CMOS ACTIVE PIXEL SENSORS THAT INCORPORATE CONCEPTS OF FAULT-TOLERANCE AND ACTIVE BACKGROUND ILLUMINATION DISCRIMINATION

by

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ABSTRACT

This thesis addresses how to increase the manufacturing yields of CMOS Active Pixel Sensors and improve upon detectors of optical scanners.

To increase manufacturing yields of APS, the APS is split into two subpixels, so that when one half of the pixel is defective, the entire APS will not fail. The other subpixel will still be functional and through current summing, a multiplication be used for recovery.

Current optical scanners suffer from two drawbacks. The laser is at about the same intensity as the ambient illumination and local surface imperfections cause laser to be reflected at odd angles. To address this issue, the dual-output APS has been designed. It consists of one APS cell with two readout structures such that the background illumination is integrated on one side while the background and laser is integrated onto another side. A subtraction would reveal the laser in preference to the background.
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## GLOSSARY

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$\varphi_m$</td>
<td>Work function of gate material (metal or polysilicon)</td>
</tr>
<tr>
<td>$\varphi_s$</td>
<td>Work function of silicon</td>
</tr>
<tr>
<td>ADC</td>
<td>Analog-to-Digital Converter</td>
</tr>
<tr>
<td>APS</td>
<td>Active Pixel Sensor</td>
</tr>
<tr>
<td>$c$</td>
<td>Speed of light ($3 \times 10^8$ m/s)</td>
</tr>
<tr>
<td>CCD</td>
<td>Charge-Coupled Devices</td>
</tr>
<tr>
<td>CDS</td>
<td>Correlated Double Sampling</td>
</tr>
<tr>
<td>CMC</td>
<td>Canadian Microelectronics Corporation</td>
</tr>
<tr>
<td>CMOS</td>
<td>Complementary Metal Oxide Semiconductor</td>
</tr>
<tr>
<td>CTE</td>
<td>Charge Transfer Efficiency</td>
</tr>
<tr>
<td>D-APS</td>
<td>Duo-Output Active Pixel Sensor</td>
</tr>
<tr>
<td>DS</td>
<td>Double Sampling</td>
</tr>
<tr>
<td>DSLR</td>
<td>Digital Single Lens Reflex</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Conduction energy</td>
</tr>
<tr>
<td>$E_F$</td>
<td>Fermi energy</td>
</tr>
<tr>
<td>$E_G$</td>
<td>Bandgap energy</td>
</tr>
<tr>
<td>$E_i$</td>
<td>Intrinsic energy</td>
</tr>
<tr>
<td>$E_v$</td>
<td>Valence energy</td>
</tr>
<tr>
<td>FPN</td>
<td>Fixed Pattern Noise</td>
</tr>
<tr>
<td>FT-APS</td>
<td>Fault Tolerance Active Pixel Sensor</td>
</tr>
<tr>
<td>$\hbar$</td>
<td>Planck's constant ($6.626 \times 10^{-34}$ J s)</td>
</tr>
<tr>
<td>IBM</td>
<td>International Business Machines Corporation (a commercial enterprise)</td>
</tr>
<tr>
<td>JPL</td>
<td>Jet Propulsion Lab</td>
</tr>
<tr>
<td>LED</td>
<td>Light Emitting Diode</td>
</tr>
<tr>
<td>$L_n$</td>
<td>Minority carrier diffusion length where the minority carriers are holes</td>
</tr>
<tr>
<td>$L_p$</td>
<td>Minority carrier diffusion length where the minority carriers are electrons</td>
</tr>
<tr>
<td>MOS</td>
<td>Metal-Oxide-Semiconductor</td>
</tr>
<tr>
<td>MOS-C</td>
<td>Metal-Oxide Semiconductor Capacitor</td>
</tr>
<tr>
<td>$M_{\text{RESET}}$</td>
<td>Reset transistor</td>
</tr>
<tr>
<td>$M_{\text{RS}}$</td>
<td>Row-Select Transistor</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
</tr>
<tr>
<td>--------</td>
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<tr>
<td>$M_{SF}$</td>
<td>Source-Follower Transistor</td>
</tr>
<tr>
<td>NASA</td>
<td>National Aeronautics and Space Administration (USA)</td>
</tr>
<tr>
<td>NMOS</td>
<td>N-doped Metal-Oxide Semiconductor transistor</td>
</tr>
<tr>
<td>PD</td>
<td>Photodiode</td>
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<tr>
<td>PG</td>
<td>Photogate</td>
</tr>
<tr>
<td>PMOS</td>
<td>P-doped Metal-Oxide Semiconductor transistor</td>
</tr>
<tr>
<td>PRNU</td>
<td>Photo Response Non-Uniformity</td>
</tr>
<tr>
<td>SH</td>
<td>Stuck High</td>
</tr>
<tr>
<td>SL</td>
<td>Stuck Low</td>
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<tr>
<td>SLR</td>
<td>Single Lens Reflex</td>
</tr>
<tr>
<td>SOC</td>
<td>System-On-a-Chip</td>
</tr>
<tr>
<td>QE</td>
<td>Quantum Efficiency</td>
</tr>
<tr>
<td>RCA</td>
<td>Radio Corporation of America (a commercial enterprise)</td>
</tr>
<tr>
<td>TFT</td>
<td>Thin-Film Transistor</td>
</tr>
<tr>
<td>TSMC</td>
<td>Taiwan Semiconductor Manufacturing Company (a commercial enterprise)</td>
</tr>
<tr>
<td>TX</td>
<td>Transfer Gate</td>
</tr>
<tr>
<td>$V_A$</td>
<td>Applied voltage for a p-n junction</td>
</tr>
<tr>
<td>$V_{bi}$</td>
<td>Built-in potential</td>
</tr>
<tr>
<td>VDD</td>
<td>Power supply voltage</td>
</tr>
<tr>
<td>$V_{GB}$</td>
<td>Voltage between the gate and the bulk/substrate</td>
</tr>
<tr>
<td>$V_{THN}$</td>
<td>Threshold voltage for NMOS</td>
</tr>
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CHAPTER 1 : INTRODUCTION

Many manufacturing-related industries require high precision dimensional control over their products. For example, the thickness of textile, paper or plastics need to be consistent and tightly controlled. This is also true of piston rings, roller bearings and pieces of lumber. These industries require a form of measurement that would not require contact, makes measurements at a high rate, allow high precision and good accuracy.

One solution is to use a combination of lasers, detectors and algorithms to extract the dimensions of the object [3][4]. There are three common techniques for non-contact dimensional measurement using lasers, namely (1) the scanned-beam method, (2) optical triangulation method and (3) diffraction-based method. Of these 3 methods, optical triangulation is the most common technique, where a triangle is formed from three points, the laser, the object and the detector. The angle of the laser and the detector is known, which gives the angle of the object. By knowing all these angles as well as the distance between the laser and the detector, one can use the sine law to obtain the distance between the laser and the object. Thus if the object is moved in a known fashion, then one can get a profile of the object. Typically what is done is that there is an array of lasers in a line and detectors that are used to speed up the process. The location of each laser and detector and the sequence at which the measurements are taken needs to be known.
1.1 Motivation for Research

Hermary Optoelectronics Inc. (referred to as Hermary from now on), the company that is sponsoring our research, is in the business of designing optical scanners to be used in the industrial automation, automobile manufacturing, tire manufacturing, food processing, as well as, process and industrial controls [5]. All of Hermary’s products are based on optical triangulation and the type of detector used is the Charge-Coupled Device (CCD).

Laser profilometry scanners are machines that are used to measure the distance of all points on scanned objects to thus create a profile of the item. From these “point clouds”, the scanned object is then reconstructed by using 3D polygonal meshes to create a full computer model of the external structure. There are many algorithms for meshing and volume reconstruction, which are very mathematically intensive and algorithmically intensive in nature, and requires optimization, usually both in the algorithms itself and the implementation. However, these reconstruction algorithms are beyond the scope of this
thesis. Instead, this thesis will concentrate on the scanner hardware itself. What is important for this thesis is that the accuracy of these reconstructions is vitally dependent on the precision of the scanner data.

1.1.1 Current Scanning Technology

An industrial scanner can be categorized into two general categories, namely the contact and non-contact scanners. Contact scanners depend on some tactile feedback to form the geometry of the scanned object. Non-contact scanners depend on some other types of mechanisms other than tactile feedback. The advantages of the contact scanners are that they are usually cheaper while the non-contact scanners depend on some sort of radiation source and detector scheme which could typically be more expensive. The disadvantages of the contact scanners are wear and tear due to physical contact, the need for physical contact, susceptibility to vibration and potential damage to contact probe. The disadvantages of non-contact scanners are that it is very hard to scan large objects, rough surfaces, objects that might be at a certain distance where one has no control over the distance and the fact that non-contact scanning typically requires a moderately clean environment. On the other hand, the non-contact scanners are more reliable because there is no physical contact, thus wear-and-tear which characterizes the contact system is non-existent in the non-contact systems. Non-contact scanners will be the main focus of this thesis. More specifically, the non-contact scanners described by this thesis will be used for laser profilometry where the laser used for scanning and the ambient light intensity where the scanning takes place are at approximately the same order of magnitude.

A non-contact scanner can be categorized into two general categories, namely active scanners and passive scanners. Active scanners depend on emission of some sort of
radiation or waves (light, sound, or radio) that is reflected off the object that is scanned and then detected by the scanners. Passive scanners depend on ambient radiation of some sort that can then be detected by the scanners. Active scanners are typically more expensive than passive scanners and usually more accurate because of the higher signal-to-noise ratio associated with active scanners. Therefore, this thesis will address the active scanners.

Active scanners depend typically on laser as a radiation source to be reflected off the scanned object. Lasers are easier to employ and more accurate than radar or acoustic signals except where the object is either at long distance or embedded within other materials. There are two principal methods to scan objects, namely the time-of-flight method and the optical triangulation method. The time-of-flight method depends on the known speed of light and the time it takes for the light to be reflected off the object. The distance between the object to be scanned and the detector is given by half of the product of speed of light and the time taken for the light to be detected. The disadvantages of time of flight method is the relatively fast detector needed, the short pulse time in the range of ps detection to get mm accuracy, making it expensive and difficult.

The optical triangulation method can be shown in Figure 1. There are several variants of the method but all of them work on these two basic mathematical formulae, first the sine rule and second the cosine rule:

\[
\text{Sine rule: } \frac{A}{\sin a} = \frac{B}{\sin b} = \frac{C}{\sin c}
\]

\[
\text{Cosine rule: } A^2 = B^2 + C^2 - 2BC \times \cos a
\]

Where \(A, B, C\) are the lengths of the three sides of the triangle and \(a, b, c\) are the opposite
angles to the capitalized letter that denotes the lengths of the three sides of the triangle as shown in the abstract optical triangle in Figure 1. The distance between the image sensor array and the laser source, $A$, is known, and the angle of the laser source corner, $c$, is known, as well as the the image sensor array corner angle, $b$. $c$ is known because the laser source and the image sensor array are fixed in positions, while $b$ is known by looking at the image sensor output when the laser spot is in the field of view. This also means that $a$ can be calculated from knowing $b$ and $c$ because the sum of the angles of the triangle is $180^\circ$. $A$, $a$, $b$ and $c$ are known and between the simultaneous applications of the sine rule and cosine rule, there are three linear equations and two unknowns, $B$ and $C$, so these two unknowns can be solved for. In other words, the distance between the object and the scanner is known as well as its angle of reflection, thus making reconstruction possible.

It is quite obvious that a lens system is needed for the detector and a very coherent and non-dispersive source of light such as a laser is necessary for the measurement to be of any reasonable accuracy.

1.1.2 Problems with Current Scanning Technology

Currently most industrial scanners employ CCDs as detectors. However, there are problems with current scanning technologies that make an improvement desirable. Firstly, there is the problem of low signal-to-noise ratio when the laser is used in an environment where there could be multiple ambient light sources, or where typical illumination levels are very high, such as direct sunlight, or where the light distribution is not uniform that might cause shadowing of the object. The laser source has to be limited in power in industrial scanners where there might also be human operators because there are safety standards that limit the laser to 1 mW (Class 3a lasers). This is equivalent to
sunlight on a clear day which has an intensity of 1000 W/m² or 0.1 W/cm². This is done to assuage safety concerns. Furthermore, there might be other laser sources at about the same level of intensity as the laser being used for scanning, so stray laser sources could incur problems of how to differentiate one from the other.

Secondly, the current CCD or even standard APS scanning systems have a problem when the object being scanned has local surface imperfections. For example, where there are great changes in reflectivity or absorption because of either the presence of different types of materials, or surface structures such as dimples or deep holes, or where the surface is acting as a mirror When there are deep holes, then that particular point being scanned will not be seen, and when there are surfaces that are very shiny, it could fool the scanning system that the point is actually much closer than reality. In both cases, the point is not scanned correctly. Mirror surfaces that have high reflection coefficient reflects the beam away from the detector and will not illuminate the surface so it is seen by the camera system as being much closer than reality. Depending on the resolution of the scanning done as well as the size of these imperfections and the applications at hand, the results could either be still accurate, harmlessly inaccurate or disastrously inaccurate.
Figure 2: Problems with current scanning technology, example when there is a deep hole and a shiny spot

The real issue is that if one tries to identify the location of the surface based on some sort of intensity measurements based on the reflected light, then changes in surface absorption due to material properties or just local surface shapes make it difficult to carry out an accurate surface profile scanning. The extreme cases of this change in surface absorption are the presence of deep holes on the surface or extremely shiny surface, shown in Figure 2. In the presence of deep holes, there is no reflected light. In the presence of shiny surfaces, the intensity of the reflected light onto the detectors are brighter than expected and cause it to appear closer than it really is. These are the simplest cases. In reality, because of the presence of non-uniform illumination or multiple sources of light, one surface can be in the shadow and one under bright illumination, causing the same effects as if there were deep holes and shiny surfaces.
Figure 3 shows the confusion that might arise when ray of light were reflected off a dimple as opposed to just a smooth surface. For example, in Figure 3, there are two rays of light, A1 and A2, which would be the path of light travel if the light rays were reflecting off a smooth surface, while R1 and R2 are the rays of light that were reflected off a dimple. The result in this contrived case is an interchange of the light reflected from the dimples, causing confusion by the detector as to the position of the dimples. Even though, this is a contrived case, one can imagine what happens when there are more than two rays of light and more than two dimples.

1.1.3 Background Light Discrimination Problem

Hermay’s interest is to be able to deploy optical scanning solutions which do not require high-intensity laser and yet can still be effectively detected. High-intensity lasers are a hazard to workers whose eyes might possibly be in the path of the laser beam. Consequently, Hermay is obligated by law and health concerns, to use low-intensity lasers so that workers do not require protective laser goggles. However, the lower the intensity of the laser, the more difficult it is to detect. For example, allowed maximum
laser intensity levels are only 0.01 W/cm² less than bright sunlight levels which can reach up to 0.1 W/cm² [6]. To compound the problem, an array of lasers has been incorporated into the optical scanner. The resulting issue is that of the optical scanner distinguishing between laser from one source and laser from another source. Also, traditional solid-state scanners such as CCD’s are not able to perform well when there are dark areas combined with shiny parts (e.g., wet areas) on the objects being scanned due to the CCD limited dynamic range. Figure 4 illustrates the profile seen by the detector array when the object being scanned contains both dark and shiny areas in addition to the laser spots used as part of the profile. The problem requires a separation of the laser spot from the background light and one cannot use thresholding because this can be fooled by background related effects, such as shiny spots and dark areas due to surface depressions.

![Surface Profile](image)

**Figure 4**: Surface profile of a realistic surface to be scanned showing how dynamic range of CCD can be exceeded because of the irregularities on the surface

Furthermore, when there are natural strong light sources like sunlight, this causes the signal-to-background-light-ratio to decrease drastically, which causes differentiation of the laser spot from the unwanted sources to be more difficult. For example, in Figure 4, one can see that some parts of the shiny spot intensity profile is of the same height as that of the normal spot, while some parts of the dark area are almost at the same level of
intensity as the normal spot's profile too. This highlights the difficulty in using thresholding where the normal spot cannot be readily differentiated from the shiny spot or the dark spot.

We have attempted to solve Herrmary's problem by focusing on the detectors. To this end, we have selected CMOS photodetector, also known as CMOS image sensor (CIS) or Active Pixel Sensor (APS), as our solution. From this point on, I would refer to the detectors as APS.

The first solution of encoding the laser at a particular frequency does not preclude the use CCD detectors and standard APS detectors, which are commercially present currently. However, the local pixel processing cannot be done using current technologies unless the image readout rate is faster than the laser encoding and one has access to significant computing power which would allow analysis of frame to frame image set. CCD technologies have no local pixel processing capability as each pixel is a photosensing element and also act as part of the transfer mechanism to the periphery. However, APS has the potential of local pixel-level processing because each pixel is a photosensing element and does not have the role of charge transfer of the signal to the periphery. This ensures that active circuitry can be incorporated into the pixel in APS to effect local processing and thus potentially address the second issue.

The local pixel processing being proposed consists of two parts, namely the elimination of the effects of ambient light intensity from the laser scanner detector and phase sensitive detection of laser after laser has been frequency encoded.

The simple elimination of ambient light intensity depends on integrating the scene when laser and ambient are present and when only the ambient is present. The
difference between these two integration outputs will provide you with the position of the laser light. This method is simple but does not work well when the laser signal used is weak or the ambient light intensity is much brighter than the laser and also requires the use of very sensitive and thus expensive imaging array.

The simple method of synchronizing integration of scene with laser and ambient and that of ambient alone and its consequent subtraction allows for phase sensitive detection of laser light. This scheme only works well when the number of integrations used is large so that the subtraction will yield a noticeable result. Since large number of integrations needs to be used, this also means that the scene needs to be relatively static at least for the period of time the integration is performed.

APS detectors will be described in more detail in chapter 2.

1.2 Fault-Tolerant Pixel

Another problem that will be addressed in this thesis is that as digital cameras start to rival that of the resolution of traditional films, the array size increases and the pixel size decreases at the same time. The economics accompanying this natural technical transition requires lowering defects at fabrication time so as to keep yields high, thus improving profit margin for the semiconductor companies involved. Besides defects during fabrication, there are also reliability and lifetime issues involved with semiconductors. Even though perfect and functional at fabrication, these imaging chips might degrade once used in the field.

As digital cameras start to rival that of the resolution of traditional films, the array size increases and the pixel size decreases at the same time. The economics
accompanying this natural technical transition requires lowering defects at fabrication time so as to keep yields high, thus improving profit margin for the semiconductor companies involved. There are many comments in digital camera discussion groups on pixel failures happening to cameras in the field.

1.2.1 Types of Defects

Defects in APS are categorized into three main classes on the basis of the final output signal [41]: (1) optical stuck high: optical signal saturates one pixel, which means that it reads out a bright saturated pixel all the time (e.g. photodiode malfunction or not fully formed), (2) optical stuck low: absence of optical signal on one pixel (e.g. readout transistor gate shorted to VDD, photodiode shorted to VDD, reset gate shorted to VDD), and (3) low sensitivity: photo-sensing element is blocked (e.g. by a dust particle or remnants of residual fabrication material). It is important to note that term “optical stuck high” refers to the apparent optical signal being high, that is the pixel being saturated (or maximum signal) under all conditions, rather than the internal electrical condition of the circuit. Optical stuck high has several possible sources such as that cathode of the photodiode being electronically stuck low so that the source follower is turned off or that the source follower is dead or that the photodiode is partially blocked by an opaque contaminant or that the reset transistor is always switched on. In the same spirit, “optical stuck low” refers to the pixel being dark (black) under all conditions. There are several causes, such as the cathode of the photodiode to be electronically stuck high or the source follower to be shorted to the power supply or the reset transistor is always switched off or the source and drain of the readout transistor is always open.

These defects in the APS can be identified by exposing the APS array to a light
field illumination (uniform lighting near the saturation of the pixels) and a dark field illumination (totally dark) [41]. By illuminating the entire sensor array to saturation, the light field image identifies low sensitivity, stuck-at-low, and completely malfunctioning pixels. A no-light image (dark field) locates base noise level and stuck-at-high pixels. Thus, a dark spot on a light field illumination test reveals an electronically stuck high pixel while a bright spot on a dark field illumination reveals an electronically stuck low pixel or the source follower to be shorted to the power supply or the reset transistor is always switched off or the source and drain of the readout transistor is always open.

There are 3 operating modes for the fault tolerant APS, when the pixel is working, when one half of the pixel is stuck low and when one half of the pixel is stuck high. The likelihood of defects and the type of defects depends heavily on the pixel layout design. For example, a reset metal line is more likely to be shorted to ground if it is located nearer to a ground interconnect than to a VDD power supply which is located further away.

1.2.2 Yield and Defects in Semiconductor Fabrication

For many regular IC designs, chip area tends to shrink with time, due to design improvements to shrink the area. This shrinkage, combined with design adjustments to remove defect tendencies, results in chip yield rising with time. Movement to smaller dimension technologies shrinks areas generally, or keeps the area the same while expanding circuit capabilities. As a given fabrication technology matures, the yield does increase. However for digital imagers the needed chip area depends on the optics of the system, and the optics is often kept constant over several camera models. Furthermore current commercial imager sizes are smaller for reasonable costs than is needed for many
camera systems. For example most Digital SLR’s use smaller 25x15 mm imagers than allowed for by the lens design, 36x24 mm. The cost of these larger sensors is so high that it is restricted to the much more expensive professional cameras. Hence commonly imager chip area stays constant, and as the technology geometry shrinks, the pixel size decreases and pixel count goes up. Hence the yield of perfect sensors increases more slowly when compared to regular chips. Furthermore, unlike most IC’s where a defect makes the chip inoperable, pixel failures degrade imager performance.

It has been found that IC fabrication generates clustering of defects and thus follows the Negative Binomial probability distribution model [31]:

\[ D_{total} = D_0 \prod_{j=1}^{k} \left(1 + \frac{\lambda_j A}{\alpha_j}\right)^{-\alpha_j} \]  

where \( D_{total} \) refers to the total number of defects in a chip, \( D_0 \) is a factor that accounts for systematic non-random defects, \( \lambda_j \) is the number of defects per unit area of fault type \( j \) in a chip, \( \alpha_j \) is a clustering parameter that accounts for clustering effects of defects of type \( j \) in a chip, typically in the order of 0.3 to 5 and \( A \) is the total area of the chip. To get an estimate of \( \lambda \), the number of defects per unit area, assume that a 10 megapixel camera for Digital SLR, is 25 mm \( \times \) 15 mm and assuming that most of the area is occupied by the array, and that there is 1 defect per megapixel, this works out to a \( \lambda \) of \( 2.67 \times 10^2 \) defects per mm\(^2\).

In addition to the types of defects that could occur during fabrication, there are also the types of defects that occur during operation of the device especially in harsh environments. One of the most important damages that can happen to the pixels is due to radiation. Radiation damage can cause traps to form at the surface of the pixel, thus
causing the pixels to become “hot”, which mean that these pixels give high output values even in the dark. Another common type of damage to aluminium interconnect is what is termed metal migration when a current passes through the interconnect over long periods of time.

In this thesis we will study a new pixel design, a Fault Tolerant Active Pixel Sensor (FT-APS) which should significantly reduce the occurrence of defective pixels both at fabrication, and compensate for the occurrences of defects that develop after fabrication.

1.3 Organization of Thesis

The thesis will be organized in the following way:

Chapter 2: Active Pixel Sensor (APS) gives a history of image sensor development. A description of the underlying theory behind photo-sensing in semiconductor device physics will be given. Then a technology adopted earlier for image sensing called the CCDs will be given. The alternative technology that is currently being developed named APS will then be introduced. The figures of merit for both types of image sensors will be discussed.

Chapter 3: Experimental APS Design will discuss new pixel designs, the first being the fault-tolerant photodiode-based APS (FT-APS), the second being the dual-output photogate-based APS (D-APS). The FT-APS is designed to increase yield of APS, while the D-APS is designed to be detectors in non-contact laser-based active scanning system, where the laser is of almost the same intensity as the ambient light because of safety reasons. The D-APS is designed specifically to meet the needs of Hermary, while
it is hoped that the FT-APS can be used in Hemyary’s design to increase effective yield and also in ordinary APS.

Chapter 4: Experimental Setup is a description of the experimental setup for testing the fault-tolerant photodiode-based APS (FT-APS) and testing the dual-output photogate-based APS (D-APS) would be given. This will give the reader an idea as to how the experimental measurements that produce the results discussed in this thesis are made.

Chapter 5: Experimental Results of Fault-Tolerant Photodiode-Based APS shows the experimental results of the fault-tolerant photodiode-based APS and its viability will be statistically accessed.

Chapter 6: Theory of Phase Sensitive Operation of Dual-Output Photogate APS (D-APS) introduces the theory of operation of D-APS by giving the structure of the D-APS and the simulations that suggest that the concept might work as proposed.

Chapter 7: Experimental Results of Dual-Output Photogate-Based APS gives the results of the D-APS experiment that has been carried out in the lab.

Chapter 8: Conclusion gives a summary of the thesis and further work that could be done to prove the concepts of FT-APS and D-APS.
CHAPTER 2 : ACTIVE PIXEL SENSOR (APS)

At the heart of this thesis is the Active Pixel Sensor (APS), which is a relatively recent development in solid-state scanning technology. As a result, it would be useful to discuss the background necessary to understand the APS. The history of image sensors and the context in which image sensors are invented will also be given. Then the uses of the image sensors will be discussed. I will provide the basic physics for understanding the operation of image sensors, including photodetector physics, which encompasses photodiode and MOS capacitor and important figures of merit for photodetectors. Two types of image sensors are included in this chapter, namely the CCD and the APS. Two different types of APS would be described, namely the photodiode-based and the photogate-based APS.

This thesis looks at modifications of the standard APS design to create new sensor features. APS has recently becoming more mature in the industry with CCD being its predecessor in solid-state imaging. CCD has a reputation of having extremely good imaging performance but is expensive and power-hungry and thus more suitable for scientific applications like astrophysics. While APS has a reputation of having low power consumption and less superior imaging performance, it is cheaper and thus more suitable for portable consumer electronics like digital still cameras and cell phone cameras.
2.1 Uses of Image Sensors

Image sensors are used mostly in the following broad categories: general imaging, machine vision, scientific and military [2].

General imagery includes applications like digital still cameras, mobile phone cameras and video cameras. These range from the simple, like those in disposable cameras to sophisticated ones used for professional broadcasting. The defining criteria for these cameras are mainly image quality which is dictated by dynamic range, resolution, and often color accuracy. High dynamic range is needed for many scenes with a wide range of light levels, while high resolution is needed for object identification. Gamma correction algorithms are necessary for the obtained images to appeal to the human vision system. Essentially, these algorithms are used to control the overall brightness of the picture by controlling the ratio of red to green to blue colors. An example of such a high end camera is the Canon EOS 5D, which utilizes a 12.8 megapixel CMOS image sensor having an area of 35.8 mm × 23.9 mm, with pixels having a size of approximately 8.2 μm × 8.2 μm, achieving a maximum frame rate of 3 frames/sec.

Machine vision camera includes applications like locating, inspecting, gauging, counting, identifying, recognizing and motion tracking. Since these cameras are used in real-time environments, the emphasis is on extremely high frame rates. They usually employ algorithms that are very application-specific. These cameras might need to operate in relatively harsh environments which are not human-friendly, for example, in space where there is ionizing radiation, in high radiation facilities, and extreme temperature conditions (hot or cold). An example of such a camera is the MT9M413 designed by Micron which can operate up to 500 frames/s with a resolution of 1,280
(Horizontal) \times 1,024 (Vertical), a total of 1.3 Mpix at 10-bit color resolution with pixel size of 12.0 \mu m \times 12.0 \mu m [23] and has an optimized dynamic range of 59 dB, whereas that for most conventional camera APS, it is 45 dB.

Scientific imagers are used in a variety of applications, like that used in optical and infrared astronomy for detecting distant stars or astronomical objects, or identifying molecules to study chemical reactions. The design concerns for these imagers are low noise, high responsivity, large dynamic range and high resolution. High frame rate is usually not critical for many scientific applications. Princeton Instruments, for example, specialize in such cameras, although they are in CCD formats. For example, Princeton Instruments recommend one CCD for astronomy purposes. It has 2048 \times 2048 resolution with 13.5 \mu m \times 13.5 \mu m pixel size and has a quantum efficiency of 95\% which is achieved with 100\% fill factor and cryogenic or air cooling [24]. Fill factor is defined as the percentage of the pixel area that can be actively used for converting photons to electron-hole pairs. This is typically not 100\% because of the presence of interconnect in CCD.

Military applications often entail detecting, recognizing and identifying targets at long distances, necessitating, high resolution, low noise systems and more extreme temperature and radiation conditions. Since these cameras have to perform under rugged conditions, they are becoming prevalent in military applications, where they can be mounted on unmanned reconnaissance vehicles in hostile and climatically unfavourable conditions.
2.2 History of Image Sensor

MOS image sensors as monolithic solid-state image sensors preceded the research and development of both CCD and APS. Fossum has described the history of solid-state image sensors in [7], which is useful for understanding the context in which APS has been invented.

- In 1963, Morrison described a MOS image sensor that could find the position of a light source [8]. It exploited photoconductivity properties of silicon to do so.

- In 1964, IBM designed and made the scanistor [9], which consists of photosensitive n-p-n junctions connected to a resistor network producing an electrical output proportional to the light intensity.

- In 1966, Westinghouse implemented a $50 \times 50$ array of monolithic phototransistors [10]. These three devices detect instantaneous light intensities and as a result suffer from low signal-to-noise ratio (SNR), compared to most current imagers which integrate the signal leading to higher SNR.

- In 1967, Weckler at Fairchild first proposed the idea of photon integration mode operation of p-n photodetectors [11]. The sensor is preset to a certain voltage and light would then stimulate photocurrent in a reverse-biased p-n junction. This photocurrent, would in turn, discharge the preset voltage that exist on the p-n junction’s parasitic capacitance. Readout uses a PMOS switch and the output is converted into voltage by employing resistors. By using
photon integration, Weckler’s idea improved the SNR of the image sensor. As you can see later on, this fundamental principle forms the foundation for the design of APS.

- In 1967, RCA made a $180 \times 180$ array of thin-film transistor (TFT) solid state image sensor [12], which integrated scanning readout circuitry, allowing it to address and readout individual pixels made of p-n photodetectors.

- In 1968, Weckler [13] reported a $100 \times 100$ array of photodiodes utilizing the principle of photon integration.

- In 1968, Noble [14] reported a solid-state image sensor that uses photodiodes that are on the surface and buried, as well as the first implementation of using MOS source follower (common-drain amplifier) for in-pixel unity gain and the use of charge-integrating amplifiers for readout that is used so widely now in APS systems.

- In 1969, Chamberlain [15] elaborated the explanation of how Noble’s solid-state image sensor functions. Chamberlain wrote a rigorous theoretical paper where he gives a mathematical and semiconductor physics analysis, which laid the groundwork for the foundation of image sensors.

- Also in 1969, Boyle and Smith of Bell Labs, invented the CCD (see section 2.4) [16][17][18]. The operation of the CCD is based on MOS capacitor
devices where photoelectrons can be collected and transferred to an output device.

- In 1970, Fry discussed one of the most serious problems with MOS image sensors, which is Fixed Pattern Noise (FPN) [19]. FPN is a type of noise that exists spatially in the image and is fixed regardless of illumination. Major causes of FPN are due to variations in doping concentrations for the pixels and mismatches of transistor characteristics. Today, FPN is also one of the major issues in the wide acceptance of APS.

- In the 1970s and 1980s, a number of companies invested a lot of effort in improving CCD performance. CCD was chosen as the technology of choice over MOS because of its smaller FPN, smaller pixel size, higher optical fill factor, reduced dark current, high charge transfer efficiency and low smearing. At the same time, research on MOS image sensors was intermittent at best and the quality of images obtained by MOS image sensors during that period never came close in terms of quality to those obtained by CCD at that time [20]. This is because in the 1980s, the minimum feature size in a CMOS technology was about 5 μm. This means that the interconnect lines for a MOS image sensor would take up much of the sensor array, and the pixel would have a prohibitively low fill factor. The FPN was higher in the MOS image sensor because the threshold voltage in the CMOS technology was less highly controlled due to immature process. In this way, CCD became entrenched as the better solid-state image sensor. Also during this time, Hitachi and
Matsushita persevered in the research and development of MOS image sensors [21][22]. Hitachi eventually gave up this line of research.

- In the early 1990s, two independent needs stimulated new research of CMOS image sensors. The first reason can be attributed to the requirement of image sensors to integrate on-chip control circuitry, reducing number of parts, improving reliability and thus lowering cost. The second reason is due to the need for low-power image sensors that has improved performance, and that at the same time, is radiation-insensitive. This is mainly for deep space exploration by NASA [20]. The CMOS image sensor in its modern form, the Active Pixel Sensor (see section 2.5) was designed by the US Jet Propulsion Lab (JPL), which was then transferred to commercial companies. One of the main researchers at JPL, Eric Fossum, started a company named Photobit, which was subsequently acquired by Micron. This CMOS image sensor exploited the fabrication technology of its times, when process improvements have reduced process variations thus reducing FPN, the major cause of acceptance of CMOS image sensors.

Today, CMOS image sensors are widely accepted as replacements for CCDs in low power applications like cell phones and high end digital still cameras. Companies like Canon have used APS in their high end digital still cameras. An example of this is the Canon EOS 5D, which utilizes a 12.8 megapixel CMOS image sensor.

