CENSORED LATENT EFFECT AUTOREGRESSION MODEL ON CANADIAN UNEMPLOYMENT RATE: A NON-LINEAR APPROACH

by

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Abstract

Non-linear economic methods are often employed to describe the asymmetry of unemployment rates in the industrialized countries during the postwar period. Perhaps most common are Markov switching models. However, this project employs a different approach, a so called “censored latent effects autoregression” (CLEAR) model to describe the data. After specification tests, a parsimonious model is suggested to produce a good fit of the data. Some applications are discussed and proposed. In the end, the forecast comparisons with a linear competitor AR model and non-linear competitor Markov-switching model are conducted, where CLEAR model outperforms the AR model, but does not show advantages over Markov-switching model.
Acknowledgements

First of all, I want to thank my examining committee: Prof. Ken Kasa, Prof. David Andolfatto, for the tremendous support and valuable advice they have given me in this paper. They always provided guidance and help to me which carry me through all the difficulties. I admire them both immensely. I also want to thank Prof. Simon Woodcock for his patience in answering my questions no matter how busy he is and great help in programming in Matlab.

Secondly, I want to thanks all my friends, who support me always. Many thanks to Frank, Sam, Quinton, Nemo, Yilin. I love you all.
Dedication

To my parents, for your love

To lulu, for my love
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1. Introduction

The post-war unemployment rate in many industrialized countries often display two features: during the expansionary period, the unemployment decreases slowly, while during the recessionary period, the unemployment increases sharply. This noticeable asymmetry of unemployment motives many recent studies both theoretically and empirically. David (1997) provides some economic intuition behind the asymmetry of unemployment. He shows the asymmetry can be derived from the labor-matching model through the individual actions naturally.¹

And many other studies proposed modifications to linear models to capture the asymmetry. Two classes of nonlinear models are often employed in an effort to describe the data adequately. One class of these models is the (smooth) switching regime time series model; see Terasvirta (1994), Granger and Terasvirta (1993) and Tong (1990). This model allows for different dynamic structures across regimes. The regimes can be defined by some exogenous variables or some time series of interest.

An alternative model is the Markov switching model, introduced by Goldfeld and Quandt (1973) and Lindgren (1978), and popularized by Hamilton (1989). The Markov switching

¹ Suppose there are a group of workers, who is doing a kind of “fringe work”, which means only worth of doing during the economic upturn, but will be destroyed during economic downturn. When the works are destroyed, they could not be brought back immediately. So the workers need to invest extra time in searching for new jobs.
model assumes transitions between regimes follow an unobserved Markov process. The transition probabilities are usually assumed to be constant over time.

This project follows another approach, called “Censored Latent Effects Autoregression” (CLEAR model) suggested by Franses and Papp (2002). In their paper, the CLEAR model shows many advantages in forecasts relative to a variety of linear and nonlinear competitor models. The main idea of CLEAR model is that recessionary periods of unemployment are determined by a large and positive exogenous shock to the series, i.e., a few large positive innovations. The CLEAR model is distinguished from Balke and Fomby (1994) in that it does not use dummy variables to account for these shocks, but instead assumes the innovations can be predicted by a leading indicator variable.

The CLEAR model distinguishes itself from other nonlinear model in several other respects: first, it uses a latent variable to capture the asymmetry of the unemployment rate, not different specifications across regimes as in regime-switching models. This enables the CLEAR model to produce good fits with only a small set of parameters. Second, it gives an explicit description of dynamics of the unemployment rate (AR process), not the generalized impulse response function, which makes the computation easier.
The project is organized as follows: Section 2 introduces the CLEAR model and specification. Section 3 discusses the estimation procedure and main results. Section 4 gives some applications of the CLEAR model, and Section 5, concludes.
2. The CLEAR model

2.1 Model specification

Following Franses and Papp (2002), I construct a CLEAR model of order \( p \) for the Canadian unemployment rate time series \( \{ y_t \}_{t=1}^{T} \), where \( y_t \) is the log of 100 times the unemployment rate.

\[
y_t = \sum_{i=1}^{p} \alpha_i y_{t-i} + x_t' \gamma + v_t + \varepsilon_t
\]

(1)

and where \( \varepsilon_t \sim \text{NID} (0, \sigma^2_\varepsilon) \), \( y_{p-1}, \ldots, y_1 \) are fixed, and \( v_t \) is a censored latent variable used to capture the large positive innovation during recessions. Define \( v_t \) as:

\[
v_t = \begin{cases} x_t' \beta + u_t & \text{if } x_t' \beta + u_t \geq 0 \\ 0 & \text{if } x_t' \beta + u_t < 0 \end{cases}
\]

(2)

With \( x_t \) a (2x1) vector, is the leading indicator\(^2\) of Canadian economy, including an intercept, and \( \beta \) and \( \gamma \) are the parameter vectors. \( u_t \sim \text{NID} (0, \sigma^2_u) \); and we assume \( \varepsilon_t \) and \( u_t \) are uncorrelated.

