing from the set of worlds in which Norm is tall. It turns out that every world in $C_\alpha$ satisfies that requirement, for they can all be given a 6'4" boundary for tallness and turn up in $C_\alpha$ (as $c_3$), but not in $\{c_1,c_2\}$, where Norm is tall. Danny's case is even simpler since the set of worlds in which he is tall is empty. Start with $C_\alpha$ and keep the worlds that can be modified in any way and stay in $C_\alpha$ but not in the empty set. The part in italics is trivial, so the requirement is to keep the worlds from $C_\alpha$ that can be modified and remain members of $C_\alpha$. The result is $C_\alpha$.

It can now be shown that the semantic contribution of not in both not tall and not definitely tall is that the proposition following the restrictor clause in tall and definitely tall is negated. Designing a $\lambda$-abstraction that handles this, i.e., that preserves every other $\lambda$ that follows it and preserves the restrictor clause in the set, but negates the proposition that follows, proved difficult. So I choose the easy way out and propose the following postulate:

$$[[\phi]] = \lambda x_1 \cdots \lambda x_n \{ \Sigma x_1 \cdots x_n : \Psi x_1 \cdots x_n \} \leftrightarrow
[[\text{not } \phi]] = \lambda x_1 \cdots \lambda x_n \{ \Sigma x_1 \cdots x_n : \neg \Psi x_1 \cdots x_n \}$$

How does all this work with definitely not tall? The answer is simple, provided that we make another assumption which I think is intuitive. Negation has so far been defined according to the postulate above, and the only change made to the semantics of definitely was in the description of the restrictor clause in the set. The semantics of definitely not tall should then follow directly from the semantics of definitely operating on the value of not tall:

$$[[\text{definitely not tall}]] = [[\text{definitely}]]([[\text{not tall}]])) = \lambda x \lambda c : \forall d. (c[d/a] \in C \rightarrow c[d/a] \in \alpha(x)(C)) ([[\text{not tall}]])$$

\[ = \lambda x \lambda C. \{ c \in C : \forall d : c[d/[[\text{not tall}]]] \in C \rightarrow c[d/[[\text{not tall}]]] \in [[\text{not tall}]](x)(C) \} \]

This works perfectly well if we accept that not tall can be assigned a delineation in every possible world, where the numerical value is the same as that associated with tall. This does not strike me as particularly demanding; all one needs to add is that the greater-than relation that holds between tall and the tallness cut-off changes direction with the addition of not, viz. if s is the boundary, then all individuals with equal or greater height than s are tall, and all with less height are not tall.
CHAPTER 2. BACKGROUND

2. BACKGROUND

Individual Height \([-\text{tall}]/\text{not definitely tall}\]

<table>
<thead>
<tr>
<th>Individual</th>
<th>Height</th>
<th>([-\text{tall}])</th>
<th>([-\text{D-tall}])</th>
<th>([-\text{D-tall}] \cap [-\text{D-tall}])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Paul</td>
<td>6'5&quot;</td>
<td>\emptyset</td>
<td>(C_\alpha)</td>
<td>\emptyset</td>
</tr>
<tr>
<td>Norm</td>
<td>6'3&quot;</td>
<td>(C_\alpha)</td>
<td>(C_\alpha)</td>
<td>(C_\alpha)</td>
</tr>
<tr>
<td>Danny</td>
<td>5'7&quot;</td>
<td>(C_\alpha)</td>
<td>\emptyset</td>
<td>\emptyset</td>
</tr>
</tbody>
</table>

Table 2.1: Dynamic computation of \(\neg\text{D-tall} \land \neg\text{D-tall}\) with varying heights

Combining these definitions to compute the value of \(\text{not definitely not tall}\) yields

\[
\text{[not definitely not tall]} = \\
\lambda x \lambda C.\{c \in C : \exists d. (c[d/\text{not tall}] \in C) \land c[d/\text{not tall}] \notin [\text{not tall}](x)(C)\}
\]

which, when applied to an individual \(x\) and a context \(C\), returns every element of \(C\) that can be modified and still remain in \(C\) but not in the set of worlds in which \(x\) is not tall. If applied to \(C_\alpha\), the function will return \(C_\alpha\) for Paul, since the set of worlds in which Paul is not tall is empty, which makes the second conjunct trivially true. Norm is not tall in \(C_3\), so there exists a degree that can be used to modify \(C_1\) and \(C_2\) to obtain a world that belongs to \(C_\alpha\) but not to \([\text{not tall}](n)(C_\alpha)\), allowing each of \(C_1\), \(C_2\), and \(C_3\) to survive the update. For Danny, the output is \(\emptyset\) because Danny is \(\text{not tall}\) in every element of \(C_\alpha\), so it is impossible to find a degree modification that produces a world in \(C_\alpha\) but not in \(\{C_1, C_2, C_3\}\).

Based on the semantics developed for \(\text{not definitely tall}\) and \(\text{not definitely not tall}\), I will now show that this system captures the same idea that underlies the equivalence \(I\phi = \neg D\phi \land \neg D\neg\phi\) in supervaluation. To be borderline-\(\phi\) is to be not definitely \(\phi\) and not definitely not \(\phi\) either. The only borderline case of tallness in \(C_\alpha\) is Norm, so to say that Norm is borderline - i.e. that he is not definitely tall and not definitely not tall - is completely true and should therefore return \(C_\alpha\). Since neither Paul nor Danny is borderline, the result should be \(\emptyset\). The values of \(\text{not definitely tall}\) and \(\text{not definitely not tall}\) are shown in Table 2.1. I assume that APs are conjoined via set intersection, i.e. that \([\land] = \lambda x \lambda y. X \cap Y\).

2.4 Truth-value Gaps: Experimental Evidence?

Through a series of experimental studies, Bonini et al. (1999) found statistically significant results that may be taken to promote the truth-value gap view, which, as shown in the previous section, is a prediction of the supervaluationary approach and is also in direct opposition
to the subvaluationists’ claims. Their main objective was to measure the boundaries that their subjects thought were appropriate for ascribing gradient properties like tall, long (for movies), tardy (for appointments), etc. Though they conducted 6 experiments in total, each experiment was a slight variant of a common theme. In measuring the boundaries for tall, for instance, their subjects were given the following instructions and asked the question that follows:\footnote{The instructions and questions were originally written in Italian and translated to English in the cited publication. All participants were undergraduate students in Italian universities, hence the use of the metric system.}

When is it true to say that a man is ‘tall’. Of course, the adjective ‘tall’ is true of very big men and false of very small men. We’re interested in your view of the matter. Please indicate the smallest height that in your opinion makes it true to say that a man is ‘tall’.

It is true to say that a man is ‘tall’ if his height is greater than or equal to \_

\text{centimeters}.

[... ] Please indicate the greatest height that in your opinion makes it false to say that a man is ‘tall’.

It is false to say that a man is ‘tall’ if his height is less than or equal to \_

\text{centimeters}.

Other variations omitted “true” and “false” from either question and elicited responses for

When is a man tall? Of course, very big men are tall and very small men are not tall. We’re interested in your view of the matter. Please indicate the smallest height that in your opinion makes a man tall.

A man is tall if his height is greater than or equal to \_

\text{centimeters}.

A man is not tall if his height is less than or equal to \_

\text{centimeters}.

It would support the truth-value glut approach if the average height below which men were no longer ‘tall’ turns out lower than the height above which men were no longer ‘not tall’. This, if indeed attested in the statistics, would show that people view borderline cases
of height as tall and not tall simultaneously. If, on the other hand, the height below which men were no longer ‘tall’ is found to be significantly higher than that above which men ceased to be ‘not tall’, then that would show that there is a gap between the two values in which men were judged to be neither tall nor not tall. The results in Bonini et al. (1999) were in line with the second scenario. They find that the value of the former, on average, is 178.30 cm, while the average of the latter is 167.22 cm. These were the results from the questionnaire that included the words ‘true’ and ‘false’. The more direct version (seen above) also produced gaps: 181.49 cm for the first value, and 160.48 cm for the second. They argue, however, that the difference in gap-size presents problems for truth-value gap theories. In a truth-gap theory, such as supervaluationism, statements like “‘n cm is tall’ is true” on the one hand, and statements like “n cm is tall” on the other, could be thought to have different truth values, hence leading to differing gap-sizes, or be thought to have identical truth values, in which case the gaps would be the same. Bonini et al. argue that gap theorists should expect greater gaps in the former’s case because “one should answer No to the former question for more values of n than for the latter.” (pg. 389). This I find difficult to understand because answering no to either question suggests that the particular value of n in question is not high enough to make tall(n) superttrue. And so, if supertallness is required to make n qualify as tall, then, as a gap-theorist, I must allow that the two statements be treated equally, i.e. that the truth conditions for both “‘n cm is tall’ is true” and “n cm is tall” be identical. But here, Bonini et al. find another flaw: if the difference between the two statements is rejected, then “n cm is tall” should have the same truth-value as “‘n cm is tall’ is true,” and “n cm is not tall” should have the same truth-value as “n cm is tall’ is false.” So “n cm is tall’ is not false” – the negation of “‘n cm is tall’ is false” – should have the same truth-value as “n cm is not not tall”. Now take a borderline height m. A truth-gap theorist must say that “m cm is tall’ is false”, and therefore that “m cm is not tall”. He must also say that “m cm is not tall” is false” – because m is borderline – and therefore that “m cm is not not tall”. So, for an average height m, the theorist would have to admit that “m is not tall and m is not not tall”, which they claim is contradictory. Of course, this piece of criticism assumes a single type of predicate negation wherein, even on a truth-value gap approach, the negation of a predicate holds just in case the predicate is false, which is certainly true in classical predicate logic but not when truth-value gaps, or non-classical truth values, are present. I offer further commentary on this conclusion after I outline my own experimental findings.
Chapter 3

Experimental Data

3.1 Outline

My goal in this chapter is to motivate (*), not, I must emphasize, as the most prominent of the response patterns in the data, but rather as an experimentally observed challenge to the predictions of supervaluations, subvaluations, and degree theories.

