A GAMING FRAMEWORK FOR MODELLING
COMPETITIVE SERVICE INDUSTRIES

by

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ABSTRACT

Due to the competitive nature of service industries, firms are often required to make sound business decisions in short periods. Errors in marketing and operations strategies can result in loss of time and money. Although computer simulation can aid in evaluating potential business models before they are deployed, the problem of making intelligent decisions becomes central to modelling rational behaviour of firms.

A multi-agent based gaming framework is proposed around a market model for service providers, where decisions as to how to allocate revenue are made using a multi-criteria optimization approach. Kalman filtering is investigated as a means for estimating unknown parameters within the model, and basic consumer behaviour heuristics are implemented for reacting to market conditions.

The study demonstrates that although a more sophisticated business model implementation is necessary to exhibit realistic behaviour, based on initial evaluation, the framework comprising its core technologies is capable of facilitating such models.

Keywords: Multi-agent system; Multi-criteria optimization; Kalman filter; Gaming; Simulation.
"Success is not final, failure is not fatal: it is the courage to continue that counts."

~Winston Churchill
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# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Approval</td>
<td>ii</td>
</tr>
<tr>
<td>Abstract</td>
<td>iii</td>
</tr>
<tr>
<td>Quotation</td>
<td>iv</td>
</tr>
<tr>
<td>Acknowledgements</td>
<td>v</td>
</tr>
<tr>
<td>Table of Contents</td>
<td>vi</td>
</tr>
<tr>
<td>List of Figures</td>
<td>vii</td>
</tr>
<tr>
<td>List of Tables</td>
<td>viii</td>
</tr>
<tr>
<td>1 Introduction</td>
<td>1</td>
</tr>
<tr>
<td>1.1 Objectives</td>
<td>2</td>
</tr>
<tr>
<td>1.1.1 Market Decision Making</td>
<td>2</td>
</tr>
<tr>
<td>1.1.2 Identifying Parameters of Optimality Criteria</td>
<td>2</td>
</tr>
<tr>
<td>1.1.3 Modelling Consumer Behaviour</td>
<td>2</td>
</tr>
<tr>
<td>1.2 Chapter Overview</td>
<td>3</td>
</tr>
<tr>
<td>2 Problem Definition</td>
<td>4</td>
</tr>
<tr>
<td>2.1 Research Classification</td>
<td>4</td>
</tr>
<tr>
<td>2.2 Competitive Business Simulation Domain</td>
<td>6</td>
</tr>
<tr>
<td>2.2.1 Validation of Modelling Methodology</td>
<td>8</td>
</tr>
<tr>
<td>2.2.2 Research Questions</td>
<td>8</td>
</tr>
<tr>
<td>2.3 Multi-Agent Systems</td>
<td>9</td>
</tr>
<tr>
<td>2.3.1 Overview</td>
<td>9</td>
</tr>
<tr>
<td>2.3.2 Application to Competitive Markets</td>
<td>11</td>
</tr>
<tr>
<td>2.3.3 Approaches to Simulation</td>
<td>18</td>
</tr>
<tr>
<td>2.3.4 Design Considerations</td>
<td>21</td>
</tr>
<tr>
<td>2.3.5 Implementation Considerations</td>
<td>23</td>
</tr>
<tr>
<td>2.4 Chapter Summary</td>
<td>26</td>
</tr>
<tr>
<td>3 Competitive Business Model</td>
<td>27</td>
</tr>
<tr>
<td>3.1 Modelling Behaviour of Firms</td>
<td>31</td>
</tr>
<tr>
<td>3.1.1 Budget Allocation</td>
<td>34</td>
</tr>
<tr>
<td>3.1.2 Optimality Criteria</td>
<td>36</td>
</tr>
<tr>
<td>3.2 Consumer Behaviour Model</td>
<td>38</td>
</tr>
<tr>
<td>3.2.1 Complexity Considerations</td>
<td>43</td>
</tr>
<tr>
<td>3.2.2 Consumer State Transition Model</td>
<td>43</td>
</tr>
<tr>
<td>Section</td>
<td>Title</td>
</tr>
<tr>
<td>---------</td>
<td>-------</td>
</tr>
<tr>
<td>3.3</td>
<td>Chapter Summary</td>
</tr>
<tr>
<td>4</td>
<td>Simulation Framework</td>
</tr>
<tr>
<td>4.1</td>
<td>Functional Design</td>
</tr>
<tr>
<td>4.1.1</td>
<td>Consumer Behaviour Implementation</td>
</tr>
<tr>
<td>4.2</td>
<td>Multi-Criteria Decision Making</td>
</tr>
<tr>
<td>4.2.1</td>
<td>Concepts</td>
</tr>
<tr>
<td>4.2.2</td>
<td>Method Classifications</td>
</tr>
<tr>
<td>4.2.3</td>
<td>Pareto Set Construction and Sequential Elimination Algorithm</td>
</tr>
<tr>
<td>4.3</td>
<td>Parameter Estimation</td>
</tr>
<tr>
<td>4.3.1</td>
<td>Extended Kalman Filter</td>
</tr>
<tr>
<td>4.3.2</td>
<td>Filter Implementation</td>
</tr>
<tr>
<td>4.4</td>
<td>Chapter Summary</td>
</tr>
<tr>
<td>5</td>
<td>Application of the Proposed Framework</td>
</tr>
<tr>
<td>5.1.1</td>
<td>Filter Verification</td>
</tr>
<tr>
<td>5.1.2</td>
<td>Kalman Filter Convergence</td>
</tr>
<tr>
<td>5.1.3</td>
<td>Multi-Criteria Decision Making Trial</td>
</tr>
<tr>
<td>5.2</td>
<td>Competitive Market Scenario</td>
</tr>
<tr>
<td>5.2.1</td>
<td>Simulation Properties</td>
</tr>
<tr>
<td>5.2.2</td>
<td>Player Decisions</td>
</tr>
<tr>
<td>5.2.3</td>
<td>Market Response</td>
</tr>
<tr>
<td>5.2.4</td>
<td>Considering Idling Consumers</td>
</tr>
<tr>
<td>5.3</td>
<td>Players with Opposing Goals</td>
</tr>
<tr>
<td>5.4</td>
<td>Kalman Filter vs. Fixed Parameters Scenario</td>
</tr>
<tr>
<td>5.5</td>
<td>Chapter Summary</td>
</tr>
<tr>
<td>6</td>
<td>Conclusions</td>
</tr>
<tr>
<td>6.1</td>
<td>General Discussion</td>
</tr>
<tr>
<td>6.2</td>
<td>Future Directions</td>
</tr>
<tr>
<td>Appendix</td>
<td></td>
</tr>
<tr>
<td>Bibliography</td>
<td></td>
</tr>
</tbody>
</table>
LIST OF FIGURES

Figure 1: A Spiral Model ................................................................. 9
Figure 2: Client/Server Architecture .................................................. 10
Figure 3: High-level Multi-Agent Structure .......................................... 12
Figure 4: Expanded Market Geography .............................................. 13
Figure 5: Internal Agent Structure .................................................... 15
Figure 6: Learning Agent ................................................................ 15
Figure 7: World Model .................................................................... 29
Figure 8: Firm Budget Distribution .................................................... 35
Figure 9: Consumer State Transition .................................................. 44
Figure 10: Competitive Service Market Agent Model .............................. 48
Figure 11: Two Criteria Pareto-frontier .............................................. 52
Figure 12: Non-dominance and Weak Efficiency .................................... 53
Figure 13: Continuous and Disconnected Pareto Frontier ......................... 54
Figure 14: Ideal and Nadir points ..................................................... 55
Figure 15: Simple Weighting Method Linear Indifference Curves ............. 58
Figure 16: Simple Weighting Method with Different Weights ................. 59
Figure 17: Convex Pareto Frontier .................................................... 60
Figure 18: Non-Convex Pareto Frontier ............................................. 61
Figure 19: Example of Possible Non-Convexity in Market Game ............ 62
Figure 20: Pareto Set Construction Algorithm ...................................... 63
Figure 21: Sequential Elimination Algorithm .................................... 65
Figure 22: Kalman Filter Application ................................................. 68
Figure 23: Discrete Kalman Filter Operation ..................................... 71
Figure 24: Extended Kalman Filter Operation ................................... 73
Figure 25: Market Gaining Effort Parameter Estimation ......................... 75
Figure 26: High-Level Algorithm for Jacobian Calculation ..................... 77
Figure 27: Derivative Approximation ............................................... 77
Figure 28: Extended Kalman Filter Gain with Varying Process Noise ........ 80
Figure 29: Filter Convergence (0 Process Noise) .................................. 81
Figure 30: Filter Convergence (0.1 Process Noise) ............................... 81
Figure 31: Filter Convergence (10 Process Noise) ................................. 82
Figure 32: Filter Convergence (100 Process Noise) ............................... 82
Figure 33: Average Parameter Estimation Error ................................... 84
Figure 34: Decision Space Illustration ................................................................. 85
Figure 35: Criteria Space Illustration ................................................................. 86
Figure 36: Promotion (with Borrowing, 0.5 vs. 0.5) ................................................ 88
Figure 37: Quality (with Borrowing, 0.5 vs. 0.5) .................................................... 88
Figure 38: Borrowing (0.5 vs. 0.5) ............................................................ 89
Figure 39: Consumer Loyalty (0.5 vs. 0.5) ......................................................... 90
Figure 40: Market Share (0.5 vs. 0.5) .............................................................. 90
Figure 41: Short-term Risk (0.5 vs. 0.5) ............................................................ 91
Figure 42: Promotion (with Borrowing, 0.2 vs. 0.7) ................................................ 93
Figure 43: Quality (with Borrowing, 0.2 vs. 0.7) .................................................... 93
Figure 44: Borrowing (0.2 vs. 0.7) ............................................................ 94
Figure 45: Consumer Loyalty (0.2 vs. 0.7) ......................................................... 94
Figure 46: Market Share (0.2 vs. 0.7) .............................................................. 95
Figure 47: Risk Factor (0.2 vs. 0.7) ............................................................... 95
Figure 48: Short-term Risk (0.2 vs. 0.7) ............................................................ 96
Figure 49: Promotion (with Filter vs. without Filter) ............................................. 98
Figure 50: Quality (with Filter vs. without Filter) .................................................. 99
Figure 51: Borrowing (with Filter vs. without Filter) ............................................ 99
Figure 52: Consumer Loyalty (with Filter vs. without Filter) ............................. 100
Figure 53: Risk Factor (with Filter vs. without Filter) ......................................... 100
Figure 54: Short-term Risk (with Filter vs. without Filter) ................................... 101
Figure 55: Market Share (with Filter vs. without Filter) ...................................... 101
Figure 56: Spending on Promotion (0.2 vs. 0.1) .................................................. 105
Figure 57: Spending on Quality (0.2 vs. 0.1) ...................................................... 105
Figure 58: Consumer Loyalty Factor (0.2 vs. 0.1) ............................................. 106
Figure 59: Market Share (0.2 vs. 0.1) .............................................................. 106
Figure 60: Spending on Promotion (0.2 vs. 0.3) .................................................. 106
Figure 61: Spending on Quality (0.2 vs. 0.3) ...................................................... 107
Figure 62: Consumer Loyalty Factor (0.2 vs. 0.3) ............................................. 107
Figure 63: Market Share (0.2 vs. 0.3) .............................................................. 107
Figure 64: Spending on Promotion (0.2 vs. 0.6) .................................................. 108
Figure 65: Spending on Quality (0.2 vs. 0.6) ...................................................... 108
Figure 66: Consumer Loyalty Factor (0.2 vs. 0.6) ............................................. 108
Figure 67: Market Share (0.2 vs. 0.6) .............................................................. 109
Figure 68: Spending on Promotion (0.5 vs. 0.1) .................................................. 109
Figure 69: Spending on Quality (0.5 vs. 0.1) ...................................................... 109
Figure 70: Consumer Loyalty Factor (0.5 vs. 0.1) ............................................. 110
Figure 71: Market Share (0.5 vs. 0.1) .............................................................. 110
Figure 72: Spending on Promotion (0.5 vs. 0.3) .................................................. 110
Figure 73: Spending on Quality (0.5 vs. 0.3) ...................................................... 111
Figure 74: Consumer Loyalty Factor (0.5 vs. 0.3) ............................................. 111
Figure 75: Market Share (0.5 vs. 0.3) .............................................................. 111
Figure 76: Spending on Promotion (0.5 vs. 0.6) .................................................. 112
Figure 77: Spending on Quality (0.5 vs. 0.6) ...................................................... 112
Figure 78: Consumer Loyalty Factor (0.5 vs. 0.6) ......................................................... 112
Figure 79: Market Share (0.5 vs. 0.6) .............................................................................. 113
Figure 80: Spending on Promotion (0.5 vs. 0.8) .............................................................. 113
Figure 81: Spending on Quality (0.5 vs. 0.8) ................................................................. 113
Figure 82: Consumer Loyalty Factor (0.5 vs. 0.8) ......................................................... 114
Figure 83: Market Share (0.5 vs. 0.8) .............................................................................. 114
Figure 84: Spending on Promotion (0.9 vs. 0.3) .............................................................. 114
Figure 85: Spending on Quality (0.9 vs. 0.3) ................................................................. 115
Figure 86: Consumer Loyalty Factor (0.9 vs. 0.3) ......................................................... 115
Figure 87: Market Share (0.9 vs. 0.3) .............................................................................. 115
Figure 88: Spending on Promotion (0.9 vs. 0.8) .............................................................. 116
Figure 89: Spending on Quality (0.9 vs. 0.8) ................................................................. 116
Figure 90: Consumer Loyalty Factor (0.9 vs. 0.8) ......................................................... 116
Figure 91: Market Share (0.9 vs. 0.8) .............................................................................. 117
Figure 92: Spending on Promotion (0.5 vs. 0.5) .............................................................. 117
Figure 93: Spending on Quality (0.5 vs. 0.5) ................................................................. 117
Figure 94: Consumer Loyalty Factor (0.5 vs. 0.5) ......................................................... 118
Figure 95: Market Share (0.5 vs. 0.5) .............................................................................. 118
Figure 96: Promotion (with Borrowing, 0.5 vs. 0.6) ......................................................... 119
Figure 97: Quality (with Borrowing, 0.5 vs. 0.6) .............................................................. 119
Figure 98: Borrowing (0.5 vs. 0.6) .................................................................................. 119
Figure 99: Consumer Loyalty (with Borrowing, 0.5 vs. 0.6) ......................................... 120
Figure 100: Market Share (with Borrowing, 0.5 vs. 0.6) ................................................. 120
Figure 101: Risk Factor (0.5 vs. 0.6) .............................................................................. 120
Figure 102: Short-term Risk (0.5 vs. 0.6) ........................................................................ 121
Figure 103: Promotion (Idling Consumers, 0.7 vs. 0.7) .................................................. 121
Figure 104: Quality (Idling Consumers, 0.7 vs. 0.7) ........................................................ 122
Figure 105: Borrowing (Idling Consumers, 0.7 vs. 0.7) .................................................. 122
Figure 106: Consumer Loyalty (Idling Consumers, 0.7 vs. 0.7) ..................................... 122
Figure 107: Market Share (Idling Consumers, 0.7 vs. 0.7) ............................................. 123
Figure 108: Risk Factor (Idling Consumers, 0.7 vs. 0.7) ................................................ 123
Figure 109: Short-term Risk (Idling Consumers, 0.7 vs. 0.7) ........................................ 123
LIST OF TABLES

Table 1: Consumer Cognitive Processing ................................................................. 41
Table 2: Kalman Filter Error Estimation ................................................................. 83
1 INTRODUCTION

This research is aimed at the design and development of a gaming framework for multi-agent simulations – specifically for the application of modelling the decision making behaviour of various markets in the service industry.

Conventionally, market behaviour in economic systems has been described mathematically, with firms having access to all relevant information, and analyzed at equilibrium conditions. Deviations from these ideal circumstances are omitted as exogenous effects [van den Bergh et al., 2002], or uncertainties. However, many more factors go into the dynamics of representing market behaviour, including the market microstructure (i.e. the organization of the market being modelled), and the firms’ specific circumstances (e.g. attitude towards risk). [van den Bergh et al., 2002]. A more versatile modelling method is needed to represent the diverse set of behaviours, perceptions, and decision rules of multiple stakeholders in a market. We look at the use of the multi-agent paradigm as a possible alternative.

A distributed multi-agent system could for example help to simulate a competitive network service market in a metropolitan area. The proposed architecture is expected to provide the core technologies necessary to facilitate a business model and determine the effectiveness of a service provider’s marketing strategies, as a means of business decision improvement. In this study the scope is limited to competitive oligopolistic service industries. Although we develop a general approach, for better clarity in what follows, we refer to this market using telecommunications business as a backdrop. Still, we believe that our method is equally applicable to other sectors of the economy, including such network service industries as airlines and financial services.
The global effort is to understand how multi-agent systems can be designed and implemented to support business market models, with focus on the technologies that allow for automated decision making. The framework is built around a yet untested model for firm behaviour that has been stripped down to its fundamental function of budget allocation so as to reduce complexity and enable better analysis of the implemented technologies.

1.1 Objectives

This research is positioned around the investigation of three particular problems in the simulation of competitive service industries, stemming from [Kyrylov et al., 2003] and [Kyrylov & Bonanni, 2006]. These include market decision making and modelling of consumer behaviour. The third problem, stemming from the former, is that of identification of unknown parameters of optimality criteria.

1.1.1 Market Decision Making

In order for firms to better allocate operational costs in seeking optimal gains, they must effectively balance various criteria which may consist of gains and risk. We identify possible criteria and propose a method for decision making, i.e. distribution amongst operational expenses.

1.1.2 Identifying Parameters of Optimality Criteria

In the existing business model [Kyrylov & Bonanni, 2006], optimality criteria contain several unknown parameters that must be estimated. We investigate Kalman filtering as a possible technique for converging on these parameters.