---

\(^2\) Leading indicator series is not necessarily a formal one. It can be constructed arbitrarily. Generally, as long as it can indicate the business cycle of Canadian economy, it makes sense. A more relevant series to unemployment rate would be useful, but not necessary.
2.2 Estimation

Estimation of the parameter $\theta = (\alpha, \gamma, \beta, \sigma_x, \sigma_y)$ can be done by maximum likelihood.

To derive the likelihood function, I need to find the density function of $y_t$, given its past.

We know, if given the past and $v_t$, the density function is:

$$f(y_t | Y_{t-1}, x_t, v_t; \theta) = \frac{1}{\sigma_x \sqrt{2\pi}} \exp\left(-\frac{1}{\sigma_x^2} (y_t - \sum_{i=1}^p y_{t-i} - x_t \gamma - v_t)^2\right)$$

So given the past only, the density function is:

$$f(y_t | Y_{t-1}, x_t; \theta) = \text{Pr}[v_t = 0 | x_t; \theta] f(y_t | Y_{t-1}, x_t, v_t, \theta) |_{v_t = 0}$$

$$+ \int_{-\infty}^{\infty} \frac{1}{\sigma_u \phi(u_t)} f(y_t | Y_{t-1}, x_t, v_t, \theta) |_{v_t = 0} \Phi(u_t) du_t$$

(3)

Where $\text{Pr}(v_t = 0 | x_t; \theta) = \int_{-\infty}^{\infty} \frac{1}{\sigma_u} \phi(u_t / \sigma_u) du_t = \Phi(-\frac{x_t \beta}{\sigma_u}) = \Phi_t$  (4)

And $Y_{t-1} = (y_{t-1}, \ldots, y_t)$;

Log likelihood function can be derived as the sum of the logarithms of the density functions of $y_t$ given in (3). In other words,

$$L(Y_T | X_T; \theta) = \sum_{t=1}^{T} \ln(f(y_t | Y_{t-1}, x_t; \theta))$$

(5)

Where $Y_T = (y_T, \ldots, y_t)$ and $X_T = (x_T, \ldots, x_t)$. The maximum likelihood estimates for $\theta$ can be obtained by the standard optimization algorithms. BHHH by Berndt et al. (1974) is used in this project. The average outer product of the scores is used to construct the standard errors of the parameter estimates. In addition, the H/SC

---

3 It is argued that employing BFGS algorithm can achieve better result.
standard errors for estimated parameters are also reported.\textsuperscript{4} The gradient vectors for the estimation and some useful results to simplify the numerical calculations are included in the appendix.

\textbf{2.3 Inference on the unobserved } $v_t$

In this part, I provide two results for latent variable $v_t$. Intuitively, if we know the values of $Y_t$, we can do inference on $v_t$ conditional on $Y_t$. In fact, the probability that $v_t = 0$ given $Y_t$ equals

$$
\Pr(v_t = 0 \mid Y_t, x_t; \theta) = \frac{\Pr(v_t = 0 \mid x_t; \theta)f(y_t \mid Y_{t-1}, x_t, v_t; \theta) \mid _{v_t=0}}{f(y_t \mid Y_{t-1}, x_t; \theta)} \tag{6}
$$

And the expected value of the shock $v_t$, given the values of $Y_t$, equals

$$
E[v_t \mid Y_t, x_t; \theta] = \frac{\int \int_{x_t, \beta} (x_t; \beta + u_t) \frac{1}{\sigma_u} \phi\left(\frac{u_t}{\sigma_u}\right)f(y_t \mid Y_{t-1}, x_t, v_t; \theta) \mid _{v_t=0, \beta+u_t} du_t}{f(y_t \mid Y_{t-1}, x_t; \theta)} \tag{7}
$$

\textbf{2.4 Diagnostic tests}

Specification tests are conducted following Wooldridge (1991). Let the difference between $y_t$ and the conditional mean of $y_t$ be

$$
e_t = y_t - E[y_t \mid Y_{t-1}, x_t; \hat{\theta}]
$$

$$
= y_t - \sum_{i=1}^{p} \hat{\beta}_i y_{t-i} - x_t \hat{\gamma} - E[v_t \mid x_t; \hat{\theta}] \tag{8}
$$

Where

$$
E[v_t \mid x_t; \hat{\theta}] = E[v_t \mid v_t = 0, x_t; \hat{\theta}] \Pr[v_t = 0 \mid x_t; \hat{\theta}] + E[v_t \mid v_t > 0, x_t; \hat{\theta}] \Pr[v_t > 0 \mid x_t; \hat{\theta}]
$$

