INVESTIGATING TRANSITION MATRICES ON U.S. RESIDENTIAL BACKED MORTGAGE SECURITIES

by

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(PO HU)

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Abstract

The purpose of our research is to expand on the work of Kavvathas (2001) that studies credit rating transition probabilities for corporate bonds. This paper, for the period of 1991-2007 will be focused on rating transition matrices for US residential mortgage-backed securities (RMBS). In particular, we extend their techniques to a different data set and more recent time period by estimating credit rating transition matrices through the cohort method and the time-homogeneous duration method. In addition, we apply an alternative approach to calculate the average transition matrices.

Keywords: Credit risk; rating transitions; transition matrices
Dedication

We wish to dedicate this project to our families for their supports.
Acknowledgements

We wish like to thank Dr. Anton Theunissen for his guidance, feedbacks, and helpful comments. We would also like to thank Dr. Andrey Pavlov for being a reader on the project.
# Table of Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Approval</td>
<td>ii</td>
</tr>
<tr>
<td>Abstract</td>
<td>iii</td>
</tr>
<tr>
<td>Dedication</td>
<td>iv</td>
</tr>
<tr>
<td>Acknowledgements</td>
<td>v</td>
</tr>
<tr>
<td>Table of Contents</td>
<td>vi</td>
</tr>
<tr>
<td>List of Figures</td>
<td>vii</td>
</tr>
<tr>
<td>List of Tables</td>
<td>viii</td>
</tr>
<tr>
<td>1. <strong>Introduction</strong></td>
<td>1</td>
</tr>
<tr>
<td>2. <strong>Literature Review</strong></td>
<td>2</td>
</tr>
<tr>
<td>3. <strong>Data and Methodology</strong></td>
<td>3</td>
</tr>
<tr>
<td>3.1 The Data</td>
<td>3</td>
</tr>
<tr>
<td>3.2 Methodology</td>
<td>3</td>
</tr>
<tr>
<td>3.2.1 The Cohort Method</td>
<td>3</td>
</tr>
<tr>
<td>3.2.2 The Duration Method</td>
<td>4</td>
</tr>
<tr>
<td>4. <strong>Results and Discussion</strong></td>
<td>6</td>
</tr>
<tr>
<td>4.1 Simple Average Calculation</td>
<td>6</td>
</tr>
<tr>
<td>4.1.1 Simple Average (1991-2007)</td>
<td>6</td>
</tr>
<tr>
<td>4.1.2 Simple Average (1991-2007)</td>
<td>8</td>
</tr>
<tr>
<td>4.2 Weighted Average Calculation</td>
<td>9</td>
</tr>
<tr>
<td>4.2.1 Weighted Average (1991-2007)</td>
<td>11</td>
</tr>
<tr>
<td>4.2.2 Weighted Average (1991-2007)</td>
<td>12</td>
</tr>
<tr>
<td>5. <strong>Conclusion</strong></td>
<td>14</td>
</tr>
<tr>
<td><strong>Appendices</strong></td>
<td>15</td>
</tr>
<tr>
<td>Appendix A: Derivation of the Estimator of Homogeneous Transition Intensities using Duration Method</td>
<td>15</td>
</tr>
<tr>
<td>Appendix B: Comparison of Transition Matrices</td>
<td>17</td>
</tr>
<tr>
<td>Appendix C: US RMBS 2000-2007: M_{SVD}</td>
<td>18</td>
</tr>
<tr>
<td>Appendix D: Matlab Code - Cohort Method</td>
<td>19</td>
</tr>
<tr>
<td>Appendix E: Matlab Code - Duration Method</td>
<td>26</td>
</tr>
<tr>
<td><strong>References</strong></td>
<td>33</td>
</tr>
</tbody>
</table>
List of Figures

Figure 1: US RMBS 1991-2007: Weights of Active Securities ......................................................... 10
Figure 2: US RMBS 2000-2007 $M_{SVD}$ .................................................................................. 18
List of Tables

Table 1: Cohort method: Simple Average 1-year (NR adjusted) transition matrix (1991-2007) ................................................................. 6
Table 2: Duration method: Simple Average 1-year (NR adjusted) transition matrix (1991-2007) ................................................................. 7
Table 3: Cohort method: Simple Average 1-year (NR included) transition matrix (1991-2007) ................................................................. 8
Table 4: Duration method: Simple Average 1-year (NR included) transition matrix (1991-2007) ................................................................. 8
Table 6: Cohort method: Weighted Average 1-year (NR adjusted) transition matrix (1991-2007) ................................................................. 11
Table 7: Duration method: Weighted Average 1-year (NR adjusted) transition matrix (1991-2007) ................................................................. 11
Table 8: Cohort method: Weighted Average 1-year (NR included) transition matrix (1991-2007) ................................................................. 12
Table 9: Duration method: Weighted Average 1-year (NR included) transition matrix (1991-2007) ................................................................. 12
Table 10: Cohort method (NR adjusted) transition matrix (2006) ................................................................. 17
Table 11: Cohort method (NR adjusted) transition matrix (2002) ................................................................. 17
1. Introduction

