ROBUST ESTIMATION WITH APPLICATION TO FAILURE DETECTION AND IDENTIFICATION (FDI) IN DYNAMICAL SYSTEMS

by

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Title of Thesis/Project/Extended Essay

"Robust Estimation with Application to Failure Detection and Identification (FDI)in Dynamical Systems"

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A new methodology for robust estimation and instrument failure detection and identification (FDI) in linear dynamical systems subject to plant parameter variations or uncertainties is presented. The proposed fault tolerant control system is based upon a single robust estimator that can simultaneously estimate the unmeasurable state variables of the system for the purpose of control, and provide necessary information to a detection logic device capable of detecting, and isolating actuator and sensor failures. In addition, it is possible to identify the shape and magnitude of the failure with this approach. The algorithm allows for fast and accurate fault detection while accounting for the structural uncertainties and parameter variations in the system's dynamical model. At the heart of the approach is the robust estimator for which necessary and sufficient conditions to its existence is given.

The overall FDI strategy is verified through simulation studies performed on the control of a VTOL Helicopter in the vertical plane.
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DEDICATION

To my parents
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CHAPTER ONE
INTRODUCTION

Instrument failure detection, isolation and accommodation (FDIA) plays a very important role in modern control engineering as early detection of failures can prevent system breakdown or serious damage.

Among the various FDIA techniques there is a class of model-based methods that use the idea of analytical redundancy. Traditional hardware redundancy is generated by multiple like instruments performing the same task. The analytical redundancy is more attractive since it offers FDIA capabilities without a need for costly and bulky redundant hardware. The underlying idea of the analytical redundancy techniques is that the system's dynamical model, along with the input and output measurements can be used to generate quantities called residuals or innovation process. Under no failure conditions and assuming that the system model is exactly known the residuals would be zero or close to it, otherwise nonzero residuals would indicate presence of a fault. For detailed survey of various analytical redundancy techniques the reader is referred to Willsky (1976), Isermann (1984), Merrill (1985), Frank (1987, 1990), and Stengel (1990).

One of the earliest works on the application of analytical redundancy technique to FDIA was due to Beard (1971) who used the concept of "detection filter" for FDIA purposes. The approach was later enhanced and modified by Jones (1973). Later Clark (1975, 1978) proposed a dedicated observer scheme (DOS) where reduced order observers were dedicated to individual sensors. Assuming that the system is completely observable through each single output, state
estimates could be obtained by each dedicated observer. If no faults occur all the state estimates are identical. However, if some of the sensors are faulty, alarm can be triggered by comparing the estimates with the actual outputs in a certain logical fashion. Since the introduction of DOS, other approaches based on it utilizing some kind of detection functions or statistical tests have been proposed (Frank (1980), Saif and Villaseca (1986, 1987a,b)). Among various advantages in using a detection function, one is the simplicity that it introduces in the FDIA logic.

In general, the plant parameters are not exactly known in practice. Often some of the plant parameters might be changing or unknown. Estimator (e.g., observer or Kalman filter) design, however, would require a full knowledge of the system's dynamics as well as the availability of the system's inputs and outputs. As a result, in an uncertain system the residuals are affected by both failures and parameter uncertainties. Therefore, in such situations FDIA based on the approaches described earlier would not be possible. In such situations there would be a need for a robust FDIA methodology.

If the range of the parameter variation is small and given, it may be possible to determine a suitable threshold such that any value of the residuals beyond it would trigger an alarm (Clark (1978)). However, this approach is not general and is highly application dependent. The determination of the suitable thresholds may also require extensive simulation studies of the particular system.

There are other alternatives to FDIA under system's uncertainties and parameter variations. One approach is based on the concept of sensitivity analysis. Here, the estimators need to be designed such that the residuals are insensitive to the system uncertainties and
sensitive to possible failures. Generally in such an approach there would be a need to reach a compromise between the number of misses and false alarms. The limitations of this method are that first, the parameter variations should be small, and second, it is hard to make the compromise between failures that go undetected and false alarms. An approach to compensate for these drawbacks was proposed by Frank and Keller (1980). In that scheme, two observers are dedicated to each sensor. One of the observers is designed to be robust to the plant parameter variations and instrument malfunctions, whereas the other is made to be insensitive to parameter variations only, but sensitive to instrument failures. The fault detection logic then processes the residuals generated by each of the two observers in order to detect faults under uncertainties. The estimator design is not a trivial task in this scheme, and the method is limited to single input single output (SISO) systems.

Another alternative to FDIA under uncertainties in the system’s model is based on the theory of unknown input observer (UIO). UIO theory was used for the problem of actuator failure detection and identification (Phatak and Viswanadham (1988), Park and Stein (1988)). Schomig (1989) used the UIO for the sensor failure detection in the Boeing 929 Jetfoil boat. Since the fault detection scheme reported in this work falls under this class of schemes, the UIO theory and the related fault detection approaches are discussed in more detail in the upcoming chapters.

Unknown input observers are a class of estimators designed to provide accurate estimation of the system’s state in the face of unknown exogenous plant disturbances in the system’s dynamical model. In recent years there have been a considerable interest in the design
of UIOs because of their widespread applications. The theory of UIO was first examined by Basile and Marro (1969). It was followed by Wang, et. al. (1975), Kudva, et. al. (1980), Kurek (1983), Yang and Wilde (1988) and Park and Stein (1988). Kurek also gave necessary and sufficient conditions for the existence of UIO. The design of UIO generally requires geometric approaches or solution of complicated matrix equations. Recently, Guan and Saif (1989), and Saif and Guan (1990) provided a very straightforward yet elegant approach to the design of UIOs. The design algorithm is believed to be computationally simpler and more direct than the previous aforementioned works.

In this thesis a robust approach for FDIA in linear uncertain dynamical systems is presented. The approach is based on the fact that the model of the linear dynamical system under plant parameter variations and uncertainties can be transformed into such a form that would make the application of the UIO theory for state estimation purposes possible. Equipping this with a FDIA strategy would result in the desired fault tolerant system.

In Chapter Two an overview of the overall thesis is presented. The objective of this chapter is to provide a somewhat informal discussion of the overall work. This chapter will also be of use to those readers who wish to get a good idea of this work without reading the entire thesis. The bulk of the theoretical considerations and the complete discussions and algorithms are given in Chapters Three and Four.

In Chapter Three the unknown input observer theory is reviewed and the algorithm developed by Saif and Guan (1990), and Guan and Saif (1989) is derived and proved.

In Chapter Four the UIO theory is used for simultaneous detection of actuator and sensor failures along with identification of the
magnitude and shape of the failures in uncertain systems.

In Chapter Five simulation studies are performed on the model of a helicopter, whose dynamics is subject to change with loading and flight conditions. Simulation results show that the performance of the estimator and the FDIA logic is quite satisfactory.

Finally, in Chapter Six the contributions of this work are summarized and the FDI approach taken is discussed. We conclude by outlining the extensions and possibilities for future research work.
CHAPTER TWO
RELATED WORK AND PRELIMINARY ANALYSIS

2.1) Problem Formulation

The detection and diagnosis of faults in complex process plants is one of the most important tasks assigned to the computers supervising such plants. The early indication of incipient failure could prevent system deteriorations or breakdowns which could otherwise result in material damage and even human fatalities. Similarly, failure detection and isolation has become a critical issue in the operation of high performance ships, submarines, airplanes, and space vehicles, where crew safety, successful mission completion, and significant investment are at stake. In the following, we will overview the various steps that need to be taken in attacking such problem.

2.2) FDIA Problem

There are two major approaches to the problem of failure detection and isolation:

1) Methods that do not use any mathematical model of the plant

Examples of such simple approaches are: limit checking, hardware redundancy, knowledge based (expert system) approaches (although some expert system approaches are model-based), etc. However, these methods need either bulky hardware or a great deal of prior knowledge of the system. Moreover, generally these approaches can not detect soft failures, or identify the shape and magnitude of the failures.

2) Methods that use a mathematical model of the plant

A broad class of failure detection and isolation methods make
explicit use of the plant's mathematical model. Most of these methods are based on the concept of analytical redundancy, and employ estimation techniques via use of observers or Kalman filters. Contrary to hardware redundancy, where redundant measurements from each sensor are compared, here the values of the estimated variables from the sensor measurements are used as redundant information for the fault detection purpose.

2.2.1) FDIA Tasks

The task of failure detection is accomplished in two steps (see Fig. 2.1):
1) generation of redundant information by estimation, and
2) decision making.

In general, multiple observers are required to generate redundant information about the system. This redundant information is generally in terms of the state estimates. The values of the estimated variables from the sensor measurements are compared to their respective measured values. Then quantities called residuals are generated so that the residuals are zero (or close to zero) if and only if no faults occur. These residuals are then processed in a logical framework (here called decision making), with the aim of detecting and isolating the failures that caused their abnormal behavior. More precisely, the decision making consists of the following tasks:
1) Failure detection, i.e., the indication that something is going wrong in the system.
2) Failure isolation, i.e., the determination of the faulty component(s) and their removal from the system.
3) Failure identification, i.e., the determination of the size and/or
Figure 2.1 - Block Diagram of a FDIA System
the shape of the failure.

4) *Failure Accommodation*, i.e., the reconfiguration of the system so that it can continue to function without interruption.

In this work, we only focus on the first three of the above tasks. Clearly, detection is an absolute must in any practical system and isolation is equally as important. One may think that identification of the failure shape may be helpful, but that it may not worth the extra effort it requires for computing it. However, the author believes that failure identification is indispensable to prevent false alarm (see section 2.3.1).

2.2.2) Nature of Failures

There are variety of factors that may affect the proper function of a system, such as instrument failures, disturbances, noise, component breakdown or changes, etc. Our discussions are confined to three cases and we think that they are general enough to serve as a basis for further discussion.

1) *Instrument faults*- These faults can cause discrepancies between the measured and true values of plant output or input variables. Among such faults the most important and likely ones are sensor and actuator failures. As an example, an actuator malfunction can be declared when there exist a discrepancy between the intended (computed) control and its realization by the actuator.

2) *Disturbances*- Disturbances can be either unmeasurable inputs acting on the plant or unmeasurable outputs contaminating the plant output.
3) **Plant parameter uncertainties or changes**- The uncertainties may be due to modeling errors, lack of knowledge about some of the plant parameters, parameter variations, etc. The reason for parameter changes could be that some components are deteriorating due to aging or that some of them are time-varying.

It should be noted that generally in this study disturbances are considered to be deterministic (e.g., a bias or soft drift) or jumps occurring at random intervals with random amplitudes.

Note also that the above representations may not describe some practical failure situations in a natural way. As an example, a complete sensor failure (zero output) would have to be described as a variable bias (equal to true value of the output) or as a parameter change (some parameters in the system's dynamical representation changing to zero).

Before we introduce an overview of our new FDI scheme, we first give a brief review of a notable FDI approach, the dedicated observer scheme (DOS) of Clark (1975, 1978). As was pointed out in Chapter One the original DOS does not take into account uncertainties, which is its main shortcoming. In the next section through an example we will illustrate some shortcomings associated with the detection logic used in such approaches.