1.1.3 Modelling Consumer Behaviour

Consumer behaviour represents a reactive model where consumers exhibit transition between a given set of states. The aim here is to fill in a gap present in the existing approach and implement reasonable heuristics for assigning these transitions.
1.2 Chapter Overview

Chapter 2 offers an overall scope of the problem as well as background information on multi-agent systems and their application to competitive market modelling and simulation. Chapter 3 outlines the competitive business and consumer models. Chapter 4 describes the framework implementation. Chapter 5 details the simulation outcomes, and Chapter 6 provides concluding remarks and potential for future research.
2 PROBLEM DEFINITION

The simulation of market behaviour is an expanding research area. Today, market modelling and simulation is used in applied research, education, training, and to a lesser extent in real-life decision making [Raczynski, 2006], [Marchi, 2005]. New techniques are being explored due to the fact that the use of traditional computational mathematics has shown weaknesses in its ability to predict certain market patterns; particularly those that result from the choices made by interacting stakeholders in a market [Streltchenko et al., 2003]. Intelligent multi-agent systems have been looked at as a possible means of facilitating the interactions between such stakeholders; i.e. agents with possibly conflicting objectives.

The advantage for using multi-agents is that simulated markets can be populated with agents that are completely different with respect to their decision models, and the consequences based on investor behaviour can be analyzed. This is much more difficult with traditional simulation techniques.

In our study we use multi-agent simulation in an effort to observe generic market trends in the competitive service industry that would justify our chosen use of technologies to support the complexities of firm decision making.

2.1 Research Classification

Our research does not attempt to challenge any existing knowledge in the field of artificial intelligence or computing science, but rather it is seeking to build upon what is presently known about multi-agent systems, and attempting to find a viable solution to the proposed objectives.
Although there is clearly a strong applied component, there is also an important conceptual element. This stems from the underlying theory that machines can be utilized to effectively model human decision making behaviour.

This research effort is primarily exploratory in nature, although there is also a predictive component. It is predictive in the sense that it seeks to reliably determine the actions of various agents and the outcomes that these actions have on their environment and each other. It is exploratory in the sense that it also seeks to understand how technology can be utilized in business simulation games, in order to improve the decision making process. In addition, the problem of market modelling simulation currently has no closely tied benchmark with which to compare against and its true scope is yet unclear.

In order to explain the impact and benefits that such a simulation would pose one must understand what software agents are and how they are typically deployed. Due to the varying roles that agents can play, it is difficult to provide a formal definition. However agents do share several general characteristics:

- Autonomy – agents are able to operate on their own
- Social ability – agents are able to communicate with each other
- Reactivity – agents operate within an “environment”, and make decisions based on their own perception of this environment
- Proactivity – agents have objectives, and may initiate actions to achieve their goals

Essentially, an agent is a computer system that, in addition to having these properties, is either conceptualized or implemented using concepts that are more usually applied to humans [Shoham, 1993]. In a multi-agent system, as the name suggests, many such intelligent agents interact with each other.

This research has potential for future social value. For example, the technology could be developed into a business game. This could serve as an educational tool for training decision makers in improving competitive strategies and business decisions [Dobson & Kyrylov, 2004].
The conceptual value is also of benefit to the artificial intelligence research community. The framework potentially can serve to demonstrate how multi-agents can be implemented and effectively utilized for modelling firms, which can aid in future research.

2.2 Competitive Business Simulation Domain

Business modelling allows for business processes to be expressed in forms that are suitable for analysis of their properties. This can aid managers in better understanding why certain phenomena occur within the business process. However, model complexities and time constraints can limit the ability of firms to fully test business strategies before actually deploying them, which can result in costly errors. Automated computer simulation can help alleviate these constraints and can be used in the competitive service industries to help in forecasting market trends as well as risk management.

Risk management implies that firms must balance their efforts of gaining returns against certain pitfalls. Therefore an automated market simulation game must be capable of selecting actions on behalf of firms that lead to performance that reflects the attitude of the firm. For example, a firm may choose to be more aggressive in gaining market share at the cost of assuming greater risk. This could manifest through the borrowing of funds in order to spend more on business operations. Since all firms are operating in the same market, their individual goals may also conflict with each other. This hints at the need for some kind of multi-criteria decision making module that can select optimal alternatives against various criteria.

The actions taken are the investments made by firms that may include expenditures in various operational costs and loans. Since the competitive service industry we are dealing with is oligopolistic in nature, firms would likely be forecasting the behaviour of their competitors when making decisions since they would have significant effect on the market. Thus, the decision making module within the simulation should be able to account for this information.

The existing market model [Kyrylov & Bonanni, 2006] that we base our framework around has introduced the market gaining optimality criterion that is necessary for firms to utilize the perceptions of their competitors' actions to compute the state of the market. This criterion
however contains several unknown constants reflecting uncertainties that may exist in the
decision space. In the simulation domain, we would like to represent these parameters as
accurately as possible so as to improve the decision making process. That is, we would like to
obtain an estimate of the true state of the criterion even though we cannot make a direct
measurement. Since the criteria in the market domain are most certainly non-linear, the method
for estimating the parameters should be able to handle such processes (i.e. those objective
functions that firms wish to optimize).

In the service industries, decisions as to how to invest in operational expenses are all
grounded towards drawing in clients and maintaining the consumer base. When we look at
industries such as telecommunications, firms are all offering very similar products at comparable
price points. They rely on promotions and quality of service to attract clientele. These factors
should then affect how consumers behave and react in the market. In addition, consumers’
decisions may be affected by their overall experience with one firm or another. This implies that
they should be designed to incorporate some form of memory of their experiences. For example,
if a customer experiences multiple outages in Internet service with their provider over a period of
time, they may eventually switch to another company offering similar service.

A global consideration we have to make with regards to all of the framework components
is that of the computing power required to run market scenarios reliably and in timely fashion.
Though conducting a performance evaluation of the implemented algorithms is beyond the extent
of this study, our main goal here is just to scale our framework with reasonable expectations with
respect to simulation run-times.

Finally, we want the outcomes of the competitive service industry game simulations to
exhibit some simple trends that are reflective of the real world. The most fundamental of which
is that increased spending on operational costs leads to better consumer satisfaction and thus
greater gains. Likewise, increased borrowing of funds would lead to greater risk.
2.2.1 Validation of Modelling Methodology

We expect that market behaviours as those we have described would manifest as a result of games played against each other by the firms. This form of modelling methodology involves comparing exhibited behaviour against our qualitative assertions. The validity of these assertions stems from common sense. They serve as rough guides as to what kind of tendencies should be demonstrated by the market at large. We reiterate that our focus is on the technologies that may support competitive service industry games, and not on the intricacies of the market models themselves.

2.2.2 Research Questions

Having outlined the various requirements of the market simulation domain, we arrive at several questions that the framework is aimed at addressing. The first of which is what kind of general technical framework is capable of supporting such a market game? We have multiple actors that are pursuing both individual goals as well as competing against each other. In addition, the simulation should be autonomous such that human interaction is only required for calibration and analysis.

The second question is how to model rational decisions by firms based on criteria that may involve conflicting goals? While firms attempt to distribute their budget to gain market share, they also should consider the risk associated with the costs involved. The implementation should be flexible enough to support any type of modelled criteria, as well as firms’ individual attitudes (i.e. partiality) towards those criteria.

The third question is how to optimally estimate unknown parameters existing within the optimality criteria used for decision making? The method to achieve this should seek to minimize the possible estimation error, while also being computationally efficient.

The fourth question is how to model fundamental features of consumer behaviour in a service industry? Given that there are computational limitations, the framework cannot model individual consumers. Rather, it may encompass the general reactions that the consumer base as a whole would exhibit as a response to the actions taken by service providers.
In order to better demonstrate the nature of the problem addressed in this thesis, below we provide an overview of multi-agent systems and highlight the features that are most important to this study.

2.3 Multi-Agent Systems

2.3.1 Overview

Before delving into the theory of multi-agent systems, we provide a brief overview of the design process for our framework to help offer a context at the technology level. Design, development, and testing of the architecture follows loosely from the standard spiral model of the software development lifecycle [Boehm, 1986].

Figure 1: A Spiral Model

![Spiral Model Diagram]

This is an iterative approach whereby results of trial simulations are fine tuned at each stage. From a full scale prototype, inputs of the model could be adjusted to reflect suggestions made by market analysts to enhance the realism of the simulations. For preliminary analysis, an existing economic model that is reduced to include limited optimality criteria and other parameters is used to dictate system behaviour.

As a potentially distributed system, the overall high-level design would be comprised of various structural components, including a basic game server and clients. A database and some form of report generator would be used for statistical analysis.
The server application would simulate aspects of the telecommunication market and facilitate the following:

- actions executed by players
- register the entire model process parameters sufficient for the evaluation of players strategy effectiveness
- support the entry and update of the model parameters
- include the program which implements the algorithms reflecting the business logic as market subjects
- execute simulation runs in different modes of operation
- support user interface for entering data and viewing parameters of the model and its major components

Each client application represents a single market player. The client application must:

- get the current market information from the server
- forecast changes of market state, evaluate possible alternatives, and action decisions (decision must be rational according to the criteria defining the player strategy)
- simulate the player behaviour on the market
• include a program which implements the algorithms reflecting the business logic of telecommunication service providers as market subjects

A database would be used to store conditionally-constant model input parameters and results of simulation runs. The report generator would allow for graphical results of simulation runs.

With respect to resources, as computing power is finite, several model simplifications must be made. In addition, for future research the aid of a business analyst would serve to play a critical role in refinement of the system, as well as tailoring to different applications.

2.3.2 Application to Competitive Markets

In order to design the agents, we must first provide a context by which they will operate. As a simulation, boundaries as well as an overall scope must be defined. To alleviate complexities that may arise in designing such a simulation, we construct the architecture around a multi-player game like scenario in which the following properties hold:

• The game involves two players
• Players have attributes which may change throughout the course of a game
• Players take turns in which various actions are made available to them
• The game ends after a user specified number of turns

Essentially, each turn represents a fixed period of time. Due to the simulated nature of the game, each player alternates moves, however at each turn the actions are based on prior turn history. For example, consider the third turn of a two player game. After player 1 makes his/her actions, player 2 will follow - but player 1’s actions at turn three will have no effect on that of player 2’s – i.e. player 2 will make moves based only on the events at steps 1 and 2. Agents have full knowledge of their own prior history; however they only have perceived knowledge of their opponents.

The actions made by players are also based on perceptions of an environment. The environment is a global space by which all agents operate within.
At each turn, the agents will collect information from the environment. Each agent has its own perception of the state of the environment. These percepts are then processed by the agent which generates a set of actions to perform. Upon executing the actions, the environment is changed, as are each agent’s perceptions of the environment. Thus we can see that with N agents, we must maintain N+1 states of the environment; i.e. the true view of the environment, along with each agent’s perceived view. In terms of the real world aspect, this is logical, because telecommunications companies may not have all the information at hand about what their rivals are investing in, in terms of infrastructure etc. Therefore, they may wish to spend revenue on gaining a clearer view of the state of the environment.

The environment is made up of firms’ perceived state of knowledge of their competitors as well as the market state. The market state is essentially the customer base comprising consumers exhibiting reactive behaviours. These behaviours manifest themselves such that each individual customer may subscribe for the service offered by some firm, keeps using that service or leaves and possibly defects to a competitor. Thus player actions can affect the inflow and outflow of consumers. From an algorithmic standpoint, this can be viewed as a two dimensional grid, with each cell representing a certain population density and fixed area (as shown in Figure 4).
The geographical area is necessary in order to determine the range of service that various facilities can cover. In particular, for the telecommunication services market, these facilities include unmanned cable/DSL ‘offices’ and/or towers with wireless transponders.

In order to process sensory input (i.e. percepts), agents have defined goals that may continuously change along the course of a game. These goals must be pre-defined at the start of a game, but agents may change their goals based on perceptions from the environment. The selection of actions could be based on the entire percept sequence history, i.e. all actions taken to date, or a specified number of prior turns, depending on the selected goal. Possible long-term goals include gaining market share, maximizing profits, and minimizing expenses.

The definition of a goal can essentially be abstracted as a performance measure. This is the criterion for success of an agent’s behaviour [Russell & Norvig, 2003]. If the environment responds with a desirable sequence of states based on an agent’s actions, then the agent has performed well. Obviously, we would like to maximize the performance measure, however this is not the sole criterion for an optimally functioning agent. For an agent to perform rationally, i.e. do what we want it to do, there are four main contributing factors:

- The performance measure
- The agent’s prior knowledge of the environment
- The available actions
- The agent’s percept sequence to date
Once we know what environment the agents are operating upon to achieve their goals, we can specify a task environment. This is basically the problem to which we want the agents to be solutions. The task environment is made up of the performance measure, environment, and actuators and sensors. The properties of the task environment are as follows:

- **Fully vs. partially observable** – *Do sensors detect all aspects of the environment?* The environment is partially observable because the agent’s sensors may not detect all aspects that are relevant to the choice of action.

- **Deterministic vs. stochastic** – *Is the environment’s next state determined completely by the current state and actions taken?* The environment is stochastic from the point of view of the agent in the sense that companies cannot predict the market behaviour exactly. In terms of the model architecture though it is deterministic because the environment is determined by the current state and the actions of the players.

- **Episodic vs. sequential** – *Episodic agent’s experiences are independent of each other.* The environment is sequential because the current agent decision could affect all future decisions and short term actions could have long term effects on the world model.

- **Static vs. dynamic environments** – *Dynamic environments are able to change while agents are deciding what to do.* In the real world scenario this would clearly be dynamic as markets do not wait for firms before acting – however for simplicity the simulation model will be static – hence the ‘turn-based game’ architecture.

- **Discrete vs. continuous** – The system is discrete due to the finite number of percepts and actions.

- **Single agent vs. multi-agent** – Here we have a competitive multi-agent environment because players have conflicting goals.
Figure 5 (above) outlines the basic structure of an agent. Aside from utilizing current percepts and the percept history, they rely on the goal information to describe the desirable states. In addition to goals, a utility function may be necessary to explicitly quantify the measure of their performance. However, this would not change the overall structure of the agent. Also, a mechanism for learning may be incorporated in order to allow the agent to automatically improve its decisions over the course of the game. This would involve the addition of a feedback module that informs the agent’s learning component of its progress with respect to a fixed performance standard.

Figure 6: Learning Agent
An agent can only learn if it tries actions that may not always be considered ‘optimal’. To account for this, a problem generator module would be needed in order to suggest possible ‘exploratory’ actions that would lead to alternate pathways. While these paths may be sub-optimal, they could also potentially lead to much better actions in the long term.

All of the decision making processes of an agent occur within the black-box shown in the high-level multi-agent structure. Some mechanism must be in place to decipher what moves to take at each step in the game. It is the job of this mechanism or agent-function to map any given percept sequence into an action or set of actions. For our real-world simulation, such actions could include investing revenue on promotions, expansion (building new facilities), maintaining quality of service, modifying services (price, content), paying dividends, and/or borrowing funds. In addition, an agent must be able to decipher when is the best time to invest in market research, which would effectively expand its view of the environment. Due to the large number of parameters, all of these actions have the potential to be far too complex for a computer or even a parallel multi-processor system. Although we would like the agent to select the optimal action parameters to achieve its goals, we are constrained by the given computing power, time constraints, and resource constraints.

There are many AI techniques that are geared towards searching for an optimal path – i.e. a sequence of states connected by a sequence of actions that leads to a goal state. However, most of these methods are typically oriented for scenarios with observable and deterministic environments. Also, the effectiveness of such algorithms is highly dependent on the number of steps ahead that can be forecasted and maintained in memory. This is clearly not an appropriate direction for our purposes.

Due to the multi-agent nature of the simulation, it is not possible to look ahead many steps with a great deal of reliability since other agents are constantly changing the environment with their own actions. Although agents have their own perception of the environment, and can make logical, educated, guesses as to what their competitors may be likely to do next, the probability of finding a good solution decreases exponentially the further we attempt to look ahead in the search tree. This is because we would only be making guesses as to what state the
environment would be in from a proposed action. In contrast, consider a game of chess. Here, there is no guess work at all – we know exactly what the state of the game board will be if we move piece A to position B.

With games like chess we could also attempt to apply a recursive minimax like search algorithm [Russell & Norvig, 2003] to follow the paths resulting in the alternating turns of the players assuming that each player is playing optimally. This form of adversarial search strategy is aimed at maximizing the goal utility, while minimizing the utility of opponent turns. However, even with optimizations like alpha-beta pruning which attempt to cut down on the size of the search tree, this is not applicable to a full game of chess, as the search space would still be enormously large. Thus searching decision trees in chess is only possible for just several moves ahead. With a simulation like the market game, searching a decision tree is even less feasible because moves made by the players may involve multiple actions with multiple parameters. In addition, market games of this class may involve several players, thus increasing the complexity of the decision tree at an exponential rate. This effectively rules out the possibility of applying an algorithm dependant on decision tree search.

A more logical approach is to construct an agent that utilizes a set of heuristics, and gradually fine tune the heuristic parameters based on trial and error. An online search agent is best suited for this purpose. With an online agent, computations and actions are interleaved. First, an action is taken, and then the agent observes the environment and computes the next action. This form of agent is also ideal for the incorporation of a form of self-learning as mentioned earlier. This is because actions to achieve the strategy goals are used by the agent as a means of exploration of the environment. An online agent has three key pieces of information:

- A list of actions allowed in a particular state, $s$
- The step-cost function which returns the cost of moving from a state $s$ to $s'$
- Goal tests to determine whether the current state $s$ is a goal state

To provide greater context, the major difference between an online search and an offline search lies in the fact that the agents cannot access the successors of a state unless they actually
try all the actions in that state. With the online algorithm, only the nodes of the game tree that are being occupied can be expanded for further exploration. The key simplicity for applying this method to the market game is that there is no need to backtrack through the search to find a better path, since actions cannot be effectively ‘undone’. Instead, heuristics can be applied to determine whether an agent is progressing in a counter-intuitive manner. Take for example a scenario in which a player continually loses customer base in a particular region over a certain period of time. Then, the cost of service and/or quality of service can be measured against that of the competitors and alternate courses of action can be recommended.

Such a heuristic based approach to decision making could result in finding model parameters that are close to their ‘true’ state in a relatively short period of time, while also maintaining a high level of performance of the simulation itself.

2.3.3 Approaches to Simulation

The purpose of simulation is to capture and model some facet of reality. Since there are many applications for simulation, it is important to discuss what approaches can be taken in various instances. Traditionally, simulations have been useful for representing engineering and physical systems. Social systems have not been targeted for simulation nearly as often due to the many complexities of human behavioural interaction. However, modelling such systems even at a reduced scope can provide many useful insights. There are three main approaches to social systems simulations [Nagendra-Prasad & Chartier, 2000]:

- Game theoretic simulations – Here, agents make strategic decisions by reasoning about each other and the potential outcomes of their actions based on what information they have access to.