Following a description of the experimental methology (Section 3.2), I begin my investigation of the results by showing the percentages of “classical” answers to the questionnaire (3.3.1). In Section 3.3.2 I highlight some findings that, in contrast to the conclusions of Bonini et al. (1999), promote truth-value gaps approaches, and in Section 3.3.3 I show the statistical correlations that point in the direction of (*).

3.2 The Experiment

The survey used for this study consisted of 20 True/False questions. The participants were presented with a synthesized image of 5 suspects in what looks like a police line-up (see Figure 3.1). The suspects appear to be 5'4", 5'11", 6'6", 5'7", and 6'2", and are shown in the picture in that order\(^1\). The suspects were given the numbers (1-5) as names, which were printed on their faces in the image. These numbers were used to refer to the suspects in the questionnaire.

\(^1\) The suspects were purposely not sorted by height. There were no other restrictions on their order aside from that.
Once the participants were shown the picture, the sheet containing the 20 questions was handed out in hard-copy. The checkboxes next to each question were labeled “True”, “False”, and “Can’t tell”. For each suspect, (#1 for example), there were 4 corresponding questions (4 questions $\times$ 5 suspects = 20 questions. There were no filler questions):\footnote{The survey actually contained an extra question per suspect which said “$x$ is tall and $x$ is not tall”. The number of true responses were analyzed and found to be significantly lower than those for the simpler contradiction “$x$ is tall and not tall”. The initial plan was to see if there was any difference between sentential conjunction and predicative conjunction in contradictory statements, but I chose to exclude these results from the final analysis because the question, I felt, was badly worded; given a referent’s name, e.g. #1, it may be anaphorically unconventional to make a statement such as “#1 is tall and #1 is not tall”, and this may have contributed to the decrease of true responses.}

#1 is tall.

#1 is not tall.

#1 is tall and not tall.

#1 is neither tall nor not tall.

In order to minimize the effect of order on the subjects' responses, each sheet was printed with the questions randomly ordered. This was done in every copy of the survey, so no two copies had the same order of questions. A total of 76 subjects participated.
CHAPTER 3. EXPERIMENTAL DATA

The data collection was done on the Simon Fraser University campus, and all participants were undergraduate Simon Fraser University students. 63.2% were native speakers of English. The overall distribution of English fluency among the participants is shown in Table 3.1.

<table>
<thead>
<tr>
<th>Level</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fluent</td>
<td>77.6%</td>
</tr>
<tr>
<td>Advanced</td>
<td>13.2%</td>
</tr>
<tr>
<td>Intermediate</td>
<td>6.6%</td>
</tr>
<tr>
<td>Beginner</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

Table 3.1: Command of English among Participants

57.9% of the participants were between 20 and 25 years of age, 21.2% were under 20, and 19.7% were over 25.

3.3 Results

The results of this study can be presented in a number of ways, but because I intend to discuss the issues that relate to truth-value gaps and truth-value gluts, I shall focus on the numbers relevant just to that. A prototypical gap-like response, for instance, would include *false* to each of ‘is tall’, ‘is not tall’, and ‘is tall and not tall’, and *true* to ‘neither tall nor not tall’. A glut-like response would include *true* to ‘is tall’, ‘is not tall’, and *false* ‘is tall and not tall’ and ‘is neither tall nor not tall’.\(^3\) Patterns like these will for the most part be considered for each suspect, with special attention to #2, #4, and #5 (heights 5'11", 5'7", and 6'2", respectively). Occasionally, however, I will show percentages of answer patterns across the entire survey and not just specifically to groups of suspects. I will clarify these as they come up, beginning with classical answers.

3.3.1 The Classicists

I define the set of classical responses as the two variants shown in Table 3.2. There were only 3.9% classical answers across the entire questionnaire, which is to say that 3.9% of

\(^3\)To expect ‘neither tall nor not tall’ to be true in truth-gap theories is quite intuitive, but later I shall briefly explain why it must actually be *false* in supervaluations.
our subjects obeyed classicality through and through. Figure 3.2 shows the percentage of classical answers by suspect. (If it is of any help, I suggest that Figure 3.2 be read as follows: 52.6% of the participants answered classically when asked about suspect #1, i.e. 5'4"; 34.2% answered classically with asked about 5'7", etc. This may help with reading the remaining charts in this chapter).

<table>
<thead>
<tr>
<th>Question</th>
<th>Classical 1</th>
<th>Classical 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>X is tall</td>
<td>True</td>
<td>False</td>
</tr>
<tr>
<td>X is not tall</td>
<td>False</td>
<td>True</td>
</tr>
<tr>
<td>X is tall and not tall</td>
<td>False</td>
<td>False</td>
</tr>
<tr>
<td>X is neither tall nor not tall</td>
<td>False</td>
<td>False</td>
</tr>
</tbody>
</table>

Table 3.2: Classical answers

Figure 3.2: The percentages of “Classical” responses plotted against suspect height.

Figure 3.3 shows the percentages, by suspect, of classicality under a less strict definition. A classical response here is simply an answer in which the simple predicates “is tall” and “is not tall” are given different answers: either the first is true and the second is false, or the other way around, regardless of what the answers were to the complex predicates “tall and not tall” and “neither tall nor not tall”. The numbers show, again, that classicality is least respected in the borderline range.

3.3.2 Who is Borderline?

Figures 3.2 and 3.3 suggests that the participants had more difficulty maintaining classicality for suspects #2, #4, and #5, (whose heights are 5'11", 5'7", and 6'2", respectively), the
Figure 3.3: “Almost classical” responses by suspect height.

most difficult case being #2. This suggests that #2 is borderline, as it is in the borderline range that classicality is expected to face the most resistance. Figures 3.4 and 3.5 provide more support to this. In Figure 3.4(a), the percentages for true responses to X is tall are shown to increase with height, starting with 1.3% at 5'4”, reaching the median value of 46.1% at 5'11”, and peaking at 98.7% at 6'6”. Conversely, the percentage of false responses, seen in 3.4(b) begins with a ceiling of 98.7% at 5'4” and drops to 1.3% at 6'6”, passing the median at 5'11” with a value of 44.7%.

Figure 3.4: The Truth and Falsity of X is tall

Figure 3.5(a) shows the percentage of true responses to X is not tall, which also reaches the median at 5'11”, this time at 25.0%, and peaks at 5'4” at 94.7% and drops to 0.0% at 6'6”. The percentage of false responses to X is not tall is shown in 3.5(b): 3.9% at 5'4”, a median of 67.1% at 5'11”, and a maximum of 100.0% at 6'6”.

Now that the borderlinehood of #2 is established, it is interesting to note that denying
a proposition (at least when it comes to vague expressions) is more popular than asserting
the negated counterpart. This is evident if the charts in Figures 3.4 and 3.5 are compared
diagonally: at the top left (Figure 3.4(a)), the number of true responses for X is tall are
seen, and Figure 3.5(b) (bottom right), shows the number of false responses for X is not tall.
Classically, these two expressions are equivalent, i.e., for any individual a and predicate P,
P(a) ↔ ¬¬P(a), so if T(P(a)) and F(P(a)) were to read ‘P(a) is true’ and ‘P(a) is false’,
respectively, then (classically) T(¬¬P(a)) holds just in case F(P(a)) holds and, likewise,
F(¬¬P(a)) holds just in case T(P(a)) holds. Comparing 3.4(a) with 3.5(b), and 3.5(a) with
3.4(b) shows, in every pair, that the percentages for the latter’s responses are higher than
those for the former. The differences are summarized in Table 3.3.

