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THE USE OF MANIPULATIVE MATERIAL
TO FACILITATE TRANSFER OF RATIO AND
PROPORTION FROM MATHEMATICS TO SCIENCE

by

Robert John Aitchison
B.Sc., University of British Columbia, 1968

A THESIS SUBMITTED IN PARTIAL FULFILLMENT OF
THE REQUIREMENTS FOR THE DEGREE OF
MASTER OF SCIENCE (EDUCATION)
in the Department of
Education

ROBERT JOHN AITCHISON 1976
SIMON FRASER UNIVERSITY
April 1976

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ABSTRACT

Aitchison, Robert John

The Use of Manipulative Material to Facilitate Transfer of Ratio and Proportion from Mathematics to Science

The purpose of this study was to determine whether the use of manipulative material would improve students' ability to apply mathematics to science problems and to perceive more clearly the relationship between mathematics and science as measured by cognitive and attitude instruments respectively. An additional purpose was to determine whether any differences could be attributed to the sex of the individual and if there was evidence of interaction between sex and treatment.

Fifty-seven grade nine science students were randomly divided into two groups and taught the mathematics of ratio and proportion, one by the use of manipulative materials, the other by a conventional lecture presentation without such materials.

Each group received two hours of instruction. The two groups were compared as to their respective attitude scores and as to their ability to apply mathematics to science problems. To reflect shifts in attitude, a Likert-type instrument was administered before and after the experimental period. Cognitive measures consisted of a mathematics test on ratio and proportion and a science test mainly
of chemistry problems requiring the application of the principles of ratio and proportion. The mathematics test was given both before and after the experimental period. The latter test was used as the measure of transfer of mathematics to science.

Null hypotheses were formulated as to the results of the experiment on both attitudes and cognitive achievement for the effects of treatment, sex and interaction.

All null hypotheses were accepted. Thus a major result of this study was a failure to show that the use of manipulative materials improves either the attitude of students towards the interrelationship of mathematics and science or the cognitive ability to apply the mathematical principles of ratio and proportion to the solution of science problems. Similarly, there was no indication that the sex of the student influences either the attitude or the cognitive ability, nor was there any indication that one of the two methods of instruction was more appropriate for girls than for boys.

The obvious conclusion from the results of the study is that neither method of instruction is superior in facilitating the transfer of the mathematics of ratio and proportion to science problems. However, this conclusion must be tempered by the limitations of the study. It was further concluded that the study of transfer of training was not aided by the two major theories on transfer (Thorndike's identical element theory and Judd's generalization theory)
identified through the literature review. It was felt
instead, that transfer could best be understood in terms of
the conditions and limitations of a particular line of
investigation.
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CHAPTER I

INTRODUCTION

Background

It is the opinion of the writer that many students in the junior high school have difficulties in science courses because of their failure to perceive clearly the relationship between the principles they learn in mathematics and the application of these principles to the sciences. In other words, they fail to transfer a knowledge of mathematics to the solving of science problems. This opinion has been based on observation of student behavior during several years of experience in teaching at the junior high school level. During these years, the failure of students to apply mathematical principles was observed, for example, in their attempts to solve chemistry problems dealing with formulas and physics problems involving simple machines. It was particularly noticeable where an understanding of ratio and proportion was required.

The problem of transfer from mathematics to science was found to be of general concern in the literature on science teaching (Gleason (1968); Bowen (1966)). Thus the writer felt that this problem was a worthwhile study to pursue and decided to undertake a closer investigation of the problem.
While working closely with individual students experiencing difficulties in applying mathematics to science problems, the writer made two observations which appeared to have particular significance. One was that the use of concrete manipulative material seemed to help them to overcome difficulties in applying mathematics to science. The other observation suggested the existence of an attitudinal barrier, in that the students tended to regard mathematics and science as two distinct and unrelated disciplines.

The first of these observations led to a concern to find out whether carefully planned and structured use of manipulative materials could lead students to have a better understanding of mathematical principles and, in addition, a greater ability to transfer their knowledge of these principles to the solving of science problems than they would have otherwise.

A further concern was to find out whether there was a significant difference between the responses of boys and girls to the use of manipulative materials. The writer held the opinion, derived originally from his teaching experience and reinforced by studies made by others such as Howe (1971), that boys tend to be more at ease with such materials and therefore respond to them in a more active, a more aggressive, and a more exploratory manner than girls do. Thus an interest arose as to whether the use of concrete manipulative materials might be more appropriate for one sex than the other.
The second observation, relating to the attitudinal barrier, appeared to have an easy explanation. In our junior high schools, mathematics and science are usually taught in isolation from each other, with teachers often specializing in one or the other of the two subjects. Further, an examination of curriculum guides issued by the British Columbia Department of Education since 1920 reveals that for decades little thought has been given to the integration of the two disciplines. In 1969, for example, the aims and objectives of the grade nine algebra course included no reference to the desirability of using algebra in practical situations (See Appendix A). As for science, the 1969 curriculum guide listed no general aims. Rather, it presented specific aims for individual science lessons. For example, one aim was expressed as, "The proportions (by mass) in which substances will react with each other can be found from the equation describing the reaction." Although the guide made frequent reference, as in this example, to such mathematical terms as ratio and proportion, no mention was made of the necessity for teaching these concepts, nor was there any suggestion that teachers of mathematics and teachers of the sciences might, or should, integrate their work.

The failure of curriculum guides to suggest, let alone prescribe, the integration of the two disciplines seems an omission difficult to understand in view of what had been written in much of the literature concerned with the teaching of mathematics and science. Such literature, which is
reviewed in the second chapter advocates strongly that the practical application of mathematics should be emphasized, and it would seem to the writer that an important area of such application would be in the sciences.

Thus the attitude held by many students that mathematics and science are unrelated seems to be consistent with the manner in which these subjects are presented in curriculum guides. The writer became curious to determine whether there were means by which this attitude could be changed.

The Problem

The general purpose of this study was to determine whether the use of concrete manipulative materials would increase the ability of students to transfer their knowledge of mathematics to science problems and whether using these materials would change the attitude towards the inter-relationship of mathematics and science.

In the hypotheses formulated for the study, differences in treatment (the use of manipulative material versus lecture presentation) and differences in sex constitute the main effects. The interaction effects of these two independent variables are also included in the hypotheses.

The null hypotheses to be tested are as follows:

(A) There will be no significant difference in the transfer of mathematics to science due to treatment, sex, or interaction as measured by a science test administered at the conclusion of the treatment period.
There will be no significant difference in attitude due to treatment, sex, or interaction as measured by a Likert-type attitude scale administered at the conclusion of the treatment period.

**Plan of Study**

Detailed procedures used for the investigation are given in Chapter Three. Briefly, the approach was as follows.

Two groups of students were randomly selected. Both groups studied the mathematics of ratio and proportion, one group being taught mainly with manipulative materials and the other mainly through lectures. All students were tested on their knowledge of ratio and proportion immediately before and immediately after the experimental period. At the conclusion of the experimental period, both groups were given a science test that required the application of the mathematics previously studied, namely the mathematics of ratio and proportion. A Likert-type instrument to measure students' attitude toward the relationship between mathematics and science was also administered before and after the treatment period. Comparisons between the two groups were made on the basis of these measures.

**Summary and Implications**

In summary, this study implemented the foregoing procedures for the purposes of:

(A) determining whether involvement with manipulative material would improve students’ competency in applying mathematics to science.
determining whether involvement with manipulative material would alter students attitude towards the inter-
relationship of mathematics and science.

For each of these purposes a general hypothesis was formulated. Both employed treatment and sex as main effects and both were stated in null form.¹

An acceptance of hypothesis A would imply that both methods of instruction produced significantly the same transfer ability, that both sexes possessed significantly the same transfer ability and that neither method of instruction was more appropriate for one sex than for the other. On the other hand, rejection of this hypothesis would imply that one or more of the following occurred: that one method of instruction was superior to the other in facilitating transfer, that one sex was superior to the other in the ability to transfer or that sex was the factor determining which method of instruction was superior.

Similarly, an acceptance of Hypothesis B would mean in regard to the attitude of students towards the interrelation-ship of mathematics and science, that both methods of instruction produced significantly the same degree of change

¹The possible acceptance or rejection of any hypothesis is determined when statistical techniques indicate that there either are, or are not, significant differences between the mean scores of each group on each main effect being tested. For example, if any hypothesis is rejected, then the difference in the means is significant at that level of confidence. The statistical technique used in this study is the analysis of co-variance yielding a F-ratio. The confidence level is .05.
of attitude, that both sexes exhibited significantly the same attitudes, and that neither method of instruction effected a significantly greater degree of change in the attitude of one sex than of the other. On the other hand, rejection of Hypothesis 8 would mean that one or more of the following occurred: that one method of instruction was superior to the other in effecting a change of attitude, that one sex perceived the interrelationship of math and science more clearly than the other, or that the superiority of either method of instruction was dependent on the sex of the student.

