Lameduck Leadership: The Effects of Interim Leadership on Party Discipline

by

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Abstract

While party discipline has been studied extensively under standard conditions, there has to date been no work done on the impact of interim leadership on the incentives of legislators and in particular on the frequency of their dissent. Given the prevalence of interim leadership in both federal and provincial governments in Canada, this represents a significant gap in the literature. This paper seeks to explore the impact of interim leadership on incentives by developing a formal model of those incentives, based on existing work on regular leadership. It also discusses two different approaches to modelling sanctions, advancement, policy preferences, and policy outcomes, showing the impact of treating these as continuous choices as compared to treating them as dichotomous options. I find the policy position of the future leader has more importance under the assumption of continuous variables than the assumption of dichotomous variables.

**Keywords:** party discipline, Canadian politics, interim leadership, formal theory, game theory
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Chapter 1

Introduction

While a significant amount of work has been done on party discipline and parliamentary legislative behaviour, no attention has been given to how the absence of a permanent party leader affects this. With political parties frequently changing their leadership following defeat, there is a lot of time spent with interim leaders in place. Interim leaders are temporary leaders, only in power until the next more permanent leader assumes office. In the Canadian context, this occurs when a party is waiting for a leadership race to conclude and select a new permanent leader. With the NDP’s recent success, there are now three parties vying for power; with an addition party comes an increase in the expected amount of time one party will be without permanent leadership. Following the 2011 election, both the NDP and Liberal Party held leadership races, and then again following the 2015 election both the NDP and Conservatives held leadership races. This frequency of interim leadership is compounded by provincial legislatures, which each have several parties that themselves may have interim leaders. The sheer number of parties involved makes it likely that at any given moment some fraction of the country has at least one party with an interim leader in one parliament governing them.

In order to investigate the influence of leader status on legislative behaviour, the LEADS (Loyalty Elicited through Advancement, Discipline, and Socialization) model developed by Christopher Kam (2009) is particularly useful. Part of this framework gives a formal model that predicts dissent as a function of sanctions and advancement. Such a model creates a foundation on which I will add interim leadership. This paper will develop several adapted forms of the LEADS formal model accounting for different assumptions, including variations on how variables operate, showing the impact of continuous and dichotomous variables. It further tests several assumptions about how actors could act in the game, making allowances for more cooperation due to the repeated nature of the game.

Ultimately, I expect when the future leader is sufficiently far from the interim leaders’ policy ideals dissent will be most common, while when the future leader’s ideal point is close to that of the interim leader we should expect to see less dissent. This expectation, however, is contingent on the variables acting as if they were continuous rather than dichotomous.
The model with continuous variables also suggests that when the future leader holds a similar but not identical view to the interim leader, the party’s ultimate position will be at the future leader’s ideal point rather than the interim leader’s ideal point.
Chapter 2

Party Discipline

2.1 The Desire to Dissent

The first question that should be answered is why dissent exists in the first place. This will be foundational to the incentives of actors later on in the formal game, as well as to discussion of how leaders prevent actors from dissenting. While parties are often treated as unitary beings that can act as a whole, it is clear that they are comprised of actors and those actors sometimes have incentives that differ from the incentives of the party. This section will discuss the potential for policy disagreement and as well as the electoral benefits of dissent.

2.1.1 Preferences

The work on party discipline emerged as a natural continuation of work on the formation of political parties with party discipline being initially seen as a product of party formation. The mechanism behind this natural co-occurrence between formation and discipline is called partisan sorting (Godbout and Hoyland, 2017). Conceptually, legislators move to parties that already share similar policy positions but dissent when they disagree with party leadership on a specific issue. Parties naturally only form in a group where there is a great deal of ideological consistency, but this consistency will only go so far and so on certain issues there will be disagreement and dissent.

Further research supports the idea that discipline is a naturally occurring phenomenon. In the absence of parties, ideological whipping came about in Australia when MPs were asked to vote on the question of same-sex marriage and given the opportunity to vote against their party lines without repercussion (Plumb, 2015). The result of the voting was reasonably in line with the political parties that exist in the legislature, supporting the notion that parties are ideological constructs resulting from partisan sorting. The natural state of affairs for parties is to accept some level of disagreement from their members on less important issues while still capturing broad agreement on most core beliefs (Rose, 2014, p. 4).
Recall that in an environment of party discipline, preferences are a root cause of legislators’ desire to dissent. By contrast, the desire to remain in line with the party is a function of party discipline and structure (Koenker, 1997). Koenker, who was himself an MLA, found himself disagreeing with a policy from his government and initially found the pressure from his party sufficient to remain quiet. Ultimately, however, despite the significant pressure from within the party, he dissented on the issue when he was confronted by his constituents (Koenker, 1997).

Finally, evidence presented by Christopher Kam both supports and questions if dissent is preference driven (2001). While his study revealed that some portion of legislative dissent could be explained by preferences, there was still a remaining portion that it did not account for. This is similar to the story, painted by Koenker (1997), that while preferences might sometimes drive a desire to dissent, there are other associated costs that impact the decision making process.

This is generally where the literature has settled itself, with most scholars agreeing that knowledge of both preferences and party affiliation are central to explaining the absence or presence of dissent (Willumsen, 2017; Ansolabehere et al., 2001). This immediately rules out the possibility of a model that is centered entirely around preferences.

### 2.1.2 Electoral Benefits

Beyond just preferences, there are additional pressures to dissent created by the electoral circumstances of a legislator. There is a significant amount of research that suggests a strong personal brand within a constituency increases the vote share of the incumbent, and that dissent is correlated to the strength of this brand (Carey and Shugart, 1995; Kam, 2009; Rogers, 2017; Tavits, 2009).

While not a result of dissent, the process for private members’ bills in the Canadian House of Commons provides direct evidence of the impact of independent action from one’s party (Loewen et al., 2014). The ability to present such a bill is determined by lottery, creating a natural experiment to see what impact they have on election results. The evidence shows that being chosen to present a private members’ bill provides a member with a significant increase in their vote share that corresponds to an increase in their chance of being elected.

The literature discusses the causal arrow between personal brand and dissent in both directions. The majority of the work talks about the electoral benefit of dissent (c.f. Rogers, 2017). The benefits are nicely summarized by Campbell et al. 2016 who see dissent as a valence signal to constituents that the legislator shares beliefs with the constituents, has strong personal integrity, and also more generally helps to increase the visibility of the legislator within the constituency. Kam 2009 does not find evidence that it is particularly effective at those first two goals but finds that it does increase the name recognition of the incumbent to a degree that is electorally beneficial.
Further evidence was found in the United Kingdom, where during World War II, legislators who faced a more competitive district for re-election were more likely to dissent, showing their efforts to build their local brand (Rasmussen, 1970).

Comparative literature suggests that this would also be felt in Canada, particularly after the introduction of the Reform Act, which reduced the party leader’s ability to close nominations and make them uncontested. Research shows that the more fiercely these nominations are contested, the greater the depth of dissent will be in Parliament (Carey, 2007). This is further supported by the work showing electoral systems impact the level of party unity (Hix, 2004). Under a single-member plurality system, such as the one Canada currently has, legislators rely on the support of a single local constituency and the evidence shows that that reliance leads to lower levels of party unity and more dissent.

There is however some possibility that the causal arrow primarily works in the opposite direction and that strong personal brand creates the opportunity for dissent (Tavits, 2009). Here it is argued that existing personal brand is what predicts future dissent, and this is in part shown in the relationship between past municipal politics experience (seen as a source of name recognition and brand within the community) and higher levels of dissent.

2.2 Toeing the Party Line

We have now seen that preferences alone do not explain the prevalence of dissent because they would predict more dissent than actually occurs. Add to this the fact that legislators also benefit from an increased chance of re-election if they dissent and we are left with the question of how parties keep their membership in line.

2.2.1 Advancement

The first of these potential costs associated with dissent that legislators consider is its impact on their chances of advancement within their caucus. Although legislators have a desire for re-election, they also have a desire for promotion and increased personal power and status (Crisp et al., 2009). Being in Parliament is not their final goal and once they attain that first rung, they continue to search for opportunities to advance higher.

