Study of $^{28}\text{Mg}$ and $^{22}\text{Ne}$ using fusion-evaporation and Doppler shift techniques

by

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Abstract

Investigations of nuclear structure via high precision measurements of energies and lifetimes of excited nuclear states has been the focus of an ongoing experimental program carried out at TRIUMF, Canada’s particle accelerator centre, using the TIGRESS Integrated Plunger (TIP) infrastructure. As part of this work, methods have been developed to determine energies and lifetimes of excited states in nuclei populated using the fusion-evaporation reaction mechanism and measured using the Doppler-shift attenuation method (DSAM). These methods include a comparison of experimental data to Monte-Carlo simulations in order to determine lifetimes of observed states. Methods were validated using data collected for the benchmark nucleus $^{22}\text{Ne}$ at the ISAC-II facility at TRIUMF. Additionally, an array of 128 CsI(Tl) charged particle detectors has been constructed at Simon Fraser University. This new ‘CsI ball’ array replaces a 24-detector array used in previous TIP experiments and offers improved charged particle detection efficiency due to its increased solid angle coverage (nearly $4\pi$ in the lab frame).

A subset of the CsI ball array was used to study excited states of $^{28}\text{Mg}$ in an experiment at ISAC-II/TRIUMF, with the goal of investigating the evolution of nuclear shells and searching for evidence of the lowering in energy of $pf$ negative parity orbitals predicted in this region. For the first time $^{28}\text{Mg}$ was investigated using a fusion-evaporation reaction, leading to preferential population of states at high spin and excitation energy where the influence of the $pf$ negative parity orbitals is expected. Analysis methods developed for $^{22}\text{Ne}$ were applied to the $^{28}\text{Mg}$ DSAM data to extract lifetimes of observed states. 3 new excited states of $^{28}\text{Mg}$ were identified. Multiple candidates for negative parity states were also observed, providing an explicit indication of $pf$ negative parity orbital population.

Underlying principles and theories of nuclear structure, a technical overview of the CsI ball array and other TIP infrastructure, along with results from the $^{22}\text{Ne}$, CsI ball commissioning, and $^{28}\text{Mg}$ experiments are presented. A comparison of the data to the predictions of various theoretical models is shown and implications for the structure of $^{28}\text{Mg}$ are discussed.

**Keywords:** nuclear structure; fusion-evaporation; gamma-ray spectroscopy; Doppler shift lifetime methods
Dedication

Hi, mom!
I would like to thank my senior supervisor, Dr. Krzysztof “Kris” Starosta for teaching me the value of having high standards and for his willingness to endlessly debate the philosophical side of what we do. As you might imagine, Kris had some input on this thesis.

I also want to thank my supervisory committee members - Greg Hackman for his strong support of this work from within TRIUMF (including providing theory contacts for the $^{28}$Mg work and arranging beamtime for commissioning the CsI detectors), and Loren Kaake for his chemistry related questions and insight during committee meetings.

Also, the detailed comments and suggestions provided by Gordon Ball were a big help in the process of preparing papers on $^{28}$Mg and the CsI ball, and are reflected in this thesis as well.

Much of the time invested in this project was spent analyzing and interpreting data at SFU. I’ve had the privilege of working alongside many excellent researchers and soon-to-be-researchers. In particular, I had many discussions over the years with Aaron Chester on analysis methods and details, and much of the code written for this project could be considered a joint effort between us. Aaron, along with Rachel Ashley, Jen Pore, and Phil Voss, helped mentor me in my early years in the program, and I want to thank them for their advice and inspiration (not to mention their help with running experiments). Hopefully I can pass some of this on to the next generation of students. My former colleagues Thomas Domingo and Usman Rizwan may be pleased to hear that 3 hour group meetings are no longer a thing here, that the LN2 system works again, and that the neutron generator might get licensed soon™ - the progress of Western Civilization marches on. Thanks for your help on the many experiments over the years (both the successes and the ones not mentioned here). I should also thank the imminent Dr. Fatima Garcia for all the cupcakes (except the fruit ones). And to Corina’s other students: Kenneth, Isaiah, Kevin, Aimee, Kurtis, thanks for making it look like we actually get work done in the lab and for answering my dumb questions - I’ll look forward to the art gallery exhibition feat. your finished level schemes.

Melanie and Frank: everything is in your hands now, good luck (lol). Since this paragraph is rapidly becoming a meme, I’d just like to reiterate that none of this could have been done the way it was without everyone’s help.

Outside of the academic realm, I’ve been fortunate to have the weirdly unconditional support of friends and family. I’d like to thank Mano, Cameron, Shelton, and Andrei, as well as Richard, Mark, Julia, and the army of extended relations out there who might be reading this for some reason. Shout-outs to my parents who let me live for free until I could figure out what to do with my life. Surrey WHAT?
Epigraph

“Are we going to eat?”
“What are we going to eat?”
“Where are we going to eat?”
“What did it look like?”

The 4 stages of civilization according to Kris Starosta, PhD

Editor’s note: possibly cyclical.

“It’s published, so it must be true.”

Kris Starosta, PhD

“There is no hope.”

Kris Starosta, PhD
Feb 9, 2017

“There is always hope.”

Aragorn, son of Arathorn
The Lord of the Rings: The Two Towers (New Line Cinema ver., 2002)
Not a PhD
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# List Of Abbreviations and Symbols

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<th>Abbreviation</th>
<th>Definition</th>
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<tr>
<td>$A$</td>
<td><strong>Nucleon number</strong> - Sum of neutron and proton numbers ($N + Z$).</td>
</tr>
<tr>
<td>AMeV</td>
<td><strong>Mega electron volt per nucleon</strong> - Measure of energy for accelerated beams. The total energy divided by the number of nucleons $A$ in the beam species.</td>
</tr>
<tr>
<td>DAQ</td>
<td><strong>Data Acquisition</strong> - The electronic system(s) used to acquire data during an experiment.</td>
</tr>
<tr>
<td>DSAM</td>
<td><strong>Doppler Shift Attenuation Method</strong> - Method for measuring lifetimes of excited nuclear states on the femtosecond scale.</td>
</tr>
<tr>
<td>eV</td>
<td><strong>Electron volt</strong> - Unit of measure for energy. $1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$.</td>
</tr>
<tr>
<td>FWHM</td>
<td><strong>Full width at half maximum</strong> - The full width of a peak at half of its maximum value, used to quantify the width of spectral lines.</td>
</tr>
<tr>
<td>HPGe</td>
<td><strong>High Purity Germanium</strong></td>
</tr>
<tr>
<td>ISAC</td>
<td><strong>Isotope Separator and Accelerator</strong> - Experimental facility at TRIUMF.</td>
</tr>
<tr>
<td>$N$</td>
<td><strong>Neutron number</strong></td>
</tr>
<tr>
<td>keV</td>
<td><strong>Kilo electron volt</strong> - $10^3$ eV.</td>
</tr>
<tr>
<td>MeV</td>
<td><strong>Mega electron volt</strong> - $10^3$ keV.</td>
</tr>
<tr>
<td>OLIS</td>
<td><strong>Off-line ion source</strong> - Source for stable beams at TRIUMF.</td>
</tr>
<tr>
<td>RDM</td>
<td><strong>Recoil Distance Method</strong> - Method for measuring lifetimes of excited nuclear states on the picosecond scale.</td>
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TIGRESS

TRIUMF-ISAC Gamma-Ray Escape Suppressed Spectrometer - An array of HPGe detectors at TRIUMF for gamma-ray spectroscopy.

TRIUMF

TRI University Meson Facility (obsolete acronym) - Canada’s particle accelerator centre, located in Vancouver.

Z

Proton (atomic) number

Unless otherwise indicated, uncertainties of all stated quantities are reported at the 1σ level. Uncertainties are typically given in parenthesis eg. 1.9(6) × 10^2 fs corresponds to 1.9 × 10^2 fs ± 0.6 × 10^2 fs.

Spins $I$ and parities $\pi$ of ground and excited states in nuclei are indicated using $I^\pi$ notation, with subscripts specifying the first, second, etc. state (in terms of excitation energy) of a given spin-parity. For example, $I^\pi = 2^+_1$ denotes the lowest energy level in a nucleus with spin $I = 2$ and positive parity. Brackets are used to indicate tentatively assigned quantities (quantities outside of brackets are firmly assigned). For example, $I^\pi = (0, 4)^-$ corresponds to a level tentatively assigned as either $I = 0$ or $I = 4$, with firmly assigned negative parity.
Chapter 1

Introduction and Theory

The properties of nuclei - such as the energies and mean lifetimes of excited states, and the strengths of transitions between these states - give clues about their structure and the fundamental forces binding them together. For instance, assuming a simple model of a spherical nucleus bound by a strong short range force, protons and neutrons can be organized into shells analogous to the electron shells in chemistry. These shells are observed in experiment, as nuclei at shell closures are remarkably stable and difficult to excite out of their ground state. For example: $^{16}\text{O}$, with closed shells for protons ($Z = 8$) and neutrons ($N = 8$) has a first excited state at over 6 MeV [1], whereas in typical nuclei away from shell closures (including those studied in this thesis) the first excited state is typically at 1.5 MeV or lower. The structure of nuclei is therefore revealed through observation of their fundamental properties.

Electromagnetic transition rates are one such fundamental property. Excited states in nuclei often have lifetimes in the range of $10^{-12}$ to $10^{-15}$ seconds, a timescale shorter than the time resolution of modern detectors (picoseconds for LaBr fast scintillators) and data acquisition systems (which are clocked in the MHz to GHz range). Doppler-shift techniques such as the Doppler Shift Attenuation Method (DSAM) [2] and Recoil Distance Method (RDM) [2, 3] allow for measurement of these short lifetimes by correlating the time of flight of the nucleus to the energy of the gamma ray observed when the nucleus decays from its short lived excited state(s).

The experimental program at the ISAC-II facility [4] at TRIUMF, Canada’s particle accelerator centre, allows for gamma-ray spectroscopy and Doppler shift lifetime measurements using the TRIUMF-ISAC Gamma-Ray Escape Suppressed Spectrometer (TIGRESS) [5], a Compton-suppressed high purity germanium (HPGe) clover array which is used for detection of gamma rays with high position and energy resolution. TIGRESS is operated in conjunction with the TIGRESS Integrated Plunger (TIP) infrastructure developed at SFU [6] which is used to implement DSAM and RDM experiments. TIP also provides
means to operate various ancillary charged particle detectors, including an array of CsI(Tl) scintillators which was developed as part of this thesis.

With over 3,000 nuclides and over 150,000 nuclear states known [7], it is not necessarily obvious where experimenters should focus when conducting investigations of nuclear structure. However despite this large body of knowledge there are still many measurements which have not been made, and clear paths which lead to new observables are illustrated qualitatively in the coordinate system of Figure 1.1. At the origin of Figure 1.1 are the properties of the ground and low-lying states of stable nuclides, which are generally well known and described by existing nuclear theories. The extreme regions of Figure 1.1 represent more exotic systems which provide stringent tests of nuclear theories, but are difficult to access experimentally. For instance, in order to access nuclei far from stability with extreme proton/neutron ratios (often quantified by the isospin projection $T_z = (Z - N)/2$ it is often necessary to use reactions involving radioactive beams, necessitating the development of facilities capable of providing these beams. An alternative approach (taken in this thesis) is the study of species closer to stability at high excitation energy $E_{ex}$ and/or angular momentum $I$ where little data is available. This approach can be done using stable beams but often presents a significant challenge in terms of data analysis, since the reaction channel of interest is often a small portion of the overall data. Regardless of the approach used, the study of exotic nuclear systems presents significant experimental challenges but also the opportunity for new insights into nuclear structure.

This thesis focuses on studies of two neutron-rich isotopes of intermediate mass: stable $^{22}$Ne and radioactive $^{28}$Mg. The properties of $^{22}$Ne are well known and have been the target of multiple previous studies, so this nucleus was used as a benchmark for development of experimental and data analysis techniques. On the other hand, $^{28}$Mg is more exotic and lies near an interesting region of the nuclear landscape known as the ‘island of inversion’ which is discussed in depth in Section 1.1. There is limited information available for $^{28}$Mg, in particular about its properties at high angular momentum and excitation energy. The main objective of study in this thesis is therefore the measurement of these properties, which along with the known properties of other nuclei in this region provides a test of theoretical models of the nuclear interaction.

### 1.1 Present context for these studies

Theoretical models of the nuclear interaction are necessary for the understanding of problems where it is prohibitively difficult to obtain experimental data. For example, the mechanism of the astrophysical $r$-process - largely responsible for the production of elements

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1. Isospin is a quantum number distinguishing nucleons (protons and neutrons) governed by a formalism analogous to the formalism handling spin 1/2 systems in quantum mechanics. Each nucleon is assigned isospin $T = 1/2$, with protons having isospin projection $T_z = +1/2$ and neutrons having isospin projection $T_z = -1/2$. The isospin projection of the nucleus is then $T_z = (Z - N)/2$ [8].
Figure 1.1: Possible regimes for the study of nuclear structure, in terms of excitation energy $E_{ex}$, angular momentum $I$, and isospin projection $T_z = (Z - N)/2$. The yrast line specifies states with the lowest excitation energy $E_{ex}$ for a given angular momentum $I$. Exotic systems which provide stringent tests for nuclear theories are found by moving away from the origin.

Heavier than iron in the universe - strongly depends on reaction rates for neutron capture on isotopes which presently are impossible to produce in the laboratory, as well as beta-decay rates of the same isotopes and/or their isomeric states. The same problem exists for the interactions of fission neutrons with heavy fission fragments produced in nuclear reactors. Nuclear theories also seek to answer questions about the nature of matter - for instance to understand the limits of nuclear stability, and to determine whether long-lived isotopes of transactinide or super-heavy elements exist.

However, significant challenges exist for nuclear theorists. When developing a model of the nuclear interaction, two problems must be solved: the nature of the interaction between nucleons, and the challenge imposed by modeling a system of many interacting nucleons. Unfortunately, the nature of the interaction responsible for binding the nucleus together is not understood particularly well. Experimental evidence shows that the nuclear interaction is strongly attractive between nucleons over a short range (on the order of 1 fm, the size of a nucleon), repulsive between nucleons at a shorter range (< 0.5 fm, this prevents the nucleus from collapsing), is charge-independent, and depends on the spin$^2$ and orbital angular momentum of the nucleons in question. However, an exact form of the interaction is not known [8]. The other major challenge is the ‘many-body problem’ - since nuclei are comprised of many nucleons each exhibiting interactions with all neighbouring nucleons,

$^2$Spin is an intrinsic property of the nucleon. It is a component of the total angular momentum of the nucleon separate from orbital angular momentum, as discussed in Section 2.1.2.
direct calculations of the properties of nuclei would still be very computationally intensive even if an exact form of the interaction was known.

The nucleon-nucleon interaction is now understood to be a residual interaction resulting from the fundamental interaction (the strong interaction) which binds together the quarks that comprise individual nucleons. In chemistry, this is analogous to the Van der Waals forces between molecules which are residual interactions resulting from the fundamental interaction (the electromagnetic interaction) binding individual molecules. The fundamental theory of quantum chromodynamics (QCD) is a comprehensive description of the strong interaction between quarks and the force carrier particles known as gluons [9]. Presently, QCD has been extended to systems of a few nucleons (e.g. $^2\text{H}$, $^3\text{He}$) [9, 10]. More complex systems may be investigated by assuming effective interactions between two or more nucleons with symmetries which are consistent with QCD and with coupling constants fixed by experimental data at low mass. These effective interactions form the basis of ‘effective field theories’ which can then be extended to higher mass systems than is possible with the full QCD treatment.

In the context of nuclear physics, both QCD and effective field theories are referred to as ab-initio models, where the goal is to directly calculate nuclear properties from first principles. Ab-initio models do not rely on experimental data beyond the coupling constants which are derived from precision measurements of systems of only a few nucleons. This is in contrast to the phenomenological approach where the form of the interaction is derived by examining broad trends in experimental data, typically in a specific region of the nuclear landscape with the goal of predicting properties of nuclei in and around that region. Due to this simplified approach, phenomenological models can be used to calculate properties of systems with very high nucleon numbers, whereas ab-initio effective field theories are presently applicable only to light mass nuclei up to nickel [11]. However, since phenomenological models are fundamentally rooted in comparison to existing data, their utility as a predictive tool away from the region encompassed by the fit is limited. For this reason, a long-term goal of the nuclear physics community is to extend ab-initio models to high mass systems [10].

A separate problem from the form of the nuclear interaction is the computational challenge of calculating the properties of systems with many individually interacting nucleons. This ‘many-body problem’ can be mitigated by making simplifying assumptions about the interactions between nucleons. The nuclear shell models (discussed in Section 1.1.1) achieve this by considering only the interactions between a subset of ‘valence’ nucleons with a potential well which represents the average of the true interactions with other nucleons, often with added corrections to describe interactions between individual valence nucleons. The form of these interactions and the potential well may be derived using either ab-initio or phenomenological approaches. However, depending on the simplifying assumptions made...
(including choice of valence nucleons or ‘model space’), the predictions made by these models can vary significantly.

Regardless of the approach used it is beneficial to validate nuclear models against experimental data for nuclei which provide an ‘edge’ or ‘corner case’ that would challenge a simplified picture of the nuclear interaction. Data on nuclei which exhibit abrupt changes in observable quantities (eg. energies of excited states, electromagnetic transition rates) compared to their neighbours are particularly useful. One of the nuclei studied in this thesis is \( ^{28}\text{Mg} \), which lies near a region known as the ‘island of inversion’ (discussed in Section 1.1.3) where the ordering of neutron shells for nuclei within the ‘island’ is inverted compared to the neighbouring nuclei. Experimental data near the ‘island of inversion’ is particularly useful for constraining shell models which are challenged by significant changes in proton and neutron single particle energies in this region. Robust theoretical models should be able to simultaneously explain both the inverted and non-inverted configurations which are observed for different nuclei in and around the ‘island of inversion’.

1.1.1 Nuclear shell model

Experimentally, nuclei with certain nucleon ‘magic’ numbers have been observed to have greater stability than neighbouring nuclei, as evidenced by the relatively large numbers of stable isotopes with these proton or neutron numbers [12]. The nucleon shells may be reproduced using a simple model which assumes a potential in the form of a ‘nuclear potential well’ that takes into account the combined interactions of all nucleons. The form of the potential well is often phenomenological - in the simplest models, a harmonic oscillator or square well potential is often used as these allow for easy calculation of energy levels. However more sophisticated potentials may be used. Woods and Saxon famously derived a flat-bottomed finite potential based on the mass distributions of heavy nuclei inferred from the results of proton scattering experiments, which takes the form [13]:

\[
U(r) = \frac{U_0}{1 + \exp((r - R_0)/a)},
\]

where \( U_0 \) determines the depth of the potential well, \( R_0 \) specifies on the size of the nucleus, and \( a \) specifies the surface diffuseness of the nucleus. The Woods-Saxon and other similar potentials are deep within the volume of the nucleus to represent the binding of nucleons provided by the attractive but short-ranged nuclear interaction. The flat bottom of the potentials implies that there is no net force on nucleons inside the nucleus, due to cancellation of the short-range attractive forces between a nucleon and the surrounding nucleons. For protons, the effect of Coulomb repulsion causes the bottom of the well to be raised in energy [8]. Assuming that the nucleus is a sphere of uniform charge, the Coulomb term \( U_C(r) \) can be written:
Figure 1.2: Woods-Saxon potential wells of protons and neutrons, plotted for $^{40}$Ca (left) and $^{208}$Pb (right) using Equation 1.1 with the additional $U_C(r)$ term of Equation 1.2 for Coulomb repulsion between protons. The parameters of Equation 1.1 are fixed to the recommended values $U_0 = (-51 + 33(N - Z)/A)$ MeV, $R_0 = 1.27A^{1/3}$ fm, and $a = 0.67$ fm from Ref. [14].

\[ U_C(r) = \frac{kZ_1Z_2}{2R_0} \left(3 - \frac{r^2}{R_0^2}\right), \quad \text{when } r \leq R_0 \]
\[ = \frac{kZ_1Z_2}{r}, \quad \text{when } r > R_0, \]  

where $k = 1.44$ MeV fm. The overall Woods-Saxon potentials for protons and neutrons are plotted in Figure 1.2 for $^{40}$Ca and $^{208}$Pb.

Discrete single-particle energy levels $E_{sp}$ corresponding to nucleon orbitals may be obtained by solving the Schrödinger equation:

\[ H_{sp} |\psi_{sp}\rangle = E_{sp} |\psi_{sp}\rangle, \]
\[ H_{sp} = -\frac{\hbar^2}{2m} \Delta + U_{sp}, \]  

where $H_{sp}$ is the single-particle Hamiltonian corresponding to the interaction of a single particle (nucleon) with the potential $U_{sp}$. For a spherically symmetric potential $U(r)$, the single particle wavefunction $\psi_{sp}$ may be split into radial and angular terms [14]:

\[ |\psi_{sp};nlm\rangle = R_{nl}(r)Y_{lm}(\theta, \phi), \]

where $n$ is the principal quantum number, $l$ is the azimuthal (orbital angular momentum) quantum number, $m$ is the magnetic quantum number, and the solution for the radial component $R_{nl}(r)$ varies depending on the form of the potential $U(r)$. As a result, the
energies of single-particle orbitals will also vary depending on the form of $U(r)$. The angular component of the wavefunction represented by the spherical harmonics $Y_{lm}(\theta, \phi)$ is common as long as the potential is spherically symmetric.

When solving Equation 1.3 using a harmonic oscillator or a flat-bottomed function for the nuclear potential $U_{sp}$, large energy gaps between the orbitals obtained roughly correspond to observed magic numbers for light nuclei. However above $N, Z = 20$, the magic numbers are not reproduced. This discrepancy was resolved by adding a ‘spin-orbit coupling’ contribution which splits each orbital into two orbitals with energies depending on the alignment of the spin and orbital angular momenta of the contained nucleons [15, 16]. The spin-orbit contribution $U_{ls}$ modifies the nuclear potential:

$$U_{sp} = U(r) - U_{ls},$$

$$U_{ls} \propto \frac{dU(r)}{dr} \hat{l} \cdot \hat{s},$$

The spin-orbit interaction provides a phenomenological method to account for the spin dependence of the nucleon-nucleon two-body interaction, and is primarily a surface effect as indicated by the $dU(r)/dr$ term in Equation 1.5 [14]. The magnitude of the spin-orbit contribution may be related to the sum of the orbital and spin angular momentum operators $\hat{j} = \hat{l} + \hat{s}$:

$$\hat{j} = \hat{l} + \hat{s},$$

$$\hat{j}^2 = \hat{l}^2 + \hat{s}^2 + 2\hat{l} \cdot \hat{s},$$

$$U_{ls} \propto \hat{l} \cdot \hat{s} = \frac{1}{2}(\hat{j}^2 - \hat{l}^2 - \hat{s}^2),$$

$$\langle \psi_{sp}; j = l + 1/2, s = 1/2 | U_{ls} | \psi_{sp}; j = l + 1/2, s = 1/2 \rangle \propto \frac{1}{2}(j(j+1) - l(l+1) - s(s+1)).$$

As shown in Equation 1.6, the expectation value of $U_{ls}$ increases with $j$. Since the intrinsic spin of a nucleon is $s = 1/2$, there are two possible values of the total angular momentum for a given $l$ value: $j = l \pm 1/2$. Equation 1.6 can be further simplified under these conditions:

$$\langle \psi_{sp}; j = l + 1/2, s = 1/2 | U_{ls} | \psi_{sp}; j = l + 1/2, s = 1/2 \rangle \propto \frac{1}{2}l,$$

$$\langle \psi_{sp}; j = l - 1/2, s = 1/2 | U_{ls} | \psi_{sp}; j = l - 1/2, s = 1/2 \rangle \propto -\frac{1}{2}(l+1).$$

Combining the result of Equation 1.7 with Equation 1.5, the spin-orbit contribution results in single particle states with total angular momentum $j = l + 1/2$ (aligned spin
Figure 1.3: Single-particle orbitals obtained for a square-well potential with and without the spin-orbit coupling correction. Red lines indicate shell gaps obtained without spin-orbit coupling, and magenta lines indicate shell gaps only obtained with spin-orbit coupling. Similar results are obtained for other flat-bottomed potentials [17].

and orbital angular momentum) being lowered in energy relative to the states with total angular momentum \( j = l - 1/2 \) [14]. This correction reproduces all magic numbers observed for stable nuclei. A diagram of the resulting single-particle orbitals is shown in Figure 1.3 for a square-well potential. Energy levels are labelled\(^3\) based on the principal quantum number \( n \) and the orbital and spin angular momentum quantum numbers \( l \) and \( s \).

The simple shell model discussed above has a fundamental weakness in that it only describes the interactions of individual nucleons with a mean field represented by the potential wells. This simplified description is satisfactory to explain the observed proton and neutron magic numbers for stable nuclides. However, as discussed in Section 1.1, a mean field is only a simplified version of the true interaction between individual pairs and groups of nucleons. In Equation 1.3, a nucleon interacts only with the mean field and not with other nucleons, and so the Hamiltonian \( H_{sp} \) is written for a single particle. This approach is not sufficient to describe the many-body system with multiple nucleons occupying states in the potential well. In such a case the two-body potentials \( U_{j_1j_2} \) arising from the interactions between nucleons in orbitals with total angular momentum \( j_1 \) and \( j_2 \) need to be considered. In order to account for these interactions, a many-body Hamiltonian \( H \) must instead be constructed:

\(^3\)For example, \( 1f_{7/2} \) corresponds to \( n = 1, l = 3, j = l + s = 7/2 \). Values of \( l \) are specified using the spectroscopic notation of \( s \) for \( l = 0 \), \( p \) for \( l = 1 \), \( d \) for \( l = 2 \), \( f \) for \( l = 3 \), \( g \) for \( l = 4 \), \( h \) for \( l = 5 \)...
\[ H |\psi\rangle = E |\psi\rangle, \]
\[ H = \sum_{j_1} \left( -\frac{\hbar^2}{2m_{j_1}} \Delta_{j_1} + \frac{1}{2} \sum_{j_1,j_2} U_{j_1,j_2} \right) = \sum_{j_1} \left( -\frac{\hbar^2}{2m_{j_1}} \Delta_{j_1} + U_{sp,j_1} \right) + U_R, \]
\[ U_R = \frac{1}{2} \sum_{j_1,j_2} U_{j_1,j_2} - \sum_{j_1} U_{sp,j_1}, \]

where \( H \) is the true many-body Hamiltonian containing all interactions between individual nucleons, \( H_{sp,j_1} \) is the single-particle Hamiltonian of Equation 1.3 for nucleons in the orbital with total angular momentum \( j_1 \), and \( U_R \) describes all ‘residual’ interactions between pairs of nucleons in the orbitals \( j_1 \) and \( j_2 \) - the correction required to obtain the full many-body Hamiltonian \( H \) from the sum of single-particle Hamiltonians\(^4\).

One might instead attempt to construct a many-body Hamiltonian as the sum of the single particle Hamiltonians of Equation 1.3:
\[ H' = \sum_{j} H_{sp,j}. \]

In this case the total energy of a given state of the nucleus would be the sum of single particle energies:
\[ E' = \sum_{j} E_{sp,j} n_j, \]

where \( E_{sp,j} \) is the single particle energy of nucleons in orbital \( j \) and \( n_j \) is the number of nucleons in orbital \( j \) (an integer between 0 and \( 2j + 1 \)). In order for \( E' \) to be a good approximation of the true energy \( E \), it is necessary that \( U_R \approx 0 \) (i.e. nucleons only interact with the mean field \( U_{sp} \)). However as discussed below, in nuclei away from stability changes in orbital energies and the ordering of orbitals are observed, implying \( U_R \neq 0 \). Consequently, the single particle behaviour represented by Equation 1.3 is not sufficient to properly determine the energies of states in these nuclei.

The correspondence between the shell model with and without residual interactions can be partially maintained by introducing the concept of effective single particle energies \( E_{esp} \).

Here, the single particle energies \( E_{sp} \) are first determined using the single-particle model of Equation 1.3, and then perturbed using the residual interactions \( U_R \) from the many-particle

\(^4\)Though not considered in this model, three-body and higher order residual interactions can also be put into \( U_R \) at the cost of added computational complexity.
model of Equation 1.8. The residual interaction \( U_R \) can be decomposed into monopole and higher order terms \([18, 19]\):

\[
U_R = U_R^{(\lambda=0)} + U_R^{(\lambda=1)} + \ldots
\] (1.11)

The contribution to the single particle energies from the interaction \( U_R \) originates from the expectation value \( \langle \psi | U_R | \psi \rangle \) (where the wavefunction \( \psi \) is the many-body wavefunction of Equation 1.8), which can be evaluated for each term of the many-body Hamiltonian shown in Equation 1.11. In Refs. \([19, 20]\) it is shown that the contribution of the monopole \((\lambda = 0)\) term can be simply expressed in terms of the numbers of protons and neutrons occupying the orbital(s) of interest and the diagonal matrix elements of the original interaction \( U_R \). Taking the monopole contribution into account, the effective single particle energies \( E_{esp} \) are written \([18, 19]\):

\[
E_{esp,j1} = E_{sp,j1} + \langle \psi | U_R^{(\lambda=0)} | \psi \rangle,
\]

\[
= E_{sp,j1} + \sum_{j2} \bar{E}(j1,j2)n_{j2},
\]

\[
\bar{E}(j1,j2) = \frac{\sum_J (2J + 1) \langle \psi; j1j2J | U_{R,pn} | \psi; j1j2J \rangle}{\sum_J (2J + 1)} \text{ (proton-neutron)},
\]

\[
\bar{E}(j1,j2) = \frac{\sum_J (1 - \delta_{j1j2}(-1)^{2j1-J})(2J + 1) \langle \psi; j1j2J | U_{R,nn} | \psi; j1j2J \rangle}{\sum_J (2J + 1)} \text{ (like-nucleon)},
\] (1.12)

where \( E_{sp,j1} \) is the single particle energy (obtained without considering residual interactions) of nucleons in the orbital with total angular momentum \( j_1 \), \( n_{j2} \) is the number of nucleons occupying the orbital with total angular momentum \( j_2 \) (any real number between 0 and \( 2j_2 + 1 \)), \( J \) is the total (coupled\(^5\)) angular momentum of the nucleons in the orbitals \( j_1 \) and \( j_2 \) which participate in the residual interaction \( U_R \), and \( \bar{E}(j1,j2) \) is the angular momentum averaged interaction energy between nucleons in the two orbitals. The angular momentum averaging takes into account the existence of \( 2J + 1 \) degenerate states of angular momentum \( J \), with all possible \( J \) values being taken into account by summing over \( J \). For the like-nucleon (proton-proton or neutron-neutron) interaction, an additional term is present in \( \bar{E}(j1,j2) \) which prevents the interacting nucleons from occupying the same state in violation of the Pauli exclusion principle. Equation 1.12 contains only the \( \lambda = 0 \) component of \( U_R \), but not the higher order terms as these cannot be written directly in terms of the nucleon occupancies \( n_j \) \([19, 20]\).