CMOS image sensors accounted for more than 60% of total image sensor market in 2006 up from 45% in 2003. IC Insights, a respected company specializing in market
research for the semiconductor industry, stated that CMOS image sensors will claim more than 80% of the total revenues and over 90% of the units in 2008[32]. It is probably reasonable to conclude that CMOS image sensors will succeed commercially.

2.3 Fundamentals of Photodetectors

This section is a review of the physical mechanism that makes sensitivity to light possible in silicon semiconductor. From these humble beginnings, we will then explain the p-n junction, as well as the MOS capacitor, and the mode of operation which allows it to detect light and convert it into a measurable electrical voltage. A scheme will then be designed to allow each one of these devices to operate well in an array, leading up naturally to the APS.

2.3.1 Solid-State Photo-detection

There are a variety of solid-state photo-detectors using different materials such as silicon, and more generally the compound semiconductors like Al$_x$Ga$_{1-x}$As (direct gap for x<0.4), CdTe, GaP, InP and GaAs to name a few[25]. However, silicon detectors dominate the visible light image sensor market because of the low cost of fabrication, due to the dominance of silicon in regular IC fabrication. Compound materials are mostly used for infrared detectors, and emitters such as light emitting diodes or laser diodes.

Figure 5 outlines the electromagnetic spectrum[26]. The visible part occupies a very small portion of the entire range, from 400 nm (blue) to 750 nm (red) and silicon detectors work well in this electromagnetic range.
As the light impinges on a piece of silicon, some reflection happens at the interface, while most of the light gets transmitted to different depths of silicon. The absorption is defined as:

\[ A = 1 - R - T \tag{4} \]

where \( A \) is the absorption, \( R \) is the reflection and \( T \) is the transmission. The absorption depends on a number of factors, most notably wavelength (color) and temperature.

The light intensity at depth \( x \), \( I(x) \), is related to the unreflected light intensity at the interface by the Beer-Lambert Law [27]:

\[ I(x) = I_0 e^{-\alpha x} \tag{5} \]

where \( I_0 \) is the intensity of light at the surface, \( \alpha \) is the absorption coefficient and \( x \) is the distance from the surface to the depth of penetration of light.
Figure 6 shows the absorption coefficient $\alpha$ of silicon against the wavelength of light and its reciprocal which is defined as the penetration depth. The penetration depth is the depth at which the light intensity has dropped to $1/e$ or 37% of its original intensity at the interface. In silicon this is approximately 0.2 $\mu$m for blue light (wavelength of 400 nm) and on the other side of the spectrum for red light is 10 $\mu$m (wavelength of 750 nm).

![Absorption Coefficient vs Wavelength](image)

**Figure 6: Absorption coefficient of silicon and its reciprocal, the penetration depth after R. Hornsey [28]**

### 2.3.2 Photo-Absorption in Silicon

Photo-absorption is a process whereby electrons and holes (carriers) are created through impinging photons on the silicon semiconductor lattice that has energy greater than the bandgap energy of silicon. This happens when the energy of the photons, is enough to create electron-hole pairs (see Figure 7(a)). In silicon, the bandgap energy ($E_c$) is 1.12 eV at room temperature (see Figure 7(b)). An intuitive and pictorial representation of photo-generation is given in Figure 7. Figure 7(a) shows the atomic view of photo-absorption, while Figure 7(b) illustrates the energy band view of said phenomenon.
There are two types of bandgap energy, direct and indirect bandgap. Direct bandgap implies that when carriers traverse across the bandgap energy from the valence band to the condition band as shown in Figure 7(b), no phonons (acoustic packets) are produced or absorbed, whereas in indirect bandgap materials, carriers traversing the bandgap require phonons to be produced or absorbed. The implication of this for photodetectors is that indirect bandgap material like silicon is much less efficient in photo-absorption whereas GaAs is more efficient.

![Figure 7: (a) Atomic view of photo-absorption, (b) Energy band view of photo-absorption after G.W. Neudeck [29]](image)

The energy of photons is given as such [25]:

\[
E = h \cdot f = \frac{h \cdot c}{\lambda} = \frac{1.24}{\lambda (\mu m)} \text{ (eV)}
\]  

where \( h = 6.626 \times 10^{-34} \, \text{J} \cdot \text{s} \) is the Planck’s constant, \( f \) is frequency, \( c = 3 \times 10^8 \, \text{m/s} \) is the speed of light and \( \lambda \) is the wavelength. From this relationship, it can be seen that light of longer wavelength has less energy and so red light would have less energy compared to blue light. Since the bandgap energy of silicon is 1.12 eV at room temperature, the longest wavelength radiation that has sufficient energy to raise an electron from the...
conduction to the valence band, and thus can be detected, is 1.11 µm. Any wavelength shorter than 1.11 µm will be more energetic and cause electron-hole pair to be generated. In some cases, when the energy is high, more than one electron-hole pair can be created. This also means that silicon photodetectors are sensitive to infrared light, visible light as well as anything that is very energetic like gamma radiation and so on.

2.3.3 Photodiode

The photodiode that is used for detecting visible and near-infrared light are usually diode that is operated in reverse-biased mode [25]. To understand the photodiode, one has to understand the operation of a diode, because potentially diodes are naturally sensitive to light. A photodiode is essentially a diode that has been specifically made more sensitive for light detection.

The p-n junction silicon diode is formed by creating a p-doped silicon above or below an n-doped silicon [29]. When a p-n junction is not subjected to external stimulation like applied voltage, temperature gradients and electromagnetic fields, the p-n junction is said to be in an equilibrium state. The energy band diagram of a p-n junction in equilibrium is shown in Figure 8. When the diode is formed, the Fermi energy $E_F$, of the p-doped and n-doped semiconductor aligns itself. The reason this happens comes from the definition of Fermi energy, which is the probability that the energy state is occupied by a carrier is in the middle of the band under equilibrium conditions. This alignment of Fermi energy, $E_F$, causes the bending of the conduction band $E_C$ and the valence band $E_V$ such that a transition occurs and joins the bands of the two differently doped semiconductors as shown in Figure 8.
Before reaching equilibrium conditions, the holes would diffuse from p-doped to the n-doped region, while the electrons would diffuse from the n-doped to the p-doped region. While this is happening, the p-side and the n-side have a built-in electric field (which produces $V_{bi}$), that causes a flow of carriers in the opposite direction to diffusion. This movement is defined as carrier drift. Diffusion is a process driven by carrier concentration gradient difference while drift is driven by electric field. Under equilibrium conditions, the carrier drift is balanced by the carrier diffusion.

In the context of Figure 9, forward bias corresponds to applied voltage, $V_A > 0$ and reverse bias corresponds to $V_A < 0$. In a forward biased diode, the effective potential barrier is $q(V_{bi} - V_A)$, thus increasing diffusion current components while the drift components remain the same as that in equilibrium conditions. Consequently, there is a net forward current. In a reverse biased diode, the effective potential barrier is $q(V_{bi} + V_A)$, thus decreasing diffusion current components while the drift components remain the same as that in equilibrium conditions. Consequently, there is a very small current, and after a fraction of time, reverse biased voltage drops to zero.

**Figure 8: The diode in equilibrium conditions after G.W. Neudeck [29]**
2.3.4 Operation of Photodiode

When light impinges on the depletion region of the photodiode and causes photo-generation, electron-hole pairs are generated, which are then separated by the built-in electric field. $L_n$ and $L_p$, as shown in Figure 9, are the minority carrier diffusion lengths for electrons and holes respectively, which is the average distance the minority carriers travel in the bulk before undergoing recombination. Any minority carriers generated within the bulk region but within the minority carrier diffusion length will be attracted to the depletion region by virtue of its electric field. This implies that a larger depletion region will create a bigger photo-sensitive volume.

In order to increase the depletion region width, there are two general methods. One is to operate the photodiode in the reverse biased mode, and the other one would be to introduce an intrinsic region in the middle of the photodiode. An intrinsic region is a region where the semiconductor is not doped, thus producing even more photo-sensitive area. This special type of photodiode is referred to as a PIN photodiode. Since this
requires a specialized fabrication process, we will not discuss this in the context of the thesis.

In standard CMOS technology, this photodiode would be formed from the p-substrate with an n+ implant at the surface, forming a vertical photodiode. With the scaling of technology to smaller dimensions, the increased substrate doping would decrease minority diffusion carrier length and mobility of minority carriers, thus reducing the effective charge collection region [30]. It is very doubtful that standard CMOS process of today would continue to be used in CMOS imagers to come, as shrinking of CMOS occurs.

In the case of the vertical photodiode made from TSMC 0.18 μm technology, which has an n+ doped-p substrate component, this depends on the dopant concentration of the n-doped part at the top part of the photodiode, the temperature, the spectral response of semiconductor and the volume formed by depletion region with the addition of approximately one diffusion length away from the depletion region of the semiconductor, which forms the volume of the charge collection region. In the case of the photogate, it depends on the thickness of the photogate material made from polysilicon that has been taken care not to be salicided (because salicide is not very transparent), the temperature and the volume of the charge collection region, which is similarly, the volume formed by the depletion region plus one diffusion length away from the depletion region. In many modern circuits, salicide covers the polysilicon layer to reduce the effective sheet resistance of polysilicon, and since it has high absorption coefficient, this defeats the purpose of the pixel collecting light-induced photoelectrons and have to be removed.
2.3.5 MOS Capacitor

The MOS capacitor (MOS-C) is the basis of CCDs as well as photo-gate CMOS devices as shown in Figure 10. Its operation is best understood by first understanding the flatband condition of MOS-C and then the effects of biasing.

![Figure 10: MOS gate for p-type silicon after G. Holst [2](image)](image)

Under the flatband condition for an ideal MOS capacitor (MOS-C), the created energy difference between the work function of the gate material (metal or polysilicon), \( q\phi_m \), and that of the semiconductor, \( q\phi_s \), is zero, that is \( q\phi_{ms} = 0 \). It is also assumed that there are no charge carriers and charges in the oxide, that there is no tunnelling of the oxide under dc biasing conditions, the semiconductor region is uniformly doped and that

![Figure 11: Energy band diagram of an ideal MOS capacitor under flatband condition \( V_G = 0 \) V after S. Wolf [34](image)](image)
the gate layer is entirely of the same potential [34]. For the purpose of discussion, a p-type semiconductor is used because the TSMC 0.18 μm technology uses a p-substrate.

Let us consider qualitatively what happens when a slowly decreasing negative voltage is applied at the metal of the MOS-C. Under this condition, holes are attracted onto the interface of the oxide and p-type semiconductor. This region of operation is referred to as the accumulation condition. Since there are more holes at the surface, this implies that the $E_F$ is closer to $E_V$ at the surface than in the bulk causing a bending of $E_F$. The bandgap energy being a constant at a certain temperature means that $E_C$ is similarly bent, as shown in Figure 12(a).

When a positive charge that is slowly increasing is applied to the MOS-C, where $V_{GB} > 0V$ but $V_{GB} < V_T$, this repels the holes in the p-substrate, leaving some part of the p-substrate to with negative ions as shown in Figure 12(b). When this happens, since the acceptors are ionized negatively, $E_F$ is closer to $E_C$, causing a downward bending of the energy bands. This condition of absence of carriers is known as depletion. As the positive
voltage at the metal is increased more, it starts attracting electrons that are thermally generated or photo-generated, thus causing there to be more electrons than ionized acceptors at the surface. As this happens, $E_F$ crosses over $E_i$, which is closer to $E_C$ than $E_i$ at the surface. This time when the positive voltage at the metal is increased further, the bands will continue to bend downwards until $E_i$ at the surface is eventually below $E_F$. At this point, the electron density at the surface is the same as that of the density of the ionized acceptor, and this is called strong inversion.

The inversion process provides the inversion layer where the electrons that are photo-absorbed can be collected. This is because the inversion layer effectively creates a potential well which allow for the integration of electrons. Unfortunately, this also attracts electrons that are thermally generated. Furthermore, in reality there are charge centres at the oxide layer due to presence of impurities during fabrication, thus causing another additional source of potential noise in MOS-C. This will be seen to be the basis of operation for CCD and photogate devices.

2.3.6 Important Figures of merit for Photo-detectors

Definition of these figures of merit uses the form created by S. Sze [25].

Full-well capacity is defined as the total capacity of the semiconductor to store photo-electrons usually in the depletion region as well as approximately a diffusion length away from the depletion region.

Spectral response is the responsivity of the photodetectors to light of different wavelengths. This is a function of the photodetector element, the temperature at which the photodetector is at, the size of the charge collection region, as well as the semiconductor material which the photodetector is made from.
2.4 Charge-Coupled Device (CCD)

Charge-Coupled Device (CCD) was invented by Boyle and Smith in 1969 at Bell Labs. The basic structure of a CCD is the MOS-C as shown in Figure 10. The CCD is essentially MOS-C type photodetector with a “bucket brigade” whose architecture has three basic functions: (a) charge collection, (b) charge transfer, and (c) conversion of charge into a readout voltage. By controlling the voltages on the gate of each individual MOS-C, charges can be collected or transferred. The charges are created by photons impinging the depletion region of the bulk semiconductor.

![Three-phase CCO](image)

Figure 13: Three-phase CCO. The gates must overlap for efficient charge transfer after G. Holst [2]

2.4.1 Operation of the CCD

The first CCD was a three-phase one like that in Figure 13. The gates must overlap for efficient charge transfer to occur, which is a very central point in the functioning of a CCD. The operation of the three-phase CCD can be illustrated in Figure 14. Assuming randomly that there is a position in the CCD referred to as Well 1, Well 2 and Well 3 associated with the three controlling gate voltages \( V_1 \), \( V_2 \) and \( V_3 \) as shown in Figure 14. First during the exposure phase (a) \( V_1 \) is pulsed, to typically 10 V, while the other gate voltages are at 0V. This causes the depletion region or potential well to form just under the gate controlled by \( V_1 \), which when exposed to light generates
photoelectrons stored in this potential well. Where $V_2$ and $V_3$ are not pulsed to a non-zero voltage, potential wells are not formed even though there are photons impinging upon this depletion region too. The photoelectrons generated under the gates controlled by $V_2$ and $V_3$ immediately recombines with the p-substrate or get swept away by the electric field into the p-silicon substrate which is grounded. Subsequently, in the readout phase (b) $V_2$ is pulsed to 10V, while $V_1$ remains at 10V, which causes the photoelectrons to be spread between potential wells of $V_1$ and $V_2$. Then $V_1$ is pulsed to 5V, which causes the potential well to be reduced, thus spilling some of the photoelectrons from potential well $V_1$ to potential well $V_2$. This is part of the charge transfer. In the next step of the transfer phase (c), where $V_1$ is pulsed low, the photoelectrons are fully transferred from Well 1 to Well 2. The reason this is done as opposed to just directly pulsing Well 1 to 10V while Well 2 is 0V, and then in the next phase pulsing Well 1 to 0V and Well 2 to 10V is because the charge transfer is not going to be complete as the charge would be shared between the two wells. There is lag, which causes some of the photoelectrons not to be transferred directly to the next potential well in this scenario. This is why the three-phase CCD was designed in this way. Well 2 and Well 3 are then pulsed in the same fashion as Well 1 and Well 2.

After that, the photoelectrons are transferred in this very fashion, aptly called the bucket brigade, to the end of the column. The last row contains yet another bucket brigade, which transfers all the photoelectrons from each pixel in the CCD to a charge amplifier that converts the photoelectron charge to a voltage. This entire CCD transfer is orchestrated by these three clock phases.
Figure 14: Charge transfer in a three-phase CCD after G. Holst [2]
2.4.2 Important Figures of Merits for CCD

From the description of the operation of the CCD in the previous section, one can deduce several figures of merits that will be discussed in this section.

One of the most important figures of merits for CCD, especially the larger ones, is the charge transfer efficiency (CTE). It defines how well the photoelectrons are transferred from one potential well to another. For example, for an array whose size is $1000 \times 1000$, the charge packet furthest from the readout node has to traverse across 2000 pixels because it would have to be readout through the 1000 shiftout registers. When many transfers exist, for example 2000 pixels then the CTE must be high. Let us assume that the net transfer efficiency for the entire array has to be at least 0.99, then the effective CTE for each pixel has to be very high at 0.99999. The CTE is primarily affected by the concentration of surface states and the clock frequency.

Dark current is also another important figure of merit for CCD. The sources of dark current are primarily caused by thermal generation in the depletion region, thermal generation and diffusion in the neutral bulk material and thermal generation due to surface states. Dark current densities vary very widely among manufacturers ranging from 0.1 nA/cm$^2$ to 10 nA/cm$^2$ [2]. Dark current is a big concern when the integration time is small or at low-light levels as the signal-to-noise ratio can be degraded because of a lower signal.

The spectral response of the CCD array depends on the material with which the CCD is made from as well as the gate material overlaying the pixels and the temperature at which the CCD is at. This has been discussed in great details in Section 2.3.1. One has
to consult this for the application at hand, since individual application requires sensitivity at different wavelengths.

Another figure of merit is the fill factor, which is the percentage of the photosensitive area to the entire size of the die, which might include circuitry. The larger the fill factor, the greater the net photosensitivity of the entire CCD. The fill factor is typically 25% because of interconnect. A microlens is a miniature lens that is placed on top of the pixel and can cause the fill factor to approach 100% if the quality of the microlens placed on top of each pixel is good and if a good lens is used at the package level that matches the chief ray angle of the microlens and that of the package lens.

CCDs are fabricated with a great deal of care but even then defects are still possible. The defect specification of the CCD is indispensable especially for applications such as astronomy. This is usually specified as the maximum limit on the number of defective pixels. It is typical that the smaller the upper limit of the number of defective pixels, the more expensive the CCDs. CCD is very sensitive to defects. Even a single point defect can block the transfer gate and cause an entire column to fail.

Finally, noise is an important issue with CCDs. Noise is statistical in nature and highly temperature sensitive because the noise sources are usually related to thermal processes. As such noise sources can exist in the pixel itself, the readout chain and the charge amplifier. This is also why the charge amplifier is designed with great care as the CCD array is only as good as the charge amplifier.

Finally, the cause of most concern for most portable devices like digital cameras, cell phone cameras and other consumer devices is that the power consumption of the CCD is large. More importantly, the CCD array needs to be powered on all the time.
during operation and therefore the power consumption is large. Also because of the high voltages associated with CCDs and the complex clocking schemes that drive large lengths of poly gates which are highly capacitive in nature at very high speeds. This has been one of the major reasons the industry has been looking at our next candidate, the Active Pixel Sensor.

2.5 Active Pixel Sensor (APS)

An Active Pixel Sensor (APS) is a CMOS based integrating sensor where each photo-detecting element has a built-in amplifier in it. This is important especially when the array is huge and the signals that need to be driven out of the array cannot be diminished by the relatively huge capacitance of interconnect, in a reasonable amount of time. The photo-detecting element of the APS can be either photodiode or photogate based. Several seminal papers on APS are [20] and [37].

APS has the advantage of being cheaper to fabricate since they can utilize standard CMOS process. It also has the advantage of being cheaper at the system level since the CMOS process can be used to integrate analog circuitry such as ADC and digital image processing circuitries. One of the most important advantages is that APS arrays consume much less power than CCDs, thus making it a very attractive choice in portable applications that use batteries. Another important advantage is that the CTE of a large APS array does not need to be large because each pixel is read out individually unlike that in CCD, allowing it to be a larger resolution image sensor.

However, APS has the disadvantage of having poorer noise characteristics, like FPN. Furthermore, because of its fabrication using standard CMOS process, the pixels also typically have lower dynamic range. Due to the added in-pixel circuitry, it typically
has a lower fill factor than a CCD pixel. Furthermore, the APS has a poorer blue response than CCDs, one that might affect its acceptance in digital still camera market.

![Photodiode-based APS diagram](image)

**Figure 15: Photodiode-based APS after Fossum [37]**

### 2.5.1 Photodiode-Based APS

The photodiode-based APS uses a reverse-biased pn junction to collect the photocurrent on the gate of the source follower transistor as shown in Figure 15. The photodiode-based APS consists of 3 transistors, $M_{\text{RESET}}$ (reset transistor), $M_{\text{SF}}$ (source follower transistor) and $M_{\text{RS}}$ (row select transistor), as well as a photodiode. External to the APS is a biasing circuit for the source follower, which is $M_{\text{BIAS}}$, creating the current...
sink for the source follower. The source follower is basically a common drain amplifier which approaches that of unity gain and acts as a voltage buffer, that has high input impedance and low output impedance. The low output impedance is necessary to drive the relatively large capacitance of the output column line to reduce loading effect.

In order to understand the operation of the photodiode-based APS, it is necessary to look at the timing diagram of a typical operational cycle. At the beginning of a read cycle, the reset transistor ($M_{\text{RESET}}$) is turned on and pre-charges the gate of the source follower transistor ($M_{\text{SF}}$) to a known voltage. The photodiode then discharges the $M_{\text{SF}}$ gate for a fixed integration period before the final output is read out by enabling the row select transistor ($M_{\text{RS}}$). The output voltage ($V_{\text{OUT}}$) is a linear function of the integrated photocurrent. When $M_{\text{RS}}$ is always on, the output voltage can be shown in Figure 16. The function of the row select transistor would be clear in the context of an array. In an array, the row select transistor would be used to select the row in which the pixel would be located, assuming that a column-readout architecture is used, where one biasing transistor for the source follower is used for each column.

![Figure 16: Timing operation of the photodiode-based APS after Fossum [37]](image-url)
Figure 17: Typical operation cycle of the photodiode-based APS when row select transistor is always on.

Assume that during reset, $V_{DD}$ is applied to the gate of $M_{RESET}$. Because there is very little photocurrent to bias the $M_{RESET}$, it will be operating in the linear region and act as a switch. The voltage at the gate of $M_{SF}$ is approximately $V_{DD} - V_{THN}(M_{RESET})$, where $V_{THN}(M_{RESET})$ is the threshold voltage of the reset transistor, when the photocurrent has not discharged that node. As that node is discharged by the photocurrent that occurs, the gate voltage of $M_{SF}$ decreases at a rate that is proportional to the product of integration time and the amount of photocurrent. The change in voltage at this node is given by:

$$\Delta V = \frac{Q_{total}}{C_{total}} \quad (7)$$

$$C_{total} = C_{gate} + C_{diode} + C_{parasitic} \quad (8)$$

where $Q_{total}$ is the total amount of charge that has been collected by the photodiode and $C_{total}$ is the total capacitance at said node, which is mainly the photodiode reverse-biased capacitance (depletion capacitance), but also includes the parasitic capacitance of the transistors and the gate capacitance of the source follower, which is typically 1/10 of the photodiode reverse-biased capacitance. This causes a drop in voltage because electrons...
are the charge carriers and the amount of voltage drop is proportional to total amount of
charge collected, which is a function of quantum efficiency, integration time and light
intensity. The larger the drop in voltage, the larger the conversion gain.

This voltage swing is measured by sampling the output voltage after integration
and the next reset cycle and differencing the two output voltages. This is termed double
sampling (DS) because the reset and integration readouts happen in different cycle. On
the other hand, Correlated Double Sampling (CDS) is a readout scheme where the reset
and the integration readouts happen in the same cycle and is less susceptible to noise. The
reason it is difficult to do CDS here is because if the reset is first sampled and then held
in the capacitor, the integration time might be too long for the sampling capacitor to hold
the charge with reasonable confidence of fidelity, before the integration readout is
performed.

This output voltage is then read out through the source follower when row select
transistor is switched on, which causes the source follower transistor to be momentarily
biased by the column current sink. The maximum and minimum output voltage swing,
$V_{OUT(MAX)}$ and $V_{OUT(MIN)}$, in this case for the source follower transistor to be in saturation
is:

$$V_{OUT(MAX)} = V_{DD} - 2 \cdot V_{THN}$$

(9)

$$V_{OUT(MIN)} = V_{DS,SAT} \left( M_{BIAS} \right) = V_{BIAS} - V_{THN} \left( M_{BIAS} \right)$$

(10)

In reality, this output voltage swing might be limited by the photodiode’s dynamic range.
2.5.1.1 Important Figures of merit for the photodiode-based APS

Important figures of merit for the APS would include conversion gain, fill factor, reset noise, Fixed Pattern Noise (FPN), Photo-Response Non-Uniformity (PRNU), and flicker noise.

FPN is due to pixel-to-pixel responsivity variations when illumination is not present. The main cause of FPN is due to threshold voltage variations of transistors in the APS due to variations in dopant concentrations. The threshold voltage variation in the reset transistor, the source follower transistor and that of the column biasing transistor play the major role in FPN. Another source of FPN is non-ideal clock distribution scheme which clocks some parts of the array at different times than other parts of the array, which cause slight differences in integration times either because of resistive drops or slew-rate limitations on the reset line or row select line.

PRNU is due to responsivity variations when illumination is present. This is due to the variations in semiconductor spectral response, which in the case of the photodiode-based APS could be due to n+ dopant concentration varying across the APS array and crystal defects in semiconductor in different parts of the array. In the case of the photogate, variations in thickness of polysilicon gate material could cause PRNU.

The conversion gain is given by:

\[ Conversion \ gain = \frac{Gq}{C} \]  

(11)

where \( G \) is the source follower gain which is typically close to 1, \( q \) is the charge of an electron and \( C \) is the capacitance of the sensing node, which is mainly the photodiode capacitance in the case of the photodiode-based APS and the floating diffusion.
capacitance in the case of the photogate-based APS. C, the capacitance of the sensing node is the sum of the photodiode capacitance and the gate capacitance of the source follower. The photodiode capacitance is typically 10 times larger than that of the gate capacitance of the source follower. The larger the conversion gain, the higher the output voltage swing. But the larger output voltage swing might not be well accommodated by the dynamic range of the output circuitry and needs to be adjusted to that of the dynamic range of gain stages and ADC that lies further in the pipeline of the image signal chain.

Fill factor is defined as the ratio of the total photosensitive area to the total area of the sensor, which includes circuitry. The larger the fill factor, typically the better the sensor array is. This is an indication of the compactness of the supporting circuitry.

There are two types of noise in an APS array, the random noise and the pattern noise. The random noise is true electrical noise while pattern noise is spatial artifacts that look like patterns to the eyes of the observer. Examples of random noise are shot noise, thermal noise (dark current), reset noise, flicker noise or 1/f noise, while pattern noise can be divided into that which is in the absence of illumination, FPN, and that which is illumination-dependent, the PRNU.

Reset noise is due to the uncertainty caused during reset, which cannot be removed by double sampling as in the photodiode-based APS. The reset noise is caused by the parasitic resistance and sensing node capacitance, forming an RC low-pass circuit that has the following rms reset noise in terms of number of electrons:

$$\langle n_e^2 \rangle = \frac{kTC}{q}$$ (12)
where \( n_r \) is the number of electrons due to reset noise, \( k \) is the Planck’s constant, \( T \) is the absolute temperature, \( C \) is the capacitance of the sensing node including parasitics and \( q \) is the electronic charge. As a result, the only effective way of decreasing reset noise in the photodiode-based APS would be by having lower sensing node capacitance or using smaller-area pixels. This of course increases the conversion gain, while at the same time increasing the dynamic range of the entire APS.

Flicker noise or 1/f noise arises from the presence of traps at the silicon and silicon dioxide interface. Flicker noise is less as the frequency increases. This problem cannot be alleviated completely but can be minimized by reducing the area of the gates of the transistors. Since the \( W/L \) of the source follower transistor affects its gain, it is much better to just minimize \( L \) of the transistor, so as not to sacrifice the gain.

Finally, power consumption is very much less than in CCDs. On top of that, there is leveraging of the fabrication process for CMOS technology that can be amortized over many products whereas a CCD process line is typically only used for manufacturing CCDs and nothing else. Thus ADCs and other types of circuitry can be integrated onto the same die which reduces the price of an entire imaging system.

### 2.5.2 Photogate-Based APS

The photogate based APS works in a similar way in terms of the readout transistor circuitry as the photodiode based APS. The difference lies in the photo-detecting element, which is a photogate. The photogate is a variant of the MOS capacitor and can be understood as such. The only difference is that there is a transfer gate (TX), where the charge collected in the MOS-C is then transferred to the floating diffusion, which is connected to the gate of the source follower transistor, as shown in Figure 18. The
photogate-based APS is essentially a single-stage CCD transfer with a single pixel. Note that the capacitance is smaller than that of the photodiode APS because there is only the depletion capacitance of the potential well, which is typically smaller than that for the photodiode. Also, the photogate being a piece of polysilicon on top of the pixel and is somewhat translucent, can impact the quantum efficiency of the pixel beneath.

A typical operational cycle of the photogate-based APS is shown in Figure 19 and Figure 20. First, the photo-electrons are integrated under the photogate when a positive voltage is applied on PG. Then the floating diffusion is reset, pre-charging it to a certain voltage, effectively emptying the charge. Then a ground potential or a low voltage is applied to the PG while a high voltage is applied to TX, which causes the potential well of the MOS-C to be emptied out into the floating diffusion. This phase is called the
transfer process. A ground potential or low voltage is then applied to TX, which will empty out the charge remaining in the floating diffusion, while the voltage at PG is high again to allow for the next integration cycle, while a readout of the charge is performed.

In the photogate based APS, true Correlated Double Sampling (CDS) is performed as opposed to just double sampling, resulting in a more accurate measurement, because in this case, it is possible to subtract the reset and integration readout in the same cycle. The photogate is made of polysilicon that has not been salicided, as a result, there is the disadvantage of loss of light due to optical absorption by the photogate material, resulting in lower quantum efficiency for the photogate.

![Figure 19: Typical operational cycle of a photogate-based APS after Fossum [37]](image-url)
2.5.2.1 Important Figures of merit for the APS

Important figures of merit for the APS would include conversion gain, fill factor, reset noise, Fixed Pattern Noise (FPN), Photo-Response Non-Uniformity (PRNU), and flicker noise and other figures of merit just like in photodiode-based APS discussed in Section 2.5.1.1.

Reset noise is due to the uncertainty caused during reset, which cannot be removed by double sampling as in the photodiode-based APS. The photogate-based APS, on the other hand, can employ Correlated Double Sampling (CDS), therefore minimizing reset noise.

The photogate-based APS has a poorer blue response in terms of its spectral response because of the photogate which is essentially a layer of polysilicon above the substrate. This reduces the penetration depth of light in the silicon because some of the light is absorbed in the polysilicon layer instead of the silicon.
The conversion gain of the photogate-based APS is typically lower because of the higher capacitance provided at the gate of the source follower due to the presence of the diffusion capacitance. Typically, the full well capacity of the photogate-based APS is higher than that of the photodiode-based APS. The photodiode-based APS is more common in research because it has higher quantum efficiency and better fill factor.

2.6 Conclusion

It is true that APS presents many advantages in consumer devices which require low power. However, CCDs are still holding ground in certain scientific applications like astronomy where they are very entrenched. However, these are extremely narrow applications that have sales of limited quantities. On the other hand, the future of APS seems certain in the field of consumer devices as some who did careful characterization has affirmed that APS has equalled or even surpassed CCD in terms of picture quality [32].

However, we proposed a solution that would make APS even more competitive by increasing the yield of APS as covered in Chapter 3. This proposal involves designing a fault-tolerant APS that would by the definition of being fault-tolerance, increase the yield in the presence of manufacturing defects.

Another important advantage of the APS is that it allows for greater flexibility in modifying the image sensor for niche applications that is called for by Hermary, that is the specialized problem of detecting laser in the presence of other light sources at almost the same intensity. This could not have been done at the pixel level in the CCD, because each pixel in the CCD is not only capturing photons but is also responsible for charge
transfer. In the case of the APS, each pixel has its own charge transfer node and charge collection region. Thus we will also describe this pixel design in Chapter 3.
CHAPTER 3 : EXPERIMENTAL APS DESIGN

The APS designs tested in this thesis will be given in this chapter. The first is the standard types of APS found in the literature, and is used as a baseline APS which will serve as a comparison with other published results [7]. The standard APS designs include the photodiode-based APS and the photogate-based APS. The experimental APS designs include the Fault-Tolerant APS (FT-APS) and Dual-Output APS (D-APS). In this thesis the experiments the FTAPS is photodiode-based while the D-APS is derived from the photogate APS. In fact, this alternative has been covered by my colleague, Desmond Cheung in his thesis [33].

3.1 Technology Utilized for APS Designs

The technology used for all the designs in this thesis is provided by Canadian Microelectronics Corporation (CMC). The CMC is using Taiwan Semiconductor Manufacturing Company (TSMC) to fabricate the chips using their 0.18 μm technology developed circa 2002.

The decision to use the 0.18 μm technology was quite involuntary. We would have preferred to use the older 0.25 μm technology because it has a double-poly process and better transistor characteristics especially in terms of leakiness, as well as being cheaper, however it was phased out at the beginning of the thesis. The 0.13 μm technology was too unstable at the time and we knew it had very leaky transistors that would have degraded the performance of the APS. At the same time, it is a very expensive process technology that at the time could only be justified for leading edge
mixed-signal cores or digital/SOC (System-On-a-Chip) designs.

There are several noteworthy things about the selected technology (0.18 μm), namely

1. a single-poly process with a salicide top to reduce the sheet resistance of the poly layer, so that there is less voltage drooping when the gate is farther away from the controlling signal source,

2. it uses two power supplies, 1.8V and the 3.3V, requiring two different categories of transistors, aptly named the 1.8V and the 3.3V transistor,

3. it is an n-well process where the corresponding substrate is of the p-type,

4. it is a 6-metal aluminium process optimized for routing digital lines in complex digital designs and

5. this is a process which has been optimized for digital designs while the analog characteristics of the process has been somewhat compromised because of the single-poly.

