THE SIZE EFFECT DUE TO INACCURATE MEASUREMENT OF BETA

by

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ABSTRACT

The small firm effect has been a recognized anomaly of modern capital market theory for over a quarter of a century. It stems from the predication that small firms outperform large firms on a risk adjusted basis. The following paper demonstrates that the size effect is not an anomaly, but rather that it is a product of incorrect risk assessment. Specifically, using longer return intervals to measure risk provide for a better explanation of the variation of stock returns across size portfolios. We test monthly and annual betas over two periods and determine that indeed annual betas provide for better a measure of risk of an asset.
ACKNOWLEDGEMENTS

I would like to specifically thank Dr. Andrey Pavlov and Dr. Rob Grauer for teaching me two of the most complicated and interesting courses I have taken during my academic career. Without courses in Econometrics and Capital Markets, not only would I have not had the tools necessary to write this paper, but I would not have acquired the essential knowledge and perspective necessary to excel in my education and career in finance.
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1 INTRODUCTION

The size effect, also known as the small firm effect has been a topic of discussion that has brought empiricists and theorists head to head for over a quarter of a century. This study reviews the findings of empiricists that regard the size effect as an unexplained anomaly of modern capital market theory and theorists that regard the anomaly as simply a statistical artefact. We begin by considering the historical context of the size effect from its roots with Reinganum's (1981) use of the size effect as an attack on the Arbitrage Pricing Theory (APT), defended by Roll and Ross (1980) and review the argument by Roll (1981) where he contends that there is a downward bias in the variance and betas of small firms when returns are measured over short time intervals. Short return intervals bias the measurement of beta because non-synchronous trading leads to higher auto-correlation of returns, especially in small stocks (Scholes and Williams, 1977). We look closely and pattern our empirical work in this paper after a study by Handa, Kothari and Wasley (1989), who demonstrate that the covariance between an asset and the market does not increase proportionately as return interval is increased (Handa, Kothari and Wasley (1989). They put forth an empirical methodology for determining whether monthly or annual betas provide for the best risk measure.

Our empirical work considers data from 1941 to 1982, the period studied by Handa, Kothari and Wasley (1989), and data from 1941 to 2005, to determine whether our overall findings have greater consequence in recent periods. The methodology is aimed at determining whether or not longer beta measurement intervals provide for returns and betas that provide for more reasonable results holding the CAPM true, and whether or not longer beta measurement intervals better explain the cross-section of monthly stock returns.
better explains the cross-section of stock returns. In doing so, we examine the average intercept, average $\beta$ coefficient as, as well as average $R^2$ value of the regressions to determine best fit. We mirror Handa, Kothari and Wasley (1989) by running monthly multiple regressions of returns on both monthly and annual betas to determine through examination of the average coefficients, which are contributing more to the explanation of the cross-section of portfolio returns. We then examine the results between the two time periods to determine whether our findings are more relevant in recent periods. This study will show that annual betas provide for a better measure of risk than do monthly betas. Furthermore, this study shows that annual betas provide for a better measure of risk in recent periods.
2 LITERATURE REVIEW

2.1 Foundations of the Size Effect

The size effect has been put through rigorous testing over the last quarter century. Sophisticated empirical tests came as a challenge to the early Arbitrage Pricing Theory (APT) (Roll & Ross, 1980). Roll & Ross purport that according to the APT, the return on an asset is equal to its expected return, plus the sensitivity to a series of common factors to those securities. According to the APT, the return on an asset can be determined by its $b_{ik}$ sensitivity to the $\tilde{\delta}_k$ factors. This theory is tested similarly to the Capital Asset Pricing Model (CAPM) of beta to the market risk premium. In effect the $b_{ik}$ estimates are proxy for systematic risk first introduced by the CAPM. This puts into perspective the sheer magnitude of the size effect as a directly observed pricing anomaly that is readily traded on in everyday practice, but fails to disappear and therefore a direct threat to the foundations of modern capital market theory. If the size effect can indeed bypass the APT, where small stocks outperform large stocks despite common factors, and CAPM theory, where small stocks outperform large stocks risk adjusted, where risk is measured by the beta sensitivity to the market risk premium, the “size effect” could indeed be the most significant continuous, lasting and directly observed opponent of efficient market asset pricing techniques.

