ASSET PRICING IN A DYNAMIC GENERAL EQUILIBRIUM FRAMEWORK

by

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Title of Project: Quantifying The Equity Premium In A Dynamic General Equilibrium Setting

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DATE: Dec. 02/03
Abstract

A dynamic general equilibrium model of stock and bond prices is developed which yields a quantifiable equity premium. Unlike previously developed asset pricing models, the current study incorporates the innovation of wages and profit components. The current study goes further to embed the correlation between the innovations of wages and profits and quantifies a risk premium marginally different from the one observed in the market for the period under study.
Acknowledgements

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1. Introduction

1.1 Background

The historical size of the U.S. equity premium has puzzled economists since the 1980s. Economists have long assumed that the relative size of the premium is primarily a measure of the compensation that investors demand for taking on the riskier investment option. The standard asset-pricing model has been unable to account for an equity premium as large as the seven percentage points recorded in the market data. With reasonable levels of risk aversion and other standard assumptions, the model predicts a premium around 0.25 of a percentage point (Mehra and Prescott 1985, Hansen and Jagannathan 1991). This discrepancy between data and economic theory has come to be known as the “Equity Premium Puzzle.” The size of the equity premium has suggested that perhaps something not inherent in the standard economic models is the determining factor in the size of the risk premium.

Historically, as recorded by Mehra and Prescott (1985), the average return on equity has far exceeded the average return of short-term virtually default free debt. Over the ninety-year period of 1889 to 1978 the authors note that average real annual yield on the Standard and Poor 500 Index was seven percent, while the average yield on short-term debt was less than one percent. The six percent risk premium puzzled the authors and brought forth the phrase “risk free rate puzzle,” which questioned the abnormally low return by bonds in relation to stock returns. Clement (2001) comments that economic theory suggests that stocks should pay investors a bit more than bonds, to reward them for enduring the higher risk of the stock market, but that additional risk should give stock investors no more than a one percent premium. In fact, similarly to Mehra and Prescott, Clement (2001) notes that according to the Center for Research in Security Prices, between 1926 and 1999, the mean of the annual stock returns was around seven percent higher than bond yields.
Jagannathan, McGrattan and Scherbina (2000) have suggested that the factors not considered in the standard asset pricing models are perhaps market imperfections such as the inability of investors to fully ensure against major risks outside the organized stock markets, such as shocks to their labor income; the significant direct and indirect costs that investors face in order to make transactions; and investors’ incomplete knowledge of existing investment opportunities. Similarly, Sanford and Shiller (1981) argue that asset prices vary too much to be explained by variations in per capita consumption.

Jagannathan, McGrattan and Scherbina (2000) claim that equity investment imperfections have been mitigated in the last three decades, a consequence of the latest technological improvements which have made it increasingly easier for investors to access information, communicate and transact with others and enforce contractual obligations. Using recent data and the Gordon dividend discount model to value equity prices, the authors demonstrate that the U.S. equity premium has declined significantly during the last three decades. The authors claim that the average equity premium over the last 30 years was just 0.7 of a percentage point.

The current study will compare two dynamic asset-pricing models in their ability at quantifying the risk premium for the time period, 1959 to 1995. The first model developed will informally replicate work done by Mehra and Prescott (1985) whereby asset prices are quantified through innovations in the real endowment of aggregate output. One should note that Mehra and Prescott used the growth in consumption, not output. This paper, however, will assume that production and consumption are equal. The second model will decompose the output endowment model into correlated wage and dividend components under the assumption that this segregation would give rise to a more realistic premium. The wage and dividend components will be calibrated to reflect real market wages and real profit values found in macroeconomic data in the United States for the periods 1959 to 1995. The assumption is that the inability of Mehra and Prescott’s model to account for the large equity premium lies in the model’s choice of the endowment variable.
The composition of this paper is as follows. The current section, section one, summarizes past research and various attempts at quantifying the equity premium. Market data on real bond and equity returns is presented in part two of section one. Section two will describe the statistical properties of data used in the output endowment model (Model A) and the decomposed model (Model B), where output has been divided into wage and dividend innovations. Section three will present the two models and report the findings. The last section will provide advice on future research and possible extensions for the current study.

1.2 Market Data on Bond and Equity Returns

Table One summarizes equity and bond yields in the U.S. for the period under study. The proxy for equity returns, as used by numerous authors, is the S&P 500, which is a market-value weighted index that encompasses 500 of the major U.S. firms. The risk free asset considered is the one-year U.S. Treasury bill, a default free U.S. Government security. The equity premium was calculated as the difference between the real S&P 500 yields and the real one-year Treasury bill yields. The average inflation rates have also been presented to allow the reader to compute the nominal portfolio returns and the nominal bond yields. The risk premium, defined as the difference between the equity and bond returns, should be independent of the inflation rates.
Table 1-1  
Returns on U.S. Stocks and Bonds  
Annual Averages in Percent, 1959-2002

