MODELING AND PERFORMANCE ANALYSIS OF PUBLIC SAFETY WIRELESS NETWORKS

by

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B.Eng., Soochow University, China, 2000

THESIS SUBMITTED IN PARTIAL FULFILLMENT OF THE REQUIREMENTS FOR THE DEGREE OF

MASTER OF APPLIED SCIENCE

In the
School
of
Computing Science

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SIMON FRASER UNIVERSITY

Spring 2005

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ABSTRACT

Public safety wireless networks (PSWNs) play a vital role in operations of emergency agencies such as police and fire departments. In this thesis, we describe analysis and modeling of traffic data collected from the Emergency Communications for Southwestern British Columbia (E-Comm) PSWN.

We analyze network and agency call traffic and find that lognormal distribution and exponential distribution are adequate for modeling call holding time and call inter-arrival time, respectively. We also describe a newly developed wide area radio network simulator, named WarnSim. We use WarnSim simulations to validate the proposed traffic model, evaluate the performance of the E-Comm network, and predict network performance in cases of traffic increase. We also provide recommendations on allocating E-Comm network resources to deal with the increased traffic volume.
DEDICATION

To my father Kejun Jia, my mother Shen’an Song, and my wife Zhen Yuan.
ACKNOWLEDGMENTS

There are few things in the world that a person could accomplish alone. Writing this thesis is certainly not one of them. I would like to express my special thanks to my senior supervisor Dr. Ljiljana Trajković for the continuously guidance and everlasting support she has been giving me throughout my graduate studies.

My sincere gratitude goes to Dr. Qianping Gu, for being my supervisor and being my advising voice on my thesis. Furthermore, I would like to express appreciation to Dr. Uwe Glässer and Dr. Jian Pei for being on my examining committee and for giving me valuable suggestions on my thesis.

Finally, I would like to thank all members of the Communication Networks Laboratory at Simon Fraser University for numerous and enlightening discussions.
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## GLOSSARY

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDF</td>
<td>Cumulative Density Function</td>
</tr>
<tr>
<td>E-Comm</td>
<td>Emergency Communications for Southwestern British Columbia Inc.</td>
</tr>
<tr>
<td>ECDF</td>
<td>Empirical Distribution Function</td>
</tr>
<tr>
<td>EDACS</td>
<td>Enhanced Digital Access Communications System</td>
</tr>
<tr>
<td>FIFO</td>
<td>First In First Out</td>
</tr>
<tr>
<td>GoS</td>
<td>Grade of Service</td>
</tr>
<tr>
<td>GVRD</td>
<td>Great Vancouver Regional District</td>
</tr>
<tr>
<td>K-S GoF test</td>
<td>Kolmogorov-Smirnov Goodness-of-Fit Test</td>
</tr>
<tr>
<td>LMR</td>
<td>Land Mobile Radio</td>
</tr>
<tr>
<td>MLE</td>
<td>Maximum Likelihood Estimation</td>
</tr>
<tr>
<td>PDF</td>
<td>Probability Density Function</td>
</tr>
<tr>
<td>PAMR</td>
<td>Public Access Mobile Radio</td>
</tr>
<tr>
<td>PCS</td>
<td>Personal Communication Services</td>
</tr>
<tr>
<td>PMR</td>
<td>Private Mobile Radio System</td>
</tr>
<tr>
<td>PSTN</td>
<td>Public Switched Telephone Network</td>
</tr>
<tr>
<td>PSWN</td>
<td>Public Safety Wireless Network</td>
</tr>
<tr>
<td>PTT</td>
<td>Push-to-Talk</td>
</tr>
<tr>
<td>WARN</td>
<td>Wide Area Radio Network</td>
</tr>
<tr>
<td>WarnSim</td>
<td>Wide Area Radio Network Simulator</td>
</tr>
</tbody>
</table>
CHAPTER 1: INTRODUCTION

1.1 Motivation

Sharing vital voice information via radio in a timely and reliable manner is critical for operations of various public safety agencies, such as law enforcement, fire departments, and emergency medical services. Hence, public safety wireless networks (PSWNs) employed for on-scene communications play an important role in ensuring public safety.

The public safety community has identified the limited and fragmented radio spectrum as one of the key issues that adversely affects public safety wireless communications [1]. In the United States, the SAFECOM program was recently established to serve as the umbrella program within the Federal Government to help local, state, and federal public safety agencies improve public safety response through more effective and efficient wireless communications [2]. The emergency wireless communication service providers are also concerned with the spectrum shortage and the high cost of deploying and operating voice channels. For example, emergency agencies covering Vancouver share only eleven wireless voice channels. Therefore, understanding, analyzing, evaluating, and optimizing public safety wireless networks are a valuable research topic.

We use a modeling/simulation approach and a customized simulator named WarnSim to evaluate and predict the performance of the public safety wireless network (PSWN) deployed in British Columbia, Canada.
1.2 Objectives

The objectives of this project are to develop statistical models for call traffic in the E-Comm network, to evaluate performance of the E-Comm network in terms of channel utilization and call blocking probability, and to predict future performance of the E-Comm network via a simulation approach.

1.3 Organization of thesis

This thesis is organized as follows. We introduce the structure of the E-Comm network in Chapter 2. Analysis of the E-Comm traffic data is presented in Chapter 3. In Chapter 4, we describe statistical modeling of call holding and call inter-arrival times, two components of a statistical model for call traffic. In Chapter 5, We introduce the design and implementation of a customized simulator for wide area radio networks, named WarnSim. Simulations performed with WarnSim and the simulation results are presented in Chapter 6. Conclusions and future work are given in Chapter 7. Scripts for Maximum Likelihood Estimation (MLE) are given in Appendix A.
CHAPTER 2: BACKGROUND

In this Chapter, we introduce the history of E-Comm. We also describe the network architecture and the traffic data collected from the E-Comm network.

2.1 Introduction to E-Comm

2.1.1 History of E-Comm

On June 14th, 1994, the Stanley Cup riots took place in downtown Vancouver, Canada. Several emergency agencies, including Vancouver police, fire department, the RCMP, and BC Ambulance, were mobilized to handle the emergency situation. After the disturbance, it was discovered that communications between these emergency agencies were highly inadequate. One reason was that these emergency agencies operated their own wireless radio networks that were not interconnected. Moreover, emergency radio channels utilized by the police and fire departments needed significant capacity upgrading. To solve these problems, the government of British Columbia initiated a new emergency communication project. This project evolved into E-Comm in 1997.

E-Comm is the regional Emergency Communications Centre for Southwest British Columbia. E-Comm provides wide area radio dispatching services for a variety of emergency agencies throughout the Greater Vancouver Regional District (GVRD), the Sunshine Coast Regional District, Whistler, and Pemberton [3]. Until 2004, the E-Comm project has a total investment of $160 million CAD and an annual operating budget of $41 million CAD [4].
2.1.2 E-Comm network architecture

E-Comm's wireless radio network utilizes the Enhanced Digital Access Communications System (EDACS), manufactured by M/A-Com [5]. Its architecture is shown in Figure 2.1. It contains a central system controller (network switch), a management console, several repeater sites (base stations), one or more fixed user sites (dispatch consoles), a private branch exchange (PBX) gateway to the public switched telephone network (PSTN), and thousands of mobile users. Individual systems (cells) that cover separate geographic regions (City of Vancouver, City of Burnaby) in the E-Comm network are interconnected by high-speed data links. System events and call activities in the network are recorded by base stations and are forwarded every hour through the data gateway to the central database.

Figure 2.1: System architecture of EDACS.
2.1.3 Simulcast system

EDACS may operate in one of the five modes: single site system, voted system, simulcast system, single channel system, and multi-site system. Systems (cells) in the E-Comm network are simulcast systems that employ multiple repeaters to transmit and receive identical audio and data information over the same carrier frequency. For example, there are five simulcast repeaters covering the City of Vancouver. Simulcast systems are employed when it is necessary to use a limited number of frequencies to cover an area too large for a single repeater. The simulcast system may also provide higher signal strength and better fault tolerance.

2.1.4 Transmission trunking and message trunking

Communications within the E-Comm network are conducted via transmission trunking rather than via traditional message trunking. With message trunking, a radio channel is assigned to a call for the entire duration of the conversation. Because gaps of silent periods usually exist during a conversation, occupying the channel during the entire conversation time is inefficient in terms of channel utilization. With transmission trunking, the radio channel is automatically released as soon as the caller completes the sentence and releases the push-to-talk (PTT) button. The next speaker may begin a call by pushing the PTT button on his/her radio device. The system then checks the channel availability and assigns required number of channels to the call. Transmission trunking is 20–25% more efficient than message trunking [5]. However, there is a price for the high channel utilization in the transmission trunking mode. Overheads due to channel assigning time and channel dropping time are added to each transmission because the processes of channel assigning and channel dropping are repeated for every press of the
PTT button. The continuously available high-speed control channel in the E-Comm network alleviates this overhead problem. The control channel uses a dedicated radio channel that supports 9.6 kbps digital signalling. The channel assigning and channel dropping times are 0.25 seconds and 0.16 seconds, respectively.

2.1.5 Push-to-talk mechanism

The communication mechanism in the E-Comm network is push-to-talk (PTT). It implies that a user who wishes to initiate a call needs to press a button on the radio device to request one or more channels. The central switch locates users in the specific talk group and checks the availability of channels in systems covering the talk group. The caller receives an audible signal if there are free channels to establish the call. Otherwise, the call request is queued.

2.1.6 Talk group and group calls

In the E-Comm network, talk groups are defined at various levels (agency level, fleet level, and sub-fleet level) for better coordination of operations. Each radio device belongs to one or more talk groups and may be switched between talk groups.

Group call is the standard call type in the E-Comm network. Call recipients are members of a talk group. The advantage of a group call is that it eliminates the need for radio users to know the target device number in order to reach an individual user. Users may call a target group without having to know the current members of the group.

It is important to note that in the E-Comm network, a single call might use several channels simultaneously. If all users of a talk group reside within a single system, the network controller will allocate one free channel to the call. However, if members of a
talk group are distributed across several systems, the network controller will allocate to the call a free channel in each system.

2.1.7 Systems and channels of the E-Comm network

E-Comm has eleven systems deployed in the GVRD as of December 2003. Table 2.1 shows the system ID, system coverage, and number of deployed channels in each system.

Table 2.1: Number of channels deployed in each E-Comm system as of December 2003.