Definition of Terms

For the purposes of this study, the terms, "transfer of learning" and "manipulative material" are employed according to the following definitions.

Transfer of Learning: the extent to which performance in, or experience with, one task influences performance in some subsequent task. In this study the original task is the acquisition of skills in the mathematics of ratio and proportion. The subsequent task is the application of these skills to chemistry problems.

Manipulative Material: concrete objects which students are able to handle physically during the course of a classroom period. The manipulative material used in the main study were Cuisenaire Rods with which students acquired skills in the mathematics of ratio and proportion. Previous to this study a pilot study used various manipulative materials such as paperclips and washers.
Limitation of Study

The subjects for this study were fifty-seven students in two grade nine general science classes. These classes were considered to be heterogeneous in composition. The junior secondary school which they attended is located in a suburban area of greater Vancouver. Generally the socio-economic level of the community could be described as lower middle class.

The facilities for this study were limited to the confines of one general science laboratory classroom with which the students were already familiar.

Each of the two treatment groups into which the subjects were randomly divided received only two one-hour periods of instruction during the treatment period. All instruction was given by the same teacher.

A final limitation of this study is the fact that transfer is measured in a comparative manner. Treatment and sex are used as main effects, and results can only be stated in terms of them. For instance one treatment may be superior to the other in facilitating transfer although transfer may occur using both. The quantitative or absolute measure of transfer is not dealt with in this study.

Further details regarding limitations as to time intervals and compositions of groups are given in the third chapter.
CHAPTER II

SUPPORTIVE LITERATURE

Introduction
Before this study was undertaken, literature on topics relevant to it was reviewed. General theories regarding the transfer of learning as well as literature on the particular problem of the transfer or application of mathematics were reviewed and are reported in this chapter. A selective review on the topic of teaching ratio and proportion and the use of manipulative materials was also conducted previous to initiating the experiment.

The Teaching of Mathematics and Science in the Modern Junior High School

A review of some educational journals in the fields of mathematics and science assured the writer that the problem he had encountered with his students was far from unique. In many articles, educators complained of the inability of their students to apply mathematics to science. (Rising (1967); Bowen (1966); Kullman (1966)).

However, these educators did not agree as to the solution to the problem. Their opinions were divided into two major categories. One group of educators such as Gleason (1968) recommended the total integration of the two disciplines. However others, such as Rising (1967) strongly advocated a heavy emphasis on the interrelationship of the two disciplines, but would not agree to their total integration.
The latter felt that many students have functional competence in mathematics hidden just below the surface although they cannot recognize and solve the same problems outside of the mathematics classroom. Rising (1967:29-31) recommends "Setting up communication lines between the science and the mathematics departments" and urges "both mathematics and science teachers to read or even teach each other's current programme."

Similarly, Kullman (1966:645-649) would keep the disciplines separate, but have mathematics and science teachers more aware of the importance of relating the two in order to emphasize the interrelationship in their respective classrooms. He suggests:

> The responsibility of both science and mathematics teachers is to help them (the students) to see the possibilities of using mathematics in describing the real world.

Others who agreed with Gleason felt that students would profit by a fully integrated mathematics-science programme. Gleason (1968a:118-128) states that

> Tomorrow's child may think of mathematics and science as being a single subject not because they are identical but because his school will probably present them in an integrated form.

He advocated that instead of having separate mathematics and science classes trying to keep pace one with the other, it would be far better to have an integrated class devoted to both subjects and emphasizing when the needs arise, whichever discipline seems appropriate.

Thus it was clear that although two solutions to
the problem of transfer of mathematics to science problems existed, the general objective of these two solutions was identical. It was to emphasize to a much greater degree the interrelationship of mathematics and science.

The Teaching of Ratio and Proportion

Since the writer's study was to be concerned primarily with the transfer of a knowledge of ratio and proportion to the solving of science problems, it was interesting to find indications in the literature on the teaching of ratio and proportion that these concepts are considered the mathematical principles most essential in the study of secondary school science. Gleason (1968b:54-60) comments, for example,

... the director of a well-known curriculum project stated that a major breakthrough in the teaching of science in high school would occur if only the students would master ratio and proportion.

The writer was particularly interested in a number of studies on student understanding of ratio and proportion conducted by R. Karplus (1970) in which an examination on ratio and proportion that involved the manipulation of materials was devised and students' responses were categorized on a scale measuring sophistication of response ranging from guessing to proportionate reasoning. The test was administered to students between the ages of nine and eighteen. Disappointed in the results, Karplus and Peterson (1970:813-820) concluded that
Successful proportionate reasoning is not achieved earlier than the last years in high school, even though the subjects of ratio and proportion make their appearance in most mathematical programs in junior high school. It seems, therefore, that there is a serious gap between secondary school mathematics and science and the students' reasoning ability.

Two years later, Karplus administered the same examination to a sampling of the students previously tested to see in what way these students had "matured" in their responses. He was again disappointed, the students not having "matured" to the extent he had expected. However, one of his comments at this stage is of particular interest to the present study, for he says, (Karplus and Karplus (1972:735-742))

It would seem that the investigation of levers or balances, shadows, electric circuits, solutions, densities, and similar items could help students develop proportionate reasoning.

This remark supports the notion of teaching ratio and proportion with concrete materials.

Karplus's final study (Karplus 1973) was an extension of the two just described to include other types of physical situations involving the concepts of ratio and proportion. His conclusions were again very pertinent to concerns of this study. He says:

Mathematics programs in the junior high school appear to contribute little toward proportional reasoning even though ratio and proportion are topics usually taken up. One reason probably is that ratios are introduced as fractions and proportions as equivalent fractions, to be handled by certain standard procedures of multiplication and division. Yet the essence of ratio is that it may associate six paper clips with four buttons, twelve yellow candles with twenty reds, and fifteen centimeters of one length of string with ten centimeters.
of another. Curricula make little effort to interpret ratio, proportion, and the related division process in terms of such correspondences of measurements. In this use of division, the concept of remainder has no place.

It seems to us that the preoccupation with integers in both traditional and modern mathematics curricula leads to serious disadvantages when students have to use numbers to describe physical objects and have to carry out operations on these numbers. Furthermore, Cuisenaire rods, systems of coupled pulleys, and other mechanical devices that embody proportions would appear to be valuable teaching devices that could help students engage in proportional reasoning on the concrete level before being faced with similar problems on the abstract level. Combined math-science activities with such an emphasis should be considered seriously for the upper elementary or junior high school grades. At the same time, it may be that the foundations of the mathematics program in the primary grades will have to be rethought as well.

Although Karplus focuses strongly on the topic of ratio and proportion, he seems to agree with the general sentiments regarding the emphasis on the correlation and integration of mathematics and science expressed by Bowen (1965), Rising (1967), and Gleason (1968).

The Use of Manipulative Materials

Other educators besides Karplus, have provided information on the use of manipulative materials for the acquisition of mathematical skills. Lovell (1972), Dienes (1960) and Gattegno (1963) are three such people whose writings on this topic were investigated.

Lovell (1972:277-283) states that knowledge cannot be acquired directly from the blackboard, textbook, or file by mere perception or acquired by drill; rather these general ways of knowing have to be actively constructed by the child through interaction with the environment.
He goes on to suggest that...

...the opportunity for pupils to act on physical materials be provided and to use games in the manner suggested by Dienes. It is the abstraction from actions performed on objects and not the objects themselves that aid forward knowledge of mathematical ideas. Not until flexible formal operational thought is available in mathematics can the latter be learned using words and symbols only and intuitive data dispensed with.

Dienes (1963) to whom Lovell refers above, is a strong advocate of the use of manipulative materials. He provided literature on various mathematical topics ranging from those studied in primary school to much more advanced topics studied in college. Although much of his content was irrelevant for the writer's immediate concerns, his approach to any mathematical concept was most encouraging. It always recommended procedures involving the use of concrete manipulative materials. In his theories on the learning of mathematics, he criticizes "traditional" mathematics programs for inhibiting the degree of transfer by not providing such materials. Clearly identifying himself with the Piagetian concept of intellectual stages of development, Dienes would have manipulative materials used in sequentially ordered levels of experience. Kennedy (1973) states that

At the first level, the materials are used by children in an unstructured, undirected manner. According to Dienes, this is a necessary preliminary step before the materials can be organized on

\[1\] It is impossible to discuss Piaget in detail in this thesis. However an excellent account of Piaget's work as it relates to Dienes is provided in Key & Post (1973).
manipulation in an organized way. At the second level, the child’s actions with the materials becomes directed toward some end. The materials are ordered and classified and symbolic representations, principally words, begin to be used in the production of ideas. At the third level, relationships and structural patterns are perceived by the child and insights are developed and strengthened. In all these levels, the interactions between children and materials are the crucial components of concept development. In essence Dienes gives strong support to the view that constructive thinking must precede analytic thinking (Piaget’s period of formal operations); otherwise there is nothing to analyze.

