DOWNSIDE RISK-ADJUSTED PERFORMANCE MEASURES: CAN THEY RECOGNIZE BANKRUPT ASSET ALLOCATION STRATEGIES?

by

Xiao Fang He
Bachelor of Engineering, Shenzhen University, 1993

and

Tsun Yin Joseph Kwok
Bachelor of Science, Simon Fraser University, 2002

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APPROVAL

Name: Xiao Fang He & Tsun Yin Joseph Kwok

Degree: Master of Arts

Title of Project: Downside Risk-adjusted Performance Measures: Can They Recognize Bankrupt Asset Allocation Strategies?

Supervisory Committee:

______________________________
Dr. Robert R. Grauer
Senior Supervisor
Endowed University Professor, Faculty of Business Administration

______________________________
Dr. Peter Klein
Second Reader
Professor, Faculty of Business Administration

Date Approved: July 18, 2007
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ABSTRACT

Recent work by Dr. Robert R. Grauer has shown that traditional performance measures, such as Jensen's Alpha and the Sharpe Ratio, are unable to distinguish between perfect-foresight and bankrupt investments. In this paper, we test the ability of various downside risk-adjusted performance measures to identify bankrupt performance. Using zero percent as a minimal acceptable level of return, the Sortino ratio, the upside potential ratio and the portfolio performance index can distinguish bankrupt strategies from non-bankrupt ones. However, the ranking based on these measures over non-bankrupt strategies are inconsistent with the ranking of accumulated wealth over the 1934-1999 period.

Keywords: Downside risk, measurement of investment performance, bankruptcy

Subject Terms: Investment analysis, risk, risk management
DEDICATION

Joseph dedicates this paper to his parents, Susanna and Anthony, and his brother Simon. He would like to thank them for their love and continual support through his life, especially during the last twelve months of graduate studies.

Crystal dedicates this paper to her loving husband, Geoffrey and her son Billy, the angel in her life. Without their caring support, it would not have been possible to finish the last twelve months of graduate studies.
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We would also like to thank Dr. Peter Klein, who serves as our second reader of this project. He provides insightful comments from a practitioner’s point of view. With the help of Mr. Todd Brulhart of KCS Fund Strategies Inc., he also compiled the monthly return data on various hedge fund indices for our empirical tests at the early stages of our project. However, since we have decided to focus on strategies that generate extremely negative investment returns, the results of these tests are not reported in this paper.

Furthermore, we would like to thank Mr. Nick Macleod, who provided a clear explanation and the derivation of the Adjusted Sharpe ratio (Johnson, Macleod and Thomas 2002) via e-mail.
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1 INTRODUCTION

Portfolio choice among a set of investment opportunities with uncertain returns has been the subject of many studies over the past fifty years. Harry Markowitz (1959) set the foundation for mean-variance portfolio choice. Markowitz’s theory formed the basis of one of the most prevalent asset pricing theory of all times – the Sharpe (1964) and Lintner (1965) Capital Asset Pricing Model (CAPM). One of the earliest applications of the CAPM was the development of a series of performance measurements for fund managers. These measures have become very popular among academics and practitioners. For instance, the Jensen (1968) performance measure is based on the regression

\[ r_{pt} - r_{lt} = \alpha_p + \beta_p (r_{mt} - r_{lt}) + \epsilon_{pt} \]  

(1)

where \( r_{pt} \) is the portfolio return, \( r_{lt} \) is the risk-free lending rate, \( r_{mt} \) is the market return and \( \beta_p \) is the measure of systematic risk. The intercept \( \alpha_p \), which is commonly known as Jensen’s alpha, measures the portfolio’s deviation from the securities market line. On the other hand, Sharpe (1966) developed the Sharpe ratio:

\[ Sh_p = (\bar{r}_p - \bar{r}_L) / \hat{\sigma}_p \]  

(2)

where \( \bar{r}_p \) is the average excess return on the portfolio, \( \bar{r}_L \) is the average risk-free lending rate, and \( \hat{\sigma}_p \) is the sample standard deviation of the portfolio return. This measures excess return per unit of the portfolio’s total risk.
However, Grauer (2007a) showed that these performance measures cannot correctly rank strategies with extremely positive abnormal performance, and cannot distinguish between them and the destructively negative performance of bankrupt mean-variance asset-allocation strategies. Roll (1978) showed that Jensen's alpha (and any performance measures that based on the securities market line) is ambiguous and is not a robust performance measure, because beta, the measure of systematic risk, and alpha, depend on the proxy chosen for a market portfolio. These, along with other criticisms of these traditional performance measures, motivate the search for new ones that are unambiguous and are robust even under extreme returns environment.

Downside risk represents the uncertainty that investment returns would fall below a certain minimal acceptable level. Recent studies suggest that downside risk is a better notion of risk, compared to variance, which considers the volatility of returns both above and below this minimal acceptable level. As such, various performance measures incorporating downside risk have been proposed. Some of these measures involve replacing the standard deviation in the denominator of the Sharpe Ratio by a measure of downside risk, while some involve computations that are more complex. The effectiveness of these measures has been empirically demonstrated in previous studies. In this project, we study whether these measures are robust under an extreme return environment, such as bankruptcy. In particular, we determine the wealth generated by eight mean-variance efficient portfolios, the global minimum-variance portfolio and the tangency portfolio estimated from four different estimators of the means with short sales permitted. We define a bankrupt portfolio as one with cumulative wealth that is zero. Using zero percent as a minimal acceptable level of return, we conclude that some of
these measures, namely the Sortino ratio, the upside potential ratio and the portfolio performance index can identify the bankrupt strategies. However, the ranking based on these measures is inconsistent with the ranking of cumulative wealth for those non-bankrupt portfolios.

This paper proceeds as follows. We will begin with the various critiques of these measures in Section 2. In Section 3, we highlight the need to incorporate downside risk in the evaluation of investment performance. We then describe the downside risk-adjusted performance measures proposed in academic literature and seminal work in Section 4. Section 5 describes the data and Section 6 provides the result of our empirical analysis. Section 7 concludes the paper.
2 CRITIQUES OF TRADITIONAL PERFORMANCE MEASURES

Despite their popularity among practitioners, traditional performance measures developed based on CAPM received much criticism from academia.

