LOW FREQUENCY NOISE STUDIES IN MOSFETS:
THEORY AND EXPERIMENTS

by

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ABSTRACT

Low frequency noise in MOSFETs continues to be a very active area of research, especially as devices are shrunk to sub-micron dimensions. Low frequency noise is also increasingly important in both digital and analog integrated circuits because of the need to improve sensitivity and minimize noise. Therefore, theoretical and experimental research was conducted to examine the detailed low frequency noise characteristics in MOSFETs in my thesis work.

In the thesis project, we propose a new unified simulation-oriented flicker noise model for MOSFETs that is valid from the linear to the saturation region. The model is physically-based, and it incorporates both number fluctuation and mobility fluctuation theories which are widely individually used to explain low frequency noise in MOSFETs. We have done measurements, modelling and theoretical analysis using various n-channel MOSFETs operating from the linear to the saturation modes, and at varying temperatures. The experiments at high temperatures (from 298K to 423K) also provide a unique way to study the two fluctuation mechanisms responsible for low frequency noise in MOSFETs. The experimental noise spectra were modelled using the new theory developed, and good agreement between the experimental results and the theoretical calculations were obtained.

Finally, the new combined model considers not only the individual carrier number fluctuation, but also considers mobility fluctuations from bulk phonon-lattice scattering, surface Coulombic scattering induced by oxide traps, and surface roughness scattering which is important in strong inversion mode of operation of MOSFETs. The field-dependence of Hooge parameter is also taken into consideration.

In addition, the project includes an analytic and physical 1/f noise model for short-channel MOSFETs in saturation and deep saturation. By considering the velocity saturation effect in short-channel MOSFETs, an analytic and physical 1/f noise model
for MOSFETs in both saturation and deep saturation based on the mobility fluctuation noise expression is proposed. This model uses detailed one-dimensional distributions of both carrier number and channel potential variation with channel position to calculate the gate referred noise spectral density in both saturation and deep saturation modes of MOSFET operation. These analyses have been corroborated with MINIMOS simulations, and they are in agreement with the experimental results obtained.

A complete investigation of noise in MOSFETs is important in integrated circuit engineering, and it is also very useful for basic research on low-frequency fluctuations. The mechanisms of trapping and phonon fluctuations in MOSFETs inspire us to propose a new concept of multiphonon fluctuation in trapping and detrapping which considers the roles both of traps, and of phonons in 1/f fluctuations in semiconductors.
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Table of Contents

APPROVAL ........................................................................................................... i
ABSTRACT ............................................................................................................... ii
ACKNOWLEDGEMENTS ......................................................................................... iv

Chapter 1 Introduction ................................................................................................. 1

Chapter 2 Flicker Noise Mechanisms in Semiconductor Devices ................................. 4
  §2.1 McWhorter’s Surface Modulation Noise Mechanism ..................................... 6
  §2.2 Hooge’s Mobility Fluctuation Mechanism ......................................................... 7
  §2.3 Multiphonon Fluctuation in Trapping and Detrapping ...................................... 12
    §2.3.1 Phonon-Number Fluctuation Model .............................................................. 13
    §2.3.2 Derivation of Hooge’s Empirical 1/f Noise Relation ................................... 16
  §2.4 Other Mechanisms Leading to 1/f Noise ......................................................... 20
    §2.4.1 1/f Noise from Dislocations in Semiconductors ......................................... 20
    §2.4.2 1/f Noise from Quantum Theory ................................................................. 22
    §2.4.3 1/f Noise from Temperature Fluctuation ..................................................... 23

Chapter 3 Flicker Noise in MOSFETs ........................................................................... 27
  §3.1 Number Fluctuation Theory and Models for MOSFETs ................................. 27
  §3.2 Mobility Fluctuation Theory and Models for MOSFETs ................................. 37
  §3.3 Noise in Short-Channel MOSFETs ................................................................. 39

Chapter 4 New Unified Simulation-Oriented Flicker Noise Model in MOSFETs .......... 47

Chapter 5 Experimental System .................................................................................. 64
  §5.1 Devices Studied ............................................................................................... 64
  §5.2 DC Measurements in MOSFETs ...................................................................... 66
  §5.3 High Temperature Low Frequency Noise Measurements ................................ 66
  §5.4 Charge Pumping Measurements ..................................................................... 70

Chapter 6 Results and Discussions ............................................................................ 72
  §6.1 Low Frequency Noise in MOSFETs at High Temperatures ........................... 72
    §6.1.1 D.C. Characteristics .................................................................................... 73
    §6.1.2 Low Frequency Noise ................................................................................ 79
  §6.2 Verification of New Noise Model with Other Published Results ...................... 97
    §6.3 Verification of Multiphonon Fluctuation Expression ....................................... 106

Chapter 7 Conclusions ............................................................................................... 111

References .............................................................................................................. 113
Table of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>Figure 2.1</td>
<td>Interaction of Electrons with Fluctuating Phonons</td>
<td>8</td>
</tr>
<tr>
<td>Figure 2.2</td>
<td>Lattice Vibration's Effect on Bands</td>
<td>10</td>
</tr>
<tr>
<td>Figure 2.3</td>
<td>Multiphonon Emission and Absorption</td>
<td>13</td>
</tr>
<tr>
<td>Figure 2.4</td>
<td>Barrier Height Fluctuation Due to Dislocation</td>
<td>20</td>
</tr>
<tr>
<td>Figure 2.5</td>
<td>Electrons' &quot;Streamline Flow&quot; Assumption</td>
<td>22</td>
</tr>
<tr>
<td>Figure 3.1</td>
<td>The Coordinate System for n-channel MOSFETs</td>
<td>28</td>
</tr>
<tr>
<td>Figure 3.2</td>
<td>Traps' Distribution in MOSFETs</td>
<td>29</td>
</tr>
<tr>
<td>Figure 3.3</td>
<td>Energy Band Diagram in n-channel MOSFETs</td>
<td>31</td>
</tr>
<tr>
<td>Figure 3.4</td>
<td>&quot;Two-Region Model&quot; for Short-Channel MOSFETs</td>
<td>40</td>
</tr>
<tr>
<td>Figure 3.5</td>
<td>Distribution of Number of Electrons in Channel</td>
<td>42</td>
</tr>
<tr>
<td>Figure 3.6</td>
<td>Distribution of Channel Potential</td>
<td>43</td>
</tr>
<tr>
<td>Figure 4.1</td>
<td>Three Scattering Mechanisms in MOSFETs</td>
<td>53</td>
</tr>
<tr>
<td>Figure 5.1</td>
<td>Flow Chart of High Temperature Measurement</td>
<td>65</td>
</tr>
<tr>
<td>Figure 5.2</td>
<td>Biasing Circuit for Gain Measurement</td>
<td>67</td>
</tr>
<tr>
<td>Figure 5.3</td>
<td>Biasing Circuit for Noise Measurement</td>
<td>69</td>
</tr>
<tr>
<td>Figure 5.4</td>
<td>Conventional Charge Pumping Technique</td>
<td>71</td>
</tr>
<tr>
<td>Figure 6.1</td>
<td>D.C. Characteristics for Test MOSFETs</td>
<td>76</td>
</tr>
<tr>
<td>Figure 6.2</td>
<td>Threshold Voltages versus Temperature</td>
<td>77</td>
</tr>
<tr>
<td>Figure 6.3</td>
<td>Effective Mobility versus Temperature</td>
<td>78</td>
</tr>
<tr>
<td>Figure 6.4</td>
<td>Gate Referred Noise versus Frequency</td>
<td>80</td>
</tr>
<tr>
<td>Figure 6.5</td>
<td>Drain Current Noise versus Drain Voltage</td>
<td>83</td>
</tr>
<tr>
<td>Figure 6.6</td>
<td>Drain Noise versus Drain Voltage at 10Hz</td>
<td>85</td>
</tr>
<tr>
<td>Figure 6.7</td>
<td>Drain Noise versus Drain Voltage at 14Hz</td>
<td>86</td>
</tr>
<tr>
<td>Figure 6.8(a)</td>
<td>Gate Noise versus Drain Voltage at 10Hz</td>
<td>87</td>
</tr>
<tr>
<td>Figure 6.8(b)</td>
<td>Gate Noise versus Drain Voltage at 14Hz</td>
<td>88</td>
</tr>
<tr>
<td>Figure 6.9(a)</td>
<td>Gate Noise versus Drain Voltage at 14Hz</td>
<td>89</td>
</tr>
<tr>
<td>Figure 6.9(b)</td>
<td>Gate Noise versus Drain Voltage at 114Hz</td>
<td>90</td>
</tr>
<tr>
<td>Figure 6.10(a)</td>
<td>Drain Noise versus Temperature</td>
<td>93</td>
</tr>
<tr>
<td>Figure 6.10(b)</td>
<td>Drain Noise versus Temperature</td>
<td>94</td>
</tr>
<tr>
<td>Figure 6.11</td>
<td>Charge Pumping Measurements</td>
<td>96</td>
</tr>
<tr>
<td>Figure 6.12</td>
<td>Simulation and Experimental Results</td>
<td>101</td>
</tr>
<tr>
<td>Figure 6.13</td>
<td>Simulation and Experimental Results</td>
<td>102</td>
</tr>
<tr>
<td>Figure 6.14</td>
<td>Simulation and Experimental Results</td>
<td>103</td>
</tr>
<tr>
<td>Figure 6.15</td>
<td>Simulation and Experimental Results</td>
<td>104</td>
</tr>
<tr>
<td>Figure 6.16</td>
<td>Simulation and Experimental Results</td>
<td>105</td>
</tr>
<tr>
<td>Figure 6.17</td>
<td>Comparison of $\alpha_h$ Values between Theory and Experiments</td>
<td>108</td>
</tr>
<tr>
<td>Figure 6.18</td>
<td>Comparison of $\alpha_h$ Values between Theory and Experiments</td>
<td>109</td>
</tr>
<tr>
<td>Figure 6.19</td>
<td>$\alpha_h$ from New Expression and Dislocation Theory</td>
<td>110</td>
</tr>
</tbody>
</table>
## Table of Tables

Table 2.1 1/f Noise in Electronics System .......................................................... 5
Table 2.2 Materials Obeying Hooge’s Empirical Relation .......................... 9
Table 4.1 Noise versus Device Parameters in MOSFETs ............................... 47
Table 4.2 Noise versus External Parameters in MOSFETs ......................... 48
Table 4.3 New Model’s Validity and Limitations ........................................... 62
Table 6.1 Device Parameters for Test MOSFETs ........................................... 75
Table 6.2 Listing of Hooge Parameter and Trap Density ............................... 92
Table 6.3 Device Parameters in Published Data ............................................. 97
Table 6.4 Fitting Parameters Used in Hung’s Model ....................................... 98
Table 6.5 Alpha and Trap Density Values for Test Devices .......................... 98
Table of Important Symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_{ox}$</td>
<td>Gate oxide capacitance per unit area (F/cm²)</td>
</tr>
<tr>
<td>$C_d$</td>
<td>Depletion capacitance per unit area (F/cm²)</td>
</tr>
<tr>
<td>$C_{it}$</td>
<td>Interface state capacitance per unit area (F/cm²)</td>
</tr>
<tr>
<td>$C_i$</td>
<td>Inversion charge capacitance per unit area (F/cm²)</td>
</tr>
<tr>
<td>$f$</td>
<td>Frequency (Hz)</td>
</tr>
<tr>
<td>$c$</td>
<td>Frequency index</td>
</tr>
<tr>
<td>$T$</td>
<td>Temperature (K)</td>
</tr>
<tr>
<td>$E_c$</td>
<td>Critical electric field (V/cm)</td>
</tr>
<tr>
<td>$w$</td>
<td>Draw gate channel width (µm)</td>
</tr>
<tr>
<td>$q$</td>
<td>Electron charge (C)</td>
</tr>
<tr>
<td>$N_t$</td>
<td>Oxide trap density (cm³eV⁻¹)</td>
</tr>
<tr>
<td>$k$</td>
<td>Boltzmann’s constant (eV/K)</td>
</tr>
<tr>
<td>$\hbar$</td>
<td>Planck’s constant (eV s)</td>
</tr>
<tr>
<td>$x$</td>
<td>Position along the channel length (µm)</td>
</tr>
<tr>
<td>$y$</td>
<td>Position along the channel width (µm)</td>
</tr>
<tr>
<td>$z$</td>
<td>Position along the channel depth (µm)</td>
</tr>
<tr>
<td>$N_{it}$</td>
<td>Interface trap density (cm²eV⁻¹)</td>
</tr>
<tr>
<td>$\Delta N_t$</td>
<td>Number of carriers in the oxide traps in the volume element $\Delta x \Delta y \Delta z$</td>
</tr>
<tr>
<td>$\Delta N_T$</td>
<td>Number of Traps in the volume element $\Delta x \Delta y \Delta z$</td>
</tr>
<tr>
<td>$N_T(E_f)^{\text{eff}}$</td>
<td>Effective Oxide trap density (cm³eV⁻¹)</td>
</tr>
<tr>
<td>$\Delta I_D(x)$</td>
<td>Drain current fluctuation at position $x$ (A)</td>
</tr>
<tr>
<td>$\Delta I_D$</td>
<td>Total drain current fluctuation in channel (A)</td>
</tr>
<tr>
<td>$g_m$</td>
<td>Transconductance (A/V)</td>
</tr>
<tr>
<td>$\bar{I}$</td>
<td>Average drain-to-source current (A)</td>
</tr>
<tr>
<td>$I_D$</td>
<td>Drain-to-source current (A)</td>
</tr>
<tr>
<td>$L$</td>
<td>Channel length (µm)</td>
</tr>
<tr>
<td>$L'$</td>
<td>Channel position from source at which the potential is equal to $V_{GS} - V_T$ (µm)</td>
</tr>
<tr>
<td>$L''$</td>
<td>Channel position from source at which the numbers of free electrons in the channel and the electrons in depletion region are equal (µm)</td>
</tr>
<tr>
<td>$N$</td>
<td>Total carrier number in the sample</td>
</tr>
<tr>
<td>$\bar{N}$</td>
<td>Total average carrier number in the channel</td>
</tr>
<tr>
<td>$N_A$</td>
<td>Substrate doping concentration (cm³)</td>
</tr>
<tr>
<td>$n(x)$</td>
<td>The number of electron per unit length along the channel (cm⁻¹)</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$S_{\mu}(f)$</td>
<td>Noise power spectral density due to mobility fluctuation ($cm^4/V^2s$)</td>
</tr>
<tr>
<td>$S_I(f)$</td>
<td>Current noise spectral density ($A^2/Hz$)</td>
</tr>
<tr>
<td>$S_R(f)$</td>
<td>Resistance noise spectral density ($\Omega^2/Hz$)</td>
</tr>
<tr>
<td>$S_{Ip}(f)$</td>
<td>Drain current noise spectral density ($A^2/Hz$)</td>
</tr>
<tr>
<td>$S_{Ip}(f)(DS)$</td>
<td>Drain current noise spectral density in deep saturation ($A^2/Hz$)</td>
</tr>
<tr>
<td>$S_V(f)$</td>
<td>Voltage noise spectral density ($V^2/Hz$)</td>
</tr>
<tr>
<td>$S_{VC}(f)(DS)$</td>
<td>Voltage noise spectral density in deep saturation ($V^2/Hz$)</td>
</tr>
<tr>
<td>$S_{VC}(f)$</td>
<td>Equivalent gate voltage noise spectral density ($V^2/Hz$)</td>
</tr>
<tr>
<td>$\bar{V}$</td>
<td>Average channel potential ($V$)</td>
</tr>
<tr>
<td>$V(x)$</td>
<td>Channel potential at $x$ when the source and body are grounded ($V$)</td>
</tr>
<tr>
<td>$V_{DS}$</td>
<td>Drain voltage ($V$)</td>
</tr>
<tr>
<td>$V_{GS}$</td>
<td>Gate voltage ($V$)</td>
</tr>
<tr>
<td>$V_{TR}$</td>
<td>MOSFET threshold voltage ($V$)</td>
</tr>
<tr>
<td>$V_{DSSat}$</td>
<td>Saturation drain voltage ($V$)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Effective Hooge’s parameter</td>
</tr>
<tr>
<td>$\alpha_{H}$</td>
<td>Hooge’s dimensionless constant</td>
</tr>
<tr>
<td>$\alpha_{ph}$</td>
<td>Hooge’s parameter due to phonon scattering</td>
</tr>
<tr>
<td>$\alpha_{sr}$</td>
<td>Hooge’s parameter due to surface-roughness scattering</td>
</tr>
<tr>
<td>$\tau_{n1}$</td>
<td>Minimum of relaxation time of oxide traps (s)</td>
</tr>
<tr>
<td>$\tau_{n2}$</td>
<td>Maximum of relaxation time of oxide traps (s)</td>
</tr>
<tr>
<td>$\tau_{m1}$</td>
<td>Minimum of relaxation time of mobility fluctuation (s)</td>
</tr>
<tr>
<td>$\tau_{m2}$</td>
<td>Maximum of relaxation time of mobility fluctuation (s)</td>
</tr>
<tr>
<td>$\tau_{ph}$</td>
<td>Phonon’s relaxation time (s)</td>
</tr>
<tr>
<td>$\tau'$</td>
<td>Phonon’s correlation time (s)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Attenuation coefficient of the electron wave function ($cm^{-1}$)</td>
</tr>
<tr>
<td>$\varepsilon_{Si}$</td>
<td>Permittivity of silicon ($F/cm$)</td>
</tr>
<tr>
<td>$\mu_{w}$</td>
<td>Channel mobility without impurity scattering ($cm^2/Vs$)</td>
</tr>
<tr>
<td>$\mu_{ph}$</td>
<td>Channel Mobility affected by phonon scattering ($cm^2/Vs$)</td>
</tr>
<tr>
<td>$\mu_{sr}$</td>
<td>Channel Mobility affected by surface-roughness scattering ($cm^2/Vs$)</td>
</tr>
<tr>
<td>$\mu_{ph}$</td>
<td>Channel Mobility affected by oxide traps’ scattering ($cm^2/Vs$)</td>
</tr>
<tr>
<td>$\mu_{eff}$</td>
<td>Effective Channel mobility ($cm^2/Vs$)</td>
</tr>
<tr>
<td>$\mu_0$</td>
<td>Channel mobility and very low electric fields ($cm^2/Vs$)</td>
</tr>
<tr>
<td>$v_s$</td>
<td>Saturation velocity ($cm/s$)</td>
</tr>
<tr>
<td>$\phi_F$</td>
<td>Fermi level potential ($V$)</td>
</tr>
<tr>
<td>$\theta_0$</td>
<td>Vertical field mobility degradation constant ($V^{-1}$)</td>
</tr>
<tr>
<td>$\theta_1$</td>
<td>Mobility velocity saturation effect constant ($\mu m.V^{-1}$)</td>
</tr>
</tbody>
</table>
Chapter 1  Introduction

The need for quieter device operation is in sharp conflict with the decrease of operating signal versus increase of noise. The serious contradiction between the continuous reduced supply operating signal and the sharp increased noise due to the aggressive scaled-down structures in VLSI integrated circuits leads to a very active study of noise in electronic devices, especially the metal-oxide-semiconductor field-effect transistors (MOSFETs).

MOSFETs are finding more important and more widespread utilization not only in the area of digital circuits but also in the area of analog ones such as A/D converters, memories, and telecommunication circuits. One of the key requirements of analog circuits is the capability of a sufficiently large dynamic range which translates directly in the requirement for a large swing and a low noise.

In the integrated circuits MOSFETs tend to generate much more noise than their BJT and JFET counterparts, especially in the low frequency region where the flicker (1/f) noise dominates. Noise in MOSFETs increases when they are operating from high frequency to low frequency region.

Noise in MOSFETs can be simulated by many general-purpose simulation programs, but their results are rather unreliable because of the inaccuracy of the noise models on which the simulators are based. SPICE, one of the most extensive utilized simulation programs, uses an oversimplified flicker noise expression. Therefore, there is an urgent need for a more accurate model for circuit simulators to optimize the low frequency noise performance in MOSFETs in different operating regions and at different temperatures.