<table>
<thead>
<tr>
<th>Expression/truth</th>
<th>5'4&quot;</th>
<th>5'7&quot;</th>
<th>5'11&quot;</th>
<th>6'2&quot;</th>
<th>6'6&quot;</th>
</tr>
</thead>
<tbody>
<tr>
<td>T(tall(X))</td>
<td>1.3%</td>
<td>5.3%</td>
<td>46.1%</td>
<td>80.3%</td>
<td>98.7%</td>
</tr>
<tr>
<td>F(¬tall(X))</td>
<td>3.9%</td>
<td>17.1%</td>
<td>67.1%</td>
<td>82.9%</td>
<td>100.0%</td>
</tr>
<tr>
<td></td>
<td>2.6%</td>
<td>11.8%</td>
<td>21.0%</td>
<td>2.6%</td>
<td>1.3%</td>
</tr>
<tr>
<td>T(¬¬tall(X))</td>
<td>94.7%</td>
<td>78.9%</td>
<td>25.0%</td>
<td>9.2%</td>
<td>0.0%</td>
</tr>
<tr>
<td>F(tall(X))</td>
<td>98.7%</td>
<td>93.4%</td>
<td>44.7%</td>
<td>10.5%</td>
<td>1.3%</td>
</tr>
<tr>
<td></td>
<td>4.0%</td>
<td>14.5%</td>
<td>19.7%</td>
<td>1.3%</td>
<td>1.3%</td>
</tr>
</tbody>
</table>

Table 3.3: Truth/falsity difference in tall and ¬tall: row after each pair shows difference.
CHAPTER 3. EXPERIMENTAL DATA

These results present a counterargument to the findings of Bonini et al. (1999). Recall that their argument against truth-gap theories rests on the claim that gaps could not account for the difference between the height \( n \) at which \( "n \) is tall" and \( "n \) is tall' is true." They argue that truth-gaps theories must expect either the average value of the former to be lower than the latter, or the two values to be equal, but since they find the average value for the former significantly higher, they abandon the truth-gap approach and instead promote their epistemic hypothesis. But if, for a given height \( n \) that qualifies as borderline-tallness, the falsity of \( "n \) is tall" and \( "n \) is not tall" is agreed upon by more people than the truth of \( "n \) is not tall" and \( "n \) is tall", then the supervaluationary truth-gap approach is vindicated because of the difference it predicts between the assertion of a proposition and the denial of its negation. It may be objected here that my use of the word 'denial' is misplaced, since the available answer in the questionnaire was 'false' and not 'wrong', 'deny', or 'disagree' etc.. This charge would have been evaded if, instead of 'false', the questionnaire had included 'not true' as an answer, which it unfortunately did not. But suppose that a participant was in disagreement with a statement and the only three options (as in this questionnaire) were ‘true’, ‘false’, and ‘can’t tell’, then it is obvious that the participant would have to check ‘false’. So, if it is legitimate to consider ‘false’ as a sign of denial in this case, and I think it is, then the truth-gap approach is supported because it can be denied that an individual of borderline height is tall (or not tall) without asserting that the individual is not tall (or tall); in a gap, neither a predicate nor its negation holds, but both can be denied.\(^4\)

3.3.3 Contradictions and Borderline Cases: Gaps vs. Gluts

Of particular interest are the results related to cases of borderline height in relation to the questions "X is tall and not tall" and "X is neither tall nor not tall". As is shown in Figures 3.6-3.7, the numbers of true responses to each of these states increased when the suspect's height was closer to average, peaking at 44.7% and 53.9%, respectively, for the 5'11" suspect. The number of false responses followed a complementary pattern, decreasing as the heights approached 5'11" and reaching a minimum of 40.8% and 42.1% at that midpoint.

The correlation between the answers to these two questions is shown in Tables 3.4-3.9.

\(^4\)According to a \( \chi^2 \) test for independence, the chance of the difference (between denial and assertion) in the case of #2 being drawn from the same distribution is less than 5%: \( \chi^2(2) = 8.22; p < 0.05. \)
CHAPTER 3. EXPERIMENTAL DATA

Figure 3.6: The Truth and Falsity of "X is tall and not tall"

Figure 3.7: The Truth and Falsity of "X is neither tall nor not tall"
Tables 3.4-3.6 show the distribution of $F$, $C$, and $T$ responses to “$X$ is tall and not tall” when a $T$ response is given to “$X$ is neither tall nor not tall”. The purpose is to show that neither, the truth of which can justify a truth-value gap, coincides in many cases of borderline-height with both, which, when true, suggests a truth-value glut.

Note that the correlation is highest at $5'11"$ (Table 3.4), where 53.7% of those who thought it was true that the suspect was neither tall nor not tall also thought it was true that he was tall and not tall. The correlation slightly lowers (to 50.0%), for $6'2"$ (Table 3.5), and takes a drop to 25.0% at $5'7"$, 20.0% at $6'6"$ and 9.5% at $5'4"$ (tables not shown for the last two).

Similar results are seen in Tables 3.7-3.9, which show the distribution of truth for “$X$ is neither tall nor not tall” when “$X$ is tall and not tall” is true. Table 3.7 shows that 64.7% of those who thought $5'11"$ was tall and not tall also thought that he was neither, and Table 3.8 shows that 63.6% followed the same pattern for $6'2"$. The correlation lowers to 37.5%
at 5'7" (Table 3.9), 45.5% at 5'4", and 25% at 6'6" (tables not shown).

<table>
<thead>
<tr>
<th>Both</th>
<th>neither</th>
<th>% within</th>
<th>% overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
<td>35.5%</td>
<td>15.8%</td>
</tr>
<tr>
<td>T</td>
<td>C</td>
<td>0.0%</td>
<td>0.0%</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>64.7%</td>
<td>28.9%</td>
</tr>
</tbody>
</table>

Table 3.7: Distribution of neither tall nor not tall when both is true. Height = 5'11"

<table>
<thead>
<tr>
<th>Both</th>
<th>neither</th>
<th>% within</th>
<th>% overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
<td>31.8%</td>
<td>9.2%</td>
</tr>
<tr>
<td>T</td>
<td>C</td>
<td>4.5%</td>
<td>1.3%</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>63.6%</td>
<td>18.4%</td>
</tr>
</tbody>
</table>

Table 3.8: Distribution of neither tall nor not tall when both is true. Height = 6'2"

<table>
<thead>
<tr>
<th>Both</th>
<th>neither</th>
<th>% within</th>
<th>% overall</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>F</td>
<td>50.0%</td>
<td>10.5%</td>
</tr>
<tr>
<td>T</td>
<td>C</td>
<td>12.5%</td>
<td>2.6%</td>
</tr>
<tr>
<td>T</td>
<td>T</td>
<td>37.5%</td>
<td>7.9%</td>
</tr>
</tbody>
</table>

Table 3.9: Distribution of neither tall nor not tall when both is true. Height = 5'7"

At this point one may wonder why the correlation, if it bears any theoretical significance, should diminish in non-borderline cases, for if a subject should, for some reason, find it true that a certain height counts as neither tall nor not tall, then shouldn’t there be a (close to) 50% chance of that subject thinking that the same height is tall and not tall, as the statistics for borderline cases suggest? The answer is yes, but I suspect that the samples in such cases are too small for statistical analysis. Only 5.3% of 76 (four people), for example, thought it was true that 6'6" was tall and not tall. To expect two of the four to also think it true that 6'6" is neither tall nor not tall (thus following the borderline pattern) is probably unrealistic, considering the general distractions that may affect the participants.

In any case, the correlation between the truth of neither and the truth of both is clearly present in the borderline range of tall if one considers 5'11" and (possibly) 6'2" to qualify
as borderline. More support for this connexion is found in the correlation between both and the individual predicates tall and not tall. Figure 3.8 shows that 32.4% of those who thought it was true that 5'11" was ‘tall and not tall’ also thought it was false that 5'11" was tall and false that 5'11" was not tall. The ratio decreases to 18.8% at 5'7" and 18.2% at 6'2" and drops to 9.1% at 5'4" and 0.0% at 6'6".

The correlation in the other direction is shown in Figure 3.9, which shows the percentage of true responses to ‘tall and not tall’ when the conjuncts ‘tall’ and ‘not tall’ are false. Note that the ratio is highest at 6'2", where every subject who answered false to both ‘tall’ and ‘not tall’ also answered true to ‘tall and not tall’. The ratio is lower at 5'11" (68.8%), which might come as a surprise since 5'11" seemed more “borderline” than 6'2". But the difference could originate in the overall number of subjects: the 68.8% in the case of 5'11" represents a total of 16 people, while the 100.0% comprises only 4.

![Figure 3.8: The falsity of 'tall' and 'not tall' when 'tall and not tall' is true.](image)

In comparison, the percentage of false responses to ‘tall and not tall’ when ‘tall’ and ‘not tall’ are false amounted to 25.0% at 5'11" and 0.0% at 6'2". The numbers of true/true combinations to ‘tall’ and ‘not tall’ when ‘tall and not tall’ was judged true were 2.9% for 5'11" and 9.1% for 6'2".