2.3) **The Dedicated Observer Scheme (DOS)**

In recent years, a variety of analytical redundancy approaches have been proposed for sensor failure detection, isolation and accommodation (FDIA) in dynamical control systems (see references cited in Chapter One). These approaches are more attractive than the traditional hardware redundancy techniques since they offer FDIA capabilities
without a need for costly and bulky redundant hardware. Moreover, they are capable of detecting soft (small drift and bias) failures in addition to hard failures. The underlying idea of these techniques is that the system's dynamic model, along with the input and output measurements can be used to generate redundant information about the system by using estimation techniques. These information are then processed and monitored in a logical decision process for FDIA purposes.

Among various approaches that have been proposed for FDIA, one of the notable ones is the dedicated observer scheme (DOS) of Clark (1975,1978). In the DOS a single observer is dedicated to each sensor output. Then assuming that the system is observable from each output, each dedicated observer estimates the state of the system using the output of sensor, and the system's input. If all the sensors are healthy then their estimate after a short transient period should converge to the actual state of the system. However, if one of the sensors fail then the state estimate from its dedicated observer will be in disagreement with the estimates from the remaining dedicated observers, and therefore this would provide a means for detecting sensor failures. Figure 2.2 shows the basic structure of the FDI subsystem using this method.
Figure 2.2 - The Dedicated Observer Scheme (DOS)
As an example, should a fault develop in instrument 1, then \( y_1 \) will be faulty, and so \( \dot{x}_1 \) will be erroneous. However, \( \dot{x}_2, \dot{x}_3, \ldots, \dot{x}_m \) will remain identical to \( x \) since they are not affected by instrument 1. The failure of instrument 1 is thus identified through a comparison of \( \dot{x}_1 \) with the corresponding, unaffected, vector \( \dot{x}_2, \dot{x}_3, \ldots, \dot{x}_m \). We note that since the detection is based on a "majority vote" logic then \( m \) must be more than three for the FDIA to be possible. That is, our set of instruments must contain at least three sensors. This is by the way one of the shortcomings of the approach.

2.3.1) Logic for Fault Detection

The desired function of the FDI subsystem is to register an alarm indicating that an incipient fault has occurred in a specific instrument immediately following the advent of that fault. There are many possibilities and the logic design is dependent on the design of the observer. The following approach was developed by Clark (1978), which uses dedicated reduced order observers. For illustration, we assume that the system is of fourth order and that the system has four output measurements. Therefore there are four dedicated observers, each of which is of third order. Defining the following vector function

\[
\Delta y_i = \begin{pmatrix}
\Delta y_{1i} \\
\Delta y_{2i} \\
\Delta y_{3i} \\
\Delta y_{4i}
\end{pmatrix} = \begin{pmatrix}
y_1 - \hat{y}_{1i} \\
y_2 - \hat{y}_{2i} \\
y_3 - \hat{y}_{3i} \\
y_4 - \hat{y}_{4i}
\end{pmatrix}
\]

(2.1)

where \( \hat{y}_{ji} \) is the estimated output of instrument \( j \) produced by the \( i \)th observer and \( y_j \) is simply the \( j \)th instrument output. Defining the
scalar function $\Delta_1(t)$ as

$$\Delta_1(t) = |(\Delta y_j^1)(\Delta y_k^1)(\Delta y_\ell^1)|$$

$$i=1, 2, 3, 4; \ j \neq i, k \neq i, \ell \neq i$$

Thus we have, for example,

$$\Delta_2(t) = |(\Delta y_1^2)(\Delta y_3^2)(\Delta y_4^2)|$$

The term $(\Delta y_2^2)$ is not included in $\Delta_2(t)$ because it is identically zero since in observer 2 we happen to have $y_2^2 - y_2$. When there is only one faulty instrument, for example instrument 2, it is easy to see that $\Delta_1(t)$, $\Delta_3(t)$, $\Delta_4(t)$ will be nonzero but $\Delta_2(t)$ will equal to zero. When there are two faulty instruments, for example, instrument 1 and instrument 2, then $\Delta_1(t)$ and $\Delta_2(t)$ will both be nonzero, and there will be no false alarm for instrument 3 and 4 because $\Delta_3(t)$ will remain zero by virtue of $\Delta y_3^3(t)$ being zero, and similarly $\Delta_4(t)$ will be zero by virtue of $\Delta y_4^4(t)$. However, three simultaneously faulty instruments (say instruments 1, 2, 3) could not be identified without causing a false alarm on the fourth instrument. That is, all of the four scalar functions would be nonzero while in fact one of them should be zero.

There is a major drawback with the detection logics such as the one described above. Through simulations the author discovered that the detection logics such as the one reviewed above can not distinguish a spike (momentary failure) from a persistent sensor failure. To illustrate our point consider the following example:

EXAMPLE 2.1- Consider the following stable linear system

$$\begin{pmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{pmatrix} =
\begin{pmatrix}
-1 & -3 & 1 \\
1 & -3 & 1 \\
-1 & 1 & -6
\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
x_3
\end{pmatrix} +
\begin{pmatrix}
1 \\
2 \\
1
\end{pmatrix} u$$
where the input \( u \) is a unit step signal. It is assumed that this system has only one faulty sensor \( (y_1) \) modeled by the term containing the variable \( f \) in the output equation. We will consider two different failure conditions (shapes) for \( f \) as illustrated by Figure 2.3. It can be seen that in case (b) there is obviously a persistent sensor failure while in case (a) we merely have a momentary spike which in most situations may be tolerated. We are going to show that the failure detection logic described previously can not distinguish between the two conditions, and will have similar responses in both situations.

**DESIGN OF THE DEDICATED OBSERVERS**

a) Dedicated Observer (DO) 1

The output equation is given by

\[
y_1 = [1 \ 0 \ 0] x
\]

A reduced observer could be designed. Since the design is well known we only give the results. By assigning the eigenspectrum of the observer at \([-10 \ -11]\), we obtain the observer dynamics as

\[
\begin{pmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{pmatrix} = \begin{pmatrix}
-19.41 & 6.47 \\
-12.24 & -1.59
\end{pmatrix} \begin{pmatrix}
x_1 \\
x_2
\end{pmatrix} + \begin{pmatrix}
7.47 \\
5.41
\end{pmatrix} u + \begin{pmatrix}
73.17 \\
68.53
\end{pmatrix} y_1
\]

b) Dedicated Observer 2 and 3

Their respective output equations are

\[
y_2 = [0 \ 1 \ 0] x
\]
(a) A momentary spike in the output measurement

(b) A constant bias in the output measurement

Figure 2.3 - Two Different Failure Models
and

\[ y_3 = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} x \]

By assigning the closed loop poles of the observer 2 and 3 at the same locations as observer one we obtain the following observers' dynamics

\[
\begin{pmatrix}
\dot{x}_1 \\
\dot{x}_2 \\
\dot{x}_3
\end{pmatrix} = 
\begin{pmatrix}
-28 & -24 \\
10 & 5 \\

\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2 \\
x_3
\end{pmatrix} + 
\begin{pmatrix}
-48 \\
23 \\

\end{pmatrix} u + 
\begin{pmatrix}
-314 \\
163 \\

\end{pmatrix} y_2
\]

\[
\begin{pmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{pmatrix} = 
\begin{pmatrix}
17 & -21 \\
36 & -38 \\

\end{pmatrix}
\begin{pmatrix}
x_1 \\
x_2
\end{pmatrix} + 
\begin{pmatrix}
-17 \\
-33 \\

\end{pmatrix} u + 
\begin{pmatrix}
-320 \\
-471 \\

\end{pmatrix} y_3
\]

After designing the three DOs, we can use the functions defined in (2.2) to detect and isolate the failure. Since only sensor 1 \(y_1\) is faulty, according to the previous discussions, both \(\Delta_2(t)\) and \(\Delta_3(t)\) are zero and \(\Delta_1(t)\) would be nonzero. For case (a), where the output measurement error signal is merely a spike, its \(\Delta_1(t)\) response is shown in Figure 2.4. For case (b), the response is shown in Figure 2.5. We can see that both responses are very similar although in the first case the sensor is momentarily at fault which may be tolerated, whereas in the second case the sensor failure persists, and an alarm should be triggered.

From the above discussion it is clear that there is a need for a more intelligent fault detection scheme. Namely one that can detect number of simultaneous failures under plant parameter variations and uncertainties. In addition, it is of interest to us to distinguish between a persistent and a momentary type of failure. It is precisely for this reason that we think identification of the failure shape could prove to be useful. Notice that in the above example if one could
Figure 2.4 - Detection Function Response for a Momentary Jump

Figure 2.5 - Detection Function Response for a Persistent Failure
identify the shape of the failure, one could see that the scenario described in case (a) could be tolerated and therefore there would be no need for alarm. The fault detection logic presented in this work has such capabilities.

2.4) Overview of the Proposed Approach

In this thesis, we will present a new approach to the failure detection problem as described above. In the proposed scheme, only a single estimator is required for the purpose of feedback control as well as detection and isolation of failures. The failures could be either actuator or sensor failures. In addition, the proposed algorithm will enable us to identify the shape of the failure in a very simple fashion. As a matter of fact this is accomplished as a by product of the system formulation and the estimates obtained through the estimator.

The proposed scheme relies on the theory of state estimation for linear systems subject to exogenous unknown inputs and the unknown input observer (UIO) theory. In the following without going through the details of the UIO design a brief mathematical overview of the failure detection scheme is given.

2.4.1) General Model Structure

Consider a linear time-invariant system with both instrument failures and plant uncertainties. This system can be described as follows

\[ \dot{x} = A x + B u + D v \]  

(2.3)
where \( x \in \mathbb{R}^n \) is the state vector, \( u \in \mathbb{R}^q \) is the input, \( v \in \mathbb{R}^m \) is the unmeasurable input disturbance which can be considered as actuator failure, \( y \in \mathbb{R}^p \) is the measurable output vector, and \( f \in \mathbb{R}^r \) is the unmeasurable additive disturbance which can be considered as sensor faults. The nominal values of the plant matrices \( A \), and \( B \) are known, but otherwise, they are allowed to contain unknown elements. We argue that this system can be transformed into a known system with unknown inputs and unknown outputs only. This is done in Chapter Four by basically decomposing every uncertain matrix in the system's description in (2.3) as the sum of two matrices. One of these matrices is exactly known and represents the nominal value of the particular matrix, while the other contains uncertainties. Then the system's state space description is rewritten as a part that is completely known and represents the nominal system plus an additional part that accounts for the uncertainties. This additional part is obtained by lumping all the uncertain part of the system matrices together and treating them as additional unknown input to the system. From the foregoing discussion it is clear that the uncertainties can be handled and it is possible to rewrite the dynamics of an uncertain system in the form of (2.3-4) with matrices \( A \) and \( B \) completely known. Therefore, for the sake of simplicity and the discussion in this section assume that the system plant parameters in (2.3-4) are completely known.

2.4.2) Model Transformation and Fault Identification

If we can obtain the state estimates, then the instrument failures can be identified with a simple algorithm, as shown later in this
section. However, it would be fairly difficult to directly design an UIOs for system (2.3), and (2.4). Fortunately, we can show that system in (2.3) and (2.4) can be transformed into an augmented system with unknown inputs only. Then we can apply the UIO algorithm developed in Chapter Three for the purpose of estimating the state variables.