2.1 Inability to Distinguish between Perfect-Foresight and Bankrupt Strategies

Grauer (2007a) investigates whether various performance measures can distinguish between perfect-foresight and bankrupt strategies. First, he benchmarks these measures using seven perfect-foresight asset-allocation strategies. The first three strategies' returns do equal to the maximum of the Treasury bill return or the return on either (1) the CRSP value-weighted index; (2) the S&P 500 Index, small stocks and long-term government bond return; and (3) twelve value-weighed industry indices. The next three strategies extend upon the first three, allowing each to be levered with a margin requirement and at a risk-free borrowing rate that is different from the Treasury-bill return. The seventh strategy invests 40% of wealth in the market and 60% of wealth in T-bills in down markets, and vice versa in up markets. He shows that the Sharpe ratio fails to identify the strategy that results in highest cumulative wealth at the end of 1999 arising from an investment of $1 at the beginning of 1934. It ranks such strategy as the fifth best, while ranking the third-best strategy as the most superior one. Grauer attributes the result to the fact that standard deviation does not measure risk well when an investment never loses.
Next, he benchmarks the performance measures using the performance of the global minimum variance portfolio of risky assets, the tangency portfolio and six mean-variance efficient portfolios in an industry rotation setting. Grauer (2007b) shows that, without short sales constraints, some of these portfolios can lose more than 100% in at least one of the 264 quarters in the 1934-1999 period. Focussing on the case where portfolio means are estimated from dividend yields and risk free rates and "shrunk" to historic means, Grauer (2007a) demonstrates that the traditional performance measures do not distinguish between bankrupt and non-bankrupt strategies. For instance, Sharpe ratios for all strategies are approximately equal, and that the Jensen’s alpha for the bankrupt strategies are higher than that for the perfect foresight strategies. These results motivate our search for other performance measures that may clearly identify bankrupt and non-bankrupt strategies.

2.2 Ambiguity of Securities Market Line-Based Performance Measures

Another desirable quality of a good portfolio selection criterion is that different users of this criterion must obtain the same judgement given the same set of portfolios. However, Roll (1978) suggests that any performance selection criteria that are based on the securities market line are ambiguous. Using Jensen’s alpha as the selection criterion, the ranking provided by the three judges in the fictional investment competition differ significantly because they use different indices as proxies for the unobservable market. This can be attributable to the use of securities market lines that correspond to different market proxies. As an extreme case, the judge that uses a mean-variance efficient market was unable to construct a ranking because all portfolios lie on the securities market line.
Using mathematical analysis, the author concluded, “individual differences in portfolio selection ability cannot be measured by the securities market line criterion.” (Roll 1978, p.1060)

2.3 Problem with Negative Excess Return

Vinod and Reagle (2005) use the following example to demonstrate an old unresolved problem with the Sharpe ratio, where the ranking provided by this measure can be inconsistent with economic theory. Suppose two portfolios have a common mean excess return of -1% with distinct standard deviation, 1 and 10, respectively. The first portfolio would have a Sharpe ratio of -0.0101 while the latter portfolio would have a Sharpe ratio of -0.001. The Sharpe ratio methodology suggests that the latter portfolio is preferred because of its larger magnitude. However, economic theory would suggest that a portfolio whose mean excess return is negative and has a larger standard deviation is “doubly undesirable”. In this case, choosing the portfolio with a greater Sharpe ratio would be inconsistent with economic theory.

2.4 Violation of CAPM Market Efficiency Assumption

The CAPM is based on a market efficiency assumption that all investors know the components of the market portfolio, and forms homogeneous expectation of asset prices based on such identical information. This implies that, if the market is mean-variance efficient, one cannot obtain a higher expected return or lower standard deviation than the market portfolio unless it is implemented with superior information. However, Leland (1999) demonstrates that one can increase the Sharpe ratio of the market portfolio by implementing a simple dynamic strategy or its options equivalent. He assumes a market
portfolio that increases by 25% per annum with 80% probability or falls by 20% per
annum with 20% probability over a two-year period. A portfolio with an initial
allocation of 60/40 stock-to-cash ratio has the same expected return over the same two-
year period. However, one can achieve a higher Sharpe ratio, if the portfolio is strategic
re-balanced at the end of year 1. Equivalently, this strategy can be implemented by
writing a two-year call option on the market at the beginning of year 1 and investing the
proceed, along with the initial wealth, in the market portfolio. Since a higher Sharpe ratio
can be attained without superior information, the notion that the market is efficient can be
rejected.

2.5 Failure to Reflect Investment Objective

Many institutional investors have a mandate to earn a minimum acceptable return
(MAR). In this context, good volatility reflects the risk that portfolio returns are greater
than MAR, and bad volatility reflects the risk that the portfolio underperforms with
respect to the MAR. Since standard deviation measures deviation from the mean, it fails
to distinguish between good and bad volatility. As such, Sortino and Price (1994) argue
that Sharpe ratio, which uses standard deviation as a risk measure, is inadequate. As
shown in the authors’ example, a fund manager who invests solely in short term
Treasuries that have mean return of 5% may have a very low standard deviation, but his
portfolio would be exposed to bad volatility should the MAR be at a high quantile of the
portfolio’s return distribution. On the other hand, the returns of a well-diversified
portfolio with its MAR at a low quantile of the return distribution may have a larger
standard deviation, but is less exposed to bad volatility. Moreover, Sortino and Price
argue that the reference point for the excess return should reflect the stated or implied
objective of the investors, who are more concerned with the risk exposure of the strategy taken to achieve or exceed their MAR. As such, the appropriate reference point for evaluating excess return should be the MAR, not the risk-free interest rate.
3 MOTIVATION FOR CONSIDERATION OF DOWNSIDE RISK

In an effort to find a better risk-adjusted performance measure, researchers attempt to modify existing measures or develop new measures that incorporate downside risk. The concept of downside risk can be dated back to the 1950s, where Roy (1952) suggests that investors set some minimum acceptable return that will conserve capital, and Markowitz (1959) suggests that only downside risk measure serves as a proper notion of risk in the absence of normality. Recent empirical studies show that non-normalities can arise for different reasons. Stutzer (2000) attributes non-normalities to large asymmetrical economic shocks, investment in options and other derivatives with inherently asymmetrical returns, and limited liability effects on asset returns. Johnson, Macleod and Thomas (2002) suggest that non-normalities in hedge fund distributions are due to “fat tails brought about by imperfect convergence to a normal distribution”, and that hedge funds take on overpriced risks associated with rare events, thus effectively implementing short option strategies. This “makes it difficult to interpret differences in risk-adjusted performance measures as differences in forecasting skills.” (Plantinga, van der Meer and Sortino, 2001).

