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ABSTRACT

Experimental excitation functions have been measured by radiochemical means for a number of reactions proceeding through the compound nucleus Ge\textsuperscript{68}. The target-projectile pairs were: He\textsuperscript{4}+Zn\textsuperscript{64}, C\textsuperscript{12}+Fe\textsuperscript{56}, and O\textsuperscript{16}+Cr\textsuperscript{52} leading to the following reactions: \((\alpha,\gamma)\), \((x,p)\), \((x,n)\), \((x,pn)\), and \((x,2n)\). Recoil ranges were measured for products of the alpha-induced reactions in order to determine those reactions which proceeded by compound nucleus formation and decay. It has been found that the "independence hypothesis" is verified for reactions induced by different target-projectile pairs but proceeding through compound nuclei of nearly equal angular momentum.

Excitation functions have been calculated with the SFU IBM System 360/40 computer via the statistical theory of nuclear reactions according to a formalism containing the explicit dependence of nuclear emission probabilities on angular momentum. Probabilities for \(\gamma\)-ray emission were calculated according to the single-particle model coupled with enhancement functions chosen to reflect experimentally observed collective effects.

Agreement between experiment and calculations was found to be good for the Zn\textsuperscript{64}+He\textsuperscript{4} excitation functions. The "high energy tails" of the excitation functions were found to be accounted for by effects of \(\gamma\)-ray competition with particle emission when the latter was suppressed by angular
momentum effects. Agreement in the Fe$^{56}$ + C$^{12}$ case was found to be very encouraging, considering the complexity of the target-projectile system and attendant theoretical difficulties.
RESUME

Les fonctions d'excitation expérimentales ont été mesurées par les moyens radiochimiques pour plusieurs réactions qui ont lieu comme résultat de la formation du noyau composé Ge\(^{68}\). Les paires de projectile-cibles étaient: He\(^{4}\)+Zn\(^{64}\), C\(^{12}\)+Fe\(^{56}\), et O\(^{16}\)+Cr\(^{52}\) qui produisent les réactions suivantes: (\(\alpha,\gamma\)), (\(x,p\)), (\(x,n\)), (\(x,pn\)), et (\(x,2n\)). On a mesuré les distances d'atomes reculant pour les produits des réactions alpha-induites pour déterminer ces réactions qui ont résulté de la formation et de la décomposition de noyaux composés. On a vérifié "l'hypothèse d'indépendance" pour les réactions, induites par de différentes paires de projectile-cibles, mais qui résultent de noyaux composés d'à peu près la même quantité de mouvement angulaire.

On s'est servi de l'ordinateur SFU IBM Système 360/40 pour calculer les fonctions d'excitation par moyen de la théorie statistique de réactions nucléaires et selon le formalisme, qui contient la dépendance explicite, des probabilités d'émission nucléaire, du quantité de mouvement angulaire. On a calculé les probabilités d'émission de rayons \(\gamma\) selon le modèle à une seule particule en conjonction avec les fonctions d'augmentation, qui ont été choisies pour démontrer les effets collectifs observés par de différents expérimentateurs.
L'accord entre l'expérience actuelle et les calculs pour les fonctions d'excitation Zn$^{64}_{6}$+He$^{4}_{4}$, était bon. On pourrait expliquer la persistance de grandes valeurs pour la section efficace aux haute énergies par les effets de la concurrence entre les rayons γ et l'évaporation de particules quand celle-la fut supprimée par les effets de quantité de mouvement angulaire. L'accord au cas de Fe$^{56}_{56}$+C$^{12}_{12}$ était très encourageant, si l'on tient compte de la complexité du système de projectile-cibles et des difficultés théoriques.
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I. INTRODUCTION

A. Nuclear Reaction Mechanisms

1. Compound Nucleus Reactions

The term nuclear reaction is applied to a variety of processes involving the collisions of nucleons or groups of nucleons.

One of the simplest types of nuclear reaction is the interaction of a neutron with a target nucleus. Early workers, measuring the variation of neutron reaction cross-sections over small bombarding energy ranges, found large fluctuations (termed "resonances"), the widths (~0.1 eV) of which are small compared to the spacing (1 - 10^3 eV) between them. Bethe attempted to explain neutron reactions in terms of a potential-well model, but was unable to account for the narrowness of the resonances. The postulate that the resonances corresponded to many-particle excited states of the product nucleus rather than virtual single-particle states of a neutron in a nuclear potential well led to the conclusion by Bohr and by Breit and Wigner that the incoming neutron rapidly shared its energy with the nucleons of the target nucleus to form a "compound nucleus".

In classical terms, Bohr visualised the reaction as a two-step process in which the bombarding energy is first distributed among all of the nucleons to form a metastable state which may then decay in a number of ways.
The excitation energy, if reconcentrated on a nucleon or group of nucleons near the surface of the nucleus as a result of random nucleon-nucleon collisions may supply enough energy to eject the particle or particles in question, or the compound nucleus could de-excite by the emission of gamma rays. In the case of higher mass nuclides, de-excitation could also occur by division of the compound nucleus into two fragments of nearly equal mass, a process called fission.

Bohr concluded that the compound nucleus lifetime must be long compared to nuclear relaxation times\(^3\), but the validity of the model has been found to extend to short-lived reaction systems\(^5\). The two steps, compound nucleus formation and decay, were assumed to be independent; that is, the decay of a compound nucleus would depend only on the constants of motion of that particular compound nucleus, not on how it was formed. This is the so-called independence hypothesis; one consequence of it is that the probability for a given reaction may be factored into two parts, one corresponding to the formation probability (the cross-section for capture of the incoming particle) and a second to the decay probability as follows:

\[
\sigma (a,b) = \sigma_{\text{cap}} (a) \left[ \frac{P_b}{\sum P_i} \right]
\]

The quantity in square brackets (the branching ratio) is the quotient of the probability for de-excitation via the reaction channel "b" (i.e., to form a particular residual...
nucleus and particle with a given kinetic energy) and the probability sum over all possible exit channels.

The principle of detailed balance must hold for compound nucleus reactions, that is, the reaction system must be symmetric with respect to the exchange of entrance and exit channels. Thus,

$$\frac{\sigma(a, b)}{\lambda_a^2} = \frac{\sigma(b, a)}{\lambda_b^2}$$

Where \(\lambda_a\) and \(\lambda_b\) are the reduced De Broglie wave lengths of the system in the entrance and exit channels6-7.

2. Direct Interactions

A necessary condition in compound nucleus reactions is that energy equilibration be achieved after the incoming particle enters the target nucleus. However, a second group of reactions proceeds via a quite different mechanism in which much less than complete projectile momentum is transferred to the compound nucleus. These processes are called "direct interactions". In one form, for instance, the incoming particle may enter the target nucleus, excite a small number of nucleons to bound excited or unbound states and retain enough energy to leave the nucleus.

Another well characterised form of direct interaction is the Oppenheimer-Phillips reaction8. This reaction involves the interaction of a deuteron with a nucleus resulting in the absorption by the nucleus of a
neutron to populate single particle neutron states accompanied by the emitting of a high energy proton (with the proton energy perhaps exceeding the initial deuteron energy).

Compound nucleus reactions and direct interactions may be thought of as two opposite extreme mechanisms for low energy nuclear reactions. A particular reaction may proceed by an intermediate mechanism with more characteristics of one extreme than the other.

3. Experimental Distinction of Reaction Mechanisms

The predominant mechanism may often be deduced from experimental results. In the case of a compound nucleus reaction, the cross-section for formation of a specific product nucleus (say by means of an \((\alpha,n)\) reaction) should increase rapidly with bombarding energy until the threshold for a competing reaction (e.g., the emission of two nucleons instead of one) is reached; at this point the cross-section for the \((\alpha,n)\) reaction will begin to fall as the competing reaction cross-section rapidly rises. This follows from the strong dependence of the excitation energy of the compound nucleus on bombarding energy.

On the other hand, the incoming particle in a direct interaction deposits an excitation energy which is weakly related to the bombarding energy, thus rapid changes in the cross-section with the bombarding energy due to the competition described above would not be expected.
The statistical model predicts that the emission of particles from the compound nucleus is a stochastic process, and, therefore, that a compound nucleus (with zero angular momentum) emits particles isotropically. However, if a compound nucleus has a non-zero angular momentum, particles will be emitted preferentially in the equatorial plane. Classically, the compound nucleus may be thought of as a flywheel, emitting particles tangentially in its plane of rotation. The axis of rotation is perpendicular to the beam direction, hence preferential emission will be found at 0° and 180° to the beam direction in the center-of-mass system, with the overall distribution symmetric about 90°.

Most direct interactions, however, will show preferential emission at small angles to the incident beam direction as a result of the fact that a substantial fraction of the incident particle momentum is transferred directly to the emitted particles.

The emitted particle energy spectrum in the case of compound nucleus reactions is expected to be Maxwellian (except for coulomb barrier effects), but the energy spectrum for direct interactions will be more intense at higher energies, and often peaks may be resolved, corresponding to the low-energy single particle states which have been excited in the target nucleus.

The residual nuclei of a nuclear reaction will have more recoil energy in the compound nucleus case, since the linear momentum of the bombarding particle is transferred completely to the compound system.
If a projectile of energy $E_a$ and mass $A_a$ strikes a target of mass $A_t$ to form a compound nucleus which emits a particle to form a residual nucleus with mass $A_r$ and recoil energy $E_r$, then,

$$E_r = \frac{E_a A_a A_r}{(A_a + A_t)^2} = CE_a$$  \hspace{1cm} (1-a)

where $C$ is a constant. Furthermore, from experiment it is found that the recoil range in matter $R_r = C'E_r$ so that $R_r = C''E_a$, or a plot of the range of recoiling nuclei vs. the bombarding energy should be a straight line.

This result would not be expected in direct interactions, on the other hand, since the amount of energy transferred to recoiling nuclei is weakly related to the bombarding energy and is always less than $E_r$ calculated from equation (1-a). Having found proportionality between range and energy over a specific bombarding energy region, one may resort to theoretical range-energy relationships to calculate a recoil energy corresponding to an experimental range and then compare the energy with that calculated for complete momentum transfer. (see appendix I). Confidence in the theoretical calculation is gained by comparing theory to experiment at the reaction threshold where full momentum transfer must be achieved for the reaction to proceed.
B. The Statistical Model

1. Weisskopf Evaporation Formula

Bethe\textsuperscript{10}, Weisskopf\textsuperscript{11}, and Weisskopf and Ewing\textsuperscript{12} developed early statistical models to deal with compound nucleus reactions. Weisskopf\textsuperscript{11}, comparing the emission of nucleons by an excited nucleus to the evaporation of molecules by a liquid drop (an analogy first suggested by Frenkel\textsuperscript{13}), performed the thermodynamic derivation which follows. If one considers a nucleus B with energy $E_B$ and a neutron with energy between $\epsilon$ and $\epsilon + d\epsilon$ and velocity $v = \left( \frac{2\epsilon}{M} \right)^{\frac{1}{2}}$ enclosed in a volume $\Omega$, the probability $P_C$ per unit time that the neutron will be captured to form nucleus A with energy between $E_A$ and $E_A + d\epsilon$ is given by

$$P_C = \sigma (E_A, \epsilon) \frac{v}{\Omega}$$  \hspace{1cm} (1-b)

where $\sigma (E_A, \epsilon)$ is the mean cross-section for the collision of the neutron with nucleus B ($E_B = E_A - E_O - \epsilon$) producing compound nucleus A ($E_A$), and $E_O$ is the binding energy of the neutron to nucleus B. The probability for the reverse process is obtained by dividing $P_C$ by the number $(\omega_A(E_A)d\epsilon)$ of states into which the neutron can be captured and multiplying by the number $(\omega_B(E_A-E_O-\epsilon))$ of states into which A($E_A$) can decay. In the volume $\Omega$, there are

$$\frac{\Omega}{2 \pi^2 \hbar^3} (2m\epsilon)^{\frac{3}{2}} d\epsilon$$
translational states which may be occupied by the neutron
($g$ is the spin degeneracy of the neutron), so that by
detailed balance

$$ P_n (\varepsilon) \, d\varepsilon = P_c \frac{w_B(E_B)}{w_A(E_A)} \frac{\Omega g m}{2 \pi^2 \hbar^3} (2 \varepsilon m)^{1/2} \, d\varepsilon $$

or

$$ P_n (\varepsilon) \, d\varepsilon = \sigma(E_a, \varepsilon) \frac{g m \varepsilon}{\pi^2 \hbar^3} \frac{w_B(E_B)}{w_A(E_A)} \, d\varepsilon \quad (2-b) $$

The entropy of a nucleus with energy between $E$ and $E + dE$
is defined by

$$ S = \log \omega(E) $$

Then,

$$ P(\varepsilon)d\varepsilon = \sigma(E_a, \varepsilon) \frac{g m E}{\pi^2 \hbar^2} e^{S_B(E_A - E_0 - \varepsilon) - S_A(E_A)} \, d\varepsilon $$

If $E_A \gg E_0$ and $E_A \gg \varepsilon$ and $\omega_A$ and $\omega_B$ are assumed to be
identical functions ($\omega_A(E) = \omega_B(E)$), a Taylor expansion on
$S_B$ yields

$$ S_B(E_A - E_0 - \varepsilon) = S_A(E_A) - (E_0 + \varepsilon) \left( \frac{dS_A}{dE} \right)_{E_A} $$

Also $\frac{dS_A}{dE} = \frac{1}{T_A(E)}$ where $T_A(E)$ is the temperature (in units of
$kT$) at which the most probable energy of the body $A$ is
equal to $E$. Thus,

$$ P(\varepsilon)d\varepsilon = \sigma(E_a, \varepsilon) \frac{g m}{\pi^2 \hbar^3} e^{-E_0/T_A(E)} \varepsilon e^{-\varepsilon/T_A(E_A)} \quad (3-b) $$

Which is the usual form of Weisskopf's evaporation formula.
2. The Fermi Gas Model

Little is known about the densities of levels in highly excited nuclei; the variation of level density with energy has been derived from appropriate models. A widely used formulation may be derived from the Fermi gas model, in which the nucleus is described as a collection of degenerate fermions enclosed in a sphere of radius $R$. Under these restrictions, one may use the principles of statistical thermodynamics to derive a level density for the system. The number of states occupied by nucleons with momentum less than $P_F$ (the momentum corresponding to the "Fermi energy") is given by

$$N = \frac{2}{(2\pi\hbar)^3} \frac{4}{3} \pi \Omega P_F^3 \left(\frac{\Omega}{3\pi R^3}\right)$$

At complete degeneracy (ie., the ground state),

$$P_{F,\text{proton}} = (3 \pi^2)^{\frac{1}{3}} \hbar \left(\frac{Z}{\Omega}\right)^{\frac{1}{3}}$$

$$P_{F,\text{neutron}} = (3 \pi^2)^{\frac{1}{3}} \hbar \left(\frac{A-Z}{\Omega}\right)^{\frac{1}{3}}$$

Where $Z$ and $A$ are the proton and mass numbers respectively. The total kinetic energy of the protons considered as a degenerate gas at absolute zero is given by

$$E_0^{\text{protons}} = \frac{2\Omega}{(2\pi\hbar)^3} \int_0^{P_F} \frac{p^2}{2m} 4 \pi p^2 dp$$

$$= \frac{\pi}{3} \frac{5}{3} \frac{5}{10} \left(\frac{Z}{\Omega}\right)^{\frac{8}{3}} Z \frac{R^2}{m}$$

$$= \frac{3}{5} \frac{P_F^2}{2m} Z = \frac{3}{5} E_F Z$$

($m$ is the mass of the proton)
If the Fermi gas is heated to a temperature $T$ (i.e., the nucleus is excited), the excitation energy may be related to the temperature of the gas using formulas valid for a perfect gas of fermions of high degeneracy. One obtains 

for protons

$$ (E_T - E_0)_{\text{protons}} = \frac{\pi^2}{3} \frac{k}{E_F} T^2 $$

where $k$ is Boltzmann's constant. Substituting the value of $E_F$ calculated from the $P_F$ of equation (4-b),

$$ (E_T - E_0)_{\text{protons}} = \frac{\pi^2}{72} \frac{\Omega m}{h^2} \left( \frac{Z}{\Omega} \right)^{\frac{3}{2}} (kT)^2 $$

Similarly, for neutrons

$$ (E_T - E_0)_{\text{neutrons}} = \frac{\pi^2}{72} \frac{\Omega m}{h^2} \left( \frac{A-Z}{\Omega} \right)^{\frac{3}{2}} (kT)^2 $$

Summing the neutron and proton energies

$$ (E_T - E_0)_{\text{total}} = 0.03A^{\frac{3}{2}} \left[ Z^{\frac{1}{3}} + (A-Z)^{\frac{1}{3}} \right] (kT)^2 $$

where $a$ is called the Fermi gas constant. The density of states at a given energy is related to the entropy of the gas by

$$ S = k \log \frac{\rho(E)}{\rho(0)} $$

or,

$$ \rho(E) = \rho(0) e^{s/k} \quad (5-b) $$
Thermodynamically,

\[
S(T) = \int_0^T \frac{dE}{T} = 2ak^2 \int_0^T T \frac{dT}{T} = 2ak^2 T = 2(ak^2 E)^{1/2} \quad (6-b)
\]

Equating quantities in equations (5-b) and (6-b)

\[
k \log \frac{\rho(E)}{\rho(0)} = 2(ak^2 E)^{1/2}
\]

or

\[
\frac{\rho(E)}{\rho(0)} = e^{2(aE)^{1/2}}
\]

\[
\rho(E) = Ce^{2\sqrt{aE}} \quad (7-b)
\]

Many calculations have been performed using a level density of the above form (equation (7-b)) where \( C \) and \( a \) were assumed to be adjustable parameters which were varied to give a best fit to experimental data.\(^{16,17}\)

The Fermi gas level density expression must, however, be corrected for nucleon angular momentum pairing. Weisskopf and Ewing\(^{12}\) suggested adjustment of the pre-exponential constant as follows:

\( C_{\text{odd-odd}} = 4C_{\text{even-even}} \) and \( C_{\text{even-odd}} = C_{\text{odd-even}} = 2C_{\text{even-even}} \)

Hurwitz and Bethe\(^{18}\) concluded that it is more realistic to consider odd-even effects on level densities as arising from displacements of the ground-state energies caused by nucleon pairing.
Thus, the energy appearing in the level density expression must be measured from a virtual ground state, displaced upward from the true ground state by an amount equal to the pairing energy. For even-even nuclei, the proton and neutron pairing energies cause an additive displacement, thus,

\[ \delta_{oe} \approx \delta_{eo} \quad \text{and} \quad \delta_{ee} \approx 2 \delta_{oe} \]

Cameron\textsuperscript{19} has calculated pairing energies from a comparison of results of his semi-empirical mass equation with measured atomic masses. Newton\textsuperscript{20} has taken shell effects into account by calculating a characteristic value of \( a \) for each proton and neutron number.

Dostrovsky, Fraenkel, and Friedlander\textsuperscript{21} have corrected for shell effects by including an additional energy shift which is positive just before and negative after closed shells.

3. Capture and Inverse Cross-Sections

A neutron, striking a target with impact parameter \( b \), if captured, will form a system with an angular momentum vector normal to the direction of the relative momentum \( \mathbf{p} \) of the neutron and target and of magnitude

\[ L = pb = \frac{\hbar b}{\lambda} \quad (8-5) \]

Where \( \lambda \) is the reduced De Broglie wave length of the system in the entrance channel. The angular momentum is quantized in units of \( \hbar \), so that

\[ L = n\hbar \quad n = 0,1,2,3, \ldots, n_{\text{max}} \quad (9-6) \]
The maximum value of $i$ is limited by the maximum value of the impact parameter which is the sum of radii of the neutron and target ($b_{\text{max}} = R$), and from combination of equations (8-b) and (9-b),

$$b_{\text{max}} = i_{\text{max}} \lambda$$
or

$$i_{\text{max}} = R / \lambda$$

Classically, one may think of the probability for formation of an intermediate system with a specific angular momentum as corresponding to the probability that the neutron will hit the nucleus with a velocity and an impact parameter corresponding to that particular value, thus\textsuperscript{9}.

$$\sigma_i = \pi \lambda^2 \left[ (i+1)^2 - i^2 \right]$$

$$= \pi \lambda^2 \left( 2i + 1 \right)$$

(10-b)

The total cross-section is the sum of these partial cross-sections over $i$,\textsuperscript{9}.

$$\sigma_t = \sum_{i=0}^{i_{\text{max}}} \sigma_i$$

$$= \pi \lambda^2 \sum_{i=0}^{i_{\text{max}}} \left( 2i + 1 \right)$$

(11-b)

$$= \pi \lambda^2 (i_{\text{max}} + 1)^2$$

$$= \pi (\lambda + R)^2$$
Equation (11-b) represents an upper limit for $\sigma_r$ since, in reality, not all collisions will result in capture. Barrier penetration (or transmission) coefficient must be introduced into the sum as follows:

$$\sigma_r = \pi \lambda^2 \sum_{t=0}^{t_{\text{max}}} (2t+1) T_t$$

(12-b)

where

$$T_t \leq 1$$

In the case of a charged particle interacting with a target nucleus, the value of the relative momentum at the point of contact will be

$$p = (2\mu)^{\frac{1}{2}}(\varepsilon-B)^{\frac{1}{2}} = (2\mu \varepsilon)^{\frac{1}{2}}(1-\frac{B}{\varepsilon})^{\frac{1}{2}}$$

Where $\varepsilon$ is the energy of charged particle, $\mu$ is the reduced mass, and the coulomb barrier, $B$, is given by

$$B = \frac{Z_X Z_T e^2}{R}$$

In this case, the upper limit for the cross-section may be shown to be

$$\sigma_r = \pi R^2 \left( 1 - \frac{B}{\varepsilon} \right)$$

(13-b)

Equation (12-b) is valid for charged particle cross-sections, provided that the coulombic as well as the centrifugal barrier has been included in the calculation of the transmission coefficients.