The haze of global economy has not completely cleared off since the current explosion of sub-prime mortgage in the United States. Back to two years ago, the U.S. housing market started to experience a significant downturn. According to the report from Standard & Poor’s (2007), a large number of U.S. Residential Mortgage-Backed Securities (RMBS) were downgraded during the last half year of 2007. Many researchers have investigated rating transitions since they are the core of credit risk management in financial market. The purpose of our research is to expand on the work of Kavvathas (2000) that studies credit rating transition probabilities for corporate bonds. This paper will be focused on rating transition matrices for the U.S. RMBS. In particular, we extend their techniques to a different data set and more recent time period by illustrating two techniques for estimating credit rating transition matrices – the cohort method and the time-homogeneous duration method. This paper shows that the duration approach yields greater default probabilities for high credit ratings and low credit rating compared to the cohort approach. In addition, we find that the duration method generates smaller downgrade and upgrade probabilities.

The outline of our paper is as follows. In Section 2, we review some academic literature on credit transition matrices. Section 3 describes the data and methodology. The results are presented and discussed in Section 4. Finally, we will summarize our work and draw the conclusion in the Section 5.
2. Literature Review

There have been many researches discussing various techniques for estimating transition matrices.

Lando and Skodeberg (2002) compare transition matrices through the cohort method and the non-parametric duration method using the data from 1981 to 1997. They find higher default probabilities for CCC rating category in the duration case. They also confirm that the default probabilities are non-zero for the AAA rating category using either the maximum likelihood estimator in the time-homogeneous case or the time-inhomogenous case. Furthermore, they find significant downgrade momentum (except for the categories of BB, CCC+ and CCC) which increases the downgrade intensity by a factor of three.

Kavvathas (2000) conducts many extensive empirical investigations into credit rating transition probabilities. Kavvathas concentrates on US corporate bonds and uses S&P data from 1981 to 1998. They examine term structure and equity factors, reporting that an increase in nominal short and long and real interest rates, low equity returns and high equity return volatility are usually associated with high downgrade probabilities. In addition, Kavvathas presents many extensions to capture rating momentum and economic cycles. They discover that macroeconomic variables and industry effects are significant to rating intensities.

In a more recent study, Jobst and Gikes (2005) find that duration approach significantly increases default probabilities for investment grade. They also confirm that the default probabilities for non-investment grade ratings are generally higher compared to the cohort method.

Currently, most studies are investigated on the ratings for corporate bonds.
Data and Methodology

3. Data and Methodology

3.1 The Data

In the paper, we use the data set from Standard & Poor’s CreditPro. The raw data covers 18 years of rating history from 1990 to 2007, totaling 51,258 U.S. RMBS and 28,838 rating transitions. However, we observe sample period from 1991 to 2007, omitting 401 observations of the first year data set. The reason is that the first year data ignores default securities by only recording the active securities. Such biased selection can substantially affect the results of default probabilities. The ratings are listed based on 23 categories including NR. The highest rating is AAA and the lowest is C. Each of the categories of AA, A, BBB, BB, B and CCC then contains two modifiers “+” or “-”. Finally, there are a default category denoted D and a not rated category denoted NR. For our results, we use full categories for simplicity.

3.2 Methodology

Broadly there are the two approaches for modeling rating transitions, the cohort method and duration method.

3.2.1 The Cohort Method

The most common and straight-forward approach to estimate the transition matrices is the cohort method. It observes the starting states and the ending states for each security. The probabilities of rating transitions are calculated through dividing the number of transitions by the number of securities from the original ratings. We denote \( n_i(t) \) the total number of securities in rating \( i \) at time \( t \), and \( n_{ij}(0, t) \) the number of transitions of the securities from rating \( i \) at time 0 to rating \( j \) at time \( t \). The maximum likelihood estimator of \( q_{ij} \) is
Data and Methodology

\[ \hat{Q} = e^{\hat{\Lambda}} \]

The cohort method conceals a significant drawback. It captures only the terminal information in each year, but ignores the intra-year information which can be important for transition matrices.