<table>
<thead>
<tr>
<th>System ID</th>
<th>Coverage</th>
<th>Number of channels</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Vancouver</td>
<td>13</td>
</tr>
<tr>
<td>2</td>
<td>Burnaby</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td>Maple Ridge</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>Langley</td>
<td>6</td>
</tr>
<tr>
<td>5</td>
<td>Seymour</td>
<td>6</td>
</tr>
<tr>
<td>6</td>
<td>Port Coquitlam</td>
<td>8</td>
</tr>
<tr>
<td>7</td>
<td>Richmond</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>Mission</td>
<td>5</td>
</tr>
<tr>
<td>9</td>
<td>Surrey</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>South Surrey</td>
<td>8</td>
</tr>
<tr>
<td>11</td>
<td>Bowen Island</td>
<td>4</td>
</tr>
</tbody>
</table>

2.1.8 Mobility of radio devices and call handover

Mobility of radio devices and call handover are two major concerns for micro-cell cellular networks. However, they are of little importance in the E-Comm network. The E-Comm network is a wide area radio network with each system covering a citywide area. Because an emergency call lasts 3.8 seconds on average, there is only a negligible probability that one radio device moves between two systems during such a short time.
2.2 A sample of the E-Comm traffic data

All call activities occurred in the E-Comm network are recorded by base stations and are stored in the central database. Thus, the central database is the ideal location for collecting network wide traffic data that contain call activity information from all systems. Table 2.2 shows a sample of raw call traffic data collected from the E-Comm network. The 26 fields of the raw traffic data table are explained in Table 2.3.
Table 2.2: A sample of raw traffic data collected from the E-Comm network.

<table>
<thead>
<tr>
<th>Event.UTC</th>
<th>Duration.ms</th>
<th>Network_Id</th>
<th>Node_Id</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003-05-01 00:00:09.620</td>
<td>1990</td>
<td>1</td>
<td>33</td>
</tr>
<tr>
<td>2003-05-01 00:00:10.020</td>
<td>1990</td>
<td>1</td>
<td>33</td>
</tr>
<tr>
<td>2003-05-01 00:00:11.607</td>
<td>1990</td>
<td>1</td>
<td>33</td>
</tr>
<tr>
<td>2003-05-01 00:00:12.010</td>
<td>1990</td>
<td>1</td>
<td>33</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>System.Id</th>
<th>Channel_Id</th>
<th>Slot_Id</th>
<th>Caller</th>
<th>Callee</th>
</tr>
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<tbody>
<tr>
<td>7</td>
<td>4</td>
<td>NULL</td>
<td>9999</td>
<td>1111</td>
</tr>
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<td>4</td>
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<td>9999</td>
<td>1111</td>
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<table>
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<tr>
<th>Call_Type</th>
<th>Call_State</th>
<th>Call_Direction</th>
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<th>Digital_Call</th>
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<table>
<thead>
<tr>
<th>Voice_Call</th>
<th>Mcp</th>
<th>Confirmed_Call</th>
<th>Msg_Trunked_Call</th>
<th>Preempt_Call</th>
</tr>
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<table>
<thead>
<tr>
<th>Primary_Call</th>
<th>Queue_Depth</th>
<th>Queue_Pri</th>
<th>Multi_System_Call</th>
<th>Caller_Bill</th>
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<tbody>
<tr>
<td>1</td>
<td>NULL</td>
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<td>0</td>
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<table>
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<tr>
<th>Callee_Bill</th>
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<tr>
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<td>0</td>
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</tbody>
</table>
### Table 2.3: Descriptions of data fields in the E-Comm traffic data table.

<table>
<thead>
<tr>
<th>Traffic data fields of interests in this thesis:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Event.UTC</td>
<td>Call arrival time</td>
</tr>
<tr>
<td>Duration_ms</td>
<td>Call holding time (call duration)</td>
</tr>
<tr>
<td>System_Id</td>
<td>System ID in which a call occurred</td>
</tr>
<tr>
<td>Channel_Id</td>
<td>Channel ID in which a call was using</td>
</tr>
<tr>
<td>Caller</td>
<td>ID of a radio device that initiates a call</td>
</tr>
<tr>
<td>Callee</td>
<td>ID of a radio device that receives a call</td>
</tr>
<tr>
<td>Queue_Depth</td>
<td>Number of calls waiting in the queue</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Traffic data fields that are not used in this thesis:</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Network_Id</td>
<td>Constant “1”</td>
</tr>
<tr>
<td>Node_Id</td>
<td>Constant “33”</td>
</tr>
<tr>
<td>Slot_Id</td>
<td>Constant “NULL”</td>
</tr>
<tr>
<td>Call_Type</td>
<td>Call type (group/individual/emergency/group set/system all/Morse code/test/paging/scramble data/sys login/start emergency/cancel emergency)</td>
</tr>
<tr>
<td>Call_State</td>
<td>Call state (channel assign/channel drop/key/un-key/digits/over digits/queue/busy/deny/convert to callee)</td>
</tr>
<tr>
<td>Slot_Id</td>
<td>Constant “NULL”</td>
</tr>
<tr>
<td>Call_Direction</td>
<td>Indicator of making or receiving a call</td>
</tr>
<tr>
<td>Voice_Call</td>
<td>Indicate if the call contains voice information</td>
</tr>
<tr>
<td>Digital_Call</td>
<td>Indicate if the call is from digital device or analog device</td>
</tr>
<tr>
<td>Interconnect_Call</td>
<td>Indicate if it is call to PSTN</td>
</tr>
<tr>
<td>Multi_System_Call</td>
<td>Indicate if it is a multi-system call</td>
</tr>
<tr>
<td>Confirmed_Call</td>
<td>Indicate if it is a confirmed call</td>
</tr>
<tr>
<td>Msg_Trunked_Call</td>
<td>Indicate if the call is message trunking or transmission trunking</td>
</tr>
<tr>
<td>Preempt_Call</td>
<td>Indicate if it is a pre-empt call</td>
</tr>
<tr>
<td>Primary_Call</td>
<td>Indicate if it is a real call or group set signal</td>
</tr>
<tr>
<td>Queue_Pri</td>
<td>The priority of a call in a queue</td>
</tr>
<tr>
<td>Mcp</td>
<td>Indicator of multiple channel partition</td>
</tr>
<tr>
<td>Caller_Bill</td>
<td>Indicate who (caller or callee) will pay for the call</td>
</tr>
<tr>
<td>Callee_Bill</td>
<td>Indicate who (callee or caller) will pay for the call</td>
</tr>
<tr>
<td>Reason_Code</td>
<td>Additional information</td>
</tr>
</tbody>
</table>
CHAPTER 3: TRAFFIC DATA ANALYSIS

In this Chapter, we first describe the procedure for pre-processing the E-Comm raw traffic data. We then analyze the call traffic data and describe their characteristics. To illustrate the evolution of the E-Comm network, we also compare the E-Comm traffic data collected from year 2002 with the traffic data collected from year 2003.

3.1 Traffic data pre-processing

We analyze three sets of call traffic data from the E-Comm network. These sets contain (year-month-day) 2 days (2001-11-01 to 2001-11-02), 30 days (2002-02-09 to 2002-03-10), and 92 days (2003-03-01 to 2003-05-31) of call traffic data.

In the raw traffic data, one row represents one event that occurred in the E-Comm network. A single call usually generates two or more events. For example, if one call covers Systems 1, 2, and 3, it will generate a channel-assigning event and a channel-dropping event in all three systems. Thus, this call generates six events in the E-Comm network and it is recorded in 6 rows in the raw traffic data table. For the sake of analysis, it is desirable to combine these duplicated rows and use only one row to represent one call. Furthermore, it is necessary to delete redundant and irrelevant system events from the raw traffic data table. We follow six steps to pre-process raw traffic data:

Step 1: Delete redundant and irrelevant data rows. Keep only rows that represent genuine voice calls.
Step 2: Ignore columns (data fields) that are irrelevant to our research: Network_Id, Node_Id, Slot_Id, Interconnect_Call, Caller_Bill, Callee_Bill, Voice_Call, Msg_Trunked_Call, Reason_Code, Digital_Call, Primary_Call, Call_Direction, Multi_System_Call, Preempt_Call, Queue_Depth, and Confirmed_Call.

Step 3: Combine data rows that represent a single call into one row. Update fields Systems and Channels.

Step 4: Add a new field named Caller_Agency into the traffic data table.

Step 5: Retrieve caller agency ID information based on Caller ID and update field Caller_Agency.

Step 6: Sort calls by the timestamp field Event.UTC (call arrival time).

A sample of the processed traffic data corresponding to the raw data shown in Table 2.2 is shown in Table 3.1. The data row in Table 3.1 depicts a call made by caller 9999 to callee 1111 at 2003-05-01 00:00:09.620. The call lasted for 1,990 ms and covered Systems 1 and 7. The call employed Channels no. 3 and no. 4 in Systems 1 and 7, respectively. The caller belonged to Agency 5.

<table>
<thead>
<tr>
<th>Call arrival time</th>
<th>Duration (ms)</th>
<th>Caller agency</th>
</tr>
</thead>
<tbody>
<tr>
<td>2003-05-01 00:00:09.620</td>
<td>1,990</td>
<td>5</td>
</tr>
<tr>
<td>Caller</td>
<td>Callee</td>
<td>System ID</td>
</tr>
<tr>
<td>9999</td>
<td>1111</td>
<td>1, 7</td>
</tr>
</tbody>
</table>

Table 3.1: A sample of the processed call traffic data.
3.2 Traffic data characteristics

3.2.1 Hourly and daily call arrival rates

We plot the hourly call arrival rate of 2002 and 2003 traffic data to visually examine the pattern of the hourly call arrival rate (number of calls that arrived within one hour). Figures 3.1 and 3.2 show cyclic patterns of the hourly call arrival rates in 2002 and 2003 datasets, respectively.

![Figure 3.1: Plot of the hourly call arrival rate over 30 days in year 2002.](image-url)
We consider 2,208 hours (92 days) of continuous traffic data in the 2003 dataset and use Fourier analysis to analyze hourly and daily call arrival rates in frequency domain [6]. The power spectrum of hourly and daily call arrivals are shown in Figures 3.3 and 3.4, respectively.
Figure 3.3: Power spectrum of hourly call arrival from the 2003 dataset indicates a strong daily cyclic pattern.

Figure 3.4: Power spectrum of daily call arrival from the 2003 dataset indicates a strong weekly cyclic pattern.

Further analysis results shown in Figure 3.5 indicate that the busiest hour in a day is around midnight, while the least amount of traffic is expected between 2 pm and 3 pm.
Furthermore, Thursday is the busiest day of a week, while Monday has the least number of calls, as shown in Figure 3.6.

The observations of the cyclic patterns of the call arrival rates are important because they provide useful guidelines for the E-Comm operators to schedule regular network maintenance tasks, such as database backup, to be performed when the network has least traffic.