\(^5\)For instance, coupling of nucleons in the 1d\(_{5/2}\) orbital \((j_1 = 3/2)\) and a the 1f\(_{7/2}\) orbital \((j_2 = 7/2)\) can lead to possible values of \( J = 2, 3, 4, 5 \) from the vector sum of \( j_1 \) and \( j_2 \).
1.1.2 The tensor force

An important component of the nucleon-nucleon residual interaction $U_R$ is the tensor force, which is the component of the interaction which depends on both the relative position of nucleons and orientation of their spins (in contrast to vector forces which only depend on the relative orientation of vector quantities$^6$ and scalar forces which only depend on the relative position$^7$). An analogy can be made between the nuclear tensor force and the attraction and repulsion between dipole magnets, which is also position and orientation dependent as shown in Figure 1.4. In a two-nucleon system, the nuclear tensor force is known to prefer systems with aligned spin ($S = 1$) and with nucleon positions aligned along the axis parallel to the spin vectors, based on the observed ground state properties of the deuteron (aligned nucleon spin, prolate shape [21]).

Consider the interaction between a proton and a neutron, each with orbital angular momentum $l$. The total angular momentum of each is either $j = j_> = l + 1/2$ or $j = j_< = l - 1/2$. In either case it is possible to arrange the nucleons such that their spins are aligned, as is preferred by the tensor force. In the case where both nucleons have the same total angular momentum (either $j = j_>$ or $j = j_<$), aligned spins implies that the orbital angular momenta are also aligned, resulting in low relative momentum between the two nucleons. According to the uncertainty principle$^8$, this causes the spatial wave function describing the relative motion of the nucleons to be broadly distributed along the direction of orbital motion, resulting in spatial alignment of the nucleons along the axis orthogonal to the spin vectors as shown in Figure 1.5. On the other hand, when one nucleon has total angular momentum $j>$ and the other has total angular momentum $j<$, aligned spins implies that the orbital angular momenta are unaligned, maximizing the relative angular momentum between the two nucleons. This narrows the spatial wave function along the direction of orbital motion, resulting in an increased likelihood of the nucleons being spatially aligned along the axis parallel to the spin vectors as shown in Figure 1.5. Since this is the preferred ‘deuteron-like’ orientation, the result of the tensor force is an attraction between protons and neutrons with the opposite spin-orbit alignment (protons with $j = j_>$ and neutrons with $j = j<$, or vice versa) compared to protons and neutrons with both $j = j_>$ or $j<$ [22, 23]. This attractive tensor force is the key driver of neutron shell evolution in the region of the ‘island of inversion’ studied in this thesis [24].

$^6$For example the spin-orbit coupling which depends on the relative orientation of a single nucleon’s orbital and spin angular momenta.

$^7$For instance, the interaction of a single particle with a potential well.

$^8$\(\Delta j \Delta \phi \geq \hbar/2\) for angular momentum $j$ and for the arc of orbital motion subtending the angle $\phi$. This arc specifies the distribution of the spatial wave function describing the relative motion of the two nucleons.
Figure 1.4: Schematic of the tensor interaction between two dipole magnets. The attractive or repulsive nature of the interaction depends on both the position and orientation of the magnets.

Figure 1.5: Schematic of the tensor interaction between two nucleons, according to Ref. [22]. The grey shaded area corresponds to the spatial distribution of the two-body wavefunction of the interacting nucleons. The tensor force is attractive between nucleons with total angular momentum $j_> = l + 1/2$ and $j_< = l - 1/2$. 
1.1.3 Island of inversion

As discussed above, the location and/or position of shell gaps can change when moving away from stability, for instance due to differences in the contribution of the residual interactions described in Equation 1.12 as a function of the number of nucleons \( n_j \) in a given orbital. Nuclei far from stability are good test cases to determine the limits of applicability for theoretical models which predict this shell evolution.

One region of interest in the nuclear chart is a so-called ‘island of inversion’ which occurs around the neutron-rich nucleus \(^{32}\text{Mg}\), with 12 protons and 20 neutrons. As 20 is one of the magic numbers shown in Fig 1.3, one might expect the ground state of \(^{32}\text{Mg}\) to contain neutrons up to the \(^{1}\text{s}_{1/2}\) and \(^{1}\text{d}_{3/2}\) orbitals, with a large energy gap between the ground and first excited states (as seen in other closed-shell nuclei such as \(^{40}\text{Ca}\)). However for \(^{32}\text{Mg}\), the first excited state is at a relatively low energy of 885.3(1) keV \cite{25}, and as first measured in 1975 by Thibault et al. \cite{26} the ground state binding energies of nuclei in this region are anomalously low compared to the predictions of model calculations in the \(sd\) shell\(^9\). The ground state of \(^{32}\text{Mg}\) is now understood to contain neutrons in the \(^{2}\text{p}_{3/2}\) and \(^{1}\text{f}_{7/2}\) orbitals above the \(N = 20\) shell gap \cite{27}, which model calculations suggest is partially due to the reduction of this shell gap far from stability, as well as the effect of proton-neutron residual interactions \cite{28}.

The evolution of the \(N = 20\) shell gap in this region is understood to arise from the relative attraction between the proton \(^{1}\text{d}_{5/2}\) orbital and the neutron \(^{1}\text{d}_{3/2}\) and \(^{1}\text{f}_{7/2}\) orbitals, which is stronger for the neutron \(^{1}\text{d}_{3/2}\) orbital \cite{24} due to the attractive tensor force discussed in Section 1.1.2. This effect is responsible for lowering of the neutron \(^{1}\text{d}_{3/2}\) orbital with respect to the neutron \(^{1}\text{f}_{7/2}\) orbital which widens the \(N = 20\) shell gap, and is strongest when the proton \(^{1}\text{d}_{5/2}\) orbital is fully occupied \((Z \geq 14)\). For lower \(Z\) the opposite is true and the \(N = 20\) shell gap is reduced, instead opening a gap at \(N = 16\) for neutron rich oxygen \((Z = 8)\) isotopes \cite{29, 30}. Investigations of the \(N = 16\) shell gap for nuclei near \(^{24}\text{O}\) are an area of active investigation, however these nuclei are difficult to produce experimentally as they are far from stability.

Another region under active study is centred on nuclei with partial proton \(^{1}\text{d}_{5/2}\) orbital occupation where neither the \(N = 20\) gap is fully closed nor is the \(N = 16\) gap fully opened. In this region, which includes the neutron rich Na \((Z = 11)\) and Mg \((Z = 12)\) isotopes, the \(N = 20\) shell gap is still reduced in comparison to the \(Z \geq 14\) case. The \(N = 20\) ‘island of inversion’ lies in this region. Although the ‘island of inversion’ represents a region of the nuclear chart for which the \(N = 20\) magic number is ‘broken’, it should be noted that the gap in effective single particle energies between the \(sd\) and \(pf\) shell is still predicted to be on the order of 5 MeV at \(^{32}\text{Mg}\) \cite{31}. This energy gap is offset by correlation energy.

\(^9\) Throughout this thesis and the referenced literature, the \(sd\) shell refers to the nuclear shell comprised of the \(^{1}\text{d}_{5/2}, 2s_{1/2}, \) and \(^{1}\text{d}_{3/2}\) orbitals below the \(N, Z = 20\) shell gap. The \(pf\) shell refers to the \(^{1}\text{f}_{7/2}, 2p_{3/2}, 1f_{5/2}, \) and \(^{2}p_{1/2}\) orbitals above the \(N, Z = 20\) shell gap. See Figure 1.3.
gained from residual nucleon-nucleon interactions in $^{32}\text{Mg}$, including those resulting from the deformation of the nucleus associated with breaking of the closed neutron shell. As a result, partial neutron occupation of the $fp$ shell becomes the most energetically favourable (ie. ground state) configuration [31].

As experimental data in this region is scarce, recent experiments have continued to test the limits of the $N = 20$ ‘island of inversion’ for Na [32] and Mg [24, 33] isotopes. An additional area of investigation is the structure of nuclei outside of the ‘island of inversion’ when these nuclei are excited to high energies. For nuclei with $N < 20$, the $N = 20$ shell gap may still be investigated by exciting neutrons into the relevant single-particle orbitals above the shell gap ($1f_{7/2}$, $2p_{3/2}$, etc.). Such studies have previously been carried out for the $N = 14$ nuclei $^{26}\text{Mg}$ [34] and $^{30}\text{Si}$ [35]. This thesis provides a similar investigation of the $N = 16$ species $^{28}\text{Mg}$. The goal of this study was to provide detailed and high precision measurements of properties of the excited states in this nucleus, in order to constrain theoretical models describing the shell structure near the ‘island of inversion’. Details of the $^{28}\text{Mg}$ experiment are found in Chapter 5.

Although the $N = 20$ island was the first to be identified, it is now understood that similar ‘islands of inversion’ exist in regions of the nuclear chart where other magic numbers are broken (for instance, at $^{11}\text{Li}$, for $N = 8$ [27]).

1.1.4 Models used near the island of inversion

Based on the general principles outlined in Section 1.1, it is possible to generate any number of theoretical models to describe nuclear structure. The studies of $^{28}\text{Mg}$ in this thesis include a comparison of experimental data to calculations using specific state-of-the-art models. An overview of the characteristics of the particular models used is given in Table 1.1.
Table 1.1: Description of theoretical models referenced in this thesis.

<table>
<thead>
<tr>
<th>Interaction Name</th>
<th>Description</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>USDB</td>
<td>A phenomenological shell model considering one and two-body interactions in the $sd$ shell ($8 &lt; N \leq 20$, $8 &lt; Z \leq 20$). Model parameters are based on a fit to experimentally obtained level energy data from 77 nuclei in the $sd$ shell. USDB is a refined version of the earlier USD interaction [36, 37], taking advantage of experimental and computational advances to allow more data to be used in the fit.</td>
<td>[36]</td>
</tr>
<tr>
<td>SDPF-U</td>
<td>A phenomenological shell model based on the earlier SDPF-NR interaction [39], assuming an inert $^{16}$O core with valence protons in the $sd$ shell ($8 &lt; Z \leq 20$), and valence neutrons in the $sdpf$ shell ($8 &lt; N \leq 40$). Model parameters are fit to reference level energy data for isotopes of Si through Ca. Proton-proton interactions use matrix elements derived from USD [39].</td>
<td>[38]</td>
</tr>
<tr>
<td>SDPF-MU</td>
<td>A shell model in the $sdpf$ valence space which uses components of various existing phenomenological models, including USD for interactions in the $sd$ shell. SDPF-MU introduces a monopole-based universal interaction term $V_{MU}$ [41] which is responsible for modelling cross-shell interactions due to the tensor force described in Section 1.1.2, including those which give rise to the ‘island of inversion’.</td>
<td>[40]</td>
</tr>
<tr>
<td>IM-SRG</td>
<td>In-Medium Similarity Renormalization Group. An ab-initio shell model approach (ie. a shell model using two-nucleon ($NN$) and possibly higher order effective interactions) which allows for decoupling of ‘core’ and ‘valence’ space Hamiltonians from a starting $A$-body Hamiltonian, simplifying the calculations. For calculations of $sd$ shell nuclei the core is typically $^{16}$O, with higher shells forming the valence space.</td>
<td>[42]</td>
</tr>
<tr>
<td>CCEI</td>
<td>Coupled-Cluster Effective Interaction. An ab-initio shell model approach in which the $A$-body interaction Hamiltonian is expanded into a series of ‘valence cluster’ Hamiltonians corresponding to core, one-body, two-body, and higher order terms.</td>
<td>[43]</td>
</tr>
<tr>
<td>SA-NCSM</td>
<td>Symmetry-Adapted No-Core Shell Model. An ab-initio shell model approach, where ‘no-core’ indicates that all $A$ nucleons are treated as active (ie. no inert core is used, no concept of a valence space outside of the core). The SA-NCSM differs from other no-core shell models in terms of organization of basis states - in this case, using an ‘SU(3) scheme’ which maps directly onto the three-dimensional harmonic oscillator basis (see Section 3.2 of Ref. [44]) and can be used to directly express nuclear states in terms of configurations with deformed shapes.</td>
<td>[44]</td>
</tr>
</tbody>
</table>
Chapter 2

Experimental Background and Techniques

2.1 Properties of excited states in nuclei

2.1.1 Excitation energy

The excitation energy $E_{ex}$ of a nuclear state refers to the energy of the state relative to the ground state of the same nucleus. In most nuclear systems, discrete states may be observed with excitation energy ranging from a few keV to a few MeV. Gamma-ray spectroscopy allows for the measurement of excitation energies since the energy of a gamma ray emitted in an electromagnetic transition between two states is nearly equal to the energy difference between the states (see Section 2.3).

Excitation energies can also be calculated using shell models such as those discussed in Section 1.1.1. In these models, the energies of individual states are represented by the eigenvalues of the many-body Hamiltonian, as defined in Equation 1.8. These energies can be represented as the sum of single particle energies of all nucleons in the system with additional corrections resulting from residual interactions. Energies calculated in this manner can then be expressed relative to the calculated energy of the lowest lying (ground) state to obtain excitation energies of the state(s) of interest.

2.1.2 Spin

The terms ‘spin’ and ‘total angular momentum’ are often used interchangeably in nuclear structure studies. Confusingly, the ‘total angular momentum’ or ‘spin’ of a nucleon has two components, one of which is also called ‘spin’:

- Orbital angular momentum: arising from orbital motion of a body around a central point (eg. orbit of a planet around a star, nucleon around other nucleons). Orbital
angular momentum is described by the operator $\hat{L} = \hat{r} \times \hat{p}$ and denoted by the quantum number $l$.

- Spin angular momentum: the ‘intrinsic’ component of the total angular momentum not associated with orbital motion$^1$. Spin is described by operators represented by Pauli matrices and for nucleons is denoted by the quantum number $s = 1/2$.

The ‘total angular momentum’ or ‘spin’ $\hat{j}$ of a nucleon depends on the operators $\hat{l}$ and $\hat{s}$ (which in the shell model vary depending on the shell that the nucleon occupies as discussed in Section 1.1.1) [17]:

$$\hat{j} = \hat{l} + \hat{s}$$
$$j = l \pm 1/2.$$  \hfill (2.1)

Since the quantum number $l$ is always an integer, the quantum number $j$ is always half integer. The operator of the ‘nuclear spin’ or total angular momentum $\hat{I}$ of the nucleus is obtained from the sum of the angular momenta operators $\hat{j}$ of the individual nucleons [17]:

$$\hat{I} = \sum \hat{j}.$$ \hfill (2.2)

As nucleons carry half-integer spin, nuclei with an odd number of nucleons (eg. $^{13}$C) have half-integer spin, while nuclei with an even number of nucleons (eg. $^{16}$O) have integer spin. Moreover, in the ground state nucleons of the same type with identical $l$ and $s$ pair with anti-aligned angular momenta, resulting in the quantum number $I = 0$ for nuclei with even values of $N$ and $Z$ [17].

2.1.3 Parity

The parity of a nucleus in a given state is a property of its nuclear wavefunction $\psi$ describing whether the wavefunction is changed upon reflection of all spatial coordinates (the parity operation). In Cartesian coordinates, the parity operation maps $x \to -x$, $y \to -y$, and $z \to -z$. In spherical coordinates the mapping is $r \to r$, $\theta \to \pi - \theta$, and $\phi \to \phi + \pi$. For a parity operation changing all spatial coordinates $\vec{R} \to -\vec{R}$, then [21]:

- If $\psi(\vec{R}) = \psi(-\vec{R})$, the nuclear wavefunction $\psi$ is said to have even or positive parity.
- If $\psi(\vec{R}) = -\psi(-\vec{R})$, the nuclear wavefunction $\psi$ is said to have odd or negative parity.

Experimental observations indicate that parity is a conserved quantity in the strong and electromagnetic interactions [8]. For electromagnetic transitions such as those studied

$^1$The name originates from the classical description of an object rotating about an axis, however this description is not applicable to systems such as nuclei and nucleons, where typical values of the spin angular momentum would result in classical motion on the equator exceeding the speed of light.
in this thesis, the conservation of parity leads to selection rules which govern the electric and/or magnetic character of the observed radiation depending on the initial and final state (see Section 2.3), which can be used to infer the spin and parity of these states [8].

Quantum numbers for spin $I$ and parity $\pi$ are often expressed together as a fundamental property of a given nuclear state called 'spin-parity' $I^\pi$. For example, a state with spin-parity $I^\pi = 3^-$ has spin $3\hbar$ and negative parity.

2.2 Population of excited states in nuclei

Many techniques exist to access excited states in nuclei and measure properties relating to nuclear structure. When designing an experiment there are many considerations to take into account, depending on the specific nuclei and properties being investigated. These include but are not limited to: feasibility of populating the desired nucleus via a given technique, rate of population, and technical considerations.

2.2.1 Common reaction mechanisms

The nucleus of interest may be produced either as the direct result of a reaction between a beam and a target, or as the decay product of another nucleus. The latter case is often convenient for studies involving gamma-ray spectroscopy. In decay experiments such as those using the GRIFFIN spectrometer [45] at TRIUMF, the parent nucleus is delivered to the experimental station as a low energy beam where the species of interest is separated from contaminants based on its mass-to-charge ratio. The beam is then stopped and the decays of the parent nucleus are observed. The required beam intensity to the experimental station is relatively low, since all of the parent nuclei can decay, resulting in a high yield of the daughter nucleus being studied. Additionally, since the parent nucleus is allowed to decay at rest, gamma rays observed from the daughter nucleus are not Doppler shifted, simplifying the analysis procedure. However the maximum energy of excited states available for decay study is limited by the $Q$ value of the decay of the parent to the daughter species$^2$.

In-beam reaction mechanisms such as fusion-evaporation$^3$ or Coulomb excitation$^4$ are necessary to populate the states which are unreachable by decay reactions. However these methods typically require delivery of accelerated beams at significantly higher rates than are

---

$^2$\(Q\) represents the mass-energy difference between reactants and products in a reaction, a consequence of mass-energy conservation based on the mass-energy equivalence \(E = mc^2\). For instance in the beta decay process, a neutron transforms into a proton or vice versa, accompanied by emission of an electron \(e^-\) or positron \(e^+\) and an electron anti-neutrino \(\bar{\nu}_e\) or neutrino \(\nu_e\) to conserve charge and lepton number. The reaction formulae are \(\frac{A}{2}X_N \rightarrow \frac{A}{2}+1Y_{N-1}+\epsilon^-+\bar{\nu}_e\) or \(\frac{A}{2}X_N \rightarrow \frac{A}{2}-1Y_{N+1}+e^++\nu_e\), so \(Q = M_X c^2 - M_Y c^2 - m_e c^2\) (assuming negligible neutrino mass).

$^3$Fusion of the beam and target species to form an excited compound nucleus, which partially de-excites by emitting light particles such as protons, neutrons, and alpha particles to populate the nucleus of interest.

$^4$Inelastic scattering of the beam and target species, resulting in excitation of the beam or target species via the Coulomb interaction.
Table 2.1: Qualitative comparison of reaction mechanism suitability for an experiment studying properties of excited states in one or more nuclei.

<table>
<thead>
<tr>
<th>Reaction Mechanism</th>
<th>Fusion-evaporation</th>
<th>Coulomb excitation</th>
<th>β decay</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of reaction channels</td>
<td>Many</td>
<td>One</td>
<td>One</td>
</tr>
<tr>
<td>$E_{ex}$ of states observed</td>
<td>Variable</td>
<td>Variable</td>
<td>Fixed (Q value)</td>
</tr>
<tr>
<td>$I$ of states observed</td>
<td>High</td>
<td>Variable</td>
<td>Low</td>
</tr>
<tr>
<td>Number of states observed</td>
<td>Many</td>
<td>Limited</td>
<td>Many</td>
</tr>
<tr>
<td>Min. required beam rate</td>
<td>$\sim 10^7$ s$^{-1}$</td>
<td>$\sim 10^5$ s$^{-1}$</td>
<td>$\sim 1$ s$^{-1}$</td>
</tr>
</tbody>
</table>

Figure 2.1: Qualitative comparison of typical excitation energies $E_{ex}$ and spins $I$ of excited states populated by various reaction mechanisms.

required for beta decay experiments, since in-beam reactions often have low cross sections which results in low yield of the desired reaction products.

In-beam-reactions are often written using the notation $a(b,c)d$, where $a$ is the target species, $b$ is the beam species, $c$ is the ejectile, and $d$ is the reaction product. For instance, the reaction $^{12}$C($^{18}$O,2α)$^{22}$Ne corresponds to production of $^{22}$Ne via fusion of an $^{18}$O beam and $^{12}$C target, followed by evaporation of 2 alpha particles ($^4$He nuclei).

Table 2.1 contains a qualitative comparison of various reaction mechanisms indicating advantages and disadvantages of each method. Figure 2.1 shows a comparison of typical excitation energies $E_{ex}$ and total angular momentum $I$ of excited states populated by the same reaction mechanisms. The yrast line shown in Figure 2.1 defines the lowest possible excitation energy for given angular momentum $I$. States on the yrast line are known as yrast states, and there are no states below the yrast line.

2.2.2 Fusion-evaporation

Fusion-evaporation is the primary reaction mechanism used in this thesis, particularly in the studies of $^{22}$Ne and $^{28}$Mg presented in Chapters 4 and 5. As the name suggests, fusion-
evaporation involves the fusion of beam and target nuclei, followed by particle evaporation from the compound system. In order for fusion-evaporation to occur, the beam energy must be large enough to overcome the Coulomb repulsion between beam and target nuclei\(^5\). Following fusion, a compound system is formed with excitation energy \(E_{ex}\) given by:

\[
E_{ex} = E_b + Q_{\text{fusion}} - E_c,
\]

(2.3)

where \(E_b\) and \(E_c\) are the beam and compound kinetic energies in the lab frame at the time of the reaction, with \(E_c\) determined using conservation of momentum\(^6\). Following fusion, the constituent nucleons of the compound system reach thermal equilibrium (this is in contrast to direct reactions where the intermediate system does not reach an equilibrated state). The excitation energy of the equilibrated compound system is often sufficient to allow for prompt evaporation of neutrons and/or light charged particles, which proceeds with the types, energies, and directions of emitted particles depending on the density of available final states. Following particle evaporation, a residual nucleus remains with up to a few MeV of excitation energy (which is insufficient for further particle evaporation). The residual nucleus may then de-excite to its ground state via gamma-ray emission. As will be discussed in Section 2.3, these gamma-rays may be used to characterize the structure and properties of the residual nucleus. The differing time scales for particle evaporation (\(\sim 10^{-19}\) s) and gamma-ray emission (10\(^{-15}\) to 10\(^{-9}\) s) reflect rate differences for processes dependent on the strong and electromagnetic interactions, respectively. A schematic of the fusion-evaporation reaction mechanism is shown in Figure 2.2.

---

\(^5\)Fusion will occur for beam energies above the Coulomb barrier \(U_C(r)\) defined in Equation 1.2, where \(r\) is the radius at which the nuclear and electromagnetic force have equal magnitude and opposite direction. This radius can be approximated using \(r = R_0 A_1^{1/3} + R_0 A_2^{1/3} + 2\) fm, \(R_0 = 1.2\) fm. Using this approximation for \(^{12}\)C and \(^{18}\)O, the required energy is 8.8 MeV in the centre of mass.

\(^6\)As the target is stationary, the compound must have the same momentum in the lab frame as the incoming beam. Assuming classical momentum \(p = \sqrt{2mE_k}\), then \(m_bE_b = m_cE_c\), and so \(E_c = \frac{m_b}{m_b + m_t} E_b\), where \(m_t\) is the mass of the target species (\(m_c = m_b + m_t\), neglecting the mass defect which is small compared to the compound mass \(m_c\)).
Figure 2.3: Schematic indicating the bias to population of states at high spin via fusion-evaporation. Left: impact parameter $b$ is proportional to angular momentum of the final system. Right: high impact momentum is favoured in fusion, since low impact parameters only occur when near the ‘bulls-eye’ which has low cross-sectional area.

Although the cross-section of fusion-evaporation reactions populating nuclei far from stability is small (necessitating very intense beams and/or long run times for experiments), fusion-evaporation gives access to excited states at very high excitation energy and spin. As shown in Figure 2.3, the angular momentum $L$ of the beam-target system depends on the impact parameter $b$ or spatial separation of the beam and target nuclei in the direction perpendicular to the beam momentum $\vec{p}$ in the center of mass frame:

$$|\vec{L}| = |\vec{r} \times \vec{p}| = p \cdot r \sin \theta = pb.$$  (2.4)

For a beam dispersed over a cross-sectional area significantly larger than the target nucleus\(^7\), large impact parameters and therefore high spin are preferred, since a low impact parameter implies a head-on collision (hitting the ‘bullseye’, as shown in Figure 2.3) with a region which has relatively low cross-sectional area. This preferential population of states at high spin is useful for experiments studying nuclear structure as these states are often inaccessible via other reaction mechanisms. For example, the study of $^{28}$Mg presented in Chapter 5 of this thesis uses the energies of specific high spin states observed via fusion-evaporation to probe the evolution of nuclear shell gaps in this region.

### 2.3 Electromagnetic transitions

A nucleus may transition from an excited configuration to a lower energy configuration via the electromagnetic interaction, accompanied by emission of a gamma ray with energy $E_\gamma$

\(^7\)Experimentally, this is always the case. Typical beam spot sizes achieved for experiments in this work are $\sim 2$mm in diameter, compared to nuclear radii of a few fm.
corresponding to the difference in energy between these configurations\(^8\). Detection of these gamma rays allows for identification of a nucleus based on its known excited states, or the identification of previously unobserved excited states. Additionally, as discussed in section 2.5.4 the peak shape observed for a gamma ray corresponding to a specific transition can yield information about its transition rate.

### 2.3.1 Multipole radiation

Observable properties of electromagnetic transitions in nuclei such as the transition energy and rate can be used to infer information about the structure of the nuclear states connected by these transitions. In order to understand this concept it is necessary to examine the properties of electromagnetic transitions and the radiation (photons) which accompanies them. This section summarizes derivations which are performed in detail in Chapter 1 of Ref. [46]. As in Ref. [46], equations in this section are presented in natural units (ie. \(c, \hbar, m_e = 1\)).

The electromagnetic interaction is governed by the Maxwell equations, which can be expressed in terms of a vector potential \(\vec{A}(\vec{r}, t)\) and a scalar potential \(\phi(\vec{r}, t)\):

\[
\begin{align*}
\vec{B} &= \vec{\nabla} \times \vec{A} \\
\vec{E} &= -\frac{\partial \vec{A}}{\partial t} - \vec{\nabla} \phi
\end{align*}
\]  

(2.5)

where \(\vec{B}\) and \(\vec{E}\) express the magnetic and electric field vectors, respectively. It is possible to expand the vector potential \(\vec{A}(\vec{r}, t)\) in terms of a series of plane waves:

\[
\vec{A}(\vec{r}, t) = \sum_{\vec{p}, \tau} (\vec{A}_{\vec{p}\tau}(\vec{r}, t)a_{\vec{p}\tau} + \vec{A}^*_{\vec{p}\tau}(\vec{r}, t)a^*_{\vec{p}\tau}),
\]

\[
\vec{A}_{\vec{p}\tau}(\vec{r}, t) = \sqrt{\frac{2\pi}{VE_{\gamma}}} \vec{e}_\tau e^{i(\vec{p}\cdot\vec{r} - E_{\gamma}t)},
\]

(2.6)

where \(a_{\vec{p}\tau}\) and \(a^*_{\vec{p}\tau}\) are expansion coefficients, \(\vec{e}_x\) and \(\vec{e}_y\) are orthogonal unit vectors which are both perpendicular to the photon propagation direction \(\vec{k}\), and the expression for \(\vec{A}_{\vec{p}\tau}(\vec{r}, t)\) is derived for a single photon of momentum \(\vec{p}\), energy \(E_{\gamma}\), and helicity\(^9\) \(\tau\) in a box with volume \(V\).

\(^8\)Minus the recoil energy \(E_r\) of the nucleus, which is a small correction: \(E_r = \frac{E^2_{\gamma}}{2m_c}\), where \(m\) is the mass of the nucleus.

\(^9\)Helicity represents the projection of spin angular momentum \(\vec{s}\) onto linear momentum \(\vec{p}\). Here spin angular momentum refers to the component of total angular momentum that is not the orbital angular momentum \(\vec{L}\), since the projection of \(\vec{L}\) onto \(\vec{p}\) is 0 by definition (\(\vec{L} = \vec{r} \times \vec{p}\)).
The scalar potential $\phi(\vec{r}, t)$ of Equation 2.5 may be separately solved in spherical coordinates [46]:

$$\phi(\vec{r}, t) = f_L(E_\gamma r)Y_{LM}(\theta, \phi)e^{-iE_\gamma t}, \quad (2.7)$$

where $L$ and $M$ are the azimuthal and magnetic quantum numbers, $Y_{LM}(\theta, \phi)$ are spherical harmonic functions and $f_L(E_\gamma r)$ are either spherical Bessel or Hankel functions (which may be chosen to obtain either standing wave solutions or outgoing/incoming spherical wave solutions). The quantum numbers $L$ and $M$ are equivalent to the quantum numbers shown earlier in Equation 1.4, however the notation presented here differs in order to make a distinction between $L$ and $M$ which are used to describe electromagnetic fields, and $l$ and $m$ which are used to describe single particle states in nuclei.