- System Dynamics – Typically, system dynamics uses a feedback structure whereby relationships among primitives are captured using differential equations.

- Agent-based Modelling – Here, the domain is modelled as a set of behaviours. In a more sophisticated approach, agents can improve their behaviours by learning through their interactions with one another.
System Dynamics has been explored in much more depth than other techniques, mainly due to the fact that it has been around longer. Hence, there are more tools available that have been thoroughly tested to aid in the modelling process. However, the agent based approach alleviates many of the difficulties involved in System Dynamics, especially in the social sciences domain where different sub systems may be pursuing conflicting goals. For one, due to their behavioural centred structure, agent systems can offer more natural representations of the problem domain. System Dynamics has difficulty in converting actions and ‘sense and respond’ behaviours into feedback systems [Nagendra-Prasad & Chartier, 2000]. In addition, agent-based models are able to more easily map relationships between entities at different levels of abstraction. For example, in the telecommunications industry, one may wish to explore how a firm’s quality of service efforts leads to behavioural changes in an individual consumer. This is essentially taking a property at a macroscopic point of view, and tying it to an entity with a more microscopic focus. With System Dynamics, models are more geared towards parameters that are aggregated at the same level.

There are several advantages and disadvantages to utilizing an agent based model as described in [Twomey & Cadman, 2002]. Here we provide a brief summary of advantages and weaknesses:

Advantages:

- System Assumptions – System is not constrained to exhibit a particular behaviour.
- Realism – Agents are interacting entities that can be made to an arbitrarily realistic level of complexity.
- Natural representations – Agent based models are generally quite intuitive, as they have a structural correspondence with their target system.
- Heterogeneity – The models are driven by the diversity in the agent attributes. Traditional models usually require homogeneous modules.
• Bounded rationality – Agents need not act in perfectly rational fashion. They may be given limited information and abilities, and can be modelled with social/habitual limitations as well.

• Communication and social networking – Agents can share information and imitate behaviours. This is generally not possible with traditional mathematical/equation-based models due to the high degree of complexity.

• Object-oriented design – Agent based modelling is well suited to the object-oriented paradigm for implementation which is the mainstream paradigm in software systems development.

• Maintenance and refinement – New agents, attributes and behaviours can be easily incorporated without disturbing the model.

Weaknesses:

• Data problems – The process-based approach to modelling injects a potential lack of adequate data. New scenarios may be needed to account for the various potentialities of agent-based simulation.

• Identifying rules of behaviours – Capturing the appropriateness of the mechanisms defining the agent behaviour can be complex. A positive is that this forces design assumptions to be made explicit.

• Programming skills – Building an agent-based model requires a good understanding of object-oriented programming.

• Computational time – Depending on the number of agents being modelled and the number of parameters involved, these systems can be computationally intensive.

• Unrealistic model expectations – New users can over estimate the predictive ability of such systems. Complex adaptive systems can exhibit chaos, which make long-term predictions unrealistic. However in such cases, the exploratory
value of an agent-based model is still not to be overlooked. By modifying various behaviours, the model could be observed to tip in one direction or the other. This may not otherwise be an obvious qualitative measurement.

- Lack of prescriptive ability – There is no mechanism to force a model from one state to another. Simulations can be run under different scenarios (i.e. where the user inputs information such as initial conditions to the system) in an effort to arrive at a particular desired outcome. However because of the complex nature of the interactions among agents, there is no obvious way to control what specific sequence of interactions are necessary to observe that particular event.

Our framework benefits from the multi-agent paradigm in that it is a logical approach to conceptualize human decision makers as agents. This allows not only for natural representation of firms interacting in a market, but also supports better flexibility in design as complex decision making behaviour can more easily be incorporated in the future. Since our study is exploratory and qualitative in nature, we are able to avoid some of the weaknesses of multi-agent systems, namely those dealing with the scope of implementation. However, as the framework develops to incorporate agents that are more multifaceted, computational resources would likely become more of a factor.

2.3.4 Design Considerations

When designing an agent, there are three broad categories to consider – those with high, medium, or low fidelity [Twomey & Cadman, 2002]. This essentially refers to an agent’s level of abstraction. In the case of low fidelity, all agents typically employ the same behaviours and attributes. In the strictest sense, this is not really considered a multi-agent system. With medium fidelity, agents are calibrated at a distributed level; i.e. observations of groups of agents can be made to capture properties of individual agents. With high fidelity, there is an explicit effort to capture internal behaviour of the agents. Here, an agent may learn and adapt to changes in the environment over time. Such agents are also known as ‘cognitive’ agents.
In the case of the competitive service market game, we are dealing with a kind of mix between medium and high fidelity agents. The agents exhibit medium fidelity in the sense that their attitudes are set in order to calibrate the model. Yet they demonstrate high fidelity in that they make an effort to learn from the state of the environment when making decisions.

Agents are also designed internally to exhibit a certain level of adaptive ability. A hierarchical classification scheme for an agent’s level of ability to adapt has been proposed in [van den Bergh et al., 2002]. Here we provide a brief summary:

- **Weak adaptation** – The mapping from an agent’s percepts to actions remains fixed throughout the simulation.
- **Semi-weak adaptation** – The sets of percepts and actions are fixed, however the function that maps them can change.
- **Semi-strong adaptation** – An agent’s goals can be modified.
- **Strong adaptation** – Agents can modify their intentions and strategies for reaching their goals, as well as their set of actions.

The possible mechanisms outlined in [van den Bergh et al., 2002] for achieving these levels of adaptation are:

- **Imitation** – Copying the actions or goals of another agent.
- **Reaction** – Reactions are responses to particular events.
- **Reactive learning** – Here, agents use received feedbacks (i.e. past experience) to make future decisions.
- **Generative learning** – Agents are able to adapt by anticipating changes in the environment.
- **Evolution** – Components of agents are modified gradually.

In the market game we employ semi-weak adaptation because agents maintain the same goals throughout the simulation. This is to maintain a form of consistency while observing the emergent trends. This is achieved through both reactive and generative learning where agents use
feedback from the market and attempt to predict the behaviour of the environment. Though in our implementation, the generative learning ability is somewhat limited because players only have a fixed perception of how their competitors behave, i.e. they cannot make an explicit attempt at gaining more accurate information as to the true state of the environment.

2.3.5 Implementation Considerations

The internal structure or decision making process of an agent may employ any type of artificial intelligence technique, including neural networks, fuzzy systems, genetic algorithms, and rule-based heuristics. Which approach is best depends on the particular application. The key is that they all share the ability to perform nonlinear mapping from their inputs to their outputs, since the function that maps an agent’s percepts to actions could also be nonlinear [van den Bergh et al., 2002]. An in depth look at these A.I. techniques is beyond our scope; however we explore the few multi-agent simulations that have been developed in the social sciences, as a backdrop for our framework.

One such simulation is that of TalentSim [Nagendra-Prasad & Chartier, 2000]. TalentSim is a prototype of a decision support tool that allows organizations to observe how changes in workforce practices can be matched with business strategies. The theory is that organizational changes will affect employee performance due to possible lack of processes that support the employee transition to the new culture, or simply that the new culture does not fit well with the individual employee. The obvious benefit of simulation in this case is that clients can see how their decisions would potentially affect their workforce before actually implementing any policies.

An interesting point about TalentSim is that it uses System Dynamics at the individual agent level, but models with agent-based behavioural methods at the interaction level. To bridge the gap between the differential equation based model and the rule-based model, Fuzzy set theory is used, where by relationships of continuous member functions can be converted into discreet decision rules. This demonstrates that the design of a model need not conform strictly to one type of simulation or another. Combinations of techniques are often advantageous – the key is to properly identify which situations call for which types of models.
In TalentSim, employees are given various attributes and performance indicators are used to determine how well the employee is responding to a given process. Interactions between employee and company agents can lead to changes in morale and other attributes. For the most part, these rules are defined heuristically.

There have also been multi-agent simulations at a larger scale. ASPEN, developed at Sandia National Laboratories is a very ambitious project that attempts to create a microeconomic model of the U.S. economy [Basu et al., 1998]. Due to the huge number of agents needed in order to represent the various entities (households, firms from various industries, government, and financial sector agents), the simulation is run on a massively parallel Paragon computer. The heavy computing requirements are one of the reasons we have elected not to model our consumer population as individual agents.

Another difficulty with ASPEN is that since it attempts to gain insight into the economy as a whole at the macroscopic level, the level of detail required when modelling from the microscopic perspective is vast. Agent based systems are more useful for observing general trends when the scale and forecasting intensity are smaller, so that a) the user can have more confidence in outcomes, and b) there is a closer and more qualitative relationship between the causes and effects.

Firm decision making in ASPEN is based on a set of probabilities that are adjusted through trial-and-error. Trends such as price, sales, and profits for each firm are monitored, and depending on whether these criteria are rising or falling, each agent is given a probability vector that determines what action to take next (e.g. raise prices, lower prices, maintain price). Then, based on what happens to the trends in the future, the probability vector for that state is adjusted to reflect what improved decision to make if that scenario occurs again.

In contrast, with our framework an explicit market model has been defined with criteria functions. Therefore, agents attempt to make decisions by weighting these criteria and selecting the optimum alternative.

There are other multi-agent simulations in the field of financial market forecasting e.g. [Streltchenko et al., 2003], though most of them are designed so that unsuccessful agents are
replaced by more successful ones during the simulation. As detailed in [van den Bergh et al., 2002], this approach is less effective for analysis because it does not take into account agents' different attitudes towards risk and time horizon.

There are few agent-based model building tools available. Most simulations have been built from the ground up. An experimental tool-box known as SWARM has been in development and aims to reduce overhead by offloading simulation run time steps that would otherwise be implemented through looping on an object-oriented platform [Terna, 1998]. CUBES is an example of a simulation built using the SWARM engine to model consumer populations. CUBES utilizes a genetic algorithm to adjust simulation parameters for building a pool of consumer agents [Ben Said et al., 2002]. Our particular approach to parameter estimation for model calibration is quite different as explained in section 4.3. Additional examples of multi-agent simulation, including more details on CUBES are also discussed in a later section when we explore consumer behaviour in more detail.

A commonality with all of these agent-based simulations is that verification and validation is mostly done through qualitative measurements. This is reasonable when dealing in problem domains (such as those in the social sciences) where it is difficult to analyze data quantitatively. With these types of simulations, the focus is on capturing general reasonable behavioural patterns rather than processing large amounts of data. Also, the modelling process is iterative where refinements are made as the system matures. The overall design of multi-agent simulations involves identifying the entities involved, the behaviours and attributes associated with them, and the interactions with their behaviours [Nagendra-Prasad & Chartier, 2000]. In addition, the problem domain should have a defined time horizon, i.e. a scope for the period of simulation (e.g. short term/long term). This allows for more accurate modelling of other attributes within the system.

As we have explored, existing multi-agent systems in the economics and social sciences fields have typically involved agents with goals that do not necessarily conflict with each other. This is a key difference with our framework. In our model, agents are competing with each other directly while trying to satisfy their individual needs. Simulations with competitive agents do
exist in other domains however, and are most common in sports games where players and/or teams are matched against each other. An example is the Robotic World Cup (RoboCup) Initiative which is a set of guidelines for simulating soccer. The initiative encourages others to build their own ‘teams’ using artificial intelligence techniques. In fact, the approach to multi-criteria decision making (MCDM) taken in [Kyrylov, 2006], by which the soccer player in possession of the ball decides where best to ‘kick’ the ball on the field, is a very similar approach to the one we take for implementing decisional behaviour of our agents. We discuss this algorithm in detail in a later section.

2.4 Chapter Summary

In this chapter, we have provided an overall context for our study and outlined our research questions. We have made a case for the use of the multi-agent paradigm as the basis of our framework. In addition, we have looked at previous work done in domains similar to that which we are exploring in order to serve as a background and better evaluate the peculiarities of design and implementation with respect to simulation of competitive service markets.
3 COMPETITIVE BUSINESS MODEL

In this section, we provide a high-level overview of the entire competitive business model as published in our paper, [Kyrylov et al., 2003]. This will serve as an expanded form of our implementation and provide perspective on how the framework could support a more complex model. The current implementation (detailed in chapter 4) is based on simplified subsets of the firm decisional model and consumer behavioural model presented here.

To simulate the decision making processes in the service/telecommunications industry we utilize a strategy game approach, where the players are service providers. Existing business strategy games are heavily reliant on human users that are making decisions for the playing parties, which can lead to time constraints. To alleviate this problem, human role-players can be replaced with active intelligent agents.

Players offer a set of services, which are only targeted at clients having fixed locations, i.e. residents. Examples of such services are land-line telephone, cable TV, and Internet access.

Services of the same type offered by different players can be based on different technologies. For example, high-speed Internet access can be offered over coax cable or over copper wires using the ADSL technology. Different services can share the same technology, e.g. TV can be delivered using the same technology as for the Internet. The assumption is that each customer chooses only one provider of a service type.

Customers make their choices of whether or not to subscribe to an offered service, or discontinue using it. In cases where more than one provider is offering the same service in the same region of availability, those customers who are already using that service can defect to the other provider(s) if they find it more attractive a solution.
The simulation scenario spans over several years and therefore can be regarded as an exercise in long-term business analysis. The simulation is run in steps, having short duration in terms of business operations (1-3 months). In each such period, players create operations plans and execute them. In particular, each player selects budget options, such as investing in promotion, quality of service, expansion, and market research. In addition, players can borrow money and/or decide what amount to pay back in order to reduce debt. These expense items are further split between market segments by product and location to ensure normal ongoing business operations.

In order to model rational behaviour, players optimize their actions on each step with respect to the anticipated outcomes. The outcomes are evaluated using a set of optimality criteria, such as profit, market share, or risk. Since players in general may be pursuing different strategic goals, they can rely on different sets of criteria. Besides depending on the player strategy, these criteria may have different priorities. Rivalry in the marketplace is explicitly addressed in this model. Each player maintains a perceived world model which it uses for evaluating the moves made by the competitors. The results of these evaluations are an essential part of the criteria used for optimizing player actions.

Among random factors present in the simulation is the imprecision of the world model used by the players for decision making. Its accuracy depends on the investment in market research and mainly affects the perceived actions by the competitors. The updates to this information are subject to random delays, omissions, and errors. The magnitude of this negative impact decreases with spending on the market research. Still, players would never have the complete and precise picture of the environment.

For obtaining simulation results, we set up a scenario in which players have different attitudes towards risk. A scenario is a set of data specifying the policies, players, services offered, and their initial states and goals. These goals are expressed in terms of optimality criteria and their priorities. As a result, we obtain the information about the expected performance of market players and measure the volatility of the outcomes under different external conditions, such as policies and/or player strategies.
The purpose of the world model is to represent the information which is necessary for modelling the player actions and the resulting changes to the environment.

Figure 7: World Model

The World Model

- Consumer Base
  - Consumer Behavior
  - Perceived World Model
- Inflow
  - Latent
  - Attracted by promotion
  - Defectors from other providers
- Outflow
  - Latent
  - Non-latent

This data is mainly concerned with consumer behaviour and the actions by other players as they are perceived by the agent.

From an expanded point of view, the consumer base is calculated for each cell (see Figure 4) and measures the fraction of residents who are potentially willing to buy telecommunications services, but have not yet done so. It is modelled as a function of four parameters: percentage of existing subscribers, household income, minimal price currently available on the market, and the migration factor. The greater the percentage of existing subscribers, the closer the market is to the saturation point, and the smaller the consumer base is in a given cell. The level of saturation, however, is determined by the average household income. In wealthy neighbourhoods the saturation level is rather high, but in poor locations it is lower. This level is also driven by the minimal price, since fewer households can afford buying telecommunications services if the price is too high. The migration factor creates demand by newly arriving residents. This steady demand would always be present even when the saturation level has been reached.

However, our model implementation is simplified to factor only a single cell where the total population is fixed and individual consumer attributes such as household income are omitted.
The consumer behaviour model describes actions made by individual households on a given simulation step in a given cell regarding a service \( s \), provided by player \( p_j \). From the service-player combination viewpoint, on each simulation step there is inflow and outflow of consumers and their balance reflects the current subscribers in the given cell. Therefore, the consumer behaviour model is split into two sub-models, inflow and outflow, which in turn are split by the player and offered services.

The outflow is made of consumers who elect to cancel subscription for service \( s \), offered by player \( p_j \). There are two different components to consider. The first, latent outflow is driven by migration, and therefore players have no control over it. The second, non-latent outflow results from customer dissatisfaction and depends on how player \( p_j \) performs compared to that of the competitors. The model for determining the number of subscribers leaving their provider takes into account various dynamics including price, quality of service, and promotional effort by player \( p_j \) compared with the competitors. On each simulation step some subscribers discontinue their relationship with player \( p_j \) and are added to a pool. This pool is then distributed between all the players acting on the marketplace.

The inflow of new customers is composed of latent inflow, consumers attracted by promotion, and the defectors from other players. The latent inflow is driven by the telecommunications company reputation though word of mouth. We assume that this inflow is proportional to the consumer base and is a function of the total number of subscribers using service \( s \), offered by player \( p_j \). Even in the absence of any promotion, firms could attract new subscribers through social networking, whose influence grows with the total number of subscribers. The new consumer inflow attracted by promotion directly depends on actions by all players. It grows with the total spending by player \( p_j \) on the promotion of service \( s \), and the spending on maintaining quality of this service. This dependency is highly nonlinear, since the fundamental economics law of diminishing returns applies. The promotional effort by player \( p_j \) is measured as it compares with the spending by rivals. Price difference is an additional factor where rivalry is significant. If player \( p_j \) increases cost, its promotional effort and spending on quality of service would result in reduced new consumer inflow compared to a competitor charging the lowest price among all service providers.
The defector inflow is the result of distribution of the pool created by the households who had discontinued their subscriptions and decided to select a new service provider. We model this inflow by calculating the attractiveness factor for each provider and distribute the defector pool proportionally to it. Attractiveness is driven by the nonlinear combination of the price difference, promotion effort, and spending on quality of service by each provider compared to the rivals.

