CONSTANT PROPORTION DEBT OBLIGATIONS:
GENUINE MONEY MACHINES OR ARTIFACTS OF
AMBITIOUS MODELING?

by

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We model Constant Proportion Debt Obligations (CPDOs) within a Monte Carlo simulation framework in order to examine their sensitivity to various assumptions and parameters. We find that, although we are able to generate exceptionally low default rates under certain assumptions that are commonly used in the industry literature, the sensitivity of default rates and other performance measures to these assumptions is high. Moreover, given the questionable plausibility of some of these assumptions, it is not clear that CPDOs truly carry the same low risk as similarly-rated high investment grade products.

**Keywords:** Constant Proportion Debt Obligation; CPDO; Structured Finance; Structured Credit; Credit Derivatives
DEDICATION

To our families and spouses.
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**GLOSSARY AND ABBREVIATIONS**

<table>
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<tr>
<td><strong>Basis Points (bps)</strong></td>
<td>(1 \text{ bps} = 0.01%).</td>
</tr>
<tr>
<td><strong>Cash-In Barrier (CIB)</strong></td>
<td>The present value of all promised future liabilities of the CPDO. It is the minimum note value required for the CPDO to cash-in.</td>
</tr>
<tr>
<td><strong>Cash-In Event</strong></td>
<td>An event where the CPDO unwinds its risky exposure because it has accumulated enough value to pay all of its future liabilities.</td>
</tr>
<tr>
<td><strong>Cash-Out Event</strong></td>
<td>An event where the CPDO unwinds its risky exposure because the note value has migrated below some predetermined cash-out threshold.</td>
</tr>
<tr>
<td><strong>Cash-out Threshold</strong></td>
<td>A predetermined note value, usually expressed as a percentage of the notional amount, below which the CPDO cashes-out.</td>
</tr>
<tr>
<td><strong>Cash Deposit</strong></td>
<td>Synonymous and used interchangeably with <em>Cash Deposit Account</em>.</td>
</tr>
<tr>
<td><strong>Cash Deposit Account (CDA)</strong></td>
<td>The CPDO's cash account to which interest payments, protection premiums, and realized MtM gains are credited; and from which coupon payments and realized MtM losses are debited. Cash Deposit Account = Note Value - Unrealized Gains + Unrealized Losses.</td>
</tr>
<tr>
<td><strong>CDX Indices</strong></td>
<td>A family of CDS Indices that contain North American and emerging market companies as reference entities. New series of CDX indices are typically issued every six months.</td>
</tr>
<tr>
<td><strong>CDX.NA.IG (5-year)</strong></td>
<td><em>CDX North American Investment Grade (5-year)</em>. A specific, highly liquid type of CDX Index on which protection is commonly sold in first generation CPDO structures. The 125 equally weighted reference entities are all North American and of investment grade quality. Each series expires 5.25 years after its roll date.</td>
</tr>
<tr>
<td><strong>Constant Proportion Debt Obligation (CPDO)</strong></td>
<td>An adjustable-leverage structured credit product that pays a regular floating coupon to an investor while taking leveraged exposure to one or more CDS indices. The exposure is typically rolled-over every six months to the newest series of the CDS indices.</td>
</tr>
<tr>
<td>Term</td>
<td>Definition</td>
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<td>-------------------------------------------</td>
<td>---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td><strong>Contracted Spread</strong></td>
<td>The agreed-upon spread at which a protection buyer pays a protection seller for default protection.</td>
</tr>
<tr>
<td><strong>Credit Default Swap (CDS)</strong></td>
<td>A type of credit derivative where a protection seller receives regular payments from a protection buyer in return for providing protection against a credit event on the part of a specified reference entity. In the event of a credit event, the protection seller pays the protection buyer the deliverable obligations of the reference entity, and the CDS contract is terminated.</td>
</tr>
<tr>
<td><strong>Credit Default Swap (CDS) Index</strong></td>
<td>A standardized portfolio of single-name credit default swaps. The two main families of CDS indices are CDX and iTraxx.</td>
</tr>
<tr>
<td><strong>Credit Default Swap (CDS) Index Spread</strong></td>
<td>The market-observed spread at which the CDS index in question is trading. Usually annualized and expressed in basis points, it is the premium which a protection buyer would have to pay a protection seller for $1 of notional protection.</td>
</tr>
<tr>
<td><strong>Credit Event</strong></td>
<td>An event, such as a reference entity default, downgrading or bankruptcy, that affects the payoffs of a credit derivative linked to that reference entity.</td>
</tr>
<tr>
<td><strong>Credit Exposure</strong></td>
<td>Unless mentioned otherwise, synonymous and used interchangeably with Risky Exposure.</td>
</tr>
<tr>
<td><strong>Credit Spread</strong></td>
<td>Unless mentioned otherwise, synonymous and used interchangeably with Credit Default Swap (CDS) Index Spread.</td>
</tr>
<tr>
<td><strong>Entity</strong></td>
<td>Unless mentioned otherwise, synonymous and used interchangeably with Reference Entity.</td>
</tr>
<tr>
<td><strong>First Generation CPDO</strong></td>
<td>An unmanaged CPDO. This type of CPDO is the basis of analysis in this study.</td>
</tr>
<tr>
<td><strong>Gap Risk</strong></td>
<td>The risk that losses on an investment exceed 100% of the notional invested.</td>
</tr>
<tr>
<td><strong>Investment Grade (IG)</strong></td>
<td>A term used to describe a credit-risky investment product with a credit rating of BBB or higher as per the Standard &amp; Poor’s ratings system.</td>
</tr>
<tr>
<td><strong>iTraxx Indices</strong></td>
<td>A family of CDS Indices that contain European, Asian, and Australian companies as reference entities. New series of iTraxx</td>
</tr>
</tbody>
</table>
indices are typically issued every six months.

**iTraxx Europe (5-year)**
A specific, highly liquid type of iTraxx Index on which protection is commonly sold in first generation CPDO structures. The 125 equally weighted reference entities are all European and of investment grade quality. Each series expires 5.25 years after its roll date.

**Junk**
A term used to describe a credit-risky investment product with a credit rating not classified under investment grade.

**Leverage**
The number by which the notional investment is multiplied to arrive at the risky exposure of the CPDO.

**LIBOR**
*London Interbank Offered Rate.* The interest rate at which banks can borrow funds from other banks in the London interbank market. LIBOR is used as a proxy for the risk-free interest rate in this study.

**Mark-to-Market (MtM) Gain**
An increase in the CPDO note value due to favourable movements in spreads.

**Mark-to-Market (MtM) Loss**
A decrease in the CPDO note value due to reference entity credit events or unfavourable movements in spreads.

**Note Value (NV)**
The current market (or net asset) value of the CPDO. Unlike the cash deposit account, the note value always reflects unrealized MtM gains/losses: 

\[
\text{Note Value} = \text{Cash Deposit Account} + \text{Unrealized Gains} - \text{Unrealized Losses}
\]

**Notional**
In general, the initial investment in a CPDO structure by an investor. In some circumstances, it may also refer to the amount of notional default protection sold on a CDS Index (although this is usually labelled as *Risky Exposure*).

**Principal**
Unless mentioned otherwise, synonymous and used interchangeably with *Notional*.

**Reference Entity**
An underlying firm or organization whose debt is linked to a credit derivative.

**Risky Exposure**
The total dollar amount of notional protection sold by the CPDO on the CDS indices at any particular time.

**Roll Date**
The date at which a new CDS index series is issued. For the CDX.NA.IG (5-year) and iTraxx Europe (5-year) indices, the roll
dates are March 20 and September 20 of each year.

**Roll-Down / Roll-Over**

The process whereby, on a roll date, the following two transactions are simultaneously performed: i) default protection is bought back on the previously on-the-run CDS index series, and ii) default protection is sold on the new on-the-run series.

**Roll-Down Benefit**

The profit earned from performing the roll-down process on a roll date.

**Second Generation CPDO**

A managed CPDO. This type of CPDO is not examined in this study.

**Shortfall**

The amount by which the cash-in barrier exceeds the note value. Shortfall = Cash-in Barrier – Note Value.

**Spread**

Unless mentioned otherwise, synonymous and used interchangeably with *Credit Default Swap (CDS) Index Spread*.

**Structure**

A structured investment product such as a CPDO. Note: *structure* is often used interchangeably with *CPDO* in this study.
1. INTRODUCTION

The Constant Proportion Debt Obligation (CPDO), one of the most innovative yet controversial products in the brief history of structured credit markets, was first developed by investment bank ABN Amro in 2006. The product received much hype due to the fact that it offered investors a AAA rating not only on principal, but also on coupons that were as high as LIBOR+200 basis points (bps) – spreads previously unheard of for AAA products. The structure, which essentially takes a leveraged exposure to one or more credit default swap (CDS) indices in order to meet its obligations, follows a complex and dynamic strategy that some have lauded as a ground-breaking design in financial engineering. At the same time, given the tremendous implications of such a product, many have questioned exactly how the structure is able to generate such high-paying yet highly-rated coupons. While some would claim that the answer lies in the leverage mechanism and the mean-reverting tendency of credit spreads, others have more bluntly argued that the structure is unworthy of its high credit rating. More specifically, some believe that the underlying modeling assumptions used in structuring CPDOs are problematic and seriously bring into question the ability of CPDOs to survive adverse market conditions.

In this study, we conduct an in-depth analysis of CPDO modeling and performance. The purpose of the study is primarily to answer the question posed in the

---

1 For example, CPDOs may become highly attractive to institutional investors for whom the AAA rating is essential due to Basel II regulatory capital requirements.
title; i.e. are CPDOs authentic, revolutionary money machines, or are they merely an artificial construct based on flawed modeling assumptions? As a related exercise, we also conduct a sensitivity analysis of the structure’s performance to key parameters.

Section 2 outlines in broad strokes the CPDO strategy including cash inflows and outflows, cash-in and cash-out events, the leverage mechanism, and the index roll-down. We also present two sample CPDO simulation paths, as well as an overview of the important distinction between first generation and second generation CPDOs. Section 3 provides a detailed exposition of our Monte Carlo modeling approach, as well as the performance measures and base case employed in our simulations. In Section 4, we summarize and discuss the simulation results, with an emphasis on the sensitivity of CPDO performance to the most important assumptions and parameters in the modeling framework. Finally, in Section 5, we conclude the study.
2. THE CPDO STRATEGY²

Over the life of the CPDO, the structure’s performance and strategy depends partially on deterministic parameters such as the gearing factor and leverage cap, but primarily on the evolution of stochastic variables including credit spreads, interest rates, reference entity defaults, and transaction costs. In this section we paint a broad overview of the key features of the CPDO strategy including cash inflows and outflows, cash-in and cash-out events, the leverage mechanism, and the roll-over process. Figure 1 presents a schematic diagram of the CPDO strategy.³ For a technical description, our quantitative modeling approach is presented in Section 3.

2.1. Cash Inflows and Outflows

In return for a notional down payment, which (net of upfront fees) is initially deposited in the cash deposit account, an investor is promised regular coupon payments of LIBOR plus a fixed spread, as well as redemption of the notional at the contract’s maturity.⁴ In order to meet its liabilities, the structure takes a leveraged exposure to the credit markets by selling protection on one or more credit default swap (CDS) indices of investment grade reference entities, usually the 5-year CDX North American Investment Grade Index (CDX.NA.IG) and the 5-year iTraxx Europe Index. Hence, two of the structure’s primary cash inflows consist of i) premiums received for selling protection on

² This and the following sections make extensive use of industry jargon, some of which may be unfamiliar or ambiguous to the reader. We encourage the reader to consult the Glossary and Abbreviations section (page xi) for an exhaustive list of terms and definitions.