The following proposition is essential for the transformation:

Proposition 2.1- Any signal $f$ can be considered as an output of any given dynamical system with a certain input $\xi$. In fact, for any $f$ and $A_f$, if we choose $\xi$ as

$$\dot{\xi} = f - A_f f,$$

we have

$$\dot{f} = A_f f + \xi. \quad (2.5)$$

Augmenting (2.3) and (2.5), we have the following augmented system:

$$\begin{bmatrix} \dot{x} \\ \dot{f} \end{bmatrix} = \begin{bmatrix} A & 0 \\ 0 & A_f \end{bmatrix} \begin{bmatrix} x \\ f \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix} u + \begin{bmatrix} D & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} v \\ \xi \end{bmatrix} \quad (2.6)$$

$$y = \begin{bmatrix} C & E \end{bmatrix} \begin{bmatrix} x \\ f \end{bmatrix} \quad (2.7)$$

In principle, it is now possible to design an UIO for the above system. Once the estimate of the augmented state $\begin{bmatrix} x \\ f \end{bmatrix}$ is obtained, we can immediately identify the sensor failures. This is simply done by checking the components of the estimated vector $f$. Any nonzero component of this vector would indicate presence of a fault in the corresponding sensor. Moreover, the nonzero value gives the estimate of the fault magnitude. Hence, we can not only detect a faulty sensor, but also identify the shape of the fault. Next, our task is to identify the
actuator failures. Since both $\dot{x}$ and $\dot{f}$ are available through estimation, the only unknown element in (2.3) is the unknown input vector $v$. Solving the first equation of (2.3) (see section 4.3.2), we have

$$x(t) = \exp(A \, t) \left( \exp(-A \, t) \, x(t_0) + \int_{t_0}^{t} (B \, u + D \, v) \, dr \right) \quad (2.8)$$

In chapter four we prove that if the sampling interval is small enough we can solve from this equation for $v$ in terms of $x$ and $u$. Once $v$ is found, again presence of any nonzero component in $v$ would indicate the corresponding faulty actuator. Figure 2.6 shows the overall block diagram of this scheme.

To summarize, the various steps involved in the design of the fault detection, isolation (FDI) system can be briefly outlined as follows:

1) A system with both plant parameter uncertainties and instrument faults (actuator and/or sensor failures) is transformed into a completely known system with instrument faults, and some additive unknown input disturbances.

2) The model of the system with both actuator and sensor failures is then transformed into a higher dimensional model containing actuator faults only.

3) At this point the obtained model is suitable for employment of the UIO. The instrument faults along with the state of the system are estimated through an UIO.

4) The estimated variables are used for control and FDI purposes.

Detailed discussion of each of the above design steps are presented in the next two chapters.
Figure 2.6 - Failure Detection Using UIO
3.1) Previous Work on UIO

In this chapter we only consider the problem of state estimation for a known system. That is, the plant parameters are assumed to be exactly known and no parameter variations is allowed.

Consider the following system in state space form with some unknown plant disturbances acting on the system:

\[ \dot{x} = Ax + Bu + Dv \]
\[ y = Cx. \]

Without loss of generality, assume that \( C \) has a special structure given by \([0 \ I]\). This is not a restrictive assumption, since as long as \( C \) is of full rank, there will always exist a similarity transformation that if performed on the system will bring the matrix \( C \) of the transformed system into this special structure (Chen 1984). Furthermore, we also assume that \( D \) is of full rank. This is not a restrictive assumption either. If \( \text{Rank}(D) = m_1 \leq m \), then \( D \) is expressed as \( D = \bar{D} N \) (see the matrix decomposition in Section 4.2.2), where \( \bar{D} \) has full rank \( m_1 \). Then \( \hat{v} = Nv \) could be defined as the new unknown input vector.

As mentioned in the first chapter, a few algorithms have been proposed for the design of full order and reduced order UIOs.

Basically, the central idea of these approaches can be described as follows: Consider a full order observer only, which can be described
as a dynamical system having the following general structure:

\[
\begin{align*}
\dot{z} &= Nz + Ly + Gu \\
\dot{x} &= z - Ey
\end{align*}
\]

(3.3)  \hspace{1cm} (3.4)

where \( z \in \mathbb{R}^n \), and matrices \( E, G, L \) and \( N \) have appropriate dimensions.

The following design procedure is used for finding suitable matrices \( N, L, G, \) and \( E \). To do this, define the error vector

\[
q = \dot{x} - x.
\]

(3.5)

Then

\[
\begin{align*}
\dot{q} &= \dot{x} - x \\
&= \dot{z} - E \dot{y} - x \\
&= (Nz + Ly + Gu) - (EC + I) (Ax + Bu + Du + Dv)
\end{align*}
\]

Replacing variable \( z \) with \( (q + Ey + x) \), we have

\[
\begin{align*}
\dot{q} &= -Nq + (N + LC - A - ECA + NEC)x \\
&\quad + (G - B - ECB)u - (D + ECD)v
\end{align*}
\]

(3.6)

In order to estimate the state of the system accurately, it is required for the estimation error \( q \) to approach zero in an asymptotic fashion, independent of the inputs and outputs. Therefore, the following equations must hold:

\[
\begin{align*}
(N + LC - A - ECA + NEC) &= \mathbf{0} \\
G - B - ECB &= \mathbf{0} \\
D + ECD &= \mathbf{0}
\end{align*}
\]

(3.7)  \hspace{1cm} (3.8)  \hspace{1cm} (3.9)
in which case (3.6) reduces to

\[ \dot{q} = N \dot{q} \quad (3.10) \]

The design procedure is then to find a stable matrix \( N \) as well as matrices \( I, G, \) and \( E \) that will simultaneously satisfy (3.7-9). This, however, is by no means an easy task. The problem will be even more complicated when designing a reduced order UIO, since the observer dynamics would become:

\[ \dot{z} = Nz + Ly + Gu \quad (3.11) \]

\[ \dot{x} = Mz - Ey \quad (3.12) \]

where \( M \) is a \( n \times p \) matrix. In this case, it is not easy to deduce equation (3.6). Instead, we will run into the concept of generalized inverse of a matrix (Kurek 1983).

3.2) A Novel Approach to UIO Design

In this section, we will give a novel approach to design of a reduced order UIO. The main advantage of the proposed scheme over the other schemes is that we do not need to solve any matrix equations. The design procedure only involves matrix addition and multiplications, and is therefore numerically attractive. In addition, the problem of designing an UIO can be transformed into the problem of designing a standard reduced order Luenberger observer.

3.2.1) A Necessary Condition

The following theorem states a necessary condition for the
existence of UIO.

**Theorem 1.** A necessary condition for the existence of any order observer for system (3.1) and (3.2) is that

1. \( p \geq m \)
2. \( \text{R} \text{ank}(D_2) = m \)

where

\[
D = \begin{pmatrix} D \\ D_2 \end{pmatrix}
\]  
(3.13)

\( D_1 \) is a \((n-p) \times m\), and \( D_2 \) is \(p \times m\) matrices.

**Proof:**

Any observer can be described as a dynamical system having the following general structure:

\[
\begin{align*}
\dot{z} &= G z + H u + L y \\
\dot{x} &= M z + N y
\end{align*}
\]  
(3.14, 3.15)

where \( z \in \mathbb{R}^k \), \( n-p \leq k \leq n \), and matrices \( G, H, L, M, \) and \( N \) have appropriate dimensions. Define the error vector

\[
q = x - \dot{x}
\]  
(3.16)

Suppose an observer exists, then there must exist a stable matrix \( Q \) such that

\[
\dot{q} = Q q
\]  
(3.17)

Combining (3.13)-(3.17), we get

\[
\dot{x} - (M \dot{z} + N \dot{y}) = x - (M z + N y)
\]
Therefore

\[(I - NC) \dot{x} - M \dot{z} = (I - NC) x - M z\]

Eliminating the derivative of \(x\) and \(z\), we have

\[(I - NC) (Ax + Bu + Dv) - M (Gz + Hu + Ly) =
(I - NC) x - M z\]

It follows that

\((PA - QP - MLG) x + (PB - MH) u + PDv + (QM - MG) z = 0\) \hspace{1cm} (3.18)

where

\[P = I - NC\] \hspace{1cm} (3.19)

Since \(x, u, v\) and \(z\) are independent vectors, equation (3.18) holds if and only if the following equations are satisfied

\[PA - QP - MLG = 0\] \hspace{1cm} (3.20)
\[PB - MH = 0\] \hspace{1cm} (3.21)
\[QM - MG = 0\] \hspace{1cm} (3.22)
\[PD = 0\] \hspace{1cm} (3.23)

Recalling that the special structure of \(C\) allows us to write (3.23) as

\[D - N [O \ I] \begin{pmatrix} D_1 \\ D_2 \end{pmatrix} = 0\]

and therefore

\[ND_2 = D\] \hspace{1cm} (3.24)
Since D is of full rank, N is solvable if and only if the rank of D_2 is equal to m. This implies that p ≥ m. □

3.2.2) Observer Design Algorithms

In this section, it is assumed that the necessary condition of section (3.2.1) holds. The design of UIO will be carried out for each of the following two cases independently:

CASE 1) p > m
CASE 2) p = m

CASE 1) Define

\[ y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \]

where \( y_1 \in \mathbb{R}^{p-m} \) and \( y_2 \in \mathbb{R}^m \). Then, the state vector \( x \) can be written as

\[ x = \begin{bmatrix} x_1 \\ y_1 \\ y_2 \end{bmatrix} \]

where \( x_1 \in \mathbb{R}^{n-p} \) is the vector that needs to be estimated.

Using the above notations, the system described in (3.1) and (3.2) can be written in a partitioned form as

\[
\dot{x} = \begin{pmatrix} A_1 \\ A_2 \\ A_3 \end{pmatrix} x + \begin{pmatrix} B_1 \\ B_2 \\ B_3 \end{pmatrix} u + \begin{pmatrix} D_1 \\ D_2 \\ D_3 \end{pmatrix} v \quad (3.25)
\]

\[
y = \begin{bmatrix} 0 & I \end{bmatrix} x \quad (3.26)
\]

where \( A_i, B_i \) and \( D_i, i=1,2,3 \), have the corresponding dimension with \( x_1, y_1 \) and \( y_2 \).
Notice that the matrix components \[
\begin{pmatrix}
D_2 \\
D_3
\end{pmatrix}
\] defined in (3.25) are newly defined partition of the previously defined \( D_2 \) in (3.13). Therefore, from condition 2 of theorem 1

\[
\text{Rank}\left( \begin{pmatrix} D_2 \\ D_3 \end{pmatrix} \right) = m
\]

Without any loss of generality, assume that \( D_3 \) is invertible. Consider now the following matrix operator \( T \):

\[
T = \begin{pmatrix}
I & 0 & -D_1 D_2^{-1} \\
0 & I & -D_2 D_3^{-1} \\
0 & 0 & I
\end{pmatrix}
\quad (3.27)
\]

Premultiplying (3.25) with \( T \), we have:

\[
\begin{pmatrix}
\dot{x}_1 - D_1 D_2^{-1} \dot{y}_2 \\
\dot{y}_1 - D_2 D_3^{-1} \dot{y}_2
\end{pmatrix} = \begin{pmatrix}
A_1 - D_1 D_2^{-1} A_5 \\
A_2 - D_2 D_3^{-1} A_3
\end{pmatrix} \begin{pmatrix}
x_1 \\
y_1
\end{pmatrix} + \begin{pmatrix}
0 \\
0
\end{pmatrix} (3.28)
\]

\[
\begin{pmatrix}
B_1 - D_1 D_2^{-1} B_3 \\
B_2 - D_2 D_3^{-1} B_3 \\
B_3
\end{pmatrix} u + \begin{pmatrix}
0 \\
0 \\
D_3
\end{pmatrix} v
\]

