MONEY, DEBT AND TAXATION IN A MODEL OF GOVERNMENT FINANCE

by

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Money, Debt and Taxation in a Model of Government Finance

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ABSTRACT

It has long been understood that the state can use control over the issue of money in order to generate revenue for itself. A large number of works in economics have dealt with various aspects of this form of government finance. Much of the recent literature has ignored this aspect of monetary policy in favour of the view that control over the money supply allows the state to conduct or attempt to conduct countercyclical stabilization policy aimed at minimizing fluctuations in employment and output. At the same time, the literature has been enriched by works on debt policy and optimal tax composition.

This work integrates the revenue from monetary expansion, debt policy and optimal taxation considerations into a positive model of optimal government finance. The model developed generates many of the results found in the literature in a straightforward way by employing techniques for optimization over time. One prediction of the model is opposite to what would be observed if countercyclical stabilization policy was attempted and thus a comparative evaluation of motives is possible. Results of a cross-sectional study are basically consistent with the hypothesis that governments use their control over their central banks in order to gather revenues. Using data for a number of countries, longitudinal results indicate that the monetary bases in these countries behave in a manner consistent with stabilization motives though there is
also evidence of revenue motives. Despite a growing consensus that monetary stabilization policy has not worked, governments are evidently motivated to pursue such policy.
DEDICATION

To the memory of Doreen Wilkinson.
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CHAPTER I
A MODEL OF OPTIMAL PUBLIC FINANCE

What is raised by printing notes is just as much taken from the public as a beer duty or an income tax. (A) government can live by this means when it can live by no other. It is the form of taxation which the public finds hardest to evade and even the weakest government can enforce, when it can enforce nothing else.¹

Ever since the establishment of the Federal Reserve System, every chairman of the Federal Reserve Board, indeed, I suspect every member of the board, has proclaimed that the Federal Reserve will not be an engine of inflation. Yet the Federal Reserve System was an engine of inflation during both world wars and has been one in peacetime since at least 1960.²

Introduction

Central banks of numerous countries throughout the world, despite stated objectives of maintaining the value of their currencies, have acted as engines of inflation. Recently, Stanley Fischer has shown that the central banks of all countries for which data are available have been generating revenues for their governments by increasing the supply of central bank money.³ Competing explanations for this behavior

³ Fischer (1982), pp. 308-12. In some countries the importance of monetary finance is debatable while in others as much as one-tenth of total revenues may be obtained through monetary expansion. The highest figure calculated by Fischer is for Argentina where, in recent times, the average of the change in the monetary base divided by total revenue (change in the base plus all other revenues) is about 46%.
may be roughly grouped into two broad categories. The first views inflation as a purposeful means of taxation while the second views inflation as the undesirable consequence of attempts at macroeconomic stabilization."

Since ancient times it has been recognized that the state could collect seigniorage if it held control over the issue of coinage and could maintain the status of that coinage as money. By increasing its issue of coinage the state could immediately increase its command over real goods and services. Together with taxing commodities and the selling monopoly licenses, taxing the holding of money has long been an example of the way in which the state can use its advantage in holding, awarding and enforcing property rights in order to collect revenues.

Specifically, through its monopoly of legal tender, the government is able to force holders of currency to make interest-free loans; similarly, through its monopoly of central banking, the government is able to force commercial bank holders of commercial bank deposits to make an interest-free loan directly to the central bank and indirectly to itself. Alternatively, the government is able through monopoly to impose a tax on holding these assets.

Several authors, including Cagan (1956), Bailey (1956), Tower (1971) and Friedman (1971), have considered the revenue maximizing rate of monetary expansion in partial equilibrium

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4 In different articles, authors such as Milton Friedman and Robert J. Gordon each have presented both views.


frameworks. Others such as Phelps (1973) and Robert J. Gordon (1975) have used general equilibrium approaches to the question of socially optimum rates of inflation. Phelps demonstrated that in a model where other goods and services are taxed, a negative rate of inflation, as suggested by the optimum quantity of money argument, would generally not be optimal according to the Ramsey rule.7 Gordon presented a model in which inflation was used as a means of taxation and as such was a substitute for other taxes such as the taxation of income. Optimality required that, in a static framework, the marginal costs of each form of revenue collection be equal. Further, in Gordon's model the short-run marginal cost curve was less elastic in the case of income taxes than in the case of taxation through monetary expansion. Governments faced with urgent needs for additional revenues would turn to inflationary finance rather than the administratively and politically more costly increases in other forms of taxation.

Gordon's elegant model is unfortunately marred by a reliance on sociological rather than price-theoretic arguments. Specifically, Gordon chose to consider inflationary finance as politically less costly than other forms because the public was assumed to be unable to correctly perceive the inflation as the outcome of a particular choice of means of government finance. The specific example presented by Gordon is the financing of the war in Vietnam. While it is safe to say that the United States

administration would have met with vocal opposition from congress had it moved that taxes be increased to pay for the war and thus pursued financing of the war by other means, Gordon's analysis seems less adapted to other, more popular wars during which monetary finance and other taxes were used together. In the U.S. as in other countries throughout history war as well as other acute revenue needs have been associated with the use of inflationary finance.  

The view that inflation is the undesirable result of attempts to achieve other governmental policy objectives has attracted attention in recent years. One line of argument, often associated with Milton Friedman, is that inflation is the costly and unexpected product of unwise though well-intentioned efforts on the part of government to manage the macroeconomy. He recommends a policy to limit severely the ability of governments and central bankers to alter the rate of money supply growth. In a historical paper, Thomas Sargent (1982) tells of how at the end of the Austrian hyperinflation reforms which increased the state's power to raise taxes lessened the incentive to print additional fiat money. At the same time accords signed between

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9 See Friedman (1977). Friedman (1982) presents a different view however.
Austria and Britain, France, Italy and Czechoslovakia led to the establishment of an independent central bank that would not advance notes to the spending authorities. Thus the costs, to the state, of monetary finance were raised. Sargent also notes that in 1924, at the end of the Polish hyperinflation, there was a move towards both a balanced budget and the establishment of an independent central bank that would not provide the state with unsecured loans.\(^\text{10}\)

A second, essentially Keynesian argument also regards inflation as a result of monetary stabilization policy. In this case inflation is regarded as a necessary cost of policies used to achieve real wage (and price) flexibility in the face of nominal rigidities. Work by Robert Gordon (1981) and Stanley Black (1982) are examples of such arguments. Essentially, sticky nominal wages and prices in manufacturing and service sectors are seen as preventing the proper allocation of resources over time. In particular, unemployment may occur in industries where real wages are above the market clearing levels. Inflation is supplied by the monetary sector in an attempt to provide downward flexibility to real wages and prices without necessitating that nominal values be reduced.

In Black as in Barber and McCallum (1982), factors such as the size (and strength) of the unionized labour force, the frequency and severity of strikes and the lack of representation

of organized labour in the nation's government ultimately lead to higher inflation rates. Some of this work, for example Gordon (1982), addresses the problem of stopping inflation, yet it is not clear that a zero rate of inflation is optimal. Suggestions to attack the problem of inflation through the imposition of wage and price controls or taxed based incomes policies may merely represent attempts to alter the way in which inflationary pressures impart costs on the economy. The burden may be transformed, but the impact may well be increased by such policies. In any event, the effective taxation of money remains.

The literature on inflation as a means of taxation may be divided into two main categories. Work involving the use of partial equilibrium analysis has been concerned primarily with the revenue maximizing rate of inflation and with the deadweight losses associated with a tax on money balances. Other work involving general equilibrium models has examined the use of monetary expansion as a tax in a world where other taxes are also collected and has been concerned with deriving the socially optimal rate of inflation given the existence of other forms of taxation. Lately, it has been suggested that, while the government can obtain resources through anticipated and unanticipated increases in the money supply, this does not account for all of the recent inflation experienced in the United States. Some inflation, it is suggested, seems to be the result of countercyclical monetary policy. 11

11 See Barro and Gordon (1983b).
Several questions arise in connection with the use of inflationary finance. How does the optimal finance mix change over time in response to changing conditions? Is there a relationship between debt and expansion of the monetary base? Why do some countries evidently rely on monetary finance more than others? Are there identifiable factors which account for the degree of independence or dependence of the central bank? The desire to respond to these questions prompts the building of a general model of optimal public finance.

Barro (1979, 1984b) in modelling the determination of the public debt does not deal with currency issue, but chooses to aggregate this form of finance with all other taxes. There is little reason according to Barro to separate monetary finance from other taxes. Since his purpose in these papers is to model the determination of the quantity of publicly held debt, this may be reasonable. Barro has also noted that the magnitude of countercyclical debt growth is larger than implied by the tax-smoothing hypothesis he puts forward. Thus there is evidence that attempts have been made in fiscal and monetary stabilization.\(^{12}\) Isolating currency issue from other forms of taxation does provide the opportunity to investigate the central bank's behavior pertaining to its role in government finance and to gather additional evidence relating to the financial versus stabilization motives behind fiscal policy and monetary base

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\(^{12}\) See Barro (1979) pp. 941-942 and Barro (1984a). Barro's (1979) seminal article does consider the effect of persistent inflation on the nominal deficit, but does not explicitly cover monetary policy. The inflation is taken to be exogenous.
management. A prediction of the finance model, that monetary base growth would move procyclicly, is at odds with the predictions of standard stabilization models.

In this chapter, a model of revenue collection is proposed which incorporates generalized cost of collection functions for each form of revenue. The state is assumed to minimize the present value of the costs of collecting sufficient revenues. The implications of this approach include a prediction that there will be an optimal long-run rate of monetary base expansion and that this growth rate will be linked to the level of government expenditure. This growth rate will also be associated with the explicit taxation of income and commodities, but not necessarily with debt sales to the public. In the long-run, growth of government expenditure, money holding or income provides a role for debt financing as does a difference between the government's and the public's rate of discount. In the short-run, deviations from trends in government expenditure and national income will likely be associated with bond financing and only less so with explicit taxation. It is also predicted that short-run extraordinary revenue needs will be associated with increases in the stock of central bank supplied money unless legal constraints exist to prevent this.

The model predicts that the rate of monetary base expansion will be closely tied to government expenditures. Other variables, for example unemployment rates, are predicted to influence monetary base expansion to the extent that they affect
the level of government expenditures. This suggests a comparative test of monetary growth theories: (i) monetary expansion as a tax versus (ii) monetary growth as a purely discretionary policy response to changes in output or employment.

The second section of this chapter examines the revenue collection methods under consideration. The third section develops a model of the financing of a given level of government expenditure in a long-run setting and the problem of financing given short-run deviations from trends. The last section discusses the predictions of the model. The positive policy implication of the model is clear; less inflation can be obtained through either reduced government expenditures or higher rates of explicit taxation of income and commodities other than money.

Revenue Collection

Revenue from Monetary Finance

Real revenue can be collected by the simple act of the state increasing its issue of noninterest bearing debt (the monetary base) with which it can purchase real goods and services. Since the nongovernment sector supplies the goods and services, a supply it would not have offered otherwise, the burden of monetary finance is met when the government purchases the goods and services introducing the newly created money into the
economy. This revenue may or may not be equal to the inflation tax as usually defined for several reasons.

There has been some debate over the correct calculation of the inflation tax. The tax is alternatively calculated as the level of real balances times: (i) the inflation rate, (ii) the nominal interest rate or (iii) the growth rate of nominal balances. For example, Alvin Marty measures the revenue using the nominal interest rate times the level of real balances. Marty states,

...(W)e may ask how one tax can substitute for another to meet a fixed government revenue constraint; included in our arsenal of taxes is the inflation tax at different rates of inflation. Within this framework, the inflation tax revenue is measured by the stock of real cash balances $M/P$ multiplied by the tax rate, the money rate of interest rather than this stock multiplied by the growth of the money supply - this latter being the traditional Chicago definition. The clearest rationale for this new measure was expounded by Phelps, although a richer variety of models was presented by Phelps and Burmeister in an earlier article.13

Phelps observed that the term "inflation tax" had no meaning to some because different authors meant different things by it. Phelps, thus aware of the problem, explained his choice in the following way.

In my inflation policy book, I used the nominal interest rate multiplied by real balances, $(\pi+r)M/P$ - where $r$ is the real interest rate. In this concept, the proceeds of the inflation tax are metaphorical in the sense that the tax does not produce a visible flow of currency into the treasury like that produced by ordinary taxes or a currency printing press in the treasury basement. However..., the proceeds of the tax as conceived here are measured in terms of the "saving" in other (nonmetaphorical) tax revenues. Needless to say, these various measures generally differ from one another, and

not by inessential constants, save in the case of a stationary state with zero real rate of interest.\textsuperscript{14}

The two definitions differ, but are related. Assume that $\pi$, $r$ and $i$ refer to instantaneous rates of inflation, real interest and nominal interest and that these are, for simplicity, constant over time. Consider the present value of the Phelps-Marty inflation tax flows

$$\int_0^\infty \pi \text{me}^{-rt} \, dt = (i/r)m = m + (\pi/r)m.$$ 

This equals

$$m + \int_0^\infty \pi \text{me}^{-rt} \, dt$$

the value of outstanding real balances plus the present value of the flows of inflation taxation as measured by $\pi m$. The Phelps-Marty measure $im$, amortizes the revenue received by the initial creation of existing real balances and adds this to the per-period flows of revenue, $\pi m$ (which, in fact, are flows of currency in or out of the treasury).\textsuperscript{15}

\textsuperscript{14} Phelps (1973), p. 70.

\textsuperscript{15} It can be demonstrated that Phelps and Marty really used the Friedman-Bailey measure of the revenue flow from inflationary finance. Phelps for example, presents the expression

$$T + \pi M/P = \Theta(t) + (i-\pi)\Delta(t)$$

where $T$ is real taxation, $\Theta(t)$ is a constant at any time $t$, $\Delta = M/P + D^*/P$ and $D^*$ is the stock of publicly held debt. Phelps refers the second term on the left as "the seignorage received by the government from the central banks exchange of interestless cash for earning assets from the public." Substitution and simplification result in the expression

$$T + \pi M/P = \Theta(t) + rD^*/P.$$ 

The real net flow of revenue from inflation is $\pi M/P$. If a policy change resulted in a zero rate of inflation real taxes would have to be changed by the amount $\pi M/P$ above. Holding the nominal rate of interest equal to zero (not taxing money in Phelps' terms) is really undertaking an increased expenditure by effectively paying interest on money and there will be an actual flow of currency out of the treasury.
The transmission mechanism connecting growth in the monetary base with inflation may be complex particularly if the state imposes controls on prices or controls on broader monetary aggregates. Revenue collection may not be reflected in inflation for some time after the expansion of the monetary base due to the influence of expectations on the price level or because are used to suppress the inflation.\textsuperscript{16}

Consider an economy in which the rate of monetary growth is such that the rate of inflation is zero. Say the government suddenly introduces a large volume of newly created money into the economy. The revenue gained would not be zero (the tax base times the tax rate as described above). The government will have generated for itself revenue equal to the real value of the newly created money at the time the new money is introduced into the economy. The additional money whether absorbed in the form of higher real balance holdings or through a new higher price level generates revenue for the state. Next, consider the case described by Auernheimer (1974) in which a transition is made between one rate of inflation and a correctly perceived new higher rate. What is the revenue generated in this case? Say the nominal interest rate rises from $i_0$ to $i_1$ while real balances held falls from $m_0$ to $m_1$. The revenue collected before the

\textsuperscript{16} Suppressed inflation will involve additional costs in terms of the additional resources used by the government to carry out the suppression and the dead-weight losses associated the misallocation of resources which occurs when the ability to allocate according to price information is impaired. The excess administration costs represent an additional government expenditure that must be financed as any other.
change would be \( \pi m_0 \) while after the change the revenue would be \( \pi m_1 \). During the period of the change the revenue collected would equal \( \pi m_0 + (m_1 - m_0) \). That is, it would be necessary to adjust the revenue calculation to include the revenue reducing effect of the reduction in real balances held. Finally, consider a surprise one-time increase in the nominal money supply as described by Brennan and Buchanan (1981). Say the nominal money stock rose from \( M_0 \) to \( M_1 \). As this event is not initially perceived, real balances held rise from \( m_0 \) to \( m_1 \) while the interest rate remains at \( i_0 \). Before the event the revenue gathered equals \( \pi m_0 \). When the one-time increase in \( M \) occurs, the real revenue gathered becomes \( \pi m_0 + (m_1 - m_0) \). During the period in which the price level rises and real balances return to \( m_0 \), the revenue generated will be \( \pi m_1 + (m_0 - m_1) \). Afterwards the revenue will equal \( \pi m_0 \) once again. In each of these cases the amount of revenue generated by monetary finance will be equal to the real value of the money introduced into the economy at the time that money is introduced. Measuring the revenue flow in this way is completely general in that it is applicable under all circumstances.

The above cases suggest that equal increases in the nominal stock of government issued money may generate different revenue.

\[ (m_1 - m_0) = \Delta M/P_0 = (M_1 P_1 - M_0 P_1)/(P_1 P_0) = m_0 ((P_1/P_0) - 1) = m_0 (\Delta P/P_0). \]

The last term represents the gain to the state or equivalently, the reduction in the purchasing power of the public's previously held balances due to the induced rise in the price level.

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17 The term \((m_1 - m_0)\) is equal to the effect of the eventual induced inflation on the purchasing power of previously held real balances; a tax on wealth held in the form of money. Note that where \( P \) refers to the initial price level, \( (m_1 - m_0) = \Delta M/P_0 = (M_1 P_1 - M_0 P_1)/(P_1 P_0) = m_0 ((P_1/P_0) - 1) = m_0 (\Delta P/P_0) \). The last term represents the gain to the state or equivalently, the reduction in the purchasing power of the public's previously held balances due to the induced rise in the price level.
flows under different circumstances. It has been suggested that
surprise increases in the stock of money could be used to obtain
substantial amounts of real resources by imposing a surprise ex
post tax on money balances. A government with a sufficiently
short time horizon may be tempted to use such a misperceived
form of taxation. The important issue is not the amount of
revenue that can be obtained since an equal increase in the real
value of new money issued will generate the same revenue, but
rather is the possibly lower political costs involved in such
misperceived taxation. This case is somewhat like that suggested
by Robert Gordon in which the government uses a poorly
understood form of taxation (monetary finance) instead of taxes
which the public can easily recognize and react to effectively
because of the lower political costs to the state. A surprise
tax will lower the wealth of the public and it is not clear that
the public will, with memory and foresight, not ultimately hold
the state responsible for the decline. As in Gordon's case, the
relatively smaller political costs can only be maintained if the
public does not come to understand how they have been made worse
off.

Rather than calculate various versions of the inflation-tax,
the measure which will be adopted here will be the amount of
revenue collected by creating and introducing new central bank
money into the economy. The real value of this money at the time
of its introduction, which equals the real value of goods and

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18 See Brennan and Buchanan (1981) for example.
services purchased by the government, is the measure of the amount of wealth the government has receives from the public which will be used and will be termed seigniorage. This measure will not be sensitive to the state of the public's expectations, the exact way the money is introduced (i.e. as a lump-sum or as a continual increase in the growth rate) and the possible imposition of controls.