<table>
<thead>
<tr>
<th>Period</th>
<th>Inflation Rate</th>
<th>Real S&amp;P 500 Yield</th>
<th>Real T-bill Yield</th>
<th>Equity Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>1959-69</td>
<td>1.46</td>
<td>7.0</td>
<td>2.78</td>
<td>4.23</td>
</tr>
<tr>
<td>1970-79</td>
<td>7.2</td>
<td>2.83</td>
<td>0.17</td>
<td>2.66</td>
</tr>
<tr>
<td>1980-89</td>
<td>5.66</td>
<td>3.60</td>
<td>2.71</td>
<td>0.89</td>
</tr>
<tr>
<td>1990-1996</td>
<td>3.15</td>
<td>4.23</td>
<td>1.47</td>
<td>2.76</td>
</tr>
<tr>
<td>Average Yields</td>
<td>4.44</td>
<td>4.41</td>
<td>1.78</td>
<td>2.63</td>
</tr>
</tbody>
</table>
2. Model Calibration and Data

2.1 Output endowment model

This section will use a model similar to Mehra and Prescott's (1985) output endowment model to price equity and debt using current macroeconomic data found in U.S. for the 1959-1995 period. In 1985, Mehra and Prescott used a dynamic general equilibrium model to price equity and bond returns using standard macroeconomic data. In their attempt to replicate the seven percent market premium, the authors calibrated their model's main parameter to reflect the fluctuations in real consumption found in numerous countries. In their model no attempt was made at segregating the consumption variable into different more stochastic components. Using this endowment economy of total real consumption, the authors were unsuccessful at quantifying the equity premium to match the one observed in the market for the 1889-1978 period under study.

In replicating Mehra and Prescott's asset pricing model, the current study will not use real consumption, but will use innovations in real output. In this output endowment model, the current study assumes that output evolves according to a first order Markov process where \( y_{t+1} = (1 - \rho_y)\bar{y} + \rho_y y_t + \epsilon \). The variable \( \bar{y} \), denotes some mean level of output and the persistence parameter, \( \rho_y \) corresponds to the correlation between \( y_t \) and \( y_{t+1} \). In order to calibrate the parameters of the endowment model, to reflect the current market, data for United States was downloaded from the Saint Louis Fed Website for the period of 1959 to 2002. Data on the real Gross Domestic Product is from the U.S. Department of Commerce: Bureau of Economic Analysis. The data is seasonally adjusted and appears in quarterly rates and billions of chained 1992 dollars. The data was first logged and then differenced from the HP trend line, producing the cyclical component of the real output variable.
The theoretical variance of real output is assumed to be time independent meaning that the variance should be equal in periods t and t+1. Imposing \( \text{Var}(y, t) = \text{Var}(y, t+1) \) yields a standard deviation for real output defined as

\[
\sigma_e \sqrt{1 - \rho_y^2},
\]

where \( \sigma_e \) is the standard deviation of the error component. The standard deviation of the error components is found by running an OLS AR(1) regression and recording the standard deviation of the resulting residuals. Table Two summarizes the cyclical properties of the real output variable and the theoretical standard deviation measure to be used in the endowment model.

<table>
<thead>
<tr>
<th>Std of Output Cycle</th>
<th>( \bar{y} )</th>
<th>( \rho_y )</th>
<th>( \sigma_y )</th>
<th>Theoretical Std of Output Cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.58 %</td>
<td>10.0</td>
<td>0.862</td>
<td>0.00807</td>
<td>1.56 %</td>
</tr>
</tbody>
</table>

One possible explanation as to the failure of the standard endowment model to replicate the high equity premium rests in the volatility of the output variable. It is assumed that the volatility of the output variable alone cannot account for the volatility in equity prices and their returns. From the table above the cyclical fluctuation in the logged output variable, as measured by the standard deviation, was calculated as 1.58 percent. However, U.S. corporate data downloaded from the U.S. Department of Commerce: Bureau of Economic Analysis, demonstrates that the equivalent standard deviation measure for actual corporate dividends was much higher indeed, at 3.57 percent. One should question whether or not cyclical fluctuations in real output are able to account for the magnitude of corporate dividend fluctuations. It is for this reason that the current study has extended the endowment model by segregating output into stochastic wage and dividend components in hopes of improving the calibration of the volatility parameters and therefore the actual results.
2.2 Dividend and Wage Model

Unlike the previous model, where the output endowment was used to price assets and returns, in this section output will be decomposed into wage and dividend components. In this model, agents do not have a claim to the entire output, but only to a fraction of output. This fraction of output will represent firm profits or dividends, as the current study assumes that firms have do not retain any earnings, but distribute all profits in the form of dividends. The endowment over time will exhibit a first-order Markov process. Let next period’s dividends and wages and their respective probabilities be defined as:

\[ d_{t+1} = (1-\alpha_1)\bar{d} + \alpha_1 d_t + e_{d,t} \quad \text{where} \quad e_{d,t} = \epsilon(+\sigma_d, -\sigma_d) \quad \text{and} \quad \text{Prob}[e_d = \sigma_d] = 1/2 \]

\[ w_{t+1} = (1-\alpha_2)\bar{w} + \alpha_2 w_t + e_{w,t} \quad \text{where} \quad e_{w,t} = \epsilon(+\sigma_w, -\sigma_w) \quad \text{and} \quad \text{Prob}[e_w = \sigma_w] = 1/2 \]