From the scalar potential solutions of Equation 2.7, solutions to the vector potential $\vec{A}(\vec{r}, t)$ may also be derived in spherical coordinates:

$$\vec{A}^{(E)}_{LM}(\vec{r}, t) = \frac{1}{E_\gamma \sqrt{L(L+1)}}(\vec{\nabla} \times \vec{L})\phi_{LM}(\vec{r}, t),$$

$$\vec{A}^{(M)}_{LM}(\vec{r}, t) = \frac{1}{\sqrt{L(L+1)}}\vec{L}\phi_{LM}(\vec{r}, t), \quad (2.8)$$

where $(E)$ and $(M)$ denote electric and magnetic multipole solutions. The solutions of Equation 2.8 represent spherical waves carrying $L$ units of orbital angular momentum with projection $M$.

The plane wave solution for $\vec{A}^{\vec{p}\tau}_{\vec{r}t}(\vec{r}, t)$ shown in Equation 2.6 can be expressed in terms of the spherical solutions $\vec{A}^{(E,M)}_{LM}(\vec{r}, t)$ of Equation 2.8:

$$\vec{A}^{\vec{p}\tau}_{\vec{r}t}(\vec{r}, t) = -\sqrt{2\pi} \sum_{LM} i^L D_{LM\tau}^{(L)}(\vec{e}_z \rightarrow \vec{k})\sqrt{2L+1}([\vec{A}^{(E)}_{LM}(\vec{r}, t) + \tau \vec{A}^{(M)}_{LM}(\vec{r}, t)]. \quad (2.9)$$

where $D_{LM\tau}^{(L)}(\vec{e}_z \rightarrow \vec{k})$ are Wigner D-matrices describing rotation from the $\vec{z}$ direction to the direction $\vec{k}$ in which the electromagnetic plane wave travels, and $\tau$ is the helicity of plane wave.

The Hamiltonian $\hat{H}_{int}$ describing the interaction between the nucleus and electromagnetic radiation field within a volume $V$ is:

$$\hat{H}_{int} = -\int \vec{j}(\vec{r})\vec{A}(\vec{r}, t)dV,$$

$$\vec{j}(\vec{r}) = \sum_k e_k \vec{p}_k \delta(\vec{r} - \vec{r}_k), \quad (2.10)$$
where the index $k$ enumerates the particles of the material system (i.e. nucleons in the nucleus) which interact with the radiation field, with position and momentum vectors $\vec{r}_k$ and $\vec{p}_k$. The Hamiltonian of Equation 2.10 can be used to describe an electromagnetic transition between two states $|\psi_i\rangle$ and $|\psi_f\rangle$ which is accompanied by emission of a single photon, such as those observed in internal transitions in nuclei.

The probability of emitting a single photon with momentum $\vec{p}$ and helicity $\tau$ can be expressed in terms of the initial ($|\psi_i\rangle|0\rangle$) and final ($|\psi_f\rangle|1\rangle\vec{p}\tau\rangle$) state of the photon as well as photon annihilation and creation operators $\hat{a}_{\vec{p}\tau}$ and $\hat{a}_{\vec{p}\tau}^+$:

$$\langle 1\vec{p}\tau, \psi_f | \hat{H}_{\text{int}} | \psi_i, 0 \rangle = \int \langle \psi_f | j(\vec{r}) \bar{A}^*_{\vec{p}\tau}(\vec{r}, t) | \psi_i \rangle e^{iE_f t} dV, \quad (2.11)$$

where the amplitudes $a_{\vec{p}\tau}$ and $a^*_{\vec{p}\tau}$ of Equation 2.6 have been replaced by $\hat{a}_{\vec{p}\tau}$ and $\hat{a}_{\vec{p}\tau}^+$, respectively and the $e^{iE_f t}$ term has been added to conserve energy, as discussed in Ref. [46]. Only the $\bar{A}^*_{\vec{p}\tau}(\vec{r}, t)$ term from Equation 2.6 remains due to the definition of the creation and annihilation operators (i.e. $\langle 1\vec{p}\tau | \hat{a}_{\vec{p}\tau}^+ | 0 \rangle = 1$, $\langle 1\vec{p}\tau | \hat{a}_{\vec{p}\tau} | 0 \rangle = 0$).

Fermi’s golden rule describes the transition rate between initial and final energy eigenstates $\psi_i$ and $\psi_f$ of a quantum mechanical system:

$$\lambda = \frac{2\pi}{\hbar} \left| \langle \psi_f | \hat{H}_{\text{int}} | \psi_i \rangle \right|^2 \rho(E_f), \quad (2.12)$$

with $\rho(E_f)$ representing the density of final states. The matrix element shown in Equation 2.11 can then be used to calculate the transition rate for emission of a single photon into a solid angle $d\Omega$:

$$\lambda(I_i \rightarrow I_f, \theta, \phi) \ d\Omega \propto \int \langle \psi_f | j(\vec{r}) \bar{A}^*_{\vec{p}\tau}(\vec{r}, t) | \psi_i \rangle^2 \ dV. \quad (2.13)$$

The $\bar{A}^*_{\vec{p}\tau}(\vec{r}, t)$ term is a plane wave which may be expressed in terms of a multipole series expansion, as shown in Equation 2.9. Equation 2.13 indicates that the transition rate varies with the angle. This is manifested as an angular distribution of emitted photons when considering an ensemble of many events corresponding to a transition between two states $I_i$ and $I_f$. The exact form of the angular distribution depends on the multipoles $EL$ or $ML$ which participate in the transition. The use of photon angular distributions to characterize observed electromagnetic transitions is discussed in Section 2.5.5.

By integrating Equation 2.13 over the solid angle, cross terms corresponding to interference between different multipoles cancel out$^{10}$, leading to an expression for the total transition rate:

$^{10}$The plane wave term $\bar{A}^*_{\vec{p}\tau}(\vec{r}, t)$ of Equation 2.13 is expressed using a series of spherical harmonic terms $Y_{LM}(\theta, \phi)$, as previously shown in Equations 2.7 through 2.9. Spherical harmonics are orthonormal (i.e. $\int Y_{LM} Y^*_{L'M'} d\Omega = \delta_{LL'} \delta_{MM'}$), so the squared sum of Equation 2.13 reduces to the sum of squares shown in Equation 2.14.
$$\lambda(I_i \rightarrow I_f) = \frac{8\pi}{2I_i + 1} \sum_{E_{\text{L or ML}}} \frac{E_{\gamma}^{2L+1}}{[(2L + 1)!!]^2} \frac{L + 1}{L} \frac{1}{|\langle \psi_f | |\mathcal{M}(E \text{L or ML})| |\psi_i\rangle|^2}. \quad (2.14)$$

where $I_i$ refers to the spin of the initial state $\psi_i$, and $E$ or $M$ refer to the electric or magnetic multipole terms of the plane wave series expansion seen in Equation 2.9. The multipole operator terms of Equation 2.14 are:

$$\mathcal{M}(E \text{L}, M) = \int \rho(\vec{r}) r^L Y_{LM}(\theta, \phi) dV,$$

$$\mathcal{M}(M \text{L}, M) = \frac{-i}{L + 1} \int \vec{j}(\vec{r}) L r^L Y_{LM}(\theta, \phi) dV, \quad (2.15)$$

where $\vec{j}(\vec{r})$ is the current density operator of Equation 2.10 and the charge density operator $\rho(\vec{r})$ is:

$$\rho(\vec{r}) = \sum_k e_k \delta(\vec{r} - \vec{r}_k). \quad (2.16)$$

### 2.3.2 Selection rules

The results of equations 2.14 and 2.15 allow for characterization of nuclear states based on the electromagnetic transitions connecting these states. As shown in Equation 2.14, the transition rate for a given multipole $L$ depends on the photon energy by a factor of $E_{\gamma}^{2L+1}$, indicating that high energy gamma rays are expected to correspond to transitions with short mean lifetimes. Furthermore, the multipolarity $L$ of a given transition is restricted by conservation of angular momentum, leading to a selection rule based on the spins of the initial and final states $I_i, I_f$ of the system:

$$|I_i - I_f| \leq L \leq I_i + I_f. \quad (2.17)$$

Parity selection rules may also be established. Since the integral over all space of a negative parity ($\pi = -1$) function is 0, the transition rates described in Equation 2.14 are non zero only if $\pi(\psi_f^* \cdot \pi(\mathcal{M}(E \text{L or ML}) \cdot \pi(\psi_i) = 1$. This condition leads to the following rules for electric and magnetic multipole terms of the series expansion:

$$\pi(\psi_i)\pi(\psi_f) = (-1)^L \text{ for } E \text{L (electric multipole terms)},$$

$$\pi(\psi_i)\pi(\psi_f) = (-1)^{L+1} \text{ for } M \text{L (magnetic multipole terms)}. \quad (2.18)$$
The angular momentum and parity selection rules of Equations 2.17 and 2.18 may be used to infer information about nuclear states based on the electromagnetic transitions connecting them, and vice versa. For instance, for a transition between nuclear states of spin-parity \( I_i^\pi = 2^+ \) and \( I_f^\pi = 0^+ (2^+ \rightarrow 0^+) \), \( \pi(\psi_i)\pi(\psi_f) = 1 \) (no parity change), \(|J_i - J_f| = 2\), and \( J_i + J_f = 2 \), so from Equations 2.17 and 2.18 the only possible multipolarity is E2.

The selection rules of Equations 2.17 and 2.18 also allow for the possibility of mixed multipoles. For example, a \( 3^+ \rightarrow 2^+ \) transition can proceed via the \( M_1, E_2, M_3, E_4, \) or \( M_5 \) multipoles, though in practice the \( M_1 \) and \( E_2 \) modes will dominate as higher order modes proceed at comparatively low rates (as shown in Appendix B.3). The mixing ratio \( \delta \) defines the relative contribution of \( L \) and \( L + 1 \) multipoles for a given transition [21]:

\[
\delta = \frac{\langle \psi_f | \mathcal{M}(E \text{ or } M, L + 1) | \psi_i \rangle}{\langle \psi_f | \mathcal{M}(M \text{ or } E, L) | \psi_i \rangle}.
\] (2.19)

As will be shown in Section 2.5.5, the dominant multipole(s) which contribute to a transition and the value of \( \delta \) may be measured from the angular distribution of photons emitted in the transition. Using this information, it is often possible to infer the spins and/or parities of states connected by the transition using the selection rules.

**2.3.3 Reduced transition probability**

Dropping the natural unit convention of Section 2.3.1, the transition rates of Equation 2.14 can be rewritten [8, 47]:

\[
\lambda(EL, I_i \rightarrow I_f) = \frac{8\pi(L + 1)e^2b_L}{L[2L + 1]!!} \left( \frac{E_\gamma}{\hbar c} \right)^{2L+1} B(EL, I_i \rightarrow I_f)
\] (2.20)

for electric transitions of multipolarity \( L \) and

\[
\lambda(ML, I_i \rightarrow I_f) = \frac{8\pi(L + 1)\mu_N^2b_L^{-1}}{L[2L + 1]!!} \left( \frac{E_\gamma}{\hbar c} \right)^{2L+1} B(ML, I_i \rightarrow I_f)
\] (2.21)

for magnetic transitions of multipolarity \( L \). \( E_\gamma \) is the energy of the gamma ray emitted in the transition, \( e \) is the elementary charge, \( \mu_N = e\hbar/2m_e c \) is the nuclear magneton, \( \hbar c = 197 \) MeV fm, and \( b = 10^{-24} \text{ cm}^2 \). \( B(EL, I_i \rightarrow I_f) \) and \( B(ML, I_i \rightarrow I_f) \) are quantities known as reduced transition probabilities. The reduced transition probability corresponds to the part of the electromagnetic transition rate which is independent of the transition energy \( E_\gamma \) and in particular depends on the structure of the wavefunctions \( \psi_i \) and \( \psi_f \) which are connected by the transition. A consequence of Equations 2.20 and 2.21 is that for a given energy \( E_\gamma \) the transition rate is expected to decrease with increasing multipolarity \( L \), since the factor \( E_\gamma/\hbar c << 1 \) for transitions of a few MeV or less.

By rearranging Equations 2.20 and 2.21, it is possible to calculate the reduced transition probability \( B(EL) \) or \( B(ML) \) of a transition if the transition rate \( \lambda \), transition energy \( E_\gamma \),
and multipolarity $EL$ or $ML$ of the transition are known (assuming negligible multipole mixing).

Reduced transition probabilities are often reported in Weisskopf units (W.u.), which are a means to determine the degree to which a transition involves a single nucleon. These units originate from a model by Weisskopf [48] in which transition probabilities were estimated assuming a transition involving a single proton in a nucleus represented by a spherical mean field. Details of Weisskopf’s model are given in Appendix B.3. Comparison of the observed transition rate $\lambda_{\text{obs}}$ to the value predicted by Weisskopf’s single particle model $\lambda_{\text{sp}}$ yields a reduced transition probability in Weisskopf units:

$$B(EL; ML; I_i \rightarrow I_f)(\text{W.u.}) = \frac{\lambda_{\text{obs}}}{\lambda_{\text{sp}}}.$$  \hspace{1cm} (2.22)

If $B(EL; ML; I_i \rightarrow I_f) \approx 1$ W.u., then the transition is said to be single – particle in nature (in agreement with Weisskopf’s single-particle model), for instance the excitation of a nucleon between shells in the shell model. If $B(EL; ML; I_i \rightarrow I_f) >> 1$ W.u., then the transition is collective, involving the excitation or de-excitation of multiple nucleons. Examples of collective excitation modes include rotations and vibrations of the nucleus.

### 2.4 Detection of reaction products

As discussed in Section 2.2.2, the fusion-evaporation reaction process is accompanied by prompt emission of nucleons and light ions (such as protons, neutrons, and alpha particles) followed by gamma rays. It is therefore necessary to be able to detect both the nucleons/light ions (in order to determine the reaction channel being studied) as well as gamma rays (in order to study the level structure of the species of interest).

#### 2.4.1 Charged particle interactions with matter

Charged particles such as protons and alpha particles travelling through an absorber will undergo Coulomb interactions with orbital electrons from atoms in the absorber. Interactions with nuclei are also possible, but are a small contribution compared to Coulomb interactions due to the short range of nuclear interactions [49]. These interactions result in deposition of the charged particle’s energy in the absorber medium.

For some experimental techniques (including the Doppler shift lifetime techniques used in this thesis) the rate of stopping of ions in matter is important. The energy loss over a path length $dx$ is known as the linear stopping power $S$ [49]:

$$S = -\frac{dE}{dx}.$$ \hspace{1cm} (2.23)
Figure 2.4: Schematic plot of specific energy loss \((S = -dE/dx)\) for a charged particle as a function of distance travelled in an absorber.

The Bethe formula shown below can be used to calculate stopping powers assuming the charged particle velocity \(v\) is large compared to the velocities of orbital electrons\(^{11}\) in the absorber\(^{49}\):

\[
S = \frac{4 \pi e^4 z^2 N_V Z}{m_0 v^2} \left[ \ln \frac{2m_0 v^2}{I} - \ln \left(1 - \frac{v^2}{c^2}\right) - \frac{v^2}{c^2} \right],
\]

where \(m_0 = 0.511 \text{MeV}/c^2\) is the rest mass of the electron, \(z\) and \(Z\) are the atomic numbers of the charged particle and absorber, \(N_V\) is the number density of the absorber, and \(I\) is the average excitation and/or ionization energy of the absorber. \(I\) must be determined experimentally for a given absorber. Equation 2.24 shows that the stopping power increases with the square of the charge of the primary particle, and in the non-relativistic case \((v \ll c)\) is proportional to \(1/v^2\) or the inverse of the energy of the primary particle. Thus as the particle loses energy in the absorber, the stopping power increases. Equation 2.24 does not take into account charge exchange which occurs at low \(v\). As a positively charged particle slows, it is able to pick up electrons from the absorber, reducing its charge and therefore the rate of energy loss in the absorber. As shown in Figure 2.4, the stopping power \(S = -dE/dx\) for a particle traversing an absorber reaches a maximum (the Bragg peak) near the end of the track, which is a result of the competing effects of Equation 2.24 and electron exchange with the absorber.

Charged particles may also undergo Rutherford scattering off of absorber nuclei, in which the Coulomb repulsion between the projectile and target nucleus cause the projectile to scatter at an angle which depends on the initial projectile trajectory. The differential cross

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\(^{11}\)From the classical Bohr model (see Appendix B.1), \(v \approx Z \alpha c/n\) for an electron in the \(n\)th energy level \((\alpha \approx 1/137)\). Typical recoil velocities in fusion-evaporation experiments are on the order of a few percent \(c\), so approximation of the Bethe formula is not always valid.
section for Rutherford scattering varies strongly with the angle $\theta$ at which the projectile is scattered [8]:

$$\frac{d\sigma}{d\Omega} = \frac{2Z_T Z_P e^2}{4m_P v_{P,cm}^2 \sin^4(\theta/2)},$$

(2.25)

where $Z_T$ and $Z_P$ are the atomic numbers of the target and projectile, $m_P$ is the mass of the projectile, $v_{P,cm}$ is the initial velocity of the projectile in the center of mass, and $e$ is the elementary charge. As Equation 2.25 shows, the cross section decreases sharply for large values of the scattering angle $\theta$ which significantly alter the trajectory of the projectile.

The result is that a very small portion of nuclear scattering events have a large impact on the stopping process, which for example can significantly change the observed Doppler shift of gamma rays emitted from the projectile (an important consideration for Doppler shift methods such as the DSAM, see Section 2.5.4). Since nuclear stopping results in a low number of events which have a large impact it is difficult to parametrize this effect in an equation, as opposed to the electronic stopping parametrized in Equation 2.24 which is the result of many events that each have a small impact on the projectile. In order to account for nuclear stopping in the data in this thesis, Monte Carlo methods were used to simulate the effect of nuclear stopping on an event-by-event basis for comparison with the experimental data, using the methods outlined in Section 3.2.

Since charged particles have finite kinetic energy, Equation 2.24 implies that they must also have finite range in an absorber. The range $R$ can be calculated:

$$R = \int_{E_0}^{E_0} \left( \frac{dE}{dx} \right)^{-1} dE,$$

(2.26)

with initial energy $E_0$ of the primary particle. As charged particles with higher $Z$ stop more quickly, it is possible to design an absorber with thickness such that lighter particles (eg. protons, alpha particles) are transmitted while heavier ions are stopped. This is useful in fusion-evaporation experiments where the aim is to prevent high intensity scattered beam from entering charged particle detectors but allow light reaction products through. In the $^{28}$Mg thin target data presented in Chapter 5 of this thesis, a ‘catcher foil’ with thickness in-between the stopping range of alpha particles and beam particles was used for this purpose.

### 2.4.2 Gamma-ray interactions with matter

Electromagnetic transition energy and rate measurements such as those performed in this thesis rely on detection of gamma rays emitted during the transition. Gamma rays entering a detector may interact via one of the mechanisms listed below, resulting in partial or full deposition of the gamma ray’s energy.
Photoelectric absorption

Photoelectric absorption is a process in which a gamma ray deposits its full energy in a single interaction with an orbital electron in the detection medium [50]. The electron is ejected from the atom with energy:

\[ E = E_\gamma - E_b, \]  

(2.27)

where \( E_\gamma \) is the energy of the incoming gamma ray and \( E_b \) is the binding energy of the ejected electron. For gamma rays emitted following internal transitions in nuclei with energy on the keV or MeV scale, \( E_\gamma \gg E_b \) and \( E \approx E_\gamma \). A schematic of this process is shown in Figure 2.5. The electron ejected as a result of photoelectric absorption deposits its energy fully within the detector medium, for example in a semiconductor by exciting neighbouring electrons into the conduction band. The cross section of photoelectric absorption increases with the atomic number \( Z \) of the absorber and decreases with gamma-ray energy \( E_\gamma \) [50]:

\[ \sigma \propto \frac{Z^4}{E_\gamma^3}. \]  

(2.28)

As illustrated in Figure 2.6, the photoelectric absorption process generates a single **photopeak** in the gamma-ray energy spectrum which roughly corresponds to the energy of the original gamma ray, according to Equation 2.27.

Compton scattering

Compton scattering and pair production are processes in which the gamma ray does not deposit its full energy in a single interaction with the detection medium.

Compton scattering occurs when the gamma ray deposits part of its energy in the detection medium via excitation of an electron, which results in the ejection of the electron and scattering of the gamma ray as indicated in Figure 2.5. The energy of the scattered gamma ray \( E_{\gamma,f} \) and the ejected electron \( E_e \) depend on the scattering angle \( \theta \) (see Appendix B.2):
\[ E_{\gamma,f} = \frac{E_{\gamma,i}}{1 + \left(\frac{E_{\gamma,i}}{m_e c^2}\right)(1 + \cos \theta)}, \quad (2.29) \]

\[ E_e = E_{\gamma,i} - E_{\gamma,f}. \quad (2.30) \]

The energy \( E_e \) recorded in the detector as a result of a single Compton scattering interaction varies significantly depending on the scattering angle \( \theta \). Compton scattering therefore produces a continuum of energies in the gamma-ray energy spectrum, as illustrated in Figure 2.6. For gamma rays at lower energies, the Compton continuum from higher energy gamma rays can contribute significant background, reducing the signal-to-noise ratio and possibly obscuring photopeaks with sufficiently low intensity.

**Pair production**

Pair production occurs when some of the energy of the incoming gamma ray is converted to mass according to \( E = mc^2 \) following interaction with an absorber nucleus [50]. If the gamma ray has energy 1.022 MeV or larger, an electron and positron pair may be produced as each has rest mass \( 1.022/2 = 0.511 \) MeV. To conserve momentum, the electron and positron are emitted at 180\(^\circ\) with respect to each other (back-to-back) in the center of mass frame. Following emission both electron and positron will lose energy via excitation and/or ionization of the surrounding medium. As the positron nears rest it will interact with an electron and annihilate, producing two 0.511 MeV gamma rays emitted back-to-back. One or both of these 0.511 MeV gamma rays may escape the detector, resulting in peaks in the gamma-ray spectrum at 0.511 MeV (single escape peak, where one of the 0.511 MeV gamma rays escapes detection) and 1.022 MeV (double escape peak, where both 0.511 MeV gamma rays escape detection) below the photopeak for the original gamma ray, as illustrated in Figure 2.6. A schematic of the pair production process is shown in Figure 2.5.

**2.4.3 Radiation detection systems**

**Semiconductor detectors**

Semiconductor materials such as silicon and germanium are often used in charged particle and/or gamma ray detectors. Compared to insulators, these materials are characterized by the small energy gap between the valence and conduction bands for electrons (\( \sim 1 \) eV for silicon, \( \sim 0.7 \) eV for germanium), such that free electron and hole pairs may be produced by depositing a relatively small amount of energy in the detection medium.

Semiconductor diode detectors measure the charge, or number of electron-hole pairs produced from deposition of energy by ionizing radiation. Since the average energy required to produce a single electron-hole pair or ‘information carrier’ is small (\( \sim 3 \) eV for silicon or germanium [49], larger than the band gap since not all interactions between the incident
particle\textsuperscript{12} and the detection medium are sufficient to create electron-hole pairs), the number of ‘information carriers’ produced per event is high and the statistical dispersion between signals corresponding to events of the same energy is very low. As a result, semiconductor detectors are able to provide very good energy resolution.

The energy required to excite electrons across the band gap is low enough to allow for some thermal excitation at room temperature. The probability per unit time to generate an electron-hole pair through thermal excitation is \[ p(T) = C T^{3/2} e^{-\frac{E_g}{2kT}}, \] (2.31)

where $E_g$ is the band gap energy, $T$ is the temperature, $C$ is a constant which depends on the specific material, and $kT \approx 0.025$ eV at 20°C. In particular, germanium detectors cannot be operated at room temperature due to the large thermally-induced current resulting from excitation of electrons across the small band gap\textsuperscript{13}. As such, germanium detectors are operated at cryogenic temperatures, usually achieved via active cooling using liquid nitrogen \[49].

It should be noted that the observed variance in the number of ‘information carriers’ (responsible for the energy resolution) produced from the interaction of ionizing radiation

\textsuperscript{12}For gamma rays interacting with a semiconductor detector, the incident particles are electrons produced from the interactions listed in Section 2.4.2.

\textsuperscript{13}At 20°C, the factor $e^{-E_g/2kT}$ of Equation 2.31 is $\sim 7 \times 10^{-13}$ for germanium ($E_g \approx 0.7$ eV) and $\sim 8 \times 10^{-20}$ for silicon ($E_g \approx 1.1$ eV). When the probability of thermal excitation is weighted by the number of electrons in the detector medium (on the order of the Avagadro constant), it is evident that thermal excitations are a significant consideration in the operation of germanium detectors.
with semiconductor detectors is usually lower than would be expected if the formation of electron-hole pairs followed a Poisson model, in which individual events are independent and occur with a fixed probability. The Fano factor $F$ is an empirical measure of the deviation of this variance $\sigma^2_{\text{obs}}$ from the expected variance using Poisson statistics $\sigma^2_{P}$ (which here is equal to the number of electron-hole pairs produced):

$$ F = \frac{\sigma^2_{\text{obs}}}{\sigma^2_{P}}. \quad (2.32) $$

Various measurements of the Fano factors for silicon and germanium have given values between 0.05 to 0.15, with a historical trend toward smaller values in more recent measurements (perhaps owing to refined procedures for estimating systematic uncertainties which affect the observed peak width) [49]. Further discussion of the specific properties and operating characteristics of semiconductor detectors may be found in Refs. [49] and [51].

**Scintillation detectors**

Scintillation detectors make use of a scintillator material to convert the kinetic energy of ionizing radiation into detectable light. Scintillator materials are broadly categorized as organic or inorganic. In general, organic scintillators have faster response times\(^{14}\), while inorganic scintillators have better light output and the most linear energy response (linearity of the amount of light produced with respect to energy of interacting radiation).

*Fluorescence* is the process responsible for prompt generation of detectable light in a scintillator. Photons are emitted from the scintillator as it de-excites following excitation by interacting radiation. For excited levels separated by a few eV (corresponding to the level spacing found in some organic molecules), de-excitation is accompanied by emission of visible light\(^{15}\). Inorganic materials with level separation which is too large to fluoresce in the visible range are often modified by adding an activator impurity, which serves to modify the level structure at local sites in the crystal lattice such that fluorescence is possible. For example, CsI(Tl) scintillation detectors, used in this thesis for charged particle detection, use cesium iodide activated with thallium [49].

Compared to semiconductor detectors, scintillators are often used in applications which require high detection efficiency but do not require high energy resolution. High detection efficiency is obtained by using materials with high $Z$, as suggested by Equation 2.26 for charged particles and Equation 2.28 for gamma rays. High-$Z$ scintillators include CsI(Tl) ($Z = 53$ for I, 55 for Cs) or bismuth germanate ($Z = 83$ for Bi). The lower energy resolution is a consequence of not all of the energy of the ionizing radiation being converted into scintillation light (since not all de-excitation of a scintillator material necessarily passes

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\(^{14}\)Decay constants of many plastic and liquid scintillators are on the order of a few ns, whereas most inorganic scintillators are an order of magnitude or more slower. There are exceptions, for instance BaF$_2$ is an inorganic scintillator where the scintillation light contains a fast component with 0.6 ns decay time [49].

\(^{15}\)Photon energies $E = hc/\lambda$ corresponding to visible light are in the range 1.65 - 3.10 eV.
through levels with the appropriate energy spacing), resulting in fewer ‘information carriers’ produced per unit of energy deposited in the detector when compared to semiconductor detectors (∼15 eV per photon for CsI(Tl) [49], although as discussed in Section 2.4.4 not all photons are collected by the photomultiplier or photodiode, and furthermore the relative resolution of semiconductors compared to scintillators is improved by the Fano factor).

### 2.4.4 Charged particle detection

In fusion-evaporation experiments, charged particle detection allows for discrimination of individual reaction channels, each of which are accompanied by a specific combination of evaporated particles such as protons, neutrons, and alpha particles. Time coincidence (discussed in Section 2.5.1) and particle identification (PID, discussed below) techniques can allow for increased sensitivity to the reaction channel of interest.

For most applications involving detection of charged particles, silicon diode detectors are considered to be the best available option. In particular, silicon diodes are known to provide very good energy and timing resolution compared to competing systems, with the latter being advantageous for the time coincidence method discussed in Section 2.5.1. Principles of operation of silicon diode detectors are discussed in Ref. [49]. However for the experiments presented in this thesis, CsI(Tl) scintillator detectors were used instead, primarily due to their superior charged particle identification (PID) capability from analysis of signal pulse-shapes as discussed below.

**CsI(Tl)**

Thallium activated cesium iodide (CsI(Tl)) is a scintillator which has a light yield similar to other inorganic scintillators such as NaI(Tl), however the peak wavelength of light emitted (540 nm) is poorly matched to the response of most photomultiplier tubes which peak at lower wavelengths [49, 52] or photodiodes which peak at higher wavelengths [52]. Therefore in practice, CsI(Tl) is often quoted as having low light yield. This low light yield results in poor energy resolution. Furthermore, the decay time for light emission is relatively slow compared to most inorganic scintillators, on the order of several µs, an order of magnitude slower than NaI(Tl) [49].