Evidence does suggest that when legislators lose the hope for advancement they begin to dissent more. One study has found that when a legislator is removed from Cabinet or it is otherwise made clear that they will not be able to advance, they increase their dissent (Benedetto and Hix, 2007).

This desire to attain office manifests itself through support for leaders, because legislators hope that that support will improve their chances of promotion (Kam, 2006). Members of the party leadership are the only people who are able to make advancement into Cabinet, Shadow Cabinet, or more senior committee assignments possible for regular legislators and they are therefore the primary source of advancement available to legislators once in office.
This thus makes legislators supporting them, through voting the way the leader wants them to vote, a key source of party discipline.

This incentive to adhere to party discipline is very dependent on the size of the parliamentary caucus that a member is part of as well (Eguia, 2010). Larger caucuses have less promotion to offer to each individual member. As such, the expected value of remaining loyal is lowered while the expect value of dissent is not changed. This means that for the average member of a larger caucus, the cost of dissent is lower than that of the average member of a small caucus.

There is also evidence to suggest that the desire for advancement has a smaller impact on discipline that one might expect. Evidence from the UK shows that voting behaviour in an MP’s final term does not seem to change, even though they would appear to have no hope for advancement anymore (Willumsen and Goetz, 2016). They do, however, show up to vote less, which could be hiding the fact that they are changing the pattern of votes they support. It is possible that rather than vote in favour of motions they do not support, they change their strategy to start skipping those votes. Another explanation that we will see later is that socialization is an important force in the minds of legislators, and those retiring will be the most socialized to the parliamentary environment so perhaps there are counteracting forces in place. What is clear here is that legislators were not vocally dissenting to a significantly greater degree after having made the decision to retire.

2.2.2 Sanctions

Leaders also have the more direct ability to elicit loyalty through the use of sanctions. This is clearly an effective approach as legislators view dissent to be a very costly activity, and often find that even if they want to dissent they are unable to justify it (Koenker, 1997; Guay, 2002).

It is unclear whether the ability of a party leader to discipline their membership has changed significantly since the adoption of the Reform Act, which was a targeted attempt to limit the power of leaders (Geisler, 2015). Geisler does not believe it will be hugely impactful on party discipline in Canada; it mostly only hurts leaders’ ability to deny re-nomination but this is not a huge portion of how legislators are kept in line.

Hix (2004), however, would likely disagree with this. He argues "if the electoral system is party centered or if the party leadership controls the selection of candidates, the politician is likely to vote with her party leadership and against the wishes of local party elites and voters in her district" (Hix, 2004, p. 219). This leaves the actual impact of the Reform Act as an open question. It is possible that party leadership has indeed been weakened in Canada, but it is equally possible that party leaders are able to shift their methods of discipline to recover any control they might have lost.
2.2.3 Socialization

The final pressure on legislators that dissuades them from dissenting is the influence of socialization. More time in office reduces legislators propensity to dissent as they become more accustomed to the norms of the institution in which they work. Evidence of this can be seen in the impact that length of service in parliament had during World War II, where those in who had served in parliament for longer were less likely to dissent (Rasmussen, 1970).

This socialization is a direct function of time, with legislators in office longer becoming more socialized. Once elected, legislators begin to shift their frame of reference towards that of their colleagues in parliament (Kornberg, 1967). This makes them more concerned about going against the norms of parliament, including maintaining party unity. However, the process may not be uniform across all actors and will be impacted by the nature of the individual legislator (Crowe, 1983).

2.3 LEADS model

Most comprehensive discussions of dissent and party discipline do acknowledge the importance of many factors coming together (c.f. Andeweg and Thomassen, 2011; Willumsen, 2017). Andeweg and Thomassen discuss how although it may seem like this leaves party discipline over-determined, particularly given the many different mechanisms available within each of these categories of advancement, discipline, and socialization, it may simply be a case of different tools being used in different cases.

Christopher Kam presents the LEADS model as a combination of the various incentives (2009). As part of that model, he derives a formal game played out between the leader and legislator where the leader uses sanctions and advancement to dissuade the legislator from dissenting.

There is one caveat that should be noted here, which is that while party leadership benefits from unity within a party on the macro scale, they also benefit from re-election of individual members (Fujimura, 2012). This means minor amounts of dissent may, in fact, be in the best interest of a party leader if they mean ensuring a legislators re-election even if it is at the cost of party unity. Kam notes this and argues that "party leaders are sensitive to the electoral pressures face by their MPs, but they are equally aware of the costs of disunity to the party" (Kam, 2009, p. 26).
Chapter 3

Theory

This section seeks to extend the LEADS model both formally and informally to the case of interim leadership in Canada. To do so, I will first consider the informal theoretical implications of interim leadership and then second apply those considerations to a formal model of decision making.

3.1 LEADS Under Interim Leadership

When taking the model proposed by Kam and applying it to interim leadership, there are some several natural consequences from the transition that must be considered.

First, there is an inherent limitation to the study of interim leadership in Canada: interim leaders generally only occur in opposition. This means that all considerations are limited to the case where the legislator is in opposition. However, this itself does not change the incentive structure of the members. Christopher Kam acknowledges that in opposition, it remains important to present a unified party because if they do not, there is still real electoral damage that can result and so while dissent will not result in the end of a government, it is still costly to the party leadership (2009).

We must then consider how interim leadership affects the three portions of the LEADS model: advancement, discipline, and socialization. Socialization is easiest because there should be no major impact on it by virtue of a leader being interim. Legislators will still have had as much time to socialize in parliament regardless of whether they are in a party with a permanent leader or an interim leader. Interim leadership is often indicative of a party having just lost an election. However, a lost election, as opposed to a won election, will not bring about a large influx of new legislators. If there is a change in the composition of the legislature, controlling for the years of service of each legislator would account for any changes from this, although there are no changes to socialization predicted by interim leadership.

In terms of advancement, it is unlikely that the interim leader will have any ability to offer important advancement in the party. This comes from two different factors. First, the
future leader is not bound by any decision made by the interim leader and any advancement made can be undone and replaced. Second, this first factor is compounded by the fact that there are generally a number of temporary advancements made under interim leadership when the more senior members resign from Shadow Cabinet to run in the leadership race and those vacancies must be filled. Advancements offered by an interim leader will likely be temporary and once the future leader assumes office, the benefit is likely lost. For our model, we can therefore put advancement at the discretion of the future leader alone.

Discipline is more complicated. There are immediate disciplinary measures that remain in the hands of the interim leader but longer-term discipline, such as the withholding of a nomination, is the prerogative of the future leader. Thus when legislators consider their likelihood to be disciplined, they are forced to consider both the reaction from the interim leader, as well as the reaction from the future leader. Here, it is important to note that the legislator should expect the future leader to care about this dissent, even though it will have occurred before their time as leader. Leaders consult many different individuals before making the decision to promote a legislator to cabinet, including asking their whips to confirm individuals voting record (Crowe, 1986, p. 162).

### 3.2 Defining the Variables

Before moving forward with a discussion of the formal model, we stop to consider how each relevant variable should be characterized. The nature of variables determines whether we are dealing with a set of discrete outcomes or rather a continuous set of outcomes. This consideration will significantly impact the way in which the game can be modelled. Ultimately, however, it seems that there is no clear choice and that there may be some merit to considering both a game of discrete values and a game of continuous values.

Of the variables we are interested in, dissent is the most limited. Practically, dissent looks as though it is categorical with many different discrete options existing, perhaps to the degree that it can be treated as a continuous variable. At the lower end of the dissent spectrum, legislators can speak vocally against the bill on limited occasions while on the opposite extreme they can vote against it or even put forward motions in an attempt to alter or change the outcome. We could include in this spectrum abstention on votes, although there is some debate as to how much that constitutes dissent versus simply being the result of other processes (c.f. Willumsen and Öhberg, 2017).

