AN ANALYSIS OF THE WELFARE EFFECTS OF TARIFF PROTECTION UNDER PERFECT MONOPOLISTIC COMPETITION

by

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AN ANALYSIS OF THE WELFARE EFFECTS OF TARIFF PROTECTION UNDER PERFECT MONOPOLISTIC COMPETITION

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ABSTRACT

This paper presents a detailed examination of the domestic welfare effects of asymmetric tariff protection under perfect monopolistic competition as presented in two representative papers by A.J. Venables (1982) and K. Lancaster (1984). The two domestic models are presented along with the specific assumptions necessary for the models to produce their results. Because Lancaster's (1984) model is relatively unknown, a discussion of the fundamentals of his "characteristics" approach is included. In both models, particular attention is paid to the conditions necessary for positive welfare effects from increased tariffs. Once the argument for positive welfare effects under these assumptions is made, a discussion of the possibility of these effects occurring as well as the consideration of the world necessary for these effects to occur is presented. The discussion shows that the positive effects hold only in the models’ restrictive world and that even within the models’ world, the positive effects occur only under special conditions.
This project is dedicated to my parents, Nora and Donald, and the rest of my large and loving family.
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An Analysis of the Welfare Effects of Tariff Protection
Under Perfect Monopolistic Competition

This paper examines the effects of positive tariffs on domestic welfare in small open economies where imported goods compete with domestic goods produced within a perfectly monopolistically competitive industry. Within a monopolistically competitive industry, production is characterized by increasing returns to scale and heterogeneous goods so that the quantity produced and the differentiation between each good are determined by free entry and profit maximization. The welfare properties of these economies have been studied by Krugman (1979), Dixit and Norman (1980), Venables (1982) and using the characteristics approach, Lancaster (1984).

These papers have indicated the possibility that positive tariffs may increase domestic welfare through two effects: the tariff can serve to increase the quantities of domestically produced goods within the monopolistically competitive industry, enabling domestic producers to more fully realize economies of scale or the tariff may increase the number and variety of goods available to domestic consumers by increasing the number of firms producing differentiated goods.

This paper will present the two different models developed in Venables (1982) and Lancaster (1984) and focuses its attention on two major points: first, what are the necessary conditions within the model for positive tariffs to increase welfare and second, do these models provide
justification for positive tariffs in trade policy.

**Section 1: Introduction**

Although the models presented represent two distinct approaches to the questions outlined above, the economies developed in both models are similar in many respects. Both models include two sectors within the domestic economy; one sector producing an homogeneous good under constant returns to scale and another monopolistically competitive sector producing differentiated goods under increasing returns to scale. The monopolistically competitive sector is assumed to be unable to export its production and therefore produces only for the domestic consumers. The foreign economy produces differentiated goods that are imported into the domestic economy to compete directly with the domestically produced differentiated goods. Consumers are assumed to have diverse preferences and maximize their utility while producers maximize their profit.

The paper has the following structure: Section 2 will outline Venables (1982) model and analyze the optimal tariff derived. Section 3 will present Lancaster's characteristics approach to this economy and examine the welfare effects of tariffs within his model. Section 4 will conclude the paper by discussing whether the two models presented provide justification for the use of positive tariffs in domestic trade policy.

**SECTION 2: Venables (1982)**

This section assumes a small open two sector economy which exports a numeraire (homogeneous) good and imports foreign differentiated goods which compete directly with domestically produced
differentiated goods. The optimal tariff as described by Venables is presented and discussed.

The commodities produced by the domestic monopolistically competitive industry will be labeled $i = 1, \ldots, n$, the imported differentiated goods are labeled $j = 1, \ldots, m$. The homogeneous good is labelled $Y$. The marginal cost of the $i$th commodity is given by $c_i$ and the fixed cost is labelled $F_i$. The unit cost of the $j$th commodity is given by $c_j$. The quantity and price are denoted by $x_i (x_j)$ and $p_i (p_j)$. It is assumed that each firm produces only one good and production in the monopolistically competitive sector takes place under increasing returns to scale. It is further assumed that all domestically produced differentiated goods are produced under symmetry which requires that $c_1 = c_2 = \cdots = c_n$ and $F_1 = F_2 = F_n$ for all $i = 1, \ldots, n$. Also, the demand characteristics are assumed to be the same for all domestic goods so that $x_i = x_n$ and $p_i = p_n$ for all $i = 1, \ldots, n$. Production of the homogeneous good is characterized by perfect competition and constant returns to scale.

