A DYNAMIC PROGRAMMING APPROACH TO OPTIMAL FOREST STAND
MANAGEMENT REGIMES IN COASTAL BRITISH COLUMBIA

by

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A DYNAMIC PROGRAMMING APPROACH TO OPTIMAL FOREST STAND MANAGEMENT

REGIMES IN COASTAL BRITISH COLUMBIA.

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15 April 1988

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The forest sector of British Columbia has historically been dominant in the province's economy. As the mature stands of timber are exhausted, second growth stands will take over as the primary source of timber to fuel the forest economy. The opportunity exists to manage these stands in such a way so as to provide the greatest benefit to the people of British Columbia.

One way to measure these benefits is by determining the net present value of the logs grown on the stand, and then reducing this value by subtracting the planting, management and logging costs. To determine which management regime maximizes these benefits, dynamic programming (DP) is used.

DP has emerged as a powerful approach to stand level problems in recent years. DP is a recursive optimization approach that simplifies complex problems by transforming them into a sequence of smaller simpler problems. DP is used here to determine the optimal management regime for a stand of coastal Douglas-fir including initial planting density, precommercial thinning, timing and intensity, fertilization timing and the optimal rotation length.

Tree growth is simulated using the Tree and Stand Simulator (TASS) while costs and revenue functions reflect conditions on the B.C. coast. Results were obtained in a deterministic setting with both full and constrained funding and in a stochastic setting where stand growth could be influenced by plantation failure, site specific factors, fire or pests. The effects of uncertain future prices and management funding were also investigated.

The results obtained in the deterministic case show forest
management to be unprofitable given current costs and prices. It becomes profitable if one assumes increased real prices in the future or if firms are given mature timber today in expectation of increased future volumes. In the stochastic environment there was limited evidence of decreased management and shorter rotations (than the deterministic case) being optimal. In general, stand values were increasing sufficiently to warrant the risk of losing the stand by fire or pests. Uncertain funding was shown to have a greater effect on optimal regimes and forest sector investments.
ACKNOWLEDGEMENT

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Chapter 1

INTRODUCTION

1.1 HISTORICAL PERSPECTIVE

The forest sector has long been the dominant component of British Columbia's economy. It consistently accounts for nearly half of the total manufacturing shipments in the province and close to 60% of provincial exports. Approximately 86,000 persons or 7% of British Columbia workers are directly employed in the forest sector and about twice this number are indirectly dependent on the forest sector for their livelihood in service industries such as transportation, capital repair and construction, material and supply (Environment Canada 1983). In non-metropolitan areas outside the Lower Mainland and the southern part of Vancouver Island there are many communities and regions which are dependent on the forest resource as their major or only economic base (White et al. 1986).

The B.C. forest sector was based and has been maintained on an endowment of high quality timber. While the benefits of silviculture treatments such as thinning have been known for some time, their practice has been limited as there was always another stand of virgin timber 'over the next hill'. This seemingly endless supply of timber and the belief that cutover lands would regenerate naturally meant lands were not managed for a second crop of trees and large scale replanting projects were not undertaken as they were viewed as uneconomic (Heaps 1985).

Interest in better forest management and in the benefits of silvicultural efforts grew between 1945 and 1970 but the belief was still general that there was plenty of wood to meet the needs of an expanding industry. Through the 1970s and particularly in this decade the finite nature of the old-growth forest has become evident. Reports by Reed (1978)
and the first Forest and Range Resource Analysis (British Columbia Ministry of Forests 1980) highlighted this fact. The latter discussed the 'fall down' effect where present levels of harvest could not be maintained with existing harvesting and silvicultural practices. The severe impact that decreased harvests could have on the provincial economy has led to a significant increase in silvicultural activity in the province and a keen interest in second growth forests and the impact of forest management practices on them.

This augmentation in silvicultural activity has come largely from the public purse through increased provincial spending (see British Columbia Ministry of Forests (1986) for projected expenditures and activities over the next five years), Federal-Provincial cost-shared agreements (Canada 1985) and provincial stumpage payment credits for silvicultural activities undertaken by forest industry firms. It is in the interest of all parties concerned that these funds be spent in an economically efficient manner. This dissertation will increase the information base on the economics of second growth stands to aid in the efficient allocation of funds to silvicultural activities in these stands.

1.2 OBJECTIVES

The end result of any optimization exercise is to provide a prescription for obtaining the greatest returns from scarce resources. The primary objective of this dissertation is to determine forest management regimes which, in both constrained and unconstrained settings, will bring about the highest return from a forest stand. The topic is not new to forest economics as optimal rotations and optimal regimes have long been (and continue to be) a subject of much debate and interest in the literature (Samuelson 1976, Brodie and Haight 1985, Reed 1986). Hence
this work will contribute to the literature by synthesizing some important elements of the literature into an approach which will be applicable to British Columbia. The results will provide insight into how forest management activities are affected by factors such as our limited knowledge of how stands respond to treatments, the risk of attack from pests or fire and limited uncertain funding.

1.3 SCOPE AND PROCEDURE

In its simplest form the problem of the forest manager begins with the manager facing a bare plot of land on which he wishes to establish a stand of Douglas-fir (Pseudotsuga menziesii (Mirb.) Franco)¹. The manager wishes to maximize the financial (as opposed to volume) return from the stand over an infinite number of timber crops. The lengthy period of optimization can be justified in either of two ways. It can be assumed that the Crown is the manager and is interested in maximizing returns to the public over all generations or that a private land owner plans to leave the land as a bequest and wants to leave the site in a state that all future generations may benefit from equally. To meet this objective the manager must make a number of decisions on how the stand is to be managed. He must decide whether to leave the site to regenerate naturally or to plant seedlings. In the latter case he will need to determine the number of seedlings to plant. As the stand grows he will need to decide when and how much to space the juvenile stand and whether fertilization should be undertaken. As the trees achieve a size where they are suitable for conversion into products such as lumber, the

¹It is assumed that the forestry is the best use and that Douglas-fir is the best species for the site. The issue of alternate uses for the land is beyond the scope of the study.
manager must consider thinning some trees to allow the other trees to grow more freely and also to obtain some return from the trees removed. Finally the manager will have to determine the right time to harvest the site and prepare it for the next crop. In economic terms the problem is sometimes regarded as capital being invested in a timber growing machine. The product (timber) produced by the machine becomes part of the machine and maintenance (management activities) is carried out on the machine to ensure optimal performance. The ultimate decision is when to harvest the machine and terminate production. The solution to the forest manager's problem must stipulate both the 'maintenance schedule' and the optimum harvest date.

The manager's problem is further complicated by the fact that he does not have perfect foresight. He does not have perfect knowledge about how trees grow or how they respond to fertilization and thinning. He does not know if funding will be available when 'maintenance' is called for on his schedule. He cannot know exactly when a pest might attack the stand or the damage it may inflict. It is on the optimal management regime when faced with these uncertainties and on the expected return from the stand that this research will focus.

The results of the research will be useful in a number of ways. Beyond suggesting to managers an optimal regime of management activities for a second-growth Douglas-fir stand on the B.C. Coast, it will be of interest to budget planners as it investigates the effects of funding constraints and continuity. This will be done by comparing scenarios at different levels of uncertainty. Returns to some aspects of forest research will also be investigated. Governments have the ability to provide a measure of funding continuity while research can reduce
uncertainty associated with growth and with pest attack. As the stand level solutions are the basic unit in forest planning, those presented here will also be of interest to forest level planners to aid in harvest scheduling and yield planning.

The starting point for this work will be to develop a stand model in a deterministic setting to serve as a base case to which subsequent results can be compared. The optimal regime will be obtained using dynamic programming (DP) which has emerged as a powerful approach to stand level problems in recent years (Brodie and Haight 1985). This case assumes full information about stand growth and unlimited funds to carry out the work necessary to achieve optimality.

The next stage will incorporate possible constraints on the funding available to the manager but assume he still has full knowledge of the effects of his actions. The funding constraints will be in two forms. The first will constrain funding to less than the amount needed to carry out the optimal regime. Solutions to this problem will reveal the most profitable management activities. The second will allow for possible uncertainty on the part of the manager as to whether or not he will obtain the required funding in each period to carry out the optimal program. This form will begin to show how a lack of funding continuity effects returns on forest management investments.

The final stage will model risk and uncertainty² as a Markov Process (MP) and utilize the DP algorithm to solve for the optimal regime. While MPs do not model uncertainty explicitly, scenarios

²These terms are employed in the Knightian sense. Risk refers to decision making with known probabilities of occurrence while uncertainty refers to complete ignorance of the probabilities.
reflecting management's response to uncertainty can be run with the MP model, when both risk and uncertainty are present. For instance, growth could be modelled as risk while funding levels are uncertain over the same period.

The technique of DP has been shown to be useful in forest management problems. The literature is still quite small and many useful applications of DP in forestry, particularly in B.C., have not been investigated. This work will contribute to that body of literature as it meets the following objectives:

a) The application of DP in a B.C. forest setting
b) Using the decomposition method to determine the effect of budget constraints and uncertain funding on the optimal regime
c) Demonstrating the financial consequences of uncertain stand growth. Previous DP models have focused on volume.

These objectives will be met as the previously discussed stages are carried out.

1.4 ORGANIZATION

Chapter 2 will briefly discuss the technique for DP and review its application to stand level problems. Included in this chapter will be a discussion of how DP improved upon the early solutions to stand optimization problems and the limitations of the existing DP literature.

Chapter 3 outlines the growth, price and cost components of the study. This data will include a statement on assumptions made regarding the discount rate (a highly controversial topic in forestry analysis). Forest management options included in the study are also discussed in this chapter.

Chapter 4 contains the solution to the base case deterministic
model and some of its variations. This chapter will also discuss the form of the DP in terms of state descriptors and stage length.

The Markov model is presented in Chapter 5. Solutions to the model reveal the impact of risk and uncertainty on optimal regimes as they are compared to the solutions obtained in the previous chapter.

Chapter 6 contains policy implications of the research findings as well as a summary of findings, a discussion of further applications of the results and ideas for future research in the area.
Chapter 2

DYNAMIC PROGRAMMING AND ITS APPLICATION TO STAND LEVEL OPTIMIZATION

2.1 INTRODUCTION

The goal of this chapter is to acquaint the reader with the technique of dynamic programming and its application to forestry. This will be accomplished by reviewing the basic features which must characterize a problem which is to be solved using DP. A discussion will follow relating why DP is proving to be a useful tool in stand level optimization problems. This will be followed by a review of the literature where DP has been applied to forestry problems with emphasis on stand level optimization applications.

2.2 CHARACTERISTICS OF DYNAMIC PROGRAMMING PROBLEMS

Dynamic programming is essentially an optimization approach that simplifies complex problems by transforming them into a sequence of smaller simpler problems (Bradley et al. 1977). Not all problems however can be broken down and simplified. Following the presentation in Hillier and Lieberman (1980) the basic features which characterize DP problems are discussed below:

1. The problem can be divided into stages with a policy decision required at each stage. Normally in DP problems, stages represent time periods in a planning horizon but they can represent anything which divides the problem up into sections with a decision required at each section, e.g. legs of a journey. The policy decision is what action to take at each time period, stopping point, etc. A further characteristic of DP problems is that the sequence of policy decisions are interrelated.

2. Each stage has a number of states related to it. The states are a description of the various possible conditions the system may be in at
a given stage. It is a description of the system. It could be a stock inventory level, a degree of machine wear or a description of a forest stand in terms of number of trees, basal area and stand age. The state description is a vital component of the DP formulation. It must be detailed enough to accurately describe the system being studied but simple enough to limit the number of states at each stage. As the number of states at each stage grows the problem is beset with the 'curse of dimensionality' and becomes more difficult if not impossible to solve. For instance the forest could be described by number of trees, basal area, tree height, stand age, soil characteristics, years since last thinning and so on but by doing so each stage would have a very large number of states and the problem could not be solved. Instead detail is sacrificed to allow a solution to be obtained.

3. Each time a policy decision is taken it transforms the current state into a state associated with the next stage. The new state entered into may be determined by both the policy decision and a probability distribution. For instance, given a level of product inventory (a state variable), the decision (a stage) of how much to produce today (the policy decision) will influence how much inventory (the state) we have at the next stage. A decision to thin a stand at age fifteen will influence the characteristics of that stand at age twenty. The probability distribution recognizes that we do not know with certainty what the stand will look like. From this description it should be evident that some DPs (including those used here) can be set up as networks and solved as linear programs (see Hillier and Lieberman (1978) chapter 5). In Figure 2.1 below the columns refer to the
stages while the nodes correspond to states. Policy decision moves us through the network. Contributions to the objective function from the various policy decisions are represented by values assigned to the branches which connect the nodes.

4. The optimal policy for all remaining stages is entirely independent of policies adopted in previous stages. This features of DPs is commonly called the principle of optimality. Wagner (1978, p. 266) puts it as follows:

"An optimal policy must have the property that regardless of route taken to enter a particular state, the remaining decisions must constitute an optimal policy for leaving that state".

It is this property that allows DPs to be broken up into a series of smaller, simpler problems. We will see in the next section how this

Figure 2.1 The network form of a DP
property places some important constraints on the formulation of problems involving policies or actions which have a lasting effect such as forest management operations.

5. The solution procedure must begin by finding the optimal policy for either the first or last stage.

These techniques are called forward induction and backward induction respectively. Backward induction is more commonly used but the forestry DP literature (Brodie et al. 1978) favors forward induction. Both methods will be used here and a discussion of the pros and cons of each is contained in the next section.

6. Finally a recursive optimization procedure can be developed "which builds to a solution of the overall N-stage problem by ... sequentially including one stage at a time and solving one stage problems until the overall optimum has been found" (Bradley et al. p. 462).

While the recursive relationship will vary according to the problem, a general formulation suitable to the deterministic problems investigated herein is described below.

A general forward recursive function is:

\[ f_n^*(s) = \max \{ f_{n+1}^* (x_{n+1}) + c_{sx} \} \]

where: \( f_n^*(s) \) is the return from being in any state in the set of all feasible states from the first stage of the problem to the present stage \( (n) \) and state \( (s) \).

\( x_{n+1} \) is any state in the set of all feasible states from which the current state \( s \) can be reached

and \( c_{sx} \) is the return associated with going from state \( x_{n+1} \) to the current state \( s \).
The problem is solved by sequentially choosing the maximizing values of $x_{n+1}$.

This process is described in detail in Chapter 4.

2.3 FORESTRY AND DYNAMIC PROGRAMMING

The previous section outlined the basic features which characterized DP problems. The purpose of this section is to demonstrate how problems of optimal forest management fit the outlined framework.

Many DP problems are divided into stages by dividing up an extended period of time into equal sections. Stand optimization problems are suited to be divided up into time segments of one, five, ten or any other number of years with an action or policy decision such as thin, fertilize, harvest, do nothing and so on required at each stage.

As mentioned in the previous section, the state variable should describe the forest in a simple way. A balance between describing the forest as accurately as possible and limiting the number of states so as to be able to solve the problem in a reasonable time must be struck. Previous work (see the next section) has limited the description of the forest stand to age, number of trees per hectare, basal area per hectare and sometimes one other variable such as time since last thinning or type of thinning. As well as limiting the number of variables which describe the forest, it is also important to discretize those variables. Kao (1980) shows that the number of trees per hectare must be put into groups of 37 or more trees per hectare so that all diameter classes of trees can be considered for thinning. Similarly basal area is described in intervals of say half metres squared per hectare. Kao (1980) describes the effects of varying these intervals. The key factor in determining the interval width is the same as that for determining the number of variables.
used per state i.e., it must not be so fine that it creates an inoperable number of states or so broad that it does not distinguish between forest stands that should be classed differently.

Policy decisions in forest stand problems involve forest management decisions such as degree of juvenile spacing, level of fertilization, degree of thinning and eventually the decision to harvest and to return to bare land or an initial stocking density. These decisions move us from an existing state to a state associated with the next stage. As with the descriptions of states, these decisions are made in intervals such as comparing juvenile spacings of ten, twenty and thirty percent.

The Principle of Optimality places an important constraint on forest stand problems. The effects of an action taken today must be completely described by the state variables. This is exemplified by fertilization. Let us assume that the state variables being used are number of trees per hectare and basal area and the stages are time intervals of 10 years. Say in year 30 a stand is fertilized. This action accelerates growth and in year 40 the stand is again described by the number of trees and basal area per hectare. The Principle of Optimality requires that all the effects of the fertilization be completed in the 10 year period. Another way of stating this is that a stand with N trees per hectare and G basal area per hectare today which has been fertilized in the past must grow from now on in the same way as a stand with the same number of trees and basal area which has not been fertilized. If the time interval between stages is not sufficient to guarantee this condition, then an additional state variable is required to described the stand as fertilized.
When a single tree distance-dependent growth model is used the configuration of the trees will also affect stand growth. As noted, the Principle of Optimality requires stands in the same state to grow in the same way. Number of trees and basal area will not be sufficient descriptors if stand configurations vary. The non-random element which will have the greatest effect on where the trees are in the stand and in particular how close one tree may be to another is the initial stocking density of the stand. Stands with the same initial density can be run repeatedly with the same initial tree locations. This will prevent differences in growth caused by tree location. Where initial density is an important consideration, it will need to be added to the list of state descriptors.

Forward recursions have been favored in the forestry DP literature largely because they do not require "a separate pass through the network ... for each candidate rotation" (Brodie et al. 1978, p. 517). Forward recursions require only paths of interest to be searched. The positive side of this technique is that it minimizes the solution time and provides optimal paths to rotations shorter than those being investigated. It fails however to provide optimal regimes for states not on the optimal path. The recursion must be restarted at the 'non-optimal' state and the problem resolved. Backward recursions provide optimal regimes for all states considered in the problem. Within a large problem such as most forestry problems are and where the optimal path is of primary importance the time savings involved in forward recursions are significant when only one pass through the network is required. If the user is primarily interested in what happens when we move off the optimal path or if the problem being studied is relatively small then backward recursions appear to be superior. The deterministic model in Chapter 4 will be solved via a
forward recursion while the stochastic problems in subsequent chapters must be solve via backward recursion. The mathematical programming software IMPS (See Appendix 1) will be used to solve the stochastic problems.

Forest stand optimization questions fit the framework for DP. Caution must be exercised to assure that the Principle of Optimality is adhered to and that problem is not too large to be solved within a reasonable time. It must also be remembered that the growth model does not provide exact stand growth but only an approximation based on limited information. Characteristics such as species, age, site quality, density, management regimes or tree quality are related directly through equations to stand parameters such as number of trees, average diameter, height, basal area and volume (Mitchell 1980). It can not be expected that our solutions will be more precise than our data. The growth model TASS which underlies the models in Chapters 4 and 5 is discussed in section 3.2.

2.4 PREVIOUS SOLUTIONS

The problem of optimal forest management regimes has been (and continues to be) solved by methods other than DP. Indeed, the early DP literature (such as Amidon and Akin 1978) used the same problems solved earlier by other means to verify their solutions. The three dominant solution methods other than DP are:

1) Marginal Analysis

2) Control Theory

3) Comparative Simulation

Marginal analysis was one of the earliest approaches to the stand optimization problem. Early approaches include USDA Forest Service (1963), Duerr and Christiansen (1964) and Chappelle and Nelson (1964).
Schreuder (1971) criticized the latter and noted some weaknesses in the marginal analysis approach. As employed in forestry problems the method involves comparing the marginal value growth percent of the stand with the marginal cost of capital for each consecutive time period. Then the optimal stocking level is determined. Schreuder states that the main problem with this approach is that it may not provide a global optimum. This is because it takes into account only one period at a time and thus does not consider the interdependencies of the periods. This approach excludes precommercial thinnings (defined as thinning which cost more than they bring in the same period) which have been shown to be economic over the life of a stand. Schreuder also says that marginal analysis becomes very difficult if not impossible to use when prices and costs vary over time.