However, it is not always going to be true that parties are opposed to each kind of dissent and if that is the case, treating it as continuous becomes more problematic. This is exemplified by the disagreement on the impact of legislators vocalizing disagreement with one of their party’s policies to their constituency. Some research has suggested that vocal dissent hurts a party’s ability to claim competency on a given issue since when a party has internal disagreement on an issue, it is not clear to voters that they have a plan to deal
with that issue (Greene and Haber, 2015). On the other side of the argument is the notion that party leadership can benefit from minor forms of dissent because it can help a party speak to a portion of the electorate who disagree with that party (Campbell et al., 2016). It can also help a party make its position more ambiguous or help to bring an issue into the spotlight and draw more attention to that issue generally, all of which can be beneficial to a party’s electoral chances in the next election.

This makes dissent a tricky case both to operationalize and to develop a formal model around. On the one hand, we should be cautious and avoid type I errors so as to not over diagnose dissent. For instance, if we included abstentions as a form of dissent we would often see cases of supposed dissent that are in reality something as simple as a scheduling conflict. However, we also want to avoid missing dissent where it does occur, particularly in a system like Canada where the costs of dissent are high and more extreme forms of dissent will be particularly rare. If we only include votes against party lines as dissent we risk seeing almost no dissent even when there is significant vocal opposition from within a party.

The other variables are less theoretically difficult to deal with although practically they might be equally complicated to measure as continuous or pseudo-continuous variables. Taking policy preferences and positions first, at an abstract theoretical level it seems natural to treat these as continuous objects. That said, once an abstract model is applied to a specific case, it is likely that the options available will not actually be continuous.

As for sanctions and advancement, it seems reasonable to model them as continuous because of the wide variety of advancements and sanctions possible but that does not mean they actually are continuous. It is clear that those variables are both actually categorical in practice. That said, for both of these variables there are many different categories and within each category are sanctions and advancements of different classes (Bailer, 2018). Thus it is not unreasonable to treat it as a continuous variable in a model.

In general, this leaves a trade off with neither option being without merit. Thus we should apply both assumptions to the model, first by showing the formal result of treating all the variables as continuous measures, the approach used by Kam (2009). I will then apply a new approach to model each variable as binary, only allowing them to exist at the extremes of their ranges from the first model. While this is not a perfect representation of the variables in either case, it should provide insight into incentives of the players under interim leadership.

### 3.3 Continuous Variables

There are two ways to think about the game that is being played out, even when only considering continuous variables. The first that will be presented is treating the Member of Parliament, the interim leader, and the future leader each as separate rational actors.
The second will instead treat the future leader as an actor whose nature is not known. This split provides insight into the two separate ways a leadership race can play out. If, in the race, there is a clear winner and it is extremely unlikely anyone else will win, the Member of Parliament should be able to know the nature of the future leader and act in accordance with that expectation. However, if the leadership race is more split, the nature of the future leader will be less clear to the MP and so they will need to treat the future leader in a more random way. This topic will be addressed in greater detail below.

### 3.3.1 Knowing the Future Leader

The players will be denoted as follows: the legislator will be $MP$, the interim leader will be $I$, and the future leader will be $F$.

$MP$ has strategy $R$ available to them, which is the amount of dissent they choose to exercise. This is constrained by $R \in [0,1]$, which is to say it is a number from 0 to 1 with 0 denoting no dissent and 1 denoting complete dissent, or the most dissent possible. Any given amount of dissent $r$ is associated with a cost to the party of $r^2$ that is paid by all of $MP$, $I$, and $F$.

$I$ has strategies $X, S_I$, where $X$ refers to the policy position taken by the party and $S_I$ refers to the sanctions put on the dissenting $MP$ by the interim leader. $X$ is constrained by $X \in [0,1]$, which is to say it, like $R$, is chosen from a range of 0 to 1 by $I$. In this case, 0 is defined as the preferred policy position of $MP$ while 1 is defined as the preferred policy position of $I$. It would generally be irrational to choose a policy position on the far side of 1 because it would both be less acceptable to $MP$ while not being more acceptable to $I$. $S_I$ is the level of sanctions placed on $MP$ by $I$ and it is associated with a cost $S_I^2$ for the party leadership.

The game at this point is reasonably similar to the game presented in the original LEADS model, however it changes when we move on to $F$, the future leader. $F$ has $A, S_F$ available to them where $A$ is the level of advancement offered to $MP$ and $S_F$ is the level of sanctions placed on $MP$. $S_F$ is also associated with $S_F^2$, a cost for the party, and $A$ is associated with $A^2$, a cost to the party for advancement $A$. $F$ also has an ideal point in the policy discussion that is denoted by $X_F$.

\[
U_{MP} = -x + x r - r^2 - (r s_I) - (r s_F) + (1 - r) a \tag{3.1}
\]

\[
U_I = (1 - r) x - r^2 - S_I^2 \tag{3.2}
\]

\[
U_F = -|x_F - (1 - r) x| - r^2 - a^2 - S_F^2 - S_I^2 \tag{3.3}
\]
First, the utility of MP is a function of $x$, $r$, $s$, and $a$. The closer $x$ is to 0, the more utility MP gains (recall 0 is defined as the ideal policy position of MP). The next term, $xr$, captures the idea that they gain more utility from dissenting the further the policy is from their ideal point. There is no inherent benefit to dissent if the proposed policy position matches MP’s ideal point. The next term, $-r^2$ refers to the potential for dissent to hurt party unity and through that to hurt MP’s electoral chances.

Note that $r^2$ is a squared term. This model uses squared terms for the costs associated to actions to ensure that the costs are lower than the direct impact of those actions. All the variables with associated costs are bounded by 0 and 1, and so any number within the allowed range squared ends up being smaller than it was before being squared. This is practical because if actions have a higher cost than effect, they would naturally not be beneficial.

The next terms, $-rs_I$ and $-rs_F$, both refer to the impact of the sanctions on MP, which are directly proportionate to the level of dissent that MP chooses to use. The more MP dissents, the stronger sanctions $s_I$ and $s_F$ will be. MP also suffers a reduction in their advancement potential inversely proportional to the level of dissent they choose, as denoted by $(1 - r)a$.

Second we consider the utility function for $I$. The first term, $(1 - r)x$, denotes the utility gained from the policy position $I$ chose, but allows for it to be diluted by the impact of $r$ in moving it closer to 0. $I$ also suffers the direct cost of $r^2$, imposed by the dissent of MP. Finally they suffer the cost of $s_I^2$, which is the cost of placing sanctions on MP. Note that cost $S^2_F$ is not included in this utility function. This is a tenuous exclusion but stems from the fact that the interim leader will be most focused on the performance of the party for their duration, but $S^2_F$ is inherently a cost that only occurs after $I$ has left their position.

Lastly there is $F$ who suffers all the costs, $r^2$, $a^2$, $s_I^2$ and $s_F^2$. They suffer all these costs because they are concerned with the party’s long term interests and so these costs, each of which is related to the party as a whole, impact them. They also have the most complex term for their utility gains from the policy result. They lose utility the further $x$ is from $x_F$, with no regard for which side of $x x_F$ falls on (as reflected by the absolute value), but since this is affected by the dissent $r$ of MP, this must also be taken into account. This results in the more complex form seen above.

As is required by formal models, several assumptions are made in translating real action into a simplified model. Perhaps the most important assumption made by this model is the ordering of actions. Here, I have been required to make a significant assumption beyond that made in the original model: I have chosen when the future leader acts. I made this decision under the assumption that the behaviour of the future leader occurs on a higher level and that they are not specifically reacting to every position a party takes. This is because the future leader has other non-legislative priorities that increase the cost of paying very close attention to the process under discussion. Instead they have more broadly taken stances on
whether to support the interim leader or to dissent. This assumption is most natural in the
dichotomous version of this game described below as we can think of it as either the future
leader supporting the current regime broadly or supporting an alternative.

