ANALYSIS AND STRATEGIES FOR MANAGING INTEREST RATE RISK IN NORTH AMERICAN MARKETS

by

Michael Craig
CFA, CFA International, 2003
B.Comm., University of British Columbia, 1999

PROJECT SUBMITTED IN PARTIAL FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF ARTS

In the
Faculty of
Business Administration

Financial Risk Management Program

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SIMON FRASER UNIVERSITY

Summer 2006

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APPROVAL

Name: Michael Craig

Degree: Master of Arts (Spec)

Title of Project: Analysis and Strategies for Managing Interest Rate Risk in North American Markets

Supervisory Committee:

Dr. Christophe Péignon
Senior Supervisor
Assistant Professor of Finance

Dr. Daniel R. Smith
Second Reader
Assistant Professor of Finance

Date Approved: July 27, 2006
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ABSTRACT

This paper presents a strategy for estimating the factors driving the Canadian and US term structures of interest rates using principal component analysis. I also estimate the factors for a combination of the US and Canadian term structures. I find that the factors are stable through time when using various overlapping and non-overlapping sub-periods. These factors pertain to the level, steepness and curvature effects of the yield curve. This research has applications in immunization and liability management as it provides evidence that US fixed income securities can be used to partially offset Canadian denominated liabilities and principal components can be used rather than bond durations when analysing and matching exposures. I also model forecasted weekly factor volatility against weekly absolute factor changes using a GARCH(1,1) model. The results are promising as the GARCH model does provide some insight into actual volatility. This has important implications for the risk management of interest sensitive securities as it provides an alternative to value at risk models and other scenario analysis techniques.
I dedicate this paper to my wonderful wife Rana. Thank you for your love, patience and support throughout these last twelve months. I love you very much.
ACKNOWLEDGEMENTS

I would like to thank Dr. Christophe Pérignon for his guidance, insights and for sharing his wisdom in an area of finance for which I previously had limited knowledge of. You have helped me achieve a goal for which I will be forever grateful. I would also like to thank Dr. Daniel Smith for his valuable comments and critiques of this paper.

I would also like to thank my parents who have always challenged me to consider my ideas from all vantage points.
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1 INTRODUCTION

The management of interest rate sensitive securities to changes in the shape of the yield curve has been the subject of extensive study. This paper looks into the practicality of using principal component analysis (hereafter referred to as PCA) to measure exposures to the Canadian yield curve, the US yield curve as well as a combined North American yield curve. The impetus for analyzing the interest rate risk to both the U.S. and Canadian markets is due the fact that Canadian liquidity is drying up due to consistent Federal Government budget surpluses while the U.S. government continues to run large deficits. Thus from a Canadian managers perspective the United States is a logical market to invest in due to it's close proximity and strong correlation with the Canadian market. The paper will follow in the same spirit as Litterman and Scheinkman's paper in 1991. Litterman and Scheinkman (1991) found that changes in fixed income portfolios consisting of Treasury bonds could be explained by three factors, or attributes of the yield curve. They referred to these as the level, steepness and curvature of the yield curve and applied their research to hedging portfolios. This paper will follow this in that it will be applied to hedging portfolios, however I will first look at the success of this strategy as it applies to Canadian interest rate sensitive securities and then to a combination of North American government securities. This area of research is of interest because currently there are no foolproof methods to manage a mixture of US and Canadian fixed income securities. Although there is a positive correlation between the US and Canadian term structures of interest rates this correlation is far from perfect. This poses a difficult task for investment and risk managers because they are now faced with multiple sources of interest rate risk and it begs the questions - should the investment manager use the straightforward modified duration approach and simply report the modified duration of U.S. and Canadian bonds separately, thus exposing the pool of assets to
changes in the slope and curvature of the respective yield curves? Alternatively should the manager use a key rate duration approach (Ho 1992) and measure the interest rate risk to each point in the yield curve, thus bombarding the manager with two separate vectors of key rate durations. Clearly neither of these two methods is alluring.

This paper will use a “building block” approach as I present my findings. First I will analyse the success of using PCA to manage Canadian and U.S. interest rates over an extended period of time. I will then apply the same methodology to a combination of the two yield curves. The results indicate the first 3 to 4 factors in the reduced set of orthogonal factors can explain the majority of variability in the data.

