EVALUATING VALUE AT RISK MODELS AT CANADIAN COMMERCIAL BANKS

by

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Approval

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Abstract

This research paper analyzes the internal Value-at-Risk (VAR) models of the “Big Five” Canadian commercial banks by empirically investigating the banks’ actual non-anonymous daily VAR and profit-and-loss (P&L) data. These data points were extracted from graphs included in the banks’ own annual report. Out of the 4340 trading days analyzed in this study, there are 47 exceptions (days when the actual loss exceeds the disclosed VAR); whereas, the expected figure is 43 exceptions at a 99% confidence interval. During a financial crisis, as is the case for the study’s time line, this internal VAR model demonstrates an inconsistent result among our sample banks.

For example, the number of exceptions was found to be significantly variable between banks, with two banks, BMO and RBC, experiencing 26 and 16 exceptions respectively, while BNS and TD only have one day with a loss exceeding the VAR. We doubt whether the internal model precisely evaluate VAR, so we conduct alternative method such as Historical simulation model and GARCH(1,1) model to calculate the banks’ VAR, we conclude Historical simulation model is best among those models based on the test results.

Key words: Canadian Commercial Bank, Value at Risk, Historical simulation and GARCH model, Back testing
Dedication

I wish to dedicate this paper to my dearest parents and my girlfriend for their endless support. Also I wish to dedicate this paper to all my professors, who coach and support me during the period I study in Canada.

Xiaoyi Chen

I wish to dedicate this paper to my dearest parents for their love and support. I also want to dedicate this paper to my teacher and classmates, who always support me during my graduate study in Simon Fraser University.

Duo Zheng
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1. Introduction

Under current regulations, because banks normally calculate the 99% Value-at-Risk (VAR) on a daily basis, the CEOs of commercial banks generally are concerned with that last 1% of their profit-and-loss (P&L) distribution. As a market risk measure, VAR is an indispensable tool since it allows risk managers to summarize the projected maximum loss over a target horizon (such as a single day or a single week) at a specified confidence level (typically 95% or 99%). VAR also allows banks to quantitatively measure downside risk, as well as to summarize various factors for single dollar amounts, including diversification, leverage effects, and the risk analysis of potential negative market price movements. VAR is considered a significant reference measure for market risk even though Artzner (1999) criticized elements of its mathematical properties and other researchers claim that its use entails potentially destabilizing effects on the economy.

Upon implementing the Amendment to the Capital Accord to Incorporate Market Risks (Basel Committee on Banking Supervision, 1996a), banks heavily involved in trading must now calculate a market risk capital charge employing either the banks’ own internal risk model, measurement, or a Committee-developed standard operating procedure. Banks are required to set aside sufficient capital to offset potential losses from their trading activities if they use the internal model approach. However, supervisory regulators must first accept that the bank’s internal model demonstrates high efficiency and accuracy by conducting a back testing of its output through the use of a year’s worth of historical data (Basel Committee on Banking Supervision, 2004). Additionally, to properly utilize the Market Risk Amendment, a daily computation of VAR must be conducted daily using a 99th percentile, one-tailed confidence interval. The bank’s model is also required to accurately identify and account for any unique risks due to associated market factors. The Basel Committee, though, does not require banks to use any single variant of the VAR model. Banks are allowed to employ any model based on established VAR-calculation methods such as variance-covariance matrices, historical simulations, or Monte Carlo simulations. Commercial banks often use far more complex
computing methods and for market risk management, major commercial banks are now developing large scale, multifaceted VAR models.

In recent years, as the commercial banks’ trading accounts increase significantly, another question raised is whether analysts and investors should use VAR disclosures to compute the banks’ risk levels regarding their trading portfolios? Jorion (2002) analyzed the relation between disclosures for trading VAR using a small sampling of American commercial banks, against their respective trading revenues’ variability. His study’s empirical results indicate because they can help predict trading revenue variability, VAR disclosure should be considered informative and relevant. At the same time, VAR disclosure has a demonstrated correlation with banks’ trading P&L, yet the banks’ VAR models continue to be reasonably accurate.

Domestic regulators ensure that all banks within their jurisdictions are regularly computing and disclosing their VAR forecasts in compliance with international standards. In America, the regulation requiring such market risk disclosures is set out in the SEC’s 1997 Financial Reporting Release Number 48. North of the border, the Office of the Superintendent of Financial Institutions Canada (OSFI) has made similar VAR calculation compulsory since 1997. Since the late nineties, VAR figures have also been publicly disclosed, thus fulfilling three key objectives. VAR calculations are supposed to be all-inclusive market risk assessment figures borne by a given bank. Its use lowers the information asymmetry between the bank and the market. Next, VAR estimates tell banks how much reserve capital must be set aside to provide the necessary cushion to cover the firms’ cumulative losses if negative market conditions occur. For regulators, the third important reason for disclosing VAR figures is to allow them to test out the bank’s VAR model through back testing those data points. The basic premise is that if the bank’s VAR model works accurately, exceptions, for example when actual losses exceed the disclosed calculated VAR, should be very limited no matter which statistical model is used. For instance, if the calculated VAR assumes a one-day holding period and a 99% confidence level, then the predicted frequency that trading losses
will exceed VAR measures is ten times every thousand trading days.

Our study will examine the commercial banks’ VAR models’ accuracy and attempt to determine how the five major Canadian banks investigated in this research project are evaluating their VAR? In depth examination of the five major Canadian banks’ actual VAR and P&L data help to answer this question. For instance, out of the 4340 trading days analyzed in this research, 47 exceptions were identified as opposed to the expected 43 number of exceptions with 99% VAR. This empirically derived result goes counter to the widely accepted perception that banks are intentionally understating their cumulative market risks in order to minimize their market risk capital charges. However, when we investigate each bank, the result is quite inconsistent.