The disadvantages listed above are offset by the particle identification (PID) capabilities of CsI(Tl) based detectors. The response of the detector defined by the rise/decay time of the waveform depends on the type of interacting particle [49], with dependence on both the atomic number Z and mass number A of the interacting particle [53]. This property is understood to arise from the existence of two scintillation components with decay times of 0.68 µs and 3.34 µs at room temperature [54], with the amplitude of the faster component

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16 Energy resolution for a specific detector also depends on its light collection properties, which depends on the specific construction of the detector. Typical energy resolutions obtained for CsI(Tl) detectors used in this work are on the order of 10% FWHM/centroid, as discussed in Section 3.3.
Figure 2.7: Left: Typical waveform of a CsI(Tl) detector used in an experiment at TRIUMF, with a waveform fit consisting of multiple components with different mean lifetimes. Lifetimes are fixed, and the particle is identified through the ratio of slow and fast rise time components $A_S/A_F$ [6]. Right: Typical PID plot showing the amplitude ratio plotted against particle energy on an event-by-event basis for a fusion-evaporation experiment in which both protons and alpha particles are emitted. Regions corresponding to each particle type are labelled. For the waveform fits used in this thesis, $\tau_F = 0.64 \mu s$ and $\tau_S = 3.80 \mu s$.

depending on the interacting particle type [55]. CsI(Tl) detector waveforms may be fit using a sum of exponential terms as shown in Figure 2.7, with the relative amplitudes of the ‘slow’ and ‘fast’ components of the waveform fit providing information on the type of interacting particle.

A further advantage of inorganic scintillators such as CsI(Tl) compared to semiconductors such as Si is increased tolerance to high radiation exposure. The performance of Si detectors is reduced following exposure on the order of $10^9$ hits per cm$^2$ or more, due to radiation-induced damage to the crystal structure. This level of radiation exposure can occur rapidly if an accelerated beam such as those used in experiments at TRIUMF is scattered into the detector position. As a result, the use of Si detectors results in additional constraints in terms of detector positioning and shielding when compared to CsI(Tl) scintillators.

By coupling CsI(Tl) scintillators to small Si photodiodes such as the Hamamatsu S3590-08 used in the TIP CsI arrays discussed in Sections 3.1.4 and 3.3, it is possible to build very compact detectors which can be closely packed into large detector arrays, allowing for increased position sensitivity for detected particles compared to arrays employing bulky photomultiplier tubes.

### 2.4.5 Gamma-ray detection

High purity germanium (HPGe) detectors represent the present state-of-the-art for high resolution gamma-ray spectroscopy, required for the nuclear structure studies presented in this thesis. As discussed in Section 2.4.3, semiconductors such as HPGe are preferred...
for studies where high energy resolution is required, due to the efficient conversion of the energy of ionizing radiation into ‘information carriers’. As a detection medium, germanium has an advantage over other semiconductors such as silicon due its high density and atomic number, which results in higher detection efficiency since there are an increased number of electrons for gamma rays to interact with compared to other materials [17]. Principles of HPGe detectors are discussed in detail in Ref. [49]. TIGRESS, an array of HPGe detectors at the ISAC-II facility at TRIUMF, was used for the studies presented in this thesis and is discussed further in Section 3.1.2.

**Compton suppression**

As discussed in Section 2.4.2, a gamma ray may scatter in the detection volume and only partially deposit its energy. As illustrated in Figure 2.6, these events can correspond to a significant fraction of the overall number (only the photopeak corresponds to full energy deposition). These events are common since HPGe detectors have high energy resolution but poor detection efficiency, so scattered gamma rays often escape the detector without any further interaction. When performing spectroscopy on a source with more than one gamma ray energy, the low energy background from partial energy deposits of high energy gamma rays can obscure photopicks of lower energy gamma rays, reducing the signal-to-noise in the resulting spectrum.

To counter this effect, HPGe detectors are often surrounded by Compton suppression shields - auxiliary detectors designed with poorer energy resolution but high efficiency, typically scintillators using bismuth germanate (BGO). If gamma rays which scatter out of the main HPGe volume are detected by the suppression shields, the event may be identified as a partial energy deposit and discarded. This process reduces the background contribution from Compton scattering, allowing for improved sensitivity of the detection system.

In order to obtain data which is Compton suppressed as discussed above, it is necessary to determine whether detection events occur in both an HPGe detector and surrounding BGO shields within a short time window. This implies that both detector timing and an algorithm for determining time-coincident events (as discussed in Section 2.5.1) must be implemented.

**Add-back**

Like Compton suppression, add-back is a method used to improve signal-to-noise for HPGe detection systems, and often the two methods are used together. State-of-the-art HPGe arrays such as TIGRESS (discussed in Section 3.1.2) often contain many HPGe crystals packed in close proximity. When a gamma ray interacts with one crystal and then scatters into and interacts with an adjacent crystal, it is possible to misinterpret the event as...
the detection of two gamma rays, each with energy equal to the energy deposited in the corresponding crystal.

Add-back algorithms attempt to intelligently reconstruct events using both the proximity between detectors and the time between detector hits to determine whether an event containing energy deposits in two or more detection volumes corresponds to detection of a single gamma ray or multiple gamma rays. When multiple energy deposits are determined to correspond to a single gamma ray, the energies are summed. The resulting effect on the gamma ray spectrum is a reduction in the intensity of the Compton scattering background and an increase in the intensity of the photopeak at high energy. This effect can be quantified using an ‘add-back factor’ $F_{AB}$:

$$F_{AB} = \frac{N_{AB}}{N}. \quad (2.33)$$

where $N_{AB}$ and $N$ are the number of counts in the photopeak of interest with and without add-back, respectively. Typical values of $F_{AB}$ for individual HPGe clover detectors are between 1 and 2 with larger values obtained for high energy gamma rays [56].

### 2.5 Experimental techniques and considerations

#### 2.5.1 Time coincidence method

Typically, nuclear structure studies involving detection of gamma-rays and/or charged particles require the ability to distinguish reaction and/or decay events of interest from events induced by contaminants or background radiation. One method for achieving this is the time coincidence method. If the reaction being studied is known to produce two or more signals, the time difference between these signals can be used to determine whether or not they originate from the same event. For example, most fusion-evaporation reactions produce multiple charged particles followed by multiple gamma rays over timescales of nanoseconds or shorter. These reactions can be reliably separated from background events by only considering events that contain multiple particles and gamma-rays detected over a short time period. Time coincidence logic can be performed by the data acquisition hardware at the time of acquisition (online), or in software to filter existing data (offline).

When using in-beam reactions, the beam may be pulsed over a set time interval rather than delivered at a constant rate, with pulsed beam delivery often mandated due to the properties of the accelerator system. Beam pulses can be used to assist in the determination of time coincident events. When plotting the detection times of events from different detection systems a structure corresponding to events originating from different beam pulses is observed. Timing separation can then be performed by only retaining events corresponding to the diagonal region of the plot in which both detections originate from the same beam pulse, as shown in Figure 2.8.
In this thesis, the time coincidence method was used in the analysis of data used to commission the TIP CsI ball array (Section 3.3), as well as the fusion-evaporation studies of $^{22}$Ne and $^{28}$Mg (Chapters 4 and 5). Figure 2.9 shows a schematic of the logic used to identify coincidences between detected gamma rays (from TIGRESS data) and charged particles (from CsI scintillator array data). The specific timing windows are chosen based on the detector systems used, with the timing windows for different detection systems offset in time to compensate for differences in signal processing time between the systems. Timing window length and offset values used in this thesis are listed in the corresponding sections in Chapters 3 through 5.

2.5.2 Digital data acquisition systems

Data taken for the studies in this thesis used the digital data acquisition system described in Ref. [57]. Digitization and storage of detector waveforms allows for the particle identification technique of Section 2.4.4 to be used to determine the type of charged particle interacting with a CsI(Tl) scintillator on an event-by-event basis. Other offline analysis techniques benefit from storage of detector waveforms, including waveform fitting for the purposes of time correlation and removal of pile-up signals in which the waveform from one detection event overlaps with the waveform from another (ie. near-simultaneous detections within the same detector).

Additionally, digital data acquisition systems may be designed to support higher numbers of data channels than are practically feasible with analog systems. For instance, the TIGRESS data acquisition system used at TRIUMF is capable of simultaneously supporting hundreds of detector signals and performing time correlation logic between these signals in real time. Such a capability is necessary for online separation of events of interest from background events during in-beam reaction studies, as was performed in the $^{22}$Ne and $^{28}$Mg experiments discussed in Chapters 4 and 5.
Figure 2.9: Generic algorithm for identification of time coincidences between event fragments in two different detection systems (represented by circles and squares). Event fragments from the first detection system are retained if they fall within a time window with respect to the first event fragment from the same detection system. Event fragments from the second detection system are retained only if they fall within a time window which is offset with respect to the first time window.

The primary disadvantage of a digital system is related to time resolution. Typical clock speeds for these systems are in the MHz to GHz range (with the data acquisition system used in this thesis clocked at 100 MHz), resulting in timing resolutions on the µs to ns scale. This time resolution may be insufficient to distinguish time correlated detector hits from a single event from random time coincidences of separate events, especially in experiments where the event rate is high.

2.5.3 Identification of excited nuclear states

In gamma-ray spectroscopy studies such as those in this thesis, the presence of gamma rays implies the population of excited nuclear states which de-excite via gamma ray emission. As higher energy excited states often decay to lower lying excited states (rather than decaying directly to the ground state of the nucleus), determining the excitation energy of observed states requires knowledge of the decay scheme. By examining time coincidences between gamma rays using the method of Section 2.5.1, it is possible to build a decay scheme under the assumption that gamma rays observed simultaneously all belong to a cascade depopulating a specific excited state. As discussed in Section 2.5.2, the timing resolution of digital data acquisition systems is coarse to the extent that an entire cascade will often be detected as part of a single event. The energy of the excited state is then the sum of the
energies of gamma rays observed in time coincidence within a cascade, with corrections for the recoil of the nucleus following gamma ray emission.

2.5.4 DSAM

The Doppler Shift Attenuation Method (DSAM) is a technique used to measure the lifetimes of excited nuclear states in the range of a few femtoseconds to a few picoseconds.

In an experiment implementing DSAM, the reaction producing the nucleus of interest is such that the excited nucleus recoils away from the reaction target at high speed. The nucleus then enters a backing attached to the target, where it slows down and stops over the course of a few picoseconds. As the nucleus slows, it de-excites and emits gamma ray(s). The observed energy $E_{\gamma}$ of the gamma ray(s) emitted are Doppler shifted from the true energy $E_0$ by a factor $D$ which depends on the speed of the source nucleus $\beta = v/c$ at the time of gamma-ray emission and its angle $\theta$ with respect to the detector:

$$E_{\gamma} = E_0 D$$

$$D = \sqrt{1 - \beta^2} \approx 1 + \beta \cos \theta$$

An excited state which decays with a mean lifetime comparable to the stopping time in the target backing will emit gamma rays with a distribution of Doppler shifts. Then for this transition, the gamma-ray detector(s) will not record a photopeak corresponding to a single discrete energy but rather a lineshape corresponding to distribution of Doppler shifted energies. The specific lineshape observed depends on the mean lifetime of the transition, as well as the choice of target backing. For instance, good sensitivity to lifetimes below 0.5 ps may be obtained using a gold backing due to its high density and atomic number. Target backings with lower density and/or atomic number may provide increased sensitivity to lifetimes on the order of 1 ps. Secondary effects on the stopping may result from the physical form of the target backing (eg. amorphous or crystalline). A schematic is shown in Figure 2.10.

The lifetime of the transition being studied can be roughly determined based on the centroid of the lineshape, or to high precision using Monte-Carlo methods to simulate the expected lineshape for different lifetimes. Monte-Carlo simulation code was developed specifically for the work in this thesis, and is discussed in section 3.2.

2.5.5 Spin-parity assignment

It is possible to identify the spin-parity of a level of interest from the multipolarity of a transition between that level and another level of known spin-parity, by applying the spin
and parity selection rules of Equations 2.17 and 2.18. There are various methods available for determining the multipolarity of an observed transition:

**From the angular distribution of gamma rays**

The probability of photon being emitted from a source at a given angle with respect to the angular momentum vector $I$ of the radiating system varies with the angle [58]. Many sources of radiation (eg. stationary sources used in beta decay studies) are unoriented, with the $I$ vector of each nucleus oriented at random, resulting in isotropic emission of photons. For in-beam reactions such as those done in this work, orientation of the reaction products is achieved since the beam itself is oriented, resulting in an anisotropic distribution of photons which can be written in terms of the multipolarity $L$ of the corresponding electromagnetic transition [58]:

$$W(\theta) = \frac{d\Omega}{4\pi} \sum_{\lambda=\text{even}}^{2L} B_\lambda(I_i) A_\lambda(\delta) P_\lambda(\cos \theta). \quad (2.35)$$

where $\theta$ is the angle with respect to the beam axis and $P_\lambda$ are the Legendre polynomials which specify the spatial distribution. These are weighted by orientation parameters $B_\lambda(I_i)$ which depend on the orientation of the source angular momenta with respect to the quantization axis defined by the beam, and distribution coefficients $A_\lambda(\delta)$. The orientation parameters are defined:

$$B_\lambda(I_i) = \sqrt{2I_i + 1} \rho_0^\lambda(I_i). \quad (2.36)$$
\[ \rho_0^\lambda(I) = \frac{\sum_{m=-I}^{+I} (-1)^{I+m} \langle I - mIm|\lambda0|I - mIm|\lambda0 \rangle e^{-\frac{m^2}{2\sigma^2}}}{\sum_{m=-I}^{+I} e^{-\frac{m^2}{2\sigma^2}}}. \]  

(2.37)

where \( e^{-\frac{m^2}{2\sigma^2}} \) defines a Gaussian distribution of magnetic substates \( m \) with width \( \sigma \), where \( m = 0 \) represents alignment orthogonal to the quantization axis. Increasing de-orientation of sources(s) with respect to the quantization axis results in larger values of \( \sigma \).

The distribution coefficients are defined:

\[ A_\lambda(\delta) = \frac{F_\lambda(LLf_II_i) + 2\delta F_\lambda(LL + 1f_II_i) + \delta^2 F_\lambda(L + 1L + 1f_II_i)}{1 + \delta^2}. \]

(2.38)

where \( L \) is the multipolarity of the transition (eg. 2 for a transition of E2 character) and \( F_\lambda \) are coefficients defined in Ref. [58] which only depend on \( L \) and the initial and final spin.

The quantity \( \delta \) represents the amplitude mixing ratio between the \( L \) and \( L + 1 \) multipoles, as defined in Section 2.3.2.

By fitting Equation 2.35 to experimental data, it is often possible to infer the spin \( I_i \) or \( I_f \) of one of the states connected by an observed electromagnetic transition, and/or the mixing ratio \( \delta \) for the emitted gamma ray(s).

Equation 2.35 is often parametrized in terms of the amplitudes of the individual terms. Truncating the series to include terms only up to \( L = 2 \), the following expression may be obtained:

\[ W(\theta) = a_0 + a_2 P_2(\cos \theta) + a_4 P_4(\cos \theta). \]  

(2.39)

where the weighting coefficients \( a_0, a_2 \) and \( a_4 \) correspond to the \( \lambda = 0, 2, 4 \) terms of Equation 2.35. As noted in Ref. [59], qualitative arguments for the multipolarity of a transition \( (L \leq 2) \) may be made based on the signs of the \( a_2 \) and \( a_4 \) parameters. This approach is useful when the statistics obtained in an experiment are insufficient to firmly assign spin-parity to the state of interest. For pure E2 transitions\(^{17}\) between high spin states \( a_2 \) is positive and \( a_4 \) is negative, while for mixed M1/E2 transitions various values are possible for \( a_2 \) and \( a_4 \) depending on the initial and final spins. Because of this ambiguity, the signs of \( a_2 \) and \( a_4 \) cannot be used to firmly assign spin-parity values. However as shown for the \(^{28}\)Mg data for this thesis in Section 5.1, they can be used to argue for or against the E2 nature of a given transition.

\(^{17}\)For example, rotational bands containing pure E2 transitions with no mixing are commonly observed in nuclei, with the spin-parity sequence \( 0^+, 2^+, 4^+, 6^+, \ldots \) (where higher spin corresponds to higher excitation energy).
From the decay scheme

As indicated in Section 2.3.3 and the transition rate estimates of Appendix B.3, the lifetime of a transition depends strongly on its multipolarity, with higher order transitions being hindered significantly compared to lower order transitions. Using this principle, it is possible to use lifetime measurements along with the observed decay scheme to infer the multipolarity of a transition and therefore spin-parity information for the state(s) connected by the transition, especially if a state is unusually short or long lived compared to otherwise similar state(s).

As an example, if transitions between a level of interest and two levels with known spin-parity $I_f^\pi = 2^+, 4^+$ are observed, and the mean lifetime of the level of interest is less than $\sim 1$ ps, then the spin of the level of interest is most likely between $I_i = 2$ and $I_i = 4$. The main assumption used to arrive at the conclusion is that the observed transitions must have a multipolarity of E2 or lower due to the short lifetime of the transition\(^{18}\). Then based on this assumption the change in spin between the initial and final state must be 2 or less for each transition, restricting the spin of the initial state to $I_i = 2, 3, 4$. For E2 transitions no parity change is allowed based on the selection rules of Equation 2.18, restricting the spin-parity of the initial state to $I_i^\pi = 2^+, 3^{\pm}, 4^+$.

\(^{18}\)According to the Weisskopf estimates shown in Appendix B.3, higher order transitions should result in lifetimes $> 1$ ps for transitions of a few MeV or less.
Chapter 3

Development of Experimental Infrastructure

3.1 Pre-existing infrastructure

The TIP experimental program has been active for some time prior to this work, with initial commissioning studies published in 2014 [6]. The following sections discuss the experimental infrastructure and techniques established prior to this work.

3.1.1 Beam delivery at TRIUMF

The experiments in this thesis utilize accelerated beams of $^4$He (for detector commissioning, see Section 4.2), $^{18}$O (for the fusion-evaporation experiments of Chapters 4 and 5), and $^{36}$Ar (for further detector commissioning). As these beams are stable\(^1\) and low in mass, they have been provided by the TRIUMF off-line ion source (OLIS) [60].

As described in Ref. [60], the OLIS terminal contains microwave, surface, and multi-charge ion sources, any of which can be chosen based on their suitability for a given experiment. The microwave ion source (MWIS) is the longest operating of the OLIS sources (since 1995) and is considered to be very reliable, capable of operating for months without maintenance. Microwave power at 2.45 GHz is used to ionize a magnetically confined plasma. MWIS is used to provide mainly $+1$ and some $+2$ ion beams from stable isotopes in gaseous, liquid, or solid form. The surface ion source (SIS) is used to extract group I and II elements with low ionization potentials such as Na. This is achieved by extracting species which have been ionized on the surface of a tantalum or rhenium/tungsten tube which is externally heated by an oven to a desired temperature in the range 25-2000°C. The multi-charge ion

\(^1\)Beams of radioactive species are also produced at TRIUMF using the isotope separation on-line (ISOL) method in which protons produced by the 500 MeV TRIUMF cyclotron are reacted with target materials to produce a variety of species from which the ions of interest are separated and delivered to the experimental station [4].
source (MCIS) integrates a commercially available electron cyclotron resonance ion source [61] with the existing OLIS terminal, which is used to obtain ions with high charge states by multiple ionization. High charge states are required when producing species with high $A$ in order to meet the mass-to-charge acceptance criteria ($3 < A/Q < 6$) of the Drift Tube Linac secondary accelerator serving the intermediate and high energy experimental stations at ISAC-I and ISAC-II. The OLIS has been used in past experiments to deliver various stable ion beams ranging from $^1\text{H}$ up to intermediate mass species of $A \approx 80$ [62].

The specific operating conditions and parameters of the OLIS and beam delivery systems used in the experiments in this thesis were left to the discretion of the ISAC Operations Group at TRIUMF.

Due to the properties of the Drift Tube Linac, beams delivered from the OLIS to the high energy experiments of ISAC-II arrive in pulses separated by $\sim 85$ ns. Beam pulses are used to assist with time correlation of events in experimental data as discussed in Section 2.5.1.

3.1.2 TIGRESS

The TRIUMF-ISAC Gamma-Ray Escape Suppressed Spectrometer (TIGRESS) [5] is an array of up to 16 HPGe clover$^2$ detectors at TRIUMF/ISAC-II. As detailed in Ref. [49], HPGe detectors represent the state-of-the-art for high resolution gamma-ray spectroscopy.

The 16 detector positions of TIGRESS are fixed at specific positions on a mounting frame. A maximum of 4 detectors may be mounted at $\theta = 45^\circ$ with respect to the beam axis, up to 8 detectors may be mounted at $\theta = 90^\circ$, and 4 detectors may be mounted at $\theta = 135^\circ$. The clover geometry of the detectors places individual crystals at approximately $\theta \pm 10^\circ$ with respect to the detector angle(s), effectively providing 6 ‘rings’ of detectors which can be used to study the angular distribution of gamma rays emitted from the center of the array.

The HPGe clover detectors of the array are equipped with Compton suppression shields of bismuth germanium oxide (BGO), a scintillator which provides high detection efficiency but low energy resolution for gamma rays. The purpose of the suppression shields is to veto events which result in only a partial energy deposit in the HPGe cores. If a gamma-ray scatters out of an HPGe core into a BGO shield, the TIGRESS data acquisition system [57] is able to recognize the event as a partial energy deposit in the core, and the event may be discarded, reducing the background contribution in the recorded spectrum.

The nearly $4\pi$ coverage of TIGRESS allows for high overall gamma-ray detection efficiency, which is on the order of 10% depending on the positioning of the HPGe cores and

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$^2$‘Clover’ describes the close-packed configuration of HPGe crystals which share a single cryostat. Each of the 16 detectors in TIGRESS contains 4 HPGe crystals as described in Ref. [5]. The large number of closely packed crystals increases the overall detection efficiency of the array, as well as the position/angle granularity achievable for individual detection events.
suppression shields. Additionally, angular distributions of gamma rays for spin-parity measurements of excited nuclear states may be determined based on the relative counting rates of detectors at different positions.

3.1.3 TIP target device

As the name implies, the TIGRESS Integrated Plunger (TIP) program relies on experimental devices designed to be operated alongside TIGRESS. For DSAM measurements, a target device (shown in Figure 3.1) is used which provides two stationary target positions in addition to beam tuning apertures. The targets are mounted on a ‘target wheel’ - a semi-circular mounting bracket attached to a rod which can be rotated from outside of the vacuum chamber, allowing an operator to switch between targets without breaking vacuum. The tuning apertures on the target wheel may be used prior to an experimental run to assist the beam operator with focusing the beam. Typically, TIP experiments (including those in this thesis) are run once the tune achieves 80% or more beam transmission through the 2 mm aperture on the target wheel. This practice ensures that a significant fraction of the delivered beam arrives on target and with minimal spatial dispersion.

3.1.4 CsI wall

In most TIP experiments, the reaction of interest represents a small fraction of all reactions which occur. For example, in an experiment populating a reaction channel of interest via fusion-evaporation, there may be side reactions involving other mechanisms (for example,
elastic scattering, Coulomb excitation of the beam or target) which contaminate the resulting gamma-ray spectrum. Detection of charged particles in time coincidence with reaction events (using the methods described in Section 2.5.1) provides additional information which allows the experimenter to select data corresponding to specific reaction(s) of interest. For example, if the experimenter wants to study $^{22}$Ne produced via the $^{12}$C($^{18}$O,$2\alpha$)$^{22}$Ne fusion-evaporation reaction, gamma-rays corresponding to $^{22}$Ne will be found in coincidence with two alpha particles, since these are the charged particles evaporated from the compound nucleus during the reaction, whereas gamma rays corresponding to other nuclei will be in coincidence with other charged particles. It is therefore important to have charged particle detectors in addition to gamma-ray detectors when running such an experiment.

A 24-element wall of CsI(Tl) scintillator detectors, shown in Figure 3.2, is available for charged particle detection and identification using the methods discussed in Section 2.4.4 [6]. The array may be mounted downstream from the target position, where it covers forward angles of $16^\circ$ to $42^\circ$ in the lab frame. The consequence of this small angular coverage is low detection efficiency, as a significant number of charged particles may be emitted in the backward direction, depending on the reaction used. The reduction in event rate due to low detection efficiency becomes more significant in experiments which require coincident detection of multiple charged particles.

The majority of TIP experiments prior to late 2016 were run using the wall array [63, 64], including the $^{22}$Ne data presented in this thesis. In order to improve the charged particle detection efficiency in TIP experiments to allow for studies of weaker reaction channels far
from stability, a ‘CsI ball’ array with near-complete spherical coverage has been built and commissioned as part of this thesis. This array is discussed further in Section 3.3.

3.2 GEANT4 simulations

It is often the case that quantities of interest cannot be directly observed from experimental data, but are instead inferred based on knowledge of the experimental setup and physical processes at work in the experiment. For instance, when measuring lifetimes of excited nuclear states using DSAM as described in Section 2.5.4, the observation (the measured lineshape of the gamma ray emitted when depopulating the state) depends on the reaction process that populates the state as well as the specific experimental setup: the positions of detectors, composition and thickness of reaction target. In order to encapsulate these effects in the data analysis, detailed simulations have been developed using GEANT4 [65, 66], a C++ framework for developing applications which simulate physical interactions using Monte-Carlo methods.

Initially, the simulation code was developed for the $^{22}$Ne data analysis discussed in Section 4.1, using version 9.4 of the GEANT4 framework. As part of this work, the TIGRESS array, CsI(Tl) detector wall, and TIP target geometry were implemented in the simulations as shown in Figure 3.3. As $^{22}$Ne was populated via the $^{12}$C($^{18}$O,$2\alpha$)$^{22}$Ne fusion-evaporation reaction in the experimental data, a fusion-evaporation mechanism was also defined as a custom physics process in GEANT4 and implemented in the simulation code. This mechanism is described in Section 3.2.1.

Further refinements to the simulation code were developed for the $^{28}$Mg data analysis discussed in Chapter 5, including implementation of the CsI ball array in the simulations and porting of the code to version 10.3 of the GEANT4 framework.

Results of the simulations, including track information and energy deposits in the HPGe and CsI(Tl) detectors were stored in a tree structure of the ROOT framework [67], allowing the data to be further processed to generate charged-particle and gamma-ray energy spectra using various sorting schemes without the need for re-simulation. Additional data analysis and processing code was developed for these tasks and is discussed in Appendix C.

3.2.1 Fusion-evaporation mechanism

The following steps describe the fusion-evaporation mechanism as implemented:

- Once the primary beam particle fired by the GEANT4 particle gun reaches a randomized linear depth in the target, a compound system is formed from the fusion of beam and target nuclei. The compound excitation energy $E_{\text{ex}}$ is calculated using Equation 2.3.
Figure 3.3: TIP and TIGRESS geometry implemented in GEANT4. The square CsI(Tl) detector wall is visible at the center.

- Individual particles are evaporated from the compound system, isotropically in the center of mass. The portion of the compound excitation energy $\delta E_{\text{ex}}$ going into each evaporated particle is chosen using a statistical distribution (discussed in Section 3.2.2), with the restriction that the total kinetic energy of the evaporated particle $E_{\text{part}} = \delta E_{\text{ex}} + Q_{\text{evap}}$ cannot be negative. Recoil of the residual nucleus due to conservation of momentum during the evaporation process is also calculated during this step.

- The leftover residual nucleus is propagated, and emits either a single gamma ray or a cascade of gamma rays with user-defined energy and lifetime. Gamma-ray emission is restricted to residual nuclei with remaining excitation energy $E_{\text{ex}}$ sufficient to emit the cascade as defined. Gamma rays are emitted isotropically in the center of mass.

- When a simulated gamma ray is emitted from a moving residual nucleus, GEANT4 determines the Doppler shift factor $D$ and the Doppler-shifted energy $E_{\gamma}$ as defined in Equation 2.34.

When using the example of $^{12}\text{C}(^{18}\text{O},2\alpha)^{22}\text{Ne}$ fusion-evaporation studied in Section 4.1 with $E_\delta = 48$ MeV, the compound energy $E_\text{c}$ is 28.80 MeV and $Q_{\text{fusion}} = 23.65$ MeV, resulting in a total excitation energy $E_{\text{ex}} = 42.85$ MeV for the $^{30}\text{Si}$ compound system. For
subsequent alpha particle evaporation the $Q$ values are negative, $Q_{\text{evap}} = -10.643$ MeV for the first alpha particle, and $Q_{\text{evap}} = -10.615$ MeV for the second alpha particle, resulting in less excitation energy being available to the residual $^{22}{\text{Ne}}$ nucleus than the initial 42.85 MeV available to the compound system.

### 3.2.2 Particle evaporation

The distribution governing the portion of the compound nucleus excitation energy $\delta E_{\text{ex}}$ used in the evaporation and kinetic energy of emitted charged particles was modelled as a Gaussian with a high energy exponential tail:

$$P(x = \delta E_{\text{ex,\alpha}}) = \frac{1}{2} \left( \frac{1}{\sigma \sqrt{2\pi}} e^{\left(\frac{x-\mu}{2\sigma^2}\right)^2} + \lambda e^{-\lambda x} \right)$$

This form was chosen as it was able to closely reproduce the charged particle energy distributions observed during experiment (see Section 4.1.2). The three free parameters $\mu$, $\sigma$, and $\lambda$ of Equation 3.1 were determined on a per-experiment basis by varying them on a grid and comparing the simulated data at each grid point to the experimental data, using $\chi^2$ minimization to simultaneously find the best fit value for all three parameters. Specific parameter fits obtained for the $^{22}{\text{Ne}}$ and $^{28}{\text{Mg}}$ data are discussed in Chapters 4 and 5. More information on the grid fitting method is given in Appendix A.1.

### 3.2.3 Lifetime determination from gamma-ray lineshapes

When studying a particular nucleus, lifetimes of excited states were determined by simulating gamma-ray lineshapes for each transition observed, varying the lifetime of the state of interest in the simulations. Each simulated lineshape was then compared with the experimental data using a $\chi^2$ goodness of fit test, with the value of $\chi^2$ minimized between the experimental data and the simulated data by applying a scaling factor and quadratic background term.

After determining the corresponding value of $\chi^2$ for each simulated lifetime, the best fit lifetime corresponding to minimum $\chi^2$ and its associated uncertainty were determined using the method of Appendix A.1.

An example of the use of this method is shown for the $^{22}{\text{Ne}}$ data in Section 4.1. Figure 4.9 shows the best fit lineshape for a simulated transition in $^{22}{\text{Ne}}$ compared to the experimental data.

