OPTIMAL EXCHANGE MARKET INTERVENTION

by

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OPTIMAL EXCHANGE MARKET INTERVENTION

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ABSTRACT
This thesis examines the appropriate justification for, and optimal methods of, government intervention in foreign exchange markets. It begins with a critique of the principal arguments for and against intervention. Those arguments which have traditionally been accepted as possible criteria for an optimal intervention rule are rejected. Instead, the purpose of intervention is taken to be the minimization of unpredictable exchange rate movement. The thesis also deviates from earlier approaches to optimal intervention by recognizing the operating constraint imposed by finite levels of foreign exchange reserves.

An optimal intervention rule is then derived, given this operating constraint. The problem is formulated as an inventory or reserve management problem. The optimal reserve management rule specifies the reserve behaviour desired ex ante by the foreign exchange authority in terms of optimal limits on ex post reserve behaviour. The main feature of these limits is that the strength of current intervention to resist exchange rate adjustment should decrease the further reserves have moved away from some initial or target level.

The ex post exchange rate behaviour produced by this reserve management rule is then examined using simulation techniques in a Canadian setting. Ex post performance is considered in relation to both the primary objective and to additional concerns, namely, the cost of the implied average
reserve holding and the possible cost(s) of deviation from a freely floating exchange rate. The results show that there is a tradeoff between the rule's ability to smooth unsystematic exchange rate variation and its ability to follow the general trends in a freely floating exchange rate. They also show that in the absence of perfect future information, the returns to additional reserve holding diminish and may eventually become negative.
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While this thesis may reflect the help given me by others, the remaining errors are, of course, my responsibility alone.
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I. INTRODUCTION

In the 1960s and early 1970s, as it became increasingly obvious that the Bretton Woods agreement could not survive, there was much debate concerning the implications of a system of more flexible exchange rates. In addition, there were many proposals put forward for the implementation of such a system.¹ The debate on the implications of flexibility revived, among other things, the arguments for and against freely floating exchange rates. There was a general consensus, however, that while greater flexibility was desirable to relieve the strains then impinging on the system,² this flexibility should not extend to a free float, at least in the short term. Indeed, many of the proposals for implementing greater flexibility involved various rules for governing or controlling permissible exchange rate movements. And since any regime other than a free float requires intervention in foreign exchange markets, these proposals were in effect rules for exchange market intervention.

Since 1971 the Bretton Woods agreement has not been replaced by any similar long term international agreement on the

¹ Halm (1970), for example, provides a vivid record of such debate.

² Triffin (1960), (1972), Mundell (1968), Cooper (1972), Posner (1972) and Kenen (1973) provide descriptions and analysis of the strains caused by 'confidence problems' and asymmetries of adjustment. Johnson (1970), Krause (1971) and Cooper (1976) give alternative views on the importance of the strains caused by short term speculation.
limits of exchange rate flexibility. The fact that no such agreement has been ratified can be attributed to several factors.

The first was incompatibility of objectives. The debate concerning the benefits or otherwise of exchange rate management was carried out on two levels. On one hand, there were the arguments which focused on the welfare implications of such management for an individual country. On the other hand, there were those arguments which focused on the necessary conditions for the coordination of individual countries' actions and the smooth functioning of the international monetary system as a whole. The support for a new international agreement on exchange rate management generally came from those who believed that rules of conduct were necessary for smooth functioning of the system. However, the arguments, both political and economic, used to support this belief presumed to a greater or lesser extent that such rules would thwart the desires of individual countries. Given the political climate of the time, and in particular the strong feelings of nationalism stirred by leaders such as De Gaulle, the lack of consensus on the form of a new agreement was not surprising.³

The second impediment to a new international agreement was rejection of unequal treatment. The search for a replacement to the Bretton Woods system involved more than a search for a

³ See, for example, Krause (1970) and Cooper (1970) on these points.
common objective - it required a system of methods or rules to achieve that aim. Given the necessity of compromise on objectives, it was felt that a set of rules could minimize the scope for chiselling, or at least provide some standard by which to monitor each country's performance. However, even if agreement could be reached on a single objective or weighting of competing objectives, the recognition of differences between countries implied differences in the methods of achieving that aim. Many countries were unwilling, again for political reasons, to accept the institutionalization of unequal treatment within the terms of a long term agreement. 5

The last impediment to a new international agreement concerned choice of rules. Even among those who supported the concept of a new international agreement there were doubts as to the ability of any particular set of rules to achieve its aim - whether in terms of promoting international harmony or increasing the welfare of individual countries - while avoiding the pitfalls of the Bretton Woods system. One of the major flaws of that agreement was the difficulty of applying the clause on 'fundamental disequilibrium'. This clause allowed any country to devalue when its established parity was deemed to be in

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* In the 1960s and early 1970s discussion centred on the differences or asymmetries between surplus and deficit countries and on the special role played by the United States as the world's reserve centre.

5 See Corden (1981) for this point.
'fundamental disequilibrium'. Nevertheless, the IMF articles contained no clear definition of the term. More importantly, authorities with less than perfect knowledge and foresight faced the problem of identifying exactly what the true equilibrium exchange rate should be. As a result they also found difficulty in distinguishing a disequilibrium that was truly permanent and fundamental, in the sense of arising from secular growth or structural change, from one that arose through temporary changes and would be subsequently corrected without further action. This problem was not unique to the Bretton Woods system. Those who attempted to formulate new proposals for exchange rate management faced the same dilemma. The problem was to design rules which could achieve their aim without presuming, firstly, a knowledge of the true equilibrium exchange rate or secondly, a recognition of the distinction between trend, cycle and 'white noise' from any viewpoint other than hindsight. So while there is no international agreement on exchange rate management, the international monetary system is 

6 The United States, in its position as the world's reserve centre and 'n th country', was of course an exception.  

7 It can be argued that this difficulty in recognition led to delays in changing parities until enough evidence had accumulated to indicate a disequilibrium of crisis proportions. The adjustment, when it finally came, was therefore more traumatic than it need have been. Another reason for delays was that parity adjustments were seen as a blow to national prestige. It is probable that these factors were not independent.  

8 See, for example, Cooper (1970), Tosini (1977) and Bigman (1980) on this point.
nevertheless now characterized by 'managed floating', where the term in its broadest sense can be applied to any exchange rate regime lying between the extremes of a free float or perfect fixity. Since 1971, individual countries have been free to choose the regime they will adopt, a freedom legitimized in the Jamaican Agreement among IMF members in 1976. And while the regimes have differed widely in form, most have involved limited flexibility. They have been designed to allow more frequent, though not necessarily larger, changes than those which occurred under the 'fundamental disequilibrium' clause of the Bretton Woods agreement. However, none have allowed market forces unfettered sway.\(^9\)

Some countries have adopted a nominally floating exchange rate, but their foreign exchange authorities have at various times engaged in discretionary intervention in foreign exchange markets.\(^10\) Other countries have adopted a group float,\(^11\) setting limits on relative movements between members while allowing fluctuations against non-member currencies. Still others\(^12\) have pegged to a 'basket' of currencies, rather than to a single currency. The effect of this is that fluctuations against each individual basket currency tend on average to be smaller than if the country were pegged to one of them alone and the others were

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\(^9\) Chile is a possible exception.

\(^10\) Included in this category are Canada and the United States.

\(^11\) An example is the European 'snake' arrangement.

\(^12\) Austria and New Zealand are examples.
floating against it. Nevertheless, these countries can institute de facto floating by changing the weights applied to the components of the basket.\textsuperscript{13} Finally, there has been a tendency for pegged rates to be adjusted more frequently.\textsuperscript{14}

While managed exchange rates have been the order of the day since 1971, the heat of discussion concerning the twin issues of the justification for exchange rate management, and its appropriate form and extent, has dropped considerably. Instead, the focus of theoretical work has turned to the determinants of freely floating exchange rates. The advances in this field have been great.\textsuperscript{15} Bigman and Taya (1980, p.x) summarize the state of the art as follows:

The analysis of exchange rates in the short run has raised some serious questions in regard to many of the intuitively attractive criteria that have been put forward in advocating the floating system. Thus the analysis has considerably dampened any naive enthusiasm for full flexibility. Furthermore, it appears to have strengthened the arguments which were applied in the 1960s and early 1970s against freely floating exchange rates.

If recent analysis shows that the theoretical issues have not changed substantially since the breakdown of the Bretton Woods agreement, have not the policy issues also remained the same? If recent analysis confirms the 'undesirable' properties

\textsuperscript{13} This point has been made by Roper and Turnovsky (1980).

\textsuperscript{14} Heller (1977) discusses the factors influencing a country's choice of institutional arrangement.

\textsuperscript{15} References to this work are contained in Chapter 2.
of freely floating exchange rates, does it also confirm the presumption, clearly accepted by most countries since 1971, that exchange rates should be managed? Finally, if exchange rate management is justified, do not the problems which prevented the adoption of a post-Bretton Woods accord now deserve answers? This thesis proposes to address these questions.

The Jamaican Agreement of 1976 defines the institutional framework of the current international monetary system. Since individual countries are now free to choose their own exchange rate regime, some of the problems which faced the would-be architects of a Bretton Woods replacement are no longer relevant. At least from an analytical viewpoint, it is now possible to separate the issue of what is optimal for an individual country from the issues which concern the smooth functioning of the international monetary system as a whole. Therefore, it rests with individual countries to define their objectives, although in an interdependent world a single country must of necessity take account of other countries' actions when deciding how to achieve these goals. Similarly, the institutional framework is now flexible enough to allow 'unequal methods' to evolve, without institutionalizing them in a formal agreement. Nevertheless, an individual country is still faced with the problem of finding methods or rules which do not presume omniscience. Thus the policy issues towards which a

16 The Jamaican Agreement does allow for IMF surveillance of individual members' policies, albeit in a very general and flexible manner.
A decade of analysis and experience can be applied are as follows: by what criteria should a country choose its exchange rate regime, is intervention therefore justified, and if so, what is its appropriate form and extent?

There has been some progress in the literature towards deriving optimal rules for exchange rate management. The focus continues to be on rules, rather than discretionary intervention. The reason no longer lies in the desire for a standard for international enforcement, but rather in a recognition of the superiority of rules over discretionary policy from an individual country's viewpoint. In this respect, the literature applies new theoretical developments to old problems. However, it fails in several important areas to recognise the relevance of the debate from the time of the Bretton Woods breakdown.

Firstly, the recent literature deals inadequately with the justification for intervention. It generally starts from an uncritical acceptance of some 'intuitively attractive' criterion for judging exchange rate regimes. This criterion generally focuses on what is optimal from an individual country's viewpoint, rather than on what is required to promote international harmony. However, much of the debate at the time

17 See, for example, Boyer (1978), Cox (1980), Frenkel (1980) and Roper and Turnovsky (1980).

18 This aspect of policy making is discussed in the rational expectations literature. See, for example, Lucas (1972) and Sargent and Wallace (1976).
of the Bretton Woods breakdown can be traced to disagreement about the appropriate objectives of exchange rate policy, even for an individual country. This disagreement was not successfully reconciled then and has not been fully addressed since. Therefore, this thesis begins with a critical review of the debate. A careful examination of the welfare economic foundations of many of the criteria that have been recently adopted shows them to be specious. By contrast, it is shown that exchange market intervention is justified, but for reasons which differ from those adopted to date.

The recent literature on optimal exchange rate management also fails to address some of the problems which have long surrounded the appropriate form and extent of intervention. By adopting the lessons and techniques of other fields\textsuperscript{19} the literature can deal explicitly with the foreign exchange authority's lack of perfect current and future information. However, it frequently fails to recognise what Krause (1970, P.223) has called 'operating constraints', constraints which arise when a country has several competing objectives and whose recognition can well determine the ultimate choice of rule.

The most simple operating constraint, one which is universally ignored in the recent literature, is that international reserves are finite. This constraint must be

\textsuperscript{19} Again, the literature on rational expectations is relevant.
recognised if rules are to be made operational. Therefore, this thesis derives an optimal intervention rule which takes into account this constraint.

The foreign exchange authority may have additional concerns, each of which imposes an additional operating constraint. One example is the constraint imposed by the cost of holding foreign exchange reserves for intervention purposes. Another is that imposed by the possible cost(s) of whatever systematic divergence from a freely floating exchange rate the intervention rule produces. This thesis examines the extent to which the authority can, by judicious choice of the policy parameters which the above rule makes available, influence the outcome with respect to these additional constraints. Thus the margins of choice are defined by which the authority can meet its additional concerns.

The exposition proceeds as follows. Chapter 2 reviews the arguments for and against exchange rate management, and indicates the contribution of several recent theoretical developments to this debate. Primarily, the monetary approach to the balance of payments and its converse, the monetary or asset view of exchange rate determination, have helped to clarify a number of issues. Of the various justifications for exchange rate management, one is chosen as the appropriate objective of exchange market intervention. The objective chosen is the

20 Mexico's recent problems underscore the importance of this constraint.
minimization of unpredictable exchange rate movement.

Chapter 3 derives an intervention rule which will achieve the chosen objective subject to the constraint that reserve levels are finite. The problem is formulated as an inventory or reserve management problem. The resulting optimal rule specifies the reserve behaviour desired ex ante by the foreign exchange authority in terms of optimal limits on ex post reserve behaviour.

In order to evaluate this reserve management rule in terms of competing objectives or constraints, its implications for ex post exchange rate behaviour must be determined. Since an analytical solution proves difficult, the implications for exchange rate behaviour are determined using simulation techniques. The simulation environment is developed in Chapter 4. This chapter derives an equilibrium relationship between reserve level and exchange rate behaviour and reports the results of estimating this relationship in a Canadian context.

Chapter 5 uses the results of simulating the optimal reserve management rule in a Canadian context to examine its implications in terms of several competing objectives or constraints. It also considers the effects when the foreign exchange authority varies the decision parameters on which the rule depends. From these results, it defines those remaining margins of choice by which the foreign exchange authority can meet its additional concerns.
Chapter 6 summarizes the main theoretical findings and restates their policy implications.
II. JUSTIFICATION FOR INTERVENTION

This chapter addresses the following interrelated questions: by what criteria should an individual country choose its exchange rate regime, and is intervention therefore justified. These questions have not received satisfactory attention to date. However, they must be answered before an optimal intervention policy can be derived.

The discussion of criteria by which to judge exchange rate regimes has its genesis in the now familiar debate on fixed versus freely floating exchange rates. Much of the dispute over the relative merits of these extreme regimes was founded in a disagreement over the appropriate objectives of exchange rate policy. Since the breakdown of the Bretton Woods agreement, a country's options have been widened to include any regime intermediate between complete fixity and a free float. However, much of the confusion over the appropriate objectives of exchange rate policy persists.

This chapter reviews the arguments for and against freely floating exchange rates in order to firstly isolate, and secondly evaluate, the implicit criteria used to judge exchange rate regimes. These arguments have proceeded on both political and economic levels. Within the framework of the economic arguments for and against flexibility, this chapter also considers more recent contributions from the literature on optimum currency areas, rational expectations and choice under
uncertainty, contributions which have examined more fully the costs and benefits of exchange rate flexibility. A careful examination of the welfare economic foundations of the arguments for intervention in foreign exchange markets shows that many of them are based on inappropriate objectives. These objectives are not the appropriate targets for exchange rate policy in the well-defined sense that to achieve them by using such policy is a second best solution - the first best solution lies elsewhere. Finally, this chapter identifies an objective for which exchange market intervention represents a first best solution.

A Review of Political Arguments

The classic political arguments for freely floating exchange rates have followed several lines. It has been argued that freely floating exchange rates are desirable because they promote national autonomy. In particular, freely floating exchange rates allow countries to select their preferred mixture of inflation and unemployment (their preferred point on the so-called Phillips curve) free from the constraints imposed by external balance and without the need to consider the stabilization policies of other countries.¹ Johnson (1970) has also argued that this freedom from balance of payments constraints would promote the dismantling of quantitative trade

¹ Since the exchange rate moves to clear the foreign exchange market without the need for reserve changes and their associated domestic monetary impact, the government is then free to determine the nominal quantity of money, and is not bound to use demand management policies to maintain external balance.
restrictions and thus allow full exploitation of the gains from trade. In addition, Milton Friedman has frequently argued that society benefits from the absence of any government control over the free market mechanism. Such control may be bestowed in order to achieve the most laudable of social aims, but when placed in the hands of fallible men can be used to restrict individual freedom.

Against these, it has been argued that the freedom by government to move an economy along the Phillips curve is open to abuse for short term political gain. The discipline imposed on national governments under a fixed exchange rate regime mitigates against irresponsible vote buying. It can also promote a high degree of international consultation and cooperation. Within such an environment, the dangers of competitive 'beggar thy neighbour' policies are minimized.

These political issues are implicit in many of the proposals for reform of the Bretton Woods system that were put forward in the 1960s and early 1970s. However, the lack of consensus at this level was one reason why the attempt to find a replacement for the Bretton Woods agreement founded. These issues are still important, but by themselves they provide little guidance on the issue of intervention versus non-intervention. The perceived desirability of national

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2 See, for example, Marsh, Krause, Peller and Willett in Halm (1970). These arguments can also be found in Mikesell and Goldstein (1975), Ethier and Bloomfield (1975) and Tosini (1977).
autonomy, as opposed to international cooperation, will continue to change as the winds of nationalism blow more or less strongly. It seems unlikely that mere choice of exchange rate regime can greatly influence the course of these winds.\(^3\) Furthermore, the possibility of abuse of power by governments is always present, though presumably a democratic system is designed to minimize such abuse.\(^4\) Whether the opportunity for such abuse should be minimized by following a free float requires a more careful and comprehensive study of all the costs and benefits, political and otherwise, that alternative actions may bestow.

**A Review of Economic Arguments For Flexibility**

The classic economic argument for freely floating exchange rates has been put most forcefully by Friedman (1953) and Sohmen (1969). These writers note that the exchange rate is merely a price - the price of foreign currency. Demands for and supplies of foreign currency are derived demands and supplies which arise as individuals make or receive payment for foreign goods and assets. In a world of free markets, the equilibrium exchange rate is that which equilibrates the demands for and supplies of

\(^3\) The extent to which choice of exchange regime can influence the degree of autonomy a national government enjoys is itself open to question, as will be discussed in further detail later.

\(^4\) Halm (1969) and Johnson (1970) have noted that the argument for fixed exchange rates based on the promotion of domestic and international responsibility itself largely disappears once parity adjustments are possible.
foreign exchange. It will, like all other prices, be determined in a general equilibrium system comprising supplies of and demands for all goods and assets, both domestic and foreign. Furthermore, in the absence of externalities, the resulting competitive equilibrium will be Pareto optimal. World resource allocation will be efficient and world welfare will be maximized.

This argument is a powerful one against both fixed exchange rates and any degree of intervention. Proponents of this argument maintain that the existence of market imperfections is by itself insufficient justification for intervention. If a market distortion exists, the appropriate action is to directly remove the distortion. Thus exchange market intervention can apparently be justified only if it can be shown that a freely floating exchange rate regime leads to externalities or market failures.

A Review of Economic Arguments Against Flexibility

The most influential arguments against freely floating exchange rates during the 1960s and early 1970s were those which indicated why the regime would lead to 'excessive' exchange rate fluctuations. Among these arguments were those that pointed to the J-curve phenomenon, the possibility of destabilizing speculation and the difficulties of transition to such a regime. Implicit in these arguments was the idea that exchange rate fluctuations were disruptive and should therefore be smoothed.
Recent arguments have taken for granted the existence of fluctuations, 'excessive' or otherwise, in freely floating exchange rates and have examined more closely the costs and benefits associated with them. Important contributions have come from the literature on optimum currency areas, rational expectations and choice under uncertainty. The arguments have generally identified the costs and benefits of exchange rate volatility in terms of its implications for either employment or consumption.

Each type of argument is now examined in order to evaluate its implicit welfare economic foundation.

Reasons for Excessive Exchange Rate Instability

(i) The J-curve

The J-curve phenomenon can arise when the short run price elasticities of demand for imports are very low. This raises the possibility that in the short run, under fixed exchange rates, a depreciation will increase rather than decrease import spending and cause a deterioration rather than an improvement in the trade balance. One explanation for low short run elasticities is that importers are frequently locked into long term contracts. Another is that, even without such contractual

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5 The exact conditions for this to occur have been formalized by Robinson (1947), Haberler (1949) and Machlup (1964) in what has come to be called the Marshall-Lerner condition. The term 'J-curve' describes the resulting time path of the trade balance as it firstly deteriorates and then improves.
arrangements, it takes time to learn about price changes and make and implement decisions based on them. However, over time, as decisions are implemented and contractual arrangements are reviewed, import elasticities increase and the more usual response of the trade balance to a depreciation is restored.

Prior to 1971, the presumption was that the J-curve phenomenon would, under a system of flexible exchange rates, produce wider than 'normal' exchange rate fluctuations to offset perverse short run movements in the trade account. Some academics and many bankers and businessmen saw such wide fluctuations as undesirable because the uncertainty they engendered would be inimical to international trade and capital flows.

Some writers have recently examined more closely the interaction of trade and capital accounts under freely floating exchange rates. They have shown that indeed such exchange rate volatility can occur, especially when the trade account moves either not at all or perversely in response to exchange rate

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6 The empirical study by Dornbusch and Krugman (1976) supports the proposition that while there is considerable price responsiveness in the long run, the short run elasticities are small and the adjustment lags can be substantial. See also the evidence in Houthakker and Magee (1969), Junz and Rhomberg (1973) and Magee (1975).

7 At the forefront of such studies are the proponents of the asset market view of exchange rate determination. See, for example, the articles in the Scandinavian Journal of Economics, Vol. 78, No. 2, May 1976.
changes. Recent analysis therefore confirms the earlier presumption that under flexible exchange rates, a J-curve phenomenon increases exchange rate fluctuations over time.

There are several reasons why such exchange rate fluctuations need not be as severe as originally thought. One reason is that these fluctuations may simply reflect an inappropriate domestic policy mix. Another reason is that the fluctuations may be dampened rather than amplified by speculative activity. However, the exchange rate fluctuations that are produced by the J-curve phenomenon have been viewed as undesirable because of the uncertainty they engender.

Proponents of freely floating exchange rates would claim that market rigidities underlying the 'J-curve' criticism of flexible exchange rates constitute a market imperfection which should be removed directly, rather than offset by fixing or managing the exchange rate. However, whether such market rigidities constitute a true market imperfection is itself open to question. If these rigidities are the result of contractual arrangements, the theory of information and contracts suggests various price theoretic reasons why such contracts might be

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8 Dornbusch (1976) has shown that when exchange rate expectations are sticky, exchange rate overshooting in the face of a monetary expansion occurs in order to keep expected asset returns in line. Dornbusch and Krugman (1976) note that this overshooting will increase the more perversely the trade account reacts to the exchange rate in the short run.

9 See Dornbusch and Krugman (1976) on this point.

10 This possibility has been examined by Britton (1970) among others, and most recently by Driskill and McCafferty (1980).
made. This being the case, the issue becomes one of whether exchange rate variability is of itself undesirable and, furthermore, whether it is something that individuals are unable to optimally protect themselves against by appropriate contractual arrangements. This is a question which will be considered in more detail later in the chapter.

(ii) Destabilizing Speculation

Proponents of freely floating exchange rates saw speculators as fulfilling a vital role of smoothing the exchange rate variation produced by exogenous shocks. A second, recurrent criticism of freely floating exchange rates during the 1960s and early 1970s was that speculation could play a destabilizing rather than stabilizing role. The picture painted\(^{11}\) was one of massive international movements in speculative capital, motivated by expectations of exchange rate changes, setting forces in motion to make such expectations self-justifying. Exchange rate changes could then feed expectations of further changes and create wild cumulative swings and chaos on foreign exchange markets.

This view was challenged by Johnson (1970, p. 104), who claimed:

...speculators who engage in genuinely destabilizing speculation, that is, whose speculations move the exchange rate away from rather than toward its equilibrium level, will consistently lose money, because they will consistently be buying when the rate is 'high' and selling when it is 'low' by comparison with its equilibrium value.

\(^{11}\) See, for example, Fellner et al. (1966) and Krause (1971).
This challenge was as much a comment about an individual's ability to fight the rest of the market as it was about the possibility of homogenous destabilizing expectations. The debate at the time was unable to completely allay the fears aroused by this latter possibility. The underlying fear seems to be that speculation can, at least in the short term, divorce exchange rate determination from the underlying real forces in the economy and turn the foreign exchange market into what Keynes described the stock market as being - a casino.  

Recent analysis has shown that, even if exchange rate expectations are formed independently of real forces in the economy, there are limits on the extent to which speculators can or will wish to muster the resources to counteract those forces for significant periods of time. This argument suggests limits to the persistence of speculative capital flows, but does not necessarily imply a limit on their size or impact in the short run. This impact depends in turn on the nature of expectations formation. However, recent treatments of expectations stress that it is not rational to form expectations in a way that leads

12 See Keynes (1976, P. 159).

13 The asset market approach to exchange rate determination stresses that capital flows will be maintained only until induced changes in interest rates, prices and other variables ensure stock asset equilibrium. This process may occur more quickly than goods market adjustment.

14 See Bigman and Taya (1980) for recent asset market approaches to the issues of instability, overshooting and 'disorderly market conditions' with freely floating exchange rates.
to consistently wrong predictions. Furthermore, unbiased exchange rate predictions are ensured if expectations are formed, not in some ad hoc manner, but with an understanding of the underlying process of exchange rate determination. The resulting analysis appears to reemphasise the general equilibrium nature of exchange rate determination and the signalling function of the price mechanism. However, it does not rule out the possibility of wild exchange rate swings driven by an arbitrary element in expectations that are nevertheless unbiased and therefore self-justifying.\textsuperscript{16}

There is no doubt that speculative activity based on exchange rate expectations can of itself cause variations in the exchange rate, interest rates and prices which can have real effects elsewhere in the economy, at least in the short run. Whether such fluctuations should be seen as 'illegitimate' in some sense, thus justifying intervention, is arguable. If, on the other hand, these fluctuations are seen as undesirable per se, then intervention, to the extent that it can reduce them, may be justified. A later section examines the costs of these fluctuations.

(iii) Transition Costs

A third argument against freely floating exchange rates, while admitting the benefits of this regime, pointed to the

\textsuperscript{15} See Lucas (1972), Sargent and Wallace (1976) and Barro (1976), among others, for seminal works on rational expectations formation.

\textsuperscript{16} See Flood and Garber (1980) on this point.
likelihood of enormous transition costs. The chaos caused by abrupt adoption of floating rates by many countries would undoubtedly spill over into domestic markets. The social cost of this upheaval was judged to outweigh such benefits of the regime as may eventually accrue. Certainly the limited historical experience with truly free exchange rates would tend to support this view. However, such comparison may be unfair. Adoption of a free float, as in several European countries after WWI, generally followed widespread social and economic upheaval. Under these conditions, freely floating exchange rates could not be expected to operate smoothly to provide a general panacea for more fundamental problems. Finally, if the costs are associated with the abruptness of transition to freely floating exchange rates, they could be avoided by a gradual transition.