On the other hand, recent researches show that investors do not seek the highest return for a given level of risk. Instead, “most investors have a preference for positively skewed returns, which implies that prices in equilibrium reflect more than mean and variance.” (Leland 1999). In particular, they seek upside potential with downside
protection. Prospect Theory in behavioural finance, developed by Kahnemann and Tversky (1979), explains the asymmetry in investor preferences. When asset returns are asymmetric, a portfolio strategy that maximizes the Sharpe ratio fails to capture such preferences. Sharpe (1994) acknowledges this and agrees that the use of additional or substitute measures may be required.
4 DOWNSIDE RISK-ADJUSTED PERFORMANCE MEASURES

To address the shortcomings of the traditional performance measures and to address the need to incorporate downside risk, alternative performance measures have been proposed. Most of them represent modifications to the Sharpe ratio using downside deviation, a notion motivated by the works of Bawa (1975) and Fishburn (1977). This section describes some of these measures and cites examples that show their superiority over the Sharpe ratio.

4.1 Symmetric Downside-Risk Sharpe Ratio

Sharpe ratio uses the standard deviation of portfolio return as the denominator. Instead, Ziemba (2005) proposes the symmetric downside-risk Sharpe ratio (SDRShₚ), which can be calculated as follows:

\[ SDRShₚ = \frac{(\bar{r}_p - \bar{r}_L)}{\sqrt{2\hat{\sigma}^2_{DZ}}} \]  

(4)

where \( \bar{r}_p \) and \( \bar{r}_L \) are as in the Sharpe ratio and \( \hat{\sigma}^2_{DZ} \) is defined as

\[ \hat{\sigma}^2_{DZ} = \frac{\sum r_p^2}{n-1} \]

(5)

where \( n \) is the number of returns observations. The motivation of this measure is to reward portfolio’s positively skewed excess return by reducing the denominator in the
ratio, generating a higher ratio for ranking purposes, thus reflecting the investors' preference.

Ziemba demonstrates the superiority of this measure by comparing the two ratios' ability to recognize the relative difference in wealth generated by the two funds, the Ford Foundation and Berkshire Hathaway, a fund run by Warren Buffet. Using quarterly data from June 1977 to March 2000, Ziemba plots the value of $1 invested on June 30, 1977 and finds that there is little difference in value between the two funds during this period. However, the Sharpe ratio is much higher for Ford Foundation (0.970 vs. 0.773 for Berkshire Hathaway), failing to recognize that an investor’s preference over these two funds should be similar. On the other hand, the symmetric downside-risk Sharpe ratio for these two investments are approximately equal (0.920 for Ford Foundation and 0.917 for Berkshire Hathaway). Hence, this ratio could be a better indicator of accumulated wealth.

4.2 Downside Sharpe Ratio

To solve the problem of incorrect ranking due to the presence of negative excess return, and to reflect investors’ aversion against downside risk, Vinod and Reagle (2005) propose the downside Sharpe ratio (DSh\(_p\)), which can be calculated as follows:

\[
DSh_p = \frac{[\bar{r}_p - \bar{r}_f] + AF}{DSD_p}
\]  

where \([\bar{r}_p - \bar{r}_f]\) is the mean excess return over the risk-free lending rate, and AF is a common add factor that makes the numerator in the downside Sharpe ratio for all portfolios in consideration greater than 0; the portfolio ranking would be correct if
The downside standard deviation of the portfolio, DSDₚ, is defined as follows:

\[
DSDₚ = \sqrt{\frac{\sum_{t=1}^{n} w_i (\bar{r}_p - \bar{r}_f)(\bar{r}_p - \bar{r}_f)'}{\sum_{t=1}^{n} w_i}}
\]

(7)

where \(w_i\) is the weight assigned to each observation. The weight is set to zero for an observed return that is less than zero, and the weight for non-negative return can be set arbitrarily. For our empirical analysis in Section 6, we assume equal weight (\(w_i=1\)) for all non-negative returns.

### 4.3 Adjusted Sharpe Ratio

Johnson, Macleod and Thomas (2002) propose “an adjustment to the Sharpe ratio that compensates for distortions arising from non-normality while preserving its desirable properties and ease of interpretation.” The adjusted Sharpe ratio (ASHₚ) is the Sharpe ratio that is implied by the fund’s observed downside deviation, assuming the returns were normally distributed. It establishes a reference level for downside deviation, and tells us what the ratio of downside to standard deviation “should be”, for a return series with the observed Sharpe ratio.

Macleod derived the mathematical relationship between the ratio of downside variance to variance and the adjusted Sharpe ratio (ASHₚ) by substituting the formula for normal distribution

\[
f(r_p) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{(r_p - \mu)^2}{2\sigma^2}}
\]
downside deviation around the risk-free rate: \( \sigma_{DL}^2 = \int_{-\infty}^{\alpha} (r_p - r_L)^2 f(r_p) dr_p \), and defining the Sharpe ratio to be \( \frac{\mu - r_L}{\sigma} \). The adjusted Sharpe ratio can then determined by solving the following equation:

\[
\frac{\sigma_{DL}^2}{\sigma^2} = [1 + ASh_p^2][1 - \Phi(ASH_p)] - ASh_p \times \phi(ASH_p)
\]  

where \( \Phi(\cdot) \) and \( \psi(\cdot) \) represent cumulative distribution function and probability distribution function of the standard normal distribution, respectively, and \( \sigma_{DL}^2 \) defined as:

\[
\sigma_{DL}^2 = \frac{\sum_{t=1}^{n} (r_p - r_L)^2}{n-1}
\]  

with \( r_L \) representing the risk-free lending rate. This calibration process penalizes departures from normality associated with excess downside deviation, and rewards those associated with a reduced downside deviation. This adjustment can be quite severe, as demonstrated in the author’s example, where a hedge fund’s Sharpe ratio of 2.56 was adjusted to 0.79 after taking into account downside deviation.