In order to calibrate the volatility parameters to replicate those seen in the real world, data for United States was downloaded from the Saint Louis Fed Website for the period of 1959 to 1995. Data on real corporate profits is from the U.S. Department of Commerce: Bureau of Economic Analysis. It is seasonally adjusted in annual rates and appears in billion of dollars. Data on the real Gross Domestic Product is from the U.S. Department of Commerce: Bureau of Economic Analysis. The data is seasonally adjusted and appears in quarterly rates and billions of chained 1992 dollars. The GDP deflator-chain type price index was found in U.S. Department of Commerce: Bureau of Economic Analysis. It is also seasonally adjusted and the index base year was also 1992. Note that all the data was left in quarterly frequency. A made-up proxy was used for the value of wages. Once real profits have been deducted from total real output, the leftover could be approximated to represent return to labor and debt. As this study assumes that all corporate debt is high grade, the wage variable used throughout the current study pertains to the real wage bill and real return on debt. The data was logged and the cyclical components of each series, using the HP filter, were derived. The descriptive statistics of the three variables for the period 1959-1995 appear in Table Three below.
Table 2-2
Properties of Dividend, Wage and Output Cycles

<table>
<thead>
<tr>
<th></th>
<th>WAGE_CYCLE</th>
<th>DIVIDEND_CYCLE</th>
<th>OUTPUT_CYCLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>-1.85E-13</td>
<td>-0.001</td>
<td>-2.84E-08</td>
</tr>
<tr>
<td>Median</td>
<td>-0.000</td>
<td>0.007</td>
<td>-0.000</td>
</tr>
<tr>
<td>Maximum</td>
<td>0.041</td>
<td>0.193</td>
<td>0.038</td>
</tr>
<tr>
<td>Minimum</td>
<td>-0.053</td>
<td>-0.299</td>
<td>-0.047</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.017</td>
<td>0.100</td>
<td>0.015</td>
</tr>
<tr>
<td>Observations</td>
<td>176</td>
<td>176</td>
<td>176</td>
</tr>
</tbody>
</table>

The cyclical fluctuations in dividends, output and wages over the sample period are presented in Figure One below. The figures below show the difference between the actual observations on the variables and the long-run trend component of each series. The percent standard deviation from trend is the unit of measure, found on the vertical axis for all three graphs. From the figures below, one can see that the dividend series is more volatile than the real output series below it, suggesting to the reader that dividends, measured by corporate profits, are more sensitive to business cycles and therefore could approximate actual corporate dividends more closely.
Figure 2-1 (a), (b), (c)
Volatility of Wage, Dividend and Output Cycles
Measured in percent standard deviation from trend

(a) 

(b) 

(c) 

PROFIT_CYCLE

OUTPUT_CYCLE
One should note that the volatility, as measured by standard deviation, of the dividend component was estimated at 10 percent, whereas the standard deviation of the output component was measured at 1.58 percent. Recall that using U.S. data on actual corporate dividends, the standard deviation measure was 3.68 percent. This is precisely the reason for splitting up total output into wage and dividend components, as the dividend component in the current study displays more variation than the output component. By this decomposition, the current study hopes that the calibration of the volatility parameters is enhanced which will manifest itself into superior estimation of the risk premium.

To quantify the standard deviation components of the innovations in (1) and (2) and compute values for the coefficients of interest, a VAR system was created. In this model, as the innovation of wages and dividends is assumed to be correlated therefore it would not have been appropriate to run two separate OLS regressions. Instead a VAR system was used in which all the lagged endogenous variables appear as exogenous in both equations. The VAR took the following form

\[
\begin{align*}
    d_{t+1} &= \alpha_{11}d_t + \alpha_{12}w_t + e_d \\
    w_{t+1} &= \alpha_{21}d_t + \alpha_{22}w_t + e_w
\end{align*}
\]

The VAR estimation using the system above can be written as:

\[
    z_{t+1} = Az_t + e_t \quad \text{where} \quad z_{t+1} = \begin{bmatrix} d_{t+1} \\ w_{t+1} \end{bmatrix}, \quad z_t = \begin{bmatrix} d_t \\ w_t \end{bmatrix}, \quad e_t = \begin{bmatrix} e_d \\ e_w \end{bmatrix} \quad \text{and} \quad A = \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix}
\]

The complete results of the VAR output are found in the appendix. Table Four summarizes the VAR coefficients, the standard deviations of VAR error components and the correlation between the wage and dividend error components for the period under study.
The correlation between the dividend and wage error components is also recorded in the table above. A measure of the theoretical standard deviations was also found for both variables. Using the estimated coefficients from the VAR system above and the standard deviation of the individual error components, the theoretical standard deviation along with each variable’s mean was simulated. The simulation results showed that the standard deviations for the dividend and wage variables were 4.58 and 3.38 percent.
3. Model Extensions and Results

3.1 Agents and Behavior

This model will build on the work done by Mahra and Prescott (1985) and Lucas (1978). The model works with an endowment economy where capital and labor inputs are treated as exogenous. Considered is an economy with a single consumer, interpreted as a representative "stand in" for a large number of identical consumers. All the agents live infinite lives and their choice problem is to maximize their utility in each and every period, which is a function of their consumption. Each agent in the economy is faced with wanting to maximize the quantity

\[ \max E_t \sum_{t=0}^{\infty} \beta^t U(c_t) \]

where \( c_t \) is a stochastic process representing consumption of a single good, \( \beta \in (0,1) \) is a discount factor that agents apply to the utility derived from future consumption, \( U(c_t) \) is a current period utility function derived from consuming \( c_t \), and \( E_t \) is the expectations operator. For each and every period, \( t \), the utility function will take the following form

\[ U(c_t) = c_t^{(1-\alpha)} / (1-\alpha) \quad \text{where} \quad \alpha \geq 0, \quad U'(c_t) > 0 \quad \text{and} \quad U''(c_t) < 0. \]