Taking the first two rows of (3.28), which are two equations independent of the unknown inputs, and defining

\[
\tilde{A}_1 = A_1 - D_1 D_3^{-1} A_3
\]
\[ \hat{B}_1 = B_1 - D_1 D^{-1}_3 B_3 \]

we have

\[ \dot{x}_1 - D_1 D^{-1}_3 \dot{y}_2 = \hat{A}_1 x + \hat{B}_1 u \]  \hspace{1cm} (3.29)

and

\[ \dot{y}_1 - D_2 D^{-1}_3 \dot{y}_2 = \hat{A}_2 x + \hat{B}_2 u \]  \hspace{1cm} (3.30)

Let

\[ \hat{A}_i = [ \hat{A}_{i1} \hat{A}_{i2} \hat{A}_{i3} ] \]  \hspace{1cm} (3.31)

where \( \hat{A}_{i1} \in \mathbb{R}^{n-p} \), \( \hat{A}_{i2} \in \mathbb{R}^{p-m} \), and \( \hat{A}_{i3} \in \mathbb{R}^m \). Then, (3.29) and (3.30) will become

\[ \dot{x}_1 - D_1 D^{-1}_3 \dot{y}_2 = \hat{A}_{i1} x + \hat{A}_{i2} y_1 + \hat{A}_{i3} y_2 \]

and

\[ \dot{y}_1 - D_2 D^{-1}_3 \dot{y}_2 = \hat{B}_2 u - \hat{B}_1 u \]

Rearrange the above two equations into the following form

\[ \dot{x}_1 = \hat{A}_{i1} x_1 + r \]  \hspace{1cm} (3.32)

\[ z = \hat{A}_{i2} x_1 \]  \hspace{1cm} (3.33)

where

\[ r = \hat{A}_{i2} y_1 + \hat{A}_{i3} y_2 + D_1 D^{-1}_3 \dot{y}_2 + \hat{B}_1 u \]

\[ z = \dot{y}_1 - D_2 D^{-1}_3 \dot{y}_2 - \hat{A}_{22} y_1 - \hat{A}_{23} y_2 - \hat{B}_2 u \]
Since (3.32) and (3.33) contain no unknown elements, a conventional Luenberger observer can be designed for this system. Assume the following dynamics for the observer

\[
\dot{x}_1 = (\hat{A}_{11} - M \hat{A}_{21}) \dot{x}_1 + r + M z
\]  

(3.34)

Substitute \( r \) and \( z \) into (3.34) to get

\[
\dot{x}_1 = (\hat{A}_{11} - M \hat{A}_{21}) \dot{x}_1 + (\hat{A}_{12} - M \hat{A}_{22}) y_1 + (\hat{A}_{13} - M \hat{A}_{23}) y_2
\]

\[+ (\hat{B}_1 - M \hat{B}_2) u + ((D_1 - M D_2) D_3^{-1} \dot{y}_2 + M \dot{y}_1)
\]  

(3.35)

However, equation (3.35) contains the derivatives of the outputs, which are not available. This problem can be circumvented by defining a new variable as

\[w = \dot{x}_1 - ((D_1 - M D_2) D_3^{-1} y_2 + M y_1)
\]  

(3.36)

then

\[
\dot{w} = (\hat{A}_{11} - M \hat{A}_{21}) (w + (D_1 - M D_2) D_3^{-1} y_2 + M y_1)
\]

\[+ (\hat{A}_{12} - M \hat{A}_{22}) y_1 + (\hat{A}_{13} - M \hat{A}_{23}) y_2 + (\hat{B}_1 - M \hat{B}_2) u
\]

or

\[
\dot{w} = (\hat{A}_{11} - M \hat{A}_{21}) w + G y_1 + H y_2 + L u
\]  

(3.37)

where

\[G = \hat{A}_{12} - M \hat{A}_{22} - \hat{A}_{11} M + M \hat{A}_{21}^M
\]  

(3.38)

\[H = \hat{A}_{13} - M \hat{A}_{23} + (\hat{A}_{11} - M \hat{A}_{21})(D_1 - M D_2) D_3^{-1}
\]  

(3.39)
From (3.37), it is easy to see that the necessary and sufficient condition for a stable observer to exist is given in the following theorem.

**Theorem 2.** The necessary and sufficient condition for existence of an observer capable of estimating the state of the dynamical system given in (3.1) and (3.2) is that the pair \((\hat{A}_{11}, \hat{A}_{21})\) given in (3.31) must be completely observable. Additionally, if the above condition is satisfied, then the eigenspectrum of the closed loop observer can be placed at arbitrary locations as long as complex conjugate eigenvalues appear in pairs.

The proof of the above theorem is well known and is given in Chen (1984). Combining \(y_1\) and \(y_2\) into \(y\) will change equation (3.37) into the following convenient form.

\[
\dot{w} = (\hat{A}_{11} - \hat{A}_{21}) w + [ G H ] y + Lu \tag{3.41}
\]

Given \(w\), the state estimate \(\dot{x}\) could be reconstructed as follows:

\[
\dot{x} = \begin{pmatrix} \dot{x}_1 \\ y \end{pmatrix}
\]

\[
= \begin{pmatrix} I & 0 \\ 0 & I \end{pmatrix} w + \begin{pmatrix} N \\ I \end{pmatrix} y \tag{3.42}
\]

where

\[
N = \begin{bmatrix} M & (D_1 - M D_2) D_3^{-1} \end{bmatrix}
\]

**Case 2) \( p = m \)**
Partition the system described in (3.1) and (3.2) into the following form:

\[
\begin{align*}
\dot{x}_1 &= A_{11} x_1 + A_{12} y + B_1 u + D_1 v \\
\dot{y} &= A_{21} x_1 + A_{22} y + B_2 u + D_2 v
\end{align*}
\]  

(3.43)  

(3.44)

Since \( p = m \), it follows that \( D_2 \) is a square matrix, and according to theorem 1, it is invertible.

**Theorem 3.** If \( p = m \), the eigenspectrum of a minimal order UIO could not be arbitrarily assigned. The necessary and sufficient condition for the existence of a stable minimal order UIO with fixed eigenspectrum is that the following matrix must be stable

\[
Q = (A_{11} - D_1 D_2^{-1} A_{21})
\]

**Proof:**

Dynamics of an estimator capable of estimating \( x_1 \) can be described by the following equations:

\[
\begin{align*}
\dot{w} &= G w + H u + L y \\
\dot{x}_1 &= M w + N y
\end{align*}
\]  

(3.45)  

(3.46)

If we compare the observer in (3.45), and (3.46) with that of (3.14), and (3.15), we notice that the only difference between them lies in their estimate equations (i.e. in (3.46) and (3.15)). One is estimating \( x \) and the other is estimating \( x_1 \). As will be seen shortly, the observer in (3.45), and (3.46) is much easier to design. To prove the theorem and provide a design procedure, define the estimation
error as

\[ q = \dot{x}_1 - \ddot{x}_1 \]  

Suppose a stable observer exists, then, there must exist a stable matrix \( Q \) such that

\[ \dot{q} = Qq \]  

Combining (3.43)-(3.48), will result in

\[
(A_{11} - NA_{21} - Q) x_1 + (QM - MG) w + (D_1 - ND_2) v + \\
(A_{12} - ML - NA_{22} + QN) y + (B_1 - MH - NB_2) u = 0
\]

From the above equation it follows that

\[
A_{11} - NA_{21} - Q = 0 \tag{3.49}
\]

\[
QM - MG = 0 \tag{3.50}
\]

\[
A_{12} - ML - NA_{22} + QN = 0 \tag{3.51}
\]

\[
B_1 - MH - NB_2 = 0 \tag{3.52}
\]

\[
D_1 - ND_2 = 0 \tag{3.53}
\]

From (3.53), we get

\[
N = D_1 D_2^{-1} \tag{3.54}
\]

Substituting (3.54) into (3.49), will result in

\[
Q = A_{11} - D_1 D_2^{-1} A_{21} \tag{3.55}
\]
From (3.55), it can be seen that the error dynamics are fixed by the system and cannot be altered by the designer. If $Q$ is not stable, the state of the dynamical system cannot be estimated, and thus no observer exists. On the other hand, if $Q$ turns out to be a stability matrix, then the state of the dynamical system in (3.1) and (3.2) can be estimated. This can be done as follows:

Taking

$$M = I$$

from (3.54-56), we get

$$G = Q$$

$$L = A_{12} - N A_{22} + Q N$$

$$H = B_1 - N B_2$$

Given the above, if the number of unknown inputs are equal to the number of outputs, the state of the dynamical system in (3.1), and (3.2) can be estimated using the estimator given in (3.45), and (3.46)

### 3.3) Summary

In this chapter we presented a computationally attractive approach to the design of reduced order unknown input observers. It was proved that under certain necessary condition it is possible to design such observers. It was also proved that the eigenvalues of the estimator can be arbitrarily assigned if only if the number of outputs is greater than the number of unknown inputs, and certain observability condition is satisfied. If the number of unknown inputs is equal to that of outputs, the eigenspectrum of the estimator can not be arbitrarily
assigned, however, an estimator with fixed eigenspectrum may exist.
The UIO design is simply not possible if the number of unknown inputs
is greater than that of outputs.

In the next chapter, this UIO design is generalized and is applied
to an uncertain system with both unknown inputs and unknown outputs.
Based on this generalized UIO design, a procedure for detecting and
isolating both actuator and sensor failures in uncertain dynamical
systems would be presented.
CHAPTER FOUR
FAILURE DETECTION AND IDENTIFICATION SCHEME

4.1) A New FDI Scheme

In Chapters One and Two we reviewed some of the more pertinent approaches to FDIA problem. Specifically, in Chapter Two the dedicated observer scheme (DOS) was illustrated in some detail and various shortcomings of the approach was pointed out. In this chapter we will present a new and more powerful methodology for FDI problem. The approach presented here as opposed to the DOS and its extensions is much more efficient in terms of its computational and decision making complexity, and is capable of detecting failures under more practical and general circumstances. That is, fault detection in sensors and actuators is possible in uncertain systems. At the heart of this FDI strategy is a single estimator that can simultaneously estimate the states of the system as well as detect, isolate, and identify the actuator and sensor failures. Since the proposed scheme can estimate the exact shape of the failure, theoretically, the false/missed alarm rate will drop to zero. In section 4.2, we will explain how a certain or uncertain system with both actuator and sensor failures can be augmented into a known system, which is of the form given in (3.1), and (3.2) (i.e., without unknown output disturbances or sensor failures). Hence, the U1O theories established in Chapter Three can be applied. Then in section 4.3, we will explain how faults can be detected based on the developments of section 4.2.
4.2) Building the System's Model

In the developments and the design of the UIO in Chapter Three the major assumption that was made throughout was that the dynamical system in (3.1-2) was known perfectly, and no parameter variations were to take place. In practice, however, there may be disturbances, sensor, or actuator failures affecting the plant's state and output trajectories. In addition, some of the plant parameters may be unknown or time variant. In this and the next section we will consider each of these effects one at a time. This way we will incorporate each of these effects and build up our system's model one step at a time until we arrive at a model of a practical dynamical system that accounts for all of these effects.