### 3.4 Summary

This chapter was intended to show, firstly, that the borderline range of the gradient predicate “tall” fell within a truth-value gap, and, secondly, that there is experimental support for (*), a pattern in which a contradictory conjunction is judged to be true despite the falsity of its individual conjuncts. What may be considered a gap under a framework like supervaluation...
Figure 3.9: The truth of 'tall and not tall' when 'tall' and 'not tall' are false.

can in fact coincide with a glut-like region, and exciting as this discovery may be, I will keep from proposing my very own truth-value "glap" framework in my excited haste and instead attempt to reconcile both truth-gap and truth-glut theories using the Gricean conversational principles. On to Chapter 4.
Chapter 4

Implications

An objector may complain that I am taking the contradiction too literally, i.e. that tall and not tall doesn't really mean tall and not tall; it just means borderline. So, to look for a compositionally logical explanation for (*) is unnecessary and perhaps even gratuitous. The objector may corroborate his charge with the observation that tall on its own is false and ¬tall is also false. But I believe that there are some observations that still point in the direction of compositionality. First, one must dismiss the possibility that (tall ∧ ¬tall) is an idiomatic expression. If these conjunctions were indeed idioms, then we would expect a much larger percentage of true responses for the borderline suspect, since it would take the subjects no effort to interpret the question and at once judge the familiar idiom to be true of men of average height. Second, I do not believe that φ ∧ ¬φ is the type of expression that is commonly used to mean borderline-φ, and I say this simply because it is not something that is heard in casual everyday conversation. Furthermore, even if these contradictory expressions evolved as substitutes for borderline, their evolution itself is enough reason to investigate the rationale that, not only makes such constructions possible, but also produces borderline as a semantic output. This, I find, is strong evidence that these expressions are not treated by speakers as opaque units, but are rather interpreted according to the individual words and the logical connectives that combine them.

How is it then possible for a contradictory conjunction to compositionally be true despite the falsity of both of its conjuncts? In the remainder of this section, I re-evaluate the methods

\footnote{This probably requires further research, but a simple Google search yielded nothing but formal logic results for the search query “tall and not tall”.
}
that were outlined in Chapter 2, and finally propose a way in which they may be applied to our results.

4.1 Multi-valued Logics

Aside from the theoretical problems that were shown to surround fuzzy logic and degree theories of truth, our experimental results cast further doubts on their appropriateness in dealing with vagueness.

At first sight the patterns shown in Chapter 3 can be taken to support the fuzzy approach. For one thing, the percentages of true responses to the simple statements 'is tall' and 'is not tall' were gradient with respect to height, in the former's case varying from 1.3% to 98.7% as height increased, and from 94.7% down to 0.0% in the latter's. Similarly, false responses showed gradual decline for 'is tall' from 98.7% at 5'4" to 1.3% at 6'6", and gradual increase from 3.9% to 100.0% for 'is not tall'. In addition, the percentage of true responses to the contradictory conjunction 'is tall and not tall' reached its maximal value in the middle of height spectrum, at 5'11", where it reached a value of 44.7%. Recall that an expression of the form $\phi \land \neg \phi$ can never be truer than 0.5 in fuzzy logic, which occurs only when $[\phi] = 0.5$, i.e., when $\phi$ is borderline; if $[\phi] = 0.5$, then

$$[\neg \phi] = 1 - [\phi] = 0.5$$

$$[\phi \land \neg \phi] = \min([\phi], [\neg \phi]) = \min(0.5, 0.5) = 0.5.$$  

Similarly, false responses for the same conjunction reached their minimum at 5'11" (40.8%), and increased gradually in either direction. More support comes from the similar patterns for true and false responses for 'neither tall nor not tall'. Assuming that, for any two predicates $\Phi$ and $\Psi$, 'neither $\Phi$ nor $\Psi$' can be logically represented as $\neg(\Phi \lor \Psi)$, then the truth of $\neg(\Phi \lor \neg \Phi)$ reaches its maximum at 0.5, just when its falsity reaches the minimum of 0.5, as is (roughly) seen in Figures 3.6 and 3.7.

Despite these observations, I think it is mistaken to take the correspondences as testimony to the success of degree theories of truth. Firstly, no definition of conjunction
that I am aware of produces a higher truth value that either of its conjuncts. Aside from defining conjunction as the lesser value of the two conjuncts (in the tradition founded by Lukasiewicz (1970), and endorsed in Tye (1994), Zadeh (1975) and Machina (1972, 1976),) Goguen (1969) offers a variant in which $\phi \land \psi$ is the product of the truth-values of $\phi$ and $\psi$, resulting in an even smaller degree of truth for the conjunction. The same is applicable to Edgington’s probabilistic model, in which the value of $\phi \land \psi$ is defined as the verity of $\psi$ – i.e. degree of its closeness to complete truth – given the verity of $\phi$, multiplied by the verity of $\phi$. This, as I mentioned in Section 2.2.1 (pg. 9), guarantees the falsity of contradictions, since the probability of $\phi$ given $\neg \phi$ is 0. The only way to account for (\star) in any of these systems is to drop $\land$-elimination, which is present in all of them. I also think that, in order for the experimental data to support degree theories, truth must be considered collectively rather than individually. The resemblance shown between the statistical patterns and the predictions of fuzzy logic is made significant only if one assumes that degrees of truth can be measured through consensus, without examining the individual combinations of answers between, say, conjunct and conjunction, disjunct and disjunction, etc. as was done in the previous chapter.

4.2 Supervaluation

As explained in Chapter 1, one of the advantages of the supervaluationary method is its preservation of LNC and LEM; the former holds because contradictions are false in every (classical) precisification and therefore superfalsely, and the latter is supertrue and thus true. What follows is that no truth value is assigned to a predicate (or its negation) in borderline cases, making it false that ‘$X$ is tall’ and false that ‘$X$ is not tall’ if $X$ is borderline-tall, since neither claim is true in all precisifications. This, of course, assumes that false responses can signify denial, a claim made and motivated on 41.

The question now is whether supervaluationary theory allows the truth of ‘neither tall nor not tall’. Intuitively this is expected because a proposition that lack either of the values true and false must be neither true nor false, but this does not guarantee that, for a predicate $\Phi$ and individual $a$, $a$ can be neither $\Phi$ nor not $\Phi$. If ‘neither tall nor not tall’ is to be formally translated as $\neg(tall \lor \neg$-tall), then for any individual, whether borderline-tall or not, the negated disjunction must be superfalsely, since LEM is supertrue. The falsity of ‘is tall and not tall’ must also hold in supervaluations because of LNC. But it was shown in
Figure 3.9 that 68.8% of those who thought ‘5’11” is tall’ and ‘5’11” is not tall’ were false also thought ‘5’11” is tall and not tall’ was true.

And so, truth as supertruth does predict the falsity of both ‘is tall’ and ‘is not tall’ in borderline cases, but never the simultaneous truth of ‘is tall and not tall’ and ‘is neither tall nor not tall’.

4.3 Subvaluation

The predictions of subvaluation find very little experimental evidence also. The predicates ‘is tall’ and ‘is not tall’ must simultaneously be true, and ‘is tall and not tall’ and ‘is neither tall nor not tall’ must both false, in order for the theory to find support. But only 3.9% of the participants thought it was true that 5’11” was ‘tall’ and ‘not tall’, and 5.3% had the same opinion of the suspect standing 6’2”. Of the first group, one third thought it was true that the suspect was ‘tall and not tall’ and another third thought it was false.

4.4 Interlude

Let us now pause and think about how these theories can be used to define the semantic values of gradable adjectives. Suppose that the model $\mathcal{M}$ consists of the quadruple $\langle \mathcal{U}, \mathcal{P}, \leq, V \rangle$, where $\mathcal{U}$ is the domain of discourse, $\mathcal{P}$ the set of specification points, $\leq$ the ordering relation between members of $\mathcal{P}$, and $V$ the evaluation function. Then $\text{tall}$ is defined in supervaluation as

$$[[\lambda p \ a \text{ is tall}]]_{sp} = \{ x : \forall p \in \mathcal{P} [x \in V^+(\text{tall}, p)] \}$$

and the subvaluationary equivalent, assuming that the same structure underlies the framework, i.e. that $\mathcal{M} = \langle \mathcal{U}, \mathcal{P}, \leq, V \rangle$, is spelled out as

$$[[\lambda p \ a \text{ is tall}]]_{sb} = \{ x : \exists p \in \mathcal{P} [x \in V^+(\text{tall}, p)] \}$$

which, when used sententially, yield

$$[[s \ a \text{ is tall}]]_{sp} = 1 \iff V(a) \in \{ x : \forall p \in \mathcal{P} [x \in V^+(\text{tall}, p)] \}$$

and, mutatis mutandis,

$$[[s \ a \text{ is tall}]]_{sb} = 1 \iff V(a) \in \{ x : \exists p \in \mathcal{P} [x \in V^+(\text{tall}, p)] \}$$
The derivations become complicated with the addition of not. Negation finds many different definitions not only among differing logical systems, but also in the syntax/semantics of natural language. A thorough discussion of the differences is found in Horn (1989) and Wansing (2001). A central difference in this discussion is that between predicate negation and predicate-term negation, which I will refer to as weak and strong negation, respectively. The weak negation of a predicate assignment \( \varphi(a) \) (which I will write as \( \sim \varphi(a) \)) holds just in case \( \varphi(a) \) is not true, while the strong negation of \( \varphi(a) \), i.e. \( \neg \varphi(a) \), holds just in case \( \varphi(a) \) is false. Classically, the two formulas are equivalent, since \( \varphi(a) \) is false just in case \( \varphi(a) \) is not true. But in a 3-valued logic, for example, \( \neg \varphi(a) \) would hold iff \( [\varphi(a)] = 0 \), while \( \sim \varphi(a) \) would hold iff \( [\varphi(a)] = 0 \) or \( [\varphi(a)] = I \), where \( I \) is an intermediate truth-value.²