With the above considerations, my thesis project works on the theoretical and experimental studies on low frequency noise in MOSFETs.
In general, two theories have been used to explain the origin of low frequency noise in MOSFETs. The number fluctuation theory attributes flicker noise in MOSFETs to the random trapping and detrapping processes of carriers in the oxide traps near the Si-SiO₂ surface. It is assumed that the channel in MOSFETs can exchange charges with the oxide traps by tunneling via interface states. The other is mobility fluctuation theory which considers the flicker noise to result from the fluctuation of bulk mobility.

We proposed a combined low frequency noise model in MOSFETs which considers the individual carrier number fluctuation and mobility fluctuations from bulk phonon-lattice scattering, surface Coulombic scattering induced by oxide traps and surface roughness scattering which is important in strong inversion. The model is verified by the experimental results which also proves in a unique way that the two fluctuation mechanisms are existing for MOSFETs. It is in good agreement with the experimental results done by us as well as other published experimental data.

The high-temperature performance of MOSFETs has attracted several studies, especially since MOSFETs are finding increasingly more applications in high-temperature electronics. The low noise performance of MOSFETs at high temperatures is very important for the production of a wide range of advanced high-temperature MOS integrated circuits, particularly for analog circuits which are much more sensitive to design details such as operating temperature range. To optimize the low frequency noise performance in MOSFETs at different temperatures, a careful study and accurate simulation of it is urgently needed.

Our thesis project focuses on the low-frequency noise in MOSFETs not only due to its importance in the scaled-down structures in integrated circuits and the wide-spread utilization of MOS devices in analog electronics, but also due to the fact that it provides a possible way to research flicker noise which is a universal phenomenon existing in most physical systems. A complete investigation of noise in MOSFETs is important to IC engineering and is also very useful for basic research in the
low-frequency fluctuations. The mechanisms of trapping and phonon fluctuations in MOSFETs make us get the idea of multiphonon fluctuation in trapping and detrapping which considers the roles both of traps and of phonons in 1/f fluctuation in semiconductors.

The study on multiphonon fluctuation starts from the Langevin method and derives a phonon-number fluctuation model. Then using the trapping noise concept and multiphonon transition process, we proposed a new explanation on Hooge’s empirical 1/f noise relation. In the relation, the Hooge’s empirical constant is related to the number of traps and the emitted largest and smallest phonon wave vectors.

Chapter 2 introduces the flicker noise mechanisms in semiconductor devices including a new study on Hooge’s empirical relation with multiphonon 1/f fluctuation in trapping and detrapping. Chapter 3 uses the flicker noise mechanisms studied in chapter 2 to describe flicker noise in MOSFETs. Chapter 4 described a new unified simulation-oriented flicker noise model in MOSFETs. In chapter 5, a description of the experimental system to conduct noise measurements and charge pumping measurements. Chapter 6 presented and discussed most of the important results obtained. Finally chapter 7 gave the conclusions from this thesis project.
Chapter 2  Flicker Noise Mechanisms in Semiconductor Devices

Flicker noise is a fluctuation phenomenon which shows a power spectral density inversely proportional to the frequency, within a wide range of frequency. The power spectral density \( S_x(f) \) could be expressed as,

\[
S_x(f) \propto f^{-c}
\]  

(2.1)

where \( c \) could range from 0.8 to 1.4 but is typically very close to unity, and where \( X \) represents the fluctuating quantity of interest such as current, voltage and resistance.

The main characteristic of flicker noise is its \( 1/f^c \) spectrum; and since \( c \) is typically very close to unity, then for this reason flicker noise is often used synonymously with \( 1/f \) noise. Flicker noise is typically observed out to frequencies approximately \( 10^5 \) Hz, after which white noise becomes predominant. The lowest frequency of flicker noise has been measured was down to \( 10^6 \) Hz, but this is not absolute lowest frequency. Flicker noise is considered to be a stationary random process [Wolf78] and measurements on the variance of flicker noise have verified the statement. Another characteristic feature of flicker noise is that there is typically an \( I^2 \) dependence of the power spectral density \( S_x(f) \). These observations have led to the conclusion that \( 1/f \) noise originates in resistance fluctuations. In terms of temperature dependence, \( 1/f \) noise depends on temperature since the current depends on temperature. Besides this indirect temperature influence, many studies show a direct temperature dependence, for example [Eber77].

Flicker noise spectrum was first observed by Johnson in 1925 in vacuum tubes and he used the term "flicker noise" [Ziel79] because of the flicker phenomenons observed in the cathode current of vacuum tubes. Schottky gave the first interpretation on flicker noise in 1926 and showed that flicker noise was more strongly suppressed by
space charge than shot noise in 1937. Christensen and Pearson were the first to measure flicker noise in carbon microphones and in carbon contacts. Later, flicker noise was found in a great variety of other components and devices. Table 2.1 shows the components in which 1/f noise are observed in electronics system. Besides electrical system, 1/f fluctuation has been observed in physical, technological, biological, economic and transportation systems [Mush76].

<table>
<thead>
<tr>
<th>Electronics System</th>
<th>Signal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistor</td>
<td>Current</td>
</tr>
<tr>
<td>Carbon film</td>
<td>Current</td>
</tr>
<tr>
<td>Metal film</td>
<td>Current</td>
</tr>
<tr>
<td>Semiconductor</td>
<td>Current</td>
</tr>
<tr>
<td>Superconductor</td>
<td>Flux Flow</td>
</tr>
<tr>
<td>Vacuum Tube</td>
<td>Current</td>
</tr>
<tr>
<td>Junction diode</td>
<td>Current</td>
</tr>
<tr>
<td>Schottky diode</td>
<td>Current</td>
</tr>
<tr>
<td>Zener diode</td>
<td>Current</td>
</tr>
<tr>
<td>BJT</td>
<td>Current</td>
</tr>
<tr>
<td>MOSFET</td>
<td>Current</td>
</tr>
<tr>
<td>Quantum well diode</td>
<td>Current</td>
</tr>
</tbody>
</table>

*Table 2.1* 1/f Noise in Electronics System

The universal existence of 1/f fluctuation has stimulated research to question: whether 1/f noise is a general fluctuation phenomenon inherent in the collective motion of particles, or whether it is a property of a specific system? To understand why different mechanisms give the identical spectrum, we have to identify the mechanisms that
generate 1/f spectrum for different systems.

In this chapter, we introduce mechanisms leading to 1/f noise in semiconductor devices. Then we will review 1/f noise mechanisms in MOSFETs in chapter 3.

§2.1 McWhorter’s Surface Modulation Noise Mechanism

In 1957, McWhorter proposed a surface modulation noise mechanism [McWh57] to explain 1/f noise in semiconductors. The proposal is based on the following assumptions:

(1) Interface traps located in the oxide layer near the semiconductor-oxide interface exchange carriers with the conduction and valence bands of bulk semiconductor by Shockley-Read-Hall (SRH) process [Fu72].

(2) Oxide traps within the distance of $0 \sim 100\AA$ to the interface have a wide range of relaxation time distribution when they exchange carriers with bulk semiconductor by tunneling and SRH processes. The relaxation time constant of traps can be expressed as
\[
\tau = \tau_0 \exp (\gamma z)
\] (2.1.1)
where $z$ is the distance of trap in cm to the interface, and $\gamma = 10^8 \text{cm}^{-1}$.

(3) The fluctuation of carrier number $\delta N^2$ is proportional to the carrier number $N$, as $\delta N^2 = bN$, with $b$ related to the oxide traps density.

Based on the above assumptions, the carrier number fluctuation power spectrum density can be derived as
\[
S_N(f) = \frac{bN}{f^{\gamma \ln(\tau_{\text{max}} / \tau_{\text{min}})}}
\] (2.1.2)
where $\tau_{\text{max}} = \tau(z_0)$, and $\tau_{\text{min}} = \tau(0)$. If oxide traps are distributed uniformly between $z=0$ to $z=z_0$, then $\gamma$ is approximately 1, otherwise $\gamma$ can vary from 0.8 to 1.4.
McWhorter’s surface modulation mechanism is extensively used to qualitatively analyze, and to quantitatively calculate 1/f noise in semiconductor devices. It is widely used in the interpreting 1/f noise in MOSFETs, in spite of some limitations. To date, there have been many experimental results to support McWhorter’s model. For example, the gate referred input noise $S_{vc}(f)$ of MOSFETs is proportional to the interface density $N_{it}$ [Ghib87], [Reim84], [Fang86a], and [Steg84]. However, McWhorter’s model cannot easily be used to explain 1/f noise in many bulk materials. It cannot explain why 1/f noise results from dislocation defects. For example, it was found by Macrae [Macr62] that the removal of the adsorbed gases and the surface oxide on silicon filaments did not have an appreciable effect on either the magnitude or the frequency dependence of the 1/f noise from the filament. To solve some of the deficiencies of McWhorter’s model, Hooge proposed a phenomenological relation that is now described.

§2.2 Hooge’s Mobility Fluctuation Mechanism

In 1969, Hooge proposed an empirical relation of the power spectral density $S_f(f)$ of current fluctuations, and $S_R(f)$ of resistance fluctuations for semiconductors and other materials [Hoog69] that is given by

$$\frac{S_f(f)}{f^2} = \frac{S_R(f)}{R^2} = \frac{\alpha_H}{Nf} \tag{2.2.1}$$

where $N$ is the total number of free charge carriers, and $\alpha_H$ is a dimensionless quantity that was experimentally found to be about $2 \times 10^{-3}$. The Hooge’s formula represents a bulk effect of noise mechanism.

Flicker noise in terms of mobility fluctuation has been investigated for more than two decades. The empirical expression (2.2.1) was later extended in [Hoog79] where it was shown that
Eqn.(2.2.2) is obtained under the assumption that the mobility-fluctuation noise is caused by interaction of electrons with slowly fluctuating longitudinal acoustical phonon populations which is schematically shown in figure 2.1.

\[ \frac{S_v(f)}{V^2} = \frac{S_{\mu}(f)}{\mu^2} = \alpha_H \left( \frac{\mu}{\mu_{\text{latt}}} \right)^2 \frac{1}{fN} = \frac{\alpha}{fN}. \]  

(2.2.2)

\[ \alpha \text{ is dependent on the material studied and is given by} \]

\[ \alpha = \left( \frac{\mu}{\mu_{\text{latt}}} \right)^2 \alpha_H, \]  

(2.2.3)
with $\mu$ being the observed effective mobility, and $\mu_{\text{lat}}$, the value that the mobility would have had if no impurity scattering had been present [Van80b]. Hooge's original expression was based on the assumption that only lattice scattering causes 1/f noise in the mobility of the charge carriers [Hoog79]. However, it was shown in figure 2.1 [Vand80a] that competing scattering mechanisms other than lattice scattering led to a reduction of measured 1/f noise.

Hooge's formula has been examined and verified by many experiments. The materials which obey the Hooge's empirical relation [Hoog72] [Voss76] [Vand74] are shown in table 2.2.

<table>
<thead>
<tr>
<th>Point Contacts</th>
<th>Ag, Al, Au, Brass, Cr, Cu, Pb, Pt, Sn, Steel, and Mn</th>
</tr>
</thead>
<tbody>
<tr>
<td>Metal Films</td>
<td>Ag, Au, Bi, Cu, Sn</td>
</tr>
<tr>
<td>Semiconductor materials</td>
<td>n- and p- type Ge, Si, GaAs, InSb, n-GaP, and Li-doped MnO</td>
</tr>
</tbody>
</table>

Table 2.2 Materials Obeying Hooge's Empirical Relation.

The measured $\alpha_\mu$ is in the range of $1 \times 10^{-3}$ to $9 \times 10^{-3}$. Hooge's empirical 1/f noise relation has been used by many researchers as an aid to study and to describe 1/f noise in homogeneous metals and semiconductors. However, when the Hooge's formula has been used in interpreting 1/f noise for MOSFETs, it is found that the $\alpha_\mu$ is $10^3$ to $10^4$ times smaller for experiments and predictions using the Hooge's empirical relation to be in agreement.

In view of the above, one can agree that 1/f noise in semiconductors could be explained using number fluctuations as well as mobility fluctuations. The number
fluctuation concept has a definite physical model from which it originates while the
mobility fluctuation model is based on empirical observations. In 1981, Jindal and
Van der Ziel [Jind81] firstly proposed a phonon fluctuation model for flicker noise
in elemental semiconductors. It believes that phonon population fluctuates randomly,
and this fluctuation has a g-r noise spectrum. Through electron-phonon interaction,
the g-r noise spectrum is transferred to electron mean free path. As we all know,
lattice vibration can modulate the positions of electron conduction and valence
bands. Phonon-lattice vibration’s effect on electron conduction and valence bands is
demonstrated in figure 2.2 below.

Fig 2.2 Lattice Vibration’s Effect on Bands

By calculating the mean free path formulation for electrons, an expression of
spectral density in mean free path fluctuation is given by [Jind81] as
To get the pure 1/f noise, from the expression, it is required that $p_{\text{el}}^* < 1/\nu > l$. From eqn(2.2.4), the noise spectral density was determined and was obtained,

$$\frac{S_t(f)}{f^2} = \frac{\alpha_0^2}{\ln(\tau_t/\tau_0)f}$$  

(2.2.4)

with $\mu = (e/m^*) < 1/\nu > l$. From eqn(2.2.4), the noise spectral density was determined and was obtained,

$$\frac{S_{\mu}(f)}{f^2} = \frac{S_t(f)}{Nf^2} = \frac{\alpha_0^2}{\ln(\tau_t/\tau_0)fN} = \frac{\alpha_0}{fN}$$  

(2.2.5)

where $\alpha_0 = \alpha_0^2/\ln(\tau_t/\tau_0)$ and $\tau_t/\tau_0 = 10^{12}$. With the assumption of $\alpha_0 = 0.2$, eqn. (2.2.5) expression yields $\alpha_0 = 1.4 \times 10^{-3}$.

The model [Jind81] firstly provides a theoretical explanation for bulk effect 1/f noise in elemental semiconductors based on mobility fluctuations. However a more detailed determination of the fitting factor $\alpha_0$, and its dependence on doping is still to be investigated. Also, it is doubtful that phonon scattering can be fluctuating at the low frequencies typically for 1/f noise.

More recently, using the amplitude spectrum($S_R$) of lattice harmonic oscillators, a closed-form expression for $\alpha_{th}$ is derived in [Dier91], in which

$$S_R = \frac{-4 + 2\eta}{5 + 2\beta - \eta} AB^{(1 + 2\beta + \eta)(4 - 2\eta)f(1 + 2\beta + \eta)(2 - \eta)}.$$  

(2.2.6)

To get the pure 1/f noise, from the $S_R$ expression, it is required that

$$\frac{(1 + 2\beta + \eta)}{(2 - \eta)} = -1, \quad \text{or} \quad \beta = -3/2 \quad \text{or} \quad \eta = -\infty$$  

(2.2.7)

This model considers the essential difference between the phonon-interaction time constant, and the phonon correlation time, as expressed by

$$\tau^* = \tau (kT/\hbar\omega)^2.$$  

(2.2.8)

The lattice oscillator’s correlation time $\tau^*$ is extended sharply for large $kT/\hbar\omega$, compared to the phonon transition time constant $\tau$. The weak points are the
empirical assumptions that have been made: the spectrum of the energy level of a single lattice vibration; the speculative choice of the parameters $\beta$ or $\eta$ to tune the result to a pure $1/f$ spectrum.

§2.3 Mutliphonon Fluctuation in Trapping and Detrapping

McWhorter's noise mechanism proposes that $1/f$ noise spectrum is induced by oxide traps. Hooge's mobility fluctuation mechanism could for $1/f$ noise spectrum could be explained as due to phonon-number fluctuation. Here we pose the following question, is there any relation between the above two mechanisms?

In electron trapping and detrapping in extrinsic semiconductors, there could be three kinds of processes involved: radiative transition, multiphonon transition, and Auger transition. However, multiphonon process plays a decisive role in the electron transition, as previously pointed out in [Ye84]. Multiphonon emission most likely happens for indirect gap semiconductors [Kime75], which is shown in figure 2.3 below.
Here, we proposed to study multiphonon fluctuation mechanisms in trapping and detrapping. The explanation begins with the Langevin method and derives a phonon-number fluctuation model. Then using the trapping noise model and multiphonon transition process, a new $1/f$ noise relation is derived. In the final noise spectral density relation, the Hooge’s empirical constant $\alpha_H$ is related to the number of traps and the values largest and smallest phonon wave vectors that are emitted [Zhu92a].

§2.3.1 Phonon-Number Fluctuation Model

It has been explained in [Jind81] that the number of acoustic phonons per mode fluctuates with a generation-recombination (g-r) noise spectrum. This was later experimentally verified in [Mush90]. In [Jind81], it was stated that the
number of phonons for a given mode randomly fluctuates, and that the g-r noise displayed by the phonon population fluctuation per mode is characterized by the relaxation time of the phonon. Phonons’ energies are expressed as \((n + \frac{1}{2})(\hbar \omega)\), with \(n\) varying between 0 and \(\infty\). Each phonon associated with a mode has energy \(\hbar \omega\). The energy of each phonon mode is fluctuating around its equilibrium value, and its power spectral density could be derived from the Langevin differential equation [Ziel86]. The appearance and disappearance of phonons per mode could be described by the Langevin differential equation of the form

\[
\frac{d\Delta n_p}{dt} = -\frac{\Delta n_p}{\tau_p} + H(t)
\]

(2.3.1.1)

in which \(\Delta n_p\) is the fluctuation in the number of phonons per mode, \(H(t)\) is a random noise term, and \(\tau_p\) is the lifetime of the increased phonons.

Using eqn. (2.3.1.1), we can derive the power spectral density of phonon-number fluctuation per mode \(S_{\Delta n_p}(f)\) to be

\[
S_{\Delta n_p}(f) = \frac{S_H(0)\tau_p^2}{1 + \omega^2\tau_p^2}
\]

(2.3.1.2)

with \(S_H(0)\) being the power spectrum density of white noise, and \(\omega\) the angular frequency. Since the mean square fluctuation of \(n_p\) could be obtained from

\[
\overline{\Delta n_p^2} = \int_{0}^{\infty} S_{\Delta n_p}(f) df = S_{\Delta n_p}(0)\tau_p \int_{0}^{\infty} \frac{\tau_p df}{1 + \omega^2\tau_p^2} = \frac{S_H(0)\tau_p}{4},
\]

(2.3.1.3)

then using eqns. (2.3.1.2) and (2.3.1.3), we get that

\[
S_{\Delta n_p} = 4\overline{\Delta n_p^2} \frac{\tau_p}{1 + \omega^2\tau_p^2}.
\]

(2.3.1.4)

The experiments show that the phonon’s g-r noise is not correlated from mode to mode [Mush90]. Therefore, we can superpose \(4\overline{\Delta n_p^2}\tau_p/(1 + \omega^2\tau_p^2)\) for all
phonon modes. Now considering the case of phonons with \( Q \) modes and \( Q \) time constants, we get

\[
S_{\Delta N_p} = \frac{Q}{\sum_{i=1}^{Q} 4\Delta n_{pi}^2} \frac{\tau_{pi}}{1 + \omega^2 \tau_{pi}^2}
\]

(2.3.1.5)

where \( \tau_{pi} \) is the time constant associated with the \( i \)th phonon. Therefore,

\[
\Delta N_p^2 = \int_{0}^{\infty} S_{\Delta N_p}(f)df = \frac{Q}{\sum_{i=1}^{Q} 4\Delta n_{pi}^2} \int_{0}^{\infty} \frac{\tau_{pi}df}{1 + \omega^2 \tau_{pi}^2} = \frac{Q}{\sum_{i=1}^{Q} \Delta n_{pi}^2}.
\]

(2.3.1.6)

Eqn. (2.3.1.5) can be now be written as

\[
S_{\Delta N_p} = \frac{\Delta N_p^2}{\sum_{i=1}^{Q} g(\tau_{pi})} \frac{\tau_{pi}}{1 + \omega^2 \tau_{pi}^2} \quad \text{with} \quad g(\tau_{pi}) = \frac{\Delta n_{pi}^2}{\sum_{i=1}^{Q} \Delta n_{pi}^2}
\]

(2.3.1.7)

or

\[
S_{\Delta N_p} = \frac{\Delta N_p^2}{\int_{0}^{\infty} g(\tau_p)\tau_p d\tau_p} \quad \text{with} \quad \int_{0}^{\infty} g(\tau_p)d\tau_p = 1
\]

(2.3.1.8)

In eqns. (2.3.1.7) and (2.3.1.8), \( S_{\Delta N_p}(f) \) represents the power spectra density for phonons of all modes, \( \Delta N_p^2 \) represents the phonons' number fluctuation for phonons with all modes, and \( g(\tau_p) \) is the phonon time distribution of all modes.