In the utility function, \( \alpha \) is the coefficient of relative risk aversion, which is a measure of the concavity of the utility function or the disutility of fluctuations in consumption. One should note however that this utility function is a special case of constant relative risk aversion. In the literature, the parameter of risk aversion is assumed to be greater than zero and less than infinity.
3.2 Output Endowment Model and Results

This section is a brief summary of Mehra and Prescott’s (1985) endowment economy\(^1\). As previously mentioned the authors assume that innovations in total output govern the asset prices and agents in the economy receive a dividend payment, which is equal to the endowment of total output. The assets are priced over the space of total output and the only shock considered comes from output fluctuations. The solution to maximizing the function \( U(c_t) + E_t(c_{t+1})\beta \) by choice of next period’s bond and equity holdings \((s_{t+1}, b_{t+1})\) and subject to the budget constraint and general equilibrium restrictions, such that \( y_t = c_t \), yields the following equity and bond pricing functions:

\[
\begin{align*}
(6) \quad p_t U'(c_t) &= E_t(p_{t+1} + d_{t+1})U'(c_{t+1})\beta \quad \text{and} \\
(7) \quad q_t U'(c_t) &= E_t U'(c_{t+1})\beta
\end{align*}
\]

Looking at the asset pricing functions, one can conclude that higher current income, decreases the marginal utility of current consumption and therefore the effect bids upward the asset prices, ceteris peribus. In the bond pricing function, the expectation of a high future income, puts downward pressure on bond prices today as agents can afford to decrease their savings in expectation of higher future income. In the equity pricing function, as future income is conditional on today’s income, if the current income is high and for that reason agents expect future income to be high as well, then there are two opposing effects on equity prices today. Since equity stocks are normal goods, agents will increase their demand of equity shares today, in expectation of high future income. However, expecting a high future income decreases the marginal utility of future consumption and therefore decreases demand for equity shares today. Using the currently derived equity pricing function, the final effect on the price of equity shares is ambiguous.

---

\(^1\) The current study assumes that the reader is familiar with Mehra and Prescott’s endowment model.
Recall that the innovation of the output variable was defined as

\[ y_{t+1} = (1 - \rho_y) \tilde{y} + \rho_y y_t + e_{y,t} \quad \text{where} \quad e_{y,t} = \epsilon (+\sigma_y, -\sigma_y) \quad \text{and} \quad \text{Prob}[e_y = \sigma_y] = 1/2 \]

In each and every period the state of the economy is such that the probability of high or low output is one half. Denoting next period’s high and low output as \( y_H \) and \( y_L \) respectively, the evolution of output is constructed such that with a probability of one half, future output will yield a high return denoted as

\[ y'_{H} = (1 - \rho_y) \tilde{y} + \rho_y y + \sigma_y, \]

and similarly with a probability of one half, future output will take on a lower yield denoted as

\[ y'_{L} = (1 - \rho_y) \tilde{y} + \rho_y y - \sigma_y. \]

Using the probabilities from above, the state space for the equity and the bond pricing functions is defined as

\[ p(y) = \beta(y)^\alpha [(0.5)p(y'_H) + d_H][y'_H]^{-\alpha} + (0.5)[p(y'_L) + d_L][y'_L]^{-\alpha} \]

\[ q(y) = \beta(y)^\alpha [(0.5)(y'_H) + (0.5)(y'_L)] \]

Prior to quantifying the asset prices and returns, one must first develop an output grid over which the asset prices will be computed. In creating a grid for the output variable one must define the lower and upper bounds of the grid. The following general formula was used: \( [Y, \bar{Y}] = Ybar \pm \text{std}(Y)(\text{number of standard deviations}) \).

The mean of the output variable was normalized to one so \( Ybar = 10.0 \). The number of standard deviations is arbitrarily chosen to be two and the standard deviation of the output variable corresponds to the theoretical standard deviation estimated in section two. Table Five summarizes the components used in estimating the bounds for the output grid space.
Prior to reporting the simulation results, the last two parameters were calibrated. The time discount parameter, beta, should be expected to take a value of 0.9 - 0.99 which is a number commonly used in the literature. Donaldson and Mehra (7) use a discount factor of 0.90 in their paper, "Inter-temporal Asset Pricing," however this model will use a higher discount rate of 0.99 used by Mehra and Prescott (1985). The risk aversion parameter, alpha, can take on any value greater than zero. Interestingly, a value of one for the risk aversion parameter simplifies the utility function into a logarithmic function. This model will take on a small risk aversion value of 2.0, however sensitivity analysis will be conducted to see how the risk premium responds to changes in the risk aversion parameter.