### 3.2.4 Lifetime determination from overlapping lineshapes

In regions of experimental data where lineshapes corresponding to two or more transitions overlap in energy, each transition was simulated separately over a range of lifetime values, and the value of $\chi^2$ was minimized between the sum of the simulated data for each transition
3.3 CsI ball

3.3.1 Motivation and design

Until 2016, studies using TIP at TRIUMF (including the work done on $^{22}$Ne in this thesis, see Section 4.1) have used the 24-element wall of CsI(Tl) scintillator detectors discussed in Section 3.1.4. The limited angular coverage and resulting low detection efficiency of the wall prompted development of a spherical ‘ball’ array to replace it. The CsI ball design, developed in a collaboration between SFU and the TRIUMF detector laboratory, consists of 128 detectors arranged in 10 rings with nearly $4\pi$ coverage with respect to the source position. In comparison to the $16^\circ$ to $42^\circ$ lab angle coverage of the CsI wall, the new CsI ball coverage is $3.2^\circ$ to $157.65^\circ$. Detector segmentation (in terms of solid angle per detector $\Omega_{det}$) is increased at forward angles in order to reduce the count rate per detector and avoid pileup of detector signal, which is necessary for forward detectors since products are boosted towards forward angles in the lab frame based on the center of mass motion of the beam-target system. Design dimensions of the CsI ball array are listed in Table 3.1. Drawings of the design are shown in Figure 3.4.
Table 3.1: Design dimensions and angular coverage of the CsI ball array. Each ring contains \( N_{det} \) detectors with a CsI(Tl) crystal of thickness \( \Delta t \) at a radius \( r \) defined as the distance from the center of the array to the CsI(Tl) - light guide interface [6].

<table>
<thead>
<tr>
<th>Ring</th>
<th>( N_{det} )</th>
<th>CsI(Tl) ( \Delta t ) (mm)</th>
<th>( r ) (mm)</th>
<th>( \theta_{ring} ) (deg)</th>
<th>( \Omega_{det} ) (msr)</th>
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</thead>
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<td>36.6</td>
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</tr>
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</tr>
<tr>
<td>9</td>
<td>8</td>
<td>1.2</td>
<td>57.2</td>
<td>148.5</td>
<td>130.5</td>
</tr>
</tbody>
</table>

As part of the work in this thesis, the CsI ball array was constructed at SFU between the summers of 2016 and 2017 with the assistance of the SFU science machine and electronics shops, based on the pre-existing SFU/TRIUMF design outlined above. During this time, several commissioning experiments were run using the ball detectors at TRIUMF, one of which became the basis for the \(^{28}\text{Mg}\) data shown in Chapter 5.

### 3.3.2 Detector construction

CsI ball detectors were constructed at Simon Fraser University in a process adapted from Ref. [6]. Each detector in the array used CsI(Tl) scintillator material (purchased from Hilger Crystals) with a Thallium doping concentration of \( \sim \)1000 ppm as in Ref. [6]. The thickness of CsI(Tl) material used varied from 1.2 to 5 mm depending on the detector ring as listed in Table 3.1, with downstream ring detectors using thicker material to allow for stopping of charged particles boosted to high energy by the centre of mass motion of the beam-target system. Each CsI(Tl) crystal was coupled to a silicon PIN diode (Hamamatsu S3590-08) via an acrylic light guide. The details of the construction procedure are listed below:

- For each detector, a light guide was machined from UV plastic material (Saint-Gobain BC-800). The purpose of the light guide is to funnel the light produced in the CsI scintillation process to a photodiode which produces the detector signal. The shape of the light guide is related to the specific geometry of the detector, and depends on the relative size of the CsI(Tl) crystal used with respect to the photodiode. Figure 3.5 shows a typical light guide as well as photodiodes used for the CsI ball.

- Light guides were coupled to the CsI(Tl) crystals and photodiode using optical-grade epoxy (Saint-Gobain BC-600) at room temperature. Special care was taken to ensure
the epoxy was free of bubbles, typically by placing mixed epoxy under vacuum for 1 hour prior to application.

- The coupled CsI(Tl) crystal assemblies were machined to the correct thickness and geometry, matching the light guide geometry. When fly cutting the surface of the CsI(Tl) crystals, special care had to be taken as it was found that the choice of cutting direction impacted the brightness/dullness of the surface finish due to the orientation of the cutter with respect to the crystal structure. Additional care was taken when machining the crystal shapes as the soft CsI(Tl) material is significantly more prone to cracking and/or fracturing than the light guide plastic.

- Exterior detector faces were painted using reflective paint (Saint-Gobain BC-620), in order to increase light collection via the light guide. The paint causes light produced in the scintillation process to be reflected from the sides of the detector rather than escape it, increasing the light collected by the photodiode and therefore the signal-to-noise ratio of the detector.

- Detectors were wrapped in 25 µm thick aluminized Mylar. The conductive outer surface of the Mylar was connected to the ground pin of the photodiode in order to prevent charge buildup on the surface of the detector. Following mylar wrapping, the sides of the detector are wrapped in Teflon tape to prevent tearing of the mylar and to further increase light collection.

Figure 3.6 shows an example of the detectors which result from the above construction process. The detectors may be further wrapped in absorber material for experiments where it is necessary to prevent scattered beam from entering the detectors - for instance, fusion-evaporation with heavy ion beams where the channel of interest is identified by detection of protons or alpha particles.
Electronics

The major challenges for signal readout from a CsI ball array are the high number of channels and poor signal-to-noise properties of CsI(Tl) scintillators. To address these issues, purpose-built preamplifier boards were designed and developed at the SFU Science Technical Centre electronics shop. High density Nuclear Instrumentation Module standard (NIM) modules were produced with 16 preamplifier channels per module, allowing the full CsI ball readout to be achieved using 8 modules. The design of the 16-fold preamplifier boards is shown in Figure 3.7 alongside a finished module. CR-110 charge sensitive preamplifier modules by Cremat [68] were used. Following preamplification, a bandpass filter with bandwidth 1 kHz - 12 MHz was used for noise reduction, and the resulting signal was subsequently amplified and output from the module.

Three copies of the amplified output signal are available from the rear panel of each preamplifier module, in order to facilitate additional signal processing. The decay time constant of the preamplifiers was set to 50 µs in order to obtain a count rate similar to that of the TIGRESS array, which sets an upper limit on the rate of TIGRESS-CsI coincident events obtainable in an experiment. With a longer time constant the CsI ball count rate is lowered, possibly resulting in missed coincidences with TIGRESS. A shorter time constant
would result in decreased pileup, however this can also result in distortion of the pulse shape which may adversely affect particle identification (PID) and energy evaluation (based on the amplitude of the pulse).

Coaxial cables were fabricated to carry signals from the detectors to a feedthrough in the TIP chamber, and from the external side of the feedthrough to the preamp modules. Internal cables were vacuum rated. Cables and feedthroughs were designed such that individual detector signals did not share a common ground, as this was identified as a significant source of noise during the design process.

Detector array assembly

Energy resolution and light collection characteristics of completed detectors were tested at SFU using an $^{241}$Am source as detailed in Section 4.2.1. Detectors which showed a proper signal were assembled into the final array using a 3D-printed support structure designed by the TRIUMF Detector Facility Group and built at the Simon Fraser University Machine Shop. As the spatial tolerance in the original design of the array ($\sim 0.5$ mm between individual detectors) was found to be larger than necessary due to high precision achieved in the machining process, side faces of each detector were wrapped with Kapton tape as padding.

When preparing for an experiment, space constraints in the target chamber necessitate the cabling of individual detectors prior to mounting the array in the chamber, and individual detectors in the array remain inaccessible until the full array is removed from the chamber. Figure 3.9 shows the array after insertion into the target chamber and cabling. Due to the spherical coverage of the array, the reaction target is not visible upon insertion into the array, however the position of the TIP target holder is fixed by multiple alignment pins and mounting screws which center the target in the array.
Figure 3.9: Assembled and cabled CsI ball array in the TIP target chamber at the SFU Machine Shop (left) and on the TIGRESS beamline at TRIUMF (right).

The first in-beam tests of the CsI ball at TRIUMF utilized ring 0 through 3 of the array (for a total of 38 detectors) and were run in September and December 2016 - these tests included the $^{28}$Mg data of Chapter 5. Afterwards, work continued on construction of the remaining detectors. The full ball was completed in October 2017 and is shown in assembled form in Figure 3.8. The final CsI ball commissioning tests (see Section 4.2) were completed in June 2018.
Chapter 4

Commissioning Experiments

4.1 Test of DSAM technique via study of $^{22}\text{Ne}$

Levels in $^{22}\text{Ne}$ were studied using the $^{12}\text{C}(^{18}\text{O},2\alpha)^{22}\text{Ne}$ fusion-evaporation reaction, in an experiment run as part of the commissioning process of the TIP device and the TIGRESS digital data acquisition system (DAQ) [57]. The $^{18}\text{O}$ beam was delivered to the experiment at 48 MeV, resulting in an initial recoil velocity of 0.045$c$ for the $^{30}\text{Si}$ compound nucleus. The DSAM target consisted of a 28.79 mg/cm$^2$ (14.9 µm thick) gold backing evaporated onto a 433 µg/cm$^2$ (~2 µm thick) natural carbon film as discussed in Ref. [69]. The thickness of the gold backing was sufficient to stop heavy residual nuclei generated following fusion reactions on the target while allowing light evaporated ions through to the downstream CsI(Tl) detectors. The target assembly was installed on the TIP device (discussed in Section 3.1.3) at the center of the target chamber. 12 of the 16 TIGRESS clover detectors were run in a 0/8/4 configuration: 0 detectors at the 45° position with respect to the beam axis (these positions were occupied by TRIUMF auxiliary neutron detectors undergoing separate testing), 8 detectors at the 90° position, and 4 detectors at the 135° position. The clover arrangement of the germanium crystals in the TIGRESS detectors allowed for additional position sensitivity of detected gamma rays, resulting in four effective angular bins at approximately 80°, 100°, 125°, and 145°. The 24 element CsI wall (discussed in Section 3.1.4) was mounted 51.7 mm downstream from the reaction target for charged particle detection. In order to gain additional selectivity, the TIGRESS DAQ trigger logic was set to only accept events containing coincident triggers in two clover units and two separate charged particle detectors. The experimental setup is shown in Figure 4.1.

Energy calibration of the CsI(Tl) detector array was performed for alpha particles via a Rutherford scattering experiment. Alpha particle beams at fixed energies over the range 5 - 30 MeV were scattered on a 0.3 mg/cm$^2$ (0.34 µm thick) $^{58}\text{Ni}$ target. Energies of alpha particles scattered into each detector were computed using the kinematics calculator in the LISE++ software package [70] to obtain calibration data. Energy and relative efficiency
calibrations of the TIGRESS HPGe clover detectors were performed using $^{56}$Co and $^{152}$Eu source data.

4.1.1 Data Analysis

In order to reduce contributions from background radiation and competing reaction mechanisms, data recorded during the experiment was sorted into a set containing events in which all particles and gamma rays were detected in a narrow time coincidence window compared to the $\sim 1$ $\mu$s window provided by the TIGRESS DAQ. CsI(Tl) hits arriving more than 90 ns later than the first CsI(Tl) hit and HPGe hits arriving more than 60 ns later than the first HPGe hit were dropped. Furthermore, a restriction that the first HPGe hit and the first CsI(Tl) hit arrive between 250 ns and 390 ns of each other was enforced. The optimal coincidence window sizes were determined via inspection of detector hit timestamps in the experiment data, and arise from the specific timing implementation in the DAQ.

Following separation of time-correlated events, reaction channel discrimination was obtained via pulse-shape analysis of CsI(Tl) waveforms, as discussed in Section 2.4.4. A separated data set corresponding to formation of the $^{22}$Ne residual nucleus following the $^{12}$C($^{18}$O,2$\alpha$)$^{22}$Ne reaction was obtained by gating on events containing 2 alpha particle hits in the CsI(Tl) wall.

Compton suppression and add-back of coincident hits in adjacent crystals of individual TIGRESS clovers was applied as discussed in Section 2.4.5. A gamma-ray spectrum corresponding to population of $^{22}$Ne was projected from the time-correlated particle gated data, and is shown in Figure 4.2 alongside the equivalent spectrum without particle gating.

4.1.2 Evaporated particle distribution

The GEANT4 simulations described in Section 3.2 were tuned to match the experimentally observed energy and angular distribution recorded by the CsI wall detector array. The energy
Figure 4.2: Separated (top) and unseparated (bottom) gamma-ray spectra, when using the particle gating scheme corresponding to production of $^{22}\text{Ne}$. The similarity of the two spectra indicates that the majority of events recorded in this experiment correspond to $^{22}\text{Ne}$ population.
Table 4.1: Best fit parameters of the $\delta E_{ex}$ distribution for alpha particles observed in the $^{12}$C($^{18}$O,2$\alpha$)$^{22}$Ne reaction.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fit Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>14.57(7)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1.20(10)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.47(2)</td>
</tr>
</tbody>
</table>

and angular distributions were treated separately: first the simulated energy distribution was matched to the experimental data on a detector-by-detector basis, then the angular distribution was matched by biasing the simulated data such that the number of simulated counts in each detector was proportional to the experimental data with a common scaling factor.

Best fit values of the energy distribution parameters $\mu$, $\sigma$, and $\lambda$ discussed in Section 3.2.2 were determined using the grid search method outlined in Appendix A.1. When determining $\chi^2$ values for each point on the grid, only events with a threshold energy of 3 MeV or higher were considered. This threshold was used because the $E_{ex,\alpha}$ distribution of Equation 3.1 otherwise fails to reproduce the shape of the experimental CsI(Tl) energy spectra at low energies. This is most likely an artifact of the specific trigger used to select CsI(Tl) events in the experimental data acquisition hardware and/or the offline CsI(Tl) event waveform fitting. At low particle energies on the order of a few MeV, CsI(Tl) signal amplitudes become comparable to background noise.

The best fit parameters describing the alpha particle $\delta E_{ex}$ distribution are listed in Table 4.1. A simulated spectrum of alpha particle energies prior to stopping was generated using these parameter values and compared to the same spectrum obtained from calculations using the fusion-evaporation reaction code PACE4 [71] with default parameters, shown in Figure 4.3. Overall, PACE4 slightly under predicts alpha particle energies compared to the GEANT4 simulation, which was tuned to best reproduce the experimental data.

In order to match the experimental angular distribution of alpha particle events in the CsI(Tl) array, the CsI(Tl) detectors were first indexed into 5 rings of detectors at equal angles from the center of the array. The number of events in each pair of CsI(Tl) detector rings (pairs were considered since each event contained 2 alpha particle hits) was then determined for both the experimental data and simulated data using the optimal values of $\mu$, $\sigma$, and $\lambda$ determined above. The simulated data was biased using weights $w_{i,j}$:

$$w_{i,j} = \frac{N_{exp,i,j}}{N_{sim,i,j}} \times \frac{N_{sim,1,1}}{N_{exp,1,1}}$$

(4.1)

where $w_{i,j}$ is the weight for events containing a hit in CsI(Tl) detector rings $i$ and $j$, and $N_{sim,i,j}$ and $N_{exp,i,j}$ are the total number of counts in the ring pair in the unbiased simulated and experimental data, respectively. The weights calculated using Equation 4.1
Figure 4.3: Comparison of alpha particle lab energy spectrum predicted by PACE4 (dark) and simulated data (light). Only alpha particles emitted at lab angles of 16° to 42° are considered, corresponding to the coverage of the CsI(Tl) wall used in the experiment.

ranged between $w_{1,1} = 1.00$ and $w_{5,5} = 0.43$, such that the simulated data was biased towards alpha particle hits in low angle detectors. This suggests that the overall angular distribution of alpha particles emitted from the compound nucleus is non-isotropic in the center of mass and biased towards low angles.

A comparison between simulated particle energy spectra and the experimental data is shown in Figure 4.4. The simulated data was generated using the best fit parameters for $\mu$, $\sigma$, and $\lambda$, and biased with the weights of Equation 4.1.

4.1.3 Doppler reconstruction

In order to aid determination of the gamma-ray energies and lifetimes of observed transitions in $^{22}$Ne, the experimental data was analyzed on an event-by-event basis in an attempt to extract the Doppler shift factor $D$ corresponding to a given event. Sorting events by the $D$ value rather than by detector angle (with respect to the beam axis) allows for clearer separation of gamma-ray lineshape components corresponding to different Doppler shifts, since the momentum of the $^{22}$Ne nucleus is not necessarily directed along the beam axis.

First, the momentum of the residual nucleus $\vec{p}_{\text{res}}$ was determined:

$$\vec{p}_{\text{res}} = \vec{p}_{\text{beam}} - \sum_i \vec{p}_{\alpha i}, \quad (4.2)$$

where $\vec{p}_{\text{beam}}$ is the momentum of the incoming beam, based on the known beam energy, mass, and direction, and $\vec{p}_{\alpha i}$ is the momentum of the $i$th alpha particle detected in the CsI(Tl) array. Velocity vectors were approximately determined based on the center position of the detector and the detected particle energy. Equation 4.2 assumes that all particles
emitted by the compound nucleus are detected, that the momenta of gamma rays emitted from the residual nucleus are small compared to $\vec{p}_{\text{res}}$, and that there is no significant stopping of alpha particles in the target backing.

Knowing $\vec{p}_{\text{res}}$ and the position of the detector in which a gamma ray was detected in the same event, the angle $\theta$ between the source and detector was approximated:

$$\cos \theta = \frac{\vec{p}_{\text{res}} \cdot \vec{p}_\gamma}{|\vec{p}_{\text{res}}||\vec{p}_\gamma|},$$

(4.3)

where $\vec{p}_\gamma$ is the momentum of the gamma ray, based on the position of the HPGe crystal in which the gamma ray was detected and the energy of the detected gamma ray.

The speed of the residual nucleus $\beta$ was determined from its momentum $\vec{p}_{\text{res}}$ and its mass. Then the overall Doppler shift factor $D$ was calculated from $\beta$ and $\cos \theta$, using Equation 2.34.

### 4.1.4 Gamma-ray energy determination

In order to constrain the Monte-Carlo simulations used to measure lifetimes of observed transitions in $^{22}\text{Ne}$, the experimentally observed gamma ray energies corresponding to de-population of these transitions were determined. The excellent energy resolution of HPGe detectors makes them ideal for the measurement of gamma ray energies of stationary sources. However since the lineshapes measured in a moving source (DSAM-like) experiment consist of gamma rays emitted with a range of Doppler shifts, the unshifted energy is not immediately apparent.

Using the Doppler reconstruction method as discussed above, it is possible to generate 2D histograms of the observed gamma ray energy $E_\gamma$ plotted against the $\cos \theta$ values calculated using Equation 4.3. Then according to Equation 2.34, all gamma rays belonging to
Figure 4.5: Plot of $E_\gamma$ vs $\cos \theta$ for the events near the transition depopulating the 5523.2 keV level of $^{22}$Ne, before (left) and after (middle) shearing the data about the slope of the initial fit line. A separate transition with a different rate is also seen near the bottom of the $E_\gamma$ vs $\cos \theta$ plots. Right: $E_\gamma$ spectrum projected from the sheared data with background and Gaussian peak shape indicated.

A given transition should form a line with slope $\beta E_0$ and intercept $E_\gamma = E_0$ at $\cos \theta = 0$.

Figure 4.5 shows a plot of events in the region of the transition depopulating the 5523.2 keV level of $^{22}$Ne.

It is possible to fit a line to the distribution of events defining the transition. If events belonging to the transition are symmetrically distributed about the fit line for a given angle $\theta$, then the intercept of the line is an estimate of the unshifted energy $E_0$ of the transition. The statistical uncertainty in this estimate may be determined using a confidence interval for the linear fit. In practice, events are not symmetrically distributed about the fit line, typically due to the lineshape containing a stopped component and/or interference from other nearby transitions.

A method to account for interference from other transitions is to fit a line to a region of the data which only contains events belonging to the transition of interest by gating in a selected range of $E_\gamma$, $\cos \theta$, or both. The data may then be sheared along the $E_\gamma$ axis by the slope of the initial fit, and a 1-D gamma ray spectrum projected out on the $E_\gamma$ axis. One may then fit the transition to a Gaussian peak shape, taking into account background contributions from adjacent transitions.

In longer-lived transitions where a stopped component is visible in the lineshape (such as the $4^+ \rightarrow 2^+$ transition in Figure 4.2), this stopped component will interfere with the shifted component. When multiple components belonging to the same transition are observed, the unshifted energy may still be determined by gating on events in a window which is symmetric in $\cos \theta$ about $\cos \theta = 0$, projecting out a spectrum in $E_\gamma$ and fitting the centroid $E_0$ of the resulting peak. Since this peak is comprised of multiple components, each symmetrically distributed about the centroid $E_0$, it may be broad, reducing the statistical accuracy with which the centroid may be determined. This method is best applied to transitions with high statistics available, since the $\cos \theta$ gate will discard a large number of events.
4.1.5 Observed and unobserved feeding between transitions

For transitions depopulating low-lying levels which are fed by transitions depopulating higher-lying levels, the observed lineshape depends both on the lifetime of the transition of interest and the lifetimes of the transitions feeding it. For high-lying states where no observed feeding is present, one cannot disentangle the effects of unobserved feeding\(^1\) on the lineshape from the experimental data and can therefore only fit the lineshape to obtain an ‘effective lifetime’ of the transition which neglects any correction for the effects of feeding.

In order to determine an accurate lifetime for low-lying states in \(^{22}\)Ne where feeding from higher states was observed, additional simulations of the observed feeding branch were performed, where the mean lifetime of the higher state was fixed to the best fit value determined using the lineshape analysis methods of Section 3.2.3 and 3.2.4 and the lifetime of the low-lying state was left free to vary. Only gamma rays in time coincidence with the transition between the feeding state and the state of interest were retained in the gamma-ray spectrum, resulting in an experimental lineshape corresponding only to events belonging to the observed feeding branch. This lineshape was then compared to the simulated data for the feeding branch containing the transition of interest, and the best fit lifetime \(\tau\) for the low-lying transition determined using the method of Section 3.2.3.

For the sake of comparison, lifetimes with and without the feeding correction described above were generated for each transition with observed feeding. When not using the feeding correction, the simulations assumed that the transition of interest was directly populated, and the effective lifetime was determined using the method of Section 3.2.3 with ungated experimental data.

4.1.6 Statistical uncertainty due to electronic stopping powers

The speed distribution of the residual nucleus upon gamma ray emission (and therefore the observed energy distribution of gamma rays due to the Doppler effect) depends strongly on the stopping of the nucleus inside the target and target backing material.

By default, GEANT4 computes electronic stopping using stopping power tables derived from ICRU Report 73 [72]. It is possible to use data from other stopping models by modifying the built-in data tables. A comparison of the default stopping in GEANT4 to stopping using stopping powers generated by the 2013 revision of the SRIM code [73] was performed. Figure 4.6 shows a GEANT4 simulation of the speed distributions of a \(^{22}\)Ne nucleus being stopped in a thick amorphous \(^{197}\)Au backing, chosen to match the target backing from the experiment\(^2\). The speed distributions were determined as a function of

\(^1\)It is possible that an observed level may be fed via transitions from unobserved levels at higher energy. These transitions may not be observed if their intensity is low compared to background events or if their energy is outside the sensitive range of the detection system.

\(^2\)The target backing used in the \(^{22}\)Ne experiment was amorphous since it was produced by physical vapour deposition as discussed in Ref. [69].
Figure 4.6: GEANT4 comparison of $^{22}\text{Ne}$ nucleus speed as a function of time spent recoiling through a $^{197}\text{Au}$ backing, for ICRU 73 and SRIM-2013 stopping powers. Results are averaged over $10^6$ events, $1\sigma$ bounds in the speed distributions are denoted by error bars.

time for both stopping power data sets. The initial energies of the $^{22}\text{Ne}$ nuclei were given a Gaussian distribution with centroid value 10.9 MeV and full width at half maximum 5.7 MeV, approximately matching the energy distribution of residual nuclei seen in GEANT4 simulations when using the best fit alpha particle energy parameters obtained in Section 4.1.2.

Figure 4.6 shows that the $^{22}\text{Ne}$ nucleus is stopped in $^{197}\text{Au}$ after 900 fs regardless of the stopping powers used, implying that lifetime sensitivity cannot be obtained for levels which are significantly longer-lived than 900 fs. In general, the speed distributions of Figure 4.6 show significant overlap at times below 300 fs. This indicates that the gamma-ray energy distribution for transitions with lifetimes below 300 fs is not strongly affected by the choice of ICRU 73 or SRIM-2013 stopping powers. Transitions falling in the intermediate range of lifetimes between 300 fs and 900 fs will yield gamma-ray energy distributions which depend strongly on the choice of stopping powers.

In order to quantify the dependence of best fit transition lifetimes on the stopping powers used, lineshapes were generated using GEANT4 for a 1 MeV gamma ray emitted from the recoiling $^{22}\text{Ne}$ nucleus at various mean lifetimes using both ICRU 73 and SRIM-2013 stopping powers. Simulated lineshapes were compared between stopping powers using the method of Section 3.2.3 to determine the percentage difference in lifetimes which gave the best agreement in lineshapes generated with the two sets of stopping powers. This percentage difference is plotted in Figure 4.7 as a function of the lifetime used when generating the lineshape with ICRU 73 stopping powers.
Figure 4.7: Plot of the percentage difference in lifetimes resulting in the best overlap between lineshapes generated using ICRU 73 and SRIM-2013 stopping powers in GEANT4, as a function of the lifetime used with the ICRU 73 stopping data.

All simulation results reported in Section 4.1.7 use the default GEANT4 electronic stopping power dataset based on ICRU Report 73. The systematic uncertainty in lifetime for a given transition in $^{22}$Ne due to stopping was taken as the percentage difference shown in Figure 4.7 determined for the lifetime of the transition of interest.

4.1.7 Results

Mean lifetimes $\tau_{\text{mean}}$ and energies of levels in $^{22}$Ne with excitation energy up to and including 8976 keV were investigated. Reported level energies include a correction for the recoil energy of the $^{22}$Ne nucleus in addition to the observed gamma-ray energies. A partial level scheme is shown in Figure 4.8. Results are summarized in Tables 4.2 and 4.3. Table 4.4 shows a comparison of lifetimes determined with and without the feeding correction described in Section 4.1.5, for levels in which feeding is observed.

Figure 4.9 shows a comparison between experimental data and simulated lineshapes for the $4^+$ level at 3357 keV. Figure 4.10 shows the outcome of the $\chi^2$ goodness of fit test for this level, and for the $(4)^+$ level at 5523 keV. Similar fits were obtained for all lifetimes measured in this study.

This study was not sensitive to the lifetime of the yrast $2^+$ level of $^{22}$Ne with observed level energy 1274.4(2) keV. Previous RDM studies [75, 76] suggest a lifetime on the order of 5 ps for this level. As previously discussed in Section 4.1.6, 5 ps is significantly longer than the expected stopping time for $^{22}$Ne in this study.
Figure 4.8: Partial level scheme showing selected excited levels of $^{22}$Ne observed in the experimental data (relative intensities of gamma rays are not indicated).

Table 4.2: Measured gamma-ray energies for select observed $^{22}$Ne transitions.

<table>
<thead>
<tr>
<th>Evaluated level energy (keV) [74]</th>
<th>$I^\pi$ [74]</th>
<th>Measured Values</th>
<th>Gamma energy (keV)</th>
<th>Level energy (keV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1274.537(7)</td>
<td>$2^+$</td>
<td>1274.4(2)$^a$</td>
<td>1274.4(2)</td>
<td></td>
</tr>
<tr>
<td>3357.2(5)</td>
<td>$4^+$</td>
<td>2082.0(2)$^a$</td>
<td>3356.6(3)</td>
<td></td>
</tr>
<tr>
<td>5146.0(9)</td>
<td>$2^-$</td>
<td>3869.7(3)$^a$</td>
<td>5144.5(4)</td>
<td></td>
</tr>
<tr>
<td>5523.3(6)</td>
<td>$(4)^+$</td>
<td>2165.5(3)$^b$</td>
<td>5522.2(4)</td>
<td></td>
</tr>
<tr>
<td>6311.0(10)</td>
<td>$(6)^+$</td>
<td>2955.2(3)$^{a,c}$</td>
<td>6312.0(4)</td>
<td></td>
</tr>
<tr>
<td>6345.1(10)</td>
<td>$4^+$</td>
<td>2989.3(9)$^{a,c}$</td>
<td>6346.1(9)</td>
<td></td>
</tr>
<tr>
<td>7423.0(9)</td>
<td>$(5)^+$</td>
<td>1900.5(4)$^{a,c}$</td>
<td>7422.8(6)</td>
<td></td>
</tr>
<tr>
<td>8976(3)</td>
<td>-</td>
<td>1923.6(10)$^{a,c}$</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

$^a$From gating events near $\cos \theta = 0$.

$^b$From 1-D gamma-ray spectrum shear projection method.

$^{c}$Overlapping peaks - simultaneous fit.
Table 4.3: Measured mean lifetimes for select observed $^{22}$Ne transitions.

<table>
<thead>
<tr>
<th>Evaluated level energy (keV) [74]</th>
<th>$I^\pi$ [74]</th>
<th>$\tau_{\text{mean}}$ (fs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3357.2(5)</td>
<td>4$^+$</td>
<td>290(50)$^a$</td>
</tr>
<tr>
<td>5146.0(9)</td>
<td>2$^-$</td>
<td>1100(200)</td>
</tr>
<tr>
<td>5523.3(6)</td>
<td>(4)$^+$</td>
<td>30(10)$^a$</td>
</tr>
<tr>
<td>6311.0(10)</td>
<td>(6)$^+$</td>
<td>70(5)$^b$</td>
</tr>
<tr>
<td>6345.1(10)</td>
<td>4$^+$</td>
<td>11(6)$^b$</td>
</tr>
<tr>
<td>7423.0(9)</td>
<td>(5)$^+$</td>
<td>&lt; 16$^b$</td>
</tr>
<tr>
<td>8976(3)</td>
<td>-</td>
<td>&lt; 6$^b$</td>
</tr>
</tbody>
</table>

$^a$Corrected for feeding by gating on the 7423 keV level.