All of the aforementioned sub-models concern the probability distribution of different choices made by the individual consumer rather than the actual number of consumers making a specific choice, which is random. To obtain these actual numbers, we could implement a model where each individual household is an active agent exhibiting similar probabilistic behaviour. However, because this would probably be too costly in terms of required computational effort, we elected to calculate the expected integral effect – i.e. aggregate the consumers as a whole rather than model them as individuals.

The perceived world model is the part of the world model which is available for a given player. Realistically, player perceptions would be dependent on spending on market research. Players know the actions performed by their rivals, such as their budget items and the location of the facilities built. The data would become available to players with delay, which decreases with spending on market research. The data is used to estimate the state of the market in each cell which is then applied towards decision making. With increased spending on market research, the quality of these decisions would improve.

### 3.1 Modelling Behaviour of Firms

Here we outline the various components of the model which comprise the internal structure of agents representing firms in the competitive marketplace. The agent decisions should be made on the basis of several forecasted optimality criteria, which should reflect different aspects of the game environment, such as profits and risks. Rivalry in the marketplace is another complicating factor. Due to the size of the decision-making problem, we must address the issue of containing its computational complexity. This complexity results from the large size of the player decision space. On each simulation step players exhibit their behaviour in executing actions. Action parameters include budget items split by offered services, and the location of new
elements of the infrastructure to be built on a given step. With $S$ offered types of service and $B$ budget items (i.e. items of expense for firms) we get $(S \times B)$ dimensions of the decision space. When combined with thousands of possible territory expansion options, such as the number of new facilities and their locations, this would lead to a multi-dimensional decision space of unmanageable proportions.

A standard way to contain complexity is to use a hierarchical decision making process. The highest level comprises dividing the operational budget between several general spending items. These include promotion, quality of service, expansion, market research, dividends distribution, product pricing, and borrowing or paying back loans.

On the second level these items, if applicable, are split between market segments by offered service. The third level concerns only the expansion budget. The decisions made on this level are the number of new facilities to be built and their particular locations.

To simulate rational player behaviour, decisions on each level are made using multi-criteria optimization techniques. On the highest level, the set of optimality criteria used by each player and their priorities are determined by player strategic goals. If a player should attempt to fight its competitors in order to gain market share, the set of criteria would include the anticipated market share and, if other criteria are also used, maximal priority would be assigned to the first one.

Possible optimality criteria include market share, the risk factor, profit, and consumer loyalty. On each simulation step the players compute the predicted values of some or all of these criteria using a modification of the Cobb-Douglas model, a standard approach for representing production functions in microeconomics. The parameters of this model reflect the perceptions by firms and could be regarded as 'prior knowledge'. These parameters are unknown and therefore must be estimated. The estimation problem is complicated by the fact that, in addition to the player effort, the effort applied by the competing firms as it is perceived by the player, needs to be taken into account.

With the player agents seeking to establish optimality criteria with unknown variables, there must also be some way to parameterize these values. The research in [Twomey & Cadman,
2002] offers two methods for this. The first is to use survey data and field research. The other is to use an initial best guess and then calibrate the values with the model output. Since the market conditions in the simulation may change on any given cycle, it would be logical to use the latter approach. Also, there would be a great deal of complexity involved in simply analyzing field data and filtering out irrelevant information. In our model, we want to calibrate the unknown parameters of the optimality criterion for market share such that an estimate can be made as to how firms should plan their next course of action.

Without the ability to fine tune model parameters, a simulated player would tend to make suboptimal decisions unless precise enough parameter estimates are obtained. This is contradictory to real world scenarios, where the prior knowledge is always present, allowing humans to make rational decisions from the first step.

Consequently, we utilize an extended Kalman filter (described in detail in section 4.3) in an effort to achieve these estimates. This is a standard technique for stochastic estimation [Jazwinski, 1970]. We use the nonlinear version of the Kalman filtering technique which allows for parameter estimation of systems that are not fully defined – in our case, the market share performance criterion.

In the business simulation game, the initial values of the Kalman filter parameters are estimated using an arbitrary number of warm-up simulation runs. During this warm up, players gradually gain the ability to forecast the optimality criterion with enough precision from the first step of the game. Thus we achieve the effect of prior knowledge that is indeed present in real life.

As we have mentioned, there is a need for firms to make optimal decisions based on various criteria that may counter balance one another, i.e. rewards and risks. For solving this multi-criteria optimization problem for each player, we use a heuristic search algorithm, first proposed in [Kyrylov, 2006]. The algorithm is based on the discretization of the decision space into a finite, yet relatively large number of options that are close enough in the continuous decision space. Decision making is split in two phases. First, the set of non-dominated alternative decisions is sought which is referred to as the Pareto set [Ehrgott, 2005]. Second, the
optimal solution is found by applying different criteria in a random sequence and eliminating poor options sequentially. The frequency with which each criterion is applied in the elimination process is selected with respect to the priority assigned to the criteria. More important criteria are applied more frequently. The last remaining option is the optimal solution sought. With $N$ alternatives, the complexity of finding the Pareto set is $O(N^2)$. With $K$ elements in this set, the complexity of the exhaustive search of the solution is $O(K^2)$. As $N \gg K$, we would say that this algorithm is fairly simple in terms of computations, even though the total number, $N$, of options to be tested can be of the order of tens of thousands. With the parallel execution of player clients on different computers, this computational complexity could be tolerated.

Once the high-level budget decisions have been made, the budget items, such as promotion, quality of service, and expansion spending are distributed by the player between offered services. Then the expansion budget for each service is implemented in the particular placement of the new facilities. Budget distribution across services is executed using a similar multi-criteria optimization method which is used for high-level budget decision making.

### 3.1.1 Budget Allocation

Our framework is based on the economic model for budgeting decisions of telecommunication service providers presented in [Kyrylov & Bonanni, 2006]. Here, we provide a synopsis of the key components. Our model implementation comprises a subset of these components (as later described) for the purpose of verification, though the framework supports extensions for future research.

The model is comprised of the following characteristics:

1. In each planning period, players (or firms) allocate budget towards high-level expenses including facilities expansion, dividend distribution, and net borrowing, as well as operating level expenses including promotion, and quality of service.

2. Firms use a set of decision criteria including expected market share, expected profit, expected consumer loyalty. They also have a measure of anticipated risk.

3. Optimality criteria take into account counter-actions of competing firms.
4. Formal method for selecting a single alternative from the Pareto optimal set which accounts for player strategy (i.e. aggressive vs. defensive).


**Figure 8: Firm Budget Distribution**

Firms maintain consumer loyalty through spending on quality of service $Q$. Expansion $E$ refers to facilities that firms can build to increase their service area. This, along with spending on promotion $P$ will serve to attract new customers. Spending on market research $M$, would allow agents to gain a more accurate picture of the environment as well as their perception of their competitors. The five decision variables ($E$, $D$, $P$, $Q$, and $M$) shown in Figure 8 conform to the following linear budget constraint [Kyrylov & Bonanni, 2006]:

$$R = U + P + Q + M + E/(1-r) + D/(1-r), \text{ where } r = \text{tax rate}$$

When net borrowing $B$ is taken into account, the budget constraint can be adjusted:

$$B = F_1 - L - F_2 = F_1 - h F_1 - F_2, \text{ where } F_1 = \text{borrowing}, h = \text{interest rate}$$

$$R + F_1 = U + P + Q + M + F_2 + E/(1-r) + D/(1-r) + L$$

The decision model is simplified by using normalized variables defined as fractions of the budget:
\[ p = P/C0, \quad q = Q/C0, \quad m = M/C0, \quad e = E/(1-r)C0, \quad d = D/(1-r)C0, \] where \( C0 = \) net budget.

\[ p + q + m + e + d = 1, \] (where each decision factor \( 0 \leq x \leq 1 \))

The borrowing factor can also be normalized as follows:

\[ b = 1 + B/(R-U), \] where \( 0 < b \leq b_{\text{max}}, \, b_{\text{max}} > 1 \)

The borrowing factor acts as a scalar for the net budget. When \( b = 1 \), the net borrowing \( B \) is 0. Less than 1 and the firm is paying back its debt; more than 1 and the debt is increasing.

Stochastic and non-stochastic ambiguity is present in the model in that the firms cannot make accurate predictions due to unknown actions of their competitors, and they are uncertain of the consumer behaviour. With increased spending on market research, firms could reduce the degree of these uncertainties.

For the time horizon, long term forecasts are more imprecise, however short term decisions can lead to more uncertain future outcomes. Ideally, a multi-layered hierarchal model would allow for firms to balance their decisions, however for the purpose of this investigation, we utilize a single tier approach.

### 3.1.2 Optimality Criteria

In each planning period, firms must decide on the percentage of budget to allocate towards the decision variables \( b, \, p, \, q, \) etc. in a way that is optimal with respect to decision criteria. Their current state is based on the past decisions they have made, and their current decisions take into consideration the actions of their competitors. Criteria are split into rewards and risks, with rewards comprised of such factors as expected market share, profit, and consumer loyalty. Risk is governed by a single criterion, which should be minimized.

A modification of a Cobb-Douglas production function is used to define the optimality criteria since it can model non-linear functions and it adheres to the microeconomics law of diminishing returns – that with increased spending in a particular area, the marginal reward should be non-decreasing. Further details of the economic justification are given in [Kyrylov &
Here, we cover two main optimality criteria that are used in our implementation: anticipated market share, and risk.

Market share is dependent on the market-gaining effort applied by the \(i^{th}\) player, \(W_{1i}\), compared with the effort of all the competitors, \(\sum_k W_{1k}\). Since each player has only a perceived value of it’s competitors, \(W_{1k}^{(i)}\), the market share, \(u_{1i}\), is defined as:

\[
u_{1i} = \frac{W_{1i}}{\sum_k W_{1k}^{(i)}}
\]

An assumption is made that market gaining effort increases monotonically with the decision variables, which leads to the following modified Cobb-Douglas function [Kyrylov & Bonanni, 2006]:

\[
W_{1i} = (bp + p_{0i})^{a_p} (bq + q_{0i})^{a_q} (be + e_{0i})^{a_e} (bm + m_{0i})^{a_m} g_1(\pi_i),
\]

where \(a_{si}\) are sensitivity parameters (which are all non-negative and add to 1), and \(x_{0i}\) are uncontrollable parts of the normalized budget items - all of these items must be estimated. Uncontrollable items may be attributed to factors that are not necessarily directly related to budget allocated to the particular expense. For example, a consumer may have his/her own individual attitude towards a particular promotion, regardless of how heavily the firm invests in it. The variable \(g\) is the pricing factor, though in our implementation we focus only on the promotion and quality of service factors.

Risk is a very vague criterion, measured largely by the borrowing factor, and is attributed mostly by a firm’s own actions, i.e. the competitor’s decisions do not affect a player’s risk factor. We factor past borrowing as a decaying residual that cumulatively has effect on risk. It is cumulative in the sense that with each new time cycle, the borrowing in the past accumulates to account for some fraction of the current level of risk. This is a very simple model, but it allows us to make a direct link to the borrowing factor which would aid in qualitative analysis of the
simulation scenarios. In the real world firms would be required to pay back loans in regular periods, however we make the simplification to omit this factor and leave it to future research, because it would require implementing additional criteria such as financing/pricing, making the complexity much greater. Still, we can view risk as an independent optimality criterion to compare against gains.

\[
\text{Risk} = u_{2t} = (1-a)^0 b_n + (1-a^1)b_{n-1} + \cdots + (1-a)^n b_1, \quad \text{where } 0 < a < 1, \tag{2}
\]

and \( n \) is the current number of actions taken.

When we compute the derivative of the risk factor (with respect to time), we arrive at a measure for short-term risk. This is useful in looking at risk in a non-cumulative sense, and gives a better view of the rate of change such that the firms can be compared against each other more intuitively.

The economic model in [Kyrylov & Bonanni, 2006] suggests a decision space that is entirely convex. However, this is only true in a limited range of conditions. So we cannot make this assumption as our framework is to serve as a facilitator for variations or other potential models. Therefore, the problem of optimization cannot be reduced to simply aggregating the criteria as a weighted sum and producing a single utility function. Our particular approach to solving this problem satisfies our requirement of being generic enough to handle essentially any type of modelled criteria. The technique is described in detail in section 4.2.

### 3.2 Consumer Behaviour Model

Here, we provide some background on methods used in modelling consumer behaviour. Since the requirement of our framework is only to implement some basic, yet essential, reactive behaviour that would lend itself to exhibiting general market trends in the simulation, our model is not as sophisticated as some of those we describe. Yet this section will help give a better understanding as to the advantages and disadvantages of various techniques in relation to our study. We believe that our approach employs some features that could allow for modelling more realistic behaviours in the future.
Traditional methods for modelling consumer behaviour in a telecommunications marketplace, such as statistical and equation-based modelling, are typically linear in structure. This linearity refers to the relatively stable nature of the market [Twomey & Cadman, 2002]. Due to the presence of substantial non-linearity in such markets, these techniques are not always sufficient for making confident predictions of consumer reactions.

The bottom-up approach to designing agent-based systems is one of the key advantages over traditional methods for market modelling. A typical model may observe the effects that a quantifiable change in quality of service or the price of a product would have on the consumer market. However, in an agent-based system, the model focuses on the individual consumers and how their interactions lead to the system behaviour [Twomey & Cadman, 2002]. Interactions include dealing with the environment as well as with other agents.

Consumer models can account for behavioural attitudes from various disciplines. While our model focuses on very basic aspects of marketing and branding, more advanced factors can be facilitated, such as economic and social conditions, as well as personality traits. BT Laboratories has conducted research in the area of social networking to study consumer adoption of telecommunications services [Twomey & Cadman, 2002]. Here, word of mouth is a key contributor to how consumers learn about various services that are available. The correlation between the interaction of consumers and the adoption of telecommunications services can be derived much more intuitively with an agent based system due to the bottom-up modelling approach mentioned earlier. In our model, this is difficult to implement because of computing requirements. Implementing millions of consumers as individual agents would be costly in terms of the amount of memory required. Again, this is why we elect to model consumers as a single group rather than as separate entities.

Another consumer model that is focused on testing the effects of marketing strategies is the ‘Customer Behaviour Simulator’ (CUBES) [Ben Said et al., 2001]. Here, behavioural attitudes of consumers are modelled with the intent of observing real world phenomena at the macro level. In addition, CUBES simulates the brand reactions to changes in the market to see what effect this may have on individual consumer attitudes.
The research in [Ben Said et al., 2001] presents a consumer theoretical framework that revolves around three disciplines – psychology, economy, and sociology. The common field that ties them together is that of marketing. From the psychological perspective, consumers compare brands and make purchase decisions. Brand loyalty can be established at this level. From the sociological perspective, consumers are observed to be influenced by their own social class. The networks formed through various groups (such as family/friends, age groups) can have significant effect on purchasing decisions. Finally, the economic discipline relates to consumer reaction to factors such as price.

In the CUBES simulation, consumers’ opinions are affected either positively or negatively based on a set of external stimuli such as rumours and recommendations. These are known as the consumer behaviour setting [Foxall, 1999]. They comprise the surroundings in which consumers act. The stimuli are applied to consumer attributes as weights that are tied to the specific brand from which the stimulus originated. Thusly, each consumer forms its own ‘opinions’ on each brand in the market which change over time based on the stimuli response.

The three main attributes that affect an individual consumer agent’s behaviour are ‘opinion leader’, ‘attitude’, and ‘involvement’ [Ben Said et al., 2002]. Other agents may seek advice of an opinion leader. They are considered to be savvy on particular market segments. The attitude is a preset attribute that defines whether or not a consumer has a positive position on a product. Involvement captures the level of consumer interest cased by a product. This can increase a consumer’s awareness of competing brands and possibly develop brand loyalty.

A point of difficulty with CUBES is that the market model must output the specific stimuli that are input into the consumer model. Since the market model present in CUBES is in itself very general, plugging in a model that caters to a specific industry may not be easily accomplished. This further emphasizes our need to implement our own basic heuristic that would better reflect a response to the choices made by firms and allow us to observe the general trends that emerge as a result.

An additional example of a consumer model that focuses on behaviour dictated by psychological factors is that presented in [Janssen & Jager, 2001]. In this model, the consumers
switch between various strategies based on shifts in their preferential habits. The model separates the social (exogenous) and individual (endogenous) preference changes of consumers. The idea here again, is that a consumer’s partiality towards a certain product can be influenced by others who are consuming the product. The paper identifies two effects driving consumers to change their preference. The first is socialization, which as we have explained is the contact a consumer has with others. The second is exposure, i.e. the repeated consumption of a product. Exposure can be viewed as the strengthening between consumer stimulus response of product and preference. These two properties are implemented in the model such that if a certain fraction of those agents within a consumer’s social network defect to a different product, that consumer is probable to follow. Likewise, with the exposure effect, as an agent repeatedly consumes a product, its need satisfying capacity is increased, thus introducing a sort of resistance to behavioural change. This is independent from the actual satisfaction level of the product.

Within the social networks, two properties can be observed - the ‘small-world’ and ‘clustering’ effects. The small-world effect basically means that all agents within even a large consumer population are connected to each other in some way. In other words, information within the network can propagate very quickly from one consumer to the next. The clustering effect refers to the tendency of consumer’s social circles to overlap. These phenomena can be modelled using a lattice in which the links between nodes are rewired with a set of probabilities, as described in [Janssen & Jager, 2001].

In the model, the consumer’s decisional behaviour is based on their attributes of need satisfaction, and uncertainty, as shown in the table below.

<table>
<thead>
<tr>
<th></th>
<th>Need Satisfaction</th>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Repetition</strong></td>
<td>High</td>
<td>Low</td>
</tr>
<tr>
<td><strong>Deliberation</strong></td>
<td>Low</td>
<td>Low</td>
</tr>
<tr>
<td><strong>Imitation</strong></td>
<td>High</td>
<td>High</td>
</tr>
<tr>
<td><strong>Social Comparison</strong></td>
<td>Low</td>
<td>High</td>
</tr>
</tbody>
</table>

Table 1: Consumer Cognitive Processing

The degree of uncertainty comes from the difference between the actual and expected outcomes of the agent behaviour. When consumers are satisfied with the product and their level of uncertainty is low, they will continue to consume the product. With ‘deliberation’, consumers
are most likely to consider all possible outcomes of their actions, i.e. they attempt to optimize their goal of maximizing need satisfaction (within their individual boundaries, such as the time horizon). In [Janssen & Jager, 2001], this optimization is achieved by using a simple maximisation rule, in contrast with our model (with respect to firm decision making), which requires that multiple criteria be weighed since they cannot be aggregated into a single utility function.