Brodie et al. (1978) provide a brief review of the important contributions to the stand optimization problem by writers working in a control theoretic framework. Naslund (1969) formulated the problem but provided no solution. Anderson (1976) formulated a solution which showed consistency with the traditional Faustman model (Samuelson 1976) and other more general dynamic models but again no actual solutions are shown. Clark and de Pree (1979) provide numerical results forming the problem as one of linear control. McDonough and Park (1975) and Dixon and Howitt (1980) are reported in Cawrse et al. (1984) to have obtained local optimum solutions using iterative techniques. As evidenced by the papers noted here, the major shortcoming of this approach is the difficulty in obtaining numeric solutions to problems. Cawrse et al. (1984) attempt to overcome this difficulty by employing a variational solution technique. Their solution however, would only be effective for simple one equation models of stand growth.
Comparative simulation has been very popular in recent years. Simulators 'grow' a stand to a given user age, thin as requested, grow again and so on. A path is followed to some rotation age. Paths are compared and the highest permitted one is considered best. Complete enumeration of every possible path through a network is not feasible and even approaching that lofty goal is time and budget consuming. For examples and discussion of comparative simulation techniques see Randall (1977), Reukema and Bruce (1977) and Sleavin (1983).

The method of solution to stand optimization problems which has shown the greatest promise in recent years is dynamic programming. Dynamic programming has the potential to overcome the difficulties noted in each of the solutions above. It is a multiperiod approach. It provides global optimal solutions. Variable factor and product prices can be allowed for. Every possible path need not be searched to obtain the solution. In sum, dynamic programming is a useful tool for this problem because it is more efficient than simulation and simpler and more versatile than marginal analysis and control theory.

The DP literature will be reviewed chronologically. This is appropriate since applications of DP to forestry have become more sophisticated over time and the review can lead into what is being presented in this dissertation.

Hool (1966) provided the first North American\(^1\) application of DP to a forest production problem. Using data from the Darlington Woods, Indiana Continuous Forest Inventory System, he defines his states by

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\(^1\)He cites a Japanese paper that appears to be the first DP application to forestry.
groupings of volumes, tree counts and by whether or not the stand had been thinned. Stages are divided into two-year intervals over a total time period of 16 years. The choices left the decision makers are undisturbed growth, thinning and harvesting with the latter being divided into selection harvest and clearcuts. The problem is then solved as a Markov chain using the general recurrence relation:

\[ V^*(i) = \max \left\{ \sum_{k=1}^{m} p_{ij}^k [r_{ij}^k + V^*(j)] \right\} \]

where
- \( i \) represents states of which there are \( n \)
- \( m \) represents stages
- \( p_{ij}^k \) represents the probability of a transition from state \( i \) to state \( j \) given decision \( k \)
- \( r_{ij}^k \) represents the return associated with the transition from \( i \) to \( j \) given decision \( k \)

\( V_{m-1}(j) \) represents the optimal accumulated expected returns given that you start from state \( j \) in stage \( m-1 \).

\( V_m^*(i) \) represents the optimal expected return of being in state \( i \) at stage \( m \).

Hool uses the above to obtain the optimal activity policy for each state and then develops sequences of woodlot development-activity prescriptions by assuming that once the optimal policy decisions have been made that the most likely transitions occur. He goes on to calculate the mean number of transitions each state is expected to remain unchanged, the mean number of transitions the woodlot in a given state requires in order to go to another state by undisturbed growth and the probability of a state existing if the stand were allowed to grow undisturbed for a long period of time. It is a Markovian problem since the probability and return values are specified to be independent of time and the probability
of transition from $S_i$ to $S_j$ depends only on $S_i$ and not on the history of the system.

While the Hool paper is valuable in demonstrating a forestry application of the Markov chain, it is applied to a very restricted problem. The number of states is small and time horizon is not only finite (16 years) but very short for a forestry problem. The technique could, however, be generalized and used for larger problems.

Two USDA Forest Service researchers provided the first widely circulated application of a deterministic DP to a forest stand problem (Amidon and Akin 1968). The authors use DP to confirm the results obtained through marginal analysis by Chappelle and Nelson (1964). They use a backward recursion and therefore solve the problem for several different rotations. They restrict the decisions to be made on their sawtimber stand to a light thinning that leaves a greater volume (1 MBF) of sawtimber at the start of the next stage 5 years hence or a heavier thinning that leaves the stand with 1 MBF less sawtimber at the start of the next stage. This highly restricted problem is solved by using the following recursive structure:

$$ T(x,y) = \max \{D(x,y) + T(x+1, y+1); I(x,y) + T(x+1, y-1)\} $$

where

- $x$ refers to stand age in 5 year intervals
- $y$ refers to sawtimber volume in 1 MBF intervals
- $D(x,y)$ = decrement in growing stock value from $x+1$, $y+1$ to $x$, $y$
- $I(x,y)$ = increment in growing stock value from $x+1$, $y-1$ to $x$, $y$
- $T(x,y)$ = optimal total value from $(x,y)$ up to the rotation age

and

$$ T(n,y) = \text{initial condition, i.e., final harvest value for the rotation corresponding to } n. $$
The results confirmed the work of Chappelle and Nelson and note the more flexible nature of DP in handling changes such as product prices increasing with time. The shortcoming of the work was the highly restricted set of thinning options and the simple description of the stand in terms of sawtimber volume alone.

Gerard Schreuder (1971) recasts a continuous time model as a discrete time dynamic program to obtain an optimal thinning schedule and rotation age. This recasting simplifies the solution of the problem. The stages are time periods and states are described by the total volume per acre. The decision variable is net cut (cut minus growth). The problem is solved by backward recursion and improves upon the Amidon and Akin solution by including the cost of land. The weak point of this paper was also an oversimplified state space but it was valuable in demonstrating the flexibility of the DP approach compared to other solutions.

Lembersky and Johnson (1975) and Lembersky (1976) use a Markovian decision process to optimize timber production in the face of uncertainties. The uncertainties stem from the growth and natural mortality behavior of the trees in the latter paper while the former includes uncertain future prices. Lembersky and Johnson (1975) seek to maximize the financial return from the forest while Lembersky (1976) is interested in maximizing average annual volume. According to Lembersky (1976):

"The two criteria receive separate treatments because there are fundamental differences between Markov Decision Processes (MDP) with average undiscounted rewards (here volumes) and MDP with discounted summed rewards (dollar revenues in Lembersky and Johnson). The differences are not in the underlying basic
structure, but result from mathematical properties of aggregated expected rewards” (p. 69).

The properties differ because undiscounted returns tend to infinity over an infinite horizon and therefore must be averaged to provide meaningful comparisons. Discounted returns are almost always finite and therefore comparable².

The paper expands on the work of Hool (1966) by stretching it to an infinite horizon and including product market behavior. The state space includes average dbh, number of trees per hectare and relative market price. By not including an indicant of thinning, the authors are assuming that whether the stand reaches its current state naturally or by thinning it will grow the same thereafter. Forty-eight forest states are considered and given 5 relative market values, the problem consists of 240 states in total. Fifteen management actions are considered ranging from leaving the stand undisturbed to clearcutting and replanting at various densities. Optimal actions are then derived for each state.

The strong point of these papers is that they recognize risk in forest production problems. The shortcoming is that the range of states and management decisions, as in Hool's work, is very limited.

J. Douglas Brodie, his colleagues and students at Oregon State University have made a number of valuable contributions to the DP and forestry literature. Brodie et al. (1978) using data on yield curves from Bulletin 201 The Yield of Douglas-fir in the Pacific Northwest (McArdle and Others 1961) use a forward recursion to simultaneously determine optimal stocking levels and rotation. The stage intervals are 10 years

²Future discussion will focus on Lembersky and Johnson (1975) since financial rewards are the focus of this dissertation.
and the stands are described by volume per acre. A major improvement over previous work is the number of states and therefore policy decisions included here. The authors consider interval units of 100 cubic feet per acre (7 cubic metres per hectare). At each state all actions which could achieve a lower stocking level are considered using the recursive function described by Kao (1980) as:

\[
T(x+1, y) = \max_{y'} \{T(x, y') + P(y', y)\}
\]

where \(y'\) is any node stocking level in the current stage that can reach stocking level \(y\) in the next stage.

\(P(y', y)\) is the net revenue obtained from the transition from state \(y'\) to \(y\).

\(T(x+1, y)\) is the total value of the optimal schedule up to \((x+1, y)\).

\(T(x, y')\) is total revenue of the optimal schedule up to the stage being considered.

Since forest growth is continuous and DP requires states to be described in discrete terms, some stands required a mandatory thinning so as to fit into a state. As well, thinnings are only considered every ten years. These shortcomings were overcome in Brodie and Kao (1979) and Kao and Brodie (1979)\(^3\).

Brodie and Kao (1979) noted that the primary shortcoming of Brodie et al. (1978) was "the model could not explicitly treat the accelerated diameter growth associated with more intensive thinning." (p. 665). This is overcome by replacing the yield tables of the previous paper with the biometric model DFIT (Bruce et al. 1977) which simulates

\(^3\)This paper is discussed in section 4.2.
the growth of Douglas-fir stands. The incorporation of tree size is important as it affects selling prices and logging costs. In order to incorporate tree size into the DP solution the state descriptors are altered. The stand age remains one descriptor but the volume descriptor has been replaced by two descriptors, the number of trees and basal area. The solution is again obtained by using a forward recursion and the algorithm DOPT. DOPT is simply DFIT with a DP solving algorithm integrated into it. This allows for the greatest amount of variability seen in the literature to date. Intervals of 15 trees, 4 square feet of basal area and 10 years are used in the paper. The optimal value function is defined as the value of the present net worth (PNW) "path" from regeneration to age t, number of trees N, and basal area G for the stand:

\[ f_t(N, G) = \max_{n, g} \left( \frac{P_t(T) - L_t(T) - C}{(1+i)^t} + f_{t-10}(n, g) \right) \]  

(Brodie and Kao 1979, p. 668-669)

where

- \( P_t \) is the selling price of logs of average diameter \( D \) at breast height
- \( T \) is the volume of thinnings [including mortality captured]
- \( L_t(D) \) logging costs given average diameter \( D \)
- \( C \) is a fixed entry cost which is incurred if thinning or harvest take place
- \( i \) is the interest rate
- \( (n, g) \) is the set of feasible basal areas and number of trees at age \((t-10)\) from which the current level of \( N \) and \( G \) can be reached.

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*Basal area is a recognized measure of stand density. It is defined as the area of the cross section at breast height of a single tree or (as in our case) of all trees in a stand. It is measured on a per hectare basis.*
The starting condition is:

\[ f_{30}(N,G) = R \]

where \( R \) represents the value cost of all regeneration and other treatments before age 30.5

This paper represented the culmination of work in the field of deterministic DPs related to forestry. Kao (1980) expands on the work in his dissertation and considers more management variables than just thinning. Riitters, Brodie and Hann (1982) investigate the optimization of timber production and grazing in ponderosa pine but "the structure of the DP algorithm essentially is the same ...: (p. 519). Hann et al. (1983) used the same structure to study initial planting density and precommercial thinning.

DOPT is used again in Riitters et al. (1982) to compare volume versus value maximization. Sleavin (1983) develops a similar model for the growth simulator DFSIM. Martin (1978) and Haight et al. (1984) utilize single-tree simulators as opposed to whole stand simulators such as DFIT and DFSIM but modify the output to permit the computational format to remain essentially the same (Brodie and Haight 1985).

While this research was going on at Oregon State, a few other papers of note linking DP and forestry appeared independent of that main body of research. The most fundamental of these were Chen et al. (1980a). The state variable used is basal area and the decisions are thinning by amount of basal area. De Kluyver et al. (1980) determine optimal stand management using a two-stage approach for a forest consisting of many stands. A DP is used to find the most efficient regime

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5The authors simulate a naturally regenerated stand to age 30 and make management decisions from that point on.
for each stand, then a multiple objective linear program is used to
determine the optimal policies for the whole forest. The state space is a
yield distribution over various products such as veneer, saw logs or
pulpwood. The problem solved is very restricted with only eight different
regimes being investigated. The paper has very limited practical
application.

With the exception of the papers that constructed DPs as
Markovian decision processes (Hool (1966); Lembersky and Johnson (1975);
and Lembersky (1976)) the questions of risk and uncertainty have not been
taken into account in the papers that we have reviewed. The risk of fire
is handled in a Markov process DP by Martell (1979). He shows that the
soil expectation value and the optimal rotation fall as the risk of fire
increases. Reed and Errico (1985) reach a similar conclusion. For a more
general review of how to handle risk and uncertainty in stand level
problems one must go back to a member of the Oregon State group mentioned
above. Two papers by Chiang Kao (1982, 1984) investigate optimal stocking
levels and rotation under risk and uncertainty respectively.

Under deterministic assumptions when we know the level of growing
stock\(^6\), growth is a fixed value. When risk is present, growth becomes
a random variable and is characterized by a probability function. In Kao
(1982) stocking level is used as the state variable and volume is
maximized. There is a probability of being in each state and an expected
thinning associated with being in that same state.

If we are in state A and desire to be in state Y' then the
probability of being in state Y' is the cumulative probability of having a
growing stock greater than that level in the next stage. The stand could

\(^6\)The level of growing stock in cubic metres per hectare in a stand.
then be thinned back to that level and the amount of thinning expected can be calculated. It should be stressed that the structure of network nodes and the basic method of solving the problem remain unchanged from the deterministic case. However, more calculations are needed as each node must now carry information about the probability of which node would be reached in the next stage.

The recursion used by Kao to solve this problem is the following:

\[ R_{n+1}(Y) = \max_{X} \{P_{n+1}(Y|X) \cdot P(X) \cdot [T_{n+1}(Y|X) + C_n(X) + Y] \} \]

where \( R_{n+1}(Y) \) is the expected total volume which can be captured from state \( Y \) (a stocking level) at stage \( n+1 \).

\( X \) is the decision variable. It is the stocking level we choose to be in at stage \( n \) to get to state \( Y \) at stage \( n+1 \).

\( P_{n+1}(Y|X) \) is the probability of being in state \( Y \) in stage \( n+1 \) conditional on being in state \( X \) in stage \( n \).

\( P_n(X) \) is the probability of being in state \( X \) in stage \( n \).

\( T_{n+1}(Y|X) \) is the expected volume of thinning associated with making transition from state \( X \) in stage \( n \) to state \( Y \) in stage \( n+1 \).

\( C_n(X) \) is the total expected thinnings associated with being at state \( X \) in stage \( n \).

\( Y \) are the thinnings already accumulated up to stage \( n \).

Note that the recursion is composed of two parts. The first part concerns the probabilities of being in the desired states and is obtained
by multiplying the probability of being in state Y at stage n+1 (conditional on being in state X) by the probability of being in state X at stage n. The second is composed of the expected thinnings volume contained (as defined above) in state Y. These two parts are multiplied together to yield the expected total volume associated with being in state Y.

The recursion used by Kao is quite awkward and confusing. The optimal value for X is eventually used to generate values for \( P_{n+1}(Y) \) and \( C_{n+1}(Y) \). The reader is referred to Kao's text for greater detail on this work.

While the method employed here allows for more flexibility in state description (it could even be described as quasi-continuous) than the earlier Markov decision process approaches it would be much more difficult to employ if financial results were desired. The state space would need to be expanded to include number of trees and a proxy for tree size such as basal area. This would expand the elements of risk and more probabilities would need to be included. The "curse of dimensionality" would soon become a factor. For these reasons the probabilistic DPs in Chapter 5 will be based on Markov processes.

Kao (1984) focuses on optimal stocking levels and rotation under uncertainty. Kao stresses that pure uncertainty ("... the complete ignorance of the probabilities attached to each outcome" (p. 922)) does not really exist since we have at least a vague idea of possible outcomes or at least of impossible outcomes. Kao modifies the definition of uncertainty to "... no probability is known exactly" (p. 923). Kao estimates growth from a growth function derived as the stand develops. Probabilities of future states can also be estimated from the growth function and thus the problem is transformed from one of uncertainty to one of risk and the recursion equation
of Kao (1982) can be used. In this process of adaptive optimization the optimal stocking levels and rotation are recalculated each time we have more information about the true growth function. The results show that under uncertainty expected returns are less than under any of the levels of risk used in Kao (1982). The optimal rotation, however, is 70 years which is longer than the rotation at the highest level of risk which was 50 years. My closer investigation of the uncertainty solution shows that the level of risk is actually less in Kao's uncertainty case than in the highest risk case.

2.5 CONCLUSION

The structure of forest stand optimization problems has made them an ideal candidate for being solved by DP. In particular, it is the fact that the solutions require a series of interrelated decisions that makes DP particularly suitable. The body of literature reviewed in this chapter shows that DP has proved very useful in solving these kinds of problems. The papers also testify that the fundamentals have been covered and that the field is now open for more sophisticated applications of DP to forestry as described in Chapter 1. The improvement of previous work begins in the next chapter where cost and revenue models are devised.
Chapter 3
GROWTH, COST AND REVENUE ASSUMPTIONS

3.1 INTRODUCTION

The literature reviewed to this point has displayed how DP can be applied to forest stand optimization problems. Since the goal of these papers was to show how DP could be utilized rather than to provide accurate site specific prescriptions, the cost and revenue assumptions and to a lesser extent the growth assumptions were at times highly simplified. This is particularly true of the earliest papers. Now that DP has been established as a viable technique, it is appropriate to apply it in a more realistic setting. This can be accomplished by using growth, cost and revenue assumptions that reflect the B.C. coastal situation to some reasonable degree. The purpose of this chapter is to outline those assumptions.

The following section reviews the growth assumptions including thinning types, site characteristics and assumptions with regard to effects of fertilization, fire and pests along with a brief outline of the growth model TASS.

3.2 GROWTH ASSUMPTIONS

DP stand optimization models have been driven by two basic growth mechanisms. The early work relied on yield tables (Amidon and Akin 1968; Martell 1979; Brodie et al. 1978) while the later work (Kao 1979; Brodie and Kao 1979; Riitters et al. 1982) has taken advantage of stand projection models such as DFIT (Bruce et al. 1977), DFSIM (Curtis et al. 1981) and PPINE (Hann 1980). The model chosen for this paper is TASS (Mitchell 1975).

TASS falls in the category of single tree - distance dependent
simulators (Munro 1974; Mitchell 1980). In particular TASS is a crown stand model. It simulates the development of the crown and the bole of individual trees in considerable detail. The sum (or average in the case of dbh and height) of the tree statistics generated make up the stand statistics. The centre of activity in TASS is the crown. It responds to growing space constraints through death in the lower branches. This recession of the crown ceases when competitors are removed through mortality or thinning creating additional space for crown expansion. External influences on the tree such as those brought about by pest attack, pruning or fertilization are transmitted through the crown.