3.2.2 The Duration Method

To avoid the cohort method problems, Lando and Skodeberg (2002) propose duration method to estimate the intensity matrices \( \hat{\Lambda} \), which require the knowledge of the precise transition time during the rating years. The duration method can be based on two assumptions: time-homogeneity and time-inhomogeneity. The assumption of time homogeneity implies that the intensity matrices are constant over time. The estimator of time-homogeneous transition intensities is

\[ \hat{\lambda}_{ij} = \frac{m_{ij}(T)}{\int_0^T n_i(s)ds} \]

where \( m_{ij}(T) \) is the total number of transitions from rating \( i \) to rating \( j \), \( i \neq j \) over the interval \([0, T]\). See Appendix A for the details. The \( m_{ij}(T) \geq n_{ij}(T) \), because \( m_{ij}(T) \) includes the transitions entering rating \( i \) during the interval. The denominator \( \int_0^T n_i(s)ds \) is the total amount of time that securities stay in the rating \( i \).

There is a much more complex estimation based on the assumption of time-inhomogeneous transition intensity. By using the Nelson-Aalen estimator, Lando and Skodeberg (2000) propose

\[ \hat{A}_{ij}(t) = \sum_{T_{ij}(k) \leq t} \frac{1}{n_i(T_{ij}(k) -)} \]
Data and Methodology

where \( T_{ij}(k) \) is the time of the \( k \)-th transition from rating \( i \) to rating \( j \), and \( n_t(t-\cdot) \) is the number of \( i \) rated securities, evaluated exactly before time \( t \). And the formula \( \hat{A}_{ij}(t) = \int_0^t \lambda_{ij}(s) \, ds \) applies to convert the Nelson-Aalen estimator to intensity estimators. The probability density of the transition time from rating \( i \) to rating \( j \) for each rating state:

\[
f(t_{ij}) = \lambda_{ij} e^{-(t_{ij}-t_{ij}^{\cdot}) \sum_{k \neq i} \lambda_k}
\]

Lando and Skodeberg (2002) reveal that the time-inhomogeneous estimator is not much different to the time-homogeneous estimator. Therefore, we restrict our investigation to cohort method and time-homogeneous duration method.
4. Results and Discussion

4.1 Simple Average Calculation

We obtain the average transition matrix by simply calculating the mean of yearly transition matrices.

\[
\hat{q}_{i,j} = \overline{q_{i,j,k}}
\]

4.1.1 Simple Average (1991-2007) one-year transition matrices: NR-adjusted

To begin our study, we first calculate transition matrices on a NR-adjusted base. Table 1 reports the average one year transition matrix by the cohort method. Table 2 presents the one year transition matrix by the time-homogeneous duration method.

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
<th>B</th>
<th>CCC</th>
<th>CC</th>
<th>C</th>
<th>D</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAA</td>
<td>99.536</td>
<td>0.436</td>
<td>0.013</td>
<td>0.012</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.004</td>
</tr>
<tr>
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<td>6.481</td>
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<td>0.225</td>
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<td>0.016</td>
<td>0.055</td>
<td>0.035</td>
<td>0.000</td>
<td>0.000</td>
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</tr>
<tr>
<td>A</td>
<td>1.821</td>
<td>3.955</td>
<td>91.687</td>
<td>1.934</td>
<td>0.303</td>
<td>0.022</td>
<td>0.253</td>
<td>0.000</td>
<td>0.000</td>
<td>0.024</td>
</tr>
<tr>
<td>BBB</td>
<td>0.182</td>
<td>1.766</td>
<td>2.618</td>
<td>93.479</td>
<td>0.968</td>
<td>0.377</td>
<td>0.208</td>
<td>0.115</td>
<td>0.000</td>
<td>0.288</td>
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<tr>
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<td>0.034</td>
<td>0.042</td>
<td>1.447</td>
<td>3.754</td>
<td>90.909</td>
<td>1.090</td>
<td>1.253</td>
<td>0.439</td>
<td>0.000</td>
<td>1.033</td>
</tr>
<tr>
<td>B</td>
<td>0.130</td>
<td>0.042</td>
<td>0.029</td>
<td>1.102</td>
<td>3.207</td>
<td>91.701</td>
<td>1.691</td>
<td>0.421</td>
<td>0.000</td>
<td>1.678</td>
</tr>
<tr>
<td>CCC</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>41.539</td>
<td>11.985</td>
<td>1.861</td>
<td>44.615</td>
</tr>
<tr>
<td>CC</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>57.814</td>
<td>0.000</td>
<td>42.186</td>
</tr>
<tr>
<td>C</td>
<td>0.000</td>
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<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>100.000</td>
</tr>
<tr>
<td>D</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>100.000</td>
</tr>
</tbody>
</table>

Table 1: Cohort method: Simple Average 1-year (NR adjusted) transition matrix (1991-2007)
Results and Discussion

By comparing the two methods, several major differences stand out:

First of all, the default probabilities of investment grade are non-zero under both cases. However, the duration method generates greater default probabilities for high credit ratings (AAA, AA and A) compared to the cohort approach.