In a test of the APT, Reinganum (1981) shows that if common factors are first identified in securities and then grouped into control portfolios of those factors and returns are observed at year Y-1. In year Y, excess daily returns are calculated by subtracting daily security returns at year Y from the control portfolio returns at year Y-1. These portfolios are then divided into 10 groups measured by market value. It was found under all of three, four and five factor models,
that the small stock portfolio did indeed outperform the large stock portfolio. Furthermore, Reinganum found that each of 10 size portfolios had a higher excess return than the adjacent higher market cap portfolio. This would fully support Reinganum’s contention, and it has been well observed that small stocks outperform large stocks and therefore the 1st decile of common stocks based on market value should beat the 10th decile, and also that the 4th beats the third and the 5th beats the 4th and the 6th beats the 5th and so on for each portfolio. The following graphically represents Reinganum’s findings for each grouping from MV1 (smallest) to MV10 (largest);

Figure 2.1 Size Portfolio Returns Under Reinganum’s 3 Factor Model

![Graph of Size Portfolio Returns](image)

Adapted from Table 1 (Reinganum, 1981 p. 318)
This overwhelming consistency under each of the 3, 4, and 5 factor analysis almost is conducive to the argument that the size factor is the only explanation of stock returns. Surely these findings warranted examination of the data and econometric methods implicit in Reinganum’s findings.
In response to Reinganum’s findings and in defense of the APT, Roll (1981) shows that the fault in Reinganum’s findings had nothing to do with small firms mysteriously earning higher returns than large firms, but rather that there is an inherent downward bias on the variance and betas of small stocks when returns are measured over shorter time intervals because small stocks are more thinly traded. Roll uses an equally weighted index of the New York and American listed common stocks are compared to the S&P 500. The equally weighted index as a proxy for small stocks as “a value weighted index such as the S&P 500 index is obviously more heavily invested in large firms than is an equally-weighted index” (Roll, 1981). Roll studied $\beta E|S$, which is an OLS regression of the equally weighted index returns on the S&P 500. He found that the resulting beta is 0.879. This actually insinuates that using daily data, small stocks are actually less risky than large stocks. At the other end of the spectrum, semi-annual data presents a beta of 1.48 for the same securities. To parallel this, the variance of the equal weighted portfolio divided by the variance of the S&P 500 portfolio - $\sigma_E^2 / \sigma_S^2$, is 1.05 using daily data, consistent with equal risk, to 3.166 using semi-annual data. Interestingly, the mean difference in the annualized return for the equally weighted portfolio less the annualized return for the S&P 500 under each “holding period” has a mean of 12.5% and is relatively constant through holding periods, having a standard deviation of 0.2%. The following graph illustrates the significance that Roll presents with respect to the holding period or time interval of return calculation (Roll, 1981);
It is clear from these results that longer holding periods show a much higher risk than when considering daily data alone. The premise for the argument that daily data presents a downward bias on $\beta$ and $\sigma^2$ is that daily returns are compounded daily and therefore have a higher auto-correlation with each other. This is more prominently observed in small stocks, as small stocks experience a greater level of non-synchronous trading. For empirical results, see Roll, 1981.
It is apparent from Roll's findings that not only is there significant autocorrelation of returns, a challenge to the assumption that returns are independent of one another, but that the equally weighted index has much higher instances of auto-correlation. The rationale for why this is is that small stocks are more thinly traded. This occurs because if security does not trade on a particular day, that security's implicit true return is captured on the following trading day (Roll, 1981).

It should be considered that there is an inherent data gathering issue with using semi-annual data in security analysis given the limited amount of time that there is data available. As it is now the year 2006, it can be argued that there are another approximately \((2005-1981) \times 2 = 48\) more data points available since Roll's article. Nevertheless, in a discipline that boasts the most accurate and abundant data, using semi-annual data to dispute the size effect is open to much scrutiny because there simply isn't enough data available for formidable analysis. The size effect according to this rationale could very well have been put to rest at least until the next millennium.