So, on a supervaluationary interpretation of a vague predicate \( \varphi \)

\[
[\sim \varphi(a)]_{sp} = 1 \text{ iff } \forall p \in \mathcal{P} [V(a) \in V^+(\varphi, p)] \\
= 1 \text{ iff } \exists p \in \mathcal{P} [V(a) \notin V^+(\varphi, p)] \\
\therefore [\sim \varphi(a)]_{sp} = 1 \text{ iff } D\varphi(a)
\]

where \( D \) stands for the definitely operator. The strong negation \( \neg \varphi(a) \) is derived through the usual semantics of supertruth/superfalsity, i.e. that

\[
[-\varphi(a)]_{sp} = 1 \text{ iff } \forall p \in \mathcal{P} [V(a) \notin V^+(\varphi, p)] \\
\therefore [-\varphi(a)]_{sp} = 1 \text{ iff } D\neg \varphi(a)
\]

To define weak negation in subvaluationary semantics one must recall that truth is truth in at least one admissible precisification, and falsity likewise. So, if the negation of a proposition held just in case the proposition itself did not, then for a predicate \( \varphi \)

\[
[\sim \varphi(a)]_{sb} = 1 \text{ iff } \exists p \in \mathcal{P} [V(a) \in V^+(\varphi, p)] \\
= 1 \text{ iff } \forall p \in \mathcal{P} [V(a) \notin V^+(\varphi, p)] \\
\therefore [\sim \varphi(a)]_{sb} = 1 \text{ iff } D\neg \varphi(a)
\]

²In fear of causing any confusion, I want to mention that I do not include sentential negation among either of the negation types I discuss here. Both my strong and weak negations, \( \sim \) and \( \neg \), can be prefixed to a predicate \( \varphi \) and assigned a semantic value accordingly: \( [\sim \varphi] \) is the complement of \( [\varphi] \), i.e. the set of individuals of whom \( \varphi \) is false and borderline; and \( [\neg \varphi] \) is the antiextension of \( \varphi \), the set of individuals of whom \( \varphi \) is false.
and the strong negation $\neg\Phi(a)$ holds just in case it sub-holds, i.e.

$$[[\neg\Phi(a)]]_s^T = 1 \text{ iff } \exists p \in \mathcal{P} [V(a) \notin V^+(\Phi, p)]$$

$$\therefore [[\neg\Phi(a)]]_s^T = 1 \text{ iff } \neg D\Phi(a)$$

The question here is which of these two interpretations, assuming either a super- or a sub-valuationary framework, corresponds to negation in natural language. Negated propositions in supervaluation, for example, are typically considered true iff their unnegated equivalents were superfalse, but even if one assumes a supervaluationary semantics of vague expressions, one still finds cases that require the weak interpretation of the negative. Consider a scenario in which Alice and Jim are persuading their friend Maggie to go out with Norm. Having never met Norm, Maggie inquires about his height. Alice says "he is not tall" in response, but Jim objects - in defense of his friend's physical stature - and says "well, he's not not tall. He's average." Here the two nots are not interpreted the same way, whether strongly or weakly. If they were both strong, assuming supervaluation, then 'not tall' would hold iff Norm's tallness was superfalse, or if his not-tallness was supertrue. So 'not (strong-)not tall' holds just in case Norm's not-tallness was superfalse, i.e. if his tallness was supertrue. The double weak negation also cancels itself trivially, for if 'not tall' held just in case 'tall' didn't, then Norm's not-tallness holds just in case he was not super-tall. His not-not-tallness would hold just in case his not-tallness did not, but his not-tallness holds just in case he was supertall. So the two negations serve no purpose in discourse if they are both interpreted as strong or weak. The only way for 'not not tall' to be informative is for the two negative particles to be interpreted differently; once weakly and the other strongly. In Norm's case, 'not not tall' can be logically represented as $\sim \neg \text{tall}$ or as $\neg \sim \text{tall}$.

$$[[\neg \text{tall}(n)]]_{sp} = 1 \text{ iff } D\neg \text{tall}(n)$$

$$\therefore [[\sim \neg \text{tall}(n)]]_{sp} = 1 \text{ iff } \neg D[\neg \text{tall}(n)]$$

$$=[[\sim \text{tall}(n)]]_{sp} = 1 \text{ iff } \neg D\text{tall}(n)$$

$$\therefore [[\sim \sim \text{tall}(n)]]_{sp} = 1 \text{ iff } D[\neg D\text{tall}(n)]$$

$$=[[\neg \text{tall}(n)]]_{sp} = 1 \text{ iff } D\text{tall}(n)$$
So $\sim \sim \text{tall}(n)$ is also trivial since it only produces the “classically” double-negated predicate, but $\sim \sim \text{tall}(n)$ is compatible with both $D\text{tall}(n)$ and, more importantly, with $I\text{tall}(n)$, i.e. $D\sim \sim \text{tall}(n) \land \sim D\text{tall}(n)$.

If the distinction between the two negations is indeed warranted, as I claim, then it must be the case that a sentence containing a singly-negated predicate is ambiguous. ‘Norm is not tall’ can either mean $\sim \text{tall}(n)$ or $\sim \sim \text{tall}(n)$, each of which is subject to the different truth-conditions shown above. But note that weak negation in supervaluationary semantics is logically equivalent to strong negation in subvaluationary semantics, and strong supervaluationary negation is equivalent to strong negation in subvaluation:

$$\left[\sim \Phi(x)\right]_{sp} = \left[\sim \Phi(x)\right]_{sb}$$
$$\left[\sim \Phi(x)\right]_{sp} = \left[\sim \Phi(x)\right]_{sb}$$

which means that the truth conditions associated with negation in either framework can be obtained with the other, bringing their strong similarities into focus.

### 4.5 Combining Sub- and Super-valuationism

On their own, neither supervaluation nor subvaluation can be used to explain the results in (*), but I propose that a combination of the two systems is workable. Earlier I discussed the ambiguity of negated propositions in both frameworks. What I suggest is that this ambiguity arises not only in negated expressions, but in any expression containing a vague predicate. If in $\Phi(x)$ $\Phi$ is gradient, then the expression’s semantic value is underspecified for definiteness: $\Phi(x)$ is true either when it is supertrue, or when it is subtrue. This proposal, I believe, is supported by the lack of an overt linguistic equivalent of the operators $D$ and $V$ in natural language sentences like ‘Norm is tall’. By ‘is tall’ our intention can be represented either as $D\text{tall}(x)$ or as $V\text{tall}(x)$. Of the two interpretations, the former seems to me the most compliant with Grice’s maxims, most obviously the first sub-maxim of QUANTITY:

“make your contribution as informative as is required (for the current purposes of the exchange).”