³ See Appendix 2 for all figures.

⁴ Most, if not all, first generation CPDO structures pay quarterly coupons and have a maturity of ten years.
the CDS indices, and ii) mark-to-market (MtM) gains achieved if the market spread decreases relative to the contracted spread, where the contracted spread is equal to the initial market spread when protection was first sold.\textsuperscript{5} Another (usually less significant) cash inflow is that of overnight interest earned on the cash deposit account, typically at a risk-free LIBOR rate. Conversely, the structure’s cash outflows consist of i) regular, promised coupon payments to the investor, and ii) MtM losses sustained if either the market spread increases relative to the contracted spread, or credit events occur on the reference entities that comprise the CDS indices. There may also be periodic administration fees charged by the dealer facilitating the structure. An essential feature of the structure is that cash outflows continue to be debited from the cash deposit even when the cash deposit has fallen below the initial value of notional invested (subject to the structure not cashing-out; see Section 2.2). Hence, there is no guarantee of notional redemption at maturity.

2.2. Cash-In and Cash-Out Events

There are two types of events following which the CPDO prematurely unwinds or terminates its risky exposure before maturity. The first is known as a cash-in event, defined by a point in time when the structure has accumulated enough value to pay all of its future liabilities; namely, the remaining coupon payments and notional redemption at maturity. Equivalently, the structure is said to have cashed-in at a particular time when the CPDO note value is greater than or equal to the time-dependent cash-in barrier – a measure of the present value of the structure’s liabilities. Yet another way of expressing a

\textsuperscript{5} Intuitively, MtM gains (losses) are realized when spreads decrease (increase) because the structure is receiving protection premiums in excess of (below) those required by the market.
cash-in event is that the structure’s shortfall is less than or equal to zero, where shortfall measures the marginal amount required for the CPDO to cash-in; mathematically, \( \text{shortfall} = \text{cash-in barrier} - \text{note value} \). If cash-in occurs, the structure immediately discontinues selling default protection and is exposed only to a risk-free asset until maturity. It should be noted that if the CPDO does not cash-in at some point during its life (including possibly at maturity), then the CPDO defaults on its obligations.

A second event where the structure unwinds is known as a \textit{cash-out event}. A cash-out occurs if the note value migrates below some predetermined threshold during the life of the structure. For example, if the notional value of the CPDO is 100 and the \textit{cash-out threshold} is 5\%, then the structure cashes-out if the note value falls below 5 at any time. The purpose of the cash-out threshold is to protect the issuer from \textit{gap risk}: the risk that losses on the structure exceed 100\% of the notional invested.\(^6\) If cash-out occurs, the structure immediately discontinues selling default protection, and the remaining balance in the cash deposit (if any) is invested at the risk-free rate until maturity. It should be noted that the cash-out threshold only mitigates gap risk; it does not render the issuer immune to gap risk. For instance, if a structure has not yet cashed-out there is always the (albeit remote) possibility that a sudden downturn in market conditions may cause a large once-off \textit{MtM} loss that exceeds the note value preceding the downturn.

The destiny of a CPDO contract, however, is not limited to either a cash-in or cash-out event. The third and final possibility is that the structure neither cashes-in nor cashes-out; it simply fails to redeem the notional investment when the contract expires at

---

\(^6\) Equivalently, gap risk is the risk that the note value becomes negative.
maturity. In this event, the investor receives all of the promised coupons but suffers a loss on the notional investment.

2.3. Leverage Mechanism

The leveraged exposure to the CDS indices (or simply leverage) is a function primarily of the structure’s performance. If the structure has performed well; i.e. the shortfall has decreased, then leverage is reduced in order to mitigate potential future losses. Recall that the CPDO’s only obligations are to pay the pre-specified floating coupons and to return the notional investment at maturity, implying that the structure has no incentive to generate abnormal profits. Therefore, if the structure is performing well and is approaching the point where it can pay off all its liabilities with certainty (i.e. the cash-in barrier), then leverage is lowered so as to mitigate, if not prevent, the loss of realized gains. On the other hand, if the structure is performing poorly, leverage is raised in order to generate higher nominal protection premiums to make up for the incurred losses. For this reason, the leverage mechanism has often been compared to the “gambler’s fallacy” or “gambler’s ruin” strategy whereby an investor, despite losing money, keeps increasing his bets because he believes that eventually he will recover his lost money. The fundamental difference between the CPDO leverage strategy and plain gambling, however, is that (with CPDOs) this philosophy may hold some legitimacy if credit spreads are mean-reverting – as historical data appear to suggest.

There are a couple caveats related to the general leverage strategy described above. Firstly, leverage is normally capped at a pre-specified, fixed level in order to limit

---

7 The CPDO leverage strategy is the direct opposite to that of Constant Proportion Portfolio Insurance (CPPI) structures.
the risky exposure of the structure in adverse market conditions, where a further increase in leverage would excessively heighten the probability of a cash-out event and would amplify gap risk.\textsuperscript{8} A common leverage cap in first generation CPDO structures is 15; i.e. the maximum amount of notional protection sold (or risky exposure) is 15 times that of the principal value of the CPDO. Secondly, the target leverage is not necessarily implemented every time it changes because the transaction costs of doing so would be overbearing. Typically, CPDO strategies only rebalance their risky exposure when the target leverage moves outside predetermined bands around the existing leverage. For example, if the existing leverage is 10 and the rebalancing lower and upper bounds are (typical values of) 0.75 and 1.25, respectively, then the risky exposure will only be adjusted if the target leverage falls outside the range [7.5, 12.5]. Monitoring of the target leverage is done on a regular, usually daily basis.

2.4. Index Roll-Down

The final key aspect of the CPDO strategy is the index roll-down or roll-over. In order to provide a full description of this procedure, some background institutional details about the underlying CDS indices are helpful. The CDS indices on which most CPDO structures typically sell protection are those in the CDX and iTraxx families. The CDX indices are composed of North American and emerging market reference entities, whereas the iTraxx indices are typically composed of European, Asian, and Australian reference entities. Every six months, for most CDX and iTraxx indices, a new series is issued on a roll date. For example, the 5-year CDX.NA.IG and 5-year iTraxx Europe

\textsuperscript{8} Some structures employ a dynamic leverage cap that is a function of both the note value and index spread level.
indices, the two indices most commonly used in CPDO structures, have roll dates of March 20 and September 20 in each year.\(^9\) The primary reason that new series are issued every six months is to help ensure that the on-the-run series are composed of the most liquid investment grade entities.\(^{10}\) Typically, entities that have either i) been downgraded below investment grade status, ii) experienced a credit default or similar credit event, or iii) become illiquid, are removed from a series at the next roll date. The overall effect is that the on-the-run indices are generally more liquid and carry less credit risk than the off-the-run indices.

The index roll-over or roll-down is a fundamental aspect of the CPDO strategy whereby, on each roll date, the following two transactions are simultaneously performed: i) default protection is bought back on the previously on-the-run CDS index series, and ii) default protection is sold on the new on-the-run series. For example, at the initiation date of the CPDO contract (assume this is also a roll date), protection is sold on the CDS indices specified in the CPDO contract. After six months, protection is purchased for the series on which the structure had been selling protection up until that time. Note that unless the market spread at this time is exactly equal to the initial spread at time zero, there will be a *MtM* gain or loss from the transaction that will be realized in the cash deposit account. In addition, at the six-month mark, the structure enters into new contracts whereby it sells protection on the new, on-the-run CDS index series. This process is repeated every six months until either cash-in, cash-out, or the maturity of the CPDO structure.

\(^9\) The “5-year” maturity labels are slight misnomers. The maturities of these indices are actually 5.25 years. For example, the 5-year iTraxx Europe series issued on March 20, 2007, expires on June 20, 2012.

\(^{10}\) The average Standard and Poor’s rating on the indices is in the ‘BBB+’ to ‘A-’ range.
There are three important reasons why the roll-down process is part of the CPDO strategy: i) reduced default risk, ii) increased liquidity, and iii) roll-down benefits. Firstly, reference entity default risk is reduced by the roll-down process because those entities which migrate below investment grade status – i.e. those entities that carry the highest default risk – are removed from the index every six months. Moreover, the probability of an entity migrating from investment grade status – the status of all entities on the roll date – to default within six months is exceptionally low. Secondly, as mentioned earlier, the on-the-run series are by far the most liquid series as reflected in historical bid-ask spreads, implying that transaction costs are less of a concern for CPDOs that always hold on-the-run series. Thirdly, since the term structure of credit spreads is typically upward sloping – reflecting the belief that, ceteris paribus, default probabilities are smaller over shorter periods of time – there is often a profit to be made simply by rolling-down the CDS index series. These profits are known as roll-down benefits. Some, however, have questioned the validity of the roll-down benefit assumption. We explore this argument in Section 4.2.2.

2.5. Sample CPDO Simulation Paths

Figures 2 and 3 present two sample CPDO simulation paths – one cash-in and one cash-out. These figures are helpful for developing intuition on the co-evolution of four key variables: leverage, note value, spread, and cash-in barrier. As seen in the figures, the spread and note value generally move in opposite directions to each other (as explained in Section 2.1). Leverage and note value similarly move in opposite directions, subject to

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11 This strategy is analogous to that of "riding the yield curve" in the bond markets.
the leverage cap of 15 as well as rebalancing bounds. There is further noise generated as a result of credit defaults and, to a lesser extent, interest rate fluctuations.

2.6. First Generation Versus Second Generation CPDOs

In this study we only examine first generation CPDOs. First generation CPDOs are those that were first issued in the fall of 2006. The primary distinction between first generation CPDOs and their second generation brethren, introduced in 2007, is that first generation CPDOs are not managed. In other words, the dynamic strategy to be followed for the entire lifetime of the structure is fully defined at time zero. In particular, the structure’s roll-over strategy as well as the formula for determining leverage do not change, even if, in the future, it should become clearly apparent that the predetermined strategy is suboptimal. For this and other reasons discussed later in the paper, newer generation CPDOs are typically managed. While we only examine first generation CPDOs in this study, most of the basic principles and results can be applied to second generation CPDOs.

\[\text{No, the title of this section is not a reference to an intergalactic battle from a Star Wars movie!}\]

\[\text{Furthermore, unless mentioned otherwise, any reference to CPDO in this paper implicitly means first generation CPDO.}\]
3. MODELING METHODOLOGY

The modeling of CPDOs is complex and tedious. Necessary steps include interest rate, spread, and reference entity default simulation, as well as the calculation of various interrelated measures (e.g. leverage, note value) at every discrete point in time. Most challenging of all, the spread and default simulation must be done in a joint, consistent manner. A reasonable modelling framework should reflect the market’s anticipation of high defaults in high spread environments, and vice-versa.

Moreover, given the complex, stochastic interaction between many variables, a Monte Carlo simulation framework is required to analyze the structure.\(^{14}\) For each scenario (see Table 2), we conduct 10,000 simulations and analyze the results based on a number of key performance measures described in Section 3.2.\(^{15}\) In order to examine the sensitivity of the performance measures to various parameters and assumptions, the scenarios (34 in total) generally adjust one or a few parameters from the base case described in Section 3.3.