These aspects of the process affect the APS designs in various ways. The single-poly process has several impacts such as greater difficulty in forming a good capacitor by using a poly-poly capacitor, instead an metal-insulator-semiconductor (MIS) capacitor needs to be formed which is of much less capacitance per area. A double poly process allows for photogates that could be formed like those in CCDs, with overlapping photogates allowing for higher CTE. Also, the fact that the poly is salicided requires that the salicide mask be turned off. This is because salicide is optically opaque, thus drastically reducing transmission of light to the photosensitive area of the APS. The
photodiode even though not having any overlaying poly layer is still exposed to saliciding in the fabrication process. The photogate uses a poly layer and so would be affected by the salicide process. This means that special design layer combinations must be used to ensure that the photosensitive area is not salicided. This is done by ensuring that the resist protection oxide (RPO) layer is switched on in the layout.

The availability of the two power supplies, 1.8V and 3.3V is quite an important characteristic. The 1.8V can be used for the digital sections for speed characteristics and lower power consumption, whereas the 3.3V can be used by the analog sections of the chip for increased dynamic range. This means that the APS could potentially use a 3.3V power supply to improve dynamic range. However, this was not done because at the time of the design, we have not found a way to instantiate the 3.3V transistors in the design tool, which would be required for the 3.3V supply. As a result, the design depended on only 1.8V power supply, which is unfortunate. The difference between the 1.8V and 3.3V transistor is in the thickness of the oxides, so that it can withstand these voltages reliably during operation. The fact that it is a p-substrate means that the photodiodes can be formed several ways, either using n+/p-sub or n-well/p-sub photodiodes, whereas the photogate is limited to using a p-substrate. In general, electrons are collected instead of holes. Collecting electrons instead of holes is good for the reason that electrons have higher mobility than holes in silicon and in general.

The 6-metal process allows complex routing in the APS array itself because there are a number of signals there, namely the power supplies, the reset signal, the row select signal and the connection between the photodiode/photogate to the gate of the source follower. The fact that this process is optimized for digital logic is not as important in this
thesis as in a real product where there are image processing functionalities built-in together other digital control blocks. Since this is not true for the thesis, this point has little relevance in our context.

3.2 Standard APS Designs

The standard APS designs are used to replicate what others had done and verify that the results from our designs match those published in the literature. This serves to act as a control for our experimental APS designs for the TSMC 0.18μm technology. There are two types of standard APS that has been designed, the photodiode-based APS and the photogate-based APS.

In all of the APS layouts, great care is taken to make sure that there are connections to ground on the p-substrate reasonably close to all of the circuitry of the APS. This is to ensure that there is little or hopefully no crosstalk between the pixels, which has been ensured by the design rules for TSMC 0.18 μm technology. The ground contacts also make sure that the p-substrate is always connected to grounds everywhere on the chip so that there is no possibility of p-n junctions being forward biased, a condition often called latch-up. Latch-up is usually destructive as the forward biased p-n junctions conduct huge amounts of currents enough to burn the relatively thin substrate.

Several different variations in the APS layout are used, while the sizes range from approximately 5 μm × 5 μm to 10 μm × 10 μm. The shapes of the APS are mostly rectangular but some are square too. Also, the dimension of the transistors in the layout varies in accordance to the differing layout. Several different currents were used in the column current sink for the source follower.
3.2.1 Standard Photodiode-Based APS

There are different possible versions of photodiode-based APS' (refer to Figure 15), the n+/p-sub and n-well/p-sub varieties. The n+/p-sub photodiode has the depletion region very close to the surface of the silicon, thus having a better blue response whereas the n-well/p-sub photodiode has the depletion region much deeper into the silicon thus causing it to have poorer blue response. What ended up being used for the thesis are the n+/p-sub photodiode type because they are better able to absorb different wavelengths of light. The layout of the standard photodiode-based APS example can be shown in Figure 21 and a photomicrograph of the same design realized on TSMC 0.18 μm technology is shown in Figure 22. The fill factor of this design is 85%. The fill factor is defined as the percentage of the pixel area that can be used for actively converting photons to electron-hole pairs. In the APS, this is typically not 100% because there are interconnect and at least 3 active transistors. These designs are what the other researchers have used in the past, and are therefore referred to as standard photodiode-based APS designs and will be used as a control for comparison with the experimental APS that would be designed for this thesis. Proving that this standard design works and doing some characterization on it will give us a standard for the newer designs that are being investigated for the thesis.
3.2.2 Standard Photogate-Based APS

The standard photogate-based APS requires a poly layer above the p-substrate and is essentially a one-stage CCD transfer, as can be seen in Figure 23. The fill factor of the photogate APS in our designs is 45% and tend to be lower than that of the photodiode-based APS. This is because in order to make sure that the salicide layer does not get deposited onto the photogate, as described in Section 3.1, there are design rules that limit the gate area. This is a standard design of the photogate-based APS patterned after those used by other researchers such as Fossum [20]. It will serve as a control for comparison with the experimental photogate-based APS designed for this thesis.
Figure 23: Cross section of photogate-based APS after Fossum [37]

Figure 24: The standard photogate-based APS
3.2.3 Experimental APS Designs

In the following sections new APS designs which are the focus of this thesis are described. These are experimental and have not been reported by any other research groups. The first one is the Fault-Tolerant APS (FT-APS) design and has been designed as an aid in improving manufacturing yields. The second one is the dual-output APS (D-APS). It had been conceived to address the problems of laser profilometers found by Hermary Optoelectronics described in Section 3.4.

In all the cases of the experimental APS designs, the standard designs of the current section will be used as a baseline.

3.3 Fault-Tolerant APS (FT-APS)

As digital cameras start to rival that of the resolution of traditional films, the array size increases and the pixel size decreases at the same time. The economics accompanying this natural technical transition requires lowering defects at fabrication time so as to keep yields high, thus improving profit margin for the semiconductor companies involved.
3.3.1 Fault-Tolerant Photodiode-Based APS

To correct these defects, most commercial products and researchers have resorted to software techniques. These include interpolation by nearest-eight neighbors or more sophisticated heuristic algorithms that ensure that the interpolation is done only on the nearest-eight neighboring pixels that are definitely not faulty pixels. Most cameras employ a variant of the heuristic algorithm that takes care of the case where one of the nearest-eight neighboring pixels is, for example to exaggerate the point, optically stuck low, while all the other pixels are very bright. This would cause a corruption of the pixel value that needs to be corrected and can cause misinformation. Now even these heuristic algorithms can only be guaranteed within a certain degree of probability to give correct pixel values and there is still a non-negligible probability that there is misinformation. For example, if the real pixel value is bright, but that particular pixel of interest is optically stuck low, then if every other pixel surrounding is dark, then the interpolation mechanism will reveal a dark pixel where a bright one is supposed to be. This is critical in some applications like astronomy, when the “bright object of interest” happens to be on the pixel which is optically stuck low.

Hence, software corrections only partially correct for pixel failures, and still degrade imager performance. The problem is that in sensor arrays, unlike IC’s, the information has a position-dependence. Hardware defect avoidance techniques commonly used in other digital circuits are not useful in the case of imagers. This is because defect and failure modes in Active Pixel Sensor are substantially different from that in conventional electronic circuitry. For electronic devices, defects are identified and located by using methods such as quiescent current ($I_{DDQ}$) and Built-In Self-Test (BIST). On the other hand, defects in image sensors are located by electronic as well as imaging
tests. Some defects, like particles or metal layers covering part of a sensor, are considered optically faulty, even though this would not be considered electronically faulty in conventional circuits. A concept called sparing [35] in redundancy terminology involves implementing fault-tolerance by including more electronic components. This is only good in arrays like memory arrays where position dependence does not make a difference and the introduction of spare rows and columns is good only for those elements at the edges. What is needed is another concept called local redundancy [35], which is useful in sensor array where the same environment can be brought to all the redundant elements. In the case of the thesis, the approach for redundancy will be that of local redundancy.

In order to solve this problem, Chapman and Audet introduced redundancy in Active Pixel Sensors (APS) and then simulated them [38][39]. Subsequently, Koren and Chapman showed that combining hardware fault-tolerance with software correction led to Active Pixel Sensor array that could be made virtually immune to defects [40][41].

To achieve redundancy we split the photodiode (PD) and readout transistor (M2) into two parallel segments (PD.a, PD.b, M2.a, and M2.b) where the outputs are summed together to create a more reliable pixel. Figure 26 shows the schematic of a fault-tolerant photodiode while Figure 27 shows the layout of the APS in which the reset transistor is duplicated (M1.a and M1.b). The duplicating of reset and row select transistors are optional but this would increase the defect tolerance of the pixel. In the presence of defects, as has been explained in Section 1.2.1 where one of the two sub-pixels fails, roughly half of the signal can still be detected. Thus multiplying by factor of two would result in the full signal [38][39]. In this thesis, the emphasis is on redundancy for
photodiode APS. Redundancy for the photogate APS has been investigated by D. Cheung [46].

Figure 26: Schematic of fault-tolerant photodiode-based APS

Figure 27 shows the layout design of the fault-tolerant photodioide APS. Figure 28 is the picture taken with an optical microscope after fabrication.
The photodiode-based fault-tolerant APS is based on current summing from the two branches and therefore does not require a biasing transistor. A standard APS requires biasing of the source follower but in the case of the FT-APS, a summing of the currents has been used and therefore does not require biasing. One can use a suitably sized resistor for doing the readout or a summing amplifier. The summing is obtained by the addition of the currents from the transistor M2.a and transistor M2.b into a node, which in this
case is the summing amplifier. If one side is stuck high or stuck low and the other side is not defective, which could be found through dark field illuminations and light field illuminations respectively, the half-defective pixel could still operate. A simple multiplication factor of two will render the pixel functional again, which does not cost a lot as a left shift one time in the digital domain is a very cheap operation.

The probability that both sides of the fault-tolerant APS would be defective at the same time would be very small. If for example, the probability of defect on half-pixel is \( p = 0.01\% \). Then assuming the probability of the two sides of the half-pixel failing are independent events in the first order approximation. Thus using standard binominal probability, the combined probability of both half-pixels failing is

\[
P_{\text{both}} = p^2 = 0.0001 \times 0.0001 = 10^{-6}\%
\]  

Typically defect rate is in the order of 1 per megapixel. Assume that there are 10 defects on average in a 10 megapixel array. These 10 defects are real and can be very obvious under high uniform illumination conditions, however, with the introduction of FT-APS into this 10 megapixel array, assuming that these 10 defects are not very close to each other, or if they are close, they cluster together into one half-pixel, then the number of defective pixels is nearly 0. This illustrates the usefulness of this idea in terms of reduction in defect count. However, if both sides do have defects, software correction methods could compensate, for example, by using interpolation [41].

The disadvantage of the fault-tolerant APS is that the fill factor is less because of more metal interconnects that are required as well as having twice the number of transistors. However most of a typical APS pixel area is taken by the photodiode or the row/column lines, and transistors are much smaller. In the thesis, since no effort has been
made to make sure that the standard APS designs and the fault-tolerant APS designs are of the same size, and no effort has been made to make the design extremely compact which would take a lot of time, fill factor comparisons cannot be made. However, previous studies have shown that the reduction factor in fill factor is approximately 5% [35].

3.3.2 Simulation

Simulation of the implemented fault-tolerant photodiode APS has been carried out using Hspice in TSMC 0.18 μm technology. To simulate the integration operation, the photodiode is substituted with a suitable model. A simulation model for the photodiode is proposed by Swe and Yeo in [42]. The model is shown in Figure 29,

![Photodiode model](image)

Figure 29: Photodiode model

where $I_p$ is photocurrent generated by the photodiode, $C_J$ is the junction capacitance, $R_J$ is the junction resistance, and $R_S$ is the series resistance. The photocurrent per area is calculated by the following equation [25]:

\[
I_p = \frac{1}{\frac{1}{I_{ph}} + \frac{1}{I_{ph}} + \frac{1}{I_{ph}}}
\]
\[ I = \frac{q \eta P}{h \nu} = \frac{q \eta P}{hc / \lambda} \]  \tag{14}

where,

\[ I = \text{photo current per area} \]

\[ q = \text{electronic charge} = 1.602 \times 10^{-19} \text{C} \]

\[ \eta = \text{quantum efficiency} \]

\[ P = \text{optical power} \]

\[ h = \text{Planck's constant} = 6.6262 \times 10^{-34} \text{Js} \]

\[ \nu = \text{frequency} \]

\[ c = \text{speed of light} = 3 \times 10^8 \text{m/s} \]

\[ \lambda = \text{wavelength} \]

The junction capacitance, \( C_j \), is the parasitic capacitance of the photodiode. This depletion capacitance changes as the voltage at the cathode changes during integration period. From [29], the depletion capacitance of a photodiode is:

\[ C_{jdisp} = \frac{c_j \cdot A_D}{1 + \frac{V_{obs}}{pb} \cdot m_j} + \frac{c_{jsw} \cdot P_D}{1 + \frac{V_{obs}}{pbsw} \cdot m_{jsw}} \]  \tag{15}

where,

\[ c_j = \text{zero-bias depletion capacitance} \]

\[ c_{jsw} = \text{sidewall zero-bias depletion capacitance} \]

\[ A_D = \text{Area of diode} \]

\[ P_D = \text{Periphery of diode} \]

\[ V_{obs} = \text{Voltage across diode} \]

\[ pb = \text{built-in potential} \]

\[ m_j = \text{grading coefficient} \]

\[ pbsw = \text{built-in potential of the sidewall} \]

\[ m_{jsw} = \text{grading coefficient of the sidewall} \]
Typical capacitance value of a photodiode of size $10\mu m \times 10\mu m$ in CMOS 0.18 micron technology is approximately $7\ fF$. The series resistance, $R_s$, consists of the resistance of the non-depleted silicon and contacts resistance [25] and it is:

$$R_s = \frac{(W_s - W_d)\rho}{A} + R_c$$  \hspace{1cm} (16)

The junction resistance, $R_J$, varies with the current of the photodiode [25]:

$$R_J = \frac{nkT}{qJA_J} = \frac{nkT}{ql}$$ \hspace{1cm} (17)

Before interpreting the results of the simulation, it might be useful to consider how the output signal of the FT-APS or the APS in general for that matter, is obtained. Referring to Figure 30, the output signal of the FT-APS or APS in general is derived through the subtraction of the output reset voltage and the readout of the integrated signal for the same integration cycle.

Normal operation was simulated by applying integration on both sub-pixels. For the optically stuck low case, an electrical stuck high is applied at the gate of the readout transistor. Whereas for the optically stuck high case, an electrical stuck low is applied at the gate of the readout transistor. In both cases, the other sub-pixel is subjected to the normal integration operation. Using the method of obtaining the APS output in Figure 30, one can see through Figure 31, the result was that the output voltage drop for stuck low or stuck high, is half of the normal operation, as expected.
3.4 Dual Output Photogate-Based APS (D-APS) for Laser Profilometry

There are solutions to problems of current scanning technologies as stated in Section 1.1.2 that others have tried. One attempted solution was to use colored filters that would remove most of the background light but this does not effectively address the high
reflectivity of the shiny spot. Another solution involves thresholding, which basically sets a lower and upper limit on what could be detected, in order to remove the effects of shiny spots as well as dark spots, but since the scene is typically not well controlled and the object to be measured typically has widely varying reflectivity, this method is impractical. A third solution that others have used was to detect transition. This method is not very useful because when reflective and dark areas of the reflected laser beam coincide and sum or subtract, it might appear that “transitions” have appeared when in fact, there were none. Yet another attempt to eliminate the problem is to use different widths for the laser spots. However, this is quite difficult and depending on the reflectivity of the spots on the material to be scanned, the widths might have been drastically changed by the time they reach the detector.

To address the first of these problems, a method of encoding the laser pulse at a particular frequency has been proposed. Thus one can differentiate the uniquely encoded laser pulse laser sources that are present from the unencoded background light. This can be seen in Figure 32 where the frequency encoding of the laser means that the laser is pulsed at a certain frequency on and off. So in time 1, the laser pulse is on, and in time 2, the laser pulse is off. When the laser pulse is on, the laser intensity is added on top of the reflected intensity, while when the laser is off, the reflected intensity from the object is the only light intensity present. A subtraction of the light integration results in time 1 from that of time 2 reveals the presence of the laser spot. If the laser beams are encoded differently, then they can be separated from the ambient light and from each other, thus marking each laser beam as coming from a specific laser source which has a known laser location. Secondly, to address the issue of local surface imperfections, one can introduce
the idea of locality of processing at the pixel level so that local effects are locally processed.

Figure 32: Timing diagram illustrating the reflected light level from an object during the ON phase and OFF phase of a cycle

3.4.1 The Dual-Output Photogate-Based APS (D-APS) for Laser Profilometry

Laser profilometers are machines that are used to measure the distance of all points on scanned objects to create a profile of the item. From these “point clouds”, the scanned object is then reconstructed by using 3D polygonal meshes, thus creating a 3D surface of the object. This thesis will concentrate on the detectors needed for these laser profilometers and a type of detector called the Dual-Output Photogate-Based APS (D-APS) has been proposed.

The thesis proposes the design of an APS to create ambient light elimination close to the pixel level and phase sensitive detection of laser encoded frequency light. The dual-output APS (D-APS) has been designed to effect this local processing and is shown in Figure 33. This design in and by itself does not show how to locate the laser beams irrespective of the illumination conditions or the reflective surface conditions. Instead,
the key is to operate this design with certain timing parameters of the transfer gate of the photogate APS. The timing needed for a simple ambient light elimination is quite different from that of phase sensitive detection. The timing for phase sensitive detection is more complicated and this will be explained in greater detail in the next section, Section 3.4.2.

Figure 33: Cross section of the dual-output photogate-based APS
It can be seen from Figure 33 that this is a modified version of the standard photogate APS. Essentially, there is only one photogate and two transfer gates and output transistors. The reset gates shares the same reset signal, even though there are two
physical reset transistors. There is an n+ diffusion between the PG and each one of the TX gates. The TX gates are between the n+ diffusion and the floating diffusion. The floating diffusion consists of a p-n junction for each side.

3.4.2 Intuitive Idea of Operation of the Dual-Output Photogate-Based APS

The basic concept of the operation of the D-APS is to have a pixel with 2 separately integrated outputs. In its simplest form, by controlling the pixel integration in synchronization with the laser being on and off it is possible to separate the laser signal from all background or other illuminations by the subtraction of the integrated outputs when the laser is on and when it is off as shown in Figure 32. The operation of the dual-output photogate APS can be seen in Figure 35. The reason a dual-output photogate APS was used instead of a dual-output photodiode APS is because of the presence of floating diffusions where charges can be summed up, whereas the photodiode APS does not have storage nodes for the summation of charge. This allows us to vary several parameters to be adjusted (1) number of integrations for each reset cycle, (2) reset pulse width, (3) photogate pulse width and (4) transfer pulse width as well as their corresponding voltages, namely the (5) reset pulse voltage, (6) photogate pulse voltage and (7) transfer pulse voltage. As will be shown in Chapter 6, this enables a phase sensitive detection of the laser light signal.

Figure 36 shows the of the operation of the dual-output photogate APS across one integration cycle using by means of the potential wells and transfer of charges.

Integration 1 and Integration 2 refer to the same signal, the PG signal, but has been differentiated because at the end of Integration 1, TX.1 is activated and at the end of Integration 2, TX.2 is activated. This means that after Integration 1, the photoelectrons
are moved to side 1 of the dual-output photogate APS and conversely for Integration 2 into side 2.

Within each reset cycle, assuming \( n \) number of integrations, the number of PG pulses is \( 2n \) before the next reset occurs, while there are \( n \) number of TX.1 and TX.2 each alternately activated in each reset cycle.

![Figure 35: Operation of Dual-Output Photogate APS (only the timing and phase relationship is important here, the amplitudes are not to scale)](image)

In this way, the sum of the foreground light intensity-induced photoelectrons and background light intensity can be collected in one output diffusion and the background light intensity-induced photoelectrons can be collected in another output diffusion. A subtraction of the two outputs is done to obtain \( D \), the laser or foreground photoelectron voltage, thus effectively eliminating background photoelectrons and thus illumination. This is done conveniently by means of an op-amp in a difference configuration.
Figure 36: Typical semiconductor operation of dual-output photogate APS across one TX.1 transfer and one TX.2 transfer or equivalently across one integration cycle
3.5 Design and Implementation of APS Chips

The APS chips are test platforms for the experimental APS designs. In the course of this thesis, three APS chips have been fabricated. Each chip is about 1.5 mm × 1.5 mm and has been fabricated through Canadian Microelectronic Corporation in TSMC 0.18 μm technology. The chips have been designed using Cadence Virtuoso Layout tool and the digital parts have been synthesized using Synopsys tools. A block diagram of a representative APS chip design is shown in Figure 37.

![Figure 37: Block diagram of a representative APS chip design](image)

These chips generally contain a 6-to-64 row decoder, a 3-to-8 column decoder, an APS array, I/O ring, I/O pads and power supply pads. Also included are independent standalone APS cells that are bonded to I/O pads inside the chip itself so that if there is anything wrong with the bonding or the I/O ring design and layout of the power supplies then the entire chip will not be wasted as these standalone APS then can be micro-probed. There are also alignment crosses at the corners of the chips to assist machine alignment if necessary.
Figure 38 and Figure 39 are meant as examples of the photomicrograph and layout of the general experimental APS chips. The experimental APS cells of interest are the fault-tolerant photodiode-based APS (FT-APS) and the dual-output photogate-based APS (D-APS), although the experimental APS chips contain many other different types of APS cells, for example, various shapes of photodiodes, various shapes of photogates, classical photodiode APS, classical photogate APS, dual-output photodiode APS, dual-output photogate APS and photodiodes made from n+/p-sub and n-well/p-sub junctions.

![Photomicrograph of ICFSFSD1, the first experimental APS chip (1.5×1.5 mm)](image-url)
Figure 39: Layout of ICFSFSD2, the second experimental APS chip (1.5×1.5 mm)

3.6 Conclusion

The fault-tolerant photodiode-based APS (FT-APS) has been described as a method to improve yield while not reducing the performance of the sensor. Furthermore, the operation has been described and simulation performed. These are just theoretical simulations but the experimental setup will be described in Chapter 4 and the results of the experiment on FT-APS will be described in Chapter 5.

The dual-output photogate-based APS has been discussed as a viable solution to address current limitations on scanning technology. Its theoretical operation has been described and simulations have been performed to prove its viability. The next step is to verify functionality on real silicon. The experimental setup for this verification process will be detailed in Chapter 4 and the experimental results will be given in Chapter 7.
The experimental platform in which these experimental APS has been built has also been described. There were three chips that had been fabricated for the experiments. Unfortunately, there was only time to test two of them, and the other one has been left as an experimental platform for future work.

In the next chapter, the experimental setup for this thesis will be described.
CHAPTER 4: EXPERIMENTAL SETUP

The experimental setup is concerned with testing individual pixels of the new designs, instead of a pixel array. This is because we would like to characterize the performance of the FT-APS and D-APS pixels to see whether they will serve the functionality of the design it has been intended for. Further work would be required after verification of the experimental APS for entire arrays of such pixels.

4.1 Introduction to Experimental Laser Setup

To test the Fault Tolerant APS we need to simulate faults in one side of the pixel while allowing the other to operate properly. The method chosen in this work to do this is by using a combination of overall illumination and a focused laser beam to adjust light levels on one pixel half relative to the other. With this, both stuck lows and stuck high conditions can be created, and the pixel behaviour characterized.

The initial experiments tested the normal (defect free) operation of the fault-tolerant APS by focusing a laser spot on the center of the correct sensor as shown in Figure 40, creating equal amounts of illumination on each sub-pixel. Then measurements of pixel output versus illumination power density were taken over about two decades of light intensity. The illumination levels are kept within the APS’ linear operation region.

To test a sub-pixel that is optically stuck low, only one-half of the fault-tolerant APS is illuminated with the laser spot over a range of powers while the other side is in the dark as shown in Figure 40. This is possible because the size of each sub-pixel is approximately $5 \, \mu m \times 10 \, \mu m$, whereas the laser spot is approximately $2.5 \, \mu m$ in
diameter. Very little signal was observed on adjacent APS' even at saturation conditions on the illuminated pixel showing little crosstalk among the APS. To create an optically stuck high condition, one-half of the redundant APS is illuminated with laser at an intensity that just saturates that sub-pixel, while the entire pixel is uniformly illuminated with the microscope light. This creates the same effect as if one of the gates of the output transistor was grounded (or the transistor is not functioning).

To create an optically stuck high sub-pixel, the laser spot is positioned within that sub-pixel and it is ensured that the sub-pixel reaches saturation, while the microscope light is then used as a source of illumination for the other sub-pixel of the same FT-APS as shown in Figure 40. The microscope light is then varied from dark to the point of saturation of the non-defective sub-pixel.

Figure 40: Laser position in a fault-tolerant pixel for (a) normal pixel and half stuck low (b) normal pixel and half stuck high

4.2 Laser Table Setup for Optical Exposure of APS

The nature of the Fault Tolerant APS testing is that we need to control the exposure level within each half of the pixel. The laser table setup enables this research to effectively control the illumination on individual pixels, or to a small area of the pixel, without illuminating any other pixels. This is important, for example, in testing the fault-tolerant APS, as only one half of the pixel needs to be illuminated and the other needs to
be either in flood illumination for simulating optically stuck high pixels or in the dark to simulate optically stuck low pixels. Also because of the precise location and size of the laser spot, one can also study the effects of crosstalk by illuminating one pixel and observing the outputs of the adjacent pixels. This gives an idea of how much electrical crosstalk there might be between adjacent pixels.

![Schematic diagram of the experimental setup](image)

**Figure 41: Laser table setup shows argon laser is focused on sample after Tu [47]**

A schematic view of the entire experimental setup is shown in Figure 41. The actual experimental setup is shown in Figure 42 while a more closeup of the APS chip holder and lens system is illustrated in Figure 43. As can be seen from this the setup contains an x-y table and a z-table on top of an anti-vibration table. There is an argon laser that is firmly attached to this table with an optical bench for the mirrors and lens to direct and focus the light and laser onto the target. Using objective lenses ranging from 1x to 50x, a laser spot can be focused to a range of sizes, from 2μm to 100μm to cover a single
pixel to an array of pixels. Aligning the laser beam is done manually by observing the pixel using the TV camera system on a microscope. As shown in Figure 41, there is also a light source that is attached to the microscope that is controllable by the light control. This setup is primarily used in testing the fault-tolerant APS. In order to create different illuminations for different sections of the chips, one can use a combination of the laser light and the microscope light. There is a CCD camera attached to the microscope that is aimed at the pixel on the APS chip, where the field of view can be seen on a monitor because of a link between the camera and a PC. This PC also has the controller software for the x-y table and the z-table. The microscope light can be used for background illumination, while the laser can be used for the laser illumination. The microscope light can be varied from dark to bright to allow for different background elimination.

Figure 42: Laser XYZ table for optical testing of APS

Figure 43: Close-up picture of APS chip under focused laser testing
4.3 Electrical Measurement Setup

The APS chips that have been designed are mixed-signal designs that incorporate digital control for reading out the analog pixel values. In order to control the readout of the APS chips, one has to control the row decoder and column decoder (see Section 3.5), which ensures that the output of the APS in the right location is then passed into the appropriate output pads. It is very important that the power-up sequence of the chips be in the right order. It has been found that that the column decoder has to be selected before the row decoder. The reason for this is that the groups of APS columns are powered up by the column decoder. Assuming that the row decoder is powered up first, this means that the selected APS columns have not been powered on. The row decoder enables the row select gates of the selected APS columns whereupon the power up of the APS, a huge current spike surges through between the power supply and the current sink, burning some transistors in the path. On top of that, one has to synchronize the readout sequence of the pixels with the analog pixel output, so that the appropriate output can be captured at the right time, based on the appropriate triggering sequence of events.
<table>
<thead>
<tr>
<th>Signal</th>
<th>Input/Output</th>
<th>Analog/Digital</th>
<th>Number of lines</th>
</tr>
</thead>
<tbody>
<tr>
<td>Row select</td>
<td>Input</td>
<td>Digital</td>
<td>5</td>
</tr>
<tr>
<td>Column select</td>
<td>Input</td>
<td>Digital</td>
<td>3</td>
</tr>
<tr>
<td>Reset.1</td>
<td>Input</td>
<td>Digital</td>
<td>1</td>
</tr>
<tr>
<td>Reset.2</td>
<td>Input</td>
<td>Digital</td>
<td>1</td>
</tr>
<tr>
<td>TX.1</td>
<td>Input</td>
<td>Digital</td>
<td>1</td>
</tr>
<tr>
<td>TX.2</td>
<td>Input</td>
<td>Digital</td>
<td>1</td>
</tr>
<tr>
<td>Row Select.1</td>
<td>Input</td>
<td>Digital</td>
<td>1</td>
</tr>
<tr>
<td>Row Select.2</td>
<td>Input</td>
<td>Digital</td>
<td>1</td>
</tr>
<tr>
<td>Power</td>
<td>Input</td>
<td>Analog</td>
<td>1</td>
</tr>
<tr>
<td>APS Outputs</td>
<td>Output</td>
<td>Analog</td>
<td>8</td>
</tr>
</tbody>
</table>

The APS chips have been bonded into a 40-pin DIP package, which has then been mounted on a breadboard with the appropriate wiring as shown in Figure 44. When used with the laser system this breadboard is mounted on the laser table so measurements can be made while it is illuminated by the laser. To reduce the potential crosstalk and inductive coupling to a minimum, a universal connector connects the breadboard wiring to a terminal block that is then connected to two National Instruments data acquisition cards, the PCI6024E and PCI6713, which are inside the control PC. The electrical aspect of the experimental setup for testing of APS chips is shown in Figure 45.

The PC is installed with LabVIEW software that controls the reset of the APSs', the column and row decoder, as well as the power supply to the APS chips. The output of the APS chip is directed towards a Tek TDS3000B series digital oscilloscope. The results
of the APS output are then displayed on the oscilloscope and then is downloaded onto the computer in csv format, which is then processed using Microsoft Excel spreadsheet.

**Figure 44: Wiring of experimental setup for testing of APS chips after B. Wang and G. Liaw [48]**

![Wiring Diagram](image)

**Figure 45: Electrical aspect of experimental setup for testing of APS chips**

4.4 Software Control of APS Measurements

Instead of just the timing sequence of the decoders, there are timing sequences that are very critical to the operation of the APS in general. This involves the more intricate timing and voltages of the following signals, photogate signal, reset signal,
transfer signal and readout or row select signal. This controls the exposure of the individual pixel.

Referring to Figure 46, first, the individual pixel is selected by choosing the correct inputs for the row decoder and column decoder. Then in this particular experimental setup because of the fact that sample-and-hold circuits are not included in the design, the row select transistor is switched on so that the output can be observed. A photogate signal (B) is issued to create a potential well for photoelectron collection for a
time referred to as the integration time, while at the same time, a reset pulse (A) is issued, precharging the floating diffusion and gate of the source follower. The effect of the reset signal on the output can be observed through the output of the appropriate pads, and after the integration time desired controlled by the program has passed, the transfer gate (D) is turned on, transferring the background and the foreground photoelectrons to the floating diffusion for the readout. This marks the end of the first half of the integration cycle. The second half of the integration cycle then starts with the photogate signal (C) being applied, this time integrating the background photoelectrons only, which then get transferred by a second transfer pulse (E). After the second transfer pulse, the integration cycle of the D-APS ends. The readout or data acquisition occurs during this transfer. Several parameters are of interest, namely when each of the photogate, reset, transfer pulses are issued, the duration of the pulse and the voltages of the pulse. All of this affects the output of the individual pixels, although this will not be investigated in great details as optimizing the performance of the pixels is not in the scope of this thesis.

In order to control the experimental setup, a graphical programming language called LabVIEW from National Instruments is used. This provides a front-end GUI as well as acquisition logic and control logic. A screenshot of the LabVIEW GUI can be seen in Figure 47. This is similar to software used for testing the fault-tolerant APS. Most of this experimental setup had been built by two colleagues, Benjamin Wang and Gary Liaw [48]. Two acquisition cards are used because of bandwidth limitations. This acquisition program has been written primarily by Dr. Chinheng Choo, who has kindly and generously donated his expertise for our project.
4.5 Optical Testing for the Fault-Tolerant APS

The basic concept of testing the fault-tolerant photodiode-based APS is to separately control the illumination on one sub-pixel of the redundant APS relative to the other sub-pixel as shown in Figure 40. In this way, it is possible to create the effect of both optically stuck low and stuck high conditions. This is accomplished by using a combination of a focused argon laser source and a field illumination light source. On a computer-controlled submicron X-Y-Z positioning table with an accuracy of 0.05 μm [44] with vibration isolation, a mirror and microscope lens system directs the laser beam to focus onto the correct APS. The laser beam is directed using a 50× objective lens, creating a laser spot approximately 2.0μm in diameter allowing the illumination to be confined within a fraction of the sub-pixel. The size of the sub-pixel is 2 μm × 12 μm. Aligning the laser spot is done manually by observing the pixel using the TV camera system on the same microscope. The microscope light itself provides a controllable uniform illumination across the sub-pixels. This combination of laser and microscope light allows for the generation of any illumination from dark to saturating light levels separately on each sub-pixel. The illumination source is an Argon laser that has a wavelength of 514 nm (bluish-green) made by Coherent Inc.
Electronically, the test setup must first address the correct APS, the fault-tolerant photodiode APS being one of the many sensors on the chip, and control the timing signal to obtain readout of the device. As shown in Figure 45 and Figure 47, LabVIEW software is used for controlling the row/column addressing and reset/readout timing for the desired APS. An op-amp TLC2274ACD, acting as a current-to-voltage converter, sums the currents from the two sub-pixels. The resulting signals are acquired using the Tektronix TDS2014 digital oscilloscope. What is commonly done is that the APS is read out only after an integration cycle.