### 4.4 Sortino Ratio

Based on the arguments presented in Section 2.5, Sortino and Price (1994) construct the Sortino ratio, defined as

\[
SR_p = \frac{\bar{r}_p - r_{MAR}}{DD_p}
\]  

where \( \bar{r}_p \) represents the average return of the portfolio, \( r_{MAR} \) is the minimum acceptable rate of return, and \( DD_p \) is the downside deviation of the portfolio. This ratio focuses on the downside risk, making it a more suitable measure for investors who are more concerned with losses than with gains.
where the numerator is the excess portfolio return over the investor's MAR, and the denominator is the downside deviation (DD):

\[ DD_p = \sqrt{\frac{\sum_{r_p > r_{MAR}} (r_p - r_{MAR})^2}{n}} \]  

where \( n \) is the total number of observations.

To demonstrate this ratio, the Sortino and Price compare the performance of two fund managers, Twentieth Century Growth Fund (20\(^{th}\) Century) and RCM Growth Equity Fund (RCM), against the Frank Russell 2000, NYSE index, NASDAQ, Small-cap index, World index and 90-day T-bills using data for the ten years ending December 1992. The MAR was set to 8%, a typically accepted return for a defined-benefit pension plan at that time. The Sharpe ratio ranked the T-bills first and 20\(^{th}\) Century fourth, while the Sortino ratio ranked 20\(^{th}\) Century second and T-bills last. Should investors make decisions based on the Sharpe ratio instead of the Sortino ratio, the authors claim that such strategy “guarantees failure to accomplish their goal.”

Despite wide recognition by the academics and industry professionals, there were doubts about the appropriateness of the Sortino ratio. In the endnotes of Leland (1999), the Sortino ratio is referred to as an “ad hoc” approach that is not “grounded in capital market equilibrium theory and may themselves spuriously identify superior or inferior managerial performance.” Pedersen and Satchell (2002) provide theoretical foundation by showing that this ratio satisfies the maximum principle under certain circumstances. The maximum principle defines a performance measure as one that would rank portfolios both for individual investors and for the representative investor who holds the market
portfolio. This measure should be maximized by the representative investor; if the maximum is exceeded, the presence of special information or skills can be inferred. For a ratio of excess expected return to a risk measure (such as the Sharpe ratio and the Sortino ratio) to satisfy the maximum principle, the market portfolio must be efficient and the efficient frontier must be linear. Pedersen and Satchell show that the latter condition would be satisfied under the following conditions. First, the portfolio’s MAR is endogenously determined by the following formula:

\[ t = r + wt^* \]  \hspace{1cm} (12)

where \( t \) is the MAR, \( r \) is the risk-free return, \( w \) is the weight of the risky (market) portfolio held by the representative investor, and \( t^* \) is a constant that is less than or equal to zero. This implies that the MAR is less than the risk-free rate. Secondly, the representative investor has a utility function that takes the form:

\[ U(\tilde{r}_p) = \tilde{r}_p \quad \text{if} \quad \tilde{r}_p \geq t \quad \text{and} \]

\[ U(\tilde{r}_p) = \tilde{r}_p - c(t - \tilde{r}_p)^2 \quad \text{if} \quad \tilde{r}_p < t \]  \hspace{1cm} (13)

where \( \tilde{r}_p \) is the portfolio return, \( t \) is the MAR and \( c \) is a positive constant. Under these assumptions, the Sortino ratio is a theoretically sound measure of relative investment performance.

### 4.5 Upside Potential Ratio

Recognizing recent research results in behavioural finance that shows investors seek upside potential with downside protection, Sortino, Van der Meer and Plantinga...
(1999) developed a performance measure that is consistent with this attitude, namely, the 
Upside Potential Ratio (UPR):

\[
UPR_p = \frac{\sum_{r_p > r_{MAR}} (r_p - r_{MAR})}{DD_p}
\]

(14)

where \( n \) is the total number of observations, and DD is the downside deviation as defined in Equation 11.

To investigate the difference between the Sharpe ratio and UPR, the authors conducted two empirical studies. In Sortino et al (1999), the authors compared the ranking of six Dutch mutual funds and various Dutch and International indices. The rankings based on Sharpe ratio and the UPR are significantly different. As an extreme case, the fund that is ranked first out of 18 funds by the Sharpe ratio is ranked 11th by the UPR. This suggests that UPR better portraits the preference of an institutional investor with the goal to maximize upside potential. Subsequently, in Plantinga, Van der Meer and Sortino (2001), the authors calculate the Sharpe ratio and the UPR for 810 mutual funds in the EURONEXT market\(^1\) using monthly data from January 1994 to December 1999. Two sets of UPRs are constructed: UPR\(_0\), which uses a MAR of 0\%, and UPR\(_{rf}\), which uses the risk-free rate as the MAR. They then calculate the rank correlation between the Sharpe ratio and the two sets of UPRs. Contrary to the results of the first study, the authors find that the correlations are very high (97.2\% for correlation between Sharpe ratio and UPR\(_{rf}\), and 97.6\% between Sharpe ratio and UPR\(_0\)). They attribute the high correlation to the fact that the returns distribution is normal, as they cannot reject

\(^1\) The EURONEXT market is the result of a merger of the stock exchanges in Belgium, France and the Netherlands.
normality in 90.5% of the funds in their sample at 5% significance level. Regressing the difference in ranking between Sharpe ratio and UPR_{rf} against skewness of fund return distribution, the authors show a positive linear relation at 5% significance level and with a high $R^2$. Based on these findings, they conclude that the differences in ranking between the two performance measures can be attributed to skewness of the returns distribution.