In accordance with the maturity of standard first generation CPDOs, we assume a term to maturity of ten years for all contracts. We also make the standard assumptions that i) coupon payments are made to the investor on a quarterly basis, ii) premiums from selling protection on the CDS indices are collected on a quarterly basis, and iii) premium

\(^{14}\) Interestingly, while nearing the completion of this study, we discovered that UBS Investment Research recently published a closed-form approach to CPDO modeling (Varloot et al., 2006).

\(^{15}\) For most scenarios, 10,000 simulations are sufficient to obtain reliable results. There are only a handful of scenarios, in which CPDO default rates are exceptionally low, where a larger number of simulations would be preferable for calculating highly accurate default rates. For our purposes, however, extremely high accuracy is not required.
collection and coupon payment dates coincide at the start of every quarter. Without loss of generality, we also conduct all simulations and calculations at a quarterly frequency; therefore, the strategy is evaluated at four discrete time points per year. Section 3.1 presents a detailed overview of the modeling approach for each simulation. See Figure 3 for a schematic diagram of the approach presented in Section 3.1.

3.1. Modeling Requirements and Specifications for Each CPDO Simulation

3.1.1. Interest Rate and CDS Index Spread Simulation

The simulation of the interest rate and, in particular, the CDS index spread, are essential steps in CPDO modelling. The simulated interest rate is the three-month LIBOR rate, a commonly-used proxy for the risk-free rate. Unlike some industry research papers which simulate the CDS spread of each individual reference entity or ratings category, we simulate the spread of only a single CDS index on which the CPDO is selling protection. In doing so, we implicitly assume that i) the structure is selling protection on only one CDS index,\(^{16}\) and ii) at every point in time, the simulated spread is of the same, identical maturity. Since the CDS indices on which first generation CPDOs sell protection typically have maturities of 5.25 years on their roll dates (see Footnote 11), this is the maturity assumed in our spread simulation. Furthermore, we assume that the starting spread of the index series at each roll date is exactly equal to the final simulated spread from the previous series.\(^{17}\)

\(^{16}\) Or, equivalently, we assume that the structure is selling protection on multiple CDS indices, and the single simulated spread is a weighted average spread.

\(^{17}\) This assumption is made more plausible by the fact that we are, at all points in time, simulating the 5.25-year spread. Therefore, we need not be concerned with the term structure of spreads in making this assumption.
The model adopted for the simulation of both the interest rate and the CDS index spread is the Cox-Ingersoll-Ross (CIR) model (Cox et al., 1985), which features a mean reversion property as well as volatility that varies with the square root of the level of the current simulated interest rate or spread. This model is chosen because historical data generally indicate that both interest rates and credit spreads tend to i) revert towards a long term mean, and ii) exhibit volatility that is proportional to their current level. In continuous time, the CIR model is expressed as:

\[
dr_t = \alpha_r (\mu_r - r_t) dt + \sigma_r \sqrt{r_t} dW_t^r \quad \text{for the three-month LIBOR rate } r_t; \quad \text{and}
\]

\[
dS_t = \alpha_S (\mu_S - S_t) dt + \alpha_S \sqrt{S_t} dW_t^S \quad \text{for the 5.25-year CDS index spread } S_t,
\]

where \(\alpha_r\) and \(\alpha_S\) represent the mean reversion speeds, \(\mu_r\) and \(\mu_S\) the long term means, \(\sigma_r\) and \(\sigma_S\) the constant multiples of local volatility, and \(dW_t^r\) and \(dW_t^S\) are Standard Brownian Motions; i.e. \(W(t+h) - W(t) \sim N(0,h)\), where \(N\) denotes the normal distribution.

For the purpose of simulation, the corresponding discrete representations are:

\[
r_{t+1} = r_t + \alpha_r (\mu_r - r_t) \Delta t + \sigma_r \sqrt{r_t} \sqrt{\Delta t} W_t^r \quad \text{for the three-month LIBOR rate } r_t; \quad \text{and}
\]

\[
S_{t+1} = S_t + \alpha_S (\mu_S - S_t) \Delta t + \sigma_S \sqrt{S_t} \sqrt{\Delta t} W_t^S \quad \text{for the 5.25-year CDS index spread } S_t,
\]

where \(\alpha_r, \alpha_S, \mu_r, \mu_S, \sigma_r, \sigma_S\) are all annualized parameters, \(\Delta t\) equals 0.25 years, and \(W_t\) is a standard normal random variate. See Appendices 3 and 4 for detailed information on spread and interest rate parameter estimation and simulation.
3.1.2. Reference Entity Credit Default Simulation

Reference entity credit defaults or credit events are simulated in a standard reduced form framework under the usual assumption of a Gaussian copula dependence structure. In brief, the default time for each entity $i$ follows an exponential distribution:

$$F(t) = 1 - \exp\left[\int_0^{t} \lambda_i(y)dy\right],$$

where parameter $\lambda_i$ denotes the instantaneous hazard rate. For a detailed description of the reduced form and copula methodologies, see Lando (2004), Li (2000), and Sklar (1973).

Since the CPDO sells protection on a new CDS index every six months, reference entity defaults must be simulated over each six-month period for the entire life of the CPDO. In doing so, we make two key assumptions. Firstly, we assume that all reference entities, over a particular six-month period, are of the same credit quality that remains constant throughout the period. Equivalently, the reference entities each have an identical hazard rate for the six-month period that may be viewed as the “average” index hazard rate; i.e. $\lambda_i(t,t+0.5) = \lambda(t,t+0.5) \forall i$. While clearly the assumption of homogeneous hazard rates across firms is in violation of reality, we believe that there is little loss in making this assumption over the long term (10 years) and over many simulations (10,000).

Secondly, in order to reflect the market’s anticipation of high (low) defaults in high (low) spread environments, we assume that the constant, “average” (annualized) hazard rate over a particular six-month period, $\lambda(t,t+0.5)$, can be inferred from the simulated CDS

---

18 We simulate defaults for 250 equally weighted reference entities in each six-month period. This corresponds to two indices of 125 entities – the typical number of entities in each of the 5-year CDX.NA.IG and 5-year iTraxx Europe indices.
index spread at the start of that period. This is achieved via the generic CDS formula for constant hazard rates (Lando, 2004):

\[ \lambda(t, t + 0.5) = \frac{S(t, t + 0.5)}{1 - R}, \]

where \( S(t, t+0.5) \) is the time \( t \) simulated (annualized) spread assumed to be constant until time \( t+0.5 \) (i.e. for the next 6 months), and \( R \) is the exogenous entity recovery rate in the event of default (assumed to be identical for all entities and constant over time).\(^{19}\) We believe this approach is an improvement over much of the industry literature which assumes that default risk is constant over the ten-year life of the CPDO.

It is important to note that, under this approach for credit default simulation (which is the approach in our base case), we are using risk-neutral probabilities of default. This is in direct contrast to much of the industry literature which uses objective, or empirical probabilities of default.\(^{20}\) Therefore, in some simulation scenarios, it may be of interest to observe the effect of using objective vis-à-vis risk-neutral default probabilities. Since there is no established method to convert between the two, we take an ad hoc approach. To convert risk-neutral probabilities of default into those that are “objective,” as we do in some scenarios, we employ a hazard rate coefficient (HRC) of 0.25; i.e. we multiply the spread-implied hazard rate at every time point by a factor of 0.25. The justification for this crude approach is as follows. If we assume that all entities

\(^{19}\) We recognize the limitations in using this approach when the current spread differs substantially from its long term mean. For example, if the current spread is well below the long term mean, we would expect the spread to migrate upwards over the next six months. In this situation, we would underestimate the risk of default over the six-month period. This being said, even though instantaneous hazard rates in our model are not reflective of the spread’s long term mean, it is not clear that the long-term mean is generally reflected in market prices.

\(^{20}\) For example, some studies in the industry literature simulate defaults by way of ratings transition simulation. Rating agency credit ratings are all based on objective probabilities of default.
(or the average entity) in the CDS index maintain a constant rating of BBB+ for the next ten years, then the constant six-month objective probability of default for each entity is approximately 0.1% (Wong et al., 2007, p. 14). Over ten years, this implies an expected number of entity defaults of \((250 \text{ firms}) \times (20 \text{ six-month periods}) \times (0.1\% \text{ six-month probability of default}) = 5\) defaults. This is approximately one quarter of the average number of entity defaults generated in our simulations under the risk-neutral measure, implying that we should employ a hazard rate coefficient of 0.25 to approximately convert risk-neutral default probabilities into objective default probabilities. Furthermore, an objective expectation of five entity defaults over ten years is roughly consistent with results from the industry literature.

3.1.3. Leverage and Risky Exposure Determination

The leverage at time \(t\), \(\text{Leverage}_t\), determines the risky notional exposure at time \(t\), \(\text{Risky Exposure}_t\), to the CDS index on which the CPDO is selling default protection. Recall that the basic mechanism of the CPDO's leverage strategy is such that when the structure is performing poorly (well), the CPDO raises (lowers) its leverage, subject to a cap and rebalancing bounds (see Section 2.3). The first step in calculating \(\text{Leverage}_t\) is to determine the target leverage at time \(t\), \(\text{Target Leverage}_t\). The only difference between the two measures is that \(\text{Leverage}_t\) takes into consideration rebalancing bounds, whereas \(\text{Target Leverage}_t\) does not. Although there are slight variations in some structures, the formula most commonly used for \(\text{Target Leverage}_t\) (and the formula we use) is given by:
\[
Target \text{ Leverage}_i = \text{Min} \left[ \frac{\text{Shortfall}_i + \text{Cushion} \cdot \text{Notional}}{\text{PV(Premium Flow)}} \cdot \text{Gearing Factor}, \text{Cap} \right],
\]
where \( \text{Shortfall}_i \) measures the amount required for the CPDO to cash-in, \( \text{Notional} \) is the structure's initial investment or principal value, \( \text{Cushion} \) is a predetermined fraction of the notional (usually 5-10%), \( \text{PV(Premium Flow)} \) is the present value of future premiums earned from an unlevered exposure to the CDS index for the remaining life of the CPDO, \( \text{Gearing Factor} \) is a predetermined constant, and \( \text{Cap} \) is the fixed, maximum allowable leverage.\(^{21}\) We address each of these variables/parameters in further detail below.

As discussed in Section 2.2, \( \text{Shortfall}_i \) is the amount by which the cash-in barrier exceeds the note value at time \( t \). Recall that when \( \text{Shortfall}_i \) is less than or equal to zero, the cash-in barrier is triggered and the risky portfolio is unwound. Mathematically, \( \text{Shortfall}_i \) is defined as:

\[
\text{Shortfall}_i = \text{Cash-in Barrier}_i - \text{Note Value}_i,
\]
where \( \text{Cash-in Barrier}_i \) is the present value of the CPDO's total remaining liabilities, and \( \text{Note Value}_i \) is the market value of the CPDO. \( \text{Cash-in Barrier}_i \) is calculated as:

\[
\text{Cash-in Barrier}_i = \text{Notional} \cdot \left[ 1 + \frac{\text{Fixed Spread}}{4} \cdot \sum_{i=1}^{40-4t} B \left( t, t + \frac{i}{4} \right) \right],
\]
where \( \text{Fixed Spread} \) is the fixed (annualized) coupon payment to the investor above LIBOR, \( B \left( t, t + \frac{i}{4} \right) \) is the risk-free discount factor from time \( t \) to \( t + \frac{i}{4} \), and \( \text{Notional} \) is as previously defined.\(^{22}\)

\(^{21}\) By convention, we set the initial (time zero) leverage equal to the \( \text{Cap} \).