The use of manipulative materials must, according to Dienes, also accompany the appropriate situation or problem. For example, Dienes states that

They (materials) can be very valuable as part of mathematical laboratories where they will be of use in conjunction with other situations, other problems and other materials. In isolation they tend to produce associative learning rather than abstractive learning.

Thus, although Dienes views mathematics as an art form to be studied for its intrinsic value, he also views the use of manipulative material as a means of maximizing transfer of learning. Dienes is not convinced that the abstractive learning of concepts that he considers to facilitate transfer can be realized by formal blackboard instruction. He says

Juggling with symbols is no more abstract than juggling with coloured balls and if the mathematical situations are not learned as generalities their essence will not be known and situations similar to them will not be recognized. In other words transfer will simply not take place.

Dienes’ philosophy regarding the use of manipulative materials emphasizes a fundamental principle called multiple embodiment. Based on the notion that children learn at
different rates and in different ways, this principle implies that every concept should be presented in as many different ways as possible. Thus Dienes advocates a multitude of physical concrete situations which could embody the same mathematical concept.

Gattegno (1960) provided much information for the writer on the use of manipulative materials, especially the use of Cuisenaire rods. In a series of volumes entitled "For the Teaching of Mathematics," he provides strong philosophical arguments in a general way for his notions on mathematics learning. In these discussions, Gattegno directs the reader to particular instances where he employs the use of concrete manipulative materials. It is very apparent that Gattegno's philosophy on mathematics learning quite naturally employs such use.

His philosophy is reflected in his view of the role of the mathematics teacher. Gattegno (1960) sees each lesson "as a complex experiment in a human laboratory" and feels that the teacher's position in the room is not that of an authoritative figure but rather that of a scientist who, like the children with whom he is working, is involved in the process of learning. The "subordination of teaching to learning," part of a title of another of his works, is perhaps the most appropriate and concise way that his philosophy could be stated.

Gattegno (1970) basess this philosophy of learning on what he calls an "awareness" that every human being
possesses and suggests that children possess tremendous capacity to learn if learning situations are provided that can "draw out" this awareness. He states that mathematics teaching becomes the task of making students aware of themselves as the basis of reaching the dynamics of mathematical relationships and offering the situations that involve all sorts of these relationships.

Previous to this statement, Gattegno (1963) had also said that much of the 'latent' potential of children's ability to perform mathematical operations can be realized through manipulative materials. He writes:

It is well known that an immense difference in the teaching of arithmetic has resulted from the introduction of Cuisinaire rods. Whereas before, interminable hours of repetition were needed to ensure the remembering of the links between whole numbers, now children of five can work on fractions and at seven can demonstrate that they have the mathematical experience of other pupils of twelve or more.

My prolonged and detailed study of this success prompts me to say quite unequivocally that it is due to the fact that these rods place within the grasp of the child the most primitive notions we know in mathematics and thus which we only met at the end of our own process through an ever-deepening abstraction.

These manipulative materials are, in Gattegno's opinion, extremely appropriate in facilitating an optimum learning situation that can develop the "awareness in children." For instance, he suggests that inherent in the rods are the many different mathematical situations that could arise from the single mathematical topic of addition. He states:

Consider for example, the fact that, generally speaking, addition is taught for the first months all by itself, that subtraction comes later, followed much
later by multiplication and division and later still by fractions while in the study of the situation offered by the Cuisenaire materials all this is met at one and the same time.

Although Dienes and Gattegno both advocate the use of manipulative materials, they differ significantly in their philosophical basis for such use. As has been pointed out, Dienes advocates the carefully structured use of materials in a manner that tends to parallel the stages of intellectual development suggested by Piaget. On the other hand, Gattegno, by referring to a latent awareness in all children that could be realized under proper conditions seems to disregard any readiness to learn (Piaget’s formal stages). Also the manner in which these two educators recommend the use of manipulative materials can be contrasted. Dienes suggests that many physical situations should be provided to arrive at an understanding of the same concept to accommodate differences in the rates and manner in which children learn. Gattegno, on the other hand, provides a situation which tends to make children aware of a vast quantity of mathematical concepts. It is the writer’s belief that these two different approaches are related to the respective philosophies of these two educators.

Transfer of Learning

The major concern of this paper is the transfer of the learning of certain mathematical concepts to science problems. Reference has already been made to the topic of
transfer in the discussion of Dienes' philosophy concerning the use of manipulative materials. It is the intention of the writer to now discuss some general theories on the transfer of learning apart from the particular problem dealt with in this study (mathematics to science) and then to later focus on the relevance of these theories to problems encountered in this study.

Theories on Transfer. The literature on transfer of learning presents two main theories, the identical element theory and the generalization theory. The two have been the centre of considerable controversy this century in the fields of psychology and education.

Before the beginning of this century it seemed to be generally assumed that learning any given subject would increase one's general ability to learn. The learning of certain subjects was credited with being able to enhance the ability to learn certain others. Learning Latin, for example, was thought to improve one's ability to learn any language, and learning mathematics was thought to increase reasoning powers generally.

This notion was disputed by Thorndike (1921:352-364) who first proposed the theory of identical elements. He maintained that

By doubling a boy's reasoning operation in arithmetical problems we do not double it for formal grammar or chess or economic history or the theories of evolution.

He was not denying that transfer of learning occurs but felt that the important questions to be answered
concerned the extent to which transfer occurs and the conditions under which it occurs. He proposed

...that a change in one function alters any other only in so far as the two functions have as factors identical elements. The change in the second function is in amount that due to the change in the elements common to it and the first. To take a concrete example, improvement in addition will alter one's ability in multiplication because addition is absolutely identical with a part of multiplication and because certain other processes, e.g., eye movements and the inhibition of all save arithmetical impulses, are in part common to the two functions.

Thorndike defined identical elements as "mental processes which have the same cell action in the brain as their physical correlate."

It would seem, from the example that he gives in the above quotation, what Thorndike has in mind is what Gagne (1968) called vertical transfer. Shulman (1970:23-71) explains:

Vertical transfer refers to the manner in which the learning of a subordinate capability serves to facilitate the mastery of some subsequent learning at a higher level of the same hierarchy.

Shulman quotes Gagne (1968:177-191) as saying that

...transfer occurs because of the occurrence of specific identical (or highly similar) elements within developmental sequences.

and maintains therefore that Gagne is essentially a proponent of Thorndike theory.

According to the generalization theory proposed by C.H. Judd (1927), transfer of learning goes beyond the "particular experience" to which it is limited according to Thorndike's theory. Sandiford (1963:7-25) explains Judd's theory as follows:
Judd's generalization of experience resolves itself into nothing more nor less than the formation of specific language habits having applicability to situations other than those in which they were learned.

Sandiford goes on to say that in the teaching of mathematics new problems must be expressed in the language forms of old problems whose solution is already known. This is the kind of operation he calls generalization of experience.

Both theories have their supporters in more recent literature on the subject of transfer.

As has been pointed out, Gagne is seen as echoing Thorndike's theory of identical elements. He distinguishes between two kinds of transfer - vertical transfer, the definition of which has been given previously and lateral transfer which Shulman (1970:23-71) explains thus.

Lateral transfer refers to the manner in which the learning of a capability in one domain can facilitate the mastery of some parallel capability in another domain. These parallel capabilities would be at the same levels of their respective learning hierarchy.

Among modern educational theorists who support Judd's theory of generalization, Bruner (1957) is perhaps the best known. Of Bruner's position, Shulman (1970:23-71) says

Bruner stresses the lateral transfer of broad principles and strategies from one domain or topic to another. He gives examples such as the concept of conservation or balance. Is it not possible to teach balance of trade in economics so that, when ecological balance is considered, pupils see the parallel? This could then be extended to balance of power in political science, or to balancing equations. In his view, learning by discovery leads to the ability to discover, that is, to the development of broad inquiry competencies in students.
An example of the application of the generalization theory is the existence of process-oriented science courses in modern junior high schools. By minimizing content and emphasizing the enquiry process, these courses, it is hoped, will promote an attitude of general scientific enquiry into many kinds of problems.

Recent Thoughts on Transfer. Current psychological studies on the subject of transfer, (Wittrock (1975), Campione and Brown (1973)), although making casual reference to these general theories do not attempt to relate any theory to their work. Ellis (1969) states that the reason for this lies in the fact that

These early theories of transfer, perhaps better described as points of view rather than theories in the more formal sense, represent historical viewpoints which guided much of early research in transfer. They were all stated in rather general language thus making them difficult to test in a rigorous fashion.