\textsuperscript{4} H/SC SE is constructed following Wooldridge J (1991).
So the corresponding fit of the model is

\[ 0 + x_i \hat{\theta} (1 - \Phi_i) + \sigma_u \phi_i, \]

(9)

Where \( \phi_i = \phi(-x_i \beta / \sigma_u); \)

So the corresponding fit of the model is

\[
E[y_t \mid Y_{t-1}, x_t; \hat{\theta}] = \sum_{i=1}^{p} \tilde{\alpha}_i y_{t-i} + x_i \tilde{\gamma} + E[v_t \mid x_t; \hat{\theta}; \]

(10)

Note, the residuals \( e_t \) are heteroscedastic as the conditional variance of \( y_t \) is

\[
V[y_t \mid Y_{t-1}, x_t; \theta] = \sigma^2_e + V[v_t \mid x_t; \theta]
\]

(11)

Where \( V[v_t \mid x_t; \theta] = \sigma^2 (1 - \Phi_i) + x_i \beta E[v_t \mid x_t; \theta] - E[v_t \mid x_t; \theta]^2 \)

(12)

**LM test for AR(p) serial correlation in the residuals:**

Following Wooldridge (1991), the robust LM test for AR(p) serial correlation in the residuals can be done by the following method:

Test the hypothesis:

\( H_0 = \) no serial correlation;

Against 2 alternative hypotheses:

\( H_1 = 1 \text{st order of serial correlation}; \)

\( H_i = 1 \text{st to } 5 \text{th order of serial correlation}; \)

Step 1: regress \((e_{t-1}, \ldots, e_{t-p})\) on the gradient of the \( E[y_t \mid Y_{t-1}, x_t; \hat{\theta}] \) with respect to \(( \alpha, \gamma, \beta, \sigma_u )\) evaluated at the MLE estimated values. Save the residuals \( w_i = (w_{i,1}, \ldots, w_{i,p}); \)

---

5 Following the result of the expectation of truncated normal. Please refer to Greene. (2001).
Step 2: regress 1 on \(w, e_i\); save the SSR (the sum of squared residuals) from this regression;

The test statistic for AR(p) serial correlation then equals (T-P)-SSR. It is asymptotically \(\chi^2\) (p) distribution.

The gradient vector is:

\[
\frac{\partial E[y_t | x_t; \theta]}{\partial \beta} = (1 - \Phi_t) x_t,
\]

\[
\frac{\partial E[y_t | x_t; \theta]}{\partial \sigma_u} = \phi_t,
\]

\[
\frac{\partial E[y_t | x_t; \theta]}{\partial \gamma} = x_t,
\]

\[
\frac{\partial E[y_t | x_t; \theta]}{\partial \alpha_i} = y_{t-i}
\]

To increase the power of the LM test, we correct the heteroscedasticity, by using \(\sqrt{s_t}\) as the weight, where \(s_t = V[y_t | y_{t-1}, x_t, \hat{\theta}]\);

**LM test for ARCH (Q) effects in the residuals:**

Test the hypothesis:

- \(H_0\) : No ARCH (Q) effects

Against 2 alternative hypotheses;

- \(H_1 = 1\)st order of ARCH;

- \(H_1' = 1\)st to 5\(^{th}\) order of ARCH;
Step 1: regress \((e_{t-1}^2, \ldots, e_{t-Q}^2)\) on the gradient of \(V[y_t | Y_{t-1}, x_t; \theta]\) with respect to \((\sigma^2, \beta, \sigma_x)\) evaluated at MLE estimates. Save the residuals from this regression \(w_t = (w_{t,1}, \ldots, w_{t,Q})\).

Step 2: regress 1 on \(w_t(e_t^2 - 1)\); save the SSR;

The LM test for ARCH (Q) effect is \((T-Q)\text{-SSR}\), which is distributed asymptotically \(\chi^2(Q)^6\),

The gradient vector here is:

\[
\frac{\partial V[y_t | x_t; \theta]}{\partial \beta} = (\sigma_x \phi_t + E[v_t | x_t; \theta] + x_t \beta (1 - \Phi_x) - 2E[v_t | x_t; \theta](1 - \Phi_x))x_t
\]

\[
\frac{\partial V[y_t | x_t; \theta]}{\partial \beta} = 2\sigma_x (1 - \Phi_x) - 2E[v_t | x_t; \theta] \phi_t
\]

\[
\frac{\partial V[y_t | x_t; \theta]}{\partial \beta} = 2\sigma_x
\]

---

^6 Formally, I should also correct the heteroskedasticity to get a better performance of the test.
3. Estimation and Results