Table Six below presents the risk premium estimated by the output endowment model. This model is an informal replication of the Mehra and Prescott’s (1985) model using current U.S. data. As one can see, when the risk aversion parameter has been standardized to two, the application of Mehra and Prescott’s asset pricing model to current market data failed to replicate the visible equity premium. In order to come close at replicating the market premium of 2.63 percent, the risk aversion parameter had to be increased to a value of eight.
Table 3-2
Risk Premium - Output Endowment Model
Average of Annualized Real Returns in Percent

<table>
<thead>
<tr>
<th></th>
<th>Output Endowment Model When $\alpha = 2.0$</th>
<th>Output Endowment Model When $\alpha = 5.0$</th>
<th>Output Endowment Model When $\alpha = 8.0$</th>
<th>Annual Market Returns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity Return</td>
<td>4.294</td>
<td>5.208</td>
<td>6.984</td>
<td>4.41</td>
</tr>
<tr>
<td>Bond Return</td>
<td>4.1078</td>
<td>4.198</td>
<td>4.432</td>
<td>1.79</td>
</tr>
<tr>
<td>Risk Premium</td>
<td><strong>0.186</strong></td>
<td><strong>1.00</strong></td>
<td><strong>2.552</strong></td>
<td><strong>2.63</strong></td>
</tr>
</tbody>
</table>

When the risk aversion parameter is set to two, the equity premium is estimated to be less than one fifth of a percent. Decomposing the premium, the estimated real annual equity return predicted by the model is marginally different from the mean return seen in the market data. However, the mean return predicted by the model for the risk free asset was 4.10 percent, whereas market data shows a mean return of 1.79 percent. In conclusion, the endowment model using current data is facing a similar problem past models have encountered. The deviation in the model and market data comes from the abnormally low bond yields, as seen in the market data. Mehra and Prescott in 1985 found similar deviations in the risk free rate and coined the phrase, “risk free rate puzzle.” The question then becomes, with such abnormally low bond yields seen in the market, why would not the investors simply react by increasing demand for equity shares and drive down their returns, lessening the equity premium gap? In the following section, developed is a model that decomposes output into correlated wage and dividend components in hopes of predicting a more realistic equity premium.
3.3 Dividend and Wage Model and Results

In this section, developed is a model that treats production as exogenous. The model assumes that there is a fixed amount of labor and capital in the economy. Agents in the economy are employed by existing firms and all agents are owners of all equity shares of these firms. In return for supplying labor, each agent receives some wage. The dividend payment is the return on equity holdings and is equal to total firm profits, as retained earnings do not exist in this model. In a simplified sense, one could think of agents being paid in wages, denoting their return to labor and dividends, denoting their return to capital. Each agent will be endowed one equity share at the beginning of time, which will entitle the agent to a claim against the future dividend flow.

In response to uncertainty regarding the consumption smoothing behavior, the agent can save by consuming less in periods of high income and using the excess funds to make investment decisions. The most basic constraint associated with such a model of an endowment economy is seen in literature and is one where consumption and investments must equal the agent's income in each and every period.

\[ y_t = w_t + s_t d_t + b_t \]

where

\[ S_t = p_t (s_{t+1} - s_t) \]
\[ D_t = q_t b_{t+1} \]

In the constraint above, \( c_t \) is the agent's consumption at time \( t \), \( y_t \) is the agent's total income, \( S_t \) is the amount of savings invested in the form equity purchases from period \( t \) to period \( t+1 \), and \( D_t \) is savings invested in discount bonds. In every period, each unit of equity yields a dividend, \( d_t \) to its owner at the beginning of period \( t \). Each agent begins \( t \) with \( s_t \) shares and the initial dividend of the consumption good. Let \( p_t \) be the price of equity in period \( t \), measured in units of consumption goods per equity and let \( w_t \) represent the real wage received for supplying labor to firms. The price of a risk free
bond at time $t$ is denoted as $q_t$. The risk free bond is a discount bond and does not pay dividends. The bond matures in the following period and yields a gross rate of return measured in consumption goods of $\theta$, where $\theta = 1/q_t$. Unlike equity shares, the individual is not endowed any bond holdings at the initial time period. Suppose now we let all the individuals in the economy choose between equity and bond purchases. Constraint (11) can be rewritten as

\begin{equation}
(12)\quad c_t = (d_t + p_t)s_t + b_t - p_t s_{t+1} - q_t b_{t+1} + w_t
\end{equation}

The solution to maximizing the function \{ $U(c_t) + E_t(c_{t+1})\beta$ \} by choice of next period's bond and equity holdings ($s_{t+1}$ and $b_{t+1}$) and subject to the budget constraint (4) is characterized by two first order conditions. Solutions to desired equity and bond holdings take the respective forms:

\begin{align*}
(13)\quad & p_t U'(c_t) = E_t(p_{t+1} + d_{t+1})U'(c_{t+1})\beta \\
(14)\quad & q_t U'(c_t) = E_t U'(c_{t+1})\beta
\end{align*}

The pricing functions above can be decomposed into marginal benefits of current and future consumption. In (13) and (14) the marginal benefit of current consumption must equal the discounted marginal benefit of expected future consumption, which is the right hand side of equity and bond pricing functions.

Next step calls for the imposition of the general equilibrium clearing conditions and the constant relative risk aversion form of the utility function. In equilibrium, consumption and output in each and every period must be equal to one another simply because all agents in the economy are identical and there is only one source of consumption good. In equilibrium $c_t = d_t + w_t = y_t$, allowing one to derive the equity pricing function

\begin{equation}
(15)\quad p_t = E_t(p_{t+1} + d_{t+1})\beta\left[\frac{d_t + w_t}{d_{t+1} + w_{t+1}}\right]^\alpha
\end{equation}
The equity pricing function (15) is a function of the following parameters: $p_{t+1}, d_t, w_t, \alpha, \beta$. Let's briefly comment on the behavior of the pricing function. As the expected price of equity $p_{t+1}$ increases, the agents increase their demand for equity shares today in expectation of a higher rate of return, measured by $RE_t = \frac{E(p_{t+1} + d_{t+1})}{p_t} - 1$. The higher the dividends received in the current period, the higher the current stock price as stocks are considered to be normal goods. Current wages have a similar interpretation as current dividends.