Costs (and Benefits) of Exchange Rate Instability

More recent approaches to the choice of exchange rate regime begin by noting that the J-curve phenomenon or destabilizing speculation are not the only causes of instability over time. They may imply that the adjustment of a freely floating exchange rate, as it responds to a single exogenous shock, displays volatility that is in some sense 'excessive'. However, such shocks are rarely isolated. Under any exchange

17 The word 'exogenous' is not used in this thesis to denote shocks from foreign sources. It relates to those elements which are 'givens' to a country, elements which may therefore include its own endowments and preferences.
rate regime, a country must continuously adjust to exogenous
shocks of various kinds. Much of the theoretical work over the
last decade has concentrated on the way in which exogenous
shocks from various sources cause adjustment and fluctuations in
a freely floating exchange rate. However, that section of the
literature which argues from the existence of fluctuations in a
freely floating exchange rate to the desirability or otherwise
of exchange market intervention generally does so using the
following line of argument.

The way in which exogenous shocks are transmitted, both
domestically and internationally, of course depends on which
markets clear 'instantaneously' and which clear 'over time',
which goods and assets are internationally traded, and whether
prices or quantities do the adjusting. Nevertheless, the shocks
ultimately lead to fluctuations over time in at least some of
the things on which welfare depends. Given the usual concavity
assumptions about utility functions, (expected) utility will,
ceteris paribus, be a decreasing function of the variance of its
arguments. Therefore, any exchange rate regime which leads to
smaller variation over time in these arguments will, ceteris
paribus, lead to a higher level of welfare.\textsuperscript{18}

Many recent contributions to the literature on optimal
exchange rate management can be interpreted as attempts to find
\textsuperscript{18} If a policy allows a lower variance of some arguments only at
the expense of increasing the variance of others, then welfare
implications can be made only by reference to the underlying
utility functions. This point is made by Lapan and Enders
(1980), but is frequently ignored elsewhere.
an exchange rate regime which can reduce such variation and increase welfare. However, all of these contributions have focused on either employment or consumption as an argument of some implicit social welfare function. We now evaluate each of these approaches in turn.

(i) Employment

The view that different exchange rate regimes have different implications for the level of, and degree of variation in, employment essentially makes choice of exchange rate regime one aspect of domestic stabilization policy for an open economy. As such, all of the recent debate concerning the desirability or feasibility of such stabilization policy becomes relevant. Frequently, however, these considerations have been ignored.

One strand of the optimum currency area argument uses considerations of variability in the level of employment to argue for freely floating exchange rates. When domestic wages are sticky and factors not fully mobile internationally, exogenous shocks must lead to fluctuations over time in the aggregate quantity of employment if external balance is to be maintained with a fixed exchange rate. Under flexible exchange rates, by comparison, depreciation can substitute for domestic

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19 The seminal works on optimum currency areas are by Mundell (1961), (1973), McKinnon (1963) and Kenen (1969). The literature considers the costs and benefits of both fixed and freely floating exchange rates in terms of their implications for the optimum size of currency areas - areas within which exchange rates are fixed and between which exchange rates are flexible. To the extent that the costs and benefits are are functions of the size of a currency area, the optimum currency area is one for which the costs and benefits are balanced at the margin.
deflation and appreciation for domestic inflation. Variation over time in the quantity of employment is seen as detrimental to welfare, either directly because welfare depends on employment, or indirectly because the variation in real income required for external balance conflicts with other domestic policy targets.\textsuperscript{20} Thus flexible exchange rates are preferable.\textsuperscript{21}

Some of the recent attempts\textsuperscript{22} to derive optimal exchange market intervention rules have also been based on the objective of stabilizing income over time. Also implicit in these attempts is the judgement that exogenous shocks, and the variations they cause in the level of employment, justify such intervention.

Recent work in the theory of inflation and unemployment has examined more closely the conditions under which the level of unemployment can deviate for any period of time from the so-called natural rate,\textsuperscript{23} even in the face of exogenous shocks. Such deviations depend crucially on price and/or wage rigidities derived from what might broadly be called informational inefficiencies, including money illusion.

\textsuperscript{20} This latter argument is the economic analogue of the political argument concerning national autonomy.

\textsuperscript{21} More recent comparisons of fixed and freely floating exchange rates on the basis of income variation are contained in Chan (1979) and Kaminow (1979).

\textsuperscript{22} See Boyer (1978), Henderson (1979), Cox (1980) and Roper and Turnovsky (1980).

\textsuperscript{23} The natural rate is, crudely speaking, that rate implied by the frictional unemployment associated with job search.
As early as 1963, McKinnon countered the optimum currency area argument by noting that a depreciation under flexible exchange rates imposes as much of a loss in real income as a monetary contraction or increase in taxation under fixed exchange rates. Depreciation feeds domestic inflation by increasing the price of traded goods and, at a given wage level, thereby reduces real income. This allows the expenditure reduction necessary to eliminate the exchange rate pressure only if workers do not use the real income reduction to negotiate a higher wage. McKinnon therefore concludes that the argument for flexible exchange rates based on its ability to free domestic policy tools for stabilization purposes must therefore rest on the fact that workers are more likely to suffer money illusion in the face of exchange rate changes than in the face of monetary contraction or taxation.24

More recently, the theory of rational expectations, in both its weak and strong forms, has suggested that the government's ability to exploit money illusion is at best limited to the short run. The essence of the weak form of rational expectations has been put succinctly by Friedman (1976, p. 231):

...since you can't fool all of the people all of the time, the true long-run Phillips curve is vertical.25

Friedman notes, however, that the natural rate of unemployment

24 This of course is more plausible the smaller the attempted deception - that is, the larger and therefore less open the economy.

25 The strong form of rational expectations suggests that even the short run Phillips curve may be vertical.
implied by a vertical Phillips curve is not an irreducible minimum - it can be reduced by eliminating frictions in the labour market.\textsuperscript{26} Furthermore, if the government can exploit superior information to reduce short run income variation around the natural rate, it can achieve the same result by simply disseminating that information.\textsuperscript{27}

Recent discussions of exchange market intervention have tended to sidestep altogether these issues raised in the context of stabilization policy for a closed economy.\textsuperscript{28} One reason is that, prior to the early 1970s, a country's exchange rate regime was not seen as an object of choice - it was merely a given within which domestic monetary and fiscal stabilization policy had to operate. From this perspective, furthermore, the discussion of stabilization in an open economy has been dominated by the assignment approach. The idea here was to assign to each target a single instrument. In general terms, however, the best policy mix may or may not turn out to be a simple one-to-one assignment. In this context, the issue of national autonomy and 'freeing of instruments' can be seen to be

\textsuperscript{26} It is possible, however, that these rigidities reflect a rational reaction by workers and employers to the uncertainty engendered by exogenous shocks. See the literature on quasi-contracts.

\textsuperscript{27} See Barro (1976) for this point.

\textsuperscript{28} Boyer (1978, P. 1048), for example, explicitly dismisses rational expectations formation as irrelevant to the issue of optimal exchange rate stability.
something of a red herring, as McKinnon's comments indicate.29 Conversely, anything which may have implications for a government's ability to achieve the target, such as the possibility of rational expectations formation, cannot be ignored.

The whole issue of the effectiveness of active stabilization policy remains unsettled and is still evoking a large body of literature. The recognition that an economy is open does add exchange rate management policy to the government's arsenal, although as the above comments make clear, it may be that this policy instrument cannot be operated independently. However, recent analysis has raised serious questions about the usefulness of trying to stabilize income by monetary, fiscal or exchange rate methods. It has generally confirmed the argument that if real income varies in the face of exogenous shocks because of market rigidities, then the first best, and possibly the only solution is to remove these rigidities directly.

(ii) Consumption

The view that different exchange rate regimes have different implications for the degree of variation in consumption introduces issues of saving behaviour and intertemporal choice into the discussion of optimal exchange rate management.

29 These points are recognised by Boyer (1980), who nevertheless argues for a specialized assignment, given imperfect information and costs of intervention.
One strand of the optimum currency area argument notes that when there is sufficient wage and price flexibility to ensure full employment, but when full stock equilibrium in money markets is achieved only over time, a fixed exchange rate system can allow a smoothing of fluctuations in expenditure relative to output that would not otherwise occur. Implicit in this approach is the judgement that since welfare depends on real consumption, which need not equal real income, intervention which can reduce the variation in real consumption over time is justified.

This argument is implicit in Mundell (1973, P.115):

...a harvest failure, strikes or war in one of the countries causes a loss of real income, but the use of a common currency (or foreign exchange reserves) allows the country to run down its currency holdings and cushion the impact of the loss, drawing on the reserves of the other country until the cost of adjustment has been spread efficiently over the future. If, on the other hand, the two countries use separate monies with flexible rates the whole loss (with a qualification) has to be borne alone.

It has also been formalized by Fischer (1977) in the context of evaluating fixed versus flexible exchange rates, and by Frenkel (1980) in the context of determining the optimal degree of exchange market intervention. These writers have also shown that the appropriate exchange rate regime will in general depend on the source of the exogenous shocks facing an economy. In particular, Fischer (1977) shows that if the shocks are primarily real, then fixed exchange rates are preferable. If,

\[ \text{---These are defined as shifts in aggregate supply.} \]
however, disturbances are nominal, flexible exchange rates lead to a lower variation in real consumption over time. Similarly, if disturbances originate overseas, then flexible exchange rates are preferable.

However, these conclusions can be seen as arising from a market imperfection, one which prevents international capital mobility. Helpman and Razin (1979), among others, have pointed out that in the simple model used by Fischer (1977) and Frenkel (1980), an economy with flexible exchange rates and no international capital movements is essentially closed. There are no channels by which it can borrow to finance consumption in the face of a harvest failure. Under fixed exchange rates, by comparison, foreign borrowing and lending, with the accompanying transfer of real resources, is permitted through the foreign exchange authority.

Clearly capital is not completely immobile internationally. Nevertheless, those who argue that intervention is justified on the grounds that it can still reduce the variation in real consumption over time must first answer another question — namely, why it is that private individuals, by their own

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31 These are defined as shifts in aggregate money demand.

32 This latter conclusion confirms the insulating properties of flexible exchange rates discussed previously.

33 Mundell (1973) is aware of the way in which capital flows affect his conclusion.

34 This asymmetry is also implicitly recognised by Laffer (1973) and in the literature on currency substitution — see, for example, Brillembourg and Schadler (1979, p. 515).
actions, cannot smooth their consumption streams to the optimum extent.

This question has recently be addressed by reference to the literature on choice under uncertainty. Implicit not only in these arguments for intervention based on consumption, but also in some of those based on employment, is consideration of the extent to which choice of exchange rate regime can affect the sharing of risk. The welfare implications of risk sharing are as follows. Under the usual assumption of risk aversion, individuals face a tradeoff between the expected level of real income (or consumption) and its variance or riskiness. To extent that the exogenous shocks to various goods supplies or asset returns are negatively correlated, diversification allows a reduction in risk for a given level of expected real income (consumption). Furthermore, the theory of choice under uncertainty suggests individuals will diversify to the most

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35 See, for example, the seminal work by Tobin (1958). Extensions into the theory of international finance have been made by Solnik (1974) and Grauer, Litzenberger and Stehle (1976). Applications to international trade include Ruffin (1974a), (1974b) and Mayer (1976).

36 See, for example, Mundell (1973) for an exposition in these terms. The risk aspect is seen even more clearly when there is some degree of non-homogeneity within, as well as between, countries. Mundell (1961) considers geographical non-homogeneity and Lapan and Enders (1980) consider demographic non-homogeneity. They then examine how risk sharing between regions or generations can substitute for risk sharing between countries.

37 Such diversification behaviour can sometimes be broken into two components - the insurance and the gambling motives normally associated with hedging and speculation.
efficient extent - they will act so as to ensure the lowest possible risk for a given expected return. If the government can do better, it can only be because it can affect a transfer of real resources between individuals, regions or countries which could not otherwise take place, and therefore allow further diversification. 38

This points to the fundamental difficulty with these arguments for intervention. Risk sharing between individuals, regions or countries will not be carried out to the optimum extent when there are barriers to the transfer of real resources. These barriers could prevent the transfer of either goods, factors or financial assets. 39 Their removal, rather than intervention, will allow individuals to increase risk sharing and hence welfare.

The Justification for Intervention

A third strand of the optimum currency area argument has focused on the implications of different exchange rate regimes for the usefulness of money. 40 Firstly, the smaller and more

38 In the language of the literature, the government can contribute toward the achievement of 'complete' markets, a situation which allows a Pareto optimal allocation of risk.

39 Grauer, Litzenberger and Stehle (1976) have shown that, under certain conditions, the efficient allocation of risk does not require free trade in goods and services, so long as financial assets are freely mobile.

40 The first two strands have generally lead to the conclusion that the benefits of having flexible exchange rates between currency areas increase, the smaller and more open is each currency area. The third strand provides the quid pro quo.
numerous are separate currency areas, the higher are the costs associated with collecting information about foreign prices and converting currencies in the course of trade. Secondly, the fewer members there are in a currency area, the smaller are the savings available by pooling foreign exchange reserves. Finally, fluctuating exchange rates at the border of the currency area cause fluctuations in the price level within the currency area. The smaller and more open the currency area, the larger the exchange rate and price level fluctuations are likely to be. These fluctuations themselves impose a cost. The area's currency will have a low liquidity value and individuals will seek to accumulate assets whose real value is more stable.

This third strand of argument has been extended in order to argue that for any country, irrespective of whether it constitutes an optimum currency area, these costs of exchange rate flexibility by themselves justify intervention in foreign exchange markets. Grubel (1973), (1977) has argued that even under the most ideal circumstances these costs remain, and further, that they constitute an externality. To illustrate this

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*1 See Mundell (1968), (1973).


*3 See McKinnon (1963).

*4 Fischer (1977) also judges that price level fluctuations are undesirable. He argues that the costs arise because price level fluctuations could lead to output changes via a Phillips curve relationship. Hence his argument is a variant of the one that focuses on employment levels, rather than part of the current argument.
point, he portrays the following scenario.

Wages and prices are assumed fully flexible and factors internally mobile. Hence full employment is maintained in the face of exogenous shocks. International capital mobility allows individuals to choose their desired consumption path over time. The foreign exchange effects of exogenous shocks are accommodated by allowing the exchange rate to float freely. The resulting exchange rate fluctuations cause instability in the domestic price of traded goods, and hence instability in the domestic price level. These price and exchange rate fluctuations will be dampened by speculative activity, but will not be entirely eliminated.\(^5\) Hence some instability remains.

In this scenario, there are no social costs associated with fluctuating employment or consumption. If factor reallocations occur instantaneously and costlessly, then there are also no social costs associated with fluctuating relative prices. Nevertheless, the costs associated with fluctuations in the price level remain. Grubel (1973, P.354) comments that, in particular:

The lower yield of services from money induces reduced holdings. The public makes do with less efficient substitutes and moves closer to the system of barter. In the process, some of the benefits from specialization in a money economy are sacrificed.

\(^5\) The reason is as follows. Price changes over time provide an opportunity for intertemporal arbitrage. The benefits of successful arbitrage remain so long as prices are less than fully stable. However, efficiency requires that speculation be carried only to the point where the benefits are equated at the margin with the forecasting, inventory storage and risk premium costs.
Grubel then asserts that the costs of price level instability constitute an externality - a cost to society which is not taken into account in private cost-benefit calculations. Hence intervention in foreign exchange markets is justified.

This argument raises a plethora of issues which lie at the heart of monetary and capital theory. The first involves consideration of what constitutes the yield of services from money, and whether price fluctuations do in fact lower the marginal yield and induce lower holdings. Most contributions in this area have concentrated on the demand, rather than the supply, side of the market.

The classic reasons for holding money have been characterized as the 'transactions' and 'asset' motives. These motives explain, respectively, why money rather than barter is used to carry out transactions at a given point in time, and why money rather than some interest-earning asset is held to transfer purchasing power through time. Different strands of monetary and capital theory have indicated how the existence of price fluctuations over time alters the margin of choice between money as a store of wealth and money as a medium of exchange.

An analysis of money holding behaviour which attempts to integrate money's role as a medium of exchange and store of value is contained in the inventory-theoretic models of Baumol (1952) and Miller and Orr (1966).\(^6\) Their cost minimization

\(^6\) They derive an optimum average money holding where the brokerage costs associated with holding interest-earning assets are balanced against money's opportunity cost in terms of foregone interest.
approach leads to an optimum average nominal money holding which will increase with the size of variation in nominal payments flows, a variance which itself may result from fluctuating prices.

On the other hand, recent work in the theory of intertemporal choice under uncertainty, particularly in the finance literature, has investigated asset preferences when assets, including money, are the means of transferring purchasing power through time. In this case, an increase in price level variation leads to an increase in the variance of the real return on money and will, ceteris paribus, lead to a reduction in the demand for that asset.

Therefore, these two theories, which by no means cover all of the subtleties of motive behind money holding, suggest that while the existence of price level variation alters the margin of choice between money in its various roles, the effect on the quantity of money demanded is ambiguous. Furthermore, neither suggests that the existence of price fluctuations over time imposes an external cost.

However, both these approaches abstract from certain aspects of money holding behaviour which are particularly relevant for the present purpose. For example, both the finance

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47 When the price level is stable, money is a riskless asset and will be held in a diversified portfolio despite a zero nominal return. See Tobin (1958). When the price level fluctuates, money becomes a risky asset whose demand then depends on the extent to which its return provides protection against variation in other security returns, or against variation in aggregate real wealth. See Grauer, Litzenberger and Stehle (1976).
literature and the inventory-theoretic models distinguish between money and other interest-bearing nominal assets. However, neither approach makes an explicit distinction between assets whose returns are denominated in nominal terms and those whose returns are denominated in real terms.\footnote{This latter distinction is conceptually possible in the paradigm of choice under uncertainty, but its inclusion imposes severe analytical difficulties.} The fact that this is ignored could be crucial, for it is at this margin of choice that price fluctuations would be expected to bite in terms of money holding behaviour. Furthermore, neither approach takes explicit account of the way in which the use of money, in the words of Hirshleifer (1970, P. 134),

...enlarges productive and consumptive opportunities by reducing 'transactions costs', imperfections of the exchange process, from what they would be under barter.

Klein (1974a), (1977) has noted that these deficiencies can be corrected by placing greater emphasis on the money supply process - or more correctly, the production process by which a given stock of real money balances supplies the flow of services for which it is held. Indeed, it is certain aspects of these monetary services to which Hirshleifer refers. Most analysis of the demand for money implicitly assumes that the marginal productivity of monetary services from a given real money stock is constant. Klein (1974a), (1977) explores the nature of monetary equilibrium when the 'quality' of money is not constant, but at least partially under the control of the suppliers of money. From this analysis, he determines the nature...
of the cost associated with price variability.

The holders of real money balances demand these because of the services they provide. The suppliers of money can meet an increase in this demand either by supplying more nominal money balances, whose real value they cannot guarantee, or by increasing the quality of the existing stock of nominal money balances. Equilibrium will occur at both a positive implicit price of monetary services and at a positive level of expenditure on the quality of money. If money is competitively supplied, a decrease in the quality of a particular supplier's money will not change the equilibrium price of monetary services but will drive demand for this money to zero. If money is not competitively supplied, a decrease in the quality of money will raise the implicit price of monetary services and lead to lower demand for real money balances on this count. However, it will also increase the quantity of real balances required to produce a given service stream and increase the demand for real money balances on this count. With two additional assumptions, it can be shown that the latter effect will dominate - a decrease in the quality of money will lead to an increase in the

49 Klein (1974a) argues that it is because different monies are distinguishable by their quality that the competitive supply of money would not lead to an infinite price level.

50 See Klein (1974a) for these points.

51 These assumptions are that the demand for money is interest inelastic, as most empirical studies find, and that the flow of services is proportional to the real money stock, with the constant of proportionality being a measure of quality.
equilibrium quantity of real money balances demanded.52

These considerations have particular relevance to the issue of exchange market intervention. One factor which adds quality to a given quantity of real money balances is predictability. An increase in price variability reduces the quality of money by making its future real value more uncertain. Given that money is not competitively supplied, but that the central monetary authority acts as the sole, or at least dominant, producer, an increase in price variability can be expected to increase the quantity of real money balances demanded.53 If one were to follow Grubel (1973) in drawing welfare conclusions based on what happens to the real quantity of money demanded, then it would appear that an increase in price variation increases welfare. However, Klein's analysis shows that these welfare conclusions can be just as misleading as those drawn by observing the nominal quantity of money.54 An increase in price variability reduces welfare, despite an increase in the quantity of real balances demanded, because it leads to an increase in the implicit price of monetary services and a fall in the consumer surplus derived from the monetary service flow. Therefore, the argument does ultimately support Grubel's (1973) proposition that price variability imposes a cost.

52 See Klein (1977) for these points.

53 Klein (1977) provides empirical support for this proposition.

54 Klein (1977) notes how the confusion between nominal stocks, real stocks and service flows persists in some of the literature on the optimum quantity of money.
However, Klein's analysis does not suggest that this cost is an external cost. His analysis shows that the quality of money can command a positive implicit price. To the extent that a central monetary authority can produce quality through exchange market intervention, either because exchange rate stabilization directly promotes 'monetary confidence' or because it smooths price level variation and enhances predictability, market efficiency requires that such activity be carried out at some positive level. If there is an externality or market imperfection at all, it lies in the fact that the monetary authority, as a monopolistic supplier, has an incentive to underinvest in quality. However, the question of market imperfections arising from a possibly natural monopoly in the provision of money can be separated from the justification for exchange market intervention, an argument which need not appeal to market imperfections at all. As Klein (1974a, P.544) himself notes,

...these...considerations argue for fixed exchange rates, but not necessarily for a dominant money.

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55 If expectations are formed rationally, the latter is more likely to be the case.

56 One reason for this is that an increase in inflation, to the extent that it reduces future price predictability and increases the quantity of money demanded, thereby increases the tax base from which the government's inflation tax is drawn.

57 Klein (1974a) attributes the naturalness of the monopoly to the diminishing marginal cost of providing monetary confidence, as the optimum currency area argument suggests, and to the extremely high cost of switching to an alternative dominant money, as indicated by the continued use of established money during many hyperinflations.
In short, market efficiency requires some expenditure on the production of quality, or investment in 'brand name capital', irrespective of whether money is supplied competitively or monopolistically. In an open economy context, this implies that the cost of holding international reserves should be incurred in order to use them for exchange rate stabilization purposes.

Nevertheless, this does not necessarily argue for absolutely and permanently fixed exchange rates. The feature which is crucial to the quality of money is its predictability. As Klein (1974a) notes, predictability may be facilitated as much by a constant rate of depreciation of the exchange rate or inflation of the price level as by a constant exchange rate or price level.\(^5\) Indeed, the experience under the Bretton Woods system suggests that exchange rate movements are required if reserves are not to be exhausted. Therefore, the argument is one for an exchange rate regime of the type that proved so elusive in the 1960s and early 1970s - one which can smooth exchange rate and price fluctuations without prejudging levels. Indeed, this will be the focus of the remaining chapters. Does such an exchange rate regime exist?

To summarize, the economic argument for fully flexible exchange rates is strong. Its strength derives from its basis in considerations of efficiency, considerations which lie at the heart of economic theory. By contrast, many arguments for

\(^5\) A regime which allows exchange rate or price trends also allows the price mechanism to perform its signalling function, an aspect of 'efficient' regimes for which Corden (1981) argues.
intervention in foreign exchange markets have been based on the existence of some market imperfection. Recent theoretical analysis has reaffirmed the importance of instability, overshooting and 'disorderly market conditions' with freely floating exchange rates. To the extent that this imposes a social cost in terms of variations in employment or consumption levels, the first best solution is the removal of the underlying market imperfections.

However, to the extent that exchange rate variations induce price level variations that affect the quality or 'usefulness' of money, efficiency demands some expenditure by the supplier(s) of money on quality-producing activity. Intervention in foreign exchange markets which reduces price variation or increases price predictability is therefore justified because it ensures efficiency at a margin ignored by the proponents of freely floating exchange rates - the demand for and supply of money. Therefore, subsequent chapters take the minimization of exchange rate and price variation as the appropriate objective of exchange market intervention.
III. OPTIMAL EXCHANGE MARKET INTERVENTION

Consider the following problem. A country has a fixed stock of foreign exchange reserves. In the absence of exogenous shocks, the exchange rate in that country would be constant at some level. In the face of exogenous shocks, but in the absence of intervention, the exchange rate would fluctuate around that level. The foreign exchange authority wishes to minimize variation in the exchange rate, but cannot fix the rate completely because there are limits to the amount of reserves that the country can hold. If official international borrowing is ruled out, then reserves cannot be negative. An upper limit is set where the country has accumulated all the reserves in the world, although clearly such strong economic and political pressure would be brought to bear in this situation that the effective upper bound could be much lower. Given that reserve levels cannot move in a completely unrestricted way, what reserve management rule should the authority follow so as to minimize the variation in the exchange rate?

At this stage it is useful to draw an analogy. This problem of reserve management is similar to the problem of hydroelectric power generation. Consider a hydro dam which has behind it a lake or reservoir of a fixed size and fed by a river. If the dam

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1 When managed floating is used in an n-country world to smooth each country's price level, n-1 countries can do this indirectly by stabilizing their exchange rates. The nth country must stabilize its price level directly.
has no spillway, then the only outlet for the water flowing into
the reservoir is through the turbines. Suppose that the power
authority wishes to minimize variation in electricity supply. If
the rate of flow of the river is constant, the authority can
simply match the rate of flow through the turbines, and hence
the supply, to the rate of flow into the reservoir. By
elementary physics, the level of water in the reservoir would
also be constant. However, suppose that there is a random
component to the inflow owing to random rainfall in the
catchment area. The authority can absorb the random component of
the inflow by allowing the level of the reservoir to vary, while
maintaining a constant rate of electricity generation. However,
it can only do so as long as the level of the reservoir stays
within its maximum and minimum levels, determined by the floor
of the reservoir and the height of the dam. Otherwise it must
increase or decrease the rate of flow through the turbines. The
problem is one of determining the optimal management of the
reservoir level.

Both problems are inventory management problems. Clearly,
both have a dynamic aspect. It would be feasible to allow the
reserve/reservoir level to vary so as to fully offset all random
shocks until the reserve/reservoir limits are reached. However,
there is no guarantee that while the exchange rate/electricity
supply is then allowed to adjust, subsequent shocks will
immediately return reserves/the reservoir to more normal levels.
Thus it may be better to allow a smaller adjustment in the
exchange rate/electricity supply at an earlier stage and so avoid prolonged fluctuations in the exchange rate/electricity supply later.

The following model of reserve management recognises the dynamic aspect of the problem and thus yields an optimal reserve management policy which leans against the wind in this fashion. Furthermore, the size of the exchange rate adjustment which is permitted increases as reserves approach their limits.

A Simple Model of Reserve Management

The model begins with an assumption about the structure of the exogenous shocks. Suppose that these are such that, under a fixed exchange rate regime, reserves would follow a random walk. From one period to the next, they would either increase by an amount $M$, say, with probability $p$ or decrease by $M$ with probability $(1-p)$. At this level of abstraction, it is also assumed that the fixed exchange rate is set at its 'correct' permanent or equilibrium level. In terms of the hydro analogy, this means that the rate of electricity generation matches the average rate of water inflow. The model considers only that

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2 This assumption was used by Miller and Orr (1966) in their inventory-theoretic model of the demand for money by firms. The subsequent description of reserve behaviour closely follows their approach, although the optimization problem differs.