The unemployment rate used in this paper is from 1976.1 to 2004.8. Figure 1 shows the series of the natural log of 100 times the unemployment rate. Log form of unemployment rate is taken to reduce the variance of the time series. The OECD Composite Leading Indicator is used as the leading indicator. (http://www.oecd.org/). A detailed description of this leading indicator is available at the website. See also, a short description is in the appendix;

Figure 3.1 Unemployment Rate
3.1 **Estimation procedure**

The sample period is from 1976.1 to 2004.8. Earlier observations are used as starting values. $y_t$ is the log form of the unemployment rate. The leading indicator series is non-stationary and bi-annual differencing is used to get the stationary series, denoted as $x_t$ (including an intercept). Figure 3.2 shows the bi-annual difference of the leading indicator series.

**Figure 3.2  Bi-annual Difference Leading Indicator Series**

---

7 I also try monthly difference, annual difference. But the performance of bi-annual series is best.
To determine the order for the CLEAR model, the first step is to get the order for the AR part. This is done by minimizing the BIC of Schwarz (1972), allowing $p$ to vary from 1 to 12. The result is $p$ equals 4. The second step is to get the lag order for leading indicator. An ARX model (basically, it is a AR model with an exogenous variable, here, is $x_t$), is used, by varying lag order from 1 to 12, and using MLE to compare likelihood to pick up the largest one. The lag order turns out to be 1.

Next, we begin to estimate the model. It is hard for us to get the analytic result for the estimates for the parameters. Numerical method is employed to estimate the parameters. In this paper, I use the BHHH algorithm. A key advantage of the BHHH algorithm is that calculation of the Hessian is not required. Detailed discussion is in the appendix.

Potential misspecification is next examined. For the AR serial correlation in the residuals, LM test is performed for the first order and the first to the fifth order serial correlation. In addition, LM test is used again for the ARCH effect. The results are in table 3.1.

---

8 To calculate Hessian analytically is really a messy job in this project.
Table 3.1  The Test Report

<table>
<thead>
<tr>
<th>Test Type</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$LM_{ser} (1)^a$</td>
<td>1.8108(0.19)</td>
</tr>
<tr>
<td>$LM_{ser} (5)^b$</td>
<td>8.3731(0.13)</td>
</tr>
<tr>
<td>$LM_{ARCH} (1)^c$</td>
<td>0.14675 (0.70)</td>
</tr>
<tr>
<td>$LM_{ARCH} (5)^d$</td>
<td>0.2066(0.999)</td>
</tr>
<tr>
<td>$LR_{\gamma=0}$</td>
<td>30.054 (0.00)</td>
</tr>
<tr>
<td>$LR_{\sigma^2=0}$</td>
<td>0.3217(0.57)</td>
</tr>
<tr>
<td>$LR_{\tau_{i-1}=0}$</td>
<td>1.4506(0.23)</td>
</tr>
<tr>
<td>Log Likelihood</td>
<td>5234.22</td>
</tr>
</tbody>
</table>

*a* Robust LM test for 1st order serial correlation  
*b* Robust LM test for 1st to 5th order serial correlation  
*c* Robust LM test for 1st order ARCH effects  
*d* Robust LM test for 1st to 5th order ARCH effects  
*e* LR test for nonlinearity  
*f* LR test for neglected dynamics

The results in the table allow me to say safely that the model does not suggest misspecification in the conditional mean and variance.

So I continue to test for non-linearity. LR test is used to test $\beta = 0$ and $\sigma^2 = 0$.

Following the Wolak (1989), the test is asymptotically distributed as $\frac{1}{2} \chi^2(2) + \frac{1}{2} \chi^2(3)$. The 95 percentile is 7.80, and the test statistic is more than 30, which does not allow me to reject the non-linearity.
Next, LR test is employed to test the $\gamma = 0$. It turns out that I failed to reject the null hypothesis, which is consistent with intuition. The leading indicator itself does not enter the dynamics of the unemployment rate. It affects the unemployment rate only indirectly by affecting the probability of a “positive innovation” during the recession.