The paper continues with an analysis of the eigenvalues produced by PCA. While the eigenvectors are relatively constant through various periods the eigenvalues vary significantly. I have attempted to model the volatility of yield factors using various GARCH models and have found promising results.

The contributions of this paper are twofold. First, from a liability matching perspective the paper provides a technique for investment managers to purchase a combination of US and Canadian bonds to offset Canadian liabilities while being able to control the net risk of these two offsetting portfolios. The second contribution is that the paper shows that using a basic parsimonious GARCH(1,1) model a risk manager has some ability to forecast the volatility of the factor levels.
2 MODEL DEVELOPMENT

2.1 Principal Component Models of US, Canadian and Combined Interest Rates

I am interested in modelling the US, Canadian and Combined North American yield curve dynamics where $Y$ is the current yield at $t$ and $\Delta Y$ is the change in yield between $t$ and $t-1$.

For a complete discussion with regards to using the change in the level and modelling the variance covariance matrix of the term structures please refer to Lardic et al. (2001). I first take the levels for the entire data set and divide the data into sub-periods. I take the matrices of yield changes, denoted as:

$$\Delta Y = (Y_{t,k} - Y_{t-1,k})_{1 \leq k \leq K}$$

where we have $k$ maturities. In this model this is zero coupon maturities which combine to form the yield curve and $t$ observations. Using this data we develop covariance matrixes, $\Sigma$, for the various data samples. Following the methodology of Pérignon and Villa (2002) I take the sample covariance matrix and decompose it into the corresponding eigenvalues and eigenvectors. The following is the decomposition of the covariance matrix.

$$\Sigma_{\Delta Y} = AA'$$

Here I am denoting the eigenvectors as $A$ and eigenvalues as $\Lambda$. $\Sigma$ represents the covariance matrix for the yield changes during the period sampled. The eigenvectors are column vectors and are used to calculate the yield factors. The eigenvalues are diagonal matrices in which the left and right quadrants of the matrix are zeros and the diagonal contains the values. The analysis is
repeated three times, first I look at the Canadian data, second the US data and thirdly I run the tests for the combined data. The percent explained by each of N factors, starting with the first factor, can be calculated using the following formula:

$$\lambda_i / \sum_{i=1}^{N} \lambda_i$$

(3)

which is the percentage explained by $i^{th}$ factor where $N$ equals the total number of factors.

2.2 Estimation of the Level, Steepness and Curvature Volatility for the Three Data Series

In the second part of this paper I develop a model to forecast the volatility of the yield factors. I take weekly yield levels from the entire sample and then derive the covariance matrix, eigenvectors and eigenvalues. Hence I have the relationship $\Sigma_{n,n} = AA'$. The matrix is larger; however we are only concerned with the first three columns of vectors in the matrix which I will denote as:

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

(4)

From this the level effect (LT), steepness effect (ST) and curvature effect (CT) can be generated as follows:
For the volatility estimation I will use a simple GARCH(1,1) model. I do an in sample estimation of the beta coefficients. Once these coefficients have been calculated I use them to estimate the weekly volatility for the yield factors using the same methodology as used by Pérignon and Smith (2005). This is then compared to the values generated using equation (6).

\[ LT_{t,k} = Y_{t,k} \times a_{1,t,k} \]
\[ ST_{t,k} = Y_{t,k} \times a_{2,t,k} \]
\[ CT_{t,k} = Y_{t,k} \times a_{3,t,k} \]

(6)

\[ \sigma^2_{LT} = \beta_{0L} + \beta_{1L} \sigma_{LT-1} + \beta_{2L} e^2_t \]

(7)

\[ \sigma^2_{ST} = \beta_{0S} + \beta_{1S} \sigma_{ST-1} + \beta_{2S} e^2_t \]

(8)

\[ \sigma^2_{CT} = \beta_{0C} + \beta_{1C} \sigma_{CT-1} + \beta_{2C} e^2_t \]

(9)
3 DATA AND EMPIRICAL RESULTS

The model uses Canadian zero-coupon Canada bond yields obtained from the bank of Canada’s website at www.bankofcanada.ca. The US Treasury zero-coupon data was obtained from the St. Louis Federal Reserve at http://stlouisfed.org. Both data sets cover the 1986:01 to 2005:12 period and consist of daily data points. For the most part the data is clean except for a period of approximately one month where the Canadian rates were not updated. To generate the factors I use a vector consisting of 8 elements. These elements are the following yield curve points, {3M, 6M, 1Y, 2Y, 3Y, 5Y, 7Y, 10Y} where M indicates month and Y indicates year.