3 The random walk assumption shows in the strongest way possible that randomness of disturbances is consistent with cumulative change. Even when the random walk is driftless, so that $p=1/2$, the variance of reserve levels tends to infinity. The following analysis can be easily generalized so that each trial occurs within some fraction $(1/k)$ of a period.
random variation in reserve levels which arises from random shocks whose impact is over and above the systematic forces which would determine what the equilibrium exchange rate would be. Therefore, it is assumed that the shocks themselves have no systematic effect on reserve levels in the sense that, from one period to the next, reserves either increase or decrease with equal probability. In what follows, it is therefore assumed that $p = (1-p) = 1/2$.

Therefore, the starting point is this reserve behaviour under a fixed exchange rate regime - that is, a driftless random walk with step size $\$M$. One implication of such behaviour is that reserve depletion or excessive accumulation is an eventual certainty. However, by choosing a smaller step size $\$m$, in the same direction as $\$M$, the foreign exchange authority can exercise control over reserve movements and can thereby avoid the possibility of reserve depletion or excessive accumulation. In this case, by definition, the exchange rate can no longer remain fixed at its 'correct' equilibrium level. However, it is assumed, firstly, that the exchange rate will now vary around its permanent or equilibrium value, and secondly, that this exchange rate variation around its permanent rate will be smaller, the closer is $\$m$ to $\$M$.

The next section considers what those systematic forces might be by developing a simple model of exchange rate determination. The third section considers the implication of introducing exchange rate expectations into this model.

See footnote 3 in this chapter.
Therefore, this section examines only the reserve management aspect of the optimal intervention problem. The implications of the choice of \( m \) for actual exchange rate behaviour are examined more closely in later sections, where an explicit model of exchange rate determination is introduced. However, a result which can be derived from almost any model of exchange rate determination is that, when reserve changes are smaller than, but in the same direction as, those required to fix the exchange rate completely, then the exchange rate will adjust by an amount that is smaller than, but in the same direction as, the adjustment in a freely floating rate. It is therefore assumed, in this section, that exchange rate variation can be minimized by 'leaning' as strongly 'against the wind' as reserve levels will allow. The problem of minimizing exchange rate variation is simply taken to be making \( m \) as close to \( M \) as possible, while the reserve management aspect imposes the constraint that reserve levels cannot, now or in the future, move outside of certain limits. To return to the hydro analogy, the problem in this section is taken to be determining the period-by-period change in reservoir levels that would absorb as much of the random component of rainfall as possible, subject to the constraint that the reservoir level can never go beyond the physical limits of reservoir capacity.

At this stage a distinction is drawn between reserve management and hydro management. The information available to the foreign exchange authority regarding the nature of the
random exogenous shocks is less than the information available to the power authority regarding rainfall. It has been argued so far that the objective of policy is to make reserve/reservoir changes as close as possible to those changes which would occur with complete intervention. The question here is whether the authorities know ahead of time what that full-intervention change would be. A hydro authority can obtain almost immediate reports, or at least fairly accurate short term forecasts, on the size and duration of the single disturbance it faces—rainfall variation in the catchment area. The foreign exchange authority, by contrast, faces shocks from a number of different sources. Even with knowledge of those shocks, the authority may not be able to anticipate the effect they would have on reserves under a fixed exchange rate regime. If the authority could always know what that change would be, it could use that information when setting its optimal management rule. Without this knowledge, the best it can do is to calculate some maximum desired reserve change ex ante, then wait to see whether this change is sufficient to fix the exchange rate ex post, or whether it leans against the wind.\(^6\)

The limitation on reserve levels places a constraint on the choice of the reserve change \(m\). There is a critical lower bound on reserves, namely zero, and some critical upper bound \(h\), which define a country's 'ruin'. The duration (in periods) of a 

\[^6\] Formally, this means that the ex ante optimal step size \(m\) is not a function of \(M\), but that ex post, the reserve change will be the smaller of \(m\) and \(M\).
random walk which starts from an initial or target reserve level \( z \), takes step sizes \( m \) each period, and terminates at either \( h \) or zero is a random variable whose expectation is given by:  

\[
ED = \frac{z(h-z)}{m^2} \tag{1}
\]

The authority's problem is to make the current reserve change or step size \( m \) as large as possible, subject to the constraint that this step size, were it to continue in future periods, lead to a random walk whose expected duration is at least as long as the authority's own planning horizon \( H \). By solving the constraint for \( m \), the maximum permissible step size in the current period is given by:  

\[
m = \left[ \frac{z(h-z)}{H} \right]^{1/2} \tag{2}
\]

However, if the prospect of ruin is to be avoided, then clearly this step size cannot continue in future periods. One way to view the problem is to note that at the beginning of the next period, the authority will have as its starting point an initial reserve level which has moved away from \( z \) and towards one of the critical levels by an amount \( m_1 \), where \( m_1 \) is the current permissible reserve change. If the authority is then to choose a new reserve change \( m_2 \) such that, if \( m_2 \) were to continue  

\[ \text{***************} \]

\footnote{This formula is appropriate only for the case where \( p=1/2 \), as was assumed above. See Feller (1957, pp. 348-9) for a proof of the result.}

\footnote{This formula assumes that shocks hit, and intervention occurs, once per period. When shocks hit and intervention occurs \( k \) times per period, the denominator becomes \( Hk \) instead of \( H \). Thus the desired step size depends only on the number of shocks that hit during the planning horizon, and is not sensitive to how a 'period' is defined.}
in future periods, it would lead to a random walk whose expected duration is also at least as long as \( H \), then \( m_2 \) will be smaller than \( m_1 \) in situations where the new starting point is closer to one of the barriers than the old starting point \( z \). This assumption about foreign exchange authority behaviour implies that the authority recognises the dynamic aspect of its reserve management problem by employing piecewise static optimization.\(^9\) It leads to a reserve management rule whereby the current reserve change depends on how close the previous period's reserve level is to the barriers.\(^10\)

A similar approach is to view the current reserve change as being dependent on, not how close reserves are to the barriers, but how far they are from the initial starting point \( z \). Consider again the authority's strategy in the second period. Suppose it were to choose the maximum allowable reserve change \( m_2 \) such that, had \( m_2 \) begun in the first period and continued into the future, it would lead to a random walk whose expected duration is at least as long as \( H+1 \) when measured from the first period, or \( H \) when measured from the current period. This second view of foreign exchange authority behaviour again corresponds to piecewise static optimization. However, it is not exactly

\[^9\] The optimal control formulation proves to be difficult to solve analytically, since it involves Kuhn-Tucker conditions. However, it can be argued that, in practical terms, the information required to implement a fully dynamic optimal control solution would far exceed that available to a foreign exchange authority.

\[^{10}\] The 'optimal' reserve change would then be given by \( m = \left[ R(t-1)[h-R(t-1)]/H \right]^{1/2} \).
equivalent to the first view, since the first period's reserve change will actually have been $m_1$, not $m_2$. In this sense, this second approach begins by ignoring the past history of reserve changes and thus will not absolutely rule out reserve depletion unless some cutoff is imposed to ensure $m=0$ once the boundaries are reached. The issue of cutoff is discussed further in Appendix I, where it is shown that once such cutoff is imposed, the two approaches do yield similar results. And while the second approach may seem more awkward than the first, it will be seen that because it initially makes the 'time' aspect of the problem explicit, it allows the generalization of the rule which is introduced in the second section of this chapter.

Using this second approach, the 'optimal' reserve management rule appears to depend on time:

$$m(t) = \left[ \frac{z(h-z)}{(H+t)} \right]^{1/2} \quad (3)$$

However, what is crucial is not the passage of time directly, but that as more time passes, the further reserves can conceivably have walked away from their initial level $z$ and towards the critical levels $h$ or zero. To state this point another way, if reserves happen to have walked back to their initial level $z$, $t$ can be reset to zero and the whole process started anew.\(^{11}\) Therefore, what matters is not real time, but the minimum time required to achieve any particular sized deviation in reserves from target. This is how the time variable

\(^{11}\) Feller (1957, p. 86-88) has an interesting example of the sensitivity of random walk behaviour to the point chosen as $t=0$. 53
t will be interpreted in this section.\textsuperscript{12}

If a relationship could be obtained between any particular deviation $|R(t-1)-z|$ in reserves $R$ from target, and the minimum time $t$ required to achieve that deviation, then the above reserve management rule could be restated in terms of deviations — that is, in terms of how far reserves are from their initial or target level $z$. Such a relationship can be obtained by noting that the magnitude of the current deviation from target is just the sum of past reserve changes $m(t)$, where these changes in turn depend on time.\textsuperscript{13} The derivation of this relationship between time and deviations is given in Appendix I, where it is then substituted into the above expression for $m(t)$. A cutoff is then imposed to ensure $m=0$ once the boundaries are reached, and the reserve management rule can then be expressed in the approximately linear form:

$$m^* = |R(t)-R(t-1)| = A - B|R(t-1)-z|$$

where $A = \frac{\sqrt{z(h-z)}}{\sqrt{4(H+1)^2}}$ and $B = \frac{1}{2(H+1)}$ \hspace{1cm} (4)

Thus this approach leads to an optimal desired reserve change $m^*$ which depends on how far reserves are from their initial or target level. Furthermore, the larger the current deviation, the smaller the current desired reserve change so the more a current shock must be absorbed by exchange rate rather

\textsuperscript{12} The next section examines a second sense in which time matters.

\textsuperscript{13} Furthermore, 'minimum time' implies that all past reserve changes must have been in the same direction, so there is no ambiguity about the direction of successive step sizes $m$. 

54
than reserve changes. Thus the rule specifies that the strength of intervention to resist exchange rate movement should decrease, the further reserves have moved away from target.

During the time of the Bretton Woods breakdown, Marsh (1970) put forward a proposal for reform which involved a reversal of the Bretton Woods system. He suggested that reserve levels, rather than exchange rate levels, be restricted to lie within a narrow band around parity. The reserve management policy just developed is perhaps closest to a reversal of the band-and-crawl proposals which seemed to gain widest acceptance at the time. These proposals required the exchange rate to be kept within a band around an adjustable parity, with various rules being suggested to govern how the parity should move. The problem with such proposals for exchange rate management is that if the implied constraint on exchange rate flexibility is binding at all, then reserve levels must absorb whatever additional incipient exchange rate pressure arises. Such proposals dictate when the foreign exchange authority should enter the market, but once in, the authority must face that pressure, even to the point of ruin. By contrast, the reserve management rule of this section requires reserves to be kept

\[ \frac{\partial M^*}{\partial |R(t-1) - z|} \] is negative.

Indeed, he argued that such a feature was a necessary and sufficient condition for stability in the international monetary system.

See, for example, Johnson (1970) and Willett (1970).
within a band around an adjustable level, that level being whatever reserves were in the previous period and the band being narrower, the further is that level from target. The authority in effect uses the rule to calculate the maximum quantity of reserves it will sell should the exchange rate begin to rise during the period, and the maximum quantity it will buy should the exchange rate begin to fall during the period. This policy therefore dictates when the foreign exchange authority should leave, rather than enter, the market. The rule ensures that the authority always leaves before the point of ruin, and indeed if ruin is already close enough, then the authority may not enter the market at all. 17 Like the band-and-crawl proposal, it limits exchange rate flexibility, but unlike the band-and-crawl proposal, it eliminates the possibility of ruin by allowing the exchange rate to absorb all remaining pressure and in that sense, find its own level.

The reserve management rule limits the strength of intervention by allowing reserve levels to fluctuate by an amount that is inversely related to how far reserves are from their initial or target level. It was argued earlier, however, that because of insufficient information, the foreign exchange authority cannot tell ahead of time whether these fluctuations would be sufficient to fix the exchange rate completely, or

17 As presented, the rule ensures \( m^*=0 \) exactly at the point of ruin. However, the critical barriers \( s_0 \) and \( s_h \) can be redefined or rescaled so that they represent reserve levels close to, but just short of, those which would cause real political or economic ruin.
whether they would simply lean against the wind. The rule itself merely gives a statement of intent, and its implications for ex post reserve behaviour will also depend on the additional assumptions made about its implementation.\(^{18}\) This can be illustrated using Figure I.

At the beginning of the period, the authority uses the rule to calculate the maximum permissible reserve change \(m^*\) for that period, on the basis of how far total reserve levels are from the target level \(z\). The line J-K-L shows this optimal ex ante reserve change \(m^*\) as a function of how far reserves have deviated from target. Suppose, as before, that during the period, the exchange market receives a shock which would require a reserve change of size \(M\) in order to keep the exchange rate fixed at its previous level. The line D-E shows the reserve change \(M\) which is required to fix the exchange rate.

If the reserves carried forward from the previous period are not far from target and lie between the points \(X\) and \(Y\), then the current permissible reserve change \(m^*\) is more than sufficient to fully absorb the shock. The authority can therefore meet all contracts at the prevailing exchange rate without violating the rule, and the exchange rate can remain fixed at its previous level.\(^{19}\) If the reserves carried forward

\(^{18}\) Formally, the requirement that \(|R(t)-R(t-1)|=\min(m^*, M)\) is now imposed ex post.

\(^{19}\) If, however, the shock were very large relative to the target stock of reserves, the line D-E would lie above the point \(K\) and there would be no such region of 'automatic' fixity.
Figure I - **Optimal Reserve Change and the Deviation of Reserve Levels from Target**

Reserve Change

\[ |R(t) - R(t-1)| \]

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**Legend:**

- **J**
- **G**
- **X**
- **Y**
- **L**
- **K**
- **D**
- **E**
- **M**
- **F**

\[ m^* = A - B |R(t-1) - z| \]

**Reserve Level:**

\[ R(t-1) \]

**Initial Level:**

\[ z \]
happen to be a little further from target, in the range J-X or Y-L, then the current permissible reserve change is less than required to fully absorb the shock. Suppose, for example, that reserve levels have fallen below target and are at the point G. If the shock of size \( M \) implies a further reduction in reserves, then the rule requires the authority to meet this excess demand for foreign exchange at the prevailing exchange rate only to the extent of \( F-G \). Therefore, by definition, the exchange rate must also adjust, but by less than it would under a free float. In this case, the policy leans against the wind. Suppose, on the other hand, that the shock of size \( M \) is in the opposite direction, requiring reserve accumulation in order to absorb it. At the beginning of the period, the authority does not know in which direction the shock will be. According to the ex ante optimal rule, it should limit this reserve accumulation, since it could equally have been reserve depletion. However, implementation of the rule clearly requires monitoring during the period and therefore provides an opportunity for ex post adjustment. In subsequent chapters, it is assumed that when shocks would take reserves back to target, then no limitation is placed on such movement and the exchange rate remains fixed at its previous level.\(^20\) Therefore, in the range J-X or Y-L the authority is assumed to lean only against those winds which would take reserves further from target. Once the reserves

\(^20\) However, with reserves below target, for example, accumulation would be limited if it were great enough to take reserve levels into the range Y-L.
carried forward reach the points J or L, and the current shock would take them further towards ruin, the current permissible reserve change becomes zero and the exchange rate floats completely in the current period.

To recapitulate slightly, this section began with the assumption that the problem of minimizing exchange rate variation could be solved by leaning against the wind. It was then shown that with finite reserve levels, the reserve management aspect of the problem would dictate when, and how strongly, to lean against the wind. The result is a rule which in some instances leans completely and thereby fixes the exchange rate at its previous level, in other instances leans not at all and allows the exchange rate to float freely, and in yet other instances leans partially and allows some mixture of exchange rate adjustment and reserve accommodation. At this stage, the practical implications of leaning against the wind deserve discussion.

The exact dynamics, during the period, of the mixture of 'price' and 'quantity' adjustment will depend in part on the institutional arrangements under which the leaning is carried out. For expositional purposes, the description so far has suggested that quantity adjustment will occur before price adjustment. In practice, however, the authority will have to refrain from intervening until some (small) exchange rate adjustment makes clear the direction of the current shock. The exact sequence will then depend on whether the authority
immediately buys (or sells) foreign exchange reserves up to the full amount permitted by the rule, or whether it buys (or sells) some fraction of the permitted amount and waits to see whether the exchange rate reverts to its previous level or adjusts further in the same direction, before committing the remainder of its allowed resources. In general terms, the implication is that, even if the exchange rate is successfully 'fixed' end-of-period, it may not remain absolutely stable during the period. These practical implications of leaning against the wind are merely noted here. Subsequent effort will examine the effects of the above rule, one which dictates how strongly to lean, but will assume that this leaning occurs in an idealized fashion. In particular, it will be assumed that when exchange rates are fixed end-of-period, they are also fixed during the period.

The above reserve management rule sets the strength of current intervention, ceteris paribus, according to how far reserves have deviated from target. Now turn to the additional determinants of the strength of intervention, as given by the comparative statics of the rule. Firstly, for a given deviation in reserves from target, the current reserve change can be larger, and so absorb more of the current period's exogenous

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21 If it were to commit the full amount when reserves were in the range X-Y in Figure I, it would reverse the direction of the exchange rate adjustment from what it would have been under a free float. However, as Sohmen (1969) notes, such 'mistakes' can still lead to exchange rate behaviour which is smoother than under a free float.
shock, the further are the critical reserve levels from the initial or target level.22 Secondly, the longer is the planning horizon $H$, or the more cautious is the foreign exchange authority, the smaller is the current permissible reserve change.23 Most importantly, but not surprisingly, the larger is the initial reserve level $z$, the more the foreign exchange authority can absorb a given shock in reserve changes rather than exchange rate changes.24 However, this last result has an important proviso — the initial or target reserve level must be less than half way between the critical upper and lower bounds for this result to hold. The significance of the proviso can best be seen by noting that if $z$ were itself an object of choice, a target rather than an arbitrary initial point, then the value which would make $m^*$ as large as possible would be one half way between the two critical bounds.25 Alternatively, suppose both $h$ and $z$ increase together in such a way that $h=2z$.

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22 The partial derivative of $m^*$ with respect to $(h-z)$ is positive. In terms of Figure I, the point K moves upwards as $(h-z)$ increases.

23 The partial derivative of $A$ with respect to $H$ is positive, for $H>1$, and the partial derivative of $A/B$ with respect to $H$ is zero. In Figure I, the point K moves downward as $H$ increases.

24 The partial derivative of $m^*$ with respect to $z$ is positive, for $z$ less than $h/2$. In Figure I, the point K moves up as $z$ increases toward $h/2$.

25 This can be seen from equation (3). In equation (4), cutoff has been imposed in a way that assumes $z=h/2$. This 'optimal' $z$ is smaller than that obtained by Miller and Orr (1966) because they consider the opportunity cost of holding reserves as well as some penalty for breaking the barriers and thus introduce an incentive to economize on reserves.
In terms of the hydro analogy, this is equivalent to increasing the size of the reservoir. In this case, the current permissible reserve change \( m^* \) unambiguously increases.\(^{26}\)

To summarize the results of this section, the reserve management problem outlined at the beginning of this chapter has a dynamic aspect. When this aspect is recognised, albeit in a piecewise static fashion, then the optimal reserve management strategy allows the strength of intervention to vary according to how far reserves have deviated from target. Furthermore, since the rule, as a statement of intent, does not prejudge what strength would be required to fix the exchange rate completely, it may in fact do so ex post when reserves are close to target. For reserve levels a little further from target, this optimal strategy will merely lean against the wind. Finally, if reserves are close to 'ruin' levels, the exchange rate will float freely in that period.

**Equilibrium and the Private Sector**

In the previous section, the discussion of reserve management began with the assumption that the exchange rate, under a fixed regime, was set at its 'correct' permanent or equilibrium level. In terms of the hydro analogy, it was assumed a priori that the average rate of water inflow into the reservoir was matched by the rate of electricity generation.

\(^{26}\) In the expression for \( m^* \), the term \( z(h-z) \) is replaced by \( z^2 \). The partial derivative of this new \( m^* \) with respect to \( z \) is positive.
This assumption allowed the artifice of characterizing a fixed exchange rate regime as one where reserve gains and losses of equal size occurred with equal probability. The implicit assumption was therefore that the reserve management rule would minimize exchange rate variation around its long run equilibrium rate.

This section considers the implications of relaxing that assumption and examines the case where reserve gains and losses need not be of equal size, nor occur with equal probability. Instead, the simple characterization of reserve behaviour under a fixed exchange rate regime is replaced by an economic model describing explicitly the impact that exogenous shocks would have on the exchange rate in the absence of intervention, or on reserves under fixed exchange rates. This section then reexamines the ex post exchange rate and reserve behaviour resulting from the operation of the reserve management rule in this new environment. The first question is whether the simple rule of the previous section can indeed ensure minimal variation in the exchange rate, as was assumed earlier. A second question is whether this minimal variation will continue to occur around the correct long run equilibrium, even though that equilibrium need not be constant, but may itself contain a trend component.

Finally, an amended version of the simple rule is derived. However, in the same way that the simple rule, as a statement of intent, did not assume prior knowledge of the size or direction of those reserve changes required to fix the exchange rate, the
amended version does not presume prior knowledge of the true long run equilibrium, nor, equivalently, the true probabilities of reserve gains and losses.\textsuperscript{27}

The first step is to make explicit the way in which exogenous shocks affect exchange rate and/or reserve behaviour through the requirements for equilibrium in the private sector. Consider the simple system:\textsuperscript{28}

\begin{align*}
\text{Demand for real money balances:} & \quad \frac{Md}{P} = f(x) \quad (6) \\
\text{Supply of nominal money balances:} & \quad Ms = eR + D \quad (7) \\
\text{Monetary equilibrium condition:} & \quad Md = Ms \quad (8) \\
\text{Purchasing power parity:} & \quad P = eP^* \quad (9)
\end{align*}

where $Md$ and $Ms$ are nominal money demand and supply, $P$ and $P^*$ are the domestic and foreign price levels, $e$ is the exchange rate defined as the price of foreign currency, $R$ is foreign exchange reserves denominated in foreign currency, $D$ is the domestic component of the monetary base,\textsuperscript{29} and $x$ captures all other influences on the demand for real money balances.

This system gives the relationship between reserve changes and exchange rate changes that must be satisfied in order to

\textsuperscript{27}For this reason, the amended version is not derived in a generalized random walk framework.

\textsuperscript{28}This system, like the Fischer (1977) model discussed in Chapter 2, assumes that foreign borrowing and lending occurs only through the foreign exchange authority. In this environment, a decision to intervene is a decision to allow such borrowing and lending to take place. The effects of privately initiated capital flows and currency substitution are introduced in Chapter 4.

\textsuperscript{29}For simplicity, this abstracts from the effect of a fractional reserve banking system.
preserve monetary equilibrium in the private sector in the face of exogenous disturbances. By solving this system for $R$ in terms of $e$ and differentiating with respect to time (where the dot notation represents time derivatives), this relationship is:

$$ \dot{R} = g'(x)x - D/e + (D/e^2)e $$

(10)

where the determinant $P^*$ has been subsumed into $x$, so that $P^*f(x)$ has been written as $g(x)$. For the purposes of exposition, this relationship will be assumed, in the neighbourhood of some point $x=x_0$, $e=e_0$, to take the approximately linear form:

$$ \dot{R} = f(e, x) = \alpha e + \xi x $$

(10')

where the determinant $D$ has also been subsumed into $x$.

This relationship shows how disturbances in the real demand for money impinge on the exchange market, and gives the amount by which the exchange rate and/or reserve level would move in the face of these shocks. This can be illustrated in Figure 11, where the line $AB$ shows the above relationship for a shock which increases the real demand for money. Under fixed exchange rates, reserves would fully accommodate the shock and accumulate by an amount $R = \xi x$, as shown at point $A$. In this case the nominal supply of money would increase to match the demand. At the other extreme of freely floating exchange rates, reserves would not change but the exchange rate would appreciate by an amount $e = -(\xi/\alpha)x$, as shown at point $B$. In this case the real value of a fixed nominal supply of money would increase to match the demand. By choosing to lean against the wind, the foreign exchange authority picks an intermediate point $X$, say, in order
Figure II - Requirements for Private Sector Equilibrium

Reserve Change $R$

$R = \alpha e + \beta x \ (x > 0)$

$R = \alpha e + \beta x \ (x < 0)$

Exchange Rate Change $e$
to limit to $XC$ the amount of the reserve adjustment that accompanies the particular shock, but must also accept an exchange rate change of the amount $XD$ imposed by the requirements of monetary equilibrium in the private sector. The line $PQ$ likewise describes the authority's options in a situation where the exogenous shock decreases the real demand for money. The previous section considered the special case where the configurations $PQ$ and $AB$ occurred with equal probability and where the reserve changes $OA$ and $OP$ were of equal size. That restriction no longer applies.

Thus the above relationship shows the tradeoffs that the foreign exchange authority faces when it manages the exchange rate, by giving the environment within which an intervention rule must operate. An intervention rule describes the locus of chosen points $X$ for all possible types of shock. Ex post exchange rate and reserve behaviour is determined by the interaction of these components.

Now consider the optimal reserve management rule from the previous section. This rule specifies permissible reserve movements which, because they lean against the wind, can be expressed as a fraction $\phi$ of the amount by which reserves would move were the exchange rate fixed: $30$

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$30$ From the previous section, the intervention parameter can also take the values 1 and 0, corresponding to complete fixity and a free float. The following discussion concentrates on the implications for exchange rate behaviour when it is known that the parameter lies in the closed interval $[0,1]$, but does not consider in a formal way the dependence of its exact value on reserve levels.
The previous section argued that because of constraints on reserve levels, the optimal value of $\phi$ in turn depends on how far reserves have deviated from their initial or target level. The effects can be illustrated using Figure II. Suppose, for example, that the exogenous disturbances consistently lead to tradeoff configurations such as AB, so that any rule which leans against the wind leads to reserve accumulation. However, the longer this persists, the greater reserve levels will be and the weaker will be the strength of further intervention. In terms of Figure II, the optimal point $X$ will move towards $B$, the point at which the exchange rate floats freely. Likewise, the longer that configurations such as $PQ$ persist, the closer the optimal point will move towards $Q$ rather than $P$.\textsuperscript{31}

Now that both the private sector behaviour and the authority's reserve management rule have been specified, they can be solved for ex post exchange rate changes:\textsuperscript{32}

$$\dot{e} = (\phi/\alpha) (\phi - 1) x$$

(12)

This shows that, as in the previous section, ex post exchange rate behaviour depends on the pattern of exogenous determinants.

\textsuperscript{31} Roper and Turnovsky (1980) use a similar presentation to depict their optimal intervention rule, but because they ignore the constraint imposed by finite reserve levels, their equivalent of the chosen point $X$ does not move over time.

\textsuperscript{32} It is a non-trivial problem to derive an analytical solution for ex post exchange rate levels, since the parameter $\phi$ in turn depends on the endogenous variable $R$. However, this equation does allow consideration of ex post exchange rate variation, as measured by the variance of exchange rate changes.
in the same way that electricity generation depends on rainfall. In particular, exchange rate behaviour will have a systematic and an unsystematic component according as the exogenous determinants have a systematic and an unsystematic component. However, for a given set of exogenous determinants, systematic or otherwise, ex post exchange rate behaviour also depends on the intervention policy of the foreign exchange authority. Now turn to the questions raised at the beginning of this section.