³A more detailed account of the similarities of supervaluation and subvaluation is provided in Akiba (1999).
If it is assumed by the interlocutor(s) - our experiment subjects in this case - that this principle is observed, then it is expected that by ‘tall’ we meant to convey the most informative reading possible, which to the hearer must correspond to that definition of ‘tall’ that s/he thinks all (or most) people would agree upon and, also, that s/he assumes that I, the speaker, think all (or most) people would agree upon (context and other distorting factors being fixed.) The closest match to this description is the super-interpretation, i.e. that ‘is (not) tall’ is read as $D(-)tall$. So, when the question arises as to whether a person standing 5’11” is tall (or not tall,) the addressee - who may reasonably be expected to comply with the Gricean principles - is very likely to say ‘false’. When the claim ‘is tall and not tall’ is made regarding the same suspect, only the sub-interpretation can be used informatively because the statement would otherwise be contradictory; the conjunction of $[AP\ tall]$ and $[AP\ not\ tall]$, on a supervaluationary account, will produce an empty set if conjunction is represented as set intersection in the semantics:

\[
[[AP\ tall]]_{sp} = \{x : \forall p \in \mathcal{P} [x \in V+(tall, p)]\}
\]
\[
[[AP\ not\ tall]]_{sp} = \{x : \forall p \in \mathcal{P} [x \notin V+(tall, p)]\}
\]
\[
.\ ... \ [[AP\ tall\ and\ not\ tall]]_{sp} = \{x : \forall p \in \mathcal{P} [x \in V+(tall, p)]\} \cap \{x : \forall p \in \mathcal{P} [x \notin V+(tall, p)]\}
= \emptyset
\]

The sub-interpretation, on the other hand, will produce just the set of individuals that are borderline, i.e. the ones of whom the predicate $Itall$ (i.e. $\neg Dtall \land \neg D\neg tall$) holds:

\[
[[AP\ tall]]_{sb} = \{x : \exists p \in \mathcal{P} [x \in V+(tall, p)]\}
\]
\[
[[AP\ not\ tall]]_{sb} = \{x : \exists p \in \mathcal{P} [x \notin V+(tall, p)]\}
\]
\[
.\ ... \ [[AP\ tall\ and\ not\ tall]]_{sb} = \{x : \exists p \in \mathcal{P} [x \in V+(tall, p)]\} \cap \{x : \exists p \in \mathcal{P} [x \notin V+(tall, p)]\}
\neq \emptyset
\]

The (trivial) falsity of the super-interpretation of the phrase ‘tall and not tall’ is obviously in violation of Grice’s maxim of QUALITY, and so the sub-interpretation is then invoked in order for the statement to be informative.\textsuperscript{4}

\textsuperscript{4}Kamp and Partee (1995) briefly discuss the possibility of using QUALITY to reassess the meaning of contradictory statements involving vague predicates.
CHAPTER 4. IMPLICATIONS

A point that could be made here is that this solution is not really a combination of supervaluation and subvaluation at all, but rather a promotion of the former over the latter. I am, after all, using super-tallness as the desired interpretation in the case of 'tall' and of 'not tall', and I am using the falsity of 'tall and not tall' to flout quality and consequently reinterpret the conjunction. But note that, if the sub-interpretation was not available to begin with, then there is no guarantee for the conjunction to be made meaningful. It is the very availability of the subvaluationary alternative that not only makes it possible to interpret the otherwise contradictory expression, but also assigns it the reading that corresponds to 'borderline'.

So, if the definite interpretation arises from pragmatic, rather than semantic, factors, then one expects the use of gradable adjectives to lend itself to some of the same conversational principles that govern, for instance, the use of modals and quantifiers, i.e. to what has become known as scalar implicature. Consider the following examples:

(1) You should help Bobo with his marital problems.

(2) Many/some of our students work within the department.

The use of the “should” in (1) implicates “not must”, and “many/some” in (2) implicates “not every”. But neither does “should” entail “not must” nor does “many/some” entail “not every”. The implicatures associated with “should” and “many/some” can in fact be cancelled, as seen in (1’) and (2’).

(1’) You should help Bobo with his marital problems. In fact, you must help Bobo with his marital problems.

(2’) Many/some of our students work within the department. In fact, all our students do.

Note also that placing the stronger statement first results in ungrammaticality:

(1") You must help Bobo with his marital problems. #In fact, you should help Bobo with his marital problems.

---

One may wonder, here, whether it is ever possible to utter a falsehood. Is it always possible to repair a blatantly contradictory expression and make it meaningful? Obviously not. Take a non-gradient property like prime. A number can never be prime and not prime, and there is very little that quality can do to save the utterer of such an expression from embarrassment. Similarly – and with more relevance to the present discussion – can Shaquille O’Neal be considered “tall and not tall”? Not according to my analysis, because the conjunction is meaningful only if the sub-interpretation is used on both “tall” and “not tall”, and in that case the non-empty intersection of the two sets coincides with the borderline range of height, far, far from where Shaq stands. In this case a falsehood is uttered, but not without meaningfulness.
(2") All our students work within the department. #In fact, many/some of our students do.

(1") and (2") fail pragmatically because the first sentence in each utterance logically entails the second, i.e. $\forall x P x \equiv \exists x P x$ and $\Box \phi \equiv \Diamond \phi$.\(^6\)

The prediction, then, is that it is possible to cancel the implicature associated with an expression whose semantic value is equivalent (roughly) to $\forall \text{tall}$ (the sub-interpretation) and instead communicate $\text{Dtall}$ (the super-interpretation), and not the other way around, which is borne out (I cannot help but add (4), from the well-known cheese shop scene in Monty Python’s Flying Circus):

(3) Bobo is kinda tall. In fact, he’s quite tall.

(3') Bobo is quite tall. #In fact, he’s kinda tall.

(4) Palin (not wanting to sell the Camembert to Cleese): It’s a bit runny, sir.

Cleese: Oh, I like it runny!

Palin: Well, as a matter of fact, it’s very runny sir.

An alternative account of (*) could be given using an adaptation of the Strongest Meaning Hypothesis (SMH) (Dalrymple et al., 1998). Under SMH, a reciprocal sentence $S$ is felicitous in a context $c$ that supplies non-linguistic information $I$ iff the set $\mathcal{S}_c$ has a member that entails every other member, where $\mathcal{S}_c$ is the set of propositions that are consistent with $I$ and are obtained by interpreting the reciprocal in one of the ways provided in their taxonomy. The taxonomy consists of 6 quantified expressions that range from a universal-like strong form down to a semi-existential form. Given a felicitous usage of $S$ as dictated above, “the use of $S$ in $c$ expresses the logically strongest proposition in $\mathcal{S}_c$.” The adaptation I am offering may seem simplistic, but will probably do for the present case. In order for SMH to account for (*), the definite interpretation of a sentence like “a is tall” would need to be included in the set $\mathcal{S}_c$ that corresponds to that sentence. This is because the definite reading, $\text{Dtall}$, is the stronger of the two possible readings, $\text{Dtall}$ and $\forall \text{tall}$ – both of which are consistent with the non-linguistic information provided in $I$ – and it is the definite

---

\(^6\)The semantics of denotic modals involve more than simply prefixing the operators $\Box$ and $\Diamond$, but the point relevant here is made without a detailed treatment of such semantics.
CHAPTER 4. IMPLICATIONS

reading that we expect to emerge from uttering "a is tall" and "a is not tall". SMH would predict exactly what we observe in these two cases, because the definite reading is the one that entails every other member of \( \mathcal{R} \). SMH would also predict the truth of "tall and not tall" given the definite and the indefinite readings as options of interpreting the predicates "tall" and "not tall". The set \( \mathcal{R} \) that would be associated with the contradictory statement should include the propositions obtainable from interpreting the adjectives as either definite or indefinite, provided that these resulting propositions are consistent with \( I \) and \( S \). The proposition that would result from using the definite readings in the contradiction would be inconsistent with any non-linguistic piece of information, because it is contradictory. This would consequently disqualify \( Dtall \) from belonging to \( \mathcal{R} \). Using the indefinite interpretation, on the other hand, could be consistent with \( I \) because its truth is contingent. I therefore conclude that SMH could be extended to account for (\(*\)), but I feel that a more detailed treatment is needed to verify this.

In any case, I conclude – at least based on the support I offered for the Gricean account – that a possible resolution to the rivalry between the super- and sub-valuationary approaches to vagueness can come to a compromise in which both interpretations are semantically available, and in which their emergence is pragmatically governed.

4.6 Consequences

4.6.1 Comparatives

An advantage of the fuzzy approach to vague adjectives is its simple solution to comparative constructions. Because truth is assigned from values ranging from 0 to 1, then an individual \( a \) is more \( \Phi \) than \( b \) iff \([\Phi(a)] > [\Phi(b)]\). Of course, this raises problems with the assignment of truth values because no expression \( \Phi(a) \) can be completely true – i.e. be such that \([\Phi(a)] = 1\) – if it were conceivable that another individual \( b \) was more \( \Phi \) than \( a \), for in that case, \([\Phi(b)] \neq [\Phi(a)]\) because \([\Phi(b)] = [\Phi(a)] = 1\).

A similar problem can come up in a super/subvaluationary treatment of vague expressions if the semantics of comparatives were formulated such that, for any predicate \( \Phi \) and two individuals \( a \) and \( b \)

\[
\text{'}a\text{' is } \Phi\text{-er than } b' = 1 \text{ iff } \{p \in \mathcal{P} : V(b) \in V^+(\Phi, p)\} \subset \{p \in \mathcal{P} : V(a) \in V^+(\Phi, p)\}
\]

i.e. ‘a is \( \Phi \)-er than \( b \)’ iff the set of precisifications in which \( b \) is \( \Phi \) is a proper subset of the set
CHAPTER 4. IMPLICATIONS

of precisifications in which \(a\) is \(\Phi\). This formulation is presented in Kamp (1975). Its flaw is noted in Klein (1980), who points out that if a six-niner, say, were admitted into every admissible way of making the predicate tall precise, then a seven-footer must be admitted into every one of those same precisifications also. So then neither set of precisifications is a proper superset of the other, making neither \(b\) nor \(a\) taller than the other.