To do some analysis of neglected dynamics, I add next lag of leading indicator, LR test (setting the coefficient to 0) shows as in the table 3. I could not reject the null hypothesis. So, it is safe for me to assume there is no neglected dynamics of leading indicator variable.\(^9\)

Based on the results of these tests, I begin to do the final version of estimation:

Imposing $\gamma = 0$, and re-estimating the model, the estimates for the parameters are found as showed in table 3.2:

Table 3.2  Estimation Result

<table>
<thead>
<tr>
<th></th>
<th>Estimated value</th>
<th>Standard error</th>
<th>H/SC SE&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y_{t-1})</td>
<td>0.94998</td>
<td>0.056567</td>
<td>0.030296</td>
</tr>
<tr>
<td>(y_{t-2})</td>
<td>-0.080228</td>
<td>0.083927</td>
<td>0.038831</td>
</tr>
<tr>
<td>(y_{t-3})</td>
<td>0.20491</td>
<td>0.083134</td>
<td>0.038528</td>
</tr>
<tr>
<td>(y_{t-4})</td>
<td>-0.20999</td>
<td>0.055024</td>
<td>0.029574</td>
</tr>
<tr>
<td>(\beta)</td>
<td>-0.008</td>
<td>0.00031797</td>
<td>0.0010881</td>
</tr>
<tr>
<td>(\sigma_e)</td>
<td>0.04999</td>
<td>0.0019589</td>
<td>0.080085</td>
</tr>
<tr>
<td>(\sigma_u)</td>
<td>0.048198</td>
<td>0.12454</td>
<td>0.050041</td>
</tr>
<tr>
<td>Intercept</td>
<td>0.30001</td>
<td>0.030977</td>
<td>0.0172</td>
</tr>
</tbody>
</table>

<sup>a</sup> H/SC SE: heteroskedasticity/serial correlation robust standard error. Following Wooldridge (1991, p12) (a brief description of process to calculate H/SC SE is included in the appendix).

The summation of the coefficients in the AR part is 0.864672, which is not very close to 1. So Unit root test is not conducted.
4. Application and Interpretation

4.1 Fit of the model

The final version of the CLEAR model in this paper has 8 parameters. The sign of the leading indicator is negative, as expected. It means decreases in the leading indicator variable (normally positively correlated with GDP), positively affect the probability of a positive shock to the unemployment rate. Figure 4.1 shows the fitted value from the CLEAR model for the unemployment rate, I could say, it fit the data well.

Figure 4.1 Fitness of the Data
And for the residuals, Figure 4.2:

**Figure 4.2 Residuals Plot**

![Residuals Plot](image)

So, combined the two graphs above, I could safely say, my model give a good description of the data;

4.2 Business cycle analysis

Figure 4.3 is the Conditional expectation of the censored latent effect $E[v_i | Y_i, x_i; \hat{\theta}]$; where $E[v_i | Y_i, x_i; \hat{\theta}]$ is defined in (6);
Figure 4.3 Conditional Expectation $E[v_t \mid Y_t, x_t; \hat{\theta}]$;

Figure 4.4 shows the conditional probability that $v_t > 0$. Like a Markov switching model, the conditional probability can be interpreted as an indicator of the regimes of business cycle of the Canadian economy. If we define recessions by 6 consecutive months where $\Pr(v_t > 0 \mid Y_t, x_t; \theta) > 0.5$ (defined in equation (7)) A peak can be defined by the last observation before the recession. And a trough can be defined by the last observation of the recession.\(^{10}\) Table 4.1 shows the “peak” and “trough” from the CLEAR model:

\(^{10}\) Analytic result for conditional probability and expectation is given in appendix.
Note, it is a little different from the official report. CLEAR model does a good job in suggesting the recession of 81-82 and the recession of 90-92. But it does suggest another recession 94-95.

Unemployment itself is not a good indicator of Canadian economy. Unemployment rate for males over 20 years old is found more useful sometimes. Due to time constraints, I could not use this data set to redo the paper. Another suggestion is to add more exogenous variables. Following Hamilton (1993), Tatum (1988) and Mork (1989), oil prices may have explanatory power. By same reason, I could not do this.

Also, other leading indicators of the Canadian economy may have more power. The leading indicator series used in this paper is not for the quantitive analysis. I should find another leading indicator series which have a better explaining power. I tried the one constructed by CIBC, but, it did more poorly than this one.

4.3 NRU (Natural Rate of Unemployment) dynamics

If we believe the AR part of Clear model is stationary, the equilibrium unemployment rate, can serve as a dynamic NRU. Which is

\[ E[y_i | x_i; \theta] = (1 - \sum_{i=1}^{p} a_i)^{-1} * (x_i; \gamma + E[v_i | x_i; \theta]) \]

Figure 4.5 is to give a description of the series of this dynamic NRU.
NRU varies within the interval [5.12, 11.5]. The mean of NRU during the 80s is 7.7505; and during the 90's is 7.5894, which is consistent with what many economist believe.\(^\text{12}\)

### 4.4 Forecast

In this part, I will compare the performance in forecasting of the CLEAR model with the competitor models. I compare the result from CLEAR model and two competitors, a

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\(^{12}\) Andrew B. Abel, Ben S.Bernanke, Gregor W. Smith, Macroeconomics 2003, Addison Wesley, p92,'...Although there is no single official measure of the nature rate of unemployment, many economists believe that the natural rate of unemployment was....increased gradually to about 8% in the 1980s, then fell to 7% in the 1990s.
simple linear AR model and nonlinear simple Markov-switching model. The AR model is the AR part of the CLEAR model, with the order 4:

\[ y_t = \sum_{p=1}^{4} \alpha_p y_{t-p} + \varepsilon_t \]

And for Markov-switching model, I employ the one used in Hamilton (1989):

\[ y_t = \alpha_1 s_t + \alpha_0 + z_t \]

Where \( z_t \) follows ARMA(4,0) process.