Figure 3.5: Average hourly call arrival rate over 24 hours.
3.2.2 Call holding time

We also analyze the hourly average of call holding time (call duration). As shown in Figure 3.7, call holding time does not demonstrate a strong cyclic pattern.
We use Fourier analysis to analyze the hourly average of call holding time in the frequency domain. Analysis results shown in Figure 3.8 indicate that the average hourly call holding time has a weekly cyclic pattern.

![Power spectrum of average hourly call holding time](image)

Figure 3.8: Power spectrum of average hourly call holding time indicates a weekly cyclic pattern.

3.2.3 Agencies and call arrival rates

The E-Comm network currently serves over ten emergency agencies including Vancouver Police, RCMP, and BC Ambulance. Each emergency agency has a unique agency ID. In this thesis, we use agency IDs to represent emergency agencies. The mapping between agency ID and agency name is not disclosed due to security concerns.

Emergency agencies and their average daily call arrival rate are listed in Table 3.2. We can observe that few heavy user agencies account for most of the call traffic in the E-Comm network. For example, call traffic generated by Agencies 2 and 5 accounts
for more than 80% of the overall traffic in both 2002 and 2003. The call traffic increasing rates vary significantly among user agencies.

Table 3.2: Agencies and average daily call arrival rates.

<table>
<thead>
<tr>
<th>Agency ID</th>
<th>Average daily call arrival rate</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Year 2002</strong></td>
<td><strong>Year 2003</strong></td>
</tr>
<tr>
<td>n/a</td>
<td>1,360</td>
<td>118</td>
</tr>
<tr>
<td>1</td>
<td>93</td>
<td>307</td>
</tr>
<tr>
<td>2</td>
<td>27,803</td>
<td>27,659</td>
</tr>
<tr>
<td>3</td>
<td>1,191</td>
<td>1,266</td>
</tr>
<tr>
<td>4</td>
<td>855</td>
<td>894</td>
</tr>
<tr>
<td>5</td>
<td>25,994</td>
<td>48,915</td>
</tr>
<tr>
<td>6</td>
<td>n/a</td>
<td>438</td>
</tr>
<tr>
<td>7</td>
<td>38</td>
<td>34</td>
</tr>
<tr>
<td>8</td>
<td>214</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>9,439</td>
<td>11,692</td>
</tr>
<tr>
<td>11</td>
<td>1,184</td>
<td>656</td>
</tr>
<tr>
<td>13</td>
<td>212</td>
<td>145</td>
</tr>
<tr>
<td>14</td>
<td>78</td>
<td>60</td>
</tr>
<tr>
<td>15</td>
<td>n/a</td>
<td>&lt;1</td>
</tr>
<tr>
<td>20</td>
<td>n/a</td>
<td>&lt;1</td>
</tr>
<tr>
<td>21</td>
<td>&lt;1</td>
<td>2,999</td>
</tr>
<tr>
<td>Total</td>
<td>68,462</td>
<td>95,184</td>
</tr>
</tbody>
</table>

The difference of call arrival rates among user agencies suggests that modeling call traffic on agency level is more accurate than modeling traffic for the entire network. Moreover, heavy user agencies should receive additional attention in traffic modeling.

3.2.4 Systems and call arrival rates

The E-Comm network consists of eleven systems. E-Comm system IDs and average daily call arrival rate of each system are listed in Table 3.3. Table 3.3 indicates
that the overall call traffic increased by 37% from 2002 to 2003, which implies that additional channels are required in 2003 in order to maintain the same Grade of Service (GoS). We can also observe that the change of average daily call arrival rate in different systems varies significantly, which implies that each system needs individual assessment when expanding its wireless channels.

![Table 3.3: Systems and average daily call arrival rates.](image)

<table>
<thead>
<tr>
<th>ID</th>
<th>Coverage</th>
<th>Average daily call arrival rate</th>
<th>Change</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Vancouver</td>
<td>53,183</td>
<td>7%</td>
</tr>
<tr>
<td>2</td>
<td>Burnaby</td>
<td>19,581</td>
<td>40%</td>
</tr>
<tr>
<td>3</td>
<td>Maple Ridge</td>
<td>7,050</td>
<td>17%</td>
</tr>
<tr>
<td>4</td>
<td>Langley</td>
<td>7,705</td>
<td>138%</td>
</tr>
<tr>
<td>5</td>
<td>Seymour</td>
<td>4,241</td>
<td>26%</td>
</tr>
<tr>
<td>6</td>
<td>Port Coquitlam</td>
<td>15,336</td>
<td>48%</td>
</tr>
<tr>
<td>7</td>
<td>Richmond</td>
<td>18,752</td>
<td>27%</td>
</tr>
<tr>
<td>8</td>
<td>Mission</td>
<td>5,929</td>
<td>-4%</td>
</tr>
<tr>
<td>9</td>
<td>Surrey</td>
<td>12,714</td>
<td>151%</td>
</tr>
<tr>
<td>10</td>
<td>South Surrey</td>
<td>7,288</td>
<td>92%</td>
</tr>
<tr>
<td>11</td>
<td>Bowen Island</td>
<td>7,960</td>
<td>-49%</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>159,740</td>
<td>37%</td>
</tr>
</tbody>
</table>

### 3.3 Busy hours

PSWNs are designed to meet the GoS standards during busy hours. Industry Canada requires that less than 3% of calls could be queued for an average call holding time during busy hours [7].

The average hourly call arrival rates for the 2002 and 2003 datasets are 2,853 and 3,966, respectively. Table 3.4 shows that the call arrival rate during busy hours is
approximately twice the average call arrival rate. For the statistical modeling of call traffic, we only consider busy hour traffic.

<table>
<thead>
<tr>
<th>Busy hour (BH)</th>
<th>Period</th>
<th>Number of calls</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st BH in 2002</td>
<td>2002-03-01 04:00 – 05:00</td>
<td>5506</td>
</tr>
<tr>
<td>2nd BH in 2002</td>
<td>2002-02-28 22:00 – 23:00</td>
<td>5417</td>
</tr>
<tr>
<td>3rd BH in 2002</td>
<td>2002-03-01 22:00 – 23:00</td>
<td>5399</td>
</tr>
<tr>
<td>1st BH in 2003</td>
<td>2003-05-29 22:00 – 23:00</td>
<td>8922</td>
</tr>
<tr>
<td>2nd BH in 2003</td>
<td>2003-05-15 02:00 – 03:00</td>
<td>8664</td>
</tr>
<tr>
<td>3rd BH in 2003</td>
<td>2003-05-08 21:00 – 22:00</td>
<td>8471</td>
</tr>
</tbody>
</table>
In this Chapter, we first introduce the modeling/simulation approach and describe traditional models for evaluating radio networks. We then describe the process of modeling call holding time and call inter-arrival time. We also address the call coverage issue in the E-Comm network. Two statistical modeling techniques, the Kolmogorov-Smirnov goodness-of-fit (K-S GoF) test and the Maximum Likelihood Estimation (MLE), are also described.

4.1 Modeling/simulation approach

It is important to precisely estimate the maximum network capacity of public safe wireless networks and predict network performance in the future. In other words, it is important for the network operators to know answers for questions such as “How many mobile radios we can put into our network while keeping the busy hour call blocking probability under three percent?” and “How many additional channels are needed when call volume is increased by twenty percent?”

Generally speaking, there are four approaches to study a system and to answer these questions [8]: (1) experiment with the deployed system, (2) experiment with a physical model of the system (test bed), (3) use an analytical approach to experiment with a mathematical model of the system, and (4) use simulation to experiment with a mathematical model of the system.
There are three ways to evaluate and predict the E-Comm network performance: utilize existing formulas, employ a modeling/analytical approach, and/or use a modeling/simulation approach. E-Comm operators currently use existing formulas such as Erlang B and Erlang C to evaluate their network performance. However, we will show in Section 4.2 that this approach has several limitations and cannot provide reliable estimation results.

One way to predict future network traffic is to study the past traffic and extract past patterns and relationships. These patterns and relationships form a model. The modeling process provides a means of creating simplified representations of the "real network". Once the mathematical model of the target network is built, we may either use an analytical approach or a simulation approach to evaluate and predict the network performance. In this thesis, we adopt the modeling/simulation approach because of its capability to handle complex mathematical models.

To build the E-Comm traffic model, we first study the traffic data collected from the E-Comm network. We then select several candidate distributions, estimate their parameters, identify the most suitable distribution, and build a statistical model that represents the usage of the E-Comm network.

### 4.2 Introduction to traditional traffic models

#### 4.2.1 Erlang models

Erlang B and Erlang C models are widely used by call traffic engineers to plan the network resources, such as number of phone lines in Public Switched Telephone Network (PSTN), Private Branch Exchange (PBX) capacity of call centers, and number of radio
channels in cellular networks [10]. Erlang models are named after the Danish mathematician A. K. Erlang.

Erlang is used as a dimensionless unit in traffic engineering to measure traffic intensity. It measures continuous use of one circuit. For example, call traffic generated by a one hour long call is one Erlang. Call traffic generated by two 30 minutes long calls is also one Erlang.

Mathematical representations of Erlang B and Erlang C models are given by Equations (4-1) and (4-2), respectively. In both equations, \( N \) is the total number of resources in the system, such as channels or lines, and \( A \) is the total traffic volume measured in Erlang.

In Erlang B model:

\[
P_B = \frac{A^N}{\sum_{x=0}^{N} \frac{N!}{x!} A^x}, \tag{4-1}
\]

\( P_B \) is the probability that a call request will be rejected due to lack of resources.

In Erlang C model:

\[
P_C = \frac{A^N N}{\sum_{x=0}^{N-1} \frac{A^x}{x!} + \frac{A^N N}{N! (N-A)}}, \tag{4-2}
\]

\( P_C \) is the probability that a call request will be delayed in obtaining a resource.

Erlang B and Erlang C models assume (1) infinite number of call sources or that the number of call sources is much larger than the number of available channels, (2)
Poisson distributed call arrival, and (3) exponentially distributed call holding time. The Erlang B model further assumes that (4) a blocked call will be rejected immediately and (5) a user will not retry the same request after being rejected. The Erlang C model further assumes that (6) a blocked call will be put into a FIFO queue with infinite size and that (7) a user will not abandon the call request until the call is successfully completed.

4.2.2 Failure of Erlang models

In the E-Comm network, although assumptions (1) and (2) are likely to be valid, assumptions (3) through (7) are not applicable. Moreover, Erlang models were originally developed for traditional networks where there is no group call, which implies that caller and callee have a one-to-one relationship. However, calls in the E-Comm network are all group calls (one-to-many caller-to-callee relationship) and most calls use more than one channel simultaneously. These indicate that Erlang models are not suitable for the E-Comm network.