The interpretation of the parameter governing the expected dividends is more complex. There are two opposing effects on stock prices with respect to future dividends. The wealth effect tells us that higher expected dividends result in higher expected income, and therefore agents can afford to increase consumption today. The wealth effect puts upward pressure on stock prices. However, the substitution effect indicates that higher future dividends increase future income and therefore the marginal utility of future consumption decreases, causing consumption of equity shares in the current period to decrease. The final effect on stock prices depends on the magnitude of the two effects. However, take note that the higher the risk aversion parameter, $\alpha$, the larger is the relative size of the wealth effect. This makes intuitive sense as an increase in the risk aversion parameter stimulates a higher demand for savings, which will in turn bid up stock prices. Lastly, all else being equal, an increase in expected wages, increases expected income and decreases the marginal utility of future consumption, decreasing the price of equity today.

Imposing the specific form of the utility function and the general equilibrium conditions to (6) yields the following bond pricing function

$$q_t = E_t \left[ \frac{d_t + w_t}{d_{t+1} + w_{t+1}} \right]^\alpha \beta$$
Notice the price of the risk free asset depends on the following parameters: $d_t$, $w_t$, $\alpha$, $\beta$. Demand for savings increases, causing the price of the discount bond to increase and therefore the return. The price of a risk free bond increases as the agent's current dividend or wage incomes increase. Higher expected dividends put downward pressure on bond prices as higher expected future income decreases the marginal utility of consumption, thereby decreasing consumption of bonds today. Similarly interpretation is applied to the expectation of future wages. In conclusion, in response to higher current income, measured by current wages and dividends, agents demand more bonds and therefore bid down the return to bonds measured as, $1/q_t$. In response to an expected higher future income, agents decrease their demand for bonds today and drive up their yields.

Once the general asset pricing functions have been defined, the next step is to introduce probabilities and the process of wage and dividend innovations. Recall that in this model, the endowment over time will exhibit a first-order Markov process.

\begin{equation}
(17) \quad d_{t+1} = (1 - \alpha_t)\bar{d} + \alpha_t d_t + e_{d,t} \text{ where } e_{d,t} = \varepsilon (+\sigma_d, -\sigma_d) \text{ and } \text{Prob}[e_{d,t}=\sigma_d] = 1/2
\end{equation}

\begin{equation}
(18) \quad w_{t+1} = (1 - \alpha_2)\bar{w} + \alpha_2 w_t + e_{w,t} \text{ where } e_{w,t} = \varepsilon (+\sigma_w, -\sigma_w) \text{ and } \text{Prob}[e_{w,t}=\sigma_w] = 1/2
\end{equation}

In each period, the state of the world can be such that the probability of a high or low dividend yield or a high or low labor return is equal to one half. The time path of dividends is constructed such that with a probability of one half, future dividends will yield a high payment denoted by $d'_{H} = (1 - \alpha_1)\bar{d} + \alpha_1 d_t + \sigma_d$, and similarly with a probability of one half, future dividends will take on a lower yield denoted as $d'_{L} = (1 - \alpha_1)\bar{d} + \alpha_1 d_t - \sigma_d$. Similar time path interpretation was applied to wages creating the variables $w'_H$ and $w'_L$. Note that the parameters, $\sigma_d$ and $\sigma_w$ are parameters governing the transition of wages and dividends. If one assumes that the
notation for next periods high and low dividends is $d_H^t$ and $d_L^t$ respectively, and similarly for wages, then the equity pricing function can be rewritten to form:

$$(19) \quad p(d, w) = [w_i + d_i]^a \beta[(\omega_1)[p(d_H^t, w_H^t) + d_H^t][d_H^t + w_H^t]^{-a} + (\omega_2)[p(d_L^t, w_H^t) + d_L^t][d_L^t + w_L^t]^{-a} + (\omega_3)[p(d_H^t, w_L^t) + d_H^t][d_H^t + w_L^t]^{-a} + (\omega_4)[p(d_L^t, w_L^t) + d_L^t][d_L^t + w_L^t]^{-a}]$$

where, for example, the term $p(d_H^t, w_H^t)$ denotes next period's equity price as a function of next period's high dividend and wage yields. The bond pricing function is derived in similar fashion. The pricing function for discount bonds is a little less complex as future bond prices do not influence current prices.

$$(20) \quad q(d, w) = \beta[(d + w)^a [(\omega_1)(d_H^t + w_H^t) + (\omega_2)(d_H^t + w_L^t) + (\omega_3)(d_L^t + w_H^t) + (\omega_4)(d_L^t + w_L^t)]$$

In the equity and bond pricing functions from above, the parameter, $\omega$ governs the actual correlation between wage and dividend error components. In order to calibrate this parameter, the following procedure was used: $\omega_1 = \omega_4 = 0.5\gamma$ and $\omega_2 = \omega_3 = 0.5(1-\gamma)$, whereby $corr(e_d, e_w) = (2\gamma -1)$. In the current study there appeared to be a negative correlation between the error components in the order of $-0.375$. As a result the gamma parameter was set to 0.3125 yielding the following weights; $\omega_1 = \omega_4 = 0.156$ and $\omega_2 = \omega_3 = 0.34385$.