Secondly, the default probabilities for low credit ratings are smaller for the duration method. For example, the CCC default probability of cohort method is 1.5 times as big as that of the duration method. Note that other low credit rating such as CC is not indicative because there are only 54 CC-rating securities out of 51,258.

Moreover, both matrices are diagonally dominant – in other words, there are high probabilities that securities maintain their ratings. But the probabilities of no migration are mostly higher for the duration method. We also find that the duration approach yields smaller downgrade probabilities (the entries right to the diagonal) and upgrade probabilities (the entries left to the diagonal). Therefore, as the cohort method ignores the intra-year transitions it may overestimate both upgrade and downgrade probabilities.

<table>
<thead>
<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
<th>A</th>
<th>BBB</th>
<th>BB</th>
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<th>CCC</th>
<th>CC</th>
<th>C</th>
<th>D</th>
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<td>99.671</td>
<td>0.301</td>
<td>0.007</td>
<td>0.013</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.000</td>
<td>0.008</td>
</tr>
<tr>
<td>AA</td>
<td>0.119</td>
<td>99.738</td>
<td>0.049</td>
<td>0.048</td>
<td>0.006</td>
<td>0.006</td>
<td>0.025</td>
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<td>0.000</td>
<td>0.010</td>
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<tr>
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<td>99.134</td>
<td>0.147</td>
<td>0.107</td>
<td>0.034</td>
<td>0.053</td>
<td>0.000</td>
<td>0.000</td>
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<td>BBB</td>
<td>0.061</td>
<td>0.087</td>
<td>0.139</td>
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<td>BB</td>
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<td>0.401</td>
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<td>B</td>
<td>0.276</td>
<td>0.029</td>
<td>0.459</td>
<td>0.184</td>
<td>0.248</td>
<td>97.268</td>
<td>0.726</td>
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<td>32.305</td>
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<td>0.000</td>
<td>0.000</td>
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</tr>
<tr>
<td>D</td>
<td>0.000</td>
<td>0.000</td>
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<td>0.000</td>
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Table 2: Duration method: Simple Average 1-year (NR adjusted) transition matrix (1991-2007)
4.1.2 Simple Average (1991-2007) one-year transition matrices: NR-included

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<tr>
<th></th>
<th>AAA</th>
<th>AA</th>
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<th>B</th>
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<th>CC</th>
<th>C</th>
<th>D</th>
<th>NR</th>
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<td>AAA</td>
<td>86.764</td>
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<td>0.012</td>
<td>0.011</td>
<td>0.000</td>
<td>0.000</td>
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<td>0.000</td>
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<tr>
<td>AA</td>
<td>5.955</td>
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<td>0.000</td>
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<td>0.205</td>
<td>0.114</td>
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<td>8.305</td>
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Table 3: Cohort method: Simple Average 1-year (NR included) transition matrix (1991-2007)

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Table 4: Duration method: Simple Average 1-year (NR included) transition matrix (1991-2007)

We now include the NR category in the cohort and duration methods. Table 3 and Table 4 report the results when the transitions from and to NR are observed. We find that including the NR transition has little influence on the results of the duration method. For the cohort case, the default probabilities and the probabilities of no migration become smaller. Our explanation is that the cohort method ignores the amount of time that securities spend in each rating state before they
Results and Discussion

migrate to NR. Hence, the less efficient cohort method underestimates the default probabilities and probabilities of no migration.

4.2 Weighted Average Calculation

In previous results, we equally averaged the probability matrices. However, Table 5 shows that the number of data is unevenly distributed as most securities receive their ratings after the year 2000. Thus, we count the numbers of active securities in each year and calculate their weights by

\[ w_k = \frac{N_k}{\sum_{i=1}^{k} N_i} \]

The weights of active securities are shown in the Figure 1.

<table>
<thead>
<tr>
<th>Initial Rating Year</th>
<th>Numbers of securities</th>
<th>Weights in Total</th>
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<tbody>
<tr>
<td>1991</td>
<td>455</td>
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<tr>
<td>1992</td>
<td>619</td>
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<tr>
<td>1993</td>
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<tr>
<td>1994</td>
<td>502</td>
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<tr>
<td>1995</td>
<td>370</td>
<td>0.73%</td>
</tr>
<tr>
<td>1996</td>
<td>305</td>
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</tr>
<tr>
<td>1997</td>
<td>521</td>
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</tr>
<tr>
<td>1998</td>
<td>776</td>
<td>1.53%</td>
</tr>
<tr>
<td>1999</td>
<td>518</td>
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</tr>
<tr>
<td>2000</td>
<td>573</td>
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<tr>
<td>2001</td>
<td>1466</td>
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<tr>
<td>2002</td>
<td>3004</td>
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<td>2003</td>
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<td>2004</td>
<td>7364</td>
<td>14.48%</td>
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<td>2005</td>
<td>10464</td>
<td>20.58%</td>
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<td>11085</td>
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<td>7341</td>
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<tr>
<td>Total</td>
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</table>

*Table 5: US RMBS 1991-2007: Numbers of Securities Entering Rating*
Furthermore, the comparison of transition matrices for 2002 and 2006 indicates that transition probabilities are not constant over the rating histories (Appendix B). Moreover, the variation by calculating average singular value metrics confirms that transition probabilities fluctuate over our sample period (Appendix C).