4.3 Traffic modeling on agency level

We showed previously in Table 3.2 that the majority number of calls is initiated by few user agencies. We further investigate the number of calls made by each agency during the top three busiest hours in 2002 and 2003 datasets.
Table 4.1: Aggregated number of calls during the top three busiest hours in 2002 and 2003.

<table>
<thead>
<tr>
<th>Agency ID</th>
<th>Year 2002</th>
<th>Percentage</th>
<th>Year 2003</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>n/a</td>
<td>72</td>
<td>0.4%</td>
<td>17</td>
<td>0.1%</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>0.0%</td>
<td>19</td>
<td>0.1%</td>
</tr>
<tr>
<td>2</td>
<td>7,222</td>
<td>44.2%</td>
<td>7,976</td>
<td>30.6%</td>
</tr>
<tr>
<td>3</td>
<td>363</td>
<td>2.2%</td>
<td>464</td>
<td>1.8%</td>
</tr>
<tr>
<td>4</td>
<td>60</td>
<td>0.4%</td>
<td>180</td>
<td>0.7%</td>
</tr>
<tr>
<td>5</td>
<td>6,289</td>
<td>38.5%</td>
<td>14,183</td>
<td>54.4%</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>0.0%</td>
<td>119</td>
<td>0.5%</td>
</tr>
<tr>
<td>7</td>
<td>4</td>
<td>0.0%</td>
<td>0</td>
<td>0.0%</td>
</tr>
<tr>
<td>8</td>
<td>22</td>
<td>0.1%</td>
<td>0</td>
<td>0.0%</td>
</tr>
<tr>
<td>10</td>
<td>1,878</td>
<td>11.5%</td>
<td>2,278</td>
<td>8.7%</td>
</tr>
<tr>
<td>11</td>
<td>308</td>
<td>1.9%</td>
<td>161</td>
<td>0.6%</td>
</tr>
<tr>
<td>13</td>
<td>28</td>
<td>0.2%</td>
<td>25</td>
<td>0.1%</td>
</tr>
<tr>
<td>14</td>
<td>71</td>
<td>0.4%</td>
<td>37</td>
<td>0.1%</td>
</tr>
<tr>
<td>15</td>
<td>0</td>
<td>0.0%</td>
<td>0</td>
<td>0.0%</td>
</tr>
<tr>
<td>20</td>
<td>0</td>
<td>0.0%</td>
<td>0</td>
<td>0.0%</td>
</tr>
<tr>
<td>21</td>
<td>0</td>
<td>0.0%</td>
<td>598</td>
<td>2.3%</td>
</tr>
<tr>
<td>Total</td>
<td>16,322</td>
<td>100%</td>
<td>26,057</td>
<td>100%</td>
</tr>
</tbody>
</table>

Table 4.1 indicates that during busy hours, more than 80% of the calls are initiated by Agencies 2 and 5. Since these two heavy user agencies have larger impact on the network than the remaining of the agencies, to simplify the traffic modeling process, we regroup the agencies into three new agencies and model their call traffic. Table 4.2 shows the mapping between the original and the new agency IDs.

Table 4.2: Regrouped agencies and the original agencies.

<table>
<thead>
<tr>
<th>Virtual agency ID</th>
<th>Original agency ID</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>2</td>
</tr>
<tr>
<td>B</td>
<td>5</td>
</tr>
<tr>
<td>C</td>
<td>(all other agencies)</td>
</tr>
</tbody>
</table>
4.4 Modeling call holding time

Before we start describing the modeling of call holding time, we introduce two related statistical modeling techniques: Kolmogorov-Smirnov goodness-of-fit (K-S GoF) test and Maximum Likelihood Estimation (MLE).

4.4.1 Maximum likelihood estimation

Introduced by R. A. Fisher in 1920s, MLE [9] is currently the most popular method of estimating distribution parameters. The goal of MLE is to find the parameters that make the candidate distribution fit the observed data best.

Let \( f(x | \theta) \) be the probability distribution function, where \( \theta = (\theta_1, \theta_2, ..., \theta_m) \) is the parameter vector. Likelihood of \( \theta \) is defined as

\[
\text{like}(\theta) = \prod_{i=1}^{n} f(x_i | \theta). \tag{4-3}
\]

Since the maximum product is difficult to calculate and logarithm is an increasing function, we use the log-likelihood \( l(\theta) \) instead. The parameter \( \theta \) that maximizes \( l(\theta) \) will also maximize \( \text{like}(\theta) \):

\[
l(\theta) = \log \left( \prod_{i=1}^{n} f(x_i | \theta) \right) = \sum_{i=1}^{n} \log (f(x_i | \theta)) \tag{4-4}
\]

We use the Poisson distribution as an example to demonstrate how MLE works. The Probability Density Function (PDF) for the Poisson distribution is

\[
p(x) = \frac{\lambda^x e^{-\lambda}}{x!}.
\]
The log-likelihood can be deduced as

\[ l(\lambda) = \sum_{i=1}^{n} \left( x_i \log \lambda - \lambda - \log x_i ! \right) \]

\[ = \log \lambda \sum_{i=1}^{n} x_i - n\lambda - \sum_{i=1}^{n} \log x_i ! \]  

(4-5)

To find the maximum of \( l(\theta) \), we find its derivative and set it to zero:

\[ \frac{\partial l(\lambda)}{\partial \lambda} = \frac{1}{\lambda} \sum_{i=1}^{n} x_i - n \]

\[ = 0 \]  

(4-6)

Equation (4-6) implies \( \lambda = \bar{X} \). However, in most other cases, \( l(\theta) \) is not deducible mathematically. We use a software tool called \textit{mle} [12] to estimate the parameters.

4.4.2 Kolmogorov-Smirnov goodness-of-fit test

Goodness-of-fit test (GoF) is used to statistically verify whether or not a sample dataset \( X_1, X_2, \ldots, X_n \) follows a particular distribution with distribution function \( F \). In other words, the GoF test is used to test the following null hypothesis \( H_0 \):

\( H_0: \ X_1, X_2, \ldots, X_n \) are independent random variables with a specific distribution \( F(x) \).

The Chi-square test [13] and the Kolmogorov-Smirnov test [13] are the most popular goodness-of-fit tests. The Chi-square test is suitable for testing discrete random variables, while the K-S test is more suitable for testing continuous random variables. Since our objects (call holding time and call inter-arrival time) are both modeled using continuous random variables, we use the K-S test.
The K-S GoF test is based on the empirical distribution function $F_e$. Suppose that we have $n$ independent random variables $X_1, X_2, ..., X_n$. Since the order of these variables does not affect the underlying distribution function, we further assume that $X_1, X_2, ..., X_n$ are in an increasing order. The Empirical Distribution Function (ECDF) $F_e$ is defined as:

$$F_e(x) = \frac{\text{total number of } i \text{ where } Y_i \leq x}{n}. \quad (4-7)$$

The Kolmogorov-Smirnov test statistic $D$ is defined as:

$$D = \text{Supremum} \left| F_e(x) - F(x) \right|$$

$$= \text{Supremum} \left\{ \text{Supremum} \left\{ F_e(x) - F(x) \right\}, \text{Supremum} \left\{ F(x) - F_e(x) \right\} \right\} \quad (4-8)$$

$$= \text{Supremum} \left\{ \frac{i}{n} - F(Y_i), F(Y_j) - \frac{j-1}{n}, j = 1, ..., n \right\}. \quad (4-8)$$

Equation (4-8) states that a large value of $D$ indicates a poor fit of $F_e(x)$ and $F(x)$. $p$-value, defined as $p$-value $= P_{\bar{d}} \{ D \geq d \}$, is an indicator of how likely $F(x)$ is accepted. Null hypothesis $H_0$ is likely to be accepted when $p$-value $> 0.01$ and $H_0$ is very likely to be accepted when $p$-value $> 0.05$.

4.4.3 Survey of existing models for call holding time

Over the years, a number of probability models have been developed for call holding time in various networks such as PSTN and cellular networks. Bolotin shows [14] that the exponential distribution underestimates the proportion of short calls in traditional PSTNs. Bolotin further shows [15] that the call holding time in traditional PSTNs follows lognormal distribution with a mean value of 200-300 seconds. In [16]–
Jordan and Barceló show that Erlang-\(jk\) and mixture of lognormals fit very well voice holding time in Public Access Mobile Radio (PAMR) systems with mean value of 20–40 seconds. Using field data from cellular telephony systems, Jedrzycki and Leung find that the lognormal distribution provides a better fit for the channel holding time [20]. In [21], [22], Orlik and Rappaport proposed a model called the Sum of the Hyper-Exponential (SOHYP) to model the channel holding time in data used in [20]. Fang et al., proposed the Hyper-Erlang distribution [23]–[25] to model cell residence time that may be used to better characterize channel holding time in mobile computing and Personal Communication Services (PCS) networks.

Traffic data from PSWNs are rarely available due to their confidential nature and, hence, few models of call holding time have been developed for such networks. The large difference between mean call holding times in PSWNs and call holding times in PSTN/Cellular networks suggests that call holding time models developed for other networks might not be suitable for PSWNs. A recent study of E-Comm’s aggregated traffic shows that the call holding time during busy hours fits the lognormal distribution [27]. However, to the author’s knowledge, there is no published work dealing with modeling call traffic in PSWNs on user agency level.

4.4.4 Model comparison and selection

In this Section, we investigate the call holding time of E-Comm traffic on the user agency level. The candidate distributions to be tested and their PDFs are listed in Table 4.3.
Table 4.3: Candidate distributions for call holding time.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Probability density function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential</td>
<td>( f(x) = \frac{1}{\beta} e^{-\frac{x}{\beta}} )</td>
</tr>
<tr>
<td>Lognormal</td>
<td>( f(x) = \frac{1}{x\sqrt{2\pi\sigma^2}} e^{-\frac{(\ln(x) - \mu)^2}{2\sigma^2}} )</td>
</tr>
<tr>
<td>Gamma</td>
<td>( f(x) = \beta^{-k} \frac{x^{k-1}}{(k-1)!} e^{-\frac{x}{\beta}} )</td>
</tr>
</tbody>
</table>

To fit and compare candidate distributions with E-Comm traffic data, we first extract 500 sequential data samples (call holding times) from the traffic data table. We then select a candidate distribution and use MLE to estimate its parameters. The K-S GoF test is then used to evaluate the candidate distribution. When \( p \)-value is greater than 0.03, the candidate distribution is likely to be accepted. Otherwise, the candidate distribution is likely to be rejected.

We fit candidate distributions with call holding times of Agencies A, B, and C during the top three busiest hours in the 2002 dataset. Estimated distribution parameters and K-S test results are listed in Table 4.4. Cells with \( p \)-value > 0.03 are marked in grey. K-S test results (\( p \)-values) shown in Table 4.4 indicate that the lognormal distribution is ideal to model the call holding time for Agency A.
Table 4.4: Test of candidate distributions for call holding time of Agency A during busy hours in the 2002 dataset.