If $M$ represents the endowment of the economy, and there are $n$ domestic goods and $m$ foreign goods the supply of the numeraire commodity can be written as

$$Y = M - n(c_i x_i + F_i) - m c_j x_j$$

where $m c_j x_j$ is the total cost of the imports and $n(c_i x_i + F_i)$ is the resource cost for domestically produced goods. Equation (1) is the resource (budget) constraint for the economy. If we introduce a tariff ($t_j$) on the foreign imports, the unit cost of imports ($c_j$), which is considered
exogenous, may differ from its market price \((p_j)\) in the following manner:

\[(2)\quad p_j(1-t_j) = c_j\]

If we assign \(T_j = t_jp_jx_j\) as the total tariff revenue from the \(j\)th good, and \(T_i\) as the total tax collected from the \(i\)th domestically produced commodity, the budget constraint can be written

\[(3)\quad Y = M - np_ix_i - mp_jx_j + nT_i + mT_j\]

Demands for the goods are derived from a social welfare function which is assumed to take the form

\[(4)\quad U = U(Y, V(x_1, \ldots, x_n; x_{j}, \ldots, x_m))\]

The subutility function \(V\) is assumed to be additively separable, so that the social utility function can be written as,

\[(4a)\quad U = U(Y, n\nu_i(x_i) + m\nu_j(x_j))\]

where \(\nu_i (\nu_j)\) is a function of \(x_i (x_j)\) with \(\nu_i(0)=0\) and \(\nu_i(x_i)\) increasing, strictly concave and twice differentiable for all \(i=1, \ldots, n\).

The equilibrium for this economy occurs where the social welfare function is maximized subject to the budget constraint (eq.(3)) being satisfied\(^1\). The optimal tariff problem for this economy is to find the tariff \(t_j\) that maximizes social welfare subject to the economy being in equilibrium and for a given level of domestic taxes \(t_i\). Equation (2) shows that with \(c_j\) exogenous, varying \(t_j\) is equivalent to varying \(p_j\); Venables therefore uses \(p_j\) as the control variable.

\(^1\) The details of this maximization are given in Venables(1982) pages 230-31.
As \( p_j \) is varied, there are three effects on domestic welfare: the quantity of imports \( (x_j) \) changes, the number of foreign products offered for sale may change and the number of domestic firms producing may change. These effects are discussed below.

First, the quantity of imported commodities changes as \( p_j \) is varied. This is determined by the demand function for imports.

The second effect occurs when quantities of imported goods change. If quantities of imports change and foreign firms face increasing returns to scale, some foreign firms may realize post-tariff non-zero profit. Entry or exit will therefore occur in the foreign economy and the number of imported commodities will vary. The domestic economy has no control over the response of foreign producers to the change in quantity therefore the foreign producers' response is exogenous to the domestic economy. Venables summarises the response of the number of imported goods, \( m \), to changes in \( x_j \) by the following elasticity

\[
N = \frac{(x_j/dx_j)/(m/dm)}{} = \frac{dx_j}{dx_jm} > 0
\]

The third effect of varying \( p_j \) is the response of domestic production to the tariff. In this model, the equilibrium \( x_i \) and \( p_i \) are unchanged when the closed economy is opened to international trade; only the number of domestic firms changes with the introduction of imported goods. The domestic \( x_i \) and \( p_i \) therefore remain unaffected by changes in the tariff\(^2\).

---

\(^2\) This situation arises from the particular model chosen by Venables (1982). He shows in his proposition 3.1 that the demand equations describing \( x_i, p_i \) are the same under autarky and free trade equilibrium, therefore changes in \( p_j \), (a variable not found under autarky),
In my opinion, this represents a weakness in the model in that changes in relative prices of domestic and imported differentiated goods should affect domestic firms' output and hence price decisions. In this model however this is not the case.

Domestic production can be affected by the tariff however, in that the number of commodities produced domestically, $n$, will change as $p_j$ changes. This response of the number of domestically produced goods, $n$, to changes in $p_j$ is determined within the model.

As mentioned earlier, the optimal tariff is found by maximizing social welfare for a given level of domestic tax subject to the economy being in equilibrium. Venables shows that the solution to the optimal tariff problem is given by the following:

cannot affect $x_i, p_i$.

3. Again, since the open and closed economy domestic demand functions are exactly similar, the lagrangean multiplier found in the first order conditions is unchanged with the introduction of trade. The marginal rate of substitution between $Y$ and $V(x_1,\ldots,x_n)$, (which is the inverse of the lagrangean multiplier) is therefore unchanged. This means that when $p_j$ changes, the number of domestic firms, $n$, must change in such a manner as to hold the multiplier constant. The response of $n$ to changes in $p_j$ is therefore determined.

Section 2.1: Necessary Conditions for Positive Welfare Effects

Before proceeding with the welfare analysis of the expression for the optimal tariff, two things should be noted. First, since the tariff can not change the price of domestic goods, there can be no positive (negative) effects on domestic welfare arising from lower (higher) post-tariff prices for domestically produced goods. Changes in domestic welfare can therefore only be affected by changes in the number of goods offered to domestic consumers and changes in the price of imports. Because of these factors, the discussion of the optimal tariff in this model will basically rely on two elements: the elasticity \( N \), (the response of the number of foreign firms, \( m \), to changes in \( x_i \)), and the exogenous level of domestic tax \( t_i \).