The first question is whether the simple reserve management rule can continue to ensure minimal variation in the exchange rate. The solution above shows that any policy which leans against the wind reduces the total amount of exchange rate variation relative to what it would have been without intervention. To the extent that the simple reserve management rule leans as strongly as possible, given the constraint imposed by finite reserves, it thereby does ensure minimal total variation in the exchange rate, given this constraint.\textsuperscript{33}

The second question is whether this minimal variation continues to occur around the correct long run equilibrium exchange rate. In other words, does the simple rule reduce total exchange rate variation by operating only on the unsystematic component of exchange rate behaviour, or does it also affect the systematic component? This question encapsulates the problem faced by the would-be architects of a replacement to the Bretton

\textsuperscript{33} From the above analysis, the variance of exchange rate changes is proportional to \((\phi - 1)^2\) and is lower, the closer is \(\phi\) to unity.
Woods agreement. The lack of agreement on rules for exchange rate management, as opposed to reserve management, was precisely because the level of the exchange rate, on average through time, cannot be a decision variable. Instead, the exchange rate must be allowed to move with the trends in the exogenous determinants, as it would without intervention. Otherwise, not only is the country practically ensuring eventual reserve depletion or excessive accumulation, but it is also accruing whatever costs are associated with having the wrong exchange rate for prolonged periods of time. The reserve management rule of the previous section eliminates the possibility of reserve depletion. The question here is whether it can also track the general trends in a freely floating exchange rate.

The above solution shows that exchange rate behaviour will on average be the same with intervention as without it so long as reserve levels are on average the same with intervention as without it. In other words, the simple reserve management rule will minimize the unsystematic component of exchange rate behaviour without affecting its systematic component so long as reserves are constant, on average, over time.34 The corollary is that the foreign exchange authority does not need to know the long run average or equilibrium exchange rate, so long as it has a way of ensuring that reserves are on average equal to their

34 In terms of the above analysis, the management rule will not alter the systematic component if \( E[\frac{d}{dt}((1-\phi)x)] = E[\frac{d}{dt}x] \), where it is now recognised that, with the constraints, \( \phi \) also varies over time. This will hold if \( E[\frac{d}{dt}(\phi x)] = 0 \), or, via equation (11), if the expected reserve change is zero.
initial or target level. Now the simple reserve management rule allows reserve changes as close as possible to those which would occur under full intervention, while reserve levels are in effect restricted so as to lie within a band around their initial or target level (the interval J-L in Figure I). However, this does not ensure that reserves are equal, on average, to that initial or target level.

Therefore, it can be shown that when the simplifying assumptions of the previous section are relaxed, the reserve management rule derived in that section can continue to minimize total exchange rate variation, but that this minimal variation need not occur around the correct long run equilibrium. In other words, there is no guarantee that the rule reduces total exchange rate variation by operating only on the unsystematic component of exchange rate behaviour. This can be further illustrated with a couple of examples.

In the previous section, it was assumed that reserve gains and losses of equal size occurred with equal probability. Suppose now that reserve losses are more likely than gains. If the rule operated so as to ensure that reserve losses were also smaller, on average, than the gains, then reserves could still on average be constant and the exchange rate could still on average be the same as it would without intervention. When ex post adjustment of the rule is allowed during implementation, so that partial leaning occurs only against those winds which would take reserves further from target, then an element of inbuilt
asymmetry is introduced which allows this sort of reserve behaviour. If the greater frequency of reserve losses forces reserves increasingly below target, then permissible further downward movements become smaller while upward movements, to the extent that they occur, are constrained only when they would take reserves above target.

However, there are situations in which even this asymmetry is insufficient to ensure that the exchange rate is on average the same as it would be were reserves fixed at their target and the exchange rate free to float. Consider the case where reserve losses occur with certainty. The simple rule would allow reserves to fall to their lowest permissible level (point J on Figure I), at which point no further reserve changes would take place and the exchange rate would float freely. However, there is then no mechanism to take reserves back to target. In this instance the exchange rate would be systematically different from that which would occur with reserves fixed on target.

Nevertheless, the simple rule can be amended so as to rule out this latter possibility. Consider again the argument of the previous section. This developed an optimal ex ante reserve change which apparently depended on time, the crucial point being that as more time passed, the further reserves could conceivably have moved away from target. Therefore, time was interpreted as the minimum possible time required to achieve any particular sized deviation in reserves from target. In the above example, however, the exchange rate deviates systematically from
a freely floating rate precisely because the time spent by reserves away from target is far greater than the minimum time required to achieve a deviation of that size. If reserves could eventually be brought back to target, then this systematic deviation from a freely floating exchange rate could eventually be eliminated.

This argument suggests an alternative interpretation of the time variable in equation (3). It suggests that it is not real time, or even minimum time, which matters, but rather, the total time that reserves have spent on one side or the other of target. In this section, that is how the time variable will be interpreted. However, this total time can be split into two components. Let \( t \) be the minimum time required to achieve a given deviation from target, as before, and let \( T \) be the excess time, over and above this minimum, that a deviation on one side or the other has persisted. Therefore, the total time that reserves have spent on one side or the other of target is the sum of \( t \) and \( T \), and equation (3) can be amended to become

\[
m(t,T) = \left[ z(h-z)/(H+t+T) \right]^{1/2} \tag{13}
\]

thus giving the optimal ex ante reserve change as a function of time in the two senses in which it matters. This reserve change is now smaller than that allowed by equation (3).

However, if reserves are eventually to be brought back to target, then not only must any reserve change of size \( m \) be smaller than that allowed by the simple rule, it may also have to be in the opposite direction. Consider the above example
where reserve losses occur with certainty. The authority may initially accommodate this excess demand for foreign exchange by running down its stock of reserves. However, the authority must at some point begin buying, rather than selling, foreign exchange, thereby adding to the excess demand, rather than accommodating it, if those reserves are to be brought back to target. Eventually, for some finite value of excess time $T$, the permissible reserve change must have a magnitude $m$ which is negative, so as to be in the opposite direction from the current deviation in reserves from target. Therefore, this approach not only requires the imposition of some cutoff so as to rule out reserve depletion or excessive accumulation, it also requires the imposition of a cutoff to ensure that reserves can be brought back to target in this fashion.

Once again, a relationship can be obtained between any particular deviation $|R(t-1)-z|$ in reserves from target and the minimum time $t$ required to achieve that deviation. The derivation of this relationship is given in Appendix I, where it is then substituted into the above expression for $m(t,T)$. A cutoff is then imposed, as before, to ensure that $m=0$ when the reserve boundaries are reached. A second cutoff is also imposed to ensure that reserves will eventually be brought back to target for a finite value of $T$. The amended version of the

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35 More exactly, a relationship $t=t(X,T)$ is derived from which both minimum time and excess time can be derived. Minimum time is given by $t(X,0)$ and excess time by $T=T_0-t(X,0)$, where $T_0$ is the total number of periods that reserves have spent on one side or the other of target.
optimal reserve management rule can then be expressed in the approximately linear form:

\[ m^* = A - B|R(t-1)-z| - CT \]  \hspace{1cm} (14)

where \( C = \frac{1}{2} \frac{z(H-z)}{4(H+1)^3} \) and \( A \) and \( B \) are the same as previously.

This amended rule can be visualized in terms of Figure I by noting that as reserves spend increasing amounts of time on one side or the other of target, the lines J-K-L, which show the optimal ex ante reserve change, move inwards toward the origin. As the lines move in, the current deviation in reserves may fall outside the current critical points J and L, in which case the direction of intervention is no longer dictated by the direction of the current shock. The authority must adjust its reserves in the direction that will bring them back towards the new critical bounds, and towards target.\(^{36}\) In the limit, the lines converge to the origin and reserves are forced back to target. One implication, however, is that the foreign exchange authority no longer need always lean against the wind. If reserves are to be forced back to target, the authority may find itself betting with, rather than against, the market.

This amended rule ensures that a small deviation in reserves from target cannot persist forever. As such, it ensures that in at least some situations where the long run average

\(^{36}\) The size of that adjustment, however, may still depend on the direction of the shock. Suppose ex post adjustment of the rule is allowed during implementation. If the current shock would take reserves back towards target anyway, this movement can be unrestricted.
exchange rate has a trend component, intervention activity will not cause systematic deviations from that trend. Furthermore, the amended rule, like its simple counterpart, imposes only very simple information requirements and does not assume that the foreign exchange authority knows the true equilibrium trend.

The results of this section can be summarized as follows. When the simplifying assumptions of the previous section are relaxed, the simple reserve management rule derived in that section can continue to minimize total exchange rate variation. However, there is no guarantee in theoretical terms that it does so by operating only on the unsystematic component of exchange rate behaviour. A counterexample showed that it can produce ex post exchange rate behaviour which differs systematically from that produced were reserves fixed at target and the exchange rate free to float. From consideration of this counterexample, an amended version of the simple reserve management rule was derived.

From a policy perspective, however, the deviations produced by either rule are important only if they impose a cost—whatever cost is associated with having the wrong exchange rate for prolonged periods of time. Furthermore, such costs matter only when they outweigh the benefits obtained by the reduction in unsystematic exchange rate variation, with its associated increase in exchange rate predictability. From a policy perspective, therefore, what matters is the rate of tradeoff between the reduction in unsystematic variation and the impact
on systematic adjustment. This in turn depends on how the reduction in total exchange rate variation is divided ex post between the systematic and unsystematic components of exchange rate behaviour. A priori, the division can be expected to depend on the size of the unsystematic component of the exogenous shocks relative to their systematic component. However, for either version of the reserve management rule, the exact division cannot easily be determined analytically. Nor is it a simple matter to determine how the division is affected by the target quantity of reserves. Therefore, subsequent chapters lay the groundwork for an empirical examination of these questions using simulation techniques in a Canadian context.

**Expectations - Variation vs. Unpredictability**

The previous section examined the simple reserve management rule using a simple model of exchange rate determination as a framework for discussion. That model, however, abstracted from the influence of exchange rate expectations. This section examines the implications for the simple version of the rule when such expectations are introduced.

The introduction of expectations raises a question not previously raised by simple considerations of exchange rate determination. The argument developed in the previous chapter to justify exchange market intervention was that exchange rate and

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37 Formally, an analytical solution requires solving equation (10), together with either equation (4) or equation (14), as a simultaneous differential equation system.
price level unpredictability was deleterious to welfare, the reason being that it reduced the usefulness or quality of money. Therefore, it is not exchange rate variation per se which matters, but exchange rate unpredictability. In general terms, however, what constitutes unpredictability depends on the sophistication of the people doing the predicting. In order to determine which component of exchange rate behaviour is unpredictable or unanticipated, the mechanism by which predictions or expectations are formed must be known.

As a casual observation, Klein (1974a) notes that prediction may be facilitated as much by a constant depreciation as by a constant exchange rate level. Klein's observation can be interpreted as implying that people can identify and predict a constant exchange rate trend. Reduction of unpredictability therefore involves reducing unpredictable exchange rate variation around that trend. However, when these expectations are correct, then the predictable component of exchange rate behaviour coincides with its actual systematic movement. There is then no difference between the unpredictable component and the unsystematic component of exchange rate variation.

This argument can be generalized. When expectations are rational, then exchange rate behaviour will have a systematic or equilibrium component which will reflect all that is known or predictable ex ante about the process of exchange rate determination, given the information currently available. There will also be a component of variation around that equilibrium.
exchange rate which will reflect all that is unknown or unpredictable about the process of exchange rate determination. Therefore, if the simple reserve management rule can minimize unsystematic variation in the exchange rate around its systematic equilibrium component, then it will also minimize the unpredictable component of exchange rate behaviour. The question remaining, once again, is whether a rule which leans against the wind can minimize the unsystematic component without affecting the systematic component.

This question is examined in Appendix II, where the simple model of exchange rate determination is extended to include the expectations of future exchange rates and a rational expectations solution is then derived. The above question is in many ways easier to answer than previously. With rational expectations, the public’s presumed knowledge of the structure of the exogenous shocks, though not their exact value, itself has a bearing on the systematic component of exchange rate behaviour, with or without intervention. Therefore, the answer does not depend on the structure of the shocks. Furthermore, Appendix II shows that the simple reserve management rule affects only the unpredictable component of exchange rate behaviour. By leaning against the wind, it reduces this component without affecting the systematic component.

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38 As in the previous section, the rule is characterized by an intervention parameter lying in the closed interval [0,1].
An intuitive explanation of this result can be given by reference to the earlier literature on the implications of rational expectations in a macroeconomic context. Sargent and Wallace (1976), for example, have argued that when expectations are rational, there is no feedback rule that the monetary authority can use to systematically direct the level of unemployment to some desired level. Unemployment will simply revert to the natural rate, as it would without countercyclical policy, unless unanticipated shocks in the form of unanticipated inflation cause short term deviations. Such arguments have been used to support Milton Friedman's constant monetary growth rule rather than interventionary policy. However, Shiller (1978) and Roper and Turnovsky (1980), for example, note that the monetary authority may be able to reduce the variation in unemployment around its natural rate and offset the impact of unanticipated shocks. In the same way, the simple reserve management rule can reduce the impact that unanticipated shocks have on the exchange rate, although when expectations are rational, it cannot influence the systematic, anticipated component of exchange rate behaviour. This remains the same as it would without intervention.

Furthermore, as is also confirmed in Appendix II, the amount by which the reserve management rule reduces the impact

39 Another reason for the advocacy of a constant monetary growth rule is the belief that monetary policy operates on unemployment with a long and variable lag, but the same cannot be said of the operation of reserve changes on the exchange rate.
of unanticipated shocks depends on how long those shocks, once they hit, are expected to last. At one extreme, when the shock is expected to last for only one period, the reserve management rule can have its strongest effect on limiting the unpredictable exchange rate adjustment. It reinforces the limits already placed on that adjustment by the expectation of the shock's future elimination. Whereas previously, the setting of limits on cumulative reserve movements allowed the reserve management rule to screen out temporary exchange rate movement while allowing at least some permanent adjustment to take place, this automatic selection is now reinforced by the public's own knowledge of the nature of the shocks. However, the longer the shock is expected to last, the larger would be the immediate exchange rate adjustment in the absence of intervention. The rule would then be leaning against a stronger wind, and its impact is diluted accordingly.

The analysis in Appendix II yields one final result. Consider the argument reviewed in the previous chapter which saw intervention as aimed at eliminating the possibility of destabilizing speculation. There it was argued that this view was based on a spurious distinction between the equilibrium exchange rate consistent with underlying 'real' forces and the equilibrium sullied, as it were, by (unreal?) asset portfolio adjustments. However, it was also noted that there was no guarantee that rational expectations could not be destabilizing, in the sense of producing explosive exchange rate movements.
Appendix II shows that so long as domestic monetary policy is not 'too far' out of line with the underlying exogenous determinants of the private sector's willingness to hold money, then rational expectations will not be destabilizing. Furthermore, this condition does not depend on exchange market intervention, or the lack of it. This suggests, therefore, that at least in the context of the simple exchange rate model under consideration, the simple reserve management policy developed in this section will not, by itself, induce instability.

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This is defined as the behaviour of the domestic component of the monetary base.
IV. PRIVATE SECTOR BEHAVIOUR - THEORY AND ESTIMATION

The previous chapter derived both simple and amended versions of a reserve management rule which were designed to minimize exchange rate variation while recognizing that there are limits to cumulative reserve movement. The problem was formulated as an inventory or reserve management problem. The resulting rules specify the reserve behaviour desired ex ante by the foreign exchange authority in terms of optimal limits on ex post reserve behaviour.

However, the rules were derived under a very simple assumption about ex post reserve behaviour in the absence of such optimal intervention. This assumption was then replaced by a more general relationship, which showed that while the authority might choose the amount of reserve adjustment that accompanies any particular shock, the amount of exchange rate adjustment is determined as well by private sector behaviour and the requirements for monetary equilibrium. Within this more general environment, the rules' impact on ex post exchange rate and reserve behaviour was difficult to determine analytically, at least in situations where individuals did not have sufficient information to form strictly rational exchange rate expectations. In particular, while it could be shown that the simple rule continued to minimize total exchange rate variation, the relative impacts of either rule on the systematic and unsystematic components of exchange rate behaviour could not
easily be determined. Therefore, this question requires further examination on an empirical level.

The chapter which follows reports the results of an empirical examination, using simulation techniques, in a Canadian context. However, in order to perform that simulation, the simple model of private sector needs to be developed in two ways. On a theoretical level, the source and impact of the exogenous shocks hitting the exchange market must be identified more precisely. Secondly, the private sector behavioural equation must be estimated in the same context in which the simulations are performed. This chapter reports these developments, and thus describes the simulation environment.

The extended model of private behaviour is monetarist, as before, in the sense that it regards asset demands and the adjustment to portfolio equilibrium as playing an important part in exchange rate determination, although international trade in goods is no longer ignored. In addition, many of the assumptions, reflecting some of the judgements made in Chapter 2, are distinctly monetarist in flavour. The important assumptions are:

(i) domestic price level flexibility.
(ii) a single traded good.
(iii) perfect goods arbitrage between countries.
(iv) perfect capital mobility, but not perfect substitutability, between domestic and foreign bonds.
(v) the possibility of currency substitution.
Price level flexibility ensures that full employment is maintained. This assumption reflects the judgement that variation in the level of employment, to the extent that it does occur, does not provide a justification for foreign exchange market intervention. The assumption of a single traded good means that structural effects and the differential impact of exogenous shocks on traded and non-traded goods sectors, or import and export sectors, cannot be studied. Again, this simplification reflects the judgement that such structural effects, to the extent that they have welfare implications because of market imperfections, are not the proper concern of the foreign exchange authority. Perfect international arbitrage of a single traded good implies that the domestic and foreign price levels are linked through the purchasing power parity condition. In contrast, the assumption of less than perfect substitutability between domestic and foreign bonds means that the standard interest parity condition is not invoked. This latter condition presumes risk neutrality. The current formulation therefore allows that there may be important components of the overall return to bond holding, particularly risk premia, which are reflected in uncovered interest rate differentials. If all risk sharing occurs through capital markets, there will be no comparable risk premia attached to goods prices. Finally, it is assumed that individuals can hold both domestic and foreign money. Thus currency substitution by private individuals is possible.
From these assumptions, an equation is derived which describes the relationship between exchange rate and reserve changes that is consistent with stock and flow equilibrium in the private sector. This equation is therefore analogous to the simple description of private sector behaviour from the previous chapter, although, through a supply and demand system, the exogenous influences are now identified explicitly.

The parameters of this equation are then estimated in a Canadian context. Therefore, Canada becomes the 'domestic' country and its major trading partner, the United States, is treated as the 'foreign' country or the 'rest of the world'. The parameter estimation draws on data from both these countries over the period July 1970 to March 1979, a period in which the exchange rate between them floated, though not freely. Furthermore, it is assumed that the optimal intervention rule operates as a short term policy - the decision on policy settings is taken monthly, rather than quarterly or yearly, in order to smooth short term exchange rate variation. Therefore, monthly data are used in estimating parameter values for the private sector behavioural equation.

The discussion proceeds in several parts. Firstly, the required equation is derived from a demand and supply framework. Next the appropriate estimation techniques are selected and the results, obtained under several alternative techniques and formulations, are reported. Finally, the observed estimates are compared with a priori beliefs about their values.
Theoretical Framework

Private sector reaction to reserve and exchange rate changes derives from the requirements for equilibrium in the markets for goods and assets. Begin by considering the demands for those goods and assets.

Individuals are assumed to choose current real consumption $c$, nominal holdings of domestic money and bonds $Md$ and $Bd$ (denominated in domestic currency), and holdings of foreign money and bonds $Mf$ and $Bf$ (denominated in foreign currency), so as to maximize a utility function of the form

$$ U = U(c, Md/P, eMf/P, Bd/P, eBf/P) = U(c, md, mf, bd, bf) $$

where $e$ is the exchange rate, defined as the domestic currency price of foreign currency, and $P$ is the domestic price level.

The maximization is subject to a wealth constraint

$$ w = w = \frac{Md + Bd + eMf + eBf}{P} = \frac{md + bd + mf + bf}{P} $$

and an income constraint

$$ y + rBd + aeMf + (r* + a)eBf = c $$

where $y$ is real income from human sources, $r$ and $r*$ are the domestic and foreign nominal interest rates and $a$ is the expected rate of exchange depreciation. ¹

In this timeless formulation of the choice problem, utility depends on the real value of the stock holdings of nominal

¹ This approach is similar to that of Klein (1974b) and Bordo and Choudhri (1982).
assets, including money. The utility function can be regarded as an indirect utility function, approximating the outcome of some explicit intertemporal maximization in which nominal assets may or may not yield direct non-pecuniary services, but certainly appear in the intertemporal budget constraint as a means of transferring purchasing power through time. The above utility function can also reflect differences in attitude towards domestic and foreign nominal assets, particularly those which arise from attitudes toward risk. Therefore, this general utility function can capture individual preferences with respect to both the consumption-savings decision and the portfolio composition decision.

The wealth constraint takes the standard form. The income constraint states that current real consumption is limited by the sum of real incomes from human and financial sources. The income and wealth constraints can be combined, by eliminating domestic bond holdings, to give

\[ y + rw = c + rmd + (r-a)mf + (r-r*-a)bf \]  

(18)

The left hand side of this equation is a measure of the real income stream derived from both human and non-human sources. This must equal current real 'consumption' of goods and assets, where the assets are weighted by their opportunity cost relative to domestic bonds.

Maximization of utility subject to this combined budget constraint gives the demand equations for real consumption and real asset holdings as functions of real human income, real
financial wealth and these opportunity costs. These demand equations can then be rearranged so as to be functions of real human income, real financial wealth, nominal interest rates and the expected rate of depreciation:

\[
\begin{align*}
    c &= c(r, r^*, a, y, w) \\
    \text{md} &= l(r, r^*, a, y, w) \\
    \text{mf} &= m(r, r^*, a, y, w) \\
    \text{bd} &= n(r, r^*, a, y, w) \\
    \text{bf} &= q(r, r^*, a, y, w)
\end{align*}
\]  

Without assuming an explicit functional form for the utility function, the following signs can be imputed to the partial derivatives of the demand functions:

\[
\begin{align*}
    c1 &> 0; \ c2 < 0; \ c3 < 0; \ c4 > 0; \ c5 > 0 \\
    l1 &< 0; \ l2 < 0; \ l3 < 0; \ l4 > 0; \ l5 > 0 \\
    m1 &< 0; \ m2 < 0; \ m3 > 0; \ m4 > 0; \ m5 > 0 \\
    n1 &> 0; \ n2 < 0; \ n3 < 0; \ n4 > 0; \ n5 > 0 \\
    q1 &< 0; \ q2 > 0; \ q3 > 0; \ q4 > 0; \ q5 > 0
\end{align*}
\]  

where \(c_i, l_i, m_i, n_i\) and \(q_i\) are partial derivatives of the corresponding demand functions with respect to the \(i\)th argument. This assignment reflects the assumption that an increase in the opportunity cost of a particular asset decreases demand for that asset and increases demand for its substitutes. It also assumes that 'own' opportunity cost effects dominate 'cross' effects.\(^2\)

Finally, increases in real human income or real financial wealth are assumed to have positive effects on all demands. The signs of these partial derivatives are important in that they determine the expected signs of the coefficients attached to the

\(^2\) For example, the domestic interest rate \(r\) affects the demand for domestic money \(\text{md}\) because it affects, not only money's own opportunity cost, but also the opportunity costs of two of its substitutes - foreign money and foreign bonds. It is assumed that the first effect dominates.
estimating equation.

If foreigners face a similar choice problem, an analogous argument gives foreign demands for the consumption good \( c^* \), for foreign money and bonds \( mf^* \) and \( bf^* \) and for domestic money and bonds \( md^* \) and \( bd^* \). If it is assumed that foreigners have the same beliefs about the exchange rate as domestic residents, then \( a \) represents, to foreigners, the expected rate of appreciation of their currency. With this in mind, the foreign demand equations can be written as

\[
\begin{align*}
c^* &= c^*(r, r^*, a, y^*, w^*) \\
md^* &= m^*(r, r^*, a, y^*, w^*) \\
mf^* &= m^*(r, r^*, a, y^*, w^*) \\
b^* &= m^*(r, r^*, a, y^*, w^*) \\
bfd^* &= q^*(r, r^*, a, y^*, w^*)
\end{align*}
\]

and the following signs can be imputed to the partial derivatives:

\[
\begin{align*}
c^1 &< 0; c^2 > 0; c^3 > 0; c^4 > 0; c^5 > 0 \\
l^1 &< 0; l^2 < 0; l^3 < 0; l^4 > 0; l^5 > 0 \\
m^1 &< 0; m^2 < 0; m^3 > 0; m^4 > 0; m^5 > 0 \\
n^1 &< 0; n^2 < 0; n^3 < 0; n^4 > 0; n^5 > 0 \\
q^1 &< 0; q^2 > 0; q^3 > 0; q^4 > 0; q^5 > 0
\end{align*}
\]

So far no explicit functional form has been assumed for the utility functions or demand equations in each country. However, subsequent analysis will make use of the partial derivatives of the demand equations, representing as they do the income, interest etc. sensitivities of goods and asset demands. In addition, it will be assumed that these sensitivities are at least approximately constant. Therefore, the following analysis is consistent with any demand system which yields partial derivatives of this form.
The choice problem for a single individual in each country has now been outlined. If that individual is a 'representative' individual, then the demand equations derived above can also be used to characterize aggregate demands in each country. It remains to specify the supplies of goods and assets and to invoke market clearing conditions.

It is assumed that there is sufficient wage and price flexibility in each country to ensure that full employment is maintained. Therefore, the level of output of the single consumption good in each country will be that produced by the fully employed labour force. Hence the terms $y$ and $y^*$, which now measure aggregate real income from human sources, also measure aggregate real output in each country, and are treated as exogenous.

The supply of domestic money is a constant fraction $k$ of the domestic monetary base. This base has a domestic and a foreign component. The domestic component $D$ comprises domestic government bonds held by the banking system. The foreign component comprises the domestic currency value of foreign exchange reserves $eR$, where $R$ measures the foreign currency value of these reserves and $e$ is the exchange rate. Hence the domestic money supply can be written:

\[ \text{domestic money supply} = k \times \text{domestic monetary base} \]

---

3 Since the analysis abstracts from reproducible capital, the real return to capital need not be added in order to measure total output.

4 It is assumed that domestic banks do not hold foreign bonds as reserves.
\[ M = k(eR + D) \]  \hspace{1cm} (23)

For simplicity, it is assumed that \( k = 1 \).

The supply of foreign money is similarly given by:

\[ M^* = k^*(R^*/e + D^*) \]  \hspace{1cm} (24)

where here all magnitudes except \( R^* \) are denominated in foreign currency and, again, it is assumed that \( k^* = 1 \).

Finally, supplies of domestic bonds \( B \) and foreign bonds \( B^* \) are assumed to be exogenously given. The analysis abstracts from the issuing of debt instruments by private individuals, so that all bonds are government bonds. The total stock of bonds outstanding is therefore determined by current and past government budget deficits, whose value is set by considerations outside the model. In addition, it is assumed that the government can control the number of bonds \( D \) held by the banking system - that is, it can control, by open market operations, the domestic component of the monetary base.