Equations such as (13-b) have often been used for the approximate calculation of inverse reaction cross-sections and capture cross-sections.
However, the development of the optical model (sec. I-C(1.)) has made possible more detailed calculation of transmission coefficients and several optical model computer programs are available for this purpose.23-25.
C. Modern Compound Nucleus Theories

1. The Optical Model

When considering the interactions of incoming particles with target nuclei, an exact treatment of the many-body problem is prohibitively complicated; many simplifying assumptions are necessary for all but two-body cases. Most treatments of particle-nucleus interactions neglect all structure of the nucleus and adopt a model which will describe the nuclear properties of interest.

Bethe described the nucleus as a real potential well, characterised only by its radius and depth. However, several features of this model were found to be inadequate; the magnitude of predicted neutron capture cross-sections, the wide spacing of resonances, and the slow change of cross-sections with energy were not in agreement with experimental results.

Compound nucleus theories, alone, were also found to be inadequate, since no account is taken of direct interaction contributions.

The addition of an imaginary term to the potential of the well in Bethe's model was found to extend its range and accuracy. Early work by Serber and by Fernbach, Serber, and Taylor resulted in the proposal that the elastic scattering of neutrons be compared with the scattering of a wave by a refracting and absorbing sphere of refractive
Where the wave-length of the light corresponds to the energy of the neutrons \((E = h\nu)\). The nuclear potential is of the form

\[ V_{\text{nuc}} = V + iW \]

Where \(V\) is the average single particle real potential deflecting particles from straight-line motion, and \(W\) is the "absorption potential" representing the particles lost in the formation of compound nuclei\(^{28}\). The nuclear potential is a function of the radial distance, \(r\), between the centers of the interacting bodies, and spin-orbit interactions must be considered (except where both particles are of zero spin), leading to the following form:

\[
V_{\text{nuc}}(r) = Vf(r) + iWg(r) + (\frac{\hbar}{\mu c})^2(V_{\text{so}} + iW_{\text{so}})(\frac{df}{dr})\sigma \cdot \iota
\]

Where \(V\) and \(W\) are the depths of the real and imaginary potentials, \(V_{\text{so}}\) and \(W_{\text{so}}\) are the real and imaginary spin-orbit potentials, \(\sigma\) is the Pauli spin operator, \(\iota\) is the orbital angular momentum in units of \(\hbar\), and \(f(r)\) and \(g(r)\) are form factors\(^{29}\).

According to the Woods-Saxon\(^{30}\) configuration

\[
f(r) = [1 + \exp \left( \frac{r-R}{a} \right)]^{-1}
\]

\[(c-1a)\]
Where $R$ is the sum of the nuclear radii of the interacting particles ($R = r_o(A_T^{1/3} + A_X^{1/3})$) and $a$ is a parameter which measures the real radial diffusness.

Two different assumptions have been made about the form factor $g(r)$.\textsuperscript{29}:

(1.) $g(r) = f(r)$ This assumption corresponds to volume absorption.

(2.) Surface absorption.

In the latter case (a purely phenomenological approach) the imaginary form factor is usually assumed to be Gaussian.

$$g(r) = \exp\left[ \frac{-(r-R)^2}{b^2} \right] \quad (c-1b)$$

Where $b$ corresponds to the imaginary radial diffusness, but is usually treated as an adjustable parameter. The real form factor is usually taken from the Woods-Saxon configuration in both cases.\textsuperscript{29}.

The total potential is

$$V_T(r) = V_C(r) + \frac{1(i+1)\hbar^2}{2\mu r^2} + V_{nuc}(r) \quad (c-2)$$

Where the first two terms correspond to coulombic and centrifugal potentials. The transmission coefficients for equation (12-b) may be calculated from optical model phase shifts $(\delta_t)\textsuperscript{5}$.

$$T_l(\varepsilon) \equiv 1 - |e^{21\delta_1 \varepsilon}|$$
The phase shifts being obtained by solution of the Schrödinger equation:

\[
[-\left(\frac{\hbar^2}{2M}\right)v^2 + V(r)]\psi = E\psi
\]

using the \(V(r)\) of equation (c-1).

Since, if a spin-orbit interaction is used, the calculated transmission coefficients will be angular momentum dependent, the form of the summation to obtain the cross-section will be dictated by the way in which the angular momenta are coupled. If intermediate coupling, (see appendix II) is used, the angular momentum distribution of the compound nuclear states is given by \(5\).

\[
\sigma_C(I, \varepsilon, J_C) = \frac{\pi \hbar^2 (2J_C+1)}{(2s+1)(2I+1)} \sum_{S=\mid I-S \mid}^{I+S} \sum_{T=\mid J_C-S \mid}^{J_C+S} |T_t(\varepsilon)|^2
\]

For cases where \(I = s = 0\), (c-4) reduces to

\[
\sigma_C(\varepsilon, J_C) = \pi \hbar^2 \sum_t (2t+1) T_t(\varepsilon) \quad (c-5)
\]

which is identical in form to equation (12-b) except that \(\sigma_C\) and \(T_t\) now refer to a specific angular momentum. The total reaction cross-section of equation (12-b) is given by

\[
\sigma_r = \sum_{J=0}^{\infty} \sigma_C(\varepsilon, J_C)
\]
2. Angular Momentum Considerations

Since the impact parameter is not unique in nuclear reactions, compound nuclei will be formed with a distribution in angular momentum which is a function of the nature and energy of the reacting system. Equation (2-b) which was derived without reference to angular momentum, may be rewritten to include angular momentum effects as follows:

\[ P(E_F, J_F; E_C, J_C) \, d\varepsilon = \frac{(2s_x + 1)}{\pi^2 \hbar^3} \cdot \varepsilon \cdot w(E_F, J_F) \cdot \sigma(E, J_C, J_F) \frac{w(E_C, J_C)}{w(E_C, J_C)} \, d\varepsilon \]

(c-6)

Where, for clarity, the subscripts A and B have been changed to C and F, signifying "compound" and "final" respectively. The spin degeneracy, \( g \), has been replaced by its equivalent, \( 2s_x + 1 \), where \( s_x \) is the spin of the particle \( x \) in the entrance channel.

The total probability \( \Gamma_x \) for emission of particle \( x \) from a compound nucleus with exitation energy \( E_C \) and angular momentum \( J_C \) is the probability given in equation (c-6), summed over all possible \( J_F \) and integrated over all possible energies of emission.

\[ \Gamma_x (E_C, J_C) = \frac{(2s_x + 1)\mu}{\pi^2 \hbar^3} \int_{E}^{\infty} \sum_{J_F} \sigma_x (E, J_C, J_F) \frac{w(E_F, J_F)}{w(E_C, J_C)} \, d\varepsilon \]

(c-7)

The cross-section for a particular reaction, e.g. \( X + b \rightarrow Y + x \) is given by

\[ \sigma(b,x) = \sum_{J_C} \sigma_{cap}(E_b, J_C, I) \frac{\Gamma_x J(E_F, J_F)}{\sum_i \Gamma_i J(E_F, J_F)} \]

(c-8)
Where $I$ is the spin of the target nucleus and the sum over $I$ includes all possible particles which may be emitted by the compound nucleus. It can be seen that factoring of the formation and decay probabilities is no longer possible, so that the independence hypotheses as previously stated is no longer valid. The independence hypothesis is valid for a particular $J_c$ however, since

$$\sigma(b,x) = \sum_{J_c} \sigma_{J_c}(b,x)$$

where

$$\sigma_{J_c}(b,x) = \sigma_{cap}(\varepsilon_b, J_c, I) \frac{\Gamma_{xJ}(E_F, J_F)}{\sum \Gamma_{1J}(E_F, J_F)}$$

The capture and inverse cross-sections in equations (c-8) and (c-6) may be calculated as in section C-1. The level density will also be affected by the inclusion of angular momentum considerations. From the general theoretical grounds of statistical mechanics, the dependence of the level density on angular momentum is expected to be

$$\exp \left[ -\frac{\text{rotational energy}}{\text{temperature}} \right]$$

If $M$ is the total angular momentum projection on a Z-axis and the number of particles, $N$, is large, the statistical central limit theorem predicts that the distribution of
M will be Gaussian

\[ w(M) \propto \exp \left[ \frac{-M^2}{2\sigma^2} \right] \]

or

\[ w(E,M) = \left[ 2\pi\sigma^2 \right]^{\frac{3}{2}} w(E) \exp \left[ \frac{-M^2}{2\sigma^2} \right] \quad (c-9) \]

Where \( \sigma^2 = N \langle m^2 \rangle \) is the mean square deviation of \( M \), \( m \) being the projection of the angular momentum of a single particle. The projections of the total angular momentum are \( M = J, J-1, \ldots, -J \), and

\[ w(E,M) = \sum_{J=|M|}^{J_{\text{max}}} w(E,J) \]

so that

\[ w(E,J) = w(E,M=J) - w(E,M=J+1) \quad (c-10) \]

Since \( w(E,M=J) \) and \( w(E,M=J+1) \) may be expanded in a Taylor series about \( M = J + \frac{1}{2} \),

\[ w(E,M=J) = w(E,J+\frac{1}{2}) + (M-J-\frac{1}{2}) \frac{\partial w}{\partial M} \bigg|_{M=J+\frac{1}{2}} + \cdots \]

\[ = w(E,J+\frac{1}{2}) - \frac{1}{2} \frac{\partial w}{\partial M} \bigg|_{M=J+\frac{1}{2}} + \cdots \]

\[ w(E,M=J+1) = w(E,J+\frac{1}{2}) + \frac{1}{2} \frac{\partial w}{\partial M} \bigg|_{M=J+\frac{1}{2}} + \cdots \]

it follows, (neglecting terms \( \leq \frac{1}{96} \frac{\partial^2 w}{\partial M^2} \bigg|_{M=J+\frac{1}{2}} \)) from (c-10) and (c-9) that

\[ w(E,J) \approx \frac{\partial}{\partial M} \left[ w(E,M) \right]_{M=J+\frac{1}{2}} \]

\[ = w(E) \left( \frac{2J+1}{2(2\pi)^{\frac{3}{2}}} \left( \frac{\hbar}{\sigma} \right)^3 \exp \left[ \frac{-\hbar^2 (J+\frac{1}{2})^2}{2\sigma^2} \right] \right) \quad (c-11) \]
The derivation of equation (c-9) depends on use of the central limit theorem which requires that the number of particles involved be large, but Lang has added further theoretical justification of the Gaussian form by the following derivation. For a Fermi gas with a density of single particle states $g$ and the following equation of state

$$E = at^2 - \frac{3}{2}t + \frac{M^2}{2\langle m^2 \rangle g}$$

the density of states is given by

$$w(E, M) = w(E - \frac{M^2}{2\langle m^2 \rangle g}, 0)$$

that is, the rotational and thermal energies are thought of as separate. Since

$$\log[w(E, M)] \approx \log[w(E, 0)] - \frac{M^2}{2\langle m^2 \rangle g} \frac{\partial}{\partial E} \log[w(E, 0)]$$

and

$$\frac{1}{3} = \frac{\partial}{\partial E} \log [w(E, 0)]$$

we obtain

$$w(E, M) = w(E, 0) \exp \left[ -\frac{M^2}{2\langle m^2 \rangle} \frac{1}{3} \right]$$

The quantity $c$ is the moment of inertia in units of $\hbar$ ($c = \frac{I}{\hbar}$) and is related to the spin cut off parameter by

$$\sigma^2 = ct$$
Where $t$ is the thermodynamic temperature related to the "nuclear temperature" $\mathcal{J}$ by

$$\mathcal{J} \approx t + \frac{a}{4} \alpha^{-1}$$

For a Fermi gas with the equation of state given by (c-12),

$$w(E) = w(E,0) = \frac{1}{12} \pi^2 a^{-\frac{1}{4}} \left(E + \frac{3}{2} t\right)^{-\frac{5}{4}} \exp \left(\frac{2}{aE}\right)$$

and, substituting into (c-11),

$$w(E,J) = \frac{\sqrt{2}}{48} a^{\frac{3}{2}} \left(\frac{\hbar}{2I}\right)^{\frac{3}{2}} (E + \frac{3}{2} t)^{-2} (2J + 1) \exp \left[2(aE)^{\frac{1}{2}} \frac{2J(J+1)}{2It}\right]$$

Bethe and others have arrived at similar forms for the level density. This form is unrealistic, however, in that it predicts the same proportion of high and low angular momentum states at all energies, whereas in reality, higher $J$ states appear only at higher energies. Lang has attempted to correct for the superfluity of high $J$ states at low energies by introducing a higher order term in $M$ as follows:

$$w(E,J) = \frac{w(E,0)}{2\pi \left(\frac{\hbar}{2I}\right)^{\frac{1}{2}} \frac{3M^2}{5gt}} \exp \left[-\frac{M^2}{2It} - \frac{M^4}{(2It)^2 \cdot 5gt}\right]$$

Sarantites and Pate have derived a level density of the following form:

$$w(E,J) = \frac{\sqrt{2}}{48} \left(\frac{\hbar}{2I}\right)^{\frac{3}{2}} a^{\frac{1}{2}} (E + t - E_r)^{-2} (2J + 1) \exp \left\{2 \left[a(E-E_r)\right]^{\frac{1}{2}}\right\}$$

(c-13)
where

\[ E_T = \frac{\hbar^2 J(J+1)}{2I} \]

This expression results in a much sharper decrease in the number of levels just before the highest value of \( J \) allowed (defined by \( E = \frac{\hbar^2 J_{\text{max}}(J_{\text{max}}+1)}{2I} \)), but does not appreciably relieve the persistence of high \( J \) levels at lower energies.

It can be seen that the value chosen for \( I \) greatly influences the angular momentum dependence of the level density. More will be said about the choice of \( I \) in the following section.

3. The Superconductor Analogy

The Fermi gas model (sec. B-2) is among the simplest of macroscopic models of the nucleus, since the nucleons are assumed not to interact. It is unrealistic, however to assume that the motions of the nucleons are completely independent within a finite nucleus.

Theories developed to describe the motion of electrons in metals have shown that even small interactions between fermions of a system may result in changes in the macroscopic properties of the system. Bardeen, Cooper, and Schrieffer\(^{37}\) have shown that correlations between pairs of electrons arise from interactions with lattice vibrations in a crystal. For energies near the Fermi surface, electrons with equal and opposite angular momenta may pair to form
quasi-bound states, thus lowering the total energy of the system and causing the appearance of a gap in the originally continuous energy spectrum.\textsuperscript{38}

The observed energy gap in the spectra of even-even nuclei led to the suggestion by Bohr, et. al.\textsuperscript{39} that nucleon correlations exist which are similar to the fermion correlations of the electrons of a superconductor. The theories of superconductivity have been applied extensively to the nuclear case\textsuperscript{40,41,42} where lattice interactions are replaced by forces between the nucleons which lead to angular momentum pairing. The strength of the nucleon interactions are proportional to the correlation function $\Theta$. Lang\textsuperscript{42} has shown that the energy gap is equal to $2\Theta$ in even-even nuclei which is approximately equal to $2\Delta$, where $\Delta$ is the energy difference between the ground state masses of even and odd nuclei corrected for surface, coulomb, and symmetry energies. A more detailed set of calculations by Vonach, et. al.\textsuperscript{43} led to the value

$$\Theta = 1.3\Delta$$

For the ground state, the theories of Bardeen, Cooper, and Schrieffer\textsuperscript{37} give a condensation energy\textsuperscript{43}:

$$C_0 = \frac{1}{4}g\Theta^2 = 0.47 \text{ atc}^2$$
where

\[ a = \frac{1}{6} n^2 g \]

\[ t_c = \frac{4 \theta}{7} \]

It is assumed that \( 2\theta_0 \) is the energy gap for \( t = 0 \). "g" is the single particle level density of protons and neutrons for a nucleus without residual interactions. The quantity \( C_0 \) decreases with energy and reaches zero at \( U_c \), the energy corresponding to the critical temperature, \( t_c \), at which all pairs are broken. Above \( t_c \), the nucleus may be described as a Fermi gas with a ground state energy shifted upward by an amount equal to the condensation energy. The critical energy is given by \( 43 \).

\[ U_c = at_c + C_0 = 1.47 at_c^2 \]

Below \( U_c \) the concept of a level density begins to lose meaning, although theoretical forms have been derived.\(^{55,56}\).

In the nuclear case, superconductor theories have been extended to describe changes in the rotational moment of inertia, which is expected to decrease with energy below \( t_c \). Belyaev\(^{40}\) and others\(^{39,44}\) have derived approximate forms for the rotational moment of inertia as a function of energy below the critical energy. At higher energies, the moment of inertia is expected to be equal to the moment of inertia of a rigid body of the nuclear dimensions.
4. Isobaric Spin

Since the proton and the neutron are often thought of as different quantum states of the same particle (the nucleon), it is convenient to introduce a quantity which specifies charge state. This variable, the isobaric spin or \( i \)-spin, takes on the values \( t = +\frac{1}{2} \) and \( t = -\frac{1}{2} \) for protons and neutrons respectively. An isobaric spin operator, \( \mathbf{s} \), may be introduced which has properties analogous to the Pauli spin operator, \( \sigma \); in fact, the quantum mechanical behavior of \( t \) parallels that of \( s \), the ordinary spin (however, spin-space and \( i \)-spin-space are completely independent).

The total \( i \)-spin of a nucleus is:

\[
T = \sum_{i=1}^{A} t_i
\]

which has \( Z \) component

\[
T_Z = \frac{1}{2}(Z - N)
\]

For light nuclei (\( A \approx 50 \)), \( T \) is a good quantum number, but as \( Z^2/A \) increases or as the neutron excess increases, \( T \) becomes less valid, in the former case because of coulombic effects on commutation properties, and in the latter because different single particle shells are being filled by neutrons and protons.
Isobaric spin conservation is important in the reactions between elementary particles and light nuclei. In the scattering of deuterons by helium, an intermediate system with $T = 0$ is expected. The first three excited states of Li$^6$ are the following:\(^7\):

$$
\begin{align*}
J &= 3^+ \quad T = 0 \quad 2.19 \text{ MeV} \\
J &= 0^+ \quad T = 1 \quad 3.57 \text{ MeV} \\
J &= 2^+ \quad T = 0 \quad 4.52 \text{ MeV}
\end{align*}
$$

Resonances are found corresponding to the $T = 0$ states, but the $T = 1$ state is not formed as a compound nucleus.\(^7\).

The exact mass and energy regions of applicability of isobaric spin conservation are at present undefined, since most experiments to date only test relative isobaric spin relationships between neighboring isobars (charge exchange reactions), and little information is available for more complicated systems.\(^4\)\(^7\).

Studies of alpha particles and protons emitted from the compound nucleus Cu$^{63}$ produced by alpha ($T = 0$) bombardment of Co$^{59}$ ($T = -5/2$) and proton ($T = \frac{1}{2}$) bombardment of Ni$^{62}$ ($T = -2$) have yielded results, the anomalies in which have been discussed in terms of isobaric spin arguments\(^4\)\(^8\),\(^4\)\(^9\).; however, these arguments are still in the formative stage, and the validity of isobaric spin conservation at such high energies is doubtful.
D. Present Work.

The present work is a radiochemical study of reactions proceeding through the compound nucleus Ge$^{68}$ excited to energies of 10-50 MeV.

Previous studies have dealt with production of the compound nucleus Ge$^{68}$ by bombardment of Zn$^{64}$ with He$^4$ ions, but have not included product range measurement to determine predominant reaction mechanisms. Different formalisms of the statistical model have been employed with varied success in describing the magnitude of the observed cross-sections. Calculations by Porile, which did not include angular momentum dependence, required variation of the level density parameter between 0.8 and 2.8 MeV$^{-1}$ to obtain agreement with experiment. From the shapes and high energy tails of some of the excitation functions Porile concluded that a sizable direct interaction contribution was probably present or that the statistical model was not completely applicable to this case.

Dostrovsky, Friedlander, and Fraenkel required anomalously low ($a = A/40$) level density parameters to obtain agreement between their non angular momentum dependent statistical model calculations and the data of Porile.

Esterlund and Pate employed a primitive angular momentum dependent formalism with a variable moment of inertia. In order to obtain agreement with the data of Porile, a moment of inertia of 1.2 times the rigid body
value was required; this value is in obvious conflict with physical possibility.

Saratites and Pate$^{31}$ used Monte Carlo techniques and an angular momentum dependent formalism in their calculations. For calculations involving the emission of one particle by the compound nucleus, excellent agreement was obtained with the data of Porile$^{50}$ using a realistic value of the level density parameter and the rigid body moment of inertia. For the two particle case, agreement was less satisfactory.

In light of the anomalies observed in previous studies of this reaction system, a reinvestigation of the reactions involved was undertaken. In order to detect any major direct interaction contributions, recoil ranges were measured. Also, the same compound nucleus was formed in three different ways; namely, He$^4$ bombardment of Zn$^{64}$, C$^{12}$ bombardment of Fe$^{56}$, and O$^{16}$ bombardment of Cr$^{52}$. Cross-sections were measured as function of energy for the following reactions:

<table>
<thead>
<tr>
<th>Product</th>
<th>Target</th>
<th>Zn$^{64}$</th>
<th>Fe$^{56}$</th>
<th>Cr$^{52}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ge$^{67}$</td>
<td>($\alpha$,n)</td>
<td>(C$^{12}$,n)*</td>
<td>(O$^{16}$,n)*</td>
<td></td>
</tr>
<tr>
<td>Ga$^{67}$</td>
<td>($\alpha$,p)</td>
<td>(C$^{12}$,p)</td>
<td>(O$^{16}$,p)*</td>
<td></td>
</tr>
<tr>
<td>Ge$^{66}$</td>
<td>($\alpha$,2n)</td>
<td>(C$^{12}$,2n)</td>
<td>(O$^{16}$,2n)*</td>
<td></td>
</tr>
<tr>
<td>Ga$^{66}$</td>
<td>($\alpha$,pn)</td>
<td>(C$^{12}$,pn)</td>
<td>(O$^{16}$,pn)*</td>
<td></td>
</tr>
<tr>
<td>Ge$^{68}$</td>
<td>($\alpha$,\gamma)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(* only upper limits on the cross-sections were measurable)
At the energies of the measurements, the average angular momentum of the compound nucleus is expected to differ in the three cases when considered at equal excitation energies, providing a test for the angular momentum dependence of the excitation functions. Also, direct interaction contributions are expected to differ substantially for three such different reaction systems.