That the seigniorage measure, $\frac{\dot{M}}{P}$, is applicable to any circumstance can be shown. Starting with the quantity equation,

$$\ln M + \ln V = \ln P + \ln y$$

where $M$ is the nominal stock of government issued money, $V$ is its velocity of circulation, $P$ is the price level and $y$ is the level of real income. Differentiation with respect to time yields,

$$\frac{\dot{M}}{M} = \frac{\dot{P}}{P} + \frac{\dot{y}}{y} - \frac{\dot{V}}{V}$$

or

$$\frac{\dot{M}}{P} = M \frac{\dot{P}}{P} + \frac{\dot{y}}{y} - \frac{\dot{V}}{V}$$

or, equivalently,

$$\frac{\dot{M}}{P} = m_\pi + mx - m\frac{\dot{V}}{V}$$

where $\pi$ is the rate of inflation and $x$ is the growth rate of real income. If real income and velocity are unchanged, then

$M/P = m\pi$ measures the "inflation tax" collected. If $x$ is positive, the state can gain revenue without causing inflation by issuing new money at a rate equal to $x$, assuming the income elasticity of demand is unity. In this instance the increase in real resources being directed to the state would be identically proportional to the increase in gross and net income of the public. Seigniorage does not have to take the form of an "inflation tax" since revenues can be obtained without necessarily generating inflation. Finally, some issue could be absorbed through a reduction in velocity without causing inflation at a rate equal to the growth rate of issue in excess of the growth rate of real income. If the state issued (and spent) a sum of newly created money which was subsequently hoarded by the public, the state would still have received a transfer equal in value to the goods or services purchased with the new issue. In short, the real value of the newly created money, measured when that money is introduced into the economy, is the measure of the real revenue obtained. The amount of revenue collected due to the state's monopoly over the supply of currency will be

$$s = \dot{M}/P = \dot{m} + m\pi$$

where $s$ denotes seigniorage, $m$ the change in real balances over some period and $\pi$ the inflation rate.\(^2\)

The most important feature of monetary finance from an applications standpoint is the relative ease with which it can

\(^2\) In discrete units $s = (M - M_{t-1})/P$. Simple manipulation reveals that $s = \Delta m + \pi M_{t-1}/P_{t-1}$ where $\pi = (P - P_{t-1})/P_{t-1}$.  

16
be adopted unless purposeful steps are taken to insulate the central bank from the spending authorities. In dramatic situations such as war, little constraint may be placed on the spending authorities by the central bank which dutifully accommodates the government's demand for revenue. War has been associated with large increases in the government's supply of money for centuries. Further, monetary finance is a difficult form of taxation to avoid since the burden will fall on all of those who hold any domestic currency. Attempts to avoid the tax would necessitate the use of a non-state issued currency by a large number of domestic residents.

Bond Finance

In his famous treatise on taxation and the economy, David Ricardo argued that the notion that debt financing of government expenditures imposed less of a burden on the economy than tax financing was incorrect. Ricardo wrote,

Taxes which are levied on a country for the purpose of supporting war, or for the ordinary expenses of the state, and which are chiefly devoted to the support of unproductive labourers, are taken from the productive industry of the country; and every saving which can be made from such expenses will generally be added to the income, if not to the capital of the contributors. When, for the expenses of a year's war, twenty millions are raised by means of a loan, it is the twenty millions which are withdrawn from the productive capital of the nation. The million per annum which is raised by taxes to pay the interest of this loan, is merely transferred from those who pay to those who receive it, from the contributor to the tax, to the national creditor. The real expense is the twenty millions, and not the interest which must be paid for it. Whether the interest

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be paid or not be paid, the country will be neither richer nor poorer. Government might at once have required the twenty millions in the shape of taxes; in which case it would not be necessary to raise annual taxes to the amount of a million. This, however, would not have changed the nature of the transaction.\textsuperscript{22}

Ricardo did not consider debt and taxes as perfect substitutes however, rather he contended that debt financing was undesirable because the public would undersave. As individuals they feel that the debt presented a smaller tax bill than a tax levy equal to the amount of the debt. Ricardo preferred the use of taxation which still allowed those who wished to maintain their current non-tax expenditures to borrow from other individuals. The "delusion" as Ricardo referred to it seems to rest upon the public believing that the interest and principle of the debt will be paid-back by the government without the use of additional taxation in the future.

A similar result would occur if the public discounted future taxes at a higher rate than it paid on its debt. If the public as a whole is aware of the implied future taxation and has access to a well-functioning private loan market, then one could go beyond Ricardo and conclude that there is in fact no difference between current taxation and debt financing by government. Both would be perceived as having equal effects on wealth since the public would discount the future taxes implicit in the government debt issue.

\textsuperscript{22} ibid pp. 244-5.
James Buchanan and Richard Wagner (1977) attacked what they regarded as the excessive use of government debt to finance growing levels of government expenditures. In particular they attacked the assumption contained in many texts and held, according to the authors, by policy makers that debt financing can be undertaken at little cost to the economy.\(^2\) In their book, Buchanan and Wagner labelled the possible equivalence of debt and taxes the "Ricardian theorem" or the "Ricardian equivalence theorem" though they duly note that Ricardo did not consider the two means of gathering revenue as equivalent.\(^2\)

Under certain restrictions preventing differences in the distribution of tax liabilities among individuals, Buchanan and Wagner state,

\[
\text{(T)he equivalence theorem can be generalized beyond the straightforward tax-debt comparison. In its most inclusive variant, the theorem would assert that the particular way in which government extracts resources from the citizen is irrelevant for either private or public choice. Tax finance may be replaced by debt finance; either may be replaced by money creation; an income tax may be replaced by a sales tax. So long as this outlay remains the same in each case, and so long as this outlay is shared among persons in the same way, there are no effects on final outcomes. The theorem rests on the basic presumption that the representative}\\
\]

\(^2\) Textbooks have commonly presented the result that debt financed government expenditure is more expansionary than tax financed expenditure. Where "crowding out" is considered, its importance has often been minimized or monetization, to prevent the interest rate from increasing, has been suggested as a remedy.

\(^2\) The authors cite an earlier paper by Gerald O'Driscoll (1977) which deals with what O'Driscoll referred to as "The Ricardian Non-equivalence Theorem". Ricardo did not consider taxes and bonds to be equivalent. Buchanan criticizes Barro's "...attempt to apply the Ricardian theorem, without reference to Ricardo." (Buchanan and Wagner p. 136.) Ricardo however did not subscribe to such an equivalence theorem.
decision maker has perfect knowledge about how changes in the means of financing government will affect his own net worth. If in such a setting for analysis, the alternatives are presented so as to ensure that the arithmetical value of the fiscal charge is identical under varying instructional forms, it is no wonder that the precepts of rationality dictate indifference among them.25

The requirement of perfect knowledge is unnecessary as all that is really required is that individuals rationally anticipate future taxes. This aside, the quotation appears to be accurate in its conclusion that all forms of finance will be equivalent under certain conditions. Indeed it has been rigorously shown that taxes on goods, on income and on wealth are equivalent in certain cases.26 Different forms of finance are close substitutes assuming that the state can ensure payment of taxes, holding of money and purchases of debt by the public. Since these different forms of finance are each recognized distinctly, they cannot be equivalent. A theory of public debt issue and taxation has been provided by Robert Barro. In Barro's model, taxation and debt are close but not perfect substitutes and thus the state is not indifferent between the two means of finance.

Barro (1974a), presented a model in which one generation cares not only for its own future well-being, but also the well-being of generations that will follow. He argued that

25 ibid p. 136.

government bonds need not represent net wealth to the present generation.\textsuperscript{27} If the public has access to capital markets on the same terms as the government and so can borrow (or lend) in order to alter the timing of disbursements, and if the current generation matches increased future tax liabilities implied by government bonds with increased savings with which future generations can meet those liabilities, government bonds and current lump sum taxation would be equivalent in at least their first-order (i.e. non-distributive) effects. Barro also supplied evidence in support of his hypothesis.\textsuperscript{28} In a later paper, Barro states,

With the corresponding absence of a shift in perceived wealth consumer demand would not be stimulated by the movement from taxes to debt issues. It follows that the

\textsuperscript{27} It is interesting to note that the intergenerational argument was presented by Ricardo in 1820. He wrote: "It would be difficult to convince a man possessed of 20,000 £, or any other sum, that a perpetual payment of 50 £, per annum was equally burdensome with a single tax of 1000 £. He would have some vague notion that the 50 £ per annum would be paid by posterity, and would not be paid by him; but if he leaves his fortune to his son, and leaves it charged with a perpetual tax, where is the difference whether he leaves him 20,000 £, with the tax or 19,000 £, without it? This argument of charging posterity with the interest of our debt, or relieving them with a portion of such interest, is often used by otherwise well informed people, but we confess we see no weight in it." David Ricardo, "The Funding System", in P. Sraffa ed., The Works and Correspondence of David Ricardo Cambridge, Cambridge University Press 1962 p. 187.

\textsuperscript{28} Other controversial propositions connected to the questions of debt were put forward by Barro. These include: (i) that increased nominal deficits did not necessarily lead to inflation, but inflation did lead to increased nominal deficits so as to keep the real debt constant, (ii) that real deficits increase in response to transitory revenue needs of government and then decline as the government experiences transitory revenue increases, and (iii) that only surprise levels of government deficit (and of expenditure) have a real impact on the economy.
supply of private loanable funds would rise one to one with the cut in current taxes (increase in current disposable income), so that the extra government demand for funds implied by its debt issue would be fully absorbed by the private sector without an increase in the rate of return. Under these circumstances — when public debt issue leaves unchanged the value of perceived wealth — the crowding out of private investment would not arise.29

Indeed, debt is a form of current "taxation"; only the actual mechanics of making the collection is deferred. The state obtains the revenue currently and the public experiences the transfer of resources currently.

A problem with the strict interpretation of the equivalence theorem is that both taxes and debt financing are in fact used simultaneously. If the two are taken to be equivalent the amount that each will be used is indeterminate. In order to model the selection of means of finance in a choice-theoretic framework, the two forms of finance cannot be considered equivalent. Barro (1979) develops a model in which increases in bonds issued are linked to transitory deviations in expenditures or tax receipts. The key to Barro's model is the convexity of the tax collection cost function with respect to tax levels for a given level of income. Thus perfect lump sum taxes are assumed not to exist.30

Sales of debt to the public are used to spread the cost of collecting additional taxes over time in order to avoid sharply increased costs associated with large increases in current tax rates. Since the initial exogenous level of debt could be taken

29 Barro (1978a) p. 238.

30 Barro's model rests on the general principle that convex functions are necessary to yield deterministic solutions.
to be zero, Barro not only determines debt sales to the public, but also the size of the accumulated debt. By assuming that costs of collecting taxes are an increasing function of the level of taxes to be collected per unit of real income earned and no costs apply to bond finance except the future tax collection costs, it is shown that it is optimal to issue debt rather than increase taxes during periods when there is a short-fall of revenues. When there is a short-run surplus of revenues, the debt is retired.

The argument can be generalized to a situation where there is a cost of issuing bonds as long as this cost is not prohibitive. Given a temporary need for additional funds, debt will be issued to the point that the marginal discounted sum of future tax collection costs and any administration costs associated with the bond sales equals the additional cost of collecting the revenue through current taxation. The Barro model can be extended further to include certain specific forms of taxation. Two such forms are the taxation of income and the taxation of money holding. In Barro (1977) a model of the choice between these two forms is described.

Barro does not completely dismiss the possibility that bonds may be perceived as a form of net wealth due to inadequate private capital markets as might exist in undeveloped economies.

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The given level of debt could also be negative. The state could hold a stock of privately issued debt which it sells as needed to avoid short-run excess revenue needs. Barro does not explicitly mention this possibility though his model does allow for such a case.
If bonds are perceived as net wealth their existence would be easily explained. Why more or all financing is not undertaken with the use of bonds would then be an important problem. The distinguishing feature of Barro's approach is that it does not rely on the government having cost advantages in the issuing of debt relative to the private sector. The convexity of the cost of tax collection function is alone sufficient to determine a unique level of debt and amount of current deficit or surplus. If individuals differ in their desired time paths of payment there is an opportunity for private debt to be exchanged. The co-existence of government debt and private debt issue is not inconsistent with Barro's model. Bonds will present an element of net wealth in the short-run in the sense that there is a cost savings realized by society in the use of bonds for short-term excess revenue needs as opposed to raising current taxes. This paper will adopt the cost of taxation function used by Barro (1979).

While it has been noted that Barro has not simply resurrected the argument made by Ricardo, it is perhaps interesting to notice how Irving Fischer's view of bond finance compares to the current thinking of Barro and others. Fischer wrote in 1930,

The public loan for consumption is exemplified in the war loans and the loans to anticipate future revenues. A government receives its income chiefly in taxes, and in

---

The model does not rely on intergenerational altruism. Increasing per unit costs of collecting taxes is sufficient to produce a model in which, at the margin, government bonds are not net wealth. See the model below.
some cases only once a year, whereas its outgo occurs day by day and month by month. It thus happens that a government is alternatively accumulating a large surplus and suffering a large deficit. The inconvenient effects of this have often been commented on, especially in this country, where the Treasury for half a century was relatively independent of such institutions of credit between the governments and certain central banks as have long existed in England, and exist now in this country. The government may correct the irregularities in its income stream by borrowing for current expenses in anticipation of taxes.... The opposite process may be employed. The government may lend at interest by depositing funds in banks to draw interest until needed for disbursements, or what amounts to receiving interest, it may, by buying its own bonds or redeeming them for a sinking fund, save interest which would otherwise have to be paid. But this last operation is normally employed only when the funds are not needed later for disbursements.

The public productive, or business loan, is exemplified in loans for the purpose of constructing railroads, ...the erection of government buildings, the improvement of roads, bridges and harbours, ...municipal waterworks or schoolhouses. In all such cases it is usual to finance the enterprise by issuing bonds. The reason is that these improvements constitute an extraordinary cost, similar to the expense of a war, which if undertaken without the issue of bonds would cause a temporary and inconvenient depression in the income of the taxpayers. They...prefer to avoid such a fluctuating income stream, and to secure instead a more uniform one. This uniformity is secured by the loan, which so far as they are concerned, spreads the expenditures over part or all of the period during which the public improvement is expected to last.33

The only major difference between this and Barro's argument concerning the role of debt is that Barro would not argue that bond financing smooths income flows, but rather that bond financing smooths net income flows by smoothing the deadweight cost of tax collection. The use of debt to finance public projects was correctly seen by Fischer as being due to the

extraordinary size of the expenditure.

Buchanan (1975) argues that large capital projects should be debt financed since is debt financing allows payments for the projects to be made over time as the benefits are received from the project. 34 This argument is inadequate. Say a country built ten millions worth of schoolhouses or roads each and every year. These public assets last for twenty years at which time they are replaced. There would be no reason in such a case for the expenditures to be financed with twenty year bonds or bonds of any maturity. Since the level of expenditure is constant each year, current explicit taxation would be preferred if there was even a small cost involved in issuing debt. This argument would even apply to a period of expected prolonged warfare. It is not what government is spending on, but rather the ordinary or extraordinary magnitude of the expenditure that dictates the optimal means of finance. 35

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35 Buchanan and Flowers note that war has often been associated with large deficits. This apparent contradiction is handled by judicious choice of definition. "In one sense, the extraordinary spending necessitated by war takes the form of capital investment in the whole social structure, the benefits of which can be expected to endure permanently if the war is won." Why this argument can not be extended to policemen's wages, educational funding and subsidies to the fine arts is unclear except that the size of such expenditures are seldom extraordinary.
Explicit Taxation

For simplicity it is assumed that taxation of commodities and of income can be considered as one means of collecting revenue and monetary finance another. It may be less costly in the short-run to alter some forms of taxation than others. The precise cost functions would depend in part on the legislative restraints that are relevant in a short-run context.

The fundamental problem is why tax? The alternative would be to borrow in perpetuity and allow private bequests to deal with the problem of relieving the burden placed on future generations. The net effect, without additional costs would be the same as with explicit taxation. Phelps (1973) provides the explanation that too much bond financing would severely 'crowd-out' investment. Referring to a classroom discussion on the topic, Phelps provides the following answer to the question of why taxation is used,

If public expenditures were unaccompanied by positive tax rates, the resources diverted to public use would come mainly from the capital goods sector; there would be no fiscal restraint on consumption or leisure to accommodate the public expenditure so as to spare capital formation.\(^{36}\)

In the previous section on bond finance it was noted that, if the public recognized and discounted the future tax liabilities implied by government debt, and increased their savings accordingly so as to provide for the future tax levies, there would be no 'crowding-out' as described by Phelps. Bond issues

\(^{36}\) Phelps (1973), p.69.
would, under such conditions, place a fiscal constraint on consumption just as taxation would. The effect of bond sales on leisure would not be as suggested in the quotation.

The assumptions that future flows are discounted at a positive rate and that there is a convex collection cost function for taxation are sufficient to determine the level of debt. Let $T$ denote tax revenues including seigniorage and $c$ the collection costs (a function of the dissipation of income) at time $t$. Let $g$, $r$, $d$ and $y$ represent real values of expenditure, the interest rate, debt and income respectively. For all points in time, $g+rd≤y-c$ and $T+b≤y-c$. Since collection costs reduce net income in the future, they will be discounted at the same rate as income even in the absence of intergenerational altruism.37

Thus,

$$\int_0^\infty (g(t)+rd(t-1))e^{-rt} \, dt = \int_0^\infty (T(t)+b(t))e^{-rt} \, dt$$

so,

$$g(t)+rd(t-1) = T(t)+b(t) ≤ y(t) - c(t) = \text{net } y(t)$$

and

$$d(t) = \int_0^\infty b(t)e^{-rt} \, dt = \int_0^\infty \text{net } y(t)e^{-rt} \, dt.$$ 

Selfishness will not produce an infinite or undetermined amount of debt. Debt policy will still be important. Maximizing the net worth of the economy and thus the transfer to the current generation requires the minimization of the present value of

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37 A selfish generation would seek to maximize the amount of net wealth that could be extracted from future generations through the issue of debt conditional on the debt remaining liquid. Future costs are still important if the bonds are to be sold to future generations.
collection costs. This implies that some taxation would be applied in the current period. The important difference would be in the initial level of debt. With altruism the initial desired level of debt would be zero at the beginning of time, while without altruism it would equal the capitalized value of income less collection costs. In the absence of altruism, the current generation would attempt to transfer future generations' wealth to themselves.

Choice Among Means of Finance

Steady-state Case

In the following model a government faces the problem of financing a given level of expenditure which, for the purposes of this analysis, will be taken as exogenous. The objective of a self-interested government is to collect the required revenue with the lowest possible total cost in terms of expenditure on revenue collection. Initially, a zero rate of real growth in all exogenous variables is assumed and cross effects between forms of taxation are not considered. This involves no serious loss of generality.

Each form of finance involves some dissipation of real resources. Record keeping, printing, filing, enforcement and administration involve the use of resources which could otherwise be allocated to other activities. That is, a pure lump-sum tax without deadweight cost is assumed not to exist. A
government whether benevolent or not and whether democratic or not would seek to minimize these costs so as to maximize net revenues. In addition, taxation and monetary finance will result in some deadweight welfare losses being suffered by the public. A benevolent government may wish to minimize such losses. Alternatively, in the case of a totally selfish state, these costs may be reflected in political costs faced by the state; costs which can be reduced to some extent by increased expenditures on public relations, manipulation or coercion. To the degree that the state faces and reacts to such political costs, the welfare losses should be considered as part of the cost of government finance. The cost of any form or revenue collection is assumed to be an increasing function of the amount of real revenue collected.

The government faces a budget constraint of the following form:

\[ g(t) = p(t) + b(t) + s(t) - rd(t-1) \]

for a period \( t \), where \( s(t) = M(t)/P(t) = (dM(t)/M(t)dt)m \) is the seigniorage collected, \( g \) represents real government expenditures, \( p \) real explicit taxation, \( b(t) = d(D(t)/P(t))/dt \) is the real deficit defined as the change in real indebtedness, \( r \) the real interest rate, \( d \) the stock of real government debt held by the public, \( M \) the nominal magnitude of the monetary base and value of the \( m \) the real monetary base.\(^{38}\) It is assumed that the

\(^{38}\) An alternative definition of the real deficit is \( d' = dD/P \). In this case the budget constraint should read, \( g = p + s + b - id' \), where \( i \) is the nominal interest rate paid on the debt. The appendix shows that the two forms of specification are equivalent. The
government does not generate any real income through productive activity and must gather all revenue from the nongovernment sector. Relaxing this assumption involves only the reinterpretation of g and p. Income generating endeavours are defined as nongovernmental while transfer activities are considered government activities.

The real cost of collecting revenues are expressed by a set of homogeneous cost functions patterned after Barro (1977, 1979). With subscripts suppressed, the generalized cost function for explicit taxation, seigniorage and bond financing is,

\[ C(p,s,b;y) \]

where \( C_p, C_s, C_b, C_{pp}, C_{ss}, C_{bb} > 0 \) and \( C_{py}, C_{sy}, C_{by}, C_{ps} < 0 \).^39

The cost of bond finance requires some elaboration. It is assumed that some current costs of administration including distribution and policing are attached to the issuance of debt. It is assumed that these costs are proportional to \( d \), the real magnitude of the debt, and thus to \( b \), the real deficit. The present value of future costs of collecting revenues should be included in the cost function and this is also reflected in the \( b \) term. Current issue costs would have to be less than the costs of current taxation if debt is to be used at all.