Since experiments show that the fluctuation of phonon numbers in thermal equilibrium has a 1/f power spectrum density [Mush90], then we can write that

\[
g(\tau)d\tau = d\tau/\tau \{\ln(\tau_{max}/\tau_{min})\} \quad \text{for} \quad \tau_{min} \leq \tau \leq \tau_{max}. \]

Therefore, eqn. (2.3.1.5) becomes

\[
S_{\Delta N_p}(f) = \frac{\Delta N_p^2}{f \ln(\tau_{max}/\tau_{min})} \quad \text{for} \quad \frac{1}{\tau_{max}} \leq \omega \leq \frac{1}{\tau_{min}}
\]

(2.3.1.9)

in which

\[
f_{min} = \frac{1}{4\tau_{max}} \quad \text{and} \quad f_{max} = \frac{1}{\pi^2 \tau_{min}}.
\]

(2.3.1.10)
The phonon number fluctuation has a $g$-$r$ noise spectrum in which the time constant is related to the phonon wave vector. From [Jindal81], the point defect scattering time is related to the wave vector $q$ by $\tau \approx 1/q^4$. Therefore, from this $\tau - q$ relation, we get

$$\frac{d\tau}{\tau} \approx -4\frac{dq}{q},$$

and using

$$g(\tau)d\tau = \frac{d\tau/\tau}{\ln(\tau_{\text{max}}/\tau_{\text{min}})} = \frac{dq}{q \ln(q_{\text{max}}/q_{\text{min}})},$$

where $\tau_{\text{max}}$ is the longest lifetime of phonon, and $\tau_{\text{min}}$ is the shortest lifetime of phonon. Also $q_{\text{max}}$ is the largest phonon wave vector, and $q_{\text{min}}$ is the smallest phonon wave vector. Usually in the largest phonon wave vector $q_{\text{max}}$ is less than $\sqrt{3}\pi/a$, and the smallest phonon wave vector $q_{\text{min}}$ is larger than $\pi/(P_{\text{max}}a)$. $P_{\text{max}}$ is the total number of all possible phonon modes which is determined by the material and its dimension.

### §2.3.2 Derivation of Hooge’s Empirical 1/f Noise Relation

For electrons' trapping and detrapping, the expression below applies [Ye84],

$$C_{NF} = \gamma_{NF}N_F(N_t - N_{Fi}) \quad \text{and} \quad E_{NF} = \beta_{NF}N_{Fi},$$

where $C_{NF}$ is the electron trapping rate, $E_{NF}$ the electron release rate from traps, $\gamma_{NF}$ the electron capture coefficient, $\beta_{NF}$ the electron release coefficient, $N_F$ the total number of free electrons, $N_t$ the number of traps, and $N_{Fi}$ the trapped electron number.
For the equilibrium situation, we have that
\[ \gamma_{NF}N_{F}^{0}(N_{i}-N_{F}^{0}) = \beta_{NF}N_{F}^{0} \]  
(2.3.2.2)
so that,
\[ \beta_{NF} = \frac{\gamma_{NF}N_{F}^{0}(N_{i}-N_{F}^{0})}{N_{F}^{0}}. \]  
(2.3.2.3)

In (2.3.2.3), \( \gamma_{NF} \) is determined by the parameters of the material and is a function of temperature, while \( \beta_{NF} \) is related to \( N_{i}, E_{F} \) the Fermi level, and \( E_{i} \) energy level of a trap. Using
\[ N_{F,i} = \frac{N_{i}}{1+e^{(E_{i}-E_{F})/kT}} \]  
(2.3.2.4)
and defining \( \lambda \) as
\[ \lambda = \frac{1}{1+e^{(E_{i}-E_{F})/kT}} \quad \text{with} \quad N_{i} = \lambda N_{F,i} \]  
(2.3.2.5)
and then using the Master equation (see eqn. 2.54, p.20 in [Zie186]) and eqn. (2.3.2.5), we have that
\[ \tau = \frac{1}{E'_{NF}-E'_{NF}} = \frac{1}{\beta_{NF}+\gamma_{NF}N_{F}} \]  
(2.3.2.6)
from which we can derive
\[ \overline{\Delta N_{F}^{2}} = \gamma_{NF}N_{F} \frac{1}{\beta_{NF}+\gamma_{NF}N_{F}} = N_{i}(1-\lambda)\lambda. \]  
(2.3.2.7)

In electron trapping and detrapping from traps, there could be three kinds of processes involved as previously explained, radiative transition, multiphonon transition, and Auger transition. The multiphonon process plays a decisive role in the electron transition [Ye84]. Since multiphonon emission processes are believed
to be a mechanism responsible for traps' capture effects in semiconductors [Henr77] [Uren85] [Kirt89] [Schu91]. The multiphonon emission and absorption process is shown in figure 2.3 [Kime78].

Assuming that the rate of phonon generation (or removal) is proportional to the rate of trapped charge variation, that is

\[
\frac{d\Delta N_p}{dt} = C_1 \frac{d\Delta N_F}{dt} + C_2.
\]  

Therefore, \( C_2 \) represents the rate of phonon generation (or removal) due to interactions other than trapping changes (such as phonon-phonon, phonon-electron, etc), and further assuming that phonon density fluctuation is dominated by trapping, then we get,

\[
\Delta N_p = C\Delta N_F
\]  

where \( \Delta N_F \) is the fluctuation of free electrons due to traps, \( \Delta N_p \) the difference between increased phonon number and phonon number at equilibrium due to electron number fluctuation. \( C \) could be explained as transition probability which depends on the energy difference between electrons and traps, and temperature.

In the phonon-assisted electron emission from traps or capture by traps, the change in the net number of phonons produced is proposed to be proportional to \( \Delta N_F \). This is reasonable because a change \( \Delta N_F \) in the number of trapped electrons introduces a proportional change in the number of phonons. The phonons' modes are different from each other in the multiphonon transition.

From eqn. (2.3.2.9), we get

\[
\overline{\Delta N_p^2} = C^2 \overline{\Delta N_F^2} \quad \text{and} \quad S_{\Delta N_p}(f) = C^2 S_{\Delta N_F}(f)
\]  

Combining eqns. (2.3.1.9), (2.3.1.12) and (2.3.2.10), we have
where $\tau_{max}^p$ is the phonon’s longest relaxation time, and $\tau_{min}^p$ the phonon’s shortest relaxation time.

Introducing eqn. (2.3.2.6) into (2.3.2.11), the expression for $S_{NF}(f)$ is obtained

$$
\frac{S_{NF}(f)}{\Delta N_p^2} = \frac{S_{NF}(f)}{\Delta N_F^2} = \frac{1}{f \ln(q_{max}/q_{min})} = \frac{1}{f \ln(q_{max}/q_{min})} \tag{2.3.2.11}
$$

$$
in \frac{1}{\tau_{max}^p} \leq \omega \leq \frac{1}{\tau_{min}^p}
$$

Defining

$$
\alpha_H = \frac{N_F \lambda (1 - \lambda)}{N_F \ln(q_{max}/q_{min})} \tag{2.3.2.13}
$$

we finally get the Hooge’s empirical relation as

$$
\frac{S_{NF}(f)}{N_F^2} = \frac{N_F \lambda (1 - \lambda)}{fN_F^2 \ln(q_{max}/q_{min})} = \frac{\alpha_H}{fN_F} \tag{2.3.2.14}
$$

$\alpha_H$ as defined in eqn. (2.3.2.13) was obtained under the assumption of a single trap involved in the phonon-assisted electron trapping and detrapping processes. However, in extrinsic semiconductors, there are usually several traps, for example in silicon, there is a continuous U-shaped distribution of traps in the band gap with peak close to the band edges. Therefore, we can write the generalized expression for $\alpha_H$ from eqn. (2.3.2.13) as

$$
\alpha_H = \sum_{i} \frac{N_i \lambda_i (1 - \lambda_i)}{N_F \ln(q_{max}/q_{min})} \tag{2.3.2.15}
$$
From eqn. (2.3.2.15), we note that \( \alpha_H \) is related to the maximum and minimum phonon wave vectors, the number of traps and their energy levels, and the Fermi level. Calculations of \( \alpha_H \) from the new expression are in agreement with published experimental results. \( \alpha_H \) values predicted by the expression is also in good agreement with that predicted by the dislocation theory which will be discussed in the following.

§2.4 Other Mechanisms Leading to 1/f Noise

§2.4.1 1/f Noise from Dislocations in Semiconductors

A model using dislocations has been proposed to explain 1/f noise in semiconductors in [Mom55] [Morr90] [Morr92b]. In this model, 1/f noise arises from electron capture and emission over a double layer barrier, either a surface barrier at an interface, or a cylindrical barrier around a dislocation as schematically indicated in figure 2.4 below.

![Fig 2.4 Barrier Height Fluctuations Due to Dislocation](image)

Fig 2.4 Barrier Height Fluctuations Due to Dislocation
The barrier height fluctuations are associated with states of dislocations shifting their location. The barrier $qV_{bn}$ could be very large if the neighboring state is near a trapped state, but small if the state is far from the trapped species. Fluctuations in the barriers $qV_{bn}$ in a Debye interval will cause fluctuations in the rate of electron trapping. In turn, these rate fluctuations will lead to fluctuations in carrier density and in the diameter of the dislocation pipe. The fluctuations in diameter will cause fluctuations in the effective mobility of the carriers. Finally the corresponding mobility fluctuations are used to obtain $1/f$ noise [Mor92c].

This dislocation theory for $1/f$ noise is supported by experimental results. It was found by Sugawara [Mor92a] that the noise figure increases rapidly for dislocation densities $> 10^9 m^{-2}$. Another example is that of Mihaila and Amberiadis and Wu et al who induced dislocations in the base of a bipolar transistor by a high phosphorus concentration in the emitter, and found $1/f$ noise increasing rapidly with dislocation density above $10^9 m^{-2}$. However, the calculated Hooge parameter by the dislocation mechanism is about a factor of 10 too low. The dislocation model also neglects the electron density fluctuations which could be an important factor for the origin of $1/f$ noise. In addition, the physical details and consequences of how the mobility is described well by a "streamline flow" mechanism and any effect of overlap of the space charge cylinders shown in Figure 2.5 are still needed.
§2.4.2 1/f Noise from Quantum Theory

A quantum theory approach of 1/f noise was proposed by Dr. Handel in 1975 who took into account that bremsstrahlung radiation is emitted when moving charges are scattered [Hand75]. In this quantum 1/f noise theory, bremsstrahlung means emission or absorption of any massless particles which lead to infrared divergence in the lowest significant order of the perturbation theory, such as photons and phonons. In general, elementary physical processes such as scattering, recombination, tunneling, of charged current carriers have a certain bremsstrahlung amplitude and all with infrared-divergent coupling to the carriers. The quantum 1/f noise theory has been verified by some experiments in BJT and diodes. For metals Hooge’s constant to be approximately $2 \times 10^{-3}$ in the low temperature limit by using the quantum 1/f noise model. But more various experimental support is still required.
§2.4.3 1/f Noise from Temperature Fluctuation

The resistance fluctuations in devices and electrical materials has been explained in [Clar74] due to thermal equilibrium temperature fluctuations which modulate the temperature dependent resistance. It was found that manganin with a negligible temperature coefficient c of the resistance does not show 1/f noise. The other materials with some temperature coefficient c have 1/f noise. It was assumed by them that spatially and temporally uncorrelated temperature protuberances $\delta T$ with the variance

$$\delta T^2 = kT^2/C_v$$

(2.4.3.1)

where $C_v$ is the specific heat, and k is Boltzmann's constant. Introducing certain spatial correlations between the protuberances, one finds the spectrum fitting to a 1/f noise spectrum of the form,

$$S_T(f) = \frac{kT^2/C_v}{(3 + 2\ln(L/w))f}$$

(2.4.3.2)

where $L/w$ is the ratio of the length to width in the sample.

Since the temperature fluctuation could cause carrier mobility fluctuation, this probably could be used to explain the Hooge's mobility model for 1/f noise. Carrier mobility can be expressed,

$$\mu = CT^d$$

(2.4.3.3)

where C is a temperature independent constant, and for metals, $d = 1$, where as for semiconductors, $d = 3/2$. $C_v = 3N_0k$ with $N_0$ being the total number of atoms of the sample. Since we can write that

$$\frac{\delta \mu}{\mu} = d\frac{\delta T}{T}, \text{ then we get that}$$

$$\frac{S_\mu(f)}{R^2} = \frac{S_T(f)}{\mu^2} = d^2S_T(f) \frac{1}{T^2}.$$
Finally combining these equations, we get

\[
\frac{S_R(f)}{R^2} = \frac{d^2}{3(3+2\ln(L/w))} \frac{1}{fN_0} = \frac{d^2n}{3(3+2\ln(L/w))} \frac{1}{fN} = \frac{\alpha}{fN} \tag{2.4.3.6}
\]

with \( \alpha = \frac{d^2n}{3(3+2\ln(L/w))} \)

where \( n \) is the free electron number per atom and is usually equal to 1. If \( L/w = 10^2, d = 1, n = 1 \), then, \( \alpha = 10^{-3} \), which is in agreement with the Hooge’s mobility fluctuation model. The temperature fluctuation model could explain some experimental results. For MOSFETs with very large power density, their power will convert into thermal energy which can cause temperature fluctuations.

Temperature fluctuation or phonon population fluctuation will modulate the carrier current and induces excess noise. This is why the MOSFETs has higher 1/f noise than BJT and JFET. Moreover, since the thermal conduction coefficient of SiO₂ is much less than of Si, then the thermal resistance at interface of SiO₂-Si is larger than in the bulk silicon. Also, the energy dissipated at interface of SiO₂-Si is much larger than energy caused by drift current. Therefore, noise induced by electron recombination at surface must be greater than noise induced by electron recombination at bulk, which in turn is much greater than that the noise induced by drift current.

After studying the above mechanisms leading to 1/f noise, we could draw the following conclusion. The mechanisms reviewed above could explain 1/f noise in semiconductors from different points of view, and they could be fundamentally divided into two kinds of origin: mobility fluctuation and number fluctuation. The 1/f noise from dislocation and temperature fluctuation in fact can be sorted into mobility fluctuation, and the multiphonon emission mechanism in
trapping and detrapping is the combination of number fluctuation and mobility fluctuation. Handel's quantum 1/f noise hasn't found wide application to date and so is not described for this.

If a resistance of a sample with length of L and mobility μ is given,

\[ R = \frac{L^2}{q \mu N} \]  

the fluctuation of resistance is contributed by:

1. the fluctuation of carrier number

\[ \frac{\delta R}{R} = -\frac{\delta N}{N} \quad \text{and} \quad \frac{S_R(f)}{R^2} = \frac{S_N(f)}{N^2}, \quad \text{and} \]  

2. the fluctuation of mobility

\[ \frac{\delta R}{R} = -\frac{\delta \mu}{\mu} \quad \text{and} \quad \frac{S_R(f)}{R^2} = \frac{S_\mu(f)}{\mu^2}. \]  

Considering the two situations, and defining

\[ \frac{S_N(f)}{N^2} = \frac{\alpha_{HN}}{fN} \quad \text{and} \quad \frac{S_\mu(f)}{\mu^2} = \frac{\alpha_{H\mu}}{fN} \]  

we get,

\[ \alpha_H = \alpha_{H\mu} + \alpha_{HN} \]  

When the carrier number fluctuation dominates, we have that

\[ \alpha_H = \alpha_{HN} \]  

and when the mobility fluctuation dominates, we have that

\[ \alpha_H = \alpha_{H\mu}. \]  

Therefore, the physical meaning of the above equations is that the flicker noise can be explained either by mobility fluctuation or number fluctuation. It is believed that mobility fluctuation is caused by the interaction of carriers with
slowly fluctuating longitudinal acoustical-phonon populations. Number fluctuation is contributed by the fluctuation in the number of carriers across the conduction channel induced by oxide traps.

Compared McWhorter’s surface mechanism to Hooge’s bulk mechanism, we get,

\[ \alpha_{HN} = \frac{b}{\ln(\tau_{max}/\tau_{min})}. \]  

(2.4.3.14)

Since b depends on the interface state density, then \( \alpha_{HN} \) is not a constant, and it heavily depends on the structure of devices and their fabrication technique.

One can therefore agree that the bulk mobility fluctuation will become a dominant mechanism after the surface carrier number fluctuation and the surface mobility fluctuation induced by scattering of oxide traps is reduced to a very low level. In fact, it has been reported in [Park82b] [Ziel88] that data on some MOSFETs with normal inversion channel agree better with the number-fluctuation model, whereas other MOSFETs with a built-in channel made by shallow ion implantation are in better agreement with the mobility fluctuation model.

Therefore, it is believed that flicker noise from mobility fluctuation is a fundamental noise which arises from the 'unavoidable' phonon-scattering. It is related to the basic operation of a device and cannot be eliminated as far as the operation of the device is concerned. The number fluctuation is non-fundamental noise which can be reduced by reducing the number of traps or defects through improved fabrication technique. It can therefore be removed without affecting the operation of the device.
Chapter 3  Flicker Noise in MOSFETs

After careful study of the origin of various flicker noise mechanisms, it becomes clear that the flicker noise of MOSFETs is contributed by both number fluctuation and mobility fluctuation. To achieve a good understanding of flicker noise in MOSFETs, we have to discuss the number fluctuation and mobility fluctuation models for MOSFETs in detail, and this is done in this chapter. The discrimination and unification of two mechanisms will be done in Chapter 4.

§3.1 Number Fluctuation Theory and Models for MOSFETs

Since McWhorter first proposed the surface number fluctuation model for noise in semiconductor devices in 1957 [McWh57], many researchers have applied it to explain flicker noise in MOSFETs. Presently, it is widely accepted that flicker noise in MOSFETs under most operating conditions is partly caused by the random trapping and detrapping process of charges by the oxide traps near the Si-SiO$_2$ surface [Fu72] [Reim84] [Klaa71]. For MOSFETs, the carrier density fluctuations is thought to be caused by interaction of free carriers with oxide traps via interface states. We firstly review Van Der Ziel’s microscopic model [Ziel88b] for MOSFETs from first principles, and then analyze Reimbold’s theory [Reim84] and Klaassen’s model [Klaa71] for different operating conditions of MOSFETs. Then, we will introduce some important number fluctuation models which are recently developed.

Figure 3.1 show the coordinate system for n-channel MOSFETs used in our analysis. With suitable notation changes, the analysis can also be applied to the p-channel MOSFETs as well.
In figure 3.1 x is in the length(L) direction of MOSFET, y is the width(w) direction, and z is perpendicular and pointing into the oxide.

As is well known, there are numerous traps distributed in the SiO₂ surface to within a distance of about 100Å from the Si-SiO₂ interface. These traps are defined as oxide and interface traps, and are schematically shown in figure 3.2. Interface traps are located at the Si-SiO₂ interface, and they are also called fast surface states since they can swiftly capture or emit carriers by SRH. An interface state with energy level at $E_T$ within the energy bandgap will act as a SRH trapping center. Oxide traps are distributed within about 100Å region from the interface. They first
exchange carriers with interface states by tunneling and then the interface states transit these carriers to Si's valence or conduction bands by SRH process. They are also called slow surface states since they capture or emit carriers relatively slowly.

![Diagram of MOSFETs with traps](image)

Fig 3.2 Traps' Distribution in MOSFETs.