When the market clearing conditions are invoked, it must be remembered that the markets for all goods and assets extend beyond the borders of any one country. In the goods market, this means that total goods demand equals total goods supply:

\[ c + c^* = y + y^* \]  \hspace{1cm} (25)

or that a trade deficit (dissaving) in one country is balanced by a trade surplus (saving) in the other. Similarly, the supply of domestic money equals the sum of demands by domestic
residents and foreigners:  
\[
\frac{M}{P} = \frac{eR + D}{P} = \frac{md + md^*}{P} \tag{26}
\]
Likewise, the supply of foreign money equals the sum of demands by foreigners and domestic residents:
\[
\frac{M^*}{P^*} = \frac{R^*/e + D^*}{P^*} = \frac{mf + mf^*}{P^*} \tag{27}
\]
where \(P^*\) is the foreign price level. The supply of domestic bonds equals the sum of holdings by domestic residents, foreigners and the domestic banking system:
\[
\frac{B}{P} = \frac{bd + bd^* + D}{P} \tag{28}
\]
The supply of foreign bonds equals the sum of holdings by domestic residents, foreigners and the foreign banking system:
\[
\frac{B^*}{P^*} = \frac{bf + bf^* + D^*}{P^*} \tag{29}
\]
The market clearing conditions already incorporate an implicit assumption of purchasing power parity:
\[
P = eP^*. \tag{30}
\]
It is further assumed that the foreign price level is exogenously given — either because the domestic country is small by comparison, or because foreign prices are some (unspecified) function of foreign monetary and fiscal policy initiatives.

Finally, the total world supply of foreign exchange reserves is

A market clearing condition of this form presumes that both domestic and foreign holders of domestic currency hold these deposits in domestic banks. Hence the analysis abstracts from so-called Eurocurrency deposits.

Otherwise the clearing condition for domestic money, for example, would contain the term \([eP^*/P]md^*\) instead of \(md^*\).
assumed fixed (in foreign currency terms)

R + eR* = R₀ \hspace{1cm} (31)

so as to be consistent with the arguments of the previous chapter.

The goods market clearing condition describes world flow savings equilibrium. It is a direct statement of Walras law applied to flows. The next four conditions describe world stock portfolio equilibrium. But by the world wealth constraint,

\[ w + w* = md + md* + mf + mf* + bd + bd* + bf + bf* \]  \hspace{1cm} (32)

only three of these four stock conditions are independent. Therefore, a total of four independent market clearing conditions could, together with the demand equations, determine equilibrium values for four endogenous variables. However, there are eight endogenous variables in the model – r, r*, w, w*, a, e, R and P.\(^7\) Nevertheless, a 'reduced form' in e, R, a and P can be derived which describes that relationship between these variables consistent with stock and flow equilibrium. This would then be the required estimating equation.

In order to simplify the derivation, several additional assumptions are made. Firstly, Walras law is not invoked directly to eliminate one stock condition, but rather, all four conditions are used, together with the flow savings condition, on the understanding that satisfaction of the world wealth constraint places certain 'adding up' constraints on the

\(^7\) Now M, M* and R* are also endogenous, but equation (23) gives M, and equations (24) and (31) give M* and R*.
Now consider the two bond market clearing conditions. For any given set of values for \( w, w^*, a, e, R \) and \( P \), there will be solution values for the domestic and foreign interest rates \( r \) and \( r^* \) which ensure that these two conditions are satisfied and that the total stock of each country's bonds is willingly held. Therefore, in the remaining three market clearing conditions, \( r \) and \( r^* \) can be treated as givens, but on the understanding that their given values are consistent with stock equilibrium in the bond markets. And to the extent that changes in \( r \) and \( r^* \) are subsequently hypothesized that are independent of changes in \( D \) and \( D^* \), it is understood that there must be accommodating changes in \( B \) and \( B^* \) in order for these changes to be consistent with continuous bond market equilibrium. 8

Finally, the total foreign money supply \( M^* \) is regarded as being given, rather than being composed of an exogenous domestic component and an endogenous reserve component. This assumption implies an independent foreign monetary policy. 9

The remaining three equilibrium conditions, together with the demand equations, give an equilibrium relationship relating the endogenous variables \( e, R, a \) and \( P \) to the exogenous

8 This simplifying assumption does not imply the view that interest rates are determined 'solely in the bond markets'. Simplification could equally have been achieved by 'exogenizing' \( e \) and \( R \), thus giving an evaluation of interest rate policies!

9 It also means that, for the sake of simplicity, the issue of international coordination of domestic monetary policies is ignored.
variables (now including r and r*) and the parameters of the demand functions. Differentiation of the equilibrium conditions and demand functions with respect to time gives a relationship relating the time paths of e, R, a and P to the time paths of the exogenous variables and to the income, interest etc. sensitivities of the demand functions. And if the system starts from equilibrium, the time paths of e, R, a and P will be equilibrium time paths.

The derivation is carried out in Appendix III. Formally, the goods market and foreign money market clearing conditions are used to eliminate the domestic and foreign wealth terms from the domestic money market clearing condition. The result is an expression of the form

$$\frac{d e_R}{dt} = -\frac{d D}{dP} + b1 \frac{d M^*}{dtP} + b2r + b3r^* + b4a + b5y + b6y^* \tag{33}$$

where the b_i coefficients are, in general, quite complex functions of the income, interest etc. sensitivities of the domestic and foreign goods and money demand functions.\(^\text{10}\) The purchasing power parity condition has not yet been invoked throughout, so that the domestic price level P also appears explicitly. Purchasing power parity is invoked during simulation in its absolute form, without time differentiation, to tie the system down and ensure that the time paths of e, R, a and P are equilibrium time paths.

\(^{10}\) These expressions are examined in the third section of this chapter.
This equation was derived from the requirement that supply equal demand in the markets for goods and assets. It gives the interaction between reserve and exchange rate changes that is necessary to meet these equilibrium requirements in the face of shocks to exogenous supplies and to the exogenous determinants of demand. It is therefore analogous to the simple description of private sector behaviour from the previous chapter, where through the supply and demand system, the exogenous influences have now been identified explicitly.

**Estimation Problems and Results**

The above equation is not the form which is estimated. Firstly, its dependent variable, measuring the change in the real value of international reserves, is the sum of two components. The first measures the impact of changes in nominal reserves or the domestic price level and is given by \( e \frac{d}{dt} \left( \frac{R}{P} \right) \). The second measures the impact of exchange gains and losses and is given by \( e \left( \frac{R}{P} \right) \). Girton and Roper (1977) argue that the second component, to the extent that it increases the domestic money supply, should be attributed to the domestic component of the monetary base. Therefore, only the first component is used as regressand, while the second component is subsumed into the domestic monetary regressor. Furthermore,

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11 And while this formulation, involving the time derivative of expectations, may at first sight seem peculiar, it parallels a perhaps more familiar concept found in models of a closed economy, where changes in money holding depend on changes in the expected rate of inflation.
while the model suggests that the coefficient of this regressor is -1, its value is left unrestricted during the estimation. As several writers have noted, an observed magnitude smaller than one is consistent with less than instantaneous adjustment to equilibrium in the market for domestic money.

Secondly, the equation is estimated with a constant term and eleven seasonal dummy variables for the months January through November. As usual, the estimated coefficients of the dummy variables can be interpreted as the amount by which, ceteris paribus, the change in the real value of international reserves in the associated months differs from that in December. However, since the model suggests that the true constant is zero, the estimated constant can also be interpreted as a kind of seasonal dummy — the amount by which, ceteris paribus, the change in the real value of international reserves in December differs from that which would occur without seasonal influences.

With these modifications, the estimating equation can be written, in discrete time form, as

\[
RR(t) = c_0 + c_1 DD(t) + c_2 MM^*(t) + c_3 rr(t) + c_4 rr^*(t)
+ c_5 aa(t) + c_6 yy(t) + c_7 yy^*(t) + \Sigma diDUM_i(t)
\]  

where

- \( RR(t) = e(t-1) \delta[R(t)/P(t) ] \)
- \( DD(t) = \delta[D(t)/P(t) ] \)
- \( MM^*(t) = \delta[M^*(t)/P^*(t) ] \)
- \( rr(t) = \delta[r(t) ] \)
- \( rr^*(t) = \delta[r^*(t) ] \)
- \( aa(t) = \delta[a(t) ] \)
- \( yy(t) = \delta[y(t) ] \)
- \( yy^*(t) = \delta[y^*(t) ] \)

(34)

(35)

12 See, for example, Dornbusch (1976).
and where $D_{UMi}$ denotes the seasonal dummy for the ith month and $\text{del}[.]$ denotes the difference operator.

The monthly data used for the estimation are obtained from 'International Financial Statistics', published by the International Monetary Fund.\(^{13}\)

Canadian international reserves are measured in millions of US dollars, end of period. Since the analysis is primarily concerned with intervention activity and has abstracted from exogenous changes in world reserves, the data series is adjusted to exclude two important sources of exogenous increase - gold price revaluation profits\(^{14}\) and SDR allocations.

The Canadian exchange rate is measured in Canadian dollars per US dollar, end of period.

The domestic component of the Canadian monetary base, measured in millions of Canadian dollars, is calculated by subtracting the end of period Canadian dollar value of international reserves, described above, from the end of period Canadian monetary base.

The US monetary base is measured in billions of US dollars, end of period. Previously, no distinction was made between the foreign monetary base and the foreign money supply. Here the former is chosen so as to keep the foreign monetary aggregate consistent with the domestic money aggregates.

\(^{13}\) This source was chosen over primary sources because the estimation requires data for both Canada and the United States on a comparable basis.

\(^{14}\) Gold is valued at $(US)35$ per ounce throughout.
The Canadian and US price indices are the monthly average consumer price indices, with bases $1975 = 1$. Consumer prices indices are chosen over alternative indices on the assumption that, in a model of asset preference, the behaviour of wealth holders is dominated by consumers rather than firms, so that the consumer price index best reflects the price of the goods into which most assets in the economy would be converted.

The Canadian and US interest rates are measured as simple averages of the rate for three month Treasury bills, a short term rate, and the yield for government bonds with maturities of ten years or more, a long term rate, where each of these component rates have been converted from a rate per annum to a rate per month. The averaging of Treasury bill and government bond rates reflects the return to a bond portfolio that is diversified, at least term-wise. The conversion from annual to monthly rates places the interest rate measures on the same theoretical footing as the expected rate of exchange depreciation. Both reflect the gains available from month to month.

The preferred measures for Canadian and US real income are monthly indices of GNP at constant prices. Unfortunately, these figures are not available in the 'International Financial Statistics' for the entire period of estimation. Therefore,

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15 The conversion is carried out by regarding the annual rates as a geometric mean of monthly rates. Hence the conversion takes account of compounding, but ignores the penalties associated with early redemption.
monthly seasonally adjusted indices of industrial production, with bases 1975 = 100, are used as an empirical proxy. This choice can be defended on the grounds that industrial production is generally more labour intensive than agriculture, for example, so that the index reflects the product available for distribution to a significant proportion of the individuals in primary and secondary industries. However, the omission of the service industries may be an important one, especially if income in this sector is either more volatile than, or negatively correlated with, that in the industrial sector.

Finally, an empirical measure is required for \( \Delta \alpha \), the change in the expected rate of exchange depreciation. The reason it is included, even though subsequent simulations abstract from expectations, is that any private sector behavioural equation estimated from historical data without an expectations term clearly incorporates specification error. Now the standard approach, which invokes interest rate parity to empirically estimate the way in which expectations have historically been formed, is not adopted. Unless uncertainty is treated explicitly, this approach confounds forecast errors with risk premia.\(^{16}\)

The private sector behavioural equation can be estimated, however, using any pre-specified expectations mechanism. Several alternative specifications are tried, so as to see how sensitive

\(^{16}\) For the same reason, the forward parity condition is not used to estimate expectations.
the parameter estimates are to possible misspecification of the expectations variable. The first formulation is that expectations are static. Under this scheme $a_a$ does not appear in the estimating equation. The second formulation is that expectations are regressive, an assumption that has become widely accepted as a 'useful characterization' of exchange rate expectations. The normal exchange rate, to which these regressive expectations return, is taken to be a 6 month moving average of past market rates. This case, however, leads to a familiar identification problem - the measurement of the underlying behavioural parameter is confounded by an unspecified speed of adjustment parameter. The third formulation is that expectations are adaptive. Klein's method for direct estimation of distributed lag models is adopted. The advantage of this method is that, by grid search, a maximum likelihood estimate of the speed of adjustment parameter is obtained, thus avoiding the identification problem mentioned earlier. For the sake of brevity, these results are not reported in full and only the results associated with the best value for the speed of

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17 In each case, the assumption about expectations formation is converted to the appropriate measure of $a_a$, the change in the expected rate of exchange depreciation. It is assumed throughout that the current exchange rate is known.

18 See, for example, Roper and Turnovsky (1980).

19 See Maddala (1977, pp. 361-2) for a discussion of this method and the properties of the estimates it produces.
adjustment parameter are presented.\textsuperscript{20} The final formulation is
that expectations are rational. The unbiasedness property of
rational forecasts is used to replace an expected future rate by
its true value, so that the regression residuals also contain
the forecast errors - errors which by the assumption of
rationality are serially uncorrelated with a (conditional)
extpectation of zero.

The parameters of the private sector behavioural equation
are estimated, using the alternative formulations for \( aa \), by
ordinary least squares (OLS) regression. However, there is an
important and obvious source of simultaneous equation bias -
some regressors are endogenous in the overall model of exchange
rate determination. The full model comprises this estimating
equation, the purchasing power parity condition, and an exchange
market intervention rule or reaction function. Within this
model, the exchange rate and the domestic price level are
endogenous, so that the regressors \( DD \) and \( aa \) in which they
appear are also endogenous. Therefore, the private sector
behavioural equation is also estimated using the two-stage least
squares (2SLS) technique.\textsuperscript{21}

Implementation of this technique, however, requires an
assumption to be made about the type of intervention policy in
\textsuperscript{20} In a grid search over values 0.1, 0.2 ..., 0.9 the value 0.1
provided the smallest sum of squared residuals.

\textsuperscript{21} The regressor \( DD \) is a non-linear function of \( P \), so that \( DD \)
need not be contemporaneously linearly correlated with the
disturbance term, even though \( P \) may be. Nevertheless, the 2SLS
instruments are applied to \( DD \) as well as to \( aa \).
use during the period of estimation. This assumption determines which variables can be used as 2SLS instruments. It is doubtful that Bank of Canada actions during the 1970s can accurately be described by the optimal rule of the previous chapter. Instead, it is assumed that, during the period of estimation, exchange rate policy was formed using current and past values of reserves and the exchange rate as indicator variables. A general lag structure is assumed for this reaction function, and 15 lags on both the exchange rate and the reserve level are therefore used as instruments.

Table I shows the regression results produced by both OLS and 2SLS estimation of the private sector behavioural equation, using alternative formulations of the expectations term \( \alpha \) and 102 monthly observations from September 1970 to February 1979. The table gives the coefficient estimates for each regressor, with the magnitudes of the associated t statistics reported in parentheses. As noted previously, the theoretical framework suggests that the coefficient of \( DD \), the change in the real value of the domestic component of the Canadian monetary base,

\[ \frac{\Delta D}{\Delta t} \]

This number of lags leaves sufficient degrees of freedom for the Klein method to be legitimate.

The remaining instruments are the exogenous regressors, including seasonal dummies, the foreign price level, and the lagged endogenous variable \( e(t-1)R(t-1)/P(t-1) \) which appears on the left hand side of the estimating equation.

All empirical results in this thesis use the TROLL (Time-shared Reactive On-Line Laboratory) econometric package, maintained by the Centre for Computational Research in Economics and Management Science at MIT.
Table I - **Private Sector Behavioural Equation**

Dependent Variable = Change in Real Value of Canadian Reserves

<table>
<thead>
<tr>
<th>Expectations Formulation &amp; Estimation Technique</th>
<th>Independent Variables</th>
<th>2</th>
<th>R</th>
<th>DW</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>*</td>
<td>DD</td>
<td>MM*</td>
<td>rr</td>
</tr>
<tr>
<td>Static -OLS</td>
<td>-0.94</td>
<td>-2.55</td>
<td>778.27</td>
<td>-1912.48</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.35)</td>
<td>(0.60)</td>
<td>(1.45)</td>
</tr>
<tr>
<td>-2SLS</td>
<td>-0.96</td>
<td>-2.62</td>
<td>773.98</td>
<td>-1893.97</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.78)</td>
<td>(0.62)</td>
<td>(1.44)</td>
</tr>
<tr>
<td>Regressive-OLS</td>
<td>-0.81</td>
<td>-0.35</td>
<td>576.51</td>
<td>-1530.81</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3.77)</td>
<td>(0.09)</td>
<td>(1.20)</td>
</tr>
<tr>
<td>-2SLS</td>
<td>-0.91</td>
<td>-1.73</td>
<td>685.81</td>
<td>-1721.83</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.43)</td>
<td>(0.43)</td>
<td>(1.36)</td>
</tr>
<tr>
<td>Adaptive -OLS</td>
<td>-0.89</td>
<td>-2.64</td>
<td>847.41</td>
<td>-1465.88</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.31)</td>
<td>(0.66)</td>
<td>(1.66)</td>
</tr>
<tr>
<td>-2SLS</td>
<td>-0.92</td>
<td>-2.69</td>
<td>821.60</td>
<td>-1584.59</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.32)</td>
<td>(0.66)</td>
<td>(1.60)</td>
</tr>
<tr>
<td>Rational -OLS</td>
<td>-0.89</td>
<td>-2.96</td>
<td>562.66</td>
<td>-1598.08</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(2.37)</td>
<td>(0.73)</td>
<td>(1.10)</td>
</tr>
<tr>
<td>-2SLS</td>
<td>-0.94</td>
<td>-2.85</td>
<td>670.64</td>
<td>-1737.68</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(1.02)</td>
<td>(0.69)</td>
<td>(1.27)</td>
</tr>
</tbody>
</table>

* Estimates for DD tested against -1.
should be \(-1\). The \(t\) statistic reported for the DD coefficient is therefore appropriate for a test against \(-1\).\(^{25}\) The \(t\) statistics reported for all other coefficients are the usual ones associated with tests of significance. The critical value for one-sided \(t\) tests at the 5 percent level is \(1.671\). Durbin-Watson statistics lying between \(1.78\) and \(2.22\) indicate no first order autocorrelation in the regression residuals.\(^{26}\)

For brevity, the coefficient estimates for the seasonal dummy variables are not reported. In all cases, however, they are significant, even at the one percent level, and uniformly negative. Similarly, the estimates of the constant term are not reported. It is invariably significant and positive.\(^{27}\) The results for the remaining coefficients likewise show an insensitivity to either the expectations formulation or estimation technique adopted. One exception occurs with the coefficient of the expectations variable itself, but it should be remembered that for different formulations, this estimate measures different things. Only the estimates in the adaptive and rational formulations provide a direct estimate of the underlying behavioural parameter.

\(^{25}\) In all cases, the coefficient is significantly different from zero, even at the one percent level.

\(^{26}\) The hypothesis of no autocorrelation is accepted, even for the OLS regressions. This contrasts with the results of similar estimation exercises by Girton and Roper (1977) and Ujiie (1978), where first order error correction was required.

\(^{27}\) These results together suggest that the change in the real value of Canadian reserves has been systematically higher in December than in any other month.
Theory vs. Evidence

The signs and degrees of significance of the estimated coefficients are now considered in relation to the theoretical model of private sector behaviour.

Firstly, the coefficient of the Canadian monetary variable DD is not significantly different from -1, at least under 2SLS estimation. There has been a presumption that models which incorporate the assumptions of purchasing power parity and instantaneous stock and flow equilibrium are appropriate only to the longer run. The fact that the coefficient of DD is close to -1 when estimated using monthly data runs counter to this presumption.

Secondly, the coefficient of the US monetary variable MM* is uniformly negative, but not significant. This lack of significance can be explained a priori by the possibility of currency substitution. With this possibility, the coefficient is found, in Appendix III, to equal \((c_5c_5^*-c_5^5c_5^*)/(m_5c_5^*-c_5^5m_5^*)\), where \(c_5\) and \(c_5^*\) are the wealth sensitivities of Canadian and US demands for the consumption good, \(15\) and \(15^*\) are the wealth sensitivities of Canadian and US demands for Canadian money, and \(m_5\) and \(m_5^*\) are the wealth sensitivities of Canadian and US demands for US money. With currency substitution, US currency can be held in the wealth portfolios of US and Canadian citizens. It is therefore not surprising that the impact of an increase in the real quantity of US money depends on the wealth
sensitivities of citizens in both countries towards that money, relative to the wealth sensitivities of its substitutes - that is, the impact depends on the pattern of portfolio adjustment in both countries. Since all sensitivities are assumed positive, however, the coefficient cannot be signed unambiguously. By comparison, when currency substitution is not possible, the appropriate expression for the coefficient is given by 

\[-(c5*/c5m*)\] and is unambiguously negative.\(^{28}\)

This comparison shows that when international trade in domestically 'produced' assets, as well as in goods, is allowed, the pattern of gains from trade in those assets depends on the pattern of differences in preferences between countries. The lack of significance in the estimated coefficient of the US monetary variable may simply reflect that the difference in preferences between Canada and the United States is not particularly strong. However, under the intuitively appealing assumptions that \(c5 = c5^*\), but that in each country the wealth sensitivity with respect to the home currency exceeds that with respect to the offshore currency,\(^{29}\) then the coefficient is negative, as observed.

The signs of the remaining coefficients are now established a priori. In Appendix III, the coefficient of each regressor is

\(^{28}\) Without currency substitution, \(m5\) and \(l5^*\) equal zero.

\(^{29}\) Such an assumption can be justified on the grounds that the offshore currency is held primarily for transactions purposes and, as such, is sensitive more to changes in income than to anything else.
found to be as follows:

\[ rr : (l1+l1^*) - (m1+m1^*)b1 + (c1+c1^*)Y \]  
\[ \text{where } b1 \text{ is the coefficient of } M^*, \text{ given above, and} \]

\[ Y = [15m5*-m515^*/m5c5*-c5m5^*] \]

\[ rr^*: (l2+l2^*) - (m2+m2^*)b1 + (c2+c2^*)Y \]

\[ aa : (l3+l3^*) - (m3+m3^*)b1 + (c3+c3^*)Y \]

\[ yy : 14 - m4b1 - (1-c4)Y \]

\[ yy^*: 14^* - m4*b1 - (1-c4^*)Y \]

where the \( li, li^*, mi, mi^*, ci \) and \( ci^* \) parameters are the sensitivities of money and goods demands with respect to changes in the associated regressor.

These expressions measure the impact of the associated regressor on the real value of Canada's international reserves. Reserve flows between countries reflect the below-the-line money flows which balance the above-the-line flows of goods and bonds. Therefore, the impact of these regressors depends on the portfolio adjustment in Canadian and US holdings of Canadian money, as reflected in the \( li \) and \( li^* \) terms, and in Canadian and US holdings of US money, as reflected in the \( mi \) and \( mi^* \) terms.

The impact also depends on adjustments in portfolio size, an outcome of the consumption/savings decision and reflected in the \( ci \) and \( ci^* \) terms.\(^{30}\)

---

\(^{30}\) This can be seen most clearly in the expressions for the coefficients of \( yy \) and \( yy^* \), where the terms \((1-c4)\) and \((1-c4^*)\) represent the Canadian and US propensities to save out of income from human sources.
Earlier, reasonable assumptions were suggested under which \( b_1 \) would be negative. Under the same conditions, the term \( Y \) is also negative. Finally, the signs imputed in the first section of this chapter to the income, interest, etc. sensitivities of goods and money demands suggest the following signs for the remaining terms:

\[
\begin{align*}
(11+11^*) < 0; & \quad (m1+m1^*) < 0; \quad c1 > 0; \quad c1^* < 0 \\
(12+12^*) < 0; & \quad (m2+m2^*) < 0; \quad c2 < 0; \quad c2^* > 0 \\
(13+13^*) < 0; & \quad (m3+m3^*) > 0; \quad c3 < 0; \quad c3^* > 0 \\
14 > 0; & \quad m4 > 0; \quad (1-c4) > 0 \\
14^* > 0; & \quad m4^* > 0; \quad (1-c4^*) > 0
\end{align*}
\]

(41)

Now these assumptions are still not sufficient to remove all sign ambiguity for the coefficients of the income and interest rate regressors. A problem remains with the conflicting signs of the \( c_i \) and \( c_i^* \) terms. Nevertheless, if it is assumed that adjustments in portfolio composition dominate adjustments in portfolio size, then negative signs are obtained a priori for the coefficients of the domestic and foreign interest rate variables \( r_r \) and \( r_r^* \), and positive signs a priori for the domestic and foreign income variables \( y_y \) and \( y_y^* \).31 Once again, these are the signs which would be allocated without the assumption of currency substitution.32

31 The sign of the coefficient for the expectations variable \( a_a \) still defies a priori definition, but its observed positive value is consistent with the situation where the impact of expectations on Canadian holdings of US currency outweighs the impact on US holdings of Canadian currency.

32 The corresponding coefficients can be found by setting \( m_i \) and \( i_i^* (i=1,\ldots,5) \) to zero in the expressions (36) through (40).
The empirical results obtained from both OLS and 2SLS estimation are therefore generally consistent with the model of private sector behaviour. The estimated coefficients of \( rr^* \), \( yy \) and the monetary aggregates \( DD \) and \( MM^* \) are invariably of the correct sign. With the exception of the coefficient for \( MM^* \), discussed above, they are also generally significant. The coefficients of \( rr \) and \( yy^* \) have the wrong sign, but are invariably insignificant. The anomalous result for the variable \( rr \) can be explained by noting that, over the last ten years, the Canadian interest rate has tended to be an instrument of exchange rate policy, a feature not captured by the earlier assumption on Bank of Canada conduct during that period.

The description of private sector behaviour examined in this chapter bears similarities to the ones derived in Girton and Roper (1977), Ujiie (1978), Roper and Turnovský (1980) and almost any approach which uses the domestic stock money equilibrium condition, possibly combined with other conditions, as an organising structure within which to discuss balance of payments and/or exchange rate determination. In particular, it is similar to those derived in models of exchange market pressure.\(^{33}\) However, the signs which have been allocated a priori to the coefficients of the estimating equation differ in many cases from those which would have been allocated in such exchange market pressure models. The differences arise from the

\(^{33}\) See, for example, the seminal article by Girton and Roper (1976).
present treatment of money demand in a general equilibrium context.

Models of exchange market pressure generally consider only the requirement of stock money equilibrium. They arbitrarily subtract a foreign from a domestic monetary equilibrium condition to obtain an equation explaining the sum of reserve and exchange rate movement in terms of the determinants of domestic and foreign money demand. On the basis of this arbitrary mathematical operation, the regressors associated with the foreign country are given signs opposite to those of their domestic counterparts.\footnote{In particular, the coefficient of the foreign money supply is assumed positive and the coefficients of foreign income and interest rates are assumed to have signs opposite to their domestic counterparts.} Nevertheless, recent empirical estimation of such equations, especially in a short run context, have produced 'poor' results, an outcome which has been attributed to the inappropriateness of the underlying assumption of continuous stock equilibrium.\footnote{See, for example, Hacche and Townend (1981).}

By contrast, the present derivation takes account of the requirement for flow savings equilibrium as well as for stock asset equilibrium.\footnote{It therefore implicitly involves adding the foreign to the domestic monetary equilibrium condition, so as to obtain an expression for the level of world asset accumulation to combine with the flow condition that net world saving be zero. This was implicitly recognised in some of the seminal analysis of the global monetarist approach to the balance of payments. See, for example, Dornbusch (1973).} Therefore, the effect that an increase in

---

\footnote{In particular, the coefficient of the foreign money supply is assumed positive and the coefficients of foreign income and interest rates are assumed to have signs opposite to their domestic counterparts.}
the real US money supply, for example, has on the real value of Canadian reserves reflects, not an arbitrary mathematical operation, but the requirements for that real foreign supply increase to be willingly held by US and Canadian residents.\textsuperscript{37}

The implication is that the anomalous short run empirical results obtained from models which take account only of stock money equilibrium can be explained, not by assuming lagged adjustment, as Hacche and Townsend (1981) suggest, but by also taking account of flow savings equilibrium, as has been done here.