The isotopic spin is, however, invariant and plays no role for the three target projectile systems studied, since projectiles with zero isotopic spin are impinging on targets with isotopic spin -2 in all three cases considered.

A Fortran IV coding of statistical model calculations was undertaken for an IBM system 360/40 computer. Large memory capacities have made possible direct calculations and summation of reaction probabilities, and this method was chosen, rather than Monte Carlo techniques which have been used earlier.
II. Experimental Procedures

A. Targets

Thin metal foil targets were prepared from the materials described in Table I. Materials enriched in an isotope of interest were used in those cases where activities would be produced via reactions with other constituents of the natural isotopic mixture and would interfere with the radioassay of the nuclide of interest.

1. Zinc

Zinc targets were prepared by vacuum deposition of metallic zinc onto thin aluminum foils. The aluminum surface had to be pretreated; aluminum metal which has been exposed to air always has an oxide coating and zinc will not readily condense onto non-metallic surfaces. This difficulty may be overcome by using high vapor intensities, but deposits thus formed are granular and uneven. In the case of zinc enriched in Zn, this would have been especially troublesome, since only small amounts of the enriched isotope were available.

It has been found, however, that a thin layer (~1 Å) of silver will provide nucleating sites for zinc on a non-metallic surface. Therefore, the aluminum foils were pre-treated by evaporating a thin deposit of silver, the thickness of which was determined by means of a quartz crystal and frequency monitor, the frequency of a crystal being proportional to the mass deposited upon it.
<table>
<thead>
<tr>
<th>Element</th>
<th>Isotope</th>
<th>Natural (^{56})</th>
<th>Enriched (^{57})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cr</td>
<td>50</td>
<td>4.31</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>52</td>
<td>83.76</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>53</td>
<td>9.55</td>
<td>—</td>
</tr>
<tr>
<td></td>
<td>54</td>
<td>2.38</td>
<td>—</td>
</tr>
<tr>
<td>Fe</td>
<td>54</td>
<td>5.82</td>
<td>0.1 ± 0.02</td>
</tr>
<tr>
<td></td>
<td>56</td>
<td>91.66</td>
<td>99.7 ± 0.1</td>
</tr>
<tr>
<td></td>
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<td>0.1 ± 0.02</td>
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<td>66</td>
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<td>0.14 ± 0.04</td>
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<tr>
<td></td>
<td>70</td>
<td>0.62</td>
<td>&lt;0.01</td>
</tr>
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</table>
In all cases, the overall average thickness of the silver deposit was less than the atomic radius, and the amount deposited was known to $\pm 10\%$. The zinc deposits evaporated onto a foil thus pretreated were examined by means of a microscope fitted with a calibrated eyepiece, and the grain size was found to be much less than the thickness of the deposit in all cases.

High purity, commercially available, natural zinc foils of .0005 inch thickness were used at higher energies when the beam energy loss in such foils was less than 1 MeV.

2. Enriched Iron

Iron foils, enriched in Fe$^{56}$, were prepared by electroplating iron onto aluminium foil from basic tartrate solutions. The methods of Blann et. al.$^{59}$ were modified by lowering the pH to 9.5 to avoid reaction of the basic solution with the aluminium substrate. A 5 cm. diameter tantalum disc was used as the cathode at a distance of 0.5 cm. from the aluminium foil anode. Foils were cut from the center of the plated area to avoid variations in the deposit thickness at the edges of the foils.

3. Natural Iron and Chromium

Natural iron and chromium foils were prepared by vacuum evaporation onto aluminium foil using standard techniques.$^{54}$
4. Uniformity

The uniformity of all foils was checked by accurately weighing segments of the foil which had been cut with a small die of known area (7.55 mm$^2$). None of the foils used for bombardment had a thickness which varied by more than 3%.

B. Bombardments

1. He$^4$

Bombardments were performed at the Brookhaven National Laboratory 60-inch Cyclotron, the University of Washington (Seattle) 60-inch Cyclotron, and the Washington University 45-inch Sector-focused Cyclotron.

Incident alpha-particle beam energies were measured via the observed ranges in stacks of aluminium plus blue cellophane foils. An amount of aluminium was used corresponding to most of the range, with blue cellophane used downstream for the final few percent. A brief (~1 μA-sec) passage of charged particles through blue cellophane causes ionization-induced bleaching, and the thickness beyond which bleaching no longer occurs corresponds to the range of the incident beam. The energy was then calculated from the previously measured range-energy relationship for aluminium and blue cellophane.
The incident beam energies were found to be:

- U. of Washington 60-inch Cyclotron: 41.6 ± 0.5 MeV
- Brookhaven N.L. 60-inch Cyclotron: 41.0 ± 0.3 MeV
- Washington U. 45-inch Cyclotron: 25.7 ± 0.3 MeV

The target configuration used on the U. of Washington 60-inch cyclotron consisted of a water-cooled aluminium plate onto which were clamped the foil stacks to be irradiated. The beam itself was not well collimated; therefore collimation at the target was effected by means of a thick aluminium mask clamped around the edges of the foil stack. In most cases, a special target chamber was used to avoid overheating of the target foils. This so-called "bell jar" assembly, (see figure 1.) allowed a flow of helium to pass across the face of the target stack, but lowered the incident beam energy to 37.4 ± 0.5 MeV.

A similar target block was used on the Washington U. 45-inch cyclotron, but no special cooling was required, because of smaller beam intensity and more efficient water-cooling of the block.

In both of the above cases, the beam intensity was measured by means of the production of Zn⁶⁵ in copper foils. The excitation function for the combined Cu⁶³ (α,2n) Ga⁶⁵ and Cu⁶³ (α,pn) Zn⁶⁵ reactions was determined in a separate experiment (see appendix III) and found to be in agreement with the results of Houck and Miller⁶¹.
The projectile energy at various positions through the foil stack was calculated using the range-energy curves due to Atkinson and Willis\textsuperscript{62}. The range of alpha particles in zinc was determined relative to aluminium by a series of blue cellophane experiments in which different portions of the beam were intercepted by zinc or aluminium degraders before entering the cellophane stack. From the relative positions of the stopped beams in the stack, the range in zinc was calculated from the range difference and beam energy.

In the Washington University bombardments, during which the Zn\textsuperscript{64} (α,γ) Ge\textsuperscript{68} excitation function was determined, only thin foils (<2 mg/cm\textsuperscript{2}) were used. The more accurate incident energy determined from the average of an extensive series of range measurements by Reeder\textsuperscript{63} was used in calculations (25.9 ± 0.1 MeV).

A detailed description of the Brookhaven target assembly and Faraday cup used to monitor the beam intensities is given elsewhere\textsuperscript{64}.

2. Heavy Ion (O\textsubscript{16} and C\textsubscript{12})

Bombardments were performed at the Yale University Heavy Ion Linear Accelerator. The most probable velocity of the fully accelerated beam particles is a constant (independent of the charge of the ions) as is the energy per projectile mass unit (10 MeV). To insure uniform energy, the C\textsubscript{12} and O\textsubscript{16} beams (consisting of charge +5 ions) were
passed through an analyzing magnet before being brought into focus in the experimental area.

A typical one inch diameter target stack consisted of one target foil (~250 µg/cm²) preceded by an appropriate thickness of beam-degrading aluminium foils and followed by several thin (~200 µg/cm²) aluminium catchers. This stack was clamped onto a brass cylinder which slipped into a 1½ inch x 6 inch Faraday cup (see figure 2). The charge from the Faraday cup was integrated by a calibrated Cary electrometer. The efficiency of the Faraday cup in retaining secondary electrons had been shown to be 100% under the influence of a strong magnetic field.

The energy of the beam particles at various positions through the target stack was determined from the range-energy curves of Northcliffe.

C. Chemistry and Radiation Assay Techniques

1. Chemical Separations

Standard radiochemical procedures were adapted for use on all targets in this work. Each target was dissolved in a 6N HCl solution containing known amounts of Ge(IV) and Ga(III) carrier plus Zn, Ni, Co, and Mn holdback carriers. To insure that the germanium and gallium activities were in the appropriate oxidation states, a few drops of 40% H₂O₂ solutions were added.

Germanium activities were separated by distillation as GeCl₄ from the above solution. The gaseous GeCl₄ was
dissolved in water and the germanium was precipitated as GeS₂ with H₂S, filtered and mounted for counting. Chemical yields were usually greater than 90%.

Following distillation of the germanium activity, the remaining solution was extracted with isopropyl ether to remove the gallium activities into the ether phase as GaCl₄. The ether phase was washed several times with 6N HCl before the gallium activity was back-extracted with water. The water solution was then buffered to pH 7.0 with NH₃-NH₄Cl before addition of a few drops of saturated 8-hydroxyquinoline solution to precipitate the gallium activity as Ga(C₅H₆NO)₃; this was heated to expel water and mounted for counting. Chemical yields varied between 45 and 70%.

2. Radiation Assay

Ge⁶⁶ (2.4 hours) - A 7.6 cm x 7.6 cm NaI(Tl) detector was employed and the intensity of the 381.4-keV full energy peak was measured. Efficiency data due to Heath were employed. The relative gamma-ray intensity was taken as 334 ± 0.45 photons/disintegration. In some cases this nuclide was allowed to decay to its gallium daughter which was then assayed.

Ge⁶⁷ (19 minutes) - Allowed to decay to Ga⁶⁷ which was assayed.

Ge⁶⁸ (280 days) - The 511-keV annihilation radiation from the decay of 68 - minute Ga⁶⁸ in secular
equilibrium was assayed via a 7.6 cm x 7.6 cm NaI(Tl) detector. The anihilation radiation detection efficiency was calibrated with a Na$^{22}$ standard source of known disintegration rate which was calibrated by conventional$^9$. 511-511 keV gamma-ray coincidence measurements. Positron emission was taken to occur in 87.9$^%$ of the Ga$^{68}$ disintegrations. (see appendix IV)

Ga$^{66}$ (9.5 hour) - Anihilation radiation was assayed via 7.6 cm x 7.6 cm NaI(Tl) detectors. Positron emission was taken to occur in 56.54$%$ of the disintegrations$^{73}$. The decay of the nuclide was followed in some cases by means of beta proportional counters described elsewhere$^{65}$. 

Ga$^{67}$ (78 hour) - The 91.22-93.26 keV and 184.5-206 keV gamma-ray peaks were assayed via a 7.6 cm x 7.6 cm NaI(Tl) detector. The relative gamma-ray intensities were taken as 0.473 photons/disintegration and 0.274 photons/disintegration respectively$^{74}$. Representative gamma-ray spectra were inspected for impurity activities via Ge(Li) detectors described elsewhere$^{71}$. A series of measurements of the above specified kind spaced in time was taken for each sample and the resulting decay curves were subjected to least squares analysis via CLSQ$^{75}$, the Brookhaven computer program for radioactive decay analysis.
III Treatment of Data

A. Recoil Range Calculations

1. Influence of Particle Emission on Recoil Ranges

In a compound nucleus reaction, the energy $E_R$ imparted to the recoiling excited compound nucleus will be determined by the conservation of linear momentum in the target-projectile system.

\[ E_R = \frac{E_b A_b A_R}{(A_b + A_T)} \]

The subsequent emission of particles by the compound nucleus (symmetric about 90° to the beam direction in center of mass coordinates) will result in a distribution of recoil energies about an average value close to, but not equal to, that of the recoiling compound nucleus. Blann\textsuperscript{76,77} has calculated that the emission of a proton by a mass 59 compound nucleus will result in no more than a 10-15% decrease from the recoil energy of the compound nucleus.

In this work recoil ranges were corrected for effects of particle emission using the approximations of Blann\textsuperscript{76,77}, namely:

(1) The evaporation energy spectrum is replaced with a unique average energy.

(2) The evaporated particle angular distribution is replaced with 50% of the particles emitted forward and 50% emitted backward along the beam direction.
Detailed calculations by Blann\textsuperscript{76,77} have shown that these approximations produce in the recoil ranges of the product when $\alpha$-particles are emitted from $\text{Ni}^{59}$ a less than 1\% error and a less than 7\% error respectively.

In the present calculations, average ranges were corrected by a maximum of 16\% for emission of two nucleons. It was assumed that the average neutron and proton emission energies were 3.2 and 6.5 MeV respectively. In calculations of Blann\textsuperscript{77}, a factor of two change in the assumed average particle energies was found not to effect the calculated recoil velocities by more than 1.5\%.

2. Effects of Target Thickness

Projected ranges were calculated using the formalism described in Appendix I-1 (namely equation I-8) which takes into account the variation of the reaction cross-section within the target due to degradation of the beam energy. This correction never exceeded 35\% in the cases considered.

3. Scattering Corrections

The experimentally measured projected ranges were converted to total path lengths using the data of Lindhard, Scharff, and Schiøtt\textsuperscript{78}. (see Appendix I-2). This correction was normally approximately one third of the projected range. Effects due to scattering of recoils at the target-catcher foil interface were assumed to be small and were ignored.
B. Excitation Function Calculations

1. Cross Section Calculations

During the bombardment, the number of product nuclei, \( N \), present at time \( t \) is:

\[
N = \frac{R}{\lambda} (1 - e^{-\lambda t})
\]

where \( R \) is the rate of formation and \( \lambda \) is the decay constant of the product nuclei. The rate of formation of nuclei in a reaction with cross section \( \sigma \), induced by \( I \) particles per unit time impinging upon the target (of superficial atomic density \( N_T \)) is:

\[
R = \sigma N_T
\]

and if \( I \) is constant throughout the bombardment,

\[
\sigma = \frac{N}{N_T I (1 - e^{-\lambda t})}
\]

The quantity \( \sigma \) (which is often, as at present, that of interest) may be extracted if all the quantities on the right hand side of the equation are known or measured.

A complication may arise when one measures the number of nuclei of a particular species to determine the cross section for a particular reaction. For example, the product of the \( \text{Zn}^{64}(\alpha,n) \) reaction, \( \text{Ge}^{67} \), decays by positron emission with a 19-minute half life to an isotope, 77-hour \( \text{Ga}^{67} \) which is also the product of the \( \text{Zn}^{64}(\alpha,p) \) reaction. In this
case, the amount of Ga\(^{67}\) produced both during the bombardment and before chemical separation of the gallium fraction must be calculated and subtracted from the measured total to obtain the amount of Ga\(^{67}\) formed directly in the reaction of interest. This calculation, which is described in detail elsewhere\(^9\), was performed also for the Zn\(^{64}\)(a, pn)Ga\(^{66}\) reaction and the corresponding heavy ion cases.

If the beam current varies during the bombardment (as it did in the HILAC bombardments), \(R\) will not be constant. Under these conditions, the bombardment may be divided into smaller time intervals, \(\Delta t_i\), during which the rate of formation, \(R_i\), is approximately constant. Under these conditions:\(^9:\)

\[
N = \frac{1}{\lambda} \sum_{i=1}^{n} R_i (1 - e^{-\lambda \Delta t_i}) e^{-(t-t_i)}
\]

where \(t_i\) is the time at the end of the \(i^{th}\) interval.

2. Excitation Energy

In order to compare the excitation functions for various target projectile systems forming the compound nucleus Ge\(^{68}\), it is convenient to convert the bombarding energies into corresponding excitation energies of the compound nucleus.

\[
E^* = E_{cm} + E_{BE}
\]
where $E^*$ is the excitation energy of the compound nucleus, 
$E_{cm}$ is the center of mass energy of the target projectile system, and $E_{BE}$ is the binding energy of the projectile to the target. $c$ is the velocity of light.

The projectile, target, and compound nucleus masses ($A_b$, $A_T$, and $A_{cn}$ respectively) were obtained from the mass table of Mattauch, et al. 79.

3. Coulomb Barriers and Reaction Thresholds

Coulomb barrier energies, $V$, were calculated via the approximation of spherical nuclei in contact 80., i.e.

$$V = \frac{Z_b Z_T e^2}{R_b + R_T}$$

where $e$ is the charge of the electron and other symbols have their usual meaning with $b$ and $T$ designating the bombarding particle and target nucleus respectively.

Reaction threshold energies were calculated as differences between the compound nucleus and product nucleus binding energies as tabulated by Mattanch, et al. 81.

Because of the high coulomb barrier for the $^{12}O$ and $^{16}O$ reaction systems, all measurements were taken on these systems at bombarding energies far above the thresholds of the reactions studied (see table II).
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<th>System</th>
<th>V(MeV)</th>
<th>$E_{BE}$(MeV)</th>
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<td>Cr$^{52}$ + o$^{16}$</td>
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<td>Fe$^{56}$ + C$^{12}$</td>
<td>24.6</td>
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<td>Zn$^{64}$ + He$^{4}$</td>
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<table>
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<td>Ge$^{68}$ → Ga$^{67}$+p</td>
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<td>Ge$^{68}$ → Ga$^{67}$+p+n</td>
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<tr>
<td>Reaction Threshold (MeV)*</td>
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* All energies refer to excitation energies in Ge$^{68}$. 
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<tr>
<th>Compound Nucleus</th>
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<th>Binding energy</th>
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</tr>
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</table>
IV Experimental Results

A. Radiative Capture

The experimentally measured \( Zn^{64} (\alpha, \gamma) Ge^{68} \) excitation function is shown in figure 3. The previously measured data of Porile\(^50\) are shown on the same figure for comparison. The heavy lines were drawn to fit the data of the present work.

Above 17 MeV no \((\alpha, \gamma)\) cross-sections were measurable because of the interfering \( Ge^{68} \) activity produced via the \((\alpha, 2n)\) reaction on \( Zn^{66} \).

The four lowest energy points give a fairly good definition of the \( Zn^{64} (\alpha, \gamma) Ge^{68} \) excitation function. Differences between these results and those of Porile\(^50\) may be explained in terms of different energy resolution in the two sets of experiments. The measurements of Porile\(^50\) were achieved by degrading a full energy beam of \( 41.0 \pm 0.5 \) MeV to energy values in the region of interest. The present data was measured at energies resulting from degradation of a \( 25.9 \pm 0.1 \) MeV\(^63\) full energy beam. The maximum error in the energy values plotted for the present work is not expected to exceed 0.5 MeV, while the uncertainty in the energy values of Porile\(^50\) is expected to be many times this value. It can be seen from figure 5 that Porile's cross-section value at the highest energy he employed may have included the effects of interfering activity from the \( Zn^{66} (\alpha, 2n) Ge^{68} \) reaction, while his lowest energy cross-section may have extended into the energy region below the
coulomb barrier (as may the lowest energy cross-section in the present work).

The error bars on the cross-section magnitudes in this work are mainly a measure of the uncertainty in detector efficiencies during the assay of the various samples. Chemical yields were greater than 96% in most cases. The absolute uncertainty for any of the points shown in figure 3 is not expected to exceed 15% and the relative uncertainty of one value with respect to others is 5%.

Unfortunately, recoil ranges for the $\text{Zn}^{64}(\alpha,\gamma)\text{Ge}^{68}$ reaction were unmeasurable in these experiments. Long irradiation times were necessary for the production of useful activity of 280 day $\text{Ge}^{68}$, and only a small fraction of the $\text{Ge}^{68}$ recoils were found in the catcher foils as a result of low recoil energies.

Aside from known collective effects$^{82-88}$, the majority of radiative capture events are expected to proceed via a compound nucleus mechanism and complete momentum transfer. Also, the absence of either a proton or neutron excess in the bombarding particle would further hinder any possibility of direct capture$^{89}$.

The $\text{Cr}^{52}(^{16}\text{O},\gamma)\text{Ge}^{68}$ and $\text{Fe}^{56}(^{12}\text{C},\gamma)\text{Ge}^{68}$ excitation functions were unmeasurable in the present work.
B. Reactions Involving the Emission of One Nucleon by the Compound Nucleus

1. Zn$^{64}$ (α,n) Ge$^{67}$

The Zn$^{64}$ (α,n) Ge$^{67}$ excitation function from the present study is shown in figure 4, together with the data of Porile$^{50}$. The heavy line was drawn to fit the data of the present work.

Although the positions in energy of the excitation function peaks are in reasonable agreement for the two sets of measurements, the low and high energy ends differ considerably. Above a bombarding energy of 30.8 MeV, interference from the Zn$^{66}$ (α,3n) Ge$^{67}$ reaction is possible and may be present in the two highest energy points measured in this work.

The error bars on the cross-section magnitudes in figure 4 are due mainly to a combination of uncertainties from radioactivity assay statistics and detector efficiencies as well as uncertainties in chemical yields. The uncertainty in the magnitude of any of the cross-sections does not exceed 20% and the relative uncertainty of the points is approximately 7%.

Ranges for the product of this reaction were experimentally unmeasurable directly in the present work, because of the low activities in the catcher foils. However, the long duration of bombardment for the production of Ga$^{67}$ by the (α,p) reaction resulted in a substantial
amount of Ga\textsuperscript{67} formed indirectly (through the (\alpha, n) reaction followed by \beta^+ decay), so that the ranges measured by determining the fraction of Ga\textsuperscript{67} in the catcher foils are actually combined ranges for the products of the (\alpha, n) and (\alpha, p) reaction. These ranges are presented and discussed in the following section.

2. Zn\textsuperscript{64} (\alpha, p) Ga\textsuperscript{67}

The Zn\textsuperscript{64} (\alpha, p) Ga\textsuperscript{67} excitation function for the present work is shown in figure 5 together with the data of Porile\textsuperscript{50}. It can be seen that agreement between the two sets of data is within experimental error throughout the entire energy range of the measurements.

Sources of error in the present work include uncertainties in the chemical yields, beam currents, target thickness, incident and degraded particle energy, and radioactivity assay statistics and efficiencies. The latter uncertainties account for most of the magnitude of the error bars on the cross-sections of figure 5. These error bars represent less than 20\% uncertainties in all cases. The relative uncertainties are less than 10\%.

The combined Zn\textsuperscript{64} (\alpha, n) Ge\textsuperscript{67} and Zn\textsuperscript{64} (\alpha, p) Ga\textsuperscript{67} ranges are also shown in figure 5. Within experimental error, the ranges are directly proportional to bombarding energy. The ranges at the highest energy have the largest uncertainty and may contain an appreciable direct interaction component.
In figure 6, the experimental ranges are shown plotted together with the range-energy curve calculated by means of the formalism of Lindhard, Scharff, and Schiøtt (see Appendix I). Agreement between the data and theory is within experimental error.