Our analysis for interaction between oxide traps and interface states starts with the following equations described by Van der Ziel[Ziel86],

\[
\frac{d\Delta N_i}{dt} = g(\Delta N_i) - r(\Delta N_i), \quad g(\Delta N_i) = a\Delta N_i(\Delta N_T - \Delta N_i)
\]

and

\[
r(\Delta N_i) = b\Delta N_i(\Delta N_u - \Delta N_i)
\]  

(3.1.1)
where \( \Delta N_T = n_T(E)\Delta E\Delta x\Delta y\Delta z \) means the number of traps in the volume element \( \Delta x\Delta y\Delta z \) with energy \( \Delta E \), \( \Delta N_u = n_u(E)\Delta E\Delta x\Delta y \), and \( \Delta N_i \) and \( \Delta N_e \) the number of carriers in oxide traps and interface traps respectively.

At equilibrium, we substitute \( \Delta N_s = \Delta N_{s0} + \delta \Delta N_{e} \) and \( \Delta N_i = \Delta N_{i0} + \delta \Delta N_i \) to get
\[
d\delta \Delta N_i/dt = -a \Delta N_u + \Delta g(t) - \Delta r(t).
\] (3.1.2)

From equation (3.1.2), which is the typical Langevin noise equation, we can get the fluctuation of trapped electrons' number given by
\[
\overline{\delta \Delta N_i^2} = \Delta N_T f_i(1 - f_i)
\] (3.1.3)

Langevin noise equation also gives the trapped electron number's noise spectrum density as below
\[
S_{\Delta N_i}(f) = 4N_T(E)\Delta E\Delta x\Delta y f_i(1 - f_i)\frac{\tau}{1 + \omega^2\tau^2}.
\] (3.1.4)

In [Ziel86], a uniform distribution in traps for \( 0 < z < z_i \), and zero traps outside is assumed, and traps' relaxation time's relation with \( z \) is assumed as follows
\[
\tau = \tau_0 \exp(\gamma z), \quad \tau_1 = \tau_0 \exp(\gamma z_1) \quad \text{and} \quad \gamma = 10^8 \text{cm}^{-1}
\]

Then,
\[
\frac{dz}{z_1} = \frac{d\tau/\tau}{\ln(\tau_1/\tau_0)}
\] (3.1.5)

Combined (3.1.4) and (3.1.5), it is obtained,
\[
S_{\Delta N_i}(f) = \frac{N_T(E)\Delta E\Delta x\Delta y}{\gamma f_i(1 - f_i)} f_i(1 - f_i) \quad \text{for} \quad \frac{1}{\tau_1} < \omega < \frac{1}{\tau_2}
\] (3.1.6)
Since \( f_t(1-f_t) \) has a sharp peak at \( E = E_t \) as shown in figure 3.3, then we can define \( N_T(E_t)_{\text{eff}} \) by relation,

\[
N_T(E_t)_{\text{eff}} = \int_{-\infty}^{\infty} N_T(E)f_t(1-f_t)dE/kT.
\] (3.1.7)

Substituting (3.1.7) in (3.1.6) and integrating along \( y \), we got,

\[
S_{\text{AN}}(x, f) = \frac{N_T(E_t)_{\text{eff}}kTW\Delta x}{\gamma_f}
\] (3.1.8)

The above formula is based on the assumption \( S_{\text{AN}}(x, f) = S_{\Delta N}(x, f) \), which is only valid when MOSFETs are in strong inversion mode of operation.

---

**Fig 3.3 Energy Band Diagram in n-channel MOSFETs**
For arbitrary inversion state, [Reim84] derived $\Delta S_{ID}(x,f)$ as

\[
\Delta S_{ID}(x,f) = \frac{I_D^2 N_T(E_f)_{\text{eff}} kT w \Delta x}{\Delta N^2} \left( \frac{\delta \Delta N}{\delta \Delta N_i} \right)^2
\]  \hspace{1cm} (3.1.9)

with

\[
\frac{\delta \Delta N}{\delta \Delta N_i} = \frac{-C_n}{C_d + C_{it} + C_\alpha + C_n}, \hspace{1cm} (3.1.10)
\]

and where $C_d$, $C_\alpha$, $C_{it}$, and $C_n$ are capacitances from the depletion region, the oxide, interface traps, and channel charge respectively. Using $C_n = q^2 N/kT$ where $N$ is the electron density in the channel per unit area and integrating the above equation along the channel and the width, we get a general valid for weak and strong inversion at low drain bias as,

\[
S_{ld}(f) = \frac{q^4 I_D^2 N_T(E_f)_{\text{eff}}}{w LkT \gamma(C_d + C_\alpha + C_{it} + q^2 N/kT)^2 f}.
\]  \hspace{1cm} (3.1.11)

For MOSFETs in weak inversion $C_n = q^2 N/kT \ll C_d + C_\alpha + C_{it}$, so that we get

\[
S_{ld}(f) = \frac{q^4 I_D^2 N_T(E_f)_{\text{eff}}}{w LkT \gamma(C_d + C_\alpha + C_{it})^2 f}.
\]  \hspace{1cm} (3.1.12)

From (3.1.12), we note that $S_{ld}/I_D^2$ is constant as long as $C_n$ and $N_i$ variations versus biases remain small. The statement is verified by experiments in [Reim84] in which a plateau of the noise of MOSFETs in weak inversion was observed.

For MOSFETs in strong inversion $C_n = q^2 N/kT \gg C_d + C_\alpha + C_{it}$, so that

\[
S_{ld}(f) = \frac{q^4 I_D^2 N_T(E_f)_{\text{eff}}}{w LkT \gamma(q^2 N/kT)^2 f}.
\]  \hspace{1cm} (3.1.13)

Since the model at strong inversion treats $N_T(E_f)_{\text{eff}}$ as a constant, then $S_{ld}/I_D^2$ strongly depends on gate bias through $1/N^2$. This is in disagreement with Reimbold’s experiments which gave a much less dependence on $(V_G-V_t)$ at low bias. To improve this model, Klaasen proposed an expression,
where \( k \) is treated as a constant, and at strong inversion \( N_{1}(E_{i})_{\text{eff}} \) is not a constant, but it varies with \( \Delta N \), which, in turn, varies as \( (V_{G}-V_{T}-V(x)) \). With \( I_{D} = q\mu(\Delta N/\Delta x) dV/dx \), the \( S_{ID} \) expression is written as

\[
S_{ID} = \frac{1}{L^{2}} \int_{0}^{L} \Delta S_{ID}(x,x',t) dx' dx
\]

or

\[
S_{ID} = \frac{kI_{D}}{L^{2}f} \int_{0}^{L} q\mu(\Delta N/\Delta x)(dV/dx) \frac{1}{\Delta N} \Delta x dx = \frac{q\mu kI_{D}V_{D}}{L^{2}f}
\]

(3.1.15)

(3.1.16)

The gate input noise expressions can now be derived as

\[
S_{vG} = \begin{cases} 
\frac{qk}{C_{aw}L_{f}}(V_{G}-V_{T}-\frac{1}{2}V_{D}) & \text{in linear region} \\
\frac{qk}{2C_{aw}L_{f}}(V_{G}-V_{T}) & \text{in saturation region}
\end{cases}
\]

(3.1.17)

These expressions were experimentally verified in [Klaa71] where \( S_{vG} \) at saturation varied as \( (V_{G}-V_{T}) \), and it was inversely proportional to the input capacitance at the zero bias.

After basic number fluctuation model was built for MOSFETs by Van der Ziel, Reimbold, and Klaassen, many modifications on it have been made with consideration of mobility's field dependence, trap's energy dependence or its time constant distribution [Ghib87] [Hend88] [Klei90] [Fang91] [Wong91]. Here we introduce some important models which have been recently developed.

Based on Reimbold's expression (3.1.13), an improved drain current noise spectra was given by Ghibaudo in 1987 by considering the gate bias dependence of the effective mobility.
Applying the above equation under the strong inversion condition $C_a = q^2/N/kT >> C_d + C_{ox} + C_a$, the expression will be of the form,

$$S_{id}(f) = \frac{q^2kTq^2N_T(E_f)_{eff}}{fwL\gamma C_{ox}^2(V_G - V_T)^2 (1 + \theta(V_G - V_T))^2}.$$

(3.1.19)

In 1988, Hendriks re-evaluated (3.1.13) with consideration of field dependence of mobility and the energy dependence of the traps, and these are now described.

1. The oxide trap density is independent of energy and the mobility is independent of electric field. The drain current noise spectrum density is given by

$$S_{id} = \frac{q^2\mu N_T(E_f)_{eff}}{f\nu L^2 C_{ox}} \ln\left(\frac{V_G - V_T}{V_G - V_T - V_D}\right).$$

(3.1.20)

2. The oxide trap density is independent of energy level but the mobility depends on the electric field. $\mu = (\mu_0)/(1 + E/E_c)$ is taken into account with $E_c$ being critical field of electrons, they get

$$S_{id} = \frac{q^2\mu_0 N_T(E_f)_{eff}}{f\nu L^2 C_{ox}} \ln\left(\frac{V_G - V_T}{V_G - V_T - V_D}\right) - \frac{I_D V_D}{\mu_0 w C_{ox} E_c (V_G - V_T) (V_G - V_T - V_D)}. \quad (3.1.21)$$

3. The mobility is independent of electric field and the oxide trap density depends on the position of the Fermi level. Usually a U-shaped distribution is observed in the band gap where the density of states increases towards both bands edge [Schu83]. The effective trap density is related to the number of induced electrons per unit of length given by

$$N_T(E_f,x)_{eff} = \frac{qG_0 N(x)}{w C_{ox}} = G_0 (V_G - V_T - V(x)). \quad (3.1.22)$$

in which, $G_0$ is a proportionality constant. With (3.1.22), they get
Consideration of both the oxide trap density's energy-dependence and mobility's field-dependence. $S_{ID}$ is given by

\[ S_{ID} = \frac{q^2 \mu_0 G_o I_D}{f \gamma l^2 C_{ox}}. \]  

Later on, a detailed number fluctuation theory on MOSFETs around saturation region was done by Kleinpenning. It was pointed out that $\Delta N^2 = N_T$ violate the physical law $\Delta N^2 \leq N$ around saturation where $n(L) < n_T$. It is quite clear that the number fluctuation $\delta n$ must be less than the trap density $n_T(E_T)$ if the total number of carriers is less than $n_T(E_T)$. Therefore $n_T$ is replaced by $n_T n(x)/(n_T + n(x))$.

Now, the $S_{ID}$ expressions under saturation are as follows,

\[
S_{ID} = \begin{cases} 
\frac{2I_D^2 N_T}{f \gamma \tau_o N^2} \ln \left( \frac{N}{N_T} \right) & N \gg N_T \gg N_0 \\
\frac{2I_D^2 N_T}{f \gamma \tau_o N^2} \ln \left( \frac{N}{N_0} \right) & N \gg N_0 \gg N_T 
\end{cases}
\]  

where $N_0 = kT(C_d + C_s + C_{ox})/q^2$ and $N = C_{ox}(V_G - V_T - V(x))/q$.

The theories of 1/f noise in MOSFETs introduced above are generally used to describe the flicker noise phenomena based on the assumption of a traps' time constant of the form of $\tau = \tau_0 \exp(\gamma z)$.

A 1/f noise theory which is appropriate for strong inversion in the linear, sub-saturation and saturation regions, as well as for weak inversion in the whole range of drain voltage is proposed [Fang91] on the assumption that a trapping time constant is given by,
\[ \tau(E_a) = \tau_0 \exp(E_a/kT). \]  

(3.1.26)

where \( E_a \) is the activation energy dependent on the trap level, the conduction band level, etc. Therefore, the \( S_{ID} \) can be written as

\[ S_{ID}(f) = K_0 \int_{r_1}^{2} \int_{0}^{d} \int_{0}^{L} N_T(x, z, E_T) f_T(E_T)(1 - f_T(E_T)) \ \frac{x}{(C_{ox} + C_i + C_n + C_d)^2} \ \frac{\tau(E_a)}{1 + 4\pi^2 f^2 \tau^2(E_a)} D(\tau) dx dz d\tau \]

(3.1.27)

where \( K_0 = \frac{4I_D^2\beta^2 q^2}{L^2w} \delta^2 \).

Here, \( \beta = q/kT \) and \( \delta = 1 \) for weak inversion, \( \delta = 1/2 \) for strong inversion.

After integration, the following expressions can be obtained

\[
S_{ID} = \begin{cases} 
\frac{K_f I_D^2 N_T(E_T) \beta^2}{f n^2} & \text{in weak inversion} \\
\frac{K_f I_D^2 N_T(E_T)}{f (V_G - V_T)^2} \ln \frac{V_G - V_T + 2n/\beta}{V_G - V_T - V_D/2} & \text{in linear region (} V_D < V_{DS} \text{)} \\
\frac{K_f I_D^2 N_T(E_T)}{f V_D(V_G - V_T - V_D/2)} \ln \frac{V_G - V_T + 2n/\beta}{V_G - V_T - V_D} & \text{in subsaturation region (} V_D \leq V_{DS} \text{)} \\
\frac{K_f I_D^2 N_T(E_T)}{f LV_{DS}(V_G - V_T - V_{DS}/2)} \ln \frac{V_G - V_T + 2n/\beta}{V_G - V_T - V_{DS}} & \text{in saturation region (} V_D \geq V_{DS} \text{)} 
\end{cases}
\]

(3.1.28)

where \( K_f = q^2d_i/4LwC_{ox}^2 \) and \( n = (C_d + C_i + C_{ox})/C_{ox} \).  

(3.1.29)

Note that for integrating (3.1.27), the assumption of \( f_{\text{min}} = 0 \) and \( f_{\text{max}} = \infty \) has to be made. The parameter of the distance in the oxide over which the traps are distributed \( d_i \) is also difficult to be determined.

It was found in [Well86] that the trapping or detrapping time constant is energy-activated and could be either enhanced or inhibited by applied gate voltage.
Based on the studies of the different time constants of trapping and detrapping, a new expression on drain current noise spectra in MOSFETs was developed in [Wong91].

\[
S_{ID}(\omega) = \frac{(\pi/2)I_D^2kTN_T(E_f)_{eff}\tau_{oc}^{-\kappa}}{\sin(\pi \kappa/2)LW \omega^\kappa} \left\{ \frac{q^2}{C_{ox} V_G^2} - \frac{(\lambda_t + \lambda_d)q^2z}{\varepsilon_{ox} C_{ox}^2 V_G} \right\}
\]  

(3.1.30)

where \( \tau_{oc} \) is the proportionality constant for characteristic time constant which is related to proportionality constant for trapping and detrapping processes. \( \kappa \) is the frequency index of the noise spectrum. \( z \) is the traps location measured from the Si-SiO\(_2\) interface, \( \lambda_t \) and \( \lambda_d \) are the gate voltage sensitivity of trapping and detrapping processes respectively.

This model including terms \( C_{ox}^{-1} \) and \( C_{ox}^{-2} \) can explain the two kinds of results reported on the dependence noise power on oxide capacitance, the drain current power spectral density is proportional to \( C_{ox}^{-2} \), whereas the equivalent noise power measured by other groups is proportional to \( C_{ox}^{-1} \). It also could predict the temperature dependence of the frequency index.

§3.2 Mobility Fluctuation Theory and Models for MOSFETs

Based on Hooge’s empirical bulk relation, a bulk origin of low frequency noise in MOSFETs was first described in [Vand80a] [Vand80b]. From the agreement between the experimental results and the model calculations, it was concluded that the 1/f noise in some MOSFETs is a bulk effect caused by mobility fluctuations. To apply Hooge’s empirical relation to a more complicated structure like MOSFET, we have to integrate over the whole sample, all the small volumes on which the empirical relation can be applied. To that end an expression for the drain current is given,

\[
I_D(t) = q\mu(E)N(x)E(x) + H(x,t)
\]  

(3.2.1)
where \( \mu \) is the mobility, \( N \) is the number of electrons per unit of length, \( E \) is the electric source-drain field, \( H \) is a Langevin noise source and \( x \) is the position with respect to the source. By linearizing eqn (3.2.1) and integrating from source to drain, the spatial cross-spectral intensity of the drain current noise is given,

\[
S_{I_D}(f) = \frac{1}{L^2} \int_0^L \int_0^L S_H(x_1, x_2, f) dx_1 dx_2. \tag{3.2.2}
\]

In the Hooge model, the 1/f noise is caused by mobility fluctuations. The Langevin source function is given by

\[
H(x, t) = qN(x)E(x)\Delta \mu(x, t) = I_D \Delta \mu(x, t) / \mu(x) \tag{3.2.3}
\]

where \( \Delta \mu(x, t) \) is the fluctuations in mobility. The spatial cross-spectral density of \( H(x,t) \) is given by

\[
S_H(x_1, x_2, f) = \frac{I^2}{\mu^2(x)} S_{\mu}(x_1, x_2, f) = \frac{\alpha(x, E) I^2}{fN(x)} \delta(x_1 - x_2). \tag{3.2.4}
\]

On condition that \( \mu(x), \delta \mu(x)/dE \) and \( \alpha(x, E) \) are independent of \( x \), the expression (3.2.2) can be rewritten as

\[
S_{I_D}(f) = \frac{\alpha I^2}{fL^2} \int_0^L \frac{dx}{N(x)} \quad (\text{General Equation}) \tag{3.2.5}
\]

\[
= \frac{\alpha \mu I_D V_D}{fL^2} \quad (\text{Simplified Equation with } \alpha, \mu = \text{constant}). \tag{3.2.6}
\]

According to Van der Ziel, the above equation combined with \( g_m = (\mu w C_{ox} / L) V_D \), and \( S_{VG} = S_{I_D}(f)/g_m^2 \) yields the gate input referred noise spectrum

\[
S_{VG} = \begin{cases} 
q \frac{\alpha H}{C_{ox} w L f} \left( V_G - V_T - \frac{1}{2} V_D \right) & \text{with } I_D = \mu w \frac{C_{ox}}{L} \left( (V_G - V_T) V_D - \frac{1}{2} V_D^2 \right) \\
\frac{q \alpha H}{2C_{ox} w L f} (V_G - V_T) & \text{with } I_D = \frac{\mu w C_{ox}}{2L} (V_G - V_T)^2
\end{cases} \tag{3.2.7}
\]
Little is known about the spatial dependence of $\alpha$, and in some cases such as small geometry devices, a constant Hooge parameter $\alpha$ can be expected. The consideration of field-dependence of $\alpha$ will be taken into account in our new unified simulation-oriented flicker noise model in MOSFETs described in Chapter 4.

Considering the gate voltage dependence of effective mobility in strong inversion, $\mu_{\text{eff}} = \mu_0/\{1 + \theta(V_G - V_T - V(x))\}$, $S_{ID}$ for linear and saturation in MOSFETs has been given [Ghib90] as

$$S_{ID} = \begin{cases} \frac{\alpha_H I_D \mu_0 q}{fL^2} \log \left( \frac{1 + \theta(V_G - V_T)}{1 + \theta(V_G - V_T - V_D)} \right) & V_D < V_{DS} \\ \frac{\alpha_H I_D \mu_0 q}{fL^2} \left\{ \frac{1}{\theta} \log(1 + \theta(V_G - V_T) + (V_D - (V_G - V_T))) \right\} & V_D \geq V_{DS} \end{cases}$$

(3.2.8)

§3.3 Noise in Short-Channel MOSFETs

Recently a new 1/f noise expression for MOSFETs in saturation and deep saturation based on mobility fluctuation was described in [Deen91b]. It considers the profile of the electron number $N(x)$ per unit length along the channel and takes into account the velocity saturation region near the drain to evaluate the Hooge’s empirical relation as the Langevin source term of noise. The new model uses detailed one-dimensional expressions of $N(x)$ and channel potential $V(x)$ varying with channel position $x$ to calculate the gate referred noise spectral density $S_{vG}(f)$ in both saturation and deep saturation. The analytic variation of both $N(x)$ and $V(x)$ with $x$ has been corroborated with MINIMOS simulations.