Unfortunately, researchers, including our group, do not have public access to actual daily VARs and P&L in a machine-readable format. Daily VARs are not, in fact, private information, but neither are VARs really public information since this data is often disclosed simply in graphical format as part of the banks’ annual reports. Therefore, the accepted term that describes this type of financial information is “seemingly public”. To cope with this lack of ready-to-use banking data, the research team developed a unique data extraction technique that enables us to extract VAR and daily P&L data points from graphs found in the banks’ annual reports.

The remainder of this paper is organized in the following manner. Empirical analysis of actual VAR and P&L data from our sample of the five major Canadian banks is presented in the next section. Results from back testing each banks’ actual VARs is included. The third section includes our comparisons of the banks’ VARs to simple VAR estimates derived from historical simulations and GARCH modeling techniques. Finally, a summary of our findings and conclusions can be found in Section 4.
2. Empirical analysis

2.1. Data

The actual VARs and P&Ls of five major Canadian commercial banks for the period November 1, 2005 to October 31, 2009 was analyzed. The five banks are Royal Bank of Canada (RBC), Toronto-Dominion Bank (TD), Bank of Nova Scotia (BNS), Bank of Montreal (BMO) and Canadian Imperial Bank of Commerce (CIBC). Canadian commercial banks were chosen for this study since, unlike their American counterparts, Canadian financial institutions include VAR and P&L data in graph-format as standard procedure in their annual reports.

The P&L variable is a measure of the expected daily gain or loss incurred by a bank’s trading portfolio. In order to remain consistent with daily VAR forecasts that set portfolio changes during the holding period at zero, the P&L measures hypothetical portfolio value changes that would hypothetically be incurred if each end-of-day position stays the same. In our sample, all banks compute hypothetical P&Ls, except for the Bank of Nova Scotia and Toronto-Dominion Bank that report actual P&Ls. We extracted daily VARs and P&L data from graphs posted in the banks’ annual reports. For each bank, we retrieve actual VARs and P&Ls using an Adobe photo based application that allows us to convert the annual reports’ graph format into a time series of daily values (See Appendix I for data extraction detail). In order to ensure data reliability and accuracy, we drew our graphs based on the extracted data by using excel, and visually comparing them with the graphs in the annual report. Since our excel-created graphs bore striking similarity to the original annual report graphs, we concluded that our data is authentic. However, if the original graph is either inaccurate or incomplete, this data analysis method cannot be used. For example, in our case, we could not use this methodology for TD since this bank did not disclose actual P&L in its 2006 and 2007 annual reports. Despite this limitation, the number of observations is sufficiently large enough to implement our back testing procedure and the sample size is robust enough to construct our
historical simulation and GARCH models.

Descriptive statistics

<table>
<thead>
<tr>
<th></th>
<th>BMO</th>
<th>BNS</th>
<th>CIBC</th>
<th>RBC</th>
<th>TD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: sample banks</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Market Cap (2009)</strong></td>
<td>30,813.33</td>
<td>50,447.52</td>
<td>26,168.50</td>
<td>79,953.19</td>
<td>56,698.23</td>
</tr>
<tr>
<td><strong>Total Assets (2009)</strong></td>
<td>388,458.00</td>
<td>496,516.00</td>
<td>335,944.00</td>
<td>654,989.00</td>
<td>557,219.00</td>
</tr>
<tr>
<td><strong>Stock return (06-09)</strong></td>
<td>-4.31%</td>
<td>1.29%</td>
<td>-3.20%</td>
<td>5.17%</td>
<td>1.52%</td>
</tr>
<tr>
<td><strong>Stock volatility (06-09)</strong></td>
<td>33.03%</td>
<td>31.95%</td>
<td>35.30%</td>
<td>32.67%</td>
<td>31.32%</td>
</tr>
<tr>
<td><strong>revenue from Canada (2009)</strong></td>
<td>69.72%</td>
<td>-</td>
<td>96.18%</td>
<td>59.41%</td>
<td>68.05%</td>
</tr>
<tr>
<td><strong>revenue from US (2009)</strong></td>
<td>25.49%</td>
<td>-</td>
<td>3.02%</td>
<td>26.45%</td>
<td>21.87%</td>
</tr>
<tr>
<td><strong>Panel B: VaR</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Moving Window</strong></td>
<td>N/A</td>
<td>300 Days</td>
<td>N/A</td>
<td>500 Days</td>
<td>259 Days</td>
</tr>
<tr>
<td><strong>Confidence Level</strong></td>
<td>99%</td>
<td>99%</td>
<td>99%</td>
<td>99%</td>
<td>99%</td>
</tr>
<tr>
<td><strong>Time Horizon</strong></td>
<td>1 day</td>
<td>1 day</td>
<td>1 day</td>
<td>1 day</td>
<td>1 day</td>
</tr>
<tr>
<td><strong>Start date</strong></td>
<td>Nov 1,2005</td>
<td>Nov 1,2005</td>
<td>Nov 1,2005</td>
<td>Nov 1,2005</td>
<td>Nov 1,2007</td>
</tr>
<tr>
<td><strong>Total observations</strong></td>
<td>1000</td>
<td>786</td>
<td>1018</td>
<td>1030</td>
<td>506</td>
</tr>
</tbody>
</table>

Table 1

Notes:
This table presents some descriptive statistics for Bank of Montreal (BMO), Bank of Nova Scotia (BNS), Canadian Imperial Bank of Commerce (CIBC), Royal Bank of Canada (RBC), and Toronto-Dominion Bank (TD). Panel A highlights each respective banks’ market capitalization, total assets, annualized stock returns, annualized standard deviations of the stock returns and both domestic and American-generated revenues. Panel B highlights each respective bank’s internal VAR models, the lengths of the estimation window used to extract daily VARs, the start dates and end dates of our sample and the total number of observations.