### 4.6 Portfolio Performance Index

The five aforementioned performance measures are similar to the Sharpe ratio, in the sense that they are all reward-to-variability ratios, each using different measures of reward and variability. In contrast, the Portfolio Performance Index proposed by Stutzer (2000) is the exponential rate of decay that minimizes the probability that excess portfolio return is less than zero, defined mathematically as follows:

$$\text{prob}[(r_p - r_{MAR}) \leq 0] = \frac{c}{\sqrt{n}} e^{-I_p c}$$

(15)

where $c$ is a constant that depends on the return distribution. Like the UPR, this measure stems from the behavioural hypothesis, which reflects the desire to avoid underperformance against a MAR, but emphasizes growth in excess of the benchmark at the same time. Given past return data, the performance index ($I_p$) can be computed using an optimizer:

$$I_p = \max_\theta \left\{ -\ln \frac{1}{n} \sum_{i=1}^{n} e^{\theta(r_i - r_{MAR})} \right\}$$

(16)$^2$

---

$^2$ A slight variation of this index is presented in Foster and Stutzer (2004), where $r_p$ and $r_{MAR}$ are replaced by $\ln (1 + r_p)$ and $\ln (1 + r_{MAR})$, respectively. We do not adopt this version, as the some portfolio returns of bankrupt strategies are less than 100% and would result in undefined terms.
If the excess portfolio return is normally distributed, Stutzer claims that the performance index equals one-half of the squared Sharpe ratio. In this case, the rankings given by the Portfolio Performance Index and the Sharpe ratio are identical. Furthermore, when returns are not normally distributed, Equation (16) can be re-arranged to show the equivalence of this measure to expected constant absolute risk aversion (CARA) utility. Since CARA utility has a positive third derivative, this index’s ranking would reflect investors’ preference for skewness\(^3\). This provides a behavioural foundation relevant for long-horizon institutional investors, like pension fund or mutual fund managers, that is free of unspecified parameters.

To compare the Portfolio Performance Index and the Sharpe ratio, Stutzer (2000) constructs a portfolio of 23 randomly selected stocks, and determines the optimal weight in each asset by maximizing the Sharpe ratio and by maximizing the Performance Index. An investor who maximizes the Sharpe ratio would place a lower weight on a stock with highest excess skewness than the one who maximizes the Performance Index. In contrast, an investor who maximizes the Sharpe ratio would place a higher weight on the stock with the lowest excess skewness than the one who maximizes the Portfolio Performance Index. This shows that this measure can better captures the investor’s preference of stocks with higher skewness.

\(^3\) However, as suggested by Brulhart and Klein (2006), scaled measures such skewness and kurtosis are not relevant measures of extreme returns. Instead, the unscaled third moment and fourth moment are consistent with the terms of the Taylor series expansion of an investor’s utility function.
5 DATA

Our empirical tests are performed using the returns from five sets of portfolios from Grauer (2007b and 2007c) over the 1934-1999 period, generated using a mean-variance optimizer.

Each of the first four sets consists of eight mean-variance efficient portfolios, a global minimum-variance portfolio and a tangency portfolio. The mean-variance efficient portfolios are determined using power utility functions with risk tolerance parameters $T=1/(1-\gamma)$, where $\gamma$ ranges from -50 to 0.9. The four sets differ by the methods used to estimate the ex-ante mean return for the optimizer. They include the historic means, Jorion’s (1985, 1986 and 1991) Bayes-Stein (BS) estimator, Sharpe (1964)-Lintner (1965) CAPM means, and an estimate of the mean vector forecasted using dividend yields and risk-free interest rates. (Please refer to Grauer (2007b) for the description of the means and the estimation methodology.) The investment universe for these portfolios consists of twelve value-weighted industries. Lending takes place at the T-bill rate and the borrowing rate is assumed to be 1% higher than the call money rate. The data for the underlying assets come from several sources. The returns for the value-weighted industry groups are constructed from the returns on individual New York Stock Exchange firms contained in the Center for Research in Security Prices’ (CRSP) monthly returns database. The firms are combined into twelve industry groups based on the first two digits of the firms' SIC codes. (Grauer, Hakansson and Shen (1990) contains a detailed description of the industry data.) The risk-free asset is assumed to be 90-day U.S. Treasury bills.
maturing at the end of each quarter. For 1934-1976, the call money rates are obtained from *Survey of Current Business*, where the *Wall Street Journal* is the source for later periods.

The fifth set of portfolios consists of ten mean-variance efficient portfolios, with $\gamma$ ranges from -50 to 0.9. The ex-ante mean return is estimated using the dividend yields and risk-free interest rates. The investment universe consists of common stocks, long-term government bonds, and small stocks. Unlike the first four sets of portfolios, short sales are precluded in this case. Ibbotson Associates' database is the source of the returns for common stocks (Standard & Poor's 500 Index), long-government bonds, and small stocks. The margin requirements for stocks are obtained from the Federal Reserve Bulletin, and the initial margins are set at 10% for government bonds.
6 EMPIRICAL ANALYSIS

Our analysis consists of three components. First, we calculate the summary statistics of the first four sets of portfolios to study their characteristics. Then we replicate the results in the work of Grauer (2007a and 2007b) to show that Jensen’s alpha and Sharpe ratio is inadequate in distinguishing bankrupt strategies. Next, we compare the downside risk-adjusted performance measure for each portfolio to test whether these measures can separate bankrupt strategies apart from non-bankrupt strategies. We also evaluate the ranking ability of these measures among non-bankrupt portfolio. We then provide possible explanations for the results. To conclude this section, we confirm our results using the fifth set of portfolios which generate substantial accumulated wealth.

Table 1, which appears on page 31, shows the summary statistics of the returns for the first four sets of portfolios. For each portfolio, we calculate the mean, standard deviation, skewness and kurtosis for the returns. We also tabulate the minimum and maximum return over the observation period, and the number of periods during which investment return is negative. Furthermore, we calculate the wealth accumulated for each portfolio through the observation period, with an initial investment of $1 at the beginning of the first quarter of 1934. Bankruptcy occurs if the investment return in any period equals or less than -100%. No further wealth is accumulated after bankruptcy; however, investment returns generated by the mean-variance optimizer after bankruptcy is included in the calculation of the sample statistics shown in Table 1 and the performance measures calculated in our subsequent analysis.
From utility theory and modern portfolio theory, we know that a person with a
der higher risk appetite (larger risk tolerance parameter) would invest in a portfolio with
higher ex-ante mean and ex-ante standard deviation (not shown in Table 1). Assuming
that historical returns serve as good representation of future returns, such preference
should be reflected in the ex-post results. From Table 1, we can see that, regardless of
the means estimator used, the ex-post means and standard deviations are higher for larger
value of $\gamma$ (and hence larger value of the risk tolerance parameter). Over the long run,
this also implies the higher possibility of extremely large cumulative wealth or
bankruptcy. This is again reflected in each set of portfolios, where accumulated wealth
increases then drops to zero as $\gamma$ increases.