This new model, which we term the "two-region model", divides the channel into two parts as shown in figure 3.4. Region I, the linear region, is the conductive channel from the source junction edge at $x=0$ to the velocity saturation point at
Region II, the velocity saturation region, is the depleted part of the channel $x=L''$ to the drain junction edge at $x=L$. Note that the distance between $L'$ and $L''$ was exaggerated to show their difference in figure 3.4.

**Short-Channel MOSFET**

In region I, the expression for the number of electrons per unit length along the channel [Klei90] is

$$n(x) = (V_{GS} - V_T - V(x))wC_{ox}/q$$  \hspace{1cm} (3.3.1)$$

with

$$V(x) = (1 - (1 - x/L)^{1/2})(V_{GS} - V_T).$$  \hspace{1cm} (3.3.2)$$

Fig 3.4 "Two-Region Model" for Short-Channel MOSFETs
Therefore \( n(x) = (V_{GS} - V_T)(1-x/L')^{1/2}WC_{ox}/q \) (3.3.3)

\( L' \), defined as \( L - \Delta L \) is given by [10]

\[
L' = L - (2\varepsilon_s/qN_A)^{1/2}(\sqrt{\phi_D} + (V_{DS} - V_{Dsat}) - \sqrt{\phi_D}).
\] (3.3.4)

In eqn. (3.3.4), \( \phi_D \) is given by \( \phi_D = \varepsilon_sE_i^2/2qN_A \), and \( E_i \) is the value of the channel field intensity in the x direction at the left end of the depletion region. Note that \( \phi_D \) can be a fitting parameter that is chosen for the best fit between the experimental data and resulting expressions for \( I_D \).

In region II, the carriers are considered to travel at the saturation velocity \( v_x \).

\( n_0 \) is therefore constant and can be determined from

\[
n_0 = I_D/qv_x = n(L'') = C_{ox}w(V_{GS} - V_T - V(L''))/q
\] (3.3.5)

with \( L'' = L'\left(1 - \left(\frac{\mu_{ef}(V_{GS} - V_T)}{2v_xL'}\right)^2\right) \). (3.3.6)

The comparison of \( N(x) \) and \( V(x) \) with MINIMOS simulations for 1.2\( \mu \)m MOSFET at \( V_{DS} = 4V \) and \( V_{GS} = 2V \) are shown in figure 3.5 and 3.6. Good agreement between the calculated \( N(x) \) and \( V(x) \) and simulated MINIMOS values are obtained. These results clearly demonstrate the need to carefully consider the variation of both \( N(x) \) and \( V(x) \) with biasing voltages in developing noise models for the MOSFETs.
Figure 3.5 Distribution of number of electrons $N(x)$ per unit length as a function of channel position from the source to the drain. NM represents the new model. Results from MINIMOS simulation are also shown.
Figure 3.5 Distribution of channel potential $V(x)$ as a function of channel position from the source to the drain. NM represents the new model. Results from MINIMOS simulation are also shown.
Details of the derivation of $L''$ are given in [Deen91b]. To obtain $S_{ID}$, we first derive an expression for $\Delta I_D$ by using [Klei90]

$$\Delta I_D(x) = \frac{I_D}{\mu_{eff}} x \Delta \mu_{eff}(x) + q \mu_{eff} d_n \{n(x) \Delta V(x)\}/dx,$$  \hspace{1cm} (3.3.7)

where eqn. (3.3.7) holds on the condition that $\mu_{eff}$ is independent of $x$. In the depletion region beyond the pinch-off point, electrons travel at saturation velocity, and are therefore not affected by $E(x)$ and $\mu_{eff}(x)$, so $\Delta I_D$ is given by

$$\Delta I_D = \frac{1}{L''} \int_0^{L''} \Delta I_D(x)dx = \frac{I_D}{\mu_{eff} L''} \int_0^{L''} \Delta \mu_{eff}(x)dx + \frac{q \mu_{eff} n(L'') \Delta V(L'')}{L''}.$$ \hspace{1cm} (3.3.8)

From the eqn. (3.3.1), we can rewrite $n(x)$ by expanding the $V_T$ term [Pulf89] to get

$$n(x) = \frac{wC_{ox}}{q} (V_{GS} - V_{FB} - 2\phi_F - V(x)) - \frac{W}{q} \sqrt{2\varepsilon_S q N_A (2\phi_F + V(x))}.$$ \hspace{1cm} (3.3.9)

Therefore, $\Delta n(L'')$ can now be evaluated as

$$\Delta n(L'') = -\frac{w \Delta V(L'')}{q} \left( C_{ox} + \frac{1}{2} \frac{2\varepsilon_S q N_A}{V(L'') + 2\phi_F} \right) = \Delta I_D/L''/q\nu_x.$$  \hspace{1cm} (3.3.10)

Defining

$$Z = -\nu_x w(C_{ox} + 1/2(2\varepsilon_S q N_A/V(L'') + 2\phi_F)^{3/2}),$$ \hspace{1cm} (3.3.11)

we obtain $\Delta V(L'') = \Delta I_D/L''/Z$. Defining $Z' = L'' - \mu_{eff} I_D/Z\nu_x$, and combining the above eqns. (3.3.8) to (3.3.11), we get

$$\Delta I_D = \frac{I_D}{\mu_{eff} Z'} \int_0^{L''} \Delta \mu_{eff}(x)dx.$$ \hspace{1cm} (3.3.12)

Converting eqn. (3.3.12) into spectral noise densities, we have

$$S_{ID}(f) = \frac{I_D^2}{(\mu_{eff} Z')^2} \int_0^{L''} S_{\mu}(x)dx = \frac{I_D^2}{Z^2} \int_0^{L''} S_{\mu}(x)dx.$$ \hspace{1cm} (3.3.13)

Using the relationship from [Jind81] that
\[
\frac{S_p(x)}{\mu^2} = \frac{\alpha}{fn(x)}
\]  

(3.3.14)

and combining eqns. (3.3.13) and (3.3.14), we obtain

\[
S_{ID}(f) = \frac{\alpha I_D^2}{fZ^2} \int_0^{L''} \frac{dx}{n(x)}.
\]  

(3.3.15)

Using eqn. (3.3.3) in eqn. (3.3.15), \( S_{ID}(f) \) can be evaluated as

\[
S_{ID}(f) = \frac{2\alpha I_D^2 q L'(1-(1-L''/L')^{1/2})}{fZ^2 \omega C_{ox}(V_{GS} - V_T)}.
\]  

(3.3.16)

The drain current expression is given as

\[
I_D = \frac{w \mu_{eff} C_{ox}(V_G - V_i)^2}{2L'}
\]  

(3.3.17)

and \( g_m \) as

\[
 g_m = \left. \frac{\partial I_D}{\partial V_{GS}} \right|_{V_{DS}} = \frac{2I_D}{(V_{GS} - V_T)}.
\]  

(3.3.18)

Therefore, the gate referred 1/f noise voltage spectral density is

\[
S_{VG}(f) = S_{ID}/g_m^2 = \frac{\alpha q (V_{GS} - V_T) L'(1-(1-L''/L')^{1/2})}{2fZ^2 \omega C_{ox}}.
\]  

(3.3.19)

A MOSFET is in the deep saturation when \( V_{D_{sat}} \geq L'(1-D)\nu_x/\mu_{eff} \). In this case, the drain current (excluding hot-carrier effects) is the maximum possible value and is given by

\[
I_D = \nu_x w C_{ox} V(L'').
\]  

(3.3.20)

Therefore, \( L'' \) can now be obtained [Deen91b] as
From eqn. (3.3.16) and $g_m = I_D/(V_{GS} - V_T)$, we obtain the gate-referred noise spectral density in deep saturation as

$$L'' = \frac{\mu_{eff}(V_{GS} - V_T)}{V_x} \left( 2 - \frac{\mu_{eff}(V_{GS} - V_T)}{V_x L'} \right). \quad (3.3.21)$$

The new model is derived from the fundamental equation of $1/f$ noise. It is simple and shows good agreement with the experimental results. In addition, we have studied the effects of the profile of $N(x)$ and $V(x)$ on the flicker noise in MOSFETs in saturation and deep saturation. The new model also reduces to the previously published model [Zie186] when appropriate approximations are made, i.e. if $L'' = \tilde{L} = L$, then eqns. (3.3.18) and (3.3.22) reduce to the original model.
Chapter 4  New Unified Simulation-Oriented Flicker Noise Model in MOSFETs

Although many models including number fluctuations and mobility fluctuations have been investigated on low frequency noise in MOSFETs, inconsistencies still exist between the theoretical modelling and experimental results. Here, we categorize the experimental results on low frequency in MOSFETs in table 4.1 and table 4.2 below,

<table>
<thead>
<tr>
<th>$S_{D}/I_{D}^{2}$</th>
<th>Device Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>↑</td>
<td>Length($L$)</td>
</tr>
<tr>
<td>↓</td>
<td>Width($w$)</td>
</tr>
<tr>
<td>↑</td>
<td>Trap density($N_{i}(E_{F})_{eff}$)</td>
</tr>
<tr>
<td>↑</td>
<td>↓ by $C_{ox}^{2}$</td>
</tr>
<tr>
<td>↑</td>
<td>↓ by $C_{ox}$ Oxide capacitance</td>
</tr>
</tbody>
</table>

* Table 4.1 Noise versus Device Parameters in MOSFETs

(↑ means increase, and ↓ means decrease)
In the past, there have been much results that have supported both the number fluctuation and mobility models for 1/f noise in MOSFETs. Because of this, a unified model combining both mobility and number fluctuation has been proposed to explain the experimental noise results in different MOSFETs.

The first unified MOSFET noise model is described in [Grab88], and this is simply a sum of mobility model and number fluctuation model given by,

### Table 4.2: Noise versus External Parameters in MOSFETs

<table>
<thead>
<tr>
<th>Noise Parameter</th>
<th>External Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{VG}$</td>
<td>Independence on $V_G$ in Some Experiments</td>
</tr>
<tr>
<td>$\uparrow$</td>
<td>$V_G \uparrow$ for Other Experiments</td>
</tr>
<tr>
<td>$\uparrow$</td>
<td>$V_D \downarrow$ for Linear Mode of Operation</td>
</tr>
<tr>
<td>$\uparrow$</td>
<td>Independence on $V_D$ for Saturation Mode of Operation</td>
</tr>
<tr>
<td>$S_{ID}/I_D^2$</td>
<td>Temperature (K) $\uparrow$ in Some Experiments</td>
</tr>
<tr>
<td>$\uparrow$</td>
<td>Temperature (K) $\downarrow$ in Some Experiments</td>
</tr>
<tr>
<td>Frequency index</td>
<td>Temperature(K), Gate Voltage (V)</td>
</tr>
<tr>
<td>$S_{ID}/I_D^2$</td>
<td>Fluctuate around Unity or Depend on Devices</td>
</tr>
</tbody>
</table>
with $K$ being $N_0 \gamma_0 \gamma_0^2 k_0/2$, $N_0$, the scattering centre density at the interface, $\gamma_0$, coefficient of phonon scattering on scattering centres, $\delta n_q$, the fluctuation in density of phonons on $\delta n_q$, $\gamma_2$, coefficient of transfer of $\delta n_q$, $\tau_{n1}$ and $\tau_{n2}$, minimum and maximum of relaxation time of traps, and $\tau_{\mu1}$ and $\tau_{\mu2}$, minimum and maximum of relaxation time of mobility fluctuation.

This model has neglected the strong correlation between the two fluctuation mechanisms, especially the mobility fluctuation of oxide traps' scattering caused by number fluctuation. Also, the application of this model is quite limited since the first part is only valid for strong inversion mode, while the second one is only good for ohmic mode of operation.

Another unified model was recently described in [Hung90a] and this model incorporated both the number fluctuation and surface mobility fluctuation mechanisms, and is valid for both linear and saturation operating mode of the MOSFETs. Considerations of the carrier number fluctuation and the mobility fluctuation which are coupled by oxide traps' scattering led to the drain current noise spectra given by

\[
S_{vc}(f) = \frac{q^2 kT N_{n0}(E_f)}{f \omega L c_0^2 \ln(\tau_{n2}/\tau_{n1})} + \frac{q^2 K (C_{ox}/q)^n V_{GT}^n (1 + \theta V_{GT})^2}{f \omega L c_0^2 \mu_0^2 \ln(\tau_{\mu2}/\tau_{\mu1})}
\]  

(4.1)

By introducing

\[
N_i(E_f) = A + BN + CN^2
\]  

(4.3)

into the above equation, the drain current noise spectra can then be written as
where $A$, $B$, and $C$ are fitting parameters related to the interface trap density around the Fermi level, and

$$
N_0 = C_{ox}(V_G - V_T0)/q,
$$

(4.5a)

$$
N_L = C_{ox}(V_G - V_T0 - aV_D)/q
$$

(4.5b)

With fitting parameter $A$, $B$, and $C$, the model showed good agreement to their experimental results.

However, this model requires three fitting parameters which is generally too much for modelling of low frequency noise in MOSFETs. The model also only considered mobility fluctuation caused by Coulombic scattering effect of the fluctuating oxide traps. Since the mobility fluctuation of MOSFETs is also contributed by surface-roughness scattering and phonon scattering which are very important in strong inversion, then these should be included in the model.

However, in order to build a unified model, we should firstly research in more detail on the trapping and scattering mechanisms in MOSFETs. Then low frequency noise in MOSFETs should be modelled from the first principles. As indicated in [Ziel88], devices with a small surface-to-volume ratio have bulk 1/f noise, whereas devices with a large surface-to-volume ratio have surface 1/f noise, unless the surface is very well-passivated.

Experiments reported by [Ambe83] on p-type substrate diffused and ion-implanted MOSFETs that there are three sources of 1/f noise,

1. number fluctuation 1/f noise which comes from the interaction of carriers with the surface oxide.

2. bulk mobility fluctuation 1/f noise which is due to the fluctuation in the bulk mobility.
(3) modulation 1/f noise. When the surface is inverted, electrons in the inversion layer begin to interact with oxide traps which gives rise to fluctuations in the surface potential. The surface potential in turn gives a 1/f modulation of the surface mobility, or a direct modulation of the bulk resistance.

In MOSFETs on a p-type substrate, the interaction of holes with traps in the surface oxide diminishes, and the surface 1/f noise due to number fluctuation of holes is gradually eliminated when $V_0$ passes the flat-band situation and the holes are repelled from the surface. However, there is still some residual 1/f noise left that can be attributed to hole mobility 1/f noise.

When an appreciable inversion layer is formed in and the surface part of the channel becomes n-type, electrons in the inversion layer begin to interact with surface traps and produce a surface potential fluctuating in a 1/f fashion. The surface potential fluctuating has two effects.

1) Direct resistance modulation. The fluctuating surface potential modulates the width of the inversion layer, and so the resistance of the channel-region, resulting in extra noise.

2) Surface mobility modulation. The fluctuating surface potential modulates the scattering of the holes in the inversion layer and this causes surface-controlled mobility fluctuations.

Experiments have been performed to prove in a unique way that the two existing noise sources, that is (1) number fluctuation and (2) bulk mobility fluctuation for the MOSFET, can be observed simultaneously, and that there is also (3) surface mobility fluctuation resulting from a surface potential fluctuation. For diffused MOSFETs, it was observed that the origin of 1/f noise switches from fluctuations in the bulk mobility to surface mobility fluctuation with the increase of gate voltage [Ambe83].
For ion-implanted n-channel MOSFETs, it was observed that the noise decreases at the onset of weak inversion. Since an inversion layer begins to be formed at the surface at that point, it was concluded that the decrease in noise is caused by elimination of trapping 1/f noise, because the holes can no longer reach the surface to interact with oxide traps. When this surface noise was eliminated, there was some 1/f noise remaining. It was attributed to bulk 1/f noise proposed by Hooge. The merit of this experiment is that it shows how surface 1/f noise can be turned off and on.

With further increase of gate voltage, the noise increased strongly again when strong inversion sets in. The reason is again that the electrons in the inversion layer can now interact with oxide traps and produce a surface mobility fluctuation. Special attention should be paid to the fact that these researches [Hung90] observed some correlation between bulk mobility and surface mobility for ion-implanted MOSFETs.

In the experiments in [Sury88], it was reported that low frequency noise in p-channel MOSFETs was caused by fluctuations in both the density and the surface mobility of the channel carriers through the modulation of the surface potential and the scattering rate respectively.

Both theories and experiments [Sury88] [Ziel86] have shown that when free carriers in the inversion layer are captured by the oxide traps, the local surface potential at the vicinity of the trap is changed, resulting in the fluctuation of the carrier density in the channel. Since the traps acted as scattering centers, the trapping and detrapping processes also result in the fluctuation of the carrier mobility due to the charging and discharging of the traps. Experiments reported in [Vand89] have verified that low frequency noise in MOSFETs drops when the conduction path changes from the surface to the bulk by varying gate voltages.

For MOSFETs, we should consider the mobility fluctuation induced by Coulombic scattering of the oxide traps [Hung91a], and the mobility fluctuation modulated
by phonon and surface roughness scattering in the channel [Sun80]. Taking these three factors into account, and using Matthiessen's rule [Hart80] to determine the effective mobility, we get

$$\frac{1}{\mu_{\text{eff}}} = \frac{1}{\mu_{\text{ox}}} + \frac{1}{\mu_{\text{sr}}} + \frac{1}{\mu_{\text{ph}}}$$

(4.6)

where the total mobility $\mu_{\text{eff}}$ is made up of contribution from the oxide traps scattering mobility $\mu_{\text{ox}}$, the surface roughness mobility $\mu_{\text{sr}}$, and the phonon scattering mobility $\mu_{\text{ph}}$ shown schematically in figure 4.1.

*Figure 4.1 Three Scattering Mechanisms in MOSFETs.*
Phonon scattering due to the various modes of lattice vibration including surface acoustic phonons and optical phonons which plays an important role in determining the mobility at strong inversion [Ezaw74]. The mobility fluctuation caused by phonon scattering can also be expected in surface inversion layers [Miko82] [Sun80].

Surface-roughness scattering is primarily due to the deviation of the interface from an ideal plane in MOSFETs. This type of scattering is important under strong inversion conditions because the strength of the interaction is governed by the distance of the carriers from the surface. The closer the carriers are to the surface, the stronger the scattering due to surface roughness [Sun80]. The mobility fluctuation is also contributed by the surface potential roughness fluctuation which arises from carriers non-uniformities in the channel, as already described in [Aoki77] [Cheng73] [Cheng74].