Further provides a description of many possible variables that are more than just points of view or opinions but rather are stated conditions very specific in nature on which transfer tasks can relate to. For instance, such variables that influence transfer are prior learning, the type of factor being transferred (general or specific), and the kinds and number of "association" which exist between the original and transfer tasks, etc. In other words the study of transfer cannot be neatly divided into one of the two packages suggested in the previous description of the transfer theories.

Thus it becomes the writer's belief that transfer
can be discussed in a more meaningful way if it is directly related to these specific conditions rather than general theories. The intent of this study is to provide a detailed description of the conditions under which transfer was tested (Chapter III) and to report the results of this study (Chapter IV) as they relate specifically to these conditions. In the last chapter, conclusions are provided which also relate specifically to the conditions and limitations of the study.
CHAPTER III

PROCEDURES

Setting

This study was done in a large junior high school (pop. 850) in School District #43, Coquitlam, B.C. Since the school year was divided into two semesters, it was possible to carry out pilot experiments during the first semester to determine whether any features should be changed before implementing the main experiment during the second semester.

Format of the Experiments

Selection of Subjects. As subjects, the writer chose the students of science classes assigned to him by the school administration. The classes thus formed were considered to be heterogeneous groups. Both grade nine and grade ten students were involved in the pilot studies, but only grade nine students in the main study.

Preparatory to dividing the subjects into the treatment and control groups, each subject was arbitrarily assigned an identification (I D) number. Next, the non-verbal intelligence quotients\(^1\) (N.V.I.Q.) of the subjects were used to stratify the group, the name of the subjects

\(^1\) Derived from Lorge-Thorndike Intelligence Tests administered during the previous school year.
being arranged in descending order according to their N.V.I.Q.'s and the appropriate I D number assigned to each. Thus a list of I D numbers from the highest N.V.I.Q. to the lowest was generated. This list was then divided into three sections, one section comprising the I D's of the top third of the list, another the I D's of the middle third, and another the I D's of the bottom third. Two I D numbers were next taken manually in rotation from each of the three sections and placed in two separate piles. This procedure was continued until all the I D numbers had been placed in one of the two piles. One pile was then designated as containing the I D numbers of the students who would use manipulative materials. The second pile would contain the I D numbers of those students that would experience the lecture presentation.

The Pre-Experimental Period. At the beginning of the semester and therefore approximately a month before the experimental period, all subjects were administered a test designed to evaluate the attitude or perception of the students as to the relationship between mathematics and science. For about the next four weeks of the semester the students in both groups were exposed to elementary junior high school chemistry which required very little mathematical understanding. Topics under consideration during this period included differentiation of matter into mixtures, elements and compounds, chemical symbols, chemical formulae, atomic and molecular weight, physical and chemical
properties, and the balancing of chemical equations. All of these concepts can be mastered at the junior high level with the addition of integers being the only arithmetic required.

The next topic to be considered was the solution of "weight-weight" problems, which require an understanding of ratio and proportion. Because the difficulties described in Chapter I would normally occur at this point, the experimental period was now initiated.

**The Experimental Period.** At the beginning of the experimental period, a mathematics test examining the students' knowledge of ratio and proportion was administered. The attitude test given at the beginning of the semester was also readministered. Two one-hour periods following these tests constituted the treatment period. No chemistry was taught. Instead, the students were introduced to mathematical problems involving ratio and proportion.

One group of students used manipulative materials to solve the problems. During the pilot studies of the first semester, the manipulative materials were similar to those used by Karplus (1970) - paper clips to measure heights and washers to be placed inside squares to measure areas. During the main study in the second semester, Cuisenaire rods were used exclusively.

The students in the other group used no manipulative materials. Instead, they received conventional lecture-blackboard demonstration instruction, after which they attempted to solve the same ratio and proportion problems as
At the conclusion of the experimental period three tests were administered. One was a repetition of the mathematics test on ratio and proportion originally administered at the beginning of the experimental period. A second was a science test containing problems requiring the application of the principles of ratio and proportion for their solution. The third was a repetition of the attitude test given at the beginning of the semester and at the beginning of the experimental period.

**Summary of Format.** The entire format consisting of both the non-treatment and experimental periods and various tests (mathematics, science and attitude) can best be summarized by a diagram illustrating the complete order of events.

|-----------------------|------|------|------|---------|------|------|------|---------|--------------|-----------|-----------|--------------|-------------|--------|

**FIGURE I  FORMAT OF STUDY**
**Logic of Design**

Figure I illustrates the manner in which the experiment was carried out. It can be seen that the two groups receive lessons in mathematics by two different treatments. All students have been pre-tested on their mathematics ability prior to the treatment so that the investigator has an indication of their ability to do ratio and proportion without any formal instruction on his part. The amount of instructional input by the investigator can be measured by the post-test on mathematics which is the same measure as the pre-test but which is administered after the experimental period. The results of this measure will thus indicate to the investigator any significant difference between the two groups on their ability to perform the mathematics of ratio and proportion as a result of two different experimental treatments. Similarly, the final science measure will indicate possible differences in the transfer ability of students due to the same effect. The attitude test administered before and after the experimental treatment should indicate possible changes in attitude as a result of the treatment period.

A treatment period of two hours was considered to be of appropriate duration because the writer's experience had indicated that the principles of ratio and proportion and the application of them to chemistry problems could be adequately taught in two hours. In addition, relatively short time duration is a further control on minimizing
intervening variables that would arise due to history and maturation.

Previous to the experimental period and the administration of the pre-tests both groups were exposed to the manipulative materials used by one group in the experiment. This procedure served to familiarize the students who would eventually use the manipulative materials with the equipment. If this procedure was not done, it was felt that these students would be at a severe disadvantage spending an unnecessary portion of their time during the experimental period familiarizing themselves with the equipment and not working on the appropriate subject matter (ratio and proportion). Thus their result on the post-tests would not be valid. Although this initial exposure of both groups to manipulative materials was done for convenience, it was felt that this procedure would in no way interfere with the results obtained by the group of students not to be exposed to manipulative materials during the experimental period because the subject matter in this preliminary procedure was unrelated to the experiment.

The Pilot Studies

Grade Ten Class, Semester I. The first group of students exposed to the experiment were grade ten students. No hypotheses were made and no statistical analysis was done on the results. The study was done mainly to familiarize the writer with the experimental procedure and to
expose errors or difficulties before attempting the main study.

For a number of reasons this group of students was thought not to provide meaningful data. Not only was the population too small, both groups only containing ten subjects, but there was also excessive absenteeism. In addition, the students were at various levels of progress in their study of chemistry, some of them having already studied the topic to be learned during the treatment period. Finally, the writer's inexperience with the procedures to be used with the group experiencing manipulative materials was considered to limit the credibility of any results.

**Grade Nine Class, Semester I.** The writer felt that Grade Nine students would be better subjects for the study than Grade Ten students because, to the best of his knowledge, none of the students had been previously exposed to the chemistry which would be involved in the study. The students in the two Grade Nine classes selected for the second pilot study were more heterogeneous in ability and their range of ages was less than was the case in the Grade Ten class. Also absenteeism was minimal.

The purpose of this second pilot study was to obtain valuable information as to possible flaws in the experiment and to establish the reliability of the various tests to be used in it.
The Mathematics Measure. (Appendix C) The mathematics measure on ratio and proportion consisted of a series of items which became progressively more difficult. The first items dealt with simple proportion. The final items consisted of practical problems requiring the use of ratio and proportion, such as conversions of percentages into raw scores. Such problems were dealt with to a limited extent during the experimental period.

The Science Measure. (Appendix D) All questions on the science test required the application of a knowledge of ratio and proportion. They were chemistry problems of the type familiarly known as "weight-weight" problems, the study of which usually follows the study of the balancing of equations. The students were given no indication that they needed to apply the mathematics they had just learned to the solution of these problems. To the best of the writer's knowledge, they had never before seen the precise type of problem comprising the test. In short, what was being tested was the students' ability to apply mathematics to chemistry problems rather than their ability to understand the chemistry involved.

The Attitude Measure. (Appendix E) As already indicated, the attitude measure was given three times. It was given at the beginning of the semester, and therefore some weeks before the beginning of the experimental period, because the writer wanted to obtain the initial uncontaminated
attitudes of the students towards the interrelationship of mathematics and science. A positive shift in attitude by the treatment group during the experimental period but not earlier would help to establish that the vital factor in the change was the use of manipulative materials.

The attitude consisted of thirty statements to which the students could choose a response ranging from "strongly agree" to "strongly disagree." Ten statements did not relate to the attitude that was being tested because the writer did not want the students to be aware that this was his main interest in the test. The statements that related directly to the attitude being tested were stated in both positive and negative forms. For example, one statement would suggest that mathematics and science are related and another statement would suggest that they are not related. Consequently, in analyzing the data, the responses to results of some statements had to be reversed, and responses to non-relevant statements were ignored.