### 2.2 The Size Effect Survives Thin Trading and Autocorrelation

It is true that Roll did not directly test his hypothesis (Reinganum, 1982) but used equal weighted versus value-weighted securities as a proxy. Conversely Reinganum (1982) did have the sufficient data to estimate the OLS beta's of each of his market value portfolio's and found that his OLS beta estimates purport that the smallest size portfolio was less risky than the largest size portfolio. To mitigate this, Reinganum used the Aggregated Coefficients (AC) method of estimating beta as it yielded more realistic beta estimates. The difference between the AC beta for the small (MV1) portfolio and the large (MV10) portfolio is 0.72. Reinganum's small (MV1) stock portfolio had an average daily return of 0.14% representing about 42% return annually versus a 6% annual return for the large (MV10) portfolio. Therefore, holding the CAPM true, a 0.72 difference in Beta yielding a 36% difference in returns would imply a market risk premium of about 50% (Reinganum, 1982).
In response to Roll’s critique, Reinganum uses daily data compounded to yield quarterly data and finds that a difference in beta of about 1 for a quarterly return difference of 9.5%. Using the same logic, holding the CAPM true, this would imply a market risk premium of about 44%. According to Reinganum, Underestimation of risk due to autocorrelation of returns of small stocks due to thin trading volumes does explain some of the performance of small stocks over large stocks, but not all of it. Nevertheless, the continued existence of this phenomenon without any other explanation would quickly be traded away and the size effect would soon disappear. It does not, however disappear.

Whether the size effect is a product of thin trading or not is not be the primary area of interest. In fact what needs to be considered is if we are not currently measuring risk effectively, then what is a better measure? Handa, Kothari and Wasley (1989) test the time series means of monthly second-pass regression coefficients using monthly and annual betas. They find that longer intervals provide for better measures of beta. They test the period from 1941-1982 and various intervals over this period. They test both monthly and annual betas in the cross-sectional regression each month to explain the variation in size portfolio returns. Specifically, they test:

\[
R_{pt} = \gamma_{0t} + \gamma_{1t}b_{\text{monthly}, pt-1} + \gamma_{2t}b_{\text{annual}, pt-1} + \epsilon_{pt}
\]  \hspace{1cm} (1.1)

They test specific time periods under OLS and GLS estimation. They find little difference between OLS and GLS results because the “gain in efficiency of the GLS estimator over the OLS estimator increases with cross-correlation among residuals and decreases with co linearity among the explanatory variables” (Handa, Kothari and Wasley, 1989). Their OLS results for the period between 1941 and 1982 are presented in the following table.
Table 2.1 Handa, Kothari and Wasley’s Monthly versus Annual Betas Test, 1989

<table>
<thead>
<tr>
<th>Months</th>
<th>$\gamma_0$</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
<th>Adj. R² (average)</th>
</tr>
</thead>
<tbody>
<tr>
<td>504</td>
<td>0.0019</td>
<td>0.0011</td>
<td>0.0103</td>
<td>37.22</td>
</tr>
<tr>
<td></td>
<td>(0.60)</td>
<td>(0.26)</td>
<td>(3.87)</td>
<td></td>
</tr>
</tbody>
</table>

Considering the much higher test-statistic on $\gamma_2$, this study finds that annual betas do a better job as explanatory variables for returns and therefore provide a better measure of risk than do monthly betas.

---

3 MONTHLY AND ANNUAL BETA’S

The small firm effect is not a question of whether small firms outperform large firms. It is more a question of whether the risk of small firms is measured correctly. If it is the case that small firms experience thinner trading and therefore the inherent autocorrelation of returns cause beta’s to be incorrectly measured, then a longer time interval should be used. To determine whether monthly or annual betas should be used, we first determine whether small firms outperform large firms over two time periods, 1941-1982 and 1941-2005. Next we determine which measurement best explains returns in the CAPM setting.

3.1 Data

Securities used in this study are grouped into 10 market value deciles. Betas are estimated using the following regression equation, specifically:

\[ R_{pt} - r_f = \alpha_{pt} + \beta_{pt} (R_{mt} - r_f) + \epsilon_{pt} \]  \hspace{1cm} (2.1)

Monthly betas are calculated using monthly excess returns from 1941 to 1982 and from 1941 to 2005. Annual betas are calculated using annual excess returns from 1941 to 1982 and 1941 to 2005. The period 1941 to 1982 is used for comparison against the findings of Handa, Kothari and Wasley (1989). Data was sourced from the Ken French data library using portfolios formed on size. “The portfolios are constructed at the end of
June in each year using the June market equity and NYSE breakpoints\(^2\)." "The portfolios for July of year t to June of t+1 include all NYSE, AMEX, and NASDAQ stocks\(^3\)."