Social comparison involves comparing one’s past actions with those of similar agents and selecting the best alternative. When both the need satisfaction and level of uncertainty are high, consumers tend to imitate the behaviour of similarly minded ones. This requires less computation than with social comparison since agents do not examine the potential outcomes of their own actions.

Clearly, the agents perform more rational or reasoned behaviour when their satisfaction levels are low. This is logical, since if they were already satisfied with their consumption, it would be counterproductive to expend the additional effort to reach decisions. With high levels of uncertainty, consumers are most likely to utilize the social information available to them when making decisions.

In our model, we are dealing with a specific industry in which few new services are introduced into the market. In this case consumers are more likely to exhibit the repeater or possibly the imitator condition, rather than deliberation and social comparison due to the relatively high level of need satisfaction. In such a scenario, consumers are acting more as a group because they do not need to exhibit as complex individual cognitive processing. We are more concerned with how consumers may react to changes in the services being offered, rather than how they respond with respect to their own preferences. And indeed the results of the [Janssen & Jager, 2001] study confirmed that preferential change in consumers has minimal effect on market dynamics. This is one reason we opt to implement consumers as a collective market.
3.2.1 Complexity Considerations

The agent-based approach is also not practical in some situations. A major drawback is the significant amount of computing power necessary to model each consumer individually. Also, when the objective is to obtain a feasible decision support tool, the computational complexity of such models often has to be reduced [Engel et al., 1995]. For these reasons, we take an alternate approach while maintaining the modularity of the multi-agent paradigm. Here, the consumers are modelled as a single group rather than individuals and are included as a part of the world model as opposed to acting on the environment as external agents. This is essentially exhibiting the property of low fidelity as discussed earlier because all consumers operate in the same manner. They react to the player’s actions (i.e. promotional and quality of service effort), changing state on each step in the simulation.

To keep the size of the problems addressed in this study manageable, we deliberately left the criteria of pricing out of consideration, leaving it for future research. One reason for doing so is that, as described in [Ben Said et al., 2001], in some markets, pricing may not be regarded as a determining factor as this does not play a major part in the purchasing process. This is especially true in the telecommunications industry, where competing firms offer the same services at comparable price levels. For some telecommunications services, prices are regulated by governments. As an example, consider two telecom companies existing in the same market, one offering DSL, and the other offering cable Internet service. As they are comparatively priced, consumers are more likely to base their decisions on such factors as availability, reliability, speed, and bandwidth. These are operationally efficient (OE) companies which gain advantage through more robust execution, rather than product companies who excel in innovation, or customer intimate (CI) companies that are geared towards specialized services [Nagendra-Prasad & Chartier, 2000].

3.2.2 Consumer State Transition Model

In order for consumers to react to the decisions made by firms, they need a mechanism that will allow them to adopt new services or change service providers.
In [Kyrylov & Bonanni, 2006] it is assumed that consumers make probabilistic selections as to their possible state (shown in Figure 9) on each simulation step. This method models consumers individually; however consumers do not make use of the history of actions taken by firms in considering their next state. This is not the most realistic approach because consumers often base their decisions on their overall experience with a company. Thus, a potential enhancement would be to incorporate memory of quality of service such that depending on a particular consumer’s experience, they may decide to defect and not return to the firm they were subscribing to, unless that firm makes a significant attempt at improving quality in the future. Behaviours depending on the state of consumer memory cannot be represented in the form of a simple state-transition model as shown in Figure 9.

Since our approach focuses on modelling consumers as a group, we provide a simple heuristic to model basic consumer behaviour based on a function of the actions taken by firms that takes into account a form of memory. This model is later outlined in section 4.1.

3.3 Chapter Summary

In this chapter we have described a high-level view of the competitive business model, including the optimality criteria used by firms, and the budget expenses that they must decide upon which in turn lead to the evaluation of those criteria. We have also taken a closer look at previous work done in modelling consumer behaviour and how it relates to our study. Having
explored the competitive business model we now have a context for the design and implementation of technologies in our framework.
4 SIMULATION FRAMEWORK

Here we present both a high-level view of the framework in its current implementation (i.e. with simplifications made to the model previously described in chapter 3), as well as a detailed description of the methods used for multi-criteria decision making, and parameter estimation – the two main components of the simulation.

4.1 Functional Design

In order to make the main principle of modelling the framework more intuitive to understand and evaluate, we make a number of simplifications. The game involves just two firms that offer a single similar service in the market. This is still sufficient to model the main peculiarities of both consumer and firm’s behaviour in the competitive market that we are interested in. Firms differ in personality by their attitude towards risk. Once a feasible solution set has been calculated (i.e. a set of budget decisions), an alternative is selected based on the firm’s level of bias towards one criterion or the other (i.e. gain or risk).

Simulation steps are deliberately made rather short in order to better capture the customer base dynamics. Actions are thus taken on every 3rd simulation cycle. We can think of the simulation step as one month, leaving up to the firms taking actions on a quarterly basis, which is normal practice in business simulations. We limit our decision variables to promotion and quality, with the borrowing factor serving as a scaling factor. Thus we have just two independent variables. Yet this is still sufficient for our study, as the decision space has more than one dimension. The decision variables are selected from a discrete set conforming to the constraint that the sum of promotion and quality is less than or equal to the maximum borrowing factor:
\[ p + q \leq b_{\text{max}} \]  

The Cobb-Douglas production function is reduced to include only the relevant promotion and quality of service variables as:

\[ W_i = (bp + q_{i,0})^{\alpha} (bq + q_{i,0})^{1-\alpha} \]  

Note that since the sum of the two sensitivity parameters totals 1, this effectively reduces the number of unknown parameters in this version of the Cobb-Douglas function from four to three (for a total of six including the perceived parameters of the competitor). Precisely determining the perceived market gaining effort requires creating a perceived model of the competitors for each firm. To avoid complex calculations in this study, we deliberately simplified the model by assuming that the perceived values of the competitor equal the actual values on the previous simulation cycle that are observable by the market players. For the first cycle it is just assumed that the firm’s opponent is taking the same actions as itself. In reality, a firm may increase its knowledge of what its competitors are doing by investing in market research. However, as this would require a more sophisticated model of the firm, we have left this improvement for the future.
Figure 10 (above) displays an internal view of the multi-agent structure. To each player, the environment consists of the state of the market (i.e., the consumer base) along with the perception of how the competitors are behaving. These give the player a current world view. The firm then passes this knowledge through the Kalman filter to obtain its own parameters for computing market gaining effort. The multi-criteria decision making module is then used to select how to distribute the total budget amongst operational costs. A number of viable alternatives may be generated, however a single solution is selected based on the goal preference of the individual agent. For example, a firm may wish to maximize market share with little regard to how risk is modelled or vice versa.

After each player has taken their actions, the consumers react accordingly.

4.1.1 Consumer Behaviour Implementation

Here we discuss the implementation of consumer behaviour for reacting to actions undertaken by firms. To make the interpretation of the simulation results easier, we also introduce several simplifying assumptions. Although all of these assumptions do not appear
realistic, they could be removed in what follows, once a clear understanding is achieved of how
the research questions of this study could be resolved.

Consumers are modelled as a group and transition from one of two states including the
option to either remain with the current provider, or switch to another provider. If a consumer
leaves some provider, he/she would tend to avoid returning to it for some time. As the simulation
is consisting of only two providers, some consumers after defecting from both providers may be
idling on the marketplace waiting until their ‘bad memories’ dissipate; then these consumers may
choose one of the firms again. This forms an important difference from the early state-transition
model of consumer behaviour; we feel this technique contributes to the state of the art of business
modelling and simulation.

In all computational experiments, the market is initially divided with half of the consumer
population aligned with either firm. Since we are only focusing on some small grouped segment
of the population from a typical metropolitan area, we eliminate transitions involving new
consumers entering the market, and those leaving due to relocation etc. This is in effect a closed
world assumption. Also, in the communication services market we do not expect that consumers
would be idling in the marketplace as such services typically need to be readily accessible.

Below we explain the anticipated behaviour of our model. In what follows, these
expectations will be used as benchmarks to evaluate whether these behaviours have actually been
achieved in the model or not.

Market share is essentially driven by the actions taken by firms. This includes budget
spent on promotion and quality of service. Our model factors in the concept of ‘memory’, where
consumers may react to their perception of the collective actions taken by a firm over a period of
time. We implement heuristics based on two basic principles – that consumers tend to defect
from providers due to poor quality of service, and second that consumers are attracted by
promotion. The inverse of consumer defection essentially yields consumer loyalty due to positive
quality of service.

Consumer reaction to these two factors would typically vary by response rate.
Promotions tend to have an immediate effect followed by drop off, where as quality of service
has more of a tapered off effect – i.e. past actions have some residual significance on consumer reaction. Hence, to help establish a pool of defectors, a coefficient is computed based on the firms’ history of spending, with more recent actions having greater weight, this effectively represents a consumer loyalty factor:

\[
\text{Consumer loyalty} = (1-a)^0 q_n + (1-a)^1 q_{n-1} + \cdots + (1-a)^n q_1, \quad \text{where} \quad 0 < a < 1, \quad \text{and} \quad n \text{ is the number of actions taken thus far.} \tag{5}
\]

A firm with higher cumulative quality of service will not lose consumers at a higher rate than its competitor. Since we are using a closed world model with two players, we expect that a smaller percentage of consumers would defect from the firm investing the greater amount on quality. Of course, in the real world there would be some latent defection rate regardless of spending on quality, though in this study we neglect this factor. Therefore, the ratio of consumer loyalty between firms is used to compute the percentage of defectors for the firm with worse service.

The pool of defectors is then redistributed based on the direct comparative level of spending on promotion between the firms for the most immediate action. For example, if firm 1 spends a net budget of 70% on promotion and firm 2 spends 60%, firm 1 will take 55% of the pool of defectors and firm 2 will take 45%. In this way, consumers demonstrate their level of attraction to promotions by firms, and the closed world model is maintained.

4.2 Multi-Criteria Decision Making

Firms allocate budget towards operational expenses (promotion and quality) with respect to their optimality criteria. In our case, this means attempting to maximize market share, while minimizing risk. This is a trade off, and as such there may be no single solution that will optimize both criteria simultaneously. These types of problems in which multiple objectives are incomparable and/or in conflict with each other require efficient solutions that will find the best compromise between all objectives. Such solutions are also referred to as non-inferior, non-dominated, or Pareto-optimal.
Multi-criteria optimization has been applied to many fields including engineering and economics. There are several well established techniques that cater to finding Pareto-optimal sets, though selection of one appears mostly to be a matter of design preference. Still, some methods have limitations and thus are not suited for all applications. Our discussion follows from [Liu et al., 2003], and [Ehrgott, 2005].

4.2.1 Concepts

The problem of multi-objective optimization can be formalized as simultaneously minimizing the \( n \) objectives \( \Phi_i(p) \), where \( i = 1, \ldots, n \), of a variable vector \( p \) in a decision/variable space \( F \), such that:

\[
p^* = \arg \min_{p \in F} (\Phi_1(p), \Phi_2(p), \ldots, \Phi_n(p)).
\] (6)

The variable space \( F \) encompasses the attainable/feasible set, i.e. the set of alternatives of the decision problem, \( p = [x_1, \ldots, x_m] \). The criteria space contains the image of the feasible set substituted into the objective functions.

Since it is generally not possible to minimize all objective/criteria functions due to inherent conflict, the solution is to search for Pareto-optimality, that has the property where by no single criterion function can be reduced without increasing at least one other objective function. A variable \( p^* \) in the feasible set of decisions \( F \) is said to be efficient if and only if there does not exist any other point \( p \in F \) such that:

a) \( \Phi_i(p) \leq \Phi_i(p^*) \) for all \( i = 1, \ldots, n \)

b) \( \Phi_j(p) < \Phi_j(p^*) \) for at least one \( j \).

If \( p^* \) is efficient, then \( \Phi(p^*) \) is called a non-dominated point. Within the Pareto-optimal set, there are two classifications of solutions. The first are solutions known as weakly efficient; and within this set there is a subset of solutions that also satisfy the additional condition of being
strictly efficient. From the above definitions, these are conditions b) and a) respectively. Also note that the objective functions here are assumed for minimization.

In addition, a feasible solution \( \hat{p} \in F \) is considered properly efficient if it is efficient and has bounded tradeoffs among the objectives:

For all \( i \) and \( p \in F \) satisfying \( \Phi_i(p) < \Phi_i(\hat{p}) \), there exists an index \( j \) where \( \Phi_j(\hat{p}) < \Phi_j(p) \) such that,

\[
\frac{\Phi_j(\hat{p}) - \Phi_j(p)}{\Phi_j(p) - \Phi_j(\hat{p})} \leq M, \text{ where } M > 0 \text{ is a real number.}
\]

Figure 11: Two Criteria Pareto-frontier

Figure 11 (above) illustrates a Pareto-frontier (non-dominated set) in the criteria space for two objective functions. Points in the attainable/feasible set (shaded region) that are not on the boundary are sub-optimal because they are dominated by at least one other point, i.e. both \( f_1(p) \) and \( f_2(p) \) can be improved (reduced). For any point on the Pareto frontier, if \( f_i(p) \) is to be decreased, then \( f_2(p) \) must be increased, and vice versa.

Elaborating on the concept of non-dominance we state the following comparisons with respect to the decision and criteria spaces:
• Either \( p^1 \succ p^2 \) if and only if \( \Phi(p^1) \leq \Phi(p^2) \) (Strict inequality for at least one objective)

• Or \( p^1 \prec p^2 \) if and only if \( \Phi(p^1) \geq \Phi(p^2) \) (Strict inequality for at least one objective)

• Or \( p^1 \sim p^2 \) if and only if \( \Phi(p^1) = \Phi(p^2) \)

• Or \( p^1 \preceq p^2 \) if and only if \( \Phi(p^1) \prec \Phi(p^2) \).

The comparison \( p^1 \succ p^2 \) denotes that \( p^1 \) dominates \( p^2 \). Stated alternatively, \( \Phi(p^1) \) is better than the given evaluation \( \Phi(p^2) \) on at least one objective function and it is not worse on any of the others.

For \( p^1 \sim p^2 \), this indicates that neither solution is dominating the other, i.e. a solution performs better on some criteria than the other, and worse on others. The symbol ‘\( \sim \)’ signifies indifference.

**Figure 12: Non-dominance and Weak Efficiency**

![Figure 12](image)

Figure 12 (above), point \( p^3 \) is dominated by point \( p^1 \) because \( p^1 \) is located in the cone in \( \mathbb{R}^n \) with vertex \( p^3 \); the sides of which are parallel to the coordinate subspaces \( \mathbb{R}^{n-1} \). Point \( p^4 \) is not efficient because it is weakly dominated by \( p^3 \) in that it is worse on the first criterion, but equal on
the second. We can see that \( p^1 \prec p^3 \) because \( p^3 \) is better than \( p^1 \) on criterion 2, and worse on criterion 1. Therefore, both solutions are considered part of the Pareto set. Also note that minimization of the objective functions is assumed.

For completeness, we give the formal definitions of a cone and convexity as stated in [Ehrgott, 2005]:

A subset \( C \subseteq \mathbb{R}^n \) is called a cone, if \( \alpha d \in C \) for all \( d \in C \) and for all \( \alpha \in \mathbb{R}, \alpha > 0 \).

A cone \( C \) in \( \mathbb{R}^n \) is called:

- Nontrivial or proper if \( C \neq \emptyset \) and \( C \neq \mathbb{R}^n \),
- Convex if \( \alpha d^1 + (1 - \alpha)d^2 \in C \) for all \( d^1, d^2 \in C \) and for all \( 0 < \alpha < 1 \),
- Pointed if for \( d \in C, d \neq 0, -d \notin C \), i.e., \( C \cap (-C) \subseteq \{0\} \).

Figure 13: Continuous and Disconnected Pareto Frontier

Figure 13 (above) illustrates a feasible set in continuous space. Points on segment \( \overline{AB} \) are all weakly efficient with point \( B \) being strictly efficient. Curve \( \overline{CD} \) comprises efficient solutions. Together, segment \( \overline{AB} \) and curve \( \overline{CD} \) make up the Pareto frontier. All points in the cone \( P_1 \) are dominated by point \( p \), which is non-dominated itself because cone \( P_2 \) does not contain
any points from the feasible set. In addition to being disconnected, the set of non-dominated points need not be convex.

We now introduce *ideal* and *nadir* points as the lower and upper bounds respectively on non-dominated points [Ehrgott, 2005]:

The ideal point \( P^I = (P_1^I, \ldots, P_n^I) \) is given by \( P_i^I = \min_{p \in F} \Phi_i(p) \).

The nadir point \( P^N = (P_1^N, \ldots, P_n^N) \) is given by \( P_i^N = \max_{p \in F} \Phi_i(p) \).

Since these indicate the range of attainable values for a Pareto set, they can be useful in finding a preferred solution from the set of efficient solutions. The efficient set of solutions does not need to be calculated to compute the ideal point, which makes finding it relatively simple.

**Figure 14: Ideal and Nadir points**

\[\begin{align*}
  f_1(p) \\
  f_2(p)
\end{align*}\]

In summary, we find that the elements of multi-criteria optimization problems are:

- The feasible set \( F \),
- The objective function vector \( \Phi = (\Phi_1, \ldots, \Phi_n) : F \to \mathbb{R}^n \),
- The objective space \( \mathbb{R}^n \),
• The ordered set \((\mathbb{R}^n, \preceq)\),

• The model map \(\theta\).

The first three elements make up the data of the problem. The model map essentially provides a link between the objective space and the ordered set. From this, we get our aforementioned definition of minimization (Eq. 6).

### 4.2.2 Method Classifications

There are three main categories of multi-criteria decision making methods [Huang & Masud, 1979]:

- *a priori* articulation of preferences
- *a posteriori* articulation of preferences
- Progressive articulation of preferences

With a priori methods, individual objectives are aggregated into a single utility function, effectively reducing the problem to a single objective before optimization occurs. Computational efficiency with these algorithms is good, because optimization happens only once. In a posteriori methods, a non-inferior solution set is first computed, and then a single solution is selected from that set based on preference trade-offs. For progressive methods, decision making and optimization occur concurrently. That is, on each cycle preference information is utilized to generate best alternatives.