The five sample banks are the largest, most dominant commercial banks in Canada, also known as the “Big Five”. The largest bank is the Royal Bank of Canada, with assets totaling $655 billion on deposits of $398.2 billion. RBC’s 1,197 branches and 71,186 employees slightly outnumber the 1,116 Toronto-Dominion Bank branches with their 65,930 employees. TD bank possesses $557.2 billion in assets and $391 billion in deposits, compared to number three, the Bank of Nova Scotia, with $496.5 billion in assets and $350 billion in deposits. BNS also possesses 1,019 branches with 607,802 employees. In fourth place is the Bank of Montreal (BMO) with $388.5 billion in assets, $236.2 billion in deposits but only 900 branches. It employs 36,173 individuals throughout its operation, as opposed to Canadian
Imperial Bank of Commerce (CIBC) 41,941 employees. CIBC though only has $335.9 billion in assets, $223.1 billion in deposits spread across its 1,072 branches (Appendix 1). Among our sample banks, BMO and CIBC exhibit negative stock return, -4.31% and -3.2% respectively. The stock volatility of our sample banks ranged from 31.32% to 35.03%, with CIBC generating the most volatile stock returns and TD having the most stable stock returns. Although the lack of information about BNS’s revenue preclude us from drawing any conclusions about geographic revenue generation, the rest of our sample banks are heavily reliant on the North American market. For instance, almost 100% of CIBC’s revenues originate from the North-American market. Table 1 also indicates that all sample banks compute and disclose one-day ahead VARs with 99% confidence level on a daily basis. Depending on the banks, we get daily historical data encompassing between 2 to 4 years with a total number of trading days from 506 to 1030. The long sample periods and high number of data points proved advantageous when employing back testing procedures, as they result in more powerful statistical tests.

Figure 1
Figure 2

Scotia 06-09(Million)

Figure 3

CIBC 06-09(Million)
Figure 4

Figure 5
Notes: This table presents some descriptive statistics on VAR (panel A) and profit and loss (panel B) for each sample bank. Panel A lists each respective bank’s expected number of exceptions and actual number of exceptions. Panel B lists each respective bank’s percentage of days with some loss and the 99th percentile of the empirical profit and loss distribution. Posted means, standard deviations, and 99th percentiles are stated in million of Canadian dollars. The sample period is November 1, 2005 and October 31, 2009.

We plot in Figure 1-5 “Big Five’s” daily, one-day ahead, 99% VARs (in red) and daily P&Ls (in blue). It illustrates several important features of the actual VAR and P&L data, such as the high volatility of daily P&Ls. Also, our sample group demonstrates a degree of similarity with regard to fluctuations in P&L movement across all studied banks, although ranges within those fluctuations varied greatly between banks. Compared to studies conducted by Perignon, Deng, and Wang during relative “boom” times, our results graphically illustrate how all five banks suffered from large numbers of negative P&Ls and every bank had several days’ worth of sharp P&L drops. VAR forecasts also fluctuate from one day to next, reflecting a daily rebalancing of the banks’ trading portfolios and volatility shocks. Most importantly for this study, the number of exceptions is significantly variable between banks even though the overall industry-wide exceptions rate falls within the expected 99% confidence interval. In our sample, two banks, BMO and RBC, experience 26 and 16 exceptions respectively, while both BNS and TD only have one day with a loss exceeding the VAR. When compared to the
expected number of exceptions in a sample of 4340 trading days of 99% VARs (43 exceptions), the total number of exceptions (47) is approximately equal to the expected result. Our results contrast with those of Perignon et al., in their study of six Canadian commercial banks’ VAR. Using 2 to 6 years sample period, they reported only two exceptions out of total 7354 trading days, leading them to conclude that the VAR were overstated. One possible explanation for the demonstrated VAR differences between the two studies may be the global economic downturn that occurred during our chosen sample period. According to our graph, most extreme points happen during 2007 to 2009, falling within the timeframe of the recent global collapse of major financial institutions, the billions dollars taxpayer bailouts of banks and the worldwide stock market crashes.

Table 2 has some descriptive statistics on actual VAR and P&L data. According to Panel A (information related to VAR), TD has the largest number (44.79) for the mean in absolute value while CIBC has the smallest number (10.05) in absolute value. TD also has the largest sigma (18.32) while CIBC has the smallest (3.6). VARs are all left-skewed (corresponding to the negative skewness data points) in sample banks, but they also exhibit fatter tails than normal distribution.

### 2.2 Back testing

As mentioned above, the number of actual exceptions are either too high or too low comparing to expected exceptions, which means the banks’ internal models are not quite suitable for those banks. To confirm this “rough” conclusion, firstly, we apply standard coverage test to banks’ disclosed VAR and P&L to see how accuracy the banks’ internal VAR models are.

Statistically speaking, the back testing is consisted of three likelihood ratio tests: unconditional coverage test, independence test, and conditional coverage test. Just as its name
implies, the unconditional test ignores “conditioning” or time variation in data, while the independence test consider the situation of exceptions over successive days. So the conditional test just sums the statistic results from the previous two tests.