Therefore, we average the probability matrices based on the weights of data over 17 years. In particular, we use the numbers of rated securities as the input data for calculating the weights. The average transition probabilities matrix is modified as follows

\[ \tilde{q}_{i,j} = \sum_{k=1}^{l} q_{i,j,k} \cdot w_k \]
### Results and Discussion

#### 4.2.1 Weighted Average (1991-2007) one-year transition matrices: NR-adjusted

<table>
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*Table 6: Cohort method: Weighted Average 1-year (NR adjusted) transition matrix (1991-2007)*

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*Table 7: Duration method: Weighted Average 1-year (NR adjusted) transition matrix (1991-2007)*
4.2.2 Weighted Average (1991-2007) one-year transition matrices: NR-included

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Table 8: Cohort method: Weighted Average 1-year (NR included) transition matrix (1991-2007)

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Table 9: Duration method: Weighted Average 1-year (NR included) transition matrix (1991-2007)
By calculating the transition matrices on a weighted average base, we find the default probabilities for investment grade ratings are slightly higher. Generally, the results of Table 5 to Table 8 show that the differences between the duration and cohort method on the weighted average base are similar to the ones on the simple average base.

According to the Central Limit Theorem in statistics, the larger the data observed, the more precise information will be extracted from the samples. Given that US RMBS securities are unevenly distributed, the securities before 2000 are over-weighted on the simple average base because of small sample size. Thus, we prefer to adopt weighted average calculation for more accuracy.
5. **Conclusion**

In this work we estimate rating transition matrices for US RMBS securities for the period of 1991 to 2007. Similar to the findings for corporate bonds, our results do not report significant change on the new data set and time period. Comparisons between the cohort method and the time-homogeneous duration method reveal that the latter yields greater default probabilities for high credit ratings and lower default probabilities for low credit ratings. In addition, we find that the duration method generates smaller downgrade and upgrade probabilities.

Furthermore, we conduct an alternative approach for calculating the average transition matrices. We suggest that, in the case of unevenly distributed data set, the weighted average calculation approach is more efficient and accurate.
Appendices

Appendix A: Derivation of the Estimator of Homogeneous Transition Intensities using Duration Method

Lando and Skodeberg (2000) propose the direct maximum-likelihood estimator for the generator matrices $\hat{\lambda}$ in time-homogeneous case. This estimator process requires the precise information in time at which the ratings transitions take place.

We use the following notations:

$m_{ij}$ = number of transitions to rating $j$ from rating $i$

$t_{ij}$ = time of the security transits from rating $i$ to rating $j$

$t'_{ij}$ = time of the security entered rating $i$ and finally transits to rating $j$

$m_{ii}$ = number of the securities stay in the same ratings

$t'_i$ = time of security entered rating $i$ but not transitioning

$m$ = total number of the securities entered rating $i$

$\lambda_{ij}$ = transition intensity from rating $i$ to rating $j$

For time homogeneous case, the $\hat{\lambda}_{ij}$ is a fixed intensity estimator. The probability density of the transition time from rating $i$ to rating $j$ for each rating state:

$$f(t_{ij}) = \lambda_{ij} e^{-\lambda_{ij}(t_{ij})} \sum_{k=i}^j \lambda_{ik}$$
For the time in $[t_{ij}', t]$, otherwise 0.

The likelihood function for a portfolio of securities occupying rating i at some time during $[0, t]$ is the product off the above density functions:

$$L = \left( \prod_{i=1}^l \prod_{j=1}^{m_{ij}} \lambda_{ij} e^{-(t_{ij} - t_{ij}') \sum_{k=1}^l \lambda_{ik}} \right) \prod_{i=1}^l e^{-(t_i - t_i') \sum_{k=1}^l \lambda_{ik}}$$

Taking the log of L

$$\ln L = \sum_{i=1}^l \sum_{j=1}^{m_{ij}} \left( \ln \lambda_{ij} - (t_{ij} - t_{ij}') \sum_{k=1}^l \lambda_{ik} \right) - \sum_{i=1}^l \left( (t_i - t_i') \sum_{k=1}^l \lambda_{ik} \right)$$