Discrimination of candidate solutions can be done through weights, priorities, or goal values. Weighting coefficients assign a level of importance to objectives, with respect to an overall utility measure. Priorities represent the order by which the set of objectives should be optimized. Goal values indicate the desired level of performance for each criterion. In contrast to weights and priorities which are expressed numerically, goals can be represented by specific conditions such as maintaining a certain level of performance on an objective.

In general, finding a best compromise solution \(p'\) is a problem of maximizing the utility of the set of all objectives, as:
\[
\begin{align*}
p' &= \arg \max (u(\Theta(p))) = \arg \max (u(\Phi_1(p), \cdots, \Phi_n(p))).
\end{align*}
\]

Where an explicit utility function can be constructed, single objective solutions can be applied. However this is often not possible. There exist several algorithms for multi-objective optimization. Though a complete technical breakdown of each is beyond our scope, we briefly mention some of the commonly used methods, and then further elaborate on the fundamental simple weighting method. More details can be found in [Liu et al., 2003] and [Ehrgott, 2005].

The method of weighted sums attaches weighting coefficients to the objectives, effectively producing a single scalar objective. Where the solution set is non-convex, this method may be ineffective. A technique that can alleviate some of the convexity problems is the \(\varepsilon\)-constraint method. Here, a \textit{main} objective is minimized while the others are expressed as inequality constraints. Disadvantages are that users must select appropriate values of the constraint in order to find a feasible solution; and that hard constraints are often inadequate for representing design objectives. Another method known as 'minimax', attempts to find a best compromise by minimizing the deviation between feasible solutions and the ideal point (see pg. 55), given a set of weights.

We now describe the general process of applying the simple weighting method for generating efficient solutions as in [Liu et al., 2003]. A two criterion optimization problem can be stated as:

- Minimize \(\Phi_1(p)\),
- Minimize \(\Phi_2(p)\),
- where \(p \in F\).

Then, given that compensation among the two criteria is allowed, and that the image of the feasible set in the criteria space is convex, we can rewrite the problem as a single utility function as:
Minimize: \( \Phi(p) = \omega_1\Phi_1(p) + \omega_2\Phi_2(p) \),

where \( \omega_1 \geq 0 \), \( \omega_2 \geq 0 \) are weighting factors.

Dividing the equation by \( \omega_1 \) (given that \( \omega_1 > 0 \)) and designating \( \omega = \frac{\omega_2}{\omega_1} \) yields:

\[
\Phi(p, \omega) = \Phi_1(p) + \omega\Phi_2(p).
\]

Since the feasible set is convex, the optimal solution here will also be efficient. Which particular solution is generated varies with the selection of \( \omega \).

**Figure 15: Simple Weighting Method Linear Indifference Curves**

The objective function essentially represents the line given by:

\[
\Phi_1 + \omega\Phi_2 = \alpha, \text{ or }
\]

\[
\Phi_2 = -\frac{1}{\omega} \Phi_1 + \frac{\alpha}{\omega}, \text{ where } \alpha \text{ is constant.}
\]
Clearly we can see that the slope is $-\frac{1}{\omega}$, and the vertical intercept is $\frac{\alpha}{\omega}$. Figure 15 (above) demonstrates the same line at two separate positions in the feasible objective space. Since all points on the lines have the same weighted objective value, the lines are referred to as linear indifference curves. Thus, points $B$ and $C$ are equally efficient (or equally inefficient as the case may be).

The solution is found by moving the indifference line (in parallel) towards the origin until it becomes tangent to the feasible set. This is represented by point $A$ in the figure, and we can see that it is indeed efficient.

Setting different weights basically has the effect of rotating the indifference line so that different solutions are found.

**Figure 16: Simple Weighting Method with Different Weights**

In figure 16 (above), $\omega^1$ is set to $\omega > \omega^1 > \omega$. This yields the solution at point $D$. While the weight of the second objective function is increased, it comes at the expense of the first criterion. Conversely, with the weight $\omega^2$ set to $\omega > \omega^2 > 0$, more weight is attached to minimizing the first objective at the cost of increasing the second, and we get the solution at point $E$. 

59
With a weight set to 0, the decision maker is essentially ignoring the second objective and solely attempting to optimize the first. Likewise, with weight set to $\infty$, the first objective is not considered and only $\Phi_1(p)$ is minimized.

Optimization problems with more objective functions are solved in a similar fashion. Though we reiterate that in a non-convex criteria space, this technique may not be capable of generating efficient solutions from the Pareto frontier.

### 4.2.3 Pareto Set Construction and Sequential Elimination Algorithm

In a continuous space, there may be an infinite number of efficient solutions. Even in a discrete space, the possibility of a large number of solutions is high, due to the conflicting objectives. Hence, the process of decision making involves two main steps:

1. Constructing the set of efficient solutions
2. Finding the best compromise solution from the Pareto set

As previously mentioned, we can have a Pareto set that is continuous or discrete, connected, or disconnected, linear or non-linear. In addition, the type of algorithm we apply to our problem is largely dictated by whether or not its efficient set exhibits convexity.

**Figure 17: Convex Pareto Frontier**
For a Pareto frontier to be considered convex, any given pair of points within the set must be connectable by a line segment that does not intersect with the curve generated by the efficient set of solutions. An example of this is shown in Figure 17 (above).

Conversely, in a non-convex Pareto set, there exists at least two points where the straight line connecting those points does cross the Pareto set. This is illustrated in Figure 18 (below). It can be seen that although segment $\overline{AB}$ does not cross the non-dominated set, segment $\overline{CD}$ does, thus making the set non-convex.

**Figure 18: Non-Convex Pareto Frontier**

Though we would like our algorithm to handle cases of non-convexity regardless, we can also show that generation of a convex Pareto frontier cannot be guaranteed for each firm on each decision cycle. In Figure 19, we demonstrate a case where a non-convex Pareto set is formed. The figure is a screenshot of an application that solves the decision making problem for a single firm. By manually setting the values of the constant sensitivity parameters, we can see the resultant Pareto frontier shown in red colour in the criteria space. The area in the decision space corresponding to the Pareto set is also highlighted in red.

Worth noting, is that although the multi-criteria decision making problem is inherently continuous in nature, we elect to use a discrete approximation of the decision space (in turn leading to a discrete criteria space) in order to greatly reduce computing complexity.
The graph on the left side of the figure (decision space) shows the decision variables of quality and promotion, and the graph on the right side shows the image of the feasible set in the criteria space (objective/criteria space), with objectives of risk and gain; (risk is modelled using additional decision variables in the Java applet, though this is just for testing purposes). Notice that since we are minimizing the objective functions, we plot risk against negative gain.

The red points indicate efficient solutions. By inspection, we can see that it is not possible for the two non-dominated points in the criteria space (marked by the two arrows) to be connected by a line without crossing the Pareto frontier, thus suggesting non-convexity.

Figure 19: Example of Possible Non-Convexity in Market Game

In addition to being non-convex, we can classify our multi-criteria decision making problem as discrete (since we are discretizing the continuous space), finite, connected (in that the interval between each successive discrete point on the Pareto frontier is the same), and non-linear.
Before determining the Pareto set, we approximate the feasible set in the decision space with a fixed number of points. In this case, the term feasible refers to constraints applied by the decision variables. Since we are not dealing with firms taking out loans or paying back debt, the maximum borrowing factor is fixed at 1 (i.e. neutral). This means that spending on promotion and quality cannot exceed 100% of the net budget; hence the triangular shape of the decision space. For example, in Figure 19, the maximum borrowing factor is 2.5, so if we consider the point at 2.5 on the x-axis (promotion), we can see that there is no budget left for quality. After the discrete set of points in the decision space is generated (i.e. the set of potential solutions), each point is mapped over to the objective space to form a feasible set. From there, optimization process begins which consists of determining the Pareto set and finding the optimal point in it.

The Pareto set construction involves searching the finite feasible set and comparing each element to the remaining ones, removing those that are dominated. The algorithm is given in figure 20 (below).

**Figure 20: Pareto Set Construction Algorithm**

- **Input:** Feasible set vector

  Loop through each element i in the feasible set
  
  Loop through each element j in the feasible set
  
  Loop through each criterion
  
  If either element (given i≠j) is >= (i.e. worse) on the objective, mark it as such (increment value).

  If either element is worse than the other on both criteria (dominated point), then remove it from the vector and decrement the loop indices.

Return Pareto set vector

Note that the algorithm also eliminates all weakly dominated points. All points that are better than all other points on at least one criterion are non-dominated and remain part of the efficient (Pareto) set.
The final step is to select a point from the Pareto set that represents a best compromise solution. This is based on the additional information about the firm’s attitude to risk and gains; with \( n \) criteria it is expressed as asset of \( n \) weights. This in effect represents their level of partiality to the objective functions. The weights are defined as:

\[ w_1 + w_2 + \ldots + w_n = 1, \text{ where } n \text{ is the number of criteria, and } w \geq 0. \] (8)

Since in our case the problem is not necessarily convex, we employ an algorithm known as ‘the sequential elimination of the poorest alternative’. With \( K \) points in the discreet approximation of the Pareto set, this algorithm guarantees finding the balanced (optimal) solution in \( K-1 \) iterations, no matter what the shape the original infinite Pareto frontier has. Originally this method has been applied towards optimization of ball passing decisions in a digital soccer game [Kyrylov, 2006].

The idea underlying this algorithm is in eliminating the least suitable point from the Pareto set by considering just one criterion on each iteration. Since all criteria are applied in turns, the elimination process ends in a balanced solution if the size of the set \( K \) is substantially greater than the number of criteria \( n \).

The weights are essentially the probabilities that a specific criterion will be selected for minimization on the current iteration. Hence they give each firm an individual strategy characteristic. For example, one player may wish to take high risk, while the other plays more cautiously. These attitudes are initialized at the beginning of the game and remain fixed throughout. With greater weight assigned to the risk factor, higher-risk alternatives would be more likely to be eliminated, leaving lower risk alternatives for potential selection. Thus risk-aversive strategies are characterized by higher weights of the risk factor.

The sequential elimination of the poorest alternative algorithm is given in Figure 21.
Figure 21: Sequential Elimination Algorithm

\[ S := P; \]
Loop \((k := 1 \text{ to } K-1)\) (where \(K\) is the number of elements in the Pareto set)

- With probability \(w_j\), randomly select the \(j\)th criterion;
- Find element \(x \in S\) having the maximal value of \(g_j(x)\);
- Remove \(x\) from \(S\);
Return last element in \(S\)

Where the set of all alternatives is denoted by \(X\), \(x \in X\) is a decision vector. The Pareto set mapped back to the decision space is then given by \(P \subseteq X\), and the criteria functions are designated as \(g_1(x) \ldots g_n(x)\).

On each cycle, the poorest alternative of the selected criterion is removed from the set. Obviously, those objectives with higher probabilities (those that firms are more biased towards) will be selected more frequently. For example, consider a firm that has been initialized with a weight of 0.3. This gives a 70% chance that the criterion for maximizing gain will be selected for optimization. Likewise there would be a 30% chance of minimizing risk. Whichever criterion is selected, the point with the maximum value (i.e. the worst alternative) is eliminated from contention.

This is repeated until only one decision vector remains. The computational complexity of the algorithm is \(O(K^2)\). Another attractive feature is that it does not rely on any prior information about the objective functions.

4.3 Parameter Estimation

As previously described, the market model we have presented (chapter 3) requires several unknown parameters of market gaining effort (Eq. 4) be estimated for each player. In real life, these parameters stand for the prior knowledge possessed by the decision makers. In our model, we use the parameter estimates obtained as described below.

To reiterate, our requirements (section 2.2) are that the estimator be optimal, deterministic, and non-linear. Optimality refers to the need to minimize the degree of error in
estimation. The estimator should be deterministic in that it should take advantage of multiple
noisy measurements input into the system as feedback. The requirement for non-linearity is
obvious, as the process to be modelled (market gaining effort) is intuitively a nonlinear function.

As the model parameters of our interest cannot be measured explicitly, we are using an
implicit method of estimation. Thus we define the problem of estimating the implicit effect of
disturbance \( d \) on process \( y \) from the measured effect of disturbance on \( x, d \), as in [Brosilow &
Joseph, 2002]:

Process modelled as:

\[
\begin{align*}
x(s) &= A^T(s)d(s) + p_x(s)u(s), \\
y(s) &= b^T(s)d(s) + p_y(s)u(s),
\end{align*}
\]

where

- \( x(s) \) is the \((p \times 1)\) vector of secondary measurements,
- \( y(s) \) is the scalar primary output variable to be controlled,
- \( u(s) \) is the manipulated (scalar) variable,
- \( A^T(s) \) is the \((p \times n)\) matrix of transfer functions,
- \( a_{ik}(s) \) is the transfer function relating inputs \( i \) and output \( j \),
- \( b^T(s) \) is the \((1 \times n)\) vector of transfer functions relating \( y(s) \) and \( d(s) \),
- \( p(s) \) is the transfer function matrix relating \( y(s) \) and \( u(s) \),
- \( d(s) \) is the random input disturbances process.

In effect, the vector of measurements refers to our unknown parameter vector (which is a
set of constants) and the output is the market share optimality criterion. Random disturbances
may come in the form of uncertainties with respect to the market. We assume these disturbances
to be white, and distributed normally from a zero mean. We also assume that there is no
correlation between the process and measurement noise. Basically, firms are able to make
educated guesses as to the predicted state of the market based on the past. These measurements
may not be entirely accurate because firms do not have a precise view of their opponents' actions.
Thus we need a way of using these measurements to make an optimal estimate of the system/process (market share criterion) which cannot be measured directly. This can be thought of as a black-box where by firms can see the output of the system, i.e. the market share, but they cannot directly observe the internal states of the system, i.e. the parameters of the market share function. A technique for filtering out the inherent disturbances and arriving at a best estimate of the process is known as the Kalman filter.

Before delving into the subject of the Kalman filter, we provide definitions for mean and covariance with respect to matrix algebra from [Lay, 2003], since we refer to these terms throughout our discussion.

Let $[X_1 \ldots X_N]$ be a $p \times N$ matrix of observations (where $p$ represents the number of coordinates of each vector. The sample mean of observation vectors $X_1, \ldots, X_N$ is:

$$M = \frac{1}{N}(X_1 + \cdots + X_N).$$

For $k = 1, \ldots, N$, let $\hat{X}_k = X_k - M$.

The columns of the $p \times N$ matrix $B = [\hat{X}_1 \cdots \hat{X}_N]$ have sample mean of zero. Then, the $p \times p$ covariance matrix $S$ is given by:

$$S = \frac{1}{N-1} B B^\top.$$

For elements $S = [s_{ij}]$, where $i=j$ the scalar values represent the variances, i.e. the spread of the values on $x_j$ of a vector $X$ that varies over the set of observation vectors. Where $i \neq j$, the elements represent the correlations between $x_i$ and $x_j$.

Traditionally, the Kalman filter has been used for such applications as aerospace, radio communications, embedded systems, robot localization, and trajectory control [Simon, 2001]; less frequently, it has also been used for economic growth forecasting [Bomhoff, 1996], and other fields. By using the Kalman filter in the field of business modelling and simulation, we thus further extend the area of application of this advanced technique.
The Kalman filter uses a set of mathematical equations to provide recursive means to estimate the state of a process so as to minimize the mean of the squared error. It supports estimations of past, present, and future states, and does not require the precise nature of the system to be known [Welch & Bishop, 1995]. In what follows, we discuss the discrete Kalman filter as described in [Ribeiro, 2004], [Welch & Bishop, 1995], [Sorenson, 1980], [Maybeck, 1979], [Miller & Leskiw, 1987], and [Simon, 2006].

**Figure 22: Kalman Filter Application**

The Kalman filter uses smoothening in that it optimally combines two unbiased estimates of the state vector to obtain an improved estimate. One is the measurement and the other is the previous known state. Unbiased estimates are those where the expected value is the same as that of the quantity being estimated. One of the key features of Kalman filters is that they can incorporate all information that can be passed to them to estimate the variables of interest. This includes all available measurements (with any degree of precision). To do this, it uses knowledge of the system/measurement dynamics, noise, measurement errors, and uncertainty in the models. Due to the recursive nature of the filter, previous data does not need to be kept in storage and reprocessed each time a measurement is taken. This is in contrast to other solutions such as the Wiener filter [Brown & Hwang, 1992], which operates directly on all data for every estimate.
For this reason, the Kalman filter also provides us with a practical solution in terms of implementation.

We now modify the previous process formulation to fit the general filtering model:

Let \( x(k+1) = f(x(k), u(k), w(k)), \)

\( z(k) = h(x(k), v(k)) \)

be state dynamics of a general non-linear time-varying system, where

- \( x \in \mathbb{R}^n \) is the system state vector
- \( f(...) \) defines the system’s dynamics,
- \( u \in \mathbb{R}^m \) is the control vector,
- \( w \) is a vector containing system error sources,
- \( y \in \mathbb{R} \) is the observation vector,
- \( h(...) \) is the measurement function,
- \( z \) is a vector representing measurement errors.

Given:

- \( f, h, \) noise characterization, initial conditions,
- set of controls, \( u(0), u(1), ..., u(k-1), \)
- set of measurements, \( y(1), y(2), ..., y(k), \)

Find:

- best estimate of \( x(k) \).

The state, \( x \in \mathbb{R}^n \), of a process to be estimated by the Kalman filter is governed by the following linear stochastic difference equation:

\[
x_k = Ax_{k-1} + Bu_{k-1} + w_{k-1}, \tag{9}
\]
with measurement $z \in R^n$: $z_k = Hx_k + v_k$ \hspace{1cm} (10)

The variables $w_k$ and $v_k$, represent the process and measurement noise respectively. Though we do not know the precise values of the noise at each time step, we can still approximate the state and measurement vectors without them. The noise is white, independent, and has normal probability distribution. The covariance matrices are then defined as:

$p(w) \sim N(0,Q)$,

$p(v) \sim N(0,R)$, where

$N(m,P)$ denotes a normal distribution with mean $m$, and covariance $P$.