<table>
<thead>
<tr>
<th>Distributions</th>
<th>Distribution parameters and K-S test results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Busy hour 1</td>
</tr>
<tr>
<td>Exponential</td>
<td></td>
</tr>
<tr>
<td></td>
<td>β = 3683.6</td>
</tr>
<tr>
<td></td>
<td>p-value = 0</td>
</tr>
<tr>
<td>Lognormal</td>
<td>σ = 8.0538</td>
</tr>
<tr>
<td></td>
<td>μ = 0.5505</td>
</tr>
<tr>
<td></td>
<td>p-value = 0.0832</td>
</tr>
<tr>
<td>Gamma</td>
<td>β = 1108.0657</td>
</tr>
<tr>
<td></td>
<td>k = 3.3246</td>
</tr>
<tr>
<td></td>
<td>p-value = 0.0002</td>
</tr>
</tbody>
</table>

K-S test results shown in Table 4.5 indicate that although the lognormal distribution does not fit the call holding time of Agency B at 3% significance level all the time, it is nevertheless adequate to model it call holding time.

Table 4.5: Test of candidate distributions for call holding time of Agency B during the top three busy hours in the 2002 dataset.

<table>
<thead>
<tr>
<th>Distributions</th>
<th>Distribution parameters and K-S test results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Busy hour 1</td>
</tr>
<tr>
<td>Exponential</td>
<td></td>
</tr>
<tr>
<td></td>
<td>β = 4269.78</td>
</tr>
<tr>
<td></td>
<td>p-value = 0</td>
</tr>
<tr>
<td>Lognormal</td>
<td>σ = 8.0882</td>
</tr>
<tr>
<td></td>
<td>μ = 0.7256</td>
</tr>
<tr>
<td></td>
<td>p-value = 0.0258</td>
</tr>
<tr>
<td>Gamma</td>
<td>β = 2141.4626</td>
</tr>
<tr>
<td></td>
<td>k = 1.994</td>
</tr>
<tr>
<td></td>
<td>p-value = 0</td>
</tr>
</tbody>
</table>

K-S test results shown in Table 4.6 indicate that no candidate distribution fits the call holding time of Agency C at satisfying significance levels. However, the lognormal distribution still performs better than other candidate distributions.
Table 4.6: Test of candidate distributions for call holding time of Agency C during the top three busy hours in the 2002 dataset.

<table>
<thead>
<tr>
<th>Distributions</th>
<th>Distribution parameters and K-S test results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Busy hour 1</td>
</tr>
<tr>
<td>Exponential</td>
<td>$\beta = 3505.58$</td>
</tr>
<tr>
<td></td>
<td>p-value = 0</td>
</tr>
<tr>
<td>Lognormal</td>
<td>$\sigma = 7.8759$</td>
</tr>
<tr>
<td></td>
<td>$\mu = 0.8154$</td>
</tr>
<tr>
<td></td>
<td>p-value = 0</td>
</tr>
<tr>
<td>Gamma</td>
<td>$\beta = 1848.5619$</td>
</tr>
<tr>
<td></td>
<td>$k = 1.8965$</td>
</tr>
<tr>
<td></td>
<td>p-value = 0</td>
</tr>
</tbody>
</table>

Because the two heavy user agencies (Agencies A and B) account for 80–90% of the total call traffic and their call holding time follows lognormal distribution, we adopt the lognormal distribution to model call holding time in simulations.

4.5 Modeling call inter-arrival time

4.5.1 Survey of existing models for call inter-arrival time

The Poisson model for call arrival time is widely used in PSTNs [28]. It implies that the inter-arrival time is exponentially distributed. Exponentially distributed call inter-arrival time was originally assumed in cellular networks. It was shown [29]–[31] that exponential distribution is only suitable for modeling inter-arrival times of “fresh” (new) calls in micro-cellular networks. However, exponential distribution is not suitable for modeling all (new and hand-over) call inter-arrivals in micro-cellular networks due to the frequent call handovers.
The E-Comm network is a macro-cellular network where call handover rarely happens. Call inter-arrival time in the E-Comm network was found to be exponentially distributed [27].

4.5.2 Model comparison and selection

In this Section, we investigate the call inter-arrival time of E-Comm traffic on the user agency level. Candidate distributions are listed in Table 4.7.

Table 4.7: Candidate distributions for call inter-arrival time.

<table>
<thead>
<tr>
<th>Distribution</th>
<th>Probability density function</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exponential</td>
<td>$f(x) = \frac{1}{\beta} e^{-\frac{x}{\beta}}$</td>
</tr>
<tr>
<td>Lognormal</td>
<td>$f(x) = \frac{1}{x\sqrt{2\pi\sigma^2}} e^{-\frac{(\ln(x) - \mu)^2}{2\sigma^2}}$</td>
</tr>
<tr>
<td>Gamma</td>
<td>$f(x) = \beta^k x^{k-1} e^{-\frac{x}{\beta}} (k-1)!$</td>
</tr>
</tbody>
</table>

The method we employ to evaluate candidate distributions of call inter-arrival time is similar to the method used to evaluate call holding time. Estimated distribution parameters and K-S test results are listed in Tables 4.8–4.10. Cells with $p$-value $> 0.03$ are marked in grey.

The results shown in Tables 4.8 and 4.9 indicate that both exponential distribution and gamma distribution are adequate to model the call inter-arrival time of Agencies A and B. This is expected since the gamma distribution is a more general form of exponential distribution. We use the exponentially distributed call inter-arrival time in later simulations because of its simplicity.
Results shown in Table 4.10 indicate that although Gamma distribution may occasionally model the distribution of call inter-arrival time of the light user Agency C, no candidate distribution is likely to be accepted. We assume exponentially distributed call inter-arrival time for Agency C in simulations because Agency C accounts for only approximately 10% of the call traffic and has much less impact on E-Comm network than Agencies A and B.
Table 4.10: Test of candidate distributions for call inter-arrival time of Agency C during the top three busy hours in the 2002 dataset.

<table>
<thead>
<tr>
<th>Distributions</th>
<th>Distribution parameters and K-S test results</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Busy hour 1</td>
</tr>
<tr>
<td>Exponential</td>
<td>$\beta = 4941.8036$</td>
</tr>
<tr>
<td></td>
<td>p-value $= 0$</td>
</tr>
<tr>
<td>Lognormal</td>
<td>$\sigma = 7.7809$</td>
</tr>
<tr>
<td></td>
<td>$\mu = 1.2987$</td>
</tr>
<tr>
<td></td>
<td>p-value $= 0.0001$</td>
</tr>
<tr>
<td>Gamma</td>
<td>$\beta = 6050.6496$</td>
</tr>
<tr>
<td></td>
<td>k $= 0.8168$</td>
</tr>
<tr>
<td></td>
<td>p-value $= 0$</td>
</tr>
</tbody>
</table>

### 4.6 Call coverage pattern

The call coverage pattern is important in simulating the E-Comm network because all calls in the E-Comm network are group calls and most of them cover more than one system. As previously discussed in Section 2.1.6, we might underestimate the call blocking probability if we ignore the group calls and investigate only one system.

The call coverage pattern in the E-Comm network is largely determined by the organizations of talk groups and the radio device deployment of user agencies. Factors that affect the call coverage pattern are unknown. In simulations of the E-Comm network, we assume that the pattern of call coverage and the corresponding probability of occurrences remain unchanged. A sample of call coverage pattern is shown in Table 4.11.

Table 4.11: A sample of call coverage pattern of Agency A during the top three busy hours in 2003.

<table>
<thead>
<tr>
<th>Call coverage</th>
<th>No. of occurrences</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>System No. 1, 2, 3</td>
<td>45</td>
<td>1.35%</td>
</tr>
<tr>
<td>System No. 1, 3, 8, 10, 11</td>
<td>69</td>
<td>2.07%</td>
</tr>
<tr>
<td>System No. 3, 5, 7, 9, 10</td>
<td>236</td>
<td>7.08%</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>
CHAPTER 5: WARNSIM: A SIMULATOR FOR PSWN

In this Chapter, we introduce the design, implementation, and validation of WarnSim.

5.1 Overview

Several network simulation tools are currently available for simulating packet switched networks: OPNET [32], ns-2 [33], and J-SIM [34]. However, to the author’s knowledge, there is no simulation software tool publicly available for simulating circuit switched networks such as PSWNs. Moreover, it is inefficient to use a simulator designed for packet switched networks to mimic and simulate circuit switched networks. Hence, we developed WarnSim, an effective, flexible, and easy to use simulation tool to simulate PSWN systems.

WarnSim is a publicly available tool designed to simulate circuit switched radio networks [35], [36]. It can simulate various network sizes (number of cells in a network) and cell capacities (number of channels in cells). Traffic traces are easily imported or can be modeled with a variety of statistical distributions. These features make WarnSim preferable for simulating wide area radio networks, such as PSWNs.

WarnSim is developed using Microsoft Visual C# .NET, which has a comparable performance to Java [37]. WarnSim works on Windows platforms with .NET framework support and comes with a graphical user interface.
5.2 Modules and call flowchart

WarnSim consists of seven modules, shown in Figure 5.1.

![Diagram of WarnSim modules]

Figure 5.1: WarnSim: (a) module hierarchical diagram and (b) relationship diagram.

*Network module* models the cells of a PSWN. A network module may consist of several cells. Each cell has one or more channels with the attribute “busy duration”. A channel is free if its “busy duration” is zero. Otherwise, the channel is occupied for the duration of time specified by the “busy duration”. The network module distributes calls to cells, tracks the status of the system (number of free or occupied channels), and periodically updates the channel status in every cell.
*Call traffic module* prepares call traffic for simulation. It can import traffic trace files from text files/databases or generate call traffic based on user-defined distributions and combine them during a single simulation run.

*Call admission control module* uses the calls from the call traffic module and checks with the network module to determine if there are available channels for a call to be established. It also manages the retrying mechanism of blocked calls.

*Simulation configuration module* keeps track of call queuing mechanism and parameters, such as the maximum call queuing time, simulation duration, and simulation granularity.

*Simulation process module* uses a global timer to control and synchronize the operation of the WarnSim modules.

*Simulation statistics module* collects real-time and summary statistics (number of calls, blocked calls, call blocking probability, channel utilization, cumulative number of calls, cumulative blocked calls, cumulative call blocking probability, and cumulative channel utilization) from other modules. It is also used to display and visualize simulation results.