4.2.1) Effect of Instrument Failures

We start in this section by assuming that the plant parameters are exactly known. Here we will only account for the effect of sensor and actuator failures. A linear dynamical systems accounting for such failures can be described as follows:

\[ \dot{x} = Ax + Bu + Dv \]  \hspace{1cm} (4.1)

\[ y = Cx + Ef \]  \hspace{1cm} (4.2)

where it is assumed that the matrices C, D and E are of full rank. From (4.1-2) it can be seen that the system is not in the proper form given by (3.1-2), which is required if an UIO is to be employed. Fortunately, this problem can be remedied by showing that system (4.1-2) can be transformed into the form (3.1-2). To do this we will first give a
simple but useful fact.

**Proposition 1.** For any piece-wise differential vector \( f \in \mathbb{R}^r \) (with a finite number of discontinuities) and a \( r \times r \) matrix \( A_f \), there will always exists an input vector \( \xi \in \mathbb{R}^r \) such that

\[
\dot{f} = A_f f + \xi \tag{4.3}
\]

The proof is immediate by simply taking

\[
\xi = f - A_f f \quad \blacksquare
\]

It should be mentioned that since the calculations by computer are discrete, the number of discontinuities are always finite. Hence in practice discontinuities do not pose any problem.

With the above fact, it can easily be shown that (4.1-2) can be transformed or rewritten in the form of (3.1-2). To do this we will simply augment (4.1) by (4.3) to get

\[
\begin{pmatrix}
\dot{x} \\
\dot{f}
\end{pmatrix} =
\begin{pmatrix}
A & 0 \\
0 & A_f
\end{pmatrix}
\begin{pmatrix}
x \\
f
\end{pmatrix} +
\begin{pmatrix}
B \\
0
\end{pmatrix} u +
\begin{pmatrix}
D & 0 \\
0 & I
\end{pmatrix}
\begin{pmatrix}
u \\
\xi
\end{pmatrix} \tag{4.4}
\]

\[
y = [ C \quad E ] \begin{pmatrix} x \\ f \end{pmatrix} \tag{4.5}
\]

The essence of this theorem is that we model the disturbance vector (sensor failure) at the output as an output from a certain system with unknown inputs. Note that \( A_f \) could be arbitrarily chosen as long as it is stable. System (4.4-5) will be referred to as the **augmented system**.

From theorem 3.1 and the augmented system (4.4-5), it is not difficult to deduce the following lemma.
Lemma 1. A necessary condition for the existence of any order UIO for system (4.1-2) is as follows:

1. \( p \geq m + r \)
2. \( \text{Rank} \left( \begin{bmatrix} C & D & E \end{bmatrix} \right) = m + r \)

We now give further discussion about this Lemma. Recalling that the special structure of \( C \) allows us to write condition (2) as

\[
\begin{bmatrix} C & D & E \end{bmatrix} = \begin{bmatrix} 0 & I \end{bmatrix} \begin{bmatrix} D_1 & D_2 \end{bmatrix} E
\]

where \( D_1 \) is a \((n-p)\times m\) matrix and \( D_2 \) is \( pxm \). Therefore

\[
\begin{bmatrix} C & D & E \end{bmatrix} = \begin{bmatrix} D_2 & E \end{bmatrix}
\]

Hence, condition (2) is equivalent to

\[
\text{Rank} \left( \begin{bmatrix} D_2 & E \end{bmatrix} \right) = m + r \quad (4.6)
\]

Since \( D_2 \) is \( pxm \) and \( E \) \( pxr \), equation (4.6) holds only if \( D_2 \) and \( E \) have ranks \( m \) and \( r \) respectively. In other words, both \( D \) and \( E \) must have full ranks. Therefore, condition (1) requires that the number of unknown inputs plus the number of unknown outputs be less than or equal to the number of measurable outputs. Condition (2) further requires that the unknown input \( v \) must be fed back to the state derivative vector through a full rank matrix and that the unknown output \( f \) must be fed back to the output vector through a full rank matrix. In addition, these two matrices must be right prime (have independent columns).

Remark 1 - Suppose that a reduced order UIO for (4.1-2) exists, then the order of the UIO depends the difference between the number of total
outputs and the number of unknown outputs. It does not depend on the number of unknown inputs.

As an example, assuming that the number of state variables is \( n \), that of measurable outputs \( p \), and that of unknown outputs \( r \). Then the order of a reduced UIO is \( n-p+r \), which is obvious from (4.5).

4.2.2) Uncertainty Effects

The previous discussion was restricted to systems whose dynamics are known perfectly. This is because estimator design algorithms (e.g., observer or Kalman filter) require a complete knowledge of the system's dynamics as well as measurements of the system's input and output. However, often in practice the dynamics of control systems are not known with high degree of certainty. In this section, system uncertainties will be considered, and their effects will be incorporated into our model.

Consider the system described in (4.1-2). We assume that plant matrices \( A \) and \( B \) contain uncertain elements. In the following we will transform this uncertain system into a known system with unknown inputs. To do this, the following definition is introduced.

**Definition 1.** The uncertainty indicator matrix of any \( n \times m \) matrix \( A \) is defined as \( A_{i} = (a_{1}, a_{2}, \ldots, a_{k}) \), which is an \( n \times k \) matrix. Here \( k \) is the number of rows that contain unknown elements. The \( j \)th column of \( A_{i} \) has zero entries except for the \( a_{j} \)th entry which has a value of one.

As an example if \( A \) is a \( 4 \times 4 \) matrix and there are uncertain elements in the second and third rows (therefore \( k=2 \)), then \( a_{1}=2 \) and \( a_{2}=3 \). Note that from the definition we have

\[
0 < a_{1} < a_{2} \ldots < a_{k} < n
\]
Define the uncertain part of $A$ as

$$
\Delta A = \begin{bmatrix}
\Delta A_1 \\
\vdots \\
\Delta A_n
\end{bmatrix}
$$

(4.7)

where $\Delta A_i$ is the uncertainty vector in the $i$th row. Assume that only $\Delta A_{a_1}, \Delta A_{a_2}, \ldots, \Delta A_{a_k}$ have non-zero increments. Then (4.7) can be written as

$$
\Delta A = \Pi_A^{a_1, a_2, \ldots, a_k}
$$

(4.8)

To simplify the notation, we denote (4.8) as

$$
\Delta A = \Pi(A) \Delta A
$$

(4.9)

Similar definitions will apply to matrix $B$. Then, the system (4.1-2) with parameter variations can be presented by the following system model

$$
\dot{x} = A x + B u + D v + \Pi(A) \Delta A x + I(B) \Delta B u
$$

(4.10)

$$
y = C x + E f
$$

(4.11)

Equivalently

$$
\dot{x} = A x + B u + D^* v^*
$$

(4.12)

$$
y = C x + E f
$$

(4.13)

where
Recall that Lemma 1 requires that the matrix \( D^* \) be of full rank. This however may not be automatically satisfied. To illustrate this point and the previous discussion we will give the following example.

**EXAMPLE 4.1** - Consider the following uncertain system

\[
\dot{x} = A x + B u + D v
\]

Assume that the nominal matrices are known to be

\[
A_0 = \begin{bmatrix} 2 & 4 & 5 \\ 5 & 3 & 7 \\ 4 & 7 & 2 \end{bmatrix}, \quad B_0 = \begin{bmatrix} 1 \\ 6 \\ 2 \end{bmatrix}, \quad D_0 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}
\]

Assume that the actual systems matrices are given by

\[
A = \begin{bmatrix} 3 & 2 & 4 \\ 5 & 3 & 7 \\ 6 & 8 & 9 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 6 \\ 2 \end{bmatrix}
\]

Notice that the assumption here is that we know which elements of these matrices are uncertain. Therefore, the uncertain matrices can be written in terms of their uncertainty indicator matrix as
Hence we have
\[
\begin{pmatrix}
1 & -2 & -1 \\
0 & 0 & 0 \\
2 & 1 & 7 \\
\end{pmatrix}
\Delta A
= \begin{pmatrix}
1 & 0 \\
0 & 0 \\
0 & 1 \\
\end{pmatrix}
I(A)
\begin{pmatrix}
1 & -2 & -1 \\
2 & 1 & 7 \\
\end{pmatrix}
\Delta A
\]

\[
\begin{pmatrix}
1 & -3 \\
0 & 0 \\
0 & 0 \\
\end{pmatrix}
\Delta B
= \begin{pmatrix}
1 \\
0 \\
0 \\
\end{pmatrix}
I(B)
\begin{pmatrix}
1 & -3 \\
\end{pmatrix}
\Delta B
\]

Hence we have
\[a_1 - 1, a_2 - 3, \text{ and } b_1 = 1\]

According to (4.15) we have
\[
D^* = \begin{pmatrix}
1 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 \\
\end{pmatrix}
\Delta A \times
\begin{pmatrix}
v \\
\Delta A \times \\
\Delta B \times \\
\end{pmatrix}
\begin{pmatrix}
u \\
\Delta A \times \\
\Delta B \times \\
\end{pmatrix}
\]

Hence \(D^*\) is obviously not a full rank matrix as is required for existence of a UIO. To alleviate this problem, we introduce the following matrix decomposition proposition.

**Proposition 2: Matrix decomposition.** Any \(p \times q\) matrix \(A\), whose rank is \(r\), can be decomposed as follows

\[
A = B C
\]  \hspace{1cm} (4.16)

where \(B\) is a \(p \times r\) full rank matrix, and \(C\) is a \(r \times q\) full rank matrix.
Theorem: According to singular value decomposition theorem (Noble and Daniel (1977)), matrix A can decomposed as

\[ A = GDH \]  \hspace{1cm} (4.17)

where G and H are orthogonal \( p \times p \) and \( q \times q \) matrices and D is a non-negative definite matrix with the form as

\[ D = \begin{pmatrix} \Sigma & 0 \\ 0 & 0 \end{pmatrix} \]

Define \( r \times r \) matrix \( \sigma \) such that

\[ \sigma^2 = \Sigma \]

Then, matrix A be expressed as

\[ A = G \begin{pmatrix} \sigma \\ 0 \end{pmatrix} \begin{pmatrix} \sigma & 0 \end{pmatrix} H \]

Define

\[ B = G \begin{pmatrix} \sigma \\ 0 \end{pmatrix} \quad \text{and} \quad C = \begin{pmatrix} \sigma & 0 \end{pmatrix} H \]

Then, \( B \) is a \( p \times r \) full rank matrix and \( C \) is a \( r \times q \) full rank matrix.

Using the above result, matrices \( D^* \) can be decomposed as

\[ D^* = \tilde{D} \tilde{N} \]  \hspace{1cm} (4.18)

where \( \tilde{D} \) and \( \tilde{N} \) have full ranks.

Now going back to our example, we obtain...
\[
\begin{pmatrix}
1 & 1 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0
\end{pmatrix} -
\begin{pmatrix}
1 & 0 \\
0 & 0 \\
0 & 1
\end{pmatrix}
\begin{pmatrix}
1 & 1 & 0 & 1 & 1 \\
0 & 0 & 1 & 0 & 0
\end{pmatrix}
\]

\[
D^* \quad \tilde{D} \quad \tilde{N}
\]

Therefore in such cases where \( D^* \) is rank deficient (4.12-13) can be presented as

\[
\dot{x} = A x + B u + \tilde{D} (\tilde{N} v^*)
\]

(4.19)

\[
y = C x + E f
\]

(4.20)

where \((\tilde{N} v^*)\) can now be considered as the new unknown input.

Considering the developments of the previous section, we conclude that the UIO algorithms developed previously can now be applied. However, an important remark is in order.