Maximizing with respect to a particular $\lambda_{ij}$ and take the first order conditions then set it to zero

$$\frac{\partial \ln L}{\partial \lambda_{ij}} = \frac{m_{ij}}{\lambda_{ij}} \sum_{i=1}^l \sum_{j=1}^{m_{ij}} (t_{ij} - t_{ij}') - \sum_{i=1}^l (t_i - t_i') = 0$$

Therefore, by solving the above partial differentiation equations, we can get the estimator for at maximum likely:

$$\hat{\lambda}_{ij} = \frac{m_{ij}}{T}$$

$$T = \sum_{i=1}^l \sum_{j=1}^{m_{ij}} (t_{ij} - t_{ij}') - \sum_{i=1}^l (t_i - t_i')$$
Appendix B: Comparison of Transition Matrices

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<th>AA</th>
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<th>BBB</th>
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Table 10: Cohort method (NR adjusted) transition matrix (2006)

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Table 11: Cohort method (NR adjusted) transition matrix (2002)

The comparison indicates that probabilities of no migration and default probabilities are not constant for different years.
Appendix C: US RMBS 2000-2007: $M_{SVD}$

We compute the average of the singular values of the mobility matrix $M_{SVD}$ for each year by following Jafry and Schuermann (2004).

$$M = Q - I$$

$$s(M) = \sqrt{\text{eig}(M'M)}$$

$$M_{SVD} = \frac{\sum_{j=1}^{N} (\sqrt{e_j(M'M)})}{N}$$

where $M$ and $Q$ represent the mobility matrix and transition matrix, respectively.

Figure 2: US RMBS 2000-2007 $M_{SVD}$

Figure 2 plots the singular value metric for the 8 annual migration matrices (2000-2007). The horizontal line represents the average of the $8 M_{SVD}$ values. This figure shows significant amount of variations over time.
Appendix D: Matlab Code - Cohort Method

% Cohort Method RMBS

% This code is designed to estimate the transition matrix of the credit ratings

clear all

close all

clec

load RMBS_Raw.mat

% preset an 3-D transition matrix by zeros formatting: (Rating@t-1,Rating@t,28-years)

Q=nan(23,23,18);

Q(:,;)=0; % set all cells to zero

% preset a annual rating matrix from 01-Jan-1991 to 01-Jan-2008

RMBS_Raw(:,18:19)=nan;

Ratings=nan(length(RMBS_Raw),19);

Ratings(:,1)=RMBS_Raw(:,1);

% preset a date table

NewYearDays=[727199,727564,727930,728295,728660,729025,729391,729756,730121,730486,730852,731217,731582,731947,732313,732678,733043,733408,733774];

Observations_initial=[401,455,567,502,370,521,776,518,578,1466,3004,4927,7364,10464,11085,7341];

%datedisp(NewYearDays)
% Set Parameters

% "No Ratings" NR=1 (NR adjusted) NR=0 (NR included)
NR=1;

% "Rating Modifiers" RM=1 (without RM) RM=0 (with RM)
RM=1;

% "Weighted Matrix" Weighted=1 (Weighted Average) Weighted=0 (Equal Average)
Weighted=0;

tic
for idx=1:length(RMBS_Raw(:,1))
    jdx=1;
    p=RMBS_Raw(idx,jdx*2+1);
    Position_p=find(p<NewYearDays);
    q=RMBS_Raw(idx,jdx*2+3);
    if isnan(q)==1 & isnan(p)~=1
        Ratings(idx,Position_p+1:end)=RMBS_Raw(idx,jdx*2);
    end
    while isnan(p)~=1 & isnan(q)~=1
        Position_p=find(p<=NewYearDays);
        Position_q=find(q<=NewYearDays);
    end
end
Ratings(idx,Position_p(1)+1:Position_q(1)+1:end)=RMBS_Raw(idx,jdx*2);

jdx=jdx+1;

p=RMBS_Raw(idx,jdx*2+1);

q=RMBS_Raw(idx,jdx*2+3);

% Recheck with the "tail"

if isnan(q)==1

    switch RMBS_Raw(idx,jdx*2)
    case 22 % Default as the last state
        Ratings(idx,Position_q(1)+1:end)=22; %??? Default forever?
    case 23 % No Ratings as the last state
        Ratings(idx,Position_q(1)+1)=23;
    otherwise % Lasting same ratings until present
        Ratings(idx,Position_q(1)+1:end)=RMBS_Raw(idx,jdx*2+2);
    end

end

end

end

end

toc

% Adjusting the Rating Modifiers
if RM

    for idx=3:3:18

        Ratings(Ratings==idx-1)=idx;

        Ratings(Ratings==idx+1)=idx;

    end

end

% Calibrate the Migration Matrices

tic

Ratings_form(:,1)=1:23;