3.3.2 Uncertainty

To solve the previous game, there is a required assumption that $MP$ knows $F$ such that
they can develop a best response to $F$’s best response. As described above, this assumption
does not necessarily hold so the game must be adapted to also function in situations where
$MP$ cannot predict with certainty who $F$ will be.

This situation has significant ramifications for Equation 3.3. While costs to party lead-
ership will remain reasonably constant regardless of who assumes office, the first term based
on the distance between the final policy and $F$’s ideal point will become impossible to pre-
dict. The more competitive the leadership race is, the more random this term will ultimately
be. At the extreme, $MP$ can have no expectations about its value outside of the mean of the
policy points for every candidate in the race, which as the number of candidates grows
should approach the mean view of the party.

Thus when $MP$ and $I$ are choosing their strategies, they are making decisions with
the intention of maximizing their expected utility given the distribution of potential future
leader types. Crucially, these leaders vary from each other on the value of $X_F$.

3.3.3 Solving the Game

The perfect information game presented above can be solved as seen in Appendix A. To
put the results back into the context of dissent, the analysis in the Appendix shows that for
values of $X_F$ that are large enough, the interim and future leaders play such that dissent
levels are low and the final policy outcome is the same as the ideal outcome of the future
leader. If $X_F$ is sufficiently small, $F$ minimizes the impact of the policy instruments to
encourage as much dissent as possible, but the interim leader’s policy position remains
important and the outcome occurs somewhere between the future leader’s ideal point and
the interim leader’s ideal point.

This means that when a future leader is sufficiently far from the interim leader’s policy
ideals dissent will be most common while when the future leader’s ideal point is close
to the interim leader we should expect to see no dissent. This, of course, is under the
assumption that members of a party can confidently predict who the next leader will be. If
they cannot, the situation will be significantly different. In those cases, each of the candidates
for leader can attempt to play the game as normal but individual legislators must treat it
as a distribution of potential outcomes based on who the candidates are from which they
form an expectation. The larger the field of candidates for the leadership race, the more
likely they will have an average policy position among them at the average policy position of
the party. Thus with larger numbers of candidates for leader, parties will generally see less
dissent, without having to account for the position of each individual candidate (although there will be individual cases where this does not hold).

3.4 Binary Variables

With binary variables used instead of the continuous variables presented above, we are left with a much simpler game. Given that the decisions are all binary, the potential number of outcomes is simply \(2^N\) with \(N\) being the number of decisions there are. Thus this game only has \(2^5\) or 32 potential results and so can be solved simply by playing out the extensive form game and applying backwards induction. The only issue that then remains is how one should expect an actor to react to a situation where their expected utility is the same regardless of their decision. This section will consider three different options: a randomized approach, a generous approach in favour of the interim leader, and a generous approach in favour of the future leader.

The first approach is the standard game theoretic approach. Because actors are rational, they should be indifferent between options that provide them the same utility and given the model presented above, there are numerous situations where the utilities are equal. In these cases, the choice is effectively random, with an equal chance of both outcomes occurring. Appendix B shows in more detail how the formal results of this model (and the other versions of this model discussed below) are arrived at. The model leaves us with two cases: either the future leader has the same policy position as the interim leader, or they have the same policy position as the legislator.

If \(F\) has the same policy position as the legislator, they do everything they can to encourage dissent, which in this case means no use of sanctions or threat of lost advancement. Once they have done that, the interim leader becomes indifferent to the different options. Then, looking back at how likely this is to end in dissent, there is only a one in eight chance of dissent occurring in this model.

On the other hand, if \(F\) has the same policy position as the interim leader, they are indifferent to either set of actions. The majority of outcomes following from this do result in no dissent, which makes theoretical sense. In fact, only one scenario results in dissent and while this model shows that result is possible through indifference there is only a one in thirty two chance that this option ends up being selected.

Overall, it is clear that dissent is quite unlikely to occur under this model, although it is always still possible.

We can then look at a different version of the model where actors are generous. First, we should pause to consider why rational actors might still base their decision on helping someone else even though it has no impact on their utility according to the utility functions I have presented above. Because this is a repeated game, occurring every time there is a
vote that an actor may want to dissent on, actors are often able to coordinate over time (Willumsen, 2017).

To model that cooperation, the next two models will have the interim and future leader both choose the option that benefits the other most when they themselves are indifferent. That leaves two clear versions: one where the legislator is generous to the interim leader and one where they are generous to the future leader. Detailed results of both models are in Appendix B, however looking at their results, the first thing is that both models appear to generate the same result. That result is that no actor is applies sanctions, offers advancement, or engages in dissent and the position of the party is set at that of the interim leader. Effectively, this suggests that the party will be perfectly disciplined without any actual use of sanctions. Interestingly, this result is not actually impacted by the policy position of $F$.

This model largely eliminates the importance of knowing who the future leader actually is. In the more generous models, it is completely irrelevant because $F$ acts the same regardless of their policy position. Thus any possible future leader would act that way. Then for the more strict model the behavior of all non-$F$ actors remains a coin toss regardless of the nature of $F$. It is slightly more likely that actors will end up playing out the single scenario that results in dissent if $x_F = 0$, but in both cases it is unlikely. If actors are unsure what the value of $x_F$ is and thus hedging their actions, dissent will be unlikely. If they know the value of $x_F$, dissent will still be unlikely regardless of its value.

Regardless of assumptions, with binary variables it seems that dissent becomes very unlikely (verging on impossible). In the majority of outcomes, the party’s position remains at the position of the interim leader, regardless of the position of the future leader.
Chapter 4

Discussion

4.1 Model Selection

Without empirical tests, it is hard to make a definitive statement about which model is superior. Both types are natural consequences of the nature of variables that was assumed in their creation. It does seem that we can rule out specifically the two generous models as not being universally accurate, but beyond that any kind of specification is difficult.

The two approaches do generally agree that under interim leadership we should expect to see very little dissent if the expected future leader is similar to the interim leader but if they are different, there is a more substantial chance of dissent. Interestingly, the continuous approach says that level of dissent is entirely based on the preferences of the future leader while the binary approach says that there is an element of chance (which could simply end up being the result of situation-specific exogenous factors).

The most important difference between the models is where the party’s ultimate position ends at. When using binary variables, the model is largely insensitive to the preferences of the future leader and it remains at $x = 1$, which is to say the party adopts the position of the interim leader. When the variables are treated as continuous, this changes. For a range of $X_F$, the future leader is able to manipulate the policy position to their ideal point and for smaller values of $X_F$ they are still able to draw some concessions in the policy position.

However, this is not to say that the models suggest the interim leader is powerless. On the contrary, the interim leader effectively determines the range of possible positions while the future leader determines which of those positions to take. According to the continuous model, once the position of the future leader is sufficiently far from the interim leader, the policy stops moving and becomes static. This makes the interim leader a powerful moderating factor during their time as leader.
4.2 Generalization

While this model has been both derived from and applied to the Canadian context in this paper, there is merit in considering whether the general power structure between actors seen here is applicable in other contexts. Although interim leadership in the Canadian style is quite uncommon, because this model takes such a simplistic view of the relationship between actors it can be applicable in any leadership transition scenario, given the assumptions of the model are met.

For example, consider transitions of power in the United States. After every presidential election there is a four month period where there is both a sitting President and a President-Elect. This is not a perfect comparison as in Canada the levers of sanctioning and advancement that party leaders hold are much more compelling than those held by the President in the United States, but when there is a transition of power between two presidents of the same party this model of behaviour should be applicable. A further caveat is that often the transition is also between parties and so this model is not applicable.

It therefore generalizes to a narrow model of lameduck leadership that is only applicable when the lameduck leader is of the same party as the incoming leader and it only applies to legislators also from that party. As the US example demonstrates, this is a very specific set of circumstances for presidential systems. In parliamentary systems that choose their leader within the parliamentary caucus, this type of leadership is no longer possible as they will never face an interim or lameduck leader.