\(^{22}\) See Appendix 5 for the CIR discount factor formulas.
The present value of the future premium flow at time $t$, $PV(Premium\ Flow)_t$, is the present value of future protection premiums earned from an unlevered exposure to the CDS index for the remaining life of the CPDO. It is calculated under the assumption that all future premiums are constant and equal to the spread simulated for the current contract period. The applied formula is:

$$PV(Premium\ Flow)_t = \frac{S_{c,t}}{4} \cdot Notional \cdot \sum_{i=1}^{40\times4} \left( B\left(t,t+\frac{i}{4}\right)H\left(t,t+\frac{i}{4}\right) \right),$$

where $S_{c,t}$ is the (annualized) **contracted spread** equal to the current simulated CDS index spread if $t$ is a roll date (or the simulated spread from the most recent roll date if $t$ is not a roll date),

$H\left(t,t+\frac{i}{4}\right)$ is the risky discount factor from time $t$ to $t+\frac{i}{4}$ that takes into consideration the uncertainty of the premiums, and all other variables are as previously defined. It is worth noting that $PV(Premium\ Flow)_t$ is used only for the purpose of determining target leverage; it has no direct effect on other variables.

The leverage formula contains three constants. The **Cushion** multiplied by the **Notional** (on the numerator of the target leverage formula) exists only for practical purposes. Intuitively, if it did not exist, then as the shortfall approaches zero, the target leverage would also approach zero, implying that the structure would not be able to sell enough credit exposure to generate the imminent cash-in event. The **Gearing Factor**, which typically has a value between one and three (two in our base case), is simply an arbitrary constant determined by the CPDO issuer to adjust the leverage and overall risk.

---

23 In other words, the contracted spread only changes every six months.

24 See Appendix 5 for the formula used to calculate the risky discount factor.
of the structure. Increasing the gearing factor increases the target leverage, and vice versa. Finally, the Cap is the fixed, maximum allowable leverage of the structure. As described in Section 2.3, it exists primarily to reduce the probability of irreversible losses in adverse market conditions.

Having calculated Target Leverage, Leverage is determined by taking into consideration rebalancing bounds. Recall from Section 2.3 that, since leverage rebalancing has associated transaction costs, a leverage rebalancing range is imposed whereby the structure only rebalances its risky exposure when Target Leverage moves outside predetermined bands around Leverage. Specifically, leverage is adjusted to the new target level only when the new target leverage moves above the upper bound or below the lower bound of the rebalancing range. More formally, redefining Leverage as Lev and Target Leverage as TLev, Leverage can be defined using the indicator function I():

\[
Leverage_t = Lev_t = TLev_t \cdot \left[ I\{TLev_t \leq \varepsilon_L Lev_{t-1}\} + I\{TLev_t \geq \varepsilon_U Lev_{t-1}\} \right] \\
+ Lev_{t-1} \cdot I\{TLev_t > \varepsilon_L Lev_{t-1}\} \cdot I\{TLev_t < \varepsilon_U Lev_{t-1}\},
\]

where \(\varepsilon_L\) is the rebalancing lower bound and \(\varepsilon_U\) is the rebalancing upper bound. For example, our base case values are \(\varepsilon_L = 0.75\) and \(\varepsilon_U = 1.25\).

Finally, Risky Exposure, the total dollar amount of notional protection sold by the CPDO on the CDS index at time \(t\), is calculated by multiplying the leverage by the notional amount of the CPDO:

---
25 Some structures employ a time-dependent gearing factor; however, we ignore this complication.
\[ \text{Risky Exposure}_t = \text{Leverage}_t \cdot \text{Notional}. \]

3.1.4. Cash Deposit Account and Note Value Calculation

The Cash Deposit Account (or simply Cash Deposit) and the Note Value are two dynamic variables that reflect the performance of the CPDO structure. Though both record gains and losses, there are slight differences in their calculation that reflect the distinction between realized and unrealized mark-to-market cash flows. The Cash Deposit records only realized Mark-to-Market (MtM) gains and losses, whereas the Note Value records both realized and unrealized MtM gains and losses; therefore, the Note Value reflects the true market value of the structure.\(^{26}\)

To summarize the basic relationship between the two variables:

\[
\text{Note Value}_t = \text{Cash Deposit Account}_t + \text{Unrealized Gains}_t - \text{Unrealized Losses}_t.
\]

At the beginning of each quarter, the cash deposit is credited by i) premiums from selling credit default protection on the CDS index, ii) realized MtM gains, and iii) interest earned from investing the previous period’s cash deposit at the three-month LIBOR rate. Meanwhile, the cash deposit is debited by i) promised coupon payments to the investor, and ii) realized MtM losses. We ignore any upfront or periodic administration fees.

The premium received in each quarter for selling credit default protection is based on the contracted (annualized) spread \(S_{c,t}\) and the previous period’s risky exposure,\(\text{Risky Exposure}_{c,t-1}\). The premium received at the beginning of time \(t\), for selling protection during the previous period, is calculated as:

\(^{26}\) We later provide the distinction between realized and unrealized MtM gains/losses.
\[ \text{Premium}_t = \frac{S_{t-1}}{4} \cdot \text{Risky Exposure}_{t-1}. \]

The dollar amount of the coupon paid to the investor at the beginning of each quarter is simply a function of last period’s LIBOR (i.e. in arrears), the fixed spread above LIBOR, and the CPDO’s notional value:

\[ \text{Coupon}_t = \frac{1}{4} (\text{LIBOR}_{t-1} + \text{Fixed Spread}) \cdot \text{Notional}. \]

The investor’s notional investment (or remaining notional investment in the event of default) is also returned after ten years – the maturity of the structure.

*Mark-to-Market* gains and losses depend on many variables including spreads, interest rates, and transaction costs, as well as *roll-down coefficients* and the period in which *MtM* is calculated. The formulas for calculating *MtM*, are:

\[ \text{MtM}_t = \frac{1}{4} (S_{t-1} - RDC_1 \cdot S_t - tcost) \cdot \text{Risky Exposure}_{t-1} \cdot \sum_{i=1}^{40} B \left( \frac{t \cdot i}{4} \right) H \left( \frac{t \cdot i}{4} \right) \]

\[ - \text{Default Losses}_t, \quad \text{if } t \text{ is a roll date, and} \]

\[ \text{MtM}_t = \frac{1}{4} (S_{t-1} - RDC_2 \cdot S_t) \cdot \text{Risky Exposure}_{t-1} \cdot \sum_{i=1}^{40} B \left( \frac{t \cdot i}{4} \right) H \left( \frac{t \cdot i}{4} \right) \]

\[ - \text{Default Losses}_t, \quad \text{if } t \text{ is not a roll date}, \]

where \( S_{t-1} \) is the previous period’s simulated spread, \( S_t \) is the current simulated spread, \( RDC_1 \) is the roll-down coefficient on roll dates, \( RDC_2 \) is the roll-down coefficient on non roll dates, \( tcost \) is the constant transaction cost associated with the roll-down process, \( \text{Default Losses} \) are any losses incurred due to reference entity defaults, and all other variables are as previously defined.
The roll-down coefficients, $RDC_1$ and $RDC_2$, are used if we wish to impose roll-down benefits on the model by multiplying the simulated spread at each time point by a coefficient ($\leq 1$) that is a function of the term to maturity of the index series. Since it is assumed that we are always simulating the 5.25-year spread (see Section 3.1.1), our model cannot account for a non flat term structure of credit spreads unless we artificially modify the simulated spread at each time point. Typically, $RDC_1 > RDC_2$ because, on roll dates, the term to maturity of the off-the-run index series (i.e. the series on which protection is bought back) is 0.25 years shorter than it was at the previous time point. Although this approach is very much ad hoc, it is the same as that employed in much of the industry literature.

The transaction cost on the roll date, $t_{cost}$, is simply the bid-ask spread of the off-the-run CDS index series on the roll date. For simplicity, we assign a constant value to this parameter for the entire life of the CPDO. Note that we do not apply a transaction cost for leverage rebalancing events in between roll dates.\(^{27}\)

Default losses at time $t$ depend primarily on the number of simulated reference entity credit defaults in the previous period:

$$\text{Default Losses}_t = \frac{\text{No. of Ref. Entity Defaults}_{t-1}}{250} \cdot (1 - R) \cdot \text{Risky Exposure}_{t-1},$$

where $R$ is the recovery rate, 250 are the number of equally weighted underlying reference entities in the CDS index (see Section 3.1.2), and Risky Exposure$_{t-1}$ is as previously defined. Since the simulated default times may occur at any time point

\(^{27}\)There should not be much loss of generality in not applying a transaction cost for leverage rebalancing events in between roll dates. If anything, this slightly biases the results in favour of better CPDO performance; however, it is hard to believe that this bias would be significant if even detectable.
between each quarter, any default loss that does occur is accrued at the risk-free rate of interest to the end of the quarter in which the loss occurs. Without loss of generality, in the event of a reference entity default, we do not directly adjust the notional amount of protection sold on the index as a result of that entity being removed from the index.\(^{28}\)

As discussed earlier, it is necessary to distinguish between realized and unrealized MtM gains/losses. We assume that MtM, on the roll date is always realized immediately in the Cash Deposit Account because contracts are settled when protection is bought back on the old CDS index series at that date. MtM gains or losses on non roll dates, however, are unrealized in the Cash Deposit Account. The only exception is Default Losses, which are realized in every period where reference entity defaults occur because, with standard CDS indices, payments by protection sellers are usually made fairly soon following credit events. To summarize:

\[
\begin{align*}
\text{Realized } MtM, &= MtM, & \text{if } t \text{ is a roll date,} \\
\text{Realized } MtM, &= \text{Default Losses}, & \text{if } t \text{ is not a roll date,} \\
\text{Unrealized } MtM, &= 0, & \text{if } t \text{ is a roll date, and} \\
\text{Unrealized } MtM, &= MtM, + \text{Default Losses}, & \text{if } t \text{ is not a roll date.}
\end{align*}
\]

To sum up, \(CDA_t\), the Cash Deposit Account at time \(t\), is calculated as:

\[
CDA, = CDA,_{t-1} \cdot \left(1+\frac{1}{4} \cdot \text{LIBOR}_{t-1}\right) - \text{Coupon}, + \text{Premium}, + \text{Realized MTM},.
\]

\(^{28}\)In reality, if a reference entity default occurs, the amount of notional protection sold on the index decreases by the weight of the defaulted entity in the index. For example, assuming an initial notional protection of $1,000 and a equally weighted index of 250 reference entities, then, if there is a default, the amount of notional protection sold immediately decreases by 0.4% to 99.6% * $1,000 = $996. Ignoring this technicality makes practically no difference to our simulation results.
*Note Value*, the net asset value of the CPDO at time $t$, is calculated as:

$$
notevalue_t = CDA_t + \text{Unrealized MTM}_t.
$$

### 3.1.5. Cash-in or Cash-out Events

As described in Section 2.2, cash-ins and cash-outs are two types of events following which the CPDO prematurely unwinds or terminates its risky exposure before maturity. A cash-in event occurs if and only if, at any time $t$, $\text{Shortfall}_t$ is less than or equal to zero. If a cash-in occurs, the structure immediately discontinues selling default protection and hedges its remaining liabilities. It does so, firstly, by purchasing a series of risk-free zero coupon bonds to pay the remaining fixed coupon strips above LIBOR. The face value of each zero equals the amount of the fixed, quarterly coupon strip above LIBOR. Under the risk-neutral measure, the prices of these bonds are assumed to be based exactly on the CIR risk-free discount factors (see Appendix 5). Once these risk-free zero coupon bonds are purchased to hedge the remaining fixed coupon strips above LIBOR, the remaining cash deposit account balance is exactly equal to the notional amount invested.\(^{29}\) This notional amount is then invested at the three-month LIBOR rate that exactly matches the floating LIBOR strip payments to the investor. This implies that the remaining cash balance at maturity, following the final coupon payment, is exactly equal to the notional redemption value. Overall, having gone through this hedging argument, the main modeling implications are that, following a cash-in: i) the balance of the *Cash Deposit Account* is equal to the *Notional* for the remaining life of the CPDO, and ii) the *Note Value* is at all future points in time equal to the *Cash-in Barrier*.