While a number of models have the capability of generating the data required for this study, TASS was selected for a number of reasons. The most compelling reason was the fact that it is a model based primarily on B.C. data and thus could best simulate growth in coastal B.C. Secondly, it is the growth model used by the B.C. Ministry of Forests. This will make this study more practical than if another model was used. Thirdly, the model was available for use at no direct cost, and finally, it has not been used in previous optimization studies of this type. Comparison of Douglas-fir yields generated from TASS and DFSIM have shown that these independently developed models using different methods and data have produced similar results. TASS stands tend to have less mortality and therefore grow more volume but smaller trees (Mitchell 1986). The overall result is that stands grown by TASS result in lower financial returns than stands grown with DFSIM. This is discussed in more detail in Chapter 4 and Appendix 2.

TASS allows for many management options and environmental factors to be considered. These include variable planting density, thinning,
fertilization, pruning, deer browsing, defoliation and site quality. To include all possible combinations and variations would create too large a problem and so the set of options has been restricted to those primarily of interest to B.C. decision makers. These priorities have been revealed through existing and proposed government programs and activities. (See Ministry of Forests 1985 and Ministry of Forests 1986). In this light the effects of pruning and deer browsing will not be considered.

Two initial planting densities will be compared. Current interest in this topic is evidenced by Smith (1986).

Both precommercial thinning (PCT) (also known as juvenile spacing) and commercial thinning will make up part of the study¹. TASS allows for a variety of thinnings and this set has also been restricted. Stands may be thinned by removing a certain basal area, a certain volume, a percentage of trees, particular individual trees or strips of trees. Trees can be removed starting with those of smallest diameter (thinning from below) or the largest diameter (thinning from above). As well, limits may be set on what diameter trees should be left or removed. While TASS has the valuable feature of keeping track of diameter distributions, it was decided to remain consistent with previous work (notably Kao 1979) and remove trees by a number of trees. Thinning will commence with the smallest trees. Thinning from above could be a source of future study and has been investigated in different types of problems by Haight et al. (1985).

Simple models will be used to simulate fire and pest attacks.

¹Precommercial thinning is defined as a thinning where there are not immediate returns from the trees removed. Immediate rewards from a commercial thinning may or may not exceed the logging costs.
Expected losses from fire will be compared under various ignition probabilities. Expected losses from a serious pest infestation will be obtained by calculating expected losses generated from a series of possible scenarios. In this way we will also be able to learn how the returns to the stand can be increased as we become more knowledgeable about the timing of major pest attacks.

Only one site quality will be used in the study. In as much as low (poor) sites do not receive much attention from managers and top quality sites are rare, the site quality chosen is in the good to medium range. This is appropriate since forest management work in the province is concentrated on the good and medium sites.

The method used by Kao (1979) with respect to fertilization will be followed in this study. Kao utilized the work of Turnbull and Peterson (1976a, b) where response to fertilization is a function of age, basal area, number of trees and site. These are the state variables employed both by Kao and this paper making the adaptation quite easy. TASS allows for user specified responses to fertilization so there is no problem in combining the models. The version of TASS used in this work does not contain its own predictive response to fertilization.

In sum, the growth assumptions have attempted to strike a balance between accurately depicting the growth, management and environment on the B.C. coast with the need to create a soluble problem. The constraints placed on the problem presented here have tried to represent the norm. Further work could investigate possible benefits of different management approaches.

3.3 COST ASSUMPTIONS

A vital component of stand optimization studies are the
assumptions with regard to cost. In many of the studies reviewed in the
previous chapter, these assumptions have been highly simplified, glossed
over, not specified or as in the case of volume based studies simply
ignored. As the focus of those studies was the DP component it is not
surprising that limited attention was paid these cost elements; they were
peripheral to the primary goals of the studies. In this study, greater
attention will be given to the cost (and revenue) elements as the study
aims to utilize DP rather than simply show how it can be used. In other
words, the results obtained are the focus of the study rather than how
they were obtained.

The subsections which follow describe how the various cost
elements in the study were obtained. These were far more difficult to
obtain and less site specific than I had hoped on commencing the study.
There is clearly a need in B.C. for research into cost equations for a
variety of silvicultural activities. For instance, data do not exist on
planting costs by density. Similarly, there is no concrete data on
juvenile spacing by intensity given an original stand density and stand
characteristics. The required data to create such equations has been
collected by the B.C. Forest Service but a shortage of manpower has
prevented any work being done in the area. It will certainly be the focus
of research as the dollars available for forest management work become
more scarce and the desire to allocate them efficiently increases.

3.3.1 SITE PREPARATION

Following harvest, a site must be cleaned up in order to allow
for prompt regeneration. This includes activities such as surveys, slash
burning and falling residual stems.

The B.C. Ministry of Forests (1985) Annual Report reports the
average cost of silvicultural activities on Crown land each year. The average cost for preparing land for planting in 1984-85 was $172 per hectare for the province as a whole and between $151 and $162 per hectare in coastal regions.

Site preparation costs can also be estimated from the Vancouver Stumpage Appraisal Guide. This method is used by Cooney (1981). Heaps (1985) also uses Cooney’s figures. These amount to $0.02 per cubic metre for spot slash burning costs, $0.01 per cubic metre for residue and surveys and a residual falling allowance of $15-20 per hectare. These costs could be updated by using the Ministry of Forests trend factor which updates an historic cost base to the year of reporting. It differs from an inflation factor in that it is specific to costs associated with removing timber.

Because of the narrow range of site preparation values included in the BCMOF annual report and its relative simplicity, a roughly weighted average of costs in the Prince Rupert and Vancouver Regions will be used. Specifically a site preparation cost of $155 per hectare will be used. This figure will be used on all rotations except the first. That is, it is assumed that the decision maker is commencing with bare land suitable for planting.

3.3.2 PLANTING

Two levels of initial spacings will be considered in this study. Dense stands tend to increase in volume quickly in the early years but this advantage dissipates due to high mortality. The surviving trees tend to have considerable vigor as they are sorted by natural selection. Less dense stands do not have this advantage but are less expensive to establish and the average diameter of the tree is greater than for more
dense stands.

In most of the previous DP studies planting costs have been treated as a simple fixed value as only one initial stocking density was considered. Site specific considerations, such as slope, were ignored. Exceptions to this were Lembersky and Johnson (1975) and Hann (1983) where a variety of initial densities were considered. In these papers equations were presented which correlated planting cost with initial density. Lembersky and Johnson used an equation developed from Buongiorno and Teeguarden (1973). Specifically

\[ \text{Planting Cost per hectare} = 52.5 + 0.14p \]

where \( p > 0 \) is the number of trees planted and costs are converted to 1985 Canadian dollars. Hann presents separate costs for four different planting levels. When these points are joined they can be expressed by the equation

\[ \text{Planting Cost per hectare} = 112.09 + 0.14p. \]

A third source for planting cost information was a survey of planting contracts by A.G. Fraser of the Canadian Forestry Service and W.G. Howard of the Industry Development and Marketing Branch of the B.C. Ministry of Forests (Fraser and Howard 1987). Their survey of twenty-seven contracts across B.C. revealed the following relationship between planting cost and initial density:

\[ \text{Planting Cost per hectare} = 57.92 + 0.14p. \]

The variable component of the three studies are remarkably similar. The variation in intercept between the first two studies is understandable given the ten year time difference between the studies and the effects of inflation over that period. The intercept in the Fraser and Howard study seems somewhat low. This could have been influence by
the overall small sample size and by the fact that only a third of the data points fell between 420 and 1000 trees per hectare while the other two thirds were between 1060 and 1500. The results of the survey are further put in question by the average silvicultural cost data in the BCMOF Annual Report (1985). That places average planting costs in B.C. at $297 per hectare. That would place the intercept at about $140 given that about 1100 trees are planted per acre on average. Support for a somewhat lower intercept can be found in planting costs associated with the Canada-British Columbia Forest Resource and Development Agreement (FRDA) which placed average planting costs over the first year of the agreement at $223 per hectare which would mean an intercept at $69. The weighted average planting costs on the coast do not vary much from the provincial averages. Vancouver region costs tend to be higher than Prince Rupert region costs. The weighted average of costs from the BCMOF Annual Report (1985) and the first year of FRDA are $293 and $227 respectively. By calculating the weighted average of these two figures, it will be assumed that the cost of planting 1100 trees (exclusive of seedling cost) in a coastal stand is $236. This means that the equation used to determine planting costs at various densities will be:

\[
\text{Planting Cost per hectare} = 96 + 0.14 \, \text{p.}
\]

The equations discussed in this section are displayed graphically in Figure 3.1.

3.3.3 PRECOMMERCIAL THINNING

Precommercial thinning (PCT) or juvenile spacing as it is also known, removes inferior trees from dense plantations between 10 and 15 years old. This leaves room for accelerated growth in the biggest, healthiest best formed trees. Trees that are felled are left on the ground to decay and return nutrients to the soil.
The cost of PCT is influenced by various factors. A cost function for PCT would be increasing in the age and initial density of the stand and in the intensity of the thinning. Site specific factors such as slope and density of undergrowth also influence cost. Most previous DP work has ignored the variables affecting the cost of PCT and has treated it as an average or fixed cost since they contained a single PCT. Hann (1983) considers a variety of PCT levels but still uses an average cost. Lembersky and Johnson (1975) is the only DP paper reviewed which recognized some cost variation. The equation, which was developed from Schroedel (1971) is:

\[ \text{Thinning Cost per hectare} = 97.50 + 0.0037 \times t_r \]

where \( t_r \) is the number of trees removed per hectare.
Lembersky and Johnson consider removing between 375 and 2435 trees so the cost difference at these two extremes is only $8 per hectare given the equation above. Even though this is expressed in 1970 U.S. funds, it is little different than using a fixed cost.

Using a fixed cost may lead to some biases in the results when an array of PCT options are being considered. The potentially more expensive options of heavier thinnings, postponed thinnings and thinning larger average diameter stands become more attractive. Ideally the cost of PCT would be determined by a cost function taking into account the factors reviewed. A vigorous search of the literature failed to uncover such a function. Discussion with the BCMOF revealed that they are interested in such a function and have sufficient contract data to generate it, however manpower shortages have prevented it from being calculated.

To avoid some of the biases inherent in a fixed PCT cost, Table 3.1 will be used to determine the cost assigned to a given PCT operation.

<table>
<thead>
<tr>
<th>Intensity of Thinning</th>
<th>Average Diameter</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Small (&lt;8 cm)</td>
<td>Medium (8&lt; x &lt;13 cm)</td>
</tr>
<tr>
<td>Light</td>
<td>325</td>
<td>450</td>
</tr>
<tr>
<td>Medium</td>
<td>450</td>
<td>600</td>
</tr>
<tr>
<td>Heavy</td>
<td>575</td>
<td>650</td>
</tr>
</tbody>
</table>

Table 3.1 Assumed Precommercial Thinning Costs

While no specific function is used to generate the figures in the table, they are based on average PCT costs presented in the BCMOF Annual Report (1985) and from first year costs under FRDA. The weighted average for the coastal regions under FRDA was approximately $700 per hectare and $600 per hectare in the Annual Report. The difference can be explained by
the fact that more of the thinnings under FRDA occurred in the Prince Rupert regions where thinning tended to be more expensive than in the Vancouver region. Other factors influencing the table are the report by Berg (1970) noting the steep increase in time required to fell larger trees in a PCT operation even when the diameter differences are small and contact with the BCMOF with regard to the intensity and average diameter cut of present PCT operations. They revealed that most thinnings currently taking place on Crown land could be classed as heavy thinnings of medium sized stands. The thinnings are heavy because they are on naturally regenerated stands which tend to be much more dense than planted stands. As more realistic functions become available in the future, they will replace the data used in this table. Unfortunately, these functions are not available at this writing.

3.3.4 FERTILIZATION

Fertilizer is applied to forest stands in an effort to increase the diameter, height, gross volume and merchantable volume. Through improved yields obtained by fertilization it is hoped that crop rotation lengths can be reduced. No fertilization has taken place on Crown lands in recent years as available funds have been utilized instead in basic silviculture (BCMOF 1985). Funds from FRDA will permit fertilization treatments to be carried out at least over the next few years. Private firms are also renewing fertilization efforts (Vanc. Sun, Nov. 17, 1986).

Fertilizer is normally applied by helicopter. The last annual report which reported on fertilization costs calculated the average cost at $227 per hectare (BCMOF 1985). According to the Sun article cited above, the expected cost of work to be carried out this year is $160 per hectare. This figure will be used in this study.
3.3.5 COMMERCIAL THINNING

Commercial thinning seeks to allow accelerated growth in the residual stand just as PCT does. The former takes place when the trees that are removed are large enough to be sold for revenues which partially offset the cost of logging. Little if any commercial thinning has been practised in recent years in B.C. as it has been considered uneconomic by the timber industry.

Given the lack of this activity, costs were once again difficult to obtain. Previous Canadian and American studies (Heaps 1985; Brodie and Kao 1979) have used data contained in the U.S. Bureau of Land Management Timber Production Costs Schedule (1979). This data has become somewhat dated. As well it is not sensitive to various intensities of thinnings and is reflective of costs in western Oregon rather than on the B.C. coast. Kao (1979) derives a stump to truck commercial thinning cost function from the work of Sessions (1979). The cost is based on mean diameter and volume removed. This is a modification of the function used for total logging cost in Kao (which is also based on Sessions) which bases costs on mean diameter and volume present per hectare. Again the data is dated and based on conditions in western Oregon.

To avoid fabricating a cost function for the B.C. coast and using dated data from another region, a function providing the net return to commercial thinning will be constructed based on the work of Fight, Ledoux and Ortman (1984), Williams (1987) and Gasson and Williams (1986). Fight et al. used a simulator to develop cost equations for thinning small second-growth stands for all phases of logging from felling to loading on a truck. Where appropriate the equations are functions of average stand diameter and volume removed. The authors combine the equations to provide
a table which displays tree to truck costs by stand volume and average
diameter in 1983 U.S. dollars per thousand cubic feet. This table was
then converted to 1985 Cdn. dollars per cubic metre. In this form,
however, the model is still reflecting Oregon costs. The model was
brought into line by using a tree-to-truck costs model developed by
Williams (1986). This model was designed for clearcuts rather than
thinnings but common points with Fight's model do exist. These common
points are found where Fight looks at heavy thinnings of dense stands.
The assumption was then made that while the relative costs between B.C.
and Oregon operations differ they move in the same way. That is the
ratios between points on an Oregon table and a B.C. table would be the
same but the absolute numbers would differ with B.C. costs being higher.
Table 3.2 displays the results.

Fixed costs were estimated to be $35 per cubic metre. These were
estimated from material contained in Morrison et al. (1985).

The revenue function is drawn from Gasson and Williams (1986).
It is discussed in more detail in Section 3.4.

While the method used here falls short of providing a pure B.C.
net revenue for commercial thinning, it provides a function that allows
for the impact of harvesting small stems and low volumes. Figure 3.2
highlights the fact that commercial thinnings are not profitable in the
short run except when the harvests are large enough to be clearcuts. It
is evident that if commercial thinning is to be profitable, it will have
to be through the gains obtained at harvest. That is, it will need to be
profitable in the same way as PCT.
Table 3.2 Commercial thinning tree-to-truck costs per cubic metre by average stand diameter and harvested volume per hectare

<table>
<thead>
<tr>
<th>DBH cm</th>
<th>35</th>
<th>70</th>
<th>105</th>
<th>140</th>
<th>350</th>
<th>700</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.24</td>
<td>62</td>
<td>57</td>
<td>54</td>
<td>52</td>
<td>47</td>
<td>46</td>
</tr>
<tr>
<td>20.32</td>
<td>52</td>
<td>47</td>
<td>45</td>
<td>43</td>
<td>38</td>
<td>37</td>
</tr>
<tr>
<td>25.40</td>
<td>45</td>
<td>40</td>
<td>37</td>
<td>36</td>
<td>32</td>
<td>31</td>
</tr>
<tr>
<td>30.48</td>
<td>39</td>
<td>33</td>
<td>31</td>
<td>30</td>
<td>26</td>
<td>26</td>
</tr>
<tr>
<td>35.56</td>
<td>34</td>
<td>29</td>
<td>27</td>
<td>25</td>
<td>22</td>
<td>21</td>
</tr>
<tr>
<td>40.64</td>
<td>30</td>
<td>25</td>
<td>23</td>
<td>21</td>
<td>19</td>
<td>18</td>
</tr>
<tr>
<td>45.72</td>
<td>37</td>
<td>29</td>
<td>25</td>
<td>23</td>
<td>19</td>
<td>18</td>
</tr>
<tr>
<td>50.80</td>
<td>34</td>
<td>25</td>
<td>22</td>
<td>20</td>
<td>16</td>
<td>15</td>
</tr>
<tr>
<td>55.88</td>
<td>32</td>
<td>24</td>
<td>20</td>
<td>18</td>
<td>15</td>
<td>14</td>
</tr>
<tr>
<td>60.96</td>
<td>32</td>
<td>23</td>
<td>19</td>
<td>17</td>
<td>14</td>
<td>13</td>
</tr>
</tbody>
</table>

3.3.6 HARVESTING

The cost of harvesting a forest stand is the sum of the cost of many operational phases. Roads must be constructed and maintained. The trees must be felled and bucked, yarded and loaded onto trucks. The logs must be hauled to the mill or to dump sites in preparation for barging/towing. Other costs associated with logging include crew transportation, engineering, scaling costs as well as administration and overhead.

Models exist which estimate logging costs as a function of parameters such as species harvested, stand diameter, volume per hectare, slope, terrain, obstacles and so on. Cooney (1981), Morrison et al. (1985) and Williams (1986) have produced models of recovery costs for the B.C. coast based on the Coast Lumber Appraisal Manual. This is the manual used by BCNOF to estimate logging costs on the B.C. coast for the purpose of calculating stumpage under a residual system.
Figure 3.2 Net commercial thinning costs by cubic metre per hectare
The phases noted above will be divided into four groups for cost estimation: development costs, tree-to-truck costs, hauling costs and other. Factors affecting each of these are discussed below.

The cost of developing a site for logging can be expressed as a function of side slope, road construction costs, number and size of bridges and culverts, haul distance for ballast and the volume of timber per hectare (Williams 1986). With the exception of volume of timber, all these factors will be fixed to reflect an average stand. The average development costs on the coast in 1985 was $3600/ha. Development costs per cubic metre will be viewed as a decreasing function of volume. So as the cubic metres in a stand increases a lesser amount per cubic metre will be attributed to development costs.

Tree-to-truck costs will be estimated using the model contained in Williams (1986). It estimates tree-to-truck costs per cubic metre as a function of species, average stand diameter, volume per hectare, slope, terrain and logging system. Slope and terrain will be fixed along with species (Douglas-fir) and the breakdown of the logging system. The logging system is broken down to reflect the proportion of high-lead and grapple yarding system used on the coast. Average stand diameter and volume per hectare become the TASS outputs which drive this phase of the model. Increasing stand diameter or volume per hectare decreases the cost per cubic metre of this phase.

Hauling costs per cubic metre are a function of haul distance, road conditions and load size. To these costs must be added water transportation costs which averaged $7.88 in 1985. Total transportation costs will be assumed at the 1985 coastal average of 46% while the terrain will be assumed as even.
and crew related costs will be assumed at $15/m³. These conditions will be considered fixed to represent average conditions. Future work should consider a range of logging costs. Other costs such as scaling, administration, overhead and allowance for profit and risk will also be treated as fixed costs.