First, consider the case where \( N = 0 \). This means that the domestic economy cannot influence the number of foreign goods it imports. The optimal tariff becomes

\[
(6) \quad 1 - t_i = \frac{1 - t_i}{1 + N} \left( \frac{x_i v_i(x_i)}{v_i(x_i)} \right) 1 + N \quad \frac{v_i(x_i)}{v_i(x_i)} \times i \frac{v_i(x_i)}{v_i(x_i)}
\]

Since \( v_i(x_i) \) is an increasing function of \( x_i \) it can be shown that \( v_i(x_i) > x_i v_i(x_i) \) and the corresponding fraction on the right-hand side is less than one. The result is the following; when \( N = 0 \) and \( t_i > 0 \) then the optimal tariff is greater than zero and positive tariffs increase welfare. The reasoning behind the positive tariff is that since the number of foreign products will not change with the introduction of tariffs, positive tariffs should be employed to provide a price incentive to increase the number
of domestic producers. The tariff would represent an indirect subsidy to domestic producers.

Earlier in his paper, Venables derives an expression for the domestic taxation level, $t_i$, that would ensure the optimal number of domestically produced commodities. Venables found that a subsidy given by the expression

$$ (8) \quad t_i = 1 - \frac{v_i(x_i)}{x_i v'_i(x_i)} < 0 $$

produced the optimal number of domestic commodities. If this expression for $t_i$ is substituted into the expression for the optimal tariff (equation (7)) the optimal tariff is found to be equal to zero. The reasoning behind this result is simply that if the optimal number of domestic firms are operating and the number of foreign firms cannot be changed, a positive tariff would only serve to increase the price of imports and decrease welfare.

**In this model when $N = 0$, positive tariffs increase welfare only when the number of domestic firms has not been optimised.**

When $N > 0$ a decrease in the quantity of foreign goods ($x_j$) decreases the number of foreign firms exporting to the domestic economy. If the level of domestic tax has been set at the optimum level given in equation (8) the expression for the optimal tariff when $N > 0$ becomes

$$ (9) \quad 1 - t_j = \frac{1}{1 + N} \frac{1}{1 + N} \frac{v_j(x_j)}{x_j v'_j(x_j)} $$

Using $v_i/x_i v'_i > 1$, the right hand side is greater than one, and the optimal

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5. Ibid. section 2.4 pages 229-30.
tariff must be negative. Therefore, when \( N > 0 \) and the number of domestic firms has been optimised, the optimal tariff is negative (subsidy to imports).

In this case since \( n \) has already been optimised, the negative tariff (import subsidy) serves to increase the number of foreign commodities available to domestic producers thereby increasing domestic welfare.

If domestic production has not been optimised by domestic tax levels, then the optimal tariff can no longer be unambiguously determined. For example, a positive tariff will decrease both \( x_j \) and \( m \). This will decrease domestic welfare but at the same time provide an indirect subsidy to domestic producers allowing more domestic production. Increasing the number of domestic producers will increase welfare. The optimal tariff will be determined by the resultant of these two forces.

We have therefore shown the following; if the number of domestic firms has been optimised, positive tariffs can only serve to decrease domestic welfare. Furthermore, when the number of domestic firms has not been optimised, it has been shown that domestic subsidies can provide the same effect as positive tariffs on imports. Therefore positive welfare effects can be realized through domestic subsidies or tariffs on imports. No argument has been made as to which of these policy instruments is preferable.

Section 3: Lancaster(1984)

Section 3.1: The Characteristics Approach

Lancaster begins the analysis by reintroducing the 'characteristics'
approach, developed in earlier papers\(^6\), where goods are seen not as entities but rather as bundles of properties or characteristics. Individuals are assumed to be interested in the characteristics that a good possesses, rather than the good itself, so that the demand for \textbf{goods} is derived and indirect - the demand depending on preferences for the \textbf{characteristics} embodied within each good.

The analysis begins with production. The economy consists of two sectors - a homogeneous sector and a differentiated goods sector. The homogeneous sector is assumed to be competitive and produces under constant returns to scale. The differentiated sector is assumed to possess technology that permits variations in the mix of characteristics (specification) embodied within a good and gives economies of scale in the production of large quantities of any single specification. For simplification, it is assumed that differentiated goods can be produced to any specification within some relevant range and that for any given level of inputs (resources), the amount of one characteristic within a good can be increased only by decreasing the amount of a different characteristic within that good. The production technology for goods containing two characteristics can be summarized by the unit product differentiation curve (PDC) shown in figure 1(a). The PDC's are assumed to be homothetic (same shape) but allow for increasing returns to scale as long as all

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\(^6\). Earlier papers include the complete model in Lancaster(1979) and the subsequent use of this model in Lancaster(1980) and (1982).
products within the group of differentiated products possess the same economies of scale.