REMARK 2 - In such situations where \( D^* \) is not of full rank, the above matrix decomposition can be used for designing an estimator. However, actuator fault detection using the simple approach presented next would not be possible. This point is further discussed in section 4.3.2.

4.3) FDI Scheme

In most of the FDIA techniques, the various tasks involved are performed in different stages. That is, generally some sort of a residual or innovation process is monitored for detecting presence of a fault. Once it is determined that a fault exist another decision making process is triggered to identify the faulty component(s). Often this is the most complicated task to be performed since various components have to be checked for their health. After the faulty component is
identified, the isolation and accommodation has to take place in that order. In our approach, we detect, identify, and additionally estimate the shape of the failure all at once. This obviously results in a great simplification of the FDI process and makes our approach more appealing for real time applications, where faults are to be detected, identified, and accommodated as quickly as possible.

The algorithm of Phatak, and Viswanadham (1988), and Viswanadham, and Srichander (1987) uses a bank of estimators for detecting actuator failures. If sensor failure detection is desired as well they suggest using another bank of estimators, in addition to the previous bank of estimators that was used for actuator failure detection. This approach would therefore require a large number of estimators for MIMO systems, and would result in a great deal of complexity. In addition, the approach is still not able to identify the shape of the failures. Park and Stein (1988) gave an algorithm based on a single estimator that can simultaneously estimate the state of the system and also can be used for actuator failure detection. Their proposed estimator uses the derivatives of the measured signals, and from the practical point of view is not very desirable. To the best of our knowledge there has not been an attempt for designing a FDI system for uncertain system capable of detecting multiple and simultaneous sensor and actuator failures using a single estimator.

In this section we will present such a FDI methodology. To carry out the development of this section, consider the following representation of the uncertain system in (4.19-20), resulting from augmentation of (4.3) with (4.19)
Note that our model building task is now completed. The system model (4.21-22) includes sensor, actuator, and parameter uncertainty effects. This is the desired model for the remainder of our study.

In the next section, we will first describe how the sensor failures are detected and their shape is identified. The approach for actuator failure detection and identification is described afterward.

4.3.1) FDI for Sensors

It almost takes no effort in identifying sensor failures. Assume that the UIO for the system (4.21-22) exist, and the estimate of the state of this dynamical system is obtained. Therefore, monitoring the state estimates (i.e. \( \hat{x} \) and \( \hat{\theta} \)) would provide an immediate means for the detection of sensor failures. That is, \( \hat{\theta} \) would provide us with the estimate of every component of the failure vector \( \theta \). The logic is very simple, under no sensor failures all the components of \( \theta \) should be zero or close to it. A nonzero component of the \( \hat{\theta} \) would be indicative of a presence of the corresponding sensor fault. The value of the nonzero component is the estimate of the fault magnitude. Similarly, presence of two nonzero entries in \( \hat{\theta} \) would indicate two sensor faults and so on.

4.3.2) FDI for Actuators

The actuator failure identification takes some additional efforts.
We first introduce the following theorem.

Consider the following standard time invariant system

\[ \dot{x} = A x + B u \quad (4.23) \]

For the above system we can prove the following theorem.

**Theorem 1** Denote the value of \( x \) and \( u \) at time \( kT \) by \( x(k) \) and \( u(k) \), where \( T \) is the sampling interval. If \( T \) is small enough, then, for system (4.23), given \( x(k) \), the input \( u(k) \) can be calculated as follows

\[ u(k) = (B^T B)^{-1} B^T S(k) \quad (4.24) \]

where

\[ S(k) = A \left( e^{A T} - I \right)^{-1} (x(k+1) - e^{A T} x(k)) \]

**Proof:**

The value of \( x \) at time \( (k+1) \) is given by

\[ x(k+1) = e^{A[(k+1)T]}(e^{-A(kT)}x(k)) + \int_{kT}^{(k+1)T} (e^{-A\tau} B u(\tau))d\tau \]

\[ = e^{A[(k+1)T]}(e^{-A(kT)}x(k) + (e^{-A(kT)} - e^{-A[(k+1)T]}) A^{-1} B u(k)) \]

\[ = e^{AT} x(k) + (e^{AT} - I) A^{-1} B u(k) \]

Therefore

\[ x(k+1) - e^{AT} x(k) = (e^{AT} - I) A^{-1} B u(k) \]

Generally \( (e^{AT} - I) \) is invertible. Hence we have
\[
A \left( e^{AT} - I \right)^{-1} \left( x(k+1) - e^{AT} x(k) \right) = B u(k)
\]

Letting

\[
S(k) = A \left( e^{AT} - I \right)^{-1} \left( x(k+1) - e^{AT} x(k) \right)
\]

we have

\[
s(k) = B u(k) \tag{4.25}
\]

Without loss of generality, assume that \( B \) has full rank. Then the pseudo inverse of \( B \), \( (B^T B)^{-1} \), will exist. Taking pseudo inverse of \( B \) on both sides of (4.25), we obtain the desired result.

From the above theorem it is now obvious that using the estimate \( \hat{x} \) for \( x \) (assuming that no failures occur during the observer transient period), and the knowledge of the control input \( u \), it is possible to compute for \( \hat{N} \hat{v}^* \). From the (4.21) we have

\[
\dot{x} = A x + B u + \hat{D} \left( \hat{N} \hat{v}^* \right)
\]

Therefore using (4.24-25) we have

\[
s(k) = B u(k) + \hat{D} \left( \hat{N} \hat{v}^* \right)
\]

Hence

\[
s(k) - B u(k) = \hat{D} \left( \hat{N} \hat{v}^* \right)
\]

Finally since \( \hat{D} \) has full rank we have

\[
\hat{N} \hat{v}^* = (\hat{D}^T \hat{D})^{-1} \hat{D}^T \left( s(k) - B u(k) \right) \tag{4.26}
\]

Hence, we obtain the estimate of \( \hat{N} \hat{v}^* \), denoted by \( \hat{N} \hat{v}^* \). The next
problem is that whether or not we can get an estimate of $v^*$ from $(\tilde{N} \hat{v}^*)$. If we can, then from (4.15) we can get $\hat{v}$. The following theorem gives a necessary and sufficient condition to this question.

**Theorem 2** The necessary and sufficient condition for the unknown variables $v^*$ to be identifiable is that matrix $D^*$ in (4.14) to have full rank.

**Proof:** Since $\tilde{N} \hat{v}^*$ is available, define

$$v = \tilde{N} \hat{v}^*$$

(4.27)

Obviously, we can obtain $\hat{v}^*$ if and only if the left pseudo inverse of $\tilde{N}$ exist. However, it is well known that the left pseudo inverse of a matrix exists only when the matrix has full column rank. Rewriting (4.18) as

$$D^* = \tilde{D} \tilde{N}$$

Suppose $D^*$ has dimension $n \times m$ with rank $r$ ($r \leq m, n$). Then $\tilde{N}$ will have dimension $r \times m$. $\tilde{N}$ has full column rank if and only if $r \leq m$. Therefore we have

$$r = m$$

Hence $D^*$ has full rank.

Suppose that $D^*$ has full rank. Then we do not need to decompose $D^*$ any more. Hence

$$\dot{x} = A \dot{x} + Bu + D^* v$$

Similar to the previous deduction we have
\[
\hat{v}^* = (\hat{\mathbf{D}}^T \hat{\mathbf{D}}^*)^{-1} \hat{\mathbf{D}}^T (s(k) - B u(k))
\] (4.26)

Similar to the approach for sensor failure detection and identification, the actuator failures can now be detected and identified. This is simply accomplished by inspecting the components of the vector \( \hat{v} \), which itself is one vector component of \( \hat{v}^* \) as is given in (4.15). Once again a nonzero component of the \( \hat{v} \) vector would be an indicative for the corresponding actuator failure. Multiple and simultaneous actuator failures are also easily detectable, as in the sensor failure case.

It should be reiterated again that sensor failure detection and identification is always possible by the above methodology. However, detection and identification of actuator failures does depend on the invertability of \( \hat{N} \). If this condition is satisfied then both actuator and sensor failures can be detected and identified.

4.4) Summary

In this chapter, we presented a FDI scheme in which only one unknown input observer is needed for both state estimation and failure detection purposes. Simultaneous actuator and sensor failures as well as system uncertainties are allowed in this scheme. The simultaneous failures can not only be detected and isolated, but also be their shape and magnitude can be identified. It was shown that if the conditions for existence of the UIO are satisfied, then the sensor failures can be detected and identified. Additionally, under an additional condition, actuator failures can also be detected and identified by using a simple detection logic.
The FDI approach presented will be applied to the model of a physical system in the next chapter in order to verify the applicability and capabilities of the proposed scheme.
In this chapter the theoretical results of the previous chapters are used in a numerical problem in order to show the applicability of the proposed FDI scheme. The system considered is the VTOL helicopter for which a 4th order dynamical model was obtained and verified by Narendra and Tripathi (1973).

The dynamics of a helicopter in the vertical plane is described as follows (Narendra and Tripathi 1973):

\[
\begin{align*}
\dot{x} &= \begin{pmatrix}
-.0366 & .0271 & .0188 & -.4555 \\
.0482 & -1.0100 & .0024 & -.0208 \\
.1002 & .3681 & -.7070 & 1.4200 \\
.0000 & .0000 & 1.0000 & .0000
\end{pmatrix} x + \begin{pmatrix}
.4422 & .17 & 61 \\
3.5446 & -7.59 & 22 \\
-5.5200 & 4.49 & 00 \\
.0000 & .00 & 00
\end{pmatrix} u \\
&= A_1 x + B_1 u
\end{align*}
\]

(5.1)

where

\(x_1\) --horizontal velocity;

\(x_2\) --vertical velocity;

\(x_3\) --pitch rate;

\(x_4\) --pitch angle;

\(u_1\) --collective (here taken as unit step);

\(u_2\) --longitudinal cyclic (unit step).

The above parameters are nominal values obtained at the air speed of 135 knots. However, as the air speed and loading changes, all the elements of the first three rows of both matrices also change. The most significant changes take place in the elements \(a_{32}\), \(a_{34}\) and \(b_{21}\).
Therefore, in the following discussion all other elements are assumed to be constant. The parameter variations (uncertainties) are assumed to be as follows:

\[
\Delta a_{32} = .5, \quad \Delta a_{34} = 2, \quad \Delta b_{21} = 2
\]

As can be seen from the above, compared to the nominal values of these parameters, the uncertainties are relatively large. Since the eigenvalues of the nominal matrix \( A \) are \(-2.0727, -.2325, .2758+.2576j\) and \(.2758-.2576j\), the system is unstable and needs to be stabilized by feedback action. We place the closed loop poles of the stabilized system at \(-4, -2, -2.4\) and \(-3\) by using a state feedback control law. The controller gain that would achieve the desired closed loop pole locations was computed to be

\[
k = \begin{pmatrix}
-16.5042 & -1.3111 & .5789 & 9.5658 \\
-14.6120 & -1.1087 & -.0657 & 7.1505
\end{pmatrix}
\]

For the sake of simplicity, we use the actual states for state feedback control. Thus, the stabilized system would become

\[
x = \begin{pmatrix}
-9.0477 & -.7476 & .2632 & 5.0337 \\
.0000 & .0000 & 1.0000 & .0000
\end{pmatrix}
x + \begin{pmatrix}
.4422 & .1 & 761 \\
3.5448 & -7.5 & 022
\end{pmatrix} u
\]

\[
\begin{pmatrix}
-20.8922 & -.6381 & -4.1075 & -1.2774
\end{pmatrix}
\]

(5.2)

It is assumed that the output equation is given by

\[
y = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 1 & 1 & 1
\end{pmatrix} x
\]

(5.3)
To demonstrate the feasibility of our scheme, we will consider four separate cases:

1) Both matrices A and B contain unknown elements. Only sensor three is faulty. This is to demonstrate that a sensor failure could be detected and identified in an uncertain system.