On the basis of the analysis of the range data for the mixed \((\alpha,n)\) and \((\alpha,p)\) products, it may be concluded that both of these reactions proceed primarily via the compound nucleus mechanism.

3. \(\text{Fe}^{56} (\text{C}^{12},p) \text{Ga}^{67}\) and \(\text{Fe}^{56} (\text{C}^{12},n) \text{Ge}^{67}\)

Only the sum of the cross-sections for these two reactions was measurable in the present work. The cross-sections obtained are shown in figure 7 plotted as a function of the Ge\(^{68}\) excitation energy together with the sum of the experimentally measured Zn\(^{64}\) \((\alpha,p) \text{Ga}^{67}\) and Zn\(^{64}\) \((\alpha,n) \text{Ge}^{67}\) cross-sections.

Errors in the experimentally measured cross-sections arise from the same sources discussed in the previous section, but in the present work, uncertainties in the beam energy and small non-constant beam currents provide the major sources of error. The uncertainties in the cross-section magnitudes are about 60%, and the energy uncertainty is about ±1.5 MeV.

The cross-sections for the \(\text{C}^{12}\) induced reactions are seen to be in agreement within experimental error with an extrapolation of the \(\alpha\)-induced excitation function.
This agreement is not unexpected, since the compound nucleus angular momenta resulting from the two reaction systems are calculated to be very nearly equal in this energy region. In figure 8, the average Ge$^{68}$ angular momentum as calculated with ABACUS-2*23,90. is plotted vs. excitation energy for the three reaction systems: Zn$^{64}$ + He$^4$, Fe$^{56}$ + C$^{12}$, and Cr$^{52}$ + O$^{16}$. At the two bombarding energies studied for the C$^{12}$ case, the average compound nucleus angular momentum is expected to differ by less than 2$\hbar$ from the corresponding alpha cases. It is not surprising, therefore, that behavior similar to that first observed by Ghoshal$^{91}$ is seen for these two reaction systems.

No ranges were measured, but it may be assumed that C$^{12}$-induced reactions resulting in products close to the mass of the compound nucleus proceed predominantly by a compound nucleus mechanism, since a direct mechanism whereby the incident C$^{12}$ nucleus imparts the majority of its momentum to one or two nucleons of the target nucleus, is quite improbable$^{92}$.

4. Cr$^{52}$ ($^{16}$O,n) Ge$^{67}$ and Cr$^{52}$ ($^{16}$O,p) Ga$^{67}$

Upper limits determined for the magnitude of the sum of the Cr$^{52}$ ($^{16}$O,n) Ge$^{67}$ and Cr$^{52}$ ($^{16}$O,p) Ga$^{67}$ cross-sections are shown plotted as a function of Ge$^{68}$ excitation energy in figure 7. The lower energy cross-section was measured at an energy below the coulomb barrier and the low value indicated by the experimental limit is
expected. The higher energy cross-section is seen to be below
the value one would expect by extrapolating the $\alpha$ and $C^{12}$
excitation functions to this energy. This apparent violation
of the independence hypothesis is expected, however, since
the average angular momentum of the compound nucleus formed
by $O^{16}$ bombardment of $Cr^{52}$ is lower at this energy than
either the $\alpha$ or $C^{12}$ induced average angular momentum
(see figure 8). Excitation functions for systems with
higher angular momenta are expected to be shifted to higher
energies, and this is seen to be consistent with the present
experimental results.
C. Reactions Involving the Emission of Two Nucleons by the Compound Nucleus.

1. Zn\(^{64}\)(a,2n)Ge\(^{66}\)

The cross-sections measured for the Zn\(^{64}\)(a,2n)Ge\(^{66}\) reaction are shown in figure 9, plotted together with the data of Porile\(^{50}\)\(^{9}\). The magnitudes of the latter cross-sections were adjusted by a factor corresponding to the replacement of the rather approximate \(\text{Ga}^{66}\) positron branching ratio, \(\approx 66\%\)\(^{93}\), used by Porile in his \(\text{Ga}^{66}\) assay, with the more accurate value \(56.54\%\)\(^{73}\), now available. The solid line in figure 11, was drawn to fit the present data.

The main discrepancy between the two sets of data exists at energies between the threshold and the peak of the excitation function. The present data are expected to be correct to within \(\pm 20\%\) in cross-section magnitude and within \(\pm 0.8\) MeV in energy. The relative accuracy of the cross-sections is expected to be better than \(\pm 10\%\).

The recoil ranges measured for the product of this reaction are also shown in figure 9, plotted on a linear ordinate scale vs. bombarding energy.

Figure 10, shows the experimental and calculated recoil ranges plotted against the calculated average recoil energy for the compound nucleus. The solid line was calculated using the formalism of Lindhard, Scharff, and Schiött\(^{78}\)\(^{7}\). The experimentally measured projected ranges agree well within experimental error with the calculated projected ranges.

This analysis provides strong evidence that the Zn\(^{64}\)(a,2n)Ge\(^{66}\) reaction proceeds predominantly via the compound nucleus mechanism.
The cross-sections measured for the $\text{Zn}^{64}(\alpha, \text{pn})\text{Ga}^{66}$ reaction are shown in figure 11 plotted together with the data of Porile$^{63}$, which were again adjusted to correct for the calibration error as described in the previous section. The solid line was drawn to fit the present data.

The discrepancy between the two sets of data appears to be a displacement along the energy axis. The present data consist of a combination of data measured via bombardments at two different cyclotrons (Brookhaven, and University of Washington). The Brookhaven experiments also resulted in points on the $\text{Zn}^{64}(\alpha, \text{p})\text{Ga}^{67}$ excitation function which agreed with the data of Porile$^{50}$ (see section IV-B-4). The present data are expected to be accurate to within $\pm 20\%$ in cross-section magnitude and within $\pm 0.8\text{MeV}$ in energy. The relative accuracy of the cross-sections is expected to be better than $\pm 10\%$.

The recoil ranges measured for the product of this reaction are also shown in figure 11. plotted vs. bombarding energy on a linear ordinate scale. The two highest energy ranges appear to break from the linearity established by the lower energy points.

Figure 12 shows the recoil ranges plotted against the calculated average recoil energy of the compound nucleus. The solid line was calculated using the formalism of Lindhard, Schaff, and Schiøtt.$^{78}$ The higher energy disparity between the calculated ranges and experimental ranges is quite pronounced in this graph.
On the basis of this analysis, it may be deduced that the Zn$^{64}$(α,pn)Ga$^{66}$ reaction proceeds predominantly via the compound nucleus mechanism at lower energies (<32MeV). At higher energies, the lower ranges would indicate the onset of a noncomplete momentum transfer process (although this assumption is based essentially on the validity of one range measurement.)

3. Fe$^{56}$(C$^{12}$,2n)Ge$^{66}$ and Fe$^{56}$(C$^{12}$,pn)Ga$^{66}$

The cross-sections measured for the Fe$^{56}$(C$^{12}$,2n)Ge$^{66}$ and Fe$^{56}$(C$^{12}$,pn)Ga$^{66}$ reactions are shown plotted together as a function of C$^{12}$ bombarding energy in figure 13., the dotted and solid lines were drawn to fit the cross-sections for the respective reactions.

The Fe$^{56}$(C$^{12}$,2n)Ge$^{66}$ and Zn$^{64}$(α,2n)Ge$^{66}$ cross-sections are plotted as a function of Ge$^{68}$ excitation energy in figure 14., with the dotted and solid lines drawn to fit the two respective excitation functions.

The lowest energy (C$^{12}$,2n) cross-section is seen to be in agreement with the (α,2n) excitation function, but, unfortunately, the higher energy portion of the (α,2n) excitation function is not sufficiently well-defined to enable a meaningful comparison with the higher energy (C$^{12}$,2n) cross-sections.

The Fe$^{56}$(C$^{12}$,pn)Ga$^{66}$ and Zn$^{64}$(α,pn)Ga$^{66}$ cross-sections are shown plotted vs. Ge$^{68}$ excitation energy in figure 15. Although the high energy portion of the (α,pn) excitation function is more well-defined experimentally in this case, some uncertainty arises
from the apparent presence of a non-compound nucleus reaction component as discussed in section IV-C-2. The triangles of figure 15. represent an upper limit to the compound nucleus contribution to the cross-section. These upper limits were determined using the approximation that all catcher activity resulted from compound nuclear processes; target foil activities (and the corresponding cross-sections) were calculated with equation I-5 (Appendix I) using the range values calculated by means of the formalism of Lindhard, Scharff, and Schiøtt\(^{78}\). Again, the lowest energy heavy ion cross-section is seen to be in agreement with the \((a, pn)\) excitation function as defined by either the measured or corrected cross-sections, but little can be inferred about agreement with the higher energy heavy ion cross-sections.

A comparison of the alpha- and \(^{12}C\)-induced results in the form of cross-section ratios is deferred to section V-C-2 to follow.

4. \(Cr^{52}(0^{16}, 2n)Ge^{66}\) and \(Cr^{52}(0^{16}, pn)Ga^{66}\).

Only upper limits were measurable for the \(Cr^{52}(0^{16}, 2n)Ge^{66}\) and \(Cr^{52}(0^{16}, pn)Ga^{66}\) reactions. These limits are plotted in figures 14. and 15., respectively. The decreased magnitude of these cross-sections when compared to the corresponding \(^{12}C\)- and \(a\)-induced excitation functions may be explained by arguments analogous to those used in section IV-B-4.
V. Statistical Model Calculations

A. Formalism

The formalism used for the calculation of reaction cross-sections is based on the statistical model theory discussed in sections I-B and I-C.

1. Particle Emission

The probability for transitions between nuclear states was calculated via equation (c-6), namely:

\[ P(E_F, J_F; E_C, J_C) \, \mathrm{d}e = \frac{(2s+1)}{\pi^2 \lambda^3} \, \epsilon \mu \sigma (\epsilon, J_C, J_F) \, \frac{\omega(E_F, J_F)}{\omega(E_C, J_C)} \, \mathrm{d}e \]

Where

\[ \omega(E, J) = \sqrt{\frac{2}{48}} \, a^3 \, \lambda^3 \, (E + \frac{3}{2} \epsilon)^{-2}(2J+1)\exp \left[ -2(aE)^{\frac{1}{3}} - \frac{\epsilon^2 J(J+1)}{21t} \right] \]

and

\[ \sigma(\epsilon, J_C, J_F) = \frac{\pi}{(2s+1)(2J_F+1)} \sum_{S=|J_F-S_X|}^{J_F+S_X} \sum_{t=|J_C-S|}^{J_C+S} T_t(\epsilon) \]

For non-interacting free nucleons confined within a nucleus of radius \( R = r_o A^{\frac{1}{3}} \), the Fermi gas model predicts the level density parameter to be the following:

\[ a = 2\left(\frac{\pi}{3}\right)^{\frac{4}{3}} \frac{m_r^2}{h^2} \, A \]

Experimental evidence, as interpreted by Lang\(^{94}\), indicates the value \( a = \frac{A}{8.0} \, \text{MeV}^{-1} \) (corresponding to a radius parameter \( r_o = 1.15 \, \text{fm} \)) which was used in this work.

The energy used in the level density expression was corrected for nucleon pairing with the pairing energies of Cameron\(^{19}\). Since Ge\(^{68}\) lies far from shell closure, no
shell correction was used.

For each excitation energy, $E^*$, a maximum value of the angular momentum, $J_{\text{max}}$, is expected\textsuperscript{95} to exist above which the level density is zero. If the excitation energy is partitioned into thermal and rotational energies and the rotational energy is taken to be\textsuperscript{96,33},

$$E_{\text{rot}} = \frac{J(J+1)\hbar^2}{2I}$$

it follows that

$$E^* = \frac{J_{\text{max}}(J_{\text{max}}+1)\hbar^2}{2I}$$

For a rigid spherical body, the moment of inertia, $I_r$, is given by\textsuperscript{29},

$$I_r = \frac{2}{5} AR^2$$

For a nucleus with a constant pairing energy between particles in doubly occupied pair states (see section I-C-3) the moment of inertia is expected to be considerably less than the rigid body value for energies less than a neutron binding energy.\textsuperscript{40,42,44} For energies greater than a neutron binding energy, the rigid body moment of inertia is expected to be a good approximation to the nuclear moment of inertia.\textsuperscript{40,42}

To reflect these expectations, the moment of inertia was taken to vary between some value at zero energy ($I_0$), and the rigid body value ($I_r$) at higher energies as follows:

$$I = I_r (1 - \text{be}^{-0.693E/c})$$

where

$$I_r (1 - b) = I_0$$
It was found that the magnitude of the assumed value for $I_0$ (which determines $b$) could be changed from $0.04 I_T$ to $0.4 I_T$ with an effect of only a few per cent on the calculated cross-sections. The value $I_0 = 0.04 I_T$ or $b = 0.96$ was used throughout the calculations. The calculation results were found, however, to be quite sensitive to the value chosen for $c$, which controls the rate of approach to rigidity with energy; this was taken as 3.0 MeV, resulting in a moment of inertia equal to 93% of the rigid body value at the neutron binding energy for $^{68}$Ge (see figure 16.)

Below a certain energy (which was also taken as 3 MeV, but was unrelated to the above choice), the level density becomes unrealistically large when compared with experiment$^{97,98}$, and was replaced with an extrapolation to zero energy, namely:

$$w'(E,J) = \left( \frac{E}{3.0} \right)^2 w(E=3.0,J) + \left( \frac{3.0-E}{3.0} \right) w_0$$

Here $w_0$ is the "level density" at zero energy which was chosen from the trends of experimentally measured level densities.$^{97,98}$

The optical model parameters (see equations c-1, c-1-a, and c-1-b) used in calculating transmission coefficients are summarized in Table III. Parameters for p-, n-, and α-penetration were chosen for optimum agreement with the tabulations of Hodgson$^{99}$. The parameters for the $^{12}$C and $^{16}$O cases are those suggested by Vogt.$^{100}$
<table>
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<th>$V$</th>
<th>$W$</th>
<th>$a, b$</th>
<th>$r_0$</th>
<th>$V_{so}$</th>
<th>$W_{so}$</th>
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<td>-11 MeV</td>
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<td>1.10f</td>
<td>0</td>
<td>0</td>
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<tr>
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<td>-100 MeV</td>
<td>-10 MeV</td>
<td>0.6f</td>
<td>1.10f</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Cr$^{52}$ + O$^{16}$ → Ge$^{68}$</td>
<td>-100 MeV</td>
<td>-10 MeV</td>
<td>0.6f</td>
<td>1.10f</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Ge$^{68}$ → Ge$^{67}$ + n</td>
<td>-52.5 MeV + 0.6E</td>
<td>-2.5 - 0.3E</td>
<td>0.6f</td>
<td>1.10f</td>
<td>-9.5 MeV</td>
<td>0</td>
</tr>
<tr>
<td>Ge$^{68}$ → Ga$^{67}$ + p</td>
<td>-55.0 MeV + 0.7E</td>
<td>-0.50 - 0.45E</td>
<td>0.65f</td>
<td>1.10f</td>
<td>-9.5 MeV</td>
<td>0</td>
</tr>
</tbody>
</table>

* See equation c-1, c-1(a), and c-1(b) for the meaning of the symbols.
All transmission coefficients were calculated with ABACUS - 2*, the optical model program of Auerbach as revised by Donnelly for use on the University of British Columbia IBM 7040 computer. It has been found that the coulomb wave functions used in ABACUS - 2 may not give proper convergence in energy regions near coulomb barriers or at low excitation energies. The revisions in ABACUS - 2* include changes in the coulomb wave functions, however, and proper convergence is expected with these changes.

2. Gamma Ray Emission

The probabilities for gamma ray emission were calculated using the formalism derived from the single particle model as follows:

\[ P_\gamma (E_F, J_F; E_C, J_C) = C_1 (\varepsilon_\gamma) \frac{w(E_F, J_F)}{w(E_C, J_C)} \varepsilon_\gamma^{2i+1} \]

(A-1)

where \( i \) corresponds to the multipolarity of the emitted gamma ray of energy

\[ \varepsilon_\gamma = E_C - E_F \]

The factors \( C_1 (\varepsilon_\gamma) \) (which have been assumed to be constants by previous authors) were altered from the single particle estimates to take into account experimentally observed collective effects not predicted by the single particle model.
Dipole gamma rays show in all nuclei a broad absorption resonance of 4 - 10 MeV full width at half maximum peaked at about 20 MeV. This so-called giant dipole resonance was taken to be of the Breit-Wigner form as follows:

\[ C_i(\varepsilon) = C_i'f(\varepsilon) \quad (i=1) \]

with

\[ f(\varepsilon) = \frac{E_d^2 + \frac{1}{4}\Gamma^2}{(\varepsilon - E_d)^2 + \frac{1}{4}\Gamma^2} \]

where \( E_d \) is the energy at the peak of the resonance and \( \Gamma \) is the full energy width at half maximum. The factor \( C_i' \) (\( i=1 \)) is taken to be a constant the magnitude of which is determined by empirical fitting of the calculation results to the measured \((\alpha,\gamma)\) excitation function. The hydrodynamic model predicts that the energy of the resonance peak will vary as \( A^{-\frac{1}{3}} \), and the following form is consistent with experimental results, giving for \( Ge^{68} \) the value 20 MeV. The resonance width was chosen, consistent with experimental evidence, to be 5 MeV. It has been found that calculated cross-sections are "relatively insensitive" to variations of 1 MeV in the resonance peak energy and of 2 MeV in the resonance width.

Other gamma-ray resonances are known to exist, but not enough experimental characterization is available.
to warrant their inclusion in the present calculations.

Many experimental electric quadrupole (E2-) transition intensities are found to be strongly enhanced over single particle estimates.\(^{29}\). According to the single-particle model alone, one would expect the dipole gamma-ray emission rate to be \(10^2\) to \(10^3\) times faster than the quadrupole gamma-ray emission rate,\(^{102, 103}\) but compilations of experimentally known reduced gamma-ray emission rates\(^{108}\) show many E2 emission rates are \(10^2\) to \(10^3\) times faster than single-particle estimates. This effect has been explained\(^{86}\) in terms of rapid de-excitation through intra-rotational band cascades. An attempt was made in some of the present calculations to reflect this behavior by introducing an enhancement factor into the quadrupole term for gamma emission as follows:\(^{29}\).

\[
C_i(\epsilon_\gamma) = C_i' \frac{J_c(J_c+1)(J_c-2)}{2(2J_c-1)} \left[ \frac{\Delta E(J_c-J_c-2)}{\Delta E(J_c-J_c-1)} \right]^{5}
\]

where \(C_i'\) is again taken to be a constant. The quantities \(\Delta E\) correspond to level spacings at the excitation energy \(E^*\). The above expression was not allowed to exceed

\[
C_i(\epsilon_\gamma) = C_i' Z^2
\]

the theoretical upper limit.\(^{88}\).

For quadrupole transitions not involving a change of two units of angular momentum, no enhancement factor was used, i.e.

\[
C_i(\epsilon_\gamma) = C_i'
\]
Since the present calculations did not include parity considerations, no distinction could be drawn between electric quadrupole and magnetic quadrupole transitions. The latter are not expected to be enhanced.

Although the above form for the enhancement factor has some theoretical grounds, it was used only as a rough attempt to reflect experimentally observed effects, and is in no way to be considered a quantitative treatment of the problem.

The factors \( C_i(\epsilon, \gamma) \) for \( i = 3 \) and \( i = 4 \) were taken to be constants. Higher multipoles than that corresponding to \( i = 4 \) were not considered. The constants \( C_i' \) were chosen to reflect the single-particle estimates for each multipolarity and, since the single particle model predicts smaller contributions from higher multipoles, these constants were rather arbitrarily taken to decrease by factors of ten for each unit increase in \( i \). The magnitude of these constants was set by fitting of the calculated value of the \((\alpha, \gamma)\) cross-section to the experimentally measured value.
B. SFUSMAP - Program Logic

The calculation was performed via direct calculation and summation of the various transition probabilities, according to a procedure similar to that of Blann.109.

The initial emitting system was taken to be a compound nucleus with a unique excitation energy 
\( E^* = E_{cm} + E_{BE} \) and a distribution in angular momentum, \( \sigma(Z,A,E^*,J) \), characteristic of the reaction through which it was formed.

The possible modes of de-excitation considered were gamma, alpha, proton, and neutron emission. Arrays dimensioned in residual energy and angular momentum were considered for the product nuclei formed by these respective modes of de-excitation, namely \( \sigma(Z,A,E,J) \), populated by gamma emission by the compound nucleus \( (Z,A) \); \( \sigma(Z-2,A-4,E',J') \), populated by alpha emission; \( \sigma(Z-1,A-1,E'',J'') \), populated by proton emission; and \( \sigma(Z,A-1,E''',J''') \), populated by neutron emission. To conserve computer space these arrays were divided into bins \( 2\hbar \) units wide in angular momentum and 2 MeV wide in energy. Reducing the energy width to 1 MeV was found to effect the outcome of sample calculations by less than 5%.

The initial angular momentum population of the compound nucleus \( (\sigma(Z,A,E^*,J)) \) was calculated via ABACUS - 2*90. (see preceding section) and read in as input data to the program. The relative probabilities for population of bins in the arrays for the several daughter
nuclei were calculated for each angular momentum value of the compound nucleus, multiplied in turn by the capture cross-section leading to each particular compound nucleus angular momentum value, and summed to give the total population cross-section of the daughter arrays by the decay of the compound nucleus. Thus, for neutron emission,

$$\sigma(Z,A-1,E',J') = \sum_{J} \sigma(Z,A,E',J) \frac{P_n(E^*,J;E',J')}{\sum_{E} \sum_{J'} P_1(E,J;E',J')}$$

The populations $\sigma(Z,A-2,E,J)$, $\sigma(Z-1,A-1,E,J)$, and $\sigma(Z-1,A-2,E,J)$ are then considered in order.