Reliability and Validity of the Measures. In order to establish the reliability of the measures an item analysis was conducted. This procedure considered the correlation of each item of each measure with the total score on that measure. Where items were unrelated to the total score they were replaced (justification of limiting values given in Chapter IV). This replacement of items was found to be necessary only for the attitude measure. Thus reliability of test items as reported in Appendix (C) shows results on
the cognitive measures during Semester 1 and the attitude measures during Semester 2.  

A measure of the internal consistency of each measure was also obtained as an indication of the reliability of the measure. This procedure involved finding the intercorrelation of all items on the measures to obtain the overall consistency of the measure.

Another procedure used to establish reliability was the administration of the same test on two occasions. This procedure could be done on both the attitude and mathematics measure but not on the science measure due to the nature of the format of the study. The intercorrelation of the two successive administrations of the same exam gave what is termed a test-retest reliability. This reliability was expected to be less than the reliability coefficient indicating internal consistency as the experimental treatment would naturally introduce variables that were designed to alter the results on the second administration of the measure.

The validity of the measure was obtained by soliciting the opinions of both colleagues and university personnel. In this process each measure was given to the above personnel and their subjective opinion of the intent of each measure was sought. Relative agreement amongst the interviewed personnel was sought as an end result of this exercise.

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1 Simon Fraser's University's computer centre was used for this purpose. The program implemented for reliability was the Scale Analysis Program from the University of Colorado.
Treatment of Results of Measures. To obtain results for this study, means and standard deviations were calculated. Three statistical procedures were used on the results. A t-test was used to obtain information on the significance of the difference in the pre and post math measure. Analysis of variance was used to find any significant difference between the two treatment groups on both the pre and post math measures. Analysis of covariance was used on the dependent variable in the study—namely the science test measuring transfer of math to science and the final attitude test measuring a change in the understanding of the interrelation of math and science due to the experimental treatment. The covariates for this procedure were the pre and post mathematics measure for the science test and the first two administrations of the attitude test for the final attitude test.

The t-test and analysis of variance employed only the main effect of treatment whereas the analysis of covariance considered the two main effects of treatment and sex.
CHAPTER IV

RESULTS

In this chapter the results of the procedures listed in Chapter III are reported. The results of the Pilot Studies are reported in the first section and are followed by a section dealing with the validity and reliability of the measures. A third section compares the results of the mathematics pre-test with those of the mathematics post-test. Next the responses to the null hypotheses stated in Chapter I are dealt with, and then an interpretation of the results is provided as a conclusion to the chapter.

Pilot Studies  Semester I

As stated in the chapter on procedures, the two pilot studies carried out during the initial stages of the research were for the purpose of testing the experimental procedure prior to conducting the main study. No statistical analyses were carried out; they were reserved for the main study carried out during the second semester. However, these pilot studies were of great value to the investigator in that they clearly illustrated possible "pitfalls" in the experimental procedures. For example, they indicated that care had to be taken to insure that the students were fully aware of the exercises with manipulative material; otherwise, the exercise might appear irrelevant to them and the results
of the experiment would be suspect. An examination of the

techniques used in the manipulative material treatment indi-
cated that too much emphasis was put on the instruction
involved in the exercises, that is, the students seemed to
spend too much time trying to understand the object of the
exercise. Further, the pilot studies clearly pointed out the
necessity of an equal amount of teacher input to each group
of students experiencing treatment. Students using the

manipulative materials received a lesser amount of teacher
instruction on ratio and proportion. They were simply
provided with a question booklet which directed them to
perform certain activities using various manipulative
materials. In general, they experienced frustration because
they did not understand the reasons for manipulating material
in the manner prescribed. For example, they were asked to
measure the height of an object in large paper clips and
then in small paper clips. They were then asked to predict
the height of another object in small paper clips. Appendix
B gives in detail all the activities which the students were
told to undertake. The entire exercise seemed irrelevant to
them.

These observations were used to shape the final

experiment implemented during the second semester. The

major changes were the rejection of the question booklet
and the restriction of manipulative materials to Cuisinaire
rods. Instead of the students being left to work independ-
ently from written instructions involving a variety of
manipulative material, they were given direct instruction by the teacher in the purpose and use of the Cuisinaire rods.

The pilot studies also served another purpose. Prior to the pilot studies the writer had submitted plans of his experiment to his colleagues and senior advisor soliciting their opinions as to whether the two treatments were indeed different. The responses were generally positive, that is, these people found the treatments to be in their opinion distinctly different in nature. During the course of the pilot studies, the writer's observations reassured him that the two treatments were in fact different.

Reliability and Validity of the Measures (Main Study)

As indicated earlier, an item analysis for each measure was conducted. It (see Appendix C) shows the reliabilities of most test items to be in the range of approximately .3 to .6.¹

The reliability of each measure as a whole is shown in the following table in which the numbers appearing in the diagonal indicate the internal consistencies of the measures. The other numbers indicate the intercorrelation between pairs of measures.

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¹In simple terms, an item analysis establishes the correlation of each item on a measure with the measure as a whole. The writer determined that these correlations were acceptable according to the procedures described by Scott (1968) and employed in the social psychology department at Boulder, Colorado, after consultation with the advisor for his study.
Table I

RELIABILITIES AND INTERCORRELATION OF COGNITIVE AND ATTITUDE MEASURES

<table>
<thead>
<tr>
<th>Measure</th>
<th>Pre-test (Math)</th>
<th>Post-test Math</th>
<th>Science</th>
<th>Attitude Measure (1)</th>
<th>Attitude Measure (2)</th>
<th>Attitude Measure (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-test (Math)</td>
<td>.84</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Post-test (Math)</td>
<td>.69</td>
<td>.92</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Science</td>
<td>.48</td>
<td>.41</td>
<td>.87</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Attitude Measure #1</td>
<td>.38</td>
<td>.21</td>
<td>.41</td>
<td>.80</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Attitude Measure #2</td>
<td>.43</td>
<td>.38</td>
<td>.22</td>
<td>.63</td>
<td>.80</td>
<td></td>
</tr>
<tr>
<td>Attitude Measure #3</td>
<td>.39</td>
<td>.32</td>
<td>.33</td>
<td>.63</td>
<td>.69</td>
<td>.82</td>
</tr>
</tbody>
</table>

The reliability figures indicating internal consistency are in the range of .8 to .9. Statistical references (Ary (1972)) indicate that a desirable internal consistency of a measure is .8. This result is true for all measures used in this study. On the other hand, the correlations between the same tests given at different stages of the study (for example, the mathematics pre-test and post-test) are lower (.6 to .7). The latter correlations also indicate a degree of reliability because they indicate the stability of the test. The lower correlations between the repeated measures are acceptable considering that the period of time
between the administrations of the measures (i.e., the treatment period) was designed to produce a change and hence would tend to lower the correlations between the measures.

The Results of the Mathematics Pre-tests and Post-tests

A t-test was used to establish whether there were significant differences between the pre-test and post-test on mathematics. The results are recorded in the following table.

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Pre-test Mean</th>
<th>Pre-test Standard Deviation</th>
<th>Post-test Mean</th>
<th>Post-test Standard Deviation</th>
<th>T Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manipulative Materials</td>
<td>21.35</td>
<td>8.58</td>
<td>29.83</td>
<td>8.31</td>
<td>7.19*</td>
</tr>
<tr>
<td>Lecture Presentation</td>
<td>20.38</td>
<td>7.6</td>
<td>29.07</td>
<td>8.2</td>
<td>7.18*</td>
</tr>
</tbody>
</table>

* significant at .001

The t-test results indicate that the means of the post-test differ significantly from the pre-test for both methods of treatment. In fact, both treatments raised the scores on the mathematics measure approximately one standard deviation. The following figure (Figure II) illustrates graphically the elevation of scores.
The above figures are raw scores and consequently the indicated gain must be analyzed with the reservation that some error of measurement could exist due to history, maturation, etc.

Next, an analysis of variance calculation between the scores obtained by the manipulative material treatment group compared with the lecture treatment group on both the pre-test and post-test mathematics measure was computed. It indicated no significant difference due to treatment. Thus both methods of instruction have resulted in significantly the same improvement in mathematics.