2) Only matrix A contains unknown elements. Actuator two and sensor three are faulty. This is to demonstrate that this scheme can distinguish actuator failures from sensor failures and detect and identify them simultaneously.

3) Aircraft is flying at the nominal flight conditions. Therefore, there are no parameter uncertainties. Actuator two and sensors three and four are faulty. This is to demonstrate the proposed approach is capable of detecting simultaneous sensor failures in addition to actuator failures.

4) The aircraft is flying under nominal flight conditions. Additionally, there are are no actuator failures. But all the sensors have failed. This is to demonstrate a unique capability of this scheme that the majority vote based do not possess.

CASE 1) \( \Delta a_{32} = .5, \Delta a_{34} = 2, \Delta b_{21} = 2 \) and \( f = .2, \) for \( t \geq 2 \) (sec) (soft bias failure).

Assuming sensor failure distribution matrix \( E \) is given by

\[
E = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}^T
\]

The closed loop system under state feedback control is given by
Using (4.3-5) and choosing \( A_f = -4 \), the augmented system of (5.4-5) would be given as follows:

\[
\begin{align*}
\dot{x} &= \begin{pmatrix}
-9.4477 & -7.7476 & 2.632 & 5.0337 \\
52.1659 & 2.7452 & 5.5332 & -24.4221 \\
0.0000 & 0.0000 & 1.0000 & 0.0000
\end{pmatrix} x \\
&\quad + \begin{pmatrix}
0.4422 & 0.1761 \\
3.5446 & -7.5922 \\
-5.5200 & 4.4900 \\
0.0000 & 0.0000
\end{pmatrix} u + \begin{pmatrix}
0 \\
0 \\
1 \\
0
\end{pmatrix} \nu^* \\
\end{align*}
\]

\[
(5.4)
\]

\[
y = \begin{pmatrix} 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 1 & 1 & 1 \end{pmatrix} x + \begin{pmatrix} 0 \\
0 \\
0 \\
1 \end{pmatrix} f
\]

\[
(5.5)
\]

where

\[
\nu^* = \begin{pmatrix}
[0.5 & 0.2] x \\
[2 & 0] u
\end{pmatrix}
\]
Note that the output equation is not in the desired form \[ \begin{bmatrix} 0 & 1 \end{bmatrix} \]. In order to achieve this form, consider the a similarity transformation on the system (5.6-7), where the transform operator is given by

\[
T = \begin{bmatrix}
0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 \\
0 & 1 & 1 & 1 & 0
\end{bmatrix}
\]

Since the number of actual outputs of the system is four and the augmented system is of fifth order, the minimal order observer required to estimate the inaccessible state variable of the system is of dimension one. Placing the pole of the minimal order observer at \( p = -8 \) and applying the algorithms in Chapter 3, we construct the observer as

\[
\dot{w} = -8w + [2.858 \ 0.4832 \ 0 \ 0.6138]y + [-0.6489 \ -0.2584]u
\]

\[
z = \begin{bmatrix}
1.4673 \\
0 \\
0 \\
0 \\
0
\end{bmatrix}w + \begin{bmatrix}
1.4673 & 1 & 1 & -1 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}y
\]

where

\[
z = T \begin{bmatrix}
\dot{x} \\
\dot{\xi}
\end{bmatrix}
\]

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Thus we are able to obtain the estimate of $x$ and $f$ simultaneously. The results are shown in Figures 5.1-5. Figures 5.1-4 display the actual state trajectory of the system versus the estimates (dotted lines). It can be seen from these figures that the observer's estimates converge very fast. Figure 5.5 illustrates that the failure is detected immediately and is identified accurately.

CASE 2) \[ \Delta a_{32} = .5, \Delta a_{34} = 2, \Delta b_{21} = 0, \]

\[ f = .2, \text{ for } t \geq 2 \text{ (sec), and } \]

\[ v = .5 \text{ for } t \geq 5 \text{ (sec)} \]

Assume that the first actuator and the third sensor are faulty. Then

\[
D = \begin{bmatrix} 0.4422 & 3.5446 & -5.5200 & 0 \end{bmatrix}^T 
\quad E = \begin{bmatrix} 0 & 0 & 1 & 0 \end{bmatrix}^T 
\]

Using (4.12-15), we model the system as

\[
\dot{x} = \begin{bmatrix} -9.9477 & -7.7476 & 0.2632 & 5.0337 \\
52.1659 & 2.7452 & 5.5532 & -24.4221 \\
0.0000 & 0.0000 & 1.0000 & 0.0000 
\end{bmatrix} x 
+ \begin{bmatrix} 0.4422 & 0.1761 \\
3.5446 & -7.5922 \\
-5.5200 & 4.4900 \\
0.0000 & 0.0000 
\end{bmatrix} u 
+ \begin{bmatrix} 0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 
\end{bmatrix} v^* 
+ \begin{bmatrix} 0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 
\end{bmatrix} f 
\]

\[
y = \begin{bmatrix} 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 1 & 1 & 1 
\end{bmatrix} x 
+ \begin{bmatrix} 0 \\
0 \\
0 \\
0 
\end{bmatrix} f 
\]

where

\[
v^* = \begin{bmatrix} v \\
\begin{bmatrix} 0 & .5 & 0 & 2 \end{bmatrix} x 
\end{bmatrix} 
\]
Figure 5.1 - Actual and Estimated Value of Horizontal Velocity

Figure 5.2 - Actual and Estimated Value of the Vertical Velocity
Figure 5.3 - Actual and Estimated Pitch Rate

Figure 5.4 - Actual and Estimated Pitch Angle
Figure 5.5 - Detection of a Soft Failure in Sensor 3 (Case I)
Using (4.3-5) and choosing $A_f = -4$, the augmented system of (5.8-9) is obtained as follows

$$
\begin{pmatrix}
\dot{x} \\
\dot{f}
\end{pmatrix} =
\begin{pmatrix}
-9.9477 & -0.7476 & 0.2632 & 5.0337 & 0.0000 \\
5.3145 & 2.7452 & 5.5332 & -0.5200 & 0.0000 \\
26.0922 & 2.6361 & -4.1975 & -0.4197 & 0.0000 \\
0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 & -4.0000 \\
\end{pmatrix}
\begin{pmatrix}
x \\
f
\end{pmatrix} +
\begin{pmatrix}
-0.4422 & 0.1761 \\
3.5446 & -7.5922 \\
-0.5200 & 4.4999 \\
0.0000 & 0.0000 \\
0.0000 & 0.0000 \\
\end{pmatrix}
\begin{pmatrix}
u^*
\end{pmatrix}
$$

(5.10)

$$
y =
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 \\
0 & 1 & 1 & 1 & 0
\end{pmatrix}
\begin{pmatrix}
x \\
f
\end{pmatrix}
$$

(5.11)

Once again since the output equation (5.11) is not in the desired form, the system (5.10-11) is transformed using the following transformation matrix

$$
T =
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 1 & 1 & 1 & 0
\end{pmatrix}
$$

Similar to CASE 1, it is concluded that the minimal order observer has dimension one. Placing the pole of the minimal order observer at -10 and applying the algorithms in Chapter 3, we construct the observer as

$$
\dot{w} = -10w + [7.1435 \ 1.5255 \ 0 \ 1.0612]y + [0 \ -1.2430]u
$$
The simulation results are shown in Figures 5.6-11. Once again the state estimates are shown in Figures 5.6-9, and Figures 5.10-11 illustrate that the failures are detected and identified immediately.

Notice that when the sensor faults occur, there is little effect in the future state trajectories (see Figures 5.1-4). However, when the actuator fault occurs, it does produce an obvious impact on the state trajectories, causing instantaneous deviations from the normal values (see Figures 5.6-9). It may therefore be concluded that actuator failures would have a more adverse effect on the behavior of this system.

CASE 3) \[ \Delta a_{32} = 0, \Delta a_{34} = 0, \Delta b_{21} = 0, \]

\[ f = [.2 \ 0.4]^T, \text{ for } t \geq 5 \text{ (sec)}, \text{ and} \]

\[ v = .5 \text{ for } t \geq 5 \text{ (sec)} \]

\[ D = [0.4422 \ 3.5446 \ -5.5200 \ 0]^T \]

\[ E = \begin{pmatrix} 0 & 0 & 0 & 1 \end{pmatrix}^T \]

To see whether multiple faults can be detected and identified, in this case, the actuator fault and simultaneous sensor faults occur all at the same time.

Since the VTOL is flying under nominal flight conditions, there are no system uncertainties and we have
Figure 5.6 - Actual and Estimated Horizontal Velocity

Figure 5.7 - Actual and Estimated Vertical Velocity
Figure 5.8 - Actual and Estimated Pitch Rate

Figure 5.9 - Actual and Estimated Pitch Angle
Figure 5.10 - Detection of a Soft Failure in Sensor 3 (Case II)

Figure 5.11 - Detection of a Soft Failure in Actuator 1 (Case II)
Choosing the augmented system for (5.12-13) is obtained as follows

$$\begin{pmatrix}
-0.9447 & -0.7478 & 0.2632 & 5.0337 \\
52.1659 & 2.7452 & 5.5532 & -24.4221 \\
0.0000 & 0.0000 & 1.0000 & 0.0000
\end{pmatrix} x + \begin{pmatrix}
-0.4422 & -0.3546 \\
-7.5922 & 4.4900 \\
-5.5200 & -5.5200 \\
0.0000 & 0.0000
\end{pmatrix} u + \begin{pmatrix}
-0.4422 \\
3.5446 \\
-5.5200 \\
0.0000
\end{pmatrix} v \quad (5.12)$$

$$y = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 1 & 1 & 1 & 0
\end{pmatrix} x + \begin{pmatrix}
0 & 0 & 0 \\
0 & 0 & 1 \\
0 & 0 & 1
\end{pmatrix} f \quad (5.13)$$

Choosing

$$A_r = \begin{pmatrix}
-4 & 0 \\
0 & -5
\end{pmatrix}$$

the augmented system for (5.12-13) is obtained as follows

$$\begin{pmatrix}
\dot{x} \\
\dot{f}
\end{pmatrix} = \begin{pmatrix}
-0.9447 & -0.7478 & 0.2632 & 5.0337 & 0.0000 & 0.0000 \\
52.1659 & 2.7452 & 5.5532 & -24.4221 & 0.0000 & 0.0000 \\
26.0922 & 2.6361 & -4.1975 & -19.2774 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 1.0000 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\
0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & -5.0000
\end{pmatrix} \begin{pmatrix}
x \\
f
\end{pmatrix}$$

$$+ \begin{pmatrix}
-0.4422 & -0.3546 \\
-7.5922 & 4.4900 \\
-5.5200 & -5.5200 \\
0.0000 & 0.0000 \\
0.0000 & 0.0000 \\
0.0000 & 0.0000
\end{pmatrix} u + \begin{pmatrix}
-0.4422 & 0 \\
3.5446 & 0 \\
-5.5200 & 0 \\
0 & 0 \\
0 & 1 \\
0 & 0
\end{pmatrix} v \quad (5.14)$$

$$y = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 1 & 1 & 0 & 1
\end{pmatrix} \begin{pmatrix}
x \\
f
\end{pmatrix} \quad (5.15)$$
The following transformation matrix is used to bring the system output equation into the desired form

\[ T = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 \end{pmatrix} \]

The necessary UIO in this case is of second order. Placing the closed loop poles of the minimal order observer at -12 and -13, and applying the algorithms of Chapter 3, we construct the observer as

\[ \dot{w} = \begin{pmatrix} 44.6785 \\ 60.4735 \end{pmatrix} w + \begin{pmatrix} 44.9833 \\ 59.6784 \end{pmatrix} u + \begin{pmatrix} 0 \\ 0 \end{pmatrix} \begin{pmatrix} 5.6237 \\ 7.6670 \end{pmatrix} u + \begin{pmatrix} -0.9198 \\ -2.4483 \end{pmatrix} y \]

\[ z = \begin{pmatrix} 1.5220 \\ -0.2971 \end{pmatrix} w + \begin{pmatrix} 1.5200 \\ 0.2971 \end{pmatrix} y \]

The results of the simulation are shown in Figures 5.12-18. It can be seen once again that the failure detection and identification tasks have been accomplished in this case.