4.6 Optical Testing for the Dual-Output APS

For testing the dual-output APS, the laser is not used as a flood illumination of the entire chip area, not just one pixel, is needed. The laser system is designed to illuminate a single pixel at a time, with a stable illumination level. Furthermore, the lab in which the experiments were carried out is shared by a number of graduate students and equipment shortage made a separate LED setup absolutely necessary. The setup for testing the dual-output APS is much simpler and uses an LED that is attached to a fixture which ensures it is in the dark and just directly above the APS chip being tested. This involves using a function generator to drive the LED for simulation of the foreground (laser) and background (ambient) illumination. To ensure that the APS chip is not disturbed by other light sources, the APS chip is placed inside a wooden cavity, where the LED is fitted into a drilled hole on the wooden block (see Figure 48). To ensure that the LED illumination is reasonably constant, the light cone of the LED has been positioned with reasonable
care such that the center of the light cone is optically aligned with the center of the APS chip.

Figure 48: Dual-output experimental APS setup after D. Cheung [33]

The purpose of testing the dual-output APS is to investigate the how much difference in light intensity can be distinguished by the dual-output photogate APS. To this end, there is an added advantage in the LED experimental setup in that it is relatively simple to superimpose the background and foreground light onto the same LED just for the purpose of simulating laser in the presence of ambient light. The superposition is done with the help of a function generator where the offset voltage is the background illumination and the foreground illumination is a rectangular wave riding on top of the offset. In reality, the laser and the ambient light sources are separate. The dominant frequencies of the environment can be characterized and then the integration frequency of the D-APS can be adjusted so as to maximize the elimination of the dominant mode of background illumination frequencies. For example, in a setting where the dominant mode
of illumination is powered by 60 Hz AC, the dominant background frequency is 120 Hz, due to half-wave rectification of the AC power.

The LED calibration setup and specifications are shown in Figure 49 and respectively. The characterization results of the LED are shown in Figure 50. This is used for controlling the light intensity by using a function generator’s output voltage. This will give us an idea of how the relationship of the LED light intensity varies with voltage applied. The calibration setup of the LED is done to mimic the way the experimental setup is made. There is a 1K resistor in series with the LED as shown in Figure 49.

![Figure 49: Calibration Setup for LED](image)

<table>
<thead>
<tr>
<th>Part No.</th>
<th>LTL-4221N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lens</td>
<td>Red Diffused</td>
</tr>
<tr>
<td>Source Color</td>
<td>Hi-Eff Red</td>
</tr>
<tr>
<td>Peak Emission</td>
<td>635 nm</td>
</tr>
<tr>
<td>Wavelength</td>
<td></td>
</tr>
<tr>
<td>Spectral Line</td>
<td>40 nm</td>
</tr>
<tr>
<td>Half-Width</td>
<td></td>
</tr>
</tbody>
</table>

It should be noted that LED’s are manufactured with significant tolerances and that these light intensity values should be taken as typical and not exact. The light intensity values are guaranteed to be within ±15% tolerance. The characteristic of the LED is taken as the most accurate approximation possible and the values have been used with such an understanding. Figure 50 shows the actual measured characteristics of the LED using the setup shown in Figure 49. 128 measurements of the light intensity were taken and averaged.
The amplitudes of the background light intensity are chosen to be a fraction of the foreground light intensity as shown in Error! Reference source not found. Let this ratio be denoted as \( I_{\text{ratio}} \). Two background light intensities are used, namely 2 \( \mu \text{W} \) and 20 \( \mu \text{W} \), depending on the integration time. The integration time is fixed by the frequency of the reset cycle. The frequencies used are 100 Hz and 1 kHz respectively. The fraction is denoted as the ratio of the background light intensity to that of the foreground and is used to simulate signal-to-noise ratio. For example, a fraction of \( \frac{1}{2} \) when the background light intensity is 2 \( \mu \text{W} \), imply that the foreground laser intensity used is 1 \( \mu \text{W} \) and so on.

Figure 51: LED voltages used to simulate combination of background light and laser

The zero fraction is used as a form of control where there is supposed to be zero background light illumination and no foreground illumination. This is the simple case...
which needs to be checked if we are to be sure everything works as expected. Progressively, fractions of 0.5, 0.833, 0.933, 0.9633 and 1 are used (see Table 3). Figure 51 shows the LED voltage waveform that is used to simulate a combination of background light and laser. In this case, the fraction of background LED light intensity to foreground LED light intensity is 1. Note that the fractions are not linearly increased but is somewhat chosen to “logarithmically” increase. This is because the 0.5 fraction is used to check for gross functionality while the higher fractions are chosen to see whether the dual-output APS can detect foreground up to the point where the background is of the same intensity as the foreground. This is of much more concern at the higher fractions where the signal-to-noise ratio is lower.

Table 3: The background intensity which is expressed as a fraction of the foreground intensity

<table>
<thead>
<tr>
<th>Fraction of Background LED Light Intensity to Foreground LED Light Intensity (I_{ratio})</th>
<th>Background Light Intensity Used (nW) When Foreground Light Intensity Used is 2000 nW</th>
<th>Background Light Intensity Used = 20000 nW</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>\frac{1}{2}=0.5</td>
<td>1000</td>
<td>10000</td>
</tr>
<tr>
<td>\frac{1}{2}+\frac{1}{3}=\frac{5}{6}=0.833</td>
<td>1700</td>
<td>17000</td>
</tr>
<tr>
<td>\frac{1}{2}+\frac{1}{3}+\frac{1}{10}=0.933</td>
<td>1900</td>
<td>19000</td>
</tr>
<tr>
<td>\frac{1}{2}+\frac{1}{3}+\frac{1}{10}+\frac{3}{100}=0.9633</td>
<td>1930</td>
<td>19300</td>
</tr>
<tr>
<td>1</td>
<td>2000</td>
<td>20000</td>
</tr>
</tbody>
</table>

4.7 Conclusion

This chapter has discussed the experimental setup for testing FT-APS and D-APS. The laser table setup has allowed for the control of illumination of sub-pixels of FT-APS,
while the electrical setup controls the power-up sequence of the FT-APS. The software controls perform mainly timing of the reset pulses, while the optical setup simulates optically stuck high and optically stuck low conditions for testing the viability of the FT-APS.

For the D-APS, the electrical setup allows the safe power-up of the pixels, while the software controls mainly the timing of the photogate, reset and transfer pulses. Optically, an LED broad illumination is used for illumination purposes where both the foreground and background are superimposed by a function generator onto the LED. This allows for testing the viability of rejecting background illumination for the D-APS.

Chapter 5 and Chapter 6 discuss the experimental results for the FT-APS and D-APS respectively.

In particular, Chapter 5 shows that the FT-APS can in fact withstand optically stuck high and optically stuck low faults to give half of the functionality of the entire FT-APS when only half of the pixel is functioning. It is also shown that the signal can be recovered reasonably well with a multiplication factor of 2.

Chapter 6 shows an analysis of how D-APS allows rejection of background light in favour of laser of the desired frequency.
CHAPTER 5: EXPERIMENTAL RESULTS OF FAULT-TOLERANT PHOTODIODE-BASED APS

The fault-tolerant photodiode-based APS has been designed to compensate for defects that develop in the field and to increase manufacturing yields and thus decrease the cost of producing APS cameras in general. It is the purpose of this chapter to investigate the viability of this design.

The testing of the FT-APS in this thesis involves injection by optical simulation: two fault types: optically stuck high and optically stuck low. While these defects are injected, the optical response of the entire FT-APS is studied by varying the illumination on the other sub-pixel where the defect is not introduced. By measuring the optical response of the combined working and defective sub-pixels this section creates equations that allow us to recover the actual response of defect free pixel.

5.1 Typical Experimental Results of Fault-Tolerant APS

Chapter 4 described in detail the experimental setup required for testing the FT-APS on the laser XYZ table. In order to simulate the expected faults, we will create optically stuck-low sub-pixels in some tests and optically stuck-high sub-pixels in others, as has been discussed in Section 4.5.

A typical experimental result from the oscilloscope is shown in, Figure 52 where the saturation illumination results for the entire defect free pixel is shown. The reset pulse and the pixel output during the entire integration period are observed to confirm correct operation. This is similar to the simulation results in Figure 31, with the output
voltage polarity reversed. There are three distinct output stages: during the reset period (in Figure 52 0-0.2 msec) the output gate/photodiode becomes fully charged, in the linear integration period (in Figure 52 0.2-0.8 msec) where the signal is accumulated, and, under high illumination conditions, when the APS goes into saturation (in Figure 52 0.8-1 msec). It can be seen that the output is quite linear for most of the integration region before the onset of saturation starts to take effect. This also illustrates the illumination setting and operation for the stuck high condition.

![Figure 52: Fault tolerant APS timing cycle, reset (top) and output of FT-APS (bottom) to optical exposure with respect to time](image)

**5.2 Optically Induced Defects in Fault-Tolerant APS**

To study the optical response of the FT-APS, it has to be subjected to the following optical tests, (1) testing under regular illumination, (2) tests with one sub-pixel that is optically stuck high while the other sub-pixel is functional, and (3) tests with one sub-pixel that is optically stuck low while the other sub-pixel is functional. The experimental results are then compared to results that are obtained using the classical photodiode APS. This is used to validate that the FT-APS operates in normal conditions and when one sub-pixel is defective compared to the more well-known results of classical
photodiode APS that are readily available in many papers such as that shown in [7].

There are two ways to induce or simulate the defects that might be present in the APS and in particular the FT-APS. The first one which is the simpler of the two methods involves electrical simulation where the gates of one of the source follower of the APS is tied low (optically stuck high case) and where the gates of one of the source follower of the APS is tied high (optically stuck low case). The disadvantages of the method of electrically simulating the faults are that they involve separate pixel designs, where the optically stuck high pixel and the optically stuck low pixel need 2 different designs and that make permanent changes to the pixel so that it cannot be used for any other purposes other than the optically stuck test.

The second one, which is used in this thesis, involves optical simulation. In the optically stuck high case, the microscope light is illuminated on the entire pixel such that the entire output of the FT-APS approaches that of half of the saturated output, which has been determined beforehand by characterization of the FT-APS with differing light intensities. The characterization of the entire FT-APS has been done by using laser directed to the center of the whole FT-APS. This is made possible because whereas the entire FT-APS is 13 μm x 10 μm. One can position the laser as such by examining the output of the CCD camera and adjusting the x-y controller. The other sub-pixel has been characterized by using the laser to illuminate it. This is possible because the laser diameter is approximately 2 μm (refer to Figure 40), whereas each half-pixel is approximately 6 μm x 8 μm. One can position this laser spot quite precisely in the center of the sub-pixel by looking at the output of the CCD camera. In the optically stuck low case, the entire FT-APS is in the dark with the exception of the center of the sub-pixel.
which has been illuminated by the laser spot. Plotting the results of these tests gives in Figure 53 the output voltage versus illumination levels for normal APS operation, in Figure 54, the optically stuck low (one side dark) and in Figure 55 the optically stuck high condition (one side bright). My colleague, Desmond Chueng has investigated the fault-tolerant photodiode-based APS by using electrically-induced defects [33], giving similar results to those presented here. Using linear regression analysis, the linear regression equations and the slopes of the output voltage versus illumination intensity were measured at:

Linear regression equation for output voltage versus illumination intensity

Normal operation of redundant APS: $y = 0.0582x$

One sub-pixel stuck low operation of redundant APS: $y = 0.0288x$

One sub-pixel stuck high operation of redundant APS: $y = 0.0325x + 1.603$

Slope of output voltage versus illumination intensity (Note that the slope errors are the errors of the linear regression fit)

Normal operation of redundant APS: $0.112 \pm 0.0035 \text{ V/W/m}^2$

One sub-pixel stuck low operation of redundant APS: $0.064 \pm 0.0015 \text{ V/W/m}^2$

One sub-pixel stuck high operation of redundant APS: $0.045 \pm 0.0021 \text{ V/W/m}^2$

For the stuck low condition the sensitivity is $0.571 \pm 0.0313$ of that of the fully operational pixel, or 14% higher than the expected ratio of 0.5. This means the illuminated sub-pixel is showing more output than expected. In the stuck high condition, the ratio is $0.402 \pm 0.0313$, so the unstuck half is 20% less sensitive than expected.
Figure 53: Relation of output voltage to illumination intensity for normal pixels

Figure 54: Relation of output voltage to illumination intensity for optically stuck low pixels
Figure 55: Relation of output voltage to illumination intensity for optically stuck high pixels

Note that when using the optically stuck high condition, one concern comes to mind, in that the adjacent pixels are also illuminated. There might be concern with blooming, where the photoelectrons from one cell spill over to the other cell thus causing electrical crosstalk or even within the subpixels of the same FT-APS. This is a valid concern and had been addressed by an experiment where an FT-APS has been illuminated whereas those adjacent have not been illuminated. Under this condition, the APS that is adjacent to those illuminated showed no signs of non-negligible output. This shows that there are no signs of crosstalk. By the same token, crosstalk should not be a problem for subpixels of the same FT-APS. This should come as no surprise since the size of the FT-APS and subsequently the sub-pixels are relatively large at about 6 μm × 8 μm each and therefore has a much larger well capacity.
5.3 Analysis of Fault-Tolerant APS Optical Response

A summary of the experimental results showing sensitivity of the fault-tolerant photodiode-based APS can be shown in Table 4. The experiment tested 4 FT-APS pixels. It can be seen that the stuck high case (see Figure 55) is more sensitive than that of the stuck low case (see Figure 56). Also, there is a deviation from a multiplication of 2 that is needed for compensation. However, it is important to note that this deviation lies within the experimental errors and therefore suggests that the experimental accuracy needs to be improved in the future for more careful studies. Experimentally, the stuck low case requires a multiplication of 2.02 while the stuck high case requires a multiplication of 1.79. If further tests prove this correct it could make implementation more complex since multiplications other than 2 increase computational overhead. However imagers already have significant computational requirements so this effect may not be large.

Table 4: Summary of sensitivity of fault-tolerant photodiode-based APS

<table>
<thead>
<tr>
<th>Fault tolerant APS operating modes</th>
<th>Sensitivity (V/W/m²)</th>
<th>Sensitivity ratio of non-defective to single defect</th>
<th>Difference from expected value of 2</th>
<th>Expected error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-Defective (Normal)</td>
<td>0.0582 ± 0.00204</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Single Defect – Stuck Low</td>
<td>0.0288 ± 0.00100</td>
<td>2.02</td>
<td>0.02</td>
<td>0.141</td>
</tr>
<tr>
<td>Single Defect – Stuck High</td>
<td>0.0325 ± 0.00105</td>
<td>1.79</td>
<td>0.21</td>
<td>0.121</td>
</tr>
<tr>
<td>Difference between SH and SL Ratio</td>
<td>0.23</td>
<td>N/A</td>
<td>0.262</td>
<td></td>
</tr>
</tbody>
</table>

The error represents the uncertainties in both the illumination levels and the output measurements. Higher error occurs in the stuck high condition because two light sources are being used, each with their own margins of error. First, it is important to note
that under all three defect conditions, namely optically stuck high, optically stuck low and low sensitivity, the redundant APS is behaving linearly (after removing any background level).

The actual output voltage versus illumination power shown in Figure 56 (normal case), Figure 57 (stuck low case) and Figure 58 (stuck high case) on each sub-pixel is shown with the resulting slopes:

Linear regression equation for output voltage versus illumination power

Normal operation of redundant APS: \( y = 1.3855x \)

One sub-pixel stuck low operation of redundant APS: \( y = 1.3609x \)

One sub-pixel stuck high operation of redundant APS: \( y = 1.3613x + 2.1658 \)

Slope of output voltage versus illumination power (Note that the slope errors are the errors of the linear regression fit)

Normal operation of redundant APS: \( 1.39 \pm 0.043 \text{ V/nW} \)

One sub-pixel stuck low operation of redundant APS: \( 1.36 \pm 0.004 \text{ V/nW} \)

One sub-pixel stuck high operation of redundant APS: \( 1.36 \pm 0.004 \text{ V/nW} \)
Figure 56: Relation of output voltage to illumination power for normal pixels

Figure 57: Relation of output voltage to illumination power for optically stuck low pixels
Figure 58: Relation of output voltage to illumination power for optically stuck high pixels

The slopes for the 3 cases are approximately the same because when the illumination power is constant say 1 nW, then the number of photons generated are the same, and the number of electron-hole pairs generated are theoretically the same. Note how the normally operating redundant pixel is slightly less sensitive than the half-illuminated version. Under this condition, they should be identical suggesting that some of the photoelectrons are lost in the small space separating the two sub-pixels. This explains the more than 0.5 ratio value for the stuck low condition. The stuck high condition is still clearly showing an unexpected smaller slope, which is larger than the expected error and unexplained at this time. While the source of these differences from the expected values is being investigated, it is important to note that the redundant APS generates linear behavior under all conditions. Deviations in the slope can be easily compensated for by a calibration correction if they are predictable.
5.4 Conclusion

Experimental results of the operation of the redundant photodiode APS have been presented. The results show that the fault-tolerant photodiode APS works in practice, and most importantly, it behaves linearly under both normal and defective conditions. There are some deviations that are within the limits of experimental errors that the author believes may be attributed to errors in the experimental setup. However, the stuck high condition is less sensitive than was expected in the experiment that was carried out. This suggests that a multiplication by other than 2 as compensation for the optically stuck high case would add complexity and since there are complex image processing chips used, this difference might not be too bad. But one can say that the errors for the compensation by multiplication of 2 is within the experimental errors and more careful measurement methods would be required.

Future work will involve measuring the performance of more pixels and different designs to confirm both the linearity and to investigate the differences between the stuck high and low conditions in greater details. Also, a potentially more accurate way for determining the performance of the FT-APS would be to electrically cause an optical stuck high and optical stuck low on one side of the FT-APS in design, and this would reduce the uncertainties from the illumination sources, possibly reducing the amount of experimental errors, thus giving more accurate results. However, it should be noted that the faults that are caused by electrical simulation have their own problems. For example, for the defective pixels that are electrically shorted to either power supplies or grounds, there are race conditions that might be caused in the summing part of the circuitry. The advantage of the optical simulation of faults is that it does not require extra pixel design
but involves more elaborate experimental setup.

In the next chapter, we will discuss phase sensitivity theory for Dual-Output Photogate APS (D-APS), which would solve the problem of background discrimination in preference for foreground.
CHAPTER 6 : THEORY OF PHASE SENSITIVE OPERATION OF DUAL-OUTPUT PHOTOGATE APS (D-APS)

One of the targets of this thesis is the design and investigation of specialized pixels which detected and encoded laser beam on a surface against the background ambient light illumination. This chapter investigates the theory of an operational cycle that should turn the dual-output APS (D-APS), shown in Figure 59, into a phase sensitive optical detector.

Figure 59: A schematic view of dual-output photogate-based APS

6.1 Introduction to Theory and Simulation of Dual-Output Photogate APS (D-APS)

In section 3.4.2 the dual output APS (D-APS) was introduced with the concept of a simple two cycle operation, where one side integrates during a background light period, and the other during the laser illumination phase. This allows the subtraction of the background from the illumination. However the photogate D-APS allows multiple
integration cycles to be summed for each cycle before the readout. This chapter shows how this ability should enable phase sensitive detection where the encoded laser signal can be differentiated from a changing background light. For example if the background light is changing quickly, for example with a 60 Hz flicker as would occur with standard AC illumination, a simple subtraction may not be enough.

Figure 60: Laser signal modulated in a sinusoidal fashion in the time-domain

Figure 61: Laser signal modulated in a sinusoidal fashion in the time-domain but slightly out of phase with the integration cycle

Assume that the laser intensity is modulated in a sinusoidal fashion in the time-domain as shown in Figure 60 and Figure 61. This is a laser signal that has a dc offset the same size as the amplitude of the sinusoidal wave. In such a case, if the integration time is synchronized with half the period starting from when the sinusoidal wave is increasing from the zero value plus the dc offset, and an integration cycle is executed every half a period, then the difference of the sum of the even integrated cycle (integration cycle 0, 2
and so on) and the sum of the odd integrated cycle (integration cycle 1, 3 and so on) is the maximum as compared to the same sinusoidal wave when the integration cycle is out of phase (compare Figure 60 and Figure 61). Phase is the timing relationship between the start of the first detector integration cycle and the peak of the illumination source or laser source. Note that one can have one D-APS integration cycle which consists of subtracting results of integration cycle 0 and results of integration cycle 1 or one can have two D-APS integration cycles which correspond to subtraction of results of integration cycle 0 and 2 from that of the results of integration cycle 1 and 3. In general, one can have as many integration cycles as one like by extension of the above definition. This intuitive insight forms the basis of the proposed method. Translated into physical reality, this means that the modulated laser intensity is sampled by the detector which can be used to integrate on the even cycle and the odd cycle separately. When the detector is integrating the even and the odd cycle with a different phase than that of the sinusoidal-modulated laser, then the difference between the integration of the even cycle and the odd cycle, from now on, referred to as $D$, would be less than when detector and laser are in phase.

The definition of phase of 0, 45, 90, 135, 180, 225, 270 and 315 are given here as well as in Figure 62. Basically, phase of 0 means that the very first Integration 1 cycle is synchronized with the start of sinusoidal background intensity at 0°, whereas phase of 45 means that the very first Integration 1 is 45° out of phase with the 0° starting point of the sinusoidal background intensity and so on.

It should also be noted that for Figure 62, $n = 2$ and the background frequency is half of the foreground frequency. For the purpose of the experiment, the foreground
frequency used is 100 Hz and 1000 Hz, while the background frequency is correspondingly 50 Hz and 500 Hz respectively.

Figure 62: Foreground-Background phase relationship (amplitude is not to scale, the diagram is important for its phase and timing relationship only, the background frequency is half of the foreground frequency). The foreground light pulse has a duty cycle of 32.5%. The duty cycles of integration 1 and integration 2 are also 32.5%.
Finally, it is worthwhile to note that the phase and phi are not exactly related in the same way. Phase was a term that was used to describe the timing relationship between the start of the integration cycle and the start of the sinusoidal light intensity. Phi was a term that was used to describe the same thing except in simulations. Basically, the way to understand how to translate between phi and phase is that when the integration cycle starts at the peak of the sinusoidal light intensity, then even though phase is 90, phi is 0. The basic translation scheme between phase and phi is given by Figure 62 and in the formula which is stated as follow: phi = phase - 90 or phi = phase - 270. The maximum value of D occurs when phi = 0. It is also worthwhile to note that the laser encoding frequency can be synchronized with the integration clock but with an external light source, there is no explicit relationship between phase of laser and integration clock.

Instead of just attempting to detect the laser intensity, the detector would be phase sensitive given that the frequency of the encoding of the laser and the phase of the laser and integration clock is a priori known. Even though the signals that are used for simulation are sinusoidal, in reality, the laser is powered on and off in rectangular pulses. However, the same principles apply and the integration cycle is rectangular in nature, so the sinusoidal simulations would give the worst case condition when the laser pulse is slewing and not exactly in its ideal rectangular formats.

In terms of physical reality, this means that the modulated laser intensity is sampled by the detector which can be used to integrate on the even cycle and the odd cycle separately. When the detector is integrating the even and the odd cycle with a different phase than that of the sinusoidal-modulated laser, then the difference between the integration of the even cycle and the odd cycle, from now on, referred to as D, will be
a function of the phase difference. Note that there are a number of parameters that one can see when translating the situation into physical reality, the frequency with which the detector is detecting, the frequency with which the laser is switched on and off and the phase difference between the detector frequency and laser frequency, assuming that the detector frequency is an integer multiple of the laser frequency.

We will now do a robust simulation to prove that this intuitive method is mathematically correct. The simulation platform used is Maple. The target of the simulation with the the creation of plots, such as Figure 63 which shows the difference between the total mathematically summed values of the even integration cycles and that of the odd integration cycles, the value of $D$, when plotted against the frequency of the illumination. In these the laser frequency corresponds to $x = 1$, in other words it is the same as the frequency at which the two integration cycles of the detector occur. These simulations will also look at how light which is cycling at other frequencies, (e.g. a 60 Hz background illumination) will be detected. Hence these figures will plot the summed output $D$ for various frequencies after $n$ summations of the integration cycles. So for example in this figure, since the nominal laser frequency in this particular simulation is 1 kHz, $x = 0.9$ would correspond to light of 900 Hz. Since these results will depend on the phase difference between the detector and the illumination being looked at, plots such as Figure 63 and Figure 64 will show the result for several phases, with each phase plotted as a different colour. Generally two plots will be shown, Figure 63(a) which covers the range from $x=0$ to 2 or from DC to twice the frequency of the laser, and Figure 63(b) which looks at a much smaller frequency range.
One can see from Figure 63(a) that there is a peak at $x = 1$, that is when the detector frequency is twice that of the laser encoding frequency, the value of $D$ can be differentiated very easily and this peak drops off very quickly as can be seen in Figure 63(b). The lower frequency peak nearest to $x = 1$ is 0.008, which is much less than the peak value at $x = 1$, which is 0.064. Figure 64(a) shows that the peaks are weaker when the phase is not 0. The waveforms for the different phase are of the same shape as that of phase is 0. It can be seen that when the phase is not 0, the values of $D$ is always less than when $x = 1$ and phase = 0. Figure 64(b) shows that the peak value of $D$ nearest $x = 1$ is 0.056 which is lower than the peak value of $D$ when $x = 1$, which is 0.064, a difference of 0.008. This is a 12.5% difference which is significant.

The other parameter specific to each plot is $n$, the number of summations taken by the detector. One can easily see that the maximum value occurs when $x = 1$, that is when the frequency of the detector sampling the laser is the same frequency as the modulation of the laser by a sinusoidal wave when phase difference is absent.
Figure 63: $bg = \cos(2\pi 50t)$, Simulated response of double sided APS to light of frequencies varying with the laser encoding signal. This shows the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light in phase with the laser (a) signal range 0 to 2 times encoding frequency, (b) detail of signals 0.9 to 1.1 of encoding frequency. This is simulation for 100 total integration cycles.

- Black [$\varphi = 0$], red [$\varphi = \pi/4$], pink [$\varphi = \pi/2$], orange [$\varphi = 3\pi/4$], yellow [$\varphi = \pi$], green [$\varphi = 5\pi/4$], blue [$\varphi = 3\pi/2$], violet [$\varphi = 7\pi/4$]

Figure 64: $bg = \cos(2\pi 50t)$, Simulated response of dual-output APS shows (a) the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light when not in phase with the laser encoding frequency and signal range ratio from 0.95 to 1.05, (b) detail of signals when light is not in phase with the laser encoding frequency ratio from 0.9 to 1. This is simulation for 100 total integration cycles.
6.2 Simulation of Dual-Output Photogate APS (D-APS)

The D-APS needs to be operated with a laser encoded frequency in an ambient light setting together with an integration cycle of the same frequency on each side of the D-APS with the exception that the integration cycle for each side is 180° out of phase. The laser has a DC offset level where the sinusoidal signal rides upon and has a frequency component. There is a phase difference between the detector sampling and the integration cycle. The D-APS is operated with a certain number of integration cycles. Looking at the summary of the operation of the D-APS, the parameters of simulation are given as follow (referred to as the reference simulation parameter set):

- \( w_{laser} = 2\pi \times 1000 \) (rad/s)
- \( w = 2\pi \times 1000 \) (rad/s)
- \( dc = 1 \)
- \( bg = \cos(60 \times t) \)
- \( \varphi = 0 \)
- \( n = 100 \)
- \( fg = dc + \sin(x \times w_{laser} \times t + \varphi) \)
- \( total = fg + bg = dc + \sin(x \times w_{laser} \times t + \varphi) + \cos(60 \times t) \)
- \( phase1 = \int_{r2}^{total} \int_{r1}^{n \times \frac{\pi}{w}} \)
- \( phase2 = \int_{r1}^{total} \int_{r2}^{n \times \frac{\pi}{w}} \)
- \( Difference = D = \sum_{i=0}^{n-1} phase1 - \sum_{i=0}^{n-1} phase2 \)

For a complete understanding of the simulation parameters and the formulas, please refer to Appendix 1.1.

The results prove that for one set of simulation parameter, the reference simulation parameter set, the intuitive idea presented is correct. Only the results of the simulations will be presented in this chapter. The actual maple program which shows the
simulations and more detailed results are presented in Appendix 1, Appendix 2: and Appendix 3:

6.2.1 Simulation of Dual-Output Photogate APS (D-APS) when background amplitude is changed

Before looking at the results presented in Figure 65 and Figure 66, it would be useful to try to understand how to interpret these figures. Figure 65(a) shows the simulated response of the D-APS to light of frequencies varying with the laser encoding signal. It shows the differential output of side 1 and side 2 of the D-APS for 100 integrations per side varying with laser light frequencies expressed as a fraction of the laser encoding frequency which has been set to 1 kHz for laser light that is in phase with the integration cycle. Figure 65(b) is a magnified version of Figure 65(a). Both these figures show that the peak differential output of the D-APS occurs when the laser is synchronized with the integration cycle and when each integration cycle is half of the laser period on each side of the D-APS. It is also noteworthy that this peak is significantly higher (approximately 6 times) than the next highest peak. Figure 66(a) shows the simulated response of the differential output of the D-APS when 100 integration cyles were used, and when the differential output of the D-APS is plotted against the laser frequencies which has been expressed as a fraction of the laser encoding frequency of 1 kHz. The waveforms of different colors show the differential output with differing phase. Figure 66(b) is a magnification of Figure 66(a). It shows that the peak differential output occurs when laser is in phase with the integration cycle and when the laser is of the same frequency as the integration cycle, in other words, when integration cycle and laser are totally synchronized.
In summary, one can see from Figure 65(a), (b) that when there is no phase difference between the modulated laser light and the integration cycles, that the maximum $D$ occurs when $x = 1$, that is, where the laser and the detector frequency are the same. When a phase difference is introduced, the maximum $D$ still occurs when $x = 1$ and $\varphi = 0$, as shown in Figure 66(a), (b). Table 6 shows the values of $D$ under the different conditions. In general, the higher the number of integrations for a given background of constant amplitude, the higher the value of $D$. 
Figure 65: \( bg=1000 \cos(2\pi 50t) \), Simulated response of double sided APS to light of frequencies varying with the laser encoding signal. This shows the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light in phase with the laser (a) signal range 0 to 2 times encoding frequency, (b) detail of signals 0.9 to 1.1 of encoding frequency.

- Black \([\varphi=0]\), red \([\varphi=\pi/4]\), pink \([\varphi=\pi/2]\), orange \([\varphi=3\pi/4]\), yellow \([\varphi=\pi]\), green \([\varphi=5\pi/4]\), blue \([\varphi=3\pi/2]\), violet \([\varphi=7\pi/4]\)

Figure 66: \( bg=1000 \cos(2\pi 50t) \), Simulated response of dual-output APS shows (a) the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light when not in phase with the laser encoding frequency and signal range ratio from 0 to 1, (b) detail of signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05
Also, the higher the background amplitude is, the higher the value of $D$. The values of $D$ are maximum for a given condition when $x=1$ and $\varphi=0$. This is less true as the number of summations become less. One can see this when $n=2$, where the $D$ values at $x=0.999$ is slightly more than that at $x=1$ and $\varphi=0$ as can be seen in Section 6.2.2.

Now consider what happens when the intensity of the background illumination is increased. When one compares, Figure 63 and Figure 64 with Figure 65 and Figure 66, where the foreground light intensity remains constant but the background amplitude has been multiplied by 1000, one can see that there is not much difference in terms of the results of $D$. This is due to the extremely large number of summations used.

What is important about this result is that it clearly indicates that the output is both a function of the frequency and the phase relationship between the detector and any illustration. This is the basis of any phase sensitive detection system.

6.2.2 Simulation of Dual-Output Photogate APS (D-APS) when number of summations is changed

The number of summation cycles, $n$, will clearly affect the detection results. This section simulates how this change affects the output $D$. The results of the simulation results and graphs used in the following figures are given in Appendix 1.3.