4.3 Limitations

4.3.1 Limitations of the Formal Model

As already discussed, one of the main limitations of the models presented is that the variables are not properly captured as either binary or continuous entities. In treating them as either we are seriously simplifying reality. While there are justifications for both approaches, the ideal approach would be some method of quantifying every possible outcome because they are clearly finite and repeating the backwards induction process for that complex model.

However, such a model featuring a huge number of categorical options is well beyond the scope of this paper and is quite possibly not more informative because of the number of assumptions that would be necessary for it.

This difficulty is to some degree a by-product of the larger issue that rational choice approaches are inherently a substantial simplification of reality because they distil the complexity of human decision making into a process that only takes into account rationality. Even after adding in some ability for cooperation to the binary model, we remain several layers of abstraction away from reality. In every case where we may want to apply this model
there will be quirks that affect actors’ decision making: personal relationships, heuristics, imperfect information, etc.

What this model does do is provide a sense of what incentives are acting on every legislator regardless of their individual differences. Every time there is even a slight disagreement between the interim leader and a legislator these forces come into play and while the result may not always play out exactly as presented here, on the whole behaviour should tend towards these underlying forces.

4.3.2 Limitations of Empirical Testing

Of course the most significant limitation of the theoretical approach taken in this paper is that it does not provide any empirical evidence for the claims. Rational choice provides structured predictions for how actors will behave in different situations, but without empirical testing we cannot say that they do behave that way.

There are many difficulties that will be encountered when attempting to test this. For many of these concepts, measurement will be extremely difficult. Measuring sanctions and advancement are themselves difficult enough, but they represent one of the smaller challenges in this case. Policy preferences are particularly difficult to measure as there is a strong incentive to present preferences for certain things even if they are not genuine. There is a great deal of literature that discusses the difficulties of measuring this and many different solutions have been proposed (c.f. Griffin, 2008; Loewenberg, 2008). This is a significant obstacle that will need to be overcome for empirical tests.

The other key issue is in measurement of dissent. As discussed above, dissent is difficult enough to conceptualize and in doing so, the main thing that emerges as a true representation of dissent is the presence of a vote against party lines. The issue that arises is that records of votes are subject to inherent selection bias (Carrubba et al., 2006). There are a great number of votes that occur that are not recorded in divisions because they are not important or close. Only the most important and consequential votes are recorded. Thus for the votes that are recorded, the incentives of party leaders to maintain discipline are highest. This will naturally deflate the amount of observed dissent observed using this measure.
Chapter 5

Conclusion

In short, it seems clear that there are three main forces that limit the extent of dissent: the fear of sanctions, the hope for advancement, and the pressure of socialization. Of these, the fear of sanctions and hope for advancement are both strongly impacted by interim leadership. There is no theorized direct impact on socialization stemming from the introduction of an interim leader.

When we treat sanctions, advancement, dissent, and policy options as continuous, we are left in a situation where there is very little dissent unless the future leader has a policy preference that breaks significantly from the preferences of the interim leader. In these cases the policy of the party ends up at the same point as the future leader’s ideal point. When the variables are only allowed to fluctuate between their maxima and minima, which is to say they are treated as binaries, the importance of the future leader’s policy position largely vanishes. That said, given the model with continuous variables, the interim leader still plays a strong moderating influence on the future leader.

The next steps to take in the study of interim leadership should be to empirically test these predictions. Only by testing them can we truly know whether the assumptions presented in this paper are accurate or if they must be rethought.
Bibliography


Appendix A

Formal Model With Continuous Variables

A.1 Solving for MP

The game as presented leads to an extensive form game, which is solved through a maximization problem played out sequentially. In this case, $F$ plays first choosing $s_F$ and $a$, then $I$ plays $x$ and $s_I$, and finally $MP$ responds with a given level of $r$. The game can thus be solved like most long form games through backwards induction, but because of the continuous nature of the utility curves for each player and simultaneously the constraints on the values it must be done in a more complex way.

We begin with the formula for the utility of $MP$:

$$U_{MP} = -x + xr - r^2 - rs_I - rs_F + (1 - r)a$$  \hspace{1cm} (A.1)

$MP$ maximizes their utility, ignoring the constraint $0 \leq r$, by setting $r$ such that $\frac{\partial U_{MP}}{\partial r} = 0$, meaning that $MP$ cannot increase their utility by unilaterally changing the value of $r$. We find:

$$\frac{\partial U_{MP}}{\partial r} = x - 2r - s_I - s_F - a$$  \hspace{1cm} (A.2)

Here we see the beginning of a pattern that is consistent in the entire game where $s_F$ and $a$ always appear together in the same quantity and are in fact that same value. Thus we will substitute:

$$b = s_F + a$$  \hspace{1cm} (A.3)

$$\frac{\partial U_{MP}}{\partial r} = x - 2r - s_I - b$$  \hspace{1cm} (A.4)
We can then set that to 0 and solve for \( r \) to get \( r^* \), a formula for the optimal response \( r \) (provided that this value satisfies the constraint) to any given values of \( x, s_I, \) and \( b \):

\[
     r^* = \frac{x - b - s_I}{2}
\]  

(A.5)

**A.2 Solving for I**

Now recall the equation for \( U_I \), who sets \( x \) and \( s_I \). They know \( MP \) will respond with \( r^* \) and so they choose their actions to maximize \( U_I \) given both the knowledge of how \( MP \) will respond and the knowledge that \( r \) is constrained by \( R \in [0, 1] \). This gives the equation:

\[
     U_I = (1 - r)x - r^2 - s^2_I
\]  

(A.6)

Substituting in \( r^* \) for \( r \) we get:

\[
     U_I = (1 - \frac{x - b - s_I}{2})x - (\frac{x - b - s_I}{2})^2 - s^2_I
\]  

(A.7)

The unconstrained version of this problem then seeks to maximize:

\[
     U_I = -\frac{3}{4}x^2 + x(b + s_I + 1) - \frac{b^2}{4} - \frac{bs_I}{2} - \frac{5s^2_I}{4}
\]  

(A.8)

In Figure A.1 below we show the set of possible choices for \( MP \).

![Figure A.1: Set of strategies available to I. The line marks \( s_I = x - b \), where \( b \in [0, 2] \). Note that if \( b > 1 \), the line will be outside the strategy set for I.](image)
The line $x = s_I - b$ represents the boundary between two distinct regions. Above it lies the area where $r = 0$ because $x - b - s_I < 0$, making the constraint on $r$ binding and forcing it to 0. Below it lies the area where $r > 0$.

For all $b$ we must find the maximum of $U_I$ over the whole square. We do so by maximizing separately within the region $r = 0$ and the region $r > 0$ and then comparing the two maxima to see which is bigger. A first step to this process would be to find when there is an unconstrained maximum of the problem (i.e. when a maximum occurs inside the triangle). This is found at the location where the partial derivatives of $U_I$ are set to 0.

$$\frac{\partial}{\partial x} = -\frac{3x}{2} + 1 + b + s_I \tag{A.9}$$

$$\frac{\partial}{\partial s_I} = x - b - \frac{5s_I}{2} \tag{A.10}$$

We can then solve for $x$ in the first equation and substitute that value into the second equation. Solving that equation again gives a value for $s_I$ in terms of only $b$ and that can be substituted into the first equation. This gives the following values for $x$ and $s_I$:

$$x = \frac{8b + 10}{11} \tag{A.11}$$

$$s_I = \frac{4 + b}{11} \tag{A.12}$$

These equations are only valid for $b \leq \frac{1}{5}$ as anything above that would mean $x > 1$ which violates the constraints of the game. If $b > \frac{1}{5}$, we know that the maximum value for the triangle portion of the strategy set occurs on an edge, as the only critical point (the solution where the partial derivatives are 0) occurs outside of the triangle. We can then add these values for $x$ and $s_I$ back into the equation for $U_I$ and get the following:

$$U_I = \frac{b^2 + 8b + 5}{11} \tag{A.13}$$

Returning to the remainder of the square, we can reasonably and easily rule out a large portion of the strategies $I$ could take. In fact, every point on the rest of the square is dominated by points along the line $s_I = x - b$. This is because for any value above that line, $I$ could either increase $x$ or decrease $s_I$ without suffering any increase in $r$. This leaves a much simpler maximization problem over the triangle with sides $s_I = 0$, $x = 1$, and $s_I = x - b$.