3.3.7 DISCOUNT RATE

The question of the proper discount rate to be used in the context of forestry investments has enjoyed a long debate in the literature. Recent reviews of the subject by Fraser (1985) and Heaps (1985) support a range between 5% and 10% real with Heaps leaning towards the lower figure because although there are substantial risks inherent in the growth of a forest stand (failure, pests, fire, etc.) these become negligible across many stands. Five percent will be used in this study for the base case, while other rates will be used at times for comparison. A higher discount rate would affect the results by favoring shorter less profitable rotations.

3.4 REVENUE

The revenue function is based on the work of Gasson and Williams (1986). The value of a stand is a function of the species contained in the stand and the average produced log volume. Log volume is a function of mean diameter and is adjusted for yarding losses, current utilization standards and is net of "logs left due to defect, size, or breakage" (Gasson and Williams 1986, p. 2). The authors also estimate the expected breakdown of log grades from each species and calculate the relative value of those grades from information on the Vancouver Log Market. The prices used in the study are average log market prices over the years 1980 to 1985. From this information a single value can be calculated which represents the
value per cubic metre obtained based on the average stand diameter and the
species present in the stand. This computation has been set up in a form
compatible with Lotus so the calculations can easily be made on a personal
computer.

This method is consistent with other recent DP studies in that it
reflects the benefits accruing to harvesting logs of larger diameter.
This long range benefit of thinning can thus be accounted for.

3.5 CONCLUSION

The cost and revenue functions presented were chosen to strike a
balance between providing sufficient detail to accurately reflect reality
without making the problem solvable only at great expense. The functions
also try to present the average case. This is particularly true of
certain components of logging cost. Finally, where detail or exact cost
has not been included, care has been taken to insure the functions respond
in the right direction to changes in variables. An example of this is in
precommercial thinning. The result of these efforts has been to provide a
more detailed cost and revenue picture than has been provided by other DP
papers in the field while still maintaining enough simplicity to allow
solutions to be obtained within a reasonable time span.

The functions discussed in this chapter will be used to obtain
net stand values in Chapters 4 and 5. Justification for maximizing net
stand value is contained in the next chapter.
Chapter 4

OPTIMAL STAND MANAGEMENT REGIMES IN A DETERMINISTIC SETTING

4.1 INTRODUCTION

Most of the forestry-DP literature reviewed in Chapter 2 contained deterministic models of the type to be developed in this chapter. While we do not live in a deterministic world, models of this type are important as a base case to which we can compare our results when we enter the more realistic realm of stochastic models in the next chapter.

This chapter outlines the assumptions within the model and discusses the management options included and the method of solution. Results from the solution of the model when unlimited funding is available and when budget constraints are in place are also provided.

4.2 MODEL ASSUMPTIONS AND METHOD OF SOLUTION

In a deterministic model all factors influencing the solution of the model are known and constant. Specifically the stand grows in a predictable fashion. There are no shocks in the form of periods of exceptionally good (or bad) growing conditions, random pest attacks or fires. When a stand is thinned or fertilized, it responds in a consistent and predictable manner. Prices are assumed constant as are the costs of silvicultural activities and timber harvesting. The discount rate is also known and constant.

Assumptions must also be made with regard to the method of solution of the model. This refers to factors such as the stage interval and the various thinning and basal area intervals to be considered.

The role of the stage interval in a forestry-DP problem is considered by Kao and Brodie (1979) and to a lesser extent by Roise (1986).
Reducing the stage interval from ten years to one year increased the soil expectation value of the stand by about one percent according to Kao and Brodie. Roise increased the soil expectation value by thirty-nine percent by reducing the stage interval from 15 years to 10 years. Given the small gains obtained by reducing the interval to less than 10 years, an interval of 10 years is used in this study.

States will be described by age, number of trees and basal area. Kao (1980) discusses the impact of various intervals of number of trees and basal areas. A small interval of say, fifteen trees provides more accuracy for a given basal area interval but increases computing time. Increased precision can be obtained by decreasing both tree interval and basal area interval. Kao (1980) shows that reducing the basal area interval from a 3.7 m$^2$ to .37 m$^2$ increases the soil expectation value by about five percent. The greatest benefits come in reducing the interval from 3.7 m$^2$ to 1.9 m$^2$. Kao states that any interval between .37 m$^2$ to 1.9 m$^2$ should be suitable along with a tree interval of 15 trees. Larger basal area intervals were shown to significantly reduce computing time. Smaller tree and basal area intervals also reduce the occurrence of artifact effects. Such an effect occurs when more than one alternative is found to lead to a state. The alternative with the greatest cumulative net worth is selected to represent that state. It is possible however, that an alternative not selected could contribute more in future periods and ultimately would have been the best choice by a small amount. Kao (1980) and Brodie and Kao (1979) stress the differences in value will be very small. These errors could only be eliminated by using continuous states. They can be minimized by using large tree intervals or by using a combination of small tree intervals and small basal area intervals.
Following Kao (1980) the latter method will be used here and intervals of 25 trees and 0.5 m² will be used to classify the states.

In the cases involving juvenile spacing and commercial thinning a given number of the stems will be removed beginning with the shortest trees. Thinnings will begin at 25 trees per hectare and increase up to 250 trees per hectare for stands planted at 550 stems per hectare and up to 600 trees per hectare for stands with an initial density of 1100 stems.

Two initial densities will be considered. Each will be run separately to avoid adding to the state space. The densities considered will be 1100 and 550 trees per hectare. The impact of adding 222 kilograms per hectare of fertilizer at 30, 40, 50 and 60 years will also be evaluated.

The unconstrained deterministic models are solved by means of forward induction as discussed in Chapter 2. Specifically they take on the following form:

\[ T(x+1,y) = \max_{y'} \{T(x,y') + R(y',y)\} \]

where \( y' \) is any node stocking level in the current stage that can reach stocking level \( y \) in the next stage.

\( R(y',y) \) is the discounted revenue or cost associated with the transition from \( y' \) to \( y \).

\( T(x,y') \) is the total discounted net revenue up to the stage being considered.

\( T(x+1,y) \) is the total discounted value of the optimal schedule up to \( (x+1,y) \).

This recursion can be modified into more conventional notation. If we replace the \( x \) representing the stage by \( t \) and use the superscripts \( j \) and \( k \)
to represent states, the recursion can be written as:

$$T(y_{t+1}^j) = \max_{y_t^k} T(y_t^k) + R(y_t^k, y_{t+1}^j)$$

By assuming that $j$ and $k$ represent consecutive time periods, the $t$s can be suppressed for ease of reading to yield:

$$T(y_j^j) = \max_{y_k^k} T(y_k^k) + P(y_k^k, y_j^j)$$

Now if we let $w_i^i$ represent the value of being in state $i$ then the $y$s can be suppressed, the recursion can be written as:

$$w_j^j = \max_{y_k^k} R(k, j) + w_k^k$$

A state $k$ is being chosen from all states which lead to state $j$ in the next stage. That $k$ is chosen so that the value of being in that state ($w_k^k$) combined with the return from making the transition from $k$ to $j$ provides with the maximum possible value ($w_j^j$) of being in state $j$.

4.2.1 THE OBJECTIVE FUNCTION

A discussion of what we are maximizing is important at this point. Since 95% of the forest land in B.C. is in provincial Crown land, it is the Crown that will make the decision of what should be maximized.

It could be argued that the Crown would maximize stumpage or the return to itself from the forest (Heaps 1985). This, however, is a very narrow view of how the forests can contribute to the welfare of the citizens of British Columbia. The forests must also be managed to allow for a strong forest industry which provides many benefits (employment, tax revenue, investment) to the people of British Columbia. A strong forest

1These benefits may not be as great in the case of a foreign firm which transfers a portion of its profits outside the country.
industry relies on stands of high value. It can be argued that the returns to British Columbians can be maximized by growing stands of maximum value (Lewis 1976). Stands increase in value as they contain characteristics which make them valuable to the forest industry. Over the long term stumpage returns will be high when stands are of high value and the industry is strong. Under the new Comparative Value Timber Pricing System to calculate stumpage effective October 1, 1987, the provincial objective for forest revenue will be broadly related to the gross sales value of products produced by the forest industry (British Columbia Forest Service 1987). More valuable stands will bring the Crown stumpage greater revenue over time.

It can be seen that the Crown's goals of getting a return from its resource in the form of stumpage and creating an environment for a strong forest industry can be met by growing stands of maximum value. Therefore, the objective function used will be net stand value where net stand value is defined as the expected value of the logs produced by the stand, net of harvesting, stand tending and planting costs.

4.3 AN EXAMPLE

A small modified example extracted from the solution to the problem with an initial stand density of 550 trees per hectare will help the reader visualize how a DP is solved by forward induction and what makes it more efficient than simulation.

This is shown in Figure 4.1.

Given the network form of this problem it can be solved using a simple shortest route recursion such as:

$$Y_i = \max_j \{R_{ij} + y_j\}$$
where the values of y are values of the nodes and the $R_{ij}$ are the costs of traversing the arcs.

The main point of interest in the example occurs when the decision maker decides how to get to state 6. The figure shows that it is less expensive to pass through state 3 than state 4 so this is the optimal path. This implies that the path 1-2-4-6 need never be considered again. That is, the level of PCT at age 10 that caused the transition from state 2 to state 4 is eliminated and we do not have to look at any events beyond
age 30 along that path. This highlights the benefit of DP over simulation. Simulation would continue to use that path for comparison of regime values while DP has saved time and effort by eliminating the run sooner. While this is a very simple example it does show the method of solution for the larger DP.

The reader should also note the network structure of the problem. It could also be solved by linear programming algorithms for networks. The importance of this observation will be seen in section 4.5.

4.3 RESULTS

The results are presented in two ways reflecting a single rotation on a site and an infinite series of rotations. The first will be the Net Present Value (NPV) return while the second is the soil expectation (SE) value. These can be calculated using the following formulae:

\[
\text{NPV}_r = \frac{Y_r + T_b (1+i)^{r-b} - C_c (1+i)^{r-c}}{(1+i)^r}
\]

\[
\text{SE}_r = \frac{Y_r + T_b (1+i)^{r-b} - C_c (1+i)^{r-c}}{(1+i)^r - 1}
\]

where, 

- \(Y\) = net yield at rotation age
- \(T\) = net value of commercial thinnings
- \(b\) = ages of commercial thinnings
- \(C\) = cost of treatments (including planting, spacing and fertilization)
- \(c\) = ages at which cost is incurred
- \(r\) = length of rotation
- \(i\) = interest rate
A simpler way to obtain the SE value once the NPV has been calculated follows:

$$SE = \frac{NPV_r (1+i)^r}{r(1+i)^r - 1}$$

Since i and r are known the calculation simply becomes NPV times a single number.

While the above equations define NPV and SE they do not reflect how they are obtained in a DP setting. At each stage there is a node which contains the maximum NPV for that rotation (r) if harvest were to take place at that age. The SE value is obtained by taking the NPV value and adding to it an infinite series of similar rotations. This can be expressed as:

$$SE_r = NPV_r (1 + \alpha^r + \alpha^{2r} + \ldots)$$

This reduces to:

$$SE_r = \frac{NPV_r}{1-\alpha^r}$$

So the maximum value can be found according to:

$$SE^* = \max_{r = 10, 20 \ldots} \frac{NPV}{1-\alpha^r}$$

and $\alpha$ is the discount rate.

The initial results paint a discouraging picture of the benefits of silviculture investment in B.C. Using the 5% discount rate discussed in the previous chapter, a plot of bare land of high medium quality devoted to timber growing could not provide a positive return on investment given average logging conditions in 1985. Furthermore, over

\footnote{TASS does not consider competition the seedlings may face from weeds and/or unwanted hardwood species. B.C. experience has shown that removal of these is essential for a stand to thrive.}
<table>
<thead>
<tr>
<th>Rotation Age</th>
<th>Initial Density</th>
<th>Optimum Regime¹</th>
<th>NPV $/ha</th>
<th>SE $/ha</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>550</td>
<td>Thin by 250 at 20</td>
<td>-428.15</td>
<td>-469.05</td>
</tr>
<tr>
<td>50</td>
<td>1100</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>60</td>
<td>550</td>
<td>No action</td>
<td>-243.09</td>
<td>-256.84</td>
</tr>
<tr>
<td>60</td>
<td>1100</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>70</td>
<td>550</td>
<td>No action</td>
<td>-121.47</td>
<td>-125.59</td>
</tr>
<tr>
<td>70</td>
<td>1100</td>
<td>Thin by 750 at 30</td>
<td>-381.13</td>
<td>-394.07</td>
</tr>
<tr>
<td>80</td>
<td>550</td>
<td>No action</td>
<td>-84.69</td>
<td>-86.46</td>
</tr>
<tr>
<td>80</td>
<td>1100</td>
<td>Thin by 750 at 20</td>
<td>-327.13</td>
<td>-333.99</td>
</tr>
<tr>
<td>90*</td>
<td>550</td>
<td>No action</td>
<td>-82.72</td>
<td>-83.76</td>
</tr>
<tr>
<td>90</td>
<td>1100</td>
<td>No action</td>
<td>-203.97</td>
<td>-206.53</td>
</tr>
<tr>
<td>100</td>
<td>550</td>
<td>No action</td>
<td>-97.46</td>
<td>-98.21</td>
</tr>
<tr>
<td>100*</td>
<td>1100</td>
<td>No action</td>
<td>-194.53</td>
<td>-196.02</td>
</tr>
<tr>
<td>110</td>
<td>1100</td>
<td>No action</td>
<td>-202.39</td>
<td>-203.34</td>
</tr>
</tbody>
</table>

Notes: 1. Blanks indicate trees were too small to obtain an accurate return on harvest figure.
   * Optimal rotation ages by initial density

Table 4.1 NPV and SE values by rotation age and initial density

One or an infinite number of rotations (see Table 4.1) losses could be minimized by planting a stand and then ignoring it until harvest². The following subsections investigate the impact each of the decision variables had on stand growth and the optimal level of each of these variables. An effort is also made to reconcile the fact that many dollars are invested in silviculture each year when returns appear to be negative.
4.3.1 INITIAL DENSITY

Initial densities of 550 and 1100 trees per hectare were considered. Stands planted at the lower density tended to grow faster, mature sooner (i.e., lower rotation age) and produce plots of trees of greater average diameter. The higher returns were beyond those which could be explained by the lower cost of planting alone ($166 versus $236) and lend credence to the views recently expressed by Smith (1986) about the benefits of wide spacing.

4.3.2 PRECOMMERCIAL THINNING

In general, stands which were precommercially thinned produced larger trees, matured sooner and provided greater revenues at harvest than unthinned stands. This is consistent with the results obtained for initial density. With a 5% discount rate however, these benefits did not exceed the cost of obtaining them.

Regardless of initial density, earlier thinnings lead to greater revenues at harvest time than the same intensity of thinning at a later date. Though thinning later is more expensive in pure dollar terms and has less growth impact than earlier thinnings, when discounting is taken into effect later thinnings become the preferred alternatives in terms of NPV and SE. (See Table 4.2).

The optimal intensity of thinning also varied depending on initial density and whether harvest revenue or financial return is optimized. The optimum financial return was achieved at a lower intensity than the maximum level of harvest revenue when initial density was 1100 trees per hectare (See Table 4.3).
Table 4.2 Impact of thinning age on harvest return, NPV and SE values

<table>
<thead>
<tr>
<th>Initial density (trees/ha)</th>
<th>Thinning Intensity (trees removed/ha)</th>
<th>Stand age at Thinning</th>
<th>Harvest Age at Thinning</th>
<th>Harvest Revenue ^1 $/ha</th>
<th>NPV $/ha</th>
<th>SE $/ha</th>
</tr>
</thead>
<tbody>
<tr>
<td>1100</td>
<td>250</td>
<td>10</td>
<td>100</td>
<td>6052</td>
<td>-389</td>
<td>-392</td>
</tr>
<tr>
<td>1100</td>
<td>250</td>
<td>20</td>
<td>100</td>
<td>6018</td>
<td>-313</td>
<td>-315</td>
</tr>
<tr>
<td>1100</td>
<td>250</td>
<td>30</td>
<td>100</td>
<td>5521</td>
<td>-269</td>
<td>-271</td>
</tr>
<tr>
<td>1100</td>
<td>500</td>
<td>10</td>
<td>90</td>
<td>5393</td>
<td>-445</td>
<td>-451</td>
</tr>
<tr>
<td>1100</td>
<td>500</td>
<td>20</td>
<td>90</td>
<td>5229</td>
<td>-340</td>
<td>-345</td>
</tr>
<tr>
<td>1100</td>
<td>500</td>
<td>30</td>
<td>90</td>
<td>4511</td>
<td>-284</td>
<td>-288</td>
</tr>
<tr>
<td>550</td>
<td>200</td>
<td>10</td>
<td>80</td>
<td>5635</td>
<td>-252</td>
<td>-254</td>
</tr>
<tr>
<td>550</td>
<td>200</td>
<td>20</td>
<td>80</td>
<td>5488</td>
<td>-225</td>
<td>-226</td>
</tr>
<tr>
<td>550</td>
<td>200</td>
<td>30</td>
<td>80</td>
<td>4633</td>
<td>-217</td>
<td>-218</td>
</tr>
</tbody>
</table>

Note 1: Harvest age is optimal for the regime considered.

Table 4.3 Optimal thinning intensities for maximizing harvest revenue, NPV or SE

<table>
<thead>
<tr>
<th>Initial density (trees/ha)</th>
<th>Thinning Intensity (trees removed/ha)</th>
<th>Stand age at Thinning</th>
<th>Harvest Age</th>
<th>Harvest Revenue $/ha</th>
<th>NPV $/ha</th>
<th>SE $/ha</th>
</tr>
</thead>
<tbody>
<tr>
<td>550</td>
<td>175</td>
<td>30</td>
<td>80</td>
<td>5084</td>
<td>-208</td>
<td>-212</td>
</tr>
<tr>
<td>550</td>
<td>175</td>
<td>20</td>
<td>80</td>
<td>5688</td>
<td>-221</td>
<td>-225</td>
</tr>
<tr>
<td>1100</td>
<td>200</td>
<td>30</td>
<td>100</td>
<td>5521</td>
<td>-269</td>
<td>-271</td>
</tr>
<tr>
<td>1100</td>
<td>600</td>
<td>10</td>
<td>100</td>
<td>9455</td>
<td>-517</td>
<td>-521</td>
</tr>
</tbody>
</table>

4.3.3 COMMERCIAL THINNING

It was noted in the section on commercial thinning costs in Chapter 3 that the absence of this activity in British Columbia made it difficult to obtain reliable cost data. The results obtained here verified that commercial thinning is not a viable activity on second growth coastal stands. Thinning the trees at age 40 and greater is a much more expensive exercise than PCT due to the size of the trees. The revenue obtained from the trees does not approach this increased cost. In addition to revenue from thinned trees commercial thinning operations also have a cost advantage from occurring later in the stand's life and thereby being
subject to greater discounting; this also failed to outweigh the increased
costs of thinning larger trees. In sum, commercial thinning alone or in
combination with PCT did not produce any of leading regimes in terms of
net present value or harvest revenue.

4.3.4 FERTILIZATION

Fertilization was allowed to impact the stand as described by
Turnbull and Peterson (1976a, b). Fertilized stands tended to produce
more total volume, have a greater average diameter and produce greater
harvest revenue. Despite this, fertilization treatments did not pay for
themselves at a 5% discount rate. Financial losses from fertilization
were minimized when it was applied later (age 60) as opposed to age 30.
Harvest values also peaked with fertilizer applied at age 60.