Coulombic scattering is due to charged centers, and it includes interface-state charge N_{it}, and oxide trapped charge N_{ox}. It was reported [Hung90A] that the oxide traps that are contributing to the flicker noise from the surface mobility fluctuation by Coulombic scattering. $1/\mu_{ox}$ depends nearly linearly on oxide traps [Hart80] and could be expressed as

$$\frac{1}{\mu_{ox}} = \eta N_i$$

where $\eta$ is the scattering parameter of the oxide traps including fast and slow traps, and $N_i$ is the trap density. Since $\eta$ is a function of the local carrier density due to screening effect given by

$$\eta = \frac{\pi m_e e^3}{16 e^2_{av} kT}$$

where $e_{av} = (e_{Si} + e_{ox})/2$ is the average dielectric constant of Si and SiO$_2$ and $m_e$ is the electron effective mass [Sah72]. Then the usual value for $\eta$ at room temperature is 2.7 $\times$ 10$^{-15}$. 
After differentiating equation (4.6), we obtain

\[ -\frac{\delta \mu_{\text{eff}}}{\mu_{\text{eff}}^2} = -\frac{\delta \mu_{\text{ph}}}{\mu_{\text{ph}}^2} - \frac{\delta \mu_{\text{sr}}}{\mu_{\text{sr}}^2} + \eta \delta N. \]  

(4.9)

Using Van der Ziel's MOSFETs' coordinate system shown previously in figure 3.1 and considering a section of channel with width \( w \) and length \( \Delta x \), the fluctuations in the channel carrier number and mobility can be given as,

\[ \frac{\delta I_D}{I_D} = \frac{\delta \Delta N}{\Delta N} + \frac{\delta \mu_{\text{eff}}}{\mu_{\text{eff}}}. \]  

(4.10)

Multiplying eqn. (4.9) throughout by \(-\mu_{\text{eff}}\) and incorporating it into eqn. (4.10), we get

\[ \frac{\delta I_D}{I_D} = \frac{\delta \Delta N}{\Delta N} - \eta \delta N \mu_{\text{eff}} + \frac{\delta \mu_{\text{ph}}}{\mu_{\text{ph}}^2} \mu_{\text{eff}} + \frac{\delta \mu_{\text{sr}}}{\mu_{\text{sr}}^2} \mu_{\text{eff}}. \]  

(4.11)

With \( \Delta N = Nw\Delta x \), \( \Delta N_i = N_iw\Delta x \) and \( R = (\delta \Delta N)/(\delta \Delta N_i) \), we can get

\[ \frac{\delta I_D}{I_D} = \left( \frac{R}{N} - \eta \mu_{\text{eff}} \right) \frac{\delta \Delta N_i}{w\Delta x} + \frac{\delta \mu_{\text{ph}}}{\mu_{\text{ph}}^2} \mu_{\text{eff}} + \frac{\delta \mu_{\text{sr}}}{\mu_{\text{sr}}^2} \mu_{\text{eff}}. \]  

(4.12)

Defining \( \alpha_1 = \alpha_{\text{ph}} (\mu_{\text{eff}}/\mu_{\text{ph}})^2 \) and \( \alpha_1 = \alpha_{\text{sr}} (\mu_{\text{eff}}/\mu_{\text{sr}})^2 \), we get that eqn. (4.12) reduces to

\[ \frac{\delta I_D}{I_D} = \left( \frac{R}{N} - \eta \mu_{\text{eff}} \right) \frac{\delta \Delta N_i}{w\Delta x} + \frac{\alpha_1}{\alpha_{\text{ph}}} \frac{\delta \mu_{\text{ph}}}{\mu_{\text{eff}}} + \frac{\alpha_1}{\alpha_{\text{sr}}} \frac{\delta \mu_{\text{sr}}}{\mu_{\text{eff}}}. \]  

(4.13)

From the number fluctuation,

\[ \frac{\delta I_D}{I_D} \bigg|_{\Delta N_i} = \left( \frac{R}{N} - \eta \mu_{\text{eff}} \right) \frac{\delta \Delta N_i}{w\Delta x} \]  

(4.14)

So that

\[ S_{\Delta \rho}(x, f) \bigg|_{\Delta N_i} = \left( \frac{I_D}{w\Delta x} \right)^2 \left( \frac{R}{N} - \eta \mu_{\text{eff}} \right)^2 S_{\Delta N_i}(x, f) \]  

(4.15)
From [Ziel86] on number fluctuation, and with $\tau = \tau_0 \exp(\gamma z)$, we have,

$$S_{\Delta N_i(x,f)} = \int_{E_v}^{E_C} \int_{0}^{w} \int_{0}^{T_{ex}} 4N_i(E,x,y,z) \Delta x f_i(1-f_i) \frac{\tau(E,x,y,z)}{1 + \omega^2 \tau^2(E,x,y,z)} dz dy dE$$

(4.16)

$$= N_i(E_p,x) \frac{kT_w \Delta x}{\gamma f}$$

(4.17)

where $\gamma$ is the attenuation coefficient of the electron wave function in the oxide with a typical value about $10^8 \text{ cm}^{-1}$, and $N_i(E_p)_{\text{eff}}$ is the effective oxide trap density.

From the mobility fluctuations,

$$S_b(f)_{\mu} = \frac{1}{L^2} \int_{0}^{L} S_{\Delta \mu}(x,f) \Delta x dx$$

$$= \frac{kTqI_D \mu_{\text{eff}}}{\gamma f L^2} \int_{0}^{V_{bs}} N_i(E_f)_{\text{eff}} \left(1 - \eta \mu_{\text{eff}} \frac{R}{N}\right)^2 \frac{R^2}{N} dV$$

(4.18)

From the mobility fluctuations,

$$\frac{\delta I_D}{I_D} \bigg|_{\mu_p} = \frac{\alpha_p}{\alpha_e} \frac{\delta \mu_p}{\mu_{\text{eff}}}$$

(4.19)

$$\frac{\delta I_D}{I_D} \bigg|_{\mu_s} = \frac{\alpha_s}{\alpha_e} \frac{\delta \mu_s}{\mu_{\text{eff}}}$$

(4.20)

Using Hooge's empirical equation given by

$$\frac{S_{\mu}(x)}{\mu_{\text{eff}}^2} = \frac{(\alpha_p + \alpha_s)}{fn(x)}$$

(4.21)

and using the relation $S_{I_D/I_D^2} = S_b/\mu^2$ and the Langevin spatial cross spectral density equation, we get

$$S_{b}(f)_{\mu} = \frac{I_D^2}{L^2} \int_{0}^{L} \frac{(\alpha_p + \alpha_s)}{fn(x)} dx$$

(4.22)

$$= \frac{I_D^2}{L^2} \int_{0}^{L} \alpha(x,E) dx$$

(4.23)
where \( \alpha = \frac{\alpha_1(1/\alpha_{ph} + 1/\alpha_{ne})}{\text{. Therefore, the total noise is given by (4.13), (4.18) and (4.23) and is,}
\[
S_D(f) = \frac{kTqI_D\mu_{\text{eff}}}{\gamma fL^2} \int_0^{V_{GS}} N_i(E_f)_{\text{eff}} \left(1 - \eta \mu_{\text{eff}} N R^{-1}\right)^2 \frac{dV}{N} + \frac{I_D^2}{L^2} \int_0^L \alpha_{\text{xe}}(x) \, dx. \quad (4.24)
\]

For the device in strong inversion, it has been reported in [Reim84] that \( R = C_n/(C_d+C_n+C_{ox}+C_{ib}) \), and \( C_n = q^2 N/kT \gg (C_d+C_n+C_{ox}+C_{ib}) \). Therefore, we get that (4.24) is reduced to
\[
S_D(f) = \frac{kTqI_D\mu_{\text{eff}}}{\gamma fL^2} \int_0^{V_{GS}} N_i(E_f)_{\text{eff}} \frac{(1 - \eta \mu_{\text{eff}} N)^2}{N} \, dV + \frac{I_D^2}{L^2} \int_0^L \alpha_{\text{xe}}(x) \, dx. \quad (4.25)
\]

It should be noted that the effective Hooge parameter has an electric field dependence [Klei81b] given by
\[
\alpha_1 = \frac{\alpha}{(1 + E/E_c)^2} \quad (4.26)
\]
and this is taken into account in the integration of eqn. (4.25). Also the effective trap density \( N_i(E_f)_{\text{eff}} \) depends on the position of the Fermi level, and the trap density could have a non-uniform energy distribution across the band-gap. Usually a U-shaped distribution [Schu80] is observed in the band gap where the density of states increases towards both bands edges. It can be assumed that the energy dependence is such that around the Fermi level, the effective trap density is proportional to the number of induced electrons per unit length [Park82], so that
\[
N_i(E_f)_{\text{eff}} = \frac{qG_0N}{wC_{ox}} = G_0(V_{GS} - V_t - V(x)) \quad (4.27)
\]
where \( G_0 \) is a proportionality constant, and the number of electrons in the channel [Klei90] is
\[
N = \frac{C_{ox}}{qL} (V_{GS} - V_t - V(x)) \quad (4.28)
\]
where \( V(x) \) is the channel potential of MOSFETs. Using the ideas and equations above, we now write the noise equations for the various modes of operation of the MOSFET.

(1) The **number fluctuation in linear region** for \( V_{DS} < V_{DS_{sat}} \)

\[
S_{\nu}^{\text{linear}} = \frac{kTqI_D\mu_{eff}}{2fL^2} \int_0^{V_{DS}} N_i(E) \left( 1 - \eta \mu_{eff} N \right) \frac{1}{N} dV
\]

(4.29)

\[
S_{\nu}^{\text{linear}} = \frac{kTqI_D\mu_{eff}}{2fL^2} \int_0^{V_{DS}} G_0(V_{GS} - V - V_i) \left( \frac{1}{N} - 2\eta \mu_{eff} + N\eta^2 \mu_{eff}^2 \right) dV
\]

(4.30)

\[
S_{\nu}^{\text{linear}} = \frac{kTqI_DG_0}{2fL_{eff}^2} \left( \frac{q \mu_{eff} V_{DS}}{C_{ox}} - \eta \mu_{eff}^2 (2(V_{GS} - V_i) - V_{DS}) V_{DS} + \eta^2 \mu_{eff}^3 C_{ox} \left( \frac{(V_{GS} - V_i)^3 - (V_{GS} - V_i - V_{DS})^3}{3q} \right) \right)
\]

(4.31)

(2) The **mobility fluctuation in linear region** for \( V_{DS} < V_{DS_{sat}} \)

Using the \( \alpha \) model given by eqn. (4.26), and considering the horizontal electrical field' effect on effective mobility \( \mu = \mu_{eff}(1 + E/E_c) \) and assuming a uniform distribution of \( \alpha \), we get

\[
S_{\mu}^{\text{linear}}(f) = \frac{I_D^2}{L^2} \int_0^L \frac{\alpha(x,E)}{f \mu(x)(1 + E/E_c)^2} dx
\]

(4.32)

and using

\[
E = \frac{I_D}{wC_{ox}(V_{GS} - V_i - V))\mu_{eff}} \quad \text{and} \quad \beta = \frac{I_D}{wC_{ox}\mu_{eff}E_c},
\]

(4.33)

we finally get,


\[ S_{\mu}^{\text{linear}}(f) = \frac{q\alpha_{(0)}}{L^2} \int_0^{V_{DS}} \frac{dV}{f(1+I_D/(wE_c\mu_{\text{eff}}C_{ox}(V_{GS}-V_t-V))^3} \]

\[ = \frac{q\alpha_{(0)}I_D\mu_{\text{eff}}}{fL^2} \left( \frac{2y^3 + 3\beta^3 - 12\beta^2(y + \beta)}{2(y + \beta)^2} - 3\beta \ln(y + \beta) \right)^{y=V_{GS} - V_t} \quad (4.35) \]

(3) The number fluctuation in saturation region, for \( V_{DS_{\text{sat}}} < V_{DS} \)

\[ S_{\mu}(f)_{N_i}^{\text{sat}} = \frac{kTqI_D\mu_{\text{eff}}}{\gamma fL^2} \int_0^{V_{DS}} G_0(V_{GS}-V_t-V) \left( \frac{1}{N_i} - 2\eta\mu_{\text{eff}} + N\eta^2\mu_{\text{eff}}^2 \right) dV \quad (4.36) \]

\[ = \frac{kTqI_DG_0}{\gamma fL^2} \left( \frac{\mu_{\text{eff}}(V_{GS} - V_t)}{C_{ox}} - \eta\mu_{\text{eff}}^2(V_{GS} - V_t)^2 + \frac{\eta^2\mu_{\text{eff}}^2C_{ox}}{3q}(V_{GS} - V_t)^3 \right) \]

(4.37)

(4) The mobility fluctuation in saturation region for \( V_{DS_{\text{sat}}} < V_{DS} \)

Noting that the mobility-fluctuation based noise model for MOSFETs in the saturation region \( S_{\mu}^{\text{sat}}(f) \) is given by

\[ S_{\mu}^{\text{sat}}(f) = \frac{I_D^2}{fL^2} \int_0^L \frac{\alpha(x,E)}{n(x)} dx. \quad (4.38) \]

The derivation of eqn. (4.38) has already been described in chapter 3. The current noise spectral density is given by

\[ S_{\mu}^{\text{sat}}(f) = \frac{q\alpha_{(0)}I_D\mu_{\text{eff}}}{fL^2} \int_0^L \frac{dx}{n(x)(1+E/E_c)^2} \quad (4.39) \]

\[ = \frac{2\alpha_{(0)}I_D^2q}{fLwC_{ox}L(V_{GS} - V_t)} \left( 1 + H - \frac{H^2}{1 + H^2} + 2H \ln \frac{H}{H + 1} \right) \quad (4.40) \]

with \( H = (V_{GS} - V_t)/(2LE_c) \) and \( \alpha_{(0)} \) is Hooge effective parameter at very low field.

(5) The mobility fluctuation in saturation region for short-channel MOSFETs with \( V_{DS_{\text{sat}}} << V_{DS} \)
Noting that the mobility-fluctuation based noise model for MOSFETs in the
saturation region $S_{\mu}^{sat}(f)$ is given by

$$S_{\mu}^{sat}(f) = \frac{I_D^2}{fZ^2} \int_0^L n(x) \frac{\alpha(x,E)}{1 + E/E_c} \, dx.$$  \hspace{1cm} (4.41)

The derivation of eqn. (4.41) has already been described in chapter 3 and full
details are also given in [Zhu92b]. The current noise spectral density is given by

$$S_{\mu}^{sat}(f) = \frac{q\alpha_0 I_D \mu_{\text{eff}}}{fZ^2} \int_0^L \frac{dx}{n(x)(1 + E/E_c)^2}$$ \hspace{1cm} (4.42)

$$= \frac{\alpha_0 I_D^2 q}{fZ^2 W C_{ox} E_c H'} (1 + H' - \frac{H'^2}{1 + H'^2} + 2H' \ln \frac{H'}{H' + 1})$$ \hspace{1cm} (4.43)

with $H = (V_{GS}-V_i)/(2L'E_c)$ and $\alpha_0$ is Hooge effective parameter at very low field.

Combining eqns. (4.30) - (4.43), the physically-based unified model for 1/f noise
in MOSFETs in linear and saturation regions is finally obtained and is now described.

<table>
<thead>
<tr>
<th>For linear region, i.e. for $V_{DS} &lt; V_{DS_{sat}}$, we have</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_{J_D}^{linear}(f) = \frac{\alpha_0 I_D q \mu_{\text{eff}}}{f_{\text{eff}}^2} \left( \frac{2y^3 + 3\beta^3 - 12\beta^2(y + \beta) - 3\beta \ln(y + \beta)}{2(y + \beta)^2} \right) \bigg</td>
</tr>
<tr>
<td>$+ \frac{kTqI_D G_0}{f_{\text{eff}}^2} \left( \frac{q \mu_{\text{eff}} V_{DS}}{C_{ox}} - \frac{\eta_2^2 \mu_{\text{eff}}^2 (2(V_{GS} - V_i) - V_{DS}) V_{DS}}{3q} + \frac{\eta_2^2 \mu_{\text{eff}} C_{ox}}{3q} (V_{GS} - V_i)^3 - (V_{GS} - V_i - V_{DS})^3 \right)$</td>
</tr>
</tbody>
</table>

(4.44)
For on-set saturation region, that is for $V_{DS_{sat}} < V_{DS}$, we have

$$S_{ID_{on-sat}}(f) = \frac{2\alpha_{x0} f_{D} q}{fL_{eff}wC_{ox}(V_{GS}-V_{t})} \Big(1 + H - \frac{H^2}{1 + H^2} + 2H \ln \frac{H}{H+1} \Big)$$

$$+ \frac{kT q I_{D} G_{0}}{\gamma fL_{eff}^2} \Bigg( q \mu_{eff} C_{ox} (V_{GS}-V_{t}) - \eta \mu_{eff}^2 (V_{GS}-V_{t})^2 + \frac{\eta^2 \mu_{eff}^3 C_{ox}}{3q} (V_{GS}-V_{t})^3 \Bigg)$$

(4.4.45)

For saturation region in short-channel MOSFETs, that is for $V_{DS_{sat}} << V_{DS}$, we have

$$S_{ID_{sat}}(f) = \frac{\alpha_{x0} f_{D} q}{fL_{eff}^2 C_{ox} E_{c} H} \Big(1 + H' - \frac{H'^2}{1 + H'^2} + 2H' \ln \frac{H'}{H'+1} \Big)$$

$$+ \frac{kT q I_{D} G_{0}}{\gamma fL_{eff}^2} \Bigg( q \mu_{eff} C_{ox} (V_{GS}-V_{t}) - \eta \mu_{eff}^2 (V_{GS}-V_{t})^2 + \frac{\eta^2 \mu_{eff}^3 C_{ox}}{3q} (V_{GS}-V_{t})^3 \Bigg)$$

(4.4.46)

For weak inversion region, that is $C_{n} << (C_{d} + C_{x} + C_{ox} + C_{w})$

$$S_{ID}(f) = \frac{kT q I_{D} \mu_{eff}}{\gamma fL^2} \int_{0}^{V_{DS}} N_{x}(E_{x}) \Big(1 - \eta \mu_{eff} \frac{R^2}{N} \Big)^2 \frac{R^2}{N} dV + \frac{f_{D}^2}{L^2} \int_{0}^{L} \alpha_{x(E)} dx$$

$$- \frac{kT q I_{D} \mu_{eff}}{\gamma fL^2} \int_{0}^{V_{DS}} N_{x}(E_{x}) \left\{ 1 - \eta \mu_{eff} \left( \frac{C_{a}}{C_{ox} + C_{d} + C_{a} + C_{w}} \right) \right\} \frac{C_{n}(C_{ox} + C_{d} + C_{a} + C_{w})^2}{N} dV$$

$$+ \int_{0}^{L} \frac{f_{D}^2}{L^2} \alpha_{x(E)} dx$$

(4.4.47)
The advantages of the new physical-based incorporated and simulated-oriented low frequency noise model in MOSFETs in all operating regions are listed in the table 4.3.

<table>
<thead>
<tr>
<th>Fluctuation Mechanisms New Model Based on</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number Fluctuation</td>
</tr>
<tr>
<td>Correlation between Two Mechanisms</td>
</tr>
<tr>
<td>Mobility Fluctuation</td>
</tr>
<tr>
<td>Modelling Consideration</td>
</tr>
<tr>
<td>Modelling Parameters</td>
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<td></td>
</tr>
<tr>
<td>Simulation Purpose</td>
</tr>
<tr>
<td>Experiment Verification</td>
</tr>
<tr>
<td>-------------------------</td>
</tr>
<tr>
<td>Simulation Test</td>
</tr>
<tr>
<td>Future Research</td>
</tr>
</tbody>
</table>

Table 4.3 New Model's Validity and Limitations
Chapter 5 Experimental System

In this chapter, the flow chart with which all the experiments were performed is described. Experiments on DC characterization, and low frequency noise spectra and charge pumping measurements of MOSFETs were conducted with an automated microelectronics measurement system that was implemented for the microelectronics research group at school of engineering science, Simon Fraser University. It has been proven experimentally that the system is reliable and is fairly easy to use. The overall measurement flow chart is outlined in figure 5.1

§5.1 Devices Studied

The MOSFETs used in our low frequency noise experiments were packaged in dual-in-line 24-pin ceramic packages. The gate electrodes were degenerately doped n-type polysilicon, and the devices were implanted with Boron ions for threshold voltage adjustment. The gate oxide thickness was 25nm, and the effective surface substrate doping of the n-MOSFET was $2 \times 10^{16} \text{cm}^{-3}$. The MOSFETs in the test structure were individual devices without any input or output protection circuits. Test devices with a drawn channel width of 10 $\mu\text{m}$ and effective channel lengths of 0.9 to 3.2 $\mu\text{m}$
Flow Chart of High Temperature Measurement

Start

I-V Measurement

Device GOOD? No Disconnect Device

Yes

Apply Temperature

Delay

Noise/Gain and CP in Linear, Saturation

Final T Measured? No

Yes

Fig 5.1 Flow Chart of High Temperature Measurement
§5.2  DC Measurements in MOSFETs

D.C. measurements were performed to ensure that the devices had good I-V characteristics (low gate and substrate currents), both before and after the low frequency noise measurements. All transistor parameters such as threshold voltage \( V_T \), effective mobility \( \mu_{\text{eff}} \) and transconductance \( g_M \) were obtained from these D.C. measurements. The D.C. measurements are performed with an automated HP4145B Semiconductor Parameter Analyzer (SPA). Computer control programs developed for the D.C. measurements are SPA. The syntax of using this program is:

```
SPA or SPA1 <name of configuration file> <name of output file>
```

Most of the D.C. parameters are extracted from a data set obtained with the drain fixed at 0.1 V, the source and substrate connected to ground potential and a varying gate voltage from 0 to 5 V in steps 0.01V. The resolution of the HP4145 SPA is about 0.1 pA.