The definition of a single specification can take one of two forms: the ratio of characteristics given by the ray from the origin to a point on the PDC curve or the arc length along the PDC from one extreme to a single specification divided by the entire arc length of the PDC. Since specifications with zero amounts of one unit are possible in some relevant ranges, the arc length definition of specifications is preferable. This arc length specification measure allows us to 'straighten out' the PDC and use linear measures along it to define specifications. The transformed PDC curve is shown in figure 1(b).

Individual utilities are described by indifference curves in characteristics space as shown by the IC curve in figure 1(a). The indifference curves for characteristics are assumed to possess the conventional properties normally assumed for goods. In the characteristics approach, individual preferences reflect preferences for characteristics and not preferences for goods. Preferences for goods are derived from the underlying characteristic preferences and technology. The optimal transfer of characteristics occurs where the IC curve is tangent to the PDC curve

---

7. The arc length specification is also preferable because of assumed homotheticity in PDC's for different resource levels, and the fact that the length along each PDC is normalized for the length of the entire PDC. In this way the arc measure gives equal changes in specification for equal distances along the PDC.
as shown in figure 1(a).

If the preference mapping for individuals is to be transferred to the linear PDC shown in figure 1(b), the IC curves must undergo a similar transformation. Also, in general all possible specifications will not be produced so that some individuals will be unable to make an optimal transfer because their most preferred specification (MPS) is not available to consume. These two facts are combined to produce a compensating function, \( h(v) \), which describes a relationship between a given quantity of an individuals MPS and the quantity of an available good that will provide the same level of welfare for that person.

Lancaster’s model is expanded to many individuals with diverse preferences by making the assumption of uniformity. The uniformity assumption assumes three things: that every individual’s indifference curve is symmetric about the point of tangency with the PDC, that the MPS’s for the individuals are assumed to be distributed evenly, continuously and uniformly along the specification spectrum and that all individuals can be represented by the same compensating function.

The market demand equilibrium for this model occurs where consumers maximize their utility subject to a budget constraint based on income and prices. The supply side equilibrium requires some further assumptions.

First, each firm produces only one good but is free to choose and vary the specification without cost. Second each firm uses variables under its control (price and specification) to maximize profit taking the other
firms reactions as exogenous. Third, there is no collusion among firms and fourth, there are no barriers to entry and exit. Finally, firms are assumed to possess full information of market demand and consumers possess full information on specifications and prices.

These assumptions along with the uniformity assumption allow for a Nash equilibrium to be established where, with certain end-of-spectrum assumptions, the following conditions occur:

1. an equal spacing between produced specifications;
2. the same prices for all goods;
3. the same quantities of all goods, the same marginal and average costs for all firms and the same elasticities in each market.
4. each firm will earn zero economic profit.

This Nash equilibrium where firms operate in two equal half-markets in either direction is shown in figure 2(a).

Section 3.2: Interleaving and Asymmetric Protection

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8. The assumption that treats other firms prices and specifications as exogenous is the assumption used in a Nash equilibrium.

9. A fall-off in the density of consumers near the ends of the specification spectrum could be used to keep the two end firms (which enjoy one regular half market as well as another half market where there is no competition with other firms) at the same price and quantity as interior firms.
The strongest argument that Lancaster (1984) makes for positive welfare effects from positive tariffs is the case where domestic and imported differentiated goods are arranged on the spectrum in an ‘interleaved’ distribution and that any tariffs levied against imports are asymmetric, i.e. no retaliatory tariffs occur. The interleaving of imports and domestic goods along the spectrum is shown in figure 2(b). Simply, it represents a distribution where domestic and foreign goods alternate along the specification spectrum so that any domestic good has a foreign good as its closest neighbour. The Nash equilibrium in this market is shown in figure 2(c) where the domestic market is shared equally by foreign and domestic producers who produce at the same costs, charge the same price and face the same demand conditions.

Since each firm is identical except for specification, each half market is identical and the effects of a tariff on the entire spectrum can be analysed by focusing on one representative half market. The general and explicit derivation of aggregate demand for Lancasters model is left for the appendix. This paper is more interested in what happens to the Nash equilibrium when
the domestic economy levies a positive tariff on imports.

**Section 3.3: The Effects of a Tariff**

In order to understand the effects of a tariff on this economy the properties of the aggregate demand function must be understood. The aggregate demand function $Q(P', P, D)$ has the following properties (subscripts denote partial derivatives):

$$Q_{P'} > 0, \quad Q_P < 0, \quad Q_D > 0$$

where $P'$ is the price of imported good, $P$ is the price of domestic good and $D$ is the distance between the neighbouring specification. The elasticity of demand has the following properties:

$$E_{P'} < 0, \quad E_P > 0, \quad E_D < 0.$$ 

These properties will become important in later analysis.