4.4 ANALYSIS

The results presented in the previous section lead one to
question investments in silviculture. The results suggest that they
simply do not pay for themselves. In this section, the results will be
reconciled with what we see in British Columbia, i.e., substantial
investment in silviculture.

First of all, the model and the cost assumptions should be
reviewed. Perhaps thinning costs have been overestimated or the model
understates the impacts of forest management. It is also possible that
the revenue model does not fully reflect the premiums that will be paid
for large diameter logs once the old growth stands have disappeared. As
discussed in Chapter 3, work is needed to fine tune silvicultural cost

---

3Light commercial thinnings at age 60 were among the leaders in terms
of NPV but this occurs because the best option is to do nothing. Light
commercial thinning cut into this value and decrease harvest revenue but
not enough to bring them down to the next best alternative.
data. As costs charged by contractors tend to vary substantially depending on market conditions, the figures presented here represent average conditions. Growth models will improve as more second growth stands mature, but all in all the data appear reasonable.

A part of the model that requires closer investigation is the discount rate. A lower discount rate decreases the financial benefits accruing to later silvicultural treatments and reduces the penalty on harvest returns. Hence regimes compared under lower discount rates would differ from higher discount rate regimes in four distinct ways:

1) Overall they would be more profitable
2) Management costs would be accepted earlier in the regime
3) Optimal regimes are more likely to include silvicultural work
4) Optimal harvest age may be later.

The table below confirms point one and suggests merit in points three and four. A closer look at the results shows that although "no action" remained optimal, its advantage over the next best regimes was only 20% (at 550 trees per hectare) at 2%, compared with 98% at 3%, and 147% at 5%. A similar trend exists at 1100 trees per hectare. This lends credence to point three. Point four is verified by the fact that for individual regimes the optimal rotation length never fell as the discount rate fell and increased in one case. Comparison of non-optimal regimes showed that as the discount rate fell, regimes with earlier management actions moved up to a rank higher than regimes with later management actions thus confirming the second point.

Separate runs were conducted to measure the financial impact of fertilization at discount rates below 5%. Fertilization proved to be a viable option when the discount rate fell to 2%. 
Table 4.4 Effect of the discount rate on optimal regime and returns

The reasons for using the 5% rate were discussed earlier. The results presented here show why it is a topic of heated debate as a movement of a single percentage point can radically alter the results.

If the costs, returns and discount rate are accepted as reasonable we must look elsewhere to reconcile the results obtained with the level of forest management witnessed on both private and Crown land in B.C. A possible solution lies in laws and regulations affecting silvicultural work on provincial Crown land. Under Section 88 of the B.C. Forest Act, firms may:

"Where, under an agreement made under this or the former Act,
(a) a person performs on Crown land work that is not considered in the determination of stumpage rates by:
   (i) constructing a logging access road.
   (ii) applying reforestation or other silvicultural treatment, or
   (iii) carrying out another responsibility.
(b) the work is approved in advance by the regional manager.
(c) all or part of the expense of performing the work, or a formula for ascertaining the expense, is approved by the
regional manager, and

(d) the work is performed to the satisfaction of the regional manager, the expense that is approved, or that is ascertained according to the formula, shall be applied as a credit against stumpage payable by the person in respect of timber harvested in a prescribed area of the Province."

(British Columbia 1986, p. 45).

In simpler terms, silvicultural work may be carried out on Crown land with the cost being deducted from the stumpage payable by the firm or individual. The firm may pay x dollars in stumpage to the Crown or carry out one of the activities outlined above. This would increase the level of silviculture carried out by firms only if they have a reasonable expectation of obtaining a positive return from investing the funds in silviculture. The return need only be positive since the return on stumpage payments is 0, except where the payments decrease the firms taxable income. Silvicultural treatments would decrease this by the same amount.

The results show that investments in silviculture have negative returns. Regardless of how silvicultural work is paid for, it has the same effect on stand values. Given a 5% discount rate and existing silvicultural costs, net stand values are negative. Section 88 provides higher valued stands to industry at no cost to them and this explains why they would be willing to carry out this work as it improves their profit potential. It does not explain however, why the Crown invests in silviculture.

Federal government assistance may be another factor in encouraging firms and the Crown to invest in silviculture. Silvicultural
costs are allowed as deductions from income in computing taxable income (Boulter 1984) and as such their real cost is 40% less than the actual cost of treatment. This transfer increases the net stand value associated with planting a stand to $100 per hectare or $17 per hectare for stands with initial densities of 1100 and 550 stems per hectare respectively. It does not make investment in reforestation and stand tending profitable.

The second case to be reviewed is when the federal government contributes to planting and stand tending costs on provincial Crown land through agreements such as the Forest Resource Development Agreement (FRDA). Through FRDA the federal government pays 50% of the cost of approved backlog reforestation and intensive forest management projects. These investments can not be justified at current cost and revenue levels. Even when stand tending costs are reduced by 50% the optimal regime is plant and leave. Sensitivity analysis showed that at an initial density of 550 stems per hectare spacing costs would have to be reduced to about $50 per hectare to be chosen as part of the optimal regime. Note, however, that the return from these regimes would still be negative. The outlook was brighter for fertilization where costs only needed to be reduced by about 55% to bring them into the optimal regime.

The planting of backlog reforestation sites where site rehabilitation and planting costs average $700-$800 is also not profitable at current costs and revenues even when the federal government is paying 50% of the cost.

The other factor which can influence the real cost of silvicultural treatment is known as the allowable cut effect (ACE). In brief, the ACE permits the expected increase in mature timber at harvest
resulting from a silvicultural investment today to be partially recouped today. For example, a reforestation project estimated to provide 750 m$^3$ in 55 years could provide an additional 13.6 m$^3$ additional harvest from mature stands today (Fraser 1985). That is, the government would increase the Annual Allowable Cut (AAC) of firms engaging in silviculture activities or a private firm may cut more of its stock today in anticipation of adequate supplies to meet its future needs. This approach has some limitations. Sufficient mature forests must be available over the life of the project to allocate additional timber to the firm. It has also been shown (Fraser 1985) that the allowable cut effect encourages reforestation projects over spacing and fertilization projects since these bring a greater increase in allowable cut today. Indeed, it is questionable whether spacing adds volume at all.

An increase in cut today could also be viewed as a decrease in the cost of a silviculture project. For instance, Fraser provides an example where a spacing/fertilization project provides an additional 195 m$^3$ at harvest time. The allowable cut effect would be 4.4 m$^3$ per year over the 44 years until harvest. If we assume that the additional harvest has a mean dbh of 50 cm and a net value therefore (according to our value and costs model) of about $5/m^3$, a flow of $22 per year would accrue over the 44 year period. This would be divided between the firm and the Crown. Discounted at a rate of 5% the net present value (NPV) of this flow is $389 which helps offset the approximate $700 cost of the project. If this flow is taxed at a full 40% the NPV is still $233 while the net (tax benefits considered) cost of the project would be about $420. The table below demonstrates the impact of the allowable cut effect on the optimal regime where the initial density is 1100 trees per hectare and in particular on the return to silvicultural investment.
When the allowable cut effect is considered the optimal action is still to leave the trees to grow after planting. The difference is that the additional mature timber the firms are allowed to cut makes planting

<table>
<thead>
<tr>
<th>Rotation Age (years)</th>
<th>Optimal Regime</th>
<th>ACE NPV $/ha</th>
<th>NPV of Treated Stand Including ACE NPV</th>
</tr>
</thead>
<tbody>
<tr>
<td>70</td>
<td>No action</td>
<td>2545</td>
<td>2092</td>
</tr>
<tr>
<td>80*</td>
<td>No action</td>
<td>2636</td>
<td>2340</td>
</tr>
<tr>
<td>90</td>
<td>No action</td>
<td>2315</td>
<td>2111</td>
</tr>
<tr>
<td>100</td>
<td>No action</td>
<td>2241</td>
<td>2046</td>
</tr>
</tbody>
</table>

Notes 1. ACE = Allowable Cut Effect
2. * = Optimal Regime

Table 4.5 Optimal regimes when Allowable Cut Effect is considered

the trees very profitable for the firm. The no action alternative provides the greatest ACE benefits because it provides the most volume. Note that the second column shows that the return from the ACE and the new stand is less than from the ACE alone. The newly planted stand does not provide a positive return on its own but is planted because it produces very high stand values for the firm. The promise of additional volumes of mature timber today should encourage firms to reforest stands even on Crown lands. There is evidence of some of this taking place in British Columbia (Vancouver Sun, May 23, 1987).

While planting stands on the basis of ACE is profitable from the firm's perspective and helps the government maintain a viable forest industry in the province, the ACE benefits that stem from spacing and fertilization are not as clear. Runs on the TASS simulator show that spacing decreases or provides very small increases in the volume of planted stands at maturity though the logs are of greater size. Fertilization consistently provides small increases in volume. For
instance, a stand on a good-medium site planted at 1100 stems per hectare and thinned 500 stems creates only 4 m³ of additional volume. This does not justify the cost of the project.

Provincial policy allows for greater benefits to firms who undertake spacing projects than TASS results appear to justify. Section 52 of the Forest Act states that when a firm conducts an intensive management activity that "the gain in productivity ... attributable to the additional silviculture treatment" (British Columbia 1981) can be added to the firms AAC. The Policy Manual which accompanies the Forest Act allows for productivity gains of 10% for spacing projects and 3% for fertilization projects on medium sites.

Given these benefits, management of Crown owned stands by forest firms becomes not only part of the optimal regime but creates positive net stand values both for stands planted at 550 stems per hectare (Table 4.6).

<table>
<thead>
<tr>
<th>Initial Density</th>
<th>Optimal Regime</th>
<th>NPV</th>
<th>-40%</th>
<th>-50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>1100</td>
<td>Thin by 200 at 30 Cut at 100</td>
<td>-59.53</td>
<td>-26</td>
<td>-17.65</td>
</tr>
<tr>
<td>550</td>
<td>Thin by 175 at 30 Cut at 80</td>
<td>50.31</td>
<td>108.16</td>
<td>122.63</td>
</tr>
</tbody>
</table>

Table 4.6 Optimal regimes for spacing and fertilizing projects with provincially established productivity gains

Caution must be exercised in interpreting these values since they are based on government policy rather than actual biological results. If the Crown does not have the additional volumes, then clearly there will be long term problems in maintaining this policy.

Another factor which may motivate the Crown and private firms to invest in silviculture is an expected increase in the premium paid for
large logs. This would favor regimes with thinning and fertilization and if the premium were sufficient could even mean positive returns.

The allowable cut effect provides an incentive for dishonest action on the part of the firms. Note that at the optimal rotation age of 80 years the NPV from the allowable cut effect exceeds the return from harvest and the allowable cut effect. It proves profitable for the firm to harvest a stand at a loss to earn the bonus that comes with planting the stand. The incentive exists to commit to the plan which offers the greatest bonus but actually carry out the plan that maximizes harvest return or to postpone the harvest of the stand. It is evident that a full implementation of the allowable cut effect would require long term careful monitoring of firms' activity.

The preceding discussion has shown that given certain government incentives that silvicultural investments can be worthwhile for a private firm operating on Crown land. More often, the impact on net stand value of silviculture investments is negative. A question which begs asking though the answer is beyond the scope of this work is: Is the government spending its funds in an optimal manner through these incentives to the forest industry and other investments in silviculture? This would be a difficult question to answer since the hard economics of financial returns to government are blurred by such questions as job creation, regional needs and community stability but given the diminishing old growth forest it deserves investigation.

4.5 OPTIMIZATION UNDER BUDGET CONSTRAINTS

The results presented to this point have been based on the assumption that funds were available to carry out any regime desired. In some cases these were private funds, in others it was assumed they were
credits against stumpage. The private fund case with a budget constraint is a non-starter except when the allowable cut effect is considered. This is because the optimal regime calls for no management action. To create an interesting case, we will assume a discount rate of 0%. This will ensure that a variety of activities are profitable and the decision maker will be forced to choose between them. In this way the budget constraint dilemma facing a forest manager can be illustrated.

4.5.1 METHODOLOGY

Conceptually, the simplest way to include a budget constraint in a dynamic programming problem would be to add a 'budget remaining' element to the state space. This is not desirable however as it enlarges the problem and would reduce the number of regimes which would be eliminated as we recurse forward through the problem. This can be seen by referring once again to Figure 4.1 earlier in the chapter. Since different amounts of money are spent on each path that reaches state 6, the paths would no longer lead to a common state*. Instead of one run being eliminated they both must be carried on at the expense of additional computer time. Over the full problem, this could become a significant concern. As well, by eliminating many common paths the benefits of the DP approach are at least undermined and at worst eliminated. The worst case scenario would see all common paths eliminated thereby reducing the problem to simulation.

A more reasonable and less clumsy approach is to take advantage of the linear structure of the problem. It was noted in Chapter 2 and can be seen from Figure 4.1 that the DP can be set up as network problems or LPs. There are LP algorithms available to solve network problems. For

*Since the budget remaining values assigned would be over a range, it is conceivable that the paths would still converge in some cases.
example, the problem outlined on page 51 could be set up as a shortest
routeproblem. What follows is the dual of that problem.

\[
\text{Max } W^L = \sum_k \sum_j R_{kj} X_{kj}
\]

\[
\text{s.t. } \sum_j X_{sj} = 1 \quad \text{Initial Mode}
\]

\[
\text{s.t. } \sum_k X_{jk} = \sum_{l} X_{lj} = 0 \quad \text{All Interior Nodes}
\]

\[
1 = \sum_j X_{jL} \quad \text{Final Node}
\]

\[
X_{jk} \geq 0 \quad \text{for all arcs in the network}
\]

where

- \( W^L \) is the value of last node expressed as the sum of all arcs in the
  network times the decision variable \( X_{kj} \).
- \( X_{kj} \) is defined as the decision to traverse from node \( k \) to \( j \). It is a
  0-1 variable which equals 1 if the arc is to be traversed and 0 if not.
- \( S \) refers to the initial node.
- \( L \) refers to the final node.

There is one network node for each state in the DP. The \( R_{kj} \)
refer to the arcs and the discounted returns associated with each arc in
the network. The constraints force you to choose a way out of a node if
you choose to enter it and to choose only one way in and out. For
instance, if the arc \( X_{23} \) is part of the network then there must be a
 corresponding arc \( X_{12} \). These will each take on the value of 1 and the
constraint will sum to 0.

The budget constraint can easily be added to this problem in the
following form:

\[
\sum_k L_{kj} X_{kj} \leq B
\]
where \( L_{kj} \) is the budget used in traversing the arc \( k_j \).

\( X_{kj} \) is as defined previously.

and \( B \) is the total budget allocation.

In terms of the forestry problem, the arcs in the network represent costs of management actions or returns to harvest, the nodes as stated represent the various states that the system could be in as a result of the forest management decisions which are made. The goal is to find the longest path through the network or maximize the sum of management costs and harvest returns.

The problem could also be formulated as an LP by taking the dual of the formulation just given. It takes the form:

Minimize \( W^S - W^L \)

s.t. \( -W^k + W^J \geq R_{kj} \) for all arcs.

This problem could be solved by letting the node \( W^S = 0 \) and solving for the other \( W^S \) sequentially. This is like dynamic programming in that it involves precedence. By setting \( W^S = 0 \) we would be solving the problem by a forward recursion. This structure is equivalent to the simple shortest route formulation on page 51.

Structuring the problem in either of these ways creates a large LP. The LP would grow for each policy option considered. It is not important that we create this model to solve the budget constraint problem but only that we show that it can be done. This accomplished, we can exploit the linear structure to solve the problem using the decomposition method.

The decomposition method divides a problem into two parts, one with 'easy' constraints and one with 'complicating' constraints. In our problem the complicating constraint is the budget constraint as it is not
explicitly considered in the DP. The two parts of the problem become the original DP and an LP generated from solutions of the DP. Normally, both parts are LPs but since the DP has an inherent linear structure as shown above it is suitable for use in decomposition. Further, any solution to the subproblem (i.e., the problem without the complicating constraint or in our case, the DP) is a potential solution to the master problem (i.e., the problem containing the complicating constraint or in our case, the budget constraint) and is called a proposal in decomposition terminology.

To initiate the problem solution two subproblem proposals must be generated. These are then combined with weights. The proposals can be generated in a number of ways. These are reviewed in the next section. Table 4.7 below outlines the information used to solve the problem.

<table>
<thead>
<tr>
<th>Proposal 1</th>
<th>Proposal 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Budget used, L1</td>
<td>Budget used, L2</td>
</tr>
<tr>
<td>Objective value</td>
<td>Objective value</td>
</tr>
</tbody>
</table>

Weights

\[ \frac{\lambda_1}{\frac{w_1}{w_2}} \]

Table 4.7 Weighting subproblem proposals

If the weights are greater than or equal to zero and sum to one then a weighted combination of the two proposals will also satisfy the subproblem constraints and is therefore also a proposal. This is known as the Representation Property and is discussed in detail in Bradley et al. (1977). This composite proposal can be made best for the overall problem by solving the optimization problem:

\[ \text{maximize} \quad \sum_{i=1}^{n} \lambda_i \times \text{value of proposal}_i \]

\[ \text{subject to} \quad \sum_{i=1}^{n} \lambda_i = 1 \]

\[ \lambda_i \geq 0 \quad \text{for all } i \]

*The format for this table is drawn from Bradley, et al. (1977).*
Max $W = \sum_i W_i \lambda_i$ \hspace{1cm} \text{Optimal shadow prices}

subject to

$\sum_i L_i \lambda_i \leq B$

$\sum_i \lambda_i = 1$

$\lambda_i \geq 0$

where $W_i$ is the objective value of subproblem $i$ or the value for management regime $i$. $L_i$ is the amount of budget used in subproblem $i$ for management regime $i$. $B$ is the available budget for forest management.

The first constraint is the budget constraint and the remaining constraints define the $\lambda$s as weights. Again following Bradley, we can consider the effect of introducing any new proposal or management regime to be considered with the two already introduced. If we assume that each unit of the new proposal contributes $P_1$ to the objective function and uses $r_1$ of the budget the problem can be written as:

Max $T_1 \lambda_1 + T_2 \lambda_2 + P_1 \lambda_3$

$L_1 \lambda_1 + L_2 \lambda_2 + r_1 \lambda_3 \leq B$

$\sum \lambda_i = 1$

all $\lambda_i \geq 0$

We now wish to discover if this (or any) new proposal or management will increase the objective function. From the theory of the simplex method it can be shown that the solution to this problem is optimal for the overall problem if every column in the master problem prices out at greater than or equal to zero. We need only price out this new activity by determining its reduced cost coefficient $\bar{p}_1$. This is obtained by applying the prices:
\[ \bar{p}_1 = p_1 - \pi r_1 - \sigma \]

where \( p_1 \) and \( r_1 \) are obtained by solving the subproblem (or in our case the DP) and the \( \sigma \) and \( \pi \) are obtained from the previous solution of the modified problem. This continues until no new proposal or management regime improves the solution to the modified problem.

The decomposition algorithm finds the optimal solution by generating new proposals for the weighting problem (the one containing the complicating constraint) with each solution of the DP. The DP solutions differ because the silvicultural costs are multiplied by the shadow price \( \pi \) obtained in solving the LP above and subtracted from their original levels. These original levels would be the objective function coefficients in an LP formulation or the cost associated with policies involving management actions in the DP. The new shadow price to be used is obtained from the shadow price \( \pi \) of the weighted budget constraint.