$^b$Overlapping lineshapes - simultaneous fit.

Table 4.4: Measured mean lifetimes for select observed $^{22}$Ne transitions, with and without correction for feeding.

<table>
<thead>
<tr>
<th>Evaluated level energy (keV) [74]</th>
<th>$I^\pi$ [74]</th>
<th>Measured lifetime (fs)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\tau_{\text{mean}}$</td>
<td>$\tau_{\text{no feeding}}$</td>
</tr>
<tr>
<td>3357.2</td>
<td>290(50)</td>
<td>310(50)</td>
</tr>
<tr>
<td>5523.3</td>
<td>30(10)</td>
<td>27(2)</td>
</tr>
</tbody>
</table>

Figure 4.9: Lineshape of the 3357 keV level of $^{22}$Ne from experimental data (black) and simulated data (red) using the best fit mean lifetime of 290 fs. Data is gated on the 7423 keV level to correct for feeding.
The energy of the level with evaluated level energy 8976(3) keV was not remeasured, since the energy of the level to which it decays via the observed 1923.6(10) keV gamma ray could not be measured. This lower-lying level has evaluated level energy 7051(3) keV, and is depopulated via a strong gamma ray at 5776 keV [74]. At this high energy, the gamma ray detection efficiency of TIGRESS was sufficiently low that the 5776 keV gamma ray could not be distinguished from background events, hence the energies of the 7051(3) keV level and levels feeding it could not be measured in this study.

4.1.8 Determination of systematic uncertainties

When measuring gamma-ray energy values, the statistical uncertainties from the Gaussian peak shape fits were combined with a systematic uncertainty arising from calibration of the TIGRESS HPGe clovers, which was determined by comparing peak positions of gamma rays observed in calibrated $^{56}$Co source data taken during the experiment with the tabulated gamma-ray energies. Based on this analysis, the upper limit of this systematic uncertainty was taken as 0.2 keV for all transitions observed. All systematic and statistical errors were added in quadrature to determine the total uncertainty in energy for a given gamma ray.

Table 4.5 reports the separate fit-related statistical and stopping-related systematic uncertainties associated with measured lifetimes in $^{22}$Ne. The 1σ statistical uncertainty bounds
Table 4.5: Statistical and systematic uncertainties for lifetimes measured in Table 4.3.

<table>
<thead>
<tr>
<th>Evaluated level energy (keV) [74]</th>
<th>$\delta\tau_{stat}$ (fs)</th>
<th>$\delta\tau_{sys}$ (fs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3357.2(5)</td>
<td>10</td>
<td>50</td>
</tr>
<tr>
<td>5146.0(9)</td>
<td>160</td>
<td>180</td>
</tr>
<tr>
<td>5523.3(6)</td>
<td>5</td>
<td>8</td>
</tr>
<tr>
<td>6311.0(10)</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>6345.1(10)</td>
<td>6</td>
<td>0.3</td>
</tr>
<tr>
<td>7423.0(9)</td>
<td>11</td>
<td>0.9</td>
</tr>
</tbody>
</table>

determined using the method of Appendix A.1 were typically on the order of a few fs, resulting in larger percentage uncertainties for short-lived states (typically at higher excitation energies) while for long-lived states the uncertainty was dominated by the stopping process.

Many previous DSAM studies of this nucleus assume errors on the order of 5% to 10% in electronic and nuclear stopping powers [77, 78, 79]. From the simulations of the stopping process outlined in Section 4.1.6, this study suggests that the uncertainty due to stopping powers is lower than 10% for transitions with lifetimes between 10 and 100 fs, and higher than 10% for longer-lived transitions. As a result, the reported uncertainties for the longer-lived levels at 3357 and 5146 keV are relatively large compared to existing literature.

4.1.9 Discussion

As Table 4.2 shows, the measured level energies in $^{22}$Ne agree well with the evaluated data. The main factor contributing to uncertainty in these measurements was the aforementioned 0.2 keV systematic uncertainty due to detector calibration.

In general, measured lifetimes obtained for observed states of $^{22}$Ne agree well with evaluated data regardless of whether observed feeding was taken into account when determining lifetimes. Uncertainties in lifetimes were dominated by the systematic error in stopping.

Overall, both energy and lifetime measurements are in good agreement with the existing literature on $^{22}$Ne, demonstrating the validity of the analysis methods developed in this thesis, and their applicability to the analysis of $^{28}$Mg data carried out in Chapter 5.

4.2 CsI ball detector commissioning

The CsI ball detector array discussed in Section 3.3 was commissioned in a series of experiments at SFU and TRIUMF following its construction at SFU.

4.2.1 Energy resolution and light collection

CsI ball detectors were tested individually using a $^{241}$Am source, which emits intense alpha particles at 5485.56 and 5442.80 keV energy during its decay to $^{237}$Np. A source-detector
distance of 51.7 mm was used for all detectors, which were tested individually using a common preamplifier and data acquisition system. Typical width / centroid resolution of the resulting peak in the energy spectrum (resolution was insufficient to separate the two alpha energies) ranged from 7-20% depending on the detector tested, with an average value of 12.1%. Typical resolution of detectors in the CsI wall are 10%, however various factors can affect the resolution and light collection characteristics, including the geometry of the light guides (which depends on the detector ring) and/or optical imperfections in the interfaces between the CsI(Tl) crystal, light guide, and PIN diode. In general, the surface areas of the CsI(Tl) crystals are larger with respect to the PIN diode for the ball detectors than with the wall detectors. This is expected to adversely affect resolution and light collection, as light produced from the interaction of charged particles with the ball detectors is more likely to escape the light guide prior to reaching the PIN diode due to the detector geometry. A plot of energy resolution and peak centroid values (representing a relative measurement of the light collected) for the CsI ball detectors is shown in Figure 4.11. Overall, the energy resolution is not strongly correlated to the light collection, suggesting that the creation of ‘information carriers’ in the scintillation process is largely independent of the light collection process, with the former process being the limiting factor affecting the energy resolution.

Figure 4.11: Resolution and light collection characteristics of individual CsI ball array detectors.
Table 4.6: Pb absorber thicknesses used for each ring of the array and the associated transmission and detection thresholds (calculated using the ELAST code [80]), assuming a 3 MeV minimum energy for charged particle detection. All energy values are in the lab frame.

<table>
<thead>
<tr>
<th>Ring</th>
<th>Absorber thickness (mg/cm²)</th>
<th>(^4)He threshold (MeV)</th>
<th>(^{36})Ar threshold (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Transmission</td>
<td>Detection</td>
<td>Transmission</td>
</tr>
<tr>
<td>0</td>
<td>18</td>
<td>5.38</td>
<td>7.25</td>
</tr>
<tr>
<td>1</td>
<td>17</td>
<td>5.15</td>
<td>7.05</td>
</tr>
<tr>
<td>2</td>
<td>17</td>
<td>5.15</td>
<td>7.05</td>
</tr>
<tr>
<td>3</td>
<td>16</td>
<td>4.92</td>
<td>6.85</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
<td>4.92</td>
<td>6.85</td>
</tr>
<tr>
<td>5</td>
<td>16</td>
<td>4.92</td>
<td>6.85</td>
</tr>
<tr>
<td>6</td>
<td>15</td>
<td>4.67</td>
<td>6.65</td>
</tr>
<tr>
<td>7</td>
<td>15</td>
<td>4.67</td>
<td>6.65</td>
</tr>
<tr>
<td>8</td>
<td>14</td>
<td>4.42</td>
<td>6.44</td>
</tr>
<tr>
<td>9</td>
<td>14</td>
<td>4.42</td>
<td>6.44</td>
</tr>
</tbody>
</table>

4.2.2 Commissioning runs at TRIUMF

The charged particle detection efficiency of the CsI ball array was measured in two separate experiments using TIP and TIGRESS at TRIUMF. The first experiment employed a \(^{196}\)Pt\((\alpha,\alpha)\)^{196}\)Pt* Coulomb excitation reaction, and the second used \(^{58}\)Ni\(^{({^{36}}\)Ar,xpyn}\) fusion-evaporation. The CsI array detectors were wrapped in Pb absorbers during this group of experiments in order to prevent scattered \(^{36}\)Ar beam from entering the detectors. Absorber thicknesses are listed in Table 4.6.

4.2.3 Efficiency from Coulomb excitation

In the Coulomb excitation experiment, the data acquisition system was set to trigger upon gamma-ray detection, and the efficiency of the CsI ball was taken as the ratio of the number of events containing a gamma ray at 355 keV corresponding to Coulomb excitation of \(^{196}\)Pt and an alpha particle hit in the CsI array to the number of events containing a 355 keV gamma ray only. A 2.0 mg/cm² (0.93 µm thick) enriched \(^{196}\)Pt target was used. The CsI ball array efficiency was measured at \(^4\)He beam energies of 18.8, 14.6, and 10.2 MeV in order to determine the effect of the Pb absorbers on detection of scattered alpha particles. As this was a coincidence measurement, the measured efficiency \(\epsilon_{\text{meas}}\) was a function of the timing gate width due to the contribution of random coincidences with background events. The true efficiency of the CsI array was determined by assuming a time-random background and varying the timing gate width. For large gate widths, the background contribution to the efficiency was assumed to be linear with gate width and this contribution was fitted, with the value at zero width representing the true efficiency \(\epsilon_{\alpha}\) in the absence of background. Figure 4.12 shows the timing spectrum with timing gates superimposed for the data corresponding to 18.8 MeV \(^4\)He beam energy. Figure 4.13 shows the extraction of \(\epsilon_{\alpha}\) for the same data.
Figure 4.12: TIGRESS vs. CsI array timing plot for the $^{196}$Pt Coulomb excitation experiment with 18.8 MeV $^4$He beam energy. The central peak corresponds to time-coincident events in the two detection systems, while events outside of the peak correspond to random coincidences with background events. Timing gates of 50 and 350 ns width are indicated.

Table 4.7: Measured alpha particle detection efficiencies $\epsilon_\alpha$ for the CsI ball array in the $^{196}$Pt Coulomb excitation experiment.

<table>
<thead>
<tr>
<th>$^4$He beam energy (MeV)</th>
<th>$\epsilon_\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10.2</td>
<td>0.558(10)</td>
</tr>
<tr>
<td>14.6</td>
<td>0.62(2)</td>
</tr>
<tr>
<td>18.8</td>
<td>0.689(4)</td>
</tr>
</tbody>
</table>

Table 4.7 lists alpha particle detection efficiencies $\epsilon_\alpha$ for the CsI ball array obtained in this experiment as a function of $^4$He beam energy. As the beam energy increases, the detection efficiency $\epsilon_\alpha$ increases due to increased penetration of scattered alpha particles through the Pb absorbers.

4.2.4 Efficiency from fusion-evaporation

A fusion-evaporation experiment was run using an enriched 0.3 mg/cm$^2$ (0.34 µm thick) $^{58}$Ni target and $^{36}$Ar beam at 170 MeV. An online 2 particle - 2 Compton suppressed gamma ray trigger was used in order to separate background events from fusion-evaporation events. Channels observed at high intensity included $^{89}$Mo and $^{89}$Nb, corresponding to $4p1n$ and $5p$ evaporation from the compound nucleus $^{94}$Pd, respectively. Figure 4.14 shows the gamma-ray spectra obtained from TIGRESS before and after gating on the $5p$ channel corresponding to $^{89}$Nb.
Figure 4.13: Measured efficiency $\epsilon_{\text{meas}}$ of the CsI ball array in the $^{196}\text{Pt}$ Coulomb excitation experiment with 18.8 MeV $^4\text{He}$ beam energy, as a function of timing gate width. The true alpha efficiency $\epsilon_\alpha$ is obtained by extrapolating the linear background contribution to zero gate width.

The proton detection efficiency $\epsilon_p$ of the CsI ball array was determined by examining gamma rays corresponding to known particle evaporation channels under different particle gating conditions. The number of counts observed for a specific transition under a specific particle gate depends on $\epsilon_p$ and a ‘gating efficiency’ $\epsilon_g$ which indicates the proportion of particles which are properly identified by the particle gate. For gamma rays belonging to the $np$ evaporation channel, the number of counts $C_{n,x}$ in a given peak in the gamma ray spectrum obtained when setting an $xp$ gate is:

$$C_{n,x} = A_{n,x} \sum_{y=m}^{n} \binom{n}{y} \epsilon_p^y (1 - \epsilon_p)^{n-y} \cdot \binom{y}{x} \epsilon_g^x (1 - \epsilon_g)^{y-x},$$

where $m$ is the minimum possible number of detected particles required in an event, and $A_{n,x}$ is a scaling factor. When applying an $xp$ gate offline, the number of required particles and the trigger multiplicity are usually the same (ie. $m = x$, an event in the 3p gate must have at least 3 detected particles). However, for the data presented here an online trigger was used requiring detection of at least 2 particles at the time of data acquisition, so $m = \max(x, 2) \geq 2$.

The two multiplicative terms of equation 4.4 indicate the probability of detecting $y$ particles if $n$ were emitted, and the probability of $x$ particles falling into the particle gate.
Figure 4.14: Gamma-ray spectrum summed over TIGRESS detectors before (top) and after (bottom) gating on events in coincidence with 5 protons. Following gating, the $^{58}\text{Ni}(^{36}\text{Ar},5\text{p})^{89}\text{Nb}$ reaction channel is isolated.
Table 4.8: Measured proton detection efficiencies $\epsilon_p$ for the CsI ball array in the $^{58}$Ni($^{36}$Ar,xp) fusion-evaporation experiment.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\epsilon_p$</td>
<td>0.635(2)</td>
</tr>
<tr>
<td>$\epsilon_g$</td>
<td>0.909(2)</td>
</tr>
<tr>
<td>$\epsilon_{p,corr}$</td>
<td>0.698(3)</td>
</tr>
</tbody>
</table>

if $y$ were detected. In the case where $m = x$ (ie. $x \geq 2$), the series in equation 4.4 simplifies to a single term:

$$C_{n,x} = A \left( \frac{n}{x} \right) e^x (1 - \epsilon)^{n-x}. \quad (4.5)$$

where the overall efficiency $\epsilon = \epsilon_p \epsilon_g$ is the product of the particle detection efficiency $\epsilon_p$ and the gating efficiency $\epsilon_g$, the latter of which describes the fraction of particles of interest which are included in the appropriate gate. A proof is shown in Appendix B.4. Equation 4.5 implies that in order to uniquely determine the values of $\epsilon_p$ and $\epsilon_g$ (instead of the product $\epsilon_p \epsilon_g$), data with $x < m$ must be examined. A $\chi^2$ minimization was performed to determine the best fit values of $A_{n,x}$, $\epsilon_p$, and $\epsilon_g$:

$$\chi^2 = \sum_{n,x} \left( \frac{O_{n,x} - C_{n,x}}{\sigma_{O_{n,x}}} \right)^2. \quad (4.6)$$

where $O_{n,x}$ is the observed number of counts in the np channel when setting an xp gate on the experimental data, with 1σ uncertainty $\sigma_{O_{n,x}}$.

The proton detection efficiency determined in this experiment via minimization of the $\chi^2$ defined in equation 4.6 is reported in Table 4.8, with $\epsilon_{p,corr}$ representing the overall proton efficiency corrected for the gating efficiency. Figure 4.15 shows the fit to the data.

The GEANT4-based simulation code discussed in Section 3.2 was used to simulate the $^{58}$Ni($^{36}$Ar,4p)$^{90}$Mo fusion-evaporation reaction incorporating realistic geometry for the $^{58}$Ni target, TIGRESS, and the CsI ball array. A geometric proton efficiency $\epsilon_{geo} = 0.8705(3)$ was obtained from the simulations. Therefore the measured proton efficiency $\epsilon_{p,corr} = 0.698(3)$ indicates that in a realistic experimental setting approximately 80% of protons incident on a CsI detector in the array are detected. The largest contribution to the observed reduction in efficiency is shadowing from the TIP target holder. During the experiments the shadow fully covered ring 6 of the array. When omitting ring 6 from the GEANT4 simulations, the geometric efficiency becomes $\epsilon_g = 0.7748(4)$, and the observed efficiency is 90% of the geometric efficiency. The remaining 10% difference may arise from nondetection of protons due to incomplete light collection, for instance in events where the particle hit is near a detector edge.
Figure 4.15: Fusion evaporation data for various transitions in $^{89}$Nb and $^{89}$Mo, fitted to equation 4.4 as a function of the number of particles in the proton gate $x$.

When simulating the geometric proton efficiency for the 24-element CsI wall array of Section 3.1.4 instead of the CsI ball, a value $\epsilon_{\text{geo,wall}} = 0.1814(3)$ is obtained, indicating that in a realistic fusion-evaporation experiment, the CsI ball provides more than a factor of 4 gain in particle detection efficiency over the wall array, even when considering the effect of shadowing by the TIP target holder. For detection of events accompanied by emission of multiple charged particles, the gain is approximately $4^m$, when $m$ is the charged particle multiplicity.
Chapter 5

Study of $^{28}\text{Mg}$

An experiment to investigate the structure of the neutron-rich nucleus $^{28}\text{Mg}$ was run in September 2016 at TRIUMF/ISAC-II. In this experiment, a $^{12}\text{C}(^{18}\text{O},2p)^{28}\text{Mg}$ fusion-evaporation reaction was used to populate $^{28}\text{Mg}$ at high spin and with excitation energy up to its neutron separation energy. 13 of the total 16 TIGRESS clover detectors were run using a 4/5/4 configuration (giving maximal coverage at the 45° and 135° positions of the array, where sensitivity to the Doppler effect is maximized), along with 38 of the CsI ball detectors making up the most downstream 4 rings of the array (representing the portion of the array which had been constructed at the time of the experiment, shown in Figure 5.1). The triggering condition required coincident detection of 2 charged particles and 2 gamma rays by the TIGRESS DAQ. An online Compton suppression algorithm was used to discard TIGRESS events identified as containing partial energy deposition at the time of data acquisition. As in the $^{22}\text{Ne}$ experiment, the purpose of the trigger condition was to discriminate background events from fusion-evaporation events (which are accompanied by emission of several charged particles and gamma rays from de-excitation of the compound system, as discussed in Section 2.2.2).

![Figure 5.1: Pictures of the partial CsI ball array setup used in the $^{28}\text{Mg}$ experiment.](image-url)
The experiment was split into two parts, with the first part using a DSAM target consisting of a 433 \(\mu g/cm^2\) (\(\sim 2 \mu m\) thick) carbon layer with a thick 28.79 mg/cm\(^2\) (14.9 \(\mu m\)) gold backing in order to facilitate lifetime measurements of excited states in \(^{28}\)Mg via DSAM \[69\]. The second part used a self-supporting thin target foil of 500 \(\mu g/cm^2\) (2.5 \(\mu m\)) diamond-like carbon produced by Micromatter \[81\] separated from a gold catcher foil positioned 2 mm downstream from the target. The purpose of the thin-target experiment was to obtain increased sensitivity to excited states populated in the reaction via gamma-ray spectroscopy. When using a thin target, the majority of the reaction products emerge from the target and recoil into vacuum prior to de-excitation and gamma-ray emission. Any gamma rays emitted while the source crosses the gap between the target and catcher in vacuum are detected with a constant Doppler shift, as the source crosses the gap at constant velocity. Knowing the recoil velocity, the Doppler shift can be corrected for, allowing for true gamma-ray energies to be reconstructed for use in coincidence spectroscopy. The thickness of the gold layer was comparable in both the DSAM and thin targets in order to obtain similar detection efficiencies in the downstream CsI(Tl) array in both parts of the experiment.

An \(^{18}\)O beam at 48 MeV (2.67 AMeV, resulting in an initial recoil velocity of 0.045c for the \(^{30}\)Si compound nucleus) with a rate of \(\sim 10^{10}\) particles/s was delivered to the experiment from the TRIUMF OLIS. The total experiment duration was 3 days, with the DSAM target running for 40.5 hours and the thin target running for 21 hours. Energy and relative efficiency calibration of the TIGRESS array was performed using standard \(^{56}\)Co and \(^{152}\)Eu sources, with these calibrations extrapolated for gamma rays at higher energy than the 3.451 MeV line in \(^{56}\)Co. Energy calibration of the CsI(Tl) array was performed using a triple-alpha source containing the alpha particle emitters \(^{239}\)Pu, \(^{241}\)Am, and \(^{244}\)Cm.

### 5.1 Data Analysis

Data corresponding to fusion-evaporation was separated from the background using sequential time gates on TIGRESS and CsI ball events followed by a 2-proton PID gate from CsI(Tl) waveform fits. To reduce contributions from random background events, only events containing 2 CsI ball hits within 200 ns of each other, 2 TIGRESS hits within 120 ns of each other, and a TIGRESS and a CsI ball hit within 140 ns of each other were retained. The specific timing windows were chosen based on the observed timing resolution for each detector array used in the experiment.

The 2-proton PID gate was not sufficient by itself to completely isolate \(^{28}\)Mg, as events corresponding to the 2p1n (\(^{27}\)Mg), 2p2n (\(^{26}\)Mg), 1p (\(^{29}\)Al) and 1p1n (\(^{28}\)Al) channels remained due to a combination of factors including lack of neutron detection capability, high reaction cross-sections for population of these contaminant channels, and high beam rate causing a small fraction of events containing one proton to be detected simultaneously as a single event containing two protons. Gamma-gamma coincidences were used in order to
further isolate $^{28}\text{Mg}$, wherein only events containing at least one gamma-ray with energy corresponding to a known gamma-ray in $^{28}\text{Mg}$ (typically the decay of the first excited state, with gamma-ray energy 1473.54(10) keV [82] was used) were retained. Add-back of partial gamma-ray energy deposits in neighbouring HPGe cores was performed when evaluating gamma-ray energies in order to improve the signal-to-noise ratio.

Doppler reconstruction of the thin-target data was performed using the method of Section 4.1.3, taking into account the incoming beam momentum and the position and energy at which charged particles and gamma rays were detected. As only alpha particle calibration data was available for the CsI ball, the proton energies detected in the array were approximated as the equivalent alpha particle energies determined via the triple-alpha source calibration. To improve the energy resolution of the Doppler corrected data, the beam momentum was multiplied by a factor of 0.95 in the calculation to account for slowing of the beam in the target prior to reaction. The Doppler reconstruction procedure generated gamma-ray spectra with typical FWHM-to-position resolution of $\sim 1\%$ (compared to $\sim 0.25\%$ obtained for $^{56}\text{Co}$ calibration source data). The increased peak width in the reconstructed data corresponds to a distribution of recoil velocities due to slowing of the residual nucleus in the thin carbon reaction target, as well as the finite size of individual detectors in the CsI and TIGRESS arrays.

Gamma-ray cascades in $^{28}\text{Mg}$ were identified by gating on known gamma rays in $^{28}\text{Mg}$ and projecting out spectra of gamma rays in time coincidence. Overall, 17 gamma-rays were identified, corresponding to 14 excited levels in $^{28}\text{Mg}$. Of these, 8 gamma rays and 3 levels were newly identified. Relative intensities of gamma rays were derived from the thin target gamma-ray spectra using an efficiency calibration of the TIGRESS array derived from $^{56}\text{Co}$ source data. The level scheme established for $^{28}\text{Mg}$ is shown in Figure 5.2.

The mean lifetimes $\tau_{\text{mean}}$ of observed levels in $^{28}\text{Mg}$ were determined with the GEANT4 simulations discussed in Section 3.2 using a similar method to that used for $^{22}\text{Ne}$. Compared to the $^{22}\text{Ne}$ data, the main differences in the analysis method for $^{28}\text{Mg}$ were the use of the CsI ball array geometry rather than the CsI wall geometry, and the use of experimental thin-target data to determine the energy distribution of evaporated particles and the resulting velocity distribution of the residual nucleus (rather than energy calibrated CsI(Tl) data, which was not available for this experiment). Additionally, the likelihood ratio $\chi^2$ method developed in Ref. [83] for low-statistics data was used rather than the Neyman $\chi^2$ method to evaluate the best fit lifetime values, owing to the fact that $^{12}\text{C}({^{18}\text{O}},2\alpha)^{28}\text{Mg}$ is a much weaker reaction channel than $^{12}\text{C}({^{18}\text{O}},2\alpha)^{22}\text{Ne}$.

The velocity distribution of the residual nucleus was inferred by noting that the Doppler shifted energy distribution observed in the thin-target data for each TIGRESS ring depends on this velocity distribution (an extreme Doppler shift results in a larger observed gamma-ray energy difference between detector rings for a given transition). The particle distribution parameters $\mu$, $\sigma$, and $\lambda$ discussed in Section 3.2.2 were determined using a grid minimiza-
Figure 5.2: Scheme of observed levels and gamma rays in $^{28}$Mg. Arrow widths specify relative intensities of gamma rays. Spin-parity values are taken from Table 5.3 and Ref. [82].
Table 5.1: Fit and adopted parameters of the $\delta E_{ex}$ distribution for protons observed in the $^{12}$C($^{18}$O,2$p$)$^{28}$Mg reaction.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Fit Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu$</td>
<td>19(4)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>2(2)</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>0.05(3)</td>
</tr>
</tbody>
</table>

Table 5.2: Listing of gamma ray/charged particle detector groups corresponding to unique TIGRESS/CsI(Tl) detector combinations with a range of average Doppler shift factors $D = E_{det}/E_0$ for gamma rays detected in the TIGRESS core derived from the GEANT4-based simulations.

<table>
<thead>
<tr>
<th>Group</th>
<th>$D$</th>
<th>No. of detector combinations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$1.032 &lt; D$</td>
<td>6370</td>
</tr>
<tr>
<td>2</td>
<td>$1.018 &lt; D \leq 1.032$</td>
<td>4286</td>
</tr>
<tr>
<td>3</td>
<td>$1.0 &lt; D \leq 1.018$</td>
<td>5990</td>
</tr>
<tr>
<td>4</td>
<td>$0.980 &lt; D \leq 1.0$</td>
<td>7330</td>
</tr>
<tr>
<td>5</td>
<td>$0.968 &lt; D \leq 0.980$</td>
<td>3657</td>
</tr>
<tr>
<td>6</td>
<td>$D \leq 0.968$</td>
<td>6333</td>
</tr>
</tbody>
</table>

The angular distribution of charged particles detected in the CsI(Tl) detector array was matched between experimental and simulated data using the weighting method of Section 4.1.2 and Equation 4.1. Detector rings used for biasing were taken as defined in Table 3.1, with detector rings 0 through 3 used in this experiment.

As in Ref. [83], each unique combination of a TIGRESS core and two CsI(Tl) detectors was classified into a group according to the average simulated Doppler shift factor $D = E_{det}/E_0$ of a gamma ray detected in the TIGRESS core for an event containing these detectors, and lineshapes were compared to the experimental data on a group-by-group basis, as shown in Figure 5.3. Average Doppler shift factors contained in each group are listed in Table 5.2. This method of grouping the data allows for clean separation of individual components of the lineshape.

Table 5.3 lists the properties of $^{28}$Mg measured in this experiment. Reported level energies include a correction for the recoil energy of the $^{28}$Mg nucleus in addition to the observed
Figure 5.3: Comparison of lineshapes from data and GEANT4 simulations for the gamma rays depopulating the 4021.4(3) and 6528.2(9) keV levels in $^{28}\text{Mg}$, shown for each detector group. $\tau_{\text{mean}}$ values of 180 fs for the 4021.4(3) keV level and 190 fs for the 6528.2(9) keV level were used. The total simulated lineshape contains contributions from the decay of both levels (since the 6528.2(9) keV level decays to the 4021.4(3) keV level which then further decays, its lineshape contains two gamma rays), as well as feeding of the 4021.4(3) keV level from the 5184.3(5) and 6528.2(9) keV levels.
Figure 5.4: Background subtracted gamma-ray coincidence spectra showing observed levels in $^{28}\text{Mg}$, after gating on the $4^+_1 \rightarrow 2^+_1$ transition (top) and the $2^+_1 \rightarrow 0^+_1$ transition (bottom), from Doppler corrected thin target data. Inset figures show the high energy region of the corresponding spectrum.
Table 5.3: List of $^{28}\text{Mg}$ levels and gamma rays observed in this work, and measured mean level lifetimes.

<table>
<thead>
<tr>
<th>Level energy (keV)</th>
<th>Gamma rays (keV)</th>
<th>$I_{\gamma,\text{rel}}$</th>
<th>$I^\gamma$</th>
<th>$\tau_{\text{mean}}$ (fs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1473.63(9)</td>
<td>1473.59(9)</td>
<td>1.000(15)</td>
<td>$2^+$</td>
<td>$&gt; 1.5 \times 10^3$</td>
</tr>
<tr>
<td>4021.4(3)</td>
<td>2547.6(3)</td>
<td>0.65(2)</td>
<td>$4^+$</td>
<td>1.8(3) x 10^2</td>
</tr>
<tr>
<td>4556.3(12)</td>
<td>3082.5(12)</td>
<td>0.119(16)</td>
<td>-</td>
<td>$&lt; 42$</td>
</tr>
<tr>
<td>4879(2)</td>
<td>3405(2)</td>
<td>0.023(7)</td>
<td>-</td>
<td>$&lt; 26$</td>
</tr>
<tr>
<td>5172.9(7)</td>
<td>3698(3), 1151.5(6)</td>
<td>0.08(2), 0.031(10)</td>
<td>$3^-$</td>
<td>$2.6^{+1.6}_{-1.3} \times 10^2$</td>
</tr>
<tr>
<td>5184.3(5)</td>
<td>1163.1(4)</td>
<td>0.120(9)</td>
<td>$(4^+)$</td>
<td>$21^{+15}_{-14}$</td>
</tr>
<tr>
<td>5475(3)</td>
<td>4001(3)</td>
<td>0.018(8)</td>
<td>-</td>
<td>$&lt; 5.0 \times 10^2$</td>
</tr>
<tr>
<td>5672(3)</td>
<td>4198(3)</td>
<td>0.048(10)</td>
<td>-</td>
<td>$1.1(4) \times 10^2$</td>
</tr>
<tr>
<td>6139.0(12)</td>
<td>4664.9(12)</td>
<td>0.032(9)</td>
<td>$(0,4)^-$</td>
<td>$&gt; 1.0 \times 10^3$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$&lt; 2.5 \times 10^5$</td>
</tr>
<tr>
<td>6528.2(9)</td>
<td>1356.3(12), 2506.7(8)</td>
<td>0.021(5), 0.115(12)</td>
<td>$(4,5)^-$</td>
<td>1.9(6) x 10^2</td>
</tr>
<tr>
<td>7203(3)</td>
<td>3181(3), 5723(5)</td>
<td>0.033(10), 0.009(4)</td>
<td>$(2^+,3,4^+)$</td>
<td>$&lt; 3.8 \times 10^2$</td>
</tr>
<tr>
<td>7747(2)</td>
<td>3726(2)</td>
<td>0.033(8)</td>
<td>(5)</td>
<td>$&lt; 85$</td>
</tr>
<tr>
<td>7929.3(12)</td>
<td>3907.6(12)</td>
<td>0.060(11)</td>
<td>$(6^+)$</td>
<td>$&lt; 42$</td>
</tr>
<tr>
<td>8438(5)</td>
<td>4416(5)</td>
<td>0.024(10)</td>
<td>(5,6)</td>
<td>$&lt; 1.5 \times 10^3$</td>
</tr>
</tbody>
</table>

*Data from Ref. [82]. All other observations from this work. *Limit reported to a 90% confidence level.
*Limit estimated based on lineshape (see text).