Now consider the effects of a unilateral tariff of level $t$ levied against imports of the differentiated goods from the foreign country. Since producers abroad have the same marginal costs as the domestic producers, the tariff serves to increase the effective marginal cost of foreign producers and consequently, to maximize profit, the foreign firms increase their price to equate marginal revenue with the new higher
marginal cost. The market widths for imports decrease as consumers whose MPS's lie between foreign and domestic goods switch towards the relatively cheaper domestic goods. The size of markets for domestic goods increases and each domestic firm's elasticity of demand decreases ($E_p < 0$). The price of domestic goods will therefore rise initially in order to equate the domestic firms' steeper marginal revenue curve with their original marginal cost curve. Both domestic and foreign prices increase, however the foreign price rises farther because the effective foreign marginal cost has increased relative to domestic marginal cost.

Domestic firms experience both an increase in quantity and an increase in price. Profits, which were originally zero, are now realized by domestic firms and the existence of these positive profits will attract new firms into the industry. Lancaster makes two assumptions at this point; first, that foreign firms continue to operate in the domestic economy as long as revenue covers variable cost plus the tariff, and second that imports, even after entry, will remain interleaved with domestic goods.

The entry of new firms brings with it an increase in the product differentiation (within the domestic industry) and product variety (domestic
plus imported goods). This serves to decrease the distance $D$ between goods along the product spectrum which increases the elasticity of demand for all firms. As the elasticity of demand increases the demand curve becomes more horizontal and the markup (distance between price and marginal cost) is reduced. Lancaster therefore concludes that "...the prices of both local goods and imports will fall relative to the post-tariff, pre-entry stage, and the price of home goods will be less than under free trade.

The equilibrium in this case possesses the following characteristics:

a) The degree of product differentiation and product variety is increased by tariff.

b) The price of domestic goods is lower as a result of protection. Although there are more firms within the spectrum each earning zero profit, the domestic markets are wider than pre-tariff markets due to the relatively higher price of foreign goods. Due to economies of scale, the larger output for each firm leads to a lower average cost and hence a lower zero profit price.
c) The price of foreign goods is higher (by the amount of the tariff) as a result of the protection.

Lancaster therefore argues that if tariffs were returned to purchasers of foreign goods, "... the purchasers of both home goods and imports would be better off as a result of the tariff.10.

It should be noted however that Lancasters' argument points only to the possibility of a positive welfare effect. Although his specific model generates lower post-tariff equilibrium prices and increased product variety, in general, the effects of a tariff on the price of domestic goods in equilibrium will depend on the cross-price effects observed between domestic and foreign goods. This point will be shown in the next section.

Section 3.4: Necessary Conditions for Positive Welfare Effects11.

Let NR(P,P') represent revenue less assigned variable costs (net revenue) from domestic market sales and NR'(P,P') the net revenue form sales of imports. These can be represented by,


11. This derivation comes directly from Lancaster(1984) pp. 150 to 151.
\begin{align}
(10.\text{a}) \quad & NR(P,P') = (P - m)Q(P,P') \\
(10.\text{b}) \quad & NR'(P',P) = (P - m - t)Q'(P',P)
\end{align}

where \( m \) is the marginal cost. Firms choose \( P \) (\( P' \)) to maximise \( NR \) (\( NR' \)) so the first order conditions take the form
\begin{align}
(11.\text{a}) \quad & NR_p = Q + (P - m)Q_p \\
(11.\text{b}) \quad & NR_{p'} = Q' + (P' - m - t)Q_{p'}
\end{align}

where subscripts denote partial derivatives. (The subscript 2 will denote cross price derivatives). The effect of a small change in the tariff is given by
\begin{align}
(12.\text{a}) \quad & \frac{dP}{dt} = \frac{(Q_p'NR_{p2})}{(NR_{pp}NR_{p'p'} - NR_{p2}NR_{p'2})} \\
(12.\text{b}) \quad & \frac{dP'}{dt} = \frac{(Q_{p'}NR_{pp})}{(NR_{pp}NR_{p'p'} - NR_{p2}NR_{p'2})}
\end{align}

The denominators in 12.a and 12.b are positive if the direct effects outweigh the cross effects. This will be assumed here. \( NR_{pp} < 0 \) is necessary for a stable equilibrium and \( Q_{p'} < 0 \), therefore \( dP/dt > 0 \) necessarily and tariffs must increase the equilibrium price of imports.

The effect on domestic price will depend on the cross effect \( NR_{p2} \).