Lagrangian or Kuhn-Tucker methods might be used here. However this problem requires optimization over time and a definition used in the text is simpler.\(^{38}\)

\(^{39}\) Cross effects do not substantially alter the results below and are not included in their derivations.
stock-flow relationship exists between the current deficit and the accumulated debt. Standard Lagrangian techniques are not generally applicable to dynamic problems since the values of the Lagrangian multipliers will usually not be constant through time. Hamiltonian techniques represent a simple extension to Lagrangian methods and one which allows for the possibility of corner solutions at particular points in time. The optimization problem is more easily handled in a continuous-time framework.

The objective is to minimize the present-value of collection costs,

\[ I = \int_{0}^{\infty} C(p(t), s(t), b(t); y(t)) e^{-zt} \, dt \]

is the cost function in general form and \( z \) is the rate of discount used by the government subject to the continuous constraints,

\[ g(t) = p(t) + s(t) + b(t) - rd(t) \]

and

\[ b(t) = \dot{d}(t) \]

where \( r \) is the real rate of interest paid on debt and \( b(t) = \dot{d}(t) \) is the stock-flow relationship between real deficits and real debt. This is a transformation of the problem in which the state attempts to maximize the present value of net revenues (expenditures) \( g-c \), where \( g \) is a constant, or net wealth (discounted future income less collection costs) in the economy. Where it is understood that \( p, s, b \) and \( y \) are functions of time,

\[^{40}\text{See Dixit (1976) and Intriligator (1971) for introductions to techniques of dynamic optimization in economics.}\]
the objective function may be written as,

\[ \text{Min } I = \int_{0}^{\infty} C(p, s, b; y) e^{-zt} \, dt \]

subject to

\[ \dot{d} = b = g - p - s + rd \]

and

\[ y, g > 0. \]

The problem may be rewritten in Hamiltonian form as,

\[ H = C(p, s, b; y) + v(g - p - s + rd) \]

where \( v(t) = u(t) e^{zt} \).

The conditions for a minimum are:

\[ H_p = C_p - v = 0, \]

\[ H_s = C_s - v = 0, \]

\[ H_b = C_b = 0, \]

\[ \dot{v} = zv - Hd = (z - r)v \]

and

\[ \dot{H} = 0. \]

These conditions, together with the budget constraint determine the optimal levels of \( p, s, \) and \( b. \)

If \( z = r \) then \( \dot{v} = 0 \) so the marginal costs of explicit taxation and seigniorage be equated for each form used and should be constant over time for each form used as the shadow price, \( v, \) is constant over time. If the marginal cost of a form of revenue collection exceeds that of the others and the total revenue requirement is met, then that form will never be used under the

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\(^{1}\) Since \( C_p = C_s = v, \) \( C_p \dot{p} = C_s \dot{s} = \dot{v}. \) If \( z = r \) then \( \dot{v} = 0, \) implying that \( \dot{p} \) and \( \dot{s} \) are equal to zero.
assumptions above. Only if cost functions or revenue requirements change sufficiently, will we see a new means of revenue collection being introduced.\textsuperscript{42}

Further, with subscripts suppressed, if \( z=r \), then

\[ C_p = C_s = v > 0, \]

implying that \( p \) and \( s \) will be constant over time. Bond finance will be everywhere dominated by current explicit taxation and monetary base expansion since debt financing implies that \( p \) and \( s \) will not be constant over time which in turn implies that cost minimization is not obtained.\textsuperscript{43} This suggests, as in Barro, that there is no room for debt financing of expenditures unless cost functions are expected to change over time or unless there are transitory fluctuations in revenue needs. That is, the real level of the debt will remain constant at its initial level (which, for simplicity, may be considered zero). There may be nominal surpluses or deficits if there is deflation or inflation, but only in order to keep the real level of accumulated debt constant. Since the real debt is unchanged there are no real surpluses or deficits. Thus, in the steady-state case bonds would represent negative net-wealth to the extent that there are specific costs involved in issuing and servicing the bonds.

\footnote{The emergence of income tax is a very recent historical phenomenon; one facilitated by the refinement of accounting practices to reduce calculation and enforcement costs.}

\footnote{If \( z=r \) so that \( \dot{v} = 0 \) and \( \dot{p} = \dot{s} = 0 \), then the condition \( \dot{H} = 0 \), written explicitly is, \( H = v(b) = 0 \). Now since \( b = g - p - s + rd \), \( v(b) = v(g - p - s + rb) = 0 \) since \( d = b \). However \( g = p = s = 0 \) in a steady state while \( v > 0 \) so \( b = 0 \) and \( rb = 0 \) thus \( b = 0 \).}
If \( z > r \), that is the rate at which the government discounts the future is greater than the real rate paid on government issued debt, then \( p \) and \( s \) will not be held constant over time. The government would find it advantageous to issue debt (future taxes) instead of relying on current forms of taxation (including seigniorage). In this case deficit finance has a role. The marginal cost of each means of finance will not be kept constant over time. The rate of increase in explicit taxation and seigniorage will be \( z - r \) times the inverse of the absolute convexity of the cost function.\(^4\) The difference between \( z \) and \( r \) may be due to imperfections in (private) capital markets or because government assets are perceived to be less risky than private assets.\(^5\) The question remains as to why the debt would not be quickly increased to capitalize on the government's advantage (i.e. to where \( z = r \)).

**Growth in Exogenous Variables**

In the preceding section it was assumed that income, real balances and expenditures were constant over time. In this section these variables will be assumed to be growing at constant rates. Let income grow at a rate of \( x \), the demand for real balances at a rate of \( x \) and government expenditures at a rate of \( w \). These rates may be positive, negative or zero and may or may not equal one another. At time \( t \),

\[
\dot{p}(z-r)(Cp/Cpp) > 0. \quad \text{Also} \quad \dot{s}(z-r)(Cs/Css) > 0. \quad \text{It is implied that} \quad \dot{p}, \dot{s} \text{ and } b \text{ are all greater than zero.}
\]

\(^4\) Cox (1983) uses this assumption in a recent article.
\[ y(t) = y_0 e^{xt} \]

and

\[ g(t) = g_0 e^{wt}. \]

The optimization problem is still one of minimizing the present value of revenue collection costs subject to constraints on the budget and growth in the exogenous variables. Formally, where it is understood that the variables are functions of time,

\[
\text{Min } \int_0^\infty C(p,s,b;y)e^{-zt} \, dt
\]

subject to:

\[
\dot{d} = b = g - s + rd,
\]

\[
\dot{y} = xy
\]

and

\[
\dot{g} = wg.
\]

The resulting Hamiltonian function is,

\[
H = C(p,s,b;y) + v_1 (g - p - s + rd) + v_2(xy) + v_3(wg)
\]

where \( v_i(t) = u_i(t)e^{zt} \), the future value of the shadow price. The conditions for a minimum are:

\[
H_p = C_p - v_1 = 0,
\]

\[
H_s = C_s - v_1 = 0,
\]

\[
H_b = C_b = 0,
\]

while

\[
\dot{v}_1 = zv_1 - H_d = (z - r)v_1,
\]

\[
\dot{v}_2 = zv_1 - H_y = (z - x)v_2 - Cy,
\]

\[
\dot{v}_3 = zv_1 - H_g = (z - w)v_3 - v_1
\]

and

\[
\dot{H} = 0.
\]

\[ ^{a6} \text{ Stability requires that } x < z \text{ and } w \leq x. \]
Interpretation of these conditions and their implications is best handled in a step by step manner.

Explicitly, along the optimal time path,
\[ \dot{H} = C_p \dot{p} + C_s \dot{s} + C_y \dot{y} + \dot{v}_1(b) + \dot{v}_1(b) + \dot{v}_2(xy) + v_2(xy) + \dot{v}_3(wg) + v_3(wg) = 0. \]
If \( x = w = 0 \), this problem reduces to the one discussed in the previous section. The appropriateness of deficit financing depends on the difference between the rate of interest on bonds and the government's own rate of discount.

To consider the impact of growth in income alone, let \( x \neq 0 \) while expenditures do not grow \( (w = 0) \) and the government's rate of discount equals the private rate \( (z = r) \). The above equation then reduces to,
\[ \dot{H} = C_p \dot{p} + C_s \dot{s} + C_y \dot{y} + \dot{v}_1(b) + \dot{v}_2(xy) + v_2(xy) = 0. \]
Recalling that,
\[ \dot{y} = xy, \]
and
\[ b = g - p - s + rd \]
while \( d = b \), substitution and simplification lead to an expression for the level of the current deficit,
\[ b = -xyv_2/v_1. \]
Since a higher level of income lowers the cost of obtaining a given amount of revenue, all else equal, the shadow cost of \( xy \), \( v_2 \), is negative. This implies that \( b \) has the same sign as \( x \), that is a positive rate of growth implies positive deficits. Say income is growing. As income increases the marginal costs of collecting taxes (both explicit and monetary), would fall unless
the level of taxation was increased. Debt financing is used to defer the actual collection of taxes so as to keep the marginal costs constant.\textsuperscript{47}

Nissar Liviatan (1982) criticizes Barro for not including money in his public debt model. The model above generates all of Leviatan's important results and reveals his criticism as weak in that he assumes economic growth. His assertion that debt is non-neutral rests fundamentally not on the existence of money, but on the assumption of a positive rate of growth. In the model above as in Barro's own model, debt is not neutral in a growing economy as it is used to transfer wealth from the future to the present so as to reduce tax collection costs and thus increase the net wealth of the economy.\textsuperscript{48}

To consider the effect of growth in (planned) government expenditure, let $w \neq 0$ while $x=0$ and $z=r$. The solution for the level of deficits is,

$$b = -\frac{wgv_3}{v_1}.$$  

Since the impact of $g$ on costs is positive, $v_1$ is positive and $b$ will have the opposite sign to $w$, the growth rate of expenditures. Since increasing expenditures requires increasing revenues collected, changes in the marginal collection costs

\textsuperscript{47} Consider the optimal condition for explicit taxation $C_p - v_1 = 0$ or $C_p = v_1$. Differentiation with respect to time gives $C_{pp} \dot{p} + C_{py} \dot{y} = v_1 = 0$ if $z=r$. Now since $C_{pp}>0$, $y>0$ and $C_{py}<0$, the implication is that $\dot{p}>0$. Specifically, $\dot{p} = -(C_{py}/C_{pp})\dot{y} = -(\partial p/\partial y)\dot{y}$. If, as in Barro, the tax rate $p/y$ is to be held constant over time (i.e. $C_{py} = -C_{pp}$), then $(\partial p/\partial y)=1$ and $\dot{p} = \dot{y}$.

cannot be avoided without the use of assets. As long as expenditures are growing the implication is that governments should run surpluses (buy assets) in anticipation of larger disbursements in the future rather than relying solely on contemporaneous increases in taxation.

For the general case where income and government expenditures are growing and where there may be a difference between the private and government discount rates, the $H=0$ condition can be solved to give the size of the deficit,

$$b=x^2y-z/r(x^2y+(v_2/v_1)xy+(v_3/v_1)wg).$$

Letting $z=r$ this becomes,

$$b=-x/v_1(v_2y+v_3g)$$

the sign of which is ambiguous because while $v$ is negative, $v$ is positive. In general, neutral growth has unclear implications for the level of deficits. If, in a long-run sense (where the cost function is homogeneous of degree zero), it is optimal to keep the tax rates constant over time so as to keep collection costs constant, there would be no effect on deficits. The optimality condition, that the present value of taxes equals that of expenditures, implies that if expenditures are growing at the same rate as income so will taxation, a zero deficit would be optimal.⁴⁹

In summary, anticipation of larger levels of income in the future implies the use of deficit financing while anticipation

---

⁴⁹ This implies that $v_2=-v_3$. Such a condition seems reasonable since a change in the tax to income ratio whether due to a change in income or expenditure should have an equal effect.
of increases in expenditures implies the accumulation of real budget surpluses. These implications depend upon the assumption that the growth rates are anticipated. The assumption that the rates of growth are constant was made strictly for convenience and the results should hold qualitatively for other growth patterns.

Temporary Revenue Needs

Suppose the state requires a temporary source of additional revenue to finance extraordinary expenditures. The situation may be described by the budget constraint

\[(g(t) + g'(t)) = p(t) + b(t) + s(t) - rd(t-1) + p'(t) + s'(t) + b'\]

where \(g'\) represents the temporary, one period additional financing requirement and \(p', s', b'\) the additional use of explicit taxation, seigniorage and debt sales to the public.

The objective of the state remains the minimization of the total cost of collecting the required revenue, but the cost function considered should be the short-run cost function where fixed factors may exist. The short-run cost function may be written

\[C(p, s, b; y, Kp^*, Km^*, Kb^*)\]

where \(Kp\) and \(Km\) refer to the inelastic supply of collection and avoidance capital as described in Gordon (1975) and Barro (1977) and \(Kb\) is capital in debt sales. For example, the difficulty in changing tax rates and enforcing collection in the short run is

---

\(^50\) Essentially the same analysis would hold in the case of a temporary shortfall in tax revenues due to a fall in income.
reflected in \( K_e \) while the ease with which seigniorage may be increased or decreased is captured by \( K_m \). The levels of capital are chosen so as to minimize net collection costs for expected long-run revenue requirements. These optimal levels are indicated by asterisks.

If the transitory revenue requirement is expected to be extremely short lived little augmentation of the levels of capital will be made. In the limit, where the transitory revenue requirement lasts for only a moment, no change in the capital stocks would occur. This would also be the case if transitory revenue requirements (or changes in income) were considered to be random deviations around the mean. The optimal level of capital, chosen so as to minimize the expected present value of collection costs, would not be altered with each fluctuation. The optimal mix of revenue sources would depend fully on the short-run marginal cost functions.

The additional amount to be financed, \( g' \), will be met by increments to the three sources of revenue. Thus,
\[
g'(t) = (p'(t) + s'(t)) + b'(t).
\]
Optimal finance requires that the marginal costs of explicit taxation and seigniorage collection be equated throughout time. The increments to explicit taxation and seigniorage should therefore be made permanent and constant over time. This is done by letting the current increase in explicit taxation and seigniorage collection together equal the future interest payments implied by the current deficit. So
\( (p'(t)+s'(t)) = rb' \)

and the current deficit will be

\( b'(t) = g'(t)/(1+r). \)

Deficit finance has a role to play in smoothing tax collection costs over time. Taxation and seigniorage will remain at \( p+p' \) and \( s+s' \) indefinitely while in the future deficits will be zero. The additional real debt issued this period will not be retired unless an offsetting shock occurs in the future.

In Figure 1.1, the short-run marginal cost curves lie above their long-run counterparts except at the points where the revenue to be collected equals the long-run levels. In the long-run bonds are dominated by explicit taxation or seigniorage or both. In Figure 1.1, both explicit taxation and seigniorage are depicted as being used in the long-run. The total revenue collected, \( p+s \), suffices to cover all revenue requirements \( g \) plus any interest due on accumulated debt.51 In the short-run, a transitory increase in required revenue would be met by increases the amount of revenue obtained through at least one source of finance. In Figure 1.1, an increase in the use of all three forms is depicted. A symmetrical result would be obtained if there was a transitory decrease in required revenue. If the transitory decline is of sufficient size, the surplus will be used to retire outstanding debt or to accumulate privately issued debt instruments.

\[ \]

51 In the long-run, any accumulated debt would neither be added to or diminished since \( b=0 \). New debt may be issued, but only to replace maturing issues.
Figure 1.1: Long-run and Short-run Revenue Collection
The specific marginal cost curves drawn in Figure 1.1 are hypothetical in that the short-run curve for seigniorage collection might be relatively less elastic than the curve for explicit taxation. This might be the case if, for example, the central bank of a country was separated from the spending authorities and rules prohibiting the growth of the monetary base in excess of a certain rate were in effect. Some forms of explicit taxation such as excise taxes might be increased in the short-run with a relatively small increase in cost to the state. Figure 1.2 shows the time paths of each form of revenue collection in response to a change in expenditure given an initial level of debt outstanding and where the temporary increase in $g$ is later offset by a temporary decrease. It is assumed for illustrative purposes that in this economy that both explicit taxation and monetary finance will be used in both the long and short runs.
Figure 1.2: Time Paths of Revenue Collections
The elasticities of the marginal cost curves are matters for empirical investigation. The major conclusion of this section is that bond finance is predicted to be used when either: (i) there are differential growth rates between the private economy and the state's expenditure due to different long-run trends or short-run deviations from trends or (ii) there is a difference between the rate of discount used by the state and the real rate of interest the state pays holders of its bonds.
Summary

This chapter has considered deficits and monetary expansion as tools of public finance. Following a discussion of the literature on deficit and monetary finance, a simple model of optimal public finance was developed and used to derive testable implications concerning the use of taxation, deficits and monetary expansion.

In most published work, inflation is considered undesirable. In a model in which monetary expansion is one of several substitutable sources of government revenues, reliance on monetary finance may be preferable to increases in other forms of taxation. Consideration of the increase in other taxes that would accompany a reduction in the rate of monetary expansion is missing in much of the welfare cost of inflation literature. One prediction which is implied by the model used in this paper is that monetary finance will be used relatively more in countries where income or commodity taxation is difficult to use. One aspect of monetary finance is that the extent to which it is used can be altered quickly unless predetermined institutional arrangements are such as to make it difficult to change the growth rate of the monetary base in a discretionary manner. In countries where it is difficult to collect other forms of taxation, we might expect to see few such impediments to the use of short-term monetary finance.
There has been a considerable amount of debate in the literature concerning the role of deficit finance. To assume that government bonds constitute a form of net wealth is to assume either the lack of a well-functioning private capital market or myopia on the part of the public. It would seem to be preferable to explain the existence of government bonds without the need to use such assumptions. If the model fails to explain empirical observations, then there is a role for such restrictive assumptions in a new model of debt finance. Assuming that there are private capital markets that function at least as well as state operated loan systems and assuming that the public is not myopic with respect to future taxes implied by the issuing of government bonds, raises the problem of how the levels of debt and taxes are determined. Following Barro, the model used in this paper determines the optimal size of the deficit and the level of taxation broadly defined as the result of minimizing the cost of government finance. The model predicts that deficits will appear if income is growing, real balance holdings are growing, government expenditures are falling over time, there are transitory increases in government expenditures, or the government's rate of discount is less than that of the public. This last case, where the government's rate of discount is less than the rate paid on government bonds, represents the case where government debt is a form of net wealth in the economy.
The extent to which each form of revenue collection will be used depends upon the circumstances and the specifics of the cost conditions facing the government. The model in this paper represents an attempt to discover general principles, a starting point for an empirical study of how actual economies meet the financial needs of government through the use of taxation, monetary expansion and the sale of bonds.
The model of government finance presented above treats monetary expansion, debt issue and other taxes as (close) substitutes. Monetary expansion and debt issue are considered to be specific forms of taxation without additional properties significantly affecting real income or the interest rate. The model rests on two foundations: (i) the monetary base can be and is used as a revenue collection mechanism and (ii) government debt (deferred tax collection) is a close substitute for current tax levies. In each case there exists a relevant set of empirical papers.

This survey is not intended to be exhaustive, but is intended to provide a basis for further empirical work by criticizing what has been done and indicating what remains to be done. Most importantly, little has been done on the topic of monetary finance using the monetary base as opposed to broader aggregates, as the variable to be explained. In addition, little work on debt and taxes has been done using data for countries other than the U.S.
Monetary Finance

Deficits and Inflation: Tests of Money Supply Determination

A widely expressed view among business spokesmen and opposition politicians is that deficits contribute to inflation if not actually cause it. Buchanan and Wagner (1977) express such a view in their condemnation of "Keynesian" deficit financing of government expenditures. They argued that the Federal Reserve would monetize a portion of the deficit in order to prevent the interest rate from increasing too much and leading to an excessive crowding out of private investment. This line of argument is in stark contrast to a hypothesis in which government bonds are not treated in aggregate as a form of net wealth. According to such a hypothesis, interest rates need not rise following an increase in the deficit (given a level of government expenditure). Money supply growth would be determined by finance considerations or stabilization policies and would not be determined by the deficit. The view that deficits will be monetized translates into a view that deficits cause increases in the money supply and lead to inflation as a result. Two of the several attempts to test such a proposition are discussed below.