Looking at the maximum return and the minimum return of each portfolio, we can
see that as $\gamma$ increases, the maximum return and minimum return both approach extreme
levels. For instance, for the set of portfolios estimated using historical means, maximum
quarterly return soars up to an almost unbelievable 9739% while the minimum return
falls to negative 6193% when $\gamma$ equals to 0.9. We also see the number of negative returns
increases with gamma, hence reflecting a higher possibility of bankruptcy.

As noted in Section 3, the non-normality of investment returns motivates the use
of downside-risk adjusted performance measures. We test the normality of returns by
performing the Jarque-Bera normality test for each portfolio. The p-values for all
portfolios equal zero or are very close to zero. As such, we conclude that the hypothesis
of normality can be rejected at any reasonable confidence level, and confirms that
downside-risk adjusted performance measures are more suitable.
The next step in our analysis is to evaluate the investment performance using traditional performance measures, namely Jensen’s alpha and Sharpe ratio. We first replicate the results in Grauer (2007a) and the figures are shown in the third and fourth column of Table 2, Panel B on page 34. We compare these with the corresponding figures in Panels A, C and D and draw the same conclusion as Grauer (2007a). Portfolio returns generated using the same set of mean estimator has approximately the same Sharpe ratios, and as such, cannot identify bankrupt strategies from non-bankrupt ones. If superior investment performance is being judged by the level of accumulated wealth, Jensen’s alpha would fail, as it ranks bankrupt strategies ahead of non-bankrupt strategies consistently.

At this time we test each of the downside risk-adjusted performance measure described in Section 4 to evaluate their ability to identify bankrupt strategies. First we consider the symmetric downside risk Sharpe ratio. For the historical means portfolio (Table 2, Panel A), the ratio is highest for the tangency portfolio, which has an accumulated wealth of zero. For the dividend-yield risk-free means portfolio, the difference between the ratio for the bankrupt portfolios (0.20 for $\gamma=-5$) and non-bankrupt portfolios (0.20 for MV portfolio) is small. For CAPM means portfolios and BS means portfolios, a bankrupt strategy yields a higher ratio (0.29 and 0.26, respectively) than the MV portfolio (0.20). We can conclude that this ratio is not a good candidate for identifying bankrupt strategies.

Next we consider the downside Sharpe ratio and the adjusted Sharpe ratio. Aside from the portfolio generated by dividend-yield risk-free rate means, the downside Sharpe ratios are identical for bankrupt and non-bankrupt strategies. Although the non-bankrupt
strategies appear to have a higher downside Sharpe ratio in the dividend-yield risk-free means portfolios, the ratios are actually distorted by the use of an add factor, defined in Section 4.3. If we manually set the add factor to zero, the downside Sharpe ratio would, again, be very similar for each portfolio. This is consistent with Vinod and Reagle's warning that the add factor should only be used for ranking purposes only. The performance for adjusted Sharpe ratio is similarly poor, where the ratios are virtually identical for bankrupt and non-bankrupt portfolios.

A common attribute for the next three measures is that the user can choose the MAR at which the measures are calculated. For the purpose of our test, we choose the risk-free rate, which implies that the investor's objective is to earn at least as much as holding Treasury bills, and a return of zero, implying an objective of capital preservation. The Sortino ratio with risk-free rate as the MAR does not distinguish bankrupt strategies from non-bankrupt ones. In all four sets of portfolios, there is no difference between the Sortino ratios for all levels of \( \gamma \). However, using a MAR of zero yields significantly better results. Aside from the historical means portfolio (Table 2, Panel A), the non-bankrupt portfolios have a higher Sortino ratio than the bankrupt ones. For example, from Table 2, Panel B, we see that the Sortino ratios for the non-bankrupt portfolios (0.79, 0.52, 0.37) are distinguishably higher than those for bankrupt portfolios (between 0.27 and 0.32). Similar patterns can be found for the CAPM means and BS means portfolios in Panel C and Panel D.

Similar to the Sortino ratio, the upside potential ratios that use the risk-free rate as the MAR show no difference between bankrupt portfolios and non-bankrupt ones. However, if we take zero as the MAR, the ratios perform much better. From Table 2,
Panel B, we can see that the non-bankrupt portfolios generated by dividend-yield risk-free means have higher upside potential ratio than the bankrupt ones. For the portfolio with $\gamma=-50$, $\gamma=-10$ and the MV portfolio, the ratios are 1.18, 0.79, and 0.96, respectively. All three portfolios have higher ratio than the bankrupt portfolios in the same set. Hence we conclude that upside potential ratio with MAR set to zero can correctly identify non-bankrupt and bankrupt strategies.

As explained in Section 4.6, the portfolio performance index is the exponential rate of decay that minimizes the probability of negative excess portfolio return with respect to a MAR. Like the Sortino and upside potential ratio, the measures of all portfolios are almost the same if we take the risk-free rate as MAR. If we set the MAR to be zero, the portfolio performance index shows the difference between bankrupt and non-bankrupt portfolios. A higher figure corresponds to a non-bankrupt strategy, whereas a lower figure indicates a bankrupt strategy.

While the last three measures can distinguish bankrupt portfolios if the MAR is set to zero, we observe that these measures do not rank the portfolios correctly according to their cumulative wealth for all four sets of portfolios. Taking historical means portfolios as an example, the correct ranking for the $\gamma=-50$, $\gamma=-10$ and the MV portfolios should be [3,1,2]. However, the ranking by Sortino ratio is [1,4,3]. The rankings by upside potential ratio and portfolio performance index are both [1,3,2], which is a complete reverse of the correct ranking. The same observation can be made over other three sets of portfolios as well. Furthermore, we observe that the ratios for the highest ranking portfolio by wealth are often closest to the ratios for bankrupt portfolios. Take the CAPM means portfolios (Table 2, Panel C) for example. The Sortino ratio, the
upside potential ratio and the portfolio performance index (with MAR equals zero) for the portfolio with highest wealth ($\gamma=1$) are 0.51, 0.81 and 0.04. In comparison, the bankrupt $\gamma=0$ portfolio’s ratios are 0.46, 0.77 and 0.03. This raises questions as to where the line can be drawn between bankrupt portfolios and those that provide superior returns.