The traditional coverage test is focusing on unconditional test, which is to see whether the exceptions of bank disclosed VAR statistically equal to the expected exceptions. The expected number of exceptions for 99% VAR over T trading days is 0.01*T. Knowing this, we use the following formula to do this test:

\[ LR_{uc} = -2 \ln \left( (1-p)^{T-N} p^N \right) + 2 \ln \left( \left( 1 - \frac{N}{T} \right)^{T-N} \left( \frac{N}{T} \right)^N \right) \]

\[ LR_{uc} \sim \chi^2_1 \]

In this formula, \( p \) is the expected violation rate, \( T \) is the total trading days (observations), \( N \) is the number of exceptions, and \( LR_{uc} \) stats is following chi-square distribution with one degree of freedom.

Although the unconditional test is very straightforward and easy to implement, there are two flaws:

1. The TYPE 1 and TYPE 2 error are significant when the test applies
2. The test does not consider exception clustering (exception dependence)

So, then we apply the conditional coverage test, which can solve above two problems:

\[ LR_{ind} = -2 \ln \left( (1 - \pi)^{T_{00}+T_{01}+T_{10}+T_{11}} \right) + 2 \ln \left( (1 - \pi_0)^{T_{00}+T_{01}} (1 - \pi_1)^{T_{10}+T_{11}} \right) \sim \chi^2_1 \]

\[ LR_{cc} = LR_{uc} + LR_{IND} \sim \chi^2_2 \]

Where \( T_{ij} \) is the number of days that state \( j \) happens in one day while state \( i \) happened one day before that day. 0 means no exception, while 1 means exception. \( \pi_i \) is a conditional probability of exception conditional on state \( i \) in previous day. So, in this case, \( \pi_0 = T_{0i}/(T_{00} + T_{01}) \), and \( \pi_1 = T_{1i}/(T_{10} + T_{11}) \). \( LR_{IND} \) Stats are following chi-square distribution with one degree of freedom, and \( LR_{cc} \) stats is following chi-square distribution with two degree of freedom. One more thing should be mentioned is that the null hypothesis
for $LR_{IND}$ is the exception days are independent each other.

<table>
<thead>
<tr>
<th>Bank</th>
<th>Trading day</th>
<th>Expected exceptions</th>
<th>Actual exceptions</th>
<th>$LR_{UC}$</th>
<th>$LR_{IND}$</th>
<th>$LR_{CC}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BMO</td>
<td>1000</td>
<td>10</td>
<td>36</td>
<td>25.7483</td>
<td>3.6729</td>
<td>29.4212</td>
</tr>
<tr>
<td>BNS</td>
<td>786</td>
<td>8</td>
<td>1</td>
<td>7.816</td>
<td>0.0029</td>
<td>7.816</td>
</tr>
<tr>
<td>CIBC</td>
<td>1018</td>
<td>10</td>
<td>3</td>
<td>4.3618</td>
<td>0.0221</td>
<td>4.3839</td>
</tr>
<tr>
<td>RBC</td>
<td>1010</td>
<td>10</td>
<td>16</td>
<td>5.6999</td>
<td>1.0301</td>
<td>6.724</td>
</tr>
<tr>
<td>TD</td>
<td>506</td>
<td>5</td>
<td>1</td>
<td>3.1501</td>
<td>0.005</td>
<td>3.2551</td>
</tr>
</tbody>
</table>

Table 3

We summarize the test results in Table 3. Again, the first column indicates the bank disclosed VAR for BMO and RBC are relatively small, while the disclosed VAR for the other three banks are very big. The $LR_{UC}$ stats demonstrate all banks' actual number of exceptions does not statistically equal to expected number of exceptions at 90% confidence level, which is consistent with our “rough” conclusion based on actual exceptions. For the $LR_{IND}$ stats, only BMO's exception days are not statistically independent. The $LR_{CC}$ stats accept TD’s and CIBC’s internal VAR model.

In order to further analyze the efficiency and accuracy of banks’ internal model, we also compute the all banks’ VAR by another two widely used models. These two models are depending on disclosed banks’ P&L only.

The first model is the historical simulation model. According to the “big five” banks' annual reports, the historical simulation model is the most popular model used by big commercial banks. We apply the “quantile” function in MATLAB to sort the daily P&L in ascending order, and if there are 200 observations (99% confidence interval), 2 losses will be bigger than VAR. We choose 200 days moving window for BMO, CIBC, and RBC, and we choose 105 days moving window for TD and Scotia due to smaller dataset. So the formula for BMO, CIBC,
and RBC to calculate historical simulation VAR is:

$$\text{quantile}(BMO/CIBC/RBC(i: 199 + i), [0.01])$$

The formula for TD and Scotia is:

$$\text{quantile}(TD/Scotia(i: 104 + i), [0.01])$$

The second model is the famous GARCH model, which is far more complicated than Historical simulation model. Empirical evidence says the GARCH (1, 1) is totally enough to deal with most situations, and it is more practical and easier to implement comparing to other GARCH model. The first step also the key step of using GARCH model is to calculate the variance by following formula:

$$h_t = \alpha_0 + \alpha_1 \cdot r_{t-1}^2 + \beta \cdot h_{t-1}$$

Because this formula is very hard to implement manually, we install the GARCH toolbox in MATLAB to do this complex computation. The main code is following:

```
“fattailed_garch(data,1,1,'NORMAL')”
```

Then we use the parametric VAR formula to calculate the VAR:

$$VAR(\text{mean}) = W_0 \times (\alpha \sigma \sqrt{\Delta t})$$

Where $\sigma_t$ equals to $h_t$ that we have got in the previous formula, and $\alpha$ is 2.33 for 99% confidence interval. $\sqrt{\Delta t}$=1 day.
Table 4