### 3.2 Do Small Firms Outperform Large Firms?

We calculate the mean returns for each portfolio in over 1941-1982 and from 1941 to 2005 and compare them to the respective betas of those portfolios. To determine if small firms outperform large firms and if small firms are riskier than large firms as measured by beta in the CAPM, we present returns and betas in table 2.

<table>
<thead>
<tr>
<th>Portfolio</th>
<th>Mean Excess Monthly Return</th>
<th>Monthly Beta</th>
<th>Annual Beta</th>
</tr>
</thead>
<tbody>
<tr>
<td>1941-1982</td>
<td>MV1 1.65 1.29 1.86</td>
<td>MV2 1.15 1.28 1.56</td>
<td>MV3 1.08 1.24 1.42</td>
</tr>
<tr>
<td></td>
<td>MV4 1.02 1.20 1.39</td>
<td>MV5 0.94 1.16 1.30</td>
<td>MV6 0.88 1.15 1.23</td>
</tr>
<tr>
<td></td>
<td>MV7 0.85 1.14 1.23</td>
<td>MV8 0.80 1.09 1.10</td>
<td>MV9 0.73 1.03 1.03</td>
</tr>
<tr>
<td></td>
<td>MV10 0.56 0.98 0.93</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1941-2005</td>
<td>MV1 1.41 1.15 1.66</td>
<td>MV2 0.97 1.23 1.42</td>
<td>MV3 0.95 1.21 1.30</td>
</tr>
<tr>
<td></td>
<td>MV4 0.89 1.19 1.30</td>
<td>MV5 0.88 1.17 1.22</td>
<td>MV6 0.83 1.13 1.15</td>
</tr>
<tr>
<td></td>
<td>MV7 0.86 1.13 1.15</td>
<td>MV8 0.78 1.10 1.06</td>
<td>MV9 0.76 1.03 1.00</td>
</tr>
<tr>
<td></td>
<td>MV10 0.60 0.99 0.95</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^2\) Ken French Data Library

\(^3\) Ken French Data Library
It is apparent that small firms (the smallest being MV1) do outperform large firms (the largest being MV10). MV1 has an average monthly return of 1.65% while MV10 has an average monthly return of 0.56% for the period between 1941 and 1982. From the period 1941-2005, MV1 had an average monthly return of 1.41% and MV10 had an average monthly return of 0.60%. The difference in the beta estimate between MV1 and MV10 is quite large as well.

There seems to be a natural progression of returns as we move from large firms to small firms, though the increasing mean returns from MV10 to MV1 is not perfect, as we see MV7 has a higher mean return than MV6 in the period 1941-2005. Nevertheless, this relationship between size and mean return is readily visible here. The relationship between beta and size also has a similar progression. MV1 has a higher monthly and annual beta than MV10 in both periods, although MV1 in the period 1941 to 2005 has a lower monthly beta than would be expected.

According to the Capital Asset Pricing Model, the expected excess return on an asset is the product of the expected excess return on the market and the beta of the asset. Specifically,

$$E(R_p) - r_f = [E(R_m) - r_f] \beta_p$$  \hspace{1cm} (3.1)

where $E(R_p)$ represents expected returns. Substituting mean returns for expected returns,

$$\overline{R}_p - \overline{r}_f = (\overline{R}_m - \overline{r}_f) \beta_p$$  \hspace{1cm} (3.2)

Reinganum dismissed the size effect because substituting the difference in mean return and beta and between the large firm portfolio and the small firm portfolio, in equation 3.2 implied a market risk premium that was too large. Using this logic, if a longer measurement interval provides for a more accurate measure of risk, then the difference in mean return and annual beta should provide for an implied market risk premium that is more reasonable.
Using Reinganum’s method of holding the CAPM true by substituting the difference in annual return and betas between the small and the large size portfolio in equation 3.2 implies a market risk premium of 47.6% for the period between 1941 and 1982 and 54.2% for the period between 1941 and 2005 using monthly betas. This is very much in line with Reinganum’s findings using daily data and again, the implied market risk premium is simply not plausible. Using annual betas, the implied market risk premium for the periods 1941 to 1982 and 1941 to 2005 are 15.8% and 15.3% respectively. This still seems to be slightly high, but it is not completely implausible.