60 observations are left out to do the forecast. And table 4.2 reports the root mean squared forecast error (RMSE), and the mean absolute percentage errors (MAPE).

<table>
<thead>
<tr>
<th>Model</th>
<th>RMSE</th>
<th>MAPE</th>
</tr>
</thead>
<tbody>
<tr>
<td>CLEAR Markov-switching model</td>
<td>0.20576</td>
<td>0.091699</td>
</tr>
<tr>
<td>AR</td>
<td>1.0129</td>
<td>0.46417</td>
</tr>
<tr>
<td>Markov-switching model</td>
<td>0.25254</td>
<td>0.089756</td>
</tr>
</tbody>
</table>

From the above report, the CLEAR model performs better than AR model, under RMSE and MAPE criteria.

CLEAR model outperforms Markov-switching model under criterion RMSE, but loses under MAPE.

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13 From economic intuition mentioned in last part, leading indicator does not affect the unemployment rate directly. So I do not employ ARX version model.
Finally, I do an encompassing test. Let $f_c$ be the forecast from CLEAR model, and $e_c$ to be the forecast error, and $f_{ar}$ be the forecast from AR model, $f_m$ to be the forecast from Markov-switching model. The forecasts from CLEAR model are said to encompass the forecasts from the AR (Markov-switching) model if the coefficient $\delta_0$ in the auxiliary regression model is $0^{14}$:

$$e_c = \delta_0 (f_{ar(m)} - f_c) + \delta_1 \frac{\partial e_c}{\partial \theta} |_{\theta=\hat{\theta}} + \eta_c$$

If $\delta_0$ is not 0, then the forecasting performance of the CLEAR model can be improved by adding some features of the AR (Markov-switching) model. To test for $\delta_0 = 0$, I use F-test. The result is

<table>
<thead>
<tr>
<th>Table 4.3</th>
<th>Encompassing Test Results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>F-statistic</td>
</tr>
<tr>
<td>AR</td>
<td>1.243337</td>
</tr>
<tr>
<td>Markov-switching</td>
<td>17.23505</td>
</tr>
</tbody>
</table>

So the forecasts from CLEAR model encompass the AR model, but does not encompass Markov-switching model.

So I continue to check whether Markov-switching model encompasses CLEAR or not.

But result turns to be:

| F-statistic   | 7467.366   | Probability | 0.000000 |

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$^{14}$ Please refer to Clements and Hendry (1993, p 634)
So the result about the encompassing test between CLEAR and Markov-switching model is ambiguous.
5. Conclusion

In this project, I employ a censored latent effects autoregression model to describe Canadian unemployment rate data. The result is satisfactory in terms of fitness. Specification tests do not suggest any misspecification problems. Based on this model, I conduct an analysis of the Canadian business cycle. A time path of the dynamics of the natural rate of unemployment is suggested. In the end, forecast comparison shows that CLEAR outperforms linear competitor, AR model. But comparing to Markov switching model, the result is ambiguous.
Appendix

Gradient vector for the estimation; results from this part follow Franses and Paap(2002).

\[
\frac{\partial \ln f_t}{\partial \alpha_i} = \frac{1}{f_t} \left[ \Phi_i \frac{e_t}{\sigma_u} f(y_t \mid Y_{t-1}, v_t; \theta) \right]_{y_t=0} \\
+ \int_{-\infty}^{\infty} \frac{1}{\sigma_u} \phi(u_t) \frac{e_t - x_t \beta - u_t}{\sigma_u^2} f(y_t \mid Y_{t-1}, v_t; \theta) \left|_{y_t=x_t \beta + u_t} \right. du_t \right] y_{t-1}
\]

\[
\frac{\partial \ln f_t}{\partial \gamma} = \frac{1}{f_t} \left[ \Phi_i \frac{e_t}{\sigma_u} f(y_t \mid Y_{t-1}, v_t; \theta) \right]_{y_t=0} \\
+ \int_{-\infty}^{\infty} \frac{1}{\sigma_u} \phi(u_t) (\frac{e_t - x_t \beta - u_t}{\sigma_u^2}) \left|_{y_t=x_t \beta + u_t} \right. du_t \right] x_i
\]