The formation cross section for a particular nucleus is taken as the sum of the bins in the array populated by gamma emission in that nucleus. This approximation will overestimate the formation cross-section slightly, since some states populated by gamma emission may still emit particles; however, the contribution of these states to the total cross-section will be small (<0.1%) as a result of the fact that gamma emission is normally several orders of magnitude less probable than particle emission when the latter is energetically allowed.

Thus, the cross-section for formation of the product nucleus resulting from emission of two neutrons by the compound nucleus is,

$$\sigma(x,2n) = \sum_{E'} \sum_{J'} \left[ \sum_{E} \sum_{J} \sigma(Z,A-2,E,J) \frac{P_\gamma(E,J;E',J')}{\sum_{i=\gamma,\alpha,p,n} \sum_{E} \sum_{J'} P_i(E,J;E',J')} \right]$$
The de-excitation process is only considered up to and including emission of three particles. This limitation is a result of lack of computer space only, and the program may easily be expanded to include the consideration of further particle evaporation.

The output of each set of calculations contains the population distribution for each nucleus considered both before and after emission of one gamma-ray, as well as the final formation cross-sections. A more detailed description of the computer program is given in Appendix V.
C. Calculation Results

1. General Features of Calculated Results

Sample results for residual nuclei resulting from particle emission are shown in figures 17, 18, and 19. The calculated quantities $\sigma(Z,A,E,J)$ are shown plotted vs. $E$ and $J$ in the form of contour diagrams, with successive contours representing factors of ten in cross-section magnitude. The emitting system in all three cases is Ge$^{68}$ excited to 40 MeV and with 20\hbar units of angular momentum. The vertical dotted line represents the angular momentum of the emitting compound nucleus, and the horizontal dotted line represents the excitation energy of the emitting nucleus minus the binding energy of the emitted particle. The region of no states corresponds to $J$ values greater than $J_{\text{max}}$ as defined in section V-A.

In figures 17, and 18, representing the results of neutron and proton emission respectively, the average change in angular momentum is approximately 2.5\hbar in the direction of lower angular momentum. This decrease in angular momentum is a result of the greater availability of levels at $J$ values lower than 20\hbar as compared to the availability at higher $J$ values.

Figure 19, representing the results of alpha particle emission, shows an average decrease in angular momentum of 6\hbar. Furthermore, the average change in angular momentum is seen to increase sharply with increasing alpha energy. This effect is a consequence of the fact that...
higher energy alpha particles are able to carry off (or carry in) many more units of angular momentum (cf. figure 8). This same effect is also seen for neutrons and protons, but to a much lesser extent because of the smaller masses of these particles. In the region bordering the rotational cutoff \( J = J_{\text{max}} \), these trends are quite important.

The star in figure 20 represents a state in a compound nucleus and the vertical components of the arrows correspond in length to the binding energy of an emitted particle. For purposes of illustration, states in the daughter nucleus are represented on the same diagram. The horizontal and curved dotted lines represent a displacement of one binding energy from zero energy and the rotational cutoff, respectively. It is seen that for the compound nucleus in a state represented by the star, particle emissions with a zero or positive change in the nuclear angular momentum are inhibited (dotted arrow), but that emissions resulting in negative charges in the nuclear angular momentum are allowed (full arrow). Unless the change in angular momentum is very large indeed, the energy of the emitted particle will, of necessity, be low. For low energy particles, transmission coefficients for higher \( z \) values are very small, so that the corresponding emission probability will be small. Under these conditions, gamma ray emission is expected\(^{109,104}\) to compete favorably with particle emission.
Figures 21 and 22 illustrate a particular case where population of states near the rotational cutoff is an important consideration. In the upper right hand corner of the diagram, the cross-section (in millibarns) for population of a given compound nucleus angular momentum value during bombardment is shown plotted against angular momentum (in units of \( \hbar \)). The cross-section distribution is seen to be peaked at higher \( J \) values for the Fe\(^{56}\) + \( \text{C}^{12} \) case (figure 21) than for the corresponding Zn\(^{64}\) + He\(^4\) case (figure 22). The contour diagrams (which are identical to those described previously, except that now the contours represent the sum of contributions from each angular momentum value along the accompanying \( \sigma_J \) distribution in the emitting compound nucleus) reflect this angular momentum difference even after emission of two nucleons. It can be seen, particularly in the Ge\(^{66}\) and Ga\(^{66}\) product nuclei, that the population of states along the rotational cutoff is much more dense in the Fe\(^{56}\) + \( \text{C}^{12} \) case. Since, therefore, these two product nuclei have a much higher probability for gamma ray emission in the Fe\(^{56}\) + \( \text{C}^{12} \) case, one would expect the Fe\(^{56}\)(\( \text{C}^{12}\),pn) and Fe\(^{56}\)(\( \text{C}^{12}\),2n) reaction cross-sections to be larger at this energy than the corresponding Zn\(^{64}\)(\( \alpha\),pn) and Zn\(^{64}\)(\( \alpha\),2n) cross-sections which will be depleted by particle emission. For the same reason, reaction thresholds for emission of three nucleons should be shifted to higher energies for the higher angular momentum case. Corresponding shifts to higher energies are expected for excitation functions corresponding to reactions involving evaporation of one particle by the compound nucleus.
An even more pronounced case of highly populated states in the vicinity of the rotational cutoff is seen in figure 23 for the system $Cr^{52} + O^{16}$ at $64\text{MeV}$ excitation.

The effects of gamma ray competition with particle emission are well illustrated by the calculated excitation functions shown in figure 24.

The differences in slope between the two excitation functions below $40\text{MeV}$ are due mainly to differences in the total reaction cross-section. Above $40\text{MeV}$, however, where both total reaction cross-sections change slowly with energy, the slope of the $Fe^{56}(C^{12}, pn)Ga^{66}$ excitation function increases in relation to the $Zn^{64}(\alpha, pn)Ga^{66}$ excitation function; this is due to effects such as that just described following the rapidly increasing average angular momentum of the $C^{12} + Fe^{56}$ system as compared to the $He^{4} + Zn^{64}$ system (see figure 8).

2. Comparisons of Calculations with Experiment.

The calculated $Zn^{64}(\alpha, \gamma)Ge^{68}$ excitation function is shown in figure 25 together with the present experimental data. The magnitude of the calculated excitation function was set by adjustment of the gamma strength constants ($C_{i}'$) in formula V-A-1. The apparent discrepancy in energy is probably due to inaccuracy in the values used for particle binding energies\(^{81}\) which are subject to large uncertainties.\(^{20,81,110}\) The magnitudes of the gamma strength constants obtained for $Ge^{68}$ from this calculation were assumed to be identical for neighboring nuclei.
The calculated $^{64}\text{Zn}(\alpha,n)^{67}\text{Ge}$ excitation function is shown in figure 26 plotted together with the experimental cross-sections. Agreement is seen to be quite good above 20 MeV, whereas agreement is poor for lower energy values. This disparity is probably due to inaccuracy in the neutron binding energy used and concomittant inaccuracy of the reaction threshold.

The $^{64}\text{Zn}(\alpha,p)^{67}\text{Ga}$ excitation function is shown in figure 27. Although the cross-section magnitudes are generally lower than those measured, the excitation function shape is seen to reflect the shape of the experimental excitation function quite well.

The $^{64}\text{Zn}(\alpha,2n)^{66}\text{Ge}$ excitation function is shown in figure 28. Again there appears to be an energy shift between the experimental and calculated excitation functions which may probably be attributed to inaccurate binding energies. Otherwise, both the shape and the magnitude of the calculated excitation function reflect those of the experimental excitation function quite well.

The calculated $^{64}\text{Zn}(\alpha,pn)^{66}\text{Ga}$ excitation function is shown in figure 29. The calculated excitation function shape is seen to closely reproduce that of the experimentally measured one, but, as in the $(\alpha,p)$ case, the calculated cross-section magnitudes are low. This suggests possible inaccuracies in the calculated proton transmission
coefficients, although the disparity may be due to other causes.

The calculated excitation function for the sum of the \( \text{Fe}^{56}(\text{C}^{12},n)\text{Ge}^{67} \) and \( \text{Fe}^{56}(\text{C}^{12},p)\text{Ga}^{67} \) excitation functions are shown in figure 30 plotted together with the experimental sum cross-sections. There is a more serious lack of agreement in this case.

The calculated \( \text{Fe}^{56}(\text{C}^{12},2n)\text{Ge}^{66} \) excitation function is shown in figure 31. Agreement is seen to be good between the calculated excitation function and the two higher energy cross-section values. The calculation is seen not to be in agreement with the lowest energy cross-section value. This lack of agreement is very likely due to inaccuracy of the coulomb barrier as calculated by ABACUS - 2*.

The rigid sphere in contact approximation (see section III-B) predicts a much lower coulomb barrier energy, and this prediction appears to be in better agreement with experiment.

The calculated \( \text{Fe}^{56}(\text{C}^{12},pn)\text{Ga}^{66} \) excitation function is shown in figure 32. Again, agreement with experiment is good for the two highest energy points, but poor for the lowest energy point. Again, the inaccurate coulomb barrier probably accounts for this discrepancy.

Some ambiguity arises when comparing cross-section magnitudes because of the different probabilities of forming the compound nucleus for different target projectile systems. This ambiguity is removed when one considers reaction cross-section ratios, however.
Figure 33 shows the calculated and experimental ratios of the Zn$^{64}$($\alpha$,pn)Ga$^{66}$ cross-section to the Zn$^{64}$(α,2n)Ge$^{66}$ cross-section as well as the (C$^{12}$,pn)/(C$^{12}$,2n) ratios plotted vs. excitation energy. The discrepancy between the calculated ($\alpha$,pn)/(α,2n) ratio and experiment is probably to be attributed to binding energy errors. At higher energies the calculated and experimental ratios are seen to merge within experimental error. The calculated and experimental (C$^{12}$,pn)/(C$^{12}$,2n) ratios are seen to agree well within experimental error.

It is interesting to note that the experimental ratios form a continuous curve for the two systems. The coincidence of the two sets of data in the region from 36 - 40MeV where the angular momenta of the two systems is very closely matched (cf. figure 8) provides strong evidence that the predominant mechanism for both of these reactions is compound nucleus formation and decay and is a convincing verification of the independence hypothesis. Since any direct interaction component would not be expected to be of the same magnitude for two such differing target-projectile systems, it must be concluded that the highest energy range measurement for the Zn$^{64}$(α,pn)Ga$^{66}$ reaction (see figure 11.) is probably in error.
VI. Conclusions

It may be stated, on the basis of this work, that, contrary to previous observations, the compound nucleus model provides a satisfactory account of the reactions of alpha particles with $\text{Zn}^{64}$. This conclusion is based on experimental recoil range evidence as well as the agreement of calculated excitation functions with experiment. The "high energy tails" of the measured excitation functions may be explained in terms of angular momentum effects and gamma ray competition with particle emission, rather than by assuming large direct interaction contributions to the reaction mechanism. Furthermore, all calculation parameters employed were based on available independent experimental evidence, rather than adjusted to achieve best agreement between calculation and experiment as has been done by other workers.$^{34,44,63,66}$

The main source of difficulty in performing calculations of excitation functions was found, in the present work, to be lack of reliable optical model parameters as well as uncertainty in particle binding energies.

The outcome of the $\text{Fe}^{56}+\text{C}^{12}$ calculations seems encouraging in light of the fact that one would expect the approximations inherent in the optical model to be an oversimplification for such a complex target-projectile system. The success of the statistical model on this reaction system would suggest further such experiments for investigation of angular momentum effects (see Appendix VI).
The treatment of gamma ray emission used in this work is in obvious need of refinement. The single particle estimates are inadequate, and the nature of collective enhancement of gamma ray emission probabilities is still a largely unexplored field. The rather empirical approach used in this work was seen to be adequate only to a first approximation.

In conclusion, it may be stated that, provided one has a reasonably accurate knowledge of the constants of motion of a compound nucleus formed by a given target-projectile system, the decay of this compound nucleus may be accurately described by the statistical model.
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<th>$\sigma(\alpha,\gamma)$</th>
<th>$\sigma(\alpha,p)$</th>
<th>$\sigma(\alpha,n)$</th>
<th>$\sigma(\alpha,pn)$</th>
<th>$\sigma(\alpha,2n)$</th>
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<th>$E_a$ (MeV)</th>
<th>$\sigma(\alpha,\gamma)$</th>
<th>$\sigma(\alpha,p)$</th>
<th>$\sigma(\alpha,n)$</th>
<th>$\sigma(\alpha,pn)$</th>
<th>$\sigma(\alpha,2n)$</th>
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(All cross-sections are expressed in millibarns)
TABLE V

Zn$^{64}$ and He$^4$ Recoil Ranges.

<table>
<thead>
<tr>
<th>$E_a$(MeV)</th>
<th>$R_0[(a,n)+(a,p)]$</th>
<th>$R_0[(a,pn)]$</th>
<th>$R_0[(a,2n)]$</th>
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</table>

(All ranges are expressed in $\mu$g/cm$^2$.)
TABLE VI

\textbf{Fe}^{56} \text{ and } \textbf{C}^{12} \text{ Cross-sections}

<table>
<thead>
<tr>
<th>( E_{\text{C}^{12}} \text{(MeV)} )</th>
<th>( E* \text{(MeV)} )</th>
<th>( \sigma(\text{C}^{12},n)+\sigma(\text{C}^{12},p) )</th>
<th>( \sigma(\text{C}^{12},pn) )</th>
<th>( \sigma(\text{C}^{12},2n) )</th>
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</thead>
<tbody>
<tr>
<td>36.9</td>
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<td>- - -</td>
<td>17.2</td>
<td>5.3</td>
</tr>
<tr>
<td>75.4</td>
<td>68.1</td>
<td>- - -</td>
<td>&lt;0.5</td>
<td>&lt;0.9</td>
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</tbody>
</table>

(all cross-sections are expressed in millibarns)
TABLE VII

Cr\textsuperscript{52} and O\textsuperscript{16} Cross-sections

<table>
<thead>
<tr>
<th>E\textsubscript{016}(MeV)</th>
<th>E\textsuperscript{*}(MeV)</th>
<th>(\sigma(\text{O\textsuperscript{16},n}) + \sigma(\text{O\textsuperscript{16},p}))</th>
<th>(\sigma(\text{O\textsuperscript{16},pn}))</th>
<th>(\sigma(\text{O\textsuperscript{16},2n}))</th>
</tr>
</thead>
</table>
| \(\begin{array}{c}
39.2 \\
47.5 \\
131.0
\end{array}\) | \(\begin{array}{c}
36.4 \\
42.7 \\
106
\end{array}\) | \(\begin{array}{c}
<1.8 \\
<2.7 \\
<2.6
\end{array}\) | \(\begin{array}{c}
<0.23 \\
<0.34 \\
<0.33
\end{array}\) | \(\begin{array}{c}
<.061 \\
<.090 \\
<.088
\end{array}\) |

<table>
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<tr>
<th>E\textsubscript{016}(MeV)</th>
<th>E\textsuperscript{*}(MeV)</th>
<th>(\sigma(\text{O\textsuperscript{16},α2n}))</th>
<th>(\sigma(\text{O\textsuperscript{16},apn}))</th>
<th>(\sigma(\text{O\textsuperscript{16},3pn}))</th>
</tr>
</thead>
</table>
| \(\begin{array}{c}
39.2 \\
47.5 \\
131.0
\end{array}\) | \(\begin{array}{c}
36.4 \\
42.7 \\
106
\end{array}\) | \(\begin{array}{c}
<3.1 \\
<2.1 \\
<3.0
\end{array}\) | \(\begin{array}{c}
<0.70 \\
<0.48 \\
<0.69
\end{array}\) | \(\begin{array}{c}
<0.28 \\
<0.19 \\
<0.27
\end{array}\) |

(all cross-sections are expressed in millibarns)
UNIVERSITY OF WASHINGTON
"BELL JAR" TARGET ASSEMBLY
(Schematic)

FIGURE 2
$\text{Zn}^{66}(\alpha,2n)\text{Ge}^{68}$

$\text{Zn}^{64}(\alpha,\gamma)\text{Ge}^{68}$

$x$ This work.
$\circ$ Data of Porile (Phys. Rev. 115 939 [1959])
\(\text{Zn}^{64}(\alpha,n)\text{Ge}^{67}\)

- This work
- Data of Porile
  (Phys. Rev. 115, 939 [1959])
$\text{Zn}^{64}(\alpha, p)\text{Ga}^{67}$

- This work
- Data of Porile (Phys. Rev. 115, 939 [1959])
- $R_0$

![Graph showing the cross-section ($\sigma$) in $\text{mb}$ and range in $\mu g/cm^2$ as a function of $E_\alpha$ (Mev).]
Range of Recoiling Mass - 67 Ions in Zinc

\[ \times \text{Zn}^{64} (\alpha, p) \text{Ga}^{67} + \text{Zn}^{64} (\alpha, n) \text{Ge}^{67} \]

---

**Figure 6**

Range (\(\mu g/cm^2\)) vs. \(E_R\) (Mev)
Figure 7

- $\sigma(\alpha, p) + \sigma(\alpha, n)$
- $\sigma(C^{12}, p) + \sigma(C^{12}, n)$
- $\sigma(O^{16}, p) + \sigma(O^{16}, n)$
- upper limit only

$V_{He^4,Zn^{64}}$, $V_{C^{12},Fe^{56}}$, $V_{O^{16},Cr^{52}}$
Average Angular Momentum vs. Excitation Energy for the Compound Nucleus Ge$^{68}$

- $\text{He}^4 + \text{Zn}^{64}$
- $\text{C}^{12} + \text{Fe}^{56}$
- $\text{O}^{16} + \text{Cr}^{52}$

$\bar{J} (\hbar)$ vs. $E^* (\text{Mev})$
$^{64}\text{Zn} (\alpha, 2n) ^{66}\text{Ge}$

$^{64}\text{Zn} (\alpha, 2n) ^{66}\text{Ge}$

Figure 9

- $\sigma(\alpha, 2n)$ - This work
- $\sigma(\alpha, 2n)$ - Data of Porile

(Phys. Rev. 115, 939 [1959])

CORRECTED FOR CALIBRATION ERROR
(SEE TEXT)

$E_\alpha$ (MeV)

$R_0$ ($\mu g/cm^2$)
Range of Recolling Germanium 66 Ions in Zinc

$\times Zn^{64} (\alpha, 2n) Ge^{66}$

LSS

$E_R$ (MeV)

Range ($\mu g/cm^2$)
$\text{Zn}^{64}(\alpha, \text{pn}) \text{Ga}^{66}$

$\times \sigma(\alpha, \text{pn})$ — This work

$\bullet \sigma(\alpha, \text{pn})$ — Data of Porile

(Phys. Rev. 115 939 [1959])

CORRECTED FOR CALIBRATION ERROR (SEE TEXT)

$R_0$

$E_\alpha \text{ (Mev)}$

$E_\alpha \text{ (Mev)}$

FIGURE 11
Ranges of Recoil Gallium Ions in Matter

- Zn\(^{64}\) (α, pn) Ga\(^{66}\)

LSS

Range (μg/cm\(^2\))

\(E_R\) (Mev)
\[ \sigma \text{ (mb)} \]

\[ E_{C^{12}} \text{ (MeV)} \]

\[ \times \text{Fe}^{56}(C^{12}, \text{pn}) \text{Ga}^{66} \]

\[ \circ \text{Fe}^{56}(C^{12}, 2\text{n}) \text{Ge}^{66} \]
Figure 1

- Zn$^{64}$ ($\alpha$, 2n) Ge$^{66}$
- Fe$^{56}$ (C$^{12}$, 2n) Ge$^{66}$
- Cr$^{52}$ (O$^{16}$, 2n) Ge$^{66}$

Upper limit

$\varepsilon^*$ (Mev)

$\sigma$ (mb)

$V_{\text{Cl}^{12}, \text{Fe}^{56}}$ $V_{\text{O}^{16}, \text{Cr}^{52}}$
$^{2n}B_4(a, pn)\ z~^6}\ C(N)\ -\ \text{see text}$

Excitation Energy (Mev)

\[ \sigma (\text{mb}) \]

- $^{64}\text{Zn} (a, pn) \text{Ga}^{66}$
- $^{64}\text{Zn} (a, pn) \text{Ga}^{66}$
- $\sigma_{\text{max}}$ (C.N.) — see text
- $^{56}\text{Fe} (C^{12}, pn) \text{Ga}^{66}$

$V_{C^{12}, Fe^{56}}$
VARIATION OF THE MOMENT OF INERTIA WITH ENERGY FOR Ge$^{68}$

FIGURE 16

$\frac{\delta}{\hbar^2} = \frac{\epsilon_r}{\hbar^2}$

$0 \rightarrow E^* \text{ (MeV)} \rightarrow 0$
FIGURE 13

Region of no states
\[ Zn^{64}(\alpha, pn) Ga^{66} \]

\[ Fe^{56}(C^{12}, pn) Ga^{66} \]
Figure 25

$Zn^{64}(\alpha, \gamma) Ge^{68}$

- Calculated
- Experimental

$\sigma (\mu b)$ vs $E_{cm}$ (MeV)

$10^2$  $10^3$

10  20  30  40  50  60

$E_{cm}$ (MeV)
\[ Zn^{64}(\alpha, n) Ge^{66} \]

- **Experimental**
- **Calculated**
$\text{Zn}^{64}(\alpha, p) \text{Ga}^{67}$

- Experimental
- Calculated

$E_{cm} (\text{Mev})$

$\sigma (\text{mb})$
Fig. 28

$\sigma (\text{mb})$

$E_{cm} (\text{Mev})$

$Zn^{64}(\alpha, 2n)Ge^{66}$

○ Experimental
- Calculated
$Zn^{64}(\alpha, pn)Ga^{66}$

- Experimental
- Calculated

$\sigma$ (mb)

$E_{cm}$ (MeV)
\( \sigma \) (mb)

\( 10^{-1} \) to \( 10^{0} \)

\( E^* \) (Mev)

30 to 65

FIGURE 30

\( \text{Fe}^{56}(C^{12}, n) \text{Ge}^{67} \)

\( \text{Fe}^{56}(C^{12}, p) \text{Ge}^{67} \)

- Calculated
- Experimental
$^{56}$Fe($^{12}$C,2n)$^{66}$Ge

○ Experimental

— Calculated

$E_{ex}$ (MeV)

$\sigma$ (mb)

$V_{^{56}Fe,^{12}C}$

$10^{-1}$ $10^{0}$ $10^{1}$
\( \text{Fe}^{56} (\text{C}^{12}, \text{pn}) \text{Ga}^{66} \)

× Experimental

— Calculated

\( \sigma \) (mb)

\( E_{\text{ex}} \) (MeV)

\( V_{\text{Fe}^{56}, \text{C}^{12}} \)
\[ \frac{\sigma(\alpha, \text{pn})}{\sigma(\alpha, 2n)} \]
- Experimental
- Calculated

\[ \frac{\sigma(\text{C}^{12}, \text{pn})}{\sigma(\text{C}^{12}, 2n)} \]
- Experimental
- Calculated

**FIGURE 33**

Reaction Ratio vs. E (MeV)
REFERENCES

1. References to early neutron reaction studies may be found in the work cited in footnote 5.


3. N. Bohr, Nature 137, 344 (1936)

4. G. Breit and E. Wigner, Phys. Rev. 49, 519 (1936)

5. E. Vogt, "The Statistical Theory of Nuclear Reactions" (to be published)


10. H.A. Bethe, Rev. Mod. Phys. 2, 69 (1937)

11. V.F. Weisskopf, Phys. Rev. 52, 295 (1937)


14. E. Segre, Nuclei and Particles (W.A. Benjamin, Inc., Amsterdam, 1964)


17. K.J. Le Couteur, Nuclear Reactions (ed. P.M. Endt and M. Demeur, North Holland Publishing Co., Amsterdam, 1959) - Other early references may be found in the works cited in footnote 6.