Between monthly and annual betas, we see that annual betas are much higher than monthly betas for small firms with betas higher than one. Conversely, annual betas are lower than monthly betas for large firms with betas less than 1. This is consistent with the findings of Handa, Kothari and Wasley (1989). It indeed seems that smaller measurement intervals tend to over-estimate the beta risk of firms with betas that are less than one, which generally are larger firms, and underestimate the beta risk of firms with betas that are greater than one, which are generally smaller firms. In effect, the spectrum of systematic risk is squeezed using smaller time intervals for measurement. Monthly betas are generally used to explain monthly returns, the following study attempts to determine whether monthly or annual betas better explain the cross-section of monthly stock returns.
4 METHODOLOGY AND RESULTS

4.1 Methodology

Considering that the greatest difference in estimation of beta occurs in the small firm portfolio, it is apparent that a truer measure of risk need be determined to better understand the risk profile of the smallest stocks. We begin by regressing mean monthly returns on monthly betas using the following regression where there are essentially 10 data points, one for each portfolio.

\[
\bar{R}_p - \bar{r}_f = \gamma_0 + \gamma_1 \beta_{Monthly,p} + \varepsilon_p
\]  

(4.1)

The initial results for mean returns on monthly betas between 1941 and 1982 are presented in table 4.1.

<table>
<thead>
<tr>
<th>Interval</th>
<th>(\gamma_0)</th>
<th>(\gamma_1)</th>
<th>(R^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Monthly Betas</td>
<td>-1.99</td>
<td>2.56</td>
<td>0.78</td>
</tr>
<tr>
<td></td>
<td>(-3.57)</td>
<td>(5.33)</td>
<td></td>
</tr>
</tbody>
</table>

The parameter estimates are accurate but the test-statistics are not. To mitigate this, we test the significance of monthly and annual betas by regressing returns on monthly betas and returns on annual betas in each month. One regression of average means on betas will produce the average coefficients of the second pass regressions, only with the second pass regressions, the test statistics will be more accurate. To demonstrate, for the
period of 1941-1952, we run 504 regressions, one for each month between years 1941
and 1982 inclusive. Each month, we regress the observed betas over the entire period for
each portfolio on each portfolio’s return in that month. We run the second pass
regressions to determine the correct standard errors. Table 4.2 shows that the mean
coefficient for the 504 second pass regressions of returns on betas equals that of the
single regression of portfolio means on betas in table 4.1. The t-statistics of course are
different and the average R-squared is different from the single regression R-squared.

We compare the mean second pass coefficients to determine whether significant
differences arise. Secondly, we employ a similar test to that of Handa, Kothari and Wasley
(1989) using multiple regressions of returns on monthly and annual betas each month, only in our
test; stationary betas are used over the entire time period.

4.2 Second Pass Regressions on Monthly and Annual Betas

We test monthly and annual betas in second pass regressions of monthly returns on
monthly and annual betas to see which explain the most variation. We test the mean coefficients
for the following regressions:

\[ R_{pt} - r_\beta = \gamma_0 + \gamma_1 \beta_{Monthly,pt} + \varepsilon_{pt} \]  \hspace{1cm} (4.2)

and

\[ R_{pt} - r_\beta = \gamma_0 + \gamma_1 \beta_{Annual,pt} + \varepsilon_{pt} \]  \hspace{1cm} (4.3)

We are mainly concerned with the significance of \( \gamma_1 \). If the test-statistic for \( \gamma_1 \) is higher
using annual betas than it is using monthly betas, then it can be determined that annual betas are a
better measure of risk and vice versa. Because we are comparing two sets of regressions, we are also interested in the measure of fit, R-squared and the minimization of the intercept. If annual betas are a better predictor of returns than are monthly betas, it can be determined that the R-squared measure would be higher using annual betas than monthly betas. Similarly, if annual betas are a better predictor of returns than are monthly betas, it can be determined that the significance of the intercept would be suppressed at the 95% level. Nevertheless, measure of fit and minimization of the intercept are secondary to the significance of the test-statistic for \( \gamma_1 \).