Ratings_2years=nan(length(Ratings),2);

for kdx=1:length(NewYearDays)-1

    % Import Rating data as two years rolling window

    Ratings_2years= Ratings(:,kdx+1:kdx+2);

    % Exclude the NaN points in the matrix

    Ratings_2years=Ratings_2years(~any(isnan(Ratings_2years),2),:);

    % Exclude NR ratings

    if NR

        Ratings_2years=Ratings_2years(any((Ratings_2years(:,1)~=23 & Ratings_2years(:,2)~=23),2),:);

    end
% Summary the moving year transition by filter stays and trans

Ratings_2years_stays=Ratings_2years(any(Ratings_2years(:,1)==Ratings_2years(:,2),2),:);

Ratings_2years_trans=Ratings_2years(any(Ratings_2years(:,1)~=Ratings_2years(:,2),2),:);

% Count all data stays in the same ratings during the moving window

Count_stays=histc(Ratings_2years_stays(:,1),1:23);

% Store the number of Active Rating Securities

Observations_Act_Raw=Ratings_2years(:,1);

Observations_Act(kdx)=length(Observations_Act_Raw(Observations_Act_Raw~=22 & Observations_Act_Raw~=23));

% Fill the Q matrices

for idx=1:23

    for jdx=1:23

        % Fill Q with the observations of changed ratings

        Count_trans=length(Ratings_2years_trans(any((Ratings_2years_trans(:,1)==idx & Ratings_2years_trans(:,2)==jdx),2),:));

        Q(idx,jdx,kdx)=Count_trans;

        % Fill Q with the observations of unchanged ratings

        if idx==jdx

            Q(idx,jdx,kdx)=Count_stays(idx);

        end

    end

end
if sum(Q(idx,:,kdx))~=0

    % Convert Q into probabilities
    Q(idx,:,kdx)=Q(idx,:,kdx)/sum(Q(idx,:,kdx));

    % Check the Q matrix stability
    check_SMO(idx,kdx)=sum(Q(idx,:,kdx));

    end

end

Default(:,kdx)=Q(:,end-1,kdx);
end
toc

% Calculate the weights for each year

% Yearly_Weights=Observations_initial(2:end)/sum(Observations_initial(2:end));

Yearly_Weights=Observations_Act(2:end)/sum(Observations_Act(2:end));

% Average the Q matrix by annulizing migration matrix

tic

for idx=1:23

    for jdx=1:23

        if Weighted

            % Continue with the rest of the code

        end

    end

end
Q_weighted(1:17)=Q(idx,jdx,1:17);

Q_weighted=Yearly_Weights.*Q_weighted;

Q_hat(idx,jdx)=mean(Q_weighted);

else

Q_hat(idx,jdx)=mean(Q(idx,jdx,1:17));

end

end

end

if sum(Q_hat(idx,:))~=0

Q_hat(idx,:)=Q_hat(idx,:)/sum(Q_hat(idx,:));

end

end

toc
Appendix E: Matlab Code - Duration Method

% Duration Method RMBS

% This code is designed to estimate the transition matrix of the credit ratings

clear all

close all

clc

load RMBS_Raw.mat

% preset an 3-D transition matrix by zeros formatting: (Rating@t-1,Rating@t,28-years)

L=nan(23,23,18);

L(:,:,)=0; % set all cells to zero

% preset a annual rating matrix from 01-Jan-1991 to 01-Jan-2008

RMBS_Raw(:,18:19)=nan;

Ratings=nan(length(RMBS_Raw),29);

Ratings_2years=nan(length(RMBS_Raw),4);

Ratings(:,1)=RMBS_Raw(:,1);

% preset a date table

NewYearDays=[727199,727564,727930,728295,728660,729025,729391,729756,730121,730486,730852,731217,731582,731947,732313,732678,733043,733408,733774];

Observations=[401,455,619,567,502,370,305,521,776,518,578,1466,3004,4927,7364,10464,11085,7341];
tic
for idx=1:length(RMBS_Raw(:,1))
    jdx=1;
    p=RMBS_Raw(idx,jdx*2+1);
    Position_p=find(p<NewYearDays);
    q=RMBS_Raw(idx,jdx*2+3);
    if isnan(q)==1 & isnan(p)~=1
        Ratings(idx,Position_p+1:end)=RMBS_Raw(idx,jdx*2);
    end
    while isnan(p)~=1 & isnan(q)~=1
        Position_p=find(p<=NewYearDays);
end
Position_q = find(q <= NewYearDays);

Ratings(idx,Position_p(1)+1:Position_q(1)) = RMBS_Raw(idx,jdx*2);

jdx = jdx + 1;

p = RMBS_Raw(idx,jdx*2+1);

q = RMBS_Raw(idx,jdx*2+3);