<table>
<thead>
<tr>
<th>Bank</th>
<th>Trading day</th>
<th>Expected exceptions</th>
<th>Actual exceptions</th>
<th>( LR_{UC} )</th>
<th>( LR_{IND} )</th>
<th>( LR_{CC} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>BMO</td>
<td>2000</td>
<td>10</td>
<td>14</td>
<td>3.73</td>
<td>0.1994</td>
<td>4.2294</td>
</tr>
<tr>
<td>BNS</td>
<td>786</td>
<td>8</td>
<td>0</td>
<td>13.6885</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CIBC</td>
<td>2018</td>
<td>10</td>
<td>5</td>
<td>1.4421</td>
<td>0.0526</td>
<td>1.5637</td>
</tr>
<tr>
<td>RBC</td>
<td>2090</td>
<td>10</td>
<td>10</td>
<td>0.3343</td>
<td>0.2442</td>
<td>0.5785</td>
</tr>
<tr>
<td>TD</td>
<td>506</td>
<td>5</td>
<td>1</td>
<td>3.2901</td>
<td>0.005</td>
<td>3.2551</td>
</tr>
</tbody>
</table>

Notes: For back test, we choose 10% confidence level, so the critical value for \( LR_{UC} \) and \( LR_{IND} \) is 2.71 (following chi-square distribution with one degree of freedom). For \( LR_{CC} \), the critical value is 4.61 (following chi-square distribution with two degrees of freedom).

Table 5

The test stats for these two alternative models are summarized in Table 4 and Table 5. In unconditional coverage test, historical simulation model only rejects BMO and RBC, while GARCH model rejects BMO, Scotia and TD. On the other hand, the independence test does not reject any banks for two models. But due to the feature of historical simulation method, its independence test’ stats is much higher. Finally, as we expect, historical simulation model only rejects BMO and RBC in conditional coverage test due to these two banks do not pass unconditional test. One thing we want to emphasize here is there are two reasons for GARCH model having “better” performance in conditional coverage test than that of Historical simulation model: one is that the actual exceptions for Scotia bank based on GARCH is Zero! Another reason is GARCH’s independence test stats are small because GARCH model is reactive to the P&L shocks (High independence). To be brief, the test results based on Historical simulation and GARCH model are better than that of banks’ internal model, and the Historical simulation model is even better than GARCH model for the banks we analyze.
2.3 Measuring VAR Error Percentage

In this part, our aim is to see the relationship between banks’ internal model and HS model or GARCH model. The following eight graphs show the error percentage between banks’ DVAR and the VAR computed by two alternative models. Once again, we use 2006-2009 annual reports’ data, and we choose 200 days moving window for BMO, CIBC and RBC, 105 days moving window for TD and Scotia. So the “real” starting point is 4th quarter of 2006 for BMO, CIBC and RBC, 3rd quarter of 2008 for TD, and 3rd quarter of 2006 for Scotia.

The formulas we use to compute Error Percentage are:

For DVAR vs GARCH model:

\[ Error \ Percentage = \frac{DVAR - GARCHvar}{GARCHvar} \]

For DVAR vs HS model:

\[ Error \ Percentage = \frac{DVAR - HSvar}{HSvar} \]

If the \textit{Error Percentage} is positive, the DVAR is overstated (comparing to GARCH or HS model), otherwise it is underestimated.
The figure clearly shows that before the 3rd quarter of 2007, the DVAR is overestimated, but the degree of overestimation is decreasing. After then, the DVAR is understated comparing to HS model.
Since the coefficients for SCOTIA are always above 0, the DVAR is overstated, especially in the 1st quarter of 2007.
DVAR vs GARCH

Scotia Error Percentage (DVAR vs GARCH)

Figure 8

The coefficients fluctuate heavily through the time. In 2nd quarter of 2007 and 4th quarter of 2007, the DVAR is overestimated, but it is underestimated in other time period.
Overall, the coefficients for CIBC’s HS model are moving smoothly. Before the 2nd quarter of 2008, the DVAR is overstated, especially in the 1st quarter of 2008. However, DVAR is underestimated since the 2nd quarter of 2008, and it is hugely overestimated in the 4th quarter of 2009.
**DVAR vs GARCH**

Contrary to the HS model, the coefficients for CIBC’s GARCH model are below zero before the financial year 2008, and above zero in the whole 2008 financial year, and below zero again afterwards.

**RBC**

**DVAR vs HS Simulation**
The coefficients for RBC’ HS model is decreasing all the time. The DVAR is approaching the “Normal” value in the 4th quarter of 2007. From 2nd quarter of 2008, the DVAR is underestimated.

**TD**

**DVAR vs HS Simulation**

Like Scotia’s HS model, the coefficients for TD’ HS model are all above zero. In 4th quarter of 2009, the DVAR is heavily overstated.
Generally speaking, the coefficients for TD’GARCH model are experiencing an up-down process. From the 3rd quarter of 2008 to 2nd quarter of 2009, the DVAR is overestimated, and the rate of inflation reaches its peak at the end of 1st quarter of 2009. However, the DVAR is under estimated in last 2 quarters of 2009.