Table 4.2 Means of Second Pass OLS Coefficients: 1941-1982

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Interval</th>
<th>( \gamma_0 )</th>
<th>( \gamma_1 )</th>
<th>Avg. R²</th>
</tr>
</thead>
<tbody>
<tr>
<td>1941-1982</td>
<td>Monthly Betas</td>
<td>-1.99</td>
<td>2.56</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-2.86)</td>
<td>(3.69)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Annual Betas</td>
<td>-0.43</td>
<td>1.07</td>
<td>0.51</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-1.34)</td>
<td>(3.73)</td>
<td></td>
</tr>
<tr>
<td>1941-2005</td>
<td>Monthly Betas</td>
<td>-0.85</td>
<td>1.54</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-1.39)</td>
<td>(2.52)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Annual Betas</td>
<td>-0.25</td>
<td>0.94</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(-0.77)</td>
<td>(3.24)</td>
<td></td>
</tr>
</tbody>
</table>

For the period 1941 to 1982, the value of \( \gamma_1 \) using monthly betas is 2.56 and using annual betas is 1.07. The test statistics on \( \gamma_1 \) for monthly and annual betas is 3.69 and 3.73 respectively and therefore are too close to one another to make any distinctive conclusions. The R-squared value is slightly higher using the annual betas than with the monthly betas, but again, the values are too close to distinctively select the annual beta estimates over the monthly. Note however that because the test-statistic on the intercept \( \gamma_0 \) is much lower when using annual betas.
than when using monthly betas and is therefore better minimized. In aggregate, there is some evidence that annual betas provide more explanatory power than do monthly betas. Nevertheless, the evidence is far from conclusive.

The period from 1941 to 2005 provides for much clearer results. The average $\gamma_1$ for monthly betas is 1.54 and 0.94 using annual betas. Both monthly and annual betas provide for significant test-statistics on $\gamma_1$ at the 95% level, although the test-statistic of 3.24 on $\gamma_1$ using annual betas is higher than the 2.52 test-statistic on $\gamma_1$ using monthly betas by a margin that would attest to annual betas being conclusively a better risk measure. Furthermore, the average R² value using monthly betas is 0.38, while that using annual betas is 0.48. This provides a wide enough margin in average R² value to conclude that using annual betas better explain the variation in stock returns of the size portfolios. As with the test between 1941 and 1982, $\gamma_0$ is better minimized when using annual betas than when using monthly betas, as the test-statistic on the average intercept using monthly betas is -1.39 and is -0.77 on annual betas. This finding provides clear evidence that from the period of 1941 to 2005, annual betas have much more explanatory power than do monthly betas. Nevertheless, we continue this study by testing both monthly and annual betas in the same regression equation.

4.3 Multiple Regression on Monthly and Annual Betas

Monthly betas are generally used to explain monthly returns, the following study attempts to determine whether monthly or annual betas better explain the cross-section of stock returns. We estimate the following regression monthly:

$$R_{pt} - r_f = \gamma_0 t + \gamma_{t1} b\text{monthly} \cdot pt + \gamma_{t2} b\text{annual} \cdot pt + \epsilon_{pt}$$

(4.4)
Again, each month we estimate the regression of monthly and annual betas on returns. We then use the mean coefficients and test which is most significant. Because this model estimates a multiple regression, we use the average adjusted R-square value to determine the average fit of the regressions. For the period from 1941 to 1982 we run 504 regressions and for the period of 1941 to 2005 we run 780 regressions. As is the case with the previous study, the mean coefficients are identical to that when running a single regression of mean returns on betas. The results are provided in the following table.

Table 4.3 Means of Second Pass Monthly and Annual Beta Coefficients

<table>
<thead>
<tr>
<th>Interval</th>
<th>$\gamma_0$</th>
<th>$\gamma_1$</th>
<th>$\gamma_2$</th>
<th>Avg. Adj. $R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1941-1982</td>
<td>0.3</td>
<td>-1.06</td>
<td>1.45</td>
<td>0.54</td>
</tr>
<tr>
<td></td>
<td>(0.57)</td>
<td>(-1.21)</td>
<td>(2.98)</td>
<td></td>
</tr>
<tr>
<td>1941-2005</td>
<td>0.38</td>
<td>-0.77</td>
<td>1.13</td>
<td>0.58</td>
</tr>
<tr>
<td></td>
<td>(0.85)</td>
<td>(-1.47)</td>
<td>(3.41)</td>
<td></td>
</tr>
</tbody>
</table>