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1 See Tobin (1980) Chapter 3 for a synopsis of the public debt debate.
As a response to Buchanan and Wagner's book, Robert Barro (1978b) presented empirical evidence to the effect that: (a) if real deficits are calculated then real surpluses are about as common as real deficits over a period of several decades; (b) sizable real deficits are associated with war, but real surpluses follow in years of peace; and (c) deficits do not lead to increases in the money supply. The first two items are presented in the form of descriptive statistics for the post 1860 U.S. economy. The last item is presented in the form of results of regressions of money supply growth on several explanatory variables. The equation is,

\[
\Delta \ln(M1) = a_0 + a_1 \Delta \ln(M1_t) + a_2 \Delta \ln(M1_{t-1}) + a_3 \ln(U_t/(1-U_t)) + a_4 \ln(G/G^*) + a_5 S/P GNP^*
\]

where

- $M1$ is the annual average of $M1$,
- $U$ is the unemployment rate,
- $G$ is real government expenditure,
- $G^*$ is normal real government spending, $\ln G^* = 0.2 \ln G + 0.8 \ln G^*_{t-1}$,
- $S$ is the nominal budget surplus,
- $P$ is the price level deflator

and

- $GNP^*$ is trend real GNP.

The lagged money growth terms are included to capture any persistence effects while the last term is intended to represent Buchanan and Wagner's hypothesis that deficits lead to money
supply growth. The third and fourth terms are intended to capture effects of stabilization policies and government financial motivations for increasing the money supply. Barro runs three different regressions each over two intervals 1941-76 and 1946-76. In the first equation the budget surplus variable is excluded while in the second equation the expenditure relative to normal variable is excluded. In order to facilitate a critical examination of Barro's results, I reproduce the results below.
Table 2.1: Barro's Results
Sample: 1941-76  Dependent Variable: $\Delta \ln(M)$

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<td>0.082</td>
<td>0.095</td>
<td>0.072</td>
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<tr>
<td></td>
<td>(0.027)</td>
<td>(0.031)</td>
<td>(0.072)</td>
</tr>
<tr>
<td>$\Delta \ln(M_{1.1})$</td>
<td>0.41</td>
<td>0.60</td>
<td>0.33</td>
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<tr>
<td></td>
<td>(0.14)</td>
<td>(0.15)</td>
<td>(0.14)</td>
</tr>
<tr>
<td>$\Delta \ln(M_{1.2})$</td>
<td>0.21</td>
<td>0.00</td>
<td>0.33</td>
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<td></td>
<td>(0.12)</td>
<td>(0.13)</td>
<td>(0.14)</td>
</tr>
<tr>
<td>$\ln(U_{1.1}/(1-U_{1.1}))$</td>
<td>0.026</td>
<td>0.027</td>
<td>0.026</td>
</tr>
<tr>
<td></td>
<td>(0.009)</td>
<td>(0.010)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>$\ln(G/G^*)$</td>
<td>0.072</td>
<td>0.133</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.016)</td>
<td>(0.034)</td>
<td></td>
</tr>
<tr>
<td>$S/P$ GNP$^*$</td>
<td></td>
<td>-0.24</td>
<td>0.32</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.09)</td>
<td>(0.16)</td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.77†</td>
<td>0.70†</td>
<td>0.80†</td>
</tr>
<tr>
<td>D.W.</td>
<td>1.90</td>
<td>1.93</td>
<td>1.86</td>
</tr>
<tr>
<td>S.E.</td>
<td>0.0147†</td>
<td>0.0170†</td>
<td>0.141†</td>
</tr>
</tbody>
</table>

Sample: 1946-76

<table>
<thead>
<tr>
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<tr>
<td></td>
<td>(0.029)</td>
<td>(0.032)</td>
<td>(0.025)</td>
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<tr>
<td>$\Delta \ln(M_{1.1})$</td>
<td>0.47</td>
<td>0.63</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
<td>(0.16)</td>
<td>(0.15)</td>
</tr>
<tr>
<td>$\Delta \ln(M_{1.2})$</td>
<td>0.19</td>
<td>-0.06</td>
<td>0.50</td>
</tr>
<tr>
<td></td>
<td>(0.16)</td>
<td>(0.13)</td>
<td>(0.17)</td>
</tr>
<tr>
<td>$\ln(U_{1.1}/(1-U_{1.1}))$</td>
<td>0.027</td>
<td>0.024</td>
<td>0.034</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.011)</td>
<td>(0.009)</td>
</tr>
<tr>
<td>$\ln(G/G^*)$</td>
<td>0.073</td>
<td>0.184</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.027)</td>
<td>(0.044)</td>
<td></td>
</tr>
<tr>
<td>$S/P$ GNP$^*$</td>
<td></td>
<td>-0.12</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.16)</td>
<td>(0.23)</td>
</tr>
<tr>
<td>$\bar{R}^2$</td>
<td>0.60</td>
<td>0.50</td>
<td>0.71</td>
</tr>
<tr>
<td>D.W.</td>
<td>1.78</td>
<td>1.81</td>
<td>1.79</td>
</tr>
<tr>
<td>S.E.</td>
<td>0.0149</td>
<td>0.0167</td>
<td>0.0131</td>
</tr>
</tbody>
</table>
Notes:
†Barro applied a weighting scheme to observations for the 1941-45 period. The \( R^2 \) statistics take this into account. The standard error of the estimate is applicable to the 1946-76 period. Table reproduced from Barro (1978b) p. 577. Standard errors in parentheses.
Barro interprets these results as suggesting that the variation in government expenditures from normal, rather than the budget surplus, is the important factor influencing money growth. Barro states,

The third equation, which includes (current versus normal federal expenditures) and (the budget surplus) simultaneously, demonstrates that the main explanatory power from the state of the federal budget derives from the expenditure relative to normal variable and not from any independent information contributed by observation of the surplus.... Basically similar results apply to the 1946-76 period. In fact, the (surplus) variable is insignificant over this sample even when the (expenditure) variable is omitted.\(^2\)

Several points need to be raised concerning these regressions and their interpretation:

(1) As discussed in Hamburger and Zwick (1981), Barro did not properly align his data. Barro's money growth figures were calculated using annual averages while the fiscal measures are end of year figures. After replacing Barro's money growth series with one calculated with end of year figures, Hamburger and Zwick found evidence that there may have been a policy shift towards Keynesian deficit financing in the post war period as described by Buchanan and Wagner and that monetary growth was affected.

(2) In equation 5 above, the surplus variable is indeed insignificant, but it is curious that when both the expenditure and surplus variables are included (equations 3 and 6) the surplus variable is significant. Further, it is significant and has the "wrong" sign. With the likelihood of multicollinearity

\(^2\) Barro (1978b) p. 578. Terms in parentheses are my replacements for Barro's symbols.
between transitory expenditure increases and deficits, as predicted by Barro's determination of debt model, the t statistics alone may not be a reliable guide to the significance of explanatory variables. A test involving standard errors of the estimates which can be converted to a comparison of R statistics is a more satisfactory guide to the importance of additional variables in a regression equation. The null hypothesis is that an additional variable does not add significantly to the explanatory power of the equation. The test statistic is,

$$F(Q-K,N-Q) = \left( \frac{R_q^2 - R_k^2}{1 - R_k^2} \right) \left( \frac{(N-Q)}{(Q-K)} \right)$$

where

$$N$$ is the number of observations (34 and 29),

$$K$$ is the number of regressors in the basic equation (4),

and

$$Q$$ is the number of regressors in the extended equation including the additional explanatory variable(s) (5).

Comparing the first and third regressions above, the statistic calculated is,

$$F(1,33) = \left( \frac{.80 - .77}{1 - .80} \right) (33/1) = 4.95,$$

while comparing the fourth and sixth equations,

$$F(1,28) = \left( \frac{.71 - .60}{1 - .71} \right) (28/1) = 10.6.$$ 

Both are significant at a 5% level of confidence.³ The null hypothesis is rejected in both cases and, as a result, the surplus variable should not be dismissed as being unimportant.

³ A similar experiment in which the expenditure variable is considered to be the additional explanatory variable also results in significant statistics.
There is a potential "errors in variables" problem in that the dependent variable, the lagged dependent variables and the surplus variable all include the change in high-powered or base money as one of their components.4

Barro intends to run, "...regressions using as a dependent variable the annual growth rate of money, \( DM = \log(M_{t}) - \log(M_{t-1}) \)." The problem is that the difference in the natural logarithms, the continuous compounding growth rate is only an approximation of the actual annual growth rate; an approximation that is close for small rates, but is increasingly inaccurate as the rates become larger.5 If \( M_{t} = (1+\gamma)M_{t-1} \), where \( \gamma \) is the annual growth rate and \( M_{t} = e^{\Gamma}M_{t-1} \), where \( \Gamma \) is the average continuous growth rate over the year, then \( \Gamma = \ln(1+\gamma) \). The approximation is a logarithmic transformation of the discretely measured annual growth rate and not a linear one. Thus each moment of a distribution of annual growth rates would be affected if the approximation is used in place of the actual growth rates.6

\[ \frac{\Delta M}{M_{1}} = 1\% \Rightarrow \Delta \ln(M_{1}) = 0.995\% \]
\[ \frac{\Delta M}{M_{1.1}} = 1\% \Rightarrow \Delta \ln(M_{1}) = 0.995\% \]
\[ \frac{\Delta M}{M_{1.2}} = 10\% \Rightarrow \Delta \ln(M_{1}) = 9.5\% \]

Barro notes that there may be such a problem. See Barro (1978b) footnote 13.

5 The value of \( \Gamma \) can be expressed as an infinite series \( \Gamma = \ln(1+\gamma) = \gamma - \gamma^{2}/2 + \gamma^{3}/3 + \Sigma(-1)^{n-1}\gamma/N \). Neglecting higher order terms, \( \Gamma \approx \gamma - \gamma^{2}/2 \). If \( \gamma \) is a very small number \( \gamma = \Gamma \), but as \( \gamma \) becomes larger \( \gamma \) underestimates \( \Gamma \) by a factor of approximately \( \gamma^{2}/2 \). The mean of a distribution of \( \Gamma \)'s, \( \bar{\Gamma} = \gamma - \gamma^{2}/2 \), where \( \gamma \) is the mean of the distribution of actual growth rates and \( \sigma^{2} \) is the variance of that distribution. The error in the mean of \( \Gamma \) is a function of both the mean and variance of the actual distribution. The nonlinear logarithmic transformation reduces the magnitude of larger numbers proportionately more than that of smaller numbers. The variance of a distribution of \( \Gamma \)'s may be
In order to examine the effects of the use of the logarithmic approximation, Barro's equation was run over the 1946-76 period using Barro's own figures for money supply growth and the annual growth rate calculated as $\Delta M_1/M_{1-1}$, using end of year figures. Barro's unemployment rate figures and his series for the difference of real government expenditures from normal where also used, but his series for the surplus and calculated values for trend real GNP were not readily obtainable so real GNP and price level deflator series were obtained and trend real GNP (GNP*) and the S/P GNP* variable as described by Barro were calculated. The results are presented below in Table 2.2.

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(cont'd) expressed as, $\sigma_2^2 = \sigma_1^2 + \sigma_3^2 (1 - \sigma_3^2/4) + (\bar{\gamma}^2 + \sigma_3^2)(\bar{\gamma} - \sigma_3^2)/2$. The first and third terms in parentheses will, in all but exceptional cases, be negative.
Table 2.2: Reestimation of Barro's Model  
Sample: 1946-76 Dependent Variable: Money Growth  

Part A: Money Growth Calculated by ln(M1)-ln(M1.1)

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<tr>
<td></td>
<td>(0.029)</td>
<td>(0.032)</td>
<td>(0.029)</td>
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<tr>
<td>Δln(M1.1)</td>
<td>0.490</td>
<td>0.609</td>
<td>0.465</td>
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<tr>
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<td>(0.158)</td>
<td>(0.168)</td>
<td>(0.165)</td>
</tr>
<tr>
<td>Δln(M1.2)</td>
<td>0.166</td>
<td>-0.129</td>
<td>0.147</td>
</tr>
<tr>
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<td>(0.155)</td>
<td>(0.125)</td>
<td>(0.160)</td>
</tr>
<tr>
<td>ln(U1/(1-U1))</td>
<td>0.029</td>
<td>0.023</td>
<td>0.027</td>
</tr>
<tr>
<td></td>
<td>(0.010)</td>
<td>(0.011)</td>
<td>(0.010)</td>
</tr>
<tr>
<td>ln(G/G*)</td>
<td>0.077</td>
<td>0.071</td>
<td>0.027</td>
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<td></td>
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<td>(0.029)</td>
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<tr>
<td>S/P GNP*</td>
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<td>(R^2)</td>
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<td>0.64</td>
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<td>D.W.</td>
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<tr>
<td>S.E.</td>
<td>0.0144</td>
<td>0.0159</td>
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Part B: Money Growth Calculated by (M1-M1.1)/M1.1

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<td>0.060</td>
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<tr>
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<td>(0.193)</td>
</tr>
<tr>
<td>ΔM1.1/M1.2</td>
<td>0.420</td>
<td>0.343</td>
<td>0.312</td>
</tr>
<tr>
<td></td>
<td>(0.175)</td>
<td>(0.188)</td>
<td>(0.193)</td>
</tr>
<tr>
<td>ΔM1.2/M1.3</td>
<td>0.001</td>
<td>-0.092</td>
<td>0.005</td>
</tr>
<tr>
<td></td>
<td>(0.193)</td>
<td>(0.150)</td>
<td>(0.190)</td>
</tr>
<tr>
<td>ln(U1/(1-U1))</td>
<td>0.014</td>
<td>0.009</td>
<td>0.013</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.016)</td>
<td>(0.016)</td>
</tr>
<tr>
<td>ln(G/G*)</td>
<td>0.043</td>
<td>0.033</td>
<td>0.039</td>
</tr>
<tr>
<td></td>
<td>(0.039)</td>
<td>(0.039)</td>
<td></td>
</tr>
<tr>
<td>S/P GNP*</td>
<td></td>
<td>-0.320</td>
<td>-0.281</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.215)</td>
<td>(0.221)</td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.30</td>
<td>0.32</td>
<td>0.34</td>
</tr>
<tr>
<td>D.W.</td>
<td>2.05</td>
<td>1.86</td>
<td>1.93</td>
</tr>
<tr>
<td>S.E.</td>
<td>0.0220</td>
<td>0.0217</td>
<td>0.0218</td>
</tr>
</tbody>
</table>
Notes:
Part B: Money supply series from The International Monetary Fund, International Financial Statistics various issues. These are year end figures. Other data as in Part A above. Standard errors in parentheses.
There are two observations that must be made. The first two regressions are very similar to those in Barro. In the first equation, only Barro's data are used and any discrepancy in the results is due to the different regression programs being used. In the second equation, a surplus to price level times trend real GNP ratio is calculated and used. The results are quite similar to Barro's. In the third equation however, the results are strikingly dissimilar. The coefficient for the surplus variable is insignificant and negative as it is in the second equation and as predicted by Barro's model. The F statistic, 

\[ F(1,24) = \frac{(0.64 - 0.63)/(1 - 0.64)}{(24/1)} = 0.67, \]

is, of course, insignificant. The null hypothesis that the surplus variable does not add to the explanatory power of the equation clearly cannot be rejected. The difference between these results and Barro's has something to do with the surplus variable. The calculation of trend real GNP is not described in detail in Barro's article so a simple regression of real GNP on time from 1941 to '76 was run and the coefficient estimates used to calculate a trend series.

When the discretely calculated annual growth rates in the money stock (using year end figures) are used in place of Barro's series, the results are statistically poor. Perhaps this is due to a problem with the data so as a final check, the fact

\[ \text{See Table 2.2, equations 4 and 5.} \]

\[ \text{Subsequently, I found Barro's original specification of G in an earlier article. Given the results in Table 2.3 below, I did not recalculate G}^* \text{ myself.} \]
that $\Delta M_1/M_1_i = \exp(\Delta \ln M_1) - 1$ was used to convert Barro's data to the discretely calculated equivalents. The results, reported in Table 2.3, are also poor. Noting that equation 1 in Table 2.3 consists entirely of Barro's data and considering all of the results together, the empirical evidence may be described as weak. The use of logs is not invalid, but care should be exercised in interpreting the resulting statistics.\footnote{See Maddala (1977) Chapter 12.}
Table 2.3: Reestimation of Barro's Results
Sample: 1947-76
Growth Rates Calculated as exp(x)-1
Barro's Data

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>0.305</td>
<td>0.137</td>
<td>0.229</td>
</tr>
<tr>
<td></td>
<td>(0.323)</td>
<td>(0.308)</td>
<td>(0.327)</td>
</tr>
<tr>
<td>∆M₁.₁/M₁.₂</td>
<td>0.405</td>
<td>0.344</td>
<td>0.308</td>
</tr>
<tr>
<td></td>
<td>(0.178)</td>
<td>(0.189)</td>
<td>(0.194)</td>
</tr>
<tr>
<td>∆M₁.₂/M₁.₃</td>
<td>-0.006</td>
<td>-0.110</td>
<td>-0.013</td>
</tr>
<tr>
<td></td>
<td>(0.187)</td>
<td>(0.148)</td>
<td>(0.186)</td>
</tr>
<tr>
<td>U₁/(1-U₁)</td>
<td>0.304</td>
<td>0.116</td>
<td>0.220</td>
</tr>
<tr>
<td></td>
<td>(0.344)</td>
<td>(0.326)</td>
<td>(0.349)</td>
</tr>
<tr>
<td>G/G*</td>
<td>0.042</td>
<td></td>
<td>0.033</td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td></td>
<td>(0.038)</td>
</tr>
<tr>
<td>S/P GNP*</td>
<td></td>
<td>-0.307</td>
<td>-0.265</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.218)</td>
<td>(0.224)</td>
</tr>
<tr>
<td>Ř²</td>
<td>0.23</td>
<td>0.32</td>
<td>0.34</td>
</tr>
<tr>
<td>D.W.</td>
<td>2.03</td>
<td>1.84</td>
<td>1.92</td>
</tr>
<tr>
<td>S.E.</td>
<td>0.0220</td>
<td>0.0217</td>
<td>0.0218</td>
</tr>
</tbody>
</table>

Notes: Data from Barro. Standard errors in parentheses.
Gerald Dwyer (1982) examined the empirical evidence relating to three hypotheses concerning the observed correlation between deficits and inflation in the United States. These hypotheses are: (1) that bonds are treated as net wealth by the public so that an increase in the deficit leads to a higher level of aggregate demand; (2) that the central bank will attempt to hold down interest rates by monetizing a portion of the deficits; and (3) that the government will increase the nominal supply of bonds when anticipating inflation so as to hold the real magnitude of the national debt constant. Dwyer presents evidence in the form of both descriptive and inferential statistics.

In terms of descriptive evidence, Dwyer plots the real deficit (the change in the real value of outstanding government debt) and the portions of the deficit purchased by the public and the Federal Reserve over time. There appears to be a close correlation between the movement of the total deficit and the portion acquired by the public. There is little correlation between total real deficits and the monetized portion of those deficits. In fact the calculated correlation coefficient turns out to be -0.21! Real deficits and the monetized portion thereof do not appear to have been connected. Further, it is revealed that on average over the 1952 to 1981 period, real deficits averaged out to be close to zero. This evidence is consistent with Barro's hypothesis.
The inferential evidence appears in the form of Sim's tests. A vector autoregression,
\[ X_t = \alpha_0 + \sum_{k=1}^{\infty} \alpha_k X_{t-k} + \epsilon_t \]
where \( X \) is a six variable vector containing the proportional increases in the price level, nominal income, nominal money stock, government debt held by the Federal Reserve, government debt held by the public and the interest rate on three month Treasury bills.\(^{10}\)

Of central interest are two tests performed by Dwyer. The first is a test of the hypothesis that government bonds are not considered to be net wealth while the second is a test of the hypothesis that anticipated inflation leads to a growth in the nominal stock of government debt. In order to test the first hypothesis, Dwyer constrains the coefficients of the four lagged growth of debt variables to be zero. Dwyer then tests to see if the explanatory power of the five equations excluding the equation for debt itself are changed significantly. If lagged growth of debt does not contribute significantly to the determination of the other variables, then the hypothesis that debt is not treated as aggregate net wealth cannot be rejected. Dwyer finds that the evidence does not allow the hypothesis to be rejected.\(^{11}\) The test of the second hypothesis involves

---

\(^{10}\) The price level is measured by the consumer price index and both M1 and M2 are used as measures of the money stock. The value of \( k \) was set at four following some experimentation. Proportional growth rates were calculated discretely as \( \Delta x/x \).