Table 5.4: Statistical and systematic uncertainties for lifetimes reported in Table 5.3. Levels with $\tau_{\text{mean}}$ reported as a limit are excluded.

<table>
<thead>
<tr>
<th>Level energy (keV)</th>
<th>$\delta \tau_{\text{stat}}$ (fs)</th>
<th>$\delta \tau_{\text{sys}}$ (fs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4021.4(3)</td>
<td>20</td>
<td>19</td>
</tr>
<tr>
<td>5172.9(7)</td>
<td>+130</td>
<td>100</td>
</tr>
<tr>
<td>5184.3(5)</td>
<td>+14</td>
<td>2</td>
</tr>
<tr>
<td>5672(3)</td>
<td>40</td>
<td>13</td>
</tr>
<tr>
<td>6528.2(9)</td>
<td>50</td>
<td>40</td>
</tr>
<tr>
<td>7203(3)</td>
<td>240</td>
<td>50</td>
</tr>
</tbody>
</table>

gamma-ray energies. Intensities of gamma rays $I_{\gamma,\text{rel}}$ are listed relative to the intensity of the 1473.59(9) keV gamma ray. $1\sigma$ statistical and systematic uncertainties $\delta \tau_{\text{stat}}$ and $\delta \tau_{\text{sys}}$ for lifetime values are listed in Table 5.4. For lifetimes reported as a limit, the 90% confidence bounds are reported. Figure 5.4 shows gamma-ray coincidence spectra demonstrating the quality of the data and providing evidence for the existence of the identified levels.

Spins of observed levels were investigated using experimentally observed gamma-ray angular distributions, constructed from the number of counts observed in all TIGRESS detector cores at a fixed angle from the beam axis. Correction of the angular distributions for the relative detection efficiency of each of the TIGRESS rings was performed by normalizing the angular distribution of the gamma ray of interest to that of a 2754 keV gamma ray observed in beta decay data taken during this experiment on a per-ring basis. This normalized angular distribution was then fitted to Equation 2.39 to determine the $a_2$ and
Table 5.5: Table of $a_2$ and $a_4$ coefficients obtained from angular distributions of gamma-rays observed in this work.

<table>
<thead>
<tr>
<th>Gamma-ray energy (keV)</th>
<th>$a_2$</th>
<th>$a_4$</th>
<th>$a_2(a_4 = 0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1163.1(4)</td>
<td>0.6(5)</td>
<td>0.1(6)</td>
<td>0.5(3)</td>
</tr>
<tr>
<td>2506.7(8)</td>
<td>0.0(4)</td>
<td>0.1(5)</td>
<td>-0.1(3)</td>
</tr>
<tr>
<td>2547.6(3)</td>
<td>0.20(10)</td>
<td>-0.19(13)</td>
<td></td>
</tr>
<tr>
<td>3082.5(12)</td>
<td>0.7(5)</td>
<td>0.6(5)</td>
<td></td>
</tr>
<tr>
<td>3726(2)</td>
<td>-0.3(6)</td>
<td>0.0(7)</td>
<td>-0.3(4)</td>
</tr>
<tr>
<td>3907.6(12)</td>
<td>0.2(4)</td>
<td>-0.3(5)</td>
<td>0.36(19)</td>
</tr>
<tr>
<td>4664.9(12)</td>
<td>-0.5(8)</td>
<td>-0.2(10)</td>
<td>-0.4(5)</td>
</tr>
</tbody>
</table>

Figure 5.5: Angular distributions of the 2547.6(3) keV, 3726(2) keV, and 3907.6(12) keV gamma rays observed in this work, normalized to the relative efficiency of each ring of TIGRESS cores. Fits to Equation 2.39 are shown. The angular distribution of the 2547.6(3) keV gamma ray is consistent with a $4^+ \rightarrow 2^+$ E2 transition, whereas the other angular distributions are further discussed in Sections 5.1.3 and 5.1.4.

$a_4$ coefficients which depend on the multipolarity of the transition as discussed in Section 2.5.5. Selected angular distribution data is plotted in Figure 5.5, and tabulated values of $a_2$ and $a_4$ are shown in Table 5.5. For some high-lying transitions with low statistics, the only information provided by the angular distribution was whether the distribution was peaked at $90^\circ$ ($a_2 < 0$) or at $0^\circ$ ($a_2 > 0$). In these cases, the value of $a_2$ with $a_4$ fixed to zero is also shown in Table 5.5.

The tentative spin assignments for levels in $^{28}$Mg listed in Table 5.3 were made based on $a_2$ and $a_4$ values for gamma rays depopulating the level, level excitation energies, the observed decay scheme, and/or transition rate arguments. Multipolarities of all observed transitions were assumed to be E2 or lower with the exception of the 4664.9(12) keV gamma ray depopulating the 6139.0(12) keV level, discussed in Section 5.1.7.
5.1.1 Details of observed levels

Several new levels and gamma rays were observed at high excitation energy near the neutron separation energy $S_n = 8503.4(20)$ keV. The strong presence of $^{27}$Mg ($2p1n$ channel) in the unseparated data provides additional evidence that $^{28}$Mg was populated at energies close to $S_n$ in this experiment. Based on the measured gamma-ray intensities, 64(7)% of the population of the $4^+_1$ state and 98(4)% of the population of the $2^+_1$ state originates from observed feeding by higher lying states.

Some low-spin levels in $^{28}$Mg including excited $0^+$ states previously observed at 3862 and 5702 keV [82] were not observed in this experiment. This is likely because the fusion-evaporation process preferentially populates levels with high spin, as discussed in Section 2.2.2.

Detailed results obtained for select levels of interest (in particular those for which spin-parities are measured or for which there were additional complications to the analysis) are provided below.
5.1.2 8438 keV level

A gamma ray at 4418 keV was tentatively observed in a previous study by Keyes et al. [87] in which it was tentatively assigned to a $6^+_1 \rightarrow 4^+_1$ transition from a level at 8438 keV. In the present work a level is inferred at 8438(5) keV from the presence of a gamma ray at 4416(5) keV, confirming the previous observation. Other candidates for the yrast $6^+$ state in $^{28}$Mg are also observed at level energies of 7929.3(12) and 7747(2) keV. All of these levels are observed in coincidence with the $4^+_1 \rightarrow 2^+_1$ and $2^+_1 \rightarrow 0^+_1$ transitions, and lie at excitation energies in general agreement with the $I(I+1)$ level spacing predicted assuming the nucleus acts as a rigid rotor. However, the energy systematics and level spacing between the $2^+_1$, $4^+_1$, and $6^+_1$ levels in neighbouring isotopes and isotones (shown in Figure 5.6) suggest that the $6^+_1$ level in $^{28}$Mg lies between 7.5 and 8 MeV excitation energy, excluding the 8440(7) keV level.

Due to insufficient statistics, the lifetime and spin-parity of this level could not be directly determined from the data. However, as there is no apparent stopped line for the 4416(5) keV gamma ray in the DSAM data, the lifetime of the level is inferred to be shorter than 1.5 ps based on comparison to the low-intensity stopped line observed for a 4664.9(12) keV gamma ray depopulating a level at 6139.0(12) keV (discussed further in Section 5.1.7). From a comparison of this lifetime limit to the Weisskopf estimates of Appendix B.3, the multipolarity of the observed transition from this level is assumed to be $L = 2$ or lower.

The spin of the 8438(5) keV level is therefore restricted to $I = (5, 6)$, since no transitions from this level to the known low-lying $2^+$ and $3^-$ levels are observed. If the 8438(5) keV level were low spin, M1 and/or E2 transitions to these low-lying levels would be expected to dominate the branching.

5.1.3 7929 keV level

A previously unobserved gamma ray at 3907.6(12) keV was assigned to a transition from a new level at 7929.3(12) keV to the $4^+_1$ level in $^{28}$Mg. Although this gamma ray is expected to overlap with the single-escape peak of the 4418 keV gamma ray, the intensity of the escape peak predicted from GEANT4-based simulations developed for the TIGRESS spectrometer in Ref. [63] is < 10% the intensity of the observed peak, hence a new transition was inferred. A lifetime limit $\tau_{\text{mean}} < 42$ fs was determined for this level from the lineshape simulations. Simulations were performed with both the gamma ray energy $E_\gamma$ and effective lifetime $\tau_{\text{mean}}$ as free parameters. The minimum $\chi^2$ value simultaneously corresponding to the best $E_\gamma$ and $\tau_{\text{mean}}$ values were found by fitting the $\chi^2$ surface in two dimensions, as shown in Figure 5.7. The limit $\tau_{\text{mean}} < 42$ fs was adopted based on the 90% confidence bound of the $\chi^2$ surface, assuming a minimum at 0 ps lifetime.

The level excitation energy of 7929.3(12) keV suggests that the newly observed state is a candidate for the yrast $6^+$ level in $^{28}$Mg. The $a_2$ and $a_4$ values determined for the angular
distribution of the 3907.6(12) keV gamma ray in Table 5.5, particularly the positive \(a_2\) value, suggest that this gamma ray corresponds to an E2 transition. When measuring the angular distribution, data from the thin and thick target parts of the experiment was combined, as there were no gamma rays observed to overlap with the 3907.6(12) keV gamma ray in the \(^{28}\text{Mg}\) data and the observed Doppler shift of this gamma ray was consistent in both sets of data due to the short lifetime of the transition.

Based on the angular distribution of the 3907.6(12) keV gamma ray (shown in Figure 5.5), the 7929.3(12) keV level is the best candidate in this data for the yrast \(6^+\) state in \(^{28}\text{Mg}\). The case for a \(6^+\) assignment is further supported by the systematics of neighbouring isotopes and isotones shown in Figure 5.6, and the preference for population of high-spin states by the fusion-evaporation process. The 7929.3(12) keV state is therefore tentatively assigned \(I^\pi = 6^+\).

### 5.1.4 7747 keV level

A previously unobserved gamma ray at 3726(2) keV was assigned to a transition from a new level at 7747(2) keV to the \(4_1^+\) level in \(^{28}\text{Mg}\). A lifetime limit \(\tau_{\text{mean}} < 85\) fs was determined for this level using the GEANT4-based lineshape simulations. When gating on the \(2_1^+ \rightarrow 0_1^+\) transition, the lineshape of the 3726(2) keV gamma ray overlapped with the 3698(3) keV gamma ray depopulating a level observed at 5172.9(7) keV, so the lifetimes of the two levels were determined simultaneously.

Due to its excitation energy of 7747(2) keV, this level is a candidate for the yrast \(6^+\) level in \(^{28}\text{Mg}\). When gating on the \(2_1^+ \rightarrow 0_1^+\) transition, the gamma ray at 3726(2) keV overlapped with another gamma ray observed in \(^{28}\text{Mg}\) at 3698(3) keV, so the angular distribution of this gamma ray was studied using a gate on the \(4_1^+ \rightarrow 2_1^+\) transition, which removed the
overlapping gamma rays but reduced statistics significantly. As shown in Table 5.5, the $a_2$ and $a_4$ values are inconclusive, but do not favour an E2 transition ($a_2 > 0$). Therefore the previously discussed level at 7929.3(12) keV is a stronger candidate for the yrast $6^+$ state in $^{28}$Mg. Using the arguments applied to the 8438(5) keV level, it is argued that the spin of this level is greater than $4\hbar$. As the 7929.3(12) keV level is assigned $I^\pi = (6^+)$ and is assumed to be yrast, the 7747(2) keV is assigned $I^\pi = (5)$.

### 5.1.5 7203 keV level

A gamma ray at 3181(3) keV was observed in coincidence with the $2^+_1 \rightarrow 0^+_1$ and $4^+_1 \rightarrow 2^+_1$ transitions and a gamma ray at 5723(5) keV in coincidence with the $2^+_1 \rightarrow 0^+_1$ transition, from which we deduce the existence of a level at 7203(3) keV. The possibility of the 3181(3) keV gamma ray being a single escape peak for the gamma ray at 3698(3) keV was ruled out by its observation in coincidence with the $4^+_1 \rightarrow 2^+_1$ transition. In order to accurately measure the branching of transitions depopulating the 7203(3) keV level, the contamination of a single escape peak was taken into account when determining the intensity of the 3181(3) keV gamma ray by measuring the intensity relative to that of the 1473.59(9) keV gamma ray when observed in coincidence with the $4^+_1 \rightarrow 2^+_1$ transition. After performing this correction, the intensity of the 5723(3) keV gamma ray was determined to be 27(13)% of the 3181(3) keV gamma ray.

The level energy is consistent with that of a level at 7200.3(6) keV previously observed via beta decay of $^{28}$Na [88]. However the placement of gamma rays differs compared to the previous work, which does not show this level decaying to the $4^+_1$ or $2^+_1$ levels, indicating that the level observed in this work is new and that there are most likely two levels near 7.2 MeV in $^{28}$Mg. A lifetime limit $\tau_{\text{mean}} < 380$ fs was determined for this level via the GEANT4-based lineshape simulations. From the lifetime and the observed decay scheme, the spin-parity of this level is assigned $I^\pi = (2^+, 3^+, 4^+)$ under the assumption that the multipolarity of observed transitions is E2 or lower.

### 5.1.6 6528 keV level

A previously unobserved gamma ray at 2506.7(8) keV was identified in coincidence with the $2^+_1 \rightarrow 0^+_1$ and $4^+_1 \rightarrow 2^+_1$ transitions. In addition, a low-intensity gamma ray at 1356.3(12) keV was identified in coincidence with the $2^+_1 \rightarrow 0^+_1$ and $3^-_1 \rightarrow 2^+_1$ transitions as shown in Figure 5.8. Thus the existence of a level with energy 6528.2(9) keV is inferred. This level energy agrees with that of a level previously observed at 6516(15) keV from magnetic spectrograph measurements of Ref. [89].

The 2506.7(8) keV gamma ray partially overlaps the 2547.6(3) keV gamma ray of the $4^+_1 \rightarrow 2^+_1$ transition in the thick target data. Additionally, the 2506.7(8) keV gamma ray corresponds to a transition to the $4^+_1$ level at 4021.4(3) keV, such that the observed lineshape
of the 2547.6(3) keV gamma ray is affected by the feeding from the 2506.7(8) keV gamma-ray transition. Due to the overlap and feeding scheme, the lifetimes of the 4021.4(3) and 6528.2(9) keV levels were determined simultaneously using the method of Section 3.2.4. Lineshapes corresponding to direct population of the 4021.4(3) keV level, feeding of the 4021.4(3) keV level from the 6528.2(9) keV level, and feeding of the 4021.4(3) keV level from another level observed at 5184.3(5) keV were simulated. These branches represent all statistically significant observed feeding of the 4021.4(3) keV level. The resulting best-fit lifetimes were $1.9(6) \times 10^2$ fs for the 6528.2(9) keV level and $1.8(3) \times 10^2$ fs for the 4021.4(3) keV level.

The $a_2$ and $a_4$ angular distribution coefficients shown in Table 5.5 for the 2506.7(8) keV gamma ray depopulating this level were inconclusive for spin-parity assignment. Based on the observed transitions to the $3^-_1$ and $4^+_1$ levels, the spin of the level is assumed to be 4 or 5. A $4^+$ assignment can be excluded based on non-observation of a transition to the $2^+_1$ level ($E_\gamma = 5055$ keV). The intensity limit of this hypothetical transition was determined to be $< 2.7$ % that of the observed transition to the $4^+_1$ level, which would lead to an extremely low E2 transition strength of $B(E2) < 8.39 \times 10^{-3}$ W.u. A $5^+$ assignment can also be excluded since such an assignment would cause the observed transition to the $3^-_1$ level to have an extremely large M2 transition strength $B(M2) = 9(3) \times 10^2$ W.u. The level is therefore assigned $I^\pi = (4,5)^-$. Of the two values, the $I^\pi = 5^-$ case is favoured based on the systematics of $4^-_1$ and $5^-_1$ levels in neighbouring isotopes and isotones shown in Figure 5.6, since the observed level energy of 6528.2(9) keV relative to the $I^\pi = 3^-_1$ level at 5172.9(7) keV is consistent with the observed spacing between $5^-_1$ and $3^-_1$ levels in neighbouring nuclei, whereas existing experimental data is insufficient to infer such a trend for $4^-_1$ levels.

5.1.7 6139 keV level

A level at 6139.0(12) keV was identified based on observation of a gamma ray at 4664.9(12) keV in coincidence with the $2^+_1 \rightarrow 0^+_1$ transition. This level energy agrees with that of a level previously observed at 6135(15) keV from magnetic spectrograph measurements of Ref. [89]. The lineshape of the 4664.9(12) keV gamma ray shown in Figure 5.9 indicates that it is stopped in the DSAM target data, corresponding to a mean lifetime limit of $> 1.0$ ps for this level. When combined with the high energy of the gamma ray, a very small transition strength is implied. As shown in Figure 5.9, all high energy gamma rays except 4664.9(12) keV are washed out when summing the DSAM target data over Doppler shift groups, indicating that only the 4664.9(12) keV gamma ray is emitted at rest. The presence of this gamma ray in the Doppler corrected thin target data and in coincidence with the $^{28}$Mg $2^+_1 \rightarrow 0^+_1$ transition indicates that the 4664.9(12) keV line belongs to $^{28}$Mg and does not originate from beta decay or a contaminant reaction channel.
Figure 5.8: Background subtracted gamma-ray coincidence spectrum following gating on the 1356.3(12) gamma ray in $^{28}$Mg.

An upper limit on the lifetime of this level may be estimated, since a stopped line is not observed in the thin target data. Since the thin target has a stopper foil mounted $\sim$2 mm downstream from the target foil, the absence of a stopped line indicates that the decay occurs before the $^{28}$Mg nucleus is able to traverse this distance. Assuming that the nucleus exits the target with speed 0.04$c$ (based on GEANT4 simulations) and that the target-stopper separation does not exceed 3 mm, an upper limit of 250 ps is determined for the 6139.0(12) keV level.

The angular distribution coefficients shown in Table 5.5 for the 4664.9(12) keV gamma ray are inconclusive for spin-parity assignment, however a gamma-ray multipolarity of (M2) is assigned based on the single-particle Weisskopf estimates, which for multipolarities below M2 give lifetimes significantly lower than 1 fs at this gamma-ray energy. The (M2) assignment is further supported by existing lifetime measurements for other levels, as all high energy E1, M1 and E2 transitions in $^{28}$Mg have been observed with lifetimes significantly shorter than 1 ps, implying that the observed transition is higher order than E2. Additionally, the 6139.0(12) keV level cannot be positive parity, since this would imply the existence of short-lived M1 and E2 transitions to other known positive parity states in $^{28}$Mg which are ruled out by the long lifetime of the observed transition. In principle if the transition is M2 it should be out-competed by fast E1 transitions to positive parity states, however this is also ruled out by the long lifetime of the observed transition. Either there is no final state to which an E1 transition may occur, or any E1 transitions which occur are hindered in rate compared to the M2 transition. Previously E1 transitions in $^{28}$Mg have been shown to be hindered by a factor of $10^{-3} - 10^{-4}$ with respect to the Weisskopf estimate in the
Figure 5.9: Lineshape of the 4664.9(12) keV gamma ray observed in background subtracted thin target and summed DSAM target data.

case of the 5172.9(7) keV level, as discussed in Ref. [88] and as indicated by the measured lifetime of this level in Table 5.3. This hindrance, combined with the reduced energy of a hypothetical E1 transition compared to the observed 4664.9(12) keV (M2) transition, may explain the non-observation of competing transitions from the 6139.0(12) keV level.

An M2 assignment for the transition to the $2^+_1$ level implies a spin-parity of $0^-\text{ or } 4^-$ for the 6139.0(12) keV level. The $0^-$ case is favoured based on the observed decay scheme, since no transition is observed from this level to the $3^-$ level at 5172.9(7) keV. The upper limit for the intensity of a transition to the $3^-_1$ level was established as 11% relative to the observed transition to the $2^+_1$ level. If the spin-parity of the 6139.0(12) keV level is $4^-$, this would imply a $B(M1; 4^- \to 3^-) < 3.47 \times 10^{-3}$ W.u., corresponding to an M1 transition which would be significantly weaker than other known M1 transitions in $^{28}\text{Mg}$ (with the weakest possible of the known transitions corresponding to depopulation of the 4879(2) keV level with a lower limit $B(M1; 2^+_1 \to 2^+_1) > 2.12 \times 10^{-2}$ W.u., using the data of Ref. [82] and the revised lifetime limit reported in Table 5.3). However, the $4^-$ case cannot be fully excluded based on this evidence, as very small $B(M1)$ values have been measured in the neighbouring isotope $^{30}\text{Si}$ [86]. Furthermore, the level energy of 6139.0(12) keV is consistent with the systematic trend of $4^-_1$ levels in neighbouring isotones shown in Figure 5.6, and preferential population of high spin states is expected using fusion-evaporation. Therefore, this level is assigned $I^\pi = (0, 4)^-$. 

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5.1.8 5184 keV level

A level at 5184.3(5) keV was identified from a gamma ray at 1163.1(4) keV in coincidence with the $2^+_1 \rightarrow 0^+_1$ and $4^+_1 \rightarrow 2^+_1$ transitions. A transition from this level to the $2^+_1$ level ($E_\gamma \approx 3711$ keV) is not ruled out as it may be obscured by gamma rays observed at 3698(3) and 3726(2) keV. An intensity limit $<28\%$ is established for the 5184.3(5) keV $\rightarrow 2^+_1$ transition relative to the 5184.3(5) keV $\rightarrow 4^+_1$ transition. A mean lifetime of $21^{+15}_{-14}$ fs was determined for this level based on the lineshape simulations.

The angular distribution coefficients shown in Table 5.5 are inconclusive for determining the spin-parity of this level, however the short lifetime precludes a negative parity assignment as E1 transitions are known to be hindered in $^{28}$Mg based on the lifetime of the $3^-_1$ level. Moreover, as the decay scheme indicates strong branching to the $4^+_1$ level rather than the $2^+_1$ level, the spin of this level is assumed to be $\geq 4$. The level energy of 5184.3(5) keV is much lower than predicted values for the yrast $5^+$ and $6^+$ levels from shell model calculations shown in Section 5.2.1, therefore this level is assigned $I^\pi = (4^+)$. 

5.1.9 5173 keV level

A level at 5172.9(7) keV was identified from a gamma ray at 1151.5(6) keV in coincidence with the $2^+_1 \rightarrow 0^+_1$ and $4^+_1 \rightarrow 2^+_1$ transitions and a gamma ray at 3698(3) keV in coincidence with the $2^+_1 \rightarrow 0^+_1$ transition. The level energy and gamma rays are consistent with a previously observed level at 5171.3(4) with spin-parity $3^-_1$ \cite{82}. The intensity of the 1151.5(6) gamma ray was measured at $41 \pm 18\%$ the intensity of the 3698(3) keV gamma ray, in agreement with the value of $38 \pm 2\%$ from the evaluated data \cite{82}. This value assumes that the geometrical coverage of the solid angle by TIGRESS detectors is sufficient to average any effects of angular distribution in the summed spectra used for analysis.

A mean lifetime of $2.6^{+1.6}_{-1.3} \times 10^2$ fs was determined for this level based on lineshape simulations for the 3698(3) keV gamma ray. When gating on the $2^+_1 \rightarrow 0^+_1$ transition, the lineshape of the 3698(3) keV gamma ray overlapped with the 3726(2) keV gamma ray depopulating the level at 7747(2) keV, so the lifetimes of the two levels were determined simultaneously. The lifetime of the 5172.9(7) keV level determined in this work is consistent with the previous evaluated value of $1.6(13) \times 10^2$ fs \cite{82}.

5.1.10 4556 keV level

A level at 4556.3(12) keV was identified based on observation of a gamma ray at 3082.5(12) keV in coincidence with the $2^+_1 \rightarrow 0^+_1$ transition. Two levels have been previously observed in $^{28}$Mg near this energy, at 4554.6(5) keV ($I^\pi = 2^+$, $E_\gamma = 3082.6(13)$ keV) and 4561.0(5) keV ($I^\pi = 1^+$, $E_\gamma = 3087.3(5)$ keV) \cite{82}. Based on the gamma-ray energy observed in this work it is concluded that the observed level is consistent with the 4554.6(5) keV level in Ref.
Table 5.6: Reported measurements of the mean lifetime $\tau_{\text{mean}}$ of the 4021.4(3) keV level in $^{28}\text{Mg}$.

<table>
<thead>
<tr>
<th>Study</th>
<th>$\tau_{\text{mean}}$ (fs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>P. Fintz et al. [90]</td>
<td>96(30)</td>
</tr>
<tr>
<td>T. R. Fisher et al. [91]</td>
<td>210(70)</td>
</tr>
<tr>
<td>This work</td>
<td>180(30)</td>
</tr>
</tbody>
</table>

[82]. A lifetime limit $\tau_{\text{mean}} < 42$ fs was determined for this level via the GEANT4-based lineshape simulations.

5.2 Interpretation of $^{28}\text{Mg}$ data

5.2.1 Collectivity in the yrast band

For the $4^+_1$ level at 4021.4(3) keV, the present evaluated lifetime of $1.5(5) \times 10^2$ fs reported in Ref. [82] was based on two conflicting measurements by Fintz et al. [90] and Fisher et al. [91], as summarized in Table 5.6. This disagreement has implications for the structure of the yrast band, as the measurement of Fintz et al. implies increasing $B(E2; I_i \rightarrow I_i - 2)$ transition strength with increasing spin - consistent with a rotational or vibrational model, while the measurement of Fisher et al. implies the opposite. As shown in Table 5.6, the lifetime measured in this work agrees with that of Fisher et al. with a large reduction in uncertainty, indicating that collective behaviour is weak in the yrast band.

Figure 5.10 shows a comparison of measured $B(E2; I_i \rightarrow I_i - 2)$ values to calculations using various phenomenological models in the $sd$ and $sdpf$ shells, including the USDB [36], SDPF-U [38] and SDPF-MU [40] interactions. $B(E2; I_i \rightarrow I_i - 2)$ values were also calculated using various ab initio approaches: in-medium similarity renormalization group (IM-SRG) [42] in the $sd$ shell with $\hbar \omega = 24$ MeV, the coupled-cluster effective interaction (CCEI) [43] using 13 major harmonic oscillator shells with $\hbar \omega = 20$ MeV, as well as the symmetry adapted no-core shell model (SA-NCSM) [44] using the NNLOpt chiral potential [92] in 9 major harmonic oscillator shells with $\hbar \omega = 15$ MeV. Effective charges for the proton $e_p = 1.5e$ and neutron $e_n = 0.5e$ were used in all calculations except the SA-NCSM which used no effective charges$^1$. Uncertainties for the SA-NCSM calculations are plotted in Figure 5.10, with lower and upper limits corresponding to calculations in 9 and 11 major harmonic oscillator shells respectively. Taking these uncertainties into consideration, the SA-NCSM calculations are in good agreement with the $B(E2; 4^+_1 \rightarrow 2^+_1)$ value measured in this study, and are within the limits of the other measured $B(E2)$ values.

---

$^1$Effective charges, though nonphysical, are often used as a phenomenological parameter in models to compensate for residual nucleon-nucleon interactions not accounted for in the model. A goal of ab-initio models is to provide accurate predictions without the use of effective charges (i.e. $e_p = 1.0e$, $e_n = 0$).
It is evident that most of the models used fail to correctly predict the observed $B(E2; I_i \rightarrow I_i - 2)$ trend, usually by over-predicting the $B(E2; 4^+_1 \rightarrow 2^+_1)$ value. A possible explanation comes from the ab initio SA-NCSM calculations, which are shown in Figure 5.11 with calculated levels organized into rotational bands based on the contributions of various nuclear shapes. The ground state band in particular is dominated by a major triaxial ($\gamma = 30^\circ$) shape and two major prolate ($\gamma < 30^\circ$) shapes. Table 5.7 shows the contributions of these shapes to the individual states in the yrast band. In the $4^+_1$ and $6^+_1$ levels the two prolate shapes occur with nearly equal probability, resulting in increased competition between these two configurations. This in turn hinders transitions to the lower lying levels which do not feature the same degree of mixing between the two prolate configurations. The hindered transition rate results in a lowered $B(E2; 4^+_1 \rightarrow 2^+_1)$ value, as indicated in Figure 5.10 for the SA-NCSM calculations.