The final question to be answered in this discussion of methodology is: How can we be sure that the algorithm causes convergence to the optimal solution? The answer is that because the master problem has a linear structure and could be written as an LP as shown earlier in the chapter, the decomposition algorithm inherits convergence from the simplex method of solving LPs. New variables are introduced via the subproblem in the same way as variables are introduced into the basis in a standard LP. In both cases it is assured that the new variable has a positive reduced cost i.e., it will add to the objective function. So from LP theory it is known that the master problem is solved in a finite number of steps. It is now clear why it was necessary to establish that the DP used in the problem had a linear structure.

The solution provided is equivalent to solving the problem as a
simple LP. This provides at worst only an approximation of the solution. An integer solution using the BALAS or Branch and Bound algorithm would give the best answer. The difference between solutions is known as the Duality Gap. Given the discrete nature of our problem and the relatively large differences in costs and values at different stages, the Duality Gap is not an obstacle to the efficient solution of the problem.

The bounds on the objective function can be calculated at each iteration as we move towards convergence. This is particularly useful in problems which approach the optimal solution very slowly. We may wish to terminate the algorithm rather than continue with an expensive run if we are satisfied with how close we are to the optimal solution. These bounds are explained below.

From Bradley et al. (1977) we learn that the solution to every subproblem proposal \( V_j \) provides a dual feasible solution to that problem. By the weak duality property of LP we know that each of these feasible solutions to the dual provides an upper bound to primal objective function \( Z^* \). It was shown earlier that the solution to every restricted master problem \( Z^j \) must be a feasible solution to the original problem. Therefore:

\[
Z^* \geq Z^j
\]

At each iteration \( Z^j \) increases and approaches \( Z^* \). The upper bound can be expressed in terms of the optimal shadow prices \( \pi_i^j \) and \( \sigma^j \) to the restricted master problem since they solve its dual problem. The final form becomes:

\[
Z^j \leq Z^* \leq Z^j - \sigma^j + v^j
\]

These bounds will be illustrated in the solution in Section 4.5.2.

While the discussion has focused on the deterministic case, the
decomposition procedure displayed could also be used in a stochastic case. The simple shortest route recursion in Section 4.3 could be rewritten to portray a random process:

\[ Y = \max \left\{ \sum_{i} P R + \sum_{ij} P Y \right\} \]

where \( d \) is a decision (thin, harvest, etc.) and \( P \) equals 1 in the deterministic case. When \( P \) does not equal 1 then the results are expected values. This however, does not effect the decomposition procedure outlined.

4.5.2 SOLUTION

To illustrate a solution to the problem with a budget constraint, let us assume a discount rate of 0 and a budget constraint of $600.

As noted in the previous section our first task is to obtain a pair of initial solutions to create the restricted master problem. The first can be obtained by simply solving the DP without regard for the budget constraint. From this we obtain:

<table>
<thead>
<tr>
<th>( V^1 )</th>
<th>Budget used</th>
<th>Optimal Regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>$9125</td>
<td>$721</td>
<td>Thin by $600 at age 10 and fert. at age 60.</td>
</tr>
</tbody>
</table>

It is not important that this result is not feasible in terms of the budget constraint. The second solution can simply be chosen from among known feasible solutions to the problem. A more frequently used and flexible means of obtaining a second solution is to use an artificial variable in the restricted master problem. While it is commonly called on artificial variable it is actually a surplus variable. It is like an artificial

---

\(^7\)For example, the solution plant and leave could be used for the second solution. The harvest revenue and planting cost can be obtained from the IMPS input file without difficulty.
variable in that once it leaves the basis it will never re-enter. Its purpose in this formulation is to force the selection of a feasible regime as a second initial feasible solution.

It becomes:

\[
\begin{align*}
\text{Max } & 0 \lambda_1 - A_1 \\
\text{S.t. } & 721\lambda_1 + S_1 - A_1 = 600 \\
& \lambda_1 = 1
\end{align*}
\]

Solution

\[
\begin{align*}
\lambda_1 &= 1 \\
Z &= -121 \\
A_1 &= 121
\end{align*}
\]

A new subproblem proposal can be generated but since we are using an artificial variable the profit (cost) contribution for every variable is 0. The new subproblem objective function is obtained from:

\[
P_1 = 0 - \sum \pi r_1 - \sigma
\]

Since the \(r_1\) are the coefficients on the planting and management activities the new objective function for the subproblem (or the immediate returns in the DP) contains the original cost coefficients but all the harvest return coefficients (immediate returns) are 0. It is clear that the result in this case will be to plant only since there are no returns to management activities. The solution is:

<table>
<thead>
<tr>
<th>(V^2)</th>
<th>Budget used</th>
<th>Optimal Regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-236$</td>
<td>$236$</td>
<td>Plant and cut age 100.</td>
</tr>
</tbody>
</table>

This provides a feasible regime so the artificial variable can be removed. The \(V^2\) value used is not -236 but 5218 (5454-236) since this is the actual return from the regime called for. The restricted master problem becomes:
Max \( 9125 \lambda_1 + 5218\lambda_2 \)  \[ \text{Shadow Prices} \]

\[ \pi = 8.05567 \quad \sigma = 3316.86 \]

S.t. \( 721\lambda + 236\lambda_2 \leq 600 \)
\[ \sum \lambda_i = 1 \]
all \( \lambda_i \geq 0 \)

**Solution**

\[ \lambda_1 = 0.750515 \quad \lambda_2 = 0.249485 \]

\[ z^j = 8150.26 \]

With two solutions in place we attempt to improve the solution. The new objective function for the subproblem is obtained from:

\[ P_2 = (\sum \text{Harvest Returns} - \sum \text{Management Costs}) - \sigma \]

In a DP setting this means that each management cost immediate return \((r_i)\) must be multiplied by \(\pi + 1\). For example the immediate return from being in State 1 (bare land) and planting is -236. In an LP setting this would be found in the objective function as -236 \(X_{12}\). It would appear in the new objective function as -236 \(X_{12}\) \((8.05567 \times 236\) \(X_{12}\)) or -2137\(X_{12}\). This reduces to -236\(X_{12}\) \((1 + \pi)\) and thus all immediate returns also appearing in the budget constraint can simply be multiplied by \(1 + \pi\).

Solving the DP with these new immediate returns provides the following solution:

<table>
<thead>
<tr>
<th>Value</th>
<th>Budget used</th>
<th>Optimal Regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3316.86</td>
<td>$236</td>
<td>Plant and cut age 100.</td>
</tr>
</tbody>
</table>

It now must be determined if this new solution will aid the maximization process. We find that \(P_2\) does not contribute since:

\[ P_2 = 3316.86 - \sigma = 3316.86 - 3316.86 = 0 \]

The fact that optimal solution has already been reached is confirmed by
checking the bounds on the objective function:
\[ z_j^* \leq z^* \leq z_j^* + y_j - g_j \]

\[ 8150.26 \leq z^* \leq 8150.26 + 3316.86 - 3316.86 \]

\[ z^* = 8150.26 \]

The optimal solution then is \( \lambda_1 = .750515 \) and \( \lambda_2 = .249485 \). This means the subproblem solutions should be implemented in these proportions. In other words, 75% of the land should be planted, thinned, and fertilized and the other 25% should be planted and left until harvest time.

The optimal shadow price \( y \) on the budget constraint can be manipulated to provide the marginal return from additional dollars provided for forest management. An additional dollar allocated today will provide a return of \( y \) or \( \$8.06 \) at harvest time in 100 years. This translates into an annualized return of 2.1%. The optimal shadow prices thus provide valuable information for determining returns to investments in silviculture.

4.6 CONCLUSION

The base case solution showed that with no incentives and a discount rate greater than 3% that planting and managing a stand could not produce a positive financial return.

Incentives such as those provided through the ACE were sufficient to provide positive returns. The ACE provided the greatest incentive for private investment. This has been borne out by recent reports of privately funded renewal programs on Crown land outside of a tree farm license (Vancouver Sun, 23 May 1987). The limitations of the ACE noted in the chapter can not be over emphasized.

Other findings within the chapter included the benefits of wider
initial spacing and the returns to spacing and fertilization combined. Juvenile spacing alone and particularly commercial thinning were never chosen as part of an optimal regime. Heavy spacing at a young age (10 or 20) combined with fertilization late in the rotation (age 60) proved to be the most viable management regime when the discount rate was lowered or the ACE was in place. At higher discounts rates (5%) later thinnings (Age 30) proved superior. Future work should be undertaken to validate these results using other stand growth models suitable to the area.

A method to employ a budget constraint within a DP framework was also displayed. This should be able to be practicably applied to other forestry-DP models.

It is clear from the results of this chapter that even in a deterministic setting that incentives are necessary to encourage private forest investment unless one is very optimistic about the course of future prices. The need for incentives is reinforced in the next chapter where the probability of pests, fire and other shocks is considered.
CHAPTER 5

IMPACT OF RISK AND UNCERTAINTY ON OPTIMAL REGIMES

5.1 INTRODUCTION

The results presented in the previous chapter assumed that the manager had perfect knowledge of stand growth regardless of the management action chosen. It was also assumed that even if funding was constrained he was at least certain of the amount which was available to him so he could use it in an optimal fashion. One factor which contributed to the world of certainty was that attacks by pests or fire were not considered. The analysis presented in this chapter is based on a more realistic environment of risk and uncertainty.

Five cases with a stochastic DP formulation will be investigated in an effort to obtain more realistic results. The first case considers the possibility of plantation failure where a planted stand must be replanted. In the second case, growth will be assumed to proceed in a stochastic manner. That is, the stand may grow slower than assumed in the previous chapter. The third and fourth cases involve the possible catastrophic destruction of a stand by fire or damage to the stand by pests. Finally the questions of funding and price uncertainty will be investigated. Risk and uncertainty will be defined in the subsection which follows. The next section will provide a theoretic discussion of stochastic DPs. The chapter will follow the order of the cases outlined above.

5.1.1 RISK AND UNCERTAINTY

The outcomes in the previous chapter were deterministic or certain. This is known as decision making under certainty.
Moving away from certainty we approach various degrees of indeterminateness. The one closest to certainty is when we know exactly what the probability is of each of a number of outcomes for a given decision. An example of this would be if we know that there is a 1% chance that a stand in a given state will be destroyed by fire if not harvested in a given period. Technically, the decision to harvest or not harvest is known as decision making under risk, which will be the context used in this chapter.

At the other end of the indeterminateness scale is decision making under uncertainty. This exists when there is complete ignorance about the probabilities. This seldom exits in its pure form. For instance, we may not know the probability that a stand will grow into a given state in the next period but we do know some states that it will not grow into. For our purposes uncertainty will be defined as an ignorance of the probabilities over a range of reasonable outcomes.

In between these two degrees exists adaptive decision making where "information gleaned from an ongoing process generates estimates of probabilities transforming the indeterminateness from uncertainty to risk" (Nemhauser 1966). This method was employed by Kao (1984) in a forestry setting. Adaptive decision making will not be used in this paper but a brief discussion of its shortcomings in a setting of funding uncertainty is included in Section 5.4.

In all runs it will be assumed that the manager is risk neutral. This implies that the criterion for choosing the best outcome will be Expected Net Present Value (ENPV) in all cases involving risk. The criterion for choosing in cases of uncertainty is outlined in Section 5.5.
5.2 THE STOCHASTIC MODEL

The recursion used in Chapter 4 for the deterministic model can be modified for use in stochastic DP. The form the recursion takes is that of a probabilistic shortest route problem. As well, the problem is written only in terms of states rather than states and stages. This follows convention. Finally, the problem is solved via backward recursion rather than forward recursion.

Following Wagner (1975) we can modify our previous deterministic model to a stochastic DP. Furthermore, this stochastic DP will be a Markov process if certain generalizations are made. First of all, a decision $d$ in state $i$ can result in the system moving into one of several states rather than only one state as in the deterministic case. This can be written as a conditional probability

$$p(j|i,d)$$

which refers to the probability that we move to state $j$ from state $i$ given decision $d$, where the decisions are spacing, fertilization and harvesting. To be a first-order Markov process, each of these conditional probabilities must embody the Markov property that the probability depends only on the existing state regardless of the history of the system. That is, no matter what decisions (thinning, fertilization) brought us to state $i$, the probability of going to state $j$ is the same. We assume a forest stand will grow the same way no matter how it comes to be as it is. This was detailed in Chapter 2. While the discussion here focuses on first-order Markov processes, it should be remembered that higher order Markov processes can be modified and modeled as first-order processes.

Secondly, $R(j',k)$, the immediate return of going to $k$ from $j'$, must now be expressed as a random process to reflect the fact that it is
now an expected return with the expected return varying with the
management decision taken. The cost or return on any arc of this random
process can be expressed as $C(j|i,d)$. This is the cost or return of going
from $i$ to $j$ under decision $d$ with probability $(p(j|i,d))$. Hence the
expected cost or return to starting in state $i$ and making decision $d$ is
$C_{id}$ which is:

$$C_{id} = \sum_{j} C(j|i,d)p(j|i,d)$$

$j$ represents all states reachable from state $i$.

The third property noted by Wagner is the acyclic property which
means that no node will be visited more than once in any run of the
system. As age is one of the state variables, this property is easily
met in our forestry problem. This makes the problem non-ergodic, that is
the trapping system does not allow us to return to any state once we
leave it.

The stochastic recursion can now be in the general form:

$$Y(n) = \max_{i \in (D_i)} \left[ \sum_{j=0}^{i-1} \alpha p(j|i,d) Y(n+1) + C_{id} \right]$$

where $D(i)$ is a decision set at node $i$ and $\alpha$ is the 10 year discount
factor.

Wagner (1975) goes on to show that the Markov chains can also be
constructed as LPs. This would allow them to be used as subproblems in a
decomposition procedure such as the one shown in Chapter 4.

The analysis can be extended to cases with an unbounded horizon
and solved using successive approximation in value space (Howard 1960;
Wagner 1975). If the problem approaches steady state then the time
subscripts can be dropped and policy iteration can be used. Since the
probabilities and values are changing through time, value iteration is
preferred. Over an unbounded horizon the acyclic property need not be postulated and there is no terminal node. $y_i$ simply represents the expected present value of starting at Node i and using an optimal strategy into infinity. Because of the acyclical nature of the problems being solved here, the acyclic property holds by default. We can find the expected present value of being in any state of the forest and following the optimal management regime indefinitely. All examples in this chapter were solved using the software package IMPS discussed in Appendix 1.

5.3 PLANTATION FAILURE

Whenever a stand is planted there is a risk that many of the seedlings planted will die and the stand will not be sufficiently stocked to be considered planted. This determination is usually made within five years after planting and requires that the stand be replanted. Pearse et al. (1986) estimate the probability of this occurring at 8%. In Table 5.1 outcomes are presented for a range of probabilities with a discount rate of 5%, a rotation age of 100 years and with the manager responsible for all costs. The discount rate is not a particularly important factor in this type of analysis since the original planting cost occurs in the first year and therefore is not discounted. Therefore the losses expressed in the table are equal to the losses that would be expected from a lower discount rate since failed plantations are not replanted. Over an infinite series of rotations the losses would have a greater impact on the low discount rate cases.

If a manager could avoid losses from plantation failure he would be willing to pay a premium to the planter who could provide him with a crop that would have a better chance of survival than he is obtaining presently. For example, if the present failure rate is 8% then the
manager would be willing to pay up to $14 dollars more for a "no risk" planting job. He would pay up to $8 more to reduce the probability to 4%. The argument also holds in reverse. The manager would be willing to accept a higher level of risk in return for a less expensive planting job. Figure 5.1 below displays the "no risk" and "acceptable" planting cost boundaries over a number of probabilities.

A factor that is ignored in the above analysis is the cost of preparing the site for replanting. Inclusion of this cost would increase the losses expected from plantation failure and affect the premiums and discounts shown in Figure 5.1.

The risk of plantation failure clearly impacts on the expected return to firms from a single rotation. The risk should have a significant effect on the prices charged by tree planters. Those able to guarantee a lower level of risk should be able to charge a premium for their services. This has implications for the British Columbia Forest Service "lowest bidder" rules. Accepting the lowest bid only potentially drives high quality planters from the market and forces everyone to achieve only the minimum planting standards set out by the province.

<table>
<thead>
<tr>
<th>PPF</th>
<th>ENPV</th>
<th>LOSS</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-195</td>
<td>-</td>
</tr>
<tr>
<td>.04</td>
<td>-201</td>
<td>6</td>
</tr>
<tr>
<td>.08</td>
<td>-209</td>
<td>14</td>
</tr>
<tr>
<td>.12</td>
<td>-216</td>
<td>21</td>
</tr>
<tr>
<td>.16</td>
<td>-224</td>
<td>29</td>
</tr>
<tr>
<td>.20</td>
<td>-232</td>
<td>37</td>
</tr>
<tr>
<td>.30</td>
<td>-256</td>
<td>61</td>
</tr>
<tr>
<td>.40</td>
<td>-283</td>
<td>88</td>
</tr>
<tr>
<td>.50</td>
<td>-315</td>
<td>120</td>
</tr>
</tbody>
</table>

TABLE 5.1 Expected losses by probability of plantation failure (PPF)
5.4 STAND VALUE

Stand values used thus far in this analysis have been based on current prices and the assumption that stands develop just as TASS simulates. More realistic scenarios would look at anticipated net revenues 80 to 100 years in the future and also consider how the results generated by TASS may differ in a real forest setting.
5.4.1 TASS Yield Levels

Mitchell and Cameron (1985) discuss factors which may cause TASS to overestimate the volume in a stand at rotation age. These include unproductive areas such as marshes and rock outcrops on a site, chronic losses caused by pests or insects which reduce yields without killing trees, measurement error and other factors which may make one site more productive than another. According to Mitchell and Cameron these factors can cause TASS to overstate the yield by 5% to 25%. Forest management practices tend to reduce losses to the lower figure.

While the exact probabilities of a stand suffering a particular loss is not known, a scenario can be created which will approximate the impact of the growth inhibiting agents discussed above. It will be assumed that unmanaged stands have a greater potential for loss and that as a stand ages, the potential chronic losses increase and therefore total losses increase. The probabilities used in runs to measure the effect of risk of yields falling short of expected values are shown below in Table 5.2. The losses occur throughout the life of the stand but only impact on stand values when the stand is harvested. Therefore, the impacts are lumped into the ages at which harvest is likely to occur.

While these probabilities can not reflect exact values, they reflect how the factors cited above could affect a stand. The range of probabilities is sufficient to show the impact risk can have on stand values and provides a more realistic picture than simply using TASS values or assuming a single level of loss for all regimes.
To determine the impact of stand losses the model was run at interest rates of 5%, 3% and 2%. Runs beyond the base case are included since these included management regimes originally and therefore will provide more insight into the sensitivity of optimal regimes to non-catastrophic stand losses. The results are presented in Table 5.3.