\(^{29}\) This result may be gleaned from the *Cash-in Barrier*, formula.
A cash-out event occurs if and only if, at any time $t$, Note Value, is less than or equal to the Cash-out Threshold multiplied by the Notional, where the Cash-out Threshold is some fixed, predetermined fraction of the notional (usually 5-10%). If a cash-out occurs, then: i) the structure immediately discontinues selling default protection, ii) coupon payments to the investor are ceased, and iii) the remaining balance in the Cash Deposit Account (if any) is invested at the risk-free LIBOR rate until maturity. The Note Value is at all future points in time equal to the Cash Deposit Account.

3.2. Performance Measures

We employ three key CPDO performance measures in our 34 simulation scenarios. The first is the CPDO default rate, or the proportion of simulations in which the CPDO defaults on any of its obligations over its ten-year lifetime. The CPDO default rate is thus a ten-year, first cent, cumulative probability of default. Clearly, a high default rate is an indication of poor CPDO performance. The second measure used is the average realized present value of the CPDO, or the average realized $PV[CPDO]$. The present value of the CPDO for each simulation is calculated by discounting the amount of each realized investor payment (coupons and principal) by the discount factors based on the realized LIBOR process, and then summing these discounted investor payments. Intuitively, the average value across all simulations can be interpreted as the CPDO
"price" under the risk-neutral measure. Theoretically, using the risk-neutral probability measure and risk-neutral discount factors (with zero transaction costs), the average realized PV[CPDO] should equal the notional value of 100; i.e. it should price to par. We were therefore surprised when, under these assumptions, the average realized PV[CPDO] tended to differ significantly from the notional value of 100. One explanation may lie in the fact that we calculate MtM gains and losses using risky (i.e. not risk-neutral) discount factors. Another explanation, as suggested by one reviewer, lies in our inference of the 6-month hazard rate from the 5.25-year spread; ideally, we should be using a shorter maturity spread for this purpose. If we employ a hazard rate coefficient of 0.5 to account for the differing maturities of the hazard rate and the simulated spread (but not employing objective default probabilities), we find that (with zero transaction costs) the average realized PV[CPDO] is approximately equal to the notional value of 100.

This price is of supreme interest because it can be interpreted as the actual worth of the CPDO structure to an investor at time zero. Surprisingly, this study is the only known study to calculate this measure; most other industry research focuses exclusively on the cash-in rate (or, equivalently, the default rate). Clearly, a high average realized PV[CPDO] is an indication of strong performance by the CPDO structure. Finally, we are also interested in the average principal recovery given default of the CPDO at maturity. A low average principal recovery might confirm suspicions that, perhaps due to the leverage mechanism, CPDOs have low recoveries in the event of default.

We note that the distribution of values that determine the performance measures, and not only the average value, is often of interest. Therefore, in the simulation results discussion, we may make reference to the shape of the associated distributions. In the case of the default rate, which is derived from a series of binary variables (i.e. default or no default), there is no related distribution of interest. Instead, we often make reference to the cash-in time distribution; i.e. the distribution of cash-in times, conditional on cash-in having occurred. We also note that for the conditional distributions – i.e. the principal recovery given default and cash-in time distributions – these are only of interest if there are a sufficient number of simulations where the conditional requirements are satisfied.
For example, for the principal recovery given default distribution, any inference based on a scenario with extremely low default rates (< 0.1% for 10,000 simulations) is unreliable.

3.3. Simulation Base Case

The regular base case parameters and inputs are presented in Table 1. By and large, all values are representative of standard industry measures. The CPDO strategy parameters, including the fixed coupon above LIBOR, gearing factor, rebalancing bounds and leverage cap, all take values similar to those found in actual first generation CPDO structures. The roll date transaction cost is approximated by the historical average bid-ask spread for the first six months of a selection of 5-year CDX.NA.IG and 5-year iTraxx Europe series. The values of the interest rate process parameters have been estimated from historical three-month LIBOR data (see Appendix 4). The values of the spread process parameters, however, have been manually chosen to be consistent with the parameters typically used in the industry literature. Estimation of the spread process parameters from historical CDS index data would be inappropriate because CDS indices have only been in existence since 2004, and have not yet experienced a major downturn in the credit markets (see Appendix 3). We note that the initial credit spread of 30 bps is the approximate value of the 5-year CDX.NA.IG and 5-year iTraxx Europe indices as of June, 2007. The correlation and recovery parameters associated with the reference entities in the CDS index also take on standard values.

There are two notable exceptions to the generally standard values of our parameters. First, we refrain from imposing roll-down benefits in our base case; i.e. both

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31 It is worth noting that, generally speaking, the bid-ask spread does not follow any noticeable trend over the first six months of the lives of these indices.
roll-down coefficients, $RDC_1$ and $RDC_2$, are set equal to one. The reason we do not impose roll-down benefits in our base case is primarily because, if we did so, it would be somewhat inconsistent with our default simulation process where we assume a constant spread over each six-month period. Nevertheless, in some (non base case) scenarios, we do impose roll-down benefits to examine the sensitivity of CPDO performance to these parameters. Second, as discussed in Section 3.1.2, in our base case, we assume risk-neutral probabilities of default for the simulation of reference entity default times. Therefore, our hazard rate coefficient ($HRC$) is set equal to one.

Later, we introduce a second base case, known as the *industry base case*, where objective probabilities of default, roll-down benefits, and lower transaction costs are all implemented. For ease of exposition, we delay the complete description of the industry base case until Section 4.2.3.
4. SIMULATION RESULTS AND DISCUSSION

4.1. Base Case Analysis

The base case (Scenario 1, Figure 4) results in an extremely high default rate for the CPDO structure (84%) and an average realized PV[CPDO] (68.9) that is significantly lower than the initial notional investment of 100. In the event of default, the average principal recovery is only 32% and the proportion of cash-outs is rather high at 35%. As a reflection of the high cash-out rate, 42% of defaults result in a notional recovery of less than 10%. Overall, even when taking into consideration the likelihood that we negatively bias the CPDO’s performance due to our assumptions of risk-neutral probabilities of default and a flat term structure of credit spreads, the performance of the CPDO structure in our base case is astonishingly poor.32

4.2. Reconciling the Results with the Industry Literature

Since our base case CPDO default rate of 84% is in clear disagreement with the known industry literature, we feel compelled to explore the reasons for this significant divergence. Since our CPDO strategy parameters (coupon above LIBOR, leverage cap, gearing factor, etc) as well as our spread and interest rate process parameters are all fairly standard, we believe there can only be two major explanatory factors accounting for the

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32 It is worth noting that the default rate and average realized PV[CPDO] are equivalent performance measures in the sense that, for almost all scenarios, they generally yield similar conclusions relating to the overall performance of the CPDO. Interestingly, over all 34 scenarios, the sample correlation coefficient between the two measures is almost perfectly negative (-0.98).
difference in results: i) our use of risk-neutral instead of objective probabilities of default, and ii) our assumption of a flat term structure of credit spreads.

4.2.1. Adjusting the Base Case for "Objective" Probabilities of Default

In the base case, ignoring the occurrence of the CPDO cashing-in or cashing-out, there are on average approximately 22 individual entity defaults in the CDS index (on which the structure is selling protection) over the ten-year period. In other words, over all the 20 six-month periods in which credit defaults are simulated for each CPDO contract, on average 22 firms default over the entire ten-year period. If we assume that all of the entities in the relevant CDX and iTraxx indices will always be of investment grade quality at the index roll date, then our results imply that, on average, at least one firm will migrate from investment grade quality to default in each six-month period for the next ten years. While recent experience suggests that such an outcome is highly unlikely – indeed, there has never been a default within the first six months in any of the CDX and iTraxx investment grade series – these indices have not existed long enough to experience a major downturn in the credit markets. Generally speaking, over a ten-year period, where a severe systematic downgrade in the credit quality of many firms is more than a remote possibility, we can expect to see at least a few episodes where firms migrate from investment grade to default within six months. For example, over the past ten years, at least two companies – namely, Enron and WorldCom – deteriorated from investment grade quality to bankruptcy overnight. Moreover, in the event of systematic downgrades

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33 The oldest CDX and iTraxx series have only been in existence since 2004. The last major downturn in the credit markets was in 2002.
in credit quality, it is possible that not all of the entities in the relevant CDS indices will be of investment grade quality on series roll dates.

In any event, the average figure of 22 entity defaults per 10 ten years in our model arises from the use of risk-neutral probabilities of default implied by the current level of the simulated spread. Therefore, to observe the effect of using "objective" default probabilities, we employ a hazard rate coefficient of 0.25 (see Section 3.1.2).

After performing this adjustment, there is a significant improvement in the CPDO’s performance (Scenario 2, Figure 5). Compared to the base case, where risk-neutral default probabilities are used, there is a dramatic 50% reduction in the CPDO’s default rate when we use "objective" default probabilities. This improvement is a direct result of a far smaller number of average entity defaults (7.3). There is also a substantial improvement in the average realized PV[CPDO] (102.3 vs. 68.9) as well as in the average principal recovery given default (67.4 vs. 31.7). Clearly, if we assume that objective probabilities of default are smaller than risk-neutral probabilities of default, then our base case significantly underestimates the CPDO’s performance. However, even when using objective default probabilities, the 32% default rate on the CPDO structure still qualifies it as junk status. This suggests that further adjustments are necessary in order to obtain the AAA default probabilities that are found in the industry literature; in particular, we need to re-examine our assumption of a flat term structure of credit spreads.
4.2.2. Adjusting for an Upward Sloping Term Structure of Credit Spreads Using Roll-Down Benefits

It is suggested in the industry literature that the realization of roll-down benefits is a major contributor to the CPDO's ability to fulfil its obligations (see Section 2.4). As previously discussed, we do not impose roll-down benefits in our base case because it would be inconsistent with our default simulation procedure. Nevertheless, we again take an ad hoc approach in an attempt to explore the effect of roll-down benefits. This approach, which is actually the same as that employed in much of the industry literature, imposes roll-down benefits on the model by multiplying the simulated spread at each time point by a roll-down coefficient \((\leq 1)\) that is a function of the term to maturity of the index series. See Section 3.1.4 for a complete description of this approach.