### 5.2.2 Influence of the neutron $pf$ shell

Figure 5.12 shows a comparison of measured level energies in $^{28}$Mg to calculations using the USDB [36], SDPF-U [38], and SPDF-MU [40] phenomenological shell models. The USDB calculations only consider nucleon excitations within the $sd$ shell, hence only positive parity states are predicted by this model. The SDPF-U and SDPF-MU calculations allow for
Figure 5.11: Comparison of experimentally determined level energies in $^{28}\text{Mg}$ to calculated values derived from *ab initio* SA-NCSM calculations, with $\hbar\omega = 15$ MeV in 9 major harmonic oscillator shells. Bands are labelled based on the major shape contributions to the lowest $0^+$ state, which remain roughly the same for the higher spin states (see also Table 5.7). Energies of excited $0^+$ levels are taken from Ref. [82], all other levels were observed in this work.

Table 5.7: Contributions of major triaxial and prolate shapes to yrast states in $^{28}\text{Mg}$, based on *ab initio* SA-NCSM calculations.

<table>
<thead>
<tr>
<th>$I^\pi$</th>
<th>triaxial</th>
<th>prolate</th>
<th>prolate</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^+_1$</td>
<td>0.40</td>
<td>0.25</td>
<td>0.15</td>
</tr>
<tr>
<td>$2^+_1$</td>
<td>0.40</td>
<td>0.23</td>
<td>0.16</td>
</tr>
<tr>
<td>$4^+_1$</td>
<td>0.40</td>
<td>0.21</td>
<td>0.18</td>
</tr>
<tr>
<td>$6^+_1$</td>
<td>0.40</td>
<td>0.19</td>
<td>0.21</td>
</tr>
</tbody>
</table>
excitation of a single neutron to the pf shell in order to predict energies of negative parity states.

It is evident that calculations using the USDB interaction are able to reproduce energies of low-lying positive parity levels, as was previously indicated in a study by Kura et al. [88]. This result affirms the conclusion of Kura et al. that the low-lying levels of $^{28}\text{Mg}$ are dominated by sd-shell configurations and hence that $^{28}\text{Mg}$ is outside of the ‘island of inversion’ discussed in Section 1.1.3. Figure 5.12 further shows that the USDB interaction is able to reproduce level energies of some high-lying levels and indicates that the $I^\pi = (5)$, $(6^+)$ levels above 7 MeV are likely positive parity and predominantly sd configurations. However, the presence of levels with negative parity at high excitation energy indicates excitation of neutrons to the pf shell above the $N = 20$ shell gap and/or hole excitations from the p shell below the $N, Z = 8$ shell gap. In general, the former case might be expected due to the proximity of $^{28}\text{Mg}$ to the island of inversion. The higher spin $I^\pi = 3^-, (4, 5^-)$ levels are more likely to contain significant population of orbitals with high total angular momentum in the pf shell, with $I^\pi = 5^-$ not possible from single hole excitation into the sd shell$^2$.

The nature of the $I^\pi = (0, 4)^-$ level is less clear. According to Ref. [93], $I^\pi = 0^-_1$ states in the sd shell are understood to transition from predominantly hole $(p - sd)$ to predominantly particle $(sd - pf)$ configurations with increasing $N$ and $Z$. The SDPF-MU calculations of Figure 5.12 suggest the latter case, since the excitation energy of the observed $I^\pi = (0, 4)^-$ level agrees well with that of the predicted $0^-_1$ state arising from a particle $(sd - pf)$ configuration. If the spin-parity of the level is instead $4^-$, the orbital population may be similar to the $I^\pi = 3^-, (4, 5^-)$ levels, as indicated in Figure 5.13 which shows calculated neutron occupancies for negative parity yrast states using the SDPF-MU interaction. In these calculations the $3^-_1$, $4^-_1$, and $5^-_1$ levels are understood to arise predominantly from neutron excitation to the $f_{7/2}$ subshell, while the $0^-_1$ level mostly contains neutrons in the p orbitals. This prediction supports the $0^-$ case for the experimentally observed $I^\pi = (0, 4)^-$ level, since the similar predicted neutron occupancies between the $4^-_1$ and $3^-_1$ should result in a strong M1 transition between these levels, and such a transition is not observed in the data (the branching limit for a hypothetical transition to the $I^\pi = 3^-$ level is established in Section 5.1.7). Experimental limits for B(M1) and B(M2) values are compared to the predicted values in Table 5.8, indicating that the observed limits are consistent with either the $0^-$ or $4^-$ case for the SDPF-U calculations, but only with the $0^-$ case for the SDPF-MU calculations. The prediction of similar neutron occupancies for the $3^-_1$, $4^-_1$, and $5^-_1$ levels is further supported by the experimental data since a $(5^-) \rightarrow 3^-$ transition is observed.

The $I^\pi = (0, 4)^-$ level has some additional implications for the structure of $^{28}\text{Mg}$. As is shown in Figure 5.12, there are predicted levels with $I^\pi = 1^-, 2^-$ lower in energy than

$^2$Below the $N, Z = 20$ shell gap, the maximum spin that a particle-hole pair may couple to is $I = 3/2 + 5/2 = 4$. 98
Figure 5.12: Comparison of experimentally determined yrast level energies in $^{28}\text{Mg}$ to calculated values derived from various interaction models in the $sd$ and $sd_{p_{f}}$ shells. Negative parity levels are shown in light grey. Connections are drawn between calculated levels and their closest candidates in the experimental data. The known $I^\pi = 1^+$ level in $^{28}\text{Mg}$ is taken from Ref. [82], all other levels were observed in this work. The observed $I^\pi = 3^-, (4^+)$ levels near 5.2 MeV are slightly offset in energy for readability.
Figure 5.13: Calculated neutron occupancies in the \( pf \) shell using the SDPF-MU interaction.

Table 5.8: Calculated reduced transition probabilities for select transitions from the \( 0^- \) and \( 4^- \) levels, compared to experimental limits obtained for the observed \( I^\pi = (0, 4)^- \) level.

<table>
<thead>
<tr>
<th>( I^\pi_i \rightarrow I^\pi_f )</th>
<th>( B(EL \text{ or } ML, I_i \rightarrow I_f) ) (W.u.)</th>
<th>SDPF-U</th>
<th>SDPF-MU</th>
<th>Limit from expt.</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 0^- \rightarrow 2^+_1 ) (M2)</td>
<td>2.66E-1</td>
<td>4.04E-1</td>
<td></td>
<td>( \geq 7.91E-3 ), ( \leq 2.19 )</td>
</tr>
<tr>
<td>( 4^- \rightarrow 2^+_1 ) (M2)</td>
<td>5.12E-1</td>
<td>5.08E-1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( 4^- \rightarrow 3^-_1 ) (M1)</td>
<td>3.50E-4</td>
<td>1.82E-2</td>
<td></td>
<td>( \leq 3.47E-3 )</td>
</tr>
</tbody>
</table>

the \( I^\pi = 0^-_1, 4^-_1 \) levels in the SDPF-U and SDPF-MU models. However, no \( I^\pi = 1^-_1, 2^-_1 \) levels have been firmly identified in \(^{28}\text{Mg}\), and no candidate levels are observed in this study, despite the possibility of strong M1 and/or E2 transitions from the \( I^\pi = (0, 4)^- \) level to these levels. In particular, there is a previously observed \( I^\pi = 1^- \) (unknown parity) level at 5193.1(5) keV which may be a candidate for the \( 1^-_1 \) state [82], however that level was not observed in this work. If the 5193.1(5) keV level is \( I^\pi = 1^- \), a strong \( 0^- \rightarrow 1^- \) M1 transition may be expected, and the absence of this transition in the experimental data may indicate that the \( I^\pi = (0, 4)^- \) level is \( 4^- \). However, if the 5193.1(5) keV level is \( I^\pi = 1^+ \), then it is consistent with the \( 0^- \) case for the \( I^\pi = (0, 4)^- \) level, due to the hindrance of E1 transitions as discussed in Section 5.1.7. Overall, these ambiguities may be resolved in future experiments by firm assignment of the spin-parity of the levels in question (obtainable via high statistics gamma-ray angular distribution measurements and/or improved branching ratio limit measurements).

As was discussed in Section 1.1.1, it is possible to determine effective energy levels of nuclear orbitals \( E_{\text{esp},j} \) by taking into account the monopole component of the two-body residual interactions between nucleons. These effective single particle energies for protons and neutrons obtained using the SDPF-MU interaction are shown in Figure 5.14. This figure indicates that the \( N = 20 \) shell gap is smaller for neutrons than it is for protons, and
therefore suggests that the lowest lying negative parity states arise from neutron excitation rather than proton excitation.

5.2.3 For the future

The aforementioned spin-parity assignments for high-lying levels in $^{28}\text{Mg}$, while supported by the observed decay scheme and model calculations, could not be conclusively determined in this work and may be an area of interest for future studies. Conclusive spin-parity assignments for high-lying levels in $^{28}\text{Mg}$ and heavier species approaching the ‘island of inversion’ can provide an unambiguous signature of shell evolution around the $N = 20$ gap, and would be an appropriate objective for future studies in this region.

For some of the longer-lived levels in $^{28}\text{Mg}$ ($\tau_{\text{mean}} > 1$ ps), lifetime measurements were limited by the sensitivity of the DSAM technique. Lifetimes of these states may be measured using TIP at TRIUMF by employing the Recoil Distance Method (RDM) instead of the DSAM. These measurements would make use of an existing TIP plunger device, which has previously been demonstrated in lifetime measurements of the $2^+_1 \rightarrow 0^+_1$ transitions in $^{84}\text{Kr}$ [83] and radioactive $^{94}\text{Sr}$ [64]. A high precision measurement of the $2^+_1 \rightarrow 0^+_1$ transition rate in $^{28}\text{Mg}$ using TIP can provide an additional test of the \textit{ab-initio} SA-NCSM calculations shown in Figure 5.10, which suggest a significantly lower $B(E2; 2^+_1 \rightarrow 0^+_1)$ value than has been measured in previous studies. Additionally, a high precision lifetime measurement of the long-lived $I^\pi = (0, 4)^-$ state observed in this work would further constrain the transition strength limits discussed in Section 5.1.7, perhaps allowing the spin-parity of the level to be firmly assigned.
Although it is apparent that the $^{12}\text{C}(^{18}\text{O},2p)^{28}\text{Mg}$ fusion-evaporation reaction gives access to previously unobserved states at high excitation energy, the low statistics obtained in this work following over 60 hours of intense beam on target indicate a very low cross section for this reaction. Future investigations of $^{28}\text{Mg}$ isotopes should also consider alternate reaction mechanisms. For instance, a future investigation of the long-lived $I^\pi = (0, 4)^-$ state at 6139.0(12) keV may consider the $^{26}\text{Mg}(t,p)^{28}\text{Mg}$ transfer reaction employed in Ref. [89].

Fusion-evaporation remains a good choice for the study of more exotic neutron-rich nuclei in this region such as $^{30,31,32}\text{Mg}$, in particular for population of high-spin states which are difficult to access via other reactions. Such studies may be achieved by using neutron-rich radioactive ion beams such as $^{19,20}\text{O}$ and/or a $^{14}\text{C}$ enriched target ($^{14}\text{C}$ is a naturally occurring radioisotope with a half-life of 5700 years). It is possible to directly access the island of inversion via fusion evaporation using a reaction such as $^{14}\text{C}(^{20}\text{O},2p)^{32}\text{Mg}$, however such an experiment would require development of appropriate targets and neutron-rich oxygen beams at high intensity, neither of which has yet been attempted at SFU and TRIUMF.
Chapter 6

Conclusion and Outlook

The work in this thesis began with the development of analysis software and procedures in order to study electromagnetic transition rates in nuclei populated by fusion-evaporation reactions. $^{22}\text{Ne}$, a well characterized stable species, was studied in order to benchmark the GEANT4 simulation code and other tools developed as a part of this project. In this study effects including charged particle energies, feeding of low-lying states from high-lying states, and the effect of stopping models were investigated in order to determine transition rates in $^{22}\text{Ne}$ to high precision. Overall, the lifetimes measured in this study agreed well with previous measurements, validating the GEANT4 simulations and external tools.

Construction and commissioning of the TIP CsI ball array has been completed. The high charged particle detection efficiency of the array allows for increased reaction channel sensitivity in all experiments, and will be particularly beneficial for fusion-evaporation experiments populating nuclei far from stability with high charged particle multiplicity. Future experiments at TRIUMF will couple the CsI ball with the TIP plunger device, allowing for measurements of lifetimes longer than 1 ps using the RDM. Presently planned TIP experiments include plunger measurements of $^{20}\text{Mg}$ near the proton drip line, $N = Z$ $^{68}\text{Se}$, and DSAM measurements of $^{55}\text{Ni}$ and $^{55}\text{Co}$ near the $N = Z = 28$ double shell closure. Further investigation of $^{28}\text{Mg}$ using the TIP plunger device may also be considered, as outlined in Section 5.2.3.

As part of the initial experimental campaign using the CsI ball array, the neutron-rich nucleus $^{28}\text{Mg}$ has been studied at high spin and excitation energy using a fusion-evaporation reaction. Three new levels were identified at 7203(3) keV, 7747(2) keV, and 7929.3(12) keV based on $\gamma - \gamma$ coincidence spectroscopy. Arguments using the branching ratios of observed transitions and transition strengths measured via the DSAM allowed for tentative spin-parity assignments for most observed levels, including two new assignments with negative parity.

A revised $\text{B(E2; } 4_1^+ \rightarrow 2_1^+ \text{)}$ value for $^{28}\text{Mg}$ was obtained in this study, indicating reduced collectivity in the yrast band compared to the previous results of Fintz et al.
agreement with Fisher et al. Based on a comparison to ab initio SA-NCSM calculations, it is speculated that the reduction in collectivity which is observed at high spin results from competition between major prolate configurations in both the $4^+_1$ and $6^+_1$ states.

Comparison of the experimental level energies to model calculations in the $sd$ and $sdpf$ shells indicates that the positive parity levels of $^{28}$Mg are dominated by $sd$ configurations while at high excitation energy the influence of the intruder orbitals of the $pf$ shell is significant. Calculations using the SDPF-MU interaction are able to reproduce the observed negative parity states by single neutron excitation into the $pf$ shell. Based on these calculations, the $0^-_1$ level is understood to arise predominantly from excitation to the $p$ shell, with other negative parity states resulting predominantly from excitation to the $f$ shell. This prediction is well-supported by the observation of a long-lived $I^\pi = (0, 4)^-$ level with energy and transition strengths which are consistent with the predicted $0^-_1$ level.

Overall, an initial campaign of fusion-evaporation experiments using TIP and TIGRESS to study $sd$ shell nuclei has been completed. As part of this work, a large amount of experimental infrastructure has been developed including charged particle detectors and analysis procedures and code. Gamma ray spectroscopy and lifetime measurements have been performed for $^{22}$Ne and $^{28}$Mg, with the latter study providing new information on the influence of intruder orbitals approaching the $N = 20$ ‘island of inversion’.
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Appendix A

Analysis Techniques

A.1 $\chi^2$ minimization on a grid

In this work, it was often necessary to minimize a merit function (such as the $\chi^2$ goodness of fit statistic between simulated and experimental data) as a function of one or more variables. For one independent variable (such as the lifetime of the transition of interest), this was done by generating simulated data at discrete values of the independent variable and fitting the data to a cubic function (Equation A.1), and finding the local minimum.

$$f(x) = a_1 x^3 + a_2 x^2 + a_3 x + a_4$$  \hspace{1cm} (A.1)

When fitting $\chi^2$ data, the 1σ uncertainty bounds about the minimum value $\chi^2_{\text{min}}$ correspond to values of $\chi^2_{\text{min}} + 1$ [94].

For two independent variables $x$ and $y$ (such as the two lifetimes of overlapping transitions), a grid of data points was generated for discrete pairs of values $(x, y)$. This data was fit to a bi-variate cubic polynomial (Equation A.2), and the local minimum found.

$$f(x, y) = a_1 x^3 + a_2 y^3 + a_3 x^2 y + a_4 y^2 x + a_5 x^2 + a_6 y^2 + a_7 x y + a_8 x + a_9 y + a_{10}$$  \hspace{1cm} (A.2)

When fitting $\chi^2$ data, the 1σ uncertainty bounds correspond to an ellipse about the minimum value where $\chi^2 = \chi^2_{\text{min}} + 2.3$ [94].

For three independent variables $x$, $y$, and $z$, a grid of data points was generated for discrete combinations of values $(x, y, z)$. This data was fit to a tri-variate paraboloid (Equation A.3), and the local minimum found.

$$f(x, y, z) = a_1 x^3 + a_2 y^3 + a_3 z^3 + a_4 x^2 y + a_5 y^2 x + a_6 z^2 + a_7 z x^2 + a_8 y^2 z + a_9 z x^2 + a_{10} x y z + a_{11}$$  \hspace{1cm} (A.3)
\[ f(x, y, z) = a_1x^2 + a_2y^2 + a_3z^2 \\
+ a_4xy + a_5xz + a_6yz \\
+ a_7x + a_8y + a_9z + a_{10} \] (A.3)

When fitting \( \chi^2 \) data, the 1\( \sigma \) uncertainty bounds correspond to an ellipsoid about the minimum value where \( \chi^2 = \chi^2_{min} + 3.53 \) \cite{94}.

Fitting of data using the above methods is implemented in the Gridlock code described in Appendix C.
Appendix B

Derivations

B.1 Orbital electron speeds

Using the classical Bohr picture of electron orbits around the nucleus, the Coulomb force between an orbital electron and the nucleus with atomic number $Z$ can be equated to a centripetal force:

$$\frac{m_e v^2}{r} = \frac{Z e^2}{4\pi \epsilon_0 r^2}, \quad (B.1)$$

where $v$ is the speed of the electron, $m_e$ is the electron mass, $r$ is the electron-nucleus separation distance, $e$ is the elementary charge, and $\epsilon_0$ is the electric constant or permittivity of free space. The angular momentum of the electron is:

$$L = m_e v r. \quad (B.2)$$

The angular momentum $L$ may be expressed in terms of the wavelength $\lambda$ of the electron using the de Broglie equation:

$$\lambda = \frac{h}{m_e v}, \quad (B.3)$$

where $h$ is the Planck constant. Under the condition that the circumference of the $n$th electron orbit contains $n$ wavelengths ($2\pi r = n\lambda$), combining Equations B.2 and B.3 gives:

$$L = m_e v r = \frac{hr}{\lambda} = \frac{nh}{2\pi} = nh, \quad (B.4)$$

$$m_e v = \frac{nh}{r}.$$
where $h = h/2\pi$. Substituting the expression for $m_e v$ from Equation B.4 into Equation B.1 leads to an expression for the orbital electron speed $v$:

\[
\frac{n\hbar v}{r^2} = \frac{Ze^2}{4\pi\varepsilon_0 r^2},
\]

\[
v = \frac{Ze}{n} \frac{e^2}{4\pi\varepsilon_0 \hbar c},
\]

\[
v = \frac{Z\alpha c}{n},
\]

(B.5)

where $\alpha = e^2/4\pi\varepsilon_0 \hbar c$ is the dimensionless fine structure constant (approximately $1/137$).

### B.2 Compton Scattering

Compton scattering of gamma-rays is analogous to elastic scattering, however the interacting photons are massless with kinetic energy and momentum:

\[
E_\gamma = h\nu,
\]

(B.6)

\[
p_\gamma = \frac{E_\gamma}{c} = \frac{h\nu}{c},
\]

(B.7)

where $h$ is Planck’s constant and $\nu$ is the frequency in $s^{-1}$ of the photon. Since the energy scale is large (typically on the order of 1 MeV for gamma-ray interactions) and on the order of the electron mass (0.511 MeV), relativistic kinematics must be used. The energy of the electron is then:

\[
E_e = \sqrt{\left(p_e c\right)^2 + \left(m_e c^2\right)^2}.
\]

(B.8)

Choosing the reference frame such that the electron is initially at rest ($p_e = 0$), the expression above simplifies to the rest energy of the electron. The energy and momentum conservation expressions then become:

\[
E_{\gamma,i} + m_e c^2 = E_{\gamma,f} + \sqrt{\left(p_{e,f} c\right)^2 + \left(m_e c^2\right)^2},
\]

(B.9)

\[
\vec{p}_{\gamma,i} = \vec{p}_{\gamma,f} + \vec{p}_{e,f}.
\]

(B.10)

The final energy of the photon after scattering $E_{\gamma,f}$ may be determined via rearrangement of Equations B.9 and B.10:
\[ p_{e,f}^2 c^2 = (E_{\gamma,i} - E_{\gamma,f} + m_e c^2)^2 - (m_e c^2)^2, \]  
\[ p_{e,f}^2 c^2 = (h\nu_i - h\nu_f + m_e c^2)^2 - (m_e c^2)^2. \]  
\( (B.11) \)

\[ \vec{p}_{e,f} = \vec{p}_{\gamma,i} - \vec{p}_{\gamma,f}. \]  
\( (B.12) \)

\( p_{e,f}^2 \) may be expanded via substitution with Equation B.12:

\[ p_{e,f}^2 = (\vec{p}_{\gamma,i} \cdot \vec{p}_{\gamma,i} - \vec{p}_{\gamma,f} \cdot \vec{p}_{\gamma,f}), \]  
\[ = p_{\gamma,i}^2 + p_{\gamma,f}^2 - 2 p_{\gamma,i} p_{\gamma,f} \cos \theta, \]  
\( (B.13) \)

where \( \theta \) is the angle between the initial and final gamma-ray momenta (i.e. the scattering angle of the gamma ray). Multiplying the result of Equation B.13 by \( c^2 \) and equating to the right side of Equation B.11 gives:

\[ (h\nu_i)^2 + (h\nu_f)^2 - 2 h^2 \nu_i \nu_f \cos \theta = (h\nu_i - h\nu_f + m_e c^2)^2 - (m_e c^2)^2. \]  
\( (B.14) \)

Expanding the right side of Equation B.14 and cancelling terms gives the expression:

\[ h^2 \nu_i \nu_f \cos \theta = h^2 \nu_i \nu_f - m_e c^2 [h\nu_i - h\nu_f]. \]  
\( (B.15) \)

Solving Equation B.15 for \( E_{\gamma,f} = h\nu_f \) gives the result:

\[ E_{\gamma,f} = h\nu_f = \frac{(h\nu_i)(m_e c^2)}{m_e c^2 + (h\nu_i)(1 + \cos \theta)}, \]  
\[ = \frac{E_{\gamma,i}}{1 + (E_{\gamma,i}/m_e c^2)(1 + \cos \theta)} \]  
\( (B.16) \)

### B.3 Weisskopf estimates

As was discussed in Section 2.3.3, transition probabilities have been estimated by Weisskopf [48] assuming a transition involving a single proton in a nucleus represented by a spherical mean field. In this model, the angular momentum of the proton is zero in the final state and \( L \) in the initial state. Additionally, the radial wavefunction of the proton is assumed to be constant within the volume of the nucleus, and zero outside. The resulting single-particle estimates for the reduced transition probabilities \( B(EL) \) and \( B(ML) \) of Equations 2.20 and 2.21 are [47]:

115
\[ B(EL)_{SP} = \frac{1}{4\pi b^2} \left( \frac{3}{3+L} \right)^2 R^{2L}, \]
\[ B(ML)_{SP} = \frac{10}{\pi b^{L-1}} \left( \frac{3}{3+L} \right)^2 R^{2L-2}, \]

where the nuclear radius \( R = R_0 A^{1/3}, \) \( R_0 = 1.2 \) fm, and \( b = 10^{-24} \) cm\(^2\) = 100 fm\(^2\). The reduced transition probabilities shown above may be substituted into Equations 2.20 and 2.21 to obtain single-particle estimates for the half-life of electric (\( EL \)) and magnetic (\( ML \)) transitions of multipolarity \( L \) [47]:

\[ t_{1/2}(EL)_{SP} = \frac{\ln 2}{2(L+1)c^2 R^{2L}} \left( \frac{3+L}{3} \right)^2 \left( \frac{hc}{E_\gamma} \right)^{2L+1}, \]
\[ t_{1/2}(ML)_{SP} = \frac{\ln 2}{80(L+1)\mu_N^2 R^{2L-2}} \left( \frac{3+L}{3} \right)^2 \left( \frac{hc}{E_\gamma} \right)^{2L+1}. \] (B.18)

Evaluating constants in the above equations leads to simple expressions for the estimated half-life as a function of the mass number \( A \) and gamma-ray energy \( E_\gamma \) in keV:

\[ t_{1/2}(E1) = \frac{\ln 2}{\lambda(E1)} = \frac{6.76 \times 10^{-6}}{E_\gamma A^{2/3}}. \] (B.19)
\[ t_{1/2}(M1) = \frac{\ln 2}{\lambda(M1)} = \frac{2.20 \times 10^{-5}}{E_\gamma^3}. \] (B.20)
\[ t_{1/2}(E2) = \frac{\ln 2}{\lambda(E2)} = \frac{9.52 \times 10^6}{E_\gamma^5 A^{4/3}}. \] (B.21)
\[ t_{1/2}(M2) = \frac{\ln 2}{\lambda(M2)} = \frac{3.10 \times 10^7}{E_\gamma^5 A^{2/3}}. \] (B.22)

As Equations B.19-B.22 indicate, the transition probability depends strongly on the gamma-ray energy \( E_\gamma \). Moreover, the dependence is larger (to the fifth power) for \( L = 2 \) transitions than it is for \( L = 1 \) transitions. As a result, for transitions with very large \( E_\gamma \), \( L = 2 \) transitions become more likely since the mechanism with the lowest \( t_{1/2} \) value will dominate. At low energies, \( E1 \) or \( M1 \) transitions are expected to dominate, depending on whether there is a parity change in the transition. It is common for a transition to be mixed between \( M1 \) and \( E2 \) (if there is no parity change) or \( E1 \) and \( M2 \) (if parity changes) if the half-lives for each transition mode are similar. Transitions of higher order than \( E2 \) (ie. \( M2, E3, M3, E4, \) etc.) are rare as they are outcompeted by lower-order transitions at typical \( E_\gamma \) values of a few MeV or lower, however they can exist in cases where the selection rules prevent lower-order transitions from occurring.
B.4 Particle gating combinatorics

Take the special case of equation 4.4 where \( m = x \):

\[
C_{n,x} = A_{n,x} \sum_{y=x}^{n} \binom{n}{y} \epsilon_p^{y} (1 - \epsilon_p)^{n-y} \cdot \binom{y}{x} \epsilon_g^{x} (1 - \epsilon_g)^{y-x}. \tag{B.23}
\]

The binomial coefficients may be combined:

\[
\binom{n}{y} \binom{y}{x} = \frac{n!}{y!(n-y)!} \cdot \frac{y!}{x!(y-x)!} \cdot \frac{(n-x)!}{(n-y)!(y-x)!} \tag{B.24}
\]

Equation B.23 may then be written:

\[
C_{n,x} = A_{n,x} \sum_{y=x}^{n} \binom{n}{y} \epsilon_p^{y} \epsilon_g^{x} (1 - \epsilon_p)^{n-y} (1 - \epsilon_g)^{y-x} \tag{B.25}
\]

where \( \epsilon = \epsilon_p \epsilon_g \). From the binomial theorem:

\[
\sum_{i=0}^{j} \binom{j}{i} a^{j-i} b^{i} = (a + b)^{j}. \tag{B.26}
\]

substituting \( i = y - x \), \( j = n - x \) with \( j - i = n - y \), \( a = 1 - \epsilon_p \), and \( b = \epsilon_p (1 - \epsilon_g) \), equation B.25 may be re-written:

\[
C_{n,x} = A_{n,x} \epsilon^{x} \sum_{y=x}^{n} \binom{n}{y} \epsilon_p^{y} \epsilon_g^{x} (1 - \epsilon_p)^{n-y} (1 - \epsilon_g)^{y-x} \tag{B.27}
\]

Hence the sum of equation 4.4 collapses to a single term under the condition \( m = x \).
Appendix C

Code

All analysis programs developed for this work are open source and are publicly available online via GitHub. The main source code repository is located at: https://github.com/SFUNUSC, with local backups available at SFU. The following table lists descriptions and locations for each code:

Table C.1: Overview of code developed for the work in this thesis.

<table>
<thead>
<tr>
<th>Name</th>
<th>Description</th>
<th>Language</th>
<th>Location</th>
</tr>
</thead>
<tbody>
<tr>
<td>TIPsort</td>
<td>TIP/TIGRESS data sorting and analysis code</td>
<td>C</td>
<td><a href="https://github.com/SFUNUSC/TIPsort">https://github.com/SFUNUSC/TIPsort</a></td>
</tr>
<tr>
<td>TIP Fusion-Evaporation</td>
<td>Fusion-Evaporation simulation code</td>
<td>C++</td>
<td><a href="https://github.com/SFUNUSC/TIP_Fusion_Evaporation">https://github.com/SFUNUSC/TIP_Fusion_Evaporation</a></td>
</tr>
<tr>
<td>TopSpek</td>
<td>Gamma-ray lineshape $\chi^2$ comparison tool</td>
<td>C</td>
<td><a href="https://github.com/SFUNUSC/TopSpek">https://github.com/SFUNUSC/TopSpek</a></td>
</tr>
<tr>
<td>Gridlock</td>
<td>Multivariate data fitting tool</td>
<td>C</td>
<td><a href="https://github.com/SFUNUSC/Gridlock">https://github.com/SFUNUSC/Gridlock</a></td>
</tr>
<tr>
<td>FileConvTools</td>
<td>File format conversion tools</td>
<td>C++</td>
<td><a href="https://github.com/SFUNUSC/FileConvTools">https://github.com/SFUNUSC/FileConvTools</a></td>
</tr>
<tr>
<td>AngDist</td>
<td>Angular distribution calculator</td>
<td>FORTRAN</td>
<td><a href="https://github.com/SFUNUSC/AngDist">https://github.com/SFUNUSC/AngDist</a></td>
</tr>
</tbody>
</table>

Note that the above code has been developed and tested on Linux-based systems, and may not properly compile or run on other platforms. Some codes contain external dependencies that are required to compile them, which are detailed in their respective README files available online.

Additionally, fitting of gamma-ray spectra and gamma-gamma coincidence matrices was performed using the RadWare software package, which is available at http://radware.phy.ornl.gov.