**Table 5.2** Assumed risk of yield loss at rotation age

<table>
<thead>
<tr>
<th></th>
<th>AGE 80 PROB.</th>
<th>DECREASE IN STAND VALUE</th>
<th>AGE 90 PROB.</th>
<th>DECREASE IN STAND VALUE</th>
<th>AGE 100 PROB.</th>
<th>DECREASE IN STAND VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>NO MANAGEMENT</td>
<td>.5</td>
<td>.10</td>
<td>.5</td>
<td>.15</td>
<td>.5</td>
<td>.2</td>
</tr>
<tr>
<td></td>
<td>.5</td>
<td>.15</td>
<td>.5</td>
<td>.25</td>
<td>.5</td>
<td>.25</td>
</tr>
<tr>
<td>FERTILIZATION ONLY</td>
<td>.5</td>
<td>.05</td>
<td>.5</td>
<td>.10</td>
<td>.5</td>
<td>.15</td>
</tr>
<tr>
<td></td>
<td>.5</td>
<td>.10</td>
<td>.5</td>
<td>.15</td>
<td>.5</td>
<td>.20</td>
</tr>
<tr>
<td>THINNED ONLY</td>
<td>.34</td>
<td>.02</td>
<td>.34</td>
<td>.05</td>
<td>.34</td>
<td>.07</td>
</tr>
<tr>
<td></td>
<td>.33</td>
<td>.05</td>
<td>.33</td>
<td>.10</td>
<td>.33</td>
<td>.15</td>
</tr>
<tr>
<td></td>
<td>.33</td>
<td>.10</td>
<td>.33</td>
<td>.15</td>
<td>.33</td>
<td>.20</td>
</tr>
<tr>
<td>THINNED AND FERTILIZED</td>
<td>.5</td>
<td>.02</td>
<td>.5</td>
<td>.05</td>
<td>.5</td>
<td>.10</td>
</tr>
<tr>
<td></td>
<td>.5</td>
<td>.05</td>
<td>.5</td>
<td>.10</td>
<td>.5</td>
<td>.15</td>
</tr>
</tbody>
</table>

**Table 5.3** Optimal regimes and returns in deterministic and stochastic settings

<table>
<thead>
<tr>
<th>DISCOUNT RATE</th>
<th>OPTIMAL REGIME</th>
<th>NPV($/HA)</th>
<th>DIFF $/HA</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DETERMINISTIC</td>
<td>STOCHASTIC</td>
<td>DETM.</td>
</tr>
<tr>
<td>5</td>
<td>Harvest at 100</td>
<td>-195</td>
<td>-208</td>
</tr>
<tr>
<td>3</td>
<td>Harvest at 100</td>
<td>47</td>
<td>-43</td>
</tr>
<tr>
<td>2</td>
<td>Harvest at 100; thin by 600 at 20; fert at 60</td>
<td>690</td>
<td>458</td>
</tr>
</tbody>
</table>
The most obvious result in the stochastic setting is lower returns as expected since the original TASS runs assumed uninhibited growth of the stand and therefore represented theoretical maximums. The stochastic runs should also have shown a preference for regimes including management activity since these removed some of the risk. This is not borne out by the results but on the other hand it is not contradicted, since the optimal stochastic regime never calls for less management than the deterministic regime. It can be assumed that regimes including management activity produced values which were closer in value to those with no management activity when the runs were conducted in the stochastic setting as opposed to the deterministic setting. Finally the stochastic runs should have favoured harvest at an earlier date. No evidence was found to support or contradict this premise. The optimal regimes all called for harvest at the same age in both settings. This is a function of the high increases in stand value from year 70 to year 100.

5.4.2 Catastrophic loss

Stand value can be severely reduced or even wiped out entirely by a catastrophic event such as fire. Martell(1980), Routledge(1980) and Reed and Errico (1985) have looked at the impact of fire on expected soil expectation (ESE) values. A common shortcoming of these papers in their failure to incorporate realistic cost information. In all papers it has been assumed that the probability of fire is constant at all ages. Consequently it leads to decreased soil expectation values and shorter rotations. Reed and Errico also examine the variable hazard case where young stands are more likely to be destroyed by fire than older stands. Each author noted that the soil expectation values fell sharply as the probability of fire increased. The results from two of the papers are
summarized below in Table 5.4. While showing similar results Reed and Errico's presentation was not compatible with this table.\(^1\)

<table>
<thead>
<tr>
<th>Ignition Probability</th>
<th>Percentage loss in ESE value due to fire risk reported by Martell and Routledge with a 5% discount rate.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.005</td>
<td>Martell 12.6 Routledge 17.4</td>
</tr>
<tr>
<td>0.01</td>
<td>Martell 19.8 Routledge 37.8</td>
</tr>
<tr>
<td>0.02</td>
<td>Martell 35.8 Routledge 89.5</td>
</tr>
</tbody>
</table>

These losses can be reduced if salvage value is considered (see Reed and Errico 1985). The salvage model would also be suitable for analyzing the impact of epidemic pest attacks where pests kill the trees in a stand but the wood is still salvageable for a period of time.

The impact of both the probability of fire and of an epidemic pest attack are modeled here. The fire model will follow Martell and Routledge in that it will be assumed that there is a constant probability of fire throughout the life of the stand. The model will be run for several probabilities and for simplicity it will be assumed that no salvage values are obtained. The probabilities used exceed the very low actual fire probabilities on the B.C. coast (two tenths of 1% is the average area burned between 1974 and 1983).

The results of the fire model are shown in Table 5.5. The value in repeating the work of Martell and Routledge here is that the results apply to a B.C. coastal setting and use realistic cost and revenue data.

---

\(^1\) Reed and Errico's examples shows the SE falling from about $6/ha to about $-6/ha as the risk of fire increases from 0 to .01. This is not based on realistic cost information.
The model was run for both a single rotation case and over an infinite rotation. In the single rotation case it was assumed that if the stand was destroyed by fire that it was lost and not replanted. For the infinite rotation case it was assumed that the stand was replanted in the period following loss at a cost of $391 which includes the planting and site preparation costs. This allowed for a destroyed stand to eventually be harvested if it were not continually destroyed by fire. This explains why the expected values for the infinite rotation tend to be lower values than those of a single rotation. All runs were based on the deterministic runs first presented in Chapter 4. This permits the isolation of the effects of fire. The only exception to this is that the runs show the additional impact of the risk of plantation failure as this tends to exceed the risk of fire (see Section 5.2).

As expected the expected value of the stand decreased as the probability of destruction by fire increased. The decreases became more pronounced as the discount rate was reduced. This result could also be anticipated since the stands had greater value as they aged and therefore the catastrophic loss of a stand causes a greater loss. Another way of stating this is that the stand value is destroyed only by fire in the low discount case rather than by fire and discount rate as in the higher discount rate cases.

The results do not replicate the findings of other authors who found that the optimal rotation age of the stand fell as the risk of fire increased because cutting earlier reduces the risk of losing the stand to fire. They also do not discuss the impact of fire on optimal stand regimes. The rotation age falls because the expected value of the stand is reduced when the probability of fire is increased. The reduction
expected in stand value from the possibility of fire must be sufficient to fall above the expected loss from cutting sooner. Since the model did not select an earlier rotation age the expected value must have continually been higher at each stage for each level of probability. This means that the gain from waiting for 10 years exceeded the expected losses from fire. The most likely explanation for these results comes from the form of the model compared to the papers cited earlier. First of all the model generates the result that management does not pay at higher interest rates. The resulting unmanaged stand increases in value significantly between the years 90 and 100. It should be remembered that the increase comes from two sources: larger trees and lower logging costs from logging larger trees. This results in a more than doubling of stand value between these years and this more than offsets the expected loss by fire. Further runs determined that at a discount rate of 6% or if the probability of fire is increased to .023 per annum the rotation age did fall to 90 years.

At a discount rate of 2% management actions are called for when the probability of fire is 0 or .001 per annum. As the risk of fire increases the management actions are no longer optimal. The risk of fire has decreased the expected returns from waiting another 10 years to harvest and this decrease is sufficient to make the discounted benefits of forest management fall below the discounted costs.

The final notable result from Table 5.5 is that when the annual fire probability is .02 that the SE value actually declines with the discount rate. This can only be occurring because the high cost of site preparation and planting which occur with greater frequency in the high probability case are given increased weight as the discount rate falls since they occur earlier in the life of the stand. As the discount rate
<table>
<thead>
<tr>
<th>Discount Rate</th>
<th>Prob. of Fire</th>
<th>ENPV</th>
<th>ESE</th>
<th>Optimal Regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>No action</td>
<td>-496.90</td>
<td>161.63</td>
<td>0.02</td>
</tr>
<tr>
<td>0.001</td>
<td>No action</td>
<td>-38.38</td>
<td>5.00</td>
<td>0.01</td>
</tr>
<tr>
<td>0.005</td>
<td>No action</td>
<td>178.69</td>
<td>69.003</td>
<td>0.005</td>
</tr>
<tr>
<td>0.01</td>
<td>No action</td>
<td>80.539</td>
<td>505.93</td>
<td>0.01</td>
</tr>
<tr>
<td>0.02</td>
<td>No action</td>
<td>77.68</td>
<td>616.02</td>
<td>0.02</td>
</tr>
<tr>
<td>0.001</td>
<td>No action</td>
<td>-4.42</td>
<td>25.08</td>
<td>No action</td>
</tr>
<tr>
<td>0.005</td>
<td>No action</td>
<td>16.92</td>
<td>16.92</td>
<td>No action</td>
</tr>
<tr>
<td>0.01</td>
<td>No action</td>
<td>-83.33</td>
<td>225.33</td>
<td>0.02</td>
</tr>
<tr>
<td>0.02</td>
<td>No action</td>
<td>36.36</td>
<td>36.36</td>
<td>No action</td>
</tr>
<tr>
<td>0.001</td>
<td>No action</td>
<td>-179.24</td>
<td>231.90</td>
<td>0.02</td>
</tr>
<tr>
<td>0.005</td>
<td>No action</td>
<td>33.33</td>
<td>33.33</td>
<td>No action</td>
</tr>
<tr>
<td>0.01</td>
<td>No action</td>
<td>-79.24</td>
<td>277.68</td>
<td>0.02</td>
</tr>
<tr>
<td>0.02</td>
<td>No action</td>
<td>33.33</td>
<td>33.33</td>
<td>No action</td>
</tr>
</tbody>
</table>

TABLE 5.5: Expected NPV and ESE and optimal regime for various risks of fire and selected discount rates.
falls the values do not decrease proportionately but are weighted more heavily in the earlier years. The increased impact of these negative values swamps the increase in the harvest value of the stand.

It is clear from the results shown above and from the papers of Martell, Routledge and Reed and Errico that it is important to take into account the risk of fire when determining the expected values to be obtained from second growth stands. This is particularly true if one believes a low discount rate is appropriate as the future values lost have greater value than in the high discount case.

The second form of catastrophic loss to be discussed is an attack by pests. It is assumed that the stand must be harvested shortly after the attack. Indeed, it is B.C. Ministry of Forests policy to harvest "all beetle attacked timber as soon as possible so that the timber is utilized before it starts to deteriorate." (Traunt 1986). If this attack is anticipated it could affect the way in which a stand is managed since, as was shown in Chapter 4, managed stands tend to mature more quickly. An important impact of catastrophic pest attacks which is beyond the scope of this study is the impact it can have on forest level plans. The need to harvest a stand now that was scheduled to be harvested in 30 years can have significant affects on the total harvest schedule and the values obtained from the forest.

The pest attack problem will be modeled in a simple fashion but this will still be effective in showing three things:

1. The impact of anticipated pest attacks on the expected value of the stand.

2. The impact of anticipated pest attacks on the optimal management regime.
3. The returns to research from correctly predicting pest attacks and hence implementing appropriate management regimes. The deterministic optimization model used in Chapter 4 will be modified and run to show the impact of the probability of a pest attack in a single 10 year period or over an extended period and the resulting optimal regimes. Returns from harvesting the infested stands will be assumed at 90% of their full values (Giles 1986) and that losses due to fungal decay are 0\(^1\). A very low discount rate of 1% will be used to ensure there are a number of profitable regimes to choose from. The results are displayed in Table 5.6. Stand value fell as the probability of pest attack increased and as the period in which the pest might attack was extended. The greatest impact occurred when the pest attack was expected between years 70 and 80 and harvest would be required in year 80. By this time the stand was 20 years from the optimal harvest age and far from its optimal value. Expected losses were much smaller when the pest attack was expected later and the stand had neared or reached its potential maximum value.

The probability of pest attack had only a marginal affect on the optimal management regime. Only when the probability of attack was projected in year 70 or year 80 alone did the regime change and then only the age of thinning was modified. One would have expected the optimal harvest to fall in order to avoid the potential for loss by pest but this failed to occur in all cases. This means that the expected increase in stand value was greater in every case than the expected loss due to pests

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\(^1\) Fungal decay can destroy a dead tree in 2 to 3 years if it is not properly stored. Where this decay can be avoided a 20 year old dead tree can retain over 70% of its value (Giles 1986).
This represents the optimal planned regime. If a pest attacks, this regime would not be followed but rather the stand would be harvested as explained in the text.

Table 5.6 Expected returns from stands at risk from pest attacks at harvest

<table>
<thead>
<tr>
<th>%</th>
<th>$</th>
<th>Return</th>
<th>Expected</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>6</td>
<td>1362</td>
<td>1525</td>
</tr>
<tr>
<td>7</td>
<td>6</td>
<td>2388</td>
<td>2100</td>
</tr>
<tr>
<td>9</td>
<td>6</td>
<td>1643</td>
<td>600</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>3721</td>
<td>100</td>
</tr>
<tr>
<td>8</td>
<td>6</td>
<td>2961</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>1735</td>
<td>600</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>1210</td>
<td>600</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>2254</td>
<td>600</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>6226</td>
<td>600</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>2419</td>
<td>600</td>
</tr>
<tr>
<td>3</td>
<td>6</td>
<td>269</td>
<td>600</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>1262</td>
<td>600</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>2266</td>
<td>600</td>
</tr>
<tr>
<td>1</td>
<td>6</td>
<td>9256</td>
<td>600</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>244</td>
<td>600</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>2255</td>
<td>600</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>2788</td>
<td>600</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>2794</td>
<td>600</td>
</tr>
<tr>
<td>5</td>
<td>6</td>
<td>2888</td>
<td>600</td>
</tr>
</tbody>
</table>

70, 80 and 90 attack in years

Probability of pest
and early harvesting. Two of the assumptions of the analysis had a significant influence on this result. First of all, the low discount rate of 1% placed only a small penalty on waiting another ten years to harvest. A larger penalty or cost brought on by a larger discount rate could have altered the results. Secondly, the stands were harvested at 90% of value if the pest attacked. Once again this is a small penalty that the results indicate a manager would be willing to bear over the probabilities and discount rate used here.

The results show that there is significant value in research that can predict when a pest is likely to attack a stand. Note for example the large differences in the expected returns between stands that do, and do not, have the probability of pest attack in year 70. While all results in the cases run here have positive NPVs, the magnitude of the differences could, under different assumptions, make the difference between the expected value of the stand being positive or negative. In terms of percentage of stand value lost, losses from stands which might be attacked in year 70 range from 21% to 58% of stand value while stands that can only be attacked later have losses ranging from 5% to 22% of stand value. The results would be even more dramatic if the probability of attack was considered at earlier ages of a stand's life.

While the analysis here has been highly restricted and simplified, it has been shown that DP can handle problems where risk of pest attack is a concern. Where more detailed information is available on the likelihood of attack and the expected impact of that attack on stand values, useful optimal regimes could be derived.
5.5 PRICE AND FUNDING UNCERTAINTY

5.5.1 Price Uncertainty

Another factor which will impact on stand values is the future trend of forest product prices. In their Markov model Lembersky and Johnson (1975) assumed a general upward trend in prices (2.7%/year) and that market fluctuations retained their present character. This is equivalent to assuming that the probability of making a transition between stages to a state characterized by a certain level of prices is the same today as it will be far into the future. This is a bold assumption which puts the problem into the category of Knightian risk rather than uncertainty where it more probably belongs.

While there is general agreement that real forest product prices are likely to increase in the years ahead, the magnitude of this increase and the path that it will take are unknown. The best approach in this case is to determine the optimal regimes under various scenarios of future price trends and base decisions on the results. The level of uncertainty in forecasting future prices is very high and almost reduces planting and management to a particular regime to an act of faith. For instance, if a decision maker were to assume that real log prices and real harvesting costs were to increase by 2% per year then the returns from the first future harvest would be the same as if we had reduced the discount rate for harvest returns by 2%. The problem could be run at the lower discount rate but the cost of forest management activities would have to be adjusted upwards so that they would still be effectively discounted at the original discount rate. In this way future prices can be increased without affecting current management costs. Of course if we are anticipating changes in forest management costs then this too would have
to be modeled using a scenario approach.

The impact of future forest product price increases can be seen quite readily from the results showing the impact of lower discount rates. Just as lower discount rates make investment in forestry and more particularly in forest management activities profitable, so projected price increases have the potential to make forestry a profitable investment. If, however future prices are expected to remain constant or decrease then forestry investment becomes profitable to firms only if the discount rate falls or some mechanism (federal grants or subsidies) exists to cover forest management costs, including planting. This emphasizes the importance to firms of knowing what funding may be available in the future as they make plans today. The impact of uncertain future funding is discussed in Section 5.5.2.

While Kao (1984) states that the standard approaches for decision making under uncertainty are not applicable to forest stocking control type problems, at least four of five standard approaches do appear to be suitable in this case. Briefly stated the five approaches are:

1. **Maximax criterion** - selects the best possible prize offered by any strategy. This is the strategy for optimists.

2. **Wald criterion** - maximin criterion which chooses the strategy which ensures the worst case scenario will be the best of the worst outcomes that could occur. For example, if three strategies have as their worst outcome 5, 10 and 20, the decision maker would choose the one with 20 regardless of other possible outcomes.
3. Savage Criterion - minimizes regret, chooses the smallest maximum regret where regret is defined as the difference between the best and the worst outcome.

4. Laplace Criterion - assumes that states are equally likely to occur and calculates expected value and chooses the strategy which maximizes expected value.

5. Hurwicz criterion - requires the choice of a "coefficient of optimism (pessimism)" to weight the maximum and minimum payoffs. This is regarded as a criterion best used by firms that can afford losses (Thierauf and Klekamp 1975). This criterion is not used here because there is no reason to choose one weight over another.

Table 5.7 displays the ENPV of three regimes given four possible average annual price increases over a single rotation. The regime chosen will depend on the decision makers attitude to risk. The optimist or "risk lover" would seek the highest price and manage the stand expecting to reap $268/ha when prices increase at an annual average of 3%. A risk-averse decision maker would want to avoid the worst possible outcome. This can only be accomplished by not planting at all. The same choice would be made under the Savage criterion. The risk-neutral decision maker using the Laplacian criterion would plant the stand but would not undertake thinning and fertilization. The no management regime has the highest expected net present value.

These results are consistent with our earlier findings at planting becoming profitable at a lower discount rate. The manager interested only in a financial gain would plant only if he expects future
discount rates to be 2-3 percent lower or if he expects future prices to increase at an annual rate of 2-3 percent over the next rotation period.

<table>
<thead>
<tr>
<th>Regime</th>
<th>Annual Percentage Price Increase</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
</tr>
<tr>
<td>No Planting</td>
<td>0</td>
</tr>
<tr>
<td>Plant but no management</td>
<td>-129</td>
</tr>
<tr>
<td>Plant, thin by 600 at age 20, fertilize at age 60</td>
<td>-318</td>
</tr>
</tbody>
</table>

Table 5.7 Expected returns given various future price increase and regimes and a 4% discount rate.