The effect of imposing roll-down benefits, with all other parameters equal to the base case, is staggering. Using standard industry values for the roll-down coefficients of \(RDC_1 = 0.93\) (roll date) and \(RDC_2 = 0.97\) (non roll date), not only does the CPDO default rate decrease dramatically to \(11\%\), but the average realized \(PV[CPDO]\) (104.2) also exceeds the notional investment (Scenario 3, Figure 6). Interestingly, there is not much effect on the principal recovery given default distribution. With slightly more optimistic values for the roll-down coefficients (Scenario 4, Figure 7), the marginal improvement in the CPDO's performance catapults the structure to high investment grade status (0.4% default rate). In light of these results, one might surmise that the effect of imposing roll-
down benefits is even more substantial than that of switching from risk-neutral to “objective” probabilities of default (as in Scenario 2).³⁴

Overall, the evidence that the CPDO’s performance is heavily influenced by its ability to earn roll-down benefits is clear. This result is somewhat discomforting, for two reasons. Firstly, one study (Linden et al., 2007) has shown that if an investor held a long position in highly correlated proxies for the CDX and iTraxx investment grade indices over the past ten years, the investor would have experienced a significant amount of negative roll-down benefits. Moreover, there have been a number of periods in the history of bond markets where credit spread curves have been inverted; i.e. short-term spreads have exceeded long term spreads. These periods have typically occurred during high volatility and high spread environments that the CDX and iTraxx indices have yet to experience. Secondly, there is a widely-held concern that CPDOs may themselves move the credit markets in a manner such that roll-down benefits are diminished or eliminated due to liquidity-related constraints. Since, by and large, first generation CPDOs are contractually obligated to buy protection on the same off-the-run indices and sell protection on the same new on-the-run indices every six months, such large-scale identical trading may eliminate the gap between the 4.75 and 5.25-year spreads due to supply and demand forces. Moreover, to the further detriment of the CPDO strategy, other market participants may position themselves in order to take advantage of these predictable movements. In light of these veritable concerns, it is certainly discomforting that CPDO performance rests so heavily on the realization of roll-down benefits.

³⁴ Although this may well be the case, we avoid the temptation of explicitly stating as such for two reasons: i) the relative impact of the effects may be largely influenced by the values of our base case parameters; and ii) the methods we have used to impose both the “objective” probabilities of default and the roll-down benefits are somewhat simplistic and slightly inconsistent with our default simulation process.
4.2.3. Adjusting the Base Case for Both “Objective” Default Probabilities and an Upward Sloping Credit Spread Term Structure: The “Industry Base Case”

In this scenario (Scenario 5) we examine the joint effect of using “objective” default probabilities and imposed roll-down benefits on the structure (using the same methods as described in Sections 4.2.1 and 4.2.2). Furthermore, as other research documents have done, we also reduce the roll date transaction cost to half the bid-ask spread.\(^{35}\) In combination, these three adjustments form an “industry base case” that should sway the results heavily in favour of strong performance for the CPDO structure. The purposes of creating this scenario are i) to make our simulation results more comparable to the industry literature where these three assumptions are widely implemented, and ii) to see if we can generate the extremely low, AAA-associated default rates that are found in the industry literature.

Not surprisingly, the simulations results (Figure 8) reveal an extremely low default rate (0.04%). Interestingly, this corresponds closely to a 10-year AAA default rate. Moreover, the average realized PV[CPDO] (109.2) well exceeds the notional value and, as we might expect, is very close to the initial cash-in barrier of 109.4.\(^{36}\) Even when we stress this “industry base case” by raising the coupon payments to the investor to 200 bps (Scenario 7) and 250 bps above LIBOR (Scenario 8, Figure 9), the default rates in these scenarios are still exceptionally low (although the cash-in time distribution does shift proportionately to the right). In short, it is clear that the use of objective default

\(^{35}\) As confirmed in Scenario 6, whether we use half or the full bid-ask spread as the transaction cost (given that we have already converted the default probabilities and imposed the roll-down benefits) has practically no effect on the CPDO’s performance.

\(^{36}\) The reason that the average realized PV[CPDO] should be very close to the initial cash-in barrier when the CPDO default rate is so low is because the expected PV of a (near risk-free) structure paying LIBOR+120 should be (almost) the same as the PV of a risk-free floating rate note paying LIBOR+120 (i.e. the initial cash-in barrier).
probabilities in combination with imposed roll-down benefits are essential for designing a CPDO structure that produces simulation results representative of a high investment grade product.

4.3. General Sensitivity Analysis

In this section we conduct a more general sensitivity analysis to examine the effect of various input parameters on CPDO performance. The approach that we take is to vary one parameter at a time from our base case described in Section 3.3. In some scenarios, we use the "industry base case" described in Section 4.2.3 as the base case from which we vary one parameter. We note that our focus in this section is geared more towards qualitative instead of quantitative results.37

4.3.1. Changing the Coupon Payment

As changing the size of coupon payments to the investor would affect the size of the CPDO's liabilities, we would also expect the CPDO's performance to change in response to different coupon payments. Intuitively, higher (lower) coupon payments would increase (decrease) the structure's liabilities at every point in time and should i) result in a higher (lower) default rate, and ii) shift the cash-in distribution to the right (left). This intuition is confirmed in Scenarios 9 and 10, where we change the coupon payments to 200 bps (Figure 10) and 40 bps (Figure 11) above LIBOR, as well as in Scenarios 7 and 8 (Figure 9), where we alter the coupon payments from the industry base

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37 By "qualitative focus" we mean that the reader is more likely to see a statement such as "increasing the mean reversion speed of the spread process worsens the structure's performance" as opposed to "increasing the mean reversion speed by a factor of 2.5 increases the probability of default by 8.65%." The reason for this approach is that the actual numbers depend heavily on the other parameters being held constant as well as our assumptions related to risk-neutral default probabilities and the credit spread term structure.
In Scenarios 9 and 10, where there are a sufficient number of defaults, it appears that there is not a significant effect on the principal recovery default as a result of changing the coupon payment.

The effect of changing the coupon payment on the average realized PV[CPDO] is less obvious. While a higher coupon would indisputably raise the average realized PV[CPDO] if the structure were risk-free, with a risky structure there exists a trade-off because a higher coupon makes the CPDO more likely to default on the principal redemption and later coupon payments. When using the regular base case, where the default rate is high to begin (84%), it appears that the benefit of reducing the default rate from paying a lower coupon outweighs the cost of receiving a lower coupon (Scenarios 9 and 10). The results are the complete opposite using the industry base case in Scenarios 7 and 8, where the default rate is extremely low to begin (0.04%).

4.3.2. Changing the Initial Spread

The value of the starting CDS index spread at the CPDO’s initiation date, \( S_0 \), relative to the long term mean of the spread process, \( \mu_s \), should have a significant effect on CPDO performance. Intuitively, a low initial spread implies a relatively high amount of \( MtM \) losses during the structure’s life since the spread will tend to drift upward towards its long term mean, and vice versa for a high initial spread.\(^{39}\) A low initial spread also implies that the structure receives relatively low protection premiums and therefore

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\(^{38}\) While the cash-in distribution always shifts to the right when we increase the coupon payments from the “industry base case” (Scenarios 7 and 8), the default rate curiously does not monotonically increase (see Table 3). Given that the industry base case has such an exceedingly low default rate to begin with – four defaults from 10,000 simulations – we attribute the aforementioned anomaly to noise in the Monte Carlo process. We would expect the results to conform to intuition if we performed a much higher number of simulations (50,000+).

\(^{39}\) Recall that spread increases (decreases) cause \( MtM \) losses (gains) for CPDOs.
makes it more difficult for the CPDO to cover its shortfall. On the other hand, in the context of our risk-neutral default simulation process where we infer the hazard rate every six months from the spot simulated spread, a low initial spread also implies fewer reference entity defaults over the lifetime of the CPDO.

Changing the initial spread from 30 bps to that of the long term mean, 80 bps (Scenario 14, Figure 14), dramatically reduces the default rate and increases the mean realized PV[CPDO]. The default rate is reduced (and the mean realized PV[CPDO] is increased) even further when the starting spread is raised above the long term mean to 120 bps (Scenario 15, Figure 15). In addition, there does not appear to be a significant effect on principal recovery given default as a result of changing the initial spread. Overall, when there is a low (high) initial spread, it appears that the \(MtM\) losses (gains) due to spread increases (decreases) far outweigh any possible gains (losses) due to fewer (more) reference entity defaults.

4.3.3. Changing the Long Term Mean of the Spread Process

The interpretation of results when we alter the long term mean of the spread process, \(\mu_s\), is analogous to the interpretation from changing the initial spread. That is, when the long term mean is lowered, the CPDO's performance improves because there are fewer entailed \(MtM\) losses (or more entailed \(MtM\) gains) from drifting towards to a lower long term mean, and vice versa when the long term mean is raised. This intuition is confirmed in Scenarios 16 and 17 (Figures 16 and 17), where all three performance measures are better when using a lower long term mean.
4.3.4. Changing the Spread Volatility

We also explore the effect on the CPDO’s performance of changing the CDS index spread volatility, $\sigma_s$ (rather, since we are using a CIR process, the \textit{constant multiple of local volatility}). On the surface, the effect of changing the spread volatility is not obvious. A higher spread volatility, for example, raises the probability of favourable \textit{and} unfavourable extreme events such as large $MtM$ gains (from sudden negative spread jumps) and large $MtM$ losses (from sudden positive spread jumps).

The simulation results reveal that the effect of changing the spread volatility depends on the base case used. Using our regular base case, where the default rate is already very high (84%), increasing the volatility notably improves the CPDO’s performance as reflected in lower default rates and higher values of the average realized $PV[CPDO]$ (Scenarios 20-22, Figures 20-22).\footnote{However, in the regular base case, the average principal recovery given default worsens in response to higher volatility.} This result is the direct opposite when we use the industry base case as the simulation base case, where the default rate of 0.04% is exceptionally low to begin (Scenarios 23-25, Figures 23-25). That being said, the relative effect on the performance measures of altering the volatility in the two base cases is far more significant in the regular base case than in the industry base case. The only result in common from changing the volatility of the two base cases is that, in both base cases, increasing the volatility shifts the cash-in distribution to the left; i.e. cash-in times generally occur much earlier, when they do occur. In fact, the cash-in time distribution becomes highly skewed to the right.

It is apparent, therefore, that the overall effect of changing the spread volatility depends on the performance of the CPDO in the base case; i.e. the original scenario from
which the volatility is varied. In our regular base case, where any given simulation path is most likely to end in default, raising the spread volatility heightens the chances of cashing-in because there is now a greater probability of sudden, large MtM gains that were far less likely under the volatility in the base case. Although there is also a greater probability of sudden, large MtM losses, this effect is relatively insignificant because the simulation path was highly likely to end in default anyway.\textsuperscript{41} This effect is the opposite in our industry base case where a higher volatility can only be detrimental to a simulation path that, to begin with, is highly likely to cash-in.

4.3.5. Changing Default Correlation

We also examine the effect of changing the pairwise default correlation among the reference entities in the CDS index, \( \rho \). Higher (lower) correlation implies fewer (more) periods in which default losses occur; however, when losses do occur, they tend to be relatively large (small). The effect that higher or lower correlation should have on the CPDO’s performance is not, at first, obvious.