This approach, however, assumes a semantics of comparatives that is built on top of \textquoteleft parasitic on\textquoteright, in the words of Keefe (2000) – a semantics for positives. Morphologically, the approach is tenable, since positive (bare) adjectives are simpler than the comparative (inflected/modified) forms. But there is no reason to assume that comparatives, and therefore their semantics, are conceptually more complex than positive forms. Sapir (1944), for instance, suggests that knowledge of quantifiable concepts presupposes knowledge of grading, in the comparative sense. A semantics of bare positives, he argues, requires knowledge of the relevant comparison class and subsequent application of comparative semantic analysis on that class, implying that the type of reasoning associated with comparison is required for using bare positives. The idea was adopted and applied (in varying forms) in Cresswell (1976), Klein (1980), Kamp and Partee (1995), and Kennedy (2007), among many others. So, if there is any conceptual motivation for treating comparatives as semantically simpler than positives, then an approach which includes reference to specification spaces (or collections of precisifications) in the evaluation of comparative constructions may be replaced with an approach in which reference is made to a single precisification. It is assumed in super- and sub-valuationary frameworks that every specification point is associated with a sharp divider that separates the positive extension (of the predicate in question) from the negative extension, and even without an entirely clear explanation regarding the nature of these dividers, I feel that a framework that relies on multiple possibilities of sharpening a vague predicate can be utilized to sharpen the predicate and use the sharpening for comparatives. What I have in mind is some sort of accessibility that permits the assignment of a divider at random within a hypothetical precisification, and then searching through these assignments to determine if it is possible to include an individual in the positive extension of the predicate and simultaneously exclude the other. \(\textquoteleft a\) is \(\Phi\)-er than \(b\) \textquoteleft iff it is possible to assign a sharp divider \(d\) to a complete specification point \(p\) such that \(\Phi(a)\) holds and \(\Phi(b)\) does not. Let \(p_d\) represent a complete specification point \(p\) whose divider is updated to \(d\),
then

\[ 'a \text{ is } \Phi\text{-er than } b' = 1 \text{ iff } \exists d | \forall p \in \mathcal{P} \left[ V(a) \in V^+(\Phi, p_d) \land V(b) \notin V^+(\Phi, p_d) \right] \]

This proposal not only evades the problem that Klein (1980) finds in the use of subsets to compute the semantics of comparatives, but also matches the claim that comparatives are semantically simpler than plain positives. Barker (2002) follows a similar route in his approach to the problem, to which I turn in the next subsection.

### 4.6.2 Consequences for Barker's Dynamic Approach

Barker offers the following as a semantic value for the comparative morpheme:

\[ [-\text{er}] = \lambda \alpha \lambda x \lambda y \lambda C. \{ c \in C : \exists d. (c[d/\alpha] \in \alpha(y)) \land (\neg c[d/\alpha] \in \alpha(x)) \} \]

which, given an adjective \( \alpha \), a pair of individuals \( x \) and \( y \), and a context \( C \), returns the set of worlds in \( C \) which can be modified according to some degree \( d \) and admit \( y \) into the extension of \( \alpha \) but not \( x \). A strong advantage of this solution is that it agrees with the morphology and builds the semantics of comparatives from the semantics of positive adjectives in combination with the comparative morpheme, without running into the problem alluded to in Klein (1980). But the machinery of Barker's system operates on degrees not only for comparative constructions, but also for uninflected positive forms (see Section 2.3.2).

Whether or not it is problematic for Barker to rely on degrees is not my main concern. I am rather interested in applying his system to handle the pattern seen in (\(*\)). The solution I proposed in Section 4.5 was a pragmatically driven combination of both super- and subvaluationary semantics. Barker claims that "deep connections" underlie supervaluations and his dynamic approach, adding that "supervaluations approximate a dynamic semantics." The parallels between the two techniques are certainly evident, since both are built on possibilities of precisifying gradable adjectives and abstracting over these sharpenings. But because Barker's approach makes no mention of truth values, there is no room for implementing a subvaluationary alternative in the semantics of unmodified gradable adjectives, and without such an implementation it would not be possible to make the sub-interpretation available and hence account for the pattern wherein both \('x \text{ is } \alpha'\) and \('x \text{ is not } \alpha'\) are false, and \('x \text{ is } \alpha \text{ and not } \alpha'\) is true. On the dynamic approach, any statement of the form \('x \text{ is } \alpha \text{ and not } \alpha'\) is necessarily assigned the empty set as value, since the phrase consisting of the positive adjective \( \alpha' \) will be the complement of its negated sister \( \neg \alpha' \).
CHAPTER 4. IMPLICATIONS

Since my experimental findings suggest that a vague adjective $\alpha$ can be read as $D\alpha$ and $\neg\alpha$, I want to propose a dynamic translation of the sub-reading, i.e. the indefinite interpretation as an addition to Barker's semantics. The definite reading is formalized in Barker as

$$[D\alpha] = \lambda x \lambda C. \{ c \in C : \forall d. (c[d/\alpha] \in C) \rightarrow c[d/\alpha] \in \alpha(x)(C) \}$$

The indefinite reading, or the sub-reading, was shown on pg. 22 to be logically equivalent to $\neg D\neg \alpha$, which was extended to Barker's system on pg. 32 as

$$[\neg \alpha] = [\text{not definitely not } \alpha]$$
$$= \lambda x \lambda C. \{ c \in C : \exists d. (c[d/\neg \alpha] \in C) \land c[d/\neg \alpha] \notin [\neg \alpha](x)(C) \}$$
$$= \lambda x \lambda C. \{ c \in C : \exists d. (c[d/\alpha] \in C) \land c[d/\alpha] \notin [\alpha](x)(C) \}$$

The dynamic approach is advantageous because it allows for context update in situations where the standard associated with a gradable adjective is unknown (recall the example where "Paul is tall" is used to inform a listener of what counts as tall in a context that he is not familiar with). But supposing that the context is fixed, then it is plausible that by saying "Paul is tall" one means that Paul is definitely tall, that is, that Paul is tall in every possible way of making the adjective tall precise, as governed by informativeness (see Section 4.5). But it is also possible to convey the indefinite reading meaningfully in the contradiction tall and not tall, which requires the availability of the value assigned to $[\neg tall]$ above.

4.6.3 Multi-dimensional adjectives

I have so far applied my argument - and those of the super- and subvaluationists - to adjectives that vary along a uni-dimensional scale, e.g. 'tall', 'bald', etc. But it is obvious that many, if not most, adjectives are scalar along multiple dimensions. 'Big', for example, may be considered a combination of height and weight, for describing people, height, length, and width, for describing objects, etc.\(^7\) Goguen (1969), who takes a fuzzy approach to vagueness, places the components of multi-dimensional adjectives in an ordered sequence and assigns truth values according to a combination of component values.

\(^7\)Things are obviously not so simple for more "natural" adjectives like intelligent, religious, etc., though I do think that these can be divided into components also, e.g. witty, knowledgeable, etc. for intelligent.
A super-/subvaluationary adaptation of this idea would involve, as an initial step, assigning each n-dimensional adjective \( \alpha \) a conjunction of its sub-components as semantic value:

\[
[\alpha] = [\alpha_1] \wedge [\alpha_2] \wedge \cdots \wedge [\alpha_n],
\]

where each \( \alpha_i \) is substituted for its own sub-components and computed accordingly, or, if \( \alpha_i \) is uni-dimensional, its semantic value is assigned according to super-/subvaluations. But this predicts that, for a multi-dimensional predicate \( \Phi \) and an individual \( a \), \([\Phi(a)]\) is false given the definite falsity of any one of the conjuncts \( \alpha_i \). A cuboid, for example, would count as definitely not big if it is definitely not tall, or not high, even if it was definitely long and definitely wide, and I don’t think this prediction is correct. The distribution of components needs to be tweaked, in this case, so that the assignment of a composite predicate to an individual depends on some sort of a majority count over the individual components. In addition, the approach would need to capture the difference in the weight that may be assigned, or thought to be assigned, to each of the components of an adjective. So, for example, to be “big” for a person might have more to do with weight than height.\(^8\)

A step towards solving these issues is to join the groups of specification points associated with each \( \alpha_i \) in \( \alpha_1 \cdots \alpha_n \) together, i.e. \( \mathcal{P}_1 \cup \mathcal{P}_2 \cdots \cup \mathcal{P}_n \), so that, for any multi-dimensional predicate \( \Phi \) composed of \( \alpha_1 \cdots \alpha_n \)

\[
[\Phi(a)]^M = 1 \iff \forall p \in \bigcup(\mathcal{P}_1 \cdots \mathcal{P}_n) | V(a) \in V^+(\Phi)].
\]

But this is obviously problematic because the precisifications associated with each individual \( \alpha_i \) do not delineate the (anti)extensions for \( \Phi \) itself. Unlike the conjunctive approach shown above, however, this formulation would assign no truth value to the statement ‘this cuboid is big’ if it was definitely long, definitely wide, but definitely not high, because in some precisifications in \( \bigcup(\mathcal{P}_1 \cdots \mathcal{P}_n) \), the cuboid belongs in the positive extension of \text{big}, but in others it does not. The prediction according to the pragmatic argument I presented is that ‘the cuboid is big’ implicates that the cuboid is definitely long, wide, and high, but can be interpreted subvaluationally, i.e. that in some element of \( \bigcup(\mathcal{P}_1 \cdots \mathcal{P}_n) \), the cuboid belongs in the positive extension of \text{big}.