The Results of the Science Measure and Attitude Measure

The means and standard deviation of all measures including the science and attitude measure are indicated in the following tables. They refer to both main effects—treatment and sex. In the last column of each table are the scores that refer directly to the null hypotheses of this study. Consequently a discussion of the statistical analyses on the scores related to these hypotheses follows the tables.

\[ F_{pre-test} = 0.0848, \quad F_{post-test} = 0.0131 \]

These are not significant at 0.05.
### TABLE III

MEANS AND STANDARD DEVIATIONS OF ALL COGNITIVE MEASURES CONSIDERING BOTH MAIN EFFECTS EMPLOYED IN THE STUDY

<table>
<thead>
<tr>
<th>Effect</th>
<th>Mathematics Pre-test Mean</th>
<th>Mathematics Pre-test S.D.</th>
<th>Mathematics Post-test Mean</th>
<th>Mathematics Post-test S.D.</th>
<th>Science Measure Mean</th>
<th>Science Measure S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Manipulative</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Material</td>
<td>21.35</td>
<td>8.58</td>
<td>29.83</td>
<td>8.32</td>
<td>11.59</td>
<td>6.72</td>
</tr>
<tr>
<td>Lectures</td>
<td>20.38</td>
<td>7.63</td>
<td>29.07</td>
<td>7.86</td>
<td>11.86</td>
<td>7.92</td>
</tr>
<tr>
<td>Sex</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Boys</td>
<td>20.25</td>
<td>7.20</td>
<td>28.80</td>
<td>7.81</td>
<td>10.97</td>
<td>7.51</td>
</tr>
<tr>
<td>Girls</td>
<td>21.50</td>
<td>9.11</td>
<td>20.23</td>
<td>8.38</td>
<td>12.62</td>
<td>7.01</td>
</tr>
</tbody>
</table>

### TABLE IV

MEANS AND STANDARD DEVIATIONS OF ALL ATTITUDE MEASURES CONSIDERING BOTH MAIN EFFECTS EMPLOYED IN THE STUDY

<table>
<thead>
<tr>
<th>Effect</th>
<th>Attitude test #1 Mean</th>
<th>Attitude test #1 S.D.</th>
<th>Attitude test #2 Mean</th>
<th>Attitude test #2 S.D.</th>
<th>Attitude test #3 Mean</th>
<th>Attitude test #3 S.D.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treatment</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Manipulative</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Material</td>
<td>70.34</td>
<td>9.75</td>
<td>70.83</td>
<td>9.63</td>
<td>72.35</td>
<td>9.06</td>
</tr>
<tr>
<td>Lectures</td>
<td>70.88</td>
<td>8.43</td>
<td>70.21</td>
<td>8.96</td>
<td>70.86</td>
<td>9.60</td>
</tr>
<tr>
<td>Sex</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Girls</td>
<td>71.61</td>
<td>9.84</td>
<td>71.70</td>
<td>9.50</td>
<td>72.00</td>
<td>9.58</td>
</tr>
<tr>
<td>Boys</td>
<td>69.48</td>
<td>8.37</td>
<td>69.64</td>
<td>9.05</td>
<td>71.29</td>
<td>9.17</td>
</tr>
</tbody>
</table>
In the first chapter the two hypotheses for the study were stated in null form. Hypothesis A referred to the effect of treatment, sex and interaction on the transfer skills of the students. Similarly Hypothesis B referred to the results of these same effects on the attitude regarding the inter-relationship of math and science. All of these effects on both the cognitive and attitude results were tested statistically using the analysis of covariance described in the third chapter. The following table includes the main effects, and the F value, for both the cognitive and attitude levels.

**TABLE V**

**F RATIO VALUES FOR COGNITIVE AND ATTITUDE MEASURES**

<table>
<thead>
<tr>
<th>Main Effect (Cognitive Measure)</th>
<th>F-Ratio</th>
<th>Main Effect (Attitude Measure)</th>
<th>F-Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>treatment</td>
<td>.15856</td>
<td>treatment</td>
<td>.78645</td>
</tr>
<tr>
<td>sex</td>
<td>.32688</td>
<td>sex</td>
<td>.42910</td>
</tr>
<tr>
<td>interaction</td>
<td>.52176</td>
<td>interaction</td>
<td>1.83900</td>
</tr>
</tbody>
</table>

At confidence level .05 all F ratio values in the above table were too low in order that the null hypotheses could be rejected. For example, the effect of treatment on the cognitive ability to transfer mathematics to science has a F ratio value of .15856, which is much lower than the critical value of between 4 and 4.08 which it would have to exceed to have a probability (less than .05) that any difference reflected by the instruction method is due to chance.
alone. In fact the probability that any difference is due to chance alone is in this example close to .7.

**Concluding Statements - Interpretation of Results**

The results of this study indicated that over an instructional period of two hours, students can be taught the mathematics of ratio and proportion so as to elevate significantly their mean scores on the same measure given after the instructional period. However, the different treatments did not produce significantly different results, i.e., both methods appear to be equally adequate in accomplishing the learning of ratio and proportion. Similarly, both treatments produced significantly the same transfer results.

Thus, in the context of the logic of the design, it is seen that in this study a two hour instructional period was able to elevate the means of both treatment groups significantly on the post math test independent of treatment used. However the treatment effect (manipulative materials vs. lectures) on the cognitive measure of both mathematics and science did not produce significantly different results. Also sex and the interaction of sex and treatment were not significant effects in the study.
Summary of Results

(1) All measures possessed internal reliability greater than .8, which was interpreted as acceptable according to literature cited.

(2) For both treatment groups there was a significant increase in the mean on the mathematics measure after the experimental period according to a t-test using confidence level .001.

(3) There was no significant difference between the groups exposed to the manipulative material and the groups exposed to the lecture treatment on the mean scores of their post math measure. (Confidence level .05).

(4) There was no significant difference in the transfer of mathematics to science due to treatment, sex, or interaction as measured by a science test. (Confidence level .05).

(5) There was no significant difference in attitude due to treatment, sex, or interaction as measured by a Likert-type attitude scale. (Confidence level .05).
CHAPTER V

CONCLUSIONS AND IMPLICATIONS

Results

The statistical evidence revealed no support for the notion that the ability of junior high school students to transfer the mathematical principles of ratio and proportion to scientific problems can be improved by the use of concrete manipulative materials as employed in this study. Similarly, the statistical evidence revealed no support for the notion that the students' attitude regarding the relationship of mathematics and science could be improved by the use of such materials, nor for the notion that one sex applies mathematics to science or understands the inter-relationship of the two disciplines better than the other. The interaction of the two factors, treatment and sex was also found not to be significant, that is, neither treatment appears to be more appropriate for one sex than for the other.

The results indicated that both treatments were effective in elevating the mathematics scores. However, neither treatment was more effective in accomplishing this goal. Both treatments produced the same outcomes on the mathematics post-test and on the science test measuring the transfer of mathematics to science.
Explanation of Results and Conclusions

The findings of the study could be explained by the suggestion that both methods of instruction work equally well in facilitating transfer. This suggestion is in fact correct considering the contextual framework of the study. However, this explanation is inconsistent with the ideas and theories expounded in this paper. Consequently it is the writer's belief that another explanation more closely related to theory of transfer of training exists.

There is evidence in the literature (Ellis (1969)) to suggest that the more closely the transfer task is related to the original task, the more easily transfer of learning can occur. Thus it is felt by the writer that perhaps the gap between the tasks used in this study was too large to comment on the significance of manipulative materials on transfer. Expansion on this point is given in the last section of this chapter.

Thus although these results on the transfer of mathematics to science were contrary to the expectations of the writer, whose intuition, supported by the literature on the teaching of mathematics and science, had led him to believe that the group exposed to manipulative materials would show superiority in their ability to transfer, they have not convinced him that the use of manipulative materials is useless or undesirable. Although the performance of the group experiencing manipulative materials to that of the group experiencing the lecture presentation was not superior,
neither was it inferior. It is conceivable that other benefits such as student independence and interest could accrue from the use of such materials.

Based on the experience of doing the study and the attempt to base the experiment on some type of conceptual framework, two additional conclusions are worthy of consideration.

(1) In general, the study of the transfer of training in this study is not aided by the theories of Thorndike and Judd (see Chapter II) and can best be discussed, studied, and understood considering only the conditions and limitations of a particular line of investigation such as the work of Wittrock (1968).

(2) In particular, the limitations of the study are such that the results must be considered tentative and further research is necessary.

Reasons for Conclusions

The first conclusion of this study can only be made considering that the study did not measure absolute or quantitative transfer but rather a comparison of transfer using two different treatments. Also relevant to this first conclusion is the fact that these theories were not examined in the experimental procedure of this study. Thus the two theories were not useful in providing a framework
to the study and were consequently hard to relate to the transfer investigated in this study. In particular it could not be positively concluded that the transfer in this study was one of "identical elements"—common to both mathematics and science or one of generalization (refer to Chapter II). This opinion on the part of the writer is also reinforced by his reading of recent studies on transfer such as Wittrock (1968). These studies are never placed in a theoretical context. Instead only the specific conditions and limitations are stated. However, at some point in time one might expect a theory of transfer to arise from empirical research, but at the moment this does not appear to be the case.

The second conclusion in suggesting that the results are tentative, infers that the results are not generalizable. That is, they can only be discussed relevant to the limitations imposed on the study. Thus arises the suggestion that under different conditions, different results might be expected. For example, it is felt by the writer that the separation of the mathematics and the chemistry may have been too great for transfer to occur, particularly with the short period allotted for the treatment.
Suggestions for Further Research

Growing out of the description relating to the last conclusion, further research might attempt to narrow the separation between the two disciplines. For instance a comparison could be made between the teaching of mathematics separated from chemistry to the teaching of mathematics "alongside" of the science. That is, the mathematics needed for a chemistry problem could be taught directly as it is needed, to solve the chemistry problem. Also the mathematics could be taught by the two methods employed in this study.