As revealed in Scenarios 26-29 (Figures 26-29), the impact of changing correlation is remarkably similar to that of changing the spread volatility. That is, increasing correlation improves CPDO performance in our regular base case (where default rates are already high) and worsens CPDO performance in the industry base case (where default rates are already low).\textsuperscript{42} In all cases, however, it appears that the effect of changing correlation is quantitatively not as significant as altering the spread process parameters.

\textsuperscript{41} This is somewhat analogous to the effect of higher volatility on the value of vanilla call options.

\textsuperscript{42} As with raising volatility in the regular base case, the only exception is the principal recovery given default, which worsens in the regular base case when correlation is raised.
4.3.6. Other Effects: Transaction Costs, Mean Reversion Speed of the Spread Process, Interest Rate Process Parameters, Leverage Cap, and Gearing Factor

As evident in the simulation results for Scenarios 11-13 (Figures 12 and 13), the effect of the roll date transaction cost on CPDOs is unambiguous. Increasing (decreasing) the transaction cost results in higher (lower) default rates, lower (higher) values for the average realized PV[CPDO], and lower (higher) principal recoveries given default. Interestingly, the quantitative effect of changing the transaction cost is far more significant in the regular than in the industry base case.

The effect of changing the mean reversion speed, $\alpha_S$, is not intuitively obvious. If the initial spread is below the long term mean (as in our base case), then a higher mean reversion speed implies a faster expected drift to the long term mean, which implies relatively high $MtM$ losses early in the CPDO’s life but a longer period in which to make up for those losses. A lower mean reversion speed, on the other hand, implies smaller $MtM$ losses that occur, however, for a longer period of time. The results (Scenarios 18-19, Figures 18-19) indicate that a higher (lower) mean reversion speed results in poorer (better) CPDO performance; however, the effect compared to other spread process parameters does not appear to be quantitatively as significant.

Intuitively, the effect of the interest rate (or LIBOR) process parameters on CPDO performance should not be that significant because any change in the interest rate level would have a similar effect on both the structure’s assets and liabilities (which are both, for the most part, floating assets and liabilities with respect to the interest rate). However, the effect of changing interest rates on the assets and liabilities is not exactly equal because a portion of the liabilities is fixed; i.e. there is a fixed spread above LIBOR (120 bps in our base case) which must be paid to the investor as a portion of every coupon. In
this sense, an increase (decrease) in the interest rate level will have a slightly favourable (negative) effect on the CPDO; however, this effect will likely not be too significant since the fixed portion of the structure’s liabilities is generally much smaller than the floating portion of the structure’s liabilities. In Scenarios 30-31, it can be seen that the effect of changing the volatility and mean reversion speed of the interest rate process has little effect on overall CPDO performance.

The effects of changing the leverage cap and the gearing factor do affect CPDO performance but not as materially as changing other parameters, as indicated in Scenarios 32-34.\textsuperscript{43}

\textsuperscript{43} Unfortunately, due to time and computational constraints, we were unable to construct further scenarios in which to better examine the effects of the leverage cap and gearing factor.
5. CONCLUSION

We find that, although there are significant differences between our regular base case results and the standard results found in the industry literature, these differences can be explained by different probabilities of defaults and assumptions relating to the term structure of credit spreads. More specifically, it is clear that the use of “objective” default probabilities as well as imposed roll-down benefits are critical for designing high investment grade CPDOs that also pay relatively high coupons. However, given the questionable plausibility of the roll-down benefit assumption (as discussed in Section 4.2.2.), it is somewhat troubling that CPDO performance rests so heavily on this assumption.44

The sensitivity analysis reveals, firstly, that CPDO performance is most sensitive to the coupon size, initial spread, spread long term mean, and spread volatility. More generally, the analysis also reveals that the sensitivity to most parameters is quantitatively larger in the regular base case (where performance is poor to begin) than in the “industry” base case (where performance is strong to begin). This result helps explain the supposed resilience of the structure as found in the industry literature, where performance of the structure as a whole is, by default, already in the high investment grade range. In the event that the structure has been modelled incorrectly and is actually more likely to default on its obligations than industry research suggests, then the structure will also be

44 In light of this observation, it is apparent that second generation CPDOs improve upon first generation CPDOs by giving managers the option to roll-down on each roll date; i.e. rolling-down is not mandatory on every roll date.
more sensitive to various parameters; in particular, those mentioned above. Therefore, not only would a CPDO generally perform worse if it has been modeled incorrectly, but it would also be more “volatile” in the sense that its sensitivity to various parameters would be greater. Moreover, given the complexity of CPDO modeling and the associated high model risk, it is not implausible that there have been modeling errors in the industry research.

Overall, in light of these conclusions, it is not clear that CPDOs truly carry the same low risk of similarly-rated high investment grade products. It will certainly be of interest to see if first generation CPDOs are able to weather the storms that the credit markets will almost certainly provide over the next ten years.
APPENDICES

Appendix 1. Tables

Table 1. Base Case Parameters

<table>
<thead>
<tr>
<th>CPDO Strategy Parameters</th>
<th>Base Case (Scenario 1)</th>
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<tbody>
<tr>
<td>Notional ($)</td>
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<tr>
<td>Coupon above LIBOR (bps)</td>
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<td>Coupon frequency (per year)</td>
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<td>Rebalancing upper bound</td>
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<td>Initial/capped leverage (x notional)</td>
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<td>Cash-out threshold (fraction of notional)</td>
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<td>Roll-down coefficient (roll date)</td>
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LIBOR Process Parameters (annualized)

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<tr>
<td>$\alpha_r$ (CIR)</td>
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<tr>
<td>$\sigma_r$ (CIR)</td>
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Spread Process Parameters (bps, annualized)

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<td>$\mu_S$ (CIR)</td>
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<tr>
<td>$\alpha_S$ (CIR)</td>
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<td>$\sigma_S$ (CIR)</td>
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Default Simulation Parameters

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<td>Recovery (fraction of exposure)</td>
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<td>$\rho$</td>
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<td>Hazard rate coefficient</td>
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Table 2. Description of Scenarios

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<tr>
<td>2</td>
<td>Hazard Rate Coefficient: $HRC = 0.25$</td>
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<td>3</td>
<td>Roll-down Coefficient: $RDC_1 = 0.93$ (roll date), $RDC_2 = 0.97$ (non roll date)</td>
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<td>4</td>
<td>$RDC_1 = 0.838$, $RDC_2 = 0.869$</td>
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<td>Industry Base Case: $tcos = 0.26$; $RDC_1 = 0.93$, $RDC_2 = 0.97$; $HRC = 0.25$</td>
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<tr>
<td>6</td>
<td>$RDC_1 = 0.93$, $RDC_2 = 0.97$; $HRC = 0.25$</td>
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<td>Coupon above LIBOR (bps): $c = 200$</td>
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<td>10</td>
<td>Coupon above LIBOR (bps): $c = 40$</td>
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<td>11</td>
<td>Trans. cost on roll date (bid-offer, bps): $tcos = 0$</td>
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<td>12</td>
<td>Trans. cost on roll date (bid-offer, bps): $tcos = 0.26$</td>
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<tr>
<td>13</td>
<td>Trans. cost on roll date (bid-offer, bps): $tcos = 1$</td>
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<tr>
<td>14</td>
<td>Initial credit spread: $S_0 = 80$</td>
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<td>15</td>
<td>Initial credit spread: $S_0 = 120$</td>
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<tr>
<td>16</td>
<td>Long term mean of credit spread (CIR): $\mu_S = 40$</td>
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<tr>
<td>17</td>
<td>Long term mean of credit spread (CIR): $\mu_S = 120$</td>
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<tr>
<td>18</td>
<td>Mean reversion speed of credit spread (CIR): $\alpha_S = 0.2$</td>
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<tr>
<td>19</td>
<td>Mean reversion speed of credit spread (CIR): $\alpha_S = 1$</td>
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<td>20</td>
<td>Constant multiple of credit spread volatility (CIR): $\sigma_S = 1.5$</td>
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<tr>
<td>21</td>
<td>Constant multiple of credit spread volatility (CIR): $\sigma_S = 4$</td>
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<td>22</td>
<td>Constant multiple of credit spread volatility (CIR): $\sigma_S = 6$</td>
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<tr>
<td>26</td>
<td>Correlation of entities in the credit spread index: $\rho = 0.1$</td>
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<tr>
<td>27</td>
<td>Correlation of entities in the credit spread index: $\rho = 0.5$</td>
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<tr>
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<td>Constant multiplier of interest rate volatility (CIR): $\sigma_r = 0.1$</td>
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<td>32</td>
<td>Gearing factor = 1.5</td>
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<td>33</td>
<td>Gearing factor = 2.5</td>
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<td>34</td>
<td>Initial/capped leverage (x notional) = 20</td>
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Table 3. Simulation Results

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<th>Scenario</th>
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<td>4.45 3.47</td>
<td>44.67 29.28 1.06</td>
<td>9.92 20.43</td>
</tr>
</tbody>
</table>

For each scenario, Mean, Std dev and Skew are the simple mean, standard deviation and skewness, respectively, of the recorded measures for the 10,000 simulations. Therefore, to find the standard error of the mean realized PV[CPDO] estimate, for example, one would need to divide Std dev by the square root of the number of simulations.
Appendix 2. Figures

Figure 1. Schematic Diagram of CPDO Strategy
Figure 2. Sample CPDO Simulation Paths

Cash-in in Year 4

Cash-out in Year 8

Figure 3. Schematic Diagram of Modeling Approach

- Interest Rate Simulation ➔ Leverage and Risky Exposure Determination ➔ Cash-in
- CDS Index Spread Simulation ➔ Cash Deposit and Note Value Calculation ➔ Neither Cash-in Nor Cash-out
- Ref. Entity Default Simulation ➔ Cash-out
Figure 4. Scenario 1 – Regular Base Case

Cash-in Time Distribution

Realized PV[CPDO] Distribution

Principal Recovery (Given Default) Distribution

49
Figure 5. Scenario 2 – Regular Base Case Except Hazard Rate Coefficient = 0.25

Cash-in Time Distribution

Realized PV[CPDO] Distribution

Principal Recovery (Given Default) Distribution
Figure 6. Scenario 3 – Regular Base Case Except Roll-Down Coefficients = 0.97 (non roll date), 0.93 (roll date)
Figure 7. Scenario 4 – Regular Base Case Except Roll-Down Coefficients = 0.87 (non roll date), 0.84 (roll date)
Figure 8. Scenario 5 – Industry Base Case: Regular Base Case Except i) Roll-Down Coefficients = 0.97 (non roll date), 0.93 (roll date), ii) Hazard Rate Coef. = 0.25, and iii) T-Cost = 0.26
Figure 9. Scenario 8 – Industry Base Case Except Coupon Above LIBOR = 250

Cash-in Time Distribution

Realized PV[CPDO] Distribution

Principal Recovery (Given Default) Distribution
Figure 10. Scenario 9 – Regular Base Case Except Coupon Above LIBOR = 200 bps
Figure 11. Scenario 10 – Regular Base Case Except Coupon Above LIBOR = 40 bps

Cash-in Time Distribution

Realized PV[CPDO] Distribution

Principal Recovery (Given Default) Distribution
Figure 12. Scenario 11 – Regular Base Case Except Transaction Cost on Roll Date = 0 bps