From equation 10.a we can derive the expression for \( NR_{p2} \),
\begin{align}
(13) \quad & NR_{p2} = Q_2 + (P - m)Q_{p2}
\end{align}
Here $Q_2 = >0$ and $Q_{p2} < 0$ (generally) so that $NR_{p2}$ can take on any sign. The price of domestic goods need not decrease with the introduction of the tariff.

Welfare may still be increased by increasing the number of firms operating. Since entry will occur if positive profits occur, we can see the effect of a tariff on the number of firms in the following manner. The profit per firm is given by the sum of net revenue from home and foreign markets less the fixed cost. This is given by

\[
(14) \text{Profit}(P,P') = NR(P,P') + NR'(P,P') - FC.
\]

Therefore a small change in the tariff yields the following,

\[
(15) \frac{d(\text{Profit})}{dt} = NR_2[dP'/dt] + NR'_2[dP/dt] - Q'.
\]

since $NR_p, NR'_p = 0$ at revenue maximizing prices and $dNR'/dt = -Q$.

If cross effects are zero the $NR_2, NR'_2 = 0$ then profits are necessarily negative so that fewer firms will exist post tariff. If the cross effects are not zero then $NR_2, NR'_2$, and $dP'/dt$ are positive, $-Q'$ is negative and $dP/dt$ can be negative or positive, profits can therefore be positive or negative. **Positive welfare effects from positive tariffs are only possible in Lancaster's model and can (but need not) occur only when the**
cross price effects are non-zero.

Lancaster recognizes this fact when he states, "... although the result given previously, that profits initially rise from the effect of the tariff in the ‘interleaved’ case, is certainly true for the specific model analysed, and the model itself has features that would be generally accepted as representative, the result cannot be regarded as truly general."

Section 4: Discussion

Lancasters’ Model

The discussion related to Lancasters’ model can be classified into two groups: arguments against the possibility of a world in which tariffs can increase welfare, and descriptions of the world necessary for these effects to occur. The first set of arguments question whether the conclusions of the model can follow from the assumptions Lancaster makes, and/or are additional assumptions necessary to complete the model. The second set of arguments accepts Lancasters’ model and instead describe the world necessary for his assertions to occur. In describing the world necessary for Lancaster’s conclusions, the critics hope to show that Lancaster’s world and the actual world are very
different. If the two worlds can be shown to differ, then Lancaster’s conclusions, although interesting, are no longer useful when formulating real world policies.

We begin with arguments against the possibility of welfare improving tariffs of which there are basically three. The first begins with Lancaster’s assumption of post-tariff interleaving. Lancaster asserts that positive profits made by domestic firms will attract new firms into the industry. Increasing the number of domestic firms must change the spacing between domestic firms along the product spectrum and that implies that foreign firms that continue to sell in the domestic market must change the specifications of their products. Also, to obtain interleaving with the increased number of domestic firms one must see an equal increase in the number of foreign firms selling in the domestic market. Although there may exist situations and strategies which could give rise to these occurrences, Milner (1986) points out that all of this must be achieved by a domestic economy that is assumed to be too small to affect the foreign economy. Even if it is assumed that the foreign country produces a wide range of products, it is not clear why it becomes attractive for more foreign firms
to sell in the post-tariff domestic market. The post-tariff markets for foreign goods deliver the same effective price, offer less width, yet attract more foreign firms. It seems reasonable to assume that the foreign firms that can operate in the post tariff market, could have entered before the tariff was imposed since the effective marginal cost has not changed. A further description of the foreign industry necessary for positive domestic welfare effects would present a more complete model.

A second argument in this set, as characterized by Greenaway(1985) and Milner(1986), suggests that the model has completely ignored the costs to the new firms, of introducing the new products domestically. By excluding these start-up costs, any consideration of the net welfare benefits of the tariff is biased upward. Presumably, one is interested in the net effects (positive price effects less the costs of introduction) when one considers the cost/benefits of protection. Lancaster however does not consider this in his model. A complete analysis of the net welfare effects of a tariff should therefore include an assumption about the costs of introduction.

The third argument arises from James and Stewart’s(1981)
suggestion that increases in product variety may decrease social welfare if the ‘old’ products become unavailable. Although their argument is based on high and low income characteristics and pertinent to less developed countries, they do bring to attention the possibility of an ‘optimal’ level of product variety. In Lancasters’ model, the optimal level of product variety exists when every individual can purchase a good that exactly conforms to their MPS so that the number of firms equals the number of consumers - in general, any situation that increases the number of products offered for sale will increase welfare. What James and Stewart’s (1981) arguments suggest is that there may be further considerations (such as the characteristics of new products, the number of existing products, or the maximum discernable width between products) in the optimization that Lancaster has not modeled.