3.1 Canadian Term Structure

I first analyse the Canadian data when estimating the main factors that drive the term structure. Using the daily changes in yields from the data set I estimate various variance covariance matrices for a variety of sub-periods. These matrices are proxies for the “true” covariance which is of course unobservable. As discussed in the model specification section these covariance matrices are decomposed into their corresponding eigenvalues and eigenvectors. I run two analyses, in the first analysis I take four equal subsets of the data, compute covariance matrices for each subset and then decompose each subset into its eigenvalues and eigenvectors. In the second analysis I take ten year subsets of the data, calculate the covariance matrix, eigenvalues and eigenvectors and then move the window of data forward by one year.
Figure 1  Level Factor Loadings for Canadian Interest Rates - Separate Periods

Figure 2  Steepness Factor Loadings for Canadian Interest Rates - Separate Periods

Figure 3  Curvature Factor Loadings for Canadian Interest Rates - Separate Periods
Figure 4  Level Factor Loadings for Canadian Interest Rates - Rolling Periods

Figure 5  Steepness Factor Loadings for Canadian Interest Rates - Rolling Periods

Figure 6  Curvature Factor Loadings for Canadian Interest Rates - Rolling Periods
The first factor, displayed in exhibits 1 and 4, has virtually the same affect across entire maturity spectrum and is relatively consistent. This can be interpreted as the level factor, as yields rise (fall) in a parallel (consistent) manner across the entire yield curve all securities will fall (rise) in value by an equal percentage.

The second factor, displayed in exhibits 2 and 5, shows the factor loadings as positive at the short end of the yield curve, while decreasing sharply to a range of $-0.2$ to $-0.4$ for longer maturity points. This can be interpreted as the steepness factor. This makes intuitive sense; as yields widen (tighten) at the long end relative to the short end you would expect longer dated securities to under-perform (outperform) shorter dated securities.

The third factor, displayed in exhibits 3 and 6, shows the factor loadings as positive at the short and long and negative at the middle of the yield curve. This is known as the curvature or “butterfly effect”. We observe this when the short and long bonds out perform the belly or vice versa. The results from all periods are very consistent and therefore we can ascertain that this factor is very stable through time. The percentages of variance explained by the three factors are displayed in the following two tables.

| Table 1 | Percentage of Variance Explained by the First Three Factors for Canadian Interest Rates |
|---------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|
|         | Sub-Period | Factor | 86-90 | 91-96 | 97-00 | 01-06 |
|         | 86-90 | 91-96 | 97-00 | 01-06 |
| Level | 56.6% | 64.5% | 59.6% | 78.7% |
| Steepness | 32.4% | 22.7% | 27.9% | 14.0% |
| Curvature | 4.9% | 6.6% | 8.7% | 5.2% |
| Total % Explained | 94.9% | 93.8% | 96.2% | 97.9% |
Table 2  Percentage of Variance Explained by the First Three Factors for Canadian Interest Rates

<table>
<thead>
<tr>
<th>Sub-Period</th>
<th>Factor</th>
<th>86-95</th>
<th>87-96</th>
<th>88-97</th>
<th>89-98</th>
<th>90-99</th>
<th>91-00</th>
<th>92-01</th>
<th>93-02</th>
<th>94-03</th>
<th>95-04</th>
<th>96-05</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level</td>
<td>59.1%</td>
<td>58.9%</td>
<td>57.7%</td>
<td>59.2%</td>
<td>60.4%</td>
<td>62.5%</td>
<td>63.5%</td>
<td>65.3%</td>
<td>65.2%</td>
<td>63.7%</td>
<td>64.3%</td>
<td></td>
</tr>
<tr>
<td>Steepness</td>
<td>29.0%</td>
<td>29.4%</td>
<td>30.6%</td>
<td>28.8%</td>
<td>27.3%</td>
<td>24.5%</td>
<td>23.9%</td>
<td>23.0%</td>
<td>23.6%</td>
<td>25.1%</td>
<td>23.8%</td>
<td></td>
</tr>
<tr>
<td>Curvature</td>
<td>6.1%</td>
<td>6.2%</td>
<td>6.1%</td>
<td>6.3%</td>
<td>6.5%</td>
<td>7.3%</td>
<td>7.1%</td>
<td>6.9%</td>
<td>7.1%</td>
<td>7.9%</td>
<td>8.4%</td>
<td></td>
</tr>
<tr>
<td>Total % Explained</td>
<td>94.2%</td>
<td>94.5%</td>
<td>94.4%</td>
<td>94.3%</td>
<td>94.2%</td>
<td>94.3%</td>
<td>94.6%</td>
<td>95.2%</td>
<td>96.0%</td>
<td>96.7%</td>
<td>96.6%</td>
<td></td>
</tr>
</tbody>
</table>