\(^{11}\) In unconstrained regressions the coefficients of the lagged growth of debt variables are not significantly different from zero in all but the equation for the current growth of debt.
determining whether or not the same lagged variables that determine other dependent variables also determine the current growth in the debt. For example, if lagged money growth helps determine the current inflation rate, then lagged money growth should also help determine the growth in the nominal stock of debt according to Barro's hypothesis. As Dwyer notes this is a weak test because cyclical policy involving the use of deficit financing would produce the same correlation between variables as would the behavior described by Barro. Dwyer's results are consistent with the hypothesis that anticipated inflation leads to increases in the level of the nominal debt. Lagged values of the growth in the nominal debt do not appear to significantly affect the rate of inflation variable, but lagged values of inflation do appear to affect the size of nominal deficits.

As has been noted in the literature, including Dwyer's article itself, we must be careful not to misinterpret the results of Granger and Sims tests. The "causality" involved only represents intertemporal correlations between some variable and lagged values of other variables; correlations which do not appear if the assumed dependent-independent relationship is inverted. Dwyer's results do not rule out alternative hypotheses. For example, it may be that expected (future) deficits cause inflation. The observed correlation between past inflation and current deficits does not allow us to reject such a hypothesis. Nevertheless Dwyer's results are not inconsistent
with Barro's hypothesis that future tax liabilities are discounted. While there may be no strong evidence in support of Barro according to his critics, there is also no evidence that refutes his hypothesis.
Three Cross-Sectional Studies of Inflationary Finance

The evidence presented below suggests that all states generate revenues through seigniorage to varying degrees. The proportion of total revenue raised by monetary finance varies markedly between countries. Two of the three papers surveyed attempt to provide some explanation of the extent to which seigniorage will be relied upon while the other presents some interesting figures on the use of seigniorage around the world.

Bade and Parkin

Robin Bade and Michael Parkin (1980) investigate the relationship between the monetary policies of twelve industrialized countries and the degree of statutory independence between the central bank and the spending authorities in each country. They state that,

The central banks of Australia, Belgium, France, Italy, the Netherlands, Sweden and the United Kingdom are subservient to the central governments in those countries in which the formulation and conduct of monetary policy: those of Germany, Japan, Switzerland and the United States have varying degrees of independence from central government; that of Canada underwent a change in status in 1967 and, prior to that date, was largely independent of government but since then has had its legislated powers diminished.¹²

Bade and Parkin are concerned with three features of monetary policy: (i) the rate of inflation; (ii) the variability of inflation; and (iii) the responsiveness of policy to economic shocks. For the purpose of this survey, we will consider

¹² Bade and Parkin p. 3.
Independence, in Bade and Parkin, is determined by the legal relationship between the authorities directly in charge of central bank operations and the legislative and spending authorities. The closeness of the relationship involves the extent of collaboration in the formulation and execution of monetary policy, the extent to which the legislative authorities appoint those who control the central bank's operations and the degree of financial control the spending authorities have over the central bank.

Bade and Parkin present statistics derived from International Monetary Fund data which are intended to reveal any relationship between central bank independence and inflation. A simplified version of one of their tables is presented below.
Table 2.4: Average Inflation and Variability 1972-79

<table>
<thead>
<tr>
<th>Country</th>
<th>Inflation Rate</th>
<th>Variability</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>%</td>
<td>Rank</td>
</tr>
<tr>
<td>1. Dependent Central Banks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Australia</td>
<td>11.1</td>
<td>(10)</td>
</tr>
<tr>
<td>Belgium</td>
<td>7.9</td>
<td>(5)</td>
</tr>
<tr>
<td>Canada</td>
<td>8.5</td>
<td>(6)</td>
</tr>
<tr>
<td>France</td>
<td>9.7</td>
<td>(8)</td>
</tr>
<tr>
<td>Italy</td>
<td>14.2</td>
<td>(12)</td>
</tr>
<tr>
<td>Netherlands</td>
<td>7.4</td>
<td>(3)</td>
</tr>
<tr>
<td>Sweden</td>
<td>8.9</td>
<td>(7)</td>
</tr>
<tr>
<td>U.K.</td>
<td>13.8</td>
<td>(11)</td>
</tr>
<tr>
<td>2. Relatively Independent Central Banks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>9.7</td>
<td>(8)</td>
</tr>
<tr>
<td>U.S.A.</td>
<td>7.6</td>
<td>(4)</td>
</tr>
<tr>
<td>3. Independent Central Banks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>5.0</td>
<td>(1)</td>
</tr>
<tr>
<td>Switzerland</td>
<td>5.0</td>
<td>(1)</td>
</tr>
</tbody>
</table>

Notes: See Bade and Parkin p. 24. The original Source was International Financial Statistics, Yearbook, 1980. Independence refers to the degree of policy cooperation between the central bank and the government spending authorities in the formulation and conduct of policy.
Bade and Parkin state,

It is interesting that the two most independent central banks, Germany and Switzerland, have delivered a lower inflation rate than the intermediate central banks of Japan and the United States. The mean inflation rate of the eight government dominated central banks is 10.2 percent with a standard deviation of 2.6 percent; thus, although the U.S. and Japanese rates are well within two standard deviations of that mean, those of Switzerland and Germany are exactly two standard deviations below the mean. On the basis of these facts, we would tentatively conclude that there is an association between the degree of central bank policy independence and the average rate of inflation.\(^{13}\)

It is difficult to draw strong conclusions from such a small sample and for such a limited time period however. The inflation rate, measured by the growth in the consumer price index, may confuse inflation and one time price increases for specific goods. The average rate of inflation for the eight year period may also reflect events which occurred prior to 1972, or expectations of future events or even events which are not directly connected to central banks and the money supply. The authors, for example, cite the case of Japan. Japan did not attempt to prevent fuel prices from rising in 1973 and as a result the price level index increased dramatically, but briefly. Such an event is not to be confused with central bank generated inflation however. It appears that little can be said about the variability of inflation except that Japan had an extremely variable rate of inflation relative to the other countries in the sample. Bade and Parkin attribute this to the oil price increases in the 1970's as described above.

\(^{13}\) Bade and Parkin, p. 28.
The standard deviation in the inflation rate may not represent the most meaningful measure of varying inflation. A standard deviation of 4% is relatively more important, one would assume, in a country with an average inflation rate of 12% than in one with an average inflation rate of 120%. In Table 2.5, the coefficients of variation are given. We can observe that two of the more independent central banks have relatively more variable inflation than the others. The average coefficient of variation for countries other than Japan and Switzerland is 0.3. Japan and Switzerland have markedly higher relative variability in their inflation rates than the other countries. The U.S. and Germany, on the other hand do not. It is difficult to draw meaningful conclusions about central bank independence and inflation variability on the basis of this sample.
### Table 2.5: Relative Variability of Inflation 1972-79

**Relative Variability of Inflation 1972-79**

<table>
<thead>
<tr>
<th>Country</th>
<th>Coeff. of Variation</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1. Dependent Central Banks</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Australia</td>
<td>0.3</td>
<td>(2)</td>
</tr>
<tr>
<td>Belgium</td>
<td>0.4</td>
<td>(3)</td>
</tr>
<tr>
<td>Canada</td>
<td>0.2</td>
<td>(1)</td>
</tr>
<tr>
<td>France</td>
<td>0.2</td>
<td>(1)</td>
</tr>
<tr>
<td>Italy</td>
<td>0.3</td>
<td>(2)</td>
</tr>
<tr>
<td>Netherlands</td>
<td>0.3</td>
<td>(2)</td>
</tr>
<tr>
<td>Sweden</td>
<td>0.2</td>
<td>(1)</td>
</tr>
<tr>
<td>U.K.</td>
<td>0.4</td>
<td>(3)</td>
</tr>
<tr>
<td><strong>2. Relatively Independent Central Banks</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>0.7</td>
<td>(4)</td>
</tr>
<tr>
<td>U.S.A.</td>
<td>0.4</td>
<td>(3)</td>
</tr>
<tr>
<td><strong>3. Independent Central Banks</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>0.3</td>
<td>(2)</td>
</tr>
<tr>
<td>Switzerland</td>
<td>0.7</td>
<td>(4)</td>
</tr>
</tbody>
</table>

**Notes:**
Table based on Bade and Parkin p.24. Reproduced as Table 2.4 above.
Stanley Fischer (1982) calculates average seigniorage rates for a large number of countries. Table 2.6 reproduces some of Fischer's calculations. There appears to be no discernable relationship between central bank independence and the tax placed upon the population of each country by monetary expansion. The independent German central bank supplies the German government with proportionately more of its total revenue requirement than the totally dependent banks of Australia, Belgium, Canada, France and the U.K. Independence, as defined by Bade and Parkin, does not seem to capture an important aspect of central bank behavior namely the role the bank plays in governmental finance. A fuller theory of central banking appears to be necessary.

One hypothesis that might be considered here is that in a country such as the United States, the system of government might make explicit tax increases more difficult for an administration to pursue. Thus the administration would rely more heavily on monetary finance (which it has more direct control over) than in parliamentary nations which are not characterize by the same separations of power and checks and balances. The figures in Table 2.6 do not support such a hypothesis.
Table 2.6: Fischer's Calculation of Seigniorage Rates 1973-79

<table>
<thead>
<tr>
<th>Country</th>
<th>Inflation Rate</th>
<th>Seigniorage Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>%</td>
<td>% of all revenues</td>
</tr>
<tr>
<td>1. Dependent Central Banks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Australia</td>
<td>6.0</td>
<td>2.4</td>
</tr>
<tr>
<td>Belgium</td>
<td>9.2</td>
<td>3.5</td>
</tr>
<tr>
<td>Canada</td>
<td>9.2</td>
<td>3.4</td>
</tr>
<tr>
<td>France</td>
<td>10.7</td>
<td>1.0</td>
</tr>
<tr>
<td>Italy</td>
<td>16.4</td>
<td>16.0</td>
</tr>
<tr>
<td>Netherlands</td>
<td>7.9</td>
<td>N.A.</td>
</tr>
<tr>
<td>Sweden</td>
<td>10.3</td>
<td>N.A.</td>
</tr>
<tr>
<td>U.K.</td>
<td>16.1</td>
<td>2.8</td>
</tr>
<tr>
<td>2. Relatively Independent Central Banks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Japan</td>
<td>11.3</td>
<td>12.9</td>
</tr>
<tr>
<td>U.S.A.</td>
<td>8.0</td>
<td>2.7</td>
</tr>
<tr>
<td>3. Independent Central Banks</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Germany</td>
<td>4.7</td>
<td>4.8</td>
</tr>
<tr>
<td>Switzerland</td>
<td>4.0</td>
<td>N.A.</td>
</tr>
</tbody>
</table>

Notes:
The seigniorage rate equals the change in the monetary base divided by total government revenue from all sources. From Stanley Fischer (1982) pp. 308-309.
Lawrence Kenny (1983) presents a cross sectional study of the determinants of inflation rates based on a model of optimal tax composition. The model includes an income tax, which is costly to collect, and inflation resulting from monetary base expansion. Inflation is costly in that it diminishes households' utilities. It is expected that factors which make income (and other) taxes easier to collect will reduce a government's reliance on inflationary finance while factors which make it easier to tax money holdings will lead to an increased use of inflation as a means of gathering revenue. Kenny views inflation as a means of redistribution within the population in addition to a means of governmental finance. Kenny assumes the direction of redistribution is from the poor towards the rich. As such, it is predicted that inflation rates will tend to be lower in socialist countries concerned with even distribution of wealth and higher in dictatorships which tend to redistribute in favour of the wealthy.15

Kenny's results conform in large part to the predictions made, but there are notable exceptions. Recorded employment is expected to have a negative impact on inflation since other taxes should be easier to collect from those on record as being

14 Kenny notes that there are other sources of inflation such as changes in the money supply multiplier or the velocity of circulation, but does not pursue this point.

15 Kenny does not provide evidence that those are in fact the motives of such governments.
employed as opposed to those who are underground. The coefficient for this variable turns out to be positive and insignificant. Kenny's results also show that use of a country's currency abroad has no significant effect on the inflation rate and that the variable for the severity of war used, battle deaths to population, has only a marginally significant impact. Both results are surprising. Income growth, which is predicted to increase the demand for real balances (the tax base) and thus the use of inflationary finance, does not appear to have a significant effect.

Kenny's results did indicate that other variables related to the tax bases (recorded income and real balances held) and to the redistribution aspect of inflationary finance matter. Variables for trade and tourism which are used to capture the openness of the economy appear to have negative impacts on the inflation rate. Recorded per capita GDP also has a negative impact. It appears that if it is less costly to collect explicit taxes less inflationary finance will be used. Factors which decrease average real balance holdings such as population density (a proxy for the proximity of people and their banks) has the expected negative effect. The proportion of the population living in urban areas, which Kenny argues leads to greater "...anonymity, specialization, (and) greater money holdings and thus to higher inflation," has a positive impact on the inflation rate. Basically, Kenny's results indicate that nondemocratic countries which he considers to favour transfers
of wealth to the rich inflate relatively more than average while socialist countries inflate relatively less.

Kenny also found that inflation rates rose sharply after the Bretton Woods system was abandoned in 1971, but he concludes that the move away from pegging currencies to the U.S. dollar alone does not explain all of the increase in inflation rates.

Kenny's empirical work focuses on the rate on inflation as an indicator of the use on inflationary finance as opposed to the flow of revenue into the government from monetary base expansion. As noted earlier in the discussion of Fischer's (1981) work, the two measures do not always correspond. It would be desirable to isolate factors which would determine when and how heavily governments would seek revenue from monetary expansion. Inflation rates reveal to some extent that governments have used monetary expansion in the past or are expected to do so in the future.
Debt as a Tax

Tests of Debt-Tax Equivalence

In recent times much has been said of the deficits of the United States, Canada and other countries. Some, such as Martin Feldstein and Robert McNamara, with impressive academic and governmental credentials warn of the effect deficits will have on current and future generations. That deficits crowd out current investment and impoverish future generations appears to be the consensus of the business world as a whole. Traditional conservatives, Keynesians and even the lefoot have been critical of the deficits being run in many jurisdictions. Yet there is little or no evidence to support such cries of alarm. The phenomenon of crowding out found its place in the macroeconomics syllabus in the years before large-scale empirical investigation was feasible and remains a notion that many accept but few have ever attempted to evaluate. Pointing out the lack of empirical evidence to support the contention that debt crowds out investment and places a special burden on future generations in a way which is fundamentally different from the levying of increased present taxes, Milton Friedman suggests that, "Unexamined repetition works wonders."  

Consider a period in which a government lowers taxes while holding the total volume of government expenditures constant.

---

16 Time, March 5, 1984 pp. 54-61. Nowhere in the article is the distinction between real and nominal deficits drawn.

The shortfall in current revenue is made up through the increased sale of government debt. A rational public, according to the tax discounting hypothesis, if it correctly perceives the future taxes implied by the bonds issued, would increase savings in anticipation of those taxes by using the current increase in disposable income to buy the new, additional government debt. The implication is that total consumption would not change and suggests that a test of the debt-tax equivalence (future-tax discounting) hypothesis is possible by examining the effect of deficits on consumption. According to the tax-discounting hypothesis, the increase in savings should match the deficit and interest rates should be unaffected. This suggests a further means of testing the hypothesis. Of the three studies surveyed below, the first examines the effects of deficits on consumption, the second examines the effects of deficits on interest rates and the third study indicates that a strong intergenerational bequest motive exists. The existence of such a motive adds support to Barro's tax-discounting hypothesis since current generations may well be motivated to offset increased future tax burdens placed on future generations by deficits through increased current savings.

**Kochin**

Levis Kochin (1974) includes the federal deficit and change in the monetary base in a simple formulation of the aggregate consumption function. Assuming that consumption depends on permanent disposable income and that permanent disposable income
is calculated by taking a weighted average of past incomes where the weights are assumed to decline at a constant percentage rate, the Koyck transformation results in a consumption function of the form,

\[ C = a + bY_d + c_1C_{-1} , \]

where \( C \) is consumption and \( Y_d \) is disposable income.\(^8\) To this equation Kochin adds the current value of the federal deficit which will be denoted by Deficit. The function estimated is

\[ C = a + bY_d + d\text{Deficit} + c_1C_{-1} . \]

The discounting hypothesis predicts that \( d \) will be zero and thus a \( t \)-test is performed on \( d \) to determine if it is significantly different from zero. Kochin's results are:

\[
\begin{align*}
C &= 5.56 + 2.83Y_d - 0.224\text{Deficit} + 0.643C_{-1} , \\
&\quad (1.81) \quad (3.79) \quad (2.56) \quad (5.12)
\end{align*}
\]

\[ R^2=0.9989 \quad \text{S.E.}=2.23 \quad \text{D.W.}=0.68 \quad \text{Period 1952 to 1971} \]

Standard errors in parentheses.

The coefficient in question turns out to be negative but insignificant indicating that the tax discounting hypothesis cannot be rejected. Kochin then adds the change in the monetary base as an additional regressor. As changes in the base are hypothesized to be simply another form of taxation, its coefficient is expected to be zero given a constant deficit. This coefficient turns out to be positive but insignificant.

\[ \text{------------------} \]

\[ ^8 \text{For example, let } C=a'+bY_d+c_1bY_d_{-1}+c_1^2bY_d_{-2}+c_1^3bY_d_{-3}+\ldots, \text{ be the permanent income based consumption function. The Koyck transformation (subtracting } c_1C \text{ from both sides and simplifying) yields, } C=(1-c_1)a'+bY_d +c_1C_{-1}. \text{ Except where lags or leads are indicated, time subscripts will not be written.} \]
There are some points that must be considered before Kochin's results can be interpreted as being consistent with or, more properly, not inconsistent with the tax discounting hypothesis. First of all, the variables used are all nominal. This presents difficulties since while the national accounts calculation of the real deficit is simply the nominal deficit deflated by a price index, the economically meaningful calculation of the real deficit would be the change in the real value of outstanding government debt. With inflation or deflation the two calculations will not be the same nor need they change in the same direction.\(^{19}\) If we are concerned with real variables, Kochin's work would have to be redone using the economic version of the real deficit. A second problem is that the inclusion of a lagged dependent variable raises the possibility of a potentially difficult estimation problem.\(^{20}\) Kochin does not calculate a Durbin-H statistic or analyze the residuals from his regressions in order to test for the possible existence of autocorrelation. Kochin, on the basis of poor Durbin-Watson statistics, estimates the equations in first-difference form, but this is not a generally successful method of circumventing estimation problems.\(^{21}\) Thirdly, the

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\(^{19}\) See Robert Barro (1984a) for a comparison of the two versions of the real deficit.

\(^{20}\) Specifically, the Durbin-Watson statistic may be biased towards two if there is autocorrelation among the residuals. This would make the statistic unreliable. In addition, the existence of autocorrelation would bias the coefficient estimates and render the t-statistics unreliable.

\(^{21}\) Kochin did not difference the change in the base variable. No reason is given in the paper.
change in the monetary base variable is later included along with the deficit variable in order to capture the effect of an increase in the base given the size of the total deficit. Kochin's equation is
\[ C = a + bY + d\text{Deficit} + e\text{Base} + c_1C, \]

However, the change in the base represents the monetized portion of the deficit and this portion of the deficit now appears twice in the equation. The equation actually being run is
\[ C = a + bY + d(\text{Def}^* + \Delta\text{Base}) + e\Delta\text{Base} + c_1C, \]
or
\[ C = a + bY + d'\text{Def}^* + d'\Delta\text{Base} + e\Delta\text{Base} + c_1C, \]
where Def* is the nonmonetized federal deficit. It would appear that the coefficient of the change in base variable, e, is biased downwards.