CASE 4) \( \Delta a_{32} = 0, \Delta a_{34} = 0, \Delta b_{21} = 0, \)

\[ f = [0.2 \ 0.4 \ \sin(t-2) \ \cos(t-2)]^T, \text{ for } t \geq 2 \text{ (sec)} \]
Figure 5.12 - Actual and Estimated Horizontal Velocity

Figure 5.13 - Actual and Estimated Vertical Velocity
Figure 5.14 - Actual and Estimated Pitch Rate

Figure 5.15 - Actual and Estimated Pitch Angle
Figure 5.16 - Detection of a Soft Failure in Sensor 3 (Case III)

Figure 5.17 - Detection of a Soft Failure in Sensor 4 (Case III)
Figure 5.18 - Detection of a Soft Failure in Actuator 1 (Case III)
The system that has neither uncertainties nor actuator failure could be described as

$$E = I_{4 \times 4}$$

Choosing the augmented system for (5.17-18) is obtained as

$$\dot{x} = \begin{pmatrix} -9.9477 & -2.7476 & 2.6321 & 5.0337 \\ 52.1859 & 2.7452 & 5.5532 & -24.4221 \\ 26.0922 & 2.6361 & -4.1975 & -19.2774 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 \end{pmatrix} x + \begin{pmatrix} .4422 \\ 3.5446 \\ 2.5200 \\ 0.0000 \end{pmatrix} u + \begin{pmatrix} .1861 \\ -7.5922 \\ 4.4900 \\ 0.0000 \end{pmatrix} \xi$$

(5.17)

$$y = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix} x + \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} f$$

(5.18)

Choosing

$$A_r = \begin{pmatrix} -4 & 0 & 0 & 0 \\ 0 & -5 & 0 & 0 \\ 0 & 0 & -6 & 0 \\ 0 & 0 & 0 & -7 \end{pmatrix}$$

the augmented system for (5.17-18) is obtained as

$$\begin{pmatrix} \dot{x} \\ f \end{pmatrix} = \begin{pmatrix} -9.9477 & -2.7476 & 2.6321 & 5.0337 \\ 52.1859 & 2.7452 & 5.5532 & -24.4221 \\ 26.0922 & 2.6361 & -4.1975 & -19.2774 \\ 0.0000 & 0.0000 & 1.0000 & 0.0000 \end{pmatrix} \begin{pmatrix} x \\ f \end{pmatrix} + \begin{pmatrix} 0_{4 \times 4} \\ 0_{4 \times 4} \end{pmatrix} (5.19)$$
Use the following transformation matrix to get the output equation (5.20) in the desired form:

$$y = \begin{pmatrix} 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ f \end{pmatrix} \quad (5.20)$$

In (5.19) since the number of unknown input equals to the number of outputs. We cannot arbitrarily assign the eigenspectrum of reduce UIO. Applying the algorithms in Chapter 3 (when $p = m$), we construct the observer with fixed eigenspectrum as:

$$T = \begin{pmatrix} \mathbb{I}_{4 \times 4} & \mathbb{O}_{4 \times 4} \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{pmatrix}$$

The eigenspectrum of this observer are [-4, -2, -2.4, -3] and hence the observer is stable. The results of the simulation study are shown in
Figures 5.19-26. It is clear that the FDI scheme can detect all failures and identify them accurately.

The following remark gives some additional insight into the dynamics of the estimator designed in this last case study.

REMARK 1 - It should be noted that the eigenvalues of the resulting closed loop estimator in Case 4, are the same as the stabilized system. This is not coincidental. To clarify this consider the following argument:

A completely known system with all faulty sensors can be described as

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx + I \xi
\end{align*}
\]  

The augmented state equation for (5.21-22) is

\[
\begin{pmatrix}
\dot{x} \\
\dot{f}
\end{pmatrix} = \begin{pmatrix} A & 0 \\ 0 & A_r \end{pmatrix} \begin{pmatrix} x \\ f \end{pmatrix} + \begin{pmatrix} B \\ 0 \end{pmatrix} u + \begin{pmatrix} 0 \\ I \end{pmatrix} \xi
\]

According the system partition in (3.43-44), we have

\[
D_1 = 0 \quad \text{and} \quad D_2 = I
\]

The \( A_{11} \) and \( A_{21} \) defined in (3.43-44) would now be \( A \) and \( A_r \) respectively. Hence the matrix \( Q \) defined in (3.55) would be given by

\[
Q = A - D_1 D_2^{-1} A = A
\]

Therefore, the following useful conclusion can be reached for this particular system. That is, a stable observer for system (5.14) exists if and only if the system itself is stable. It should be noted that then that the aircraft considered in this example is open loop.
Figure 5.21 - Actual and Estimated Pitch Rate

Figure 5.22 - Actual and Estimated Pitch Angle
Figure 5.23 - Detection of a Soft Failure in Sensor 1 (Case IV)

Figure 5.24 - Detection of a Soft Failure in Sensor 2 (Case IV)
Figure 5.25 - Detection of a Soft Failure in Sensor 3 (Case IV)

Figure 5.26 - Detection of a Soft Failure in Sensor 4 (Case IV)
unstable, and an UIO for it can not be designed. However, since in this case the basic assumption is that the system contains no parameter variations, it is possible to stabilize the system using state feedback control law before designing the UIO.
CHAPTER SIX

CONCLUSIONS AND RECOMMENDATIONS

Two contributions were made in this research study:

1) A simple but elegant approach to the design of unknown input observers was presented. Algorithms, as well as necessary and sufficient conditions for the existence of such observers was stated. Essentially, it was proved that an unknown input observer will not exist if the number of unknown inputs is greater than the system outputs. It was shown, however, that if the number of unknown inputs is equal to that of outputs, then a stable estimator with fixed eigenspectrum may exist. Algorithm for designing such an estimator was outlined. Finally, it was proved that if the number of unknown inputs is less than that of outputs, an estimator can be designed under certain observability conditions. In addition, in this case the designer would have freedom to assign the eigenspectrum of the estimator at arbitrary locations. Furthermore, the design methodology was generalized to make it applicable to uncertain systems with both unknown plant and output disturbances.

The main advantage of the UIO design methodology presented here over the previous proposed approaches is that the proposed UIO design algorithm is very straightforward, and is computationally very attractive. Unlike many of the previous approaches that require solving simultaneous matrix equations, the proposed design for the most parts requires simple matrix addition and
multiplication.

2) A new fault detection and identification (FDI) scheme based on the above UIO theories developed in this work was presented. In order to accomplish the FDI, a suitable model which took into account the system's uncertainties, parameter variations, actuator and sensor failure effects was developed for the dynamical systems. The model building was carried out in stages, and at each stage different effects were incorporated into the system model. During this process a novel method for transforming an uncertain system into a known system with unknown inputs and unknown outputs was presented. This enabled us to use techniques suited to known systems for analyzing and designing controller and estimator for the uncertain system.

To illustrate the effectiveness of the approach, simulation tests were run on the fourth order dynamical model of VTOL Helicopter. The results of the simulation study revealed that the approach proposed is effective in detecting instrument failures in a variety of combinations, and under plant parameter uncertainties.

With the support of our simulations, we believe that the FDI scheme presented in this work has the following advantages over some of the other existing approaches:

a) The approach can not only detect and isolate simultaneous instrument failures, but can also actually identify the exact shape of the failures.
b) A large number of fault detection approaches require a bank of filters for detecting sensor, and another bank of filters for detecting actuators. The FDI scheme reported here uses only one estimator that reconstructs the state of the system for the purpose of control, as well as detecting and identifying failures.

c) The FDI system can detect both sensor as well as the actuator failures. In addition, simultaneous failures can easily be detected.

d) Since the algorithm is capable of identifying the exact shape of the failures, false alarms would not be registered for momentary faults that clear themselves. Hence, the false alarm rate will drop down considerably, and more meaningful decisions can be made.

e) Since the fault detection logic is not based on majority vote ruling, even if all the sensors have failed this scheme will still work. In addition, all the sensor failures can be simultaneously identified. This is a property that the approaches based on majority vote ruling such as DOS do not possess.

f) Estimation technique was used to identify instrument failures directly. This resulted in the detection logic simplification. This simplification makes the FDI approach attractive for real time applications where these tasks need to be performed very fast.
The strengths of the FDI approach were listed in the above. On the other side of the coin, there are some issues that this simple approach can not handle at this time, and they are subjects of future investigations. These are:

a) It was pointed out and was proved in Chapter Four that if the matrix $D^*$ is not of full rank, then actuator failure detection would not be possible. There is a need to investigate what can be done in such situations to detect actuator failures.

b) Assume that the state $x \in \mathbb{R}^n$, the unknown input $v \in \mathbb{R}^m$, the unmeasurable output $f \in \mathbb{R}^r$, and the measurable output $y \in \mathbb{R}^p$. Then according to the developments of Chapters Three and Four, we know that at least $p-r-m$ rows in $A$ and $B$ matrices are allowed to contain uncertain elements. However, this is not necessary and is only a sufficient condition. At present, we do not know the upper bound on this.

c) For certain systems with large amount of structural uncertainties and not enough available output measurements, it would be impossible to satisfy the existence condition for the UIO. In such situations the present approach will fail altogether. If an UIO does not exist, it is neither possible to control, nor to detect and identify the instrument failures. This is perhaps the most limiting and challenging obstacle to overcome at present. We believe that in such cases it may still be possible to accomplish the FDI tasks. However, most probably there would be a need for
more than one estimator performing FDI tasks as was the case in this study. The problem is therefore to determine how this can be done, and what is the minimum number of estimators needed for getting the job done.

d) Failure accommodation was not dealt with in this work. For a truly fault tolerant control system, it is necessary to have the controller reconfiguration capability after the faults are detected. This is an important issue that needs to be investigated.

e) Variety of systems in practice exhibit nonlinear behavior, therefore, extensions and applications to nonlinear, and bilinear systems is natural.
REFERENCES


[20] Saif, M., and Guan, Y., (1990), "Decentralized State Estimation in Large-Scale Interconnected Dynamical Systems", revised and resubmitted to *Automatica*.