The three factors explain the majority of the term structure variation. The first table displays the percentage of variance explained for the separate, non-overlapping periods. The percentages ranged from 93.8% to 97.9%. However the explanatory power of the three factors did jump around. The level factor begins at 56.6% in the mid to late 1980s and moves to 78.7% in the 2000s. The opposite affect can be observed for the steepness factor. It starts at 32.4% in the mid to late 1980s and drops to 14.0% in the 2000s. Similar results and fluctuations can be observed when conducting the rolling analysis. The percentage of variance ranges between 94.22% and 96.56%, however the level factor has increased throughout the period from 59% to 64% while the steepness factor has weekend from 29% to 23.84%. Thus over the past 20 years bond returns have been increasing determined by level changes and conversely the explanatory power of the steepness factor has decreased. These results are consistent with the ones obtained by Clare et al. (2000). In their paper they compared the relationships between Germany, the United Kingdom, and the United States and found that while the overall covariance of these markets are stable, the components of the covariance can vary over time.

3.2 US Term Structure

I now turn to the US term structure of interest rates. Again I use daily observations and select sub-periods from a twenty year sample of data. I use four non-overlapping sub periods as well as a rolling analysis in which I take samples of ten years of data and roll the period one year forward per sample. The US data shows similar behaviour to the Canadian data and appears to be
even more consistent when viewed over many sub-periods. That is, the principal factor loadings of the US data appear to be more constant than their Canadian counterparts.

Figure 7  Level Factor Loadings for US Interest Rates – Separate Periods

Figure 8  Steepness Factor Loadings for US Interest Rates – Separate Periods
Figure 9  Curvature Factor Loadings for US Interest Rates – Separate Periods

Figure 10  Level Factor Loadings for US Interest Rates – Rolling Periods

Figure 11  Steepness Factor Loadings for US Interest Rates – Rolling Periods
As with the analysis of the Canadian factor loadings, I again look at the first three factor loadings. Beginning with the level factor, we notice that the factor loadings range from 0.12 to 0.42 for the separate period sub-samples and 0.15 to 0.42 for the rolling period analysis. Moreover, there seems to be less variability of the factor structure for the US data compared to the Canadian data, which indicates that the factor structure is more stable in the US.

As with the Canada statistics, the steepness factor loadings are negative at the short part of the maturity curve and positive at the long end. The curvature chart details the positive factor loadings at the short and long part of the yield curve and negative in the middle. The results are all similar to those obtained in Canada; however as with the level factor the charts show less variability compared to the Canadian data indicating that the second and third factors are also more stable in the US as compared to Canada. The percentage of variance explained by the three factors is presented in the following two tables.
Table 3  Percentage of Variance Explained by the First Three Factors for US Interest Rates - Separate Periods

<table>
<thead>
<tr>
<th>Factor</th>
<th>86-90</th>
<th>91-96</th>
<th>97-00</th>
<th>01-06</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level</td>
<td>81.8%</td>
<td>84.9%</td>
<td>80.0%</td>
<td>84.2%</td>
</tr>
<tr>
<td>Steepness</td>
<td>11.4%</td>
<td>9.3%</td>
<td>12.4%</td>
<td>10.0%</td>
</tr>
<tr>
<td>Curvature</td>
<td>2.7%</td>
<td>2.6%</td>
<td>3.2%</td>
<td>3.0%</td>
</tr>
<tr>
<td>Total % Explained</td>
<td>95.8%</td>
<td>96.9%</td>
<td>95.6%</td>
<td>97.2%</td>
</tr>
</tbody>
</table>