*Random variable module* generates random numbers and random variables. We adopted the MT random number generator [38], which is suitable for stochastic simulations. We also implemented seven types of random variables: uniform, exponential, gamma, normal, lognormal, loglogistic, and Weibull.
A high level diagram illustrating the call flow mechanism of WarnSim is given in Figure 5.2. The basic functionality of WarnSim is described by the pseudo-code, as shown in Figure 5.3.
Figure 5.2: High level diagram of the WarnSim simulator.
Load call traffic traces from the call generator or text files/databases;
Save call traffic traces into FutureCallList and sort them by CallArrivalTime;

for (SimTime=SimStartTime; SimTime < SimEndTime; SimTime++)
{
    Update network status (system/channel status) and statistics;
    Move calls with (CallArrivalTime<SimTime) from FutureCallList/FeedbackCallList to MainCallQueue;

    for (each call in MainCallQueue)
    {
        if (there are enough channels in network to establish that call)
            Call is assigned to network and is removed from MainCallQueue;
        else if (CallArrivalTime-SimTime >= MaxQueuingTime)
        {
            if (Call feedback mechanism is enabled)
                Generate a feedback call and add it to FeedbackCallList;
            else
                Call is discarded;
        }
    }
}
5.3 WarnSim simulation steps

Most simulations with WarnSim include five main steps:

Step 1: Network topology setup

add/remove/configure systems

load/save network topology

Step 2: Traffic sources setup

add/remove/configure traffic sources from a text file or a database

add/remove/configure call traffic generators

load/save traffic sources

Step 3: Simulation parameters configuration

configure the length (time span) of simulation

configure calls queuing mechanism

configure dropped calls retrying mechanism

configure statistics collection interval

Step 4: Run simulation

run/stop simulation

pause simulation to view intermediate simulation results

Step 5: Analyze simulation results

list/save simulation results
5.4 WarnSim interface

WarnSim includes a graphical user interface. Figures 5.4–5.8 show four WarnSim screens during a simulation experiment. Systems in WarnSim are configured in a star network topology, as shown in Figure 5.4 (a). Figure 5.4 (b) shows the WarnSim network configuration screen capturing network information (cells and number of channels). A popup window used to configure the system ID, system name, and number of channels is also shown in Figure 5.4 (b). Figures 5.5 and 5.6 show the configuration screens for traffic sources. A list of traffic sources and popup windows used to import/generate/configure traffic traces are also shown. Figures 5.7 (a) and (b) capture the simulation results screens for channel utilization vs. simulation time. Figures 5.8 (a) and (b) show the simulation results for discarded calls vs. simulation time.
Figure 5.4: WarnSim: (a) network topology and (b) network configuration screen.
Figure 5.5: WarnSim traffic source configuration screen: importing traffic.

Figure 5.6: WarnSim traffic source configuration screen: generating traffic.
Figure 5.7: WarnSim simulation results screens for channel utilization of Vancouver system during: (a) a sample week in 2002 and (b) a sample week in 2003. The graphs show the running average calculated over ten-minute intervals.
Figure 5.8: WarnSim simulation results screens for the cumulative number of blocked calls in Vancouver system during: (a) a sample week in 2002 and (b) a sample week in 2003.
5.5 Validation of WarnSim simulator

We performed several steps to validate WarnSim. We first use build-in K-S GoF functions provided by MATLAB [39] and S-PLUS [40] to test the random variable generator implemented in WarnSim. All K-S GoF tests return high p-values (> 0.1), which validates the WarnSim generated random variables.

We then use various artificial testing traces to validate the deterministic WarnSim modules, such as call admission control module and simulation statistics module. A simple example of the testing scenario is to let 11 calls simultaneously reach a system with 10 channels. As expected, one call will be queued due to the lack of a free channel.

We further validate WarnSim by comparing the prediction results of Erlang B model with WarnSim simulation results. As shown in Table 5.1, call blocking probabilities match very well.

<table>
<thead>
<tr>
<th></th>
<th>Erlang B model</th>
<th>WarnSim</th>
</tr>
</thead>
<tbody>
<tr>
<td>Network configuration</td>
<td>10 phone lines</td>
<td>1 system with 10 channels</td>
</tr>
<tr>
<td>Call traffic volume</td>
<td>10 Erlangs</td>
<td>10 Erlangs</td>
</tr>
<tr>
<td>Call holding time</td>
<td>exponentially distributed</td>
<td>exponentially distributed with mean value of 180 seconds</td>
</tr>
<tr>
<td>Call inter-arrival time</td>
<td>exponentially distributed</td>
<td>exponentially distributed with mean value of 18 seconds</td>
</tr>
<tr>
<td>Dealing with blocked calls</td>
<td>blocked calls neither queued nor retried</td>
<td>Max Queuing Time = 0; blocked calls are not retried</td>
</tr>
<tr>
<td>Call blocking probability</td>
<td>21.5%</td>
<td>17% – 27%; average=21.86%; (10 simulation runs)</td>
</tr>
</tbody>
</table>
5.6 Scalability and performance of WarnSim

WarnSim provides good flexibility in choosing the network size (the number of cells in the network) and cell capacities (the number of channels in each cell). In our simulation experiments, WarnSim successfully handled sample networks with 100 cells and 50 channels in each cell. They are approximately ten times larger than the size of the E-Comm network and are sufficient to model any deployed PSWNs.

WarnSim simulation tests are performed on a windows workstation equipped with Intel Pentium 4 2.8GHz CPU and 512Mbyte memory. The operating system is Microsoft Windows XP Professional with .NET framework 1.1 supports. On this platform, a simulation with one hour of busy hour traffic from the E-Comm PSWN lasts less than one minute. The high performance of WarnSim enables users to quickly test various network configurations and call traffic, and to efficiently perform trace driven simulations and evaluate network performances during busy hours.
CHAPTER 6: SIMULATION AND PREDICTION

In this Chapter, we use WarnSim to validate the proposed traffic model and to evaluate and predict E-Comm network performance.

6.1 Validation of the proposed traffic model

To validate the proposed traffic model, we first use WarnSim to generate traffic traces and compare them with the collected E-Comm traffic data. WarnSim distribution parameters listed in Table 6.1 are used for the call generator to model busy hour traffic of the 2003 dataset.

<table>
<thead>
<tr>
<th></th>
<th>Agency A</th>
<th>Agency B</th>
<th>Agency C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call holding time</td>
<td>Lognormal ( \sigma = 8.05 )</td>
<td>Lognormal ( \sigma = 8.09 )</td>
<td>Lognormal ( \sigma = 7.88 )</td>
</tr>
<tr>
<td></td>
<td>( \mu = 0.55 )</td>
<td>( \mu = 0.73 )</td>
<td>( \mu = 0.82 )</td>
</tr>
<tr>
<td>Call inter-arrival time</td>
<td>Exponential ( \beta = 1354 )</td>
<td>Exponential ( \beta = 761 )</td>
<td>Exponential ( \beta = 3480 )</td>
</tr>
</tbody>
</table>

In the simulation scenarios, systems and channels shown in Table 6.2 reflect the configuration of the E-Comm network during the busiest hour in 2003.

<table>
<thead>
<tr>
<th>System ID</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channels</td>
<td>10</td>
<td>7</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>7</td>
<td>8</td>
<td>4</td>
<td>7</td>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>
Simulation results listed in Table 6.3 indicate that the proposed traffic model performs adequately when used to evaluate call blocking probability and channel utilization in the E-Comm network.

Table 6.3: Comparison of network performance: using collected traffic (actual) vs. model generated traffic (simulated).

<table>
<thead>
<tr>
<th>System ID</th>
<th>Actual blocking probability (%)</th>
<th>Actual channel utilization (%)</th>
<th>Simulated blocking probability (%)</th>
<th>Simulated channel utilization (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.9 – 3.5</td>
<td>57 – 65</td>
<td>2.9 – 3.9</td>
<td>53 – 56</td>
</tr>
<tr>
<td>2</td>
<td>0.0 – 0.6</td>
<td>29 – 48</td>
<td>0.8 – 1.1</td>
<td>34 – 37</td>
</tr>
<tr>
<td>3</td>
<td>0.0</td>
<td>11 – 14</td>
<td>0.0</td>
<td>11</td>
</tr>
<tr>
<td>4</td>
<td>0.0 – 0.4</td>
<td>21 – 23</td>
<td>0.4 – 1.3</td>
<td>21 – 26</td>
</tr>
<tr>
<td>5</td>
<td>0.0</td>
<td>4 – 17</td>
<td>0.0 – 1.1</td>
<td>10 – 11</td>
</tr>
<tr>
<td>6</td>
<td>0.0 – 0.3</td>
<td>19 – 42</td>
<td>0.1 – 0.3</td>
<td>27 – 29</td>
</tr>
<tr>
<td>7</td>
<td>0.0 – 0.4</td>
<td>25 – 34</td>
<td>0.0 – 0.2</td>
<td>25 – 27</td>
</tr>
<tr>
<td>8</td>
<td>0.0</td>
<td>8 – 11</td>
<td>0.0</td>
<td>9 – 10</td>
</tr>
<tr>
<td>9</td>
<td>0.3 – 0.5</td>
<td>37 – 43</td>
<td>1.1 – 2.0</td>
<td>36 – 39</td>
</tr>
<tr>
<td>10</td>
<td>0.0</td>
<td>16 – 26</td>
<td>0.1 – 0.2</td>
<td>20 – 22</td>
</tr>
<tr>
<td>11</td>
<td>0.0</td>
<td>6 – 10</td>
<td>0.0</td>
<td>6 – 8</td>
</tr>
</tbody>
</table>

6.2 Evaluation of E-Comm network performance

We can evaluate the Grade of Service (GoS) of PSWN by performing WarnSim simulations using genuine traffic data. In this Section, we first describe the simulation scenarios and source traffic traces. We then describe the evaluation results of two GoS standards in the E-Comm network: call blocking probability and channel utilization.

6.2.1 Network performance during busy hours

Three simulation scenarios are setup to evaluate the GoS of the E-Comm network during the top three busiest hours in 2003. Systems and channels reflect the
corresponding configurations of the E-Comm network, as shown in Table 6.4. The source traffic traces are extracted from the top three busiest hours in 2003 dataset and are transformed into the format shown in Table 3.1. Maximum queuing time of blocked call is set to zero. Simulation statistics including channel utilization, total number of calls, number of blocked calls, and call blocking probability for each system are collected by WarnSim. Simulation results shown in Tables 6.5–6.7 indicate that System 1 is the busiest system in the E-Comm network in terms of both hourly call arrival rate and call blocking probability. Few calls may be blocked in Systems 2, 4, 6, 7, and 9. In other systems, calls never got blocked and channel utilizations are relatively low.

Table 6.4: System IDs and number of channels during the top three busy hours in the 2003 dataset.