Again, this approach fails because it hinges on precisifying the composite adjective, e.g. \text{big}, in the specification points associated with its components, \text{tall}, \text{wide}, and \text{long}. Instead,

\(^8\)This would be inescapably susceptible to the sorites paradox and would, therefore, require a vague metalanguage, which will be vague itself, and so on.
I propose the following:

\[
\lbrack \Phi_{a_1\cdots a_n}(a)\rbrack_{sp} = 1 \text{ iff } \lbrack a_1(a)\rbrack^\# \land \cdots \land \lbrack a_n(a)\rbrack^\# \text{ (or iff } \forall i \in [1, n] \lbrack a_i(a)\rbrack^\#) \\
= 0 \text{ iff } \lbrack \neg a_1(a)\rbrack^\# \land \cdots \land \lbrack \neg a_n(a)\rbrack^\# \text{ (or iff } \forall i \in [1, n] \lbrack \neg a_i(a)\rbrack^\#) \\
\text{undefined otherwise.}
\]

\[
\lbrack \Phi_{a_1\cdots a_n}(a)\rbrack_{sb} = 1 \text{ iff } \lbrack a_1(a)\rbrack^\# \lor \cdots \lor \lbrack a_n(a)\rbrack^\# \text{ (or iff } \exists i \in [1, n] \lbrack a_i(a)\rbrack^\#) \\
= 0 \text{ iff } \lbrack \neg a_1(a)\rbrack^\# \lor \cdots \lor \lbrack \neg a_n(a)\rbrack^\# \text{ (or iff } \exists i \in [1, n] \lbrack \neg a_i(a)\rbrack^\#)
\]

According to this definition, the truth of multi-dimensional predicate assignment requires, in the supervaluationary version, that every component predicate hold of the individual in question, and likewise the falsity. The subvaluationary alternative requires only that one component predicate hold of its argument. This permits the truth of statements like ‘the cuboid is big and not big’, since the subvaluationary reading will find some component predicates that hold of the cuboid, e.g. length and width, and some others that do not, e.g. height. Whether or not this is an advantage to be sought is not clear, of course, but it seems plausible given the patterns seen in the experimental data.
Chapter 5

Further Research

Many ideas suggest themselves as follow-up on the experimental part of this study. Some of these were hinted at in earlier parts of this thesis. In my review of Bonini et al. (1999), for example, I argued against their findings and in doing so I assumed that false was a satisfactory sign of denial, or disagreement. It would be interesting to re-run the experiment I conducted but with true/not true, or agree/disagree as answer options instead of true/false. Another possibility would be to replace the answer options with a small set of integers to represent degrees of agreement/truth, e.g. 1-7 or 1-5. I predict that the overall statistics from this latter test resemble a fuzzy graph if considered globally, that is, if the mean value was taken for each response independently. But to further confirm (*) one would need to count the number of participants that chose a higher degree of agreement/truth to the contradictory conjunction than both its conjuncts individually and compare it, for instance, to the number of those who gave it a lower value, an equal value, etc.

Greg Coppola (personal correspondence) suggested adding the following query to the survey:

Sheila likes to date tall men. Would Sheila date #2?

The answer to this question could be compared with the answers to “is tall”, “is not tall”, etc. of the respective suspect. My solution predicts that there be high correlation between the responses to the predicate “is tall” and the answers to Coppola’s question, because in both cases the super-interpretation is assigned to the predicate “tall”, due to Gricean informativeness. This would have to be confirmed (or disconfirmed) experimentally.

Another idea, suggested by Morgan Mameni, is to add to the line-up another suspect,
CHAPTER 5. FURTHER RESEARCH

say #6, who has the exact same height as #2 (our borderliner), and then elicit true/false responses for “#2 is tall and #6 is not tall”. I predict that a significantly greater number of false responses be given to this latter statement than the number of false responses to “#2 is tall and not tall”, despite the fact that both suspects are of the same height. Whether or not my account of (*) predicts this is not clear. It could be argued (as suggested by Nancy Hedberg) that the question regarding both suspects is likely to generate more false responses because of its violation of the maxim of MANNER; the statement can be thought to be more obscure than the other because it misleads the hearer into thinking that #2 is taller than #6, which cannot be said of the statement “#2 is tall and not tall”.

It would also be interesting to adjust the questionnaire for use with children and ask them, with the use of puppets or some such thing, similar questions on tallness, baldness etc.. This would allow one to observe how tolerant children are to contradictions, and also measure their (un)willingness to assign a predicate or its negation to an individual in the predicate’s borderline range. The idea of testing implicature in child language comes from Noveck (2001). Insights into adjusting the current experiment for use with kids can be found in his work.

Another way of refining this study is to include more contextual information in the questionnaire. In my analysis of the results, I did not attempt to reconcile the inconsistency of (*) on the basis of comparison-class differences. This is because I believe that, given a very specific comparison class, e.g. basketball players, the player who is borderline-tall for a basketball player will still cause some to answer in the way seen in (*): it will be false that he is tall for a basketball player, false that he is not tall for a basketball player, but true that he is tall and not tall for a basketball player. It would be interesting to confirm this experimentally.

Finally, and most obviously, my experiment was entirely lacking in adjectival variety, and in linguistic diversity. Future replications of this study could, perhaps, use a wider selection of predicates, e.g. “bald”, “red”, “long”, “late”, etc. and perhaps re-write the questionnaire in other languages.
Appendix A

Sample Question Sheet

<table>
<thead>
<tr>
<th>#1 is tall</th>
<th>True □</th>
<th>False □</th>
<th>Can't tell □</th>
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<tbody>
<tr>
<td>#2 is tall</td>
<td>True □</td>
<td>False □</td>
<td>Can't tell □</td>
</tr>
<tr>
<td>#5 is not tall</td>
<td>True □</td>
<td>False □</td>
<td>Can't tell □</td>
</tr>
<tr>
<td>#4 is neither tall nor not tall</td>
<td>True □</td>
<td>False □</td>
<td>Can't tell □</td>
</tr>
<tr>
<td>#5 is tall and not tall</td>
<td>True □</td>
<td>False □</td>
<td>Can't tell □</td>
</tr>
<tr>
<td>#1 is tall and #1 is not tall</td>
<td>True □</td>
<td>False □</td>
<td>Can't tell □</td>
</tr>
<tr>
<td>#4 is tall and #4 is not tall</td>
<td>True □</td>
<td>False □</td>
<td>Can't tell □</td>
</tr>
<tr>
<td>#3 is neither tall nor not tall</td>
<td>True □</td>
<td>False □</td>
<td>Can't tell □</td>
</tr>
<tr>
<td>#5 is neither tall nor not tall</td>
<td>True □</td>
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<td>False □</td>
<td>Can't tell □</td>
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<td>#4 is tall and not tall</td>
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<td>True □</td>
<td>False □</td>
<td>Can't tell □</td>
</tr>
<tr>
<td>#2 is not tall</td>
<td>True □</td>
<td>False □</td>
<td>Can't tell □</td>
</tr>
<tr>
<td>#3 is tall</td>
<td>True □</td>
<td>False □</td>
<td>Can't tell □</td>
</tr>
<tr>
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<td>True □</td>
<td>False □</td>
<td>Can't tell □</td>
</tr>
<tr>
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<td>True □</td>
<td>False □</td>
<td>Can't tell □</td>
</tr>
<tr>
<td>#4 is not tall</td>
<td>True □</td>
<td>False □</td>
<td>Can't tell □</td>
</tr>
<tr>
<td>#3 is tall and #3 is not tall</td>
<td>True □</td>
<td>False □</td>
<td>Can't tell □</td>
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<tr>
<td>#1 is neither tall nor not tall</td>
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<td>False □</td>
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<tr>
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<td>True □</td>
<td>False □</td>
<td>Can't tell □</td>
</tr>
<tr>
<td>#2 is tall and not tall</td>
<td>True □</td>
<td>False □</td>
<td>Can't tell □</td>
</tr>
<tr>
<td>#4 is tall</td>
<td>True □</td>
<td>False □</td>
<td>Can't tell □</td>
</tr>
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<td>True □</td>
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<td>#5 is tall</td>
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<td>False □</td>
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Figure A.1: Sample Question Sheet (Scale 0.7)
Bibliography


BIBLIOGRAPHY


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