Thus a very structured experiment could be designed to accommodate the above consideration. For example, the suggestion that the transfer to be expected by students of the age group in this study should be more closely related to direct teaching rather than the broad transfer of a topic taught in isolation could be investigated by comparing the results of students exposed to either one of these treatments. The actual method of instruction in either the direct or isolated teaching approach could be either treatments used in this study. (Lecture presentation or manipulative materials.) Thus four possible situations exist. They would be the following: (1) Direct teaching of ratio and proportion for science using manipulative materials. (2) Isolated teaching of ratio and proportion for science using manipulative materials. (3) Direct teaching of ratio and proportion not using manipulative materials (Lectures
used instead). (4) Isolated teaching of ratio and proportion for science not using manipulative materials.

The above possibilities are displayed in the figure following.

<table>
<thead>
<tr>
<th>Manipulative Materials</th>
<th>Lectures</th>
</tr>
</thead>
<tbody>
<tr>
<td>Direct Teaching</td>
<td>Direct Teaching</td>
</tr>
<tr>
<td>Manipulative Materials</td>
<td>Lectures</td>
</tr>
<tr>
<td>Indirect or Isolated Teaching</td>
<td>Isolated Teaching</td>
</tr>
</tbody>
</table>

FIGURE III A POSSIBILITY FOR FUTURE RESEARCH

From this figure the following 6 comparisons could be made: (1) Between manipulative materials and lectures using direct teaching, (2) Between manipulative materials and lectures using isolated teaching, (3) Between direct and isolated teaching using manipulative materials, (4) Between direct and isolated teaching using lectures, (5) Between manipulative materials using direct teaching and lectures using isolated teaching, (6) Between lectures using direct teaching and manipulative materials using isolated teaching.

Another limitation of the study that could similarly be altered to provide a further insight into the topic of transfer is related to the format of its experimental design. At no place in this study is a quantitative measure of transfer obtained. Rather, only comparisons are made between effects of treatment and sex on transfer abilities.
It is intuitively believed by the writer that transfer had occurred using both treatments. In order to measure the quantity of transfer, another experiment would have to be designed to compare a group experiencing either treatment to a group experiencing no treatment at all. A pre-science measure would have to be given to both groups (treatment group and non treatment group) and the post science measure could gauge the effect of the treatment on transfer. Thus comparisons on this measure could be made between the treatment and non treatment group. The differences between the scores of these two groups on the post science measure could give a quantitative indication of the transfer that occurred.

Implications for Teaching

Thus although this study provides no evidence to reject the hypotheses stated in the first chapter, it certainly has suggested other research studies that could be attempted. From the point of view of teaching as opposed to research, this study is considered beneficial. For instance it is intuitively felt by the writer that Cuisinaire rods have great value in the teaching of science courses that require mathematics. It is felt that the use of these manipulative materials could best be done directly in the science classes as inferred in the first suggestion for further research.

This study also helped to crystallize the writer's opinion regarding the integration of junior high school
science and mathematics courses. In full agreement with such references as Gleason (1968) cited in the literature chapter, the writer feels that the isolated manner in which the study of mathematics and science is traditionally treated, is artificial and subject to change. It is the writer's feeling that many of the historic problems concerning the transfer of learning between mathematics and science could be reduced if the two disciplines were integrated.

A final benefit provided by this study relates directly to the writer's attitude regarding expectations of student achievement. This study has reinforced the writer's opinion that the transfer problem is indeed a barrier to the total understanding of many science ideas. The literature has also provided opinions similar to the writer's on this problem. (e.g. Rising (1967)) Thus a reassuring feeling on the part of the writer that this barrier is a universal one, would tend to allow instruction in the sciences to continue in the spirit of empathy and understanding. It is the writer's opinion that if this study has even partially tended to achieve this result, it has been of benefit to his teaching of science.
AIMS AND OBJECTIVES OF GRADE NINE ALGEBRA
CURRICULUM GUIDE 1969

A To lay rigorous foundations for a study of algebra.
B To introduce the algebra of a real number system.
C To introduce such set language as is immediately useful.
D To introduce real co-ordinate geometry.
Station #1

Examine the equipment at this station.

Read carefully the instructions at this station.

After performing the tasks asked in the instructions, attempt to answer the following questions:

1. Make a guess as to how many small paper clips could be put end to end to measure Mr. Short's height.

2. Now measure Mr. Short's height in small paper clips to check your guess. How many?

3. Is there a way of getting the correct answer for the number of small paper clips for Mr. Short without guessing or measuring? Please explain answer as fully as possible.

4. Divide the height of Mr. Tall in large paper clips with the height of Mr. Short in large paper clips. Show work.

5. Divide the height of Mr. Tall in small paper clips with the height of Mr. Short in small paper clips. Show work.
6. Comment on question (4) and (5).

7. Divide the height of Mr. Tall in large paper clips with the height of Mr. Tall in small paper clips.

8. Divide the height of Mr. Short in large paper clips with the height of Mr. Short in small paper clips.

9. Comment on questions (7) and (8).

10. Can you now answer question 3 better. If so, do so here!

11. Can you now find 2 different ways of being able to predict Mr. Short's height in small paper clips if you knew:
   - Mr. Tall's height in large paper clips.
   - Mr. Tall's height in small paper clips.
   - Mr. Short's height in large paper clips.
   Explain using your measurements.

12. Go to Station (2).
Station #2
Examine the equipment at this station.

Read carefully the instructions at this station.

After performing the tasks asked in the instructions, attempt to answer the following questions.

QUESTIONS

1. Make a guess as to how long the diagonal of the medium square is (in centimetres).

2. Measure the diagonal of the medium square with your metre stick in centimetres.

3. Is there a way of getting the correct answer without measuring or guessing. Please explain as fully as possible.

4. Divide the length of the side of the large square by the side of the medium square.

5. Divide the length of the diagonal of the large square by the diagonal of the medium square.

6. Comment on (4) and (5).
7. Divide the length of the side of the large square by the diagonal of the large square.

8. Divide the length of the side of the medium square by the diagonal of the medium square.

9. Comment on questions (7) and (8).

10. Can you now answer question 3 better. If so, do so here.

11. Can you now find 2 different ways to predict the size of the diagonal of the medium square if you know:
   1. The length of the side of the large square.
   2. The length of the side of the medium square.
   3. The length of the diagonal of the large square.
   Explain using your measurements.

12. Now without measuring or guessing, predict the size of the diagonal of the small square.

13. Go to Station #3.
Station #3

Examine the equipment at this station.

Read carefully the instructions at this station.

After performing the tasks asked in the instructions, attempt to answer the following questions.

1. Count the number of tacks in bag #2.

2. Count the number of paper clips in bag #2.

3. Can you find out how many paper clips are in bag #2 without counting knowing that bag #2 is identical to bag #1 except in size and weight. Show work and explain answer.

4. Divide the number of tacks in bag 1 with the number of tacks in bag 2.

5. Divide the number of paper clips in bag 1 with the number of paper clips in bag 2.

6. Comment on question (4) and (5).

7. Divide the number of tacks in bag 1 with the number of paper clips in bag 1.
8. Divide the number of tacks in bag 2 with the number of paper clips in bag 2.

9. Comment on (7) and (8).

10. Can you now answer question (3) better. If so, do so here.

11. Can you give 2 different ways of predicting the number of paper clips in bag 2 if you know:
   1. The number of tacks in bag 1.
   2. The number of tacks in bag 2.
   3. The number of paper clips in bag 1.

Now without counting predict the following and show your work.

12. Number of paper clips in bag 3.

13. Number of nails in bag 2.

14. Number of nails in bag 3.

15. If an identical bag to bag 1, 2 and 3 but different in weight and size is used and if this bag has 57 paper clips, how many tacks and nails does it have?

16. Go to any station you wish to check your answer.
APPENDIX C

MATH

Division

Fill in the squares.

1. \( \frac{2}{5} = \square \)

\[ \frac{5}{12} = \square \]

3. \( \frac{2}{8} = \square \)

4. 4 is to 7 as 8 is to \( \square \).

5. 2 is to 6 as \( \square \) is to 24.

6. \( \square \) is to 15 as 2 is to 5.

7. \( \frac{2}{5} = \square \)

8. \( \frac{4}{5} = \square \)
Math

9. \[
\frac{8}{12} = \frac{5}{?}
\]

10. \[
\frac{4}{?} = \frac{7}{15}
\]

11. 39.37 inches = 100 cm.

How many inches are equal to 125 cm.? 