Table 4  Percentage of Variance Explained by the First Three Factors for US Interest Rates – Rolling Periods

<table>
<thead>
<tr>
<th>Factor</th>
<th>86-95</th>
<th>87-96</th>
<th>88-97</th>
<th>89-98</th>
<th>90-99</th>
<th>91-00</th>
<th>92-01</th>
<th>93-02</th>
<th>94-03</th>
<th>95-04</th>
<th>96-05</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level</td>
<td>82.7%</td>
<td>82.6%</td>
<td>83.9%</td>
<td>84.3%</td>
<td>83.4%</td>
<td>82.5%</td>
<td>82.1%</td>
<td>82.2%</td>
<td>82.4%</td>
<td>82.2%</td>
<td>82.3%</td>
</tr>
<tr>
<td>Steepness</td>
<td>10.7%</td>
<td>11.0%</td>
<td>9.8%</td>
<td>9.6%</td>
<td>10.3%</td>
<td>10.7%</td>
<td>11.3%</td>
<td>11.2%</td>
<td>11.1%</td>
<td>11.2%</td>
<td>10.9%</td>
</tr>
<tr>
<td>Curvature</td>
<td>2.5%</td>
<td>2.5%</td>
<td>2.5%</td>
<td>2.5%</td>
<td>2.7%</td>
<td>3.0%</td>
<td>3.0%</td>
<td>3.0%</td>
<td>3.0%</td>
<td>3.1%</td>
<td>3.2%</td>
</tr>
<tr>
<td>Total % Explained</td>
<td>96.0%</td>
<td>96.1%</td>
<td>96.3%</td>
<td>96.4%</td>
<td>96.4%</td>
<td>96.2%</td>
<td>96.4%</td>
<td>96.4%</td>
<td>96.5%</td>
<td>96.4%</td>
<td>96.4%</td>
</tr>
</tbody>
</table>

Similar to the findings by Bliss (1997), the percentage of the variation explained by the first three factors ranges from 95.8% to 97.2% for the separate sample analysis and 96.0% to 96.5% for the rolling analysis. Moreover, the percentage explained by each factor remained fairly constant for both the rolling and separate period analysis. The percent explained by the level factor was approximately 20% higher than the corresponding level factor for the Canadian data while the steepness and curvature factors explained much less of the variation relative to the Canadian steepness and curvature factors. Of particular interest is that while the percentage explained by US factors is stable, the percentage explained by the level and steepness factors in Canada seems to be increasing and decreasing respectively. Thus the explanatory percentages of the yield factors in Canada appear to have been gravitating to those reported in the US.

3.3 Combined North American Term Structure

I now turn to a “North American” curve in which I combine both US and Canadian interest rates. As before, I start by creating a variance covariance matrix of the 16 yield curve
points (8 Canadian, 8 US) for the same sub-periods of data which were used in the Canadian and US analysis. I then proceed to decompose the matrix into corresponding eigenvalues and eigenvectors.

**Figure 13** Level Factor Loading, Combined Interest Rates – Separate Periods

![Level Factor Loading Graph](image)

**Figure 14** Steepness Factor Loading, Combined Interest Rates – Separate Periods

![Steepness Factor Loading Graph](image)
Figure 15  Curvature Factor Loading, Combined Interest Rates - Separate Periods

Figure 16  Level Factor Loading, Combined Interest Rates - Rolling Periods

Figure 17  Steepness Factor Loading, Combined Interest Rates - Rolling Periods
The first principal component, inferred as the level component is displayed in exhibits 17 and 20. Notice that all Factor loadings are above zero but are not as consistent as either the US or Canadian first principal component because the level factors are not as consistent across the maturity spectrum. This being said, the traces are fairly stable and there does not appear to be a wide disparity when using the 10 year rolling sub-periods. The separate sub-periods are not nearly as stable and do exhibit some volatility, however the factor loadings are all positive indicating that the direction is correct.