5.5.2 Funding Uncertainty

A longstanding problem on the Canadian forest scene is funding uncertainty. The lack of knowledge about where funds will come from to plant and tend stands which replace today's old-growth forest has been cited as a major deterrent to forest sector investment. Mechanisms to fund forestry in Canada has been the focus of a major Federal-Provincial study (Thorne, Stevenson Kellog 1981) and remain a concern as evidenced by discussions at the Forestry Forum held in Vancouver in February of last year where one of the recommendations was that the Canadian Council of Forest Ministers:

"... maintain its search for other appropriate funding mechanisms that enhance continuity and stability in the long term planning and conduct of essential forest management ..." (Canadian Council of Forest Ministers 1986 p.27)

The forest sector looks to government as a source from which this consistent flow of funds may come. When funds are committed and then
withdrawn a sub-optimal level of forest investment results. This was revealed in a more general case by Kydland and Prescott (1981) in a discussion of rules versus discretion for monetary and fiscal policy. The authors show that people make decisions based not only on policies in place today but on the policies they expect to be in place in the future. The man choosing to build his house on a floodplain without the presence of a dam would not do so without the knowledge that the government's policy was to build dams when people inhabit a floodplain. An inventor may be reluctant to invent if the patent protection in place when he undertakes his work is expected to be removed in the near future. Optimal levels of investment can be obtained when the decision makers know the policies that affect them today and will affect them in the future. For example, a government which promises funds for stand tending to private firms who plant today will generate a certain level of planting activity. If in five years the government removes those funds then it is likely that the level of planting in the previous five years was greater than the optimal level given that the policy was going to change. In sum, decision makers need to see the whole picture before they can invest optimally in planting and tending the second-growth crop.

Earlier, risk and uncertainty were defined. The case to be discussed here is one of uncertainty. Decision makers cannot look at past history and estimate the probability that the government will withdraw a funding program in the future. Instead they can look at the consequences of the government withdrawing a program 10, 20 or 30 years in the future. The optimal stand management regime will depend on how the decision maker (in this case a private firm) chooses to react to the uncertain situation.

The impact of funding uncertainty on forest management regimes
will be tested by running an experiment with the following assumptions:

1. The firm is responsible for the cost of planting the trees and is told that stand tending costs will be covered by a rebate to stumpage from other stands presently being cut.

2. All costs are as outlined in Chapter 3 and the discount rate is 2%.

3. The goal of the firm is to maximize the stand value in each stand over the rotation in which the stand is planted.

The results contained in Table 5.8 display the value of a single stand under three regimes when the government abandons the rebate plan at stand age 9 or 19. The strategy chosen by a given firm would depend on that firm's attitude towards risk. For example a risk taker would probably follow the maximax route while a risk averse firm may want to minimize regret or possibly choose the Wald criterion. The Laplace criterion could be the choice of the risk neutral firm.

<table>
<thead>
<tr>
<th>Regime selected</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year of Possible Funding Cutoff</td>
<td>No Cutoff</td>
<td>Cutoff</td>
<td>No Cutoff</td>
</tr>
<tr>
<td>No cutoff</td>
<td>-</td>
<td>517</td>
<td>-</td>
</tr>
<tr>
<td>9</td>
<td>517</td>
<td>517</td>
<td>637</td>
</tr>
<tr>
<td>19</td>
<td>517</td>
<td>517</td>
<td>1109</td>
</tr>
</tbody>
</table>

TABLE 5.8 Impact of funding cutoffs on selected regimes.

Notes: Regimes 1 No action; cut at 100
2 Thin by 600 at 10; fert. at 60; cut at 100
3 Thin by 600 at 20; fert. at 60, cut at 100

\(^3\) The regimes chosen were the regimes chosen by the model at a 2% discount rate with and without the assumption of Section 88 being present.
While the three regimes were simply selected for the example outlined below, the decomposition procedure used in Chapter 4 could have been used to generate a set of stand values. The budget constraint could be divided by time periods to reflect the possibility of the budget being cut off. For example, the possibility of the budget being cut off in year 9 could be set up with the following set of constraints.

\[
\sum \ell e_i \lambda_i \leq B_1 \\
\sum \ell m_i \lambda_i \leq B_2
\]

where \( \ell e_i \) equals the money spent in years 0 to 19 for each solution \( i \).

\( B_1 \) represents the budget available to year 19.

\( \ell m_i \) equals the money spent in years later than year 19.

\( B_2 \) represents the budget available after year 19.

The problem could then be solved by allowing \( B_1 \) to cover only the cost of planting and \( B_2 \) to be 0. This simulates funding being cut off at year 9. The problem could be solved again letting \( B_1 \) be high enough to cover all management possibilities in the first 19 years and setting \( B_2 \) at 0. This simulates funding being cut off at year 19. Finally, an unconstrained run would simulate no budget restrictions. This approach would provide another set of values to which one could apply the criteria noted above.

The impact of uncertainty is brought out by the different optimal regimes chosen under the various decision criteria. The optimal regime under certainty would be regime 2 with an expected return of $1158. This regime would also be selected by the optimist or risk taker under the maximax criterion and by the rationalist under the Laplace criterion. A more conservative decision maker who desires a minimum degree of uncertainty or who wants to avoid the lowest possible outcome would choose
differently. The Savage criterion which minimizes regret and provides a maximum of certainty would be satisfied by choosing regime 1 in which no management actions are carried out. The maximin or Wald criterion is met by regime 3 which has the highest minimum outcome.

In this example uncertain funding is shown to potentially affect management in a variety of ways. Under the Savage criterion investment in management would be zero. Under the Wald criterion, investment is postponed for 10 years. An important effect which is not brought out in the example but that can not be ignored is that the lower expected returns brought on by uncertainty may cause the decision maker to abandon the planting decision altogether. The decision maker could opt to invest elsewhere or leave the industry completely if greater returns are expected from such a decision. In the forest sector continuity of government funding is particularly important since most of the harvesting activity takes place on Crown land and government is the primary sponsor of forest management activity. Failure by governments to send clear signals with respect to the funding mechanisms which will be in place in the long term can only inhibit a socially optimal level of management being undertaken by firms on Crown and even private land. The problem is compounded when one considers the impact that insufficient forest management activity has on the future wood supply. The eventual result is a shrinking wood processing industry which would bring with it a decrease in economic activity in many other sectors of the economy and in particular damage or destroy the economies of some of the many communities and regions which depend solely on the forest sector for their economic base.

There are two other points which emphasize how funding uncertainty impacts on decision making in forestry. First of all, funding
uncertainty affects all stands equally and simultaneously. The uncertainty is not spread over a number of stands as it is in the case of risk to fire, pests, varying climatic conditions and so on. The decision maker may look over his forest as a whole or the stands included in his allowable cut and estimate his losses due to these factors. The risk is spread over all stands and a fire in one stand does not destroy the other stands. On the other hand a decision to cut funding or uncertainty about whether funding will or will not be present affects the decision he makes with regard to all his stands. He could of course prioritize stands and treatments in light of cutbacks to programs but the lack of knowledge about whether a program will or will not be in place can not be isolated as other hazards can be.

Secondly, in the face of funding uncertainty adaptive optimization is not an appropriate decision technique when the goal is to obtain an optimal level of forestry investment. For example, the adaptive optimizer would wait until year 10 in our previous example and if the program were cut off they would wait until year 20 to perform the thinning and thereby would be optimizing given existing conditions. This approach is clearly not suitable for the long range planning needed in forestry. As well, it is unlikely that the initial level of investment in forestry would be at a socially optimal level if there is uncertainty about what funding policies will be in place ten or twenty years into the rotation. This reinforces the comments of Kydland and Prescott (1981) noted earlier in the section that optimality can only be achieved when decision makers have a clear view of what is on the policy horizon.

5.6 CONCLUSION

The impact of risk and uncertainty on optimal stand regimes and
their value was brought out in this chapter. In most cases the impacts were clear. Stand values unequivocally fell as risk increased. The rotation age either fell or remained constant but never increased in the presence of risk. These points confirmed what was already well established in the literature.

The level of management was affected differently by different kinds of risk. Tending a stand reduced the risk of chronic losses due to pests and site specific factors and this favoured regimes with management components. However when the risk due to fire, for example, increased sufficiently then options with management components were found not to return the cost of the investment.

Given that risk lowered the expected returns from forestry investment, the level of investment in the presence of risk will be lower than in an environment of certainty.

The impact of risk from nature did not have as great an affect on optimal regimes as shown in other papers. In particular optimal rotation lengths and management levels were not highly sensitive to small increases in risk. This implied that stand values were increasing at a level which warranted leaving the stand until the next period even though the risk of losing the stand was increasing. This was brought about by the two factors affecting the stand’s value as it matured: increased log value and decreased logging costs. Low discount rates retained these increased values and encouraged longer rotations than cases run at high discount rates. The repercussions of risk are also diminished when they are considered over a large number of stands. The argument has been made that given a large number of stands spread over a large area a very long time horizon that risks from the market and from nature are minimal at the
forest level (Heaps 1985).

The same can not be said for uncertain funding. As noted in the chapter it can not be spread over a large number of stands because elimination of a program affects all young stands. The importance of funding continuity is especially important in light of the finding that except at very low discount rates the future stand values were not sufficient to motivate present management actions. This means that unless a firm is highly optimistic about the path of future prices incentives beyond future harvesting rights need to be firmly in place to motivate a firm to invest on Crown land or even on their own private land. The incentives could take the form of stumpage rebates, tax concessions additional harvest allocations when management takes place and so on. It is important however that whatever mechanism is chosen to encourage forest investment to the level which the government deems optimal, it must not only be in place in the present but agents must have a reasonable degree of assurance that the program will remain in place. Failure to do so will seriously inhibit forestry investment. Risks from nature can be accounted for and reduced by agents; risks of uncertain funding do not occur in any predictable fashion and have such a wide impact that they must be considered the more dangerous of the two shocks which can hit the forest sector.
CHAPTER 6

SUMMARY AND CONCLUSIONS

6.1 INTRODUCTION

The purpose of this chapter is to review the findings of the previous chapters, provide some concluding remarks and outline the possibilities for further work in this field. The chapter follows in that order.

6.2 SUMMARY

Dynamic programming was shown to be a powerful algorithm for solving stand optimization problems. A stand model was created for solution using this technique. The model was based on the single tree distance dependent crown based stand growth model TASS and cost and revenues which reflected 1985 conditions in Coastal British Columbia. The base case solution showed that planting a stand of Douglas fir on bare land could not provide a positive return when the discount rate was greater than 3%. Positive returns could also be obtained by assuming real price increases of 2% annually and fixed management costs. Incentives such as stumpage rebates and the Allowable Cut Effect also made planting a plot of bare land a profitable activity.

The first deviation from the base case was to constrain the budget available for forest management activities. The problem was solved using decomposition and produced results which outlined what proportion of the stand should be treated with each regime. Decomposition was effective in preserving the benefits of DP while determining the impacts of a linear constraint.

The shocks of plantation failure, stochastic growth, fire, pests and funding uncertainty were considered in Chapter 5. These reduced the
expected stand value, shortened rotations and in most cases made management activities less profitable. The optimal regimes were not however highly sensitive to changes in the probability of these shocks occurring. Stand values increased sufficiently in the years preceding harvest to warrant the risk of being hit with one of the aforementioned shocks. Funding uncertainty was determined to be a particularly important potential shock. It had much more impact on what level of management took place and even on whether or not the stand should be planted at all. It was also noted that funding uncertainty could affect all stands simultaneously where other shocks would only affect some stands.

6.3 CONCLUSION AND FURTHER STUDY

While the study has shown the effectiveness of the DP algorithm, the impact of shocks (both natural and manmade) and the profitability of forest stands under various conditions, the study falls short of being able to provide clear policy direction. The work here could be built upon to provide valuable information for policy planners.

6.3.1 Areas for further study

To improve the efficiency of obtaining stand level optimal regimes the TASS growth model should be integrated with the DP algorithm as has already been done with DFSIM (DOPT) and some other stand simulators. The stand solutions could also be improved with improved information on management costs. This is particularly true of thinning costs. A model is needed which predicts thinning costs given at least stand characteristics and density of thinning. For comparison runs simulating natural regeneration should be compared with the results obtained here. Better data on survival rates and regeneration delays would be necessary to include this option. Finally an improved stand
model would include more sophisticated and realistic fire and pest probabilities. These improvements to the stand model would certainly not be impossible to implement.

The stand results obtained here should be compared with results obtained using a variety of logging conditions. The impacts of non-timber values and using shadowed priced labor for forest management would also be of interest.

Given an improved stand model with a more efficient means of obtaining optimal solutions the next step would be to obtain forest level solutions. Decomposition again appears to be a technique with much potential in this area (Williams (1976); Nazereth (1973)). The impacts of stumpage rebates, allowable cut effects, budget constraints and uncertain management funding would be significant contributions to the existing literature and provide a useful model to forest planners.
LIST OF REFERENCES


Dixon, B.L. and R.E. Howitt. 1980. Resource production under uncertainty:


McDonough, J.M. and D.E. Park, Jr. 1975. A discrete maximum principle solution to an optimal control formulation of timberland and


The dissertation is highly quantitative in nature. As such, the computer was an important tool in obtaining many of the results presented here. The following is a brief discussion of how the computer was used and how effectively it performed in conducting this research.

The Tree and Stand Simulator (TASS) was used to predict stand growth. As the system was already installed at the Pacific Forestry Centre, I was able to use the model at no cost. Mitchell and Cameron (1985) state that typical runs take 20 to 60 seconds of processing time at a cost of $15-50. Hundreds of runs were conducted. While TASS provided the biological data necessary, it requires the user to start each run at year 0. As DP requires information for ten year intervals, it would have been useful to have been able to specify other starting ages. This would have reduced the total computing time required considerably.

Tree-to-truck logging costs and log values were obtained from models developed for Lotus 1-2-3 by Doug Williams (1987). These required only stand condition information and provided costs and revenues in dollars per cubic metre. These models were effective and efficient in rapidly (a few seconds) providing the cost revenue information necessary for input in the DP model.

The DPs and LPs were solved using the mathematical programming system IMPS (Love 1986). IMPS requires only the appropriate commands to generate the results. While processing time of the runs is not available, even the largest stochastic DPs containing 300-400 states were solved in about 5 minutes of real time. The program is written in Fortran 77 and can be adapted to operate in any interactive environment.
The results presented in this dissertation rely heavily on the assumptions made about tree growth, management and logging costs and revenues. These were discussed in detail in Chapter 3. The purpose of this Appendix is to show how results can vary when one modifies the assumptions noted above. This will be done comparing the results obtained here with results obtained by Heaps (1985). Heaps' work is appropriate for comparison because his results vary significantly from the results obtained here. Specifically, he finds very positive returns to planting bare land. The comparison will focus on three areas of interest - tree growth, logging costs and revenue. The case considered for comparison will be the planting of 1100 trees per hectare on Site 125 with no management.

Heaps uses the stand simulator DFSIM. Table A.1 compares the biological data from ages 30 to 100 for TASS and DFSIM. The key results which drives the two apart is the stems per hectare figure. TASS begins with 1100 stems per hectare while the DFSIM run contained 988 stems. By age 30 the higher mortality rate in the TASS stand has brought the stems per hectare figures closer together. After age 30 however a steady self-thinning takes place in the DFSIM stand while relatively few stems die in the TASS stand until after year 80 when heavy mortality takes place. The growth statistics between the two stands differ in much the same way as we would expect to see a thinned and an unthinned stand differ. The DFSIM stand produces slightly more volume and trees of larger dbh. Revenue premiums for higher dbh logs and lower logging costs associated with larger logs and fewer stems partly explain Heaps' more profitable stands. This will be looked at in more detail later.
Table A.2 displays differences in revenue per cubic metre and costs per cubic metre between the two papers. Heaps' revenues are much higher as they include the processing of logs into lumber. When the processing cost is removed from the logging cost component a significant gap appears in logging costs. Heaps' results come from Cooney (1979). One of the reasons for such a large difference is the six year gap between the studies of Cooney and those by Williams which were used for this study (see Chapter 3). If we assume a modest inflation rate of 4% annually, the gap closes substantially. Another factor that drives the figures apart is the logging conditions assumed. Heaps assumes good conditions with a 10% slope while the work in this paper assumes average conditions and a 46% slope.

Table A.3 show the differences in SE values and the source of the differences by rotation age. Revenue and logging cost disparities carry about equal weight in account for the gap in SE values. The revenue differences were expected because of the additional processing but the logging cost differential can only be explained by differences in the models of Cooney and Williams as well as the factors noted above.

Table A.4 displays the amount of the gap between the figures of Heaps and White that can be explained because of the different growth models used. The differences become more important later in the life of a stand.

In sum, the differences can be traced to the growth function, the revenue function and the logging cost function. This exercise has pointed out the need for improved research in these areas to provide researchers with a common base on which to study the economics of forest management.
<table>
<thead>
<tr>
<th>AGE</th>
<th>Merch.</th>
<th>DBH</th>
<th>Stems/ha</th>
<th>Merch.</th>
<th>DBH</th>
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<tbody>
<tr>
<td></td>
<td>MAI</td>
<td>TASS</td>
<td>DFSIM</td>
<td>TASS</td>
<td>DFSIM</td>
</tr>
<tr>
<td>30</td>
<td>6.9</td>
<td>958</td>
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<tr>
<td>40</td>
<td>9.6</td>
<td>958</td>
<td>10.2</td>
<td>753</td>
<td>383</td>
</tr>
<tr>
<td>50</td>
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<td>904</td>
<td>12.2</td>
<td>625</td>
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<td>13.2</td>
<td>534</td>
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<tr>
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<td>11.3</td>
<td>495</td>
<td>12.7</td>
<td>324</td>
<td>1129</td>
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Table A.1 Biological data for TASS and DFSIM planted at 1100 and 988 stems respectively on Site 125.

<table>
<thead>
<tr>
<th>AGE</th>
<th>Revenue per cubic metre</th>
<th>Costs per cubic metre</th>
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<tbody>
<tr>
<td></td>
<td>White</td>
<td>Heaps</td>
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<tr>
<td>60</td>
<td>38.35</td>
<td>60.00</td>
</tr>
<tr>
<td>70</td>
<td>40.24</td>
<td>62.58</td>
</tr>
<tr>
<td>80</td>
<td>41.50</td>
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<tr>
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<td>44.04</td>
<td>65.43</td>
</tr>
<tr>
<td>100</td>
<td>45.09</td>
<td>66.33</td>
</tr>
</tbody>
</table>

Table A.2 Comparison of revenue and costs per cubic metre obtained by White and Heaps.
### Table A.3 Breakdown of cost and revenue differences in White and Heaps by rotation age.

<table>
<thead>
<tr>
<th>AGE</th>
<th>Total Revenue Difference</th>
<th>Logging Difference</th>
<th>Other$^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>-871.05</td>
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<tr>
<td>100</td>
<td>-196.42</td>
<td>79.32</td>
<td>-200.38</td>
</tr>
</tbody>
</table>

$^1$ Other includes differences in site preparation costs, planting costs and method of discounting.

### Table A.4 S.E. values and percentage of the difference between White and Heaps which can be explained when DFSIM growth figures are used with the cost and revenue functions of White.

<table>
<thead>
<tr>
<th>AGE</th>
<th>DFSIM - White S.E. Values</th>
<th>Difference with Heaps</th>
<th>Percentage of difference reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
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<td>691.76</td>
<td>35</td>
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</tr>
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