18. H. Hurwitz & H.A. Bethe, Phys. Rev. 81, 898 (1951)


22. E.g. preceding footnote and Dostrovsky, Rabinowitz, and Burns, Phys. Rev. 111, 1659 (1958)


26. R. Serber, Phys. Rev. 72, 1008 (1947)


29. M.A. Preston, Physics of the Nucleus, (Addison-Wesley, Reading, Massachusetts, 1963)


31. D.G. Sarantits and B.D. Pate, Nuclear Physics A93, 545 (1967)

32. I. Kanestrøme, Nuclear Physics 83, 380 (1966)

33. D.W. Lang, Nuclear Physics 77, 545 (1966)


41. A. B. Migdal, Nuclear Physics 12, 655 (1959)

42. D.W. Lang, Nuclear Physics 42, 353 (1963)

43. H.K. Vonach, R. Vandenbosch, & J.R. Huizenga, Nuclear Physics 60, 70 (1964)


45. W. Heisenberg, Z Physik 77, 1 (1932)


49. J.M. D'Auria and J.M. Miller, Nuclear Physics (to be published)


52. M. Cogneau and L. Gilly, Nuclear Physics 73, 122 (1965)

53. R.A. Esterlund and B.D. Pate, Nuclear Physics 69, 401 (1965)

55. J.F. O'Hanlon and R.R. Haering, personal communication.
56. Natural Abundances are from the Chart of the Nuclides, prepared by David T. Goldman, Knolls Atomic Power Laboratory, Schenectady, N.Y., 9th ed. (1966)
57. Enriched isotopes were obtained from Oak Ridge National Laboratory, Oak Ridge, Tennessee, who performed the isotopic analysis.
60. J. B. Cumming, personal communication (1967).


83. R. V. Carlson and P. J. Daly, Nuclear Physics A102, 177 (1967).


87. E. V. Verdieck and J. M. Miller, to be published.


89. J. M. Miller, personal communication (1968).


100. E. Vogt, personal communication (1967).


122. L. Kowalski, J. C. Jodogne, and J. M. Miller, (to be published).

Appendix I
Recoil Range Theories.

1. Recoil Range measurements.

Recoil ranges may be studied in a variety of ways, the simplest of which is to measure projected ranges along the beam direction. For a given reaction at a specified energy, the observed ranges of nuclei recoiling from the target into some stopping medium will center about an average value $R_0$. Deviations from this average value, called range straggling, arise from a number of sources:

(1.) Straggling from effects of the nuclear reaction, i.e., resultant velocities of individual recoiling atoms will differ because of non-unique alignment of the velocity vectors of the emitted particles. ($\rho_n$)

(2.) Straggling inherent in the stopping process. ($\rho_s$)

(3.) Straggling caused by finite target thickness, i.e., the individual recoiling nuclei originate from different depths in the target and escape with different degrees of the full range. ($\rho_w$)

(4.) Straggling caused by inhomogeneities in the stopping medium. ($\rho_f$)

The overall range straggling parameter is the resultant of parameters corresponding to the above combined in quadrature

$$\rho^2 = \rho_n^2 + \rho_s^2 + \rho_w^2 + \rho_f^2$$

and the range straggling is given by $R_0\rho$. If the distribution
in range values is Gaussian, then

\[ P(R) \, dR = \frac{1}{R_0 \sqrt{2\pi}} \exp \left( -\frac{(R-R_0)^2}{2R_0^2} \right) \, dR \]  

(I-1)

Assuming uniform production of recoils across a layer of target atoms of thickness \( W \), the fraction of recoils which remain in the target is

\[ F_W = \frac{1}{(2\pi)^{3}R_0 W} \int_0^W \int_0^W \exp\left( -\frac{(r-s-R_0)^2}{2R_0^2} \right) \, dr \, ds \]  

(I-2)

where \( s \) is the distance from the edge of the layer to the point at which the recoiling atom originates and \( r-s \) is the distance that it travels. The integration of the above equation leads to the following

\[ F_W = \frac{\sqrt{2R_0}}{W} \left( \mathfrak{H}(\frac{R_0-W}{\sqrt{2R_0^2}}) - 2\mathfrak{H}(\frac{R_0}{\sqrt{2R_0^2}}) + \mathfrak{H}(\frac{R_0+W}{\sqrt{2R_0^2}}) \right) \]  

(I-3)

where

\[ \mathfrak{H}(y) = \frac{1}{2\sqrt{\pi}} \exp(-y^2) - \frac{1}{2} \left[ 1 - I(y) \right] y \]

and

\[ I(y) = \frac{2}{\sqrt{\pi}} \int_0^y \exp(-u^2) \, du \]

\( F_W \) may be approximated by the first term for \( W \geq R_0 \)

\[ F_W \approx \frac{\sqrt{2R_0}}{W} \mathfrak{H}(\frac{R_0-W}{\sqrt{2R_0^2}}) \]  

(I-4)

and if the thickness is many times the average range, (D-4) reduces to

\[ R_0 = W(1-F_W) \]  

(I-5)
If one bombards a thick target backed by a thick catcher foil, $F_W$ is calculable from the activities observed in the target and catcher foils,

$$1 - F_W = \frac{A_C}{A_T + A_C}$$  \hspace{1cm} (I-6)

and the calculation of $R_o$ follows.

In practice, recoiling atoms will not be produced uniformly across the target thickness. If one assumes a linear variation of cross-section over the energy range corresponding to the target thickness, i.e.,

$$\sigma_s = \frac{(\sigma_W - \sigma_o)s}{W} + \sigma_o$$  \hspace{1cm} (I-7)

where $\sigma_o$ is the cross-section at the target-catcher interface ($s=0$) and $\sigma_W$ is the cross-section at the opposite surface ($s=W$), and all recoiling ions are assumed to be formed within their range, $R_o$, from the surface, then

$$1 - F_W = \frac{A_C}{A_T + A_C} = \int_0^R \frac{\sigma_s ds}{\sigma_s + \sigma_o} = \frac{R}{W} \cdot \frac{2\sigma_W + (\sigma_W - \sigma_o)R_o}{\sigma_o + \sigma_W}$$

For small differences between $\sigma_o$ and $\sigma_W$ ($|\sigma_W - \sigma_o| < \sigma_o$), and $R \ll W$,

$$R_o = (1 - F_W)W \left(\frac{\sigma_o + \sigma_W}{2}\right)$$  \hspace{1cm} (I-8)

that is, the range must be corrected by a factor corresponding to the ratio of the average cross-section in the foil to the cross-section at the target-catcher interface.
Such thick target, thick catcher, or integral, range experiments provide a simple means for determining ranges if the variation of cross-section is small across available target thicknesses and the distribution in ranges is known to be Gaussian. If these conditions are not met, however, a differential method may be appropriate.

A convenient differential method for measuring average projected ranges in the beam direction is to bombard thin targets \((W<<R_0)\) backed by several thin catchers. If \(F_t\), the fraction of activity that passes through thickness \(t\), is plotted on a probability scale vs. \(t\), the \(t\) value corresponding to \(F_t = \frac{1}{2}\) defines \(R_0\). The \(t\) value for \(F_t\) equals 0.0787 specifies \(R_0(1+\sqrt{2}\rho)^{1/2}\).

Also, any deviation from Gaussian distribution about the average projected range may be observed directly in this type of experiment. However, one must be studying a product which is produced in high yield and the average range must be relatively large \((>200\text{mg/cm}^2)\) in order for this type of differential measurement to be experimentally feasible.

2. The Stopping Process.

In order for interpretation of the results of recoil range measurements to be possible, the nature of the interactions of recoiling atoms with matter must be known. According to the formulation due to Bohr\(^{115}\), the stopping process is dependent on the velocity \((v)\) of the moving atom.
If \( v \) is greater than the orbital velocities of the electrons of the stopping atoms, stopping is mainly by interaction with these electrons. If \( v \) is less than the orbital electron velocity, stopping is by interaction with the atoms as a whole (e.g. with the crystal lattice of the stopping medium.) In the latter case, Bohr derived the following formula\(^{115}\) for \( R_o \) expressed in \( \text{mg/cm}^2 \) and \( E \) in MeV.

\[
R_o = 0.600 \frac{A_s(A_s+AR)}{AR} \frac{(Z_R^{3/2}+Z_S^{3/2})^{3/2}}{Z_S^{3/2} Z_R^{1/2}} E
\]

which holds when the recoiling particle masses are much larger than the stopping atomic masses \((A_R >> A_s)\).

In general, the stopping power may be represented (neglecting channeling effects) as the following\(^{116}\).

\[
\frac{dE}{dX} = k\left( E^{\frac{2}{3}} + c_1 \right)
\]

The first term corresponds to electronic stopping and the second to elastic (atomic) stopping. Lindhard, Scharff, and Schiøtt\(^{78}\) (LSS) have derived a general stopping theory where atomic stopping was described by a Thomas-Fermi potential and the proportionality constant, \( k \), for the electronic stopping term was given as\(^{78}\).

\[
k = \epsilon_e \frac{0.0793 Z_R^{\frac{1}{3}} Z_S^{\frac{1}{3}} (A_R+A_S)^{\frac{3}{2}}}{(Z_R^{\frac{3}{2}}+Z_S^{\frac{3}{2}})^{3/4} A_R^{\frac{3}{4}} A_S^{\frac{3}{4}}}
\]

(I-9)

\[
\epsilon_e = Z_R^{\frac{1}{6}}
\]
The LSS theory makes use of dimensionless range and energy parameters, \( \rho \) and \( \epsilon \), given by

\[
\begin{align*}
\rho &= \frac{\rho A_n^2\lambda^2 \rho}{(\rho \lambda + \rho n_S)^2} \\
\epsilon &= E A R [Z \rho Z_S e^2 (\rho \lambda + \rho n_S)]^{-1} \\
a &= a_0 \{0.8853 (Z \rho \lambda + Z_S \lambda)^{-\frac{1}{2}}\}
\end{align*}
\]

Where \( N \) is the number of stopping atoms per unit volume, \( e \) is the electronic charge, and \( a_0 \) is the first Bohr radius in the hydrogen atom. The LSS calculations produce a set of \( \rho - \epsilon \) curves, each characterised by a value of \( k \). The straggling, or mean square deviation from the mean, is given by plotting the quantity

\[
\frac{(\rho \lambda + \rho n_S)^2}{\lambda^2} \frac{\Delta \rho^2}{R^2}
\]

vs. \( \epsilon \) for different values of \( k \). The range along the total path length of the recoiling atom is related to the projected range by the following correction factor given by LSS.

\[
R = R_o \frac{1}{2} \left[ -1 - 3 + (5 + u) \right] \cdot \frac{1 + u}{2u^2} \arccos \left( \frac{1 - u}{1 + u} \right)
\]

where \( u = A_S / A_R \). LSS have given curves of \( \frac{1}{u} \frac{R - R_o}{R_o} \) and \( \frac{R}{R_o} - 1 \) vs. \( \epsilon \) for several values of \( k \).

At low energies

\[
\frac{R}{R_o} \approx 1 + \frac{u}{3}
\]

although several inadequacies exist in the theories of LSS, they have, in general, provided agreement within
experimental error with the type of integral and
differential ranges measured by the previously described
experiments.
Appendix II

Coupling Schemes.

For the reaction \( X + b \rightarrow Y + x \), the target and projectile will have a mutual orbital angular momentum, \( \vec{l} \), as will the products, \( \vec{l}' \). \( X \) and \( Y \) will have spins \( \vec{l} \) and \( \vec{l}' \) and orbital angular momenta \( \vec{L} \) and \( \vec{L}' \), respectively, and \( b \) and \( x \) will have spins \( \vec{s} \) and \( \vec{s}' \), respectively. Angular momentum must be conserved; however, the method of addition of these vectors depends on the type of interaction assumed or "coupling" scheme used\(^{29}\).

(a) \( j-j \) Coupling.
\[
\vec{J} = \vec{l} + \vec{L} \quad \vec{J}' = \vec{l}' + \vec{L}'
\]
\[
\vec{J} = \vec{t} + \vec{s} \quad \vec{J}' = \vec{t}' + \vec{s}'
\]
\[
\vec{J} + \vec{J}' = \vec{J}_c = \vec{J}' + \vec{J}'
\]

(b) \( L-S \) (Russel Saunders) Coupling.
\[
\vec{S} = \vec{l} + \vec{s} \quad \vec{S}' = \vec{l}' + \vec{s}'
\]
\[
\vec{L}_1 = \vec{L} + \vec{t} \quad \vec{L}_2 = \vec{L}' + \vec{t}'
\]
\[
\vec{S} + \vec{L}_1 = \vec{J}_c = \vec{S}' + \vec{L}_2
\]

(c) Intermediate Coupling.
\[
\vec{J} = \vec{l} + \vec{L} \quad \vec{J}' = \vec{l}' + \vec{L}'
\]
\[
\vec{S} = \vec{J} + \vec{s} \quad \vec{S}' = \vec{J}' + \vec{s}'
\]
\[
\vec{S} + \vec{t} = \vec{J}_c = \vec{S}' + \vec{t}'
\]

The quantity \( S \) is called the channel spin\(^{29}\).
If the intermediate coupling scheme is used, the distribution in angular momentum of intermediate states is given by:

\[
\sigma_c(I, \epsilon, J_c) = \frac{\pi \hbar^2 (2J_c+1)}{(2s+1)(2I+1)} \sum_{s=|I-s|}^{(I+s)} \sum_{s=|J_c-s|}^{(J_c+s)} T_t(\epsilon)
\]
Appendix III.

The Cu\textsuperscript{63}(\alpha,pn)Zn\textsuperscript{65} and Cu\textsuperscript{63}(\alpha,2n)Ga\textsuperscript{65} Reactions.

The excitation function for the production of Zn\textsuperscript{65} in natural copper foils via the Cu\textsuperscript{63}(\alpha,2n)Ga\textsuperscript{65} and Cu\textsuperscript{63}(\alpha,pn)Zn\textsuperscript{65} reactions was measured by the bombardment of 3.64 mg/cm\textsuperscript{2} natural copper foils\textsuperscript{58} backed by thick aluminium catchers and interspersed between aluminium degrading foils. The beam intensity was monitored via the production of Ga\textsuperscript{67} in zinc foils by the Zn\textsuperscript{64}(\alpha,n)Ge\textsuperscript{67} and Zn\textsuperscript{64}(\alpha,p)Ga\textsuperscript{67} reactions.

All 15 minute Ga\textsuperscript{65} activity was allowed to decay to 245 day Zn\textsuperscript{65}. A 7.6 cm x 7.6 cm NaI(Tl) detector was employed to monitor the intensity of the 511-kev and 1115-kev peaks. Efficiency data due to Heath\textsuperscript{70} were employed. Positron emission was taken to occur in 51.7\%\textsuperscript{74} of the decays and the intensity of the 1115-kev peak was taken as 0.49 photons/disintegration\textsuperscript{74}. The disintegration rates determined separately from the two peaks were found to agree within a few percent in all cases.

The measured excitation function is shown in figure 34.
Appendix IV.

Characterization of 280 day Ge$^{68}$.

The activity of 280 day Ge$^{68}$ was determined by radioactivity assay of its 67 minute daughter after secular equilibrium had been established. Ge$^{68}$ decays by pure electron capture, and therefore does not emit any easily detectable radiation. The radiations emitted during the decay of Ga$^{68}$ are easily detectable, however.

The identity of Ga$^{68}$ was established by a series of measurements immediately after separation of GeCl$_4$ from the dissolved target (several months after the irradiation to eliminate shorter lived Ge activities), followed by analysis of the radioactive growth curve. Figure 35 is such a curve. The crosses denote measured activity plotted versus time from the end of the separation to the time of the measurement of the activity. The open circles represent the difference between measured activity and the equilibrium activity. The half life was measured from the decay curve defined by the open circles in order to establish the identity of this isotope, and was found in all cases to agree well with the value measured in an independent study of the nuclide Ga$^{68}$. 

Appendix V.

SFUSMAP

The Fortran VI statistical model program, SFUSMAP, consists of a main program, four subprograms, and a function subroutine.

a.) MAIN.

The main program reads the input data and performs the summations and multiplications of the various probabilities for populations of the E-J grids corresponding to the various nuclear states. Most of the written output from the program is executed by MAIN.

b.) Subroutine WRITE.

This subprogram was written as a result of core space restrictions on the main program and is used to write out the E-J matrix ONE.

c.) Subroutine BRANCH.

Probabilities for the population of final states are calculated for a specific compound nucleus energy and angular momentum by the subprogram. Most of the statistical model mathematics are performed by this subroutine which calls the remaining two subroutines and the function subroutine.

d.) Subroutine FURG.

Transmission coefficients (which are read in as data) are interpolated and summed to give inverse reaction cross-sections by this subroutine.
e.) Subroutine BIND.

This subroutine returns a value of the binding energy of a particular particle for a given compound nucleus.

f.) Function ALEVEL.

This function calculates the level density corresponding to a particular E and J.

A listing of SFUSMAP immediately follows this discussion. The program has been revised substantially since this listing to remove programming steps which were included to avoid inadequacies in early versions of the SFU system 360/40 compiler programs.

A typical input data deck consists of:

(1) 36 cards containing binding energies (punched in columns 1-6) for the nine nuclei surrounding (and including) the compound nucleus at lower neutron and proton numbers.

(2) 108 cards, each containing 8 transmission coefficients for alpha penetrations (punched in each group of 10 columns). Each group of four cards represents one energy value with each value separated by 2 MeV. This data designates the matrix TLA (J,JXE) where J and JXE signify angular momentum and energy respectively.

(3) 135 cards, each containing eight proton transmission coefficients as above. Each group of five cards represents one energy value. This data populates the matrix TPL(J,L,JXE) where L signifies orbital angular momentum.
(4) 135 cards containing neutron transmission coefficients, populating the matrix TLM \((J,L,JXE)\).

(5) 1 card containing the four single-particle \(\gamma\)-transmission strength constants \((C_i')\) punched in the first four groups of 10 columns.

(6) An eight card packet which contains:

(a) The center of mass energy of the target-projectile system (punched in columns 1-8).

(b) The binding energy of the projectile and the target (columns 2-8).

(c) 5 cards containing the capture cross-sections for population of each \(J\) state from \(J=0\) to \(J=39\) helium-39 for the energy specified by the first card in the packet (a). The data is punched in each group of 10 channels.

(d) The angular momentum value above which the capture cross-section is zero.