§5.3  High Temperature Low Frequency Noise Measurements

The test setup used for measuring low frequency noise in MOSFETs at high temperatures is shown in figure 5.2 and figure 5.3. The oven is also programmable. Therefore the test setup is all automated.

The circuit in figure 5.2 is to measure the gain of the device and a low noise amplifier is used in order to obtain the gate referred voltage spectral density (equivalent noise source at the gate).
Gain Measurement for MOSFETs

The circuit in Figure 5.3 is designed to measure the low frequency noise in MOSFETs. Biasing is provided by batteries and resistor networks made from low noise metal film resistors and wire wound potentionmeters.

MOSFETs' low frequency noise were measured from 273K to 423K at drain bias voltages ranging from 0.1 to 4.0V, and with fixed gate bias of 2.5V. During the measurements, the MOSFETs were placed in an programmable oven and the temperature was slowly increased to the desired value. The devices were thermally anchored to the oven base, and measurements were made after each temperature was
stable for at least 30 minutes, so as to minimize errors due to temperature variations or instabilities. The measurements were repeated on cooling to check for hysteresis due to temperature effects, but no measurable effects were found.

For the noise experiments, a dynamic signal analyzer, an ultra-low noise amplifier and a programmable frequency synthesizer were used. A computer program for taking the noise measurement was developed. The program is called ZNOISE and ZGAIN. This program is capable of controlling the automated measurement system to take the noise data as well as measure the gain data of device under test. The syntax of using this program is,

ZNOISE  <noise.cfg>  <name of output file>
ZGAIN  <gain.cfg>  <name of output file>
Noise Measurement for MOSFETs

Since for gain measurements the programmable frequency synthesizer generates the reference gain level, then the gain floor is measured at every gain measurement. The net gain is obtained by subtracting the gain floor from the output. Biasing was provided by batteries and resistor networks made from low noise metal film resistors and wire wound potentiometers. Since the noise levels being measured are very low, the noise must be amplified through a low noise amplifier. It is necessary to ensure that the 1/f noise of the amplifier is lower than that of the device under test. Therefore the test structures resistance was replaced by a resistor of the value equal to the MOSFETs' equivalent resistance to get the noise floor of the test.
system. For our experimental noise measuring system, the noise floor of the system was approximately -160dBV (\(=10^{-16}V^2/Hz\)) and the noise power for all device measurements was at least 10dBV higher than the noise floor of the system.

To minimize errors, the results are averaged at least 300 times. Also noise measurements were found to be repeatable to within +/- 0.5 dBV of the average value. Since the noise data used in analysis are usually between -90dBV and -120dBV, there is only a very small error in noise power at the analyzed frequency range.

§5.4 Charge Pumping Measurements

The charge pumping technique was first described in [Brug69] in which they applied periodic square pulses to the gate of a MOSFET whose source and drain were connected together. The average substrate current was monitored. When the device is pulsed into inversion, the surface becomes deeply depleted and electrons will flow from the source and drain regions into the channel, where some of them will be captured by the interface states. When the gate pulse is driving the surface back into accumulation, the mobile charge drifts back to the source and drain due to the high potential at these two terminals. But the charges trapped in the interface states will recombine with the majority carriers from the substrate and give rise to a net flow of negative charge into substrate.

Our charge pumping experiments were performed with the semiconductor parameter analyzer and a programmable waveform generator. The experimental setup is shown in figure 5.4.
In the measurements reported here, conventional charge pumping was done in order to the interface state density, even though more elaborate charge pumping experiments have been performed in our group [Li90]. A computer control program called CPSPA was implemented to conduct the automated charge pumping measurements. The syntax of using the program,

\[
\text{CPSPA } <\text{cp.cfg}>
\]

Using these experimental techniques, many results were obtained and some of the more important results are described in the next chapter.
Chapter 6 Results and Discussions

§6.1 Low Frequency Noise in MOSFETs at High Temperatures

The high-temperature performance of MOSFETs has attracted several studies, especially since MOSFETs are finding increasingly more applications in high-temperature electronics. The low noise performance of MOSFETs at high temperatures is very important for the production of a wide range of advanced high-temperature MOS integrated circuits, particularly for analog circuits which are much more sensitive to operating conditions such as voltage or current biases and operating temperatures. To optimize the low frequency noise performance in MOSFETs at high temperatures, a careful study and accurate simulation of it is urgently needed.

A detailed study of the temperature dependence of low frequency noise in MOS-transistors has been described in [Chri68a] where it was reported that the low frequency noise dependence on temperature is different for p-channel and for n-channel MOSFETs. In a p-channel MOSFET, the magnitude of the noise increases with temperature, while the magnitude of the noise decreases with increasing temperature for n-channel MOSFETs [Chri68b].

Recently, 1/f noise in the drain current in n-channel (100) Si-MOSFETs in the temperature range of 4.2 to 295K as a function of drain voltage was reported in [Hend87]. Here [Hend87], it was concluded that two noise sources contribute to the 1/f noise, which are a McWhorter-like number fluctuations noise and a mobility fluctuations noise at temperatures above T=130K, but between 4.2 and 130K, the 1/f noise is dominated by number fluctuation noise. From this temperature-dependent 1/f noise study, it was concluded that the contribution of the mobility fluctuation increases but the contribution of the number fluctuations decreases with increasing
temperature. Also, the experimental temperature dependence of 1/f fluctuations in Germanium and Silicon between 80 and 300K has been described in [Biss83]. There, it was found that the Hooge parameter \( \alpha_H \) strongly depends on temperature. When the temperature increases, \( \alpha_H \) increases, which means that the mobility fluctuation increases.

Since many important parameters of semiconductor devices such as interface state density and carrier mobility, and \( \alpha_H \) are very temperature dependent [Prul89], it is expected that the variation of these parameter with temperature will play an important role in determining the variation of low frequency noise with temperature. For low frequency noise arising from number fluctuations, trap density at different energy levels would be "probed" as the temperature changes. This, coupled with their different densities could result in complicated changes in number-fluctuation dominating 1/f noise. Moreover, the carrier mobility fluctuation could also result in a complicated temperature dependence 1/f noise in MOSFETs. A better understanding on low frequency noise at different temperatures is obviously needed.

Because most of previous research on low frequency noise in semiconductor devices were reported at either room temperature, or at low temperatures [Hafe89] [Hend87], we are motivated to study low frequency noise in MOSFETs at high temperatures. Measurements, modelling and theoretical analysis have been done using n-channel MOSFETs at temperatures from 298K to 423K and operating from linear to saturation region and the results are reported.

§6.1.1 D.C. Characteristics

Figure 6.1 shows the \( I_{DS} - V_{GS} \) characteristics for a 1.7\( \mu \)m n-MOSFET at four temperatures between 273K and 373K. Similar results were also obtained for the 1.2\( \mu \)m n-MOSFET at the six temperatures between 273K and 423K in step of
25K and these results are also shown in figure 6.1.

The decrease of drain current with the increase of temperature is due to the decrease of carrier mobility in the channel. In figure 6.2, we show the variation of threshold voltage $V_t$ with temperature for both devices at varying drain biases. As expected, $V_t$ decreased with increasing temperature in an almost linear fashion. This $V_t$-Temperature relationship can be approximated by the expression [Tsiv87]

$$V_t(T) = V_t(T_r) - k(T - T_r)$$  \hspace{1cm} (6.1.1.1)

where $T$ is absolute temperature, $T_r$ is room temperature and $V_t(T_r)$ is the threshold voltage at room temperature. The average of $k$ is 1.6mV/K for 1.2μm device, and 2.75mV/K for 1.7μm device, and all $k$ values for both devices are listed in table 6.1.

Figure 6.3 shows that the variation effective carrier mobility with temperature for 1.7μm device in which $\mu_{eff}$ decreases with increasing temperature. The $\mu_{eff}$-Temperature relation could be simply modelled by the expression,

$$\mu = \mu(T_r) \left( \frac{T}{T_r} \right)^{\beta}$$  \hspace{1cm} (6.1.1.2)

where $\mu(T_r)$ is the carrier mobility at room temperature and $\beta$ is a device fitting parameter. Similar results were also obtained for the 1.2μm device and table 6.1. shows the $\beta$ values for both the 1.2μm and 1.7μm devices at the four drain biasing voltages used.
Table 6.1 Listing of $k$ and $\beta$ values for the 1.2µm and 1.7µm MOSFETs. The average $k$ ($\overline{k}$) values for both devices are also listed.

<table>
<thead>
<tr>
<th>$V_{DS}(V)$</th>
<th>1.2µm</th>
<th>1.7µm</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k(mV/K)$</td>
<td>$\beta$</td>
<td>$k(mV/K)$</td>
</tr>
<tr>
<td>0.1</td>
<td>1.40</td>
<td>2.3</td>
</tr>
<tr>
<td>1</td>
<td>1.60</td>
<td>2.3</td>
</tr>
<tr>
<td>2.5</td>
<td>1.80</td>
<td>1.5</td>
</tr>
<tr>
<td>4</td>
<td>1.60</td>
<td>1.3</td>
</tr>
<tr>
<td>$\overline{k}$</td>
<td>----</td>
<td></td>
</tr>
</tbody>
</table>

The temperature increase should tend to increase the drain current by considering the term $(V_{GS}-V_{T}(T))$, and to decrease it by considering the term $\mu(T)$ for the device in saturation region and using the results in Figure 6.2 and 6.3 shown previously. From the experimental results, we note that the decrease of $\mu(T)$ with temperature dominates over $V_{GS}-V_{T}(T)$ for the $I_{DS}-V_{GS}$ curves measured, and shown previously in Figure 6.1.
Fig 6.1 D.C. Characteristics for Test MOSFETs
Fig 6.2 Variation of threshold voltage $V_t$ with Temperature for both 1.2μm and 1.7μm MOSFETs. Note that $V_t$ values at different drain biases are also shown. Values for the fitting parameter $k$ using eqn. (6.1.1) are given in table 6.1.
Fig 6.3 Variation of effective mobility with Temperature for the 1.7μm MOSFET at four drain biases. A similar $\mu_{\text{eff}}$-Temperature variation was found for the 1.2μm device and values for the fitting parameter $\beta$ using eqn. (6.1.1.2) are given in table 6.1.
§6.1.2 Low Frequency Noise

Figure 6.4 shows that the input referred gate noise spectra $S_{VG}(f)$ (where $S_{VG} = S_{1b}/g_M^2$ and $g_M$ is the device’s transconductance) for $1.7\mu m$ at $V_{GS}=2.5$V and $V_{DS}=4.0$V are proportional to $1/f^\beta$ with $\beta$ close to 1 at the frequencies ranging from 3Hz to 100kHz and at four temperatures. The measurements of $S_{VG}(f)$ for $1.2\mu m$ at $V_{GS}=2.5$V and $V_{DS}=4.0$V showed that it was proportional to $1/f^\beta$ with $\beta$ close to 1 at the lower frequencies up to around 100Hz at all six temperatures used.

The noise spectra in figure 6.4 for $1.7\mu m$ device were also modelled as a combination of $1/f$ noise and generation-recombination (GR) noise. From the decomposed GR noise spectra, it is obtained that the trapping time constants at each temperature were determined, and from Arrhenius-like plots, the activation energies and capture cross-section were calculated. It is found that the activation energies in the range of 20 to 110mV and capture cross-sections in the range of $10^{-20}$ to $10^{-21}$ cm$^2$. These low capture cross sections indicated that the traps are negatively charged. For the $1.2\mu m$ device, no evidence for generation-recombination noise was found.
Fig 6.4 $S_{VG}(f)$ versus Frequency for the 1.7μm device at temperatures from 298K to 373K and at $V_{GS} = 2.5V$. These spectra were decomposed into 1/f and g-r noise components. The 1/f noise component was fitted using the theory developed in chapter 4.
Before modelling the 1/f noise for 1.2μm and 1.7μm devices at different temperatures and different bias points, the relative contribution of the number fluctuation or mobility fluctuation noise mechanisms in devices should be determined. Using Klaassen’s theory [Klaa72] for high-inversion MOSFETs’ 1/f noise, it is possible to discriminate the number fluctuation noise and mobility fluctuation noise. From [Klaa72], $S_{1D}(f)$ is given by

$$S_{1D}(f) = \frac{q\mu_{\text{eff}}\alpha_H I_D}{L^2 f} \int_0^{V_{DS}} \frac{dV}{dx} \frac{dV}{dx} = \frac{q\mu_{\text{eff}}\alpha_H I_D V_{DS}}{L^2 f}$$

(6.1.2.1)

where $\alpha_H$ is effective Hooge parameter, $\mu_{\text{eff}}$ is effective channel mobility, and the other parameters have usual meanings. Klaassen’s model holds for both number-fluctuation and mobility-fluctuation sources of 1/f noise. They differ only in the value of $\alpha_H$. For number fluctuation, $\alpha_H$ is believed to be constant, whereas for mobility fluctuation, $\alpha_H$ depends on the electric field. Therefore, using the expressions

$$\mu_{\text{eff}} = \frac{\mu_0}{1 + E/E_c}, \quad \text{and} \quad \alpha_H = \frac{\alpha_{H0}}{1 + (E/E_c')^2}$$

with eqn. (6.1.2.1), we get for the number-fluctuation model, that

$$S_{1D}(f) = \frac{q\mu_0 \alpha_H I_D}{L^2 f} \int_0^{V_{DS}'} \frac{dV}{1 + E/E_c'}$$

(6.1.2.2)

and for the mobility-fluctuation model, that

$$S_{1D}(f) = \frac{q\mu_0 \alpha_H I_D}{L^2 f} \int_0^{V_{DS}'} \frac{dV}{(1 + E/E_c')(1 + (E/E_c')^2)}$$

(6.1.2.3)

In eqns. (6.1.2.2) and (6.1.2.3), $E_c$ is the critical field for electron velocity saturation, $E'_c$ is the critical field for the saturation value of $\alpha_H$, $\alpha_{H0}$ is the Hooge’s dimensionless constant, and $\mu_0$ is the channel mobility at very low electric fields.
When considered as a function of $V_{DS}$, $S_{ID}(f)$ in at a fixed $V_{DS}$ in eqn. (6.1.2.2) saturates at the saturation voltage $V_{DS\text{sat}}$ of the MOSFET, whereas $S_{ID}(f)$ in eqn. (6.1.2.3) goes through a clear maximum well before MOSFET saturation voltage is reached. In this way discrimination between the two models becomes possible [Ziel86].

To discriminate the number fluctuation noise and mobility fluctuation noise for 1.2$\mu$m and 1.7$\mu$m devices, figure 6.5 is plotted to show that $S_{ID}(f)$ versus $V_{DS}$ at a fixed $V_{GS}=2.5\text{V}$ and at temperatures of 323 and 373K. From this graph, it is clearly seen that $S_{ID}(f)$ for 1.7$\mu$m device saturates when $V_{DS\text{sat}}$ is reached, but $S_{ID}(f)$ for 1.2$\mu$m device goes through a maximum before $V_{DS\text{sat}}$ is reached. Therefore, we concluded that the mobility fluctuation dominates the 1/f noise in 1.2$\mu$m device and the number fluctuation dominates the 1/f noise in 1.7$\mu$m device. Later, these conclusions will be further verified using the analysis on their noise characteristics with varying high temperatures.
Fig 6.5 $S_{ID}(f)$ versus Drain Voltage for both 1.2$\mu$m and 1.7$\mu$m MOSFETs at temperatures of 323K and 373K. The symbols are the experimental data and the lines are used to connect these data to show $S_{ID}(f)$ for 1.7$\mu$m device saturates at the saturation voltage $V_{DS_{sat}}$, whereas $S_{ID}(f)$ for 1.2$\mu$m device shows a maximum well before $V_{DS_{sat}}$ is reached.
Using eqns. (4.4.44), (4.4.45) and (4.4.46) in chapter 4, we calculate the \( S_{b}(f) \) for 1.2\( \mu \)m and 1.7\( \mu \)m devices at high temperatures and at several frequencies. Figure 6.6 shows good agreement between the calculated and experimental results at \( f=10Hz \) and at all four temperatures and four drain bias voltages. Similar good agreement was obtained between experiment and calculated results at other frequencies like at \( f=14Hz \) shown in figure 6.7.

To show the improvement of the new model over the previous model described in Klaassen model, figure 6.8(a) and figure 6.8(b) shows that the variation of \( S_{VG} \) as a function of drain voltage for a 1.7\( \mu \)m device at four temperatures between 298K and 373K and at \( f=10Hz \) and \( f=14Hz \). It can be seen that in the linear region, the new model’s \( S_{VG} \) versus \( V_{DS} \) slope is steeper, and it fits the data better than the old model because of the inclusion of the field dependence in the Hooge’s parameter. In the saturation region, \( S_{VG} \) in the new model increases slightly faster and is in better agreement with the experimental results than the old model given earlier by eqn. (6.1.2.1). The reason for this improved fitting in the new model is because we have taken into account the fact that mobility fluctuation results from three kinds of scattering mechanisms, and we have also considered the short-channel velocity saturation effect, the field dependence of Hooge parameter \( \alpha \) and the energy dependence of oxide trap density.

Similar good agreement was obtained between experiment and calculated results for 1.2\( \mu \)m at \( f=14Hz \) and \( f=114Hz \) shown in figure 6.9(a) and 6.9(b).
Fig 6.6 Variation of $S_{ID}(f)$ at $f=10\text{Hz}$ with Drain Voltage for the 1.7$\mu$m MOSFET at temperatures from 298 to 373K. The symbols are the experimental data and the lines are calculated results using the new model. Note the good agreement between calculated and experimental results.
Fig 6.7 Variation of $S_{ID}(f)$ at $f=14\text{Hz}$ with Drain Voltage for the 1.7\text{$\mu$m} MOSFET at temperatures from 298 to 373K. The symbols are the experimental data and the lines are calculated results using the new model. Note the good agreement between calculated and experimental results.
Fig. 6.8(a) Variation of $S_{VG}(f)$ at $f=10\text{Hz}$ with Drain Voltage for the 1.7$\mu$m MOSFET at temperatures from 298 to 373K. Note the reasonably better agreement between calculated(solid lines) and experimental results than previously used model. The dashed lines show the simulation results from eqn. (6.1.2.1).
Fig. 6.8(b). Variation of $S_{VG}(f)$ at $f=14\text{Hz}$ with Drain Voltage for the 1.7$\mu$m MOSFET at temperatures from 298 to 373K. Note the reasonably better agreement between calculated(solid lines) and experimental results than previously used model. The dashed lines show the simulation results from eqn. (6.1.2.1).
Fig. 6.9(a) Variation of $S_{V_{GS}}(f)$ at $f=14\text{Hz}$ with Drain Voltage for the 1.2µm MOSFET at temperatures from 373 to 423K. Note the reasonably better agreement between calculated(solid lines) and experimental results than previously used model. The dashed lines show the simulation results from eqn. (6.1.2.1).
Fig. 6.9(b). Variation of $S_{\text{Ve}}(f)$ at $f=114\text{Hz}$ with Drain Voltage for the 1.2$\mu$m MOSFET at temperatures from 373 to 423K. Note the reasonably better agreement between calculated(solid lines) and experimental results than previously used model. The dashed lines show the simulation results from eqn. (6.1.2.1).
Re-examining figure 6.8(a) and 6.8(b) again, we note that the variation of $S_{ID}(f)$ for a 1.7µm device at a fixed drain bias decreases as the temperature increases. This variation in $S_{ID}(f)$ with temperature also verifies our earlier conclusion that $S_{ID}(f)$ in the 1.7µm device is dominated by number fluctuation. This could be explained as follows. First assume that the trap density has a U-shaped distribution is across the band gap and it increases towards both bands edges. Then since the effective trap density depends on the position of the Fermi level, and when the temperature increases, the Fermi level decreases towards the mid-gap, the net result is that the number of traps at the Fermi level decreases with increasing temperature.