Figure 67(a) shows that when the number of summations is 1, one can see that there is a peak value of $D$ when $x$ is near 1, that is, the detector frequency is twice that of the laser encoding frequency, at 0.00128. $x$ is the ratio of the frequency to the fundamental frequency of the laser encoding frequency. However, upon closer look from Figure 68(a) shows that the peak value occurs when $x = 0.98$. The width of the peak measured at the zero crossing is 0.52, whereas the lower frequency peak closest to $x = 1$
is 0.00026. Figure 67(b) shows the same waveform as in Figure 67(a) but zoomed in the x-axis. Figure 68(a) shows that as the phase changes and as $x$ changes too, the value of $D$ still remains highest when phase = 0 when compared to the other phases. However, from this figure, one can also see that the highest value of $D$ does not occur at $x = 1$. It occurs at $x = 0.98$ as has been stated before. Figure 69(a) shows that the peak value of $D$ occurs when $x = 1$ and its value is 0.0035 when the number of summations is 2 and the peak width at the zero crossing has reduced to 0.22 from 0.52, a reduction of more than 50%. The lower frequency peak nearest to $x = 1$ has a value of 0.0008. Figure 69(b) shows the zoomed in picture of Figure 69(a) in the x-axis. Figure 70(a), (b) shows that when the phase changes and when $x$ changes, the peak value of $D$ still occur at $x = 1$ and when the phase is 0. It can also be seen that the value of $D$ falls off very quickly when phase is not 0 while $x$ remains at 1. Similar interpretations can be applied to Figure 71, Figure 72, Figure 73, Figure 74, Figure 75, Figure 76, Figure 77, Figure 78 which show the effects of the number of summations on $D$. Figure 67 (n=1), Figure 69 (n=2), Figure 71 (n=5), Figure 73 (n=10), Figure 75 (n=50), Figure 77 (n=100), show that as total number of integrations is increased, one can see that the peak value of differential output when $x = 1$, that is when the laser encoding frequency is exactly the same as the integration frequency of the D-APS, gets higher. Figure 68 (n=1), Figure 70 (n=2), Figure 72 (n=5), Figure 74 (n=10), Figure 76 (n=50), Figure 78 (n=100), shows that as the total number of integrations increases, the value of differential output when laser encoding cycle is exactly in phase with integration cycle at exactly the same frequency, is getting higher. However, Figure 67 (n=1), shows that the peak value of $D$ occur not when $x = 1$ and $\phi = 0$, but at a value when $x = 0.9$ and $\phi = \pi/4$. Likewise, Figure 70 (n=2), shows that the
peak value of \( D \) occurs when \( x = 1 \) but when \( \varphi = \pi/4 \). This is due to the low number of integration cycles used in the process. All these values can be summarized in Table 5. This table summarizes very succinctly that as the number of summation cycles increases, the value of \( D \) increases at \( x = 1 \), while the width of the peak decreases making differentiation between \( x = 1 \) and when \( x \) is not 1 simpler. Lower frequency peak nearest to \( x = 1 \) increases with increasing number of summations. One can conclude from this table that increasing number of summation cycles makes it easier to discriminate the background in preference to the laser illumination.

<table>
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<tr>
<th>( n )</th>
<th>Value of ( D ) at ( x = 1 )</th>
<th>Width of peak at ( x = 1 )</th>
<th>Lower frequency peak nearest to ( x = 1 )</th>
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Figure 67: $n=1$, Simulated response of double sided APS to light of frequencies varying with the laser encoding signal. This shows the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light in phase with the laser (a) signal range 0 to 2 times encoding frequency, (b) detail of signals 0.9 to 1.1 of encoding frequency.

Figure 68: $n=1$, Simulated response of dual-output APS shows (a) the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light not in phase with the laser encoding frequency and signal range ratio from 0 to 1, (b) detail of signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05.
Figure 69: n=2, Simulated response of double sided APS to light of frequencies varying with the laser encoding signal. This shows the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light in phase with the laser (a) signal range 0 to 2 times encoding frequency, (b) detail of signals 0.9 to 1.1 of encoding frequency.

- Black [φ=0], red [φ=π/4], pink [φ=π/2], orange [φ=3π/4], yellow [φ=π], green [φ=5π/4], blue [φ=3π/2], violet [φ=7π/4]
Figure 70: \( n=2 \), Simulated response of dual-output APS shows (a) the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light when not in phase with the laser encoding frequency and signal range ratio from 0 to 1, (b) detail of signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05.

![Graph](image1.png)

(a) ![Graph](image2.png)

(b)

Figure 71: \( n=5 \), Simulated response of double sided APS to light of frequencies varying with the laser encoding signal. This shows the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light in phase with the laser (a) signal range 0 to 2 times encoding frequency, (b) detail of signals 0.9 to 1.1 of encoding frequency.

![Graph](image3.png)

(a) ![Graph](image4.png)

(b)

- Black \( \varphi=0 \), red \( \varphi=\pi/4 \), pink \( \varphi=\pi/2 \), orange \( \varphi=3\pi/4 \), yellow \( \varphi=\pi \), green \( \varphi=5\pi/4 \), blue \( \varphi=3\pi/2 \), violet \( \varphi=7\pi/4 \)

Figure 72: \( n=5 \), Simulated response of dual-output APS shows (a) the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light when not in phase with the laser encoding frequency and signal range ratio from 0 to 1, (b) detail of signals when light is not in phase with the laser encoding frequency ratio...
from 0.95 to 1.05f signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05.

Figure 73: **n=10**, Simulated response of double sided APS to light of frequencies varying with the laser encoding signal. This shows the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light in phase with the laser (a) signal range 0 to 2 times encoding frequency, (b) detail of signals 0.9 to 1.1 of encoding frequency.

Figure 74: **n=10**, Simulated response of dual-output APS shows (a) the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light when not in phase with the laser encoding frequency and signal range ratio from 0 to 1, (b) detail of signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05.
Figure 75: \( n=50 \), Simulated response of double sided APS to light of frequencies varying with the laser encoding signal. This shows the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light in phase with the laser (a) signal range 0 to 2 times encoding frequency, (b) detail of signals 0.9 to 1.1 of encoding frequency.

Figure 76: \( n=50 \), Simulated response of dual-output APS shows (a) the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light when not in phase with the laser encoding frequency and signal range ratio from 0 to 1, (b) detail of signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05 when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05.
Figure 77: $n=100$, Simulated response of double sided APS to light of frequencies varying with the laser encoding signal. This shows the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light in phase with the laser (a) signal range 0 to 2 times encoding frequency, (b) detail of signals 0.9 to 1.1 of encoding frequency.

- Black [$\phi=0$], red [$\phi=\pi/4$], pink [$\phi=\pi/2$], orange [$\phi=3\pi/4$], yellow [$\phi=\pi$], green [$\phi=5\pi/4$], blue [$\phi=3\pi/2$], violet [$\phi=7\pi/4$]

Figure 78: $n=100$, Simulated response of dual-output APS shows (a) the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light when not in phase with the laser encoding frequency and signal range ratio from 0 to 1, (b) detail of signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05
6.3 Summary of Simulation Results

Looking at Figure 84 to Figure 119 and Table 6, one can also see that the value of $D$ is in general maximum when $x=1$ and when $\varphi=0$, with the exception when $n=2$. Once again, this is because of the low number of integrations used. This is not true as $n$, the number of integrations become asymptotically becomes big.

In general, the simulation results show the feasibility of this scheme of creating a phase sensitive detection system from the value of $D$. As the number of summations increase asymptotically, the value of $D$ is the maximum under the same conditions when $x=1$ and $\varphi=0$. As a result, this means that given that the laser frequency and the summation cycles of the photo-detector are in synchronicity, one can differentiate between a laser of the same frequency and phase as the one that is desired by inputting the frequency and phase information for the photo-detector, thus creating an effect of detecting the laser no matter what the background amplitude is.

A clear extension of this is that if there are several lasers, each with its own frequency, then the detector can be tuned to detect each frequency separately. Thus by using several frequencies of exposure cycles many different laser illuminations could be identified. The laser spots can be separated by its encoding frequency because if the detector frequency is two times that of the laser encoding frequency and the detector and laser frequencies are in phase, then the value of $D$ will be very high, whereas if the detector frequency is not two times that of the laser frequency or not in phase then the value of $D$ will be very low. The detector frequency is decided by choosing which laser frequency is to be detected, while the phase can be enforced by synchronization of the
laser frequency with that of the detector frequency. As a result, different laser spots can be separated based on the value of $D$.

The rest of the thesis will be aimed at proving that this is true experimentally under the conditions when the number of integration cycles is fixed at $n=2$, while the background amplitude is slowly increased until it reaches the same amplitude as that of the laser light. Carrying out the experiment for larger values of $n$ will be done in the future.
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Table 6: Values of $D$ under various conditions
6.4 Conclusion

It has been shown through simulations that the concept of background elimination through synchronizing laser encoding cycle with the integration cycle works in simulations. The next chapter will detail the experimental results for the case when the number of summation cycles used is 2. This will show that the concept works and is thus extensible for higher number of integration cycles.
CHAPTER 7 : EXPERIMENTAL RESULTS OF DUAL-OUTPUT PHOTOGATE-BASED APS

The experimental dual-output photogate-based APS was designed to test the concept of background elimination that had been discussed in Chapter 6. The experiment involves measuring the output of the D-APS when varying foreground and background illuminations are used. The illuminations are driven by function generators, where the background is driven by the offset voltage and the foreground is driven by the swing of the voltages around the offset voltage. The foreground and background are superimposed together on an LED just for “simulation” purposes. In reality, the laser and the ambient light sources would usually be separate. The dominant frequencies of the environment can be characterized and then the integration frequency of the D-APS can be adjusted so as to maximize the elimination of the background illumination. One of the most common background illumination frequency in North America is 120 Hz because the sources like light bulb produce light at the twice frequency of 60 Hz AC power.

7.1 Explanation of Notations

There were 4 dual-output photogate APS that were tested and had the values of $D$ measured. For these 4 values of $D$, one set does not include dark current compensation and one set includes dark current compensation. Dark current compensation basically uses the same timing waveforms as the one without dark current compensation but the experiment is carried out in the dark. The values are read out on sideA and sideB
(Figure 46) in two integration cycles before reset is applied, giving \( \text{Value(sideA, Dark)} \) and \( \text{Value(sideB, Dark)} \).

It was found that charge transfer asymmetry exists between side A and side B. This is undesirable because what is integrated on side A and side B cannot be subtracted readily to give the correct value. As a result, a method has been devised to cancel out the charge transfer asymmetry. Assuming that foreground and background light is integrated on sideA and background light is integrated on sideB for two summation cycles before reset is applied, giving \( \text{Value(sideA)} \) and \( \text{Value(sideB)} \). Then, the value for \( D \) is given as such when dark current compensation is included:

![Figure 79: Timing diagram of entire experimental setup from the perspective of the data acquisition program for a D-APS showing one integration cycle only](image)
\[ D = [\text{Value}(\text{sideA}) - \text{Value}(\text{sideA}, \text{Dark})] - [\text{Value}(\text{sideB}) - \text{Value}(\text{sideB}, \text{Dark})] \quad (18) \]

and as such when dark current compensation is not included:

\[ D = \text{Value}(\text{sideA}) - \text{Value}(\text{sideB}) \quad (19) \]

Some of the results of the values of \( D \) are negative which is highly unintuitive. Unfortunately, the reason is as yet unknown and will not be investigated in this thesis. To counteract this tendency of the values of \( D \) being negative, the following is done when dark current compensation is included:

\[ D_1 = [\text{Value}(\text{sideA}) - \text{Value}(\text{sideA}, \text{Dark})] - [\text{Value}(\text{sideB}) - \text{Value}(\text{sideB}, \text{Dark})] \quad (20) \]

where foreground and background light is integrated on sideA while background light is integrated on sideB. Let

\[ D_2 = [\text{Value}(\text{sideB}) - \text{Value}(\text{sideB}, \text{Dark})] - [\text{Value}(\text{sideA}) - \text{Value}(\text{sideA}, \text{Dark})] \quad (21) \]

where foreground and background light is integrated on sideB while background light is integrated on sideA.

When dark current compensation is not included then,

\[ D_1 = \text{Value}(\text{sideA}) - \text{Value}(\text{sideB}) \quad (22) \]

where foreground and background light is integrated on sideA while background light is integrated on sideB. Let

\[ D_2 = \text{Value}(\text{sideB}) - \text{Value}(\text{sideA}) \quad (23) \]

where foreground and background light is integrated on sideB while background light is integrated on sideA. The values of \( D \) are then given by the following regardless of whether dark current compensation is included or not: \( D = D_1 + D_2 \). The idea of this is
that if there is asymmetric transfer charge efficiency to sideA and sideB, the asymmetry will be cancelled out. First, the foreground and background are integrated on sideA and the background on sideB. Secondly, the background and foreground are integrated on sideB and background on sideA. After the results from the first part and second part are obtained, the summing of the first result and the second result cancels out the asymmetry.

7.2 Experimental Results

It must be noted that the experimental results here are restricted only to the case where the total number of summation cycles is 2 and not for any other cases. The other cases which include more summation cycles would be carried out in the future by my colleagues. It is expected that since 2 cycles of summation is the minimum number that would be needed to include even and odd cycle for background subtraction, that this would give the worst performance case. Please keep this in mind as this number of integration cycles gives the fundamental mode and functionality of the dual-output photogate-based APS and it is the fundamental results that we are interested in primarily in this thesis.

7.2.1 Overview of Analysis of Results

In order to understand the next section, the overall strategy of how the data is going to be analyzed will be given.

First, the value of the summation cycle, $D$, will be shown to be insensitive to background intensity with or without dark compensation. This is desirable as this means that this method would be immune to holes and shiny spots. This will be proven by ANOVA analysis.
Second, the value of summation cycle, $D$, will be shown to be sensitive to phase.

Third, the value of summation cycle, $D$, will be shown to be sensitive only to phase and not the presence or absence of dark compensation.

Fourth, understanding that the value of $D$ is only sensitive to phase and not presence or absence of dark compensation, all the values of measurements that are obtained with and without dark compensation can be lumped together for the purpose of analysis, thus giving more data for analysis. And it is here that it can be shown that for phase $= 90$ or $\phi = 0$, there is great ease in separating the value of $D$ when detector frequency is in sync with laser encoding frequency from that of other phases.

7.2.2 Analysis of Results

The data is very complicated and has been tabulated in Appendix 3. In this chapter we will discuss the statistical analysis of the results, done by using ANOVA (ANalysis Of VAriance) analysis.

7.2.2.1 Insensitivity to background intensity with or without dark compensation

For the D-APS to be successful it is necessary to test that the subtraction of the foreground from the background illumination will generate a signal that is independent of the background intensity. In this section we will compare the $D$ value for a range of background illuminations while the foreground value is held constant, where the phase difference is zero (i.e. the detector and the foreground illumination are synchronized. For purposes of compactness, only the value of $D_j$ without dark compensation with respect to background intensity is shown in Figure 80. To explain the results shown in Figure 80, one has to understand that the ANOVA (Analysis of Variances) of the data compares the variances between the individual distributions as grouped by foreground intensity and the
variance of the means of the individual distributions. If the variance of the means of the individual distributions is smaller than the variances of the individual distributions, this means that the data statistically does not depend on the foreground intensity.

Since the Prob > F is 0.13 which is greater than the significance value of 0.05 which is traditionally taken to be the threshold of significance, the variances of the means of the individual distributions is greater than the variances of the individual distributions. This statistically means that there is no statistical dependence between the value of $D_f$ and the foreground intensity. This can also be seen at the very bottom of Figure 80, where the data for each background intensities (denoted by Level column) are all categorized in the group A and thus cannot be distinguished statistically from values of $D_f$ for other background intensities.

The conclusion for the above ANOVA analysis is that the values of $D_1$, $D_2$, $D_3$ and $D_4$ with and without dark compensation are statistically independent of the background intensities. This corresponds to the expected result for the D-APS and confirms that it successfully subtracts the background signal from the illumination, leaving only the added light in the foreground illumination.
Figure 80: D-APS removal of background light: ANOVA analysis of $D_1$ without dark compensation with respect to background intensity where Level denotes background intensity.
7.2.2.2 Sensitivity to phase

As noted in Chapter 6 the multiple summation of the D-APS should result in a phase sensitive detection of the Forground light. Since a summation of $n=2$, as shown in The simulations of Figure 70 of Chapter 6, changes in phase between the D-APS cycles and the Forground illumination will significantly change the signal. Further ANOVA analysis of these values with respect to phase reveals a statistical dependence with respect to phase. An example of such an ANOVA analysis given for $D_i$ without dark compensation is shown in Figure 81. One can see the statistical dependence of $D_i$ without dark compensation on phase at the bottom of Figure 81, where the values of $D_i$ without dark compensation are classified into 4 groups, namely Group A, B C and D, which means that these 4 groups have statistically different values.

The conclusion of this preliminary study is that it can be shown that the values of $D_1$, $D_2$, $D_3$ and $D_4$ with and without dark compensation are statistically dependent on the phase through similar ANOVA analysis. There is not enough data points that are being used to say what the dependence of these values with phase, but in the next study, more number of points will be used to evaluate the phase sensitivity.

Since the values of $D$ are statistically only dependent on the phase and not the background intensity, this simplifies the analysis of the results considerably. Also this is exactly what is desired if the operation of the D-APS where its operation only depends on the phase and less strongly on the background intensity. Thus this is confirming the simulations that the D-APS will act as a phase sensitive detector.
Figure 81: D-APS phase sensitive detection: ANOVA analysis of D1 without dark compensation with respect to phase where Level denotes the phase.
7.2.2.3 Sensitivity to phase regardless of presence or absence of dark compensation

This section will test the effectiveness of dark compensation for the D-APS and its effect on phase sensitive detection. Measurements were made at various background levels and phase differences with and without using dark compensation as part of the measurement. Another ANOVA analysis shown in Figure 82 shows that the values of $D$, with and without dark compensation, are not statistically different for presence or absence of dark compensation, but it does show that the values of $D$ statistically vary with phase. This can be shown by the fact that "0_DC" and "0_NDC" signifying 0 phase with dark compensation and no dark compensation are in the same group and the values of any phase with dark and no dark compensation are in the same group. "45_DC" and "45_NODC" are in the same group, "90_DC" and "90_NODC" are in the same group, "135_DC" and "135_NODC" are in the same group, "180_DC" and "180_NODC" are in the same group, "225_DC" and "225_NODC" are in the same group, and "315_DC" and "315_DC" are in the same group. The conclusion of this study is that with and without dark compensation, the phase sensitivity is the same for values for $D$. Thus dark compensation does not affect the ability to do the phase sensitive detection.
Figure 82: Test of D-APS dark compensation operation: ANOVA analysis of values of D with dark compensation ("_DC") and without dark compensation ("_NDC") with respect to phase where Level denotes phase with or without dark compensation
7.2.2.4 Value of Summation Cycle when phase = 90 or phi = 0 can be easily distinguished from the value when other phases are used

Since dark compensation has null effect on the measurements we can use those measurements both with and without it in an analysis of the effect of phase. This allows greater statistical accuracy to be done on the 0 and 90° phase measurements of the D-APS. With this in mind, one can treat the values of $D$ with and without dark compensation as being the same and thus for every phase, there are 80 values that can be used, since the foreground intensity and whether there is dark compensation or no dark compensation are statistically insignificant factors. The practical implication of the Central Limit Theorem is such that with 80 values for each data point, one can say that the distribution is relatively normal.

Another ANOVA analysis shown in Figure 83 shows that the maximum of the values of $D$ statistically occur when phase is 90 or phi = 0. Statistically, this means that the D-APS has the maximum value of $D$ when phase is 90 or phi = 0, as has been simulated. From looking at Figure 83 and the level as well as the category letter, one can see that the values of $D$ are not sensitive to differences between phase = 0 and phase = 315, and not sensitive to differences between phase = 180, phase = 225 and phase = 270.

In conclusion, what is important in this study is that when phase = 90 or phi = 0, the values of $D$ are statistically significant from the other values and can therefore be distinguished easily from values of $D$ when different phases are used.
Figure 83: ANOVA analysis of values of D with respect to phase where Level refers to phase
7.3 Conclusion

Statistically speaking, one can see that the D-APS will have a maximum output value when phase = 90 or phi = 0 under the experimental conditions of 2 summation cycles. It is notable that there is no statistical dependence of the output of the D-APS on the foreground intensity, which means that the foreground intensity could be very low and yet the D-APS would be able to discriminate between the foreground against the background signal. In practical terms, this would allow the scanner to detect the foreground preferentially over the background based more strongly on phase of the foreground rather than its intensity.

It is expected that this would work out much better with more cycles of summation. This has proven that the idea of the dual-output photogate-based APS (D-APS) is very promising and worthwhile investigating further. This work would be carried out by my colleagues in the future.

Essentially, this shows that local processing at a pixel level to discriminate against background illumination is viable and solves two fundamental problems in modern optical scanning technology. The first problem is that of the limited intensity of the laser which causes limited signal-to-noise ratio. This is not a problem at all since the experiments have shown that the laser can be of the same intensity as the ambient light. Secondly, the local processing abilities mean that surface local imperfections on the object to be scanned will not cause a problem because there the background signal is integrated in one cycle whereas the background and foreground signals are integrated onto the other side, so that the stray signals will stay in the background and can be subtracted off.
CHAPTER 8 : CONCLUSION

The thesis has investigated two novel APS designs, namely the fault-tolerant photodiode-based APS (FT-APS) and the dual-output photogate-based APS (D-APS). There was an introduction to the concept of photo-sensing in semiconductors, CCDs and APSs in general. Also discussed was the need for the FT-APS and D-APS to address current limitations in imaging technology. FT-APS was designed to improve manufacturing yields of APS in general and bring about economies of production in higher megapixel cameras where yields are typically low because of no fault-tolerance. Also covered were current scanning technologies, of which the main concern was non-contact active scanner where the D-APS was designed to make sure that scanning technologies can operate when laser used is of the same intensity as the ambient light and also ensuring that presence of surface local imperfections do not corrupt scanning results.

8.1 Fault-Tolerant Photodiode-Based APS (FT-APS)

The FT-APS experimental results have ascertained the gross functionality of the concept of fault-tolerance as incorporated into the APS. It has been shown experimentally that each subpixel of the FT-APS acts separately with nearly half as much sensitivity as the entire FT-APS. In the presence of single defects which basically render half-pixel non-functional, the experiment suggests that a multiplication of 2.02 for the optically stuck low case and 1.79 for the optically stuck high case. Since they are near the expected value of 2 and in the interest of economical computation, a simple multiplication of two would be used to recover the results.
To find out which side of the pixel is defective, one method would be to use near saturation bright illumination (light field) to discover optical stuck low faults and use dark illumination (dark field) to discover optical stuck high cases. Also in the presence of an entirely faulty FT-APS which is very unlikely, one would have to resort to software interpolation of the surrounding FT-APS.

8.1.1 Future Work

It has been shown that even though compensation of multiplication by two in presence of single defect in an FT-APS gives reasonable accuracy, the stuck low case needs a multiplication of less than two, which means that it is less sensitive. This would complicate compensation for sure. However, there are speculations that these are caused by the fact that the stuck high case is overly sensitive because of photoelectrons spilling into the other side. To eliminate this possibility, the following FT-APS design could be used in the future, that of separating the half-pixels at varying distances and making sure that the output is not affected at various distances of separation. If there is a correlation between sensitivity and separation distance, then design can be undertaken to ensure less crosstalk between subpixels, by building some sort of barrier, like surrounding the subpixels with ground, which might reduce fill factor somewhat.

Furthermore, one could probably conceive of a different readout circuitry based on voltages instead of current summing.

Also of interest is extending this idea of the FT-APS into larger arrays and actually quantifying its performance in terms of imaging metrics, like picture quality, color fidelity and so on.
Further yield modeling work could also be done to quantify the amount of economic savings so as to justify any negative points that the FT-APS has and whether these negative points can be tolerated in the complicated mixture of economics of production and technical fidelity.

8.2 Dual-Output Photogate-Based APS

The purpose of the D-APS is to produce an imager where an added laser signal can be detected independent of the amount of background illumination on the scene. This was confirmed in the experiments of Chapter 7. In addition, it has been shown experimentally with two cycles of summation that statistically it would be possible to distinguish the values of summation that happen when the laser encoding frequency is in sync with the detector frequency, thus enabling phase sensitive detection. This is important as the background illumination in many locations is not a steady signal but rather has an AC value. Usually, there is one dominant mode of ambient illumination which is the electrical lights powered by 60 Hz AC in North America, giving an illumination frequency of 120 Hz. When placed in another location which is running with a different frequency of power supply, the frequency of the integration cycle needs to be adjusted accordingly. Of course, the phase between the laser and the summation cycle has to be adjusted to be 90 or phi of 45 for this to be successful. And the experiment clearly shows that this scheme works even when the laser is of the same intensity as that of the ambient light.

Obviously, new experiments have to be done to verify that this scheme will succeed with reasonably accuracy and precision when more cycles of summation is used. One can safely say that it will probably not take too many summation cycles before
converging upon the correct answer. This reduces computational burden and the number of cycles of integration representing the levels of accuracy needed can be traded off for more real-time like abilities, depending on the application at hand.

One could envision that in a real optical scanning system, the D-APS and the frequency-encoded laser are driven by a central controller that ensures that the summation cycles and the laser are synchronized so as to optimize performance.

8.2.1 Future Work

It would be imperative to extend the experiment to include more cycles of integration and see how fast the answer converges.

Also, there is a hypothesis that there is leakage between the two floating diffusions and the photogate area, thus potentially corrupting the value of $D$. One way to test this would be to make sure that the transfer gates are much longer. This would ensure that corruption of $D$ because of diffusion leakage is not as predominant.

Another interesting experiment would be to change the transfer voltage from 0 V to something small like 0.1 V or something like this, as this could reduce the amount of transfer lag as well as charge accumulated in the floating diffusion, diffusing back to the photogate area.

There has been a trend of moving towards blue lasers. Photogate APS are not known for its blue sensitivity. However, it could be interesting to check the foreground discrimination scheme with blue light as well.

Building of a linear scanner using D-APS and checking it in a real-time optical scanning system would be the eventual proof of the concept and would be very
interesting to see. One could possibly build in circuitry necessary to minimize number of external components and bring the cost of the entire optical scanning system down.

### 8.3 Summary

This chapter has shown the conclusions of both the FT-APS and the D-APS. Future work has also been discussed pertaining to each type of novel APS design. However, future work regarding the APS test platform could be given as well and they will be given here.

It might be interesting to include the control circuitry of the test platform so that there is less that needs to be done off-chip by LabVIEW. For example, including a sample-and-hold circuit, a crow-bar readout circuitry and an ADC would simplify processing to the digital domain.

A spectral performance analysis of the two types of APS designs would probably be very important in gauging how useful it will be in a range of applications. Quantifying other pixel performance parameters might be important, some of which are signal-to-noise ratio, quantum efficiency, conversion gain, dynamic range of the photosensing element and supporting circuitry and FPN among many other things.
APPENDICES

Appendix 1: Simulation of the Dual-Output Photogate-Based APS

This section shows the simulations that are performed in Maple to verify the theoretical idea for the operation of the dual-output photogate-based APS. The first section shows what happens when the background light intensity is varied while number of integration cycles is kept constant, the second section covers the case when the number of integration cycles is varied while the background light intensity is kept constant, the third section covers the case of varying background intensity when the number of integration cycles is two and the encoding frequency of laser (foreground light) is 100 Hz. The third section is very specific because this thesis addresses this exact issue and future work is underway to verify the operations in more general terms. Finally, the last section of the simulation shows a graphical summary of the all the simulation results.

1.1 Explanations of Formulas used in Simulations

In order to understand the simulations in this section, one has to understand some of the mathematical variables used. First, it is important to understand the meanings of the variables used. However, one has to have already understood Chapter 6 before one can understand these variables.

\[ w = \text{detector sampling frequency} \]

\[ w_{\text{laser}} = \text{laser encoding frequency} \]

\[ \phi = \text{phase offset} \]

\[ n = \text{number of summation cycles} \]
de = DC offset of the foreground signal

bg = background illumination

x = fraction of the nominal laser encoding frequency

Beginning with this set of variables, one can then attempt to understand the formulas that are used.

The first formula describes the foreground light intensity:

\[
fg = dc + \sin(xw_{\text{laser}}t + \phi)
\]

where the foreground light intensity (the laser intensity) is described as having a dc offset because the foreground light can have sinusoidal components and the total light intensity can never be negative. The sinusoidal component of the foreground light intensity is described as a sine wave which has as its argument the multiplication of the laser encoding frequency, the fraction of the laser encoding frequency and time, which is then summed with the phase difference, \(\phi\).

The background light is given by:

\[
bg = \cos(2\pi50t)
\]

where 50 Hz is assumed to be the modulation of the background light.

The total light is given by total:

\[
total = fg + bg
\]

where the total light intensity is the summation of the foreground and background light intensity.

The 2 integration phases of the summation cycle is denoted as phase1 and phase2.
phase1 = \int(total, t=(2i\pi/w .. (2i+1)\pi/w)

phase2 = \int(total, t=(2i\pi/w .. 2(i+1)\pi/w)

where phase1 is the integration of the total light intensity but on every first half of the ith period of the total light intensity and phase2 is the integration of the total light intensity but on every second half of the ith period of the total light intensity.

The difference is the subtraction of the sum of phase1 and phase2 over the total number of summation cycles, giving the value of \( D \).