Finally, one of the sets of coefficient estimates must be chosen for use in the subsequent empirical examination of the reserve management rules. As has already been noted, the coefficient estimates are generally insensitive to either the expectations formulation or the estimation technique. The equation chosen is the OLS estimation which incorporates rational expectations formation. Not only does this equation have the advantage of avoiding identification problems, but it also provides OLS estimates which are closest to most of the 2SLS estimates. Appeal can then be made to the robustness of OLS estimates in the presence of other estimation problems and to the fact that OLS estimates provide predictions which compare favourably with those from alternative estimation techniques.

\textsuperscript{37} It must come at least partly at the expense of private holdings of Canadian currency, since it is assumed that portfolio substitution dominates the wealth-induced increase in saving.
V. RULE PERFORMANCE UNDER SIMULATION

Chapter 3 derived two versions of a reserve management rule which were designed to minimize unsystematic exchange rate variation and enhance predictability, while allowing systematic adjustment to take place so as recognise the operating constraint imposed by finite reserve levels. It was then argued that the foreign exchange authority may have additional concerns, one being the cost(s) of whatever divergence from a freely floating exchange rate such intervention activity produces. A second possible concern is the cost of acquiring and holding foreign exchange reserves for intervention purposes. Either cost could mitigate against the achievement of the primary objective.

One approach to such multiple, conflicting objectives would be to reformulate the optimal intervention rule, either by viewing the additional concerns as placing additional operating constraints on intervention activity, or by giving them a weighting in some hypothesized social welfare function. The approach adopted in this chapter is to examine how the foreign exchange authority can, by judicious choice of the policy parameters which the existing reserve management rule makes available, influence the outcome with respect to these additional concerns.

This chapter therefore compares the different versions' ability to track the general trends in a freely floating
exchange rate, relative to their success in enhancing exchange rate predictability. It also examines how either version's smoothing ability is affected by alternative target reserve levels. It therefore identifies margins of choice, over versions of the rule and over target reserve levels, by which the foreign exchange authority can meet its additional concerns. The precise choice can then be made once costs and benefits have been imputed ex post to these outcomes.

It was found in Chapter 3, however, that these outcomes could not easily be determined analytically. In particular, the simple version of the rule was derived under a simplifying assumption about the nature of the exogenous shocks hitting the exchange market. These were assumed to be such that, when the exchange rate was fixed at its 'correct' permanent or equilibrium level, reserves would follow a random walk. It was also assumed that a reserve management rule which leaned against the wind would minimize exchange rate variation around its permanent or equilibrium rate. When the first of these simplifying assumptions was relaxed, it could be shown that the simple reserve management rule could indeed minimize total exchange rate variation. However, there was no guarantee in theoretical terms that it would do so by operating only on the unsystematic component of exchange rate behaviour, at least for those situations where individuals did not have sufficient information to form strictly rational exchange rate expectations. The amended version was derived from consideration
of a particular counterexample. For either version of the rule, however, its relative impact on the systematic and unsystematic components of exchange rate behaviour could not be determined analytically in all situations. Nor was it a simple matter to determine how these impacts might be affected by alternative target reserve levels.

This chapter therefore examines these questions empirically, using simulation techniques. As in previous chapters, Canadian experience during the 1970s provides the setting. Rule performance is then examined in the face of a wide variety of exogenous shocks.

Nevertheless, any evaluation of the reserve management rules is meaningless unless it can be shown that they outperform existing policies. During the 1970s, the Canadian dollar floated against the US dollar, though not freely, thus suggesting some intervention policy was being followed by the Bank of Canada. Therefore, each aspect of rule evaluation is carried out, not only for a wide variety of hypothetical shocks, but also for those shocks which hit the Canadian dollar in the 1970s. In this way, the performance of the reserve management rules can be compared with the performance of the Bank of Canada's policies, as embodied in actual exchange rate behaviour, over that period.

The first section of this chapter describes the economic environment within which the simulations are performed. The second section describes the simulation results. It compares the performance of the reserve management rules, both alone and
relative to Bank of Canada actions, according to the above three concerns - enhancement of predictability, tracking ability, and performance under alternative target reserve levels. The third section summarizes these results, discussing them in relation to what has already been established about the rules' impact on ex post exchange rate and reserve behaviour, and in relation to the policy tradeoffs which they suggest.

The Simulation Environment

The previous two chapters developed separately the components of a full model of ex post exchange rate and reserve behaviour, within which to examine the performance of the optimal rules. The full model is as follows:

**Intervention Rule:**

\[ R = F(t) \]  

**Private Sector Behaviour:**

\[ \frac{d}{dt} \frac{eR}{P} = b_1 dD + b_2 dM^* + b_3 r + b_4 r^* + b_5 y + b_6 y^* \]

**Purchasing Power Parity:**

\[ P = eP^* \]

The endogenous variables are the exchange rate \( e \), the reserve level \( R \) and the domestic price level \( P \). The policy variable is \( F \). The exogenous variables are the domestic component of the monetary base \( D \), the foreign money supply \( M^* \), the foreign price level \( P^* \), the domestic and foreign interest rates \( r \) and \( r^* \) and the domestic and foreign real income levels \( y \) and \( y^* \).
The form in which the intervention rule is written allows characterization, by alternative choices of \( P \), of not only the simple and amended versions of the optimal rule, but also the benchmark of freely floating exchange rates. The private sector behavioural equation is identical to that developed in the previous chapter, except that the expectations term has been omitted. Chapter 3 showed that when expectations are rational, the performance of any rule which leans against the wind can be determined analytically. Evaluation is problematic only when individuals do not have sufficient information to form fully rational expectations. The omission of the expectations term is equivalent to assuming that expectations are static.\(^1\)

In a simulation experiment, all parameter values and time profiles of the exogenous variables are pre-specified and the system is solved for the time profiles of the endogenous variables. The simulation environment can therefore be described in terms of the parameter and exogenous variable values chosen.

The previous chapter reported the parameter estimates for the private sector behavioural equation, obtained from Canadian and US monthly data over the period September 1970 to February 1979.

The specification of the policy variable depends on the rule to be simulated. In the case of the benchmark regime, the parameter specification is straightforward. For flexible

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\(^1\) Such expectations may nevertheless be rational, depending on the structure of the exogenous variables.
exchange rates, the setting \( F(t) = 0 \) is used. The rule then specifies that no reserve change occur in any period.

The specification of the parameter values for the simple and amended versions of the optimal rule is slightly less straightforward. The rules specify the reserve behaviour desired ex ante by the foreign exchange authority, in terms of optimal limits on ex post reserve changes. Alternatively, define upper and lower bounds \( R_l \) and \( R_u \) on the current reserve level, one each side of the target level \( z \), whose distance from \( z \) is determined by two things - the deviation from \( z \) to date, plus the current permissible change in the same direction:

\[
R_l = z - |R(t-1)-z| - m(t) \tag{45}
\]

\[
R_u = z + |R(t-1)-z| + m(t) \tag{46}
\]

where \( m(t) = A - B|R(t-1)-z| \) for the simple version

\[
= A - B|R(t-1)-z| - CT \text{ for the amended version} \tag{47}
\]

and where \( A, B \) and \( C \) are policy parameters, \( |R(t-1)-z| \) measures the current deviation from target and \( T \) measures the excess time, over and above the minimum, that reserves have spent on one side or the other of target.

The rule allows any reserve change sufficient to fix the exchange rate, so long as it does not take reserve levels outside these bounds. Call the current reserve level which would

\[-------------------------\]

2 The placement of limits around \( z \), rather than \( R(t-1) \), assumes that there is ex post adjustment of the ex ante rule during the period. See the discussion in the first section of Chapter 3.
be consistent with no change in exchange rates Re. The rule then specifies that the current reserve level should be

$$ R = F^* = R_l \text{ if Re is less than } R_l $$
$$ Ru \text{ if Re is greater than } Ru $$
$$ Re \text{ otherwise} $$

(48)

In the above general form, the setting $F(t) = F^*-R(t-1)$ can then be used.

It remains to specify values for the policy parameters A, B and C. Chapter 3 derived expressions for these as functions of the target reserve level $z$, the upper limit on feasible reserve levels $h$ and the length of the authority's own planning horizon $H$. The choice of values for $z$, $h$ and $H$ is now described.

One purpose of the simulation experiments is to see whether, under various exogenous shocks, the simple and amended versions of the rule produce exchange rate behaviour which differs systematically from that produced were reserves fixed at target and the exchange rate free to float. As is explained shortly, these simulations begin with all the exogenous variables taking the values they had in Canada in March 1979, the first period beyond the estimation interval. Under flexible exchange rates, reserves remain at this level, so that is the value chosen for the target reserve level $z$. This value of $---------------- $3 Re(t)$ is found by setting $e(t)=e(t-1)$ in the private sector behavioural equation.

4 It was noted in Chapter 3 that while calculation of the reserve limits does not require prior knowledge of Re, implementation without this knowledge does impose practical difficulties. Here it is assumed that leaning against the wind occurs in an idealized fashion, essentially by assuming that the foreign exchange authority does know Re ahead of time.

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$(US)4.55\ billion\ is\ also\ 'representative',\ in\ the\ sense\ of\ being\ very\ close\ to\ the\ average\ reserve\ level,\ $(US)4.75\ billion,\ maintained\ in\ Canada\ over\ the\ period\ July\ 1970\ to\ March\ 1979.

In\ Chapter\ 3,\ it\ was\ suggested\ that\ if\ the\ target\ reserve\ level\ were\ itself\ an\ object\ of\ choice,\ then\ it\ would\ be\ set\ midway\ between\ the\ upper\ bound\ h\ and\ lower\ bound\ zero\ on\ feasible\ reserve\ levels.\ However,\ it\ was\ also\ suggested\ that\ the\ upper\ and\ lower\ bounds\ could\ be\ rescaled\ so\ that\ they\ represented\ reserve\ levels\ close\ to,\ but\ just\ short\ of,\ those\ which\ would\ cause\ real\ political\ or\ economic\ ruin.\ These\ two\ propositions\ are\ accepted\ in\ this\ chapter.\ The\ result\ is\ that\ the\ term\ z(h-z),\ which\ appears\ in\ the\ expressions\ for\ A\ and\ C,\ is\ replaced\ by\ the\ term\ (z-b)^2,\ where\ (z-b)\ measures\ the\ distance\ from\ target\ to\ either\ of\ the\ rescaled\ bounds.\ It\ also\ means\ that\ A/B\ equals\ (z-b),\ which,\ under\ the\ simple\ rule,\ becomes\ the\ maximum\ cumulative\ deviation\ in\ reserves\ from\ target\ that\ is\ allowed\ before\ a\ free\ float\ is\ instituted.\ Choosing\ a\ value\ for\ h\ now\ becomes\ a\ question\ of\ choosing\ a\ value\ for\ (z-b).\ The\ value\ chosen\ is\ $(US)3.32\ billion.\ This\ in\ turn\ implies\ a\ value\ $(US)1.23\ billion\ for\ the\ lower\ bound,\ and\ $(US)7.87\ billion\ for\ the\ upper\ bound.\ While\ these\ bounds\ allow\ wider\ fluctuations\ in\ reserve\ levels\ than\ those\ experienced\ in\ Canada\ during\ the\ 1970s\ [$(US)2.95-5.65\ billion],\ they\ ensure\ that\ reserves\ are\ stopped\ short\ of\ complete\ depletion.
Finally, a value for the length of the planning horizon \( H \) is required. While it is obviously difficult to obtain a direct estimate of this parameter, the value chosen is at least consistent with observed reserve behaviour in Canada during the 1970s. The choice of \( H \) will determine the maximum of permissible period-by-period changes in reserve levels, as reflected in the size of the policy parameter \( A \). In terms of Figure I in Chapter 3, it will determine the height of the point \( K \). At first sight, it would seem reasonable to assume that \( H \) takes the value of an electoral term – 48 months. However, such a value, when combined with the above assumptions about \( z \) and \( (z-b) \), implies a maximum permissible period-by-period change that is far smaller than the maximum of the monthly changes experienced in Canada over the last ten years. However, a value \( H=3 \), together with the values just chosen for \( z \) and \( (z-b) \),\(^5\) imply that the maximum of permissible period-by-period changes is \$\text{US}0.41\) billion. This value is approximately one standard deviation above the mean monthly reserve change in Canada in the 1970s.\(^6\)

A second purpose of the simulation experiments is to examine the effects of alternative policy parameter settings,

\(^5\) These values for \( z \), \( (z-b) \) and \( H \) produce values for \( A \), \( B \) and \( C \) of \( A=415.427 \), \( B=0.125 \) and \( C=51.928 \). However, the value for \( B \) actually used during simulation was \( 0.10825 \), and resulted from a heuristic solution for \( m \) that was slightly different from that presented in Appendix I.

\(^6\) The monthly changes in reserve levels in Canada between July 1970 and March 1979 had a mean of \$\text{US}0.17\) billion and a standard deviation of \$\text{US}0.24\) billion. However, there were only 9 changes during this period that were larger than \$\text{US}0.41\) billion.
derived from alternative values for \( z \), \( (z-b) \) and \( H \). The planning horizon \( H \) is assumed to be an internal decision parameter, not subject to amendment. The target reserve level \( z \) and the distance \( (z-b) \) to the upper and lower bounds together determine the size of the reserve pool (or reservoir, in the language of Chapter 3) available to the foreign exchange authority. When \( z \) is varied, the assumption that it lies midway between the upper and lower bounds is also maintained. Although the upper and lower bounds are also rescaled, they are not changed equipropotionately with \( z \), so that larger values of \( z \) truly imply a larger reserve pool. The optimal rules are then simulated using values for \( z \) that are half, one-and-a-half and twice the size chosen above. These results are then compared with those obtained were reserves fixed at these alternative target levels and the exchange rate free to float.

Now turn to the time profiles of the exogenous variables. In Chapter 3, it was argued that the performance of the rules would depend on the relative size and structure of the systematic and unsystematic components of the exogenous shocks. In a simulation context, the shocks are introduced through the exogenous variables. Since there are two types of exogenous variable in the model, nominal and real, shocks are introduced

\[ A = 207,714 \text{ and } C = 25,964. \]  
\[ A = 623,141 \text{ and } C = 77,693. \]  
\[ A = 830,855 \text{ and } C = 103,857. \]

\( B \) depends only on \( H \) and is unaffected by these changes.

---

7 When \( z = \$ (US) \) 2.28 billion and \( (z-b) = \$ (US) 1.66 billion \), then \( A = 207,714 \) and \( C = 25,964 \). When \( z = \$ (US) 6.83 billion \) and \( (z-b) = \$ (US) 4.99 billion \), then \( A = 623,141 \) and \( C = 77,693 \). When \( z = \$ (US) 9.10 billion \) and \( (z-b) = \$ (US) 6.65 billion \), then \( A = 830,855 \) and \( C = 103,857 \).
to each separately to see whether rule performance is dependent on the source of the shock. For each source of shock, both a systematic and unsystematic component to its movement over time is specified. Three types of systematic movement and three types of unsystematic movement are hypothesized, and for each source, these are tried in different combination. The size of each component, however, is chosen so as to be roughly consistent with Canadian experience during the 1970s. Thus the saccharine syndrome is avoided.8 The structure of the exogenous shocks is now described in more detail.

The three types of systematic movement are no change, once-for-all change and continuous change. The three types of unsystematic 'error' superimposed on these are independent, autocorrelated and random walk errors. The time profile of an exogenous shock is generated according to

\[ X(t) = X_0 + v_0 d(t) + v_1 t + v_2 u(t) \]  \hspace{1cm} (49)

where \( u(t) = \phi u(t-1) + \nu V(t) \) \hspace{1cm} (50)

\( X_0 \) is some initial or constant level for the associated exogenous variable \( X \). The variable \( d(t) \) is a 0-1 dummy variable which allows the introduction of a once-for-all change, and \( t \) is a time trend which allows the introduction of continuous change. The scaling parameters \( v_0 \) and \( v_1 \) determine the size of these systematic changes. The unsystematic component is introduced through the variable \( u(t) \), which is generated in turn by the

\[ \text{-------------} \]

8 Anything would be carcinogenic if consumed at the rate of 50 tons a day for 30 years.
standard normal random variable $RV$. A value for $\rho$ of zero then gives serially independent errors, $\rho=0.5$ is chosen to generate autocorrelated errors and $\rho=1.0$ generates random walk errors. The scaling parameter $v_2$ determines the size of this unsystematic movement.

Values for the scaling parameters $v_0$, $v_1$ and $v_2$ are chosen from Canadian experience as follows:

(i) Nominal Disturbances.

The first kind of disturbance is one to the domestic component of the Canadian monetary base $D$. Over the period July 1970 to March 1979, its behaviour can roughly be described as variation around an upward trend. A regression of $D$ on a constant term and a time trend $t$ over this period gives the following results:

$$D = -2260.13 + 98.63t$$

$$R^2 = 0.96 \quad DW = 0.26 \quad SER = 628.928$$

Therefore, $v_1=98.63$ fixes the size of a nominal trend. Once-for-all nominal change is set by $v_0=1297.25$, the maximum change in $D$ in Canada over the period July 1970 to March 1979. No systematic change occurs when $v_0$ and $v_1$ are zero.

The size of a nominal independent error term is set by $v_2=628.928$, the standard error of the above regression. For autocorrelated nominal errors, a value $v_2=544.668$ ensures that
their error variance is the same as for the independent errors.9 Although the error variance is undefined for random walk errors, their 'size' is set by $v_2=226.432$, the average size of changes in $D$ in Canada over the period July 1970 to March 1979.

(ii) Real Disturbances.

The interest rate variables $r$ and $r^*$, the real income variables $y$ and $y^*$ and the foreign real money supply variable $M^*/P^*$ enter the model in exactly the same way - changes in each appear linearly in the private sector behavioural equation.10 Therefore, for a given disturbance in any one of these exogenous variables, there is an equivalent disturbance in any one of the others which would have the same impact on the dependent variable of that equation. In terms of examining the effects of different sources of disturbance, it does not matter which of these variables is chosen. Their effects are equivalent, in the sense just described, and can loosely be called 'real' effects.11 For simulation purposes, the domestic real income variable is chosen as the vehicle for disturbances. The choice, while arbitrary, reflects a tradition of concern with the effects of 'a harvest failure, strikes or war.'12 Nevertheless,

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9 This calculation uses the relationship $V(u) = V(RV)/(1-\rho^2)$.

10 The foreign price level $P^*$ also appears in the purchasing power parity condition, but changes in $M^*/P^*$ can be hypothesized that are independent of changes in $P^*$.

11 While the interest rate variables are in nominal terms, the difference between them can be regarded as having a real effect.

12 See Mundell (1973, p.115).
the scaling parameters are chosen so that the size of variation in $\dot{y}$ is the same as that obtained by combining all the historical variation in the real variables $\dot{y}$, $\dot{y}^*$, $\dot{r}$, $\dot{r}^*$ and $\ddot{d}/\ddt(M^*/P^*)$, assuming at the same time that these separate influences are always reinforcing.

Therefore, $v_1 = \Sigma b_k / b_6 = 0.042082$ fixes the size of a real trend, where $k_i$ is the average change in each of these variables over the period July 1970 to March 1979, and $b_i$ is its associated regression coefficient. A once-for-all real change is set by $v_0 = 16.75624$, calculated in the same way but where $k_i$ now represents the maximum change in each variable. No systematic real change occurs when $v_0$ and $v_1$ are zero.

The size of a real independent error term is set by $v_2 = 3.925$, calculated as above but where $k_i$ represents the standard deviation of changes in each variable. For autocorrelated real errors, a value $v_2 = 3.399$ ensures that their error variance is the same as for the independent errors. Again, the error variance is undefined for random walk errors, but their 'size' is determined by $v_2 = 0.1$, a value which gives roughly the same exchange rate variance as with the other types of error, given the sample set of observations on RV.

With this simulation environment, each rule is simulated under every possible combination of systematic and unsystematic disturbance. Each simulation is given the same starting point, with the exogenous variables taking the values they had in Canada in March 1979. In subsequent periods, the source of the
disturbance takes the values just described, while all the other exogenous variables remain at their starting values. These values are:

\[
\begin{align*}
D &= D_0 = \text{(Can) 8838.37 million} \\
M^* &= M^*o = \text{(US) 142.4 billion} \\
Y^* &= Y^*o = 1.297 \\
r &= r_0 = 0 \quad \text{where } r = 0.8674 \\
r^* &= r^*o = 0 \quad \text{where } r^* = 0.7415 \\
y &= y_0 = 0 \quad \text{where } y = 120.6 \\
y^* &= y^*o = 0 \quad \text{where } y^* = 129.9
\end{align*}
\]

(52)

The same sequence of observations on RV is used in all simulations and has a mean of -0.0295 and a standard deviation of 0.9537. Each simulation proceeds for just over ten years, a period which, for concreteness, is called April 1979 to June 1989, and generates 123 monthly observations on the endogenous variables.

Finally, in order to compare the reserve management rules with Bank of Canada policies during the 1970s, each management rule is simulated when the exogenous variables take their actual values from the period July 1970 to March 1979. The resulting simulations generate 105 monthly observations on the endogenous variables.
The Simulation Results

This section compares the performance of the reserve management rules, both alone and relative to Bank of Canada actions, according to three concerns—enhancement of predictability, tracking ability, and performance under alternative target reserve levels.

(i) Enhancement of Predictability.

How much of the total impact of the intervention rules falls on the unsystematic component of exchange rate behaviour? In particular, do the reserve management rules produce unsystematic exchange rate variation that is significantly lower than that produced under a free float, given the wide variety of disturbances?

To answer these questions, a measure of unsystematic exchange rate variation is required. The variance of the exchange rate is inappropriate because it measures total variation, not unsystematic variation. At the very least, any trend component must be screened out before unsystematic variation can be measured. Therefore, the variance of exchange rate changes is used as a crude measure of unsystematic variation. This by definition is a measure of variation around the mean exchange rate change—that is, around the trend.

A standard F-test on equality of variances is then performed to compare the unsystematic variation produced by each

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13 As Taylor (1982) notes, a large variance need not be inconsistent with some notions of stability.
intervention rule with that produced under a free float.\textsuperscript{14} The resulting $F$ statistics are reported in Table II for both the various kinds of hypothesized disturbance and for the disturbances actually experienced in Canada during 1970-79. In each case, those instances are flagged where the rules produce unsystematic variation that is \textit{not} significantly smaller than under a free float.

Table II shows that in almost all cases, both versions of the reserve management rule significantly reduce the unsystematic variation that would otherwise occur, with the simple rule outperforming the amended version on this score. Furthermore, an argument can be made to dismiss the five exceptions on empirical grounds.

It was argued \textit{a priori} in Chapter 3 that the rules' success could be expected to depend on the relative sizes of systematic and unsystematic variation in the disturbance. Now in the above cases where the rules perform poorly, the size of unsystematic variation in the underlying hypothetical disturbance is significantly smaller than that experienced in Canada during the 1970s, and in that sense unrealistic. However, in other instances where the hypothetical unsystematic variation is

\textsuperscript{14} This test assumes the simulated sample variances are independent. Now the same set of observations on RV is used in each simulation so as to facilitate visual comparison. Dependence between sources of disturbance does not ensure dependence between variances, however, since the basic disturbance is run through different filters. The issue of independence is thus difficult to establish without an analytical solution.
Table II - **Comparison of Unsystematic Exchange Rate Variation**

*F*-test on Equality of Variances Between Intervention and Free Float

<table>
<thead>
<tr>
<th>Simulated Disturbance</th>
<th>Intervention Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Source</strong></td>
<td><strong>Systematic Component</strong></td>
</tr>
<tr>
<td>Nominal Constant</td>
<td>Independent</td>
</tr>
<tr>
<td></td>
<td>Random Walk</td>
</tr>
<tr>
<td></td>
<td>Independent</td>
</tr>
<tr>
<td>Autocorr.</td>
<td>7.21</td>
</tr>
<tr>
<td>Random Walk</td>
<td>4.77</td>
</tr>
<tr>
<td>Step</td>
<td>Independent</td>
</tr>
<tr>
<td></td>
<td>Autocorr.</td>
</tr>
<tr>
<td>Random Walk</td>
<td>1133.16</td>
</tr>
<tr>
<td>Real Constant</td>
<td>Independent</td>
</tr>
<tr>
<td></td>
<td>Autocorr.</td>
</tr>
<tr>
<td></td>
<td>Random Walk</td>
</tr>
<tr>
<td>Trend</td>
<td>Independent</td>
</tr>
<tr>
<td></td>
<td>Autocorr.</td>
</tr>
<tr>
<td></td>
<td>Random Walk</td>
</tr>
<tr>
<td>Step</td>
<td>Independent</td>
</tr>
<tr>
<td></td>
<td>Autocorr.</td>
</tr>
<tr>
<td></td>
<td>Random Walk</td>
</tr>
<tr>
<td>Historical 1970-79***</td>
<td>61.37</td>
</tr>
</tbody>
</table>

*Not significantly different from zero at the 5% level*

**F*-statistic very large

***Bank of Canada policy over this period, as embodied in actual exchange rate behaviour, produced F=7.28
smaller than historical experience, both versions of the rule significantly reduce that variation. Therefore, there is a margin of safety within which the rules perform well, despite small unsystematic variation.\textsuperscript{15}

Somewhat as an aside, the five exceptions just dismissed on empirical grounds are all associated with real disturbances. One difference between the real and nominal shocks is that levels have been prespecified for the nominal variable, but changes for the real variable. The real disturbance therefore has an additional, inbuilt element of autocorrelation. Consideration of both types of shock provides a richer variety of exchange rate patterns on which to base conclusions. However, the real shocks alone invariably produce unsystematic exchange rate variation that is unrealistically small.\textsuperscript{16} By contrast, the nominal disturbances alone can produce a range of unsystematic exchange rate variation in the neighbourhood of historical experience. Therefore, only the latter kind of disturbance is used when examining alternative target reserve levels.

\textsuperscript{15} The comparison of hypothetical and historical unsystematic shocks is performed using F-tests on the unsystematic variation thereby produced in a freely floating exchange rate. For the five exceptions, F statistics in the range 15.32 to 2147.00 are obtained, showing the unsystematic variation in the simulated disturbances to be significantly smaller than historical experience. In the other cases of small variation, F statistics in the range 2.97 to 76.61 are obtained, suggesting some margin of safety within which the rules can perform well.