\[
\frac{\partial \ln f_t}{\partial \sigma_x} = \frac{1}{f_t} \left[ \Phi_i \left( \frac{e_t^2}{\sigma_u^2} - \frac{1}{\sigma_u^2} \right) f(y_t \mid Y_{t-1}, v_t; \theta) \right]_{y_t=0} \\
+ \int_{-\infty}^{\infty} \frac{1}{\sigma_u} \phi(u_t) \left( \frac{e_t - x_t \beta - u_t}{\sigma_u^2} \right)^2 \left|_{y_t=x_t \beta + u_t} \right. du_t \right] x_i
\]

\[
\frac{\partial \ln f_t}{\partial \beta} = \frac{1}{f_t} \left[ \int_{-\infty}^{\infty} \frac{1}{\sigma_u} \phi(u_t) \frac{e_t - x_t \beta - u_t}{\sigma_u^2} f(y_t \mid Y_{t-1}, v_t; \theta) \left|_{y_t=x_t \beta + u_t} \right. du_t \right] x_i
\]

\[
\frac{\partial \ln f_t}{\partial \gamma} = \frac{1}{f_t} \left[ \frac{x_t \beta}{\sigma_u^2} \frac{e_t}{\sigma_u^2} f(y_t \mid Y_{t-1}, v_t; \theta) \left|_{y_t=0} \right. \right] \\
\int_{-\infty}^{\infty} \frac{u_t^2}{\sigma_u^2} \phi(u_t) f(y_t \mid Y_{t-1}, v_t; \theta) \left|_{y_t=x_t \beta + u_t} \right. du_t \right]
\]

Where \( e_t = y_t - \sum_{i=1}^{g} \alpha_i y_{t-i} - x_t \gamma \)

So the gradient vector per observation is
So the gradient vector of the log likelihood function equals:

\[ g_i(\theta) = \left( \frac{\partial \ln f_i}{\partial \alpha_i}, \ldots, \frac{\partial \ln f_i}{\partial \alpha_p}, \frac{\partial \ln f_i}{\partial \gamma}, \frac{\partial \ln f_i}{\partial \sigma_e}, \frac{\partial \ln f_i}{\partial \beta}, \frac{\partial \ln f_i}{\partial \sigma_u} \right) \]

So the gradient vector of the log likelihood function equals:

\[ G(\theta) = \sum_{i=1}^{T} g_i(\theta) \]

The MLE estimation can be done using the BHHH algorithm.

**Useful results for numerical computation**

In this part, to simplify the numerical calculation, I present some results I found useful to construct the gradient vector:

If let \( a = y_i - \sum_{i=1}^{p} a_i y_{i-i} - x_i (\gamma + \beta) \); \( b = -x_i \beta \) and \( \sigma^2 = \frac{\sigma_u^2 \sigma_e^2}{\sigma_u^2 + \sigma_e^2} \)

Define \( A = \int \frac{1}{\sigma_u} \phi(\frac{u_i}{\sigma_u}) \frac{1}{\sigma_e} \phi(\frac{u_i - a}{\sigma_e}) du_i \)

\[ = \frac{\sigma}{\sigma_u \sigma_e \sqrt{2\pi}} \exp\left( \frac{a^2}{2\sigma_e^2} \right) \left( \int (\Phi(\frac{\sigma_e^2 a - b}{\sigma}) - 1) \right) \]

\[ B = \int \exp\left( -\frac{1}{2} \left( \frac{u_i}{\sigma_u} \right)^2 + \left( \frac{u_i - a}{\sigma_e} \right)^2 \right) du_i \]

\[ = A \sigma_u \sigma_e (2\pi) \]

\[ C = \int u_i \exp\left( -\frac{1}{2} \left( \frac{u_i}{\sigma_u} \right)^2 + \left( \frac{u_i - a}{\sigma_e} \right)^2 \right) du_i \]

\[ = E + B \frac{a}{\sigma_e} \]

\[ D = \int u_i^2 \exp\left( -\frac{1}{2} \left( \frac{u_i}{\sigma_u} \right)^2 + \left( \frac{u_i - a}{\sigma_e} \right)^2 \right) du_i \]
\[ E = \exp \left\{ -\frac{1}{2\sigma^2} \left( b^2 - \frac{2a\sigma^2}{\sigma_e^2} b + \frac{\sigma^2}{\sigma_e^2} a^2 \right) \right\} \]