The number of data packets which may be used is not limited by the program. The last card in the deck should contain a zero punched in place of the energy.
C  SFUSMAP, THE COMPUTER MOLLER.
  0001  DIMENSION GAMMA(4), CL(4)
  0002  DIMENSION P(4,9)
  0003  DIMENSION TLA(32,27), TLP(2,20,27), TLN(2,20,27)
  0004  REAL ONE
  0005  DIMENSION ONE(30,20), TN(3,20), THREE(3,20), FIVE(11), G0
  0006  DIMENSION FOUR(30,20), SIG(9)
  0007  DIMENSION SFRBAR(30,20,4)
  0008  REAL NINE
  0009  DIMENSION NINE(30,20)

  0010  COMMON R,AN,ZH,IX,EF
  0011  COMMON U
  0012  COMMON CL
  0013  COMMON TLA,TLP,TLN
  0014  COMMON SFRBAR
  0015  READ(5,1)(B(I,IX,ICN),I=1,4),ICN=1,9)

  0016  1 FORMAT(1F6.3)
  0017  READ(5,2) ((TLA(J,JXE),J=1,32),JXE=1,27)
  0018  READ(5,2) ((TLP(J,L,JXE),J=1,2),L=1,20),JXE=1,27)
  0019  READ(5,2) ((TLN(J,L,JXE),J=1,2),L=1,20),JXE=1,27)
  0020  READ(5,6) (GL(L),L=1,4)
  0021  6 FORMAT(4E10.3)

  0022  2 FORMAT(8F10.3)
  0023  3 FORMAT(8E10.3)
  0024  4 FORMAT(2F8.3,13,E20.8)
  0025  5 FORMAT(8F15.8)
  0026  9 FORMAT(2F12.8,5X,2F12.8,5X,2F12.8,5X,2F12.8)
  0027  7 FORMAT(1F10.5)

  0028  2008 FORMAT(E20.8)
  0029  200 FORMAT(4E20.8)

  CONTROL PROGRAM STARTS HERE

  0030  FDG=1.0E-07
  0031  DIMENSION SIGCAP(40)
  0032  READ(5,7) ECM

  0033  READ(5,7) Q
  0034  READ(5,3) (SIGCAP(JCN),JCN=1,40)
0035       READ(5,7) RR
0036       XY=PB
0037       7515 CONTINUE
0038       WRITE(6,7) ECM
0039       WRITE(6,7) Q
0040       WRITE(6,7) RR
0041       AA=ECM+Q
0042       DO 3000 I=1,30
0043           DO 3000 J=1,20
0044           JEF=1
0045           JJF=J
0046           ONE(JEF,JJF)=0.0
0047           TWO(JEF,JJF)=0.0
0048           THREE(JEF,JJF)=0.0
0049           FIVE(JEF,JJF)=0.0
0050           FOUR(JEF,JJF)=0.0
0051           NINE(JEF,JJF)=0.0
0052           3000 CONTINUE
0053           ICN=1
0054           SIG(ICN)=0.0
0055           4004 CONTINUE
0056           DO 10 K=1,30
0057               DO 10 J=1,20
0058               DO 10 I=1,4
0059               JEF=K
0060               JJF=J
0061           IX=I
0062           SFURAR(JEF,JJF,IX)=0.0
0063           10 CONTINUE
0064           JCA=BB+1.
0065           SFURAB=0.0
0066           DO 20 IA=1,4
0067           IX=IA
0068           AN=68.
0069           ZN=32.
0070           CALL BRANCH(GAMMA,AA,BB)
0071           SFURAB=SFURAB+GAMMA(IX)
0072           20 CONTINUE
0073           IF((SFURAB).LE.0.0) GO TO 12002
0074           DC 2002 ID=1,30
0075  DO 2002  IE=1,20
0076   DO 2002  IF=1,4
0077    JEF=1D
0078    JJF=1E
0079    IX=1F
0080 SFUPAR(JEF, JJF, IX)=SFUPAR(JEF, JJF, IX)/SFUPAR
0081 2002 CONTINUE
0082 12002 CONTINUE
0083    IX=1
0084   DJ 2003 IG=1,3C
0085   DC 2003 IH=1,20
0086    JEF=1G
0087    JJF=IH
0088 ONE(JEF, JJF)=ONE(JEF, JJF)+SFUPAR(JEF, JJF, IX)*SIGCAP(JCN)
0089 ICN=1
0090  SIG(IGN)=SIG(IGN)+SFUPAR(JEF, JJF, IX)*SIGCAP(JCN)
0091 2003 CONTINUE
0092   IX=4
0093   DO 2004 IC=1,30
0094   DO 2004 IP=1,20
0095    JEF=1C
0096    JJF=1P
0097  TWO(JEF, JJF)=TWO(JEF, JJF)+SFUPAR(JEF, JJF, IX)*SIGCAP(JCN)
0098 2004 CONTINUE
0099   IX=3
0100   DO 2005 IQ=1,3C
0101   DO 2005 IK=1,20
0102    JEF=1Q
0103    JJF=1R
0104  FOUR(JEF, JJF)=FOUR(JEF, JJF)+SFUPAR(JEF, JJF, IX)*SIGCAP(JCN)
0105 2005 CONTINUE
0106   IX=2
0107   DC 2106 IS=1,30
0108   DO 2106 IT=1,20
0109    JEF=IS
0110    JJF=IT
0111  NINE(JEF, JJF)=NINE(JEF, JJF)+SFUPAR(JEF, JJF, IX)*SIGCAP(JCN)
0112 2106 CONTINUE
0113    PR=PR-1.
0114    IF((BB).GE.O.C) GO TO 4004
0115
0115  \text{IGN}=1
0116  \text{WRITE}(6,4) \text{ECM},Q,\text{IGN},\text{SIG}(	ext{IGN})
0117  \text{WRITE}(6,100)
0118  \text{CALL WRITE(UDE)}
0119  \text{WRITE}(6,102)
0120  \text{WRITE}(6,204)
0121  \text{DC 2104} I=1,30
0122  \text{JEF}=I
0123  \text{WRITE}(6,203)(\text{JEF},(\text{TWO} (\text{JEF,JJF}),\text{JJF}=1,10))
0124  2104 \text{CONTINUE}
0125  \text{WRITE}(6,100)
0126  \text{WRITE}(6,205)
0127  \text{DO 2105} I=1,30
0128  \text{JEF}=I
0129  \text{WRITE}(6,203)(\text{JEF},(\text{TWO} (\text{JEF,JJF}),\text{JJF}=11,20))
0130  2105 \text{CONTINUE}
0131  \text{WRITE}(6,103)
0132  \text{WRITE}(6,204)
0133  \text{DO 2111} I=1,30
0134  \text{JEF}=I
0135  \text{WRITE}(6,203)(\text{JEF},(\text{FOUR} (\text{JEF,JJF}),\text{JJF}=1,10))
0136  2111 \text{CONTINUE}
0137  \text{WRITE}(6,100)
0138  \text{WRITE}(6,205)
0139  \text{DO 2112} I=1,30
0140  \text{JEF}=I
0141  \text{WRITE}(6,203)(\text{JEF},(\text{FOUR} (\text{JEF,JJF}),\text{JJF}=11,20))
0142  2112 \text{CONTINUE}
0143  \text{WRITE}(6,100)
0144  \text{DO 2119} I=1,30
0145  \text{JEF}=I
0146  \text{WRITE}(6,203)(\text{JEF},(\text{NINE} (\text{JEF,JJF}),\text{JJF}=1,10))
0147  2119 \text{CONTINUE}
0148  \text{WRITE}(6,100)
0149  \text{DO 2113} I=1,30
0150  \text{JEF}=I
0151  \text{WRITE}(6,203)(\text{JEF},(\text{NINE} (\text{JEF,JJF}),\text{JJF}=11,30))
0152  2113 \text{CONTINUE}
0153  \text{IGN}=2
0154  \text{DO 9800} I=1,30
DO 9800 J=1,20
0156 JEF=1
0157 JJF=J
0158 ONE(JEF, JJF)=0.0
0159 9800 CONTINUE
0160 ENSUM=G.C
0161 SIG(IN)=0.0
0162 JA=1
0163 8500 CONTINUE
0164 JB=1
0165 2006 CONTINUE
0166 DO 3001 I=1,30
0167 DC 3001 J=1,20
0168 DC 3001 K=1,4
0169 JEF=I
0170 JJF=J
0171 IX=K
0172 SFURAR=(JEF, JJF, IX)=0.0
0173 3001 CONTINUE
0174 SFURAR=0.0
0175 DO 4000 JC=1,4
0176 IX=JC
0177 AN=47.
0178 ZN=32.
0179 EEF=2*JA-1.75
0180 FJ=40.0-2*JB
0181 JJEFF=EEF/2+1
0182 JJJEFF=FJ/2.0+1.5
0183 FURAR=IN(G(JJEFF, JJJEFF))
0184 IF((FURAR).LT.FDG) GO TO 12019
0185 IF((JA-3).LT.C) GC TO 700
0186 CALL BRANCH(AMC, EEF, FJ)
0187 SFURAB=SFURAB+GAMMA(IX)
0188 4000 CONTINUE
0189 IF((SFURAB).LT.FDG) GC TO 700
0190 GO TO 701
0191       7CC IX=1
0192       SFUBAR(JJEF,JJF,IX)=1.0
0193       GO TO 2109
0194       7C1 CONTINUE
0195       ENSUM=ENSUM+SFUBAR
0196       DO 2010 I=1,30
0197       DO 2010 J=1,20
0198       DC 2010 K=1,4
0199       JEF=I
0200       IX=K
0201       JJF=J
0202       SFUBAR(JEF,JJF,IX)=SFUBAR(JEF,JJF,IX)/SFUBAR
0203       2010 CONTINUE
0204       2109 CONTINUE
0205       DC 2007 JD=1,30
0206       DO 2107 JE=1,20
0207       JEF=JD
0208       JJF=JE
0209       IX=1
0210       SIG(IGN)=SIG(IGN)+SFUBAR(JEF,JJF,IX)*FUBAR
0211       ONE(JEF,JJF)=ONE(JEF,JJF)*SFUBAR(JEF,JJF,IX)*FUBAR
0212       IX=4
0213       THREE(JEF,JJF)=THREE(JEF,JJF)+SFUBAR(JEF,JJF,IX)*FUBAR
0214       IX=3
0215       FIVE(JEF,JJF)=FIVE(JEF,JJF)+SFUBAR(JEF,JJF,IX)*FUBAR
0216       2007 CONTINUE
0217       12019 CONTINUE
0218       JS=JB+1
0219       IF((JB-20),L.E.0) GO TO 2006
0220       8520 CONTINUE
0221       JA=JA+1
0222       IF((JA-30),L.E.0) GO TO 8500
0223       8519 CONTINUE
0224       ENORP=ENSUM
0225       WRITE(6,4) ECP, Q, IGN, SIG(IGN)
0226       WRITE(6,10+)
0227       WRITE(6,204)
0228       DO 2107 I=1,30
0229       JEF=I
0230       WRITE(6,201) (JEF,(THREE(JLF,JJF),JJE=1,10))
```plaintext
0231 2107 CONTINUE
0232 WRITE(6,100)
0233 WRITE(6,205)
0234 DO 2108 I=1,30
0235 JEF=I
0236 WRITE(6,203)(JEF,THREE(JEF,JF),JF=11,20)
0237 2108 CONTINUE
0238 WRITE(6,105)
0239 WRITE(6,204)
0240 DO 2115 I=1,30
0241 JEF=I
0242 WRITE(6,203)(JEF,FIVE(JEF,JF),JF=1,10)
0243 2115 CONTINUE
0244 WRITE(6,100)
0245 WRITE(6,205)
0246 DO 2116 I=1,30
0247 JEF=I
0248 WRITE(6,203)(JEF,FIVE(JEF,JF),JF=11,20)
0249 2116 CONTINUE
0250 WRITE(6,109)
0251 CALL WRITE(ONE)
0252 ICN=3
0253 ENSUM=G.0
0254 SIG(ICN)=0.0
0255 DO 9801 I=1,30
0256 DO 9801 J=1,20
0257 JEF=I
0258 JJF=J
0259 ONE(JEF,JJF)=0.0
0260 9801 CONTINUE
0261 IA=1
0262 8502 CONTINUE
0263 IA=1
0264 5006 CONTINUE
0265 DO 5001 I=1,30
0266 DO 5001 J=1,20
0267 DO 5001 K=1,4
0268 JEF=I
0269 JJF=J
0269 IX=K
```
<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>0271</td>
<td>SFURAB(JEF, JJE, IX) = 0.0</td>
</tr>
<tr>
<td>0272</td>
<td>5001 CONTINUE</td>
</tr>
<tr>
<td>0273</td>
<td>SFURAB = C.0</td>
</tr>
<tr>
<td>0274</td>
<td>DO 5000 IC = 1, 4</td>
</tr>
<tr>
<td>0275</td>
<td>IX = IC</td>
</tr>
<tr>
<td>0276</td>
<td>AN = 4.6</td>
</tr>
<tr>
<td>0277</td>
<td>ZN = 32.</td>
</tr>
<tr>
<td>0278</td>
<td>EEF = 2 * IA - 1.75</td>
</tr>
<tr>
<td>0279</td>
<td>FJ = 40.0 - 2 * IR</td>
</tr>
<tr>
<td>0280</td>
<td>JJTF = EEF / 2 + 1</td>
</tr>
<tr>
<td>0281</td>
<td>JJJF = FJ / 2.0 + 1.5</td>
</tr>
<tr>
<td>0282</td>
<td>SFURAB = THREE(JJTF, JJJF)</td>
</tr>
<tr>
<td>0283</td>
<td>IF((SFURAB).LT.FUG) GO TO 15011</td>
</tr>
<tr>
<td>0284</td>
<td>IF((IA-3).LT.C) GO TO 7C2</td>
</tr>
<tr>
<td>0285</td>
<td>CALL BRANCH(GAMMA, EEF, FJ)</td>
</tr>
<tr>
<td>0286</td>
<td>SFURAB = SFURAB + GAMMA(IX)</td>
</tr>
<tr>
<td>0287</td>
<td>5000 CONTINUE</td>
</tr>
<tr>
<td>0288</td>
<td>IF((SFURAB).LT.FUG) GO TO 7C2</td>
</tr>
<tr>
<td>0289</td>
<td>GO TO 703</td>
</tr>
<tr>
<td>0290</td>
<td>7C2 IX = 1</td>
</tr>
<tr>
<td>0291</td>
<td>SFURAB(JJE, JJJF, IX) = 1.0</td>
</tr>
<tr>
<td>0292</td>
<td>GO TO 15010</td>
</tr>
<tr>
<td>0293</td>
<td>703 CONTINUE</td>
</tr>
<tr>
<td>0294</td>
<td>ENSUM = ENSUM + SFURAB</td>
</tr>
<tr>
<td>0295</td>
<td>DO 5010 I = 1, 30</td>
</tr>
<tr>
<td>0296</td>
<td>DO 5010 J = 1, 20</td>
</tr>
<tr>
<td>0297</td>
<td>DO 5010 K = 1, 4</td>
</tr>
<tr>
<td>0298</td>
<td>JEF = 1</td>
</tr>
<tr>
<td>0299</td>
<td>JJE = J</td>
</tr>
<tr>
<td>0300</td>
<td>IX = K</td>
</tr>
<tr>
<td>0301</td>
<td>SFURAB(JJE, JJJF, IX) = SFURAB(JJE, JJJF, IX) / SFURA2</td>
</tr>
<tr>
<td>0302</td>
<td>5010 CONTINUE</td>
</tr>
<tr>
<td>0303</td>
<td>15010 CONTINUE</td>
</tr>
<tr>
<td>0304</td>
<td>DO 5007 ID = 1, 30</td>
</tr>
<tr>
<td>0305</td>
<td>DO 5007 IF = 1, 20</td>
</tr>
<tr>
<td>0306</td>
<td>JEF = 10</td>
</tr>
<tr>
<td>Line</td>
<td>Content</td>
</tr>
<tr>
<td>------</td>
<td>---------</td>
</tr>
<tr>
<td>0307</td>
<td>JF = IF</td>
</tr>
<tr>
<td>0308</td>
<td>IX = 1</td>
</tr>
<tr>
<td>0309</td>
<td>ICN = 3</td>
</tr>
<tr>
<td>0310</td>
<td>SIG(ICN) = SIG(ICN) + SFUBAR(JEF, JF, IX) * FUPA</td>
</tr>
<tr>
<td>0311</td>
<td>ONE(JEF, JF) = CNE(JEF, JF) + SFUBAR(JEF, JF, IX) * FUPA</td>
</tr>
<tr>
<td>0312</td>
<td>CONTINUE</td>
</tr>
<tr>
<td>0313</td>
<td>CONTINUE</td>
</tr>
<tr>
<td>0314</td>
<td>I8 = I8 + 1</td>
</tr>
<tr>
<td>0315</td>
<td>IF((I8 - 30), LE, 0) GO TO 5006</td>
</tr>
<tr>
<td>0316</td>
<td>CONTINUE</td>
</tr>
<tr>
<td>0317</td>
<td>IA = IA + 1</td>
</tr>
<tr>
<td>0318</td>
<td>IF((IA - 30), LE, 0) GC TO 8502</td>
</tr>
<tr>
<td>0319</td>
<td>CONTINUE</td>
</tr>
<tr>
<td>0320</td>
<td>ENORM = ENSUM</td>
</tr>
<tr>
<td>0321</td>
<td>WRITE(6, 4) ECM, Q, ICN, SIG(ICN)</td>
</tr>
<tr>
<td>0322</td>
<td>WRITE(6, 110)</td>
</tr>
<tr>
<td>0323</td>
<td>CALL WRITE(ONE)</td>
</tr>
<tr>
<td>0324</td>
<td>ICN = 4</td>
</tr>
<tr>
<td>0325</td>
<td>ENSUM = C.C</td>
</tr>
<tr>
<td>0326</td>
<td>SIG(ICN) = 0.0</td>
</tr>
<tr>
<td>0327</td>
<td>DO 9802 I = 1, 30</td>
</tr>
<tr>
<td>0328</td>
<td>DO 9802 J = 1, 20</td>
</tr>
<tr>
<td>0329</td>
<td>JEF = I</td>
</tr>
<tr>
<td>0330</td>
<td>JF = J</td>
</tr>
<tr>
<td>0331</td>
<td>ONE(JEF, JF) = 0.0</td>
</tr>
<tr>
<td>0332</td>
<td>CONTINUE</td>
</tr>
<tr>
<td>0333</td>
<td>JA = 1</td>
</tr>
<tr>
<td>0334</td>
<td>CONTINUE</td>
</tr>
<tr>
<td>0335</td>
<td>CONTINUE</td>
</tr>
<tr>
<td>0336</td>
<td>6005</td>
</tr>
<tr>
<td>0337</td>
<td>DO 6001 I = 1, 30</td>
</tr>
<tr>
<td>0338</td>
<td>DO 6001 J = 1, 20</td>
</tr>
<tr>
<td>0339</td>
<td>DC 6001 K = 1, 4</td>
</tr>
<tr>
<td>0340</td>
<td>JEF = I</td>
</tr>
<tr>
<td>0341</td>
<td>JF = J</td>
</tr>
<tr>
<td>0342</td>
<td>IX = K</td>
</tr>
<tr>
<td>0343</td>
<td>SFUPAP(JEF, JF, IX) = 0.0</td>
</tr>
<tr>
<td>0344</td>
<td>CONTINUE</td>
</tr>
<tr>
<td>0345</td>
<td>SFURAB = 0.0</td>
</tr>
<tr>
<td>0346</td>
<td>DD 6000 JC = 1, 4</td>
</tr>
</tbody>
</table>
0347  IX=JC
0348  AN=67.
0349  ZN=3L.
0350  FEF=2*JA-1.75
0351  FJ=40.0-2*JB
0352  JJEF=EEF/2+1
0353  JJJF=FJ/2.0+1.5
0354  FUBAR=FOUR(JJEF, JJJF)
0355  IF((FUPAR).LT.FEG) GO TO 16011
0356  IF((JA-3).LT.C) GO TO 7C4
0357  CALL BRANCH(GAMMA, EEF, FJ)
0358  SFURAB=SFURAB+GAMMA(IX)
0359  6000 CONTINUE
0360  IF((SFURAB).LT.FDG) GC TC 704
0361  GO TO 705
0362  7C4 IX=1
0363  SFURAB(JJEF, JJJF, IX)=1.0
0364  GO TO 16010
0365  705 CONTINUE
0366  ENSUM=ENSUM+SFURAB
0367  DO 6010 I=1,30
0368  DO 6010 J=1,20
0369  DO 6010 K=1,4
0370  JEF=I
0371  JJF=J
0372  IX=K
0373  SFURAP(JEF, JJF, IX)=SFURAP(JEF, JJF, IX)/SFURAB
0374  6010 CONTINUE
0375  16010 CONTINUE
0376  DO 6007 IU=1,30
0377  DO 6007 IE=1,20
0378  JEF=ID
0379  JJF=IE
0380  IX=4
0381  FIVE(JEF, JJF)=FIVE(JEF, JJF)+SFURAP(JEF, JJF, IX)*FUBAR
0382  IX=1
0383  SIG(ICON)=SIG(ICON)+SFURAP(JEF, JJF, IX)*FUBAR
0384  ONE(JEF, JJF)=ONE(JEF, JJF)+SFURAP(JEF, JJF, IX)*FUBAR
0385  6007 CONTINUE
0386  16011 CONTINUE
0387 JR = JR + 1
0388 IF((JR - 20) .LE. 0) GO TO 6006
0389 8521 CONTINUE
0390 JA = JA + 1
0391 IF((JA - 30) .LE. 0) GO TO 8501
0392 8511 CONTINUE

0393 ENORM = ENSUM
0394 WRITE(6, 4) ECN, Q, ICA, SIG(IGN)
0395 WRITE(6, 106)
0396 CALL WRITE(ONE)
0397 WRITE(6, 107)
0398 WRITE(6, 204)

0399 DO 2306 I = 1, 30
0400 JEF = I
0401 WRITE(6, 203) (JEF, (FIVE(JEF, JJF), JJF = 1, 10))
0402 2306 CONTINUE
0403 WRITE(6, 100)
0404 WRITE(6, 205)

0405 DO 2307 I = 1, 30
0406 JEF = I
0407 WRITE(6, 203) (JEF, (FIVE(JEF, JJF), JJF = 11, 20))
0408 2307 CONTINUE

0409 ICN = 5
0410 ENSUM = 0.0
0411 SIG(IGN) = 0.0
0412 DC 9803 I = 1, 30
0413 DC 9803 J = 1, 20
0414 JEF = J

0415 JJF = J
0416 ONE(JEF, JJF) = 0.0
0417 9803 CONTINUE
0418 IA = 1
0419 9502 CONTINUE
0420 IA = 1

0421 9006 CONTINUE
0422 DO 9001 I = 1, 30
0423    DG 9001 J=1,20
0424    DO 9001 K=1,4
0425    JEF=J
0426    JJF=J

0427    IX=K
0428    SFURAR(JEF, JJF, IX)=0.0
0429    9001 CONTINUE
0430    SFURAB=0.0
0431    DO 9000 IC=1,4
0432    IX=IC

0433    AN=66.
0434    ZN=31.
0435    EEF=2*IA-1.75
0436    FJ=40.0-2*IF
0437    JJE=EFF/2+1
0438    JJFJ=FJ/2.0+1.5

0439    FUBAR=FU5E(JJF, JJFJ)
0440    IF((FUBAR).LT.FCG) GC TO 10011
0441    IF((IA-3).LT.0) GO TO 706
0442    CALL BRANCH(OMA, EEF, FJ)
0443    SFURAB=SFURAB+GAMMA(IX)
0444    9000 CONTINUE

0445    IF((SFURAB).LT.FCG) GO TO 706
0446    GC TO 707
0447    706 IX=1
0448    SFUBAR(JJEF, JJFJ, IX)=1.0
0449    GO TO 1901C
0450    707 CONTINUE