We offer two explanations for the above anomalies. First, the denominator for both the Sortino ratio and the upside potential ratio is the downside deviation. A larger downside deviation would result in lower ratios. The magnitude of downside deviation depends on two factors: the number of returns that are below MAR (which controls the number of terms in the summation) and the magnitude of these returns. From Table 1, Panel A, we find the number of negative returns increases with the level of accumulated wealth. The portfolio with $\gamma=-50$ has least negative returns, while portfolio with $\gamma=-10$ and MV portfolio have 25 and 21 more negative returns. As such, downside deviation for this portfolio is low, resulting in the highest rank by all three measures. On the other hand, the minimum return for portfolio with $\gamma=-10$ is -52.61, which is much lower than the other two portfolios. This results in a high downside deviation and low ratios. Analyzing the remaining panels in Table 1 would give similar insights to the other sets of portfolios.

Our second explanation relates to the intended objectives for these measures. The portfolio performance index intends to identify the portfolio with the least probability of its return falling below the MAR. However, such portfolio is likely not going to generate a high level of wealth, as this portfolio would be more conservatively invested and lie towards the lower-left corner of the mean-variance space. A good example is the mutual funds used in Foster and Stutzer’s (2004) empirical analysis. The Fidelity Equity Income
Fund ranks 11th by the portfolio performance index, while the Van Kamp Emerging Growth Fund ranks 32nd by the same measure. The ranking by average log return, however, are slightly higher for the latter fund. An examination of the composition of the portfolio would provide clear explanation. The Fidelity fund invests only 68% in common stocks, whereas 91% of Van Kamp fund’s assets is in this asset class. It is clear why the more risk-averse Fidelity fund results in a higher ranking under portfolio performance index, but not by cumulative wealth. From the above, we conclude that the portfolio performance index is not an appropriate measure to be used for ranking portfolios by cumulative wealth, just like the downside Sharpe ratio with an arbitrary add factor is not appropriate for identifying bankrupt portfolios.

To confirm our findings above, we perform our empirical set on the fifth set of portfolios from Grauer (2007c). Each of these portfolios accumulates positive wealth over the 1934-1999 period, some of which are significantly greater than the previous four sets of portfolios. Table 3 shows the summary statistics of sample data of margin strategies. We are not surprised at these statistics: the accumulated wealth increases up with the investor’s risk appetite (the larger risk tolerance parameter). While the mean return and the standard deviation of the return increases with γ, it does not reach the extreme high levels for some of the bankrupt strategies demonstrated earlier. For instance, for γ=0.9, the mean quarterly return and the standard deviation are only 7.26% and 21.31%, respectively. Moreover, the maximum return for each portfolio is larger than the absolute value of minimum returns. The number of negative returns for each portfolio is lower than most of the portfolios in Table 1.
Table 4 on page 37 shows the various performance measures for these portfolios. Focusing on the Sortino ratio, the upside potential ratio and the portfolio performance index with MAR set to zero, we see that they are distinguishably higher than the corresponding measures for bankrupt strategies in Table 2. At the same time, the inability to rank portfolios is more evident; the rankings by each measure are exactly the reverse from the rankings by cumulative wealth. It is interesting to note that, except for the cases where $\gamma$ is non-negative, Jensen’s alpha does a better job in ranking portfolios by wealth.
7 CONCLUSION

In this paper, we have tested six downside risk-adjusted performance measures’ ability to distinguish bankrupt portfolios from non-bankrupt ones. The symmetric downside risk Sharpe ratio, the downside Sharpe ratio and the adjusted Sharpe ratio are not effective measures for this purpose. On the other hand, the Sortino ratio, the upside potential ratio and the portfolio performance index for bankrupt portfolios are substantially different from the non-bankrupt ones if the MAR is set to zero. We also observe that these three measures are unable to correctly rank the non-bankrupt portfolios according to their cumulative wealth over the 1934-1999 period.

Investors make portfolio selection decisions based on different criteria. Some investors prefer ultimate wealth accumulation and can tolerate higher downside risk, while some requires an investment that is higher than a certain minimum acceptable level and more concerned about investment safety. One must consider his investment objective when choosing the appropriate performance measure, or may expose himself to higher than expected risk.

A notable aspect of our study is that the portfolio returns are generated by the mean-variance optimizer using different estimates of the means. Portfolios with returns as extreme as the ones shown in this paper are not commonly observed. However, one may wish to perform our empirical test using data from a set of individual hedge funds, where some bankrupted and some prospered. Additional research may be conducted with other risk-adjusted performance measures, such as value-at-risk, Omega and Kappa.
Table 1
Summary Statistics for Mean-Variance Efficient Portfolios, Global Minimum-Variance Portfolios, and Tangency Portfolios Estimated from Historical Means When Short Sales Are Permitted

The investment universe consists of twelve value-weighted industry groups in the 1934-1999 periods. Quarterly portfolio revision with a 32-quarter estimation period is employed. The arithmetic average return and standard deviation of return are measured in percent per quarter. The standard deviation used in constructing the Sharpe ratio is measured in units of excess return. Wealth is the cumulative wealth at the end of 1999 arising from an investment of $1 at the beginning of 1934. A mean-variance efficient portfolio is defined in term of gamma of a power utility function $u(w) = (1/\gamma)w^\gamma$, $\gamma < 1$. The corresponding risk tolerance parameter employed in the mean-variance optimizer is given by $T = 1/(1-\gamma)$.