The period from 1941 and 1982 provides for an average intercept coefficient of 0.30, having a test-statistic of 0.57. This provides for evidence that the regression minimized the intercept to a clearly level that is clearly insignificant with 95% confidence. The average value of the coefficient $\gamma_1$ is -1.06 with a test statistic of -1.21, which too is insignificant at the 95% level. The average value of the coefficient $\gamma_2$ is 1.45 with a test-statistic of 2.98. Average adjusted $R^2$ is 0.54, which provides for a healthy measure of fit. According to these findings, it is clear that annual betas do a better job of explaining return variation between 1941 and 1982.
The period between 1941 and 2005 provides for an average intercept of 0.38, having a test statistic of 0.57. Again, the model minimizes the intercept to a level that is not significant at the 95% level. The average value of the coefficient $\gamma_1$ is -0.77 with a test statistic of -1.47, again being insignificant at the 95% level. The average coefficient for $\gamma_2$ is 1.45, with a test-statistic of 3.41, which is significant at the 95% level. The average adjusted $R^2$ for this period is 0.58, which again provides for more than adequate measure of fit. These findings re-enforce the argument that annual betas are a better measure of measuring the risk of firms, as they better explain the variation in stock returns.

4.4 Comparison of Periods

Considering the results of our tests, we find that the use of annual over monthly betas is truer in recent periods. The results of the single variable OLS regression on monthly and annual betas, we found that annual betas provided for more significant results in the 1941 to 2005 period than in the 1941 to 1982 period. The mean coefficient on annual betas from 1941 to 1982, although significant, did not provide for superior explanation of variation in stock returns over that of monthly betas as the difference between $\gamma_1$ for annual betas and $\gamma_1$ for monthly betas is only 0.04. Conversely, in the period from 1981 to 2005, the mean coefficient on annual betas had a much higher level of significance than did that of monthly betas. In this period, the difference in the test-statistic is 0.72 in favour of annual betas.

When we run the monthly multiple regressions on both monthly and annual betas, we find results that lead to a similar conclusion. Comparing the results for $\gamma_1$ we find that although the absolute value of the test statistic is higher in the period between 1941 and 1982, it is more significantly negative in that the regression is essentially pushing the value of $\gamma_1$ down to make
room for $\gamma_2$. Similarly, we find that the test statistic for $\gamma_2$ is higher in the period from 1941 to 2005 than it is in the period from 1941 to 1982.

Considering these findings, it should be considered that going forward, that we consider this to be a new reality. Monthly betas are not explaining stock returns as well as they used to and annual betas appear to be becoming more encompassing as a truer measure of the risk of a security.
5 CONCLUSION

Do small firms earn higher average returns than do large firms? Yes, of course they do. The question presented here is a question whether small firms outperform large firms on a risk-adjusted basis. In order to effectively ascertain risk, two things must be true: 1) we must have the correct risk measure and 2) we must measure it correctly. At the foundation of modern capital markets theory, we have the premise that in order to realize higher return; investors must take on higher risk. If this were not the case, prices would adjust. Because investors are opportunistic, and opportunity is finite by definition, the small firm effect could not have possibly lasted for over a quarter of a century without being traded away.

Looking closer at the arguments in this paper, it becomes clear that proponents of both sides of this argument are very much on the same side. From the very beginning we see Reinganum arguing the size effect in an attempt to disprove the APT. Roll’s response was that this may be due to autocorrelation of returns arising from thin trading, and thus putting a downward bias on the variance of small stock returns. Both sides of this coin were to some degree in agreement in that the APT model may not either 1) contain the correct risk measures and/or 2) measure them correctly. We see similar results from either side of the CAPM, where one side sees the CAPM as being false because of the size effect, while the other purports that the betas of small stocks are underestimated. Both agree that indeed, at least one of the two components necessary to ascertain risk is being violated.

We consider risk of an asset in the CAPM framework to ascertain whether monthly or annual betas better explain the monthly cross-section of stock returns. Our findings suggest that monthly returns are determined better by annual betas than that of monthly betas across size portfolios. This finding suggests that traditional methods of calculating beta are sub-optimal and
new methods need be considered and tested to ensure that mis-measurement is not wrongly perceived as anomalies of the capital markets. Not only do we find that a larger beta measurement interval better explains the cross-section of stock returns, but also that longer intervals provide for a better explanatory variable in recent history than in the past. Therefore, whether or not this is caused by autocorrelation of returns due to thin trading, or some other characteristic of small firms that is mitigated by using longer intervals for risk measurement, we can at least determine that it is prudent to consider longer intervals. In the absence of a revolutionary capital market model that fully explains the variation in stock returns, we are left with only the option to make adjustments to the ones currently available to us in a manner that better estimates that variation.
REFERENCE LIST


Ken French Data Library