Yawitz and Meyer (1976) criticize Kochin for misspecifying the test equation. They note that the value of outstanding government debt may change due to changes in the interest rate or because of open market operations. The change in the market value of outstanding government debt is a more meaningful statistic than the budgetary deficit. Yawitz and Meyer contend that Kochin errs in using the budgetary deficit as a regressor rather than using separate variables for the stocks of government debt and private net wealth. Redoing Kochin's equation with private wealth (A) included yields the result
\[ C = 12.79 + 0.213Y + 0.0265A + 0.0124\text{Deficit} + 0.544C, \]
\[ (2.65) (3.33) (1.98) (0.107) (4.08) \]
Further, Yawitz and Meyer generate an estimate of the market value of the outstanding U.S. Government debt. The equation

$$C = 0.75 + 0.03A + 0.05\text{Debt}$$

(17.31) (3.60) (3.88)

does not provide evidence for the tax-discounting hypothesis. Rather it appears as though debt is treated as a form of net wealth. The authors caution that since the value of the debt did not vary greatly over the test period, the results should not be taken as definitive.

Precisely why using stocks of wealth and not deficits (changes in the stock of government debt) is correct is not elaborated upon by Yawitz and Meyer. Their equation contains both stocks and flows as explanatory variables. The magnitude of additional tax liabilities is the value of the deficit over a period of time.

Evans

A cliometric examination of the effect of deficits on U.S. interest rates has been conducted by Paul Evans (1985). Three periods in U.S. history characterize by war and large federal deficits were studied. The periods were 1858-69 (spanning the Civil War), 1814-1920 (spanning World War I) and 1938-50
For the Civil War and World War I periods, Evans ran regressions of the form:

$$I_t = B_0 + B_1 GR_t + B_2 DR_t + B_3 MR_t + \eta_t$$

where

$I$ is alternatively the commercial paper rate (CPR), the railroad bond rate (RRBR) and the ex post real commercial paper rate. For the Civil War period, the New England municipal bond rate was also used;

$GR$ is the ratio of Federal Government expenditure to trend real national income;

$DR$ is the ratio of federal deficits to trend real national income;

$MR$ is the ratio of paper currency in circulation to trend real national income. This is designed to capture monetization of the deficits; and

$\eta$ is a random error term.

In every case the level of real federal spending has a positive effect on interest rates and in most regressions the coefficient $B_1$ is significant.\(^{22}\) The coefficient $B_3$ was negative and significant. These two results are consistent with the IS-LM framework and the optimal government finance framework.

\(^{22}\) For the World War I regressions the exogenous variables were included in the form of distributed lags with lags running from zero to six periods. The nonlagged values of $B_1$ were positive as were the sums of the lagged values of $B_1$.\(^{22}\)
In every regression, the coefficient B2 has a negative sign and frequently the variable is significant. This leads Evans to remark that these results, "...provide strong evidence against the conventional paradigm....If anything, interest rates would have been lower, had the Union levied lower taxes, yielding a larger deficit."\textsuperscript{23}

In the case of the World War II period, the government imposed binding price controls, quantity rationing and pegged interest rates. As a result there was little movement in interest rates possible during the war. However, it was also the case that little movement in rates occurred before or in the years immediately following the war. The existence of pegging does render the type of equation described above inappropriate. Since there was a black market in ration tickets which used currency, Evans uses the demand for money as an indirect test of the effects of deficits. Deficits, Evans finds, are associated with declines in the demand for real balances and do not appear to lead to increased aggregate demand and increased demand for real balances.

After a study of the recent years 1979 to 1983, for which he does not find a positive relationship between deficits and interest rates, Evans concludes that the evidence is consistent with the tax discounting hypothesis. Increases in government indebtedness are met with increases in private savings in anticipation of future taxes which offsets any positive pressure

\textsuperscript{23} Evans, p. 72.
that deficits would place on interest rates. Evans notes that many macroeconomists are not willing to accept Barro's model because they do not accept the idea that the public can accurately foresee and discount the implied future tax liabilities and because they doubt the existence of a strong intergenerational bequest motive.\(^2^4\) Evans states,

The assumption of accurate foresight of future tax liabilities does indeed seem implausible. One should however, judge the utility of an assumption by its predictive and explanatory power and not by its realism. The phenomena detailed above are consistent with accurate foresight but not with the conventional paradigm (IS-LM).\(^2^5\)

Two points may be considered here. First, much of economics attempts to explain or predict complex behavior. Often the reasons for this behavior is not articulated by those engaged in it. Most if not all of the assumptions used to do this task can be criticized as being unrealistic, but they are necessary. Second, what appears to be realistic or unrealistic may itself reflect no more than previously held theories which are commonly held but seldom examined.\(^2^6\) A novel hypothesis will seem unrealistic until it becomes widely accepted.

As to the problem of whether an intergenerational bequest motive exists and is strong, Evans cites the work of Kotlikoff


\[^2^5\] Evans, p. 85

\[^2^6\] Crowding-out itself is an example of conventional wisdom that only recently has been examined. To some, the IS-LM model with its mixture of statics and dynamics and assumption of myopia seems very unrealistic.
and Summers (1981) which suggests that there are intergenerational bequests and that, in the U.S. at least, they are enormous in size. By definition, total life-time consumption equals total life-time income less bequests. Life-cycle wealth can be computed as savings accumulated to finance the remainder of an age group's life-time consumption. Using data for the period 1900-74, Kotlikoff and Summers estimate life-cycle wealth to be roughly one trillion dollars. Total wealth of the U.S. economy at the end of 1974 was estimated to be $4.154 trillion. About 75 percent of net wealth in the U.S. is estimated to consist of intergenerational transfers.

It would appear that in the U.S. there has been a strong bequest motive in effect. Even if the proportion of transfers to total net wealth is lower, the same suggestion holds for other countries. It appears that a bequest motive does exist.
Barro's Tests of Debt Determination

In his important article, Robert Barro (1979) built a model of rational public debt determination, derived predictions and presented empirical test results. Barro's testing involves the effects of transitory and permanent expenditure requirements on the level of the United States national debt. Barro's fundamental regression equation is,

$$\ln(B/B^*) = a_0 + a_1 \Pi + a_2 (P(G-G*)/B^*) + a_3 (\ln(Y/Y*))(PG + rB/B^*)$$

where

- $\Pi$ is the (adaptively) expected rate of inflation,
- $B$ is the nominal stock of publicly held debt on December 31,
- $B^* = \sqrt{BB_1}$ is the average debt outstanding during the year $t$,
- $\Pi$ is the average anticipated rate of inflation for year $t$ based on lagged values of monetary growth and the unemployment rate and on current interest rates and federal spending,
- $P$ is the average price level (GNP deflator),
- $G$ is real government expenditure,
- $G^*$ is normal government expenditure for year $t$ calculated by a distributed lag of expenditure with an adaption coefficient of 0.2 and modified to account for the long-term 5.6 per cent average growth of $G$,
- $Y$ is real aggregate income (real GNP),
- $Y^*$ is normal income for year $t$ calculated by use of a trend line and
- $r$ is the real interest rate calculated by subtracting the
anticipated rate of inflation from the nominal interest rate. In his estimations Barro used $B_*$, in place of $B^*$ since $B$ would have appeared on both sides of the equation.

The constant term, $a_0$, should reflect the growth rate of real income, $\Delta Y/Y\Delta t$, as long as income and government expenditures grow at the same rate. If the growth rate of government expenditures, $\Delta G/G\Delta t$, exceeds that of income, the constant term would be lowered by an amount proportional to the difference in the growth rates. The coefficient $a_1$ should equal unity according to Barro. Real factors would determine the the real magnitude of the debt and inflation, all else equal, should result in a proportional increase in the magnitude of the nominal debt so as to keep the real debt constant. The coefficient $a_2$ is expected to be positive and Barro anticipates a value between 0.8 and 1.0. Temporary positive (negative) deviations in government expenditures would have a positive (negative) effect on the quantity of debt outstanding. On the other hand, temporary positive (negative) deviations in income from trend would have a negative (positive) impact on the debt. Barro anticipated a coefficient value between -1.0 and -0.8.

Barro's results were revealing. Over twelve separate regressions for 1941-76, 1948-76, 1941-47, 1922-40, 1922-29 and 1930-40, with and without $a_1$ constrained to be one were run. The results were quite robust. The coefficient for the anticipated inflation rate variable was not significantly different from one at the 5% level of confidence. The transitory expenditure
coefficient, \( a_2 \), did turn out to be within the expected 0.8 to 1.0 range. The coefficient for deviations in real income, \( a_3 \), was significantly less than minus one in almost every regression run. "Hence," states Barro, "there is an indication that the magnitude of typical debt response has exceeded the amount that would be dictated purely from efficient public finance considerations."\(^{27}\)

In order to test the dual tax-discounting and debt as a buffer hypothesis on which his model is based, Barro added \( B.//(P_. Y_.) \) and \( PG*/B. \), to the basic equation above. When the first variable was added in order to test for an autoregressive tendency in the level of debt, its coefficient was not significantly different from zero indicating that the level of debt did not have a 'memory' but moves randomly with fluctuations in income and government expenditure. When the second variable, the ratio of average spending to average debt, was added to the equation its coefficient was not significantly different from zero. A significant coefficient would have been inconsistent with Barro's contention that changes in the level of debt are a means of buffering increases in the costs of collecting revenues in order to finance transitory expenditures.

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\(^{27}\) Barro (1979) p. 963.
Summary

There is evidence that governments use their control over the issuance of central bank money to generate revenue. Long-term, cross-sectional studies suggest that monetary finance is used throughout the world, but little work has been done concerning the short-term use of monetary finance. Empirical research has typically dealt with the rate of inflation or growth of the money supply (i.e. M1) rather than with the growth of central bank money. Since it is through expansion of central bank issued money that the government obtains a flow of revenues, it would be desirable to study monetary base behavior. It was been noted, particularly by Barro, that while the money supply (M1) is affected by government expenditure, it also moves countercyclically indicating that stabilization policy has been pursued. Whether or not the monetary base behaves in a like manner and whether or not the same holds for countries other than the U.S. remains to be discovered.

The empirical literature concerning the public debt contains a number of papers which support the hypothesis that government debt is treated as deferred taxes by the economy.\(^2^8\) Evidence consistent with the tax-discounting hypothesis, suggests that debt and deficits do not have special impacts upon aggregate demand and the interest rate above and beyond those of government expenditure and taxation. In particular, there is  

\(^2^8\) There are exceptions of course particularly in the work of Martin Feldstein. See Feldstein (1984) for example.
evidence that interest rates are affected by the total level of government expenditure and finance and not by the specific means of finance chosen. Like the work on monetary finance, most of the research in the area of debt and taxation has been limited to U.S. data.
CHAPTER III
A CROSS-SECTIONAL STUDY OF SEIGNIORAGE COLLECTION

Introduction

Empirical papers in the area of monetary finance typically deal with the determination of the inflation rate or the M1 definition of money rather than with the determination of the monetary base itself.¹ If there is a nonzero rate of economic growth or if the M1 multiplier is a function of the expected or actual rate of inflation, then the rate of inflation would not equal the seigniorage rate measured on a cash basis. It would be desirable therefore to study the behavior of the monetary base itself for it is the act of issuing new central bank money that results in (cash based) revenue flows to the government. In most countries a central bank exists as a government controlled monopoly which produces revenues for the state. The question at hand is what determines how much the state will use the bank as a tax collector.

The optimal government finance model yields predictions about changes in the use of seigniorage in a country over time as a response to changing conditions. The model predicts that seigniorage will increase with the level of trend government expenditure, will decrease if trend income is expected to rise and will increase if real income is temporarily above its trend

¹ See the survey of empirical work above.
Seigniorage collection will move in harmony with other taxes so as to maintain equal marginal collection costs between all forms of feasible taxation. The model does not provide detail on the determinants of the cost function however.

A number of factors have been suggested which will affect the government's use of monetary finance. Nichols (1974) proposes a number of features that would make inflationary finance less costly to collect in the sense that the resulting rate of inflation can be minimized. If the government restricts the existence of financial assets which act as a close substitute for money then a given rate of monetary expansion will yield more revenue. Nichols' first principle was stated, "The inflation that results from a given government deficit will be lower than otherwise if the availability of close substitutes for money is curtailed." Nichols' second principle is that, "The rate of inflation will be lower if the holding of foreign currency is prohibited." If the deadweight cost of seigniorage collection increases with the seigniorage rate and ultimately with the difference between the inflation rate and what it otherwise would be, then Nichols' principles are relevant to a

Nichols (1974) pp. 423-424. Bryant and Wallace propose that the government's ability to restrict the supply of small denomination, interest yielding financial instruments allows the government to effectively practice price discrimination. That is that government money is sold at a price which exceeds the competitive price. See Bryant, John and Neil Wallace, "A Price Discrimination Analysis of Monetary Policy," Review of Economic Studies Vol. 51, No. 165, April 1984, pp. 279-288. There are small denomination, interest bearing assets allowed in modern economies, namely bank deposits. These must be backed by reserves of central bank money however and as such do not escape the tax.
government deciding on the composition of the total tax levy. Nichols also notes that the ability of a government to sell interest bearing debt which is a good substitute for private financial assets reduces the government's reliance on monetary finance. Gordon (1975) makes essentially the same suggestion, but from the opposite perspective. According to Gordon if short-run revenue requirements cannot be met by the sale of bonds to the public, then a government is likely to use monetary finance. A government that lacks or is perceived to lack the ability to administer the issuance of debt will have to rely on current taxation including seigniorage. Gordon suggests that the short run costs of monetary finance are likely less than the costs of raising other tax rates. In particular, Gordon cites the financing of the war in Vietnam by the U.S. administration. It is proposed that the unpopularity of the war made the issue of debt or increased explicit taxation very costly and as a result monetary finance was relied upon more heavily than usual.

Some factors affecting the collection costs of seigniorage as opposed to the use of other taxes have been hypothesized by Gordon. In a more mobile society the collection of income and other taxes will be more costly. More complex markets which make use of record keeping the conduct of business will be more easily taxed in an explicit manner. Increased urbanization may be associated with more complex trading arrangements and more explicit taxation as a result. Kenny (1982) however suggests the
opposite. Kenny hypothesizes that increased urbanization leads to more anonymous and specialized trading and more cash holding. Thus, he expects to see a greater reliance on monetary finance in more urbanized countries since seigniorage taxes cash intensive trading effectively. It could be argued as effectively that if urbanization results in less personal familiarity among traders, there will be less underground trading. Agreements to deceive the tax collector will be harder to enforce in impersonal market places and if trade credit is granted there will be a greater need for government enforcement of these contracts resulting in a greater use of record keeping. It is suggested here that urbanized economies will be subject to proportionately less monetary taxation than the economies of less urbanized nations.

Gordon and Kenny both argue that war will lead to a higher rate of inflation. War may make the collection of some forms of taxation relatively more costly due to social disruption and the increased opportunity cost of tax collectors' time. Combined with an increased need for revenues the effect is to increase the reliance on seigniorage as a means of taxation. Kenny suggests that the form of government is also important because the inflation tax is regressive and as such less appealing to democratically elected governments.3

3 Earl Thompson (1981) argues quite the opposite in that he regards inflationary finance as a tax which affects the wealthy relatively more than the less wealthy. As a redistribution device, he favours the use of monetary expansion. See Thompson, Earl "Who Should Control the Money Supply", American Economic Review Vol. 71, No. 2, March 1977, pp. 356-361.
Other factors affecting the reliance on seigniorage may also be considered. A dictatorial government might be considered as being one which is more costly to displace. As such it would recognize smaller costs from inflationary finance. It is hypothesized here that the reliance on seigniorage will be lower in industrialized nations due to greater use of trade credits and associated record keeping. It is hypothesized here that the reliance on seigniorage will be lower in countries where fewer goods and services are prohibited by law since such prohibitions lead to the establishment of cash intensive underground markets. Finally, it is suggested that since children in most economies perform income generating work for their families and that such income is often nonmonetary and nonmarket or is frequently "off the books", that nations with proportionally more children will rely more heavily on inflationary finance.

The Equations

Two questions may be asked. First, what determines how much a country will use monetary finance absolutely and second, what determines the reliance on monetary finance relative to other forms of revenue gathering? Two cross-sectional regression equations are used:

\[ \Delta Base/GNP = b_0 + b_1(G/GNP) + b_2(X/GNP) + b_3(Growth) + b_4(GNPgrowth) + b_5(Pop15) + b_6(Urban) + b_7(Industry) + b_8(Politics) + b_9(GNP/Pop) + b_{10}(Pegged) \]

and
(2) $\Delta \text{Base}/G = b_0 + b_1 (G/\text{GNP}) + b_2 (X/\text{GNP}) + b_3 \text{Growth} + b_4 \text{GNPgrowth} + b_5 \text{Pop15} + b_6 \text{Urban} + b_7 \text{Industry} + b_8 \text{Politics} + b_9 \text{GNP/Pop} + b_{10} \text{Pegged}$. 

The variables and their expected signs are as follows:

$\Delta \text{Base}/\text{GNP}$ is the change in the monetary base divided by the gross national product.

$\Delta \text{Base}/G$ is the change in the monetary base divided by government revenues which must equal the total amount of cash revenues collected.

$G/\text{GNP}$ is the value of government expenditure divided by the GNP. In Equation (1) this variable is expected to have a positive sign since higher levels of expenditure will require a higher level of revenue collection including the use of seigniorage. In Equation (2) the effect is ambiguous.

$X/\text{GNP}$ is the level of exports relative to GNP which is designed to serve as a proxy for the degree of openness of the country. It is expected to have a negative effect on the use of seigniorage because international goods flows are more easily monitored and taxed by the state.

$\text{Growth}$ is the five-year growth rate of real government expenditure. Data availability set a limit of five years on the calculation of this variable. The model predicts that a higher rate of growth of trend expenditure will lead to higher levels of current taxation including the use of seigniorage. In Equation (1) the variable is expected to have a positive effect while in Equation (2) the effect is ambiguous.

$\text{GNPgrowth}$ is the five-year growth rate of real gross national
product. The optimal government finance model predicts that growth in trend income would lead to a deferral of taxation through the increased issuance of debt. A negative sign for this variable is expected in Equation (1) while in the case of Equation (2) the effect is ambiguous.²

Pop15 is the proportion of the population aged fifteen years or less. It is predicted to have a positive effect in both equations.

Urban is the proportion of the population living in United Nations defined urban areas. It is predicted to have a negative effect on the use of seigniorage and thus to have a negative sign in both equations.

Industry is a dummy variable for industrial or manufacturing based economies as classified by World in Figures. Industry is equal to 1 if the country is considered to be industrialized. It is expected to have a negative sign in both equations.

Politics is a dummy variable for the existence of a democratic (plural party) republic or parliamentary system of government. Following Kenny, it is expected to have a negative effect.

GNP/Pop is the level of GNP per capita in thousands of U.S. dollars in 1979. Following Kenny it is expected to have a positive effect on the use of seigniorage since relatively higher levels of income results in a higher demand for real balances.

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² Following an argument made by Kenny, monetary finance may be made less costly to the state if monetary finance is a regressive form of taxation and growth reduces the impact of such a regressive tax. If this is so then this variable should have a negative effect in Equation (2).
Pegged is a dummy for countries which, according to the International Monetary fund have pegged exchange rate regimes. Its expected sign is ambiguous.
### Table 3.1: Cross-Sectional Results

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<th>Variable</th>
<th>Value</th>
<th>t Statistic</th>
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</thead>
<tbody>
<tr>
<td>Constant</td>
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<td>-0.83507</td>
</tr>
<tr>
<td>G/GNP</td>
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<tr>
<td>X/GNP</td>
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<td>Ggrowth</td>
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<td>GNPgrowth</td>
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<tr>
<td>GNP/Pop</td>
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<tr>
<td>Pegged</td>
<td>0.00825</td>
<td>0.57068</td>
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$R^2=0.42043$

$F(10/46)=3.337$

### Equation (1) Dependant variable $\Delta$Base/GNP

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<th>Variable</th>
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<tr>
<td>Pegged</td>
<td>0.00813</td>
<td>0.15941</td>
</tr>
</tbody>
</table>

$R^2=0.36678$

$F(10/46)=2.664$

**Notes:** The series $\Delta$BASE/GNP, $\Delta$BASE/G and G/GNP are averages for the years 1977, 78 and 79. Government financial statistics are not very reliable. It is possible that errors and omissions in one year's figures are compensated for in the next year. Averaging was used to overcome such problems. Other variables...
are for the year 1979. Data for the monetary base, government expenditure, gross national product, exports and price levels from *International Financial Statistics Yearbook* lines 14, 82 plus 83, 99a, 90c and 64 respectively. Population figures and dummies for industrial economy and political system from *World in Figures*. Urban/total population variable from *World Statistics in Brief*.