Cash-in Time Distribution

Realized PV[CPDO] Distribution

Principal Recovery (Given Default) Distribution
Figure 13. Scenario 13 – Regular Base Case Except Transaction Cost on Roll Date = 1 bps
Figure 14. Scenario 14 – Regular Base Case Except Initial Spread = 80 bps

Cash-in Time Distribution

Realized PV[CPDO] Distribution

Principal Recovery (Given Default) Distribution
Figure 15. Scenario 15 – Regular Base Case Except Initial Spread = 120 bps
Figure 16. Scenario 16 – Regular Base Case Except Long Term Mean = 40 bps

Cash-in Time Distribution

Realized PV[CPDO] Distribution

Principal Recovery (Given Default) Distribution
Figure 17. Scenario 17 – Regular Base Case Except Long Term Mean = 120 bps

Cash-in Time Distribution

Realized PV[CPDO] Distribution

Principal Recovery (Given Default) Distribution
Figure 18. Scenario 18 – Regular Base Case Except Alpha = 0.2

Cash-in Time Distribution

Realized PV[CPDO] Distribution

Principal Recovery (Given Default) Distribution
Figure 19. Scenario 19 – Regular Base Case Except Alpha = 1

Cash-in Time Distribution

Realized PV[CPDO] Distribution

Principal Recovery (Given Default) Distribution
Figure 20. Scenario 20 – Regular Base Case Except Sigma = 1.5

Cash-in Time Distribution

Realized PV[CPDO] Distribution

Principal Recovery (Given Default) Distribution

65
Figure 21. Scenario 21 – Regular Base Case Except Sigma = 4

Cash-in Time Distribution

Realized PV[CPDO] Distribution

Principal Recovery (Given Default) Distribution
Figure 22. Scenario 22 – Regular Base Case Except Sigma = 6

Cash-in Time Distribution

Realized PV[CPDO] Distribution

Principal Recovery (Given Default) Distribution
Figure 23. Scenario 23 – Industry Base Case Except Sigma = 1.5

**Cash-in Time Distribution**

- Cash-in Time (year)
- Relative Frequency
- No cash-in

**Realized PV[CPDO] Distribution**

- Realized PV[CPDO] ($)
- Relative Frequency

**Principal Recovery (Given Default) Distribution**

- Principal Recovery Given Default ($)
- Relative Frequency
Figure 24. Scenario 24 – Industry Base Case Except Sigma = 4

Cash-in Time Distribution

Realized PV[CPDO] Distribution

Principal Recovery (Given Default) Distribution
Figure 25. Scenario 25 – Industry Base Case Except Sigma = 6

Cash-in Time Distribution

Realized PV[CPDO] Distribution

Principal Recovery (Given Default) Distribution
Figure 26. Scenario 26 – Regular Base Case Except Rho = 0.1

Cash-in Time Distribution

Realized PV[CPDO] Distribution

Principal Recovery (Given Default) Distribution
Figure 27. Scenario 27 – Regular Base Case Except Rho = 0.5

Cash-in Time Distribution

Realized PV[CPDO] Distribution

Principal Recovery (Given Default) Distribution
Figure 28. Scenario 28 – Industry Base Case Except Rho = 0.1

Cash-in Time Distribution

Realized PV[CPDO] Distribution

Principal Recovery (Given Default) Distribution
Figure 29. Scenario 29 – Industry Base Case Except Rho = 0.5

Cash-in Time Distribution

Cash-in Time (year)

Realized PV[CPDO] Distribution

Realized PV[CPDO] ($) 

Principal Recovery (Given Default) Distribution

Principal Recovery Given Default ($)
Appendix 3. CDS Index Spread Process

A.3.1. CDS Index Spread Model

We model the 5.25-year CDS index spread $S_t$ using the Cox-Ingersoll-Ross (CIR) model (Cox et al., 1985):

$$dS_t = \alpha_S(\mu_S - S_t)dt + \sigma_S\sqrt{S_t}dW_t^S,$$

where $\alpha_S$ represents the mean reversion speed, $\mu_S$ the long term mean, $\sigma_S$ the constant multiple of local volatility, and $dW_t^S$ is a Standard Brownian Motion; i.e. $W(t+h) - W(t) \sim N(0,h)$, where $N$ denotes the normal distribution.

In contrast to the Vasicek and other constant volatility processes, volatility in the CIR model is proportional to the level of the spread. One advantage of the continuous model is that the simulated spread is always positive as long as $2\alpha_S\mu_S \geq \sigma_S^2$.

A.3.2. Estimation of CDS Index Spread Process Parameters

The parameters of the spread process are estimated by maximum likelihood. The regression equation is given by:

$$S_{t+1} = S_t + \alpha_S(\mu_S - S_t) + \sigma_S\sqrt{S_t} \epsilon_t^S.$$

CDX.NA.IG (5-year) and iTraxx Europe (5-year) index data are used to estimate the parameters in the model. CDX.NA.IG (5-year) daily data, from March 3, 2006 to June 19, 2007 (four series), are collected from Bloomberg, and iTraxx Europe (5-year) daily data over the same period are collected from the International Index Company.\footnote{www.intindexco.com} For each index, only data from the on-the-run series is used. For each day in the sample
period, the spread that is used is the end-of-day mid-spread. Since credit protection on first generation CPDOs typically references 50% iTraxx Europe (5-year) and 50% CDX.NA.IG (5-year), each value used in the regression is comprised of a 50-50 weighting of the two indices for each day. A time series of these values is shown in Figure 30. In all, 302 sample days are employed to estimate the parameters $\alpha_S$, $\mu_S$, and $\sigma_S$.

![Figure 30. Historical “Average” CDS Index Spread Time Series](image)

The estimated daily parameters (in bps) are given by $\mu_s^* = 28.28$, $\alpha_s^* = 0.0521$ and $\sigma_s^* = 0.0091$. We convert these parameters into annualized values as follows:

$$\mu_s = \mu_s^* = 28.28, \quad \alpha_s = \alpha_s^* \cdot 252 = 13.14, \quad \text{and} \quad \sigma_s = \sigma_s^* \cdot \sqrt{252} = 0.1444.\quad \text{47}$$

The estimation results indicate that our sample data are featured by extremely high mean reversion, extremely low volatility, and an unrealistically low long term mean. These results are not surprising after viewing Figure 30. The major reason for these unrealistic parameters is that the collected historical data span a short period of time where spreads were unusually low and had unusually low volatility. Hence, the estimated parameters are clearly unrepresentative of the spread movements that one might reasonably expect over

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47 We make the standard assumption of 252 trading days per year.
a ten-year period, based at least on historical bond spreads. Therefore, we choose arbitrary parameters for the spread process that are commonly used in industry research papers: $\mu_S = 80$, $\alpha_S = 0.4$, and $\sigma_S = 2.72$. As shown in Figure 31, these parameters produce somewhat reasonable ten-year simulation paths for the spread.

A.3.3. CDS Index Spread Simulation

The simulation is conducted according to the following difference equation:

$$S_{t+1} = S_t + \alpha_S (\mu_S - S_t) \Delta t + \sigma_S \sqrt{S_t} \sqrt{\Delta t} W_t^S,$$

where $\alpha_S$, $\mu_S$, and $\sigma_S$ are the annualized parameters given in Section A.3.2, $\Delta t$ equals 0.25 years, and $W_t^S$ is a standard normal random variate. The initial credit spread is set equal to 30 bps – the approximate value of the 5-year CDX.NA.IG and 5-year iTraxx Europe indices as of June, 2007.

Figure 31. Sample CDS Index Spread Simulation Paths
Appendix 4. Interest Rate (LIBOR) Process

A.4.1. Interest Rate Model

Interest rate process estimation and simulation are very similar to that for the spread process described in Appendix 3; therefore, this section focuses mainly on the differences. We model the three-month LIBOR rate $r_t$ also using the Cox-Ingersoll-Ross (CIR) model:

$$dr_t = \alpha_r (\mu_r - r_t)dt + \sigma_r \sqrt{r_t} dW^r_t,$$

where $\alpha_r$ represents the mean reversion speed, $\mu_r$ the long term mean, $\sigma_r$ the constant multiple of local volatility, and $dW^r_t$ is a Standard Brownian Motion.

A.4.2. Estimation of Interest Rate Process Parameters

The parameters of the interest rate process are estimated by maximum likelihood. The regression equation is given by:

$$r_{t+1} = r_t + \alpha_r (\mu_r - r_t) + \sigma_r \sqrt{r_t} \epsilon_t^r.$$
The data used are the three-month US dollar LIBOR rate (at monthly intervals and on a monthly basis), retrieved from the Economagic website. The sample period is from January, 1987 to June, 2007. A historical time series graph of the data is shown in Figure 32.

Figure 32. Historical 3-Month LIBOR Time Series

![Historical 3-Month LIBOR Time Series](image)

The estimated monthly parameters are: $\mu' = 0.0435$, $\alpha = 0.0045$ and $\sigma_r = 0.0102$. We convert these parameters into annualized values as follows:

$$
\mu = \mu' = 0.0435, \quad \alpha = \alpha' \cdot 12 = 0.0542 \quad \text{and} \quad \sigma = \sigma' \cdot \sqrt{12} = 0.0355.
$$

A.4.3. Interest Rate Simulation

The simulation is conducted according to the following difference equation:

$$
r_{t+1} = r_t + \alpha_r (\mu_r - r_t) \Delta t + \sigma_r \sqrt{r_t} \sqrt{\Delta t} W'_t,
$$

where $\alpha_r$, $\mu_r$, and $\sigma_r$ are the annualized parameters given in Section A.4.2, $\Delta t$ equals 0.25 years, and $W'_t$ is a standard normal random variate. The initial LIBOR rate is set equal to

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0.05, the approximate value as of June, 2007. Four sample LIBOR simulation paths are presented in Figure 33.

**Figure 33. Sample 3-Month LIBOR Simulation Paths**

Appendix 5. Risk-Free and Risky Discount Factor Formulas

The risk-free present value factor, $B(t,T)$, is calculated using the bond price formula for a Cox-Ingersoll-Ross process (Cairns, 2004). This formula is calculated as the expected present value of one dollar under the risk-neutral measure. The formula is given by:
\[ B(t, T) = \exp \left[ A_c \cdot (T - t) - B_c \cdot (T - t) \cdot r(t) \right], \]

where \( A_c(T - t) = \frac{2\alpha \mu}{\sigma^2} \ln \left( \frac{2\gamma \exp \left( \frac{(\gamma + \alpha)(T - t)}{2} \right)}{(\gamma + \alpha)(\exp(\gamma(T - t)) - 1) + 2\gamma} \right), \]

\[ B_c(T - t) = \frac{2\left( \exp(\gamma(T - t)) - 1 \right)}{(\gamma + \alpha)(\exp(\gamma(T - t)) - 1) + 2\gamma}, \quad \gamma = \sqrt{\alpha^2 + 2\sigma^2}, \]

\( r(t) \) is the spot simulated interest rate, and \( \alpha, \mu, \) and \( \sigma \) are all estimated annualized parameters of the CIR process (see Appendix 4).

The risky discount factor, \( H(t, T) \), takes into account the market-implied probability of default and is calculated as:

\[ H(t, T) = \exp \left[ \frac{-S(t) \cdot (T - t)}{1 - R} \right], \]

where \( S(t) \) is the spot simulated spread, and \( R \) is the recovery rate of the reference entities in the CDS index (Wong et al., 2007, p. 16).
REFERENCE LIST