The matrix $Q$ represents covariance of the state random component change, and $R$ gives the measurement noise covariance.

The matrix $A$ is of dimension $n \times n$, and relates the state at time step $k - 1$ to the state at the current step. The matrix $B$ is an optional control input to the state $x$. The matrix $H$ is of dimension $m \times n$, and relates the state to the measurement $z_k$.

The filter works by taking the noisy measurements and applying corrections, then using the corrected estimates to make new predictions for the next time step. So there are essentially two phases to the filter – the “time update”, where $a$ priori predictions are made of the current state and error covariance for the next step, and the “measurement update”, where the predictions are adjusted $a$ posteriori to account for errors in the prediction. The measurement updates provide feedback to the system in that they incorporate the new measurements to improve the $a$ priori estimates. Figure 23 demonstrates the discrete Kalman filter algorithm.
The error covariance matrix $P_k$ tells us how much confidence there is in the state:

$$E[(x_k - \hat{x}_k)(x_k - \hat{x}_k)^T] = P_k,$$

$$p(x_k \mid z_k) \sim N(\hat{x}_k, P_k).$$

Note that the a posteriori state estimate reflects the mean (or first moment) of the state distribution.

Since $H$ converts the state to a measurement, the difference between the actual measurements and the measurement prediction gives us the measurement prediction error, $(z_k - H\hat{x}_k)$ or 'innovation' vector. Given this vector, the Kalman gain specifies by how much the state should be adjusted. Thus, the corrected a posteriori state estimate $\hat{x}_k$ is drawn from the a priori estimation made on the last time step $\hat{x}_k$ combined with the weighted difference between the actual measurement and the measurement prediction $K_k(z_k - H\hat{x}_k)$.
The \((n \times m)\) Kalman gain matrix is essentially a 'blending' factor that minimizes the \textit{a posteriori} error covariance \(P_k\). Rewriting the equation we get:

\[ K_k = \frac{P_k^{-1}H^T}{HP_k^{-1}H^T + R}, \tag{11} \]

As the measurement error covariance \(R\) approaches zero, the gain \(K\) weights the measurement innovation (residual) more heavily:

\[ \lim_{R_k \to 0} K_k = H^{-1}. \]

Also, as the \textit{a priori} estimate error covariance \(P_k^-\) approaches zero, the gain weights the residual less heavily:

\[ \lim_{P_k^- \to 0} K_k = 0. \]

What this means is that the actual measurement \(z_k\) is trusted more as the measurement error covariance tends to zero, and the predicted measurement is trusted less. Likewise, \(z_k\) is trusted less as the \textit{a priori} estimate error covariance tends to zero, and the predicted measurement is trusted more.

### 4.3.1 Extended Kalman Filter

We have discussed the discrete Kalman filter for parameter estimation. However this method works only on a linear stochastic difference equation [Welch & Bishop, 1995], and as is evident, we are dealing with a non-linear process. The solution is to linearize the movement model about the current mean and covariance – this technique is known as the extended Kalman filter (EKF) which we now describe following the references listed in the previous section.

The linearization involves using partial derivatives from the movement model (i.e. the process and measurement functions) to extract the estimates. This is achieved with the use of Jacobian matrices [Magnus & Neudecker, 1999].
A Jacobian matrix is a matrix of first-order partial derivatives of a vector based function. It gives the best linear approximation to a differentiable function near a given point.

Given a function $F: \mathbb{R}^n \rightarrow \mathbb{R}^m$ with components $y_1(x_1, \ldots, x_n), \ldots, y_m(x_1, \ldots, x_n)$, the Jacobian matrix of $F$ is the $m \times n$ matrix:

$$
\begin{bmatrix}
\frac{\partial y_1}{\partial x_1} & \cdots & \frac{\partial y_1}{\partial x_n} \\
\vdots & \ddots & \vdots \\
\frac{\partial y_m}{\partial x_1} & \cdots & \frac{\partial y_m}{\partial x_n}
\end{bmatrix}
$$

Figure 24 shows the Kalman filter algorithm, modified to handle a non-linear process.

**Figure 24: Extended Kalman Filter Operation**

### Time Update ("Predict")

1. Project the state ahead
   $$\hat{x}_k^- = f(\hat{x}_{k-1}, u_{k-1}, 0)$$

2. Project the error covariance ahead
   $$P_k^- = A_k P_{k-1}^T A_k^T + W_k Q_{k-1} W_k^T$$

### Measurement Update ("Correct")

1. Compute the Kalman gain
   $$K_k = P_k^T H_k^T (H_k P_k^T H_k^T + V_k R_k V_k^T)^{-1}$$

2. Update estimate with measurement $z_k$
   $$\hat{x}_k = \hat{x}_k^- + K_k (z_k - h(\hat{x}_k^-, 0))$$

3. Update the error covariance
   $$P_k = (I - K_k H_k) P_k^-$$

The new process is directed by non-linear functions $f$, and $h$:

$$x_k = f(\hat{x}_{k-1}, u_{k-1}, w_{k-1}), \quad (12)$$
\[ z_k = h(x_k, v_k) . \] (13)

Where \( f \) relates the previous state to the current state, and \( h \) relates the state to the measurement.

To linearize the movement model, we form approximations of the actual state and measurement vectors as [Welch & Bishop, 1995]:

\[ x_k \approx \hat{x}_k + A(x_{k-1}, \hat{x}_{k-1}) + Ww_{k-1}, \] (14)

\[ z_k \approx \hat{z}_k + H(x_k, \hat{x}_k) + Vv_k, \] (15)

Where\(^1\):

- \( A \) is the Jacobian matrix of partial derivatives of \( f \) with respect to \( x \)

\[ \Rightarrow A_{(x,u)} = \frac{\partial f_{(x,u)}}{\partial x} (\hat{x}_{k-1}, u_{k-1}, 0) \]

- \( W \) is the Jacobian matrix of partial derivatives of \( f \) with respect to \( w \)

\[ \Rightarrow W_{(x,w)} = \frac{\partial f_{(x,w)}}{\partial w} (\hat{x}_{k-1}, u_{k-1}, 0) \]

- \( H \) is the Jacobian matrix of partial derivatives of \( h \) with respect to \( x \)

\[ \Rightarrow H_{(x,v)} = \frac{\partial h_{(x,v)}}{\partial x} (\hat{x}_0, 0) \]

- \( V \) is the Jacobian matrix of partial derivatives of \( h \) with respect to \( v \)

\[ \Rightarrow V_{(x,v)} = \frac{\partial h_{(x,v)}}{\partial v} (\hat{x}_0, 0) \]

\(^1\) Note that time step subscripts are omitted for the Jacobian matrices; however they do change on each cycle.

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74
Worth noting is that the Jacobian matrix $H_k$ affects the Kalman gain such that only the relevant part of the measurement is carried over. This means that it will only magnify the part of the residual that affects the state, even if there is not a one-to-one mapping between the measurement and the state [Welch & Bishop, 1995].

### 4.3.2 Filter Implementation

We reiterate that by estimating the unknown parameters of market gaining effort, we simulate the prior knowledge that in real life a firm possesses about the world in which it is acting. Even though it is not always precise, this knowledge is normally in place. So we want to obtain estimates of the four sensitivity parameters from: $W = (bp + p_0)^{x_s} (bq + q_0)^{x_t}$, which are $p_0$, $a_p$, $q_0$, and $a_q$. These represent that prior knowledge. The measured parameter in our case is the market share gained as a result of certain sets of actions by the firms.

The filter is run at the beginning of the simulation (before the main loop) for each player. Players take trial actions at each recursive level, and measurements of the market share are taken. The trial actions are generated randomly in order to uniformly fill the space of all feasible actions. The procedure ends when reasonably good precision of the parameter estimates is achieved.

**Figure 25: Market Gaining Effort Parameter Estimation**
If during the simulation the difference between the actual market share and expected market share grows (to a user defined amount), the filter re-computes the parameters. Since there is no movement in the model i.e. the parameters remain constant, the filter complexity is slightly reduced in that the transition matrix $A$ is simply equal to the identity.

Before estimating the parameters, the following initialization is used:

For function $f = (bp + x_o)^n (bq + x_2)^n$, the state vector

$$x = [p_0 \ a_p \ q_0 \ a_q]^T.$$  \hspace{1cm} (16)

Due to the constraint that $a_p + a_q = 1$, we can simply set $a_q = 1 - a_p$, and reduce the state vector to three elements.

Since our certainty in the model is low, we initialize the error covariance matrix as:

$$P = \begin{bmatrix} \infty & 0 & 0 & 0 \\ 0 & \infty & 0 & 0 \\ 0 & 0 & \infty & 0 \\ 0 & 0 & 0 & \infty \end{bmatrix}.$$  

The variances are set to a large amount (usually the maximum integer value allotted on a given platform), and there are no correlations between the parameters.

Noise levels are adjustable, and are set at the discretion of the user.

Although we are dealing with a process containing a single function, we have not limited our implementation by hard coding the Jacobian matrix calculations. The framework should facilitate possible future modifications to the economic models. Thus, our calculations are close approximations for the partial derivatives. Matrix arithmetic is done using the Java Matrix Package (JAMA). The high level algorithm is shown in Figure 26.
Figure 26: High-Level Algorithm for Jacobian Calculation

- Input: Number of functions, Parameter vector, \( \Delta \)

Loop through each function, \( f \)

Loop through each parameter, \( p \)

Loop through updated parameter, \( u \)

\[
\text{if}(c == p) \{
\text{updated}[u] = \text{parameter}[u] + (\text{param}[u] \times \Delta);
\}
\]

\text{else} \{
\text{updated}[u] = \text{parameter}[u];
\}

\[
\text{JacobianMatrix}[f][p] = \frac{\text{calculateFunction}(f, \text{updated}) - \text{calculateFunction}(f, \text{parameter})}{(\Delta \times \text{parameter}[p])}.
\]

For each function we are computing the derivative with respect to each parameter. These elements then make up the Jacobian matrix as described previously.

Figure 27: Derivative Approximation

For example, the derivative of the \( i \text{th} \) function of vector \( h \) with respect to the \( j \text{th} \) parameter of vector \( x \) is given by:

\[
\frac{\partial h^{(i)}(x)}{\partial x^{(j)}} \approx \frac{h^{(i)}(x^{(i)}, \ldots, x^{(j)}(1 + \Delta), \ldots, x^{(m)}) - h^{(i)}(x^{(i)}, \ldots, x^{(j)}, \ldots)}{x^{(j)}(1 + \Delta) - x^{(j)}}
\]

(17)
4.4 Chapter Summary

In this chapter we described the overall architecture of the competitive service market game including the internal workings of firms and consumer behaviour. Moreover, we have established a multi-agent framework that conforms to the requirements of our simulation domain. We have also provided the theory behind the methods applied for multi-criteria decision analysis and parameter estimation that allow for firms to make optimal decisions with respect to their objective functions.
5 APPLICATION OF THE PROPOSED FRAMEWORK

Our effort in this section is to verify our multi-criteria decision making model and Kalman filter operation. To test the current implementation of the framework, we then simulate various scenarios where players with different attitudes towards optimality criteria are pitted against each other and observe their decisions and the consumer reaction.

5.1 Method Verification and Validation

Referring back to our simulation domain requirements, we want the outcomes of the competitive service industry game simulations to display broad trends that are suggestive of the real world. Aggressive behaviour should lead to increased gains in market share, and alternatively passive players should be able to lessen risk. Firms with similar attitudes should be observed to be more competitive with each other.

Before setting up our sample gaming simulation scenarios we run tests to ensure that the Kalman filter is converging on the parameters and that the multi-criteria decision making algorithm is selecting Pareto-optimal alternatives.

5.1.1 Filter Verification

Since we do not know the actual values of the parameters and there is no movement in the business model (i.e. the parameters remain constant), we can verify convergence of the filter through comparing the average estimation error. Therefore, we devise an independent example of the filter with movement in order to demonstrate its properties and functional correctness of our implementation. The system is modelled as follows:
\[
\begin{bmatrix}
    p_{x+1} \\
    v_{x+1}
\end{bmatrix} =
\begin{bmatrix}
    1 & T \\
    0 & 1
\end{bmatrix}
\begin{bmatrix}
    p_x \\
    v_x
\end{bmatrix} +
\begin{bmatrix}
    T^2/2 \\
    T
\end{bmatrix} a_x,
\]
where \( p \) represents position and \( v \) gives velocity.

The value of \( T \) represents the time between two consecutive steps, and \( a_x \) factors noise.

Figure 28 shows the Kalman gain properties with increasing amounts of process noise. As we would expect, with a higher degree of noise, the Filter puts greater credence on the measurement since there is less confidence in the estimation. We can see that with process noise of 100, the filter gain settles close to 0.6, meaning that it is relying heavily on the measurement. With minimal process noise, the gain easily converges to zero, indicating that adjustments due to the measurements have minor significance.

Figure 28: Extended Kalman Filter Gain with Varying Process Noise
Figure 29: Filter Convergence (0 Process Noise)

Extended Kalman Filter (Process Noise \( w=[0;0] \))

![Graph showing filter convergence with 0 process noise. The graph displays position over time steps, with actual and estimated measurements plotted.]

Figure 30: Filter Convergence (0.1 Process Noise)

Extended Kalman Filter (Process Noise \( w=[0.1;0] \))

![Graph showing filter convergence with 0.1 process noise. The graph displays position over time steps, with actual and estimated measurements plotted.]
Figure 31: Filter Convergence (10 Process Noise)

Extended Kalman Filter (Process Noise $w=[10;0]$)

- Actual
- Estimated
- Measurement

Time Step

Figure 32: Filter Convergence (100 Process Noise)

Extended Kalman Filter (Process Noise $w=[100;0]$)

- Actual
- Estimated
- Measurement

Time Step
The four figures above (Fig. 29, 30, 31, 32) demonstrate the filter convergence for each of the variations in process noise, which meets qualitative expectations. Clearly, as the noise is increased the process becomes more erratic, and thus estimations are relying heavily on measurements that are tightly spaced around the skewed process. When process noise is relatively low (as in the first three graphs) and the filter gain settles near zero, little stock is placed in the measurement error and the filter converges close to the actual process.

Thus we can surmise that the developed implementation is valid for practical purposes.

5.1.2 Kalman Filter Convergence

The table and figure below (Table 2, Fig. 33) illustrate the average estimation error of the six unknown model parameters by the extended Kalman filter across 200 simulation runs. We can see that by the sixth step in the filter cycle, the filter has achieved near convergence. This is expected, since there is no present noise or movement in the model, the filter should converge after $n$ recursive cycles, where $n$ represents the number of parameters to be estimated.

Table 2: Kalman Filter Error Estimation

<table>
<thead>
<tr>
<th>Step</th>
<th>Average Estimation Error</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>157.8441</td>
</tr>
<tr>
<td>2</td>
<td>147.071</td>
</tr>
<tr>
<td>3</td>
<td>92.19721</td>
</tr>
<tr>
<td>4</td>
<td>3.088113</td>
</tr>
<tr>
<td>5</td>
<td>2.126457</td>
</tr>
<tr>
<td>6</td>
<td>1.10E-08</td>
</tr>
<tr>
<td>7</td>
<td>3.00E-09</td>
</tr>
<tr>
<td>8</td>
<td>2.68E-09</td>
</tr>
</tbody>
</table>
Modelling of process noise is beyond our scope; however this could manifest itself in a number of ways as external market conditions can skew the perception of market gaining effort.

5.1.3 Multi-Criteria Decision Making Trial

The two figures below (Fig. 34, 35) illustrate the feasible area in the decision space and its pattern mapped into the criteria space. The yellow triangle is the decision found within the multi-criteria decision making module. Note that the maximum borrowing factor is set to 1 implying that the sum of the promotion and quality of service decision variables should be less than or equal to 1; (no extra funds are borrowed, yet some funds are diverted to the outside if \( p+q<1 \)). Thus, in this case although borrowing is a decision parameter it can essentially be neglected since firms do not borrow greater than a factor of 1. This allows us to view promotion and quality from a neutral standpoint when comparing the spending between firms in the simulations.

Within the discrete, finite decision space, the mapped Pareto set is highlighted, along with the final selection. The criteria space shows a convex set where we can observe visually that the frontier is being accurately computed. Through manually tuning the parameters outside of the main simulation, we can graphically verify the functional correctness of the algorithm.
Although an investigation of the computational performance of the decision making algorithm is beyond our scope, we found that the decision space had to be limited to less than five thousand discrete points in order to observe smooth and timely operation on our test platform. The number of decision variables and objective functions present would also obviously limit the execution speed. This could potentially hinder the achievable level of realism given the high degree of complexity involved in accurately modelling such criteria as risk.

Figure 34: Decision Space Illustration
5.2 Competitive Market Scenario

Now that we have established the functionality of the technologies composing the framework, we run several scenarios to observe the qualitative trends that emerge. The purpose of our first scenario is to see if multi-criteria decision making can be applied towards selection of budget items that results in competitive market behaviour amongst firms.

5.2.1 Simulation Properties

The simulated game consists of two players/firms who each have the same goals. A goal is identified as a firm’s partiality towards the criteria of gaining market share and minimizing risk. In this case, each firm has a weight of 0.5, meaning that there is a 50% chance of selecting either criterion (risk or negative gain) for minimization once the Pareto set has been computed. Again, the lower the weight, the greater tendency the player has towards maximizing market share. Conversely, with higher weights, players should tend to spend less on operations.
There are 100 simulation cycles, on each of which firms make budget decisions. Market response is given in the form of consumer loyalty, market share, and risk. The maximum borrowing factor is set to 2, meaning that firms may spend up to twice their budget at the cost of assuming greater risk.

We note that in our current implementation, simulating two firms is sufficient for evaluating the competitive nature of goal-based agents. Adding a third firm would allow for observation of more complex dynamics in terms of consumer reactions; however this is not necessary for the inspection of basic market response.

Also, we currently model two decision variables (operational expenses) and two optimality criteria (risk and gain). Having opposing criteria (i.e. those that cannot necessarily be minimized simultaneously) allows us to analyze the balance between the two. Criteria that do not conflict with each other effectively defeats the purpose of our multi-criteria decision making methods, since they could simply be aggregated into a single utility.

Likewise, players taking actions comprising two decision variables is adequate for examining the distribution of budget for firms. Incorporating more variables serves only to increase the dimension and complexity of the decision space, and does not add value towards analysis of the MCDM properties.