% Recheck with the "tail"

if isnan(q) == 1
    switch RMBS_Raw(idx,jdx*2)
    case 22 % Default as the last state
        Ratings(idx,Position_q(1)+1:end) = 22; % ??? Default forever?
    case 23 % No Ratings as the last state
        Ratings(idx,Position_q(1)+1) = 23;
    otherwise % Lasting same ratings until present
        Ratings(idx,Position_q(1)+1:end) = RMBS_Raw(idx,jdx*2+2);
    end
    end
end
end
toc
% Adjusting the Rating Modifiers

if RM

    for idx=3:3:18

        Ratings(Ratings==idx-1)=idx;

        Ratings(Ratings==idx+1)=idx;

    end

end

% Loop the Duration Method yearly transition matrix

tic

for kdx=1:length(NewYearDays)-1

    % Filter the data to rolling two years with detail of transition records

        Ratings_2years= Ratings(:,kdx+1:kdx+2);

    % Transited records

        Position_Trans=find(isnan(Ratings_2years(:,1))==0 & isnan(Ratings_2years(:,2))==0 &
                            Ratings_2years(:,1)~=Ratings_2years(:,2));

    for idx=1:length(Position_Trans)

        o=find(RMBS_Raw(Position_Trans(idx),:)<=NewYearDays(kdx+1) &
               RMBS_Raw(Position_Trans(idx),:)>NewYearDays(kdx));

        Ratings_2years(Position_Trans(idx),3)=RMBS_Raw(Position_Trans(idx),o(1))-NewYearDays(kdx);

    end

end
% Exclude the NaN points in the matrix %Ext.: Exclude NR ratings

Ratings_2years=Ratings_2years(~any(isnan(Ratings_2years),2),:);

% Exclude NR ratings

if NR

    Ratings_2years=Ratings_2years((Ratings_2years(:,1)~=23 & Ratings_2years(:,2)~=23),:);

end

Ratings_2years_stays=Ratings_2years((Ratings_2years(:,1)==Ratings_2years(:,2)),:);

Ratings_2years_trans=Ratings_2years((Ratings_2years(:,1)~=Ratings_2years(:,2)),:);

Count_stays=histc(Ratings_2years_stays(:,1),1:23);

Ratings_2years(Ratings_2years==0)=nan;

% Store the number of Active Rating Securities

Observations_Act_Raw=Ratings_2years(:,1);

Observations_Act(kdx)=length(Observations_Act_Raw(Observations_Act_Raw~=22 & Observations_Act_Raw~=23));

% Fill the L matrices

for idx=1:23

    for jdx=1:23

        Ratings_2years_trans_ij=Ratings_2years_trans((Ratings_2years_trans(:,1)==idx & Ratings_2years_trans(:,2)==jdx),:);

        Count_trans=size(Ratings_2years_trans_ij);

        if idx~=jdx & sum(Ratings_2years_trans_ij(:,3))~=0...
\begin{verbatim}
\texttt{\%t=\text{sum}(Ratings\_2\text{years}\_\text{trans}\_ij(:,3)/(\text{NewYearDays}(kdx+1)\text{-NewYearDays}(kdx)));

L(idx,jdx,kdx)=\text{Count\_trans}(1)/\text{sum}(Ratings\_2\text{years}\_\text{trans}\_ij(:,3)/(\text{NewYearDays}(kdx+1)\text{-NewYearDays}(kdx)));

\text{elseif idx==jdx}
\text{\%L(idx,jdx,kdx)=0;}

L(idx,jdx,kdx)=\text{Count\_stays}(idx);

end}

end}

\text{\%if sum(L(idx,:,kdx))\neq0}
\text{\% L(idx,:,kdx)=L(idx,:,kdx)/sum(L(idx,:,kdx));}

\text{\%end}

end}

\text{Default(:,kdx)=L(:,end-1,kdx);}

end}

toc

\text{\% Calculate the weights for each year}

\text{\%\%Yearly\_Weights=Observations\_initial(2:end)/sum(Observations\_initial(2:end));}

\text{Yearly\_Weights=Observations\_Act(2:end)/sum(Observations\_Act(2:end));}
\end{verbatim}
tic

for idx=1:23
    for jdx=1:23
        if Weighted
            L_weighted(1:17)=L(idx,jdx,1:17);
            L_weighted=Yearly_Weights.*L_weighted;
            L_hat(idx,jdx)=mean(L_weighted);
        else
            L_hat(idx,jdx)=mean(L(idx,jdx,1:17));
        end
    end
    if sum(L_hat(idx,:))~=0
        L_hat(idx,:)=L_hat(idx,:)/sum(L_hat(idx,:));
    end
end

toc
References