0451    ENSU=ENSU+SFURAB
0452    DC 9010 I=1,30
0453    DC 9010 J=1,20
0454    DO 9010 K=1,4
0455    JEF=J
0456    JJF=J

0457    IX=K
0458    SFURAR(JEF, JJF, IX)=SFURAR(JEF, JJF, IX)/SFURAB
0459    9010 CONTINUE
0460    19010 CONTINUE
0461    DO 9007 IE=1,30
0462    90 5007 IE=1,20
JEF=1D
JJC=1E
IX=1
ICN=5

0467  ONE(JEF,JJC)=CNE(JEF,JJC)+SFUBAR(JEF,JJC,IX)*FLBAR
0468  SIG(ICN)=SIG(ICN)+SFUBAR(JEF,JJC,IX)*FLBAR
0469  9007 CONTINUE
0470  15011 CONTINUE
0471  IR=IR+1
0472  IF((IB-20).LE.0) GO TO 9006

0473  9522 CONTINUE
0474  IA=IA+1
0475  IF((IA-30).LE.0) GO TO 9502
0476  9512 CONTINUE
0477  WRITE(6,108)
0478  CALL WRITE(ICN)

0479  ENORM=ENSUM
0480  WRITE(6,4) ECM, C, ICN, SIG(ICN)
0481  ICN=1
0482  WRITE(6,4) ECM, C, ICN, SIG(ICN)
0483  ICN=4
0484  WRITE(6,4) ECM, C, ICN, SIG(ICN)

0485  ICN=3
0486  WRITE(6,4) ECM, C, ICN, SIG(ICN)
0487  ICN=2
0488  WRITE(6,4) ECM, C, ICN, SIG(ICN)
0489  ICN=5
0490  WRITE(6,4) ECM, C, ICN, SIG(ICN)

0491  REAC(5,7) ECM
0492  IF((ECM).LT.1.0) GO TO 11111
0493  READ(5,7) C
0494  REAC(5,3)(SIGCAP(JCN),JCN=1,10)
0495  READ(5,7) PB
0496  XY=PB

0497  GO TO 7515
0498  11111 CONTINUE
0499  100 FORMAT(1HL///)
0500  101 FORMAT(1HL,6X,5HPOPULATION OF STATES IN GERMANY 67 AFTER CENSUS
1EMISSION///)
0501  102 FORMAT(1HL,6X,5HPOPULATION OF STATES IN GERMANY 67 AFTER CENSUS
11ion of a neutron by the compound nucleus

103 FORMAT (1H1, 6X, 87Hpopulation of states in gallium 67 after emission
in of a proton by the compound nucleus.///)

104 FORMAT (1H1, 6X, 87Hpopulation of states in germanium 66 after neutron
emission.///)

105 FORMAT (1H1, 6X, 85Hpopulation of states in gallium 66 after proton e
emission from states in germanium 67///)

106 FORMAT (1H1, 6X, 85Hpopulation of states in gallium 67 after gamma e
mission///)

107 FORMAT (1H1, 6X, 96Hpopulation of states in gallium 66 after neutron
emission from states in gallium 67 (additive///)

108 FORMAT (1H1, 6X, 55Hpopulation of states in gallium 66 after gamma e
mission///)

109 FORMAT (1H1, 6X, 58Hpopulation of states in germanium 67 after gamma e
mission///)

110 FORMAT (1H1, 6X, 58Hpopulation of states in germanium 66 after gamma e
mission///)

203 FORMAT (//L2, 5X, 1CE12.3)

8505 FORMAT (2F10.5)

204 FORMAT (3HJEF, 10X, 3HJC, 6X, 3HJ#1, 9X, 3HJ#2, 9X, 3HJ#3, 9X, 3HJ#4, 9X, 3HJ#5, 9X, 3HJ#6, 9X, 3HJ#7, 9X, 3HJ#8, 9X, 3HJ#9///)

205 FORMAT (3HJEF, 9X, 4HJ#10, 3X, 4HJ#11, 3X, 4HJ#12, 3X, 4HJ#13, 3X, 4HJ#14, 3X, 4HJ#15, 3X, 4HJ#16, 3X, 4HJ#17, 3X, 4HJ#18, 3X, 4HJ#19///)

206 FORMAT (3HJEF, 9X, 4HJ#20, 3X, 4HJ#21, 3X, 4HJ#22, 3X, 4HJ#23, 3X, 4HJ#24, 3X, 4HJ#25, 3X, 4HJ#26, 3X, 4HJ#27, 3X, 4HJ#28, 3X, 4HJ#29///)

207 FORMAT (3HJEF, 9X, 4HJ#30, 3X, 4HJ#31, 3X, 4HJ#32, 3X, 4HJ#33, 3X, 4HJ#34, 3X, 4HJ#35, 3X, 4HJ#36, 3X, 4HJ#37, 3X, 4HJ#38, 3X, 4HJ#39///)

STOP

DEBUG UNIT(6), INIT(ENORM)

AT 9001

END

TOTAL MEMORY REQUIREMENTS 007700 BYTES
SUBROUTINE BRANCH(GAMMA, AA, 33)
DIMENSION GAMMA(4), CL(4)
DIMENSION E(4, 9)
DIMENSION TLA(32, 27), TLP(2, 23, 27), TLH(2, 23, 27)
DIMENSION SFURAR(30, 2C, 4)
REAL JF, JR
COMMON B, AN, ZN, IX, EF
COMMON U
COMMON CL
COMMON TLA, TLP, TLH
IF((AA) .LE. 0.0) GO TO 33
AB = .96
CC = 3.0
RI = 15.58
C = (1. - AB * EXP(-.693 * CC * AA)) * 31
A JR = -.5 + 0.5 * SQRT(1.0 + 0.5 * AA * CC)
IF((BB - AJR), GT, 0.0) GO TO 33
COMMON SFURAR
GAMMA(I X) = 0.0
SFUSU = 0.0
CONTINUE
DIMENSION PN(5), PZ(5)
PZ(1) = -1.36
PZ(2) = 0.0
PZ(3) = -1.09
PZ(4) = 0.0
PZ(5) = -1.47
PZ(6) = 0.0
PN(1) = -1.46
PN(2) = 0.0
PN(3) = -1.44
PN(4) = 0.0
PN(5) = -1.46
PN(6) = 0.0
ACN = 68.
0037  ZCN=32.
0038  IZ=ZCN-ZN+1.
0039  IN=ACN-AN-ZCN+Z+N+1.
0040  IF(IA=3.0) 4000,4009,4001

0041  4003  E=3.0+PN(IN)+PZ(IZ)
0042  W=ALEVEL(E,EP)*AA*2.0/9.0+(3.0-FA)*10.0
0043  GO TO 4002
0044  4001  E=FA+PN(IN)+PZ(IZ)
0045  W=ALEVEL(E,EP)
0046  4002  CONTINUE

0047  GG TC 17
0048  33  GAMMA(IX)=0.0
0049  GO TO 20
0050  CONTINUE
0051  17  GO TO (4,5,6,7),IX
0052  4  BX=0.0

0053  DZ=0.0
0054  DA=0.0
0055  GO TO 9
0056  5  CALL BIND(BX)
0057  DZ=2.0
0058  DA=4.0

0059  GO TO 8
0060  6  CALL BIND(BX)
0061  DZ=1.0
0062  DA=1.0

0063  GO TO 8
0064  7  CALL BIND(BX)
0065  DZ=0.0
0066  CA=1.0
0067  CONTINUE
0068  8  U=AA-BX

0069  IF(U<0.0) GO TO 16
0070  GO TO 33
0071  19  CONTINUE
0072  AN=AN-CA
0073  ZN=ZN-DZ
0074  FACTA=AN*DA/(AN+DA)

0075  IZ=ZCN-ZN+1.
0076  IN=ACN-AN-ZCN+ZN+1.
0077    DO 111 I=1,30
0078    DO 2 K=1,20
0079    EF=2*I-1.75
0080    XF=AA-EF-BX

0081    IF((XE).LT.0.0) GO TO 3
0082    JEF=EF/2+1
0083    C=1.-AB*EXP(-.693*CC*EF)*RI
0084    JR=-0.5*0.5*SQRT((1.0+.5*EF*C)
0085    JF=40-2*K
0086    JF=JF/2.0+1.5

0087    IF((JF-JR).GT.0.0) GO TO 2
0088    IF((JF).LT.0.0) GO TO 2
0089    CALL FURG1(XE,XF,GR,JF,TLA,TLP,TLN,SGMA,AA,IZ)
0090    IF(EF-3.0) 3000,3000,3001
0091    3000  E=3.0+PNI(IN)+PZ(IZ)
0092    YAJ=SIGMA*(ALVEL(E,JF)*EF**2.0/G,0+10.0*(3.0-EF))

0093    GO TO 3002
0094    3001  E=EF+PNI(IN)+PZ(IZ)
0095    YAJ=SIGMA*ALVEL(E,JF)
0096    3002  CONTINUE
0097    SFUBAR(JEF,JF,IX)=FACTA*X*YAJ/X
0098    IF((IX).EQ.2) GO TO 3012

0099    EF=2*I-0.75
0100    XE=AA-EF-BX
0101    IF((XE).LT.0.0) GO TO 3012
0102    IF((XE).LT.0.0) GO TO 3C12
0103    CALL FURG1(XE,XF,PP,JF,TLA,TLP,TLN,SGMA,AA,IZ)
0104    IF(EF-3.0) 3010,3C10,3C11

0105    3010  CONTINUE
0106    E=3.0+PNI(IN)+PZ(IZ)
0107    YAJ=SIGMA*(ALVEL(E,JF)*EF**2.0/G,0+10.0*(3.0-EF))
0108    GO TO 3013
0109    3011  CONTINUE
0110    E=EF+PNI(IN)+PZ(IZ)

0111    YAJ=SIGMA*ALVEL(E,JF)
0112    3013  CONTINUE
0113    SFUBAR(JEF,JF,IX)=SFUBAR(JEF,JF,IX)*FACTA*XYJ/X/2.0
0114    3012  CONTINUE
0115    SFUSUM=SFUSUM+SFUBAR(JEF,JF,IX)
0116    2 CONTINUE
CONTINUE
3  GAMMA(I)=5FUSUM
GO TO 20
CONTINUE
U=AA-BX
IZ=ZCA-ZN+1.
IN=ACN-AN-ZCN+ZN+1.
DO 13 LL=1,4
L=LL
DO 11 I=1,30
EF=2*1-1/75
XE=AA-EF
IF((XE+2*0.,LT.0.0)) GO TO 13
IF((XE),GT.0.0) GO TO 7560
XE=0.20
EF=AA-XE
7000 CONTINUE
JEF=EF/2+1
C=(1.-AB*EXP(-.693*CC*EF))<21
JR=-0.5*C.5*SQR(T(1.0+8.*EF*C))
DJ=L
JD=BR+DJ+0.5
LF=PB-DJ+0.5
DO 12 K=LF,JD
JF=K
JJF=JF/2+1
IF((JF),LT.0.0) GO TO 12
IF((JF-JR),GT.0.0) GC TO 12
IF(FF-EF,GT.0.0) GC 2000,2002,2001
2000 E=3.0+PN(11)+PZ(12)
YAJ=ALEVEL(E,JF)*EF**2.0/S.0+(3.0-EF)**1.0
GC TO 2002
2001 E=EF+PN(11)+PZ(12)
YAJ=ALEVEL(E,JF)
2002 CONTINUE
IF((L),EQ.1) GC TO 5000
IF((B),LT.4.0) GC TO 202
IF((AA),LT.3.0) GC TO 202
IF((L),EQ.2) GC TO 200
202 CONTINUE
<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0157</td>
<td>GO TO 5001</td>
<td>CONTINUE</td>
</tr>
<tr>
<td>0158</td>
<td>5000 CONTINUE</td>
<td></td>
</tr>
<tr>
<td>0159</td>
<td>GIDIP=41C.26/((20.1-1L)*26.25)</td>
<td></td>
</tr>
<tr>
<td>0160</td>
<td>SFUBAR(JEF, JJF, IX)=SFUBAR(JEF, JJF, IX)+GIDIP*CL(L)<em>Z**2</em>(2.07+1.)</td>
<td></td>
</tr>
<tr>
<td>0161</td>
<td>YAJ</td>
<td>W</td>
</tr>
<tr>
<td>0162</td>
<td>5001 CONTINUE</td>
<td></td>
</tr>
<tr>
<td>0163</td>
<td>SFUBAR(JEF, JJF, IX)=SFUBAR(JEF, JJF, IX)+CL(L)<em>Z**2</em>(2.07+1.)*YAJ</td>
<td>W</td>
</tr>
<tr>
<td>0164</td>
<td>5002 CONTINUE</td>
<td></td>
</tr>
<tr>
<td>0165</td>
<td>SFUSUM=SFUSUM+SFUBAR(JEF, JJF, IX)</td>
<td></td>
</tr>
<tr>
<td>0166</td>
<td>GC TC 204</td>
<td></td>
</tr>
<tr>
<td>0167</td>
<td>200 CONTINUE</td>
<td></td>
</tr>
<tr>
<td>0168</td>
<td>DELJ=ABS(JF-PR)</td>
<td></td>
</tr>
<tr>
<td>0169</td>
<td>IF((DELJ).EQ.2.0) GC TC 201</td>
<td></td>
</tr>
<tr>
<td>0170</td>
<td>GC TO 202</td>
<td></td>
</tr>
<tr>
<td>0171</td>
<td>201 CONTINUE</td>
<td></td>
</tr>
<tr>
<td>0172</td>
<td>CONJ=2.<em>((2.<em>BB+1.</em>)/(BB+1.</em>)<em>((BB-1.</em>)**2-1.*))</td>
<td></td>
</tr>
<tr>
<td>0173</td>
<td>WD=1.0/ALVEL(F,PR)</td>
<td></td>
</tr>
<tr>
<td>0174</td>
<td>WE=1.0/YAJ</td>
<td></td>
</tr>
<tr>
<td>0175</td>
<td>ETWO=CONJ*(WD/(WD+WE))**5</td>
<td></td>
</tr>
<tr>
<td>0176</td>
<td>SFUBAR(JEF, JJF, IX)=SFUBAR(JEF, JJF, IX)+CL(L)<em>Z**2</em>(2.07+1.*YAJ</td>
<td>W</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0177</td>
<td>ETMAX=ZN**2</td>
<td></td>
</tr>
<tr>
<td>0178</td>
<td>IF(ETWO-ETMAX).GT.0.0) GC TO 253</td>
<td></td>
</tr>
<tr>
<td>0179</td>
<td>GO TO 251</td>
<td></td>
</tr>
<tr>
<td>0180</td>
<td>250 ETWO=ETMAX</td>
<td></td>
</tr>
<tr>
<td>0181</td>
<td>251 CONTINUE</td>
<td></td>
</tr>
<tr>
<td>0182</td>
<td>GO TO 5002</td>
<td></td>
</tr>
<tr>
<td>0183</td>
<td>204 CONTINUE</td>
<td></td>
</tr>
<tr>
<td>0184</td>
<td>12 CONTINUE</td>
<td></td>
</tr>
<tr>
<td>0185</td>
<td>11 CONTINUE</td>
<td></td>
</tr>
<tr>
<td>0186</td>
<td>13 CONTINUE</td>
<td></td>
</tr>
<tr>
<td>0187</td>
<td>GAMMA(I)=SFUSUM</td>
<td></td>
</tr>
<tr>
<td>0188</td>
<td>20 RETURN</td>
<td></td>
</tr>
<tr>
<td>0189</td>
<td>END</td>
<td></td>
</tr>
</tbody>
</table>

TOTAL MEMORY REQUIREMENTS 0C1356 BYTES
FORTRAN IV G LEVEL 0, MOD C

ALEVEL

DATE = 6/3/17

3/13/71

0001 FUNCTION ALEVEL(E,J)
0002 REAL M
0003 REAL J
0004 CC=3.0
0005 A=1.96
0006 R=15.58
0007 CCC=CC*5.0
0008 IF(E-CCC)4,5,5
0009 5 C=R I
0010 GO TO 6
0011 4 C=11.-A*B*EXP(-.693*CC*C)/C
0012 6 CCONTINUE
0013 2 A=8.5
0014 FACT=5.232
0015 T=(1.5+SQR(2.25+4.*A*E))/2.*A
0016 10 CCONTINUE
0017 M=J+.5
0018 ET=1E+.5*T
0019 ALEVEL=FACT*2.*E*EXP(-.5*SQR(A*E)-M)/E
0020 3 RETURN
0021 END

TOTAL MEMORY REQUIREMENTS 000252 BYTES
SUBROUTINE FURG(X,E,B,F,J,L,T,TL,T,L,SIGMA,AN,ZR)

DIMENSION TLA(32,27), TLP(2,20,27), TLN(2,20,27)

REAL JF

JIX=1X-1

JXE=XE

JXEAR=JXE/2+1

JXEAV=JXEAR+1

XEA=2*(JXEAV-1)

INTER=0.5*(X-E-XEA)

GO TO (2,1,1), JIX

1 S=0.0

DA=1.0

GO TO 3

2 S=0.5

IF((X-E),LE,5.0) GO TO 7

DA=4.0

GO TO 7

3 JS=ABS(JF-S)

JJS=JF+S

FACTA=.566*(2.*PB+1.)*(AN+DA)/(2.*JF+1.)*(2.*S+1.)*AN*DA*XE

CONTINUE

SIGSUM=C*C

DO 4 JJ=JS,JJS

4 BJ=JJ

LL=ABS(2*PJ)

LLL=BB+BJ

TALLY=C*C

DO 5 L=LL,LLL

5 J=JJ-JS+1

GO TO (11,12,13), JIX

11 IF((L-32),CT,6) GO TO 6

TLX=TLA(L,JXFA)-(TLA(L,JXFA)-TLA(L,JXFA))&INTER

GO TO 14

12 IF((L-20),CT,6) GO TO 6

TLX=TLP(J,L,JXFA)-(TLP(J,L,JXFA)-TLP(J,L,JXFA))&INTER

GO TO 14

13 IF((L-20),CT,6) GO TO 6
0037 TLX = TLX(J, L, JXEA) - (TLN(J, L, JXEA) - TLN(J, L, JXEA(L))) * INTER
0038 14 TALLY = TALLY + TLX
0039 5 CONTINUE
0040 6 SIGSUN = SIGSUN + TALLY

0041 4 CONTINUE
0042 SIGMA = SIGSUN * FACTA
0043 GO TO 8
0044 7 SIGMA = 0.0
0045 8 CONTINUE
0046 RETURN

0047 END

FORTRAN IV, LEVEL 9, WCO 0
FURG
DATE = 63/17
31/12/71

TOTAL MEMORY REQUIREMENTS 000760 BYTES
SUBROUTINE BND(X)
DIMENSION B(4,9)
COMMON B, AN, ZN, IX, EF
ACN=69.0
ZCN=32.0
ICA=ACN-AN+2.0*(ZCN-ZN)+1.0
BX=B(IX,ICN)
RETURN
END

TOTAL MEMORY REQUIREMENTS 000196 BYTES
Appendix VI. The $\text{Ar}^{40} + \text{Si}^{28}$ Target-Projectile System.

Extension of the study of the compound nucleus Ge$^{68}$ to reactions induced by the target-projectile pair Si$^{28}$ and Ar$^{40}$ would provide a test of the effects of extremely high angular momentum on the decay of this compound nucleus.

The pertinent data for this target-projectile system are contained in table VIII. The large negative binding energy makes possible the extension of measurements to much lower Ge$^{68}$ excitation energies than were possible for either the C$^{12}$ or O$^{16}$ systems. However, the high projectile energies necessary to form the compound nucleus might present a problem, since recent work$^{122}$ has shown that the cross-section for complete fusion processes above 100 MeV decreases with increasing energy of the bombarding particle. Attempts were made$^{122}$ to explain this behavior by considering a sharp cut-off in the spin distribution of the compound nucleus. Collisions which would lead to higher compound nucleus spins were assumed to result instead in noncomplete fusion processes.

Obvious experimental difficulties would be expected for this system (e.g. low beam currents and beam energy uncertainty), however theoretical problems might be even more formidable. For instance, the optical model description of nuclear interactions will likely prove to be inadequate for a complex system such as this. A model which considers interactions with two potential wells might be more appropriate.
Also, for higher angular momentum states, fission may become a predominant mode of de-excitation for the compound nucleus $^{123}_{\text{nucleus}}$.

The reactions studied would be those leading to products one or two mass units removed from the compound nucleus, in order to avoid reactions which would proceed predominantly via direct interaction mechanisms.
## Table VIII

<table>
<thead>
<tr>
<th>V(MeV)</th>
<th>(E_{BB}(\text{MeV})^{79})</th>
<th>(E_{Ar^{40}}(\text{MeV})^{\dagger})</th>
<th>(E^*(\text{MeV})^{\dagger})</th>
</tr>
</thead>
<tbody>
<tr>
<td>37.7</td>
<td>-10.042</td>
<td>91.5</td>
<td>27.7</td>
</tr>
</tbody>
</table>

\(\dagger\) Listed values are minimum energies required to overcome the coulomb barrier.

<table>
<thead>
<tr>
<th>Element</th>
<th>Isotope</th>
<th>Natural Abundance (atomic %) (^{56})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Si</td>
<td>28</td>
<td>92.21</td>
</tr>
<tr>
<td></td>
<td>29</td>
<td>4.70</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>3.09</td>
</tr>
<tr>
<td>Ar</td>
<td>36</td>
<td>0.337</td>
</tr>
<tr>
<td></td>
<td>38</td>
<td>0.063</td>
</tr>
<tr>
<td></td>
<td>40</td>
<td>99.60</td>
</tr>
</tbody>
</table>
$\text{Cu}^{63}(\alpha, 2n)\text{Ga}^{65} + \text{Cu}^{63}(\alpha, \text{pn})\text{Zn}^{65}$

FIGURE 34
$^{68\text{Ge}} \rightarrow \epsilon^{68\text{Ga}} \rightarrow \beta^{+}^{68\text{Zn}}$

$t_{1/2} = 66.5$ m.

$A_{\text{Ga}^{68}}$

$A_{\text{Ga}^{68}}^{\text{max}} - A_{\text{Ga}^{68}}$

FIGURE 35