The second factor plots much like the US and Canadian second principal component with the factor loading being positive at the short end of the curve and negative at the long end of the curve. There appears to be some volatility of the traces between the 6 month and 2 year maturity points but afterwards the curves are extremely similar. What is interesting is that only the short Canadian maturities are positive while all other maturity points, including the US short maturities are negative. Rather than observing two distinct charts, one for the Canadian maturities and one for the US maturities I observe a fairly continuous curve. Moreover, the last separate sub-period seems to be the inverse of the previous three sub-periods. Thus in the last 5 years the opposite affect has occurred relative to the previous 15 years.
Finally I turn to the third principal component or “curvature” factor loading. As with the second principal component this chart is very similar to the individual US and Canadian charts. The factors are positive at the short and long ends of the curve and negative at the middle, which is the effect we would assume based on the curve exhibiting a butterfly movement. Oddly enough, however, is the fact that while the mid range Canadian maturities exhibit negative factor loadings the US loadings are all fairly consistent across all maturities. This could be due to the minimal affect of the curvature factor for the US term structure while the affect is 4-5 times greater in Canada.

The fourth yield factor did exhibit some significance thus has been included for completeness. The fourth yield factor for the separate and rolling periods are displayed next.

Figure 19  Country Factor Loading, Combined Interest Rates – Separate Periods
The fourth yield factor is inconsistent when analysing separate, non-overlapping periods but is very consistent when the data samples are overlapping. This may be due to the central banks in each country having interest rate policies that are similar in some periods but then slightly diverge in other periods. Referring to Exhibit 24, it appears that the fourth factor has a positive factor loading for the short maturities for both countries but then declines as the maturities increase, thus the effects of the fourth factor have similar affects for the term structures in both countries.

To conclude the analysis of the combined or “North American” yield curves I present the percentage of variance explained by the first four principal components. This differs from the analysis of the individual interest rate curves in that there appears to be some significance of the fourth principal component which likely represents the systematic influences of each country’s central bank. I therefore have deemed this fourth principal component as the country effect.
Table 5  Percentage of Variance Explained by the First Four Yield Factors for Combined North American Interest Rates – Separate Periods

<table>
<thead>
<tr>
<th>Sub Period</th>
<th>Factor</th>
<th>86-90</th>
<th>91-96</th>
<th>97-00</th>
<th>01-06</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level</td>
<td>49.5%</td>
<td>56.7%</td>
<td>55.3%</td>
<td>72.7%</td>
<td></td>
</tr>
<tr>
<td>Steepness</td>
<td>28.6%</td>
<td>20.7%</td>
<td>18.6%</td>
<td>10.2%</td>
<td></td>
</tr>
<tr>
<td>Curvature</td>
<td>9.3%</td>
<td>9.9%</td>
<td>12.0%</td>
<td>6.8%</td>
<td></td>
</tr>
<tr>
<td>Country</td>
<td>4.3%</td>
<td>5.0%</td>
<td>5.2%</td>
<td>4.5%</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>91.7%</td>
<td>92.3%</td>
<td>91.1%</td>
<td>94.2%</td>
<td></td>
</tr>
</tbody>
</table>

Table 6  Percentage of Variance Explained by the First Four Yield Factors for Combined North American – Rolling Periods

<table>
<thead>
<tr>
<th>Sub Period</th>
<th>Factor</th>
<th>86-95</th>
<th>87-96</th>
<th>88-97</th>
<th>89-98</th>
<th>90-99</th>
<th>91-00</th>
<th>92-01</th>
<th>93-02</th>
<th>94-03</th>
<th>95-04</th>
<th>96-05</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level</td>
<td>52.1%</td>
<td>51.9%</td>
<td>50.6%</td>
<td>52.2%</td>
<td>52.9%</td>
<td>54.6%</td>
<td>55.2%</td>
<td>56.4%</td>
<td>58.8%</td>
<td>58.1%</td>
<td>60.2%</td>
<td></td>
</tr>
<tr>
<td>Steepness</td>
<td>25.0%</td>
<td>25.1%</td>
<td>26.6%</td>
<td>24.7%</td>
<td>23.4%</td>
<td>21.2%</td>
<td>20.9%</td>
<td>20.8%</td>
<td>18.2%</td>
<td>18.4%</td>
<td>15.9%</td>
<td></td>
</tr>
<tr>
<td>Curvature</td>
<td>10.0%</td>
<td>10.3%</td>
<td>10.4%</td>
<td>10.7%</td>
<td>11.0%</td>
<td>10.9%</td>
<td>10.8%</td>
<td>9.9%</td>
<td>10.1%</td>
<td>10.5%</td>
<td>10.4%</td>
<td></td>
</tr>
<tr>
<td>Country</td>
<td>4.6%</td>
<td>4.6%</td>
<td>4.6%</td>
<td>4.7%</td>
<td>4.8%</td>
<td>5.0%</td>
<td>4.9%</td>
<td>4.8%</td>
<td>4.9%</td>
<td>5.3%</td>
<td>5.4%</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>91.7%</td>
<td>91.9%</td>
<td>92.3%</td>
<td>92.3%</td>
<td>92.0%</td>
<td>91.7%</td>
<td>91.7%</td>
<td>92.0%</td>
<td>92.1%</td>
<td>92.3%</td>
<td>92.0%</td>
<td></td>
</tr>
</tbody>
</table>