<table>
<thead>
<tr>
<th>System ID</th>
<th>No. of channels in busy hour 1</th>
<th>No. of channels in busy hour 2</th>
<th>No. of channels in busy hour 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>2</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>5</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>6</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
<td>8</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>4</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>9</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>10</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>
Table 6.5: Simulation results of the busiest hour in the 2003 dataset.

<table>
<thead>
<tr>
<th>System ID</th>
<th>Channel utilization</th>
<th>Total No. of calls</th>
<th>No. of blocked calls</th>
<th>Blocking Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>57%</td>
<td>5,070</td>
<td>95</td>
<td>1.9%</td>
</tr>
<tr>
<td>2</td>
<td>48%</td>
<td>2,907</td>
<td>16</td>
<td>0.6%</td>
</tr>
<tr>
<td>3</td>
<td>13%</td>
<td>426</td>
<td>0</td>
<td>0.0%</td>
</tr>
<tr>
<td>4</td>
<td>23%</td>
<td>1,133</td>
<td>5</td>
<td>0.4%</td>
</tr>
<tr>
<td>5</td>
<td>17%</td>
<td>490</td>
<td>0</td>
<td>0.0%</td>
</tr>
<tr>
<td>6</td>
<td>42%</td>
<td>2,602</td>
<td>9</td>
<td>0.3%</td>
</tr>
<tr>
<td>7</td>
<td>31%</td>
<td>2,150</td>
<td>0</td>
<td>0.0%</td>
</tr>
<tr>
<td>8</td>
<td>11%</td>
<td>391</td>
<td>0</td>
<td>0.0%</td>
</tr>
<tr>
<td>9</td>
<td>43%</td>
<td>2,804</td>
<td>12</td>
<td>0.4%</td>
</tr>
<tr>
<td>10</td>
<td>26%</td>
<td>1,368</td>
<td>0</td>
<td>0.0%</td>
</tr>
<tr>
<td>11</td>
<td>10%</td>
<td>301</td>
<td>0</td>
<td>0.0%</td>
</tr>
</tbody>
</table>

Table 6.6: Simulation results of the second busiest hour in the 2003 dataset.

<table>
<thead>
<tr>
<th>System ID</th>
<th>Channel utilization</th>
<th>Total No. of calls</th>
<th>No. of blocked calls</th>
<th>Blocking Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>65%</td>
<td>5,790</td>
<td>205</td>
<td>3.5%</td>
</tr>
<tr>
<td>2</td>
<td>29%</td>
<td>1,676</td>
<td>0</td>
<td>0.0%</td>
</tr>
<tr>
<td>3</td>
<td>14%</td>
<td>491</td>
<td>0</td>
<td>0.0%</td>
</tr>
<tr>
<td>4</td>
<td>21%</td>
<td>949</td>
<td>0</td>
<td>0.0%</td>
</tr>
<tr>
<td>5</td>
<td>7%</td>
<td>197</td>
<td>0</td>
<td>0.0%</td>
</tr>
<tr>
<td>6</td>
<td>19%</td>
<td>1,185</td>
<td>0</td>
<td>0.0%</td>
</tr>
<tr>
<td>7</td>
<td>25%</td>
<td>1,773</td>
<td>0</td>
<td>0.0%</td>
</tr>
<tr>
<td>8</td>
<td>9%</td>
<td>324</td>
<td>0</td>
<td>0.0%</td>
</tr>
<tr>
<td>9</td>
<td>37%</td>
<td>2,453</td>
<td>12</td>
<td>0.5%</td>
</tr>
<tr>
<td>10</td>
<td>22%</td>
<td>1,202</td>
<td>0</td>
<td>0.0%</td>
</tr>
<tr>
<td>11</td>
<td>7%</td>
<td>204</td>
<td>0</td>
<td>0.0%</td>
</tr>
</tbody>
</table>
6.2.2 Number of channels and Grade of Service

We now investigate the relationship between the number of available channels and the GoS (channel utilization and call blocking probability). The second busiest hour in 2003 dataset is chosen because it has the highest call blocking probability in System 1 among the top three busiest hours in the 2003 dataset. While keeping the number of channels in the remaining systems unchanged, we vary the number of available channels in System 1 from 1 to 14. WarnSim simulation results are collected from System 1. In this scenario, the maximum queuing time of blocked calls is set to zero.

Simulation results shown in Table 6.8 and Figure 6.1 depict the relationship between the number of channels and the call blocking probability. When the number of channels increases from 1 to 8, call blocking probability drops to 12% at a near constant rate as shown in Figure 6.1. It then decreases slowly to zero when the number of channels reaches 14. The channel utilization drops slowly when the number of channels increases.

<table>
<thead>
<tr>
<th>System ID</th>
<th>Channel utilization</th>
<th>Total No. of calls</th>
<th>No. of blocked calls</th>
<th>Blocking probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60%</td>
<td>5,484</td>
<td>169</td>
<td>3.0%</td>
</tr>
<tr>
<td>2</td>
<td>34%</td>
<td>2,131</td>
<td>1</td>
<td>0.0%</td>
</tr>
<tr>
<td>3</td>
<td>11%</td>
<td>459</td>
<td>0</td>
<td>0.0%</td>
</tr>
<tr>
<td>4</td>
<td>21%</td>
<td>1,045</td>
<td>0</td>
<td>0.0%</td>
</tr>
<tr>
<td>5</td>
<td>4%</td>
<td>106</td>
<td>0</td>
<td>0.0%</td>
</tr>
<tr>
<td>6</td>
<td>25%</td>
<td>1,677</td>
<td>1</td>
<td>0.1%</td>
</tr>
<tr>
<td>7</td>
<td>34%</td>
<td>2,141</td>
<td>8</td>
<td>0.4%</td>
</tr>
<tr>
<td>8</td>
<td>8%</td>
<td>353</td>
<td>0</td>
<td>0.0%</td>
</tr>
<tr>
<td>9</td>
<td>37%</td>
<td>2,316</td>
<td>8</td>
<td>0.3%</td>
</tr>
<tr>
<td>10</td>
<td>16%</td>
<td>960</td>
<td>0</td>
<td>0.0%</td>
</tr>
<tr>
<td>11</td>
<td>6%</td>
<td>178</td>
<td>0</td>
<td>0.0%</td>
</tr>
</tbody>
</table>
from 1 to 8 and then it drops at a constant rate. Based on these results, we can determine the number of necessary channels in order to meet a certain Grade of Service.

Table 6.8: Relationship between number of channels, call blocking probability, and channel utilization. Maximum queuing time is set to zero.

<table>
<thead>
<tr>
<th>No. of channels</th>
<th>Channel utilization (%)</th>
<th>Blocking probability (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>88</td>
<td>86.5</td>
</tr>
<tr>
<td>2</td>
<td>87</td>
<td>73.6</td>
</tr>
<tr>
<td>3</td>
<td>87</td>
<td>61.0</td>
</tr>
<tr>
<td>4</td>
<td>85</td>
<td>49.5</td>
</tr>
<tr>
<td>5</td>
<td>83</td>
<td>38.0</td>
</tr>
<tr>
<td>6</td>
<td>81</td>
<td>27.9</td>
</tr>
<tr>
<td>7</td>
<td>78</td>
<td>19.5</td>
</tr>
<tr>
<td>8</td>
<td>74</td>
<td>12.3</td>
</tr>
<tr>
<td>9</td>
<td>69</td>
<td>6.8</td>
</tr>
<tr>
<td>10</td>
<td>64</td>
<td>3.4</td>
</tr>
<tr>
<td>11</td>
<td>59</td>
<td>1.3</td>
</tr>
<tr>
<td>12</td>
<td>54</td>
<td>0.4</td>
</tr>
<tr>
<td>13</td>
<td>50</td>
<td>0.1</td>
</tr>
<tr>
<td>14</td>
<td>46</td>
<td>0.0</td>
</tr>
</tbody>
</table>
6.2.3 **Maximum queuing time and Grade of Service**

An effective way to lower the call blocking probability without adding new channels is using call queuing mechanism. The call queuing mechanism implies that instead of dropping a call immediately after it failed to obtain a free channel, the call is placed in a waiting queue for a certain amount of time. The call is dropped if it cannot get the required channels after waiting for the amount of time specified by the maximum queuing time.

We simulate scenarios where maximum queuing time increases from 0 to 1.5 seconds and collect call blocking probability statistics. The second busiest hour in the 2003 dataset is chosen for simulation. Simulation results shown in Table 6.9 and Figure 6.2 indicate that increasing the maximum queuing time can effectively lower the call
blocking probability. The call blocking probability of System 1 is lowered by more than 80% when the maximum queuing time is increased from 0 to 1 second.

Table 6.9: Maximum call queuing time vs. call blocking probability.

<table>
<thead>
<tr>
<th>Maximum queuing time (ms)</th>
<th>Call blocking probability (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.4</td>
</tr>
<tr>
<td>100</td>
<td>2.7</td>
</tr>
<tr>
<td>200</td>
<td>2.3</td>
</tr>
<tr>
<td>300</td>
<td>1.8</td>
</tr>
<tr>
<td>400</td>
<td>1.6</td>
</tr>
<tr>
<td>500</td>
<td>1.4</td>
</tr>
<tr>
<td>600</td>
<td>1.0</td>
</tr>
<tr>
<td>700</td>
<td>0.9</td>
</tr>
<tr>
<td>800</td>
<td>0.7</td>
</tr>
<tr>
<td>900</td>
<td>0.6</td>
</tr>
<tr>
<td>1,000</td>
<td>0.5</td>
</tr>
<tr>
<td>1,100</td>
<td>0.4</td>
</tr>
<tr>
<td>1,200</td>
<td>0.3</td>
</tr>
<tr>
<td>1,300</td>
<td>0.3</td>
</tr>
<tr>
<td>1,400</td>
<td>0.3</td>
</tr>
<tr>
<td>1,500</td>
<td>0.2</td>
</tr>
</tbody>
</table>
6.3 Prediction of the E-Comm network performance

In this Section, we predict the E-Comm network performance in terms of call blocking probability in cases of increased call traffic volume. Two case scenarios are investigated. The system IDs and the number of channels for both scenarios are shown in Table 6.10. The maximum queuing time in both scenarios is set to zero.

Table 6.10: System IDs and number of channels.

<table>
<thead>
<tr>
<th>System ID</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>Channels</td>
<td>10</td>
<td>7</td>
<td>4</td>
<td>5</td>
<td>3</td>
<td>7</td>
<td>8</td>
<td>4</td>
<td>7</td>
<td>6</td>
<td>3</td>
</tr>
</tbody>
</table>

6.3.1 Simulation scenario one

We first consider a sample scenario where the number of calls made by Agency A increases by 100% from the 2003 busy hours. WarnSim distribution parameters for the call generator are listed in Table 6.11.
Table 6.11: Scenario one: distribution parameters for the WarnSim call generator.