From previous discussions, we concluded that the low frequency noise in the 1.2µm MOSFET was due to mobility fluctuations and in generally increased with temperature and noise in the 1.7µm MOSFET was due to number fluctuations and in generally decreased with temperature. These are shown in figure 6.10(a) and 6.10(b) in which $S_{ID}(f)$ shows an increasing trend with temperature for 1.2µm MOSFET and an decreasing trend with temperature for 1.7µm MOSFET. The lines drawn are calculations using the parameter values listed in table 6.2 below. As shown, there is reasonable agreement between experiment and calculated results for this device.

As defined in the new model, $\alpha_{(0)}$ is the dimensionless Hooge parameter which is independent of electric field. $G0$ is the proportional constant of the effective trap density with the channel potential as the expression $N_i(E_f)_{eff} = G_0(V_{GS} - V_t - V(x))$. $G0'$s unit is cm$^3$eV$^{-1}$V$^{-1}$ and independent of the electric field in the channel. Both $\alpha_{(0)}$ and $G0$ are varying with temperature.
Table 6.2: Listing of $\alpha_{(0)}$ and $G_0$ as a function of Temperature for both MOS-FETs.

<table>
<thead>
<tr>
<th>L</th>
<th>1.2$\mu$m</th>
<th>1.7$\mu$m</th>
</tr>
</thead>
<tbody>
<tr>
<td>T (K)</td>
<td>$\alpha_{(0)}$</td>
<td>$\alpha_{(0)}$</td>
</tr>
<tr>
<td>298</td>
<td>4.0x10$^{-5}$</td>
<td>5.6x10$^{-4}$</td>
</tr>
<tr>
<td>323</td>
<td>3.5x10$^{-5}$</td>
<td>3.1x10$^{-4}$</td>
</tr>
<tr>
<td>348</td>
<td>5.5x10$^{-5}$</td>
<td>2.2x10$^{-4}$</td>
</tr>
<tr>
<td>373</td>
<td>1.75x10$^{-4}$</td>
<td>2.2x10$^{-4}$</td>
</tr>
<tr>
<td>398</td>
<td>2.3x10$^{-4}$</td>
<td>-----</td>
</tr>
<tr>
<td>423</td>
<td>2.9x10$^{-4}$</td>
<td>-----</td>
</tr>
</tbody>
</table>
Fig 6.10(a) Variation of $S_{1D}(f)$ with Temperature for the 1.2µm MOSFET at temperatures from 298 to 423K. The symbols are the experimental data and the lines are the calculated results using the new unified model.
Fig 6.10(b) Variation of $S_{D}(f)$ with Temperature for the 1.7μm MOSFET at temperatures from 298 to 398K. The symbols are the experimental data and the lines are the calculated results using the new unified model.
Charge pumping experiments were conducted to get the value of \( N_i \). The charge pumping current can be found from the basic Groeseneken equation [Groe84]. The total charge involved in the charge pumping process can be determined by the integration over the energy interval swept by the gate pulses, and is given by

\[
Q_{ss} = A_{eff} q \int N_i(E) dE = A_{eff} q \bar{N}_i \Delta \psi_s
\]  

(6.1.2.4)

These charges are being moved at frequency \( f \), and they form the charge pumping substrate current \( I_{cp} \), given by

\[
I_{cp} = f Q_{ss} = f A_{eff} q \bar{N}_i q \Delta \psi_s
\]  

(6.1.2.5)

where \( A_{eff} \) is the effective area of the gate, \( \bar{N}_i \) the interface trap density, and \( \Delta \psi_s \) the total sweep of the surface potential about \( 2\phi_F \).

The oxide trap density can be calculated on the assumption that the oxide traps are distributed within \( z \) (around 100Å) distance from the interface,

\[
N_i = \frac{I_{cp}}{q f A_{eff} q \Delta \psi_s z}
\]  

(6.1.2.6)

\( N_i \)'s value, \( 1.72 \times 10^{17} \text{ cm}^{-3}\text{eV}^{-1} \), from the charge pumping experiment is in good agreement with the constant trap’s density \( G_0 \) obtained from the noise measurements. The charge pumping result is shown in figure 6.11.
Fig 6.11 The result of $N_t$ from the charge pumping experiment is in good agreement with the constant trap's density $G_0$ obtained from the noise measurements.
§6.2 Verification of New Noise Model with Other Published Results

To further test the validity of the new simulation-oriented noise model, we have to simulate the noise characteristics of both n-channel and p-channel MOSFETs. We also have to compare the simulation results with experimental results published in other literatures. In [Hung90A] and [Hung90B], a lot of flicker noise experimental data were presented with detailed device parameters. These noise data including n-channel and p-channel MOSFETs are digitized from the literatures.

These digitized experimental data are the drain current noise power at 100Hz and plotted as a function of gate bias. A detailed device parameters given in [Hung90A] and [Hung90B] are extremely helpful for us to accurately simulate the flicker noise in their MOSFETs. The MOSFETs’ parameters used are listed in table 6.3.

<table>
<thead>
<tr>
<th>MOSFETs</th>
<th>Type</th>
<th>Process</th>
<th>L(μm)</th>
<th>W(μm)</th>
<th>$T_{ox}$(nm)</th>
<th>$N_{sub}$(cm$^{-3}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>C7_1W10L5</td>
<td>N</td>
<td>CMOS</td>
<td>4.5</td>
<td>9.5</td>
<td>50.0</td>
<td>1.0x10$^{15}$</td>
</tr>
<tr>
<td>P2W4L5_10</td>
<td>P</td>
<td>PMOS</td>
<td>5.0</td>
<td>4.0</td>
<td>8.8</td>
<td>1.0x10$^{14}$</td>
</tr>
<tr>
<td>N3_2W5L5_0</td>
<td>N</td>
<td>NMOS</td>
<td>4.5</td>
<td>4.5</td>
<td>8.6</td>
<td>5.0x10$^{17}$</td>
</tr>
<tr>
<td>STXW20L3</td>
<td>N</td>
<td>CMOS</td>
<td>1.9</td>
<td>20.0</td>
<td>28.5</td>
<td>2.6x10$^{16}$</td>
</tr>
<tr>
<td>STXW20L175</td>
<td>N</td>
<td>CMOS</td>
<td>0.65</td>
<td>20.0</td>
<td>28.5</td>
<td>2.6x10$^{16}$</td>
</tr>
</tbody>
</table>

Table 6.3: Listing of Device Parameters in Published Data
By using a unified model presented in [Hung90a] and [Hung90b], the simulation process has to choose the following fitting parameters to meet the experimental results shown in table 6.4.

<table>
<thead>
<tr>
<th>Devices(μm)</th>
<th>A(cm⁻²eV⁻¹)</th>
<th>B(cm⁻¹eV⁻¹)</th>
<th>C(cm eV⁻¹)</th>
<th>N⁺(cm⁻²)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5μm NMOS (CMOS)</td>
<td>1.5x10¹⁶</td>
<td>5.1x10⁴</td>
<td>-1.4x10⁻⁸</td>
<td>2.0x10¹⁰</td>
</tr>
<tr>
<td>5.0μm PMOS</td>
<td>9.9x10¹⁴</td>
<td>2.4x10³</td>
<td>1.4x10⁻⁸</td>
<td>2.0x10¹⁰</td>
</tr>
<tr>
<td>4.5μm NMOS (NMOS)</td>
<td>1.0x10¹⁶</td>
<td>-2.2x10⁴</td>
<td>2.6x10⁻⁸</td>
<td>2.0x10¹⁰</td>
</tr>
<tr>
<td>1.9μm NMOS</td>
<td>4.1x10¹⁶</td>
<td>3.9x10⁴</td>
<td>-4.6x10⁻⁹</td>
<td>2.0x10¹⁰</td>
</tr>
<tr>
<td>0.65μm NMOS</td>
<td>6.7x10¹⁶</td>
<td>9.4x10³</td>
<td>-1.6x10⁻⁹</td>
<td>2.0x10¹⁰</td>
</tr>
</tbody>
</table>

Table 6.4: Listing of Fitting Parameters by Hung's Model to Simulate Published Data

Using the new unified simulation-oriented model proposed in chapter 4, the good simulation results have been achieved. Instead of using three fitting parameters A, B, and C, the new simulation model uses the density of oxide traps and Hooge parameter shown in table 6.5.

<table>
<thead>
<tr>
<th>Devices(μm)</th>
<th>α₀(0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.5μm NMOS (CMOS Technology)</td>
<td>3.0x10⁻⁵</td>
</tr>
<tr>
<td>5.0μm PMOS (PMOS Technology)</td>
<td>1.0x10⁻⁶</td>
</tr>
<tr>
<td>4.5μm NMOS (NMOS Technology)</td>
<td>1.0x10⁻⁷</td>
</tr>
<tr>
<td>1.9μm NMOS (CMOS Technology)</td>
<td>8.0x10⁻⁷</td>
</tr>
<tr>
<td>0.65μm NMOS (CMOS Technology)</td>
<td>7.0x10⁻⁷</td>
</tr>
</tbody>
</table>

Table 6.5: Listing of α₀(0) and G0 for different devices.
\(\alpha_{(0)}\) used in the new model is field independent and is \(10^3\) or \(10^4\) smaller than the \(\alpha_{eff}\) which is around \(2 \times 10^3\). But \(\alpha_{(0)}\) used here is reasonable agreement with published Hooge parameter used in many previous models. \(G_0\) is related to the effective trap density \(N_t(E_f)_{eff}\) as \(N_t(E_f)_{eff} = G_0(V_{GS} - V_t - V(x))\). \(V(x)\) is the channel potential.

Using eqn. (4.4.44), (4.4.45) and (4.4.46) to simulate the results in [Hung90b]. It shows the good agreement between the simulation and experimental results of a 4.5\(\mu\)m NMOS device fabricated by CMOS technology in figure 6.12. The symbols are digitized data and lines are simulation results based on the new model. All device parameters used in the simulation process is according to the table 6.3. The experimental data are extracted both in the operation mode of linear region and in the operation mode of saturation. Therefore the model including linear and saturation are used to simulate the results. The figure 6.12 is plotted as drain current noise spectrum density versus gate voltages. Every line represents a different drain voltage from 0.3V to 4V. The symbols are the published experimental data and the lines are the calculated results from the new model.

With suitable notation changes, the new simulation-oriented model could be applied to PMOS devices. It can be clearly seen that the good agreement between the simulation and experimental results of a 5.0\(\mu\)m PMOS device fabricated by a PMOS Technology in figure 6.13. As the gate voltage negatively increases, the drain current noise spectrum density also increases for a fixed drain voltage. The model also predicts the same trend.

The trend is also verified in figure 6.14 which shows the reasonable agreement between the simulation and experimental results of a 4.5\(\mu\)m NMOS device fabricated by a submicrometer NMOS technology.
Using the new unified model to the experimental results from the 1.9\textmu m NMOS device fabricated by a CMOS technology, it shows a disagreement between the simulation and experimental results.

It could be seen in figure 6.16 that some disagreement between the simulation and experimental results of a 0.65\textmu m NMOS device fabricated by a CMOS technology.

This disagreement shows some limitations of the model. As indicated in the beginning of the chapter 4, some experimental results demonstrate that the drain current noise spectrum density is independent of the variation gate voltage. That means the effective oxide density is independent of the gate bias. However, in our model we predict the relation of effective oxide density as 

\[ N_e(E_f)_{\text{eff}} = G_0(V_{GS} - V_t - V(x)). \]

This is why discrepancy between the experimental results and simulation results originates. If the constant effective oxide density is used at the beginning of derivation, a better agreement between the experimental results and simulation could be achieved.

In order to solve the limitation, the model could be improved to use two kinds of dependence relation of effective oxide density on gate voltage for two different devices.
Fig 6.12 Comparison of simulation and experimental results on the drain current noise power of a 4.5μm NMOS device fabricated by CMOS technology. The symbols are the published experimental data and the lines are the calculated results from the new model.
Fig 6.13 Comparison of simulation and experimental results on the drain current noise power of a 5.0μm PMOS device fabricated by a PMOS technology. The symbols are the published experimental data and the lines are the calculated results from the new model.
Fig 6.14  Comparison of simulation and experimental results on the drain current noise power of a 4.5μm NMOS device fabricated by a submicrometer NMOS technology. The symbols are the published experimental data and the lines are the calculated results from the new model.
Fig 6.15  Comparison of simulation and experimental results on the drain current noise power of a 4.5μm NMOS device fabricated by a PMOS technology. The symbols are the published experimental data and the lines are the calculated results from the new model.
Fig 6.16 Comparison of simulation and experimental results on the drain current noise power of a 0.65μm NMOS device fabricated by a CMOS technology. The symbols are the published experimental data and the lines are the calculated results from the new model.
§6.3 Verification of Mutliphonon Fluctuation Expression

The new expression of Hooge's parameter in eqn. (2.3.2.15) is now compared to some published experimental results and another noise theory for mobility fluctuation noise. Firstly, \( \alpha_H \) calculated as a function of doping concentration in figure 6.17 is shown that the \( \alpha_H \) obtained from eqn. (2.3.2.15) is in agreement with the experimental results reported in [Biss83]. The crosses are experimental \( \alpha_H \) value for p-type Ge at 300K as a function of doping concentration. The dashed and solid lines are calculated from the new Hooge's parameter formula with assuming three traps' energies 190, 200 and 210meV respectively. The theoretical \( \alpha_H \) value is scaled down by using the ratio of number between traps and doping concentration as 0.3, and the ratio of maximum and minimum phonon's wave vector as \( \sqrt{3} \times 10^9 \).

Figure 6.18 shows that the variation of \( \alpha_H \) as a function of temperature from 77 to 300K. The dots represent the experimental results [Clev87] from boron-doped silicon with doping concentration of \( 3 \times 10^{17} \text{ cm}^{-3} \). The three groups of symbols shown in the figure are \( \alpha_H \) values measured from samples with different annealing processes. The group of the highest \( \alpha_H \) values are from silicon samples after an annealing process at 450C, the middle group of triangles are \( \alpha_H \) values measured on the sample after annealing process at 650C, and the group with lowest \( \alpha_H \) values are from silicon samples after an annealing process at 750C.

The theoretical \( \alpha_H \) values for different processes are in agreement with measurement data. The \( \alpha_H \) values are calculated based on an assumption of uniform distribution of energy levels of the traps for simplicity and the ratio of maximum and minimum phonon's wave vector as \( \sqrt{3} \times 10^9 \). The highest group of \( \alpha_H \) values are obtained with traps' energy levels from 82 to 120meV at the temperature range of 80 to 300K. The ratio of traps to doping concentration is chosen as 0.009 to scale the experimental \( \alpha_H \) values. The middle group of \( \alpha_H \) values are obtained with traps'
energy levels from 95 to 380meV. The lowest group of $\alpha_H$ values are obtained with traps' energy levels from 140 to 520meV. The ratio of traps to doping concentration is chosen as $1.5 \times 10^{-4}$ to fit the experimental $\alpha_H$ values. From these calculations, it seems that annealing Si samples results in lower $\alpha_H$ values and that deeper traps participate in the phonon trapping and detrapping. This is analogous to g-r noise where deeper traps results in lower noise, compared to shallower traps because deeper traps have much less probability to capture and emit carriers from the conduction band.

Figure 6.19 compares calculated results from eqn. (2.3.2.15) with the dislocation theory's predictions of $\alpha_H$ described in [Morr92a]. The symbols are the $\alpha_H$ values predicted by dislocation theory in n-type silicon as a function of doping concentration. The solid line is calculated data with a trap energy level at 330meV and the ratio of traps to doping concentration is chosen as 0.027. As shown in this figure, the agreement between our calculations based on phonon-assisted electron trapping and detrapping and calculations based on dislocation theory [Morr92a] is quite good. This could be expected since dislocations produce traps and these traps in conjunction with multiphonon absorptions and emissions are the source of noise in our theory.

Calculations of $\alpha_H$ from the new expression in eqn. (2.3.2.15) are in agreement with published experimental results. The model predicts that the magnitude of the 1/f noise is more sensitive to defects and lattice damage than mobility, and this has been experimentally proven in [Clev87]. In [Clev87], it is suggested that more defects and lattice damage generate more 1/f noise. Also, the $\alpha_H$ values predicted by this theory is in good agreement with that predicted by the dislocation theory [Morr92a].
Fig 6.17 $\alpha_n$ values (dashed and solid lines) obtained from the new expression are in agreement with the experimental results (dots) for p-type Ge at 300K as a function of doping concentration.
Fig 6.18 $\alpha_H$ values (dashed and solid lines) are in agreement with the experimental results (dots) for p-type Si with $3 \times 10^{17}$ cm$^3$. The group of the highest $\alpha_H$ values are from silicon samples after an annealing process at 450C, the middle group are from silicon samples after annealing process at 650C, and the group with lowest $\alpha_H$ values are from silicon samples after an annealing process at 750C.
Fig 6.19 \( \alpha_H \) values (solid line) obtained from the new expression are in agreement with the predicted \( \alpha_H \) values (dots) by dislocation theory for n-type Si at 300K as a function of doping concentration.
Chapter 7 Conclusions

The major contribution of this thesis research is the development of a unified low-frequency noise model for MOSFETs based on physical principles. The model can be easily used as a simulator to predict the noise characteristics of MOSFETs over a wide range of operating conditions and device parameter values. It has been tested using our own experiments as well as published experimental results and good agreement has been obtained in all tests to date. The derived model is an extension of all previously published models and compared to other unified models, only physical fitting and also fewer fitting parameters are required.

The proposed combined noise model incorporates the detailed mobility mechanisms in MOSFETs and is valid from the subthreshold, to linear, to saturation and even deep saturation modes of operation. The model extension in both saturation and deep saturation, in particular, is a significant importance over other models in that it considers in detail both the carrier number and the electric field distribution in the channel and develops analytical expressions for them. These analytical expression gave good agreement with results from the 3-D device simulator MINIMOS. In addition, the saturation and deep saturation model considered the detailed electrical dimensions of the devices using a "two-region" consideration for the channel.

Unlike previous unified models, the oxide trap density has been experimentally obtained from low frequency noise calculations, and this trap density is in good agreement to that obtained from the charge pumping measurements. This research is also significant in that it adds to existing noise results by presenting detailed noise results for MOSFETs at high temperatures from 298 to 423K and a wide range of operating bias conditions. For example, the detailed results in MOSFETs at temperatures from 273K to 423K are from several devices of channel lengths 1.2 and 1.7μm.
The operating conditions of the devices were such that a variation from linear to saturation mode was obtained using a fixed gate voltage of 2.5V and varying the drain voltage from 0.1 to 4V.

Results of the gate-input referred noise spectral densities $S_{\nu_0}$ in the frequency range of 3Hz to 100kHz for 1.2$\mu$m device and of 3Hz to 100Hz for 1.7$\mu$m device were proportional to $1/f^\beta$ with $\beta$ close to 1. For fixed biasing conditions, $S_{\nu_0}(f)$ increases with temperature for the 1.2$\mu$m device, but the opposite trend was observed for the 1.7$\mu$m devices. For both devices, we found that $S_{\nu_0}(f)$ were lower in the linear region compared to the saturation region at all measured temperatures.

From the theoretical analysis of distinguishing the number-fluctuation and mobility-fluctuation noise mechanisms, it was found that the flicker noise for the 1.2$\mu$m device is most likely dominated by mobility-fluctuation, but for the 1.7$\mu$m, it is mainly dominated by number-fluctuation. After considering the mobility-fluctuation due to Coulombic scattering induced by fluctuations in number of carriers due to carrier trapping and detrapping, and also mobility-fluctuation caused by phonon and surface roughness scattering, a new combined noise model was derived and successfully fitted to our experimental results. From measurements, modelling and calculations for high-temperature low frequency noise in n-channel MOSFETs, a better understanding on low frequency noise at high temperatures in MOSFETs is developed.

The experimental results for 0.9$\mu$ and 1.2$\mu$m short-channel MOSFETs for the saturation region of operation are in good agreement with the proposed model for short-channel MOSFETs with the consideration of velocity saturation effect. MINIMOS simulation results on channel potential and carrier density also support the expressions where the model is based. Finally, the research on trapping noise and multiphonon transition concept is helpful in understanding $1/f$ fluctuations in semiconductors.
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