Notice that the percentage of variance explained ranges from 91.1% to 94.2% for the separate period analysis and 91.67% to 92.28% for the rolling period analysis. Also it appears that the percentage explained by the level factor has rapidly increased in the separate period analysis and it gradually increased for the rolling period analysis. The precise opposite can be observed for the steepness factor as it decreases in both separate and rolling period analysis. The curvature factor and country factors remain fairly constant for the entire analysis. This overall consistency of the factors is appealing, as it serves as evidence to the possibility of using these factors to manage a combination of US and Canadian fixed income assets against a Canadian benchmark using 4 factors or metrics.

3.4 GARCH Model Analysis

The second part of this paper uses weekly levels taken throughout the entire 20 year sample and attempts to model the weekly factor movements using a univariate GARCH(1,1) model. The theory behind this reasoning is that changes in factors will depend on the size of the
previous value observed and the observed volatility. That is, when factors are large then the weekly changes are assumed to be larger than if the factors are smaller. The following charts model dynamics of the conditional volatility relative to the actual factor level for the level, steepness and curvature.

Figure 21  Absolute Level Change vs. Level Factor Volatility – Combined Term Structures

Figure 22  Absolute Steepness Change vs. Steepness Factor Volatility – Combined Term Structures
These models provide some insight into the expected volatility. Although the magnitude of the volatility at times is not as precise as I would hope for, it does provide a decent directional measure. In periods where the factor spikes the variance can be seen as increasing as well. It is important to note that I am using a basis parsimonious GARCH(1,1) to model the volatilities. The following table provides the statistical results from the three models.

Figure 24 Statistical output from level, steepness and curvature volatility models

<table>
<thead>
<tr>
<th>R-Squared</th>
<th>Level</th>
<th>Steepness</th>
<th>Curvature</th>
</tr>
</thead>
<tbody>
<tr>
<td>12.5%</td>
<td>14.0%</td>
<td>4.8%</td>
<td></td>
</tr>
<tr>
<td>β1</td>
<td>0.013</td>
<td>0.001</td>
<td>0.007</td>
</tr>
<tr>
<td>β2</td>
<td>0.160</td>
<td>0.117</td>
<td>0.111</td>
</tr>
<tr>
<td>β3</td>
<td>0.805</td>
<td>0.886</td>
<td>0.781</td>
</tr>
</tbody>
</table>

Future research could involve using a bivariate GARCH model in which the covariance matrix of Canadian and US interest rate volatilities is used (Bivariate model).
4 CONCLUSION

This paper provides an alternative approach to managing the risks involved with holding portfolios of Canadian, US and Canadian and US interest rate sensitive securities. I find the eigenvectors are relatively stable through time for the first three factors (four when considering combining the term structures). When looking at the term structures individually I find that the first three principal components explain at least 96% of the variability of interest rates as compared to explaining 91% of the variability when combining the curves together. This research could benefit asset liability managers such as pension funds and insurance companies as it provides a tool for them to manage the risk of holding a Canadian/US portfolio of bonds versus a Canadian only benchmark, therefore allowing them to tap into a much deeper, liquid and diverse market.

This paper also provides an approach to managing the changes in the factors by proposing a GARCH model to model the volatility of the first 3 factors of the combined North American curve. The results appear to be promising, considering a basic GARCH model was used. The benefit of this is that it provides risk and asset liability managers a method of developing scenarios with regards to possible fluctuations in the volatility of levels. Further research could involve developing a bivariate GARCH method in which the variance covariance matrix between US and Canadian volatility could be used in determining levels for each country as well as the combined factor.
Reference List