1.2 Varying Background Light Intensity

This particular section addresses the simulation for varying background light intensity when foreground illumination remains constant, encoding frequency of the foreground laser frequency is 1000 Hz and the total number of integration cycles is fixed at 100. This set of simulation gives an idea of the robustness of the method in rejecting background light intensity of various levels up to when the background light intensity is the same as that of the intensity of the foreground laser illumination.

\[ n = 100, \; dc = 1, \; f = 1000, \; bg = 0.5x \]

```
> restart;
w = sampling frequency, phi = phase offset, n = # of samplings, dc = DC offset, bg = background light level
> \( w_\text{laser} := 2\pi \times 1000 \);
> n := 100;
> dc := 1;
> bg := \cos(2\pi \times 50 \times t);
> \( w := 2\pi \times 1000 \);
> \( n := 100 \);
> \( dc := 1 \);
> bg := \cos(100 \pi t) \)
> \( w := 2000 \pi \)

fg := dc + sin(x \times w \times t + \phi);
> \( fg := dc + \text{sin}(x \times w_\text{laser} \times t + \phi) \);
\( fg := 1 + \text{sin}(2000 x \pi t + \phi) \)
x = fraction of signal frequency to the sampling frequency
> \( \text{total} := \text{fg} + bg \);
```
total := 1 + \sin(2000 \pi t + \phi) + \cos(100 \pi t) \\
> \text{phase1} := \int(\text{total}, t = (2i\pi/w \ldots (2i+1)\pi/w)) ; \\
\text{phase1} := -\frac{1}{2000} \left( -\pi x + \cos(2x\pi i + x\pi + \phi) - 20 \sin\left(\frac{1}{10} \pi (2i + 1)x\right) \right) / (x\pi) \\
\text{phase2} := \int(\text{total}, t = (2(i+1)\pi/w \ldots 2(i+1)\pi/w)) ; \\
\text{phase2} := \frac{1}{2000} \left( \pi x - \cos(2x\pi i + 2x\pi + \phi) + 20 \sin\left(\frac{1}{10} \pi (i + 1)x\right) \right) / (x\pi) \\
> \text{Difference} := \sum(\text{phase1}, i = 0 \ldots n-1) - \sum(\text{phase2}, i = 0 \ldots n-1) ; \\
> \text{plot}(\text{eval}(\text{Difference}, \phi = 0), x = 0.9 \ldots 1.1) ; \\

> \text{fg} := d + \sin(x \cdot \text{w laser} \cdot t + \text{evaln}(\phi)) ; \\
\text{fg} := 1 + \sin(2000 \pi t + \phi) \\
> \text{plot}(\text{eval}(\text{Difference}, \phi = 0), x = 0.05 \ldots 1.95) ; \\

> \text{plot}([\text{eval}(\text{Difference}, \phi = 0), \text{eval}(\text{Difference}, \phi = 0.25\pi), \\
\text{eval}(\text{Difference}, \phi = 0.5\pi), \text{eval}(\text{Difference}, \phi = 0.75\pi), \\
\text{eval}(\text{Difference}, \phi = \pi), \text{eval}(\text{Difference}, \phi = 1.25\pi), \\
\text{eval}(\text{Difference}, \phi = 1.5\pi), \text{eval}(\text{Difference}, \phi = 1.75\pi)]) ;
\textbf{x}=0.95..1.05, \texttt{color=[black, red, pink, orange, yellow, green, blue, violet, navy]);}

\textbf{plot(\{eval(Difference, phi=0), eval(Difference, phi=0.25*Pi), eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi), eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi), eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)\}, x=0.9..1, color=[black, red, pink, orange, yellow, green, blue, violet, navy]);
> simplify(eval(eval(Difference, phi=0*Pi), x=0.005));
  .1273239545 10^-9
> simplify(eval(eval(Difference, phi=0*Pi), x=0.01));
  .8594366926 10^-10
> simplify(eval(eval(Difference, phi=0*Pi), x=0.05));
  -.7384789358 10^-10
> simplify(eval(eval(Difference, phi=0*Pi), x=0.1));
  -.4042535554 10^-10
> simplify(eval(eval(Difference, phi=0*Pi), x=0.5));
  0.
> simplify(eval(eval(Difference, phi=0*Pi), x=0.99));
  -.8644846372 10^-9
> simplify(eval(eval(Difference, phi=0*Pi), x=0.999));
  .05961466421
> simplify(eval(eval(Difference, phi=0*Pi), x=0.9999));
  .06366197789
> simplify(eval(eval(Difference, phi=0*Pi), x=0.99999));
  .06366197789
> simplify(eval(eval(Difference, phi=Pi/4), x=0.5));
  .1082253613 10^-9
> simplify(eval(eval(Difference, phi=Pi/4), x=0.9999));
  .04501595588
> simplify(eval(eval(Difference, phi=3*Pi/4), x=0.5));
  .2021267777 10^-10
> simplify(eval(eval(Difference, phi=3*Pi/4), x=0.9999));
  -.04501567670
>.6366197789e-1;
  .06366197789
\[ .1082253613e-9 + .4501595588e-1 = 0.04501595599 \]
\[ .2021267777e-10 - .4501567670e-1 = -0.04501567668 \]

\( n = 100, \ dc = 1, \ f = 1000, \ bg = 5x \)

\[ \text{restart;} \]

\( w = \text{sampling frequency, phi = phase offset, n = # of samplings, dc = DC offset, bg = background light level} \)

\[ w = 2\pi \]
\[ n = 100 \]
\[ dc = 1 \]
\[ bg = 10\cos(2\pi \cdot 50\cdot t) \]

\( f_x = \text{fraction of signal frequency to the sampling frequency} \)

\[ \text{plot(eval(Difference, phi=0), x=0.9..1.1);} \]
\[ f_g := 1 + \sin(2000 \pi t + \phi) \]

\[ \text{plot}(\text{eval}(\text{Difference}, \phi=0), x=0.05..1.95); \]

\[ \text{plot}([\text{eval}(\text{Difference}, \phi=0), \text{eval}(\text{Difference}, \phi=0.25*\pi), \text{eval}(\text{Difference}, \phi=0.5*\pi), \text{eval}(\text{Difference}, \phi=0.75*\pi), \text{eval}(\text{Difference}, \phi=\pi), \text{eval}(\text{Difference}, \phi=1.25*\pi), \text{eval}(\text{Difference}, \phi=1.5*\pi), \text{eval}(\text{Difference}, \phi=1.75*\pi)], x=0.95..1.05, \text{color}=[\text{black}, \text{red}, \text{pink}, \text{orange}, \text{yellow}, \text{green}, \text{blue}, \text{violet}, \text{navy}]); \]
> plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi),
> eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi),
> eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi),
> eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)],
> x=0.05..0.95, color=[black, red, pink, orange, yellow, green, blue,
> violet, navy]);

> plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi),
> eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi),
> eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi),
> eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)],
> x=0.9..1, color=[black, red, pink, orange, yellow, green, blue, violet, navy]);
\texttt{\textbackslash simplify} (\texttt{\textbackslash eval} (\texttt{\textbackslash eval} (\texttt{\textbackslash Difference}, \texttt{\phi}=0*\texttt{Pi}), \texttt{x}=0.005)); \\
\hspace{1cm} 0.1209577567 \times 10^{-9} \\
\texttt{\textbackslash simplify} (\texttt{\textbackslash eval} (\texttt{\textbackslash eval} (\texttt{\textbackslash Difference}, \texttt{\phi}=0*\texttt{Pi}), \texttt{x}=0.01)); \\
\hspace{1cm} -0.3183098861 \times 10^{-11} \\
\texttt{\textbackslash simplify} (\texttt{\textbackslash eval} (\texttt{\textbackslash eval} (\texttt{\textbackslash Difference}, \texttt{\phi}=0*\texttt{Pi}), \texttt{x}=0.05)); \\
\hspace{1cm} -0.7957747154 \times 10^{-10} \\
\texttt{\textbackslash simplify} (\texttt{\textbackslash eval} (\texttt{\textbackslash eval} (\texttt{\textbackslash Difference}, \texttt{\phi}=0*\texttt{Pi}), \texttt{x}=0.1)); \\
\hspace{1cm} -0.4106197531 \times 10^{-10} \\
\texttt{\textbackslash simplify} (\texttt{\textbackslash eval} (\texttt{\textbackslash eval} (\texttt{\textbackslash Difference}, \texttt{\phi}=0*\texttt{Pi}), \texttt{x}=0.5)); \\
\hspace{1cm} 0. \\
\texttt{\textbackslash simplify} (\texttt{\textbackslash eval} (\texttt{\textbackslash eval} (\texttt{\textbackslash Difference}, \texttt{\phi}=0*\texttt{Pi}), \texttt{x}=0.99)); \\
\hspace{1cm} -0.8641952646 \times 10^{-9} \\
\texttt{\textbackslash simplify} (\texttt{\textbackslash eval} (\texttt{\textbackslash eval} (\texttt{\textbackslash Difference}, \texttt{\phi}=0*\texttt{Pi}), \texttt{x}=0.999)); \\
\hspace{1cm} 0.05961466421 \\
\texttt{\textbackslash simplify} (\texttt{\textbackslash eval} (\texttt{\textbackslash eval} (\texttt{\textbackslash Difference}, \texttt{\phi}=0*\texttt{Pi}), \texttt{x}=0.9999)); \\
\hspace{1cm} 0.06366197789 \\
\texttt{\textbackslash simplify} (\texttt{\textbackslash eval} (\texttt{\textbackslash eval} (\texttt{\textbackslash Difference}, \texttt{\phi}=\texttt{Pi}/4), \texttt{x}=0.5)); \\
\hspace{1cm} 0.1192070524 \times 10^{-9} \\
\texttt{\textbackslash simplify} (\texttt{\textbackslash eval} (\texttt{\textbackslash eval} (\texttt{\textbackslash Difference}, \texttt{\phi}=\texttt{Pi}/4), \texttt{x}=0.9999)); \\
\hspace{1cm} 0.04501595590 \\
\texttt{\textbackslash simplify} (\texttt{\textbackslash eval} (\texttt{\textbackslash eval} (\texttt{\textbackslash Difference}, \texttt{\phi}=3*\texttt{Pi}/4), \texttt{x}=0.5)); \\
\hspace{1cm} 0.1352817016 \times 10^{-10} \\
\texttt{\textbackslash simplify} (\texttt{\textbackslash eval} (\texttt{\textbackslash eval} (\texttt{\textbackslash Difference}, \texttt{\phi}=3*\texttt{Pi}/4), \texttt{x}=0.9999)); \\
\hspace{1cm} -0.04501567673 \\
\texttt{\textbackslash simplify} (\texttt{\textbackslash eval} (\texttt{\textbackslash eval} (\texttt{\textbackslash Difference}, \texttt{\phi}=\texttt{Pi}/4), \texttt{x}=0.9999999999)); \\
\hspace{1cm} -0.6366197789e-1; \\
\hspace{1cm} 0.06366197789
\[
\begin{align*}
&0.1192070524e-9 + 0.4501595590e-1; \\
&0.1352817016e-10 - 0.4501567673e-1; \\
&-0.04501567672
\end{align*}
\]

\( n = 100, \ dc = 1, \ f = 1000, \ bg = 50x \)

\textarea{restart;}

\( w = \) sampling frequency, \( phi = \) phase offset, \( n = \# \) of samplings, \( dc = \) DC offset, \( bg = \) background light level

\textarea{w := 2*Pi*1000; \ n := 100; \ dc := 1; \ bg := 100*cos(2*Pi*50*t); \ w := 2*Pi*1000;}

\( fg := dc + \sin(x*w*t+phi); \)

\textarea{fg := dc + \sin(x*w_laser*t+phi);}

\( x = \) fraction of signal frequency to the sampling frequency

\textarea{total := fg + bg;}

\( total := 1 + \sin(2000 \pi t + \phi) + 100 \cos(100 \pi t) \)

\textarea{phase1 := int(total, t=(2*i*Pi/w .. (2*i+1)*Pi/w));}

\( phase1 := \frac{1}{2000} \left( x \pi - \cos(2 x \pi i + x \pi + \phi) + 2000 \sin(\frac{1}{20} \pi (2 i + 1)) x \right) + \cos(2 x \pi i + \phi) - 2000 \sin(\frac{1}{10} \pi i) x \right)/(x \pi) \)

\textarea{phase2 := int(total, t=(2*i+1)*Pi/w .. 2*(i+1)*Pi/w);}

\( phase2 := \frac{1}{2000} \left( x \pi - \cos(2 x \pi i + 2 x \pi + \phi) + 2000 \sin(\frac{1}{10} \pi (i + 1)) x \right) + \cos(2 x \pi i + x \pi + \phi) - 2000 \sin(\frac{1}{20} \pi (2 i + 1)) x \right)/(x \pi) \)

\textarea{Difference := sum(phase1, i=0..n-1) - sum(phase2, i=0..n-1);}

\textarea{plot(eval(Difference, phi=0), x=0.9..1.1);}
\[ f_g := d c + \sin(x \cdot w_{laser} \cdot t + \text{evaln}(\phi)) \]

\[ f_g := 1 + \sin(2000 \pi \cdot t + \phi) \]

\[ \text{plot}([\text{eval}(\text{Difference}, \phi=0), \ x=0.05 \ldots 1.95]) \]

\[ \text{plot}([\text{eval}(\text{Difference}, \phi=0), \ \text{eval}(\text{Difference}, \phi=0.25\pi), \ \text{eval}(\text{Difference}, \phi=0.5\pi), \ \text{eval}(\text{Difference}, \phi=0.75\pi), \ \text{eval}(\text{Difference}, \phi=\pi), \ \text{eval}(\text{Difference}, \phi=1.25\pi), \ \text{eval}(\text{Difference}, \phi=1.5\pi), \ \text{eval}(\text{Difference}, \phi=1.75\pi)], \ x=0.95 \ldots 1.05, \ \text{color}=[\text{black}, \text{red}, \text{pink}, \text{orange}, \text{yellow}, \text{green}, \text{blue}, \text{violet}, \text{navy}]) \]
\begin{verbatim}
> plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi), eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi), eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi), eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)], x=0.05..0.95, color=[black, red, pink, orange, yellow, green, blue, violet, navy]);

> plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi), eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi), eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi), eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)], x=0.9..1, color=[black, red, pink, orange, yellow, green, blue, violet, navy]);
\end{verbatim}
\[
\text{simplify}\left(\text{eval}\left(\text{eval}\left(\text{Difference}, \phi=0\pi\right), x=0.005\right)\right); \\
\text{.3183098862} \times 10^{-10}
\]
\[
\text{simplify}\left(\text{eval}\left(\text{eval}\left(\text{Difference}, \phi=0\pi\right), x=0.01\right)\right); \\
\text{.1400563499} \times 10^{-9}
\]
\[
\text{simplify}\left(\text{eval}\left(\text{eval}\left(\text{Difference}, \phi=0\pi\right), x=0.05\right)\right); \\
\text{-1.101352206} \times 10^{-9}
\]
\[
\text{simplify}\left(\text{eval}\left(\text{eval}\left(\text{Difference}, \phi=0\pi\right), x=0.1\right)\right); \\
\text{-3.246760839} \times 10^{-10}
\]
\[
\text{simplify}\left(\text{eval}\left(\text{eval}\left(\text{Difference}, \phi=0\pi\right), x=0.5\right)\right); \\
\text{0.}
\]
\[
\text{simplify}\left(\text{eval}\left(\text{eval}\left(\text{Difference}, \phi=0\pi\right), x=0.99\right)\right); \\
\text{-0.8636808244} \times 10^{-9}
\]
\[
\text{simplify}\left(\text{eval}\left(\text{eval}\left(\text{Difference}, \phi=0\pi\right), x=0.999\right)\right); \\
\text{0.05961466416}
\]
\[
\text{simplify}\left(\text{eval}\left(\text{eval}\left(\text{Difference}, \phi=0\pi\right), x=0.9999\right)\right); \\
\text{0.06366197789}
\]
\[
\text{simplify}\left(\text{eval}\left(\text{eval}\left(\text{Difference}, \phi=\pi/4\right), x=0.5\right)\right); \\
\text{0.1136366294} \times 10^{-9}
\]
\[
\text{simplify}\left(\text{eval}\left(\text{eval}\left(\text{Difference}, \phi=\pi/4\right), x=0.9999\right)\right); \\
\text{0.04501595588}
\]
\[
\text{simplify}\left(\text{eval}\left(\text{eval}\left(\text{Difference}, \phi=\pi/4\right), x=0.99999\right)\right); \\
\text{0.1384648004} \times 10^{-10}
\]
\[
\text{simplify}\left(\text{eval}\left(\text{eval}\left(\text{Difference}, \phi=\pi/4\right), x=0.999999\right)\right); \\
\text{0.04501567673}
\]
\[
\text{.6366197789e-1}; \\
\text{0.06366197789}
\]
n = 100, dc = 1, f = 1000, bg = 500x

> restart;
> w := sampling frequency, phi = phase offset, n = # of samplings, dc = DC offset, bg = background light level
> w_laser := 2*Pi*1000; n := 100; dc := 1; bg := 1000*cos(2*Pi*50*t);
w := 2*Pi*1000;

\[
\begin{align*}
  w_laser & := 2000 \pi \\
  n & := 100 \\
  dc & := 1 \\
  bg & := 1000 \cos(100 \pi t) \\
  w & := 2000 \pi
\end{align*}
\]

fg := dc + sin(x*w*t + phi);
> fg := dc + sin(x*w_laser*t + phi);

\[
fg := 1 + \sin(2000 \pi t + \phi)
\]

x = fraction of signal frequency to the sampling frequency

> total := fg + bg;

\[
\begin{align*}
  total & := 1 + \sin(2000 \pi t + \phi) + 1000 \cos(100 \pi t)
\end{align*}
\]

> phase1 := int(total, t = (2*i*Pi/w .. (2*i+1)*Pi/w));

\[
\begin{align*}
  phase1 & := - \frac{1}{2000} \left( -x \pi + \cos(2 \pi i + x \pi + \phi) - 20000 \sin \left( \frac{1}{20} \pi (2 i + 1) \right) x \\
  & \quad - \cos(2 \pi i + \phi) + 20000 \sin \left( \frac{1}{10} \pi i \right) x \right) / (x \pi)
\end{align*}
\]

> phase2 := int(total, t = (2*i+1)*Pi/w .. 2*(i+1)*Pi/w);

\[
\begin{align*}
  phase2 & := \frac{1}{2000} \left( x \pi - \cos(2 \pi i + 2 \pi x + \phi) + 20000 \sin \left( \frac{1}{10} \pi (i + 1) \right) x \\
  & \quad + \cos(2 \pi i + x \pi + \phi) - 20000 \sin \left( \frac{1}{20} \pi (2 i + 1) \right) x \right) / (x \pi)
\end{align*}
\]

> Difference := sum(phase1, i = 0 .. n-1) - sum(phase2, i = 0 .. n-1);
> plot(eval(Difference, phi=0), x = 0.9 .. 1.1);
\[ f_g := 1 + \sin(2000 \pi t + \phi) \]

\[ \text{plot}(\text{eval}(\text{Difference}, \phi=0), x=0.05..1.95) ; \]

\[ \text{plot}([\text{eval}(\text{Difference}, \phi=0), \text{eval}(\text{Difference}, \phi=0.25*\pi), \text{eval}(\text{Difference}, \phi=0.5*\pi), \text{eval}(\text{Difference}, \phi=0.75*\pi), \text{eval}(\text{Difference}, \phi=\pi), \text{eval}(\text{Difference}, \phi=1.25*\pi), \text{eval}(\text{Difference}, \phi=1.5*\pi), \text{eval}(\text{Difference}, \phi=1.75*\pi)], x=0.95..1.05, \text{color=[black, red, pink, orange, yellow, green, blue, violet, navy]}); \]
> plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi),
  eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi),
  eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi),
  eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)],
  x=0.05..0.95, color=[black, red, pink, orange, yellow, green, blue,
  violet, navy]);

> plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi),
  eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi),
  eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi),
  eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)],
  x=0.9..1, color=[black, red, pink, orange, yellow, green, blue, violet, navy]);
\begin{verbatim}
> simplify(eval(eval(Difference, phi=0*Pi), x=0.005));
  -.4456338406 10^{-10}
> simplify(eval(eval(Difference, phi=0*Pi), x=0.01));
  -.3501408748 10^{-10}
> simplify(eval(eval(Difference, phi=0*Pi), x=0.05));
  -.5538592018 10^{-10}
> simplify(eval(eval(Difference, phi=0*Pi), x=0.1));
  -.3405915782 10^{-10}
> simplify(eval(eval(Difference, phi=0*Pi), x=0.5));
  0.
> simplify(eval(eval(Difference, phi=0*Pi), x=0.99));
  -.8616873685 10^{-9}
> simplify(eval(eval(Difference, phi=0*Pi), x=0.999));
  .05961466421
> simplify(eval(eval(Difference, phi=0*Pi), x=0.9999));
  .06366197789
> simplify(eval(eval(Difference, phi=Pi/4), x=0.5));
  .1104535305 10^{-9}
> simplify(eval(eval(Difference, phi=Pi/4), x=0.99999999));
  .04501595588
> simplify(eval(eval(Difference, phi=3*Pi/4), x=0.5));
  .1941690306 10^{-10}
> simplify(eval(eval(Difference, phi=3*Pi/4), x=0.99999999));
  -.04501567668
> .6366197789e-1; 
  .06366197789
\end{verbatim}
1.3 Varying Number of Summation Cycles

This particular section addresses the simulation for varying number of summation cycles when foreground illumination remains constant, the encoding frequency of the foreground laser frequency is 1000 Hz and the background intensity remains constant. This set of simulation gives a measurement of the robustness of the method in rejecting background light intensity as a function of the number of integration cycles.

\[ n = 1, \; dc = 1, \; f = 1000, \; bg = 0.5 \]

```
> restart;
> w := sampling frequency, phi = phase offset, n = # of samplings, dc = DC offset, bg = background light level
> w_laser := 2*Pi*1000; n := 1; dc := 1; bg := cos(2*Pi*50*t); w := 2*Pi*1000;
> n := 1
> dc := 1
> bg := cos(100 * Pi * t)
> w := 2000 * Pi

fg := dc + sin(x * w * t + phi);
> fg := dc + sin(x * w_laser * t + phi);
> fg := 1 + sin(2000 * x * Pi * t + phi)

x = fraction of signal frequency to the sampling frequency
> total := fg + bg;
> total := 1 + sin(2000 * x * Pi * t + phi) + cos(100 * Pi * t)

> phasel := int(total, t = (2*i*Pi/w..(2*i+1)*Pi/w));
> phasel := \( \frac{1}{2000} \left( x \pi - \cos(2 x \pi i + x \pi + \phi) + 20 \sin\left(\frac{1}{20} \pi (2 i + 1)\right)x \right) + \cos(2 x \pi i + \phi) - 20 \sin\left(\frac{1}{10} \pi i\right)x/(x \pi) \)

> phase2 := int(total, t = (2*i+1)*Pi/w..2*(i+1)*Pi/w);
```
\[
\text{phase2} := \frac{1}{2000} \left( x \pi - \cos(2 \pi i + 2 \pi + \phi) + 20 \sin \left( \frac{1}{10} \pi (i + 1) \right) x \
+ \cos(2 \pi i + x \pi + \phi) - 20 \sin \left( \frac{1}{20} \pi (2 i + 1) \right) x \right) / (x \pi)
\]

\>
\text{Difference} := \text{sum} (\text{phase1}, \ i=0..n-1) - \text{sum} (\text{phase2}, \ i=0..n-1);
\>
\text{plot} (\text{eval} (\text{Difference}, \ \phi=0), \ x=0.9..1.1);

\>
\text{fg} := \text{dc} + \sin(x \times \text{w}\_\text{laser}\times t + \text{evaln(\phi)});
\>
f_g := 1 + \sin(2000 \times \pi \times t + \phi)

\>
\text{plot} (\text{eval} (\text{Difference}, \ \phi=0), \ x=0.05..1.95);

\>
\text{plot} ([\text{eval} (\text{Difference}, \ \phi=0), \ \text{eval} (\text{Difference}, \ \phi=0.25*\pi), \ \text{eval} (\text{Difference}, \ \phi=0.5*\pi), \ \text{eval} (\text{Difference}, \ \phi=0.75*\pi), \ \text{eval} (\text{Difference}, \ \phi=\pi), \ \text{eval} (\text{Difference}, \ \phi=1.25*\pi), \ \text{eval} (\text{Difference}, \ \phi=1.5*\pi), \ \text{eval} (\text{Difference}, \ \phi=1.75*\pi)], \ x=0.95..1.05, \ \text{color}=[\text{black}, \ \text{red}, \ \text{pink}, \ \text{orange}, \ \text{yellow}, \ \text{green}, \ \text{blue}, \ \text{violet}, \ \text{navy}]);
> plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi),
eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi),
eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi),
eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)],
x=0.05..0.95, color=[black, red, pink, orange, yellow, green, blue, violet, navy]);

> plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi),
eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi),
eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi),
eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)], x=0.9..1,
color=[black, red, pink, orange, yellow, green, blue, violet, navy]);
simplify(evaleval(Difference, phi=0*Pi), x=0.005));
.4408240509 10^-5
simplify(evaleval(Difference, phi=0*Pi), x=0.01));
-.3437830804 10^-5
simplify(evaleval(Difference, phi=0*Pi), x=0.05));
-.0006515239674
simplify(evaleval(Difference, phi=0*Pi), x=0.1));
-.0001359058556
simplify(evaleval(Difference, phi=0*Pi), x=0.5));
.00001226109246
simplify(evaleval(Difference, phi=0*Pi), x=0.99));
.0006548354857
simplify(evaleval(Difference, phi=0*Pi), x=0.999));
.0006495134046
simplify(evaleval(Difference, phi=0*Pi), x=0.99999999));
.0006488808712

n = 2, dc = 1, f = 1000, bg = 0.5x

> restart;
w = sampling frequency, phi = phase offset, n = # of samplings, dc = DC offset, bg = background light level
> w_laser := 2*Pi*1000; n:=2; dc:=1; bg:=cos(2*Pi*50*t); w:=2*Pi*1000;
w_laser := 2000 Pi
n := 2
dc := 1
bg := cos(100 Pi t)
\[ w = 2000 \pi \]

\[ fg := dc + \sin(x \cdot w \cdot t + \phi); \]

\[ fg := 1 + \sin(2000 \pi t + \phi) \]

\( x \) = fraction of signal frequency to the sampling frequency

\[ total := fg + bg; \]

\[ total := 1 + \sin(2000 \pi t + \phi) + \cos(100 \pi t) \]

\[ phase1 := \text{int}(total, t=(2\star i \cdot \pi / w .. (2\star i+1) \cdot \pi / w)); \]

\[ phase1 := \frac{1}{2000} \left( x \pi - \cos(2 \pi i + x \pi + \phi) + 20 \sin\left( \frac{1}{20} \pi (2 i + 1) \right) x \right) \]

\[ + \cos(2 \pi i + \phi) - 20 \sin\left( \frac{1}{10} \pi i \right) x \right) / (x \pi) \]

\[ phase2 := \text{int}(total, t=(2\star i+1) \cdot \pi / w .. 2\star (i+1) \cdot \pi / w); \]

\[ phase2 := \frac{1}{2000} \left( x \pi - \cos(2 \pi i + 2 \pi x + \phi) + 20 \sin\left( \frac{1}{10} \pi (i + 1) \right) x \right) \]

\[ + \cos(2 \pi i + x \pi + \phi) - 20 \sin\left( \frac{1}{20} \pi (2 i + 1) \right) x \right) / (x \pi) \]

\[ \text{Difference} := \text{sum}(phase1, i=0..n-1) - \text{sum}(phase2, i=0..n-1); \]

\[ \text{plot}(\text{eval}(\text{Difference}, \phi=0), x=0.9..1.1); \]

\[ fg := dc + \sin(x \cdot w \_laser \cdot t + \text{evaln}(\phi)); \]

\[ fg := 1 + \sin(2000 \pi t + \phi) \]

\[ \text{plot}(\text{eval}(\text{Difference}, \phi=0), x=0.05..1.95); \]
> plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi),
    eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi),
    eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi),
    eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)],
  x=0.95..1.05, color=[black, red, pink, orange, yellow, green, blue,
  violet, navy]);
plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi), eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi), eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi), eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)], x=0..1, color=[black, red, pink, orange, yellow, green, blue, violet, navy]);

> simplify(eval(eval(Difference, phi=0*Pi), x=0.005));
0.0003214621917

> simplify(eval(eval(Difference, phi=0*Pi), x=0.01));
0.0001650827835

> simplify(eval(eval(Difference, phi=0*Pi), x=0.05));
-0.0009940503541

> simplify(eval(eval(Difference, phi=0*Pi), x=0.1));
-0.001918949897

> simplify(eval(eval(Difference, phi=0*Pi), x=0.5));
0.0004784416841

> simplify(eval(eval(Difference, phi=0*Pi), x=0.99));
0.01330457008
simplify (eval (eval (Difference, phi=0*Pi), x=0.999));
0.001322323633

simplify (eval (eval (Difference, phi=0*Pi), x=0.99999999));
0.001321083725

\[ \text{n} = 5, \text{dc} = 1, f = 1000, \text{bg} = 0.5x \]

restart;
w = sampling frequency, phi = phase offset, n = # of samplings, dc = DC offset, bg = background light level

\[ \text{w}_{\text{laser}} := 2\pi \times 1000; \text{n} := 5; \text{dc} := 1; \text{bg} := \cos (2\pi \times 50 \times t); \text{w} := 2\pi \times 1000; \]
\[ \text{w}_{\text{laser}} := 2000 \pi \]
\[ \text{n} := 5 \]
\[ \text{dc} := 1 \]
\[ \text{bg} := \cos (100 \pi t) \]
\[ \text{w} := 2000 \pi \]

\[ \text{fg} := \text{dc} + \sin (x \times \text{w}_{\text{laser}} \times t + \phi) ; \]
\[ \text{fg} := \text{dc} + \sin (x \times \text{w}_{\text{laser}} \times t + \phi) ; \]
\[ \text{fg} := 1 + \sin (2000 \times \pi \times t + \phi) \]
\[ x = \text{fraction of signal frequency to the sampling frequency} \]
\[ \text{total} := \text{fg} + \text{bg} ; \]
\[ \text{total} := 1 + \sin (2000 \times \pi \times t + \phi) + \cos (100 \pi t) \]

\[ \text{phase1} := \text{int} (\text{total}, t = (2i \pi / \text{w}..(2i+1) \pi / \text{w})) ; \]
\[ \text{phase1} := \frac{1}{2000} \left ( x \pi - \cos (2 \pi x i + x \pi + \phi) + 20 \sin \left ( \frac{1}{20} \pi (2 i + 1) \right ) x + \cos (2 \pi i + \phi) - 20 \sin \left ( \frac{1}{10} \pi i \right ) x \right )/(x \pi) \]

\[ \text{phase2} := \text{int} (\text{total}, t = (2i+1) \pi / \text{w}..2(i+1) \pi / \text{w}) ; \]
\[ \text{phase2} := \frac{1}{2000} \left ( x \pi - \cos (2 \pi x i + 2 x \pi + \phi) + 20 \sin \left ( \frac{1}{10} \pi (i + 1) \right ) x + \cos (2 \pi i + x \pi + \phi) - 20 \sin \left ( \frac{1}{20} \pi (2 i + 1) \right ) x \right )/(x \pi) \]

\[ \text{Difference} := \text{sum} (\text{phase1}, i = 0..n-1) - \text{sum} (\text{phase2}, i = 0..n-1) ; \]
\[ \text{plot} (\text{eval} (\text{Difference}, \phi = 0), x = 0.9..1.1) ; \]
\[ f_g := \text{dc} + \sin(x \cdot w_{\text{laser}} \cdot t + \text{evaln}(\phi)) \]
\[ f_g := 1 + \sin(2000 \pi t + \phi) \]

\[ \text{plot(} \text{eval(Difference, phi=0), x=0.05..1.95);} \]

\[ \text{plot([} \text{eval(Difference, phi=0), eval(Difference, phi=0.25*Pi), eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi), eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi), eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)], x=0.95..1.05, color=[black, red, pink, orange, yellow, green, blue, violet, navy]);} \]
> plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi), eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi), eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi), eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)], x=0.05..0.95, color=[black, red, pink, orange, yellow, green, blue, violet, navy]);
\texttt{n = 10, dc = 1, f = 1000, bg = 0.5x}

\texttt{restart;}

\texttt{w = sampling frequency, phi = phase offset, n = \# of samplings, dc = DC offset, bg = background light level}

\texttt{w \_laser := 2*Pi*1000; n := 10; dc := 1; bg := cos(2*Pi*50*t); w := 2*Pi*1000; w \_laser := 2000 \pi}

\texttt{n := 10}

\texttt{dc := 1}

\texttt{bg := cos(100 \pi t)}

\texttt{w := 2000 \pi}
\[ fg := dc + \sin(x \cdot w_{\text{laser}} \cdot t + \phi) \]

\[ fg := 1 + \sin(2000 \cdot x \cdot \pi \cdot t + \phi) \]

\[ x = \text{fraction of signal frequency to the sampling frequency} \]

\[ \text{total} := fg + bg; \]

\[ \text{total} := 1 + \sin(2000 \cdot x \cdot \pi \cdot t + \phi) + \cos(100 \cdot \pi \cdot t) \]

\[ \text{phase1} := \int_{(2 \cdot i \cdot \pi / w)}^{(2 \cdot (i + 1) \cdot \pi / w)} \left( x \cdot \frac{1}{2000} \left( x \cdot \pi - \cos(2 \cdot x \cdot \pi \cdot i + x \cdot \pi + \phi) + 20 \cdot \sin \left( \frac{1}{20} \cdot \pi \cdot (2 \cdot i + 1) \right) x \right) \right) + \cos(2 \cdot x \cdot \pi \cdot i + \phi) - 20 \cdot \sin \left( \frac{1}{10} \cdot \pi \cdot i \right) x \right) / \left( x \cdot \pi \right) \]

\[ \text{phase2} := \int_{(2 \cdot i + 1) \cdot \pi / w}^{(2 \cdot (i + 1) \cdot \pi / w)} \left( x \cdot \frac{1}{2000} \left( x \cdot \pi - \cos(2 \cdot x \cdot \pi \cdot i + 2 \cdot x \cdot \pi + \phi) + 20 \cdot \sin \left( \frac{1}{10} \cdot \pi \cdot (i + 1) \right) x \right) \right) + \cos(2 \cdot x \cdot \pi \cdot i + x \cdot \pi + \phi) - 20 \cdot \sin \left( \frac{1}{10} \cdot \pi \cdot (2 \cdot i + 1) \right) x \right) / \left( x \cdot \pi \right) \]

\[ \text{Difference} := \sum_{i=0}^{n-1} \text{phase1} - \sum_{i=0}^{n-1} \text{phase2}; \]

\[ \text{plot}(\text{eval}(\text{Difference}, \phi=0), x=0..1.1); \]

\[ \text{fg} := dc + \sin(x \cdot w_{\text{laser}} \cdot t + \text{evaln(\phi)}); \]

\[ \text{fg} := 1 + \sin(2000 \cdot x \cdot \pi \cdot t + \phi) \]

\[ \text{plot}(\text{eval}(\text{Difference}, \phi=0), x=0.05..1.95); \]
> plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi),
    eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi),
    eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi),
    eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)],
   x=0.95..1.05, color=[black, red, pink, orange, yellow, green, blue, violet, navy]);
> plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi), eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi), eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi), eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)], x=0.9..1, color=[black, red, pink, orange, yellow, green, blue, violet, navy]);

> simplify(eval(eval(Difference, phi=0*Pi), x=0.005)); 0.004237748832
> simplify(eval(eval(Difference, phi=0*Pi), x=0.01)); 0.003540721833
> simplify(eval(eval(Difference, phi=0*Pi), x=0.05)); 0.005010306202
> simplify(eval(eval(Difference, phi=0*Pi), x=0.1)); 0.005010306177
> simplify(eval(eval(Difference, phi=0*Pi), x=0.5)); 0.005010306216
> simplify(eval(eval(Difference, phi=0*Pi), x=0.99)); 0.006516202266
n = 20, dc = 1, f = 1000, bg = 0.5x

> restart;
> w := sampling frequency, phi = phase offset, n = # of samplings, dc = DC offset, bg = background light level
> w_laser := 2*Pi*1000; n := 20; dc := 1; bg := cos(2*Pi*50*t); w := 2*Pi*1000;
> w_laser := 2000 \pi
> n := 20
> dc := 1
> bg := cos(100 \pi t)
> w := 2000 \pi

fg := dc + sin(x*w_laser*t + phi);
> fg := dc + sin(x*w_laser*t + phi);
> fg := 1 + sin(2000 x \pi t + \phi)

x = fraction of signal frequency to the sampling frequency
> total := fg + bg;
> total := 1 + sin(2000 x \pi t + \phi) + cos(100 \pi t)

> phase1 := int(total, t = (2*i*Pi/w .. (2*i+1)*Pi/w));
> phase1 := \frac{1}{2000} \left( x \pi - \cos(2 x \pi i + x \pi + \phi) + 20 \sin\left(\frac{1}{20} \pi (2 i + 1)\right) x 
+ \cos(2 x \pi i + \phi) - 20 \sin\left(\frac{1}{10} \pi i\right) x \right)/(x \pi)

> phase2 := int(total, t = (2*i+1)*Pi/w .. 2*(i+1)*Pi/w);
> phase2 := \frac{1}{2000} \left( x \pi - \cos(2 x \pi i + 2 x \pi + \phi) + 20 \sin\left(\frac{1}{20} \pi (i + 1)\right) x 
+ \cos(2 x \pi i + x \pi + \phi) - 20 \sin\left(\frac{1}{20} \pi (2 i + 1)\right) x \right)/(x \pi)

> Difference := sum(phase1, i = 0 .. n-1) - sum(phase2, i = 0 .. n-1);
> plot(eval(Difference, phi=0), x = 0.9 .. 1.1);
> fg := dc + sin(x * w_laser * t + evaln(phi));
   \[ fg := 1 + \sin(2000 \pi \frac{x}{t} + \phi) \]

> plot(eval(Difference, phi=0), x=0.05..1.95);

> plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi), eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi), eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi), eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)], x=0.95..1.05, color=[black, red, pink, orange, yellow, green, blue, violet, navy]);
> plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi), eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi), eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi), eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)], x=0.05..0.95, color=[black, red, pink, orange, yellow, green, blue, violet, navy]);

> plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi), eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi), eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi), eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)], x=0.9..1, color=[black, red, pink, orange, yellow, green, blue, violet, navy]);
n = 30, dc = 1, f = 1000, bg = 0.5x

> restart;

w = sampling frequency, phi = phase offset, n = # of samplings, dc = DC offset, bg = background light level

> w_laser := 2*Pi*1000; n := 30; dc := 1; bg := cos(2*Pi*50*t); w := 2*Pi*1000;

> simplify(eval(eval(Difference, phi=0*Pi), x=0.005));
-0.001469493823

> simplify(eval(eval(Difference, phi=0*Pi), x=0.01));
-0.002377836252

> simplify(eval(eval(Difference, phi=0*Pi), x=0.05));
0.6366197722 10^{-12}

> simplify(eval(eval(Difference, phi=0*Pi), x=0.1));
-0.1782535362 10^{-10}

> simplify(eval(eval(Difference, phi=0*Pi), x=0.5));
0.