The above arguments dispute the assumptions and conclusions asserted in Lancaster’s model; these arguments, however, do not represent all of the questions that have arisen in response to this model. The literature has also produced objections that dispute the applicability of Lancaster’s model to real world situations.
The main point that arises from this line of criticism is that the assumptions that Lancaster makes in order to arrive at his results are perhaps too restrictive for his model to be useful in determining tariff policy. For example, the assumption of post-tariff interleaving, as discussed above, is crucial to the realization of positive welfare effects. However, little theoretical or empirical evidence is suggested for explaining why this might happen. The same is true for the initial assumption of pre-tariff interleaving. They are simply assumed to occur.

Of course, this presents the question whether the assumption of 'perfect interleaving' is necessary for Lancaster's assertions. One can envision many different arrangements of foreign and domestic firms along the product spectrum; from a 'split' arrangement with foreign firms on one side, domestic on the other, to a random placement of firms which would on average leave 50% of the domestic firms interleaved. In his paper, Lancaster describes the effects of a tariff on both the 'interleaved' and 'split' arrangements since they represent the most and least interactive structures. The interleaved arrangement therefore presents the most favorable structure for positive welfare effects - any other arrangement
would have less interaction and lowers positive effects. As well, since domestic firms are assumed similar in every respect except for the specification of their product, a problem arises when some domestic firms increase profit while other domestic firms profits are unaffected. In some non-symmetric arrangements, the equilibrium structure of firms may become unstable so that a stable Nash equilibrium may not exist.

Another of Lancaster's simplifying assumptions is that the industry producing the differentiated product is assumed to possess economies of scale even though, from a strictly theoretical point of view, constant or decreasing returns to scale are equally valid assumptions. Although arguments have been made for the existence of increasing returns to scale, the further assumptions that they are possessed industry-wide and are equally enjoyed throughout the domestic industry, can be questioned.

Further simplifying assumptions include:

(i) free entry and exit with single product firms producing dividable goods,

(ii) perfect costless information for both consumers and producers. There are no search costs incurred by consumers
with the introduction of new products, nor any asymmetric information as to characteristic quality or quantity,
(iii) constant costs in the foreign economy,
(iv) no collusion between firms (no concerted lobby for protection).

When one adds on the additional assumptions necessary for Lancaster's development such as economies of scale, even distribution of consumers preferences, pre- and post-tariff interleaving, and no tariff retaliation from the foreign economy, it can be seen that the model Lancaster has developed must be used with caution when formulating tariff policies.

Venables' Model

The objections to Venables model are numerically smaller than those to Lancasters' model, however they are just as serious. The main objection stems from the absence of price effects on domestic production. Since the autarky and free trade solutions for domestic price and quantity are exactly the same, the introduction of imports can only change the number of domestic firms producing. The equilibrium solution suggests that domestic firms do not adjust the price or quantity at which they produce regardless
of the level of the tariff on imported goods. Therefore if domestic firms are operating on a zero profit level before the tariff, if price and quantity do not change, profit does not change and hence there is no signal for more domestic firms to enter the market.

Venables model therefore presents only a portion of the real effect of a tariff. The price or quantity effects on the domestic goods of any tariff cannot be used to counter the welfare effects of increasing the number of goods available to the domestic consumer. It is common sense that increases in the variety of goods does not come without a cost. Since we are interested in the net welfare effects of tariffs, these costs should be included in the analysis.

Section 5: Conclusions

Although there exist considerable objections to both of the models presented, the idea that unreciprocated tariffs could decrease the price of domestically produced goods under increasing returns to scale remains intuitively appealing. It should be noted that none of the objections have completely denied the possibility of a world in which there may exist positive welfare gains from tariffs. Although their results are in no sense
general and not directly related to real world economies, the models can claim at least the possibility that these gains exist in special cases. This is not to say however that the policy of small open economies should include tariffs. The effects of tariffs on production and price are in no sense clearly predictable in the real world so that the positive effects Lancaster and Venables allude to are in no way assured.

There also remains a question posed by both Greenaway(1985) and Milner(1985) on whether tariffs are the most efficient instrument of intervention for realizing the possible welfare gains. Both Greenaway and Milner state that optimal intervention analysis would lead one to anticipate that an alternative instrument, for instance production subsidies, could achieve increased product variety and lower prices of domestic goods with no increase in foreign prices i.e., the same positive effect without welfare loss due to higher import prices. Therefore, even if the possibility of positive welfare gains existed, it is not clear that tariffs are the most efficient instrument in realizing these possible gains.
APPENDIX
Lancaster's Characteristics Approach

The utility function is formulated in constant elasticity of substitution and is given as

\[ U(q,u,y) = \{aq^w h(v) - w + (1-a)y^w\}^{1/w} \quad (A.1) \]

where \( q \) is quantity of differentiated good, \( y \) is the quantity of homogenous good, \( v \) is the spacing between the specification of the good that is actually produced and the individuals most preferred specification and \( h(v) \) is the compensating function. The homogenous good \( y \) will be used as numeraire for price and income so that the individual budget constraint takes the form

\[ Pq + y = I \quad (A.2) \]