3. Conclusion

We analyze the internal Value at Risk (VAR) model for five biggest Canadian commercial banks in 2006-2009 periods. Since all the banks have been disclosing the daily P&L and total VAR since 2000, we can extract these data from banks’ annual report, and these data provide us a reliable dataset to process back testing and compute VAR by other alternative models.
Unlike our reference paper, we notice that BMO and RBC have much more exceptions (ie. \( \text{Loss} > \text{VAR} \)) than expected, while the other three banks has less than three exceptions. So we run the back testing for “big five”’s internal VAR models. The traditional unconditional test rejects all the banks’ internal model, even the modified conditional test rejects three banks’ internal model. Based on this, we think all the banks do not have the good enough internal models to evaluate their VAR.

In order to find a more suitable model for these banks, we apply Historical simulation model (with specific moving window) and GARCH (1, 1) model to calculate the banks’ VAR, and the results from unconditional test and conditional test for both models are better than the internal models, and the Historical simulation model is even better than the GARCH model according to the test results.

We also try to investigate the degree that the banks’ VAR is over/under estimated. The historical model reveals that most of banks inflate their VAR before 2008, and a little bit underestimate (or fairly estimate) the VAR after 2008. In the late of 2009, some banks inflate their VAR again. The reason is that before 2008, the effects of financial crisis is not obvious, so the banks’ P&L is still stable and not experiencing many big losses. During the period of 2008-early 2009, all the banks’ are affected by the financial crisis. The probability of big loss is rising quickly, so it is reasonable to get the conclusion that the DVAR are underestimated. As the economy is recovering in the late 2009, some banks’ DVAR is going to be overstated again. For the GARCH model, due to the model is sensitive to actual P&L shocks, the trend is hard to demonstrate.

Finally, according to the Basel Accord Amendment (1996), the higher VAR is, the higher market risk charge will be. So it is not wise for the banks to overstate their
VAR after the financial crisis because this can be costly for the banks.

Appendix

Appendix I: Procedures for Photoshop based data extraction method

(1) Find the Daily P&L and Daily VAR graphs from banks’ Annual Reports

(2) Import these graphs into Adobe Photoshop. Adjust the resolution to make the graphs clear enough to be click

(3) Convert the Photoshop scale to the graph scale

(4) Record all the data by clicking and converting in EXCEL

Appendix II: MatLab codes for each bank

For BMO

clc
clear all
close
format compact
load BANK.mat
addpath(genpath('D:\Program Files\MATLAB\R2009b\toolbox\Ucsd_garch'))

%Computing variance by GARCH method
data=BMO(201:1000,1);
[parameters, likelihood, stderrors, robustSE, ht, scores] = fattailed_garch(data,1,1,'NORMAL');
ht;

%Historical Simulation
VAR_Simulation=nan(800,1);
for i=1:800
    VAR_Simulation(i,1)=quantile(BMO(i:199+i,1),0.01);
end
VAR_Simulation;
% Garch Approach
VAR_Garch=nan(800,1);
for i=1:800
    VAR_Garch(i,1)=-norminv(0.99)*ht(i,1).^0.5;
end

VAR_Garch;
% Calculate 'hits'
hits_hs=[BMO(201:1000,1)<VAR_Simulation(:,1)];
hits_gh=[BMO(201:1000,1)<VAR_Garch(:,1)];
hits_bank=[BMO(201:1000,1)<BMO(201:1000,2)];

hitcount_hs=sum(hits_hs) % Count the number of hits
hitcount_gh=sum(hits_gh)
hitcount_bank=sum(hits_bank)

% (e) LR_uc, LR_ind and LR_cc
[lr_hs.uc lr_hs.ind lr_hs.cc]=lr_fn(hits_hs(:,1),0.01);
lr_gh.uc lr_gh.ind lr_gh.cc]=lr_fn(hits_gh(:,1),0.01);
lr_bank.uc lr_bank.ind lr_bank.cc]=lr_fn(hits_bank(:,1),0.01);

disp ('Banks' )
disp ([lr_bank.uc lr_bank.ind lr_bank.cc])
disp ('Historical Simulation' )
disp ([lr_hs.uc lr_hs.ind lr_hs.cc])
disp ('GARCH' )
disp ([lr_gh.uc lr_gh.ind lr_gh.cc])

sum(BMO(:,1)<0)/1000
quantile(BMO(:,1),0.01)

Function:
function [lruc lrind lrcc]=lr_fn(x, p)
t=length(x(:,1))-1;
n=sum(x(:,1));
t00=0;
t01=0;
t10=0;
t11=0;
for i=1:t
    if x(i,1)
        if x(i+1,1)
            t11=t11+1;
else
    t10=t10+1;
end
elseif x(i+1,1)
    t01=t01+1;
else
    t00=t00+1;
end
end
pi=(t01+t11)/t;
pi0=t01/(t00+t01);
pi1=t11/(t10+t11);
lruc=-2*log((1-p)^(t-n)*p^n)+2*log((1-n/t)^(t-n)*(n/t)^n);
lrind=-2*log((1-pi)^(t00+t10)*pi^(t01+t11))+2*log((1-pi0)^t00*pi0^t01*(1-pi)\textasciitilde t10*\textasciitilde t11);
lrcc=lruc+lrind;

For Scotia
clc
clear all
close
format compact
load BANK.mat
addpath(genpath('D:\Program Files\MATLAB\R2009b\toolbox\Ucsd_garch'))

%Computing variance by GARCH method
data=BNS(106:786,1);
[parameters, likelihood, stderrors, robustSE, ht, scores] = fattailed_garch(data,1,1,'NORMAL');
ht;

%Historical Simulation
VAR_Simulation=nan(681,1);
for i=1:681
    VAR_Simulation(i,1)=quantile(BNS(i:104+i,1),0.01);
end
VAR_Simulation;