> simplify(eval(eval(Difference, phi=0*Pi), x=0.99));
0.009732752211

> simplify(eval(eval(Difference, phi=0*Pi), x=0.999));
0.01271161275

> simplify(eval(eval(Difference, phi=0*Pi), x=0.99999999));
0.01273239557
\[ w := 2000 \pi \]

\[ fg := dc + \sin(x \cdot w \cdot \text{laser} \cdot t + \phi) \]

\[ fg := 1 + \sin(2000 \pi t + \phi) \]

\( x = \) fraction of signal frequency to the sampling frequency

\[ \text{total} := fg + bg \]

\[ \text{total} := 1 + \sin(2000 \pi t + \phi) + \cos(100 \pi t) \]

\[ \text{phase1} := \int_{t=(2 \cdot i \cdot \pi / w \ldots (2 \cdot i + 1) \cdot \pi / w)} \left( x \pi - \cos(2 x \pi i + x \pi + \phi) + 20 \sin\left(\frac{1}{20} \pi (2i + 1)\right) x \right) / (x \pi) \]

\[ \text{phase2} := \int_{t=(2 \cdot i + 1) \cdot \pi / w \ldots 2 \cdot (i + 1) \cdot \pi / w)} \left( x \pi - \cos(2 x \pi i + 2 x \pi + \phi) + 20 \sin\left(\frac{1}{10} \pi (i + 1)\right) x \right) / (x \pi) \]

\[ \text{Difference} := \text{sum} (\text{phase1}, i=0 \ldots n-1) - \text{sum} (\text{phase2}, i=0 \ldots n-1) \]

\[ \text{plot} (\text{eval}(\text{Difference}, \phi=0), x=0.9 \ldots 1.1) \]
> plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi),
>      eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi),
>      eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi),
>      eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)],
>      x=0.95..1.05, color=[black, red, pink, orange, yellow, green, blue, violet, navy]);
plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi),
     eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi),
     eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi),
     eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)], x=0.9..1,
color=[black, red, pink, orange, yellow, green, blue, violet, navy]);

> simplify(eval(eval(Difference, phi=0*Pi), x=0.005));
    0.002987722546

> simplify(eval(eval(Difference, phi=0*Pi), x=0.01));
    0.002632469518

> simplify(eval(eval(Difference, phi=0*Pi), x=0.05));
    0.005010306248

> simplify(eval(eval(Difference, phi=0*Pi), x=0.1));
    0.005010305152

> simplify(eval(eval(Difference, phi=0*Pi), x=0.5));
    0.005010306216

> simplify(eval(eval(Difference, phi=0*Pi), x=0.99));
    0.01023378296

> simplify(eval(eval(Difference, phi=0*Pi), x=0.999));
    0.01950571631
simplify (eval (eval (Difference, phi=0*Pi), x=0.99999999));

0.1959962398

n = 50, dc = 1, f = 1000, bg = 0.5x

> restart;
w = sampling frequency, phi = phase offset, n = # of samplings, dc = DC offset, bg = background light level

> w_laser:=2*Pi*1000; n:=50; dc:=1; bg:=cos(2*Pi*50*t); w:=2*Pi*1000;
w_laser := 2000 π

n := 50

dc := 1

bg := cos(100 π t)

w := 2000 π

fg:=dc+sin(x*w*t+phi);

fg := 1 + sin(2000 x π t + φ)

x = fraction of signal frequency to the sampling frequency

> total:=fg+bg;

total := 1 + sin(2000 x π t + φ) + cos(100 π t)

> phasel:=int(total, t=(2*i*Pi/w .. (2*i+1)*Pi/w));

phase1 := 1/2000 (x π - cos(2 x π i + x π + φ) + 20 sin(1/20 π (2 i + 1)) x

+ cos(2 x π i + φ) - 20 sin(1/10 π i) x)/(x π)

> phase2:=int(total, t=(2*i+1)*Pi/w .. 2*(i+1)*Pi/w);

phase2 := -1/2000 (-x π + cos(2 x π i + 2 x π + φ) - 20 sin(1/10 π (i + 1)) x

- cos(2 x π i + x π + φ) + 20 sin(1/20 π (2 i + 1)) x)/(x π)

> Difference:=sum(phasel, i=0..n-1)-sum(phase2, i=0..n-1);

> plot(eval(Difference, phi=0), x=0.9..1.1);
\[ f_\text{g} := d + \sin(x \cdot w_\text{laser} \cdot t + \text{evaln}(\phi)) \]
\[ f_g := 1 + \sin(2000 \pi t + \phi) \]

> plot(eval(Difference, phi=0), x=0.05..1.95);
\begin{verbatim}
plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi),
eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi),
eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi),
eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)],
x=0.05..0.95, color=[black, red, pink, orange, yellow, green, blue,
violet, navy]);
\end{verbatim}
\[
\text{\textbf{w laser :=} } 2\pi \times 1000; \quad n := 100; \quad dc := 1; \quad bg := \cos(2\pi \times 50 \times t); \quad w := 2\pi \times 1000; \quad n := 100; \quad dc := 1; \quad bg := \cos(100 \pi t); \quad w := 2000 \pi
\]
\[ f_g := d_c + \sin(x \cdot w \cdot t + \phi) \]

\[ f_g := 1 + \sin(2000 \cdot x \cdot \pi \cdot t + \phi) \]

\[ x = \text{fraction of signal frequency to the sampling frequency} \]

\[ \text{total} := f_g + b_g \]

\[ \text{total} := 1 + \sin(2000 \cdot x \cdot \pi \cdot t + \phi) + \cos(100 \cdot \pi \cdot t) \]

\[ \text{phase1} := \int(\text{total}, t=(2 \cdot i \cdot \pi / w \ldots (2 \cdot i+1) \cdot \pi / w)) \]

\[ \text{phase1} := \frac{1}{2000} \left( x \cdot \pi - \cos(2 \cdot x \cdot \pi \cdot i + x \cdot \pi + \phi) + 20 \sin\left( \frac{1}{20} \cdot \pi \cdot (2 \cdot i + 1) \right) \cdot x \right) + \cos(2 \cdot x \cdot \pi \cdot i + \phi) - 20 \sin\left( \frac{1}{10} \cdot \pi \cdot i \right) \cdot x / (x \cdot \pi) \]

\[ \text{phase2} := \int(\text{total}, t=(2 \cdot i+1) \cdot \pi / w \ldots 2 \cdot (i+1) \cdot \pi / w) \]

\[ \text{phase2} := \frac{1}{2000} \left( x \cdot \pi - \cos(2 \cdot x \cdot \pi \cdot i + 2 \cdot x \cdot \pi + \phi) + 20 \sin\left( \frac{1}{10} \cdot \pi \cdot (i + 1) \right) \cdot x \right) + \cos(2 \cdot x \cdot \pi \cdot i + x \cdot \pi + \phi) - 20 \sin\left( \frac{1}{20} \cdot \pi \cdot (2 \cdot i + 1) \right) \cdot x / (x \cdot \pi) \]

\[ \text{Difference} := \sum(\text{phase1}, i=0 \ldots n-1) - \sum(\text{phase2}, i=0 \ldots n-1) \]

\[ \text{plot}(\text{eval}(\text{Difference}, \phi=0), x=0.9 \ldots 1.1) ; \]

\[ \text{plot}(\text{eval}(\text{Difference}, \phi=0), x=0.9 \ldots 1.1) ; \]
plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi),
eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi),
eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi),
eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)],
x=0.95..1.05, color=[black, red, pink, orange, yellow, green, blue,
violet, navy]);

plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi),
eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi),
eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi),
eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)],
x=0.05..0.95, color=[black, red, pink, orange, yellow, green, blue,
violet, navy]);
\begin{verbatim}
> plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi),
    eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi),
    eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi),
    eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)],
    x=0.9..1, color=[black, red, pink, orange, yellow, green, blue, violet, navy]);

> simplify(eval(eval(Difference, phi=0*Pi), x=0.005));
  -1.336901522 \times 10^{-9}

> simplify(eval(eval(Difference, phi=0*Pi), x=0.01));
  0.2037183271 \times 10^{-9}

> simplify(eval(eval(Difference, phi=0*Pi), x=0.05));
  -0.6493521678 \times 10^{-10}

> simplify(eval(eval(Difference, phi=0*Pi), x=0.1));
  -0.6716338598 \times 10^{-10}

> simplify(eval(eval(Difference, phi=0*Pi), x=0.5));
  0.

> simplify(eval(eval(Difference, phi=0*Pi), x=0.99));
\end{verbatim}
n = 500, dc = 1, f = 1000, bg = 0.5x

> restart;
> w := sampling frequency, phi = phase offset, n = # of samplings, dc = DC offset, bg = background light level
> w_laser := 2*Pi*1000; n := 500; dc := 1; bg := cos(2*Pi*50*t); w := 2*Pi*1000;
> Difference := sum(phasel, i = 0 .. n-1) - sum(phase2, i = 0 .. n-1):
> plot(eval(Difference, phi = 0), x = 0.9 .. 1.1);
\[ f_g := d_c + \sin(x \cdot \text{laser} \cdot t + \text{evaln}(\phi)); \]
\[ f_g := 1 + \sin(2000 \pi t + \phi) \]

\[ \text{plot} (\text{eval}(\text{Difference}, \phi=0), x=0.05..1.95); \]

\[ \text{plot} ([\text{eval}(\text{Difference}, \phi=0), \text{eval}(\text{Difference}, \phi=0.25*\pi), \text{eval}(\text{Difference}, \phi=0.5*\pi), \text{eval}(\text{Difference}, \phi=0.75*\pi), \text{eval}(\text{Difference}, \phi=\pi), \text{eval}(\text{Difference}, \phi=1.25*\pi), \text{eval}(\text{Difference}, \phi=1.5*\pi), \text{eval}(\text{Difference}, \phi=1.75*\pi)], x=0.95..1.05, \text{color}=[\text{black, red, pink, orange, yellow, green, blue, violet, navy}]); \]
1.4 Varying Background Intensity Level when Number of Summation Cycles is 2 and Linear Frequency of Foreground Light is 100 Hz

This particular section addresses the simulation for the fundamental mode of the D-APS, that is when the number of summation cycles is 2, which is the minimum theoretically necessary to carry out any sort of subtraction while the foreground light frequency is at 100 Hz, which is the lower end of the frequency so that any speed
considerations of the D-APS will be diminished and the operation of the D-APS theoretically and practically will be at its simplest level. Given these conditions, if this case succeeds, then the concept of the D-APS will show promise and will therefore be worthy of more investigation. This is the special fundamental case in which the experiments in this thesis will be based upon. The simulation addresses the case when the background intensity is varied up to the level of the foreground intensity.

\[ n = 2, \ dc = 1, \ f = 1000, \ bg = 0 \]

```
> restart;

\> \text{w} = \text{sampling frequency,} \ \phi = \text{phase offset,} \ n = \# \text{of samplings,} \ dc = \text{DC offset,} \ bg = \text{background light level}

\> \text{w} = 2 \ \pi \cdot 100; \ n := 2; \ dc := 1; \ bg := 0 \star \cos(2 \star \pi \cdot 50 \cdot t); \ w := 2 \star \pi \cdot 100;

\> \text{w} \_ \text{laser} := 200 \ \pi

\> n := 2

\> dc := 1

\> bg := 0

\> w := 200 \ \pi

\> \text{fg} := dc + \sin(x \cdot w \_ \text{laser} \cdot t + \phi);

\> \text{fg} := dc + \sin(x \cdot \text{w} \_ \text{laser} \cdot t + \phi);

\> \text{fg} := 1 + \sin(200 \ x \ \pi \ t + \phi)

\> x = \text{fraction of signal frequency to the sampling frequency}

\> \text{total} := \text{fg} + \text{bg};

\> \text{total} := 1 + \sin(200 \ x \ \pi \ t + \phi)

\> \text{phasel} := \int(\text{total}, \ t=(2 \star i \ \pi / w .. (2 \star i + 1) \ \pi / w));

\> \text{phasel} := \frac{1}{200} \ x \ \pi - \cos(2 \ x \ \pi \ i + x \ \pi + \phi) + \cos(2 \ x \ \pi \ i + \phi)

\> \text{phase2} := \int(\text{total}, \ t=(2 \star (i+1) \ \pi / w .. 2 \star (i+1) \ \pi / w));

\> \text{phase2} := \frac{1}{200} \ x \ \pi - \cos(2 \ x \ \pi \ i + 2 \ x \ \pi + \phi) + \cos(2 \ x \ \pi \ i + x \ \pi + \phi)

\> \text{Difference} := \text{sum(phasel,} \ i=0 .. n-1) - \text{sum(phase2,} \ i=0 .. n-1);

\> \text{plot(eval(Difference,} \ \phi=0), \ x=0.9 .. 1.1);
```
\[ f_g := 1 + \sin(200 \pi t + \phi) \]

```
> plot(eval(Difference, phi=0), x=0.05..1.95);
```

```
> plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi), eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi), eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi), eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)], x=0.95..1.05, color=[black, red, pink, orange, yellow, green, blue, violet, navy]);
```
> plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi), eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi), eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi), eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)], x=0.05..0.95, color=[black, red, pink, orange, yellow, green, blue, violet, navy]);

> plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi), eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi), eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi), eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)], x=0.9..1, color=[black, red, pink, orange, yellow, green, blue, violet, navy]);
\[\text{simplify } \text{eval}\left(\text{eval}\left(\text{Difference, \phi=0*Pi}\right), x=0.005\right)\]; \quad -0.001569794860

\[\text{simplify } \text{eval}\left(\text{eval}\left(\text{Difference, \phi=0*Pi}\right), x=0.01\right)\]; \quad -0.003133588464

\[\text{simplify } \text{eval}\left(\text{eval}\left(\text{Difference, \phi=0*Pi}\right), x=0.05\right)\]; \quad -0.001472492038

\[\text{simplify } \text{eval}\left(\text{eval}\left(\text{Difference, \phi=0*Pi}\right), x=0.1\right)\]; \quad -0.002397391559

\[\text{simplify } \text{eval}\left(\text{eval}\left(\text{Difference, \phi=0*Pi}\right), x=0.5\right)\]; \quad 0.

\[\text{simplify } \text{eval}\left(\text{eval}\left(\text{Difference, \phi=0*Pi}\right), x=0.99\right)\]; \quad 0.01282612839

\[\text{simplify } \text{eval}\left(\text{eval}\left(\text{Difference, \phi=0*Pi}\right), x=0.999\right)\]; \quad 0.01274479467

\[\text{simplify } \text{eval}\left(\text{eval}\left(\text{Difference, \phi=0*Pi}\right), x=0.9999\right)\]; \quad 0.01273239557

\[\text{simplify } \text{eval}\left(\text{eval}\left(\text{Difference, \phi=Pi/4}\right), x=0.5\right)\]; \quad -0.1591549430 \times 10^{-11}

\[\text{simplify } \text{eval}\left(\text{eval}\left(\text{Difference, \phi=Pi/4}\right), x=0.99\right)\]; \quad 0.009003163815

\[\text{simplify } \text{eval}\left(\text{eval}\left(\text{Difference, \phi=Pi/4}\right), x=0.999\right)\]; \quad 0.4774648292 \times 10^{-11}

\[\text{simplify } \text{eval}\left(\text{eval}\left(\text{Difference, \phi=Pi/4}\right), x=0.9999\right)\]; \quad -0.009003163130

\[0.1273239557e-1; \quad 0.01273239557\]
n = 2, dc = 1, f = 1000, bg = 0.5x

w = sampling frequency, phi = phase offset, n = # of samplings, dc = DC offset, bg = background light level

w_laser := 2*Pi*100; n := 2; dc := 1; bg := 10000/20000*2*cos(2*Pi*50*t);
w := 2*Pi*100;

fg := dc + sin(x*w*t + phi);

x = fraction of signal frequency to the sampling frequency

total := fg + bg;

phase1 := int (total, t = (2*i*Pi/w .. (2*i+1)*Pi/w));

phase2 := int (total, t = (2*i*Pi/w .. 2*(i+1)*Pi/w));

Difference := sum (phase1, i = 0 .. n-1) - sum (phase2, i = 0 .. n-1);

plot (eval (Difference, phi = 0), x = 0 .. 1.1);
\[ f_g := d_c + \sin(x \cdot w_{\text{laser}} \cdot t + \text{evaln}(\phi)) \]

\[ f_g := 1 + \sin(200 \pi t + \phi) \]

\[ \text{plot(eval(Difference, \phi=0), x=0.05..1.95)}; \]

\[ \text{plot([eval(Difference, \phi=0), eval(Difference, \phi=0.25*Pi), eval(Difference, \phi=0.5*Pi), eval(Difference, \phi=0.75*Pi), eval(Difference, \phi=Pi), eval(Difference, \phi=1.25*Pi), eval(Difference, \phi=1.5*Pi), eval(Difference, \phi=1.75*Pi)], x=0.95..1.05, color=[black, red, pink, orange, yellow, green, blue, violet, navy]);} \]
\[
\begin{align*}
\phi &= 0.25 \pi, \\
\phi &= 0.75 \pi, \\
\phi &= 1.25 \pi, \\
\phi &= 1.75 \pi.
\end{align*}
\]

\[
\text{plot}([\text{eval}(\text{Difference}, \phi=0), \text{eval}(\text{Difference}, \phi=0.25*\pi), \\
\text{eval}(\text{Difference}, \phi=0.5*\pi), \text{eval}(\text{Difference}, \phi=0.75*\pi), \\
\text{eval}(\text{Difference}, \phi=\pi), \text{eval}(\text{Difference}, \phi=1.25*\pi), \\
\text{eval}(\text{Difference}, \phi=1.5*\pi), \text{eval}(\text{Difference}, \phi=1.75*\pi)], \\
x=0.05..0.95, \text{color}=[\text{black, red, pink, orange, yellow, green, blue, violet, navy}]);
\]

\[
\begin{align*}
0.01 & \quad 0.005 & \quad 0.001 & \quad 0 & \quad -0.001 & \quad -0.005 & \quad -0.01 \\
0.2 & \quad 0.4 & \quad 0.6 & \quad 0.8 &
\end{align*}
\]

\[
\text{plot}([\text{eval}(\text{Difference}, \phi=0), \text{eval}(\text{Difference}, \phi=0.25*\pi), \\
\text{eval}(\text{Difference}, \phi=0.5*\pi), \text{eval}(\text{Difference}, \phi=0.75*\pi), \\
\text{eval}(\text{Difference}, \phi=\pi), \text{eval}(\text{Difference}, \phi=1.25*\pi), \\
\text{eval}(\text{Difference}, \phi=1.5*\pi), \text{eval}(\text{Difference}, \phi=1.75*\pi)], \\
x=0.9..1, \text{color}=[\text{black, red, pink, orange, yellow, green, blue, violet, navy}]);
\]
simplify (eval (eval (Difference , phi=0*Pi) , x=0.005));
-0.001569794860

simplify (eval (eval (Difference , phi=0*Pi) , x=0.01));
-0.003133588464

simplify (eval (eval (Difference , phi=0*Pi) , x=0.05));
-0.001472492038

simplify (eval (eval (Difference , phi=0*Pi) , x=0.1));
-0.002397391559

simplify (eval (eval (Difference , phi=0*Pi) , x=0.5));
0.

simplify (eval (eval (Difference , phi=0*Pi) , x=0.99));
0.01282612839

simplify (eval (eval (Difference , phi=0*Pi) , x=0.999));
0.01274479467

simplify (eval (eval (Difference , phi=0*Pi) , x=0.9999));
0.01273239557

simplify (eval (eval (Difference , phi=Pi/4) , x=0.5));
-1591549430 \times 10^{-11}

simplify (eval (eval (Difference , phi=Pi/4) , x=0.99999999));
0.009003163815

simplify (eval (eval (Difference , phi=3*Pi/4) , x=0.5));
4774648292 \times 10^{-11}

simplify (eval (eval (Difference , phi=3*Pi/4) , x=0.99999999));
-0.009003162685

.1273239557e-1;
0.01273239557
\[ n = 2, \text{dc} = 1, f = 1000, \text{bg} = 0.85x \]

```maple
restart;

w = \text{sampling frequency}, \phi = \text{phase offset}, n = \# of samplings, dc = \text{DC offset}, bg = \text{background light level}

\[ w_{\text{laser}} := 2\pi \times 100; \quad n := 2; \quad \text{dc} := 1; \quad \text{bg} := \frac{17000}{20000} \times 2 \times \cos(2\pi \times 50 \times t); \]

\[ w := 2\pi \times 100; \]

\[ \text{fg} := \text{dc} + \sin(x \times w \times t + \phi); \]

\[ \text{fg} := \text{de} + \sin(x \times w_{\text{laser}} \times t + \phi); \]

\[ \text{total} := \text{fg} + \text{bg}; \]

\[ \text{phase1} := \int (\text{total}, t = (2i\pi/w .. (2i+1)\pi/w)); \]

\[ \text{phase2} := \int (\text{total}, t = (2i+1)\pi/w .. 2(i+1)\pi/w); \]

\[ \text{Difference} := \sum (\text{phase1}, i = 0 .. n-1) - \sum (\text{phase2}, i = 0 .. n-1); \]

\[ \text{plot(eval(Difference, \phi = 0), \text{x} = 0.9 .. 1.1);} \]
```
\[ f_g := d_c + \sin(x \cdot w_{\text{laser}} \cdot t + \text{evaln}(\phi)) \]

\[ f_g := 1 + \sin(200 \cdot \pi \cdot t + \phi) \]

\[ \text{plot} (\text{eval}(\text{Difference}, \phi=0), x=0.05 \ldots 1.95) ; \]

\[ \text{plot} ([\text{eval}(\text{Difference}, \phi=0), \text{eval}(\text{Difference}, \phi=0.25\pi), \text{eval}(\text{Difference}, \phi=0.5\pi), \text{eval}(\text{Difference}, \phi=0.75\pi), \text{eval}(\text{Difference}, \phi=\pi), \text{eval}(\text{Difference}, \phi=1.25\pi), \text{eval}(\text{Difference}, \phi=1.5\pi), \text{eval}(\text{Difference}, \phi=1.75\pi)], x=0.95 \ldots 1.05, \text{color}=[\text{black}, \text{red}, \text{pink}, \text{orange}, \text{yellow}, \text{green}, \text{blue}, \text{violet}, \text{navy}] ) ; \]
plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi), eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi), eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi), eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)], x=0.05..0.95, color=[black, red, pink, orange, yellow, green, blue, violet, navy]);
> simplify(eval(eval(Difference, phi=0*Pi), x=0.005));
  -0.001569794860
> simplify(eval(eval(Difference, phi=0*Pi), x=0.01));
  -0.003133588464
> simplify(eval(eval(Difference, phi=0*Pi), x=0.05));
  -0.00147292038
> simplify(eval(eval(Difference, phi=0*Pi), x=0.1));
  -0.002397391559
> simplify(eval(eval(Difference, phi=0*Pi), x=0.5));
  0.
> simplify(eval(eval(Difference, phi=0*Pi), x=0.99));
  0.01282612839
> simplify(eval(eval(Difference, phi=0*Pi), x=0.999));
  0.01274479467
> simplify(eval(eval(Difference, phi=0*Pi), x=0.9999));
  0.01273239557
> simplify(eval(eval(Difference, phi=0*Pi), x=0.99999));
  0.01273239557
> simplify(eval(eval(Difference, phi=Pi/4), x=0.5));
  -1.591549430 \times 10^{-11}
> simplify(eval(eval(Difference, phi=Pi/4), x=0.99999999));
  0.009003163815
> simplify(eval(eval(Difference, phi=3*Pi/4), x=0.5));
  4.774648292 \times 10^{-11}
> simplify(eval(eval(Difference, phi=3*Pi/4), x=0.99999999));
  -0.009003162685
> 0.1273239557e-1;
  0.01273239557
n = 2, dc = 1, f = 1000, bg = 0.95x

```maple
restart;
w = sampling frequency, phi = phase offset, n = # of samplings, dc = DC offset, bg = background light level

> w_laser := 2*Pi*100; n := 2; dc := 1; bg := 19000/20000*2*cos(2*Pi*50*t);
w := 2*Pi*100;

fg := dc + sin(x*w_laser*t + phi);
> fg := dc + sin(x*w_laser*t + phi);
f_g := 1 + sin(200 x pi t + phi)

x = fraction of signal frequency to the sampling frequency
> total := fg + bg;

total := 1 + sin(200 x pi t + phi) + 19/10 cos(100 pi t)

> phase1 := int(total, t=(2*i*Pi/w .. (2*i+1)*Pi/w));

phase1 := (1/1000) (5 x pi - 5 cos(2 x pi i + x pi + phi) + 19 sin(1/2 pi (2 i + 1)) x

+ 5 cos(2 x pi i + phi) - 19 sin(pi i) x) / (x pi)

> phase2 := int(total, t=(2*i+1)*Pi/w .. 2*(i+1)*Pi/w);

phase2 := (1/1000) (5 x pi - 5 cos(2 x pi i + 2 x pi + phi) + 19 sin(pi (i + 1)) x

+ 5 cos(2 x pi i + x pi + phi) - 19 sin(1/2 pi (2 i + 1)) x) / (x pi)

> Difference := sum(phase1, i=0 .. n-1) - sum(phase2, i=0 .. n-1);

> plot(eval(Difference, phi=0), x=0.9 .. 1.1);
```
\[ f_g := d c + \sin(x \cdot \text{w\_laser} \cdot t + \text{eval(n)}(\phi)) \]

\[ f_g := 1 + \sin(200 \pi \cdot t + \phi) \]

\[ \text{plot(eval(Difference, phi=0), x=0.05..1.05)}; \]

\[ \text{plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi), eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi), eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi), eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)], x=0.95..1.05, color=[black, red, pink, orange, yellow, green, blue, violet, navy]);} \]
> plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi),
    eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi),
    eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi),
    eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)],
  x=0.05..0.95, color=[black, red, pink, orange, yellow, green, blue,
    violet, navy]);

> plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi),
    eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi),
    eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi),
    eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)], x=0.9 .. 1,
  color=[black, red, pink, orange, yellow, green, blue, violet, navy]);
> simplify(eval(eval(Difference, phi=0*Pi), x=0.005));
> simplify(eval(eval(Difference, phi=0*Pi), x=0.01));
> simplify(eval(eval(Difference, phi=0*Pi), x=0.05));
> simplify(eval(eval(Difference, phi=0*Pi), x=0.1));
> simplify(eval(eval(Difference, phi=0*Pi), x=0.5));
> simplify(eval(eval(Difference, phi=0*Pi), x=0.99));
> simplify(eval(eval(Difference, phi=0*Pi), x=0.999));
> simplify(eval(eval(Difference, phi=0*Pi), x=0.99999999));
> simplify(eval(eval(Difference, phi=Pi/4), x=0.5));
> simplify(eval(eval(Difference, phi=Pi/4), x=0.99999999));
> simplify(eval(eval(Difference, phi=3*Pi/4), x=0.5));
> simplify(eval(eval(Difference, phi=3*Pi/4), x=0.99999999));
> .1273239557e-1;
> .01273239557
n = 2, dc = 1, f = 1000, bg = 0.965x

> restart;
> w := sampling frequency, phi = phase offset, n = # of samplings, dc = DC offset, bg = background light level
> w_laser := 2*Pi*100; n := 2; dc := 1; bg := 19300/20000*2*cos(2*Pi*50*t);
w := 2*Pi*100;

> fg := dc + sin(x*w_laser*t + phi);
> fg := dc + sin(x*w_laser*t + phi);

> total := fg + bg;
> total := 1 + sin(200 x pi t + phi) + 193/100 cos(100 pi t)

> phase1 := int(total, t=(2*i*Pi/w .. (2*i+1)*Pi/w));
> phase1 := 1/10000 (50 x pi - 50 cos(2 x pi i + x pi + phi) + 193 sin(1/2 pi (2 i + 1)) x
+ 50 cos(2 x pi i + phi) - 193 sin(pi i) x)/(x pi)

> phase2 := int(total, t=(2*i+1)*Pi/w .. 2*(i+1)*Pi/w);
> phase2 := 1/10000 (50 x pi - 50 cos(2 x pi i + 2 x pi + phi) + 193 sin(pi (i + 1)) x
+ 50 cos(2 x pi i + x pi + phi) - 193 sin(1/2 pi (2 i + 1)) x)/(x pi)

> Difference := sum(phase1, i=0 .. n-1) - sum(phase2, i=0 .. n-1);
> plot( eval( Difference, phi=0 ), x=0.9 .. 1.1 );
\[ f_g := d + \sin(x \cdot w_{\text{Laser}} \cdot t + \text{eval}\phi) ; \]

\[ f_g := 1 + \sin(200 \cdot \pi \cdot t + \phi) \]

\[ \text{plot (eval (\text{Difference}, \phi = 0), x = 0.05 .. 1.95)} ; \]

\[ \text{plot ([eval (\text{Difference}, \phi = 0), eval (\text{Difference}, \phi = 0.25 \cdot \pi), eval (\text{Difference}, \phi = 0.5 \cdot \pi), eval (\text{Difference}, \phi = 0.75 \cdot \pi), eval (\text{Difference}, \phi = \pi), eval (\text{Difference}, \phi = 1.25 \cdot \pi), eval (\text{Difference}, \phi = 1.5 \cdot \pi), eval (\text{Difference}, \phi = 1.75 \cdot \pi)], x = 0.95 .. 1.05, color = [\text{black, red, pink, orange, yellow, green, blue, violet, navy}]) ;} \]
> plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi), eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi), eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi), eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)], x=0.05..0.95, color=[black, red, pink, orange, yellow, green, blue, violet, navy]);

> plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi), eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi), eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi), eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)], x=0.9..1, color=[black, red, pink, orange, yellow, green, blue, violet, navy]);
> simplify(eval(eval(Difference, phi=0*Pi), x=0.005));
> -0.001569794860

> simplify(eval(eval(Difference, phi=0*Pi), x=0.01));
> -0.0003133588464

> simplify(eval(eval(Difference, phi=0*Pi), x=0.05));
> -0.001472492038

> simplify(eval(eval(Difference, phi=0*Pi), x=0.1));
> -0.002397391559

> simplify(eval(eval(Difference, phi=0*Pi), x=0.5));
> 0.