So:

\[ f_i = \Phi_i \phi \left( \frac{e_i}{\sigma_e} \right) \frac{1}{\sigma_e} + A \]

\[ \frac{\partial \ln f_i}{\partial \alpha_i} = \frac{1}{f_i} \left[ \Phi_i \left( \frac{e_i}{\sigma_e} \right) \phi \left( \frac{e_i}{\sigma_e} \right) + \frac{e_i - b}{\sigma_e^2} A - \frac{1}{\sigma_e \sigma_u 2\pi} C \right] y_{i-1} \]

\[ \frac{\partial \ln f_i}{\partial \gamma} = \frac{1}{f_i} \left[ \Phi_i \left( \frac{e_i}{\sigma_e} \right) \phi \left( \frac{e_i}{\sigma_e} \right) + \frac{e_i - b}{\sigma_e^2} A - \frac{1}{\sigma_e \sigma_u 2\pi} C \right] x_i \]

\[ \frac{\partial \ln f_i}{\partial \sigma_e} = \frac{1}{f_i} \left[ \Phi_i \left( \frac{e_i^2}{\sigma_e^2} - \frac{1}{\sigma_e^2} \right) \phi \left( \frac{e_i}{\sigma_e} \right) + \left( \frac{1}{\sigma_u \sigma_e 2\pi} \right) \frac{(e_i - b)^2}{\sigma_e^3} + \frac{1}{\sigma_e} s_i \right] B \]

\[ - \left( \frac{1}{\sigma_u \sigma_e 2\pi} \right) \frac{2(e_i - b)}{\sigma_e^2} C + \left( \frac{1}{\sigma_u \sigma_e 2\pi} \right) D \]

\[ \frac{\partial \ln f_i}{\partial \beta} = \frac{1}{f_i} \left[ - \frac{1}{\sigma_u \sigma_e 2\pi} \left( \frac{e_i - b}{\sigma_e^2} \right) B - \frac{1}{\sigma_u \sigma_e 2\pi} C \right] x_i \]

\[ \frac{\partial \ln f_i}{\partial \sigma_u} = \frac{1}{f_i} \left[ - \frac{b}{\sigma_u^2 \sigma_e} \phi \left( \frac{e_i}{\sigma_e} \right) + \frac{1}{2\pi \sigma_e \sigma_u^4} D - \frac{1}{2\pi \sigma_e \sigma_u^2} B \right] \]

A detailed calculation is available on request.

Some other analytic results

Follows previous part, we can get analytic expression for conditional probability and conditional expectation for latent variable \( v_i \) given \( Y_i \), ie,
Brief introduction to H/SC SE:

In time series analysis, it is often useful to report H/SC standard error for the estimated parameters. In this project, I followed the method discussed in Wooldridge (1991) to construct the H/SC SE. For a detailed discussion, please refer to Wooldridge’s paper.

Step 1: let \( e_t = y_t - \text{onestepfit} \), to get the estimated \( \sigma^2; \) \((T^{-1} \sum_{i=1}^{T} e_i^2)\), and for the usual standard errors: \( H^{-1/2}\sigma \); here \( se(\theta_i) \)

Step 2: use OLS regression; define \( g' \), gradient vector of \( E[y_t \mid Y_{t-1}, x_t; \hat{\theta}] \);

Regress \( g'_i \) on \( g'_{-i} \); where \( g'_i \) is the ith element of the \( g' \); save the residuals \( r_n \); define \( \varepsilon_t = r_n e_t \);

Step 3: the H/SC SE for \( \theta_i = (se(\theta_i)/\sigma)^2 \sum_{i=1}^{T} \varepsilon_i^2 \);

Description of leading indicator of OECD:

Glossary:
The composite leading indicator of OECD (CLI): is an aggregate time series displaying a reasonably consistent leading relationship with the reference series for the macroeconomic cycle in a country. CLI is constructed by aggregating together component series selected according to the criteria specified in component series. However, it is important to emphasize that component series are not selected according to a strict quantitative criteria based on the cross-correlation with the reference series. Thus, CLI can be used to give an early indication of turning points in the reference series but not for quantitative forecasts.

COMPONENT SERIES: Component series are economic time series which exhibit leading relationship with a reference series at the turning points. Their seasonally adjusted or raw forms are combined into a CLI. The component series are selected from a wide range of economic sectors. The number of series used for the compilation of the OECD CLIs varies for each Member country, i.e. between five and eleven series. Selection of the appropriate series for each country is made according to the following criteria: Economic significance: there has to be an a priori economic reason for a leading relationship with the reference series. Cyclical behavior: cycles should lead those of the reference series, with no missing or extra cycles. At the same time, the lead at turning points should be homogeneous over the whole period; Data quality: statistical coverage of the series should be broad; series should be compiled on a monthly rather than on a
quarterly basis; series should be timely and easily available; there should be no break in time series; series should not be revised frequently.

(Description is available at the website: http://www.oecd.org/document/1/0,2340,en_2649_201185_32694145_1_1_1_1,00.html)
References