5.2.2 Player Decisions

The three figures below (Fig. 36, 37, 38) demonstrate the spending of firms on promotion and quality of service as well as the amount of borrowing in the market game simulation.
Figure 36: Promotion (with Borrowing, 0.5 vs. 0.5)

Figure 37: Quality (with Borrowing, 0.5 vs. 0.5)
Immediately we notice a large number of oscillations in the decision variables. This could be due to the short planning periods for firms. Also, across most of our trials we observed spending on promotion to be generally higher than that of quality of service. These anomalies warrant further investigation. A point worth noting though is that isolated spikes in spending on quality of service in our simulations can also be attributed to the nature of consumer loyalty. Since past actions taken with respect to quality have an effect on the current consumer loyalty factor, a single spike in quality can allow a firm to retain consumers over the duration of several cycles, thus allowing them to spend more on promotion.

What we can see is that where players have equal goals, spending on operational expenses for both firms remains similar. This is what we would expect in order for both firms to remain competitive in the market.

As both players have weights of 0.5 in this case, their tendencies are to select Pareto optimal points that effectively balance all criteria. This is why spending on operations is roughly split at 50% for each budget item. Also, the average borrowing factor is approximately 1, meaning that although firms borrow up to the limit, they both spend under budget at points, thus indicating a desire to pay back loans (i.e. manage risk).
5.2.3 Market Response

The three figures below (Fig. 39, 40, 41) demonstrate response to the budget decisions made by firms in the form of consumer loyalty, market share, and short-term risk.

Figure 39: Consumer Loyalty (0.5 vs. 0.5)

Figure 40: Market Share (0.5 vs. 0.5)
The consumer loyalty graphs give us a picture of the current effect of the accumulation of investment in quality of service. This factor levels off at the point where firms learn how much they need to spend to remain competitive. Here we observe that both firms make consistent attempts at retaining their customer base. At the start of the simulation there is approximately a 40% spread in market share between the two firms, but this gap steadily closes as the firms adjust to each other’s spending habits. Also, there are 6 lead changes in market share over the duration of the game, further indicating the competitive nature of the firms where their goals are the same. Both firms also assume relatively the same amount of risk throughout.

Across our trials where both firms employ the same behaviours, risk remained comparatively the same for the two, and market share fluctuated on average within bounds of 0.35 and 0.65 with standard deviation of 0.07 for the course of the simulation, demonstrating competitive behaviour. Thus, we find that multi-criteria decision making can be a viable method for selection of budget items that lead to balanced criteria, where firms have similar attitudes towards those criteria.

5.2.4 Considering Idling Consumers

Due to the closed world nature of the consumer model implementation, the effect of promotion is less visible in our scenarios. Market share takes a symmetrical shape which is
expected since within our implementation promotion serves only to distribute defectors and consumers would logically switch to the alternate service provider upon defecting from their current one.

In extreme cases, consumers may be permitted to idle in the market place. In cases where this occurs, the total market share amassed by both firms would be under 100%. Realistically this would not happen in the telecommunications industry as consumers typically require steady service from one provider or another. However this effect can be achieved by simply modifying the consumer response where dissatisfaction to quality of service for both firms leads to a percentage of consumers who elect to idle in the market without service from either firm. Once a player increases its consumer loyalty factor above a given threshold, this pool of consumers would then transition to that firm. Note that with the early state-transition consumer behaviour model it would be impossible to correctly simulate such scenarios since there was no concept of ‘memory’ where consumers are able to react to past experience with quality of service.

In such cases, where all firms in the market employ overly cautious tactics, their spending would be naturally lower and quality of service would suffer. Thus there would be instances where consumers would opt to go without service until firms increase their level of quality.

5.3 Players with Opposing Goals

In this scenario, we wish to evaluate the multi-criteria decision making method’s ability to balance criteria where firms have opposing goals. To do this, we initialize player 1 with a weight of 0.2 and player 2 with a weight of 0.7. This means that player/firm 1 has a 20% chance of minimizing risk and an 80% chance of maximizing gain in selecting a final budget decision once the Pareto optimal set has been computed. This indicates the firm’s aggressive nature, and we would expect that it borrows and spends more on operations in order to maintain a higher market share.

Conversely, firm 2 has a 70% chance of selecting risk for minimization and only a 30% chance of maximizing gain. Thus, the firm would be more likely to play cautiously, and attempt to minimize risk factor.
All other initial conditions remain the same as in our competitive market scenario. Below (Fig. 42-48) we present the graphs for the decision variables and market response as in the previous scenario. In addition, we include the graph for the cumulative risk factor upon which short-term risk is based.

Figure 42: Promotion (with Borrowing, 0.2 vs. 0.7)

Figure 43: Quality (with Borrowing, 0.2 vs. 0.7)
Figure 44: Borrowing (0.2 vs. 0.7)

![Borrowing Factor graph](image)

Legend:
- Player 1 (weight: 0.2)
- Player 2 (weight: 0.7)

Figure 45: Consumer Loyalty (0.2 vs. 0.7)

![Consumer Loyalty Factor graph](image)

Legend:
- Player 1 (weight: 0.2)
- Player 2 (weight: 0.7)
Figure 46: Market Share (0.2 vs. 0.7)

Market Share

![Market Share Chart]

Figure 47: Risk Factor (0.2 vs. 0.7)

Risk

![Risk Chart]
Here, we are able to observe several trends. Notably, players who have greater tendency
to maximize market share are consistently spending more on operational costs, and also steadily
take market share away from their competitors. This is in line with what we would generally
expect since the market gaining effort relies on the basic principle of allocating budget towards
promotion and quality of service for increased gains. Those players who attempt to gain market
share need to spend more on operational expenses which is a reflection of the reactive consumer
model. To spend more, they in turn borrow more money which is why we see costs for
promotion and quality occasionally exceeding 100%.

Since consumer loyalty never drops below that of it’s competitor for player 1, it never
loses share over the course of the game. Areas where consumer loyalty tapers off (such as
simulation cycles 51-61 for player 2) indicate more sudden dry spells in spending on quality.
This is since consumers begin to respond as the rate of quality drops.

Throughout our trials, we observed that the rate at which market share is gained generally
depends on the amount of difference between firms’ individual goals. Where firms have widely
differing goals as is the case here, market share is gained more quickly for the aggressive firm.

In reality, firms would obviously have to pay back their debts (with interest). However,
since we do not include financing or pricing in our simplified model, we omit this factor; though
we can still observe the risk optimality criterion. The risk graphs give a view of the cumulative risk based on smoothened borrowing factor as we have described previously. This allows us to more easily view the ratio of risk between the players at any given time step. In the long term, risk accumulates because firms are essentially not paying back any of the funds they borrow.

It is evident that although player 1 gains market share, it does so at the cost of assuming much greater risk by borrowing significantly more than that of its competitor. We can see that player 2 makes several attempts to keep up, by borrowing funds in spikes at several points during the game, however it is not enough to overcome its passive nature and tendency to minimize risk.

This highlights a key weakness in the multi-criteria decision making method. In cases where the player personalities are further apart, there is less competitive action as the more aggressive player steals market share much more quickly. This demonstrates that although both players are making Pareto optimal decisions based on the optimality criteria, the selection of an alternative from the Pareto set has a significant impact on the outcome. In such cases, multi-criteria decision making alone may not be enough to effectively balance all criteria.

As seen in our MCDM trial, nearly 25% of all points in the image of the feasible set were Pareto optimal. The large spread of these points can lead to a more skewed budget selection if the player is extremely biased towards a particular criterion. Thus, a potential enhancement could be to allow players to dynamically change their goals during the course of the game. Cautious players may also see further gains if the risk factor considered consumer loyalty in addition to borrowing. In other words, dangerously low levels of quality of service would lead to higher risk, and force passive firms to spend more in order to better balance their criteria.

5.4 Kalman Filter vs. Fixed Parameters Scenario

Having observed the properties of the implemented multi-criteria decision making technique and consumer response model, we now turn our attention to the Kalman filter to test whether it offers an advantage. To do this, we run trials where both firms are using the same strategy (weights at 0.5), however one firm employs the extended Kalman filter for parameter estimation, and the other has fixed parameters (at 0.5). For the filter we fix both process and
measurement noise at a factor of 10. The idea is that we want some noise in the system, but not to the degree that would hinder our ability to evaluate the Kalman filter’s aptitude for converging on the parameters.

In this case, we would expect the firm with the Kalman filter to eventually take control of the market without taking on significantly greater risk than its competition. We can then compare the results to those from our first competitive market scenario, where both players implemented the filter and all other attributes were the same.

Below, Figures 49-55 illustrate the player actions and market response for a single trial run as per our previous simulations.

**Figure 49: Promotion (with Filter vs. without Filter)**

![Promotion Graph](image)
Figure 50: Quality (with Filter vs. without Filter)

![Spending on Quality](image)

Figure 51: Borrowing (with Filter vs. without Filter)

![Borrowing Factor](image)
Figure 52: Consumer Loyalty (with Filter vs. without Filter)

Figure 53: Risk Factor (with Filter vs. without Filter)
In the simulation, player 1 utilizes the filter and player 2 does not. This scenario is indicative of results obtained across the board – i.e. the agent with the filter steadily gains total market share by maintaining better consumer loyalty while borrowing only marginally more than its filter-less competitor. It generally takes about 30 cycles for the filter to establish a consistent advantage (i.e. where there are no additional lead changes in market share), although this “sweet spot” would be different depending on its competitors’ strategy. Firms may take longer to adjust to more aggressive opponents.
Across 50 trial runs, we found the filter allowed the firm to gain control of the market 100% of the time while spending an average of 10% more on operational costs.

This is in stark contrast to trials where both agents implement identical methods, as seen in our competitive market scenario. In this case, we see far less fluctuation in market share, and there are only two lead changes that occur early in the simulation. We can thus conclude that the Kalman filtering technique is a viable approach to parameter estimation within competitive market simulation.

5.5 Chapter Summary

In this chapter we have run several scenarios that have allowed us to make qualitative observations as to the emergent trends from the market game. We have verified that our multi-criteria decision making module is able to produce Pareto-optimal decisions based on the given set of criteria. In addition, the Kalman filtering technique appears to be a suitable solution for converging on the unknown parameters of the market gaining optimality criterion.

The basic trends we have been able to observe are reflective of the requirements of our simulation domain. Aggressive firms spend more and see better gains. Passive firms borrow and spend less and thus see reduced risk. Although individual attitudes appear to be a dominating factor with respect to balancing of optimality criteria, competitive behaviour manifests itself most prominently where firms employ similar goals.
6 CONCLUSIONS

6.1 General Discussion

Through exploratory study, we have presented a high-level design and preliminary assessment of a gaming framework for modelling competitive service industries. We have found that multi-agent based modelling methodology provides a suitable foundation for facilitating intelligent player behaviour in an automated simulation.

In the case of the telecommunications market example, a particular component of modelling rational behaviour is the problem of attempting to balance several rewards and risks for the purpose of distributing budget across various expenses. We have demonstrated that once a set of objectives (that are functions of the decision variables) has been defined, multi-criteria decision making theory can be applied towards selection of a Pareto-optimal solution. This solution most effectively balances optimality criteria when competing firms have similar individual goals.

In order for multi-objective optimization to be applied, there is an additional problem within the agent decision making process specific to the criteria present in the market model. Namely, that of estimating unknown parameters of the modelled non-linear system. The extended Kalman filtering algorithm serves as an effective solution to this problem as it has been proven to minimize estimation error, and is computationally efficient in its recursive nature.

The dynamics of modelling the effect of promotion and quality of service on consumer behaviour are obviously much more complex than those we have implemented. However, in building on the existing model, we have demonstrated how memory of past experiences can
affect current decisions. In addition, we have taken a more realistic approach to consumer modelling in terms of computing requirements.

With our framework, we have highlighted how the implemented technologies are able to support competitive business games by providing the necessary means for firms to estimate the actions of their competitors in order to make informed decisions that lead to more favourable output in terms of market response and mitigation of risk. Above all, we underscore the exploratory value of the agent-based paradigm. Though it may exhibit limited predictive ability, it has demonstrated that subtle modifications of behaviours can lead to the emergence of macroscopic qualitative trends that would not otherwise be obvious. This was most evident as we gradually scaled the personal attitudes of firms and found MCDM to be less effective at managing criteria when their bias towards either objective function was further apart.

At the current level of development, the framework is only in very initial stages and is not ready to be used in practical application. Though we believe that based on our early findings that we have taken a valid approach that can lead to further investigation and development.

6.2 Future Directions

In addition to expanding the market model implementation, there are a few other interesting research directions that can stem from this research. A study of computational efficiency of the algorithms would benefit the work from a practical application standpoint. Parallelizing the algorithms could allow for not only faster execution, but additional types of agents such as individual consumers and governmental agents.

Another avenue could be to see how well the framework scales to other types of models. We would like to see the framework develop to complement existing models in economics, for the benefit of business decision improvement and the research community at large.

"The real danger is not that computers will begin to think like men, but that men will begin to think like computers." ~Sydney J. Harris
APPENDIX

Additional scenarios (maximum borrowing factor fixed at 1, actions taken every 3\textsuperscript{rd} simulation cycle to improve readability):

**Figure 56: Spending on Promotion (0.2 vs. 0.1)**

![Figure 56: Spending on Promotion (0.2 vs. 0.1)](image-url)

**Figure 57: Spending on Quality (0.2 vs. 0.1)**

![Figure 57: Spending on Quality (0.2 vs. 0.1)](image-url)
Figure 58: Consumer Loyalty Factor (0.2 vs. 0.1)

![Consumer Loyalty Factor Graph](image)

Figure 59: Market Share (0.2 vs. 0.1)

![Market Share Graph](image)

Figure 60: Spending on Promotion (0.2 vs. 0.3)

![Spending on Promotion Graph](image)
Figure 61: Spending on Quality (0.2 vs. 0.3)

Figure 62: Consumer Loyalty Factor (0.2 vs. 0.3)

Figure 63: Market Share (0.2 vs. 0.3)
Figure 64: Spending on Promotion (0.2 vs. 0.6)

Figure 65: Spending on Quality (0.2 vs. 0.6)

Figure 66: Consumer Loyalty Factor (0.2 vs. 0.6)
Figure 67: Market Share (0.2 vs. 0.6)

Figure 68: Spending on Promotion (0.5 vs. 0.1)

Figure 69: Spending on Quality (0.5 vs. 0.1)
Figure 70: Consumer Loyalty Factor (0.5 vs. 0.1)

![Consumer Loyalty Factor Graph](image)

Figure 71: Market Share (0.5 vs. 0.1)

![Market Share Graph](image)

Figure 72: Spending on Promotion (0.5 vs. 0.3)

![Spending on Promotion Graph](image)
Figure 73: Spending on Quality (0.5 vs. 0.3)

Figure 74: Consumer Loyalty Factor (0.5 vs. 0.3)

Figure 75: Market Share (0.5 vs. 0.3)
Figure 76: Spending on Promotion (0.5 vs. 0.6)

Figure 77: Spending on Quality (0.5 vs. 0.6)

Figure 78: Consumer Loyalty Factor (0.5 vs. 0.6)
Figure 79: Market Share (0.5 vs. 0.6)

![Market Share Graph]

Figure 80: Spending on Promotion (0.5 vs. 0.8)

![Spending on Promotion Graph]

Figure 81: Spending on Quality (0.5 vs. 0.8)

![Spending on Quality Graph]
Figure 82: Consumer Loyalty Factor (0.5 vs. 0.8)

![Figure 82: Consumer Loyalty Factor](image)

Figure 83: Market Share (0.5 vs. 0.8)

![Figure 83: Market Share](image)

Figure 84: Spending on Promotion (0.9 vs. 0.3)

![Figure 84: Spending on Promotion](image)
Figure 85: Spending on Quality (0.9 vs. 0.3)

Figure 86: Consumer Loyalty Factor (0.9 vs. 0.3)

Figure 87: Market Share (0.9 vs. 0.3)
Figure 88: Spending on Promotion (0.9 vs. 0.8)

Figure 89: Spending on Quality (0.9 vs. 0.8)

Figure 90: Consumer Loyalty Factor (0.9 vs. 0.8)
Figure 91: Market Share (0.9 vs. 0.8)

![Market Share Graph](Image)

Figure 92: Spending on Promotion (0.5 vs. 0.5)

![Spending on Promotion Graph](Image)

Figure 93: Spending on Quality (0.5 vs. 0.5)

![Spending on Quality Graph](Image)
Figure 94: Consumer Loyalty Factor (0.5 vs. 0.5)

Figure 95: Market Share (0.5 vs. 0.5)

Additional scenario (maximum borrowing factor fixed at 2):
Figure 96: Promotion (with Borrowing, 0.5 vs. 0.6)

Figure 97: Quality (with Borrowing, 0.5 vs. 0.6)

Figure 98: Borrowing (0.5 vs. 0.6)
Figure 99: Consumer Loyalty (with Borrowing, 0.5 vs. 0.6)

![Consumer Loyalty Factor Graph](image)

Figure 100: Market Share (with Borrowing, 0.5 vs. 0.6)

![Market Share Graph](image)

Figure 101: Risk Factor (0.5 vs. 0.6)

![Risk Factor Graph](image)
Figure 102: Short-term Risk (0.5 vs. 0.6)

![Short-term Risk Graph]

Additional scenario (with potentially idling consumers due to poor QoS for both firms):

Figure 103: Promotion (Idling Consumers, 0.7 vs. 0.7)

![Promotion Graph]
Figure 104: Quality (Idling Consumers, 0.7 vs. 0.7)

Spending on Quality

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Figure 105: Borrowing (Idling Consumers, 0.7 vs. 0.7)

Borrowing Factor

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Figure 106: Consumer Loyalty (Idling Consumers, 0.7 vs. 0.7)

Consumer Loyalty Factor

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</table>
Figure 107: Market Share (Idling Consumers, 0.7 vs. 0.7)

![Market Share Graph](image)

Figure 108: Risk Factor (Idling Consumers, 0.7 vs. 0.7)

![Risk Factor Graph](image)

Figure 109: Short-term Risk (Idling Consumers, 0.7 vs. 0.7)

![Short-term Risk Graph](image)
BIBLIOGRAPHY