> simplify(eval(eval(Difference, phi=0*Pi), x=0.99));
> 0.1282612839

> simplify(eval(eval(Difference, phi=0*Pi), x=0.999));
> 0.1274479467

> simplify(eval(eval(Difference, phi=0*Pi), x=0.9999));
> 0.1273239557

> simplify(eval(Difference, phi=Pi/4), x=0.5));
> -0.1591549430 \times 10^{-11}

> simplify(eval(Difference, phi=Pi/4), x=0.99999999));
> 0.009003163815

> simplify(eval(Difference, phi=3*Pi/4), x=0.5));
> 0.4774648292 \times 10^{-11}

> simplify(eval(Difference, phi=3*Pi/4), x=0.99999999));
> -0.009003162685

> .1273239557e-1;
> 0.1273239557
\[ n = 2, \text{dc} = 1, f = 1000, \text{bg} = 1x \]

\[ \text{restart;} \]

\( w = \) sampling frequency, \( \phi = \) phase offset, \( n = \) # of samplings, \( \text{dc} = \) DC offset, \( \text{bg} = \) background light level

\[ \text{laser} := 2 \pi \times 100; \quad n := 2; \quad \text{dc} := 1; \quad \text{bg} := \cos(2 \pi \times 50 \times t); \quad w := 2 \pi \times 100; \]

\[ \text{laser} := 200 \pi \]

\[ n := 2 \]

\[ \text{dc} := 1 \]

\[ \text{bg} := \cos(100 \pi t) \]

\[ w := 200 \pi \]

\[ \text{fg} := \text{dc} + \sin(x \times \text{laser} \times t + \phi); \]

\[ \text{fg} := \text{dc} + \sin(x \times \text{laser} \times t + \phi); \]

\[ \text{fg} := 1 + \sin(200 \times \pi \times t + \phi) \]

\[ x = \text{fraction of signal frequency to the sampling frequency} \]

\[ \text{total} := \text{fg} + \text{bg}; \]

\[ \text{total} := 1 + \sin(200 \times \pi \times t + \phi) + \cos(100 \times \pi \times t) \]

\[ \text{phasel} := \text{int}(\text{total}, \quad t = (2i \times \pi / w .. (2i+1) \times \pi / w)); \]

\[ \text{phasel} := \frac{1}{200} \left[ x \pi - \cos(2 \times \pi \times i + x \pi + \phi) + 2 \sin\left(\frac{1}{2} \pi \times (2 \times i + 1)\right) x + \cos(2 \times \pi \times i + \phi) \right] - x \pi \]

\[ \text{phase2} := \frac{1}{200} \left[ x \pi - \cos(2 \times \pi \times i + 2 \times \pi \times \phi) + 2 \sin(\pi \times (i + 1)) x \right. \]

\[ + \cos(2 \times \pi \times i + x \pi + \phi) - 2 \sin\left(\frac{1}{2} \pi \times (2 \times i + 1)\right) x \] \]

\[ \text{Difference} := \text{sum}((\text{phasel}, \quad i = 0 .. n-1) - \text{sum}((\text{phase2}, \quad i = 0 .. n-1)); \]

\[ \text{plot}((\text{eval}(\text{Difference}, \quad \phi = 0), \quad x = 0.9 .. 1.1)); \]
\( f_g := d_c + \sin(x \cdot w_{\text{laser}} \cdot t + \text{evaln}(\phi)) \)

\( f_g := 1 + \sin(200 \cdot x \cdot \pi \cdot t + \phi) \)

\[
> \text{plot}(\text{eval}(\text{Difference}, \phi = 0), x = 0.05 .. 1.95);
\]

\[
> \text{plot}([\text{eval}(\text{Difference}, \phi = 0), \text{eval}(\text{Difference}, \phi = 0.25*\pi), \text{eval}(\text{Difference}, \phi = 0.5*\pi), \text{eval}(\text{Difference}, \phi = 0.75*\pi), \text{eval}(\text{Difference}, \phi = \pi), \text{eval}(\text{Difference}, \phi = 1.25*\pi), \text{eval}(\text{Difference}, \phi = 1.5*\pi), \text{eval}(\text{Difference}, \phi = 1.75*\pi)], x = 0.95 .. 1.05, \text{color} = [\text{black}, \text{red}, \text{pink}, \text{orange}, \text{yellow}, \text{green}, \text{blue}, \text{violet}, \text{navy}]);
\]
> plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi),
      eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi),
      eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi),
      eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)],
     x=0.05..0.95, color=[black, red, pink, orange, yellow, green, blue,
     violet, navy]);

> plot([eval(Difference, phi=0), eval(Difference, phi=0.25*Pi),
      eval(Difference, phi=0.5*Pi), eval(Difference, phi=0.75*Pi),
      eval(Difference, phi=Pi), eval(Difference, phi=1.25*Pi),
      eval(Difference, phi=1.5*Pi), eval(Difference, phi=1.75*Pi)],
     x=0.9..1, color=[black, red, pink, orange, yellow, green, blue, violet, navy]);
\begin{align*}
&> \text{simplify(} \text{eval(} \text{eval(}\text{Difference}, \phi=0*\pi) , x=0.005)\text{)}, \text{)}; \\
&\quad -0.001569794860 \\
&> \text{simplify(} \text{eval(} \text{eval(}\text{Difference}, \phi=0*\pi) , x=0.01)\text{)}, \text{)}; \\
&\quad -0.003133588464 \\
&> \text{simplify(} \text{eval(} \text{eval(}\text{Difference}, \phi=0*\pi) , x=0.05)\text{)}, \text{)}; \\
&\quad -0.001472492038 \\
&> \text{simplify(} \text{eval(} \text{eval(}\text{Difference}, \phi=0*\pi) , x=0.1)\text{)}, \text{)}; \\
&\quad -0.002397391559 \\
&> \text{simplify(} \text{eval(} \text{eval(}\text{Difference}, \phi=0*\pi) , x=0.5)\text{)}, \text{)}; \\
&\quad 0.0 \\
&> \text{simplify(} \text{eval(} \text{eval(}\text{Difference}, \phi=0*\pi) , x=0.99)\text{)}, \text{)}; \\
&\quad 0.01282612839 \\
&> \text{simplify(} \text{eval(} \text{eval(}\text{Difference}, \phi=0*\pi) , x=0.999)\text{)}, \text{)}; \\
&\quad 0.01274479467 \\
&> \text{simplify(} \text{eval(} \text{eval(}\text{Difference}, \phi=0*\pi) , x=0.9999)\text{)}, \text{)}; \\
&\quad 0.01273239557 \\
&> \text{simplify(} \text{eval(} \text{eval(}\text{Difference}, \phi=\pi/4) , x=0.5)\text{)}, \text{)}; \\
&\quad -1.591549430 \times 10^{-11} \\
&> \text{simplify(} \text{eval(} \text{eval(}\text{Difference}, \phi=\pi/4) , x=0.9999999)\text{)}, \text{)}; \\
&\quad 0.009003163815 \\
&> \text{simplify(} \text{eval(} \text{eval(}\text{Difference}, \phi=3*\pi/4) , x=0.5)\text{)}, \text{)}; \\
&\quad 0.4774648292 \times 10^{-11} \\
&> \text{simplify(} \text{eval(} \text{eval(}\text{Difference}, \phi=3*\pi/4) , x=0.999999999)\text{)}, \text{)}; \\
&\quad -0.009003163130 \\
&> 0.1273239557e-1; \\
&\quad 0.01273239557
\end{align*}
\[-.1591549430e-11 + .9003163815e-2;\]
\[.009003163813\]
\[-.4774648292e-11 -.9003163130e-2;\]
\[-.009003163125\]
Appendix 2: Graphical Summary of the Above Simulations

2.1 Varying Background Level (n = 100, dc = 1, f = 1000)

Figure 84: \(bg = \cos(2\pi 501t)\), Simulated response of double sided APS to light of frequencies varying with the laser encoding signal. This shows the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light in phase with the laser (a) signal range 0 to 2 times encoding frequency, (b) detail of signals 0.9 to 1.1 of encoding frequency.

Figure 85: \(bg = \cos(2\pi 501t)\), Simulated response of dual-output APS shows (a) the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light when not in phase with the laser encoding frequency and signal range ratio from 0 to 1, (b) detail of signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05.
Figure 86: $bg = 10 \cos (2\pi 50t)$, Simulated response of double sided APS to light of frequencies varying with the laser encoding signal. This shows the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light in phase with the laser (a) signal range 0 to 2 times encoding frequency, (b) detail of signals 0.9 to 1.1 of encoding frequency.

Figure 87: $bg = 10 \cos (2\pi 50t)$, Simulated response of dual-output APS shows (a) the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light when not in phase with the laser encoding frequency and signal range ratio from 0 to 1, (b) detail of signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05

- Black [$\varphi=0$], red [$\varphi=\pi/4$], pink [$\varphi=\pi/2$], orange [$\varphi=3\pi/4$], yellow [$\varphi=\pi$], green [$\varphi=5\pi/4$], blue [$\varphi=3\pi/2$], violet [$\varphi=7\pi/4$]
Figure 88: $b_g = 100 \cos(2\pi ft)$, Simulated response of double sided APS to light of frequencies varying with the laser encoding signal. This shows the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light in phase with the laser (a) signal range 0 to 2 times encoding frequency, (b) detail of signals 0.9 to 1.1 of encoding frequency.

- Black [$\varphi=0$], red [$\varphi=\pi/4$], pink [$\varphi=\pi/2$], orange [$\varphi=3\pi/4$], yellow [$\varphi=\pi$], green [$\varphi=5\pi/4$], blue [$\varphi=3\pi/2$], violet [$\varphi=7\pi/4$]

Figure 89: $b_g=100 \cos(2\pi ft)$, Simulated response of dual-output APS shows (a) the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light when not in phase with the laser encoding frequency and signal range ratio from 0 to 1, (b) detail of signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05
Figure 90: $bg=1000 \cos (2\pi 50t)$, Simulated response of double sided APS to light of frequencies varying with the laser encoding signal. This shows the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light in phase with the laser (a) signal range 0 to 2 times encoding frequency, (b) detail of signals 0.9 to 1.1 of encoding frequency.

- Black [$\varphi=0$], red [$\varphi=\pi/4$], pink [$\varphi=\pi/2$], orange [$\varphi=3\pi/4$], yellow [$\varphi=\pi$], green [$\varphi=5\pi/4$], blue [$\varphi=3\pi/2$], violet [$\varphi=7\pi/4$]

Figure 91: $bg=1000 \cos (2\pi 50t)$, Simulated response of dual-output APS shows (a) the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light when not in phase with the laser encoding frequency and signal range ratio from 0 to 1, (b) detail of signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05
2.2 Varying Number of Integration Cycles ($dc = 1, f = 1000, bg = 0.5x$)

Figure 92: $n=1$, Simulated response of double sided APS to light of frequencies varying with the laser encoding signal. This shows the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light in phase with the laser (a) signal range 0 to 2 times encoding frequency, (b) detail of signals 0.9 to 1.1 of encoding frequency.

Figure 93: $n=1$, Simulated response of dual-output APS shows (a) the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light when not in phase with the laser encoding frequency and signal range ratio from 0.9 to 1, (b) detail of signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05 signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05.
Figure 94: \( n=2 \), Simulated response of double sided APS to light of frequencies varying with the laser encoding signal. This shows the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light in phase with the laser (a) signal range 0 to 2 times encoding frequency, (b) detail of signals 0.9 to 1.1 of encoding frequency.

- Black \( [\varphi=0] \), red \( [\varphi=\pi/4] \), pink \( [\varphi=\pi/2] \), orange \( [\varphi=3\pi/4] \), yellow \( [\varphi=\pi] \), green \( [\varphi=5\pi/4] \), blue \( [\varphi=3\pi/2] \), violet \( [\varphi=7\pi/4] \)

Figure 95: \( n=2 \), Simulated response of dual-output APS shows (a) the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light when not in phase with the laser encoding frequency and signal range ratio from 0 to 1, (b) detail of signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05.
Figure 96: $n=5$, Simulated response of double sided APS to light of frequencies varying with the laser encoding signal. This shows the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light in phase with the laser (a) signal range 0 to 2 times encoding frequency, (b) detail of signals 0.9 to 1.1 of encoding frequency.

- Black [$\varphi=0$], red [$\varphi=\pi/4$], pink [$\varphi=\pi/2$], orange [$\varphi=3\pi/4$], yellow [$\varphi=\pi$], green [$\varphi=5\pi/4$], blue [$\varphi=3\pi/2$], violet [$\varphi=7\pi/4$]

Figure 97: $n=5$, Simulated response of dual-output APS shows (a) the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light when not in phase with the laser encoding frequency and signal range ratio from 0 to 1, (b) detail of signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05.
Figure 98: $n=10$, Simulated response of double sided APS to light of frequencies varying with the laser encoding signal. This shows the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light in phase with the laser (a) signal range 0 to 2 times encoding frequency, (b) detail of signals 0.9 to 1.1 of encoding frequency.

Figure 99: $n=10$, Simulated response of dual-output APS shows (a) the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light when not in phase with the laser encoding frequency and signal range ratio from 0 to 1, (b) detail of signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05.

- Black [$\varphi=0$], red [$\varphi=\pi/4$], pink [$\varphi=\pi/2$], orange [$\varphi=3\pi/4$], yellow [$\varphi=\pi$], green [$\varphi=5\pi/4$], blue [$\varphi=3\pi/2$], violet [$\varphi=7\pi/4$]
Figure 100: \( n=20 \), Simulated response of double sided APS to light of frequencies varying with the laser encoding signal. This shows the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light in phase with the laser (a) signal range 0 to 2 times encoding frequency, (b) detail of signals 0.9 to 1.1 of encoding frequency.

Figure 101: \( n=20 \), Simulated response of dual-output APS shows (a) the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light when not in phase with the laser encoding frequency and signal range ratio from 0 to 1, (b) detail of signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05 if signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05.

- Black \([\varphi=0]\), red \([\varphi=\pi/4]\), pink \([\varphi=\pi/2]\), orange \([\varphi=3\pi/4]\), yellow \([\varphi=\pi]\), green \([\varphi=5\pi/4]\), blue \([\varphi=3\pi/2]\), violet \([\varphi=7\pi/4]\)
Figure 102: $n=30$, Simulated response of double sided APS to light of frequencies varying with the laser encoding signal. This shows the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light in phase with the laser (a) signal range 0 to 2 times encoding frequency, (b) detail of signals 0.9 to 1.1 of encoding frequency.

- Black [$\varphi=0$], red [$\varphi=\pi/4$], pink [$\varphi=\pi/2$], orange [$\varphi=3\pi/4$], yellow [$\varphi=\pi$], green [$\varphi=5\pi/4$], blue [$\varphi=3\pi/2$], violet [$\varphi=7\pi/4$]

Figure 103: $n=30$, Simulated response of dual-output APS shows (a) the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light when not in phase with the laser encoding frequency and signal range ratio from 0 to 1, (b) detail of signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05f signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05
Figure 104: $n=50$, Simulated response of double sided APS to light of frequencies varying with the laser encoding signal. This shows the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light in phase with the laser (a) signal range 0 to 2 times encoding frequency, (b) detail of signals 0.9 to 1.1 of encoding frequency.

- Black [φ=0], red [φ=π/4], pink [φ=π/2], orange [φ=3π/4], yellow [φ=π], green [φ=5π/4], blue [φ=3π/2], violet [φ=7π/4]

Figure 105: $n=50$, Simulated response of dual-output APS shows (a) the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light when not in phase with the laser encoding frequency and signal range ratio from 0 to 1, (b) detail of signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05 signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05.
Figure 106: $n=100$, Simulated response of double sided APS to light of frequencies varying with the laser encoding signal. This shows the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light in phase with the laser (a) signal range 0 to 2 times encoding frequency, (b) detail of signals 0.9 to 1.1 of encoding frequency.

- Black [$\varphi=0$], red [$\varphi=\pi/4$], pink [$\varphi=\pi/2$], orange [$\varphi=3\pi/4$], yellow [$\varphi=\pi$], green [$\varphi=5\pi/4$], blue [$\varphi=3\pi/2$], violet [$\varphi=7\pi/4$]

Figure 107: $n=100$, Simulated response of dual-output APS shows (a) the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light when not in phase with the laser encoding frequency and signal range ratio from 0 to 1, (b) detail of signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05.
2.3 Varying Background Intensity Level when Number of Summation Cycles is 2 and Linear Frequency of Foreground Light is 100 Hz (n = 2, dc = 1, f = 100)

Figure 108: \( bg = \cos(2\pi 50t) \), Simulated response of double sided APS to light of frequencies varying with the laser encoding signal. This shows the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light in phase with the laser (a) signal range 0 to 2 times encoding frequency, (b) detail of signals 0.9 to 1.1 of encoding frequency.

Figure 109: \( bg = \cos(2\pi 50t) \), Simulated response of dual-output APS shows (a) the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light when not in phase with the laser encoding frequency and signal range ratio from 0 to 1, (b) detail of signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05.
Figure 110: $b_g = 1.7 \cos(2\pi 50t)$, Simulated response of double sided APS to light of frequencies varying with the laser encoding signal. This shows the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light in phase with the laser (a) signal range 0 to 2 times encoding frequency, (b) detail of signals 0.9 to 1.1 of encoding frequency.

- Black [$\varphi=0$], red [$\varphi=\pi/4$], pink [$\varphi=\pi/2$], orange [$\varphi=3\pi/4$], yellow [$\varphi=\pi$], green [$\varphi=5\pi/4$], blue [$\varphi=3\pi/2$], violet [$\varphi=7\pi/4$]

Figure 111: $b_g = 1.7 \cos(2\pi 50t)$, Simulated response of dual-output APS shows (a) the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light when not in phase with the laser encoding frequency and signal range ratio from 0 to 1, (b) detail of signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05 signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05
Figure 112: $b_g = 1.9 \cos(2\pi 50t)$, Simulated response of double sided APS to light of frequencies varying with the laser encoding signal. This shows the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light in phase with the laser (a) signal range 0 to 2 times encoding frequency, (b) detail of signals 0.9 to 1.1 of encoding frequency.

Figure 113: $b_g = 1.9 \cos(2\pi 50t)$, Simulated response of dual-output APS shows (a) the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light when not in phase with the laser encoding frequency and signal range ratio from 0 to 1, (b) detail of signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05f signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05

- Black [$\varphi=0$], red [$\varphi=\pi/4$], pink [$\varphi=\pi/2$], orange [$\varphi=3\pi/4$], yellow [$\varphi=\pi$], green [$\varphi=5\pi/4$], blue [$\varphi=3\pi/2$], violet [$\varphi=7\pi/4$]
Figure 114: \( \phi g = 1.93 \cos(2\pi 50t) \), Simulated response of double sided APS to light of frequencies varying with the laser encoding signal. This shows the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light in phase with the laser (a) signal range 0 to 2 times encoding frequency, (b) detail of signals 0.9 to 1.1 of encoding frequency.

Figure 115: \( \phi g = 1.93 \cos(2\pi 50t) \), Simulated response of dual-output APS shows (a) the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light when not in phase with the laser encoding frequency and signal range ratio from 0 to 1, (b) detail of signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05 signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05.

- Black \( [\phi=0] \), red \( [\phi=\pi/4] \), pink \( [\phi=\pi/2] \), orange \( [\phi=3\pi/4] \), yellow \( [\phi=\pi] \), green \( [\phi=5\pi/4] \), blue \( [\phi=3\pi/2] \), violet \( [\phi=7\pi/4] \)
Figure 116: $bg = \cos(2\pi 50t)$, Simulated response of double sided APS to light of frequencies varying with the laser encoding signal. This shows the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light in phase with the laser (a) signal range 0 to 2 times encoding frequency, (b) detail of signals 0.9 to 1.1 of encoding frequency.

Figure 117: $bg = \cos(2\pi 50t)$, Simulated response of dual-output APS shows (a) the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light when not in phase with the laser encoding frequency and signal range ratio from 0 to 1, (b) detail of signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05f signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05

- Black $[\phi=0]$, red $[\phi=\pi/4]$, pink $[\phi=\pi/2]$, orange $[\phi=3\pi/4]$, yellow $[\phi=\pi]$, green $[\phi=5\pi/4]$, blue $[\phi=3\pi/2]$, violet $[\phi=7\pi/4]$
Figure 118: \( bg = 0 \), Simulated response of double sided APS to light of frequencies varying with the laser encoding signal. This shows the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light in phase with the laser (a) signal range 0 to 2 times encoding frequency, (b) detail of signals 0.9 to 1.1 of encoding frequency.

- Black \( [\varphi = 0] \), red \( [\varphi = \pi/4] \), pink \( [\varphi = \pi/2] \), orange \( [\varphi = 3\pi/4] \), yellow \( [\varphi = \pi] \), green \( [\varphi = 5\pi/4] \), blue \( [\varphi = 3\pi/2] \), violet \( [\varphi = 7\pi/4] \)

Figure 119: \( bg = 0 \), Simulated response of dual-output APS shows (a) the differential output of the two APS sides for 100 integrations per side plotted versus frequencies as a harmonic of the laser encoding frequency (1000 Hz) for light when not in phase with the laser encoding frequency and signal range ratio from 0 to 1, (b) detail of signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05 signals when light is not in phase with the laser encoding frequency ratio from 0.95 to 1.05
Appendix 3: Experimental Results for Dual-Output Photogate-Based APS

\[ \text{Fg-<intensity of foreground>-bg-<intensity of background>-fg-<modulated foreground frequency>-<the side of the dual-output photogate APS where both background and foreground is integrated in>} \]

For example, \text{fg-2000nW-bg-0nW-fg-100Hz-sideA} means that the intensity of the foreground is 2000 nW, the intensity of the background is 0nW, while the foreground frequency is 100 Hz which implicitly means that the background frequency is 50 Hz, and that the side of the dual-output photogate APS which is used to integrate both foreground and background light is sideA, which implicitly means that sideB is used to integrate the background light only.

When the phase is referred to as “dark” in the tables, it means that since both foreground and background light is suppressed, there is no need for phase information. “bg-0nW” is used to refer to a phase where since the background light is suppressed and only the foreground light is integrated, that this information is not useful. It is similar to “dark” with the exception that the foreground light intensity is switched on.

“result <number> with Dark l” means result number with dark current compensation, whereas “result<number> w/o Dark l” means result number without dark current compensation.
Table 7: Values of $D$ with and without dark current taken into account where foreground light and background light signal is accumulated on side A of the dual-output photogate APS, while background light signal is accumulated on side B and foreground pulse intensity is 2000 nW while background sinusoidal intensities are 1000 nW, 1700 nW, 1900 nW, 1930 nW, 2000 nW, with different fg-bg phase relationship. There are 4 results ($D$ values) from 4 different dual-output photogate APS. The foreground pulse has a duty cycle of 32.5%.

<table>
<thead>
<tr>
<th>Condition</th>
<th>phase</th>
<th>result 1 w/o dark</th>
<th>result 2 w/o dark</th>
<th>result 3 w/o dark</th>
<th>result 4 w/o dark</th>
<th>result 1 w dark</th>
<th>result 2 w dark</th>
<th>result 3 w dark</th>
<th>result 4 w dark</th>
</tr>
</thead>
<tbody>
<tr>
<td>fg-2000nW-bg-0nW-fg-100Hz-sideA</td>
<td>bg-0nW</td>
<td>0.160</td>
<td>0.120</td>
<td>0.140</td>
<td>0.100</td>
<td>0.140</td>
<td>0.080</td>
<td>0.100</td>
<td>0.080</td>
</tr>
<tr>
<td>fg-dark-bg-dark-fg-100Hz-sideA</td>
<td>dark</td>
<td>0.020</td>
<td>0.040</td>
<td>0.040</td>
<td>0.020</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
</tr>
<tr>
<td>fg-2000nW-bg-1000nW-fg-100Hz-sideA</td>
<td>bg-phase0</td>
<td>-0.040</td>
<td>-0.040</td>
<td>-0.040</td>
<td>-0.060</td>
<td>-0.060</td>
<td>-0.080</td>
<td>-0.080</td>
<td>-0.080</td>
</tr>
<tr>
<td>fg-2000nW-bg-1000nW-fg-100Hz-sideA</td>
<td>bg-phase45</td>
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<td>0.100</td>
<td>0.140</td>
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<td>0.080</td>
<td>0.060</td>
<td>0.100</td>
<td>0.060</td>
</tr>
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<td>fg-2000nW-bg-1000nW-fg-100Hz-sideA</td>
<td>bg-phase90</td>
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<td>0.100</td>
<td>0.140</td>
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<td>0.120</td>
<td>0.060</td>
<td>0.100</td>
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</tr>
<tr>
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<td>bg-phase135</td>
<td>0.080</td>
<td>0.080</td>
<td>0.100</td>
<td>0.040</td>
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<td>0.040</td>
<td>0.060</td>
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<tr>
<td>fg-2000nW-bg-1000nW-fg-100Hz-sideA</td>
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<td>0.060</td>
<td>0.100</td>
<td>0.040</td>
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<td>0.040</td>
<td>0.060</td>
<td>0.040</td>
</tr>
<tr>
<td>fg-2000nW-bg-1000nW-fg-100Hz-sideA</td>
<td>bg-phase225</td>
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<td>0.100</td>
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<tr>
<td>fg-2000nW-bg-1000nW-fg-100Hz-sideA</td>
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<td>0.100</td>
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</tr>
<tr>
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</tr>
<tr>
<td>fg-dark-bg-dark-fg-100Hz-sideA</td>
<td>dark</td>
<td>0.020</td>
<td>0.040</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
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<td>bg-phase0</td>
<td>-0.140</td>
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<td>-0.040</td>
<td>-0.120</td>
<td>-0.160</td>
<td>-0.180</td>
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<td>-0.120</td>
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<td>bg-phase45</td>
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<td>0.080</td>
<td>0.100</td>
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Table 8: Values of $D$ with and without dark current taken into account where foreground light and background light signal is accumulated on side A of the dual-output photogate APS, while background light signal is accumulated on side B and foreground pulse intensity is 20000 nW while background sinusoidal intensities are 10000 nW, 17000 nW, 19000 nW, 19300 nW, 20000 nW, with different fg-bg phase relationship. There are 4 results ($D$ values) from 4 different dual-output photogate APS. The foreground pulse has a duty cycle of 32.5%.

<table>
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<th>result 2 w/o dark</th>
<th>result 3 w/o dark</th>
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250
Table 9: Values of $D$ with and without dark current taken into account where foreground light and background light signal is accumulated on side B of the dual-output photogate APS, while background light signal is accumulated on side A and foreground pulse intensity is 2000 nW while background sinusoidal intensities are 1000 nW, 1700 nW, 1900 nW, 1930 nW, 2000 nW, with different fg-bg phase relationship. There are 4 results ($D$ values) from 4 different dual-output photogate APS. The foreground pulse has a duty cycle of 32.5%.

<table>
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<tr>
<th>condition</th>
<th>phase</th>
<th>result 1 w/o dark I</th>
<th>result 2 w/o dark I</th>
<th>result 3 w/o dark I</th>
<th>result 4 w/o dark I</th>
<th>result 1 w dark I</th>
<th>result 2 w dark I</th>
<th>result 3 w dark I</th>
<th>result 4 w dark I</th>
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<td>fg-2000nW-bg-0nW-fg-100Hz-sideB</td>
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<td>0.224</td>
<td>0.272</td>
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<td>-0.024</td>
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<td>0.008</td>
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<td>0.248</td>
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<td>0.152</td>
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Table 10: Values of $D$ with and without dark current taken into account where foreground light and background light signal is accumulated on side B of the dual-output photogate APS, while background light signal is accumulated on side A and foreground pulse intensity is 20000 nW while background sinusoidal intensities are 10000 nW, 17000 nW, 19000 nW, 19300 nW, 20000 nW, with different fg-bg phase relationship. There are 4 results ($D$ values) from 4 different dual-output photogate APS. The foreground pulse has a duty cycle of 32.5%.

<table>
<thead>
<tr>
<th>condition</th>
<th>phase</th>
<th>result 1 w/o dark I</th>
<th>result 2 w/o dark I</th>
<th>result 3 w/o dark I</th>
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Table 11: Values of $D$ with and without dark current taken into account where foreground light and background light signal is accumulated on side B of the dual-output photogate APS, while background light signal is accumulated on side A and foreground pulse intensity is 2000 nW while background sinusoidal intensities are 1000 nW, 1700 nW, 1900 nW, 1930 nW, 2000 nW, with different fg-bg phase relationship. There are 4 results ($D$ values) from 4 different dual-output photogate APS. The foreground pulse has a duty cycle of 32.5%.

<table>
<thead>
<tr>
<th>condition</th>
<th>phase</th>
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<th>result 2 w/o dark I</th>
<th>result 3 w/o dark I</th>
<th>result 4 w/o dark I</th>
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<th>result 2 w dark I</th>
<th>result 3 w dark I</th>
<th>result 4 w dark I</th>
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<td>0.276</td>
<td>0.364</td>
<td>0.352</td>
<td>0.212</td>
<td>0.256</td>
</tr>
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<td>bg-phase0</td>
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<td>0.024</td>
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<td>0.340</td>
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<tr>
<td>bg-2000nW-fg-1000nW-fg-100Hz</td>
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<td>0.192</td>
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<tr>
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<td>bg-phase135</td>
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<td>0.140</td>
<td>0.128</td>
<td>0.188</td>
<td>0.176</td>
<td>0.124</td>
<td>0.108</td>
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<td>0.188</td>
<td>0.148</td>
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<td>0.180</td>
<td>0.156</td>
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<td>0.232</td>
<td>0.132</td>
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</tr>
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<td>0.060</td>
<td>0.056</td>
<td>0.072</td>
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<td>-0.144</td>
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<td>0.180</td>
<td>0.236</td>
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<td>0.232</td>
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<td>0.328</td>
<td>0.212</td>
<td>0.256</td>
</tr>
<tr>
<td>bg-2000nW-fg-1700nW-fg-100Hz</td>
<td>bg-phase135</td>
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<td>0.108</td>
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<td>0.072</td>
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<td>0.072</td>
<td>0.056</td>
<td>0.028</td>
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<tr>
<td>bg-2000nW-fg-1700nW-fg-100Hz</td>
<td>bg-phase225</td>
<td>0.112</td>
<td>0.092</td>
<td>0.072</td>
<td>0.064</td>
<td>0.100</td>
<td>0.084</td>
<td>0.048</td>
<td>0.088</td>
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<td>0.116</td>
<td>0.124</td>
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<td>bg-2000nW-fg-1900nW-fg-100Hz</td>
<td>bg-phase0</td>
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<td>bg-phase135</td>
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<td>0.080</td>
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<td>0.136</td>
<td>0.108</td>
<td>0.064</td>
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<td>0.056</td>
<td>0.072</td>
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<tr>
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<td>0.072</td>
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<td>0.104</td>
<td>0.092</td>
<td>0.056</td>
<td>0.068</td>
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<td>0.144</td>
<td>0.152</td>
<td>0.096</td>
<td>0.124</td>
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<tr>
<td>bg-2000nW-fg-1900nW-fg-100Hz</td>
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<td>-0.168</td>
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<td>result 3 w/ dark</td>
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</tr>
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<td>0.324</td>
<td>0.352</td>
<td>0.228</td>
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<tr>
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<td>bg-phase135</td>
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<td>0.108</td>
<td>0.100</td>
<td>0.092</td>
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<td>0.040</td>
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<td>0.068</td>
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<td>0.044</td>
<td>0.064</td>
<td>0.084</td>
<td>0.064</td>
<td>0.048</td>
<td>0.060</td>
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<td>0.072</td>
<td>0.104</td>
<td>0.112</td>
<td>0.116</td>
<td>0.068</td>
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<td>-0.084</td>
<td>-0.112</td>
<td>-0.196</td>
<td>-0.168</td>
<td>-0.080</td>
<td>-0.116</td>
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<tr>
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<td>-0.212</td>
<td>-0.228</td>
<td>-0.336</td>
<td>-0.308</td>
<td>-0.208</td>
<td>-0.232</td>
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<td>0.164</td>
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<td>0.208</td>
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<td>0.312</td>
<td>0.212</td>
<td>0.224</td>
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<td>0.008</td>
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<td>0.068</td>
<td>0.028</td>
<td>0.044</td>
<td>0.036</td>
<td>0.072</td>
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<td>-0.172</td>
<td>-0.244</td>
<td>-0.296</td>
<td>-0.144</td>
<td>-0.176</td>
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</table>
Table 12: Values of $D$ with and without dark current taken into account where foreground light and background light signal is accumulated on side $B$ of the dual-output photogate APS, while background light signal is accumulated on side $A$ and foreground pulse intensity is 20000 nW while background sinusoidal intensities are 10000 nW, 17000 nW, 19000 nW, 19300 nW, 20000 nW, with different fg-bg phase relationship. There are 4 results ($D$ values) from 4 different dual-output photogate APS. The foreground pulse has a duty cycle of 32.5%.

<table>
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<tr>
<th>condition</th>
<th>phase</th>
<th>result 1 w/o dark</th>
<th>result 2 w/o dark</th>
<th>result 3 w/o dark</th>
<th>result 4 w/o dark</th>
<th>result 1 w dark</th>
<th>result 2 w dark</th>
<th>result 3 w dark</th>
<th>result 4 w dark</th>
</tr>
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<td>bg-phase0</td>
<td>0.240</td>
<td>0.216</td>
<td>0.168</td>
<td>0.144</td>
<td>0.224</td>
<td>0.196</td>
<td>0.168</td>
<td>0.164</td>
</tr>
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<td>fg-20000nW-bg-10000nW-fg-1000Hz</td>
<td>bg-phase45</td>
<td>0.252</td>
<td>0.284</td>
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<td>0.180</td>
<td>0.236</td>
<td>0.264</td>
<td>0.208</td>
<td>0.200</td>
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<tr>
<td>fg-20000nW-bg-10000nW-fg-1000Hz</td>
<td>bg-phase135</td>
<td>0.296</td>
<td>0.312</td>
<td>0.200</td>
<td>0.168</td>
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<td>0.292</td>
<td>0.200</td>
<td>0.256</td>
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<td>0.236</td>
<td>0.168</td>
<td>0.148</td>
<td>0.236</td>
<td>0.216</td>
<td>0.168</td>
<td>0.168</td>
</tr>
<tr>
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<td>0.140</td>
<td>0.112</td>
<td>0.220</td>
<td>0.200</td>
<td>0.140</td>
<td>0.132</td>
</tr>
<tr>
<td>fg-20000nW-bg-10000nW-fg-1000Hz</td>
<td>bg-phase270</td>
<td>0.236</td>
<td>0.220</td>
<td>0.140</td>
<td>0.132</td>
<td>0.220</td>
<td>0.200</td>
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<td>0.132</td>
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<td>fg-20000nW-bg-10000nW-fg-1000Hz</td>
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<td>0.160</td>
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<td>0.220</td>
<td>0.192</td>
<td>0.160</td>
<td>0.152</td>
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<td>0.016</td>
<td>0.040</td>
<td>0.040</td>
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<td>fg-20000nW-bg-17000nW-fg-1000Hz</td>
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<td>0.092</td>
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<td>0.064</td>
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<td>0.044</td>
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<td>0.044</td>
<td>0.060</td>
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<td>0.032</td>
<td>0.044</td>
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<td>0.012</td>
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<tr>
<td>fg-20000nW-bg-17000nW-fg-1000Hz</td>
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<td>0.004</td>
<td>0.012</td>
<td>0.044</td>
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<td>bg-phase45</td>
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REFERENCE