Countries included are: Argentina, Australia, Austria, The Bahamas, Belgium, Botswana, Brazil, Cameroon, Canada, Chile, Columbia, Cyprus, The Dominican Republic, Ecuador, El Salvador, Fiji, Finland, France, (West) Germany, Greece, Guatemala, Honduras, Iceland, India, Indonesia, Ireland, Isreal, Italy, Jamaica, Japan, Jordan, Kenya, Korea, Kuwait, Malawi, Malaysia, Mauritius, Mexico, The Netherlands, Oman, Pakistan, Paraguay, Peru, The Philippines, Singapore, South Africa, Spain, Sri Lanka, Sweden, Switzerland, Tanzania, Thailand, The United Kindom, The United States, Uruguay, Venezuela and Zambia.
The results of Equation (1) are basically consistent with the optimal government finance model. Higher levels of government spending are associated with higher levels of monetary expansion though the coefficient is quite small reflecting the fact that in most countries seigniorage is a relatively small part of the entire tax collection. Openness of the economy has a negative but very insignificant impact. Growth of real expenditure has the predicted positive effect while growth of real income has a negative effect. The proportion of the population under sixteen years of age variable has the expected positive sign while the dummy for a democratic government has the expected negative sign. Per capita income has a positive effect on the use of seigniorage. The pegged exchange rate variable has an insignificant coefficient.

The results of Equation (2) are also basically consistent with the optimal government finance model. Growth in expenditure increases the current use of seigniorage while growth in income reduces it. The larger the proportion of the population under the age of sixteen, the more the nations in the sample tend to rely on monetary finance. The Industry and Politics dummy variables have expected negative coefficients. The other variables are very insignificant. The X/GNP variable in particular does appear significant in this equation.
Conclusions

The results above are consistent with the hypothesis that governments use monetary expansion as a means of finance in a manner predicted by the optimal government finance model. However, given concern about the accuracy of some of the data and the low levels of significance of many of the variables, strong statements about the factors determining the degree to which a government will rely on monetary as opposed to other means of finance really cannot be made. It does appear that democratic, industrialized and growing nations rely less heavily on seigniorage than other nations in the sample. Countries characterized by relatively high rates of growth in government spending and higher levels of per capita income appear to use monetary finance relatively more.

As is the case with all such work, this research must be taken as being cursory and explorative and certainly not definitive. Future work would benefit from a finer, more reliable data base and, above all, a more developed model of the determinants of the taxation collection cost function.

There are exceptions of course. Iceland, the oldest parliamentary democracy in existence, relies very heavily on inflationary finance. Germany also relies relatively heavily on monetary expansion as a source of revenue.
CHAPTER IV
MONEY, TAXES AND DEBT: AN EMPIRICAL INVESTIGATION

Introduction

In a model of optimal government finance, government attempts to finance current and expected future expenditures at the lowest possible present value of collection costs. The intertemporal budget constraint is:

\[ \int_0^\infty g(t)e^{-rt}dt = \int_0^\infty (p(t) + s(t))e^{-rt}dt \]

where \( g(t) \), \( p(t) \) and \( s(t) \) are the expected levels of real expenditure, conventional (nonmonetary) taxation and seigniorage respectively. If the cost of collecting revenues are monotonically inversely related to the level of real income \( y \), then \( p \) and \( s \) are monotonically positively related to \( y \). Thus,

\[ \int_0^\infty g(t)e^{-rt}dt = \int_0^\infty f(y(t))e^{-rt}dt. \]

In the absence of transitory shocks, the taxation rates \( (p(t)/y(t)) \) and \( (s(t)/y(t)) \) should be equal to long-run rates \( p*/y* \) and \( s*/y* \). Deviations of real income from trend values would be largely absorbed by deficits if the collection cost function is convex. Similarly, deviations of government expenditures from trend would be largely absorbed by deficits. Current taxation rates would increase or decrease only slightly by an amount necessary to amortize the deficit or surplus. Deficits would be positively related to recessions and transitory increases in expenditure. Taxation would be positively related to increases in income and permanent
increases in expenditure.

Barro (1979, 1984b) chose not to separate conventional taxation from seigniorage collection. He states,

I do not separate out the revenue from money creation from the government's other revenues. Rather I think of inflationary finance as a tax on the holding of money.... Then in order to focus on taxes in one period versus those in another, I combine the inflation tax with the variety of other levies (on income, sales, property, etc.) that apply at the same date. In particular, there seems to be no reason to give special treatment to the inflation tax.¹

There may be reasons which make it desirable to separate monetary from other forms of taxation. In particular it may be useful to separate seigniorage from conventional taxation. This allows us to evaluate the optimal government finance model as opposed to a more traditional stabilization model. The two models predict the same behavior for conventional taxation and deficits, but predict different behavior for the monetary base. The stabilization model predicts that in a recession, government will increase the money supply. The optimal government finance model predicts that the monetary base will not be increased as much as normal in a recession. If the money supply is a positive and stable function of the monetary base, a comparative test of models is suggested.²


² Manipulation of the money supply would involve central bank open market operations or monetization of new debt if conducted over a long enough period of time.
The Optimal Government Finance Model

The optimal government finance model suggests that the use of the three forms of finance depends upon trends in real income and government expenditure and upon deviations from those trends. In general form:

\[ s^* = s(g^*, y^*) \]
\[ p^* = p(g^*, y^*) \]
\[ d^* = d(g^*, y^*) \]

and

\[ s - s^* = s_1(g - g^*) + s_2(y - y^*), \frac{\partial s_1}{\partial g} > 0, \frac{\partial s_2}{\partial y} > 0 \]
\[ p - p^* = p_1(g - g^*) + p_2(y - y^*), \frac{\partial p_1}{\partial g} > 0, \frac{\partial p_2}{\partial y} > 0 \]
\[ b = d - d_1 = d_1(g - g^*) + d_2(y - y^*), \frac{\partial d_1}{\partial g} > 0, \frac{\partial d_2}{\partial y} < 0. \]

A stabilization model is represented as:

\[ s - s^* = s_1(y - y^*), \frac{\partial s_1}{\partial y} < 0 \]
\[ p - p^* = p_1(y - y^*), \frac{\partial p_1}{\partial y} > 0 \]
\[ b = d - d_1 = d_1(y - y^*), \frac{\partial d_1}{\partial y} < 0 \]
\[ g - g^* = g_1(y - y^*), \frac{\partial g_1}{\partial y} < 0. \]

The sign of \( \frac{\partial s^*}{\partial y} \) provides a means of discriminating between the two models.

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3 Except where lags are indicated or where confusion might arise, time subscripts will be suppressed. A subscript -1, for example, should be interpreted as \( t-1 \) where \( t \) is the current time period.
Specification

Growth will affect levels of taxation, seigniorage collection and debt issue. Since $p/y$ and $s/y$ should remain constant in the absence of transitory shocks to income or expenditure, $(p+s)/y$ will also be constant if the government wishes to collect revenues at the lowest present value of collection costs. Let income grow at a rate $x$ and government expenditures, including interest payments, grow at a rate $w$. The budget constraint,

$$(p+s)/y_0 \int_{0}^{\infty} y_0 e^{(x-r)t} \, dt = \int_{0}^{\infty} g_0 e^{(w-r)t} \, dt,$$

must hold. Thus,

$$(p+s)/y_0 = (g_0/y_0)((x-r)/(w-r)).$$

for a point in time $t$, while

$p+s = (p+s)/y_0 y = g_0 ((x-r)/(w-r)) e^{xt},$

indicating that taxation, including seigniorage, will grow over time at the same rate as income. Growth in expenditure does not affect the growth rate of taxation, but does affect the initial and subsequent levels of taxation. The effect of growth on the level of debt or size of the deficits (change in real indebtedness) is more obscure. If income and expenditure are both growing at the same rate $x$, then the deficit will also grow at the rate $x$ if there is no outstanding debt in the beginning. If income and expenditure are not growing at the same rate, the growth in the deficit is a function of the excess of the rate of

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a For most countries separate data on interest payments are not available. Including interest payments in expenditures is convenient and for most countries an empirical necessity.
income growth over that of expenditure.\(^5\)

The equation for the change in the real debt may be derived in a manner similar to that in Barro (1984b). From the budget constraint above the real deficit at time \(t\) is given by,
\[ b = g - (p + s) \]
or
\[ b = g - \left( \frac{g_0}{y_0} \right) \left( \frac{(x - r)}{(w - r)} \right) y. \]
If \(x = w = 0\),
\[ b = g - \left( \frac{g^*}{y^*} \right) y, \]
where \(g\) and denote trend (or in Barro's terminology "normal") levels of \(g\) and \(y\). The above can be rewritten to get,
\[ b = g + g^* - g^* - \left( \frac{y}{y^*} \right) g^*, \]
or
\[ b = (g - g^*) + \left( \frac{(y - y^*)}{y^*} \right) g^*. \]
If \(x\) and \(w\) do not both equal zero a growth term would have to be added. Since it is difficult to specify, it is assumed that a term \(n d\), where \(n\) is a constant, can be added to the equation to capture the effect of growth on the level of debt and on the deficit. Thus we have,
\[ b = (g - g^*) + \left( \frac{(y - y^*)}{y^*} \right) g^* + nd. \]
With subscripts suppressed this may be rewritten as,
\[ b/d.1 = n + (g - g^*)/d.1 - \left( \frac{(y - y^*)}{y^*} \right) (g^*/d.1). \]

\(^5\) From the optimal finance model,
\[ b = -x/v_1(v_2 xy + v_3 w^2 g) \]
so \(b = -x/v_1(v_2 x^2 y + v_3 w^2 g). \) The growth in \(b\) is given by,
\[ b/b = (v_2 x^2 y + v_3 w^2 g)/(v_2 x y + v_3 w g). \] If \(v_2 = -v_3\), (that is the collection costs depend on the ratios \(p/y\) and \(s/y\) only) and \(x = w\), then \(b/b = x. \) If \(x > w\), then \(b/b = (x^2 y - w^2 g)/(x y - w g). \) Since \(x > w\) and \(y > g, \) \(b/b\) would be positive. The rate of growth of the debt depends on its initial value.
This last equation is very similar to that used by Barro (1984b, 1985) and is similar to the equation used in Barro (1979). 6

The taxation equations are more cumbersome. Consider seigniorage. The trend rate $s*/y*$ planned at this point in time should be constant over time. Departures from the planned rate $s*/y*$ would come about only because of transitory revenue needs. Thus, $s=(s*/y*)y$ with $s*=f(g*,y*)$. Now consider shocks. Then,

$$ds=(s-s*)=((\partial s/\partial g)dg+(\partial s/\partial y)dy+(s*/y*)dy.$$ 

Considering that the trend value of taxation, including seigniorage, will grow at the rate of growth of trend income, the above may be amended and rewritten to yield,

$$ds=ws_1+((\partial s/\partial g)dg+(\partial s/\partial y+s*/y*)dy.$$ 

Expressing $dg$ as $g-g*$ and $dy$ as $y-y*$, we have,

$$ds/s_1=w+((\partial s/\partial g)(g-g*)/s_1+((\partial s/\partial y+s/y)(y-y*)/s_1.$$ 

A similar argument would exist for other forms of taxation. Thus the equations used in this evaluation are,

1. $\Delta s/s_1=b_0+b_1(g-g*)/s_1+b_2(y-y*)/s_1+\epsilon$,
2. $\Delta p/p_1=b_0+b_1(g-g*)/p_1+b_2(y-y*)/p_1+\epsilon$,
3. $\Delta d/d_1=b_0+b_1(g-g*)/d_1+b_2(y-y*)(g*/y*)/d_1+\epsilon$.

Since the constant should capture the effects of income growth, the variance of the residuals should not grow over time.

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6 Barro's equation is

$$\Delta d/d_1=(g-g*)/d_1-((y-y*)/y*)(g/d_1+(r-n))+n+\epsilon$$

where $n$ is the growth rate of government expenditure and income and $g$ is net of interest payments on debt.

As noted in the appendix to Chapter 1, using the definition $b=\Delta d=\Delta (D/P)$, where $D$ is the nominal value of outstanding debt and $P$ is the price level, avoids the necessity of including expected inflation as an explanatory variable.
and these equations should not exhibit heteroskedasticity. As a check on the specifications, regressions excluding the scaling factors (e.g. s.) were run and the residuals examined. The residuals showed evidence of positive heteroskedasticity.\footnote{Such specifications are sometimes criticized as being prone to spurious correlation due to the scaling factor. Maddala (1977) argues that this is not necessarily a valid criticism. See Maddala (1977) Chapter 12.}
Estimation

The advantage of the above specifications is that trend growth rate effects are captured by the constant terms if scaling variables \((s, p, \text{ and } d)\) are used. However trend values for income and expenditure must be estimated. This was done using the lag scheme,

\[ y^* = (1+\gamma)((1-\lambda)y_{-1}+\lambda y^*_{-1}) \]

with income for example. This reduces to,

\[ y^* = (1+\gamma)(1-\lambda) \sum_{i=1}^{4} ((1+\gamma)\lambda)^{i-1} y_{-i} \],

where \(\gamma\) is a long term average rate of growth and \(\lambda\) is an adjustment factor. In practice, the series was truncated after four lags to preserve observations and \(\lambda\) was varied from 0.1 to 0.9. The trend series that best mapped the actual series was used. Sensitivity analysis revealed that the results do not depend critically on the choice of \(\lambda\). This procedure was applied both to income and to real government expenditure.
Results

U.S.A.

Results using annual data for the U.S. are presented below in Table 4.1. Seigniorage is measured by the increase in government bonds held by the Federal Reserve deflated by the CPI while tax revenues are the collections reported by the Internal Revenue Service deflated. Debt refers to the deflated value of publicly held U.S. government debt. Thus the equations deal with the financing of the Federal government. Due to the accounting practices used in recording the data and the unconventional definition of real deficit chosen, the sum of seigniorage, conventional taxation and deficit is not expected to sum to the value of expenditure in practice. For this reason the regressions were estimated unconstrained.\(^8\) The results are basically consistent with the optimal finance model with the exception of the coefficient for the critical GNP shock variable in the seigniorage equation. There is evidence in support of the model which has the base moving in a countercyclical fashion in an attempt to stabilize income. The tax revenue and debt equations are not strongly significant, but do accord with the finance model. As an experiment, the growth of trend government expenditure was added as an additional explanatory variable to test the hypothesis that growth in government affects current

\(^8\) The regressions were also run with a one year lag on the income variable in order to reduce the possibility of multicollinearity. The results obtained were very similar to those reported in Table 4.1.
taxation and deficits. The variable was insignificant in both cases.
Table 4.1: U.S. Results

Period 1948 to 1983

(1) $\Delta s/s_1 = -0.540 + 0.176(g-g^*)/s_1 - 0.024(y-y^*)/s_1$

$\bar{R}^2=0.92219$
$F(2/33)=195.56$
$St. Error=1.7571$
$D.W.=1.89$
$Rho=0.2944$

(2) $\Delta p/p_1 = 0.050 + 0.260(g-g^*)/p_1 + 0.081(y-y^*)/p_1$

$\bar{R}^2=0.27516$
$F(2/33)=6.264$
$St. Error=0.0673$
$D.W.=2.03$
$Rho=-0.1587$

(3) $\Delta d/d_1 = 0.010 + 0.807(g-g^*)/d_1 - 0.003(y-y^*)(y^*/g^*)/d_1$

$\bar{R}^2=0.28748$
$F(2/33)=6.637$
$St. Error=0.0528$
$D.W.=1.61$
$Rho=0.5167$

Notes: Regressions were done using GLS to correct for first degree autocorrelation. Results for OLS were quite similar. Residuals for regressions which did not include scaling variables did exhibit evidence of heteroskedasticity. $T$-statistics in parentheses. Units are billions of 1958 Dollars.

Data Sources:
All Series from Historical Statistics of the United States updated with some series in Statistical Abstract of the U.S. Series numbers are: $s$ change in $Y_{49Z}$, $g$ $Y_{457}$, $y$ $F_{47}$, $p$ $Y_{358}$, $d$ $Y_{49Z}$. Series deflated by GNP deflator, series $F_{47}$.
The constant in the equations should reflect the long-run growth trend of income as described above. In equations (2) and (3), the constants are positive though in equation (3) the coefficient is not significant. Unexpectedly, the constant in equation (1) is negative although it too is insignificant. To some extent there seems to have been a trend away from monetary finance towards other forms of taxation. This could be due to changing collection cost functions, but the model developed earlier does not explain the determinants of these functions. The reaction to expenditure shocks is as expected. Ideally, if the data reflected the theoretical concepts of seigniorage, debt and other taxes used, then the expenditure shock variable coefficients should sum to one. In fact they sum to about 1.3, which given the nature of the data can be considered quite close to one. This adds some increased credibility to the results obtained. According to the results, an expenditure shock is financed by increased seigniorage and conventional taxation to some extent. However, as predicted by the optimal government finance model, most of the short-run financing is done with debt sales to the public.

Less can be said about the reaction to income shocks. The coefficients for the income shock variables are not expected to sum to a particular number since no budget constraint applies. Transitory income increases result in the predicted increase in conventional taxation and decrease in real government indebtedness although the coefficient in equation (3) is
significant only at low levels of confidence. As noted, a positive income shock decreases the amount of seigniorage collected contrary to the predictions of the optimal government finance model. The coefficient is small in the sense that a billion Dollar transitory increase in real income results in only a $24 million decrease in seigniorage collection, but the coefficient is statistically highly significant. This is consistent with the stabilization model. In the case of the U.S., the stabilization motive cannot be rejected while the results are strictly inconsistent with the optimal finance model.

9 There are of course any number of ex post explanations for the result which may fit inside the optimal finance framework. These will not be pursued here.
Canada

Results using annual data for Canada are presented below. The data series used corresponds as closely as possible to those used for the U.S. Not surprisingly, the results are similar to those obtained for the U.S. The results are given in Table 4.2 below.

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10 As in the case of the U.S., lagging the income variable did not result in significant changes in the results.
Table 4.2: Canadian Results

Period 1948 to 1983

(1) \( \Delta s/s_{i} = 0.917 + 0.328(g-g*)/s_{i} - 0.121(y-y*)/s_{i} \)
\[ (0.519) \quad (7.438) \quad (-9.832) \]

\( R^2 = 0.85895 \)
\( F(2/33) = 100.483 \)
St. Error = 11.2961
D.W. = 1.67
Rho = -0.0887

(2) \( \Delta p/p_{i} = 0.045 + 0.181(g-g*)/p_{i} + 0.177(y-y*)/p_{i} \)
\[ (4.592) \quad (2.715) \quad (3.154) \]

\( R^2 = 0.36130 \)
\( F(2/33) = 9.334 \)
St. Error = 0.0744
D.W. = 2.13
Rho = -0.2767

(3) \( \Delta d/d_{i} = -0.001 + 0.319(g-g*)/d_{i} - 2.035(y-y*)(y*/g*)/d_{i} \)
\[ (-0.131) \quad (1.915) \quad (-4.069) \]

\( R^2 = 0.39019 \)
\( F(2/32) = 10.238 \)
St. Error = 0.0632
D.W. = 1.84
Rho = -0.1375

Notes: Regressions were done using GLS to correct for first degree autocorrelation. Results using OLS were quite similar. T-statistics in parentheses. Units are millions of 1971 Canadian Dollars.