%Garch Approach
VAR_Garch=nan(681,1);
for i=1:681
    VAR_Garch(i,1)=-norminv(0.99)*ht(i,1).^0.5;
end
VAR_Garch;

% Calculate 'hits'
hits_hs=[BNS(106:786,1)<VAR_Simulation(:,1)];
hits_gh=[BNS(106:786,1)<VAR_Garch(:,1)];
hits_bank=[BNS(106:786,1)<BNS(106:786,2)];

hitcount_hs=sum(hits_hs) % Count the number of hits
hitcount_gh=sum(hits_gh)
hitcount_bank=sum(hits_bank)

% (e) LR_uc, LR_ind and LR_cc
[lr_hs.uc lr_hs.ind lr_hs.cc]=lr_fn(hits_hs(:,1),0.01);
lr_gh.uc lr_gh.ind lr_gh.cc]=lr_fn(hits_gh(:,1),0.01);
lr_bank.uc lr_bank.ind lr_bank.cc]=lr_fn(hits_bank(:,1),0.01);

disp ('Banks' )
disp ([lr_bank.uc lr_bank.ind lr_bank.cc])
disp ('Historical Simulation' )
disp ([lr_hs.uc lr_hs.ind lr_hs.cc])
disp ('GARCH' )
disp ([lr_gh.uc lr_gh.ind lr_gh.cc])
sum(BNS(:,1)<0)/786
quantile(BNS(:,1),0.01)

Function:
function [lruc lrind lrcc]=lr_fn(x, p)
t=length(x(:,1))-1;
n=sum(x(:,1));
t00=0;
t01=0;
t10=0;
t11=0;
for i=1:t
    if x(i,1)
        if x(i+1,1)
            t11=t11+1;
        else
            t10=t10+1;
        end
    elseif x(i+1,1)
        t01=t01+1;
    else
        pass
    end
end
else
t00=t00+1;
end

end

pi=(t01+t11)/t;
pi0=t01/(t00+t01);
pi1=t11/(t10+t11);
lruc=-2*log((1-p)^(t-n)*p^n)+2*log((1-n/t)^(t-n)*n/t^n);
lrind=-2*log((1-pi)^(t00+t10)*pi^(t01+t11))+2*log((1-pi0)^t00*pi0^t01
*(1-pi1)^t10*pi1^t11);
lrcc=lruc+lrind;

For CIBC

clc
clear all
close
format compact
load BANK.mat
addpath(genpath('D:\Program Files\MATLAB\R2009b\toolbox\Ucsd_garch'))

%Computing variance by GARCH method
data=CIBC(201:1018,1);
[parameters, likelihood, stderrors, robustSE, ht, scores] =
fattailed_garch(data,1,1,'NORMAL');
ht;

%Historical Simulation
VAR_Simulation=nan(818,1);
for i=1:818
    VAR_Simulation(i,1)=quantile(CIBC(i:199+i,1),0.01);
end
VAR_Simulation;

%Garch Approach
VAR_Garch=nan(818,1);
for i=1:818
    VAR_Garch(i,1)=-norminv(0.99)*ht(i,1).^0.5;
end
VAR_Garch;

% Calculate 'hits'
hits_hs=[CIBC(201:1018,1)<VAR_Simulation(:,1)];
hits_gh=[CIBC(201:1018,1)<VAR_Garch(:,1)];
hits_bank=[CIBC(201:1018,1)<CIBC(201:1018,2)];
hitcount_hs=sum(hits_hs) % Count the number of hits
hitcount_gh=sum(hits_gh)
hitcount_bank=sum(hits_bank)

% (e) LR_uc, LR_ind and LR_cc
[lr_hs.uc lr_hs.ind lr_hs.cc]=lr_fn(hits_hs(:,1),0.01);
[lr_gh.uc lr_gh.ind lr_gh.cc]=lr_fn(hits_gh(:,1),0.01);
[lr_bank.uc lr_bank.ind lr_bank.cc]=lr_fn(hits_bank(:,1),0.01);

disp ('Banks')
disp ([lr_bank.uc lr_bank.ind lr_bank.cc])
disp ('Historical Simulation')
disp ([lr_hs.uc lr_hs.ind lr_hs.cc])
disp ('GARCH')
disp ([lr_gh.uc lr_gh.ind lr_gh.cc])

sum(CIBC(:,1)<0)/1018
quantile(CIBC(:,1),0.01)

Function:

function [lruc lrind lrcc]=lr_fn(x, p)
t=length(x(:,1))-1;
n=sum(x(:,1));
t00=0;
t01=0;
t10=0;
t11=0;
for i=1:t
    if x(i,1)
        if x(i+1,1)
            t11=t11+1;
        else
            t10=t10+1;
        end
    elseif x(i+1,1)
        t01=t01+1;
    else
        t00=t00+1;
    end
end
pi=(t01+t11)/t;
pi0=t01/(t00+t01);
pi1=t11/(t10+t11);
\[
\text{lruc} = -2 \log((1-p)^{t-n} + p^n) + 2 \log((1-n/t)^{t-n} + (n/t)^n);
\]
\[
\text{lrind} = -2 \log((1-p)^{t00+t10} + p^{t01+t11}) + 2 \log((1-p10)^{t00} + p^{t01} + p10^{t10} + p^{t11});
\]
\[
\text{lrcc} = \text{lruc} + \text{lrind};
\]

For RBC

clc
clear all
close
format compact
load BANK.mat
addpath(genpath('D:\Program Files\MATLAB\R2009b\toolbox\Ucsd_garch'))