Once the equity and bond pricing functions were solved, the real quarterly returns were calculated which were further annualized. The formula used to calculate the holding period return on equity was, $RE_t = [E(p_{t+1} + d_{t+1})/ p_t] - 1$ and the return on discount bonds was simply calculated as $RB_t = 1/q_t$. The risk premium is calculated as the difference between the two returns.

Recall that the goal of the current study is to separate output into wage and dividend components. In order to simulate asset prices and returns, the asset pricing
functions must first be solved over a certain grid space that wages and dividends will take. In other words, one needs to specify the interval for wages and dividends that the asset prices are going to be computed over.

To calibrate the grid bounds for wages and dividends one first needs to specify the mean values for the variables found in the data. Recall that the mean value for real output for the period under study was normalized to one, denoted as \( \bar{y} = 10.0 \). Using the 1959-1995 data the mean dividend, \( \bar{d} \), was found to be 0.50, calculated by taking the mean of the real profit’s share of total output. Since there is only one other variable, the mean \( \bar{w} \) was set to 9.5. Once again to create the lower and upper bounds for the intervals of wages and dividends the following formula was utilized:

\[
[ X, \bar{X} ] = \bar{X} \pm \text{std}(X) \times \text{(number of standard deviations)}.
\]

The choice of the number of standard deviations is arbitrary. For this model I have chosen to solve the asset pricing functions over a space of two standard deviations. Having solved for the mean components and decided on the number of standard deviations, the theoretical standard deviation of dividends and wages for the periods under study were utilized from section two. The theoretical standard deviation of each variable along with the lower and upper bounds appear in Table Seven.

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**Table 3-3**  
Bounds for Dividend & Wage Variables in the Dividend and Wage Model

<table>
<thead>
<tr>
<th>Period</th>
<th>Mean Values</th>
<th>Standard Deviations for</th>
<th>Interval Bounds</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>dbar</td>
<td>wbar</td>
<td>Dividend Cycle</td>
</tr>
<tr>
<td>1959-1995</td>
<td>0.50</td>
<td>9.5</td>
<td>4.58 %</td>
</tr>
</tbody>
</table>

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The average of the simulated returns for the current study appear in Table Eight along with the actual market returns and the returns predicted by the output endowment model from the previous section. The purpose of the current study was to decompose the output endowment model into wage and dividend innovations in expectation of a more realistic risk premium. When the value of the risk aversion parameter is set to two, results reported in Table Eight fail to replicate the annual risk premium observed in the current market, however they do manage to quantify a higher risk premium than the one simulated by the output endowment model. In conclusion, the decomposition of output into correlated wage and dividend innovations proved to be more successful in quantifying the currently observed market premium. In analyzing the individual components of the estimated equity premium, while the return on equity seems to correspond to the market return, the bond yield predicted by the model seems to be highly overestimating the market yield.

### Table 3-4
Risk Premium – Dividend & Wage Endowment Model
Average of Annualized Real Returns in Percent

<table>
<thead>
<tr>
<th></th>
<th>Dividend &amp; Wage Model When $\alpha = 2.0$</th>
<th>Dividend &amp; Wage Model When $\alpha = 5.0$</th>
<th>Actual Market Returns</th>
<th>Output Endowment Model When $\alpha = 2.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity Return</td>
<td>3.937</td>
<td>6.288</td>
<td>4.43</td>
<td>4.294</td>
</tr>
<tr>
<td>Bond Return</td>
<td>2.515</td>
<td>3.412</td>
<td>1.79</td>
<td>4.1078</td>
</tr>
<tr>
<td>Risk Premium</td>
<td>1.4218</td>
<td>2.876</td>
<td>2.63</td>
<td>0.186</td>
</tr>
</tbody>
</table>
An interesting study specifically revolves on the risk aversion parameter. The parameter, $\alpha$, which measures ones’ willingness to substitute consumption between successive time periods is an important one in many fields of economics. As agents become more risk averse one would expect demand for bonds to increase and demand for equity to decrease, driving up the price of bonds and down the price of current equity shares. This in turn should put downward pressure on bond yields and upward pressure on equity returns, measured by $[E(p_{t+1} + d_{t+1})/ p_t] - 1$. Therefore, in theory, one would expect a higher equity premium associated with higher risk aversion. A value of two to five, for the risk aversion parameter, has been commonly used in literature seen in portfolio theory work done by Donaldson and Duthie (2002). Table Eight demonstrates the correlation between the equity premium and the value of the risk aversion parameter. Note that when the risk aversion parameter has been set to five, the risk premium estimated by the current study is able to replicate the market premium. However, one should also note that the reason for this is mostly a result of the higher equity returns. The demand for bonds and therefore the bond returns do not seem to be responsive to the different risk aversion parameter values and are still too high relative to the market data.