\textsuperscript{16} Using the above method of comparison, F-statistics in the range 9.18 to 2147.00 are produced by the real disturbances.
Finally, those results obtained in the face of the disturbances experienced in Canada in the 1970s show that Bank of Canada policies, as embodied in the actual exchange rate behaviour over the period, produced exchange rates which were significantly smoother than the freely floating exchange rate. However, the simple, though not the amended version of the optimal rule would have outperformed the Bank of Canada on this score.

(ii) Tracking Ability.

The question here is whether the reserve management rules produce exchange rate behaviour that differs systematically from a free float.

This question is answered as follows. A sequence of observations is calculated on the difference between a freely floating exchange rate and that rate produced by the intervention rules. A t-test is then performed to see if the average of this sequence is significantly different from zero. The associated t statistics are reported in Table III, for both the various kinds of hypothesized disturbance and for the disturbances actually experienced in Canada during 1970-79. In each case, those instances are flagged where the rules are not significantly different from a free float.

Table III shows that in many instances, the reserve management rules produce exchange rate behaviour that is significantly different from that under a free float, although the amended version tracks better than the simple version.
Table III - Comparison of Systematic Exchange Rate Variation  

*t-test on Significance of Average Exchange Rate Difference between Intervention and Free Float*

<table>
<thead>
<tr>
<th>Simulated Disturbance</th>
<th>Intervention Rule</th>
<th>Amended Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Source</strong></td>
<td><strong>Systematic Component</strong></td>
<td><strong>Unsystematic Component</strong></td>
</tr>
<tr>
<td>Nominal Constant</td>
<td>Independent</td>
<td>Autocorr.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Random Walk</td>
</tr>
<tr>
<td></td>
<td>Independent</td>
<td>Trend</td>
</tr>
<tr>
<td></td>
<td>Autocorr.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Random Walk</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Independent</td>
<td>Step</td>
</tr>
<tr>
<td></td>
<td>Autocorr.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Random Walk</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Independent</td>
<td>Nominal Constant</td>
</tr>
<tr>
<td></td>
<td>Autocorr.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Random Walk</td>
<td></td>
</tr>
<tr>
<td>Real Constant</td>
<td>Independent</td>
<td>0.67*</td>
</tr>
<tr>
<td></td>
<td>Autocorr.</td>
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<td>Random Walk</td>
<td>7.16</td>
</tr>
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<td>Independent</td>
<td>Trend</td>
</tr>
<tr>
<td></td>
<td>Autocorr.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Random Walk</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Independent</td>
<td>Step</td>
</tr>
<tr>
<td></td>
<td>Autocorr.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Random Walk</td>
<td></td>
</tr>
<tr>
<td>Historical 1970-79**</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* Not significantly different from zero at the 5% level.  
** Bank of Canada policy over this period, as embodied in actual exchange rate behaviour, produced t=12.84
However, the question of divergence cannot be answered by this statistical test alone. In the first instance, the t statistics are sensitive to the direction, as well as to the size of divergence. Thus a minute but persistent difference in the exchange rate from a free float may show as significant, simply because the size of the difference, although small, is large relative to the degree of variation in the difference. Secondly, the question of divergence hinges on the costs of divergence relative to the benefits obtained by the reduction in unsystematic exchange rate variation, as was argued in the introduction to this chapter.

Nevertheless, this apparent inconclusiveness can be put in perspective by considering whether the rules outperform the intervention policy actually followed by the Bank of Canada during the 1970s. Table III notes that over this period, there was substantial divergence between the actual rate and that which would have occurred had the exchange rate floated freely. By comparison, both the simple and amended versions of the reserve management rule track the freely floating rate much more closely.

(iii) Alternative Target Reserve Levels.

The question here is whether success in reducing unsystematic exchange rate variation is sensitive to the choice of target reserve level.

Table IV reports the unsystematic exchange rate variation produced using different target reserve settings for the optimal
rules. Table V reports the F statistics which show this unsystematic exchange rate variation, relative to that produced were reserves fixed at the alternative levels and the exchange rate free to float.\textsuperscript{17} The target reserve level used to generate the results above is denoted by $z$. This value of $(\text{US})4.55 billion is close to the average reserve level maintained in Canada over the period July 1970 to March 1979. Alternative target reserve levels are denoted relative to this benchmark. The results are produced under various hypothetical nominal disturbances, as well as under those disturbances experienced in Canada over the period July 1970 to March 1979.

Tables IV and V show that the ability of each version to reduce unsystematic exchange rate variation initially increases, the larger is the target reserve level - that is, the larger is the reserve pool with which to perform the smoothing. However, beyond a certain reserve level, each version's performance again deteriorates. For various disturbances, the simple rule usually performs best at a target reserve level of $1.5z$, while the amended rule performs best at a target reserve level of either $z$ or $1.5z$. Assuming continuity, it can thus be claimed that the optimal target reserve level lies somewhere between $z=$(US)$4.55 billion is close to the average reserve level maintained in Canada over the period July 1970 to March 1979. Alternative target reserve levels are denoted relative to this benchmark. The results are produced under various hypothetical nominal disturbances, as well as under those disturbances experienced in Canada over the period July 1970 to March 1979.

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\textsuperscript{17} Relative as well as absolute reduction is reported because the absolute figures are not strictly comparable across target reserve levels. Despite the same disturbances, the total unsystematic variation to be reduced decreases slightly as the target reserve level increases. The larger the target reserve level, the larger the average nominal money supply, the smaller the percentage change in the money supply that a given disturbance represents, and the smaller the exchange rate adjustment required to compensate.
Table IV - Target Reserves and Absolute Variation Reduction

Std. Deviation of Exchange Rate Changes x 1000

<table>
<thead>
<tr>
<th>Simulated Disturbance</th>
<th>Systematic Component</th>
<th>Target Reserve Level+</th>
<th>Unsystematic Component</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>0.5z</td>
<td>z</td>
</tr>
<tr>
<td>Simple Rule</td>
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</tr>
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<td>Random Walk</td>
<td>7.68</td>
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<tr>
<td>Trend</td>
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<td>26.13</td>
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<td>21.12</td>
<td>14.03</td>
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<td>Random Walk</td>
<td>8.74</td>
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<td>Autocorr.</td>
<td>23.38</td>
<td>13.36</td>
</tr>
<tr>
<td></td>
<td>Random Walk</td>
<td>7.09</td>
<td>0.54*</td>
</tr>
<tr>
<td>Historical 1970-79</td>
<td></td>
<td>18.02</td>
<td>7.74*</td>
</tr>
<tr>
<td>Amended Rule</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>Independent</td>
<td>33.40</td>
<td>15.84</td>
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<td></td>
<td>Autocorr.</td>
<td>22.40</td>
<td>13.14</td>
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<tr>
<td></td>
<td>Random Walk</td>
<td>10.51</td>
<td>6.84*</td>
</tr>
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<td>Trend</td>
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<td>20.05</td>
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<td>Random Walk</td>
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<td>11.21*</td>
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<tr>
<td>Step</td>
<td>Independent</td>
<td>33.49</td>
<td>20.04</td>
</tr>
<tr>
<td></td>
<td>Autocorr.</td>
<td>23.84</td>
<td>15.05</td>
</tr>
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<td></td>
<td>Random Walk</td>
<td>9.51</td>
<td>6.06*</td>
</tr>
<tr>
<td>Historical 1970-79</td>
<td></td>
<td>30.79</td>
<td>17.83</td>
</tr>
</tbody>
</table>

+ $z = \$ (US) 4.55 billion

* Minimum unsystematic exchange rate variation
Table V - **Target Reserves and Relative Variation Reduction**

F-test on Equality of Variances Between Intervention and Free Float

<table>
<thead>
<tr>
<th>Simulated Disturbance</th>
<th>Target Reserve Level+</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.5z</td>
</tr>
<tr>
<td><strong>Systematic Component</strong></td>
<td><strong>Unsystematic Component</strong></td>
</tr>
<tr>
<td><strong>Simple Rule</strong></td>
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<td>Constant</td>
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<td>Random Walk</td>
</tr>
<tr>
<td>Trend</td>
<td>Independent</td>
</tr>
<tr>
<td></td>
<td>Autocorr.</td>
</tr>
<tr>
<td></td>
<td>Random Walk</td>
</tr>
<tr>
<td>Step</td>
<td>Independent</td>
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<tr>
<td></td>
<td>Autocorr.</td>
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<tr>
<td></td>
<td>Random Walk</td>
</tr>
<tr>
<td>Historical 1970-79</td>
<td>5.28</td>
</tr>
<tr>
<td><strong>Amended Rule</strong></td>
<td></td>
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<td>Constant</td>
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<tr>
<td></td>
<td>Autocorr.</td>
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<tr>
<td></td>
<td>Random Walk</td>
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<tr>
<td>Trend</td>
<td>Independent</td>
</tr>
<tr>
<td></td>
<td>Autocorr.</td>
</tr>
<tr>
<td></td>
<td>Random Walk</td>
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<tr>
<td>Step</td>
<td>Independent</td>
</tr>
<tr>
<td></td>
<td>Autocorr.</td>
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<tr>
<td></td>
<td>Random Walk</td>
</tr>
<tr>
<td>Historical 1970-79</td>
<td>1.81</td>
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</tbody>
</table>

* z = $(US) 4.55 billion

* Maximum reduction in unsystematic exchange rate variation

** F-statistic very large
billion and 1.5z= $(US) 6.83 billion, with the optimum perhaps being smaller for the amended version than for the simple version.

The explanation for this result lies in the way the rules operate. There is an inherent conflict between the desire to minimize exchange rate variation and thus maximize reserve level variation, and the requirement that reserves remain within the limits set by reserve availability. The larger the pool of available reserves, the wider those limits, so that more of any given shock can be absorbed by reserve movement when reserves are close to targeted levels. With wider limits, however, reserves can move further away from target and remain away for longer periods. When this occurs, exchange rate adjustment must begin to substitute for reserve adjustment.18

The simulation results therefore suggest that for target reserve levels larger than $(US) 4.55-6.83 billion, these countervailing forces begin to dominate. Beyond this range, there are negative returns to additional reserve holding. The results also confirm that because the amended version incorporates stronger countervailing forces than the simple version, its negative returns occur earlier.

18 The simple version sets the current permissible reserve change m* according to A-BX, where X is the deviation in reserves from target. From Chapter 3, the partial derivative of m* with respect to z is positive when the assumption h=2z is maintained. However, dX/dz (and dT/dz) may ex post be negative.
Result Summary and Policy Conclusions

Appendix IV depicts graphically the simulation results obtained for a given target reserve level. In that Appendix, Figures V to XXII show the exchange rate behaviour produced by the reserve management rules with each hypothetical source and structure of disturbance, and Figure XXIII shows the results obtained in the face of those disturbances experienced in Canada over the period July 1970 to March 1979. This last figure also shows Bank of Canada performance, as embodied in actual exchange rate behaviour, over that period.

The simple version of the reserve management rule produces exchange rate behaviour which appears, by casual observation, to be smoother than that under a free float. Both versions operate by placing (variable) upper and lower bounds on reserve movements. So long as the disturbance is small enough to keep reserves within these bounds, the exchange rate remains fixed. When the bounds are hit, the simple version leans against the wind - it allows an exchange rate adjustment, but one smaller than which would occur under a freely floating regime. Finally, both versions allow the possibility of a free float in the face of large or persistent disturbances. In this way, the simple

---

19 An interesting point is that a once-for-all real change is not distinguishable from the variation around that change. This confirms the earlier judgement on the desirability of designing a rule which does not presume an ability to distinguish different kinds of shock. The results suggest that this may be difficult.

20 In no case did the simple version actually produce a free float, although Figure XVI shows a case where it came very close to doing so.
version reduces both the size and the number of exchange rate shocks. The results of the previous section confirm that this reduction in unsystematic exchange rate variation is statistically significant, at least for disturbances in a wide range around those experienced in Canada during the 1970s.

The amended version of the rule also appears to smooth exchange rate variation, even though it does not always lean against the wind. If reserves remain away from target for prolonged periods of time, then this version eventually forces them back, independent of the current shock. Ex post, the authority may bet with, rather than against, the market and exacerbate a single exchange rate adjustment. But as the results show, this need not be inconsistent with stability or smoothness over the longer term. The results of the previous section confirm that the reduction in unsystematic exchange rate variation is statistically significant, at least for disturbances in a wide range around historical experience. However, the results also show that, because this version places an additional constraint on reserve movement, it cannot reduce unsystematic exchange rate variation as successfully as the simple version.

Both versions of the reserve management rule have a degree of inbuilt asymmetry, which allows systematic exchange rate adjustment in the face of a trend in the underlying disturbance. Because reserves are kept within some range around target, exchange rate adjustment must eventually substitute for reserve
The amended version sets narrower limits on reserves, which explains why it appears to track the systematic adjustment of a freely floating exchange rate better than its simple counterpart. Nevertheless, the results of the previous section show that either version can produce deviations from a free float that are statistically significant.

From a policy perspective, however, such deviations matter only if they impose costs which might outweigh the benefits obtained from the reduction in unsystematic exchange rate variation, with its associated increase in exchange rate predictability. This in turn depends on the tradeoff between smoothing and tracking ability. In this regard, the simulation results can be summarized by Figure III. This gives the tradeoff produced in the face of those disturbances experienced in Canada during the 1970s. This case typifies the results produced under a wide variety of hypothetical disturbances, and also allows a comparison with Bank of Canada policies over the period.

On the horizontal axis is a measure of unsystematic exchange rate variation, normalized so that the variation produced by a freely floating exchange rate gives a value of

21 It appears from Appendix IV that the greater the degree of serial correlation in the unsystematic component of the underlying disturbance, the worse the tracking ability of either version. This is because positive serial correlation leads to short term cycles, within which reserves can persistently lie on one side or the other of target and thus reinforce any such tendency already implicit in the systematic component of the exogenous shock.
Figure III – Tracking Ability and Enhancement of Exchange Rate Predictability

Systematic Deviation from Free Float

Unsystematic Exchange Rate Variation

D-Bank of Canada
C-Simple Version
B-Amended Version
A-Free Float
unity. On the vertical axis is a measure of the systematic deviation from a freely floating exchange rate. In the face of those disturbances experienced in Canada during the 1970s, each version of the reserve management rule would have produced a certain combination of unsystematic variation and systematic deviation, as shown by the points B and C. Typically, the simple version enhances exchange rate predictability more successfully than the amended version, but produces more divergence from a freely floating exchange rate. The point A is obviously associated with freely floating exchange rates. The point D is the point produced by Bank of Canada policies, as embodied in actual exchange rate behaviour, over the period.

The line A-B-C defines the tradeoff between smoothing and tracking ability made available by different versions of the reserve management rule. The line A-D defines the tradeoff produced by Bank of Canada actions during the 1970s. Given that a foreign exchange authority would prefer to be as close to the origin as possible, the simple and amended versions of the optimal rule clearly produce a more efficient tradeoff than does Bank of Canada policy. The choice between versions, however, depends on the exact shape of the authority's indifference map. This depends in turn on the benefits the authority imputes to enhancement of exchange rate predictability, relative to the

---

22 This measure is the inverse of the F-statistics reported in Table III.

23 This measure is the t-statistic from Table III.
The primary objective of the optimal reserve management rule was the enhancement of exchange rate predictability. The above results indicate that, within the general framework of the rule, the foreign exchange authority has a margin of choice by which it can meet an additional concern. By choosing between versions of the rule, it can take account of the costs it imputes to prolonged divergence from a freely floating exchange rate.

A second question is whether, within the general framework, it has a margin of choice by which it can take account of the costs of the implied average reserve holding. This can be answered by considering how alternative target reserve levels affect rule performance.

Figure IV shows schematically the simulation results obtained under alternative target reserve levels. The horizontal axis shows alternative target reserve levels, measured in multiples of $(US) 4.55 billion, a figure very close to the average reserve level maintained in Canada during the 1970s. The vertical axis measures unsystematic exchange rate variation.

Figure IV shows that up to a point, a larger reserve pool can enhance the performance of the optimal intervention rules. Beyond that point, there are negative returns to additional reserve holding. This result reflects the inherent conflict between the desire to minimize exchange rate variation, and the
Figure IV - Target Reserve Levels and Enhancement of Exchange Rate Predictability

Unsystematic Exchange Rate Variation

Target Reserve Levels (multiples of $(US)4.55 billion)
requirement that reserves remain within the limits set by reserve availability. The larger the pool of available reserves, the wider those limits, so the more a given shock can be absorbed by reserve movement when reserves are close to targeted levels. However, the wider the limits, the further reserves can move away from their targeted levels and when this occurs, exchange rate adjustment must begin to substitute for reserve movement.

More fundamentally, the existence of negative returns to additional reserve holding can be traced to the information assumptions embodied in the optimal rules. As was discussed in the introduction to this thesis, one of the major flaws of the Bretton Woods system was the difficulty of applying the clause on 'fundamental disequilibrium'. Without perfect knowledge or foresight, authorities had difficulty distinguishing a disequilibrium that was truly permanent and fundamental from one that arose through temporary changes and would be subsequently corrected without further action. The reserve management rules do not require that the foreign exchange authority make this distinction. Despite this, the exchange rate could still be smoothed completely and indefinitely, were reserves infinite. With anything less than infinite reserves, the possibility of reserve depletion or excessive accumulation must be considered. Successful smoothing of unsystematic, reversible exchange rate movement therefore requires, not just that reserves be large, but that they be large enough to fully accommodate the shock.
before the limits of reserve depletion or excessive accumulation are reached.

With perfect information about the future, part of the choice of optimal target reserve levels would involve consideration of the quantity required to smooth the exchange rate until a trend is reversed. In this case, there would not be negative returns to additional reserve holding. Without perfect information, it cannot be known if or when a trend will be reversed. However, the constant risk of reserve depletion causes the reserve management rules to operate on the side of conservatism. Exchange rate adjustment is sometimes allowed which, in hindsight, could be seen to be unnecessary. Furthermore, larger target reserve levels allow stronger resistance to exchange rate adjustment initially, but when eventual exchange rate adjustment occurs, it must be larger and more disruptive than with smaller target reserve levels and more frequent, gradual exchange rate adjustment. With the longer view provided by the simulation experiments, it can be seen that these considerations eventually dominate. Therefore, in the absence of perfect future information, there are eventually negative returns to additional reserve holding.

From a policy perspective, however, the existence of negative returns need only be a theoretical curiosity. For the same reason that a profit maximizing firm will never choose to operate where marginal productivity is negative, an authority that wishes to maximize social welfare will never choose a
target reserve level which provides a negative marginal return. The exact choice of target reserve levels is governed by a weighing of the benefits imputed to enhancement of exchange rate predictability, relative to the costs of acquiring and holding reserves. By holding reserves for intervention purposes, the authority is foregoing the real return obtained had they been invested in productive activity. The authority should choose that target reserve level for which the imputed marginal benefit equals the marginal cost - that is, the real interest rate. At this point, the returns to additional reserve holding are still positive.

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24 This argument is developed more fully in the literature on the demand for reserves. See, for example, Clark (1970) and Grubel (1973), (1977).
VI. SUMMARY AND POLICY CONCLUSIONS

Since the breakdown of the Bretton Woods agreement in 1971, the international monetary system has been characterized by managed floating. This system is not the result of a formal international agreement. Instead it has evolved as each country has made its decision concerning the exchange rate regime it would adopt. The choice of managed floating over full flexibility is consistent with recent theoretical findings, which raise serious questions about the supposed desirability of freely floating exchange rates. However, recent experience with managed floating has also raised questions about the usefulness of those approaches to optimal exchange market intervention which are based on inappropriate objectives, or which ignore the operating constraints imposed, for example, by finite reserve levels. Instead, recent experience suggests that many of the questions raised at the time of the Bretton Woods breakdown now deserve answers. This argues strongly for a comprehensive study of exchange market intervention, in terms of its justification, and in the context of methods which recognise the operating constraints. This thesis provides such a study.

The examination begins in Chapter 2 with a review of arguments used to justify exchange market intervention. Most recent literature has argued for intervention as a way of directing either real income or real consumption towards some desired goal. However, a careful examination of the welfare
economic foundations of these arguments shows that it is generally some assumed price rigidity or market failure which is responsible for the original divergence in real income/consumption from its goal. Thus exchange market intervention on these grounds constitutes a second best solution.

The strongest argument for intervention, and the one adopted here, is one in which intervention represents, not a second best solution, but an essential ingredient to market efficiency. Intervention which smoothes variation and increases predictability of exchange rate and price levels has a positive private and social value because it contributes to the usefulness or 'quality' of a country's money. Efficiency therefore demands some level of expenditure by the supplier(s) of money on quality-producing intervention activity. Intervention is thus justified because it ensures efficiency at a margin usually ignored by the proponents of freely floating exchange rates - the demand for and supply of money.

Were foreign exchange reserves available in unlimited quantities, such exchange rate and price level variation could be reduced, and indeed eliminated, by fixing the exchange rate completely. It is a non-trivial problem, however, to design a reserve management rule which can minimize unsystematic or unpredictable exchange rate fluctuation without violating the constraint imposed by reserve availability. The problem is similar to one of hydroelectric power generation. With water
flowing into a reservoir at a fluctuating rate, what rule on reservoir levels should the authority follow so as to minimize variation in electricity generation without violating the reservoir's physical capacity? A priori, it is obvious that in order to avoid reserve depletion or excessive accumulation, some systematic exchange rate/electricity supply adjustment must on occasion be allowed to take place. The problem is further complicated, however, when the authority cannot distinguish ahead of time a shock that represents systematic movement from one that represents unsystematic variation around that movement.

The approach adopted in Chapter 3 is to begin by making a specific simplifying assumption about the stochastic behaviour of reserve levels in the absence of optimal intervention, and to derive an optimal reserve management rule based on this assumption. The result is a rule which specifies the reserve behaviour desired ex ante by the foreign exchange authority in terms of optimal limits on ex post reserve movements. These limits are set according to how far reserves have already deviated from some targeted long run average level. The further reserves are from target, the less a given shock can be absorbed by further reserve movement, so the more it must be absorbed by exchange rate adjustment. The rule does not presume, however, that the foreign exchange authority knows ahead of time what the current shock will be, nor that it knows how that private sector will respond to that shock. It therefore imposes minimal information requirements.
Chapter 3 also considers situations where the simplifying assumption behind the rule's derivation is not met. Consideration of one particular alternative scenario leads to an amended version of the simple rule. The simple version rules out reserve depletion or excessive accumulation, but does not rule out lengthy deviations in reserve levels on one side or the other of target. It does not, therefore, rule out lengthy divergence from a freely floating exchange rate. The amended version limits the duration of such deviations.

The primary objective of these reserve management rules is the enhancement of exchange rate predictability. However, the foreign exchange authority may have additional concerns, one being the cost(s) of whatever divergence from a freely floating exchange rate such intervention activity produces. A second possible concern is the cost of acquiring and holding foreign exchange reserves for intervention purposes. The study examines how the foreign exchange authority can, by exercising its choice over versions of the rule and over the target reserve level on which both depend, influence the outcome with respect to these additional concerns. However, the implications of such choices for ex post exchange rate and reserve behaviour cannot be determined analytically in all situations. These questions are therefore examined empirically, using simulation techniques. The simulation environment is described in Chapter 4 and the first section of Chapter 5, and is chosen to reflect Canadian economic conditions in the 1970s.
The simulation results in the remainder of Chapter 5 show rule performance in the face of a wide variety of exogenous shocks. The results confirm that in these situations, both versions of the rule significantly reduce exchange rate variation and enhance predictability, within the constraint imposed by reserve availability.

A second, related concern is the possible cost(s) associated with whatever divergence from a freely floating exchange rate remains. The results show that the authority can meet this concern by choosing between versions of the rule. Typically, the simple version enhances exchange rate predictability more successfully than the amended version, but produces more divergence from the systematic adjustment of a freely floating exchange rate. The precise choice of version, therefore, is governed by a weighing of the benefits imputed to enhancement of exchange rate predictability, relative to the costs imputed to divergence from a freely floating exchange rate.

Another possible concern of the foreign exchange authority is the cost of the implied average reserve holding. More specifically, the authority must choose a target reserve level with which to operate its chosen version of the rule. However, reserves are costly to acquire and hold. The results show that in the absence of perfect future information, the marginal benefits of additional reserve holding, in terms of increased exchange rate predictability, diminish and may eventually become
negative. Thus the choice of target reserve levels is governed by a weighing of the benefits imputed to exchange rate predictability, relative to the costs of acquiring and holding reserves. By holding reserves for intervention purposes, the authority is foregoing the real return obtained had they been invested in productive activity. The authority should therefore choose a target level for which the marginal benefit equals the marginal cost, namely, the real interest rate.

A final question is whether the optimal reserve management rules could have outperformed the intervention policy actually used by the Bank of Canada during the 1970s. It is not clear exactly what the aims of Bank of Canada intervention were over this period. Nevertheless, past experience of leaning against the wind has been characterized by Taylor (1982), among others, as one of

...resisting exchange rate changes. In some cases, this has lead to pegging the existing exchange rate when its equilibrium level changes. The authorities are able to hold out for a limited time but are eventually forced to allow the adjustment to take place, and lose substantial sums in the process.

Taylor argues that this characterization certainly fits the Canadian experience, although the eventual adjustment, apparent from early 1977, had as much to do with a change of monetary policy direction as anything else.

The above view sees leaning against the wind as a way of resisting long term trends. By comparison, the intervention rules developed here are aimed at resisting short term fluctuation, while allowing long term trend adjustment to take
place. This is a more ambitious aim, especially since the rules do not assume the authority has prior knowledge of the true trend.

Canadian experience subsequent to 1979 suggests that the Bank of Canada is more willing than previously to allow systematic long term adjustment in the Canadian exchange rate. However, the results show that in the 1970s there was substantial divergence between the actual rate and that which would have occurred had the exchange rate floated freely. By comparison, the simple and amended versions of the reserve management rule could have smoothed exchange rate variation to roughly the same extent, while tracking the general trends in a freely floating exchange rate much more closely. They therefore provide a more efficient tradeoff between smoothing and tracking ability than do the actual policies of the period. Therefore, if the Bank of Canada wishes to allow systematic adjustment, while still smoothing unsystematic variation and thus promoting the quality or usefulness of its money, then the reserve management rules developed here will outperform its policies of the recent past without imposing onerous information requirements.
APPENDIX I

This appendix gives the derivation of both the simple and amended versions of the optimal reserve management rule.

The simple version is derived from

\[ m(t) = \left[ z(h-z)/(h+t) \right]^{1/2} \tag{1.1} \]

where it was argued that \( t \) should measure, not real time, but the minimum time required to achieve any particular sized deviation in reserves from target. A relationship is therefore required which gives the minimum time \( t \) for a given deviation in reserves from target \( |R(t-1)-z| \), or, equivalently, the maximum deviation from target for a given time \( t \).