% Computing variance by GARCH method
data = RBC(201:1030,1);
[parameters, likelihood, stderrors, robustSE, ht, scores] = fattailed_garch(data,2,2,'NORMAL');
ht;

% Historical Simulation
VAR_Simulation = nan(830,1);
for i = 1:830
    VAR_Simulation(i,1) = quantile(RBC(i:199+i,1), 0.01);
end
VAR_Simulation;

% Garch Approach
VAR_Garch = nan(830,1);
for i = 1:830
    VAR_Garch(i,1) = -norminv(0.99) * ht(i,1)^0.5;
end
VAR_Garch;

% Calculate 'hits'
hits_hs = [RBC(201:1030,1) < VAR_Simulation(:,1)];
hits_gh = [RBC(201:1030,1) < VAR_Garch(:,1)];
hits_bank = [RBC(201:1030,1) < RBC(201:1030,2)];
hitscount_hs = sum(hits_hs) % Count the number of hits
hitscount_gh = sum(hits_gh)
hitscount_bank = sum(hits_bank)
% (e) LR_uc, LR_ind and LR_cc
[lr_hs.uc lr_hs.ind lr_hs.cc]=lr_fn(hits_hs(:,1),0.01);
[lr_gh.uc lr_gh.ind lr_gh.cc]=lr_fn(hits_gh(:,1),0.01);
[lr_bank.uc lr_bank.ind lr_bank.cc]=lr_fn(hits_bank(:,1),0.01);

disp ('Banks' )
disp ([lr_bank.uc lr_bank.ind lr_bank.cc])
disp ('Historical Simulation' )
disp ([lr_hs.uc lr_hs.ind lr_hs.cc])
disp ('GARCH' )
disp ([lr_gh.uc lr_gh.ind lr_gh.cc])

sum(RBC(:,1)<0)/1030
quantile(RBC(:,1),0.01)

Function:

function [lruc lrind lrcc]=lr_fn(x, p)
t=length(x(:,1))-1;
n=sum(x(:,1));
t00=0;
t01=0;
t10=0;
t11=0;
for i=1:t
    if x(i,1)
        if x(i+1,1)
            t11=t11+1;
        else
            t10=t10+1;
        end
    elseif x(i+1,1)
        t01=t01+1;
    else
        t00=t00+1;
    end
end
pi=(t01+t11)/t;
p00=0/(t00+t01);
p11=t11/(t10+t11);
lruc=-2*log((1-p)^t-n)p^n)+2*log((1-n/t)(t-n)*(n/t)^n);
lrind=-2*log((1-p)^t00+t10)*pi^t01+t11)+2*log((1-p10)^t00*p10^t01
*(1-pi)^t01^t11);
lrcc=lruc+lrind;
For TD

clc
clear all
close
format compact
load BANK.mat
addpath(genpath('D:\Program Files\MATLAB\R2009b\toolbox\Ucsd_garch'))

% Computing variance by GARCH method
data=TD(106:506,1);
[parameters, likelihood, stderrors, robustSE, ht, scores] =  
fattailed_garch(data,1,1,'NORMAL');
ht;

% Historical Simulation
VAR_Simulation=nan(401,1);
for i=1:401
    VAR_Simulation(i,1)=quantile(TD(i:104+i,1),0.01);
end
VAR_Simulation;

% Garch Approach
VAR_Garch=nan(401,1);
for i=1:401
    VAR_Garch(i,1)=-norminv(0.99)*ht(i,1).^0.5;
end
VAR_Garch;

% Calculate 'hits'
hits_hs=[TD(106:506,1)<VAR_Simulation(:,1)];
hits_gh=[TD(106:506,1)<VAR_Garch(:,1)];
hits_bank=[TD(106:506,1)<TD(106:506,2)];

hitcount_hs=sum(hits_hs) % Count the number of hits
hitcount_gh=sum(hits_gh)
hitcount_bank=sum(hits_bank)

% (e) LR_uc, LR_ind and LR_cc
[lr_hs.uc lr_hs.ind lr_hs.cc]=lr_fn(hits_hs(:,1),0.01);
[lr_gh.uc lr_gh.ind lr_gh.cc]=lr_fn(hits_gh(:,1),0.01);
[lr_bank.uc lr_bank.ind lr_bank.cc]=lr_fn(hits_bank(:,1),0.01);
disp ('Banks')
disp ([lr_bank.uc lr_bank.ind lr_bank.cc])
disp ('Historical Simulation')
disp ([lr_hs.uc lr_hs.ind lr_hs.cc])
disp ('GARCH')
disp ([lr_gh.uc lr_gh.ind lr_gh.cc])
sum(TD(:,1)<0)/506
quantile(TD(:,1),0.01)

Function:

```matlab
function [lruc lrind lrcc]=lr_fn(x, p)
t=length(x(:,1))-1;
n=sum(x(:,1));
t00=0;
t01=0;
t10=0;
t11=0;
for i=1:t
    if x(i,1)
        if x(i+1,1)
            t11=t11+1;
        else
            t10=t10+1;
        end
    elseif x(i+1,1)
        t01=t01+1;
    else
        t00=t00+1;
    end
end
pi=(t01+t11)/t;
pi0=t01/(t00+t01);
pi1=t11/(t10+t11);
lruc=-2*log((1-p)^(t-n)*p^n)+2*log((1-n/t)^(t-n)*(n/t)^n);
lrind=-2*log((1-pi)^(t00+t10)*pi^(t01+t11)) + 2*log((1-pi0)^t00*pi0^t01*(1-pi)^t10*pi1^t11);
lrcc=lruc+lrind;
```
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