Data Sources:
Data from Historical Statistics of Canada. Series numbers are: s change in J59, g H34, y F55, p (Total Tax Revenue) H11, d H60 minus J59. Series deflated by GNP deflation K17Z.
The constant in equation (1) is extremely large in magnitude considering that it is supposed to reflect the growth rate of real income. The T-statistic however is small and the coefficient cannot be considered significantly different from zero at a reasonable level of confidence. The differences between the values of the constants in the three equations is disturbing. Only in the case of equation (2) does the size of the constant seem reasonable.

The coefficients of the expenditure shock terms in the three equations sum to about 0.828 which, given the nature of the data, is considered to be close to one. What is striking about the response to expenditure shocks is that in Canada there appears to have been more reliance on monetary finance for short-run revenue needs than in the U.S.A. Along with this there also seems to have been less use of debt sales to the public in Canada.\footnote{There are two common arguments connected with this observation. One hypothesis is that, due to the constitutional form of government in the United States with its division of powers. An administration has to achieve the approval of the Congress and Senate in order to increase taxes or float large quantities of bonds. As in Gordon (1975), it is argued that this leads the administration to rely heavily on the Federal Reserve as a short-run revenue collector. In a parliamentary system these divisions of power do not exist. The second argument is that the Federal Reserve in the U.S. is more independent than an institution like the Bank of Canada which is owned and controlled by the spending authorities. According to this argument less use of inflationary finance should be observed in the U.S. The evidence favours the second argument.}

The reaction of conventional taxation and debt sales to the public is as predicted by both the finance and stabilization...
models, but as in the U.S. case, the reaction of seigniorage collection to a positive income shock is significantly negative. This is inconsistent with the optimal government finance model and consistent with stabilization motives.
Paraguay and Venezuela

Utilizing the International Monetary Fund's International Financial Statistics, data for Paraguay and Venezuela were obtained. These were the only South American countries for which sufficiently long series could be obtained. In the case of Paraguay, debt statistics were not available so only equations (1) and (2) could be estimated. The results for Paraguay are presented in Table 4.3 while those for Venezuela are presented in Table 4.4. It should be noted that the IMF public finance data cannot be considered highly reliable for many countries. There are frequent and serious revisions to series and where other sources do exist with which to compare IMF series there are usually large differences in the figures. Nevertheless, since the IMF is the only single source of the required data it was used.
Table 4.3: Paraguay Results

Period 1962 to 1982

\[(1) \Delta s/s_1 = 0.225 + 0.891(g-g^*)/s_1 - 0.407(y-y^*)/s_1 \]
\[= (0.846) (1.874) \quad (-3.719) \]

\[\bar{R}^2=0.55036\]
\[F(2/18)=11.016\]
\[\text{St. Error}=1.5274\]
\[\text{D.W.}=2.11\]
\[\text{Rho}=-0.2971\]

\[(2) \Delta p/p_1 = 0.074 + 0.307(g-g^*)/p_1 + 0.277(y-y^*)/p_1 \]
\[= (4.289) (1.528) \quad (0.571) \]

\[\bar{R}^2=0.1415\]
\[F(2/18)=1.483\]
\[\text{St. Error}=0.0778\]
\[\text{D.W.}=1.92\]
\[\text{Rho}=-0.0274\]

Notes: Units are millions of 1975 Guaranies.
Data Sources:
Data from International Financial Statistics. Line numbers are:
s change in line 14, g lines 82 plus 83, y line 99a, p line 81
all deflated by the CPI line 64.
Table 4.4: Venezuela Results

Period 1962 to 1982

(1) $\Delta s/s_{i}= -0.164 + 0.043(g-g^{*})/s_{i} - 0.003(y-y^{*})/s_{i}$

\[ R^2=0.87192 \]
\[ F(2/20)=68.078 \]
St. Error=2.3105
D.W.=2.07
Rho=0.2251

(2) $\Delta p/p_{i}= 0.155 + 0.892(g-g^{*})/p_{i} - 0.019(y-y^{*})/s_{i}$

\[ R^2=0.99634 \]
\[ F(2/20)=2724.10 \]
St. Error=0.2022
D.W.=1.95
Rho=-0.1256

(3) $\Delta d/d_{i}= -0.022 - 0.138(g-g^{*})/d_{i} + 0.029(y-y^{*})(y^{*}/g^{*})/d_{i}$

\[ R^2=0.20984 \]
\[ F(2/20)=2.656 \]
St. Error=4.0834
D.W.=2.00
Rho=0.0550

Notes: Units are millions of 1975 Bolivares. 
Data Sources: 
As with Table 4.3. Debt is from line 88 deflated by line 64.
Paraguay experienced a recorded annual average inflation rate of almost 15 per cent in the 1962 to 1982 period.\footnote{Calculated using the IMF data used in the regressions.} The reliance on monetary finance may be reflected in the size of the constant in equation (1) though the associated T-statistic is small. The coefficient for the expenditure shock term indicates that monetary base expansion is heavily relied upon for short-run revenue. The coefficient for the income shock term is negative, large in magnitude and significant. The question arises as to whether this indicates stabilization policy or the substitution of seigniorage for other taxes under fluctuating conditions. Without equation (3) to complete the evaluation of the model this question cannot be addressed.

The results for equation (2) for Paraguay are consistent with the optimal finance model, but the equation itself is not statistically significant.

The results for Venezuela are troublesome. The constant terms do not conform with reasonable expectations for real income growth. In equations (1) and (2) the constants are negative in sign although not significantly different from zero. In equation (2) the constant is statistically significant, but appears to be too large to reasonably reflect trend real income growth. A small but statistically significant portion of expenditure shocks are financed with seigniorage while a large proportion is financed with conventional taxation. The coefficient of the income shock term is negative but
insignificant in equation (1) while in equation (2) it is significantly negative. This result is not consistent with either the optimal finance or stabilization models. Results for equation (3) also are inconsistent with either model.\textsuperscript{13} Given the nature of the data, it is perhaps best not to place much emphasis on these results however.

\textsuperscript{13} One reason for the peculiar results might be that income and expenditure shocks are associated with each other. Regressions of one shock on the other did not produce significant results however.
Summary and Conclusions

The United States and Canada are the two countries for which the most reliable data can be readily obtained. The results reported above indicate that the use of monetary expansion does not follow the predictions of the optimal government finance model in these two countries. It appears that monetary expansion is not a tax like any other tax. Separating the revenue from monetized debt from other taxes produces results that cannot be distinguished from those predicted by a stabilization model. Whether successful or not the evidence indicates that monetary stabilization policy has been pursued in the post World War Two years. The results for the equations dealing with the behavior of tax collections and nonmonetized debt were consistent with the finance model (which is indistinguishable from a stabilization model in these cases), but the results are statistically weak.

Paraguay and Venezuela are the two countries in South America for which series of sufficient length can be obtained. While Chile and Argentina would have been interesting cases since they are often indentified with inflationary finance, insufficient data was available for estimations to be conducted. The data used is of dubious quality and the results obtained in

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\[\text{\textsuperscript{14} Central Statistical Office data for the U.K. was subject to frequent and extensive revision and cannot be considered reliable. In addition, definitions of some reported series changed at various times making a consistent set of data unobtainable.} \]
the Paraguay and Venezuela cases should be interpreted with such an understanding. The results for Paraguay reveal an extreme reliance on seigniorage collection as a revenue gathering device. Unfortunately, public debt data was not available and a complete evaluation of the optimal finance model is not possible in the case of Paraguay. The results for Venezuela conform to neither the finance or stabilization models. Whether this is due to the nature of the data or because of other factors cannot be resolved without additional data.

The observed inverse relationship between the change in monetary base expansion and deviations of real income from trend may be due to several factors including: (i) the pursuit of activist monetary stabilization policy, (ii) changes in collection costs resulting from income changes, (iii) the desire to maintain total tax revenues when income changes by substituting monetary for other forms of taxation and (iv) the pursuit of stabilization-like policies even though such policies are not expected to stabilize income and employment. These possibilities will be discussed in turn.

There exists both theoretical and empirical debate over the effectiveness of monetary and fiscal policies. Feldstein (1976, 1982), Fisher (1976) and Tobin (1980) among others maintain that money and public debt are not neutral and that there is scope for stabilization policy and have presented some empirical evidence in their support. Governments' acceptance of the view that money and debt are non-neutral would explain the results
obtained for all of the equations except in the case of Venezuela. However, there is also a sizable empirical literature suggesting that stabilization policy is ineffectual.\textsuperscript{15} Regressions of changes in seigniorage and of seigniorage (change in the base) on real income did not produce significant results. Lagging the monetary base variable did not result in significant results either.\textsuperscript{16} Using the data collected in order to estimate equation (1), no effectiveness of monetary policy could be discerned. Nevertheless, those who are to conduct stabilization policy would have an incentive to promote its use though they may also deny its potency.

It may be possible that when income falls below trend the cost of monetary finance falls sharply relative to other forms of taxation. Possibly there is a movement towards less visible forms of taxation in such circumstances. Such an ex post rationalization of results is unpalatable. As with Gordon's (1976) hypothesis concerning inflationary finance and the war in Vietnam it is difficult to accept that the public does not understand a phenomenon as ancient as seigniorage collection to be a form of taxation or at least to hold government ultimately at fault for inflation.

Barro (1977) and Melnick and Sokoler (1984) suggest that when income falls the seigniorage base (real balances held)

\textsuperscript{15} See the various papers by Barro listed in the Bibliography for examples

\textsuperscript{16} In the case of the U.S., F statistics for these regressions were below 2.0. For Canada, the F statistics were below 0.5.
falls as well and that the state increases the seigniorage rate so as to keep revenue collected constant. Three problems arise immediately in response to such a hypothesis. Firstly, the amount of revenue to be collected after the fall in demand for real balances depends not only on the seigniorage rate but on the elasticity of demand for balances as well. It is possible that after the demand for balances has decreased, the demand will be relatively elastic. Increasing the seigniorage rate would result in even lower revenue in such a case. Secondly, in the literature, a similar pattern is not suggested for other forms of taxation. Thirdly, the argument ignores the role of debt finance in smoothing tax rates. Melnick and Sokoler state that their model does not fit within a model of optimal taxation policy.

An attempt to reconcile the observations that activist monetary policy is quite ineffectual with evidence of activist monetary policy is pursued in Barro and Gordon (1983a). The stylized facts that rates of monetary growth and inflation are excessive from an efficiency point of view and that there are attempts to stabilize the economy with monetary policy are addressed by a model, adapted from Kydland and Prescott (1977) which produces an outcome in which, "...the policymaker pursues activist policy that ends up having no desirable effects - in fact, unemployment is unaltered but inflation ends up being excessive."

In the absence of a commitment to obey some known

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rule (e.g. adopt the gold standard), the authorities would have an incentive to generate unanticipated inflation up to a point where the marginal cost of additional inflation equals the marginal benefit from reduced unemployment. The public is aware of this and anticipates that the government will generate that level of inflation. It is optimal for the policymakers to in fact generate that inflation rate. A lower rate would increase unemployment and the costs faced by the state while a higher rate would result in a lower unemployment rate but higher costs to government due to the high rate of inflation. When unemployment is relatively high there would be a greater incentive to reduce unemployment by generating unexpected inflation. This is well understood by the public and this understanding is in turn understood by the government. More inflation will be generated to meet the expectation that more inflation will be generated, but there will be no effect on the level of employment. Thus there is the seemingly paradoxical coexistence of apparently activist monetary policy and expected policy ineffectiveness.

The Barro and Gordon (1983a) model generates numerous implications, but does not explain why a monetary commitment that is in everyone's interest is infeasible. The model rests fundamentally on the existence of a stable expectations augmented Phillips curve relationship between the difference in the actual and the "natural" rate of unemployment and the difference between actual and expected inflation. The possible
existence of such a relationship depends on informational asymmetries or price rigidities which are difficult to conceive of in a rational expectations world.\textsuperscript{18}

While surprise changes in monetary policy are discussed in the context of revenue generation and are described as being time inconsistent, Barro and Gordon do not consider a normal anticipated rate of monetary expansion which is calculated to gather revenue in an optimal (least cost) way as determined by an optimal government finance model. However, Barro (1983a) considers only the revenue aspects of money supply expansion. Though the Barro (1983a) model is set up differently, the resulting money supply behavior is the same as that predicted by the optimal government finance model developed above. The finance model presents an alternative cost function and policy setting which the public can understand and base expectations upon. Barro and Gordon's result is "time consistent" but "suboptimal" in that there is an excessive rate of monetary expansion and inflation. There is no clear reason to accept a Phillips curve view rather than a tax collection cost view of the problem facing government except for the empirical evidence that stabilization policy is attempted. The finance model provides a time consistent and optimal result, but one which is contrary to empirical observation.

\textsuperscript{18} Part of Barro (1984a) reviews theoretical arguments and empirical evidence for the nonexistence of the Phillips curve.
In Barro and Gordon (1983b) the expectations augmented Phillips curve is combined with revenue from unanticipated inflation as a benefit from unexpected inflation function. If unemployment might be reduced and revenue increased without deadweight losses, the government has an incentive to generate unexpected inflation subject to cost considerations. Specifically, Barro and Gordon assume that the cost of inflation faced by the government increases with the square of the actual inflation rate while the benefits increase linearly with the unexpected component of inflation. Due to the revenue aspect of unanticipated inflation, the Phillips curve is no longer necessary to produce the result that under rational expectations, "too much" expected inflation will be generated by the government. The state will always have an incentive to generate unexpected inflation and that is understood by the public hence the government's optimal rate of inflation is anticipated by the public. If the government does not in fact generate that much inflation, a loss of revenue and employment would result. The specified cost function makes this result undesirable for the government. Attempts to generate excessive inflation will result in much higher costs due to the quadratic nature of the assumed cost function and will cause the public's expectations of future rates of inflation to be revised upwards meaning that in the future the state would have to generate more inflation simply to maintain employment levels. Given the

\[\text{9 This specification thus contains the convexity necessary to determine a unique solution.}\]
resulting decrease in the revenue base and again given the specified cost function this is not a desirable policy for government to pursue.20

Like their earlier paper, the Barro and Gordon (1983b) article is built on the foundation that there is a noncooperative game being played between the state and its citizens. A mutually beneficial arrangement involving the adoption of a money supply rule does not eventually come about. It is not explained why such rules are not adopted (or readopted in the case of a metallic standard), but is only assumed that they are not. If (mutual) net gains are not exploited, there should be a good reason for this. In the context of the optimal government finance model the reason is that simple money supply growth rules are not optimal. The behavior suggested by the optimal finance model or as in Barro (1983) is an alternative to a simple rule or to noncooperation which is advantageous to both parties as long as there exists a means of disciplining authorities who do not obey the rules.

As in their earlier paper, the Barro and Gordon (1983b) assume a Phillips curve function together with rational expectations and no informational asymmetry. This part of their model is subject to the same criticism directed at their earlier article. The revenue generated by surprise inflation is assumed

20 A convincing reason for why expectations of future inflation rates are necessarily revised upward is not given by Barro and Gordon. It is not clear why a one shot revenue grab would affect expectations about the continuous rate of taxation of money balances.
to be attractive to government because unlike an income tax or other forms of taxation, such an ex post tax does not involve distortion of resource allocations. Unlike the optimal government finance model, this does not consider the political costs involved in an involuntary ex post wealth transfer. In addition in many if not most countries, governments have the legal authority to impose ex post taxes on income or wealth. This is rarely done however. We do not observe continually excessive income and and other forms of taxation as described by Barro and Gordon in countries which do not have binding taxation rules placed on the taxing authorities. The optimal finance model is built on the assumption that the cost of obtaining revenue is an increasing function of the amount of revenue collected as opposed to the rate of taxation. This specification follows from the type of cost function used in Barro's public debt and taxation papers. The Barro and Gordon models are not consistent with the earlier papers on the public debt and there has been no attempt to integrate the two theoretical frameworks. Money, in Barro and Gordon is special and the inflation tax is not a tax like any other. While Barro and Gordon's work is able to explain observed monetary behavior it fails to explain other observed phenomena. Due to theoretical problems it is not accepted as a tenable model of monetary and fiscal behavior.

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The fundamental equivalence between the budget constraint used in the discussion above and a more usual constraint based on an alternative definition of the real deficit is shown here. An implication regarding the response of the nominal deficit first suggested by Barro (1979) is revealed as well.

Beginning with a nominal budget constraint,
\[ G + iD_1 = P + S + (D - D_1) \]
where \( i = r + \pi \) is the nominal interest rate, we can deflate to obtain,
\[ g + rD_1/P + \pi D_1/P = p + S + (D - D_1)P \]
where \( (D - D_1)/P \) is one definition of the real deficit. Noting that \( (1 + \pi) = P/P_1 \), this may be rewritten,
\[ g + rD_1/P + (P/P_1)(D_1/P) = p + S + d \]
or
\[ g + rD_1/P + d = p + S + d. \]
In the limit where \( \Delta t \rightarrow 0 \), this gives,
\[ g + rd = p + s + b \]
where \( b = d \) is equal to \( d(D/P)dt \). The use of the nominal as opposed to the real interest rate in the budget constraint depends only on how the real deficit is defined. If the real deficit is defined as \( (D - D_1)/P \) (\( dD/P \) in continuous time), the nominal interest rate should be used in the budget constraint.

Abstracting from considerations of growth, the steady state inflation rate would be equal to the growth rate of the monetary
base. That is, $\pi=m/m$ and $\pi=s/m$. The budget constraint,
$$g=p+s+b-id$$
may be rewritten as
$$g=p+s+b-rd-sd(1/m).$$
The Hamiltonian formed is,
$$H=C(p,s,b;y)+v(g-p-s+rd+sd(1/m)).$$
The first order conditions include,
$$Hs=Cs+v+vd(1/m)=0$$
and
$$Hp=Cp-v=0.$$  
Thus $Cs=Cp(1+d(1/m))$ so that $Cs>Cp$ if $d>0$ along the optimal path. Included in the marginal cost of seigniorage is the effect of seigniorage on the real value of outstanding debt. Further, differentiation with respect to time yields,
$$Css=\dot{v}(1+d(1/m))+v\dot{d}(1/m)-vd(1/m^2)m.$$  
If $z=r$ then, $\dot{v}=0$. The equation above then becomes,
$$Css=\dot{v}(b/m-d\dot{m}/m^2)$$
or
$$Css=\dot{v}/m(b-ds/m).$$ 
Optimality still requires that $p=s=0$, thus we find that $b=ds/m=\pi d$. That is, the government would find it optimal to increase the deficit by the rate of inflation times the amount of outstanding debt. This is essentially the same argument made by Barro (1979). 'A tax such as seigniorage which would decrease the value of debt below the optimal level of real indebtedness---------------------------------

1 This argument assumes that there are negligible costs involved in issuing debt. If such costs are important, the argument would have to be revised. It would not be optimal to strictly maintain the level of real debt, but the real debt would be allowed to decrease somewhat.
would cause the time path of taxation to deviate from the optimal path. The government would effectively reduce current taxation and issue more debt in order to remain on the optimal path.

Surprise increases in taxation, such as surprise money creation are not considered in the model. Such surprises could happen, but under the assumptions of the model and with the existence of bonds with which taxation can be altered intertemporally, surprises are not desirable from the government's perspective. The existence of a market for government bonds provides an incentive for the government to be an honest borrower and tax collector.

Finally, let us note again the fundamental equivalence between the two methods of defining the real deficit and budget constraint. The result above, $B = \pi D_\frac{1}{1}$, in a no growth steady state, may be rewritten, $(D - D_\frac{1}{1}) = \pi D_\frac{1}{1}$. Thus $D = (1 + \pi)D_\frac{1}{1} = (P/P_\frac{1}{1})D$ and $D/P = \delta = d_\frac{1}{1} = D_\frac{1}{1}/P_\frac{1}{1}$. The major advantage of expressing the real deficit as the change in the level of real indebtedness is one of simplicity. The effect of expected inflation on nominal deficits is included in the real deficit calculation. It is not necessary in empirical work to proxy expected inflation (as done in Barro 1979, 1984b) if this definition of the real deficit is used since using $b = d$ implicitly imbeds an expected inflation term in the level of real current indebtedness.


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