The individual will choose quantities of differentiated \((q)\) and homogenous \((y)\) goods so as to maximize \( U(q,u,y) \) subject to the budget constraint. The individual’s choice satisfies the traditional first order condition that the marginal rates of substitution between the two goods is equal to the ratio of prices so that

\[ \frac{dU/dq}{dU/dy} = P/1 \]

For the functional form of the utility function \((A.1)\) the above condition becomes

\[ \frac{a/(1-a)h(v)^{(1/w - 1)}}{q^{-1/w}y^{1/w}} = P/1 \quad (A.3) \]

Solving for \( q \) and \( y \) using the first order condition \((A.3)\) and budget constraint \((A.2)\) gives the individual demand functions for \( q \) as the
following:

\[ q = IP^{-1}(1 + Ah^{w-1}p^{w-1})^{-1} \]  \hspace{1cm} (A.4)

where w is the elasticity of substitution.

The economy is made up of individuals whose preferences are the same except for their most preferred specifications (MPS). It is assumed that these MPS's are distributed continuously over the spectrum with constant density. We have also assumed economies of scale so that there is a finite number of goods produced, each good being defined by the position of the product on the spectrum of specifications.

The total market for each differentiated good is made of two half markets, one in each direction. We confine our attention to just one side of the total market. This half market consists of individuals who choose the target good rather than the neighboring good. Therefore, if P is the price of the target good and P' is the price of the neighboring good at a distance D, the half market is made up of individuals with MPS's out a distance u, where u satisfies the following condition

\[ h(u)P = h(D-u)P' \]  \hspace{1cm} (A.5)

(Note; if P = P' then u is halfway between the neighboring goods.)

The aggregate demand in the half market for the good is therefore the sum of all individual demands within a half market from the produced specification to u which is given by

\[ Q(P,P',D) = q(P,v)dv \]  \hspace{1cm} (A.6)

The development of the numerical example in Lancaster's paper directly
parallels the above general formulation. Specific functional forms will now be introduced in order to arrive at his explicit model.

We begin with the compensating function given by

\[ h(v) = 1 + v^2 \]  \hspace{1cm} (A.7)

The dividing condition (A.6) can therefore be written as

\[ P = P'\frac{[1+(D-u)^2]/[1+u^2]} . \]  \hspace{1cm} (A.8)

If we assume the elasticity of substitution \( w = 2 \) the demand function becomes

\[ q(P,v) = \frac{1}{[1 + AP(1+v^2)]} , \]  \hspace{1cm} (A.9)

where \( A \) is a constant that expresses the weight of the differentiated good in total consumption. Lancaster assume \( A = 4 \) which implies that the expenditure on the differentiated good is 20% of total expenditure when \( P = 1 \). If we choose to use \( q \) as the per capita demand for an individual with unit income and multiply \( q \) by a factor \( k \) representing the population density, we can rewrite the demand function in (A.9) as

\[ q(P,v) = \frac{k}{[4P^2(a^2+v^2)]} . \]  \hspace{1cm} (A.10)

Aggregate demand as seen in (A.6) can now be expressed explicitly as

\[ Q(P,u) = \frac{k}{4P^2} \int \frac{1}{(a^2+v^2)}dv = \frac{k}{4P^2} \arctan \frac{u}{a} \]  \hspace{1cm} (A.11)

Solving \( u \) in terms of \( P, P' \) and \( D \) from the dividing condition in (A.8) (which requires solving a quadratic) gives the explicit expression for aggregate demand for a half market as
\[ Q(P,P',D) = k \arctan \frac{2P^{1/2} [P' (P + D) (P - P')^2]^{1/2} - P' D}{4P^2 (P - P')(1 + 4P)^{1/2}} \]  

(A.12)

This is the expression Lancaster uses to derive his numerical results.
REFERENCES


Lancaster, Kelvin (1979) "Variety, Equity and Efficiency" (Oxford, Blackwell).


Figure 1(a)

- unit PDC curve shown tangent to individual indifference curve IC. Optimal transfer occurs at Y and the specification at point Y (given by ray from origin or relative arc length along PDC) is consumers most preferred specification (MPS).

Figure 1(b)

- "linear" PDC shown tangent to transformed IC curve from above diagram (shown as IC'). Note that quantity is measured by vertical distance.
- PDC shown with representative IC curves displaying uniform distribution of consumer MPSs.
- Nash equilibrium with equal spacings between firms

Figure 2(b)

- Interleaved distribution of domestic and foreign firms. $d_i$ denotes domestic firm, $f_i$ denotes foreign firm

Figure 2(c)

- The Nash equilibrium with interleaved distribution of firms. Note that quantity demanded increases as the consumer's MPS nears an available (virtual) good. The closer an individual's MPS is to an available good, the more of that good demanded by the individual.