Let us now consider several different cases for $b$. 


A.2.1 Case I

The first case will be $0 \leq b \leq \frac{1}{8}$. For this case, the formula from above can be used because the maximum is not constrained. This leaves the following values for what $I$ and $MP$ will play:

\begin{align*}
  r &= \frac{3 - 2b}{11} \quad \text{(A.14)} \\
  x &= \frac{8b + 10}{11} \quad \text{(A.15)} \\
  s_I &= \frac{4 + b}{11} \quad \text{(A.16)}
\end{align*}

A.2.2 Case II

Let us now consider the case of $\frac{1}{8} \leq b \leq \frac{3}{4}$. If $b > \frac{1}{8}$ the maximum of $U_I$ does not occur on the triangle, but is now at some point where $x > 1$.

This is sufficient information to tell us that to maximize the utility function in this range, we only need to find the maximum along the line $x = 1$. First, we know that the formula for $U_I$ is a quadratic function with a single critical point that occurs somewhere to the right of the triangle. Second, we know that because the signs on the both the $b$ and $b^2$ terms are positive we know the function has elliptical level sets (meaning that intersecting a plane with the function $U_I$ will always produce ellipses centered around the critical point).

Combining these pieces of information, we can see that the maximum on the triangle must occur at the part of the $x = 1$ side that is tangent to the most inner level set for the function $U_I$. With no other critical points, there can be nowhere else on the triangle above that first intersection.

Now we return to $U_I$ this time setting $x = 1$ we can take the derivative and solve for $s_I$ in terms of $b$. This gives:

\begin{align*}
  r &= \frac{3 - 4b}{10} \quad \text{(A.17)} \\
  s_I &= \frac{2 - b}{5} \quad \text{(A.18)} \\
  x &= 1 \quad \text{(A.19)}
\end{align*}
A.2.3 Case III

Now consider $\frac{3}{4} < b < 1$. Visualizing this from the perspective of the triangle, this is a case where the global maximum occurs both to the right of the triangle and above it. This means that the level sets of the utility function will first intersect the triangle at the top right corner for every $b$ in this range. This gives the following values for the strategies of $I$ and $MP$:

\[
\begin{align*}
\quad & r = 0 \quad \text{(A.20)} \\
\quad & x = 1 \quad \text{(A.21)} \\
\quad & s_I = 1 - b \quad \text{(A.22)}
\end{align*}
\]

A.2.4 Case IV

For this last case consider $1 \leq b$. In this case, returning to the visualization of the triangle, we can now see that it actually no longer exists. The line $s_I = x - b$ is completely outside of the strategy set of $I$ for any value of $b$ over 1. This means that the response of $MP$ to $I$ will be the same regardless ($r = 0$) and so there is no reason for $I$ not to take their ideal point as follows:

\[
\begin{align*}
\quad & r = 0 \quad \text{(A.23)} \\
\quad & x = 1 \quad \text{(A.24)} \\
\quad & s_I = 0 \quad \text{(A.25)}
\end{align*}
\]

So we now know how both $I$ and $MP$ will react to the actions of $F$. This means we can now move forward and maximize the utility of $F$.

A.3 Solving for $F$

Solving for $F$ means maximizing $U_F$ for every $x_F$. 

Now we want to substitute $b = a + s_F$, but if we take $b^2$ we get:

\[
\begin{align*}
\quad & b^2 = a^2 + s_F^2 + 2as_F \quad \text{(A.26)}
\end{align*}
\]
What we can do is substitute:

\[ a^2 + s_F^2 = b^2 - 2as_F \]  \hspace{1cm} (A.27)

This gives us:

\[ U_F = -|x_F - (1 - r)x| - r^2 - b^2 + 2as_F - s_i^2 \]  \hspace{1cm} (A.28)

We know:

\[ 0 \leq (a - s_F)^2 \]  \hspace{1cm} (A.29)

And we also know:

\[ (a - s_F)^2 = a^2 + s_F^2 - 2as_F \]  \hspace{1cm} (A.30)

And so:

\[ a^2 + s_F^2 - 2as_F = (a + s_F)^2 - 4as_F \]  \hspace{1cm} (A.31)

\[ (a + s_F)^2 - 4as_F = b^2 - 4as_F \]  \hspace{1cm} (A.32)

Because of the requirement that \( 0 \leq (a - s_F)^2 \) we also know:

\[ 4as_F \leq b^2 \]  \hspace{1cm} (A.33)

So we then know:

\[ 0 \leq as_F \leq \frac{b^2}{4} \]  \hspace{1cm} (A.34)

Note that the only other occurrence of \( b \) is in the utility function of \( MP \) and it occurs linearly meaning there is no \( as_F \) term. The relative combination of \( a \) and \( s_F \) is irrelevant to \( MP \).

So now we have \( F \) maximizing \( as_F \) constrained by the above considerations. This gives us:

\[ 2as_F = \frac{b^2}{2} \]  \hspace{1cm} (A.35)

So we can substitute that back in to \( U_F \):
This gives us:

\[ U_F = -|x_F - (1 - r)x| - r^2 - b^2 - s_f^2 \]  \hspace{1cm} (A.36)

And then finally:

\[ U_F = -|x_F - (1 - r)x| - r^2 - b^2 - s_f^2 \]  \hspace{1cm} (A.37)

We can then substitute the values for \( x, r, \) and \( s_f \) that we found above into this equation for each range of \( b \) to get the complete formula for \( U_F \):

\[
U_F = \begin{cases} 
-|x_F - (1 - \frac{3-2b}{11})| - \frac{b^2}{2} - (\frac{4+b}{11})^2 - (\frac{3-2b}{11})^2 & \text{if } 0 \leq b \leq \frac{1}{8} \\
-|x_F - (1 - \frac{3-4b}{10})| - \frac{b^2}{2} - (\frac{2-b}{5})^2 - (\frac{3-4b}{10})^2 & \text{if } \frac{1}{8} \leq b \leq \frac{3}{4} \\
-|x_F - 1| - \frac{b^2}{2} - (1-b)^2 & \text{if } \frac{3}{4} \leq b \leq 1 \\
-|x_F - 1| - \frac{b^2}{2} & \text{if } 1 \leq b 
\end{cases}
\]  \hspace{1cm} (A.38)

We now maximize each case for any value of \( x_F \), and we can then determine on what ranges of \( x_F, F \) would play which ranges of \( b \), which will ultimately leave us with equations for \( b \) for any given value of \( x_F \).

### A.3.1 Case I

Because of the absolute value in the utility function, there are five different potential places the maximum could occur for \( 0 \leq b \leq \frac{1}{8} \): the value at \( b = 0 \), the value at \( b = \frac{1}{8} \), a maximum for the derivative assuming the function inside the absolute value comes out positive, a maximum for the derivative assuming the equation inside the absolute value comes out negative, and the value where the function inside the absolute value comes out as 0.

To test these cases, all we need to do is take both the derivatives and see where the maximum occurs. We can then compare the values at those points to the values at the other cases listed above. If the maximum does not occur within the range, we know that one of the other locations is the answer. These derivatives are as follows:

\[
\frac{dU_F}{db} = \begin{cases} 
\frac{8-9b}{11} & \text{if } 0 \leq b < -\frac{21+11\sqrt{(1+4x_F)}}{8} \\
\frac{4-13b}{121} & \text{if } b = -\frac{21+11\sqrt{(1+4x_F)}}{8} \\
-\frac{163b-30}{121} & \text{if } -\frac{21+11\sqrt{(1+4x_F)}}{8} < b \leq \frac{1}{8} 
\end{cases}
\]  \hspace{1cm} (A.39)

We know to use the root of the function inside the absolute value presented above because the other root is negative over the entire range of interested and so is not useful.