<table>
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<tr>
<th>Portfolio</th>
<th>Wealth</th>
<th>Mean</th>
<th>Stddev</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Max</th>
<th>Min</th>
<th>Number of negative returns</th>
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<td>3.76</td>
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### Table 1 Continued

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<th>Skewness</th>
<th>Kurtosis</th>
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Table 2
Summary of Performance Measures for Mean-Variance Efficient Portfolios, Global Minimum-Variance Portfolios, and Tangency Portfolios Estimated from Historical Means When Short Sales Are Permitted

The investment universe consists of twelve value-weighted industry groups in the 1934-1999 periods. Quarterly portfolio revision with a 32-quarter estimation period is employed. The arithmetic average return and standard deviation of return are measured in percent per quarter. The standard deviation used in constructing the Sharpe ratio is measured in units of excess return. Wealth is the cumulative wealth at the end of 1999 arising from an investment of $1 at the beginning of 1934. A mean-variance efficient portfolio is defined in term of gamma of a power utility function $u(w) = (1/\gamma)w^{\gamma}$, $\gamma < 1$. The corresponding risk tolerance parameter employed in the mean-variance optimizer is given by $T = 1/(1 - \gamma)$.

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<th>Portfolio</th>
<th>Wealth</th>
<th>Sharpe Ratio</th>
<th>Jensen's Alpha</th>
<th>Symmetric DS Risk Sharpe Ratio</th>
<th>Downside Sharpe Ratio</th>
<th>Adjusted Sharpe Ratio</th>
<th>Sortino Ratio</th>
<th>Upside Potential Ratio</th>
<th>Portfolio Performance Index</th>
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Panel A: Historical Means
Table 2 Continued

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<th>Jensen's Alpha</th>
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<th>Downside Sharpe Ratio</th>
<th>Adjusted Sharpe Ratio</th>
<th>Sortino Ratio</th>
<th>Upside Potential Ratio</th>
<th>Portfolio Performance Index</th>
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<th>Sortino Ratio</th>
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Panel D: Bayes-Stein Means

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<th>Symmetric DS Risk Sharpe Ratio</th>
<th>Downside Sharpe Ratio</th>
<th>Adjusted Sharpe Ratio</th>
<th>Sortino Ratio</th>
<th>Upside Potential Ratio</th>
<th>Performance Index</th>
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Table 3
Summary Statistics for Mean-Variance Efficient Portfolios
Estimated from Dividend Yields and Risk-free Interest Rates Means with Margin Requirements And Short Sales are Precluded

The investment universe consists of common stocks, long-term government bonds and small stocks in the 1934-1999 periods. The margin requirements for stocks are obtained from the Federal Reserve Bulletin, and the initial margins are set at 10% for government bonds. The arithmetic average return and standard deviation of return are measured in percent per quarter. The standard deviation used in constructing the Sharpe ratio is measured in units of excess return. Wealth is the cumulative wealth at the end of 1999 arising from an investment of $1 at the beginning of 1934. A mean-variance efficient portfolio is defined in term of gamma of a power utility function $u(w) = (1/y)w^y, y < 1$. The corresponding risk tolerance parameter employed in the mean-variance optimizer is given by $T = 1/(1 - y)$.

<table>
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<th>Skewness</th>
<th>Kurtosis</th>
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<td>γ=-10</td>
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<td>6.91</td>
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<td>5883</td>
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<td>9.19</td>
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<td>46.85</td>
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<td>12.15</td>
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<td>5.90</td>
<td>60.64</td>
<td>-34.28</td>
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<td>5.47</td>
<td>15.00</td>
<td>1.26</td>
<td>9.16</td>
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<td>γ=0.5</td>
<td>323211</td>
<td>6.31</td>
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<td>14.53</td>
<td>133.15</td>
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<td>γ=0.9</td>
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<td>11.09</td>
<td>143.75</td>
<td>-76.43</td>
<td>79</td>
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Table 4
Summary of Performance Measures for Mean-Variance Efficient Portfolios
Estimated from Dividend Yields and Risk-free Interest Rates Means with Margin Requirements And Short Sales are Precluded

The investment universe consists of common stocks, long-term government bonds and small stocks in the 1934-1999 periods. The margin requirements for stocks are obtained from the Federal Reserve Bulletin, and the initial margins are set at 10% for government bonds. Quarterly portfolio revision with a 32-quarter estimation period is employed. The arithmetic average return and standard deviation of return are measured in percent per quarter. The standard deviation used in constructing the Sharpe ratio is measured in units of excess return. Wealth is the cumulative wealth at the end of 1999 arising from an investment of $1 at the beginning of 1934. A mean-variance efficient portfolio is defined in terms of gamma of a power utility function $u(w) = (1/\gamma)w^\gamma$, $\gamma < 1$. The corresponding risk tolerance parameter employed in the mean-variance optimizer is given by $T = 1/(1 - \gamma)$.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Wealth</th>
<th>Sharpe Ratio</th>
<th>Jensen Alpha</th>
<th>Symmetric DS Risk Sharpe Ratio</th>
<th>Downside Sharpe Ratio</th>
<th>Adjusted Sharpe Ratio</th>
<th>Sortino Ratio</th>
<th>Upside Potential Ratio</th>
<th>Portfolio Performance Index</th>
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<tr>
<td>$\gamma = -50$</td>
<td>32</td>
<td>0.24</td>
<td>0.06</td>
<td>0.67</td>
<td>0.28</td>
<td>0.66</td>
<td>0.61</td>
<td>3.97</td>
<td>0.95</td>
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<td>0.27</td>
<td>0.11</td>
<td>0.59</td>
<td>0.28</td>
<td>0.54</td>
<td>0.62</td>
<td>2.43</td>
<td>0.96</td>
</tr>
<tr>
<td>$\gamma = -15$</td>
<td>170</td>
<td>0.29</td>
<td>0.21</td>
<td>0.55</td>
<td>0.29</td>
<td>0.51</td>
<td>0.64</td>
<td>1.57</td>
<td>1.00</td>
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<tr>
<td>$\gamma = -10$</td>
<td>410</td>
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<td>0.30</td>
<td>0.54</td>
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<td>0.51</td>
<td>0.65</td>
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<td>0.38</td>
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<td>0.29</td>
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<td>0.63</td>
<td>1.02</td>
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<td>0.68</td>
<td>0.92</td>
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<td>0.29</td>
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<td>0.63</td>
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<td>0.47</td>
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<td>0.64</td>
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<td>0.42</td>
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<td>0.58</td>
<td>0.69</td>
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REFERENCE LIST