Another interesting study specifically centers on the correlation of the wage and dividend innovations. When the correlation between wage and dividend evolutions is extremely high such as one, agents in the economy are faced with a 50 percent gamble of landing a very high or a very low income. In order to insure against possible economic downturns, agents increase their investment demands by purchasing risk free Treasury bills. Higher demand for bonds should put downward pressure on bond yields. In principle, investing in equity shares is not desirable as stocks deliver dividend yields, which are perfectly correlated with wage innovations. In essence, investing in equity shares does not provide proper diversification. As a result, one should expect that a high correlation between dividend and wage innovations would be accompanied by a larger risk premium, however simulation results presented in table nine are mixed. One can see that as the correlation increases, the risk premium first decreases, as we get to zero correlation, and then increases, when the correlation between the innovations goes to one.
### Table 3-5
Analysis of Correlations in Innovations

<table>
<thead>
<tr>
<th></th>
<th>Dividend &amp; Wage Model When $\text{corr}(e_d, e_w) = -0.35$</th>
<th>Dividend &amp; Wage Model When $\text{corr}(e_d, e_w) = 0$</th>
<th>Dividend &amp; Wage Model When $\text{corr}(e_d, e_w) = 1.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equity Return</td>
<td>3.937</td>
<td>4.704</td>
<td>4.919</td>
</tr>
<tr>
<td>Bond Return</td>
<td>2.515</td>
<td>4.015</td>
<td>3.276</td>
</tr>
<tr>
<td>Risk Premium</td>
<td><strong>1.4218</strong></td>
<td><strong>0.6895</strong></td>
<td><strong>1.643</strong></td>
</tr>
</tbody>
</table>
4. Conclusion

The current study has come very close to replicating the market premium, with reasonable levels of risk aversion. Likewise the current study has been able provide better results then Mehra and Prescott’s 1985 model using current U.S. data. The dividend and wage endowment model has suggested that decomposing the real output variable into more volatile wage and dividend components could point the economic research of asset pricing in a direction not yet explored.

The reason why the output endowment model might have failed to predict the market premium could be a result of the calibration procedure used. Recall that the output endowment model does not calibrate the dividend innovations to the actual U.S. data on corporate dividends, but instead uses output endowment as a proxy for dividend yields. However, the use of real output as a proxy for dividends is not proper, as market data has shown that dividends vary too much to be explained by variations in aggregate output. One way to circumvent this dependence is to actually collect data on corporate dividends themselves and use that instead. However, Sanford, Shiller and LeRoy (1981) argue that asset prices vary too much to be explained by variations in dividends and furthermore show evidence that variability of stock prices indices cannot be accounted for by information regarding future dividends, since dividends just do not seem to vary enough to justify the price movements. In response to the findings above, the current study has used actual corporate profits in place of dividends. As demonstrated the variability in corporate profits is able to provide more realistic measures of asset returns and the equity premium itself.

In conclusion, while the current study has been able to produce a higher risk premium than the output endowment model, it has failed to replicate the actual nominal equity premium with reasonable levels of risk aversion. Perhaps a more finance oriented model, incorporating variables such as investment taxation, liquidity constraints, corporate theory and profits could come closer at estimating the notable equity premium.
Appendix

Dependent Variable: OUTPUT_CYCLE
Method: Least Squares
Date: 11/27/03   Time: 10:56
Sample(adjusted): 1959:2 2002:4
Included observations: 175 after adjusting endpoints

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>t-Statistic</th>
<th>Prob.</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>-9.13E-05</td>
<td>0.000612</td>
<td>-0.149248</td>
<td>0.8815</td>
</tr>
<tr>
<td>OUTPUT_CYCLE(-1)</td>
<td>0.862380</td>
<td>0.038550</td>
<td>22.37020</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

R-squared 0.743105 Mean dependent var -4.37E-05
Adjusted R-squared 0.741620 S.D. dependent var 0.015926
S.E. of regression 0.008095 Akaike info criterion -6.783739
Sum squared resid 0.011337 Schwarz criterion -6.747570
Log likelihood 595.5772 F-statistic 500.4260
Durbin-Watson stat 1.511912 Prob(F-statistic) 0.000000

Vector Autoregression Estimates
Date: 12/03/03   Time: 12:46
Sample(adjusted): 1950:2 1993:4
Included observations: 175 after adjusting endpoints
Standard errors in ( ) & t-statistics in []

<table>
<thead>
<tr>
<th>PROFIT_CYCLE</th>
<th>WAGE_CYCLE</th>
</tr>
</thead>
<tbody>
<tr>
<td>PROFIT_CYCLE(-1)</td>
<td>0.737185</td>
</tr>
<tr>
<td></td>
<td>(0.05454)</td>
</tr>
<tr>
<td></td>
<td>[13.5175]</td>
</tr>
<tr>
<td>WAGE_CYCLE(-1)</td>
<td>-0.020604</td>
</tr>
<tr>
<td></td>
<td>(0.31154)</td>
</tr>
<tr>
<td></td>
<td>[-0.06613]</td>
</tr>
<tr>
<td>C</td>
<td>-0.000329</td>
</tr>
<tr>
<td></td>
<td>(0.00518)</td>
</tr>
<tr>
<td></td>
<td>[-0.06337]</td>
</tr>
</tbody>
</table>

R-squared 0.545187 0.738048
Adj. R-squared 0.539898 0.735002
Sum sq. resid 0.808945 0.014307
S.E. equation 0.068580 0.009120
F-statistic 103.0886 242.3045
Log likelihood 222.1567 575.2196
Akaike AIC -2.504647 -6.539653
Schwarz SC -2.450394 -6.485399
Mean dependent -0.001418 -4.70E-05
S.D. dependent 0.101104 0.017717
References