<table>
<thead>
<tr>
<th></th>
<th>Agency A</th>
<th>Agency B</th>
<th>Agency C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call holding time</td>
<td>Lognormal</td>
<td>lognormal</td>
<td>lognormal</td>
</tr>
<tr>
<td></td>
<td>$\sigma = 8.05$</td>
<td>$\sigma = 8.09$</td>
<td>$\sigma = 7.88$</td>
</tr>
<tr>
<td></td>
<td>$\mu = 0.55$</td>
<td>$\mu = 0.73$</td>
<td>$\mu = 0.82$</td>
</tr>
<tr>
<td>Call inter-arrival time</td>
<td>Exponential</td>
<td>exponential</td>
<td>exponential</td>
</tr>
<tr>
<td></td>
<td>$\beta = 677$</td>
<td>$\beta = 761$</td>
<td>$\beta = 3480$</td>
</tr>
</tbody>
</table>

The simulation results shown in Table 6.12 indicate that the call blocking probability of Systems 1 visibly increases after the call traffic increase.

Table 6.12: Comparison of network performance. Scenario one: using collected traffic (original) vs. model generated traffic after the increase (predicted). Cells with call blocking probability higher than 3% are marked in grey.

<table>
<thead>
<tr>
<th>System ID</th>
<th>Original blocking probability (%)</th>
<th>Original channel utilization (%)</th>
<th>Predicted blocking probability (%)</th>
<th>Predicted channel utilization (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.9 – 3.5</td>
<td>57 – 65</td>
<td>12.5 – 13.5</td>
<td>71 – 72</td>
</tr>
<tr>
<td>2</td>
<td>0.0 – 0.6</td>
<td>29 – 48</td>
<td>0.8 – 1.1</td>
<td>35 – 37</td>
</tr>
<tr>
<td>3</td>
<td>0.0</td>
<td>11 – 14</td>
<td>0.0</td>
<td>11 – 12</td>
</tr>
<tr>
<td>4</td>
<td>0.0 – 0.4</td>
<td>21 – 23</td>
<td>0.1 – 0.2</td>
<td>22</td>
</tr>
<tr>
<td>5</td>
<td>0.0</td>
<td>4 – 17</td>
<td>0.0 – 1.8</td>
<td>7 – 11</td>
</tr>
<tr>
<td>6</td>
<td>0.0 – 0.3</td>
<td>19 – 42</td>
<td>0.3 – 0.5</td>
<td>28 – 31</td>
</tr>
<tr>
<td>7</td>
<td>0.0 – 0.4</td>
<td>25 – 34</td>
<td>0.0 – 0.1</td>
<td>28 – 30</td>
</tr>
<tr>
<td>8</td>
<td>0.0</td>
<td>8 – 11</td>
<td>0.0</td>
<td>10</td>
</tr>
<tr>
<td>9</td>
<td>0.3 – 0.5</td>
<td>37 – 43</td>
<td>1.3 – 2.0</td>
<td>39 – 40</td>
</tr>
<tr>
<td>10</td>
<td>0.0</td>
<td>16 – 26</td>
<td>0.0 – 0.1</td>
<td>21 – 22</td>
</tr>
<tr>
<td>11</td>
<td>0.0</td>
<td>6 – 10</td>
<td>0.0 – 0.8</td>
<td>7 – 8</td>
</tr>
</tbody>
</table>

Further WarnSim simulations reveal that System 1 requires 4 additional channels in order to maintain call blocking probabilities below the required 3% [7].
6.3.2 Simulation scenario two

We now consider the sample scenario where the number of calls made by Agency B increases by 100% from 2003 busy hours. WarnSim distribution parameters for the call generator are listed in Table 6.13.

Table 6.13: Scenario two: distribution parameters for the WarnSim call generator.

<table>
<thead>
<tr>
<th></th>
<th>Agency A</th>
<th>Agency B</th>
<th>Agency C</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call holding time</td>
<td>Lognormal ( \sigma = 8.05 ) ( \mu = 0.55 )</td>
<td>lognormal ( \sigma = 8.09 ) ( \mu = 0.73 )</td>
<td>lognormal ( \sigma = 7.88 ) ( \mu = 0.82 )</td>
</tr>
<tr>
<td>Call inter-arrival time</td>
<td>exponential ( \beta = 1354 )</td>
<td>exponential ( \beta = 381 )</td>
<td>exponential ( \beta = 3480 )</td>
</tr>
</tbody>
</table>

The simulation results shown in Table 6.14 indicate that the call blocking probabilities of four systems (IDs 1, 2, 4, and 9) visibly increase after the call traffic increase.

Further simulations reveal that in order to maintain call blocking probabilities of all systems below 3%, Systems 1, 2, 4, and 9 require 4, 2, 1, and 3 additional channels, respectively.
Table 6.14: Comparison of network performance. Scenario two: using collected traffic (original) vs. model generated traffic after the increase (predicted). Cells with call blocking probability higher than 3% are marked in grey.

<table>
<thead>
<tr>
<th>System ID</th>
<th>Original blocking probability (%)</th>
<th>Original channel utilization (%)</th>
<th>Predicted blocking probability (%)</th>
<th>Predicted channel utilization (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.9 - 3.5</td>
<td>57 - 65</td>
<td>12.1 - 12.6</td>
<td>71 - 72</td>
</tr>
<tr>
<td>2</td>
<td>0.0 - 0.6</td>
<td>29 - 48</td>
<td>5.1 - 7.0</td>
<td>54 - 55</td>
</tr>
<tr>
<td>3</td>
<td>0.0</td>
<td>11 - 14</td>
<td>0.1 - 0.4</td>
<td>16 - 17</td>
</tr>
<tr>
<td>4</td>
<td>0.0 - 0.4</td>
<td>21 - 23</td>
<td>1.5 - 3.7</td>
<td>35 - 39</td>
</tr>
<tr>
<td>5</td>
<td>0.0</td>
<td>4 - 17</td>
<td>1.0 - 1.3</td>
<td>16 - 18</td>
</tr>
<tr>
<td>6</td>
<td>0.0 - 0.3</td>
<td>19 - 42</td>
<td>2.1 - 2.7</td>
<td>44 - 45</td>
</tr>
<tr>
<td>7</td>
<td>0.0 - 0.4</td>
<td>25 - 34</td>
<td>0.6 - 0.8</td>
<td>38 - 40</td>
</tr>
<tr>
<td>8</td>
<td>0.0</td>
<td>8 - 11</td>
<td>0.0 - 0.3</td>
<td>16 - 18</td>
</tr>
<tr>
<td>9</td>
<td>0.3 - 0.5</td>
<td>37 - 43</td>
<td>9.0 - 10.1</td>
<td>60 - 62</td>
</tr>
<tr>
<td>10</td>
<td>0.0</td>
<td>16 - 26</td>
<td>1.3 - 1.5</td>
<td>35 - 38</td>
</tr>
<tr>
<td>11</td>
<td>0.0</td>
<td>6 - 10</td>
<td>0.3 - 0.9</td>
<td>11 - 13</td>
</tr>
</tbody>
</table>
CHAPTER 7: CONCLUSIONS AND FUTURE WORK

In this Chapter, we conclude this thesis and describe possible future work.

7.1 Conclusions and contributions

In this thesis, we described the structure of the E-Comm network and the collection of traffic data. We then analyzed the E-Comm traffic data and characterized its properties such as the daily and weekly cyclic patterns of call arrival rate and call holding time. We proposed a statistical model of call traffic on the user agency level. The lognormal distribution proved to be appropriate for modeling call holding time, while the exponential distribution was suitable for modeling call inter-arrival time. We also addressed the call coverage issue of group calls.

In order to evaluate and predict the performance of the E-Comm network, we developed a new simulation tool named WarnSim. With WarnSim simulations, we validated the proposed traffic model and evaluated the performance of the E-Comm network. We found that the E-Comm network is capable of handling busy hour traffic based on the 2002 and the 2003 E-Comm traffic data. We predicted E-Comm network performance in cases of increased call traffic volume. Simulation results showed that when call traffic generated by E-Comm heavy user agencies increases by 100% during busy hours, the E-Comm network was unable to maintain the required Grade of Service. Further WarnSim simulations suggested the E-Comm network capacity (number of channels in each system) needed to accommodate the increased call traffic.
7.2 Future work

The related network traffic prediction model, such as the cluster-based SARIMA model developed by H. Chen [41], can be adopted to work with the proposed traffic model and the WarnSim simulator to better predict future performance of the E-Comm network.

WarnSim simulator may be improved and optimized. There are seven types of random variables currently available in WarnSim. Additional random variables may be added to WarnSim when needs arise. Furthermore, the internal algorithm of WarnSim may be further optimized to achieve better performance in terms of simulation speed.

Ad hoc networks are self-organizing networks of user terminals that form without the need for prior network infrastructure. Because of their adaptable, robust, and rapidly deployable potentials, ad hoc networks are being considered for the next generation PSWNs. In [42], D. Sharp proposed a multi-level hierarchical PSWN architecture that uses a dominating-set based backbone network to interconnect geographically distributed ad hoc networks in order to meet the needs of emergency communications. WarnSim could be further developed and adopted to evaluate performance of the new generation public safety wireless networks.
APPENDIX A: MAXIMUM LIKELIHOOD ESTIMATION (MLE) SCRIPTS

A.1 MLE script for lognormal distribution

```
MLE
TITLE = "MLE for Lognormal distribution"
MAXITER = 500
EPSILON = 1E-10
DATAFILE("Sample.dat")
INPUT_SKIP = 0
DELIMITERS = ","
OUTFILE("Result.out")
DATA
  duration FIELD 1 DROPIF duration=0
END
MODEL
DATA
  PDF LOGNORMAL(duration)
  PARAM a
    LOW=0.00001
    HIGH=50
    START=0.05
  END

  PARAM b
    LOW=0.00001
    HIGH=50
    START=0.05
  END
END
RUN
  FULL
END
```

A.2 MLE script for gamma distribution

MLE
TITLE = "MLE for Gamma distribution"
MAXITER = 500
EPSILON = 1E-10
DATAFILE("Sample.dat")
INPUT_SKIP = 0
DELIMITERS = ",,"
OUTFILE("Result.out")
DATA
duration FIELD 1 DROPIF duration=0
END
MODEL
DATA
PDF GAMMA(duration)
PARAM lambda
LOW=0.000001
HIGH=1
START=0.05
END
PARAM c
LOW=0.001
HIGH=50
START=0.05
END
END
END
RUN
FULL
END
END
REFERENCES