If we look at these functions over the range presented, we can see that the first one is positive over the entire range while the third one is negative over its range. This means that there is a peak at the centre of them, which gives the maximum on this range. Thus
F plays a strategy to make the value of what is inside the absolute value 0 unless that \( b \) would fall outside of \( 0 \leq b \leq \frac{1}{8} \) and if it does the constraint becomes binding and they play the constraint.

The lower area where the constraint is binding is when, by setting \( b = 0 \), the absolute value still comes out negative. We can expand and simplify the quantity inside the absolute value above and substitute \( b = 0 \) to get the following value of \( x_F \) for which anything lower would result in the constraint on \( b \) being binding:

\[
x_F = \frac{80}{121}
\]  

(A.40)

Then the same process can get us the upper boundary for this, which occurs at \( b = \frac{1}{8} \), which gives:

\[
x_F = \frac{3}{4}
\]

(A.41)

This gives the following values of \( b \) for different values of \( x_F \):

\[
b = \begin{cases} 
0 & \text{if } x_F \leq \frac{80}{121} \\
\frac{-21+11\sqrt{(1+4x_F)}}{8} & \text{if } \frac{80}{121} \leq x_F \leq \frac{3}{4} \\
\frac{1}{8} & \text{if } \frac{3}{4} \leq x_F
\end{cases}
\]

(A.42)

A.3.2 Case II

A similar process works within the range of \( \frac{1}{8} \leq b \leq \frac{3}{4} \). For this case we have the following derivative:

\[
\frac{dU_{FII}}{db} = \begin{cases} 
\frac{4-7b}{5} & \text{if } \frac{1}{8} \leq b < \frac{10x_F-7}{4} \\
\frac{2-7b}{5} & \text{if } b = \frac{10x_F-7}{4} \\
\frac{-7b}{5} & \text{if } \frac{10x_F-7}{4} < b \leq \frac{3}{4}
\end{cases}
\]

(A.43)

For most values of \( x_F \), this is very similar to the result of Case I, however it is now possible for a critical point to occur within this range. If \( x_F \) is sufficiently large to make \( b \) larger than \( \frac{1}{8} \) while still having the absolute value come out to be positive, the maximum moves from being at the point where the absolute value comes out to 0, to be at \( b = \frac{4}{7} \). This point occurs at \( x_F = \frac{13}{17} \).

We again must check that there are no constraints on this solution with \( b \) in this range. In particular, we are interested in the lower bound, because at the upper bound, \( U_F \) is maximized at \( b = \frac{4}{7} \). If we find the lower bound by setting \( b = \frac{1}{8} \), the lower bound in this range, we get \( x_F = \frac{3}{4} \). So we get the following values of \( b \):
\[ b = \begin{cases} \frac{1}{8} \frac{10xf - 7}{4} & \text{if } xf < \frac{3}{4} \\ \frac{4}{7} & \text{if } \frac{3}{4} \leq xf \leq \frac{13}{14} \\ \frac{13}{14} & \text{if } \frac{13}{14} \leq xf \end{cases} \]  
(A.44)

### A.3.3 Case III

Maximizing \( U_F \) over the range \( \frac{3}{4} \leq b \leq 1 \) is relatively easy as the absolute value is not a function of \( b \). This means we can take a simple derivative of the function and maximize that without worrying about separate possibilities. This derivative is simply:

\[ \frac{dU_{FIII}}{db} = 2 - 3b \]  
(A.45)

Set that to 0 and solve for \( b \), we get:

\[ b = \frac{2}{3} \]  
(A.46)

But since this case is bounded by \( \frac{3}{4} \leq b \), that constraint here is binding and we take \( b = \frac{3}{4} \). This gives a maximized value of \( U_F \) of:

\[ U_{FIII} = -|xf - 1| - \frac{11}{32} \]  
(A.47)

### A.3.4 Case IV

Maximizing over \( 1 \leq b \) is again simple because the absolute value portion is again not a function of \( b \), so we can simply differentiate:

\[ \frac{dU_{FIV}}{dx} = b \]  
(A.48)

So when we set this to 0 and solve for \( b \) we will get 0. This means that again, the constraint \( 1 \leq b \) in binding and we take \( b = 1 \), leaving the following maximized value:

\[ U_{FIV} = -|xf - 1| - \frac{1}{2} \]  
(A.49)

Before moving forward, we should note that this value will always be smaller than Case III, meaning that the solution will never be \( 1 \leq b \), regardless of the value of \( xf \).
A.4 Choosing a Case

The easiest way to see how $U_F$ is maximized is to simply graph it for different values of $x_F$ and that will result in a clear picture of the strategies $F$ plays. The critical points we will investigate are $\frac{80}{121}, \frac{3}{4}$, and $\frac{13}{14}$ as these are the crucial points identified above.

Figure A.2: Utility function of $F$ for values of $x_F$ starting at $\frac{74}{121}$ and increasing by $\frac{2}{121}$ in each graph up to a maximum of $\frac{90}{121}$. The red line is the maximum for the range $0 < b \leq \frac{1}{8}$, while the green line is the maximum for the range $\frac{1}{8} \leq b \leq \frac{3}{4}$, and the blue line is the constrained maximum value of $U_F$.

First, look at the maximized values around $x_F = \frac{80}{121}$. Figure A.2 shows that below $x_F = \frac{80}{121}$ the value of $b$ will always be 0. Once it gets larger than that tipping point, $U_F$ is maximized by setting $a b$ that makes the absolute value disappear, which is evident by the maximum occurring at the top of the bump.

Figure A.3 now shows the maximum value for the range $\frac{1}{8} \leq b \leq \frac{3}{4}$ overtakes the maximum value for the lower range once $b$ grows larger than $\frac{1}{4}$.

Looking at Figure A.4, the maximum value of $U_F$ finally pulls away from the point where the maximum value disappears and starts to occur slightly below it. This is the case where there is a critical point of one of our utility functions now occurring within the allowed ranges for $b$. In particular, this is the critical point identified above that occurs at $x_F = \frac{13}{14}$.

Bringing together this information, we get the following function for the value of $b$ that maximizes $U_F$ for any given value of $x_F$:

$$b = \begin{cases} 
0 & \text{if } x_F < \frac{80}{121} \\
-\frac{21 + 11 \sqrt{1 + 4x_F}}{8} & \text{if } \frac{80}{121} \leq x_F \leq \frac{3}{4} \\
\frac{10x_F - 7}{4} & \text{if } \frac{3}{4} \leq x_F \leq \frac{13}{14} \\
\frac{13}{7} & \text{if } \frac{13}{14} \leq x_F 
\end{cases} \quad (A.50)$$
Figure A.3: Utility function of $F$ for values of $x_F$ starting at $\frac{84}{120}$ and increasing by $\frac{2}{121}$ in each graph up to a maximum of $\frac{100}{120}$. The red line is the maximum for the range $0 < b \leq \frac{1}{5}$, while the green line is the maximum for the range $\frac{1}{5} \leq b \leq \frac{3}{4}$, and the blue is the constrained maximum value of $U_F$, although in this figure it is covered at all points.

Figure A.4: Utility function of $F$ for values of $x_F$ starting at $\frac{104}{120}$ and increasing by $\frac{2}{121}$ in each graph up to a maximum of $\frac{120}{120}$. The red line is the maximum for the range $0 < b \leq \frac{1}{5}$, while the green line is the maximum for the range $\frac{1}{5} \leq b \leq \frac{3}{4}$, and the blue is the constrained maximum value of $U_F$.