Now the magnitude of the current deviation in reserves from target is determined by the sum of past reserve changes. Since \( m(t) \) is, by definition, equal to \( |R(t)-R(t-1)| \), and since \( R(0)=z \), then

\[ |R(t-1)-z| \leq \sum_{s=1}^{t-1} m(s) \tag{1.2} \]

with the exact equality holding when all reserve changes \( m \) have been in the same direction. The summation therefore gives the maximum deviation for a given time \( t \). The summation is difficult to evaluate as an explicit function of \( t \), but the result

\[ \sum_{s=1}^{t-1} m(s) \geq \int_{1}^{t} m(t) \, dt = \int_{1}^{t} \sqrt{\frac{z(h-z)}{h+t}} \, dt \tag{1.3} \]

can be used to place a lower bound on the maximum deviation for a given time. Since the maximum deviation will be an increasing function of time, this approximation will therefore also place a lower bound on the minimum time \( t \) for a given deviation \( |R(t-1)-z| \).
Proceeding with the integration, while simplifying the notation by denoting the maximum deviation $|R(t-1)-z|$ by $X$, then

$$X = 2\frac{\sqrt{z(h-z)}}{\sqrt{h+1}} (t-1)$$

$$= 2\sqrt{z(h-z)}[\sqrt{h+1} - \sqrt{h+1}]$$

This can be solved for time in terms of the maximum deviation in reserves from target $X$ as follows:

$$\sqrt{h+1} = \frac{X}{2\sqrt{z(h-z)}} + \sqrt{h+1}$$

$$h+1 = \frac{X^2}{4z(h-z)} + h + 1 + X \frac{h+1}{\sqrt{z(h-z)}}$$

$$t = 1 + X \frac{h+1}{\sqrt{z(h-z)}} + \frac{X^2}{4z(h-z)}$$

Substitution of this expression for $t$ into $m(t)$ gives

$$m^*(X) = \sqrt{z(h-z)}[H + 1 + X \frac{h+1}{\sqrt{z(h-z)}} + \frac{X^2}{4z(h-z)}]^{-1/2}$$

This can be expressed as an approximately linear function of $X$ by taking a Taylor series expansion around $X=0$. For this purpose,

$$m^*(X=0) = \frac{\sqrt{z(h-z)}}{\sqrt{h+1}}$$

$$\frac{\partial m^*}{\partial X} = -\frac{1}{2} \frac{\sqrt{z(h-z)}[H+1]}{\sqrt{z(h-z)}} - \frac{X^2}{4z(h-z)} \frac{2X}{\sqrt{z(h-z)}}$$

$$\frac{\partial m^*(X=0)}{\partial X} = -\frac{1}{2} \frac{\sqrt{z(h-z)}[H+1]}{\sqrt{z(h-z)}} - \frac{X^2}{4z(h-z)} = -\frac{1}{2(H+1)}$$

Therefore,

$$m^*(X) = \frac{A - BX}{X}$$

where $A=m^*(X=0)$ and $B=\frac{\partial m^*}{\partial X}(X=0)$.

It was noted in Chapter 3, however, that the method of piecewise static optimization which lies behind this derivation in some sense ignores the past history of reserve changes, and
that the rule therefore requires the imposition of some cutoff to ensure that $m^*=0$ when the boundaries are reached. The necessity for this can be seen from equation (1.9) which, as it stands, will not ensure that $m^*(X)=0$ for any value of $X$. While the linearized version in (1.13) at least ensures that $m^*(X)=0$ when $X=A/B$, the expressions for $A$ and $B$ given in (1.10) and (1.12) still do not ensure that this deviation $A/B$ is less than that which implies reserve depletion or excessive accumulation. Therefore, cutoff can be imposed by amending the formulas for $A$, or $B$, or both.

The essence of the rule is taken to be the feature that $m^*$ decreases as $X$ increases, as reflected in the coefficient $B$. Therefore, cutoff is imposed by rescaling $A$. Furthermore, it can be seen from (1.1) that if $z$ were also an object of choice, a target rather than an arbitrary starting point, then the value of $z$ which would make $m$ as large as possible, subject to the condition $ED > H$, would be a value $z=h/2$. In this case, $z=h-z$, as is assumed in later chapters, and $A$ must therefore be set so that $m^*(X)=0$ when $X=A/B = z = h-z$. $A$ must therefore equal

$$A = \frac{\sqrt{h-z}}{\sqrt{4(h+1)^2}}$$

(1.14)

Before turning to the amended version of the rule, the close equivalence, at least when $z=h/2$, between this version of the rule and that derived from the alternative method of piecewise static optimization will be demonstrated. The alternative view led to
\[ m' = \sqrt{R(t-1)[h-R(t-1)]} \]  \hspace{1cm} (1.15)

This can be restated directly in terms of deviations from target as follows:

\[ m'(X') = \sqrt{\frac{[R(t-1)-z+z]h-z-(R(t-1)-z)}{H}} \]  \hspace{1cm} (1.16)

\[ = \sqrt{\frac{(X'+z)(h-z-X')}{H}} \]  \hspace{1cm} (1.17)

where the deviations \( X' \) are not in absolute value form. If \( z=h/2 \), then

\[ m'(X') = \sqrt{\frac{z^2-X'^2}{H}} \]  \hspace{1cm} (1.18)

and in this case, \( m'(X')=0 \) when \( |X'|=z \), as is required to avoid reserve depletion or excessive accumulation. Taking a second order Taylor series approximation around \( X'=0 \) yields

\[ m'(X') \approx A' - B'X'^2 \quad \text{where} \quad A' = z/\sqrt{H} \quad \text{and} \quad B' = 1/(2z\sqrt{H}) \]  \hspace{1cm} (1.19)

But once again, since the approximation is taken around \( X'=0 \), some further adjustment to \( A' \) is required to ensure that the cutoff condition \( A'/B'=z \) still holds. However, the close equivalence between this and the previous approach can be seen by considering the partial derivatives of \( m \) or \( m' \) with respect to \( X \) or \( X' \). From equation (1.13),

\[ \frac{\partial m}{\partial X} = -1/[2(H+1)] \]  \hspace{1cm} (1.20)

and from equation (1.19),

\[ \frac{\partial m'}{\partial X'} = -X'/[z\sqrt{H}] \]  \hspace{1cm} (1.21)

At least for medium sized deviations in reserves from target, say in the order of \( X'=z/2 \), these partial derivatives have similar orders of magnitude.
Now turn to the amended version of the intervention rule which was presented in the second section of Chapter 3. The amended version is derived from

\[ m(t, T) = \left[ \frac{z(h-z)}{(H+t+T)} \right]^{\frac{1}{2}} \]  

(1.22)

where \( t \) should measure the minimum time required to achieve a given deviation \( X \) from target, as before, and \( T \) is the excess time, over and above this minimum, that a deviation on one side or the other of target has persisted.

A relationship between minimum time \( t \) and deviations \( X \) can be obtained as before:

\[
X = \int_{1}^{t} m(t, T) \, dt = \int_{1}^{t} \frac{z(h-z)}{\sqrt{H+t+T}} \, dt 
\]

(1.23)

\[
= 2\sqrt{z(h-z)} \left[ \frac{\sqrt{H+t+T}}{1} \right]^{t} 
\]

(1.24)

\[
= 2\sqrt{z(h-z)} \left[ \frac{\sqrt{H+T} - \sqrt{H+T+1}}{1} \right] 
\]

(1.25)

This can be solved for \( t \) in terms of the deviation in reserves from target \( X \) and excess time \( T \):

\[
\sqrt{H+T+t} = \frac{X}{2\sqrt{z(h-z)}} + \sqrt{H+T+1} 
\]

(1.26)

\[
H+T+t = \frac{X^2}{4z(h-z)} + H + T + 1 + X\sqrt{\frac{H+T+1}{z(h-z)}} 
\]

(1.27)

\[
t(X, T) = 1 + X\sqrt{\frac{H+T+1}{z(h-z)}} + \frac{X^2}{4z(h-z)} 
\]

(1.28)

Now the minimum time to achieve a given deviation in reserves from target occurs, by definition, when \( T=0 \). Therefore, the minimum time is given by \( t(X, 0) \) and an observable measure of excess time can now be defined:

\[
T = T_0 - t(X, 0) 
\]

(1.29)

where \( T_0 \) is the total number of periods that reserves have spent.
on one side or the other of target.

Substitution of the above expression for \( t(X,0) \) into \( m(t,T) \) gives

\[
m^*(X,T) = \sqrt{z(h-z)} \left[ H + T + 1 + X \frac{H+1}{\sqrt{z(h-z)}} + \frac{X^2}{4z(h-z)} \right]^{-\frac{3}{2}} \tag{1.30}
\]

This can be expressed as an approximately linear function of \( X \) and \( T \) by taking a Taylor series expansion around \( X=0, T=0 \). For this purpose,

\[
m^*(X=0, T=0) = \sqrt{z(h-z)} \sqrt{\frac{H+1}{4z(h-z)}} \tag{1.31}
\]

\[
\frac{\partial m^*}{\partial X} = -\frac{1}{2} \sqrt{z(h-z)} \left[ H+1 \right]^{-\frac{3}{2}} \left[ \frac{2X}{\sqrt{z(h-z)}} \right] \tag{1.32}
\]

\[
\frac{\partial m^*(X=0, T=0)}{\partial X} = -\frac{1}{2} \sqrt{z(h-z)} \left[ H+1 \right]^{-\frac{3}{2}} \frac{2X}{\sqrt{z(h-z)}} = -\frac{1}{2(H+1)} \tag{1.33}
\]

\[
\frac{\partial m^*}{\partial T} = -\frac{1}{2} \sqrt{z(h-z)} \left[ H+1 \right]^{-\frac{3}{2}} \frac{2X}{\sqrt{z(h-z)}} \tag{1.34}
\]

\[
\frac{\partial m^*(X=0, T=0)}{\partial T} = -\frac{1}{2} \frac{\sqrt{z(h-z)}}{\sqrt{H+1}} \tag{1.35}
\]

Therefore,

\[
m^*(X,T) = A - BX - CT \tag{1.36}
\]

where \( A = m^*(0,0) \), \( B = \frac{\partial m^*}{\partial X}(0,0) \) and \( C = \frac{\partial m^*}{\partial T}(0,0) \).

However, as in the simple case, the intercept parameter \( A \) must be rescaled so as to ensure that the cutoff condition for \( X \) is met. This condition requires \( A \) to be set so that \( m^*(X,0) = 0 \) when \( X = A/B = z \). \( A \) must therefore be

\[
A = \frac{\sqrt{z(h-z)}}{\sqrt{4(H+1)^2}} \tag{1.37}
\]

as before. However, a cutoff condition for \( T \) must also be imposed to ensure that reserves are eventually brought back to target. Equation (1.30), as it stands, will not ensure \( m^* = 0 \) for
any value of \( T \). A cutoff of this sort would imply that, eventually, a given deviation in reserves from target could not become any larger. However, the requirement that this deviation be eventually eliminated is a stronger requirement. It requires, essentially, that the critical boundaries defining ruin be brought closer to target, so that a given deviation will eventually lie outside those boundaries. The foreign exchange authority then has no choice as to the direction of intervention — it must adjust its reserves in the direction that will bring them back towards the new critical boundaries. The linearized version in (1.36) allows this when \( T \) is large enough so that \( m^* \) is negative for a given \( X \), and intervention in that period is then in a direction opposite to \( X \). Reserves will be brought fully back to target when \( m^*(0,T)=0 \), or when \( T=A/C \).

The rescaling of \( A \) therefore has implications for this implicit cutoff condition on \( T \). If, however, both \( A \) and \( C \) are rescaled by the same factor, then this cutoff condition for \( T \) will remain unchanged. Roughly speaking, this means that the cutoff which ensures reserve depletion is ruled out does not affect the length of time that small deviations are allowed to persist. Therefore, \( C \) is also rescaled so that

\[
C = \frac{1}{2} \frac{z (h-2)}{2 \sqrt{4 (h+1)}}
\]  

Finally, the method of piecewise static optimization which lies behind this derivation makes explicit the way in which deviations \( X \) depend on time. In the alternative approach of equation (1.17), the dependence of \( X \) on time is only implicitly
recognised, so that there is no obvious generalization to a notion of excess time.
This appendix derives a rational expectations solution to the simple model of exchange rate determination.1

Private sector behaviour is implicitly given by the system of equations (6) to (9) from Chapter 3. By combining these equations, one obtains

\[ R = P*f(x) - D/e = g(x) - D/e \] (2.1)

where the foreign price level \( P^* \) has been subsumed into the vector of exogenous variables \( x \). Time differentiation yields

\[ \dot{R} = g'(x) \dot{x} - \frac{D}{e} + \left( \frac{D}{e^2} \right) \ddot{e} \] (2.2)

Therefore, \( \dot{R} \) is positively related to \( \dot{x} \) and \( \dot{e} \), and negatively related to \( D \).

In the neighbourhood of some point \( x=x_0, e=e_0 \), where \( e_0 \) is the permanent value of the exchange rate,2 this relationship can be written in the approximately linear form

\[ \dot{R} = \lambda e + \beta \dot{x} - \delta D \] (2.3)

Now behaviour in the private sector depends not simply on the actual change in the exchange rate \( \dot{e} \), but rather on the expected future change \( a \). Therefore

\[ \dot{R} = \lambda a + \beta \dot{x} - \delta D \] (2.4)

where \( a = E_t e(t+1) - e(t) \) (2.5)

and where both \( e(t) \) and \( e(t+1) \) now measure deviations in the exchange rate around its permanent value. Writing \( a \) in this form

---

1 The following analysis draws heavily on Mussa (1976).
2 Later discussion examines whether such a permanent value exists.
assumes that the exchange rate is currently known.

The reserve management rule can be written in a form whereby permissible reserve changes are some fraction $\phi$ of their value under fixed exchange rates. Under fixed exchange rates, $a=0$ so that

$$\dot{R} = \delta_x - \delta D$$

(2.6)

Therefore, a reserve-based intervention rule can be written as

$$\dot{R} = \phi[\delta_x - \delta D]$$

(2.7)

Substitution of this expression into the private sector behavioural equation and conversion to discrete time notation yields

$$e(t) = \frac{(1-\phi)}{\lambda} Z(t) + E_t e(t+1)$$

(2.8)

where

$$Z(t) = \delta[x(t)-x(t-1)] - \delta[d(t)-d(t-1)]$$

(2.9)

By assuming that expectations are formed rationally and using the technique of successive substitution, it can be shown that

$$E_t e(t+1) = \frac{(1-\phi)}{\lambda} \sum_{j=1}^{\infty} E_t Z(t+j)$$

(2.10)

At this stage, the conditions can be examined under which 'exchange rate bubbles', or destabilizing expectations, are ruled out. As Flood and Garber (1980) point out, a rational solution of the above form assumes that

$$\lim_{n \to \infty} E_t Z(t+n) = 0$$

(2.11)

If this condition is not met, then rational expectations will be destabilizing. Given the definition of $Z(t)$, this condition can be interpreted as requiring that the behaviour of the domestic
component of the monetary base $D$ be not 'too far' out of line with the underlying forces $x$ which determine the private sector's willingness to hold money. Otherwise, a systematic divergence would require systematic exchange rate adjustment in all future periods. This would then be reflected, through the forward looking nature of rational expectations, in an infinitely large deviation in the current exchange rate from its permanent value.\(^3\)

In order to obtain an expression for the unanticipated component of exchange rate behaviour in terms of the intervention parameter $\phi$, a stochastic process for $Z$ must be specified. Suppose that $Z$ follows an autoregressive scheme:

$$Z(t) = u(t) \quad (2.12)$$

where $u(t) = \rho u(t-1) + v(t) \quad (2.13)$

and where $v(t)$ is a serially uncorrelated random variable with a mean of zero and people are assumed to know the structure of the disturbances. The requirement that $\rho$ be strictly less than one is sufficient to ensure that condition (2.11) is satisfied. With $Z(t)$ of this form, the rational solution for expectations becomes

$$E_t e(t+1) = \frac{\rho(1-\phi)}{\lambda(1-\phi)} u(t) \quad (2.14)$$

so that the rational solution for deviations in the exchange rate from its permanent value becomes

\[^3\] Alternatively, no single permanent value would exist.
\[ e(t) = \frac{(1-\phi)}{\lambda(1-\phi)} u(t) \] 

Therefore, even when expectations are rational, the ex ante unpredictable component of exchange rate behaviour depends on \((1-\phi)\), as before.

Finally, the strength of the influence of intervention on this unpredictable component in turn depends on \(\phi\). Firstly, when \(\phi=0\) and disturbances are expected to last for only one period, then the reserve management rule is at its most effective. As \(\phi\) increases, the disturbances are expected to last for longer periods of time. In the absence of intervention, this would be reflected in larger deviations in the current exchange rate from its permanent value, and the influence of intervention is thereby diluted.

Finally, in the simple model presented here, the case \(\phi=1\) has been ruled out because it leads to destabilizing expectations. Furthermore, when \(\phi=1\) the exogenous disturbances follow a random walk. Here, however, a random walk has different implications from those in the first section of Chapter 3. There it was used as a way of showing, in the strongest possible way, that randomness of reserve behaviour does not rule out cumulative reserve movement in the same direction. Here, the implication is that when people know the general structure of the disturbances and when these do follow a random walk, then any current disturbance, once it hits, is expected to last forever. This simple model is not rich enough to allow a distinction to be drawn between those disturbances which cause
variation in the exchange rate around its permanent value \((\rho < 1)\), as opposed to those which cause an adjustment in that permanent value itself \((\rho = 1)\). In this sense, the solution presented here has the same limitations as did the discussion in the first section of Chapter 3. However, it can be argued that, in practical terms, the distinction between a shock that lasts for 1000 years and a shock that lasts forever is an unnecessarily fine one.
In Chapter 4, the reduced expression reflecting individual savings and portfolio adjustment behaviour was given as
\[
\frac{d \varepsilon_R}{dt} = - \frac{d D}{dt} + b_1 d M^* + b_2 r + b_3 r^* + b_4 a + b_5 y + b_6 y^* (3.1)
\]
where the dot notation represents time derivatives.

Here expressions for the bi coefficients are obtained in terms of the underlying income, interest etc. sensitivities of goods and asset demands.

As explained in Chapter 4, this expression is derived from the market clearing conditions reflecting world (flow) goods equilibrium and world (stock) equilibrium for domestic and foreign money. These conditions are
\[
c + c^* = y + y^* \quad (3.2)
\]
\[
e_R/P + D/P = md + md^* \quad (3.3)
\]
\[
M^*/P^* = mf + mf^* \quad (3.4)
\]
The domestic and foreign goods and money demand functions are given by
\[
c = c(r, r^*, a, y, w) \quad (3.5)
\]
\[
c^* = c^*(r, r^*, a, y^*, w^*) \quad (3.6)
\]
\[
md = l(r, r^*, a, y, w) \quad (3.7)
\]
\[
md^* = l^*(r, r^*, a, y^*, w^*) \quad (3.8)
\]
\[
 mf = m(r, r^*, a, y, w) \quad (3.9)
\]
\[
 mf^* = m^*(r, r^*, a, y^*, w^*) \quad (3.10)
\]
Time differentiation of all these conditions with respect to time yields
\[
c + c^* = y + y^* \quad (3.11)
\]
\[
\begin{align*}
d\frac{\text{eR}}{\text{d} \ P} + d\frac{\text{D}}{\text{d} \ P} &= m\frac{\text{d}}{\text{d} \ P} + m\frac{\text{d}^{*}}{\text{d} \ P} \quad (3.12) \\
\frac{d \ M^{*}}{\text{d} \ P} &= m\frac{\text{f}}{\text{d} \ P} + m\frac{\text{f}^{*}}{\text{d} \ P} \quad (3.13) \\
c &= c1r + c2r^{*} + c3a + c4y + c5w \quad (3.14) \\
c^{*} &= c1r^{*} + c2r^{*} + c3a^{*} + c4y^{*} + c5w^{*} \quad (3.15) \\
m\frac{\text{d}}{\text{d} \ P} &= l1r + l2r^{*} + l3a + l4y + l5w \quad (3.16) \\
m\frac{\text{d}^{*}}{\text{d} \ P} &= l1r^{*} + l2r^{*} + l3a^{*} + l4y^{*} + l5w^{*} \quad (3.17) \\
m\frac{\text{f}}{\text{d} \ P} &= m1r + m2r^{*} + m3a + m4y + m5w \quad (3.18) \\
m\frac{\text{f}^{*}}{\text{d} \ P} &= m1r^{*} + m2r^{*} + m3a^{*} + m4y^{*} + m5w^{*} \quad (3.19)
\end{align*}
\]

Substitution of equations (3.16) and (3.17) into equation (3.12) gives
\[
\begin{align*}
d\frac{\text{eR}}{\text{d} \ P} + d\frac{\text{D}}{\text{d} \ P} &= (l1+l1^{*})r + (l2+l2^{*})r^{*} + (l3+l3^{*})a \\
&\quad + l4y + l4y^{*} + l5w + l5w^{*} \quad (3.20)
\end{align*}
\]

Now the \( w \) and \( w^{*} \) terms can be eliminated from this expression by using equations (3.11), (3.13), (3.14), (3.15), (3.18) and (3.19). Substitution of (3.14) and (3.15) into (3.11) and rearrangement yields
\[
\begin{align*}
c5w + c5w^{*} &= -(c1+c1^{*})r - (c2+c2^{*})r^{*} - (c3+c3^{*})a \\
&\quad + (1-c4)y + (1-c4)y^{*} \quad (3.21)
\end{align*}
\]

Substitution of (3.18) and (3.19) into (3.13) and rearrangement yields
\[
\begin{align*}
m5w + m5w^{*} &= d\frac{\text{M}^{*}}{\text{d} \ P} - (m1+m1^{*})r - (m2+m2^{*})r^{*} \\
&\quad - (m3+m3^{*})a - m4y - m4y^{*} \quad (3.22)
\end{align*}
\]

Now equations (3.21) and (3.22) can be solved simultaneously to obtain expressions for \( w \) and \( w^{*} \) in terms of \( r, r^{*}, a, y, y^{*} \) and \( \frac{d}{dt}[M^{*}/P^{*}] \). These expressions are
\[ w^* = \frac{r[c_5(m_1 + m_1^*) - m_5(c_1 + c_1^*)]}{X} \\
+ \frac{r*[c_5(m_2 + m_2^*) - m_5(c_2 + c_2^*)]}{X} \\
+ \frac{a[c_5(m_3 + m_3^*) - m_5(c_3 + c_3^*)]}{X} \\
+ \frac{y[m_4c_5 + m_5(1-c_4)]}{X} \\
+ \frac{y*[m_4*c_5 + m_5(1-c_4^*)]}{X} \\
+ \frac{d}{dt} \frac{M^*[-c_5]}{X} \quad (3.23) \]

and

\[ w = \frac{r[m_5^*(c_1 + c_1^*) - c_5^*(m_1 + m_1^*)]}{X} \\
+ \frac{r*[m_5^*(c_2 + c_2^*) - c_5^*(m_2 + m_2^*)]}{X} \\
+ \frac{a[m_5^*(c_3 + c_3^*) - c_5^*(m_3 + m_3^*)]}{X} \\
+ \frac{y[-m_4c_5^* - m_5^*(1-c_4)]}{X} \\
+ \frac{y*[-m_4*c_5^* - m_5^*(1-c_4^*)]}{X} \\
+ \frac{d}{dt} \frac{M^*[c_5^*]}{X} \quad (3.24) \]

where \( X = m_5c_5^* - c_5m_5^* \).

Substitution of these expressions for \( w \) and \( w^* \) into equation (3.20) gives, upon simplification, an equation of the form

\[ \frac{d}{dt} E_R = - \frac{d}{dt} D + \frac{b_1d}{dt} M^* + b_2r + b_3r^* + b_4a + b_5y + b_6y^* \quad (3.25) \]

where \( b_1 = \frac{(15c_5^* - c_5l_5^*)}{X} \quad (3.26) \)

\[ b_2 = (l_1 + l_1^*) - (m_1 + m_1^*)b_1 + (c_1 + c_1^*)Y \quad (3.27) \]

where \( b_1 \) is given above and \( Y = \frac{(15m_5^* - m_5l_5^*)}{X} \)

\[ b_3 = (l_2 + l_2^*) - (m_2 + m_2^*)b_1 + (c_2 + c_2^*)Y \quad (3.28) \]

\[ b_4 = (l_3 + l_3^*) - (m_3 + m_3^*)b_1 + (c_3 + c_3^*)Y \quad (3.29) \]

\[ b_5 = l_4 - m_4b_1 - (1-c_4)Y \quad (3.30) \]

\[ b_6 = l_4^* - m_4*b_1 - (1-c_4^*)Y \quad (3.31) \]
APPENDIX IV

Figures V to XXIII on subsequent pages show the exchange rate behaviour produced by simulating the effects of the optimal reserve management rule. Each figure shows the exchange rate paths produced by the two versions of the rule, together with the path produced were reserve levels fixed at target and the exchange rate free to float, for a particular source and structure of exogenous shocks. In Figures V to XXII, these sources are either real or nominal. Their associated structure is given by a systematic component, described as either constant, trend or step, and an unsystematic component, described as either an independent, autocorrelated or random walk error term. In Figure XXIII, the exogenous shocks are those which hit the Canadian dollar during the 1970s.
Figure V
Exchange Rates
Nominal Disturbance
Constant with Independent Error

Legend
● Free Float
○ Simple Rule
□ Amended Rule
Figure VI
Exchange Rates
Nominal Disturbance
Constant with Autocorrelated Error

Legend

- Free Float
- Simple Rule
- Amended Rule
Figure VII
Exchange Rates
Nominal Disturbance
Constant with Random Walk Error

Legend
- Free Float
- Simple Rule
- Amended Rule
Figure VIII
Exchange Rates
Real Disturbance
Constant with Independent Error
Figure IX
Exchange Rates
Real Disturbance
Constant with Autocorrelated Error

Legend
● Free Float
○ Simple Rule
□ Amended Rule
Figure X
Exchange Rates
Real Disturbance
Constant with Random Walk Error

Legend
- Free Float
- Simple Rule
- Amended Rule
Figure XI
Exchange Rates
Nominal Disturbance
Trend with Independent Error

Legend
● Free Float
○ Simple Rule
□ Amended Rule
Figure XIII
Exchange Rates
Nominal Disturbance
Trend with Random Walk Error
Figure XIV
Exchange Rates
Real Disturbance
Trend with Independent Error

Legend
- Free Float
- Simple Rule
- Amended Rule
Figure XV
Exchange Rates
Real Disturbance
Trend with Autocorrelated Error
Figure XVI
Exchange Rates
Real Disturbance Trend with Random Walk Error

Legend
- Free Float
- Simple Rule
- Amended Rule

Time
Exchange Rate

0.6 0.7 0.8 0.9 1.0 1.1
Figure XVII
Exchange Rates
Nominal Disturbance
Step with Independent Error
Figure XVIII
Exchange Rates
Nominal Disturbance
Step with Autocorrelated Error
Figure XIX
Exchange Rates
Nominal Disturbance
Step with Random Walk Error

Legend
- Free Float
- Simple Rule
- Amended Rule
Figure XX
Exchange Rates
Real Disturbance
Step with Independent Error

Legend
- Free Float
- Simple Rule
- Amended Rule
Figure XXI
Exchange Rates
Real Disturbance
Step with Autocorrelated Error
Figure XXII
Exchange Rates
Real Disturbance
Step with Random Walk Error

Legend
- Free Float
- Simple Rule
- Amended Rule

Exchange Rate
1.14
1.12
1.10
1.08
1.06
1.04
1.02
Time
Figure XXIII
Exchange Rate Behaviour
Historical Disturbances 1970–79

Legend
- Actual
- Free Float
- Simple Rule
- Amended Rule
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