12. How many cm are equal to 24.9 inches?

13. How many inches are equal to 85 cm?

14. Jim's final average was 75% or on the average 75 out of 100. If his final score was 120, what was the test out of?

15. For every two incorrect answers John got seven answers correct. How many correct answers did he get if he made twenty mistakes?
16. Larry got 35 marks out of a possible 36 on his science test. What was his percentage? (His mark if it were out of 100)

17. George paid $25 for a transistor radio. How many similar transistor radios could he get for $125?

18. The cost of 12 oranges is $2.25. How much do 32 similar oranges cost?

19. The cost of 7 apples is $1.26. How many similar apples can be bought for $2.00? Include fractions.

20. 13 pencils can be bought for $1.86. How much will 6 similar pencils cost?
The following equation represents the combination of hydrogen with oxygen to form water. This equation is balanced and the correct molecular weights are placed above the symbols. Please show all work in the space provided.

\[ 2H_2 + O_2 \rightarrow 2H_2O \]

(1) How many pounds of water are produced from 32 pounds of oxygen?

(2) How many pounds of oxygen are necessary to react with 12 lbs. of hydrogen?

(3) How many pounds of water are produced from 100 pounds of oxygen?
(4) How many pounds of hydrogen would produce 75 pounds of water?

(5) How many grams of oxygen are necessary to react with 9 grams of hydrogen?

\[
\begin{align*}
\text{Hg} & \quad \text{S} & \rightarrow & \quad \text{HgS} \\
200 & \quad 32 & \quad 232
\end{align*}
\]

The above equation can be represented by the following diagram:

Hg (mercury) is represented by a nail which weighs 25 milligrams and sulphur by a paper clip which weighs 4 milligrams.

Before you are a number of nails and paper clips. Study the diagrams that represents this reaction. You may use this material to answer the following questions.

(1) How many grams of sulphur will react 100 grams of mercury?
(2) How many grams of HgS are produced from 12 grams of sulphur?

(3) Draw pictures of nails and paper clips to show how you could get the answer using only the equipment in front of you.

(4) How many grams of sulphur are necessary to produce 100 grams of HgS?

(5) How many grams of mercury will react with 6 grams of sulphur?
Before you are a number of beakers. Each beaker is labelled with a different number (1-5). Answer the following questions.

(1) Water and blue solution (Beaker 1-5)

Information

<table>
<thead>
<tr>
<th>Beaker</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beaker 1</td>
<td>500 ml. of water</td>
</tr>
<tr>
<td>Beaker 2</td>
<td>25 grams of blue. Altogether 500 ml. of blue solution.</td>
</tr>
<tr>
<td>Beaker 3</td>
<td>700 ml. of water</td>
</tr>
<tr>
<td>Beaker 4</td>
<td>300 ml. of water</td>
</tr>
<tr>
<td>Beaker 5</td>
<td>Unknown amount of blue solution with same concentration as seen in Beaker 2.</td>
</tr>
</tbody>
</table>

(a) How many grams of blue are necessary to make 700 ml. of blue solution?

(b) How many grams of blue make 300 ml. of blue solution?

(c) If Beaker 5 has 12 grams of blue and its concentrate is the same as Beaker 2, then how much blue solution is there in Beaker 5?
(C) continued

(2) Beam Balance

Beam balance 1 shows 500 grams balancing 200 grams.

Beam balance 2 shows 300 grams added to the 500 grams already there.

(a) How many grams would have to be added to the 200 grams to maintain balance?

(b) If the 300 grams were added to the 200 gram side how many grams would have to be added to the 500 grams to maintain balance?
APPENDIX E

NAME ________________________
DIVISION ______________________

The following statements that you see are not facts. They are only opinions. You have an opportunity to give your opinion on the statement by indicating if you (A) strongly agree (B) agree (C) undecided (D) disagree (E) strongly disagree.

Please shade in the square A, B, C, D, or E which indicates your opinion.

1. Drinking coffee is bad for the health.   A B C D E

2. Science and Math are different types of courses and should therefore be taught by different teachers.   A B C D E

3. The strap should have never been used at school.   A B C D E

4. If a person is to be a scientist he must know math.   A B C D E

5. Science and math are related subjects.   A B C D E

6. Students should be allowed to wear outdoor clothing in class.   A B C D E

7. Scientists and Mathematicians are very different kinds of people.   A B C D E

8. If you are poor at Science you are likely to be poor at math.   A B C D E

9. A person should like doing Science even if he doesn't like doing Math.   A B C D E

10. Most teachers at Pearkes are interested in student problems.   A B C D E

11. It is not necessary to know math in order to do science.   A B C D E
12. Scientists and Mathematicians use the same type of thinking process.

13. It is not necessary to have short hair in today's world.

14. If a person is good at math he is good at science.

15. Good science students think differently than good math students.

16. You couldn't have science without math.

17. A major purpose of science is to help man live more comfortably.

18. Studying math helps you do science.

19. Scientific ideas are most easily expressed in mathematical terms.

20. Learning science skills are different than learning mathematical skills.

21. Free schools are better than traditional schools.

22. Math and science are very different.

23. People who work in laboratories do not have to use much mathematical knowledge.

24. Science and math are different subjects using different equipment and should not be taught in the same room.

25. Knowledge about numbers help a person in chemistry.

26. Poor posture brings on back ailments.
27. Teachers at Pearkes are friendly towards students.

28. You couldn't have science without math.

29. Junior high school is less interesting than elementary school.

30. Science problems and math problems do not have a lot in common.
APPENDIX F

THE MANIPULATIVE MATERIAL TREATMENT
CUISINAIRE RODS

The manipulative material used during the experimental treatment of the second semester, consisted of a number of wooden rods of ten different colours, varying in length from 1 cm. to 10 cm. Each rod had a width and depth of 1 cm. All rods of the same length had the same colour. Thus, by matching rods of different lengths a visual impression of a ratio could be obtained. For instance a white rod put against a red rod could represent a ratio of 1 to 2 or 2 to 1 depending on how it is viewed. Identical ratios to this one could be obtained by finding other rods, one of which was twice as long as the other. Most important in this discussion was the fact that very many such combinations could be found. In fact, any ratio could be represented by an infinite quantity of such matchings. Thus, the power of these materials to represent the very nature of a ratio suggested by Karplus (1972), as an association of two items in many different ways, is clearly evident.

During the first half of the first hour the students using their materials were asked to find various ways in which common ratios such as 1:3 or 2:5 could be represented. The term fraction was not used to avoid possible reference to their previous arithmetical experiences with fractions. It was the mathematical nature of ratio that was being investigated in this procedure, rather than any arithmetic compu-
tations that the students may or may not be familiar with. It was the ultimate goal of the investigator to facilitate the arrival of these arithmetic procedures through a discovery process based on a clear understanding of the mathematical concepts of ratio.

Thus, the second half of the first period was devoted to displaying all similar matched pairs (e.g., 1:2, 2:4, etc.). Students were then asked to take the longest and shortest rod of any 2 pairs and to make trains by considering the product of these two lengths. For example, the matched pair representing the ratio $\frac{2}{3}$ could be rods representing the following statement $\frac{2}{3} = \frac{4}{6}$. If a train of 2 sixes were compared to a train of 3 fours, the students should be able to see that they are of identical length.

In arithmetic, this procedure is the familiar "cross multiplication" technique of which most students in grade 9 are familiar. This term was alluded to only after the students had made the discovery that the trains were of equal length. It was important that the underlying mathematics of ratio and proportion was made evident by the use of these materials. That is, if the ratio $\frac{2}{5}$ is represented by a proportion $\frac{2}{5} = \frac{6}{15}$, the fact that 2 fifteens is equal to 6 fives can be concretely illustrated when the observation that the tripling of the smaller rod also necessitates the tripling of the larger rod in order to produce two trains of equal length is made.

By using trains a method to arrive at an unknown quantity in a proportion was discovered. For example, in
the example $\frac{2}{5} = \frac{10}{3}$ students could clearly see by using these trains, that the number of fives in a train 2 ten rods long would have to be four, and that four is twice two as ten is twice five. Thus the rationale for the cross multiplication technique could be discovered using trains.

Similar practice was then carried out on proportion that involved non-integral solutions, (i.e. $\frac{2}{3} = \gamma$) and again an arithmetic solution was discovered employing the very nature of the underlying mathematics involved in a ratio. This procedure was carried into the second hour and students were asked to solve several "fraction" questions of this type using both trains and the arithmetic method that they had discovered as a result of using this material.

THE LECTURE PRESENTATION

The lecture presentation achieved the result of arriving at the same arithmetic technique without employing manipulative materials. Although visual representations of ratio and proportion were given on the blackboard, the experience with the Cuisinaire materials that facilitated the discovery of the necessary arithmetic technique, was missing.¹

¹A tape was made of the lecture presentation used.
### Correlations of Items with Total Test Scores

#### Pre-Test

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### Appendix C
REFERENCES