With that, we can also find the equilibrium values of $r$, $x$, and $s_I$: 
\[ r = \begin{cases} \frac{3}{11} & \text{if } x_F < \frac{80}{121} \\ \frac{3 - 21 + 11 \sqrt{1 + 4 x_F}}{8} & \text{if } \frac{80}{121} \leq x_F \leq \frac{3}{4} \\ \frac{3 - 4 \sqrt{1 + 4 x_F}}{5} & \text{if } \frac{3}{4} \leq x_F \leq \frac{13}{14} \\ \frac{3 - \frac{16}{11}}{11} & \text{if } \frac{13}{14} \leq x_F \end{cases} \] (A.51)

\[ x = \begin{cases} \frac{10}{11} & \text{if } x_F < \frac{80}{121} \\ -1 + \sqrt{1 + 4 x_F} & \text{if } \frac{80}{121} \leq x_F \leq \frac{3}{4} \\ 1 & \text{if } \frac{3}{4} \leq x_F \leq \frac{13}{14} \\ 1 & \text{if } \frac{13}{14} \leq x_F \end{cases} \] (A.52)

\[ s_I = \begin{cases} \frac{4}{11} & \text{if } x_F < \frac{80}{121} \\ \frac{1 + \sqrt{1 + 4 x_F}}{8} & \text{if } \frac{80}{121} \leq x_F \leq \frac{3}{4} \\ \frac{5}{11} & \text{if } \frac{3}{4} \leq x_F \leq \frac{13}{14} \\ \frac{4 + \frac{4}{11}}{11} & \text{if } \frac{13}{14} \leq x_F \end{cases} \] (A.53)
Appendix B

Formal Model with Binary Variables

By applying values of 1 and 0 for every variable in the utility functions laid out in Equations 3.1, 3.2, and 3.3 we can get a complete table of the potential outcomes for this set of assumptions. These are set out in Figures B.1 and B.2, with B.1 showing the possible outcomes when the future leader agrees with the legislator and B.2 showing the possible outcomes when the future leader agrees with the interim leader.

B.1 Modelled with Indifference

Table B.1 shows the equilibrium results of the game for both potential values of $X_F$. In this case, when each actor makes their decision they are comparing the expected value of one choice against the expected value of the other. That expected value is calculated assuming that when an actor is indifferent, they choose each option with a probability of 0.5. This results in the expected value of that branch for each player being the mean expected value of both branches that led to that point. The values for each variable then show the likelihood that they would take an action resulting in a value of 1 for that variable (which in this case is taking that given action since their choices are to take the action or not).

To reach these results, we can perform a series of pairwise comparisons on each pair of branches stemming from $MP$ in B.1 and B.2. First we compare the values in the top two rows. This represents a potential decision that $MP$ must make after the other two actors have already made both their decisions (meaning the values of $S_F$, $S_I$, $A$, and $X$ have already been set and cannot be changed). $MP$ then makes their decision based on which option results in a higher value of $U_{MP}$. Taking ?? for example, because the bottom branch provides more utility for $MP$ than the top branch, they opt for $r = 0$. Thus when we get to this decision, before $MP$ actually acts, $I$ and $F$ can both know what their own utilities will be from this path, those being $EU_I = 0$ and $EU_F = -4$ respectively.

We can then eliminate the top branch as a potential outcome. We then repeat this process for the every pair. However, when we reach the second last line, we find that $MP$ has the
same utility regardless of their action and so in this case they are indifferent between the outcomes. In this version of the model, we expect actors who are indifferent to select their strategy at random, meaning half the time $MP$ will choose $r = 0$ and the other half the time they will choose $r = 1$. This makes it slightly more difficult for $I$ and $F$ because they do not know the ultimate outcome. However, they will still have an expected utility, which can be derived as seen in equation B.1. Here $U_{F1}$ is the utility of $F$ if $r = 1$ and $P_1$ is the probability that $r = 1$, which $U_{F0}$ and $P_0$ are those values for $r = 0$. Equation B.2 shows the same for $U_I$. 

Figure B.1: Decision tree faced in the game with the utilities for each actor, given that $x_F = 0$. 

$U_F = -4, U_I = -2, U_{MP} = -3$

$U_F = -4, U_I = 0, U_{MP} = 0$

$U_F = -3, U_I = -1, U_{MP} = 1$

$U_F = -3, U_I = -1, U_{MP} = -2$

$U_F = -2, U_I = 1, U_{MP} = 0$

$U_F = -2, U_I = -1, U_{MP} = 1$

$U_F = -2, U_I = -1, U_{MP} = -1$

$U_F = -1, U_I = 0, U_{MP} = 1$

$U_F = -3, U_I = 0, U_{MP} = 0$

$U_F = -3, U_I = -2, U_{MP} = -3$

$U_F = -3, U_I = 0, U_{MP} = -1$

$U_F = -3, U_I = -2, U_{MP} = -3$

$U_F = -2, U_I = -1, U_{MP} = 0$

$U_F = -2, U_I = -1, U_{MP} = -2$

$U_F = -2, U_I = -1, U_{MP} = 0$

$U_F = -2, U_I = -1, U_{MP} = 1$

$U_F = -2, U_I = -1, U_{MP} = -1$

$U_F = -1, U_I = 1, U_{MP} = 0$

$U_F = -1, U_I = 1, U_{MP} = 0$

$U_F = -1, U_I = 0, U_{MP} = 0$

$U_F = -1, U_I = 0, U_{MP} = 0$

$U_F = -1, U_I = 1, U_{MP} = 0$

$U_F = -1, U_I = 1, U_{MP} = 0$

$U_F = -1, U_I = 0, U_{MP} = 0$

$U_F = 0, U_I = 0, U_{MP} = 0$
Figure B.2: Decision tree faced in the game with the utilities for each actor, given that $x_F = 1$.

$$EU_F = P_1 U_{F1} + P_0 U_{F0} \tag{B.1}$$

$$EU_I = P_1 U_{I1} + P_0 U_{I0} \tag{B.2}$$

Applying these equations to the values from this pair we can find that $EU_I = 0$ and $EU_F = -1$. Thus we can collapse the two branches into one in for which the value of $R$ is
set to 0.5 because they have a 0.5 chance of dissenting but the values of the other variables remain the same and the utilities of the actors are set to their expected utilities.

We then repeat this process for every set, evaluating each pair as described above to get a new tree with only 16 outcomes where each new outcome represents the different potential game states right before MP makes their decision. This new tree also has pairs branching from I that represent the decision of I in setting the value of X.

From this new tree we can apply the same procedure done to the original tree to create an imaginary table of the potential game states before I has selected X but after they selected S_I. Of course this state is somewhat hypothetical and arbitrary as it seems to imply that I must select S_I before selecting X, which is not necessary. That said, the order here is unimportant. In both cases I is maximizing EU_I and so their ultimate choice is not impacted by this ordering. So after testing the pairs of X values, we shrink the table again to 8 final branches of expected actions (a value between 0 and 1, showing the likelihood they take a given action) and utilities.

Repeating this process two more times for the choices of F results in B.1 which shows the equilibrium for this game.

<table>
<thead>
<tr>
<th>Table B.1: Binary Game Equilibrium</th>
</tr>
</thead>
<tbody>
<tr>
<td>X_F</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
</tbody>
</table>

B.2 Modelled with Generosity to I

This model applies the same logic as the model above, with the only difference coming in how the actors behave when they are indifferent. In this case, when an actor is indifferent they choose the option that is best for I, and when I is indifferent they choose the best option for F. Note that there are no cases where both are indifferent and in fact most indifference is eliminated once the actions of MP are changed for this model. This eliminates the need for more complicated expected utility functions.

<table>
<thead>
<tr>
<th>Table B.2: Equilibrium with MP helping I</th>
</tr>
</thead>
<tbody>
<tr>
<td>X_F</td>
</tr>
<tr>
<td>-------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
</tbody>
</table>
B.3 Modelled with Generosity to F

This model is identical to the model presented directly above with the only exception being that $MP$ chooses to help $F$ when they are indifferent instead of $I$. In both models, $F$ and $I$ help each other when they would otherwise be indifferent.

<table>
<thead>
<tr>
<th>$X_F$</th>
<th>$R$</th>
<th>$X$</th>
<th>$S_I$</th>
<th>$S_F$</th>
<th>$A$</th>
<th>$U_F$</th>
<th>$U_I$</th>
<th>$U_{MP}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>1</td>
<td>-1</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
</tbody>
</table>

Table B.3: Equilibrium with $MP$ helping $F$